Transformações canônicas infinitesimais.

$$F_2\left(q,p\right)=\sum_{i=1}^{n}q_iP_i+\varepsilon G\left(q,P,t\right).$$

Temos então que

$$\begin{aligned}p_i=&\frac{\partial F_2}{\partial q_i}=P_i+\varepsilon\frac{\partial G\left(q,P,t\right)}{\partial q_i},\\ Q_i=&\frac{\partial F_2}{\partial P_i}=q_i+\varepsilon\frac{\partial G\left(q,P,t\right)}{\partial P_i},\end{aligned}$$

ou

$$\begin{aligned}P_i&=p_i-\varepsilon\frac{\partial G\left(q,p-\varepsilon\partial G/\partial q,t\right)}{\partial q_i}=p_i-\varepsilon\frac{\partial G\left(q,p,t\right)}{\partial q_i}+\mathcal{O}\left(\varepsilon^2\right),\\ Q_i&=q_i+\varepsilon\frac{\partial G\left(q,p-\varepsilon\partial G/\partial q,t\right)}{\partial p_i}\frac{\partial P_i}{\partial p_i}=q_i+\varepsilon\frac{\partial G\left(q,p,t\right)}{\partial p_i}+\mathcal{O}\left(\varepsilon^2\right).\end{aligned}$$

Definimos as variações infinitesimais como

$$\begin{aligned}\delta p_i=P_i-p_i=&-\varepsilon\frac{\partial G\left(q,p,t\right)}{\partial q_i}=\varepsilon\sum_{k=1}^{n}\left[\cancelto{0}{\frac{\partial p_i}{\partial q_k}}\frac{\partial G\left(q,p,t\right)}{\partial p_k}-\cancelto{\delta_{ik}}{\frac{\partial p_i}{\partial p_k}}\frac{\partial G\left(q,p,t\right)}{\partial q_k}\right]=\varepsilon\left\{p_i,G\right\},\\ \delta q_i=Q_i-q_i=&\varepsilon\frac{\partial G\left(q,p,t\right)}{\partial p_i}=\varepsilon\sum_{k=1}^{n}\left[\cancelto{\delta_{ik}}{\frac{\partial q_i}{\partial q_k}}\frac{\partial G\left(q,p,t\right)}{\partial p_k}-\cancelto{0}{\frac{\partial q_i}{\partial p_k}}\frac{\partial G\left(q,p,t\right)}{\partial q_k}\right]=\varepsilon\left\{q_i,G\right\}.\end{aligned}$$