
The Collected Papers of Charles Sanders Peirce

1. CONTENTS. To view the complete table of contents of **The Collected Papers of Charles Sanders Peirce**, click on the **Contents** icon on the left border of your screen. Click on plus (+) symbols in the Contents window to expand the list of entries. A minus symbol (-) indicates that a list may not be further expanded. While viewing the contents, click on **Levels** under the **Search** menu to view the Contents at the Volume, Book, or Chapter levels. While in the Contents window, you may jump to any location by double-clicking on the line of interest or click on the **Contents** icon to go back to full database view.

2. SEARCH TEXT. To execute a search, click on the **Query** icon on the left border of your screen, type in your search terms in the **Query For:** window, then click on **OK** to execute your search. To reveal reference information (e.g. page numbers) for each paragraph, click on **Hidden** under the **View** menu.

Editorial Introduction by John Deely

Past Masters Introduction

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Peirce: CP Editorial Introduction to Electronic Edition
Membra Fictæ Disjecta

(A Disordered Array of Severed Limbs)

Editorial Introduction

by John Deely

to the electronic edition of

The Collected Papers of Charles Sanders Peirce

reproducing Vols. I-VI ed. Charles Hartshorne and Paul Weiss (Cambridge, MA: Harvard University Press, 1931-1935), Vols. VII-VIII ed. Arthur W. Burks (same publisher, 1958)

1 June 1994

Peirce: CP Editorial Introduction to Electronic Edition

Charles S. Peirce (the "S" stands for "Sanders" by Baptism and later for "Santiago" as Charles' way of honoring William James) has so far best been known in academia at large as some kind of a background figure to the rise of Pragmatism, as mentor to that movement's truly well-known protagonists, William James and John Dewey. That misleading identification is in the process of changing, and the literature supporting the understanding of Peirce in the established framework of modern philosophy, particularly with its opposition of "realism" to "idealism" such as the works of Buchler, Goudge, Manley Thompson already belong to the genre of *depassé* interpretation.

Peirce: CP Editorial Introduction to Electronic Edition

It is not merely a question of the curiously underassessed fact (excepting Apel's pioneering 1970 study, *Der Denkweg von Charles S. Peirce: Eine Einführung in den amerikanischen Pragmatismus* [Frankfurt am Main: Suhrkamp Verlag], presciently retitled *From Pragmatism to Pragmaticism* for its 1981 English translation by J. M. Krois [Amherst, MA: University of Massachusetts Press]) that, despite the willingness on all sides to attribute the original coining of the term "pragmatism" as a philosophical name to him, Peirce eschewed the classical pragmatist development to the point of giving to his own position a new name, "Pragmaticism". It is a question at bottom of the principal optic through which Peirce early and ever-after came to view the problems of philosophy, the optic of "semiotic", as he called it after Locke, or the *doctrina signorum*, as both Locke and Peirce called it, both unaware of the earlier Latin Iberian development of this optic through the successive work of Domingo de Soto (with his *Summulae* or *Introductory Logic* of 1529), Pedro da Fonseca (1564) and the Conimbricenses (1607) he started, Francisco Araujo (1617), and the culminating synthesis of John Poinsett's *Tractatus de Signis* (*Treatise on Signs*) of 1632 (also a full-text data-base in this Past Masters series).

Peirce: CP Editorial Introduction to Electronic Edition

I first came to take Peirce seriously as a result of Thomas A. Sebeok's 1978 NEH Summer Seminar on semiotics as a new foundation for the sciences. In that group of seminarians there were three expert Peirceans, Jarrett E. Brock, H. William Davenport, and George A. Benedict. It soon became clear that anyone studying Peirce today on the basis of the Harvard *Collected Papers of Charles Sanders Peirce* (henceforward CP) was essentially in the position of an animal wading into a pool of piranha fish. A whole generation of young Peirce scholars had come of age under the tutelage or indirect influence of Max Fisch, the most knowledgeable of all the senior Peirce scholars, who had almost alone come to grasp the semiotic trajectory animating the entire Peircean corpus. First through Kenneth Ketner's Institute for the Study of Pragmaticism at Texas Tech University, and later through the Peirce Edition Project at IUPUI, Fisch had shown the new generation not only the importance of the unpublished Peirce manuscripts, but, equally importantly, how to read them with

semiotic eyes. Oddly enough, as an index of how much remains to be done in achieving a balanced and integral presentation of the Peircean corpus, the recent *An Introduction to C. S. Peirce* by Robert Corrington (Lanham, MD: Rowman & Littlefield, 1993) stands out as the first introduction to give semiotic a co-ordinate billing with such traditional aspects of Peircean thought as his metaphysics (yet even in this ground-breaking over-all introduction, arguably the best so far, Corrington told me that "piety toward the elders" inhibited him in annotating his bibliography).

Peirce: CP Editorial Introduction to Electronic Edition

The story of the Harvard edition titled CP, which we here re-present in electronic form, is a story fairly well known, and a sad one. Hartshorne and Weiss, along with Burks later, deserve our thanks for getting the volumes out, but we must at the same time regret the manner of their editing, which was to construct a topical scheme of their own devising under which to sort and dissect the papers left whole to Harvard through the good intentions of Josiah Royce. How Harvard abused that trust! The story, at least, is now out with the bursting upon the scene of the newly-worked (after more than thirty years of repression) biographical dissertation of Joseph Brent in the form of the book, *Charles Sanders Peirce. A Life* (Bloomington, IN: Indiana University Press, 1993). This publication is a tribute in equal parts to the writing skill and historical tenacity of its author, to the editorial genius (to say nothing of the detective skills) of Thomas A. Sebeok, and to the publishing genius of John Gallman, the Director of the Indiana University Press.

Peirce: CP Editorial Introduction to Electronic Edition

But why re-publish the CP now, just when the chronological edition of the *Writings* (henceforward W) may be getting up steam? There are several answers to this question. The first reason is that the CP is not in competition with W. The chronological edition, when completed, will become the irreplaceable standard and, if brought to completion at its current level of scholarly excellence, will remain practically unsurpassable as a hardcopy critical source. But W is, simply put, taking too long, partly in the nature of the task which, after all, however much more quickly it might have been shepherded, cannot be rushed: it needs to be done rightly, and critical editing takes time. Still, those of us alive today and interested in Peirce would like to have access to as much of his work as possible as soon as possible. At present, as far as published writings go, that still means the CP.

Peirce: CP Editorial Introduction to Electronic Edition

A second reason is that CP contains some material which, at least according to current plans, will not be included in W. That means that, for the foreseeable future, the CP will remain an independent, and at least minor, source for Peircean scholarship.

Peirce: CP Editorial Introduction to Electronic Edition

The third reason, however, is the main reason for this edition. By bringing out the CP in electronic form, we not only keep available the so-far primary published source of Peirce material, but we present it in a form that enables the user in principle to overcome the primary defect of the original publication, namely, its artificial dismemberment of the Peircean corpus. Using the invaluable tool of the Burks bibliography from the last of the eight CP volumes, which gave scholars the necessary key to reconstruct the order of the Peirce manuscripts before the CP editors dissected them and shuffled the pieces (it is amazing, between the Burks bibliography and the Robin catalogue, not to mention many lesser essays, how much Peirce scholarship has been devoted to undoing that dismemberment), we have created hypertext links which

will enable the users of the electronic edition to reconstruct and print out for themselves Peirce's manuscripts in something like their original integrity.

Peirce: CP Editorial Introduction to Electronic Edition

An illustration of this advantage of the electronic CP may be given using Peirce's c.1895 essay "That Categorical and Hypothetical Propositions Are One in Essence". According to Burks (p. 286), paragraphs CP 2.332-339, 2.278-28, 1.564-567 (c.1899), and 2.340 "are from it in this order". Using the electronic CP, a reader can reconstruct this whole and print it out as such for scholarly or classroom use. Thus the "bodily parts" of the Peircean corpus, so far as they are included in the CP, may be easily rearranged in proper order so as to appear in something closer to the light under which Peirce left them.

Peirce: CP Editorial Introduction to Electronic Edition

This illustration brings out the fourth reason for this electronic edition, namely, to stimulate self-appointed scholarly caretakers of the manuscript materials to hasten the making available of the whole of the Peirce documents in electronic form even while the critical published edition (for which there is no substitute) goes forward at its own pace. Joseph Ransdell has been tirelessly promoting the desirability of an on-line forum through the proposal of the Peirce electronic consortium and through the two Peirce bulletin boards in which he is closely involved (contact Professor Ransdell at <bnjmr@ttacs.ttu.edu> for full details of the possibilities). By presenting this edition to the scholarly world, we have done the best that was possible at this actual historical moment in bringing Peirce as so far published "on line".

Peirce: Collected Papers - PAST MASTERS Introduction
Past Masters Introduction

Below find the text of *The Collected Papers of Charles Sanders Peirce*. All footnotes have been placed at the ends of their respective volumes. We have numbered the footnotes of volumes 1-6 relative to the page (instead of using the symbols of volumes 1-6). Footnotes authored by Peirce in volumes 1-6 are identified by the letter "P" after the carat symbol (†) and before the numeral. Thus footnote †P1 is a footnote authored by Peirce (i.e. the numbered footnotes of the printed editions (CP 1-6). In volumes 7 and 8 we have followed Burks scheme.

Peirce: Collected Papers - PAST MASTERS Introduction

A number of substitutions were made for symbols. Please see the "Key to Symbols" for a complete list.

Peirce: Collected Papers - PAST MASTERS Introduction

A number of groups have been created to facilitate searches. Please see the "Groups of the database" for more information.

Peirce: Collected Papers - PAST MASTERS Introduction

A link token is found on every reference line which leads to the "Table of Cross-References." The "Table of Cross-References" correlates the bibliography with every paragraph of text of the CP. A link appears next to every bibliographic entry of

this table, which leads to the complete bibliographic record. Thus to see the complete bibliographic record which identifies the source of any particular paragraph:

a) Go to the View menu, and execute the Hidden menu item (either by moving to the item with your Arrow keys or mouse then pressing Return/Enter or by clicking on the item with your mouse). Successful execution will result in a check mark to the left of the Hidden menu item. All reference lines in the text will be unhidden. The reference lines are located at the beginning of the paragraph, and appear purplish-red on color screens.

b) Note your current paragraph number (e.g. CP 3.183); click with your mouse on top of the link to the right of "Cross-Ref:". You will be moved to a table of Cross-references for the volume in question.

c) Move to the paragraph range in the table (using your arrow keys or mouse) in which your current paragraph falls, then click on the range with your mouse. You will be moved to the record which identifies the bibliographic source of the paragraph in question.

Peirce: Collected Papers Groups
Groups of the Peirce database

A number of groups have been created to make searching the database easier. First, groups have been created from the divisions of Hartshorne, Weiss, and Burks. [Group: CP1], [Group: CP2], ... [Group: CP8] etc. identify volumes 1 through 8. Thus the search

[Group: CP3] abnumeral

would find all paragraphs in volume 3 containing the word abnumeral.

Peirce: Collected Papers Groups

A group exists for every book, chapter and section division as well. Thus the search

[Group: cp4.i.ii] good

would find all paragraphs from Volume 4, Book I, Lecture II containing the word "good."

Peirce: Collected Papers Groups

Secondly, every paragraph of the *Collected Papers* has been placed in a group which identifies the year in which the paragraph was authored. Thus the [Group: Peirce.1888] group contains all paragraphs identified in the bibliography as having been written in 1888. The search

[Group: Peirce.1888]

would find all paragraphs written by Peirce in 1888, which are in the CP. The search

[Group: Peirce.1888] abnumeral

would find all paragraphs written by Peirce in 1888 (in the CP) which contain the word "abnumeral" (if any).

Peirce: Collected Papers Groups

This chronological grouping also exists at the 5-year and 10-year level. The [Group: Peirce5.1875] and [Group: Peirce10.1880] groups contain every paragraph authored by Peirce (in the CP, identified in the bibliography) in the years 1871-1875, and 1871-1880 respectively. Thus the search

[Group: Peirce10.1890] [Group: Peirce5.1895] abnumeral

would find every paragraph containing abnumeral authored by Peirce between the years 1881-1895 (in the CP).

Peirce: Collected Papers Groups

A group has been created from the text only and footnotes only of each volume. These groups are accessible from the opening screen of the database. Thus [Group: cp1.text] contains all paragraphs of the text of volume 1, and [Group: cp3.footnotes] contains all footnotes of volume 3. A [Group: cp.text] group excludes all footnotes, introductory and explanatory material, as well as table of contents entries. (Thus the [Group: cp.text] group = [Group: cp1.text] or [Group: cp2.text] ... [Group: cp8.text].) A [Group: cp.footnotes] group contains the footnotes from all 8 volumes. Finally, a [Group: peirce] group contains all and only material authored by Peirce, from both footnotes and text.

Peirce: CP Key to Electronic Symbols: Introduction
Key to Symbols: Introduction

Many symbols which do not appear in the extended ANSI or ASCII character sets (or symbol font sets) appear in the text of the *Collected Papers*. In the Windows and Macintosh version of this database, we are creating a Peirce font set that will accurately display onscreen all symbols found in the *Collected Papers*. This new Peirce font will appear in an updated version of the database. In the meantime, below find all substitutions made, with (if necessary) an image which displays the symbol as it appears in the print edition.

Peirce: CP Key to Electronic Symbols:
Key to Symbols

All subscripts are enclosed between brackets. Thus A[1] is A followed by the subscript 1. Occasionally a bracket in the text is double-bracketed in the electronic edition, to avoid ambiguity. Thus A[[1]] would indicate that an unsubscripted 1 enclosed by brackets appears in the printed edition.

Occasionally parentheses have been introduced to disambiguate expressions made ambiguous by substituting notation. Parentheses were particularly necessary to disambiguate numerators and denominators in division from surrounding expressions.

All Greek has been transliterated and is enclosed between braces {}. This transliterated Greek will be replaced with true Greek in an updated version of this database. Standard rules for transliteration were followed with the following exceptions:

{é} = lowercase eta

{É} = uppercase eta

{ö} = lowercase omega

{Ö} = uppercase omega

The \$ sign is used to represent "some". If the the curved line appears over an expression, the \$ sign precedes the parenthesized expression.

$\overset{\smile}{A}$ = \$A

A vertical bar above a symbol or expression has been replaced with a tilde preceding the expression. Thus:

$\overline{A < B}$ = A ~< B

\bar{A} = ~A

A vertical bar underneath a symbol is represented by following the symbol with _. If the vertical bar is underneath more than one symbol, the _ sign is placed after the parenthetical expression:

$\underline{x \cup x}$ = (x \cup x)_

A dot over a symbol is represented by preceding the symbol with a dot. Thus:

$$\equiv = \equiv$$

$$\dot{\cup} = \cdot \cup$$

The remainder of the symbol-equivalents are self-explanatory:

$$\overline{=} = =,$$

$$\overline{+} = +,$$

$$\overline{+} = \cdot +$$

$$\overline{<} = <$$

$$\overline{A \overline{<} B} = A \sim < B$$

$$\overline{A \overline{=} B} = A \sim = B$$

$$\overline{A} = \sim A$$

$$\overline{\vee} = \vee$$

$$\overline{\cdot | \cdot} = \cdot | \cdot$$

$\hat{0}$

= /0\

$\frac{1}{-}$

= -(-

a • e

= a • e

Peirce: CP Texts in Chronological Order
Texts in Chronological Order

1866-1870	1871-1880	1881-1890	1891-1900	1901-1910	1911-1913
1866	1871-1875	1881-1885	1891-1895	1901-1905	1911
1867	1871	1881	1891	1901	1913
1868		1882	1892	1902	
1869	1873	1883	1893	1903	
1870		1884	1894	1904	
	1875	1885	1895	1905	Undated
	1876-1880	1886-1890	1896-1900	1906-1910	
	1876		1896	1906	
	1877	1887	1897	1907	
	1878		1898	1908	
	1879	1889	1899	1909	
	1880	1890	1900	1910	

Peirce: CP 1 Title-Page
COLLECTED PAPERS OF CHARLES SANDERS PEIRCE

EDITED BY
CHARLES HARTSHORNE

AND
PAUL WEISS
VOLUME I
PRINCIPLES OF PHILOSOPHY
CAMBRIDGE
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1931

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Peirce: CP 1 Introduction p iii
INTRODUCTION

Charles Sanders Peirce plays a unique rôle in the history of American philosophy. During his own lifetime he published no book on philosophy, and except for a relatively short period he held no university chair from which to impress his influence upon students; yet he has come to be recognized as the founder of the one distinctive movement which this country has produced.

Peirce: CP 1 Introduction p iii

Pragmatism, as it developed, followed the pattern of William James' thought and that of John Dewey rather than the conceptions of Peirce; but it was Peirce, as James and Dewey magnanimously insisted, who defined the principle of the movement and gave it the first impetus. Never indeed a leader of movements, Peirce was an originator of ideas. He clearly formulated in his writings many conceptions which are only today beginning to find recognition, and there are implications in his thought which have not yet been fully developed.

Peirce: CP 1 Introduction p iii

Articles on pragmatism represent only one phase of his work. Some of his best thought was devoted to logical problems: to the logic of classes and relations, the theory of signs, scientific method, to probability and induction, and to the logical

analysis of mathematics. In the development of exact or mathematical logic his papers represent the most important and considerable contributions in the period between Boole's *Laws of Thought* and Schröder's *Vorlesungen*. His writings on logic touch almost every point of theoretical interest in the subject.

Peirce: CP 1 Introduction p iii

His published papers, about seventy-five in number, include the series of articles on pragmatism, the logical papers, and important discussions of metaphysical problems. There are about twice as many book reviews. From these published works one may gather some suggestion of the versatility of his interests and the wide range of his studies, which included subjects as remote and unexpected as geodesy and astronomy, telepathy, criminology, and optics. But perhaps because carefully edited for publication, these papers and reviews fail to reveal as they might another side of Peirce -- his humor, freshness, pithiness of phrase, his exuberance of idea, erratic self-consciousness and self-confidence, his endless projection of vast systematic constructions, the gleams of genius described by James in his famous phrase as "flashes of brilliant light relieved against Cimmerian darkness." Only in the less formal writings does Peirce emerge as his friends at Harvard knew him in the great period of philosophy there at the turn of the century.

Peirce: CP 1 Introduction p iv

After Peirce's death in 1914, his unpublished manuscripts came into the care of the Department of Philosophy at Harvard University. They number several hundreds, not including fragments, the fruit of a long life devoted almost exclusively to philosophy and to science in a great variety of forms. These manuscripts represent all stages of incompleteness. Frequently there is no date or title, and many leaves are out of place or altogether missing. Some of them were rewritten as many as a dozen times: it is often evident that Peirce himself was not able to select the final form. Some are clearly identifiable as earlier drafts of his published papers; others one may assume to have been such drafts, although they differ from the published papers so much as to make this a matter of doubt. Often these unpublished studies contain passages, or longer portions, which impress those who have examined them as being of greater worth or clarity than those in the published articles. There are, likewise, a number of studies, often completed and of considerable length, and yet plainly unrelated to any which were printed. Sometimes they can be identified, through contemporary correspondence, as definite projects for publication which for one or another reason, never came to fruition. Often, however, there is no indication of such definite intent; he seems to have written merely from the impulse to formulate what was in his mind. Nevertheless, Peirce's studies of this kind are usually fairly continuous and systematic. If their merely private or preliminary nature is at all betrayed, this is because in them Peirce allows himself to follow out the ramifications of his topic, so that digressions appear which are inadmissible in print, but which show vividly the interconnectedness of his thought and the unsystematic character of his writings.

Peirce: CP 1 Introduction p v

Peirce possessed the system-making mind. That the merely external exigencies of his life and the indifference of publishers prevented any full-length presentation of his philosophy is a tragedy. And it is a tragedy which cannot now be set right. His system cannot be completely reconstructed; even the attempt would mean taking indefensible liberties with the manuscripts. The most that can be done is to select, with such judgment as one can command, the most important of these

unpublished papers and to compare them with his published writings on the same topic. Such selection is always difficult. Illuminating passages of great interest must be passed by because inextricably connected with other material the inclusion of which is not justified. On the other hand, because the doctrines they present are too important to be omitted, papers and fragments must often be included although one is sure that the author would not have printed them in their present condition. Often there are alternative drafts of the same study, one distinctly superior in some portion or respect; the other, in some other portion or respect. In such cases a choice is necessary, although any choice is a matter of regret.

Peirce: CP 1 Introduction p v

In general, when Peirce's thought is at its best, he writes least well. For relatively superficial and transient topics he commanded a facile style, as in the many engaging contributions to *The Nation*. And in his more serious published work, he never allowed anything to leave his hand until it had attained a certain clarity and continuity. But when he is most in earnest (the manuscripts make this evident), the systematic and detailed character of his thought impedes his pen: he is likely to fall into some harsh jargon of his own, adopted in the interests of precision. The neatly turned phrase or brief and striking statement must often be rejected, in favor of one more technically accurate, or more complicated in the interest of adequacy. It is only just, however, to recognize that there are infelicities of style which occur in some of the papers included in these volumes which Peirce himself would never have allowed to remain in the final published form.

Peirce: CP 1 Introduction p v

The more important of these manuscripts of Peirce, as well as his published papers, have now been brought together in some ten volumes which will appear in rapid succession. The first volume contains in outline his system, so far as it can be presented, his writings on scientific method and the classification of the sciences, his doctrine of the categories, and his work on ethics. The next volume deals with the theory of signs and meaning, traditional logic, induction, the science of discovery and probability; and the third volume reprints his published work on modern logic. The fourth includes his unpublished original contributions to the foundations of mathematics, logic and graphs. The fifth volume contains his papers on pragmatism. The sixth is concerned with metaphysics. It is expected that the remaining volumes will contain his writings on physics and psychology, as well as his reviews, letters and biography.

Peirce: CP 1 Introduction p vi

Nearly all the members of the Department during the last fifteen years, as well as many others who were interested in Peirce, have devoted much time to the often very intractable material of the manuscripts. But the final and laborious work of selecting, arranging and preparing the papers for the press has been done by Dr. Charles Hartshorne, formerly Instructor in Philosophy at Harvard and by Dr. Paul Weiss, who is at present an Instructor in Philosophy at this university. The Department desires to express its gratitude to the many friends who have contributed generously towards the expense of printing the volumes.

* * *

Peirce: CP 1 Introduction p vi

Wherever possible Peirce's punctuation and spelling have been retained. Titles supplied by the editors for papers previously published are marked with an *E*, while Peirce's titles for unpublished papers are marked with a *P*. Peirce's titles for previously published papers and the editors' titles for unpublished papers are not marked. Remarks and additions by the editors are inclosed in light-face square brackets. The editors' footnotes are indicated by various typographical signs, while Peirce's are indicated by numbers. Paragraphs are numbered consecutively throughout each volume. At the top of each page the numbers signify the volume and the first paragraph of that page. All references in the indices are to the numbers of the paragraphs.

HARVARD UNIVERSITY

AUGUST, 1931.

Peirce: CP 1.1 Cross-Ref:††

PREFACE †1

1. To erect a philosophical edifice that shall outlast the vicissitudes of time, my care must be, not so much to set each brick with nicest accuracy, as to lay the foundations deep and massive. Aristotle builded upon a few deliberately chosen concepts -- such as matter and form, act and power -- very broad, and in their outlines vague and rough, but solid, unshakable, and not easily undermined; and thence it has come to pass that Aristotelianism is babbled in every nursery, that "English Common Sense," for example, is thoroughly peripatetic, and that ordinary men live so completely within the house of the Stagyrte that whatever they see out of the windows appears to them incomprehensible and metaphysical. Long it has been only too manifest that, fondly habituated though we be to it, the old structure will not do for modern needs; and accordingly, under Descartes, Hobbes, Kant, and others, repairs, alterations, and partial demolitions have been carried on for the last three centuries. One system, also, stands upon its own ground; I mean the new Schelling-Hegel mansion, lately run up in the German taste, but with such oversights in its construction that, although brand new, it is already pronounced uninhabitable. The undertaking which this volume inaugurates is to make a philosophy like that of Aristotle, that is to say, to outline a theory so comprehensive that, for a long time to come, the entire work of human reason, in philosophy of every school and kind, in mathematics, in psychology, in physical science, in history, in sociology, and in whatever other department there may be, shall appear as the filling up of its details. The first step toward this is to find simple concepts applicable to every subject.†2

Peirce: CP 1.2 Cross-Ref:††

2. But before all else, let me make the acquaintance of my reader, and express my sincere esteem for him and the deep pleasure it is to me to address one so wise and so patient. I know his character pretty well, for both the subject and the style of this book ensure his being one out of millions. He will comprehend that it has not been written for the purpose of confirming him in his preconceived opinions, and he

would not take the trouble to read it if it had. He is prepared to meet with propositions that he is inclined at first to dissent from; and he looks to being convinced that some of them are true, after all. He will reflect, too, that the thinking and writing of this book has taken, I won't say how long, quite certainly more than a quarter of an hour, and consequently fundamental objections of so obvious a nature that they must strike everyone instantaneously will have occurred to the author, although the replies to them may not be of that kind whose full force can be instantly apprehended.

Peirce: CP 1.3 Cross-Ref:††

3. The reader has a right to know how the author's opinions were formed. Not, of course, that he is expected to accept any conclusions which are not borne out by argument. But in discussions of extreme difficulty, like these, when good judgment is a factor, and pure ratiocination is not everything, it is prudent to take every element into consideration. From the moment when I could think at all, until now, about forty years, I have been diligently and incessantly occupied with the study of methods [of] inquiry, both those which have been and are pursued and those which ought to be pursued. For ten years before this study began, I had been in training in the chemical laboratory. I was thoroughly grounded not only in all that was then known of physics and chemistry, but also in the way in which those who were successfully advancing knowledge proceeded. I have paid the most attention to the methods of the most exact sciences, have intimately communed with some of the greatest minds of our times in physical science, and have myself made positive contributions -- none of them of any very great importance, perhaps -- in mathematics, gravitation, optics, chemistry, astronomy, etc. I am saturated, through and through, with the spirit of the physical sciences. I have been a great student of logic, having read everything of any importance on the subject, devoting a great deal of time to medieval thought, without neglecting the works of the Greeks, the English, the Germans, the French, etc., and have produced systems of my own both in deductive and in inductive logic. In metaphysics, my training has been less systematic; yet I have read and deeply pondered upon all the main systems, never being satisfied until I was able to think about them as their own advocates thought.

Peirce: CP 1.4 Cross-Ref:††

4. The first strictly philosophical books that I read were of the classical German schools; and I became so deeply imbued with many of their ways of thinking that I have never been able to disabuse myself of them. Yet my attitude was always that of a dweller in a laboratory, eager only to learn what I did not yet know, and not that of philosophers bred in theological seminaries, whose ruling impulse is to teach what they hold to be infallibly true. I devoted two hours a day to the study of Kant's *Critic of the Pure Reason* for more than three years, until I almost knew the whole book by heart, and had critically examined every section of it. For about two years, I had long and almost daily discussions with Chauncey Wright, one of the most acute of the followers of J. S. Mill.

Peirce: CP 1.5 Cross-Ref:††

5. The effect of these studies was that I came to hold the classical German philosophy to be, upon its argumentative side, of little weight; although I esteem it, perhaps am too partial to it, as a rich mine of philosophical suggestions. The English philosophy, meagre and crude, as it is, in its conceptions, proceeds by surer methods and more accurate logic. The doctrine of the association of ideas is, to my thinking, the finest piece of philosophical work of the prescientific ages. Yet I can but pronounce English sensationalism to be entirely destitute of any solid bottom. From

the evolutionary philosophers, I have learned little; although I admit that, however hurriedly their theories have been knocked together, and however antiquated and ignorant Spencer's *First Principles* and general doctrines, yet they are under the guidance of a great and true idea, and are developing it by methods that are in their main features sound and scientific.

Peirce: CP 1.6 Cross-Ref:††

6. The works of Duns Scotus have strongly influenced me. If his logic and metaphysics, not slavishly worshipped, but torn away from its medievalism, be adapted to modern culture, under continual wholesome reminders of nominalistic criticisms, I am convinced that it will go far toward supplying the philosophy which is best to harmonize with physical science. But other conceptions have to be drawn from the history of science and from mathematics.

Peirce: CP 1.7 Cross-Ref:††

7. Thus, in brief, my philosophy may be described as the attempt of a physicist to make such conjecture as to the constitution of the universe as the methods of science may permit, with the aid of all that has been done by previous philosophers. I shall support my propositions by such arguments as I can. Demonstrative proof is not to be thought of. The demonstrations of the metaphysicians are all moonshine. The best that can be done is to supply a hypothesis, not devoid of all likelihood, in the general line of growth of scientific ideas, and capable of being verified or refuted by future observers.

Peirce: CP 1.8 Cross-Ref:††

8. Religious infallibilism, caught in the current of the times, shows symptoms of declaring itself to be only practically speaking infallible; and when it has thus once confessed itself subject to gradations, there will remain over no relic of the good old tenth-century infallibilism, except that of the infallible scientists, under which head I include, not merely the kind of characters that manufacture scientific catechisms and homilies, churches and creeds, and who are indeed "born missionaries," but all those respectable and cultivated persons who, having acquired their notions of science from reading, and not from research, have the idea that "science" means knowledge, while the truth is, it is a misnomer applied to the pursuit of those who are devoured by a desire to find things out....

Peirce: CP 1.9 Cross-Ref:††

9. Though infallibility in scientific matters seems to me irresistibly comical, I should be in a sad way if I could not retain a high respect for those who lay claim to it, for they comprise the greater part of the people who have any conversation at all. When I say they lay claim to it, I mean they assume the functions of it quite naturally and unconsciously. The full meaning of the adage *Humanum est errare*, they have never waked up to. In those sciences of measurement which are the least subject to error -- metrology, geodesy, and metrical astronomy -- no man of self-respect ever now states his result, without affixing to it its *probable error*; and if this practice is not followed in other sciences it is because in those the probable errors are too vast to be estimated.

Peirce: CP 1.10 Cross-Ref:††

10. I am a man of whom critics have never found anything good to say. When they could see no opportunity to injure me, they have held their peace. The little laudation I have had has come from such sources, that the only satisfaction I have derived from it, has been from such slices of bread and butter as it might waft my

way. Only once, as far as I remember, in all my lifetime have I experienced the pleasure of praise -- not for what it might bring but in itself. That pleasure was beatific; and the praise that conferred it was meant for blame. It was that a critic said of me that I did not seem to be *absolutely sure of my own conclusions*. Never, if I can help it, shall that critic's eye ever rest on what I am now writing; for I owe a great pleasure to him; and, such was his evident animus, that should he find that out, I fear the fires of hell would be fed with new fuel in his breast.

Peirce: CP 1.11 Cross-Ref:††

11. My book will have no instruction to impart to anybody. Like a mathematical treatise, it will suggest certain ideas and certain reasons for holding them true; but then, if you accept them, it must be because you like my reasons, and the responsibility lies with you. Man is essentially a social animal: but to be social is one thing, to be gregarious is another: I decline to serve as bellwether. My book is meant for people who *want to find out*; and people who want philosophy ladled out to them can go elsewhere. There are philosophical soup shops at every corner, thank God!

Peirce: CP 1.12 Cross-Ref:††

12. The development of my ideas has been the industry of thirty years. I did not know as I ever should get to publish them, their ripening seemed so slow. But the harvest time has come, at last, and to me that harvest seems a wild one, but of course it is not I who have to pass judgment. It is not quite you, either, individual reader; it is experience and history.

Peirce: CP 1.13 Cross-Ref:††

13. For years in the course of this ripening process, I used for myself to collect my ideas under the designation *fallibilism*; and indeed the first step toward *finding out* is to acknowledge you do not satisfactorily know already; so that no blight can so surely arrest all intellectual growth as the blight of cocksureness; and ninety-nine out of every hundred good heads are reduced to impotence by that malady -- of whose inroads they are most strangely unaware!

Peirce: CP 1.14 Cross-Ref:††

14. Indeed, out of a contrite fallibilism, combined with a high faith in the reality of knowledge, and an intense desire to find things out, all my philosophy has always seemed to me to grow. . . .

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BOOK I

GENERAL HISTORICAL ORIENTATION

CHAPTER 1

PRINCIPLES OF PHILOSOPHY*P*

LESSONS FROM THE HISTORY OF PHILOSOPHY

§1. NOMINALISM †1

15. Very early in my studies of logic, before I had really been devoting myself to it more than four or five years, it became quite manifest to me that this science was in a bad condition, entirely unworthy of the general state of intellectual development of our age; and in consequence of this, every other branch of philosophy except ethics -- for it was already clear that psychology was a special science and no part of philosophy -- was in a similar disgraceful state. About that time -- say the date of Mansel's *Prolegomena Logica*†2 -- Logic touched bottom. There was no room for it to become more degraded. It had been sinking steadily, and relatively to the advance of physical science, by no means slowly from the time of the revival of learning -- say from the date of the last fall of Constantinople.†3 One important addition to the subject had been made early in the eighteenth century, the Doctrine of Chances. But this had not come from the professed logicians, who knew nothing about it. Whewell, it is true, had been doing some fine work; but it was not of a fundamental character. De Morgan and Boole had laid the foundations for modern exact logic, but they can hardly be said to have begun the erection of the edifice itself. Under these circumstances, I naturally opened the dusty folios of the scholastic doctors. Thought generally was, of course, in a somewhat low condition under the Plantagenets. You can appraise it very well by the impression that Dante, Chaucer, Marco Polo, Froissart, and the great cathedrals make upon us. But [their] logic, relatively to the general condition of thought, was marvellously exact and critical. They can tell us nothing concerning methods of reasoning since their own reasoning was puerile; but their analyses of thought and their discussions of all those questions of logic that almost trench upon metaphysics are very instructive as well as very good discipline in that subtle kind of thinking that is required in logic.

Peirce: CP 1.16 Cross-Ref:††

16. In the days of which I am speaking, the age of Robert of Lincoln, Roger Bacon, St. Thomas Aquinas, and Duns Scotus, the question of nominalism and realism was regarded as definitively and conclusively settled in favor of realism. You know what the question was. It was whether *laws* and general *types* are figments of the mind or are real. If this be understood to mean whether there really are any laws and types, it is strictly speaking a question of metaphysics and not of logic. But as a first step toward its solution, it is proper to ask whether, granting that our common-sense beliefs are true, the analysis of the meaning of those beliefs shows that, according to those beliefs, laws and types are objective or subjective. This is a question of logic rather than of metaphysics -- and as soon as this is answered the reply to the other question immediately follows after.

Peirce: CP 1.17 Cross-Ref:††

17. Notwithstanding a great outburst of nominalism in the fourteenth century which was connected with politics, the nominalists being generally opposed to the excessive powers of the pope and in favor of civil government, a connection that lent to the philosophical doctrine a factitious following, the Scotists, who were realists, were in most places the predominant party, and retained possession of the universities. At the revival of learning they stubbornly opposed the new studies; and

thus the word *Duns*, the proper name of their master, came to mean an adversary of learning. The word originally further implied that the person so called was a master of subtle thought with which the humanists were unable to cope. But in another generation the disputations by which that power of thought was kept in training had lost their liveliness; and the consequence was that Scotism died out when the strong Scotists died. It was a mere change of fashion.

Peirce: CP 1.18 Cross-Ref:††

18. The humanists were weak thinkers. Some of them no doubt might have been trained to be strong thinkers; but they had no severe training in thought. All their energies went to writing a classical language and an artistic style of expression. They went to the ancients for their philosophy; and mostly took up the three easiest of the ancient sects of philosophy, Epicureanism, Stoicism, and Scepticism. Epicureanism was a doctrine extremely like that of John Stuart Mill. The Epicureans alone of the later ancient schools believed in inductive reasoning, which they grounded upon the uniformity of nature, although they made the uniformity of nature to consist in somewhat different characters from those Stuart Mill emphasizes. Like Mill, the Epicureans were extreme nominalists. The Stoics advocated the flattest materialism, which nobody any longer has any need of doing since the new invention of Monism enables a man to be perfectly materialist in substance, and as idealistic as he likes in words. Of course the Stoics could not but be nominalists. They took no stock in inductive reasoning. They held it to be a transparent fallacy. The Sceptics of the *Renaissance* were something like the agnostics of the generation now passing away, except that they went much further. Our agnostics contented themselves with declaring everything beyond ordinary generalizations of experience to be unknowable, while the Sceptics did not think any scientific knowledge of any description to be possible. If you turn over the pages, for example, of Cornelius Agrippa's book *De [incertitudine et] vanitate scientiarum [et artium]* [1531], you will find he takes up every science in succession, arithmetic, geometry, mechanics, optics, and after examination pronounces each to be altogether beyond the power of the human mind. Of course, therefore, as far as they believed in anything at all, the Sceptics were nominalists.

Peirce: CP 1.19 Cross-Ref:††

19. In short, there was a tidal wave of nominalism. Descartes was a nominalist. Locke and all his following, Berkeley, Hartley, Hume, and even Reid, were nominalists. Leibniz was an extreme nominalist, and Rémusat [C. F. M.?] who has lately made an attempt to repair the edifice of Leibnizian monadology, does so by cutting away every part which leans at all toward realism. Kant was a nominalist; although his philosophy would have been rendered compacter, more consistent, and stronger if its author had taken up realism, as he certainly would have done if he had read Scotus. Hegel was a nominalist of realistic yearnings. I might continue the list much further. Thus, in one word, all modern philosophy of every sect has been nominalistic.

Peirce: CP 1.20 Cross-Ref:††

20. In a long notice of Frazer's *Berkeley*, in the *North American Review* for October, 1871,†1 I declared for realism. I have since very carefully and thoroughly revised my philosophical opinions more than half a dozen times, and have modified them more or less on most topics; but I have never been able to think differently on that question of nominalism and realism. In that paper I acknowledged that the tendency of science has been toward nominalism; but the late Dr. Francis Ellingwood

Abbot in the very remarkable introduction to his book entitled "Scientific Theism" [1885], showed on the contrary, quite conclusively, that science has always been at heart realistic, and always must be so; and upon comparing his writings with mine, it is easily seen that these features of nominalism which I pointed out in science are merely superficial and transient.

Peirce: CP 1.21 Cross-Ref:††

21. The heart of the dispute lies in this. The modern philosophers -- one and all, unless Schelling be an exception -- recognize but one mode of being, the being of an individual thing or fact, the being which consists in the object's crowding out a place for itself in the universe, so to speak, and reacting by brute force of fact, against all other things. I call that existence.

Peirce: CP 1.22 Cross-Ref:††

22. Aristotle, on the other hand, whose system, like all the greatest systems, was evolutionary, recognized besides an embryonic kind of being, like the being of a tree in its seed, or like the being of a future contingent event, depending on how a man shall decide to act. In a few passages Aristotle seems to have a dim *aperçue* of a third mode of being in the *entelechy*. The embryonic being for Aristotle was the being he called matter, which is alike in all things, and which in the course of its development took on form. Form is an element having a different mode of being. The whole philosophy of the scholastic doctors is an attempt to mould this doctrine of Aristotle into harmony with christian truth. This harmony the different doctors attempted to bring about in different ways. But all the realists agree in reversing the order of Aristotle's evolution by making the form come first, and the individuation of that form come later. Thus, they too recognized two modes of being; but they were not the two modes of being of Aristotle.

Peirce: CP 1.23 Cross-Ref:††

23. My view is that there are three modes of being. I hold that we can directly observe them in elements of whatever is at any time before the mind in any way. They are the being of positive qualitative possibility, the being of actual fact, and the being of law that will govern facts in the future.

Peirce: CP 1.24 Cross-Ref:††

24. Let us begin with considering actuality, and try to make out just what it consists in. If I ask you what the actuality of an event consists in, you will tell me that it consists in its happening *then* and *there*. The specifications *then* and *there* involve all its relations to other existents. The actuality of the event seems to lie in its relations to the universe of existents. A court may issue *injunctions* and *judgments* against me and I not care a snap of my finger for them. I may think them idle vapor. But when I feel the sheriff's hand on my shoulder, I shall begin to have a sense of actuality. Actuality is something *brute*. There is no reason in it. I instance putting your shoulder against a door and trying to force it open against an unseen, silent, and unknown resistance. We have a two-sided consciousness of effort and resistance, which seems to me to come tolerably near to a pure sense of actuality. On the whole, I think we have here a mode of being of one thing which consists in how a second object is. I call that Secondness.

Peirce: CP 1.25 Cross-Ref:††

25. Besides this, there are two modes of being that I call Firstness and Thirdness. Firstness is the mode of being which consists in its subject's being positively such as it is regardless of aught else. That can only be a possibility. For as

long as things do not act upon one another there is no sense or meaning in saying that they have any being, unless it be that they are such in themselves that they may perhaps come into relation with others. The mode of being a *redness*, before anything in the universe was yet red, was nevertheless a positive qualitative possibility. And redness in itself, even if it be embodied, is something positive and *sui generis*. That I call Firstness. We naturally attribute Firstness to outward objects, that is we suppose they have capacities in themselves which may or may not be already actualized, which may or may not ever be actualized, although we can know nothing of such possibilities [except] so far as they are actualized.

Peirce: CP 1.26 Cross-Ref: ††

26. Now for Thirdness. Five minutes of our waking life will hardly pass without our making some kind of prediction; and in the majority of cases these predictions are fulfilled in the event. Yet a prediction is essentially of a general nature, and cannot ever be completely fulfilled. To say that a prediction has a decided tendency to be fulfilled, is to say that the future events are in a measure really governed by a law. If a pair of dice turns up sixes five times running, that is a mere uniformity. The dice might happen fortuitously to turn up sixes a thousand times running. But that would not afford the slightest security for a prediction that they would turn up sixes the next time. If the prediction has a tendency to be fulfilled, it must be that future events have a tendency to conform to a general rule. "Oh," but say the nominalists, "this general rule is nothing but a mere word or couple of words!" I reply, "Nobody ever dreamed of denying that what is general is of the nature of a general sign; but the question is whether future events will conform to it or not. If they will, your adjective 'mere' seems to be ill-placed." A rule to which future events have a tendency to conform is *ipso facto* an important thing, an important element in the happening of those events. This mode of being which *consists*, mind my word if you please, the mode of being which *consists* in the fact that future facts of Secondness will take on a determinate general character, I call a Thirdness.

Peirce: CP 1.27 Cross-Ref: ††

§2. CONCEPTUALISM †1

27. Many philosophers call their variety of nominalism, "conceptualism"; but it is essentially the same thing; and their not seeing that it is so is but another example of that loose and slapdash style of thinking that has made it possible for them to remain nominalists. Their calling their "conceptualism" a middle term between realism and nominalism is itself an example in the very matter to which nominalism relates. For while the question between nominalism and realism is, in its nature, susceptible of but two answers: yes and no, they make an idle and irrelevant point which had been thoroughly considered by all the great realists; and instead of drawing a valid distinction, as they suppose, only repeat the very same confusion of thought which made them nominalists. The question was whether all properties, laws of nature, and predicates of more than an actually existent subject are, without exception, mere figments or not. †P1 The conceptualists seek to wedge in a third position conflicting with the principle of excluded middle. They say, "Those universals are real, indeed; but they are only real thoughts." So much may be said of the philosopher's stone. To give that answer constitutes a man a nominalist. Are the laws of nature, and that property of gold by which it will yield the purple of Cassius,

no more real than the philosopher's stone? No, the conceptualists admit that there is a difference; but they say that the laws of nature and the properties of chemical species are results of thinking. The great realists had brought out all the truth there is in that much more distinctly long before modern conceptualism appeared in the world. They showed that the general is not capable of full actualization in the world of action and reaction but is of the nature of what is thought, but that our thinking only apprehends and does not create thought, and that that thought may and does as much govern outward things as it does our thinking. But those realists did not fall into any confusion between the real fact of having a dream and the illusory object dreamed. The conceptualist doctrine is an undisputed truism about *thinking*, while the question between nominalists and realists relates to *thoughts*, that is, to the objects which thinking enables us to know.

Peirce: CP 1.28 Cross-Ref:††
§3. THE SPIRIT OF SCHOLASTICISM †1

28. . . . [The] history of logic is not altogether without an interest as a branch of history. For so far as the logic of an age adequately represents the methods of thought of that age, its history is a history of the human mind in its most essential relation -- that is to say with reference to its power of investigating truth. But the chief value of the study of historical philosophy is that it disciplines the mind to regard philosophy with a cold and scientific eye and not with passion as though philosophers were contestants.

Peirce: CP 1.29 Cross-Ref:††

29. British logic is a subject of some particular interest inasmuch as some peculiar lines of thought have always been predominant in those islands, giving their logicians a certain family resemblance, which already begins to appear in very early times. The most striking characteristic of British thinkers is their nominalistic tendency. This has always been and is now very marked. So much so that in England and in England alone are there many thinkers more distinguished at this day as being nominalistic than as holding any other doctrines. William Ockham or Oakum, an Englishman, was beyond question the greatest nominalist that ever lived; while Duns Scotus, another British name, it is equally certain is the subtlest advocate of the opposite opinion. These two men, Duns Scotus and William Ockham, are decidedly the greatest speculative minds of the middle ages, as well as two of the profoundest metaphysicians that ever lived. Another circumstance which makes [the] logic of the British Islands interesting is that there more than elsewhere have the studies of the logic of the natural sciences been made. Already we find some evidences of English thought running in that direction, when we meet with that singular phenomenon Roger Bacon -- a man who was scientific before science began. At the first dawn of the age [of] science, Francis Bacon wrote that professedly and really logical treatise, the *Novum Organum*, a work the celebrity of which perhaps exceeds its real merits. In our own day, the writings of Whewell, Mill, and Herschel afford some of the finest accounts of the methods of thought in science. Another direction in which logical thought has gone farther in England than elsewhere is in mathematico-formal logic -- the chief writers on which are Boole, DeMorgan, and the Scotch Sir William Hamilton -- for although Hamilton was so bitter against mathematics, that his own doctrine of the quantified predicate is essentially mathematical is beyond intelligent

dispute. This fondness for the formal part of logic had already appeared in the middle ages, when the nominalistic school of Ockham -- the most extremely scholastic of the scholastics -- and next to them the school of Scotus carried to the utmost the doctrines of the *Parva Logicalia* which were the contribution of those ages to this branch of the science. And those *Parva Logicalia* may themselves have had an English origin, for the earliest known writer upon the subject -- unless the Synopsis {Aristotelous Organou} be attributed to Psellus -- was an Englishman, William Shirwood. . . . †1

Peirce: CP 1.30 Cross-Ref: ††

30. The most striking characteristic of medieval thought is the importance attributed to authority. It was held that authority and reason were two coördinate methods of arriving at truth, and far from holding that authority was secondary to reason, the scholastics were much more apt to place it quite above reason. When Berengarius in his dispute with Lanfranc remarked that the whole of an affirmation does not stand after a part is subverted, his adversary replied: "The sacred authorities being relinquished, you take refuge in dialectic, and when I am to hear and to answer concerning the ministry of the Faith, I prefer to hear and to answer the sacred authorities which are supposed to relate to the subject rather than dialectical reasons." To this Berengarius replied that St. Augustine in his book *De doctrina christiana* says that what he said concerning an affirmation is bound up indissolubly with that very eternity of truth which is God. But added: "Maximi plane cordis est, per omnia ad dialecticum confugere, quia confugere ad eam ad rationem est confugere, quo qui non confugit, cum secundum rationem sit factus ad imaginem Dei, suum honorem reliquit, nec potest renovari de die in diem ad imaginem Dei." †2 Next to sacred authorities -- the Bible, the church and the fathers -- that of Aristotle of course ranked the highest. It could be denied, but the presumption was immense against his being wrong on any particular point.

Peirce: CP 1.31 Cross-Ref: ††

31. Such a weight being attached to authority -- a weight which would be excessive were not the human mind at that time in so uneducated a state that it could not do better than follow masters, since it was totally incompetent to solve metaphysical problems for itself -- it follows naturally that originality of thought was not greatly admired, but that on the contrary the admirable mind was his who succeeded in interpreting consistently the dicta of Aristotle, Porphyry, and Boethius. Vanity, therefore, the vanity of cleverness, was a vice from which the schoolmen were remarkably free. They were minute and thorough in their knowledge of such authorities as they had, and they were equally minute and thorough in their treatment of every question which came up.

Peirce: CP 1.32 Cross-Ref: ††

32. All these characters remind us less of the philosophers of our day than of the men of science. I do not hesitate to say that scientific men now think much more of authority than do metaphysicians; for in science a question is not regarded as settled or its solution as certain until all intelligent and informed doubt has ceased and all competent persons have come to a catholic agreement, whereas fifty metaphysicians, each holding opinions that no one of the other forty-nine can admit, will nevertheless generally regard their fifty opposite opinions as more certain than that the sun will rise tomorrow. This is to have what seems an absurd disregard for others' opinions. The man of science attaches positive value to the opinion of every man as competent as himself, so that he cannot but have a doubt of a conclusion which he would adopt were it not that a competent man opposes it; but on the other

hand, he will regard a sufficient divergence from the convictions of the great body of scientific men as tending of itself to argue incompetence, and he will generally attach little weight to the opinions of men who have long been dead and were ignorant of much that has been since discovered which bears upon the question in hand. The schoolmen, however, attached the greatest authority to men long since dead, and there they were right, for in the dark ages it was not true that the later state of human knowledge was the most perfect, but on the contrary. I think it may be said then that the schoolmen did not attach too much weight to authority, although they attached much more to it than we ought to do or than ought or could be attached to it in any age in which science is pursuing a successful and onward course -- and of course infinitely more than is attached to it by those intellectual nomads, the modern metaphysicians, including the positivists.

Peirce: CP 1.33 Cross-Ref:††

33. In the slight importance they attached to a brilliant theory, the schoolmen also resembled modern scientific men, who cannot be comprehended in this respect at all by men not scientific. The followers of Herbert Spencer, for example, cannot comprehend why scientific men place Darwin so infinitely above Spencer, since the theories of the latter are so much grander and more comprehensive. They cannot understand that it is not the sublimity of Darwin's theories which makes him admired by men of science, but that it is rather his minute, systematic, extensive, strict, scientific researches which have given his theories a more favorable reception -- theories which in themselves would barely command scientific respect. And this misunderstanding belongs to all those metaphysicians who fancy themselves men of science on account of their metaphysics. This same scientific spirit has been equally misunderstood as it is found in the schoolmen. They have been above all things found fault with because they do not write a literary style and do not "study in a literary spirit." The men who make this objection cannot possibly comprehend the real merits of modern science. If the words *quidditas*, *entitas*, and *haecceitas* are to excite our disgust, what shall we say of the Latin of the botanists, and the style of any technically scientific work? As for that phrase "studying in a literary spirit" it is impossible to express how nauseating it is to any scientific man, yes even to the scientific linguist. But above all things it is the searching thoroughness of the schoolmen which affiliates them with men of science and separates them, world-wide, from modern so-called philosophers. The thoroughness I allude to consists in this, that in adopting any theory, they go about everywhere, they devote their whole energies and lives in putting it to tests *bona fide* -- not such as shall merely add a new spangle to the glitter of their proofs but such as shall really go toward satisfying their restless insatiable impulse to put their opinions to the *test*. Having a theory, they must apply it to every subject and to every branch of every subject to see whether it produces a result in accordance with the only criteria they were able to apply -- the truth of the Catholic faith and the teaching of the Prince of Philosophers.

Peirce: CP 1.34 Cross-Ref:††

34. Mr. George Henry Lewes in his work on Aristotle †1 seems to me to have come pretty near to stating the true cause of the success of modern science when he has said that it was *verification*. I should express it in this way: modern students of science have been successful because they have spent their lives not in their libraries and museums but in their laboratories and in the field; and while in their laboratories and in the field they have been not gazing on nature with a vacant eye, that is, in passive perception unassisted by thought, but have been *observing* -- that is, perceiving by the aid of analysis -- and testing suggestions of theories. The cause of

their success has been that the motive which has carried them to the laboratory and the field has been a craving to know how things really were, and an interest in finding out whether or not general propositions actually held good -- which has overbalanced all prejudice, all vanity, and all passion. Now it is plainly not an essential part of this method in general that the tests were made by the observation of natural objects. For the immense progress which modern mathematics has made is also to be explained by the same intense interest in testing general propositions by particular cases -- only the tests were applied by means of particular demonstrations. This is observation, still, for as the great mathematician Gauss has declared -- algebra is a science of the eye, †2 only it is observation of artificial objects and of a highly recondite character. Now this same unwearied interest in testing general propositions is what produced those long rows of folios of the schoolmen, and if the test which they employed is of only limited validity so that they could not unhampered go on indefinitely to further discoveries, yet the *spirit*, which is the most essential thing -- the motive, was nearly the same. And how different this spirit is from that of the major part, though not all, of modern philosophers -- even of those who have called themselves empirical, no man who is actuated by it can fail to perceive.

Peirce: CP 1.35 Cross-Ref: ††

§4. KANT AND HIS REFUTATION OF IDEALISM †1

35. Kant's whole philosophy turns upon his logic. He gives the name of logic to the greater part of his *Critic of the Pure Reason*, and it is a result of the great fault of his logical theory that he does not extend that name to the whole work. This greatest fault was at the same [time] the greatest merit of his doctrine: it lay in his sharp discrimination of the intuitive and the discursive processes of the mind. The distinction itself is not only familiar to everybody but it had long played a part in philosophy. Nevertheless, it is on such obvious distinctions that the greater systems have been founded, and [Kant] saw far more clearly than any predecessor had done the whole philosophical import of this distinction. This was what emancipated him from Leibnizianism, and at the same time turned him against sensationalism. It was also what enabled him to see that no general description of existence is possible, which is perhaps the most valuable proposition that the *Critic* contains. But he drew too hard a line between the operations of observation and of ratiocination. He allows himself to fall into the habit of thinking that the latter only begins after the former is complete; and wholly fails to see that even the simplest syllogistic conclusion can only be drawn by observing the relations of the terms in the premisses and conclusion. His doctrine of the *schemata* can only have been an afterthought, an addition to his system after it was substantially complete. For if the *schemata* had been considered early enough, they would have overgrown his whole work.

Peirce: CP 1.36 Cross-Ref: ††

36. Kant's refutation of idealism in the second edition of the *Critic of the Pure Reason* has been often held to be inconsistent with his main position or even to be knowingly sophistical. It appears to me to be one of the numerous passages in that work which betray an elaborated and vigorous analysis, marred in the exposition by the attempt to state the argument more abstractly and demonstratively than the thought would warrant.

Peirce: CP 1.36 Cross-Ref:††

In "Note 1," Kant says that his argument beats idealism at its own game. How is that? The idealist says that all that we know immediately, that is, otherwise than inferentially, is what is *present* in the mind; and things out of the mind are not so present. The whole idealist position turns upon this conception of the *present*.

Peirce: CP 1.37 Cross-Ref:††

37. The idealistic argument turns upon the assumption that certain things are absolutely "present," namely what we have in mind at the moment, and that nothing else can be immediately, that is, otherwise than inferentially known. When this is once granted, the idealist has no difficulty in showing that that external existence which we cannot know immediately we cannot know, at all. Some of the arguments used for this purpose are of little value, because they only go to show that our knowledge of an external world is fallible; now there is a world of difference between fallible knowledge and no knowledge. However, I think it would have to be admitted as a matter of logic that if we have no immediate perception of a non-ego, we can have no reason to admit the supposition of an existence so contrary to all experience as that would in that case be.

Peirce: CP 1.38 Cross-Ref:††

38. But what evidence is there that we can immediately know only what is "present" to the mind? The idealists generally treat this as self-evident; but, as Clifford jestingly says, "it is evident" is a phrase which only means "we do not know how to prove." The proposition that we can immediately perceive only what is present seems to me parallel to that other vulgar prejudice that "a thing cannot act where it is not." An opinion which can only defend itself by such a sounding phrase is pretty sure to be wrong. That a thing cannot act where it is not is plainly an induction from ordinary experience, which shows no forces except such as act through the resistance of materials, with the exception of gravity which, owing to its being the same for all bodies, does not appear in ordinary experience like a force. But further experience shows that attractions and repulsions are the universal types of forces. A thing may be said to be wherever it acts; but the notion that a particle is absolutely present in one part of space and absolutely absent from all the rest of space is devoid of all foundation. In like manner, the idea that we can immediately perceive only what is present seems to be founded on our ordinary experience that we cannot recall and reexamine the events of yesterday nor know otherwise than by inference what is to happen tomorrow. Obviously, then, the first move toward beating idealism at its own game is to remark that we apprehend our own ideas only as flowing in time, and since neither the future nor the past, however near they may be, is *present*, there is as much difficulty in conceiving our perception of what passes within us as in conceiving external perception. If so, replies the idealist, instead of giving up idealism we must go still further to nihilism. Kant does not notice this retort; but it is clear from his footnote that he would have said: Not so; for it is impossible we should so much as think we think in time unless we do think in time; or rather, dismissing blind impossibility, the mere imagination of time is a clear perception of the past. Hamilton †1 stupidly objects to Reid's phrase "immediate memory"; but an immediate, intuitive consciousness of time clearly exists wherever time exists. But once grant immediate knowledge in time, and what becomes of the idealist theory that we immediately know only the *present*? For the present can contain no time.

Peirce: CP 1.39 Cross-Ref:††

39. But Kant does not pursue this line of thought along the straight road to its

natural result; because he is a sort of idealist himself. Namely, though not idealistic as to the substance of things, he is partially so in regard to their accidents. Accordingly, he introduces his distinction of the variable and the persistent (*beharrlich*), and seeks to show that the only way we can apprehend our own flow of ideas, binding them together as a connected flow, is by attaching them to an immediately perceived persistent externality. He refuses to inquire how that immediate external consciousness is possible, though such an inquiry might have probed the foundations of his system.

Peirce: CP 1.40 Cross-Ref:††
§5. HEGELISM †2

40. The critical logicians have been much affiliated to the theological seminaries. About the thinking that goes on in laboratories they have known nothing. Now the seminarists and religionists generally have at all times and places set their faces against the idea of continuous growth. That disposition of intellect is the most catholic element of religion. Religious truth having been once defined is never to be altered in the most minute particular; and theology being held as queen of the sciences, the religionists have bitterly fought by fire and tortures all great advances in the true sciences; and if there be no true continuous growth in men's ideas where else in the world should it be looked for? Thence, we find this folk setting up hard lines of demarcation, or great gulfs, contrary to all observation, between good men and bad, between the wise and foolish, between the spirit and the flesh, between all the different kinds of objects, between one quantity and the next. So shut up are they in this conception of the world that when the seminarist Hegel discovered that the universe is everywhere permeated with continuous growth (for that, and nothing else, is the "Secret of Hegel") it was supposed to be an entirely new idea, a century and a half after the differential calculus had been in working order.

Peirce: CP 1.41 Cross-Ref:††

41. Hegel, while regarding scientific men with disdain, has for his chief topic the importance of continuity, which was the very idea the mathematicians and physicists had been chiefly engaged in following out for three centuries. This made Hegel's work less correct and excellent in itself than it might have been; and at the same time hid its true mode of affinity with the scientific thought into which the life of the race had been chiefly laid up. It was a misfortune for Hegelism, a misfortune for "philosophy," and a misfortune (in lesser degree) for science.

Peirce: CP 1.42 Cross-Ref:††

42. My philosophy resuscitates Hegel, though in a strange costume.

Peirce: CP 1.43 Cross-Ref:††
CHAPTER 2

LESSONS FROM THE HISTORY OF SCIENCE†1

§1. THE SCIENTIFIC ATTITUDE

43. If we endeavor to form our conceptions upon history and life, we remark three classes of men. The first consists of those for whom the chief thing is the qualities of feelings. These men create art. The second consists of the practical men, who carry on the business of the world. They respect nothing but power, and respect power only so far as it [is] exercised. The third class consists of men to whom nothing seems great but reason. If force interests them, it is not in its exertion, but in that it has a reason and a law. For men of the first class, nature is a picture; for men of the second class, it is an opportunity; for men of the third class, it is a cosmos, so admirable, that to penetrate to its ways seems to them the only thing that makes life worth living. These are the men whom we see possessed by a passion to learn, just as other men have a passion to teach and to disseminate their influence. If they do not give themselves over completely to their passion to learn, it is because they exercise self-control. Those are the natural scientific men; and they are the only men that have any real success in scientific research.

Peirce: CP 1.44 Cross-Ref:††

44. If we are to define science, not in the sense of stuffing it into an artificial pigeon-hole where it may be found again by some insignificant mark, but in the sense of characterizing it as a living historic entity, we must conceive it as that about which such men as I have described busy themselves. As such, it does not consist so much in *knowing*, nor even in "organized knowledge," as it does in diligent inquiry into truth for truth's sake, without any sort of axe to grind, nor for the sake of the delight of contemplating it, but from an impulse to penetrate into the reason of things. This is the sense in which this book is entitled a History of *Science*. Science and philosophy seem to have been changed in their cradles. For it is not knowing, but the love of learning, that characterizes the scientific man; while the "philosopher" is a man with a system which he thinks embodies all that is best worth knowing. If a man burns to learn and sets himself to comparing his ideas with experimental results in order that he may correct those ideas, every scientific man will recognize him as a brother, no matter how small his knowledge may be.

Peirce: CP 1.45 Cross-Ref:††

45. But if a man occupies himself with investigating the truth of some question for some ulterior purpose, such as to make money, or to amend his life, or to benefit his fellows, he may be ever so much better than a scientific man, if you will -- to discuss that would be aside from the question -- but he is not a scientific man. For example, there are numbers of chemists who occupy themselves exclusively with the study of dyestuffs. They discover facts that are useful to scientific chemistry; but they do not rank as genuine scientific men. The genuine scientific chemist cares just as much to learn about erbium -- the extreme rarity of which renders it commercially unimportant -- as he does about iron. He is more eager to learn about erbium if the knowledge of it would do more to complete his conception of the Periodic Law, which expresses the mutual relations of the elements.

Peirce: CP 1.46 Cross-Ref:††

§2. THE SCIENTIFIC IMAGINATION

46. When a man desires ardently to know the truth, his first effort will be to imagine what that truth can be. He cannot prosecute his pursuit long without finding that imagination unbridled is sure to carry him off the track. Yet nevertheless, it remains true that there is, after all, nothing but imagination that can ever supply him an inkling of the truth. He can stare stupidly at phenomena; but in the absence of imagination they will not connect themselves together in any rational way. Just as for Peter Bell a cowslip was nothing but a cowslip, so for thousands of men a falling apple was nothing but a falling apple; and to compare it to the moon would by them be deemed "fanciful."

Peirce: CP 1.47 Cross-Ref:††

47. It is not too much to say that next after the passion to learn there is no quality so indispensable to the successful prosecution of science as imagination. Find me a people whose early medicine is not mixed up with magic and incantations, and I will find you a people devoid of all scientific ability. There is no magic in the medical Papyrus Ebers. The stolid Egyptian saw nothing in disease but derangement of the affected organ. There never was any true Egyptian science.

Peirce: CP 1.48 Cross-Ref:††

48. There are, no doubt, kinds of imagination of no value in science, mere artistic imagination, mere dreaming of opportunities for gain. The scientific imagination dreams of explanations and laws.

Peirce: CP 1.49 Cross-Ref:††

§3. SCIENCE AND MORALITY

49. A scientific man must be single-minded and sincere with himself. Otherwise, his love of truth will melt away, at once. He can, therefore, hardly be otherwise than an honest, fair-minded man. True, a few naturalists have been accused of purloining specimens; and some men have been far from judicial in advocating their theories. Both of these faults must be exceedingly deleterious to their scientific ability. But on the whole, scientific men have been the best of men. It is quite natural, therefore, that a young man who might develop into a scientific man should be a well-conducted person.

Peirce: CP 1.50 Cross-Ref:††

50. Yet in more ways than one an exaggerated regard for morality is unfavorable to scientific progress. I shall present only one of those ways. It will no doubt shock some persons that I should speak of morality as involving an element which can become bad. To them good conduct and moral conduct are one and the same -- and they will accuse me of hostility to morality. I regard morality as highly necessary; but it is a means to good life, not necessarily coextensive with good conduct. Morality consists in the folklore of right conduct. A man is brought up to think he ought to behave in certain ways. If he behaves otherwise, he is uncomfortable. His conscience pricks him. That system of morals is the traditional wisdom of ages of experience. If a man cuts loose from it, he will become the victim

of his passions. It is not safe for him even to reason about it, except in a purely speculative way. Hence, morality is essentially conservative. Good morals and good manners are identical, except that tradition attaches less importance to the latter. The gentleman is imbued with conservatism. This conservatism is a habit, and it is the law of habit that it tends to spread and extend itself over more and more of the life. In this way, conservatism about morals leads to conservatism about manners and finally conservatism about opinions of a speculative kind. Besides, to distinguish between speculative and practical opinions is the mark of the most cultivated intellects. Go down below this level and you come across reformers and rationalists at every turn -- people who propose to remodel the ten commandments on modern science. Hence it is that morality leads to a conservatism which any new view, or even any free inquiry, no matter how purely speculative, shocks. The whole moral weight of such a community will be cast against science. To inquire into nature is for a Turk very unbecoming to a good Moslem; just as the family of Tycho Brahe regarded his pursuit of astronomy as unbecoming to a nobleman. (See Thomas Nash in *Pierce Penniless* for the character of a Danish nobleman.)

Peirce: CP 1.51 Cross-Ref:††

51. This tendency is necessarily greatly exaggerated in a country when the "gentleman," or recognized exponent of good manners, is appointed to that place as the most learned man. For then the inquiring spirit cannot say the gentlemen are a lot of ignorant fools. To the moral weight cast against progress in science is added the weight of superior learning. Wherever there is a large class of academic professors who are provided with good incomes and looked up to as gentlemen, scientific inquiry must languish. Wherever the bureaucrats are the more learned class, the case will be still worse.

Peirce: CP 1.52 Cross-Ref:††
§4. MATHEMATICS

52. The first questions which men ask about the universe are naturally the most general and abstract ones. Nor is it true, as has so often been asserted, that these are the most difficult questions to answer. Francis Bacon is largely responsible for this error, he having represented -- having nothing but his imagination and no acquaintance with actual science to draw upon -- that the most general inductions must be reached by successive steps. History does not at all bear out that theory. The errors about very general questions have been due to a circumstance which I proceed to set forth.

Peirce: CP 1.53 Cross-Ref:††

53. The most abstract of all the sciences is mathematics. That this is so, has been made manifest in our day; because all mathematicians now see clearly that mathematics is only busied about *purely hypothetical questions*. As for what the truth of existence may be the mathematician does not (qua mathematician) care a straw. It is true that early mathematicians could not clearly see that this was so. But for all their not seeing it, it was just as true of the mathematics of early days as of our own. The early mathematician might perhaps be more inclined to assert roundly that two straight lines in a plane cut by a third so as to make the sum of the internal angles on one side less than two right angles would meet at some finite distance on that side if

sufficiently produced; although, as a matter of fact, we observe no such tendency in Euclid. But however that may have been, the early mathematician had certainly no more tendency than the modern to *inquire into the truth of that postulate*; but quite the reverse. What he really did, therefore, was merely to deduce consequences of unsupported assumptions, whether he recognized that this was the nature of his business or not. Mathematics, then, really was, for him as for us, the most abstract of the sciences, cut off from all inquiry into existential truth. Consequently, the tendency to attack the most abstract problems first, not because they were *recognized* as such, but because such they *were*, led to mathematics being the earliest field of inquiry.

Peirce: CP 1.54 Cross-Ref:††

54. We find some peoples drawn more toward arithmetic; others more toward geometry. But in either case, a correct method of reasoning was sure to be reached before many centuries of real inquiry had elapsed. The reasoning would be at first awkward, and one case would be needlessly split up into several. But still all influences were pressing the reasoner to make use of a diagram, and as soon as he did that he was pursuing the correct method. For mathematical reasoning consists in constructing a diagram according to a general precept, in observing certain relations between parts of that diagram not explicitly required by the precept, showing that these relations will hold for all such diagrams, and in formulating this conclusion in general terms. All valid necessary reasoning is in fact thus diagrammatic.†1 This, however, is far from being obviously true. There was nothing to draw the attention of the early reasoners to the need of a diagram in such reasoning. Finding that by their inward meditations they could deduce the truth concerning, for example, the height of an inaccessible pillar, they naturally concluded the same method could be applied to positive inquiries.

Peirce: CP 1.54 Cross-Ref:††

In this way, early success in mathematics would naturally lead to bad methods in the positive sciences, and especially in metaphysics.

Peirce: CP 1.55 Cross-Ref:††

§5. SCIENCE AS A GUIDE TO CONDUCT

55. We have seen how success in mathematics would necessarily create a confidence altogether unfounded in man's power of eliciting truth by inward meditation without any aid from experience. Both its confidence in what is within and the absolute certainty of its conclusions lead to the confusion of *a priori* reason with conscience. For conscience, also, refuses to submit its dicta to experiment, and makes an absolute dual distinction between right and wrong. One result of this is that men begin to rationalize about questions of purity and integrity, which in the long run, through moral decay, is unfavorable to science. But what is worse, from our point of view, they begin to look upon science as a guide to conduct, that is, no longer as pure science but as an instrument for a practical end. One result of this is that all probable reasoning is despised. If a proposition is to be applied to action, it has to be embraced, or believed without reservation. There is no room for doubt, which can only paralyze action. But the scientific spirit requires a man to be at all times ready to dump his whole cart-load of beliefs, the moment experience is against them. The desire to learn forbids him to be perfectly cocksure that he knows already. Besides positive science

can only rest on experience; and experience can never result in absolute certainty, exactitude, necessity, or universality. But it is precisely with the universal and necessary, that is, with Law, that [con]science concerns itself. Thus the real character of science is destroyed as soon as it is made an adjunct to conduct; and especially all progress in the inductive sciences is brought to a standstill.

Peirce: CP 1.56 Cross-Ref:††
§6. MORALITY AND SHAM REASONING

56. The effect of mixing speculative inquiry with questions of conduct results finally in a sort of half make-believe reasoning which deceives itself in regard to its real character. Conscience really belongs to the subconscious man, to that part of the soul which is hardly distinct in different individuals, a sort of community-consciousness, or public spirit, not absolutely one and the same in different citizens, and yet not by any means independent in them. Conscience has been created by experience just as any knowledge is; but it is modified by further experience only with secular †1 slowness.

Peirce: CP 1.57 Cross-Ref:††

57. When men begin to rationalize about their conduct, the first effect is to deliver them over to their passions and produce the most frightful demoralization, especially in sexual matters. Thus, among the Greeks, it brought about pæderasty and a precedence of public women over private wives. But ultimately the subconscious part of the soul, being stronger, regains its predominance and insists on setting matters right. Men, then, continue to tell themselves they regulate their conduct by reason; but they learn to look forward and see what conclusions a given method will lead to before they give their adhesion to it. In short, it is no longer the reasoning which determines what the conclusion shall be, but it is the conclusion which determines what the reasoning shall be. This is sham reasoning. In short, as morality supposes self-control, men learn that they must not surrender themselves unreservedly to any method, without considering to what conclusions it will lead them. But this is utterly contrary to the single-mindedness that is requisite in science. In order that science may be successful, its votaries must hasten to surrender themselves at discretion to experimental inquiry, in advance of knowing what its decisions may be. There must be no reservations.

Peirce: CP 1.58 Cross-Ref:††

58. The effect of this shamming is that men come to look upon reasoning as mainly decorative, or at most, as a secondary aid in minor matters -- a view not altogether unjust, if questions of conduct are alone to interest us. They, therefore, demand that it shall be plain and facile. If, in special cases, complicated reasoning is indispensable, they hire a specialist to perform it. The result of this state of things is, of course, a rapid deterioration of intellectual vigor, very perceptible from one generation to the next. This is just what is taking place among us before our eyes; and to judge from the history of Constantinople, it is likely to go on until the race comes to a despicable end.

Peirce: CP 1.59 Cross-Ref:††

§7. THE METHOD OF AUTHORITY

59. When society is broken into bands, now warring, now allied, now for a time subordinated one to another, man loses his conceptions of truth and of reason. If he sees one man assert what another denies, he will, if he is concerned, choose his side and set to work by all means in his power to silence his adversaries. The truth for him is that for which he fights.

Peirce: CP 1.60 Cross-Ref:††

60. The next step which is to be expected in a logical development not interrupted by accidental occurrences will consist in the recognition that a central authority ought to determine the beliefs of the entire community. As far as morals and religion go, this plan admirably fulfills its purpose of producing uniformity. But in order that it may do this, it is desirable that there should be another less absolute authority which shall declare, not infallibly but yet with a weight of collective learning, the propositions which science from time to time puts out of reasonable doubt, and which shall aid the researches of competent investigators. The value of such services in the development of science is immense; though they are accompanied by very serious disadvantages in not allowing to unofficial studies the weight which ought to be accorded to them. The history of science is full of examples of this sort.

Peirce: CP 1.61 Cross-Ref:††

§8. SCIENCE AND CONTINUITY

61. One of the worst effects of the influence of moral and religious reasonings upon science lies in this, that the distinctions upon which both insist as fundamental are dual distinctions, and that their tendency is toward an ignoring of all distinctions that are not dual and especially of the conception of continuity. Religion recognizes the saints and the damned. It will not readily admit any third fate. Morality insists that a motive is either good or bad. That the gulf between them is bridged over and that most motives are somewhere near the middle of the bridge, is quite contrary to the teachings of any moral system which ever lived in the hearts and consciences of a people.

Peirce: CP 1.62 Cross-Ref:††

62. It is not necessary to read far in almost any work of philosophy written by a man whose training is that of a theologian, in order to see how helpless such minds are in attempting to deal with continuity. Now continuity, it is not too much to say, is the leading conception of science. The complexity of the conception of continuity is so great as to render it important wherever it occurs. Now it enters into every fundamental and exact law of physics or of psychics that is known. The few laws of chemistry which do not involve continuity seem for the most part to be very roughly true. It seems not unlikely that if the veritable laws were known continuity would be found to be involved in them. . . .†1

Peirce: CP 1.63 Cross-Ref:††

§9. THE ANALYTIC METHOD

63. The first problems to suggest themselves to the inquirer into nature are far too complex and difficult for any early solution, even if any satisfactorily secure conclusion can ever be drawn concerning them. What ought to be done, therefore, and what in fact is done, is at first to substitute for those problems others much simpler, much more abstract, of which there is a good prospect of finding probable solutions. Then, the reasonably certain solutions of these last problems will throw a light more or less clear upon more concrete problems which are in certain respects more interesting.

Peirce: CP 1.64 Cross-Ref:††

64. This method of procedure is that Analytic Method to which modern physics owes all its triumphs. It has been applied with great success in psychical sciences also. (Thus, the classical political economists, especially Ricardo, pursued this method.)†2 It is reprobated by the whole Hegelian army, who think it ought to be replaced by the "Historic Method," which studies complex problems in all their complexity, but which cannot boast any distinguished successes.

Peirce: CP 1.65 Cross-Ref:††
§10. KINDS OF REASONING †1

65. There are in science three fundamentally different kinds of reasoning, Deduction (called by Aristotle {synagōgē} or {anagōgē}), Induction (Aristotle's and Plato's {epagōgē}) and Retroduction (Aristotle's {apagōgē}, but misunderstood because of corrupt text, and as misunderstood usually translated *abduction*).†2 Besides these three, Analogy (Aristotle's {paradeigma}) combines the characters of Induction and Retroduction.

Peirce: CP 1.66 Cross-Ref:††

66. *Deduction* is that mode of reasoning which examines the state of things asserted in the premisses, forms a diagram of that state of things, perceives in the parts of that diagram relations not explicitly mentioned in the premisses, satisfies itself by mental experiments upon the diagram that these relations would always subsist, or at least would do so in a certain proportion of cases, and concludes their necessary, or probable, truth. For example, let the premiss be that there are four marked points upon a line which has neither extremity nor furcation. Then, by means of a diagram,



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we may conclude that there are two pairs of points such that in passing along the line in any way from one to the other point of either pair, one point of the second pair will be passed an odd number of times and the other point an even (or zero) number of times. This is *deduction*.

Peirce: CP 1.67 Cross-Ref:††

67. *Induction* is that mode of reasoning which adopts a conclusion as approximate, because it results from a method of inference which must generally lead to the truth in the long run. For example, a ship enters port laden with coffee. I go aboard and sample the coffee. Perhaps I do not examine over a hundred beans, but they have been taken from the middle, top, and bottom of bags in every part of the hold. I conclude by *induction* that the whole cargo has approximately the same value per bean as the hundred beans of my sample. All that induction can do is to ascertain the value of a ratio.

Peirce: CP 1.68 Cross-Ref:††

68. *Retrodution* is the provisional adoption of a hypothesis, because every possible consequence of it is capable of experimental verification, so that the persevering application of the same method may be expected to reveal its disagreement with facts, if it does so disagree. For example, all the operations of chemistry fail to decompose hydrogen, lithium, glucinum, boron, carbon, nitrogen, oxygen, fluorine, sodium, . . . gold, mercury, thallium, lead, bismuth, thorium, and uranium. We provisionally suppose these bodies to be simple; for if not, similar experimentation will detect their compound nature, if it can be detected at all. That I term *retrodution*.

Peirce: CP 1.69 Cross-Ref:††

69. *Analogy* is the inference that a not very large collection of objects which agree in various respects may very likely agree in another respect. For instance, the earth and Mars agree in so many respects that it seems not unlikely they may agree in being inhabited.

Peirce: CP 1.70 Cross-Ref:††

70. The methods of reasoning of science have been studied in various ways and with results which disagree in important particulars. The followers of Laplace treat the subject from the point of view of the theory of probabilities. After corrections due to Boole †1 and others,†2 that method yields substantially the results stated above. Whewell †3 described the reasoning just as it appeared to a man deeply conversant with several branches of science as only a genuine researcher can know them, and adding to that knowledge a full acquaintance with the history of science.

These results, as might be expected, are of the highest value, although there are important distinctions and reasons which he overlooked. John Stuart Mill endeavored to explain the reasonings of science by the nominalistic metaphysics of his father. The superficial perspicuity of that kind of metaphysics rendered his logic extremely popular with those who think, but do not think profoundly; who know something of science, but more from the outside than the inside, and who for one reason or another delight in the simplest theories even if they fail to cover the facts.

Peirce: CP 1.71 Cross-Ref:††

71. Mill denies that there was any reasoning in Kepler's procedure. He says it is merely a description of the facts.†1 He seems to imagine that Kepler had all the places of Mars in space given him by Tycho's observations; and that all he did was to generalize and so obtain a general expression for them. Even had that been all, it would certainly have been inference. Had Mill had even so much practical acquaintance with astronomy as to have practised discussions of the motions of double stars, he would have seen that. But so to characterize Kepler's work is to betray total ignorance of it. Mill certainly never read the *De Motu [Motibus] Stellae Martis*, which is not easy reading. The reason it is not easy is that it calls for the most vigorous exercise of all the powers of reasoning from beginning to end.

Peirce: CP 1.72 Cross-Ref:††

72. What Kepler had given was a large collection of observations of the apparent places of Mars at different times. He also knew that, in a general way, the Ptolemaic theory agrees with the appearances, although there were various difficulties in making it fit exactly. He was furthermore convinced that the hypothesis of Copernicus ought to be accepted. Now this hypothesis, as Copernicus himself understood its first outline, merely modifies the theory of Ptolemy so far as [to] impart to all the bodies of the solar system one common motion, just what is required to annul the mean motion of the sun. It would seem, therefore, at first sight, that it ought not to affect the appearances at all. If Mill had called the work of Copernicus mere description he would not have been *so very far* from the truth as he was. But Kepler did not understand the matter quite as Copernicus did. Because the sun was so near the centre of the system, and was of vast size (even Kepler knew its diameter must be at least fifteen times that of the earth), Kepler, looking at the matter dynamically, thought it must have something to do with causing the planets to move in their orbits. This retrodution, vague as it was, cost great intellectual labor, and was most important in its bearings upon all Kepler's work. Now Kepler remarked that the lines of apsides of the orbits of Mars and of the earth are not parallel; and he utilized various observations most ingeniously to infer that they probably intersected in the sun. Consequently, it must be supposed that a general description of the motion would be simpler when referred to the sun as a fixed point of reference than when referred to any other point. Thence it followed that the proper times at which to take the observations of Mars for determining its orbit were when it appeared just opposite the sun -- the true sun -- instead of when it was opposite the mean sun, as had been the practice. Carrying out this idea, he obtained a theory of Mars which satisfied the longitudes at all the oppositions observed by Tycho and himself, thirteen in number, to perfection. But unfortunately, it did not satisfy the latitudes at all and was totally irreconcilable with observations of Mars when far from opposition.

Peirce: CP 1.73 Cross-Ref:††

73. At each stage of his long investigation, Kepler has a theory which is approximately true, since it approximately satisfies the observations (that is, within 8',

which is less than any but Tycho's observations could decisively pronounce an error), and he proceeds to modify this theory, after the most careful and judicious reflection, in such a way as to render it more rational or closer to the observed fact. Thus, having found that the centre of the orbit bisects the eccentricity, he finds in this an indication of the falsity of the theory of the equant and substitutes, for this artificial device, the principle of the equable description of areas. Subsequently, finding that the planet moves faster at ninety degrees from its apsides than it ought to do, the question is whether this is owing to an error in the law of areas or to a compression of the orbit. He ingeniously proves that the latter is the case.

Peirce: CP 1.74 Cross-Ref:††

74. Thus, never modifying his theory capriciously, but always with a sound and rational motive for just the modification he makes, it follows that when he finally reaches a modification -- of most striking simplicity and rationality -- which exactly satisfies the observations, it stands upon a totally different logical footing from what it would if it had been struck out at random, or the reader knows not how, and had been found to satisfy the observation. Kepler shows his keen logical sense in detailing the whole process by which he finally arrived at the true orbit. This is the greatest piece of Retroductive reasoning ever performed.

Peirce: CP 1.75 Cross-Ref:††

§11. THE STUDY OF THE USELESS

75. . . . The old-fashioned political economist adored, as alone capable of redeeming the human race, the glorious principle of individual greed, although, as this principle requires for its action hypocrisy and fraud, he generally threw in some dash of inconsistent concessions to virtue, as a sop to the vulgar Cerberus. But it is easy to see that the only kind of science this principle would favor would be such as is immediately remunerative with a great preference for such as can be kept secret, like the modern sciences of dyeing and perfumery. Kepler's discovery rendered Newton possible, and Newton rendered modern physics possible, with the steam engine, electricity, and all the other sources of the stupendous fortunes of our age. But Kepler's discovery would not have been possible without the doctrine of conics. Now contemporaries of Kepler -- such penetrating minds as Descartes and Pascal -- were abandoning the study of geometry (in which they included what we now call the differential calculus, so far as that had at that time any existence) because they said it was so UTTERLY USELESS. There was the future of the human race almost trembling in the balance; for had not the geometry of conic sections already been worked out in large measure, and had their opinion that only sciences apparently useful ought to be pursued, [prevailed] the nineteenth century would have had none of those characters which distinguish it from the *ancien régime*.

Peirce: CP 1.76 Cross-Ref:††

76. True science is distinctively the study of useless things. For the useful things will get studied without the aid of scientific men. To employ these rare minds on such work is like running a steam engine by burning diamonds.

Peirce: CP 1.77 Cross-Ref:††

77. The University of Paris encouraged useless studies in the most effective way possible, by training so many men as to be almost sure of getting a large

proportion of all the minds that could be very serviceable in such studies. At the same time, it provided a sure living not only for such as were really successful, but even for those whose talents were of a somewhat inferior kind. On the other hand, like all universities, it set up an official standard of truth, and frowned on all who questioned it. Just so, the German universities for a whole generation turned the cold shoulder to every man who did not extol their stale Hegelianism, until it became a stench in the nostrils of every man of common sense. Then the official fashion shifted, and a Hegelian is today treated in Germany with the same arrogant stupidity with which an anti-Hegelian formerly was. Of course, so-called "universities," whose purpose is not the solution of great problems, but merely the fitting of a selection of young men to earn more money than their fellow citizens not so favored, have for the interests of science none of the value of the medieval and German universities, although they exercise the same baleful influence to about the same degree.

Peirce: CP 1.78 Cross-Ref:††

78. The small academies of continental Europe are reasonably free from the gravest fault of the universities. Their defect is that while they indirectly do much for their few members they extend little aid to the younger men, except that of giving a general tone of respectability to pure science.

Peirce: CP 1.79 Cross-Ref:††

79. The larger bodies give much less aid to individuals; but they begin to aid them sooner. They have a distinct though limited use when they are specialized, like the Union of German chemists. But whether the Royal Society has been as serviceable to science as the French *Académie des Sciences* may be doubted.

Peirce: CP 1.80 Cross-Ref:††

§12. *IL LUME NATURALE*

80. In examining the reasonings of those physicists who gave to modern science the initial propulsion which has insured its healthful life ever since, we are struck with the great, though not absolutely decisive, weight they allowed to instinctive judgments. Galileo appeals to *il lume naturale* at the most critical stages of his reasoning. Kepler, Gilbert, and Harvey -- not to speak of Copernicus -- substantially rely upon an inward power, not sufficient to reach the truth by itself, but yet supplying an essential factor to the influences carrying their minds to the truth.

Peirce: CP 1.81 Cross-Ref:††

81. It is certain that the only hope of retroductive reasoning ever reaching the truth is that there may be some natural tendency toward an agreement between the ideas which suggest themselves to the human mind and those which are concerned in the laws of nature.

Peirce: CP 1.82 Cross-Ref:††

§13. GENERALIZATION AND ABSTRACTION

82. The most important operation of the mind is that of generalization. There

are some exceedingly difficult questions of theoretical logic connected with generalization. On the other hand, there are some valuable lessons which evade those puzzles. If we look at any earlier work upon mathematics as compared with a later one upon the same subject, that which most astonishes us is to see the difficulty men had in first seizing upon general conceptions which after we become a little familiarized to them are quite matters of course. That an Egyptian should have been able to think of adding one-fifth and one-fifth, and yet should not have been content to call the sum two-fifths, but must call it one-third plus one-fifteenth, as if he could not conceive of a sum of fractions unless their denominators were different, seems perverse stupidity. That decimals should have been so slow in coming in, and that, when they did come, the so-called decimal point should be written as if the relation of units to tenths were somehow peculiar, while what was logically called for was simply some mark attached to the units place, so that instead of 3.14159 [what]

should have been written [was]  [Click here to view] 314159, seems very surprising. That Descartes should have thought it necessary to work problems in analytical geometry four times over, according to the different quadrants between the axes of coördinates in which the point to be determined might occur, is astonishing. That which the early mathematicians failed to see in all these cases was that some feature which they were accustomed to insert into their theorems was quite irrelevant and could perfectly well be omitted without affecting in the slightest degree the cogency of any step of the demonstrations.

Peirce: CP 1.83 Cross-Ref:††

83. Another operation closely allied to generalization is abstraction; and the use of it is perhaps even more characteristic of mathematical reasoning than is generalization. This consists of seizing upon something which has been conceived as a {epos pteroen}, a meaning not dwelt upon but through which something else is discerned, and converting it into an {epos apteroen}, a meaning upon which we rest as the principal subject of discourse. Thus, the mathematician conceives an operation as something itself to be operated upon. He conceives the collection of places of a moving particle as itself a place which can at one instant be totally occupied by a filament, which can again move, and the aggregate of all its places, considered as possibly occupied in one instant, is a surface, and so forth.

Peirce: CP 1.84 Cross-Ref:††

84. The intimate connection between generalization and continuity is to be pointed out.†1

Peirce: CP 1.85 Cross-Ref:††

§14. THE EVALUATION OF EXACTITUDE

85. For every line of scientific research there is in any given stage of its development, an appropriate standard of certitude and exactitude, such that it is useless to require more, and unsatisfactory to have less. This is a part of the doctrine of the Economy of Research. When Phoenix †2 made his celebrated survey of the route from San Francisco to the Mission of Dolores, the distance required was the sum of two parts, one of them resting on the guess of a driver, while the other was

determined at great expense to a transcendental precision. As long as one part of the distance was extremely uncertain, there was no use in spending much money in ascertaining the other part precisely. For there is a relation between the value of an increased certainty of an item of knowledge and the cost of such increase of certainty, which enables us to determine whether it is better to expend our genius, energy, time, and money upon one investigation or upon another.

Peirce: CP 1.86 Cross-Ref:††

86. If a result is to be used merely to confirm the result of an independent investigation, it may have a high value even though its probability is not very high. But if it is only to be used in combination with other results, very little will be gained by increasing its probability far beyond the probabilities of those others. Of course, knowledge that is to be put to special purposes may need to be more precise than other knowledge. Thus, it pays to determine the places of a thousand stars with the utmost accuracy, leaving hundreds of thousands only roughly located, and others only recorded upon photographs. But where a high degree of exactitude and probability is unattainable, that is no reason for refusing to accept such knowledge as we can attain. Because we cannot reach great certainty about the life and teachings of Pythagoras is no reason for sulkily dismissing the subject as one we know nothing about, as Dr. Ed. Zeller †1 would have us do.

Peirce: CP 1.87 Cross-Ref:††

§15. SCIENCE AND EXTRAORDINARY PHENOMENA

87. Science is from the nature of its procedure confined to the investigation of the ordinary course of nature. I do not mean that it cannot investigate individual objects, such as the earth. But all its explanations of such objects must be limited to the supposition that they have come about in the ordinary course of nature. A statistical result may be obtained.

Peirce: CP 1.88 Cross-Ref:††

88. We may find that such and such a proportion of calves have five legs. But we never can conclude with any probability that the ratio is strictly zero; and even if we knew that the proportion of men with golden thighs is exactly zero, that would be no argument at all against Pythagoras having had a golden thigh. For something might be true of one man, or any number of men, and yet might occur in the long run in a finite number of cases out of an infinite series. Now a finite number divided by infinity is exactly zero. That Pythagoras had a golden thigh is the testimony of history. It is asserted by Aristotle, of all possible authorities the highest, by both Porphyry and Jamblichus after Nicomachus, by Herodotus, by Plutarch, Diogenes Laertius, Aelian, Apollonius,†2 etc. This is far stronger testimony than we have for the resurrection of Jesus. Are we then to admit as a part of the science of history that Pythagoras had a golden thigh?

Peirce: CP 1.89 Cross-Ref:††

89. To do so would be to make a retroductive inference. Now a retroductive conclusion is only justified by its *explaining* an observed fact. An explanation is a syllogism of which the major premiss, or rule, is a known law or rule of nature, or other general truth; the minor premiss, or case, is the hypothesis or retroductive conclusion, and the conclusion, or result, is the observed (or otherwise established)

fact. Such an explanation, in this case, would be like this:

Every fact about Pythagoras (unless kept secret or insignificant) would be reported by his ancient biographers.

That Pythagoras had a golden thigh was a fact about Pythagoras neither secret nor insignificant.

∴ That Pythagoras had a golden thigh would be reported by all his ancient biographers.

Peirce: CP 1.90 Cross-Ref:††

90. But this syllogism may be condemned at once on the ground that it supposes we have statistical knowledge about such kinds of facts as are quite contrary to the usual course of nature. If the reply be made that it could make in regard to the reporting of the fact no difference whether it were a natural one or not, I rejoin, that granting that, it is not to the purpose. It only goes to show that there is no difference between natural and supernatural facts in this respect; from which the only just inference is that no such proposition can be known even in respect to natural facts. This, indeed, is the case. We cannot say that every remarkable public fact about Pythagoras would be reported, but only that every phenomenon would be told as it appeared to people in an almost primitive state of civilization. Nobody can think that the golden thigh was treated as a modern assayer would treat a gold brick. It was probably flexible and therefore its golden appearance was superficial. One of these days, we may find out something about the ancient Persians, Chorasmians, or Brahmins which may make this story significant. At present, it only illustrates the impossibility of science making any assertion about a fact out of the course of nature. Pythagoras was certainly a wonderful man. We have no right, at all, to say that supernal powers had not put a physical mark upon him as extraordinary as his personality. Science can no more deny a miracle than it can assert one.

Peirce: CP 1.91 Cross-Ref:††

91. But although science cannot infer any particular violation of the ordinary course of nature, it may very well be that it should find evidence that such violations are so frequent and usual that this fact is itself a part of the ordinary course of nature. For that reason, it is perfectly proper that science should inquire, for example, into the evidences of the fulfillment of prayers, etc. That is something open to experimental inquiry; and until such inquiry has been instituted nobody is entitled to any opinion whatever, or any bias, as to its result.

Peirce: CP 1.92 Cross-Ref:††

§16. REASONING FROM SAMPLES

92. Many persons seem to suppose that the state of things asserted in the premisses of an induction renders the state of things asserted in the conclusion probable. The fact that Macaulay's essay on Bacon was admired in its day shows how little the absurdity of such a position was perceived. Even John Stuart Mill holds that the uniformity of nature makes the one state of things follow from the other. He overlooks the circumstance that if so it ought to follow necessarily, while in truth no

definite probability can be assigned to it without absurd consequences. He also overlooks the fact that inductive reasoning does not invariably infer a uniformity; it may infer a diversity. I watch the throws of a die, I notice that about half are odd and half are even, and that they follow one another with the utmost irregularity. I conclude that about half of all the throws of that die are odd and that the odd and even follow one another with great irregularity. How can any principle of uniformity account for the truth of such an induction? Mill never made up his mind in what sense he took the phrase "uniformity of nature" when he spoke of it as the basis of induction. In some passages he clearly means any special uniformity by which a given character is likely to belong to the whole of a species, a genus, a family, or a class if it belongs to any members of that group. In this sense, as well as in others, overlooked by Mill, there is no doubt the knowledge of a uniformity strengthens an inductive conclusion; but it is equally free from doubt that such knowledge is not essential to induction. But in other passages Mill holds that it is not the knowledge of the uniformity, but the uniformity itself that supports induction, and furthermore that it is no special uniformity but a general uniformity in nature. Mill's mind was certainly acute and vigorous, but it was not mathematically accurate; and it is by that trait that I am forced to explain his not seeing that this general uniformity could not be so defined as not on the one hand to appear manifestly false or on the other hand to render no support to induction, or both. He says it means that under similar circumstances similar events will occur. But this is vague. Does he mean that objects alike in all respects but one are alike in that one? But plainly no two different real objects are alike in all respects but one. Does he mean that objects *sufficiently* alike in other respects are alike in any given respect? But that would be but another way of saying that no two different objects are alike in all respects but one. It is obviously true; but it has no bearing on induction, where we deal with objects which we well know are, like all existing things, alike in numberless respects and unlike in numberless other respects.† 1

Peirce: CP 1.93 Cross-Ref:††

93. The truth is that induction is reasoning from a sample taken at random to the whole lot sampled. A sample is a *random* one, provided it is drawn by such machinery, artificial or physiological, that in the long run any one individual of the whole lot would get taken as often as any other. Therefore, judging of the statistical composition of a whole lot from a sample is judging by a method which will be right on the average in the long run, and, by the reasoning of the doctrine of chances, will be nearly right oftener than it will be far from right.

Peirce: CP 1.94 Cross-Ref:††

94. That this does justify induction is a mathematical proposition beyond dispute. It has been objected that the sampling cannot be random in this sense. But this is an idea which flies far away from the plain facts. Thirty throws of a die constitute an approximately random sample of all the throws of that die; and that the randomness should be approximate is all that is required.

Peirce: CP 1.95 Cross-Ref:††

95. This account of the rationale of induction is distinguished from others in that it has as its consequences two rules of inductive inference which are very frequently violated, although they have sometimes been insisted upon. The first of these is that the sample must be a random one. Upon that I shall not dwell here. The other rule is that the character, toward the ascertainment of the proportionate frequency of which in the lot sampled [the sampling is done], must not be determined by the character of the particular sample taken. For example, we must not take a

sample of eminent men, and studying over them, find that they have certain characters and conclude that all eminent men will have those characters. We must first decide for what character we propose to examine the sample, and only after that decision examine the sample. The reason is that any sample will be peculiar and unlike the average of the lot sampled in innumerable respects. At the same time it will be approximately like the average of the whole lot in the great majority of respects.

Peirce: CP 1.96 Cross-Ref:††

96. In order to illustrate the necessity of this rule I take a random sample of eminent persons. It is quite a random one, for it consists of the first names on pages 100, 300, 500, 700, 900, of Phillips's *Great Index of Biography* [*Biographical Reference*, second edition, 1881]. The names are as follows:

Born Died

Francis Baring 1740 1810 Sept. 12

Vicomte de Custine 1760 1794 Jan. 3

Hippostrates (of uncertain age)

Marquis d'O. 1535 1594 Oct. 24

Theocrenes 1480 1536 Oct. 18

Peirce: CP 1.96 Cross-Ref:††

Now I might, in violation of the above rule of predesignation, draw the following inductions:

1. Three-fourths of these men were born in a year whose date ends in a cipher. Hence about three-fourths of all eminent men are probably so born. But, in fact, only one in ten is so born.

2. Three eminent men out of four die in autumn. In fact, only one out of four.

3. All eminent men die on a day of the month divisible by three. In fact, one out of three.

4. All eminent men die in years whose date doubled and increased by one gives a number whose last figure is the same as that in the tens' place of the date itself. In fact, only one in ten.

5. All eminent men who were living in any year ending in forty-four died at an age which after subtracting four becomes divisible by eleven. All others die at an age which increased by ten is divisible by eleven.

Peirce: CP 1.97 Cross-Ref:††

97. This rule is recognized in the requirement of physicists that a theory shall furnish predictions which shall be verified before any particular weight is accorded to it. The medical men, too, who deserve special mention for the reason that they have had since Galen a logical tradition of their own, recognize this rule, however dimly, in their working against reasoning "*post hoc, ergo propter hoc*." . . .

Peirce: CP 1.98 Cross-Ref:††
§17 THE METHOD OF RESIDUAL PHENOMENA

98. The so-called "method of residual phenomena" is so simple that it hardly calls for any remark. At any early stage of science when there are few observations of a given matter, and those rough ones, a law is made out which, when the observations come to be increased in number and made more accurate, is found not to hold exactly. The departures from this law are found themselves to follow a law which may now be shown to be true. But at a still later date it is found that this law again is interfered with, that there are still more minute departures from it, and these departures are again found to follow a law. All the successive laws so found may be real, or they may be merely empirical formulae. . . .

Peirce: CP 1.99 Cross-Ref:††
§18. OBSERVATION

99. I have already remarked that a definition of science in general which shall express a really intelligent conception of it as a living historic entity must regard it as the occupation of that peculiar class of men, the scientific men. The same remark may be extended to definitions of the different branches of science. The men who pursue a given branch herd together. They understand one another; they live in the same world, while those who pursue another branch are for them foreigners.

Peirce: CP 1.100 Cross-Ref:††

100. It will be found upon close examination that that which renders the modes of thought of the students of a special branch of science peculiar is that their experience lies in a peculiar region. And the cause of this is that they are trained and equipped to make a peculiar kind of observations. The man who is continually making chemical analyses lives in a different region of nature from other men. The same thing is even more true of men who are constantly using a microscope.

Peirce: CP 1.101 Cross-Ref:††

101. It comes to this, that sciences must be classified according to the peculiar means of observation they employ.

Peirce: CP 1.102 Cross-Ref:††

102. So too the great landmarks in the history of science are to be placed at the points where new instruments, or other means of observation, are introduced. Astronomy before the telescope and astronomy after the telescope. Prephotographic astronomy and photographic astronomy. Chemistry before the exact analytic balance, and after.

Peirce: CP 1.103 Cross-Ref:††
§19. EVOLUTION

103. The evolutionary theory in general throws great light upon history and especially upon the history of science -- both its public history and the account of its development in an individual intellect. As great a light is thrown upon the theory of evolution in general by the evolution of history, especially that of science -- whether public or private.

Peirce: CP 1.104 Cross-Ref:††

104. The main theories of the evolution of organic species are three. First, the theory of Darwin, according to which the entire interval from Moner to Man has been traversed by successive purely fortuitous and insensible variations *in reproduction*. The changes on the whole follow a determinate course simply because a certain amount of change in certain directions destroys the species altogether, as the final result of successive weakenings of its reproductive power. Second, the theory of Lamarck, according to which the whole interval has been traversed by a succession of very minute changes. But these have not taken place in reproduction, which has absolutely nothing to do with the business, except to keep the average individuals plastic by their youth. The changes have not been fortuitous but wholly the result of strivings of the individuals. Third, the theory of cataclysmal evolution, according to which the changes have not been small and have not been fortuitous; but they have taken place chiefly in reproduction. According to this view, sudden changes of the environment have taken place from time to time. These changes have put certain organs at a disadvantage, and there has been an effort to use them in new ways. Such organs are particularly apt to sport in reproduction and to change in the way which adapts them better to their recent mode of exercise.

Peirce: CP 1.105 Cross-Ref:††

105. Notwithstanding the teachings of Weismann, it seems altogether probable that all three of these modes of evolution have acted. It is probable that the last has been the most efficient. These three modes of organic evolution have their parallels in other departments of evolution.

Peirce: CP 1.106 Cross-Ref:††

106. Let us consider, for example, the evolution of standards of weights and measures. In order to define the word "pound" in the *Century Dictionary*, †1 I made a list of about four hundred pounds which had been in use in different parts of Europe -- undoubtedly a very incomplete list, for it was confined in great measure to certain provinces concerning which I was able to obtain information. Each individual pound or measuring stick is from time to time copied; and at length the old one becomes destroyed. The measure of each copy is imperceptibly larger or smaller than its immediate prototype. If then these variations cannot, by gradual summation, produce a standard much smaller without that standard being destroyed as inconvenient while no such destruction would follow upon an increase of the standard, the average of the standards will slowly grow larger by Darwinian evolution. If there were a disposition on the part of owners of pounds to file them down, so as to make them lighter, though not enough to be noticed, then these filed pounds being copied, and the copies filed, there would be a gradual lightening of the pound by Lamarckian evolution. But it is very unlikely that either of these two modes has been a considerable factor in the actual evolution of weights and measures. As long as their circumstances are unchanged, human communities are exceedingly conservative. Nothing short of the despotism of a modern government with a modern police can cause a change in weights and measures. But from time to time changes occur which cause trade to take new routes. Business has to be adapted to new conditions; and under such influences

we find all those habits of communities which are rendered unsuitable by the change become plastic enough. Then it is that a new pound or a new yard may be made which is a compromise between a desire to retain old ways and a desire to please new-comers.

Peirce: CP 1.107 Cross-Ref:††

107. In the evolution of science, a Darwinian mode of evolution might, for example, consist in this, that at every recall of a judgment to the mind -- say, for example, a judgment in regard to some such delicate question as the marriage of the clergy -- a slight fortuitous modification of the judgment might take place; the modified judgment would cause a corresponding modification of the belief-habit, so that the next recall would be influenced by this fortuitous modification, though it would depart more or less from it by a new fortuitous modification. If, however, by such summation of modifications an opinion quite untenable were reached, it would either be violently changed or would be associationally weak and not apt to be recalled. The effect of this would be in the long run that belief would move away from such untenable positions. It is possible that such a mode of influence may affect our instinctive feelings; but there can be nothing of this sort in science, which is controlled and exact. But another sort of Darwinian evolution undoubtedly does take place. We are studying over phenomena of which we have been unable to acquire any satisfactory account. Various tentative explanations recur to our minds from time to time, and at each occurrence are modified by omission, insertion, or change in the point of view, in an almost fortuitous way. Finally, one of these takes such an aspect that we are led to dismiss it as impossible. Then, all the energy of thought which had previously gone to the consideration of that becomes distributed among the other explanations, until finally one of them becomes greatly strengthened in our minds.

Peirce: CP 1.108 Cross-Ref:††

108. Lamarckian evolution might, for example, take the form of perpetually modifying our opinion in the effort to make that opinion represent the known facts as more and more observations came to be collected. This is all the time going on in regard, for example, to our estimate of the danger of infection of phthisis. Yet, after all, it does not play a prominent part in the evolution of science. The physical journals -- say, for example, Poggendorff's [*Annalen der Physik*] and *Beiblätter* -- publish each month a great number of new researches. Each of these is a distinct contribution to science. It represents some good, solid, well-trained labor of observation and inference. But as modifying what is already known, the average effect of the ordinary research may be said to be insignificant. Nevertheless, as these modifications are not fortuitous but are for the most part movements toward the truth -- could they be rightly understood, all of them would be so -- there is no doubt that from decade to decade, even without any splendid discoveries or great studies, science would advance very perceptibly. We see that it is so in branches of physics which remain for a long time without any decisive conquests. It was so, for example, in regard to the classification of the chemical elements in the lapse of time from Berzelius to Mendeléeff, as the valuable history of Venable †1 shows. This is an evolution of the Lamarckian type.

Peirce: CP 1.109 Cross-Ref:††

109. But this is not the way in which science mainly progresses. It advances by leaps; and the impulse for each leap is either some new observational resource, or some novel way of reasoning about the observations. Such novel way of reasoning might, perhaps, be considered as a new observational means, since it draws attention

to relations between facts which would previously have been passed by unperceived.

Peirce: CP 1.109 Cross-Ref:††

[I] illustrate by the discoveries of Pasteur,†2 who began by applying the microscope to chemistry. He picked out the right- and left-handed crystals of tartaric acid. The two kinds have absolutely the same properties except in regard to direction of rotation of the plane of polarization and in their chemical relations to other "optically active" bodies. Since this method of picking out individual crystals was so slow, Pasteur looked for other means. Ferments of appropriate kinds were found to have the same effect. The microscope showed these were due to living organisms, which Pasteur began studying. At that time the medical world was dominated by Claude Bernard's dictum that a disease is not an entity but merely a sum of symptoms.†3 This was pure metaphysics which only barricaded inquiry in that direction. But that was a generation which attached great value to nominalistic metaphysics. Pasteur began with the phylloxera. He found it influenced the "optical activity" of the sugar. This pointed to a ferment and therefore to an entity. He began to extend the doctrine to other diseases. The medical men, dominated by the metaphysics of Claude Bernard, raised all sorts of sophisticated objections. But the method of cultures and inoculation proved the thing, and here we see new ideas connected with new observational methods and a fine example of the usual process of scientific evolution. It is not by insensible steps.

Peirce: CP 1.110 Cross-Ref:††
§20. SOME A PRIORI DICTA

110. The last fifty years have taught the lesson of not trifling with facts and not trusting to principles and methods which are not logically founded upon facts and which serve only to exclude testimony from consideration.

Peirce: CP 1.111 Cross-Ref:††

111. Such, for example, was the dictum of Claude Bernard that a disease is not an entity -- a purely metaphysical doctrine. But the observation of facts has taught us that a disease is in many, if not most, serious cases, just as much an entity as a human family consisting of father, mother, and children.

Peirce: CP 1.112 Cross-Ref:††

112. Such was the dictum of the old psychology which identified the soul with the ego, declared its absolute simplicity, and held that its faculties were mere names for logical divisions of human activity. This was all unadulterated fancy. The observation of facts has now taught us that the ego is a mere wave in the soul, a superficial and small feature, that the soul may contain several personalities and is as complex as the brain itself, and that the faculties, while not exactly definable and not absolutely fixed, are as real as are the different convolutions of the cortex.

Peirce: CP 1.113 Cross-Ref:††

113. Such were the dicta by means of which the internal criticism of historical documents was carried to such a height that it often amounted to the rejection of all the testimony that has come down to us, and the substitution for it of a dream spun out of the critic's brain. But archeological researches have shown that ancient testimony ought to be trusted in the main, with a small allowance for the changes in the

meanings of words. When we are told that Pythagoras had a golden thigh, we are to remember that to the ancients gold did not mean a chemical element of atomic weight 197.5 and specific gravity 19.3, melting at 1045• C. and forming saline compounds of the types *AuX* and *AuX*[3]. It meant something of metallic lustre, warmer in color than electrum and cooler than copper. Dr. Schliemann's discoveries were the first socdolager that "higher criticism" received. It has since got many others.

Peirce: CP 1.114 Cross-Ref:††

114. Such was the dictum of Laplace that stones do not come from heaven.

Peirce: CP 1.115 Cross-Ref:††

115. Such were the dicta by which everything of the nature of extraordinary powers connected with psychological states of which the hypnotic trance is an example were set down as tricks. At present, while the existence of telepathy cannot be said to be established, all scientific men are obliged by observed facts to admit that it presents at least a very serious problem requiring respectful treatment.

Peirce: CP 1.116 Cross-Ref:††

§21. THE PAUCITY OF SCIENTIFIC KNOWLEDGE

116. Persons who know science chiefly by its results -- that is to say, have no acquaintance with it at all as a living inquiry -- are apt to acquire the notion that the universe is now entirely explained in all its leading features; and that it is only here and there that the fabric of scientific knowledge betrays any rents.

Peirce: CP 1.117 Cross-Ref:††

117. But in point of fact, notwithstanding all that has been discovered since Newton's time, his saying that we are little children picking up pretty pebbles on the beach while the whole ocean lies before us unexplored remains substantially as true as ever, and will do so though we shovel up the pebbles by steam shovels and carry them off in carloads. An infinitesimal ratio may be multiplied indefinitely and remain infinitesimal still.

Peirce: CP 1.118 Cross-Ref:††

118. In the first place all that science has done is to study those relations between objects which were brought into prominence and conceiving which we had been endowed with some original knowledge in two instincts -- the instinct of *feeding*, which brought with it elementary knowledge of mechanical forces, space, etc., and the instinct of *breeding*, which brought with it elementary knowledge of psychical motives, of time, etc. All the other relations of things concerning which we must suppose there is vast store of truth are for us merely the object of such false sciences as judicial astrology, palmistry, the doctrine of signatures, the doctrine of correspondences, magic, and the like.

Peirce: CP 1.119 Cross-Ref:††

119. In the next place, even within the very bounds to which our science is confined, it is altogether superficial and fragmentary. Want of knowledge of the constitution of matter and of electricity. The conservation of forces, as Helmholtz first enunciated it, untenable; whether it can be universally true in any sense is a difficult problem. To strengthen it Helmholtz greatly insisted on discontinuities -- a most

objectionable theory from every point of view. Mind quite as little understood as matter, and the relations between the two an enigma. The forces we know can be but a small part of all those that are operative. Our ignorance of small things and great, of distant times and of very slow operations. We are equally ignorant of very rapid performances which nevertheless we know to take place. Our science is altogether middle-sized and mediocre. Its insignificance compared with the universe cannot be exaggerated.

Peirce: CP 1.120 Cross-Ref:††

§22. THE UNCERTAINTY OF SCIENTIFIC RESULTS

120. It is a great mistake to suppose that the mind of the active scientist is filled with propositions which, if not proved beyond all reasonable cavil, are at least extremely probable. On the contrary, he entertains hypotheses which are almost wildly incredible, and treats them with respect for the time being. Why does he do this? Simply because any scientific proposition whatever is always liable to be refuted and dropped at short notice. A hypothesis is something which looks as if it might be true and were true, and which is capable of verification or refutation by comparison with facts. The best hypothesis, in the sense of the one most recommending itself to the inquirer, is the one which can be the most readily refuted if it is false. This far outweighs the trifling merit of being likely. For after all, what is a *likely* hypothesis? It is one which falls in with our preconceived ideas. But these may be wrong. Their errors are just what the scientific man is out gunning for more particularly. But if a hypothesis can quickly and easily be cleared away so as to go toward leaving the field free for the main struggle, this is an immense advantage.

Peirce: CP 1.121 Cross-Ref:††

121. Retrodution goes upon the hope that there is sufficient affinity between the reasoner's mind and nature's to render guessing not altogether hopeless, provided each guess is checked by comparison with observation. It is true that agreement does not show the guess is right; but if it is wrong it must ultimately get found out. The effort should therefore be to make each hypothesis, which is practically no more than a question, as near an even bet as possible.

Peirce: CP 1.122 Cross-Ref:††

§23. THE ECONOMY OF RESEARCH

122. Dr. Ernst Mach, who has one of the best faults a philosopher can have, that of riding his horse to death, does just this with his principle of Economy in science.†1 But of course there is a doctrine of the Economies of Research. One or two of its principles are easily made out. The value of knowledge is, for the purposes of science, in one sense absolute. It is not to be measured, it may be said, in money; in one sense that is true. But knowledge that leads to other knowledge is more valuable in proportion to the trouble it saves in the way of expenditure to get that other knowledge. Having a certain fund of energy, time, money, etc., all of which are merchantable articles to spend upon research, the question is how much is to be

allowed to each investigation; and *for us* the value of that investigation is the amount of money it will pay us to spend upon it. *Relatively*, therefore, knowledge, even of a purely scientific kind, has a money value.

Peirce: CP 1.122 Cross-Ref:††

This value increases with the fullness and precision of the information, but plainly it increases slower and slower as the knowledge becomes fuller and more precise. The cost of the information also increases with its fullness and accuracy, and increases faster and faster the more accurate and full it is. It therefore *may* be the case that it does not pay to get *any* information on a given subject; but, at any rate, it *must* be true that it does not pay (in any given state of science) to push the investigation beyond a certain point in fullness or precision.

Peirce: CP 1.123 Cross-Ref:††

123. If we have a number of studies in which we are interested, we should commence with the most remunerative and carry that forward until it becomes no more than equally remunerative with the commencement of another; carry both forward at such rates that they are equally remunerative until each is no more remunerative than a third, and so on.

Peirce: CP 1.124 Cross-Ref:††

124. If two or more kinds of knowledge are so related that one can replace the other so that the possession of one renders the other less profitable, this will diminish the investigation of either while increasing the investigation of all.

Peirce: CP 1.125 Cross-Ref:††

125. If two or more kinds of information are of use only as supplementing one another, that is, only when combined together, this will increase the investigations until there is little or no profit from the least profitable kind of research.

Peirce: CP 1.126 Cross-Ref:††

CHAPTER 3

NOTES ON SCIENTIFIC PHILOSOPHY

§1. LABORATORY AND SEMINARY PHILOSOPHIES †1

126. . . . The kind of philosophy which interests me and must, I think, interest everybody is that philosophy, which uses the most rational methods it can devise, for finding out the little that can as yet be found out about the universe of mind and matter from those observations which every person can make in every hour of his waking life. It will not include matters which are more conveniently studied by students of special sciences, such as psychology. Thus, everybody has remarked that there are four prominent qualities of the sense of taste, sweet, sour, salt, and bitter. But there may be other tastes, not so readily made out without special study; and in

any case tastes are conveniently studied in connexion with flavors and odors, which make a difficult experimental inquiry. Besides, the four tastes are altogether special and throw no light on the problems which, on account of their extreme generality, will naturally be examined by a class of researchers of entirely different aptitudes from those which adapt men to the discovery of recondite facts.

Peirce: CP 1.127 Cross-Ref:††

127. If anybody asks what there is in the study of obvious phenomena to make it particularly interesting, I will give two answers. The first is the one which seems to me the strongest; the other is that which nobody can fail to feel the force of. The first answer is that the spirit in which, as it seems to me, philosophy ought to be studied is the spirit in which every branch of science ought to be studied; namely, the spirit of joy in learning ourselves and in making others acquainted with the glories of God. Each person will feel this joy most in the particular branch of science to which his faculties are best adapted. It is not a sin to have no taste for philosophy as I define philosophy. As a matter of fact, however, almost everybody does feel an interest in philosophical problems, especially at that time of life at which he is spoiling for an intellectual tussle.

Peirce: CP 1.128 Cross-Ref:††

128. It is true that philosophy is in a lamentably crude condition at present; that very little is really established about it; while most philosophers set up a pretension of knowing all there is to know -- a pretension calculated to disgust anybody who is at home in any real science. But all we have to do is to turn our backs upon all such truly vicious conduct, and we shall find ourselves enjoying the advantages of having an almost virgin soil to till, where a given amount of really scientific work will bring in an extraordinary harvest, and that a harvest of very fundamental truth of exceptional value from every point of view.

Peirce: CP 1.129 Cross-Ref:††

129. This consideration touches upon the second reason for studying laboratory-philosophy (as contradistinguished from seminary-philosophy). It is that the special sciences are obliged to take for granted a number of most important propositions, because their ways of working afford no means of bringing these propositions to the test. In short, they always rest upon metaphysics. At one time, for example, we find physicists, Kelvin, Maxwell and others, assuming that a body cannot act where it is not, meaning by "where it is not" where its lines of force do not centre. At another time, we find them assuming that the laws of mechanics (including the principles of metric geometry) hold good for the smallest corpuscles. Now it is one thing to infer from the laws of little things how great things, that consist of little things, will act; but it is quite a different thing to infer from the phenomena presented by great things how single things billions of times smaller will act. It is like inferring that because in any country one man in so many will commit suicide, therefore every individual, once in such a period of time, will make an attempt at suicide. The psychical sciences, especially psychology, are, if possible, even more necessitated to assume general principles that cannot be proved or disproved by their ordinary methods of work. The philosopher alone is equipped with the facilities for examining such "axioms" and for determining the degree to which confidence may safely be reposed in them. Find a scientific man who proposes to get along without any metaphysics -- not by any means every man who holds the ordinary reasonings of metaphysicians in scorn -- and you have found one whose doctrines are thoroughly vitiated by the crude and uncriticized metaphysics with which they are packed. We

must philosophize, said the great naturalist Aristotle †1 -- if only to avoid philosophizing. Every man of us has a metaphysics, and has to have one; and it will influence his life greatly. Far better, then, that that metaphysics should be criticized and not be allowed to run loose. A man may say "I will content myself with common sense." I, for one, am with him there, in the main. I shall show why I do not think there can be any *direct* profit in going behind common sense -- meaning by common sense those ideas and beliefs that man's situation absolutely forces upon him. We shall later see more definitely what is meant. †2 I agree, for example, that it is better to recognize that some things are red and some others blue, in the teeth of what optical philosophers say, that it is merely that some things are resonant to shorter ether waves and some to longer ones. But the difficulty is to determine what really is and what is not the authoritative decision of common sense and what is merely *obiter dictum*. In short, there is no escape from the need of a critical examination of "first principles."

Peirce: CP 1.130 Cross-Ref: ††
 §2. AXIOMS †3

130. The science which, next after logic, may be expected to throw the most light upon philosophy, is mathematics. It is historical fact, I believe, that it was the mathematicians Thales, Pythagoras, and Plato who created metaphysics, and that metaphysics has always been the ape of mathematics. Seeing how the propositions of geometry flowed demonstratively from a few postulates, men got the notion that the same must be true in philosophy. But of late mathematicians have fully agreed that the axioms of geometry (as they are wrongly called) are not by any means evidently true. Euclid, be it observed, never pretended they were evident; he does not reckon them among his {koinai ennoiai}, or things everybody knows, †P1 but among the {aitémata}, postulates, or things the author must beg you to admit, because he is unable to prove them. At any rate, it is now agreed that there is no reason whatever to think the sum of the three angles of a triangle precisely equal to 180 degrees. It is generally admitted that the evidence is that the departure from 180 degrees (if there is any) will be greater the larger the triangle, and in the case of a triangle having for its base the diameter of the earth's orbit and for its apex the furthest star, the sum hardly can differ, according to observation, so much as 0.1". It is probable the discrepancy is far less. Nevertheless, there is an infinite number of different possible values, of which precisely 180 degrees is only one; so that the probability is as 1 to ∞ or 0 to 1, that the value is just 180 degrees. In other words, it seems for the present impossible to suppose the postulates of geometry precisely true. The matter is reduced to one of evidence; and as absolute precision [is] beyond the reach of direct observation, so it can never be rendered probable by evidence, which is indirect observation.

Peirce: CP 1.131 Cross-Ref: ††

131. Thus, the postulates of geometry must go into the number of things approximately true. It may be thousands of years before men find out whether the sum of the three angles of a triangle is greater or less than 180 degrees; but the presumption is, it is one or the other.

Peirce: CP 1.132 Cross-Ref: ††

132. Now what is metaphysics, which has always formed itself after the model of mathematics, to say to this state of things? The mathematical axioms being

discredited, are the metaphysical ones to remain unquestioned? I trow not. There is one proposition, now held to be very certain, though denied throughout antiquity, namely that every event is precisely determined by general laws, which evidently never can be rendered probable by observation, and which, if admitted, must, therefore, stand as self-evident. This is a metaphysical postulate closely analogous to the postulates of geometry. Its fate is sealed. The geometrical axioms being exploded, this is for the future untenable. Whenever we attempt to verify a physical law, we find discrepancies between observation and theory, which we rightly set down as errors of observation. But now it appears we have no reason to deny that there are similar, though no doubt far smaller, discrepancies between the law and the real facts. As Lucretius says,†1 the atoms swerve from the paths to which the laws of mechanics would confine them. I do not now inquire whether there is or not any positive evidence that this is so. What I am at present urging is that this arbitrariness is a conception occurring in logic, encouraged by mathematics, and ought to be regarded as a possible material to be used in the construction of a philosophical theory, should we find that it would suit the facts. We observe that phenomena approach very closely to satisfying general laws; but we have not the smallest reason for supposing that they satisfy them precisely.

Peirce: CP 1.133 Cross-Ref:††

§3. THE OBSERVATIONAL PART OF PHILOSOPHY †2

133. Every science has a mathematical part, a branch of work that the mathematician is called in to do. We say, "Here, mathematician, suppose such and such to be the case. Never you mind whether it is really so or not; but tell us, supposing it to be so, what will be the consequence." Thus arise mathematical psychology, mathematical stylometry, mathematical economics, mathematical physics, mathematical chemistry, mathematical meteorology, mathematical biology, mathematical geology, mathematical astronomy, etc., etc., etc. But there is none of these mathematical offices which constitutes quite so large a proportion of the whole science to which it is annexed as mathematical philosophy, for the obvious reason that the observational part of philosophy is a simple business, compared, for example, with that of anatomy or biography, or any other special science.

Peirce: CP 1.134 Cross-Ref:††

134. To assume, however, that the observational part of philosophy, because it is not particularly laborious, is therefore easy, is a dreadful mistake, into which the student is very apt to fall, and which gives the death-blow to any possibility of his success in this study. It is, on the contrary, extremely difficult to bring our attention to elements of experience which are continually present. For we have nothing in experience with which to contrast them; and without contrast, they cannot excite our attention. We can only contrast them with imaginary states of things; but even what we imagine is but a crazy-quilt of bits snipped off from actual experiences. The result is that roundabout devices have to be resorted to, in order to enable us to perceive what stares us in the face with a glare that, once noticed, becomes almost oppressive with its insistency. This circumstance alone would be sufficient to render philosophical observation difficult -- much more difficult, for example, than the kind of observation which the painter has to exercise. Yet this is the least of the difficulties of philosophy. Of the various hindrances more serious still, I may mention once more

the notion that it is an extremely easy thing to perceive what is before us every day and hour. But quite the worst is, that every man becomes more or less imbued with philosophical opinions, without being clearly aware of it. Some of these, it is true, may be right opinions; if he is a quite uneducated man, they doubtless will be so. But even if they are right, or nearly right, they prevent true observation as much as a pair of blue spectacles will prevent a man from observing the blue of the sky. The man will hold the right opinion, but not knowing that it might be founded upon direct observation, he will class it among articles of faith of a pretty dubious character. The more a man is educated in other branches, but not trained in philosophy, the more certain it is that two-thirds of his stock of half-conscious philosophical opinions will be utterly wrong, and will completely blind him to the truth, which he will gradually become unable so much as to conceive. I remember a really eminent French *savant*, who had sojourned for very many months in America, but who must have imbibed in his childhood the notion, then common in France, that Englishmen and Americans interject into every second sentence a certain word which the French imagine to be English. He belonged to one of the most observant of races; he was naturally a keen observer; and he was trained in an observational science; and yet, in order to assimilate himself as much as possible to American ways, he used to think it necessary to greet one every morning with a "How do you do, goddam?" and to keep it up all day. He actually believed that he had observed that such was the American style. The educated man who is a beginner in philosophy is just like that man, who (be it remembered) had been moving about in America for years; -- and by a beginner in philosophy I wish to be understood as meaning, in the case of an educated man, one who has not been seriously, earnestly, and single-mindedly devoted to the study of it for more than six or eight years. For there is no other science for which the preparatory training requires to be nearly so severe and so long, no matter how great the natural genius of the student may be. For a plain man or a boy who should be early taken in hand by an instructor capable of making him comprehend both sides of every question, the time, without doubt, can be greatly reduced, with untiring industry and energy on the pupil's part.

Peirce: CP 1.135 Cross-Ref:††
§4. THE FIRST RULE OF REASON †1

135. Upon this first, and in one sense this sole, rule of reason, that in order to learn you must desire to learn, and in so desiring not be satisfied with what you already incline to think, there follows one corollary which itself deserves to be inscribed upon every wall of the city of philosophy:

Do not block the way of inquiry.

Peirce: CP 1.136 Cross-Ref:††

136. Although it is better to be methodical in our investigations, and to consider the economics of research, yet there is no positive sin against logic in *trying* any theory which may come into our heads, so long as it is adopted in such a sense as to permit the investigation to go on unimpeded and undiscouraged. On the other hand, to set up a philosophy which barricades the road of further advance toward the truth is the one unpardonable offence in reasoning, as it is also the one to which

metaphysicians have in all ages shown themselves the most addicted.

Peirce: CP 1.136 Cross-Ref:††

Let me call your attention to four familiar shapes in which this venomous error assails our knowledge:

Peirce: CP 1.137 Cross-Ref:††

137. The first is the shape of absolute assertion. That we can be sure of nothing in science is an ancient truth. The Academy taught it. Yet science has been infested with overconfident assertion, especially on the part of the third-rate and fourth-rate men, who have been more concerned with teaching than with learning, at all times. No doubt some of the geometries still teach as a self-evident truth the proposition that if two straight lines in one plane meet a third straight line so as to make the sum of the internal angles on one side less than two right angles those two lines will meet on that side if sufficiently prolonged. Euclid, whose logic was more careful, only reckoned this proposition as a *Postulate*, or arbitrary Hypothesis. Yet even he places among his axioms the proposition that a part is less than its whole, and falls into several conflicts with our most modern geometry in consequence. But why need we stop to consider cases where some subtilty of thought is required to see that the assertion is not warranted when every book which applies philosophy to the conduct of life lays down as positive certainty propositions which it is quite as easy to doubt as to believe?

Peirce: CP 1.138 Cross-Ref:††

138. The second bar which philosophers often set up across the roadway of inquiry lies in maintaining that this, that, and the other never can be known. When Auguste Comte was pressed to specify any matter of positive fact to the knowledge of which no man could by any possibility attain, he instanced the knowledge of the chemical composition of the fixed stars; and you may see his answer set down in the *Philosophie positive*.†1 But the ink was scarcely dry upon the printed page before the spectroscope was discovered and that which he had deemed absolutely unknowable was well on the way of getting ascertained. It is easy enough to mention a question the answer to which is not known to me today. But to aver that that answer will not be known tomorrow is somewhat risky; for oftentimes it is precisely the least expected truth which is turned up under the ploughshare of research. And when it comes to positive assertion that the truth never will be found out, that, in the light of the history of our time, seems to me more hazardous than the venture of Andrée.†2

Peirce: CP 1.139 Cross-Ref:††

139. The third philosophical stratagem for cutting off inquiry consists in maintaining that this, that, or the other element of science is basic, ultimate, independent of aught else, and utterly inexplicable -- not so much from any defect in our knowing as because there is nothing beneath it to know. The only type of reasoning by which such a conclusion could possibly be reached is *retroduction*. Now nothing justifies a retroductive inference except its affording an explanation of the facts. It is, however, no explanation at all of a fact to pronounce it *inexplicable*. That, therefore, is a conclusion which no reasoning can ever justify or excuse.

Peirce: CP 1.140 Cross-Ref:††

140. The last philosophical obstacle to the advance of knowledge which I intend to mention is the holding that this or that law or truth has found its last and perfect formulation -- and especially that the ordinary and usual course of nature never can be broken through. "Stones do not fall from heaven," said Laplace,

although they had been falling upon inhabited ground every day from the earliest times. But there is no kind of inference which can lend the slightest probability to any such absolute denial of an unusual phenomenon.

Peirce: CP 1.141 Cross-Ref:††
§5. FALLIBILISM, CONTINUITY, AND EVOLUTION †1

141. All positive reasoning is of the nature of judging the proportion of something in a whole collection by the proportion found in a sample. Accordingly, there are three things to which we can never hope to attain by reasoning, namely, absolute certainty, absolute exactitude, absolute universality. We cannot be absolutely certain that our conclusions are even approximately true; for the sample may be utterly unlike the unsampled part of the collection. We cannot pretend to be even probably exact; because the sample consists of but a finite number of instances and only admits special values of the proportion sought. Finally, even if we could ascertain with absolute certainty and exactness that the ratio of sinful men to all men was as 1 to 1; still among the infinite generations of men there would be room for any finite number of sinless men without violating the proportion. The case is the same with a seven legged calf.

Peirce: CP 1.142 Cross-Ref:††

142. Now if exactitude, certitude, and universality are not to be attained by reasoning, there is certainly no other means by which they can be reached.

Peirce: CP 1.143 Cross-Ref:††

143. Somebody will suggest *revelation*. There are scientists and people influenced by science who laugh at revelation; and certainly science has taught us to look at testimony in such a light that the whole theological doctrine of the "Evidences" seems pretty weak. However, I do not think it is philosophical to reject the possibility of a revelation. Still, granting that, I declare as a logician that revealed truths -- that is, truths which have nothing in their favor but revelations made to a few individuals -- constitute by far the most uncertain class of truths there are. There is here no question of universality; for revelation is itself sporadic and miraculous. There is no question of mathematical exactitude; for no revelation makes any pretension to that character. But it does pretend to be *certain*; and against that there are three conclusive objections. First, we never can be absolutely certain that any given deliverance really is inspired; for that can only be established by reasoning. We cannot even prove it with any very high degree of probability. Second, even if it is inspired, we cannot be sure, or nearly sure, that the statement is true. We know that one of the commandments was in one of the Bibles printed with[out] a *not* in it.†1 All inspired matter has been subject to human distortion or coloring. Besides we cannot penetrate the counsels of the most High, or lay down anything as a principle that would govern his conduct. We do not know his inscrutable purposes, nor can we comprehend his plans. We cannot tell but he might see fit to inspire his servants with errors. In the third place, a truth which rests on the authority of inspiration only is of a somewhat incomprehensible nature; and we never can be sure that we rightly comprehend it. As there is no way of evading these difficulties, I say that revelation, far from affording us any certainty, gives results less certain than other sources of information. This would be so even if revelation were much plainer than it is.

Peirce: CP 1.144 Cross-Ref:††

144. But, it will be said, you forget the laws which are known to us *a priori*, the axioms of geometry, the principles of logic, the maxims of *causality*, and the like. Those are absolutely certain, without exception and exact. To this I reply that it seems to me there is the most positive historic proof that innate truths are particularly uncertain and mixed up with error, and therefore *a fortiori* not without exception. This historical proof is, of course, not infallible; but it is very strong. Therefore, I ask *how do you know* that *a priori* truth is certain, exceptionless, and exact? You cannot know it by *reasoning*. For that would be subject to uncertainty and inexactitude. Then, it must amount to this that you know it *a priori*; that is, you take *a priori* judgments at their own valuation, without criticism or credentials. That is barring the gate of inquiry.

Peirce: CP 1.145 Cross-Ref:††

145. Ah! but it will be said, you forget direct experience. Direct experience is neither certain nor uncertain, because it affirms nothing -- it just *is*. There are delusions, hallucinations, dreams. But there is no mistake that such things really do appear, and direct experience means simply the appearance. It involves no error, because it testifies to nothing but its own appearance. For the same reason, it affords no certainty. It is not *exact*, because it leaves much vague; though it is not *inexact* either; that is, it has no false exactitude.

Peirce: CP 1.146 Cross-Ref:††

146. All this is true of direct experience at its first presentation. But when it comes up to be criticized it is past, itself, and is represented by *memory*. Now the deceptions and inexactitude of memory are proverbial.

Peirce: CP 1.147 Cross-Ref:††

147. . . . On the whole, then, we cannot in any way reach perfect certitude nor exactitude. We never can be absolutely sure of anything, nor can we with any probability ascertain the exact value of any measure or general ratio.

Peirce: CP 1.147 Cross-Ref:††

This is my conclusion, after many years study of the logic of science; and it is the conclusion which others, of very different cast of mind, have come to, likewise. I believe I may say there is no tenable opinion regarding human knowledge which does not legitimately lead to this corollary. Certainly there is nothing new in it; and many of the greatest minds of all time have held it for true.

Peirce: CP 1.148 Cross-Ref:††

148. Indeed, most everybody will admit it until he begins to see what is involved in the admission -- and then most people will draw back. It will not be admitted by persons utterly incapable of philosophical reflection. It will not be fully admitted by masterful minds developed exclusively in the direction of action and accustomed to claim practical infallibility in matters of business. These men will admit the incurable fallibility of all opinions readily enough; only, they will always make exception of their own. The doctrine of fallibilism will also be denied by those who fear its consequences for science, for religion, and for morality. But I will take leave to say to these highly conservative gentlemen that however competent they may be to direct the affairs of a church or other corporation, they had better not try to manage science in that way. Conservatism -- in the sense of a dread of consequences -- is altogether out of place in science -- which has on the contrary always been

forwarded by radicals and radicalism, in the sense of the eagerness to carry consequences to their extremes. Not the radicalism that is cocksure, however, but the *radicalism that tries experiments*. Indeed, it is precisely among men animated by the spirit of science that the doctrine of fallibilism will find supporters.

Peirce: CP 1.149 Cross-Ref:††

149. Still, even such a man as that may well ask whether I propose to say that it is not quite certain that twice two are four -- and that it is even not probably quite exact! But it would be quite misunderstanding the doctrine of fallibilism to suppose that it means that twice two is probably not exactly four. As I have already remarked, it is not my purpose to doubt that people can usually *count* with accuracy. Nor does fallibilism say that men cannot attain a sure knowledge of the creations of their own minds. It neither affirms nor denies that. It only says that people cannot attain absolute certainty concerning questions of fact. Numbers are merely a system of names devised by men for the purpose of counting.†1 It is a matter of real fact to say that in a certain room there are two persons. It is a matter of fact to say that each person has two eyes. It is a matter of fact to say that there are four eyes in the room. But to say that *if* there are two persons and each person has two eyes there *will be* four eyes is not a statement of fact, but a statement about the system of numbers which is our own creation.

Peirce: CP 1.150 Cross-Ref:††

150. Still, if the matter is pressed, let me ask whether any individual here present thinks there is no room for possible doubt that twice two is four?

Peirce: CP 1.150 Cross-Ref:††

What do *you* think? You have heard of hypnotism. You know how common it is. You know that about one man in *twenty* is capable of being put into a condition in which he holds the most ridiculous nonsense for unquestionable truth. How does any individual here know but that I am a hypnotist and that when he comes out of my influence he may see that twice two is four is merely his distorted idea; that in fact everybody knows it isn't so? Suppose the individual I am addressing to be enormously wealthy. Then I ask: "Would you, in view of this possibility -- or with the possibility that you are seized with a temporary insanity, risk your entire fortune this minute against one cent, on the truth of twice two being four?" You certainly ought not to do so; for you could not go on making very many millions of such bets before you would *lose!* Why, according to my estimate of probabilities there is not a single truth of science upon which we ought to bet more than about a million of millions to one -- and that truth will be a general one and not a special fact. People say "Such a thing is as certain as that the sun will rise tomorrow!" I like that phrase for its great moderation because it is infinitely far from certain that the sun will rise tomorrow.

Peirce: CP 1.151 Cross-Ref:††

151. To return to our friends the Conservatives; these ladies and gentlemen will tell me this doctrine of fallibilism can never be admitted because the consequences from it would undermine Religion. I can only say I am very sorry. The doctrine is true; -- without claiming absolute certainty for it, it is *substantially* unassailable. And if its consequences are antagonistic to religion, so much the worse for religion. At the same time, I do not believe they are so antagonistic. The dogmas of a church may be infallible -- infallible in the sense in which it is infallibly true that it is wrong to murder and steal -- practically and substantially infallible. But what use a church could make of a mathematical infallibility, I fail to see. *Messieurs et*

mesdames les conservateurs have generally taken the lead in determining what the church should say to the novelties of science; and I don't think they have managed the business with very distinguished success so far. They have begun by recoiling with horror from the alleged heresies -- about the rotundity of the earth, about its rotation, about geology, about Egyptian history, and so forth -- and they have ended by declaring that the church never breathed a single word against any of these truths of science. Perhaps, it be just so with fallibility. For the present those knowing in divine things insist that infallibility is the prerogative of the church, but maybe bye and bye we shall be told that this infallibility had always been taken in an *ecclesiastical sense*. And that will be *true*, too. I should not wonder if the churches were to be quite agile in reformed teachings during the coming thirty years. Even one that mainly gathers in the very ignorant and the very rich may feel young blood in its veins.

Peirce: CP 1.152 Cross-Ref:††

152. But doubtless many of you will say, as many most intelligent people have said, Oh, we grant your *fallibilism* to the extent you insist upon it. It is nothing new. Franklin said a century ago that nothing was certain. We will grant it would be foolish to bet ten years' expenditure of the United States Government against one cent upon any fact whatever. But practically speaking many things are substantially certain. So, after all, of what importance is your *fallibilism*?

Peirce: CP 1.152 Cross-Ref:††

We come then to this question: of what importance is it? Let us see.

Peirce: CP 1.153 Cross-Ref:††

153. How *can* such a little thing be of importance, you will ask? I answer: after all there is a difference between something and nothing. If a metaphysical theory has come into general vogue, which can rest on nothing in the world but the assumption that absolute exactitude and certitude are to be attained, and if that metaphysics leaves us unprovided with pigeonholes in which to file important facts so that they have to be thrown in the fire -- or to resume our previous figure if that metaphysical theory seriously blocks the road of inquiry -- then it is comprehensible that the little difference between a degree of evidence extremely high and absolute certainty should after all be of great importance as removing a mote from our eye.

Peirce: CP 1.154 Cross-Ref:††

154. Let us look then at two or three of the grandest results of science and see whether they appear any different from a fallibilist standpoint from what they would to an infallibilist.

Peirce: CP 1.154 Cross-Ref:††

Three of the leading conceptions of science may be glanced at -- I mean the ideas of force, of continuity, and of evolution.

Peirce: CP 1.155 Cross-Ref:††

155. . . . The fourth law of motion was developed about forty years ago †1 by Helmholtz and others. It is called the law of the conservation of energy; but in my opinion that is a very misleading name, implying a peculiar aspect of the law under which the real fact at the bottom of it is not clearly brought out. It is therefore not suitable for an abstract and general statement, although it is a point of view which is very serviceable for many practical applications. But the law generally stated is that the changes in the velocities of particles depend exclusively on their relative positions.

Peirce: CP 1.155 Cross-Ref:††

It is not necessary now to examine these laws with technical accuracy. It is sufficient to notice that they leave the poor little particle no option at all. Under given circumstances his motion is precisely laid out for him.

Peirce: CP 1.155 Cross-Ref:††

We can from the nature of things have no evidence at all tending to show that these laws are absolutely exact. But in some single cases we can see that the approximation to exactitude is quite wonderful.

Peirce: CP 1.155 Cross-Ref:††

These laws have had a very wonderful effect upon physical sciences, because they have shown the very high degree of exactitude with which nature acts -- at least, in simple configurations. But, as I said before, the logic of the case affords us not one scintilla of reason to think that this exactitude is perfect.

Peirce: CP 1.156 Cross-Ref:††

156. The illustrious Phoenix [G. H. Derby], you remember, wrote a series of lectures on astronomy to be delivered at the Lowell Institute in Boston.†2 But owing to the unexpected circumstance of his not being invited to give any lectures at that Institution, they were ultimately published in *The San Diego Herald*. In those lectures in treating of the sun he mentions how it once stood still at the command of Joshua. But, says he, I never could help thinking that it might have wiggled a very little when Joshua was not looking directly at it. The question is whether particles may not spontaneously swerve by a very little -- less than we can perceive -- from the exact requirements of the laws of mechanics. We cannot possibly have a right to deny this. For such a denial would be a claim to absolute exactitude of knowledge. On the other hand, we never can have any right to suppose that any observed phenomenon is simply a sporadic spontaneous irregularity. For the only justification we can have for supposing anything we don't see is that it would explain how an observed fact could result from the ordinary course of things. Now to suppose a thing sporadic, spontaneous, irregular, is to suppose it departs from the ordinary course of things. That is blocking the road of inquiry; it is supposing the thing inexplicable, when a supposition can only be justified by its affording an explanation.

Peirce: CP 1.157 Cross-Ref:††

157. But we may find a general class of phenomena, forming a part of the general course of things, which are explicable not as *an irregularity*, but as the resultant effect of a whole class of irregularities.

Peirce: CP 1.157 Cross-Ref:††

Physicists often resort to this kind of explanation to account for phenomena which appear to violate the law of the conservation of energy. The general properties of gases are explained by supposing the molecules are moving about in every direction in the most diverse possible ways. Here, it is true, it is supposed that there is only so much irregularity as the laws of mechanics permit -- but the principle is there of explaining a general phenomenon by the statistical regularities that exist among irregularities.

Peirce: CP 1.158 Cross-Ref:††

158. As there is nothing to show that there is not a certain amount of absolute spontaneity in nature, despite all laws, our metaphysical pigeon-holes should not be so limited as to exclude this hypothesis, provided any general phenomena should

appear which might be explained by such spontaneity.

Peirce: CP 1.159 Cross-Ref:††

159. Now in my opinion there are several such general phenomena. Of these I will at this moment instance but one.

Peirce: CP 1.159 Cross-Ref:††

It is *the* most obtrusive character of nature. It is so obvious, that you will hardly know at first what it is I mean. It is curious how certain facts escape us because they are so pervading and ubiquitous; just as the ancients imagined the music of the spheres was not heard because it was heard all the time. But will not somebody kindly tell the rest of the audience what is the most marked and obtrusive character of nature? Of course, I mean the variety of nature.

Peirce: CP 1.160 Cross-Ref:††

160. Now I don't know that it is logically accurate to say that this marvellous and infinite diversity and manifoldness of things is a sign of spontaneity. I am a logical analyst by long training, you know, and to say this is a manifestation of spontaneity seems to me faulty analysis. I would rather say it is spontaneity. I don't know what you can make out of the meaning of spontaneity but newness, freshness, and diversity.

Peirce: CP 1.161 Cross-Ref:††

161. Let me ask you a little question? Can the operation of *law* create diversity where there was no diversity before? Obviously not; under given circumstances mechanical law prescribes *one* determinate result.

Peirce: CP 1.161 Cross-Ref:††

I could easily prove this by the principles of analytical mechanics. But that is needless. You can see for yourselves that law prescribes like results under like circumstances. That is what the word *law* implies. So then, all this exuberant diversity of nature cannot be the result of law. Now what is spontaneity? It is the character of not resulting by law from something antecedent.

Peirce: CP 1.162 Cross-Ref:††

162. Thus, the universe is *not* a mere mechanical result of the operation of blind law.†1 The most obvious of all its characters cannot be so explained. It is the multitudinous facts of all experience that show us this; but that which has opened our eyes to these facts is the principle of fallibilism. Those who fail to appreciate the importance of fallibilism reason: we see these laws of mechanics; we see how extremely closely they have been verified in some cases. We suppose that what we haven't examined is like what we have examined, and that these laws are absolute, and the whole universe is a boundless machine working by the blind laws of mechanics. This is a philosophy which leaves no room for a God! No, indeed! It leaves even human consciousness, which cannot well be denied to exist, as a perfectly idle and functionless *flâneur* in the world, with no possible influence upon anything -- not even upon itself. Now will you tell me that this fallibilism amounts to nothing?

Peirce: CP 1.163 Cross-Ref:††

163. But in order really to see all there is in the doctrine of fallibilism, it is necessary to introduce the idea of continuity, or unbrokenness. This is the leading idea of the differential calculus and of all the useful branches of mathematics; it plays a great part in all scientific thought, and the greater the more scientific that thought is; and it is the master key which adepts tell us unlocks the arcana of philosophy.

Peirce: CP 1.164 Cross-Ref:††

164. We all have some idea of continuity. Continuity is fluidity, the merging of part into part. But to achieve a really distinct and adequate conception of it is a difficult task, which with all the aids possible must for the most acute and most logically trained intellect require days of severe thought. If I were to attempt to give you any logical conception of it, I should only make you dizzy to no purpose. I may say this, however. I draw a line. Now the points on that line form a continuous series. If I take any two points on that line, however close together, other points there are lying between them. If that were not so, the series of points would not be continuous. It might be so, even if the series of points were not continuous. . . .

Peirce: CP 1.165 Cross-Ref:††

165. You will readily see that the idea of continuity involves the idea of infinity. Now, the nominalists tell us that we cannot reason about infinity, or that we cannot reason about it *mathematically*. Nothing can be more false. Nominalists cannot reason about infinity, because they do not reason logically about anything. Their reasoning consists of performing certain processes which they have found worked well -- without having any insight into the conditions of their working well. This is not logical reasoning. It naturally fails when infinity is involved; because they reason about infinity as if it were finite. But to a logical reasoner, reasoning about infinity is decidedly simpler than reasoning about finite quantity.

Peirce: CP 1.166 Cross-Ref:††

166. There is one property of a continuous expanse that I must mention, though I cannot venture to trouble you with the demonstration of it. It is that in a continuous expanse, say a continuous line, there are continuous lines infinitely short. In fact, the whole line is made up of such infinitesimal parts. The property of these infinitely small spaces is -- I regret the abstruseness of what I am going to say, but I cannot help it -- the property which distinguishes these infinitesimal distances is that a certain mode of reasoning which holds good of all finite quantities and of some that are not finite does not hold good of them. Namely, mark any point on the line A. Suppose that point to have any character; suppose, for instance, it is *blue*. Now suppose we lay down the rule that every point within an inch of a blue point shall be painted blue. Obviously, the consequence will be that the whole line will have to be blue. But this reasoning does not hold good of infinitesimal distances. After the point A has been painted blue, the rule that every point infinitesimally near to a blue point shall be painted blue will not necessarily result in making the whole blue. Continuity involves infinity in the strictest sense, and infinity even in a less strict sense goes beyond the possibility of direct experience.

Peirce: CP 1.167 Cross-Ref:††

167. Can we, then, ever be sure that anything in the real world is continuous? Of course, I am not asking for an absolute certainty; but can we ever say that it is so with any ordinary degree of security? This is a vitally important question. I think that we have one positive direct evidence of continuity and on the first line but one. It is this. We are immediately aware only of our present feelings -- not of the future, nor of the past. The past is known to us by present memory, the future by present suggestion. But before we can interpret the memory or the suggestion, they are past; before we can interpret the present feeling which means memory, or the present feeling that means suggestion, since that interpretation takes time, that feeling has ceased to be present and is now past. So we can reach no conclusion from the present but only from the past.

Peirce: CP 1.168 Cross-Ref:††

168. How do we know then on the whole that the past ever existed, that the future ever will exist? How do we know there ever was or ever will be anything but the present instant? Or stop: I must not say *we*. How do I know that anybody but myself ever existed or even I myself exist except for one single instant, the present, and that all this business is not an illusion from top to bottom? Answer: I don't know. But I am trying the hypothesis that it is real, which seems to work excellently so far. Now if this is real, the past is really known to the present. How can it be known? Not by inference; because as we have just seen we can make no inference from the present, since it will be past before the inference gets drawn.

Peirce: CP 1.169 Cross-Ref:††

169. Then we must have an immediate consciousness of the past. But if we have an immediate consciousness of a state of consciousness past by one unit of time and if that past state involved an immediate consciousness of a state then past by one unit, we now have an immediate consciousness of a state past by two units; and as this is equally true of all states, we have an immediate consciousness of a state past by four units, by eight units, by sixteen units, etc.; in short we must have an immediate consciousness of every state of mind that is past by any finite number of units of time. But we certainly have not an immediate consciousness of our state of mind a year ago. So a year is more than any finite number of units of time in this system of measurement; or, in other words, there is a measure of time infinitely less than a year. Now, this is only true if the series be continuous. Here, then, it seems to me, we have positive and tremendously strong reason for believing that time really is continuous.

Peirce: CP 1.170 Cross-Ref:††

170. Equally conclusive and direct reason for thinking that space and degrees of quality and other things are continuous is to be found as for believing time to be so. Yet, the reality of continuity once admitted, reasons are there, divers reasons, some positive, others only formal, yet not contemptible, for admitting the continuity of all things. I am making a bore of myself and won't bother you with any full statement of these reasons, but will just indicate the nature of a few of them. Among formal reasons, there are such as these, that it is easier to reason about continuity than about discontinuity, so that it is a convenient assumption. Also, in case of ignorance it is best to adopt the hypothesis which leaves open the greatest field of possibility; now a continuum is merely a discontinuous series with additional possibilities. Among positive reasons, we have that apparent analogy between time and space, between time and degree, and so on. There are various other positive reasons, but the weightiest consideration appears to me to be this: How can one mind act upon another mind? How can one particle of matter act upon another at a distance from it? The nominalists tell us this is an ultimate fact -- it cannot be explained. Now, if this were meant in [a] merely practical sense, if it were only meant that we know that one thing does act on another but that how it takes place we cannot very well tell, up to date, I should have nothing to say, except to applaud the moderation and good logic of the statement. But this is not what is meant; what is meant is that we come up, bump against actions absolutely unintelligible and inexplicable, where human inquiries have to stop. Now that is a mere *theory*, and nothing can justify a theory except its explaining observed facts. It is a poor kind of theory which in place of performing this, the sole legitimate function of a theory, merely supposes the facts to be inexplicable. It is one of the peculiarities of nominalism that it is continually supposing things to be absolutely inexplicable. That blocks the road of inquiry. But if

we adopt the theory of continuity we escape this illogical situation. We may then say that one portion of mind acts upon another, because it is in a measure immediately present to that other; just as we suppose that the infinitesimally past is in a measure present. And in like manner we may suppose that one portion of matter acts upon another because it is in a measure in the same place.

Peirce: CP 1.171 Cross-Ref:††

171. If I were to attempt to describe to you in full all the scientific beauty and truth that I find in the principle of continuity, I might say in the simple language of Matilda the Engaged, "the tomb would close over me e'er the entrancing topic were exhausted" -- but not before my audience was exhausted. So I will just drop it here. Only, in doing so, let me call your attention to the natural affinity of this principle to the doctrine of fallibilism. The principle of continuity is the idea of fallibilism objectified. For fallibilism is the doctrine that our knowledge is never absolute but always swims, as it were, in a continuum of uncertainty and of indeterminacy. Now the doctrine of continuity is that *all things* so swim in continua.

Peirce: CP 1.172 Cross-Ref:††

172. The doctrine of continuity rests upon observed fact as we have seen. But what opens our eyes to the significance of that fact is fallibilism. The ordinary scientific infallibilist -- of which sect Büchner in his *Kraft und Stoff* affords a fine example -- cannot accept *synechism*, or the doctrine that all that exists is continuous -- because he is committed to discontinuity in regard to all those things which he fancies he has exactly ascertained, and especially in regard to that part of his knowledge which he fancies he has exactly ascertained to be *certain*. For where there is continuity, the exact ascertainment of real quantities is too obviously impossible. No sane man can dream that the ratio of the circumference to the diameter could be exactly ascertained by measurement. As to the quantities he has not yet exactly ascertained, the Büchnerite is naturally led to separate them into two distinct classes, those which may be ascertained hereafter (and there, as before, continuity must be excluded), and those absolutely unascertainable -- and these in their utter and everlasting severance from the other class present a new breach of continuity. Thus scientific infallibilism draws down a veil before the eyes which prevents the evidences of continuity from being discerned.

Peirce: CP 1.172 Cross-Ref:††

But as soon as a man is fully impressed with the fact that absolute exactitude never can be known, he naturally asks whether there are any facts to show that hard discrete exactitude really exists. That suggestion lifts the edge of that curtain and he begins to see the clear daylight shining in from behind it.

Peirce: CP 1.173 Cross-Ref:††

173. But fallibilism cannot be appreciated in anything like its true significance until evolution has been considered. This is what the world has been most thinking of for the last forty years -- though old enough is the general idea itself. Aristotle's philosophy, that dominated the world for so many ages and still in great measure tyrannizes over the thoughts of butchers and bakers that never heard of him -- is but a metaphysical evolutionism.

Peirce: CP 1.174 Cross-Ref:††

174. Evolution means nothing but *growth* in the widest sense of that word. Reproduction, of course, is merely one of the incidents of growth. And what is growth? Not mere increase. Spencer says it is the passage from the homogeneous to

the heterogeneous -- or, if we prefer English to Spenserese -- *diversification*. That is certainly an important factor of it. Spencer further says that it is a passage from the unorganized to the organized; but that part of the definition is so obscure that I will leave it aside for the present. But think what an astonishing idea this of *diversification* is! Is there such thing in nature as increase of variety? Were things simpler, was variety less in the original nebula from which the solar system is supposed to have grown than it is now when the land and sea swarms with animal and vegetable forms with their intricate anatomies and still more wonderful economies? It would seem as if there were an increase in variety, would it not? And yet mechanical law, which the scientific infallibilist tells us is the only agency of nature, mechanical law can never produce diversification. That is a mathematical truth -- a proposition of analytical mechanics; and anybody can see without any algebraical apparatus that mechanical law out of like antecedents can only produce like consequents. It is the very idea of law. So if observed facts point to real growth, they point to another agency, to spontaneity for which infallibilism provides no pigeon-hole. And what is meant by this passage from the less organized to the more organized? Does it mean a passage from the less bound together to the more bound together, the less connected to the more connected, the less regular to the more regular? How can the regularity of the world increase, if it has been absolutely perfect all the time?

Peirce: CP 1.175 Cross-Ref:††

175. . . . Once you have embraced the principle of continuity no kind of explanation of things will satisfy you except that they grew. The infallibilist naturally thinks that everything always was substantially as it is now. Laws at any rate being absolute could not grow. They either always were, or they sprang instantaneously into being by a sudden fiat like the drill of a company of soldiers. This makes the laws of nature absolutely blind and inexplicable. Their why and wherefore can't be asked. This absolutely blocks the road of inquiry. The fallibilist won't do this. He asks may these *forces* of nature not be somehow amenable to reason? May they not have naturally grown up? After all, there is no reason to think they are absolute. If all things are continuous, the universe must be undergoing a continuous growth from non-existence to existence. There is no difficulty in conceiving existence as a matter of degree. The reality of things consists in their persistent forcing themselves upon our recognition. If a thing has no such persistence, it is a mere dream. Reality, then, is persistence, is regularity. In the original chaos, where there was no regularity, there was no existence. It was all a confused dream. This we may suppose was in the infinitely distant past. But as things are getting more regular, more persistent, they are getting less dreamy and more real.

Peirce: CP 1.175 Cross-Ref:††

Fallibilism will at least provide a big pigeon-hole for facts bearing on that theory.

Peirce: CP 1.176 Cross-Ref:††

BOOK II

THE CLASSIFICATION OF THE SCIENCES

PROEM: THE ARCHITECTONIC CHARACTER OF PHILOSOPHY †1

176. The universally and justly lauded parallel which Kant draws between a philosophical doctrine and a piece of architecture has excellencies which the beginner in philosophy might easily overlook; and not the least of these is its recognition of the cosmic character of philosophy. I use the word "cosmic" because *cosmicus* is Kant's own choice; but I must say I think *secular* or *public* would have approached nearer to the expression of his meaning. Works of sculpture and painting can be executed for a single patron and must be by a single artist. A painting always represents a fragment of a larger whole. It is broken at its edges. It is to be shut up in a room and admired by a few. In such a work individuality of thought and feeling is an element of beauty. But a great building, such as alone can call out the depths of the architect's soul, is meant for the whole people, and is erected by the exertions of an army representative of the whole people. It is the message with which an age is charged, and which it delivers to posterity. Consequently, thought characteristic of an individual -- the piquant, the nice, the clever -- is too little to play any but the most subordinate *rôle* in architecture. If anybody can doubt whether this be equally true of philosophy, I can but recommend to him that splendid third chapter of the Methodology, in the *Critic of the Pure Reason*.

Peirce: CP 1.177 Cross-Ref:††

177. To the cosmological or secular character of philosophy (to which, as closely connected, Kant with his unfailing discernment joins the circumstance that philosophy is a thing that has to grow by the fission of minute parts and not by accretion) is due the necessity of planning it out from the beginning. Of course, every painting likewise has its composition; but composition is not a very weighty problem, except in that kind of painting which is accessory to architecture, or is, at any rate, very public in its appeal. Indeed historical painting is one of those exceptions which go to prove the rule that in works which aim at being secular, rather than individualistic, the preliminary business of planning is particularly important and onerous.

Peirce: CP 1.178 Cross-Ref:††

178. And the reason is very plain and simple. The instincts of the lower animals answer their purposes much more unerringly than a discursive understanding could do. But for man discourse of reason is requisite, because men are so intensively individualistic and original that the instincts, which are racial ideas, become smothered in them. A deliberate logical faculty, therefore, has in man to take their place; and the sole function of this logical deliberation is to grind off the arbitrary and the individualistic character of thought. Hence, wherever the arbitrary and the individualistic is particularly prejudicial, there logical deliberation, or discourse of reason, must be allowed as much play as possible.

Peirce: CP 1.179 Cross-Ref:††

179. That is why philosophy ought to be deliberate and planned out; and that is why, though pitchforking articles into a volume is a favorite and easy method of bookmaking, it is not the one which Mr. Peirce has deemed to be the most appropriate

to the exposition of the principles of philosophy; so that, instead of making up this book by a collection of his old papers with additions, as he was urged to do, he has preferred to write it entirely anew, as if he had never before set pen to paper.†1

Peirce: CP 1.180 Cross-Ref:††
CHAPTER 1

AN OUTLINE CLASSIFICATION OF THE SCIENCES †1

180. This classification, which aims to base itself on the principal affinities of the objects classified, is concerned not with all possible sciences, nor with so many branches of knowledge, but with sciences in their present condition, as so many businesses of groups of living men. It borrows its idea from Comte's classification; namely, the idea that one science depends upon another for fundamental principles, but does not furnish such principles to that other. It turns out that in most cases the divisions are trichotomic; the First of the three members relating to universal elements or laws, the Second arranging classes of forms and seeking to bring them under universal laws, the Third going into the utmost detail, describing individual phenomena and endeavoring to explain them. But not all the divisions are of this character.

Peirce: CP 1.180 Cross-Ref:††

The classification has been carried into great detail;†2 but only its broader divisions are here given.

Peirce: CP 1.181 Cross-Ref:††

181. All science is either, A. Science of Discovery; B. Science of Review; or C. Practical Science.

Peirce: CP 1.182 Cross-Ref:††

182. By "science of review" is meant the business of those who occupy themselves with arranging the results of discovery, beginning with digests, and going on to endeavor to form a philosophy of science. Such is the nature of Humboldt's *Cosmos*, of Comte's *Philosophie positive*, and of Spencer's *Synthetic Philosophy*. The classification of the sciences belongs to this department.

Peirce: CP 1.183 Cross-Ref:††

183. Science of Discovery is either, I. Mathematics; II. Philosophy; or III. Idioscopy.†1

Peirce: CP 1.184 Cross-Ref:††

184. Mathematics studies what is and what is not logically possible, without making itself responsible for its actual existence. Philosophy is *positive science*, in the sense of discovering what really is true; but it limits itself to so much of truth as can be inferred from common experience. Idioscopy embraces all the special sciences, which are principally occupied with the accumulation of new facts.

Peirce: CP 1.185 Cross-Ref:††

185. Mathematics may be divided into *a.* the Mathematics of Logic; *b.* the Mathematics of Discrete Series; *c.* the Mathematics of Continua and Pseudo-continua.

Peirce: CP 1.185 Cross-Ref:††

I shall not carry this division further. Branch *b* has recourse to branch *a*, and branch *c* to branch *b*.

Peirce: CP 1.186 Cross-Ref:††

186. Philosophy is divided into *a.* Phenomenology; *b.* Normative Science; *c.* Metaphysics.

Peirce: CP 1.186 Cross-Ref:††

Phenomenology ascertains and studies the kinds of elements universally present in the phenomenon; meaning by the *phenomenon*, whatever is present at any time to the mind in any way. Normative science distinguishes what ought to be from what ought not to be, and makes many other divisions and arrangements subservient to its primary dualistic distinction. Metaphysics seeks to give an account of the universe of mind and matter. Normative science rests largely on phenomenology and on mathematics; metaphysics on phenomenology and on normative science.

Peirce: CP 1.187 Cross-Ref:††

187. Idioscopy has two wings: α . the Physical Sciences; and β . the Psychical, or Human Sciences.

Peirce: CP 1.187 Cross-Ref:††

Psychical science borrows principles continually from the physical sciences; the latter very little from the former.

Peirce: CP 1.188 Cross-Ref:††

188. The physical sciences are: *a.* Nomological, or General, Physics; *b.* Classificatory Physics; *c.* Descriptive Physics.

Peirce: CP 1.188 Cross-Ref:††

Nomological physics discovers the ubiquitous phenomena of the physical universe, formulates their laws, and measures their constants. It draws upon metaphysics and upon mathematics for principles. Classificatory physics describes and classifies physical forms and seeks to explain them by the laws discovered by nomological physics with which it ultimately tends to coalesce. Descriptive physics describes individual objects -- the earth and the heavens -- endeavors to explain their phenomena by the principles of nomological and classificatory physics, and tends ultimately itself to become classificatory.

Peirce: CP 1.189 Cross-Ref:††

189. The Psychical Sciences are: *a.* Nomological Psychics or Psychology; *b.* Classificatory Psychics, or Ethnology; *c.* Descriptive Psychics, or History.

Peirce: CP 1.189 Cross-Ref:††

Nomological psychics discovers the general elements and laws of mental phenomena. It is greatly influenced by phenomenology, by logic, by metaphysics, and by biology (a branch of classificatory physics). Classificatory psychics classifies products of mind and endeavors to explain them on psychological principles. At present it is far too much in its infancy (except linguistics, to which reference will be

made below) to approach very closely to psychology. It borrows from psychology and from physics. Descriptive psychics endeavors in the first place to describe individual manifestations of mind, whether they be permanent works or actions; and to that task it joins that of endeavoring to explain them on the principles of psychology and ethnology. It borrows from geography (a branch of descriptive physics), from astronomy (another branch) and from other branches of physical and psychical science.

Peirce: CP 1.189 Cross-Ref:††

I now consider the subdivisions of these sciences, so far as they are so widely separated as quite to sunder the groups of investigators who today study them.

Peirce: CP 1.190 Cross-Ref:††

190. Phenomenology is, at present, a single study.

Peirce: CP 1.191 Cross-Ref:††

191. Normative science has three widely separated divisions: i. Esthetics; ii. Ethics; iii. Logic.

Peirce: CP 1.191 Cross-Ref:††

Esthetics is the science of ideals, or of that which is objectively admirable without any ulterior reason. I am not well acquainted with this science; but it ought to repose on phenomenology. Ethics, or the science of right and wrong, must appeal to Esthetics for aid in determining the *summum bonum*. It is the theory of self-controlled, or deliberate, conduct. Logic is the theory of self-controlled, or deliberate, thought; and as such, must appeal to ethics for its principles. It also depends upon phenomenology and upon mathematics. All thought being performed by means of signs, logic may be regarded as the science of the general laws of signs. It has three branches: 1, Speculative Grammar, or the general theory of the nature and meanings of signs, whether they be icons, indices, or symbols; 2, Critic, which classifies arguments and determines the validity and degree of force of each kind; 3, Methodeutic, which studies the methods that ought to be pursued in the investigation, in the exposition, and in the application of truth. Each division depends on that which precedes it.

Peirce: CP 1.192 Cross-Ref:††

192. Metaphysics may be divided into, i, General Metaphysics, or Ontology; ii, Psychical, or Religious, Metaphysics, concerned chiefly with the questions of 1, God, 2, Freedom, 3, Immortality; and iii, Physical Metaphysics, which discusses the real nature of time, space, laws of nature, matter, etc. The second and third branches appear at present to look upon one another with supreme contempt.

Peirce: CP 1.193 Cross-Ref:††

193. Nomological physics is divided into, i, Molar Physics, Dynamics and Gravitation; ii, Molecular Physics, Elaterics and Thermodynamics; iii, Etherial Physics, Optics and Electrics. Each division has two subdivisions. The dependence of the divisions is well marked.

Peirce: CP 1.194 Cross-Ref:††

194. Classificatory physics seems, at present, as a matter of fact, to be divided, quite irrationally and most unequally, into i, Crystallography; ii, Chemistry; iii, Biology.

Peirce: CP 1.195 Cross-Ref:††

195. But crystallography is rather an offshoot from chemistry, to which it furnishes a few facts, but hardly a principle. It is highly mathematical and depends also on elaterics. Biology might be regarded (although, as a matter of fact, no such view is taken) as the chemistry of the albumoids and of the forms they assume. It is probable that all the differences of races, individuals, and tissues are chemical, at bottom. At any rate, the possible varieties of albuminoids are amply sufficient to account for all the diversity of organic forms.

Peirce: CP 1.196 Cross-Ref:††

196. Pure chemistry seems, at present, to consist of, 1, Physical Chemistry, consisting of the old chemical physics and the modern chemical dynamics; 2, Organic Chemistry, Aliphatic and Aromatic; 3, Inorganic Chemistry, consisting of the doctrine of the elements, their atomic weights, periodicity, etc., and the doctrine of compounds.

Peirce: CP 1.197 Cross-Ref:††

197. Biology is divided into, 1, Physiology; and 2, Anatomy. Physiology is closely allied to chemistry and physics. Anatomy is divided into many distinct fields, according to the nature of the forms studied.

Peirce: CP 1.198 Cross-Ref:††

198. Descriptive physics is divided into, 1, Geognosy, and, 2, Astronomy. Both have various well-known subdivisions.

Peirce: CP 1.199 Cross-Ref:††

199. Psychology is most naturally divided, according to the methods it follows, into, i, Introspectional Psychology; ii, Experimental Psychology; iii, Physiological Psychology; iv, Child Psychology.

Peirce: CP 1.199 Cross-Ref:††

This division only admits those parts of psychology which investigate the general phenomena of mind. Special psychology belongs to classificatory psychics. Both experimental and physiological psychology are dependent upon introspective psychology. But it is hard to say which of them derives most from the other. Child psychology depends on all the others. Psychology is too young a science to have any further living divisions than such as are here admitted.

Peirce: CP 1.200 Cross-Ref:††

200. Classificatory psychics is divided into, i, Special Psychology, itself consisting of, 1, Individual Psychology; 2, Psychological Heredity; 3, Abnormal Psychology; 4, Mob Psychology; 5, Race Psychology; 6, Animal Psychology; ii, Linguistics, a vast science, divided according to the families of speech, and cross-divided into, 1, Word Linguistics; 2, Grammar; and there should be a comparative science of forms of composition; iii, Ethnology, divided into, 1, the Ethnology of Social Developments, customs, laws, religion, and traditions; and, 2, the Ethnology of Technology.

Peirce: CP 1.201 Cross-Ref:††

201. Descriptive psychics is divided into, i, History proper, itself divided according to the nature of its data into, 1, Monumental History; 2, Ancient History with all other history that is drawn from few and general testimonies; 3, History drawn from a wealth of documents, as Modern History, generally. History has, beside, two cross-divisions; the one into, 1, Political History; 2, History of the Different Sciences; 3, History of Social Developments, religion, law, slavery, manners, etc.; the

other according to the different parts of the world and the different peoples whose history is studied; ii, Biography, which at present is rather a mass of lies than a science; iii, Criticism, the study of individual works of mind, itself divided into, 1, Literary Criticism; 2, Art Criticism, of which the latter is divided into many departments, as Criticism of Military Operations, Criticism of Architecture, etc.

Peirce: CP 1.202 Cross-Ref:††

202. The classification of practical sciences has been elaborated by the author, but will not here be touched upon.†1 No classification of the science of review has been attempted.

Peirce: CP 1.203 Cross-Ref:††
CHAPTER 2

A DETAILED CLASSIFICATION OF THE SCIENCES†1

§1. NATURAL CLASSES

203. Many have been the attempts at a general classification of the sciences. Dr. Richardson's little book upon the subject †P1 is quite incomplete, only enumerating one hundred and forty-six systems. They are naturally many, because not only are their purposes various, but their conceptions of a science are divergent, and their notions of what classification is are still more so. Many of these schemes introduce sciences which nobody ever heard of; so that they seem to aim at classifying, not actually existent sciences, but possible sciences. A somewhat presumptuous undertaking is that of classifying the science of the remote future. On the other hand, if classifications are to be restricted to sciences actually existing at the time the classifications are made, the classifications certainly ought to differ from age to age. If Plato's classification was satisfactory in his day, it cannot be good today; and if it be good now, the inference will be that it was bad when he proposed it.

Peirce: CP 1.203 Cross-Ref:††

This business of classifying sciences is not one to be undertaken precipitately or off-hand. That is plain. We should not begin the execution of the task until we have well considered, first, what classification is; and secondly, what science is. . . .

