Aula 9 31/03/2020

Forças conservativas, Trabalho, Energia

Potência

- Energia Cinética $$E_c = \frac{1}{2} m \overrightarrow{v}^2 = \frac{\overrightarrow{p}^2}{2m}$$
- Como muda no tempo?

$$\frac{dE_c}{dt} = m \overrightarrow{v} \,.\, \frac{d\overrightarrow{v}}{dt} = \frac{1}{m} \overrightarrow{p} \,.\, \frac{d\overrightarrow{p}}{dt}$$

- Usa segunda lei de Newton $\frac{dE_c}{dt} = \overrightarrow{v} \,.\, \overrightarrow{F}$
- Potência:
- taxa de realização de trabalho $d\tau = \overrightarrow{F} \,.\, d\vec{r}$ em dt

Potência: taxa temporal da realização de trabalho

- $\frac{dE_c}{dt} = \vec{v} \, . \, \vec{F}$

- $P = \frac{d\tau}{dt} = \vec{F} \, . \, \vec{v}$

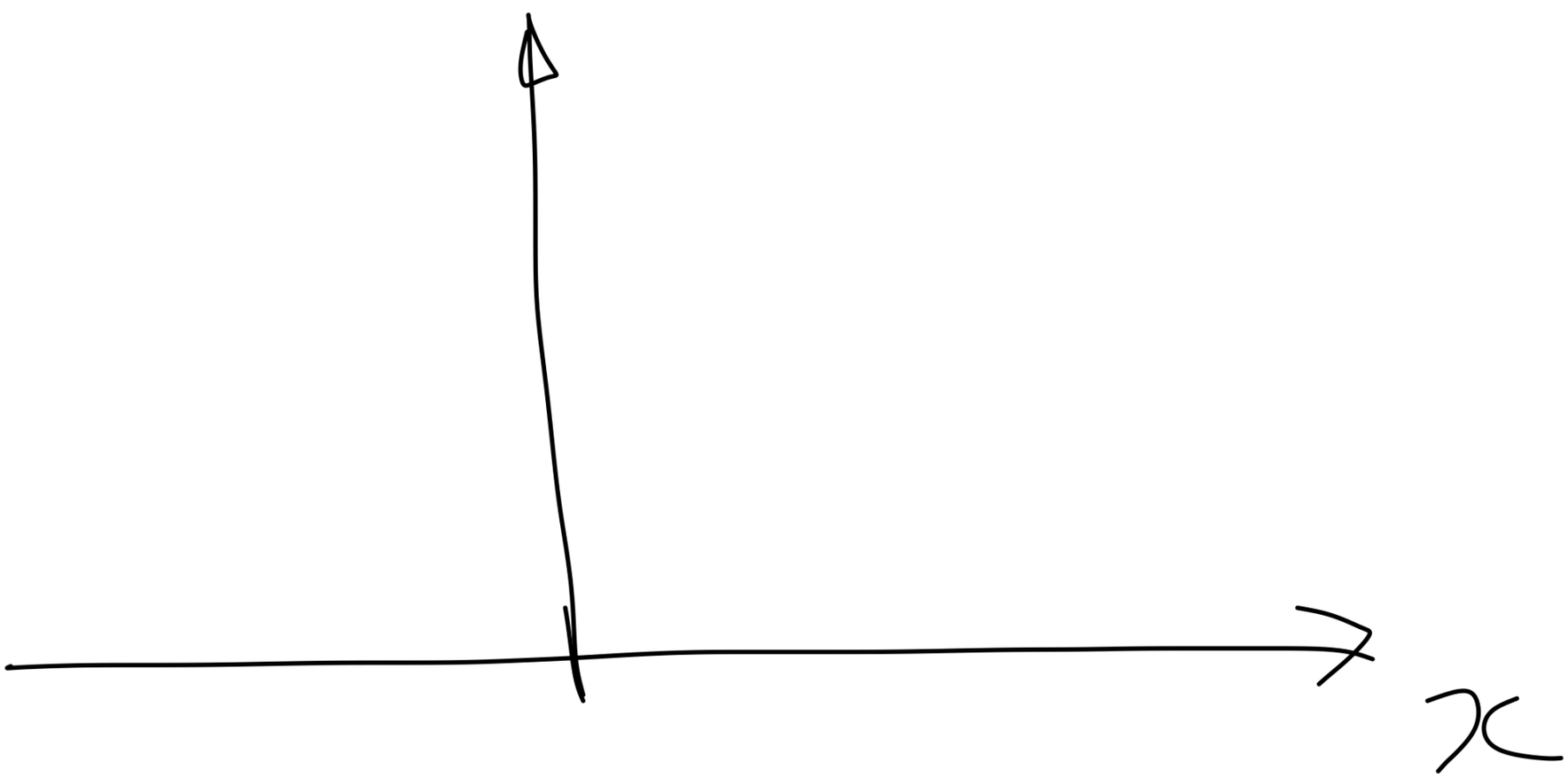
- Unidadades de Força : 1 N (Newton) = 1 kg 1m/s^ 2 : MLT^{-2}

- Energia 1 Joule = trabalho feito por F=1N em deslocamento paralelo à força de 1 metro: ML^2T^{-2}

- Potência 1W (Watt) = 1 Joule por segundo: ML^2T^{-3}

Energia Potencial

- Exemplo Oscilador Harmônico em uma dimensão
- $E = E_c + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$



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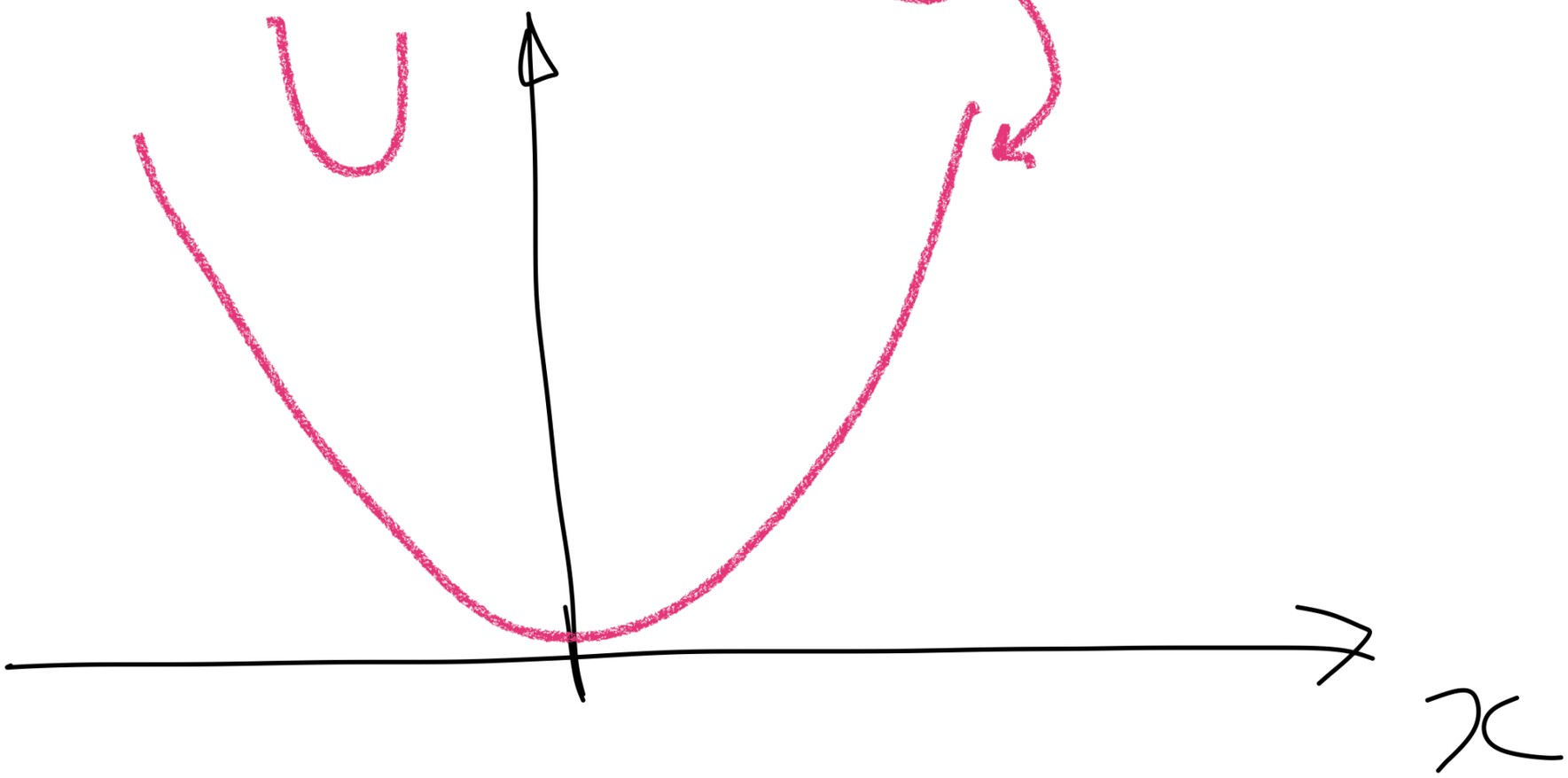

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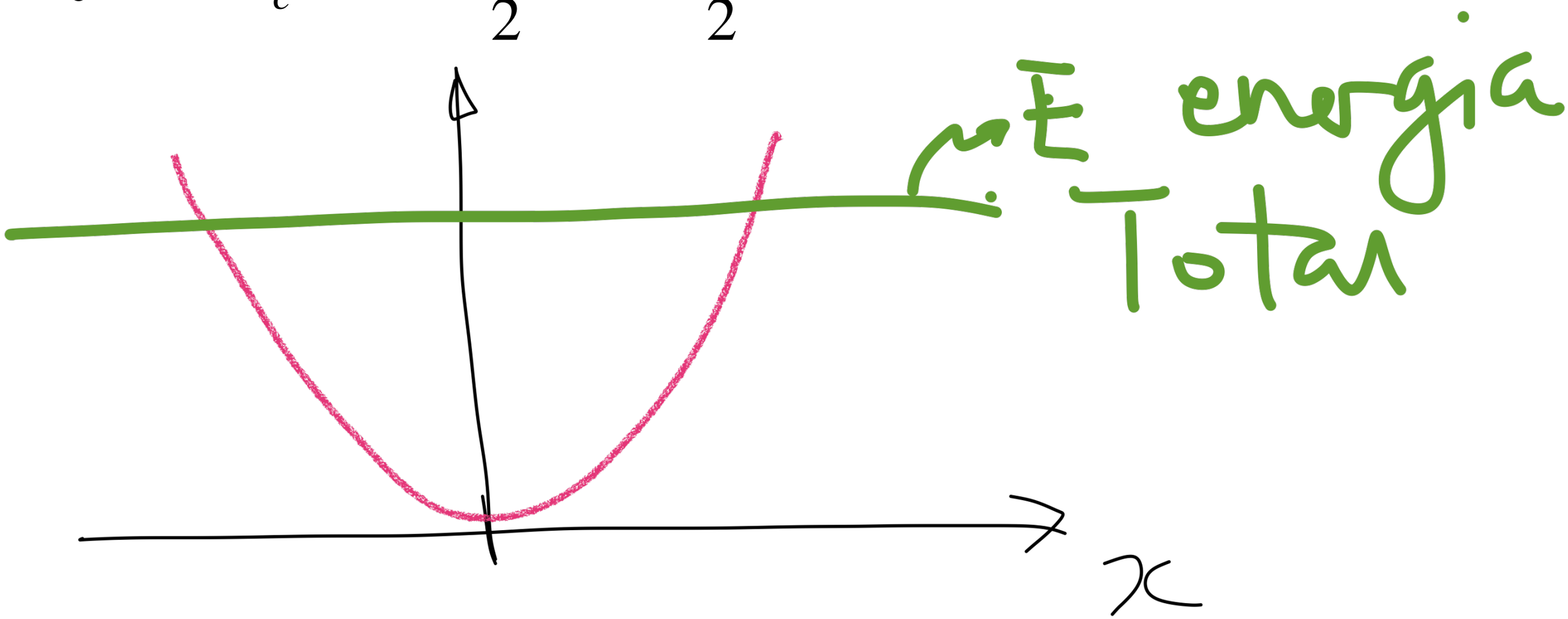

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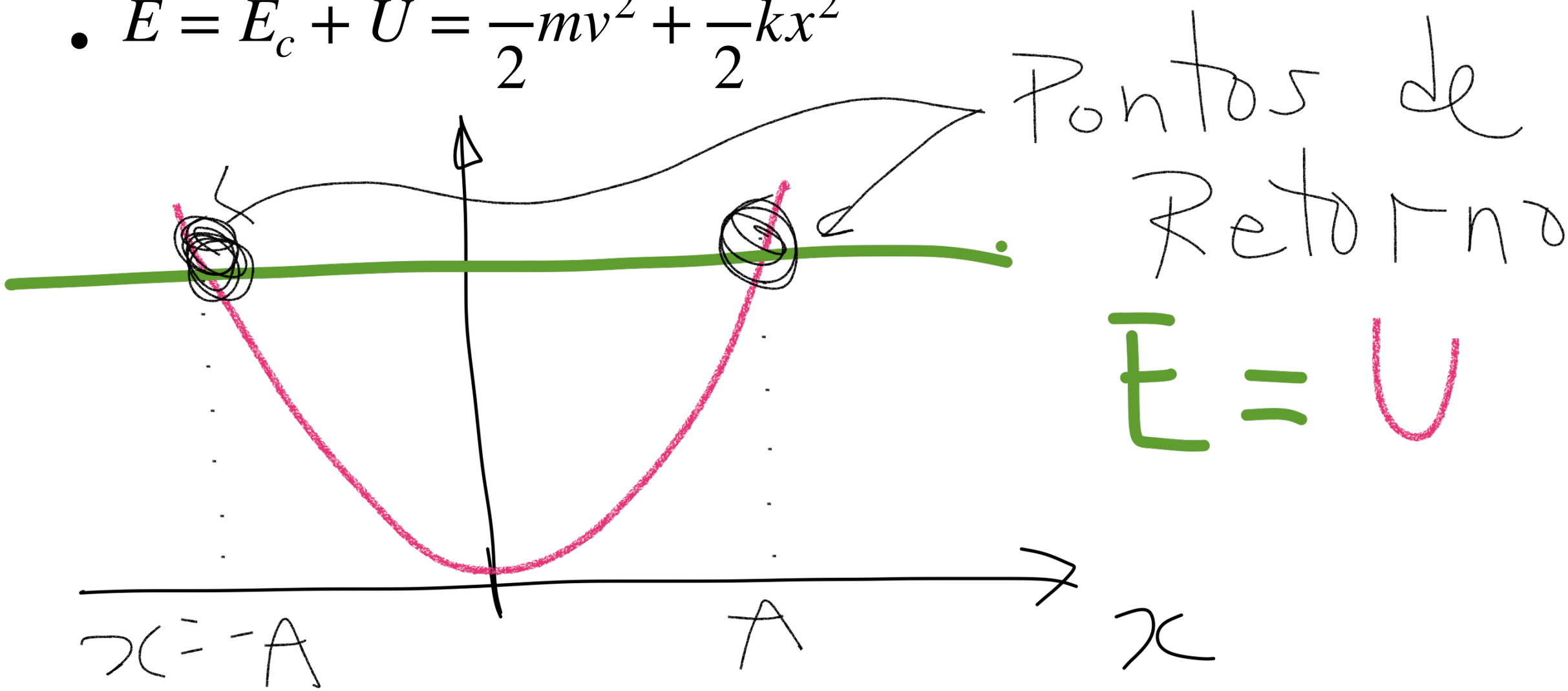

