Can the COVID-19 epidemic be managed on the basis of daily data?

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Abstract

Short answer: not much, and only with an overly cautious approach. The paper presents a simple mathematical model of the process and uses well-known results from control theory to prove that the approach taken by China and, to a slightly lesser extent, by Italy can work if the effect of delays is accounted for when taking the decision of the country lockdown, while the approach currently announced in the UK is likely to fail.

1 Introduction

The first outbreak of the COVID-19 [8] virus epidemic took place in China, starting in December 2019, possibly even earlier. The number of infected people grew exponentially until Chinese authorities declared a complete lockdown of the affected area, the 60-million-inhabitants Hubei province, including the strict confinement of people to their homes. This draconian public health measure produced the expected result: from a-posteriori analysis presented in [8], it is apparent how the actual (not measured) number of new cases stopped growing immediately, and started decaying exponentially after a few days, eventually stabilizing the total number of officially reported cases to about 80000.

A new outbreak is now starting in Europe, sparking a wide debate about what are the best public health policies to manage it.

For reasons yet to be determined, Italy has been affected first, with cases reported starting Feb 21, 2020. During the following two weeks, the number of reported cases followed a dynamics similar to that of the Chinese outbreak, with most cases so far taking place in the northern Italian regions of Lombardy and parts of neighbouring Emilia-Romagna, Piedmont, and Veneto.

The Italian government initially applied severe containment measures to some restricted areas that were heavily affected, but eventually decided to follow the Chinese approach, by closing all public places such as schools, universities, theatres, and museums, and imposing severe restrictions to social mobility, though not as harsh as in China, as people are still allowed to go to work, under certain conditions, and carry out other essential activities such as shopping for food. These measures (which some question may not yet be strong enough) have already proven effective in the Chinese case, but on the other hand impose a very severe strain on the social fabric and on the economy of the country.

Some countries, whose number of reported cases tracks the Italian one with a delay of about 7-15 days, decided to follow a similar approach, e.g. Spain.

Other countries, most notably the UK as of the time of this writing, are instead planning to try to control the outbreak by carefully managing the number of cases, adopting the so-called "Nudge Theory" to obtain the required social reaction, instead of issuing draconian, government-issued restrictions to social interaction. Their goal is to balance the trade off between two conflicting goals: on the one hand, ensure that the population reaches herd immunity as soon as possible; on the other hand, avoid to exceed the capacity of the public health system to take care of critical cases.

The goal of this paper is to give fundamental insight on the peculiar features of these control problems, using a simple mathematical model that captures the fundamental dynamics governing the process, and adopting a control-theoretical analysis framework.

The first result presented in this paper is that the Chinese and the Italian approach are feasible, but that decision makers have to be well aware of the effects of the initial exponential growth when deciding the timing for lockdown. The second result is that the Nudge-Theory based approach is infeasible, due to the unfavourable dynamic features of the process: unstable motion, delay, and uncertainty. This statement is supported by well-known basic control theory.

As a consequence, the precautionary principle should advise to adopt severe, though possibly not draconian, social isolation policies as early as possible, while rejecting the implementation of Nudge-Theory-based policies.

The paper is structured as follows: Section 2 introduces a control-oriented model of the epidemic during the time interval when the above-mentioned policies are taken, based on available daily data regarding the number of new cases. In Section 3, the two above-mentioned policies are analysed in terms of feedback control, showing in particular that the second is infeasible even under overly optimistic assumptions. Section 4 concludes the paper with some recommendations for decision makers.

2 Modelling

The mathematical theory of epidemic was started almost 100 years ago by W.O Kermack and A. G. MacKendrick in their seminal paper [4], that introduced the SIR compartmental model. Since then, their theory has been extended and refined in a large number of papers and studies, with the goal of achieving greater accuracy and predictive power. Extensions were carried out in various directions: by increasing the number of compartments, by adding more phenomena to the model (e.g. births, deaths, migrations), by distinguishing among different age cohorts, and by adding spatial structure and explicit modelling of contagion paths. Another class of epidemiologic models follow a stochastic approach, which is essential to capture some features of real epidemic that are not described correctly by deterministic compartmental models; for example, the fact that when the last infectious person has healed, the virus is eradicated and there cannot be further outbreaks.

