Deformation Plasticity Theory

Yielding occurs at the same effective stress $\bar{\sigma}$ for different stress states (1D, 2D, 3D)

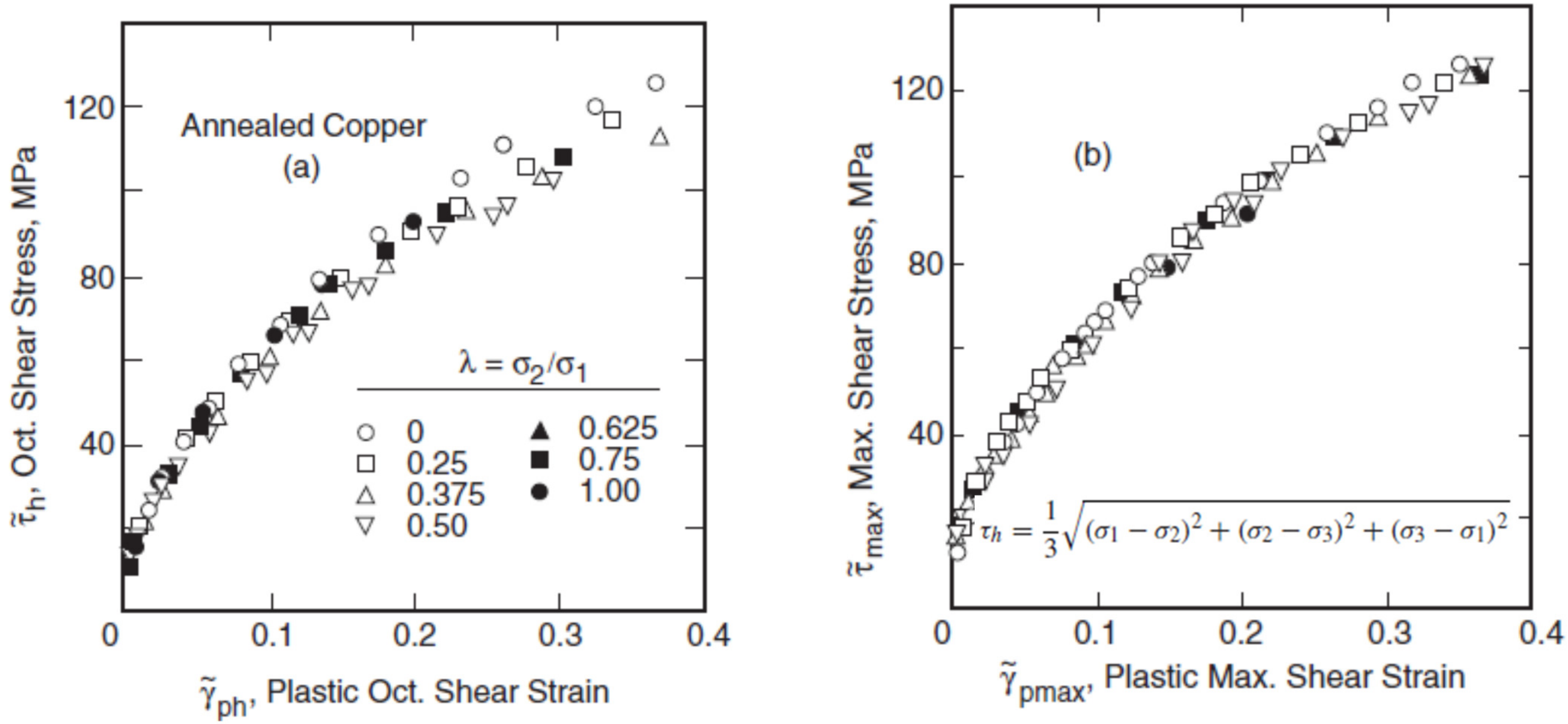


Figure 12.7 Correlation of true stresses and true plastic strains from combined axial and pressure loading of thin-walled copper tubes in terms of (a) octahedral shear stress and strain, and (b) maximum shear stress and strain. (Adapted from [Davis 43]; used with permission of ASME.)