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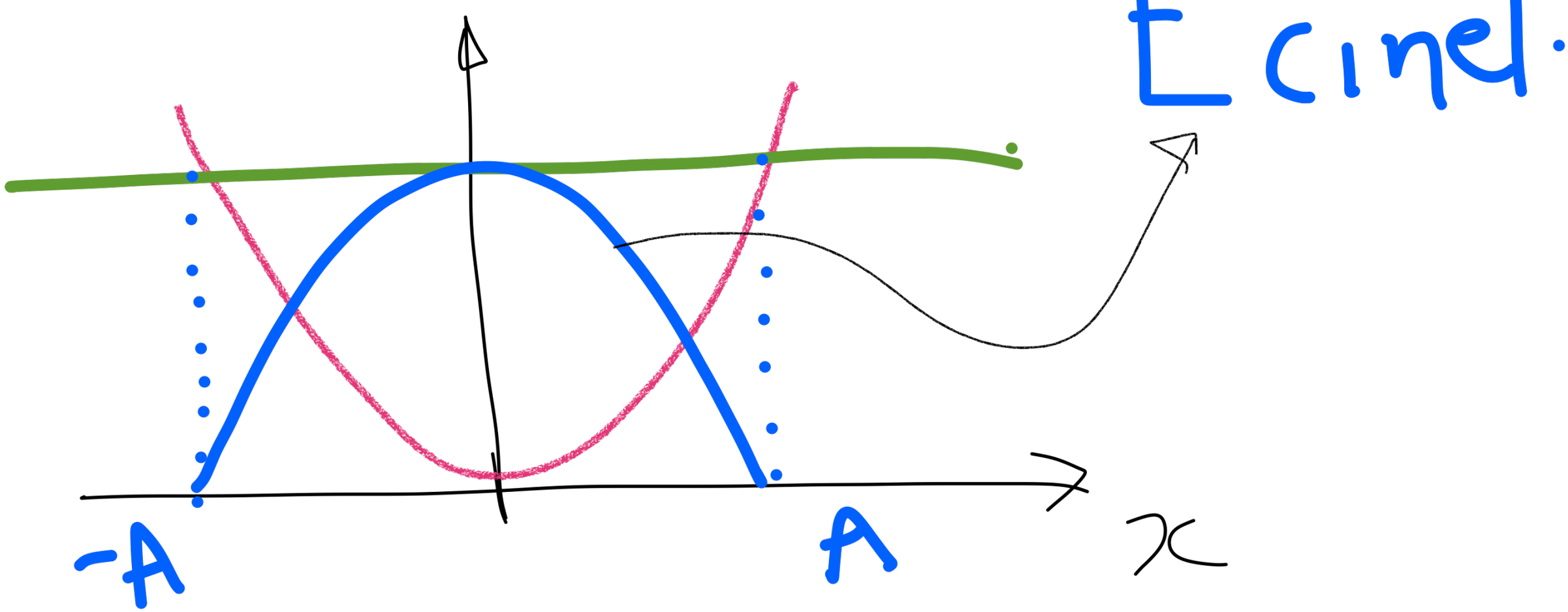


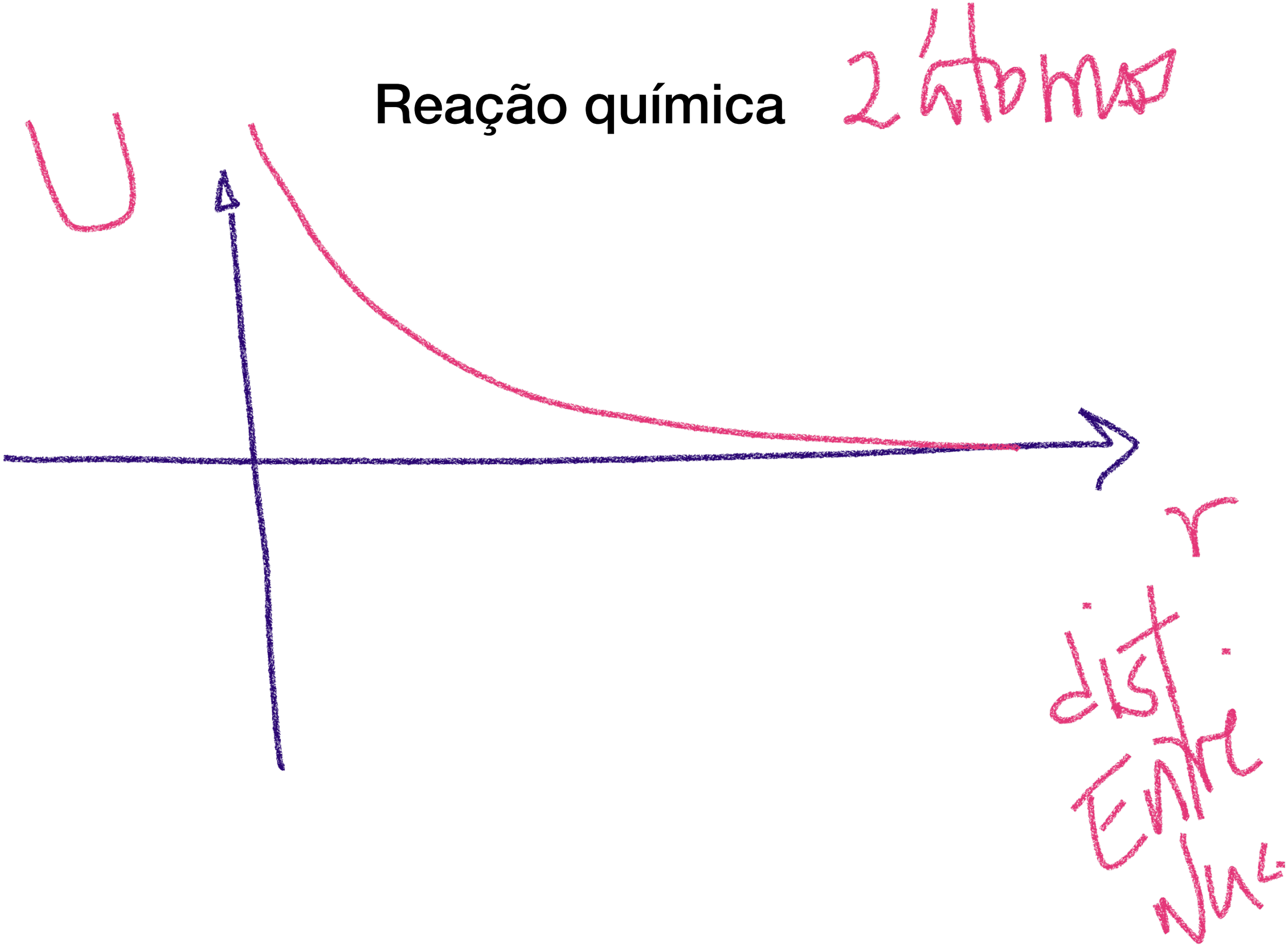
oscilador harmônico

$$E = E_c + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

$$\frac{m}{k}v^2 + x^2 = \frac{2E}{k} = A^2 \rightarrow \qquad v = \pm\,\omega A\sqrt{1 - (\frac{x}{A})^2}$$

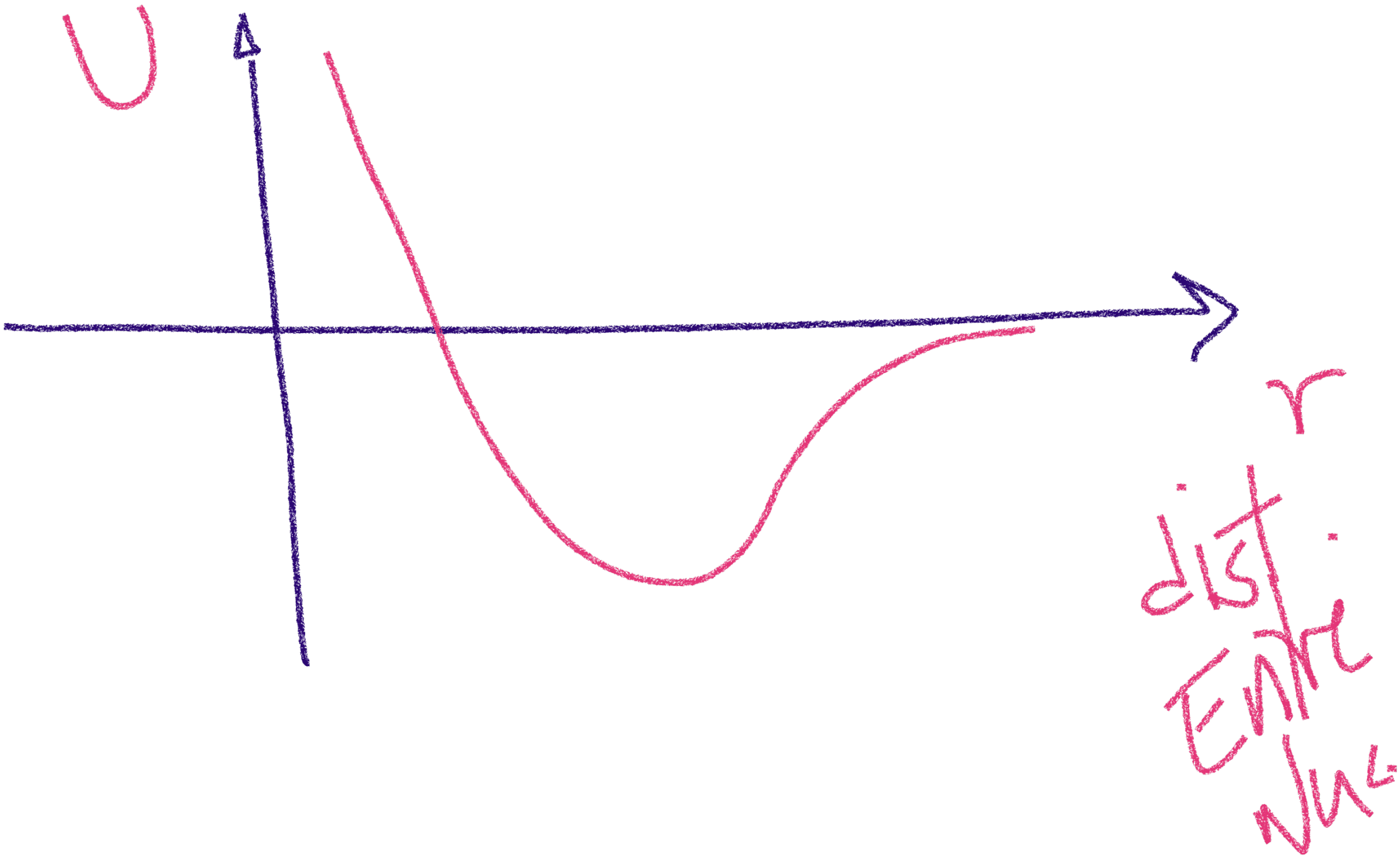
- Em geral (1 dimensão) $\qquad v = \pm\sqrt{\frac{2}{m}(E - U(x))}$

Reação química



Reação química

2 átomos



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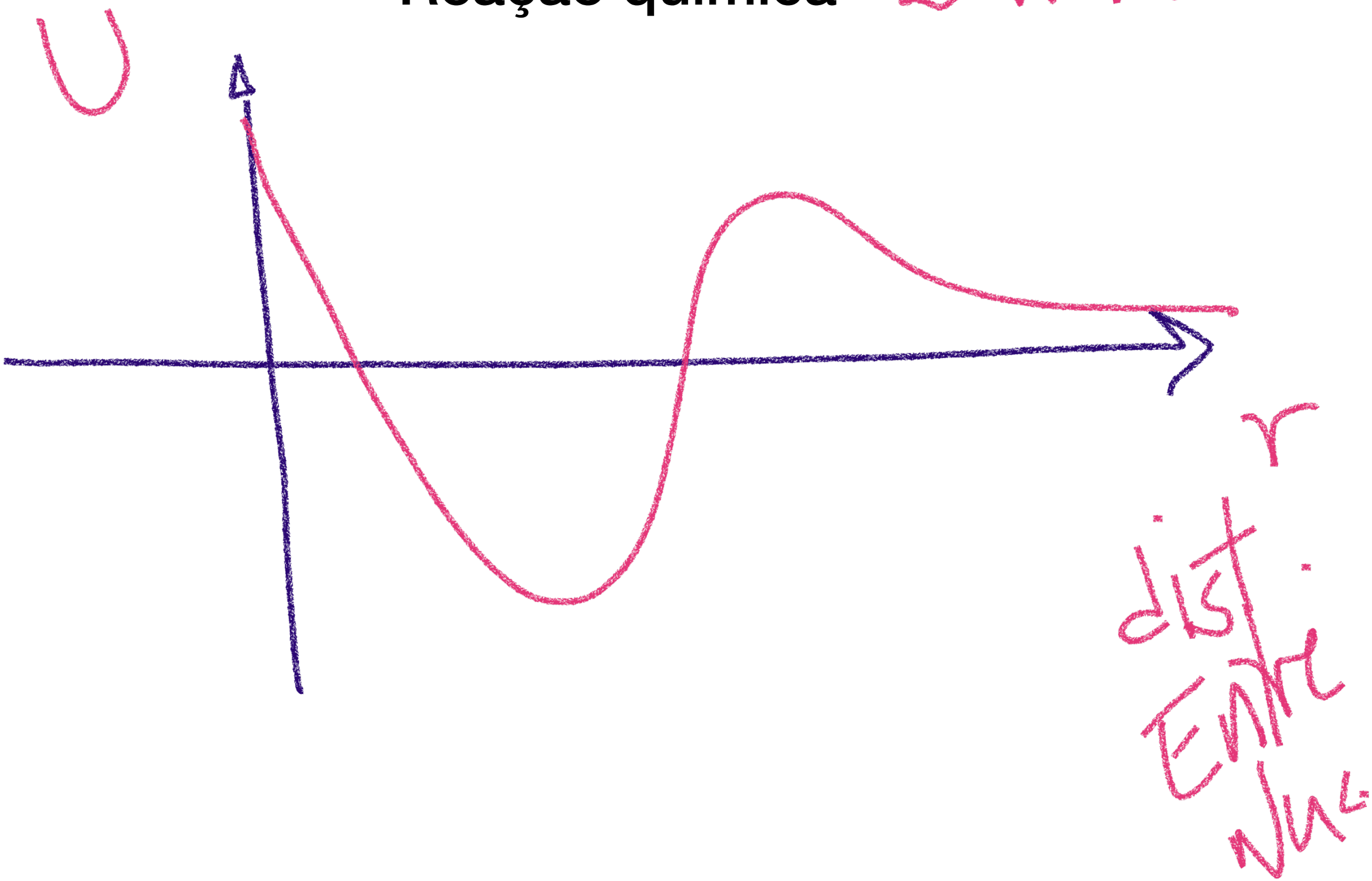