The use of such modern sophisticated models is necessary if one wants to reconstruct *a posteriori* with good accuracy the entire history of the outbreak, from the inception to the extinction of the disease, or to a steady endemic state. The goal is then to identify those parameters subject to public policy that should be changed to better manage future similar outbreaks, e.g. determining the optimal vaccination rates, or paying more attention to the spread of the disease via sewage networks, etc. On the other hand, to the author's understanding, these models were not specifically conceived to support real-time decisions on a short time horizon, and thus may lack some crucial detail in this respect.

Thus, in this paper the classical SIR model will be used, with the addition of a key feature to capture the control-relevant dynamics, namely the time delay inherent in the diagnostic and decision processes.

The basic SIR model can be formulated as follows:

$$\frac{dS}{dt} = -\frac{\beta IS}{N} \tag{1}$$

$$\frac{dI}{dt} = \frac{\beta IS}{N} - \gamma I \tag{2}$$

$$\frac{dR}{dt} = \gamma I \tag{3}$$

where N is the total population, S is the number of people susceptible to infection, I is the number of infectious people, that can transmit the disease to others, and R is the number of resistant people, which are immune to the disease because of genetics, vaccination, or immunity acquired after contracting the disease. β and γ are (possibly time-varying) parameters that account for the likelyhood of susceptible people to get infected by infectious people, and of infectious people to become resistant, per unit time. Note that, although the model is of third order, summing the three equations leads to

$$\frac{d(S+I+R)}{dt} = 0. \tag{4}$$

Hence, the last equation can be replaced by an algebraic equation, namely:

$$R(t) = N - S(t) - I(t), \tag{5}$$

making the model effectively second-order.

The COVID-19 outbreak in Europe has several specific features, that are relevant for the process modelling.

1. COVID-19 is a new virus, believed to have jumped from bats to humans in a market of Wuhan, China, at the end of 2019. So, the vast majority of European residents have never been exposed previously to it.

2. There is currently no vaccine available for COVID-19, and it may take at least 12-18 months until one becomes available, assuming this happens at all.

3. There are basically no specific and effective cures for COVID-19. Some drugs formerly used for HIV or malaria treatment may provide some limited support to help the patient fight the virus.

4. Based on data from China and Italy, the mortality rate is estimated at about 1-2% of officially reported cases. It may actually be much lower, since many subjects show no or mild symptoms, do not report to hospitals, and are thus not tested for the virus. The ratio α between positive tested cases and total infected cases is uncertain; in the case of the Hubei province outbreak it was estimated that $\alpha = 0.05$ [5].

5. A certain fraction σ of officially reported infectious cases (about 10% in Italy) ends up having severe pneumonia and respiratory difficulties, that require artificial ventilation for a period from a few days to a couple of weeks to sustain the patient's life functions, in conditions of intensive or sub-intensive care. Absent the possibility of artificial ventilation, this fraction of patients is likely to die because of respiratory failure, increasing the mortality rate dramatically.

6. The number of beds, equipment, and personnel which is required to perform artificial ventilation on patients in public health systems is calibrated based on normal needs of post-surgery care, trauma care, and care of people with rare but serious diseases like amyotrophic lateral sclerosis, with relatively small extra capacity. The number N_{ic} of COVID-19 patients that can be admitted to intensive and sub-intensive care is thus severely limited. While the actual numbers change widely by country, the typical order of magnitude is $10^{-4}N$. This number cannot be easily increased by one or more orders of magnitude during the very short time span of the initial phases of the outbreak (a few weeks).

7. The initial dynamics of the disease is very fast, with doubling times of the order of 2-3 days.

8. The moment an infected patient becomes infectious is still unclear. There are some indications that patients, who later report severe symptoms and get tested because of that, started becoming infectious two days before the onset of fever, which is a tell-tale sign prompting people to report for testing. Furthermore, in many cases people initially confuse their symptoms with seasonal influenza, which may delay the moment they are tested by many days. There is thus probably a time delay τ_t of several days on average between the moment people become infectious, and the moment they get tested. This delay is critical for the performance of decision and control.

9. The testing process also introduces a delay in the process. Although in principle instant testing kits are available, that can provide a diagnosis in a few hours, the average time to report the results may be much longer, because of logistical constraints and because of limited availability of analytical equipment. For example, the average time required to obtain the test results in Lombardy during the second week of March was about one week. This delay τ_r is also critical for the performance of decision and control.

Assuming a worst-case scenario, which is required by the precautionary principle given the number of lives at stake, Item 1 suggests to consider $S(0) = N$.