Deformation Plasticity Theory

Stress-Strain relations

$$\varepsilon_x = \varepsilon_{ex} + \varepsilon_{px}, \qquad \varepsilon_y = \varepsilon_{ey} + \varepsilon_{py}, \qquad \varepsilon_z = \varepsilon_{ez} + \varepsilon_{pz}$$

$$\gamma_{xy} = \gamma_{exy} + \gamma_{pxy}, \qquad \gamma_{yz} = \gamma_{eyz} + \gamma_{pyz}, \qquad \gamma_{zx} = \gamma_{ezx} + \gamma_{pzx}$$

Elastic strains

$$\varepsilon_{ex} = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_{ey} = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$

$$\varepsilon_{ez} = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\gamma_{exy} = \frac{\tau_{xy}}{G}, \qquad \gamma_{eyz} = \frac{\tau_{yz}}{G}, \qquad \gamma_{ezx} = \frac{\tau_{zx}}{G}$$

Plastic Strains

$$\varepsilon_{px} = \frac{1}{E_p}\left[\sigma_x - 0.5\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_{py} = \frac{1}{E_p}\left[\sigma_y - 0.5\left(\sigma_x + \sigma_z\right)\right]$$

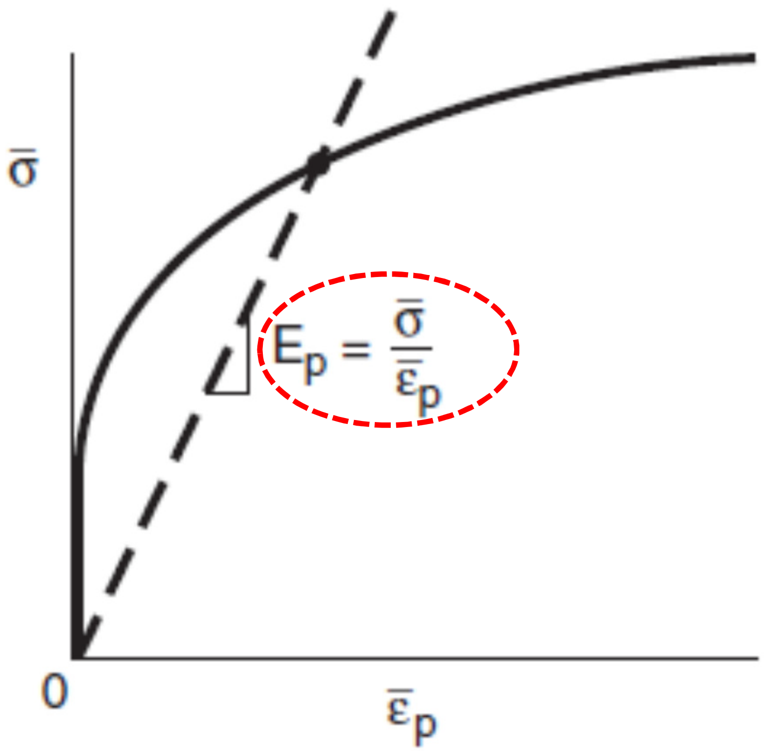
$$\varepsilon_{pz} = \frac{1}{E_p}\left[\sigma_z - 0.5\left(\sigma_x + \sigma_y\right)\right]$$

$$\gamma_{pxy} = \frac{3}{E_p}\tau_{xy}, \qquad \gamma_{pyz} = \frac{3}{E_p}\tau_{yz}, \qquad \gamma_{pzx} = \frac{3}{E_p}\tau_{zx}$$

υ=0.5 is equivalent to the assumption that plastic strains do not contribute to volume change