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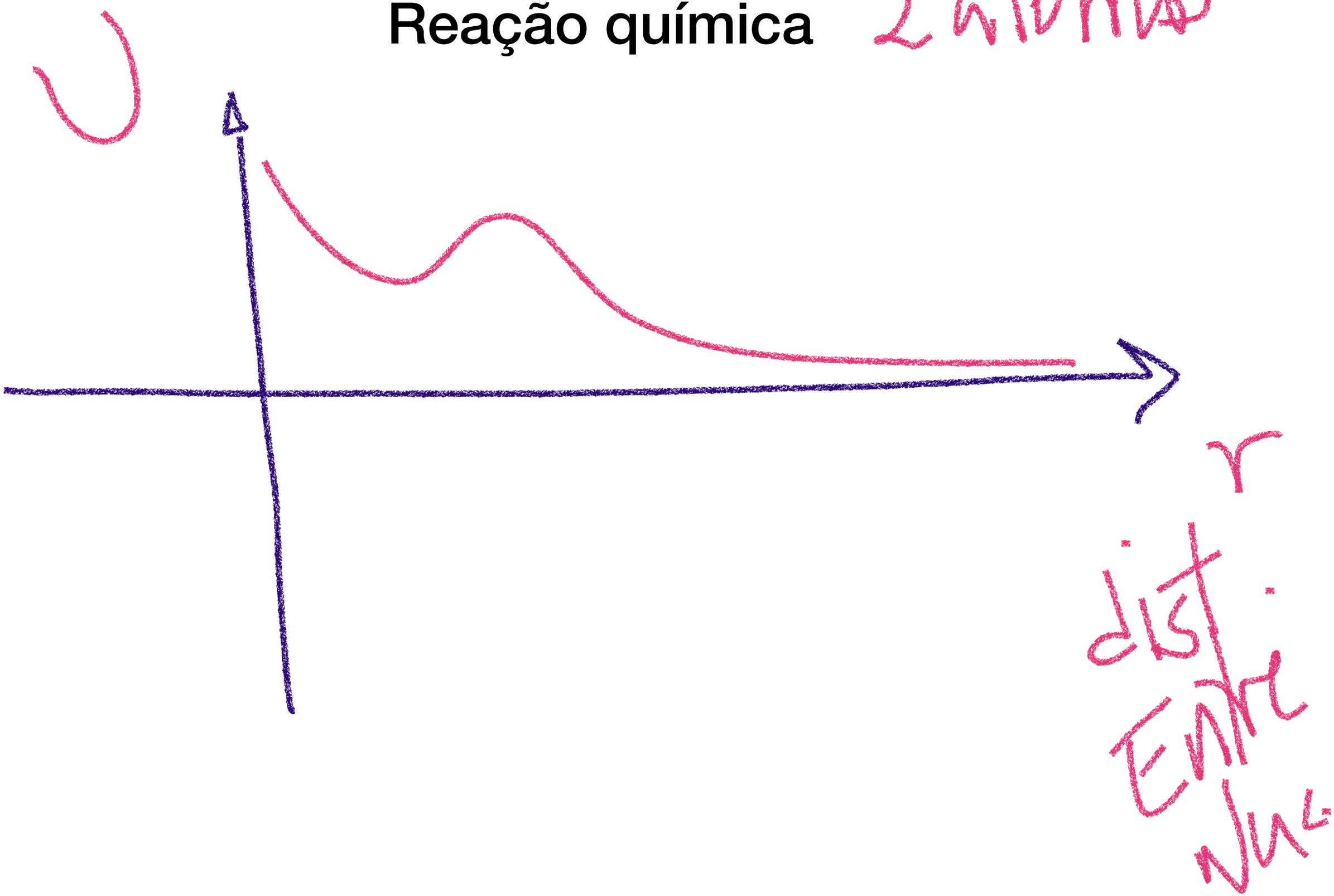

Ex: qual é o recorde mundial de salto com vara?

- Dados: o recorde dos 100 aprox 10s, o umbigo dos corredores está a 1m do chão... e podemos aplicar as leis de Newton

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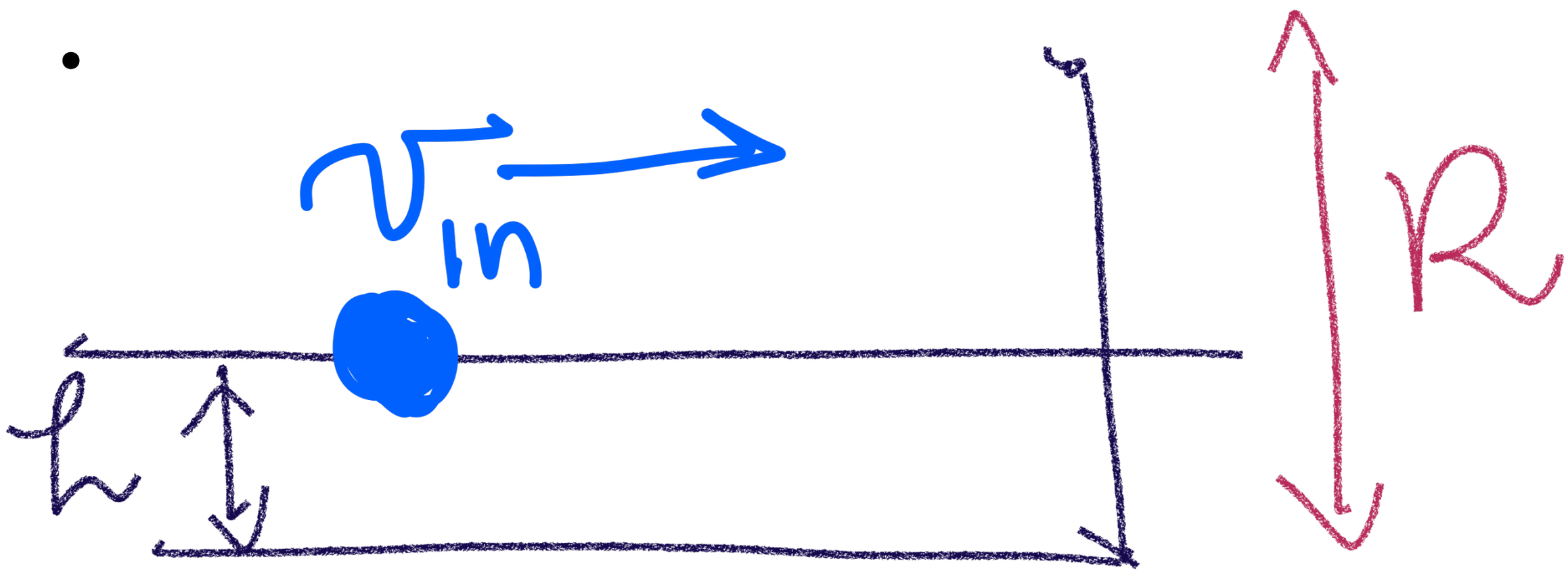
- 

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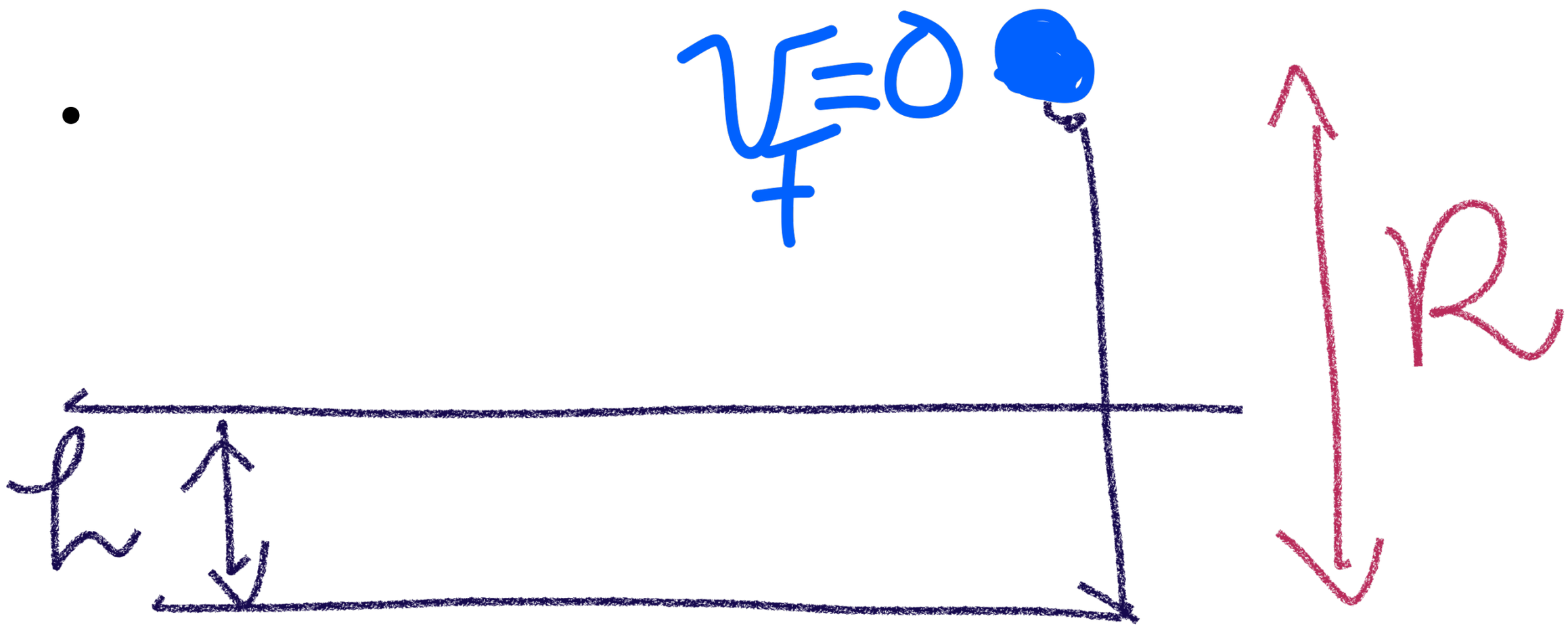

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- $E = E_c + U$

$$E = \frac{1}{2}mv_{ini}^2 + mgh$$

$$E = \frac{1}{2}mv_{fin}^2 + mgR$$

$$R = h + \frac{v_{ini}^2}{2g}$$

$$= 1m + \frac{100}{2 \cdot 10 m/s^2}\left(m/s\right)^2 = 6m$$

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Resolva usando FORÇAS

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Forças conservativas

- Matemática necessária para mais de uma dimensão.
- Derivada

$$\frac{df(x)}{dx} = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

- Derivada parcial

$$\frac{\partial f(x, y)}{\partial x} = \left(\frac{\partial f(x, y)}{\partial x}\right)_y = \lim_{\Delta x \to 0} \frac{f(x + \Delta x, y) - f(x, y)}{\Delta x}$$

Forças conservativas

- Matemática necessária para mais de uma dimensão.

- Diferencial

$$df = f(x + dx) - f(x) = \frac{df(x)}{dx}dx = (\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x})dx$$

- Diferencial (d>1) $df = f(x + dx, y + dy) - f(x, y)$

$$df = f(x + dx, y + dy) - [f(x + dx, y) - f(x + dx, y)] - f(x, y)$$

$$df = [f(x + dx, y + dy) - f(x + dx, y)] + [f(x + dx, y) - f(x, y)]$$

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$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy$$

(2d)

$$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

(3d)

Forças conservativas

- Diferencial $$df = \frac{\partial f}{\partial x}dx + \frac{\partial f}{\partial y}dy + \frac{\partial f}{\partial z}dz$$

- Gradiente de uma função escalar em três dimensões

- "Vetor" $$\nabla f(x, y, z) = \frac{\partial f}{\partial x}\hat{i} + \frac{\partial f}{\partial y}\hat{j} + \frac{\partial f}{\partial z}\hat{k} = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})$$

- $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k} = (dx, dy, dz)$

Forças conservativas

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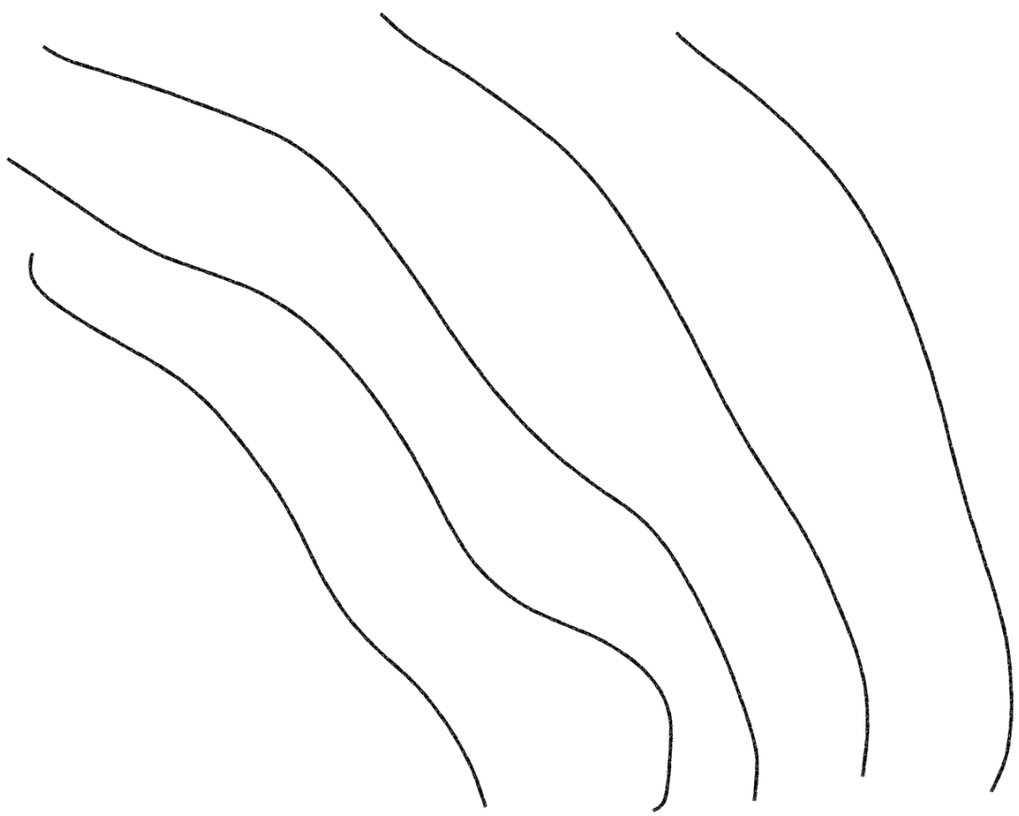
- $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k} = (dx, dy, dz)$
- Gradiente permite calcular mudança da função devida à mudança das suas variáveis

$$df = \nabla f . d\vec{r}$$

Gradiente da energia potencial

$$dU(x,y,z) = \frac{\partial U}{\partial x}dx + \frac{\partial U}{\partial y}dy + \frac{\partial U}{\partial z}dz = \nabla U \,.\, d\vec{r}$$



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(2d) Curvas

(3d) (Superf)

Equipotenciais

Gradiente da energia potencial

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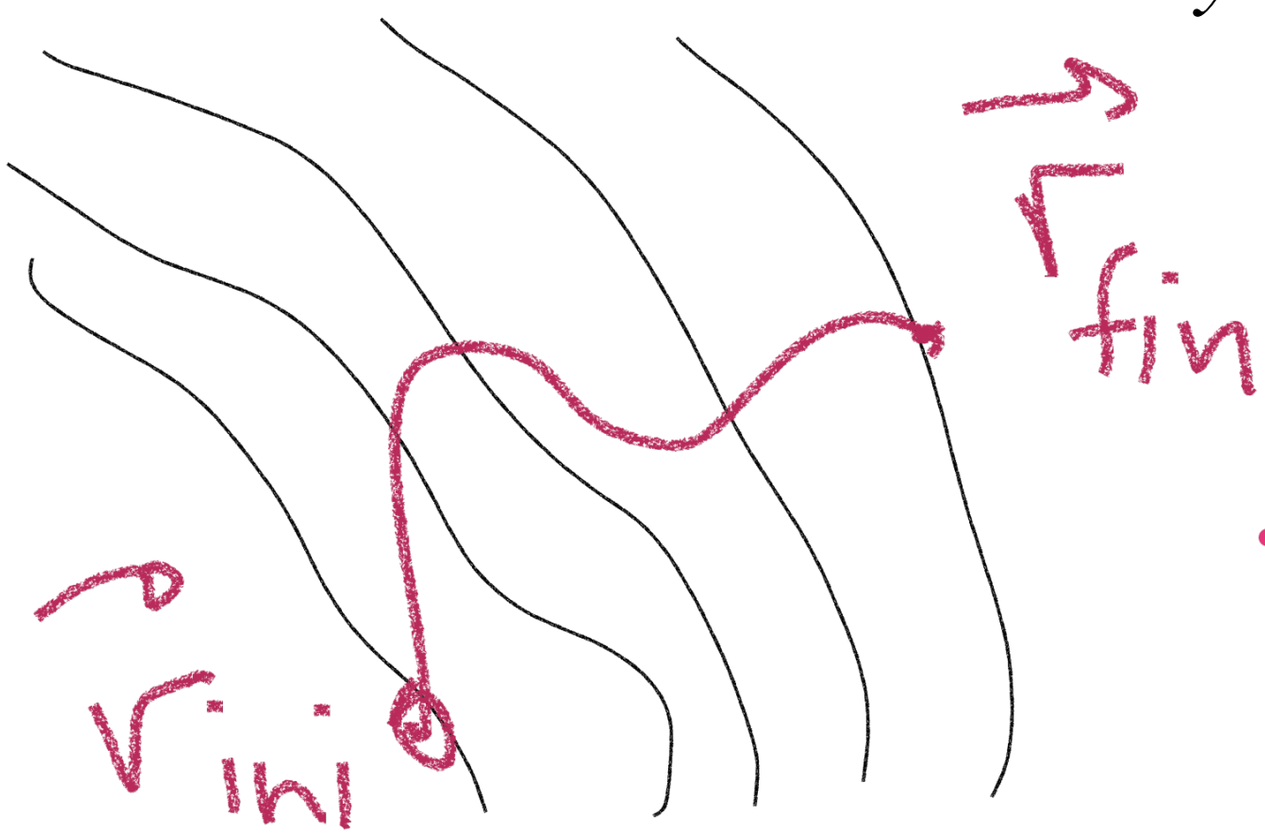

$$\sum_{\vec{r}_{ini}}^{\vec{r}_{fin}} \Delta U_i$$

Gradiente da energia potencial

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$$\Delta U = U_{fin} - U_{in} = \lim \sum_i \Delta U_i = \int_{\vec{r}_{ini}}^{\vec{r}_{fin}} dU$$