The absence of a vaccine (Item 2) implies there is no means to reduce the value of S and increase the value of R by means of vaccination campaigns.

Item 3 allows us to consider γ as a constant, at least as a first approximation, while the low mortality mentioned in Item 4, together with the short time horizon required by the model (a few weeks at most), justifies the assumption of the basic SIR model to neglect deaths (both related and unrelated to the disease), births, immigration, and emigration.

Items 5-6, coupled to Items 1-3, are crucial from the modelling point of view. When the health-care system capacity limit is reached, standard recommended triage practice is to give priority access to intensive care to younger and healthier people, which are likely to recover more quickly (freeing up the scarce resources for other patients), and who are likely to live longer once recovered, while denying access to elderly or otherwise frail individuals, see e.g. the recommendations in [3].

Once the emergency situation has arisen, these practices are of course necessary, as in war or disaster medicine. However, it is the opinion of the author that enacting a public policy with a significant risk of causing this outcome *a priori* is morally unacceptable. Hence, any acceptable control policy should ensure *a priori* that $\alpha\sigma I < N_{ic}$ at all times. With the typical numbers mentioned previously, this implies

$$I < \frac{N_{ic}}{\alpha\sigma} \approx 0.02N \tag{6}$$

It is then possible to assume *a priori* that $I \ll S$. Furthermore, during the initial phase of the outbreak, the number R of people who recover will be even smaller, so $I + R \ll S$. Since $I + R + S = N$, one can assume in Eq. (2) that S is nearly constant, and approximately equal to N. This assumption decouples Eq. (2) from Eq. (1), leading to

$$\frac{dI}{dt} = (\beta - \gamma)I. \tag{7}$$

Assuming that also β is constant, which is reasonable in case no significant social distancing measures are taken by the authorities, Eq. (7) has an exponential solution

$$I(t) = I(0)e^{(\beta-\gamma)t} = I(0)e^{rt}, \tag{8}$$

where $r = \beta - \gamma$.

In fact, Eq. (8) refers to the number of *actual* infectious cases, which is largely unknown, see Item 4. However the empirical ratio σ of patients requiring artificial ventilation mentioned in Item 5 is referred to the much lower number $I_t(t)$ of cases that will eventually get *tested positive* to the virus. Given that $I_t(t) = \alpha I(t)$, assuming ratio α to be constant, it is trivial to prove that also the dynamics of *tested positive* cases I_t will obey the same differential equation

$$\frac{dI_t}{dt} = (\beta - \gamma)I_t, \tag{9}$$

and thus have a solution

$$I_t(t) = I_t(0)e^{(\beta-\gamma)t} = I_t(0)e^{rt}. \tag{10}$$

Empirical data made available by the Department of Civil Protection of Italy for the period 24 Feb - 6 Mar 2020 [2], which probably refers to a period of about one week earlier or more, see Items 8-9, show a good fit with $r = 0.26$ day^{-1}, corresponding to a doubling time

$$T_d = \frac{log(2)}{r} \tag{11}$$

of about 2.6 days. Other countries show similar behaviour before the introduction of social distancing measures, with doubling times between 2 and 3 days.

The purpose of this model is to describe the effect of public health policy changes, in particular social distancing measures such as prohibiting large crowds, closing schools and universities, suspending religious services and public sports events, limiting people's movement to the strict necessary, quarantining people with suspect symptoms, closing non-strategic industries, etc. These measures are varied and can be applied progressively.

We can then assume that the time-varying parameter β is in fact a function of a representative manipulated variable $u(t)$, where the value of u indicates the severity of the public health measures on a scale from 0 to 1. The $\beta(\cdot)$ function is monotonously decreasing and is such that $\beta(0)$ corresponds to the maximum value of β, observed during the initial outbreak when no social restrictions are enforced, while $\beta(1) = 0$, corresponding to a total lockdown situation that reduces the likelyhood of contagion to zero. Note that this can be a bit unrealistic, even when considering the very strict policies implemented in China, since it would require people to also isolate themselves within their households.

It goes without saying that the actual shape of function $\beta(u)$ is subject to a lot of uncertainty. If harsh measures are taken, hence u is close to 1, one can expect the value of β to be drastically reduced, ensuring that $r = \beta - \gamma < 0$ with fairly good confidence. Provided that r is sufficiently smaller than zero, the number of newly infected people will decay exponentially rather quickly, irrespective of the actual exact value of r.