Deformation Plasticity Theory

Plastic Modulus



Effective total strain

$$\bar{\varepsilon} = \frac{\bar{\sigma}}{E} + \bar{\varepsilon}_p$$

Deformation Plasticity Theory

Effective Stress

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

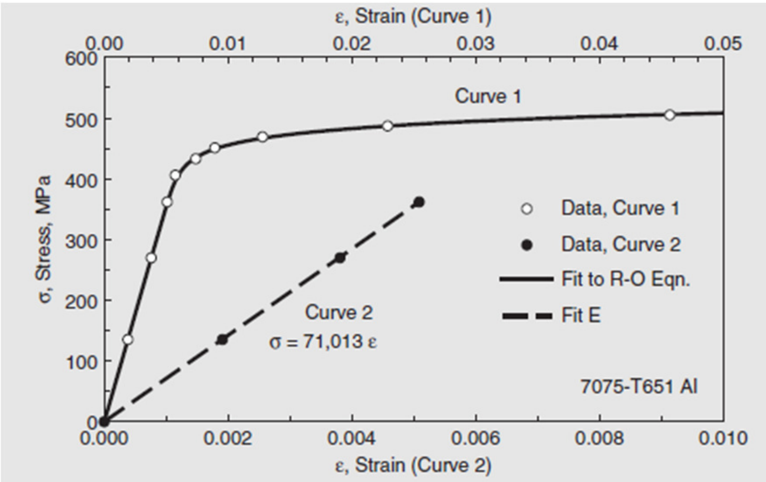
Effective Plastic Strain

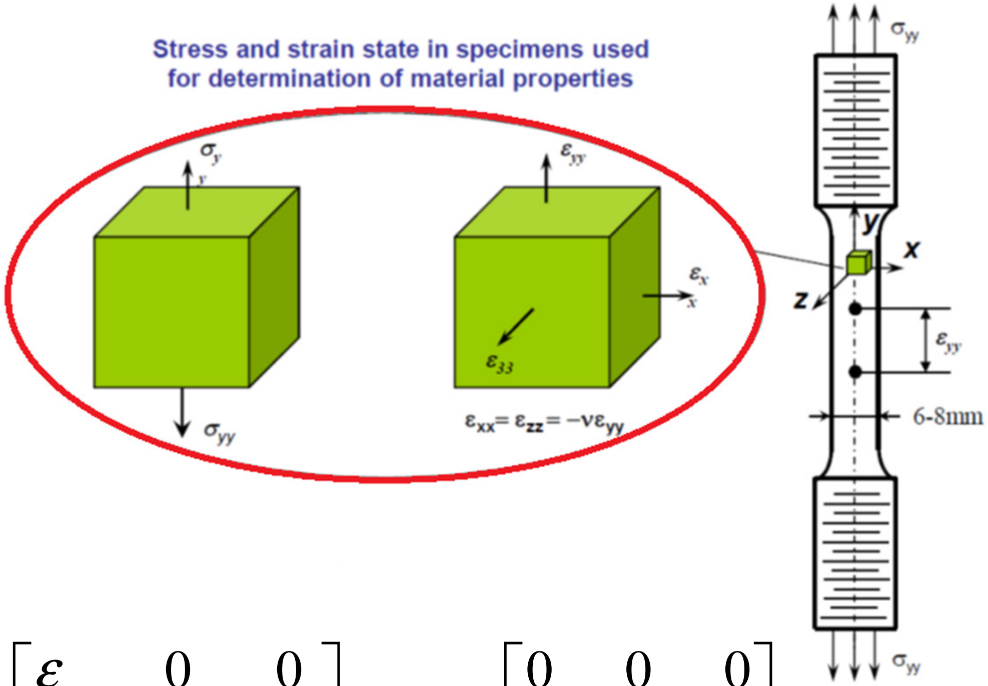
$$\bar{\varepsilon}_p = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_{p1} - \varepsilon_{p2})^2 + (\varepsilon_{p2} - \varepsilon_{p3})^2 + (\varepsilon_{p3} - \varepsilon_{p1})^2}$$