Peirce: CP 1.204 Cross-Ref:††

204. The first question then, that it seems well to consider (remembering that classification is one of the topics of logic to be dealt with more scientifically in its proper place, and that I can here only skim the surface of it) is, What is meant by a true and natural class? A great many logicians say there is no such thing; and, what is strange, even many students of taxonomic sciences not only follow this opinion, but allow it a great part in determining the conclusions of botany and zoölogy. The cause of their holding this opinion has two factors; first, that they attach a metaphysical signification to the term *natural* or *real class*, and secondly, that they have embraced

a system of metaphysics which allows them to believe in no such thing as that which they have defined a real or natural class to be. Far be it from me to wish to close any avenue by which truth may be arrived at; and if botanists and zoölogists come to the conclusion that botany and zoölogy must rest upon metaphysics, I have not a word of objection to make. Only I can tell them that metaphysics is a most difficult science, presenting more pitfalls for the uninformed than almost any, which a mere amateur at it would be foolish to fancy that he could escape. Therefore, if botany and zoölogy must perforce rest upon metaphysics, by all means let this metaphysics be recognized as an explicit branch of those sciences, and be treated in a thoroughgoing and scientific manner. Having devoted many years to it, I am entitled to my opinion upon a metaphysical question, although it may be a mistaken one; and my opinion is that it is a shallow and sciolistic metaphysics which declares a "real class," in the sense which those writers attach to the term, to be an impossible thing. At the same time, I am unable to see any need at all in positive science for considering such metaphysically real classes. To my apprehension the business of classification has no concern with them, but only with true and natural classes, in another and a purely experiential sense. For example, if I were to attempt to classify the arts, which I shall not do, I should have to recognize, as one of them, the art of illumination, and should have occasion to remark that lamps form a true, real, and natural class, because every lamp has been made and has come into being as a result of an aim common and peculiar to all lamps. A *class*, of course, is the total of whatever objects there may be in the universe which are of a certain description. What if we try taking the term "natural," or "real, class" to mean a class of which all the members owe their existence as members of the class to a common final cause? This is somewhat vague; but it is better to allow a term like this to remain vague, until we see our way to rational precision. In the case of lamps, we know what that cause is: that instinct which enables us to distinguish human productions and to divine their purpose informs us of this with a degree of certainty which it were futile to hope that any science should surpass. But in the case of natural classes the final cause remains occult. Perhaps, since phrases retain their sway over men's minds long after their meaning has evaporated, it may be that some reader, even at this day, remains imbued with the old notion that there are no final causes in nature; in which case, natural selection, and every form of evolution, would be false. For evolution is nothing more nor less than the working out of a definite end. A final cause may be conceived to operate without having been the purpose of any mind: that supposed phenomenon goes by the name of *fate*. The doctrine of evolution refrains from pronouncing whether forms are simply fated or whether they are providential; but that definite ends are worked out none of us today any longer deny. Our eyes have been opened; and the evidence is too overwhelming. In regard to natural objects, however, it may be said, in general, that we do not know precisely what their final causes are. But need that prevent us from ascertaining whether or not there is a common cause by virtue of which those things that have the essential characters of the class are enabled to exist?

Peirce: CP 1.205 Cross-Ref:††

205. The manner of distribution of the class-character will show, with a high degree of certainty, whether or not it is determinative of existence. Take, for example, the class of animals that have legs. The use of legs is clear to us, having them ourselves. But if we pass the animal kingdom in review, we see that in the majority of branches there are no such organs of locomotion; while in the others they are present throughout some whole classes, and absent throughout others; and in still others are sometimes present, sometimes absent. With such a distribution, this mode of locomotion may be so connected with the possibility of a form, that two animals of

the same order could not differ in respect to using legs; but it is evident that animals having legs do not form a natural group; for they are not separated from all others in any other important particular. We thus get a tolerably clear idea of what a natural class is: it will amply suffice for our present purpose; though we can hardly hope that it will turn out to be logically accurate. We also see that, when an object has been made with a purpose, as is, of course, the case with the sciences, no classes can be more fundamental nor broader than those which are defined by the purpose. A purpose is an operative desire. Now a desire is always general; that is, it is always some *kind* of thing or event which is desired; at least, until the element of will, which is always exercised upon an individual object upon an individual occasion, becomes so predominant as to overrule the generalizing character of desire. Thus, desires create classes, and extremely broad classes. But desires become, in the pursuit of them, more specific. Let us revert, for example, to lamps. We desire, in the first instance, merely economical illumination. But we remark that that may be carried out by combustion, where there is a chemical process kindling itself, or heat may be supplied from without in electric lighting, or it may be stored up, as in phosphorescence. These three ways of carrying out our main purpose constitute subsidiary purposes.†P1 So if we decide upon electric lighting, the question will be between incandescent and arc lighting. If we decide upon combustion, the burning matter may itself become incandescent, or its heat may serve to render another more suitable thing incandescent, as in the Welsbach burner. Here is a complication which will ordinarily be advantageous, since by not making the same thing fulfill the two functions of supplying heat to produce incandescence and of incandescing upon being heated, there is more freedom to choose things suitable to the two functions. This is a good example of that sort of natural class which Agassiz called an order; that is, a class created by a useful complication of a general plan.

Peirce: CP 1.206 Cross-Ref:††

206. Closely connected with the fact that every desire is general, are two other facts which must be taken into account in considering purposive classes. The first of these is that a desire is always more or less variable, or vague. For example, a man wants an economical lamp. Then if he burns oil in it, he will endeavor to burn that oil which gives him sufficient light at the lowest cost. But another man, who lives a little further from the source of supply of that oil and a little nearer the source of a different oil may find that different oil to be the better for him. So it is with the desires of one individual. The same man who prefers veal to pork as a general thing, may think that an occasional spare rib is better than having cold boiled veal every day of his life. In short, variety is the spice of life for the individual, and practically still more so for a large number of individuals; and as far as we can compare Nature's ways with ours, she seems to be even more given to variety than we. These three cases may be very different on their subjective side; but for purposes of classification they are equivalent.

Peirce: CP 1.207 Cross-Ref:††

207. But not only is desire *general* and *vague*, or indeterminate; it has besides a certain longitude, or *third* dimension. By this I mean that while a certain ideal state of things might most perfectly satisfy a desire, yet a situation somewhat differing from that will be far better than nothing; and in general, when a state is not too far from the ideal state, the nearer it approaches that state the better. Moreover, the situation of things most satisfactory to one desire is almost never the situation most satisfactory to another. A brighter lamp than that I use would perhaps be more agreeable to my eyes; but it would be less so to my pocket, to my lungs, and to my

sense of heat. Accordingly, a compromise is struck; and since all the desires concerned are somewhat vague, the result is that the objects actually will cluster about certain middling qualities, some being removed this way, some that way, and at greater and greater removes fewer and fewer objects will be so determined. Thus, clustering distributions will characterize purposive classes.

Peirce: CP 1.208 Cross-Ref:††

208. One consequence of this deserves particular notice, since it will concern us a good deal in our classification of the sciences, and yet is quite usually overlooked and assumed not to be as it is. Namely, it follows that it may be quite impossible to draw a sharp line of demarcation between two classes, although they are real and natural classes in strictest truth. Namely, this will happen when the form about which the individuals of one class cluster is not so unlike the form about which individuals of another class cluster but that variations from each middling form may precisely agree. In such a case, we may know in regard to any intermediate form what proportion of the objects of that form had one purpose and what proportion the other; but unless we have some supplementary information we cannot tell which ones had one purpose and which the other.

Peirce: CP 1.209 Cross-Ref:††

209. The reader may be disposed to suspect that this is merely a mathematician's fancy, and that no such case would be likely ever to occur. But he may be assured that such occurrences are far from being rare. In order to satisfy him that this state of things does occur, I will mention an incontestable instance of it; -- incontestable, at least, by any fair mind competent to deal with the problem. Prof. [W. M.] Flinders Petrie, whose reasoning powers I had admired long before his other great scientific qualities had been proved, among which his great exactitude and circumspection as a metrologist concerns us here, exhumed, at the ancient trading town of Naucratis, no less than 158 balance-weights having the Egyptian *ket* as their unit.†P1 The great majority of them are of basalt and syenite, material so unchangeable that the corrections needed to bring them to their original values are small. I shall deal only with 144 of them from each of which Mr. Petrie has calculated the value of the *ket* to a tenth of a Troy grain. Since these values range all the way from 137 to 152 grains, it is evident that the weights were intended to be copies of several different standards, probably four or five; for there would be no use of a balance, if one could detect the errors of the balance-weights by simply "hefting" them, and comparing them with one's memory of the standard weight. Considering that these weights are small, and were therefore used for weighing costly or even precious matter, our knowledge of the practice of weighing among the ancients gives us ground for thinking it likely that about half the weights would depart from their virtual standards by more, and about half by less, than, say, four or five tenths of one per cent, which, upon a *ket*, would be from half to two-thirds of a grain. Now the whole interval here is fourteen and one-half grains; and between 136.8 grains to 151.3 grains there is no case of an interval of more than a third of a grain not represented by any weight among the 144. To a person thoroughly familiar with the theory of errors this shows that there must be four or five different standards to which different ones aim to conform. . . . In order to represent these observations, I have adopted the following rough-and-ready theory; for to make elaborate calculations would, from every point of view, be a waste of time. I have assumed that there were five different standards; that the weights depart from their standards according to the probability curve; and that the probable error of a single weight is five-eighths of a grain. I assume that of the 144 weights

36 were designed to conform to a standard of 139.2 grs.

25 were designed to conform to a standard of 142.2 grs.

26 were designed to conform to a standard of 144.7 grs.

23 were designed to conform to a standard of 146.95 grs.

34 were designed to conform to a standard of 149.7 grs.

. . . I repeat that this theory has not been the subject of any but the simplest calculations. It is obvious that some such theory must be true; but to decide how near my theory probably comes to the true theory or how it ought to be modified, would be a very intricate problem for the solution of which the data are probably insufficient. It does not concern us here; our object being merely to make it clear that truly natural classes may, and undoubtedly often do merge into one another inextricably.

Peirce: CP 1.209 Cross-Ref:††

It is, I think, pretty certain that there were as many as five standards. Before the adoption of the metric system, every city throughout the greater part, if not all, the continent of Europe had its own pound, like its own *patois*. See the article "pound" in the *Century Dictionary*, †1 which was based on a list of some three hundred of such pounds whose values were known to me, a list now kept in manuscript in the Astor Library. That the same state of things must have been true in ancient Egypt may be inferred from the looseness of the tie which bound the different provinces of that empire together. Even their religions were different; so that *a fortiori* their *kets* would be so. Besides, none of the *kets* carry any authoritative mark; which is pretty conclusive proof that the central government did not intervene. It is, therefore, probable that the five standards were those of five towns with which Naucratis carried on trade. Yet virtual standards may be created in other ways. For example, where government does not insure uniformity in weights, it is usual for buyers to bring their own weights. It would thus naturally happen that some balance-weights would be manufactured for the use of buyers, and others for the use of sellers; and thus there would naturally be a tendency to the crystallization of a heavier and a lighter norm.

Peirce: CP 1.210 Cross-Ref:††

210. As for my assumption that the departures of the single weights from their virtual standards conform to the probability curve, it was only adopted as a ready way of imparting definiteness to the problem. Rich as is the store of data given by Petrie, it is insufficient, apparently, for determining the true law of those departures. If the workmen were sufficiently skillful (as I believe they would be) the departures would follow the probability curve. But if they were unskillful, it would be desirable to ascertain by what process the weights were made. The weights, being of stone, are not loaded; so that the adjustment was made by grinding, exclusively. Did the workman, then, have a balance by his side, or did he finish the weight by guesswork? In the latter case, inspection (and some sort of inspection there must, in this case, have been) would reject all weights outside a certain "tolerance," as it is called in coinage. Those that were too light would have to be thrown away. They would lie in a heap, until they reappeared to deceive a future archeologist. Petrie's weights, however, are somewhat heavier, not lighter, than independent evidence would lead us to believe the *ket* to have been. Those that were too heavy would be reground, but would for the most part still be rather heavier than the standard. The consequence would be that the [error] curve would be cut down vertically at two ordinates (equally distant, perhaps,

from the standard), while the ordinate of its maximum would be at the right of that of the standard. If the workman had a balance at hand, and frequently used it during the process of adjustment, the form of the error-curve would depend upon the construction of the balance. If it were like a modern balance, so as to show, not only that one mass is greater than another, but also whether it is much or little greater, the workman would keep in one pan a weight of the maximum value that he proposed to himself as permissible for the weight he was making; and in all his successive grindings would be aiming at that. The consequence would be a curve



[Click here to view] concave upwards and stopping abruptly at its maximum ordinate: a form easily manageable by a slight modification of the method of least squares. But most of the balances shown upon the Egyptian monuments are provided with stops or other contrivances which would be needless if the balances were not top-heavy. Such balances, working automatically, are in use in all the mints of the civilized world, for throwing out light and heavy coins. Now a top-heavy balance will not show that two weights are equal, otherwise than by remaining with either end down which may be down. It only shows when, a weight being already in one pan, a decidedly heavier weight is placed in the other. The workman using such a balance would have no warning that he was approaching the limit, and would be unable to aim at any definite value, but (being, as we are supposing, devoid of skill), would have to grind away blindly, trying his weight every time he had ground off about as much as the whole range of variation which he proposed to allow himself. If he always ground off precisely the same amounts between successive tryings of his weight, he would be just as likely to grind below his maximum by any one fraction of the amount taken off at a grinding as by any other; so that his error curve would be a



horizontal line cut off by vertical ordinates; thus, [Click here to view]. But since there would be a variability in the amount taken off between the trials, the curve would show a contrary flexure; thus,

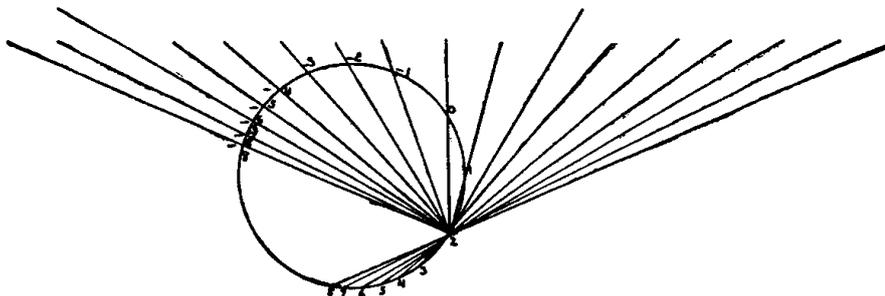


Figure 1

[Click here to view]. It must be admitted that the distribution of Petrie's *kets* is suggestive of this sort of curve, or rather of a modification of it due to a middling degree of skill.

Peirce: CP 1.211 Cross-Ref:††

211. I hope this long digression (which will be referred to with some interest when we come to study the theory of errors) will not have caused the reader to forget that we were engaged in tracing out some of the consequences of understanding the

term "natural," or "real," class to mean a class the existence of whose members is due to a common and peculiar final cause. It is, as I was saying, a widespread error to think that a "final cause" is necessarily a purpose. A purpose is merely that form of final cause which is most familiar to our experience. The signification of the phrase "final cause" must be determined by its use in the statement of Aristotle †1 that all causation divides into two grand branches, the efficient, or forceful; and the ideal, or final. If we are to conserve the truth of that statement, we must understand by final causation that mode of bringing facts about according to which a general description of result is made to come about, quite irrespective of any compulsion for it to come about in this or that particular way; although the means may be adapted to the end. The general result may be brought about at one time in one way, and at another time in another way. Final causation does not determine in what particular way it is to be brought about, but only that the result shall have a certain general character.

Peirce: CP 1.212 Cross-Ref: ††

212. Efficient causation, on the other hand, is a compulsion determined by the particular condition of things, and is a compulsion acting to make that situation begin to change in a perfectly determinate way; and what the general character of the result may be in no way concerns the efficient causation. For example, I shoot at an eagle on the wing; and since my purpose -- a special sort of final, or ideal, cause -- is to hit the bird, I do not shoot directly at it, but a little ahead of it, making allowance for the change of place by the time the bullet gets to that distance. So far, it is an affair of final causation. But after the bullet leaves the rifle, the affair is turned over to the stupid efficient causation, and should the eagle make a swoop in another direction, the bullet does not swerve in the least, efficient causation having no regard whatsoever for results, but simply obeying orders blindly. It is true that the force of the bullet conforms to a law; and the law is something general. But for that very reason the law is not a force. For force is compulsion; and compulsion is *hic et nunc*. It is either that or it is no compulsion. Law, without force to carry it out, would be a court without a sheriff; and all its dicta would be vaporings. Thus, the relation of law, as a cause, to the action of force, as its effect, is final, or ideal, causation, not efficient causation. The relation is somewhat similar to that of my pulling the hair trigger of my rifle, when the cartridge explodes with a force of its own, and off goes the bullet in blind obedience to perform the special instantaneous beginning of an act that it is, each instant, compelled to commence. It is a vehicle of compulsion *hic et nunc*, receiving and transmitting it; while I receive and transmit ideal influence, of which I am a vehicle.

Peirce: CP 1.213 Cross-Ref: ††

213. When we speak of an "idea," or "notion," or "conception of the mind," we are most usually thinking -- or trying to think -- of an idea abstracted from all efficiency. But a court without a sheriff, or the means of creating one, would not be a court at all; and did it ever occur to you, my reader, that an idea without efficiency is something equally absurd and unthinkable? Imagine such an idea if you can! Have you done so? Well, where did you get this idea? If it was communicated to you viva voce from another person, it must have had efficiency enough to get the particles of air vibrating. If you read it in a newspaper, it had set a monstrous printing press in motion. If you thought it out yourself, it had caused something to happen in your brain. And again, how do you know that you did have the idea when this discussion began a few lines above, unless it had efficiency to make some record on the brain? The court cannot be imagined without a sheriff. Final causality cannot be imagined without efficient causality; but no whit the less on that account are their modes of

action polar contraries. The sheriff would still have his fist, even if there were no court; but an efficient cause, detached from a final cause in the form of a law, would not even possess efficiency: it might exert itself, and something might follow *post hoc*, but not *propter hoc*; for *propter* implies potential regularity. Now without law there is no regularity; and without the influence of ideas there is no potentiality.

Peirce: CP 1.214 Cross-Ref:††

214. The light of these reflections brings out into distinct view characters of our definition of a real class which we might otherwise have overlooked or misinterpreted. Every class has its definition, which is an idea; but it is not every class where the *existence*, that is, the occurrence in the universe of its members is due to the active causality of the defining idea of the class. That circumstance makes the epithet *natural* particularly appropriate to the class. The word *natura* evidently must originally have meant *birth*; although even in the oldest Latin it very seldom bears that meaning. There is, however, a certain sub-conscious memory of that meaning in many phrases; just as with words from {physis}, there is the idea of springing forth, or a more vegetable-like production, without so much reference to a progenitor. Things, it may be, {phyetai} spontaneously; but *nature* is an inheritance.

Peirce: CP 1.215 Cross-Ref:††

215. Heredity, of which so much has been said since 1860, is not a force but a law, although, like other laws, it doubtless avails itself of forces. But it is essentially that the offspring shall have a *general* resemblance to the parent, not that this general resemblance happens to result from this or that blind and particular action. No doubt, there is some blind efficient causation; but it is not that which constitutes the heredity, but, on the contrary, the general resemblance.

Peirce: CP 1.216 Cross-Ref:††

216. So, then, those naturalists are right who hold that the action of evolution in reproduction produces *real* classes, as by the very force of the words it produces *natural* classes. In considering the classification of sciences, however, we have no need of penetrating the mysteries of biological development; for the generation here is of ideas by ideas -- unless one is to say, with many logicians, that ideas arise from the consideration of facts in which there are no such ideas, nor any ideas. That opinion is a superficial one, allied, on one side of it, to the notion that the only final cause is a purpose. So, those logicians imagine that an idea has to be connected with a brain, or has to inhere in a "soul." This is preposterous: the idea does not belong to the soul; it is the soul that belongs to the idea. The soul does for the idea just what the cellulose does for the beauty of the rose; that is to say, it affords it opportunity. It is the court-sheriff, the arm of the law.

Peirce: CP 1.217 Cross-Ref:††

217. I fear I may be producing the impression of talking at random. It is that I wish the reader to "catch on" to my conception, my point of view; and just as one cannot make a man see that a thing is red, or is beautiful, or is touching, by describing redness, beauty, or pathos, but can only point to something else that is red, beautiful, or pathetic, and say, "Look here too for something like that there," so if the reader has not been in the habit of conceiving ideas as I conceive them, I can only cast a sort of dragnet into his experience and hope that it may fish up some instance in which he shall have had a similar conception. Do you think, reader, that it is a positive fact that

"Truth, crushed to earth, shall rise again,"

or do you think that this, being poetry, is only a pretty fiction? Do you think that, notwithstanding the horrible wickedness of every mortal wight, the idea of right and wrong is nevertheless the greatest power on this earth, to which every knee must sooner or later bow or be broken down; or do you think that this is another notion at which common sense should smile? Even if you are of the negative opinion, still you must acknowledge that the affirmative is intelligible. Here, then, are two instances of ideas which either have, or are believed to have, life, the power of bringing things to pass, here below. Perhaps you may object that right and wrong are only a power because there are, or will be, powerful men who are disposed to make them so; just as they might take it into their heads to make tulip-fancying, or freemasonry, or Volapük a power. But you must acknowledge that this is not the position of those on the affirmative side. On the contrary, they hold that it is the idea which will create its defenders, and render them powerful. They will say that if it be that freemasonry or its foe, the Papacy, ever pass away -- as perhaps either may -- it will be precisely because they are ideas devoid of inherent, incorruptible vitality, and not at all because they have been unsupplied with stalwart defenders. Thus, whether you accept the opinion or not, you must see that it is a perfectly intelligible opinion that ideas are not all mere creations of this or that mind, but on the contrary have a power of finding or creating their vehicles, and having found them, of conferring upon them the ability to transform the face of the earth.

Peirce: CP 1.218 Cross-Ref:††

218. If you ask what mode of being is supposed to belong to an idea that is in no mind, the reply will come that undoubtedly the idea must be embodied (or ensouled -- it is all one) in order to attain complete being, and that if, at any moment, it should happen that an idea -- say that of physical decency -- was quite unconceived by any living being, then its mode of being (supposing that it was not altogether dead) would consist precisely in this, namely, that it was about to receive embodiment (or ensoulment) and to work in the world. This would be a mere potential being, a being *in futuro*; but it would not be the utter nothingness which would befall matter (or spirit) if it were to be deprived of the governance of ideas, and thus were to have no regularity in its action, so that throughout no fraction of a second could it steadily act in any general way. For matter would thus not only not actually exist, but it would not have even a potential existence, since potentiality is an affair of ideas. It would be just downright Nothing.

Peirce: CP 1.219 Cross-Ref:††

219. It so happens that I myself believe in the eternal life of the ideas Truth and Right. I need not, however, insist upon that for my present purpose, and have only spoken of them in order to make my meaning clear. What I do insist upon is not now the infinite vitality of those particular ideas, but that every idea has in some measure, in the same sense that those are supposed to have it in unlimited measure, the power to work out physical and psychical results. They have life, generative life.

Peirce: CP 1.219 Cross-Ref:††

That it is so is a matter of experiential fact. But whether it is so or not is not a question to be settled by producing a microscope or telescope or any recondite observations of any kind. Its evidence stares us all in the face every hour of our lives.

Nor is any ingenious reasoning needed to make it plain. If one does not see it, it is for the same reason that some men have not a sense of sin; and there is nothing for it but to be born again and become as a little child. If you do not see it, you have to look upon the world with new eyes.

Peirce: CP 1.220 Cross-Ref:††

220. I may be asked what I mean by the objects of [a] class *deriving their existence* from an idea. Do I mean that the idea calls new matter into existence? Certainly not. That would be pure intellectualism, which denies that blind force is an element of experience distinct from rationality, or logical force. I believe that to be a great error; but I need not stop to disprove it now, for those who entertain it will be on my side in regard to classification. But it will be urged that if that is not my meaning, then the idea merely confers upon the members of the class its character; and since every class has a defining character, any one class is as "natural" or "real" as another, if that term be taken in the sense I give to it. I cannot, however, quite admit that. Whether or not every class is or is not more or less a natural class is a question which may be worth consideration; but I do not think that the relation of the idea to the members of the natural class is simply that it is applicable to them as a predicate, as it is to every class equally. What I mean by the idea's conferring existence upon the individual members of the class is that it confers upon them the power of working out results in this world, that it confers upon them, that is to say, organic existence, or, in one word, life. The existence of an individual man is a totally different thing from the existence of the matter which at any given instant happens to compose him, and which is incessantly passing in and out. A man is a wave, but not a vortex. Even the existence of the vortex, though it does happen to contain, while it lasts, always the same particles, is a very different thing from the existence of these particles. Neither does the existence of wave or vortex consist merely in the fact that something is true of whatever particles compose them; although it is inseparably bound up with that fact. Let me not be understood as proposing any new definitions of a vortex and a wave. What I mean is this. Take a corpse: dissect it, more perfectly than it ever was dissected. Take out the whole system of blood vessels entire, as we see them figured in the books. Treat the whole systems of spinal and sympathetic nerves, the alimentary canal with its adjuvants, the muscular system, the osseous system, in the same way. Hang these all in a cabinet so that from a certain point of view each appears superposed over the others in its proper place. That would be a singularly instructive specimen. But to call it a man would be what nobody would for an instant do or dream. Now the best definition that ever was framed is, at best, but a similar dissection. It will not really work in the world as the object defined will. It will enable us to see how the thing works, in so far as it shows the efficient causation. The final causation, which is what characterizes the *definitum*, it leaves out of account. We make smoke rings. We make one pass through another, and perform various experiments, which give us an imperfect idea, yet some idea, of what a vortex really is. *How* all these things happen can be traced out from the definition. But the *rôle* that vortices really play in the universe -- no insignificant one, if all matter is built of them -- the real life of them, depends upon the idea of them, which simply finds its opportunity in those circumstances that are enumerated in the definition. Efficient causation is that kind of causation whereby the parts compose the whole; final causation is that kind of causation whereby the whole calls out its parts. Final causation without efficient causation is helpless; mere calling for parts is what a Hotspur, or any man, may do; but they will not come without efficient causation. Efficient causation without final causation, however, is worse than helpless, by far; it is mere chaos; and chaos is not even so much as chaos, without final causation; it is

blank nothing.

Peirce: CP 1.221 Cross-Ref:††

221. The writer of a book can do nothing but set down the items of his thought. For the living thought, itself, in its entirety, the reader has to dig into his own soul. I think I have done my part, as well as I can. I am sorry to have left the reader an irksome chore before him. But he will find it worth the doing.

Peirce: CP 1.222 Cross-Ref:††

222. So then, a natural class being a family whose members are the sole offspring and vehicles of one idea, from which they derive their peculiar faculty, to classify by abstract definitions is simply a sure means of avoiding a natural classification. I am not decrying definitions. I have a lively sense of their great value in science. I only say that it should not be by means of definitions that one should seek to find natural classes. When the classes have been found, then it is proper to try to define them; and one may even, with great caution and reserve, allow the definitions to lead us to turn back and see whether our classes ought not to have their boundaries differently drawn. After all, boundary lines in some cases can only be artificial, although the classes are natural, as we saw in the case of the *kets*. When one can lay one's finger upon the purpose to which a class of things owes its origin, then indeed abstract definition may formulate that purpose. But when one cannot do that, but one can trace the genesis of a class and ascertain how several have been derived by different lines of descent from one less specialized form, this is the best route toward an understanding of what the natural classes are. This is true even in biology; it is much more clearly so when the objects generated are, like sciences, themselves of the nature of ideas.

Peirce: CP 1.223 Cross-Ref:††

223. There are cases where we are quite in the dark, alike concerning the creating purpose and concerning the genesis of things; but [there are cases] where we find a system of classes connected with a system of abstract ideas -- most frequently numbers -- and that in such a manner as to give us reason to guess that those ideas in some way, usually obscure, determine the possibilities of the things. For example, chemical compounds, generally -- or at least the more decidedly characterized of them, including, it would seem, the so-called elements -- seem to belong to types, so that, to take a single example, chlorates KClO_3 , manganates $\dagger 1 \text{KMnO}_3$, bromates KBrO_3 , ruthenates $\dagger 2 \text{KRuO}_3$, iodates KIO_3 , behave chemically in strikingly analogous ways. That this sort of argument for the existence of natural classes -- I mean the argument drawn from types, that is, from a connection between the things and a system of formal ideas -- may be much stronger and more direct than one might expect to find it, is shown by the circumstance that ideas themselves -- and are they not the easiest of all things to classify naturally, with assured truth? -- can be classified on no other grounds than this, except in a few exceptional cases. Even in these few cases, this method would seem to be the safest. For example, in pure mathematics, almost all the classification reposes on the relations of the forms classified to numbers or other multitudes. Thus, in topical geometry, figures are classified according to the whole numbers attached to their *chorexis*, *cyclosis*, *periphraxis*, *apeiresis*, etc. As for the exceptions, such as the classes of hessians, jacobians, invariants, vectors, etc., they all depend upon types, too, although upon types of a different kind. It is plain that it must be so; and all the natural classes of logic will be found to have the same character.

Peirce: CP 1.224 Cross-Ref:††
§2. NATURAL CLASSIFICATIONS

224. There are two remarks more about natural classification which, though they are commonplace enough, cannot decently be passed by without recognition. They have both just been virtually said, but they had better be more explicitly expressed and put in a light in which their bearing upon the practice of classification shall be plain. The descriptive definition of a natural class, according to what I have been saying, is not the essence of it. It is only an enumeration of tests by which the class may be recognized in any one of its members. A description of a natural class must be founded upon samples of it or typical examples. Possibly a zoölogist or a botanist may have so definite a conception of what a species is that a single type-specimen may enable him to say whether a form of which he finds a specimen belongs to the same species or not. But it will be much safer to have a large number of individual specimens before him, from which he may get an idea of the amount and kind of individual or geographical variation to which the given species is subject. In proportion as the category of the class is higher, the greater will be the need of a multiplicity of examples. True, a naturalist may be so familiar with what a genus is, what a family is, what an order is, what a class is, that if you were to show him a new specimen of a hitherto unknown class, he could, with that single specimen before him, sit down and write out definitions, not only of its class, but also of its order, of its family, and of its genus, as well as of its species. Such a feat would display marvellous familiarity with what those categories [mean] in botany and in zoölogy; but intellectually it would be a performance of no high order, and the less so the greater the certainty of the conclusion. Generalization broad, luminous, and solid must enter into an intellectual performance in order to command much admiration. Such generalization, which teaches a new and clear lesson upon the truth of which reliance can be placed, requires to be drawn from many specimens. We shall endeavor, in that way, to define each class, that is to enumerate characters which are absolutely decisive as to whether a given individual does or does not belong to the class. But it may be, as our kets show, that this is altogether out of the question; and the fact that two classes merge is no proof that they are not truly distinct natural classes.

Peirce: CP 1.225 Cross-Ref:††

225. For they may, nevertheless, be genealogically distinct, just as no degree of resemblance between two men is proof positive that they are brothers. Now genealogical classification, among those objects of which the genesis is genealogical, is the classification we can most certainly rely upon as being natural. No harm will be done if, in those cases, we *define* the natural classification as the genealogical classification; or, at least, [if] we make the genealogical character one of the essential characters of a natural classification. It can not be more; because if we had before us, ranged in ancestral order, all the intermediate forms through which the human stock has passed in developing from non-man into man, it is plain that other considerations would be necessary in determining (if it admitted of determination) at what point in the series the forms begin to merit the name of human.

Peirce: CP 1.226 Cross-Ref:††

226. The sciences are, in part, produced each from others. Thus, spectroscopic astronomy has for its parents, astronomy, chemistry, and optics. But this is not the

whole genesis nor the principal part of the genesis of any broad and definite science. It has its own peculiar problem springing from an idea. That geometry derived its birth from land surveying is the tradition, which is borne out by the tradition that it took its origin in Egypt where the yearly floods must have rendered accurate surveying of special importance. Moreover, the wonderful accuracy of the dimensions of the great pyramid exhibit a degree of skill in laying out ground which could only have been attained by great intellectual activity; and this activity could hardly fail to lead to some beginnings of geometry. We may, therefore, accept with considerable confidence the tradition involved in the very name of geometry. Speaking in a broad, rough way, it may be said that the sciences have grown out of the useful arts, or out of arts supposed to be useful. Astronomy out of astrology; physiology, taking medicine as a halfway out of magic; chemistry out of alchemy; thermotics from the steam-engine, etc. Among the theoretical sciences, while some of the most abstract have sprung straight from the concretest arts, there is nevertheless a well-marked tendency for a science to be first descriptive, later classificatory, and lastly to embrace all classes in one law. The classificatory stage may be skipped. Yet in the truer order of development, the generation proceeds quite in the other direction. Men may and do begin to study the different kinds of animals and plants before they know anything of the general laws of physiology. But they cannot attain any true understanding of taxonomic biology until they can be guided by the discoveries of the physiologists. Till then the study of mollusks will be nothing but conchology. On the other hand the physiologist may be aided by a fact or two here and there drawn from taxonomic biology; but he asks but little and that little not very urgently of anything that the taxonomist can tell him and that he could not find out for himself.

Peirce: CP 1.227 Cross-Ref:††

227. All natural classification is then essentially, we may almost say, an attempt to find out the true genesis of the objects classified. But by genesis must be understood, not the efficient action which produces the whole by producing the parts, but the final action which produces the parts because they are needed to make the whole. Genesis is production from ideas. It may be difficult to understand how this is true in the biological world, though there is proof enough that it is so. But in regard to science it is a proposition easily enough intelligible. A science is defined by its problem; and its problem is clearly formulated on the basis of abstracter science. This is all I intended to say here concerning classification, in general.

Peirce: CP 1.228 Cross-Ref:††

228. Having found the natural classes of the objects to be classified, we shall then use the same methods -- probably, in most cases, the third -- in order to discover the natural classes of those classes that we have found. Is this the whole business of classification? No serious student can hold it to be so. The classes found have to be defined, naturally if possible, but if not, then at least conveniently for the purposes of science. They have not only to be defined but described, a story without an end. This applies, of course, not merely to the species or immediate classes of the objects described, but to the higher orders of classes. There may also be between the different classes relations, each of which appertains just as much to the description of any one of the set of classes to which it belongs as to any other.

Peirce: CP 1.229 Cross-Ref:††

229. In regard to the higher orders of classes, so far as concerns animals, Louis Agassiz †1 thought that he was able to characterize in general terms the different categories of classes which zoölogists talk of. That is, he undertook to say

what sort of characters distinguish *branches* from branches, *classes* from classes, *orders* from orders, *families* from families, *genera* from genera, and *species* from species. His general classification of animals has passed away; and few naturalists attach much importance to his characterizations of the categories. Yet they are the outcome of deep study, and it is a merit of them that they involve no attempt at hard abstract accuracy of statement. How can he have been so long immersed in the study of nature without some truth sticking to him? I will just set down his vague definitions and allow myself to be vaguely influenced by them, so far as I find anything in the facts that answers to his descriptions. Although I am an ignoramus in biology, I ought by this time to recognize metaphysics when I meet with it; and it is apparent to me that those biologists whose views of classification are most opposite to those of Agassiz are saturated with metaphysics in its dangerous form -- *i.e.* the unconscious form -- to such an extent that what they say upon this subject is rather the expression of a traditionally absorbed fourteenth century metaphysics than of scientific observation.

Peirce: CP 1.230 Cross-Ref:††

230. It would be useless for our purpose to copy the definitions of Agassiz †P1 had he not expressed them in the briefest terms, as follows:

Peirce: CP 1.230 Cross-Ref:††

Branches are characterized by the plan of structure;

Peirce: CP 1.230 Cross-Ref:††

Classes, by the manner in which that plan is executed, as far as ways and means are concerned; (" Structure is the watch-word for the recognition of classes." Page 145.)

Peirce: CP 1.230 Cross-Ref:††

Orders, by the degrees of complication of that structure; (" The leading idea . . . is that of a definite rank among them." Page 151.)

Peirce: CP 1.230 Cross-Ref:††

Families, by their form, as determined by structure; ("When we see new animals, does not the first glance, that is, the first impression made upon us by their form, give us a very correct idea of their nearest relationship? . . . So form is characteristic of families; . . . I do not mean the mere outline, but form as determined by structure." Pages 159, 160.)

Peirce: CP 1.230 Cross-Ref:††

Genera, by the details of the execution in special parts;

Peirce: CP 1.230 Cross-Ref:††

Species, by the relations of individuals to one another and to the world in which they live, as well as by the proportions of their parts, their ornamentation, etc.

Peirce: CP 1.231 Cross-Ref:††

231. All classification, whether artificial or natural, is the arrangement of objects according to ideas. A natural classification is the arrangement of them according to those ideas from which their existence results. No greater merit can a taxonomist have than that of having his eyes open to the ideas in nature; no more deplorable blindness can afflict him than that of not seeing that there are ideas in nature which determine the existence of objects. The definitions of Agassiz will, at least, do us the service of directing our attention to the supreme importance of bearing

in mind the final cause of objects in finding out their own natural classifications.

Peirce: CP 1.232 Cross-Ref:††
§3. THE ESSENCE OF SCIENCE

232. So much in regard to classification. Now if we are to classify the sciences, it is highly desirable that we should begin with a definite notion of what we mean by a science; and in view of what has been said of natural classification, it is plainly important that our notion of science should be a notion of science as it lives and not a mere abstract definition. Let us remember that science is a pursuit of living men, and that its most marked characteristic is that when it is genuine, it is in an incessant state of metabolism and growth. If we resort to a dictionary, we shall be told that it is systematized knowledge. Most of the classifications of the sciences have been classifications of systematized and established knowledge -- which is nothing but the exudation of living science; -- as if plants were to be classified according to the characters of their gums. Some of the classifications do even worse than that, by taking science in the sense attached by the ancient Greeks, especially Aristotle, to the word {epistémé}. A person can take no right view of the relation of ancient to modern science unless he clearly apprehends the difference between what the Greeks meant by {epistémé} and what we mean by knowledge. The best translation of {epistémé} is "comprehension." It is the ability to define a thing in such a manner that all its properties shall be corollaries from its definition. Now it may be that we shall ultimately be able to do that, say for light or electricity. On the other hand, it may equally turn out that it forever remains as impossible as it certainly is to define number in such a way that Fermat's and Wilson's theorems should be simple corollaries from the definition. I do not mean to deny that those theorems are deducible from the definition. All that is here being urged turns on the falsity of the old notion that all deduction is corollarial deduction. But, at any rate, the Greek conception of knowledge was all wrong in that they thought that one must advance in direct attack upon this {epistémé}; and attached little value to any knowledge that did not manifestly tend to that. To look upon science in that point of view in one's classification is to throw modern science into confusion.

Peirce: CP 1.233 Cross-Ref:††

233. Another fault of many classifications -- or if not a fault, it is at least a purpose very different from that which I should be bold enough to attempt -- is that they are classifications not of science as it exists, but of systematized knowledge such as the classifier hopes may some time exist. I do not believe it is possible to have that intimate acquaintance with the science of the indefinite future that the discovery of the real and natural classification of it would require. At any rate, I will make no such attempt, except in one department, and there only partially and timidly.

Peirce: CP 1.234 Cross-Ref:††

234. Let us look upon science -- the science of today -- as a living thing. What characterizes it generally, from this point of view, is that the thoroughly established truths are labelled and put upon the shelves of each scientist's mind, where they can be at hand when there is occasion to use things -- arranged, therefore, to suit his special convenience -- while science itself, the living process, is busied mainly with conjectures, which are either getting framed or getting tested. When that systematized

knowledge on the shelves is used, it is used almost exactly as a manufacturer or practising physician might use it; that is to say, it is merely applied. If it ever becomes the object of science, it is because in the advance of science, the moment has come when it must undergo a process of purification or of transformation.

Peirce: CP 1.235 Cross-Ref:††

235. A scientific man is likely in the course of a long life to pick up a pretty extensive acquaintance with the results of science; but in many branches, this is so little necessary that one will meet with men of the most deserved renown in science who will tell you that, beyond their own little nooks, they hardly know anything of what others have done. Sylvester always used to say that he knew very little mathematics: true, he seemed to know more than he thought he did. In various branches of science, some of the most eminent men first took up those subjects as mere pastimes, knowing little or nothing of the accumulations of knowledge. So it was with the astronomer Lockyer: so it has been with many naturalists. Now, did those men gradually become men of science as their stores of knowledge increased, or was there an epoch in their lives, before which they were amateurs and after which they were scientists? I believe that the answer is that, like any other regeneration, the metamorphosis is commonly sudden, though sometimes slow. When it is sudden, what is it that constitutes the transformation? It is their being seized with a great desire to learn the truth, and their going to work with all their might by a well-considered method to gratify that desire. The man who is working in the right way to learn something not already known is recognized by all men of science as one of themselves, no matter how little he is informed. It would be monstrous to say that Ptolemy, Archimedes, Eratosthenes and Posidonius were not scientific men because their knowledge was comparatively small. The life of science is in the desire to learn. If this desire is not pure, but is mingled with a desire to prove the truth of a definite opinion, or of a general mode of conceiving of things, it will almost inevitably lead to the adoption of a faulty method; and *in so far* such men, among whom many have been looked upon in their day as great lights, are not genuine men of science; though it would be foul injustice to exclude them absolutely from that class. So if a man pursues a futile method through neglect to inform himself of effective methods, he is no scientific man; he has not been moved by an intelligently sincere and effective desire to learn. But if a man simply fails to inform himself of previous work which would have facilitated his own, although he is to blame, it would be too harsh to say that he has violated the essential principles of science. If a man pursues a method which, though very bad, is the best that the state of intellectual development of his time, or the state of the particular science he pursues, would enable a man to take -- I mean, for example, such men as Lavater, Paracelsus and the earlier alchemists, the author of the first chapter of Genesis, and the old metaphysicians -- we perhaps cannot call them scientific men, while perhaps we ought to do so. Opinions would differ about this. They are, at any rate, entitled to an honorable place in the vestibule of science. A pretty wild play of the imagination is, it cannot be doubted, an inevitable and probably even a useful prelude to science proper. For my part, if these men really had an effective rage to learn the very truth, and did what they did as the best way they knew, or could know, to find it out, I could not bring myself to deny them the title. The difficulty is that one of the things that coheres to that undeveloped state of intelligence is precisely a very imperfect and impure thirst for truth. Paracelsus and the alchemists were rank charlatans seeking for gold more than for truth. The metaphysicians were not only pedants and pretenders, but they were trying to establish foregone conclusions. These are the traits which deprive those men of the title scientist, although we ought to entertain a high respect for them as mortals go;

because they could no more escape the corruptness of their aims than they could the deficiencies of their knowledge. Science consists in actually drawing the bow upon truth with intentness in the eye, with energy in the arm.

Peirce: CP 1.236 Cross-Ref:††

236. Such being the essence of science, it is obvious that its first offspring will be men -- men whose whole lives are devoted to it. By such devotion each of them acquires a training in making some particular kind of observations and experiments. (Unfortunately, his acquisition of books, instruments, laboratory, etc., depends upon qualifications in which the man of science is usually rather wanting -- as wealth, diplomacy, popularity as a teacher -- so that he is less likely to be provided with them than are men less qualified to use them for the advancement of science.) He will thus live in quite a different world -- quite a different aggregate of experience -- from unscientific men and even from scientific men pursuing other lines of work than his. He naturally converses with and reads the writings of those who, having the same experience, have ideas interpretable into his own. This society develops conceptions of its own. Bring together two men from widely different departments -- say a bacteriologist and astronomer -- and they will hardly know what to say to one another; for neither has seen the world in which the other lives. True, both use optical instruments; but the qualities striven for in a telescopic objective are of no consequence in a microscopical objective; and all the subsidiary parts of telescope and microscope are constructed on principles utterly foreign to one another -- except their stiffness.

Peirce: CP 1.237 Cross-Ref:††

237. Here, then, are natural classes of sciences all sorted out for us in nature itself, so long as we limit our classification to actually recognized sciences. We have only to look over the list of scientific periodicals and the list of scientific societies to find the families of science, ready named. I call such classes families because Agassiz tells us that it is the family which strikes the observer at first glance. To make out the genera and especially the species, closer examination is requisite; while the knowledge of orders, classes, and branches calls for a broader acquaintance with science.

Peirce: CP 1.238 Cross-Ref:††
§4. THE DIVISIONS OF SCIENCE

238. The first great division of science will be according to its fundamental purpose, making what I shall term branches of science. A modification of a general purpose may constitute a subbranch. All knowledge whatever comes from observation; but different sciences are observational in such radically different ways that the kind of information derived from the observation of one department of science (say natural history) could not possibly afford the information required of observation by another branch (say mathematics). I call groups based on such considerations *classes*, and modifications of the same nature *subclasses*. Observation is, in Agassiz's phrase, the "ways and means" of attaining the purpose of science. Of two departments of science *A* and *B*, of the same class, *A* may derive special facts from *B* for further generalization while supplying *B* with principles which the latter, not aiming so high, is glad to find ready-made. *A* will rank higher than *B*, by virtue of

the greater generality of its object, while **B** will be richer and more varied than **A**. I call groups based on these considerations *orders*, or if based on modifications of the same sort of idea, *suborders*. A given science with a special name, a special journal, a special society, studying one group of facts, whose students understand one another in a general way and naturally associate together, forms what I call a *family*. A subdivision of it on the same principle, but taken more minutely, I term a *subfamily*. I can give no such definitions of genera and species, not having carried my classification of the sciences to these minutiae. For it is to be understood that I have not first fixed my definitions of *branch*, *class*, *order*, and *family*, and then adapted the classification to those definitions, but, on the contrary, the classification was first entirely formed (except that the categories of subbranches, subclasses, and suborders had, in some cases, not been interposed, and in others had been confounded with the classes above them) before any idea of employing the terms *branch*, *class*, *order*, and *family* entered my head, and it was not until this was done that first the appropriateness of these terms struck me. I can, however, say with some confidence that I should not regard a family as constituted merely by the class of facts studied, were there no concomitant difference of procedure, giving an all-round peculiar character to the study of that subject; nor do I believe that a mere difference in the things studied could appear to me a sufficient foundation for a difference between genera. Since writing that sentence, I notice that I have made inorganic and organic chemistry *subgenera*. But, then, everybody knows that there is far more difference between inorganic and organic chemistry than that the latter studies the compounds of a somewhat peculiar element. Their whole aims and ways of thinking, as well as their manipulation, are in distinct contrast.

Peirce: CP 1.239 Cross-Ref:††

239. I recognize two branches of science: Theoretical, whose purpose is simply and solely knowledge of God's truth; and Practical, for the uses of life. In Branch I, I recognize two subbranches, of which, at present, I consider only the first, [the sciences of discovery]. Among the theoretical sciences [of discovery], I distinguish three classes, all resting upon observation, but being observational in very different senses.†P1

Peirce: CP 1.240 Cross-Ref:††

240. The first is mathematics, which does not undertake to ascertain any matter of fact whatever, but merely posits hypotheses, and traces out their consequences. It is observational, in so far as it makes constructions in the imagination according to abstract precepts, and then observes these imaginary objects, finding in them relations of parts not specified in the precept of construction. This is truly observation, yet certainly in a very peculiar sense; and no other kind of observation would at all answer the purpose of mathematics.†P2

Peirce: CP 1.241 Cross-Ref:††

241. Class II is philosophy, which deals with positive truth, indeed, yet contents itself with observations such as come within the range of every man's normal experience, and for the most part in every waking hour of his life. Hence Bentham calls this class, *coenoscopic*.†1 These observations escape the untrained eye precisely because they permeate our whole lives, just as a man who never takes off his blue spectacles soon ceases to see the blue tinge. Evidently, therefore, no microscope or sensitive film would be of the least use in this class. The observation is observation in a peculiar, yet perfectly legitimate, sense. If philosophy glances now and then at the results of special sciences, it is only as a sort of condiment to excite its own proper

observation.

Peirce: CP 1.242 Cross-Ref:††

242. Class III is Bentham's *idioscopic*†2; that is, the special sciences, depending upon special observation, which travel or other exploration, or some assistance to the senses, either instrumental or given by training, together with unusual diligence, has put within the power of its students. This class manifestly divides itself into two subclasses, the physical and the psychical sciences; or, as I will call them, physiognosy and psychognosy. Under the former is to be included physics, chemistry, biology, astronomy, geognosy, and whatever may be like these sciences; under the latter, psychology, linguistics, ethnology, sociology, history, etc. Physiognosy sets forth the workings of efficient causation, psychognosy of final causation. But the two things call for different eyes. A man will be no whit the worse physiognosist for being utterly blind to facts of mind; and if we sometimes find observation in a psychognosist, it will, unless by exception, be found not to be of a purely physical fact. Thus, a philologist may have a fine ear for language-sounds; but it is by no means pure physical resemblance which determines whether a given sound is or is not "the" Italian close *o*, for example, as it is naïvely called: it is psychical habit. In any simple physical sense the sounds not distinguished from that differ much more from one another than almost any of them do from sounds which would not be tolerated for "the" close *o*. So, this fine phonetic observation of the linguist is a knack of understanding a virtual convention. The two kinds of observation are different; but they do not seem to be quite so different as both alike are from the observation of the philosopher and the mathematician; and this is why, though I, at first, was inclined to give each of them equal rank with those classes, it has at length appeared certain that they should be placed a little lower.

Peirce: CP 1.243 Cross-Ref:††

243. I still persist in leaving unnoticed a certain subbranch of theoretical science [the sciences of review]; and as for the practical sciences,†P1 I shall merely mention a few of them, just to give an idea of what I refer to under that name. I mean, then, all such well-recognized sciences now *in actu*, as pedagogics, gold-beating, etiquette, pigeon-fancying, vulgar arithmetic, horology, surveying, navigation, telegraphy, printing, bookbinding, paper-making, deciphering, ink-making, librarian's work, engraving, etc.†1 In short, this is by far the more various of the two branches of science. I must confess to being utterly bewildered by its motley crowd, but fortunately the natural classification of this branch will not concern us in logic -- at least, will not do so as far as I can perceive.

Peirce: CP 1.244 Cross-Ref:††

244. Now let us consider the relations of the classes of science to one another. We have already remarked that relations of generation must always be of the highest concern to natural classification, which is, in fact, no more nor less than an account of the existential, or *natural*, birth concerning relations of things; meaning by birth the relations of a thing to its originating final causes.

Peirce: CP 1.245 Cross-Ref:††

245. Beginning with Class I, mathematics meddles with every other science without exception. There is no science whatever to which is not attached an application of mathematics. This is not true of any other science, since pure mathematics has not, as a part of it, any application of any other science, inasmuch as every other science is limited to finding out what is positively true, either as an individual fact, as a class, or as a law; while pure mathematics has no interest in

whether a proposition is existentially true or not. In particular, mathematics has such a close intimacy with one of the classes of philosophy, that is, with logic, that no small acumen is required to find the joint between them.

Peirce: CP 1.246 Cross-Ref:††

246. Next, passing to Class II, philosophy, whose business it is to find out all that can be found out from those universal experiences which confront every man in every waking hour of his life, must necessarily have its application in every other science. For be this science of philosophy that is founded on those universal phenomena as small as you please, as long as it amounts to anything at all, it is evident that every special science ought to take that little into account before it begins work with its microscope, or telescope, or whatever special means of ascertaining truth it may be provided with.

Peirce: CP 1.247 Cross-Ref:††

247. It might, indeed, very easily be supposed that even pure mathematics itself would have need of one department of philosophy; that is to say, of logic. Yet a little reflection would show, what the history of science confirms, that that is not true. Logic will, indeed, like every other science, have its mathematical parts. There will be a mathematical logic just as there is a mathematical physics and a mathematical economics. If there is any part of logic of which mathematics stands in need -- logic being a science of fact and mathematics only a science of the consequences of hypotheses -- it can only be that very part of logic which consists merely in an application of mathematics, so that the appeal will be, not of mathematics to a prior science of logic, but of mathematics to mathematics. Let us look at the rationale of this a little more closely. Mathematics is engaged solely in tracing out the consequences of hypotheses. As such, she never at all considers whether or not anything be existentially true, or not. But now suppose that mathematics strikes upon a snag; and that one mathematician says that it is evident that a consequence follows from a hypothesis, while another mathematician says it evidently does not. Here, then, the mathematicians find themselves suddenly abutting against brute fact; for certainly a dispute is not a rational consequence of anything. True, this fact, this dispute, is no part of mathematics. Yet it would seem to give occasion for an appeal to logic, which is generally a science of fact, being a science of truth; and whether or not there be any such thing as truth is a question of fact. However, because this dispute relates merely to the consequence of a hypothesis, the mere careful study of the hypothesis, which is pure mathematics, resolves it; and after all, it turns out that there was no occasion for the intervention of a science of reasoning.

Peirce: CP 1.248 Cross-Ref:††

248. It is often said that the truths of mathematics are infallible. So they are, if you mean practical infallibility, infallibility such as that of conscience. They appear even as theoretically infallible, if they are viewed through spectacles that cut off the rays of blunder. I never yet met with boy or man whose addition of a long column, of fifty to a hundred lines, was absolutely infallible, so that adding it a second time could in no degree increase one's confidence in the result, nor ought to do so. The addition of that column is, however, merely a repetition of $1 + 1 = 2$; so that, however improbable it may be, there is a certain finite probability that everybody who has ever performed this addition of 1 and 1 has blundered, except on those very occasions on which we are accustomed to suppose (on grounds of probability merely) that they *did* blunder. Looked at in this light, every mathematical inference is merely a matter of probability. At any rate, in the sense in which anything in mathematics is certain, it is

most certain that the whole mathematical world has often fallen into error, and that, in some cases, such errors have stood undetected for a couple of millennia. But no case is adducible in which the science of logic has availed to set mathematicians right or to save them from tripping. On the contrary, attention once having been called to a supposed inferential blunder in mathematics, short time has ever elapsed before the whole mathematical world has been in accord, either that the step was correct, or else that it was fallacious; and this without appeal to logic, but merely by the careful review of the mathematics as such. Thus, historically mathematics does not, as *a priori* it cannot, stand in need of any separate science of reasoning.

Peirce: CP 1.249 Cross-Ref: ††

249. But mathematics is the only science which can be said to stand in no need of philosophy, excepting, of course, some branches of philosophy itself. It so happens that at this very moment the dependence of physics upon philosophy is illustrated by several questions now on the *tapis*. The question of non-Euclidean geometry may be said to be closed. It is apparent now that geometry, while in its main outlines, it must ever remain within the borders of philosophy, since it depends and must depend upon the scrutinizing of everyday experience, yet at certain special points it stretches over into the domain of physics. Thus, space, as far as we can see, has three dimensions; but are we quite sure that the corpuscles into which atoms are now minced have not room enough to wiggle a little in a fourth? Is physical space hyperbolic, that is, infinite and limited, or is it elliptic, that is, finite and unlimited? Only the exactest measurements upon the stars can decide. Yet even with them the question cannot be answered without recourse to philosophy. But a question at this moment under consideration by physicists is whether matter consists ultimately of minute solids, or whether it consists merely of vortices of an ultimate fluid. The third possibility, which there seems to be reason to suspect is the true one, that it may consist of vortices in a fluid which itself consists of far minuter solids, these, however, being themselves vortices of a fluid, itself consisting of ultimate solids, and so on in endless alternation, has hardly been broached. The question as it stands must evidently depend upon what we ought to conclude from everyday, unspecialized observations, and particularly upon a question of logic. Another still warmer controversy is whether or not it is proper to endeavor to find a mechanical explanation of electricity, or whether it is proper, on the contrary, to leave the differential equations of electrodynamics as the last word of science. This is manifestly only to be decided by a scientific philosophy very different from the amateurish, superficial stuff in which the contestants are now entangling themselves. A third pretty well defended opinion, by the way, is that instead of explaining electricity by molar dynamics, molar dynamics ought to be explained as a special consequence of the laws of electricity. Another appeal to philosophy was not long ago virtually made by the eminent electrician, the lamented Hertz, who wished to explain force, in general, as a consequence of unseen constraints. Philosophy alone can pronounce for or against such a theory. I will not undertake to anticipate questions which have not yet emerged; otherwise, I might suggest that chemists must ere long be making appeal to philosophy to decide whether compounds are held together by force or by some other agency. In biology, besides the old logico-metaphysical dispute about the reality of classifications, the momentous question of evolution has unmistakable dependence on philosophy. Then again, caryocinesis has emboldened some naturalists, having certain philosophical leanings, to rebel against the empire of experimental physiology. The origin of life is another topic where philosophy asserts itself; and with this I close my list, not at all because I have mentioned all the points at which just now the physical sciences are influenced by a philosophy, such as it is, but simply because I have mentioned enough

of them for my present purpose.

Peirce: CP 1.250 Cross-Ref:††

250. The dependence of the psychological sciences upon philosophy is no less manifest. A few years ago, indeed, regenerate psychology, in the flush of her first success, not very wisely proposed to do without metaphysics; but I think that today psychologists generally perceive the impossibility of such a thing. It is true that the psychological sciences are not quite so dependent upon metaphysics as are the physical sciences; but, by way of compensation, they must lean more upon logic. The mind works by final causation, and final causation is logical causation. Note, for example, the intimate bearing of logic upon grammatical syntax. Moreover, everything in the psychological sciences is inferential. Not the smallest fact about the mind can be directly perceived as psychological. An emotion is directly felt as a bodily state, or else it is only known inferentially. That a thing is agreeable appears to direct observation as a character of an object, and it is only by inference that it is referred to the mind. If this statement be disputed (and some will dispute it), all the more need is there for the intervention of logic. Very difficult problems of inference are continually emerging in the psychological sciences. In psychology, there are such questions as free-will and innate ideas; in linguistics, there is the question of the origin of language, which must be settled before linguistics takes its final form. The whole business of deriving ancient history from documents that are always insufficient and, even when not conflicting, frequently pretty obviously false, must be carried on under the supervision of logic, or else be badly done.

Peirce: CP 1.251 Cross-Ref:††

251. The influence of philosophy upon the practical sciences is less direct. It is only here and there that it can be detected; and ethics is the division of philosophy which most concerns these sciences. Ethics is courteously invited to make a suggestion now and then in law, jurisprudence, and sociology. Its sedulous exclusion from diplomacy and economics is immense folly. We are unhappily debarred from calling this folly stupendous or egregious, because it is merely the ordinary blindness of those who profoundly believe that lies are the most wholesome of diet, who, as Edgar Poe sagaciously said, when they get home, have once locked themselves in their several chambers, have undressed, knelt down by the bedside and said their prayers, got into bed, and blown out the candle, then, at length, and not till then, indulge in one veracious wink -- the only veracious act of the day -- and lull themselves to sleep with an inward ditty that Right is a silly thing without wealth or vigor in this work-a-day world. One day man shall start up out of his slumber to see by broad daylight that that despised idea has all along been the one irresistible power. Then may begin an era when it is counted within the practical sciences, one and all -- when, in a word, a man will not design a stove nor order a coat without stopping first and sifting out his real desire -- and it is prophecy as simple as *Barbara*, that, when that comes to pass, those sciences will answer even their lower and nearest purposes far more perfectly than at present they do. So, at any rate, the student of minute logic will be forced to think.

Peirce: CP 1.252 Cross-Ref:††

252. The direct action of the special psychological and physical sciences -- the two subclasses of Class III -- upon one another seems to be slight. One cannot see how, except in some accidental or exterior way, the psychological sciences are to influence the physical sciences, unless it should be found advantageous to call upon psychology to aid the physical observer in avoiding illusions and in diminishing his errors of

observation. This, no doubt, deserves careful consideration; but I believe that, if the proper distinctions are drawn, it will be seen that as for illusions, the far better way, when it is practicable, as it almost always will be, will be to make the observations so simple and positive that no illusions can occur often enough to make any special dealing desirable; and as for errors of observation, it is best to treat them as residual phenomena like any other residual phenomena. That they are entirely physical every physicist must insist, physics being sufficiently advanced to see that all phenomena, without exception, are physical, for the purposes of physics. Soon we may hope that all psychologists, on their side, may be equally at one that all phenomena without exception are purely psychical for the purposes of psychics.

Peirce: CP 1.253 Cross-Ref:††

253. How far then are the psychical sciences influenced by physiognosy, or how far ought they to be so influenced? The theory of psychophysical parallelism would seem to imply that there is and can be no influence at all. But I must confess myself to be of the party which thinks that no psychical fact, as such, can be observed. The direct percept, as it first appears, appears as forced upon us brutally. It has no generality; and without generality there can be no psychicality. Physicality consists in being under the governance of physical, *i.e.*, efficient, causes, psychicality in being under the governance of psychical, *i.e.*, of final, causes. The percept brutally forces itself upon us; thus it appears under a physical guise. It is quite ungeneral, even antigeneral -- in its character as percept; and thus it does not appear as psychical. The psychical, then, is not contained in the percept.

Peirce: CP 1.254 Cross-Ref:††

254. "But what?" some one will say, "Does one not perceive redness? And is not redness purely a psychical matter to which nothing in the physical world corresponds?" If one must answer *yes* or *no*, in the rough, of course one must say *yes*. Yet as to there being nothing in the physical universe that corresponds to a given psychical phenomenon, the doctrine of parallelism itself disavows that opinion. Better let us say that in the present state of physical theory the peculiarity of redness finds no definite explanation. It would be an illogical presumption to say that it never can be explained. Redness, though a sensation, does not in the percept proclaim itself as such. At any rate, whether the psychical can be directly observed or not, no linguist, ethnologist, nor historian -- no psychologist, even, in an unguarded moment -- but will agree that his science rests very largely, if not quite entirely, upon physical *facts*.

Peirce: CP 1.255 Cross-Ref:††

255. This does not amount to an acknowledgment of need of help from the physical *sciences*. Some amount of such need and such help there is. It is easier detected than the dubious help received by physiognosy from psychognosy. The historian certainly depends in a measure upon physical geography. Linguistics must in the future receive substantial assistance from acoustics, in more than one direction, and from the anatomy of the vocal organs and of the ear. Besides such supplies of information, (which are relatively unimportant) psychognosy has received instruction and encouragement from the example of physiognosy in the nineteenth century. It has been helped to minute accuracy, to objectivity, to genuine love of truth as against the professor's profession of infallibility. Yet summing up all the items, the total influence is trifling compared with that of mathematics on philosophy or of both on idioscopy. Physics has, after all, supplied no principle to psychics, nor any great conception. On the contrary, every attempt to import into psychics the conceptions proper to physics has only led those who made it astray. All this confirms the justice of our rating of

these two departments as subclasses.

Peirce: CP 1.256 Cross-Ref:††

256. We can now no longer postpone the recognition of a second subbranch of theoretical science. It is a department perfectly well recognized. It belongs by virtue of its purpose to the branch of Theory; yet varies enough in its purpose from the active science to be erected into a subbranch. It is the subject of Humboldt's *Cosmos*, Comte's *Philosophie positive*, and Spencer's *Synthetic Philosophy*. It is science *en retraite*, *Wissenschaft a.D.* Its design is to sum up the results of all the theoretical sciences and to study them as forming one system. It may be called *retrospective* [or science of review], in contradistinction to *active* science.

Peirce: CP 1.257 Cross-Ref:††

257. We now come to consider groups one grade lower. Here is a point where I must confess to have hesitated. Our branches of science are distinguished by their different purposes; our classes by the fundamentally different nature of their observations. Logic suggests that *orders*, to which we next come, should be distinguished by the difference in the intellectual part of the business of the sciences under them; so that among the Physical Sciences, for example, we should have: first those which investigate the laws common to all matter; second, those which study the relations between different classes of physical objects; third, those whose objects are the understanding of different individual objects; and it is plain that a similar classification could be made in psychics. Still, although this seems *a priori* plausible, a positive guarantee that this will be a natural division is perhaps lacking. At any rate, no ground of assurance is evident. It has occurred to me that we might distribute the physical sciences into those which study objects predominantly under the dominion of force and those predominantly under the influence of final causality; giving physics and natural history. This separation would well accord with the way in which the men naturally cluster. But for that very reason, a suspicion is created that the point has not yet been reached where that cleavage should be made. Before we come to groups of men thoroughly understanding one another's work, we ought to consider groups of which the one stands in the relation of teacher of principles to another; just as, in a school, the relation of master and pupil makes a broader natural division than that between different forms or classes. . . .

Peirce: CP 1.258 Cross-Ref:††

258. Comte . . . produced a useful scale, as every candid man now confesses. It ran thus: Mathematics, Astronomy, Physics, Chemistry, Biology, Sociology. But sociology stands distinctly aloof from the others, as a psychical science. Astronomy, for Comte, meant the astronomy of his day, which was confined almost entirely to explaining the motions of the stars, and was thus directly dependent on mathematics. But our astronomy depends largely on chemistry. Striking out mathematics and sociology, which are not physical sciences, and putting astronomy where it now seems to belong, we get Physics, Chemistry, Biology, Astronomy, or perhaps

Physics

Chemistry

Biology

Astronomy

Geognosy was intended by Comte to be a subdivision of physics. But this is every way unnatural. Geognosy applies physics as well [as] biology (especially paleontology); so that a still better scheme would be

Physics

Chemistry Biology

Astronomy Geognosy

Peirce: CP 1.259 Cross-Ref:††

259. In this scheme, we see a return to my first idea. For physics, here, must mean general physics, so called; that is, the study of the laws and forces of nature. Chemistry must here be understood as the science of the different kinds of matter (which is substantially the definition of Ostwald and of Mendeléef). Thus the second lines are sciences of classes, or, say for brief, Classificatory Sciences, which of course have much more to do than merely to make schemes of classification. In the third line we find sciences descriptive and explanatory of individual objects, or individual systems; the heavens, and the earth. We may name them, by way of abridgment, Descriptive Sciences.

Peirce: CP 1.260 Cross-Ref:††

260. We may take it as settled, then, that nomological physics forms naturally the first order of the subclass of the physical sciences. But whether the others ought primarily to be divided according to the rows of the last scheme, or according to its columns is a question upon which a little further consideration needs to be bestowed. In this connection we remark that the affinity of geognosy to biology is hardly as decided as the vertical division would represent it to be. One cannot even say that chemistry is more allied to astronomy than it is to biology. Light may be thrown upon the matter by asking where crystallography and mineralogy should be placed. Nobody, surely, would hold it to be a natural classification to rank crystallography as coördinate with chemistry and biology. Nor does it belong to general nomological physics; for it is eminently a study of kinds, not of general laws. A suggestion of uniting it to biology would provoke a smile. It would seem, then, that nothing remains but to treat it as a division of chemistry, in the sense of the study of the different kinds of matter. Two great authorities, Ostwald and Mendeléef, do, in fact, so define chemistry; but I venture to assert that chemists generally do not so understand their science, and that chemical laboratories are not equipped for that study. Chemistry is, as a matter of fact, pretty closely limited to the study of reactions, to the structure of compounds, and to the behaviour of elements in combinations. A chemist, as such, does not feel himself called upon to inquire further into the properties of the different substances than is required to identify them and to make out their constitutional relations. He would, for example, think it quite beyond his province to arrest his work to determine the constants of elasticity of a substance. To ask that of him, he would say, would be a mischievous mixing of vocations. The descriptions that chemists give of crystals -- in most cases confined to their habits -- would be regarded by a crystallographer as superficial; and if they occasionally go further, it is with a view to the identification of the substances. The definition of Ostwald and Mendeléeff, then (which I myself independently gave), defines a department of science of which chemistry is but a part. Let us call the science of the kinds of matter by the name of chemology. This science will have to describe all the special properties of all kinds of

matter, and among these properties, will have to describe the shapes into which matter of different kinds grows. So, then, we may reckon crystallography as a branch of chemistry.

Peirce: CP 1.261 Cross-Ref:††

261. We cannot but remark, in passing, that a certain doubt arises here; because the study of the different kinds of crystalline form -- with their geometrical, elaterical, and optical relations to one another -- will look upon the facts of crystallization from quite another standpoint from that of the chemologist who is considering the relations of the different kinds of matter to one another. But I pass that by for the moment in order to make another remark. Suppose it were settled that that difference of the points of view of the crystallographer and the chemologist were of subsidiary importance, and that the latter's business includes the study [of] all the forms that different kinds of matter naturally assume. Then I remark that there is a certain group of chemical bodies, the albuminoids or protoplasms, of which, down to this time, the chemist can only say that they contain carbon (51 or 52 per cent), oxygen (20 to 23 per cent), nitrogen (16 to 19 per cent), hydrogen (about 7 per cent), sulphur (about 1 per cent), and probably often phosphorus and many other elements, and that there are something like fifteen thousand atoms to the molecule. These substances assume forms far more fantastical than crystals -- namely all the forms that the biologist describes; and the mathematician assures us that even if the number of atoms to the molecule is greatly less than the number which Sabanajeff †1 has determined by an approved method, there nevertheless can be no doubt that it is sufficient to afford, on general principles of chemistry, enough different kinds of protoplasm for each organ, or even cell of every individual animal or plant that ever existed on earth to have a unique kind of its own, without seriously encroaching upon the wealth of varieties of these substances. So, then, we may rationally conclude that all the variety of the biological world is due to the variety of the different kinds of chemical substances of this group, with their corresponding variety of properties and of natural figures. Thereupon in comes the logician, and for his contribution to the discussion, declares it to be absolutely impossible to frame any definite hypothesis -- however gratuitous -- which should assign any other origin to the forms of animals and plants than the chemical constitution of the protoplasm. Imagine, if you like, that separate corpuscles related to atoms as atoms are to billiard balls, are endowed with free will, so that their motions are determined by persuasion and not by the general laws of physics. That, if proved, would be a momentous discovery enough. I know not what could tend more toward the obliteration of all distinction between psychognosy and physiognosy. Nevertheless, under that state of things it would remain true that the chemical constitution of the protoplasm, about which we now make no pretension to knowing anything, although it would then turn out to be so mighty strange, containing chemical elements that would put radium to the blush -- that chemical constitution, I say, would even then be the sole determining cause of the forms of all animals and plants. So it would remain, though we were to suppose a special creative act at the birth or budding of each biological individual -- as long as there remained an approximate regularity in the action -- although this would be still more revolutionary of all our chemological conceptions. We all know the type of naturalist -- often a justly honored man of science -- who, at any suggestion that experiment can ever be of real avail in biology, pours forth a torrent in which feeling is more easily detected than logic. Some minds there are who seem to think that if A and B are radically dissimilar, it is weak to admit that they can be fundamentally unlike; although could this not be, there would be an end of natural classification. Nobody can dispute the fact that the albuminoids are radically unlike all chemical

substances whose constitution we understand.

Peirce: CP 1.262 Cross-Ref:††

262. If then we are to have an order or suborder of physiognosy consisting of the study of kinds of matter and their natural forms, it is a requirement of logic that biology should be reckoned as a family of that order or suborder. It must no doubt be admitted that the study of the kinds of matter, chemology, is one thing and the study of the kinds of forms that matter may take is another. These will be two suborders of the order of classificatory physiognosy, or the study of physical kinds. But into which of these suborders biology should be placed is not so clear. Perhaps it should constitute a third suborder.

Peirce: CP 1.263 Cross-Ref:††

263. In addition, we must recognize a third order, descriptive and explanatory of the accidents of individual systems, apart from the study of the classes to which they belong.

Peirce: CP 1.264 Cross-Ref:††

264. Is there a somewhat parallel division of psychognosy? "Anthropology" is a word sometimes used in so broad a sense that it covers all psychognosy, or would do so, were the study of animals and of non-biological manifestations of mind not arbitrarily excluded. Of anthropology, so understood, the late Dr. Brinton †1 proposed a classification, an outline of which may usefully be put before the reader. It includes, in the first place, four grand divisions of somatology, ethnology, ethnography, and archeology. The first of these is purely physical, except that it strangely includes psychology, so that it does not now concern us. The fourth is purely descriptive, and largely physical. It will afford no help. Ethnology is made to include five departments, as follows: 1. Sociology. 2. Technology, embracing the fine and useful arts. 3. Religion. 4. Linguistics. 5. Folklore. Ethnography treats of the different races of men, and is largely physical. I have no objection to admitting that zoölogy must perforce take some cognizance of the instincts of animals, just as on the other hand, it is quite evident that their minds can never be understood without taking some account of their anatomy and physiology. But for all that, if we are to admit that the study of animals' bodies is a study of efficiency, while the study of their minds is a study of finality, a distinction the truth and unescapableness [of which] will only be emphasized the more we study the different phases and facets of the subject, then we must acknowledge that those two studies of animals' minds and of animals' bodies are widely different, however much they may overlap. But in truth the overlapping is quite trifling. Very little psychology is needed by the biologist; and no very deep biology by the psychologist.

Peirce: CP 1.265 Cross-Ref:††

265. Dr. Brinton's classification is artificial. He would hardly himself have contested that judgment. Of almost any subdivision of it, it may be said that no man could judiciously devote himself to all those studies exclusively. Probably no man ever did, if we read Brinton's explanation of what each includes. But the classification has a fault even worse than that of being artificial. There can be no objection to a man's engaging at one time in tracing out final, or mental, causation, and at another time in tracing out material, or efficient, causation. But to confound these two things together is fatal. That circumstance constitutes a certain measure of justification for the warfare that has been waged, in many quarters, upon "final causes"; and it equally justifies the dislike often felt to physical explanations. Longfellow used to say that he hated sciences. I can sympathize with him. For he lived so entirely in the psychical

world, that science to him meant an endeavor to turn finality into efficiency; or as he would say, to refute poetry. It is most narrow not to consider final causes in the study of nature; but it is nonsense and utter confusion to treat them as forces in the material sense. Dr. Brinton, along with ethnologists generally, appears to be oblivious to this, throughout, and to look upon the study of the psychical from a psychical point of view as essentially inexact. To ask whether a given fact is due to psychical or physical causes is absurd. Every fact has a physical side; perhaps every fact has a psychical side. Its physical aspect -- as a mere motion -- is due exclusively to physical causes; its psychical aspect -- as a deed -- is due exclusively to psychical causes. This remains true, though you accept every doctrine of telepathy, table-turning, or what you will. If I can turn a table by the force of my will, this will simply establish the fact that something between me and the table acts just as a stick with which I should poke the table would act. It would be a physical connection purely and simply, however interesting it might be to a psychologist. But on the other hand, as my hand obeys, in a general way, my commands, clutching what I tell it to clutch, though I leave to its better judgment all the menu of just how my general order is to be carried out (and so I do with my rapier, directing its point to move so and so, but how it is done I never know), so the table-turning experiment would, I suppose, show that I could give similar general orders to the untouched table. That would be purely psychical, or final, causation, in which particulars are disregarded. Meantime, one may note that the table certainly *will* turn, if I really and truly will that it shall without being too meticulous about ways and means.

Peirce: CP 1.266 Cross-Ref:††

266. Three items only of Brinton's somatology belong to psychognosy. They are, first, prosopology, so far as that refers to the dubious science of physiognomy; second, psychology, and third, criminal-anthropology. A great part of his ethnography simply considers men as biological forms. So, too, the physical geography, geology, and paleontology that he includes. This latter I had not supposed belonged to the science of man. On the other hand much of psychognosy is omitted; such as the study of animal and vegetable instinct (both of which, especially the latter, throw much light on man's nature), theology as such (supposing there is such a science), economics, esthetics (so far as it is neither philosophy on the one hand, nor practical science on the other), and history in all its many branches (and it seems to me strange that Dr. Brinton, who makes almost everything else pertain to the science of man, should think that history does not do so), and biography.

Peirce: CP 1.267 Cross-Ref:††

267. Let us now, with Dr. Brinton's list before us, endeavor to survey psychognosy and make out its orders. In the first place, final causality, which is the object of psychical science, appears in three guises; first, quite detached from any biological organism; second, in biological individuals as vehicles; third, in societies, ranging from the family to that public which includes our indefinite "posterity." These distinctions, when we thus consider them together, impress us with a certain grandeur. It may be that this explains what, at any rate, is a fact, that the question has often pressed itself upon me whether they ought not to form the basis of the first division of the class of psychical sciences. But this would be merely, or mainly, a division according to the nature of the objects of study. We ought to classify the sciences according to their own natures; and not according to the nature of their objects in the least, except so far as this affects the nature of the studies of these objects.†P1 But before taking anything of that sort into account, we ought to look for a division based on the differences of the intellectual factor in the work of science, such as has been

found to constitute the three orders of physiognosy; to wit, the nomological, the classificatory, and the descriptive. These orders appear more and more clear, the further the subject is examined. Mind has its universal laws, operative wherever it is manifested, although these may be modified according to the mode of its incarnation or other manifestation. In studying the universal properties of mind, the student will, no doubt, have occasion to remark some of the peculiarities of different modes of manifestation of mind. It may easily happen to a young student that this study of special kinds of productions of mind comes to fascinate and absorb him far more than the thinner and abstracter science of mind's universal truths. It may happen to another student that while he makes elaborate studies of a special form of psychical fruit, he will never cease to pursue those studies with a view to their affording some clue to the general secrets of mind. Just so, a man may study the systems [of] crystals for the sake of their teachings concerning the nature of elasticity, as Rankine did, or in hopes of learning from them something about light, as Brewster did; or on the other hand, being interested in crystals and their classes, with a view to gaining a better comprehension of them, he may make studies of their cohesion, as Haüy did; and with either of these motives, he may produce a memoir which, in itself considered, might very well be classed either as a contribution to nomological physics or to crystallography. Take a larger view of his work, and there will be no possible doubt that Brewster and Rankine were physicists, while Haüy was a botanist turned crystallographer.

Peirce: CP 1.268 Cross-Ref:††

268. What shall we classifiers do with studies which might equally belong to either of two groups? Shall we, for the sake of convenience, allow a little artificiality to enter into our classification, so as to give such a study an undoubted place? That would be compromise. Now we ought always to be willing to compromise judiciously in practical matters, never in theoretical science. But if there be any such thing as natural classification, it is the truth, the theoretical truth, which is not to be sacrificed to convenience. It may be different with a classification of sciences designed to govern the alcove-arrangement of a library. That is a question for separate discussion. I will only remark here that the purposes of alcove-arrangement are so multifarious, that it will not in many cases happen that the integral convenience of any one artificial arrangement is markedly superior to that of the natural arrangement. The natural classification of science must be based on the study of the history of science; and it is upon this same foundation that the alcove-classification of a library must be based. The natural classification of science is to be a classification of men of science; and because each great man's works are published in collected form, the alcove-classification of a library must also be a classification of men. At any rate, for our convenience in this chapter the plain truth untampered with alone will answer. Where it happens that the truth is that the defining lines between natural classes are not absolutely definite, it is that truth which we want stated.

Peirce: CP 1.269 Cross-Ref:††

269. Mind has its universal mode of action, namely, by final causation. The microscopist looks to see whether the motions of a little creature show any purpose. If so, there is mind there. Passing from the little to the large, natural selection is the theory of how forms come to be adaptive, that is, to be governed by a *quasi* purpose. It suggests a machinery of efficiency to bring about the end -- a machinery inadequate perhaps -- yet which must contribute some help toward the result. But the being governed by a purpose or other final cause is the very essence of the psychical phenomenon, in general. There ought, therefore, one would think, to be under the

order of psychonomy, or nomological psychognosy, a suborder which should seek to formulate with exactitude the law of final causation and show how its workings are to be traced out.

Peirce: CP 1.270 Cross-Ref:††

270. But under this universal law of mind, there are other laws, it may be equally ubiquitous yet not so abstract. There is, first of all, the great law of association (including fusion), a principle strikingly analogous to gravitation, since it is an attraction between ideas. There are, besides, other general phenomena of mind not explicable by association. The laws of all these phenomena will be studied under a second suborder of special nomological psychology.

Peirce: CP 1.271 Cross-Ref:††

271. As a second order, we have psychotaxy, not a very good name for classificatory psychognosy or the study of kinds of mental manifestation. This order falls into two suborders, the one embracing studies of mental performances and products, the other of incarnations, or ensoulments of mind. To the latter suborder I would refer all studies of the minds of insects and (when there are any) of octopuses, of sexual characteristics, of the seven ages of human life, of professional and racial types, of temperaments and characters. To the former suborder, I would refer the vast and splendidly developed science of linguistics, of customs of all kinds, of Brinton's ethnology generally.

Peirce: CP 1.272 Cross-Ref:††

272. A third order of psychognosy is descriptive and explanatory, but not in any predominant degree inductive. Those sciences which are mainly descriptive, which tell, for example, what an explorer has found, which give accounts of systems, as metrology, chronology, numismatics, heraldry, or examine individual productions of man, will form a descriptive suborder; while those which narrate sequences of events and show how one leads to another -- History in short, whether of individuals, or of communities, or of fields of activity, or of the development of minds, or of forms of social institutions, will form a second suborder.

Peirce: CP 1.273 Cross-Ref:††

§5. THE DIVISIONS OF PHILOSOPHY †1

273. It is plain that philosophy cannot, like idioscopy, be split from top to bottom into an efficient and a final wing. For, not to mention other reasons, to philosophy must fall the task of comparing the two stems of causation and of exhuming their common root. In another way, however, philosophy falls asunder into two groups of studies to which the appellation of subclasses is alone appropriate, if we are to understand by a subclass a modification of that class-making sense in which philosophy may be said to be observational. For besides what constitutes -- in the present stage of the study, at least -- the main body of philosophy, resting exclusively upon universal experience, and imparting to it a tinge of necessity, there is a department of science which, while it rests, and can only rest, as to the bulk of it, upon universal experience, yet for certain special yet obtrusive points is obliged to appeal to the most specialized and refined observations, in order to ascertain what minute modifications of everyday experience they may introduce. If in these departments the teachings of ordinary experience took on the true complexion of

necessity, as they usually do, it would hardly be in our power to appeal to special experience to contradict them. But it is a remarkable fact that though inattentive minds do pronounce the dicta of ordinary experience in these cases to be necessary, they do not appear so to those who examine them more critically. For example, everyday experience is that events occur in time, and that time has but one dimension. So much appears necessary. For we should be utterly bewildered by the suggestion that two events were each anterior to the other or that, happening at different times, one was not anterior to the other. But a two-dimensional anteriority is easily shown to involve a self-contradiction. So, then, that time is one-dimensional is, for the present, necessary; and we know not how to appeal to special experience to disprove it. But that space is three-dimensional involves no such necessity. We can perfectly well suppose that atoms or their corpuscles move freely in four or more dimensions. So everyday experience seems to teach us that time flows continuously. But that we are not sure that it really does so, appears from the fact that many men of powerful minds who have examined the question are of the opinion that it is not so. Why may there not be a succession of stationary states, say a milliasse or so of them or perhaps an infinite multitude per second, and why may states of things not break abruptly from one to the next? Here the teachings of ordinary experience are, at least, difficult of ascertainment. There are cases where they are decidedly indefinite. Thus, such experience shows that the events of one day or year are not exactly like those of another, although in part there is a cyclical repetition. Speculative minds have asked whether there may not be a complete cycle at the expiration of which all things will happen again as they did before. Such is said to have been the opinion of Pythagoras; and the stoics took it up as a necessary consequence of their philistine views. Yet in our day, certain experiences, especially the inspiring history of science and art during the nineteenth century, have inclined many to the theory that there is endless progress, a definite current of change on the whole of the whole universe. What treasures would we not sacrifice for the sake of knowing for certain whether it really be so, or not! It is nothing to you or me, to our children, or to our remoter posterity. What concern have we with the universe, or with the course of ages? No more than my dog has in the book I am writing. Yet I dare say he would defend the manuscript from harm with his life. However, to return to the matter of progress, universal experience is rather for the notion than against it, since there is a current in time, so far as we can see: the past influences our intellect, the future our spirit, with entire uniformity. Still universal experience merely favors a guess as to larger periods.

Peirce: CP 1.274 Cross-Ref:††

274. There are two distinct questions to be answered concerning time, even when we have accepted the doctrine that it is strictly continuous. The first is, whether or not it has any exceptional instants in which it is discontinuous, -- any abrupt beginning and end. Philosophers there have been who have said that such a thing is inconceivable; but it is perfectly conceivable to a mind which takes up intelligently and seriously the task of forming the conception. Men who are ready to pronounce a thing impossible before they have seriously studied out the proper way of doing it, and especially without having submitted to a course of training in making the requisite exertion of will, merit contempt. When a man tells us something is inconceivable, he ought to accompany the assertion with a full narrative of all he has done in these two ways to see if it could not be conceived. If he fails to do that, he may be set down as a trifler. There is no difficulty in imagining that at a certain moment, velocity was suddenly imparted to every atom and corpuscle of the universe; before which all was absolutely motionless and dead. To say that there was no motion nor acceleration is to say there was no time. To say there was no action is to say there

was no actuality. However contrary to the evidence, then, such a hypothesis may be, it is perfectly conceivable. The other question is whether time is infinite in duration or not. If it has no flaw in its continuity, it must, as we shall see in chapter 4,†1 return into itself. This may happen after a finite time, as Pythagoras is said to have supposed, or in infinite time, which would be the doctrine of a consistent pessimism.

Peirce: CP 1.275 Cross-Ref:††

275. Measurement, as shall, in due course, be distinctly proved †2, is a business fundamentally of the same nature as classification; and just as there are artificial classifications in profusion, but only one natural classification, so there are artificial measurements to answer every demand; but only one of them is the natural measurement. If time returns into itself, an oval line is an icon [or analytic picture] of it. Now an oval line may be so measured as to be finite, as when we measure positions on a circle by an angular quantity, Θ , running up to 360 degrees, where it drops to 0 degree (which is the natural measure in the case of the circle); or it may be measured so that the measure shall once pass through infinity, in going round the circle, as when we project the positions on the circumference from one of them as a centre upon a straight line on which we measure the shadows by a rigid bar, as in the accompanying figure, here. This is measuring by $\tan \frac{1}{2} (\Theta - \{\text{THETA}\})$, instead of by Θ ; where $\{\text{THETA}\}$ depends upon the position of the centre of projection. Such a mode of measurement has the mathematical convenience of using every real number once and once only. It is quite possible, however, to measure so as to run over the whole gamut of numbers twice or more times. The single projection from a point



within the circle gives one repetition.

[\[Click here to view\]](#)

Peirce: CP 1.276 Cross-Ref:††

276. The question, however, is, What is the natural mode of measuring time? Has it absolute beginning and end, and does it reach or traverse infinity? Take time in the abstract and the question is merely mathematical. But we are considering a department of philosophy that wants to know how it is, not with pure mathematical time, but with the real time of history's evolution. This question concerns that evolution itself, not the abstract mathematical time. We observe the universe and discover some of its laws. Why, then, may we not discover the mode of its evolution? Is that mode of evolution, so far as we can discover, of such a nature that we must infer that it began and will end, whether this beginning and this end are distant from us by a finite number of days, hours, minutes, and seconds, or infinitely distant? In order to aid the reader in conceiving of a department of study which should make use of the discoveries of science to settle questions about the character of time as a whole, I have drawn three varieties of spirals.†1 The first of these has the equation $\Theta = (360 \bullet / \text{Log } 3) \log((r-1 \text{ inch}) / (3 \text{ inches}-r))$. Imagine each revolution round the centre of the pencil point tracing the spirals, to represent the lapse of a year or any other cycle of time; and let r , the radius vector, represent the measure of the degree of evolution of the universe -- it is not necessary to attach any more definite idea to it. Then, if the universe obeys this law of evolution, it had an absolute beginning at a point of time in the past immeasurable in years. The degree of its stage of evolution was from the very first a positive quantity, 1; which constantly increases toward 3 which it will never surpass until its final destruction in the infinitely distant future. The second spiral is not strictly logarithmic. Its equation is $\Theta = 360 \bullet \tan ((90 \bullet r) / (1 \text{ inch}))$ Here again the universe is represented improving from a stage where $r = 1$ in the infinitely distant

past to a stage where $r = 3$ in the infinitely distant future. But though this is infinitely distant when measured in years, evolution does not stop here, but continues uninterruptedly; and after another infinite series of years, $r = 5$; and so on endlessly. We must not allow ourselves to be drawn by the word "endless" into the fallacy of Achilles and the tortoise. Although, so long as r has not yet reached the value 3, another year will still leave it less than 3, yet if years do not *constitute* the flow of time, but only *measure* that flow, this in no wise prevents r from increasing in the flow of time beyond 3; so that it will be a question of fact whether or not, so far as we can make it out, the law of general evolution be such as to carry the universe beyond every fixed stage or not. It is very curious that in this case we can determine at exactly what season of the year in the infinitely distant future the value of r changes from being infinitesimally less to being infinitesimally more than 3. In the third spiral, of which the equation is $1/(r - 1/2 \text{ inch}) = 3 \log (1 + \text{anti-log } (90^\circ / (\Theta - 90^\circ)))$, the universe was created a *finite* number of years ago in a stage of evolution represented by $r = 1/2$, and will go on for an infinite series of years approximating indefinitely to a state where $r = 2$, after which it will begin to advance again, and will advance until after another infinite lapse of years it will then in a finite time reach the stage when $r = 3 \frac{1}{2}$, when it will be suddenly destroyed. This last spiral is much the most instructive of the three; but all are useful. The reader will do well to study them.

Peirce: CP 1.277 Cross-Ref:††

277. Whether it is possible to make any scientific study of such questions and of the corresponding questions concerning physical geometry is a problem into which careful inquiry will have to be made in a subsequent chapter.†1 I must assume that my reader will desire to have this difficult problem cleared up; for if he is still in that stage of intellectual development in which he holds that he has already reached infallible conclusions on certain points, *e.g.*, that twice two makes four, that it is bad manners to marry one's grandmother, that he exists, that yesterday the sun set in the west, etc., so that to hear them seriously doubted fills him with disgust and anger (a little merriment could, perhaps, hardly be suppressed, and would not imply absolute infallibility), he cannot yet gain much from the perusal of this book, and had better lay it aside. Meantime, while it is still doubtful whether or not any knowledge of this kind is attainable, in view of the extreme interest of the questions, and in view of the fact that men of no small intellectual rank are endeavoring to illuminate them, we should by all means leave, for the present, a lodging for this group of studies in our scheme of classification.

Peirce: CP 1.278 Cross-Ref:††

278. One might well ask, however, whether their proper place is in philosophy or not rather in idioscopy, since they rest in part upon special observation. Every department of idioscopy builds upon philosophy, as we have seen. How then are these studies not idioscopic? Or, if they are not that, why not treat them as the zoölogists treat the tunicates, which, being neither strictly vertebrates nor by any means worms, are held to constitute a separate branch of the animal kingdom? As to that, I confess I am a little sceptical as to the decision of the zoölogists. But keeping to our proper question, every department of idioscopy is based upon special observation, and only resorts to philosophy in order that certain obstacles to its pursuing its proper special observational inquiries may be cleared out of the way. The sciences which we are now considering, on the contrary, are based upon the same sort of general experience upon which philosophy builds; and they only resort to special observation to settle some minute details, concerning which the testimony of general experience is possibly insufficient. It is true that they are thus of a nature intermediate between

coenoscropy and idioscopy; but in the main their character is philosophical. They form, therefore, a second subclass of philosophy, to which we may give the name of *theôrics*. As inquiry now stands, this subclass has but two divisions which can hardly rank as orders, but rather as families, *chronothory* and *topotheôry*. This kind of study is in its first infancy. Few men so much as acknowledge that it is anything more than idle speculation. It may be that in the future the subclass will be filled up with other orders.

Peirce: CP 1.279 Cross-Ref:††

279. The first subclass, that of *necessary philosophy*, might be called *epistêmy*, since this alone among the sciences realizes the Platonic and generally Hellenic conception of {epistémé}.†P1 Under it, three orders stand out clearly.

Peirce: CP 1.280 Cross-Ref:††

280. The first of these is *Phenomenology*, or the Doctrine of Categories, whose business it is to unravel the tangled skein [of] all that in any sense appears and wind it into distinct forms; or in other words, to make the ultimate analysis of all experiences the first task to which philosophy has to apply itself. It is a most difficult, perhaps the most difficult, of its tasks, demanding very peculiar powers of thought, the ability to seize clouds, vast and intangible, to set them in orderly array, to put them through their exercises. The mere reading of this sort of philosophy, the mere understanding of it, is not easy. Anything like a just appreciation of it has not been performed by many of those who have written books. Original work in this department, if it is to be real and hitherto unformulated truth, is -- not to speak of whether it is difficult or not -- one of those functions of growth which every man, perhaps, in some fashion exercises once, some even twice, but which it would be next to a miracle to perform a third time.

Peirce: CP 1.281 Cross-Ref:††

281. Order II consists of the normative sciences. I wonder how many of those who make use of this term see any particular need of the word "normative." A normative science is one which studies what ought to be. How then does it differ from engineering, medicine, or any other practical science? If, however, logic, ethics, and esthetics, which are the families of normative science, are simply the arts of reasoning, of the conduct of life, and of fine art, they do not belong in the branch of theoretic science which we are alone considering, at all. There is no doubt that they are closely related to three corresponding arts, or practical sciences. But that which renders the word normative needful (and not purely ornamental) is precisely the rather singular fact that, though these sciences do study what ought to be, *i.e.*, ideals, they are the very most purely theoretical of purely theoretical sciences. What was it that Pascal †1 said? "La vraie morale se moque de la morale." It is not worth while, in this corner of the book, to dwell upon so prominent a feature of our subject. The peculiar tinge of mind in these normative sciences has already been much insisted upon. It will come out in stronger and stronger colors as we go on.

Peirce: CP 1.282 Cross-Ref:††

282. Order III consists of metaphysics,†2 whose attitude toward the universe is nearly that of the special sciences (anciently, *physics* was its designation), from which it is mainly distinguished, by its confining itself to such parts of physics and of psychics as can be established without special means of observation. But these are very peculiar parts, extremely unlike the rest.

Peirce: CP 1.283 Cross-Ref:††
§6. THE DIVISIONS OF MATHEMATICS

283. Having now attained to a pretty clear apprehension of what a natural order of science is -- deficient in distinctness though this apprehension be -- we cannot, if we have any acquaintance with mathematics, consider that class of science, without seeing that none more manifestly falls into orders than this. The hypotheses of mathematics relate to systems which are either finite collections, infinite collections, or true continua; and the modes of reasoning about these three are quite distinct. These, then, constitute three orders. The last and highest kind of mathematics, consisting of topical geometry, has hitherto made very little progress; and the methods of demonstration in this order are, as yet, little understood. The study of finite collections divides into two suborders: first, that simplest kind of mathematics which is chiefly used in its application to logic, from which I find it almost impossible to separate it †1; and secondly, the general theory of finite groups. The study of infinite collections likewise divides into two suborders; first, arithmetic, or the study of the least multitudinous of infinite collections; and second, the calculus, or the study of collections of higher multitude. Hitherto, the calculus has been entirely confined to the study of collections of the lowest multitude above that of the collection of all integral numbers. This is studied either algebraically or geometrically, or, much more commonly, and perhaps more advantageously (though it is out of fashion to think so), by the two methods combined. The traditional division of mathematics, still much used, is into geometry and algebra -- the division used by Jordanus Nemorarius †2 in the thirteenth century. It seems to me to be not only entirely artificial, but also extremely inconvenient from every point of view except the one of conforming to usage.†3

Peirce: CP 1.284 Cross-Ref:††
BOOK III

PHENOMENOLOGY

CHAPTER 1

INTRODUCTION†1

§1. THE PHANERON †2

284. Phaneroscopy is the description of the *phaneron*; and by the *phaneron* I mean the collective total of all that is in any way or in any sense present to the mind, quite regardless of whether it corresponds to any real thing or not. If you ask present *when*, and to *whose* mind, I reply that I leave these questions unanswered, never having entertained a doubt that those features of the phaneron that I have found in my mind are present at all times and to all minds. So far as I have developed this science of phaneroscopy, it is occupied with the formal elements of the phaneron. I know that there is another series of elements imperfectly represented by Hegel's Categories. But I have been unable to give any satisfactory account of them.

Peirce: CP 1.285 Cross-Ref:††

285. English philosophers have quite commonly used the word *idea* in a sense approaching to that which I give to *phaneron*. But in various ways they have restricted the meaning of it too much to cover my conception (if conception it can be called), besides giving a psychological connotation to their word which I am careful to exclude. The fact that they have the habit of saying that "there is no such idea" as this or that, in the very same breath in which they definitely describe the phaneron in question, renders their term fatally inapt for my purpose.

Peirce: CP 1.286 Cross-Ref:††

286. There is nothing quite so directly open to observation as phanerons; and since I shall have no need of referring to any but those which (or the like of which) are perfectly familiar to everybody, every reader can control the accuracy of what I am going to say about them. Indeed, he must actually repeat my observations and experiments for himself, or else I shall more utterly fail to convey my meaning than if I were to discourse of effects of chromatic decoration to a man congenitally blind. What I term *phaneroscopy* is that study which, supported by the direct observation of phanerons and generalizing its observations, signalizes several very broad classes of phanerons; describes the features of each; shows that although they are so inextricably mixed together that no one can be isolated, yet it is manifest that their characters are quite disparate; then proves, beyond question, that a certain very short list comprises all of these broadest categories of phanerons there are; and finally proceeds to the laborious and difficult task of enumerating the principal subdivisions of those categories.

Peirce: CP 1.287 Cross-Ref:††

287. It will be plain from what has been said that phaneroscopy has nothing at all to do with the question of how far the phanerons it studies correspond to any realities. It religiously abstains from all speculation as to any relations between its categories and physiological facts, cerebral or other. It does not undertake, but sedulously avoids, hypothetical explanations of any sort. It simply scrutinizes the direct appearances, and endeavors to combine minute accuracy with the broadest possible generalization. The student's great effort is not to be influenced by any tradition, any authority, any reasons for supposing that such and such ought to be the facts, or any fancies of any kind, and to confine himself to honest, single-minded observation of the appearances. The reader, upon his side, must repeat the author's observations for himself, and decide from his own observations whether the author's account of the appearances is correct or not.

Peirce: CP 1.288 Cross-Ref:††

§2. VALENCIES †1

288. There can be no psychological difficulty in determining whether anything belongs to the phaneron or not; for whatever seems to be before the mind *ipso facto* is so, in my sense of the phrase. I invite you to consider, not everything in the phaneron, but only its indecomposable elements, that is, those that are logically indecomposable, or indecomposable to direct inspection. I wish to make out a classification, or division, of these indecomposable elements; that is, I want to sort them into their different kinds according to their real characters. I have some acquaintance with two different such classifications, both quite true; and there may be others. Of these two I know of, one is a division according to the form or structure of the elements, the other according to their matter. The two most passionately laborious years of my life were exclusively devoted to trying to ascertain something for certain about the latter; but I abandoned the attempt as beyond my powers, or, at any rate, unsuited to my genius. I had not neglected to examine what others had done but could not persuade myself that they had been more successful than I. Fortunately, however, all taxonomists of every department have found classifications according to structure to be the most important.

Peirce: CP 1.289 Cross-Ref: ††

289. A reader may very intelligently ask, How is it possible for an indecomposable element to have any differences of structure? Of internal logical structure it would be clearly impossible. But of external structure, that is to say, structure of its possible compounds, limited differences of structure are possible; witness the chemical elements, of which the "groups," or vertical columns of Mendeléeff's table, are universally and justly recognized as ever so much more important than the "series," or horizontal ranks in the same table. Those columns are characterized by their several valencies, thus:

He, Ne, A, Kr, X are medads ({méden} none + the patronymic = {idés}).

H, L [Li], Na, K, Cu, Rb, Ag, Cs,-,-, Au, are monads;

G [Gl], Mg, Ca, Zn, Sr, Cd, Ba, -,-, Hg, Rd [Ra], are dyads;

B, Al, Sc, Ga, Y, In, La, -, Yb, Tc [Tl], Ac are triads;

C, Si, Ti, Ge, Zr, Sn, Co [Ce], -, -, Pc [Pb], Th, are tetrads;

N, P, V, As, Cb, Sb, Pr [Nd], -, Ta, Bi, Po [Pa], are properly pentads (as PCL[5], though owing to the junction of two pegs they often appear as triads. Their pentad character is particularly required to explain certain phenomena of albumins); O, S, Cr, Se, Mo, Te, Nd [Sm], -, W, -, U, are properly hexads (though by junction of bonds they usually appear as dyads);

F, Cl, Mn, Br, -, I, are properly heptads (usually appearing as monads);

Fe, Co, Ni, Ru, Rh, Pd, -, -, -, Os, Tr [Ir], Pt, are octads; (Sm, Eu, Gd, Er, Tb, Bz [?], Cl [Ct], are not yet placed in the table.)

Peirce: CP 1.290 Cross-Ref: ††

290. So, then, since elements may have structure through valency, I invite the reader to join me in a direct inspection of the valency of elements of the phaneron. Why do I seem to see my reader draw back? Does he fear to be compromised by my

bias, due to preconceived views? Oh, very well; yes, I do bring some convictions to the inquiry. But let us begin by subjecting these to criticism, postponing actual observation until all preconceptions are disposed of, one way or the other.

Peirce: CP 1.291 Cross-Ref:††

291. First, then, let us ask whether or not valency is the sole formal respect in which elements of the phaneron can possibly vary. But seeing that the possibility of such a ground of division is dependent upon the possibility of multivalence, while the possibility of a division according to valency can in nowise be regarded as a result of relations between bonds, it follows that any division by variations of such relations must be taken as secondary to the division according to valency, if such division there be. Now (my logic here may be puzzling, but it is correct), since my ten trichotomies of signs,†1 should they prove to be independent of one another (which is to be sure, highly improbable), would suffice to furnish us classes of signs to the number of

$$\begin{aligned}
 3^{10} &= (3^2)^5 = (10-1)^5 = 10^5 - 5 \cdot 10^4 \\
 &\quad + 10 \cdot 10^3 - 10 \cdot 10^2 \\
 &\quad + 5 \cdot 10 - 1 \\
 &= 50000 \\
 &\quad + 9000 \\
 &\quad + 49 \\
 &= 59049
 \end{aligned}$$

(*Voilà* a lesson in vulgar arithmetic thrown in to boot!), which calculation threatens a multitude of classes too great to be conveniently carried in one's head, rather than a group inconveniently small, we shall, I think, do well to postpone preparations for further divisions until there be prospect of such a thing being wanted.

Peirce: CP 1.292 Cross-Ref:††

292. If, then, there be any formal division of elements of the phaneron, there must be a division according to valency; and we may expect medads, monads, dyads, triads, tetrads, etc. Some of these, however, can be antecedently excluded, as impossible; although it is important to remember that these divisions are not exactly like the corresponding divisions of Existential Graphs,†1 which have relation only to explicit indefinites. In the present application, a medad must mean an indecomposable idea altogether severed logically from every other; a monad will mean an element which, except that it is thought as applying to some subject, has no other characters than those which are complete in it without any reference to anything else; a dyad will be an elementary idea of something that would possess such characters as it does possess relatively to something else but regardless of any third object of any category; a triad would be an elementary idea of something which should be such as it were relatively to two others in different ways, but regardless of any fourth; and so on. Some of these, I repeat, are plainly impossible. A medad would be a flash of mental "heat-lightning" absolutely instantaneous, thunderless, unremembered, and altogether without effect. It can further be said in advance, not, indeed, purely *a priori* but with

the degree of apriority that is proper to logic, namely, as a necessary deduction from the fact that there are signs, that there must be an elementary triad. For were every element of the phaneron a monad or a dyad, without the relative of teridentity †2 (which is, of course, a triad), it is evident that no triad could ever be built up. Now the relation of every sign to its object and interpretant is plainly a triad. A triad might be built up of pentads or of any higher perissad elements in many ways. But it can be proved -- and really with extreme simplicity, though the statement of the general proof is confusing -- that no element can have a higher valency than three.

Peirce: CP 1.293 Cross-Ref: ††
§3. MONADS, DYADS, AND TRIADS †1

293. A thorough study of the logic of relatives confirms the conclusions which I had reached before going far in that study. It shows that logical terms are either monads, dyads, or polyads, and that these last do not introduce any radically different elements from those that are found in triads. I therefore divide all objects into monads, dyads, and triads; and the first step in the present inquiry is to ascertain what are the conceptions of the pure monad, free from all dyadic and triadic admixtures; of the dyad (which involves that of the monad) free from all triadic contamination, and what it is that is peculiar which the dyad adds to the monad; and of the triad (which involves those of the monad and dyad) and what it is that is characteristic of the triad.

Peirce: CP 1.294 Cross-Ref: ††
§4. INDECOMPOSABLE ELEMENTS †2

294. I doubt not that readers have been fretting over the ridiculous-seeming phrase "indecomposable element," which is as Hibernian as "necessary and sufficient condition" (as if "condition" meant no more than *concomitant* and as [if] *needful* were not the proper accompaniment of "sufficient"). But I have used it because I do not mean simply *element*. Logical analysis is not an analysis into existing elements. It is the tracing out of relations between concepts on the assumption that along with each given or found concept is given its negative, and every other relation resulting from a transposition of its correlates. The latter postulate amounts to merely identifying each correlate and distinguishing it from the others without recognizing any serial order among them. Thus to love and to be loved are regarded as the same concept, and not to love is also to be considered as the same concept. The combination of concepts is always by two at a time and consists in indefinitely identifying a subject of the one with a subject of the other, every correlate being regarded as a subject. Then if one concept can be accurately defined as a combination of others, and if these others are not of more complicated structure than the defined concept, then the defined concept is regarded as *analyzed* into these others. Thus A is grandparent of B, if and only if A is a parent of somebody who is a parent of B, therefore grandparent is analyzed into parent and parent. So stepparent, if taken as not excluding parentage, is analyzed into spouse and parent; and parent-in-law into parent and spouse.

Peirce: CP 1.295 Cross-Ref:††

295. These things being premised we may say in *primo*, there is no *a priori* reason why there should not be indecomposable elements of the phaneron which are what they are regardless of anything else, each complete in itself; provided, of course, that they be capable of composition. We will call these and all that particularly relates to them *Priman*. Indeed, it is almost inevitable that there should be such, since there will be compound concepts which do not refer to anything, and it will generally be possible to abstract from the internal construction that makes them compound, whereupon they become indecomposable elements.

Peirce: CP 1.296 Cross-Ref:††

296. In *secundo*, there is no *a priori* reason why there should not be indecomposable elements which are what they are relatively to a second but independently of any third. Such, for example, is the idea of otherness. We will call such ideas and all that is marked by them *Secundan* (*i.e.*, dependent on a second).

Peirce: CP 1.297 Cross-Ref:††

297. In *tertio* there is no *a priori* reason why there should not be indecomposable elements which are what they are relatively to a second and a third, regardless of any fourth. Such, for example, is the idea of *composition*. We will call everything marked by being a third or medium of connection, between a first and second anything, *tertian*.

Peirce: CP 1.298 Cross-Ref:††

298. It is *a priori* impossible that there should be an indecomposable element which is what it is relatively to a second, a third, and a fourth. The obvious reason is that that which combines two will by repetition combine any number.†P1 Nothing could be simpler; nothing in philosophy is more important.

Peirce: CP 1.299 Cross-Ref:††

299. We find then *a priori* that there are three categories of undecomposable elements to be expected in the phaneron: those which are simply positive totals, those which involve dependence but not combination, those which involve combination.

Peirce: CP 1.299 Cross-Ref:††

Now let us turn to the phaneron and see what we find in fact.

Peirce: CP 1.300 Cross-Ref:††
CHAPTER 2

THE CATEGORIES IN DETAIL

A. FIRSTNESS

§1. THE SOURCE OF THE CATEGORIES †1

300. The *list of categories*, or as Harris,^{†2} the author of *Hermes*, called them, the "philosophical arrangements," is a table of conceptions drawn from the logical analysis of thought and regarded as applicable to being. This description applies not merely to the list published by me in 1867,^{†3} and which I here endeavor to amplify, but also to the categories of Aristotle and to those of Kant. The latter have been more or less modified by different critics, as Renouvier, and still more profoundly by Hegel. My own list grew originally out of the study of the table of Kant.

Peirce: CP 1.301 Cross-Ref:††

301. I shall not here inquire how far it is justifiable to apply the conceptions of logic to metaphysics. For I hold the importance of that question, great as it is, to be perhaps secondary, and at any rate not paramount to that of the question what such conceptions would be. I may say, however, that in my own opinion, each category has to justify itself by an inductive examination which will result in assigning to it only a limited or approximate validity.

Peirce: CP 1.302 Cross-Ref:††

§2. THE MANIFESTATION OF FIRSTNESS †4

302. The idea of First is predominant in the ideas of freshness, life, freedom. The free is that which has not another behind it, determining its actions; but so far as the idea of the negation of another enters, the idea of another enters; and such negative idea must be put in the background, or else we cannot say that the Firstness is predominant. Freedom can only manifest itself in unlimited and uncontrolled variety and multiplicity; and thus the first becomes predominant in the ideas of measureless variety and multiplicity. It is the leading idea of Kant's "manifold of sense." But in Kant's synthetic unity the idea of Thirdness is predominant. It is an attained unity; and would better have been called totality; for that is the one of his categories in which it finds a home. In the idea of being, Firstness is predominant, not necessarily on account of the abstractness of that idea, but on account of its self-containedness. It is not in being separated from qualities that Firstness is most predominant, but in being something peculiar and idiosyncratic. The first is predominant in feeling, as distinct from objective perception, will, and thought.

Peirce: CP 1.303 Cross-Ref:††

§3. THE MONAD †1

303. The pure idea of a *monad* is not that of an object. For an object is over against me. But it is much nearer an object than it is to a conception of self, which is still more complex. There must be some determination, or suchness, otherwise we shall think nothing at all. But it must not be an abstract suchness, for that has reference to a special suchness. It must be a special suchness with some degree of determination, not, however, thought as more or less. There is to be no comparison. So that it is a suchness *sui generis*. Imagine me to make and in a slumberous

condition to have a vague, unobjectified, still less unsubjectified, sense of redness, or of salt taste, or of an ache, or of grief or joy, or of a prolonged musical note. That would be, as nearly as possible, a purely monadic state of feeling. Now in order to convert that psychological or logical conception into a metaphysical one, we must think of a metaphysical monad as a pure nature, or quality, in itself without parts or features, and without embodiment. Such is a pure monad. The meanings of names of "secondary" qualities are as good approximations to examples of monads as can be given.

Peirce: CP 1.304 Cross-Ref:††
§4. QUALITIES OF FEELING †1

304. . . . Among phanerons there are certain qualities of feeling, such as the color of magenta, the odor of attar, the sound of a railway whistle, the taste of quinine, the quality of the emotion upon contemplating a fine mathematical demonstration, the quality of feeling of love, etc. I do not mean the sense of actually experiencing these feelings, whether primarily or in any memory or imagination. That is something that involves these qualities as an element of it. But I mean the qualities themselves which, in themselves, are mere may-bes, not necessarily realized. The reader may be inclined to deny that. If so, he has not fully grasped the point that we are not considering what is true, not even what truly appears. I ask him to note that the word *red* means something when I say that the precession of the equinoxes is no more red than it is blue, and that it means just what it means when I say that aniline red is red. That mere *quality*, or suchness, is not in itself an occurrence, as seeing a red object is; it is a mere may-be. Its only being consists in the fact that there *might be* such a peculiar, positive, suchness in a phaneron. When I say it is a quality, I do not mean that it "inheres" in [a] subject. That is a phaneron peculiar to metaphysical thought, not involved in the sensation itself, and therefore not in the quality of feeling, which is entirely contained, or superseded, in the actual sensation. The Germans usually call these qualities feelings, feelings *of pleasure or pain*. To me this seems to be mere repetition of a tradition, never subjected to the test of observation. I can imagine a consciousness whose whole life, alike when wide awake and when drowsy or dreaming, should consist in nothing at all but a violet color or a stink of rotten cabbage. It is purely a question of what I can imagine and not of what psychological laws permit. The fact that I can imagine this, shows that such a feeling is not *general*, in the sense in which the law of gravitation is general. For nobody can imagine that law to have any being of any kind if it were impossible that there should exist any two masses of matter, or if there were no such things as motion. A true general cannot have any being unless there is to be some prospect of its sometime having occasion to be embodied in a fact, which is itself not a law or anything like a law. A quality of feeling can be imagined to be without any occurrence, as it seems to me. Its mere may-being gets along without any realization at all.

Peirce: CP 1.305 Cross-Ref:††
§5. FEELING AS INDEPENDENT OF MIND AND CHANGE †1

305. Suppose I begin by inquiring of you, Reader, in what particulars a feeling of redness or of purple without beginning, end, or change; or an eternally sounding and unvarying railway whistle; or a sempiterna thrill of joyous delight -- or rather, such as would afford us delight, but supposed to be in that respect quite neutral -- that should constitute the entire universe, would differ from a substance? I suppose you will tell me that no such thing could be alone in the universe because, firstly, it would require a mind to feel it, which would not be the feeling itself; secondly, the color or sound and probably also the thrill of delight would consist of vibrations; thirdly, none of them could last forever without a flow of time; fourthly, each would have a quality, which would be a determination in several respects, the color in hue, luminosity, chroma, and vividness; the sound in pitch, timbre (itself highly complex), loudness, and vividness; the delight more or less sensual, more or less emotional, more or less elevated, etc.; and fifthly, each would require a physical substratum altogether disparate to the feeling itself. But I point out to you that these things are only known to us by extraneous experience; none of them are either seen in the color, heard in the sound, or felt in the visceral sensation. Consequently, there can be no logical difficulty in supposing them to be absent, and for my part, I encounter not the slightest psychological difficulty in doing so, either. To suppose, for example, that there is a flow of time, or any degree of vividness, be it high or low, seems to me quite as uncalled for as to suppose that there is freedom of the press or a magnetic field.

Peirce: CP 1.306 Cross-Ref:††
§6. A DEFINITION OF FEELING †1

306. By a feeling, I mean an instance of that kind of consciousness which involves no analysis, comparison or any process whatsoever, nor consists in whole or in part of any act by which one stretch of consciousness is distinguished from another, which has its own positive quality which consists in nothing else, and which is of itself all that it is, however it may have been brought about; so that if this feeling is present during a lapse of time, it is wholly and equally present at every moment of that time. To reduce this description to a simple definition, I will say that by a feeling I mean an instance of that sort of element of consciousness which is all that it is positively, in itself, regardless of anything else.

Peirce: CP 1.307 Cross-Ref:††

307. A feeling, then, is not an event, a happening, a coming to pass, since a coming to pass cannot be such unless there was a time when it had not come to pass; and so it is not in itself all that it is, but is relative to a previous state. A feeling is a *state*, which is in its entirety in every moment of time as long as it endures. But a feeling is not a single state which is other than an exact reproduction of itself. For if that reproduction is in the same mind, it must be at a different time, and then the being of the feeling would be relative to the particular time in which it occurred, which would be something different from the feeling itself, violating the definition which makes the feeling to be all that it is regardless of anything else. Or, if the reproduction were simultaneous with the feeling, it must be in another mind, and thus the identity of the feeling would depend upon the mind in which it was, which is other than the feeling; and again the definition would be violated in the same way. Thus,

any feeling must be identical with any exact duplicate of it, which is as much as to say that the feeling is simply a quality of immediate consciousness.

Peirce: CP 1.308 Cross-Ref:††

308. But it must be admitted that a feeling experienced in an outward sensation may be reproduced in memory. For to deny this would be idle nonsense. For instance, you experience, let us say, a certain color sensation due to red-lead. It has a definite hue, luminosity, and chroma. These [are] three elements -- which are not separate in the feeling, it is true, and are not, therefore, in the feeling at all, but are said to be in it, as a way of expressing the results which would follow, according to the principles of chromatics, from certain experiments with a color disk, color-box, or other similar apparatus. In that sense, the color sensation which you derive from looking at the red-lead has a certain hue, luminosity, and chroma which completely define the quality of the color. The *vividness*, however, is independent of all three of these elements; and it is very different in the memory of the color a quarter of a second after the actual sensation from what it is in the sensation itself, although this memory is conceivably perfectly true as to hue, luminosity, and chroma, which truth constitutes it an exact reproduction of the entire quality of the feeling.

Peirce: CP 1.309 Cross-Ref:††

309. It follows that since the *vividness* of a feeling -- which would be more accurately described as the vividness of a consciousness of the feeling -- is independent of every component of the quality of that consciousness, and consequently is independent of the resultant of those components, which resultant quality is the feeling itself. We thus learn what vividness is not; and it only remains to ascertain what else it is.

Peirce: CP 1.310 Cross-Ref:††

310. To this end two remarks will be useful. The first is that of whatever is in the mind in any mode of consciousness there is necessarily an immediate consciousness and consequently a feeling. The proof of this proposition is very instructive as to the nature of feeling; for it shows that, if by *psychology* we mean the positive, or observational, science of the mind or of consciousness, then although the entire consciousness at any one instant is nothing but a feeling, yet psychology can teach us nothing of the nature of feeling, nor can we gain knowledge of any feeling by introspection, the feeling being completely veiled from introspection, for the very reason that it is our immediate consciousness. Possibly this curious truth was what Emerson was trying to grasp -- but if so, pretty unsuccessfully -- when he wrote the lines,

The old Sphinx bit her thick lip --

Said, "Who taught thee me to name?"

I am thy spirit, yoke-fellow,

Of thine eye I am eyebeam.

"Thou art the unanswered question;

Couldst see thy proper eye,

Always it asketh, asketh;

And each answer is a lie."

But whatever he may have meant, it is plain enough that all that is immediately present to a man is what is in his mind in the present instant. His whole life is in the present. But when he asks what is the content of the present instant, his question always comes too late. The present has gone by, and what remains of it is greatly metamorphosed. He can, it is true, recognize that he was at that time, for example, looking at a specimen of red-lead, and must have seen that color, which, he perceives, is something positive and *sui generis*, of the nature of feeling. But nobody's immediate consciousness, unless when he was much more than half asleep, ever consisted wholly of a color-sensation; and since a feeling is absolutely simple and without parts -- as it evidently is, since it is whatever it is regardless of anything else, and therefore regardless of any part, which would be something other than the whole -- it follows that if the red color-sensation was not the whole feeling of the instant it has nothing in common with the feeling of the instant. Indeed, although a feeling is immediate consciousness, that is, is whatever of consciousness there may be that is immediately present, yet there is no consciousness in it because it is instantaneous. For we have seen already that feeling is nothing but a quality, and a quality is not conscious: it is a mere possibility. We can, it is true, see what a feeling in general is like; that, for example, this or that red is a feeling; and it is perfectly conceivable that a being should have that color for its entire consciousness, throughout a lapse of time, and therefore at every instant of that time. But such a being could never know anything about its own consciousness. It could not think anything that is expressible as a proposition. It could have no idea of such a thing. It would be confined to feeling that color. Thus, if you perceive that you must at the instant in question have been looking at a given specimen of red-lead, you know that that color has some resemblance to your feeling at that instant. But this only means that when the feeling gives place to comparison this resemblance appears. But there is no resemblance at all in feeling, since feeling is whatever it is, positively and regardless of anything else, while the resemblance of anything lies in the comparison of that thing with something else. . . .

Peirce: CP 1.311 Cross-Ref:††

311. Every operation of the mind, however complex, has its absolutely simple feeling, the emotion of the *tout ensemble*. This is a secondary feeling or sensation excited from within the mind, just as the qualities of outward sense are excited by something psychic without us. It seems at first glance unaccountable that a mere slight difference in the speed of vibration should make such a difference of quality as that between deep vermillion and violet blue. But then it is to be remembered that it is doubtless our imperfect knowledge of those vibrations which has led us to represent them abstractly as differing only in quantity. There is already a hint in the behavior of electrons that a lower speed and a greater one have differences which we have not been aware of. People wonder, too, how dead matter can excite feelings in the mind. For my part, instead of wondering how it can be, I feel much disposed to deny downright that it is possible. These new discoveries have reminded us how very little we know of the constitution of matter; and I prefer to guess that it is a psychic feeling of red without us which arouses a sympathetic feeling of red in our senses.

Peirce: CP 1.312 Cross-Ref:††

§7. THE SIMILARITY OF FEELINGS OF DIFFERENT SENSORY MODES †1

312. One of the old Scotch psychologists, whether it was Dugald Stewart or Reid †2 or which other matters naught, mentions, as strikingly exhibiting the disparateness of different senses, that a certain man blind from birth asked of a person of normal vision whether the color scarlet was not something like the blare of a trumpet; and the philosopher evidently expects his readers to laugh with him over the incongruity of the notion. But what he really illustrates much more strikingly is the dullness of apprehension of those who, like himself, had only the conventional education of the eighteenth century and remained wholly uncultivated in comparing ideas that in their matter are very unlike. For everybody who has acquired the degree of susceptibility which is requisite in the more delicate branches of reasoning -- those kinds of reasoning which our Scotch psychologist would have labelled "*Intuitions*" with a strong suspicion that they were delusions -- will recognize at once so decided a likeness between a luminous and extremely chromatic scarlet, like that of the iodide of mercury as commonly sold under the name of scarlet [and the blare of a trumpet] that I would almost hazard a guess that the form of the chemical oscillations set up by this color in the observer will be found to resemble that of the acoustical waves of the trumpet's blare. I am only deterred from doing so by its being apparently true that our sense of hearing is entirely analytic; so that we are totally deaf to the wave of sound as it exists, and only hear the harmonic components regardless of the phases at which vibrations of commensurable lengths are combined.

Peirce: CP 1.313 Cross-Ref:††

§8. PRESENTMENTS AS SIGNS †1

313. A mere presentment may be a sign. When the traditional blind man said he thought scarlet must be something like the sound of a trumpet, he had caught its blatancy very well; and the sound is certainly a presentment, whether the color †P1 is so or not. Some colors are called gay, others sad. The sentiment of tones is even more familiar; that is, tones are signs of visceral qualities of feeling. But the best example is that of odors, for these are signs in more than one way. It is a common observation that odors bring back old memories. This I think must be due, in part at least, to the fact that, whether from the peculiar connection of the olfactory nerve with the brain or from some other cause, odors have a remarkable tendency to *presentmentate* themselves, that is to occupy the entire field of consciousness, so that one almost lives for the moment in a world of odor. Now in the vacuity of this world, there is nothing to obstruct the suggestions of association. That is one way, namely by contiguous association, in which odors are particularly apt to act as signs. But they also have a remarkable power of calling to mind mental and spiritual qualities. This must be an effect of resemblance-association, if under resemblance-association we include all *natural* associations of different ideas. I certainly would do this; for I do not know what else resemblance can consist in.

Peirce: CP 1.313 Cross-Ref:††

A lady's favorite perfume seems to me somehow to agree with that of her

spiritual being. If she uses none at all her nature will lack perfume. If she wears violet she herself will have the very same delicate fineness. Of the only two I have known to use rose, one was an artistic old virgin, a *grande dame*; the other a noisy young matron and very ignorant; but they were strangely alike. As for those who use heliotrope, frangipanni, etc., I know them as well as I desire to know them. Surely there must be some subtle resemblance between the odor and the impression I get of this or that woman's nature.

Peirce: CP 1.314 Cross-Ref:††
§9. THE COMMUNICABILITY OF FEELINGS †1

314. Philosophers, who very properly call all things into question, have asked whether we have any reason to suppose that red looks to one eye as it does to another. I answer that slight differences there may be, but [consider the blind man imagining] red to resemble the blare of a trumpet. He had collected that notion from hearing ordinary people converse together about colors, and since I was not born to be one of those whom he had heard converse, the fact that I can see a certain analogy, shows me not only that my feeling of redness is something like the feelings of the persons whom he had heard talk, but also his feeling of a trumpet's blare was very much like mine. I am confident that a bull and I feel much alike at the sight of a red rag. As for the senses of my dog, I must confess that they seem very unlike my own, but when I reflect to how small a degree he thinks of visual images, and of how *smells* play a part in his thoughts and imaginations analogous to the part played by *sights* in mine, I cease to be surprised that the perfume of roses or of orange flowers does not attract his attention at all and that the effluvia that interest him so much, when at all perceptible to me, are simply unpleasant. He does not think of smells as sources of pleasure and disgust but as sources of information, just as I do not think of blue as a nauseating color, nor of red as a maddening one. I know very well that my dog's musical feelings are quite similar to mine though they agitate him more than they do me. He has the same emotions of affection as I, though they are far more moving in his case. You would never persuade me that my horse and I do not sympathize, or that the canary bird that takes such delight in joking with me does not feel with me and I with him; and this instinctive confidence of mine that it is so, is to my mind evidence that it really is so. My metaphysical friend who asks whether we can ever enter into one another's feelings -- and one particular sceptic whom I have in mind is a most exceptionally sympathetic person, whose doubts are born of her intense interest in her friends -- might just as well ask me whether I am sure that red looked to me yesterday as it does today and that memory is not playing me false. I know experimentally that sensations do vary slightly even from hour to hour; but in the main the evidence is ample that they are common to all beings whose senses are sufficiently developed.

Peirce: CP 1.315 Cross-Ref:††

315. I hear you say: "All that is not *fact*; it is poetry." Nonsense! Bad poetry is false, I grant; but nothing is truer than true poetry. And let me tell the scientific men that the artists are much finer and more accurate observers than they are, except of the special minutiae that the scientific man is looking for.

Peirce: CP 1.316 Cross-Ref:††

316. I hear you say: "This smacks too much of an anthropomorphic

conception." I reply that every scientific explanation of a natural phenomenon is a hypothesis that there is something in nature to which the human reason is analogous; and that it really is so all the successes of science in its applications to human convenience are witnesses. They proclaim that truth over the length and breadth of the modern world. In the light of the successes of science to my mind there is a degree of baseness in denying our birthright as children of God and in shamefacedly slinking away from anthropomorphic conceptions of the universe.

Peirce: CP 1.317 Cross-Ref:††
§10. TRANSITION TO SECONDNESS †1

317. The whole content of consciousness is made up of qualities of feeling, as truly as the whole of space is made up of points or the whole of time of instants.

Peirce: CP 1.318 Cross-Ref:††

318. Contemplate anything by itself -- anything whatever that can be so contemplated. Attend to the whole and drop the parts out of attention altogether. One can approximate nearly enough to the accomplishment of that to see that the result of its perfect accomplishment would be that one would have in his consciousness at the moment nothing but a quality of feeling. This quality of feeling would in itself, as so contemplated, have no parts. It would be unlike any other such quality of feeling. In itself, it would not even resemble any other; for resemblance has its being only in comparison. It would be a pure *priman*. Since this is true of whatever we contemplate, however complex may be the object, it follows that there is nothing else in immediate consciousness. To be conscious is nothing else than to feel.

Peirce: CP 1.319 Cross-Ref:††

319. What room, then, is there for *secundans* and *tertians*? Was there some mistake in our demonstration that they must also have their places in the phaneron? No, there was no mistake. I said that the phaneron is made up entirely of qualities of feeling as truly as space is entirely made up of points. There is a certain *protoidal* aspect -- I coin the word for the need -- under which space is truly made up of nothing but points. Yet it is certain that no collection of points -- using the word collection to mean merely a plural, without the idea of the objects being brought together -- no collection of points, no matter how abnumerable its multitude, can in itself constitute space. . . .

Peirce: CP 1.320 Cross-Ref:††

320. The phaneron does contain genuine *secundans*. Standing on the outside of a door that is slightly ajar, you put your hand upon the knob to open and enter it. You experience an unseen, silent resistance. You put your shoulder against the door and, gathering your forces, put forth a tremendous effort. Effort supposes resistance. Where there is no effort there is no resistance, where there is no resistance there is no effort either in this world or any of the worlds of possibility. It follows that an effort is not a feeling nor anything *priman* or protoidal. There are feelings connected with it: they are the sum of consciousness during the effort. But it is conceivable that a man should have it in his power directly to summon up all those feelings, or any feelings. He could not, in any world, be endowed with the power of summoning up an effort to which there did not happen to be a resistance all ready to exist. For it is an absurdity to suppose that a man could directly will to oppose that very will. A very little

thinking will show that this is what it comes to. According to such psychological analysis as I can make, effort is a phenomenon which only arises when one feeling abuts upon another in time, and which then always arises. But my psychological pretensions are little, if they exist at all, and I only mention my theory in order that contrast should impress the reader with the irrelevancy of psychology to our present problem, which is to say of what sort that is which is in our minds when we make an effort and which constitutes it an effort.

Peirce: CP 1.321 Cross-Ref:††

321. We live in two worlds, a world of fact and a world of fancy. Each of us is accustomed to think that he is the creator of his world of fancy; that he has but to pronounce his fiat, and the thing exists, with no resistance and no effort; and although this is so far from the truth that I doubt not that much the greater part of the reader's labor is expended on the world of fancy, yet it is near enough the truth for a first approximation. For this reason we call the world of fancy the internal world, the world of fact the external world. In this latter we are masters, each of us, of his own voluntary muscles, and of nothing more. But man is sly, and contrives to make this little more than he needs. Beyond that, he defends himself from the angles of hard fact by clothing himself with a garment of contentment and of habituation. Were it not for this garment, he would every now and then find his internal world rudely disturbed and his fiats set at naught by brutal inroads of ideas from without. I call such forcible modification of our ways of thinking the influence of the world of fact or *experience*. But he patches up his garment by guessing what those inroads are likely to be and carefully excluding from his internal world every idea which is likely to be so disturbed. Instead of waiting for experience to come at untoward times, he provokes it when it can do no harm and changes the government of his internal world accordingly.

Peirce: CP 1.322 Cross-Ref:††

B. SECONDNESS

§1. FEELING AND STRUGGLE †1

322. The second category that I find, the next simplest feature common to all that comes before the mind, is the element of struggle.

Peirce: CP 1.322 Cross-Ref:††

This is present even in such a rudimentary fragment of experience as a simple feeling. For such a feeling always has a degree of vividness, high or low; and this vividness is a sense of commotion, an action and reaction, between our soul and the stimulus. If, in the endeavor to find some idea which does not involve the element of struggle, we imagine a universe that consists of a single quality that never changes, still there must be some degree of steadiness in this imagination, or else we could not think about and ask whether there was an object having any positive suchness. Now this steadiness of the hypothesis that enables us to think about it -- and to mentally manipulate it -- which is a perfectly correct expression, because our thinking about the hypothesis really consists in making experiments upon it -- this steadiness, I say, consists in this, that if our mental manipulation is delicate enough, the hypothesis will

resist being changed. Now there can be no resistance where there is nothing of the nature of struggle or forceful action. By struggle I must explain that I mean mutual action between two things regardless of any sort of third or medium, and in particular regardless of any law of action.

Peirce: CP 1.323 Cross-Ref:††

323. I should not wonder if somebody were to suggest that perhaps the idea of a law is essential to the idea of one thing acting upon another. But surely that would be the most untenable suggestion in the world considering that there is no one of us who after lifelong discipline in looking at things from the necessitarian point of view †2 has ever been able to train himself to dismiss the idea that he can perform any specifiable act of the will. It is one of the most singular instances of how a preconceived theory will blind a man to facts that many necessitarians seem to think that nobody really believes in the freedom of the will, the fact being that he himself believes in it when he is not theorizing. However, I do not think it worth while to quarrel about that. Have your necessitarianism if you approve of it; but still I think you must admit that no law of nature makes a stone fall, or a Leyden jar to discharge, or a steam engine to work.

Peirce: CP 1.324 Cross-Ref:††

§2. ACTION AND PERCEPTION †1

324. [There is a category] which the rough and tumble of life renders most familiarly prominent. We are continually bumping up against hard fact. We expected one thing, or passively took it for granted, and had the image of it in our minds, but experience forces that idea into the background, and compels us to think quite differently. You get this kind of consciousness in some approach to purity when you put your shoulder against a door and try to force it open. You have a sense of resistance and at the same time a sense of effort. There can be no resistance without effort; there can be no effort without resistance. They are only two ways of describing the same experience. It is a double consciousness. We become aware of ourself in becoming aware of the not-self. The waking state is a consciousness of reaction; and as the consciousness *itself* is two-sided, so it has also two varieties; namely, action, where our modification of other things is more prominent than their reaction on us, and perception, where their effect on us is overwhelmingly greater than our effect on them. And this notion, of being such as other things make us, is such a prominent part of our life that we conceive other things also to exist by virtue of their reactions against each other. The idea of other, of *not*, becomes a very pivot of thought. To this element I give the name of Secondness.

Peirce: CP 1.325 Cross-Ref:††

§3. THE VARIETIES OF SECONDNESS †2

325. The idea of second is predominant in the ideas of causation and of statical force. For cause and effect are two; and statical forces always occur between pairs. Constraint is a Secondness. In the flow of time in the mind, the past appears to

act directly upon the future, its effect being called memory, while the future only acts upon the past through the medium of thirds. Phenomena of this sort in the outward world shall be considered below. In sense and will, there are reactions of Secondness between the *ego* and the *non-ego* (which non-ego may be an object of direct consciousness). In will, the events leading up to the act are internal, and we say that we are agents more than patients. In sense, the antecedent events are not within us; and besides, the object of which we form a perception (though not that which immediately acts upon the nerves) remains unaffected. Consequently, we say that we are patients, not agents. In the idea of reality, Secondness is predominant; for the real is that which insists upon forcing its way to recognition as something *other* than the mind's creation. (Remember that before the French word, *second*, was adopted into our language, *other* was merely the ordinal numeral corresponding to *two*.) The real is active; we acknowledge it, in calling it the *actual*. (This word is due to Aristotle's use of {*energeia*}, action, to mean existence, as opposed to a mere germinal state.) Again, the kind of thought of those dualistic philosophers who are fond of laying down propositions as if there were only two alternatives, and no gradual shading off between them, as when they say that in trying to find a law in a phenomenon I commit myself to the proposition that law bears absolute sway in nature, such thought is marked by Secondness.

Peirce: CP 1.326 Cross-Ref:††
 §4. THE DYAD †1

326. A *dyad* consists of two *subjects* brought into oneness. These subjects have their modes of being in themselves, and they also have their modes of being, as first and second, etc., in connection with each other. They are two, if not really, at least in aspect. There is also some sort of union of them. The dyad is not the subjects; it has the subjects as one element of it. It has, besides, a suchness of monoidal character; and it has suchness, or suchnesses, peculiar to it as a dyad. The dyad brings the subjects together, and in doing so imparts a character to each of them. Those characters are, in some sense, two. The dyad has also two sides according to which subject is considered as first. These two sides of the dyad form a second pair of subjects attached to the dyad; and they have their mode of union. Each of them also has a special character as a subject of the dyad.

Peirce: CP 1.326 Cross-Ref:††

This description shows that the dyad, in contrast to the monad, has a variety of features; and all these features present dyadic relations.

Peirce: CP 1.327 Cross-Ref:††

327. As an example of a dyad take this: God said, Let there be light, and there was light. We must not think of this as a verse of Genesis, for Genesis would be a third thing. Neither must we think of it as proposed for our acceptance, or as held for true; for we are third parties. We must simply think of God creating light by fiat. Not that the fiat and the coming into being of the light were two facts; but that it is in one indivisible fact. God and light are the subjects. The act of creation is to be regarded, not as any third object, but merely as the suchness of connection of God and light. The dyad is the fact. It determines the existence of the light, and the creatorship of God. The two aspects of the dyad are, first, that of God compelling the existence of

the light, and that of the light as, by its coming into existence, making God a creator. This last is in the present example merely a mere point of view, without any reality corresponding to it. That is one of the special features of the particular example chosen. Of the two aspects of the dyad, then, one is in this instance, fundamental, real, and primary, while the other is merely derivative, formal, and secondary.

Peirce: CP 1.328 Cross-Ref:††

328. I chose this instance because it is represented as instantaneous. Had there been any process intervening between the causal act and the effect, this would have been a medial, or third, element. Thirdness, in the sense of the category, is the same as mediation. For that reason, pure dyadism is an act of arbitrary will or of blind force; for if there is any reason, or law, governing it, that mediates between the two subjects and brings about their connection. The dyad is an individual fact, as it existentially is; and it has no generality in it. The being of a monadic quality is a mere potentiality, without existence. Existence is purely dyadic.

Peirce: CP 1.329 Cross-Ref:††

329. It is to be noted that existence is an affair of blind force. "The very hyssop that grows on the wall exists in that chink because the whole universe could not prevent it." No law determines any atom to exist. Existence is presence in some experiential universe -- whether the universe of material things now existing, or that of laws, or that of phenomena, or that of feelings -- and this presence implies that each existing thing is in dynamical reaction with every other in that universe. Existence, therefore, is dyadic; though Being is monadic.

Peirce: CP 1.330 Cross-Ref:††

§5. POLAR DISTINCTIONS AND VOLITION †1

330. Calling any distinction between two equally decided characters to which no third seems to be coördinate (although a neutrality separates them) a *polar* distinction, in the external world polar distinctions are few. That of past and future, with the resulting two ways of passing over a line (and consequent right-and left-handed spirals and helices, whence probably the magnetic and possibly the electric poles -- supposing the latter to be truly "polar" in our sense), with the right and left sides of our bodies, and the two sexes, seems pretty much to exhaust the list of them. Yet for the much smaller universe of psychology, polar distinctions abound, most of them referring to volition. Thus, pleasure is any kind of sensation that one immediately seeks, pain any that one immediately shuns. Right and wrong are expressly volitional. Necessity and impossibility so obviously refer to volition that the words often need qualification to show that rational modifications of them are meant. The words reasonable and perverse imply that assent is as free as choice ever is, and so proclaim their volitional strain. Roget's *Thesaurus* illustrates the great aptitude of the psychical to *polar* distinction. Any very close examination of how far this is due to volition would cause us to wander quite away from the subject of this essay. It would show that dichotomy, meaning the fact that the elements that a distinction separates are just *two* in number, is strikingly often -- perhaps that it is presumably always -- due to volition. . . .

Peirce: CP 1.331 Cross-Ref:††

331. Although the mode of consciousness we call volition, or willing,

contrasts decidedly with the mere perception that something has been done, yet it is not perfected, and perhaps does not take place at all, until something is actually effected. Trying to shove something too heavy for the man to stir nevertheless accomplishes, in considerable measure, the only thing that he directly willed to do -- namely, to contract certain muscles. In the days of table-turning we used to be commanded to sit quite away from a table, and "*with all our might*" to will that the table should move; and since the whole weight of our outstretched arms soon made our finger-tips unconsciously numb (for things are not apt to be consciously unconscious; and there were other concurring physiological effects that we did not suspect), while we were possessed of no other "might" over the table than through our muscles, we used to be speedily rewarded, by a direct consciousness of willing that the table move, accompanied by the vision of its wondrous obedience. Until it moved, we were only *longing*, not *willing*. So when certain psychologists write, chiefly in French -- a language abounding in exquisite distinctions, but one in which any analytical method of interpretation is so sure to lead to misunderstandings, that the language is not well adapted to psychology or philosophy -- about "*involuntary attention*," they can only mean one of two things, either *unpremeditated* attention or attention influenced by conflicting *desires*. Though "desire" implies a tendency to volition, and though it is a natural hypothesis that a man cannot *will* to do that which he has no sort of desire to do, yet we all know conflicting desires but too well, and how treacherous they are apt to be; and a desire may perfectly well be discontented with volition, *i.e.*, with what the man *will* do. The consciousness of that truth seems to me to be the root of our consciousness of free will. "Involuntary attention" involves in correct English a contradiction *in adjecto*.

Peirce: CP 1.332 Cross-Ref: ††
 §6. EGO AND NON-EGO †1

332. The triad, feeling, volition, cognition, is usually regarded as a purely psychological division. Long series of carefully planned self-experiments, persistent and much varied, though only qualitative, have left me little doubt, if any, that there are in those elements three quite disparate modes of awareness. That is a psychological proposition; but that which now concerns us is not psychological, particularly; namely the differences between that of which we are aware in feeling, volition, and cognition. Feeling is a quality, but so far as there is mere feeling, the quality is not limited to any definite subject. We hear of a man whose mind is jaundiced. That phrase well expresses feeling without reason. Feeling also as such is unanalyzed. Volition is through and through dual. There is the duality of agent and patient, of effort and resistance, of active effort and inhibition, of acting on self and on external objects. Moreover, there is active volition and passive volition, or inertia, the volition of reform and the volition of conservatism. That shock which we experience when anything particularly unexpected forces itself upon our recognition (which has a cognitive utility as being a call for explanation of the presentment), is simply the sense of the volitional inertia of expectation, which strikes a blow like a water-hammer when it is checked; and the force of this blow, if one could measure it, would be the measure of the energy of the conservative volition that gets checked. Low grades of this shock doubtless accompany all unexpected perceptions; and every perception is more or less unexpected. Its lower grades are, as I opine, not without

experimental tests of the hypothesis, that sense of externality, of the presence of a *non-ego*, which accompanies perception generally and helps to distinguish it from dreaming. This is present in all sensation, meaning by sensation the initiation of a state of feeling; -- for by feeling I mean nothing but sensation *minus* the attribution of it to any particular subject. In my use of words, when an ear-splitting, soul-bursting locomotive whistle starts, there is a sensation, which ceases when the screech has been going on for any considerable fraction of a minute; and at the instant it stops there is a second sensation. Between them there is a state of feeling.

Peirce: CP 1.333 Cross-Ref:††

333. As for pleasure and pain, which Kant and others have represented to be of the essence of feeling, whether it be merely because they and the section of the psychological world for which at this moment I have the presumption to speak apply the word feeling to different modifications of awareness, or whether there be a faulty analysis on the one part or the other, we certainly do not think that unadulterated feeling, if that element could be isolated, would have any relation to pain or to pleasure. For in our opinion if there be any quality of feeling common to all pleasurable experiences or components of experience, and another one quality of feeling common to all that is painful (which we are inclined to doubt, to say the least), then we hold the opinion that the one is the feeling of being attracted, the other that of being repelled, by the present state of experience. If there be two such feelings, they are feelings of states of volition. But perhaps pleasure and pain are nothing more than names for the state of being attracted and that of being repelled by present experience. Of course, feelings accompany them, but under the latter hypothesis no feeling would be common to all pleasures, and none to all pains. If we are right, the position of the hedonists is preposterous, in that they make mere feelings to be active agencies, instead of being merely conscious indications of real determinations of our subconscious volitional beings. [I may mention that their *talk* (however it may be with their thought) is further preposterous as seeming to make pain a mere privation of pleasure, although it is plain that it is pain that indicates an active, and pleasure only a passive, determination of our volitional being.]

Peirce: CP 1.334 Cross-Ref:††

334. As for volition, I would limit the term in one way and extend it in another. I would limit it to the momentary direct dyadic consciousness of an *ego* and a *non-ego* then and there present and reacting each upon the other. In one, the action is generally more active, in the other more passive; but precisely what this difference consists in I do not feel sure. I think, however, that the will to produce a change is active, the will to resist a change is passive. All sensation is essentially, by its very definition, active. The objection to this is that, according to it, the voluntary inhibition of a reflex should not give a sense of effort; and probably the definition of the distinction between the sense of externality in willing and in perception requires a supplement or other slight modification on this account. But the important point [is] that the sense of externality in perception consists in a sense of powerlessness before the overwhelming force of perception. Now the only way in which any force can be learned is by something like trying to oppose it. That we do something like this is shown by the shock we receive from any unexpected experience. It is the inertia of the mind, which tends to remain in the state in which it is. No doubt there is a marked difference between the active and intentional volition of muscular contraction and the passive and unintentional volition that gives the shock of surprise and the sense of externality. But the two are to be classed together as alike modes of double consciousness, that is, of awareness, at once and in the same awareness, of an *ego* and

a *non-ego*. . . .

Peirce: CP 1.335 Cross-Ref:††
§7. SHOCK AND THE SENSE OF CHANGE †1

335. Some writers insist that all experience consists in sense-perception; and I think it is probably true that every element of experience is in the first instance applied to an external object. A man who gets up out of the wrong side of the bed, for example, attributes wrongness to almost every object he perceives. That is the way in which he experiences his bad temper. It cannot, however, be said that he *perceives* the perversity which he wrongly attributes to outward objects.

Peirce: CP 1.336 Cross-Ref:††

336. We perceive objects brought before us; but that which we especially experience -- the kind of thing to which the word "experience" is more particularly applied -- is an event. We cannot accurately be said to perceive events; for this requires what Kant called the "synthesis of apprehension," not however, by any means, making the needful discriminations. A whistling locomotive passes at high speed close beside me. As it passes the note of the whistle is suddenly lowered from a well-understood cause. I perceive the whistle, if you will. I have, at any rate, a sensation of it. But I cannot be said to have a sensation of the change of note. I have a sensation of the lower note. But the cognition of the change is of a more intellectual kind. That I experience rather than perceive. It is [the] special field of experience to acquaint us with events, with changes of perception. Now that which particularly characterizes sudden changes of perception is a shock. A shock is a volitional phenomenon. The long whistle of the approaching locomotive, however disagreeable it may be, has set up in me a certain inertia, so that the sudden lowering of the note meets with a certain resistance. That must be the fact; because if there were no such resistance there could be no shock when the change of note occurs. Now this shock is quite unmistakable. It is more particularly to changes and contrasts of perception that we apply the word "experience." We experience vicissitudes, especially. We cannot experience the vicissitude without experiencing the perception which undergoes the change; but the concept of *experience* is broader than that of *perception*, and includes much that is not, strictly speaking, an object of perception. It is the compulsion, the absolute constraint upon us to think otherwise than we have been thinking that constitutes experience. Now constraint and compulsion cannot exist without resistance, and resistance is effort opposing change. Therefore there must be an element of effort in experience; and it is this which gives it its peculiar character. But we are so disposed to yield to it as soon as we can detect it, that it is extremely difficult to convince ourselves that we have exerted any resistance at all. It may be said that we hardly know it except through the axiom that there can be no force where there is no resistance or inertia. Whoever may be dissatisfied with my statement will do well to sit down and cipher out the matter for himself. He may be able to formulate the nature of the oppositional element in experience, and its relation to ordinary volition better than I have done; but that there is an oppositional element in it, logically not easily distinguished from volition, will, I make no doubt at all, be his ultimate conclusion.

Peirce: CP 1.337 Cross-Ref:††
C. THIRDNESS

§1. EXAMPLES OF THIRDNESS †1

337. By the third, I mean the medium or connecting bond between the absolute first and last. The beginning is first, the end second, the middle third. The end is second, the means third. The thread of life is a third; the fate that snips it, its second. A fork in a road is a third, it supposes three ways; a straight road, considered merely as a connection between two places is second, but so far as it implies passing through intermediate places it is third. Position is first, velocity or the relation of two successive positions second, acceleration or the relation of three successive positions third. But velocity in so far as it is continuous also involves a third. Continuity represents Thirdness almost to perfection. Every process comes under that head. Moderation is a kind of Thirdness. The positive degree of an adjective is first, the superlative second, the comparative third. All exaggerated language, "supreme," "utter," "matchless," "root and branch," is the furniture of minds which think of seconds and forget thirds. Action is second, but conduct is third. Law as an active force is second, but order and legislation are third. Sympathy, flesh and blood, that by which I feel my neighbor's feelings, is third.

Peirce: CP 1.338 Cross-Ref:††
§2. REPRESENTATION AND GENERALITY †1

338. The ideas in which Thirdness is predominant are, as might be expected, more complicated, and mostly require careful analysis to be clearly apprehended; for ordinary, unenergetic thought slurs over this element as too difficult. There is all the more need of examining some of these ideas.

Peirce: CP 1.339 Cross-Ref:††

339. The easiest of those which are of philosophical interest is the idea of a sign, or representation.†2 A sign stands *for* something *to* the idea which it produces, or modifies. Or, it is a vehicle conveying into the mind something from without. That for which it stands is called its *object*; that which it conveys, its *meaning*; and the idea to which it gives rise, its *interpretant*. The object of representation can be nothing but a representation of which the first representation is the interpretant. But an endless series of representations, each representing the one behind it, may be conceived to have an absolute object at its limit. The meaning of a representation can be nothing but a representation. In fact, it is nothing but the representation itself conceived as stripped of irrelevant clothing. But this clothing never can be completely stripped off; it is only changed for something more diaphanous. So there is an infinite regression here. Finally, the interpretant is nothing but another representation to which the torch of truth is handed along; and as representation, it has its interpretant again. Lo, another infinite series.

Peirce: CP 1.340 Cross-Ref:††

340. Some of the ideas of prominent Thirdness which, owing to their great importance in philosophy and in science, require attentive study are generality, infinity, continuity, diffusion, growth, and intelligence.

Peirce: CP 1.341 Cross-Ref:††

341. Let us examine the idea of generality. Every cook has in her recipe-book a collection of rules, which she is accustomed to follow. An apple pie is desired. Now, observe that we seldom, probably never, desire a single individual thing. What we want is something which shall produce a certain pleasure of a certain kind. To speak of a single individual pleasure is to use words without meaning. We may have a single experience of pleasure; but the pleasure itself is a quality. Experiences are single; but qualities, however specialized, cannot be enumerated. There are some two dozen kinds of metals well known to me. I remember to have examined lumps of those qualities. But it is only the limitation of experience which attaches that number; there is simply no end to the metallic qualities I can imagine. I can imagine an infinite variety between tin and lead, or between copper and silver, or between iron and nickel, or between magnesium and aluminum. An apple pie, then, is desired -- a good apple pie, made of fresh apples, with a crust moderately light and somewhat short, neither too sweet nor too sour, etc. But it is not any particular apple pie; for it is to be made for the occasion; and the only particularity about it is that it is to be made and eaten today. For that, apples are wanted; and remembering that there is a barrel of apples in the cellar, the cook goes to the cellar and takes the apples that are uppermost and handiest. That is an example of following a general rule. She is directed to take apples. Many times she has seen things which were called apples, and has noticed their common quality. She knows how to find such things now; and as long as they are sound and fine, any apples will do. What she desires is something of a given quality; what she has to take is this or that particular apple. From the nature of things, she cannot take the quality but must take the particular thing. Sensation and volition being affairs of action and reaction relate to particular things. She has seen only particular apples, and can take only particular apples. But desire has nothing to do with particulars; it relates to qualities. Desire is not a reaction with reference to a particular thing; it is an idea about an idea, namely, the idea of how delightful it would be for me, the cook's master, to eat an apple pie. However, what is desired is not a mere unattached quality; what is desired is that the dream of eating an apple pie should be realized in Me; and this Me is an object of experience. So with the cook's desire. She has no particular apple pie she particularly prefers to serve; but she does desire and intend to serve an apple pie to a particular person. When she goes into the cellar for the apples, she takes whatever bowl or basket comes handy, without caring what one, so long as it has a certain size, is clean, and has other qualities, but having once selected it, into that particular bowl she intends to put some apples. She takes any apples that are handy and seem good; but having taken them she means to make a pie of those apples. If she chances to see some others in the kitchen, on her return from the cellar, she will not use them for the pie, unless for some reason she changes her mind. Throughout her whole proceedings she pursues an idea or dream without any particular thisness or thatness -- or, as we say, *hecceity* -- to it, but this dream she wishes to realize in connection with an object of experience, which as such, does possess hecceity; and since she has to act, and action only relates to this and that, she has to be perpetually making random selections, that is, taking whatever comes handiest.

Peirce: CP 1.342 Cross-Ref:††

342. The dream itself has no prominent thirdness; it is, on the contrary, utterly

irresponsible; it is whatever it pleases. The object of experience as a reality is a second. But the desire in seeking to attach the one to the other is a third, or medium.

Peirce: CP 1.342 Cross-Ref:††

So it is with any law of nature. Were it but a mere idea unrealized -- and it is of the nature of an idea -- it would be a pure first. The cases to which it applies, are seconds.

Peirce: CP 1.343 Cross-Ref:††
§3. THE REALITY OF THIRDNESS †1

343. . . . It is impossible to resolve everything in our thoughts into those two elements [of Firstness and Secondness]. We may say that the bulk of what is actually done consists of Secondness -- or better, Secondness is the predominant character of what *has been* done. The immediate present, could we seize it, would have no character but its Firstness. Not that I mean to say that immediate consciousness (a pure fiction, by the way), would be Firstness, but that the *quality* of what we are immediately conscious of, which is no fiction, is Firstness. But we constantly predict what is to be. Now what is to be, according to our conception of it, can never become wholly past. In general, we may say that *meanings* are inexhaustible. We are too apt to think that what one *means* to do and the *meaning* of a word are quite unrelated meanings of the word "meaning," or that they are only connected by both referring to some actual operation of the mind. Professor Royce especially in his great work *The World and the Individual* has done much to break up this mistake. In truth the only difference is that when a person *means* to do anything he is in some state in consequence of which the brute reactions between things will be moulded [in] to conformity to the form to which the man's mind is itself moulded, while the meaning of a word really lies in the way in which it might, in a proper position in a proposition believed, tend to mould the conduct of a person into conformity to that to which it is itself moulded. Not only will meaning always, more or less, in the long run, mould reactions to itself, but it is only in doing so that its own being consists. For this reason I call this element of the phenomenon or object of thought the element of Thirdness. It is that which is what it is by virtue of imparting a quality to reactions in the future.

Peirce: CP 1.344 Cross-Ref:††

344. There is a strong tendency in us all to be sceptical about there being any real meaning or law in things. This scepticism is strongest in the most masculine thinkers. I applaud scepticism with all my heart, provided it have four qualities: first, that it be sincere and real doubt; second, that it be aggressive; third, that it push inquiry; and fourth, that it stand ready to acknowledge what it now doubts, as soon as the doubted element comes clearly to light. To be angry with sceptics, who, whether they are aware of it or not, are the best friends of spiritual truth, is a manifest sign that the angry person is himself infected with scepticism -- not, however, of the innocent and wholesome kind that tries to bring truth to light, but of the mendacious, clandestine, disguised, and conservative variety that is afraid of truth, although truth merely means the way to attain one's purposes. If the sceptics think that any account can be given of the phenomena of the universe while they leave Meaning out of account, by all means let them go ahead and try to do it. It is a most laudable and wholesome enterprise. But when they go so far as to say that there is no such idea in

our minds, irreducible to anything else, I say to them, "Gentlemen, your strongest sentiment, to which I subscribe with all my heart, is that a man worthy of that name will not allow petty intellectual predilections to blind him to truth, which consists in the conformity of his thoughts to his purposes. But you know there is such a thing as a defect of candor of which one is not oneself aware. You perceive, no doubt, that if there be an element of thought irreducible to any other, it would be hard, on your principles, to account for man's having it, unless he derived it from envioning Nature. But if, because of that, you were to turn your gaze away from an idea that shines out clearly in your mind, you would be violating your principles in a very much more radical way."

Peirce: CP 1.345 Cross-Ref:††

345. I will sketch a proof that the idea of meaning is irreducible to those of quality and reaction. It depends on two main premisses. The first is that every genuine triadic relation involves meaning, as meaning is obviously a triadic relation. The second is that a triadic relation is inexpressible by means of dyadic relations alone. Considerable reflexion may be required to convince yourself of the first of these premisses, that every triadic relation involves meaning. There will be two lines of inquiry. First, all physical forces appear to subsist between pairs of particles. This was assumed by Helmholtz in his original paper, *On the Conservation of Forces*. †1 Take any fact in physics of the triadic kind, by which I mean a fact which can only be defined by simultaneous reference to three things, and you will find there is ample evidence that it never was produced by the action of forces on mere dyadic conditions. Thus, your right hand is that hand which is toward the *east*, when you face the *north* with your head toward the *zenith*. Three things, east, west, and up, are required to define the difference between right and left. Consequently chemists find that those substances which rotate the plane of polarization to the right or left can only be produced from such [similar] active substances. They are all of such complex constitution that they cannot have existed when the earth was very hot, and how the first one was produced is a puzzle. It cannot have been by the action of brute forces. For the second branch of the inquiry, you must train yourself to the analysis of relations, beginning with such as are very markedly triadic, gradually going on to others. In that way, you will convince yourself thoroughly that every genuine triadic relation involves thought or *meaning*. Take, for example, the relation of *giving*. A *gives* B to C. This does not consist in A's throwing B away and its accidentally hitting C, like the date-stone, which hit the Jinnee in the eye. If that were all, it would not be a genuine triadic relation, but merely one dyadic relation followed by another. There need be no motion of the thing given. Giving is a transfer of the right of property. Now right is a matter of law, and law is a matter of thought and meaning. I there leave the matter to your own reflection, merely adding that, though I have inserted the word "genuine," yet I do not really think that necessary. I think even degenerate triadic relations involve something like thought.

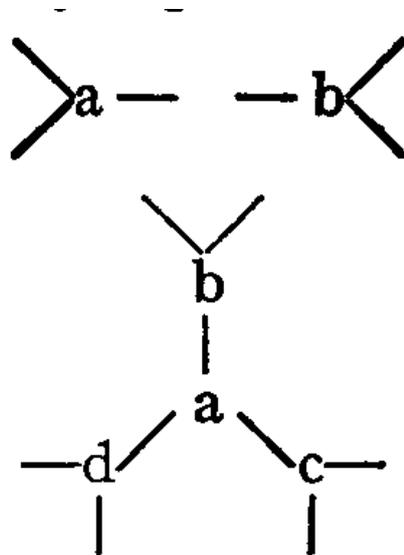
Peirce: CP 1.346 Cross-Ref:††

346. The other premiss of the argument that genuine triadic relations can never be built of dyadic relations and of qualities is easily shown. In existential graphs, a spot with one tail -- X represents a quality, a spot with two tails -- R -- a dyadic relation. †1 Joining the ends of two tails is also a dyadic relation. But you can never by such joining make a graph with three tails. You may think that a node connecting three lines of identity Y is not a triadic idea. But analysis will show that it is so. I see a man on Monday. On Tuesday I see a man, and I exclaim, "Why, that is the *very* man I saw on Monday." We may say, with sufficient accuracy, that I directly

experienced the identity. On Wednesday I see a man and I say, "That is the same man I saw on Tuesday, and consequently is the same I saw on Monday." There is a recognition of triadic identity; but it is only brought about as a conclusion from two premisses, which is itself a triadic relation. If I see two men at once, I cannot by any such direct experience identify both of them with a man I saw before. I can only identify them if I regard them, not as the *very* same, but as two different manifestations of the same man. But the idea of *manifestation* is the idea of a sign. Now a sign is something, A, which denotes some fact or object, B, to some interpretant thought, C.

Peirce: CP 1.347 Cross-Ref:††

347. It is interesting to remark that while a graph with three tails cannot be made out of graphs each with two or one tail, yet combinations of graphs of three tails each will suffice to build graphs with every higher number of tails.



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And analysis will show that every relation which is *tetradic*, *pentadic*, or of any greater number of correlates is nothing but a compound of triadic relations. It is therefore not surprising to find that beyond the three elements of Firstness, Secondness, and Thirdness, there is nothing else to be found in the phenomenon.

Peirce: CP 1.348 Cross-Ref:††

348. As to the common aversion to recognizing *thought* as an active factor in the real world, some of its causes are easily traced. In the first place, people are persuaded that everything that happens in the material universe is a motion completely determined by inviolable laws of dynamics; and that, they think, leaves no room for any other influence. But the laws of dynamics stand on quite a different footing from the laws of gravitation, elasticity, electricity, and the like. The laws of dynamics are very much like logical principles, if they are not precisely that. They only say how bodies will move after you have said what the forces are. They permit any forces, and therefore any motions. Only, the principle of the conservation of energy requires us to explain certain kinds of motions by special hypotheses about molecules and the like. Thus, in order that the viscosity of gases should not disobey

that law we have to suppose that gases have a certain molecular constitution. Setting dynamical laws to one side, then, as hardly being positive laws, but rather mere formal principles, we have only the laws of gravitation, elasticity, electricity, and chemistry. Now who will deliberately say that our knowledge of these laws is sufficient to make us reasonably confident that they are absolutely eternal and immutable, and that they escape the great law of evolution? Each hereditary character is a law, but it is subject to development and to decay. Each habit of an individual is a law; but these laws are modified so easily by the operation of self-control, that it is one of the most patent of facts that ideals and thought generally have a very great influence on human conduct. That truth and justice are great powers in the world is no figure of speech, but a plain fact to which theories must accommodate themselves.

Peirce: CP 1.349 Cross-Ref:††

349. The child, with his wonderful genius for *language*, naturally looks upon the world as chiefly governed by thought; for thought and expression are really one. As Wordsworth truly says, the child is quite right in this; he is an

"eye among the blind,

"On whom those truths do rest

"Which we are toiling all our lives to find."

But as he grows up, he loses this faculty; and all through his childhood he has been stuffed with such a pack of lies, which parents are accustomed to think are the most wholesome food for the child -- because they do not think of his future -- that he begins real life with the utmost contempt for all the ideas of his childhood; and the great truth of the immanent power of thought in the universe is flung away along with the lies. I offer this hypothetical explanation because, if the common aversion to regarding thought as a real power, or as anything but a fantastic figment, were really natural, it would make an argument of no little strength against its being acknowledged as a real power.

Peirce: CP 1.350 Cross-Ref:††

§4. PROTOPLASM AND THE CATEGORIES †1

350. Thus mathematical considerations, by which I mean study as purely *a priori* and necessary as thought can be, have suggested and indeed insisted upon a classification of the elements of the phaneron, and so of the functions of the mind, and of the nervous system, of protoplasm itself, which empirical science will find very convenient. Instead of the familiar division of Tetens or Kant which makes pleasure-pain, cognition, and volition the three categories of mental phenomena, we have feeling or quality, the action of opposition, and synthetic thought.

Peirce: CP 1.351 Cross-Ref:††

351. As to protoplasm, what the three *cenopythagorean categories*, as I call them, do, and what they are limited to doing, is to call attention to three very different

characters of this chemical body. The first is a *posse* which it has in itself; for the *priman* stops at *can-bes* and never reaches to existence, which depends on interaction, or *secundarity*. This internal power which the category merely suggests, we recognize as that of feeling. Though it is priman, it is without any doubt dependent upon the extreme complexity of the protoplasmic molecule, if the word molecule can be applied to so intricate, unstable, and ununified a system. But it is the law of high numbers that extreme complication with a great multitude of independent similars results in a new simplicity. Next there is reactive force, a twoness, which is emphasized in the nerve cells together. It is the property by which any state of high cohesiveness tends to spread through the albuminoid matter. We usually call the property contractility. Thirdly, the categories suggest our looking for a synthetizing law; and this we find in the power of assimilation, incident to which is the habit-taking faculty. This is all the categories pretend to do. They suggest a way of thinking; and the possibility of science depends upon the fact that human thought necessarily partakes of whatever character is diffused through the whole universe, and that its natural modes have some tendency to be the modes of action of the universe.

Peirce: CP 1.352 Cross-Ref:††

352. In the study of logic I have found the cenopythagorean categories unlock many a secret.

Peirce: CP 1.353 Cross-Ref:††

§5. THE INTERDEPENDENCE OF THE CATEGORIES †1

353. Perhaps it is not right to call these categories conceptions; they are so intangible that they are rather tones or tints upon conceptions. In my first attempt to deal with them,†2 I made use of three grades of separability of one idea from another. In the first place, two ideas may be so little allied that one of them may be present to the consciousness in an image which does not contain the other at all; in this way we can imagine *red* without imagining blue, and *vice versa*; we can also imagine sound without melody, but not melody without sound. I call this kind of separation *dissociation*. In the second place, even in cases where two conceptions cannot be separated in the imagination, we can often suppose one without the other, that is we can imagine data from which we should be led to believe in a state of things where one was separated from the other. Thus, we can suppose uncolored space, though we cannot dissociate space from color. I call this mode of separation *prescission*. In the third place, even when one element cannot even be supposed without another, they may oftentimes be distinguished from one another. Thus we can neither imagine nor suppose a taller without a shorter, yet we can distinguish the taller from the shorter. I call this mode of separation *distinction*. Now, the categories cannot be dissociated in imagination from each other, nor from other ideas. The category of first can be prescinded from second and third, and second can be prescinded from third. But second cannot be prescinded from first, nor third from second. The categories may, I believe, be prescinded from any other one conception, but they cannot be prescinded from some one and indeed many elements. You cannot suppose a first unless that first be something definite and more or less definitely supposed. Finally, though it is easy to distinguish the three categories from one another, it is extremely difficult accurately and sharply to distinguish each from other conceptions so as to hold it in its purity and yet in its full meaning.

Peirce: CP 1.354 Cross-Ref:††
CHAPTER 3

A GUESS AT THE RIDDLE†1P

PLAN OF THE WORK

354. Section 1. One, Two, Three. Already written.

Peirce: CP 1.354 Cross-Ref:††

Section 2. The triad in reasoning. Not touched. It is to be made as follows. 1. Three kinds of signs; as best shown in my last paper in the *Am. Jour. Math.*†2 2. Term, proposition, and argument, mentioned in my paper on a new list of categories.†3 3. Three kinds of argument, deduction, induction, hypothesis, as shown in my paper in *Studies in Logic*.†4 Also three figures of syllogism, as shown there and in my paper on the Classification of Arguments.†5 4. Three kinds of terms, absolute, relative, and conjugative, as shown in my first paper on Logic of Relatives.†6 There are various other triads which may be alluded to. The dual divisions of logic result from a false way of looking at things absolutely. Thus, besides affirmative and negative, there are really probable enunciations, which are intermediate. So besides universal and particular there are all sorts of propositions of numerical quantity. For example, the particular proposition: Some A is B, means "At least one A is B." But we can also say: At least 2 A's are B's. Also, All the A's but one are B's, etc., etc., *ad infinitum*. We pass from dual quantity, or a system of quantity such as that of Boolean algebra, where there are only two values, to plural quantity.

Peirce: CP 1.354 Cross-Ref:††

Section 3. The triad in metaphysics. This chapter, one of the best, is to treat of the theory of cognition.

Peirce: CP 1.354 Cross-Ref:††

Section 4. The triad in psychology. The greater part is written.

Peirce: CP 1.354 Cross-Ref:††

Section 5. The triad in physiology. The greater part is written.

Peirce: CP 1.354 Cross-Ref:††

Section 6. The triad in biology. This is to show the true nature of the Darwinian hypothesis.

Peirce: CP 1.354 Cross-Ref:††

Section 7. The triad in physics. The germinal section. 1. The necessity of a natural history of the laws of nature, so that we may get some notion of what to expect. 2. The logical postulate for explanation forbids the assumption of any absolute. That is, it calls for the introduction of Thirdness. 3. Metaphysics is an

imitation of geometry; and mathematicians having declared against axioms, the metaphysical axioms are destined to fall too. 4. Absolute chance. 5. The universality of the principle of habit. 6. The whole theory stated. 7. Consequences.

Peirce: CP 1.354 Cross-Ref:††

Section 8. The triad in sociology or, shall I say, pneumatology. That the consciousness is a sort of public spirit among the nerve-cells. Man as a community of cells; compound animals and composite plants; society; nature. Feeling implied in Firstness.

Peirce: CP 1.354 Cross-Ref:††

Section 9. The triad in theology. Faith requires us to be materialists without flinching.†1

Peirce: CP 1.355 Cross-Ref:††

§1. TRICHOTOMY †2*P*

355. Perhaps I might begin by noticing how different numbers have found their champions. Two was extolled by Peter Ramus, Four by Pythagoras, Five by Sir Thomas Browne, and so on. For my part, I am a determined foe of no innocent number; I respect and esteem them all in their several ways; but I am forced to confess to a leaning to the number Three in philosophy. In fact, I make so much use of threefold divisions in my speculations, that it seems best to commence by making a slight preliminary study of the conceptions upon which all such divisions must rest. I mean no more than the ideas of first, second, third -- ideas so broad that they may be looked upon rather as moods or tones of thought, than as definite notions, but which have great significance for all that. Viewed as numerals, to be applied to what objects we like, they are indeed thin skeletons of thought, if not mere words. If we only wanted to make enumerations, it would be out of place to ask for the significations of the numbers we should have to use; but then the distinctions of philosophy are supposed to attempt something far more than that; they are intended to go down to the very essence of things, and if we are to make one single threefold philosophical distinction, it behooves us to ask beforehand what are the kinds of objects that are first, second, and third, not as being so counted, but in their own true characters. That there are such ideas of the really first, second, and third, we shall presently find reason to admit.

Peirce: CP 1.356 Cross-Ref:††

356. The first is that whose being is simply in itself, not referring to anything nor lying behind anything. The second is that which is what it is by force of something to which it is second. The third is that which is what it is owing to things between which it mediates and which it brings into relation to each other.

Peirce: CP 1.357 Cross-Ref:††

357. The idea of the absolutely first must be entirely separated from all conception of or reference to anything else; for what involves a second is itself a second to that second. The first must therefore be present and immediate, so as not to be second to a representation. It must be fresh and new, for if old it is second to its former state. It must be initiative, original, spontaneous, and free; otherwise it is second to a determining cause. It is also something vivid and conscious; so only it

avoids being the object of some sensation. It precedes all synthesis and all differentiation; it has no unity and no parts. It cannot be articulately thought: assert it, and it has already lost its characteristic innocence; for assertion always implies a denial of something else. Stop to think of it, and it has flown! What the world was to Adam on the day he opened his eyes to it, before he had drawn any distinctions, or had become conscious of his own existence -- that is first, present, immediate, fresh, new, initiative, original, spontaneous, free, vivid, conscious, and evanescent. Only, remember that every description of it must be false to it.

Peirce: CP 1.358 Cross-Ref:††

358. Just as the first is not absolutely first if thought along with a second, so likewise to think the second in its perfection we must banish every third. The second is therefore the absolute last. But we need not, and must not, banish the idea of the first from the second; on the contrary, the second is precisely that which cannot be without the first. It meets us in such facts as another, relation, compulsion, effect, dependence, independence, negation, occurrence, reality, result. A thing cannot be other, negative, or independent, without a first to or of which it shall be other, negative, or independent. Still, this is not a very deep kind of secondness; for the first might in these cases be destroyed yet leave the real character of the second absolutely unchanged. When the second suffers some change from the action of the first, and is dependent upon it, the secondness is more genuine. But the dependence must not go so far that the second is a mere accident or incident of the first; otherwise the secondness again degenerates. The genuine second suffers and yet resists, like dead matter, whose existence consists in its inertia. Note, too, that for the second to have the finality that we have seen belongs to it, it must be determined by the first immovably, and thenceforth be fixed; so that unalterable fixity becomes one of its attributes. We find secondness in occurrence, because an occurrence is something whose existence consists in our knocking up against it. A hard fact is of the same sort; that is to say, it is something which is there, and which I cannot think away, but am forced to acknowledge as an object or second beside myself, the subject or number one, and which forms material for the exercise of my will.

Peirce: CP 1.358 Cross-Ref:††

The idea of second must be reckoned as an easy one to comprehend. That of first is so tender that you cannot touch it without spoiling it; but that of second is eminently hard and tangible. It is very familiar, too; it is forced upon us daily; it is the main lesson of life. In youth, the world is fresh and we seem free; but limitation, conflict, constraint, and secondness generally, make up the teaching of experience. With what firstness

"The scarfed bark puts from her native bay;" with what secondness

"doth she return,

With overweathered ribs and ragged sails."