The measured variable $I_r(t)$ of the process is the number of *reported* infected cases. As mentioned in items 8-9, the overall measurement process of reported cases is subject to an average delay of τ_t days between the onset of infectiousness and the moment the test is taken, and by another delay of τ_r days before the result of testing is available to public authorities.

The control-oriented model of the virus outbreak dynamics is thus the following:

$$\frac{dI_t(t)}{dt} = \left[\beta(u(t)) - \gamma\right] I_t(t). \tag{12}$$

$$I_r(t) = I_t(t - \tau_t - \tau_r) \tag{13}$$

where $\beta(u)$ is an uncertain function, γ is an uncertain constant parameter, and τ_t, τ_r are uncertain, possibly time-varying parameters.

3 Control

3.1 Foreword

The effects of the application of two control policies briefly outlined in the Introduction will now be analyzed, based on the model derived in the previous Section.

The common theme behind both can be summarized by the title of the famous 2003 Bode Lecture paper by Gunter Stein: "Respect the Unstable" [6]. Feedback control strategies should not be applied light-heartedly to unstable systems, particularly when a large number of human lifes is at stake, as the sobering memory of the Chernobyl disaster discussed in that paper suggests. In the case discussed in this paper, particularly deadly consequences can stem from the unique combination of unstable dynamics, time delay, and uncertainty.

3.2 Draconian measures policy

The Chinese policy, and to a slightly lesser extent, the Italian policy, can be brutally summarized *a posteriori* in the following terms: as soon as $I_r(t)$ reaches a value I_s which is scary enough to decision makers to overcome their reluctance to disrupt the social and economic life of their country, draconian containment measures are taken, which means the following discontinuous control law is applied:

$$u(t) = \begin{matrix} 0, & I_r(t) < I_s \\ 1 - \epsilon, & I_r(t) \geq I_s \end{matrix} \tag{14}$$

As long as $\epsilon \geq 0$ is small enough to ensure that $r = \beta(1 - \epsilon) - \gamma < 0$, after the threshold I_s is exceeded, the number of *tested positive* cases $I_t()$ will start decaying exponentially, until it will asymptotically reach zero in a time of about $-5/r$ days, corresponding to a steady cumulated value of infected people that will eventually heal or die. As already noted, the exact value of β is not critical in this case, as long as r is sufficiently smaller than zero.

The number of *reported* cases $I_r(t) = I_t(t - \tau_t$ - $\tau_r)$ is instead delayed by τ_t + τ_r days, so it will continue growing exponentially as e^{rt} for another τ_t + τ_r days, and only then start dropping exponentially.

Such an expected behaviour is nicely validated by the data reported for the Wuhan region in China in Fig. 1 of [8], which reports data estimated by means of interviews to the patients about the actual time of their disease onset, roughly corresponding to I_t except for a possible two-days latency period, see Item 8. In particular, while the *tested positive* cases started to drop almost immediately after the lockdown, the *reported* infectious cases continued to grow exponentially for another 10 days, before starting to decay.

A wise choice of I_s should then be such that the number of cases needing ventilation and intensive or sub-intensive care is always $\sigma I_t(t) < N_{ic}$, i.e. within the reach of the public health system, possibly with a healthy safety margin.

The potentially catastrophic risk in the implementation of such simple policy is that the residual exponential growth of *reported* cases I_r is equal to $e^{r(\tau_t+\tau_r)}$, which can be a fairly large factor. In the reported case of Italy, assuming $r = 0.27$, $\tau_t = 7$, $\tau_r = 7$, that number is about 40. Which means, decision makers who are not aware of the combined effect of exponential growth and measurement delay may severely underestimate the consequences of choosing an apparently reasonable value of I_s, say 1000 cases, while the actual value I_t, which will ultimately determine the number of dead people and the possible collapse of the public health system, is in fact a much scarier $I_t = 40000$, which may end up overwhelming the intensive care units. Hence, poorly advised decision makers may wait too much to apply draconian measures, heading straight into disaster.

The other open problem of this policy is that it is not clear if the equilibrium which is finally reached is sustainable once the draconian measures are lifted, or if there is rather a risk that a second outbreak ensues. However, even in the case this unfortunate event takes place, this policy allows to buy precious time to improve the maximum capacity of intensive and sub-intensive health care units, by building or restoring new hospitals, by procuring ventilators and intensive care beds, and by hiring and training personnel to operate them. Given enough time, these measures can strongly mitigate the final death toll and avoid the need of taking wartime-like triage decisions in severely strained hospitals.