Deformation Plasticity Theory

Effective Stress- Effective Plastic Strain

Uniaxial true stress- strain curve



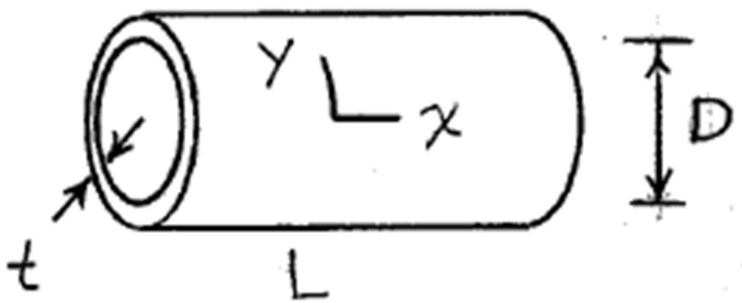


$$\varepsilon_{ij} = \begin{bmatrix} \varepsilon_{xx} & 0 & 0 \\ 0 & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \qquad \sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Deformation Plasticity Theory

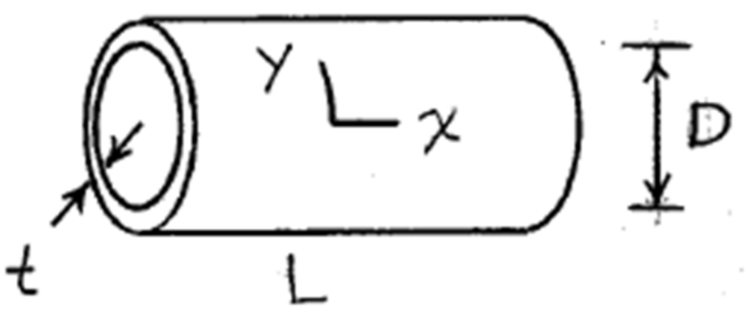
Exemplo

Um componente tubular de parede fina de radio externo *r* e espessura de parede *t* é construído de um material cuja curva tensão-deformação segue a relação de Ramberg-Osgood. Para uma dada pressão interna P, deriva um expressão para calcular a variação relativa do raio .



Deformation Plasticity Theory

Solução



Hoop Strain

$$\varepsilon_1 = \frac{\Delta(2\pi r)}{2\pi r} \longrightarrow \varepsilon_1 = \frac{\Delta r}{r}$$

Axial Strain

$$\varepsilon_2 = \frac{\Delta L}{L}$$

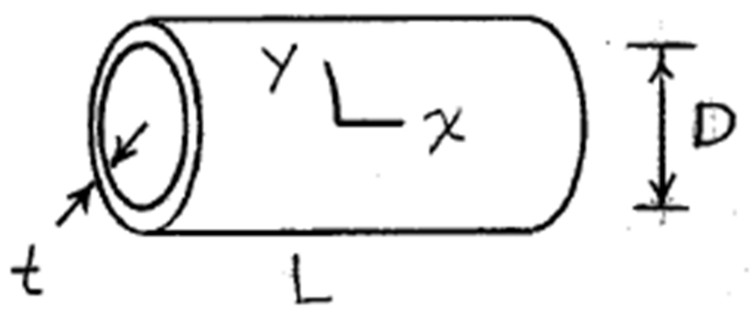
Radial Strain

$$\varepsilon_3 = \frac{\Delta t}{t}$$

Deformation Plasticity Theory

Solução

p= pressão interna



Na superfície temos estado plano de tensão

$$\lambda = \frac{\sigma_2}{\sigma_1} = 0.5$$

$$\sigma_1 = p\frac{r}{t}$$

Axial Stress

$$\frac{r}{t} \gg 1$$

$$\sigma_2 = p\frac{r}{2t}$$

Radial Stress

$$\sigma_3 = -p$$

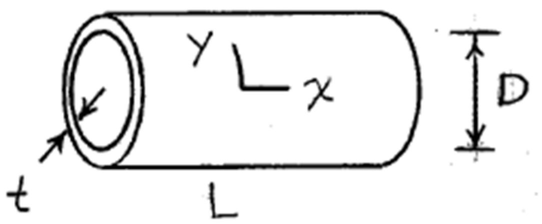
$$\sigma_3 \cong 0$$

$\lambda = \frac{\sigma_2}{\sigma_1}$ Razão de biaxialidade

Deformation Plasticity Theory

Solução

p= pressão interna



Tensão Efectiva

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

$$\lambda = \frac{\sigma_2}{\sigma_1} \qquad \sigma_3 \cong 0$$

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \lambda\sigma_1)^2 + (\lambda\sigma_1)^2 + (\sigma_1)^2}$$