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Independ. do CAMINHO

Gradiente da energia potencial

$$U_{fin} - U_{in} = \int_{\vec{r}_{ini}}^{\vec{r}_{fin}} dU = \int_{\vec{r}_{ini}}^{\vec{r}_{fin}} \nabla U \,.\, d\vec{r}$$

Se:

$$\overrightarrow{F} = -\nabla U$$

$$U_{fin} - U_{in} = \int_{\vec{r}_{ini}}^{\vec{r}_{fin}} (-\overrightarrow{F}) \,.\, d\vec{r}$$

$$U_{fin} - U_{in} = -\tau = -(E_{c,fin} - E_{c,ini})$$

Se gradiente da energia potencial é (menos) a força

$$U_{fin} - U_{in} = \int_{\vec{r}_{ini}}^{\vec{r}_{fin}} dU = \int_{\vec{r}_{ini}}^{\vec{r}_{fin}} \nabla U \,.\, d\vec{r}$$

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$$U_{fin} - U_{in} = -E_{c,fin} + E_{c,ini}$$

$$E = E_{c,fin} + U_{fin} = E_{c,ini} + U_{in}$$

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Se força pode ser escrita como gradiente de uma função

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$$U_{fin} - U_{in} = -E_{c,fin} + E_{c,ini}$$

$$E = E_{c,fin} + U_{fin} = E_{c,ini} + U_{in}$$

Exemplo: oscilador 2d

- $\overrightarrow{F} = -kx\hat{i} - ky\hat{j}$
- $U(x, y) = \dfrac{k}{2}x^2 + \dfrac{k}{2}y^2$
- $\dfrac{\partial U}{\partial x} = kx, \quad \dfrac{\partial U}{\partial y} = ky$

Exemplo: oscilador 2d

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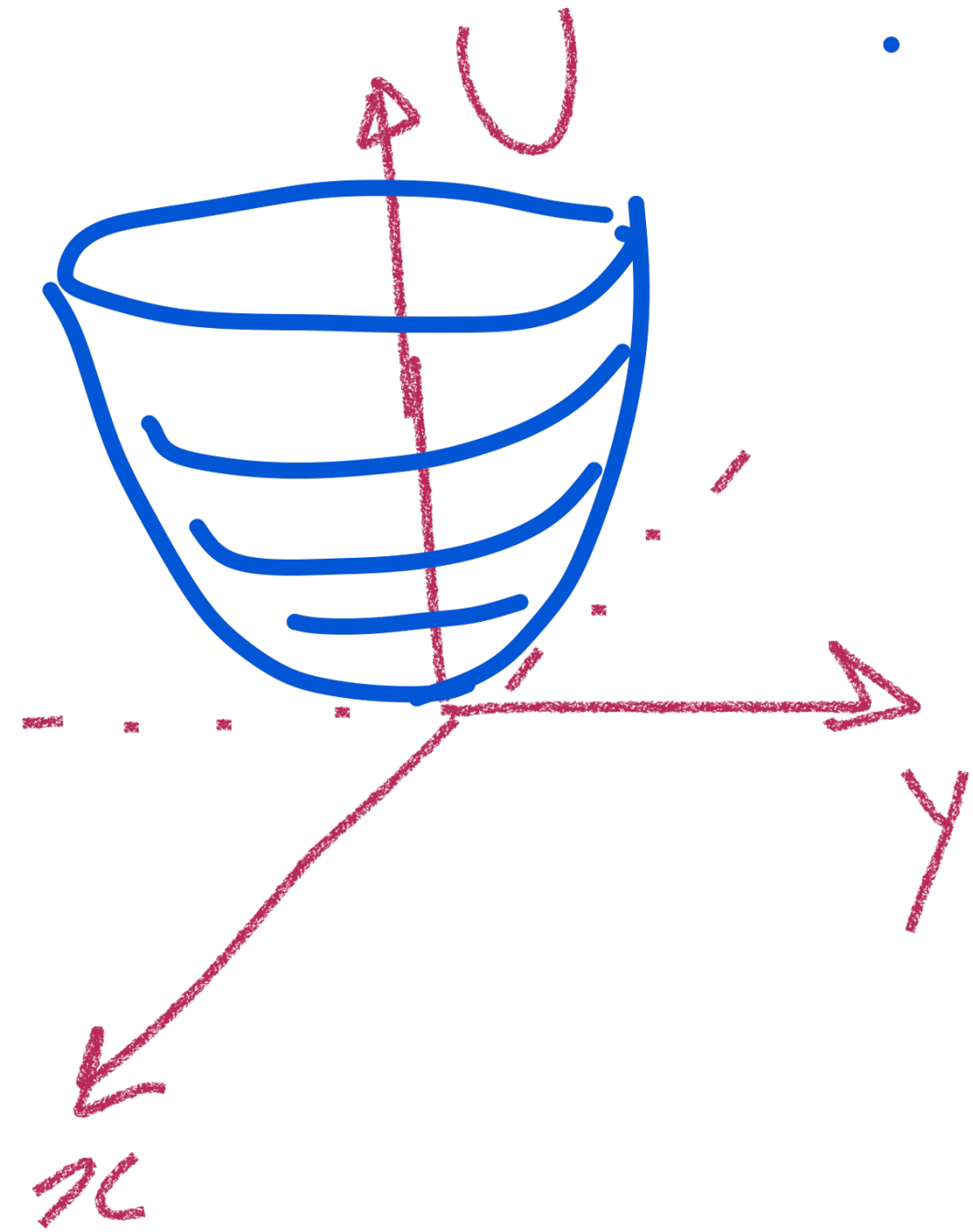

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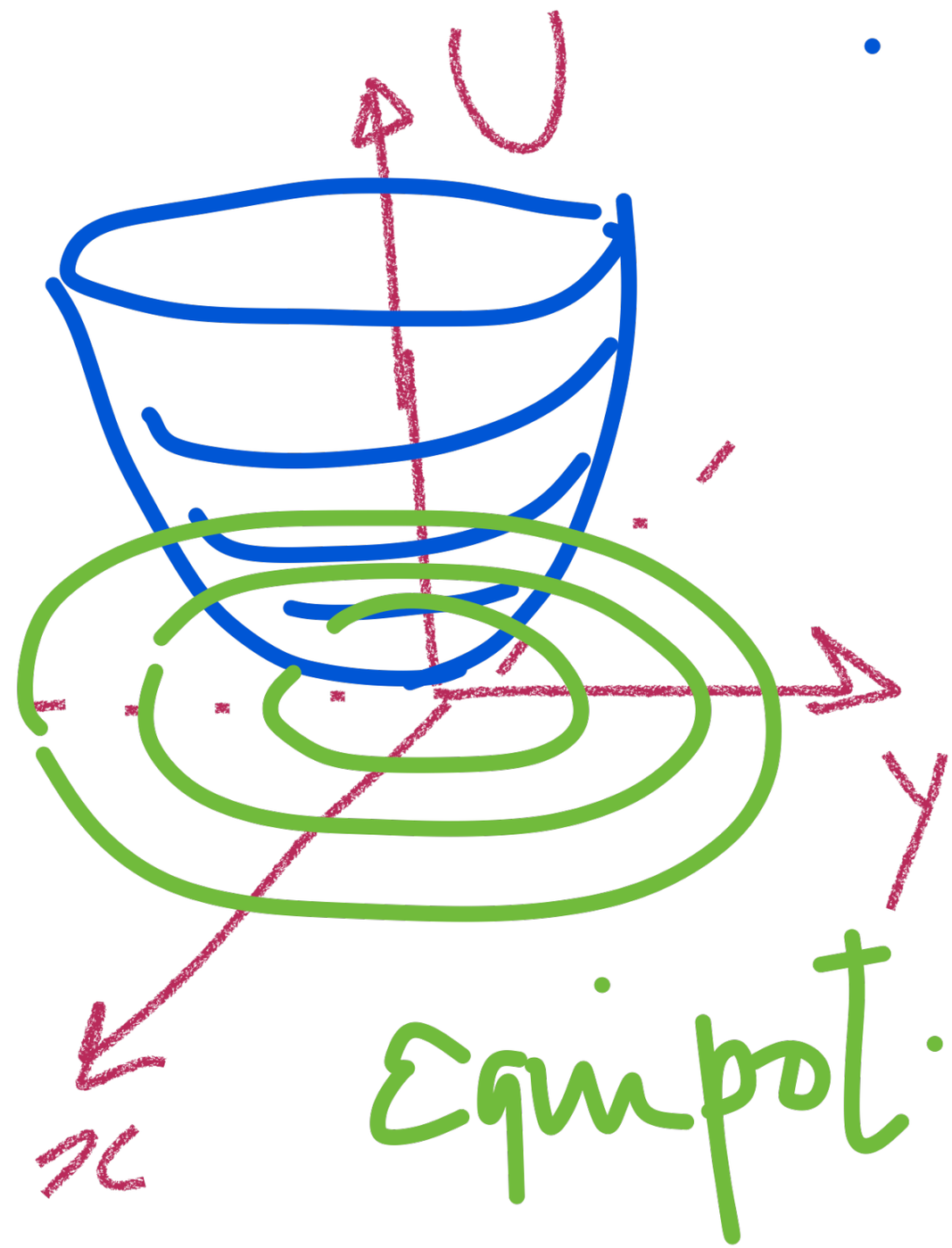

Exemplo: Newton, Coulomb 3d

$$\vec{F} = -\frac{k}{r^2}\hat{r} = -\frac{k}{r^3}\vec{r} = -\left(\frac{kx}{(x^2+y^2+z^2)^{3/2}}, \frac{ky}{(x^2+y^2+z^2)^{3/2}}, \frac{kz}{(x^2+y^2+z^2)^{3/2}}\right)$$

- $$U(x,y) = -\frac{k}{\sqrt{x^2+y^2+z^2}} = -\frac{k}{r}$$

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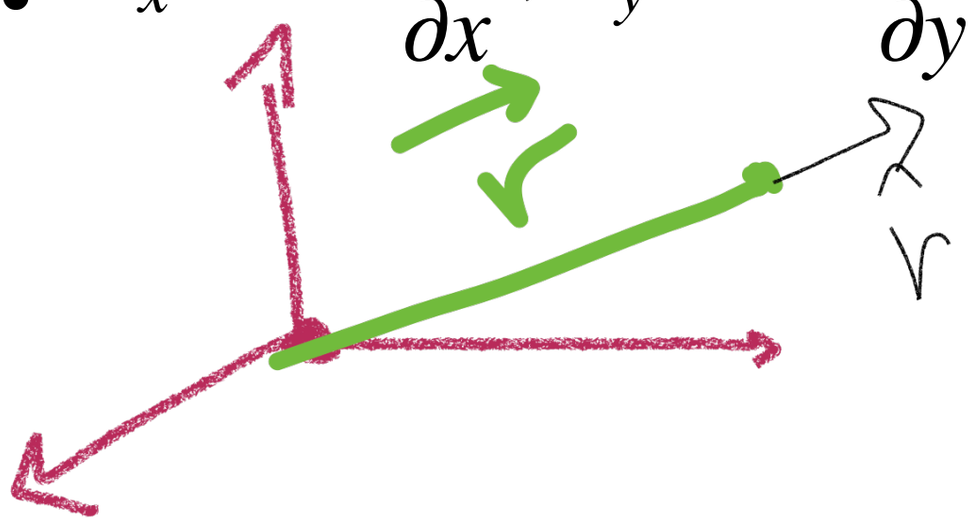