3.3 Nudge-Theory policy

3.3.1 Policy statement

Proponents of the application of the so-called Nudge-theory [7] to the management of the COVID-19 outbreak put forward the following arguments:

- in the case of the COVID-19 outbreak, the absence of a vaccine for the next 12-18 months and the relatively low mortality rate of the disease implies that, one way or the other, the majority of people should get infected and become immune, until herd immunity is achieved, at an estimated value of $R = 0.6N$;

- draconian measures are not necessary, and in fact they can be counter-productive, because they unnecessarily disrupt the social fabric and the economic system, and because people cannot be kept in confinement for too long periods without eventually becoming intolerant to them and rendering those measures ineffective;

- appropriately timed social hints (so-called "nudges"), such as suggesting only to sick people to isolate themselves for a limited amount of time (7 days), are enough to precisely steer the trajectory of the epidemic so that it can be exhausted as fast as possible, but at the same time without overcoming the capacity limits of the public health system.

- during the initial phase of the outbreak, limited social confinement measures should be adopted, to let the momentum of the epidemic build up until "the right time" comes to start slowing it down appropriately.

3.3.2 Mathematical formalization

In mathematical terms, the first step to enact this strategy is to compute a reference trajectory $I_r^0(t)$ for the reported cases $I_r(t)$, and a corresponding optimal control policy $u^0(t)$, which guarantees to reach the herd immunity condition as fast as possible, while ensuring that $\sigma I_t(t) < N_{IC}$ at all times, meaning that the health system is never exceeding its capacity limit to provide intensive and sub-intensive care to patients who need it.

Such a reference trajectory can be obtained by trial and error, or possibly by means of constrained optimal control solvers, using sophisticated models of the COVID-19 epidemic evolution, that can be much more accurate than the simple SIR model presented in the previous Section.

It is quite obvious that the unstable nature of the state trajectories while $r > 0$, i.e. before the peak of the epidemic is reached, makes a completely open-loop implementation of this policy not feasible. The reference trajectory should then be followed by adapting the adopted measures $u(t)$ in real time, based on the observed values of the reported cases $I_r(t)$. This corresponds in principle to closing a feedback loop to stabilize the unstable trajectory.

In the following sub-sections it will be shown that even a fairly sophisticated feedback control law cannot manage to stabilize the trajectory close to the reference, because of the adverse nature of the process dynamics.

This casts very serious doubts on the feasibility of the proposed approach, which will in fact use a much less sophisticated feedback control strategy to achieve the same goal.

3.3.3 Trajectory controller design

The process model, linearized around the reference trajectory $u^0(t)$, $I_r^0(t)$, reads:

$$\frac{d\Delta I_t(t)}{dt} = \beta'(u^0(t))I_t^0(t)\Delta u(t) + \left[\beta(u^0(t)) - \gamma\right]\Delta I_t(t) \tag{15}$$

$$\Delta I_r(t) = \Delta I_t(t - \tau_t - \tau_r) \tag{16}$$

where β' indicates the derivative of β with respect to u.

By making the overly optimistic assumptions that the parameters γ, σ, τ_t and τ_r are constant and perfectly known, and that the function $\beta(u)$ is time-invariant, monontonously decreasing, smooth, and perfectly known, one could compensate the very strong nonlinearity of the process behaviour, designing a linear, gain-scheduled feedback controller, that will result in a linear and (almost) time-invariant loop dynamics, and add its output to the reference trajectory $u^0(t)$ to stabilize it.

$$u(t) = u^0(t) + \frac{1}{\beta'(u^0(t-\tau_c))I_r^0(t-\tau_c)} \left[K_p\left(I_r(t-\tau_c) - I_r^0(t-\tau_c)\right) + K_d\left(\frac{dI_r(t-\tau_c)}{dt} - \frac{dI_r^0(t-\tau_c)}{dt}\right)\right], \tag{17}$$

In other words, a PD feedback control law is added to the reference trajectory, with a correction term proportional to the difference between the actual reported cases and the reference ones, and another term proportional to the difference between their rates of change. The gains are scheduled with the reference trajectory values, to eventually obtain a constant-gain loop transfer function.