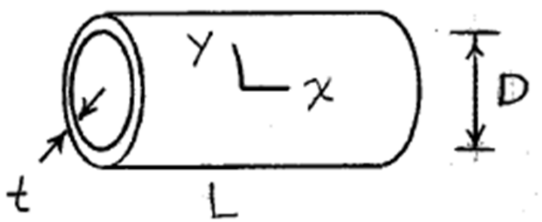
$$\bar{\sigma} = \frac{1}{\sqrt{2}}\sqrt{2\lambda^2\sigma_1^2 - 2\lambda\sigma_1^2 + 2\sigma_1^2}$$

$$\bar{\sigma} = \sigma_1\sqrt{\lambda^2 - \lambda + 1}$$

Deformation Plasticity Theory

Solução

p= pressão interna



Deformação total na direção 1

$$\varepsilon_1 = {\varepsilon_1}^e + {\varepsilon_1}^p$$

$${\varepsilon_1}^e = \frac{\sigma_1}{E}(1 - \upsilon\lambda)$$

$${\varepsilon_1}^p = \frac{1}{E_p}(\sigma_1 - 0.5\lambda\sigma_1)$$

$$\bar{\sigma} = \sigma_1\sqrt{\lambda^2 - \lambda + 1}$$

$$E_p = \frac{\bar{\sigma}}{\bar{\varepsilon}_p}$$

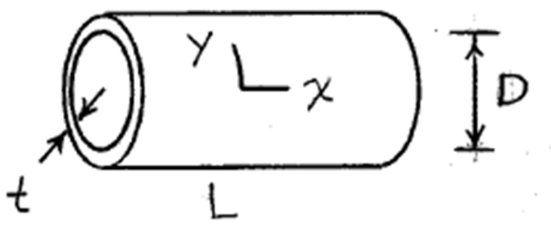
$$\bar{\varepsilon}_p = \bar{\varepsilon} - \frac{\bar{\sigma}}{E}$$

$$\bar{\varepsilon}_p = \left(\frac{\bar{\sigma}}{H}\right)^{1/n}$$

Deformation Plasticity Theory

Solução

p= pressão interna



Deformação total na direção 1

$$\varepsilon_1 = \frac{\sigma_1}{E}(1 - \upsilon\lambda) + \frac{\bar{\varepsilon}_p}{\bar{\sigma}}\sigma_1(1 - 0.5\lambda)$$

$$\varepsilon_1 = \frac{\sigma_1}{E}(1 - \upsilon\lambda) + \frac{\sigma_1}{\bar{\sigma}}\left(\frac{\bar{\sigma}}{H}\right)^{1/n}(1 - 0.5\lambda)$$

$$\sigma_1 = p\frac{r}{t}$$

$$\bar{\sigma} = \sigma_1\sqrt{\lambda^2 - \lambda + 1}$$

$$+$$

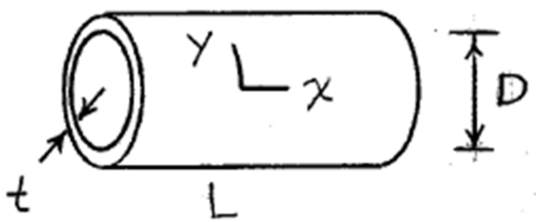
$$\varepsilon_1 = \frac{\sigma_1}{E}(1 - \upsilon\lambda) + \frac{(1 - 0.5\lambda)}{\sqrt{\lambda^2 - \lambda + 1}}\left(\frac{\bar{\sigma}}{H}\right)^{1/n}$$

$$\lambda = \frac{\sigma_2}{\sigma_1} = 0.5$$

Deformation Plasticity Theory

Solução

p= pressão interna



Deformação total na direção 1

$$\varepsilon_1 = \frac{\Delta r}{r} = p\frac{r}{Et}\left(1 - \frac{v}{2}\right) + \frac{\sqrt{3}}{2}\left(\frac{\sqrt{3}pr}{2tH}\right)^{1/n}$$