But familiar as the notion is, and compelled as we are to acknowledge it at every turn, still we never can realize it; we never can be immediately conscious of finiteness, or of anything but a divine freedom that in its own original firstness knows no bounds.

Peirce: CP 1.359 Cross-Ref:††

359. First and second, agent and patient, yes and no, are categories which enable us roughly to describe the facts of experience, and they satisfy the mind for a very long time. But at last they are found inadequate, and the third is the conception which is then called for. The third is that which bridges over the chasm between the absolute first and last, and brings them into relationship. We are told that every science has its qualitative and its quantitative stage; now its qualitative stage is when dual distinctions -- whether a given subject has a given predicate or not -- suffice; the quantitative stage comes when, no longer content with such rough distinctions, we require to insert a possible halfway between every two possible conditions of the subject in regard to its possession of the quality indicated by the predicate. Ancient mechanics recognized forces as causes which produced motions as their immediate effects, looking no further than the essentially dual relation of cause and effect. That was why it could make no progress with dynamics. The work of Galileo and his successors lay in showing that forces are accelerations by which [a] state of velocity is gradually brought about. The words "cause" and "effect" still linger, but the old conceptions have been dropped from mechanical philosophy; for the fact now known is that in certain relative positions bodies undergo certain accelerations. Now an acceleration, instead of being like a velocity a relation between two successive positions, is a relation between three; so that the new doctrine has consisted in the suitable introduction of the conception of threeness. On this idea, the whole of modern physics is built. The superiority of modern geometry, too, has certainly been due to nothing so much as to the bridging over of the innumerable distinct cases with which the ancient science was encumbered; and we may go so far as to say that all the great steps in the method of science in every department have consisted in bringing into relation cases previously discrete.

Peirce: CP 1.360 Cross-Ref:††

360. We can easily recognize the man whose thought is mainly in the dual stage by his unmeasured use of language. In former days, when he was natural, everything with him was unmitigated, absolute, ineffable, utter, matchless, supreme, unqualified, root and branch; but now that it is the fashion to be depreciatory, he is just as plainly marked by the ridiculous inadequacy of his expressions. The principle of contradiction is a shibboleth for such minds; to disprove a proposition they will always try to prove there lurks a contradiction in it, notwithstanding that it may be as clear and comprehensible as the day. Remark for your amusement the grand unconcern with which mathematics, since the invention of the calculus, has pursued its way, caring no more for the peppering of contradiction-mongers than an ironclad for an American fort.

Peirce: CP 1.361 Cross-Ref:††

361. We have seen that it is the immediate consciousness that is preeminently first, the external dead thing that is preeminently second. In like manner, it is evidently the representation mediating between these two that is preëminently third. Other examples, however, should not be neglected. The first is agent, the second patient, the third is the action by which the former influences the latter. Between the beginning as first, and the end as last, comes the process which leads from first to last.

Peirce: CP 1.362 Cross-Ref:††

362. According to the mathematicians, when we measure along a line, were our yardstick replaced by a yard marked off on an infinitely long rigid bar, then in all the shiftings of it which we make for the purpose of applying it to successive portions

of the line to be measured, two points on that bar would remain fixed and unmoved. To that pair of points, the mathematicians accord the title of the absolute; they are the points that are at an infinite distance one way and the other as measured by that yard. These points are either really distinct, coincident, or imaginary (in which case there is but a finite distance completely round the line), according to the relation of the mode of measurement to the nature of the line upon which the measurement is made. These two points are the absolute first and the absolute last or second, while every measurable point on the line is of the nature of a third. We have seen that the conception of the absolute first eludes every attempt to grasp it; and so in another sense does that of the absolute second; but there is no absolute third, for the third is of its own nature relative, and this is what we are always thinking, even when we aim at the first or second. The starting-point of the universe, God the Creator, is the Absolute First; the terminus of the universe, God completely revealed, is the Absolute Second; every state of the universe at a measurable point of time is the third. If you think the measurable is all there is, and deny it any definite tendency whence or whither, then you are considering the pair of points that makes the absolute to be imaginary and are an Epicurean. If you hold that there is a definite drift to the course of nature as a whole, but yet believe its absolute end is nothing but the Nirvana from which it set out, you make the two points of the absolute to be coincident, and are a pessimist. But if your creed is that the whole universe is approaching in the infinitely distant future a state having a general character different from that toward which we look back in the infinitely distant past, you make the absolute to consist in two distinct real points and are an evolutionist.†P1 This is one of the matters concerning which a man can only learn from his own reflections, but I believe that if my suggestions are followed out, the reader will grant that one, two, three, are more than mere count-words like "eeny, meeny, miny, mo," but carry vast, though vague ideas.

Peirce: CP 1.363 Cross-Ref:††

363. But it will be asked, why stop at three? Why not go on to find a new conception in four, five, and so on indefinitely? The reason is that while it is impossible to form a genuine three by any modification of the pair, without introducing something of a different nature from the unit and the pair, four, five, and every higher number can be formed by mere complications of threes. To make this clear, I will first show it in an example. The fact that A presents B with a gift C, is a triple relation, and as such cannot possibly be resolved into any combination of dual relations. Indeed, the very idea of a combination involves that of thirdness, for a combination is something which is what it is owing to the parts which it brings into mutual relationship. But we may waive that consideration, and still we cannot build up the fact that A presents C to B by any aggregate of dual relations between A and B, B and C, and C and A. A may enrich B, B may receive C, and A may part with C, and yet A need not necessarily give C to B. For that, it would be necessary that these three dual relations should not only coexist, but be welded into one fact. Thus we see that a triad cannot be analyzed into dyads. But now I will show by an example that a four can be analyzed into threes. Take the quadruple fact that A sells C to B for the price D. This is a compound of two facts: first, that A makes with C a certain transaction, which we may name E; and second, that this transaction E is a sale of B for the price D. Each of these two facts is a triple fact, and their combination makes up [as] genuine [a] quadruple fact as can be found. The explanation of this striking difference is not far to seek. A dual relative term, such as "lover" or "servant," is a sort of blank form, where there are two places left blank. I mean that in building a sentence round "lover," as the principal word of the predicate, we are at liberty to make anything we see fit the subject, and then, besides that, anything we please the

object of the action of loving. But a triple relative term such as "giver" has two correlates, and is thus a blank form with three places left blank. Consequently, we can take two of these triple relatives and fill up one blank place in each with the same letter, X, which has only the force of a pronoun or identifying index, and then the two taken together will form a whole having four blank places; and from that we can go on in a similar way to any higher number. But when we attempt to imitate this proceeding with dual relatives, and combine two of them by means of an X, we find we only have two blank places in the combination, just as we had in either of the relatives taken by itself. A road with only three-way forkings may have any number of termini, but no number of straight roads put end on end will give more than two termini. Thus any number, however large, can be built out of triads; and consequently no idea can be involved in such a number, radically different from the idea of three. I do not mean to deny that the higher numbers may present interesting special configurations from which notions may be derived of more or less general applicability; but these cannot rise to the height of philosophical categories so fundamental as those that have been considered.

Peirce: CP 1.364 Cross-Ref:††

364. The argument of this book has been developed in the mind of the author, substantially as it is presented, as a following out of these three conceptions, in a sort of game of "follow-my-leader" from one field of thought into another. Their importance was originally brought home to me in the study of logic, where they play so remarkable a part that I was led to look for them in psychology. Finding them there again, I could not help asking myself whether they did not enter into the physiology of the nervous system. By drawing a little on hypothesis, I succeeded in detecting them there; and then the question naturally came how they would appear in the theory of protoplasm in general. Here I seemed to break into an interesting avenue of reflections giving instructive aperçus both into the nature of protoplasm and into the conceptions themselves; though it was not till later that I mapped out my thoughts on the subject as they are presented in Section 4. I had no difficulty in following the lead into the domain of natural selection; and once arrived at that point, I was irresistibly carried on to speculations concerning physics. One bold saltus landed me in a garden of fruitful and beautiful suggestions, the exploration of which long prevented my looking further. As soon, however, as I was induced to look further, and to examine the application of the three ideas to the deepest problems of the soul, nature, and God, I saw at once that they must carry me far into the heart of those primeval mysteries. That is the way the book has grown in my mind: it is also the order in which I have written it; and only this first chapter is more or less an afterthought, since at an earlier stage of my studies I should have looked upon the matter here set down as too vague to have any value. I should have discerned in it too strong a resemblance to many a crack-brained book that I had laughed over. A deeper study has taught me that even out of the mouths of babes and sucklings strength may be brought forth, and that weak metaphysical trash has sometimes contained the germs of conceptions capable of growing up into important and positive doctrines.

Peirce: CP 1.365 Cross-Ref:††

365. Thus, the whole book being nothing but a continual exemplification of the triad of ideas, we need linger no longer upon this preliminary exposition of them. There is, however, one feature of them upon which it is quite indispensable to dwell. It is that there are two distinct grades of Secondness and three grades of Thirdness. There is a close analogy to this in geometry. Conic sections are either the curves usually so called, or they are pairs of straight lines. A pair of straight lines is called a

degenerate conic. So plane cubic curves are either the genuine curves of the third order, or they are conics paired with straight lines, or they consist of three straight lines; so that there are the two orders of degenerate cubics. Nearly in this same way, besides genuine Secondness, there is a degenerate sort which does not exist as such, but is only so conceived. The medieval logicians (following a hint of Aristotle) distinguished between real relations and relations of reason. A real relation subsists in virtue of a fact which would be totally impossible were either of the related objects destroyed; while a relation of reason subsists in virtue of two facts, one only of which would disappear on the annihilation of either of the relates. Such are all resemblances: for any two objects in nature resemble each other, and indeed in themselves just as much as any other two; it is only with reference to our senses and needs that one resemblance counts for more than another. Rumford and Franklin resembled each other by virtue of being both Americans; but either would have been just as much an American if the other had never lived. On the other hand, the fact that Cain killed Abel cannot be stated as a mere aggregate of two facts, one concerning Cain and the other concerning Abel. Resemblances are not the only relations of reason, though they have that character in an eminent degree. Contrasts and comparisons are of the same sort. Resemblance is an identity of characters; and this is the same as to say that the mind gathers the resembling ideas together into one conception. Other relations of reason arise from ideas being connected by the mind in other ways; they consist in the relation between two parts of one complex concept, or, as we may say, in the relation of a complex concept to itself, in respect to two of its parts. This brings us to consider a sort of degenerate Secondness that does not fulfill the definition of a relation of reason. Identity is the relation that everything bears to itself: Lucullus dines with Lucullus. Again, we speak of allurements and motives in the language of forces, as though a man suffered compulsion from within. So with the voice of conscience: and we observe our own feelings by a reflective sense. An echo is my own voice coming back to answer itself. So also, we speak of the abstract quality of a thing as if it were some second thing that the first thing possesses. But the relations of reason and these self-relations are alike in this, that they arise from the mind setting one part of a notion into relation to another. All degenerate seconds may be conveniently termed internal, in contrast to external seconds, which are constituted by external fact, and are true actions of one thing upon another.

Peirce: CP 1.366 Cross-Ref: ††

366. Among thirds, there are two degrees of degeneracy. The first is where there is in the fact itself no Thirdness or mediation, but where there is true duality; the second degree is where there is not even true Secondness in the fact itself. Consider, first, the thirds degenerate in the first degree. A pin fastens two things together by sticking through one and also through the other: either might be annihilated, and the pin would continue to stick through the one which remained. A mixture brings its ingredients together by containing each. We may term these accidental thirds. "How did I slay thy son?" asked the merchant, and the jinnee replied, "When thou threwest away the date-stone, it smote my son, who was passing at the time, on the breast, and he died forthright." Here there were two independent facts, first that the merchant threw away the date-stone, and second that the date-stone struck and killed the jinnee's son. Had it been aimed at him, the case would have been different; for then there would have been a relation of aiming which would have connected together the aimer, the thing aimed, and the object aimed at, in one fact. What monstrous injustice and inhumanity on the part of that jinnee to hold that poor merchant responsible for such an accident! I remember how I wept at it, as I lay in my father's arms and he first told me the story. It is certainly just that a man, even though he had no evil intention,

should be held responsible for the immediate effects of his actions; but not for such as might result from them in a sporadic case here and there, but only for such as might have been guarded against by a reasonable rule of prudence. Nature herself often supplies the place of the intention of a rational agent in making a Thirdness genuine and not merely accidental; as when a spark, as third, falling into a barrel of gunpowder, as first, causes an explosion, as second. But how does nature do this? By virtue of an intelligible law according to which she acts. If two forces are combined according to the parallelogram of forces, their resultant is a real third. Yet any force may, by the parallelogram of forces, be mathematically resolved into the sum of two others, in an infinity of different ways. Such components, however, are mere creations of the mind. What is the difference? As far as one isolated event goes, there is none; the real forces are no more present in the resultant than any components that the mathematician may imagine. But what makes the real forces really there is the general law of nature which calls for them, and not for any other components of the resultant. Thus, intelligibility, or reason objectified, is what makes Thirdness genuine.

Peirce: CP 1.367 Cross-Ref:††

367. We now come to thirds degenerate in the second degree. The dramatist Marlowe had something of that character of diction in which Shakespeare and Bacon agree. This is a trivial example; but the mode of relation is important. In natural history, intermediate types serve to bring out the resemblance between forms whose similarity might otherwise escape attention, or not be duly appreciated. In portraiture, photographs mediate between the original and the likeness. In science, a diagram or analogue of the observed fact leads on to a further analogy. The relations of reason which go to the formation of such a triple relation need not be all resemblances. Washington was eminently free from the faults in which most great soldiers resemble one another. A centaur is a mixture of a man and a horse. Philadelphia lies between New York and Washington. Such thirds may be called intermediate thirds or thirds of comparison.

Peirce: CP 1.368 Cross-Ref:††

368. Nobody will suppose that I wish to claim any originality in reckoning the triad important in philosophy. Since Hegel, almost every fanciful thinker has done the same. Originality is the last of recommendations for fundamental conceptions. On the contrary, the fact that the minds of men have ever been inclined to threefold divisions is one of the considerations in favor of them. Other numbers have been objects of predilection to this philosopher and that, but three has been prominent at all times and with all schools. My whole method will be found to be in profound contrast with that of Hegel; I reject his philosophy *in toto*. Nevertheless, I have a certain sympathy with it, and fancy that if its author had only noticed a very few circumstances he would himself have been led to revolutionize his system. One of these is the double division or dichotomy of the second idea of the triad. He has usually overlooked external Secondness, altogether. In other words, he has committed the trifling oversight of forgetting that there is a real world with real actions and reactions. Rather a serious oversight that. Then Hegel had the misfortune to be unusually deficient in mathematics. He shows this in the very elementary character of his reasoning. Worse still, while the whole burden of his song is that philosophers have neglected to take Thirdness into account, which is true enough of the theological kind, with whom alone he was acquainted (for I do not call it acquaintance to look into a book without comprehending it), he unfortunately did not know, what it would have been of the utmost consequence for him to know, that the mathematical analysts had in great measure escaped this great fault, and that the thorough-going pursuit of the ideas and

methods of the differential calculus would be sure to cure it altogether. Hegel's dialectical method is only a feeble and rudimentary application of the principles of the calculus to metaphysics. Finally Hegel's plan of evolving everything out of the abstractest conception by a dialectical procedure, though far from being so absurd as the experientialists think, but on the contrary representing one of the indispensable parts of the course of science, overlooks the weakness of individual man, who wants the strength to wield such a weapon as that.

Peirce: CP 1.369 Cross-Ref: ††
§2. THE TRIAD IN REASONING †1

369. Kant, the King of modern thought, it was who first remarked the frequency in logical analytics of *trichotomics* or threefold distinctions. It really is so; I have tried hard and long to persuade myself that it is only fanciful, but the facts will not countenance that way of disposing of the phenomenon. Take any ordinary syllogism:

All men are mortal,
Elijah was a man;
Therefore, Elijah was mortal.

There are here three propositions, namely, two premisses and a conclusion; there are also three terms, *man*, *mortal*, and *Elijah*. If we transpose one of the premisses with the conclusion, denying both, we obtain what are called the indirect figures of syllogism; for example

All men are mortal,
But Elijah was not mortal;
Therefore, Elijah was not a man.

Elijah was not mortal,
But Elijah was a man;
Therefore, some men are not mortal.

Thus, there are three figures of ordinary syllogism. It is true there are other modes of inference which do not come under any of these heads; but that does not annul the fact that we have here a trichotomy. Indeed, if we examine by itself what is by some logicians called the fourth figure, we find that it also has three varieties related to one

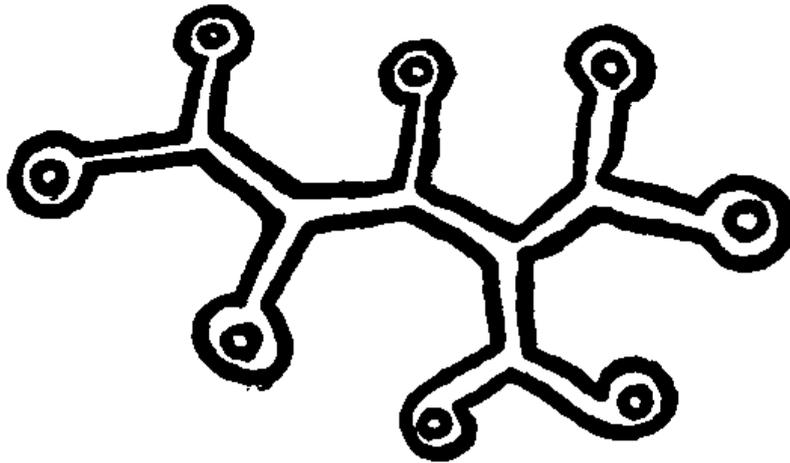
another as the three figures of ordinary syllogism. There is an entirely different way of conceiving the relations of the figures of syllogism; namely, by means of the conversion of propositions. But from that point of view also, the same classes are preserved. DeMorgan †1 has added a large number of new syllogistic moods which do not find places in this classification. The reasoning in these is of a peculiar character and introduces the principle of dilemma. Still, regarding these dilemmatic reasonings by themselves, they fall into three classes in a precisely similar manner. Again, I have shown †2 that the probable and approximate inferences of science must be classified on the very same principles, being either Deductions, Inductions, or Hypotheses. Other examples of threes in logic are statements of what is actual, what is possible, and what is necessary; the three kinds of forms, Names, †3 Propositions, and Inferences; †4 affirmative, negative, and uncertain answers to a question. One very important triad is this: it has been found that there are three kinds of signs which are all indispensable in all reasoning; the first is the diagrammatic sign or *icon*, which exhibits a similarity or analogy to the subject of discourse; the second is the *index*, which like a pronoun demonstrative or relative, forces the attention to the particular object intended without describing it; the third [or symbol] is the general name or description which signifies its object by means of an association of ideas or habitual connection between the name and the character signified.

Peirce: CP 1.370 Cross-Ref: ††

370. But there is one triad in particular which throws a strong light on the nature of all the others. Namely, we find it necessary to recognize in logic three kinds of characters, three kinds of facts. First there are *singular* characters which are predicable of single objects, as when we say that anything is white, large, etc. Secondly, there are dual characters which appertain to pairs of objects; these are implied by all relative terms as "lover," "similar," "other," etc. Thirdly, there are plural characters, which can all be reduced to triple characters but not to dual characters. Thus, we cannot express the fact that A is a benefactor of B by any descriptions of A and B separately; we must introduce a relative term. This is requisite, not merely in English, but in every language which might be invented. This is true even of such a fact as A is taller than B. If we say, "A is tall, but B is short," the conjugation "but" has a relative force, and if we omit this word the mere collocation of the two sentences is a relative or dual mode of signifying. . . .

Peirce: CP 1.371 Cross-Ref: ††

371. Let us now consider a triple character, say that A gives B to C. This is not a mere congeries of dual characters. It is not enough to say that A parts with C, and that B receives C. A synthesis of these two facts must be made to bring them into a single fact; we must express that C, in being parted with by A, is received by B. If, on the other hand, we take a quadruple fact, it is easy to express as a compound of two triple facts. . . . We are here able to express the synthesis of the two facts into one, because a triple character involves the conception of synthesis. Analysis involves the same relations as synthesis; so that we may explain the fact that all plural facts can be reduced to triple facts in this way. A road with a fork in it is the analogue of a triple fact, because it brings three termini into relation with one another. A dual fact is like a road without a fork; it only connects two termini. Now, no combination of roads without forks can have more than two termini; but any number of termini can be connected by roads which nowhere have a knot of more than three ways. See the



[Click here to view] figure, where I have drawn the termini as self-returning roads, in order to introduce nothing beyond the road itself. Thus, the three essential elements of a network of roads are *road about a terminus*, *roadway-connection*, and *branching*; and in like manner, the three fundamental categories of fact are, fact about an object, fact about two objects (relation), fact about several objects (synthetic fact).

Peirce: CP 1.372 Cross-Ref:††

372. We have seen that the mere coexistence of two singular facts constitutes a degenerate form of dual fact; and in like manner there are two orders of degeneracy in plural facts, for either they may consist in a mere synthesis of facts of which the highest is dual, or they may consist in a mere synthesis of singular facts. This explains why there should be three classes of *signs*; for there is a triple connection of *sign, thing signified, cognition produced in the mind*. There may be a mere relation of reason between the sign and the thing signified; in that case the sign is an *icon*. Or there may be a direct physical connection; in that case, the sign is an *index*. Or there may be a relation which consists in the fact that the mind associates the sign with its object; in that case the sign is a *name*^{†1} [or *symbol*]. Now consider the difference between a logical *term*, a *proposition*, and an *inference*. A term is a mere general description, and as neither *icon* nor *index* possesses generality, it must be a name; and it is nothing more. A proposition is also a general description, but it differs from a term in that it purports to be in a real relation to the fact, to be really determined by it; thus, a proposition can only be formed of the conjunction of a name and an index. An inference, too, contains a general description. . . .

Peirce: CP 1.373 Cross-Ref:††
 §3. THE TRIAD IN METAPHYSICSP

373. I will run over all the conceptions that played an important part in the pre-Socratic philosophy and see how far they can be expressed in terms of one, two, three.

Peirce: CP 1.373 Cross-Ref:††

1. The first of all the conceptions of philosophy is that of a primal matter out of which the world is made. Thales and the early Ionian philosophers busied

themselves mainly with this. They called it the {arché}, the beginning; so that the conception of first was the quintessence of it. Nature was a wonder to them, and they asked its explanation; from what did it come? That was a good question, but it was rather stupid to suppose that they were going to learn much even if they could find out from what sort of matter it was made. But to ask how it had been formed, as they doubtless did, was not an exhaustive question; it would only carry them back a little way. They wished to go to the very beginning at once, and in the beginning there must have been a homogeneous something, for where there was variety they supposed there must be always an explanation to be sought. The first must be indeterminate, and the indeterminate first of anything is the material of which it is formed. Besides, their idea was that they could not tell how the world was formed unless they knew from what to begin their account. The inductive [method] of explaining phenomena by tracing them back step by step to their causes was foreign not only to them but to all ancient and medieval philosophy; that is the Baconian idea. Indeterminacy is really a character of the first. But not the indeterminacy of homogeneity. The first is full of life and variety. Yet that variety is only potential; it is not definitely there. Still, the notion of explaining the variety of the world, which was what they mainly wondered at, by non-variety was quite absurd. How is variety to come out of the womb of homogeneity; only by a principle of spontaneity, which is just that virtual variety that is the first. †1

Peirce: CP 1.374 Cross-Ref: ††
 §4. THE TRIAD IN PSYCHOLOGY †2P

374. The line of reasoning which I propose to pursue is peculiar and will need some careful study to estimate the strength of it. I shall review it critically in the last section, but meantime I desire to point out that the step I am about to take, which is analogous to others that will follow, is not so purely of the nature of a guess as might be supposed by persons expert in judging of scientific evidence. We have seen that the ideas of one, two, three, are forced upon us in logic, and really cannot be dispensed with. They meet us not once but at every turn. And we have found reason to think that they are equally important in metaphysics. How is the extraordinary prominence of these conceptions to be explained? Must it not be that they have their origin in the nature of the mind? This is the Kantian form of inference, which has been found so cogent in the hands of that hero of philosophy; and I do not know that modern studies have done anything to discredit it. It is true we no longer regard such a psychological explanation of a conception to be as final as Kant thought. It leaves further questions to be asked; but as far as it goes it seems to be satisfactory. We find the ideas of first, second, third, constant ingredients of our knowledge. It must then either be that they are continually given to us in the presentations of sense, or that it is the peculiar nature of the mind to mix them with our thoughts. Now we certainly cannot think that these ideas are given in sense. First, second, and third are not sensations. They can only be given in sense by things appearing labelled as first, second, and third, and such labels things do not usually bear. They ought therefore to have a psychological origin. A man must be a very uncompromising partisan of the theory of the *tabula rasa* to deny that the ideas of first, second, and third are due to congenital tendencies of the mind. So far there is nothing in my argument to distinguish it from that of many a Kantian. The noticeable thing is that I do not rest here, but seek to put the conclusion to the test by an independent examination of the

facts of psychology, to see whether we can find any traces of the existence of three parts or faculties of the soul or modes of consciousness, which might confirm the result just reached.

Peirce: CP 1.375 Cross-Ref:††

375. Now, three departments of the mind have been generally recognized since Kant; they are: Feeling [of pleasure and pain], Knowing, and Willing. The unanimity with which this trisection of the mind has been accepted is, indeed, quite surprising. The division did not have its genesis in the peculiar ideas of Kant. On the contrary, it was borrowed by him from dogmatic philosophers, and his acceptance of it was, as has been well remarked, a concession to dogmatism. It has been allowed even by psychologists to whose general doctrines it seems positively hostile.†1

Peirce: CP 1.376 Cross-Ref:††

376. The ordinary doctrine is open to a variety of objections from the very point of view from which it was first delineated. First, desire certainly includes an element of pleasure quite as much as of will. Wishing is not willing; it is a speculative variation of willing mingled with a speculative and anticipatory feeling of pleasure. Desire should therefore be struck out of the definition of the third faculty, leaving it mere volition. But volition without desire is not voluntary; it is mere activity. Consequently, all activity, voluntary or not, should be brought under the third faculty. Thus attention is a kind of activity which is sometimes voluntary and sometimes not so. Second, pleasure and pain can only be recognized as such in a judgment; they are general predicates which are attached to feelings rather than true feelings. But mere passive feeling, which does not act and does not judge, which has all sorts of qualities but does not itself recognize these qualities, because it does not analyze nor compare -- this is an element of all consciousness to which a distinct title ought to be given. Third, every phenomenon of our mental life is more or less like cognition. Every emotion, every burst of passion, every exercise of will, is like cognition. But modifications of consciousness which are alike have some element in common. Cognition, therefore, has nothing distinctive and cannot be regarded as a fundamental faculty. If, however, we ask whether there be not an element in cognition which is neither feeling, sense, nor activity, we do find something, the faculty of learning, acquisition, memory and inference, synthesis. Fourth, looking once more at activity, we observe that the only consciousness we have of it is the sense of resistance. We are conscious of hitting or of getting hit, of meeting with a *fact*. But whether the activity is within or without we know only by secondary signs and not by our original faculty of recognizing fact.

Peirce: CP 1.377 Cross-Ref:††

377. It seems, then, that the true categories of consciousness are: first, feeling, the consciousness which can be included with an instant of time, passive consciousness of quality, without recognition or analysis; second, consciousness of an interruption into the field of consciousness, sense of resistance, of an external fact, of another something; third, synthetic consciousness, binding time together, sense of learning, thought.

Peirce: CP 1.378 Cross-Ref:††

378. If we accept these [as] the fundamental elementary modes of consciousness, they afford a psychological explanation of the three logical conceptions of quality, relation, and synthesis or mediation. The conception of quality, which is absolutely simple in itself and yet viewed in its relations is seen to be full of variety, would arise whenever feeling or the singular consciousness

becomes prominent. The conception of relation comes from the dual consciousness or sense of action and reaction. The conception of mediation springs out of the plural consciousness or sense of learning.

Peirce: CP 1.379 Cross-Ref:††

379. . . . We remember it [sensation]; that is to say, we have another cognition which professes to reproduce it; but we know that there is no resemblance between the memory and the sensation, because, in the first place, nothing can resemble an immediate feeling, for resemblance supposes a dismemberment and recomposition which is totally foreign to the immediate, and in the second place, memory is an articulated complex and worked-over product which differs infinitely and immeasurably from feeling. Look at a red surface, and try to feel what the sensation is, and then shut your eyes and remember it. No doubt different persons are different in this respect; to some the experiment will seem to yield an opposite result, but I have convinced myself that there is nothing in my memory that is in the least like the vision of the red. When red is not before my eyes, I do not see it at all. Some people tell me they see it faintly -- a most inconvenient kind of memory, which would lead to remembering bright red as pale or dingy. I remember colors with unusual accuracy, because I have had much training in observing them; but my memory does not consist in any vision but in a habit by virtue of which I can recognize a newly presented color as like or unlike one I had seen before. But even if the memory of some persons is of the nature of an hallucination, enough arguments remain to show that immediate consciousness or feeling is absolutely unlike anything else.

Peirce: CP 1.380 Cross-Ref:††

380. There are grave objections to making a whole third of the mind of the will alone. One great psychologist has said that the will is nothing but the strongest desire. I cannot grant that; it seems to me to overlook that fact which of all that we observe is quite the most obtrusive, namely, the difference between dreaming and doing. This is not a question of defining, but of noticing what we experience; and surely he who can confound desiring with doing must be a day-dreamer. The evidence, however, seems to be pretty strong that the consciousness of willing does not differ, at least not very much, from a sensation. The sense of hitting and of getting hit are nearly the same, and should be classed together. The common element is the sense of an actual occurrence, of actual action and reaction. There is an intense reality about this kind of experience, a sharp sundering of subject and object. While I am seated calmly in the dark, the lights are suddenly turned on, and at that instant I am conscious, not of a process of change, but yet of something more than can be contained in an instant. I have a sense of a saltus, of there being two sides to that instant. A consciousness of polarity would be a tolerably good phrase to describe what occurs. For will, then, as one of the great types of consciousness, we ought to substitute the polar sense.

Peirce: CP 1.381 Cross-Ref:††

381. But by far the most confused of the three members of the division, in its ordinary statement, is Cognition. In the first place every kind of consciousness enters into cognition. Feelings, in the sense in which alone they can be admitted as a great branch of mental phenomena, form the warp and woof of cognition, and even in the objectionable sense of pleasure and pain, they are constituents of cognition. The will, in the form of attention, constantly enters, and the sense of reality or objectivity, which is what we have found ought to take the place of will, in the division of consciousness, is even more essential yet, if possible. But that element of cognition

which is neither feeling nor the polar sense, is the consciousness of a process, and this in the form of the sense of learning, of acquiring, of mental growth is eminently characteristic of cognition. This is a kind of consciousness which cannot be immediate, because it covers a time, and that not merely because it continues through every instant of that time, but because it cannot be contracted into an instant. It differs from immediate consciousness, as a melody does from one prolonged note. Neither can the consciousness of the two sides of an instant, of a sudden occurrence, in its individual reality, possibly embrace the consciousness of a process. This is the consciousness that binds our life together. It is the consciousness of synthesis.

Peirce: CP 1.382 Cross-Ref:††

382. Here then, we have indubitably three radically different elements of consciousness, these and no more. And they are evidently connected with the ideas of one-two-three. Immediate feeling is the consciousness of the first; the polar sense is the consciousness of the second; and synthetical consciousness is the consciousness of a third or medium.

Peirce: CP 1.383 Cross-Ref:††

383. Note, too, that just as we have seen that there are two orders of Secondness, so the polar sense splits into two, and that in two ways, for first, there is an active and a passive kind, or will and sense, and second, there are external will and sense, in opposition to internal will (self-control, inhibitory will) and internal sense (introspection). In like manner, just as there are three orders of Thirdness, so there are three kinds of synthetical consciousness. The undegenerate and really typical form has not been made so familiar to us as the others, which have been more completely studied by psychologists; I shall therefore mention that last. Synthetical consciousness degenerate in the first degree, corresponding to accidental Thirdness, is where there is an external compulsion upon us to think things together. Association by contiguity is an instance of this; but a still better instance is that in our first apprehension of our experiences, we cannot choose how we will arrange our ideas in reference to time and space, but are compelled to think certain things as nearer together than others. It would be putting the cart before the horse to say that we are compelled to think certain things together because they are together in time and space; the true way of stating it is that there is an exterior compulsion upon us to put them together in our construction of time and space, in our perspective. Synthetical consciousness, degenerate in the second degree, corresponding to intermediate thirds, is where we think different feelings to be alike or different, which, since feelings in themselves cannot be compared and therefore cannot be alike, so that to say they are alike is merely to say that the synthetical consciousness regards them so, comes to this, that we are internally compelled to synthesize them or to sunder them. This kind of synthesis appears in a secondary form in association by resemblance. But the highest kind of synthesis is what the mind is compelled to make neither by the inward attractions of the feelings or representations themselves, nor by a transcendental force of necessity, but in the interest of intelligibility that is, in the interest of the synthesizing "I think" itself; and this it does by introducing an idea not contained in the data, which gives connections which they would not otherwise have had. This kind of synthesis has not been sufficiently studied, and especially the intimate relationship of its different varieties has not been duly considered. The work of the poet or novelist is not so utterly different from that of the scientific man. The artist introduces a fiction; but it is not an arbitrary one; it exhibits affinities to which the mind accords a certain approval in pronouncing them beautiful, which if it is not exactly the same as saying that the synthesis is true, is something of the same general

kind. The geometer draws a diagram, which if not exactly a fiction, is at least a creation, and by means of observation of that diagram he is able to synthesize and show relations between elements which before seemed to have no necessary connection. The realities compel us to put some things into very close relation and others less so, in a highly complicated, and in the [to?] sense itself unintelligible manner; but it is the genius of the mind, that takes up all these hints of sense, adds immensely to them, makes them precise, and shows them in intelligible form in the intuitions of space and time. Intuition is the regarding of the abstract in a concrete form, by the realistic hypostatization of relations; that is the one sole method of valuable thought. Very shallow is the prevalent notion that this is something to be avoided. You might as well say at once that reasoning is to be avoided because it has led to so much error; quite in the same philistine line of thought would that be; and so well in accord with the spirit of nominalism that I wonder some one does not put it forward. The true precept is not to abstain from hypostatization, but to do it intelligently. . . . †1

Peirce: CP 1.384 Cross-Ref: ††

384. Kant gives the erroneous view that ideas are presented separated and then thought together by the mind. This is his doctrine that a mental synthesis precedes every analysis. What really happens is that something is presented which in itself has no parts, but which nevertheless is analyzed by the mind, that is to say, its having parts consists in this, that the mind afterward recognizes those parts in it. Those partial ideas are really not in the first idea, in itself, though they are separated out from it. It is a case of destructive distillation. When, having thus separated them, we think over them, we are carried in spite of ourselves from one thought to another, and therein lies the first real synthesis. An earlier synthesis than that is a fiction. The whole conception of time belongs to genuine synthesis and is not to be considered under this head.

Peirce: CP 1.385 Cross-Ref: ††

§5. THE TRIAD IN PHYSIOLOGY^P

385. Granted that there are three fundamentally different kinds of consciousness, it follows as a matter of course that there must be something threefold in the physiology of the nervous system to account for them. No materialism is implied in this, further than that intimate dependence of the action of the mind upon the body, which every student of the subject must and does now acknowledge. Once more a prediction, as it were, is made by the theory; that is to say, certain consequences, not contemplated in the construction thereof, necessarily result from it; and these are of such a character that their truth or falsehood can be independently investigated. Were we to find them strikingly and certainly true, a remarkable confirmation of the theory would be afforded. So much as this, however, I cannot promise; I can only say that they are not certainly false; and we must be content to trace out these consequences, and see what they are, and leave them to the future judgment of physiologists.

Peirce: CP 1.386 Cross-Ref: ††

386. Two of the three kinds of consciousness, indeed, the simple and dual, receive an instant physiological explanation. We know that the protoplasmic content

of every nerve-cell has its active and passive conditions, and argument is unnecessary to show that feeling, or immediate consciousness, arises in an active state of nerve-cells. Experiments on the effects of cutting the nerves show that there is no feeling after communication with the central nerve-cells is severed, so that the phenomenon has certainly some connection with the nerve-cells; and feeling is excited by just such stimuli as would be likely to throw protoplasm into an active condition. Thus, though we cannot say that every nerve-cell in its active condition has feeling (which we cannot deny, however) there is scarce room to doubt that the activity of nerve-cells is the main physiological requisite for consciousness. On the other hand, the sense of action and reaction, or the polar sense, as we agreed to call it, is plainly connected with the discharge of nervous energy through the nerve-fibres. External volition, the most typical case of it, involves such a discharge into muscle cells. In external sensation, where the polar sense enters in a lower intensity, there is a discharge from the terminal nerve-cell through the afferent nerve upon a cell or cells in the brain. In internal volition, or self-control, there is some inhibitory action of the nerves, which is also known to involve the movement of nervous force; and in internal observation, or visceral sensation, there are doubtless transfers of energy from one central cell to another. Remembering that the polar sense is the sense of the difference between what was before and what is after a dividing instant, or the sense of an instant as having sides, we see clearly that the physiological concomitant of it must be some event which happens very quickly and leaves a more abiding effect, and this description suits the passage of a nervous discharge over a nerve-fibre so perfectly, that I do not think we need hesitate to set this phenomenon down as the condition of dual consciousness.

Peirce: CP 1.387 Cross-Ref:††

387. Synthetical consciousness offers a more difficult problem. Yet the explanation of the genuine form of that consciousness, the sense of learning, is easy enough; it is only the degenerate modes, the sense of similarity, and the sense of real connection, which oblige us to hesitate. With regard to these two degenerate forms, I am driven to make hypotheses.

Peirce: CP 1.388 Cross-Ref:††

388. When two ideas resemble one another, we say that they have something in common; part of the one is said to be identical with a part of the other. In what does that identity consist? Having closed both eyes, I open first one and then shut it and open the other, and I say that the two sensations are alike. How can the impressions of two nerves be judged to be alike? It appears to me that in order that that should become possible, the two nerve-cells must probably discharge themselves into one common nerve-cell. In any case, it seems to me that the first supposition to make, for scientific observation to confirm or reject, is that two ideas are alike so far as the same nerve-cells have been concerned in the production of them. In short, the hypothesis is that resemblance consists in the identity of a common element, and that that identity lies in a part of the one idea and a part of the other idea being the feeling peculiar to the excitation of one or more nerve-cells.

Peirce: CP 1.389 Cross-Ref:††

389. When we find ourselves under a compulsion to think that two elements of experience which do not particularly resemble one another are, nevertheless, really connected, that connection must, I think, be due in some way to a discharge of nerve-energy; for the whole sense of reality is a determination of polar consciousness, which is itself due to such discharges. For example, I recognize that a certain surface

on one side of a certain boundary is red, and on the other side is blue; or that any two qualities are immediately contiguous in space or time. If the contiguity is in time, it is by the polar sense directly that we are conscious of a dividing instant with its difference on the two sides. If the contiguity is in space, I think we have at first a completely confused feeling of the whole, as yet unanalyzed and unsynthesized, but afterward, when the analysis has been made, we find ourselves compelled, in recomposing the elements, to pass directly from what is on one side of the boundary to what is on the other. I suppose then that we are compelled to think the two feelings as contiguous because the nerve-cell whose excitation produces the feeling of one recalled sensation discharges itself into the nerve-cell whose excitation makes the feeling of the other recalled sensation.

Peirce: CP 1.390 Cross-Ref:††

390. The genuine synthetic consciousness, or the sense of the process of learning, which is the preeminent ingredient and quintessence of the reason, has its physiological basis quite evidently in the most characteristic property of the nervous system, the power of taking habits. This depends on five principles, as follows. First, when a stimulus or irritation is continued for some time, the excitation spreads from the cells directly affected to those that are associated with it, and from those to others, and so on, and at the same time increases in intensity. Second, after a time fatigue begins to set in. Now besides the utter fatigue which consists in the cell's losing all excitability, and the nervous system refusing to react to the stimulus at all, there is a gentler fatigue, which plays a very important part in adapting the brain to serving as an organ of reason, this form of fatigue consisting in the reflex action or discharge of the nerve-cell ceasing to go on one path and either beginning on a path where there had been no discharge, or increasing the intensity of the discharge along a path on which there had been previously only a slight discharge. For example, one may sometimes see a frog whose cerebrum or brain has been removed, and whose hind leg has been irritated by putting a drop of acid upon it, after repeatedly rubbing the place with the other foot, as if to wipe off the acid, may at length be observed to give several hops, the first avenue of nervous discharge having become fatigued. Third, when, from any cause the stimulus to a nerve-cell is removed, the excitation quickly subsides. That it does not do so instantly is well known, and the phenomenon goes among physicists by the name of persistence of sensation. All noticeable feeling subsides in a fraction of a second, but a very small remnant continues for a much longer time. Fourth, if the same cell which was once excited, and which by some chance had happened to discharge itself along a certain path or paths, comes to get excited a second time, it is more likely to discharge itself the second time along some or all of those paths along which it had previously discharged itself than it would have been had it not so discharged itself before. This is the central principle of habit; and the striking contrast of its modality to that of any mechanical law is most significant. The laws of physics know nothing of tendencies or probabilities; whatever they require at all they require absolutely and without fail, and they are never disobeyed. Were the tendency to take habits replaced by an absolute requirement that the cell should discharge itself always in the same way, or according to any rigidly fixed condition whatever, all possibility of habit developing into intelligence would be cut off at the outset; the virtue of Thirdness would be absent. It is essential that there should be an element of chance in some sense as to how the cell shall discharge itself; and then that this chance or uncertainty shall not be entirely obliterated by the principle of habit, but only somewhat affected. Fifth, when a considerable time has elapsed without a nerve having reacted in any particular way, there comes in a principle of forgetfulness or negative habit rendering it the less likely to react in that

way. Now let us see what will be the result of these five principles taken in combination. When a nerve is stimulated, if the reflex activity is not at first of the right sort to remove the source of irritation, it will change its character again and again until the cause of irritation is removed, when the activity will quickly subside. When the nerve comes to be stimulated a second time in the same way, probably some of the other movements which had been made on the first occasion will be repeated; but, however this may be, one of them must ultimately be repeated, for the activity will continue until this does happen, I mean that movement which removes the source of irritation. On a third occasion, the process of forgetfulness will have been begun in regard to any tendency to repeat any of the actions of the first occasion which were not repeated on the second. Of those which were repeated, some will probably be repeated again, and some not; but always there remains that one which must be repeated before the activity comes to an end. The ultimate effect of this will inevitably be that a habit gets established of at once reacting in the way which removes the source of irritation; for this habit alone will be strengthened at each repetition of the experiment, while every other will tend to become weakened at an accelerated rate.

Peirce: CP 1.391 Cross-Ref:††

391. I have invented a little game or experiment with playing cards to illustrate the working of these principles; and I can promise the reader that if he will try it half a dozen times he will be better able to estimate the value of the account of habit here proposed. The rules of this game are as follows: take a good many cards of four suits, say a pack of fifty-two, though fewer will do. The four suits are supposed to represent four modes in which a cell may react. Let one suit, say spades, represent that mode of reaction which removes the source of irritation and brings the activity to an end. In order readily to find a card of any suit as wanted, you had better lay all the cards down face up and distribute into four packets, each containing the cards of one suit only. Now take two spades, two diamonds, two clubs, and two hearts, to represent the original disposition of the nerve-cell, which is supposed to be equally likely to react in any of the four ways. You turn these eight cards face down and shuffle them with extreme thoroughness.†P1 Then turn up cards from the top of this pack, one by one until a spade is reached. This process represents the reaction of the cell. Take up the cards just dealt off, and add to the pack held in the hand one card of each of those suits that have just been turned up (for habit) and remove from the pack one card of each suit not turned up (for forgetfulness). Shuffle, and go through with this operation thirteen times or until the spades are exhausted. It will then generally be found that you hold nothing but spades in your hand.

Peirce: CP 1.392 Cross-Ref:††

392. Thus we see how these principles not only lead to the establishment of habits, but to habits directed to definite ends, namely the removal of sources of irritation. Now it is precisely action according to final causes which distinguishes mental from mechanical action; and the general formula of all our desires may be taken as this: to remove a stimulus. Every man is busily working to bring to an end that state of things which now excites him to work.

Peirce: CP 1.393 Cross-Ref:††

393. But we are led yet deeper into physiology. The three fundamental functions of the nervous system, namely, first, the excitation of cells; second, the transfer of excitation over fibres; third, the fixing of definite tendencies under the influence of habit, are plainly due to three properties of the protoplasm or life-slime

itself. Protoplasm has its active and its passive condition, its active state is transferred from one part of it to another, and it also exhibits the phenomena of habit. But these three facts do not seem to sum up the main properties of protoplasm, as our theory would lead us to expect them to do. Still, this may be because the nature of this strange substance is so little understood; and if we had the true secret of its constitution we might see that qualities that now appear unrelated really group themselves into one, so that it may be after all that it accords with our theory better than it seems to do. There have been at least two attempts to explain the properties of protoplasm by means of chemical suppositions; but inasmuch as chemical forces are as far as possible themselves from being understood, such hypotheses, even if they were known to be correct, would be of little avail. As for what a physicist would understand by a molecular explanation of protoplasm, such a thing seems hardly to have been thought of; yet I cannot see that it is any more difficult than the constitution of inorganic matter. The properties of protoplasm are enumerated as follows: contractility, irritability, automatism, nutrition, metabolism, respiration, and reproduction; but these can all be summed up under the heads of sensibility, motion, and growth. These three properties are respectively first, second, and third. Let us, however, draw up a brief statement of the facts which a molecular theory of protoplasm would have to account for. In the first place, then, protoplasm is a definite chemical substance, or class of substances, recognizable by its characteristic relations. "We do not at present," says Dr. Michael Foster †¹ (1879), "know anything definite about the molecular composition of active living protoplasm; but it is more than probable that its molecule is a large and complex one in which a proteid substance is peculiarly associated with a complex fat and with some representative of the carbohydrate group, i.e., that each molecule of protoplasm contains residues of each of these three great classes. The whole animal body is modified protoplasm." The chemical complexity of the protoplasm molecule must be amazing. A proteid is only one of its constituents, and doubtless very much simpler. Yet chemists do not attempt to infer from their analyses the ultimate atomic constitution of any of the proteids, the number of atoms entering into them being so great as almost to nullify the law of multiple proportions. I do find in the book just quoted the following formula for nuclein, a substance allied to the proteids. It is $C[29]H[49]N[9]P[3]O[22]$. But as the sum of the numbers of atoms of hydrogen, nitrogen, and phosphorus ought to be even, this formula must be multiplied by some even number; so that the number of atoms in nuclein must be two hundred and twenty-four at the very least. We can hardly imagine, then, that the number of atoms in protoplasm is much less than a thousand, and if one considers the very minute proportions of some necessary ingredients of animal and vegetable organisms, one is somewhat tempted to suspect that fifty thousand might do better, or even come to be looked upon in the future as a ridiculously small guess. Protoplasm combines with water in all proportions, the mode of combination being apparently intermediate between solution and mechanical mixture. According to the amount of water it contains, it passes from being brittle to being pliable, then gelatinous, then slimy, then liquid. Generally, it has the character of being elastico-viscous; that is to say, it springs back partially after a long strain, and wholly after a short one; but its viscosity is much more marked than its elasticity. It is generally full of granules, by which we can see slow streaming motions in it, continuing for some minutes in one way and then generally reversed. The effect of this streaming is to cause protuberances in the mass, often very long and slender. They occasionally stick up against gravity; and their various forms are characteristic of the different kinds of protoplasm. When a mass of it is disturbed by a jar, a poke, an electric shock, heat, etc., the streams are arrested and the whole contracts into a

ball; or if it were very much elongated, sometimes breaks up into separate spheres. When the external excitation is removed, the mass sinks down into something like its former condition. Protoplasm also grows; it absorbs material and converts it into the like of its own substance; and in all its growth and reproduction, it preserves its specific characters.

Peirce: CP 1.394 Cross-Ref:††

394. Such are the properties that have to be accounted for. What first arrests our attention, as likely to afford the key to the problem, is the contraction of the mass of protoplasm on being disturbed. This is obviously due to a vast and sudden increase of what the physicists call "surface tension," or the pulling together of the outer parts, which phenomenon is always observed in liquids, and is the cause of their making drops. This surface tension is due to the cohesion, or attraction between neighboring molecules. The question is, then, how can a body, on having its equilibrium deranged, suddenly increase the attractions between its neighboring molecules? These attractions must increase rapidly as the distance is diminished; and thus the answer suggests itself that the distance between neighboring molecules is diminished. True, the average distance must remain nearly the same, but if the distances which had previously been nearly equal are rendered unequal, the attractions between the molecules that are brought nearer to one another will be much more increased than those between those that are removed from one another will be diminished. We are thus led to the supposition that in the ordinary state of the substance, its particles are moving for the most part in complicated orbital or quasi-orbital systems, instead of in the chemical molecules or more definite systems of atoms of less complex substances, these particles thus moving in orbits not being, however, atoms, but chemical molecules. But we must suppose that the forces between these particles are just barely sufficient to hold them in their orbits, and that in fact, as long as the protoplasm is in an active condition, they are not all so held, but that one and another get occasionally thrown out of their orbits and wander about until they are drawn in to some other system. We must suppose that these systems have some approximate composition, about so many of one kind of particles and so many of another kind, etc., entering into them. This is necessary to account for the nearly constant chemical composition of the whole. On the other hand, we cannot suppose that the number of the different kinds is rigidly exact; for in that case we should not know how to account for the power of assimilation. We must suppose then that there is considerable range in the numbers of particles that go to form an orbital system, and that the somewhat exact chemical composition of the whole is the exactitude of a statistical average; just as there is a close equality between the proportions of the two sexes in any nation or province, though there is considerable inequality in each of the different households. Owing to the complexity of this arrangement, the moment that there is any molecular disturbance, producing perturbations, large numbers of the particles are thrown out of their orbits, the systems are more or less deranged in the immediate neighborhood of the disturbance, and the harmonic relations between the different revolutions are somewhat broken up. In consequence of this, the distances between neighboring particles, which had presented a systematic regularity, now become extremely unequal, and their average attractions, upon which the cohesion depends, is increased. At the same time, the particles thrown out of their systems shoot into other systems and derange these in their turn, and so the disturbance is propagated throughout the entire mass. The source of disturbance, however, being removed, interchanges of energy take place, in which there is a tendency to equalize the *vis viva* of the different particles, and they consequently tend to sink down into orbital motions again, and gradually something very like the original state of things is reestablished, the original

orbital systems remaining, for the most part, and the wandering particles in large proportion finding places in these systems or forming new ones. Some of these particles will not find any places, and thus there will be a certain amount of wasting of the protoplasmic mass. If the same disturbance is repeated, so far as the orbital systems remain the same as they were before, there will be a repetition of almost exactly the same events. The same kinds of particles (the same I mean in mass, velocities, directions of movement, attractions, etc.) which were thrown out of the different systems before will generally get thrown out again, until, if the disturbance is repeated several times, there gets to be rather a deficiency of those kinds of particles in the different systems, when some new kinds will begin to be thrown out. These new kinds will differently perturb the systems into which they fly, tending to cause classes of particles like themselves to be thrown out, and, in that way, the direction of propagation of the disturbance, as well as its velocity and intensity, may be altered, and, in short, the phenomenon of fatigue will be manifested. Even when the protoplasmic mass is left to itself, there will be some wandering of particles, producing regions of slight disturbance, and so inequalities of tension; and thus, streams will be set up, movements of the mass will take place, and slender processes will be formed. If, however, the mass be left to itself for a very long time, all the particles that are readily thrown out will, in all the changes that are rung on the combinations of situations and velocities in the orbital systems, get thrown out; while the others will constantly tend to settle down into more stable relations; and so the protoplasm will gradually take a passive state from which its orbital systems are not easily deranged. The food for those kinds of protoplasm that are capable of marked reaction has to be presented in chemically complex form. It must doubtless present particles just like those that revolve in the orbital systems of the protoplasm. In order to be drawn into an orbital system, a particle, whether of food matter or just thrown off from some other system, must have the right mass, must present itself at the right point, and move with the right velocity in the right direction and be subject to the right attractions. It will be right in all these respects, if it comes to take the place of a particle which has just been thrown off; and thus, particles taken in are particularly likely to be of the same material and masses and to take the same places in the orbits as those that have been shortly before thrown off. Now these particles being the exact representatives of those thrown off, will be likely to be thrown off by the same disturbances, in the same directions, and with the same results, as those which were thrown off before; and this accounts for the principle of habit. All the higher kinds of protoplasm, those for example which have any marked power of contraction, are fed with matter chemically highly complex. †1

Peirce: CP 1.395 Cross-Ref: ††

§6. THE TRIAD IN BIOLOGICAL DEVELOPMENT

395. Whether the part played by natural selection and the survival of the fittest in the production of species be large or small, there remains little doubt that the Darwinian theory indicates a real cause, which tends to adapt animal and vegetable forms to their environment. A very remarkable feature of it is that it shows how merely fortuitous variations of individuals together with merely fortuitous mishaps to them would, under the action of heredity, result, not in mere irregularity, nor even in a statistical constancy, but in continual and indefinite progress toward a better adaptation of means to ends. How can this be? What, abstractly stated, is the peculiar

factor in the conditions of the problem which brings about this singular consequence?

Peirce: CP 1.396 Cross-Ref:††

396. Suppose a million persons, each provided with one dollar, to sit down to play a simple and fair game of chance, betting for example on whether a die turns up an odd or even number. The players are supposed to make their bets independently of one another, and each to bet on the result of each throw one dollar against a dollar on the part of the bank. Of course, at the very first bet, one-half of them would lose their only dollar and go out of the game, for it is supposed that no credit is allowed, while the other half would win each \$1 and so come to be worth \$2. Of these 500,000 players, after the second throw, 250,000 would have lost, and so be worth only \$1 each, while the other 250,000 would have won, and so be worth \$3. After the third throw, 125,000, or one-half of those who had had \$1 each, would be ruined; 250,000 would be worth \$2 (namely one-half the 250,000 who had had \$1 each, and one-half the 250,000 who had had \$3 each) and 125,000 would be worth \$4 each. The further progress of the game is illustrated by the table on page 216, where the numbers of players are given having each possible sum after the first, second, third, etc., throws. It will be seen by the table that, at the end of the fourth throw, the most usual fortune is \$3, at the end of the ninth \$4, at the end of the sixteenth \$5, and in like manner at the end of the twenty-fifth it would be \$6, at the end of the thirty-sixth \$7, and so forth. Here, then, would be a continual increase of wealth, which is a sort of "adaptation to one's environment," produced by a survival of the fittest, that is, by the elimination from the game of every player who has lost his last dollar. It is easy to see that the increase of average and usual wealth comes about by the subtraction of all those small fortunes which would be in the hands of men who had once been bankrupt had they been allowed to continue betting.

Peirce: CP 1.397 Cross-Ref:††

397. Now the adaptation of a species to its environment consists, for the purposes of natural selection, in a power of continuing to exist, that is to say, in the power of one generation to bring forth another; for as long as another generation

	<i>First</i>	<i>Second</i>	<i>Third</i>	<i>Fourth</i>	<i>Fifth</i>	<i>Sixth</i>	<i>Seventh</i>	<i>Eighth</i>	<i>Ninth</i>	<i>Tenth</i>	<i>Sixteenth</i>
\$1	250,000	125,000	76,875	53,750	40,312½	31,744
2	500,000	250,000	153,750	107,500	80,625
3	250,000	182,500	138,125	107,500	86,406¼	52,292
4	125,000	122,500	107,500	92,187½
5	62,500	76,875	76,875	77,070¼	55,542
6	31,250	46,250	61,952⅞*
7	15,625	27,031¼	38,710¾	38,880
8	7,812½	15,468½
9	3,906¼	8,710¾	19,226
10	1,953⅞
11	978½	8,714
12
13
14	1,587
15
16	229
17
	15

* This figure should be 51,952⅞. Because of this error the fifth and seventh figures in the next column are each 5,000 in excess, and, with the exception of the two last items, the entire last column is wrong.

[Click here

to view]†1

is brought forth the species will continue and as soon as this ceases it is doomed after

one lifetime. This reproductive faculty, then, depending partly on direct fecundity, and partly on the animal's living through the age of procreation, is precisely what the Darwinian theory accounts for. This character plainly is one of those which has an absolute minimum, for no animal can produce fewer offspring than none at all and it has no apparent upper limit, so that it is quite analogous to the wealth of those players. It is to be remarked that the phrase "survival of the fittest" in the formula of the principle does not mean the survival of the fittest individuals, but the survival of the fittest types; for the theory does not at all require that individuals ill-adapted to their environment should die at an earlier age than others, so long only as they do not reproduce so many offspring as others; and indeed it is not necessary that this should go so far as to extinguish the line of descent, provided there be some reason why the offspring of ill-adapted parents are less likely than others to inherit those parents' characteristics. It seems likely that the process, as a general rule, is something as follows: A given individual is in some respect ill-adapted to his environment, that is to say, he has characters which are generally unfavorable to the production of numerous offspring. These characters will be apt to weaken the reproductive system of that individual, for various reasons, so that its offspring are not up to the average strength of the species. This second generation will couple with other individuals, but owing to their weakness, their offspring will be more apt to resemble the other parent, and so the unfavorable character will gradually be eliminated, not merely by diminished numbers of offspring, but also by the offspring more resembling the stronger parent. There are other ways in which the unfavorable characters will disappear. When the procreative power is weakened, there are many examples to show that the principle of heredity becomes relaxed, and the race shows more tendency to sporting. This sporting will go on until in the course of it the unfavorable character has become obliterated. The general power of reproduction thereupon becomes strengthened; with it the direct procreative force is reinforced, the hereditary transmission of characters again becomes more strict, and the improved type is hardened.

Peirce: CP 1.398 Cross-Ref:††

398. But all these different cases are but so many different modes of one and the same principle, which is the elimination of unfavorable characters. We see then that there are just three factors in the process of natural selection; to wit: first, the principle of individual variation or sporting; second, the principle of hereditary transmission, which wars against the first principle; and third, the principle of the elimination of unfavorable characters.

Peirce: CP 1.399 Cross-Ref:††

399. Let us see how far these principles correspond with the triads that we have already met with. The principle of sporting is the principle of irregularity, indeterminacy, chance. It corresponds with the irregular and manifold wandering of particles in the active state of the protoplasm. It is the bringing in of something fresh and first. The principle of heredity is the principle of the determination of something by what went before, the principle of compulsion, corresponding to will and sense. The principle of the elimination of unfavorable characters is the principle of generalization by casting out of sporadic cases, corresponding particularly to the principle of forgetfulness in the action of the nervous system. We have, then, here, a somewhat imperfect reproduction of the same triad as before. Its imperfection may be the imperfection of the theory of development.†1

Peirce: CP 1.400 Cross-Ref:††
§7. THE TRIAD IN PHYSICS

400. Metaphysical philosophy may almost be called the child of geometry. Of the three schools of early Greek philosophers, two, the Ionic and the Pythagorean, were all geometers, and the interest of the Eleatics in geometry is often mentioned. Plato was a great figure in the history of both subjects; and Aristotle derived from the study of space some of his most potent conceptions. Metaphysics depends in great measure on the idea of rigid demonstration from first principles; and this idea, as well in regard to the process as the axioms from which it sets out, bears its paternity on its face. Moreover, the conviction that any metaphysical philosophy is possible has been upheld at all times, as Kant well says, by the example in geometry of a similar science.

Peirce: CP 1.401 Cross-Ref:††

401. The unconditional surrender, then, by the mathematicians of our time of the absolute exactitude of the axioms of geometry cannot prove an insignificant event for the history of philosophy. Gauss, the greatest of geometers, declares that "there is no reason to think that the sum of the three angles of a triangle is exactly equal to two right angles."†1 It is true, experience shows that the deviation of that sum from that amount is so excessively small that language must be ingeniously used to express the degree of approximation: but experience never can show any truth to be exact, nor so much as give the least reason to think it to be so, unless it be supported by some other considerations. We can only say that the sum of the three angles of any given triangle cannot be much greater or less than two right angles; but that exact value is only one among an infinite number of others each of which is as possible as that. So say the mathematicians with unanimity.

Peirce: CP 1.402 Cross-Ref:††

402. The absolute exactitude of the geometrical axioms is exploded; and the corresponding belief in the metaphysical axioms, considering the dependence of metaphysics on geometry, must surely follow it to the tomb of extinct creeds. The first to go must be the proposition that every event in the universe is precisely determined by causes according to inviolable law. We have no reason to think that this is absolutely exact. Experience shows that it is so to a wonderful degree of approximation, and that is all. This degree of approximation will be a value for future scientific investigation to determine; but we have no more reason to think that the error of the ordinary statement is precisely zero, than any one of an infinity of values in that neighborhood. The odds are infinity to one that it is not zero; and we are bound to think of it as a quantity of which zero is only one possible value. Phoenix, in his *Lectures on Astronomy*, †2 referring to Joshua's commanding the sun to stand still, said that he could not help suspecting that it might have wiggled a very little when Joshua was not looking directly at it. We know that when we try to verify any law of nature by experiment, we always find discrepancies between the observations and the theory. These we rightly refer to errors of observation; but why may there not be similar aberrations due to the imperfect obedience of the facts to law?

Peirce: CP 1.403 Cross-Ref:††

403. Grant that this is conceivable and there can be nothing in experience to negative it. Strange to say, there are many people who will have a difficulty in

conceiving of an element of lawlessness in the universe, and who may perhaps be tempted to reckon the doctrine of the perfect rule of causality as one of the original instinctive beliefs, like that of space having three dimensions. Far from that, it is historically altogether a modern notion, a loose inference from the discoveries of science. Aristotle †1 often lays it down that some things are determined by causes while others happen by chance. Lucretius, †2 following Democritus, supposes his primordial atoms to deviate from their rectilinear trajectories just fortuitously, and without any reason at all. To the ancients, there was nothing strange in such notions; they were matters of course; the strange thing would have been to have said that there was no chance. So we are under no inward necessity of believing in perfect causality if we do not find any facts to bear it out.

Peirce: CP 1.404 Cross-Ref: ††

404. I am very far from holding that experience is our only light; Whewell's views of scientific method seem to me truer than Mill's; so much so that I should pronounce the known principles of physics to be but a development of original instinctive beliefs. Yet I cannot help acknowledging that the whole history of thought shows that our instinctive beliefs, in their original condition, are so mixed up with error that they can never be trusted till they have been corrected by experiment. Now the only thing that the inference from experience can ever teach us is the approximate value of a ratio. It all rests on the principle of sampling; we take a handful of coffee from a bag, and we judge that there is about the same proportion of sound beans in the whole bag that there is in that sample. At this rate, every proposition which we can be entitled to make about the real world must be an approximate one; we never can have the right to hold any truth to be exact. Approximation must be the fabric out of which our philosophy has to be built.

Peirce: CP 1.405 Cross-Ref: ††

405. I come now to another point. Most systems of philosophy maintain certain facts or principles as ultimate. In truth, any fact is in one sense ultimate -- that is to say, in its isolated aggressive stubbornness and individual reality. What Scotus calls the hæcceities of things, the hereness and nowness of them, are indeed ultimate. Why this which is here is such as it is; how, for instance, if it happens to be a grain of sand, it came to be so small and so hard, we can ask; we can also ask how it got carried here; but the explanation in this case merely carries us back to the fact that it was once in some other place, where similar things might naturally be expected to be. Why IT, independently of its general characters, comes to have any definite place in the world, is not a question to be asked; it is simply an ultimate fact. There is also another class of facts of which it is not reasonable to expect an explanation, namely, facts of indeterminacy or variety. Why one definite kind of event is frequent and another rare, is a question to be asked, but a reason for the general fact that of events some kinds are common and some rare, it would be unfair to demand. If all births took place on a given day of the week, or if there were always more on Sundays than on Mondays, that would be a fact to be accounted for, but that they happen in about equal proportions on all the days requires no particular explanation. If we were to find that all the grains of sand on a certain beach separated themselves into two or more sharply discrete classes, as spherical and cubical ones, there would be something to be explained, but that they are of various sizes and shapes, of no definable character, can only be referred to the general manifoldness of nature. Indeterminacy, then, or pure firstness, and hæcceity, or pure secondness, are facts not calling for and not capable of explanation. Indeterminacy affords us nothing to ask a question about; hæcceity is the *ultima ratio*, the brutal fact that will not be questioned. But every fact of a general

or orderly nature calls for an explanation; and logic forbids us to assume in regard to any given fact of that sort that it is of its own nature absolutely inexplicable. This is what Kant †P1 calls a regulative principle, that is to say, an intellectual hope. The sole immediate purpose of thinking is to render things intelligible; and to think and yet in that very act to think a thing unintelligible is a self-stultification. It is as though a man furnished with a pistol to defend himself against an enemy were, on finding that enemy very redoubtable, to use his pistol to blow his own brains out to escape being killed by his enemy. Despair is insanity. True, there may be facts that will never get explained; but that any given fact is of the number, is what experience can never give us reason to think; far less can it show that any fact is of its own nature unintelligible. We must therefore be guided by the rule of hope, and consequently we must reject every philosophy or general conception of the universe, which could ever lead to the conclusion that any given general fact is an ultimate one. We must look forward to the explanation, not of all things, but of any given thing whatever. There is no contradiction here, any more than there is in our holding each one of our opinions, while we are ready to admit that it is probable that not all are true; or any more than there is in saying that any future time will sometime be passed, though there never will be a time when all time is past.

Peirce: CP 1.406 Cross-Ref:††

406. Among other regular facts that have to be explained is law or regularity itself. We enormously exaggerate the part that law plays in the universe. It is by means of regularities that we understand what little we do understand of the world, and thus there is a sort of mental perspective which brings regular phenomena to the foreground. We say that every event is determined by causes according to law. But apart from the fact that this must not be regarded as absolutely true, it does not mean so much as it seems to do. We do not mean, for example, that if a man and his antipode both sneeze at the same instant, that that event comes under any general law. That is merely what we call a coincidence. But what we mean is there was a cause for the first man's sneezing, and another cause for the second man's sneezing; and the aggregate of these two events make up the first event about which we began by inquiring. The doctrine is that the events of the physical universe are merely motions of matter, and that these obey the laws of dynamics. But this only amounts to saying that among the countless systems of relationship existing among things we have found one that is universal and at the same time is subject to law. There is nothing except this singular character which makes this particular system of relationship any more important than the others. From this point of view, uniformity is seen to be really a highly exceptional phenomenon. But we pay no attention to irregular relationships, as having no interest for us.

Peirce: CP 1.407 Cross-Ref:††

407. We are brought, then, to this: conformity to law exists only within a limited range of events and even there is not perfect, for an element of pure spontaneity or lawless originality mingles, or at least must be supposed to mingle, with law everywhere. Moreover, conformity with law is a fact requiring to be explained; and since law in general cannot be explained by any law in particular, the explanation must consist in showing how law is developed out of pure chance, irregularity, and indeterminacy.

Peirce: CP 1.408 Cross-Ref:††

408. To this problem we are bound to address ourselves; and it is particularly needful to do so in the present state of science. The theory of the molecular

constitution of matter has now been carried as far as there are clear indications to direct us, and we are now in the mists. To develop the mathematical consequences of any hypothesis as to the nature and laws of the minute parts of matter, and then to test it by physical experiment, will take fifty years; and out of the innumerable hypotheses that might be framed, there seems to be nothing to make one more antecedently probable than another. At this rate how long will it take to make any decided advance? We need some hint as to how molecules may be expected to behave; whether, for instance, they would be likely to attract or repel one another inversely as the fifth power of the distance, so that we may be saved from many false suppositions, if we are not at once shown the way to the true one. Tell us how the laws of nature came about, and we may distinguish in some measure between laws that might and laws that could not have resulted from such a process of development.

Peirce: CP 1.409 Cross-Ref:††

409. To find that out is our task. I will begin the work with this guess. Uniformities in the modes of action of things have come about by their taking habits. At present, the course of events is approximately determined by law. In the past that approximation was less perfect; in the future it will be more perfect. The tendency to obey laws has always been and always will be growing. We look back toward a point in the infinitely distant past when there was no law but mere indeterminacy; we look forward to a point in the infinitely distant future when there will be no indeterminacy or chance but a complete reign of law. But at any assignable date in the past, however early, there was already some tendency toward uniformity; and at any assignable date in the future there will be some slight aberrancy from law. Moreover, all things have a tendency to take habits. For atoms and their parts, molecules and groups of molecules, and in short every conceivable real object, there is a greater probability of acting as on a former like occasion than otherwise. This tendency itself constitutes a regularity, and is continually on the increase. In looking back into the past we are looking toward periods when it was a less and less decided tendency. But its own essential nature is to grow. It is a generalizing tendency; it causes actions in the future to follow some generalization of past actions; and this tendency is itself something capable of similar generalizations; and thus, it is self-generative. We have therefore only to suppose the smallest spoor of it in the past, and that germ would have been bound to develop into a mighty and over-ruling principle, until it supersedes itself by strengthening habits into absolute laws regulating the action of all things in every respect in the indefinite future.

Peirce: CP 1.409 Cross-Ref:††

According to this, three elements are active in the world: first, chance; second, law; and third, habit-taking.

Peirce: CP 1.410 Cross-Ref:††

410. Such is our guess of the secret of the sphynx. To raise it from the rank of philosophical speculation to that of a scientific hypothesis, we must show that consequences can be deduced from it with more or less probability which can be compared with observation. We must show that there is some method of deducing the characters of the laws which could result in this way by the action of habit-taking on purely fortuitous occurrences, and a method of ascertaining whether such characters belong to the actual laws of nature.

Peirce: CP 1.411 Cross-Ref:††

411. The existence of things consists in their regular behavior. If an atom had no regular attractions and repulsions, if its mass was at one instant nothing, at another

a ton, at another a negative quantity, if its motion instead of being continuous, consisted in a series of leaps from one place to another without passing through any intervening places, and if there were no definite relations between its different positions, velocities and directions of displacement, if it were at one time in one place and at another time in a dozen, such a disjointed plurality of phenomena would not make up any existing thing. Not only substances, but events, too, are constituted by regularities. The flow of time, for example, in itself is a regularity. The original chaos, therefore, where there was no regularity, was in effect a state of mere indeterminacy, in which nothing existed or really happened.

Peirce: CP 1.412 Cross-Ref:††

412. Our conceptions of the first stages of the development, before time yet existed, must be as vague and figurative as the expressions of the first chapter of Genesis. Out of the womb of indeterminacy we must say that there would have come something, by the principle of Firstness, which we may call a flash. Then by the principle of habit there would have been a second flash. Though time would not yet have been, this second flash was in some sense after the first, because resulting from it. Then there would have come other successions ever more and more closely connected, the habits and the tendency to take them ever strengthening themselves, until the events would have been bound together into something like a continuous flow. We have no reason to think that even now time is quite perfectly continuous and uniform in its flow. The quasi-flow which would result would, however, differ essentially from time in this respect, that it would not necessarily be in a single stream. Different flashes might start different streams, between which there should be no relations of contemporaneity or succession. So one stream might branch into two, or two might coalesce. But the further result of habit would inevitably be to separate utterly those that were long separated, and to make those which presented frequent common points coalesce into perfect union. Those that were completely separated would be so many different worlds which would know nothing of one another; so that the effect would be just what we actually observe.

Peirce: CP 1.413 Cross-Ref:††

413. But Secondness is of two types. Consequently besides flashes genuinely second to others, so as to come after them, there will be pairs of flashes, or, since time is now supposed to be developed, we had better say pairs of states, which are reciprocally second, each member of the pair to the other. This is the first germ of spatial extension. These states will undergo changes; and habits will be formed of passing from certain states to certain others, and of not passing from certain states to certain others. Those states to which a state will immediately pass will be adjacent to it; and thus habits will be formed which will constitute a spatial continuum, but differing from our space by being very irregular in its connections, having one number of dimensions in one place and another number in another place, and being different for one moving state from what it is for another.

Peirce: CP 1.414 Cross-Ref:††

414. Pairs of states will also begin to take habits, and thus each state having different habits with reference to the different other states will give rise to bundles of habits, which will be substances.†P1 Some of these states will chance to take habits of persistency, and will get to be less and less liable to disappear; while those that fail to take such habits will fall out of existence. Thus, substances will get to be permanent.

Peirce: CP 1.415 Cross-Ref:††

415. In fact, habits, from the mode of their formation, necessarily consist in

the permanence of some relation, and therefore, on this theory, each law of nature would consist in some permanence, such as the permanence of mass, momentum, and energy. In this respect, the theory suits the facts admirably.

Peirce: CP 1.416 Cross-Ref:††

416. The substances carrying their habits with them in their motions through space will tend to render the different parts of space alike. Thus, the dimensionality of space will tend gradually to uniformity; and multiple connections, except at infinity, where substances never go, will be obliterated. At the outset, the connections of space were probably different for one substance and part of a substance from what they were for another; that is to say, points adjacent or near one another for the motions of one body would not be so for another; and this may possibly have contributed to break substances into little pieces or atoms. But the mutual actions of bodies would have tended to reduce their habits to uniformity in this respect; and besides there must have arisen conflicts between the habits of bodies and the habits of parts of space, which would never have ceased till they were brought into conformity.

Peirce: CP 1.417 Cross-Ref:††
CHAPTER 4

***THE LOGIC OF MATHEMATICS; AN ATTEMPT TO DEVELOP MY
CATEGORIES FROM WITHIN†1P***

§1. THE THREE CATEGORIES

417. Although the present paper deals with mathematics, yet its problems are not mere mathematical problems. It is not proposed to inquire into the methods of reasoning of mathematics particularly, although this subject will incidentally be touched upon. But mathematics performs its reasonings by a *logica utens* which it develops for itself, and has no need of any appeal to a *logica docens*; for no disputes about reasoning arise in mathematics which need to be submitted to the principles of the philosophy of thought for decision. The questions which are here to be examined are, what are the different systems of hypotheses from which mathematical deduction can set out, what are their general characters, why are not other hypotheses possible, and the like. These are not problems which, like those of mathematics, repose upon clear and definite assumptions recognized at the outset; and yet, like mathematical problems, they are questions of possibility and necessity. What the nature of this necessity can be is one of the very matters to be discovered. This much, however, is indisputable: if there are really any such necessary characteristics of mathematical hypotheses as I have just declared in advance that we shall find that there [are], this necessity must spring from some truth so broad as to hold not only for the universe we know but for every world that poet could create. And this truth like every truth must come to us by the way of experience. No apriorist ever denied that. The first matters which it is pertinent to examine are the most universal categories of elements

of all experience, natural or poetical.

Peirce: CP 1.418 Cross-Ref:††

418. We remark among phenomena three categories of elements.

Peirce: CP 1.418 Cross-Ref:††

The first comprises the qualities of phenomena, such as red, bitter, tedious, hard, heartrending, noble; and there are doubtless manifold varieties utterly unknown to us. Beginners in philosophy may object that these are not qualities of things and are not in the world at all, but are mere sensations. Certainly, we only know such as the senses we are furnished with are adapted to reveal; and it can hardly be doubted that the specializing effect of the evolutionary process which has made us what we are has been to blot the greater part of the senses and sensations which were once dimly felt, and to render bright, clear, and separate the rest. But whether we ought to say that it is the senses that make the sense-qualities or the sense-qualities to which the senses are adapted, need not be determined in haste. It is sufficient that wherever there is a phenomenon there is a quality; so that it might almost seem that there is nothing else in phenomena. The qualities merge into one another. They have no perfect identities, but only likenesses, or partial identities. Some of them, as the colors and the musical sounds, form well-understood systems. Probably, were our experience of them not so fragmentary, there would be no abrupt demarcations between them, at all.†1 Still, each one is what it is in itself without help from the others. They are single but partial determinations.

Peirce: CP 1.419 Cross-Ref:††

419. The second category of elements of phenomena comprises the actual facts. The qualities, in so far as they are general, are somewhat vague and potential. But an occurrence is perfectly individual. It happens here and now. A permanent fact is less purely individual; yet so far as it is actual, its permanence and generality only consist in its being there at every individual instant. Qualities are concerned in facts but they do not make up facts. Facts also concern subjects which are material substances. We do not see them as we see qualities, that is, they are not in the very potentiality and essence of sense. But we feel facts resist our will. That is why facts are proverbially called brutal. Now mere qualities do not resist. It is the matter that resists. Even in actual sensation there is a reaction. Now mere qualities, unmaterialized, cannot actually react. So that, rightly understood, it is correct to say that we immediately, that is, directly perceive matter. To say that we only infer matter from its qualities is to say that we only know the actual through the potential. It would be a little less erroneous to say that we only know the potential through the actual, and only infer qualities by generalization from what we perceive in matter. All that I here insist upon is that quality is one element of phenomena, and fact, action, actuality is another. We shall undertake the analysis of their natures below.

Peirce: CP 1.420 Cross-Ref:††

420. The third category of elements of phenomena consists of what we call laws when we contemplate them from the outside only, but which when we see both sides of the shield we call thoughts. Thoughts are neither qualities nor facts. They are not qualities because they can be produced and grow, while a quality is eternal, independent of time and of any realization. Besides, thoughts may have reasons, and indeed, must have some reasons, good or bad. But to ask why a quality is as it is, why red is red and not green, would be lunacy. If red were green it would not be red; that is all. And any semblance of sanity the question may have is due to its being not exactly a question about quality, but about the relation between two qualities, though

even this is absurd. A thought then is not a quality. No more is it a fact. For a thought is general. I had it. I imparted it to you. It is general on that side. It is also general in referring to all possible things, and not merely to those which happen to exist. No collection of facts can constitute a law; for the law goes beyond any accomplished facts and determines how facts that *may be*, but *all* of which never can have happened, shall be characterized. There is no objection to saying that a law is a general fact, provided it be understood that the general has an admixture of potentiality in it, so that no congeries of actions here and now can ever make a general fact. As *general*, the law, or general fact, concerns the potential world of quality, while as *fact*, it concerns the actual world of actuality. Just as action requires a peculiar kind of subject, matter, which is foreign to mere quality, so law requires a peculiar kind of subject, the thought, or, as the phrase in this connection is, the *mind*, as a peculiar kind of subject foreign to mere individual action. Law, then, is something as remote from both quality and action as these are remote from one another.

Peirce: CP 1.421 Cross-Ref:††

421. Having thus by observation satisfied ourselves that there are these three categories of elements of phenomena, let us endeavor to analyze the nature of each, and try to find out why there should be these three categories and no others. This reason, when we find it, ought to be interesting to mathematicians; for it will be found to coincide with the most fundamental characteristic of the most universal of the mathematical hypotheses, I mean that of number.

Peirce: CP 1.422 Cross-Ref:††
§2. QUALITY

422. What, then, is a *quality*?

Peirce: CP 1.422 Cross-Ref:††

Before answering this, it will be well to say what it is not. It is not anything which is dependent, in its being, upon mind, whether in the form of sense or in that of thought. Nor is it dependent, in its being, upon the fact that some material thing possesses it. That quality is dependent upon sense is the great error of the conceptualists. That it is dependent upon the subject in which it is realized is the great error of all the nominalistic schools. A quality is a mere abstract potentiality; and the error of those schools lies in holding that the potential, or possible, is nothing but what the actual makes it to be. It is the error of maintaining that the whole alone is something, and its components, however essential to it, are nothing. The refutation of the position consists in showing that nobody does, or can, in the light of good sense, consistently retain it. The moment the fusillade of controversy ceases they repose on other conceptions. First, that the quality of red depends on anybody actually seeing it, so that red things are no longer red in the dark, is a denial of common sense. I ask the conceptualist, do you really mean to say that in the dark it is no longer true that red bodies are capable of transmitting the light at the lower end of the spectrum? Do you mean to say that a piece of iron not actually under pressure has lost its power of resisting pressure? If so, you must either hold that those bodies under the circumstances supposed assume the opposite properties, or you must hold that they become indeterminate in those respects. If you hold that the red body in the dark

acquires a power of absorbing the long waves of the spectrum, and that the iron acquires a power of condensation under small pressure, then, while you adopt an opinion without any facts to support it, you still admit that qualities exist while they are not actually perceived -- only you transfer this belief to qualities which there is no ground for believing in. If, however, you hold that the bodies become indeterminate in regard to the qualities they are not actually perceived to possess, then, since this is the case at any moment in regard to the vast majority of the qualities of all bodies, you must hold that generals exist. In other words, it is concrete things you do not believe in; qualities, that is, generals -- which is another word for the same thing -- you not only believe in but believe that they alone compose the universe.

Consistency, therefore, obliges you to say that the red body is red (or has some color) in the dark, and that the hard body has some degree of hardness when nothing is pressing upon it. If you attempt to escape the refutation by a distinction between qualities that are real, namely the mechanical qualities, and qualities that are not real, sensible qualities, you may be left there, because you have granted the essential point. At the same time, every modern psychologist will pronounce your distinction untenable. You forget perhaps that a realist fully admits that a sense-quality is only a possibility of sensation; but he thinks a possibility remains possible when it is not actual. The sensation is requisite for its apprehension; but no sensation nor sense-faculty is requisite for the possibility which is the being of the quality. Let us not put the cart before the horse, nor the evolved actuality before the possibility as if the latter *involved* what it only *evolves*. A similar answer may be made to the other nominalists. It is impossible to hold consistently that a quality only exists when it actually inheres in a body. If that were so, nothing but individual facts would be true. Laws would be fictions; and, in fact, the nominalist does object to the word "law," and prefers "uniformity" to express his conviction that so far as the law expresses what only *might* happen, but does not, it is nugatory. If, however, no law subsists other than an expression of actual facts, the future is entirely indeterminate and so is general to the highest degree. Indeed, nothing would exist but the instantaneous state; whereas it is easy to show that if we are going to be so free in calling elements fictions an instant is the first thing to be called fictitious. But I confess I do not take pains accurately to answer a doctrine so monstrous, and just at present out of vogue.

Peirce: CP 1.423 Cross-Ref:††

423. So much for what quality is not. Now what is it? We do not care what meaning the usages of language may attach to the word. We have already seen clearly that the elements of phenomena are of three categories, quality, fact, and thought. The question we have to consider is how quality shall be defined so as to preserve the truth of that division. In order to ascertain this, we must consider how qualities are apprehended and from what point of view they become emphatic in thought, and note what it is that will and must be revealed in that mode of apprehension.

Peirce: CP 1.424 Cross-Ref:††

424. There is a point of view from which the whole universe of phenomena appears to be made up of nothing but sensible qualities. What is that point of view? It is that in which we attend to each part as it appears in itself, in its own suchness, while we disregard the connections. Red, sour, toothache are each *sui generis* and indescribable. In themselves, that is all there is to be said about them. Imagine at once a toothache, a splitting headache, a jammed finger, a corn on the foot, a burn, and a colic, not necessarily as existing at once -- leave that vague -- and attend not to the parts of the imagination but to the resultant impression. That will give an idea of a general quality of pain. We see that the idea of a quality is the idea of a phenomenon

or partial phenomenon considered as a monad, without reference to its parts or components and without reference to anything else. We must not consider whether it exists, or is only imaginary, because existence depends on its subject having a place in the general system of the universe. An element separated from everything else and in no world but itself, may be said, when we come to reflect upon its isolation, to be merely potential. But we must not even attend to any determinate absence of other things; we are to consider the total as a unit. We may term this aspect of a phenomenon the *monadic* aspect of it. The quality is what presents itself in the *monadic* aspect.

Peirce: CP 1.425 Cross-Ref:††

425. The phenomenon may be ever so complex and heterogeneous. That circumstance will make no particular difference in the quality. It will make it more general. But one quality is in itself, in its monadic aspect, no more general than another. The resultant effect has no parts. The quality in itself is indecomposable and *sui generis*. When we say that qualities are general, are partial determinations, are mere potentialities, etc., all that is true of qualities reflected upon; but these things do not belong to the quality-element of experience.

Peirce: CP 1.426 Cross-Ref:††

426. Experience is the course of life. The world is that which experience inculcates. Quality is the monadic element of the world. Anything whatever, however complex and heterogeneous, has its quality *sui generis*, its possibility of sensation, would our senses only respond to it. But in saying this, we are straying from the domain of the monad into that of the dyad; and such truths are best postponed until we come to discuss the dyad.

Peirce: CP 1.427 Cross-Ref:††

§3. FACT

427. Next, what is *fact*?

Peirce: CP 1.427 Cross-Ref:††

As before, it is not the usage of language which we seek to learn, but what must be the description of fact in order that our division of the elements of phenomena into the categories of quality, fact, and law may not only be true, but also have the utmost possible value, being governed by those same characteristics which really dominate the phenomenal world. It is first requisite to point out something which must be excluded from the category of fact. This is the general, and with it the permanent or eternal (for permanence is a species of generality), and the conditional (which equally involves generality). Generality is either of that negative sort which belongs to the merely potential, as such, and this is peculiar to the category of quality; or it is of that positive kind which belongs to conditional necessity, and this is peculiar to the category of law. These exclusions leave for the category of fact, first, that which the logicians call the *contingent*, that is, the accidentally actual, and second, whatever involves an unconditional necessity, that is, force without law or reason, *brute* force.

Peirce: CP 1.428 Cross-Ref:††

428. It may be said that there is no such phenomenon in the universe as brute

force, or freedom of will, and nothing accidental. I do not assent to either opinion; but granting that both are correct, it still remains true that considering a single action by itself, apart from all others and, therefore, apart from the governing uniformity, it is in itself brute, whether it show brute *force* or not. I shall presently point out a sense in which it does display force. That it is possible for a phenomenon in *some* sense to present force to our notice without emphasizing any element of law, is familiar to everybody. We often regard our own exertions of will in that way. In like manner, if we consider any state of an individual thing, putting aside other things, we have a phenomenon which is actual, but *in itself* is not necessitated. It is not pretended that what is here termed fact is the whole phenomenon, but only an element of the phenomenon -- so much as belongs to a particular place and time. That when more is taken into account, the observer finds himself in the realm of law in every case, I fully admit. (Nor does that conflict with tychism. †1)

Peirce: CP 1.429 Cross-Ref: ††

429. On the other hand, if the view be limited to any part of the phenomenal world, however great, and this be looked upon as a monad, entirely regardless of its parts, nothing is presented to the observer but a quality. How much, then, must we attend to, in order to perceive the pure element of fact? There are certain occurrences which, when they come to our notice, we set down as "accidental." Now, although there is really no more of the factual element in these than in other facts, yet the circumstance that we call them *par excellence* contingent, or "accidental," would lead us to expect that which distinguishes the realm of fact from the realms of quality and of law, to be particularly prominent in them. We call such facts "coincidences," a name which implies that our attention is called in them to the coming together of two things. Two phenomena, and but two, are required to constitute a coincidence; and if there are more than two no new form of relationship appears further than a complication of pairs. Two phenomena, whose parts are not attended to, cannot display any law, or regularity. *Three* dots may be placed in a straight line, which is a kind of regularity; or they may be placed at the vertices of an equilateral triangle, which is another kind of regularity. But *two* dots cannot be placed in any particularly regular way, since there is but one way in which they can be placed, unless they were set together, when they would cease to be two. It is true that on the earth two dots may be placed antipodally. But that is only one of the exceptions that prove the rule, because the earth is a third object there taken into account. So two straight lines in a plane can be set at right angles, which is a sort of regularity. But this is another rule proving exception, since $\angle AOB$ is made equal to $\angle BOC$. Now those angles are distinguished by being formed of two different parts of the line AC; so that really three things, OA, OB, and OC are considered. So much for accidental actuality. The type of brute force is the exertion of animal strength. Suppose I have long ago determined how and when I will act. It still remains to perform the act. That element of the whole operation is purely brute execution. Now observe that I cannot exert strength all alone. I can only exert my strength if there be something to resist me. Again duality is prominent, and this time in a [more] obtrusively dual way than before, because the two units are in two different relations the one to the other. In the coincidence the two phenomena are related in one way to one another. It is a monoidal dyad. But in the exertion of strength, although I act on the object and the object acts on me, which are two relations of one kind and joined in one reaction, yet in each of these two relations there is an agent and a patient, a doer and a sufferer, which are in contrary attitudes to one another. So that the action consists of two monoid dyads oppositely situated.

Peirce: CP 1.430 Cross-Ref:††

430. All this renders it quite certain that the nature of fact is in some way connected with the number two, and that of law with three or some higher number or numbers, just as we have already seen that quality is described by means of the number one. But although it is hardly more than might be expected to find that a particular category of the constituents of phenomena has a special capacity for relations of a certain form -- that some are too complex to suit this matter, while others [are] too simple to call into action its distinctive powers -- and that in that way that category comes to have an intimate affinity with a certain formal conception, yet it would certainly be astonishing if it should turn out that material constituents of phenomena were coextensive with formal ideas. We consequently wish to discover just what the connection of the dyad with fact is. We shall do well to postpone the consideration of those facts which seem to involve a triad, such as a process with beginning, middle and end, until we have examined the nature of law. For we naturally suspect, after what has been pointed out above, that where there is a threeness in a fact, there an element of generality may lurk. Putting aside then, for the present, triadic facts, we may add to the properties of fact already noticed such others as may seem worth mention, and may then turn to the consideration of duality, its properties and different formal types, so as to compare these with what is to be remarked in regard to fact.

Peirce: CP 1.431 Cross-Ref:††

431. Whenever we come to know a fact, it is by its resisting us. A man may walk down Wall Street debating within himself the existence of an external world; but if in his brown study he jostles up against somebody who angrily draws off and knocks him down, the sceptic is unlikely to carry his scepticism so far as to doubt whether anything beside the ego was concerned in that phenomenon. The resistance shows him that something independent of him is there. When anything strikes upon the senses, the mind's train of thought is always interrupted; for if it were not, nothing would distinguish the new observation from a fancy. Now there is always a resistance to interruption; so that on the whole the difference between the operation of receiving a sensation and that of exerting the will is merely a difference of degree. We may, however, learn of a fact indirectly. Either the fact was experienced directly by some other person whose testimony comes to us, or else we know it by some physical effect of it. Thus we remark that the physical effects of a fact can take the place of experience of the fact by a witness. Hence, when we pass from the consideration of the appearance of a fact in experience to its existence in the world of fact, we pass from regarding the appearance as depending on opposition to our will to regarding the existence as depending on physical effects.

Peirce: CP 1.432 Cross-Ref:††

432. There can hardly be a doubt that the existence of a fact does consist in the existence of all its consequences. That is to say, if all the consequences of a supposed fact are real facts, that makes the supposed fact to be a real one. If, for example, something supposed to be a hard body acts in every respect like such a body, that constitutes the reality of that hard body; and if two seeming particles act in every respect as if they were attracting particles, that makes them really so. This may be expressed by saying that the fact fights its way into existence; for it exists by virtue of the oppositions which it involves. It does not exist, like a quality, by anything essential, by anything that a mere definition could express. That does not help its mode of being. It might hinder it; because where there is not a unit there cannot be a

pair; and where there is not a quality there cannot be a fact; or where there is not possibility there cannot be actuality. But that which gives actuality is opposition. The fact "takes place." It has its here and now; and into that place it must crowd its way. For just as we can only know facts by their acting upon us, and resisting our brute will (I say *brute* will, because after I have determined how and when I will exert my strength, the mere action itself is in itself brute and unreasoning), so we can only conceive a fact as gaining reality by actions against other realities. And further to say that something has a mode of being which lies not in itself but in its being over against a second thing, is to say that that mode of being is the *existence* which belongs to fact.

Peirce: CP 1.433 Cross-Ref:††

433. The same conclusion can be reached by another line of thought. There are different kinds of existence. There is the existence of physical actions, there is the existence of psychical volitions, there is the existence of all time, there is the existence of the present, there is the existence of material things, there is the existence of the creations of one of Shakespeare's plays, and, for aught we know, there may be another creation with a space and time of its own in which things may exist. Each kind of existence consists in having a place among the total collection of such a universe. It consists in being a second to any object in such universe taken as first. It is not time and space which produce this character. It is rather this character which for its realization calls for something like time and space.

Peirce: CP 1.434 Cross-Ref:††

434. When we speak of a fact as *individual*, or not general, we mean to attribute to it two characters each of which is altogether peculiar to facts. One of these is the character just described, the other having a mode of being independent of any qualities or determinations, or, as we may say, having brute fighting force, or self-assertion. The individual fact insists on being here irrespective of any reason, whether it be true or not that when we take a broader view we are able to see that, without reason, it never could have been endowed with that insistency. This character makes a gulf between the *individual* fact and the *general* fact, or law, as well as between the individual *fact* and any quality, or mere possibility, which only mildly hopes it won't be intruding. But besides that character, individuality implies another, which is that the individual is determinate in regard to every possibility, or quality, either as possessing it or as not possessing it. This is the principle of excluded middle, which does not hold for anything general, because the general is partially indeterminate; and any philosophy which does not do full justice to the element of fact in the world (of which there are many, so remote is the philosopher's high walled garden from the market place of life, where fact holds sway), will be sure sooner or later to become entangled in a quarrel with this principle of excluded middle.

Peirce: CP 1.435 Cross-Ref:††

435. Thus far, in this section, attention has been called successively (but in no philosophical sequence) to six characteristic features of fact. In recollecting them, we may place at their head the circumstance that fact has distinct features, for this distinguishes it from quality although not from law. The others already examined have been as follows: second, facts are either accidentally actual or involve brute force; third, every fact has a here and now; fourth, fact is intimately associated with the dyad; fifth, every fact is the sum of its consequences; sixth, the existence of facts consists in fight; seventh, every fact is determinate in reference to every character. But in making our distribution of the elements of phenomena into quality, fact, and

law, we were led to notice additional features of fact. I continue to take them up promiscuously.

Peirce: CP 1.436 Cross-Ref:††

436. The eighth feature of fact is that every fact has a subject, which is the grammatical subject of the sentence that asserts the existence of the fact. Indeed, in a logical sense, there are two subjects; for the fact concerns two things. One of these two subjects, at least, is a thing itself of the nature of fact, or we may express this in other words by saying that the existence of this subject is a fact. This subject is a *thing*. It has its here and now. It is the sum of all its characters, or consequences. Its existence does not depend upon any definition, but consists in its reacting against the other things of the universe. Of it every quality whatever is either true or false. That this subject, whose actions all have single objects, is material, or physical substance, or *body*, not a psychical subject, we shall see when we come to consider psychical subjects in discussing the nature of law. This does not in the least contradict idealism, or the doctrine that material bodies, when the whole phenomenon is considered, are seen to have a psychical substratum.

Peirce: CP 1.437 Cross-Ref:††

437. The ninth feature of fact is that every fact is connected with a reciprocal fact, which may, or may not, be inextricably bound up with it. If one body strikes upon another, that second body reciprocally strikes upon it; and the two facts are inseparable. But if one body is hard, there must be a second body of some degree of hardness for the former to resist. Yet the annihilation of the second body would [not] destroy the hardness of the first. It would not affect it; for any other body that might grow hard at any time and the first body, remaining unaffected, would realize its hardness whenever the impact with the other should happen to occur. Here, therefore, the reciprocal fact is not so inseparable from the other. If a solid body suddenly melts, it will at once flow into the vacant parts of its vessel; and the beginning to any such consequent fact will be a change reciprocal to the first change. But there is no particular consequence which will be inseparable from the melting, perhaps. There may or may not be. So we see that the division between facts inseparable from reciprocal facts is not coincident with a division of facts into those whose reciprocal facts are separable and those whose reciprocal facts are inseparable.

Peirce: CP 1.438 Cross-Ref:††

438. The tenth feature of fact, which has just been illustrated is that its natural classification takes place by dichotomies.

Peirce: CP 1.439 Cross-Ref:††

439. The eleventh feature of dual fact is that if it involves any variation in time, this variation consists of a change in the qualities of its subjects, but never the annihilation or production of those subjects. We may, indeed, conceive of an action by which something is produced or destroyed. But either a third subject will be concerned, so that the fact is one of those the study of which we have expressly postponed, or that which is produced or destroyed will be one of those facts whose reciprocal facts are separable. If a star suddenly bursts into view, when no external subject caused it to do so, then, just as the appearance will be irrefragable demonstration that something dark was there before, so the fact itself will constitute the previous existence of its subject. For this is the only method by which we can deduce metaphysical truths. Consequently, bodies, and the subjects of facts generally, are permanent and eternal.

Peirce: CP 1.440 Cross-Ref:††

440. The twelfth feature of fact is that it is accidental. That is to say, even if it involves brute force, and though that force be governed by a law which requires the acting body continually to exert this force, yet nevertheless the individual action is not involved in the existence of the fact, but on the contrary is something that can only happen by having a subject with an independent mode of being not dependent upon this nor upon any determination whatsoever. It is something which *happens*.

Peirce: CP 1.440 Cross-Ref:††

I have taken no pains to make this promiscuous list of properties of fact complete, having only cared that it should be sufficient to enable us to compare the characters of fact with those of duality and thus ultimately to attain an understanding of why all phenomena should be composed of quality, fact, and law.

Peirce: CP 1.441 Cross-Ref:††
§4. DYADS †1

441. Let us now inquire what is involved in the conception of *two*, and in particular by what features a pair is distinguished from a single *one*, on the one hand, and from three, or any larger set, on the other.

Peirce: CP 1.442 Cross-Ref:††

442. A mathematician will be inclined to pronounce this the most ridiculously trifling question to be called a problem that could well be imagined. A pair, he may say, is just an object and an object, and that is all that is involved in this puffed-category of the *dyad*. But any logician will tell him that *that* statement, at any rate, is inaccurate. For the purposes of the logic of mathematics it is fatally inaccurate. A married couple is not a man. Neither is it a woman, and *a fortiori* it is not, at once, a man and a woman. Nor is it disjunctively either a man or a woman. It is a third object, to whose constitution, which is its nature, and therefore to its existence, too, a man is requisite and a woman is requisite. A pair is an object to whose constitution a subject and another subject are necessary and sufficient. This corresponds to a part of feature number eight of fact.

Peirce: CP 1.443 Cross-Ref:††

443. But accepting this amendment, which to his customary way of thinking is microscopic, the mathematician will be inclined to say, here is a perfect definition; and excepting a few little corollaries, there is nothing more to be said of the dyad. It behooves me, then, to clearly state what the inquiry is which I propose to institute. It is not a mathematical inquiry; because the business of the mathematician is to frame an arbitrary hypothesis, which must be perfectly distinct at the outset, so far, at least, as concerns those features of it upon which mathematical reasoning can turn, and then to deduce from this hypothesis such necessary consequences as can be drawn by diagrammatical reasoning. The present problem is one of logical analysis. Instead of setting out with a distinct hypothesis of a diagrammatic kind, we have the confused fact that a dyad is a conception of the highest utility, though we are not prepared to say exactly what its nature is, nor even, in all cases, whether a given case should properly be reckoned as a duality or not. We are somewhat in the position of a naturalist who knows that whales are large swimming animals, which spout water,

and yield blubber, spermaceti, and whalebone, but knows little else about them, and who proposes to himself to examine the anatomy and physiology of whales so as to assign them their place in the system of the animal kingdom. He does not intend to preserve the popular description nor delimitation of the class of whales. He will perhaps see reason to extend the name to some animals not popularly called whales and to refuse it to others that are so called. He will also subdivide the group, and classify it according to the facts. So far as our inquiry is a logical analysis, the greatest difference between it and that of a taxonomic biologist consists in the circumstance that we are not forced to institute special observations, because all the facts are either well known or can be ascertained by careful reflection upon those that are known.

Peirce: CP 1.444 Cross-Ref:††

444. But besides being logical in the sense of demanding a logical analysis, our inquiry also relates to two as a conception of logic. The term "logic" is unscientifically by me employed in two distinct senses. In its narrower sense, it is the science of the necessary conditions of the attainment of truth. In its broader sense, it is the science of the necessary laws of thought, or, still better (thought always taking place by means of signs), it is general semeiotic, treating not merely of truth, but also of the general conditions of signs being signs (which Duns Scotus called *grammatica speculativa*†1), also of the laws of the evolution of thought, which since it coincides with the study of the necessary conditions of the transmission of meaning by signs from mind to mind, and from one state of mind to another, ought, for the sake of taking advantage of an old association of terms, be called *rhetorica speculativa*, but which I content myself with inaccurately calling *objective logic*, because that conveys the correct idea that it is like Hegel's logic. The present inquiry is a logical one in the broad sense. It is a study of dyads in the necessary forms of signs.

Peirce: CP 1.444 Cross-Ref:††

Our method must be to observe how logic requires us to think and especially to reason, and to attribute to the conception of the dyad those characters which it must have in order to answer the requirements of logic.

Peirce: CP 1.445 Cross-Ref:††

445. We can at once see that a pair, having a structure, must present a variety of features; and this is a character in which the dyad differs markedly from the monad, which having no structure nor parts in any sense, is bare of all features except that each *one* is something peculiar. This corresponds to feature number one of fact.

Peirce: CP 1.446 Cross-Ref:††

446. A monad has no units. This sounds paradoxical, and seems to the mathematician an *aperçu* from an arbitrary point of view; but we soon see that it is the suitable point of view for logical purposes. In the pair there are unit parts; and so there are in all higher sets. Let us inquire, then, what is the function of the units of a set in the constitution of that set. We must first remark that in logic a set cannot generally be adequately represented by a diagram of a promiscuous collection of dots. Of the multitudinous examples of this in mathematics it will be sufficient to call to mind the constituents of a determinant, and how they have to be arrayed in a square block. As a general rule, the form of connection (or a part of it, at least) must be considered in logic in case a set has to be considered as such. This form of connection belongs to the set and not to its units. Now reasoning is formal. That is to say, whatever inference is sound concerning one thing or one character is sound in regard to any other thing or character whose form of connection (so far as it need be

considered) is strictly analogous to that of the former. All that has to be represented, then, for the purposes of logic, is the characters of the sets themselves; and the units need exhibit nothing except what is requisite to the exhibition of the characters belonging to sets. What, then, is the use of the units, at all? And how can they, when thus stripped of all qualities, contribute to the representation of characters of sets? The answer is that if all that were desired was to present for contemplation the character of a set, the statement of the mode of its connection in abstract terms, with no particular reference to the units, would be sufficient; and in point of fact, this is the general form which metaphysicians give to their statements, so far as the usages of speech render it convenient. But when, one set having been represented, it is desired to attach to it the representation of another set, and there is a unit or units which belong to both sets, then in order to show how the total set is composed of those two sets, it is necessary to take account of the identities of their common units. Now identity is a relation which cannot be implied by a general description of the identical things; and the descriptions of the sets, so far as they leave out the individual things, are general. Hence, it follows that the only purpose in indicating the units in the representation of the set, is in order that each of them may signify its identity with an individual of another set. The identity of different units of the same set might be similarly represented. Hence, passing from the representation of the set, to the set itself, as it is logically conceived, the only function of the units in it is to establish possible identities with the units of other sets. A unit, therefore, is something essential to a set whose existence consists in its possible identity with another unit of the same or another set. Now, identity is essentially a dual relation. That is, it requires two subjects and no more. If three objects are identical, this fact is entirely contained in the fact that the three pairs of objects are identical. Hence a unit is something whose existence consists in a possible dyad of which it is the subject. Thus, there is an element of twoness in every set. So I was right in saying that the monad has no unit, since the monad in no wise involves the dyad.

Peirce: CP 1.447 Cross-Ref:††

447. There are certain truths about quality not considered in Section 2, for the reason that they were considered as belonging under the head of the dyad. They do not concern the monad in its aspect as one, but are dyads of monads. One of these is that whatever is a possible aspect irrespective of parts has possible parts. I mean that any object presenting a quality in its purity might be further determined. Every quality is, in itself, general. Given any possible determination, there is a possible further determination. In the beginning was nullity, or absolute indetermination, which, considered as the possibility of all determination, is being. A monad is a determination *per se*. Every determination gives a possibility of further determination. When we come to the dyad, we have the unit, which is, in itself, entirely without determination, and whose existence lies in the possibility of an identical opposite, or of being indeterminately over against itself alone, with a determinate opposition, or over-againstness, besides.

Peirce: CP 1.447 Cross-Ref:††

It follows that a set considered apart from its units is a monad. In fact, in not considering the units, we allow all sets of the same general character to collect before us, and regard those sets as a monad without parts.

Peirce: CP 1.447 Cross-Ref:††

But a set considered as made up of units in a peculiar connection is a dyad if its units are two, a triad if they are three, etc. A part of the above corresponds to

feature number eight of fact.

Peirce: CP 1.448 Cross-Ref:††

448. Let us further examine the dyads of monads just mentioned. But before doing so, this designation has to be defended. It may be objected, with apparent force, that the truth just mentioned about quality is general, and that, as general, it applies to innumerable qualities and not to a pair. This is perfectly true; but then, all that we are inquiring into concerning the monad, the dyad, and plural sets, is general, and therefore, from that point of view we are not studying the monad and the dyad at all, but polyads of monads and dyads. This is true. Our thought is rational and, as such, general, or of the plural nature. But it relates to the monad and the dyad in part. So the truth just mentioned is general, but it relates to a single monad, any monad, and declares of it that a monad exists which is, for thought, equivalent to that monad further determined.

Peirce: CP 1.449 Cross-Ref:††

449. This is one of three regulative laws of logic of high importance which were enunciated by Kant in the *Critic of the Pure Reason*. †1 The other two are that there is a determination less than and included in any possible determination, and that between any two determinations, one included in the other, a third may be found. Besides these dyads, both whose subjects are monads, there are also certain dyads, one of whose subjects is a monad and the other a possible dyad, that is, a unit. And there are general laws connected with these.

Peirce: CP 1.450 Cross-Ref:††

450. The first of these is that any unit (or units) whatsoever contemplated in itself without conscious regard to its parts would, were our sense to respond to it, be seen to embody a monad. De Morgan propounded this law, so far as it is pertinent to formal logic, affirming that any collection of objects whatsoever possess universally some character which belongs to no other object at all. For, said he, they at least possess the character of being units of that collection. Considered as a *proof*, this begs the question; but considered as another way of formulating the same phenomenon, and as a way which throws some light upon it, it has its value. This coincides with the principle of excluded middle. Those objects of the universe which do not possess a given character possess another character which, in reference to that universe, is in the relation of negation to the first. Hence, it is impossible to form a single class of dyads; two classes of dyads must be formed at once. Hence, considering all the monads which can appear on the contemplation of sets of units of the universe in their monadic aspect, every single unit is determined to be one subject of a dyad which has any one of those monads as its second subject, namely it is either such a dyad as determines it to have the character of being one of the units which made up the object of the contemplation in which that monad appeared, or it is such a dyad as determines the unit to have the character belonging to all the other units of the universe.

Peirce: CP 1.451 Cross-Ref:††

451. What has been here affirmed of collections of units is equally true of collections of monads. Namely, any monads may be contemplated together, and in their monadic aspect without regard to the single monads are seen to be one monad. There is thus a relation between monads similar to the relation of a unit to a monad. But there is this difference in the two cases: a monad thus embraced under another monad is so embraced in its very mode of being, while that a unit should be embraced under a monad has no concern with the mode of existence of the unit, which lies in its brute self-identity and otherness from all the rest. It is on the contrary an adventitious

circumstance that this particular unit is embraced under this monad.

Peirce: CP 1.451 Cross-Ref:††

This corresponds to feature number seven and in part to feature number twelve of fact.

Peirce: CP 1.452 Cross-Ref:††

452. The metaphysical categories of quality, fact, and law, being categories of the matter of phenomena, do not precisely correspond with the logical categories of the monad, the dyad, and the polyad or higher set, since these are categories of the forms of experience. The dyads of monads, being dyads, belong to the category of the dyad. But since they are composed of monads as their sole matter, they belong materially to the category of quality, or the monad in its material mode of being. It cannot be regarded as a *fact* that scarlet is red. It is a *truth*; but it is only an essential truth. It is that in being which corresponds in thought to Kant's analytical judgment. It is a dyadism latent in monads.

Peirce: CP 1.453 Cross-Ref:††

453. I may pause here a moment to remark that when I say nullity consists of the possibility of the monad; that the unit consists of the possibility of the dyad, and the like, such statements have a Hegelian sound. Undoubtedly they are intrinsically of that nature. I follow an order of evolution in such phrases, the possibility evolves the actuality. So does Hegel. He reaches each category from the last preceding by virtually calling "next!" What his process [is] of making the next come and of recognizing it when it emerges is, however important it may be, yet, comparatively speaking a detail, wherein I sometimes agree with the great idealist and sometimes diverge from his footsteps -- for my own method has resulted from a more deliberate examination of the exact theory of logic (in which Hegel's age, and especially his own country, and more especially he himself were decidedly weak), and consequently has a broader form, capable of diversification to adapt itself to the special form of the germinal conception. It is not yet time to formulate it. I apply it; the reader follows it with approval if he can; and a later review will show what the laws of the procedure have unconsciously been.

Peirce: CP 1.454 Cross-Ref:††

454. The most important division of dyads is with reference to the character of their subjects. For subjects differ in regard to the nature of the dyads which they are capable of forming. They are either dyads formed merely from monads or they are dyads into which enter objects having a dyadic mode of being, that is, individual things, or units.

Peirce: CP 1.455 Cross-Ref:††

455. Dyads of the former kind are seen to subsist as soon as the two monads are regarded together; and arguing from knowledge to being (that is, merely abstracting from the imported idea of a knower), they do subsist in so far as the two monads are compossible, that is in so far as both are such monads as they are. When scarlet and red are contemplated together, the former as first, the latter as second, a certain aspect *sui generis* presents itself, like that which presents itself when toothache and ache are contemplated together, the former first, the latter second. This kind of dyadism or dyadic relation which is evolved from the very being of the subjects as soon as they are together, I call an *essential* dyadic relation, and the dyad so formed an *essential dyad*. This is the only kind of dyad that can be composed out of monads alone; because monads having no parts nor distinct features cannot,

whether singly or collectively, have any characters except those which spring directly from their several beings *sui generis*.

Peirce: CP 1.456 Cross-Ref:††

456. Dyads that are *accidental*, that is, are collective characters of their subjects adventitious to their being, must therefore concern subjects (or one subject, at least) which is not a monad, and consequently having a mode of being over and above what its mere inward suchness involves. It must have a mode of being gained by its opposition to another, that suchness does not avail to confer. What is this mode of being in its most general terms? In order that our conception of it may embrace every variety, let it begin as soon as the mode of being of the monad ends. Combine quality with quality after quality and what is the mode of being which such determinations approach indefinitely but altogether fail ever to attain? It is, as logicians have always taught, the *existence* of the individual. Individual existence whether of a thing or of a fact is the first mode of being that suchness fails to confer. Suchness, or the mode of being of the monad, is the mere possibility of an existent.

Peirce: CP 1.457 Cross-Ref:††

457. Existence is that mode of being which lies in opposition to another. To say that a table exists is to say that it is hard, heavy, opaque, resonant, that is, produces immediate effects upon the senses, and also that it produces purely physical effects, attracts the earth (that is, is heavy), dynamically reacts against other things (that is, has inertia), resists pressure (that is, is elastic), has a definite capacity for heat, etc. To say there is a phantom table by the side of it incapable of affecting any senses or of producing any physical effects whatever, is to speak of an imaginary table. A thing without oppositions *ipso facto* does not exist. Of course, the question arises, if everything that exists exists by its reactions, how does the total collection of things exist? This is a legitimate and valuable question, the answer to which brings out a new idea. But this is not the time to consider it. Our purpose of developing the complete scheme of philosophical ideas is defeated unless we take up the points one by one in their due order. That question about the totality of things throws no doubt upon the manifest truth that existence lies in opposition; and the very fact that the consideration of it would lead to a still more developed philosophy is the very reason why it must be postponed until we have mastered the conception of being through opposition.†1 Not only is this opposition essential to an individual thing or subject, but also to an individual fact. Its truth, or existence, is the sum of its effects.

Peirce: CP 1.458 Cross-Ref:††

458. *Hic et nunc* is the phrase perpetually in the mouth of Duns Scotus, who first elucidated individual existence. It is a forcible phrase if understood as Duns did understand it, not as describing individual existence, but as suggesting it by an example of the attributes found in this world to accompany it. Two drops of water retain each its identity and opposition to the other no matter in what or in how many respects they are alike. Even could they interpenetrate one another like optical images (which are also individual), they would nevertheless react, though perhaps not at that moment, and by virtue of that reaction would retain their identities. The point to be remarked is that the qualities of the individual thing, however permanent they may be, neither help nor hinder its identical existence. However permanent and peculiar those qualities may be, they are but *accidents*; that is to say, they are not involved in the mode of being of the thing; for the mode of being of the individual thing is existence; and existence lies in opposition merely.

Peirce: CP 1.459 Cross-Ref:††

459. We observe no life in chemical atoms. They appear to have no organs by which they could act. Nor can any action proper gain actuality, that is, a place in the world of actions, for any subject. Yet the individual atom exists, not at all in obedience to any physical law which would be violated if it never had existed, nor by virtue of any qualities whatsoever, but simply by virtue of its arbitrarily interfering with other atoms, whether in the way of attraction or repulsion. We can hardly help saying that it blindly forces a place for itself in the universe, or willfully crowds its way in.

Peirce: CP 1.460 Cross-Ref:††

460. No reaction among individual things can create one of those things nor destroy it; for before its existence or after it there would not be anything to react. So that the fountain of existence must be sought elsewhere.

Peirce: CP 1.461 Cross-Ref:††

461. Existence, though brought about by dyadism, or opposition, as its proper determination, yet, when brought about, lies *abstractly and in itself considered*, within itself. It is numerical identity, which is a dyadic relation of a subject to itself of which nothing but an existent individual is capable. It is to be observed that numerical identity is not empty verbiage, as the identity of a quality with itself is, but is a positive fact. This is due to the possibility of the individual's assuming different accidents. Throughout all vicissitudes its oppositions to other things remain intact, although they may be accidentally modified; and therein is manifest the positive character of identity.

Peirce: CP 1.462 Cross-Ref:††

462. The only primary essential dyadism is that between a containing monadic quality and a contained monadic quality. For qualities cannot resemble one another nor contrast with one another unless in respect to a third quality; so that the resemblance of qualities is triadic. This, however, is a point calling for reexamination in a future revision of this analysis. If I am right, there is no further logical distinction between essential dyads.

Peirce: CP 1.463 Cross-Ref:††

463. But with regard to accidental dyads, the case is far otherwise. We must at once divide them into those of which one subject is a monad, and those of which neither subject is a monad. This division is closely allied to and immediately suggested by the last. Dyads of the former kind may be termed *inherential* -- as, this thing possesses redness; those of the latter kind may be called *relational*.

Peirce: CP 1.464 Cross-Ref:††

464. An inherential dyad strongly resembles an essential dyad. Begin with any quality, as high-colored, and form an essential dyad, as *red is high-colored*. Form another with red as second subject; as, *scarlet is red*. Form another with scarlet as second subject; as, *mercuric-iodide-color is scarlet*. So we may conceive determination added to determination, and at the limit a color so specific that it can only belong to an individual object. This I say is the limit which lies just beyond the possible, but is indefinitely approachable. This limit is a dyad of inherence. It is, after all, however, radically different from the essential dyad, because the quality of the subject of inherence is a mere accident of that individual. Inherence may be regarded from another point of view. Namely, the individual subject may be conceived as brought into relation to itself by the possession of the attribute.

Peirce: CP 1.465 Cross-Ref:††

465. Relational dyads are not further divisible in regard to the metaphysical character of their subjects. But they are divisible in regard to the nature of the connection between their subjects. And, first of all, a division is suggested by the last remark concerning inherential dyads. Namely, every relational dyad is either a *dyad of identity*, in which the two subjects are existentially one and the same, or it is a *dyad of diversity*, in which the subjects are existentially two and distinct. This relational identity is not the identity of inherence, but the identity which is altogether independent of any accident or accidents. It will, however, involve such inferences as may belong to the individual and identical subject.

Peirce: CP 1.466 Cross-Ref:††

466. With this division another is closely connected; namely, a dyad of diversity may either be such that the connection between its units consists merely in their agreement or difference in respect to a monadic quality, or it may be such that the connection of the units depends upon their possessing some dyadic character or characters. This distinction is most deeply engraved into the natures of dyads. For what is a dyadic character? It is a character conferred upon one individual by another individual. It thus involves the idea of *action* or *force*, not in a narrow scientific sense, but in the sense in which we speak of the will as a force. We may say then that this division is into *qualitative* and *dynamical* diversities. Or, in place of qualitative diversity, it will perhaps be better to use the familiar phrase *partial agreements*.

Peirce: CP 1.467 Cross-Ref:††

467. Dynamic dyads are, in the first place, distinguished into those which, by virtue of the characters which they attribute to their subjects, put those two subjects into like relations each to the other, and into those which, so far as the characters they attribute to their subjects go, leave a distinction between the reciprocal relations. The former kind may be called *materially unordered*, the latter, *materially ordered*. Thus, A is one mile from B is a materially unordered relation, but A kills B is materially ordered, notwithstanding that it may happen that B also kills A.

Peirce: CP 1.468 Cross-Ref:††

468. Closely connected with this distinction is another; namely, materially ordered dyads are divisible into those in which there is no existential or intrinsic distinction between the subjects as to which is first and which second, although in stating the fact language may require us to mark one as first and the other as second, and into those in which this distinction is existential. The former may be called *formally unordered* dyads, the latter *formally ordered*. Thus, when amber is rubbed against fur, one acquires resinous and the other vitreous electricity. The dyad is thus materially ordered. But, as far as we know, neither is to be regarded as distinctively agent or first in contradistinction to the patient, or second. When, however, of two oppositely electrified bodies one attracts the other, although the second equally attracts the first, yet the two attractions are distinct dyads and the attracting body is agent, or intrinsically first, while the attracted body is intrinsically second. For one is determining and the other determined. Now the determining body is, in so far, left indeterminate; and indeterminacy, or possibility, as the character of the monad, is *first* relatively to determination, which, as essentially dyadic, is *second*.

Peirce: CP 1.469 Cross-Ref:††

469. There is no further room for distinction based upon the positions of the subjects; but the formally ordered dyads can still be divided with reference to the

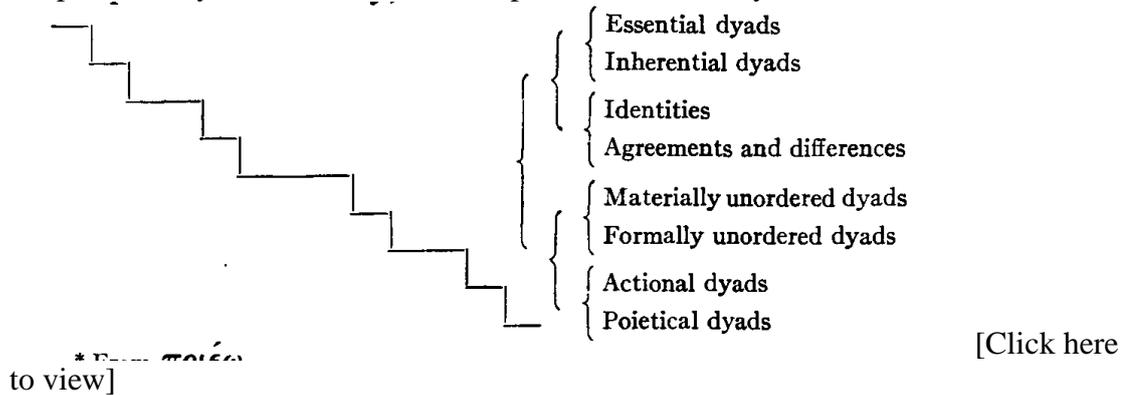
character of the dependence of one subject upon the other. Namely, this is either such that merely the monadic accidents of the second subject, or patient, are dependent upon the agent, or such that the dyadic existence of the patient is dependent upon the agent. The former may be termed *actional*, the latter *poietical*,†1 or *productive*, dyads.

Peirce: CP 1.469 Cross-Ref:††

No further distinctions seem to be relevant to the idea of the present analysis.

Peirce: CP 1.470 Cross-Ref:††

470. It will be remarked that the division is everywhere a dichotomy of the second of the two classes formed by the next preceding dichotomy. The result is that the ultimately undivided species form a staircase of successive steps. But the steps are not all equal. On the contrary, so thoroughly does twoness permeate the whole that the steps separate into successive pairs. There is also a marked distinction between the first pair of pairs and the second pair of pairs, which repeats the former with a variation. That is to say, the first pair of each of the two pairs of pairs arises from distinctions concerning the subjects, while the second pair of each pair of pairs arises from distinctions concerning the mode of connection of the subjects. The whole series of species of dyads are related like the phrases of a melody, as follows:



{ Essential dyads

{ { Inherential dyads

{

{ { { Identities

{ { { Agreements and differences

{

{ { { Materially unordered dyads

{ { { { Formally unordered dyads

{

{ { { Actional dyads

{Poietical dyads

The more this division is examined, the more clearly it will appear that it is not a fancifully imposed scheme, but springs inevitably from the evolution of the conceptions according to the general point of view adopted.

Peirce: CP 1.471 Cross-Ref:††
§5. TRIADS †1

471. We come now to the *triad*. What is a triad? It is a three. But three what? If we say it is three subjects, we take at the outset an incomplete view of it. Let us see where we are, remembering that logic is to be our guide in this inquiry. The monad has no features but its suchness, which in logic is embodied in the signification of the verb. As such it is developed in the lowest of the three chief forms of which logic treats, the term, the proposition, and the syllogism. The dyad introduced a radically new sort of element, the subject, which first shows itself in the proposition. The dyad is the metaphysical correlative of the proposition, as the monad is of the term. Propositions are not all strictly and merely dyadic, although dyadism is their prominent feature. But strictly dyadic propositions have two subjects. One of these is active, or existentially prior, in its relation to the dyad, while the other is passive, or existentially posterior. A gambler stakes his whole fortune at an even game. What is the probability that he will gain the first risk? One half. What is the probability that he will gain the second risk? One fourth; for if he loses the first play, there will be no second. It is one alternative of the prior event which divides into two in the posterior event. So if A kills B, A first does something calculated to kill B, and then this subdivides into the case in which he does kill B and the case in which he does not. It is not B that does something calculated to make A kill him; or if he does, then he is an active agent and the dyad is a different one. Thus, there are in the dyad two subjects of different character, though in special cases the difference may disappear. These two subjects are the *units* of the dyad. Each is a *one*, though a dyadic one. Now the triad in like manner has not for its principal element merely a certain unanalyzable quality *sui generis*. It makes [to be sure] a certain feeling in us. [But] the formal rule governing the triad is that it remains equally true for all six permutations of A, B, C; and further, if D is in the same relation at once to A and B and to A and C, it is in the same relation to B and C; etc.

Peirce: CP 1.472 Cross-Ref:††

472. Each of the three subjects introduces a dyad into the triad, and so does each pair of subjects. The distinctive character or quality of the triad is a monadic element. The formal law of the triad is essentially triadic. It is in that, that the threeness inheres.

Peirce: CP 1.473 Cross-Ref:††

473. Every triad is either *monadically degenerate*, *dyadically degenerate*, or *genuine*. A monadically degenerate triad is one which results from the essence of three monads, its subjects. A dyadically degenerate triad is one which results from dyads. A genuine triad is one which cannot be resolved in any such way. That orange

color is intermediate between red and yellow is a monoidally degenerate triad. So that one given quality is a compound of two others. So [that] red and green resemble violet more than they resemble each other. That red is a determination of color and scarlet of red involves a monadically degenerate triad and belongs to the class of essential triads; yet it is properly a dyadically degenerate triad where the component dyads are essential dyads. It is thus essential, but only indirectly essential. So that oranges and lemons smell alike, though it is properly only a dyad, yet may be considered as a triad, the common quality of smell being the third subject. That a citric taste and a perfume of a cologne water kind coexist in the lemon can only be regarded as a triad and not as a dyad. That A is father of B and B father of C is a genuinely dyadic degenerate triad. That A is as far north of B as B is east of C is a triad formed of two dyads of one kind and a dyad of another kind -- (I mean the similarity of the other two, but this is accidental). This is an almost, but not quite, genuine triad. A is mother of B and B is wife of C. Here the two component dyads are more independent of one another. This is a purer case of the dyadic degenerate triad.

Peirce: CP 1.474 Cross-Ref:††

474. In considering the genuine triad, it is well to notice first that the last fact supposed involves the fact that A is mother-in-law of C, which is no triad, but a dyad. Indeed, every triad, as above remarked, involves a dyad; but it is the peculiarity of the dyadic triad that this dyad only differs from the triad in the lack of particularization of the mediating subject. So, reversing the process, every dyad by a particularization evolves a dyadic triad. Thus, A murders B is a generalization of A shoots that bullet, and the bullet fatally wounds B. This is true even in regard to the dyad, A winks, which evolves the triad, A experiences a nervous irritation and the nervous irritation causes winking of the eyelid. Such an evolution may be called an *explication* of the dyad. So the monad colored is explicated in the monadic dyad, red is colored, and red is explicated in scarlet is red. A triad may be explicated into a triadic tetrad. Thus, A gives B to C becomes A makes the covenant D with C and the covenant D gives B to C.

Peirce: CP 1.475 Cross-Ref:††

475. But if we compare the monad implicated in a genuine dyad, as *red* is in "this thing is red," with that dyad, we see that the latter is more than any mere explication of red. It is the truth of what Kant called a synthetic (that is, genuinely dyadic) judgment. It involves *existence*, while *red* or any mere explication of red is but a *possibility*. Even in "something is red," which leaves wholly indeterminate what it is that is red, and consequently does not really explicate red, at all, existence is just as positive as in, "this is red." Now let us consider the triad, A makes a contract with C. To say that A signs the document D and C signs the document D, no matter what the contents of that document, does not make a contract. The contract lies in the intent. And what is the intent? It is that certain conditional rules shall govern the conduct of A and of C. There is no positive fact in this; it is only conditional and intentional. Still less, if possible, is it any mere monadic quality. It has reference to conditions of experience, involving existence, involving dyadic fact. It may be said that it is a psychical fact. This is in so far true, that a psychical fact is involved; but there is no intent unless something be intended; and that which is intended cannot be covered by any facts; it goes beyond anything that can ever be done or have happened, because it extends over the whole breadth of a general condition; and a complete list of the possible cases is absurd. From its very nature, no matter how far specification has gone, it can be carried further; and the general condition covers all that incompletable possibility.

Peirce: CP 1.476 Cross-Ref:††

476. There, then, we have an example of a genuine triad and of a triadic conception. But what is the general description of a genuine triad? I am satisfied that no triad which does not involve generality, that is, the assertion of which does not imply something concerning *every possible* object of some description can be a genuine triad. The mere addition of one to two makes a triad; and therein is contained an idea entirely indecomposable into the ideas of one and two. For addition implies two subjects added, and something else as the result of the addition. Hence, it is wrong to define two as the sum of one and one; for according to such a definition, two would involve the idea of three. The idea characteristic of two is *other*. The corresponding idea characteristic of three is *third*. *Medium* is nearly as broad, and so is *uniter*.

Peirce: CP 1.477 Cross-Ref:††

477. The genuine triad contains no idea essentially different from those of *object, other, third*. But it involves the idea of a third not resolvable into a formless aggregation. In other words, it involves the idea of something more than *all that can* result from the successive addition of one to one. This "all that can" involves the idea of *every possible* something, and therefore of generality. The genuine triad, then, must involve generality.

Peirce: CP 1.478 Cross-Ref:††

478. The world of fact contains only what is, and not everything that is possible of any description. Hence, the world of fact cannot contain a genuine triad. But though it cannot contain a genuine triad, it may be governed by genuine triads.

Peirce: CP 1.478 Cross-Ref:††

So much for the division of triads into the monadic, dyadic, and triadic of [or?] genuine triads.

Peirce: CP 1.479 Cross-Ref:††

479. Dyadic triads are obviously of two kinds, first, those which have two monadic subjects, as a high perfume and a burning taste are united in many essential oils, and secondly, those which have all their subjects individuals.

Peirce: CP 1.480 Cross-Ref:††

480. Genuine triads are of three kinds. For while a triad if genuine cannot be in the world of quality nor in that of fact, yet it may be a mere law, or regularity, of quality or of fact. But a *thoroughly* genuine triad is separated entirely from those worlds and exists in the universe of *representations*. Indeed, representation necessarily involves a genuine triad. For it involves a sign, or representamen, of some kind, outward or inward, mediating between an object and an interpreting thought. Now this is neither a matter of fact, since thought is general, nor is it a matter of law, since thought is living.

Peirce: CP 1.481 Cross-Ref:††

481. So much for the first order of subdivisions of the three classes of triads. Passing to the lower subdivisions, I find among those of the degenerate triads nothing of particular philosophical interest; though something may have been overlooked. But among the lower subdivisions of the genuine triads there is an abundance.

Peirce: CP 1.482 Cross-Ref:††

482. We first consider the first two of the three chief divisions of genuine

triads, which are the laws of quality and the laws of fact. The laws of quality are all of one type. Namely, they all simply determine systems of qualities, of which Sir Isaac Newton's law of color-mixture with Dr. Thomas Young's supplement thereto, is the most perfect known example.

Peirce: CP 1.483 Cross-Ref:††

483. The laws of fact divide themselves at the outset into those which must be true if there be any true answer to every question that has a meaning, or, as we say, into laws *logically necessary* and laws *logically contingent*. To this division another is intimately connected. Namely, of laws logically contingent the most universal are of such a kind that they must be true provided every form which by logical necessity must be thought of a given subject is also a form of its real being. Calling this kind of necessity, metaphysical necessity, we may divide laws logically contingent into laws *metaphysically necessary* and laws *metaphysically contingent*.

Peirce: CP 1.484 Cross-Ref:††

484. The general law of quality, as distinct from the classificatory system of quality (of which we can have but a fragmentary knowledge), has three clauses, relating respectively to single qualities, to pairs of qualities, and to triads of qualities. The first clause is that every quality is perfect and in itself such as it is. The second more complex law is that two qualities have one or other of two sorts of relations to one another; namely, they may be, first, independent of one another, somewhat resembling and somewhat differing from one another, or secondly, one of them may be merely a further determination of the other, this latter being essentially the first of the pair in the order of evolution, or synthesis, while it is the second of the pair in the order of involution or analysis. The third clause relates to the respects, or third qualities, in which two compared qualities agree or differ. The first of these respects is the quality of the quality, or, as we may say, its hue, in which respect the tastes of sugar and salt differ, or the pitch of sounds, or the respect in which red, blue, and green differ. The second respect is the absolute intensity of the quality, loudness in sounds, luminosity in color, strength in tastes and smells, etc. The third respect is purity, or the relative intensity of the strongest elements. It is great in high colors and in musical sounds. In some cases strength and weakness have peculiar hues. Bright colors tend toward yellow, dim colors toward violet. Very faint sounds tend toward a certain pitch. Purity and impurity may have their peculiar hues.

Peirce: CP 1.485 Cross-Ref:††

485. The general law of logic has likewise its three clauses. The monadic clause is that fact is in its existence perfectly definite. Inquiry properly carried on will reach some definite and fixed result or approximate indefinitely toward that limit. *Every subject is existentially determinate with respect to each predicate.* The dyadic clause is that there are two and but two possible determinations of each subject with reference to each predicate, the affirmative and the negative. Not only is the dyadic character manifest by the double determination, but also by the double prescription; first that the possibilities are two at least, and second that they are two at most. The determination is not both affirmative and negative, but it is either one or the other. A third limiting form of determination belongs to any subject [with regard] to [some other] one whose mode of existence is of a lower order, [the limiting case involving] a relative *zero*, related to the subjects of the affirmation and the negation as an inconsistent hypothesis is to a consistent one. The triadic clause of the law of logic recognizes three elements in truth, the idea, or predicate, the fact or subject, the thought which originally put them together and recognizes they are together; from

whence many things result, especially a threefold inferential process which either first follows the order of involution from living thought or ruling law, and existential case under the condition of the law to the predication of the idea of the law in that case; or second, proceeds from the living law and the inherence of the idea of that law in an existential case, to the subsumption of that case and to the condition of the law; or third, proceeds from the subsumption of an existential case under the condition of a living law, and the inherence of the idea of that law in that case to the living law itself.†1 Thus the law of logic governs the relations of different predicates of one subject.

Peirce: CP 1.486 Cross-Ref:††

486. The general law of metaphysics is little understood. The attention of thinkers has been so rivetted upon the question of its truth, that they have largely overlooked the importance of determining precisely what it is, even if it be not absolutely true, since it is certainly the product of natural thought and of reasoning which, however far it may be carried beyond the legitimate conclusion, is nevertheless true reasoning of a valid type. The difficulty of making here any brief statement of any value is great enough for that reason. But besides that, brief statements of a metaphysical kind can hardly be made intelligible. I can only notice some items of the law going to exhibit the threefold division of the law.

Peirce: CP 1.487 Cross-Ref:††

487. Metaphysics consists in the results of the absolute acceptance of logical principles not merely as regulatively valid, but as truths of being. Accordingly, it is to be assumed that the universe has an explanation, the function of which, like that of every logical explanation, is to unify its observed variety. It follows that the root of all being is One; and so far as different subjects have a common character they partake of an identical being. This, or something like this, is the monadic clause of the law. Second, drawing a general induction from all observed facts, we find all realization of existence lies in opposition, such as attractions, repulsions, visibilities, and centres of potentiality generally. "The very hyssop on the wall grows in that chink because the whole universe could not prevent its growing." This is, or is a part of, a dyadic clause of the law. Under the third clause, we have, as a deduction from the principle that thought is the mirror of being, the law that the end of being and highest reality is the living impersonation of the idea that evolution generates. Whatever is real is the law of something less real. Stuart Mill defined matter as a permanent possibility of sensation.†2 What is a permanent possibility but a law? Atom acts on atom, causing stress in the intervening matter. Thus force is the general fact of the states of atoms on the line. This is true of force in its widest sense, dyadism. That which corresponds to a general class of dyads is a representation of it, and the dyad is nothing but a conflux of representations. A general class of representations collected into one object is an organized thing, and the representation is that which many such things have in common. And so forth.

Peirce: CP 1.488 Cross-Ref:††

488. Passing to laws that are metaphysically contingent, that is, to such as are not necessarily involved in the literal extension to being of the necessary laws of logical truth, we may first divide these into those which impose upon the subjects of dyadic existence forms of reaction analogous to those of logic, that is, the *laws of time* (by which they evade the laws of logic in regard to contrary inferences) and those which have no relation to logic. And with this division another is closely connected, namely, the division of the latter class of laws into those which are

imposed upon objects as reacting upon one another existentially, as merely coexistent, which are the laws of *space*, and into those which are only imposed upon objects in so far as their mode of existence is in its own metaphysical nature that of a subject, that is, laws of substantial things.

Peirce: CP 1.488 Cross-Ref:††

In regard to these two divisions a long and arduous philosophical discussion is quite ineluctable. It would be quite impracticable to summarize it in the present sketch of the shapes which are assumed by the three fundamental ideas of philosophy. All that can be done is to unfold in some measure the characteristics of the view here taken.

Peirce: CP 1.489 Cross-Ref:††

489. In the first place, then, it is plain enough that the law of time is not a metaphysical law. Our logical instinct tells us that. We took as the typical example of a metaphysical law, the law that whatever exists, although its existence is a matter of brute fact, irrespective of any qualities, must definitely possess or be without each monadic quality. Now we feel instinctively that the necessity of that is altogether higher than any necessity for the junctions, between the possessions by a subject of contrary attributes, to be related to one another like premisses and conclusions, as before and after. The one is the mere existential mirror of a law of logic. It is the requirement that that which is necessarily *true* (if there be any truth) shall be a part of the existential fact, and not merely of thought. But the other requires that the mere process of thought, which logic regards as mental, and never insists upon predicating of the subject as true, shall itself be mirrored in existence. But while the law of time is not metaphysical, it is plainly, from that description of it, "next door to" a metaphysical law. This is the reason for making this division follow immediately after that into laws metaphysically necessary and contingent.

Peirce: CP 1.490 Cross-Ref:††

490. It will be very difficult for many minds -- and for the very best and clearest minds, more difficult than for others -- to comprehend the logical correctness of a view which does not put the assumption of time *before* either metaphysics or logic instead of *after* those kinds of necessity, as here arranged. But that is an objection, not to this particular item of the development, but to the general plan of it. To admit the force of the objection and carry it out to its consequences would simply result in reversing the whole order of development, making it begin with polyads, analyzing these into triads, and then finding dyads in triads, and monads in dyads. There is not only nothing erroneous in such an arrangement, but the conceptions cannot be thoroughly grasped until it has been carried out. But this is only one of two sides of the shield, both of which must be examined, and which have to be synthesized in the really philosophical view. The reason of this is, that although the view which takes the triad first is necessary to the understanding of any given point, yet it *cannot*, from the very nature of the case, be carried out in an entirely thoroughgoing manner. How, for instance, would you begin? By taking the triad *first*. You thus do, in spite of yourself, introduce the monadic idea of "first" at the very outset. To get at the idea of a monad, and especially to make it an accurate and clear conception, it is necessary to begin with the idea of a triad and find the monad-idea involved in it. But this is only a scaffolding necessary during the process of constructing the conception. When the conception has been constructed, the scaffolding may be removed, and the monad-idea will be there in all its abstract perfection. According to the path here pursued from monad to triad, from monadic

triads to triadic triads, etc., we do not progress by logical involution -- we do not say the monad involves a dyad -- but we pursue a path of evolution. That is to say, we say that to carry out and perfect the monad, we need next a dyad. This seems to be a vague method when stated in general terms; but in each case, it turns out that deep study of each conception in all its features brings a clear perception that precisely a given next conception is called for.

Peirce: CP 1.491 Cross-Ref:††

491. So far Hegel is quite right. But he formulates the general procedure in too narrow a way, making it use no higher method than dilemma, instead of giving it an observational essence. The real formula is this: a conception is framed according to a certain precept, [then] having so obtained it, we proceed to notice features of it which, though necessarily involved in the precept, did not need to be taken into account in order to construct the conception.†1 These features we perceive take radically different shapes; and these shapes, we find, must be particularized, or decided between, before we can gain a more perfect grasp of the original conception. It is thus that thought is urged on in a predestined path. This is the true evolution of thought, of which Hegel's dilemmatic method is only a special character which the evolution is sometimes found to assume. The great danger of the evolutionary procedure lies in forcing steps that are not inevitable, in consequence of not having a sufficiently distinct apprehension of the features of the conception in hand to see what it is that must immediately succeed it. The idea of time must be employed in arriving at the conception of logical consecution; but the idea once obtained, the time-element may be omitted, thus leaving the logical sequence free from time. That done, time appears as an existential analogue of the logical flow.

Peirce: CP 1.492 Cross-Ref:††

492. Time is said to be the form of inward intuition. But this is an error of the sort just considered. It confuses what is evolved from the time-idea with what is involved in it. The task of the analyst in making out the features of the time-law must begin by formulating precisely what it is which that law explicitly pretends to make subject to time. It is, in the first place, only real events that "take place," or have dates, in real time. Imaginary events, the course of a romance, are represented as having relations *like* those of time among one another, but they have no real places in time. A historical romance connects itself, more or less definitely, with real time; but that is because it "makes believe" they [the imaginary events] are real events. It is, then, only existentially real events which the law of time represents really to have places in real time. What, then, is a real event? It is an existential junction of impossible facts. A pale yellowish iron solution mixed with a pale yellow solution of ferrocyanide of potassium suddenly turns deep blue. It is requisite that its being of a pale greenish or reddish yellow, and therefore not blue, should be a fact, and that the same thing's being blue should be a fact. Those two facts are contradictory. That is, that both should be true of precisely the same subject is absurd. But that they should be true of a subject existentially identical is not absurd, since they are mere accidents of an individual thing, which, as such, has no essence, its mode of being consisting in its forcing itself into a place in the world. Still, the two accidents could not be combined with one another. That *would* be absurd. For these accidents are monadic qualities, do have essences, and these essences are disparate. Their combination would have the form of a monadic triad but would not be a possible monadic triad; for it would violate a logical law. But though the two inherences cannot be *combined*, they can be *joined*. This junction is not a monadic triad, but it is of all forms of dyadic triad that one which most closely apes the monadic triad. Had we enumerated the

divisions of dyadic triads, we should have been obliged to put this first of all. One kind of event, at least, then, is a dyadic triad of the very first kind, distinguished from the monadic triad in that it would, from the essence of the monadic qualities involved, have been contrary to a logical law, were it a monadic triad.

Peirce: CP 1.493 Cross-Ref:††

493. There are other sorts of events, somewhat more complex because the characters concerned are not simple monadic qualities. For example, A may make war upon B, that is, may pass from one sort of relation to B to another sort of relation to B. But they come to much the same thing. There is a repugnance between two monad elements. It is hardly for our present purposes worth while to undertake a long analysis in order to make the very slight correction of our definition of an event called for on this account. An event always involves a junction of contradictory inherences in the subjects existentially the same, whether there is a simple monadic quality inhering in a single subject, or whether they be inherences of contradictory monadic elements of dyads or polyads, in single sets of subjects. But there is a more important possible variation in the nature of events. In the kind of events so far considered, while it is not necessary that the subjects should be existentially of the nature of subjects -- that is, that they should be substantial things -- since it may be a mere wave, or an optical focus, or something else of like nature which is the subject of change, yet it is necessary that these subjects should be in some measure permanent, that is, should be capable of accidental determinations, and therefore should have dyadic existence. But the event may, on the other hand, consist in the coming into existence of something that did not exist, or the reverse. There is still a contradiction here; but instead of consisting in the material, or purely monadic, repugnance of two qualities, it is an incompatibility between two forms of triadic relation, as we shall better understand later. In general, however, we may say that for an event there is requisite: first, a contradiction; second, existential embodiments of these contradictory states; [third,] an immediate existential junction of these two contradictory existential embodiments or facts, so that the subjects are existentially identical; and fourth, in this existential junction a definite one of the two facts must be existentially first in the order of evolution and existentially second in the order of involution. We say the former is earlier, the latter later in time. That is, the past can in some measure work upon and influence (or flow into) the future, but the future cannot in the least work upon the past. On the other hand, the future can remember and know the past, but the past can only know the future so far as it can imagine the process by which the future is to be influenced.

Peirce: CP 1.494 Cross-Ref:††

494. Such, then, is the nature of an event. We can now go forward to an analysis of the substance of the law of time. It has three requirements, a monadic, a dyadic, and a triadic. The monadic clause in the law of time is that whatever fact or dyadic dyad exists, exists during a time, and in *this* time. The event is the existential junction of *states* (that is, of that which in existence corresponds to a *statement* about a given subject in representation) whose combination in one subject would violate the logical law of contradiction. The event, therefore, considered as a junction, is not a subject and does not inhere in a subject. What is it, then? Its mode of being is *existential quasi-existence*, or that approach to existence where contraries can be united in one subject. Time is that diversity of existence whereby that which is existentially a subject is enabled to receive contrary determinations in existence. Phillip is drunk and Phillip is sober would be absurd, did not time make the Phillip of this morning another Phillip than the Phillip of last night. The law is that nothing

dyadically exists as a subject without the diversification which permits it to receive contrary accidents. The instantaneous Phillip who can be drunk and sober at once has a potential being which does not quite amount to existence.

Peirce: CP 1.495 Cross-Ref:††

495. The dyadic requirement of the law of time is that if a subject existentially receives contrary attributes, of the two contrary states an existentially determinate one is first in the existential order of evolution and second in the existential order of involution, while the other is second in the existential order of evolution and first in the existential order of involution; and of any two events whatever, a determinate one is related to the other in this same way (although the two events are not joined, as the two states are joined in the event), unless they are independent of one another, or *contemporaneous*. Suppose I hold in my hand a leaden ball. I open my hand, the ball falls to the ground and rests there. There are three states of the ball: first, the ball is in my hand and is not on the ground; second, the ball is not in my hand and is not on the ground; third, the ball is not in my hand and is on the ground. Of the two events, the ball's leaving my hand and the ball's striking the ground, the former consists in the junction of the ball's being in my hand as first in evolution and the ball's being out of my hand as second in evolution. Hence, of the two states, the ball is in my hand but not on the ground and the ball is neither in my hand nor on the ground, the former is necessarily the first in evolution, being made so by the event. And of the two states, the ball is neither in my hand nor on the ground and the ball is not in my hand but is on the ground, the event of striking makes the former to be first in evolution. Thus, the order of the states is controlled by the nature of the events. But the events are nothing in themselves. But if the fall were instantaneous, if for example my hand intercepted at first a visual ray and were then removed, so that there were but two states -- first, the hand visible, the ground invisible; second, the hand invisible, the ground visible -- then the two events are contemporaneous. If the two states, first "P and Q," second "not P and not Q" exist, then only one of the two states "P but not Q" and "Q but not P" can exist, for the reason that it is the dyadic character of the events that decides. Thus, supposing state "P and Q" and state "neither P nor Q" to both exist, and supposing that in the event "P - not P," P is first in evolution, then the state "P and Q" must antecede the state "neither P nor Q" in evolution, and consequently in the event "Q - not Q," Q must antecede not Q in evolution. These two events, "P becomes not P" and "Q becomes not Q," may then either antecede the other in evolution, and according as one or other antecedes, one or other of the two states, "P but not Q," and "not P but Q," becomes impossible. If the two events are contemporaneous, neither being existentially determined to be first in evolution, then these two states are both impossible.

Peirce: CP 1.496 Cross-Ref:††

496. The three possible temporal relations between two instantaneous events are naturally felt by us to mirror the three possible logical relations of two propositions which can be both true or both false, but are not logically equivalent (that is, have not by logical necessity the same value, as to being true or false). Namely of two such propositions, A and B, either, first, A can be false though B is true, but B must be true should A be true, or, second, either can be false though the other be true, so that they are independent of one another, or, third, A must be true should B be true, but B can be false though A is true. It is remarkable that we should instinctively connect the first case with the temporal succession of B after A, and the third case with the temporal succession of A after B, saying, in the former case, that B would *follow* from A and, in the latter, that A would *follow* from B. For superficial

resemblances are the other way. We know what precedes in time from that which succeeds it much better than we know what is to come from that which goes before. This shows the instinct is not due to superficial resemblances. It is true that we know the conclusion later than we know the premisses; but we do not so much think of our *knowledge* as following as we do that one *fact* is logically sequent on the other. The instinct may, therefore, be presumed to be an obscure perception that temporal succession is a mirror of, or framework for, logical sequence. Thus instinct with its almost unerring certainty favors this doctrine.

Peirce: CP 1.497 Cross-Ref:††

497. That of two events not contemporaneous one should happen before the other involves a *thisness* and thus a dyadism. For as it is impossible for us to indicate or ascertain one to be first by any general quality but only by a comparison with some standard experience, so it is impossible for a distinction of first and second to be except by a dyadic force of existence. That a determinate one shall be first and the other second requires reference to some kind of standard, since right and left are, as far as any monadic quality goes, just alike. There must be a standard first and second, and for any other pair there must be some way of bringing them into experiential connection one way and not the other way with this standard. This experiential reference to a standard in knowledge corresponds to an existential dyadic connection in fact. Otherwise there would be no truth in the knowledge.

Peirce: CP 1.498 Cross-Ref:††

498. So much for the dyadic clause of the law of time. The triadic clause is that time has no limit, and every portion of time is bounded by two instants which are of it, and between any two instants either way round, instants may be interposed such that taking any possible multitude of objects there is at least one interposed event for every unit of that multitude. This statement needs some explanation of its meaning. First what does it mean to say that time has no limit? This may be understood in a topical or a metrical sense. In a metrical sense it means there is no absolutely first and last of time. That is, while we must *adopt* a standard of first and last, there is nothing in its own nature the prototype of first and last. For were there any such prototype, that would consist of a pair of objects absolutely first and last. This, however, is more than is intended here. Whether that be true or not is a question concerning rather the events in time than time itself. What is here meant is that time has no instant from which there are more or less than two ways in which time is stretched out, whether they always be in their nature the foregoing and the coming after, or not. If that be so, since every portion of time is bounded by two instants, there must be a connection of time ring-wise. Events may be limited to a portion of this ring; but the time itself must extend round or else there will be a portion of time, say future time and also past time, not bounded by two instants. The justification of this view is that it extends the properties we see belong to time to the whole of time without arbitrary exceptions not warranted by experience. Now, between any two events may be interposed not merely one event but a multitude of events greater than that statement would supply, a multitude of events as great as a multitude of objects describable. This may be really so or not, but this is the instinctive law of which we seem to be directly conscious.

Peirce: CP 1.499 Cross-Ref:††

499. By virtue of this, time is a continuum. For since the instants, or possible events, are as many as any collection whatever, and there is no maximum collection, it follows that they are more than any collections whatever. They must, therefore, be individually indistinguishable in their very existence -- that is, are distinguishable and

the parts distinguishable indefinitely, but yet not composed of individuals absolutely self-identical and distinct from one another -- that is, they form a *continuum*. A continuum cannot be disarranged except to an insignificant extent. An instant cannot be removed. You can no more, by any decree, shorten a legal holiday by transferring its last instant to the work-day that follows that feast, than you can take away intensity from light, and keep the intensity on exhibition while the light is thrown into the ash-barrel. A limited line AB may be cut into two, AC and C'B, and its ends joined, C' to A and C to B. That is to say, all this may be done in the imagination. We have a difficulty in imagining such a thing in regard to time. For in order that the time should flow continuously even in imagination from the end of one day into the beginning of a day that does historically come next, all the events must be prepared so that the states of things of these two instants, including states of gradual change, such as velocity, etc., shall be precisely the same. In the case of a line we do not think of this, although it is equally true, because we are unaccustomed to minutely dealing with the facts about single molecules and atoms upon which the cohesion of matter depends. We, therefore, see no particular difficulty in joining any end of a line to any other line's end continuously. This is as true a view as the other. As far as time itself goes, nothing prevents twenty-four hours being cut out and the day before joining continuously to the day after, were there any power that could affect such a result. In such a case, the two instants brought together would be *identified*, or made one, which sufficiently shows their want of individual *self-identity* and repugnance to all others.

Peirce: CP 1.500 Cross-Ref:††

500. Intimately connected with the division of metaphysically contingent laws into laws which impose, upon inferences of different attributes in the same subject, forms analogous to forms of thought so that they may evade laws of logic and into those laws which have no reference to thought, there is a division of these latter laws into laws which impose, upon different subjects of precisely the same qualities, forms of relationship analogous to metaphysical forms so that they may evade the laws of metaphysics, that is, laws of space, and into laws which do not concern dyads of inference but only dyads of reaction.

Peirce: CP 1.501 Cross-Ref:††

501. According to the metaphysical law of sufficient reason, alike in all respects two things cannot be. Space evades that law by providing places in which two things or any number, which are precisely alike, except that they are located in different places, themselves precisely alike in themselves, may exist. Thus, space does for different subjects of one predicate precisely what time does for different predicates of the same subject. And as time effects its evasion of the logical law by providing a form analogous to a logical form, so space effects its evasion of the metaphysical law by providing a form analogous to a metaphysical form. Namely, as metaphysics teaches that there is a succession of realities of higher and higher order, each a generalization of the last, and each the limit of a reality of the next higher order, so space presents points, lines, surfaces, and solids, each generated by the motion of a place of next lower dimensionality and the limit of a place of next higher dimensionality.

Peirce: CP 1.502 Cross-Ref:††

502. The last division of laws was a broad one. Now *a posteriori* laws are divided into those which are purely dynamical and those which are more or less intellectual, a division somewhat analogous to that of mental association into

association by contiguity and by resemblance. The former are the nomological laws of physics. So far as our present science knows them, they are as follows:

Peirce: CP 1.503 Cross-Ref:††

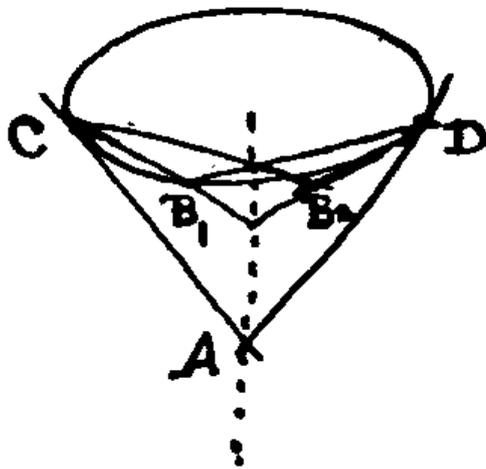
503. First, every particle, or mathematically indivisible portion of matter, when not under a force, moves along a *ray*, or line belonging to a certain family of lines such that any four of them not all cut by each of an infinite multitude of rays is cut by just two rays.

Peirce: CP 1.504 Cross-Ref:††

504. Second, there is a *firmament*, or surface, severing space into worlds; and its properties are, first, that if (A), (B), (C), (D), (E), (F), are any points in a plane †P1 section of it, the rays {AB} and {DE} will meet at a point [{AB} {DE}] which is coradial with [{BC} {EF}] and [{CD} {FA}]; secondly, no material particle ever comes to or leaves the firmament, nor does any plane fixedly connected with a particle ever move into or away from tangency with the firmament; and thirdly, if a body is *rigid*, that is, has only six degrees of freedom, so that all its radiform filaments are fixed when six of its particles are restricted to lying in fixed planes, or when six of its plane films are restricted to passing through fixed points, then, all its possible displacements are subject to the following conditions:

Peirce: CP 1.504 Cross-Ref:††

First, if two particles, A and B, of the rigid solid be situated in points [A[1]] and [B[1]] such that the ray {A[1]B[1]} has two points in the firmament, say [C[1]] and [D[1]], then A and B, however displaced, must lie in a ray that has two points in the firmament, and if any ray through [A[1]] has the two points [C[2]] and



[Click here to view] [D[2]] in the firmament, then A remaining fixed in [A[1]], B can be displaced so as to occupy the point [{{C[1]C[2]}{D[1]D[2]}} B[1]] {C[2]D[2]} or the point [{{C[1]D[2]}{C[2]D[1]}} B[1]] {C[2]D[2]}; but A and B can occupy simultaneously no pair of points which they are not necessarily able to occupy by virtue of this statement.

Peirce: CP 1.504 Cross-Ref:††

Second, if two particles, A and B, of the rigid solid are situated at points [A[1]] and [B[1]] such that the ray {A[1]B[1]} has no point in the firmament, [then] in any plane containing [A[1]] and [B[1]] let [C] and [D] be the points of tangency of rays tangent to the firmament and passing through [A[1]]. Then through

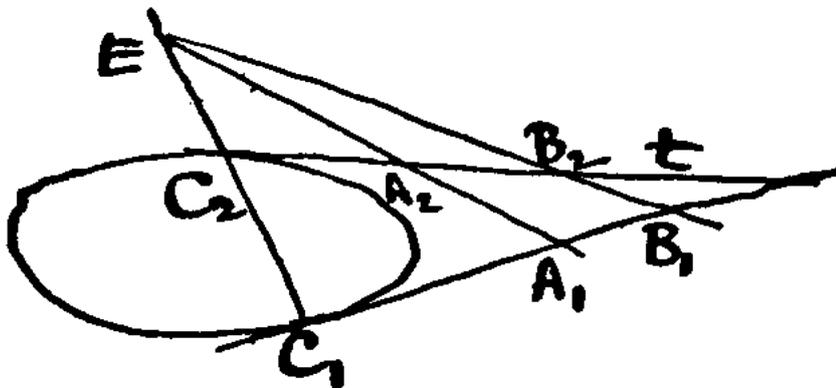


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here to view] take any ray $\{r\}$ whatever, then $\{CB\}rD\{DB\}rC\}$ will be a point where the particle B may be while A is at $[A]$.

Peirce: CP 1.504 Cross-Ref:††

Third, if two particles, A and B, of the rigid solid are situated at points $[A]$ and $[B]$ such that the ray $\{A[B]\}$ has one



[Click here

to view] point in the firmament $[C]$, [then] in any plane through $\{A[B]\}$ take any other point $[C]$ on the firmament, and take any point $[E]$ on the ray, $\{C[C]\}$. Then, if $\{t\}$ is the ray tangent to the firmament at $[C]$, A and B may be simultaneously at $\{t\{EA\}\}$ and $\{t\{EB\}\}$.

Peirce: CP 1.504 Cross-Ref:††

Every radial filament of a rigid body (supposed to fill all space) has its polar conjugate radial filament. Namely, one of these rays is the intersection of two planes tangent to the firmament, while the other passes through the two points of tangency. Every infinitesimal displacement of a rigid body is as if it were a part of a rigid body filling all space, and having two motions in one of which all the particles in one ray are fixed while all the plane films through its polar conjugate remain in the same plane, while in the other motion the reverse is the case.

Peirce: CP 1.505 Cross-Ref:††

505. Third, the effect of force upon a particle is to produce, while that force lasts, a component acceleration of the particle proportional to and in the ray of the force, and the resultant of such component accelerations is the same as if in each infinitesimal time, the different components acted successively, but each for a time equal to the whole of the infinitesimal time.

Peirce: CP 1.506 Cross-Ref:††

506. Fourth, the effect of a force between two particles is to give them opposite accelerations along the ray through them, these accelerations being inversely as certain quantities, called the *masses* of the accelerated particles, which masses are constant throughout all time.

Peirce: CP 1.507 Cross-Ref:††

507. Fifth, so far as force acts between pairs of particles regarded as mere occupiers of points, it depends upon the relative positions of the particles.

Peirce: CP 1.508 Cross-Ref:††

508. Sixth, it remains at present uncertain how the phenomena of elasticity, etc. are to be accounted for; but it is certain that all force cannot be positional attractions and repulsions. There is therefore some law additional to the last.

Peirce: CP 1.509 Cross-Ref:††

509. Seventh, all particles at a greater distance than a decimetre from one another attract one another nearly inversely as the square of the distance, the constant modulus being 6.658×10^{-8} (Boys).†1

Peirce: CP 1.510 Cross-Ref:††

510. Eighth, particles closer together are known to attract one another more strongly, and it seems probable, although it is far from proved, that there are at least two kinds of particles attracting one another differently; but here our ignorance begins to be almost complete.

Peirce: CP 1.511 Cross-Ref:††

511. Laws which connect phenomena by a synthesis more or less intellectual, or inward, are divided somewhat broadly into laws of the inward relations, or resemblances, of bodies, and laws of mind.

Peirce: CP 1.512 Cross-Ref:††

512. The laws of resemblances and differences of bodies are classificatory, or chemical. We know little about them; but we may assert with some confidence that there are differences between substances -- *i.e.*, differences in the smallest parts of bodies, and a classification based on that, and there are differences in the structure of bodies, and a classification based on that. Then of these latter we may distinguish differences in the structure of the smallest pieces of bodies, depending on the shape and size of atomicules, and differences in the manner in which bodies are built up out of their smallest pieces. Here we have a distinction between that kind of structure which gives rise to forms without power of truth [true?] growth or inorganic structures, and the chemistry of protoplasms which develop [or] living organisms.

Peirce: CP 1.513 Cross-Ref:††

513. Finally laws of mind divide themselves into laws of the universal action of mind and laws of kinds of psychical manifestation.

Peirce: CP 1.514 Cross-Ref:††

514. Thus the general scheme of the division of laws is as follows:

{ Logical law
{ { Metaphysical law
{
{ { { Law of time
{ { Law of space
{
{ { Nomological laws of physics
{ { { Classificatory laws of physics
{
{ { Nomological laws of psychics
{ Classificatory laws of psychics

Peirce: CP 1.515 Cross-Ref:††

515. We now come to thoroughly genuine triads, the third class of the third class of triads; and at this stage of the inquiry it is well that we should take our bearings and note just where we are, in order that we may lay out our course for the next advances in the discussion. The monad has no features but its suchness, which appears in logic -- let us remember that logic must be our guide throughout -- as the signification of the verb. This already receives embodiment in the lowest of the chief forms of logic, the *term*. The dyad introduced a radically different sort of element, the subject, which first shows itself in the *proposition*. The dyadic proposition has two subjects, each a sort of mimic monad, but the two [are] of different kinds, one being active, the other passive. The triad brings a third sort of element, the expression of thought, or reasoning, consisting of a colligation of two propositions, not mere dyadic propositions, however, but general beliefs; and these two propositions are connected by a common term and tend to produce a third belief. They not only tend to make the belief, but they also tend to render it true. This reason first emerges in the syllogism, which has three such colligations of premisses. Take the stock example,

All men die;

Enoch is a man;

Therefore, Enoch dies.

These propositions are not dyadic. The first is not so, because it is a rule, not a mere individual fact; the second is not so, because its second term is not a mere monadic quality, still less an individual unit, it is a class-term. The third is not so, because it is

thought as a *result*. Each pair of these three propositions is a reason tending to render the third true. The first and third do so by means of their common monadic character. The first gives *dying* as a specimen character of all men; now the third declares as a consequence that Enoch dies. This gives Enoch one character of men, and so far as Enoch's dying is a consequence goes toward making him a man. The second and third propositions tend to make the first true by means of their common dyadic subject. The second proposition declares Enoch to be a specimen of a man. The third declares as a consequence that Enoch dies. This makes one man die; and so far as Enoch's dying is a consequence tends to render it true that all men die. Finally, the first two propositions bring about the truth of the third. In this particular case they do so absolutely. They generally tend to do so in a way which ought not to be more convincing, but is more in the way in which the objective truth is conceived to result than the other two. They do so by means of their community with respect to the middle term man, a term which as combining the characters of subject and predicate has a triadic element. For combination is triadism, and triadism is combination. Just as the logical verb with its signification reappears in metaphysics as a quality, an *ens* having a *nature* as its mode of being, and as a logical individual subject reappears in metaphysics as a thing, an *ens* having *existence* as its mode of being, so the logical reason, or premiss, reappears in metaphysics as a reason, an *ens* having a *reality*, consisting in a ruling both of the outward and of the inward world, as its mode of being. The being of the quality lies wholly in itself, the being of the thing lies in opposition to other things, the being of the reason lies in its bringing qualities and things together.

Peirce: CP 1.516 Cross-Ref:††

516. In the degenerate dyad there is a metaphysical correspondent to a proposition; but it is a proposition whose two subjects are mere qualities. In the first degenerate triad there is a metaphysical correspondent to a syllogism; but it is a syllogism whose three reasons lie in mere qualities. Thus, orange color is intermediate between red and yellow. The syllogism is this:

Orange has in its own nature a certain indescribable but felt relation to red;

Yellow has a similar relation to orange; as a result,

Yellow has a similar relation to red.

Now, if yellow has a relation to orange and as a result yellow has the same relation to red, this can only be because orange has that same relation to red.

Peirce: CP 1.517 Cross-Ref:††

517. In the second degenerate triad there is likewise a metaphysical correspondent to a syllogism; but it is a syllogism whose premisses lie in mere coexistences of dyadic facts. For example:

A is the mother of B;

B is the wife of C;

it results that A is the mother-in-law of C.

Peirce: CP 1.517 Cross-Ref:††

In the genuine triad, however, there is a real law, and a real case under the law; so that the reasons are not merely reasons in form, but they really govern the truth.

Peirce: CP 1.518 Cross-Ref:††

518. But though there be a real operation of law, yet one of the three reasons may be wanting in triadic reality:

All colors are compounds of so much red, green, and blue;

Yellow is a color;

as a result, Yellow is compounded of proportions of red, green, and blue.

Peirce: CP 1.518 Cross-Ref:††

The middle term here is little more than a disjunction of qualities, differing from that only in the separate colors not being explicitly thought. Accordingly, that colors are compounded and that yellow is as a result so compounded only in form goes to make yellow a color; for in the very essence of color it is already given that yellow is a color. This triad is, therefore, only two-thirds genuine, one of its three reasons not being really operative.

Peirce: CP 1.519 Cross-Ref:††

519. A somewhat similar case arises when the middle term is a mere generalized dyadic existence.

All bodies are attracted toward one another proportionally to their masses and inversely as the square of the distance, multiplied by a fixed modulus;

The earth and moon have such and such masses and are at such a distance;

as a result, The earth and moon attract one another by so much.

Peirce: CP 1.519 Cross-Ref:††

But the last two propositions can hardly be said really to go toward making the truth of the first, since that law is nothing but the expression of the way bodies do move as facts. As far as it concerns the earth and moon it is so in the fact itself, and the earth and moon having such masses and distance as they have does not affect the brute fact, but only makes a certain proposition express that fact.

Peirce: CP 1.520 Cross-Ref:††

520. But there is a third kind of genuine triad in regard to which neither of the qualifications of their thorough genuineness applies for the reason that the result is of such a nature that it could not subsist were it not for the middle term which sustains it. A gives B to C. Say he does this by a formal legal act. Then, in this act A deprives

himself of B; he also enters into an engagement with C and by virtue of these two sides of the act of gift, and of their unity, C acquires possession of B. But this is a remote result. The immediate result is that he acquires possession of B by the gift of A and without the action of A he could not acquire that possession.

Peirce: CP 1.521 Cross-Ref:††
CHAPTER 5

DEGENERATE CASES†1

§1. KINDS OF SECONDNESS

521. Very wretched is the notion of [the categories] that can be conveyed in one lecture. They must grow up in the mind, under the hot sunshine of hard thought, daily, bright, well-focussed, and well-aimed thought; and you must have patience, for long time is required to ripen the fruit. They are no inventions of mine. Were they so, that would be sufficient to condemn them. Confused notions of these elements appear in the first infancy of philosophy, and they have never entirely been forgotten. Their fundamental importance is noticed in the beginning of Aristotle's *De Caelo*, where it is said †2 that the Pythagoreans knew of them.

Peirce: CP 1.522 Cross-Ref:††

522. In Kant they come out with an approach to lucidity. For Kant possessed in a high degree all seven of the mental qualifications of a philosopher:

1. The ability to discern what is before one's consciousness.
2. Inventive originality.
3. Generalizing power.
4. Subtlety.
5. Critical severity and sense of fact.
6. Systematic procedure.
7. Energy, diligence, persistency, and exclusive devotion to philosophy.

Peirce: CP 1.523 Cross-Ref:††

523. But Kant had not the slightest suspicion of the inexhaustible intricacy of the fabric of conceptions, which is such that I do not flatter myself that I have ever analyzed a single idea into its constituent elements.

Peirce: CP 1.524 Cross-Ref:††

524. Hegel, in some respects the greatest philosopher that ever lived, had a somewhat juster notion of this complication, though an inadequate notion, too. For if

he had seen what the state of the case was, he would not have attempted in one lifetime to cover the vast field that he attempted to clear. But Hegel was lamentably deficient in that fifth requisite of critical severity and sense of fact. He brought out the three elements much more clearly [than Kant did]; but the element of Secondness, of *hard fact*, is not accorded its due place in his system; and in a lesser degree the same is true of Firstness. After Hegel wrote, there came fifty years that were remarkably fruitful in all the means for attaining that fifth requisite. Yet Hegel's followers, instead of going to work to reform their master's system, and to render his statement of it obsolete, as every true philosopher must desire that his disciples should do, only proposed, at best, some superficial changes without replacing at all the rotten material with which the system was built up.

Peirce: CP 1.525 Cross-Ref:††

525. I shall not inflict upon you any account of my own labors. Suffice it to say that my results have afforded me great aid in the study of logic.

Peirce: CP 1.525 Cross-Ref:††

I will, however, make a few remarks on these categories. By way of preface, I must explain that in saying that the three, Firstness, Secondness, and Thirdness, complete the list, I by no means deny that there are other categories. On the contrary, at every step of every analysis, conceptions are met with which presumably do not belong to this series of ideas. Nor did an investigation of them occupying me for two years reveal any analysis of them into these as their constituents. I shall say nothing further about them, except incidentally.

Peirce: CP 1.526 Cross-Ref:††

526. As to the three universal categories, as I call them, perhaps with no very good reason for thinking that they are more universal than the others, we first notice that Secondness and Thirdness are conceptions of complexity. That is not, however, to say that they are complex conceptions. When we think of Secondness, we naturally think of two reacting objects, a first and a second. And along with these, as subjects, there is their reaction. But these are not constituents out of which the Secondness is built up. The truth is just reverse, [in] that the being a first or a second or the being a reaction each involves Secondness. An object cannot be a *second* of itself. If it is a second, it has an element of being what another makes it to be. That is, the being a second involves Secondness. The reaction still more manifestly involves the being what another makes a subject to be. Thus, while Secondness is a fact of complexity, it is not a compound of two facts. It is a single fact about two objects. Similar remarks apply to Thirdness.

Peirce: CP 1.527 Cross-Ref:††

527. This remark at once leads to another. The Secondness of the second, whichever of the two objects be called the second, is different from the Secondness of the first. That is to say it *generally* is so. To kill and to be killed are different. In case there is one of the two which there is good reason for calling the first, while the other remains the second, it is that the Secondness is more accidental to the former than to the latter; that there is more or less approach to a state of things in which something, which is itself first, accidentally comes into a Secondness that does not really modify its Firstness, while its second in this Secondness is something whose *being* is of the nature of Secondness and which has no Firstness separate from this. It must be extremely difficult for those who are untrained to such analyses of conceptions to make any sense of all this. For that reason, I shall inflict very little of it upon you -- just enough to show those who *can* carry what I say in their minds that it is by no

means nonsense. The extreme kind of Secondness which I have just described is the relation of a *quality* to the *matter* in which that quality inheres. The mode of being of the quality is that of Firstness. That is to say, it is a possibility. It is related to the matter accidentally; and this relation does not change the quality at all, except that it imparts *existence*, that is to say, this very relation of inherence, to it. But the *matter*, on the other hand, has no being at all except the being a subject of qualities. This relation of really having qualities constitutes its *existence*. But if all its qualities were to be taken away, and it were to be left quality-less matter, it not only would not exist, but it would not have any positive definite possibility -- such as an unembodied quality has. It would be nothing at all.