Equation (17) also takes into account some further delay τ_c inside the controller, that corresponds to the time which is necessary to collect the data, make decisions which are potentially disruptive for the social fabric and the economy, communicate them effectively to the public, and give time to the public to actually implement them. Modern social media allow to practically reduce the communication time, but the author believes that the other factors account for at least one/two, or possibly even more days of further delay.

The loop transfer function of this system reads:

$$L(s) = K_p \frac{1 + sT_d}{1 - (\beta(u^0(t)) - \gamma)s} e^{-s(\tau_t+\tau_r+\tau_c)}, \tag{18}$$

where $T_d = K_d/K_p$.

The loop transfer function is not strictly proper because an ideal derivative action was assumed for simplicity. In fact, introducing a high-frequency low-pass filter in the derivative action would make it strictly proper. However, it turns out from subsequent analysis that the derivative action has little use in this case and could actually be put to zero without seriously deteriorating the closed-loop behaviour, so this particular aspect is not critical.

Furthermore, the coefficient multiplying s at the denominator of $L(s)$ is not a constant during the whole transient, so technically speaking, the system is time-varying, and a standard loop transfer function cannot be computed. However, proponents of the nudge-theory approach argue that harsh measures are not really required in the initial phases of the transient, so that $\beta(u^0(t))$ won't change much during that initial period, which is the most critical due to the fast exponential increase. This allows us to assume it as (roughly) constant for the sake of this analysis.

The loop transfer function reveals the very dangerous nature of this process, which is strongly open-loop unstable and with a huge time delay. The process delay $\tau_t + \tau_r$ can be estimated between 7 and 14 days, as further confirmed by the data shown in Fig. 1 of [8], where the delay is approximately 10 days.

A further delay τ_c of one-two (or possibly more) days should be added to that, increasing the value even further, to about two weeks, or even more. This value is way larger than the observed time constant $T = 1/r$ of the unstable pole, which is 4 days in the case of Italy, and has similar values in other countries during the initial phase of the outbreak.

Furthermore, once the overly optimistic assumption of perfect knowledge of parameters is lifted, exponentially diverging uncertainty on the process gain must be added to the bill.

Anyone familiar with basic control theory will immediately recognize this situation as a guaranteed recipe for disaster [6], [1]. This can be easily shown in this context by means of Nyquist's stability criterion.

3.3.4 Stability analysis

In order to stabilize the system motion around the reference trajectory, one should ensure that the Nyquist plot makes one turn around the -1 point, which requires a sufficiently high K_p. Furthermore, in order to guarantee some robustness of the system performance against the large gain uncertainty of the process, the Bode plot of $\left|L(j\omega))\right|$ should maintain a roughly constant slope over a sufficiently wide interval around the crossover frequency ω_c, thus approximating Bode's ideal loop transfer function mentioned in [1] over a wide enough interval.

On one hand, this means that the gain K_p should be high enough to guarantee that the crossover frequency is significantly larger than the unstable pole $r = \beta - \gamma$, otherwise the property of turning around the -1 point would not be robust enough against gain uncertainty. Hence, $\omega_c > 1/T$.

On the other hand, the zero with time constant T_d should have a much higher frequency than $1/\omega_c$ for the same reason. This means that the positive contribution of the derivative term to the phase margin is by necessity limited, indicating that the derivative action is not very useful in this case and that one could actually set $K_d = 0$ for good. In other terms, the number of new daily reported cases, roughly corresponding to the derivative of the process output, seems to have a limited usefulness in the task of governing the evolution of the epidemic.

Finally, the negative phase shift introduced by the time delay should remain well below 60°, i.e. below 1 rad, to guarantee a sufficient phase margin and hence robust stability. Since this phase shift is given by $-\omega_c \tau$, one would need $\omega_c < 1/\tau$. If $\tau > T$, as it is clearly the case from the data mentioned in the previous Section, this second condition is incompatible with the $\omega_c > 1/T$ condition mentioned above. Hence the system cannot be controlled at all in a stable way.

Arguments analogous to those put forward in [1] lead to conclude that any sufficiently robust linear controller would basically suffer from the same limitations. In fact, [1], Sect. 4.6, argues that in the case of unstable pole and delay in the process transfer function, even in the most favourable case one would need $\tau < 0.326T$ to design a sufficiently stable and robust feedback controller. Given that typical values of τ are estimated between 7 and 14 days, while T is around 4-6 days, it is apparent how the task of governing the initial phase of the epidemic based on daily numbers of reported cases is utterly hopeless.