Força Central

$\vec{F} = f(r)\,\hat{r}$

Esf. Sim

Exemplo: Newton, Coulomb 3d

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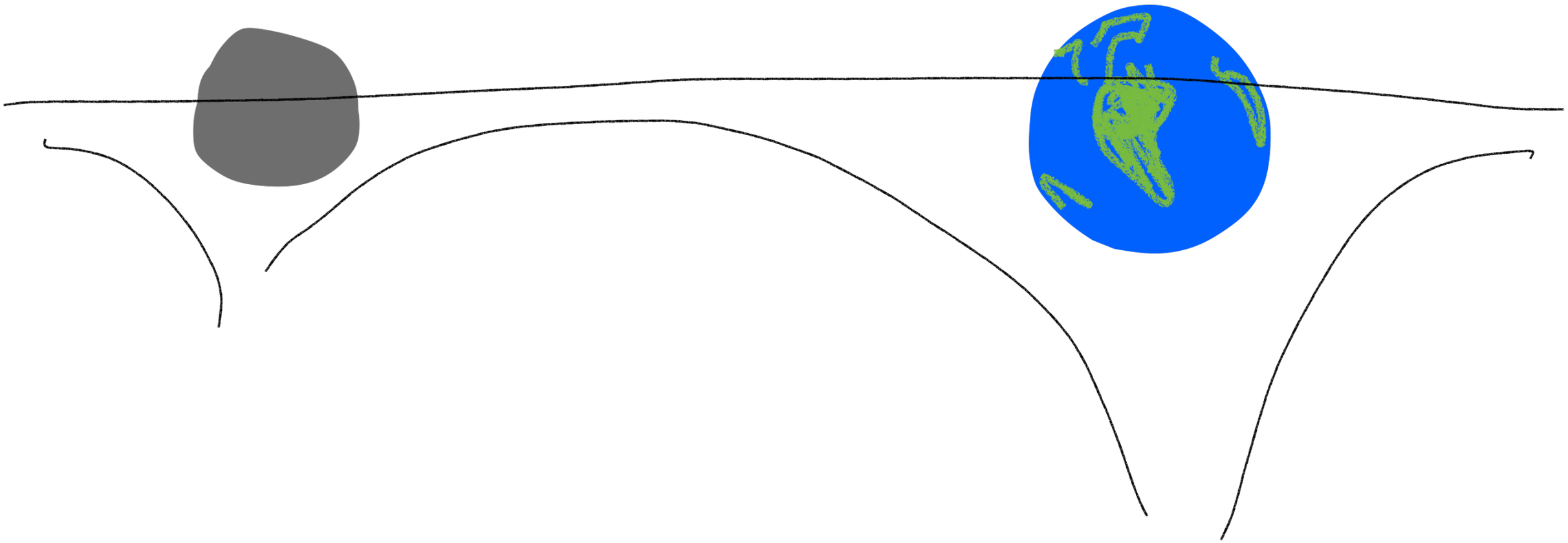
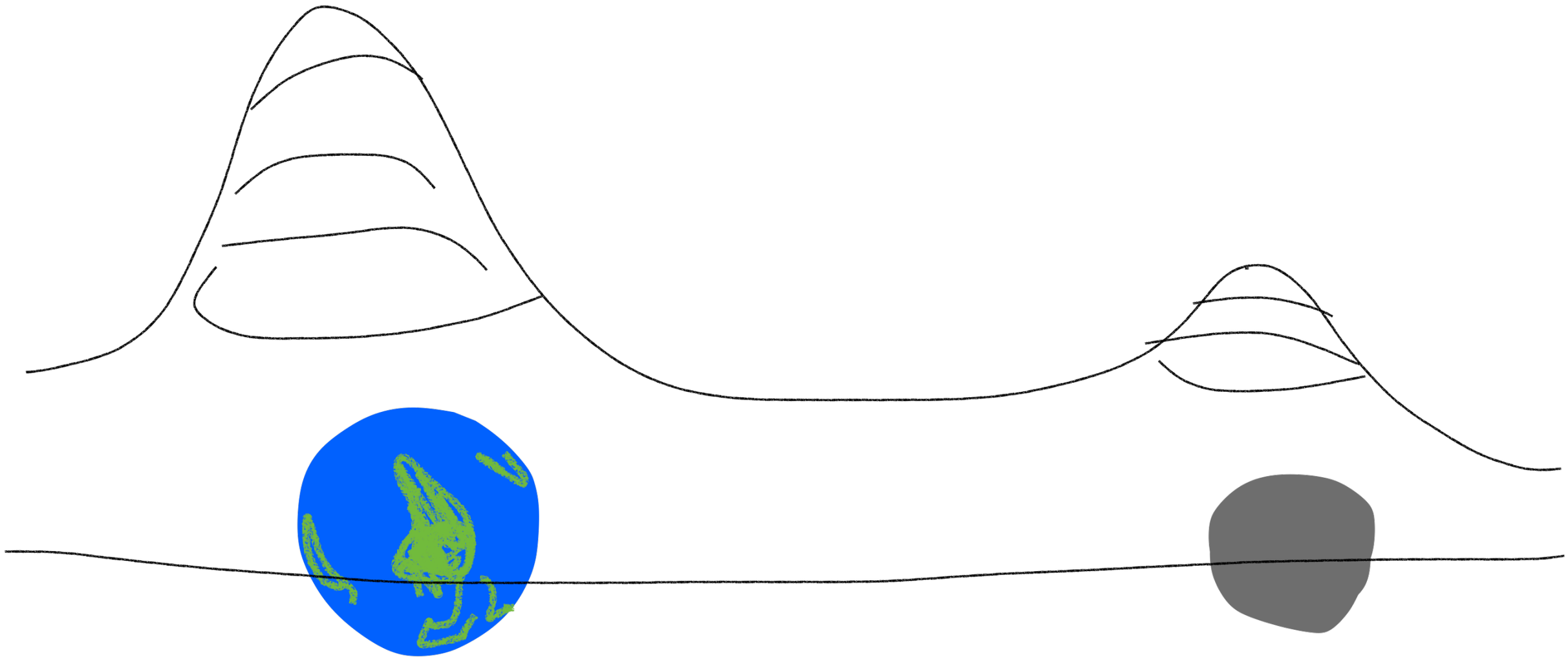
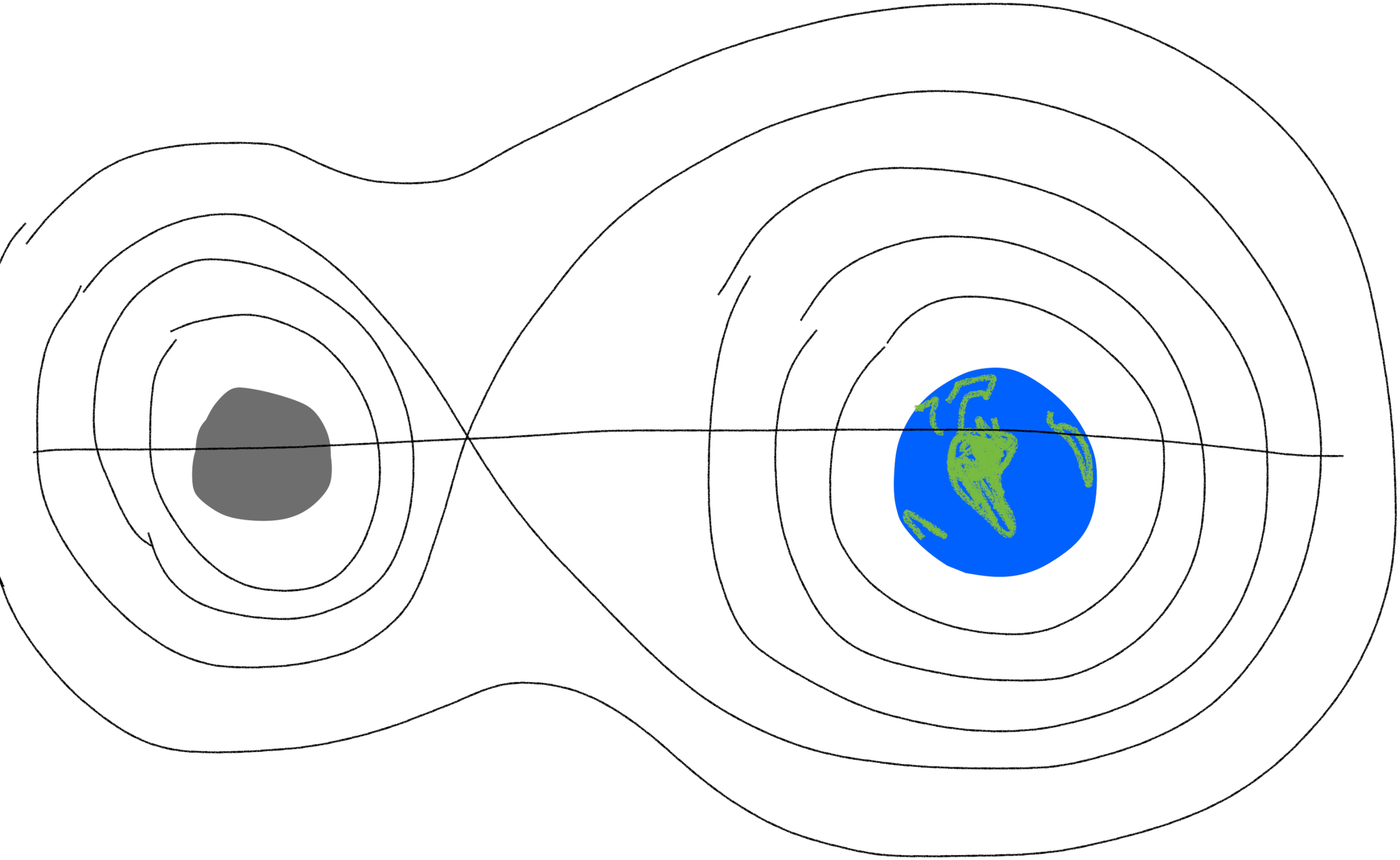
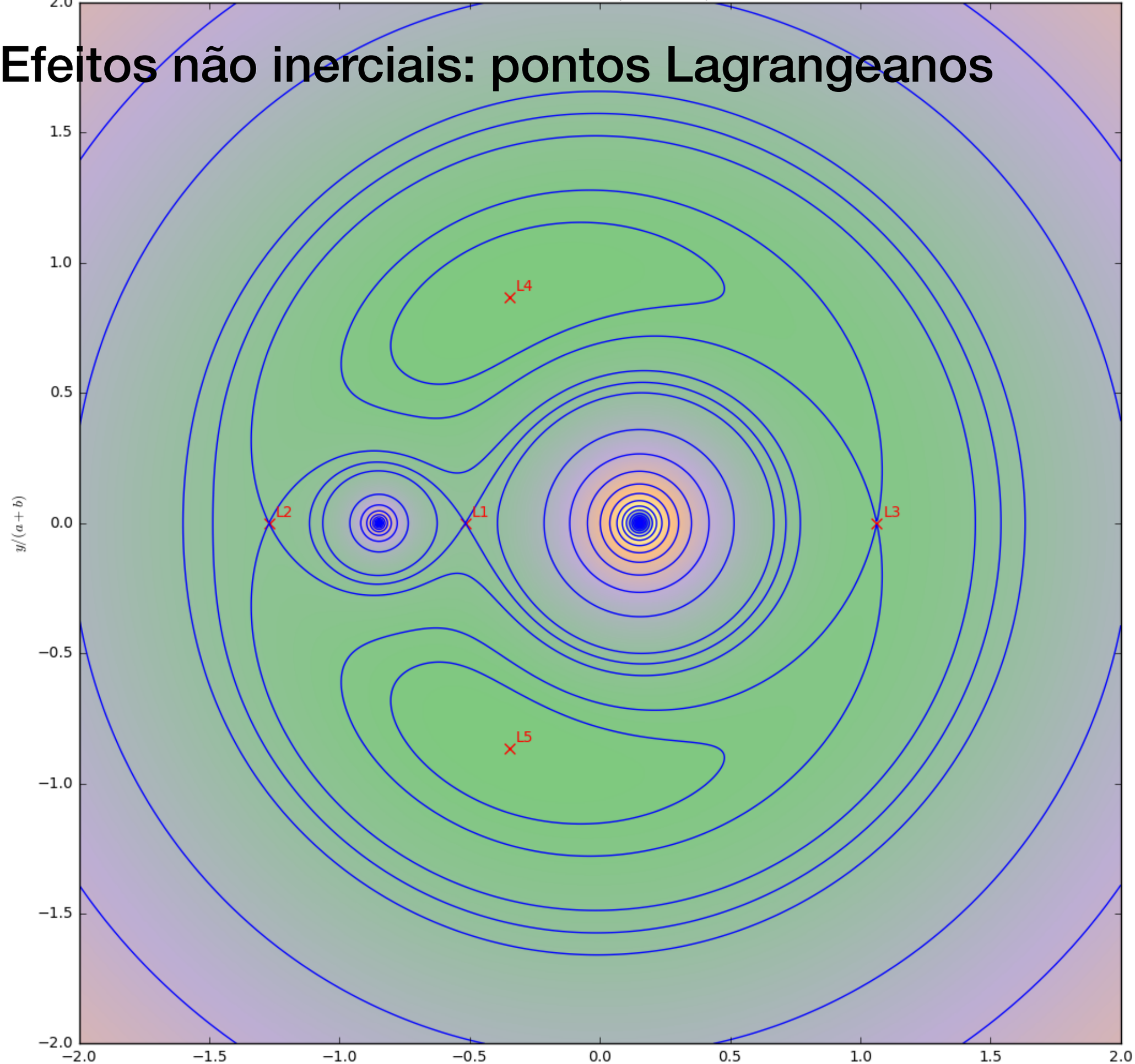
$F = GMm$, $k = \frac{-1}{4\pi\epsilon} Qq$