Peirce: CP 1.528 Cross-Ref:††

528. Thus we have a division of seconds into those whose very being, or Firstness, it is to be seconds, and those whose Secondness is only an accretion. This distinction springs out of the essential elements of Secondness. For Secondness involves Firstness. The concepts of the two kinds of Secondness are mixed concepts composed of Secondness and Firstness. One is the second whose very Firstness is Secondness. The other is a second whose Secondness is second to a Firstness. The idea of mingling Firstness and Secondness in this particular way is an idea distinct from the ideas of Firstness and Secondness that it combines. It appears to be a conception of an entirely different series of categories. At the same time, it is an idea of which Firstness, Secondness, and Thirdness are component parts, since the distinction depends on whether the two elements of Firstness and Secondness that are united are so united as to be one or whether they remain two. This distinction between two kinds of seconds, which is almost involved in the very idea of a second, makes a distinction between two kinds of Secondness; namely, the Secondness of genuine seconds, or matters, which I call genuine Secondness, and the Secondness in which one of the seconds is only a Firstness, which I call degenerate Secondness; so that this Secondness really amounts to nothing but this, that a subject, in its being a second, has a Firstness, or quality. It is to be remarked that this distinction arose from attending to extreme cases; and consequently subdivision will be attached to it according to the more or less essential or accidental nature of the genuine or the degenerate Secondness. With this distinction Thirdness has nothing to do, or at any rate has so little to do that a satisfactory account of the distinction need not mention Thirdness.

Peirce: CP 1.529 Cross-Ref:††

529. I will just mention that among Firstnesses there is no distinction of the genuine and the degenerate, while among Thirdnesses we find not only a genuine but two distinct grades of degeneracy.

Peirce: CP 1.530 Cross-Ref:††

§2. THE FIRSTNESS OF FIRSTNESS, SECONDNESS, AND THIRDNESS

530. But now I wish to call your attention to a kind of distinction which affects Firstness more than it does Secondness, and Secondness more than it does Thirdness. This distinction arises from the circumstance that where you have a triplet you have three pairs; and where you have a pair, you have two units. Thus, Secondness is an essential part of Thirdness though not of Firstness, and Firstness is

an essential element of both Secondness and Thirdness. Hence there is such a thing as the Firstness of Secondness and such a thing as the Firstness of Thirdness; and there is such a thing as the Secondness of Thirdness. But there is no Secondness of pure Firstness and no Thirdness of pure Firstness or Secondness. When you strive to get the purest conceptions you can of Firstness, Secondness, and Thirdness, thinking of quality, reaction, and mediation -- what you are striving to apprehend is pure Firstness, the Firstness of Secondness -- that is what Secondness is, of itself -- and the Firstness of Thirdness. When you contrast the blind compulsion in an event of reaction considered as something which happens and which of its nature can never happen again, since you cannot cross the same river twice, when, I say, you contrast this compulsion with the logical necessitation of a *meaning* considered as something that has no being at all except so far as it actually gets embodied in an event of thought, and you regard this logical necessitation as a sort of actual compulsion, since the meaning must actually be embodied, what you are thinking of is a Secondness involved in Thirdness.

Peirce: CP 1.531 Cross-Ref:††

531. A Firstness is exemplified in every quality of a total feeling. It is perfectly simple and without parts; and everything has its quality. Thus the tragedy of King Lear has its Firstness, its flavor *sui generis*. That wherein all such qualities agree is universal Firstness, the very being of Firstness. The word *possibility* fits it, except that possibility implies a relation to what exists, while universal Firstness is the mode of being of itself. That is why a new word was required for it. Otherwise, "possibility" would have answered the purpose.

Peirce: CP 1.532 Cross-Ref:††

532. As to Secondness, I have said that our only direct knowledge of it is in willing and in the experience of a perception. It is in willing that the Secondness comes out most strongly. But it is not pure Secondness. For, in the first place, he who wills has a purpose; and that idea of purpose makes the act appear as a *means* to an end. Now the word *means* is almost an exact synonym to the word *third*. It certainly involves Thirdness. Moreover, he who wills is conscious of doing so, in the sense of *representing* to himself that he does so. But representation is precisely genuine Thirdness. You must conceive an instantaneous consciousness that is instantly and totally forgotten and an effort without purpose. It is a hopeless undertaking to try to realize what consciousness would be without the element of representation. It would be like unexpectedly hearing a great explosion of nitroglycerine before one had recovered oneself and merely had the sense of the breaking off of the quiet. Perhaps it might not be far from what ordinary common sense conceives to take place when one billiard ball caroms on another. One ball "acts" on the other; that is, it makes an exertion *minus* the element of representation. We may say with some approach to accuracy that the general Firstness of all true Secondness is *existence*, though this term more particularly applies to Secondness in so far as it is an element of the reacting first and second. If we mean Secondness as it is an element of the occurrence, the Firstness of it is *actuality*. But actuality and existence are words expressing the same idea in different applications. Secondness, strictly speaking, is just when and where it takes place, and has no other being; and therefore different Secondnesses, strictly speaking, have in themselves no quality in common. Accordingly, existence, or the universal Firstness of all Secondness, is really not a quality at all. An actual dollar to your credit in the bank does not differ in any respect from a possible imaginary dollar. For if it did, the imaginary dollar could be imagined to be changed in that respect, so as to agree with the actual dollar. We thus see that

actuality is not a *quality*, or mere mode of feeling. Hence Hegel, whose neglect of Secondness was due chiefly to his not recognizing any other mode of being than existence -- and what he calls *existenz* is a special variety of it merely -- regarded pure being as pretty much the same as nothing. It is true that the word "existence" names, as if it were an abstract possibility, that which is precisely the not having any being in abstract possibility; and this circumstance, when you look upon existence as the only being, seems to make existence all but the same as nothing.

Peirce: CP 1.533 Cross-Ref:††

533. To express the Firstness of Thirdness, the peculiar flavor or color of mediation, we have no really good word. *Mentality* is, perhaps, as good as any, poor and inadequate as it is. Here, then, are three kinds of Firstness, qualitative possibility, existence, mentality, resulting from applying Firstness to the three categories. We might strike new words for them: primity, secundity, tertiality.

Peirce: CP 1.534 Cross-Ref:††

534. There are also three other kinds of Firstness which arise in a somewhat similar way; namely, the idea of a simple original quality, the idea of a quality essentially relative, such as that of being "an inch long"; and the idea of a quality that consists in the way something is thought or represented, such as the quality of being manifest.

Peirce: CP 1.535 Cross-Ref:††

535. I shall not enter into any exact analysis of these ideas. I only wished to give you such slight glimpse as I could of the sort of questions that busy the student of phenomenology, merely to lead up to Thirdness and to the particular kind and aspect of Thirdness which is the sole object of logical study. I want first to show you what genuine Thirdness is and what are its two degenerate forms. Now we found the genuine and degenerate forms of Secondness by considering the full ideas of first and second. Then the genuine Secondness was found to be reaction, where first and second are both true seconds and the Secondness is something distinct from them, while in degenerate Secondness, or mere reference, the first is a mere first never attaining full Secondness.

Peirce: CP 1.536 Cross-Ref:††

536. Let us proceed in the same way with Thirdness. We have here a first, a second, and a third. The first is a positive qualitative possibility, in itself nothing more. The second is an existent thing without any mode of being less than existence, but determined by that first. A *third* has a mode of being which consists in the Secondnesses that it determines, the mode of being of a law, or concept. Do not confound this with the ideal being of a quality in itself. A quality is something capable of being completely embodied. A law never can be embodied in its character as a law except by determining a habit. A quality is how something may or might have been. A law is how an endless future must continue to be.

Peirce: CP 1.537 Cross-Ref:††

537. Now in genuine Thirdness, the first, the second, and the third are all three of the nature of thirds, or thought, while in respect to one another they are first, second, and third. The first is thought in its capacity as mere possibility; that is, mere *mind* capable of thinking, or a mere vague idea. The *second* is thought playing the role of a Secondness, or event. That is, it is of the general nature of *experience* or *information*. The third is thought in its role as governing Secondness. It brings the information into the mind, or determines the idea and gives it body. It is informing

thought, or *cognition*. But take away the psychological or accidental human element, and in this genuine Thirdness we see the operation of a sign.

Peirce: CP 1.538 Cross-Ref:††

538. Every sign stands for an object independent of itself; but it can only be a sign of that object in so far as that object is itself of the nature of a sign or thought. For the sign does not affect the object but is affected by it; so that the object must be able to convey thought, that is, must be of the nature of thought or of a sign. Every thought is a sign. But in the first degree of degeneracy the Thirdness affects the object, so that this is not of the nature of a Thirdness -- not so, at least, as far as this operation of degenerate Thirdness is concerned. It is that the third brings about a Secondness but does not regard that Secondness as anything more than a fact. In short it is the operation of executing an *intention*. In the last degree of degeneracy of Thirdness, there is thought, but no conveyance or embodiment of thought at all. It is merely that a fact of which there must be, I suppose, something like knowledge is *apprehended* according to a possible idea. There is an *instigation* without any *prompting*. For example, you look at something and say, "It is red." Well, I ask you what justification you have for such a judgment. You reply, "I *saw* it was red." Not at all. You saw nothing in the least like that. You saw an image. There was no subject or predicate in it. It was just one unseparated image, not resembling a proposition in the smallest particular. It instigated you to your judgment, owing to a possibility of thought; but it never told you so. Now in all imagination and perception there is such an operation by which thought springs up; and its only justification is that it subsequently turns out to be useful.

Peirce: CP 1.539 Cross-Ref:††

539. Now it may be that *logic* ought to be the science of Thirdness in general. But as I have studied it, it is simply the science of what must be and ought to be true representation, so far as representation can be known without any gathering of special facts beyond our ordinary daily life. It is, in short, the philosophy of representation.

Peirce: CP 1.540 Cross-Ref:††

540. The analysis which I have just used to give you some notion of genuine Thirdness and its two forms of degeneracy is the merest rough blackboard sketch of the true state of things; and I must begin the examination of representation by defining representation a *little* more accurately. In the first place, as to my terminology, I confine the word *representation* to the operation of a sign or its *relation* to the object *for* the interpreter of the representation. The concrete subject that represents I call a *sign* or a *representamen*. I use these two words, *sign* and *representamen*, differently. By a *sign* I mean anything which conveys any definite notion of an object in any way, as such conveyers of thought are familiarly known to us. Now I start with this familiar idea and make the best analysis I can of what is essential to a sign, and I define a *representamen* as being whatever that analysis applies to. If therefore I have committed an error in my analysis, part of what I say about signs will be false. For in that case a *sign* may not be a *representamen*. The analysis is certainly true of the representamen, since that is all that word means. Even if my analysis is correct, something may happen to be true of all *signs*, that is of everything that, antecedently to any analysis, we should be willing to regard as conveying a notion of anything, while there might be something which my analysis describes of which the same thing is not true. In particular, all signs convey notions to *human minds*; but I know no reason why every representamen should do so.

Peirce: CP 1.541 Cross-Ref:††

541. My definition of a representamen is as follows:

A REPRESENTAMEN is a subject of a triadic relation TO a second, called its OBJECT, FOR a third, called its INTERPRETANT, this triadic relation being such that the REPRESENTAMEN determines its interpretant to stand in the same triadic relation to the same object for some interpretant.

Peirce: CP 1.542 Cross-Ref:††

542. It follows at once that this relation cannot consist in any actual event that ever can have occurred; for in that case there would be another actual event connecting the interpretant to an interpretant of its own of which the same would be true; and thus there would be an endless series of events which could have actually occurred, which is absurd. For the same reason the interpretant cannot be a *definite* individual object. The relation must therefore consist in a *power* of the representamen to determine *some* interpretant to being a representamen of the same object.

Peirce: CP 1.543 Cross-Ref:††

543. Here we make a new distinction. You see the principle of our procedure. We begin by asking what is the mode of being of the subject of inquiry, that is, what is its absolute and most universal Firstness? The answer comes, that it is either the Firstness of Firstness, the Firstness of Secondness, or the Firstness of Thirdness.

Peirce: CP 1.543 Cross-Ref:††

We then ask what is the universal Secondness, and what the universal Thirdness, of the subject in hand.

Peirce: CP 1.543 Cross-Ref:††

Next we say that Firstness of Firstness, that Firstness of Secondness and that Firstness of Thirdness, that have been described, have been the Firstness of the Firstness in each case. But what is the Secondness that is involved in it and what is the Thirdness?

Peirce: CP 1.543 Cross-Ref:††

So the Secondnesses as they have been first given are the Firstnesses of those Secondnesses. We ask what Secondness they involve and what Thirdness. And so we have endless questions, of which I have only given you small scraps.

Peirce: CP 1.543 Cross-Ref:††

The answers to these questions do not come of themselves. They require the most laborious study, the most careful and exact examination. The system of questions does not save that trouble in the least degree. It enormously increases it by multiplying the questions that are suggested. But it forces us along step by step to much clearer conceptions of the objects of logic than have ever been attained before. The *hard fact* that it has yielded such fruit is the principal argument in its favor.

Peirce: CP 1.544 Cross-Ref:††

544. The method has a general similarity to Hegel's. It would be historically false to call it a modification of Hegel's. It was brought into being by the study of Kant's categories and not Hegel's. Hegel's method has the defect of not working at all if you think with too great exactitude. Moreover, it presents no such definite question to the mind as this method does. This method works better the finer and more accurate the thought. The subtlest mind cannot get the best possible results from it;

but a mind of very moderate skill can make better analyses by this method than the same mind could obtain without it, by far.

Peirce: CP 1.544 Cross-Ref:††

Analyses apparently conflicting may be obtained by this method by different minds, owing to the impossibility of conforming strictly to the requirements. But it does not follow that the results are utterly wrong. They will be two imperfect analyses, each getting a part of the truth.

Peirce: CP 1.545 Cross-Ref:††
CHAPTER 6

ON A NEW LIST OF CATEGORIES

§1. ORIGINAL STATEMENT †1E

545. This paper is based upon the theory already established, that the function of conceptions is to reduce the manifold of sensuous impressions to unity and that the validity of a conception consists in the impossibility of reducing the content of consciousness to unity without the introduction of it.

Peirce: CP 1.546 Cross-Ref:††

546. This theory gives rise to a conception of gradation among those conceptions which are universal. For one such conception may unite the manifold of sense and yet another may be required to unite the conception and the manifold to which it is applied; and so on.

Peirce: CP 1.547 Cross-Ref:††

547. That universal conception which is nearest to sense is that of *the present, in general*. This is a conception, because it is universal. But as the act of *attention* has no connotation at all, but is the pure denotative power of the mind, that is to say, the power which directs the mind to an object, in contradistinction to the power of thinking any predicate of that object -- so the conception of *what is present in general*, which is nothing but the general recognition of what is contained in attention, has no connotation, and therefore no proper unity. This conception of the present in general, of IT in general, is rendered in philosophical language by the word "substance" in one of its meanings. Before any comparison or discrimination can be made between what is present, what is present must have been recognized as such, as *it*, and subsequently the metaphysical parts which are recognized by abstraction are attributed to this *it*, but the *it* cannot itself be made a predicate. This *it* is thus neither predicated of a subject, nor in a subject, and accordingly is identical with the conception of substance.

Peirce: CP 1.548 Cross-Ref:††

548. The unity to which the understanding reduces impressions is the unity of

a proposition. This unity consists in the connection of the predicate with the subject; and, therefore, that which is implied in the copula, or the conception of *being*, is that which completes the work of conceptions of reducing the manifold to unity. The copula (or rather the verb which is copula in one of its senses) means either *actually is* or *would be*, as in the two propositions, "There *is* no griffin," and "A griffin *is* a winged quadruped." The conception of *being* contains only that junction of predicate to subject wherein these two verbs agree. The conception of being, therefore, plainly has no content.

Peirce: CP 1.548 Cross-Ref:††

If we say "The stove is black," the stove is the *substance*, from which its blackness has not been differentiated, and the *is*, while it leaves the substance just as it was seen, explains its confusedness, by the application to it of *blackness* as a predicate.

Peirce: CP 1.548 Cross-Ref:††

Though *being* does not affect the subject, it implies an indefinite determinability of the predicate. For if one could know the copula and predicate of any proposition, as ". . . is a tailed-man," he would know the predicate to be applicable to something supposable, at least. Accordingly, we have propositions whose subjects are entirely indefinite, as "There is a beautiful ellipse," where the subject is merely *something actual or potential*; but we have no propositions whose predicate is entirely indeterminate, for it would be quite senseless to say, "A has the common characters of all things," inasmuch as there are no such common characters.

Peirce: CP 1.548 Cross-Ref:††

Thus substance and being are the beginning and end of all conception. Substance is inapplicable to a predicate, and being is equally so to a subject.

Peirce: CP 1.549 Cross-Ref:††

549. The terms "precision"†P1 and "abstraction," which were formerly applied to every kind of separation, are now limited, not merely to mental separation, but to that which arises from *attention to* one element and *neglect of* the other. Exclusive attention consists in a definite conception or *supposition* of one part of an object, without any supposition of the other. Abstraction or precision ought to be carefully distinguished from two other modes of mental separation, which may be termed *discrimination* and *dissociation*. Discrimination has to do merely with the senses of terms, and only draws a distinction in meaning. Dissociation is that separation which, in the absence of a constant association, is permitted by the law of association of images. It is the consciousness of one thing, without the necessary simultaneous consciousness of the other. Abstraction or precision, therefore, supposes a greater separation than discrimination, but a less separation than dissociation. Thus I can discriminate red from blue, space from color, and color from space, but not red from color. I can prescind red from blue, and space from color (as is manifest from the fact that I actually believe there is an uncolored space between my face and the wall); but I cannot prescind color from space, nor red from color. I can dissociate red from blue, but not space from color, color from space, nor red from color.

Peirce: CP 1.549 Cross-Ref:††

Precision is not a reciprocal process. It is frequently the case, that, while A cannot be prescinded from B, B can be prescinded from A. This circumstance is accounted for as follows. Elementary conceptions only arise upon the occasion of experience; that is, they are produced for the first time according to a general law, the

condition of which is the existence of certain impressions. Now if a conception does not reduce the impressions upon which it follows to unity, it is a mere arbitrary addition to these latter; and elementary conceptions do not arise thus arbitrarily. But if the impressions could be definitely comprehended without the conception, this latter would not reduce them to unity. Hence, the impressions (or more immediate conceptions) cannot be definitely conceived or attended to, to the neglect of an elementary conception which reduces them to unity. On the other hand, when such a conception has once been obtained, there is, in general, no reason why the premisses which have occasioned it should not be neglected, and therefore the explaining conception may frequently be prescind from the more immediate ones and from the impressions.

Peirce: CP 1.550 Cross-Ref:††

550. The facts now collected afford the basis for a systematic method of searching out whatever universal elementary conceptions there may be intermediate between the manifold of substance and the unity of being. It has been shown that the occasion of the introduction of a universal elementary conception is either the reduction of the manifold of substance to unity, or else the conjunction to substance of another conception. And it has further been shown that the elements conjoined cannot be supposed without the conception, whereas the conception can generally be supposed without these elements. Now, empirical psychology discovers the occasion of the introduction of a conception, and we have only to ascertain what conception already lies in the data which is united to that of substance by the first conception, but which cannot be supposed without this first conception, to have the next conception in order in passing from being to substance.

Peirce: CP 1.550 Cross-Ref:††

It may be noticed that, throughout this process, *introspection* is not resorted to. Nothing is assumed respecting the subjective elements of consciousness which cannot be securely inferred from the objective elements.

Peirce: CP 1.551 Cross-Ref:††

551. The conception of *being* arises upon the formation of a proposition. A proposition always has, besides a term to express the substance, another to express the quality of that substance; and the function of the conception of being is to unite the quality to the substance. Quality, therefore, in its very widest sense, is the first conception in order in passing from being to substance.

Peirce: CP 1.551 Cross-Ref:††

Quality seems at first sight to be given in the impression. Such results of introspection are untrustworthy. A proposition asserts the applicability of a mediate conception to a more immediate one. Since this is *asserted*, the more mediate conception is clearly regarded independently of this circumstance, for otherwise the two conceptions would not be distinguished, but one would be thought through the other, without this latter being an object of thought, at all. The mediate conception, then, in order to be *asserted* to be applicable to the other, must first be considered without regard to this circumstance, and taken immediately. But, taken immediately, it transcends what is given (the more immediate conception), and its applicability to the latter is hypothetical. Take, for example, the proposition, "This stove is black." Here the conception of *this stove* is the more immediate, that of *black* the more mediate, which latter, to be predicated of the former, must be discriminated from it and considered *in itself*, not as applied to an object, but simply as embodying a quality, *blackness*. Now this *blackness* is a pure species or abstraction, and its

application to *this stove* is entirely hypothetical. The same thing is meant by "the stove is black," as by "there is blackness in the stove." *Embodying blackness* is the equivalent of *black*. †P1 The proof is this. These conceptions are applied indifferently to precisely the same facts. If, therefore, they were different, the one which was first applied would fulfil every function of the other; so that one of them would be superfluous. Now a superfluous conception is an arbitrary fiction, whereas elementary conceptions arise only upon the requirement of experience; so that a superfluous elementary conception is impossible. Moreover, the conception of a pure abstraction is indispensable, because we cannot comprehend an agreement of two things, except as an agreement in some *respect*, and this respect is such a pure abstraction as blackness. Such a pure abstraction, reference to which constitutes a *quality* or general attribute, may be termed a *ground*.

Peirce: CP 1.551 Cross-Ref:††

Reference to a ground cannot be prescindend from being, but being can be prescindend from it.

Peirce: CP 1.552 Cross-Ref:††

552. Empirical psychology has established the fact that we can know a quality only by means of its contrast with or similarity to another. By contrast and agreement a thing is referred to a correlate, if this term may be used in a wider sense than usual. The occasion of the introduction of the conception of reference to a ground is the reference to a correlate, and this is, therefore, the next conception in order.

Peirce: CP 1.552 Cross-Ref:††

Reference to a correlate cannot be prescindend from reference to a ground; but reference to a ground may be prescindend from reference to a correlate.

Peirce: CP 1.553 Cross-Ref:††

553. The occasion of reference to a correlate is obviously by comparison. This act has not been sufficiently studied by the psychologists, and it will, therefore, be necessary to adduce some examples to show in what it consists. Suppose we wish to compare the letters p and b. We may imagine one of them to be turned over on the line of writing as an axis, then laid upon the other, and finally to become transparent so that the other can be seen through it. In this way we shall form a new image which mediates between the images of the two letters, inasmuch as it represents one of them to be (when turned over) the likeness of the other. Again, suppose we think of a murderer as being in relation to a murdered person; in this case we conceive the act of the murder, and in this conception it is represented that corresponding to every murderer (as well as to every murder) there is a murdered person; and thus we resort again to a mediating representation which represents the relate as standing for a correlate with which the mediating representation is itself in relation. Again, suppose we look up the word *homme* in a French dictionary; we shall find opposite to it the word *man*, which, so placed, represents *homme* as representing the same two-legged creature which *man* itself represents. By a further accumulation of instances, it would be found that every comparison requires, besides the related thing, the ground, and the correlate, also a *mediating representation which represents the relate to be a representation of the same correlate which this mediating representation itself represents*. Such a mediating representation may be termed an *interpretant*, because it fulfils the office of an interpreter, who says that a foreigner says the same thing which he himself says. The term representation is here to be understood in a very extended sense, which can be explained by instances better than by a definition. In

this sense, a word represents a thing to the conception in the mind of the hearer, a portrait represents the person for whom it is intended to the conception of recognition, a weathercock represents the direction of the wind to the conception of him who understands it, a barrister represents his client to the judge and jury whom he influences.

Peirce: CP 1.553 Cross-Ref:††

Every reference to a correlate, then, conjoins to the substance the conception of a reference to an interpretant; and this is, therefore, the next conception in order in passing from being to substance.

Peirce: CP 1.553 Cross-Ref:††

Reference to an interpretant cannot be prescinded from reference to a correlate; but the latter can be prescinded from the former.

Peirce: CP 1.554 Cross-Ref:††

554. Reference to an interpretant is rendered possible and justified by that which renders possible and justifies comparison. But that is clearly the diversity of impressions. If we had but one impression, it would not require to be reduced to unity, and would therefore not need to be thought of as referred to an interpretant, and the conception of reference to an interpretant would not arise. But since there is a manifold of impressions, we have a feeling of complication or confusion, which leads us to differentiate this impression from that, and then, having been differentiated, they require to be brought to unity. Now they are not brought to unity until we conceive them together as being *ours*, that is, until we refer them to a conception as their interpretant. Thus, the reference to an interpretant arises upon the holding together of diverse impressions, and therefore it does not join a conception to the substance, as the other two references do, but unites directly the manifold of the substance itself. It is, therefore, the last conception in order in passing from being to substance.

Peirce: CP 1.555 Cross-Ref:††

555. The five conceptions thus obtained, for reasons which will be sufficiently obvious, may be termed *categories*. That is,

Being

Quality (reference to a ground)

Relation (reference to a correlate)

Representation (reference to an interpretant)

Substance

The three intermediate conceptions may be termed accidents.

Peirce: CP 1.556 Cross-Ref:††

556. This passage from the many to the one is numerical. The conception of a *third* is that of an object which is so related to two others, that one of these must be related to the other in the same way in which the third is related to that other. Now

this coincides with the conception of an interpretant. An *other* is plainly equivalent to a *correlate*. The conception of second differs from that of other, in implying the possibility of a third. In the same way, the conception of *self* implies the possibility of an *other*. The *ground* is the self abstracted from the concreteness which implies the possibility of another.

Peirce: CP 1.557 Cross-Ref:††

557. Since no one of the categories can be prescindend from those above it, the list of supposable objects which they afford is,

What is.

Quale (that which refers to a ground)

Relate (that which refers to ground and correlate)

Representamen (that which refers to ground, correlate, and interpretant)

It

Peirce: CP 1.558 Cross-Ref:††

558. A quality may have a special determination which prevents its being prescindend from reference to a correlate. Hence there are two kinds of relation.

Peirce: CP 1.558 Cross-Ref:††

First. That of relates whose reference to a ground is a prescindible or internal quality.

Peirce: CP 1.558 Cross-Ref:††

Second. That of relates whose reference to a ground is an unprecindible or relative quality.

Peirce: CP 1.558 Cross-Ref:††

In the former case, the relation is a mere *concurrence* of the correlates in one character, and the relate and correlate are not distinguished. In the latter case the correlate is set over against the relate, and there is in some sense an *opposition*.

Peirce: CP 1.558 Cross-Ref:††

Relates of the first kind are brought into relation simply by their agreement. But mere disagreement (unrecognized) does not constitute relation, and therefore relates of the second kind are only brought into relation by correspondence in fact.

Peirce: CP 1.558 Cross-Ref:††

A reference to a ground may also be such that it cannot be prescindend from a reference to an interpretant. In this case it may be termed an *imputed* quality. If the reference of a relate to its ground can be prescindend from reference to an interpretant, its relation to its correlate is a mere concurrence or community in the possession of a quality, and therefore the reference to a correlate can be prescindend from reference to an interpretant. It follows that there are three kinds of representations.

Peirce: CP 1.558 Cross-Ref:††

First. Those whose relation to their objects is a mere community in some

quality, and these representations may be termed *likenesses*.†1

Peirce: CP 1.558 Cross-Ref:††

Second. Those whose relation to their objects consists in a correspondence in fact, and these may be termed *indices* or *signs*.†2

Peirce: CP 1.558 Cross-Ref:††

Third. Those the ground of whose relation to their objects is an imputed character, which are the same as *general signs*, and these may be termed *symbols*.

Peirce: CP 1.559 Cross-Ref:††

559. I shall now show how the three conceptions of reference to a ground, reference to an object, and reference to an interpretant are the fundamental ones of at least one universal science, that of logic. Logic is said to treat of second intentions as applied to first.†1 It would lead me too far away from the matter in hand to discuss the truth of this statement; I shall simply adopt it as one which seems to me to afford a good definition of the subject-genus of this science. Now, second intentions are the objects of the understanding considered as representations, and the first intentions to which they apply are the objects of those representations. The objects of the understanding, considered as representations, are symbols, that is, signs which are at least potentially general. But the rules of logic hold good of any symbols, of those which are written or spoken as well as of those which are thought. They have no immediate application to likenesses or indices, because no arguments can be constructed of these alone, but do apply to all symbols. All symbols, indeed, are in one sense relative to the understanding, but only in the sense in which also all things are relative to the understanding. On this account, therefore, the relation to the understanding need not be expressed in the definition of the sphere of logic, since it determines no limitation of that sphere. But a distinction can be made between concepts which are supposed to have no existence except so far as they are actually present to the understanding, and external symbols which still retain their character of symbols so long as they are only *capable* of being understood. And as the rules of logic apply to these latter as much as to the former (and though only through the former, yet this character, since it belongs to all things, is no limitation), it follows that logic has for its subject-genus all symbols and not merely concepts.†P1 We come, therefore, to this, that logic treats of the reference of symbols in general to their objects. In this view it is one of a trivium of conceivable sciences. The first would treat of the formal conditions of symbols having meaning, that is of the reference of symbols in general to their grounds or imputed characters, and this might be called formal grammar;†1 the second, logic,†2 would treat of the formal conditions of the truth of symbols; and the third would treat of the formal conditions of the force of symbols, or their power of appealing to a mind, that is, of their reference in general to interpretants, and this might be called formal rhetoric.†3

Peirce: CP 1.559 Cross-Ref:††

There would be a general division of symbols, common to all these sciences; namely, into,

Peirce: CP 1.559 Cross-Ref:††

1•. Symbols which directly determine only their *grounds* or imputed qualities, and are thus but sums of marks or *terms*;

Peirce: CP 1.559 Cross-Ref:††

2•. Symbols which also independently determine their *objects* by means of

other term or terms, and thus, expressing their own objective validity, become capable of truth or falsehood, that is, are *propositions*; and,

Peirce: CP 1.559 Cross-Ref:††

3•. Symbols which also independently determine their *interpretants*, and thus the minds to which they appeal, by premissing a proposition or propositions which such a mind is to admit. These are *arguments*.

Peirce: CP 1.559 Cross-Ref:††

And it is remarkable that, among all the definitions of the proposition, for example, as the *oratio indicativa*, as the subsumption of an object under a concept, as the expression of the relation of two concepts, and as the indication of the mutable ground of appearance, there is, perhaps, not one in which the conception of reference to an object or correlate is not the important one. In the same way, the conception of reference to an interpretant or third, is always prominent in the definitions of argument.

Peirce: CP 1.559 Cross-Ref:††

In a proposition, the term which separately indicates the object of the symbol is termed the subject, and that which indicates the ground is termed the predicate. The objects indicated by the subject (which are always potentially a plurality -- at least, of phases or appearances) are therefore stated by the proposition to be related to one another on the ground of the character indicated by the predicate. Now this relation may be either a concurrence or an opposition. Propositions of concurrence are those which are usually considered in logic; but I have shown in a paper upon the classification of arguments †1 that it is also necessary to consider separately propositions of opposition, if we are to take account of such arguments as the following:

Peirce: CP 1.559 Cross-Ref:††

Whatever is the half of anything is less than that of which it is the half:

A is half of B;

A is less than B.

Peirce: CP 1.559 Cross-Ref:††

The subject of such a proposition is separated into two terms, a "subject nominative" and an "object accusative."

Peirce: CP 1.559 Cross-Ref:††

In an argument, the premisses form a representation of the conclusion, because they indicate the interpretant of the argument, or representation representing it to represent its object. The premisses may afford a likeness, index, or symbol of the conclusion. In deductive argument, the conclusion is represented by the premisses as by a general sign under which it is contained. In hypotheses, something *like* the conclusion is proved, that is, the premisses form a likeness of the conclusion. Take, for example, the following argument:

M is, for instance, P^I, P^{II}, P^{III}, and P^{IV};

S is P^I, P^{II}, P^{III}, and P^{IV}:

∴ S is M.

Peirce: CP 1.559 Cross-Ref:††

Here the first premiss amounts to this, that "P^I, P^{II}, P^{III}, and P^{IV}" is a likeness of M, and thus the premisses are or represent a likeness of the conclusion. That it is different with induction another example will show.

S^I, S^{II}, S^{III}, and S^{IV} are taken as samples of the collection M;

S^I, S^{II}, S^{III}, and S^{IV} are P:

∴ All M is P.

Peirce: CP 1.559 Cross-Ref:††

Hence the first premiss amounts to saying that "S^I, S^{II}, S^{III}, and S^{IV}" is an index of M. Hence the premisses are an index of the conclusion.

Peirce: CP 1.559 Cross-Ref:††

The other divisions of terms, propositions, and arguments arise from the distinction of extension and comprehension. I propose to treat this subject in a subsequent paper.†1 But I will so far anticipate that as to say that there is, first, the direct reference of a symbol to its objects, or its denotation; second, the reference of the symbol to its ground, through its object, that is, its reference to the common characters of its objects, or its connotation; and third, its reference to its interpretants through its object, that is, its reference to all the synthetical propositions in which its objects in common are subject or predicate, and this I term the information it embodies. And as every addition to what it denotes, or to what it connotes, is effected by means of a distinct proposition of this kind, it follows that the extension and comprehension of a term are in an inverse relation, as long as the information remains the same, and that every increase of information is accompanied by an increase of one or other of these two quantities. It may be observed that extension and comprehension are very often taken in other senses in which this last proposition is not true.

Peirce: CP 1.559 Cross-Ref:††

This is an imperfect view of the application which the conceptions which, according to our analysis, are the most fundamental ones find in the sphere of logic. It is believed, however, that it is sufficient to show that at least something may be usefully suggested by considering this science in this light.

Peirce: CP 1.560 Cross-Ref:††

§2. NOTES ON THE PRECEDING †2†3

560. Before I came to man's estate, being greatly impressed with Kant's *Critic of the Pure Reason*, my father, who was an eminent mathematician, pointed out to me lacunæ in Kant's reasoning which I should probably not otherwise have discovered. From Kant, I was led to an admiring study of Locke, Berkeley, and Hume, and to that of Aristotle's *Organon*, *Metaphysics*, and psychological treatises,

and somewhat later derived the greatest advantage from a deeply pondering perusal of some of the works of medieval thinkers, St. Augustine, Abelard, and John of Salisbury, with related fragments from St. Thomas Aquinas, most especially from John of Duns, the Scot (Duns being the name of a then not unimportant place in East Lothian), and from William of Ockham. So far as a modern man of science can share the ideas of those medieval theologians, I ultimately came to approve the opinions of Duns, although I think he inclines too much toward nominalism. In my studies of Kant's great *Critic*, which I almost knew by heart, I was very much struck by the fact that, although, according to his own account of the matter, his whole philosophy rests upon his "functions of judgment," or logical divisions of propositions, and upon the relation of his "categories" to them, yet his examination of them is most hasty, superficial, trivial, and even trifling, while throughout his works, replete as they are with evidences of logical genius, there is manifest a most astounding ignorance of the traditional logic, even of the very *Summulae Logicales*, the elementary schoolbook of the Plantagenet era. Now although a beastlike superficiality and lack of generalizing thought spreads like a pall over the writings of the scholastic masters of logic, yet the minute thoroughness with which they examined every problem that came within their ken renders it hard to conceive in this twentieth century how a really earnest student, goaded to the study of logic by the momentous importance that Kant attached to its details, could have reconciled himself to treating it in the debonnair and *dégagé* fashion that he did. I was thus stimulated to independent inquiry into the logical support of the fundamental concepts called categories.

Peirce: CP 1.561 Cross-Ref:††

561. The first question, and it was a question of supreme importance requiring not only utter abandonment of all bias, but also a most cautious yet vigorously active research, was whether or not the fundamental categories of thought really have that sort of dependence upon formal logic that Kant asserted. I became thoroughly convinced that such a relation really did and must exist. After a series of inquiries, I came to see that Kant ought not to have confined himself to divisions of propositions, or "judgments," as the Germans confuse the subject by calling them, but ought to have taken account of all elementary and significant differences of form among signs of all sorts, and that, above all, he ought not to have left out of account fundamental forms of reasonings. At last, after the hardest two years' mental work that I have ever done in my life, I found myself with but a single assured result of any positive importance. This was that there are but three elementary forms of predication or signification, which as I originally named them (but with bracketed additions now made to render the terms more intelligible) were *qualities* (of feeling), (dyadic) *relations*, and (predications of) *representations*.

Peirce: CP 1.562 Cross-Ref:††

562. It must have been in 1866 that Professor De Morgan honored the unknown beginner in philosophy that I then was (for I had not earnestly studied it for more than ten years, which is a short apprenticeship in this most difficult of subjects), by sending me a copy of his memoir "On the Logic of Relations, etc."†1 I at once fell to upon it; and before many weeks had come to see in it, as De Morgan had already seen, a brilliant and astonishing illumination of every corner and every vista of logic. Let me pause to say that no decent semblance of justice has ever been done to De Morgan, owing to his not having brought anything to its final shape. Even his personal students, reverent as they perform were, never sufficiently understood that his was the work of an exploring expedition, which every day comes upon new forms for the study of which leisure is, at the moment, lacking, because additional novelties

are coming in and requiring note. He stood indeed like Aladdin (or whoever it was) gazing upon the overwhelming riches of Ali Baba's cave, scarce capable of making a rough inventory of them. But what De Morgan, with his strictly mathematical and indisputable method, actually accomplished in the way of examination of all the strange forms with which he had enriched the science of logic was not slight and was performed in a truly scientific spirit not unanimated by true genius. It was quite twenty-five years before my studies of it all reached what may be called a near approach toward a provisionally final result (absolute finality never being presumable in any universal science); but a short time sufficed to furnish me with mathematical demonstration that indecomposable predicates are of three classes: first, those which, like neuter verbs, apply but to a single subject; secondly, those which like simple transitive verbs have two subjects each, called in the traditional nomenclature of grammar (generally less philosophical than that of logic) the "subject nominative" and the "object accusative," although the perfect equivalence of meaning between "A affects B" and "B is affected by A" plainly shows that the two things they denote are equally referred to in the assertion; and thirdly, those predicates which have three such subjects, or correlates. These last (though the purely formal, mathematical method of De Morgan does not, as far as I see, warrant this) never express mere brute fact, but always some relation of an intellectual nature, being either constituted by action of a mental kind or implying some general law.

Peirce: CP 1.563 Cross-Ref:††

563. As early as 1860, when I knew nothing of any German philosopher except Kant, who had been my revered master for three or four years, I was much struck with a certain indication that Kant's list of categories might be a part of a larger system of conceptions. For instance, the categories of relation -- reaction, causality, and subsistence -- are so many different modes of *necessity*, which is a category of modality; and in like manner, the categories of quality -- negation, qualification, degree, and intrinsic attribution -- are so many relations of inherence, which is a category of relation. Thus, as the categories of the third group are to those of the fourth, so are those of the second to those of the third; and I fancied, at least, that the categories of quantity, unity, plurality, totality, were, in like manner, different intrinsic attributions of quality. Moreover, if I asked myself what was the difference between the three categories of quality, the answer I gave was that negation was a merely *possible* inherence, quality in degree a *contingent* inherence, and intrinsic attribution a *necessary* inherence; so that the categories of the second group are distinguished by means of those of the fourth; and in like manner, it seemed to me that to the question how the categories of quantity -- unity, plurality, totality -- differ, the answer should be that *totality*, or system, is the intrinsic attribution which results from reactions, *plurality* that which results from causality, and *unity* that which results from inherence. This led me to ask, what are the conceptions which are distinguished by negative unity, qualitative unity, and intrinsic unity? I also asked, what are the different kinds of necessity by which reaction, causality, and inherence are distinguished? I will not trouble the reader with my answers to these and similar questions. Suffice it to say that I seemed to myself to be blindly groping among a deranged system of conceptions; and after trying to solve the puzzle in a direct speculative, a physical, a historical, and a psychological manner, I finally concluded the only way was to attack it as Kant had done from the side of formal logic.

Peirce: CP 1.564 Cross-Ref:††

564. I must acknowledge some previous errors committed by me in expounding my division of signs into *icons*, *indices* and *symbols*. At the time I first

published this division in 1867 I had been studying the logic of relatives for so short a time that it was not until three years later that I was ready to go to print with my first memoir on that subject. I had hardly commenced the cultivation of that land which De Morgan had cleared. I already, however, saw what had escaped that eminent master, that besides non-relative characters, and besides relations between pairs of objects, there was a third category of characters, and but this third. This third class really consists of plural relations, all of which may be regarded as compounds of triadic relations, that is, of relations between triads of objects. A very broad and important class of triadic characters [consists of] representations. A representation is that character of a thing by virtue of which, for the production of a certain mental effect, it may stand in place of another thing. The thing having this character I term a *representamen*, the mental effect, or thought, its *interpretant*, the thing for which it stands, its *object*.

Peirce: CP 1.565 Cross-Ref:††

565. In 1867, although I had proof (duly published)†1 that there was only a third category of characters besides nonrelative characters and dual relations, yet I had not discovered that plural relations (which it had not occurred to me were sometimes not reducible to conjunctions of dual relations) constitute that third class. I saw that there must be a conception of which I could make out some features, but being unfamiliar with it in its generality, I quite naturally mistook it for that conception of *representation* which I obtained by generalizing for this very purpose the idea of a sign. I did not generalize enough, a form of error into which greater minds than mine might fall. I supposed the third class of characters was quite covered by the representative characters. Accordingly, I declared all characters to be divisible into *qualities* (nonrelative characters), *relations*, and *representations*, instead of into non-relative characters, dual relations, and plural relations.

Peirce: CP 1.566 Cross-Ref:††

566. I observed in 1867†1 that dual relations are of two kinds according as they are or are not constituted by the relate and correlate possessing non-relative characters. This is correct. Two blue objects are *ipso facto* in relation to one another. It is important to remark that this is not true of characters so far as they are dissimilar. Thus, an orange and justice are not brought into relation to one another by the disparateness of their characters. Drag them into comparison, and then they stand in the relation of dissimilarity, a relation of a quite complex nature. But as the orange and justice exist, their qualities do not constitute a relation of dissimilarity. It must not be overlooked that dissimilarity is not simple otherness. Otherness belongs to hecceities. It is the inseparable spouse of identity: wherever there is identity there is necessarily otherness; and in whatever field there is true otherness there is necessarily identity. Since identity belongs exclusively to that which is *hic et nunc*, so likewise must otherness. It is, therefore, in a sense a dynamical relation, though only a relation of reason. It exists only so far as the objects concerned are, or are liable to be, forcibly brought together before the attention. Dissimilarity is a relation between characters consisting in otherness of all the subjects of those characters. Consequently, being an otherness, it is a dynamo-logical relation, existing only so far as the characters are, or are liable to be, brought into comparison by something besides those characters in themselves.

Peirce: CP 1.567 Cross-Ref:††

567. Similarity, on the other hand, is of quite a different nature. The forms of the words *similarity* and *dissimilarity* suggest that one is the negative of the other,

which is absurd, since everything is both similar and dissimilar to everything else. Two characters, being of the nature of ideas, are, in a measure, the same. Their mere existence constitutes a unity of the two, or, in other words, pairs them. Things are similar and dissimilar so far as their characters are so. We see, then, that the first category of relations embraces only similarities; while the second, embracing all other relations, may be termed dynamical relations. At the same time, we see from the above remarks that the dynamical relations at once divide themselves into logical, hemilogical and non-logical relations. By logical relations, I mean those in respect to which all pairs [of] objects in the universe are alike; by hemilogical relations those in respect to which there is in reference to each object in the universe only one object (perhaps itself) or some definite multitude of objects which are different from others; while the alogical relations include all other cases. The logical and hemilogical relations belong to the old class of relations of reason, while relations *in re* are alogical. But there are a few not unimportant relations of reason which are likewise alogical. In my paper of 1867, I committed the error of identifying those relations constituted by non-relative characters with relations of equiparance, that is, with necessarily mutual relations, and the dynamical relations with relations of disquiparance, or possibly non-mutual relations. Subsequently, falling out of one error into another, I identified the two classes respectively with relations of reason and relations *in re*.

Peirce: CP 1.568 Cross-Ref:††
CHAPTER 7

TRIADOMANY†1

The author's response to the anticipated suspicion that he attaches a superstitious or fanciful importance to the number three, and forces divisions to a Procrustean bed of trichotomy.

Peirce: CP 1.568 Cross-Ref:††

568. I fully admit that there is a not uncommon craze for trichotomies. I do not know but the psychiatrists have provided a name for it. If not, they should. "Trichimania," [?] unfortunately, happens to be preëmpted for a totally different passion; but it might be called *triadomany*. I am not so afflicted; but I find myself obliged, for truth's sake, to make such a large number of trichotomies that I could not [but] wonder if my readers, especially those of them who are in the way of knowing how common the malady is, should suspect, or even opine, that I am a victim of it. But I am now and here going to convince those who are open to conviction, that it is not so, but that there is a good reason why a thorough student of the subject of this book †2 should be led to make trichotomies, that the nature of the science is such that not only is it to be expected that it should involve real trichotomies, but furthermore, that there is a cause that tends to give this form even to faulty divisions, such as a student, thirsting for thoroughness and full of anxiety lest he omit any branch of his subject, will be liable to fall into. Were it not for this cause, the trichotomic form would, as I shall show, be a strong argument in confirmation of the reasoning whose

fruit should take this form.

Peirce: CP 1.569 Cross-Ref:††

569. My first argument in repelling the suspicion that the prevalence of trichotomies in my system is due purely to my predilection for that form, will be that were that predilection so potent, it would inevitably have made me equally given over to the trichotomic form of classification of whatever subject I might work upon. But this is not at all the case. I once endeavored by going over the different classifications that I have made of subjects not of the special kind in which I find trichotomies to abound -- a kind which I shall define below -- to ascertain the relative frequency of different numbers of sub-classes in the divisions of classes generally, when the divisions were such as seemed to me undoubtedly to possess objective reality. I do not think my results of much value, on account of the great difference of the proportions in different kinds of subjects. Nevertheless, I will set them down. I found that among twenty-nine divisions of subjects not of the kind that specially abound in trichotomies, there would be eleven dichotomies, five trichotomies, and thirteen divisions into more than three parts. The fact that I got such a result, however rough it was, suffices to show that I have no marked predilection for trichotomies in general.

Peirce: CP 1.570 Cross-Ref:††

570. I come now to a second argument, or rather to a series of considerations not altogether foreign to what I have been saying. The warm friends who urged upon me these objections -- and nothing can be more precious to a sincere student than frank and strongly put objections -- were naturalists belonging to that family of minds to whom mathematics, even the simplest, seems a closed book. I would point out to them, or, to speak more accurately, I would tell them, that there is a world-wide difference between the divisions that one recognizes in classes whose essence one can comprehend, and the varieties that one observes from the outside, as one does those of objects of natural history, without being able to guess why they should be such as they seem to be, nor, except in the higher divisions, being at all sure that we have the full list of the parts, nor whether they result from a single division or from several, one succeeding another.

Peirce: CP 1.571 Cross-Ref:††

571. Agassiz, in his *Essay on Classification*, described well -- I do not say perfectly, but relatively, well -- what a classification of animals ought to be. But subsequent zoölogists seem to find that when he came to adjusting his idea to the facts of the animal kingdom, it did not seem to be a good fit. What wonder? It required the taxonomist to say what the idea of the Creator was, and the different manners in which the one idea was designed to be carried out. How can a creature so place himself at the point of view of his Creator?

Peirce: CP 1.572 Cross-Ref:††

572. Soon the zoölogists began to classify according to the course of evolution. No doubt this had the advantage of turning their minds to problems within the scope of science. But I venture upon the observation that, granting the perfect success of their investigation, what they so ascertain is precisely the genealogy of species. Now genealogy is not at all the same thing as logical division. Nothing renders this clearer than the studies of Galton and others upon the phenomena of the inheritance of characters. I mean that this is shown even to those who have no definite idea of what logical division is; while for those who know what it is, the studies of Galton gave emphasis and illustration to what they must have fully realized already.

Peirce: CP 1.572 Cross-Ref:††

But when my critical friends counsel me to consider the marvellous multiplicity of sub-groups into which each group of the animal kingdom is divided at each division, I accept their suggestion, and turn to Huxley's famous volume on *The Anatomy of Vertebrate Animals*. I find that he first divides this branch into three provinces: the *Ichthyopsida*, the *Sauropsida*,†P1 and the *Mammalia*. He divides each Province into Classes.†1

Peirce: CP 1.573 Cross-Ref:††
BOOK IV

THE NORMATIVE SCIENCES

CHAPTER 1

INTRODUCTION†1

573. *Normative Science*†2 forms the mid-portion of coenoscopy and its most characteristic part. . . . Logic, regarded from one instructive, though partial and narrow, point of view, is the theory of deliberate thinking. To say that any thinking is deliberate is to imply that it is controlled with a view to making it conform to a purpose or ideal. Thinking is universally acknowledged to be an active operation. Consequently, the control of thinking with a view to its conformity to a standard or ideal is a special case of the control of action to make it conform to a standard; and the theory of the former must be a special determination of the theory of the latter. Now special theories should always be made to rest upon the general theories of which they are amplifications. The present writer takes the theory of the control of conduct, and of action in general, so as to conform to an ideal, as being the midnormative science; that is, as the second of the trio, and as that one of the three sciences in which the distinctive characters of normative science are most strongly marked. He will not undertake to pronounce any other distribution of the matter of normative science to be wrong; but, according to the dissection of that matter which seems to him to separate studies as they must be separated in research, such will be the mid-normative science. Since the normative sciences are usually held to be three, Logic, Ethics, and [Esthetics], and since he, too, makes them three, he would term the mid-normative science ethics if this did not seem to be forbidden by the received acception of that term. He accordingly proposes to name the mid-normative science, as such (whatever its content may be) *antethics*, that is, that which is put in place of ethics, the usual second member of the trio. It is the writer's opinion that this *antethics* should be the theory of the conformity of action to an ideal. Its name, as

such, will naturally be *practics*. Ethics is not practics; first, because ethics involves more than the theory of such conformity; namely, it involves the theory of the ideal itself, the nature of the *summum bonum*; and secondly, because, in so far as *ethics* studies the conformity of conduct to an ideal, it is limited to a particular ideal, which, whatever the professions of moralists may be, is in fact nothing but a sort of composite photograph of the conscience of the members of the community. In short, it is nothing but a traditional standard, accepted, very wisely, without radical criticism, but with a silly pretence of critical examination. The science of morality, virtuous conduct, right-living, can hardly claim a place among the heuretic sciences.

Peirce: CP 1.574 Cross-Ref:††

574. It has been a great, but frequent, error of writers on ethics to confound an ideal of conduct with a motive to action. The truth is that these two objects belong to different categories. Every action has a motive; but an ideal only belongs to a line [of] conduct which is deliberate. To say that conduct is deliberate implies that each action, or each important action, is reviewed by the actor and that his judgment is passed upon it, as to whether he wishes his future conduct to be like that or not. His ideal is the kind of conduct which attracts him upon review. His self-criticism, followed by a more or less conscious resolution that in its turn excites a determination of his habit, will, with the aid of the sequelæ, *modify* a future action; but it will not generally be a moving cause to action. It is an almost purely passive liking for a way of doing whatever he may be moved to do. Although it affects his own conduct, and nobody else's, yet the quality of feeling (for it is merely a quality of feeling) is just the same, whether his own conduct or that of another person, real or imaginary, is the object of the feeling; or whether it be connected with the thought of any action or not. If conduct is to be thoroughly deliberate, the ideal must be a habit of feeling which has grown up under the influence of a course of self-criticisms and of hetero-criticisms; and the theory of the deliberate formation of such habits of feeling is what ought to be meant by *esthetics*.†1 It is true that the Germans, who invented the word, and have done the most toward developing the science, limit it to *taste*, that is, to the action of the *Spieltrieb* from which deep and earnest emotion would seem to be excluded. But in the writer's opinion the theory is the same, whether it be a question of forming a taste in bonnets or of a preference between electrocution and decapitation, or between supporting one's family by agriculture or by highway robbery. The difference of earnestness is of vast practical moment; but it has nothing to do with heuretic science.

Peirce: CP 1.574 Cross-Ref:††

According to this view, esthetics, practics, and logic form one distinctly marked whole, one separate department of heuretic science; and the question where precisely the lines of separation between them are to be drawn is quite secondary. It is clear, however, that esthetics relates to feeling, practics to action, logic to thought.

Peirce: CP 1.575 Cross-Ref:††

CHAPTER 2

ULTIMATE GOODS†1

575. It is pretty generally admitted that logic is a *normative* science, that is to say, it not only lays down rules which ought to be, but need not be followed; but it is the analysis of the conditions of attainment of something of which purpose is an essential ingredient. It is, therefore, closely related to an art; from which, however, it differs markedly in that its primary interest lies in understanding those conditions, and only secondarily in aiding the accomplishment of the purpose. Its business is analysis, or, as some writers prefer to say, definition.

Peirce: CP 1.575 Cross-Ref:††

The word *normative* was invented in the school of Schleiermacher.†2 The majority of writers who make use of it tell us that there are three normative sciences, logic, esthetics, and ethics, the doctrines of the true, the beautiful, and the good, a triad of ideals which has been recognized since antiquity. On the other hand, we quite commonly find the term "normative science" restricted to logic and ethics; and Schleiermacher himself states their purposes in a way that seems to give room for no third. The one, he says, relates to making thought conform to being, the other, to making being conform to thought. There seems to be much justice in this restriction. For that which renders logic and ethics peculiarly normative is that nothing can be either logically true or morally good without a purpose to be so. For a proposition, and especially the conclusion of an argument, which is only accidentally true is not logical. On the other hand, a thing is beautiful or ugly quite irrespective of any purpose to be so. It would seem, therefore, that esthetics is no more essentially normative than any nomological science. The science of optics, for example, might very well be regarded as the study of the conditions to be observed in making use of light. Under such a conception, nothing essential to optics would be omitted, nor anything foreign to it inserted. Those writers, however, who stand out for the trinity of normative sciences do so upon the ground that they correspond to three fundamental categories of objects of desire. As to that, the logician may be exempted from inquiring whether the beautiful is a distinct ideal or not; but he is bound to say how it may be with the true; and accordingly the intention of this chapter is to lay the foundation for the doctrine, which will appear more and more evident as we proceed, that that truth the conditions of which the logician endeavors to analyze, and which is the goal of the reasoner's aspirations, is nothing but a phase of the *summum bonum* which forms the subject of pure ethics, and that neither of those men can really understand himself until he perceives clearly that it is so.

Peirce: CP 1.576 Cross-Ref:††

576. I hope I shall not be thought to wander if I note one observation by the way, before formally settling down to the question. Were there nothing in reasoning more than the old traditional treatises set forth, then a rogue might be as good a reasoner as a man of honor; although a coward could not, even under such an idea of reasoning. But in induction a habit of probity is needed for success: a trickster is sure to play the confidence game upon himself. And in addition to probity, industry is essential. In the presumptive choice of hypotheses, still higher virtues are needed -- a true elevation of soul. At the very lowest, a man must prefer the truth to his own interest and well-being and not merely to his bread and butter, and to his own vanity, too, if he is to do much in science. This will appear in the logical discussion; and it is thoroughly borne out by examining the characters of scientific men and of great heuristic students of all kinds. It is a remarkable fact that, excluding idle tales about pre-socratic philosophers, all history does not tell of a single man who has considerably increased human knowledge (unless theology be knowledge) having

been proved a criminal. Of the four or five instances usually adduced, Seneca neither contributed to knowledge nor has been convicted of positive crime; Calvin was nothing but a theologian; the attacks upon Erasmus are beneath contempt; Bacon was no man of science, but only a grandiose writer, whose very style betrays him; Dr. Dodd was an ordinary commentator on the Bible; and nothing was proved against Libri. The same may be said of whispers that this or that naturalist purloined specimens in the interest of science. The lofty character of the true man of science, physical or psychical, finds not one exception among a hundred. But it is needless to go to history for cases in which relatively small obliquities have prevented eminent scientists from achieving higher successes; for they abound in the experience of everybody who knows the scientific world from within. If it were true that every fallacy were a sin, logic would be reduced to a branch of moral philosophy. This is not true. But we can perceive that good reasoning and good morals are closely allied; and I suspect that with the further development of ethics this relation will be found to be even more intimate than we can, as yet, prove it to be.

Peirce: CP 1.577 Cross-Ref:††

577. There is room for doubt whether ethics is correctly described as a normative branch of philosophy. The doctrine of rights and duties is practical rather than normative; and if we are to use the word philosophy, as I intend to do, for that part of science which rests upon so much of experience as presses in upon every man during every hour of his waking life, then it is plain that the doctrine of rights and duties, which makes heavy drafts upon wisdom, or the knowledge which comes by reflection upon the total experience of a lifetime, as well as upon a learned acquaintance with the structure of the society in which one lives, stretches far beyond the familiar ground of philosophy. But the doctrine of rights and duties is a mere superstructure upon ethics proper. This groundwork philosophy will never disavow; for it is her pride and boast, the one branch of her work in which during the last three centuries an indisputable, steady progress has been made, not put to shame by the achievements of the special sciences. I wish as much could be said of logic. Concerning what, then, have all those writers whose subtle and beautiful discussions have built up the science of ethics been mainly occupying themselves? Surely not casuistry, or the determination of what under given circumstances ought to or may be done. They have been largely busied with the analysis of conscience, which as a psychological problem, mainly, belongs among the special psychical sciences. But the more important subject of their deliberations has been, What is good? Now this is hardly a normative question: it is pre-normative. It does not ask for the conditions of fulfillment of a definitely accepted purpose, but asks what is to be sought, *not* for a reason, but back of every reason. Logic, as a true normative science, supposes the question of what is to be aimed at to be already answered before it could itself have been called into being. Pure ethics, philosophical ethics, is not normative, but pre-normative.

Peirce: CP 1.578 Cross-Ref:††

578. "If so, why this chapter?" I fear the reader will ask, and forthwith skip, as surplusage, the true life-germ of all the truths I have to unfold. "Never mind," you will say, "whether the aim which logic has in view is a good one, or not; as a matter of fact, we are interested in it. It is to learn the truth: no aim could be of more elementary simplicity. Let us turn to where we are told how to come to it." Well, if this aim is so readily comprehensible, suppose you tell me, to whom it does not seem so, what truth consists in. "Truth is the conformity of a representation to its object," says Kant.†1 One might make this statement more explicit; but for our present

purpose it may pass. It is nearly correct, so far as it is intelligible. Only, what is that "object" which serves to define truth? Why it is the *reality*: it is of such a nature as to be independent of representations of it, so that, taking any individual sign or any individual collection of signs (such, for example, as all the ideas that ever enter into a given man's head) there is some character which that thing possesses, whether that sign or any of the signs of that collection represents the thing as possessing that character or not. Very good: now only tell me what it means to say that an object possesses a character, and I shall be satisfied. But even now, in advance of our study of definition, [we can] sufficiently see that we can only reach a conception of the less known through the more known, and that consequently the only meaning which we can attach to the phrase that a thing "has a character" is that something is *true* of it. So there we are, after threading the passages of this labyrinth, already thrown out at that very conception of truth at which we entered it. Indeed, when one comes to consider it, how futile it was to imagine that we were to clear up the idea of *truth* by the more occult idea of *reality*!

Peirce: CP 1.579 Cross-Ref:††

579. Yet the logician will never be scientifically or safely equipped for his explorations until he knows precisely what it is that he is seeking. The whole doctrine of logic depends upon that to a degree one could hardly foresee. The best way will be to go back to the beginning and inquire what it is that we can be content to wish for independently of any ulterior result. For the discussions of the moralists, who have not had logic in view, are not altogether adequate for our needs. In this inquiry we are not to look for any [discussion] leading to psychology; for the anatomy and physiology of the mind, or of the brain, though they may furnish a hint now and then, can after all not tell us that anything is desirable, except for some reason; while what we wish to know is what is desirable without any reason. Psychology might, it is true, discover that there is no way whatever in [which] certain things could become objects of desire; but it can only make such a discovery by relying upon direct self-questioning, as [to] what we do or do not desire, and such premisses of psychology are here precisely the conclusions of which we are in quest. So we must make up our minds to rely entirely upon self-questioning, with here and there perhaps some secondary aid from psychology.

Peirce: CP 1.580 Cross-Ref:††

580. Such self-questioning produces no infallible response. On the contrary, consciousness may be set down as one of the most mendacious witnesses that ever was questioned. But it is the only witness there is; and all we can do is to put it in the sweat-box and torture the truth out of it, with such judgment as we can command.

Peirce: CP 1.581 Cross-Ref:††

581. What I propose now to do is to pass in review every one of the general classes of objects which anybody could suppose to be an ultimate good, and to question consciousness, first, as to whether or not each of these in turn could content us as the sole ultimate good independently of any ulterior result, and if not, whether it can be considered to be in itself a good at all, irrespective of its effects. I shall arrange my list so as to commence with the most particular satisfactions and proceed step by step to the most general. But since there are in each grade several kinds of satisfactions, I shall begin in each grade of generality with the most immediate and selfish and go on by steps to the most subservient.

Peirce: CP 1.582 Cross-Ref:††

582. I begin, then, with simple satisfactions of the moment. The most

immediate of these is the simple satisfaction of a direct instinct. I am thirsty and I want a drink. Now our shifty witness, consciousness, is very ready with her answer that a drink is good but that momentary satisfactions are not the only good. Let us not be in haste to accept either answer. Men might easily argue -- indeed, do argue -- that there can be no other good than the satisfaction of the moment's desire. But the moment I hear that word *can* used, where nothing in the world is pertinent but observations of what is, I cast the judgment aside as worthless. For "cannot be" means "not in accordance with a hypothetical construction" intended, this time, to represent human nature. But I do not care about hypothetical constructions. I just want to know whether a man *does* ever find any other satisfaction than the simple satisfaction of the moment. If there is nothing good but the simple satisfaction of this moment, and all other moments are nothing, I must be in a state of perfect satisfaction or perfect dissatisfaction. Is that so? Obviously not: I may wish for something in spite of accompanying disadvantages. Therefore, the simple satisfaction of the moment is not all. There is at least complexity. Now can the simple satisfaction of the moment be, in itself, any good at all? Here consciousness is emphatic in her reply that the drink is good, however small a good. But there can be no harm in a little cross-examination of the witness. An absolutely simple satisfaction will involve no comparison, no measure, no degree. It will be perfect, if it exists at all. Now let be supposed that it could be proved to you that, I will not say for a moment only, but for the entire duration of a millionth of a second, you were to enjoy a simple satisfaction, say that of an agreeable color sensation, with no effects whatever of any kind, and of course no memory of it. Then, since this satisfaction would be perfect and immeasurable, and would be, O Consciousness, you say, a good, at how much would you value it? How many years of purgatory would you be willing to endure for the sake of it? Come, speak up. Would you endure five minutes of toothache? For the *knowledge* that you had, or were about to have, the strange experience, perhaps. But this would be an effect. You must suppose that you were to be utterly ignorant of whether you had, or were about to have, any such feeling. Would it not be precisely the same thing as if this had happened to some other being, say to a mosquito, with this difference, that the mosquito is your neighbor, with whom you have some grain of sympathy while this isolated instant would really have no existence at all? I think I hear you murmur that an absolutely simple satisfaction would be an absurdity. Then such satisfaction is no part of the good. Still, it might be said that this result is owing to the absurd hypothesis of simplicity.

Peirce: CP 1.583 Cross-Ref:††

583. Let us, then, next consider whether the perfect and prompt satisfaction of every instinct is the only ultimate good. Though I cannot recall it at all, I think it very likely that in my childhood I read of a malevolent fairy who pronounced this curse upon an infant that during his entire life whatever wish he should conceive should be instantly gratified. If he wished for a drink, a drink should be instantly before him. If he wished it to taste differently, it should taste differently. If he was tired of sloth and wished he were working, he should be putting forth his strength. Only two things would be debarred. The first restriction should be that vague dissatisfaction, mere ennui at having his own way, should not be enough. He must form a definite wish. And secondly, wishing that his wishes should not be gratified should only be gratified until he made a positive wish. The instant he did that the satisfaction should come. I do not think that consciousness herself could have the face to pronounce this state of things good. The only alleviation of it would be the state of imbecility in which the person would be engulfed. Now I ask whether, in view of this, the mere satisfaction of an impulse can be said to be *per se* a good, at all. Under certain circumstances, the

satisfaction may be good, no doubt; but is it so *per se* and *simpliciter*? Here again, we find ourselves contemplating an absurdity. A satisfaction cannot be divorced from its circumstances. It results then that the gratification of an instinct is not *per se* any part of the good. Still, it might be said that this is only because we have supposed an impossible situation in which there was no providing for future desires.

Peirce: CP 1.584 Cross-Ref:††

584. We will, therefore, next consider whether provision for satisfying future instinctive desires is the only ultimate good, and if not, whether it is, in itself, a good at all. It will here be pertinent to remark that although the state of things last imagined would not be good for a man, yet it does not seem to differ much from the conditions under which my dog, and mare, and poultry seem to be enviably happy. Still, perhaps there may be a delusion here. The dog, I can perceive, considers that there is a heavy weight of responsibility upon him, and so do the mature poultry. Even the mare is not without this feeling; and perhaps she is not so entirely happy as the others. If there were an idiot about the place, could we regard it as an ultimate good that he should thus have all his instincts gratified? I think not. If it would produce a state of content in the poor fellow's mind, and if for any reason that were a good, then, for the sake of that effect, it is undeniable that the state of things supposed would be good: but that would not make it an ultimate good; on the contrary, it would furnish a *reason* for the sake of which it would be good. These considerations are extremely pertinent to the case we are now to consider, which is that of a person engaged busily in providing for his next day's wants, with just enough uncertainty as to his probable success to keep industrious. If, for any reason, or without an ulterior reason, it is desirable that he should be happy, and if his mental development is so low that those conditions would make him happy, as possibly they might make some creatures zoölogically human, then of course that would furnish an end as a means to which the state supposed would be good. But how is it with you, my Consciousness? Would you think it was reason enough for the creation of heaven and earth that it put you, or any other individual, into this condition of working for your living?†1

Peirce: CP 1.585 Cross-Ref:††
CHAPTER 3

AN ATTEMPTED CLASSIFICATION OF ENDS†1

585. In the *Popular Science Monthly* for January, 1901, (LVIII *et seq.*)†2 I enumerated a number of ethical classes of motives, meaning by a motive, not a spring of action, but an aim or end appearing ultimate to the agent. Any such classification may be rendered more minute by subdivisions, or broader by aggregations of classes. My endeavor was to make my enumeration about evenly specific throughout. Upon a reexamination of it, it appears to me to be sufficiently complete and systematic to afford a tolerable material to be cut up, worked over, and amplified into a satisfactory classification of ends. It is in the hope that others may be moved to interest themselves in this work and complete it, or help to complete it, that I now give an improved statement of it.

Peirce: CP 1.585 Cross-Ref:††

This statement will be facilitated and made clearer by a notation which is designed to show what the essential elements of the different ends appear to me to be.

Peirce: CP 1.586 Cross-Ref:††

586. A. A man may act in a quasi-hypnotic response to an instant command. I indicate this by the letter A.

Peirce: CP 1.586 Cross-Ref:††

B. A man may act from obedience, although not to a concrete command. I indicate this by B. In this case, he may still act as purely on the impulse of the moment as in case A. Only if he does so, while still acting from pure obedience, not from any impulse of his own, it must be a Mrs. Grundy, *a vague personification of the community*, which he obeys. I will indicate an end into which such personification enters as an element by a letter z following the capital letter.

Peirce: CP 1.586 Cross-Ref:††

Is there any way in which a man can act from pure obedience when there is no concrete command without the element z? Undoubtedly, provided he acts in obedience to a law. I will indicate that an end involves a conscious reference to a law, or general reason, by writing the figure 1 before the capital letter. We find, then, under B,

Peirce: CP 1.586 Cross-Ref:††

Bz. Acting under dread of Mrs. Grundy, without generalizing her dictum.

Peirce: CP 1.586 Cross-Ref:††

1B. Acting under awe of a law, without criticizing its obligation.

Peirce: CP 1.586 Cross-Ref:††

But cannot the elements 1 and z be combined? Cannot a man act under the influence of a vague personification of the community and yet according to a general rule of conduct? Certainly: he so acts when he conforms to custom. Only if it is mere custom and not law, it is not a case of obedience, but of *conformity to norm*, or exemplar. (I never use the word *norm* in the sense of a precept, but only in that of a pattern which is copied, this being the original metaphor.) I indicate an end which presents a norm to be conformed to by the capital letter C.

Peirce: CP 1.586 Cross-Ref:††

Conformity to a norm may take place by an immediate impulse. It then becomes instinctive imitation. But here the man does not vaguely personify the community, but puts himself in the shoes of another person, as we say. I call this putting of oneself in another's place, *retroconsciousness*. I indicate that an end essentially involves retroconsciousness by writing the letter y after the capital.

Peirce: CP 1.586 Cross-Ref:††

Conformity to a norm may also take place without either the y or the z element. Only in this case the norm must be a definite ideal which is regarded as in itself {kalos k' agathos}. I indicate an end which essentially involves the recognition of a definite ideal as universally and absolutely desirable by putting the figure 2 before the capital. Under C, then, we have the following cases:

Cy. Instinctive imitation.

1Cz. Conformity to custom.

2C. Conformity to the {kalos k' agathos}, unanalyzed.

Peirce: CP 1.587 Cross-Ref:††

587. The elements 1 and y can be combined. That is to say, a man may act from putting himself in another's place and according to a general reason furnished by that retroconsciousness. That is, he acts for the sake of that other's welfare. The object need not be a person: an estate or a plant can be treated with the same affection. But this is no longer conformity to a norm; it is *devotion* to somebody or something.

Peirce: CP 1.588 Cross-Ref:††

588. In like manner, the elements 2 and z may be combined. That is to say, a man's ultimate end may lie in a vague personification of the community and at the same time may contemplate a definite general state of things as the *summum bonum*. That is, his heart may be set upon the welfare and safety of the community. But this again is devotion, not conformity to a norm. An end the adoption of which involves devotion shall be indicated by the capital letter D.

Peirce: CP 1.588 Cross-Ref:††

Devotion may operate in a momentary impulse. In that case, the agent does not put himself in the place of the object, -- for that, without reflection, results merely †1

Peirce: CP 1.589 Cross-Ref:††

589. †2 All these distinctions would be embraced by some such scheme as the following:

Peirce: CP 1.589 Cross-Ref:††

I. The end is to superinduce upon feeling a certain quality, pleasure.

Peirce: CP 1.589 Cross-Ref:††

II. The end is to extend the existence of a subject.

1. Of something psychical, as a soul;

2. Of something physical, as a race.

Peirce: CP 1.589 Cross-Ref:††

III. The end is to realize a general ideal.

1. To bring about some general state of feeling, such as the greatest pleasure of the greatest number of persons;

2. To impress a definite subject with a definite character.

(a) This character being inward, such as altruistic sentiment;

(b) This character being outward, such as the peace and prosperity of mankind.

3. To further the realization of an ideal not definable in advance, otherwise than as that which tends to realize itself in the long run, or in some such way.

(a) This ideal being supposed to be of the inward type;

(b) This ideal being supposed to be of the outward type;

(c) This ideal being purely methodical, and thus equally capable of inward and of outward realization.

Peirce: CP 1.590 Cross-Ref:††

590. The most serious defect of this classification lies in its subdivision of rationalistic theory into only two main branches splitting upon the insignificant question of whether the end is completely attainable or not. The truth is that there have been three grand classes of rationalistic moralists who have differed from one another upon the much more important question of the mode of being of the end. Namely, there have been those who have made the end purely subjective, a feeling of pleasure; there have been those who have made the end purely objective and material, the multiplication of the race; and finally there have been those who have attributed to the end the same kind of being that a law of nature has, making it lie in the rationalization of the universe.†1

Peirce: CP 1.591 Cross-Ref:††
CHAPTER 4

IDEALS OF CONDUCT†1

591. Every man has certain ideals of the general description of conduct that befits a rational animal in his particular station in life, what most accords with his total nature and relations. If you think this statement too vague, I will say, more specifically, that there are three ways in which these ideals usually recommend themselves and justly do so. In the first place certain kinds of conduct, when the man contemplates them, have an esthetic quality. He thinks that conduct fine; and though his notion may be coarse or sentimental, yet if so, it will alter in time and must tend to be brought into harmony with his nature. At any rate, his taste *is* his taste for the time being; that is all. In the second place, the man endeavors to shape his ideals into consistency with each other, for inconsistency is odious to him. In the third place, he imagines what the consequences of fully carrying out his ideals would be, and asks himself what the esthetic quality of those consequences would be.

Peirce: CP 1.592 Cross-Ref:††

592. These ideals, however, have in the main been imbibed in childhood. Still, they have gradually been shaped to his personal nature and to the ideas of his circle of society rather by a continuous process of growth than by any distinct acts of thought. Reflecting upon these ideals, he is led to *intend* to make his own conduct conform at least to a part of them -- to that part in which he thoroughly believes. Next, he usually formulates, however vaguely, certain *rules of conduct*. He can hardly help doing so. Besides, such rules are convenient and serve to minimize the effects of future inadvertence and, what are well-named, the wiles of the devil within him. Reflection upon these rules, as well as upon the general ideals behind them, has a certain effect upon his disposition, so that what he naturally inclines to do becomes modified. Such

being his condition, he often foresees that a special occasion is going to arise; thereupon, a certain gathering of his forces will begin to work and this working of his being will cause him to consider how he will act, and in accordance with his disposition, such as it now is, he is led to form a *resolution* as to how he will act upon that occasion. This resolution is of the nature of a plan; or, as one might almost say, a *diagram*. It is a mental formula always more or less general. Being nothing more than an idea, this resolution does not necessarily influence his conduct. But now he sits down and goes through a process similar to that of impressing a lesson upon his memory, the result of which is that the *resolution*, or mental formula, is converted into a *determination*, by which I mean a really efficient agency, such that if one knows what its special character is, one can *forecast* the man's conduct on the special occasion. One cannot make forecasts that will come true in the majority of trials of them by means of any figment. It must be by means of something true and real.

Peirce: CP 1.593 Cross-Ref:††

593. We do not know by what machinery the conversion of a resolution into a determination is brought about. Several hypotheses have been proposed; but they do not much concern us just now. Suffice it to say that the determination, or efficient agency, is something hidden in the depths of our nature. A peculiar quality of feeling accompanies the first steps of the process of forming this impression; but later we have no direct consciousness of it. We may become aware of the disposition, especially if it is pent up. In that case, we shall recognize it by a feeling of *need*, of *desire*. I must notice that a man does not always have an opportunity to form a definite resolution beforehand. But in such cases there are less definite but still well-marked determinations of his nature growing out of the general rules of conduct that he has formulated; or in case no such appropriate rule has been formulated, his ideal of fitting conduct will have produced some disposition. At length, the anticipated occasion actually arises.

Peirce: CP 1.594 Cross-Ref:††

594. In order to fix our ideas, let us suppose a case. In the course of my reflexions, I am led to think that it would be well for me to talk to a certain person in a certain way. I resolve that I will do so when we meet. But considering how, in the heat of conversation, I might be led to take a different tone, I proceed to impress the resolution upon my soul; with the result that when the interview takes place, although my thoughts are then occupied with the matter of the talk, and may never revert to my resolution, nevertheless the determination of my being does influence my conduct. All action in accordance with a determination is accompanied by a feeling that is pleasurable; but, whether the feeling at any instant is felt as pleasurable in that very instant or whether the recognition of it as pleasurable comes a little later is a question of fact difficult to make sure about.

Peirce: CP 1.595 Cross-Ref:††

595. The argument turns on the feeling of pleasure, and therefore it is necessary, in order to judge of it, to get at the facts about that feeling as accurately as we can. In beginning to perform any series of acts which had been determined upon beforehand, there is a certain sense of joy, an anticipation and commencement of a relaxation of the tension of need, which we now become more conscious of than we had been before. In the act itself taking place at any instant, it may be that we are conscious of pleasure; although that is doubtful. Before the series of acts are done, we already begin to review them, and in that review we recognize the pleasurable character of the feelings that accompanied those acts.

Peirce: CP 1.596 Cross-Ref:††

596. To return to my interview, as soon as it is over I begin to review it more carefully and I then ask myself whether my conduct accorded with my resolution. That resolution, as we agreed, was a mental formula. The memory of my action may be roughly described as an image. I contemplate that image and put the question to myself. Shall I say that that image satisfies the stipulations of my resolution, or not? The answer to this question, like the answer to any inward question, is necessarily of the nature of a mental formula. It is accompanied, however, by a certain quality of feeling which is related to the formula itself very much as the color of the ink in which anything is printed is related to the sense of what is printed. And just as we first become aware of the peculiar color of the ink and afterward ask ourselves whether it is agreeable or not, so in formulating the judgment that the image of our conduct does satisfy our previous resolution we are, in the very act of formulation, aware of a certain quality of feeling, the feeling of satisfaction -- and directly afterward recognize that that feeling was pleasurable.

Peirce: CP 1.597 Cross-Ref:††

597. But now I may probe deeper into my conduct, and may ask myself whether it accorded with my general intentions. Here again there will be a judgment and a feeling accompanying it, and directly afterward a recognition that that feeling was pleasurable or painful. This judgment, if favorable, will probably afford less intense pleasure than the other; but the feeling of satisfaction which is pleasurable will be different and, as we say, a *deeper* feeling.

Peirce: CP 1.598 Cross-Ref:††

598. I may now go still further and ask how the image of my conduct accords with my ideals of conduct fitting to a man like me. Here will follow a new judgment with its accompanying feeling followed by a recognition of the pleasurable or painful character of that feeling. In any or all of these ways a man may criticize his own conduct; and it is essential to remark that it is not mere idle praise or blame such as writers who are not of the wisest often distribute among the personages of history. No indeed! It is approval or disapproval of the only respectable kind, that which will bear fruit in the future. Whether the man is satisfied with himself or dissatisfied, his nature will absorb the lesson like a sponge; and the next time he will tend to do better than he did before.

Peirce: CP 1.599 Cross-Ref:††

599. In addition to these three self-criticisms of single series of actions, a man will from time to time review his *ideals*. This process is not a job that a man sits down to do and has done with. The experience of life is continually contributing instances more or less illuminative. These are digested first, not in the man's consciousness, but in the depths of his reasonable being. The results come to consciousness later. But meditation seems to agitate a mass of tendencies and allow them more quickly to settle down so as to be really more conformed to what is fit for the man.

Peirce: CP 1.600 Cross-Ref:††

600. Finally, in addition to this personal meditation on the fitness of one's own ideals, which is of a practical nature, there are the purely theoretical studies of the student of ethics who seeks to ascertain, as a matter of curiosity, what the *fitness* of an ideal of conduct consists in, and to deduce from such definition of fitness what conduct ought to be. Opinions differ as to the wholesomeness of this study. It only concerns our present purpose to remark that it is in itself a purely theoretical inquiry,

entirely distinct from the business of shaping one's own conduct. Provided that feature of it be not lost sight of, I myself have no doubt that the study is more or less favorable to right living.

Peirce: CP 1.601 Cross-Ref:††

601. I have thus endeavored to describe fully the typical phenomena of controlled action. ***They are not every one present in every case.*** Thus, as I have already mentioned, there is not always an opportunity to form a resolution. I have specially emphasized the fact that conduct is determined by what precedes it in time, while the recognition of the pleasure it brings follows after the action. Some may opine that this is not true of what is called the pursuit of pleasure; and I admit that there is room for their opinion while I myself incline to think, for example, that the satisfaction of eating a good dinner is never a satisfaction in the present instantaneous state, but always follows after it. I insist, at any rate, that a ***feeling***, as a mere appearance, can have no real power in itself to produce any effect whatever, however indirectly.

Peirce: CP 1.602 Cross-Ref:††

602. My account of the facts, you will observe, leaves a man at full liberty, no matter if we grant all that the necessitarians ask. That is, the man ***can***, or if you please is ***compelled***, to ***make his life more reasonable***. What other distinct idea than that, I should be glad to know, can be attached to the word liberty?

Peirce: CP 1.603 Cross-Ref:††

603. Now let us compare the facts I have stated with the argument I am opposing. That argument rests on two main premisses; first, that it is unthinkable that a man should act from any other motive than pleasure, if his act be deliberate; and second, that action with reference to pleasure leaves no room for any distinction of right and wrong.

Peirce: CP 1.604 Cross-Ref:††

604. Let us consider whether this second premiss is really true. What would be requisite in order to destroy the difference between innocent and guilty conduct? The one thing that would do it would be to destroy the faculty of effective self-criticism. As long as that remained, as long as a man compared his conduct with a preconceived standard and that effectively, it need not make much difference if his only ***real*** motive were pleasure; for it would become disagreeable to him to incur the sting of conscience. But those who deluded themselves with that fallacy were so inattentive to the phenomena that they confused the judgment, after the act, that that act satisfied or did not satisfy the requirements of a standard, with a pleasure or pain accompanying the act itself.

Peirce: CP 1.605 Cross-Ref:††

605. Let us now consider whether the other premiss is true, that it is unthinkable that a man should act deliberately except for the sake of pleasure. What is the element which it is in truth unthinkable that deliberate action should lack? It is simply and solely the determination. Let his determination remain, as it is certainly conceivable that it should remain, although the very nerve of pleasure were cut so that the man were perfectly insensible to pleasure and pain, and he will certainly pursue the line of conduct upon which he is intent. The only effect would be to render the man's intentions more inflexible -- an effect, by the way, which we often have occasion to observe in men whose feelings are almost deadened by age or by some derangement of the brain. But those who have reasoned in this fallacious way have

confounded together the determination of the man's nature, which is an efficient agency prepared previously to the act, with the comparison of conduct with a standard, which comparison is a general mental formula subsequent to the act, and, having identified these two utterly different things, placed them in the act itself as a mere quality of feeling.

Peirce: CP 1.606 Cross-Ref:††

606. Now if we recur to the defendant argument about reasoning, we shall find that it involves the same sort of tangle of ideas. The phenomena of reasoning are, in their general features, parallel to those of moral conduct. For reasoning is essentially thought that is under self-control, just as moral conduct is conduct under self-control. Indeed reasoning is a species of controlled conduct and as such necessarily partakes of the essential features of controlled conduct. If you attend to the phenomena of reasoning, although they are not quite so familiar to you as those of morals because there are no clergymen whose business it is to keep them before your minds, you will nevertheless remark, without difficulty, that a person who draws a rational conclusion, not only thinks it to be true, but thinks that similar reasoning would be just in every analogous case. If he fails to think this, the inference is not to be called reasoning. It is merely an idea suggested to his mind and which he cannot resist thinking is true. But not having been subjected to any check or control, it is not deliberately approved and is not to be called reasoning. To call it so would be to ignore a distinction which it ill becomes a rational being to overlook. To be sure, every inference forces itself upon us irresistibly. That is to say, it is irresistible at the instant it first suggests itself. Nevertheless, we all have in our minds certain *norms*, or general patterns of right reasoning, and we can compare the inference with one of those and ask ourselves whether it satisfies that rule. I call it a rule, although the formulation may be somewhat vague; because it has the essential character of a rule of being a general formula applicable to particular cases. If we judge our norm of right reason to be satisfied, we get a feeling of approval, and the inference now not only appears as irresistible as it did before, but it will prove far more unshakable by any doubt.

Peirce: CP 1.607 Cross-Ref:††

607. You see at once that we have here all the main elements of moral conduct; the general standard mentally conceived beforehand, the efficient agency in the inward nature, the act, the subsequent comparison of the act with the standard. Examining the phenomena more closely we shall find that not a single element of moral conduct is unrepresented in reasoning. At the same time, the special case naturally has its peculiarities.

Peirce: CP 1.608 Cross-Ref:††

608. Thus, we have a general ideal of sound logic. But we should not naturally describe it as our idea of the kind of reasoning that befits men in our situation. How should we describe it? How if we were to say that sound reasoning is such reasoning that in every conceivable state of the universe in which the facts stated in the premisses are true, the fact stated in the conclusion will thereby and therein be true. The objection to this statement is that it only covers necessary reasoning, including reasoning about chances. There is other reasoning which is defensible as probable, in the sense that while the conclusion may be more or less erroneous, yet the same procedure diligently persisted in must, in every conceivable universe in which it leads to any result at all, lead to a result indefinitely approximating to the truth. When that is the case, we shall do right to pursue that method, provided we recognize its true

character, since our relation to the universe does not permit us to have any necessary knowledge of positive facts. You will observe that in such a case our ideal is shaped by the consideration of our situation relatively to the universe of existences. There are still other operations of the mind to which the name "reasoning" is especially appropriate, although it is not the prevailing habit of speech to call them so. They are conjectures, but rational conjectures; and the justification of them is that unless a man had a tendency to guess right, unless his guesses are better than tossing up a copper, no truth that he does not already virtually possess could ever be disclosed to him, so that he might as well give up all attempt to reason; while if he has any decided tendency to guess right, as he *may* have, then no matter how often he guesses wrong, he will get at the truth at last. These considerations certainly do take into account the man's inward nature as well as his outward relations; so that the ideals of good logic are truly of the same general nature as ideals of fine conduct. We saw that three kinds of considerations go to support ideals of conduct. They were, first, that certain conduct seems fine in itself. Just so certain conjectures seem likely and easy in themselves. Secondly, we wish our conduct to be consistent. Just so the ideal [of] necessary reasoning is consistency simply. Third, we consider what the general effect would be of thoroughly carrying out our ideals. Just so certain ways of reasoning recommend themselves because if persistently carried out they must lead to the truth. The parallelism, you perceive, is almost exact.

Peirce: CP 1.609 Cross-Ref:††

609. There is also such a thing as a general logical *intention*. But it is not emphasized for the reason that the will does not enter so violently into reasoning as it does into moral conduct. I have already mentioned the logical norms, which correspond to moral laws. In taking up any difficult problem of reasoning we formulate to ourselves a logical resolution; but here again, because the will is not at such high tension in reasoning as it often is in self-controlled conduct, these resolutions are not very prominent phenomena. Owing to this circumstance, the efficient determination of our nature, which causes us to reason in each case as we do, has less relation to resolutions than to logical norms. The act itself is, at the instant, irresistible in both cases. But immediately after, it is subjected to self-criticism by comparison with a previous standard which is always the norm, or *rule*, in the case of reasoning, although in the case of outward conduct we are too often content to compare the act with the resolution. In the case of general conduct, the lesson of satisfaction or dissatisfaction is frequently not much taken to heart and little influences future conduct. But in the case of reasoning an inference which self-criticism disapproves is always instantly annulled, because there is no difficulty in doing this. Finally, all the different feelings which, as we noticed, accompanied the different operations of self-controlled conduct equally accompany those of reasoning, although they are not quite so vivid.

Peirce: CP 1.610 Cross-Ref:††

610. The parallelism is thus perfect. Nor, I repeat, could it fail to be so, if our description of the phenomena of controlled conduct was true, since reasoning is only a special kind of controlled conduct.

Peirce: CP 1.611 Cross-Ref:††

611. What does right reasoning consist in? It consists in such reasoning as shall be conducive to our ultimate aim. What, then, is our ultimate aim? Perhaps it is not necessary that the logician should answer this question. Perhaps it might be possible to deduce the correct rules of reasoning from the mere assumption that we

have some ultimate aim. But I cannot see how this could be done. If we had, for example, no other aim than the pleasure of the moment, we should fall back into the same absence of any logic that the fallacious argument would lead to. We should have no ideal of reasoning, and consequently no norm. It seems to me that the logician ought to recognize what our ultimate aim is. It would seem to be the business of the moralist to find this out, and that the logician has to accept the teaching of ethics in this regard. But the moralist, as far as I can make it out, merely tells us that we have a power of self-control, that no narrow or selfish aim can ever prove satisfactory, that the only satisfactory aim is the broadest, highest, and most general possible aim; and for any more definite information, as I conceive the matter, he has to refer us to the esthetician, whose business it is to say what is the state of things which is most admirable in itself regardless of any ulterior reason.

Peirce: CP 1.612 Cross-Ref:††

612. So, then, we appeal to the esthete to tell us what it is that is admirable without any reason for being admirable beyond its inherent character. Why, that, he replies, is the beautiful. Yes, we urge, such is the name that you give to it, but what *is it*? What is this character? If he replies that it consists in a certain quality of feeling, a certain *bliss*, I for one decline altogether to accept the answer as sufficient. I should say to him, My dear Sir, if you can prove to me that this quality of feeling that you speak of does, as a fact, attach to what you call the beautiful, or that which would be admirable without any reason for being so, I am willing enough to believe you; but I cannot without strenuous proof admit that any particular quality of feeling is admirable without a reason. For it is too revolting to be believed unless one is forced to believe it.

Peirce: CP 1.613 Cross-Ref:††

613. A fundamental question like this, however practical the issues of it may be, differs entirely from any ordinary practical question, in that whatever is accepted as good in itself must be accepted without compromise. In deciding any special question of conduct it is often quite right to allow weight to different conflicting considerations and calculate their resultant. But it is quite different in regard to that which is to be the aim of all endeavor. The object admirable that is admirable *per se* must, no doubt, be general. Every ideal is more or less general. It may be a complicated state of things. But it must be a *single* ideal; it must have *unity*, because it is an idea, and unity is essential to every idea and every ideal. Objects of utterly disparate kinds may, no doubt, be admirable, because some special reason may make each one of them so. But when it comes to the ideal of the admirable, in itself, the very nature of its being is to be a precise idea; and if somebody tells me it is either this, or that, or that other, I say to him, It is clear you have no *idea* of what precisely it is. But an ideal must be capable of being embraced in a unitary idea, or it is no ideal at all. Therefore, there can be no compromises between different considerations here. The admirable ideal cannot be too extremely admirable. The more thoroughly it has whatever character is essential to it, the more admirable it must be.

Peirce: CP 1.614 Cross-Ref:††

614. Now what would the doctrine that that which is admirable in itself is a quality of feeling come to if taken in all its purity and carried to its furthest extreme -- which should be the extreme of admirableness? It would amount to saying that the one ultimately admirable object is the unrestrained gratification of a desire, regardless of what the nature of that desire may be. Now that is too shocking. It would be the doctrine that all the higher modes of consciousness with which we are acquainted in

ourselves, such as love and reason, are good only so far as they subserve the lowest of all modes of consciousness. It would be the doctrine that this vast universe of Nature which we contemplate with such awe is good only to produce a certain quality of feeling. Certainly, I must be excused for not admitting that doctrine unless it be proved with the utmost evidence. So, then, what proof is there that it is true? The only reason for it that I have been able to learn is that *gratification, pleasure*, is the only conceivable result that is satisfied with itself; and therefore, since we are seeking for that which is *fine* and *admirable* without any reason beyond itself, *pleasure, bliss*, is the only object which can satisfy the conditions. This is a respectable argument. It deserves consideration. Its premiss, that pleasure is the only conceivable result that is perfectly self-satisfied, must be granted. Only, in these days of evolutionary ideas which are traceable to the French Revolution as their instigator, and still further back to Galileo's experiment at the leaning tower of Pisa, and still further back to all the stands that have been made by Luther and even by Robert of Lincoln against attempts to bind down human reason to any prescriptions fixed in advance -- in these days, I say, when these ideas of progress and growth have themselves grown up so as to occupy our minds as they now do, how can we be expected to allow the assumption to pass that the admirable in itself is any stationary result? The explanation of the circumstance that the only result that is satisfied with itself is a quality of feeling is that reason always looks forward to an endless future and expects endlessly to improve its results.

Peirce: CP 1.615 Cross-Ref: ††

615. Consider, for a moment, what Reason, as well as we can today conceive it, really is. I do not mean man's faculty which is so called from its embodying in some measure Reason, or {Nous}, as a something manifesting itself in the mind, in the history of mind's development, and in nature. What is this Reason? In the first place, it is something that never can have been completely embodied. The most insignificant of general ideas always involves conditional predictions or requires for its fulfillment that events should come to pass, and all that ever can have come to pass must fall short of completely fulfilling its requirements. A little example will serve to illustrate what I am saying. Take any general term whatever. I say of a stone that it is *hard*. That means that so long as the stone remains hard, every essay to scratch it by the moderate pressure of a knife will surely fail. To call the stone *hard* is to predict that no matter how often you try the experiment, it will fail every time. That innumerable series of conditional predictions is involved in the meaning of this lowly adjective. Whatever may have been done will not begin to exhaust its meaning. At the same time, the very being of the General, of Reason, is of such a mode that this being *consists* in the Reason's actually governing events. Suppose a piece of carborundum has been made and has subsequently been dissolved in aqua regia without anybody at any time, so far as I know, ever having tried to scratch it with a knife. Undoubtedly, I may have good reason, nevertheless, to call it hard; because some actual fact has occurred such that Reason compels me to call it so, and a general idea of all the facts of the case can only be formed if I do call it so. In this case, my calling it hard is an actual event which is governed by that law of hardness of the piece of carborundum. But if there were no actual fact whatsoever which was meant by saying that the piece of carborundum was hard, there would be not the slightest meaning in the word hard as applied to it. The very being of the General, of Reason, *consists* in its governing individual events. So, then, the essence of Reason is such that its being never can have been completely perfected. It always must be in a state of incipiency, of growth. It is like the character of a man which consists in the ideas that he will conceive and in the efforts that he will make, and which only develops as the occasions actually

arise. Yet in all his life long no son of Adam has ever fully manifested what there was in him. So, then, the development of Reason requires as a part of it the occurrence of more individual events than ever can occur. It requires, too, all the coloring of all qualities of feeling, including pleasure in its proper place among the rest. This development of Reason consists, you will observe, in embodiment, that is, in manifestation. The creation of the universe, which did not take place during a certain busy week, in the year 4004 B.C., but is going on today and never will be done, is this very development of Reason. I do not see how one can have a more satisfying ideal of the admirable than the development of Reason so understood. The one thing whose admirableness is not due to an ulterior reason is Reason itself comprehended in all its fullness, so far as we can comprehend it. Under this conception, the ideal of conduct will be to execute our little function in the operation of the creation by giving a hand toward rendering the world more reasonable whenever, as the slang is, it is "up to us" to do so. In logic, it will be observed that knowledge is reasonableness; and the ideal of reasoning will be to follow such methods as must develop knowledge the most speedily. . . .

Peirce: CP 1.616 Cross-Ref: ††
CHAPTER 5

VITALLY IMPORTANT TOPICS

§1. THEORY AND PRACTICE †1

616. The early Greek philosopher, such as we read about in Diogenes Laertius, is certainly one of the most amusing curiosities of the whole human menagerie. It seems to have been demanded of him that his conduct should be in marked contrast with the dictates of ordinary common sense. Had he behaved as other men are supposed to do his fellow-citizens would have thought his philosophy had not taught him much. I know that historians possessed of "higher criticism" deny all the ridiculous anecdotes about the Hellenic sages. These scholars seem to think that logic is a question of literary taste, and their refined perceptions refuse to accept these narratives. But in truth even were taste carried to a point of delicacy exceeding that of the German professor -- which he would think was pushing it quite into that realm of imaginary quantities which lies on the other side of infinity -- it still would not weigh as logic, which is a matter of strict mathematical demonstration wherein opinion is of no weight at all.

Peirce: CP 1.617 Cross-Ref: ††

617. Now scientific logic cannot approve that historical method which leads to the absolute and confident denial of all the positive testimony that is extant, the moment that testimony deviates from the preconceived ideas of the historian. The story about Thales falling into the ditch while pointing out the different stars to the old woman is told by Plato †2 about two centuries later. But Dr. Edouard Zeller †3

says he knows better, and pronounces the occurrence quite impossible. Were you to point out that the anecdote only attributes to Thales a character common to almost all mathematicians, this would afford him a new opportunity of applying his favorite argument of objection, that the story is "too probable." So the assertion of half a dozen classical writers that Democritus was always laughing and Heraclitus always weeping "proclaims itself," says Zeller, "an idle fabrication,"†1 notwithstanding the supports it receives from the fragments. Even Zeller admits that Diogenes of Sinope was a trifle eccentric. Being a contemporary of Aristotle and one of the best-known men of Greece, his history cannot well be denied even by Zeller, who has to content himself with averring that the stories are "grossly exaggerated."†2 There was no other philosopher whose conduct according to all testimony was quite so extravagant as that of Pyrrho. The accounts of him seem to come direct from a writing of his devoted pupil, Timon of Phlius, and some of our authorities, of whom there are a dozen, profess to use this book. Yet Zeller and the critics do not believe them; and Brandis objects that the citizens of Elis would not have chosen a half-insane man high priest -- as if symptoms of that kind would not have particularly recommended him for a divine office. That fashion of writing history is, I hope, now at last passing away.

Peirce: CP 1.618 Cross-Ref:††

618. However, disbelieve the stories if you will; you cannot refuse to admit that they show what kind of man the narrators expected a philosopher to be -- if they were imaginary legends, all the more so. Now those narrators are a cloud of the sanest and soberest minds of antiquity -- Plato, Aristotle, Cicero, Seneca, Pliny, Plutarch, Lucian, Ælian, and so forth. The Greeks expected philosophy to affect life -- not by any slow process of percolation of forms, as *we* may expect that researches into differential equations, stellar photometry, the taxonomy of echinoderms, and the like will ultimately affect the conduct of life -- but forthwith in the person and soul of the philosopher himself, rendering him different from ordinary men in his views of right conduct. So little did they separate philosophy from esthetic and moral culture that the *docti furor arduus Lucreti* could clothe an elaborate *cosmogony* in noble verse, for the express purpose of influencing men's lives; and Plato tells us in many places how inextricably he considers the study of Dialectic to be bound up with virtuous living. Aristotle, on the other hand, set this matter right. Aristotle was not much of a Greek. That he was of full Greek blood is not likely. That he was not altogether a Greek-minded man is manifest. Though he belonged to the school of Plato, yet when he went there he was already a student, perhaps a personal pupil, of Democritus, himself another Thracian; and during his first years in Athens he cannot have had much intercourse with Plato, who was away at Syracuse a large part of the time. Above all Aristotle was an Asclepiades, that is to say, he belonged to a line every man of whom since the heroic age had, as a child, received a finished training in the dissecting-room. Aristotle was a thorough-paced scientific man such as we see nowadays, except for this, that he ranged over all knowledge. As a man of scientific instinct, he classed metaphysics, in which I doubt not he included logic, as a matter of course, among the sciences -- sciences in *our* sense, I mean, what *he* called theoretical sciences -- along with mathematics and natural science -- natural science embracing what we call the physical sciences and the psychical sciences, generally. This theoretical science was for him one thing, animated by one spirit and having knowledge of theory as its ultimate end and aim. Esthetic studies were of a radically different kind; while morals, and all that relates to the conduct of life, formed a *third* department of intellectual activity, radically foreign in its nature and idea, from both the other two. Now, Gentlemen, it behooves me, at the outset of this course, to confess to you that in this respect I stand before you an Aristotelian and a scientific

man, condemning with the whole strength of conviction the Hellenic tendency to mingle philosophy and practice.

Peirce: CP 1.619 Cross-Ref:††

619. There are sciences, of course, many of whose results are almost immediately applicable to human life, such as physiology and chemistry. But the true scientific investigator completely loses sight of the utility of what he is about. It never enters his mind. Do you think that the physiologist who cuts up a dog reflects, while doing so, that he may be saving a human life? Nonsense. If he did, it would spoil him for a scientific man; and then the vivisection would become a crime. However, in physiology and in chemistry, the man whose brain is occupied with utilities, though he will not do much for science, may do a great deal for human life. But in philosophy, touching as it does upon matters which are, and ought to be, sacred to us, the investigator who does not stand aloof from all intent to make practical applications will not only obstruct the advance of the pure science, but, what is infinitely worse, he will endanger his own moral integrity and that of his readers.

Peirce: CP 1.620 Cross-Ref:††

620. In my opinion, the present infantile condition of philosophy -- for as long as earnest and industrious students of it are able to come to agreement upon scarce a single principle, I do not see how it can be considered as otherwise than in its infancy -- is due to the fact that during this century it has chiefly been pursued by men who have not been nurtured in dissecting-rooms and other laboratories, and who consequently have not been animated by the true scientific *Eros*; but who have on the contrary come from theological seminaries, and have consequently been inflamed with a desire to amend the lives of themselves and others, a spirit no doubt more important than the love of science, for men in average situations, but radically unfitting them for the task of scientific investigation. And it is precisely because of this utterly unsettled and uncertain condition of philosophy at present, that I regard any practical applications of it to religion and conduct as exceedingly dangerous. I have not one word to say against the philosophy of religion or of ethics in general or in particular. I only say that for the present it is all far too dubious to warrant risking any human life upon it. I do not say that philosophical science should not ultimately influence religion and morality; I only say that it should be allowed to do so only with secular slowness and the most conservative caution.

Peirce: CP 1.621 Cross-Ref:††

621. Now I may be utterly wrong in all this, and I do not propose to argue the question. I do not ask you to go with me. But to avoid any possible misapprehension, I am bound honestly to declare that I do not hold forth the slightest promise that I have any philosophical wares to offer you which will make you either better men or more successful men.

Peirce: CP 1.622 Cross-Ref:††

622. It is particularly needful that I should say this owing to a singular hybrid character which you will detect in these lectures. I was asked in December to prepare a course of lectures upon my views of philosophy. I accordingly set to work to draw up in eight lectures an outline of one branch of philosophy, namely, Objective Logic.†1 But just as I was finishing one lecture word came that you would expect to be addressed on topics of vital importance, and that it would be as well to make the lectures detached. I thereupon threw aside what I had written and began again to prepare the same number of homilies on intellectual ethics and economics. They were wretched things; and I was glad enough to learn, when three-quarters of my task was

done, that it would be desirable that as much as possible should be said of certain philosophical questions, other subjects being put in the background. At that time, however, it was too late to write a course which should set before you what I should have greatly desired to submit to your judgment. I could only patch up some fragments partly philosophical and partly practical. Thus, you will find me part of the time offering you detached ideas upon topics of vital importance, while part of the time I shall be presenting philosophical considerations, in which you will be able to feel an undercurrent toward that logic of things concerning which I shall have an opportunity to interject scarce one overt word.

Peirce: CP 1.623 Cross-Ref:††

623. I shall have a good deal to say about right reasoning; and in default of better I had reckoned *that* as a topic of vital importance. But I do not know that the theory of reasoning is quite vitally important. That it is absolutely essential in metaphysics, I am as sure as I am of any truth of philosophy. But in the conduct of life, we have to distinguish everyday affairs and great crises. In the great decisions, I do not believe it is safe to trust to individual reason. In everyday business, reasoning is tolerably successful; but I am inclined to think that it is done as well without the aid of theory as with it. A *logica utens*, like the analytical mechanics resident in the billiard player's nerves, best fulfills familiar uses.

Peirce: CP 1.624 Cross-Ref:††

624. In metaphysics, however, it is not so, at all; and the reason is obvious. The truths that the metaphysician infers can be brought to the test of experience, if at all, only in a department of experience quite foreign from that which furnishes his premisses. Thus a metaphysician who infers anything about a life beyond the grave can never find out for certain that his inference is false until he has gone out of the metaphysical business, at his present stand, at least. The consequence is that unless the metaphysician is a most thorough master of formal logic -- and especially of the inductive side of the logic of relatives, immeasurably more important and difficult than all the rest of formal logic put together -- he will inevitably fall into the practice of deciding upon the validity of reasonings in the same manner in which, for example, the practical politician decides as to the weight that ought to be allowed to different considerations, that is to say, by the impression those reasonings make upon the mind, only with this stupendous difference, that the one man's impressions are the resultant of long experiential training, while with such training the other man is altogether unacquainted. The metaphysician who adopts a metaphysical reasoning because he is impressed that it is sound, might just as well, or better, adopt his conclusions directly because he is impressed that they are true, in the good old style of Descartes and of Plato. To convince yourself of the extent to which this way of working actually vitiates philosophy, just look at the dealings of the metaphysicians with Zeno's objections to motion. They are simply at the mercy of the adroit Italian. For this reason, then, if for no other, the metaphysician who is not prepared to grapple with all the difficulties of modern exact logic had better put up his shutters and go out of the trade. Unless he will do one or the other, I tell him to his conscience that he is not the genuine, honest, earnest, resolute, energetic, industrious, and accomplished doubter that it is his duty to be.

Peirce: CP 1.625 Cross-Ref:††

625. But this is not all, nor half. For after all, metaphysical reasonings, such as they have hitherto been, have been simple enough for the most part. It is the metaphysical concepts which it is difficult to apprehend. Now the metaphysical

conceptions, as I need not waste words to show, are merely adapted from those of formal logic, and therefore can only be apprehended in the light of a minutely accurate and thoroughgoing system of formal logic.

Peirce: CP 1.626 Cross-Ref:††

626. But in practical affairs, in matters of vital importance, it is very easy to exaggerate the importance of ratiocination. Man is so vain of his power of reason! It seems impossible for him to see himself in this respect, as he himself would see himself if he could duplicate himself and observe himself with a critical eye. Those whom we are so fond of referring to as the "lower animals" reason very little. Now I beg you to observe that those beings very rarely commit a mistake, while we ---! We employ twelve good men and true to decide a question, we lay the facts before them with the greatest care, the "perfection of human reason" presides over the presentment, they hear, they go out and deliberate, they come to a unanimous opinion, and it is generally admitted that the parties to the suit might almost as well have tossed up a penny to decide! Such is man's glory!

Peirce: CP 1.627 Cross-Ref:††

627. The mental qualities we most admire in all human beings except our several selves are the maiden's delicacy, the mother's devotion, manly courage, and other inheritances that have come to us from the biped who did not yet speak; while the characters that are most contemptible take their origin in reasoning. The very fact that everybody so ridiculously overrates his own reasoning is sufficient to show how superficial the faculty is. For you do not hear the courageous man vaunt his own courage, or the modest woman boast of her modesty, or the really loyal plume themselves on their honesty. What they are vain about is always some insignificant gift of beauty or of skill.

Peirce: CP 1.628 Cross-Ref:††

628. It is the instincts, the sentiments, that make the substance of the soul. Cognition is only its surface, its locus of contact with what is external to it.

Peirce: CP 1.629 Cross-Ref:††

629. Do you ask me to prove this? If so, you must be a rationalist, indeed. I can prove it -- but only by assuming a logical principle of the demonstration of which I shall give a hint in the next lecture.†1 When people ask me to prove a proposition in philosophy I am often obliged to reply that it is a corollary from the logic of relatives. Then certain men say, "I should like exceedingly to look into this logic of relatives; you must write out an exposition of it." The next day I bring them a manuscript. But when they see that it is full of A, B, and C, they never look at it again. Such men -- oh, well.

Peirce: CP 1.630 Cross-Ref:††

630. Reasoning is of three kinds. The first is necessary, but it only professes to give us information concerning the matter of our own hypotheses and distinctly declares that, if we want to know anything else, we must go elsewhere. The second depends upon probabilities. The only cases in which it pretends to be of value is where we have, like an insurance company, an endless multitude of insignificant risks. Wherever a vital interest is at stake, it clearly says, "Don't ask me." The third kind of reasoning tries what *il lume naturale*, which lit the footsteps of Galileo, can do. It is really an appeal to instinct. Thus reason, for all the frills it customarily wears, in vital crises, comes down upon its marrow-bones to beg the succour of instinct.

Peirce: CP 1.631 Cross-Ref:††

631. Reason is of its very essence egotistical. In many matters it acts the fly on the wheel. Do not doubt that the bee thinks it has a good reason for making the end of its cell as it does. But I should be very much surprised to learn that its reason had solved that problem of isoperimetry that its instinct has solved. Men many times fancy that they act from reason when, in point of fact, the reasons they attribute to themselves are nothing but excuses which unconscious instinct invents to satisfy the teasing "whys" of the *ego*. The extent of this self-delusion is such as to render philosophical rationalism a farce.

Peirce: CP 1.632 Cross-Ref:††

632. Reason, then, appeals to sentiment in the last resort. Sentiment on its side feels itself to be the man. That is my simple apology for philosophical sentimentalism.

Peirce: CP 1.633 Cross-Ref:††

633. Sentimentalism implies conservatism; and it is of the essence of conservatism to refuse to push any practical principle to its extreme limits -- including the principle of conservatism itself. We do not say that sentiment is *never* to be influenced by reason, nor that under no circumstances would we advocate radical reforms. We only say that the man who would allow his religious life to be wounded by any sudden acceptance of a philosophy of religion or who would precipitately change his code of morals at the dictate of a philosophy of ethics -- who would, let us say, hastily practice incest -- is a man whom we should consider *unwise*. The regnant system of sexual rules is an instinctive or sentimental induction summarizing the experience of all our race. That it is abstractly and absolutely infallible we do not pretend; but that it is practically infallible for the individual -- which is the only clear sense the word "infallibility" will bear -- in that he ought to obey it and not his individual reason, *that* we do maintain.

Peirce: CP 1.634 Cross-Ref:††

634. I would not allow to sentiment or instinct any weight whatsoever in theoretical matters, not the slightest. Right sentiment does not demand any such weight; and right reason would emphatically repudiate the claim if it were made. True, we are driven oftentimes in science to try the suggestions of instinct; but we only *try* them, we compare them with experience, we hold ourselves ready to throw them overboard at a moment's notice from experience. If I allow the supremacy of sentiment in human affairs, I do so at the dictation of reason itself; and equally at the dictation of sentiment, in theoretical matters I refuse to allow sentiment any weight whatever.

Peirce: CP 1.635 Cross-Ref:††

635. Hence, I hold that what is properly and usually called *belief*, that is, the adoption of a proposition as a {ktéma es aei} to use the energetic phrase of Doctor Carus,†1 has no place in science at all. We *believe* the proposition we are ready to act upon. *Full belief* is willingness to act upon the proposition in vital crises, *opinion* is willingness to act upon it in relatively insignificant affairs. But pure science has nothing at all to do with *action*. The propositions it accepts, it merely writes in the list of premisses it proposes to use. Nothing is *vital* for science; nothing can be. Its accepted propositions, therefore, are but opinions at most; and the whole list is provisional. The scientific man is not in the least wedded to his conclusions. He risks nothing upon them. He stands ready to abandon one or all as soon as experience opposes them. Some of them, I grant, he is in the habit of calling *established truths*;

but that merely means propositions to which no competent man today demurs. It seems probable that any given proposition of that sort will remain for a long time upon the list of propositions to be admitted. Still, it may be refuted tomorrow; and if so, the scientific man will be glad to have got rid of an error. There is thus no proposition at all in science which answers to the conception of belief.

Peirce: CP 1.636 Cross-Ref:††

636. But in vital matters, it is quite otherwise. We must act in such matters; and the principle upon which we are willing to act is a *belief*.

Peirce: CP 1.637 Cross-Ref:††

637. Thus, pure theoretical knowledge, or science, has nothing directly to say concerning practical matters, and nothing even applicable at all to vital crises. Theory is applicable to minor practical affairs; but matters of vital importance must be left to sentiment, that is, to instinct.

Peirce: CP 1.638 Cross-Ref:††

638. Now there are two conceivable ways in which right sentiment might treat such terrible crises; on the one hand, it might be that while human instincts are not so detailed and featured as those of the dumb animals, yet they might be sufficient to guide us in the *greatest* concerns without any aid from reason, while on the other hand, sentiment might act to bring the vital crises under the domain of reason by rising under such circumstances to such a height of self-abnegation as to render the situation insignificant. In point of fact, we observe that a healthy natural human nature does act in both these ways.

Peirce: CP 1.639 Cross-Ref:††

639. The instincts of those animals whose instincts are remarkable present the character of being chiefly, if not altogether, directed to the preservation of the stock and of benefiting the individual very little, if at all, except so far as he may happen as a possible procreator to be a potential public functionary. Such, therefore, is the description of instinct that we ought to expect to find in man, in regard to vital matters; and so we do. It is not necessary to enumerate the facts of human life which show this, because it is too plain. It is to be remarked, however, that individuals who have passed the reproductive period, are more useful to the propagation of the human race than to [?] any other. For they amass wealth, and teach prudence, they keep the peace, they are friends of the little ones, and they inculcate all the sexual duties and virtues. Such instinct does, as a matter of course, prompt us, in all vital crises, to look upon our individual lives as small matters. It is no extraordinary pitch of virtue to do so; it is the character of every man or woman that is not despicable. Somebody during the Reign of Terror said: *Tout le monde croit qu'il est difficile de mourir. Je le crois comme les autres. Cependant je vois que quant on est là chacun s'en tire.* It is less characteristic of the woman because her life is more important to the stock, and her immolation less useful.

Peirce: CP 1.640 Cross-Ref:††

640. Having thus shown how much less vitally important reason is than instinct, I next desire to point out how exceedingly desirable, not to say indispensable, it is for the successful march of discovery in philosophy and in science generally that practical utilities, whether low or high, should be *put out of sight* by the investigator.

Peirce: CP 1.641 Cross-Ref:††

641. The point of view of utility is always a narrow point of view. How much

more we should know of chemistry today if the most practically important bodies had not received excessive attention; and how much *less* we should know, if the rare elements and the compounds which only exist at low temperatures had received only the *share* of attention to which their *utility* entitled them.

Peirce: CP 1.642 Cross-Ref:††

642. It is notoriously true that into whatever you do not put your whole heart and soul in that you will not have much success. Now, the two masters, *theory* and *practice*, you cannot serve. That perfect balance of attention which is requisite for observing the system of things is utterly lost if human desires intervene, and all the more so the higher and holier those desires may be.

Peirce: CP 1.643 Cross-Ref:††

643. In addition to that, in philosophy we have prejudices so potent that it is impossible to keep one's *sang-froid* if we allow ourselves to dwell upon them at all.

Peirce: CP 1.644 Cross-Ref:††

644. It is far better to let philosophy follow perfectly untrammelled a scientific method, *predetermined* in advance of knowing to what it will lead. If that course be honestly and scrupulously carried out, the results reached, even if they be not altogether true, even if they be grossly mistaken, can not but be highly serviceable for the ultimate discovery of truth. Meantime, sentiment can say "Oh well, philosophical science has not by any means said its last word yet; and meantime I will continue to believe *so and so*."

Peirce: CP 1.645 Cross-Ref:††

645. No doubt a large proportion of those who now busy themselves with philosophy will lose all interest in it as soon as it is forbidden to look upon it as susceptible of practical applications. We who continue to pursue the theory must bid *adieu* to them. But so we must in any department of pure science. And though we regret to lose their company, it is infinitely better that men devoid of genuine scientific curiosity should not barricade the road of science with empty books and embarrassing assumptions.

Peirce: CP 1.646 Cross-Ref:††

646. The host of men who achieve the bulk of each year's new discoveries are mostly confined to narrow ranges. For that reason you would expect the arbitrary hypotheses of the different mathematicians to shoot out in every direction into the boundless void of arbitrariness. But you do not find any such thing. On the contrary, what you find is that men working in fields as remote from one another as the African diamond fields are from the Klondike reproduce the same forms of novel hypothesis. Riemann had apparently never heard of his contemporary Listing. The latter was a naturalistic geometer, occupied with the shapes of leaves and birds' nests, while the former was working upon analytical functions. And yet that which seems the most arbitrary in the ideas created by the two men are one and the same form. This phenomenon is not an isolated one; it characterizes the mathematics of our times, as is, indeed, well known. All this crowd of creators of forms for which the real world affords no parallel, each man arbitrarily following his own sweet will, are, as we now begin to discern, gradually uncovering one great cosmos of forms, a world of potential being. The pure mathematician himself feels that this is so. He is not indeed in the habit of publishing any of his sentiments nor even his generalizations. The fashion in mathematics is to print nothing but demonstrations, and the reader is left to divine the workings of the man's mind from the sequence of those demonstrations.

But if you enjoy the good fortune of talking with a number of mathematicians of a high order, you will find that the typical pure mathematician is a sort of Platonist. Only, he is [a] Platonist who corrects the Heraclitan error that the eternal is not continuous. The eternal is for him a world, a cosmos, in which the universe of actual existence is nothing but an arbitrary locus. The end that pure mathematics is pursuing is to discover that real potential world.

Peirce: CP 1.647 Cross-Ref:††

647. Once you become inflated with that idea, *vital importance* seems to be a very low kind of importance, indeed.

Peirce: CP 1.647 Cross-Ref:††

But such ideas are only suitable to regulate another life than this. Here we are in this workaday world, little creatures, mere cells in a social organism itself a poor and little thing enough, and we must look to see what little and definite task our circumstances have set before our little strength to do. The performance of that task will require us to draw upon all our powers, reason included. And in the doing of it we should chiefly depend not upon that department of the soul which is most superficial and fallible -- I mean our reason -- but upon that department that is deep and sure -- which is instinct.

Peirce: CP 1.648 Cross-Ref:††

648. Instinct is capable of development and growth -- though by a movement which is slow in the proportion in which it is vital; and this development takes place upon lines which are altogether parallel to those of reasoning. And just as reasoning springs from experience, so the development of sentiment arises from the soul's Inward and Outward Experiences. Not only is it of the same nature as the development of cognition; but it chiefly takes place through the instrumentality of cognition. The soul's deeper parts can only be reached through its surface. In this way the eternal forms, that mathematics and philosophy and the other sciences make us acquainted with, will by slow percolation gradually reach the very core of one's being; and will come to influence our lives; and this they will do, not because they involve truths of merely vital importance, but because they are ideal and eternal verities.

Peirce: CP 1.649 Cross-Ref:††

§2. PRACTICAL CONCERNS AND THE WISDOM OF SENTIMENT †1

649. Among the advantages which our humble cousins whom it pleases us to refer to as "the lower animals" enjoy over some of our own family is that they *never* reason about vitally important topics, and never have to lecture nor to listen to lectures about them. Docilely allowing themselves to be guided by their instincts into almost every detail of life, they live exactly as their Maker intended them to live. The result is, that they very rarely fall into error of any kind, and *never* into a vital one. What a contrast to our lives! Truly, that reason upon which we so plume ourselves, though it may answer for little things, yet for great decisions is hardly surer than a toss-up. . . .

Peirce: CP 1.650 Cross-Ref:††

650. Logic is computation, said Hobbes;†2 and those who have deepest delved

in that dreary discipline testify that all reasoning whatever involves mathematics, and laugh over the fallacies of those who attempt to reason unmathematically. Now tell me, is mathematics an occupation for a gentleman and an athlete? Is not such drudgery fit only for the lower classes? One may well be struck with pity for the masses of population concentrated in New York and living under such unnatural conditions that they are forced to think mathematically. However, it is not as if they had the tender nurture of a cultured modern Harvard, that great eleemosynary institution that Massachusetts has established to the end that the *élite* of her youths may be aided to earning comfortable incomes and living softly cultured lives. The brains of those New York plebeians are coarse, strong, laboring brains, that don't know what it is to be free from mathematics. Their conceptions are crude and vulgar enough, but their vigor of reasoning would surprise you. I have seen my [private] scholars there wrestle with problems that I would no more venture to allow the exquisitely polished intellects of a modern university to attack than I would venture to toss a cannonball into an eggshell cup.

Peirce: CP 1.651 Cross-Ref:††

651. I intend to call upon you for no reasoning in these lectures more complicated than one of Hegel's dilemmas. For all reasoning is mathematical and requires effort; and I mean to shun the guilt of overstraining anybody's powers. That is why I have selected a subject for my lectures which is not at all in my line, but which I hope may prove to be to your taste.

Peirce: CP 1.652 Cross-Ref:††

652. On vitally important topics reasoning is out of place. . . . The very theory of reasoning, were we resolutely to attack it without any dread of mathematics, would furnish us conclusive reasons for limiting the applicability of reasoning to unimportant matters; so that, unless a problem is insignificant in importance compared with the aggregate of analogous problems, reasoning itself pronounces that there is a fallacy in submitting the question to reason, at all. That must remain merely an assertion, mathematics being *taboo*. . . .

Peirce: CP 1.653 Cross-Ref:††

653. In regard to the greatest affairs of life, the wise man follows his heart and does not trust his head. This should be the method of every man, no matter how powerful his intellect. More so still, perhaps, if mathematics is too difficult for him, that is to say, if he is unequal to any intricate reasoning whatsoever. Would not a man physically puny be a fool not to recognize it, and to allow an insane megalomania to induce him to enter a match game of football? But the slightest of physical frames might as well attempt to force back a locomotive engine, as for the mightiest of mental giants to try to regulate his life advantageously by a purely reasoned-out theory.

Peirce: CP 1.654 Cross-Ref:††

654. Common sense, which is the resultant of the traditional experience of mankind, witnesses unequivocally that the heart is more than the head, and is in fact everything in our highest concerns, thus agreeing with my unproved logical theorem; and those persons who think that sentiment has no part in common sense forget that the dicta of common sense are objective facts, not the way some dyspeptic may feel, but what the healthy, natural, normal democracy thinks. And yet when you open the next new book on the philosophy of religion that comes out, the chances are that it will be written by an intellectualist who in his preface offers you his metaphysics as a guide for the soul, talking as if philosophy were one of our deepest concerns. How

can the writer so deceive himself?

Peirce: CP 1.655 Cross-Ref:††

655. If, walking in a garden on a dark night, you were suddenly to hear the voice of your sister crying to you to rescue her from a villain, would you stop to reason out the metaphysical question of whether it were possible for one mind to cause material waves of sound and for another mind to perceive them? If you did, the problem might probably occupy the remainder of your days. In the same way, if a man undergoes any religious experience and hears the call of his Saviour, for him to halt till he has adjusted a philosophical difficulty would seem to be an analogous sort of thing, whether you call it stupid or whether you call it disgusting. If on the other hand, a man has had no religious experience, then any religion not an affectation is as yet impossible for him; and the only worthy course is to wait quietly till such experience comes. No amount of speculation can take the place of experience.

Peirce: CP 1.656 Cross-Ref:††

656. Pray pardon my hopping about from one branch of my discourse to another and back again with no more apparent purpose than a robin redbreast or a Charles Lamb. Because it would hardly be logically consistent for me to arrange my matter with scrupulously logical accuracy when the very thing I am driving at is that logic and reasoning are only of secondary importance. There are two psychological or anthropological observations about our reasoning powers which it is convenient to insert here.

Peirce: CP 1.657 Cross-Ref:††

657. One is that powers of reasoning in any but the most rudimentary way are a somewhat uncommon gift, about as uncommon as a talent for music. Indeed, a much smaller number of persons actually attain to any proficiency in reasoning. But then the exercise of intricate ratiocination requires great energy and prolonged effort, while musical practice is nearly unmixed pleasure, I suppose, for those who do it well. Moreover, owing to several peculiar circumstances, good instruction in reasoning is exceedingly rare. As for what is taught in the colleges under the name of logic, oh dear, perhaps the less said the better. It is true that mathematics teaches one branch of reasoning. That is, indeed, its chief value in education. But how few teachers understand the logic of mathematics! And how few understand the psychology of the puzzled pupil! The pupil meets with a difficulty in Euclid. Two to one the reason is that there is a logical flaw. The boy, however, is conscious only of a mysterious hindrance. What his difficulty is he cannot tell the teacher; the teacher must teach him. Now the teacher probably never really saw the true logic of the passage. But he thinks he does because, owing to long familiarity, he has lost that sense of coming up against an invisible barrier that the boy feels. Had the teacher ever really conquered the logical difficulty himself, of course he would recognize just what it was, and thus would fulfill the first condition, at least, of being helpful. But not having conquered the difficulty, but only having worn out the sense of difficulty by familiarity, he simply cannot understand why the boy should feel any difficulty; and all he can do is to exclaim, "Oh, these stupid, stupid boys!" As if a physician should exclaim, "Oh, these horrid patients, they won't get well!" But suppose, by some extraordinary conjunction of the planets, a really good teacher of reasoning were to be appointed, what would be his first care? It would be to guard his scholars from that malady with which logic is usually infested, so that unless it runs off them like water from a duck, it is sure to make them the very worst of reasoners, namely, unfair reasoners, and what is worse unconsciously unfair, for the rest of their lives. The good

teacher will therefore take the utmost pains to prevent the scholars getting puffed up with their logical acquirements. He will wish to impregnate them with the right way of looking at reasoning before they shall be aware that they have learned anything; and he will not mind giving considerable time to that, for it is worth a great deal. But now come the examiner and the pupil himself. They want *results*, tangible to them. The teacher is dismissed as a failure, or, if he is allowed another chance, he will take good care to reverse the method of his teaching and give them *results* -- especially, as that is the lazy way. These are some of the causes of there being so few strong reasoners in the world. But allowing for the influence of such causes as well as we can, the fact still remains that comparatively few persons are originally possessed of any but the feeblest modicum of this talent. What is the significance of that? Is it not a plain sign that the faculty of reasoning is not of the first importance to success in life? For were it so, its absence would cause the individual to postpone marriage and so affect his procreation; and thus natural selection would operate to breed the race for vigorous reasoning powers, and they would become common. And the study of characters confirms this conclusion. For though the men who are most extraordinarily successful evidently do reason deeply about the details of their business, yet no ordinary degrees of good success are influenced -- otherwise than perhaps favorably -- by any lack of great reasoning power. We all know highly successful men, lawyers, editors, scientific men -- not to speak of artists -- whose great deficiency in this regard is only revealed by some unforeseen accident.

Peirce: CP 1.658 Cross-Ref:††

658. The other observation I desired to make about the human reason is that we find people mostly modest enough about qualities which really go to making fine men and women -- the courageous man not usually vaunting his courage, nor the modest woman boasting of her modesty, nor the loyal vain of their good faith: the things they are vain about are some insignificant gifts of beauty, or skill of some kind. But beyond all, with the exception of those who, being trained in logic, follow its rules and thus do not trust their direct reasoning powers at all, everybody else ridiculously overrates his own logic, and if he really has superior powers of reason is usually so consumed by conceit that it is far from rare to see a young man completely ruined by it; so that one is sometimes tempted to think, and perhaps truly, that it conduces not only to a man's success from a worldly point of view but to his attaining any real elevation of character to be all but a fool in this regard, provided only he be perfectly aware of his own deficiency. . . .

Peirce: CP 1.659 Cross-Ref:††

659. All those modern books which offer new philosophies of religion, at the rate of one every fortnight on the average, are but symptoms of the temporary dissolution of the Christian faith. This appears as soon as we compare them with the works of religious philosophy of the ages of faith, such as the *Summa* of St. Thomas Aquinas or the *Opus Oxoniense* of Duns Scotus -- the one reproducing without a shadow of mistrust every dogma of the Fathers of the Church, while the other displays a far stouter faith in maintaining that metaphysics has nothing to say either one way or the other concerning any question of religion, but leaves it to be decided by positive testimony or inspiration. The only old book which these modern philosophies of religion really resemble a good deal -- except that they lack its terrible earnestness -- is the *De consolatione philosophiae* and it is paying them a high compliment to say so. Boethius, you know, is utterly religionless, but he feels the need of religion and vainly tries to find a substitute for it in philosophy. His first two books are somewhat inspiring, because they breathe an unconscious religion. But as the work progresses,

reasoning enters more and more into the thought, until the last book, which resembles a modern essay much more than all the rest, is a mere diet of bran for the hungered soul.

Peirce: CP 1.660 Cross-Ref:††

660. It is hardly necessary to insist here that the highly cultured classes of Christendom -- excepting always those families which are so important as to be an object of solicitude on the part of the priests -- are nowadays nearly destitute of any religion. It was made perfectly manifest five and twenty years ago or more -- no matter for the exact date; it was at a date when men saturated with the mechanical philosophy were still hesitating to separate themselves from the church -- when John Tyndall, in the innocence of his scientific heart, proposed to measure the efficacy of prayer by experimental statistics. Instantly, the clergy, one and all, instead of meeting the proposal with the candor with which Elijah met the priests of Baal -- though by the way I notice some ingenious persons think his barrels of water were really deodorized kerosene, which for a student of the history of chemistry, would, of itself, seem to be a good enough miracle -- instead of thanking Tyndall for the idea, I say, the clergy to a man shrank back in terror, thus conclusively betraying to every eye their own utter disbelief in their own dogma. They pronounced it an impious proposition. But there was nothing more impious in it than in any other sort of inquiry into religion except this -- that they feared it would bring all "talkee-talkee" to an end. Although it must be granted that in our country the clergy are by far the most sceptical class of the community, yet where the clergy stood a generation back, the bulk of the highly educated and cultured class stands now.

Peirce: CP 1.660 Cross-Ref:††

It is a thousand times better to have no faith at all in God or virtue than to have a hemi-hypocritical faith. . . .

Peirce: CP 1.661 Cross-Ref:††
§3. VITALLY IMPORTANT TRUTHS

661. Conservatism, true conservatism, which is sentimental conservatism, and by those who have no powers of observation to see what sort of men conservatives are, is often called stupid conservatism, an epithet far more applicable to the false conservatism that looks to see on which side bread is buttered -- true conservatism, I say, means not trusting to reasonings about questions of vital importance but rather to hereditary instincts and traditional sentiments. Place before the conservative arguments to which he can find no adequate reply and which go, let us say, to demonstrate that wisdom and virtue call upon him to offer to marry his own sister, and though he be unable to answer the arguments, he will not act upon their conclusion, because he believes that tradition and the feelings that tradition and custom have developed in him are safer guides than his own feeble ratiocination. Thus, true conservatism is sentimentalism. Of course, sentiment lays no claim to infallibility, in the sense of *theoretical infallibility*, a phrase that logical analysis proves to be a mere jingle of words with a jangle of contradictory meanings. The conservative need not forget that he might have been born a Brahmin with a traditional sentiment in favor of *suttee* -- a reflection that tempts him to become a radical. But still, on the whole, he thinks his wisest plan is to reverence his deepest

sentiments as his highest and ultimate authority, which is regarding them as *for him practically infallible* -- that is, to say infallible in the only sense of the word in which *infallible* has any consistent meaning.

Peirce: CP 1.662 Cross-Ref:††

662. The opinion prevalent among radicals that conservatives, and sentimentalists generally, are fools is only a cropping-out of the tendency of men to conceited exaggeration of their reasoning powers. Uncompromising radical though I be upon some questions, inhabiting all my life an atmosphere of science, and not reckoned as particularly credulous, I must confess that the conservative sentimentalism I have defined recommends itself to my mind as eminently sane and wholesome. Commendable as it undoubtedly is to reason out matters of detail, yet to allow mere reasonings and reason's self-conceit to overslaw [over-slaugh? over-awe?] the normal and manly sentimentalism which ought to lie at the cornerstone of all our conduct seems to me to be foolish and despicable.

Peirce: CP 1.663 Cross-Ref:††

663. Philosophy after all is, at its highest valuation, nothing more than a branch of science, and as such is not a matter of vital importance; and those who represent it as being so are simply offering us a stone when we ask for bread. Mind, I do not deny that a philosophical or other scientific error may be fraught with disastrous consequences for the whole people. It might conceivably bring about the extirpation of the human race. Importance in that sense it might have in any degree. Nevertheless, in no case is it of *vital* importance.

Peirce: CP 1.664 Cross-Ref:††

664. A great calamity the error may be, *qua* event, in the sense in which an earthquake, or the impact of a comet, or the extinction of the sun would be an important event, and consequently, if it happens to lie in the line of my duty or of yours to investigate any philosophical question and to publish the more or less erroneous results of our investigations, I hope we shall not fail to do so, if we can. Certainly, any task which lies before us to be done has its importance. But there our responsibility ends. Nor is it the philosophy itself, *qua* cognition, that is vital, so much as it is our playing the part that is allotted to us.

Peirce: CP 1.665 Cross-Ref:††

665. You will observe that I have not said a single word in disparagement of the philosophy of religion, in general, which seems to me a most interesting study, at any rate, and possibly likely to lead to some useful result. Nor have I attacked any sect of that philosophy. It is not the philosophy which I hold to be baleful, but the representing it to be of vital importance, as if any genuine religion could come from the head instead of from the heart.

Peirce: CP 1.666 Cross-Ref:††

666. Somewhat allied to the philosophy of religion is the science of ethics. It is equally useless. Now books of casuistry, indeed, using the word "casuistry" not in any technical sense, but merely to signify discussions of what ought to be done in various difficult situations, might be made at once extremely entertaining and positively useful. But casuistry is just what the ordinary treatises upon ethics do not touch, at least not seriously. They chiefly occupy themselves with reasoning out the basis of morality and other questions secondary to that. Now what's the use of prying into the philosophical basis of morality? We all know what morality is: it is behaving as you were brought up to behave, that is, to think you ought to be punished for not

behaving. But to believe in thinking as you have been brought up to think defines *conservatism*. It needs no reasoning to perceive that morality is conservatism. But conservatism again means, as you will surely agree, not trusting to one's reasoning powers. To be a moral man is to obey the traditional maxims of your community without hesitation or discussion. Hence, ethics, which is reasoning out an explanation of morality is -- I will not say immoral, [for] that would be going too far -- composed of the very substance of immorality. If you ever happen to be thrown in with an unprofessional thief, the only very bad kind of thief, so as to be able to study his psychological peculiarities, you will find that two things characterize him; first, an even more immense conceit in his own reasoning powers than is common, and second, a disposition to reason about the basis of morals.

Peirce: CP 1.667 Cross-Ref:††

667. Ethics, then, even if not a positively dangerous study, as it sometimes proves, is as useless a science as can be conceived. But it must be said, in favor of ethical writers, that they are commonly free from the nauseating custom of boasting of the utility of their science.

Peirce: CP 1.668 Cross-Ref:††

668. Far be it from me to decry. Though I do hail from New York,†1 I shall hardly be mistaken for a Wall Street Philistine. A useless inquiry, provided it is a systematic one, is pretty much the same thing as a scientific inquiry. Or at any rate if a scientific inquiry becomes by any mischance useful, that aspect of it has to be kept sedulously out of sight during the investigation or else, as I shall try to show you another evening, its hopes of success are fatally cursed.

Peirce: CP 1.669 Cross-Ref:††

669. As long as ethics is recognized as not being a matter of vital importance or in any way touching the student's conscience, it is, to a normal and healthy mind, a civilizing and valuable study -- somewhat more so than the theory of whist, much more so than the question of the landing of Columbus, which things are insignificant not at all because they are useless, nor even because they are little in themselves, but simply and solely because they are detached from the great continuum of ideas.

Peirce: CP 1.670 Cross-Ref:††

670. It would be useless to enumerate the other sciences, since it would only be to reiterate the same declaration. As long as they are not looked at as practical, and so degraded to pot-boiling arts -- as our modern writers degrade the philosophy of religion, in claiming that it is practical -- for what difference does it make whether the pot to be boiled is today's or the hereafter's? They are all such that it would be far too little to say that they are valuable to us. Rather let our hearts murmur "blessed are we" if the immolation of our being can weld together the smallest part of the great cosmos of ideas to which the sciences belong.

Peirce: CP 1.671 Cross-Ref:††

671. Even if a science be useful -- like engineering or surgery -- yet if it is useful only in an insignificant degree as those sciences are, it still has a divine spark in which its petty practicality must be forgotten and forgiven. But as soon as a proposition becomes vitally important -- then in the first place, it is sunk to the condition of a mere utensil; and in the second place, it ceases altogether to be scientific, because concerning matters of vital importance reasoning is at once an impertinence toward its subject matter and a treason against itself.

Peirce: CP 1.672 Cross-Ref:††

672. Were I willing to make a single exception to the principle I thus enunciate, and to admit that there was one study which was at once scientific and yet vitally important, I should make that exception in favor of logic; for the reason that if we fall into the error of believing that vitally important questions are to be decided by reasoning, the only hope of salvation lies in formal logic, which demonstrates in the clearest manner that reasoning itself testifies to its own ultimate subordination to sentiment. It is like a Pope who should declare *ex cathedra* and call upon all the faithful to implicitly believe on pain of damnation by the power of the keys that he was *not* the supreme authority.

Peirce: CP 1.673 Cross-Ref:††

673. Among vitally important truths there is one which I verily believe -- and which men of infinitely deeper insight than mine have believed -- to be solely supremely important. It is that vitally important facts are of all truths the veriest trifles. For the only vitally important matter is *my* concern, business, and duty -- or yours. Now you and I -- what are we? Mere cells of the social organism. Our deepest sentiment pronounces the verdict of our own insignificance. Psychological analysis shows that there is nothing which distinguishes my personal identity except my faults and my limitations -- or if you please, my blind will, which it is my highest endeavor to annihilate. Not in the contemplation of "topics of vital importance" but in those universal things with which philosophy deals, the factors of the universe, is man to find his highest occupation. To pursue "topics of vital importance" as the first and best can lead only to one or other of two terminations -- either on the one hand what is called, I hope not justly, Americanism, the worship of business, the life in which the fertilizing stream of genial sentiment dries up or shrinks to a rill of comic tit-bits, or else on the other hand, to monasticism, sleepwalking in this world with no eye nor heart except for the other. Take for the lantern of your footsteps the cold light of reason and regard your business, your duty, as the highest thing, and you can only rest in one of those goals or the other. But suppose you embrace, on the contrary, a conservative sentimentalism, modestly rate your own reasoning powers at the very mediocre price they would fetch if put up at auction, and then what do you come to? Why, *then*, the very first command that is laid upon you, your quite highest business and duty, becomes, as everybody knows, to recognize a higher business than your business, *not* merely an avocation after the daily task of your vocation is performed, but a generalized conception of duty which completes your personality by melting it into the neighboring parts of the universal cosmos. If this sounds unintelligible, just take for comparison the first good mother of a family that meets your eye, and ask whether she is not a sentimentalist, whether you would wish her to be otherwise, and lastly whether you can find a better formula in which to outline the universal features of her portrait than that I have just given. I dare say you can improve upon that; but you will find one element of it is correct -- especially if your understanding is aided by the logic of relatives -- and that is that the supreme commandment of the Buddhisto-Christian religion is, to generalize, to complete the whole system even until continuity results and the distinct individuals weld together. Thus it is, that while reasoning and the science of reasoning strenuously proclaim the subordination of reasoning to sentiment, the very supreme commandment of sentiment is that man should generalize, or what the logic of relatives shows to be the same thing, should become welded into the universal continuum, which is what true reasoning consists in. But this does not reinstate reasoning, for this generalization should come about, not merely in man's cognitions, which are but the superficial film of his being, but

objectively in the deepest emotional springs of his life. In fulfilling this command, man prepares himself for transmutation into a new form of life, the joyful Nirvana in which the discontinuities of his will shall have all but disappeared.

Peirce: CP 1.674 Cross-Ref:††

674. Do you know what it was that was at the root of the barbarism of the Plantagenet period and paralyzed the awakening of science from the days of Roger Bacon to those of Francis Bacon? We plainly trace it in the history, the writings, the monuments, of that age. It was the exaggerated interest men took in matters of vital importance.

Peirce: CP 1.675 Cross-Ref:††

675. Do you know what it is in Christianity that when recognized makes our religion an agent of reform and progress? It is its marking duty at its proper finite figure. Not that it diminishes in any degree its vital importance, but that behind the outline of that huge mountain it enables us to descry a silvery peak rising into the calm air of eternity.

Peirce: CP 1.676 Cross-Ref:††

676. The generalization of sentiment can take place on different sides. Poetry is one sort of generalization of sentiment, and in so far is the regenerative metamorphosis of sentiment. But poetry remains on one side ungeneralized, and to that is due its emptiness. The complete generalization, the complete regeneration of sentiment is religion, which is poetry, but poetry completed.

Peirce: CP 1.677 Cross-Ref:††

677. That is about what I had to say to you about topics of vital importance. To sum it up, all sensible talk about vitally important topics must be commonplace, all reasoning about them unsound, and all study of them narrow and sordid.

Peirce: CP 1.1 Fn 1 p vii

†1 1 and 2 are from "A Guess at the Riddle" (c. 1898), see bk. III, ch. 3, §1 note. 3-7 and 8-14 are two fragments, c. 1897.

Peirce: CP 1.1 Fn 2 p vii

†2 See 23ff. and book III.

Peirce: CP 1.15 Fn 1 p 3

†1 From the "Lowell Lectures of 1903," Lecture IIIa.

Peirce: CP 1.15 Fn 2 p 3

†2 1851.

Peirce: CP 1.15 Fn 3 p 3

†3 1453.

Peirce: CP 1.20 Fn 1 p 6

†1 See vol. 9.

Peirce: CP 1.27 Fn 1 p 8

†1 From "Essays on Meaning," June, 1909.

Peirce: CP 1.27 Fn P1 p 9 Cross-Ref:††

†P1 It must not be imagined that any notable realist of the thirteenth or fourteenth century took the ground that any "universal" was what we in English should call a "thing," as it seems that, in an earlier age, some realists and some nominalists, too, had done; though perhaps it is not quite certain that they did so, their writings being lost. Their very definition of a "universal" admits that it is of the same generic nature as a word, namely, is: "Quod natum optimum est *praedicari* de pluribus." Neither was it their doctrine that any "universal" *itself* is real. They might, indeed, some of them, think so; but their realism did not consist in *that* opinion, but in holding that what the word *signifies*, in contradistinction to what it can be truly said of, is real. Anybody may happen to opine that "the" is a real English word; but that will not constitute him a realist. But if he thinks that, whether the word "hard" itself be real or not, the property, the character, the predicate, *hardness*, is not invented by men, as the word is, but is really and truly in the hard things and is one in them all, as a description of habit, disposition, or behavior, *then* he *is* a realist.

Peirce: CP 1.28 Fn 1 p 10

†1 From Lecture I, "Early Nominalism and Realism" of the "Lectures on British Logicians," delivered at Harvard in 1869.

Peirce: CP 1.29 Fn 1 p 11

†1 Cf. Prantl's *Geschichte der Logik*, 2 Aufl. Bd. 2, S. 266; Bd. 3, S. 10ff.

Peirce: CP 1.30 Fn 2 p 11

†2 Ibid., vol. 2, p. 72.

Peirce: CP 1.34 Fn 1 p 14

†1 *Aristotle: A Chapter from the History of Science*, London (1864).

Peirce: CP 1.34 Fn 2 p 14

†2 Quoted by Sylvester in his Presidential Address to the British Assn. in 1868. See Sylvester's *Mathematical Papers*, vol. 2, p. 654.

Peirce: CP 1.35 Fn 1 p 15

†1 35 is an unpublished, uncompleted review of T. K. Abbott's translation of Kant's *Introduction to Logic*, etc. Longmans Green & Co., 1885. 37-38 is "Notes on the Question of the Existence of an External World." c. 1890. 36 and 39 are from fragmentary alternative mss. of that same date.

Peirce: CP 1.38 Fn 1 p 17

†1 Sir William Hamilton's *Discussions on Philosophy and Literature*, ch. 2, p. 55. What Hamilton objects to is "*immediate* knowledge of the *past*" as a definition of memory.

Peirce: CP 1.40 Fn 2 p 17

†2 40 and 41-2 are from separate unidentified fragments, c. 1892.

Peirce: CP 1.43 Fn 1 p 19

†1 A manuscript of notes for a projected, but never completed, *History of Science*, c. 1896.

Peirce: CP 1.54 Fn 1 p 24

†1 See 66, 240, 369 and vol. 4, bk. II.

- Peirce: CP 1.56 Fn 1 p 25
†1 On this use of "secular" see 176.
- Peirce: CP 1.62 Fn 1 p 27
†1 See vol. 6, bk. I.
- Peirce: CP 1.64 Fn 2 p 27
†2 Cf. 4.115.
- Peirce: CP 1.65 Fn 1 p 28
†1 Cf. vol. 2, bk. III.
- Peirce: CP 1.65 Fn 2 p 28
†2 Peirce usually calls it abduction; sometimes hypothesis.
- Peirce: CP 1.70 Fn 1 p 29
†1 *Laws of Thought*, chs. 16-21.
- Peirce: CP 1.70 Fn 2 p 29
†2 Including C. S. Peirce. See Paper No. 1, vol. 3.
- Peirce: CP 1.70 Fn 3 p 29
†3 *The Philosophy of the Inductive Sciences*, 1840.
- Peirce: CP 1.71 Fn 1 p 30
†1 *Ibid.*, bk. III, ch. 2, §3.
- Peirce: CP 1.84 Fn 1 p 35
†1 See vol. 6, bk. I, ch. 7.
- Peirce: CP 1.85 Fn 2 p 35
†2 In his *Phoenixiana*, "Official Report."
- Peirce: CP 1.86 Fn 1 p 36
†1 *Der Philosophie der Griechen*, S. 279.
- Peirce: CP 1.88 Fn 2 p 36
†2 Peirce seems to have secured his authorities from Zeller's *A History of Greek Philosophy*, 1881, vol. 1, p. 328, n. 4. Zeller's references are not all accurate, and the authorities quoted are not independent. Peirce's annotated copy of this book is now, through the gift of his wife, the property of the Harvard College Library.
- Peirce: CP 1.92 Fn 1 p 39
†1 Mill's views on induction are examined in more detail in vol. 2, bk. III, ch. 9.
- Peirce: CP 1.106 Fn 1 p 42
†1 See 209. Peirce wrote the definitions of terms in mechanics, mathematics astronomy, astrology, weights and measures, logic, metaphysics, all those relating to universities, and many on psychology for the *Century Dictionary*, edition of 1889.
- Peirce: CP 1.108 Fn 1 p 45
†1 *The Development of the Periodic Law*, Easton, Pa., 1896.
- Peirce: CP 1.109 Fn 2 p 45
†2 See *Oeuvres de Pasteur*, vol. 1, p. 83, Paris, 1922.

Peirce: CP 1.109 Fn 3 p 45

†3 *Leçons de Pathologie expérimental*, 2^{me} leçon, Paris, 1872.

Peirce: CP 1.122 Fn 1 p 48

†1 See, e.g., the lecture on the "Economical Nature of Physical Inquiry" in the *Popular Scientific Lectures* (1895).

Peirce: CP 1.126 Fn 1 p 50

†1 From "Introduction showing the point of view from which Philosophy appears to the author to be an interesting subject to a man of common-sense," in the Notebook, "Sketch of Some Proposed Chapters on the Sect of Philosophy Called Pragmatism." c. 1905.

Peirce: CP 1.129 Fn 1 p 52

†1 *Metaphysics*, bk. I, 982b-3a.

Peirce: CP 1.129 Fn 2 p 52

†2 See vol. 5, bk. II, ch. 7 and bk. III, chs. 2 and 3.

Peirce: CP 1.130 Fn 3 p 52

†3 Unpaginated fragment, c. 1893.

Peirce: CP 1.130 Fn P1 p 53 Cross-Ref:††

†P1 Except the proposition that two lines cannot enclose a space, though only one of the three best manuscripts places even this in the list. But what Euclid meant was that two straight lines can have but one intersection, which *is* evident.

Peirce: CP 1.132 Fn 1 p 54

†1 *De Rerum Natura*, bk. II, 1. 216ff.

Peirce: CP 1.133 Fn 2 p 54

†2 From "The Idea of a Law of Nature among the Contemporaries of David Hume and among Advanced Thinkers of the Present Day," c. 1894.

Peirce: CP 1.135 Fn 1 p 56

†1 From unpaginated ms. "F. R. L.," c. 1899.

Peirce: CP 1.138 Fn 1 p 57

†1 19^{me} leçon.

Peirce: CP 1.138 Fn 2 p 57

†2 In 1897 Salomon August Andrée attempted to fly over the polar regions in a balloon. He died in the attempt.

Peirce: CP 1.141 Fn 1 p 58

†1 From unpaginated, untitled ms. (or mss.) which to judge from 159 was intended as part of a lecture. c. 1897.

Peirce: CP 1.143 Fn 1 p 59

†1 The "Wicked Bible" of 1631 omitted "not" from the Seventh Commandment.

Peirce: CP 1.149 Fn 1 p 61

†1 See 4.155ff.

Peirce: CP 1.155 Fn 1 p 64

†1 To judge from this, the ms. should be dated ten years earlier. But the absence of the terms and the handwriting in earlier mss., and their presence in mss. dated 1897-8 seem to indicate that the editorial dating is correct.

Peirce: CP 1.156 Fn 2 p 64

†2 *Phoenixiana*, "Lectures on Astronomy."

Peirce: CP 1.162 Fn 1 p 66

†1 See vol. 6, bk. I.

Peirce: CP 1.176 Fn 1 p 75

†1 Apparently a foreword to a volume of the *Principles of Philosophy*, c. 1896.

Peirce: CP 1.179 Fn 1 p 76

†1 However, for the only philosophical work Peirce ever completed, *The Grand Logic*, the "pitchfork" method was used. The editors, of course, were compelled to "pitchfork," though they have tried to do it according to a plan, suggested by the classifications contained in the present book.

Peirce: CP 1.180 Fn 1 p 77

†1 Pp. 5-9 of *A Syllabus of Certain Topics of Logic*, 1903, Alfred Mudge & Son, Boston, bearing the following preface: "This syllabus has for its object to supplement a course of eight lectures to be delivered at the Lowell Institute, by some statements for which there will not be time in the lectures, and by some others not easily carried away from one hearing. It is intended to be a help to those who wish seriously to study the subject, and to show others what the style of thought is that is required in such study. Like the lectures themselves, this syllabus is intended chiefly to convey results that have never appeared in print; and much is omitted because it can be found elsewhere."

Peirce: CP 1.180 Fn 2 p 77

†2 See 203ff. where, also, some modifications of the present scheme are to be found. Cf. e.g. 181 and 239.

Peirce: CP 1.183 Fn 1 p 78

†1 See 242n for Bentham's definition of this term.

Peirce: CP 1.202 Fn 1 p 82

†1 See 243.

Peirce: CP 1.203 Fn 1 p 83

†1 Section 1, ch. 2, of the "Minute Logic," 1902.

Peirce: CP 1.203 Fn P1 p 83 Cross-Ref:††

†P1 *Classification, Theoretical and Practical*, by Ernest Cushing Richardson, C. Scribner's Sons, N. Y., 1901.

Peirce: CP 1.205 Fn P1 p 86 Cross-Ref:††

†P1 I am here influenced by the *Essay on Classification* [1857] of L. Agassiz, whose pupil I was for a few months. This work appeared at a most inauspicious epoch.

Peirce: CP 1.209 Fn P1 p 88 Cross-Ref:††

†P1 *Egyptian Exploration Fund*, Third Memoir, 1886.

Peirce: CP 1.209 Fn 1 p 89
†1 p. 4657, edition of 1889.

Peirce: CP 1.211 Fn 1 p 91
†1 See *Meta*. 44 b 1 and 70 b 26.

Peirce: CP 1.223 Fn 1 p 99
†1 K[2]MnO[4].

Peirce: CP 1.223 Fn 2 p 99
†2 K[2]RuO[4].

Peirce: CP 1.229 Fn 1 p 102
†1 In the *Essay on Classification*.

Peirce: CP 1.230 Fn P1 p 103 Cross-Ref:††
†P1 Ib., quarto 1857, p. 170. The reader will perceive by the date that these ideas were put forth at a somewhat inauspicious moment.

Peirce: CP 1.239 Fn P1 p 109 Cross-Ref:††
†P1 Some catholic writers recognize sciences resting upon authority. No doubt, everybody of good sense believes some things substantially because he has been brought up to do so; but according to my conception of what science is, *that* is not science. Indeed, belief proper has nothing to do with science. [Baldassare] Lablanca [*Dialettica*, vol. II, lib. IV, c. 1, 1875] admits a class of documentary sciences. This is more plausible; although, as that author admits, documentary evidence enters into every science, while nothing can have rested wholly on documentary evidence *to the original authors of the documents*. He reckons as documentary sciences, history, linguistics, political economy, statistics, and geography. But it is quite plain that these do not form a natural group; especially since this geography must include physical geography.

Peirce: CP 1.240 Fn P2 p 109 Cross-Ref:††
†P2 Many writers of France (as Comte and Ribot), and of Germany (as Schopenhauer and Wundt), and a few in England (as Cave), have given mathematics the first place among the sciences, contrary to the doctrine of Plato and Aristotle, which has caused so many to place it below philosophy in point of abstractness. I mention this to show that I am taking no revolutionary position here: I am open to charges enough of heresy to answer to, to make me desire to avoid those that can be avoided.

Peirce: CP 1.241 Fn 1 p 110
†1 "*Coenosopic* . . . from two Greek words, one of which signifies *common* -- things belonging to others in common; the other *looking to*. By *coenosopic ontology*, then, is designated that part of the science which takes for its subject those properties which are considered as possessed in common by all the individuals belonging to the class which the name ontology is employed to designate, *i.e.* by *all individuals*." *The Works of Jeremy Bentham*, Edinburgh, 1843, viii, 83, footnote.

Peirce: CP 1.242 Fn 2 p 110
†2 "*Idiosopic* . . . from two Greek words, the first of which signifies *peculiar*. In *Idiosopic ontology*, then, we have that branch of art and science which takes for its subject such properties as are considered as peculiar to different classes

of beings, some to one such class, some to another." *Ibid.*

Peirce: CP 1.243 Fn P1 p 111 Cross-Ref:††

†P1 Some writers so little comprehend the motives of science that they imagine all sciences are directed to practical ends, as was the general understanding before the Nineteenth Century. Thus [Luigi] Ferrarese in 1828 [in his *Saggio di una nuova classificazione delle scienze*] divided all sciences into three groups according as their purpose is to maintain health, to further perfectionment, or to prevent degradation; the first and third being separated by a thin partition indeed.

Peirce: CP 1.243 Fn 1 p 111

†1 The editors have abbreviated a very long list.

Peirce: CP 1.261 Fn 1 p 122

†1 See Poggendorff's *Biographisch-literarisches Handwörterbuch* (1883-1904), bk. IV, p. 1293, for an account of Sabanajeff's work and life.

Peirce: CP 1.264 Fn 1 p 123

†1 *Anthropology: As a Science*, by D. G. Brinton, published in pamphlet form, Philadelphia, 1892.

Peirce: CP 1.267 Fn P1 p 126 Cross-Ref:††

†P1 The opposite opinion is argued explicitly by Shields in his *Philosophia Ultima*, with as much force as the position allows.

Peirce: CP 1.273 Fn 1 p 129

†1 Cf. vol. 5, bk. I, lecture V, §1.

Peirce: CP 1.274 Fn 1 p 131

†1 That part of ch. 4 does not appear to have been written, see 584n. The nature of time is discussed at some length in vol. 6.

Peirce: CP 1.275 Fn 2 p 131

†2 See 4.142ff.

Peirce: CP 1.276 Fn 1 p 132

†1 No such diagrams accompanied the ms.

Peirce: CP 1.277 Fn 1 p 134

†1 That chapter was not written. But see vol. 4, bk. I, ch. 4, and vols. 6 and 8, *passim*.

Peirce: CP 1.279 Fn P1 p 135 Cross-Ref:††

†P1 It hardly need be said that *epistemology* is quite a different thing.

Peirce: CP 1.281 Fn 1 p 136

†1 Pensée 412, édition critique de *Pensées de Pascal* par G. Michaut, Fribourg, 1896.

Peirce: CP 1.282 Fn 2 p 136

†2 See 486ff. and vol. 6.

Peirce: CP 1.283 Fn 1 p 137

†1 This is the topic of ch. 3 of the "Minute Logic"; see vol. 4, bk. I, ch. 7.

Peirce: CP 1.283 Fn 2 p 137

†2 See M. Cantor's *Geschichte der Math.* II, Kap. 43-4.

Peirce: CP 1.283 Fn 3 p 137

†3 The remainder of this section of ch. 2 of the "Minute Logic" deals in considerable detail with the subdivisions of psychics and physics, and has been relegated to vols. 7 and 8.

Peirce: CP 1.284 Fn 1 p 141

†1 According to the scheme of classification given in the preceding book, phenomenology (or phaneroscopy) is the first division of philosophy, which is, in turn, the second of the sciences of discovery. The present book, to follow that scheme, should have been preceded by one on mathematics, the first of the sciences of discovery. Peirce's positive contributions to that science, however, are too technical for the general reader and his discussions of it are too closely interwoven with the discussions of other topics to make their inclusion in the present volume feasible. Most of the contributions to mathematics are to be found in vols. 3 and 4; the discussions regarding its nature are scattered throughout all the volumes; see *e.g.* 247ff.

Peirce: CP 1.284 Fn 2 p 141

†2 284 is from the "Adirondack Lectures, 1905"; 285-287 are from "Logic viewed as Semeiotics, Introduction Number 2, Phaneroscopy," c. 1904.

Peirce: CP 1.288 Fn 1 p 142

†1 From "{pl}," c. 1908.

Peirce: CP 1.291 Fn 1 p 144

†1 See the letters to Lady Welby. These ten trichotomies are not to be confused with the ten not completely independent classes of signs given in vol. 2, bk. II. The latter originate from only three trichotomies; while the ten trichotomies yield sixty-six not completely independent classes of signs.

Peirce: CP 1.292 Fn 1 p 145

†1 See vol. 4, bk. II.

Peirce: CP 1.292 Fn 2 p 145

†2 Cf. 346.

Peirce: CP 1.293 Fn 1 p 146

†1 From "The List of Categories: A Second Essay," c. 1894. 300 and 301 precede 293 in the ms.

Peirce: CP 1.294 Fn 2 p 146

†2 "The Basis of Pragmatism," Notebook I, c. 1905.

Peirce: CP 1.298 Fn P1 p 147 Cross-Ref:††

†P1 Thus stated, the principle does not seem to extend to abnumerable multitudes. Yet it must extend to them because, after all, the abnumerable is defined by means of combinations of two, and indeed must be so, since there is no form of combination not reducible to that.

Peirce: CP 1.300 Fn 1 p 148

†1 From "The List of Categories: A Second Essay," c. 1894. 293 follows 301 in the ms.

Peirce: CP 1.300 Fn 2 p 148

†2 James Harris, in his *Philosophical Arrangements* (1775).

Peirce: CP 1.300 Fn 3 p 148

†3 See ch. 6.

Peirce: CP 1.302 Fn 4 p 148

†4 From "The List of Categories: A Second Essay, X," c. 1894.

Peirce: CP 1.303 Fn 1 p 149

†1 From "The List of Categories: A Second Essay," c. 1894. 303 follows 293 and is followed by 326 in the ms.

Peirce: CP 1.304 Fn 1 p 150

†1 From "Logic viewed as Semeiotics, Introduction Number 2, Phaneroscopy," continuing 287.

Peirce: CP 1.305 Fn 1 p 151

†1 From "An Apology for Pragmaticism," intended for the January, 1907, *Monist*. See 4.540.

Peirce: CP 1.306 Fn 1 p 152

†1 From "Phaneroscopy {phan}" intended for the January, 1907, *Monist*. See 4.540.

Peirce: CP 1.312 Fn 1 p 155

†1 From "Definition," 1910.

Peirce: CP 1.312 Fn 2 p 155

†2 Reid, *Inquiry into the Human Mind*, ch. 6, sec. II. But cf. Locke's *Essay*, bk. II, ch. 4, §5.

Peirce: CP 1.313 Fn 1 p 156

†1 From "The Basis of Pragmatism," Notebook II, c. 1905.

Peirce: CP 1.313 Fn P1 p 156 Cross-Ref:††

†P1 "As to colors, there is a somewhat serious difficulty in regarding them as presentments, because we cannot regard them as simple elements as long as they are contaminated with space-extension, which is something easily distinguishable and is also plainly not *priman*, since the space cannot of its nature be limited. Now the color not only cannot be dissociated from space, but it cannot even be prescinded from it. It can only be distinguished from it. We may, however, *neglect* the spatial element, and so reduce its emphasis indefinitely; and I am inclined to think that somehow colors may be regarded as presentments, though I cannot very clearly thread my way through the difficulty." -- From "The Basis of Pragmatism," Notebook I, c. 1905.

Peirce: CP 1.314 Fn 1 p 157

†1 An undelivered (?) passage in Lecture IV of the "Lectures on Pragmatism," 1903.

Peirce: CP 1.317 Fn 1 p 159

†1 From "Pragmatism," Fragment 2, c. 1910.

Peirce: CP 1.322 Fn 1 p 161

†1 From "Lectures on Pragmatism," II, First Draught, c. 1903.

- Peirce: CP 1.323 Fn 2 p 161
†2 See vol. 6, bk. I, ch. 2.
- Peirce: CP 1.324 Fn 1 p 162
†1 From "Lowell Lectures of 1903." Lecture III, vol. 1, 3d Draught. See 343.
- Peirce: CP 1.325 Fn 2 p 162
†2 Unidentified fragment.
- Peirce: CP 1.326 Fn 1 p 163
†1 From "The List of Categories: A Second Essay," continuing 303.
- Peirce: CP 1.330 Fn 1 p 165
†1 Unidentified fragment.
- Peirce: CP 1.332 Fn 1 p 166
†1 From "Phanerescopy or the Natural History of Concepts," c. 1905.
- Peirce: CP 1.335 Fn 1 p 169
†1 Ibid.
- Peirce: CP 1.337 Fn 1 p 170
†1 Fragment, "Third," c. 1875.
- Peirce: CP 1.338 Fn 1 p 171
†1 338 and 9 are from an unidentified fragment; 340-2 are from a fragment, "Thirdness," c. 1895.
- Peirce: CP 1.339 Fn 2 p 171
†2 Cf. vol. 2, bk. II.
- Peirce: CP 1.343 Fn 1 p 173
†1 From the "Lowell Lectures of 1903," III, vol. 1, 3d Draught. See 324 and 521.
- Peirce: CP 1.345 Fn 1 p 175
†1 *Über die Erhaltung der Kraft, Einleitung* (1847). See 1889 ed. in Ostwald's "Klassiker d.E.W." series.
- Peirce: CP 1.346 Fn 1 p 176
†1 See vol. 4, bk. II.
- Peirce: CP 1.350 Fn 1 p 178
†1 From an unidentified fragment.
- Peirce: CP 1.353 Fn 1 p 179
†1 From "One, Two, Three," c. 1880.
- Peirce: CP 1.353 Fn 2 p 179
†2 See ch. 6.
- Peirce: CP 1.354 Fn 1 p 181
†1 c. 1890. One of the drafts of this work is headed: "Notes for a Book, to be entitled 'A Guess at the Riddle,' with a Vignette of the Sphynx below the Title." This caption is followed by the remark, "And this book, if ever written, as it soon will be if I am in a situation to do it, will be one of the births of time."

Peirce: CP 1.354 Fn 2 p 181
†2 3.359ff.

Peirce: CP 1.354 Fn 3 p 181
†3 Ch. 6 below.

Peirce: CP 1.354 Fn 4 p 181
†4 Vol. 2, bk. III, ch. 8.

Peirce: CP 1.354 Fn 5 p 181
†5 Vol. 2, bk. III, ch. 2.

Peirce: CP 1.354 Fn 6 p 181
†6 Vol. 3, No. III.

Peirce: CP 1.354 Fn 1 p 182
†1 The last two sections do not seem to have been written.

Peirce: CP 1.355 Fn 2 p 182
†2 The sections of this book were originally called "chapters." There are a number of alternative versions of this section. 1 and 2 of the preface, are from one such alternative.

Peirce: CP 1.362 Fn P1 p 187 Cross-Ref:††
†P1 The last view is essentially that of Christian theology, too. The theologians hold the physical universe to be finite, but considering that universe which they will admit to have existed from all time, it would appear to be in a different condition in the end from what it was in the beginning, the whole spiritual creation having been accomplished, and abiding.

Peirce: CP 1.369 Fn 1 p 193
†1 From "One, Two, Three: Fundamental Categories of Thought and of Nature," c. 1885. This paper does not seem to form part of "A Guess at the Riddle," but is here inserted to take the place of the unwritten section 2 of the original work.

Peirce: CP 1.369 Fn 1 p 194
†1 Formal Logic, ch. 8. See also 2.568.

Peirce: CP 1.369 Fn 2 p 194
†2 See vol. 2, bk. III, chs. 2 and 5.

Peirce: CP 1.369 Fn 3 p 194
†3 Or Terms, but see 372.

Peirce: CP 1.369 Fn 4 p 194
†4 Or Arguments.

Peirce: CP 1.372 Fn 1 p 196
†1 Cf. 369.

Peirce: CP 1.373 Fn 1 p 197
†1 No more of this section seems to have been written; but see vol. 6.

Peirce: CP 1.374 Fn 2 p 197
†2 Cf. vol. 8.

Peirce: CP 1.375 Fn 1 p 199

†1 A number of manuscript pages seem to be missing here. 376-8 from "One, Two, Three: Fundamental Categories of Thought and Nature," have been substituted for them.

Peirce: CP 1.383 Fn 1 p 204

†1 Some manuscript pages seem to be missing here.

Peirce: CP 1.391 Fn P1 p 209 Cross-Ref:††

†P1 Cards are almost never shuffled enough to illustrate fairly the principles of probabilities; but if after being shuffled in any of the usual ways, they are dealt into three packs and taken up again, and then passed from one hand into the other one by one, every other one going to the top and every other to the bottom of the pack that thus accumulates in the second hand, and finally cut, the shuffling may be considered as sufficient for the purpose of this game. Whenever the direction is to shuffle, shuffling as thorough as this is meant.

Peirce: CP 1.393 Fn 1 p 210

†1 The editors have been unable to locate this quotation.

Peirce: CP 1.394 Fn 1 p 214

†1 The following note is appended to this section: "Here the Chemical Idea." For this, see vol. 6, bk. I, ch. 8.

Peirce: CP 1.397 Fn 1 p 216

†1 This figure should be $51,952 \frac{7}{8}$. Because of this error the fifth and seventh figures in the next column are each 5,000 in excess, and, with the exception of the last two items, the entire last column is wrong.

Peirce: CP 1.399 Fn 1 p 218

†1 Cf. vol. 6, bk. I, ch. 10.

Peirce: CP 1.401 Fn 1 p 219

†1 See, *e.g.*, *General Investigations of Curved Surfaces*, art. 20.

Peirce: CP 1.402 Fn 2 p 219

†2 See 156.

Peirce: CP 1.403 Fn 1 p 220

†1 *e.g.* in the *Physics* 195b, 31-198a, 13.

Peirce: CP 1.403 Fn 2 p 220

†2 Bk. II, 1. 216-93.

Peirce: CP 1.405 Fn P1 p 221 Cross-Ref:††

†P1 After the scholastics; See Eckius [?] in Petrus Hispanus 48b, nota 1.

Peirce: CP 1.414 Fn P1 p 226 Cross-Ref:††

†P1 I use substance, here, in the old sense of a thing, not in the modern chemical sense.

Peirce: CP 1.417 Fn 1 p 227

†1 c. 1896. The first four pages of the manuscript are missing.

Peirce: CP 1.418 Fn 1 p 228

†1 Cf. 313; also vol. 6, bk. I, ch. 5.

Peirce: CP 1.428 Fn 1 p 234

†1 See vol. 6, bk. I, ch. 2, and 6.102.

Peirce: CP 1.441 Fn 1 p 240

†1 Cf. vol. 3, No. XVIII.

Peirce: CP 1.444 Fn 1 p 242

†1 *Opera Omnia Collecta*, L. Durand, T. 1, pp. 45-76.

Peirce: CP 1.449 Fn 1 p 245

†1 Appendix to the "Transcendental Dialectic."

Peirce: CP 1.457 Fn 1 p 248

†1 See 6.415.

Peirce: CP 1.469 Fn 1 p 252

†1 From {poieö}.

Peirce: CP 1.471 Fn 1 p 253

†1 Cf. vol. 2, bk. II, ch. 2, §3 ff.

Peirce: CP 1.485 Fn 1 p 259

†1 Cf. 2.620 ff.

Peirce: CP 1.487 Fn 2 p 259

†2 In his *Examination of Sir Wm. Hamilton's Philosophy*, ch. 11.

Peirce: CP 1.491 Fn 1 p 262

†1 Cf. 6.302.

Peirce: CP 1.504 Fn P1 p 270 Cross-Ref:††

†P1 A plane being the surface generated by a ray restricted to cutting two fixed rays which cut one another.

Peirce: CP 1.509 Fn 1 p 272

†1 See "On the Newtonian Constant of Gravitation," *Philosophical Transactions*, London, 1895, 186A, p. 69.

Peirce: CP 1.521 Fn 1 p 277

†1 From the "Lowell Lectures of 1903," Lecture III, vol. 2, 3d Draught, following 349.

Peirce: CP 1.521 Fn 2 p 277

†2 268a 11.

Peirce: CP 1.545 Fn 1 p 287

†1 The first section of this chapter was published with this chapter heading in the *Proceedings of the American Academy of Arts and Sciences*, vol. 7, May 1867, pp. 287-298. It was intended as ch. 1 of the Grand Logic of 1893 and as Essay II of the Search for a Method, c. 1893.

Peirce: CP 1.549 Fn P1 p 288 Cross-Ref:††

†P1 **Precision.** (1) A high degree of approximation, only attainable by the thorough application of the most refined methods of science.

(2) Its earlier meaning, still more or less used by logicians, is derived from a meaning given to *praecisio* by Scotus and other scholastics: the act of supposing (whether with consciousness of fiction or not) something about one element of a percept, upon which the thought dwells, without paying any regard to other elements. Precision implies more than mere discrimination, which relates merely to the essence of a term. Thus I can, by an act of discrimination, separate color from extension; but I cannot do so by *precision*, since I cannot suppose that in any possible universe color (not color-sensation, but color as a quality of an object) exists without extension. So with *triangularity* and *trilaterality*. On the other hand, precision implies much less than dissociation, which, indeed, is not a term of logic, but of psychology. It is doubtful whether a person who is not devoid of the sense of sight can separate space from color by dissociation, or, at any rate, not without great difficulty; but he can, and, indeed, does do so, by *precision*, if he thinks a vacuum is uncolored. So it is, likewise, with space and tridimensionality.