The above stated conclusion is further strengthened if one also takes into account the substantial parametric uncertainty involved in the model.

3.3.5 Discussion

The exact details of the policy advocated by proponents of the nudge-theory approach have not been revealed at the time of this writing. What is understood is that some trajectory has been planned, and it will then be followed by changing the public health measures $u(t)$ when certain thresholds of the number of reported cases $I_r(t)$ are crossed, and possibly also considering the number of new daily reported cases, which is related to $dI_r(t)/dt$, though our analysis reveals that this information is not that useful. In other words, an extremely crude step-wise approximation of the control law presented earlier in this Section.

Of course there is no theorem that can be directly invoked to prove that such policy would not suffer of the same limitations of a carefully scheduled linear PD controller. However, limitations of control in the case of unstable processes with large delays and uncertainty are are inherent in the nature of the process dynamics and not in the fact that a linear controller is adopted. In principle, a well-designed nonlinear or time-varying controller could achieve somewhat better performance, but the large amount of uncertainty in the knowledge of the process behaviour makes this proposition completely impractical.

The situation could in principle be improved by taking all possible measures to reduce the time delay τ as much as possible, e.g. by relying on instant testing techniques that would eliminate the delay τ_r. However, the decision-making delay τ_c is unavoidable, and so is the delay τ_t, as it is not realistic to think that one can test a statistically significant and unbiased portion of the apparently unaffected population every single day with instant testing kits to obtain a reliable delay-free assessment of $I_t(t)$.

Thus, based on this analysis, there are two likely outcomes of the application of nudge-theory.

In the lucky one, the (largely unknown so far) effect of the policies applied at the beginning of the transient will be larger than expected, leading to $r < 0$

and hence to an exponential decay of new reported cases after a delay of τ days. In this case, one would obtain the same effect as in China by sheer luck, with a lot more less disruption of social and economic life, but won't achieve herd immunity, which is the other objective of the policy.

In the unlucky one, the effect of uncertainty will cause an initial uncontrolled exponential increase of reported cases and of people requiring intensive and sub-intensive care. This will be observed when it is too late to take appropriate countermeasures, potentially leading to a catastrophic failure of the public health system and to a massive increase of fatalities due to the lack of the needed ventilation support for the fraction σ of tested positive cases that is bound to develop a critical respiratory condition.

4 Conclusions

The COVID-19 virus outbreak in Europe is an unprecedented event. The present behaviour of decision makers seems to not account for (or even contradict) suggestions that clearly emerge from the application of basic systems and control theory. As shown in the previous Sections, consequences can be dramatic, as the said outbreak – from a system-theoretic viewpoint consistent with observations – features several extremely critical characteristics from a control perspective:

- fast unstable dynamics with doubling times of a few days, in the absence of sufficient social distancing measures;
- large process measurement and actuation delay, between one and two weeks;
- substantial uncertainty due to the current lack of knowledge about the virus behaviour;
- lack of effective drugs and vaccines;
- relatively small but significant number of cases that require very limited intensive or sub-intensive care units, with potential to overwhelm public health systems and force them to use triage criteria normally reserved to wartime or disaster areas.

The combination of such factors makes the decision making process particularly challenging and, as exposed herein, extremely hard – if ever possible – to manage correctly by public health experts unless some expertise on the control theory is brought into play.

From this point of view, the results presented in this paper highlight the crucial role that diagnostic time delays play when trying to control the virus outbreak, suggesting that significant effort should be undertaken to reduce them as much as possible. However, even if that is possible, residual diagnostic delays, added to the other delays in the decision process, will still make decision based on daily observation very critical, in the case of lock-down decision, or utterly infeasible, in the case of the Nudge-Theory approach.

The failure of such decision-making processes can have dire consequences, in terms of numbers of victims (particularly among the frailer members of society), disruption of public health care systems, and need by doctors to resort to decisions that are normally reserved to war time or disaster area, on a national or possibly pan-European scale.

The application of the precautionary principle, a fundamental staple of European Union legislation, suggests to avoid potentially high-risk strategies and to adopt severe containment policies to help stop the spread of the disease as early as possible, with the goal of buying time to build, organize, and operate appropriate health care infrastructure to cope with the potential consequences of the epidemic.

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