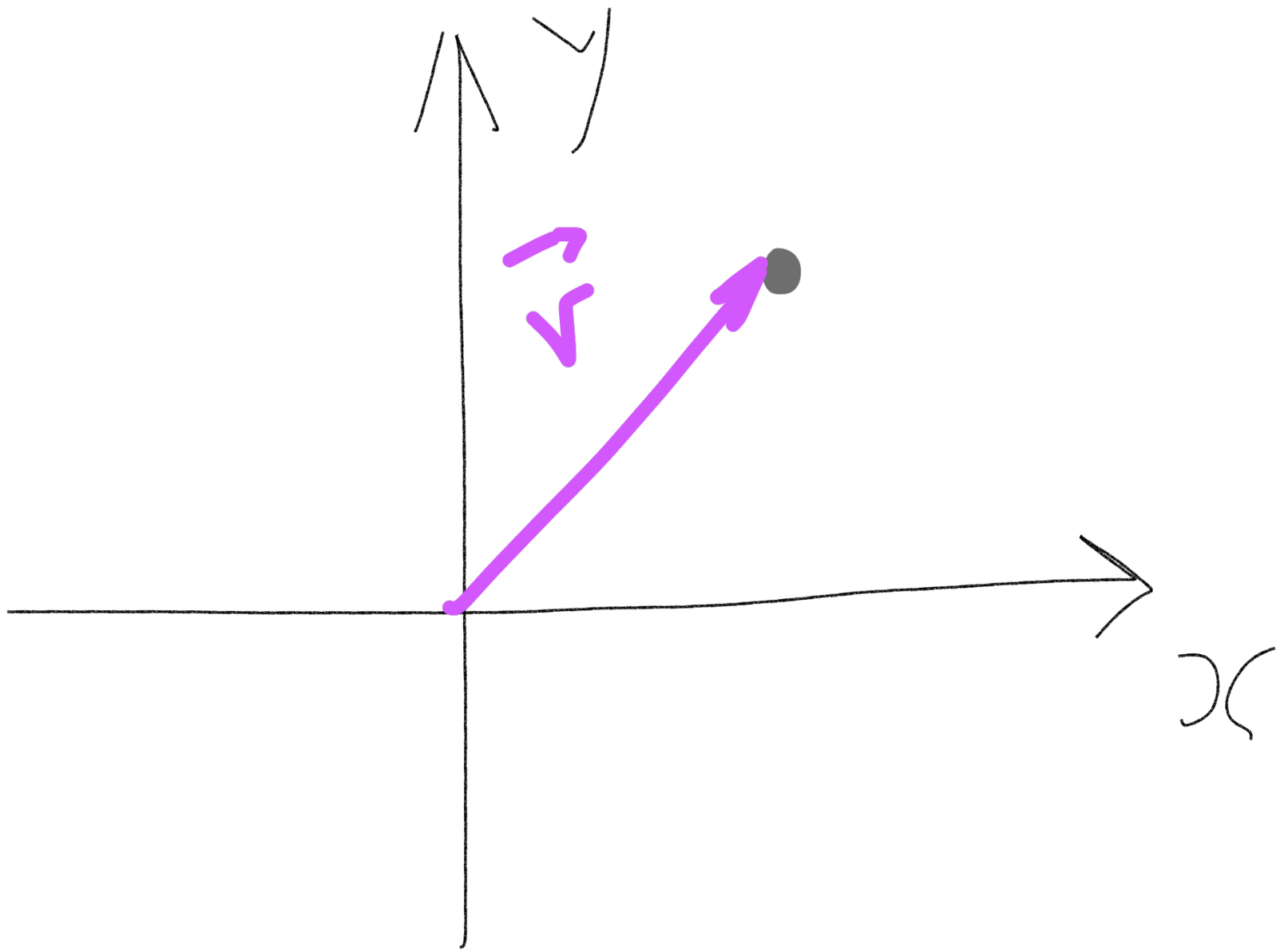




Efeitos não inerciais: pontos Lagrangeanos



Newton: coordenadas polares

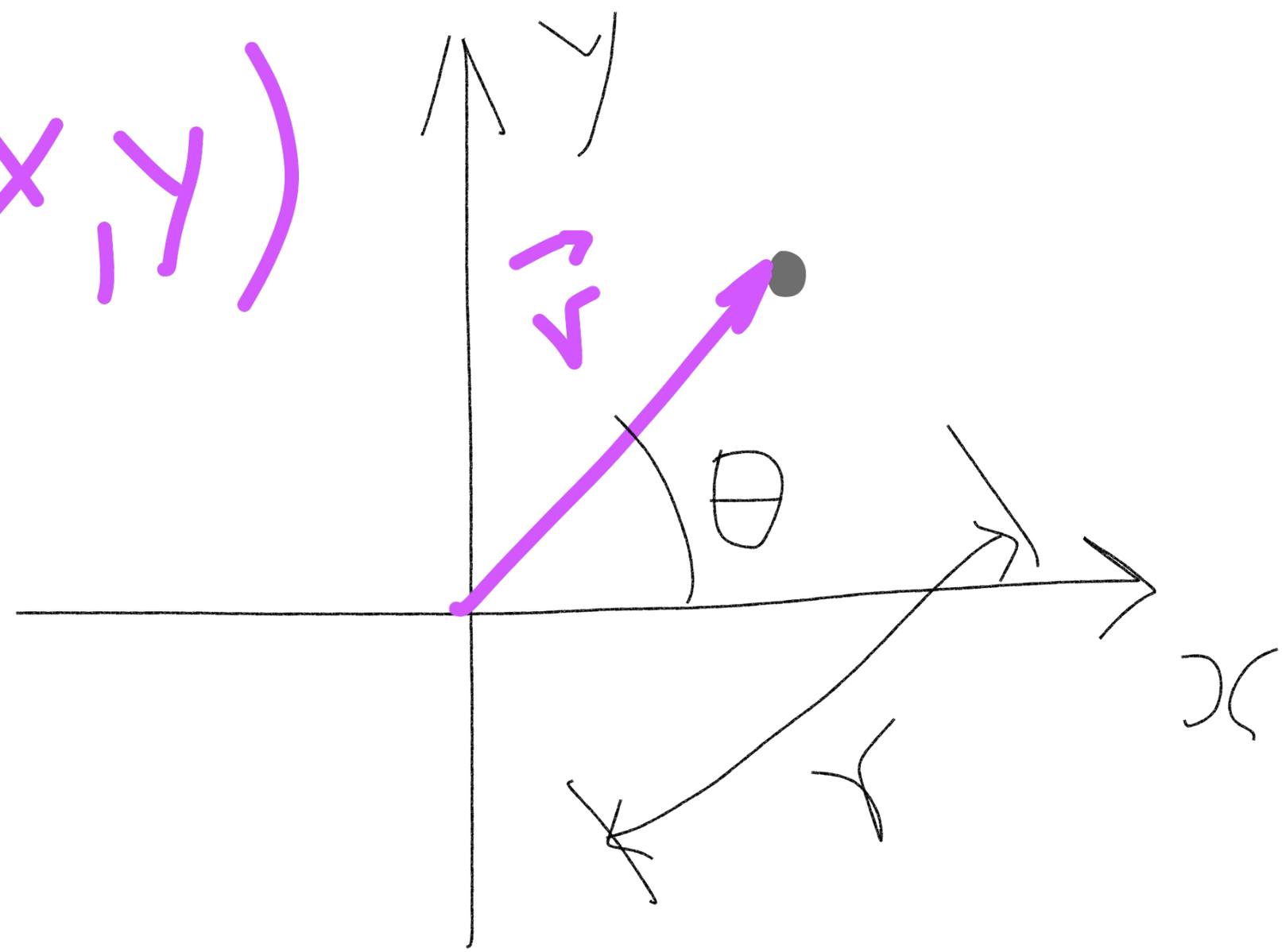


Newton: coordenadas polares

$\vec{r} = (x, y)$

ou

(r, θ)



Newton: coordenadas polares

- Representação de posição no referencial
- Dizer quanto ir para oeste e depois para norte
- Andar distância r na direção θ
- $r = \sqrt{x^2 + y^2}, \qquad \tan\theta = \dfrac{y}{x}$
- $x = r\cos\theta, \qquad y = r\sin\theta$

derivadas: coordenadas polares

- $r = \sqrt{x^2 + y^2}, \qquad \tan\theta = \dfrac{y}{x}$

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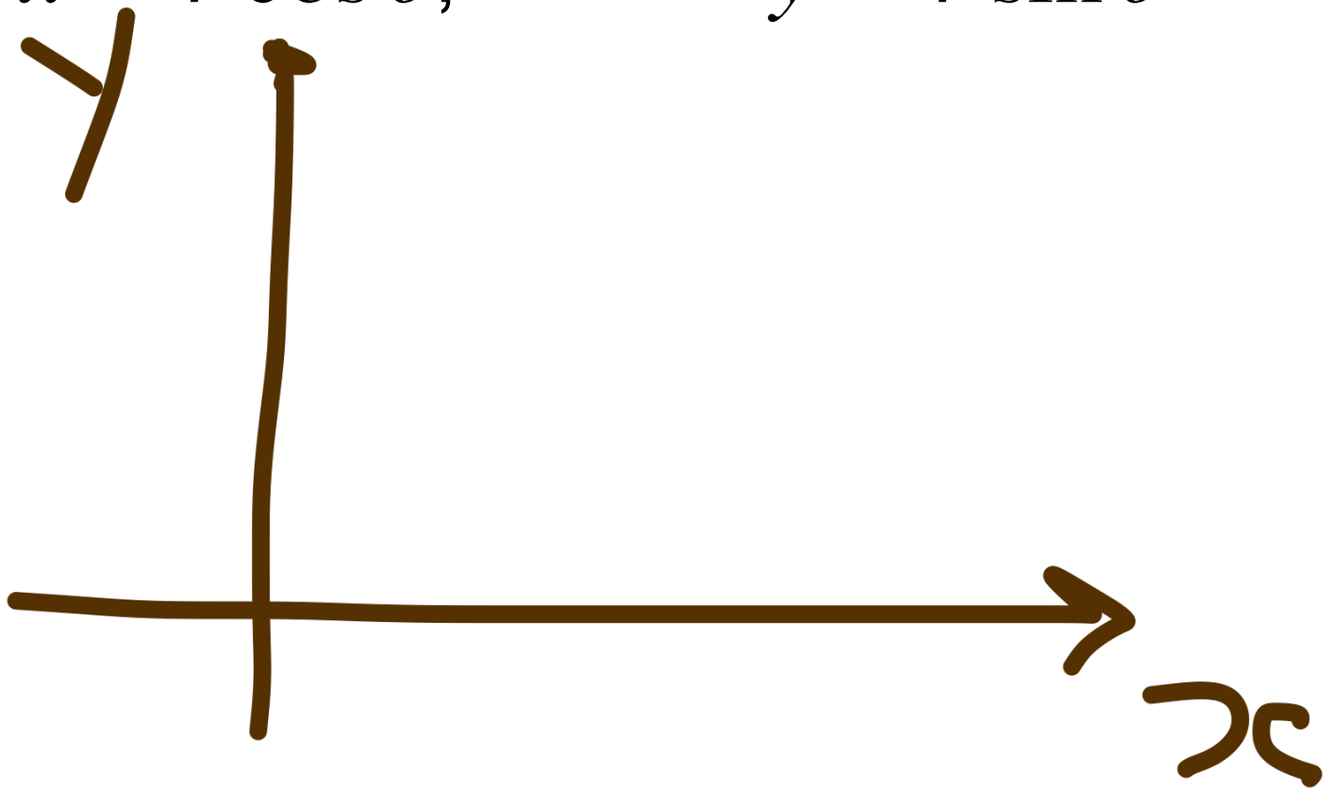

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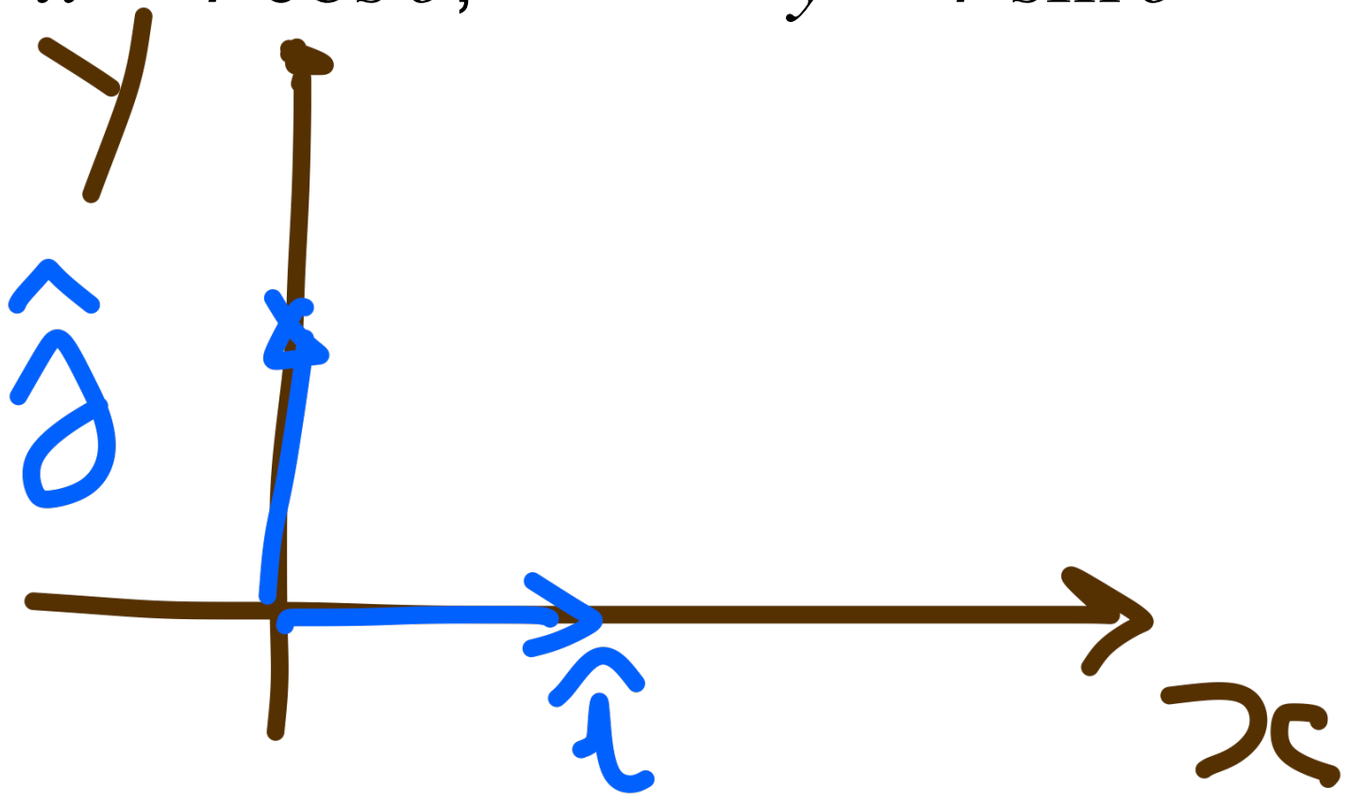
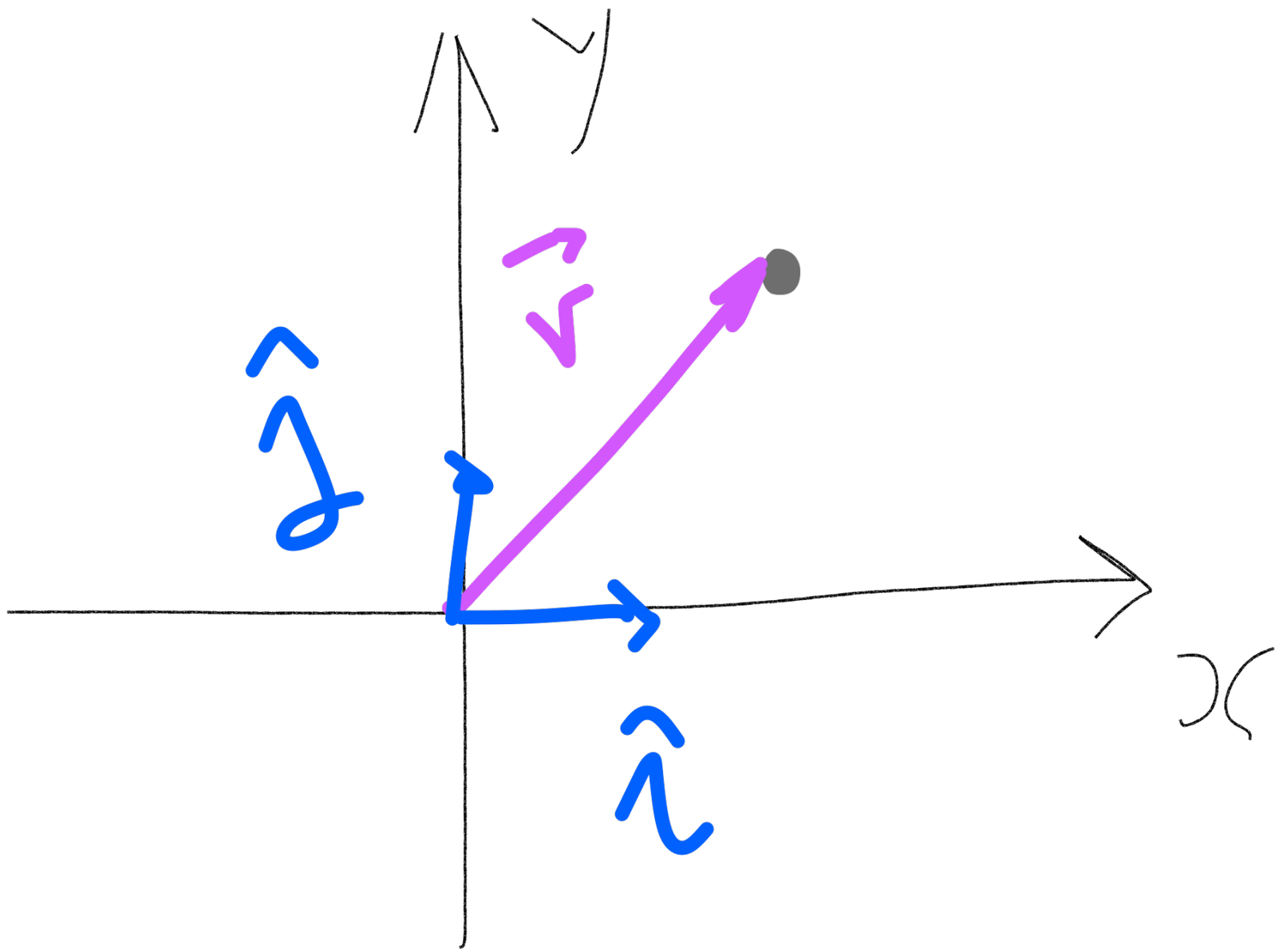