Some writers called every description of abstraction by the name *precision*, dividing precision into the real and the mental, and the latter into the negative and the positive; but the better usage named these *abstraction* divided into *real* and *intentional*, and the latter into *negative* (in which character from which abstraction is made is imagined to be *deniable* of the subject prescindend) and into *precise abstraction* or *precision*, where the subject prescindend is supposed (in some hypothetical state of things) without any supposition, whether affirmative or negative, in respect to the character abstracted. Hence, the *brocard: abstrahentium non est mendacium* (generally enunciated in connection with the *De Anima*, III, VII, 7). Scotus (in II *Physic.*, *Expositio* 20 *textus* 18) says: "Et si aliquis dicat, quod Mathematici tunc faciunt mendacium: quia considerant ista, quasi essent abstracta a motu, et materia; quae tamen sunt coniuncta materiae. Respondet, quod non faciunt mendacium: quia Mathematicus non considerat, utrum id, de quo demonstrat suas passiones, sit coniunctum materiae, vel abstractum a materia." This is not the place to treat of the many interesting logical, as well as psychological, discussions which have taken place concerning precision, which is one of the subjects which the scholastics treated in a comparatively modern way, although it leads directly to the question of nominalism and realism. It may, however, be mentioned that Scotus in many places draws a certain distinction variously designated by him and his followers (its nature and application is perhaps made as clear as anywhere in the *Opus Oxon.* III, xxii. qu. unica, "Utrum Christus fuerit homo in triduo," i.e. between the crucifixion and the resurrection), which the Thomists mostly dispute. There is some account of the matter in Chauvinus, *Lexicon* (2d ed.), under "Praecisio" . . . *Dictionary of Philosophy and Psychology*, vol. 2, pp. 323-4, Macmillan Co., New York, edition of 1911.

Peirce: CP 1.551 Fn P1 p 292 Cross-Ref:††

†P1 This agrees with the author of "De Generibus et Speciebus," *Ouvrages Inédits d'Abelard*, p. 528, [edited by V. Cousin, Paris, 1836].

Peirce: CP 1.558 Fn 1 p 295

†1 In later writings called "icons."

Peirce: CP 1.558 Fn 2 p 295

†2 In later writings an index is always taken to be but one of many kinds of signs; a sign being understood in some sense similar to that given in 540.

Peirce: CP 1.559 Fn 1 p 296

†1 See Peirce's definition in the *Century Dictionary* (1889) Intention 8; also Albertus Magnus, *Meta.* I, 1, 1, and Th. Aquinas, *Meta.* IV, 4, f. 43 v. A.

Peirce: CP 1.559 Fn P1 p 296 Cross-Ref:††

†P1 Herbart says [*Lehrbuch*, 2 A., 1te Kap., §34]: "Unsre sämtlichen Gedanken lassen sich von zwei Seiten betrachten; theils als Thätigkeiten unseres Geistes, theils in Hinsicht dessen, *was* durch sie gedacht wird. In letzterer Beziehung heissen sie *Begriffe*, welches Wort, indem es das *Begriffene* bezeichnet, zu abstrahiren gebietet von der Art und Weise, wie wir den Gedanken empfangen, produciren oder reproduciren mögen." But the whole difference between a concept and an external sign lies in these respects which logic ought, according to Herbart, to abstract from.

Peirce: CP 1.559 Fn 1 p 297

†1 Later called Speculative Grammar or Stechiology.

Peirce: CP 1.559 Fn 2 p 297

†2 Later called Critical Logic or Critic.

Peirce: CP 1.559 Fn 3 p 297

†3 Later called Speculative Rhetoric or Methodeutic.

Peirce: CP 1.559 Fn 1 p 298

†1 See vol. 2, bk. III, ch. 2.

Peirce: CP 1.559 Fn 1 p 299

†1 Vol. 2, bk. II, ch. 5.

Peirce: CP 1.560 Fn 2 p 299

†2 560-562 are from "Pragmatism," (Prag.[J]) c. 1905; 563 is from a fragment of a proposed "DI" Lecture c. 1898; 564-567 are from a fragment c. 1899.

Peirce: CP 1.560 Fn 3 p 299

†3 See also 2.340.

Peirce: CP 1.562 Fn 1 p 301

†1 "On the Syllogism IV, and the Logic of Relations," *Cambridge Philosophical Transactions*, vol. 10, pp. 331-358.

Peirce: CP 1.565 Fn 1 p 303

†1 3.93ff.

Peirce: CP 1.566 Fn 1 p 304

†1 558.

Peirce: CP 1.568 Fn 1 p 306

†1 1910.

Peirce: CP 1.568 Fn 2 p 306

†2 Apparently "The Quest of Quest -- An Inquiry into the Conditions of the Success in Inquiry (beyond the collection and observation of facts)," of which but a few pages were written.

Peirce: CP 1.572 Fn P1 p 308 Cross-Ref:††

†P1 No ingenuity can make such manufactured words invariably suggest their meaning. A tortoise may perhaps "look like a lizard," but it is hard to see how a

turkey or a heron does.

Peirce: CP 1.572 Fn 1 p 308

†1 The manuscript breaks off here.

Peirce: CP 1.573 Fn 1 p 311

†1 From the "Basis of Pragmatism." 1906. See 5.549.

Peirce: CP 1.573 Fn 2 p 311

†2 Peirce came to recognize the nature of the Normative Sciences at a very late date (c. 1903). He wrote practically nothing on esthetics (see 2.197) and linked most of his discussions of practics and ethics with those on pragmatism and logic. Logic, the third of the Normative Sciences, being the subject on which Peirce spent about sixty years of intensive study and on which he left the most manuscripts, is the special topic of Volumes Two, Three, and Four. The present book, accordingly, dealing as it mainly does with but two subjects insufficiently studied, is unusually, but necessarily, short and unsatisfactory.

Peirce: CP 1.574 Fn 1 p 313

†1 Cf. 5.130, 5.553.

Peirce: CP 1.575 Fn 1 p 314

†1 "Minute Logic," ch. 4, 1902-3.

Peirce: CP 1.575 Fn 2 p 314

†2 See 2.8n.

Peirce: CP 1.578 Fn 1 p 317

†1 See, e.g., *C. d. R. V.* A58, 320, 462.

Peirce: CP 1.584 Fn 1 p 321

†1 There are about five pages missing at this point. The manuscript then continues by repeating some of the foregoing, goes on to list a number of ultimate "ends" proposed by the early Greeks and concludes with a one-hundred-and-twenty-five-page discussion on the order, history, and contents of the Platonic Dialogues. Except for a short digression which will appear as §7-§8, ch. 11, bk. I, vol. 6, the rest of the manuscript will not be published.

Peirce: CP 1.585 Fn 1 p 322

†1 c. 1903.

Peirce: CP 1.585 Fn 2 p 322

†2 See vol. 9.

Peirce: CP 1.588 Fn 1 p 324

†1 The manuscript breaks off here.

Peirce: CP 1.589 Fn 2 p 324

†2 589-590 are from an untitled fragment, c. 1903.

Peirce: CP 1.590 Fn 1 p 325

†1 See next chapter and 5.3, 5.433, where Peirce identifies himself with the last group.

Peirce: CP 1.591 Fn 1 p 326

†1 From the "Lowell Lectures of 1903," Lecture I, vol. 1, 3d Draught;

611-615 from vol. 2, 2d Draught, which is a continuation of vol. 1, 3d Draught.

Peirce: CP 1.616 Fn 1 p 339

†1 The first lecture on "Detached Ideas on Vitally Important Topics," of 1898. It is entitled "Philosophy and the Conduct of Life."

Peirce: CP 1.617 Fn 2 p 339

†2 Theaetetus 174A.

Peirce: CP 1.617 Fn 3 p 339

†3 *Die Philosophie der Griechen*, etc. 5 Auf. 1 Teil (1892) S. 183n.

Peirce: CP 1.617 Fn 1 p 340

†1 Ib. S. 626n and S. 845n.

Peirce: CP 1.617 Fn 2 p 340

†2 Ib. 4 Auf. 2 Teil 1 Abt. (1889) S. 283n.

Peirce: CP 1.622 Fn 1 p 343

†1 See vol. 6, bk. I, ch. 7, §2.

Peirce: CP 1.629 Fn 1 p 345

†1 See "Introduction," vol. 4.

Peirce: CP 1.635 Fn 1 p 347

†1 *Fundamental Problems*, Open Court, Chicago (1891), p. 22.

Peirce: CP 1.649 Fn 1 p 351

†1 The remainder of this chapter is from an alternate version of the first lecture on Detached Ideas entitled, "On Detached Ideas in General and on Vitally Important Topics," 1898. A number of duplicate passages have been omitted.

Peirce: CP 1.650 Fn 2 p 351

†2 *Logic or Computation*, part I, ch. 1.

Peirce: CP 1.668 Fn 1 p 360

†1 Peirce was born in Cambridge, Mass., September 10, 1839, and lived there and in Milford, Pa., most of his life. For a time, however, he gave private instruction in logic at New York.

Peirce: CP 2 Title-Page
COLLECTED PAPERS OF CHARLES SANDERS PEIRCE

EDITED BY
CHARLES HARTSHORNE
AND
PAUL WEISS

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Peirce: CP 2 Introduction p iii
INTRODUCTION

Peirce liked to call himself a logician. For over fifty years he gave his best efforts to the development of logic, and there is justice in his contention that his point of view cannot be truly grasped without a mastery of his logical theories. His pragmatism, metaphysics, scientific analyses, historical studies, and even his experimental work depend in large part on these theories.

Peirce: CP 2 Introduction p iii

In the beginning Peirce planned to publish one original paper on logic every month, but he abandoned the attempt after his early papers were not well received. He continued, however, to write persistently on logical problems. His published papers and manuscripts cover almost the whole range of the subject. He seems to have read almost every available logical work, and to have paid particular attention to the logical writings of the Schoolmen and the English. Most of the manuscripts deal with logic. One is a complete book, his *Grand Logic* of 1893, for which he could not find a publisher. This is a detailed study of Aristotelian and Symbolic Logic, based largely on his own printed papers, some of which are included in the present volume. About ten years later he began to write his *Minute Logic* in twenty-three chapters, of which he unfortunately completed only three and a half. The present volume contains the first chapter and half of the second. The remainder may be found in Volume I. Volumes II, III, and IV contain the bulk of those logical papers which will be published. Volume III will include the published papers on Symbolic Logic; Volume IV, the unpublished writings on the foundations of mathematics and logic.

Peirce: CP 2 Introduction p iii

The theory of signs given in the present volume was subsequently expanded in the letters to Lady Welby, and is there shown to involve sixty-six classes of signs. Not all of these were analyzed, however. The present volume treats only of the ten classes of signs which Peirce analyzed in detail. His divisions, which reveal not only how his signs are classified, but why, for example, there is only one kind of abduction, two kinds of deduction, and three kinds of induction, are determined in part by the following principle: that which is a Second is divisible into two parts, of which one is itself divisible into two parts, etc., and that which is a Third is divisible into three parts, one of which is a First, another of which is a Second, (and thus divisible into two parts), and the last a Third, which is itself divisible into threes, and so on, apparently without end. This theory of signs is a new discipline; its application in detail he left for others. How closely he thought it to be connected with metaphysics and pragmatism can be seen from Volume I, book iii, chapter 6, and from Volume IV, book i, chapter 6.

Peirce: CP 2 Introduction p iv

The aspect of logic which seems to have interested him longest and most deeply, and which makes his studies significant even today, is scientific methodology, particularly the logic of discovery. This includes his development of the "frequency theory" of probability, his original theory of abduction, or the method of obtaining new ideas, and his novel treatment of induction which is shown to be closely related to the other two methods.

Peirce: CP 2 Introduction p iv

The papers in this volume are in some cases forty years apart, yet they do not differ as widely as might be expected, largely because of Peirce's practice of expanding, clarifying, and working over the theories he early developed. The later papers differ from the earlier in detail and in clarity rather than in point of view; and where they do so differ, Peirce has usually indicated what the difference is.

Peirce: CP 2 Introduction p iv

It has been particularly difficult to select representative papers for this volume, because of the necessity of choosing from a great number of manuscripts covering in divergent ways the same ground from the same point of view. For the theory of signs an interweaving of many papers was necessary; for the logic of discovery the published papers have been reprinted, and only those manuscripts which were directly relevant to them have been chosen. Even then the volume is disproportionately large. To have included all of his significant writings on these subjects would have made necessary a number of volumes and increased unduly the size of this edition.

* * *

Peirce: CP 2 Introduction p v

Wherever possible Peirce's punctuation and spelling have been retained. Titles supplied by the editors for papers previously published are marked with an *E*, while Peirce's titles for unpublished papers are marked with a *P*. Peirce's titles for previously published papers and the editors' titles for unpublished papers are not marked. Remarks and additions by the editors are enclosed in light-face square brackets. The editors' footnotes are indicated by various typographical signs, while Peirce's are indicated by numbers. Paragraphs are numbered consecutively throughout the volume. At the top of each page the numbers signify the volume and the first paragraph of that page. All references in the indices are to the numbers of the

paragraphs.

HARVARD UNIVERSITY

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. . . All that you can find in print of my work on logic are simply scattered outcroppings here and there of a rich vein which remains unpublished. Most of it I suppose has been written down; but no human being could ever put together the fragments. I could not myself do so.--1903

Peirce: CP 2.1 Cross-Ref:††
BOOK I

GENERAL AND HISTORICAL SURVEY OF LOGIC

ELEMENTS OF LOGIC

CHAPTER 1

CRITICAL ANALYSIS OF LOGICAL THEORIES†1

§1. LOGIC'S PROMISES*P*

1. Begin, if you will, by calling logic the theory of the conditions which determine reasonings to be secure. A conception at once more exact and more extended may be looked for in the sequel.†2 Meantime, this description will serve for our preparatory studies of the yet unvisited country into which we propose to push our explorations. Logic, then, is a theory. The end of any theory is to furnish a rational account of its object. . . . A theory directly aims at nothing but knowing. Maybe, if it be sound, it is likely, some day, to prove useful. Still, fairness forbids our making utility the criterion of the excellence of the theory. One must acknowledge that such a way of thinking would not abase logic, as it does the generality of the sciences. To judge of logic by its applications would only be to judge of it by how far it does, or does not, in a certain manner, help us to understand things. Clearly, it is the proper aim of any theory to do that. Looking at the matter on the other side, this very consideration tends to show that the touchstone of utility is superfluous and impertinent for logic. Suppose the science supports the test; what then? That would merely show that it aids the understanding. Grant that it is a sound theory, and it must have been prepared to do that, application or no application.

Peirce: CP 2.2 Cross-Ref:††

2. From this point of view, we can see that logic may be useful, apart from any direct application, by supplying us with modes of conception which are useful. There are few theories, if any, of which this is equally true.

Peirce: CP 2.3 Cross-Ref:††

3. For the most part, theories do little or nothing for everyday business. Nobody fit to be at large would recommend a carpenter who had to put up a pigsty or an ordinary cottage to make an engineer's statical diagram of the structure. In particular, applications of theories would be worse than useless where they would interfere with the operation of trained instincts. Who could play billiards by analytic mechanics? We all have a natural instinct for right reasoning, which, within the special business of each of us, has received a severe training by its conclusions being constantly brought into comparison with experiential results. Nay, we not only have a reasoning instinct, but as I shall early show,†1 we have an instinctive *theory* of reasoning, which gets corrected in the course of our experience. So, it would be most unreasonable to demand that the study of logic should supply an artificial method of doing the thinking that his regular business requires every man daily to do. . . .

Peirce: CP 2.4 Cross-Ref:††

4. Still, I repeat it, it is not in questions closely concerning a man's business that he can with any semblance of fairness look to finding the theory of reasoning helpful.†2 Such help is rather to be expected in extraordinary and unusual problems--especially in those of a speculative character, where conclusions are not readily checked by experience, and where our instinctive reasoning power begins to lose its self-confidence; as when we question what we ought to think about psychical

research,†3 about the Gospels, about difficult questions of political economy,†4 about the constitution of matter;†5 or when we inquire by what methods we can most speedily advance our knowledge of such matters.†6

Peirce: CP 2.5 Cross-Ref:††

5. But, as I said before, were direct applications of logic, such as these, never useful, instead of being frequently so, as they are, yet its indirect utility, through the useful conceptions with which it supplies us, would be immense.

Peirce: CP 2.6 Cross-Ref:††

6. Meantime, its highest and greatest value is that it affords us an understanding of the processes of reasoning. That the Platos are thoroughly right in that estimate will be more and more impressed upon our convictions as our acquaintance with the science grows.

Peirce: CP 2.7 Cross-Ref:††

7. On the other hand, we shall find reason to maintain, with Auguste Comte, that a theory cannot be sound unless it be susceptible of applications, immediate or remote, whether it be good economy so to apply it or not.†1 This is perhaps no more true of logic than of other theories; simply because it is perfectly true of all. Yet there is a special reason why it is more important to bear this point in mind in logic. Namely, logic is the theory of *right* reasoning, of what reasoning ought to be, not of what it is. On that account, it used to be called a *directive* science, but of late years Überweg's adjective *normative*†P1 has been generally substituted. It might be that a normative science, in view of the economies of the case, should be quite useless for any practical application. Still, whatever fact had no bearing upon a conceivable application to practice would be entirely impertinent to such a science. It would be easy enough--much too easy--to marshal a goodly squadron of treatises on logic, each of them swelled out with matter foreign to any conceivable applicability until, like a corpulent man, it can no longer see on what it is standing, and the reader loses all clear view of the true problems of the science. But since the relation of the theory of logic to conceivable applications of it will, by and by, come up for closer examination, we need not now consider it further.

Peirce: CP 2.8 Cross-Ref:††

§2. OF MINUTE ACCURACY

8. How shall the theory of right reasoning be investigated? The nature of the subject must be an important factor in determining the method. Before touching that, however, suppose we ask how, in the roughest sense, *any* theory ought to be investigated. Am I wrong in thinking I catch a whisper from good sense, that, for one item of the reply, a theory should be investigated *carefully and minutely*? Yet, strange to say, such a recommendation would be in flat contradiction to prevailing opinion. A month does not pass, scarce a fortnight will pass, without my attention being drawn to some new discussion by a man of strength relating to some broad, far-reaching question of science or philosophy. Every such dissertation will be sure to refer to principles of reasoning which are more or less contested. Upon the correctness of these the whole question hinges. How, then, do I find these logical principles are sought to be established? By the same severe and minute examination which the same author would approve in regard to a question of physics? Never: I am

very sure he would condemn such piddling minuteness as inappropriate to so broad a question. He proceeds slap-dash, depicting the logical situation as in a blackboard diagram rather than as in a critically accurate anatomical plate. For the most part, he has but the vaguest notion of how he has come by his principles. He has gathered them casually, after the custom of *amateurs*. It might seem to behoove every man who has occasion to lay down principles of reasoning in a grave scientific discussion to be more than an *amateur* in logic. Voluminous writers, however, on logic there are who deliberately adopt vague substitutes for any definite

method of establishing principles of reasoning.

Peirce: CP 2.9 Cross-Ref:††

9. When I was beginning my philosophical reading, my father, Benjamin Peirce,†1 forced me to recognize the extremely loose reasoning common to the philosophers. It was a matter open to the remark of every mathematician even before Weierstrass, when mathematical reasoning was far less strict than it has since become. The more recent philosophers certainly show an improvement in this respect. The metaphysics of our best contemporaries lacks but little of the rank of a science; but logical criticism has also grown more searching, and you may search the whole library of modern metaphysics from Descartes to the most accurate metaphysical reasoners †1 of today and hardly find a vital argument of an elaborate and apodictic kind that does not leave room to drive a coach and four through it.†1

Peirce: CP 2.10 Cross-Ref:††

10. The effect upon the minds of those who have been nourished on such food mainly, becomes deplorably patent to everybody who finds himself in contact with them. Their natural sense of logic is enfeebled and diseased. I am confident I detect something of this even in the majority of those young men who become known to me as only having paid particular attention to philosophy in the universities; although there are some whose logical instincts have been too robust to be so easily debilitated.

Peirce: CP 2.11 Cross-Ref:††

11. The Greek philosophers could not be persuaded that minute analysis was proper in physical science. Born Hegelian sensualists, they could not divest themselves of their belief that no worse way of getting at any comprehension of a flower could be devised than beginning by picking it to pieces, and so spoiling the flower. What was the result? Manifold have been the theories that have been successively offered, considered, and rejected, to account for the non-success of the Greeks in physics. That the vast intellect of an Aristotle, so great in zoölogy, in the science of politics, in rhetoric, in the history of philosophy; so gigantic in ethics, logic, metaphysics, and psychology, should, in physics, have sunk into abject inferiority to the cranks of modern times, the refuters of Newton, the proposers of perpetual motions, has hitherto not been adequately explained. What better account of the matter could one desire than that in physics the Hellenic element of Aristotle's nature--that Greek estheticism which forbade analysis and required that the phenomenon should be contemplated in its concreteness--here governed him? That this was the cause is shown by the fact that all the other Greeks who shared the same prejudice were equally unsuccessful; while the few who did not share it, Hipparchus, Eratosthenes, Posidonius, Ptolemy, Archimedes, were eminently successful in the physical sciences. In zoölogy, Asclepiad Aristotle, scion of a family whose every member, from the further prehistoric times, had been trained in medicine from childhood up, shared no Hellenic repugnance to dissection.†P1 Nor did that repugnance ever extend to the non-sensuous objects with which Aristotle dealt in the

sciences in which he most excelled.

Peirce: CP 2.12 Cross-Ref:††

12. In the northern Europe of the scholastic ages, the nullity of physics was due to a different cause, explicitly set forth by Roger Bacon.†1 Logic and metaphysics were studied with a considerable degree of minuteness and accuracy; so that, in spite of a barbaric civilization and other unfavorable influences, sufficiently obvious, they reached an excellence which our own generation has not been able to appreciate.

Peirce: CP 2.13 Cross-Ref:††

13. Our physical science, whatever extravagant historicists may say, seems to have sprung up uncaused except by man's intelligence and nature's intelligibility, which never could before be operative because it was not studied minutely. But modern philosophy had no such divine birth. On the contrary, it pays the usual tax upon inheritances from revolutions. It was the product of a double and triple revolution--the Renaissance, the Reformation, the Copernican revolution. It was, from the first, committed to a vendetta against all exact thinking and all punctual discussion. It has been a derelict on a vast ocean of surmise, drifting hither and thither, driven by storms, wrecked, shattered, its pieces dispersed even to opposite poles. Jetsam only is now occasionally cast up on the beaches of solid science, fraught with no cargo of wisdom except the one sad lesson of the issues of loose thinking.

Peirce: CP 2.14 Cross-Ref:††

14. Broad generalization is glorious when it is the inevitable outpressed juice of painfully matured little details of knowledge; but when it is not that, it is a crude spirit inciting only broils between a hundred little dogmas, each most justly condemning all the others. It is the usual fruit of sloth. A reader who is not disposed to work upon logic as slowly, as minutely, as laboriously as he would upon any other subject whatever--at the very least--simply will have to go without learning much about the theory of reasoning from any source.

Peirce: CP 2.15 Cross-Ref:††

15. The present volume, at any rate, is written by one who does not look upon the subject as an amusement. He will endeavor to make the doctrine present an exterior as little odious as possible, to correspond with its inward divine beauty, and to harmonize with the deep happiness the study brings. But he will not attempt to prostitute the science to the purposes of the purposeless. In order to enjoy it, it will be needful to have one's heart set on something remote from enjoyment.

Peirce: CP 2.16 Cross-Ref:††

16. The reader will often think that the writer makes far too much of microscopic distinctions. But in this science errors become enormously magnified in their effects; and stricter accuracy is for that reason rendered obligatory. No distinction--the reader may assure himself--will here be insisted upon without an adequate motive, even if it be a mistaken one.

Peirce: CP 2.17 Cross-Ref:††

17. The reader may find the matter so dry, husky, and innutritious to the spirit that he cannot believe there is any human good in it. Stuff which offers no images more enlivening than that "Socrates is a man and therefore mortal" or that of "a person who stands to some woman in the relation of a lover of every benefactor of hers," may be too desperately inhuman to be accounted by him anything but diabolic

idiocy, the product of hell's bedlam. But the fault is his. It shall not be more tedious than the multiplication table, that may be promised; and as the multiplication table is worth the pains of learning, unless one is a prince, so shall this be, even if one be a prince.

Peirce: CP 2.18 Cross-Ref:††
§3. DIFFERENT METHODS IN LOGIC^P

18. To the melancholy disintegration of philosophic thought due to loose reasoning and the aversion to minute analysis may be plausibly attributed the springing up of a swarm of different methods of establishing the truths of logic. A conspectus of the principal of these may usefully be given. The reasons for each can only be indicated in the most abridged form, and such criticisms may be added as can be made instructive in the present nascent stage of our inquiry.

Peirce: CP 2.19 Cross-Ref:††

19. First: Some writers maintain that the goodness and badness of reasonings is not merely indicated by, but is constituted and composed of the satisfaction and dissatisfaction, respectively, of a certain logical feeling, or taste, within us. This opinion is held by one of the acutest and most in fashion of the logicians of today, Christopher Sigwart, an authorized translation of whose logic is published in Swan Sonnenschein's "Library of Philosophy" in two octavos of something like three and three-fourths millions of words.†1 Before a man sits down to read them he will in prudence calculate how many words he is laying his plans to read during his lifetime in all, how many of these he will appropriate to logic, how many to any one of a dozen schools of logic, how many to any one author. Schröder, the leader of exact logic in Germany, the fourth volume of whose treatise we are impatient to receive,†2 follows Sigwart in the opinion mentioned--possibly even goes beyond him. Some readers of the English version may object that I misinterpret Sigwart, and that he holds the satisfaction of the feeling of logicality not to constitute true logicality, as I say, but merely to be the sole indication of it, in the last analysis. His phrase is "Der Glaube an das Recht dieses Gefühls ist der letzte Ankergrund aller Gewissheit überhaupt."†3 By "Recht," I here understand "rightful authority to give the decision which it claims to give"; and "Gefühl," not precisely what *feeling* means in English, but a purely subjective feeling of pleasure or pain, which renders no decision except concerning itself. The translator (with the authorization of Dr. Sigwart, who possibly does not appreciate the exact sense of the English) seems to take "Recht" in the sense of "Richtigkeit," and "Gefühl" in the vague English sense, and thus weakens the proposition. Yet even so, so long as this feeling is made the sole evidence of logicality, underlying every other, it is, to all intents and purposes, made the essence of logicality.

Peirce: CP 2.20 Cross-Ref:††

20. It may be remarked that nobody would care to study logic unless he had already made up his mind that men were so apt to err in their sense of logicality as often to reason ill, and unless he held the distinction between reasoning well and reasoning ill was that the former is conducive to the knowledge of the *truth*, and the latter not so, and that by truth is meant something not dependent upon how we feel or think it to be. Upon Sigwart's principle the distinction would be a mere distinction of

taste, or the satisfaction of a subjective feeling. This harmonizes only too well with the practice of German university professors, whose opinions are mainly founded on subjective feeling and upon fashion. In chapter 3, §6†1 we shall consider the argument by which this opinion is supported; and the reader will then be led clearly to understand how, without denying the existence of the logical sense, nor its intervention in all thought, I can maintain that it is extremely fallible, and that no appeal need or ought to be made to it in establishing the truths of logic. Judged by English standards and those which the present work aims to establish, Sigwart's teaching is calculated to undermine the vigor of reasoning, by a sort of phagedenic ulceration. So it would seem *a priori*; and *a posteriori* the impression made upon me by young reasoners who have been the most diligent students of Sigwart is that of debility and helplessness in thought.

Peirce: CP 2.21 Cross-Ref:††

21. Second: Since it must be nearly forty years since I read *La Logique* of the Abbé Gratry, a writer of subtlety and exactitude of thought as well as of elevation of reason, my account of his doctrine may not be accurate in its details. I insert it here because after feeling it seems natural to place any proposed method of basing logical principles upon direct individual experience. Now since those principles are general, only a mystical experience could give them. I cannot now assure myself that Gratry did so base any logical principles; but I remember that he considers every act of inductive reasoning in which one passes from the finite to the infinite--particularly every inference which from observation concludes that there is in certain objects of observation a true continuity which cannot be directly observed--to be due to a direct inspiration of the Holy Spirit.†2 Were that granted, however, consistency would require us to grant that the admission of a logical principle, which covers an infinity of possible inferences, is also a mystical experience.

Peirce: CP 2.22 Cross-Ref:††

22. Were I merely asked to grant that the anticipations of experience involved with (more accurately than "in") inductions cannot be accounted for except by the ancient hypothesis that man has been made in the image of his Maker, so far as his Reason goes, I should be compelled to admit this, for reasons which will be developed in good time.†1 But Gratry's doctrine is essentially different from that. Today, while I was putting my mare into her stable, in the dusk of the evening, I noticed a black streak upon the floor, which I at first took for a shadow. But upon closer inspection (for my eyes are not as good as they once were) I saw that it was a large black snake. I experienced a certain shock strong enough to enable me to perceive what that shock consisted in, namely, in a sense that the snake was there in spite of me. Now, even if I had anticipated seeing the snake, and even if, anticipating it, I had wished to see it, still, when I did come to see it, I should have experienced something of that same sense of being compelled to see it. Such a sense of compulsion, of a struggle between something within and something without, accompanies every experience whatever. How else can I distinguish between an experience and a play of fancy of extreme vividness, than by the sense of compulsion in the former case? And how can there be compulsion without resistance? Were Gratry right, then, every inductive reasoning which passes from observation of the finite and the discrete to belief in the infinite or the continuous ought to be accompanied by the sense that that belief was forced upon me, whether I will or no. That result, however, is, I believe, contradicted by observation. Instead of experiencing any such compulsion and struggle, I feel rather a sort of sympathy with nature which makes me sure that the continuity or the generality is there, somewhat as

I felt sure I understood the particular state of mind of my mare at the time I was putting her up.

Peirce: CP 2.23 Cross-Ref:††

23. Thirdly: The opinion just now referred to, that logical principles are known by an inward light of reason, called the "light of nature" to distinguish it from the "light of grace" which comes by revelation, has been the opinion entertained by the majority of careful logicians.

Peirce: CP 2.24 Cross-Ref:††

24. The phrase "light of reason," or its near equivalent, may probably be found in every literature. The "old philosopher" of China, Lao-Tze, who lived in the sixth century B.C. says for example,†P1 "Whoso useth reason's light, and turneth back, and goeth home to its enlightenment, surrendereth not his person to perdition. This is called practising the eternal." The doctrine of a light of reason seems to be inwrapped in the old Babylonian philosophy of the first chapter of Genesis, where the Godhead says, "Let us make man in our image, after our likeness." It may, no doubt, justly be said that this is only an explanation to account for the resemblances of the images of the gods to men, a difficulty which the Second Commandment meets in another way. But does not this remark simply carry the doctrine back to the days when the gods were first made in man's image? To believe in a god at all, is not that to believe that man's reason is allied to the originating principle of the universe?

Peirce: CP 2.25 Cross-Ref:††

25. The reasonings of the present treatise will, I expect, make it appear that the history of science, as well as other facts, prove that there is a natural light of reason; that is, that man's guesses at the course of nature are more often correct than could be otherwise accounted for, while the same facts equally prove that this light is extremely uncertain and deceptive, and consequently unfit to strengthen the principles of logic in any sensible degree.

Peirce: CP 2.26 Cross-Ref:††

26. But the Aristotelians, who compose the majority of the more minute logicians, appeal directly to the light of reason, or to self-evidence, as the support of the principles of logic. Grote †1 and other empiricists think that they have proved that Aristotle did not do this, inasmuch as he considered the first principles to owe their origin to induction from sensible experiences. No doubt, Aristotle did hold that to be the case, and held moreover, that the general in the particular was directly perceived, an extraordinarily crude opinion. But that process of induction by which he held that first principles became known, was according to Aristotle not to be recovered and criticized. It was not even voluntary. Consequently, if Aristotle had been asked how he knew that the same proposition could not be at once true and false, he could have given no other proof of it than its self-evidence.†P1 Grote and those who agree with him, as well as some other schools of thinkers, quite overlook the important distinction between thought that can be controlled and thought which cannot be controlled. It is idle to criticize the latter. You cannot criticize what you do not doubt; although very many philosophers deceive themselves and others into the belief that they are criticizing what they hardly pretend to doubt, and so "argue" for foregone conclusions. A proof or genuine argument is a mental process which is open to logical criticism. If, therefore, a philosopher holds that a judgment, C, has been derived from an antecedent judgment, B, by a process of thought subject to the mind's self-control, and that the judgment B has been derived from a still earlier state of mind, A, by a process of thought not controllable, he may represent the process by which B has

yielded C by a logical form of argument or proof; but the other process, by which B has been derived from A, has to be accepted as beyond criticism; and it cannot properly be represented by an argument, though it may be closely analogous to a logical argument in other respects. The truth is, that Aristotle, like all subsequent Greeks, unless we except the Epicureans, looked upon induction as logically indefensible. Consequently, where it was sub-conscious, it was treated as infallible, while it was not admitted into a scientific argument at all.

Peirce: CP 2.27 Cross-Ref:††

27. The conception of an argument or inference as a process only entitled to those designations by virtue of its being a subject of logical criticism is one which it is so important to grasp at the outset that I will here advert to another error of Aristotle's reasoning concerning first principles, which is unimportant in itself, but which leads to some further considerations on the nature of argument. Namely, Aristotle †1 argues that there must be certain first principles of science, because every scientific demonstration reposes upon a general principle as a premiss. If this premiss be scientifically demonstrated in its turn, that demonstration must again have been based upon a general principle as its premiss. Now there must have been a beginning of the process, and therefore a first demonstration reposing upon an indemonstrable premiss. This is an argument like the Achilles and Tortoise argument of Zeno, except that, instead of going forward in time, it goes backward.†1 If we were to admit that the process of thought in the mind is really composed of distinct parts, corresponding to the arguments of the logical representation of it, each requiring a distinct effort of thought, then, indeed, we should have to admit Aristotle's reasoning, unless we were prepared to admit that an endless series of distinct efforts could actually be performed in a finite time; just as we should have to admit that Achilles could never overtake the tortoise if he had to resolve to run to where the tortoise then was, and having arrived there, to form a new resolution to run to the point which the tortoise had then attained. This would involve the assumption that Achilles could not run unless he saw the tortoise ahead of him. In like manner, the assumption that the reasoning process, as it is in the mind, consists of a succession of distinct arguments, each having a previously thought premiss, involves the assumption that reasoning cannot begin with the very perceptions of sense, since in these perceptions the process of thought has not yet begun; so that they do not contain any judgments capable of being exactly represented by propositions, or assertions. If that be so, there must clearly be a first premiss. But there is no necessity for supposing that the process of thought, as it takes place in the mind, is always cut up into distinct arguments. A man goes through a process of thought. Who shall say what the nature of that process was? He cannot; for during the process he was occupied with the object about which he was thinking, not with himself nor with his motions. Had he been thinking of those things his current of thought would have been broken up, and altogether modified; for he must then have alternated from one subject of thought to another. Shall he endeavor, after the course of thought is done, to recover it by repeating it, on this occasion interrupting it, and noting what he had last in mind? Then it will be extremely likely that he will be unable to interrupt it at times when the movement of thought is considerable; he will most likely be able to do so only at times when that movement was so slowed down that, in endeavoring to tell himself what he had in mind, he loses sight of that movement altogether; especially with language at hand to represent attitudes of thought, but not movements of thought. Practically, when a man endeavors to state what the process of his thought has been, after the process has come to an end, he first asks himself to what conclusion he has come. That result he formulates in an assertion, which, we will assume, has some sort of likeness--I am inclined to think

only a conventionalized one--with the attitude of his thought at the cessation of the motion. That having been ascertained, he next asks himself how he is justified in being so confident of it; and he proceeds to cast about for a sentence expressed in words which shall strike him as resembling some previous attitude of his thought, and which at the same time shall be logically related to the sentence representing his conclusion, in such a way that if the premiss-proposition be true, the conclusion-proposition necessarily or naturally would be true. That argument is a representation of the *last part* of his thought, so far as its logic goes, that is, that the conclusion would be true supposing the premiss is so. But the self-observer has absolutely no warrant whatever for assuming that that premiss represented an attitude in which thought remained stock-still, even for an instant. If that is to be ascertained at all, it must be in some other way, probably by the physiologist. At present, such physiological surmise as we can make leans, I should say, toward the hypothesis that thought was in continual mutation. At any rate, physiology is not decisively against that hypothesis. Adopting that idea, the logical argument only represents the last part of thought, for the reason that it supposes a premiss which represents some attitude of thought which can only have resulted from thinking. Now if you only break off the last part of a time, you leave a previous time. If you break off the last part of this, you still leave a previous time; and there is no possibility of your breaking off so many last pieces that from what remains no last piece can be broken off. Hence, there is no necessity for a series of arguments representing a course of thought to have a first argument, before which there was no argument in the thought, in the only sense in which there was any argument at all, in the process of thinking. For there is no fact in our possession to forbid our supposing that the thinking-process was one continuous (though undoubtedly varied) process. At any rate, it is only the self-defence of the process that is clearly broken up into arguments. It is more than doubtful whether what we can state as an argument or inference represents any part of the thinking except in the logical relation of the truth of the premiss to the truth of the conclusion. And, moreover, the argument so considered consists in the statements in words. How nearly they represent anything really in the thought is very doubtful, and is quite immaterial. The real thinking-process presumably begins at the very percepts. But a percept cannot be represented in words, and consequently, the first part of the thinking cannot be represented by any logical form of argument. Our logical account of the matter has to start from a *perceptual fact*, or proposition resulting from thought about a percept--thinking in its own movement presumably of the same nature as that which we represent by arguments and inferences, but not so representable in consequence of a defect in that method of representation.

Peirce: CP 2.28 Cross-Ref:††

28. Descartes and others have endeavored to bolster up the light of reason by make-believe arguments from the "veracity of God," and the like. They had better not have pretended to call that in question which they intended to prove, since the proofs, themselves, call for the same light to make them evident. Besides, reason left to itself at least believes in its own pronouncements, while it refuses to sanction the pretensions to infallibility made in its behalf on the ground of sundry "veracities." The celebrated criterion of clearness and distinctness, proposed by Descartes,†1 and amended to little purpose by Leibniz,†2 was, as Hamilton says, "nothing new," since it was no more than an utterly unsuccessful attempt to define the old "self-evidence" of the axioms of reason.

Peirce: CP 2.29 Cross-Ref:††

29. The "Criterion of Inconceivability" is a touch-stone proposed for

ascertaining whether or not a proposition is necessarily true, consisting in trying whether or not its denial is inconceivable.†1 It is taken for granted that a proposition offered as necessarily true will itself be necessary, that is, it will not only be true under all circumstances [which actually do occur] but will be an assertion that something would be true under all circumstances whatsoever. Its denial, therefore, will assert merely that under some circumstances the proposition would be false. By this denial being inconceivable is meant that it is quite impossible to definitely realize in imagination a state of things in which that would be false which the denial asserts to be false. Those who believe in the criterion innocently think that in order to ascertain whether the denial is thus inconceivable, all they have to do is to try an easy experiment and so find out, at once, whether they can imagine the state of things in question, or not. But as J. S. Mill puts it, "the history of science teems with inconceivabilities which have been conquered."†2 What is required, therefore, is that "inconceivable" should mean not merely unrealizable in imagination today but unrealizable after indefinite training and education. It certainly must be admitted by every reasonable disputant, that every proposition whose denial is in that sense absolutely and eternally inconceivable is false.†3 This I will prove upon the spot. I say, then, that a reasonable disputant disputes because he hopes, or at least, goes upon the assumption that the dispute will come to something; that is to say, that both parties will at length find themselves forced to a common belief which will be definitive and final. For otherwise, why dispute? To reach a final and compulsory belief is, therefore, what the reasonable disputant aims at. But what he aims at is the truth. Therefore, by the truth he means nothing more than a finally compulsory belief. If, then, you can prove to him that a necessary proposition is such that there will be a final, unshakable compulsion preventing him from imagining it to be false, you have proved to him that it has those characters which he expresses by saying that the proposition is true. Now to say that a necessary proposition is true is to say that it is necessarily true. The proposition is therefore true by definition, if by inconceivable is meant eternally inconceivable. But a definition is not a criterion. For a criterion is a method of experiment by which something is ascertained which is a sure indication of whether or not something different, and less easy otherwise to find out, is true.

Peirce: CP 2.30 Cross-Ref:††

30. This criterion is specially deceptive for the reason that, in ordinary cases, if we ever can do a thing, we can already at least imagine some way in which it might be done. But if we cannot already imagine any way in which a proposition would be false, neither can we yet imagine any way of imagining it to be false. But for all that, the simplest little suggestion or information may at any moment put it into our power to imagine what no effort of thought could before enable us to imagine. It is like the egg of Columbus.

Peirce: CP 2.30 Cross-Ref:††

I will now give a single example of the inconceivabilities conquered with which, as Mill correctly says, the history of science teems. Euclid, some twenty-two centuries ago, laid it down as a "common notion," or axiom, evident to all men, that "a whole is greater than its part." For two millennia and more, this axiom was held to fulfill the ideal of an axiom better than any other, and when men wanted an example of an indubitable axiom, they commonly chose this. It is plain, therefore, that they could not realize in thought the truth of the contrary, try as they might. This is curious; for since Euclid's time and earlier it had never ceased to be a familiar truth that a finite magnitude added to an infinite one did not increase the latter. So, if during near 2200 years, among the millions of men who were continually declaring it

inconceivable that a part should be as great as a whole, it had ever occurred to a single one to think how it would be if the part were infinite, it would have been all up with the immaculate fame of the axiom from that moment. At length, in the last quarter of the nineteenth century, somebody, I think it was Dr. Georg Cantor,^{†1} said: The even numbers are a part only of all the integer numbers. But every whole number has a double which is the double of no other integer number. So, for every integer number there is a separate and distinct double; and thus the doubles are as many as the integer numbers. But these doubles are all even numbers; and so, the partial collection is as great as the whole collection.

Peirce: CP 2.31 Cross-Ref:††

31. . . . I may as well notice here a style of reasoning much used for the support of instinctive beliefs. I mean the transcendental method. . . . The method was the invention of Kant, and in his hands it consists in showing, by some ingenious argument--different in every case--that the logical analysis of the process which the mind must go through shows that the proposition which is to be defended is involved in the *a priori* conditions of the possibility of practical everyday experience. If Kant had performed all the work which a thorough, scientific application of his method demanded, he would have had to postpone the publication of his *Critic of the Pure Reason* for another century, at least, which would have been regrettable. It would be radically contrary to Kant's principles to base logic (in the sense in which the word is used in this present book) upon the transcendental method. On the contrary, his whole critic of the understanding is deliberately based upon a scientific logic supposed to be already established. It is singular that, notwithstanding the gigantic logical strength of the *Critic of the Pure Reason*, and notwithstanding Kant's explicit teaching that this hinges upon the scientific perfection of the underlying formal, or ordinary, logic, yet he never touches this last doctrine without betraying unmistakable marks of hasty, superficial study.

Peirce: CP 2.32 Cross-Ref:††

32. What has been said of the utter impracticability of any one man's actually executing the design of the *Critic of the Pure Reason* is a hundredfold more true of Hegel's *Logik* (which, it must be understood, little more than incidentally treats of what we call logic). That work cannot justly be regarded as anything more than a sketch, notwithstanding a depth of thought in details which I believe in more on the testimony of other readers than because I have felt it myself. Hegel is also said to pursue the transcendental method; but his method has, it appears to me, only a very general and slight resemblance to Kant's. The striking features of Hegel's procedure are only just discernible here and there in Kant. I will first describe his method generically, without adverting to certain most characteristic special features; because in this generic sense it has been adopted by many minds who would not accept it in its special form. Hegel begins, then, by assuming whatever appears most evident to an utterly unreflecting person, and sets it down. The only difference between the unreflecting person and Hegel, as he is in this mood, is that the former would consider the subject exhausted, and would pass to something else; while Hegel insists upon harping on that string until certain inevitable difficulties are met with. Hegel at once embraces these objections with the same good faith (for it is good faith, notwithstanding his being able, if he chooses, to see further) with which he assumed the original position. He pushes his objection for all it is worth--for more than it is worth, since the original position has something to say for itself, in reply. Hegel is anxious not to allow any "foreign considerations" to intervene in the struggle which ensues--that is to say, no suggestions from a more advanced stage of philosophical

development. I cannot see that it would conflict with the spirit of the general method to allow suggestions from experience, provided they are such as would be inevitable, and such as would be within the grasp of the thought which for the moment occupies the theatre.

Peirce: CP 2.32 Cross-Ref:††

From this dispute there will ultimately emerge a final, definitive opinion, due to a new way of conceiving the facts. So, at least, the disputants hope and trust; and so we find it. The first unreflective view, so far as it is not the mere accident of individual caprice or of a singular individual ill-luck, is allowed to have a certain legitimacy; but the new conception alone puts the matter in a scientific light. It must be acknowledged that if this method of thought can be carried out, it is the best possible. It must represent the history of thought, too, as far as that history is not merely accidental. Now, considering the immense multitude of minds, and that it is the normal minds who are influential, the history of thought can hardly be very seriously accidental. It is true that there will be, in history, perturbations, due to the fact that all minds are not at exactly the same stage of development. It would certainly be a most valuable method of thinking which not only should bring a man to a true thought, but to that thought which ought to be dominant with his contemporaries, and will become so in the main. But will this second conception, finality though it be, from its own a point of view, a {ktéma ex ae}, as my friend Dr. Paul Carus †1 says, remain a fixity without growth or change, such as the eternal hills are not so devoid of life as to be? No, indeed, says Hegel, and every man will echo his answer. It is of the nature of thought to grow; and now, not caring to adhere strictly to Hegel's text (as nobody animated by his spirit does, any longer), but guided rather by the history of thought, I ought to indicate in such rough way as I am prepared to do, some of the directions that growth will take.

Peirce: CP 2.33 Cross-Ref:††

33. The new conception will not be content to be restricted to the particular phenomena it was devised to explain: it will insist upon applying itself to analogous phenomena, and to others analogous to these again, without stint. For that purpose it must be widened and probably simplified and rendered more agreeable to reason. It will not be content with explaining the history of thought, but will aim to explain history in general. It will not be content with accounting for man, but will wish to grasp all the forms in the universe, which is greater than man. It will not be content with an accidental universe, but will wish to assimilate every possible universe that the mathematician can suggest. It will not be content with allowing to the unreflective view a sort of subordinate legitimacy, but will insist upon elevating it to a truth in full harmony with its own.

Peirce: CP 2.34 Cross-Ref:††

34. These tendencies are irrepressible: in the long run they will cause that which they need to come into being. But much more than that, they are thoroughly reasonable; and that which they call for ought to be. Now that which they demand above all is the fact and the admission that the world is reasonable--reasonably susceptible to becoming reasonable, for that is what it is, and all that it is, to be reasonable †2--or in other words, that man is made after his maker's image.

Peirce: CP 2.35 Cross-Ref:††

35. Such is my rude notion of what the method is to which Hegel endeavors to impart exactitude. Vague applications of it recommend themselves to my faith; but I have never met with an attempt to state a transcendental argument with precision

which began to convince me. At any rate, when I reexamine the logics of more or less Hegelian tendency which have appeared in the last quarter of a century, I must decline to allow any weight to such flummery. I do not mention earlier German works, because they are still worse.

Peirce: CP 2.36 Cross-Ref:††

36. Fourthly: Some writers make it the boast of their systems of logic that they rest upon a philosophical basis; others really use the same method, though they rather keep the fact in the back-ground, despite the good ring of saying that one's logic is philosophical. Only, if logic is to be a pavillion on the roof of metaphysics, then metaphysics cannot conveniently be made an upper story of logic, as Aristotle and Kant, the two greatest of metaphysical systematizers, would have it to be. To me, it seems that a metaphysics not founded on the science of logic is of all branches of scientific inquiry the most shaky and insecure, and altogether unfit for the support of so important a subject as logic, which is, in its turn, to be used as the support of the exactest sciences in their deepest and nicest questions.†1

Peirce: CP 2.37 Cross-Ref:††

37. Some of the most celebrated logics, however, are written from the points of view of metaphysical sects. The *Organon* of Aristotle, the title which the collection of his logical treatises received, probably at the hands of Andronicus, the Rhodian,†2 who put them together about 60 B.C.†P1 opens with a metaphysical book, the Categories, or Predicaments, although in that same treatise Aristotle evidently bases the metaphysics upon a grammatico-logical analysis of the Greek sentence. To this book was usually prefixed the treatise of Porphyry on the Predicables. About half the scholastic works on logic are commentaries on the collection of books so formed. These works, therefore, base logic on metaphysics to some extent.

Peirce: CP 2.38 Cross-Ref:††

38. Of logics which in modern times more or less take for granted special systems of metaphysics, the earliest were a series of Aristotelian treatises. Gassendi †1 wrote an Epicurean logic, but of course knew nothing of the book of Philodemus on induction which turned up, centuries later, in Herculaneum.†2 The celebrated Port Royal logic entitled *L'Art de Penser*, of Antoine Arnauld (published in 1662) and Clauberg's *Logica Vetus et Nova* [1654], were Cartesian works. Malebranche wrote his own *Recherche de la Vérité* (1674-5) which likewise professed to be Cartesian, but was in truth rather Malebranchian, like much else called Cartesian. The *Medicina Mentis* of Graf von Tschirnhaus auf Kesselwalde [1687], today a familiar name to algebraists, stole the thunder of Spinoza. Locke's philosophy was represented in the treatises of Crousaz †3 Isaac Watts,†4 and perhaps we may add of Condillac.†5 Leibnitzianism, systematized by Wolff, numbered its logics by scores. Kantianism had its Krug †6 and Esser,†7 to mention only those of whom English readers are likely to know something; and every subsequent German philosopher, Baader, Fichte, Schelling, Hegel, Schleiermacher, Schopenhauer, von Hartmann, Dühring, down to Häckel, has been followed by his train of logicians.†8 Thus, a large proportion of all the logics that have ever been written have more or less pursued this vicious order of thought.

Peirce: CP 2.39 Cross-Ref:††

39. Fifthly: Considerable controversy has taken place as to whether scientific results of psychology ought or ought not to be admitted among the premisses from which logical principles are to be deduced. Upon the affirmative side are the Wolffian

logicians †1 and many others, older and newer. J. S. Mill (*Examination of Hamilton* xx. 5th ed. p. 461) pronounces that "its [logic's] theoretic grounds are wholly borrowed from Psychology." A smaller but more powerful force is arrayed against the method. Kant holds that psychology has no influence upon logic. But then we are to remember that, according to Kant, nothing worth mention had been contributed to logic since Aristotle †2 Herbart, however, had studied the subject to the best of his high powers; and his declaration is, "In der Logik ist es nothwendig alles Psychologische zu ignorieren." Among the thoroughly unpsychological works of later date is to be mentioned Riehl's *Philosophische Kriticismus*, 2 vols., 1876-9. Venn, †3 though a follower of Mill, whose logic is representative of associationalism, may be classed among those who do not make use of scientific results of psychology. Nobody will do injustice to the present treatise by describing its position as extremely unfavorable to the use of psychology in logic.

Peirce: CP 2.40 Cross-Ref: ††

40. Of the arguments upon the affirmative side, none is stronger than the following. It is said that not to utilize in logic the results which psychologists have obtained concerning abstraction and concerning association would be a wasteful proceeding. It is also maintained that precepts setting forth how we *ought* to reason can have no rational basis except the science of the laws which describe how we *must* reason; and that, in fact, if this latter part of logic, which is, at the same time, a part of psychology, be dropped out, the purely normative part, which remains, is nothing but an auxilliary practical art.

Peirce: CP 2.41 Cross-Ref: ††

41. The general answer to all this is that it only illustrates the dire confusion brought into both sciences when they are not so understood as to be widely distinct from one another. The logicians of the period from Descartes to Kant cannot be much blamed for seeing little distinction between psychology and logic, inasmuch as the psychology of their days, whether rational or empirical, consisted in little else than a logical analysis of the products of thought, as every psychologist of our day will admit. Even the pupils of James Mill, for all the power of his extraordinary *Analysis of the Human Mind* [1829] were far from understanding psychology as it is now understood.

Peirce: CP 2.42 Cross-Ref: ††

42. Modern psychology has made an admirable beginning. Every student of exact science who is in a situation to judge of what has been accomplished in this department must applaud it with his whole heart. Yet that it is only a beginning is shown by its present tendency to turn upon its axis, without making any great advance. Matters of brain-physiology and matters of consciousness elbow one another in unsympathetic juxtaposition, in a way which can only be transitional, and is a sign for us, as well as we can look forward to conceptions not yet attained, that psychologists do not yet understand what mind is, nor what it does. I am not at all prepared to clear the matter up; but I dimly discern, I think, that the physiological view has not sufficiently affected the introspective aspect; and possibly the converse is true, also. Perhaps the introspection is mainly illusion due to quasi-logical interpretations. †1

Peirce: CP 2.43 Cross-Ref: ††

43. But let me not be understood as having much faith in the current Wundtian monism. At any rate, however, it is pretty clear that there must be an application of scientific logic in order to separate the precipitate of physiology from the filtrate of

logic. In this state of things, or in the real state of things which I am perhaps very mistakenly endeavoring to peer into, it ought, I think, to be admitted that psychological conceptions are not sufficiently matured to afford a safe foundation for any part of logic; and it seems still more evident that it is very much in the interest of psychology itself to restrain it from flowing over into the region of logic.

Peirce: CP 2.44 Cross-Ref:††

44. As for abstraction, I shall endeavor, in the course of this volume, to put before the reader reasons for thinking that it is a wholly unpsychological matter, the doctrine of which has gone far astray in consequence of the admixture of psychology with it.†2

Peirce: CP 2.45 Cross-Ref:††

45. Something like psychological association certainly appears in logic; but in order that the relation of logical association to psychological association, a relation more interesting to the psychologist than to the logician, may become clearly understood, it is desirable that the two theories should be developed separately and side by side.†1

Peirce: CP 2.46 Cross-Ref:††

46. The notion that a normative science is necessarily of the nature of a practical art, in having no independent value as a pure theory, is one which, no doubt, arises naturally enough from a superficial survey. But in another chapter †2 we shall have occasion to inquire somewhat closely into the nature of the different sciences; and we shall then find that, so far is it from being true that the normative character must necessarily be exclusively due to the branch of knowledge that possesses it being a mere concrete application to a practical need of a theory which, in its pure development, never considered that need, that, on the contrary, this character may equally have its origin in the circumstance that the science which presents it is so very abstract, so alien to any experiential lineage, that ideals alone, in place of positive facts of experience, can be its proper objects.

Peirce: CP 2.47 Cross-Ref:††

47. It is J. S. Mill †3 who insists that how we *ought* to think can be ascertained in no other way than by reflection upon those psychological laws which teach us how we *must needs* think. But here we have to distinguish the case in which the compulsion attaches to that subconscious thought over which we have no control, and the case in which it attaches to conscious reasoning. In the former case, there is no room for logical criticism, at all. But because there is nothing to be said against our thinking in a certain way, in subconscious thought, when we cannot do otherwise, it does not, at all, follow that we ought to think in that way when we have our choice between several ways of thinking. If, however, Mill refers to a compulsion attaching to conscious thought, what he no doubt has in mind is, that a person ought to think in the way in which he *would be* compelled to think, if he duly reflected, and made his thoughts clear, and brought his whole knowledge to bear. But when he asserts that in such a case there is *no other reason* to be given for thinking in a given way than barely that the thinker is under compulsion so to think, is he not applying that Criterion of Inconceivability against which we have heard him fulminate in his finest style? It is true that Mill does not say that there is no other reason in support of the conclusion, but only that there is no other reason why the reasoner ought to accept the conclusion. But this makes no pertinent difference; the arguments against the Criterion apply in this case.

Peirce: CP 2.48 Cross-Ref:††

48. As before, we are to distinguish between an absolute definitive compulsion of thought and a limited compulsion. To say that the reasoner would, if he reflected sufficiently, be under an absolute definitive compulsion to hold a certain opinion is, as was shown in the discussion of the Criterion, neither more nor less than to assert that the opinion is true. Now to say that a reasoner ought to believe something *for no other reason* than that it is true, is to say that there is no reason at all why he ought to do so. Besides, it is nonsense to say that the science of psychology establishes such a proposition, unless the subject of the belief happens to be the operations of the mind.

Peirce: CP 2.49 Cross-Ref:††

49. Nor can it be maintained that there is a distinct logic for reasonings whose conclusions are psychological. The evident truth is, that psychology never does prove a compulsion of thought of an absolute definitive kind for conscious operations of the mind. It establishes only associational compulsions; and with conquests over these Mr. Mill tells us the history of science teems.

Peirce: CP 2.50 Cross-Ref:††

50. Let us, however, come a little closer to the concrete. I do not remember any treatise on logic which tells the reader that if Sortes is a man, and all men are mortal, then Sortes *ought to be thought* mortal. Mill's *Logic* certainly says no such thing. What they all say is that Sortes must *be* mortal. Logical treatises never say anything about what "ought to be thought" as long as there is any compulsion of thought or reflection. In those cases they only speak of how the facts are. It is where there is no such compulsion that the "ought" finds room. Thus, we are told that we ought to try simple hypotheses before complex ones. But how such a maxim can be supported upon associational principles alone--which is what Mill must mean by laws showing how we must think, which have been discovered by psychology--I confess I do not see; nor do I find anything in Mill's *System of Logic* to help me to see it.

Peirce: CP 2.51 Cross-Ref:††

51. Psychology must depend in its beginnings upon logic, in order to be psychology and to avoid being largely logical analysis. If then logic is to depend upon psychology in its turn, the two sciences, left without any support whatever, are liable to roll in one slough of error and confusion.

Peirce: CP 2.52 Cross-Ref:††

52. Sixthly: Other authors, indeed, a large majority of logicians, without citing results of scientific psychology in support of the principles of logic, yet incessantly refer to *data* of psychology--or to what would ordinarily be so considered, apparent self-observations that we think so and so--as showing what the truths of logic are. All this is beside the purpose. Logic is not the science of how we *do* think; but, in such sense as it can be said to deal with thinking at all, it only determines how we *ought* to think; nor how we ought to think in conformity with usage, but how we ought to think in order to think what is *true*. That a premiss should be pertinent to such a conclusion, it is requisite that it should relate, not to how we think, but to the necessary connections of different sorts of *fact*.

Peirce: CP 2.53 Cross-Ref:††

53. Herbart made a statement very nearly correct when he said that a *thought* (Begriff), in the sense in which alone logic deals with thoughts, is not a thinking but

that which a thinking brings before the mind. The emphasis here is in the denial contained in the first clause. The rest plays no part in his treatise; and my only objection to his statement is that what the logician calls a thought, if he uses that mode of expression, cannot with pertinence signify even what is brought before the mind in thinking. If it be so brought before the mind (which seems improbable), it is of no concern to the logician.

Peirce: CP 2.54 Cross-Ref:††

54. Logic deals with the relations of knowledge and with arguments, or inferences. It may be that knowledge cannot be realized without somebody's thinking something corresponding to it, in the same sense in which color cannot be realized unless somebody sees it. "Full many a flower is born to blush unseen," seems to me to be true, nevertheless--decidedly more accurate than, "Full many a flower is of such a nature that it would blush, if light were irregularly reflected from it to my eye; but it never does blush, because I am not at this moment looking at it, or if I am, sufficient light is not shining upon it." On the same principle, I think it more accurate to say that there are stores of knowledge in a library than to say that there are books there which, if I were reading, having become acquainted with the languages they are printed in, would be conveying knowledge to my mind. Surely, knowledge is decidedly more objective than color. This, however, is, for our present purpose, a secondary point. What is more important is that it certainly seems that an inference cannot have any life unless somebody performs a thinking process. Still, the inference, or argument, so far as logic can take any cognizance of it, may be, not to say probably is, of an entirely different construction from the thinking process, as I have already pointed out,†1 in considering Aristotle's view that there must be first principles of science.

Peirce: CP 2.55 Cross-Ref:††

55. If the reader will look at any theorem of Euclid (it had better be from the first book, which alone is drawn up with great logical care), he will notice that the proposition is first enunciated in abstract terms. This is the form of embodiment of it which is suitable for storage. But having so stated it, Euclid constructs a figure, and reënunciates the proposition with reference to that figure. Since this figure may be *any* figure conforming to the conditions of the abstract statement, the new enunciation is precisely equivalent to the original one, in meaning. The difference is, that from the abstract enunciation, left abstract, no inferences could be drawn, except such trivial syllogistic ones as the traditional treatises on logic alone consider, together with certain inferences of the same trivial character which the logic of relatives sets forth. In order to reason to any good effect from or to the abstract enunciation, it is necessary to bring it into this second shape. The new enunciation, the *exposition*, as it is termed, having been obtained, certain changes are made in the figure, consisting either in moving certain parts of it, or in adding new lines, or both, whereby we are led to perceive that those same relations hold good which are expressed in the second enunciation. This pointed out in the words "which had to be demonstrated," Euclid breaks off, without having actually reached the abstract statement. But had anybody asked him what he had proved, he would have repeated that abstract enunciation. In like manner when we have finished a process of thinking, and come to the logical criticism of it, the first question we ask ourselves is "What did I conclude?" To that we answer with some *form of words*, probably. Yet we had probably not been thinking in any such form--certainly not, if our thought amounted to anything. Our whole logical criticism consists in investigating whether or not to one portion of knowledge, expressed presumably in a very different form from that in which it was thought, we can, without serious danger of error, attach a certain addition. What the

process of thinking may have been has nothing to do with this question. There may, for aught we know or care, be a hundred ways of thinking in passing from such a premiss to such a conclusion. But the question is, whether, granting that there be such a thing as truth, which can be ascertained at all, such a way of adding conclusion to premiss, as that under examination, would lead to the ascertainment of the truth by the speediest path, or not. The whole logical inquiry relates to the *truth*; now the very idea of truth is that it is quite independent of what you or I may think it to be. How we think, therefore, is utterly irrelevant to logical inquiry.

Peirce: CP 2.56 Cross-Ref:††

56. I must be excused for dwelling on this point, for no other in all logic, although it is a science of subtleties, is so hard to see. The confusion is embedded in language, leaving no words available to epigrammatize the error. Now it is not of fools exclusively, but of the greater part of the thinking world that words are the money. A celebrated treatise [by Hobbes] is entitled *Logic, or Computation*, and although not all reasoning is computation, it is certainly true that numerical computation is reasoning. But calculating machines are in everyday use; and Babbage's analytical engine would perform considerable feats in mathematics. Other logical machines have been constructed.†1 All those instruments perform inferences; and those inferences are subject to the rules of logic. If from true premisses they always yield true conclusions, what more could be desired? Yet those machines have no souls that we know of. They do not appear to think, at all, in any psychical sense; and even if we should discover that they do so, it would be a fact altogether without bearing upon the logical correctness of their operations, which we should still have to assure ourselves of in the same way we do now.

Peirce: CP 2.57 Cross-Ref:††

57. The idea I am endeavoring to convey is difficult to seize. It cannot be seized by a reader who, instead of trying to seize it, puts himself into a resolutely hostile posture of mind toward it. Does he wish to know what I mean? If so, let him postpone criticism until he clearly apprehends what it is that is to be criticized. There are many minds who will be so occupied with a certain objection that it will quite eclipse the sense of what I have been saying. They will urge that those machines do not perform any inference, at all; that it is, on the contrary, we, who, knowing how they are constructed (be it in detail or in a general way, by testimony), infer that the number which is exhibited at the end of the process must be in a certain arithmetical relation to the numbers which determined the setting and working of the machine.

Peirce: CP 2.58 Cross-Ref:††

58. It will further be urged that if those machines were to be regarded as reasoning, there are others which would reason in far higher ways. For the calculating machines only execute variations upon $1 + 1 = 2$, while there are machines which may, with as much justice, be said to resolve problems before which generations of able mathematicians have fallen back, repulsed. Such, for example, are the solids of different shapes which yacht-designers drag through the water, and thereby come to the knowledge of arcana of hydrodynamics. Blocks of wood should seem, then, on my principles to be better reasoners than the brains of Gauss and Stokes. And why stop here? Any apparatus whatever used for experimentation would be, on the same principle, a logical machine. A steam-engine would be working out, at every revolution, its problem in thermodynamics; a simple match, scratched on a box, a question that we are unequal to so much as the formulating of.

Peirce: CP 2.59 Cross-Ref:††

59. This sounds crushing. What have I to say to it all? Simply that it is absolutely just. A logical machine differs from any other machine merely in working upon an excessively simple principle which is applied in a manifold and complex way, instead of upon an occult principle applied in a monotonous way. If anybody wishes me to acknowledge that a logical machine reasons no more than any other machine, I do not know why I should not gratify him. That seems to me a matter of words. The result which the logical machine turns out has a relation to the data with which it was fed, which relation may be considered from the point of view of whether the former could be false so long as the latter are true. That is all there is in the facts of the case; and whether it is called reasoning or not I do not care. All that I insist upon is, that, in like manner, a man may be regarded as a machine which turns out, let us say, a written sentence expressing a conclusion, the man-machine having been fed with a written statement of fact, as premiss. Since this performance is no more than a machine might go through, it has no essential relation to the circumstance that the machine happens to work by geared wheels, while a man happens to work by an ill-understood arrangement of brain-cells; and if there be room for less, still less to the circumstance that a man thinks. Say, if you like, that thinking has everything to do with the life of reasoning; I still insist that it has nothing to do with the logical criticism, which is equally applicable to the machine's performances and to the man's. This is simply the question of whether or not the conclusion can be false while the premiss is true. Were it a question of whether man *can* reason ill, it might be well to examine the process and mechanism of his thinking. But there is no question that he often does reason ill; and that is the reason why we criticize reasoning and why we inquire whether or not a given way of proceeding from premiss to conclusion is conducive to the ascertainment of truth or the reverse.

Peirce: CP 2.60 Cross-Ref:††

60. Seventhly: Many of the works which come under the last two heads are more particularly distinguished by regarding logic either as consisting in, or as deduced from, a branch of knowledge which the Germans call *Erkenntnislehre*, the doctrine of cognition, or from a closely allied science which they call *Wissenschaftslehre*, the doctrine of science, or *epistemology*. . . .

Peirce: CP 2.61 Cross-Ref:††

61. Wundt at the conclusion of his *Logic* (for that work does, at last, come to an end) says that, in "the true sense of the word," *Wissenschaftslehre* takes as the proper subject of its investigations the methods and results of the special sciences, with a view to the formation of a metaphysics (*Weltanschauung*). Perhaps the clearest explanation is given by Benno Erdmann in the following words:†1 "The science whose subject is the common postulate underlying all sciences, including itself, may be designated by an expression coined by Fichte, *Wissenschaftslehre*. It is contra-distinguished from all other sciences by their being special while it is universal. Its problem may be traced back to the demand of Socrates for conceptual knowledge, or what Aristotle termed {horizesthai katholou}. It arose when the 'ideal of knowledge' awoke."

Peirce: CP 2.62 Cross-Ref:††

62. The word *Erkenntnistheorie* has no such vagueness. It is the investigation of the sense in which knowledge is possible. It is often called the theory of cognition; but I do not see why we should not retain Kant's term, *critic*, especially as he borrowed it from the English;†2 for in our language this word has been in use since Hobbes and earlier, for the science of criticism, and was admitted by Johnson

into his dictionary. The peculiar turn of meaning given to it by Kant, which makes it the critic of knowledge, or, as he would have said, the critic of the cognitive faculties, is quite admissible. Besides, the immense importance of Kant's work upon this problem imposes upon us the duty of accepting his word, as long as it is so far from being a bad one. . . .

Peirce: CP 2.63 Cross-Ref:††

63. Surely logic must begin with a critic of knowledge. That cannot be denied; for what prudent man would embark on any enterprise without first considering whether and how it could possibly succeed? Nor can it be denied that the theory of cognition is today one of the pearls of scientific psychology. But I contend that that propedeutic that is wanted for logic has no more to do with the psychological theory of cognition than logic itself is concerned with the psychical process of thinking. Even less were there room for less; since the psychological theory substantially ends where consciously controlled thought begins, with which alone logic has even an indirect connection.

Peirce: CP 2.64 Cross-Ref:††

64. It is true that students of *Erkenntnislehre* say emphatically that it is not psychology. Some call it psychologico-metaphysical. Others deny it has any psychology in it. Benno Erdmann, himself a psychologist, formulates the problem of *Erkenntnislehre* as follows: "What right have we to assume that ideas refer to objects out of the mind?" (*Mit welchen Rechten nehmen wir an, dass sich das Vorgestellte überhaupt auf ein Transscendentes bezieht?*)†1 Certainly psychology has no dealings with objects out of the mind. Still less has logic. Whether or not there is, at all, any such thing as Reality, the logician need not decide. He cannot hide from himself, any more than another man can, that objects very nearly like real things there are; and he cannot pretend to doubt it. But he sees, perhaps more clearly than other men, that approximation to reality and absolute reality itself are two different things. The mathematicians' *i*, of which the square is negative unity, *approximates* to reality. All that it is incumbent upon the logician to learn is what inferential habits are conducive to knowledge, and to positive knowledge, in case there be any reality of which it is possible to have positive knowledge, and are conducive to such semblance of positive knowledge as we can have, in case there is no perfect reality or in case otherwise true positive knowledge is impossible. But in order to solve even that problem, he has first to ascertain, in case there be any successful quest for knowledge, what the nature of knowledge would be; and for his purpose, knowledge may be something written down in a book.

Peirce: CP 2.65 Cross-Ref:††

65. The science which Berkeley, Kant, and others have developed, and which goes by the name of the theory of cognition, is an experiential, or positive science. It learns and teaches that certain things exist. It even makes special observations. But the experiential element in logic is all but nil. No doubt it is an observational science, in some sense; every science is that. Even pure mathematics observes its diagrams. But logic contents itself almost entirely, like mathematics, with considering what would be the case in hypothetical states of things. Unlike the special sciences, it is not obliged to resort to experience for the support of the laws it discovers and enunciates, for the reason that those laws are merely conditional, not categorical. The normative character of the science consists, precisely, in that condition attached to its laws. The only purpose for which it is obliged to resort to experience is to establish a few facts, without which there could be no motive for its inquiries; and these facts are so

extremely universal and atmospheric that no little acumen is required to make sure that they are anything more than empty formulae or at most hypotheses.

Peirce: CP 2.66 Cross-Ref:††

66. Logic is obliged to suppose (it need not assert) that there is knowledge embodied in some form, and that there is inference, in the sense that one embodiment of knowledge affects another. It is not obliged even so much as to *suppose* that there is consciousness. Descartes was of opinion that animals were unconscious automata. He might as well have thought that all men but himself were unconscious. To suppose them so does not annul the rules of logic. It still remains true that such and such a habit of determining one virtual store of knowledge by another will result in the concentration of actions so as to bring about definite ends. The essence of rationality lies in the fact that the rational being *will* act so as to attain certain ends. Prevent his doing so in one way, and he will act in some utterly different way which will produce the same result. Rationality is being governed by final causes. Consciousness, the feeling of the passing instant, has, as such, no room for rationality. The notion that logic is in any way concerned with it is a fallacy closely allied to hedonism in ethics.†1

Peirce: CP 2.67 Cross-Ref:††

67. Eighthly: One needs but to turn over the leaves of a few of the first logical treatises that come to hand, especially if they are English, nor will he have to search long, in order to meet with appeals to the ordinary usages of speech as determinative of logical doctrines. Some recent books are quite crowded with this type of argument. It seems usually to be employed unreflectively; but there are works in which it is deliberately laid down as the principal basis of logical science.

Peirce: CP 2.68 Cross-Ref:††

68. The greater number of those who regard this method as very efficient would seem to have in mind an extremely small group of closely similar and highly peculiar languages--say Latin and a few literary languages of modern Europe. For some of the necessities of thought which they profess to deduce in this way are violated by the Greek, the language nearest, in every sense, to these I have mentioned. Even if they took into account the entire group of Aryan and Shemitic languages--although within that group they would find modes of thought that would somewhat embarrass them, such as the usage of the Gaelic and Old Irish of putting the subject of a sentence in the genitive--yet this would be like judging of botanical possibilities by phanerogamous plants, so small and so peculiar is that group as compared with the great world of languages, the negative resemblance between which may be very roughly described by saying that they make no use of abstract ideas, but which differ as much from one another in their ways of thinking as they do from the inflected languages.

Peirce: CP 2.69 Cross-Ref:††

69. I must acknowledge that in opposition to this we find that there are a number of eminent linguists who maintain this method in its extremest possible form. But I am obliged to say that the most distinguished of these, Steinthal and Sayce, are men of great genius but not distinguished for the caution of their positions, even within their own specialties. When Sayce says that "had Aristotle been a Mexican, his system of logic would have assumed a wholly different form,"†1 I am willing to admit that there is a good deal of truth in that. It is lucky that Aristotle's only language was one that led him into as few errors as did the Greek. But so far am I from finding in this remark any encouragement to trust to the indications of language as evidence

of logical necessity, that it seems to me to go quite the other way. The mere name of Steinthal no doubt overawes many, especially among those who have not read his principal books, at least, not critically. Moreover, it must not be forgotten that it is the usual practice of these Europeans who write grammars of non-Aryan languages, violently to adapt them to the Procrustean bed of Latin grammar. Even if one fully knows how false a representation this gives, it is hard, in writing a grammar, to resist the temptation to make use of brief familiar phrases which are, after all, as nearly right as any idea one can convey without much trouble and labor both to the reader and to oneself. For that reason, it will not suffice to get one's idea of an uninflected language from any mere grammar. It is necessary to have some real, living acquaintance with it, in order to appreciate its modes of thought, especially since these will be most difficult for us to grasp.

Peirce: CP 2.70 Cross-Ref:††

70. It seems to me, then, that appeals to language can serve no other purpose than as most inadequate and deceptive evidence of psychological necessities or tendencies; and these psychological necessities and tendencies, after they are ascertained, are utterly useless for the investigation of logical questions.

Peirce: CP 2.71 Cross-Ref:††

71. Ninthly: A well-known logician,†1 whose contributions to one part of the science are valuable, but whose treatise, as a whole, represents the agnostic disease at its crisis, adopts as his definition of good and bad reasoning its tendency toward and against the stability of the existing order of society. This method is open to most of the objections that have been urged against the foregoing eight, as well as to some others.

Peirce: CP 2.72 Cross-Ref:††

72. Tenthly: A few of the medieval logicians modified the principles of logic professedly so as to make them square with the authority of the church. This procedure should not be confounded with the passion of those ardent souls who delight to believe, at the dictates of their religion, all that they know to be most degradingly absurd. This method is not that, but consists in arguing that what seems to be absurd cannot really be so, since the church declares it to be true.

Peirce: CP 2.73 Cross-Ref:††

73. It may be the part of good sense for a person to employ a method of reasoning that he or she does not understand--say, for example, the method of least squares--on the recommendation of a respectable teacher; only it is not a scientific proceeding. Authority, from the nature of things, cannot *advance* knowledge.

Peirce: CP 2.74 Cross-Ref:††

74. Eleventhly: Appeal is often made by logical writers to the history of science as supporting their views of right reasoning. Whewell placed his *Novum Organum Renovatum* [1858], a work of high logical value, almost exclusively upon the basis of this method. As a source of secondary evidence upon such general doctrines of logic as may need such help, and even of primary evidence as to the value of special modes of reasoning, this method has been (it would be disgraceful to deny it) occasionally of no little service. In making use of it, it will be necessary, of course, to take good care that the history is open to no doubt. But there are great numbers of facts of scientific history about which it is as impossible for any sane man to entertain any real doubt (unless, indeed, we include German "higher critics" among that number) as about any item of the multiplication table. We also have to ask

whether the facts are sufficiently numerous to lend any great certainty to an induction. That depends on what facts they are, as well as on how broad the induction may be that one thinks to rest upon them. It must be admitted that, as things are at present, certain special kinds of reasoning which rest mainly upon scientific experience are open to the more danger, in that those who employ them will most likely quite forget how unsafe they really are.

Peirce: CP 2.75 Cross-Ref:††

75. Twelfthly: Everyday experience, such as presses in upon every man, at every hour of his life, is open to no other doubt than that it may not have been correctly formulated in general terms. This must be the main source of what little matter of fact logic has occasion to assert. There still live men who talk of experience not yielding absolute certainty, absolute universality, absolute necessity, absolute precision. No; but there is nothing at all in our knowledge which we have any warrant at all for regarding as absolute in any particular. Absolute infallibility may belong to the pope and the ecumenical councils: it is outside my province to discuss that question. But I am quite confident it does not belong to the multiplication table. If I must make any exception, let it be that the assertion that every assertion but this is fallible, is the only one that is absolutely infallible. But though nothing else is *absolutely* infallible, many propositions are *practically* infallible; such as the dicta of conscience. As for those things which are known by everyday experience, let him doubt them who can lay his hand on his heart and say that he does doubt them. For the rest of us, it would be mendacity to say that our degree of assurance of them is unsatisfactory.

Peirce: CP 2.76 Cross-Ref:††

76. Thirteenthly: The chief source of logical truth, though never recognized by logicians, always has been and always must be the same as the source of mathematical truth. This was well shown in a little book entitled *Logische Studien* by the historian of materialism, Friedrich Albert Lange, published in 1877, having been unfortunately left unfinished at the author's death in 1875. I have found few books on logic so instructive. I do not mean that the reader will find Lange's views exactly reproduced in this work, by any means, upon a single topic. But it has influenced me considerably, and I can recommend it as one of the very few works on logic that I have found too short.

Peirce: CP 2.77 Cross-Ref:††

77. What then is the source of mathematical truth? For that has been one of the most vexed of questions. I intend to devote an early chapter of this book to it.†1 I will merely state here that my conclusion agrees substantially with Lange's, that mathematical truth is derived from observation of creations of our own visual imagination, which we may set down on paper in form of diagrams. When it comes to logical truth, I do not think the intuition is quite what Lange describes it. He holds that we imagine something like an Euler's diagram; but I do not think that necessary. There are other ways, as I shall show, among which we may take our choice.†2 Lange holds up as a model Aristotle's proof of the conversion of the universal negative proposition. As well as I can translate Aristotle's untranslatable language, his proof reads as follows:

Peirce: CP 2.77 Cross-Ref:††

"If to none of the B's the (designation) A belongs; neither will the (designation) B belong to any of the A's. For if to any, as, for example, to C, it will not be true that to none of the B's the (designation) A belongs. For C is one of the

B's."†3

Peirce: CP 2.78 Cross-Ref:††

78. It seems to me it would be much simpler to say that if No B is A, but some A is B, then we should have the two premisses of a syllogism in *Ferio*, from which we could conclude: Some A is not A, which is absurd. Hence, if No B is A, it cannot be true that some A is B; that is, it must be true that No A is B. The syllogistic form *Ferio* is

No B is A,
Some C is B,
∴ Some C is not A.

If for C we put A, we get the premisses above used. Now in my view, observation comes in to assure us that when A is substituted for C we do get these premisses, and it also enters in other similar ways.

Peirce: CP 2.78 Cross-Ref:††

Aristotle seems to reason that if we were to assume that some A is B, supposing C to represent an A that is B, we should be obliged to conclude

C is A,
C is B,
∴Some B is A.

There are reasons why I think this somewhat objectionable; but it seems to me to depend upon the same kind of observation.

Peirce: CP 2.79 Cross-Ref:††
CHAPTER 2

PARTIAL SYNOPSIS OF A PROPOSED WORK IN LOGIC†1

§1. ORIGINALITY, OBSISTENCE, AND TRANSUASION

79. This chapter's main use is to give the reader an idea of what sort of book this is to be.†2 One can see that its conceptions are unusual. We find ourselves in the

vestibule of the labyrinth. Yes, The Labyrinth--in the Vestibule only, but yet in that tremendous, only Labyrinth. Thirteen doors, not yet opened, are before us. We choose the narrowest, the least prominent, the seldomest opened of any. . . .

Peirce: CP 2.80 Cross-Ref:††

80. That a reader should deliberately seek instruction from a treatise on logic is a proof that he has already made certain observations and reflections, and has acquired certain conceptions. I propose, at the outset, to invite the reader to give one more reconsideration, perhaps a little more deliberate than he has hitherto given, to these Pre-logical Ideas, in order to see how far they are well grounded, and in order to develop them perhaps a little more and penetrate to their real significance, as far as this can be done at this stage of the inquiry.

Peirce: CP 2.81 Cross-Ref:††

81. Some mathematicians, eminent for their success in their science, and who have particularly attended to the philosophy of it, regard Mathematics as a branch of Logic.†3 This is the more worthy of notice because it might with much justice be contended that mathematics is almost, if not quite, the only science which stands in need of no aid from a science of logic. Moreover, according to the opinion defended in the present treatise, logical truth is grounded upon a sort of observation of the same kind as that upon which mathematics is grounded. For these reasons, it is desirable at once to examine the nature of the mathematician's procedure pretty thoroughly. I have reason to be confident that this study will be of aid to some of those who have no natural turn for mathematics. At the same time, I am bound to say that mathematics requires a certain vigor of thought, the power of concentration of attention, so as to hold before the mind a highly complex image, and keep it steady enough to be observed; and though training can do wonders in a short time in enhancing this vigor, still it will not make a powerful thinker out of a naturally feeble mind, or one that has been utterly debilitated by intellectual sloth.

Peirce: CP 2.82 Cross-Ref:††

82. There is another normative science which has a vital connection with logic, which has been strangely overlooked by almost all logicians. I mean Ethics. It is not necessary to be an acute reasoner in order to develop the truest moral conceptions; but I do aver, and will prove beyond dispute, that in order to reason well, except in a mere mathematical way, it is absolutely necessary to possess, not merely such virtues as intellectual honesty and sincerity and a real love of truth, but the higher moral conceptions.†1 I will not claim that the study of ethics is more directly conducive to good morals than, say, the reading of good poetry is conducive to the writing of good prose. But I will say that it affords a quite indispensable help to the understanding of logic. It is, moreover, a subtle sort of study, such as a person who is fond of logic cannot but find to his taste. . . .

Peirce: CP 2.83 Cross-Ref:††

83. Only after these topics shall have been disposed of will it become needful to take up that propedeutic to logic proper, that unpsychological *Erkenntnisslehre* at which I have hinted. I name it *Speculative Grammar*, after the title of a work by Duns Scotus having the same aim.

Peirce: CP 2.84 Cross-Ref:††

84. In announcing what I am going to say in this part of the book, I have to choose between utter unintelligibility and a lengthy anticipation of what is about to be proved, but can here only be asserted. I unhesitatingly take the latter course, since the

ideas are put into such strange forms that a double exposition will aid the reader. I begin by endeavoring to strike the dominant note of the book with such force and clearness as I can command; for it is not only the key-note, but the key, to all logic. I essay an analysis of what appears in the world. It is not metaphysics that we are dealing with: only logic. Therefore, we do not ask what really is, but only what appears to everyone of us in every minute of our lives. I analyze experience, which is the cognitive resultant of our past lives, and find in it three elements. I call them *Categories*. †1 Would I could render them to the reader as vivid, as undeniable, as rational as they are to me. They will become so, if he will give thought enough to them. They appear in myriad shapes, of which, for the purpose of introducing the reader to them, I take the first that offers itself. A definition of experience happened, just now, to flow from my pen. It was a pretty good definition, I think: suppose we set out from that. Laconically speaking, experience is *esse in praeterito*. Only, remember, once more and once for all, that we do not mean what the secret nature of the fact is, but simply what we think it to be. Some fact there is. All experience compels your acknowledgment. What, then, is the fact that is present to you? Ask yourself: it is past. A fact is a *fait accompli*; its *esse* is *in praeterito*. The past compels the present, in some measure, at least. If you complain to the Past that it is wrong and unreasonable, it laughs. It does not care a snap of the finger for Reason. Its force is brute force. So then, you are compelled, brutally compelled, to admit that there is such an element in the world of experience as brute force. What then is brute force, or what does it seem to be? We ought to find little difficulty in answering that, since we are directly conscious (or seem to be, which is all that concerns us here) of exerting it ourselves. For, no matter how good a reason we may have for an act of the will, yet when we come to the exertion itself, reason does no part of the work: it is brute action. We can make no effort where we experience no resistance, no reaction. The sense of effort is a two-sided sense, revealing at once a something within and another something without. There is binarity in the idea of brute force; it is its principal ingredient. For the idea of brute force is little more than that of reaction; and this is pure binarity. Imagine two objects which are not merely *thought* as two, but of which something is true such that neither could be removed without destroying the fact supposed true of the other. Take, for example, a husband and wife. Here there is nothing but a real twoness; but it constitutes a reaction, in the sense that the husband makes the wife a wife in fact (not merely in some comparing thought); while the wife makes the husband a husband. A brute force is only a complication of binarities. It supposes not only two related objects, but that in addition to this state of things there is a *second* subsequent state. It further supposes two tendencies, one, of the one relate, tending to change the first relation in one way in the second state; the other, of the other relate, tending to change the same relation in a second way. Both those changes are in some way combined, so that each tendency is to some degree followed, to some degree modified. This is what we mean by *force*. It is almost pure binarity. The *bruteness* will consist in the absence of any reason, regularity, or rule, which should take part in the action as a third or mediating element. Binarity is one of my categories. I do not call it a conception; for it can be given in direct perception antecedent to thought. It penetrates every part of our inner world, as every part of the universe. The emotion of it becomes more like that of brute force in proportion to the development of this element [of binarity]. Among the inner shapes which binarity assumes are those of the *doubts* that are forced upon our minds. The very word "doubt," or "dubito," is the frequentative of "duhibeo"--i.e., *duo habeo*, and thus exhibits its binarity. If we did not struggle against doubt, we should not seek the truth. Binarity equally appears in negation, and in ordinary relative terms, even in

similarity, and in a more real way in identity. The text †1 will show why individual existence is a markedly dualistic conception. Meantime, it is easy to see that only existing individuals can react against one another.

Peirce: CP 2.85 Cross-Ref:††

85. Let us now consider what could appear as being in the present instant were it utterly cut off from past and future. We can only guess; for nothing is more occult than the absolute present. There plainly could be no action; and without the possibility of action, to talk of binarity would be to utter words without meaning. There might be a sort of consciousness, or feeling, with no self; and this feeling might have its tone. Notwithstanding what William James has said, I do not think there could be any continuity like space, which, though it may perhaps appear in an instant in an educated mind, I cannot think could do so if it had no time at all; and without continuity parts of the feeling could not be synthetized; and therefore there would be no recognizable parts. There could not even be a degree of vividness of the feeling; for this [the degree of vividness] is the comparative amount of disturbance of general consciousness by a feeling. †1 At any rate, such shall be our hypothesis, and whether it is psychologically true or not is of no consequence. The world would be reduced to a quality of unanalyzed feeling. Here would be an utter absence of binarity. I cannot call it unity; for even unity supposes plurality. I may call its form Firstness, Orience, or Originality. It would be something *which is what it is without reference to anything else* within it or without it, regardless of all force and of all reason. Now the world is full of this element of irresponsible, free, Originality. Why should the middle part of the spectrum look green rather than violet? There is no conceivable reason for it nor compulsion in it. Why was I born in the nineteenth century on Earth rather than on Mars a thousand years ago? Why did I today sneeze just five hours forty-three minutes and twenty-one seconds after a certain man in China whistled (supposing this did happen)? We know perhaps why a meteorite should fall to the earth, if it gets in the Earth's way; but why should the arrangements of nature be such that this particular meteorite was in the Earth's way? All these are facts which are as they are, simply because they happen to be so. We mostly neglect them; but there are cases, as in qualities of feeling, self-consciousness, etc., in which such isolated flashes come to the front. Originality, or Firstness, is another of my Categories.

Peirce: CP 2.86 Cross-Ref:††

86. Let us now take up being *in futuro*. As in the other cases, this is merely an avenue leading to a purer apprehension of the element it contains. An absolutely pure conception of a Category is out of the question. Being *in futuro* appears in mental forms, intentions and expectations. Memory supplies us a knowledge of the past by a sort of brute force, a quite binary action, without any reasoning. But all our knowledge of the future is obtained through the medium of something else. To say that the future does not influence the present is untenable doctrine. It is as much as to say that there are no final causes, or ends. The organic world is full of refutations of that position. Such action [by final causation] constitutes evolution. But it is true that the future does not influence the present in the direct, dualistic, way in which the past influences the present. A machinery, a medium, is required. Yet what kind of machinery can it be? Can the future affect the past by any machinery which does not again itself involve some action of the future on the past? All our knowledge of the laws of nature is analogous to knowledge of the future, inasmuch as there is no direct way in which the laws can become known to us. We here proceed by experimentation. That is to say, we guess out the laws bit by bit. We ask, What if we were to vary our procedure a little? Would the result be the same? We try it. If we are

on the wrong track, an emphatic negative soon gets put upon the guess, and so our conceptions gradually get nearer and nearer right. The improvements of our inventions are made in the same manner. The theory of natural selection is that nature proceeds by similar experimentation to adapt a stock of animals or plants precisely to its environment, and to keep it in adaptation to the slowly changing environment. But every such procedure, whether it be that of the human mind or that of the organic species, supposes that effects will follow causes on a principle to which the guesses shall have some degree of analogy, and a principle not changing too rapidly. In the case of natural selection, if it takes a dozen generations to sufficiently adapt a stock to a given change of the environment, this change must not take place more rapidly, or the stock will be extirpated instead of being adapted. It is no light question how it is that a stock in some degree out of adjustment to its environment immediately begins to sport, and that not wildly but in ways having some sort of relation to the change needed. Still more remarkable is the fact that a man before whom a scientific problem is placed immediately begins to make guesses, not wildly remote from the true guess. The physicist who observes a strange phenomenon in his laboratory, for example, does not begin by wondering whether the particular aspect of the planets at that moment had something to do with it--as Ernst Mach †1 practically guesses that it is the fixed stars that keep a body moving in a right line at uniform speed--he looks about for some circumstance near at hand that may explain it. How is this marked, though excessively imperfect, divinatory power of guessing right on the part of the man and on the part of the organic stock, to be explained? Two alternatives only are open. On the one hand, we may say that there is a direct power of Reason to know how Reason will act; and that Nature is ruled by a Reasonable Power. On the other hand, we may say that the tendency to guess nearly right is itself the result of a similar experimental procedure. This involves a deeply interesting difficulty (not the mere stumbling over a *regressus ad infinitum*) which shall be touched upon before the close of this volume. As to the other hypothesis, it only concerns me here to say that as those peoples who believe in prophets look for this gift particularly among the insane, so the power here supposed would be altogether different from the operation of reasoning. Take experimental reasoning, for example. Here we have pairednesses between the experiments and the results of the experiments, consisting in the fact that the results follow the experiments in accordance with a previous hypothesis; and the nature of these pairednesses is such that they could not have existed had not a third thing, the prediction, been made. Just as a real pairedness consists in a fact being true of A which would be nonsense if B were not there, so we now meet with a Rational Threeness which consists in A and B being really paired by virtue of a third object, C. I tell my dog to go upstairs and fetch me my book, which he does. Here is a fact about three things, myself, the dog, and the book, which is no mere sum of facts relating to pairs, nor even a pairing of such pairs. I speak to the dog. I mention the book. I do those things together. The dog fetches the book. He does it in consequence of what I did. That is not the whole story. I not only simultaneously spoke to the dog and mentioned the book, but I mentioned the book to the dog; that is, I caused him to think of the book and to bring it. My relation to the book was that I uttered certain sounds which were understood by the dog to have reference to the book. What I did to the dog, beyond exciting his auditory nerve, was merely to induce him to fetch the book. The dog's relation to the book was more prominently dualistic; yet the whole significance and intention of his fetching it was to obey me. In all action governed by reason such genuine triplicity will be found; while purely mechanical actions take place between pairs of particles. A man gives a brooch to his wife. The merely mechanical part of the act consists in his laying the brooch down while uttering

certain sounds, and her taking it up. There is no genuine triplicity here; but there is no giving, either. The giving consists in his agreeing that a certain intellectual principle shall govern the relations of the brooch to his wife. The merchant in the Arabian Nights threw away a datestone which struck the eye of a Jinnee. This was purely mechanical, and there was no genuine triplicity. The throwing and the striking were independent of one another. But had he aimed at the Jinnee's eye, there would have been more than merely throwing away the stone. There would have been genuine triplicity, the stone being not merely thrown, but thrown at the eye. Here, *intention*, the mind's action, would have come in. Intellectual triplicity, or Mediation, is my third category.

Peirce: CP 2.87 Cross-Ref:††

87. There is no fourth, as will be proved.†1 This list of categories may be distinguished from other lists as the *Ceno-Pythagorean Categories*, on account of their connection with numbers. They agree substantially with Hegel's three moments. Could they be attributed to any thinker in well-known history, that would be almost enough to refute their claims to primitivity. It has occurred to me that perhaps Pythagoras brought them from Media or Aria; but careful examination has convinced me that there was not among the Pythagoreans the smallest approach to anything resembling the categories.

Peirce: CP 2.88 Cross-Ref:††

88. It is desirable that there should be technical terms for the categories. They should be expressive and not liable to be used in special senses in philosophy. The simplicity and pervasiveness of the categories render metaphorical designations quite impossible, since such a term, if at all appropriate, would contain the very category. There can be no *resemblance* to a category. A metaphorical name would probably contain the category in the first syllable, and the rest of the word would be padding. I prefer, therefore, to borrow a word, or still better, to compose one, which, etymologically, if it may be, but by similarity with familiar words, indispensably, shall suggest a number of shapes in which the category is prominent. I propose to take the following terms on probation:

Peirce: CP 2.89 Cross-Ref:††

89. *Originality* is being such as that being is, regardless of aught else.

Peirce: CP 2.89 Cross-Ref:††

Obsistence (suggesting *obviate*, *object*, *obstinate*, *obstacle*, *insistence*, *resistance*, etc.) is that wherein secondness differs from firstness; or, is that element which taken in connection with Originality, makes one thing such as another compels it to be.

Peirce: CP 2.89 Cross-Ref:††

Transuasion (suggesting *translation*, *transaction*, *transfusion*, *transcendental*, etc.) is mediation, or the modification of firstness and secondness by thirdness, taken apart from the secondness and firstness; or, is being in creating Obsistence.

Peirce: CP 2.90 Cross-Ref:††

90. Although Originality is the most primitive, simple, and original of the categories, it is not the most obvious and familiar. We have thus far considered the categories in their original aspect. We now pass to the easier study of their obsistential shapes.

Peirce: CP 2.91 Cross-Ref:††

91. In the Obsistent aspect, Originality presents itself as a Quality, which is something which is such as it is, and is so free from Obsistence as not even to be self-identical, or individual. Two Qualities which are alike, as all Qualities are, are, in so far, the same Quality. Obsistence presents itself as a Relation, which is a fact concerning a set of objects, the Relates. A Relation is either *Genuine* or *Degenerate*. A Degenerate Relation is a fact concerning a set of objects which consists merely in a partial aspect of the fact that each of the Relates has its Quality. It is a Relation of Qualities; such as that A is greater than B. Its relates may be qualities or objects possessing qualities. It may be a Similarity, which is a more Degenerate form, or a Difference, which is a less Degenerate form, or it may be mixed. A Genuine Relation is one which is not necessarily involved in its Relates having any Qualities regardless of each other. Each relate is necessarily individual, or self-identical. Various other divisions of relations will be made; and the nature of identity, otherness, coexistence, and impossibility will be specially considered. †1

Peirce: CP 2.92 Cross-Ref:††

92. Transuasion in its obsistent aspect, or Mediation, will be shown to be subject to two degrees of degeneracy. Genuine mediation is the character of a *Sign*. A *Sign* is anything which is related to a Second thing, its *Object*, in respect to a Quality, in such a way as to bring a Third thing, its *Interpretant*, into relation to the same Object, and that in such a way as to bring a Fourth into relation to that Object in the same form, *ad infinitum*. If the series is broken off, the Sign, in so far, falls short of the perfect significant character. It is not necessary that the Interpretant should actually exist. A being *in futuro* will suffice. Signs have two degrees of Degeneracy. A Sign degenerate in the lesser degree, is an Obsistent Sign, or *Index*, which is a Sign whose significance of its Object is due to its having a genuine Relation to that Object, irrespective of the Interpretant. Such, for example, is the exclamation "Hi!" as *indicative* of present danger, or a rap at the door as indicative of a visitor. A Sign degenerate in the greater degree is an Originalian Sign, or *Icon*, which is a Sign whose significant virtue is due simply to its Quality. Such, for example, are imaginations of how I would act under certain circumstances, as showing me how another man would be likely to act. We say that the portrait of a person we have not seen is *convincing*. So far as, on the ground merely of what I see in it, I am led to form an idea of the person it represents, it is an Icon. But, in fact, it is not a pure Icon, because I am greatly influenced by knowing that it is an *effect*, through the artist, caused by the original's appearance, and is thus in a genuine Obsistent relation to that original. Besides, I know that portraits have but the slightest resemblance to their originals, except in certain conventional respects, and after a conventional scale of values, etc. A Genuine Sign is a Transuasional Sign, or *Symbol*, which is a sign which owes its significant virtue to a character which can only be realized by the aid of its Interpretant. Any utterance of speech is an example. If the sounds were originally in part iconic, in part indexical, those characters have long since lost their importance. The words only stand for the objects they do, and signify the qualities they do, because they will determine, in the mind of the auditor, corresponding signs. The importance of the above divisions, although they are new, has been acknowledged by all logicians who have seriously considered them. . . .

Peirce: CP 2.93 Cross-Ref:††

93. Logic is the science of the general necessary laws of Signs and especially of Symbols. As such, it has three departments. Obsistent logic, logic in the narrow

sense, or *Critical Logic*, is the theory of the general conditions of the reference of Symbols and other Signs to their professed Objects, that is, it is the theory of the conditions of truth. Originalian logic, or *Speculative Grammar*, is the doctrine of the general conditions of symbols and other signs having the significant character. It is this department of general logic with which we are, at this moment, occupying ourselves. Transuational logic, which I term *Speculative Rhetoric*, is substantially what goes by the name of methodology, or better, of *methodeutic*. It is the doctrine of the general conditions of the reference of Symbols and other Signs to the Interpretants which they aim to determine. . . .

Peirce: CP 2.94 Cross-Ref:††

94. In consequence of every sign determining an Interpretant, which is itself a sign, we have sign overlying sign. The consequence of this, in its turn, is that a sign may, in its immediate exterior, be of one of the three classes, but may at once determine a sign of another class. But this in its turn determines a sign whose character has to be considered. This subject has to be carefully considered, and order brought into the relations of the strata of signs, if I may call them so, before what follows can be made clear.

Peirce: CP 2.95 Cross-Ref:††

§2. TERMS, PROPOSITIONS, AND ARGUMENTS

95. Symbols, and in some sort other Signs, are either *Terms, Propositions, or Arguments*.†1 A *Term* is a sign which leaves its Object, and *a fortiori* its Interpretant, to be what it may. A *Proposition* is a sign which distinctly indicates the Object which it denotes, called its *Subject*, but leaves its Interpretant to be what it may. An *Argument* is a sign which distinctly represents the Interpretant, called its *Conclusion*, which it is intended to determine. That which remains of a Proposition after removal of its Subject is a Term (a rhema) called its Predicate.†1 That which remains of an Argument when its Conclusion is removed is a Proposition called its Premiss, or (since it is ordinarily copulative) more usually its Premises. . . .

Peirce: CP 2.96 Cross-Ref:††

96. Argument is of three kinds: *Deduction, Induction, and Abduction* (usually called adopting a hypothesis). An Obsistent Argument, or *Deduction*, is an argument representing facts in the Premiss, such that when we come to represent them in a Diagram we find ourselves compelled to represent the fact stated in the Conclusion; so that the Conclusion is drawn to recognize that, quite independently of whether it be recognized or not, the facts stated in the premisses are such as could not be if the fact stated in the conclusion were not there; that is to say, the Conclusion is drawn in acknowledgment that the facts stated in the Premiss constitute an Index of the fact which it is thus compelled to acknowledge.†P1 All the demonstrations of Euclid are of this kind. Deduction is Obsistent in respect to being the only kind of argument which is compulsive. An originary Argument, or *Abduction*, is an argument which presents facts in its Premiss which present a similarity to the fact stated in the Conclusion, but which could perfectly well be true without the latter being so, much more without its being recognized; so that we are not led to assert the Conclusion positively but are only inclined toward admitting it as representing a fact of which the facts of the Premiss constitute an *Icon*. For example, at a certain stage of Kepler's

eternal exemplar of scientific reasoning, he found that the observed longitudes of Mars, which he had long tried in vain to get fitted with an orbit, were (within the possible limits of error of the observations) such as they would be if Mars moved in an ellipse. The facts were thus, in so far, a *likeness* of those of motion in an elliptic orbit. Kepler did not conclude from this that the orbit really was an ellipse; but it did incline him to that idea so much as to decide him to undertake to ascertain whether virtual predictions about the latitudes and parallaxes based on this hypothesis would be verified or not. This probational adoption of the hypothesis was an Abduction. An Abduction is Originary in respect to being the only kind of argument which starts a new idea. A Transuasive Argument, or *Induction*, is an Argument which sets out from a hypothesis, resulting from a previous Abduction, and from virtual predictions, drawn by Deduction, of the results of possible experiments, and having performed the experiments, concludes that the hypothesis is true in the measure in which those predictions are verified, this conclusion, however, being held subject to probable modification to suit future experiments. Since the significance of the facts stated in the premisses depends upon their predictive character, which they could not have had if the conclusion had not been hypothetically entertained, they satisfy the definition of a Symbol of the fact stated in the conclusion. This argument is Transuasive, also, in respect to its alone affording us a reasonable assurance of an ampliation of our positive knowledge. By the term "virtual prediction," I mean an experiential consequence deduced from the hypothesis, and selected from among possible consequences independently of whether it is known, or believed, to be true, or not; so that at the time it is selected as a test of the hypothesis, we are either ignorant of whether it will support or refute the hypothesis, or, at least, do not select a test which we should not have selected if we had been so ignorant.

Peirce: CP 2.97 Cross-Ref: ††

97. When Kepler had found that the elliptic orbit placed the planet Mars in the right longitudes, he proceeded to test the hypothesis in two ways. In the first place it had always been comparatively easy to find hypotheses approximately representing the longitudes, although not to the point of accuracy of Tycho Brahe's observations. But when these hypotheses were applied to the latitudes, it had always been found that additional hypotheses, of librations, or tiltings of the orbit of a complicated kind, having little verisimilitude, were required to come near to a representation of the latitudes. Kepler undertook the calculation of the latitudes from his elliptic theory without knowing whether the calculation would agree with the observation or not; but it was found that it did so most admirably. He then went back to the longitudes, and applied another test, of the success of which he could know nothing beforehand. What he had so far found was that the planet was at the time of observation always in the direction in which it ought to be. But was it at the right distance? This could not be quite positively ascertained. But he could take two times at which Mars had been observed, and, at which according to the elliptic theory (which in this respect could hardly be in error) it was at the same point of its orbit, but at which it was certain that the earth was at widely different points in its orbit. The orbit of the earth is so nearly circular that there could be no doubt where it was at these times. These two places and the place of Mars (supposed the same at the two times) gave a triangle of which two angles and the intermediate side (the distance between the two positions of the earth) were known (the mean distance of the sun from the earth being taken as unit of distance). From that he could calculate the distance of Mars from the sun, with no assumption except that Mars was really at the same point of his orbit, about which there could (for a reason too long to set forth here) hardly be the least doubt, whether the elliptic orbit were correct or not. By trying this at times when Mars was at the two

extremes of his orbit, and when he was at intermediate places, Kepler could get a test of the severest character as to whether the elliptic theory really flattened the orbit by the right amount or not. In the cases of the few, but well situated, pairs of observations which could be found that were suitable to this test, the accord of observation and theory was all that could be desired, and clinched the argument in the mind of every thinking person. It will be observed that the argument was very different from what it would have been if Kepler had merely taken all the observations of longitude, latitude, and parallax and had constructed from them a theory that would suit them all. That might evince no more than Kepler's extraordinary ingenuity. Nor was the last test the same that it would have been if Kepler, looking over the observations, and hunting for features of them that should suit the theory, had found this. That might only show that out of many features of the observations, some suited the theory. But his course was very different. He did not select this test because it would give a favorable result. He did not know that it would do so. He selected it because it was the test which Reason demanded should be applied. Let this course be pursued, and no theories will stand long but those which are true. But the discussion of the strength of the argument belongs to Critical Logic, and not to Speculative Grammar.

Peirce: CP 2.98 Cross-Ref:††
§3. CLEARNESS OF IDEAS^P

98. The division of all inference into Abduction, Deduction, and Induction may almost be said to be the Key of Logic.

Peirce: CP 2.99 Cross-Ref:††

99. After the thorough and careful discussion of all the above matters, involving many nice questions, including the one concerning which logicians are today disputing more than any other, many volumes having been devoted to it, I mean that of the nature of the proposition, and after every opinion has received its respectful hearing, we will come at last to the problem of Clearness, than which none in logic is more practically vital. I treated this subject in 1877,†1 and enunciated a maxim, the acceptance of which constitutes the position called Pragmatism, a question which has of late years largely occupied philosophers. My opinion remains substantially the same now †2 as then; but all those years have not passed without my learning something. I can now define the proposition more accurately, so as to close the door against those who would push the doctrine much further than I ever intended; and I can state the reasons for the rule in a manner which must be allowed to be more scientific, more convincing, and more definitory than before.

Peirce: CP 2.100 Cross-Ref:††
§4. ABDUCTION, DEDUCTION, AND INDUCTION

100. Critical Logic is then to be taken up.†3 I begin with necessary Deduction, treating it with as much completeness as possible. Yet I shall avoid wasting pages upon mere formalisms, except so far as their great familiarity entitles them to

mention. I endeavor to include every form of necessary reasoning known to me. The utility of Abstractions in reasoning, which I shall here bring out, will be a novelty. I need hardly say that ordinary syllogistic will be but a small fraction of my doctrine. The main substance of that need hardly fill a page.

Peirce: CP 2.101 Cross-Ref:††

101. I next take up †1 that immensely important branch of deductive logic, the doctrine of chances, which has been called, with little exaggeration, the logic of the exact sciences. This involves several difficult questions, of which the two chief are on the one hand, the foundation of the doctrine, together with the nature of probability, and on the other hand, the admissibility of inverse probabilities. Both of these are matters of practical importance to us all; for although few have occasion to make numerical computations of probabilities, the use of the ideas and propositions of the calculus is most widely extended, and to great advantage, while, at the same time, even the greatest mathematicians †2 have fallen into fatal practical errors both in the theory and in the application of it. The first of the two questions mentioned is by no means one to be settled at one blow. A whole nest of fallacies is hidden in it. This is why I cannot here in a few words approximately define my position so that a person acquainted with the state of discussion can get a general idea of where I stand. I may, however, say that I am one of those who maintain that a probability must be a matter of positive knowledge, or confess itself a nullity. Yet I do not go to such an extreme length of empiricism as Mr. Venn.†3 On the other hand, some very acute, but in my opinion quite untenable, positions of Mr. F. Y. Edgeworth †4 will receive examination. It is of the extremest importance to distinguish entirely different qualities commonly confounded under the name of probability. One of these, which I term "likelihood" is the most deceptive thing in the world, being nothing but the degree of conformity of a proposition to our preconceived ideas. When this is dignified by the name of probability, as if it were something on which vast Insurance Companies could risk their hundreds of millions, it does more harm than the yellow fever ever did. The probability proper is also an essentially inaccurate idea, calling for every precaution of pragmatism in the use of it, in which its inductive origin must be steadily kept in view as the compass by which we are to steer our bark safely on this ocean of probability. Induction might be accurately defined as the virtual inference of a probability, if probability could be defined without the idea of induction. When the philosophy of probability has once been put upon a sure footing, the question of inverse probabilities gives no serious difficulty. Nobody can go further than I in condemnation of this way of using probability, which completely vitiates the theory and practice of Inductive and Abductive reasoning, has set back civilization, and has corrupted ideals, to an extent so far beyond what anybody would believe possible without careful examination of the facts, that I know I must be laughed at for what seems a most ridiculous judgment. The reader would perhaps at length go with me if I could in this work enter into the history of current beliefs.

Peirce: CP 2.102 Cross-Ref:††

102. The discussion of probability naturally brings us to the interesting question of the validity of induction. I undertake to demonstrate mathematically that the validity of Induction, in the proper sense of the term, that is to say, experimental reasoning, follows, through the lemmas of probabilities, from the rudiments of the doctrine of necessary consequences, without any assumption whatever about the future being like the past, or similar results following similar conditions, or the uniformity of nature, or any such vague principle.†1 I shall set forth the reasoning in strict accuracy of form; and I defy anybody to find a flaw in it. The importance of the

question for every man is tremendous. Having fully set forth my doctrine of induction, with the very strict rules to bind it down which are necessitated by the demonstration mentioned, I pass by for the present the consideration of all other theories, and proceed at once to the study of Abduction. Upon this subject, my doctrine has been immensely improved since my essay "A Theory of Probable Inference"^{†2} was published in 1883. In what I there said about "Hypothetic Inference" I was an explorer upon untrodden ground. I committed, though I half corrected, a slight positive error, which is easily set right without essentially altering my position. But my capital error was a negative one, in not perceiving that, according to my own principles, the reasoning with which I was there dealing could not be the reasoning by which we are led to adopt a hypothesis, although I all but stated as much. But I was too much taken up in considering syllogistic forms and the doctrine of logical extension and comprehension, both of which I made more fundamental than they really are. As long as I held that opinion, my conceptions of Abduction necessarily confused two different kinds of reasoning. When, after repeated attempts, I finally succeeded in clearing the matter up, the fact shone out that probability proper had nothing to do with the validity of Abduction, unless in a doubly indirect manner. But now a number of considerations offered themselves as possibly connected with the solution of the problem, and owing to the extreme weakness of this form of inference, it was difficult to make sure that they were irrelevant. I seemed to be lost in a pathless forest, until by minute application of the first principles, I found that the categories, which I had been led to neglect from not seeing how they were to be applied, must and in fact did furnish the clue that guided me through the maze. I prefer not to make any prefacial sketch of this doctrine, but shall ask the reader to judge of it, if at all, from its full exposition. I believe it to be the most important part of the book,^{†1} whether it be estimated from the theoretical or the practical side.

Peirce: CP 2.103 Cross-Ref:^{††}

103. Having considered the three fundamental modes of argument, I now pass to the consideration of arguments which mingle the characters of these. Here belongs in the front rank the argument from analogy, followed by four ways of supporting hypotheses by uniformities, arguments from likelihood, from the resemblance of the future to the past, etc.

Peirce: CP 2.104 Cross-Ref:^{††}

104. I now go back to consider other theories of the validity of induction and hypothesis than my own. I give as complete an account of these theories as I can. I show that the arguments which their authors identify with inductions have in some cases no force at all, but lead to manifest absurdity, and that in no cases are they nearly as strong as the true inductive argument. I furthermore show that they invariably leave the doors open, theoretically and practically, to bad reasoning. In fact, there are no rules at all, for the most part, which follow as necessary corollaries from the theories. From the ordinary theories of hypotheses, on the other hand, rules do flow; but they are particularly pestiferous rules, much worse than none at all; and these unfortunately have become, through judges' charges to juries and otherwise, widely current among people who never opened a book of logic. . . .

105. All this brings us close to Methodeutic, or Speculative Rhetoric. The practical want of a good treatment of this subject is acute. It is not expected that any general doctrine shall teach men much about methods of solving problems that are familiar to them. But in problems a little remote from those to which they are accustomed, it is remarkable how not merely common minds, but those of the very highest order, stumble about helplessly. No class of thinkers can by anybody be rated higher in heuretic genius than the mathematicians; and yet see how they have boggled over comparatively simple problems of unfamiliar kinds, such as Fermat's theorems, Steiner's theorems, the problem of map-coloring, the theory of knots.

Peirce: CP 2.106 Cross-Ref:††

106. Many persons will think that there are other ways of acquiring skill in the art of inquiry which will be more instructive than the logical study of the theory of inquiry. That may be; I shall not dispute it; for it would carry me far beyond the confines of my province. I only claim that however much one may learn in other ways of the method of attacking an unfamiliar problem, something may be added to that knowledge by considering the general theory of how research must be performed. At the same time, it is this theory itself, for itself, which will here be the principal object.

Peirce: CP 2.107 Cross-Ref:††

107. In coming to Speculative Rhetoric, after the main conceptions of logic have been well settled, there can be no serious objection to relaxing the severity of our rule of excluding psychological matter, observations of how we think, and the like. The regulation has served its end; why should it be allowed now to hamper our endeavors to make methodeutic practically useful? But while the justice of this must be admitted, it is also to be borne in mind that there is a purely logical doctrine of how discovery must take place, which, however great or little is its importance, it is my plain task and duty here to explore. In addition to this, there may be a psychological account of the matter, of the utmost importance and ever so extensive. With this, it is not my business here to meddle; although I may here and there make such use of it as I can in aid of my own doctrine.

Peirce: CP 2.108 Cross-Ref:††

108. Time was when a theorem could constitute a considerable contribution to mathematical science. But now new theorems are turned out wholesale. A single treatise will contain hundreds of them. Nowadays methods alone can arrest attention strongly; and these are coming in such flocks that the next step will surely be to find a *method of discovering methods*.†1 This can only come from a theory of the method of discovery. In order to cover every possibility, this should be founded on a general doctrine of methods of attaining purposes, in general; and this, in turn, should spring from a still more general doctrine of the nature of teleological action, in general.†2

Peirce: CP 2.109 Cross-Ref:††

109. Although the number of works upon Methodeutic since Bacon's *Novum Organum* has been large, none has been greatly illuminative. Bacon's work was a total failure, eloquently pointing out some obvious sources of error, and to some minds stimulating, but affording no real help to an earnest inquirer. THE book on this subject remains to be written; and what I am chiefly concerned to do is to make the writing of it more possible.

Peirce: CP 2.110 Cross-Ref:††

110. I do not claim that the part of the present volume which deals with Speculative Rhetoric will approach that ideal. As to the other parts of my book, this prefatory chapter commits me to producing a work of great importance or to being set down a drawler of nonsense. But for the methodetic part, I only say that since my youth I have associated with strong thinkers and have never ceased to make it a point to study their handling of their problems in all its details. When I was young, no remark was more frequent than that a given method, though excellent in one science, would be disastrous in another. If a mere aping of the externals of a method were meant, the remark might pass. But it was, on the contrary, applied to extensions of methods in their true souls. I early convinced myself that, on the contrary, that was the way in which methods must be improved; and great things have been accomplished during my life-time by such extensions. I mention my early foreseeing that it would be so, because it led me, in studying the methods which I saw pursued by scientific men, mathematicians, and other thinkers, always to seek to *generalize* my conception of their methods, as far as it could be done without destroying the forcefulness of those methods. This statement will serve to show about how much is to be expected from this part of my work.

Peirce: CP 2.111 Cross-Ref: ††
§6. OBJECTIVE LOGIC †1

111. With Speculative Rhetoric, Logic, in the sense of Normative Semeiotic, is brought to a close. But now we have to examine whether there be a doctrine of signs corresponding to Hegel's objective logic; that is to say, whether there be a life in Signs, so that--the requisite vehicle being present--they will go through a certain order of development, and if so, whether this development be merely of such a nature that the same round of changes of form is described over and over again whatever be the matter of the thought or whether, in addition to such a repetitive order, there be also a greater life-history that every symbol furnished with a vehicle of life goes through, and what is the nature of it. There are minds who will pooh-pooh an idea of this sort, much as they would pooh-pooh a theory involving fairies. I have no objection to the pooh-pooh-ing of fairies, provided it be critical pooh-pooh-ing; but I wish I had the leisure to place before those gentlemen a work to be entitled The History of Pooh-pooh-ing. I think it would do them good; and make room in their minds for an essay upon the Logic of Pooh-pooh-ing. Mind, that if some forenoon, while I was in the midst of one of the most valuable of the chapters of my "Minute Logic," a rap should come at my outer door, and if, upon going to the door, I were to find two men who proposed to come in and discuss with me the principles of Mormonism or Christian Science, I should promptly recommend them to apply elsewhere. This I should do upon the same grounds upon which I declined to join the American Psychical Research Society when it was started; namely, that I thought that to do so would be to sanction a probable great waste of time, together with the placing of some men in a compromising position. In like manner, if a reader who has thought it worth while to listen to what I have had to say upon normative logic finds objective logic too remote from his interests to care to listen to any discussion of it, I shall fully approve of his allowing the leaves of my chapter upon this subject to remain uncut. But my own position is different. It lies directly in the path of my duty to consider the question critically.

Peirce: CP 2.112 Cross-Ref:††

112. The first question, then, which I have to ask is: Supposing such a thing to be true, what is the kind of proof which I ought to demand to satisfy me of its truth? Am I simply to go through the actual process of development of symbols with my own thoughts, which are symbols, and am I to find in the sense of necessity and evidence of the following of one thought upon another an adequate assurance that the course followed is the necessary line of thought's development? That is the way the question has usually been put, hitherto, both by Hegelians and by Anti-Hegelians. But even if I were to find that the sequence of conceptions in Hegel's logic carried my mind irresistibly along its current, that would not suffice to convince me of its universal validity. Nor, on the other hand, does the mere fact that I do not find a single step of Hegel's logic, or any substitute for it that I have met with, either convincing or persuasive, give me any assurance whatever that there is no such life-history. It seems to me natural to suppose that it would be far easier satisfactorily to answer the question of whether there is such a thing than to find out what particular form that life-history would take if it were a reality; and not only natural to suppose so, but made as certain by solid reasons as any such anticipation in regard to proofs could well be.

Peirce: CP 2.113 Cross-Ref:††

113. I am not one of those transcendental apothecaries, as I call them--they are so skillful in making up a bill--who call for a quantity of big admissions, as indispensable *Voraussetzungen* of logic. I am not so indulgent as Argan †1 to suppose that they can seriously expect as much as half their demands to be allowed. I reduce the indispensability of their postulates far more than that, namely, all the way from universality to the single case that happens to have come up; and even then, I do not admit that indispensability is any ground of belief. It may be indispensable that I should have \$500 in the bank--because I have given checks to that amount. But I have never found that the indispensability directly affected my balance, in the least. When a hand at whist has reached the point at which each player has but three cards left, the one who has to lead often goes on the assumption that the cards are distributed in a certain way, because it is only on that assumption that the odd trick can be saved. This is indisputably logical; and on a more critical analogous occasion there might be some *psychological* excuse, or even warrant, for a "will to believe" that such was really the case. But all that logic warrants is a *hope*, and not a belief. It must be admitted, however, that such hopes play a considerable part in logic. For example, when we discuss a vexed question, we *hope* that there is some ascertainable truth about it, and that the discussion is not to go on forever and to no purpose. A transcendentalist would claim that it is an indispensable "presupposition" that there is an ascertainable true answer to every intelligible question. I used to talk like that, myself; for when I was a babe in philosophy my bottle was filled from the udders of Kant. But by this time I have come to want something more substantial.

Peirce: CP 2.114 Cross-Ref:††

114. But whatever be the kind and degree of our logical assurance that there is any real world, external or internal, that same kind and degree of assurance we certainly have that there not only may be a living symbol, realizing the full idea of a symbol, but even that there actually is one.

Peirce: CP 2.115 Cross-Ref:††

115. I examine the question from this point of view. It certainly seems as if the mere hypothesis of such a thing as a symbol sufficed to demonstrate such a

life-history. Still, a fallacy is to be suspected. How can a mere hypothesis prove so much as this seems to prove, if it proves anything? I call in the data of experience, not exactly the every-minute experience which has hitherto been enough, but the experience of most men, together with the history of thought. The conclusion seems the same. Yet still, the evidence is unsatisfactory. The truth is that the hypothesis involves the idea of a different mode of being from that of existential fact. This mode of being seems to claim immediate recognition as evident in the mere idea of it. One asks whether there is not a fallacy in using the ordinary processes of logic either to support it or to refute it.

Peirce: CP 2.116 Cross-Ref:††

116. Aristotelianism admitted two modes of being. This position was attacked by William Ockham, on the ground that one kind sufficed to account for all the phenomena. The hosts of modern philosophers, to the very Hegels, have sided with Ockham in this matter. But now the question comes before us for reexamination: What are the modes of being? One might antecedently expect that the cenopythagorean categories would require three modes of being. But a little examination will show us that they could be brought into fairly presentable accordance with the theory that there were only two, or even only one. The question cannot be decided in that way. Besides, it would be illogical to rely upon the categories to decide so fundamental a question. The only safe way is to make an entirely fresh investigation. But by what method are we to pursue it? In such abstract questions, as we shall have already found, the first step, often more than half the battle, is to ascertain what we mean by the question--what we possibly *can* mean by it. We know already how we must proceed in order to determine what the meaning of the question is. Our sole guide must be the consideration of the use to which the answer is to be put--not necessarily the practical application, but in what way it is to subserve the *summum bonum*. Within this principle is wrapped up the answer to the question, what being is, and what, therefore, its modes must be. It is absolutely impossible that the word "Being" should bear any meaning whatever except with reference to the *summum bonum*. This is true of any word. But that which is true of one word in one respect, of another in another, of every word in some or another respect, that is precisely what the word "being" aims to express. There are other ways of conceiving Being--that it is that which manifests itself, that it is that which produces effects--which have to be considered, and their relations ascertained.

Peirce: CP 2.117 Cross-Ref:††

117. Having thus worked out a tolerable conception of Being, we turn to modes of being. But these are metaphysical conceptions. Let us first inquire how the validity of *any* metaphysical conception is to be determined. For this purpose we have only to apply the principles of Speculative Rhetoric. We sketch out the method and apply it to a few metaphysical conceptions, such as Reality, Necessity, etc. In process of doing this, we discover that all such metaphysical conceptions are but determinations of the categories, and consequently form a regular system. We also find that they can be held as valid only in approximative and imperfect senses.

Peirce: CP 2.118 Cross-Ref:††

118. But this seems to be in conflict with our conception of Being, particularly as derived from the notion of symbol; which, however, is solidly founded, too. We now begin to see the sense of talking of modes of being. They are elements of coöperation toward the *summum bonum*. The categories now come in to aid us materially, and we clearly make out three modes or factors of being, which we

proceed to make clear to ourselves. Arrived at this point, we can construct a *Weltanschauung*. From this platform, ethics acquires a new significance, as will be shown. Logic, too, shines forth with all its native nobility.†1 Common men carry this *Weltanschauung* in their breasts; and perhaps the pimp, the looting missionary, the Jay Gould, may, through the shadows of their degradation, catch now and then a purer glimpse of it, than the most earnest of citizens, the Cartises, the Emersons, the Bishop Myriels. It is beautifully universal; and one must acknowledge that there is something healthy in the philosophy of faith, with its resentment at logic as an impertinence. Only it is very infantile. Our final view of logic will exhibit it (on one side of it) as faith come to years of discretion.

Peirce: CP 2.119 Cross-Ref:††
CHAPTER 3

WHY STUDY LOGIC?P†1

§1. THE PRE-LOGICAL SCIENCES

119. The long discussion of the Classification of the Sciences to which the last section was devoted was intended to bring to light the mode of relationship of logic to other theoretical inquiries; or, at least, to make the author's opinion explicit; for as yet the truth of what has been said remains to be proved. It is, however, not a heresy but a doctrine very widely entertained, since Auguste Comte wrote, that the sciences form a sort of ladder descending into the well of truth, each one leading on to another, those which are more concrete and special drawing their principles from those which are more abstract and general.

Peirce: CP 2.120 Cross-Ref:††

120. If this be so, and if the scheme of classification of the sciences that has been proposed be correct, it will follow that there are but five theoretical sciences which do not more or less depend upon the science of logic. One of these five is Logic itself, which must contrive, by hook or by crook, to work out its own salvation without a full pre-acquaintance with its own discoveries, but which, like any other science, will lay one stone upon another in the erection of its doctrine. This is the last of the five. The first is Mathematics. Mathematics may itself be regarded as an art of reasoning. Perhaps this is not the highest conception of it. But at any rate, mathematics has no occasion to inquire into the theory of the validity of its own argumentations; for these are more evident than any such theory could be. The second of the five is that department of philosophy called Phenomenology, whose business it is simply to draw up an inventory of appearances without going into any investigation of their truth. The third is Esthetics, if I am to take the word of others that there is such a science, I myself being lamentably ignorant of it, as I fear will too plainly appear. The fourth is Ethics; certainly, one of the very subtlest of studies. The whole course of it seems to consist in painfully extricating oneself from one pitfall only

straightway to fall into another. It might seem that logic was desirable in this deliberation; but I fear that logic, as a definite theory, can be of no avail until one knows what it is that one is trying to do, which is precisely what ethics has to determine. On the contrary, that has to be settled before one can form any sound system of logic, as we shall see in due time.

Peirce: CP 2.121 Cross-Ref:††

121. All the other sciences but those five, according to the principles herein to be defended, depend upon Logic. I do not mean merely that they practice logical reasoning: they draw principles from the theory of logic. This dependence will be most direct and intimate for those sciences which stand nearest after Logic in the scheme of the sciences; yet even those which are most remote, such as History on the Psychological side, and Geology on the Physical side, have sometimes to make direct appeal to the theory of evidence. Besides that, all these Descriptive sciences must be founded upon Classificatory Sciences. Now that the classificatory sciences have to make appeal to the science of rationality, and always have done so, in order to determine what they are to think of the reality of their own classifications, will not be denied. Moreover, the Classificatory Sciences are and must be founded upon the Nomological Sciences. Here we find the psychologists, on the one side, together with Galileo, Kepler, Newton, Descartes, and all the founders of nomological physics, on the other, making direct appeal to the theory of logic. In addition to that, these Nomological Sciences cannot avoid depending upon Metaphysics. It is when they promise themselves that they will not make any metaphysical assumptions that they are most in danger of slipping too deep into the metaphysical slough for deliverance, precisely because one cannot exercise control and criticism of what one does unconsciously. At a later stage of our logical studies this dependence of nomology upon metaphysics will appear very evident. As to Metaphysics, if the theory of logic which is to be developed in this book has any truth, the position of the two greatest of all metaphysicians, Aristotle and Kant, will herein be supported by satisfactory proof, that that science can only rest directly upon the theory of logic. Indeed, it may be said that there has hardly been a metaphysician of the first rank who has not made logic his stepping-stone to metaphysics.

Peirce: CP 2.122 Cross-Ref:††

122. Such is the place of logic among the sciences; and such is its utility. Yet the reader will find that the aggregate value of all such applications will not compare with the treasure of the pure theory itself. For when he has surveyed the whole subject, he will see that the theory of logic, in so far as we attain to it, is the vision and the attainment of that Reasonableness for the sake of which the Heavens and the Earth have been created.

Peirce: CP 2.123 Cross-Ref:††
§2. TEN PRE-LOGICAL OPINIONS

123. Meantime, O Reader, not yet seeing the truth of this, why is it that you have undertaken the study of logic? You may have some excellent reasons which are peculiar to your personal relations to science and to life. But in addition to these, there are certain reasons which you must have, since they attach to the very essence of the study. Presuming that, aside from personal reasons, you desire in singleness of heart

to examine the theory of reasoning under the guidance of an older student, I remark that this very fact is evidence that you are already a much better logician than are the mass of mankind, who are thoroughly persuaded that they reason well enough already. I do not mean to say that they maintain that none of them ever reasons wrong. Far from that; though they trust to common sense as affording all the security that could be desired for reasoning, yet their adhesion is majestically unanimous to the proposition that of all the race there is but one single individual who never falls into fallacy; and their only point of difference is that each is quite sure that he himself is that man. Unfortunately, to be cocksure that one is an infallible reasoner is to furnish conclusive evidence either that one does not reason at all, or that one reasons very badly, since that deluded state of mind prevents the constant self-criticism which is, as we shall see, the very life of reasoning. Congratulations, then, from my heart go out to you, my dear Reader, whom I assume to have a sincere desire to learn, not merely the *dicta* of common sense, but what good reasoning, scientifically examined, shall prove to be. You are already an unusually good logician.

Peirce: CP 2.124 Cross-Ref:††

124. But now from what I know of you, I am led to think that you entertain certain ten opinions upon which I should like to offer some thoughts. You must almost certainly entertain these opinions, or you would not be wanting to study logic. Perhaps you and I do not think differently on most of these points. Yet it might be well to turn them over, and see what you do think, and why. Most of them are more or less controverted today.

Peirce: CP 2.125 Cross-Ref:††

125. In the first place, you would not wish to study logic unless you intended to reason; and you doubtless hold the purpose of reasoning to be the ascertainment of the truth. So it appears that you belong to the sect that maintains that there is such a thing as truth. Ought you not, then, to settle with yourself what that opinion of yours precisely amounts to, and further, what reason there is for entertaining it?

Peirce: CP 2.126 Cross-Ref:††

126. In the second place, you not only seem to fancy that there is such a thing as truth, but also that it can be found out and known, in some measure. What reason is there for that?

Peirce: CP 2.127 Cross-Ref:††

127. In the third place, you seem to think not only that some knowledge can be attained, but that it can be attained by reasoning.

Peirce: CP 2.128 Cross-Ref:††

128. In the fourth place, you seem to think that not only may reasoning lead to the truth, but that a man may be deceived by reasoning badly. This conception seems to demand scrutiny.

Peirce: CP 2.129 Cross-Ref:††

129. In the fifth place, I strongly suspect that you hold reasoning to be superior to intuition or instinctive uncritical processes of settling your opinions. What basis of fact is there for this opinion?

Peirce: CP 2.130 Cross-Ref:††

130. In the sixth place, I believe that you opine that you yourself are conscious of reasoning. I do not doubt that you are so, in some sense; but what is it precisely that you are conscious of?

Peirce: CP 2.131 Cross-Ref:††

131. In the seventh place, I am quite sure that you are already in possession of a logic, or theory of reasoning; and I wish to call your particular attention to the circumstance that in the eighth place, it would seem that though you entertain this theory of reasoning, you also entertain the opinion that your theory is wrong. How can you opine that your opinion is wrong?

Peirce: CP 2.132 Cross-Ref:††

132. In the ninth place, although you think you reason wrong--else why study logic?--you think that by this reasoning process which is wrong you can correct your method, and demonstrate by bad reasoning beyond all peradventure that your improved reasoning is perfect.

Peirce: CP 2.133 Cross-Ref:††

133. In the tenth place, you would appear to be of opinion that by improving your theory of reasoning your practice of reasoning will be improved; much as if a man subject to palpitations of the heart should expect to be cured by reading a book on physiology. Is this a sound view or not?

Peirce: CP 2.134 Cross-Ref:††

134. I think you will agree with me that these ten opinions should be revised, and if necessary, corrected, before going any further. In regard to each one, you first wish to consider precisely what the opinion is, and next what reason there may be for adhering to it. We need not carry the discussion into any deep metaphysics. Simple answers will do to begin with; only they must be distinct and explicit. Profounder insight may come later.

Peirce: CP 2.135 Cross-Ref:††
§3. THE OBJECTIVITY OF TRUTH

135. You certainly opine that there is such a thing as Truth. Otherwise, reasoning and thought would be without a purpose. What do you mean by there being such a thing as Truth? You mean that something is *SO*--is correct, or just--whether you, or I, or anybody thinks it is so or not. Most persons, no doubt, opine that for every question susceptible of being answered by *yes* or *no*, one of these answers is true and the other false. Perhaps that is carrying the doctrine to an extravagant pitch.†1 At any rate, the mere fact that you wish to learn logic would not prove that you go so far as that. It only shows that you think that *some* question--some interesting question, what one, perhaps, you are not just now prepared to say--has one answer which is decidedly right, whatever people may think about it. The essence of the opinion is that there is *something* that is *SO*, no matter if there be an overwhelming vote against it. So you plainly opine. For if thinking otherwise is going to make it otherwise, there is no use in reasoning or in studying logic.

Peirce: CP 2.136 Cross-Ref:††

136. Such is the opinion: there have been, and are today, numbers of eminent thinkers, philosophers, and even professed logicians, who have flatly denied it. One might produce stacks of *a priori* arguments against it. One might urge, for example, that there can be nothing of which God is ignorant. Even if there be no God, there can

be nothing of which an Omniscient God would be ignorant, if there were such a Being. But whatever the Omniscient may think is *ipso facto* so. Consequently, the idea of Truth, in the sense of that which is so whether the thinker thinks it so or not, must be foreign to the mind of God. Of such truth Omniscience must be ignorant, and since He is by hypothesis ignorant of nothing, there is no such thing. Another argument is that if there be anything which is so, in spite of what be thought, there may be something which cannot be thought. For it is conceivable that all who could think it were destroyed. But it is inconceivable that there should be anything that cannot be thought, for to conceive this would be to think the very thing supposed to be unthinkable. Hence it is inconceivable that there should be any Truth independent of opinions about it.

Peirce: CP 2.137 Cross-Ref:††

137. One might equally argue *a priori* in favor of the Truth. For suppose there is not any proposition which is correct independently of what is thought about it. Then if there be any proposition which nobody ever thinks incorrect, it is as correct as possible and has all the truth there is. Consider, then, the proposition: "This proposition is thought by somebody to be incorrect." Now if it is, in fact, thought by somebody to be incorrect, then it is true. For that is precisely the statement. But if it is not thought by anybody to be incorrect, it has all the truth possible, if there is no truth independent of opinion. Here, then, is a proposition which is correct whether it is thought to be so or not. Therefore, there is such a thing as a proposition correct whatever may be opinions about it. But when we come to study logic, we shall find that all such *a priori* arguments, whether *pro* or *con*, about positive fact are rubbish. This question is a question of fact, and experience alone can settle it.

Peirce: CP 2.138 Cross-Ref:††

138. Experience is that determination of belief and cognition generally which the course of life has forced upon a man. One may lie about it; but one cannot escape the fact that some things *are* forced upon his cognition. There is the element of brute force, existing whether you opine it exists or not. Somebody may object that if he did not think so, he would not be forced to think so; so that it is not an instance in point. But this is a double confusion of ideas. For in the first place, that something is, even if you think otherwise, is not disproved but demonstrated if you cannot think otherwise; and in the next place, what experience forces a man to think, of course he must think. But he is not therein forced to think that it is *force* that makes him think so. The very opinion entertained by those who deny that there is any Truth, in the sense defined, is that it is not force, but their inward freedom which determines their experiential cognition. But this opinion is flatly contradicted by their own experience. They insist upon shutting their eyes to the element of compulsion, although it is directly experienced by them. The very fact that they can and do so shut their eyes confirms the proof that fact is independent of opinion about it.

Peirce: CP 2.139 Cross-Ref:††

139. Deceive yourself as you may, you have a direct experience of something reacting against you. You may suppose that there is some substance in which *ego* and *non-ego* have alike the roots of their being; but that is beside the question. The fact of the reaction remains. There is the proposition which is so, whatever you may opine about it. The essence of truth lies in its resistance to being ignored.

Peirce: CP 2.140 Cross-Ref:††

§4. DIRECT KNOWLEDGE

140. You further opine that there is such a thing as knowledge. Your thinking there is any use in logic betrays that opinion. For you, the *non-ego* is not an unknowable thing in itself. Since the above argument for reality is that it is experienced, the same argument compels you to admit that there is knowledge; so that that branch of this second question needs no further attention. But it will be well to notice roughly in what sense this argument compels you to admit the existence of knowledge.

Peirce: CP 2.141 Cross-Ref:††

141. The knowledge which you are compelled to admit is that knowledge which is directly forced upon you, and which there is no criticizing, because it is directly forced upon you. For example, here I sit at my table with my inkstand and paper before me, my pen in my hand, my lamp at my side. It may be that all this is a dream. But if so, that such dream there is, is knowledge. But hold: what I have written down is only an imperfect description of the percept that is forced upon me. I have endeavored to state it in words. In this there has been an endeavor, purpose--something not forced upon me but rather the product of reflection. I was not forced to this reflection. I could not hope to describe what I see, feel, and hear, just as I see, feel, and hear it. Not only could I not set it down on paper, but I could have no kind of thought adequate to it or any way like it.†P1

Peirce: CP 2.141 Cross-Ref:††

Hundreds of percepts have succeeded one another while I have been setting down these sentences. I recognize that there is a percept or flow of percepts very different from anything I can describe or think. What precisely that is I cannot even tell myself. It would be gone, long before I could tell myself many items; and those items would be quite unlike the percepts themselves. In this thought there would always be effort or endeavor. Whatever is the product of effort might be suppressed by effort, and therefore is subject to possible error. I am forced to content myself not with the fleeting percepts, but with the crude and possibly erroneous thoughts, or self-informations, of what the percepts were. The science of psychology assures me that the very percepts were mental constructions, not the first impressions of sense. But what the first impressions of sense may have been, I do not know except inferentially and most imperfectly. Practically, the knowledge with which I have to content myself, and have to call "the evidence of my senses," instead of being in truth the evidence of the senses, is only a sort of stenographic report of that evidence, possibly erroneous.†P2 In place of the *percept*, which, although not the first impression of sense, is a construction with which my will has had nothing to do, and may, therefore, properly be called the "evidence of my senses," the only thing I carry away with me is the *perceptual facts*, or the intellect's description of the evidence of the senses, made by my endeavor. These perceptual facts are wholly unlike the percept, at best; and they may be downright untrue to the percept.†P1 But I have no means whatever of criticizing, correcting or recomparing them, except that I can collect new perceptual facts relating to new percepts, and on that basis may infer that there must have been some error in the former reports, or on the other hand I may in this way persuade myself that the former reports were true. The perceptual facts are a very imperfect report of the percepts; but I cannot go behind that record. As for going back to the first impressions of sense, as some logicians recommend me to do, that would be the most chimerical of undertakings.

Peirce: CP 2.142 Cross-Ref:††

142. The percepts, could I make sure what they were, constitute experience proper, that which I am forced to accept. But whether they are experience of the real world, or only experience of a dream, is a question which I have no means of answering with *absolute* certainty. I have, however, three tests which, though none of them is infallible, answer very well in ordinary cases. The first test consists in trying to dismiss the percepts. A fancy, or day-dream, can commonly be dismissed by a direct effort of will.†P2 If I find that the flow of percepts persists consistently in spite of my will, I am usually satisfied. Still, it may be a hallucination. If I have reason to suspect that it is so, I apply the second test, which consists in asking some other person whether he sees or hears the same thing. If he does, and if several people do, that will ordinarily be taken as conclusive. Yet it is an established fact that some hallucinations and illusions affect whole companies of people. There remains, however, a third test that can be applied; and it is far the surest of the three. Namely, I may make use of my knowledge of the laws of nature (very fallible knowledge, confessedly) to predict that if my percept has its cause in the real world, a certain experiment must have a certain result--a result which in the absence of that cause would be not a little surprising. I apply this test of experiment. If the result does not occur my percept is illusory; if it does, it receives strong confirmation. For example, if I and all the company are so excited that we think we see a ghost, I can try what an unimaginative kodak would say to it.†P1 So Macbeth made the experiment of trying to clutch the dagger.

Peirce: CP 2.143 Cross-Ref:††

143. All these tests, however, depend upon inference. The data from which inference sets out and upon which all reasoning depends are the *perceptual facts*, which are the intellect's fallible record of the *percepts*, or "evidence of the senses." It is these percepts alone upon which we can absolutely rely, and that not as representative of any underlying reality other than themselves.

Peirce: CP 2.144 Cross-Ref:††
§5. REASONING AND EXPECTATION

144. But since you propose to study logic, you have more or less faith in reasoning, as affording knowledge of the truth. Now reasoning is a very different thing indeed from the percept, or even from perceptual facts. For reasoning is essentially a voluntary act, over which we exercise control. If it were not so, logic would be of no use at all. For logic is, in the main, criticism of reasoning as good or bad. Now it is idle so to criticize †P2 an operation which is beyond all control, correction, or improvement.

Peirce: CP 2.145 Cross-Ref:††

145. You have, therefore, to inquire, first, in what sense you have any faith in reasoning, seeing that its conclusions cannot in the least resemble the percepts, upon which alone implicit reliance is warranted. Conclusions of reasoning can little resemble even the *perceptual facts*. For besides being involuntary, these latter are strictly memories of what has taken place in the recent past, while all conclusions of reasoning partake of the general nature of expectations of the future. What two things can be more disparate than a memory and an expectation?

Peirce: CP 2.146 Cross-Ref:††

146. The reader questions, perhaps, the assertion that conclusions of reasoning are always of the nature of expectations. "What!" he will exclaim, "can we not reason about the authorship of the Junius Letters or the identity of the Man in the Iron Mask?" In a sense we can, of course. Still, the conclusion will not be at all like remembering the historical event. In order to appreciate the difference, begin by going back to the percept to which the memory relates. This percept is a single event happening *hic et nunc*. It cannot be generalized without losing its essential character. For it is an actual passage at arms between the non-ego and the ego. A blow is passed, so to say. Generalize the fact that you get hit in the eye, and all that distinguishes the actual fact, the shock, the pain, the inflammation, is gone. It is anti-general. The memory preserves this character, only slightly modified. The actual shock, etc., are no longer there, the quality of the event has associated itself in the mind with similar past experiences. It is a little generalized in the *perceptual fact*. Still, it is referred to a special and unique occasion, and the flavor of anti-generality is the predominant one. Now, on the other hand, consider what an expectation is. Begin with something in the distant future; and dismiss from your mind mechanical and necessitarian ideas, which essentially modify the pure notion of futurity. Looking at an expectation referring to the distant future, something, for example, which you propose to do, it is very little distinguished from a mere "may be." There is a sort of picture in your imagination whose outlines are vague and fluid. You do not attach it to any definite occasion, but you think vaguely that some definite occasion there is, to which that picture does attach itself, and in which it is to become individualized. You think that at present its state of being consists in the fact that either in your will or in somebody else's, or somehow in something analogous thereto in the nature of things, some rule is determined, or nearly determined, which will probably govern the individual event when it occurs. As the time approaches, the individualizing element, which is nothing but a general notion that some individual event is to be governed by this idea, acquires more and more prominence. But the expectation ever remains a general idea which we describe to ourselves as being attached to some generally described actual occasion; whereas the memory is merely the reverberation of the shock of perception, essentially anti-general, though worn down here and there into generality by rubbing against memories of other similar occurrences. A perceptual fact is a memory hardly yet separated from the very percept. What is a conclusion of reasoning? It is a general idea to which at the suggestion of certain facts a certain general habit of reason has induced us to believe that a realization belongs. How slight is the difference between this and the description of an expectation! Yet, if we look closer, we can discern that the resemblance is nearer yet. For when we reason, we deliberately approve our conclusion. We look upon it as subject to criticism. We say to it: "Good! You will stand the fire, and come out of it with honour." Thus more than half an expectation enters into it as of its essence.

Peirce: CP 2.147 Cross-Ref:††

147. The second branch of the question, when you have decided in what your faith in reasoning consists, will inquire just what it is that justifies that faith. The simulation of doubt about things indubitable or not really doubted is no more wholesome than is any other humbug; yet the precise specification of the evidence for an undoubted truth often in logic throws a brilliant light in one direction or in another, now pointing to a corrected formulation of the proposition, now to a better comprehension of its relations to other truths, again to some valuable distinctions, etc.

Peirce: CP 2.148 Cross-Ref:††

148. As to the former branch of this question, it will be found upon consideration that it is precisely the analogy of an inferential conclusion to an expectation which furnishes the key to the matter. An expectation is a habit of imagining. A habit is not an affection of consciousness; it is a general law of action, such that on a certain general kind of occasion a man will be more or less apt to act in a certain general way. An imagination is an affection of consciousness which can be directly compared with a percept in some special feature, and be pronounced to accord or disaccord with it. Suppose for example that I slip a cent into a slot, and expect on pulling a knob to see a little cake of chocolate appear. My expectation consists in, or at least involves, such a habit that when I think of pulling the knob, I imagine I see a chocolate coming into view. When the perceptual chocolate comes into view, my imagination of it is a feeling of such a nature that the percept can be compared with it as to size, shape, the nature of the wrapper, the color, taste, flavor, hardness and grain of what is within. Of course, every expectation is a matter of inference. What an inference is we shall soon see †1 more exactly than we need just now to consider. For our present purpose it is sufficient to say that the inferential process involves the formation of a habit. For it produces a belief, or opinion; and a genuine belief, or opinion, is something on which a man is prepared to act, and is therefore, in a general sense, a habit. A belief need not be conscious. When it is recognized, the act of recognition is called by logicians a judgment, although this is properly a term of psychology. A man may become aware of any habit, and may describe to himself the general way in which it will act. For every habit has, or is, a general law. Whatever is truly general refers to the indefinite future; for the past contains only a certain collection of such cases that have occurred. The past is actual fact. But a general (fact) cannot be fully realized. It is a potentiality; and its mode of being is *esse in futuro*. The future is potential, not actual. What particularly distinguishes a general belief, or opinion, such as is an inferential conclusion, from other habits, is that it is active in the imagination. If I have a habit of putting my left leg into my trouser before the right, when I imagine that I put on my trousers, I shall probably not definitely think of putting the left leg on first. But if I *believe* that fire is dangerous, and I imagine a fire bursting out close beside me, I shall also imagine that I jump back. Conversely--and this is the most important point--a belief-habit formed in the imagination simply, as when I consider how I ought to act under imaginary circumstances, will equally affect my real action should those circumstances be realized. Thus, when you say that you have faith in reasoning, what you mean is that the belief-habit formed in the imagination will determine your actions in the real case. This is looking upon the matter from the psychological point of view. Under a logical aspect your opinion in question is that general cognitions of potentialities *in futuro*, if duly constructed, will under imaginary conditions determine *schemata* or imaginary skeleton diagrams with which percepts will accord when the real conditions accord with those imaginary conditions; or, stating the essence of the matter in a nutshell, you opine that percepts follow certain general laws. Exactly how far you hold that the percepts are determined by law is a matter of individual opinion. The mere fact that you hold reasoning to be useful only supposes that you think that *to some useful extent* percepts are under the governance of law. The habit of all loose thinkers is immensely to exaggerate the universality of what they believe, beyond all warrant. Some would boldly affirm that in all respects percepts are governed by law, a proposition which, if not logically absurd, is, at least, contrary to overwhelming evidence. Neither the mass of mankind nor even the most mechanical school of scientific speculators, of whom Laplace is the type, go to any such length. Various

opinions have more or less vogue. We shall have to examine them later.

Peirce: CP 2.149 Cross-Ref:††

149. Now, taking the opinion in a vague sense, what evidence is there that it is true? In investigating this question, the first circumstance which strikes us is that this opinion is itself of a general nature. If percepts were not subject to law, at all, it would be a matter of indifference what our general ideas were. It might be convenient to act and think according to rules; but one set of rules would be superior to another set merely as being more conveniently carried out. It is safe to say that nobody ever did believe that percepts are in no degree subject to law, whatever nominalists may have persuaded themselves that they believed. But there is no more striking characteristic of dark ages, when thought was little developed, than the prevalence of a sentiment that an opinion was a thing to be chosen because one liked it, and which, having once been adopted, was to be fought for by fire and sword, and made to prevail. Take any general doctrine you please, and it makes no difference what facts may turn up: an ingenious logician will find means to fit them into the doctrine. Ask the theologians if this is not true. As civilization and enlightenment advance, however, this style of thought tends to weaken. Natural selection is against it; and it breaks down. Whatever one's theory may be as to the invalidity of human reason, there are certain cases where the force of conviction practically cannot be resisted; and one of these is the experience that one opinion is so far from being as strong as another in the long run, though it receives equally warm support, that on the contrary, ideas utterly despised and frowned upon have an inherent power of working their way to the governance of the world, at last. True, they cannot do this without machinery, without supporters, without facts; but the ideas somehow manage to grow their machinery, and their supporters, and their facts, and to render the machinery, the supporters, and the facts strong. As intellectual development proceeds, we all come to believe in this more or less. Most of us, such is the depravity of the human heart, look askance at the notion that ideas have any power; although that some power they have we cannot but admit. The present work, on the other hand, will maintain the extreme position that every general idea has more or less power of working itself out into fact; some more so, some less so. Some ideas, the harder and more mechanical ones, actualize themselves first in the macrocosm; and the mind of man receives them by submitting to the teachings of nature. Other ideas, the more spiritual and moral ones, actualize themselves first in the human heart, and pass to the material world through the agency of man. Whether all this be true or not, it must at any rate be admitted by every candid man that he does believe firmly and without doubt that to some extent phenomena are regular, that is, are governed by general ideas; and so far as they are so, they are capable of prediction by reasoning.

Peirce: CP 2.150 Cross-Ref:††

150. I ought not to dismiss the question of the possibility of attaining a knowledge of the truth by reasoning without noticing an opinion which has of late years attained some vogue among men of science, that we cannot expect any physical hypothesis to maintain its ground indefinitely even with modifications, but must expect that from time to time there will be a complete cataclysm that shall utterly sweep away old theories and replace them by new ones. As far as I know, this notion has no other basis than the history of science. Considering how very, very little science we have attained, and how infantile the history of science still is, it amazes me that anybody should propose to base a theory of knowledge upon the history of science alone. An emmet is far more competent to discourse upon the figure of the earth than we are to say what future millennia and millionennia may have in store for

physical theories, with the atomic theory and the theory of light not yet one sleepy century old. The only really scientific theory that can be called old is the Ptolemaic system; and that has only been improved in details, not revolutionized. The most unhappy of physical theories has been that of Phlogiston; and even that was not altogether false, since something is lost from a burnt body; namely, energy.

Peirce: CP 2.151 Cross-Ref:††

§6. THE FALLIBILITY OF REASONING AND THE FEELING OF
RATIONALITY †1

151. Your studying logic is probably connected with a desire to reason *well* and to avoid reasoning *ill*. So that you probably conceive that reasoning may be either good or bad. If this is true, this matter of reasoning is a very singular function. There is no such thing as bad sight or bad hearing. True, these phrases are used; but all that is meant by "*bad*" in such locutions is *imperfect*. If a man is color-blind, he fails to perceive certain distinctions. But if there were a man to whom everything which looks to us red were to appear green, and vice versa, there would be no reason for stigmatizing such vision as bad. I detest the taste prevalent in a certain nation. Doubtless people of that nation would detest my taste; I should be heartily sorry if they did not. If either party were to go further and pronounce the other's taste to be *bad*, that would clearly imply that they made use of some other criterion than taste itself. Again, those whose sentiments I share abhor certain doctrines of certain writers upon Ethics--say, for example, those who make action the ultimate end of man.†2 I acknowledge that those writers are as much in earnest as we are, and I am quite aware that they reciprocate all our abhorrence. However it may be about taste, in regard to morals we can see ground for hope that debate will ultimately cause one party or both to modify their sentiments up to complete accord. Should it turn out otherwise, what can be said except that some men have one aim and some another? It would be monstrous for either party to pronounce the moral judgments of the other to be *BAD*. That would imply an appeal to some other tribunal.

Peirce: CP 2.152 Cross-Ref:††

152. In reasoning, however, your opinion is that we have the singular phenomenon of a physiological function which is open to approval and disapproval. In this you are supported by universal common sense, by the traditional logic, and by English logicians †1 as a body. But you are in opposition to German logicians †2 generally, who seldom notice fallacy, conceiving human reason to be an ultimate tribunal which cannot err.

Peirce: CP 2.153 Cross-Ref:††

153. Men of the best heart and purest intentions in the world have been known to commit actions contrary to their own moral principles simply because nothing in their lives had ever called their attention to the moral bearings of the kind of actions in question. In the same way, even upon the German theory, a man might reason badly from negligence, as in casting up a column of figures. Casuistry is in ill-repute, and somewhat deservedly so; and yet there are many nice questions of morals which are often discussed among men of the world; and it is a pity that there is not more discussion of that kind. I wonder some of our magazines do not open such a department. It often happens, I am sure, that a man seriously considers how it is his

duty to act in a certain case, and yet comes to a conclusion quite the reverse of that which he would reach if certain aspects of the case had not escaped him. Yet everybody agrees that in following the dictates of his conscience he acts right, and that he would, under the circumstances, have done wrong had he acted in the manner in which he would have thought it right to act if he had carried his self-discussion further. This is because right and wrong are held to be, in the ultimate analysis, subjective. It is every man's duty to enlighten his conscience as much as possible. Conscience itself requires him to do that. But still, when all is done that circumstances permit, it is his duty to act conscientiously. Does an analogous state of things hold with regard to reasoning, or does it not? Is that necessarily good reasoning which is deliberately judged by the reasoner to be good reasoning, even although further consideration would show him that it was bad reasoning? Common sense and the English logicians answer *no*. Conscience is like our Supreme Court, which intends to frame its decisions according to the principles of law. But when it has decided a point, its decision becomes law, whether the wisest counsels would have maintained it or not. For the actual law consists in that which the court's officers will sustain. But according to the English logicians it is otherwise with rationality. Every reasoning holds out some expectation. Either, for example, it professes to be such that if the premisses are true the conclusion will always be true, or to be such that the conclusion will usually be true if the premisses are true, or to be a method of procedure which must ultimately lead to the truth, or makes some other such promise. If the facts bear out that promise, then, say the English, the reasoning is good. But if the facts violate the promise, the reasoning is bad, no matter how deliberately human reason may have approved of it. For the sole purpose of reasoning is, not to gratify a sense of rationality analogous to taste or conscience, but to ascertain the Truth, in the sense of that which is SO, no matter what be thought about it. If there is no such thing as Truth in this sense, the English opinion must fall. But no mere phenomenalism or idealism which would make thought, or something analogous to thought, a factor of reality, would be in conflict with the English opinion, so long as the doctrine continued to be that there is such a thing as Truth which is independent of what you or I or any group or generation of men may opine upon the subject. We can, of course, conceive of a man who should admit that there was such Truth, but who should prefer to believe what should be agreeable to reason; but English-speaking people generally want to know the truth, and to avoid disappointment, whether the truth be agreeable to reason or not. Again, a man may very well hold that Truth, in the above sense, exists, and may desire above all things to acquire a knowledge of it; and yet he may be of the opinion that the judgments of human reason accord so accurately with that truth, that there is no surer way of reasoning right than that of reasoning as the human mind naturally approves. This very sensible opinion will call for careful scrutiny below; but it supposes our main problem here, namely, whether rationality is wholly a subjective matter, like taste, or has a subjective and an objective side, like morals, or is wholly objective, to have been already settled according to the English views. The consideration of this special variety of the English doctrine must therefore be postponed for a few pages.

Peirce: CP 2.154 Cross-Ref:††

154. I will now state the principal arguments which favor the German position. After that I will more fully expound the English position. This will enable us to comprehend better how the different German arguments hang together and to appreciate the distinctive character of the German doctrine, which it is important to appreciate, since thought having the same characteristic appears throughout the German logics. I will then set forth the arguments in favor of the English doctrine,

and I hope that, by that time, it will be plain enough what our decision should be. As a supplement to the discussion, I will consider the opinion described in the last paragraph. The reader will not deem that I am detaining him too long over so momentous a question, even although this, with the other topics of this section, must come up for more scientific treatment at a later stage of our studies.

Peirce: CP 2.155 Cross-Ref:††

155. The first German argument is that to presume to say that a physiological operation, such as is reasoning, is good or bad, is a pre-scientific conception for which there is no warrant. It belongs to the theological era of thought about nature, during the reign of which men drag the images of the saints into the street, blackguard them and threaten them if they do not cause rain to fall. Nowadays, such conduct is deemed childish. Men no longer become angry with the attraction of gravitation; and it is equally absurd to praise or blame any natural operation of man's psycho-physical organism. It certainly must be admitted that there is some force in that argument.

Peirce: CP 2.156 Cross-Ref:††

156. The second argument is that before the present question was distinctly formulated, and while men's judgments were still unbiased by it, it came to be widely, if not generally, perceived that there is a family likeness between Esthetics, Ethics, and Logic. All three of them are purely theoretical sciences which nevertheless set up norms, or rules which need not, but which ought, to be followed. Now in the case of taste, it is recognized that the excellence of the norm consists exclusively in its accordance with the deliberate and natural judgment of the cultured mind. The best opinion about morality likewise is that it has its root in the nature of the human soul, whether as a decree of reason, or what constitutes man's happiness, or in some other department of human nature. It is true that there are a few moralists who divorce the source of morality from human nature, but they are forced into a double doctrine; for they are still obliged to say that a man ought to obey his conscience, unless they abandon the very idea of morality. Nothing but the most evident necessity ought to persuade us that rationality, which is allied so closely to taste and to morals, is altogether unlike these in not being the product and expression of human reason. I will remark, at once, concerning this argument that it seems to me devoid of all force, for the reason that morality is far more objective than taste; and taste, morality, rationality, form a true sequence in this order. So that rationality ought according to the true analogy to be purely objective; taste being purely subjective, and morals half subjective, half objective. That morality is far more objective than taste is certainly the primitive judgment of common sense, to which some weight ought to be attached by those who propose to judge of reasons by natural common sense. It is true that the majority of writers on ethics in the past have made the root of morals subjective; but the best opinion is very plainly moving in the opposite direction. On the whole, therefore, the analogy of the trio is rather in favor of the English than the German doctrine.

Peirce: CP 2.157 Cross-Ref:††

157. The third argument is that science does not advance by revolutions, warfare, and cataclysms, but by coöperation, by each researcher's taking advantage of his predecessors' achievements, and by his joining his own work in one continuous piece to that already done. Now to suppose that there is any such objective entity as Rationality would be to break away from all modern thought from Descartes down. The English theory does set up such a noumenal Rationality in being dissatisfied with what appears to be rational to the normal human mind. The German might very well

say that if the present book is to be taken as an exponent of the opinions that are in the nature of things bound up with the objective view of rationality, as it professes to be, then the break of that doctrine with all modern philosophy is manifest. "If Peirce's exposition of the English doctrine is to be accepted," they might say, "and it is perhaps the only one which goes to the bottom of its philosophy, then that doctrine requires us to go back to the Aristotelian nonsense of *esse in futuro*, a conception too metaphysical for Hegel himself, which only such clouded intellects as the James Harrises and Monboddos have put up with. Something smacking very strongly of the extravagances of Wilhelmus Campallensis, who endowed abstract ideas with life, will have to be resuscitated in order to hold the parts of this doctrine together. The English thinkers themselves, after the most searching and the most friendly examination of their own philosophical classics, are today coming over to the leadership of Kant and of Hegel. According to Kant, all forms of thought, even those of metaphysical application, have their sole origin in the constitution of the human mind. Hegel has made it clear that if there is to be any philosophy, everything is to be traced back to one single elementary principle, and that the only one principle which can explain everything is thought, intellectual thought. But it is obviously an essential factor of the English doctrine of rationality that there should be a radically irrational element in Fact, an element of brute force, which this new defender of the doctrine pronounces to be anti-general!" All this has its force, without doubt, I acknowledge, as a rhetorical argument from probabilities; but it does not attack the heart of the question.

Peirce: CP 2.158 Cross-Ref: ††

158. The fourth argument, and the chief one, upon the German side, is that even if we adopt the English doctrine, still, after all, our last and only reliance has to be upon the accuracy of the natural judgments of mind as to what is rational. Practically we must assume that those judgments are infallible. For whatever criterion of rationality we may adopt, in applying that criterion it will be necessary to reason. We thus ultimately rely upon the immediate testimony of the mind as to what is good reasoning; and if we are to rely on that, it is superfluous to call in any other witness. If a given witness, A, testifies of his own knowledge to a fact, the testimony of a second witness, B, concerning the same occurrence, is of no account, if we only know his testimony at second hand, through the report which A makes of it. For the testimony of B has no value unless A tells the truth; and if A tells the truth, his own direct testimony is conclusive. This A represents human reason; B whatever other criterion of rationality may be proposed. I shall criticize this fourth argument below. But I will at once call attention to some features of it. In the first place, there is a great difference between believing evidence of a given kind and believing it to be infallible; so that if we were obliged to "trust" to the natural judgment of reason (which it will be found below that I dispute), still this would not at all amount to holding that natural judgment to be infallible. In the second place, even if the natural judgments of the mind concerning good and bad reasonings were infallible, that would not prove that rationality *consists* in the fact that certain reasonings are approved by the mind. The Pope is supposed to be infallible in his *ex cathedra* decisions as to what is pleasing to God; but it does not necessarily follow that God's being pleased has no other reality than the Pope's *ex cathedra* pronouncement. In the third place, if witness, A, testifies that he witnessed a fight in which he saw a certain John Doe aim a gun at Richard Roe and pull the trigger and shoot said Richard dead, and if he adds that another person, B, stood at his side and that B feels sure that though John Doe pulled the trigger his gun did not go off; and that Richard Roe did not fall until, a moment later, the third party, William Penn, shot him, it cannot be denied that what A tells about B does weaken his direct testimony. Whence, it follows that if A had

testified that B stood at his side and confirmed his account of what he saw in every particular, it would have strengthened his testimony. Consequently, the reasoning that because we are obliged to trust to the direct judgment of the mind concerning the validity of an argument (were this true), therefore no other criterion of that validity assured to us by another direct judgment of reasoning could have any weight whatever, is thoroughly mistaken reasoning.

Peirce: CP 2.159 Cross-Ref:††

159. A fifth argument which might be offered in support of the German doctrine, but which the defenders of that doctrine do not seem to think worth notice, appears to me to be fully as strong as any of the other four. This argument would run thus: The universe has a rational constitution. Now if all men, everywhere and always, have unhesitatingly pronounced something to be a necessity of reason, this is a very strong argument that not merely to the human mind, but to mind, as mind, it would be such a necessity. But this, if it be true, is but another aspect of the proposition that it is inherently rational. Then from this, taken in connection with the rational constitution of the universe, it will follow that a reasoner, trusting to this, cannot find that the facts will disappoint the expectation that this rationality justifies. Accordingly, that accordance of the facts with the promises of an argument which the English regard as *constituting* the rationality of the argument is, in truth, merely a corollary from the argument's being agreeable to Reason, as Reason, of which its catholic acceptance by all human minds is the sufficient evidence. Were this argument urged, my reply to it would be that a reasoner, as such, does not care whether such be the metaphysical constitution of the universe or not. Precisely what he is intent upon is that the facts shall not disappoint the promises of his argumentations. Therefore, the rationality of a reasoning, in the sense of that character of a reasoning at which the reasoner aims, does consist precisely in that necessary accord of the facts with the professions of the argument, in which the English doctrine makes it to consist.

Peirce: CP 2.160 Cross-Ref:††

160. I will now endeavor to state the English doctrine as I understand it, with such psychological additions as seem to be desirable in the present somewhat psychological discussion. It would seem, then, that in the simplest cases, the natural judgments of the mind in regard to what is logically necessary are as nearly infallible as are our ready judgments upon the simplest parts of the multiplication-table. They seem to be formed, too, in much the same way. Namely, in regard to the multiplication-table, we form our little diagram, perform our little experiment, and generalize the result under a conviction that what is true of one diagram of skeletal simplicity will be equally true, however complicated the nature of the units counted may be. So in the logical case, we imagine a simple case in which we should be called upon to act upon one rule or its denial, and we perceive that one of these rules would cause us to act at cross-purposes with ourselves. This is generalized under the conviction that it will make no difference what the nature of the case may be, so long as the logical relation remains the same. In more difficult cases, our generalizing conviction is less decided. We become doubtful. Our diagrammatic representation does not so accurately fit the case. Nevertheless, such judgment as we do form is formed precisely according to the English theory. That is to say, we consider as well as we can, how we ought to form our inference so as not to be disappointed in the result. It is simply a confused judgment of the same nature. In considering, for example, the case of a dweller upon the shores of an inland sea who should arrive at the Bay of Biscay and see the tide rise for several times successively, and asking what the nature of the reasoning should be, common sense finds itself confronted with a

problem which is too difficult for it, just as it was too difficult for Laplace and his successors. Inappropriate conceptions will be applied by common sense just as they were applied by the Laplacians. The only advantage of common sense will be that not having its attention drawn away to numerical rules, it will be a little less likely to overlook some considerations that the mathematicians overlooked in consequence of their [the considerations] being beyond the somewhat arbitrary limits of their [the mathematicians'] scales of measurement. There is probably no special instinct--using this word in a sense in which it shall embrace traditional as well as inherited habits--for rationality, such as there is for morality. Or, if there be such an instinct, it is of the most rudimentary and imperfect character. If not in all cases, at least in all cases which present any difficulty, we see natural judgments falling into the same errors into which early logicians fell. The common sense judgment concerning valid inference, so far as it is correct, seems to be nothing but a confused thinking of precisely the same nature as the English doctrine recommends. Common sense notions of logicity are in remarkable contrast to instincts in their want of obstinacy. Nothing so characterizes instincts as their persistence when all the lights of reason are against them, and this whether they are true inherited instincts or merely traditional. Well-bred people, for example, are full of traditional habits--prejudices, we call them--about manners. They may be in situations in which reason warns them that these habits are distinctly injurious to them; and still they have a difficulty in overcoming them even with a serious effort. The absence of any such phenomenon in regard to common sense judgments of logicity is striking. As soon as it appears that facts are against a given habit of reasoning, it at once loses its hold, without any such obstinate and irrational persistency as shows itself, not only in questions of morals, but even in mere matters of manners, such as *corporal pudor*. No matter how strong and well-rooted in habit any rational conviction of ours may be, we no sooner find that another equally well-informed person doubts it, than we begin to doubt it ourselves. This is plainly shown by the anger such doubt excites in us. If our own belief were not disturbed, there is no reason why we should care what others believe; and if we are quite sure that we are better acquainted with the subject than are the doubters, their doubts will in fact provoke nothing more than a good-natured smile. Contrast this with our sentiments if, in a parlor in some Pacific Island, one of the ladies should call for a dry gown and take off the only garment she had on before the company, preparatory to putting on the other, without the others appearing to see anything out of the common course in it. All sorts of shivers would run through us; but there would not be a trace of anger, for the reason that our own instincts--or rather, comparatively modern traditions--would remain absolutely undisturbed.

Peirce: CP 2.161 Cross-Ref:††

161. The English logician, therefore, not only maintains that the proper way of deciding whether a given argument is valid or not is to consider whether there is anything in the constitution of the universe and the nature of things which insures that the facts shall be such as the argument promises that they shall be, but he goes so far as to maintain that our ordinary common sense judgments respecting the validity of arguments are formed in the same way, and differ from the judgments of scientific logic only in resting upon vaguer and less distinct thought. To ask him, therefore, to abandon his method of estimating the validity of arguments in favor of the German method of appeal to natural judgments of good sense, is, in his apprehension, neither more nor less than to ask him to abandon exact thinking for a kind of thinking which differs from it in no respect except that of being loose and confused.

Peirce: CP 2.162 Cross-Ref:††

162. The Germans, as everybody knows, have put all the rest of the civilized world deeply in their debt by producing the completest treatises upon almost all departments of science. Logic is no exception. The number of deeply thought out and original systems of logic which were elaborated in Germany during the nineteenth century greatly exceeds that of such works produced in the same age in all the rest of the world. These German treatises have been great aids to me in writing the present book. This circumstance intensifies my desire to explain why it is that I not only constantly dissent from their doctrine, but am excited against them as pernicious writings. It is not their fault that they are pernicious. Contributions to philosophy might, as a general rule, almost be said to be useful in proportion as they are in error. For philosophy is still far from having reached the status of a mature science progressing in an orderly manner. It is still in that stage in which fundamental principles are disputed; and as long as that is the case, dispute is the most important business of a philosopher. But the only satisfactory refutation of a false system is that which is effected unintentionally and unconsciously by the vigorous defender of that system. He alone can make the falsity of it perfectly clear. But great genius and a great logical power are requisite for this. Few or none of the German logics are as strong as it is desirable that they should be. Still, their influence were wholesome, if it were not for the extraordinary deference which is paid to them by university men in this country and others. Deference to a treatise on logic is altogether out of place. It is an injury and almost an insult to the author. In another science this is not so true, because the author's long experience in his study justly gives a weight to his opinions beyond what can be attached to any reasons which he can express. But logic is purely a science of reasoning. The author is entitled to just so much approval as what he writes warrants, and no more. Moreover, in the present unsettled state of his science, it is his business to dispute, to give blows and take them; and it is no compliment to him to treat him tenderly. This, however, is the most innocent side of any deferential treatment of a work on logic. It is a terrible wrong to young students, who ought to be encouraged, urged, and if possible driven, to examine such works carefully, boldly, and actively--I had almost said aggressively; but that would be going too far. It is especially injurious to treat those works on logic deferentially whose doctrine is radically unsound in respect to the modes of argumentation which they approve and disapprove. This in my opinion is the case with the German treatises. At the same time, they are certainly very carefully written and well studied works. Certain general causes have operated to set their logic wrong. In the first place, the German mind is very subjective. A man must be a poor observer of men not to be able to see that. In the physical and natural sciences this tendency is held in check and prevented from doing much harm, while it has the effect of producing circumspection, care, attention to all those precautions upon which success in those sciences largely depends. There are some of the psychical sciences which are so positive and objective that a dash of subjectivity is ordinarily a wholesome corrective. But as a general rule German subjectivity appears to disadvantage in the psychical sciences. The German students of these sciences, as a class, are apt to be carried away by theories and to draw their teachings too much from the *Ichheit*, as has been commonly remarked. In mathematics subjectivity can apparently do no harm. In the present unsettled state of logic, the effect of subjectivity is lamentable. It causes appeal to be made to feeling, to prejudice, instead of to fact, and blocks any real advance. It renders logic perfectly nugatory as far as any helpfulness to science is concerned, until at length this nugatoriness crystallizes into a *laissez-faire* policy. The course of science evidently could not be placed under any intellectual constraint. Scientific men will reason as they see fit. But they would see fit to listen to the counsels of logicians, if those

counsels seemed to have a truly scientific basis. If any department of science is allowed to go on without any logical deliberation other than such as its own special studies lead it into, the result will be that, after more or less blundering, it will ultimately get its methods adapted to its temporary condition, only to plunge into new bewilderment when that temporary condition is outgrown. For the science, under such a policy, is guided by natural logicality corrected by experience. Now this natural logicality is precisely like scientific logic, except that it is thought hazily and inaccurately. It will ultimately lead to the same result, only with great waste of time and means of all sorts. Provided the truly scientific logic were put before the world, the subjective treatises could do no harm except that of occupying men's time in the examination of them, provided they were carefully examined and judged upon their merits. It is the deference paid to them because they are written by supposed-to-be renowned German professors that really does the harm.