Newton: coordenadas polares



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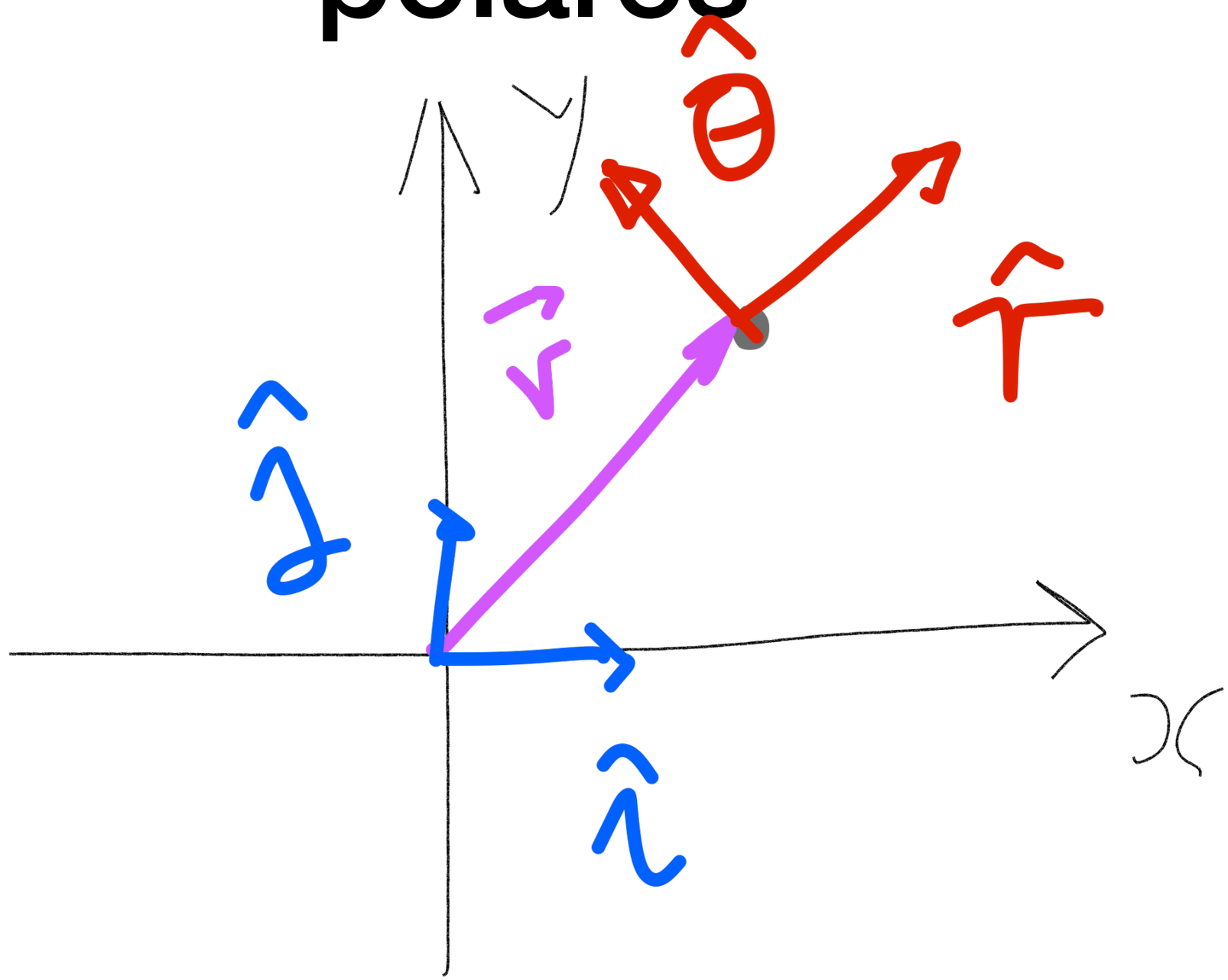

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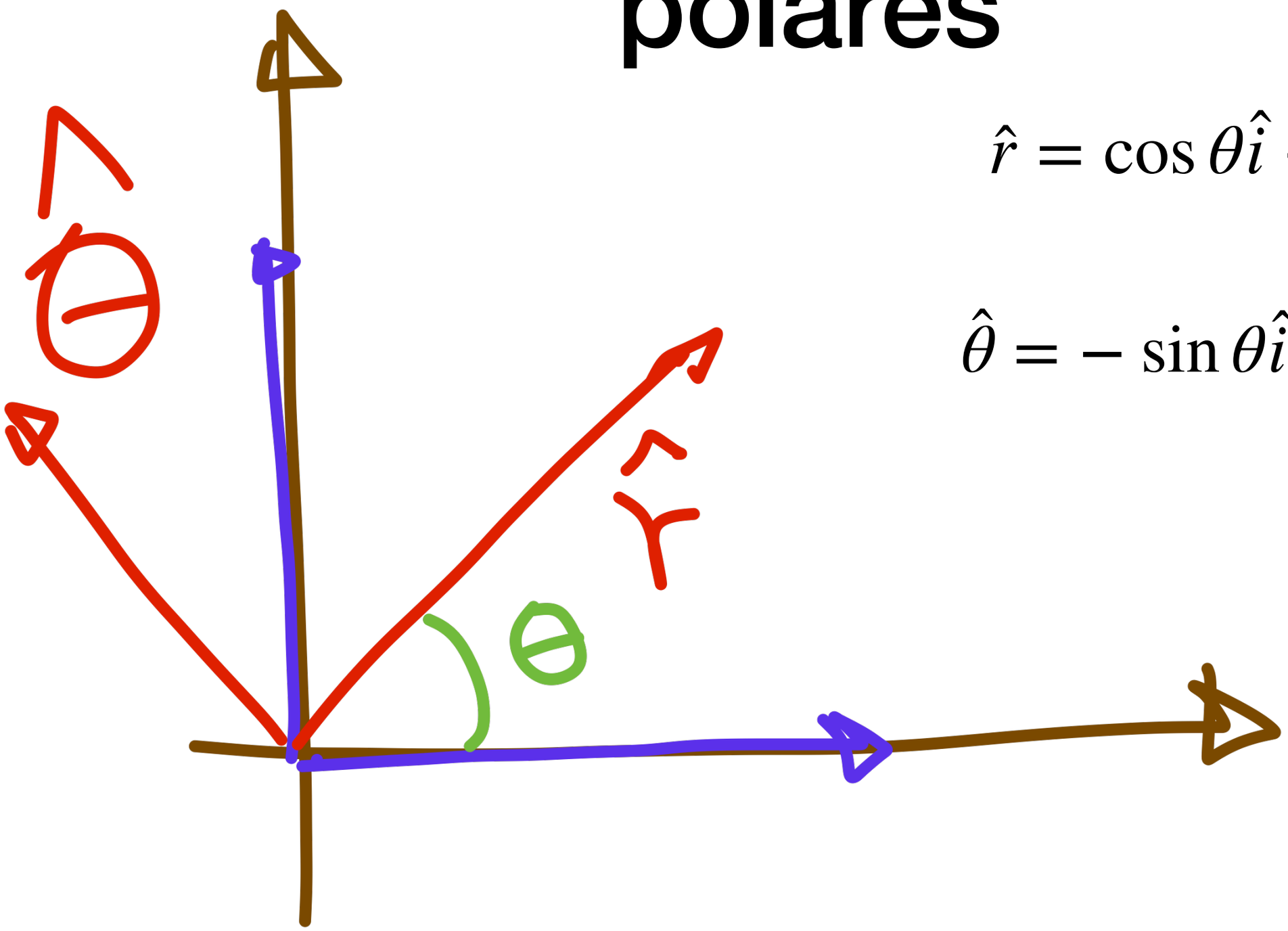
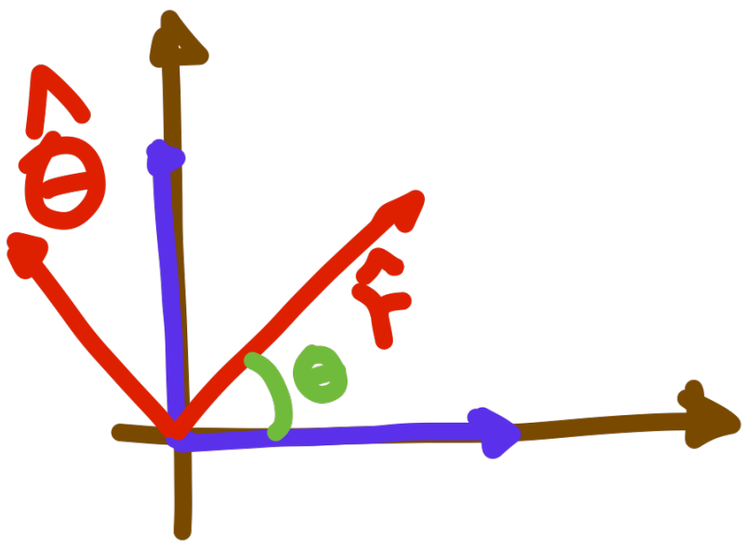


$$\hat{r} = \cos\theta\hat{i} + \sin\theta\hat{j}$$

$$\hat{\theta} = -\sin\theta\hat{i} + \cos\theta\hat{j}$$

Newton: coordenadas polares



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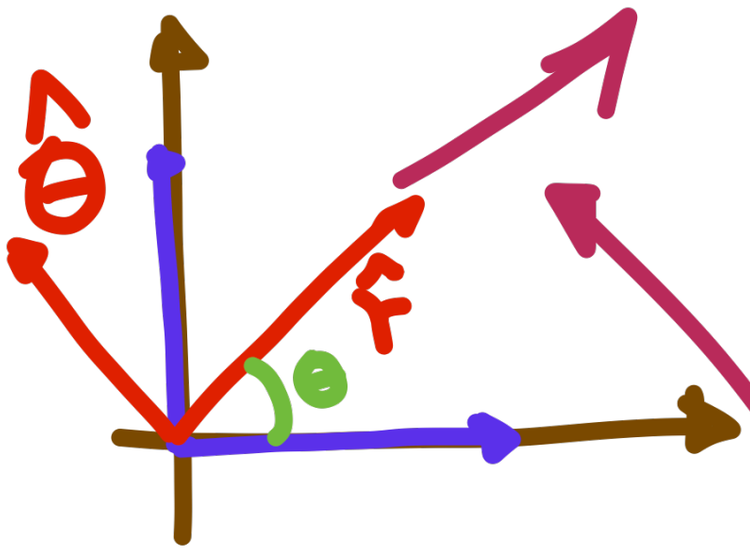
Newton: coordenadas polares



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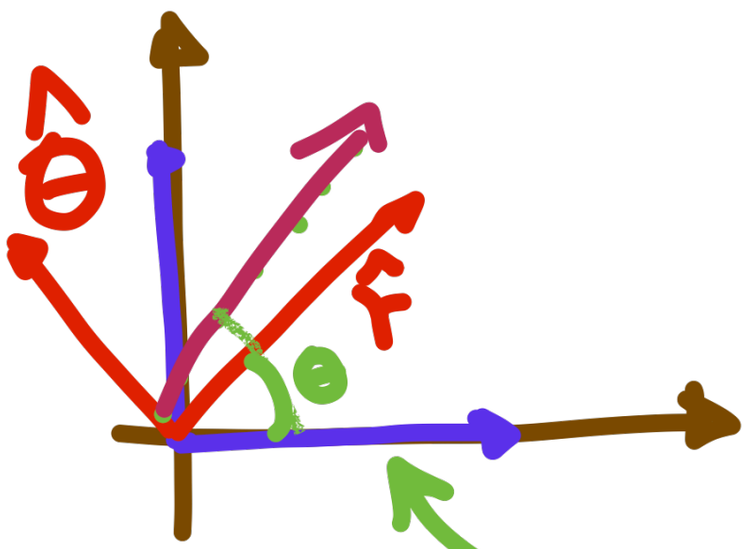

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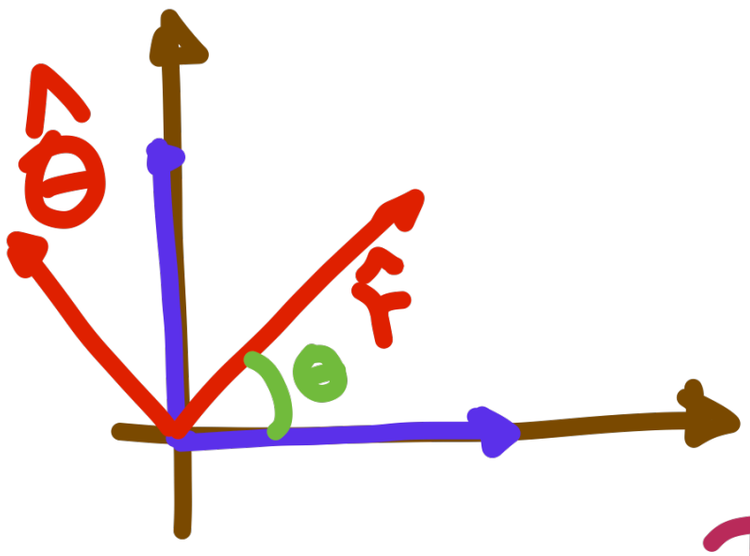

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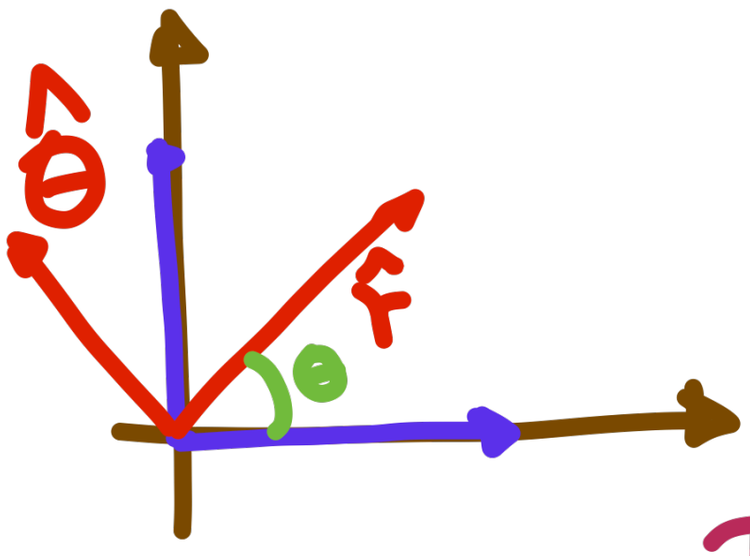
Newton: coordenadas polares



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?

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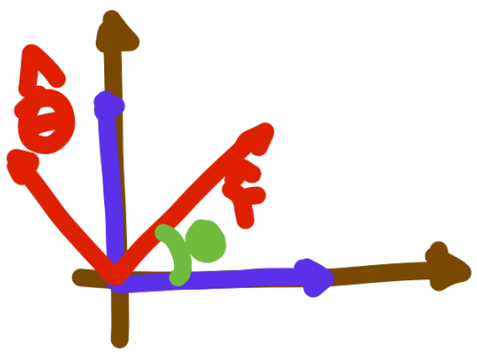

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derivadas: regra da cadeia

- $$\frac{df(\theta)}{dt} = \lim_{\Delta t \to 0} \frac{f(\theta(t + \Delta t)) - f(\theta(t))}{\Delta t}$$

$$\theta(t + \Delta t) = \theta(t) + \Delta\theta$$

- $$\frac{df(\theta)}{dt} = \lim_{\Delta t \to 0} \frac{f(\theta + \Delta\theta) - f(\theta)}{\Delta t}$$

derivadas: regra da cadeia

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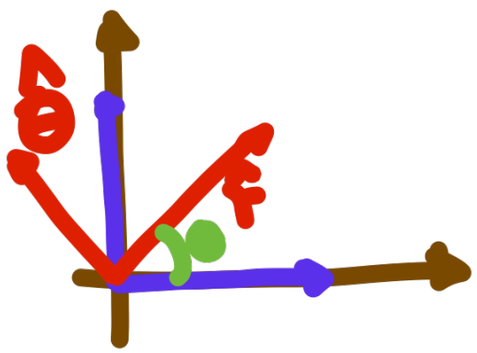
- $$\frac{df(\theta)}{dt} = \frac{df(\theta)}{d\theta} \frac{d\theta}{dt} = f'\dot{\theta}$$

derivadas: regra da cadeia

$$\frac{d}{dt}\cos\theta = -\dot{\theta}\sin\theta$$

$$\frac{d}{dt}\sin\theta = \dot{\theta}\cos\theta$$

Newton: coordenadas polares



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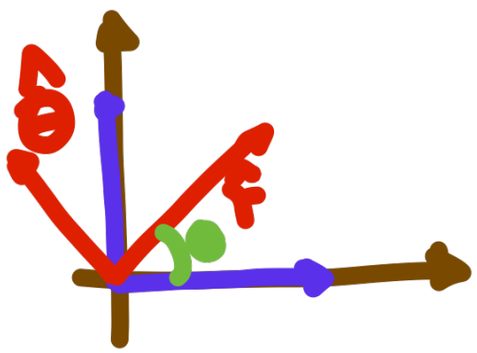
$$\frac{d\hat{r}}{dt} = \frac{d}{dt}(\cos\theta)\hat{i} + \frac{d}{dt}(\sin\theta)\hat{j} = \dot{\theta}(-\sin\theta\hat{i} + \cos\theta\hat{j})$$

$$\frac{d\hat{\theta}}{dt} = -\frac{d}{dt}(\sin\theta)\hat{i} + \frac{d}{dt}(\cos\theta)\hat{j} = -\dot{\theta}(\cos\theta\hat{i} + \sin\theta\hat{j})$$

Newton: coordenadas polares



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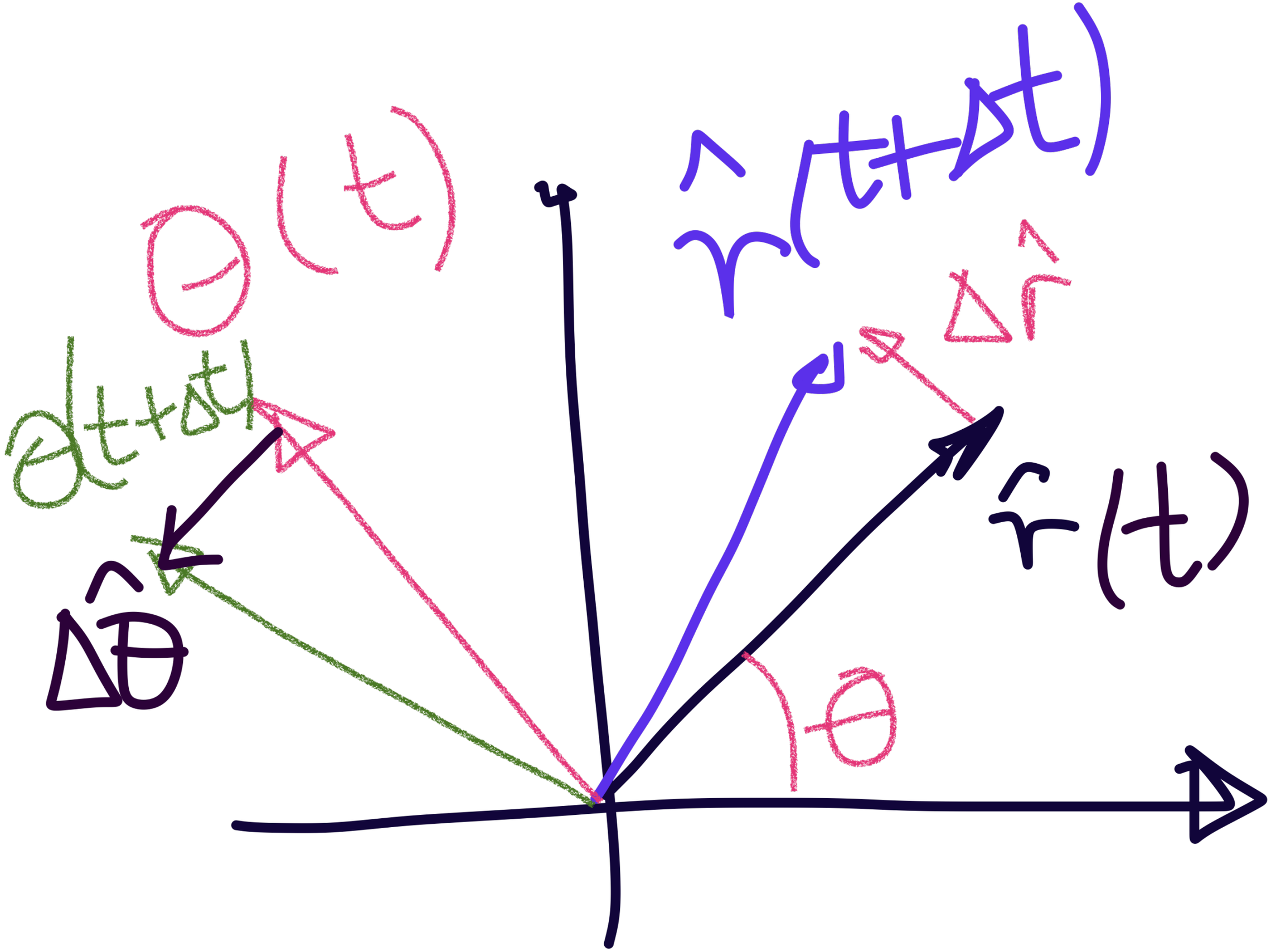
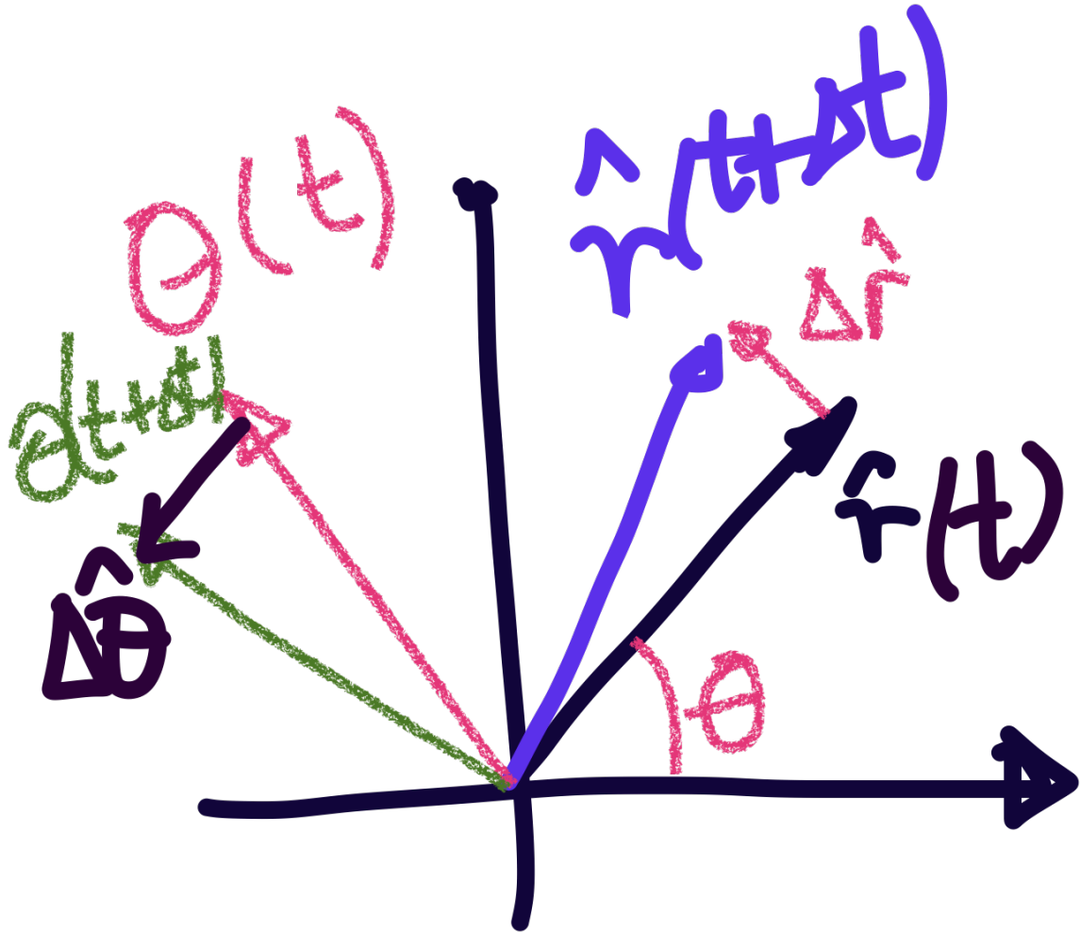
$$\frac{d\hat{r}}{dt} = \dot{\theta}(-\sin\theta\hat{i} + \cos\theta\hat{j}) = \dot{\theta}\hat{\theta}$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}(\cos\theta\hat{i} + \sin\theta\hat{j}) = -\dot{\theta}\hat{r}$$


$\hat{r}(t+\Delta t)$
$\Delta\hat{r}$
$\hat{r}(t)$
$\theta(t)$
θ
$\hat{\theta}(t+\Delta t)$
$\Delta\hat{\theta}$



$$\frac{d\hat{r}}{dt} = \dot{\theta}(-\sin\theta\hat{i} + \cos\theta\hat{j}) = \dot{\theta}\hat{\theta}$$

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$$\frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d^2\vec{r}}{dt} = \frac{d}{dt}(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta})$$

$$\frac{d^2\vec{r}}{dt} = \ddot{r}\hat{r} + \dot{r}\dot{\hat{r}} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\dot{\hat{\theta}}$$

$$\frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

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$\hat{\theta}$

$\hat{r}$

$$\frac{d\vec{r}}{dt} = \frac{dr}{dt}\hat{r} + r\frac{d\hat{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\frac{d^2\vec{r}}{dt} = \frac{d}{dt}(\dot{r}\hat{r} + r\dot{\theta}\hat{\theta})$$

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$\hat{\theta}$

$\hat{r}$

$$\frac{d^2\vec{r}}{dt} = \ddot{r}\hat{r} + \dot{r}\dot{\hat{r}} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\dot{\hat{\theta}}$$

$$\frac{d^2\vec{r}}{dt} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$\frac{d^2\vec{r}}{dt} = \ddot{r}\hat{r} + \dot{r}\dot{\hat{r}} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\dot{\hat{\theta}}$$

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CASO PARTICULAR

$\dot{r} = 0$ MOV. CIRC.

$\ddot{\theta} = 0$ unif.

$$\frac{d^2\vec{r}}{dt} = \ddot{r}\hat{r} + \dot{r}\dot{\hat{r}} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\dot{\hat{\theta}}$$

$$\frac{d^2\vec{r}}{dt} = \cancel{(\ddot{r} - r\dot{\theta}^2)}\hat{r} + \cancel{(2\dot{r}\dot{\theta} + r\ddot{\theta})}\hat{\theta}$$

caso PARTICULAR

$\dot{r} = 0$ Mov. Circ.

$\ddot{\theta} = 0$ unif.

$\vec{a} = -r\omega^2\hat{r}$

$$\overrightarrow{F} = F_r\hat{r} + F_\theta\hat{\theta} = m\frac{d^2\vec{r}}{dt}$$

$$\overrightarrow{F} = m\frac{d^2\vec{r}}{dt} = m(\ddot{r} - r\dot{\theta}^2)\hat{r} + m(2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$F_r = m(\ddot{r} - r\dot{\theta}^2)$$

$$F_\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$$

$$\vec{F} = F_r\hat{r} + F_\theta\hat{\theta} = m\frac{d^2\vec{r}}{dt}$$

Newton

$$\vec{F} = m\frac{d^2\vec{r}}{dt} = m(\ddot{r} - r\dot{\theta}^2)\hat{r} + m(2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{\theta}$$

$$F_r = m(\ddot{r} - r\dot{\theta}^2)$$

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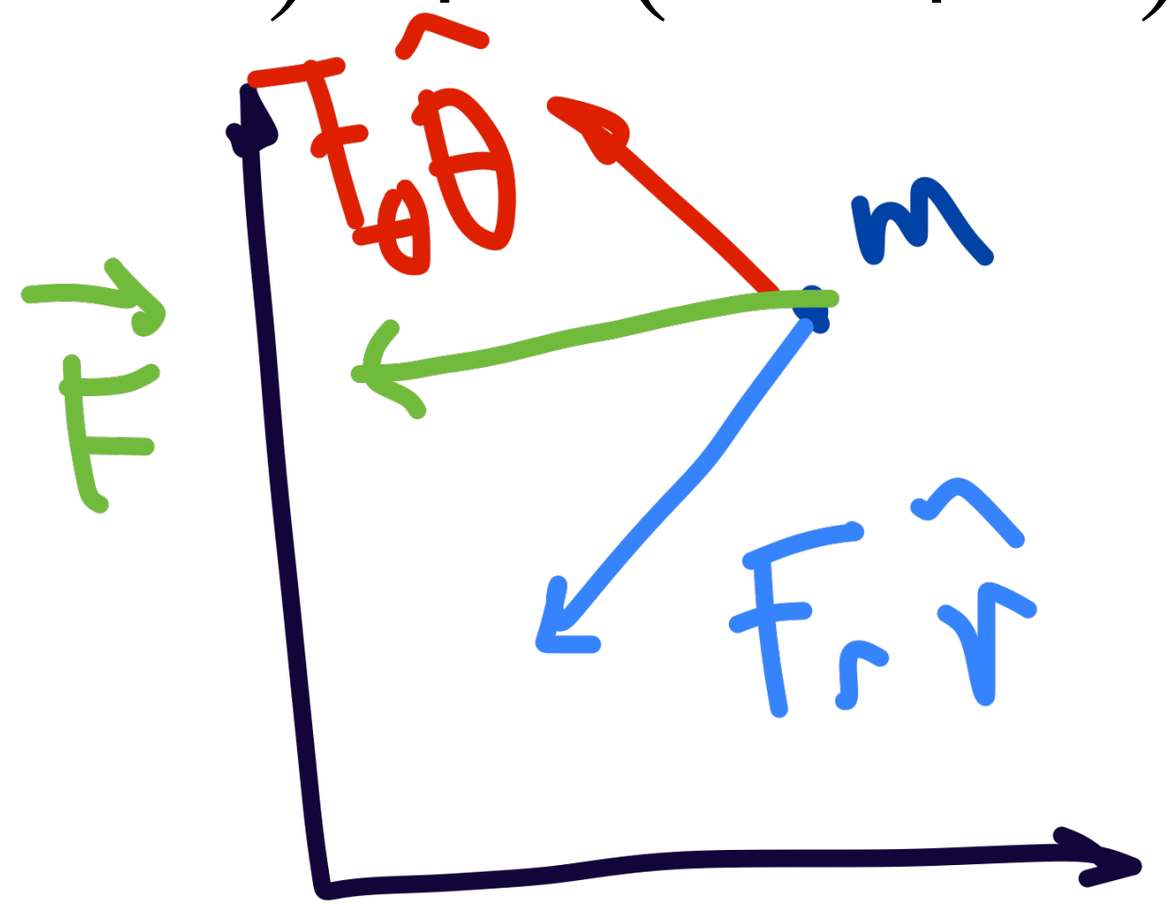

Coordenadas polares e forças centrais

$$\overrightarrow{F} = f(r,\theta)\hat{r} + g(r,\theta)\hat{\theta}$$

- Caso particular: central

$$\overrightarrow{F} = f(r,\theta)\hat{r}$$

- Isotrópico:

$$\overrightarrow{F} = f(r)\hat{r}$$

Coordenadas polares e forças centrais

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Força Central Isot

$$f(r) = m(\ddot{r} - r\dot{\theta}^2)$$

x r

$$g = 0 = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$$

$$0 = m(2\dot{r}r\dot{\theta} + r^2\ddot{\theta})$$

Mostre que

$$\frac{d}{dt}(mr^2\dot{\theta}) = m(2\dot{r}r\dot{\theta} + r^2\ddot{\theta})$$

Força Central Isot

$$f(r) = m(\ddot{r} - r\dot{\theta}^2)$$

x r

$$g = 0 = m(2\dot{r}\dot{\theta} + r\ddot{\theta})$$

$$0 = m(2\dot{r}r\dot{\theta} + r^2\ddot{\theta})$$

$$\frac{d}{dt}(\underbrace{mr^2\dot{\theta}}) = m(2\dot{r}r\dot{\theta} + r^2\ddot{\theta}) = 0$$

Mostre que

$$mr^2\dot{\theta} = l = \text{Const}$$

Força Central Isot.

$$f(r) = m(\ddot{r} - r\dot{\theta}^2)$$

$$\frac{d}{dt}\ell = \frac{d}{dt}(mr^2\dot{\theta}) = m(2\dot{r}r\dot{\theta} + r^2\ddot{\theta}) = 0$$

$$\dot{\theta} = \frac{\ell}{mr^2}$$

$$mr\dot{\theta}^2 = \frac{\ell^2}{mr^3}$$

$$f(r) = -\frac{k}{r^2} = m(\ddot{r} - r\dot{\theta}^2)$$

$$mr\dot{\theta}^2 = \frac{l^2}{mr^3}$$

$$m\ddot{r} = -\frac{k}{r^2} + \frac{l^2}{mr^3}$$