Peirce: CP 2.163 Cross-Ref:††

163. The subjective tendency of the German mind consists either in a hyperaesthesia, or else in an excessive attention to feelings in their immediacy, or more probably to both causes, each stimulating the other. The energy of the soul, being carried in that direction, is taken away from reasoning. This naturally would produce an effect which observation shows is somehow produced, namely, that the average German is not logically a very bright mind. Perhaps this contributes toward causing him to ponder very deeply any subject which occupies him; and since every man naturally talks most about subjects on which he has thought most, the German's natural dullness is in large part concealed. Nevertheless, it cannot escape a close observer. All the world over, it may be remarked that men who write logics are not on the average very good reasoners. Some men, no doubt, are led into the study of logic because their minds are naturally subtle; but there are more of whom one would guess that they must have been drawn to pay attention to logic because reasonings puzzled them which would not have puzzled an ordinarily intelligent man. These faults of reasoning to which the ordinary logics draw most attention, confusions between *all* and *some*, would seem to be so glaring that no man who really thinks could commit them, except just as he might, now and then, hastily write down that 5 and 6 make 2. The only way in which an intelligent man could *deliberately* commit any such fallacy would be by the inaccurate application to the case of some rule of logic. Now a most singular phenomenon characterizes all the German logics of the nineteenth century which I have examined--certainly, considerably over fifty of them--and distinguishes them from those of the English. It is that every one of them somewhere falls into a logical fallacy. Such things are not matters of opinion: they are indisputable. Besides, each such fault on the part of one German logician is condemned--tacitly, at any rate--by the general body of German logicians; for I am not now speaking of errors of reasoning which have been deliberately defended by this or that school of logicians: I am speaking of indefensible faults. In order to prove my assertion that German logicians are universally capable of such faults, I intend to draw up a list of some of them and append it to this volume.†1 I hope it will serve to counteract the silly deference paid in this country to German logic--an attitude which no logician worthy of the name would desire that any student should assume toward his work.

Peirce: CP 2.164 Cross-Ref:††

164. Let us now briefly reexamine the five arguments in favor of the German position. For in the light of what has been said their general character can better be appreciated. The first is that it is absurd to approve or disapprove of a physiological

operation. I heartily endorse that remark. Every man who has learned to look upon nature from the physiological point of view must share the feeling. For my part, it so burns in my heart that if I could, I would abolish almost all punishment of grown people, and all judicial approval or disapproval except of the court's own officers. Let public opinion have its approvals and disapprovals, until public opinion learns better. But as for public force, let it be restricted to doing what is necessary to the welfare of society. Punishment, severe punishment, the barbaric punishment of a prison cell, infinitely more cruel than death, is not in the least conducive to public or to private welfare. As for the criminal classes, I would extirpate them, not by the barbarous method which some of those monsters whom economics has evolved propose, but by keeping the criminals confined in relative luxury, making them useful, and preventing reproduction. It would be easy to convert them from a source of enormous expense, and perpetual injury to people, into self-supporting harmless wards of the state. The only expense would be that of losing our darling revenge upon them. As for sporadic criminals, defalcators, murderers, and the like, I would deport them to an island, and leave them to govern themselves, and deal with one another. For trifling violations of order, trifling punishments might be retained.

Peirce: CP 2.165 Cross-Ref:††

165. I grant, then, that the feeling that it is absurd to approve or disapprove a physiological operation is a good and sound one. But this very attitude implies that something is to be approved and something disapproved. This being recognized, we are led to endeavoring to define what it is that may properly be approved or disapproved. The answer is easy enough. We approve of means of bringing about purposes which we embrace, assuming it to be in our power to adopt or to reject those means. As to the purposes themselves, every man must decide for himself, though others may offer suggestions. A physiological operation takes place under nature's laws, and is beyond our control. It is, therefore, idle to approve or to disapprove of it. But in this essential respect reasoning is not a physiological operation; being a method, perfectly under our control, of attaining a definite end, that of ascertaining how future phenomena will appear. As to the purpose of a physiological operation, we know nothing, unless we may presume that it is designed to perform the function which it does in fact perform. The Germans, with their disposition to look upon everything subjectively, that is, in its immediacy as feeling, are willing that we should like or dislike reasoning, as we might a piece of music; but they look upon the natural judgment of rationality as a mere judgment of feeling, overlooking the fact that the natural man, as much as the English logician, desires to ascertain the true facts, looks upon reasoning as a means of doing this, and adopts the mode of reasoning that he does adopt, because, as well as the confused state of his ideas enables him to judge, it must bring him to the truth, as a rule.

Peirce: CP 2.165 Cross-Ref:††

I have already said enough of the second argument which seeks to assimilate Logic to Esthetics. Its subjective leaning is manifest.

Peirce: CP 2.166 Cross-Ref:††

166. To the third argument, that the English view of logic involves a rupture with all modern metaphysics, I have as yet made no reply. This argument is not directly connected with the subjective attitude of thought, but is motivated by another German tendency which has a rational foundation, though it takes a particular turn in Germany chiefly from an accidental cause, yet influenced in some degree by German subjective ways of feeling. It is quite true that the success of modern science largely

depends upon a certain solidarity among investigators. I trust that the time is not far distant when logic may enter upon that path. All my efforts have, from the beginning of my career, been directed to that desirable result; and the logic of relatives has already become, partly by my labors, an accepted division of science.†1 But such harmonious cooperation supposes that fundamental principles are settled. Until they become so, dispute has to be the method in which a given science shall make its way to the light. Unfortunately, in logic, opinion upon fundamentals is all at sixes and sevens. The natural effect of a disposition to take subjective views is to stimulate a spirit by which men fall into agreement in their philosophical sentiments from sympathy and as a matter of fashion. There is a deal of fashion in the prevalence of ideas in the German universities. The fact that a professor's revenue depends upon his estimation in the minds of young men who are incompetent to form any rational opinion concerning the value of his ideas is probably the chief cause which operates to make a good deal of irrational uniformity of opinion throughout the German universities. This is aided, however, by the sympathetic spirit of the German, which is an effect of his subjectivity. For a whole generation, ideas that were not Hegelian were looked upon throughout the German universities with the same utter contempt with which ideas that are Hegelian are now regarded in those same halls. But one and the same spirit has always been prevalent there; namely, to settle first what metaphysical ideas are agreeable to reason, that is, in effect, to the spirit of the day, and to shape the science of logic to fit those ideas. That method is no less than preposterous. The only rational way would be to settle first the principles of reasoning, and, that done, to base one's metaphysics upon those principles. Modern notions of metaphysics are not rationally entitled to any respect, because they have not been determined in that way, but on the contrary by a purely accidental circumstance, or by a circumstance which, so far as it can be looked upon as a reason, is a reason against rather than in favor of modern metaphysics. Namely, it came about in this way. In the Middle Ages, the great study in France and England, the countries that were destined to intellectual predominance in the early modern centuries, was Theology. The metaphysics of Aquinas, a modified Aristotelianism, had been immensely elaborated and deeply transformed by the vast logical genius of the British Duns Scotus, who died in 1308. The extreme intricacy of this system was felt to be an objection to it, and various attempts were made to introduce Nominalism--the simplest possible of all Logico-Metaphysical theories, if it can be sustained. These efforts finally culminated in the system of another Englishman, William Ockham. Scotism permitted assent to almost any theological dogma; it was specially favorable to those which involved mystery; it was naturally liked by ultramontanes and extreme clericals. Ockhamism, on the other hand, was naturally unfavorable to anything mysterious, not to say, to anything religious. Ockham was, if possible, more renowned as a bold opponent of Papal encroachments than as the inceptor of a new way of thinking. The result of the struggle that ensued was that when the new learning came, and science began to awaken, it was the Scotists who were in possession of the Universities. This accidental position it was, more than anything else, though the theological tendency of the doctrine aided, that caused the new scholars to look upon the Dunces as the ignorant fogies, *par excellence*, the principal enemies of all learning, progress and good sense. It was the intense hostility of the new men to fogyism, chiefly, which pushed them into a metaphysics of an Ockhamistic color. At any rate, no mistake can be greater than to suppose that Ockhamistic thought is naturally allied to the conceptions of modern science: it is anti-scientific in essence. A scientific man whose only metaphysics has been such as his own studies have suggested will be definitely adverse to the ideas of Ockham, and, so far as his simple

conceptions go, will agree with Scotus. Of course, he will know nothing of the opinions that are distinctively Scotistic, for so far as these are not technically logical they have in view theology.

Peirce: CP 2.167 Cross-Ref:††

167. In the Middle Ages the question between Scotism and Ockhamism had been closely argued. Had the conceptions of modern science been present to the minds of the disputants, the victory of the Scotists would have been more overwhelming than it was. As matters went, Ockhamism derived its chief strength from its political alliance.

Peirce: CP 2.168 Cross-Ref:††

168. So this is the way in which modern philosophy became pushed into Ockhamism. The new thinkers were incapable of the subtle thought that would have been necessary for any adequate discussion of the question. They accepted nominalistic views upon the most superficial grounds. The question soon became buried and put out of sight by new questions which overlaid it, like new papers on an encumbered study table. In that way it has happened that the question has never attracted general, acute attention among modern metaphysicians. What is there in that genealogy which entitles the prevalent metaphysics to one feather's weight of rational authority? Authority is a thing not to be lightly introduced into science. Moreover, metaphysics ought to be founded on logic. To found logic on metaphysics is a crazy scheme. It was on logical grounds, and in a treatise on logic, that Ockham himself supported his nominalism.†1 For though his system had its faults, he had a more cultivated comprehension of the architecture of philosophy than have those modern Germans who boast that their systems of logic are "philosophical," that is, founded on metaphysics.

Peirce: CP 2.169 Cross-Ref:††

169. The fourth German argument is that we must, in any case, trust to the veracity of the natural instinct for rationality, and that consequently rationality consists in that judgment. I have already completely refuted that argument; but there is one point to be made against it which I have reserved for this place, because it so well illustrates the modes of thinking of the German logicians. Suppose this case. In the course of a long country ramble, I meet a boy whom I never saw before, and inquire my way of him. I take the turn which he directs me to take, but have not gone many steps further before I am overtaken by a man who informs me that that boy's mother has been arrested upon a charge of perjury and that I am wanted to testify to her veracity. "But," I say, "I never so much as heard of the woman. I did not know she existed." "Oh," says the man, "you must have known the boy had had a mother; and since you took the turn he told you to take, you are bound to acknowledge that the mother tells the truth, for it was she who had told the boy where this road leads to." What should I say to the man? I should say, "My dear sir, if you had asked me to testify that the woman *cannot* lie, for the reason that truth *consists* in her say-so, then I should think that your demand was as unjustifiable as any ever made of me." That would be an exaggeration, since his demand would not be in truth open to all the objections to which the argument of the German logicians is open; but as far as it goes, the analogy would be perfect. Somebody shakes a pair of dice in a dice-box and asks me to guess whether his next throw will be doublets or not. Before replying I make a mental diagram of all possible throws, and relying on that, I reply that I guess the throw will not show doublets. This is unquestionably a probable inference. In making it, the only thing that I am conscious of relying upon is my mental diagram, as

representing the probable course of experience. That diagram takes the place of the boy of whom I inquired my way. But now the German logician assures me that the advice to guess against the doublets really originated in an instinct for rationality, of the existence of which I had no assurance, and have not yet; except that, as he says, the authority of the diagram must have had some origin, just as the boy must have had some mother. Although the dice have not yet been cast, and all I know is that my guess seems reasonable, I am asked to testify to the world and to myself, not only that this unheard-of instinct spoke truly on that occasion, but that it does so invariably: nay, that the truth of reasonings *consists* in that instinct's saying that they are true. Outside of a German treatise of logic, I never met with so bald a fallacy as that. So this is the sort of reasoning that it is thought not decent to controvert, unless it be with hesitancy, with the utmost humility, and with an implied acknowledgment of the impropriety of a mere American's controverting the opinion of a German.

Peirce: CP 2.170 Cross-Ref:††

170. If I may be allowed to use the word "habit," without any implication as to the time or manner in which it took birth, so as to be equivalent to the corrected phrase "habit or disposition," that is, as some general principle working in a man's nature to determine how he will act, then an instinct, in the proper sense of the word, is an inherited habit, or in more accurate language, an inherited disposition. But since it is difficult to make sure whether a habit is inherited or is due to infantile training and tradition, I shall ask leave to employ the word "instinct" to cover both cases. Now we certainly have habits of reasoning; and our natural judgments as to what is good reasoning accord with those habits. I am willing to grant that it is probable that some of our judgments of rationality of the very simplest kind have at the bottom instincts in the above broad sense. I am inclined to think that even these have been so often furbished up and painted over by reflection upon the nature of things that they are, in mature life, mostly ordinary habits. In more complicated cases, say for example, in that guess about the pair of dice, I believe that our natural judgments as to what is reasonable are due to thinking over, ordinarily in a more or less confused way, what would happen. We imagine cases, place mental diagrams before our mind's eye, and multiply these cases, until a habit is formed of expecting that always to turn out the case, which has been seen to be the result in all the diagrams. To appeal to such a habit is a very different thing from appealing to any immediate instinct of rationality. That the process of forming a habit of reasoning by the use of diagrams is often performed there is no room for doubt. It is perfectly open to consciousness. Why may not all our natural judgments as to what is good reasoning be founded on habits formed in some such ways? If it be so, the German doctrine falls to the ground; for to form a notion of right reasoning from diagrams showing what will happen, is to form that notion virtually according to the English doctrine of logic, by reasoning from the nature of things. That is to say, a habit is involuntarily formed from the consideration of diagrams, which process when deliberately approved becomes inductive reasoning. Unless there be, in addition, some immediate instinctive feeling of rationality, the German theory cannot be correct. Yet proof of the existence of such an additional instinctive feeling is not forthcoming. Not even so much as a pretended proof of it is offered, nor so much as any likelihood of it, so high do these great German logicians hold themselves above the usual obligations of scientific logic.

Peirce: CP 2.171 Cross-Ref:††

171. On the contrary, pretty strong evidence is at hand that no such instinctive feeling exists. In the first place, our natural judgments as to what is good reasoning are accompanied by a sense of evidence: one thinks one sees that the fact is so and

must be so, not merely that we cannot help thinking so. In this respect, these judgments contrast strongly with those of conscience. You and I have a horror of incest. We have been told that there is a reason for it; but that is open to doubt. Reason or no reason, however, our aversion for and horror at the idea is simply felt, without any accompanying sense of evidence. So it is with what offends our taste. "I do not like you, Dr. Fell." I have no accompanying sense of its being a well-founded feeling. In regard to a simple syllogism it is quite otherwise. It is no blind, unaccountable impulse to reason in that way that I feel. I seem to perceive that so the facts must be. This difference between judgments of taste and morals on the one hand and of *rationality* on the other can hardly be accounted for on the German theory.

Peirce: CP 2.172 Cross-Ref:††

172. If, however, as the English suppose, the feeling of rationality is the product of a sort of subconscious reasoning--by which I mean an operation which would be a reasoning if it were fully conscious and deliberate--the accompanying feeling of evidence may well be due to a dim recollection of the experimentation with diagrams. There are many other facts which point in the same way, of which I will only mention one which seems almost conclusive. This is that if we practise logical reflexion according to the English method and are thus led to see that a certain method of reasoning promises nothing more than facts must from the nature of things bear out, we do not find that we have two distinct judgments of what is rational. If I am persuaded that incest will have deplorable effects upon off-spring, I feel a distinctly duplex condemnation of the practice, the one of a cool, almost sceptical kind, the other peremptory and without apology. There are some questions about which I, and I suppose it is the same with every thinking man, find these two voices quite at odds, my reason temperately but decidedly asserting that I ought to act in one way, my instincts, whether hereditary or conventional I cannot tell, most emphatically and peremptorily, though with no pretence to rationality, giving reason the lie. That is just such a phenomenon as would naturally be anticipated. It is very surprising that I do not find any such discord in my judgments as to what is good reasoning. There are various points in which my present opinions of what is good reasoning differ diametrically from those which I entertained before I had analyzed the matter after the English doctrine. But instead of my old feeling continuing to assert itself beside my new rationalized opinion, to say that it submits with docility is not enough: I cannot detect the least trace of it remaining. If it were an immediate feeling, as the Germans suppose, it certainly would persist. The only possible explanation of its not doing so is that it was merely itself a confused conclusion of subconscious reason which feels itself superseded by clearer analysis of the same kind and along the same line.

Peirce: CP 2.173 Cross-Ref:††

173. I have discussed this matter at some length, because it is a momentous question for logic; and it seemed proper to turn it over upon different sides. But in truth the essence of the matter lies in a nutshell. Facts are hard things which do not consist in my thinking so and so, but stand unmoved by whatever you or I or any man or generations of men may opine about them. It is those facts that I want to know, so that I may avoid disappointments and disasters. Since they are bound to press upon me at last, let me know them as soon as possible, and prepare for them. This is, in the last analysis, my whole motive in reasoning. Plainly, then, I wish to reason in such way that the facts shall not, and cannot, disappoint the promises of my reasoning. Whether such reasoning is agreeable to my intellectual impulses is a matter of no sort of consequence. I do reason not for the sake of my delight in reasoning, but solely to avoid disappointment and surprise. Consequently, I ought to plan out my reasoning so

that I evidently shall avoid those surprises. That is the *rationale* of the English doctrine. It is as perfect as it is simple.

Peirce: CP 2.174 Cross-Ref:††

174. It is now scarcely worth while to notice that opinion which acknowledges the English doctrine, yet prefers the authority of natural judgments concerning reasoning to those of analytic reason. It would be all very well to prefer an immediate instinctive judgment if there were such a thing; but there is no such instinct. What is taken for such is nothing but confused thought precisely along the line of the scientific analysis. It would be a somewhat extreme position to prefer confused to distinct thought, especially when one has only to listen to what the latter has to urge to find the former ready to withdraw its contention in the mildest acquiescence.

Peirce: CP 2.175 Cross-Ref:††
§7. REASONING AND CONDUCT

175. But while there is no independent instinctive sense of logicity, it is impossible to deny that there are instinctive ways of forming opinions, especially if we continue to take instinct in that broad sense in which it will include all habits of which we are not prepared to render an account, or in one word all that goes by the name of the rule of thumb. In applying such instincts, such habits of unknown parentage, we do reason a little. But that little reasoning is based on some axiom or impression of opinion which we adopt uncritically, without any assurance that it is rational.

Peirce: CP 2.176 Cross-Ref:††

176. The popular notion is that Reason is far superior to any instinctive way of reaching the truth; and from your desire to study logic, I am perhaps warranted in presuming that such is your opinion. If so, in what respect do you hold reasoning to be superior to instinct? Birds and bees decide rightly hundreds of times for every time that they err. That would suffice to explain their imperfect self-consciousness; for if *error* be not pressed upon the attention of a being, there remains little to mark the distinction between the outer and the inner worlds.†1 A bee or an ant cannot--could not, though he were able to indulge in the pastime of introspection--ever guess that he acted from instinct. Accused of it, he would say, "Not at all! I am guided entirely by reason." So he is, in fact, in the sense that whatever he does is determined by virtual reasoning. He uses reason to adapt means to ends--that is, to his inclinations--just as we do; except that probably he has not the same self-consciousness. The point at which instinct intervenes is precisely in giving him inclinations which to us seem so singular. Just so, we, in the affairs of everyday life, merely employ reason to adapt means to inclinations which to us appear no more bizarre than those of a bee appear to him.

Peirce: CP 2.176 Cross-Ref:††

An old friend of mine once remarked to me that if a being not human were to observe mankind, he would be struck with admiration at that instinct which leads a large number of men each to contribute an insignificant sum so as to make up a fortune in the aggregate, and present it to one person chosen by lot; although certainly men who buy tickets to a lottery refrain from taking credit for their highly altruistic conduct with a modesty which does credit to their hearts. In the ordinary conduct of

everyday affairs, men really do act from instinct; and their opinions are founded on instinct in the broad sense in which I here take that term. A small dose of reasoning is necessary to connect the instinct with the occasion: but the gist and character of their conduct is due to the instinct. It is only a remarkable man or a man in a remarkable situation, who, in default of any applicable rule of thumb, is forced to reason out his plans from first principles. In at least nine such cases out of every ten, he blunders seriously, even if he manages to escape complete disaster. We shall therefore be well within bounds in pronouncing Reason to be more than a thousand times as fallible as Instinct.

Peirce: CP 2.177 Cross-Ref:††

177. Invariably follow the dictates of Instinct in preference to those of Reason when such conduct will answer your purpose: that is the prescription of Reason herself.†1 Do not harbor any expectation that the study of logic can improve your judgment in matters of business, family, or other departments of ordinary life. Clear as it seems to me that certain *dicta* of my conscience are unreasonable, and though I know it may very well be wrong, yet I trust to its authority emphatically rather than to any rationalistic morality. This is the only rational course.

Peirce: CP 2.178 Cross-Ref:††

178. But *fortunately* (I say it advisedly) man is *not so happy* as to be provided with a full stock of instincts to meet all occasions, and so is forced upon the adventurous business of reasoning, where the many meet shipwreck and the few find, not old-fashioned happiness, but its splendid substitute, success. When one's purpose lies in the line of novelty, invention, generalization, theory--in a word, improvement of the situation--by the side of which happiness appears a shabby old dud--instinct and the rule of thumb manifestly cease to be applicable. The best plan, then, on the whole, is to base our conduct as much as possible on Instinct, but when we do reason to reason with severely scientific logic. It has seemed to me proper to say this in order that I might not be understood as promising for logic what she could not perform. Where reasoning of any difficulty is to be done concerning positive facts, that is to say, not mere mathematical deduction, the aid that logic affords is most important.

Peirce: CP 2.179 Cross-Ref:††

§8. REASONING AND CONSCIOUSNESS

179. You opine that you yourself reason and are conscious of doing so. Undoubtedly, this is true in the sense that would naturally attach to the words. Yet it will be well, in beginning the study of logic, to be upon one's guard against sliding unawares from holding this to be undeniable in the ordinary sense in which it certainly is so, to holding it to be equally indubitable in another psychological sense.

Peirce: CP 2.180 Cross-Ref:††

180. Unless you are an old fox at logic such as the world does not harbor very many of, if we are to judge by what gets into print, or unless you are an extraordinarily vigorous thinker, you probably over-estimate very considerably the amount of logical reasoning which you perform. I know men justly renowned all over the civilized world as vigorous thinkers of whom I have satisfied myself that they perform little really logical thinking. They pass from premiss to conclusion of an argument, contenting themselves with remarking that that argument has the general

appearance of belonging to the same class as many arguments of which they have had experience and which turned out to lead to true conclusions, but which they have never so analyzed as to be quite sure what conditions have to be fulfilled to necessitate their validity. This may be called reasoning; and if the reasoner were to recognize its vagueness and fragility, it might even be called logical reasoning of an excessively feeble kind. An argument which professes no more than it performs is sound, however little its performance may be. The difficulty is that the men who reason so do not recognize what it is that they are doing, but take the vague likeness of their argument to others that have succeeded for a perception of its necessary rigor, consequently telling themselves that they are reasoning in a very weighty and exact manner. Now an argument which takes itself as having a sort of weight of which it is really devoid, flying false colors, is not a good argument.

Peirce: CP 2.181 Cross-Ref:††

181. Of excessively simple reasonings a great deal is done which is unexceptionable. But leaving them out of account, the amount of logical reasoning that men perform is small, much smaller than is commonly supposed. It is really instinct that procures the bulk of our knowledge; and those excessively simple reasonings which conform to the requirements of logic are, as a matter of fact, mostly performed instinctively or irreflectively.

Peirce: CP 2.182 Cross-Ref:††

182. Reasoning, properly speaking, cannot be unconsciously performed. A mental operation may be precisely like reasoning in every other respect except that it is performed unconsciously. But that one circumstance will deprive it of the title of reasoning. For reasoning is deliberate, voluntary, critical, controlled, all of which it can only be if it is done consciously. An unconscious act is involuntary: an involuntary act is not subject to control; an uncontrollable act is not deliberate nor subject to criticism in the sense of approval or blame. A performance which cannot be called good or bad differs most essentially from reasoning.

Peirce: CP 2.183 Cross-Ref:††

183. This does not imply that we must be aware of the whole process of the mind in reasoning or, indeed, of any portion of it. It is very desirable to have a clear apprehension of this distinction. We are, so to speak, responsible for the correctness of our reasonings. That is to say, unless we deliberately approve of them as rational, they cannot properly be called reasonings. But for this purpose, all that is necessary is that we should, in each case, compare premisses and conclusion, and observe that the relation between the facts expressed in the premisses involves the relation between facts implied in our confidence in the conclusion. What we call a reasoning is something upon which we place a stamp of rational approval. In order to do that, we must know what the reasoning is. In that sense, it must be a conscious act, just as a man is not bound by a contract if it can be proved that he signed it in his sleep. It must be his conscious act and deed. But for that purpose he only needs to know the character of the relation between the premisses and the conclusion. He need not know precisely what operations the mind went through in passing from the one to the other. That is a matter of detail which is not essential to his responsibility. The mind is like the conveyancer who has drawn up a deed. What books he looked into in choosing his verbiage is no concern of the person who signs, provided he knows what the paper binds him to doing.

Peirce: CP 2.184 Cross-Ref:††

184. I do not believe that, in the present state of psychology, anybody knows

much about the operations of the mind in reasoning. There is no easy way in which such knowledge can be had. There are those who say that all we have to do for this purpose is simply to watch what passes through consciousness. This word "consciousness" deceives them. It is as if someone were to argue that because science undertakes only to infer what is phenomenal, and phenomena are simply appearances, therefore, in order to make out what the processes of digestion are, it is only necessary to strip a man and watch his belly. All that we can find out by directly watching consciousness are the qualities of feeling, and those, not as they are felt, but as, after being felt, they are grouped. I can tell myself what the steps of a reasoning were; that is, I can enumerate some premisses and conclusions that were successively introduced; just as another man might give me the same information about his reasoning. It is difficult to make sure that such propositions actually passed through the mind. If they did, they were probably resting-places where the mental action ceased, and results were recorded, so that they could be recovered when the thinking should recommence.

Peirce: CP 2.185 Cross-Ref:††

185. At any rate, a knowledge of the processes of thinking, even if it were at hand, would be entirely irrelevant to that sort of knowledge of the nature of our reasonings which it is incumbent upon us to have in order that we may give them our deliberate approval.

Peirce: CP 2.185 Cross-Ref:††

These are points of importance for understanding the English, objective conception of logic.

Peirce: CP 2.186 Cross-Ref:††

§9. *LOGICA UTENS*

186. The opinions which you bring to the study of logic comprise among them a system of logic all made, although it is probably a little vague, in places. You know that this is substantially so, presuming that you are a reflective person, as you doubtless are; but you may perhaps be surprised that I should be so confident that it is so. It is simple enough, however. You would certainly not be interested in logic unless you were somewhat given to reasoning; probably not without being more or less addicted to self-observation. Now a person cannot perform the least reasoning without some general ideal of good reasoning; for reasoning involves deliberate approval of one's reasoning; and approval cannot be deliberate unless it is based upon the comparison of the thing approved with some idea of how such a thing ought to appear. Every reasoner, then, has some general idea of what good reasoning is. This constitutes a theory of logic: the scholastics called it the reasoner's *logica utens*. Every reasoner whose attention has been considerably drawn to his inner life must soon become aware of this.

Peirce: CP 2.187 Cross-Ref:††

187. He, therefore, comes to the study of logic handicapped by a conceit that he knows something about it already, but, at the same time, aided by his being able to handle questions of logic with some confidence and familiarity. He ought to endeavor to suppress his conceit while preserving his disposition to think independently.

Peirce: CP 2.188 Cross-Ref:††
§10. *LOGICA UTENS* AND *LOGICA DOCENS*

188. But the fact that you are sincerely desirous of studying logic shows that you are not altogether satisfied with your *logica utens* nor with your powers of estimating the values of arguments. Of course, there is no good of entering upon any undertaking unless one desires the sole rational purpose of that undertaking, and is consequently more or less dissatisfied with one's present condition in that respect.

Peirce: CP 2.189 Cross-Ref:††

189. It is foolish, therefore, to study logic unless one is persuaded that one's own reasonings are more or less bad. Yet a reasoning is essentially something which one is deliberately convinced is good. There is a slight appearance of contradiction here, which calls for a little logic to remove it. The substance of an opinion is not the whole opinion. It has a mode. That is to say, the opinion has been approved because it has been formed in a certain way, and of opinions formed in that way, we have the opinion that relatively few are much in error. It is for that reason that we have adopted the opinion in question. Still, we attach but a limited degree of confidence to it, being of the opinion that out of a considerable number of opinions formed in the same way, some would probably be grossly erroneous. In this way, it might happen that you should hold that a large minority of your reasonings were bad, although you were inclined to adhere to each one singly. This is the general principle. But logicians are too apt to content themselves with the statement of general principles, and to overlook peculiar effects which may arise from complications of them. The real situation in this case is too complicated to be considered to advantage; but we can illustrate the general way in which complexity may modify the effect of our general principle. Your reasonings are determined by certain general habits of reasoning, each of which has been, in some sense, approved by you. But you may recognize that your habits of reasoning are of two distinct kinds, producing two kinds of reasoning which we may call A-reasonings and B-reasonings. You may think that of the A-reasonings very few are seriously in error, but that none of them much advance your knowledge of the truth. Of your B-reasonings, you may think that so many of them as are good are extremely valuable in teaching a great deal. Yet of these B-reasonings you may think that a large majority are worthless, their error being known by their being subsequently found to come in conflict with A-reasonings. It will be perceived from this description that the B-reasonings are a little more than guesses. You will then be justified in adhering to those habits of reasoning which produced B-reasonings, by the reflection that if you do adhere to them, the evil effects of the bad ones will be mainly eliminated in course of time by opposing A-reasonings, while you will gain the important knowledge brought by the few B-reasonings that are good; whereas, if you were to discard those habits of reasoning which produced B-reasonings you would have nothing left but A-reasonings, and these could never afford you much positive knowledge. This imaginary illustration will serve to show how it might be that you should, with perfect consistency, hold your existing *logica utens* to be excessively unsatisfactory, although you are perfectly justified in adhering to it until you are in possession of a better system. Without knowing anything of your individual case, my general observation of the manner in which men reason leads me to believe it most probable that the above illustration about the A-reasonings and the B-reasonings

represents, in a general way, your condition, except that you greatly overrate the value of many of the B-reasonings, which are really little more than guesses at truth, but are, many of them, regarded by you as inductions. If this be the case, a study of logic, while making your whole thought more accurate, will enable you to rate your B-reasonings more accurately, and to substitute for about half of them reasonings that will not often deceive, while greatly improving the quality of those that will still remain more or less conjectural. This improvement will, however, be limited to logical reasonings; and of such you perhaps do not perform a great many. Those acts of the mind which chiefly depend upon instinct will remain unaffected, except that their true character will be recognized.

Peirce: CP 2.190 Cross-Ref:††

190. I am, in all this, adapting my tone to your probable state of feeling, that the principal value of logic lies in its supplying an art of reasoning; but I trust that by the time you have read this book through, you will have come to feel, as I do, that the greatest value of the science is of another and higher kind.

Peirce: CP 2.191 Cross-Ref:††

§11. THE IMPROVEMENT OF REASONING

191. In studying logic, you hope to correct your present ideas of what reasoning is good, what bad. This, of course, must be done by reasoning; and you cannot imagine that it is to be done by your accepting reasonings of mine which do not seem to you to be rational. It must, therefore, be done by means of the bad system of logic which you at present use. Some writers fancy that they see some absurdity in this. They say, "Logic is to determine what is good reasoning. Until this is determined reasoning must not be ventured upon. (They say it would be a "*petitio principii*"; but I avoid the technicality which must be unmeaning at the present stage of our inquiry, even if it really has any meaning, beyond my rendering of it.)†1 Therefore, the principles of logic must be determined without reasoning, by simple instinctive feeling." All this is fallacious. It springs from the wide-spread tendency, derived ultimately perhaps from Platonic influence, to take philosophical propositions in an exaggerated way. Let us rather state the case thus. At present, you are in possession of a *logica utens* which seems to be unsatisfactory. The question is whether, using that somewhat unsatisfactory *logica utens*, you can make out wherein it must be modified, and can attain to a better system. This is a truer way of stating the question; and so stated, it appears to present no such insuperable difficulty as is pretended. Suppose, for example, that we should show that your present logic leads to flat self-contradiction. It is true that we must, then, in a sense, trust to natural logic in its declaration that two propositions flatly contradictory, the one of the other, cannot both be true. But in doing this, we are not relying upon the exactitude of any general logical principle, in all its generality. We are only confident that the two particular propositions in question are not both true. No matter how perfect a system of logic we may ever attain, still in applying that system, we must make use of intelligence. It is a misuse of terms to say that in doing this we "trust" to anything. We simply perceive that two certain propositions in flat contradiction to one another cannot both be true. We need not commit ourselves to any general principle; we simply recognize our inability to believe both at once. We simply recognize a mathematical necessity. Mathematics is not subject to logic. Logic depends on mathematics. The recognition

of mathematical necessity is performed in a perfectly satisfactory manner antecedent to any study of logic. Mathematical reasoning derives no warrant from logic. It needs no warrant. It is evident in itself. It does not relate to any matter of fact, but merely to whether one supposition excludes another. Since we ourselves create these suppositions, we are competent to answer them. But it is when we pass out of the realm of pure hypothesis into that of hard fact that logic is called for. We then find that certain modes of reasoning are sound, because they must, by mathematical necessity, be sound, in whatever universe there may be in which there is such a thing as experience.

Peirce: CP 2.192 Cross-Ref:††

192. Nothing is more irrational than false pretence. We may all set that down as Axiom I, whether we are Artists, Practical Moralists, or Philosophers--a classification which embraces all mankind except the great world of Hypnotists--that is to say, the Medicine Men, the Confidence Men, the Horse Traders, the Diplomats, and all that lot, with which this book has no relation except that of alienity. They and the burglars are useful members of society; but they are not likely to be readers of this book. For us, who are non-hypnotists, nothing is more irrational than false pretence. Yet the Cartesian philosophy, which ruled Europe for so long, is founded upon it. It pretended to doubt what it did not doubt. Let us not fall into that vice. You think that your *logica utens* is more or less unsatisfactory. But you do not doubt that there is *some* truth in it. Nor do I; nor does any man. Why cannot men see that what we do not doubt, we do not doubt; so that it is false pretence to pretend to call it in question? There are certain parts of your *logica utens* which nobody really doubts. Hegel and his have loyally endeavored to cast a doubt upon it. The effort has been praiseworthy; but it has not succeeded. The truth of it is too evident. Mathematical reasoning holds. Why should it not? It relates only to the creations of the mind, concerning which there is no obstacle to our learning whatever is true of them. The method of this book, therefore, is to accept the reasonings of pure mathematics as beyond all doubt. It is fallible, as everything human is fallible. Twice two may perhaps not be four. But there is no more satisfactory way of assuring ourselves of anything than the mathematical way of assuring ourselves of mathematical theorems. No aid from the science of logic is called for in that field. As a fact, I have not the slightest doubt that twice two is four; nor have you. Then let us not pretend to doubt mathematical demonstrations of mathematical propositions so long as they are not open to mathematical criticism and have been submitted to sufficient examination and revision. The only concern that logic has with this sort of reasoning is to describe it.

Peirce: CP 2.193 Cross-Ref:††

193. That being settled, I propose, by purely mathematical reasoning, to show that in any world in which there is such a thing as the course of experience--an element which is absent from the world of pure mathematics--in such world a certain kind of reasoning must be valid which is not valid in the world of pure mathematics.

Peirce: CP 2.194 Cross-Ref:††

194. I have already stated in the second chapter what the course of the discussion is to be. I intend to restate it hereafter. But the interposition here of a third statement will not, by my reckoning, put the reader to any loss of time or patience. A book that goes to the bottom of an abstruse and complicated subject in such a manner that it can with profit be currently read ought to contain repetitions. Such at any rate, is the theory on which I have embarked the fortunes of this book.

Peirce: CP 2.195 Cross-Ref:††

195. I wish to show the reader how I am going to establish the doctrine of reason in making use of such imperfect reasoning powers as every reader will bring to the subject, and yet without any inadmissible assumption. What a proposition it is that those who pronounce this to be impossible virtually maintain! That imperfect reason cannot perfect itself! How then, I should be glad to be informed, did man ever pass from a state of monerism, and lower, to that of lordship of this globe? Oh, but it was not by a logical process. How so? It was by unwarrantable assumptions. What pedantry! So a man is not to be allowed to see what is reasonable unless he do so by the rules of art! {To sabbaton dia ton anthropon egeneto, ouch ho anthropos dia to sabbaton}, which being translated means, "Logic came about for the sake of reasonableness, not reasonableness for the sake of logic." Let us never lose sight of that truth, forgotten though it is, every day, in every walk of life, especially in well-regulated America!

Peirce: CP 2.196 Cross-Ref:††
§12. ESTHETICS, ETHICS, AND LOGIC

196. I shall begin with the reasoning of mathematics.†1 I shall analyze it and examine its nature. I shall not attempt to justify it. It will be time to defend it, when it shall once be deliberately doubted. Defense against sham doubt is but a blank-cartridge action. It is of no use. On the contrary, humbug is always harmful in philosophy.

Peirce: CP 2.197 Cross-Ref:††

197. Logic can be of no avail to mathematics; but mathematics lays the foundation on which logic builds; and those mathematical chapters will be quite indispensable. After them, it is my purpose to invite the reader to take up the study of Phenomenology.†2 In the derivation of this word, "phenomenon" is to be understood in the broadest sense conceivable; so that phenomenology might rather be defined as the study of what seems than as the statement of what appears. It describes the essentially different elements which seem to present themselves in what seems. Its task requires and exercises a singular sort of thought, a sort of thought that will be found to be of the utmost service throughout the study of logic. It can hardly be said to involve reasoning; for reasoning reaches a conclusion, and asserts it to be true however matters may seem; while in Phenomenology there is no assertion except that there are certain seemings; and even these are not, and cannot be asserted, because they cannot be described. Phenomenology can only tell the reader which way to look and to see what he shall see. The question of how far Phenomenology does reason will receive special attention. We shall next take up the logic of the normative sciences, of which logic itself is only the third, being preceded by Esthetics and Ethics. It is now forty-seven years ago that I undertook to expound Schiller's *Aesthetische Briefe* to my dear friend, Horatio Paine.†1 We spent every afternoon for long months upon it, picking the matter to pieces as well as we boys knew how to do. In those days, I read various works on esthetics; but on the whole, I must confess that, like most logicians, I have pondered that subject far too little. The books do seem so feeble. That affords one excuse. And then esthetics and logic seem, at first blush, to belong to different universes. It is only very recently that I have become persuaded that that seeming is illusory, and that, on the contrary, logic needs the help of esthetics. The matter is not yet very clear to me; so unless some great light should fall

upon me before I reach that chapter, it will be a short one filled with doubts and queries mainly.†2

Peirce: CP 2.198 Cross-Ref:††

198. Ethics †3: is another subject which for many years seemed to me to be completely foreign to logic. Indeed I doubted very much whether it was anything more than a practical science, or Art; although I had always been interested in ethical systems. About twenty years ago, I began to be impressed with the importance of the theory; but it has only been within five or six years that all the intimacy of its relation to logic has been revealed to me. It may very easily happen that the over-development of a man's moral conception should interfere with his progress in philosophy. The protoplasm of philosophy has to be in a liquid state in order that the operations of metabolism may go on. Now morality is a hardening agent. It is astonishing how many abominable scoundrels there are among sincerely moral people. The difficulty is that morality chokes its own stream. Like any other field, more than any other, it needs improvement, advance. Moral ideas must be a rising tide, or with the ebb foulness will be cast up. But morality, doctrinaire conservatist that it is, destroys its own vitality by resisting change, and positively insisting, This is eternally right: That is eternally wrong. The tendency of philosophers has always been to make their assertions too absolute. Nothing stands more in the way of a comprehension of the universe and of the mind. But in morals this tendency acquires triple strength. The practical side of ethics is its most obviously important side; and in practical matters, the first maxim is that everything may be exaggerated. That is the substance of Aristotle's *Ethics*. The moral spirit may very easily be carried to excess: all the more so, that the essence of that spirit is to insist upon its own absolute autocracy. All that is one side of the shield; but the other side is quite equally important. We are too apt to define ethics to ourselves as the science of right and wrong. That cannot be correct, for the reason that right and wrong are ethical conceptions which it is the business of that science to develop and to justify. A science cannot have for its fundamental problem to distribute objects among categories of its own creation; for underlying that problem must be the task of establishing those categories. The fundamental problem of ethics is not, therefore, What is right, but, What am I prepared deliberately to accept as the statement of what I want to do, what am I to aim at, what am I after? To what is the force of my will to be directed? Now logic is a study of the means of attaining the end of thought. It cannot solve that problem until it clearly knows what that end is. Life can have but one end. It is Ethics which defines that end. It is, therefore, impossible to be thoroughly and rationally logical except upon an ethical basis. Had I fully comprehended this great principle early in life, I should undoubtedly have been over-influenced by the moral spirit, and my present understanding of logic which, although it is far in advance of anybody else's, is, as compared with what it ought to be, as the scrawl on an infant's slate to a cartoon of Raphael, would have been even narrower and poorer than it now is--probably almost worthless. Before my logic was brought under the guidance of ethics, it was already a window through which much important truth could be seen, but dim with dust, distorting details by striæ. Under the guidance of ethics I took it and melted it down, reduced it to a fluid condition. I filtered it till it was clear. I cast it in the true mould; and when it had become solid, I spared no elbow-grease in polishing it. It is now a comparatively brilliant lens, showing much that was not discernible before. I believe that it will only remain to those who come after me to perfect the processes. I am as confident as I am of death that Logic will hereafter be infinitely superior to what it is as I leave it; but my labors will have done good work toward its improvement.

Peirce: CP 2.199 Cross-Ref:††

199. What I have found to be true of Ethics I am beginning to see is true of Esthetics likewise. That science has been handicapped by the definition of it as the theory of beauty. The conception of beauty is but the product of this science, and a very inadequate attempt it is to grasp what it is that esthetics seeks to make clear. Ethics asks to what end all effort shall be directed. That question obviously depends upon the question what it would be that, independently of the effort, we should like to experience. But in order to state the question of esthetics in its purity, we should eliminate from it, not merely all consideration of effort, but all consideration of action and reaction, including all consideration of our receiving pleasure, everything in short, belonging to the opposition of the *ego* and the *non-ego*. We have not in our language a word of the requisite generality. The Greek {kalos}, the French *beau*, only come near to it, without hitting it squarely on the head. "Fine" would be a wretched substitute. Beautiful is bad; because one mode of being {kalos} essentially depends upon the quality being unbeautiful. Perhaps, however, the phrase "the beauty of the unbeautiful" would not be shocking. Still "beauty" is too skin-deep. Using {kalos}, the question of esthetics is, What is the one quality that is, in its immediate presence, {kalos}? Upon this question ethics must depend, just as logic must depend upon ethics. Esthetics, therefore, although I have terribly neglected it, appears to be possibly the first indispensable propedeutic to logic, and the logic of esthetics to be a distinct part of the science of logic that ought not to be omitted. This is a point concerning which it is not desirable to be in haste to come to a decided opinion.

Peirce: CP 2.200 Cross-Ref:††

200. When our logic shall have paid its *devoirs* to Esthetics and the Ethics, it will be time for it to settle down to its regular business. That business is of a varied nature; but so far as I intend in this place to speak of it, it consists in ascertaining methods of sound reasoning, and of proving that they are sound, not by any instinctive guarantee, but because it can be shown by the kinds of reasoning already considered, especially the mathematical, of one class of reasonings that they follow methods which, persisted in, must eventually lead to the truth in regard to those problems to which they are applicable, or, if not to the absolute truth, to an indefinite approximation thereto, while in regard to another class of reasonings, although they are so insecure that no reliance can be placed upon them, it will be shown in a similar way that yet they afford the only means of attaining to a satisfactory knowledge of the truth, in case this knowledge is ever to be attained at all, doing so by putting problems into such form that the former class of reasonings become applicable to them. This prospectus of how I am to proceed is sufficient to show that there can be no ground of reasonable complaint that unwarranted assumptions are made in the course of the discussion. Nothing will be assumed beyond what every sincere and intelligent person will and must confess is perfectly evident and which, in point of fact, is not really doubted by any caviller.

Peirce: CP 2.201 Cross-Ref:††
§13. UTILITY OF LOGICAL THEORY

201. You anticipate from the study of logic important improvement in your practice of reasoning. I have already endeavored to state just how far such

improvement can be promised. Nobody will expect of theory that it should furnish skill, or render practice needless. A man of experience would be somewhat surprised to find that a writer on esthetics was an artist of power, or that a writer on ethics was a moral hero. Logicians are, as a rule, far from being the greatest of reasoners. Nevertheless, I am inclined to believe that the study of esthetics will be of benefit to an artist; though I know too little either of esthetics or of art to speak with confidence. Some writers on ethics deny that it affords any aid toward a moral life. I cannot but think that this is an exaggeration. I do not see how the study of Plato, or Shaftesbury, or Kant can fail to make men better. I doubt if the reading of Hobbes could do a man of much power of thought any harm. As for logic, I must admit that it has turned out numberless captious thinkers who do not reason nearly as well as the average woman who does not remember ever to have seen the inside of a treatise on logic. But at any rate there is no cure for such people, unless it be to teach them a rational system of logic. I am inclined to think that one reason why logicians, as a class, are such bad reasoners, as they are apt to be, is that they were interested in logic, in the first place, because they found they had great difficulty in reasoning; and while other men are repelled from the study because they find its rules so nugatory and foreign to the spirit of true science, these men have continued the study because its mechanical and piddling character fits their own calibre of mind. It is the old traditional logic which I am now alluding to, whose most stupid maxims we today so often hear appealed to in the market-place as of acknowledged authority. But it is not seriously studied. When it was so studied in the middle ages, although it naturally could not foster reasoning such as that of a modern physicist, for example, yet it seems to me clear that it did cause great numbers of men to reason much more exactly and acutely than they would have done without such study.

Peirce: CP 2.202 Cross-Ref: ††

202. For scientific reasoning about matters of science, scientific logic is indispensable.

Peirce: CP 2.203 Cross-Ref: ††
§14. LOGIC †1

203. Logic is a science which has not yet completed the stage of disputes concerning its first principles, although it is probably about to do so. Nearly a hundred definitions of it have been given. It will, however, generally be conceded that its central problem is the classification of arguments, so that all those that are bad are thrown into one division, and those which are good into another, these divisions being defined by marks recognizable even if it be not known whether the arguments are good or bad. Furthermore, logic has to divide good arguments by recognizable marks into those which have different orders of validity, and has to afford means for measuring the strength of arguments.

Peirce: CP 2.204 Cross-Ref: ††

204. An approach to such a classification is made by every man whenever he reasons, in the proper sense of that term. It is true that the contemplation of a state of things believed to be real may cause the contemplator to believe something additional, without making any classification of such sequences. But in that case he does not criticize the procedure, nor so much as distinctly reflect that it is just. He

can, consequently, not exercise any control over it. Now, that which is uncontrollable is not subject to any normative laws at all; that is, it is neither good nor bad; it neither subserves an end nor fails to do so. But it is only the deliberate adoption of a belief in consequence of the admitted truth of some other proposition which is, properly speaking, reasoning. In that case the belief is adopted because the reasoner conceives that the method by which it has been determined would either in no analogous case lead to a false conclusion from true premisses, or, if steadily adhered to, would at length lead to an indefinite approximation to the truth, or, at least, would assure the reasoner of ultimately attaining as close an approach to the truth as he can, in any way, be assured of attaining. In all reasoning, therefore, there is a more or less conscious reference to a general method, implying some commencement of such a classification of arguments as the logician attempts. Such a classification of arguments, antecedent to any systematic study of the subject, is called the reasoner's *logica utens*, in contradistinction to the result of the scientific study, which is called *logica docens*. See Reasoning.†1

Peirce: CP 2.205 Cross-Ref:††

205. That part of logic, that is, of *logica docens*, which, setting out with such assumptions as that every assertion is either true or false, and not both, and that some propositions may be recognized to be true, studies the constituent parts of arguments and produces a classification of arguments such as is above described, is often considered to embrace the whole of logic; but a more correct designation is Critic (Greek {kritiké}). According to Diogenes Laertius,†2 Aristotle divided logic into three parts, of which one was {pros krisin}). This word, used by Plato (who divides all knowledge into *epitactic* and *critic*),†1 was adopted into Latin by the Ramists, and into English by Hobbes and Locke. From the last it was taken into German by Kant, who always writes it Critik, the initial c being possibly a reminiscence of its English origin. At present it is written Kritik in German. Kant is emphatic in the expression of the wish that the word may not be confounded with critique, a critical essay (German *Kritik*).†2

Peirce: CP 2.206 Cross-Ref:††

206. It is generally admitted that there is a doctrine which properly antecedes what we have called critic. It considers, for example, in what sense and how there can be any true proposition and false proposition, and what are the general conditions to which thought or signs of any kind must conform in order to assert anything. Kant, who first raised these questions to prominence, called this doctrine transcendentale *Elementarlehre*, and made it a large part of his *Critic of the Pure Reason*. But the *Grammatica Speculativa* of Scotus is an earlier and interesting attempt. The common German word is *Erkenntnisstheorie*, sometimes translated Epistemology.

Peirce: CP 2.207 Cross-Ref:††

207. It is further generally recognized that another doctrine follows after critic, and which belongs to, or is closely connected with, logic. Precisely what this should contain is not agreed; but it must contain the general conditions requisite for the attainment of truth. Since it may be held to contain more, one hesitates to call it heuristic. It is often called Method; but as this word is also used in the concrete, methodic or methodetic would be better.

Peirce: CP 2.208 Cross-Ref:††

208. For deciding what is good logic and what bad, appeal is made by different writers to one or more, generally several, of these eight sources: to direct

dicta of consciousness, to psychology, to the usages of language, to metaphysical philosophy, to history, to everyday observation, to mathematics, and to some process of dialectic. In the middle ages appeal was frequently made to authority.

Peirce: CP 2.209 Cross-Ref:††

209. The appeal to direct consciousness consists in pronouncing certain reasoning to be good or bad because it is felt to be so. This is a very common method. Sigwart, for example, bases all logic upon our invincible mental repulsion against contradiction, or, as he calls it, "the immediate feeling of necessity" (*Logic*, §3, 2). Those who think it worth while to make any defence at all of this proceeding urge, in effect, that, however far the logician may push his criticisms of reasoning, still, in doing so, he must reason, and so must ultimately rely upon his instinctive recognition of good and bad reasoning. Whence it follows that, in Sigwart's words, "every system of logic must rest upon this principle." It is, however, to be noted that among the dicta of direct consciousness, many pronounce certain reasonings to be bad. If, therefore, such dicta are to be relied upon, man not only usually has a tendency to reason right, but also sometimes has a tendency to reason wrong; and if that be so, the validity of a reasoning cannot *consist* in a man's having a tendency to reason in that way. Some say that the validity of reasoning consists in the "definitive dictum" of consciousness; but it has been replied that certain propositions in Euclid were studied for two thousand years by countless keen minds, all of whom had an immediate feeling of evidence concerning their proofs, until at last flaws were detected in those proofs, and are now admitted by all competent persons; and it is claimed that this illustrates how far from possible it is to make direct appeal to a definitive pronouncement. Besides, say those who object to this method, all reasoning and inquiry expects that there is such a thing as the truth concerning whatever question may be under examination. Now, it is of the very essence of this "truth," the meaning of the expectation, that the "truth" in no wise depends upon what any man to whom direct appeal can be made may opine about that question. *A fortiori* it does not depend upon whether I am satisfied with it or not. It is further insisted that there can be no genuine criticism of a reasoning until that reasoning is actually doubted; and no sooner is it actually doubted than we find that consciousness has revoked her *dictum* in its favour, if she ever made any. It is, indeed, maintained that so far from true is it that every system of logic must be based upon any instinctive recognition of good and bad reasoning, that it is quite impossible for any reasoning to be based upon such recognition in respect to that same reasoning. In reasoning, a man may feel sure he is right; but to "rest" that confidence on nothing but itself is to rest it on nothing at all. If the fact that we must use our reasoning instinct in criticizing reasoning proves that we must appeal to nothing else in such criticism, it equally proves that we ought to follow the lead of that instinct without any logical control at all, which would be as much as to say that we ought not to reason at all. A man cannot criticize every part of his reasoning, since he cannot criticize the act of reasoning he is performing in the criticism, it is true. But he can criticize steps whose validity he doubts; and in doing so, ought to consider in what characters the validity of reasoning consists, and whether the reasoning in question possesses those characters.

Peirce: CP 2.210 Cross-Ref:††

210. Under an appeal to psychology is not meant every appeal to any fact relating to the mind. For it is, for logical purposes, important to discriminate between facts of that description which are supposed to be ascertained by the systematic study of the mind, and facts the knowledge of which altogether antecedes such study, and is not in the least affected by it; such as the fact that there is such a state of mind as

doubt, and the fact that the mind struggles to escape from doubt. Even facts like these require to be carefully examined by the logician before he uses them as the basis of his doctrine. But many logicians have gone much further, and have avowedly based their systems upon one or another theory of psychology. Another class of logicians have professed to base logic upon a psychological theory of cognition. Of course, if this is done, such psychological doctrine is placed above logical criticism, or, at any rate, above logical support. For if the truth of a conclusion is known only from certain premisses, it cannot be used to support those premisses. Now, it may be doubted whether psychology is not, of all the special sciences, the one which stands most in need of appeal to a scientific logic.

Peirce: CP 2.211 Cross-Ref:††

211. Appeals to the usages of language are extremely common. They are made even by those who use algebraical notation in logic "in order to free the mind from the trammels of speech" (Schröder, *Logik*, i. p. iii). It is difficult to see what can be hoped for from such a proceeding, unless it be to establish a psychological proposition valid for all minds. But to do this, it would be necessary to look beyond the small and very peculiar class of Aryan languages, to which the linguistic knowledge of most of those writers is confined. The Semitic languages, with which some of them are acquainted, are too similar to the Aryan greatly to enlarge their horizon. Moreover, even if other languages are examined, the value of any logical inferences from them is much diminished by the custom of our grammarians of violently fitting them to the Procrustean bed of Aryan grammar.

Peirce: CP 2.212 Cross-Ref:††

212. The objection which has been suggested to appeals to psychological results applies with far greater force to appeals to metaphysical philosophy, which, it will generally be conceded, can hardly take a step with security unless it rests upon the science of logic. Nevertheless, a great many logical treatises of various colors make it their boast that they are built upon philosophical principles.

Peirce: CP 2.213 Cross-Ref:††

213. Logicians occasionally appeal to the history of science. Such and such a mode of reasoning, it is said, for example, was characteristic of mediævalism or of ancient science; such another produced the successes of modern science. If logic is to be based upon probable reasonings, as some logicians maintain that it must be, such arguments, if critically examined, must be admitted to have great weight. They will naturally be out of place in a system of logic which professes to demonstrate from certain initial assumptions that the kinds of reasoning it recommends must be accepted.

Peirce: CP 2.214 Cross-Ref:††

214. There is probably room for dispute as to whether logic need assert anything at all as an absolute matter of fact. If it does not, any appeal to experience would seem to be irrelevant. If it does, still the opinion may be that such assertions of logic are of so exceedingly broad and slight a nature that the universal experience of every man's every day and hour puts them beyond all doubt--such experiences as that the world presents appearances of variety, of law, and of the real action of one thing upon another. As appearances, these things do not seem likely ever to be doubted. If logic has need of any facts, and if such facts will suffice, no objection can well be made to an appeal to them.

Peirce: CP 2.215 Cross-Ref:††

215. The boundary between some parts of logic and pure mathematics in its modern treatment is almost evanescent, as may be seen in Dedekind's *Was sind und was sollen die Zahlen* (1888, English translation 1901). There are, however, departments of logic, such as the logic of probable inference (if that be regarded a part of logic), in which appeal is sometimes made to mathematical results, such as Bernoulli's law of high numbers. It seems to be the general opinion that nothing so difficult as mathematics can be admitted into, or be appealed to by, the science of logic, which has the peculiarity of consisting chiefly of truisms.

Peirce: CP 2.216 Cross-Ref:††

216. In mathematical reasoning there is a sort of observation. For a geometrical diagram or array of algebraical symbols is constructed according to an abstractly stated precept, and between the parts of such diagram or array certain relations are observed to obtain, other than those which were expressed in the precept. These being abstractly stated, and being generalized, so as to apply to every diagram constructed according to the same precept, give the conclusion. Some logicians hold that an equally satisfactory method depends upon a kind of inward observation, which is not mathematical, since it is not diagrammatic, the development of a conception and its inevitable transformation being observed and generalized somewhat as in mathematics; and those logicians base their science upon such a method, which may conveniently be termed, and is sometimes termed, a Dialectic. Other logicians regard such a method as either extremely insecure or as altogether illusory.

Peirce: CP 2.217 Cross-Ref:††

217. The generally received opinion among professors of logic is that all the above methods may properly be used on occasion, the appeal to mathematics, however, being less generally recognized.

Peirce: CP 2.218 Cross-Ref:††

218. *Literature*: the history of logic in Western Europe, down to the revival of learning, is given by Prantl, *Geschichte der Logik im Abendlande*. Upon the points upon which this author touches, he always affords valuable information, though his judgments are peremptory and slashing. Unfortunately, he omits much which was regarded by the authors of whom he treats as most important, because he does not himself so regard it. He also omits much which would be interesting to a reader taking a broader conception of logic. It is hardly necessary to say that upon some large subjects his views are controverted. . . .

Peirce: CP 2.219 Cross-Ref:††

BOOK II

SPECULATIVE GRAMMAR^P

CHAPTER 1

THE ETHICS OF TERMINOLOGY†1

219. In order that my use of terms, notations, etc., may be understood, I explain that my conscience imposes upon me the following rules. Were I to make the smallest pretension to dictate the conduct of others in this matter, I should be reproved by the first of these rules. Yet if I were to develop the reasons the force of which I feel myself, I presume they would have weight with others.

Peirce: CP 2.220 Cross-Ref:††

220. Those reasons would embrace, in the first place, the consideration that the woof and warp of all thought and all research is symbols, and the life of thought and science is the life inherent in symbols; so that it is wrong to say that a good language is *important* to good thought, merely; for it is of the essence of it. Next would come the consideration of the increasing value of precision of thought as it advances. Thirdly, the progress of science cannot go far except by collaboration; or, to speak more accurately, no mind can take one step without the aid of other minds. Fourthly, the health of the scientific communion requires the most absolute mental freedom. Yet the scientific and philosophical worlds are infested with pedants and pedagogues who are continually endeavoring to set up a sort of magistrature over thoughts and other symbols. It thus becomes one of the first duties of one who sees what the situation is, energetically to resist everything like arbitrary dictation in scientific affairs, and above all, as to the use of terms and notations. At the same time, a general agreement concerning the use of terms and of notations--not too rigid, yet prevailing, with most of the co-workers in regard to most of the symbols, to such a degree that there shall be some small number of different systems of expression that have to be mastered--is indispensable. Consequently, since this is not to be brought about by arbitrary dictation, it must be brought about by the power of rational principles over the conduct of men.

Peirce: CP 2.221 Cross-Ref:††

221. Now what rational principle is there which will be perfectly determinative as to what terms and notations shall be used, and in what senses, and which at the same time possesses the requisite power to influence all right-feeling and thoughtful men?

Peirce: CP 2.221 Cross-Ref:††

In order to find the answer to that question, it is necessary to consider, first, what would be the character of an ideal philosophical terminology and system of logical symbols; and, secondly, to inquire what the experience of those branches of science has been that have encountered and conquered great difficulties of nomenclature, etc., in regard to the principles which have proved efficacious, and in regard to unsuccessful methods of attempting to produce uniformity.

Peirce: CP 2.222 Cross-Ref:††

222. As to the ideal to be aimed at, it is, in the first place, desirable for any branch of science that it should have a vocabulary furnishing a family of cognate words for each *scientific* conception, and that each word should have a single exact meaning, unless its different meanings apply to objects of different categories that can never be mistaken for one another. To be sure, this requisite might be understood in a

sense which would make it utterly impossible. For every symbol is a living thing, in a very strict sense that is no mere figure of speech. The body of the symbol changes slowly, but its meaning inevitably grows, incorporates new elements and throws off old ones. But the effort of all should be to keep the *essence* of every scientific term unchanged and exact; although absolute exactitude is not so much as conceivable. Every symbol is, in its origin, either an image of the idea signified, or a reminiscence of some individual occurrence, person or thing, connected with its meaning, or is a metaphor. Terms of the first and third origins will inevitably be applied to different conceptions; but if the conceptions are strictly analogous in their principal suggestions, this is rather helpful than otherwise, provided always that the different meanings are remote from one another, both in themselves and in the occasions of their occurrence. Science is continually gaining new conceptions; and every new *scientific* conception should receive a new word, or better, a new family of cognate words. The duty of supplying this word naturally falls upon the person who introduces the new conception; but it is a duty not to be undertaken without a thorough knowledge of the principles and a large acquaintance with the details and history of the special terminology in which it is to take a place, nor without a sufficient comprehension of the principles of word-formation of the national language, nor without a proper study of the laws of symbols in general. That there should be two different terms of identical scientific value may or may not be an inconvenience, according to circumstances. Different systems of expression are often of the greatest advantage.

Peirce: CP 2.223 Cross-Ref:††

223. The ideal terminology will differ somewhat for different sciences. The case of philosophy is very peculiar in that it has positive need of popular words in popular senses--not as its own language (as it has too usually used those words), but as objects of its study. It thus has a peculiar need of a language distinct and detached from common speech, such a language as Aristotle, the scholastics, and Kant endeavored to supply, while Hegel endeavored to destroy it. It is good economy for philosophy to provide itself with a vocabulary so outlandish that loose thinkers shall not be tempted to borrow its words. Kant's adjectives "objective" and "subjective" proved not to be barbarous enough, by half, long to retain their usefulness in philosophy, even if there had been no other objection to them. The first rule of good taste in writing is to use words whose meanings will not be misunderstood; and if a reader does not know the meaning of the words, it is infinitely better that he should know he does not know it. This is particularly true in logic, which wholly consists, one might almost say, in exactitude of thought.

Peirce: CP 2.224 Cross-Ref:††

224. The sciences which have had to face the most difficult problems of terminology have unquestionably been the classificatory sciences of physics, chemistry, and biology. The nomenclature of chemistry is, on the whole, good. In their dire need, the chemists assembled in congress, and adopted certain rules for forming names of substances. Those names are well-known, but they are hardly used. Why not? Because the chemists were not psychologists, and did not know that a congress is one of the most impotent of things, even less influential by far than a dictionary. The problem of the biological taxonomists has, however, been incomparably more difficult; and they have solved it (barring small exceptions) with brilliant success. How did they accomplish this? Not by appealing to the power of congresses, but by appealing to the power of the idea of right and wrong. For only make a man *really see* that a certain line of conduct is wrong, and he *will* make a

strong endeavor to do the right thing--be he thief, gambler, or even a logician or moral philosopher. The biologists simply talked to one another, and made one another see that when a man has introduced a conception into science, it naturally becomes both his privilege and his duty to assign to that conception suitable scientific expressions; and that when a name has been conferred upon a conception by him to whose labors science is indebted for that conception, it becomes the duty of all--a duty to the discoverer, and a duty to science--to accept his name, unless it should be of such a nature that the adoption of it would be unwholesome for science; that should the discoverer fail in his duty, either by giving no name or an utterly unsuitable one, then, after a reasonable interval, whoever first has occasion to employ a name for that conception must invent a suitable one; and others ought to follow him; but that whoever deliberately uses a word or other symbol in any other sense than that which was conferred upon it by its sole rightful creator commits a shameful offence against the inventor of the symbol and against science, and it becomes the duty of the others to treat the act with contempt and indignation.

Peirce: CP 2.225 Cross-Ref:††

225. As fast as the students of any branch of philosophy educate themselves to a genuine scientific love of truth to the degree to which the scholastic doctors were moved by it, suggestions similar to those above will suggest themselves; and they will consequently form a technical terminology. In logic, a terminology more than passably good has been inherited by us from the scholastics. This scholastic terminology has passed into English speech more than into any other modern tongue, rendering it the most logically exact of any. This has been accompanied by the inconvenience that a considerable number of words and phrases of scientific logic have come to be used with a laxity quite astounding. Who, for example, among the dealers in Quincy Hall who talks of "articles of *prime necessity*," would be able to say what that phrase "prime necessity" strictly means? He could not have sought out a more technical phrase. There are dozens of other loose expressions of the same provenance.

Peirce: CP 2.225 Cross-Ref:††

Having thus given some idea of the nature of the reasons which weigh with me, I proceed to state the rules which I find to be binding upon me in this field.

Peirce: CP 2.226 Cross-Ref:††

226. First. To take pains to avoid following any recommendation of an arbitrary nature as to the use of philosophical terminology.

Peirce: CP 2.226 Cross-Ref:††

Second. To avoid using words and phrases of vernacular origin as technical terms of philosophy.

Peirce: CP 2.226 Cross-Ref:††

Third. To use the scholastic terms in their anglicised forms for philosophical conceptions, so far as they are strictly applicable; and never to use them in other than their proper senses.

Peirce: CP 2.226 Cross-Ref:††

Fourth. For ancient philosophical conceptions overlooked by the scholastics, to imitate, as well as I can, the ancient expression.

Peirce: CP 2.226 Cross-Ref:††

Fifth. For precise philosophical conceptions introduced into philosophy since

the middle ages, to use the anglicised form of the original expression, if not positively unsuitable, but only in its precise original sense.

Peirce: CP 2.226 Cross-Ref:††

Sixth. For philosophical conceptions which vary by a hair's breadth from those for which suitable terms exist, to invent terms with a due regard for the usages of philosophical terminology and those of the English language but yet with a distinctly technical appearance. Before proposing a term, notation, or other symbol, to consider maturely whether it perfectly suits the conception and will lend itself to every occasion, whether it interferes with any existing term, and whether it may not create an inconvenience by interfering with the expression of some conception that may hereafter be introduced into philosophy. Having once introduced a symbol, to consider myself almost as much bound by it as if it had been introduced by somebody else; and after others have accepted it, to consider myself more bound to it than anybody else.

Peirce: CP 2.226 Cross-Ref:††

Seventh. To regard it as needful to introduce new systems of expression when new connections of importance between conceptions come to be made out, or when such systems can, in any way, positively subserve the purposes of philosophical study.

Peirce: CP 2.227 Cross-Ref:††
CHAPTER 2

DIVISION OF SIGNS

§1. GROUND, OBJECT, AND INTERPRETANT †1

227. Logic, in its general sense, is, as I believe I have shown, only another name for *semiotic* (*{sémeiōtiké}*), the quasi-necessary, or formal, doctrine of signs. By describing the doctrine as "quasi-necessary," or formal, I mean that we observe the characters of such signs as we know, and from such an observation, by a process which I will not object to naming Abstraction, we are led to statements, eminently fallible, and therefore in one sense by no means necessary, as to what *must be* the characters of all signs used by a "scientific" intelligence, that is to say, by an intelligence capable of learning by experience. As to that process of abstraction, it is itself a sort of observation. The faculty which I call abstractive observation is one which ordinary people perfectly recognize, but for which the theories of philosophers sometimes hardly leave room. It is a familiar experience to every human being to wish for something quite beyond his present means, and to follow that wish by the question, "Should I wish for that thing just the same, if I had ample means to gratify it?" To answer that question, he searches his heart, and in doing so makes what I term an abstractive observation. He makes in his imagination a sort of skeleton diagram, or outline sketch, of himself, considers what modifications the hypothetical state of

things would require to be made in that picture, and then examines it, that is, *observes* what he has imagined, to see whether the same ardent desire is there to be discerned. By such a process, which is at bottom very much like mathematical reasoning, we can reach conclusions as to what *would be* true of signs in all cases, so long as the intelligence using them was scientific. The modes of thought of a God, who should possess an intuitive omniscience superseding reason, are put out of the question. Now the whole process of development among the community of students of those formulations by abstractive observation and reasoning of the truths which *must* hold good of all signs used by a scientific intelligence is an observational science, like any other positive science, notwithstanding its strong contrast to all the special sciences which arises from its aiming to find out what *must be* and not merely what *is* in the actual world.

Peirce: CP 2.228 Cross-Ref:††

228. A sign, or *representamen*, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates I call the *interpretant* of the first sign. The sign stands for something, its *object*. It stands for that object, not in all respects, but in reference to a sort of idea, which I have sometimes †1 called the *ground* of the representamen. "Idea" is here to be understood in a sort of Platonic sense, very familiar in everyday talk; I mean in that sense in which we say that one man catches another man's idea, in which we say that when a man recalls what he was thinking of at some previous time, he recalls the same idea, and in which when a man continues to think anything, say for a tenth of a second, in so far as the thought continues to agree with itself during that time, that is to have a *like* content, it is the same idea, and is not at each instant of the interval a new idea.

Peirce: CP 2.229 Cross-Ref:††

229. In consequence of every representamen being thus connected with three things, the ground, the object, and the interpretant, the science of semiotic has three branches. The first is called by Duns Scotus *grammatica speculativa*. We may term it *pure grammar*. It has for its task to ascertain what must be true of the representamen used by every scientific intelligence in order that they may embody any *meaning*. The second is logic proper. It is the science of what is quasi-necessarily true of the representamina of any scientific intelligence in order that they may hold good of any *object*, that is, may be true. Or say, logic proper is the formal science of the conditions of the truth of representations. The third, in imitation of Kant's fashion of preserving old associations of words in finding nomenclature for new conceptions, I call *pure rhetoric*. Its task is to ascertain the laws by which in every scientific intelligence one sign gives birth to another, and especially one thought brings forth another.

Peirce: CP 2.230 Cross-Ref:††
§2. SIGNS AND THEIR OBJECTS †1

230. The word Sign will be used to denote an Object perceptible, or only imaginable, or even unimaginable in one sense--for the word '*fast*,' which is a Sign, is not imaginable, since it is not *this word itself* that can be set down on paper or pronounced, but only *an instance* of it, and since it is the very same word when it is

written as it is when it is pronounced, but is one word when it means "rapidly" and quite another when it means "immovable," and a third when it refers to abstinence. But in order that anything should be a Sign, it must "represent," as we say, something else, called its *Object*, although the condition that a Sign must be other than its Object is perhaps arbitrary, since, if we insist upon it we must at least make an exception in the case of a Sign that is a part of a Sign. Thus nothing prevents the actor who acts a character in an historical drama from carrying as a theatrical "property" the very relic that that article is supposed merely to represent, such as the crucifix that Bulwer's Richelieu holds up with such effect in his defiance. On a map of an island laid down upon the soil of that island there must, under all ordinary circumstances, be some position, some point, marked or not, that represents *qua* place on the map, the very same point *qua* place on the island. A sign may have more than one Object. Thus, the sentence "Cain killed Abel," which is a Sign, refers at least as much to Abel as to Cain, even if it be not regarded as it should, as having "*a killing*" as a third Object. But the set of objects may be regarded as making up one complex Object. In what follows and often elsewhere Signs will be treated as having but one object each for the sake of dividing difficulties of the study. If a Sign is other than its Object, there must exist, either in thought or in expression, some explanation or argument or other context, showing how--upon what system or for what reason the Sign represents the Object or set of Objects that it does. Now the Sign and the Explanation together make up another Sign, and since the explanation will be a Sign, it will probably require an additional explanation, which taken together with the already enlarged Sign will make up a still larger Sign; and proceeding in the same way, we shall, or should, ultimately reach a Sign of itself, containing its own explanation and those of all its significant parts; and according to this explanation each such part has some other part as its Object. According to this every Sign has, actually or virtually, what we may call a *Precept* of explanation according to which it is to be understood as a sort of emanation, so to speak, of its Object. (If the Sign be an Icon, a scholastic might say that the "*species*" of the Object emanating from it found its matter in the Icon. If the Sign be an Index, we may think of it as a fragment torn away from the Object, the two in their Existence being one whole or a part of such whole. If the Sign is a Symbol, we may think of it as embodying the "*ratio*," or reason, of the Object that has emanated from it. These, of course, are mere figures of speech; but that does not render them useless.)

Peirce: CP 2.231 Cross-Ref:††

231. The Sign can only represent the Object and tell about it. It cannot furnish acquaintance with or recognition of that Object; for that is what is meant in this volume by the Object of a Sign; namely, that with which it presupposes an acquaintance in order to convey some further information concerning it. No doubt there will be readers who will say they cannot comprehend this. They think a Sign need not relate to anything otherwise known, and can make neither head nor tail of the statement that every Sign must relate to such an Object. But if there be anything that conveys information and yet has absolutely no relation nor reference to anything with which the person to whom it conveys the information has, when he comprehends that information, the slightest acquaintance, direct or indirect--and a very strange sort of information that would be--the vehicle of that sort of information is not, in this volume, called a Sign.

Peirce: CP 2.232 Cross-Ref:††

232. Two men are standing on the seashore looking out to sea. One of them says to the other, "That vessel there carries no freight at all, but only passengers."

Now, if the other, himself, sees no vessel, the first information he derives from the remark has for its Object the part of the sea that he does see, and informs him that a person with sharper eyes than his, or more trained in looking for such things, can see a vessel there; and then, that vessel having been thus introduced to his acquaintance, he is prepared to receive the information about it that it carries passengers exclusively. But the sentence as a whole has, for the person supposed, no other Object than that with which it finds him already acquainted. The Objects--for a Sign may have any number of them--may each be a single known existing thing or thing believed formerly to have existed or expected to exist, or a collection of such things, or a known quality or relation or fact, which single Object may be a collection, or whole of parts, or it may have some other mode of being, such as some act permitted whose being does not prevent its negation from being equally permitted, or something of a general nature desired, required, or invariably found under certain general circumstances.

Peirce: CP 2.233 Cross-Ref:††
§3. DIVISION OF TRIADIC RELATIONS †1

233. The principles and analogies of Phenomenology enable us to describe, in a distant way, what the divisions of triadic relations must be. But until we have met with the different kinds *a posteriori*, and have in that way been led to recognize their importance, the *a priori* descriptions mean little; not nothing at all, but little. Even after we seem to identify the varieties called for *a priori* with varieties which the experience of reflexion leads us to think important, no slight labour is required to make sure that the divisions we have found *a posteriori* are precisely those that have been predicted *a priori*. In most cases, we find that they are not precisely identical, owing to the narrowness of our reflexional experience. It is only after much further arduous analysis that we are able finally to place in the system the conceptions to which experience has led us. In the case of triadic relations, no part of this work has, as yet, been satisfactorily performed, except in some measure for the most important class of triadic relations, those of signs, or representamens, to their objects and interpretants.

Peirce: CP 2.234 Cross-Ref:††

234. Provisionally, we may make a rude division of triadic relations, which, we need not doubt, contains important truth, however imperfectly apprehended, into--

Triadic relations of comparison,

Triadic relations of performance, and

Triadic relations of thought.

Triadic relations of Comparison are those which are of the nature of logical possibilities.

Triadic relations of Performance are those which are of the nature of actual facts.

Triadic relations of Thought are those which are of the nature of laws.

Peirce: CP 2.235 Cross-Ref:††

235. We must distinguish between the First, Second, and Third Correlate of any triadic relation.

Peirce: CP 2.235 Cross-Ref:††

The First Correlate is that one of the three which is regarded as of the simplest nature, being a mere possibility if any one of the three is of that nature, and not being a law unless all three are of that nature.†1

Peirce: CP 2.236 Cross-Ref:††

236. The Third Correlate is that one of the three which is regarded as of the most complex nature, being a law if any one of the three is a law, and not being a mere possibility unless all three are of that nature.1

Peirce: CP 2.237 Cross-Ref:††

237. The Second Correlate is that one of the three which is regarded as of middling complexity, so that if any two are of the same nature, as to being either mere possibilities, actual existences, or laws, then the Second Correlate is of that same nature, while if the three are all of different natures, the Second Correlate is an actual existence.†1

Peirce: CP 2.238 Cross-Ref:††

238. Triadic relations are in three ways †2 divisible by trichotomy, according as the First, the Second, or the Third Correlate, respectively, is a mere possibility, an actual existent, or a law. These three trichotomies, taken together, divide all triadic relations into ten classes [see footnote to 235]. These ten classes will have certain subdivisions according as the existent correlates are individual subjects or individual facts, and according as the correlates that are laws are general subjects, general modes of fact, or general modes of law.

Peirce: CP 2.239 Cross-Ref:††

239. There will be besides a second similar division of triadic relations into ten classes, according as the dyadic relations which they constitute between either the First and Second Correlates, or the First and Third, or the Second and Third are of the nature of possibilities, facts, or laws; and these ten classes will be subdivided in different ways.†3

Peirce: CP 2.240 Cross-Ref:††

240. It may be convenient to collect the ten classes of either set of ten into three groups according as all three of the correlates or dyadic relations, as the case may be, are of different natures, or all are of the same nature, or two are of one nature while the third is of a different nature.†1

Peirce: CP 2.241 Cross-Ref:††

241. In every genuine Triadic Relation, the First Correlate may be regarded as determining the Third Correlate in some respect; and triadic relations may be divided according as that determination of the Third Correlate is to having some quality, or to being in some existential relation to the Second Correlate, or to being in some relation of thought to the Second for something †2.

Peirce: CP 2.242 Cross-Ref:††

242. A *Representamen* is the First Correlate of a triadic relation, the Second

Correlate being termed its *Object*, and the possible Third Correlate being termed its *Interpretant*, by which triadic relation the possible Interpretant is determined to be the First Correlate of the same triadic relation to the same Object, and for some possible Interpretant. A *Sign* is a representamen of which some interpretant is a cognition of a mind. Signs are the only representamens that have been much studied.

Peirce: CP 2.243 Cross-Ref:††
§4. ONE TRICHOTOMY OF SIGNS

243. Signs are divisible by three trichotomies;†1 first, according as the sign in itself is a mere quality, is an actual existent, or is a general law;†2 secondly, according as the relation of the sign to its object consists in the sign's having some character in itself, or in some existential relation to that object, or in its relation to an interpretant;†3 thirdly, according as its Interpretant represents it as a sign of possibility or as a sign of fact or a sign of reason.†4

Peirce: CP 2.244 Cross-Ref:††

244. According to the first division, a Sign may be termed a *Qualisign*, a *Sinsign*, or a *Legisign*.

Peirce: CP 2.244 Cross-Ref:††

A *Qualisign* is a quality which is a Sign. It cannot actually act as a sign until it is embodied; but the embodiment has nothing to do with its character as a sign.

Peirce: CP 2.245 Cross-Ref:††

245. A *Sinsign* (where the syllable *sin* is taken as meaning "being only once," as in *single*, *simple*, Latin *semel*, etc.) is an actual existent thing or event which is a sign. It can only be so through its qualities; so that it involves a qualisign, or rather, several qualisigns. But these qualisigns are of a peculiar kind and only form a sign through being actually embodied.

Peirce: CP 2.246 Cross-Ref:††

246. A *Legisign* is a law that is a Sign. This law is usually established by men. Every conventional sign is a legisign [but not conversely]. It is not a single object, but a general type which, it has been agreed, shall be significant. Every legisign signifies through an instance of its application, which may be termed a *Replica* of it. Thus, the word "the" will usually occur from fifteen to twenty-five times on a page. It is in all these occurrences one and the same word, the same legisign. Each single instance of it is a Replica. The Replica is a Sinsign. Thus, every Legisign requires Sinsigns. But these are not ordinary Sinsigns, such as are peculiar occurrences that are regarded as significant. Nor would the Replica be significant if it were not for the law which renders it so.

Peirce: CP 2.247 Cross-Ref:††
§5. A SECOND TRICHOTOMY OF SIGNS

247. According to the second trichotomy, a Sign may be termed an *Icon*, an

Index, or a ***Symbol***.

Peirce: CP 2.247 Cross-Ref:††

An ***Icon*** is a sign which refers to the Object that it denotes merely by virtue of characters of its own, and which it possesses, just the same, whether any such Object actually exists or not. It is true that unless there really is such an Object, the Icon does not act as a sign; but this has nothing to do with its character as a sign. Anything whatever, be it quality, existent individual, or law, is an Icon of anything, in so far as it is like that thing and used as a sign of it.

Peirce: CP 2.248 Cross-Ref:††

248. An ***Index*** is a sign which refers to the Object that it denotes by virtue of being really affected by that Object. It cannot, therefore, be a Qualisign, because qualities are whatever they are independently of anything else. In so far as the Index is affected by the Object, it necessarily has some Quality in common with the Object, and it is in respect to these that it refers to the Object. It does, therefore, involve a sort of Icon, although an Icon of a peculiar kind; and it is not the mere resemblance of its Object, even in these respects which makes it a sign, but it is the actual modification of it by the Object.

Peirce: CP 2.249 Cross-Ref:††

249. A ***Symbol*** is a sign which refers to the Object that it denotes by virtue of a law, usually an association of general ideas, which operates to cause the Symbol to be interpreted as referring to that Object. It is thus itself a general type or law, that is, is a Legisign. As such it acts through a Replica. Not only is it general itself, but the Object to which it refers is of a general nature. Now that which is general has its being in the instances which it will determine. There must, therefore, be existent instances of what the Symbol denotes, although we must here understand by "existent," existent in the possibly imaginary universe to which the Symbol refers. The Symbol will indirectly, through the association or other law, be affected by those instances; and thus the Symbol will involve a sort of Index, although an Index of a peculiar kind. It will not, however, be by any means true that the slight effect upon the Symbol of those instances accounts for the significant character of the Symbol.

Peirce: CP 2.250 Cross-Ref:††
§6. A THIRD TRICHOTOMY OF SIGNS

250. According to the third trichotomy, a Sign may be termed a ***Rheme***, a ***Dicisign*** or ***Dicent Sign*** (that is, a proposition or quasi-proposition), or an Argument.

Peirce: CP 2.250 Cross-Ref:††

A ***Rheme***†1 is a Sign which, for its Interpretant, is a Sign of qualitative Possibility, that is, is understood as representing such and such a kind of possible Object. Any Rheme, perhaps, will afford some information; but it is not interpreted as doing so.

Peirce: CP 2.251 Cross-Ref:††

251. A ***Dicent Sign*** is a Sign, which, for its Interpretant, is a Sign of actual existence. It cannot, therefore, be an Icon, which affords no ground for an interpretation of it as referring to actual existence. A Dicisign necessarily involves, as

a part of it, a Rheme, to describe the fact which it is interpreted as indicating. But this is a peculiar kind of Rheme; and while it is essential to the Dicisign, it by no means constitutes it.

Peirce: CP 2.252 Cross-Ref:††

252. An *Argument* is a Sign which, for its Interpretant, is a Sign of law. Or we may say that a Rheme is a sign which is understood to represent its object in its characters merely; that a Dicisign is a sign which is understood to represent its object in respect to actual existence; and that an Argument is a Sign which is understood to represent its Object in its character as Sign. Since these definitions touch upon points at this time much in dispute, a word may be added in defence of them. A question often put is: What is the essence of a Judgment? A judgment is the mental act by which the judger seeks to impress upon himself the truth of a proposition. It is much the same as an act of asserting the proposition, or going before a notary and assuming formal responsibility for its truth, except that those acts are intended to affect others, while the judgment is only intended to affect oneself. However, the logician, as such, cares not what the psychological nature of the act of judging may be. The question for him is: What is the nature of the sort of sign of which a principal variety is called a proposition, which is the matter upon which the act of judging is exercised? The proposition need not be asserted or judged. It may be contemplated as a sign capable of being asserted or denied. This sign itself retains its full meaning whether it be actually asserted or not.†1 The peculiarity of it, therefore, lies in its mode of meaning; and to say this is to say that its peculiarity lies in its relation to its interpretant. The proposition professes to be really affected by the actual existent or real law to which it refers. The argument makes the same pretension, but that is not the principal pretension of the argument. The rheme makes no such pretension.

Peirce: CP 2.253 Cross-Ref:††

253. The Interpretant of the Argument represents it as an instance of a general class of Arguments, which class on the whole will always tend to the truth. It is this law, in some shape, which the argument urges; and this "urging" is the mode of representation proper to Arguments. The Argument must, therefore, be a Symbol, or Sign whose Object is a General Law or Type. It must involve a Dicent Symbol, or Proposition, which is termed its *Premiss*; for the Argument can only urge the law by urging it in an instance. This *Premiss* is, however, quite different in force (*i.e.*, in its relation to its interpretant) from a similar proposition merely asserted; and besides, this is far from being the whole Argument. As for another proposition, called the Conclusion, often stated and perhaps required to complete the Argument, it plainly represents the Interpretant, and likewise has a peculiar force, or relation to the Interpretant. There is a difference of opinion among logicians as to whether it forms a part of the Argument or not; and although such opinions have not resulted from an exact analysis of the essence of Argument, they are entitled to weight. The present writer, without being absolutely confident, is strongly inclined to think that the Conclusion, although it represents the Interpretant, is essential to the full expression of the Argument. It is usual with logicians to speak of the Premisses of an Argument, instead of the *Premiss*. But if there are more Premisses than one, the first step of the argumentation must be to colligate them into one Copulative Proposition: so that the only simple Argument of two Premisses is the Argument of Colligation. But even in this case, there are not properly two premisses. For whenever the mind is in a state ready to assert a proposition, *P*, it is already in a state of asserting a proposition, *O*, which the new proposition, *P*, only further determines; so that it is not *P*, merely, which comes to be asserted, but *OP*. In this view of the matter, there is no such thing

as an Argument of Colligation. For to say that there is would make every judgment the conclusion of an argument. But if every judgment is to be regarded as the conclusion of an argument, which is, no doubt, an admissible conception, then it is the conclusion of a quite different kind of judgment from a mere Argument of Colligation. Thus, the Argument of Colligation is a form of Argument which is introduced into logic merely in order to avoid the necessity of considering the true nature of the Argument from which a Copulative Proposition has been derived. For that reason, it seems more proper in general to speak of the "Premiss" of an Argument than of its "Premisses." As to the word *Premiss*--in Latin of the thirteenth century *praemissa*--owing to its being so often used in the plural, it has become widely confounded with a totally different word of legal provenance, the "premisses," that is, the items of an inventory, etc., and hence buildings enumerated in a deed or lease. It is entirely contrary to good English usage to spell premiss, "premise," and this spelling (whose prevalence is due perhaps to Lord Brougham, or at least chiefly supported by his insistence), simply betrays ignorance of the history of logic, and even of such standard authors as Whateley, Watts, etc.†1

Peirce: CP 2.254 Cross-Ref:††
 §7. TEN CLASSES OF SIGNS

254. The three trichotomies of Signs result together in dividing Signs into *TEN CLASSES OF SIGNS*, of which numerous subdivisions have to be considered. The ten classes are as follows:

Peirce: CP 2.254 Cross-Ref:††

First: A Qualisign [*e.g.*, a feeling of "red"] is any quality in so far as it is a sign. Since a quality is whatever it is positively in itself, a quality can only denote an object by virtue of some common ingredient or similarity; so that a Qualisign is necessarily an Icon. Further, since a quality is a mere logical possibility, it can only be interpreted as a sign of essence, that is, as a Rheme.

Peirce: CP 2.255 Cross-Ref:††

255. Second: An Iconic Sinsign [*e.g.*, an individual diagram] is any object of experience in so far as some quality of it makes it determine the idea of an object. Being an Icon, and thus a sign by likeness purely, of whatever it may be like, it can only be interpreted as a sign of essence, or Rheme. It will embody a Qualisign.

Peirce: CP 2.256 Cross-Ref:††

256. Third: A Rhematic Indexical Sinsign [*e.g.*, a spontaneous cry] is any object of direct experience so far as it directs attention to an Object by which its presence is caused. It necessarily involves an Iconic Sinsign of a peculiar kind, yet is quite different since it brings the attention of the interpreter to the very Object denoted.

Peirce: CP 2.257 Cross-Ref:††

257. Fourth: A Dicent Sinsign [*e.g.*, a weathercock] is any object of direct experience, in so far as it is a sign, and, as such, affords information concerning its Object. This it can only do by being really affected by its Object; so that it is necessarily an Index. The only information it can afford is of actual fact. Such a Sign must involve an Iconic Sinsign to embody the information and a Rhematic Indexical

Sinsign to indicate the Object to which the information refers. But the mode of combination, or *Syntax*, of these two must also be significant.

Peirce: CP 2.258 Cross-Ref:††

258. Fifth: An Iconic Legisign [*e.g.*, a diagram, apart from its factual individuality] is any general law or type, in so far as it requires each instance of it to embody a definite quality which renders it fit to call up in the mind the idea of a like object. Being an Icon, it must be a Rheme. Being a Legisign, its mode of being is that of governing single Replicas, each of which will be an Iconic Sinsign of a peculiar kind.

Peirce: CP 2.259 Cross-Ref:††

259. Sixth: A Rhematic Indexical Legisign [*e.g.*, a demonstrative pronoun] is any general type or law, however established, which requires each instance of it to be really affected by its Object in such a manner as merely to draw attention to that Object. Each Replica of it will be a Rhematic Indexical Sinsign of a peculiar kind. The Interpretant of a Rhematic Indexical Legisign represents it as an Iconic Legisign; and so it is, in a measure--but in a very small measure.

Peirce: CP 2.260 Cross-Ref:††

260. Seventh: A Dicent Indexical Legisign [*e.g.*, a street cry] is any general type or law, however established, which requires each instance of it to be really affected by its Object in such a manner as to furnish definite information concerning that Object. It must involve an Iconic Legisign to signify the information and a Rhematic Indexical Legisign to denote the subject of that information. Each Replica of it will be a Dicent Sinsign of a peculiar kind.

Peirce: CP 2.261 Cross-Ref:††

261. Eighth: A Rhematic Symbol or Symbolic Rheme [*e.g.*, a common noun] is a sign connected with its Object by an association of general ideas in such a way that its Replica calls up an image in the mind which image, owing to certain habits or dispositions of that mind, tends to produce a general concept, and the Replica is interpreted as a Sign of an Object that is an instance of that concept. Thus, the Rhematic Symbol either is, or is very like, what the logicians call a General Term. The Rhematic Symbol, like any Symbol, is necessarily itself of the nature of a general type, and is thus a Legisign. Its Replica, however, is a Rhematic Indexical Sinsign of a peculiar kind, in that the image it suggests to the mind acts upon a Symbol already in that mind to give rise to a General Concept. In this it differs from other Rhematic Indexical Sinsigns, including those which are Replicas of Rhematic Indexical Legisigns. Thus, the demonstrative pronoun "that" is a Legisign, being a general type; but it is not a Symbol, since it does not signify a general concept. Its Replica draws attention to a single Object, and is a Rhematic Indexical Sinsign. A Replica of the word "camel" is likewise a Rhematic Indexical Sinsign, being really affected, through the knowledge of camels, common to the speaker and auditor, by the real camel it denotes, even if this one is not individually known to the auditor; and it is through such real connection that the word "camel" calls up the idea of a camel. The same thing is true of the word "phoenix." For although no phoenix really exists, real descriptions of the phoenix are well known to the speaker and his auditor; and thus the word is really affected by the Object denoted. But not only are the Replicas of Rhematic Symbols very different from ordinary Rhematic Indexical Sinsigns, but so likewise are Replicas of Rhematic Indexical Legisigns. For the thing denoted by "that" has not affected the replica of the word in any such direct and simple manner as that in which, for example, the ring of a telephone-bell is affected by the person at the

other end who wants to make a communication. The Interpretant of the Rhematic Symbol often represents it as a Rhematic Indexical Legisign; at other times as an Iconic Legisign; and it does in a small measure partake of the nature of both.

Peirce: CP 2.262 Cross-Ref:††

262. Ninth: A Dicient Symbol, or ordinary Proposition, is a sign connected with its object by an association of general ideas, and acting like a Rhematic Symbol, except that its intended interpretant represents the Dicient Symbol as being, in respect to what it signifies, really affected by its Object, so that the existence or law which it calls to mind must be actually connected with the indicated Object. Thus, the intended Interpretant looks upon the Dicient Symbol as a Dicient Indexical Legisign; and if it be true, it does partake of this nature, although this does not represent its whole nature. Like the Rhematic Symbol, it is necessarily a Legisign. Like the Dicient Sinsign it is composite inasmuch as it necessarily involves a Rhematic Symbol (and thus is for its Interpretant an Iconic Legisign) to express its information and a Rhematic Indexical Legisign to indicate the subject of that information. But its Syntax of these is significant. The Replica of the Dicient Symbol is a Dicient Sinsign of a peculiar kind. This is easily seen to be true when the information the Dicient Symbol conveys is of actual fact. When that information is of a real law, it is not true in the same fullness. For a Dicient Sinsign cannot convey information of law. It is, therefore, true of the Replica of such a Dicient Symbol only in so far as the law has its being in instances.

Peirce: CP 2.263 Cross-Ref:††

263. Tenth: An Argument is a sign whose interpretant represents its object as being an ulterior sign through a law, namely, the law that the passage from all such premisses to such conclusions tends to the truth. Manifestly, then, its object must be general; that is, the Argument must be a Symbol. As a Symbol it must, further, be a Legisign. Its Replica is a Dicient Sinsign.

Peirce: CP 2.264 Cross-Ref:††

264. The affinities of the ten classes are exhibited by arranging their designations in the triangular table here shown, which has heavy boundaries between adjacent squares that are appropriated to classes alike in only one respect. All other adjacent squares pertain to classes alike in two respects. Squares not adjacent pertain to classes alike in one respect only, except that each of the three squares of the vertices of the triangle pertains to a class differing in all three respects from the classes to which the squares along the opposite side of the triangle are appropriated. The lightly printed designations are superfluous.

(I)†1	(V)	(VIII)	(X)	Rhematic	Rhematic	Rhematic	
Argument							
Iconic	Iconic	Symbol	Symbolic	Qualisign	Legisign		
Legisign	Legisign						

(II)	(VI)	(IX)	
Rhematic	Rhematic	Dicent	

| **Iconic** | **Indexical** | **Symbol** |
| **Sinsign** | **Legisign** | Legisign |

(III)	(VII)		**Rhematic**	**Dicent**
Indexical	**Indexical**			
Sinsign	**Legisign**			

(IV)	
Dicent	
Indexical	
Sinsign	

Peirce: CP 2.265 Cross-Ref:††
§8. DEGENERATE SIGNS

265. In the course of the above descriptions of the classes, certain subdivisions of some of them have been directly or indirectly referred to. Namely, beside the normal varieties of Sinsigns, Indices, and Dicisigns, there are others which are Replicas of Legisigns, Symbols, and Arguments, respectively. Beside the normal varieties of Qualisigns, Icons, and Rhemes, there are two series of others; to wit, those which are directly involved in Sinsigns, Indices, and Dicisigns, respectively, and also those which are indirectly involved in Legisigns, Symbols, and Arguments, respectively. Thus, the ordinary Dicent Sinsign is exemplified by a weathercock and its veering and by a photograph. The fact that the latter is known to be the effect of the radiations from the object renders it an index and highly informative. A second variety is a Replica of a Dicent Indexical Legisign. Thus any given street cry, since its tone and theme identifies the individual, is not a symbol, but an Indexical Legisign; and any individual instance of it is a Replica of it which is a Dicent Sinsign. A third variety is a Replica of a Proposition. A fourth variety is a Replica of an Argument. Beside the normal variety of the Dicent Indexical Legisign, of which a street cry is an example, there is a second variety, which is that sort of proposition which has the name of a well-known individual as its predicate; as if one is asked, "Whose statue is this?" the answer may be, "It is Farragut." The meaning of this answer is a Dicent Indexical Legisign. A third variety may be a premiss of an argument. A Dicent Symbol, or ordinary proposition, in so far as it is a premiss of an Argument, takes on a new force, and becomes a second variety of the Dicent Symbol. It would not be worth while to go through all the varieties; but it may be well to consider the varieties of one class more. We may take the Rhematic Indexical Legisign. *The* shout of "Hullo!" is an example of the ordinary variety--meaning, not an individual shout, but

this shout "Hullo!" in general--this type of shout. A second variety is a constituent of a Dicent Indexical Legisign; as the word "that" in the reply, "that is Farragut." A third variety is a particular application of a Rhematic Symbol; as the exclamation "Hark!" A fourth and fifth variety are in the peculiar force a general word may have in a proposition or argument. It is not impossible that some varieties are here overlooked. It is a nice problem to say to what class a given sign belongs; since all the circumstances of the case have to be considered. But it is seldom requisite to be very accurate; for if one does not locate the sign precisely, one will easily come near enough to its character for any ordinary purpose of logic.

Peirce: CP 2.266 Cross-Ref:††

§9. THE TRICHOTOMY OF ARGUMENTS

266. There are other subdivisions of some, at least, of the ten classes which are of greater logical importance. An Argument is always understood by its Interpretant to belong to a general class of analogous arguments, which class, as a whole, tends toward the truth. This may happen in three ways, giving rise to a trichotomy of all simple arguments into Deductions, Inductions, and Abductions.

Peirce: CP 2.267 Cross-Ref:††

267. A **Deduction** is an argument whose Interpretant represents that it belongs to a general class of possible arguments precisely analogous which are such that in the long run of experience the greater part of those whose premisses are true will have true conclusions. Deductions are either **Necessary** or **Probable**. Necessary Deductions are those which have nothing to do with any ratio of frequency, but profess (or their interpretants profess for them) that from true premisses they must invariably produce true conclusions. A Necessary Deduction is a method of producing Dicent Symbols by the study of a diagram. It is either **Corollarial** or **Theorematic**. A Corollarial Deduction is one which represents the conditions of the conclusion in a diagram and finds from the observation of this diagram, as it is, the truth of the conclusion. A Theorematic Deduction is one which, having represented the conditions of the conclusion in a diagram, performs an ingenious experiment upon the diagram, and by the observation of the diagram, so modified, ascertains the truth of the conclusion.

Peirce: CP 2.268 Cross-Ref:††

268. Probable Deductions, or more accurately, Deductions of Probability, are Deductions whose Interpretants represent them to be concerned with ratios of frequency. They are either **Statistical Deductions** or **Probable Deductions Proper**. A Statistical Deduction is a Deduction whose Interpretant represents it to reason concerning ratios of frequency, but to reason concerning them with absolute certainty. A Probable Deduction proper is a Deduction whose Interpretant does not represent that its conclusion is certain, but that precisely analogous reasonings would from true premisses produce true conclusions in the majority of cases, in the long run of experience.

Peirce: CP 2.269 Cross-Ref:††

269. An **Induction** is a method of forming Dicent Symbols concerning a definite question, of which method the Interpretant does not represent that from true premisses it will yield approximately true results in the majority of instances in the long run of experience, but does represent that if this method be persisted in, it will in

the long run yield the truth, or an indefinite approximation to the truth, in regard to every question. An Induction is either a *Pooh-pooh Argument*, or an *Experimental Verification of a general Prediction*, or an Argument from a *Random Sample*. A Pooh-pooh Argument is a method which consists in denying that a general kind of event ever will occur on the ground that it never has occurred. Its justification is that if it be persistently applied on every occasion, it must ultimately be corrected in case it should be wrong, and thus will ultimately reach the true conclusion. A verification of a general prediction is a method which consists in finding or making the conditions of the prediction and in concluding that it will be verified about as often as it is experimentally found to be verified. Its justification is that if the Prediction does not tend in the long run to be verified in any approximately determinate proportion of cases, experiment must, in the long run, ascertain this; while if the Prediction will, in the long run, be verified in any determinate, or approximately determinate, proportion of cases, experiment must in the long run, approximately ascertain what that proportion is. An Argument from a Random Sample, is a method of ascertaining what proportion of the members of a finite class possess a predesignate, or virtually predesignate, quality, by selecting instances from that class according to a method which will, in the long run, present any instance as often as any other, and concluding that the ratio found for such a sample will hold in the long run. Its justification is evident.

Peirce: CP 2.270 Cross-Ref:††

270. An *Abduction* is a method of forming a general prediction without any positive assurance that it will succeed either in the special case or usually, its justification being that it is the only possible hope of regulating our future conduct rationally, and that Induction from past experience gives us strong encouragement to hope that it will be successful in the future.

Peirce: CP 2.271 Cross-Ref:††
§10. KINDS OF PROPOSITIONS

271. A Dicent Symbol, or general proposition, is either *Particular* or *Universal*. A Particular Dicent Symbol is represented by its Interpretant to indicate fact of existence; as, "Some swan is black," *i.e.*, there exists a black swan. A Universal Dicent Symbol is represented by its Interpretant to indicate a real law; as "No swan is black," *i.e.*, no amount of research will ever discover a black individual among swans. A Dicent Symbol is either *Non-relative* or *Relative*. A Non-relative Dicent Symbol is not concerned with the identity of more than one individual. But this must be understood in a particular way, the proposition being first expressed in an exemplar manner. Thus "No swan is black" seems to be concerned with the identity of all swans and all black objects. But it is to be understood that the proposition is to be considered under this form: Taking any one object in the universe you please, it is either not a swan or is not black. A Relative Dicent Symbol is concerned with the identity of more than one individual, or of what may be more than one, in an exemplar expression, as, "Take any individual, A, you please, and thereafter an individual, B, can be found, such that if A is a city of over a hundred thousand inhabitants, B will be a spot on this map corresponding to A." Whether a proposition is to be regarded as non-relative or relative depends on what use is to be made of it in argument. But it does not follow that the distinction is merely one of outward guise;

for the force of the proposition is different according to the application that is to be made of it. It may here be noted as a matter of correct terminology (according to the views set forth in the second part [of the published portion] of this syllabus), †1 that a **Hypothetical Proposition** is any proposition compounded of propositions. The old doctrine is that a hypothetical proposition is either conditional, copulative, or disjunctive. But a conditional is properly a disjunctive proposition. Some propositions may equally well be regarded as copulative or disjunctive. Thus, at once, either Tully or not Cicero and either Cicero or not Tully, is the same as, either at once, Tully and Cicero or not Tully and not Cicero. Any definition may be regarded as a proposition of this sort; and for this reason such propositions might be termed **Definiform**, or **Definitory**. A copulative proposition is naturally allied to a particular proposition, a disjunctive proposition to a universal proposition.

Peirce: CP 2.272 Cross-Ref: ††

272. If parts of a proposition be erased so as to leave blanks in their places, and if these blanks are of such a nature that if each of them be filled by a proper name the result will be a proposition, then the blank form of proposition which was first produced by the erasures is termed a **rheme**. According as the number of blanks in a rheme is 0, 1, 2, 3, etc., it may be termed a **medad** (from {méden}, nothing), **monad**, **dyad**, **triad**, etc., rheme.

Peirce: CP 2.273 Cross-Ref: ††

§11. REPRESENT †1

273. To stand for, that is, to be in such a relation to another that for certain purposes it is treated by some mind as if it were that other.

Peirce: CP 2.273 Cross-Ref: ††

Thus a spokesman, deputy, attorney, agent, vicar, diagram, symptom, counter, description, concept, premiss, testimony, all represent something else, in their several ways, to minds who consider them in that way. See Sign. †2 When it is desired to distinguish between that which represents and the act or relation of representing, the former may be termed the "representamen," the latter the "representation."

Peirce: CP 2.274 Cross-Ref: ††

CHAPTER 3

THE ICON, INDEX, AND SYMBOL

§1. ICONS AND HYPOICONS †1

274. A **Sign**, or **Representamen**, is a First which stands in such a genuine

triadic relation to a Second, called its *Object*, as to be capable of determining a Third, called its *Interpretant*, to assume the same triadic relation to its Object in which it stands itself to the same Object. The triadic relation is *genuine*, that is its three members are bound together by it in a way that does not consist in any complexus of dyadic relations. That is the reason the Interpretant, or Third, cannot stand in a mere dyadic relation to the Object, but must stand in such a relation to it as the Representamen itself does. Nor can the triadic relation in which the Third stands be merely similar to that in which the First stands, for this would make the relation of the Third to the First a degenerate Secondness merely. The Third must indeed stand in such a relation, and thus must be capable of determining a Third of its own; but besides that, it must have a second triadic relation in which the Representamen, or rather the relation thereof to its Object, shall be its own (the Third's) Object, and must be capable of determining a Third to this relation. All this must equally be true of the Third's Thirds and so on endlessly; and this, and more, is involved in the familiar idea of a Sign; and as the term Representamen is here used, nothing more is implied. A *Sign* is a Representamen with a mental Interpretant. Possibly there may be Representamens that are not Signs. Thus, if a sunflower, in turning towards the sun, becomes by that very act fully capable, without further condition, of reproducing a sunflower which turns in precisely corresponding ways toward the sun, and of doing so with the same reproductive power, the sunflower would become a Representamen of the sun. But *thought* is the chief, if not the only, mode of representation.

Peirce: CP 2.275 Cross-Ref:††

275. . . . The most fundamental [division of signs] is into *Icons*, *Indices*, and *Symbols*. Namely, while no Representamen actually functions as such until it actually determines an Interpretant, yet it becomes a Representamen as soon as it is fully capable of doing this; and its Representative Quality is not necessarily dependent upon its ever actually determining an Interpretant, nor even upon its actually having an Object.

Peirce: CP 2.276 Cross-Ref:††

276. An *Icon* is a Representamen whose Representative Quality is a Firstness of it as a First. That is, a quality that it has *qua* thing renders it fit to be a representamen. Thus, anything is fit to be a *Substitute* for anything that it is like. (The conception of "substitute" involves that of a purpose, and thus of genuine thirdness.) Whether there are other kinds of substitutes or not we shall see. A Representamen by Firstness alone can only have a similar Object. Thus, a Sign by Contrast denotes its object only by virtue of a contrast, or Secondness, between two qualities. A sign by Firstness is an image of its object and, more strictly speaking, can only be an *idea*. For it must produce an Interpretant idea; and an external object excites an idea by a reaction upon the brain. But most strictly speaking, even an idea, except in the sense of a possibility, or Firstness, cannot be an Icon. A possibility alone is an Icon purely by virtue of its quality; and its object can only be a Firstness. But a sign may be *iconic*, that is, may represent its object mainly by its similarity, no matter what its mode of being. If a substantive be wanted, an iconic representamen may be termed a *hypoicon*. Any material image, as a painting, is largely conventional in its mode of representation; but in itself, without legend or label it may be called a *hypoicon*.

Peirce: CP 2.277 Cross-Ref:††

277. Hypoicons may be roughly divided according to the mode of Firstness of which they partake. Those which partake of simple qualities, or First Firstnesses, are *images*; those which represent the relations, mainly dyadic, or so regarded, of the

parts of one thing by analogous relations in their own parts, are *diagrams*; those which represent the representative character of a representamen by representing a parallelism in something else, are *metaphors*.

Peirce: CP 2.278 Cross-Ref:††

278. The only way of directly communicating an idea is by means of an icon; and every indirect method of communicating an idea must depend for its establishment upon the use of an icon. Hence, every assertion must contain an icon or set of icons, or else must contain signs whose meaning is only explicable by icons. The idea which the set of icons (or the equivalent of a set of icons) contained in an assertion signifies may be termed the *predicate* of the assertion.

Peirce: CP 2.279 Cross-Ref:††

279. Turning now to the rhetorical evidence, it is a familiar fact that there are such representations as icons. Every picture (however conventional its method) is essentially a representation of that kind. So is every diagram, even although there be no sensuous resemblance between it and its object, but only an analogy between the relations of the parts of each. Particularly deserving of notice are icons in which the likeness is aided by conventional rules. Thus, an algebraic formula is an icon, rendered such by the rules of commutation, association, and distribution of the symbols. It may seem at first glance that it is an arbitrary classification to call an algebraic expression an icon; that it might as well, or better, be regarded as a compound conventional sign. But it is not so. For a great distinguishing property of the icon is that by the direct observation of it other truths concerning its object can be discovered than those which suffice to determine its construction. Thus, by means of two photographs a map can be drawn, etc. Given a conventional or other general sign of an object, to deduce any other truth than that which it explicitly signifies, it is necessary, in all cases, to replace that sign by an icon. This capacity of revealing unexpected truth is precisely that wherein the utility of algebraical formulae consists, so that the iconic character is the prevailing one.

Peirce: CP 2.280 Cross-Ref:††

280. That icons of the algebraic kind, though usually very simple ones, exist in all ordinary grammatical propositions is one of the philosophic truths that the Boolean logic brings to light. In all primitive writing, such as the Egyptian hieroglyphics, there are icons of a non-logical kind, the ideographs. In the earliest form of speech, there probably was a large element of mimicry. But in all languages known, such representations have been replaced by conventional auditory signs. These, however, are such that they can only be explained by icons. But in the syntax of every language there are logical icons of the kind that are aided by conventional rules. . . .

Peirce: CP 2.281 Cross-Ref:††

281. Photographs, especially instantaneous photographs, are very instructive, because we know that they are in certain respects exactly like the objects they represent. But this resemblance is due to the photographs having been produced under such circumstances that they were physically forced to correspond point by point to nature. In that aspect, then, they belong to the second class of signs, those by physical connection. The case is different if I surmise that zebras are likely to be obstinate, or otherwise disagreeable animals, because they seem to have a general resemblance to donkeys, and donkeys are self-willed. Here the donkey serves precisely as a probable likeness of the zebra. It is true we suppose that resemblance has a physical cause in heredity; but then, this hereditary affinity is itself only an inference from the likeness between the two animals, and we have not (as in the case of the photograph) any

independent knowledge of the circumstances of the production of the two species. Another example of the use of a likeness is the design an artist draws of a statue, pictorial composition, architectural elevation, or piece of decoration, by the contemplation of which he can ascertain whether what he proposes will be beautiful and satisfactory. The question asked is thus answered almost with certainty because it relates to how the artist will himself be affected. The reasoning of mathematicians will be found to turn chiefly upon the use of likenesses, which are the very hinges of the gates of their science. The utility of likenesses to mathematicians consists in their suggesting in a very precise way, new aspects of supposed states of things. . . .

Peirce: CP 2.282 Cross-Ref:††

282. Many diagrams resemble their objects not at all in looks; it is only in respect to the relations of their parts that their likeness consists. Thus, we may show the relation between the different kinds of signs by a brace, thus:

{ Icons,
Signs: { Indices,
 { Symbols.

This is an icon. But the only respect in which it resembles its object is that the brace shows the classes of *icons*, *indices*, and *symbols* to be related to one another and to the general class of signs, as they really are, in a general way. When, in algebra, we write equations under one another in a regular array, especially when we put resembling letters for corresponding coefficients, the array is an icon. Here is an example:

$$a[1]x + b[1]y = n[1],$$

$$a[2]x + b[2]y = n[2].$$

This is an icon, in that it makes quantities look alike which are in analogous relations to the problem. In fact, every algebraical equation is an icon, in so far as it *exhibits*, by means of the algebraical signs (which are not themselves icons), the relations of the quantities concerned.

Peirce: CP 2.282 Cross-Ref:††

It may be questioned whether all icons are likenesses or not. For example, if a drunken man is exhibited in order to show, by contrast, the excellence of temperance, this is certainly an icon, but whether it is a likeness or not may be doubted. The question seems somewhat trivial.

283. An *Index* or *Seme*†1 ({séma}) is a Representamen whose Representative character consists in its being an individual second. If the Secondness is an existential relation, the Index is *genuine*. If the Secondness is a reference, the Index is *degenerate*. A genuine Index and its Object must be existent individuals (whether things or facts), and its immediate Interpretant must be of the same character. But since every individual must have characters, it follows that a genuine Index may contain a Firstness, and so an Icon as a constituent part of it. Any individual is a degenerate Index of its own characters.

Peirce: CP 2.284 Cross-Ref:††

284. *Subindices* or *Hyposesmes* are signs which are rendered such principally by an actual connection with their objects. Thus a proper name, personal demonstrative, or relative pronoun or the letter attached to a diagram, denotes what it does owing to a real connection with its object but none of these is an Index, since it is not an individual.

Peirce: CP 2.285 Cross-Ref:††

285. Let us examine some examples of indices. I see a man with a rolling gait. This is a probable indication that he is a sailor. I see a bowlegged man in corduroys, gaiters, and a jacket. These are probable indications that he is a jockey or something of the sort. A sundial or a clock *indicates* the time of day. Geometricians mark letters against the different parts of their diagrams and then use these letters to indicate those parts. Letters are similarly used by lawyers and others. Thus, we may say: If A and B are married to one another and C is their child while D is brother of A, then D is uncle of C. Here A, B, C, and D fulfill the office of relative pronouns, but are more convenient since they require no special collocation of words. A rap on the door is an index. Anything which focusses the attention is an index. Anything which startles us is an index, in so far as it marks the junction between two portions of experience. Thus a tremendous thunderbolt indicates that *something* considerable happened, though we may not know precisely what the event was. But it may be expected to connect itself with some other experience.

Peirce: CP 2.286 Cross-Ref:††

286. . . . A low barometer with a moist air is an index of rain; that is we suppose that the forces of nature establish a probable connection between the low barometer with moist air and coming rain. A weathercock is an index of the direction of the wind; because in the first place it really takes the self-same direction as the wind, so that there is a real connection between them, and in the second place we are so constituted that when we see a weathercock pointing in a certain direction it draws our attention to that direction, and when we see the weathercock veering with the wind, we are forced by the law of mind to think that direction is connected with the wind. The pole star is an index, or pointing finger, to show us which way is north. A spirit-level, or a plumb bob, is an index of the vertical direction. A yard-stick might seem, at first sight, to be an icon of a yard; and so it would be, if it were merely intended to show a yard as near as it can be seen and estimated to be a yard. But the very purpose of a yard-stick is to show a yard nearer than it can be estimated by its appearance. This it does in consequence of an accurate mechanical comparison made with the bar in London called the yard. Thus it is a real connection which gives the yard-stick its value as a representamen; and thus it is an *index*, not a mere *icon*.

Peirce: CP 2.287 Cross-Ref:††

287. When a driver to attract the attention of a foot passenger and cause him to

save himself, calls out "Hi!" so far as this is a significant word, it is, as will be seen below, something more than an index; but so far as it is simply intended to act upon the hearer's nervous system and to rouse him to get out of the way, it is an index, because it is meant to put him in real connection with the object, which is his situation relative to the approaching horse. Suppose two men meet upon a country road and one of them says to the other, "The chimney of that house is on fire." The other looks about him and describes a house with green blinds and a verandah having a smoking chimney. He walks on a few miles and meets a second traveller. Like a Simple Simon he says, "The chimney of that house is on fire." "What house?" asks the other. "Oh, a house with green blinds and a verandah," replies the simpleton. "Where is the house?" asks the stranger. He desires some *index* which shall connect his apprehension with the house meant. Words alone cannot do this. The demonstrative pronouns, "this" and "that," are indices. For they call upon the hearer to use his powers of observation, and so establish a real connection between his mind and the object; and if the demonstrative pronoun does that--without which its meaning is not understood--it goes to establish such a connection; and so is an index. The relative pronouns, *who* and *which*, demand observational activity in much the same way, only with them the observation has to be directed to the words that have gone before. Lawyers use A, B, C, practically as very effective relative pronouns. To show how effective they are, we may note that Messrs. Allen and Greenough, in their admirable (though in the edition of 1877 [?], too small) Latin Grammar, declare that no conceivable syntax could wholly remove the ambiguity of the following sentence, "A replied to B that he thought C (his brother) more unjust to himself than to his own friend."†1 Now, any lawyer would state that with perfect clearness, by using A, B, C, as relatives, thus:

(A)

A replied to B that he (B), thought C

(A's)

(A)

(his (B's), brother) more unjust to himself, (B) than to his

(C)

(A's)

(B's) own friend.†P1 The terminations which in any inflected

(C's)

language are attached to words "governed" by other words, and which serve to show which the governing word is, by repeating what is elsewhere expressed in the same form, are likewise *indices* of the same relative pronoun character. Any bit of Latin poetry illustrates this, such as the twelve-line sentence beginning, "*Jam satis terris.*" Both in these terminations and in the A, B, C, a likeness is relied upon to carry the attention to the right object. But this does not make them icons, in any important way; for it is of no consequence how the letters A, B, C, are shaped or what the terminations are. It is not merely that one occurrence of an A is like a previous

occurrence that is the important circumstance, but that *there is an understanding that like letters shall stand for the same thing*, and this acts as a force carrying the attention from one occurrence of A to the previous one. A possessive pronoun is two ways an index: first it indicates the possessor, and, second, it has a modification which syntactically carries the attention to the word denoting the thing possessed.

Peirce: CP 2.288 Cross-Ref:††

288. Some indices are more or less detailed directions for what the hearer is to do in order to place himself in direct experiential or other connection with the thing meant. Thus, the Coast Survey issues "Notices to Mariners," giving the latitude and longitude, four or five bearings of prominent objects, etc., and saying *there* is a rock, or shoal, or buoy, or lightship. Although there will be other elements in such directions, yet in the main they are indices.

Peirce: CP 2.289 Cross-Ref:††

289. Along with such indexical directions of what to do to find the object meant, ought to be classed those pronouns which should be entitled *selective* pronouns [or quantifiers] because they inform the hearer how he is to pick out one of the objects intended, but which grammarians call by the very indefinite designation of *indefinite* pronouns. Two varieties of these are particularly important in logic, the *universal selectives*, such as *quivis, quilibet, quisquam, ullus, nullus, nemo, quisque, uterque*, and in English, *any, every, all, no, none, whatever, whoever, everybody, anybody, nobody*. These mean that the hearer is at liberty to select any instance he likes within limits expressed or understood, and the assertion is intended to apply to that one. The other logically important variety consists of the *particular selectives*, *quis, quispiam, nescio quis, aliquis, quidam*, and in English, *some, something, somebody, a, a certain, some or other, a suitable, one*.

Peirce: CP 2.289 Cross-Ref:††

Allied to the above pronouns are such expressions as *all but one, one or two, a few, nearly all, every other one*, etc. Along with pronouns are to be classed adverbs of place and time, etc.

Peirce: CP 2.289 Cross-Ref:††

Not very unlike these are, *the first, the last, the seventh, two-thirds of, thousands of*, etc.

Peirce: CP 2.290 Cross-Ref:††

290. Other indexical words are prepositions, and prepositional phrases, such as, "on the right (or left) of." Right and left cannot be distinguished by any general description. Other prepositions signify relations which may, perhaps, be described; but when they refer, as they do oftener than would be supposed, to a situation relative to the observed, or assumed to be experientially known, place and attitude of the speaker relatively to that of the hearer, then the indexical element is the dominant element.†P1

Peirce: CP 2.291 Cross-Ref:††

291. Icons and indices assert nothing. If an icon could be interpreted by a sentence, that sentence must be in a "potential mood," that is, it would merely say, "Suppose a figure has three sides," etc. Were an index so interpreted, the mood must be imperative, or exclamatory, as "See there!" or "Look out!" But the kind of signs which we are now coming to consider are, by nature, in the "indicative," or, as it should be called, the *declarative* mood.†P1 Of course, they can go to the expression

of any other mood, since we may declare assertions to be doubtful, or mere interrogations, or imperatively requisite.

Peirce: CP 2.292 Cross-Ref:††
§3. THE NATURE OF SYMBOLS

292. A *Symbol* is a Representamen whose Representative character consists precisely in its being a rule that will determine its Interpretant. All words, sentences, books, and other conventional signs are Symbols. We speak of writing or pronouncing the word "man"; but it is only a *replica*, or embodiment of the word, that is pronounced or written. The word itself has no existence although it has a real being, *consisting in* the fact that existents *will* conform to it. It is a general mode of succession of three sounds or representamens of sounds, which becomes a sign only in the fact that a habit, or acquired law, will cause replicas of it to be interpreted as meaning a man or men. The word and its meaning are both general rules; but the word alone of the two prescribes the qualities of its replicas in themselves. Otherwise the "word" and its "meaning" do not differ, unless some special sense be attached to "meaning."

Peirce: CP 2.293 Cross-Ref:††

293. A Symbol is a law, or regularity of the indefinite future. Its Interpretant must be of the same description; and so must be also the complete immediate Object, or meaning.†P1 But a law necessarily governs, or "is embodied in" individuals, and prescribes some of their qualities. Consequently, a constituent of a Symbol may be an Index, and a constituent may be an Icon. A man walking with a child points his arm up into the air and says, "There is a balloon." The pointing arm is an essential part of the symbol without which the latter would convey no information. But if the child asks, "What is a balloon," and the man replies, "It is something like a great big soap bubble," he makes the image a part of the symbol. Thus, while the complete object of a symbol, that is to say, its meaning, is of the nature of a law, it must *denote* an individual, and must *signify* a character. A *genuine* symbol is a symbol that has a general meaning. There are two kinds of degenerate symbols, the *Singular Symbol* whose Object is an existent individual, and which signifies only such characters as that individual may realize; and the *Abstract Symbol*, whose only Object is a character.

Peirce: CP 2.294 Cross-Ref:††

294. Although the immediate Interpretant of an Index must be an Index, yet since its Object may be the Object of an Individual [Singular] Symbol, the Index may have such a Symbol for its indirect Interpretant. Even a genuine Symbol may be an imperfect Interpretant of it. So an *icon* may have a degenerate Index, or an Abstract Symbol, for an indirect Interpretant, and a genuine Index or Symbol for an imperfect Interpretant.

Peirce: CP 2.295 Cross-Ref:††

295. A *Symbol* is a sign naturally fit to declare that the set of objects which is denoted by whatever set of indices may be in certain ways attached to it is represented by an icon associated with it. To show what this complicated definition means, let us take as an example of a symbol the word "loveth." Associated with this word is an

idea, which is the mental icon of one person loving another. Now we are to understand that "loveth" occurs in a sentence; for what it may mean by itself, if it means anything, is not the question. Let the sentence, then, be "Ezekiel loveth Huldah." Ezekiel and Huldah must, then, be or contain indices; for without indices it is impossible to designate what one is talking about. Any mere description would leave it uncertain whether they were not mere characters in a ballad; but whether they be so or not, indices can designate them. Now the effect of the word "loveth" is that the pair of objects denoted by the pair of indices Ezekiel and Huldah is represented by the icon, or the image we have in our minds of a lover and his beloved.

Peirce: CP 2.296 Cross-Ref:††

296. The same thing is equally true of every verb in the declarative mood; and indeed of every verb, for the other moods are merely declarations of a fact somewhat different from that expressed by the declarative mood. As for a noun, considering the meaning which it has in the sentence, and not as standing by itself, it is most conveniently regarded as a portion of a symbol. Thus the sentence, "every man loves a woman" is equivalent to "whatever is a man loves something that is a woman." Here "whatever" is a universal selective index, "is a man" is a symbol, "loves" is a symbol, "something that" is a particular selective index, and "is a woman" is a symbol. . . .

Peirce: CP 2.297 Cross-Ref:††

297. The word *Symbol* has so many meanings that it would be an injury to the language to add a new one. I do not think that the signification I attach to it, that of a conventional sign, or one depending upon habit (acquired or inborn), is so much a new meaning as a return to the original meaning. Etymologically, it should mean a thing thrown together, just as {embolon} (embolum) is a thing thrown into something, a bolt, and {parabolon} (parabolum) is a thing thrown besides, collateral security, and {hypobolon} (hypobolum) is a thing thrown underneath, an antenuptial gift. It is usually said that in the word *symbol* the throwing together is to be understood in the sense of "to conjecture"; but were that the case, we ought to find that *sometimes* at least it meant a conjecture, a meaning for which literature may be searched in vain. But the Greeks used "throw together" ({symballein}) very frequently to signify the making of a contract or convention. Now, we do find symbol ({symbolon}) early and often used to mean a convention or contract. Aristotle calls a noun a "symbol," that is, a conventional sign.†1 In Greek, watch-fire is a "symbol," that is, a signal agreed upon; a standard or ensign is a "symbol," a watchword is a "symbol," a badge is a "symbol"; a church creed is called a "symbol," because it serves as a badge or shibboleth; a theatre ticket is called a "symbol"; any ticket or check entitling one to receive anything is a "symbol." Moreover, any expression of sentiment was called a "symbol." Such were the principal meanings of the word in the original language. The reader will judge whether they suffice to establish my claim that I am not seriously wrenching the word in employing it as I propose to do.

Peirce: CP 2.298 Cross-Ref:††

298. Any ordinary word, as "give," "bird," "marriage," is an example of a symbol. It is *applicable to whatever may be found to realize the idea connected with the word*; it does not, in itself, identify those things. It does not show us a bird, nor enact before our eyes a giving or a marriage, but supposes that we are able to imagine those things, and have associated the word with them.

Peirce: CP 2.299 Cross-Ref:††

299. A regular progression of one, two, three may be remarked in the three orders of signs, Icon, Index, Symbol. The Icon has no dynamical connection with the

object it represents; it simply happens that its qualities resemble those of that object, and excite analogous sensations in the mind for which it is a likeness. But it really stands unconnected with them. The index is physically connected with its object; they make an organic pair, but the interpreting mind has nothing to do with this connection, except remarking it, after it is established. The symbol is connected with its object by virtue of the idea of the symbol-using mind, without which no such connection would exist.

Peirce: CP 2.300 Cross-Ref:††

300. Every physical force reacts between a pair of particles, either of which may serve as an index of the other. On the other hand, we shall find that every intellectual operation involves a triad of symbols.

Peirce: CP 2.301 Cross-Ref:††

301. A symbol, as we have seen, cannot indicate any particular thing; it denotes a kind of thing. Not only that, but it is itself a kind and not a single thing. You can write down the word "star," but that does not make you the creator of the word, nor if you erase it have you destroyed the word. The word lives in the minds of those who use it. Even if they are all asleep, it exists in their memory. So we may admit, if there be reason to do so, that generals are mere words without at all saying, as Ockham †1 supposed, that they are really individuals.

Peirce: CP 2.302 Cross-Ref:††

302. Symbols grow. They come into being by development out of other signs, particularly from icons, or from mixed signs partaking of the nature of icons and symbols. We think only in signs. These mental signs are of mixed nature; the symbol-parts of them are called concepts. If a man makes a new symbol, it is by thoughts involving concepts. So it is only out of symbols that a new symbol can grow. *Omne symbolum de symbolo.* A symbol, once in being, spreads among the peoples. In use and in experience, its meaning grows. Such words as *force, law, wealth, marriage*, bear for us very different meanings from those they bore to our barbarous ancestors. The symbol may, with Emerson's sphynx, say to man,

Of thine eye I am eyebeam.

Peirce: CP 2.303 Cross-Ref:††

§4. SIGN †2

303. Anything which determines something else (its *interpretant*) to refer to an object to which itself refers (its *object*) in the same way, the interpretant becoming in turn a sign, and so on *ad infinitum*.

Peirce: CP 2.303 Cross-Ref:††

No doubt, intelligent consciousness must enter into the series. If the series of successive interpretants comes to an end, the sign is thereby rendered imperfect, at least. If, an interpretant idea having been determined in an individual consciousness, it determines no outward sign, but that consciousness becomes annihilated, or otherwise loses all memory or other significant effect of the sign, it becomes absolutely undiscoverable that there ever was such an idea in that consciousness; and in that case it is difficult to see how it could have any meaning to say that that

consciousness ever had the idea, since the saying so would be an interpretant of that idea.

Peirce: CP 2.304 Cross-Ref:††

304. A sign is either an *icon*, an *index*, or a *symbol*. An *icon* is a sign which would possess the character which renders it significant, even though its object had no existence; such as a lead-pencil streak as representing a geometrical line. An *index* is a sign which would, at once, lose the character which makes it a sign if its object were removed, but would not lose that character if there were no interpretant. Such, for instance, is a piece of mould with a bullet-hole in it as sign of a shot; for without the shot there would have been no hole; but there is a hole there, whether anybody has the sense to attribute it to a shot or not. A *symbol* is a sign which would lose the character which renders it a sign if there were no interpretant. Such is any utterance of speech which signifies what it does only by virtue of its being understood to have that signification.

Peirce: CP 2.305 Cross-Ref:††

§5. INDEX †1

305. A sign, or representation, which refers to its object not so much because of any similarity or analogy with it, nor because it is associated with general characters which that object happens to possess, as because it is in dynamical (including spatial) connection both with the individual object, on the one hand, and with the senses or memory of the person for whom it serves as a sign, on the other hand.

Peirce: CP 2.305 Cross-Ref:††

No matter of fact can be stated without the use of some sign serving as an index. If *A* says to *B*, "There is a fire," *B* will ask, "Where?" Thereupon *A* is forced to resort to an index, even if he only means somewhere in the real universe, past and future. Otherwise, he has only said that there is such an idea as fire, which would give no information, since unless it were known already, the word "fire" would be unintelligible. If *A* points his finger to the fire, his finger is dynamically connected with the fire, as much as if a self-acting fire-alarm had directly turned it in that direction; while it also forces the eyes of *B* to turn that way, his attention to be riveted upon it, and his understanding to recognize that his question is answered. If *A*'s reply is, "Within a thousand yards of here," the word "here" is an index; for it has precisely the same force as if he had pointed energetically to the ground between him and *B*. Moreover, the word "yard," though it stands for an object of a general class, is indirectly indexical, since the yard-sticks themselves are signs of the Parliamentary Standard, and that, not because they have similar qualities, for all the pertinent properties of a small bar are, as far as we can perceive, the same as those of a large one, but because each of them has been, actually or virtually, carried to the prototype and subjected to certain dynamical operations, while the associational compulsion calls up in our minds, when we see one of them, various experiences, and brings us to regard them as related to something fixed in length, though we may not have reflected that that standard is a material bar. The above considerations might lead the reader to suppose that indices have exclusive reference to objects of experience, and that there would be no use for them in pure mathematics, dealing, as it does, with ideal

creations, without regard to whether they are anywhere realized or not. But the imaginary constructions of the mathematician, and even dreams, so far approximate to reality as to have a certain degree of fixity, in consequence of which they can be recognized and identified as individuals. In short, there is a degenerate form of observation which is directed to the creations of our own minds--using the word observation in its full sense as implying some degree of fixity and quasi-reality in the object to which it endeavours to conform. Accordingly, we find that indices are absolutely indispensable in mathematics; and until this truth was comprehended, all efforts to reduce to rule the logic of triadic and higher relations failed; while as soon as it was once grasped the problem was solved. The ordinary letters of algebra that present no peculiarities are indices. So also are the letters **A**, **B**, **C**, etc., attached to a geometrical figure. Lawyers and others who have to state a complicated affair with precision have recourse to letters to distinguish individuals. Letters so used are merely improved relative pronouns. Thus, while demonstrative and personal pronouns are, as ordinarily used, "genuine indices," relative pronouns are "degenerate indices"; for though they may, accidentally and indirectly, refer to existing things, they directly refer, and need only refer, to the images in the mind which previous words have created.

Peirce: CP 2.306 Cross-Ref:††

306. Indices may be distinguished from other signs, or representations, by three characteristic marks: first, that they have no significant resemblance to their objects; second, that they refer to individuals, single units, single collections of units, or single continua; third, that they direct the attention to their objects by blind compulsion. But it would be difficult if not impossible, to instance an absolutely pure index, or to find any sign absolutely devoid of the indexical quality. Psychologically, the action of indices depends upon association by contiguity, and not upon association by resemblance or upon intellectual operations. See 1.558.

Peirce: CP 2.307 Cross-Ref:††
§6. SYMBOL †1

307. A Sign (q.v.) which is constituted a sign merely or mainly by the fact that it is used and understood as such, whether the habit is natural or conventional, and without regard to the motives which originally governed its selection.

Peirce: CP 2.307 Cross-Ref:††

{Symbolon} is used in this sense by Aristotle several times in the *Peri hermeneias*, in the *Sophistici Elenchi*, and elsewhere.

Peirce: CP 2.308 Cross-Ref:††

308. THEMA:†2 A word proposed in 1635 by Burgersdicius [Burgersdyk] in his *Logic* (I., ii., §1), for that "quod intellectui cognoscendum proponi potest"; but what he seems to mean is what Aristotle sometimes vaguely expresses by {logos}, the immediate object of a thought, a meaning.

Peirce: CP 2.308 Cross-Ref:††

It is of the nature of a sign, and in particular of a sign which is rendered significant by a character which lies in the fact that it will be interpreted as a sign. Of course, nothing is a sign unless it is interpreted as a sign; but the character which

causes it to be interpreted as referring to its object may be one which might belong to it irrespective of its object and though that object had never existed, or it may be in a relation to its object which it would have just the same whether it were interpreted as a sign or not. But the *thema* of Burgersdicius seems to be a sign which, like a word, is connected with its object by a convention that it shall be so understood, or else by a natural instinct or intellectual act which takes it as a representative of its object without any action necessarily taking place which could establish a factual connection between sign and object. If this was the meaning of Burgersdicius, his *thema* is the same as the present writer's "symbol." (See Sign.)

Peirce: CP 2.309 Cross-Ref:††
CHAPTER 4

PROPOSITIONS†1

§1. THE CHARACTERISTICS OF DICISIGNS

309. Of the three classes of the [third] trichotomy of representamens--the simple or substitutive signs, or *sumisigns* [*rhemes*]; the double or informational signs, quasi-propositions, or *dicisigns*; the triple or rationally persuasive signs, or *arguments*, or *suadisigns*--the one whose nature is, by all odds, the easiest to comprehend, is the second, that of quasi-propositions, despite the fact that the question of the essential nature of the "judgment" is today quite the most vexed of all questions of logic. The truth is that *all* these classes are of very intricate natures; but the problem of the day is needlessly complicated by the attention of most logicians, instead of extending to propositions in general, being confined to "judgments," or acts of mental acceptance of propositions, which not only involve characters, additional to those of propositions in general--characters required to differentiate them as propositions of a particular kind--but which further involve, beside the mental proposition itself, the peculiar act of assent. The problem is difficult enough, when we merely seek to analyze the essential nature of the *Dicisign*, in general, that is, the kind of sign that *conveys* information, in contradistinction to a sign [such as an icon] from which information may be derived.†P1

Peirce: CP 2.310 Cross-Ref:††

310. The readiest characteristic test showing whether a sign is a Dicisign or not is that a Dicisign is either true or false, but does not directly furnish reasons for its being so. This shows that a Dicisign must profess to refer or relate to something as having a real being independently of the representation of it as such, and further that this reference or relation must not be shown as rational, but must appear as a blind Secondness. But the only kind of sign whose object is necessarily existent is the genuine Index. This Index might, indeed, be a part of a Symbol; but in that case the relation would appear as rational. Consequently a Dicisign necessarily represents itself to be a genuine Index, and to be nothing more. At this point let us discard all

other considerations, and see what sort of sign a sign must be that in any way represents itself to be a genuine Index of its Object, and nothing more. Substituting for "represents to be" a clearer interpretation, the statement is that the Dicisign's Interpretant represents an identity of the Dicisign with a genuine Index of the Dicisign's real Object. That is, the Interpretant represents a real existential relation or genuine Secondness, as subsisting between the Dicisign and its real Object. But the Interpretant of a Sign can represent no other Object than that of the Sign itself. Hence this same existential relation must be an Object of the Dicisign, if the latter have any real Object. This represented existential relation, in being an Object of the Dicisign, makes that real Object, which is correlate of this relation, also an Object of the Dicisign.

Peirce: CP 2.311 Cross-Ref:††

311. This latter Object may be distinguished as the *Primary Object*, the other being termed the *Secondary Object*. The Dicisign in so far as it is the relate of the existential relation which is the Secondary Object of the Dicisign, can evidently not be the entire Dicisign. It is at once a part of the Object and a part of the Interpretant of the Dicisign. Since the Dicisign is represented in its Interpretant to be an Index of a complexus as such, it must be represented in that same Interpretant to be composed of two parts, corresponding respectively to its Object and to itself [the Dicisign]. That is to say, in order to understand the Dicisign, it must be regarded as composed of two such parts whether it be in itself so composed or not. It is difficult to see how this can be, unless it really have two such parts; but perhaps this may be possible. Let us consider these two represented parts separately. The part which is represented to represent the Primary Object, since the Dicisign is represented to be an Index of its Object, must be represented as an Index, or some representamen of an Index, of the Primary Object. The part which is represented to represent a part of the Dicisign is represented as at once part of the Interpretant and part of the Object. It must, therefore, be represented as such a sort of Representamen (or to represent such a sort), as can have its Object and its Interpretant the same. Now, a Symbol cannot even have itself as its Object; for it is a law governing its Object. For example, if I say, "This proposition conveys information about itself," or "Let the term 'sphinx' be a general term to denote anything of the nature of a symbol that is applicable to every 'sphinx' and to nothing else," I shall talk unadulterated nonsense. But a Representamen mediates between its Interpretant and its Object, and that which cannot be the object of the Representamen cannot be the Object of the Interpretant. Hence, *a fortiori*, it is impossible that a Symbol should have its Object as its Interpretant. An Index can very well represent itself. Thus, every number has a double; and thus the entire collection of even numbers is an Index of the entire collection of numbers, and so this collection of even numbers contains an Index of itself. But it is impossible for an Index to be its own Interpretant, since an Index is nothing but an individual existence in a Secondness with something; and it only becomes an Index by being capable of being represented by some Representamen as being in that relation. Could this Interpretant be itself there would be no difference between an Index and a Second. An Icon, however, is strictly a possibility involving a possibility, and thus the possibility of its being represented as a possibility is the possibility of the involved possibility. In this kind of Representamen alone, then, the Interpretant may be the Object. Consequently, that constituent of the Dicisign, which is represented in the Interpretant as being a part of the Object, must be represented by an Icon or by a Representamen of an Icon. The Dicisign, as it must be understood in order to be understood at all, must contain those two parts. But the Dicisign is represented to be an Index of the Object, in that the latter involves something corresponding to these parts; and it is this Secondness that

the Dicisign is represented to be the Index of. Hence the Dicisign must exhibit a connection between these parts of itself, and must represent this connection to correspond to a connection in the object between the Secundal Primary Object [*i.e.*, the primary object so far as it is dyadic in structure] and the Firstness [or quality of the primary object] indicated by the part [of the Secundal Primary Object] corresponding to the Dicisign.

Peirce: CP 2.312 Cross-Ref:††

312. We conclude, then, that, if we have succeeded in threading our way through the maze of these abstractions, a Dicisign, defined as a Representamen whose Interpretant represents it as an Index of its Object, must have the following characters:

Peirce: CP 2.312 Cross-Ref:††

First: It must, in order to be understood, be considered as containing two parts. Of these, the one, *which may be called the Subject*, is or represents an Index of a Second existing independently of its being represented, while the other, *which may be called the Predicate*, is or represents an Icon of a Firstness [or quality or essence]. Second: These two parts must be represented as connected; and that in such a way that if the Dicisign has any Object, it [the Dicisign] must be an Index of a Secondness subsisting between the Real Object represented in one represented part of the Dicisign to be indicated and a Firstness represented in the other represented part of the Dicisign to be Iconized.

Peirce: CP 2.313 Cross-Ref:††

313. Let us now examine whether these conclusions, together with the assumption from which they proceed, hold good of all signs which profess to convey information without furnishing any rational persuasion of it; and whether they fail alike for all signs which do not convey information as well as for all those which furnish evidence of the truth of their information, or reasons for believing it. If our analysis sustains these tests, we may infer that the definition of the Dicisign on which they are founded holding, at least within the sphere of signs, is presumably sound beyond that sphere.

Peirce: CP 2.314 Cross-Ref:††

314. Our definition forbids an Icon to be a Dicisign, since the proper Interpretant of an Icon cannot represent it to be an Index, the Index being essentially more complicated than the Icon. There ought, therefore, to be no informational signs among Icons. We find that, in fact, Icons may be of the greatest service in obtaining information--in geometry, for example--but still, it is true that an Icon cannot, of itself, convey information, since its Object is whatever there may be which is like the Icon, and is its Object in the measure in which it is like the Icon.

Peirce: CP 2.315 Cross-Ref:††
§2. SUBJECTS AND PREDICATES

315. All propositions are informational Symbols. Our conclusions do not prevent Dicisigns from being Symbols; but let us begin by examining whether or not our definition and conclusions apply to ordinary propositions. To fix our ideas let us set down the proposition "Tully has a wart on his nose." That is a proposition whether

it be true or not, whether anybody asserts it or not, and whether anybody assents to it or not. For an act of assertion supposes that, a proposition being formulated, a person performs an act which renders him liable to the penalties of the social law (or, at any rate, those of the moral law) in case it should not be true, unless he has a definite and sufficient excuse; and an act of assent is an act of the mind by which one endeavors to impress the meanings of the proposition upon his disposition, so that it shall govern his conduct, including thought under conduct, this habit being ready to be broken in case reasons should appear for breaking it. Now in performing either of these acts, the proposition is recognized as being a proposition whether the act be performed or not. Nor can a sound objection be grounded on the fact that a proposition is always understood as something that *might be* assented to and asserted. †P1 For our definition of the Dicisign more than recognizes the truth of that in stating that (supposing the proposition to be a Dicisign) the Interpretant of it (that is, the mental representation, or thought, which it tends to determine), represents the proposition to be a genuine Index of a Real Object, independent of the representation. For an Index involves the existence of its Object. The definition [of the Dicisign] adds that this Object is a Secondness or real fact. That this is true of ordinary "ampliative" propositions, namely, that what they mean to represent is a fact, is beyond question. But as regards explicative propositions, and especially definitions, it may be doubted. If a definition is to be understood as introducing the definitum, so that it means "Let so and so--the definitum--mean so and so--the definition," then it is a proposition in the imperative mood, and consequently, not a proposition; for a proposition is equivalent to a sentence in the indicative mood. The definition is thus only a proposition if the definitum be already known to the interpreter. But in that case it clearly conveys information as to the character of this definitum, which is matter of fact. But take an "analytical," *i.e.*, an explicative proposition; and to begin with, take the formula "A is A." If this be intended to state anything about real things, it is quite unintelligible. It must be understood to mean something about symbols; no doubt, that the substantive verb "is" expresses one of those relations that everything bears to itself, like "loves whatever may be loved by." So understood, it conveys information about a symbol. A symbol is not an individual, it is true. But any information about a symbol is information about every replica of it; and a replica is strictly an individual. What information, then, does the proposition "A is A" furnish concerning this replica? The information is that if the replica be modified so as to bring the same name before it and after it, then the result will be a replica of a proposition which will never be in conflict with any fact. To say that something never will be is not to state any real fact, and until some experience occurs--whether outward experience, or experience of fancies--which might be an occasion for a conflict with the proposition in question, it does not, to our knowledge, represent any actual Secondness. But as soon as such an occasion does arise, the proposition relates to the single replica that then occurs and to the single experience, and describes the relation between them. Similar remarks apply to every explicative proposition. The proposition "Every phoenix, in rising from its ashes, sings 'Yankee Doodle,'" will be, we may be confident, not in conflict with any experience. If so, it is perfectly true. "Every four-sided triangle is deep blue," is necessarily true, since it is impossible that any experience should conflict with it. †1 But both propositions are meaningless. Equally meaningless is any explicative proposition that is true, unless it be regarded as a proposition about a certain kind of symbol of which a replica actually occurs. If "Man is a biped" be allowed to be an explicative proposition, it means nothing unless there be an occasion in which the name "man" may be applied. If there be such an occasion, in regard to that existential individual event, it is said that the term "biped" may be applied to it. That is, on an

occasion on which the word "biped" is applied, the result will never be in conflict with any experience, real or imaginary. Thus every kind of proposition is either meaningless or has a real Secondness as its object. This is a fact that every reader of philosophy should constantly bear in mind, translating every abstractly expressed proposition into its precise meaning in reference to an individual experience. The system of existential graphs,†1 which is capable of expressing every proposition as analytically as may be desired, expresses an assertion by actually attaching an individual replica to the individual sheet, and such possible attachment is precisely what the Interpretant of a proposition represents before the proposition is asserted.

Peirce: CP 2.316 Cross-Ref:††

316. Let us now proceed to compare the conclusions from the abstract definition of a Dicisign with the facts about propositions. The first conclusion is that every proposition contains a *Subject* and a *Predicate*, the former representing (or being) an Index of the Primary Object, or Correlate of the relation represented, the latter representing (or being) an Icon of the Dicisign in some respect. Before inquiring whether every proposition has such parts, let us see whether the descriptions given of them are accurate, when there are such parts. The proposition "Cain kills Abel" has two subjects "Cain" and "Abel" and relates as much to the real Objects of one of these as to that of the other. But it may be regarded as primarily relating to the Dyad composed of Cain, as first, and of Abel, as second member. This Pair is a single individual object having this relation to Cain and to Abel, that its existence *consists* in the existence of Cain and in the existence of Abel and in nothing more. The Pair, though its existence thus depends on Cain's existence and on Abel's, is, nevertheless, just as truly existent as they severally are. The *Dyad* is not precisely the Pair. The Dyad is a mental Diagram consisting of two images of two objects, one existentially connected with one member of the pair, the other with the other; the one having attached to it, as representing it, a Symbol whose meaning is "First," and the other a Symbol whose meaning is "Second." Thus, this diagram, the Dyad, represents Indices of Cain and Abel, respectively; and thus the subject conforms to our conclusion. Next consider the subject of this proposition, "Every man is the son of two parents." This supposes a mental diagram of a pair labelled "First" and "Second," as before (or rather by symbols equivalent to these for the special purpose) but instead of the two units of the Diagram being directly considered as Indices of two existent individuals, the Interpretant of the diagram represents, that if the interpreter of the whole proposition by an act of the mind actually attaches one of the units of the diagram to any individual man, there will be an existent relation attaching the other unit to a certain pair of individuals of which, if the interpreter of the whole proposition attaches one of them specially to that unit, then the predicate will be true of that individual Dyad in the order of its members. Of course, it is not meant that the person who sufficiently understands the diagram actually goes through this elaborate process of thought, but only that this is substantially what has to be done, completely and accurately to understand the proposition. The graph of the proposition will afford help in seeing that this is so. Here, as before, the Subject represents the individual Dyad, of which the proposition is the Symbol, to be represented by an Index. If the proposition has an abstract subject, as "Redness" or "Justice," it may either be treated, after the style of the scholastics, as an *exponible*, that is, as a proposition whose real construction is disguised by a grammatical trope; or, if this does not afford the true interpretation, the proposition discourses of a universe comprising one replica each of a collection of possible symbols, somewhat indefinite, but embracing all that need be considered. We cannot say "all that are pertinent," since no collection could exhaust the possible pertinent symbols. In the case of a *conditional* proposition,†P1 "If it freezes tonight,

your roses will be killed," the meaning is that any replica of the proposition "It will freeze tonight" which may be true, coexists with a true replica of the proposition "your roses will be killed." This involves a representation of an Index just as much as does the subject of the proposition "Every rose will be killed."

Peirce: CP 2.317 Cross-Ref:††

317. Passing now to the consideration of the predicate, it is plain enough that the last proposition, or any at all like it, only conveys its signification by exciting in the mind some image or, as it were, a composite photograph of images, like the Firstness meant. This, however, does not squarely meet the question, which is not what our mental constitution causes to happen, but how the predicate represents the Firstness that it signifies.†P1 The predicate is necessarily an *Iconic Sumisign* [Rheme] (which is not always true of the subject) and as such, as we should find by a full analysis of the Sumisign, essentially signifies what it does by representing itself to represent an Icon of it. Without an analysis of the Sumisign this point must remain a little obscure.

Peirce: CP 2.318 Cross-Ref:††

318. We next come to the question whether every proposition has a Subject and a Predicate. It has been shown above that this is true of a Conditional; and it is easily seen that it is equally so of any Disjunctive. Only, an ordinary Disjunctive has such a construction that one mode of analysis of it is as good as another. That is, to say, "Either A or B is true," may equally be regarded as saying, "A replica of a Symbol is true which is not true if no replica of A is true and no replica of B is true," or as saying, "If a replica of A is not true, a replica of B is true," or as saying, "If a replica of B is not true, a replica of A is true." These come to the same thing, just as "Some X is Y," "Some Y is X," and "Something is both X and Y" come to the same thing. The most perfectly thorough analysis throws the whole substance of the Dicisign into the Predicate. A copulative proposition even more obviously has a Subject and Predicate. It predicates the genuinely Triadic relation of *tricoexistence*, "P and Q and R coexist." For to say that both A and B is true is to say that something exists which *tricoexists* with true replicas of A and B. Some logical writers are so remarkably biased or dense as to adduce the Latin sentences *fulget* and *lucet* as propositions without any subject. But who cannot see that these words convey no information at all without a reference (which will usually be Indexical, the Index being the common environment of the interlocutors) to the circumstances under which the Firstnesses they signify are asserted to take place?

Peirce: CP 2.319 Cross-Ref:††

319. The proposition should have an actual *Syntax*, which is represented to be the Index of those elements of the fact represented that correspond to the Subject and Predicate. This is apparent in all propositions. Since Abelard it has been usual to make this Syntax a third part of the proposition, under the name of the Copula. The historical cause of the emergence of this conception in the twelfth century was, of course, that the Latin of that day did not permit the omission of the verb *est*, which was familiarly, though not invariably, omitted in Greek, and not very uncommonly in classical Latin. In most languages there is no such verb. But it is plain that one does not escape the need of a Syntax by regarding the Copula as a third part of the proposition; and it is simpler to say that it is merely the accidental form that Syntax may take.

Peirce: CP 2.320 Cross-Ref:††

320. It has thus been sufficiently shown that all propositions conform to the

definition of the Dicisign and to the corollaries drawn from that definition. A proposition is, in short, a Dicisign that is a Symbol. But an Index, likewise, may be a Dicisign. A man's portrait with a man's name written under it is strictly a proposition, although its syntax is not that of speech, and although the portrait itself not only represents, but is, a Hypoicon. But the proper name so nearly approximates to the nature of an Index, that this might suffice to give an idea of an informational Index. A better example is a photograph. The mere print does not, in itself, convey any information. But the fact, that it is virtually a section of rays projected from an object *otherwise known*, renders it a Dicisign. Every Dicisign, as the system of Existential Graphs fully recognizes, is a further determination of an already known sign of the same object. It is not, perhaps, sufficiently brought out in the present analysis. It will be remarked that this connection of the print, which is the quasi-predicate of the photograph, with the section of the rays, which is the quasi-subject, is the Syntax of the Dicisign; and like the Syntax of the proposition, it is a *fact* concerning the Dicisign considered as a First, that is, in itself, irrespective of its being a sign. Every informational sign thus involves a Fact, which is its Syntax. It is quite evident, then, that Indexical Dicisigns equally accord with the definition and the corollaries.

Peirce: CP 2.321 Cross-Ref:††

321. It will be remarked that this accord, both for propositions and for informational indices, is quite irrespective of their being asserted or assented to. Now in analyses hitherto proposed, it seems to have been thought that if assertion, or at any rate, assent, were omitted, the proposition would be indistinguishable from a compound general term--that "A man is tall" would then reduce to "A tall man." It therefore becomes important to inquire whether the definition of a Dicisign here found to be applicable to the former (even though it be not "judged"), may not be equally applicable to the latter. The answer, however, comes forthwith. Fully to understand and assimilate the symbol "a tall man," it is by no means requisite to understand it to relate, or to profess to relate, to a real Object. Its Interpretant, therefore, does not represent it as a genuine Index; so that the definition of the Dicisign does not apply to it. It is impossible here fully to go into the examination of whether the analysis given does justice to the distinction between propositions and arguments. But it is easy to see that the proposition purports to intend to compel its Interpretant to refer to its real Object, that is represents itself as an Index, while the argument purports to intend not compulsion but action by means of comprehensible generals, that is, represents its character to be specially symbolic.

Peirce: CP 2.322 Cross-Ref:††

322. The above is the best analysis the author can, at present, make of the Dicisign. However satisfactory the main points of it may appear, it is not likely, on general principles, to stand without more or less amendment, though it would seem as if it could not but be pretty near to the truth. It is doubtful whether it applies fully to all kinds of propositions. This definition of the Dicisign will naturally lead one to guess that a Sumisign is any Representamen of which the Interpretant represents it as an Icon; and that the Argument or Suadisign is a Representamen of which the Interpretant represents it as a Symbol. Close examination encourages the student to believe that this is something like the truth, but so far as it has been carried, excites doubt whether this be the whole story. . . .

Peirce: CP 2.323 Cross-Ref:††

§3. DICHOTOMIES OF PROPOSITIONS

323. Indexical Dicisigns seem to have no important varieties; but propositions are divisible, generally by dichotomy primarily in various ways. In the first place, according to *Modality* or *Mode*, a proposition is either *de inesse* (the phrase used in the *Summulae*†P1 [p. 71B]) or *modal*. A proposition *de inesse* contemplates only the existing state of things--existing, that is, in the logical universe of discourse.†1 A modal proposition takes account of a whole range of possibility. According as it asserts something to be true or false throughout the whole range of possibility, it is *necessary* or *impossible*. According as it asserts something to be true or false within the range of possibility (not expressly including or excluding the existent state of things), it is *possible* or *contingent*. (The terms are all from Boëthius.)

Peirce: CP 2.324 Cross-Ref:††

324. A subject of a proposition is either *Singular*, *General*, or *Abstract*. It is singular if it indicates an otherwise known individual. It is general if it describes how an individual intended is to be selected. A general subject is (as commonly recognized) either *Universal* or *Particular* (and *Indefinite*). (These last three terms are found in Apuleius,†2 of Nero's time. But a senseless distinction between the indefinite and the particular is by the present writer unnoticed.) There is a complicated doctrine in the books as to the meaning of these terms, some kinds of universals [being taken as] asserting the existence of their subjects. The present writer makes all universals alike in not doing so. Then a *Universal* subject is one which indicates that the proposition applies to whatever individual there is in the universe or to whatever there *may be* of a general description without saying that there is any. A *Particular* subject is one which does not indicate what individual is intended, further than to give a general description of it, but does profess to indicate an existent individual at least. The order in which Universal and Particular subjects occur is material. Thus, "Some woman is adored by whatever Spaniard may exist," has its first subject "Some woman" particular, and its second "whatever Spaniard may exist" universal. But "Whatever Spaniard may exist adores some woman" has the same subjects in reverse order and so has a different meaning. It is quite conceivable that a subject should be so described as to be neither Universal nor Particular; as in *exceptives* (*Summulae*) as "Every man but one is a sinner." The same may be said of all kinds of numerical propositions, as "Any insect has an even number of legs." But these may be regarded as Particular Collective Subjects. An example of a Universal Collective subject would be "Any two persons shut up together will quarrel." A collection is logically an individual. The distinction of Universal and Particular subjects is material, not merely formal; and it seems to be (and was regarded in the middle ages as being) of essentially the same nature as the distinction of Necessary and Possible propositions.

Peirce: CP 2.325 Cross-Ref:††

325. The distinction of *Hypothetical*, *Categorical*, and *Relative* propositions is also important. At any rate, the last has some important differences from the others.

Peirce: CP 2.326 Cross-Ref:††

326. The distinction between *Affirmative* and *Negative* propositions, as applied to ordinary categorical propositions, is purely a matter of form. A process called *infinitation* (used by Abelard, *Opera hactenus Inedita*, p. 225, and constantly ever since in all Western languages to this day), consisting in prefixing *non-* to a term, converts the proposition from a negative to an *affirmative* or so-called *Infinite*

proposition. The difference between a negative and an infinite proposition is no more than that in Latin one may say *non est* or *est non*, without difference of meaning. "Socrates non est mortalis" is the usual form; but "Socrates est non mortalis" can equally be said. It must be remembered that logic has attracted to its study some of the most puerile of writers, and still continues to do so in some measure.

Peirce: CP 2.327 Cross-Ref:††

327. Finally, every proposition is either *true* or *false*. It is false if any proposition could be legitimately deduced from it, without any aid from false propositions, which would conflict with a direct perceptual judgment, could such be had. A proposition is true, if it is not false. Hence, an entirely meaningless form of proposition, if it be called a proposition, at all, is to be classed along with true propositions.

Peirce: CP 2.328 Cross-Ref:††

§4. A PRAGMATIC INTERPRETATION OF THE LOGICAL SUBJECT

328. Any symbol which may be a direct constituent of a proposition is called a *term* (*terminus*, Boëthius).†1 The logicians usually say that a categorical proposition has "two terms," its *subject* and its *predicate*, wherein, by a carelessness of expression, or by copying Aristotle,†1 they stumble upon the truth. Their usual *doctrine* is (though often not directly stated in one sentence), that such a proposition has three terms, the subject, predicate, and *copula* (Abelard).†2 The correct designation of the subject and predicate, in accord with their doctrine, is the *extremes*, which is translated from the same Greek word as *term* ({horos}). The ordinary doctrine makes the copula the only verb, and all other terms to be either proper names or general class-names. The present author leaves the *is* as an inseparable part of the class-name; because this gives the simplest and most satisfactory account of the proposition.†3 It happens to be true that in the overwhelming majority of languages there are no general class names and adjectives that are not conceived as parts of some verb (even when there really is no such verb) and consequently nothing like a copula is required in forming sentences in such languages. The author (though with no pretension to being a linguist), has fumbled the grammars of many languages in the search for a language constructed at all in the way in which the logicians go out of their way to teach that all men think (for even if they do so, that has really nothing to do with logic). The only such tongue that he has succeeded in finding is the Basque, which seems to have but two or three verbs, all the other principal words being conceived as nouns. Every language must have proper names; and there is no verb wrapped up in a proper name. Therefore, there would seem to be a direct suggestion there of a true common noun or adjective. But, notwithstanding that suggestion, almost every family of man thinks of general words as parts of verbs. This seems to refute the logicians' psychology.

Peirce: CP 2.329 Cross-Ref:††

329. A proper name, when one meets with it for the first time, is existentially connected with some percept or other equivalent individual knowledge of the individual it names. It is *then*, and then only, a genuine Index. The next time one meets with it, one regards it as an Icon of that Index. The habitual acquaintance with it having been acquired, it becomes a Symbol whose Interpretant represents it as an

Icon of an Index of the Individual named.

Peirce: CP 2.330 Cross-Ref:††

330. If you look into a textbook of chemistry for a definition of *lithium*, you may be told that it is that element whose atomic weight is 7 very nearly. But if the author has a more logical mind he will tell you that if you search among minerals that are vitreous, translucent, grey or white, very hard, brittle, and insoluble, for one which imparts a crimson tinge to an unglowing flame, this mineral being triturated with lime or witherite rats-bane, and then fused, can be partly dissolved in muriatic acid; and if this solution be evaporated, and the residue be extracted with sulphuric acid, and duly purified, it can be converted by ordinary methods into a chloride, which being obtained in the solid state, fused, and electrolyzed with half a dozen powerful cells, will yield a globule of a pinkish silvery metal that will float on gasolene; and the material of *that* is a specimen of lithium. The peculiarity of this definition--or rather this precept that is more serviceable than a definition--is that it tells you what the word lithium denotes by prescribing what you are to *do* in order to gain a perceptual acquaintance with the object of the word. Every subject of a proposition, unless it is either an Index (like the environment of the interlocutors, or something attracting attention in that environment, as the pointing finger of the speaker) or a Sub-index (like a proper name, personal pronoun or demonstrative) must be a *Precept*, or Symbol, not only describing to the Interpreter what is to be done, by him or others or both, in order to obtain an Index of an individual (whether a unit or a single set of units) of which the proposition is represented as meant to be true, but also assigning a designation to that individual, or, if it is a set, to each single unit of the set. Until a better designation is found, such a term may be called a Precept. Thus, the Subject of the proposition, "Whatever Spaniard there may be adores some woman" may best be regarded as, "Take any individual, A, in the universe, and then there will be some individual, B, in the universe, such that A and B in this order form a dyad of which what follows is true," the Predicate being "-- is either not a Spaniard or else adores a woman that is --."

Peirce: CP 2.331 Cross-Ref:††

331. Any term fit to be the subject of a proposition may be termed an *Onome*. A Categorematic term (Duns Scotus, but probably earlier) is any term fit to be the subject or predicate of a proposition. A *Syncategorematic Term* or *Syncathegreuma* (*Summulae*)†1 is a Symbol going to make up a *Categorematic* Term. The Copula seems to fall between two stools, being neither categorematic nor syncategorematic. . . .

Peirce: CP 2.332 Cross-Ref:††
§5. THE NATURE OF ASSERTION †2

332. Let us now consider in what the essential nature of *assertion* consists. I can here only restate, though in an improved form, a doctrine of *grammatica speculativa* which I first published in 1867.†3 Since that date, as my philosophical studies have progressed, I have been led half a dozen times and more to call the doctrine into serious question and to submit it to a rigid and thorough reexamination. Each reexamination, while leading to some modification more or less important, has reinstated the impeached doctrine in my estimation. I believe that I can now make a

statement of it which shall leave little to be desired. At the same time, I will take occasion to acknowledge and explain the errors of my previous statements.

Peirce: CP 2.333 Cross-Ref:††

333. In such analysis of assertion there are two kinds of reasoning which we have to employ. On the one hand, we can directly observe what is familiar to our experience of assertions and seems to be inseparable from them. Professor Schröder calls this *rhetorical* evidence; and the designation is felicitous, because the reasoning in question has the characteristics of the inferences termed *rhetorical* by the old logicians. The term also harmonizes with my name of *speculative rhetoric* for the highest and most living branch of logic. To me personally, perhaps the designation gives that sort of satisfaction which so many schools have manifested in adopting appellations invented by their opponents as depreciative. For although Professor Schröder cannot but acknowledge the value and need of this kind of reasoning, a slight shade of disesteem seems to mingle with his approval on account of its undeniable formal imperfection. Now to me this very imperfection marks the reasoning as being drawn direct from those observational sources from whence all true reasoning must be drawn; and I have often remarked in the history of philosophy, that the reasonings which were somewhat dark and formally imperfect, often went the deepest. The other kind of reasoning which I employ in the analysis of assertion consists in deducing what the constituents of assertion must be from the theory, which I accept, that truth consists in the definitive compulsion of the investigating intelligence. This is systematical; but it is only half a method. For the deductions, or quasi-predictions, from theory having been made, it is requisite to turn to the rhetorical evidence and see whether or not they are verified by observation. If we find them to be so, not only does the analysis of assertion gain evidence of being completely rounded, but the theory of truth is rendered more probable.

Peirce: CP 2.334 Cross-Ref:††

334. In every assertion we may distinguish a speaker and a listener. The latter, it is true, need have only a problematical existence, as when during a shipwreck an account of the accident is sealed in a bottle and thrown upon the water. The problematical "listener" may be within the same person as the "speaker"; as when we mentally register a judgment, to be remembered later. If there be any act of judgment independent of any registry, and if it have any logical significance (which is disputable), we may say that in that case the listener becomes identical with the speaker.

Peirce: CP 2.335 Cross-Ref:††

335. The assertion consists in the furnishing of evidence by the speaker to the listener that the speaker believes something, that is, finds a certain idea to be definitively compulsory on a certain occasion. There ought, therefore, to be three parts in every assertion, a sign of the occasion of the compulsion, a sign of the enforced idea, and a sign evidential of the compulsion affecting the speaker in so far as he identifies himself with the scientific intelligence.

Peirce: CP 2.336 Cross-Ref:††

336. Because compulsion is essentially *hic et nunc*, the occasion of the compulsion can only be represented to the listener by compelling him to have experience of that same occasion. Hence it is requisite that there should be a kind of sign which shall act dynamically upon the hearer's attention and direct it to a special object or occasion. Such a sign I call an *Index*. It is true that there may, instead of a simple sign of this kind, be a precept describing how the listener is to act in order to

gain the occasion of experience to which the assertion relates. But since this precept tells him how he is to act, and since acting and being acted on are one and the same, and thus action is also *hic et nunc*, the precept must itself employ an Index or Indices. That to which the index directs attention may be called the subject of the assertion. . .

Peirce: CP 2.337 Cross-Ref:††

337. The real world cannot be distinguished from a fictitious world by any description. It has often been disputed whether Hamlet was mad or not. This exemplifies the necessity of *indicating* that the real world is meant, if it be meant. Now reality is altogether dynamic, not qualitative. It consists in forcefulness. Nothing but a dynamic sign can distinguish it from fiction. It is true that no language (so far as I know) has any particular form of speech to show that the real world is spoken of. But that is not necessary, since tones and looks are sufficient to show when the speaker is in earnest. These tones and looks act dynamically upon the listener, and cause him to attend to realities. They are, therefore, the indices of the real world. Thus, there remains no class of assertions which involve no indices unless it be logical analyses and identical propositions. But the former will be misunderstood and the latter taken as nonsensical, unless they are interpreted as referring to the world of terms or concepts; and this world, like a fictitious world, requires an index to distinguish it. It is, therefore, a fact, as theory had pronounced, that one index, at least, must form a part of every assertion.

Peirce: CP 2.338 Cross-Ref:††

338. I term those occasions or objects which are denoted by the indices the *subjects* of the assertion. But these will not coincide with the objects denoted by the grammatical subjects. It has always been the habit of logicians to consider propositions only (or chiefly) after they have been expressed in certain standard, or canonical, forms. To treat them just as they are expressed in this or that language (as Hoppe and some others do) makes of logic a philological, not a philosophical, study. But the canonical forms chosen have been suggested by the usage of a narrow class of languages, and are calculated to lead philosophy astray. That which is called the *subject* is the noun which is in the nominative, although, even in our relatively small family of Indo-European languages, there are several in which that noun which in Latin, Greek, and the modern European languages is put in the nominative, is put in an oblique case. Witness the Irish and Gaelic. Often, too, the index is not of the nature of a noun. It may be, as we have seen, a mere look or gesture. Then again it may be so disguised, that it is impossible to say with certainty whether it be an index, at all. It helps little to appeal to the meaning of the assertion; because it is in such cases difficult to say precisely what the meaning is. Thus, in the assertion, "All men are mortal," we may say that the subject is *every man*, or we may say that it is the collection *of men*, or that *every man and some mortal* are the two subjects, or that *everything* is the subject (the predicate being "is either not man or is mortal"), or that *everything* and *humanity* and *mortality* are the three subjects, or a hundred other dispositions. But if it is desired to adopt one constant canonical form, the best rule will be to use a separate index for everything which is indifferent from a logical point of view. That is, in this case to take *everything*, *humanity*, and *mortality* as the indices.

Peirce: CP 2.339 Cross-Ref:††

339. Every subject, when it is directly indicated, as *humanity* and *mortality* are, is singular. Otherwise, a precept, which may be called its *quantifier*, prescribes

how it is to be chosen out of a collection, called its *universe*. In probable logic, the quantifiers--such as "nine out of ten," and the like--refer to an experiential course or "long run." But in necessary logic there is no reference to such a course of experience, and only two quantifiers are required; the *universal* quantifier, which allows any object, no matter what, to be chosen from the universe, and the *particular* quantifier, which prescribes that a suitable object must be chosen. When there are several quantified subjects, and when quantifications are different, the order in which they are chosen is material. It is the character of the quantifier of the *last chosen* subject which extends itself to the whole proposition. (In former statements, this last point was not clear to me.) While no other quantifiers than those two are indispensable, much more than mere brevity and convenience of writing is gained by using also two other "hemilogical"†1 quantifiers, the one permitting any object of the universe *but one* to be taken, the other restricting the liberty to one or other of a suitable two. The universe of a logical subject has always hitherto been assumed to be a discrete collection, so that the subject is an *individual* object or occasion. But in truth a universe may be continuous, so that there is no part of it of which every thing must be either wholly true or wholly false. For example, it is impossible to find a part of a surface which must be all one color. Even a point of that surface may belong indifferently to three or more differently colored parts. But the logic of continuous universes awaits investigation. . . .

Peirce: CP 2.340 Cross-Ref:††

340. In 1867 I defined a symbol as any general representamen;†1 and so far I was right. But I immediately proceeded after the traditional manner, to divide symbols into *terms, propositions and argumentations*, with the meaning that "terms" have no assertoric element, and there I was wrong, although the division itself is not so much wrong as it is unimportant. Subsequently, noticing that I had classed natural symptoms both among indices and among symbols, I restricted symbols to conventional signs, which was another error. The truth is that my paper of 1867 was perhaps the least unsatisfactory, from a logical point of view, that I ever succeeded in producing; and for a long time most of the modifications I attempted of it only led me further wrong.

Peirce: CP 2.341 Cross-Ref:††

341. Every symbol, as involving an assertion, or rudimentary assertion, is general, in the sense in which we speak of a general sign. That is, the predicate is general. Even when we say "Boz was Charles Dickens," what we mean is that "Boz was the *same* as Charles Dickens," and *sameness* is a general, even a hemilogical, relation. For a predicate is of an ideal nature, and as such cannot be a mere hecceity. In fact in the proposition "Boz is Charles Dickens," the Subjects are Boz and Charles Dickens and the predicate is "*identical with*." On the other hand, every general sign, even a "term," involves, at least, a rudimentary assertion. For what is a "term," or "class-name," supposed to be? It is something which signifies, or, to use J. S. Mills' objectionable terminology, "connotes" certain characters, and thereby denotes whatever possesses those characters. That is, it draws the attention to an idea, or mental construction, or diagram, of something possessing those characters, and the possession of those characters is kept in the foreground of consciousness. What does that mean unless that the listener says to himself, "that which is *here* (before the attention) possesses such and such characters"? That may not be quite a *proposition*, or fully an assertion, because the object of attention being in this case nothing but a mental creation, the listener does not tell himself what it is that is "here." It is, at least, not an assertion about the real world. But none the less it contains the assertoric

element, the mental copula. When a listener hears the term "light," he proceeds to *create* in his mind an image thereof, and goes through the very same process of thought which is attributed to the Elohim in the first chapter of Genesis. "And God said, Let there be light; and there was light. And God saw the light, that it was good," --that is, that the light was, in fact, what was intended to be created. It amounted to saying "that is light"! Until this process is performed, the name excites no meaning in the mind of the listener. But I object to the triad, *term, proposition, inference*, if it be regarded as all-important in logic, on the ground that common nouns, which, with their equivalents, are what is meant by terms, are mere accidental grammatical forms which happen to be very prominent in the languages most familiar to us, but which hardly exist, or at least are far from prominent, in the vast majority of tongues, and are really not needed, at all, and ought to be unknown to the *Grammatica Speculativa*. It is absurd, indeed, to erect this unnecessary part of speech into a logical form and leave the indispensable prepositions unrepresented, merely because in Indo-European languages they often appear in the form of terminations.

Peirce: CP 2.342 Cross-Ref:††

342. At the same time, it must be admitted that the proposition, "Let *l* be light," or what is the same thing "*l* is light," where *l* is not otherwise defined, is only an assertion about a fleeting idea, far less developed than the proposition "Hamlet was mad," which relates to a great creation more enduring than bronze. Take away from any proposition its quantifying signs and such an expression is what remains. Remove the quantifier from the proposition "all men are mortal," or what is the same thing, "everything is either not a man or is mortal," and we have "x is either not a man or is mortal." Remove the quantifier from the proposition "Everything has some cause," or what is the same thing "Let A be anything; then there is something, B, such that B is the cause of A," and it becomes "B is the cause of A." Such rudimentary assertions--assertions in form with no substance--precisely express the meanings of logical *terms*. In that sense, we may say that every proposition has as many terms as it has quantified subjects. Singular subjects are of a different nature. Every term is singular but indefinite. It may be affirmative or negative, according to the character of its predicate.

Peirce: CP 2.343 Cross-Ref:††

343. The copula differs from the subjects and predicate in being purely formal, and containing no special matter or complexity. No doubt, this is because we choose so to draw the lines between the different parts of the proposition as leave the copula no matter; but then there are sound reasons for so drawing those lines.

Peirce: CP 2.344 Cross-Ref:††

§6. RUDIMENTARY PROPOSITIONS AND ARGUMENTS

344. Having thus completed the analysis of the assertion, I now proceed to show, in one word, that in nearly the same sense in which a term is a rudimentary proposition, a proposition is, in its turn, a rudimentary argumentation. A term is a proposition with the subjects deprived of their forcefulness. Deprive the propositions of an argumentation of their assertiveness, and the result is an assertion. Thus, the argumentation,

Enoch was a man,
∴ Enoch was mortal,

becomes, on ceasing to assert the propositions,

If Enoch was a man, then Enoch was mortal.

Thus, the converse, at least, is true; and every so eviscerated argumentation is a proposition.

Peirce: CP 2.345 Cross-Ref:††

345. But now come almost the entire assemblage of German logicians, with Professor Schröder among them, and declare that hypothetical propositions and categorical propositions differ essentially from one another.†1 By a hypothetical proposition is meant, in that exact historical terminology which it is the great good fortune of logic to inherit, any proposition compounded of propositions. De Morgan †2 has so completely treated the subject of logical combinations, that acquaintance with his work enables us to pronounce, at once, that there are six species of simple hypotheticals ranged under two genera; and discussions by Mrs. Fabian [Ladd-] Franklin †1 and by her husband †2 show that complex hypotheticals having two members are numbered by tens of thousands. The simple species are as follows:

Genus. I. *Negative simple hypotheticals*

(not asserting nor denying either member of the hypothetical) *Species 1. Conditional propositions.* If it thunders, it *rains*.

Species 2. Disjunctive propositions. It either thunders or it *rains*.

Species 3. Repugnential propositions. It does not both thunder and rain.

Genus II. *Affirmative simple hypotheticals*

(either asserting or denying each member of the *hypothetical*)

Species 1. Independential propositions. It thunders without *raining*.

Species 2. Conjunctive propositions. It both thunders and *rains*.

Species 3. Tertial propositions. It neither thunders nor rains.

Peirce: CP 2.346 Cross-Ref:††

346. Most of these simple species were given as hypotheticals by medieval logicians. But Kant, who had arrived at the conviction that there must be three classes of propositions on every logical principle of division, having taken *categoricals* from one class, took the first two species of the negative genus of simple hypotheticals for his two other classes. But he called the conditionals, *hypothetical* propositions, so restricting that term, as, indeed, it had, by some logicians, already been restricted.

Kant was poorly equipped to draw up a table of "Functions of Judgment." Even Lambert, the greatest formal logician of those days, could not have succeeded in the task; nor could a Kant and Lambert rolled into one gigantic intellect. Kant did not even give time enough to the task to form any idea of its magnitude. But subsequent German logicians, drifting about without any exact method, and at all periods and concerning all matters, far too gregarious, official, and party-adhering in their opinions, accepted the triad of *categorical*, *hypothetical*, and *disjunctive* propositions, partly because it seemed to be recommended on the metaphysical side, and partly because they had no method which could imperatively deny any view to which they were officially inclined. But Professor Schröder, being an exact logician, could not possibly accept that triad. Nevertheless, he holds categoricals to be essentially different from all hypotheticals in the broad sense of this word. The above analysis of the term, since it makes the term a proposition, makes categorical propositions compound, or hypothetical, propositions. But we cannot pass by the deliberate opinion of such a mind as Schröder without closer examination. . . .

Peirce: CP 2.347 Cross-Ref:††

347. The quantified subject of a hypothetical proposition is a *possibility*, or *possible case*, or *possible state of things*. In its primitive sense, that which is *possible* is a hypothesis which in a given state of information is not known, and cannot certainly be inferred, to be false. The assumed state of information may be the actual state of the speaker, or it may be a state of greater or less information. Thus arise various kinds of possibility. All these varieties of possibility are *ignorantial*, or *negative*. *Positive* possibility arises when our knowledge is such as is represented by a disjunctive proposition, that either A, or B, or C, or D, etc., is true. A, B, C, D, etc., are then the positively possible cases. Thus, in playing backgammon, there are twenty-one possible throws of the dice, at each play. The aggregate of the positively possible cases is the *range* or *universe* of possibility. The speaker of a hypothetical proposition does not necessarily possess a positive disjunctive knowledge; but he can make, at any rate, a *logical* disjunction which shall be necessarily true. The quantified subject may either be universal or particular. The particular negative and universal affirmative simple hypothetical propositions will be of a different and more simple character than those which are universal negative and particular affirmative. The following table shows this.

Particular negative hypotheticals

Conditional. It may not thunder or it may rain.

Disjunctive. It may thunder or it may rain.

Repugnential. It may not thunder or it may not rain.

Universal affirmative hypotheticals

Independential. It must thunder and it cannot rain.

Conjunctive. It must thunder and it must rain.

Tertial. It cannot thunder and it cannot rain.

Universal negative hypotheticals

Conditional. In every possible case in which it should thunder it would rain.

Disjunctive. In every possible case it either thunders or rains.

Repugnential. In no possible case will it both thunder and rain.

Particular affirmative hypotheticals.

Independential. It may thunder without raining.

Conjunctive. It may thunder and rain, too.

Tertial. It may be that it will neither thunder nor rain.

Peirce: CP 2.348 Cross-Ref:††

348. In every fully developed hypothetical proposition, there is a range of possibilities. The proposition derives its characteristic from this. But the Philonians †1 maintain (and the Diodorans have usually admitted) that analysis ought to begin with the *consequentia simplex de inesse*, which is what a conditional proposition becomes for omnipotence. In other words, we are to begin by removing the quantification, and consider *singular* hypotheticals. That done, the conditional proposition becomes (according to Philonians) "In this case either it does not thunder or it rains." If we do not say what this case is, further than that it is some contemplated possibility, the singular hypothetical becomes a term. "In the case I am contemplating either it would not thunder or it would rain," amounts to, "Let us consider the case in which it either does not thunder or does rain," or to, "The case of its either raining or not thundering." The last two differ in the accidental syntax of familiar languages, but they do not differ in meaning.

Peirce: CP 2.349 Cross-Ref:††

349. In a paper which I published in 1880, †2 I gave an imperfect account of the algebra of the copula. I there expressly mentioned the necessity of quantifying the possible case to which a conditional or independential proposition refers. But having at that time no familiarity with the signs of quantification, the algebra of which I developed later, †1 the bulk of the chapter treated of simple consequences *de inesse*. Professor Schröder accepts this first essay as a satisfactory treatment of hypotheticals; and assumes, quite contrary to *my* doctrine, that the possible cases considered in hypotheticals have no multitudinous universe. This takes away from hypotheticals their most characteristic feature. It is the sole foundation of his section 45, †2 in which he notes various points of contrast, between hypotheticals and categoricals. According to this, hypotheticals are distinguished from categoricals in being more rudimentary and simple assertions; while the usual doctrine of those who maintain that there is a difference between the two forms of assertion is quite the reverse.

Peirce: CP 2.350 Cross-Ref:††

350. In several passages in Professor Schröder's first volume promises were held out that the opening section 28 of the next volume should clearly show the difference between hypotheticals and categoricals and should convict me of taking a narrow view of assertion. But when the second volume appeared that section seemed to me remarkably lame, considering the great force and exactitude of the writer's customary thought. There is so little in it that it would by itself convey quite a false idea of Professor Schröder's power as a logician.

Peirce: CP 2.351 Cross-Ref:††

351. Professor Schröder's main effort is to show that time has to be considered in the analysis of hypotheticals. But he gives no proof at all that time *must be* considered; he only shows how it may be considered. Nobody familiar with the logic of relatives needs to be told that it is easy to introduce the consideration of time, if it be desirable to do so. At any rate, when I maintain that categoricals are essentially the same as hypotheticals, I mean, essentially the same as *compound propositions*, without specially introducing the idea of time. It is evident, then, that all that discussion about time is quite beside the question in dispute; it contains a fault of logic.

Peirce: CP 2.352 Cross-Ref:††

352. The only other argument that I am able to extract from Professor Schröder's section 28 is that any two terms could be taken as subject and predicate (in the old sense) of a categorical and the result will always be true or false, while in the case of the hypothetical the result will often be nonsensical, and neither true nor false. One cannot but remark that this argument seems to be at variance with the view of section 45. According to that section a hypothetical proposition does not essentially differ from an unquantified categorical. It is there even called by Professor Schröder a particular species of categorical. But here it appears as having properties which no categorical possesses. I am confident, however, that examination will convince the reader that it has no such properties. I first remark that a proposition does not cease to be true because it is nonsensical. A proposition is false if and only if something which it either expressly asserts or implies is false; and every proposition not false is true, by the principle of excluded middle. Hence, something not an assertion, considered as an assertion, is true. We may therefore put the question of truth and falsity aside; and ask whether it be true that a hypothetical can be nonsensical, but a categorical not. The truth is that nonsensical forms are so readily made in categoricals that the usage of language has taken them up and attached meanings to them. "What I am telling you is true," and "A man is a man," are frequently heard, although these are, in the strictest sense, nonsensical. One of Professor Schröder's examples of a nonsensical proposition is "This proposition is not true." But that is easily shown to involve contradiction, that is to imply two contradictory things.†1 It therefore does imply, or mean, something. A self-contradictory proposition is not meaningless; it means too much.†2 But if Professor Schröder intends to say that a categorical proposition cannot be self-contradictory, that is equally untenable. "A is not A" refutes that.

Peirce: CP 2.353 Cross-Ref:††

353. Common nouns are primitively used to denote "sense-percepts," while clauses of hypotheticals are commonly used to denote situations which sometimes occur. One denotes the object, the other the occasion of attention. There is a psychological distinction between them. But distinctions ought not to be drawn in logic which can lead up to no discrimination between a good and a bad argumentation. For the purpose of logic, it makes no difference by what psychological

action the attention is arrested. When an analysis shall have been made of a continuous logical universe, it may come to be shown that a logical distinction ought to be drawn between such a universe and a discrete universe; and it is, perhaps, a little more natural to connect the continuous universe with a hypothetical than with a categorical. Nevertheless, in very many cases the universe of hypothetical propositions is discrete; and in very many cases the universe of categorical propositions is continuous, like the example of the colored surface, above.

Peirce: CP 2.354 Cross-Ref:††

354. There are many languages in which the simplest assertions which we make in categorical form, take, as far as we can comprehend the psychical process, hypothetical forms. There is one of these tongues a smattering of which is not an uncommon accomplishment--a smattering sufficient to carry the student into the spirit of the language--I mean the Old Egyptian. There are few words in this language which are distinctively common nouns. Every general word excites a pictorial idea. Even to the modern student, the pictorial ideograph becomes a considerable part of the idea it excites; and the influence of the hieroglyphics, the modes of expression, etc., is to make "a composite of pictures" particularly expressive in describing the idea conveyed. Now our word "*is*," the copula, is commonly expressed in Old Egyptian by a demonstrative pronoun. It is evident that this demonstrative has in such sentences the force of a relative. Where is the verb? We feel that it is contained in the general words. In short, "man is mortal" is expressed in Old Egyptian in a form which expressed the following psychological process of thinking, "What is spoken of is man, *which* what is spoken of is mortal." This is precisely the way in which the same idea is conveyed in my general algebra of logic, where, putting *h* for man and *d* for mortal, I write

$$\pi[i]h[i] \text{ -< } d[i]\dagger 1$$

Peirce: CP 2.354 Cross-Ref:††

This form equally serves for a universal categorical or a conditional proposition, and the fact that the mode of connection of the *i* with the *h* and *d*, appears a little different in the two cases from a psychological point of view, ought not to affect the logical classification.

Peirce: CP 2.355 Cross-Ref:††

355. But the reader will object that, even granting my contention that hypothetical propositions embrace all propositions, I am still far from having shown that endowing its members with assertiveness will convert it into a process of *argumentation*. I have only shown this, if I have shown it at all, in the case of universal conditional propositions. There is great force in this. The very idea of logic forces upon the logician the conception of inference, and inference involves the idea of necessary inference, and necessary inference involves the idea of the universal conditional proposition.

Peirce: CP 2.356 Cross-Ref:††

356. It remains to show in what manner I suppose the ideas of the other forms of propositions to be evolved; and this will be a chapter of what I have called "speculative rhetoric." I may begin by remarking that I use the sign -< for the sign of

inclusion. I believe I was the first to show, in 1867,^{†1} that Boole's algebra, as he left it, was unfit to express particular propositions. Following out that idea, I showed, in 1870,^{†2} before anybody else, that we needed in logic a sign corresponding to the sign \mid , but that that sign is unsatisfactory because it implies that the relation is a combination of the relations expressed by $<$ and $=$, whereas in truth, as I demonstrated, it is more simple than either. I, therefore, proposed to replace the sign \mid by $-<$, at least in logic. The sign I proposed has the advantage that it can be easily made in the composing room, while the cursive form of it is rapidly written with two strokes. On account of my priority, the sign I proposed ought, in propriety, to be retained, unless it be open to very decided objections. I shall retain it. Accordingly

$$h[i] -< d[i]$$

means that on the occasion i , if the idea h is definitively forced upon the mind, then on the same occasion the idea d is definitively forced upon the mind. On the Philonian view this is the same as to say that on the occasion i , either the idea h is not definitively forced upon the mind or on the same occasion the idea d is definitively forced upon the mind. From that hypothesis, the rules of the sign $-<$ may be mathematically deduced. I do not give them here, because my ms. containing the development was many months ago borrowed by a friend; and I have not yet quite lost hope of recovering it, and thus sparing myself the labor of repeating the work.^{†1} It follows from the omitted development, that while this sign enables us, by taking letters to denote various propositions, to express many relations, yet unless we take a letter to denote a proposition known or assumed to be false, it never enables us to express that any assertion is false. There is a very good reason for adopting the convention that

$$a -< b -< c$$

shall mean $a -< (b -< c)$, and not $(a -< b) -< c$. We are thus led to inquire what must be the meaning of

$$a -< a -< a -< a -< a -< a -< a -< a$$

without end. This series of antecedents without a final consequent are seen to be equivalent to the denial of a .^{†2} Thus, without the introduction of any other sign, but merely by the idea of an endless sequence, after we already have the idea of successive sequence, we reach the idea of negation. Thus, the conceptions involved in argumentation produce the conception of the rejection of an argumentation. Hence, we are led to generalize our idea of argumentation, from the perception that one assertion has to be admitted because another is admitted, to embrace also that process of thought in which we think that though one assertion is true yet another is not thereby necessarily true. It is not the primitive conception of argumentation, but this

generalized conception, which covers the entire field of hypotheticals. As soon as we have the idea of absurdity, we can conceive that a certain argument could logically lead to absurdity. Now an argument which can lead to absurdity is false; and an argument which is false can in some conceivable case lead to absurdity. Hence, as soon as we admit the idea of absurdity, we are bound to class the rejection of an argumentation among argumentations. Thus, as was said, a proposition is nothing more nor less than an argumentation whose propositions have had their assertiveness removed, just as a term is a proposition whose subjects have had their denotative force removed.

Peirce: CP 2.357 Cross-Ref: ††

§7. SUBJECT †1

357. Whether or not every proposition has a principal subject, and, if so, whether it can or cannot have more than one, will be considered below. A proposition may be defined as a sign which separately indicates its object. For example, a portrait with the proper name of the original written below it is a proposition asserting that so that original looked. If this broad definition of a proposition be accepted, a proposition need not be a symbol. Thus a weathercock "tells" from which direction the wind blows by virtue of a real relation which it would still have to the wind, even if it were never intended or understood to indicate the wind. It separately indicates the wind because its *construction* is such that it must point to the quarter from which the wind blows; and this construction is distinct from its *position* at any particular time. But what we usually mean by a proposition or judgment is a symbolic proposition, or *symbol*, separately indicating its object. Every subject partakes of the nature of an index, in that its function is the characteristic function of an index, that of forcing the attention upon its object. Yet the subject of a symbolic proposition cannot strictly be an index. When a baby points at a flower and says, "Pretty," that is a symbolic proposition; for the word "pretty" being used, it represents its object only by virtue of a relation to it which it could not have if it were not intended and understood as a sign. The pointing arm, however, which is the subject of this proposition, usually indicates its object only by virtue of a relation to this object, which would still exist, though it were not intended or understood as a sign. But when it enters into the proposition as its subject, it indicates its object in another way. For it cannot be the subject of that symbolic proposition unless it is intended and understood to be so. Its merely being an index of the flower is not enough. It only becomes the subject of the proposition, because its being an index of the flower is evidence that it was *intended* to be. In like manner, all ordinary propositions refer to the real universe, and usually to the nearer environment. Thus, if somebody rushes into the room and says, "There is a great fire!" we know he is talking about the neighbourhood and not about the world of the *Arabian Nights' Entertainments*. It is the circumstances under which the proposition is uttered or written which indicate that environment as that which is referred to. But they do so not simply as index of the environment, but as evidence of an intentional relation of the speech to its object, which relation it could not have if it were not intended for a sign. The expressed subject of an ordinary proposition approaches most nearly to the nature of an index when it is a proper name which, although its connection with its object is purely intentional, yet has no reason (or, at least, none is thought of in using it) except the mere desirability of giving the familiar object a designation. Among, or along with, proper names we may put abstractions,

which are the names of fictitious individual things, or, more accurately, of individuals whose being consists in the manner of being of something else. A kind of abstractions are individual collections, such as the "German people." When the subject is not a proper name, or other designation of an individual within the experience (proximate or remote) of both speaker and auditor, the place of such designation is taken by a virtual precept stating how the hearer is to proceed in order to find an object to which the proposition is intended to refer. If this process does not involve a regular course of experimentation, all cases may be reduced to two with their complications. These are the two cases: first, that in which the auditor is to take any object of a given description, and it is left to him to take any one he likes; and, secondly, the case in which it is stated that a suitable object can be found within a certain range of experience, or among the existent individuals of a certain class. The former gives the *distributed* subject of a *universal* proposition, as, "Any cockatrice lays eggs." It is not asserted that any cockatrice exists, but only that, if the hearer can find a cockatrice, to that it is intended that the predicate shall be applicable. The other case gives the *undistributed* subject of a *particular* proposition, as "Some negro albino is handsome." This implies that there is at least one negro albino. Among complications of these cases we may reckon such subjects as that of the proposition, "Every fixed star but one is too distant to show a true disk," and, "There are at least two points common to all the circles osculating any given curve." The subject of a universal proposition may be taken to be, "Whatever object in the universe be taken"; thus the proposition about the cockatrice might be expressed: "Any object in the universe having been taken, it will either not be a cockatrice or it will lay eggs." So understood, the subject is not *asserted* to exist, but it is well known to exist; for the universe must be understood to be familiar to the speaker and hearer, or no communication about it would take place between them; for the universe is only known by experience. The particular proposition may still more naturally be expressed in this way, "There is something in the universe which is a negro albino that is handsome." No doubt there are grammatical differences between these ways of stating the fact; but formal logic does not undertake to provide for more than one way of expressing the same fact, unless a second way is requisite for the expression of inferences. The latter mode is, on the whole, preferable. A proposition may have several subjects. Thus the universe of projective geometry being understood, it is a true proposition that "Whatever individuals, *A*, *B*, *C*, and *D* may be, there are individuals *E* and *F*, such that whatever individual *G* may be, there is an individual *H*, and an individual *I*, such that, if *A*, *B*, *C*, and *D* are all straight lines, then *E* and *F* are straight lines, each intersecting *A*, *B*, *C*, and *D*, and *E* and *F* are not coincident; and if *G* is a straight line, not coincident with *E*, and not coincident with *F*, and if *G* intersects *A*, *B*, and *C*, it does not intersect *D*, unless *H* is a one-sheeted hyperboloid of which *A*, *B*, *C*, and *D* are generators, and *J* is a set of generators of *H*, to which *A*, *B*, *C*, and *D* all belong"; or, in our usual phraseology, any four straight lines in space are intersected by just two different straight lines, unless these four straight lines belong to one set of generators of a one-sheeted hyperboloid. Such a proposition is called a relative proposition. The order in which the selection of individuals is made is material when the selections are different in respect to distribution. The proposition may relate to the frequency with which, in the course of ordinary experience, a generic event is of a certain species. De Morgan wishes to erect this into the general type of propositions.†1 But this is to overlook a vital distinction between probability and that which a universal proposition asserts. To say that the probability that a calf will not have more than six legs is 1, is to say that in the long run, taking calves as they present themselves in experience, the ratio of the number of those with not more

than six legs to the total number is 1. But this does not prevent there being any finite number of calves with more legs than six, provided that in the long run, that is, in an endless course of experience, their number remains finite, and does not increase indefinitely. A universal proposition, on the other hand, asserts, for example, that any calf which may exist, without exception, is a vertebrate animal. The universal proposition speaks of experience distributively; the probable, or statistical proposition, speaks of experience collectively.

Peirce: CP 2.358 Cross-Ref:††
§8. PREDICATE †2

358. The view which pragmatic logic takes of the predicate, in consequence of its assuming that the entire purpose of deductive logic is to ascertain the necessary conditions of the truth of signs, without any regard to the accidents of Indo-European grammar, will be here briefly stated. Cf. Negation.†3

Peirce: CP 2.358 Cross-Ref:††

In any proposition, i.e., any statement which must be true or false, let some parts be struck out so that the remnant is not a proposition, but is such that it becomes a proposition when each blank is filled by a proper name. The erasures are not to be made in a mechanical way, but with such modifications as may be necessary to preserve the partial sense of the fragment. Such a residue is a *predicate*. The same proposition may be mutilated in various ways so that different fragments will appear as predicates. Thus, take the proposition "Every man reveres some woman." This contains the following predicates, among others:

" . . . reveres some woman."

". . . is either not a man or reveres some woman."

"Any previously selected man reveres . . ."

"Any previously selected man is . . ."

Peirce: CP 2.359 Cross-Ref:††
§9. PREDICATION †1

359. In logic: the joining of a predicate to a subject of a proposition so as to increase the logical breadth without diminishing the logical depth.

Peirce: CP 2.360 Cross-Ref:††

360. This still leaves room for understanding predication in various ways, according to the conception entertained of the dissection of a proposition into subject and predicate. It is a question under dispute today whether predication is the essential function of the proposition. Some maintain that the proposition "It rains" involves no predication. But if it is an assertion, it does not mean that it rains in fairyland, but the very act of saying anything with an appearance of seriously meaning it is an Index (q.v.)†2 that forces the person addressed to look about to see what it is to which what

is being said refers. The "rains" recalls to his mind an image of fine up-and-down lines over the field of view; and he looks sharply out of the window, fully understanding that that visible environment is indicated as the subject where the lines of falling drops will be seen. In like manner, there is a predication in a conditional or other hypothetical proposition, in the same sense that some recognized range of experience or thought is referred to.

Peirce: CP 2.361 Cross-Ref:††

361. A few of the most frequently recurring scholastic phrases follow.

Peirce: CP 2.361 Cross-Ref:††

. . . **Analogical predication**; a rather favourite expression of Aquinas: predication in which the predicate is taken neither in its strict sense nor in an unrelated sense, but in a peculiar sense for which there is a good reason, as when a statue is said to be a man.

Peirce: CP 2.361 Cross-Ref:††

. . . **Denominative predication**: predication in which that whose nature it is to be a subject is taken as the subject, and something whose nature it is to be predicated is taken as the predicate; a predication of an accident of a substance. (It is well discussed by Scotus, *In univ. Porph.*, 9. 16, "Utrum haec sit vera, *Homo est animal*," where, as in the majority of scholastic disputations, the conclusion is foregone, and the interest lies in the formidable difficulties and how they are to be overcome.) Denominative predication, in its proper sense, is predication of an accidental concrete term of its own subject; in a broad sense, it is the predication of any concrete of a suppositum, or of any subject of less breadth; in the widest sense, it is predication of any predicate of any subject. Denominative predication may be *a posteriori* or *a priori*, as *homo est albus*, *rationale est substantia*, *homo est animal*.

Peirce: CP 2.361 Cross-Ref:††

. . . **Dialectic predication**, as defined by Aristotle (I. Top., x): the predication of a general term in a proposition which may result from an argument in a probable place, and not reducible to anything prior.

Peirce: CP 2.361 Cross-Ref:††

Direct predication: predication in the usual sense of representing that the breadth of the subject belongs to the predicate, and the depth of the predicate to the subject; or, in scholastic language, it is predication of a higher term of a lower one, of a passion of a subject, of an accident of a subject, of a mode of a quiddity, of a difference of a genus.

Peirce: CP 2.361 Cross-Ref:††

. . . **Essential predication**: in which the predicate is wholly contained in the essence of the subject. It is, therefore, in Kant's sense, an analytical judgment. But neither Kant nor the scholastics provide for the fact that an indefinitely complicated proposition, very far from obvious, may often be deduced by mathematical reasoning, or necessary deduction, by the logic of relatives, from a definition of the utmost simplicity, without assuming any hypothesis whatever (indeed, such assumption could only render the proposition deduced simpler); and this may contain many notions not explicit in the definition. This may be illustrated by the following: Man is a rational animal; hence, whatever is not a man is either, on the one hand, not rational, while either at the same time being an animal or else benefiting nothing except such objects as love nothing but fairies, or, on the other hand, is not an animal, while either

being rational or standing to whatever fairy may exist in the relation of benefiting something that loves it. Now, if it be said that that is an analytical judgment, or essential predication, neither the definition of the scholastics nor that of Kant is adequate. But if it be said that it is not an essential predication, or analytical judgment, then the accidental predication and the synthetical judgment may be a necessary consequence, and a very recondite one, of a mere definition, quite contrary to what either Kant or the scholastics supposed and built upon. Cf. Scotus (*In univ. Porph.*, 9. 12), who makes essential predication the predication of genus, species, or difference.

Peirce: CP 2.361 Cross-Ref:††

Exercised predication. The distinction between exercised and signate predication belongs to Scotus. (The passage which Prantl attributes to Antonius Andreas †1 is a quotation *verbatim* from Scotus, as often naturally happens in Prantl's *Geschichte*.) A *signate* predication is one which is *said to be* made, an *exercised* predication is one which *is* made; so that Scotus says: "A praedicari signato ad praedicari exercitum, [sive ad esse,] non tenet consequentia per se in eisdem terminis." †2 Scotus gives the following examples of the distinction, where the exercised predication is marked E, the signate S:S, **Genus praedicatur de specie; E, Homo est animal.** (The Lyons text here transposes the terms, which we give correctly.) S, *nego*; E, *non*. E, *tantum*; S, *excludo*. The abstract definition of Scotus is: "*Esse* in rebus primae intentionis, illud exercet, quod *praedicari* signat in secundis intentionibus." †3 Exercised predication is distinguished into *praedicatio de proprio supposito* and *praedicatio de subiecto*; the former is essential, the latter accidental.

Peirce: CP 2.361 Cross-Ref:††

Formal predication: predication where the predicate is in the concept of the subject, independent of any extrinsic cause or of any particular matter in qua. The difference between formal and essential predication is somewhat trivial and confused.

Peirce: CP 2.361 Cross-Ref:††

. . . Natural predication: when the subject and predicate ought to be so related according to their nature. This is substantially the definition given in many books; but it conveys little idea of how the expression is used. Natural predication is always divided into the identical and direct; non-natural predication is either *indirect*, i.e., *contra naturam*, or it is *praeter naturam*, i.e., *per accidens*. Examples of indirect predication, where the subject is related to the predicate as form to matter, are *alba est nix, animal est homo*. Examples of predication *praeter naturam*, where subject and predicate are related to some third term, as form to matter, are *album est dulce, dulce est album*. Examples of direct predication: *nix est alba, homo est animal*. Examples of identical predication: *gladius est ensis, Plato est Plato* (Conimbricenses *in Praef. Porph.*, q.i. art. 4). . . .

Peirce: CP 2.362 Cross-Ref:††

§10. QUANTITY †1

362. (In logic and mathematics.) (1) Any Accident whereby a substance has part outside of part. Cf. Quantity (2).

Peirce: CP 2.362 Cross-Ref:††

This is the old definition; and it is true to the old meaning of the word in representing quantity as much more concrete than the modern conception. Quantity (see Aristotle's *Praedicamenta*, vi) is either discrete or continuous. Continuous quantity is either magnitude or time. The old definition of mathematics as the science of quantity is misunderstood, if quantity is here taken in the modern sense; it was only meant that mathematics treated of accidents having number, magnitude, or duration. There was therefore a mathematics of music.

Peirce: CP 2.363 Cross-Ref:††

363. (2)†2 In the general modern sense, quantity is a system of serial relationships.

Peirce: CP 2.363 Cross-Ref:††

Serial relationship differs from transitive relationship merely in the point of view, and (so closely connected are the two points of view) in hardly more than the mode of expression. Now, all transitive relation is traceable to inclusion. Hence, quantity might be defined as a system of inclusions looked upon as serial. It is very important to understand that quantity is a mere system of relative ordinal relations in a linear series. Each complete determination of quantity in a given system is a "value."

Peirce: CP 2.363 Cross-Ref:††

Quantity is either counted or measured. Counted quantity may have a finite multitude of values. Of systems of quantity of denumeral multitude, the simplest is that of the integer numbers. The system of rational fractions is the only other familiarly used. These fractions can, in several ways, be arranged in their order of quantity by mere counting.

Peirce: CP 2.364 Cross-Ref:††

364. (3) Concepts, or terms, are, in logic, conceived to have *subjective parts*, being the narrower terms into which they are divisible, and *definitive parts*, which are the higher terms of which their definitions or descriptions are composed: these relationships constitute "quantity."

Peirce: CP 2.364 Cross-Ref:††

This double way of regarding a class-term as a whole of parts is remarked by Aristotle in several places (e.g., *Met.*, {D} xxv. 1023 b 22). It was familiar to logicians of every age. Thus Scotus Erigena calls logic "ars illa quae diuidit genera in species et species in genera resoluit."†1 John of Salisbury †2 refers to the distinction as "quod fere in omnium ore celebre est, aliud scilicet esse quod appellativa [i.e., adjectives and the like] *significant*, et aliud esse quod *nominant*. Nominantur singularia, sed universalia significantur." For William of Auvergne, see Prantl, III, 77. The writer has a long list of similar passages before him. But the Aristotelians had their minds upon the discrimination of different kinds of predication, and insisted that the differences of different genera are different, thus forbidding cross-divisions. Arnauld, however, in *l'Art de penser*, conceives all predicates, or all essential predicates, as alike, without distinguishing *genus* and *differentia*; and was so led to devote a short chapter (vi) to *l'étendue* and *la compréhension* before taking up the predicables. But his services in the matter have been grossly exaggerated, and it really seems to have been Kant who made these ideas pervade logic and who first expressly called them quantities. But the idea was old. Archbishop Thomson,†3 W. D. Wilson,†4 and C. S. Peirce †5 endeavor to make out a third quantity of terms. The last calls his third quantity "information," and defines it as the "sum of synthetical propositions in which the symbol is subject or predicate," antecedent or consequent.

The word "symbol" is here employed because this logician regards the quantities as belonging to propositions and to arguments, as well as to terms. A distinction of *extensive* and *comprehensive distinctness* is due to Scotus (*Opus Oxon.*, I. ii. 3): namely, the usual effect upon a term of an increase of information will be either to increase its breadth without diminishing its depth, or to increase its depth without diminishing its breadth. But the effect may be to show that the subjects to which the term was already known to be applicable include the entire breadth of another term which had not been known to be so included. In that case, the first term has gained in *extensive distinctness*. Or the effect may be to teach that the marks already known to be predicable of the term include the entire depth of another term

not previously known to be so included, thus increasing the

comprehensive distinctness of the former term. The passage of

thought from a broader to a narrower concept without change

of information, and consequently with increase of depth, is

called *descent*; the reverse passage, *ascent*. For various purposes, we often imagine our information to be less than it is.

When this has the effect of diminishing the breadth of a term

without increasing its depth, the change is called *restriction*;

just as when, by an increase of real information, a term gains

breadth without losing depth, it is said to gain extension.

This is, for example, a common effect of *induction*. In such

case, the effect is called generalization. A decrease of supposed

information may have the effect of diminishing the depth of a

term without increasing its information. This is often *called*

abstraction; but it is far better to call it *prescission*; for the word *abstraction* is wanted as the designation of an even far more

important procedure, whereby a transitive element of thought

is made substantive, as in the grammatical change of an adjective into an abstract noun. This may be called the principal

engine of mathematical thought. When an increase of real

information has the effect of increasing the depth of a term

without diminishing the breadth, the proper word for the process is *amplification*. In ordinary language, we are inaccurately

said to *specify*, instead of to *amplify*, when we add to information in this way. The logical operation of forming a hypothesis often has this effect, which may, in such case, be called *supposition*. Almost any increase of depth may be called *determination*.

Peirce: CP 2.364 Cross-Ref:††

(4) Syllogistic is sometimes regarded as the mathematics of a system of quantities consisting of but two values, *truth* and *falsity*.

Peirce: CP 2.364 Cross-Ref:††

(5) The quantity of a proposition is that respect in which a universal proposition is regarded as asserting more than the corresponding particular proposition: the recognized quantities are Universal, Particular, Singular and--opposed to these as "definite"--Indefinite. *Quantitas* is used in this sense by

Apuleius.†1

Peirce: CP 2.365 Cross-Ref:††

365. *Quantification of the Predicate*. The attachment of signs of propositional quantity to the predicates of simple propositions is called by this name. The *dictum de omni* defines the relation of subject and predicate, so that "Any *A* is *B*" is to be understood as meaning "To whatever *A* is applicable, *B* is applicable." But this definition must be modified, in order to give any room for a quantification of the predicate. If then we are to take *all* and *some* in their proper distributive senses and not in collective senses, to say that "Every man is every animal" would, as Aristotle remarks, be absurd, unless it were meant that there was but one man and one animal, and that that one man was identical with that one animal. This system has never been proposed. But Hamilton,†1 with his followers, T. S. Baynes †2 and Calderwood, take the marks of quantity in a collective sense. They thus have, as one of the propositional forms, "Some man is not some animal," which precisely denies "Every man is every animal," in the distributive sense, and is entitled to an equal standing in logic. It does not deny "All man is all animal," in the collective sense of these logicians. This system had some vogue in its day.

Peirce: CP 2.366 Cross-Ref:††

366. *De Morgan's System of Propositions*.†3 This permits the retention of the *dictum de omni*, merely applying propositional quality to the subject. We thus get the following eight forms of proposition:

-))To whatever *A* is applicable, *B* is applicable.
- (.)To whatever *A* is inapplicable, *B* is applicable.
-)-(To whatever *A* is applicable, *B* is inapplicable.
- ((To whatever *A* is inapplicable, *B* is inapplicable; i.e.,
 To whatever *B* is applicable, *A* is applicable.
- ()To something to which *A* is applicable, *B* is applicable.
- (.(To something to which *A* is applicable, *B* is inapplicable.

)-(To something to which **A** is inapplicable, **B** is applicable; i.e., To something to which **B** is applicable, **A** is inapplicable.

)(To something to which **A** is inapplicable, **B** is inapplicable.

Peirce: CP 2.366 Cross-Ref:††

The above is substantially one of De Morgan's own forms of statement, called by him onymatic.†1 There is no objection to this system; but it is an idle complication of forms which does not enable us to take account of any mode of inference that the old system does not cover. Still it does away with the figures of syllogism. But whatever the merits or demerits of the system, De Morgan developed it with logical elegance.

Peirce: CP 2.367 Cross-Ref:††

§11. UNIVERSAL †2

367. (1) This word was used in the middle ages where we should now use the word General. Another synonym was *praedicabile*: "Praedicabile est quod aptum natum est praedicari de pluribus," says Petrus Hispanus.†3 Albertus Magnus says, "Universale est quod cum sit in uno aptum natum est esse in pluribus."†4 Burgersdicius, literally translating from Aristotle, says, "Universale ({to kath' holou}) appello, quod de pluribus suapte natura praedicari aptum est," i.e., {ho epi pleionōn pephyke katégoreisthai}. When the Scholastics talk of universals, they merely mean general terms (which are said to be simple universals), with the exception here following.

Peirce: CP 2.368 Cross-Ref:††

368. (2) The five terms of second intention, or more accurately the five classes of predicates, *genus*, *species*, *difference*, *property*, *accident*, were in the middle ages (as they still are) called "the predicables." But since predicable also means fit to be a predicate, in which sense it is almost an exact synonym of universal in the first sense, the five predicables came to be often referred to as "the universals."

Peirce: CP 2.369 Cross-Ref:††

369. (3) Predicated, or asserted, in a proposition *de omni*; said to be true, without exception, whatever there may be of which the subject term is predicable. See Quantity [§10].

Peirce: CP 2.369 Cross-Ref:††

Thus "any phoenix rises from its ashes" is a universal proposition. This is called the *complex* sense of universal. The subject must be taken in the distributive sense and not in the collective sense. Thus, "All man is all redeemed," which is Hamilton's "toto-total proposition,"†5 is not a universal proposition, or assertion de omni, in the sense defined by Aristotle in the *dictum de omni*; for it means that the collection of men is identical with the collection of the redeemed, and not that each man without exception is all redeemed. Leibnitz rightly insists that a universal proposition does not assert, or imply, the existence of its subject.†1 The first reason for this is that it accords with the definition, that is, the *dictum de omni*, which is that

that is asserted universally of a subject which is said to be predicable of whatever that subject may be predicable. For this may be done without asserting that the subject is predicable of anything in the universe. The second reason is that the term *universal proposition* is a term of formal logic. Now the principal, or at least the most essential, business of formal logic is so to formulate direct syllogism as not to represent it as requiring more or less than it really does. Now the major premiss of a direct syllogism must be universal, but need not imply the existence of anything of which the subject should be predicable. Hence a form of universal proposition not asserting the existence of the subject is indispensable. Now that no second kind of universal proposition is needed will presently appear. The third reason is that it is necessary that formal logic should be provided with a form of proposition precisely denying every proposition coming under each of its simple forms. Now, if a universal proposition asserting the existence of its subject is regarded as a simple form of proposition--as, for example, "There are inhabitants of Mars and every one of them without exception has red hair"--its precise denial would be a particular proposition not asserting the existence of the subject, which would be a most singular form, hardly ever wanted, and manifestly complex, such as, "Either there is no inhabitant of Mars, or if there be, there is one at least who has not red hair." It is obviously far better to make the simple particular proposition assert the existence of its subject, "There is an inhabitant of Mars who has red hair," when the universal form will not make the same assertion, or imply it: "Whatever inhabitants of Mars there may be must, without exception, have red hair." If every particular proposition asserts the existence of its subject, then an affirmative particular proposition *implies* the existence of its predicate also. It would be a contradiction in terms to say that a proposition *asserted* the existence of its predicate, since that of which a proposition *asserts* anything is its subject, not its predicate. But perhaps it is not quite accurate to say that the particular proposition *asserts* the existence of its subject. At any rate, this must not be understood as if, in such assertion, *existence* were a predicate not implied in a proposition which does not make this assertion (see Kant, *Krit. d. reinen Vernunft*, 1st ed., 599).

Peirce: CP 2.369 Cross-Ref:††

Every proposition refers to some index: universal propositions to the universe, through the environment common to speaker and auditor, which is an index of what the speaker is talking about. But the particular proposition asserts that, with sufficient means, in that universe would be found an object to which the subject term would be applicable, and to which further examination would prove that the image called up by the predicate was also applicable. That having been ascertained, it is an *immediate inference*, though not exactly *asserted* in the proposition, that there is some *indicable object* (that is, something *existent*) to which the predicate itself applies; so that the predicate also may be considered as referring to an index. Of course, it is perfectly legitimate, and in some aspects preferable, to formulate the particular proposition thus: "Something is, at once, an inhabitant of Mars and is red haired," and the universal proposition thus: "Everything that exists in the universe is, if an inhabitant of Mars, then also red haired." In this case, the universal proposition *asserts* nothing about existence; since it must already be well understood between speaker and auditor that the universe is *there*. The particular proposition in the new form asserts the existence of a vague something to which it pronounces "inhabitant of Mars" and "red haired" to be applicable.

Peirce: CP 2.369 Cross-Ref:††

The universal proposition must be understood as strictly excluding any single exception. It is thus distinguished from the proposition "The ratio of the number of

A 's to that of the A 's that are B is as 1:1," not merely in being distributive in form instead of collective, but also in asserting much more. Thus the ratio of the multitude of all real numbers to those of them that are incommensurable is as 1:1, yet that does not prevent the commensurable numbers from existing, nor from being infinite in multitude. Were it proved that the ratio of frequency of all events to such of them as were due to natural causation was 1:1, that would be no argument whatever against the existence of miracles; although it might (or might not, according to circumstances) be an argument against explaining any given event as miraculous, if such a hypothesis can be called an explanation. Now induction may conclude that the ratio of frequency of a specific to a generic event is 1:1, in the same approximate sense in which all inductive conclusions are to be accepted. Indeed, the ratios 1:1 and 0:1 may be inductively concluded with stronger confidence in their accuracy than any other ratio can be so concluded. But under no circumstances whatsoever can induction establish the accuracy or approximate accuracy of a strictly universal proposition, or that any given series of phenomenal events is, properly speaking, general (and therefore represents a possibly infinite class), or is even approximately general. Such propositions, outside of mathematics (taking this word so as to include all definitions and deductions from them), must either be entirely unwarranted, or must derive their warrant from some other source than observation and experiment. It might conceivably be established by testimony, as, for example, by a promise by a possibly immortal being to act in a certain way upon every occasion of a certain description; and thus it would not need to be an *a priori* judgment.

Peirce: CP 2.370 Cross-Ref:††

370. (4) . . . Descartes, Leibnitz, Kant, and others appeal to the universality of certain truths as proving that they are not derived from observation, either directly or by legitimate probable inference. There is only one such passage in Descartes; and even Leibnitz, though he frequently alleges the *necessity* of certain truths (that is, their being propositions of necessary mode) against Locke's opinion, yet in only one place (the *Avant-Propos* of the *Nouveaux Essais*) distinctly adds the criterion of universality. Descartes, Leibnitz, and Kant more or less explicitly state that that which they say cannot be derived from observation, or legitimate probable inference from observation, is a universal proposition in sense (3), that is, an assertion concerning every member of a general class without exception. Descartes (*Letter xcix*) argues that no legitimate inference can be made from external phenomena to the proposition that "Things equal to the same are equal to each other," since that would be to infer a "universal" from a "particular." Leibnitz uses almost the same language: "D'où il nait une autre question, savoir, si toutes les vérités dépendent de l'expérience, c'est-à-dire de l'induction et des exemples, ou s'il y a un autre fondement . . . Or, tous les exemples qui confirment une vérité générale, de quelque nombre qu'ils soient, ne suffisent pas pour établir la nécessité universelle de cette même vérité: car il ne suit pas que ce qui est arrivé arrivera toujours de même."†1 Kant expresses himself still more unmistakably (*Krit. d. reinen Vernunft*, 2d ed., Einleitung, ii): "Erfahrung giebt niemals ihren Urtheilen wahre und strenge, sondern nur angenommene und comparative Allgemeinheit (durch Induction), so dass es eigentlich heissen muss: so viel wir bisher wahrgenommen haben, findet sich von dieser oder jener Regel keine Ausnahme. Wird also ein Urtheil in strenger Allgemeinheit gedacht, d. i. so, dass gar keine Ausnahme als möglich verstattet wird, so ist es nicht von der Erfahrung abgeleitet, sondern schlechterdings *a priori* gültig. Die empirische Allgemeinheit ist also nur eine willkührliche Steigerung der Gültigkeit, von der, welche in den meisten Fällen, zu der, die in allen gilt, wie z. B. in dem Satze: alle Körper sind schwer; wo dagegen strenge Allgemeinheit zu einem Urtheile wesentlich gehört, da zeigt diese

auf einem besonderen Erkenntnisquell derselben, nämlich ein Vermögen des Erkenntnisses *a priori*. Nothwendigkeit und strenge Allgemeinheit sind also sichere Kennzeichen einer Erkenntnis a priori, und gehören auch unzertrennlich zu einander." But notwithstanding the fact that the whole logic of all these writers, especially Kant, requires the word universal to be understood in that sense, yet there are, in the works of all of them, some passages which lend a certain colour of excuse to the stupid blunder of some interpreters who teach that by necessity they mean the irresistible psychical force with which the proposition demands our assent, and that by universality they mean catholicity, i.e., the catholic acceptance of it *semper, ubique, et ab omnibus*. Descartes in particular, and Leibnitz in some measure, perhaps even Kant (though it would be very illogical for him to do so) did more or less attach weight to the irresistible apparent evidence, and to some degree to the catholic acceptance, of propositions as tending to persuade us of their truth; but not as criteria of their origin. It is, however, to be noticed that false interpreters of Kant have used the word universal in the sense of being accepted by all men--the sense of {koinos} in the phrase {koinai ennoiai}.

Peirce: CP 2.371 Cross-Ref:††

371. The words universal and universality enter into various technical phrases:

Peirce: CP 2.371 Cross-Ref:††

. . . **Natural universal:** a natural sign predicable of a plurality of things, as smoke is a sign of fire. The nominalistic doctrine is that nothing out of the mind is universal in that sense. See Ockham, *Logica*, I. xiv *ad fin.*

Peirce: CP 2.371 Cross-Ref:††

. . . **Universal validity:** according to some logicians is the validity of such reasonings as are "calculated to operate conviction on all reasonable minds" (Hamilton, *Lect. on Logic*, xxvi). If he had omitted the word reasonable, and said "calculated to work conviction on all minds," this would not have proved they had any validity at all; for the validity of a reasoning depends upon whether it really will lead to the truth, and not upon whether it be believed that it will. Thus the word reasonable is the only pertinent word in the definition. But in fact there is no division of logical validity into universal and particular. . . .

Peirce: CP 2.372 Cross-Ref:††

§12. PARTICULAR †1

372. In untechnical language, applied to single cases coming under general heads and occurring, or supposed to occur, in experience; in this sense it is also a substantive. The particulars are the experientially known circumstances of general nature, but as they appear in the individual case.

Peirce: CP 2.373 Cross-Ref:††

373. A particular proposition is one which gives a general description of an object and asserts that an object to which that description applies occurs in the universe of discourse, without asserting that it applies to the whole universe or to everything in the universe of a specified general description; as "Some dragons breathe fire." If we hold that the particular proposition asserts the existence of something, then the precise denial of it does not assert the existence of anything; as

"No dragon breathing fire exists." It is, therefore, not true that from such precise denial any particular proposition follows, such as "Some dragon does not breathe fire." For if there is no dragon that does not breathe fire, this is false, although it may be true that there is no dragon that breathes fire.

Peirce: CP 2.373 Cross-Ref:††

For instance, from the particular proposition, "Some woman is adored by all Catholics," it follows that "Any Catholic that may exist adores a woman," i.e., "There is no Catholic that does not adore a woman," which is the precise denial of "Some Catholic non-adores all women," which is a particular proposition. From this, in turn, it follows that a woman adored by all Catholics does not exist, which is the precise denial of the first proposition, "Some woman is adored by all Catholics." Of every particular proposition the same thing is true. Thus, if "Some crow is white," it follows that "No inevitable consequence of whiteness is wanting to all crows," which is the precise denial of the particular proposition, "Some inevitable consequence of whiteness is wanting in all crows." Thus, from every particular proposition follows the precise denial of a particular proposition, but from no precise denial of a particular proposition can any particular proposition follow. But this does not extend to *a simple* particular proposition, such as "Something is white," since to say "Something is non-existent" (which the analogous treatment would yield) is an absurdity, and ought not to be considered as a proposition at all.

Peirce: CP 2.374 Cross-Ref:††
§13. QUALITY †1

374. (In grammar and logic.) (1) Take a sentence in which a common noun or adjective is predicated of a proper noun, and imagine that there is something in the reality which corresponds to the form of the proposition. Then imagine that this form of fact consists in a relation of the objective subject, or substance, to one being, the same correlate for all cases where the same noun or adjective is predicated in the same sense, and that imaginary being, whether looked upon as real, or merely as a convenience of thought, is a *quality*. Thus, if anything is beautiful, white or incomprehensible, this consists in its possessing the quality of beauty, whiteness, or incomprehensibility.

Peirce: CP 2.375 Cross-Ref:††

375. (2) But in a more proper sense the term quality will not be applied when the adjective, like *incomprehensible*, is conceived as signifying a relation. Thus, whiteness will be, in this narrow sense, a quality only so long as objects are thought as being white independently of anything else; but when this is conceived as a relation to the eye, "whiteness" is only a quality in a looser sense. Locke †1 defines quality as the power of producing an idea, which agrees with the above explanation tolerably.

Peirce: CP 2.375 Cross-Ref:††

Qualitas, having inevitably reached an excessively vague use, was in the Roman schools taken to designate almost any character or characters for which no other name was at hand. Thus arose a variety of special senses. Thus in grammar the difference between nouns which had a plural and those which had not was called a difference of quality; as was the difference between the personal pronouns and *qui*,

quis, etc.

Peirce: CP 2.376 Cross-Ref:††

376. (3) In logic: the distinction between the affirmative and the negative proposition has been called the distinction of quality in propositions by all logicians, without interruption, from Apuleius, in the second century of our era, to our own contemporaries.

Peirce: CP 2.376 Cross-Ref:††

Kant, in order to round out a triad, added a third quality, called limitative, that of "Sortes est non homo," with a distinction from "Sortes non est homo." This will not bear criticism; but Kant's authority and the force of tradition have caused it to survive. As long as the universe of characters is unlimited, it is obvious that any collection of objects have some predicate common and peculiar to them. This being the case, as ordinary syllogistic tacitly assumes it is, the distinction between affirmative and negative propositions is purely relative to the particular predicate. No doubt many logicians have assumed that negative propositions are distinguished from ordinary affirmative propositions in not implying the reality of the subject. But what, then, does "Some patriarch does not die" mean? Besides, all admit that propositions *per se primo modo* do not imply the existence of the subject, although they be affirmative. At any rate, the resulting syllogistic, if consistent, is very objectionable. If, however, the universe of characters is limited, as it is in ordinary speech, where we say that logical inconsistency and mandarin oranges have nothing in common, then the system of formal logic required will be a simple case of the logic of relatives (q.v.);†1 but the distinction of affirmative and negative propositions will become material or absolute, the forms of simple categorical propositions then being:

Any *A* possesses every character of the group β .

Any *A* wants every character of the group β .

Any *A* possesses some character of the group β .

Any *A* wants some character of the group β .

Some *A* possesses every character of the group β , etc.

Peirce: CP 2.377 Cross-Ref:††

377. (4) Quality, even in Aristotle, is especially employed to denote characters which constitute merits or demerits; and this word is remarkable for the number of specialized meanings that it bears. Since Kant it has been employed to designate the distinction of clear and obscure, or distinct and confused, etc. See the preceding topic.

Peirce: CP 2.377 Cross-Ref:††

Quality is distinguished as primary, secondary, secundo-primary, essential or substantial, accidental, manifest, occult, primitive, original, elementary, first, derived, real, intentional, imputed, passible, logical, propositional, active, alterant, affective, predicamental, etc.

Peirce: CP 2.378 Cross-Ref:††
§14. NEGATION †2

378. Negation is used (1) logically, (2) metaphysically. In the logical sense it may be used (*a*) relatively, and (*b*) absolutely. Used relatively, when applied to a proposition, it may be understood (α) as denying the proposition, or (β) as denying the predicate.

Peirce: CP 2.379 Cross-Ref:††

379. (1) In its logical sense, negation is opposed to affirmation, although, when it is used relatively, this is perhaps not a convenient contrary term; in its metaphysical sense, negative is opposed to positive (fact, etc.)

Peirce: CP 2.379 Cross-Ref:††

The conception of negation, objectively considered, is one of the most important of logical relations; but subjectively considered, it is not a term of logic at all, but is prelogical. That is to say, it is one of those ideas which must have been fully developed and mastered before the idea of investigating the legitimacy of reasonings could have been carried to any extent.

Peirce: CP 2.379 Cross-Ref:††

The treatment of the doctrine of negation affords a good illustration of the effects of applying the principle of Pragmatism (q. v.)†1 in logic. The pragmatist has in view a definite purpose in investigating logical questions. He wishes to ascertain the general conditions of truth. Now, without of course undertaking to present here the whole development of thought, let it be said that it is found that the first step must be to define how two propositions can be so related that under all circumstances whatsoever,

The truth of the one entails the truth of the other,

The truth of the one entails the falsity of the other,

The falsity of the one entails the truth of the other,

The falsity of the one entails the falsity of the other.

This must be the first part of logic. It is deductive logic, or (to name it by its principal result) syllogistic. At all times this part of logic has been recognized as a necessary preliminary to further investigation. Deductive and inductive or methodological logic have always been distinguished; and the former has generally been called by that name.

Peirce: CP 2.379 Cross-Ref:††

In order to trace these relations between propositions, it is necessary to dissect the propositions to a certain extent. There are different ways in which propositions can be dissected. Some of them conduce in no measure to the solution of the present problem, and will be eschewed by the pragmatist at this stage of the investigation. Such, for example, is that which makes the copula a distinct part of the proposition. It

may be that there are different ways of useful dissection; but the common one, which alone has been sufficiently studied, may be described as follows:

Taking any proposition whatever, as

"Every priest marries some woman to some man,"

we notice that certain parts may be struck out so as to leave a blank form, in which, if the blanks are filled by proper names (of individual objects known to exist), there will be a complete proposition (however silly and false). Such blank forms are, for example:

Every priest marries some woman to --,

-- marries -- to some man,

-- marries -- to --.

Peirce: CP 2.379 Cross-Ref:††

It may be that there is some language in which the blanks in such forms cannot be filled with proper names so as to make perfect propositions, because the syntax may be different for sentences involving proper names. But it does not matter what the rules of grammar may be.

Peirce: CP 2.379 Cross-Ref:††

The last of the above blank forms is distinguished by containing no selective word such as some, every, any, or any expression equivalent in force to such a word. It may be called a Predicate (q.v. [358]) or {rhéma}. Corresponding to every such predicate there is another, such that if all the blanks in the two be filled with the same set of proper names (of individuals known to exist), one of the two resulting propositions will be true, while the other is false; as

Chrysostom marries Helena to Constantine;

Chrysostom non-marries Helena to Constantine.

It is true that the latter is not good grammar; but that is not of the smallest consequence. Two such propositions are said to be contradictories, and two such predicates to be negatives of one another, or each to result from the negation of the other. Two propositions involving selective expressions may be contradictories; but in order to be so, each selective has to be changed from indicating *a suitable selection* to indicating *any selection that may be made*, or vice versa. Thus the two following propositions are contradictories:

Every priest marries some woman to every man;

Some priest non-marries every woman to some man.

It is very convenient to express the negative of a predicate by simply attaching a non to it. If we adopt that plan, *non-non-marries* must be considered as equivalent to *marries*. It so happens that both in Latin and in English this convention agrees with the usage of the language. There is probably but a small minority of languages of the globe in which this very artificial rule prevails. Of two contradictory propositions each is said to result from the *negation* of the other.

Peirce: CP 2.379 Cross-Ref:††

The relation of negation may be regarded as defined by the principles of contradiction and excluded middle. See Laws of Thought [bk. III, ch. 4, §15]. That is an admissible, but not a necessary, point of view. Out of the conceptions of non-relative deductive logic, such as consequence, coexistence or composition, aggregation, impossibility, negation, etc., it is only necessary to select two, and almost any two at that, to have the material needed for defining the others. What ones are to be selected is a question the decision of which transcends the function of this branch of logic. Hence the indisputable merit of Mrs. Franklin's eight copula-signs, which are exhibited as of coördinate formal rank.†1 But, so regarded, they are not properly copulas or assertions of the relation between the several individual subjects and the predicate, but mere signs of the logical relations between different components of the predicate. The logical doctrine connected with those signs is of considerable importance to the theory of pragmatism.

Peirce: CP 2.379 Cross-Ref:††

. . . *Negant* or *negative negation* is the negation effected by attaching the negative particle to the copula in the usual Latin idiom, "Socrates non est stultus," in contradistinction to *infinite* ({aoristé}), or *infinitant, negation*, which is effected by attaching the negative particle to the predicate, "Socrates est non stultus."

Peirce: CP 2.379 Cross-Ref:††

Kant revived this distinction in order to get a triad to make out the symmetry of his table of categories; and it has ever since been one of the deepest and dearest [Elec. editor: sic] studies of German logicians. No idea is more essentially dualistic, and distinctly not triadic, than negation. *Not-A* = *other than A* = *a second thing to A*. Language preserves many traces of this. *Dubius* is between *two* alternatives, yea and nay.

Peirce: CP 2.380 Cross-Ref:††

380. (2) In the metaphysical sense, negation is the mere absence of a character or relation that is regarded as positive. It is distinguished from privation in not implying anything further.

Peirce: CP 2.380 Cross-Ref:††

Spinoza's celebrated saying, of which the Schellings have made so much, "omnis determinatio est negatio," has at least this foundation, that *determinatio* to one alternative excludes us from another. The same great truth is impressed upon youth in the utterance: "You cannot eat your cake and have it too."

Peirce: CP 2.381 Cross-Ref:††
§15. LIMITATIVE †1

381. (1) Applied to a third quality of judgments, additional to affirmative and negative. The idea of such a third quality originated among the Romans from the difference between "homo non est bonus" and "homo est non bonus," the latter being the limitative. . . .

Peirce: CP 2.381 Cross-Ref:††

It is one of the numerous cases in which accidents of language have affected accepted logical forms without any good reason. Boëthius †2 and others applied the infinitation to the subject also, which De Morgan †3 has shown makes a valuable addition to logic. Wolff, †4 however, limited the modification to the predicate, without showing any serious reason for such application. Kant adopted it because it rounded out his triad of categories of quality. His defence, as reported by Jäsche, is that the negative excludes the subject from the sphere of the predicate, while the *unendliche*, limitative, or infinite judgment puts it into the infinite sphere outside the predicate. It is to be remarked that Kant regards a positive mark as differing *per se* from a negative one, and, in particular, as having a far narrower extension. Like most of the old logicians, he virtually limited the universe of marks to such as arrest our attention. If that had been explicitly and consistently done, it would have constituted an interesting particular logic, in which there would be a material and not merely formal difference between affirmative and negative facts. It is probable that Kant also understood the affirmative proposition to assert the existence of its subject, while the negative did not do so; so that "Some phoenixes do not rise from their ashes" would be true, and "All phoenixes do rise from their ashes" would be false. The limitative judgment would agree with the affirmative in this respect. This was probably his meaning, and he did not observe that his limitative judgment, "The human soul is immortal" (*nichtsterblich*), may be construed as equivalent to the conjunctive judgment, "The human soul is not mortal, and it is the human soul." No doubt Kant would have seen a world of difference between these two assertions. In that case he should have adopted a fourth quality, "The human soul is not immortal."

Peirce: CP 2.382 Cross-Ref:††
§16. MODALITY †1

382. There is no agreement among logicians as to what modality consists in; but it is the logical qualification of a proposition or its copula, or the corresponding qualification of a fact or its form, in the ways expressed by the modes *possibile*, *impossibile*, *contingens*, *necessarium*.

Peirce: CP 2.382 Cross-Ref:††

Any qualification of a predication is a mode; and Hamilton says (*Lects. on Logic*, xiv) that "all logicians" call any proposition affected by a mode a modal proposition. This, however, is going much too far; for not only has the term usually been restricted in practice, from the age of Abelard, when it first appeared, †2 until

now, to propositions qualified by the four modes "possible," "impossible," "necessary," and "contingent," with only occasional extension to any others, but positive testimonies to that effect might be cited in abundance.

Peirce: CP 2.382 Cross-Ref:††

The simplest account of modality is the scholastic, according to which the necessary (or impossible) proposition is a sort of universal proposition; the possible (or contingent, in the sense of not necessary) proposition, a sort of particular proposition. That is, to assert "*A* must be true" is to assert not only that *A* is true, but that all propositions analogous to *A* are true; and to assert "*A* may be true" is to assert only that *some* proposition analogous to *A* is true. If it be asked what is here meant by analogous propositions, the answer is--all those of a certain class which the conveniences of reasoning establish. Or we may say the propositions analogous to *A* are all those propositions which in some conceivable state of ignorance would be indistinguishable from *A*. Error is to be put out of the question; only ignorance is to be considered. This ignorance will consist in its subject being unable to reject certain potentially hypothetical states of the universe, each absolutely determinate in every respect, but all of which are, in fact, false. The aggregate of these unrejected falsities constitute the "range of possibility," or better, "of ignorance." Were there no ignorance, this aggregate would be reduced to zero. The state of knowledge supposed is, in necessary propositions, usually fictitious, in possible propositions more often the actual state of the speaker. The necessary proposition asserts that, in the assumed state of knowledge, there is no case in the whole range of ignorance in which the proposition is false. In this sense it may be said that an impossibility underlies every necessity. The possible proposition asserts that there is a case in which it is true.

Peirce: CP 2.382 Cross-Ref:††

Various subtleties are encountered in the study of modality. Thus, when the thinker's own state of knowledge is the one whose range of ignorance is in question, the judgments "*A* is true" and "*A* must be true," are not logically equivalent, the latter asserting a fact which the former does not assert, although the fact of its assertion affords direct and conclusive evidence of its truth. The two are analogous to "*A* is true" and "*A* is true, and I say so"; which are readily shown not to be logically equivalent by denying each, when we get "*A* is false" and "If *A* is true, I do not say so."

Peirce: CP 2.382 Cross-Ref:††

In the necessary particular proposition and the possible universal proposition there is sometimes a distinction between the "composite" and "divided" senses. "Some *S* must be *P*," taken in the composite sense, means that there is no case, in the whole range of ignorance, where some *S* or other is not *P*; but taken in the divided sense, it means that there is some *S* which same *S* remains *P* throughout the whole range of ignorance. So "Whatever *S* there may be may be *P*," taken in the composite sense, means that there is, in the range of ignorance, some hypothetic state of things (or it may be the unidentifiable true state, though this can hardly be the only such case) in which there either is no *S*, or every *S* there is is *P*; while in the divided sense, it means that there is no *S* at all in any hypothetic state but what in some hypothetic state or other is *P*. When there is any such distinction, the divided sense asserts more than the composite in necessary particular propositions, and less in possible universal. But in most cases the individuals do not remain identifiable throughout the range of possibility, when the distinction falls to the ground. It never applies to necessary universal propositions or to possible particular propositions.

Peirce: CP 2.383 Cross-Ref:††

383. Some logicians say that "*S* may be *P*" is not a proposition at all, for it asserts nothing. But if it asserted nothing, no state of facts could falsify it, and consequently the denial of it would be absurd. Now let *S* be "some self-contradictory proposition," and let *P* be "true." Then the possible proposition is "Some self-contradictory proposition may be true," and its denial is "No self-contradictory proposition can be true," which can hardly be pronounced absurd. It is true that those logicians usually take the form "*S* may be *P*" in the copulative sense "*S* may be *P*, and *S* may not be *P*," but this only makes it assert *more*, not *less*. The possible proposition, then, is a proposition. It not only must be admitted among logical forms, if they are to be adequate to represent all the facts of logic, but it plays a particularly important part in the theory of science. See *Scientific Method*, [vol. 7]. At the same time, according to the view of modality now under consideration, necessary and possible propositions are equipollent with certain assertory propositions; so that they do not differ from assertory propositions as universal and particular propositions differ from one another, but rather somewhat as hypothetical (i.e., conditional, copulative, and disjunctive), categorical, and relative propositions differ from one another--perhaps not quite so much.

Peirce: CP 2.383 Cross-Ref:††

According to this view, logically necessary and possible propositions relate to what might be known, without any knowledge whatever of the universe of discourse, but only with a perfectly distinct understanding of the meanings of words; geometrically necessary and possible propositions, to what a knowledge of the properties of space does or does not exclude; physical necessity, to what a knowledge of certain principles of physics does or does not exclude, etc. But when we say that of two collections one must be correspondentially greater than the other, but each cannot be correspondentially greater than the other, it has not been shown how this kind of necessity can be explained on the above principles.

Peirce: CP 2.384 Cross-Ref:††

384. The earliest theory of modality is Aristotle's, whose philosophy, indeed, consists mainly in a theory of modality. The student of Aristotle usually begins with the *Categories*; and the first thing that strikes him is the author's unconsciousness of any distinction between grammar and metaphysics, between modes of signifying and modes of being. When he comes to the *metaphysical* books, he finds that this is not so much an oversight as an assumed axiom; and that the whole philosophy regards the existing universe as a performance which has taken its rise from an antecedent ability. It is only in special cases that Aristotle distinguishes between a possibility and an ability, between a necessity and a constraint. In this, he is perhaps nearer the truth than the system of equipollencies set forth above.

Peirce: CP 2.385 Cross-Ref:††

385. Kant seems to have been the first to throw any light upon the subject. To the old distinction between logical and real possibility and necessity he applied two new pairs of terms, analytic and synthetic, and subjective and objective. The following definitions (where every word is studied) certainly advanced the subject greatly:

Peirce: CP 2.385 Cross-Ref:††

"1. Was mit den formalen Bedingungen der Erfahrung (der Anschauung und den Begriffen nach) übereinkommt, ist *möglich*."

Peirce: CP 2.385 Cross-Ref:††

"2. Was mit den materialen Bedingungen der Erfahrung (der Empfindung) zusammenhängt, ist *wirklich*.

Peirce: CP 2.385 Cross-Ref:††

"3. Dessen Zusammenhang mit dem Wirklichen nach allgemeinen Bedingungen der Erfahrung bestimmt ist, ist (existiert) *nothwendig*" (*Krit. d. reinen Vernunft*, 1st ed., 219).

Peirce: CP 2.385 Cross-Ref:††

Kant holds that all the general metaphysical conceptions applicable to experience are capable of being represented as in a diagram, by means of the image of time. Such diagrams he calls "schemata." The schema of the possible he makes to be the figure of anything at any instant. The schema of necessity is the figure of anything lasting through all time (*ibid.*, 144, 145). He further states (*ibid.*, 74, footnote; Jäsche's *Logik*, Einl. ix, and elsewhere) that the possible proposition is merely conceived but not judged, and is a work of the apprehension (*Verstand*); that the assertory proposition is judged, and is, so far, a work of the judgment; and that the necessary proposition is represented as determined by law, and is thus the work of the reason (*Vernunft*). He maintains that his deduction of the categories shows that, and how, the conceptions originally applicable to propositions can be extended to modes of being--constitutively, to being having reference to possible experience; regulatively, to being beyond the possibility of experience.

Peirce: CP 2.386 Cross-Ref:††

386. Hegel considers the syllogism to be the fundamental form of real being. He does not, however, undertake to work over, in the light of this idea, in any fundamental way, what is ordinarily called logic, but which, from his point of view, becomes merely subjective logic. He simply accepts Kant's table of functions of judgment, which is one of the most ill-considered performances in the whole history of philosophy. Consequently, what Hegel says upon this subject must not be considered as necessarily representing the legitimate outcome of his general position. His followers have been incompetent to do more. Rosenkranz (*Wissenschaft d. logischen Idee*†1) makes modality to represent the superseding of the form of the judgment and to be the preparation for that of the syllogism. In the *Encyclopädie*, Hegel's last statement, §§178-80, we are given to understand that the judgment of the *Begriff* has for its contents the totality (or, say, conformity to an ideal). In the first instance, the subject is singular, and the predicate is the reflection of the particular object upon the universal. That is, this or that object forced upon us by experience is judged to conform to something in the realm of ideas. But when this is doubted, since the subject does not, in itself, involve any such reference to the ideal world, we have the "possible" judgment, or judgment of doubt. But when the subject is referred to its genus, we get the apodictic judgment. But Hegel had already developed the ideas of possibility and necessity in the objective logic as categories of *Wesen*. In the *Encyclopädie* the development is somewhat as follows: *Wirklichkeit* is that whose mode of being consists in self-manifestation. As identity in general (the identity of *Sein* and *Existenz*) it is, in the first instance, possibility. That is to say, apparently, bare possibility, any fancy projected and regarded in the aspect of a fact. It is possible, for example, that the present Sultan may become the next Pope. But in the second movement arise the conceptions of the *Zufällig*, *Ausserlichkeit*, and "condition." The *Zufällig* is that which is recognized as merely possible: "A may be,

but *A* may not be"; but it is also described by Hegel as that which has the *Grund*, or antecedent of its being, in something other than itself. The *Aeusserlichkeit* seems to be the having a being outside the ground of its being--an idea assimilated to caprice. That which such *Aeusserlichkeit* supposes outside of itself, as the antecedent of its being, is the presupposed condition. The third movement gives, in the first instance, "real possibility." In this we find the conceptions of "fact" (*Sache*), "activity" (*Thätigkeit*), and "necessity."

Peirce: CP 2.387 Cross-Ref:††

387. Lotze and Trendelenburg represent the first struggles of German thought to rise from Hegelianism. The most remarkable characteristic of Lotze's thought is, that he not only sees no urgency for unity of conception in philosophy, but holds that such unity would inevitably involve a falsity.†1 He represents a judgment as a means of apprehending becoming, in opposition to the concept, which apprehends being; but he says that the business of the judgment is to supply the cement for building up concepts. Accordingly, he has no doctrine of modality as a whole, but merely considers three cases, between which he traces no relation. Necessity may arise either out of the universal analytic judgment, the conditional judgment, or the disjunctive judgment. By the "judgment" is meant the meaning of a proposition. Lotze finds that the meaning of the analytical judgment is illogical, since it identifies contraries. However, the meaning of this meaning is justified by its not meaning to mean that the terms are identical, but only that the objects denoted by those terms are identical. The analytic proposition is, therefore, admissible, because it is practically meant to mean a particular proposition, that is, one in which the predicate is asserted of all the particulars. And the justification of the proposition, whose use was to be to connect elements of terms, is that, meant not as it is meant, but as it is meant to be meant, these elements are identical and do not need to be connected. In this way Lotze vindicates the necessity of the analytical categorical proposition. Coming next to conditionals, by thought of the same order, he finds that, assuming that the universe of real, intelligible objects is "coherent," we may be justified in asserting that the introduction of a condition *X* into a subject *S* gives rise to a predicate *P* as an analytical necessity; and for this purpose, when it is once accomplished, it does not matter whether the ladder of the assumption of coherence remains or is taken away. Lotze treats the disjunctive proposition last, as if it were of a higher order, following Hegel in this respect. But what was excusable for Hegel is less so for Lotze, since he himself had signalized the significance of impersonal propositions, such as "it rains," "it thunders," "it lightens," whose only subject is the universe. Now, if there is any difference between "If it lightens, it thunders," and "Either it does not lighten or it thunders," it is that the latter considers the actual state of things alone, and the former a whole range of other possibilities. However, Lotze considers last the propositional form "*S* is *P*[1] or *P*[2] or *P*[3]." Properly, this is not a disjunctive proposition, but only a proposition with a disjunctive predicate. Lotze considers it a peculiar form, because it cannot be represented by an Euler's diagram, which is simply a blunder. The necessity to which it gives rise must, therefore, either be the same as the conditional necessity, or else differ from it merely by greater simplicity. For other sound objections to Lotze's theory see Lange, *Logische Studien*, ii.

Peirce: CP 2.388 Cross-Ref:††

388. Trendelenburg (*Logische Untersuch.*, xiii) maintains that possibility and necessity can only be defined in terms of the antecedent (*Grund*), though he might, perhaps, object to the translation of *Grund* by so purely formal a word as "antecedent," notwithstanding its harmony with Aristotle. If all conditions are

recognized, and the fact is understood from its entire *Grund*, so that thought quite permeates being--a sort of phrase which Trendelenburg always seeks--there is "necessity." If, on the other hand, only some conditions are recognized, but what is wanting in *Grund* is made up in thought, there is "possibility." In itself, an egg is nothing but an egg, but for thought it may become a bird. Trendelenburg will, therefore, neither admit, with Kant, that modality is originally a mere question of the attitude of the mind, nor with Hegel, whom he criticizes acutely, that it is originally objective.

Peirce: CP 2.389 Cross-Ref:††

389. Sigwart, who holds that logical questions must ultimately be decided by immediate feeling, and that the usages of the German language are the best evidence of what that feeling is, denies that the possible proposition is a proposition at all, because it asserts nothing.†1 He forgets that if a proposition asserts nothing, the denial of it must be absurd, since it must exclude every possibility. Now, the denial of "I do not know but that *A* may be true" is "I know *A* is not true," which is hardly absurd. Sigwart, it is true, in accordance with usages of speech, takes "A may be true" in what the old logicians called the *sensus usualis*, that is, for the copulative proposition "A may be true, and further A may be not true." But this does not make it assert *less*, but *more*, than the technical form. In regard to the necessary proposition, Sigwart, following his guide, the usages of speech, finds that, "A must be true" asserts less than "A is true," so that from the latter the former follows, but not at all the latter from the former. This may be true for the usages of German speech, just as such phrases as "beyond every shadow of doubt," "out of all question," and the like, in our vernacular commonly betray the fact that there is somebody who not only doubts and questions, but flatly denies, the proposition to which they are attached. Bradley †2 accepts the sensational discovery of Sigwart.

Peirce: CP 2.390 Cross-Ref:††

390. Lange (loc. cit.) thinks the matter is put in the clearest light by the logical diagrams usually attributed to Euler, but really going back to Vives. "We, therefore, here again see," he says, "how spatial intuition, just as in geometry, verifies (*begründet*) a priority and necessity."

Peirce: CP 2.391 Cross-Ref:††

CHAPTER 5

TERMS†1E

§1. THAT THESE CONCEPTIONS ARE NOT SO MODERN AS HAS BEEN REPRESENTED

391. The historical account usually given of comprehension and extension is this, "that the distinction, though taken in general terms by Aristotle,†P1 and explicitly announced with scientific precision by one, at least, of his Greek

commentators, had escaped the marvellous acuteness of the schoolmen, and remained totally overlooked and forgotten till the publication of the Port Royal Logic."†P2 I would offer the following considerations to show that this interpretation of history is not exactly true. In the first place, it is said that a distinction was taken between these attributes, as though they were previously confounded. Now there is not the least evidence of this. A German logician,†P3 has, indeed, by a subtle misconception, considered extension as a species of comprehension, but, to a mind beginning to reflect, no notions seem more unlike. The mental achievement has been the bringing of them into relation to one another, and the conception of them as factors of the import of a term, and not the separation of them. In the second place, it is correctly said that the doctrine taught by the Port Royalists is substantially contained in the work of a Greek commentator. That work is no other than Porphyry's *Isagoge*;†P1 and therefore it would be most surprising if the doctrine had been totally overlooked by the schoolmen, for whether their acuteness was as marvellous as Hamilton taught or not, they certainly studied the commentary in question as diligently as they did the Bible.†P2 It would seem, indeed, that the tree of Porphyry involves the whole doctrine of extension and comprehension except the names. Nor were the scholastics without names for these quantities. The *partes subjectives* and *partes essentielles* are frequently opposed; and several other synonyms are mentioned by the Conimbricenses. It is admitted that Porphyry fully enunciates the doctrine; it must also be admitted that the passage in question is fully dealt with and correctly explained by the mediaeval commentators. The most that can be said, therefore, is that the doctrine of extension and comprehension was not a prominent one in the mediaeval logic.†P3 [What the Port Royalists and later, still more decidedly, the Kantians, preached was the equivalence of the logical character of all essential predicates. They fused genera and differences while Porphyry, after Aristotle, is bent upon the discrimination of different kinds of predicates--1893.]

Peirce: CP 2.392 Cross-Ref:††

392. A like degree of historical error is commonly committed in reference to another point which will come to be treated of in this paper, allied, at least, as it is most intimately, with the subject of comprehension and extension, inasmuch as it also is founded on a conception of a term as a whole composed of parts--I mean the distinction of clear and distinct. Hamilton tells us: "We owe the discrimination to the acuteness of the great Leibniz. By the Cartesians the distinction had not been taken; though the authors of the Port Royal Logic came so near that we may well marvel how they failed explicitly to enounce it." (*Lectures on Logic*; Lecture IX.) Now, in fact, all that the Port Royalists say about this matter †P1 is copied from Descartes,†P2 and their variations from his wording serve only to confuse what in him is tolerably distinct. As for Leibniz, he himself expressly avows that the distinction drawn by Descartes is the same as his own.†P3 Nevertheless, it is very much more clear with Leibniz than with Descartes. A philosophical distinction emerges gradually into consciousness; there is no moment in history before which it is altogether unrecognized, and after which it is perfectly luminous. Before Descartes, the distinction of confused and distinct had been thoroughly developed, but the difference between distinctness and clearness is uniformly overlooked. Scotus distinguishes between conceiving confusedly and conceiving the confused, and since any obscure concept necessarily includes more than its proper object, there is always in what is obscurely conceived a conception of something confused; but the schoolmen came no nearer than this to the distinction of Descartes and Leibniz.

§2. OF THE DIFFERENT TERMS APPLIED TO THE QUANTITIES OF EXTENSION AND COMPREHENSION

393. Extension and comprehension are the terms employed by the Port Royalists. Owing to the influence of Hamilton, *intension* is now frequently used for comprehension;†P4 but it is liable to be confounded with intensity, and therefore is an objectionable word. It is derived from the use of cognate words by Cajetan and other early writers. *External* and *internal quantity* are the terms used by many early Kantians. *Scope* and *force* are proposed by De Morgan. *Scope* in ordinary language expresses extension, but *force* does not so much express comprehension as the power of creating a lively representation in the mind of the person to whom a word or speech is addressed. Mr. J. S. Mill has introduced the useful verbs *denote* and *connote*, which have become very familiar. It has been, indeed, the opinion of all †1 students of the logic of the fourteenth, fifteenth, and sixteenth centuries, that *connotation* was in those ages used exclusively for the reference to a second significate, that is (nearly) for the reference of a relative term (such as *father*, *brighter*, etc.) to the correlate of the object which it primarily denotes, and was never taken in Mill's sense of the reference of a term to the essential characters implied in its definition.†P1 Mr. Mill has, however, considered himself entitled to deny this upon his simple dictum,†2 without the citation of a single passage from any writer of that time.†P2 After explaining the sense in which he takes the term *connote*, Mill says: "The schoolmen, to whom we are indebted for the greater part of our logical language, gave us this also, and in this very sense. For though some of their general expressions countenance the use of the word in the more extensive and vague acceptance in which it is taken by Mr. [James] Mill, yet when they had to define it specifically as a technical term, and to fix its meaning as such, with that admirable precision which always characterized their definitions, they clearly explained that nothing was said to be connoted except *forms*, which word may generally, in their writings, be understood as synonymous with *attributes*."†3 As scholasticism is usually said to come to an end with Occam, this conveys the idea that *connote* was commonly employed by earlier writers. Yet the celebrated Prantl †P3 considers it conclusive proof that a passage in Occam's *Summa* is spurious, that *connotative* is there spoken of as a term in frequent use;†P1 and remarks upon a passage of Scotus in which *connotatum* is found, that this conception is here met with for the first time.†P2 The term occurs, however, in Alexander of Hales,†P3 who makes *nomen connotans* the equivalent of *appellatio relativa*, and takes the relation itself as the object of *connotare*, speaking of creator as connoting the relation of creator to creature.†P4 Occam's *Summa*†P5 contains a chapter devoted to the distinction of absolute and connotative names. The whole deserves to be read, but I have only space to quote the following: "Nomen autem connotativum est illud quod significat aliquid primario et aliquid secundario; et tale nomen proprie habet diffinitionem exprimentem quid nominis et frequenter oportet ponere aliquid illius diffinitionis in recto et aliud in obliquo; sicut est de hoc nomine album, nam habet diffinitionem exprimentem quid nominis in qua una dictio ponitur in recto et alia in obliquo. Unde si queratur quid significat hoc nomen album, dices quod idem quod illa oratio tota 'aliquid informatum albedine' vel 'aliquid habens albedinem' et patet quod una pars orationis istius ponitur in recto et alia in obliquo. . . . Huiusmodi autem nomina connotativa sunt omnia nomina concreta primo modo dicta, et hoc *quia talia concreta significant unum in recto et aliud in obliquo*, hoc

est dictu, in diffinitione exprimente quid nominis debet poni unus rectus significans unam rem et alius obliquus significans aliam rem, sicut patet de omnibus talibus, iustus, albus, animatus, et sic de aliis. Huiusmodi etiam nomina sunt omnia nomina relatiua, quia semper in eorum diffinitionibus ponuntur diversa idem diuersis modis vel diuersa significantia, sicut patet de hoc nomine simile. Mere autem absoluta sunt illa quae non significant aliquid principaliter et aliud vel idem secundario, sed quicquid significatur per tale nomen aequè primo significatur sicut patet de hoc nomine animal."†1 Eckius, in his comment on Petrus Hispanus, has also some extended remarks on the signification of the term *connote*, which agree in the main with those just quoted.†P1 Mr. Mill's historical statement †P2 cannot, therefore, be admitted.

Peirce: CP 2.394 Cross-Ref:††

394. Sir William Hamilton has borrowed from certain late Greek writers the terms *breadth* and *depth*, for extension and comprehension respectively.†P3 These terms have great merits. They are brief; they are suited to go together; and they are very familiar. Thus, "wide" learning is, in ordinary parlance, learning of many things; "deep" learning, much knowledge of some things. I shall, therefore, give the preference to these terms. Extension is also called *sphere* and *circuit*; and comprehension, *matter* and *content*.

Peirce: CP 2.395 Cross-Ref:††

§3. OF THE DIFFERENT SENSES IN WHICH THE TERMS EXTENSION AND COMPREHENSION HAVE BEEN ACCEPTED

395. The terms *extension* and *comprehension*, and their synonyms, are taken in different senses by different writers. This is partly owing to the fact that while most writers speak only of the extension and comprehension of concepts, others apply these terms equally to concepts and judgments (Rösling)†2, others to any mental representation (Überweg †3 and many French writers), others to cognition generally (Baumgarten †4), others to "terms" (Fowler,†5 Spalding †6), others to names (Shedden †7); others to words (McGregor †8), others to "meanings" (Jevons),†1 while one writer speaks only of the extension of *classes* and the comprehension of *attributes* (De Morgan in his Syllabus [§131]).

Peirce: CP 2.396 Cross-Ref:††

396. Comprehension is defined by the Port Royalists as "those attributes which it [an idea] involves in itself, and which cannot be taken away from it without destroying it."†2

Peirce: CP 2.396 Cross-Ref:††

It will be remembered that the *marks* of a term are divided by logicians first into the necessary and the accidental, and that then the necessary marks are subdivided into such as are strictly essential, that is, contained in the definition, and such as are called proper. Thus, it is an essential mark of a triangle to have three sides; it is a proper mark to have its three angles equal to two right angles; and it is an accidental mark to be treated of by Euclid. The definition of the Port Royalists, therefore, makes comprehension include all necessary marks, whether essential or proper.

Peirce: CP 2.397 Cross-Ref:††

397. The Port Royalists attribute comprehension immediately to any ideas. Very many logicians attribute it immediately only to concepts. Now a concept, as defined by them, is strictly only the essence of an idea; they ought therefore to include in the comprehension only the essential marks of a term. These logicians, however, abstract so entirely from the real world, that it is difficult to see why these essential marks are not at the same time all the marks of the object as they suppose it.

Peirce: CP 2.398 Cross-Ref:††

398. There can, I think, be no doubt that such writers as Gerlach †3 and Sigwart †4 make comprehension include all marks, necessary or accidental, which are universally predicable of the object of the concept.

Peirce: CP 2.398 Cross-Ref:††

Again, most German writers regard the comprehension as a sum either of concepts (Drobisch, †5 Bachmann, †6 etc.) or of elements of intuition (Trendelenburg). †7 But many English writers regard it as the sum of real external attributes (Shedden, †8 Spalding, †1 Devey, †2 De Morgan, †3 Jevons, †4 McGregor, †5 Fowler). †6 According to most writers, comprehension consists of the (necessary) attributes *thought* as common to the objects. Shedden defines it as consisting of all the attributes common to the things denoted.

Peirce: CP 2.398 Cross-Ref:††

Again, most logicians consider as marks only such as are virtually †P1 predicated; a few, perhaps, only such as are actually thought, and still fewer include those which are habitually thought. Here and there is found an author who makes comprehension include all true attributes, whether thought or not.

Peirce: CP 2.398 Cross-Ref:††

There is also a difference in the mode of reckoning up the marks. Most writers count all distinguishable marks, while a few consider coextensive marks as the same.

Peirce: CP 2.399 Cross-Ref:††

399. In the use of the term "extension" the want of a definite convention is still more marked. The Port Royalists define it as "those *subjects* to which the idea applies." †7 It would appear, therefore, that it might include mere fictions.

Peirce: CP 2.399 Cross-Ref:††

Others limit the term to *real* species, and at the same time extend it to single beings. This is the case with Watts, †8 and also with Friedrich Fischer. †9

Peirce: CP 2.399 Cross-Ref:††

Others are most emphatic in declaring that they mean by it *things*, and not species, real or imaginary. This is the case with Bachmann, †10 Esser, †11 and Schulze. †12

Peirce: CP 2.399 Cross-Ref:††

Others make it include neither concepts nor things, but singular representations. This is the case with the strict Kantian. The following table exhibits this diversity:

Extension embraces

Individual representations, according to Kant,†1 E. Reinhold,†2 etc.
Representations, according to Fries,†3 Überweg,†4 etc.
Real external things and species, according to Watts,†5 Shedden,†6 etc.
Real external individual objects, according to Bachmann,†7 Devey,†8 etc.
Things, according to Schulze,†9 Bowen,†10 etc.
Species, according to Drobisch,†11 De Morgan,†12 etc.
Objects (representations), according to Thomson,†13 etc.
Individuals, according to Mahan.†14
Concepts, according to Herbart,†15 Vorländer,†16 etc.
General terms, according to Spalding.†17
Psychical concepts, according to Strümpell.†18
Variable marks, according to Ritter.†19

Peirce: CP 2.399 Cross-Ref:††

Again, logicians differ as to whether by extension they mean the concepts, species, things, or representations to which the term is habitually applied in the judgment, or all to which it is truly applicable. The latter position is held by Herbart, Kiesewetter, etc.; the former by Duncan, Spalding, Vorländer, Überweg, etc.

Peirce: CP 2.399 Cross-Ref:††

Some logicians include only *actual* things, representations, etc., under extension (Bachmann, Fries, Herbart); others extend it to such as are merely possible (Esser, Ritter, Gerlach).

Peirce: CP 2.399 Cross-Ref:††

Finally, some few logicians speak of the two quantities as numerical, while most writers regard them as mere aggregates of diverse objects or marks.

Peirce: CP 2.399 Cross-Ref:††

[Dressler, following a hint from Beneke, distinguishes real and ideal extension and comprehension.--1893.]

Peirce: CP 2.400 Cross-Ref:††

§4. DENIALS OF THE INVERSE PROPORTIONALITY OF THE TWO QUANTITIES, AND SUGGESTIONS OF A THIRD QUANTITY

400. Until lately the law of the inverse proportionality of extension and comprehension was universally admitted.†P1 It is now questioned on various grounds.†P2

Peirce: CP 2.401 Cross-Ref:††

401. Drobisch †1 says that the comprehension varies arithmetically, while the extension varies geometrically. This is true, in one sense.

Peirce: CP 2.402 Cross-Ref:††

402. Lotze, after remarking that the only conception of a universal which we can have is the power of imagining singulars under it, urges that the possibility of determining a concept in a way corresponding to each particular under it is a mark of that concept, and that therefore the narrower concepts have as many marks as the wider ones. But, I reply, *these* marks belong to the concept in its second intention, and are not common marks of those things to which it applies, and are therefore no part of the comprehension. They are, in fact, the very marks which constitute the extension. No one ever denied that extension is a mark of a concept; only it is a certain mark of second intention.

Peirce: CP 2.403 Cross-Ref:††

403. Vorländer's †1 objection is much more to the purpose. It is that, if from any determinate notion, as that of Napoleon, we abstract all marks, all determination, what remains is merely the conception *something*, which has no more extension than Napoleon. "Something" has an uncertain sphere, meaning either this thing or that or the other, but has no general extension, since it means one thing only. Thus, before a race, we can say that some horse will win, meaning this one, that one, or that one; but by some horse we mean but one, and it therefore has no more extension than would a term definitely indicating which--although this latter would be more determinate, that is, would have more comprehension. I am not aware that those who adhere to Kant's unmodified doctrine have succeeded in answering this objection.

Peirce: CP 2.404 Cross-Ref:††

404. Überweg has the following remarks:†P1 "To the higher representation, since conformably to its definition it contains only the common elements of content of several lower representations, belongs in comparison to each of the lower a more limited content, but a wider circuit. The lower representation, on the contrary, has a richer content but narrower circuit. Yet by no means by every diminution or increase of a given content does the circuit increase or diminish, nor by every increase or diminution of a given circuit does the content diminish or increase." I am surprised that he does not explain himself further upon this point, which it is the principal object of this paper to develop.

Peirce: CP 2.405 Cross-Ref:††

405. De Morgan says:†P2 "According to such statements as I have seen, 'man, residing in Europe, drawing breath north of the equator, seeing the sun rise before those in America,' would be a more intensively quantified notion than 'man residing in Europe'; but certainly not less *extensive*, for the third and fourth elements of the notion must belong to those men to whom the first and second belong." Mr. De Morgan adopts the definitions of extension and comprehension given by the Port Royalists. According to those definitions, if the third and fourth elements necessarily belong to the notion to which the first and second belong, they are parts of the comprehension of that second notion which is composed of the first and second elements, and therefore the two notions are equal in comprehension; but if this is not the case, then the second notion can be predicated of subjects of which the first cannot, for example, of "man residing in Europe drawing breath south of the Equator"; for that there is really no such man will not affect the truth of the

proposition, and therefore the second notion is more extensive than the first.

Peirce: CP 2.406 Cross-Ref:††

406. Two logicians only, as far as I remember, Archbishop Thomson †P1 and Dr. W. D. Wilson, †P2 while apparently admitting Kant's law, wish to establish a third quantity of concepts. Neither gentleman has defined his third quantity, nor has stated what its relations to the other two are. Thomson calls his Denomination. It seems to be the same as Extension regarded in a particular way. Dr. Wilson terms his new quantity Protension; it has something to do with time, and appears to be generally independent of the other two. It is plain, indeed, that as long as Kant's law holds, and as long as logical quantities can only be compared as being more or less and not directly measured, and as long as the different *kinds* of quantity cannot be compared at all, a third quantity must be directly proportional to one or other of the known quantities, and therefore must measure the same thing, or else must be independent of the other two, and be quite unconnected with them.

Peirce: CP 2.407 Cross-Ref:††

§5. THREE PRINCIPAL SENSES IN WHICH COMPREHENSION AND EXTENSION WILL BE TAKEN IN THIS PAPER †1

407. I shall adopt Hamilton's terms, *breadth* and *depth*, for extension and comprehension respectively, and shall employ them in different senses, which I shall distinguish by different adjectives.

Peirce: CP 2.407 Cross-Ref:††

By the *informed breadth* of a term, †P1 I shall mean all the real things of which it is predicable, with logical truth on the whole in a supposed state of information. †P2 By the phrase "on the whole" I mean to indicate that all the information at hand must be taken into account, and that those things only of which there is on the whole reason to believe that a term is truly predicable are to be reckoned as part of its breadth. †1

Peirce: CP 2.407 Cross-Ref:††

If T be a term which is predicable only of S', S'', and S''', then the S' 's, the S'' 's, and the S''' 's, will constitute the informed breadth of T. If at the same time, S' and S'' are the subjects of which alone another term T' can be predicated, and if it is not known that all S''' 's are either S' or S'', then T is said to have a greater informed breadth than T'. If the S''' 's are known not to be all among the S' 's and S'' 's, this excess of breadth may be termed *certain*, and, if this is not known, it may be termed *doubtful*. If there are known to be S''' 's, not known to be S' 's or S'' 's, T is said to have a greater actual breadth than T'; but if no S''' 's are known except such are known to be S' 's, and S'' 's (though there may be others), T is said to have a greater *potential* breadth than T'. If T and T' are conceptions in different minds, or in different states of the same mind, and it is known to the mind which conceives T that every S''' is either S'' or S', then T is said to be more *extensively distinct* than T'. †P3

Peirce: CP 2.408 Cross-Ref:††

408. By the informed depth of a term, I mean all the real characters (in contradistinction to mere names) which can be predicated of it †P1 (with logical truth, on the whole) in a supposed state of information, no character being counted twice

over knowingly in the supposed state of information. The depth, like the breadth, may be certain or doubtful, actual or potential, and there is a comprehensive distinctness corresponding to extensive distinctness.

Peirce: CP 2.409 Cross-Ref:††

409. The informed breadth and depth suppose a state of information which lies somewhere between two imaginary extremes. These are, first, the state in which no fact would be known, but only the meaning of terms; and, second, the state in which the information would amount to an absolute intuition of all there is, so that the things we should know would be the very substances themselves, and the qualities we should know would be the very concrete forms themselves. This suggests two other sorts of breadth and depth corresponding to these two states of information, and which I shall term respectively the *essential*†P2 and the *substantial* breadth and depth.

Peirce: CP 2.410 Cross-Ref:††

410. By the *essential depth* of a term, then, I mean the really conceivable qualities predicated of it in its definition.

Peirce: CP 2.411 Cross-Ref:††

411. The defined term will not perhaps be applicable to any real objects whatever. Let, for example, the definition of the term T be this,

Any T is both P' and P" and P'''

then this sums up its whole meaning; and, as it may not be known that there is any such thing as P', the meaning of T does not imply that it exists. On the other hand, we know that neither P', P'', nor P''' is coextensive with the whole sphere of being. For they are determinate qualities, and it is the very meaning of being that it is indeterminate, that is, is more extensive than any determinate term. In fact, P', for example, is a real notion which we never could have except by means of its contrast to something else. Hence we must know that

Whatever is not-P' is not-T,

Whatever is not-P'' is not-T,

and Whatever is not-P''' is not-T.

Peirce: CP 2.412 Cross-Ref:††

412. Thus, if we define the *essential breadth* of a term as those real things of which, according to its very meaning, a term is predicable, not-T has an essential breadth. We may therefore divide all terms into two classes, the essentially affirmative, or positive, and the essentially negative; of which the former have essential depth, but no essential breadth, and the latter essential breadth, but no essential depth.†P1 It must be noted, however, that this division is not the same as the similar one which language makes. For example, *being*, according to this, is an essentially negative term, inasmuch as it means that which can be predicated of

whatever you please, and so has an essential breadth; while *nothing* is an essentially positive term, inasmuch as it means that of which you are at liberty to predicate what you please, and therefore has an essential depth. The essential subjects of being cannot be enumerated, nor the essential predicates of nothing.

Peirce: CP 2.413 Cross-Ref:††

413. In essential breadth or depth, no two terms can be equal; for, were that the case, the two terms would have the same meaning, and therefore, for logical purposes, would be the same term. Two terms may have unknown relations in these quantities, on account of one or other of them not being distinctly conceived.

Peirce: CP 2.414 Cross-Ref:††

414. *Substantial breadth* is the aggregate of real substances of which alone a term is predicable with absolute truth. *Substantial depth* is the real concrete form which belongs to everything of which a term is predicable with absolute truth.

Peirce: CP 2.415 Cross-Ref:††

415. General terms denote several things. Each of these things has in itself no qualities, but only a certain concrete form which belongs to itself alone. This was one of the points brought out in the controversy in reference to the nature of universals.†P2 As Sir William Hamilton [*Discussions*, Amer. Ed. p. 630] says, not even the humanity of Leibniz belongs to Newton, but a different humanity. It is only by abstraction, by an oversight, that two things can be said to have common characters. Hence, a general term has no *substantial depth*. On the other hand, particular terms, while they have *substantial depth*, inasmuch as each of the things, one or other of which is predicated of them, has a concrete form, yet have no *substantial breadth*, inasmuch as there is no aggregate of things to which alone they are applicable. In order to place this matter in a clearer light, I must remark, that I, in common with most logicians, take the copula in the sense of a sign of attribution, and not, like Hamilton, in the sense of a sign of equality in extension or comprehension. He exposes the proposition, "man is an animal," thus:

The extension of man *Subject*
equals *Copula*
a part or all of the extension of animal . *Predicate*

And thus he makes the predicate particular. Others interpret it thus:

Every man *Subject*
has all the attributes common to *Copula*
every animal *Predicate*

It is in this latter sense that the copula is considered in this paper. Now, a particular is, as has been said, an *alternative* subject. Thus, "Some S is M" means, if S', S", and S'''

are the singular S's, that "either S', or else S", or else S'", has all the attributes belonging to M." A particular term, then, has a substantial depth, because it may have a predicate which is absolutely concrete, as in the proposition, "Some man is Napoleon." But if we put the particular into the predicate we have such a proposition as this: "M has all the attributes belonging to S', or else all those belonging to S", or else all those belonging to S'". And this can never be true unless M is a single individual. Now a single individual substance is, I will not say an atom, but the smallest part of an atom, that is, nothing at all. So that a particular can have no *substantial breadth*. Now take the universal term "S." We can say, "Any S is M," but not if M is a real concrete quality. We cannot say, for instance, "Any man is Napoleon." On the other hand, we can say "Any M is S," even if M is a real substance or aggregate of substances. Hence a universal term has no *substantial depth*, but has *substantial breadth*. We may therefore divide all terms into substantial universals and substantial particulars.

Peirce: CP 2.416 Cross-Ref:††

416. Two terms may be equal in their substantial breadth and depth, and differ in their essential breadth and depth. But two terms cannot have relations of substantial breadth and depth which are unknown in the state of information supposed, because in that state of information everything is known.

Peirce: CP 2.417 Cross-Ref:††

417. In informed breadth and depth, two terms may be equal, and may have unknown relations. Any term, affirmative or negative, universal or particular, may have informed breadth or depth.

Peirce: CP 2.418 Cross-Ref:††

§6. THE CONCEPTIONS OF QUALITY, RELATION, AND REPRESENTATION, APPLIED TO THIS SUBJECT

418. In a paper presented to the Academy last May,†1 I endeavored to show that the three conceptions of reference to a ground, reference to a correlate, and references to an interpretant, are those of which logic must principally make use. I there also introduced the term "symbol," to include both concept and word. Logic treats of the reference of symbols in general to their objects. A symbol, in its reference to its object, has a triple reference:

Peirce: CP 2.418 Cross-Ref:††

First, Its direct reference to its object, or the real things which it represents;

Peirce: CP 2.418 Cross-Ref:††

Second, Its reference to its ground through its object, or the common characters of those objects;

Peirce: CP 2.418 Cross-Ref:††

Third, Its reference to its interpretant through its object, or all the facts known about its object.

Peirce: CP 2.418 Cross-Ref:††

What are thus referred to, so far as they are known, are:

First, The informed *breadth* of the symbol;

Second, The informed *depth* of the symbol;

Third, The sum of synthetical propositions in which the symbol is subject or predicate, or the *information* concerning the symbol. †P1

Peirce: CP 2.419 Cross-Ref: ††

419. By breadth and depth, without an adjective, I shall hereafter mean the informed breadth and depth.

Peirce: CP 2.419 Cross-Ref: ††

It is plain that the breadth and depth of a symbol, so far as they are *not* essential, measure the *information* concerning it, that is, the synthetical propositions of which it is subject or predicate. This follows directly from the definitions of breadth, depth, and information. Hence it follows:

First, That, as long as the information remains constant, the greater the breadth, the less the depth;

Second, That every increase of information is accompanied by an increase in depth or breadth, independent of the other quantity;

Third, That, when there is no information, there is either no depth or no breadth, and conversely.

Peirce: CP 2.419 Cross-Ref: ††

These are the true and obvious relations of breadth and depth. They will be naturally suggested if we term the information the *area*, and write--

Breadth X Depth = Area.

[Analogous to increase of information in us, there is a phenomenon of nature--development--by which a multitude of things come to have a multitude of characters, which have been involved in few characters in few things.--1893.]

Peirce: CP 2.420 Cross-Ref: ††

420. If we learn that S is P, then, as a general rule, the depth of S is increased without any decrease of breadth, and the breadth of P is increased without any decrease of depth. Either increase may be *certain* or *doubtful*.

Peirce: CP 2.420 Cross-Ref: ††

It may be the case that either or both of these increases does not take place. If P is a negative term, it may have no depth, and therefore adds nothing to the depth of S. If S is a particular term, it may have no breadth, and then adds nothing to the breadth of P. This latter case often occurs in metaphysics, and, on account of not-P as well as P being predicated of S, gives rise to an appearance of contradiction where there really is none; for, as a contradiction consists in giving to contradictory terms some breadth in common, it follows that, if the common subject of which they are predicated has no real breadth, there is only a verbal, and not a real contradiction. It is not really contradictory, for example, to say that a boundary is both within and

without what it bounds. There is also another important case in which we may learn that "S is P," without thereby adding to the depth of S or the breadth of P. This is when, in the very same act by which we learn that S is P, we also learn that P was covertly contained in the previous depth of S, and that consequently S was a part of the previous breadth of P. In this case, P gains in extensive distinctness and S in comprehensive distinctness.

Peirce: CP 2.421 Cross-Ref:††

421. We are now in condition to examine Vorländer's objection to the inverse proportionality of extension and comprehension. He requires us to think away from an object all its qualities, but not, of course, by thinking it to be without those qualities, that is, by denying those qualities of it in thought. How then? Only by supposing ourselves to be ignorant whether it has qualities or not, that is, by diminishing the supposed information; in which case, as we have seen, the depth can be diminished without increasing the breadth. In the same manner we can suppose ourselves to be ignorant whether any American but one exists, and so diminish the breadth without increasing the depth.

Peirce: CP 2.422 Cross-Ref:††

422. It is only by confusing a movement which is accompanied with a change of information with one which is not so, that people can confound generalization, induction, and abstraction. **Generalization** is an increase of breadth and a decrease of depth, without change of information. **Induction** is a certain increase of breadth without a change of depth, by an increase of believed information. **Abstraction** is a decrease of depth without any change of breadth, by a decrease of conceived information. **Specification** is commonly used (I should say unfortunately) for an increase of depth without any change of breadth, by an increase of asserted information. **Supposition** is used for the same process when there is only a conceived increase of information. **Determination**, for any increase of depth. **Restriction**, for any decrease of breadth; but more particularly without change of depth, by a supposed decrease of information. **Descent**, for a decrease of breadth and increase of depth, without change of information.†P1

Peirce: CP 2.423 Cross-Ref:††

423. Let us next consider the effect of the different kinds of reasoning upon the breadth, depth, and area of the two terms of the conclusion.

Peirce: CP 2.423 Cross-Ref:††

In the case of deductive reasoning it would be easy to show, were it necessary, that there is only an increase of the extensive distinctness of the major, and of the comprehensive distinctness of the minor, without any change in information. Of course, when the conclusion is negative or particular, even this may not be effected.

Peirce: CP 2.424 Cross-Ref:††

424. Induction requires more attention. Let us take the following example:

S', S'', S''', and S^{IV} have been taken at random from among the M's;

S', S'', S''', and S^{IV} are P:

∴ any M is P.

Peirce: CP 2.424 Cross-Ref:††

We have here, usually, an increase of information. M receives an increase of depth, P of breadth. There is, however, a difference between these two increases. A new predicate is actually added to M; one which may, it is true, have been covertly predicated of it before, but which is now actually brought to light. On the other hand, P is not *yet* found to apply to anything but S', S'', S''', and S^{IV}, but only to apply to whatever else may hereafter be found to be contained under M. The induction itself does not make known any such thing.

Peirce: CP 2.425 Cross-Ref:††

425. Now take the following example of hypothesis:

M is, for instance, P', P'', P''', and P^{IV};

S is P' P'' P''' and P^{IV}

∴S is all that M is.

Peirce: CP 2.425 Cross-Ref:††

Here again there is an increase of information, if we suppose the premisses to represent the state of information before the inferences. S receives an addition to its depth; but only a potential one, since there is nothing to show that the M's have any common characters besides P', P'', P''', and P^{IV}. M, on the other hand, receives an actual increase of breadth in S, although, perhaps, only a *doubtful* one. There is, therefore, this important difference between induction and hypothesis, that the former potentially increases the breadth of one term, and actually increases the depth of another, while the latter potentially increases the depth of one term, and actually increases the breadth of another.

Peirce: CP 2.426 Cross-Ref:††

426. Let us now consider reasoning from definition to definitum, and also the argument from enumeration. A defining proposition has a meaning. It is not, therefore, a merely identical proposition, but there is a difference between the definition and the definitum. According to the received doctrine, this difference consists wholly in the fact that the definition is distinct, while the definitum is confused. But I think that there is another difference. The definitum implies the character of being designated by a word, while the definition, previously to the formation of the word, does not. Thus, the definitum exceeds the definition in depth, although only *verbally*. In the same way, any unanalyzed notion carries with it a feeling--a constitutional word--which its analysis does not. If this be so, the definition is the predicate and the definitum the subject, of the defining proposition, and this last cannot be simply converted. In fact, the defining proposition affirms that whatever a certain name is applied to is supposed to have such and such characters; but it does not strictly follow from this, that whatever has such and such characters is actually called by that name, although it certainly *might* be so called. Hence, in reasoning from definition to definitum, there is a verbal increase of depth, and an actual increase of extensive distinctness (which is analogous to breadth). The increase of depth being

merely verbal, there is no possibility of error in this procedure. Nevertheless, it seems to me proper, rather to consider this argument as a special modification of hypothesis than as a deduction, such as is reasoning from definitum to definition. A similar line of thought would show that, in the argument from enumeration, there is a verbal increase of breadth, and an actual increase of depth, or rather of comprehensive distinctness, and that therefore it is proper to consider this (as most logicians have done) as a kind of infallible induction. These species of hypothesis and induction are, in fact, merely hypotheses and inductions from the essential parts to the essential whole; this sort of reasoning from parts to whole being demonstrative. On the other hand, reasoning from the substantial parts to the substantial whole is not even a probable argument. No ultimate part of matter fills space, but it does not follow that no matter fills space.

Peirce: CP 2.427 Cross-Ref:††
SUPPLEMENT OF 1893†1

427. The usefulness of the doctrine of logical quantity depends upon strict adherence to an exact terminology. To find the requisite terms, however, is not always easy.

Peirce: CP 2.427 Cross-Ref:††

An operation increasing the breadth of a term, whether with or without change of information, may be termed an *extension* of it. The word is most frequently employed, especially by mathematicians, to signify the application of a doctrine (perhaps with a slight modification) to a new sphere. This implies increased information. Still, the broader meaning here proposed is permitted by usage. In like manner, any diminution of breadth may be called *restriction*.

Peirce: CP 2.428 Cross-Ref:††

428. An operation increasing the depth of a term, whether with or without change of information, is known as a *determination*. The books generally give *abstraction* as the contrary of determination; but this is inadmissible. I would propose the word *depletion*. The adjective *abstract* was first used, in Latin, and in imitation of the Greek, of a geometrical form conceived as depleted of matter. Such a conception is intuitional, in the sense of being pictorial. In the seventh century, Isidorus Hispalensis defines *abstract number* in the same sense in which the phrase is still in vogue. But neither *abstract* nor any cognate word is found as a logical term until toward the close of the great dispute concerning realism and nominalism in the twelfth century, when the introduction of the term *abstraction* may be said to mark the cessation of that controversy, if not to be its most important fruit. The word is, with little doubt, a translation of the Greek {aphairesis}, although no Greek text known at that time in the West, has been adduced, from which it could have been borrowed. The etymological meaning is, of course, drawing away from; this, however, does not mean, as is often supposed, drawing the attention away from an object, but, as all the early passages in both ancient languages fully demonstrate, drawing one element of thought (namely, the form) away from the other element (the matter), which last is then neglected. But even in the very first passage in which *abstraction* occurs as a term of logic, two distinct meanings of it are given, the one the contemplation of a form apart from matter, as when we think of *whiteness*, and

the other the thinking of a nature *indifferent*, or without regard to the differences of its individuals, as when we think of a *white* thing, generally.†1 The latter process is called, also, *precision* (or better, *prescission*): and it would greatly contribute to perspicuity of thought and expression if we were to return to the usage of the best scholastic doctors and designate it by that name exclusively, restricting abstraction to the former process by which we obtain notions corresponding to the "abstract nouns."†1 Modern logicians, especially the Germans, who have been lamentably sketchy in their studies of logic, have had the idea that these abstract nouns were mere affairs of grammar, with which the logician need not concern himself; but the truth is they are the very nerve of mathematical thinking. Thus, in the modern theory of equations, the action of changing the order of a number of quantities, is taken as itself a subject of mathematical operation, under the name of a substitution. So a straight line, which is nothing but a relation between points, is studied, and even *intuited*, as a distinct thing. It would be best to limit the word abstraction to this process; but if men cannot be persuaded to do this, the next best thing is to abolish the word abstraction altogether, and denominate this process *subjectification*. It is to be remarked that modern logicians and psychologists, though they have completely changed the breadth of abstraction, commonly applying the term not to subjectification but to prescission, nevertheless retain the mediaeval definition which was intended to apply not to the latter but to the former process. Namely, they define abstraction as attending to a part of an idea and neglecting the remainder. For attention is the pure denotative application, or breadth-function, of the thought-sign; it is the part that thought plays as an index. Of course, in saying this, I do not pretend to be giving a *psychological* account of attention, even if I can be persuaded that there is any such thing as psychology, apart from logic on the one hand and physiology on the other. Attention is a certain modification of the contents of consciousness with reference to a centre. This centre is where there is a strong sense-will reaction, which imparts to the idea the nature of an index (weathercock, sign post, or other blind forcible connection between thought and thing). Now, the subject of a proposition is just such an index. Hence, the real phenomenon of attending to a quality, say white, or making that the centre of thought, consists of thinking of it as the subject to which the other elements of thought are attributes. But *prescission*, if accurately analyzed, will be found not to be an affair of attention. We cannot prescind, but can only distinguish, color from figure. But we can prescind the geometrical figure from color; and the operation consists in imagining it to be so illuminated that its hue cannot be made out (which we easily can imagine, by an exaggeration of the familiar experience of the indistinctness of hues in the dusk of twilight). In general, prescission is always accomplished by imagining ourselves in situations in which certain elements of fact cannot be ascertained. This is a different and more complicated operation than merely attending to one element and neglecting the rest. Thus, if the definition usually given of abstraction, that it is attention to a part of an idea with neglect of the rest, be accepted, the term must be applied, no longer to prescission but exclusively to subjectification.

Peirce: CP 2.429 Cross-Ref:††

429. So much for the terms expressive of increasing and diminishing logical breadth and depth, in general. In order to express an extension by depletion, and a determination by restriction, without change of information, we obviously stand in need of the words *generalization* and *specification*. Unfortunately, neither word is so used. By specification is invariably meant a determination by increased information. By generalization is sometimes meant, it is true, the extension of an idea by a

considerable modification of its depth; but still there is commonly an increase of information. Even more frequently, generalization signifies a formal increase of depth, by a general idea being predicated of cases not before synthesized; and there may or may not be an increase of breadth. Finally, generalization, by a wanton abuse of language, is often used to mean simply an induction. It seems necessary, therefore, to abandon these words for the present purpose and to content ourselves with *ascent* and *descent*.

Peirce: CP 2.430 Cross-Ref:††

430. An increase of information, in general, is, in modern speech, called a *discovery*. The old word, *invention*, was much better, since this left *discovery* to be restricted to the finding of a new thing--as the discovery of America--while the finding out of a new character was specifically called a *detection*. Thus, Oldenburg, the Secretary of the Royal Society, writes in 1672, that the dispersion of light is "the oddest, if not the most considerable, detection which hath hitherto been made into the operations of nature." It is a pity these nice distinctions have been lost. We must now speak of the *discovery of an occurrence* or *instance* and the *discovery of a property*. An imaginary increase of information is an *assumption* or *supposition*; but the former word is preferable. An increase of information by induction, hypothesis, or analogy, is a *presumption*. (A *legal presumption* is a presumption which follows an accepted rule of the courts, irrespective of the dictates of good sense.) A very weak presumption is a *guess*. A presumption opposed to direct testimony is a *conjecture*, or, if weak, a *surmise*.

Peirce: CP 2.431 Cross-Ref:††
SIGNIFICATION AND APPLICATION †1

431. These are substitute terms for what are called by Mill and others connotation and denotation; for (1) the previously well-established use of connotation was somewhat warped by Mill and his followers, and (2) these words may be applied to the corresponding properties of propositions as well as terms. The application of a term is the collection of objects which it refers to; of a proposition it is the instances of its holding good. The signification of a term is all the qualities which are indicated by it; of a proposition it is all its different implications.

Peirce: CP 2.432 Cross-Ref:††

432. Great confusion has arisen in logic from failing to distinguish between the different sorts of signification, or connotation, of a term: thus to the question, Are proper names connotative? "contradictory answers are given by ordinarily clear thinkers as being obviously correct," for the reason that they have not the same thing in mind under the term connotation. It is necessary to distinguish between; (1) the indispensable signification; (2) the banal signification; (3) the informational signification; and (4) the complete signification. (1) is so much as is contained in whatever may be fixed upon as the definition of the term--all those elements of the meaning in the absence of any one of which the name would not be applied; (2) is what "goes without saying," what is known to every one, and (3) is what there is occasion to give utterance to: these, of course, vary with the different individuals to whom the proposition is given out--that oxygen is exhilarating is informational to the student of chemistry, and banal to the teacher of chemistry (but false to those who are

familiar with the latest results of the science); (4) consists of all the valid predicates of the term in question. When I say, "The one I saw yesterday was John Peter," the indispensable signification of John Peter is simply an individual object of consciousness (usually a man, though it may be a dog, or a doll) whom it has been agreed to designate by that name; but the banal signification, to one who knows John Peter well, is very extensive.

Peirce: CP 2.433 Cross-Ref:††

433. The same characteristics apply to propositions as well as to terms: thus the complete signification (or implication) of *All x is y* is all its valid consequences, and its complete application (or range) is all those descriptions of circumstances under which it holds good--that is to say, all its sufficient antecedents.

Peirce: CP 2.434 Cross-Ref:††

434. A general term denotes whatever there may be which possesses the characters which it signifies; J. S. Mill uses, in place of signifies, the term connotes, a word which he or his father picked up in Ockham. But signify has been in uninterrupted use in this sense since the twelfth century, when John of Salisbury spoke of "quod fere in omnium ore celebre est aliud scilicet esse appellativa *significant*, et aliud esse quod, *nominant*. Nominantur singularia; sed universalia significantur."†1 Nothing can be clearer. There is no known occurrence of connote as early as this. Alexander of Hales (*Summa Theol.*, I. liii) makes *nomen connotans* the equivalent of *appellatio relativa*, and takes the relation itself as the accusative object of *connotare*, speaking of "creator" as *connoting* the relation of creator to creature. So Aquinas, *In sentent.*, I. dist. viii. q. 1, Art. 1. Subsequently, because adjectives were looked upon as relative terms, *white* being defined as "having whiteness," etc., the adjective was looked upon as connoting the abstraction, but never unless its supposed relative character was under consideration. Tataretus, for example, who wrote when the usage was fully established, will be found using such phraseology as the following: "Nulla relativa secundum se habent contrarium, cum non sint qualitates primae, sed solum relativa secundum dici, et hoc secundum esse absolutum et significatum principale eorum et non secundum esse respectivum et connotativum." Chauvin †2 (1st ed.) says: "Connotativum illud est cuius significatum non sistit in se, sed necessario ad aliud refertur, vel aliud connotat. V. g. *Rex, magister, primus*."

Peirce: CP 2.434 Cross-Ref:††

It unfortunately happened, as the above quotations show, that the precise meaning recognized as proper to the word "signify" at the time of John of Salisbury (a younger contemporary of Abelard) was never strictly observed, either before or since; and, on the contrary, the meaning tended to slip towards that of "denote." Yet even now the propriety of John's remark must be recognized.

Peirce: CP 2.434 Cross-Ref:††

A number of works were written in the middle ages, *De modis significandi*, based upon Priscian (a contemporary of Boëthius), who in turn followed Apollonius the bad-tempered, "grammaticorum princeps," who lived in the time of Hadrian and Antoninus Pius. Cf. also Thurot, *Notices et Extraits des MSS.* xxii. Pt. II, and Duns Scotus, *Works*, Lyons edit. 1.

THE GRAMMATICAL THEORY OF JUDGMENT AND INFERENCE†1

§1. JUDGMENTS

435. A judgment is an act of consciousness in which we recognize a belief, and a belief is an intelligent habit upon which we shall act when occasion presents itself. Of what nature is that recognition? It may come very near action. The muscles may twitch and we may restrain ourselves only by considering that the proper occasion has not arisen. But in general, we *virtually resolve upon* a certain occasion to act as if certain imagined circumstances were perceived. This act which amounts to such a resolve, is a peculiar act of the will whereby we cause an image, or *icon*, to be associated, in a peculiarly strenuous way, with an object represented to us by an *index*. This act itself is represented in the proposition by a *symbol*, and the consciousness of it fulfills the function of a symbol in the judgment. Suppose, for example, I detect a person with whom I have to deal in an act of dishonesty. I have in my mind something like a "composite photograph" of all the persons that I have known and read of that have had that character, and at the instant I make the discovery concerning that person, who is distinguished from others for me by certain indications, upon that index at that moment down goes the stamp of RASCAL, to remain indefinitely.

Peirce: CP 2.436 Cross-Ref:††

436. A proposition *asserts* something. That assertion is performed by the symbol which stands for the act of consciousness. That which accounts for *assertion* seeming so different from other sorts of signification is its *volitional* character.

Peirce: CP 2.437 Cross-Ref:††

437. Every assertion is an assertion that two different signs have the same object. If we ask why it should have that *dual* character, the answer is that volition involves an action and reaction. The consequences of this duality are found not only in the analysis of propositions, but also in their classification.

Peirce: CP 2.438 Cross-Ref:††

438. It is impossible to find a proposition so simple as not to have reference to two signs. Take, for instance, "it rains." Here the icon is the mental composite photograph of all the rainy days the thinker has experienced. The index, is all whereby he distinguishes *that day*, as it is placed in his experience. The symbol is the mental act whereby [he] stamps that day as rainy. . . .

Peirce: CP 2.439 Cross-Ref:††

439. In order properly to exhibit the relation between premisses and conclusion of mathematical reasonings, it is necessary to recognize that in most cases the *subject-index* is compound, and consists of a *set* of indices. Thus, in the proposition, "A sells B to C for the price D," A, B, C, D form a set of four indices. The symbol "--sells--to--for the price--" refers to a mental icon, or idea of the act of

sale, and declares that this image represents the *set* A, B, C, D, considered as attached to that icon, A as seller, C as buyer, B as object sold, and D as price. If we call A, B, C, D four *subjects* of the proposition and "--sells--to--for the price--" a *predicate*, we represent the logical relation well enough, but we abandon the Aryan syntax.

Peirce: CP 2.440 Cross-Ref:††

440. It may be asked, why may not an assertion identify the objects of any two signs whatever, as two indices? Why should it be limited to declaring the object of an *index* to be represented by an *icon*? The answer is that an assertion *may* identify the objects of any two signs whatever; yet that in every case this will amount to declaring that an *index*, or set of indices, is represented by an *icon*. For instance, let the proposition be, that William *Lamare*, the author of the book *Correctorium fratris Thomae* is really the William *Ware* who was the teacher of Duns Scotus. Here the objects of two indices are identified. But this is logically equivalent to the assertion that the icon of identity, that is, the mental composite image of two aspects of one and the same thing, represents the objects of the set of indices William *Mare* and William *Ware*.†P1 We are not, indeed, absolutely forced to regard one of the signs as an *icon* in any case; but this is a very convenient way of taking account of certain properties of inferences. It happens, too, to have some secondary advantages, such as that of agreeing with our natural metaphysics, and with our feeling in regard to subject and predicate.

Peirce: CP 2.441 Cross-Ref:††

441. As the index may be complex, so also may the icon. For instance, taking the universal selective index, *everything*, we may have an icon which is composed alternatively of two, a sort of composite of two icons, in the same way that any image is a "composite photograph" of innumerable particulars. Even what is called an "instantaneous photograph," taken with a camera, is a composite of the effects of intervals of exposure more numerous by far than the sands of the sea. Take an absolute instant during the exposure and the composite represents *this* among other conditions. Now, the two alternative icons are combined like that. We have an icon of this alternation, a composite of all the alternative cases we have thought of. The symbol asserts that one or other of those icons represents the universally selected index. Let one of the alternative icons be the idea of what is not a man, the other the idea of what is mortal. Then, the proposition will be: "Take anything you please, and it will either not be a man or will be mortal." Two signs so conjoined are said to be *aggregated*, or *disjunctively connected*, or *alternatively conjoined*. Take another example. Let the index be particularly selective. Let an icon be so compounded of two icons that in each variation of it both those icons are conjoined. For instance, let one be an icon of a Chinese, the other of a woman. Then, the combined icon will be an icon of a Chinese woman. Thus, the proposition will be: "Something can be so selected as to be at once a Chinese and a woman." Two signs so conjoined are said to be *combined*, or *conjunctively connected*, or *simultaneously conjoined*. . . .

Peirce: CP 2.442 Cross-Ref:††
§2. INFERENCE

442. It is now time to examine more carefully the nature of *inference*, or the conscious and controlled adoption of a belief as a consequence of other knowledge.

The first step of inference usually consists in bringing together certain propositions which we believe to be true, but which, supposing the inference to be a new one, we have hitherto not considered together, or not as united in the same way. This step is called *colligation*. †1 The compound assertion resulting from colligation is a *conjunctive proposition*, that is, it is a proposition with a composite icon, as well as usually with a composite index. Colligation is a very important part of reasoning, calling for genius perhaps more than any other part of the process. Many logicians refuse the name of reasoning to an inferential act of which colligation forms no part. Such an inferential act they call an *immediate inference*. This term may be accepted; but although colligation certainly gives a higher intellectuality to inference, yet its importance is exaggerated when it is represented to be of more account than the conscious control of the operation. The latter ought to determine the title of *reasoning*.

Peirce: CP 2.443 Cross-Ref: ††

443. An inference, then, may have but a single premiss, or several premisses may be united by colligation. In the latter case, they form, when colligated, one conjunctive proposition. But even if there be but one premiss, the icon of that proposition is always more or less complex. The next step of inference to be considered consists in the contemplation of that complex icon, the fixation of the attention upon a certain feature of it, and the obliteration of the rest of it, so as to produce a new icon. . . .

Peirce: CP 2.444 Cross-Ref: ††

444. Whenever one thing suggests another, both are together in the mind for an instant. In the present case, this conjunction is specially interesting, and in its turn suggests that the one necessarily involves the other. A few mental experiments--or even a single one, so expert do we become at this kind of experimental inquiry--satisfy the mind that the one icon would at all times involve the other, that is, suggest it in a special way, into which we shall soon inquire. Hence the mind is not only led from believing the premiss to judge the conclusion true, but it further attaches to this judgment another--that *every* proposition *like* the premiss, that is having an icon like it, *would* involve, and compel acceptance of, a proposition related to it as the conclusion then drawn is related to that premiss. [This is the third step of inference.] Thus we see, what is most important, that every inference is thought, at the time of drawing it, as one of a possible class of inferences. In the case of a rational inference, we *see*, in an icon which represents the dependence of the icon of the conclusion upon the icon of the premiss, about what that class of inference is, although, as the outlines of icons are always more or less vague, there is always more or less of vagueness in our conception of that class of inferences. There is no other element of inference essentially different from those which have been mentioned. It is true that changes generally take place in the indices as well as in the icon of the premiss. Some indices may be dropped out. Some may be identified. The order of selections may sometimes be changed. But these all take place substantially in the same manner in which a feature of the icon attracts attention, and must be justified in the inference by experiments upon icons.

Peirce: CP 2.444 Cross-Ref: ††

It thus appears that all knowledge comes to us by observation. A part is forced upon us from without and seems to result from Nature's mind; a part comes from the depths of the mind as seen from within, which by an egotistical anacoluthon we call *our* mind. The three essential elements of inference are, then, colligation, observation,

and the judgment that what we observe in the colligated data follows a rule.†P1

Peirce: CP 2.445 Cross-Ref:††
BOOK III

CRITICAL LOGIC

A. EXPLICATIVE REASONING

B. AMPLIATIVE REASONING

A. EXPLICATIVE REASONING

CHAPTER 1

THE ARISTOTELIAN SYLLOGISTIC†1†P1^P

§1. PRETENSIONS OF DEMONSTRATIVE REASONING

445. . . . It is easy to over-rate the importance of syllogism. Most of the older logics do that, in teaching that the substance of all reasoning lies in this. It is also easy to under-rate it; as many have done. The truth is, it is an essential element of almost all reasoning, perhaps of all.

Peirce: CP 2.445 Cross-Ref:††

A syllogism is a valid, *demonstrative*, *complete*, and *simply eliminative* argumentation.

Peirce: CP 2.446 Cross-Ref:††

446. To say that an argumentation is valid is to say that it is as truthful as it pretends to be. It is essential to reasoning, as already stated, that it should be accompanied by the reflection that it belongs to a class of reasonings, few or none of which lead from truth to falsity. All reasoning, therefore, makes a pretension; and if that pretension is true, the reasoning is *valid*.

Peirce: CP 2.447 Cross-Ref:††

447. *Demonstrative* reasoning pretends to be such that it is logically impossible for the premisses to be true while the conclusion is false. I think it would

be just to add that demonstrative reasoning further pretends:

Peirce: CP 2.447 Cross-Ref:††

First, that its premisses are logically possible, or at least that the class of propositions to which they are considered as belonging contains possibly true propositions; and,

Peirce: CP 2.447 Cross-Ref:††

Second, that its conclusion is not logically necessary, or at least, that the class of propositions to which it is considered as belonging contains propositions not necessarily true.

Peirce: CP 2.447 Cross-Ref:††

This, at least, I think would have been Aristotle's way of conceiving the matter, had the question presented itself to him. For instance, if he had been asked what he would say of this reasoning:

Chameleons assume the color of objects upon which they rest,

∴ Everything is what it is,

I think he would have said, this is no reasoning at all. Granted that the premiss cannot be true while the conclusion is false, because the conclusion cannot be false at all, reasoning pretends that there is some connection between premiss and conclusion, so that if the fact were the opposite of what is stated in the premiss, the conclusion would not necessarily be true. I think Aristotle would have made the same strictures upon an argument like this:

Some parts are greater than their wholes;

∴ The eating of green fruit proves invariably fatal.

Such an argument cannot lead from true to false, because the premiss cannot be true. But if the reasoning pretends, as it seems to do, that from one thing being in a certain relation to another, it is safe to conclude a proposition about a totally different subject, in that sense it is false.

Peirce: CP 2.448 Cross-Ref:††

448. But although I think Aristotle or any other man of good sense would take this view, I propose to reject it, and to consider both the above reasonings as sound. My reason is, that such things are of no practical importance whatever--for as long as reasoning does not lead us astray, the whole purpose of logic is fulfilled--and to admit these reasonings as sound simplifies very considerably the whole doctrine of syllogism. In this I am by no means alone. Even in ancient times, many logicians took the same ground. . . .

Peirce: CP 2.449 Cross-Ref:††

449. A *complete* argument is one which professes to be not only necessary,

but logically necessary. †P1

Peirce: CP 2.450 Cross-Ref: ††

450. An *eliminative* argumentation is one which mentions something in the premisses in two opposite ways, so that it disappears from the conclusion. When we argue:

Washington was a high-minded politician;

∴ It is possible for a politician to be high-minded,

the argumentation is not eliminative. For what is dropped is mentioned but once. Again we may argue [that in:]

All men die;

∴ Holy men (if there are any) die,

we drop nothing, but insert something. Once more we may argue:

There are women whom all men reverence,

∴ Every man reverences some woman or another.

Here we neither drop nor insert. All these non-eliminative inferences are usually called *immediate inferences* by logicians; and have received very little attention. But Barbara and all eliminative inferences are said to be *mediate*. This terminology is ill-considered. Thus, the inference:

Any two infinite planes have a line of intersection;

∴ Any three infinite planes have three lines of intersection, is not eliminative, yet it plainly arises from steps of argumentation. We say of the three planes, A, B, C, that A and B, A and C, and B and C are pairs, each having an intersection, and we count three of these pairs. No logician has ever made out, or pretended to make out, that there is any sort of elimination between three premisses. Hence undecomposable eliminative arguments (not counting mere omissions) have two premisses each.

Peirce: CP 2.451 Cross-Ref: ††

451. That the putting of the two premisses together is a distinct act of thought, so that the reasoning;

All men are mortal,

All patriarchs are men;
∴ All patriarchs are mortal,

really consists of these two steps:

All men are mortal,
All patriarchs are men;
∴ All men are mortal, and all patriarchs men;
∴ All patriarchs are mortal;

would be regarded by the ordinary logician as hair-splitting. Yet it is just in bringing the premisses together that all the difficulty lies! This preliminary uniting of the premisses is called *copulation*, or *colligation*. †P1 Even between the copulate premiss and the conclusion another stage of thought might be inserted, which will appear if we vary one of the premisses thus:

All patriarchs are men, and all sinners are mortal;
∴ If all men are sinners, all patriarchs are mortal;
∴ All patriarchs are mortal.

This last step is enthymematic. It wants, to make it logical, the expression of the leading principle, "All men are sinners." But put *men* in place of sinners, and this becomes a logical principle, not needing to be set forth as a premiss. It seems, however, that logicians recognize no difference between saying, "If all men are men, A is true," and saying directly "A is true"; and I suppose they are right. We cannot recognize logical formulae as, properly speaking, assertions. †P1

Peirce: CP 2.452 Cross-Ref: ††
§2. RULES AND CASES

452. . . . In order to get at the real why and wherefore of Aristotle's choice of propositional forms, whatever he may himself have understood it to be, we must go back to this postulate:

We cannot know anything except it be a uniformity.

Peirce: CP 2.452 Cross-Ref:††

I am not pretending the uniformity or our knowledge need be perfect. Nor am I pretending we cannot become aware of a breach of uniformity. One uniformity may constitute a breach in another. I am simply suggesting, first, that an event altogether out of order and presenting no regularity could not come to our knowledge at all and second, that only in respect to its being orderly can we know it. I do not care to insist here upon the truth of this postulate. It would take me too far afield. I merely say that if it be granted, there seems to be a good reason for Aristotle's propositional forms; but if not, I am unable to defend the system.

Peirce: CP 2.452 Cross-Ref:††

A uniformity is a consequence. All we know is that from one thing another thing follows. Those two things themselves, when they come to be carefully thought out, are seen to be consequences, and so on indefinitely. I next ask that it be granted that there is an important class of inferences which have each of them more than one premiss. Of these, considering that each premiss judges or asserts that one thing follows from another, it is plain that the simplest is: from A follows B and from B follows C; and therefore from A follows C.

Peirce: CP 2.453 Cross-Ref:††

453. . . . [In the argument, "Men are sinners, and sinners are miserable; ∴ men are miserable,"] "Sinners are miserable," must be a **Rule** without exceptions. That is, it says in effect, if you take any sinner, you will find he is miserable. The second person appropriately expresses it, because there is a second premiss which draws attention to certain sinners, and virtually picks them out. If the rule has exceptions, all I can say is, that if you let *me* pick out the sinner he will turn out miserable. If I guarantee to find a miserable sinner, of course, I guarantee *there is* a sinner in the world. But if I turn the responsibility of picking out the sinner to you, I do not guarantee you can find one. I only say if you do find one, he will turn out miserable. This is the distinction between Universal and Particular propositions.†1

The premiss

"Men are sinners,"

must refer to every character common to all sinners. No matter what the rule about sinners may be which the other premiss lays down, men are here said to be subject to that rule. If it were possible to find, in place of men, a race totally unlike sinners in every respect, the conclusion would hold that they were not miserable, if sinners were so. But that is an absurdity. For since sinners are things we can think about and speak of and reason correctly about sometimes, those other creatures would be creatures we could never think about, nor speak of, nor draw a single correct inference concerning them. For the same reason, while we can speak of angels as wanting some characters of sinners, it will not help us to assert they possess some characters of sinners, since every conceivable thing resembles every other in some respect--as for instance, in being conceivable or capable of being talked about, etc.†P1 When I speak of a common character of a class of objects, I must, for the purposes of the inference in

Barbara, mean a general rule true of all that class. Now, if I say, you can take what rule you please applicable to all sinners, and it will be found applicable to men, I am not guaranteeing that there is any general rule true of all sinners. But when I say, I could find you a rule true of all sinners that does not hold good of all women (not, for instance, of the Blessed Virgin), I have committed myself to the proposition that there is such a rule. This is the distinction between *Affirmative* and *Negative* propositions. An affirmative proposition speaks of any general rule there may be, no matter what, while a negative says there is a rule and says that such a one can be elected so as to break down if applied to a certain subject (outside the class to which the rule refers).

Peirce: CP 2.454 Cross-Ref:††

454. We thus see how syllogistic theory calls for precisely the formal distinctions of propositions that Aristotle draws, and needs no others.

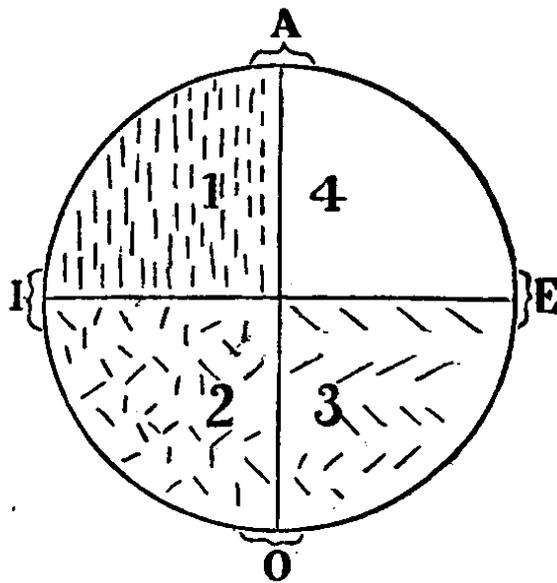
Peirce: CP 2.455 Cross-Ref:††

§3. THE QUADRANT

455. The distinction between Universal and Particular propositions is said to be the distinction in *Quantity*; that between Affirmative and Negative propositions the distinction in *Quality*. Such is the traditional terminology.†P1 But this is a terrible abuse of the important words *quantity* and *quality*, the inconvenience of which is felt in studying the *Critic of the Pure Reason*. Therefore, notwithstanding their having a generation of occupancy for every card in the whist-pack, and one for the joker too, I for one shall vote to eject them. Let us say Universals and Particulars differ in *Lexis*, Affirmatives and Negatives in *Phasis*.†P1 Lexis and Phasis are tell-way and say-way. Lexis is from {legein}, to pick out, and also to tell; it is the mode of picking out, or of reckoning. Phasis is saying, in the sense of: "What do you say? Yes or No?"; being the base of {kataphasis}, affirmation, and {apophasis}, negation. I really see no objection to them, except their novelty. For reversal of Lexis I shall use *metalexis*; for reversal of Phasis, *metaphasis*, though the meaning is nearly that of the Greek {antiphasis}.

Peirce: CP 2.456 Cross-Ref:††

456. . . . Having taken the Diodoran, in opposition to the Philonian view of validity, Aristotle must for consistency hold the universal Affirmative implies the existence of its subject. . . . He must understand: "Some philosopher's stones are red" as not asserting the existence of any philosopher's stone. . . . As the distinction between Universal and Particular propositions concerns the subject, so the distinction between Affirmative and Negative ought, for the sake of symmetry, concern the predicate; so that the difference between asserting and not-asserting the existence of the subject ought to be the distinction between Universals and Particulars, not between Affirmatives and Negatives. Universal propositions do not, while particular propositions do, imply the existence of their subjects. The following figure illustrates the precise sense here assigned to the four forms A, E, I, O.



[Click here to view]

Peirce: CP 2.456 Cross-Ref:††

In the quadrant marked 1 there are lines which are all vertical; in the quadrant marked 2 some lines are vertical and some not; in quadrant 3 there are lines none of which are vertical; and in quadrant 4 there are no lines. Now, taking *line* as subject and *vertical* as predicate,

A is true of quadrants 1 and 4 and false of 2 and 3.

E is true of quadrants 3 and 4 and false of 1 and 2.

I is true of quadrants 1 and 2 and false of 3 and 4.

O is true of quadrants 2 and 3 and false of 1 and 4.

Hence, A and O precisely deny each other, and so do E and I. But any other pair of propositions may be both true, or both false, or either true while the other is false.

Peirce: CP 2.457 Cross-Ref:††

457. Quadrant 1 includes the case in which the predicate covers the whole *universe* of discourse;†P1 so that there is this intrinsic distinction between Affirmatives and Negatives, that the latter deny their predicates to be necessary, which the former permit; just as there is this intrinsic distinction between Universals and Particulars, that the latter assert the existence of their subjects, which the former do not insist upon.

Peirce: CP 2.458 Cross-Ref:††

458. There are some languages which take the negative particle in such a sense that a repetition of it is intensive; but I shall understand the negating of a proposition to be a reversal of the above diagram across its sinister diagonal, interchanging quadrants 3 and 1, so that All S is not-not-P shall mean, All S is P. And

in like manner, I shall use the word *some*, in such a sense that a repetition of it is not sinister, but to signify a reversal of the diagram across the dexter diagonal, interchanging quadrants 2 and 4, so that Some-some-S is P shall mean All S is P. This I do for the sake of symmetry; at the same time, it is easy to give an intelligible sense to it. To say: "Every S is P" is to say: "An S, even if one of the worst cases is selected, will be identical with a P, favorably chosen." To say: "Some S is P" is to say: "An S, if not one of the worst is chosen, will be identical with a P favorably chosen." But to say: "An S, if not other than one of the worst is chosen, will be identical with a P favorably chosen," reproduces the universal. By "favorably" is to be understood, favorably to the identity, but by the "worst cases" are to be understood those most calculated to overthrow the assertion. To say: "An S, if no one of the worst is selected, will be identical with a P unfavorably chosen," implies that every P is an S, just as "Any not-S is not P" implies the same thing. So to say: "An S, even if one of the worst cases is selected, is not identical with a P not favorably selected," is as much as to say that some P is not S, just as "Some not-S is P" implies the same thing. This meaning of the word "some" certainly departs very far indeed from the ordinary usage of speech. But that is nothing: it is perfectly intelligible, and is taken so as to give balance and symmetry to the logical system, which is a matter of the utmost importance, if that system is to fulfill a philosophical function. If the main object of the syllogistic forms were in actual application, to test reasonings as to whose validity or invalidity we found it difficult to decide, as some logicians seem naively to suppose, then their close connection with ordinary habits of thought might be a paramount consideration. But in reality, their main function is to give us an insight into the inward structure of reasoning in general; and for that purpose systematic perfection is indispensable. . . .

Peirce: CP 2.459 Cross-Ref:††

459. It is a blunder on Aristotle's part to call the propositions A and E contraries merely because they may both be false, but not both true. They ought to be called *incongruous* or *disparates*, and both these terms are somewhat in use. Subcontraries (a word of Boëthius,†1 imitating the {hypenantia} of Ammonius) are propositions of opposite emphasis but, being particular, both can be true, though both cannot be false. It would be well to follow the usage of those writers who call any two propositions which can logically both be true but not both false, subcontraries. *Contradictories* (Aristotle's {antikeimena}, the word *contradictoria* comes from Boëthius)†2 are two propositions which cannot both be true nor both false, but precisely deny one another. *Subaltern* (a word found in the translation of Porphyry's *Isagoge* by Marius Victorinus in the fourth century;†3 Porphyry's word is {hypallélon}, but in the present sense first found in Boëthius)†4 is a particular proposition which follows by an immediate inference from its corresponding universal to which it is said to be subaltern.

Peirce: CP 2.460 Cross-Ref:††

460. But in my system none of the relations shown in the diagram of Apuleius [the square of opposition] are preserved, except the two pairs of contradictories. All other pairs of propositions may be true together or false together.

Peirce: CP 2.460 Cross-Ref:††

A and E, All S is P, and No S is P, are true together when no S exists, and false together when part only of the S's are P. I and O, some S is P, some S is not P, are true and false together under precisely the opposite conditions.

Peirce: CP 2.460 Cross-Ref:††

A and I, Any S is P, Some S is P, are true together when there are S's all of which are P, and are false together when there are S's none of which are P. E and O, No S is P, and Some S is not P, are true and false together under precisely the opposite circumstances. . . .

Peirce: CP 2.461 Cross-Ref:††
CHAPTER 2

ON THE NATURAL CLASSIFICATION OF ARGUMENTS†1

PART I

§1. ESSENTIAL PARTS OF AN ARGUMENT

461. In this paper, the term "argument" will denote a body of premisses considered as such.†P1 The term "premiss" will refer exclusively to something laid down (whether in any enduring and communicable form of expression, or only in some imagined sign), and not to anything only *virtually* contained in what is said or thought, and also exclusively to that part of what is laid down which is (or is supposed to be) relevant to the conclusion.†P2

Peirce: CP 2.462 Cross-Ref:††

462. Every inference involves the judgment that, if such propositions as the premisses are true, then a proposition related to them, as the conclusion is, must be, or is likely to be, true. The principle implied in this judgment, respecting a genus of argument, is termed the *leading principle* of the argument.

Peirce: CP 2.463 Cross-Ref:††

463. A *valid* argument is one whose leading principle is true.

Peirce: CP 2.464 Cross-Ref:††

464. In order that an argument should determine the necessary or probable truth of its conclusion, both the premisses and leading principle must be true.

Peirce: CP 2.465 Cross-Ref:††

§2. RELATIONS BETWEEN THE PREMISSES AND LEADING PRINCIPLE

465. The leading principle contains, by definition, whatever is considered requisite besides the premisses to determine the necessary or probable truth of the conclusion. And as it does not contain in itself the subsumption of anything under it, each premiss must, in fact, be equivalent to a subsumption under the leading

principle.

Peirce: CP 2.465 Cross-Ref:††

The leading principle can contain nothing irrelevant or superfluous.

Peirce: CP 2.465 Cross-Ref:††

No fact, not superfluous, can be omitted from the premisses without being thereby added to the leading principle,†P1 and nothing can be eliminated from the leading principle except by being expressed in the premisses. Matter may thus be transferred from the premisses to the leading principle, and *vice versa*.

Peirce: CP 2.465 Cross-Ref:††

There is no argument without premisses,†P2 nor is there any without a leading principle.

Peirce: CP 2.466 Cross-Ref:††

466. It can be shown that there are arguments no part of whose leading principle can be transferred to the premisses, and that every argument can be reduced to such an argument by addition to its premisses. For, let the premisses of any argument be denoted by *P*, the conclusion by *C*, and the leading principle by *L*. Then, if the whole of the leading principle be expressed as a premiss, the argument will become

L and *P*

∴ *C*

But this new argument must also have its leading principle, which may be denoted by *L'*. Now, as *L* and *P* (supposing them to be true) contain all that is requisite to determine the probable or necessary truth of *C*, they contain *L'*. Thus *L'* must be contained in the leading principle, whether expressed in the premiss or not. Hence every argument has, as portion of its leading principle, a certain principle which cannot be eliminated from its leading principle. Such a principle may be termed a *logical principle*.

Peirce: CP 2.466 Cross-Ref:††

An argument whose leading principle contains nothing which can be eliminated is termed a *complete*, in opposition to an *incomplete*, *rhetorical*, or *enthymematic* argument.†P1

Peirce: CP 2.467 Cross-Ref:††

467. Since it can never be requisite that a fact stated should also be implied in order to justify a conclusion, every *logical principle* considered as an assertion †1 will be found to be quite empty. The only thing it really enunciates is a rule of inference;†2 considered as expressing truth, it is nothing.†P1 On this account that method of investigating logic which works upon syllogistic forms is preferable to another, too often confounded with it, which undertakes to enunciate logical principles.

Peirce: CP 2.468 Cross-Ref:††
§3. DECOMPOSITION OF ARGUMENT †3

468. Since a statement is not an argument for itself, no fact concluded can be stated in any one premiss. Thus it is no argument to say All *A* is *B*; *ergo* Some *A* is *B*.

Peirce: CP 2.469 Cross-Ref:††

469. If one fact has such a relation to another that, if the former is true, the latter is necessarily or probably true, this relation constitutes a determinate fact; and therefore, since the leading principle of a complete argument involves no matter of fact, every complete argument has at least two premisses.†P2

Peirce: CP 2.470 Cross-Ref:††

470. Every conclusion may be regarded as a statement substituted for either of its premisses, the substitution being justified by the other premisses. Nothing is relevant to the other premisses, except what is requisite to justify this substitution. Either, therefore, these other premisses will by themselves yield a conclusion which, taken as a premiss along with the first premiss, justifies the final conclusion; or else some part of them, taken with the first premiss, will yield a conclusion which, taken as a premiss along with all the others, will again justify the final conclusion. In either case, it follows that every argument of more than two premisses can be resolved into a series of arguments of two premisses each. This justifies the distinction of *simple* and *complex* arguments.

Peirce: CP 2.471 Cross-Ref:††
§4. OF A GENERAL TYPE OF SYLLOGISTIC ARGUMENTS

471. A valid, complete, simple argument will be designated as a *syllogistic* argument.

Peirce: CP 2.472 Cross-Ref:††

472. Every proposition may, in at least one way, be put into the form,

S is *P*;

the import of which is, that the objects to which *S*, or the *total subject*, applies have the characteristics attributed to every object to which *P*, or the *total predicate*, applies.

Peirce: CP 2.473 Cross-Ref:††

473. Every term has two powers or significations, according as it is subject or predicate. The former, which will here be termed its *breadth*, comprises the objects to which it is applied; while the latter, which will here be termed its *depth*, comprises the characters which are attributed to every one of the objects to which it can be applied. This breadth and depth must not be confounded with logical extension and

comprehension, as these terms are usually taken.

Peirce: CP 2.474 Cross-Ref:††

474. Every substitution of one proposition for another must consist in the substitution of term for term. Such substitution can be justified only so far as the first term represents what is represented by the second. Hence the only possible substitutions are--

First. The substitution for a term fulfilling the function of a subject of another whose breadth is included in that of the former; and

Second. The substitution for a term fulfilling the function of a predicate of another whose depth is included in that of the former.†P1

Peirce: CP 2.474 Cross-Ref:††

If, therefore, in either premiss a term appears as subject which does not appear in the conclusion as subject, then the other premiss must declare that the breadth of that term includes the breadth of the term which replaces it in the conclusion. But this is to declare that every object of the latter term has every character of the former. The eliminated term, therefore, if it does not fulfill the function of predicate in one premiss, does so in the other. But if the eliminated term fulfills the function of predicate in one premiss, the other premiss must declare that its depth includes that of the term which replaces it in the conclusion. Now, this is to declare that every character of the latter term belongs to every object of the former. Hence, in the other premiss, it must fulfill the function of a subject. Hence the general formula of all argument must be:

M is *P*,

S is *M*,

∴*S* is *P*;

which is to be understood in this sense--that the terms of every syllogistic argument fulfill functions of subject and predicate as here indicated, but not that the argument can be grammatically expressed in this way.†P2

Peirce: CP 2.475 Cross-Ref:††
PART II †1

§1. OF APAGOGICAL FORMS

475. If *C* is true when *P* is, then *P* is false when *C* is. Hence, it is always possible to substitute for any premiss the denial of the conclusion, provided the denial of that premiss be at the same time substituted for the conclusion.†P1 Hence, corresponding to every syllogistic argument in the general form,

S is M , M is P ;

S is P .

There are two others:

It is false that S is P , M is P ; S is M , it is false that S is P ;

It is false that S is M . It is false that M is P .

Peirce: CP 2.476 Cross-Ref:††
§2. OF CONTRADICTION

476. The apagogical forms make it necessary to consider in what way propositions deny one another.

Peirce: CP 2.476 Cross-Ref:††

If a proposition be put into the general form,

S is P ,

its contradictory has, first, as its subject, instead of S , "the S now meant"†P2 or "some S "; and has, second, as its predicate, instead of P , that which differs from P or "not P ."

Peirce: CP 2.477 Cross-Ref:††

477. From these relations of contradictories--from the necessities of the logic of apagogically related arguments, therefore--arises the need of the two divisions of propositions into affirmative and negative on the one hand, and into universal and particular on the other. The contradictory of a universal proposition is particular, and the contradictory of an affirmative proposition is negative. Contradiction is a reciprocal relation, and therefore the contradictory of a particular proposition is universal, and that of a negative proposition is affirmative. The contradiction of particular and negative propositions could not be brought under the general formula, were the distinctions of affirmative and negative absolute and not merely relative; but, in fact, not-not- P is the same as P . And, if it is said that "what is now meant of the part of S meant at another time, is P ," since the part of S meant at another time is left to be determined in whatever way the proposition made at another time may determine it, this can only be true if All S is P . Therefore, if one man says "some S is not P ," and another replies, "some of that same S is P ," this second person, since he allows the first man's some S , which has not been defined, to remain undefined, in effect says that All S is P .†1

Peirce: CP 2.477 Cross-Ref:††

Whether contradictories differ in other respects than these well-known ones is an open question.

Peirce: CP 2.478 Cross-Ref:††

§3. OF BARBARA

478. Since some *S* means "the part now meant of *S*," a particular proposition is equivalent to a universal proposition with another subject; and in the same way, a negative proposition is equivalent to an affirmative proposition with another predicate.

The form, *S* is *P*,

therefore, as well as representing propositions in general, particularly represents Universal Affirmative propositions; and thus the general form of syllogism:

M is *P*, *S* is *M*;

S is *P*,

represents specially the syllogisms of the mood *Barbara*.

Peirce: CP 2.479 Cross-Ref:††

§4. OF THE FIRST FIGURE

479. Since, in the general form, *S* may be any subject and *P* any predicate, it is possible to modify Barbara by making the major premiss and conclusion negative, or by making the minor premiss and conclusion particular, or in both these ways at once. Thus we obtain all the modes of the first figure.

Peirce: CP 2.479 Cross-Ref:††

It is also possible to have such arguments as these:

Some *M* is *P*,

S has all the common characters of that part of *M* (whatever that part may be, and therefore of each and every *M*);

∴ *S* is *P*,

and

All not-*M* is *P*,

S is not *M*;

∴ *S* is *P*;

but as the theory of apagogical argument has not obliged us to take account of these peculiar modifications of subject and predicate, these arguments must be considered as belonging to Barbara. In this sense the major premiss must always be universal, and the minor affirmative.

Peirce: CP 2.479 Cross-Ref:††

Three propositions which are related to one another as though major premiss, minor premiss, and conclusion of a syllogism of the first figure will be termed respectively *Rule*, *Case*, and *Result*.

Peirce: CP 2.480 Cross-Ref:††
§5. SECOND AND THIRD FIGURES

480. Let the first figure be written thus:

Figure 1

Any *M* is *P*

is not

Any *S* is *M*

Some

Any *S* is *P*

Some is not

Peirce: CP 2.481 Cross-Ref:††

481. Then its two apagogical modifications are the second and third figures.

Figure 2

Figure 3

Any M is P	Some S is not P
is not	Any is

Some S is not P	Any S is M
Any is	Some

Some S is not M	Some M is not P
Any	is

Peirce: CP 2.482 Cross-Ref:††

482.†1 It is customary to enumerate six moods of the third figure instead of four, and the moods Darapti and Felapton appear to be omitted. But a particular proposition is asserted (actually and not merely virtually) by the universal proposition which does not otherwise differ from it; and therefore Darapti is included both under Disamis and Datisi, and Felapton both under Bocardo and Ferison. (De Morgan.)

Peirce: CP 2.483 Cross-Ref:††

483. The second figure, from the assertion of the rule and the denial of the result, infers the denial of the case; the third figure, from the denial of the result and assertion of the case, infers the denial of the rule. Hence we write the moods as follows, by allowing inferences only on the straight lines:

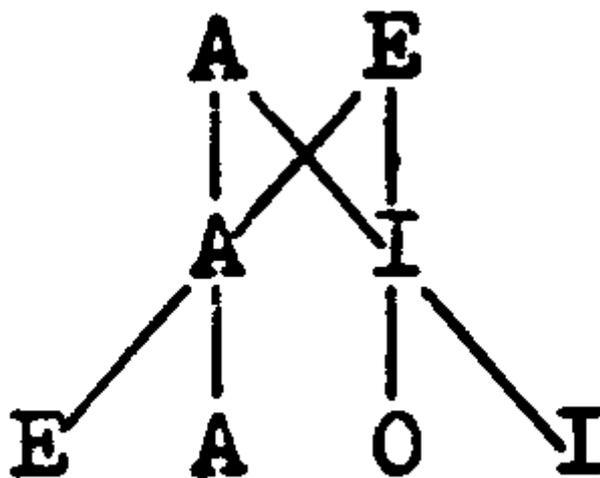


Figure 1

[Click here

to view]

Assertion of Rule,

Assertion of Case;

Assertion of Result.

Figure 1

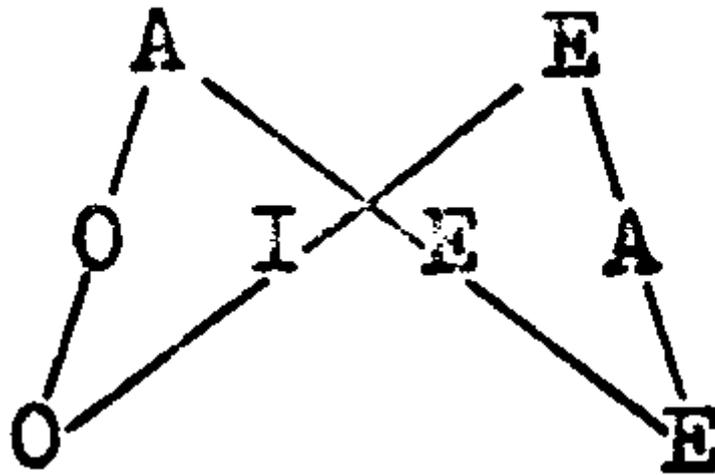


Figure 2

[Click here to view]

Assertion of Rule,

Denial of Result;

Denial of Case.

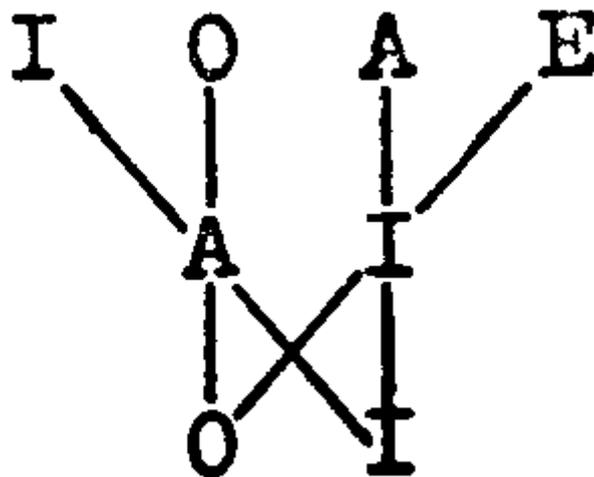


Figure 3
view]

[Click here to

Denial of Result,
 Assertion of Case;
 Denial of Rule.

Peirce: CP 2.484 Cross-Ref:††

484. The symmetry of the system of moods of the three figures is further exhibited in the following table.

Peirce: CP 2.484 Cross-Ref:††

Enter at the top the proposition asserting or denying the rule; enter at the side the proposition asserting or denying the case; find in the body of the table the proposition asserting or denying the result. In the body of the table, propositions indicated by italics belong to the first figure, those by black-letter to the second figure, and those by script to the third figure.

	I	A	E	O
E		E	A	
A	<i>I</i>	A	E	<i>O</i>
I	<i>A</i>	I	O	<i>E</i>
O		E	I	

[Click here to view]

Peirce: CP 2.485 Cross-Ref:††

485. If, as the denial of the result in the second and third figures, we put the form "Any *N* is *N*," we have:

Figure 2 *Figure 3*

No *M* is *N*, Any *N* is *N*,
 Any *N* is *N*; Some *N* is *M*;
 ∴ No *N* is *M* . ∴ Some *M* is *N*.

Peirce: CP 2.485 Cross-Ref:††

These are the formulae of the two simple conversions. Neither can be expressed syllogistically except in the figures in which they are here put (or in what is called the fourth figure, which we shall consider hereafter). If, for the denial of the result in the second figure, we put "No not- N is N " (where "not- N " has not as yet been defined) we obtain:

All M is N ,

No not- N is N ;

∴ No not- N is M .

Peirce: CP 2.485 Cross-Ref:††

In the same way, if we put "Some N is some- N " (where some- N has not been defined) for the denial of the result in the third figure, we have:

Some N is some- N ,

All N is M ;

∴ Some M is some- N .

Peirce: CP 2.485 Cross-Ref:††

These are the two ways of contraposing the Universal Affirmative.

Peirce: CP 2.486 Cross-Ref:††

486. There are two ostensive reductions of each mood of the second and third figures. I shall distinguish them as the short reduction and the long reduction. The short reduction is effected by converting or contraposing that premiss which is not the denial of the result. The long reduction is effected by transposing the premisses, contraposing or converting the denial of the result, and contraposing or converting the conclusion. The alteration thus produced in the order of the terms is shown in the following figure:

Short Reduction Long Reduction

N	M	N	N	M	{ X }
{ X }	M	{ X }	M	N	M
{ X }	N	{ X }	N	N	{ X }

Σ	π	Σ	π	Σ	P
Σ	P	P	Σ	π	Σ
P	π	P	π	π	P

Peirce: CP 2.486 Cross-Ref:††

The names bestowed by Shyreswood, or Petrus Hispanus,†1 upon the moods indicate the possibility of the short reduction in the case of Cesare and Festino of the second figure, and of Datisi and Ferison of the third figure; also the possibility of the long reduction of Camestres of the second figure and of Disamis of the third.

Peirce: CP 2.487 Cross-Ref:††

487. The short reduction of Camestres and Baroco is effected by introducing the term not- P , and defining it as that which S is when it is not P .†P1 Hence for the second premiss (Any or some S is not P) we substitute "Any or some S is not- P "; and as the first premiss, Any M is P , gives by contraposition, Any not- P is not M , the moods:

Any M is P ,

Any or some S is not P ;

∴ Any or some S is not M ,

are reduced to:

No not- P is M ,

Any or some S is not- P ;

∴ Any or some S is not M .

Peirce: CP 2.488 Cross-Ref:††

488. The short reduction of Disamis and Bocardo is effected by introducing the term some- S , defining it as that part of S which is or is not P when some S is or is not P .†P2 We can therefore substitute for the first premiss, Some S is or is not P , All some- S is or is not P ; while the second premiss, All S is M , can be contraposed into "Some M is some- S "; and thus the forms:

Some S is (*or* is not) P ,

Any S is M ;

∴ Some M is (*or* is not) P ;

are reduced to the following:

Any some- S is (*or* is not) P ,

Some M is some- S ;

∴ Some M is (*or* is not) P .

Peirce: CP 2.489 Cross-Ref:††

489. To reduce Cesare, Festino, and Baroco in the long way, it is necessary to introduce the terms not- P and some- S . Not- P is defined as that class to which any M belongs which is not P . Hence for the first premiss of Cesare and Festino we can substitute "Any M is not- P ." Some- S is defined as that class of S which is (or is not) P , when some S is (or is not) P . Hence for the second premisses of Festino and Baroco we can first substitute "Any some- S is (or is not) P "; and then, by contraposition or conversion, we obtain "Any P (or not- P) is not some- S ." Then, by the transposition of the premisses, we obtain from Cesare, which is:

No M is P , Any not- P is not S ,

Any S is P ; Any M is not- P ;

(∴ No S is M ;) ∴ Any M is not- S .

Peirce: CP 2.490 Cross-Ref:††

490. And from the conclusion of this reduced form we obtain the conclusion of Cesare by simple conversion. So Festino and its long reduction are:

Any M is not P , Any not- P is not some- S ,

Some S is P ; Any M is not- P ;

(∴ Some S is not M ;) ∴ Any M is not some- S ;

and the conclusion of Festino is obtained from that of the reduced form by a substitution which may be made syllogistically thus:

Any M is not some- S ,

Some S is some- S ;

∴ Some S is not M .

Peirce: CP 2.491 Cross-Ref:††

491. Baroco and its long reduction are:

Any M is P , Any P is not some- S ,
Some S is not P ; Any M is P ;
(\therefore Some S is not M ;) \therefore Any M is not some- S ;

and the conclusion of Baroco is obtained from the conclusion of the reduction in the same way as that of Festino.

Peirce: CP 2.492 Cross-Ref:††

492. In order to reduce Datisi, Bocardo, and Ferison in the long way, we must define Some- S as that S which is M when some S is M , and Not- P as that which some (or any) S is when it is not P . Hence, for "Some S is M " we can substitute "Any some- S is M "; and for "Some (or any) S is not P ," "Some (or any) S is not- P ." "Some S is not- P " may be converted simply; and "Any S is not- P " may be contraposed so as to become "Some not- P is some- S ." Then Datisi and its long reduction are:

Any S is P , Any some- S is M ,
Some S is M ; Some P is some- S ;
(\therefore Some M is P ;) \therefore Some P is M .

Peirce: CP 2.493 Cross-Ref:††

493. And from the conclusion of the reduction, the conclusion of Datisi is obtained by simple conversion. Ferison and its long reduction are:

Any S is not P , Any some- S is M ,
Some S is M ; Some not- P is some- S ;
(\therefore Some M is not P ;) \therefore Some not- P is M .

Peirce: CP 2.494 Cross-Ref:††

494. And from the conclusion of the reduction, the conclusion of Ferison may be obtained by a substitution whose possibility is expressed syllogistically thus:

Any not- P is not P ,

Some not- P is M ;

∴ Some M is not P .

Peirce: CP 2.495 Cross-Ref:††

495. Bocardo and its long reduction are

Some S is not P , Any S is M ,

Any S is M ; Some not- P is S ;

(∴ Some M is not P ;) ∴ Some not- P is M .

Peirce: CP 2.495 Cross-Ref:††

And the conclusion of Bocardo is obtained from that of its reduction in the same way as the conclusion of Ferison.

Peirce: CP 2.496 Cross-Ref:††

496. The ostensive reduction of the indirect or apagogical figures may be considered as the exhibition of them under the general form of syllogism:

S is M , M is P ;

∴ S is P .

But, in this sense, it is not truly a reduction if the substitutions made in the process are inferences. But although the possibility of the conversions and contrapositions can be expressed syllogistically, yet this can be done only by taking as one of the premisses:

"All N is N ,"

"Any not- N is not N ,"

or "Some N is some- N ."

Peirce: CP 2.496 Cross-Ref:††

Now, these are properly not premisses, for they express no facts; they are merely forms of words without meaning.†P1 Hence, as no complete argument has less than two premisses, the conversions and contrapositions are not inferences.†P2 The only other substitutions which have been made have been of not- P and some- S for their definitions. These also can be put into syllogistic form; but a mere modification of language is not an inference. Hence no inferences have been employed in reducing the arguments of the second and third figures to such forms that they are readily

perceived to come under the general form of syllogism.

Peirce: CP 2.496 Cross-Ref:††

There is, however, an intention in which these substitutions are inferential. For, although the passage from holding for true a fact expressed in the form "No *A* is *B*," to holding its converse, is not an inference, because, these facts being identical, the relation between them is not a fact; yet the passage from one of these forms taken merely as having *some* meaning, but not this or that meaning, to another, since these forms are not identical, and their logical relation is a fact, is an inference. This distinction may be expressed by saying that they are not inferences, but substitutions having the *form* of inferences.

Peirce: CP 2.496 Cross-Ref:††

Thus the reduction of the second and third figures, considered as mere forms, is inferential; but when we consider only what is meant by any particular argument in an indirect figure, the reduction is a mere change of wording.

Peirce: CP 2.497 Cross-Ref:††

497. The substitutions made use of in the ostensive reductions are shown in the following table, where

- e* denotes simple conversion of *E*;
- i* denotes simple conversion of *I*;
- a*[2], contraposition of *A* into *E*;
- a*[3], contraposition of *A* into *I*;
- o*[2] the substitution of "Some *S* is not *M*" for "Any *M* is not some-*S*";
- o*[3] the substitution of "Some *M* is not *P*" for "Some not-*P* is *M*";
- e*", introduction of not-*P* by definition;
- i*", introduction of some-*S* by definition.

| ***Reduction of Second Figure*** |

| Name of Mood | Short Reduction | Long Reduction |

| ***Cesare*** | *e* | *e*" *a*[2] *e* |

| ***Camestres*** | *a*[2] *e*" | *e* *e* |

| ***Festino*** | *e* | *e*" *i*" *a*[2] *o*[2] |

| ***Baroco*** | *a*[2] *e*" | *i*" *e* *o*[2] |

<i>Reduction of Third Figure</i>		
Name of Mood	Short Reduction	Long Reduction
<i>Disamis</i>	$a[3] i''$	$i i$
<i>Datisi</i>	i	$i'' a[3] i$
<i>Bocardo</i>	$a3 i''$	$e'' i o[3]$
<i>Ferison</i>	i	$i'' e'' a[3] o[3]$

Peirce: CP 2.497 Cross-Ref:††

With the exception of the substitutions i'' and e'' , which will be considered hereafter, all those which are used in the reduction of the moods of either oblique figure have the form of inferences in the same figure.†1

Peirce: CP 2.498 Cross-Ref:††

498. The so-called *reductio per impossibile* is the repetition or inversion of that contraposition of propositions by which the indirect figures have been obtained. Now, contradiction arises from a difference both in quantity and quality; but it is to be observed that, in the contraposition which gives the second figure, a change of the *quality* alone, and in that which gives the third figure a change of the *quantity* alone, of the contraposed propositions, is sufficient. This shows that the two contrapositions are of essentially different kinds, and that the reductions *per impossibile* of the second and third figures respectively involve the following formal inferences.†P1

Figure 2

The Result follows from the Case;

∴The Negative of the Case follows from the Negative of the Result.

Figure 3

The Result follows from the Rule;

∴The Rule changed in Quantity follows from the Result changed in Quantity.

Peirce: CP 2.498 Cross-Ref:††

But these inferences may also be expressed as follows:

Figure 2

P

Whatever (*S*) is *M* is not *P*;

not *P*

∴ Whatever (*S*) is *P* is not *M*.

Figure 3

S

Any some *S* is whatever (*P* or not-*P*) *M* is;

some *S*

∴ Some *M* is whatever (*P* or not-*P*) *S* is.

Peirce: CP 2.498 Cross-Ref:††

Now, the limitations in parentheses do not affect the essential nature of the inferences; and omitting them we have:

Figure 2

P

Any *M* is not *P*;

not *P*

∴ Any *P* is not *M*.

Figure 3

S

Any some S is M ;

some S

∴ Some M is S .

Peirce: CP 2.498 Cross-Ref:††

We have already seen that the former of these is of the form of the second figure, and the latter of the form of the third figure of syllogism.

Peirce: CP 2.499 Cross-Ref:††

499. Hence, it appears that no syllogism of an indirect figure can be reduced to the first figure without a substitution which has the form of the very figure from which the syllogism is reduced. In other words, the indirect syllogisms are of an essentially different form from that of the first figure, although in a more general sense they come under that form.

Peirce: CP 2.500 Cross-Ref:††
§6. THE THEOPHRASTEAN MOODS

500. It is now necessary to consider the five moods of Theophrastus, viz., *Baralippton*, *Celantes*, *Dabitis*, *Fapesmo*, *Frisesororum*. *Baralippton* is included in *Dabitis*, and *Fapesmo* in *Frisesororum*, in the same way in which *Darapti* is included in *Disamis* and *Datisi*, and *Felapton* in *Bocardo* and *Ferison*. The Theophrastean moods are thus reduced to three, viz.:

<i>Celantes</i>	<i>Frisesororum</i>	<i>Dabitis</i>
No X is Y ,	No X is Y ,	Some Y is Z ,
All Z is X ;	Some Y is Z ;	All Z is X ;

∴ Any Y is not Z . ∴ Some Z is not X . ∴ Some X is Y .

Peirce: CP 2.500 Cross-Ref:††

Suppose we have, first, a Rule; second, a Case under that rule, which is itself a Rule; and, third, a Case under this second rule, which conflicts with the first rule. Then it would be easy to prove that these three propositions must be of the form:

1. No X is Y .
2. All Z is X .

3. Some *Y* is *Z*.

Peirce: CP 2.500 Cross-Ref:††

These three propositions cannot all be true at once; if, then, any two are asserted, the third must be denied, which is what is done in the three Theophrastean moods.†1

Peirce: CP 2.500 Cross-Ref:††

These moods are resolved into one another by the contraposition of propositions, and therefore should be considered as belonging to different figures.

Peirce: CP 2.501 Cross-Ref:††

501. They can be ostensibly reduced to the first Aristotelian figure in two ways; thus,

Short Reduction Long Reduction

{B} {A}	{B} {A}	{B} {G}
{G} {B}	{G} {B}	{A} {B}
{A} {G}	{G} {A}	{A} {G}

Peirce: CP 2.502 Cross-Ref:††

502. The verses of Shyreswood †2 show how Celantes and Dabitis are to be reduced in the short way, and Frisesomorum in the long way. Celantes and its long reduction are as follows:

Any *X* is not *Y*, Any not-*X* is not *Z*,
Any *Z* is *X*; Any *Y* is not-*X*;
∴ Any *Y* is not *Z*. ∴ Any *Y* is not *Z*.

Peirce: CP 2.502 Cross-Ref:††

"Any *X* is not *Y*," becomes, by conversion, "Any *Y* is not *X*." The term "not-*X*" is then introduced, being defined as that which *Y* is when it is not *X*. Then "*Z* is *X*" becomes "Any not-*X* is not *Z*"; and, the premisses being transposed, the reduction is effected.

Peirce: CP 2.503 Cross-Ref:††

503. Dabitis and its long reduction are as follows:

Any Z is X , Any some- Z is Y ,
 Some Y is Z ; Some X is some- Z ;
 \therefore Some X is Y . \therefore Some X is Y .

Peirce: CP 2.503 Cross-Ref:††

"Some Y is Z " becomes, by conversion, "Some Z is Y ." Then the term "some- Z " is introduced, being defined as that Z which is Y if "some Z is Y ." Then "Any Z is X " becomes "Some X is some- Z ," and, the premisses being transposed, the reduction is effected.

Peirce: CP 2.504 Cross-Ref:††

504. Frisesomorum is:

Some Y is Z ,
 Any X is not Y ;
 \therefore Some Z is not X .

Peirce: CP 2.504 Cross-Ref:††

Let some- Y be that Y which is Z when some Y is Z ; and then we have:

Some Y is some- Y ,
 Any X is not Y ;
 \therefore Some some- Y is not X .

Peirce: CP 2.504 Cross-Ref:††

Then let not- X be that which any Y is when some Y is not X , and we have,

Some some- Y is not- X ,

which yields by conversion,

Some not- X is some- Y ;

and we thus obtain the reduction:

Any some- Y is Z ,
Some not- X is some- Y ;
∴ Some not- X is Z .

Peirce: CP 2.504 Cross-Ref:††

From the conclusion of this reduction, the conclusion of Frisesomorum is justified as follows:

Some not- X is Z ,
Any X is not not- X ;
∴ Some Z is not X .

Peirce: CP 2.504 Cross-Ref:††

Another mode of effecting the short reduction of Frisesomorum is this: Let not- Y be that which any X is when no X is Y , and we have:

Some Y is Z ,
Any not- Y is not Y ;
∴ Some Z is not not- Y .

Peirce: CP 2.504 Cross-Ref:††

Let some- Z be that Z which is not not- Y when some Z is not- Y , and we have,

Any some- Z is not not- Y ,

and by conversion,

Any not- Y is not some- Z .

Peirce: CP 2.504 Cross-Ref:††

Thus we obtain as the reduced form:

Any not-*Y* is not some-*Z*,

Any *X* is not-*Y*;

∴ Any *X* is some-*Z*.

Peirce: CP 2.504 Cross-Ref:††

From the conclusion of this reduction, we get that of Frisesomorum thus:

Some some-*Z* is *Z*,

Any *X* is not some-*Z*;

∴ Some *Z* is not *X*.

Peirce: CP 2.505 Cross-Ref:††

505. In either reduction of Celantes, if we neglect the substitution of terms for their definitions, the substitutions are all of the second syllogistic figure. This of itself shows that Celantes belongs to that figure, and this is confirmed by the fact that it concludes the denial of a case. In the same way, the reductions of Dabitis involve only substitutions in the third figure, and it concludes the denial of a rule.

Frisesomorum concludes a proposition which is at once the denial of a rule and the denial of a case: its long reduction involves one conversion in the second figure and another in the third, and its short reductions involve conversions in Frisesomorum itself. It therefore belongs to a figure which unites the characters of the second and third, and which may be termed the second-third figure in Theophrastean syllogism.

Peirce: CP 2.506 Cross-Ref:††

506. There are, then, two kinds of syllogism--the Aristotelian and Theophrastean. †P1 In the Aristotelian occur the first, second, and third figures, with four moods of each. In the Theophrastean occur the second, third, and second-third figures, with one mood of each. The first figure is the fundamental or typical one, and Barbara is the typical mood. There is a strong analogy between the figures of syllogism and the four forms of proposition. *A* is the fundamental form of proposition, just as the first figure is the fundamental form of syllogism. The second and third figures are derived from the first by the contraposition of propositions, and *E* and *I* are derived from *A* by the contraposition of terms, thus:

Any *S* is *P*.

Any not-*P* is not *S*. Some *P* is some-*S*.

Peirce: CP 2.506 Cross-Ref:††

O combines the modifications of *E* and *I*, just as the second-third figure combines the second and third. In the second-third figure, only *O* can be concluded, in the third only *I* and *O*, in the second only *E* and *O*, in the first either *A E I O*. Thus

A is the first figure of proposition, *E* the second, *I* the third, *O* the second-third. †P1

Peirce: CP 2.507 Cross-Ref: ††
§7. MATHEMATICAL SYLLOGISMS

507. A kind of argument very common in mathematics may be exemplified as follows:

Every part is less than that of which it is a part,

Boston is a part of the Universe;

∴ Boston is less than the Universe.

This may be reduced to syllogistic form thus:

Any relation of part to whole is a relation of less to greater,

The relation of Boston to the Universe is a relation of part to whole;

∴ The relation of Boston to the Universe is a relation of less to greater.

Peirce: CP 2.507 Cross-Ref: ††

If logic is to take account of the peculiarities of such syllogisms, it would be necessary to consider some propositions as having three terms, subject, predicate, and object; and such propositions would be divided into *active* and *passive*. The varieties in them would be endless.

Peirce: CP 2.508 Cross-Ref: ††
PART III

§1. INDUCTION AND HYPOTHESIS

508. In the syllogism:

Any *M* is *P*,

Σ'*S*' is *M*;

∴ Σ'*S*' is *P*;

where $\Sigma'S'$ denotes the sum of all the classes which come, under M , if the second premiss and conclusion are known to be true, the first premiss is, by enumeration, true. Whence we have, as a valid demonstrative form of inference:

$\Sigma'S'$ is P ,

$\Sigma'S'$ is M ;

$\therefore M$ is P .

Peirce: CP 2.508 Cross-Ref:††

This is called perfect induction. It would be better to call it formal induction.

Peirce: CP 2.509 Cross-Ref:††

509. In a similar way, from the syllogism:

Any M is $\pi'P'$,

Any S is M ;

\therefore Any S is $\pi'P'$;

where $\pi'P'$ denotes the conjunction of all the characters of M , if the conclusion and first premiss are true, the second premiss is true by definition; so that we have the demonstrative form of argument:

Any M is $\pi'P'$,

Any S is $\pi'P'$;

\therefore Any S is M .

Peirce: CP 2.509 Cross-Ref:††

This is reasoning from definition, or, as it may be termed, formal hypothesis.

Peirce: CP 2.510 Cross-Ref:††

510. One half of all possible propositions are true, because every proposition has its contradictory. Moreover, for every true particular proposition there is a true universal proposition, and for every true negative proposition there is a true affirmative proposition. This follows from the fact that the universal affirmative is the type of all propositions. Hence of all possible propositions in either of the forms,

$\Sigma S'$ is M , and M is $\pi P'$,

one half are true. In an untrue proposition of either of these forms, some finite ratio of the S 's or P 's are not true subjects or predicates. Hence, of all propositions of either of these forms which are partly true, some finite ratio more than one half are wholly true. Hence, if in the above formulæ for formal induction or hypothesis, we substitute S' for $\Sigma S'$ and P' for $\pi P'$ we obtain formulæ of probable inference. This reasoning gives no *determinate* probability to these modes of inference, but it is necessary to consider that, however weak synthetic inference might have been at first, yet if it had the least positive tendency to produce truth, it would continually become stronger, owing to the establishment of more and more secure premisses.

Peirce: CP 2.511 Cross-Ref:††

511. The rules for valid induction and hypothesis deducible from this theory are as follows:

Peirce: CP 2.511 Cross-Ref:††

1. The explaining syllogism, that is to say, the deductive syllogism, one of whose premisses is inductively or hypothetically inferred from the other and from its conclusion, must be valid.

Peirce: CP 2.511 Cross-Ref:††

2. The conclusion is not to be held as absolutely true, but only until it can be shown that, in the case of induction, S' was taken from some narrower class than M , or, in the case of hypothesis, that P' was taken from some higher class than M .

Peirce: CP 2.511 Cross-Ref:††

3. From the last rule it follows as a corollary that in the case of induction the subject of the premisses must be a sum of subjects, and that in the case of hypothesis the predicate of the premisses must be a conjunction of predicates.

Peirce: CP 2.511 Cross-Ref:††

4. Also, that this aggregate must be of different objects or qualities and not of mere names.

Peirce: CP 2.511 Cross-Ref:††

5. Also, that the only principle upon which the instanced subjects or predicates can be selected is that of belonging to M . †P1

Hence the formulæ are:

Induction.

$S' S'' S'''$, etc. are taken at random as M 's,

$S' S'' S'''$, etc. are P ;

∴ Any M is probably P .

Hypothesis

Any M is, for instance, $P' P'' P'''$, etc.

S is $P' P'' P'''$, etc.;

∴ S is probably M .

Peirce: CP 2.512 Cross-Ref:††

§2. MOODS AND FIGURES OF PROBABLE INFERENCE

512. It is obvious that the explaining syllogism of an induction or hypothesis may be of any mood or figure.

Peirce: CP 2.512 Cross-Ref:††

It would also seem that the conclusion of an induction or hypothesis may be contraposed with one of the premisses.

Peirce: CP 2.513 Cross-Ref:††

§3. ANALOGY

513. The formula of analogy is as follows:

Peirce: CP 2.513 Cross-Ref:††

S' , S'' , and S''' are taken at random from such a class that their characters at random are such as P' , P'' , P''' .

t is P' , P'' , and P''' ,

S' , S'' , and S''' are q ;

∴ t is q .

Such an argument is double. It combines the two following:

1.

S' , S'' , S''' are taken as being P' , P'' , P''' ,

S' , S'' , S''' are q ;

∴ (By induction) P', P'', P''' is q ,

t is P', P'', P''' ;

∴ (Deductively) t is q .

2.

S', S'', S''' are, for instance, P', P'', P''' ,

t is P', P'', P''' ;

∴ (By hypothesis) t has the common characters of S', S'', S''' ,

S', S'', S''' are q ;

∴ (Deductively) t is q .

Peirce: CP 2.513 Cross-Ref:††

Owing to its double character, analogy is very strong with only a moderate number of instances.

Peirce: CP 2.514 Cross-Ref:††

§4. FORMAL RELATIONS OF THE ABOVE FORMS OF ARGUMENT

514. If we take an identical proposition as the fact to be explained by induction and hypothesis, we obtain the following formulæ:

By Induction

S, S', S'' are taken at random as being M ,

S, S', S'' have the characters common to S, S', S'' ;

∴ Any M has the characters common to S, S', S'' .

By Hypothesis

M is, for instance, P, P', P'' ,

Whatever is at once P, P' , and P'' is P, P', P'' ;

∴ Whatever is at once P, P' , and P'' is M .

Peirce: CP 2.514 Cross-Ref:††

By means of the substitution thus justified, Induction and Hypothesis can be reduced to the general type of syllogism, thus:

Induction

S, S', S'' are taken as M ,

S, S', S'' are P ;

∴ Any M is P .

Reduction

S, S', S'' are P ;

Almost any M has the common characters of S, S', S'' .

∴ Almost any M is P .

Hypothesis

M is, for instance, P', P'', P''' ,

S is P', P'', P''' ;

∴ S is M .

Reduction

Whatever is, at once, P', P'', P''' is like M ,

S is P', P'', P''' ;

∴ S is like M .

Peirce: CP 2.515 Cross-Ref:††

515. Induction may, therefore, be defined as argument which assumes that a whole collection, from which a number of instances have been taken at random, has all the common characters of those instances; hypothesis, as an argument which assumes that a term which necessarily involves a certain number of characters, which have been lighted upon as they occurred, and have not been picked out, may be predicated of any object which has all these characters.

Peirce: CP 2.516 Cross-Ref:††

516. There is a resemblance between the transposition of propositions by

which the forms of probable inference are derived and the contraposition by which the indirect figures are derived; in the latter case there is a denial or change of modal quality; while in the former there is reduction from certainty to probability, and from the sum of all results to some only, or a change in modal quantity. Thus probable inference is related to apagogical proof, somewhat as the third figure is to the second. Among probable inferences, it is obvious that hypothesis corresponds to the second figure, induction to the third, and analogy to the second-third.

Peirce: CP 2.517 Cross-Ref:††
CHAPTER 3

EXTENSION OF THE ARISTOTELIAN SYLLOGISTIC

§1. ON A LIMITED UNIVERSE OF MARKS †1

517. De Morgan and his followers frequently speak of a "limited universe of discourse" in logic.†P1 An unlimited universe would comprise the whole realm of the logically possible. In such a universe, every universal proposition, not tautologous, is false; every particular proposition, not absurd, is true. Our discourse seldom relates to this universe: we are either thinking of the physically possible, or of the historically existent, or of the world of some romance, or of some other limited universe.

Peirce: CP 2.518 Cross-Ref:††

518. But besides its universe of objects, our discourse also refers to a universe of characters. Thus, we might naturally say that virtue and an orange have nothing in common. It is true that the English word for each is spelt with six letters, but this is not one of the marks of the universe of our discourse.

Peirce: CP 2.519 Cross-Ref:††

519. A universe of things is unlimited in which every combination of characters, short of the whole universe of characters, occurs in some object. In like manner, the universe of characters is unlimited in case every aggregate of things short of the whole universe of things possesses in common one of the characters of the universe of characters. Ordinary syllogistic as set forth [in 453] supposes an unlimited universe of characters.†2 The non-possession of any character is regarded as implying the possession of another character the negative of the first. Nobody has more strenuously insisted upon this than De Morgan.†1

Peirce: CP 2.520 Cross-Ref:††

520. In our ordinary discourse, on the other hand, not only are both universes limited, but, further than that, we have nothing to do with individual objects nor simple marks; so that we have simply the two distinct universes of things and marks related to one another, in general, in a perfectly indeterminate manner. The consequence is, that a proposition concerning the relations of two groups of marks is not necessarily equivalent to any proposition concerning classes of things; so that the

distinction between propositions in extension and propositions in comprehension is a real one, separating two kinds of facts; whereas in the view of ordinary syllogistic the distinction only relates to two modes of considering any fact. To say that every object of the class *S* is included among the class of *P*'s, of course must imply that every common character of the *P*'s is a common character of the *S*'s. But the converse implication is by no means necessary, except with an unlimited universe of marks. The reasonings in depth of which I have spoken, suppose, of course, the absence of any general regularity about the relations of marks and things.

Peirce: CP 2.521 Cross-Ref:††

521. I may mention here another respect in which this view differs from that of ordinary logic, although it is a point which has, so far as I am aware, no bearing upon the theory of probable inference. It is that under this view there are propositions of which the subject is a class of things, while the predicate is a group of marks. Of such propositions there are twelve species, distinct from one another in the sense that any fact capable of being expressed by a proposition of one of these species cannot be expressed by any proposition of another species.†2 The following are these forms. *S* means an object of the class *S*, while π means a quality of the group π ; and "has" means possesses as a subject and attributed; "is had by" means inheres in; "wants" and "is wanted by" are negatives of "has."

1. Every *S* has every π .
2. Some *S* has all π .
3. Every π is had by some *S*.
4. Some π is had by all *S*'s.
5. Every *S* has some π .
6. Some *S* has some π .
7. Every *S* wants every π .
8. Some *S* wants all π .
9. Every π is wanted by some *S*.
10. Some π is wanted by all *S*'s.
11. Every *S* wants some π .
12. Some *S* wants some π .†1

These are the variations produced by the different connections of the verb without taking account of *not* applied to *S* and π which would, of course, multiply them by four.

Peirce: CP 2.522 Cross-Ref:††

522. I term the above the forms of the *first order*, because they contain "has" or "is had by" "wants" or "is wanted by" once, and once only. The following is an

example of a proposition of the *second order*:

Peirce: CP 2.522 Cross-Ref:††

"Every S wants some character possessed by every P ." The following is of the *third order*:

Peirce: CP 2.522 Cross-Ref:††

"Some S wants some character had by every object that possesses all the π s."

Peirce: CP 2.522 Cross-Ref:††

The following is of the *zero order*:

"Every S is a P ."

Peirce: CP 2.523 Cross-Ref:††

523. It has been explained [in 458] that (speaking with sufficient accuracy for the purpose) "Some" means that the speaker is to select an instance, while "Every" or "Any" means that a second person is to perform the selection. Of course, it is easier to satisfy the conditions of a statement if one can select one's own examples, except for this, that he who undertakes to find an example guarantees that there is one, while if he leaves the selection to another, and there is none, his statement is not broken down. Consequently, from *Every* we can infer *Some*, provided only we have some other premiss which assures us that something exists. Thus, if and only if, we are assured some S exists we can infer 2 from 1, 8 from 7, 6 from 5, and 12 from 11. In like manner, if, and only if, we are assured some π exists we can infer 4 from 1, 6 from 3, 10 from 7, and 12 from 9.

Peirce: CP 2.524 Cross-Ref:††

524. If the first person is to select one object and a second person another, it will always be an advantage to the former to wait and see what selection the latter makes. Consequently, 3 always follows from 2. For 2, or

Some S has all π 's,

asserts that an S can be selected so that after it is selected, and the selection made known, no matter what π be selected, that S will have that π ; while 3, or

Every π is had by some S ,

asserts that no matter what π be selected, after it is selected, and the selection made known, an S can be so chosen that that S will have that π .†1 In like manner, 5 always follows from 4, 9 from 8, and 11 from 10.

Peirce: CP 2.525 Cross-Ref:††

525. Form 1 is precisely denied by 12, 2 by 11, 3 by 10, 4 by 9, 5 by 8, and 6 by 7. Forms 2 and 10, 4 and 8 are, therefore, contraries: they cannot both be true but may both be false. Forms 3 and 11, 5 and 9 are sub-contraries: both cannot be false, but both may be true.

Peirce: CP 2.525 Cross-Ref:††

Other pairs of forms in the same terms may be both false, both true, or either false while the other is true.

Peirce: CP 2.526 Cross-Ref:††

526. In this system syllogisms hold between any two propositions having a middle term of different *phasis* in the two premisses. We should therefore expect in each figure 21 different species of syllogisms. But two of these infer no more than might be inferred from weaker premisses. I am indisposed to admit them as distinct species (for these species are more natural classes than are moods). Each species by metaphasis of the three terms and two verbs gives 32 moods, and if the premisses are of different form, 64. I enumerate the species in case the middle term is a group of qualities. I state them, when convenient, in the second figure, but regard figure little. I usually take the phasis of the verb differently in the two premisses.

First Species

Every *P* has no quality but {m}'s,

Every *S* has every {m};

∴ Every *S* has every quality in any *P*.

Second Species

Every *P* has {m}'s as its only qualities,

Some *S* has all {m}'s;

∴ Some *S* has every quality of any *P*.

Third Species

Every *P* has {m}'s for its only qualities,

Every {m} is in some *S* or other;

∴ Qualities of *S*'s are alone possessed by any *P*'s.

This conclusion follows from the last.

Fourth Species

Every ***P*** has {m}'s for its only qualities,

A certain quality not {m} is in all ***S***'s;

∴ There is a quality of all ***S***'s that is absent from all ***P***'s.

Hence, again, No ***S*** is a ***P***. But this last inference would not hold if the phasis of the two premisses were not contrary.

Fifth Species

Every ***P*** has {m}'s for its sole qualities,

Every ***S*** has some quality or other beside the {m}'s;

∴ Every ***S*** possesses a quality unpossessed by any ***P***.

Hence, again, No ***S*** is a ***P***, an inference only holding on account of the antiphasis of the premisses. But the general conclusion of this species is much weaker than the last which asserts that there is a single quality which belongs to all ***S***'s and no ***P***'s; while the present inference is that each ***S*** has some quality or another absent from all ***P***'s.

Sixth Species

Every ***P*** has {m}'s for its sole qualities,

Some ***S*** has a quality not a {m};

∴ Some ***S*** has a quality unpossessed by any ***P***.

Hence, again (on account of the antiphasis) Some ***S*** is not a ***P***.

Seventh Species

Some P has $\{m\}$'s for its sole qualities,

Some S has all $\{m\}$'s;

∴ A certain S has all the qualities of a certain P .

Eighth Species

Every quality not a $\{m\}$ is absent from some P or other,

Some S has all $\{m\}$'s;

∴ Some S has all properties common to all P 's.

This conclusion follows from the last.

Ninth Species

Every quality not a $\{m\}$ is absent from some P or other,

Every $\{m\}$ is in some S ;

∴ Every property common to all P 's is in some S .

This conclusion follows from the last.

Tenth Species

According to the order of arrangement followed, I should here insert the syllogism from the premisses:

Some $\{m\}$ is absent from all P 's,

Some S has all $\{m\}$'s;

But no more can be inferred from these premisses than if the last is weakened to

Every $\{m\}$ is in some S or other.

The conclusion in either case is:

Some *S* has a quality that no *P* has,

which is precisely the conclusion of the sixth species. As remarked, owing to the antiphrasis, it further follows that

Some *S* is not a *P*.

Eleventh Species

Every *P* wants some {m},

Some *S* has every {m};

∴ Some *S* has some quality or other that each *P* wants.

Hence, further, owing to the antiphrasis, some *S* is not *P*. The general conclusion here is much weaker than that of the last species, where there was said to be a quality possessed by an *S* but unpossessed by any *P*; while here it is only stated that there is a certain *S* which has one quality that one *P* wants and perhaps a different one that others want, etc. But it is not so weak as if it only asserted that some *S* or other has each quality that any *P* wants. One of the premisses of the eleventh species follows from one of the tenth, and one of the tenth from one of the eleventh. But this does not balance matters, owing to one of these involving the middle term differently from the other.

Twelfth Species

Every *P* wants some {m} or other,

Every {m} is in some *S* or other;

∴ Every *P* wants some quality of some *S*.

In consequence of the antiphrasis, we have the further inference, Every *P* is not some *S*. The general conclusion of the twelfth species follows from that of the eleventh.

Thirteenth Species

Some quality not a {m} is absent from all *P*'s,

Some {m} is in all *S*'s;

∴Some quality of all *S*'s is other than some quality absent from all *P*'s.

We are now getting to premisses so weak that with one exception all further conclusions are spurious.†1 The present conclusion may be otherwise stated thus: The qualities absent from all *P*'s together with those in all *S*'s amount to two, at least.

Fourteenth Species

Every *P* wants some quality or other not {m},

Some {m} is in all *S*'s;

∴Some quality of all *S*'s is other than some quality or other absent from any given *P*.

This conclusion follows from the last. It does not speak of the quality of the *S*'s as being other than one quality absent from all *P*'s, but as being such that each *P* has some quality or other different from it.

Fifteenth Species

Every *P* wants some quality or other not a {m},

Every *S* has some {m} or other;

From these premisses two different conclusions follow, viz:

First, Every *S* has a quality other than some quality or other absent from each *P*.

Second, Every *P* wants a quality other than some quality or other of each *S*.

Each of these conclusions follows immediately from the conclusion of the Fourteenth Species.

Sixteenth Species

Next, according to the arrangement followed, I should put the premisses:

Some *P* wants all {m}'s,

Some *S* has some {m}.

But no inference can be drawn from these which cannot equally be drawn when the first premiss is weakened, thus:

Every {m} is absent from some *P* or other,

Some *S* has some {m};

∴ Some *S* has some quality not in some *P*.

Seventeenth Species

Some quality not {m} is absent from all *P*'s,

Some *S* has some {m};

∴ Some *S* has a quality different from some quality wanting to all *P*'s.

This spurious conclusion follows from the conclusion of the Sixth Species.

Eighteenth Species

Every *P* wants some quality or other, other than {m},

Some *S* has some {m};

∴ Some *S* has a quality other than a quality absent from each *P*.

This conclusion follows from the last.

Nineteenth Species

Some **P** wants some quality not {m},

Some **S** has some {m};

∴ Some **S** possesses a quality other than some quality absent from a **P**.

Peirce: CP 2.527 Cross-Ref:††

527. Among the above nineteen propositions, I have introduced a misstatement or two in order that the reader may have the profitable pleasure of thinking the whole matter out *in his own way*, as an exercise in reasoning. If any logician does not think the above nineteen really different kinds of inference are as important as eighteen out of the nineteen traditional moods, then I do not envy him his perspicacity.

Peirce: CP 2.528 Cross-Ref:††
§2. GENERAL CANON OF SYLLOGISMP

528. It has for ages been customary with logicians, in addition to establishing valid forms of syllogism, to put forth certain rules independent of figure, or the special forms used, which were supposed to form of themselves sufficient criteria for the ascertainment of the validity or invalidity of a syllogistic argumentation.

Peirce: CP 2.529 Cross-Ref:††

529. Most of the rules so given fail at being universally true. The only such canon that really holds is that in order that there should be a syllogistic conclusion there must be two premisses containing a middle term "distributed" in one, but not in the other, and the conclusion will be drawn by compounding the two premisses in such a way that this middle term may be dropped upon the same principle upon which it is dropped in Barbara.

Peirce: CP 2.530 Cross-Ref:††

530. A term is said to be "distributed" when it substantially fulfills the function of an antecedent, and is said to be "undistributed" when it substantially fulfills the function of a consequent. When a proposition is made an antecedent, every possible case of its being true is spoken of; and when it is made a consequent every possible state of its being false is spoken of. For to say "If **A** is true, **B** is true," is to say that every possible state of things in which **A** is true is different from every possible state of things in which **B** is false. Thus, by considering in one premiss every possible state of things in which the middle is true and in the other every possible state of things in which the middle is false, between them every possible state of things is considered. The syllogism is thus conceived under the form

{ true,

If **M** is true, **P** is { false,

{true,
 If M is false, Q is {false,

 {false {true,
 ∴ If P is {true Q is {false.

Peirce: CP 2.530 Cross-Ref:††

This shows that every term not the middle term preserves in the conclusion the same distribution it had in the premiss.

Peirce: CP 2.531 Cross-Ref:††

531. The thirteenth species of syllogism suggests such forms as the following:

The qualities of all S 's together with those absent from all P 's amount to two at least,

The qualities absent from some S 's amount to one at most;

∴ The qualities of all S 's amount to one at least.

Peirce: CP 2.531 Cross-Ref:††

This is plainly a case of the principle that the degree of particularity of the conclusion is the *algebraic sum* of those of the premisses.

Peirce: CP 2.532 Cross-Ref:††

§3. HAMILTON'S QUANTIFICATION OF THE PREDICATE †1

532. Quite clearly, the Aristotelian syllogistic omits something. Many of the old logicians would dispute this; but it is because they forget altogether the purposes of logic, not so much its practical purpose, to teach men to reason well, as its theoretical purpose to analyze reasoning and see what it consists in, so far as it consists in what cannot be false, even if all unqualified opinions are mistaken. It may be, in fact I myself maintain, that it ought to be assumed that all our knowledge has some error in it--even our knowledge that there is something real. I do not believe that anything (unless it be God) quite fulfills the idea of the real. But then, nobody can sincerely doubt that things come pretty near to being real. Those facts that there is no room to question, logic ought (as I shall show) to take account of, so far as they concern reasoning. It is true, that logic does not go into the business of doing people's reasoning for them; so far as it can be relegated to anybody, mathematics undertakes that business. Hence it is that geometry, etc. are not parts of logic, but of mathematics. . . . Perhaps I am wrong about words; that it is not the logician *qua* logician, who has

to analyze reasoning and explain the gist of it. Certainly, if a given man is usefully occupied, I do not mean to recommend him to do something else than he is inclined to undertake. But as for gentlemen who use the shibboleth of "*extralogical*"--and so offensively that shilaly was on my tongue--who seem to maintain, and many of them clearly say so, that the study of relations in general ought not be pursued by any man, I can only say that the sort of logic they are cultivating seems to me the barrenest and most trifling of things, unless it is destined to be developed into something more.

Peirce: CP 2.532 Cross-Ref:††

Until the time of Laurentius Valla, who died in 1457, no logic had ever mentioned the dilemma. The principle of excluded middle was "extralogical." This to my apprehension argues a certain narrowness on the part of the earlier logicians; and yet it is a fact admitting of no dispute that Aristotle and all the medieval logicians admitted far more into logic, than any but the most progressive party--and the party of progress is *ipso facto* perpetually in the minority--are willing to have the logical treatises of today consider. They, for example, were unanimous in thinking the general theory of relations a part of logic.

Peirce: CP 2.533 Cross-Ref:††

533. I propose in this chapter to consider one or two proposed enlargements of the Aristotelian syllogistic. Some of the proposals that have been made by the most eminent men seem to me utterly undeserving of mention, in a book like this which avoids altogether a criticism of recent contributions to logic. I barely except from these a proposal to which nothing induces me to refer except the singular celebrity it has attained, so that everybody has heard of it and expects to find something about it here, together with the fact that it will serve as a foil to bring out the merits of other proposals, and also as an instructive example of how not to do it--I allude to Sir William Hamilton's scheme of the quantified predicate. I am to a certain extent an admirer of Hamilton. I have learned much from his notes to Reid and some of his other writings. But formal logic is a mathematical subject; and Hamilton was exceptionally weak in mathematics. He wrote down eight propositions,

All *S* is all *P*, Any *S* is not any-*P*,

All *S* is some *P*, Any *S* is not some *P*,

etc., etc.,

and letting these propositions mean what the words seemed to suggest without any penetrating criticism--without a suspicion that anything of that sort was wanted--hastily drew up a silly table of inferences, which unmathematical logicians--the very ones who oppose any *rational* scheme of this sort as "extralogical"--extol to the skies. The reckless Hamilton flew like a dor-bug into the brilliant light of De Morgan's mind in a way which compelled the greatest formal logician that ever lived to examine and report upon the system. There was a considerable controversy; for Hamilton and several of his pupils were as able in controversy as they were impotent in inquiry--but De Morgan's final and unanswerable paper will be found in the *Cambridge Philosophical Transactions*, [1863] X, 428.†1

Peirce: CP 2.534 Cross-Ref:††

534. Though Hamilton never wrote a book in exposition of his system, he did better. He drew up (in 1846) a list of sixteen very precisely expressed points which such an exposition ought to contain, and offered a prize for the performance. His most attentive scholar, Thomas Spencer Baynes, gained the prize by an essay most clear, most concise, and which convinces the reader he has the matter just as Hamilton at his best conceived it. Four years later, the essay was published;†2 and Hamilton again wrote to express his unqualified approval, except only that he offers a note "showing in what respect my [his] present view differs from those stated by you in the Essay." This note goes into the smallest minutiae; so that there can be no possible mistake as to Hamilton's doctrine. Moreover, the appendices to Hamilton's *Discussions* and, much more important, the appendices to his *Lectures on Logic* contribute full verifications of every point; so that it has seldom happened in the history of philosophy that an author has left so fully satisfactory records of his thought behind him. Yet when, after Hamilton's death, De Morgan's attack came out, Baynes and all Hamilton's scholars were perfect know-nothings, could not say what he had taught at all, but thought it must have been something very different from what he and Baynes, for him, had most distinctly said, and which anybody with a head can see necessarily follows from the initial assumption.

Peirce: CP 2.534 Cross-Ref:††

The first of the sixteen requirements for the prize essay was that it should state "what logic *postulates* as a condition of its applicability." Note the tone of classroom question. No answer will gain the prize, one feels sure, unless it most precisely reproduces the author's opinion, without subtraction nor addition. Baynes' statement is admirably clear. The postulate is "that we be allowed to state in language what is contained in thought."†1

Peirce: CP 2.534 Cross-Ref:††

That the doctrine of the paragraphs was really Hamilton's doctrine, is shown, not only by all the evidences above, but by the further testimony of Archbishop Thomson and Dean Mansel, two of his foremost pupils. The very style apes Hamilton.
...

Peirce: CP 2.534 Cross-Ref:††

But though Hamilton was surer than sure that the true propositional forms were those eight, he never did succeed, nor have any of the repeaters of the shibboleth succeeded, in satisfying their own minds as to the meaning of those eight formulæ!! At one time, we find Hamilton holding, what the logic [of the paragraphs] would absolutely require, that each proposition is absolutely definite (*Lectures on Logic*, Appendix V, iv. (a)). He is not mathematician enough to see that in that case he would want either 16 or only 7 propositional forms. He himself only sees 4; he went over this statement to every class for twenty years, without seeing the absurdity of it; and it is repeated in Hamiltonian logics to this day! Every geometer knows there are only 3 nonmetrical situations which two conics can have relatively to one another (without contact). They are:

1. Coincidence,
2. Intersection,

3. Non-intersection.

Each conic may be regarded as representing the division of the universe into those things to which a term applies and those to which it does not apply. Then:

Peirce: CP 2.534 Cross-Ref:††

1. Coincidence gives two relations, between two terms, according as the two terms are on the same or opposite sides of the line. But Hamilton only recognizes the former.

Peirce: CP 2.534 Cross-Ref:††

2. Intersection gives only one relation.

Peirce: CP 2.534 Cross-Ref:††

3. Non-intersection gives four relations, two of which, however, are of the same form (the terms only being transposed). These cases Hamilton recognizes (considering them as one only), and also the case of *S*, *P*, and what is neither *S* nor *P*, making up the universe. But the case in which the different things in the universe are all either *P* or *S* exclusively or both at once, he overlooks.

Peirce: CP 2.535 Cross-Ref:††

535. Hamilton is thus shown (*a*) to be led by the nose by the words some and all, whose claim to admission into the predicate he admits without knowing what it means; and (*b*) to be utterly incapable of doing the simplest logical thinking, perfectly indispensable for the problem in hand. I will further say, (*c*) that the psychological analysis upon which the whole doctrine is built, according to which a proposition is a comparison of notions, is all wrong. It involves incidentally the absurdity that I cannot think one planet is inhabited without thinking whether they all are so, without exception. Further, (*d*) that his grand postulate is ridiculous. It is a mere trick to get us to subscribe to his humble petition to be allowed to say what he thinks, only to find that this implies that whatever a man has in thought is pertinent to his reasoning! (*e*) Moreover, De Morgan, in the memoir referred to, has shown beyond dispute that Hamilton's syllogistic involves paralogisms. Now, a man capable of a downright paralogism is an exceptionally weak reasoner; and this I think Hamilton's writings taken together show that he was. Yet in some respects his thinking powers were admirable.

Peirce: CP 2.536 Cross-Ref:††
§4. UNIVERSE OF DISCOURSE †1E

536. In every proposition the circumstances of its enunciation show that it refers to some collection of individuals or of possibilities, which cannot be adequately described, but can only be indicated as something familiar to both speaker and auditor. At one time it may be the physical universe, at another it may be the imaginary "world" of some play or novel, at another a range of possibilities.

Peirce: CP 2.536 Cross-Ref:††

The term was introduced by De Morgan in 1846 (*Cambridge Philosophical*

Transactions, viii, 380) but De Morgan never showed that he fully comprehended it. It does not seem to be absolutely necessary in all cases that there should be an index proper outside the symbolic terms of the proposition to show what it is that is referred to; but in general there is such an index in the environment common to speaker and auditor. This De Morgan has not remarked; but what he has remarked has likewise its importance, namely, that for the purposes of logic it makes no difference whether the universe be wide or narrow. The idea of a limited logical universe was adopted by Boole †1 and has been employed by all subsequent exact logicians. There is besides a universe of marks or characters, whenever marks are considered substantively, that is, as abstractions, as they commonly are in ordinary speech, even though the forms of language do not show it. Thus only, there comes to be a material difference between an affirmative and a negative proposition. For it will then alone be one thing to say that an object wants some character common to all men and another to say that it possesses every character common to all non-men. Only instead of giving *three* qualities it gives four, for the assertion may be that an object wants some character common to all non-men; a point made by ancient writers. . . .

Peirce: CP 2.537 Cross-Ref:††
CHAPTER 4

NOTES ON EXPLICATIVE REASONING

§1. LOGICAL †1

537. Irrespective of any facts except those of which logic needs to take cognizance, such as the facts of doubt, truth, falsity, etc.

Peirce: CP 2.538 Cross-Ref:††

538. *Logical possibility* is, according to usage, freedom from all contradiction, explicit or implicit; and any attempt to reform the inaccuracy would only bring confusion.

Peirce: CP 2.539 Cross-Ref:††

539. *Logical necessity* is the necessity of that whose contrary is not logically possible.

Peirce: CP 2.540 Cross-Ref:††

540. *Logical induction* is an induction based on examination of every individual of the class to which the examination relates. Thus, conclusions from a census are logical inductions. While this mode of inference is a degenerate form of induction, it also comes into the class of dilemmatic reasoning.

Peirce: CP 2.541 Cross-Ref:††

541. *Logical truth* is a phrase used in three senses, rendering it almost useless.

Peirce: CP 2.541 Cross-Ref:††

(1) The harmony of a thought with itself. Most usually so defined, but seldom so employed. So far as this definition is distinct, it makes logical truth a synonym for logical possibility; but, no doubt, more is intended. (Hamilton, *Lectures on Logic*, xxvii.)

Peirce: CP 2.541 Cross-Ref:††

(2) The conformity of a thought to the laws of logic; in particular, in a concept, consistency; in an inference, validity; in a proposition, agreement with assumptions. This would better be called *mathematical truth*, since mathematics is the only science which aims at nothing more. (Kant, *Kritik der reinen Vernunft*, First ed., 294.)

Peirce: CP 2.541 Cross-Ref:††

(3) More properly, the conformity of a proposition with the reality, so far as the proposition asserts anything about the reality. Opposed, on the one hand, to metaphysical truth, which is an affection of the *ens*, and, on the other hand, to ethical *truth*, which is telling what a witness believes to be true. (Burgersdicius, *Inst. Met.*, chap. xviii.)

Peirce: CP 2.542 Cross-Ref:††

542. . . . *Logical reasoning*. Reasoning in accordance with a Leading Principle [see 588f.] which thorough analysis, discussion, and experience have shown must lead to the truth, in so far as it is relied upon. But what Aristotle understood by a logical demonstration may be seen in his *De generatione animalium*, Lib. II, chap. viii.

Peirce: CP 2.543 Cross-Ref:††

543. . . . *Logical definition*. A strict definition by genus and specific difference. Ockham and his followers objected to the designation on the ground that the logician, as such, had no occasion to define any ordinary term, such as "man" (*Tractatus logicæ*, Pt. I, chap. xxvi).

Peirce: CP 2.544 Cross-Ref:††

§2. PURE †1

544. . . . *Pure Logic*, a phrase often used, but to which no distinct conception can be attached. The following explanation by Hamilton (*Lectures on Logic*, App. I) is as good an explanation as can be given: "The doctrine which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the forms of thought, or in the conditions of the mind itself, which is the subject in which knowledge inheres--this science may be called *formal*, or *subjective*, or *abstract*, or *pure logic*. The science, again, which expounds the laws by which our scientific procedure should be governed, in so far as these lie in the contents, materials, or objects about which logic is conversant--this science may be called *material*, or *objective*, or *concrete*, or *applied logic*." Perhaps we may say that pure logic is a logic deduced from hypotheses (which some will look upon as axioms) without any inquiry into the observational warrant for those hypotheses.

Peirce: CP 2.545 Cross-Ref:††

545. . . . *Pure probation*, or *proof*, is proof by deduction from hypotheses, or

axioms, without any inquiry into the observational warrant for those premisses. Such is the usual reasoning of geometry.

Peirce: CP 2.546 Cross-Ref:††

546. **Pure proposition, enunciation, or judgment:** a proposition de inesse, a proposition not affected by modality. The pure proposition, as something merely proposed or contemplated, might be considered as a sort of problematic proposition. But, in fact, it is generally identified with the assertory proposition. . . .

Peirce: CP 2.547 Cross-Ref:††
§3. ORGANON †1

547. Since neither the Aristotelian definition of a speculative science, nor of a practical science, nor of an art, seemed to suit logic very well, the early peripatetics and commentators denied that it was either a science or an art, and called it an instrument, {organon}; but they did not precisely define their meaning. It was negative chiefly. The collection of Aristotle's logical treatises, when it was made, thus came to be called the *Organon*. . . .

Peirce: CP 2.548 Cross-Ref:††
§4. INTENTION †2

548. Aquinas (*Summa Theologica* I, 9.53); in classical writers an act of attention (and so Aquinas, *ibid.*, I. ii, 9.38, art. 2, and elsewhere); from *in+tendere*, to stretch. Aquinas seems sometimes to use the term for a mode of being (*ibid.*, I. ii. 9.22) and sometimes for a relation (*ibid.*, I, 9.29, art. 1; 9.76, art. 3, and esp. art. 4). A concept, as the result of attention.

Peirce: CP 2.548 Cross-Ref:††

First intentions are those concepts which are derived by comparing percepts, such as ordinary concepts of classes, relations, etc. **Second intentions** are those which are formed by observing and comparing first intentions. Thus the concept "class" is formed by observing and comparing class-concepts and other objects. The special class-concept, *ens*, or what is, in the sense of including figments as well as realities, can only have originated in that way. Of relative second intentions, four are prominent--identity, otherness, co-existence, and impossibility. †3 Aquinas defined logic as the science of second intentions applied to first.

Peirce: CP 2.549 Cross-Ref:††
§5. MATERIAL LOGIC †4

549. Formal logic classifies arguments by producing forms in which, the letters of the alphabet being replaced by any terms whatever, the result will be a valid, probable, or sophistic argument, as the case may be; material logic is a logic which

does not produce such perfectly general forms, but considers a logical universe having peculiar properties.

Peirce: CP 2.549 Cross-Ref:††

Such, for example, would be a logic in which every class was assumed to consist of a finite number of individuals; so that the syllogism of transposed quantity †1 would hold good. In most cases material logic is practically a synonym of applied logic. But a system like Hegel's may also properly be termed material logic. The term originated among the English Occamists of the fourteenth century, who declared Aristotle's logic to be material, in that it did not hold good of the doctrine of the Trinity.

Peirce: CP 2.550 Cross-Ref:††

§6. LOGICAL CONTRAPOSITION AND CONVERSION †2

550. On page 148 of *Mind*, the Editor proposes to resolve the inference:

1. All *S* is *P*,
- ∴ No not-*P* is *S*.

into two steps, thus:

2. All *S* is *P*,
- ∴ No *S* is not-*P*,
- ∴ No not-*P* is *S*.

Peirce: CP 2.550 Cross-Ref:††

To this I object on the ground that both steps of the latter process depend on a property of the negative which is not essential to the validity of the inference proposed to be resolved. In the universal negative proposition, *homo non est animal*, the *non* qualifies the copula. The meaning of this qualification must, however, be defined to be such that the proposition is equivalent to *homo est non animal*, taken in such a sense that the existence of a man is not asserted. We may, therefore, substitute for the forms of inference [Nos. 1 and 2] in question,

3. All *S* is *P*,
- ∴ All not-*P* is not-*S*;

and

4. All *S* is *P*,
- ∴ All *S* is not-not-*P*,
- ∴ All not-*P* is not-*S*.

The word *not* here has two properties. The first is that it is a relative term. To say that an animal is not a plant, is to say that it is *other than* every plant, just as we might say that it was *superior to every plant*. The second property is that the relative term *not*, like *cousin of*, *similar to*, etc. is its own converse [i.e., is symmetrical]. Now the first inference [Nos. 1 and 3] does not depend on this second property since it is of a form which holds good for all relative terms whatever. Thus we may reason,

All negroes are men,
∴ Every lover of all men is a lover of all negroes.

On the other hand both of the steps of the proposed resolution [No. 4] do depend on the convertible character of negation.

Peirce: CP 2.551 Cross-Ref:††
§7. OBVERSION †1

551. Hamilton (*Lectures on Logic*, xiv, and especially Appendix V (c)) states that *Conversion* in logic is sometimes called obversion.

Peirce: CP 2.551 Cross-Ref:††

This is a surprising statement, which neither he nor his editors are able to support by citations. It is, therefore, not unlikely that Hamilton took it at second hand.

Peirce: CP 2.551 Cross-Ref:††

Bain (*Logic*, Pt. I, Bk. I, chap. iii, sec. 27) says: "In affirming one thing, we must be prepared to deny the opposite: 'the road is level,' 'it is not inclined,' are not two facts, but the same fact from its other side. This process is called obversion." Bain gives no reference. The regular scholastic name for the process he describes--a name given by Abelard (*Dialectica*, 225)--is *infinitatio*. This word is very common (see, for example, Albertus Magnus in III. *Peri hermeneias*, iii; Ockham, *Logica*, II. xii, xiii; and the index to Prantl, *Logik*, iv). But somebody may have got the notion that it was "barbarous," and have preferred to use a more classical-sounding designation.

Peirce: CP 2.552 Cross-Ref:††
§8. SYLLOGISM †2

552. (1) According to Aristotle and the great body of logicians, a simple syllogism has two premisses and a conclusion, and three terms, one of which, the "middle," disappears from the conclusion. In the writer's opinion, the limitation to non-relative premisses is contrary to usage and to propriety.

Peirce: CP 2.553 Cross-Ref:††

553. (2) The sense laid down by Aristotle at the opening of his *Analytics*, 24 b 18, and repeated with verbal accuracy at the beginning of the *Topics* and at the beginning of the *Sophistici Elenchi*, and less formally in the logical part of the *Rhetoric* (A ii, §9), namely, {syllogismos de esti logos en hōi tethentōn tinōn heteron ti tōn keimenōn ex anankés symbainēi tōi tauta einai}, "A *syllogism* is a symbol ({logos}) in which, some things having been posited, something different from the assumptions necessarily joins itself to them, by being involved in the being of the facts assumed."

Peirce: CP 2.553 Cross-Ref:††

It will be seen that, in this definition (1), no particular stress is placed upon the premisses being two in number; although Aristotle is afterwards emphatic in insisting that every syllogism has two premisses. The following inference is, on this definition, syllogistic: "A stands to a lover of *B* in the relation of benefiting everybody loved by him; hence *A* benefits *B*." The two essential characters are that the inference shall be necessary, and that the fact inferred shall be involved in the very being of the facts premissed, regardless of the manner in which those premissed facts may have become known. The operation called *colligation* by Whewell,†1 which consists in bringing the different premisses together and applying them, the one to another, or to a repetition of itself, in a particular way, wherein lies all that calls for sagacity in deductive reasoning, is then no part of the syllogism. Aristotle seems to say (1 *Anal. Pr.* xxxii) that some necessary inferences are not syllogisms; but the passage is not clear. However, if colligation is to be excluded from syllogism, then also such an inference as the following will be non-syllogistic: "Every man is an animal; hence whatever is in a given relation (as for example, 'other than') to every man is in that relation to every animal."

Peirce: CP 2.553 Cross-Ref:††

An argument consisting of a single syllogism is a monosyllogism, one of more than one a polysyllogism, called also *monosyllogistic* and *polysyllogistic proof*.

Peirce: CP 2.554 Cross-Ref:††

554. Trendelenburg, in his *Elementa Logices Aristoteleae*,†2 gives an account of the origin of the word, remarking that {syllogizesthai} is properly to conjoin in arithmetical computation or in reasoning. Thus, in the *Philebus*, 41 C, where Jowett translates "you may infer," the word means to put together two conclusions already reached and then deduce a further result. In the *Theaetetus*, 186 D, where it is said that {epistémé} does not consist {en tois pathémasi}, but {en tōi peri ekeinōn syllogismōi}, {syllogismos} is a synonym for what in the *Phaedrus*, 249 B, is called simply {logismos}, "{ek pollōn ion aisthéseōn eis hen logismōi xynairoumenon}." And Trendelenburg is of opinion that there are in Aristotle traces of the broader

meaning, as when he speaks of {ho ex epagöges syllogismos}. Others, however, think that Aristotle's language shows that he drew a distinction between {epagögé} and the syllogism from {epagögé}, the latter being valid only in case of a complete enumeration. Bonitz, however, gives other instances of Aristotle's using the word in a broad sense. The chief of these is in *Rhetoric*, A xi. §23, where Cope's †1 note quoting a similar passage in the *Poetics* is interesting. Lutoslawski points out that one of Plato's very earliest dialogues, the *Charmides*, 161 A †P1, contains a regular syllogism in Cesare introduced by the word {syllogisamenos} (160 E). †2 He also remarks †3 that in the *Philebus*, the term {meson} is used in the same technical sense as in Aristotle, and adds: "If we take into consideration that it would be entirely against Plato's view of literary composition to enumerate all possible figures of syllogism in a dialogue, as is done in Aristotle's treatise, it becomes quite possible and even probable that Aristotle's theory of syllogism was more than prepared by Plato." Of course this can be no more than a surmise, but it is a reasonable one, since Plato's mathematical mind would naturally have looked at reasoning in a formal light. At any rate it is almost incredible that any man should have struck out all that is in Aristotle's *Analytics* if the ground had not been prepared. Moreover, Aristotle everywhere exhibits a mind quite unmathematical, so that one cannot but suspect that he received some hints towards a general outline of syllogistic from some source. Yet we cannot, for an instant, doubt Aristotle's veracity in the last chapter of the *Sophistici Elenchi*, where he says of syllogistic: "Of this subject, on the other hand, there has not been a part cultivated and a part neglected, but nothing whatsoever of it has hitherto existed. . . . About syllogizing I am aware of absolutely nothing having been previously said ({peri de tou syllogizesthai pantelös ouden eichomen proteron allo legein})" †P1

Peirce: CP 2.555 Cross-Ref: ††

555. (1) The syllogistic inference may be analysed into several distinct steps, each of which shall consist either in inserting or in omitting something. It does not evidently follow that such a method must afford a simpler representation of necessary inference than to take the inference of the *modus ponens* as elementary. But in point of fact it does so. The passage from premiss to conclusion may be regarded as a permissible transformation, and best, of a transformation of the nature of an insertion; that is to say, the conclusion is superadded to the premisses; since the premisses remain true.

Peirce: CP 2.556 Cross-Ref: ††

556. Aristotle's verb {symbainei} in the definition of the syllogism †1 takes this view. The relation between the protasis and apodosis of a hypothetical proposition differs formally from the relation between a premiss and conclusion in no essential respect, except that the protasis is not positively asserted. To regard the fact *C* as necessarily following from the fact *A* is to hold that, whenever facts analogous to *A* are true, a conclusion, related to them as *C* is to *A*, will always be true. In the proposition, "If *A* is true, *C* is true," we may have regard only to the actual state of things; in any case the proposition is equivalent to "Either *A* is not true, or *C* is true." But more usually we refer to a range of possibilities, and mean that whenever any fact analogous to *A* is true, that which is related to it, as *C* is to *A*, is true. The same relation subsists between the subject and predicate of a universal proposition: to say that "All men are mortal" is to say that, taking any object *X* whatsoever, if *X* is a man, *X* is mortal. This agrees with the definition of universal predication given by Aristotle, and commonly known as the *dictum de omni*. It will be remarked that this definition does not make a universal proposition to assert the existence of its subject.

Peirce: CP 2.556 Cross-Ref:††

Now, since in any possible system of logical representation illative transformation must be performed and be recognized as permissible, it follows that any representation of a universal proposition which treats any other relation than that of the conclusion (with the premisses) to the premisses alone as the principal relation expressed by the proposition, leaves the logical analysis incomplete.

Peirce: CP 2.557 Cross-Ref:††

557. Three figures of syllogism were recognized by Aristotle, in the first of which the middle is subject of one premiss (the *major* premiss) and predicate of the other (the *minor* premiss); in the second the middle is the predicate of both premisses; while in the third the middle is subject of both premisses. Aristotle recognizes but four moods of syllogism in the first figure. Some early Peripatetic, Theophrastus it is said, added five indirect moods: *Baralipon*, *Celantes*, *Dabitis*, *Fapesmo*, *Frisesorum*. It is rumoured that Galen first constituted a fourth figure by transposing the premisses of these. About the sixteenth century this figure began to be commonly admitted, and is now almost universally so. With this, the five moods have somewhat unnecessarily changed their names. Those now most usual are *Bramantip*, *Camenes*, *Dimaris*, *Fesapo*, *Fresison*.

Peirce: CP 2.558 Cross-Ref:††

558. The logic of Relatives (q. v.;†1 see also Symbolic Logic †2) throws great light on syllogism. It shows that the copulated premisses are, as expressed in Peirce's algebra of dual relatives, in one of the three forms,

$$(x \dagger \sim y)(\$y \dagger z) \quad (x \dagger \sim y)\$y z \quad (x \sim y)\$y z.$$

These give respectively

$$x \dagger z \quad x z \quad x \top z \dagger 1.$$

The last is a so-called spurious conclusion, but such syllogisms are unscientifically excluded from consideration in almost all treatises. There remain, therefore, but two kinds of syllogism, the *universal* and the *particular*. Beginning with *Barbara*, it can be mathematically proved that every possible necessary inference from two premisses, both having the same form as the conclusion, must depend upon a relation of *inclusion* (see Schröder, *Algebra und Logik der Relative*, 337ff., where the "solution" given of transitivity is the most accurate possible definition of *inclusion*, in that general sense in which a thing need not necessarily include itself). Thus:

The *S*'s are included among the *M*'s,

The *M*'s are included among the *P*'s;

∴ The *S*'s are included among the *P*'s.

So, for example, the pseudo-syllogism $S < M, M < P, \therefore S < P$, depends partly on the fact that $S < P$ implies that the units required to measure S are included among those required to measure P , and partly on the fact that $S < P$ implies that the units required to measure P are not included among those required to measure S . Putting, therefore, $s, p,$ and m for the units required to measure $S, P,$ and M , we have, on the one hand,

The s 's are included among the m 's,

The m 's are included among the p 's;

\therefore The s 's are included among the p 's;

and, on the other hand,

The s 's are included among the m 's,

The p 's are not included among the m 's;

\therefore The p 's are not included among the s 's.

If, in order to study the differences between the different moods of syllogism, we suppose that in the universal proposition S and P are modified by relatives, these must be the same for both, in order that *Barbara* should be valid. Calling this common relative h , the form of the universal affirmative, A, can only be one of the two following:

Any h of an s is an h of a p ;

Any h of every s is an h of every p .

These differ merely as being the converse of one another. For putting k for non- h , the latter is equivalent to

Any k of a p is a k of an s .

We may, therefore, assume the first of the two forms as the form of A. Then the validity of *Celarent* requires only that the universal negative E shall have for its subject, "Any h of an s " or "Something not an h of an s ." The validity of *Darii* requires only that the predicate of the particular affirmative, I, shall be "is an h of a p ." The validity of *Ferio* requires that the subject of the particular negative, O, should

agree with that of I (in a sense not easy to explain without special notation), while its predicate should be the same as that of E, and further, that if either I or E expresses existence, so should O. The validity of *Baroko* and the moods of the second figure, generally, requires that the predicate of O (and of E) should be "is not an *h* of a *p*." The validity of *Bokardo* requires that the subject of O (and of I) should be either "Some *h* of an *s*" or else "Anything but an *h* of an *s*." The result is that there are three systems of propositions which give all the traditional moods, except *Darapti*, *Felapton*, *Baralipon*, and *Fapesmo*, which are invalid if universal propositions are understood to be the exact denials of particular propositions. These three systems are:

First System.

- A. Any *h* of an *S* is an *h* of a *P*;
- E. Any *h* of an *S* is not an *h* of a *P*;
- I. Some *h* of an *S* is an *h* of a *P*;
- O. Some *h* of an *S* is not an *h* of a *P*.

Second System.

- A. Any *h* of an *S* is an *h* of a *P*;
- E. Something not an *h* of an *S* is not an *h* of a *P*;
- I. Whatever is not an *h* of an *S* is an *h* of a *P*;
- O. Some *h* of an *S* is not an *h* of a *P*.

Third System.

- A. Any *h* of an *S* is an *h* of a *P*;
- E. Any *h* of an *S* is not an *h* of a *P*;
- I. Whatever is not an *h* of an *S* is an *h* of a *P*;
- O. Whatever is not an *h* of an *S* is not an *h* of a *P*.

Peirce: CP 2.558 Cross-Ref:††

The O of the third system might have been taken as A, and the same forms would have been reproduced in the same way with changed *h*. In the ordinary syllogistic the first system is used, and *h* is "identical with."

Peirce: CP 2.558 Cross-Ref:††

With a limited universe of marks, *h* will be a "character of."

Peirce: CP 2.559 Cross-Ref:††

559. It will be observed that *Celarent* and *Darii* introduce each an additional

principle in slight modification of *Barbara*, and *Ferio* reunites those principles. The *second* and *third* figures introduce marked additional principles, which the so-called fourth figure, i.e., the Theophrastean syllogism, reunites. In the second figure there is a perceptible difference between the mode of inference of *Camestres* and *Baroko* on the one hand, *Cesare* and *Festino* on the other; although in *reasoning* itself it is morally impossible to distinguish *Camestres* from *Cesare*. Parallel remarks apply to the third figure. Of the Theophrastean moods, *Frisesomorum* alone is peculiar, *Celantes* and *Dabitis* depending merely on the principles of the second and third figures respectively.

Peirce: CP 2.559 Cross-Ref:††

The fact that the second and third figures involve peculiar principles is shown by the fact that the modes of reduction of any mood of each involve some peculiar immediate inference.

Peirce: CP 2.559 Cross-Ref:††

Thus, the reduction of the second figure involves the conversion of E. Now this conversion can be stated in syllogistic form; but it will be a syllogism in *Cesare*, which is the typical mood of the second figure, thus:

No *M* is *N*,

Any *N* is *N*;

∴ No *N* is *M*.

So the third figure involves the conversion of I, and when this is put into syllogistic form, the syllogism is in *Datisi*, the typical mood of the third figure, thus:

Any *N* is *N*,

Some *N* is *M*;

∴ Some *M* is *N*.

Peirce: CP 2.559 Cross-Ref:††

It may be said that the convertibility of I depends upon the equiparance †1 of the relation which I expresses. But even in that case, it must be noted that there is a certain difference between the *individual* and *definite* convertibility commonly understood by equiparance and the *indefinite* conversion, properly understood, of I, which is alone required in ordinary syllogistic. This will appear below.

Peirce: CP 2.560 Cross-Ref:††

560. The second and third figures can be reduced to the first apagogically, but the principles of the reduction are different. That of the second figure is that if the *negative* (not the denial) of the result of a hypothetical assumption under a known rule is found to be true, then the negative of the subsumption is true. Thus, referring to the usual syllogism, "All men are mortal, all patriarchs are men; hence, all

patriarchs are mortal"; if the rule is correct, yet all patriarchs are immortal, it follows that all patriarchs are non-men. The statement of this principle is simply a form of the conversion of E. So the third figure may be apagogically reduced by the principle that if, a subsumption under a hypothetical rule being admitted, a partial result is found true, the partial truth of the rule follows. Thus, admitting that all patriarchs are men, if it be found that some patriarchs are mortal, it follows that at least some men are mortal. The second figure can always be reduced to the first by the simple conversion of the major premiss, this being first made negative, if necessary by infinitation. Thus,

Any M is P = Any M is not not- P ,

Any }

Some } S is not P ;

∴ Any }

Some } S is not M ,

is reduced by the conversion of the major to "No not- P is M ." So the third figure is reduced to the first by the simple conversion of the minor premiss. But there is here a difficulty, owing to our not having in our ordinary languages a term to express the reversal of the quantity of a proposition; that is to say, a single operation which shall change Any to Some, and Some to Any.

Peirce: CP 2.561 Cross-Ref:††

561. The difference between a particular and a universal proposition is that, in the former, the selection of the individual of which the predicate is asserted is transferred from the hearer to a person competent and friendly to the proposition. Let this transfer be repeated, and the choice goes back to the first party. Using "some" in the sense of prescribing such transfer, "Some-some- S is P " = Any S is P . In that way *Disamis* and Bokardo are reduced as follows. The moods are:

{ is }

Some S { } P ,

{is not}

Any S is M = Some-some- S is M ;

{ is }

∴ Some M { } P .

{ is not }

Converting "Some-some-*S* is *M*" to "Some *M* is Some-*S*," we have the syllogism of the first figure--

{ is }

Any Some-*S* { } *P*,

{ is not }

Some *M* is Some-*S*;

{ is }

∴ Some *M* { } *P*.

{ is not }

Peirce: CP 2.562 Cross-Ref:††

562. Any syllogism of the second or third figure can also be reduced by transposing the premisses, and converting the two propositions not converted in the method just described. But still the same kind of conversion continues to be required in the case of each figure, thus showing that the conversions of E and I are logically allied to the second and third figures respectively. *Camestres* and *Disamis* are the easiest so to reduce; *Festino* and *Ferison* the most difficult.

Festino.

Some *S* is *P* = No Some-*S* is not-*P*,

No *M* is *P* = Any *M* is not-*P*;

∴ Some *S* is not *M* = No Some-*S* is *M*.

Peirce: CP 2.562 Cross-Ref:††

The conversion of E gives from the first premiss "No not-*P* is Some-*S*." This with the second premiss gives "No *M* is Some-*S*." Thence another conversion of E gives "No Some-*S* is *M*," which is the conclusion.

Ferison.

Some S is M = Any Some- S is M ,

No S is P = Some-some- S is not- P ;

∴ Some M is not P = Some M is not- P .

Peirce: CP 2.562 Cross-Ref:††

The conversion of I gives from the second premiss "Some not- P is Some- S ." This with the first premiss gives "Some not- P is M ," which by another conversion of I gives "Some M is not- P ," the conclusion.

Peirce: CP 2.563 Cross-Ref:††

563. The conversion of E depends on the fact that otherness is an equiparance. If everybody loves everybody who loves him, it follows that if every man loves every woman, then every woman loves every man; and on the corresponding principle, if every S is other than every P , then every P is other than every S . The conversion of I, on the other hand, is justified by the fact that the propositional quantities of breadth and depth are inseparable. "Some S is P " = There is an embodiment of S conforming to the idea of P .

Peirce: CP 2.563 Cross-Ref:††

Now observe that we do not necessarily infer that that same P is S , but only that there is somewhere a P that is S . All that is needed for this inference is that whenever there is an embodiment of an idea conforming to a second idea, there is also an embodiment of the latter idea conforming to the idea first embodied. It is the principle that there cannot be a cork that fits a bottle unless there be a bottle that is fitted by a cork.

Peirce: CP 2.564 Cross-Ref:††

564. When we pass to probability the figures of syllogism become of an idea highly important. Thus, take the following reasoning, the nature of which we shall presently examine:

The probability of throwing doublets with a pair of dice is $1/6$,

Different throws of pairs of dice are independent;

∴ The probability of throwing precisely two doublets in six throws is just half that of throwing precisely one doublet in six throws.

Peirce: CP 2.564 Cross-Ref:††

In order to analyse this reasoning, it is necessary to note what the premisses and conclusion mean. The precise meaning of the first premiss is that we should square our actions on each single occasion to the fact that, taking throws of the dice as they occur in the course of experience, the ratio of the number of throws of doublets hitherto to the total number of throws hitherto would, if the course of experience were endless, become, after a time, permanently remote from every other ratio than $1:6$, but

would never become so from this ratio. The meaning of the second premiss is that this is so not only for all the throws, but also for every endless portion of that totality of throws whose selection is determined by the ordinal relation of the throws composing it to previous throws of any given kind; so that, where such principle of selection allows, the throws are taken as they occur in the course of experience in endless succession.

Peirce: CP 2.564 Cross-Ref:††

The necessary conclusion is that we should square our action on any occasion to the principle that taking an endless succession and entirely different sets of six throws, just as they occur in the course of experience, the ratio of the number of sets containing two doublets to the number of sets containing one doublet will at length permanently depart from every other ratio than 1:2, but will not so depart from this ratio.

Peirce: CP 2.564 Cross-Ref:††

This inference is necessary; yet it is merely probable in this sense, that we cannot be sure that the number of sets of six throws containing just one doublet would be twice the number containing two doublets in a hundred trials, or in a million, or in any other fixed number. But what is certain is that any other ratio would eventually prove decidedly wrong, while 1:2 would not. This is a probable syllogism of the first figure, since it recognizes a case as coming under a general rule, the result of which it accepts as valid in that case. Of course the reader will not fail to observe that the sense in which the reasoning is said to be a syllogism is strictly defined. It is not meant that the above is a simple syllogism.

Peirce: CP 2.565 Cross-Ref:††

565. But if it were not known exactly how often with a given set of dice doublets would be thrown, since they may be similarly or dissimilarly loaded, and if we simply threw the dice and counted the whole number of throws and the number of throws of doublets, and if, finding that in twelve hundred throws there were about two hundred doublets, we were thence to conclude that in the long run about one-sixth of the throws would be doublets, this would be reasoning in the third figure. This would be in no sense necessary, except that if the experimentation were continued endlessly, it must approximate to the true ratio at last. This form of reasoning differs from the probable syllogism of the first figure, in that there the precise value of the long-run ratio was stated in advance; while here the concluded ratio is subject to modification as experience is amplified.

Peirce: CP 2.565 Cross-Ref:††

It is inference in the third figure, since if the rule were that any ratio far from one-sixth would be satisfactory in the long run, the same ratio would probably be tolerably satisfactory in twelve hundred throws. Now, no ratio far from one-sixth is satisfactory for this set of twelve hundred throws. Hence, the rule is probably false. Now, the inference that a general rule is false because the result to which it would lead in a special case is not true is syllogism of the third figure. Although this mode of inference is not necessary, it is necessary that the method should ultimately yield an approximately correct conclusion.

Peirce: CP 2.566 Cross-Ref:††

566. A probable syllogism which appears, in consequence of a want of sharp insistence upon what is really the subject and what the predicate of propositions, to be in the second figure may be essentially of the same nature as a probable syllogism in

the third figure; but it seems to be impossible that it should really be so, since there can be no strict objective probabilities of the kind required. If, for instance, we knew that in a considerable series of sets of six throws, one doublet precisely had occurred in a set twice as often as two doublets precisely, and we were thence to conclude that the two dice were probably normal, because, although other constitutions of dice might produce the same result, yet they are very unusual constitutions, we have here a probability of quite another kind; and instead of its being certain, as in the third figure, that the method of inference would in the long run lead to an indefinite approximation to the truth, it is here quite possible that the concluded constitution of the dice will always be entirely different from the true one. The three figures are therefore quite distinct; and we see that probability lends an importance to the three figures which modern logicians have not been willing to accord to them.

Peirce: CP 2.567 Cross-Ref:††

567. Aristotle pays no attention to hypothetical syllogisms; but it is said that Theophrastus studied them. We find them treated in considerable detail by Boëthius. During the middle ages, purely formal syllogistic made no progress worth mention. About the middle of the nineteenth century some steps were taken. Sir W. Hamilton proposed a syllogistic system of no merit (see *An Essay on the New Analytic of Logical Forms*, by T. S. Baynes, Edinburgh, 1850) based on the following system of propositional forms, called the system of the thoroughgoing quantification of the predicate.

All **A** is all **B**, meaning that the whole collection of **A**'s is identical with the whole collection of **B**'s.

All **A** is some **B**, meaning that the whole collection of **A**'s is identical with part of the collection of **B**'s.

Some **A** is all **B**, meaning that a part of the collection of **A**'s is identical with the whole collection of **B**'s.

Some **A** is some **B**, meaning that a part of the collection of **A**'s is identical with a part of the collection of **B**'s.

Any **A** is not any **B**, meaning that the whole collection of **A**'s is excluded from the whole collection of **B**'s.

Some **A** is not any **B**, meaning that a part of the collection of **A**'s is excluded from the whole collection of **B**'s.

Any **A** is not some **B**, meaning that the whole collection of **A**'s is excluded

from a part of the collection of **B**'s.

Some **A** is not some **B**, meaning that a part of the collection of **A**'s is excluded from a part of the collection of **B**'s.

There is also a pretence at a second series of meanings in depth, never clearly explained. Such a system is glaringly faulty; but it had some vogue in its day. There were also some other somewhat similar systems by Archbishop Thomson,†1 Spalding,†2 etc.

Peirce: CP 2.568 Cross-Ref:††

568. De Morgan constructed various systems of syllogistic of much greater merit, although, after all, they only complicate the subject to no purpose. His principal system, in which he postulates that no term is without breadth or is coextensive with the universe, is based on the following propositional forms:

$X))Y$. All **X**'s are some **Y**'s, i.e., Any **X** is a **Y**.

$X)\cdot(Y$. All **X**'s are not (all) **Y**'s, i.e., Any **X** is non-**Y**.

$X(\cdot)Y$. Everything is either some **X** or some **Y** (or both), i.e., Any non-**X** is **Y**.

$X((Y$. Some **X**'s are all **Y**'s, i.e., Any non-**X** is non-**Y**.

$X(\cdot(Y$. Some **X**'s are not (all) **Y**'s, i.e., Some **X** is non-**Y**.

$X()Y$. Some **X**'s are some **Y**'s, i.e., Some **X** is **Y**.

$X)(Y$. Some things are not either (all) **X**'s or (all) **Y**'s, i.e., Some non-**X** is non-**Y**.

$X)\cdot)Y$. All **X**'s are not some **Y**, i.e., Some non-**X** is non-**Y**.

Peirce: CP 2.568 Cross-Ref:††

These forms arose, as one might guess, from the application of *not* to the subjects of the old four; so that it might be called the system of the thoroughgoing qualification of the subject. Whether or not this qualification of the subject can be said to be involved in the structure of any syllogism is the question upon the decision of which that of the acceptance of De Morgan's system must depend.

Peirce: CP 2.568 Cross-Ref:††

Under these conditions (figure of course disappearing) De Morgan gets eight universal syllogisms,

)))) (·))) (((·) ((((·)·(·)))·(·)·(((·)·((·);

eight *major-particular* syllogisms,

((())·(())))·)))()·((·((((·((·))((·))·);

and eight *minor-particular* syllogisms,

()))·))))((·)(((··))·(())·((·((((·((·.

Peirce: CP 2.568 Cross-Ref:††

De Morgan also takes account of complex premisses. Moreover, he developed the syllogistic of relative terms, and especially the highly important *syllogism of transposed quantity*; and he, Boole, and others studied numerically definite syllogisms.

Peirce: CP 2.569 Cross-Ref:††

569. The following technical phrases may be defined:

Arithmetical syllogism. Blundevile, in his *Arte of Logicke* (1599), in illustration of his position that "God hath prescribed certaine bounds of necessitie . . . which bounds are Syllogismes rightly made," instances this question: "If one pound of waxe be worth a groat, what is tenne pound of waxe worth? Marry tenne groates, which is proved by a Syllogisme in this manner: Every pound of waxe is worth a groat; but here is ten pound of waxe: *Ergo*, they are worth ten groats: and like as in these kinds of Syllogismes Arithmetically, the proportion which is to be judged by mans naturall knowledge, doth shew the Consequent to be infallible, even so the Consequents in other Syllogismes are shewed to be infallible, by such demonstrations as are not farre fetched, or doubtfull, but are manifest, plaine and evident."

Peirce: CP 2.570 Cross-Ref:††

570. . . . *Compound syllogism:* a hypothetical syllogism, that is, a syllogism containing a disjunctive (or conditional) or copulative premiss.

Peirce: CP 2.571 Cross-Ref:††

571. *Conditional syllogism:* a syllogism containing a conditional premiss, especially the *Modus Ponens* and *Modus Tollens*, although some logicians refuse to these arguments the name of syllogism.

Peirce: CP 2.572 Cross-Ref:††

572. . . . *Demonstrative syllogism:* a syllogism which generates science.

Peirce: CP 2.573 Cross-Ref:††

573. *Dialectic syllogism:* a syllogism which generates opinion, being made of probable and credible propositions. The phrase is as old as Aristotle; but the use to which syllogism is put does not alter its nature. It is the argumentation or reasoning, not the logical form, which is dialectical.

Peirce: CP 2.574 Cross-Ref:††

574. . . . *Direct syllogism* ({syllogismos deiktikos}): a syllogism proceeding from a *rule*, and the subsumption of a *case* under that rule, to the *result* of the rule in

that case. One of the four moods, *Barbara, Celarent, Darii, Ferio*.

Peirce: CP 2.575 Cross-Ref:††

575. . . . **Hypothetical syllogism:** a syllogism either conditional (or disjunctive) or copulative. Often used for a syllogism conditional but not disjunctive, although there is no material difference between a conditional and a disjunctive proposition. A copulative proposition is the precise denial of a conditional or disjunctive proposition; and every conditional or disjunctive syllogism will by apagogical transformation give a copulative syllogism. But many logicians refuse to consider copulative syllogisms to be syllogisms on the ground that the conclusion contains nothing not stated in the premisses taken together; just as the same logicians exclude other arguments from the class of syllogisms because their conclusions contain matter not contained in the premisses taken together.

Peirce: CP 2.576 Cross-Ref:††

576. . . . **Perfect syllogism:** a syllogism of which no part of the leading principle can be stated as a premiss and so eliminated from the leading principle; since if this is attempted it is still needed as leading principle.

Peirce: CP 2.577 Cross-Ref:††

577. . . . **Relative syllogism:** a syllogism involving relative terms. Such syllogisms have been recognized as proper subjects of logic by all logicians beginning with Aristotle.

Peirce: CP 2.578 Cross-Ref:††

578. . . . **Valid syllogism:** a syllogism whose premisses assert facts whose being is partly composed of the being of the fact asserted in the conclusion.

Peirce: CP 2.579 Cross-Ref:††

579. **Syllogism of transposed quantity:** a syllogism in which the whole quantity of one concluding term, or its contrary, is applied in a premiss to the other concluding term, or its contrary, by means of a relation of one-to-*N* correspondence. As in the following: Some *X*'s are not *Y*'s, for every *X* there is a *Y* which is *Z*; hence, some *Z*'s are not *X*'s.

Peirce: CP 2.580 Cross-Ref:††

580. **Negative syllogism:** †1 any syllogism of the second figure, or the **Modus Tollens**, where the reasoning turns upon the change of quality. The canon of syllogism, that nothing can be concluded from two negatives, is inaccurate. What is requisite, in non-relative syllogism, is that the middle term should be once distributed and once undistributed. **Darapti** and **Felapton**, which appear to violate this rule, only do so because one of the premisses, so far as it is efficient, is virtually a particular. What is requisite is, that one of the interlocutors should select the individual denoted by the middle term in one premiss and the other in the other.

Peirce: CP 2.581 Cross-Ref:††

§9. MIDDLE TERM AND MIDDLE †2

581. The adjective {mesos} is applied in Greek to a third object additional to two others, when the idea of intervening can hardly be detected. It is, therefore,

perhaps needless to seek further for Aristotle's intention in calling that term, by the consideration of which two others are illatively brought into one proposition as its subject and predicate, the middle term, or middle. It is the most important factor of Aristotle's theory of reasoning.

Peirce: CP 2.581 Cross-Ref:††

The same word means little more than third in the phrase "principle of excluded middle," which is, indeed, often called *principium exclusi tertii*. See Laws of Thought [§15]. On the other hand, something which partakes of each of two disparate natures, and renders them capable of influencing one another, is called a *tertium quid* (Aristotle's {hé trité ousia}).

Peirce: CP 2.582 Cross-Ref:††
§10. PREMISE AND PREMISS †1

582. (Two distinct words, recognized as such by older writers, but for the last century and more confounded. Premise is a legal word, derived from the French *premise*, which is a noun derived from the phrase *les choses premises*, used in inventories. Premiss is from the French noun *premiss*, and thence from the Low Latin *praemissa*, which goes back, as a substantive, to the early part of the thirteenth century. But it was hardly looked upon as a very good Latin at any time. *Propositio* replaced it, when elegance was preferred to technical accuracy): A proposition, the consideration of which has logically affected, or contributed to the determination of, a conclusion of reasoning.

Peirce: CP 2.582 Cross-Ref:††

An enthymeme is usually defined as a syllogism with a suppressed premise or premiss. Now, the expression of a train of thought may be elliptical, some thought being unexpressed in the confident anticipation that the reader, or hearer, will supply it. But in thought, a premise or premiss cannot be suppressed without ceasing to be either premise or premiss. If it be so suppressed, it enters into the leading principle of the inference. Every reasoning must proceed consciously upon some general principle, or it ceases to be a reasoning, and becomes a mere feeling of inability to think otherwise. On the other hand, when a principle of reasoning becomes by analysis distinctly apprehended and the precise effect which it has upon the conclusion understood, it becomes a *premise*, or, at least, a *premiss*. There is, therefore, no such thing as a suppressed *premiss*.

Peirce: CP 2.583 Cross-Ref:††

583. The word "premiss" became usual in the logical sense, in English, as early as Chaucer. In Wilson's *Rule of Reason* (1552) it does not occur, the phraseology there being like the following: "The double repeate, whiche is a woorde rehearsed in bothe Propositions, must not entre into the conclusion." But in Blundevile's *Arte of Logicke* (1599) we read: "A Syllogisme is a kinde of argument contayning three Propositions, whereof the two first, commonly called the premisses," etc. In Watts's and other English logics it was spelt *premiss* and *premisses*. Johnson, however, in his *Dictionary*, gives *premises* in the plural and *premiss* in the singular, as distinct words, and remarks that the latter is little used in the plural outside of technical works.†1 In such works the word spelt with two s's continued to be

employed.

Peirce: CP 2.584 Cross-Ref:††
§11. MNEMONIC VERSES AND WORDS †2

584. (1)Instrumenta novem sunt, guttur, lingua, palatum

Quattuor et dentes, et duo labra simul.†3

Peirce: CP 2.584 Cross-Ref:††

The following mnemonic verses are contained in the *Summulae Logicales* of Petrus Hispanus, but were older, perhaps very much older.

Peirce: CP 2.584 Cross-Ref:††

(2) "Quae?" ca. vel hyp., "Qualis?" ne. vel aff., u. "Quanta?" univ. par. in. vel sing.†4 (What is the substance of a proposition? categorical or hypothetical. What is its quality? negative or affirmative. What is its quantity? universal, particular, indefinite, or singular.)

Peirce: CP 2.584 Cross-Ref:††

(3)Simpliciter F~ec~i, convertitur ~Ev\$a per acci,

~Ast~o per contra: sic fit conversio tota.

Asserit ~A, negat ~E, s~ed universaliter ambae;

Asserit ~I, negat ~O, sed particulariter ambo.†5

(*E* and *I* are converted simply; *E* and *A*, *per accidens*; *A* and *O*, *per contrapositionem*.)

Peirce: CP 2.584 Cross-Ref:††

(4)Prae, contradic.; post, contra.; prae postque, subalter.

Non omnis, quidam non; omnis non, quasi nullus;

Non nullus, quidam; sed "nullus non" valet "omnis";

Non aliquis, nullus; "non quidam non" valet "omnis";

Non alter, neuter; "neuter non" praestat "uterque."

(*Non* placed before *omnis* or *nullus* gives the contradictory proposition; placed after, the contrary; both before and after, the subalternate.)

Peirce: CP 2.584 Cross-Ref:††

(5) Primus, \$Am~ab~im~us; ~Ed~ent~ul~i que, secundus;

Tertius, ~Il\$ac~e; P~urp\$ur\$e~a, reliquus.

Destruit ~u totum, s~ed ~a confirmat utrumque;

Destruit ~e dictum, destruit ~i que modum.

Omne necess\$ari~at; impossibil~e, quasi nullus;

Possibil~e, quidam; quidam non, poss~ibile non.

~E dictum negat, ~i que modum, nihil ~a, sed \$u totum.

(The first syllable of each of the four vocables *Amab~imus*, *Edent~uli*, *Il\$ace*, *Purpur\$ea*, is for the possible mode; the second for the contingent; the third for the impossible; the fourth for the necessary. The vowel *a* signifies that both mode and "dictum" are to be taken assertorically; *e*, that the dictum is to be denied; *i*, that the mode is to be denied; *u*, that both mode and dictum are to be denied. Each word refers to a line or order of equipollent modal forms.)

Peirce: CP 2.584 Cross-Ref:††

(6) Tertius est quarto semper contrarius ordo.

Sit tibi linea subcontraria prima secundae.

Tertius est primo contradictorius ordo.

Pugnat cum quarto contradicend\$o secundus.

Prima subest quartae vice particularis habens se.

Hanc habet ad seriem se lege secunda sequentem.

(The relation of "Sortem impossibile est currere" and "Sortem necesse est currere" is that of contraries; they cannot be true at once. The relation "Sortem possibile est currere" and "Sortem possibile est non currere" is that of subcontraries; they cannot be false at once. The relation of "Sortem possibile est currere" and "Sortem impossibile est currere" is that of contradictories. The relation of "Sortem possibile est non currere" and "Sortem necesse est currere" is likewise that of contradictories. "Sortem possibile est currere" follows from "Sortem necesse est currere," as does "Sortem possibile est non currere" from "Sortem impossibile est currere.")

Peirce: CP 2.584 Cross-Ref:††

(7) Sub. prae. prima, secund~a prae. bis, tertia sub. bis.

(The first figure contains the middle term as subject and predicate; the second, the middle as predicated twice; the third, the middle twice as subject.)

Peirce: CP 2.584 Cross-Ref:††

(8) *B~arb\$ar\$a, C~el~ar~ent, D\$ar\$i~i, F\$er\$i~o, B\$ar\$al~ipton,*

C~el~ant~es, D\$ab\$it~is, F~ap~esm~o, Fr~is\$es\$om~orum.

C~es\$ar\$e, C~am~estr~es, F~est~in\$o, B\$ar~ok\$o, D\$ar~apti,

F~el~apt~on, D\$is\$am~is, D~at~is\$i, B\$ok~ardo, F\$er~ison.†1

(These are original names of the syllogistic moods, which there is no sufficient reason for abandoning. The direct moods of the first figure are recognizable by their containing no sign of conversion, *s*, *p*, or *k*; the indirect moods--or moods of the fourth figure--by their having those signs attached either to the third vowel or to the first two. In the second figure, one of the signs *s*, *p* is attached to the first vowel, or to the second and third, or *k* is attached to the second. In the names of the moods of the third figure, *s* or *p* is attached to the second vowel, or to the first and third, or *k* to the first. There are also names for syllogisms with weakened conclusions or strengthened premisses, as well as for indirect moods of the first figure considered as belonging to a fourth. But the above rules will enable a reader to identify them. Thus, *Bramantip* can be nothing but *Baralipton*; while *Barbari* is *Barbara* with a weakened conclusion. *Camenes* can be nothing but *Celantes*; *Dimaris* nothing but *Dabitis*; *Fesapo* nothing but *Fapesmo*; *Fresison* nothing but *Frisesomorum*. A writer who introduces an *m* into the name of a mood containing an *s* or *p* only after its third vowel, or who omits *m* from the name of a mood having *s* or *p* after the first and second vowels, uses the fourth figure.)

Peirce: CP 2.584 Cross-Ref:††

(9) *Simpliciter vult s, verti p vero per acci.*

M vult transponi, *k* per imposs~ibile duci.

Servat maiorem variatque secunda minorem;

Tertia maiorem variat servatque minorem.

(*s*, in the name of a mood, shows that the proposition denoted by the preceding vowel is, in a preferred mode of reduction, to be converted simply; *p*, that it is to be converted *per accidens*; *m* shows that the premisses are to be transposed; *k*, that the preferred reduction is by reduction of the contradictory of the conclusion to an absurdity, this contradictory of the conclusion being, in the second figure, put in place of the minor premiss--the major being retained--and in the third figure in the place of the major--the minor being retained.)

Peirce: CP 2.584 Cross-Ref:††

A great number of other memorial words and verses have been proposed by logicians.

Peirce: CP 2.585 Cross-Ref:††
§12. REDUCTION †1

585. Reduction has been recognized as necessary by Aristotle and almost all logicians; and before Kant it was generally recognized as proving that indirect syllogisms involved two or more steps of inference. But Kant from the same premisses inferred that reasoning in the first figure is the only reasoning; and this extraordinary conclusion is reached by simply not calling immediate inferences reasoning, because they have not two premisses. On that ground, nothing ought to be called reasoning except uniting two propositions into one copulative proposition. Cf. Syllogism [§8].

Peirce: CP 2.586 Cross-Ref:††

586. Reduction is either *ostensive* or *apagogical (per impossibile)*. Ostensive reduction proceeds directly from the premisses of the syllogism to be reduced to its conclusion, by means of conversions and a direct syllogism. Apagogical reduction proceeds indirectly, by showing by direct syllogism, that from the denial of the conclusion of the syllogism to be reduced and one of the premisses, the denial of the other premiss would follow. Any syllogism of the second figure and any of the third except Darapti and Felapton (which are not valid if the universal be taken in the Philonian †2 sense) can be apagogically reduced.

Peirce: CP 2.586 Cross-Ref:††

Ostensive reduction is either short or long. Short reduction involves the conversion of one proposition only; long reduction the conversion of two propositions, and, except in the fourth figure, the transposition of the premisses.

Peirce: CP 2.587 Cross-Ref:††

587. In order to ascertain that which reduction is intended to ascertain, the nature of the different moods of syllogism, the first requisite is to recognize that we do not seek to trace out the process of reasoning; but what we seek to do is to analyse the precise logical conditions which render each mood valid. When this is recognized, it becomes easy to see and to generalize the relations expressed by the premisses and note just at what point they cease to be cogent. For this purpose, appeal must be made to the logic of Relatives (q. v.).†1 De Morgan has worked out the moods (*Cambridge Philosophical Transactions*, X, 350).

Peirce: CP 2.588 Cross-Ref:††
§13. LEADING PRINCIPLE †2

588. It is of the essence of reasoning that the reasoner should proceed, and should be conscious of proceeding, according to a general habit, or method, which he holds would either (according to the kind of reasoning) always lead to the truth, provided the premisses were true; or, consistently adhered to, would eventually approximate indefinitely to the truth; or would be generally conducive to the ascertainment of truth, supposing there be any ascertainable truth. The effect of this habit or method could be stated in a proposition of which the antecedent should describe all possible premisses upon which it could operate, while the consequent

should describe how the conclusion to which it would lead would be determinately related to those premisses. Such a proposition is called the "leading principle" of the reasoning.

Peirce: CP 2.589 Cross-Ref:††

589. Two different reasoners might infer the same conclusion from the same premisses; and yet their proceeding might be governed by habits which would be formulated in different, or even conflicting, leading principles. Only that man's reasoning would be good whose leading principle was true for all possible cases. It is not essential that the reasoner should have a distinct apprehension of the leading principle of the habit which governs his reasoning; it is sufficient that he should be conscious of proceeding according to a general method, and that he should hold that that method is generally apt to lead to the truth. He may even conceive himself to be following one leading principle when, in reality, he is following another, and may consequently blunder in his conclusion. From the effective leading principle, together with the premisses, the propriety of accepting the conclusion in such sense as it is accepted follows necessarily in every case. Suppose that the leading principle involves two propositions, *L* and *L'*, and suppose that there are three premisses, *P*, *P'*, *P''*; and let *C* signify the acceptance of the conclusion, as it is accepted, either as true, or as a legitimate approximation to the truth, or as an assumption conducive to the ascertainment of the truth. Then, from the five premisses *L*, *L'*, *P*, *P'*, *P''*, the inference to *C* would be necessary; but it would not be so from *L*, *L'*, *P'*, *P''* alone, for, if it were, *P* would not really act as a premiss at all. From *P'* and *P''* as the sole premisses, *C* would follow, if the leading principle consisted of *L*, *L'*, and *P*. Or from the four premisses *L'*, *P*, *P'*, *P''*, the same conclusion would follow if *L* alone were the leading principle. What, then, could be the leading principle of the inference of *C* from all five propositions *L*, *L'*, *P*, *P'*, *P''*, taken as premisses? It would be something already implied in those premisses; and it might be almost any general proposition so implied. Leading principles are, therefore, of two classes; and any leading principle whose truth is implied in the premisses of every inference which it governs is called a "logical" (or, less appropriately, a *formal*) leading principle; while a leading principle whose truth is not implied in the premisses is called a "factual" (or *material*) leading principle.

Peirce: CP 2.590 Cross-Ref:††

§14. *NOTA NOTÆ*†1

590. The logical principle *Nota notae est nota rei ipsius*, that is, the predicate of the predicate is the predicate of the subject, which is laid down in several places by Aristotle as the general principle of syllogism. The principal passages are as follows:

Peirce: CP 2.590 Cross-Ref:††

"When one thing is predicated of another as its subject, whatever is said of the predicate can also be said of the subject" (*Categ.*, iii. 1 b 10).

Peirce: CP 2.590 Cross-Ref:††

"Whatever is said of the predicate will hold also of the subject" (*Categ.*, v. 3 b 4).

Peirce: CP 2.590 Cross-Ref:††

"We say that something is predicated universally when nothing can be admitted as coming under the subject of which the predicate will not hold; and the same thing holds of negation" (1 *Anal. pr.*, i. 24 b 28). The term *nota notae* is from the first words of the original of this passage.

Peirce: CP 2.591 Cross-Ref:††

591. "Of whatever the species is predicated, the genus is predicable" (*Topics*, {D} i. 121 a 25).

Peirce: CP 2.591 Cross-Ref:††

Some writers (as Hamilton, *Lectures on Logic*, Appendix VII, xxii, b) imagine a distinction between the *nota notae* and the *dictum de omni*. Some have been so extravagant as to attribute the former to Kant, in whose *Falsche Spitzfindigkeit* (1762, ii) it is very likely that the precise phrase "nota notae est nota rei ipsius" first occurs, though similar phrases, such as "cui conveniunt notae eidem quoque convenit nomen," are common in Wolff's and other logics of the eighteenth century. But it is clear that in Aristotle's mind it was one principle, essentially that which De Morgan well called the principle of the "transitiveness of the copula."

Peirce: CP 2.592 Cross-Ref:††

592. Aristotle, in the last but one of the above passages, seems to regard the *nota notae* as following from the definition of universal predication. To say that "Any *S* is *P*" is to say that of whatever *S* is true, *P* is true. This amounts to deriving the transitiveness of the copula from the transitiveness of illation. If from *A* follows *B* and from *B* follows *C*, then from *A* follows *C*. This, again, is equivalent to the principle that to say that from the truth of *X* follows the truth of the consequence that from *Y* follows *Z*, is the same as to say that from the joint truth of *X* and *Y* follows *Z* [i.e., $x \rightarrow (y \rightarrow z) = xy \rightarrow z$].

Peirce: CP 2.593 Cross-Ref:††
§15. LAWS OF THOUGHT †1

593. The three formulas of identity, contradiction, and excluded middle have been widely so known, though the doctrine that they are three coördinate and sufficient laws of all thought or of all reasoning has been held by a comparatively small party which hardly survives; and it is not too much to say that the doctrine is untenable. But the designation is so familiar and convenient that those formulas may very well be referred to as "the so-called three laws of thought." The formulas have usually been stated by those who upheld the doctrine as follows:

I. *The Principle of Identity: A is A.*

II. *The Principle of Contradiction: A is not not-A.*

III. *The Principle of Excluded Middle or Excluded Third: everything is either A or not-A.*

Peirce: CP 2.593 Cross-Ref:††

It is noticeable that two of these propositions are categorical and the third disjunctive, a circumstance demanding explanation for those who hold the distinction of categorical, conditional, and disjunctive propositions to be fundamental.

Peirce: CP 2.594 Cross-Ref:††

594. The meaning of the formula of identity presents only one small difficulty. If the copula "is" be taken in the sense of "is, if it exists," then the meaning of the formula is that no universal affirmative proposition having the same term as subject and predicate is false. If, however, the copula be understood to imply existence, the meaning is that no universal affirmative proposition is false in which the same term is subject and predicate, provided that term denotes any existing object. Or, the meaning may be that the same thing is true when the subject and predicate are the same proper name of an individual. In any case, it may properly be required that the precise meaning attached to the copula should be explained; and this explanation must in substance involve one or other of the above three statements; so that in any case the principle of identity is merely a part of the definition of the copula.

Peirce: CP 2.595 Cross-Ref:††

595. In like manner, if the word "not" is to be used in logical forms, its force should be explained with the utmost precision. Such an explanation will consist in showing that the relation it expresses belongs at once to certain classes of relations, probably not more than two, in view of the simplicity of the idea. Each of these two statements may be embodied in a formula similar, in a general way, to the formulas of contradiction and excluded middle. It has, therefore, seemed to Mill and to the "exact" logicians that these two formulas ought together to constitute a definition of the force of "not."

Peirce: CP 2.596 Cross-Ref:††

596. Other writers have regarded all three laws as "practical maxims." But practically nobody needs a maxim to remind him that a contradiction, for example, is an absurdity. It might be a useful injunction to tell him to beware of latent contradictions; but as soon as he clearly sees that a proposition is self-contradictory, he will have abandoned it before any maxim can be adduced. Seeing, then, that such formulas are required to define the relation expressed by "not," but are not required as maxims, it is in the former aspect that their true meanings are to be sought.

Peirce: CP 2.597 Cross-Ref:††

597. If it is admitted that they constitute a definition, they must conform to the rules of definition. Considered as part of a definition, one of the commonest statements of the principle of contradiction, "**A non est non-A**," offends against the rule that the *definitum* must not be introduced into the definition. This is easily avoided by using the form "**A est non non-A**," "**A is not not-A**," or every term may be subsumed under the double negation of itself. If this form is adopted for the principle of contradiction, the principle of excluded middle ought to be "What is not not-A is A." If, however, we prefer to state the principle of excluded middle as "Everything is either A or not-A," then we should state the principle of contradiction as "What is, at once, A and not-A is nothing." There is no vicious circle here, since the term "nothing," or "non ens," may be formally defined without employing the particle "not" or any equivalent. Thus, we may express the principle of contradiction as follows:

Whatever there may be which is both **A** and not-**A** is **X**,
no matter what term **X** may be.

Peirce: CP 2.597 Cross-Ref:††

In either formula, A may be understood to be restricted to being an individual, or it may be allowed to be any term, individual or general. In the former case, in order to avoid conflict with the fundamental law that no true definition asserts existence, a special clause should be added, such as "if not- A there be." In the latter case, it should be stated that by "not- A " is not meant "not some A ," but "not any A ," or "other than whatever A there may be."

Peirce: CP 2.597 Cross-Ref:††

Bearing these points in mind, the formula " A is not-not- A ," or " A is other than whatever is other than whatever is A ," is seen to be a way of saying that the relation expressed by "not" is one of those which is its own converse, and is analogous to the following:

Every rose is similar to whatever is similar to whatever is a rose;

which again is similar to the following:

Every man is loved by whatever loves whatever is a man.

But if we turn to the corresponding formula of excluded middle, "Not-not- A is A ," or "Whatever is not anything that is not

any A is A ," we find that its meaning cannot be so simply expressed. Supposing that the relation r is such that it is true that

Whatever is r to whatever is r to whatever is A is A ,

it can readily be proved that, whether the multitude of individuals in the universe be finite or infinite, each individual is either non- r to itself and to nothing else, or is one of a pair of individuals that are non- r to each other and to nothing else; and conversely, if the universe is so constituted, the above formula necessarily holds. But it is evident that if the universe is so constituted, the relation r is converse to itself; so that the formula corresponding to that of contradiction also holds. But this constitution of the universe does not determine r to be the relation expressed by "not." Hence, the pair of formulas,

A is not not- A ,

Not not- A is A ,

are inadequate to defining "not," and the former of them is mere surplusage. In fact, in a universe of monogamously married people, taking any class, the A 's,

Every A is a non-spouse to whatever is non-spouse to every A ,

and

Whatever is non-spouse to whatever is a non-spouse to every A is an A .

Peirce: CP 2.598 Cross-Ref:††

598. No such objection exists to the other pair of formulas:

Whatever is both A and not- A is nothing,

Everything is either A or not- A .

Their meaning is perfectly clear. Dividing all ordered pairs of individuals into those of the form $A:B$ and those of the form $A:A$,

Peirce: CP 2.598 Cross-Ref:††

The principle of contradiction excludes from the relation "not" all of the form $A:A$,†1

Peirce: CP 2.598 Cross-Ref:††

The principle of excluded middle makes the relation of "not" to include all pairs of the form $A:B$.†2

Peirce: CP 2.598 Cross-Ref:††

From this point of view, we see at once that there are three other similar pairs of formulas defining the relations of identity, coexistence, and impossibility,†1 as follows:

Whatever is A is identical with A ; i.e.,

Identity includes all pairs $A:A$.

Whatever is identical with A is A ; i.e.,

Identity excludes all pairs $A:B$.

Whatever is A is coexistent with A ; i.e.,

Coexistence includes all pairs $A:A$.

Everything is either A or coexistent with A ; i.e.,

Coexistence includes all pairs $A:B$.

Whatever is both A and impossible with A is nothing;

i.e., Impossibility excludes all pairs $A:A$.

Whatever there may be impossible with A is A ; i.e.,

Impossibility excludes all pairs $A:B$.

Peirce: CP 2.599 Cross-Ref:††

599. Much has been written concerning the relations of the three principles to forms of syllogism. They have even been called *die Principien des Schliessens*, and have often been so regarded. Some points in reference to the meanings they have borne in such discussions require mention. Many writers have failed to distinguish sufficiently between reasoning and the logical forms of inference. The distinction may be brought out by comparing the moods Camestres and Cesare. Formally, these are essentially different. The form of Camestres is as follows:

Every P is an M ,

Every S is other than every M ;

∴ Every S is other than every P .

This form does not depend upon either clause of the definition of "not" or "other than." For if any other relative term, such as "lover of," be substituted for "other than," the inference will be equally valid. The form of Cesare is as follows:

Every P is other than every M ,

Every S is an M ;

∴ Every S is other than every P .

This depends upon the equiparance of "other than." For if we substitute an ordinary relative, such as *loves*, for "other than" in the premiss, the conclusion will be

Every S is loved by every P .

(See De Morgan's fourth memoir on the syllogism, *Cambridge Philosophical Transactions*, X. (1860) 354.) The two forms are thus widely distinct in logic; and yet when a man actually performs an inference, it would be impossible to determine that he "reasons in" one of these moods rather than in the other. Either statement is incorrect. He does not, in strict accuracy, reason in any form of syllogism. For his reasoning moves in first intentions, while the forms of logic are constructions of second intentions. They are diagrammatic representations of the intellectual relation between the facts from which he reasons and the fact which he infers, this diagram necessarily making use of a particular system of symbols--a perfectly regular and very limited kind of language. It may be a part of a logician's duty to show how ordinary ways of speaking and of thinking are to be translated into that symbolism of formal logic; but it is no part of syllogistic itself. Logical principles of inference are merely rules for the illative transformation of the symbols of the particular system employed. If the system is essentially changed, they will be quite different. As the Booleans represent Cesare and Camestres, they appear, after literally translating the algebraic signs of those logicians into words, as follows:

A that is **B** is nothing,

C that is not **B** is nothing,

∴ A that is C is nothing.

The two moods are here absolutely indistinguishable.

Peirce: CP 2.600 Cross-Ref:††

600. From the time of Scotus down to Kant more and more was made of a principle agreeing in enunciation, often exactly, in other places approximately, with our principle of contradiction, and in the later of those ages usually called by that name, although earlier more often *principium primum*, *primum cognitum*, *principium identitatis*, *dignitas dignitatum*, etc. It would best be called the *Principle of Consistency*. Attention was called to it in the fourth book of Aristotle's *Metaphysics*. The meaning of this, which was altogether different, at least in post-scholastic times, from our principle of contradiction, is stated in the so-called *Monadologie* of Leibniz (§31) to be that principle by virtue of which we judge that to be false which involves a contradiction, and the denial of the contradiction to be true. The latter clause involves an appeal to the principle of excluded middle as much as the former clause does to the formal principle of contradiction. And so the "principle of contradiction" was formerly frequently stated. But, in fact, neither is appealed to; for Leibniz does not say that the contradiction is to be made explicit, but only that it is to be recognized as an inconsistency. Interpreted too strictly, the passage would seem to mean that all demonstrative reasoning is by the *reductio ad absurdum*; but this cannot be intended. All that is meant is that we draw that conclusion the denial of which would involve an absurdity--in short, that which consistency requires. This is a description, however imperfect, of the procedure of demonstrative Reasoning (q. v.)†1 and does not relate to logical forms. It deals with first, not second, intentions.

Peirce: CP 2.601 Cross-Ref:††
§16. REGULAR PROOF †2

601. Proof which has the external form considered appropriate to making its cogency clear. The form of a regular demonstration is as follows: first, the proposition to be stated is precisely stated in general terms; second, the construction of a diagram is described conforming to the conditions of the proposition; third, the proposition is restated with reference to the construction; fourth, by means of additions to the diagram, parts of it are brought into comparison; from which it is made evident that the proposition is true of that construction. It is evident that to perfect the proof, it ought then to be shown that what is true of the particular construction will be true in every case.

Peirce: CP 2.602 Cross-Ref:††
§17. PERTINENT †3

602. In the doctrine of *obligationes*, in logic, "pertinent" is applied to a proposition whose truth or falsity would necessarily follow from the truth of the proposition to which it was said to be pertinent, and also of a term either necessarily true or necessarily false of another term to which it was said to be pertinent. Cf. *The Century Dictionary* [p. 4419, ed. of 1889].

Peirce: CP 2.603 Cross-Ref:††
§18. IMPLICIT †1

603. Said of an element or character of a representation, whether verbal or mental, which is not contained in the representation itself, but which appears in the strictly logical (not merely in the psychological) analysis of that representation.

Peirce: CP 2.603 Cross-Ref:††

Thus, when we ordinarily think of something, say the Antarctic continent, as real, we do not stop to reflect that every intelligible question about it admits of a true answer; but when we logically analyse the meaning of reality, this result appears in the analysis. Consequently, only concepts, not percepts, can contain any implicit elements, since they alone are capable of logical analysis. An implicit contradiction, or contradiction *in adiecto*, is one which appears as soon as the terms are defined, irrespective of the properties of their objects. Thus there is, strictly speaking, no implicit contradiction in the notion of a quadrilateral triangle, although it is impossible. But, owing to exaggeration, this would currently be said to involve not merely an implicit, but an explicit contradiction, or contradiction in terms.

Peirce: CP 2.604 Cross-Ref:††

604. Any proposition which neither requires the exclusion from nor the inclusion in the universe of any state of facts or kind of object except such as a given second proposition so excludes or requires to be included, is implied in that second

proposition in the logical sense of implication, †2 no matter how different it may be in its point of view, or otherwise. It is a part of the meaning of the copula "is" employed in logical forms of proposition, that it expresses a transitive relation, so that whatever inference from the proposition would be justified by the *dictum de omni* is implied in the meaning of the proposition. Nor could any rule be admitted as universally valid in formal logic, unless it were a part of the definition of one of the symbols used in formal logic. Accordingly, whatever can be logically deduced from any proposition is implied in it; and conversely. Whether what is implied will, or will not, be suggested by the contemplation of the proposition is a question of psychology. All that concerns logic is, whether all the facts excluded and required by the one proposition are among those so excluded or required by the other.

Peirce: CP 2.605 Cross-Ref: ††
§19. OBSERVATION †1

605. Attentive experience; especially, an act of voluntarily attentive experience, usually with some, often with great, effort.

Peirce: CP 2.605 Cross-Ref: ††

More or less fixity in the object is requisite. Indeed, experience supposes that its object reacts upon us with some strength, much or little, so that it has a certain grade of reality or independence of our cognitive exertion. All reasoning whatever has observation as its most essential part. Whatever else there is in the act of reasoning is only preparatory to observation, like the manipulation of a physical experiment.

Peirce: CP 2.606 Cross-Ref: ††

606. Much stress has been laid upon the distinction between "sciences of observation" and "sciences of experiment"; and undoubtedly there is a great contrast between the proceedings, let us say, of the anatomist and of the physiologist. Although the anatomist has to make many experiments (with stains, for example), yet the stress of his labour comes upon the act of observation; while the preparations for observation of the physiologist are far more elaborate, and the mere act of observation itself often very easy and coarse. The difference is, however, chiefly one of degree, and from a philosophical point of view is of quite secondary importance.

Peirce: CP 2.607 Cross-Ref: ††
§20. SPURIOUS PROPOSITION †2

607. (1) A proposition which from the constitution of the universe must be true (De Morgan, *Syllabus of Logic*, §76). (2) B. I. Gilman (*Johns Hopkins University Circular*, August, 1882), calls the conclusion from two particular premisses *spurious in the first degree*. Thus,

Some *A* is *B*,

Some *C* is not *B*;

∴Some *A* is not some *C*.

Peirce: CP 2.607 Cross-Ref:††

This asserts the existence of an *A* and of a *C*, and further, that the number of the aggregate of the *A*'s and *C*'s is at least two. The conclusion from two premisses, of which one is particular and the other spurious in the first degree, gives a conclusion spurious in the second degree. There are also anti-spurious propositions, which are the precise denials of spurious propositions of the same degree.

Peirce: CP 2.608 Cross-Ref:††
§21. OPPOSITION †1

608. One of Aristotle's *Postpredicaments*. There are said, in the book of *Categories* (cap. x), to be four kinds of opposites. Relative opposites are relate and correlate of a disquiparant relation. †2 Contrary opposites are the most unlike species of the same genus, as black and white, sickness and health. The third kind of opposition is between a habit and its privation, as sight and blindness. The fourth kind is between affirmation and negation. This passage has prevented the word opposite from taking any definite meaning in philosophy.

Peirce: CP 2.609 Cross-Ref:††
§22. INCONSISTENCY †3

609. The relation between two assertions which cannot be true at once, though it may not be a direct contradiction; as between a statement of items and a statement of their total.

Peirce: CP 2.610 Cross-Ref:††

610. A logical discrepancy, on the other hand, is a difference between two statements either difficult, or impossible to reconcile with the credibility of both. It is said to be negative if one assertion omits an inseparable part of the fact stated in another; as when one witness testifies that *A* pointed a pistol at *B*, and another that *A* shot at *B*. It is positive if one asserts what the other denies. But even then it may often be conciliable (verträglich); that is, may not prove that either statement is in other respects untrustworthy. See Bachmann, *Logik*, sec. 214 ff.

Peirce: CP 2.611 Cross-Ref:††

611. "Inconsistent" is applied to an assertion, or hypothesis, which either in itself, or in copulation with another proposition with which it is said to be inconsistent, might be known to be false by a man devoid of all information except the meanings of the words used and their syntax.

Peirce: CP 2.611 Cross-Ref:††

Inconsistent differs from contradictory in being restricted usually to propositions, expressed or implied, and also in not implying that the falsity arises

from a relation of negation. "That is John" and "It is Paul" are inconsistent, but hardly contradictory. Moreover, contradictory is also used in a peculiar sense in formal logic. Cf. Opposition.

Peirce: CP 2.612 Cross-Ref:††
§23. *REDUCTIO AD ABSURDUM*†1

612. (Latin translation of Aristotle's {apagōgē eis to adynaton.})†2 The disproof of a proposition by showing that among its consequences there is one which is impossible or simply false.

Peirce: CP 2.612 Cross-Ref:††

This mode of proof is generally considered to be unsatisfactory, as not showing on what general principle the proposition disproved is false. But it is very easy to convert any such proof into a direct proof. Take, for example, the seventh proposition of the first book of Euclid, that on the same side of the base *AB* two triangles *ABC* and *ABD* cannot exist having *AC = AD* and *BC = BD*. Euclid proves this by showing that if there were two such triangles it would follow that the angles *BDC* and *BCD* were equal and also that they were unequal. But precisely the same steps of reasoning show that if there are two triangles *ABC* and *ABD* on the same side of *AB*, and if *AC = AD*, then *BC* is unequal to *BD*, which shows that there are not two triangles having *AC = AD* and *BC = BD*, since things unequal are not equal.

Peirce: CP 2.613 Cross-Ref:††
§24. *FALLACIESE*

613. *Non sequitur*†3 (Latin for "it does not follow"). A name which belongs to the slang of the universities for the *fallacia consequentis* (called by Aristotle {hopara to hepomenon elenchos,} *De Sophist. Elen.*, 167 b I), which is, strictly speaking, a fallacy which arises from a simple conversion of a universal affirmative, or transposing a protasis and apodosis, or condition and consequent.

Peirce: CP 2.613 Cross-Ref:††

Thus, Aristotle †4 tells us that the Eleatic Melissus argued that the universe is ungenerated, since nothing can be generated by what does not previously exist. The universe, then, not being generated, had no beginning; and having no beginning, it is infinite. But, as Aristotle remarks, although everything generated has a beginning, it does not follow (*non sequitur*, {ouk ananké de touto symbainein}) that everything that has a beginning is generated. A fever, for example, is not generated. Such fallacies are extremely common. De Morgan (*Formal Logic*, 268) gives this example: "Knowledge gives power, power is desirable, therefore knowledge is desirable." But though whatever is desirable has some desirable effect, it does not follow that whatever has any desirable effect is desirable. An attack of yellow fever has the desirable effect of rendering it unlikely the patient will for a long time have another; still, it is not itself desirable.

Peirce: CP 2.613 Cross-Ref:††

But the majority of logicians not only confound this fallacy with the *post hoc, ergo propter hoc*, which Aristotle considers immediately after, but even define it as "failure in the formal adequacy of the reason" (Sidgwick, *Fallacies*, II, ii, 4), or as "the introduction of new matter into the conclusion, which is not contained in the premisses" (Hyslop, *Logic*, xviii, 2), or as "the simple affirmation of a conclusion which does not follow from the premisses" (De Morgan, *loc. cit.*), or as "any argument which is of so loose and inconsequent a character that no one can discover any cogency in it" (Jevons, *Lessons in Logic*, xxi), or "to assume without warrant that a certain conclusion follows from premisses which have been stated" (Creighton, *Introductory Logic*, §46). Very many logicians omit it altogether, which is better.

Peirce: CP 2.613 Cross-Ref:††

Aristotle, however, could not express himself more precisely: {Hōpara to hepomenon elenchos dia to oiesthai antistrephein tén akalouthésin}. That is, "from thinking that the consequentia can be converted." That is to say, thinking that because "If A, then C," therefore "If C, then A." Owing to the neglect of fallacies by the more scientific logicians, it is not easy to cite many who define the fallacy correctly. The Conimbricenses (than whom no authority is higher) do so (*Commentarii in Univ. Dialecticam* Arist. Stagir., *In lib. Elench.*, q. i. art. 4); also Eustachius (*Summa Philos.*, Tom. I, pars. III, tract. iii, disput. iii. 9.3); also Cope, an admirable student of Aristotle, in his note on the *Rhetorics*, B. cap. xxiv. See also the *Century Dictionary* under "Fallacy." [Vol. 8, p. 2728, ed. 1889.]

Peirce: CP 2.614 Cross-Ref:††

614. *Petitio Principii*†1 (Latin. This is a not very good translation of Aristotle's phrase {to ex archés} [or {en archéi}] {aiteisthai}, †2 to beg what was proposed in the beginning). It is a Fallacy of a relatively high order, inasmuch as it cannot exist unless the conclusion truly follows from the premisses. To accuse a man of begging the question is in reality a plea which virtually admits that his reasoning is good. Its only fault is that it assumes as a premiss what no intelligent man who doubted the conclusion could know to be true.

Peirce: CP 2.614 Cross-Ref:††

A very necessary, though not always sufficient, precaution against this fallacy is to ask oneself whether the reasoning rests upon any observations, or inductions from observations, or even trustworthy hypothetical inferences from observations, which really involve the conclusion, relating to those matters of experience in reference to which the conclusion is important; and if relating to those things, whether in such a way and so closely that that conclusion really can have been implicitly asserted in those premisses. For example, to take an illustration partly fanciful, a man proposes to prove the reality or possibility of clairvoyance to me by proving to me that the sum of the angles of a triangle is two right angles. If, he says, you can sit in your study and know that this is true in the most distant parts of the universe, why may not an exceptionally gifted person know many facts about what happens only a hundred miles away? Upon that, I ask myself whether geometry rests upon any observations concerning clairvoyance or anything like clairvoyance. Nay, the consequence which my arguer has pointed out seems so cogent, and yet the line of reasoning so inadmissible, that I go up to the garret to exhume my old Euclid or Legendre [*Eléments de Géométrie*] to see how it is proved that sitting in my study I can know what the angles of the triangle whose vertices are at Sirius, Arcturus, and Fomalhaut, may sum up to. I find it is done by assuming that certain propositions about space are self-evident. Now, this may be safe enough so far as that sort of

reasoning has been millions of times verified. But nothing of the sort has been, or can be, verified exactly; and for such monstrous triangles a divergence from exactitude in the formula may be large, although for terrestrial triangles it be too minute for detection. In short, I am led to see that there must be a *petitio principii* in any argument which, resting merely on common sense, concludes the exact truth of any matter of fact.

Peirce: CP 2.615 Cross-Ref:††

615. *Philosopheme*. †1 A demonstrative reasoning supposed to prove a truth from self-evident premisses.

Peirce: CP 2.615 Cross-Ref:††

It is necessarily, at best, a *Petitio Principii*; but it is the Aristotelian ideal of perfect reasoning (see Aristotle, *Top.* viii. 162 a 15; 279 a 30; 294 a 19).

Peirce: CP 2.616 Cross-Ref:††

616. *Saltus in concludendo*: †2 a paralogism which consists in proving something as an aid in proving something else and then supposing that something different has been proved. But by some writers a mere omission of an obvious step in a proof is called a saltus.

Peirce: CP 2.617 Cross-Ref:††

617. *Material Fallacy*. †3 This term originated with Whately (*Encyc. Metropolitana*, i. 218 b). Whately's material fallacies are those in which the conclusion does follow from the premisses. Therefore, excluding the multiple interrogation, which is no syllogism, of the rest of Aristotle's thirteen, only the *ignoratio elenchi* and the *petitio principii* are material.

Peirce: CP 2.617 Cross-Ref:††

Aldrich †4 had modified Aristotle's division into fallacies *in dictione* and fallacies *extra dictionem*; making a division into *Sophismata in forma argumenti* (*sicubi conclusio non legitime consequatur ex praemissis*), and *Sophismata in materia argumenti* (*sicubi legitime non tamen vere concludere videatur syllogismus*). Under the latter head he placed the *ignoratio elenchi*, the *non causa pro causa*, the *non sequitur*, and the *petitio principii*. Whately's distinction is--whether from a theoretical or a practical point of view--by far the most important that can be drawn among fallacies; so that besides the reason of priority, which ought itself to be final, the needs of the logician forbid us to depart from Whately's definition. Some logicians do not admit material fallacies among the number of fallacies, but consider them to be faults of method (Hamilton, *Lectures on Logic*, xxvi; Ueberweg, *System der Logik*, §126, §137). E. E. Constance Jones (*Elements of Logic as a Science of Propositions*, §xxvi) reduces them to formal fallacies. Hyslop (*Elements of Logic*, chap. xvii) uses the term "material" fallacy, quite unjustifiably, to include all fallacies due to something in the matter of reasoning.

Peirce: CP 2.618 Cross-Ref:††

§25. *INSOLUBILIA* †1

618. A class of sophisms in which a question is put of such a nature that, whether it be answered affirmatively or negatively, an argument unimpeachable in

form will prove the answer to be false.

Peirce: CP 2.618 Cross-Ref:††

The type is this. Given the following proposition:

This assertion is not true:

is that assertion, which proclaims its own falsity, and nothing else, true or false?
Suppose it true. Then,

Whatever is asserted in it is true,

But that it is not true is asserted in it;

∴By Barbara, That it is not true is true;

∴It is not true.

Besides, if it is true, that it is true is true. Hence,

That it is not true is not true,

But that it is not true is asserted in the proposition;

∴By Darapti, Something asserted in the proposition is not true:

∴The proposition is not true.

On the other hand, suppose it is not true. In that case,

That it is not true is true,

But all that the proposition asserts is that it is not true;

∴By Barbara, All that the proposition asserts is true;

∴The proposition is true.

Besides, in this case,

Something the proposition asserts is not true,

But all that the proposition asserts is that it is not true;

∴By Bokardo, That it is not true is not altogether true;

∴That it is true is true;

∴It is true.

Peirce: CP 2.618 Cross-Ref:††

Thus, whether it be true or not, it is both true and not. Now, it must be either true or not, hence it is both true and not, which is absurd.

Peirce: CP 2.618 Cross-Ref:††

Only two essentially distinct methods of solution have been proposed. One, which is supported by Ockham (*Summa totius logicae*, 3d div. of 3d part, cap. 38 and 45), admits the validity of the argumentation and its consequence, which is that there can be no such proposition, and attempts to show by other arguments that no proposition can assert anything of itself. Many logical writers follow Ockham in the first part of his solution, but fail to see the need of the second part. The other method of solution, supported by Paulus Venetus (*Sophismata Aurea*, sophisma 50), diametrically denies the principle of the former solution, and undertakes to show that every proposition virtually asserts its own truth. This method, therefore, denies the premiss of the antithesis that "all that the proposition asserts is that it is not true" since, like every other proposition, it also asserts its own truth, and is therefore contradictory and false, not in what it expressly asserts, but in what it implicitly asserts.†1 Some writers (as Fries)†2 hold that because every proposition asserts its own truth, therefore nothing is a proposition which asserts its own falsity. See Aristotle, *Sophisticae Elenchi*, cap. 25. Other proposed solutions of little importance are given by Paulus Venetus, *loc. cit.*†3

Peirce: CP 2.619 Cross-Ref:††
B. AMPLIATIVE REASONING

CHAPTER 5

DEDUCTION, INDUCTION, AND HYPOTHESIS†1

§1. RULE, CASE, AND RESULTE

619. The chief business of the logician is to classify arguments; for all testing clearly depends on classification. The classes of the logicians are defined by certain typical forms called syllogisms. For example, the syllogism called *Barbara* is as follows:

S is M, M is P;

Hence, S is P.

Or, to put words for letters--

Enoch and Elijah were men, all men die;

Hence, Enoch and Elijah must have died.

Peirce: CP 2.619 Cross-Ref:††

The "is P" of the logicians stands for any verb, active or neuter. It is capable of strict proof (with which, however, I will not trouble the reader) that all arguments whatever can be put into this form; but only under the condition that the *is* shall mean "*is* for the purposes of the argument" or "is represented by." Thus, an induction will appear in this form something like this:

These beans are two-thirds white,

But, the beans in this bag are (represented by) these beans;

∴The beans in the bag are two-thirds white.

Peirce: CP 2.620 Cross-Ref:††

620. But, because all inference may be reduced in some way to *Barbara*, it does not follow that this is the most appropriate form in which to represent every kind of inference. On the contrary, to show the distinctive characters of different sorts of inference, they must clearly be exhibited in different forms peculiar to each. *Barbara* particularly typifies deductive reasoning; and so long as the *is* is taken literally, no inductive reasoning can be put into this form. *Barbara*, is, in fact, nothing but the application of a rule. The so-called major premiss lays down this rule; as, for example, *All men are mortal*. The other or minor premiss states a case under the rule; as, *Enoch was a man*. The conclusion applies the rule to the case and states the result: *Enoch is mortal*. All deduction is of this character; it is merely the application of general rules to particular cases. Sometimes this is not very evident, as in the following:

All quadrangles are figures,

But no triangle is a quadrangle;

Therefore, some figures are not triangles.

Peirce: CP 2.620 Cross-Ref:††

But here the reasoning is really this:

Rule.--Every quadrangle is other than a triangle.

Case.--Some figures are quadrangles.

Result.--Some figures are not triangles.

Peirce: CP 2.620 Cross-Ref:††

Inductive or synthetic reasoning, being something more than the mere application of a general rule to a particular case, can never be reduced to this form.

Peirce: CP 2.621 Cross-Ref:††

621. If, from a bag of beans of which we know that $2/3$ are white, we take one at random, it is a deductive inference that this bean is probably white, the probability being $2/3$. We have, in effect, the following syllogism:

Rule.--The beans in this bag are $2/3$ white.

Case.--This bean has been drawn in such a way that in the long run the relative number of white beans so drawn would be equal to the relative number in the bag.

Result.--This bean has been drawn in such a way that in the long run it would turn out white $2/3$ of the time.

Peirce: CP 2.622 Cross-Ref:††

622. If instead of drawing one bean we draw a handful at random and conclude that about $2/3$ of the handful are probably white, the reasoning is of the same sort. If, however, not knowing what proportion of white beans there are in the bag, we draw a handful at random and, finding $2/3$ of the beans in the handful white, conclude that about $2/3$ of those in the bag are white, we are rowing up the current of deductive sequence, and are concluding a rule from the observation of a result in a certain case. This is particularly clear when all the handful turn out one color. The induction then is:

These beans were in this bag.-----

|

These beans are white.----- |

||

∴All the beans in the bag were white.----- ||

|||

Which is but an inversion of the deductive syllogism: |||

|||

Rule.--All the beans in the bag were white.-- | |

| |

Case.--These beans were in the bag.-----|---

|

Result.--These beans are white.-----

So that induction is the inference of the *rule* from the *case* and *result*.

Peirce: CP 2.623 Cross-Ref:††

623. But this is not the only way of inverting a deductive syllogism so as to produce a synthetic inference. Suppose I enter a room and there find a number of bags, containing different kinds of beans. On the table there is a handful of white beans; and, after some searching, I find one of the bags contains white beans only. I at once infer as a probability, or as a fair guess, that this handful was taken out of that bag. This sort of inference is called *making an hypothesis*. It is the inference of a *case* from a *rule* and a *result*. We have, then--

DEDUCTION.

Rule.--All the beans from this bag are white.

Case.--These beans are from this bag.

∴**Result.**--These beans are white.

INDUCTION.

Case.--These beans are from this bag.

Result.--These beans are white.

∴**Rule.**--All the beans from this bag are white

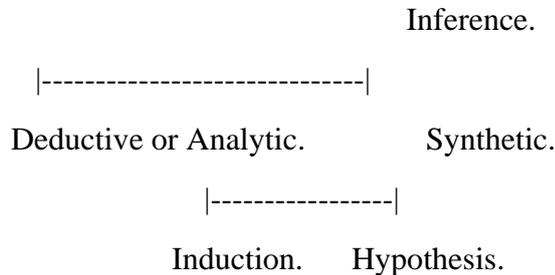
HYPOTHESIS.

Rule.--All the beans from this bag are white.

Result.--These beans are white.

∴**Case.**--These beans are from this bag.

We, accordingly, classify all inference as follows:



Peirce: CP 2.624 Cross-Ref:††

624. Induction is where we generalize from a number of cases of which something is true, and infer that the same thing is true of a whole class. Or, where we find a certain thing to be true of a certain proportion of cases and infer that it is true of the same proportion of the whole class. Hypothesis is where we find some very curious circumstance, which would be explained by the supposition that it was a case of a certain general rule, and thereupon adopt that supposition. Or, where we find that in certain respects two objects have a strong resemblance, and infer that they resemble one another strongly in other respects.

Peirce: CP 2.625 Cross-Ref:††

625. I once landed at a seaport in a Turkish province; and, as I was walking up to the house which I was to visit, I met a man upon horseback, surrounded by four horsemen holding a canopy over his head. As the governor of the province was the only personage I could think of who would be so greatly honored, I inferred that this was he. This was an hypothesis.

Peirce: CP 2.625 Cross-Ref:††

Fossils are found; say, remains like those of fishes, but far in the interior of the country. To explain the phenomenon, we suppose the sea once washed over this land. This is another hypothesis.

Peirce: CP 2.625 Cross-Ref:††

Numberless documents and monuments refer to a conqueror called Napoleon Bonaparte. Though we have not seen the man, yet we cannot explain what we have seen, namely, all these documents and monuments, without supposing that he really existed. Hypothesis again.

Peirce: CP 2.625 Cross-Ref:††

As a general rule, hypothesis is a weak kind of argument. It often inclines our judgment so slightly toward its conclusion that we cannot say that we believe the latter to be true; we only surmise that it may be so. But there is no difference except one of degree between such an inference and that by which we are led to believe that we remember the occurrences of yesterday from our feeling as if we did so.

Peirce: CP 2.626 Cross-Ref:††

§2. BAROCO AND BOCARDO; HYPOTHESIS AND INDUCTIONE

626. Besides the way just pointed out of inverting a deductive syllogism to produce an induction or hypothesis, there is another. If from the truth of a certain premiss the truth of a certain conclusion would necessarily follow, then from the falsity of the conclusion the falsity of the premiss would follow. Thus, take the following syllogism in *Barbara*:

Rule.--All men are mortal,

Case.--Enoch and Elijah were men;

∴**Result.**--Enoch and Elijah were mortal.

Peirce: CP 2.626 Cross-Ref:††

Now, a person who denies this result may admit the rule, and, in that case, he must deny the case. Thus:

Denial of Result.--Enoch and Elijah were not mortal,

Rule.--All men are mortal;

∴**Denial of Case.**--Enoch and Elijah were not men.

Peirce: CP 2.626 Cross-Ref:††

This kind of syllogism is called *Baroco*, which is the typical mood of the second figure. On the other hand, the person who denies the result may admit the case, and in that case he must deny the rule. Thus:

Denial of the Result.--Enoch and Elijah were not mortal,

Case.--Enoch and Elijah were men;

∴**Denial of the Rule.**--Some men are not mortal.

Peirce: CP 2.626 Cross-Ref:††

This kind of syllogism is called *Bocardo*, which is the typical mood of the third figure.

Peirce: CP 2.627 Cross-Ref:††

627. *Baroco* and *Bocardo* are, of course, deductive syllogisms; but of a very peculiar kind. They are called by logicians indirect moods, because they need some transformation to appear as the application of a rule to a particular case. But if, instead of setting out as we have here done with a necessary deduction in *Barbara*, we

take a probable deduction of similar form, the indirect moods which we shall obtain will be--

Corresponding to *Baroco*, an hypothesis;
and, Corresponding to *Bocardo*, an induction.

For example, let us begin with this probable deduction in *Barbara*:

Rule.--Most of the beans in this bag are white,

Case.--This handful of beans are from this bag;

∴*Result.*--Probably, most of this handful of beans are white.

Now, deny the result, but accept the rule:

Denial of Result.--Few beans of this handful are white,

Rule.--Most beans in this bag are white;

∴*Denial of Case.*--Probably, these beans were taken from another bag.

Peirce: CP 2.627 Cross-Ref:††

This is an hypothetical inference. Next, deny the result, but accept the case:

Denial of Result.--Few beans of this handful are white.

Case.--These beans came from this bag.

∴*Denial of Rule.*--Probably, few beans in the bag are white.

This is an induction.

Peirce: CP 2.628 Cross-Ref:††

628. The relation thus exhibited between synthetic and deductive reasoning is not without its importance. When we adopt a certain hypothesis, it is not alone because it will explain the observed facts, but also because the contrary hypothesis would probably lead to results contrary to those observed. So, when we make an induction, it is drawn not only because it explains the distribution of characters in the sample, but also because a different rule would probably have led to the sample being other than it is.

Peirce: CP 2.629 Cross-Ref:††

629. But the advantage of this way of considering the subject might easily be overrated. An induction is really the inference of a rule, and to consider it as the denial of a rule is an artificial conception, only admissible because, when statistical or proportional propositions are considered as rules, the denial of a rule is itself a rule. So, an hypothesis is really a subsumption of a case under a class and not the denial of it, except for this, that to deny a subsumption under one class is to admit a subsumption under another.

Peirce: CP 2.630 Cross-Ref:††

630. *Bocardo* may be considered as an induction, so timid as to lose its ampliative character entirely. Enoch and Elijah are specimens of a certain kind of men. All that kind of men are shown by these instances to be immortal. But instead of boldly concluding that all very pious men, or all men favorites of the Almighty, etc., are immortal, we refrain from specifying the description of men, and rest in the merely explicative inference that *some* men are immortal. So *Baroco* might be considered as a very timid hypothesis. Enoch and Elijah are not mortal. Now, we might boldly suppose them to be gods or something of that sort, but instead of that we limit ourselves to the inference that they are of *some* nature different from that of man.

Peirce: CP 2.631 Cross-Ref:††

631. But, after all, there is an immense difference between the relation of *Baroco* and *Bocardo* to *Barbara* and that of Induction and Hypothesis to Deduction. *Baroco* and *Bocardo* are based upon the fact that if the truth of a conclusion necessarily follows from the truth of a premiss, then the falsity of the premiss follows from the falsity of the conclusion. This is always true. It is different when the inference is only probable. It by no means follows that, because the truth of a certain premiss would render the truth of a conclusion probable, therefore the falsity of the conclusion renders the falsity of the premiss probable. At least, this is only true, as we have seen in a former paper,†1 when the word "probable" is used in one sense in the antecedent and in another in the consequent.

Peirce: CP 2.632 Cross-Ref:††

§3. RULES FOR INDUCTION AND HYPOTHESESE

632. A certain anonymous writing is upon a torn piece of paper. It is suspected that the author is a certain person. His desk, to which only he has had access, is searched, and in it is found a piece of paper, the torn edge of which exactly fits, in all its irregularities, that of the paper in question. It is a fair hypothetic inference that the suspected man was actually the author. The ground of this inference evidently is that two torn pieces of paper are extremely unlikely to fit together by accident. Therefore, of a great number of inferences of this sort, but a very small proportion would be deceptive. The analogy of hypothesis with induction is so strong that some logicians have confounded them. Hypothesis has been called an induction of characters. A number of characters belonging to a certain class are found in a certain object; whence it is inferred that all the characters of that class belong to the object in question. This certainly involves the same principle as induction; yet in a modified form. In the first place, characters are not susceptible of simple enumeration like

objects; in the next place, characters run in categories. When we make an hypothesis like that about the piece of paper, we only examine a single line of characters, or perhaps two or three, and we take no specimen at all of others. If the hypothesis were nothing but an induction, all that we should be justified in concluding, in the example above, would be that the two pieces of paper which matched in such irregularities as have been examined would be found to match in other, say slighter, irregularities. The inference from the shape of the paper to its ownership is precisely what distinguishes hypothesis from induction, and makes it a bolder and more perilous step.

Peirce: CP 2.633 Cross-Ref:††

633. The same warnings that have been given against imagining that induction rests upon the uniformity of Nature might be repeated in regard to hypothesis. Here, as there, such a theory not only utterly fails to account for the validity of the inference, but it also gives rise to methods of conducting it which are absolutely vicious. There are, no doubt, certain uniformities in Nature, the knowledge of which will fortify an hypothesis very much. For example, we suppose that iron, titanium, and other metals exist in the sun, because we find in the solar spectrum many lines coincident in position with those which these metals would produce; and this hypothesis is greatly strengthened by our knowledge of the remarkable distinctiveness of the particular line of characters observed. But such a fortification of hypothesis is of a deductive kind, and hypothesis may still be probable when such reënforcement is wanting.

Peirce: CP 2.634 Cross-Ref:††

634. There is no greater nor more frequent mistake in practical logic than to suppose that things which resemble one another strongly in some respects are any the more likely for that to be alike in others. That this is absolutely false, admits of rigid demonstration; but, inasmuch as the reasoning is somewhat severe and complicated (requiring, like all such reasoning, the use of A, B, C, etc., to set it forth), the reader would probably find it distasteful, and I omit it. An example, however, may illustrate the proposition: The comparative mythologists occupy themselves with finding points of resemblance between solar phenomena and the careers of the heroes of all sorts of traditional stories; and upon the basis of such resemblances they infer that these heroes are impersonations of the sun. If there be anything more in their reasonings, it has never been made clear to me. An ingenious logician, to show how futile all that is, wrote a little book, in which he pretended to prove, in the same manner, that Napoleon Bonaparte is only an impersonation of the sun. It was really wonderful to see how many points of resemblance he made out. The truth is, that any two things resemble one another just as strongly as any two others, if recondite resemblances are admitted. But, in order that the process of making an hypothesis should lead to a probable result, the following rules must be followed:

Peirce: CP 2.634 Cross-Ref:††

1. The hypothesis should be distinctly put as a question, before making the observations which are to test its truth. In other words, we must try to see what the result of predictions from the hypothesis will be.

Peirce: CP 2.634 Cross-Ref:††

2. The respect in regard to which the resemblances are noted must be taken at random. We must not take a particular kind of predictions for which the hypothesis is known to be good.

Peirce: CP 2.634 Cross-Ref:††

3. The failures as well as the successes of the predictions must be honestly noted. The whole proceeding must be fair and unbiased.

Peirce: CP 2.635 Cross-Ref:††

635. Some persons fancy that bias and counter-bias are favorable to the extraction of truth--that hot and partisan debate is the way to investigate. This is the theory of our atrocious legal procedure. But Logic puts its heel upon this suggestion. It irrefragably demonstrates that knowledge can only be furthered by the real desire for it, and that the methods of obstinacy, of authority, and every mode of trying to reach a foregone conclusion, are absolutely of no value.†1 These things are proved. The reader is at liberty to think so or not as long as the proof is not set forth, or as long as he refrains from examining it. Just so, he can preserve, if he likes, his freedom of opinion in regard to the propositions of geometry; only, in that case, if he takes a fancy to read Euclid, he will do well to skip whatever he finds with A, B, C, etc., for, if he reads attentively that disagreeable matter, the freedom of his opinion about geometry may unhappily be lost forever.

Peirce: CP 2.635 Cross-Ref:††

How many people there are who are incapable of putting to their own consciences this question, "Do I want to know how the fact stands, or not?"

Peirce: CP 2.635 Cross-Ref:††

The rules which have thus far been laid down for induction and hypothesis are such as are absolutely essential. There are many other maxims expressing particular contrivances for making synthetic inferences strong, which are extremely valuable and should not be neglected. Such are, for example, Mr. Mill's four methods. Nevertheless, in the total neglect of these, inductions and hypotheses may and sometimes do attain the greatest force.

Peirce: CP 2.636 Cross-Ref:††

§4. EMPIRICAL FORMULÆ AND THEORIESE

636. Classifications in all cases perfectly satisfactory hardly exist. Even in regard to the great distinction between explicative and ampliative inferences, examples could be found which seem to lie upon the border between the two classes, and to partake in some respects of the characters of either. The same thing is true of the distinction between induction and hypothesis. In the main, it is broad and decided. By induction, we conclude that facts, similar to observed facts, are true in cases not examined. By hypothesis, we conclude the existence of a fact quite different from anything observed, from which, according to known laws, something observed would necessarily result. The former, is reasoning from particulars to the general law; the latter, from effect to cause. The former classifies, the latter explains. It is only in some special cases that there can be more than a momentary doubt to which category a given inference belongs. One exception is where we observe, not facts similar under similar circumstances, but facts different under different circumstances--the difference of the former having, however, a definite relation to the difference of the latter. Such inferences, which are really inductions, sometimes present, nevertheless, some indubitable resemblances to hypotheses.

Peirce: CP 2.637 Cross-Ref:††

637. Knowing that water expands by heat, we make a number of observations of the volume of a constant mass of water at different temperatures. The scrutiny of a few of these suggests a form of algebraical formula which will approximately express the relation of the volume to the temperature. It may be, for instance, that v being the relative volume, and t the temperature, a few observations examined indicate a relation of the form--

$$v = 1 + at + bt^2 + ct^3.$$

Upon examining observations at other temperatures taken at random, this idea is confirmed; and we draw the inductive conclusion that all observations within the limits of temperature from which we have drawn our observations could equally be so satisfied. Having once ascertained that such a formula is possible, it is a mere affair of arithmetic to find the values of a , b , and c , which will make the formula satisfy the observations best. This is what physicists call an *empirical formula*, because it rests upon mere induction, and is not explained by any hypothesis.

Peirce: CP 2.637 Cross-Ref:††

Such formulæ, though very useful as means of describing in general terms the results of observations, do not take any high rank among scientific discoveries. The induction which they embody, that expansion by heat (or whatever other phenomenon is referred to) takes place in a perfectly gradual manner without sudden leaps or innumerable fluctuations, although really important, attracts no attention, because it is what we naturally anticipate. But the defects of such expressions are very serious. In the first place, as long as the observations are subject to error, as all observations are, the formula cannot be expected to satisfy the observations exactly. But the discrepancies cannot be due solely to the errors of the observations, but must be partly owing to the error of the formula which has been deduced from erroneous observations. Moreover, we have no right to suppose that the real facts, if they could be had free from error, could be expressed by such a formula at all. They might, perhaps, be expressed by a similar formula with an infinite number of terms; but of what use would that be to us, since it would require an infinite number of coefficients to be written down? When one quantity varies with another, if the corresponding values are exactly known, it is a mere matter of mathematical ingenuity to find some way of expressing their relation in a simple manner. If one quantity is of one kind--say, a specific gravity--and the other of another kind--say, a temperature--we do not desire to find an expression for their relation which is wholly free from numerical constants, since if it were free from them when, say, specific gravity as compared with water, and temperature as expressed by the Centigrade thermometer, were in question, numbers would have to be introduced when the scales of measurement were changed. We may, however, and do desire to find formulæ expressing the relations of physical phenomena which shall contain no more arbitrary numbers than changes in the scales of measurement might require.

Peirce: CP 2.638 Cross-Ref:††

638. When a formula of this kind is discovered, it is no longer called an empirical formula, but a law of Nature; and is sooner or later made the basis of an hypothesis which is to explain it. These simple formulæ are not usually, if ever, exactly true, but they are none the less important for that; and the great triumph of the

hypothesis comes when it explains not only the formula, but also the deviations from the formula. In the current language of the physicists, an hypothesis of this importance is called a theory, while the term hypothesis is restricted to suggestions which have little evidence in their favor. There is some justice in the contempt which clings to the word hypothesis. To think that we can strike out of our own minds a true preconception of how Nature acts, is a vain fancy. As Lord Bacon well says: "The subtlety of Nature far exceeds the subtlety of sense and intellect: so that these fine meditations, and speculations, and reasonings of men are a sort of insanity, only there is no one at hand to remark it."†1 The successful theories are not pure guesses, but are guided by reasons.

Peirce: CP 2.639 Cross-Ref:††

639. The kinetical theory of gases is a good example of this. This theory is intended to explain certain simple formulæ, the chief of which is called the law of Boyle. It is, that if air or any other gas be placed in a cylinder with a piston, and if its volume be measured under the pressure of the atmosphere, say fifteen pounds on the square inch, and if then another fifteen pounds per square inch be placed on the piston, the gas will be compressed to one-half its bulk, and in similar inverse ratio for other pressures. The hypothesis which has been adopted to account for this law is that the molecules of a gas are small, solid particles at great distances from each other (relatively to their dimensions), and moving with great velocity, without sensible attractions or repulsions, until they happen to approach one another very closely. Admit this, and it follows that when a gas is under pressure what prevents it from collapsing is not the incompressibility of the separate molecules, which are under no pressure at all, since they do not touch, but the pounding of the molecules against the piston. The more the piston falls, and the more the gas is compressed, the nearer together the molecules will be; the greater number there will be at any moment within a given distance of the piston, the shorter the distance which any one will go before its course is changed by the influence of another, the greater number of new courses of each in a given time, and the oftener each, within a given distance of the piston, will strike it. This explains Boyle's law. The law is not exact; but the hypothesis does not lead us to it exactly. For, in the first place, if the molecules are large, they will strike each other oftener when their mean distances are diminished, and will consequently strike the piston oftener, and will produce more pressure upon it. On the other hand, if the molecules have an attraction for one another, they will remain for a sensible time within one another's influence, and consequently they will not strike the wall so often as they otherwise would, and the pressure will be less increased by compression.

Peirce: CP 2.639 Cross-Ref:††

When the kinetical theory of gases was first proposed by Daniel Bernoulli,†1 in 1738, it rested only on the law of Boyle, and was therefore pure hypothesis. It was accordingly quite naturally and deservedly neglected. But, at present, the theory presents quite another aspect; for, not to speak of the considerable number of observed facts of different kinds with which it has been brought into relation, it is supported by the mechanical theory of heat. That bringing together bodies which attract one another, or separating bodies which repel one another, when sensible motion is not produced or destroyed, is always accompanied by the evolution of heat, is little more than an induction. Now, it has been shown by experiment that, when a gas is allowed to expand without doing work, a very small amount of heat disappears. This proves that the particles of the gas attract one another slightly, and but very slightly. It follows that, when a gas is under pressure, what prevents it from collapsing

is not any repulsion between the particles, since there is none. Now, there are only two modes of force known to us, force of position or attractions and repulsions, and force of motion. Since, therefore, it is not the force of position which gives a gas its expansive force, it must be the force of motion. In this point of view, the kinetical theory of gases appears as a deduction from the mechanical theory of heat. It is to be observed, however, that it supposes the same law of mechanics (that there are only those two modes of force) which holds in regard to bodies such as we can see and examine, to hold also for what are very different, the molecules of bodies. Such a supposition has but a slender support from induction. Our belief in it is greatly strengthened by its connection with the law of Boyle, and it is, therefore, to be considered as an hypothetical inference. Yet it must be admitted that the kinetical theory of gases would deserve little credence if it had not been connected with the principles of mechanics.

Peirce: CP 2.640 Cross-Ref:††

640. The great difference between induction and hypothesis is, that the former infers the existence of phenomena such as we have observed in cases which are similar, while hypothesis supposes something of a different kind from what we have directly observed, and frequently something which it would be impossible for us to observe directly.†1 Accordingly, when we stretch an induction quite beyond the limits of our observation, the inference partakes of the nature of hypothesis. It would be absurd to say that we have no inductive warrant for a generalization extending a little beyond the limits of experience, and there is no line to be drawn beyond which we cannot push our inference; only it becomes weaker the further it is pushed. Yet, if an induction be pushed very far, we cannot give it much credence unless we find that such an extension explains some fact which we can and do observe. Here, then, we have a kind of mixture of induction and hypothesis supporting one another; and of this kind are most of the theories of physics.

Peirce: CP 2.641 Cross-Ref:††

§5. ON THE DIFFERENCE BETWEEN INDUCTION AND HYPOTHESIS

641. That synthetic inferences may be divided into induction and hypothesis in the manner here proposed,†P1 admits of no question. The utility and value of the distinction are to be tested by their applications.

Peirce: CP 2.642 Cross-Ref:††

642. Induction is, plainly, a much stronger kind of inference than hypothesis; and this is the first reason for distinguishing between them. Hypotheses are sometimes regarded as provisional resorts, which in the progress of science are to be replaced by inductions. But this is a false view of the subject. Hypothetic reasoning infers very frequently a fact not capable of direct observation. It is an hypothesis that Napoleon Bonaparte once existed. How is that hypothesis ever to be replaced by an induction? It may be said that from the premiss that such facts as we have observed are as they would be if Napoleon existed, we are to infer by induction that *all* facts that are hereafter to be observed will be of the same character. There is no doubt that every hypothetic inference may be distorted into the appearance of an induction in this way. But the essence of an induction is that it infers from one set of facts another set of similar facts, whereas hypothesis infers from facts of one kind to facts of

another. Now, the facts which serve as grounds for our belief in the historic reality of Napoleon are not by any means necessarily the only kind of facts which are explained by his existence. It may be that, at the time of his career, events were being recorded in some way not now dreamed of, that some ingenious creature on a neighboring planet was photographing the earth, and that these pictures on a sufficiently large scale may some time come into our possession, or that some mirror upon a distant star will, when the light reaches it, reflect the whole story back to earth. Never mind how improbable these suppositions are; everything which happens is infinitely improbable. I am not saying that *these* things are likely to occur, but that *some* effect of Napoleon's existence which now seems impossible is certain nevertheless to be brought about. The hypothesis asserts that such facts, when they do occur, will be of a nature to confirm, and not to refute, the existence of the man. We have, in the impossibility of inductively inferring hypothetical conclusions, a second reason for distinguishing between the two kinds of inference.

Peirce: CP 2.643 Cross-Ref:††

643. A third merit of the distinction is, that it is associated with an important psychological or rather physiological difference in the mode of apprehending facts. Induction infers a rule. Now, the belief of a rule is a habit. That a habit is a rule active in us, is evident. That every belief is of the nature of a habit, in so far as it is of a general character, has been shown in the earlier papers of this series.†1 Induction, therefore, is the logical formula which expresses the physiological process of formation of a habit. Hypothesis substitutes, for a complicated tangle of predicates attached to one subject, a single conception. Now, there is a peculiar sensation belonging to the act of thinking that each of these predicates inheres in the subject. In hypothetic inference this complicated feeling so produced is replaced by a single feeling of greater intensity, that belonging to the act of thinking the hypothetic conclusion. Now, when our nervous system is excited in a complicated way, there being a relation between the elements of the excitation, the result is a single harmonious disturbance which I call an emotion. Thus, the various sounds made by the instruments of an orchestra strike upon the ear, and the result is a peculiar musical emotion, quite distinct from the sounds themselves. This emotion is essentially the same thing as an hypothetic inference, and every hypothetic inference involves the formation of such an emotion. We may say, therefore, that hypothesis produces the *sensuous* element of thought, and induction the *habitual* element. As for deduction, which adds nothing to the premisses, but only out of the various facts represented in the premisses selects one and brings the attention down to it, this may be considered as the logical formula for paying attention, which is the *volitional* element of thought, and corresponds to nervous discharge in the sphere of physiology.†1

Peirce: CP 2.644 Cross-Ref:††

644. Another merit of the distinction between induction and hypothesis is, that it leads to a very natural classification of the sciences and of the minds which prosecute them. What must separate different kinds of scientific men more than anything else are the differences of their *techniques*. We cannot expect men who work with books chiefly to have much in common with men whose lives are passed in laboratories. But, after differences of this kind, the next most important are differences in the modes of reasoning. Of the natural sciences, we have, first, the classificatory sciences, which are purely inductive--systematic botany and zoölogy, mineralogy, and chemistry. Then, we have the sciences of theory, as above explained--astronomy, pure physics, etc. Then, we have sciences of hypothesis--geology, biology, etc.†2

Peirce: CP 2.644 Cross-Ref:††

There are many other advantages of the distinction in question which I shall leave the reader to find out by experience. If he will only take the custom of considering whether a given inference belongs to one or other of the two forms of synthetic inference given in 623, I can promise him that he will find his advantage in it, in various ways.

Peirce: CP 2.645 Cross-Ref:††
CHAPTER 6

THE DOCTRINE OF CHANCES†1

§1. CONTINUITY AND THE FORMATION OF CONCEPTS

645. It is a common observation that a science first begins to be exact when it is quantitatively treated. What are called the exact sciences are no others than the mathematical ones. Chemists reasoned vaguely until Lavoisier showed them how to apply the balance to the verification of their theories, when chemistry leaped suddenly into the position of the most perfect of the classificatory sciences. It has thus become so precise and certain that we usually think of it along with optics, thermotics, and electrics. But these are studies of general laws, while chemistry considers merely the relations and classification of certain objects; and belongs, in reality, in the same category as systematic botany and zoölogy. Compare it with these last, however, and the advantage that it derives from its quantitative treatment is very evident.†P1

Peirce: CP 2.646 Cross-Ref:††

646. The rudest numerical scales, such as that by which the mineralogists distinguish the different degrees of hardness, are found useful. The mere counting of pistils and stamens sufficed to bring botany out of total chaos into some kind of form. It is not, however, so much from *counting* as from *measuring*, not so much from the conception of number as from that of continuous quantity, that the advantage of mathematical treatment comes. Number, after all, only serves to pin us down to a precision in our thoughts which, however beneficial, can seldom lead to lofty conceptions, and frequently descends to pettiness. Of those two faculties of which Bacon speaks,†1 that which marks differences and that which notes resemblances, the employment of number can only aid the lesser one; and the excessive use of it must tend to narrow the powers of the mind. But the conception of continuous quantity has a great office to fulfill, independently of any attempt at precision. Far from tending to the exaggeration of differences, it is the direct instrument of the finest generalizations. When a naturalist wishes to study a species, he collects a considerable number of specimens more or less similar. In contemplating them, he observes certain ones which are more or less alike in some particular respect. They all have, for instance, a certain S-shaped marking. He observes that they are not *precisely* alike, in this respect; the S has not precisely the same shape, but the differences are such as to lead

him to believe that forms could be found intermediate between any two of those he possesses. He, now, finds other forms apparently quite dissimilar--say a marking in the form of a C--and the question is, whether he can find intermediate ones which will connect these latter with the others. This he often succeeds in doing in cases where it would at first be thought impossible; whereas, he sometimes finds those which differ, at first glance, much less, to be separated in Nature by the non-occurrence of intermediaries. In this way, he builds up from the study of Nature a new general conception of the character in question. He obtains, for example, an idea of a leaf which includes every part of the flower, and an idea of a vertebra which includes the skull. I surely need not say much to show what a logical engine is here. It is the essence of the method of the naturalist. How he applies it first to one character, and then to another, and finally obtains a notion of a species of animals, the differences between whose members, however great, are confined within limits, is a matter which does not here concern us. The whole method of classification must be considered later; but, at present, I only desire to point out that it is by taking advantage of the idea of continuity, or the passage from one form to another by insensible degrees, †P1 that the naturalist builds his conceptions. Now, the naturalists are the great builders of conceptions; there is no other branch of science where so much of this work is done as in theirs; and we must, in great measure, take them for our teachers in this important part of logic. And it will be found everywhere that the idea of continuity †P2 is a powerful aid to the formation of true and fruitful conceptions. By means of it, the greatest differences are broken down and resolved into differences of degree, and the incessant application of it is of the greatest value in broadening our conceptions. I propose to make a great use of this idea †P3 in the present series of papers; and the particular series of important fallacies, which, arising from a neglect of it, †P4 have desolated philosophy, must further on be closely studied. At present, I simply call the reader's attention to the utility of this conception.

Peirce: CP 2.646 Cross-Ref: ††

In studies of numbers, the idea of continuity is so indispensable, that it is perpetually introduced even where there is no continuity in fact, as where we say that there are in the United States 10.7 inhabitants per square mile, or that in New York 14.72 persons live in the average house. †P5 Another example is that law of the distribution of errors which Quetelet, Galton, and others, have applied with so much success to the study of biological and social matters. This application of continuity to cases where it does not really exist illustrates, also, another point which will hereafter demand a separate study, namely, the great utility which fictions sometimes have in science. †1

Peirce: CP 2.647 Cross-Ref: ††

§2. THE PROBLEM OF PROBABILITY E

647. The theory of probabilities is simply the science of logic quantitatively treated. There are two conceivable certainties with reference to any hypothesis, the certainty of its truth and the certainty of its falsity. The numbers *one* and *zero* are appropriated, in this calculus, to marking these extremes of knowledge; while fractions having values intermediate between them indicate, as we may vaguely say, the degrees in which the evidence leans toward one or the other. The general problem of probabilities is, from a given state of facts, to determine the numerical probability

of a possible fact. This is the same as to inquire how much the given facts are worth, considered as evidence to prove the possible fact. Thus the problem of probabilities is simply the general problem of logic.

Peirce: CP 2.648 Cross-Ref:††

648. Probability is a continuous quantity, so that great advantages may be expected from this mode of studying logic. Some writers have gone so far as to maintain that, by means of the calculus of chances, every solid inference may be represented by legitimate arithmetical operations upon the numbers given in the premisses. If this be, indeed, true, the great problem of logic, how it is that the observation of one fact can give us knowledge of another independent fact, is reduced to a mere question of arithmetic. It seems proper to examine this pretension before undertaking any more recondite solution of the paradox.

Peirce: CP 2.648 Cross-Ref:††

But, unfortunately, writers on probabilities are not agreed in regard to this result. This branch of mathematics is the only one, I believe, in which good writers frequently get results entirely erroneous. In elementary geometry the reasoning is frequently fallacious, but erroneous conclusions are avoided; but it may be doubted if there is a single extensive treatise on probabilities in existence which does not contain solutions absolutely indefensible. This is partly owing to the want of any regular method of procedure; for the subject involves too many subtilities to make it easy to put its problems into equations without such an aid. But, beyond this, the fundamental principles of its calculus are more or less in dispute. In regard to that class of questions to which it is chiefly applied for practical purposes, there is comparatively little doubt; but in regard to others to which it has been sought to extend it, opinion is somewhat unsettled.

Peirce: CP 2.648 Cross-Ref:††

This last class of difficulties can only be entirely overcome by making the idea of probability perfectly clear in our minds in the way set forth in our last paper.†1

Peirce: CP 2.649 Cross-Ref:††
§3. ON DEGREES OF PROBABILITYE

649. To get a clear idea of what we mean by probability, we have to consider what real and sensible difference there is between one degree of probability and another.

Peirce: CP 2.649 Cross-Ref:††

The character of probability belongs primarily, without doubt, to certain inferences. Locke †2 explains it as follows: After remarking that the mathematician positively knows that the sum of the three angles of a triangle is equal to two right angles because he apprehends the geometrical proof, he thus continues: "But another man who never took the pains to observe the demonstration, hearing a mathematician, a man of credit, affirm the three angles of a triangle to be equal to two right ones, *assents* to it; *i.e.*, receives it for true. In which case the foundation of his assent is the probability of the thing, the proof being such as, for the most part, carries truth with it; the man on whose testimony he receives it not being wont to affirm anything contrary to, or besides his knowledge, especially in matters of this kind." The

celebrated *Essay Concerning Humane Understanding* contains many passages which, like this one, make the first steps in profound analyses which are not further developed. It was shown †3 in the first of these papers that the validity of an inference does not depend on any tendency of the mind to accept it, however strong such tendency may be; but consists in the real fact that, when premisses like those of the argument in question are true, conclusions related to them like that of this argument are also true. It was remarked that in a logical mind an argument is always conceived as a member of a *genus* of arguments all constructed in the same way, and such that, when their premisses are real facts, their conclusions are so also. If the argument is demonstrative, then this is always so; if it is only probable, then it is for the most part so. As Locke says, the probable argument is "*such as* for the most part carries truth with it."

Peirce: CP 2.650 Cross-Ref:††

650. According to this, that real and sensible difference between one degree of probability and another, in which the meaning of the distinction lies, is that in the frequent employment of two different modes of inference, one will carry truth with it oftener than the other. It is evident that this is the only difference there is in the existing fact. Having certain premisses, a man draws a certain conclusion, and as far as this inference alone is concerned the only possible practical question is whether that conclusion is true or not, and between existence and non-existence there is no middle term. "Being only is and nothing is altogether not," said Parmenides; and this is in strict accordance with the analysis of the conception of reality given in the last paper.†1 For we found that the distinction of reality and fiction depends on the supposition that sufficient investigation would cause one opinion to be universally received and all others to be rejected. That presupposition, involved in the very conceptions of reality and figment, involves a complete sundering of the two. It is the heaven-and-hell idea in the domain of thought. But, in the long run, there is a real fact which corresponds to the idea of probability, and it is that a given mode of inference sometimes proves successful and sometimes not, and that in a ratio ultimately fixed. As we go on drawing inference after inference of the given kind, during the first ten or hundred cases the ratio of successes may be expected to show considerable fluctuations; but when we come into the thousands and millions, these fluctuations become less and less; and if we continue long enough, the ratio will approximate toward a fixed limit. We may, therefore, define the probability of a mode of argument as the proportion of cases in which it carries truth with it.

Peirce: CP 2.651 Cross-Ref:††

651. The inference from the premiss, A, to the conclusion, B, depends, as we have seen, on the guiding principle, that if a fact of the class A is true, a fact of the class B is true. The probability consists of the fraction whose numerator is the number of times in which both A and B are true, and whose denominator is the total number of times in which A is true, whether B is so or not. Instead of speaking of this as the probability of the inference, there is not the slightest objection to calling it the probability that, if A happens, B happens. But to speak of the probability of the event B, without naming the condition, really has no meaning at all. It is true that when it is perfectly obvious what condition is meant, the ellipsis may be permitted. But we should avoid contracting the habit of using language in this way (universal as the habit is), because it gives rise to a vague way of thinking, as if the action of causation might either determine an event to happen or determine it not to happen, or leave it more or less free to happen or not, so as to give rise to an *inherent* chance in regard to its occurrence. It is quite clear to me that some of the worst and most persistent errors

in the use of the doctrine of chances have arisen from this vicious mode of expression.†P1

Peirce: CP 2.652 Cross-Ref:††
§4. THREE LOGICAL SENTIMENTSE

652. But there remains an important point to be cleared up. According to what has been said, the idea of probability essentially belongs to a kind of inference which is repeated indefinitely. An individual inference must be either true or false, and can show no effect of probability; and, therefore, in reference to a single case considered in itself, probability can have no meaning. Yet if a man had to choose between drawing a card from a pack containing twenty-five red cards and a black one, or from a pack containing twenty-five black cards and a red one, and if the drawing of a red card were destined to transport him to eternal felicity, and that of a black one to consign him to everlasting woe, it would be folly to deny that he ought to prefer the pack containing the larger proportion of red cards, although, from the nature of the risk, it could not be repeated. It is not easy to reconcile this with our analysis of the conception of chance. But suppose he should choose the red pack, and should draw the wrong card, what consolation would he have? He might say that he had acted in accordance with reason, but that would only show that his reason was absolutely worthless. And if he should choose the right card, how could he regard it as anything but a happy accident? He could not say that if he had drawn from the other pack, he might have drawn the wrong one, because an hypothetical proposition such as, "if A, then B," means nothing with reference to a single case. Truth consists in the existence of a real fact corresponding to the true proposition. Corresponding to the proposition, "if A, then B," there may be the fact that *whenever* such an event as A happens such an event as B happens. But in the case supposed, which has no parallel as far as this man is concerned, there would be no real fact whose existence could give any truth to the statement that, if he had drawn from the other pack, he might have drawn a black card. Indeed, since the validity of an inference consists in the truth of the hypothetical proposition that *if* the premisses be true the conclusion will also be true, and since the only real fact which can correspond to such a proposition is that whenever the antecedent is true the consequent is so also, it follows that there can be no sense in reasoning in an isolated case, at all.

Peirce: CP 2.653 Cross-Ref:††

653. These considerations appear, at first sight, to dispose of the difficulty mentioned. Yet the case of the other side is not yet exhausted. Although probability will probably manifest its effect in, say, a thousand risks, by a certain proportion between the numbers of successes and failures, yet this, as we have seen, is only to say that it certainly will, at length, do so. Now the number of risks, the number of probable inferences, which a man draws in his whole life, is a finite one, and he cannot be absolutely *certain* that the mean result will accord with the probabilities at all. Taking all his risks collectively, then, it cannot be certain that they will not fail, and his case does not differ, except in degree, from the one last supposed. It is an indubitable result of the theory of probabilities that every gambler, if he continues long enough, must ultimately be ruined. Suppose he tries the martingale, which some believe infallible, and which is, as I am informed, disallowed in the gambling-houses. In this method of playing, he first bets say \$1; if he loses it he bets \$2; if he loses that

he bets \$4; if he loses that he bets \$8; if he then gains he has lost $1 + 2 + 4 = 7$, and he has gained \$1 more; and no matter how many bets he loses, the first one he gains will make him \$1 richer than he was in the beginning. In that way, he will probably gain at first; but, at last, the time will come when the run of luck is so against him that he will not have money enough to double, and must, therefore, let his bet go. This will *probably* happen before he has won as much as he had in the first place, so that this run against him will leave him poorer than he began; some time or other it will be sure to happen. It is true that there is always a possibility of his winning any sum the bank can pay, and we thus come upon a celebrated paradox that, though he is certain to be ruined, the value of his expectation calculated according to the usual rules (which omit this consideration) is large. But, whether a gambler plays in this way or any other, the same thing is true, namely, that if [he] plays long enough he will be sure some time to have such a run against him as to exhaust his entire fortune. The same thing is true of an insurance company. Let the directors take the utmost pains to be independent of great conflagrations and pestilences, their actuaries can tell them that, according to the doctrine of chances, the time must come, at last, when their losses will bring them to a stop. They may tide over such a crisis by extraordinary means, but then they will start again in a weakened state, and the same thing will happen again all the sooner. An actuary might be inclined to deny this, because he knows that the expectation of his company is large, or perhaps (neglecting the interest upon money) is infinite. But calculations of expectations leave out of account the circumstance now under consideration, which reverses the whole thing. However, I must not be understood as saying that insurance is on this account unsound, more than other kinds of business. All human affairs rest upon probabilities, and the same thing is true everywhere. If man were immortal he could be perfectly sure of seeing the day when everything in which he had trusted should betray his trust, and, in short, of coming eventually to hopeless misery. He would break down, at last, as every great fortune, as every dynasty, as every civilization does. In place of this we have death.

Peirce: CP 2.654 Cross-Ref:††

654. But what, without death, would happen to every man, with death must happen to some man. At the same time, death makes the number of our risks, of our inferences, finite, and so makes their mean result uncertain. The very idea of probability and of reasoning rests on the assumption that this number is indefinitely great. We are thus landed in the same difficulty as before, and I can see but one solution of it. It seems to me that we are driven to this, that logicity inexorably requires that our interests shall *not* be limited. They must not stop at our own fate, but must embrace the whole community. This community, again, must not be limited, but must extend to all races of beings with whom we can come into immediate or mediate intellectual relation. It must reach, however vaguely, beyond this geological epoch, beyond all bounds. He who would not sacrifice his own soul to save the whole world, is, as it seems to me, illogical in all his inferences, collectively. Logic is rooted in the social principle.

Peirce: CP 2.654 Cross-Ref:††

To be logical men should not be selfish; and, in point of fact, they are not so selfish as they are thought. The willful prosecution of one's desires is a different thing from selfishness. The miser is not selfish; his money does him no good, and he cares for what shall become of it after his death. We are constantly speaking of *our* possessions on the Pacific, and of *our* destiny as a republic, where no personal interests are involved, in a way which shows that we have wider ones. We discuss with anxiety the possible exhaustion of coal in some hundreds of years, or the

cooling-off of the sun in some millions, and show in the most popular of all religious tenets that we can conceive the possibility of a man's descending into hell for the salvation of his fellows.

Peirce: CP 2.654 Cross-Ref:††

Now, it is not necessary for logicality that a man should himself be capable of the heroism of self-sacrifice. It is sufficient that he should recognize the possibility of it, should perceive that only that man's inferences who has it are really logical, and should consequently regard his own as being only so far valid as they would be accepted by the hero. So far as he thus refers his inferences to that standard, he becomes identified with such a mind.

Peirce: CP 2.654 Cross-Ref:††

This makes logicality attainable enough. Sometimes we can personally attain to heroism. The soldier who runs to scale a wall knows that he will probably be shot, but that is not all he cares for. He also knows that if all the regiment, with whom in feeling he identifies himself, rush forward at once, the fort will be taken. In other cases we can only imitate the virtue. The man whom we have supposed as having to draw from the two packs, who if he is not a logician will draw from the red pack from mere habit, will see, if he is logician enough, that he cannot be logical so long as he is concerned only with his own fate, but that that man who should care equally for what was to happen in all possible cases of the sort could act logically, and would draw from the pack with the most red cards, and thus, though incapable himself of such sublimity, our logician would imitate the effect of that man's courage in order to share his logicality.

Peirce: CP 2.654 Cross-Ref:††

But all this requires a conceived identification of one's interests with those of an unlimited community. Now, there exist no reasons, and a later discussion will show that there can be no reasons, for thinking that the human race, or any intellectual race, will exist forever. On the other hand, there can be no reason against it;†P1 and, fortunately, as the whole requirement is that we should have certain sentiments, there is nothing in the facts to forbid our having a *hope*, or calm and cheerful wish, that the community may last beyond any assignable date.

Peirce: CP 2.655 Cross-Ref:††

655. It may seem strange that I should put forward three sentiments, namely, interest in an indefinite community, recognition of the possibility of this interest being made supreme, and hope in the unlimited continuance of intellectual activity, as indispensable requirements of logic. Yet, when we consider that logic depends on a mere struggle to escape doubt, which, as it terminates in action, must begin in emotion, and that, furthermore, the only cause of our planting ourselves on reason is that other methods of escaping doubt fail on account of the social impulse, why should we wonder to find social sentiment presupposed in reasoning? As for the other two sentiments which I find necessary, they are so only as supports and accessories of that. It interests me to notice that these three sentiments seem to be pretty much the same as that famous trio of Charity, Faith, and Hope, which, in the estimation of St. Paul, are the finest and greatest of spiritual gifts. Neither Old nor New Testament is a textbook of the logic of science, but the latter is certainly the highest existing authority in regard to the dispositions of heart which a man ought to have.

Peirce: CP 2.656 Cross-Ref:††

§5. FUNDAMENTAL RULES FOR THE CALCULATION OF CHANCES^E

656. Such average statistical numbers as the number of inhabitants per square mile, the average number of deaths per week, the number of convictions per indictment, or, generally speaking, the numbers of x 's per y , where the x 's are a class of things some or all of which are connected with another class of things, their y 's, I term *relative numbers*. Of the two classes of things to which a relative number refers, that one of which it is a number may be called its *relate*, and that one *per* which the numeration is made may be called its *correlate*.

Peirce: CP 2.657 Cross-Ref:††

657. Probability is a kind of relative number; namely, it is the ratio of the number of arguments of a certain genus which carry truth with them to the total number of arguments of that genus, and the rules for the calculation of probabilities are very easily derived from this consideration. They may all be given here, since they are extremely simple, and it is sometimes convenient to know something of the elementary rules of calculation of chances.

Peirce: CP 2.658 Cross-Ref:††

658. Rule I. *Direct Calculation*.--To calculate, directly, any relative number, say for instance the number of passengers in the average trip of a street-car, we must proceed as follows:

Peirce: CP 2.658 Cross-Ref:††

Count the number of passengers for each trip; add all these numbers, and divide by the number of trips. There are cases in which this rule may be simplified. Suppose we wish to know the number of inhabitants to a dwelling in New York. The same person cannot inhabit two dwellings. If he divide his time between two dwellings he ought to be counted a half-inhabitant of each. In this case we have only to divide the total number of the inhabitants of New York by the number of their dwellings, without the necessity of counting separately those which inhabit each one. A similar proceeding will apply wherever each individual relate belongs to one individual correlate exclusively. If we want the number of x 's per y , and no x belongs to more than one y , we have only to divide the whole number of x 's of y 's by the number of y 's. Such a method would, of course, fail if applied to finding the average number of street-car passengers per trip. We could not divide the total number of travelers by the number of trips, since many of them would have made many passages.

Peirce: CP 2.658 Cross-Ref:††

To find the probability that from a given class of premisses, A, a given class of conclusions, B, follows, it is simply necessary to ascertain what proportion of the times in which premisses of that class are true, the appropriate conclusions are also true. In other words, it is the number of cases of the occurrence of both the events A and B, divided by the total number of cases of the occurrence of the event A.

Peirce: CP 2.659 Cross-Ref:††

659. Rule II. *Addition of Relative Numbers*.--Given two relative numbers having the same correlate, say the number of x 's per y , and the number of z 's per y , it is required to find the number of x 's and z 's together per y . If there is nothing which is

at once an x and a z to the same y , the sum of the two given numbers would give the required number. Suppose, for example, that we had given the average number of friends that men have, and the average number of enemies, the sum of these two is the average number of persons interested in a man. On the other hand, it plainly would not do to add the average number of persons having constitutional diseases to the average number over military age, and to the average number exempted by each special cause from military service, in order to get the average number exempt in any way, since many are exempt in two or more ways at once.

Peirce: CP 2.659 Cross-Ref:††

This rule applies directly to probabilities, given the probability that two different and mutually exclusive events will happen under the same supposed set of circumstances. Given, for instance, the probability that if A then B, and also the probability that if A then C, then the sum of these two probabilities is the probability that if A then either B or C, so long as there is no event which belongs at once to the two classes B and C.

Peirce: CP 2.660 Cross-Ref:††

660. Rule III. ***Multiplication of Relative Numbers.***--Suppose that we have given the relative number of x 's per y ; also the relative number of z 's per x of y ; or, to take a concrete example, suppose that we have given, first, the average number of children in families living in New York; and, second, the average number of teeth in the head of a New York child -- then the product of these two numbers would give the average number of children's teeth in a New York family. But this mode of reckoning will only apply in general under two restrictions. In the first place, it would not be true if the same child could belong to different families, for in that case those children who belonged to several different families might have an exceptionally large or small number of teeth, which would affect the average number of children's teeth in a family more than it would affect the average number of teeth in a child's head. In the second place, the rule would not be true if different children could share the same teeth, the average number of children's teeth being in that case evidently something different from the average number of teeth belonging to a child.

Peirce: CP 2.660 Cross-Ref:††

In order to apply this rule to probabilities, we must proceed as follows: Suppose that we have given the probability that the conclusion B follows from the premiss A, B and A representing as usual certain classes of propositions. Suppose that we also knew the probability of an inference in which B should be the premiss, and a proposition of a third kind, C, the conclusion. Here, then, we have the materials for the application of this rule. We have, first, the relative number of B's per A. We next should have the relative number of C's per B following from A. But the classes of propositions being so selected that the probability of C following from any B in general is just the same as the probability of C's following from one of those B's which is deducible from an A, the two probabilities may be multiplied together, in order to give the probability of C following from A. The same restrictions exist as before. It might happen that the probability that B follows from A was affected by certain propositions of the class B following from several different propositions of the class A. But, practically speaking, all these restrictions are of very little consequence, and it is usually recognized as a principle universally true that the probability that, if A is true, B is, multiplied by the probability that, if B is true, C is, gives the probability that, if A is true, C is.

Peirce: CP 2.660 Cross-Ref:††

There is a rule supplementary to this, of which great use is made. It is not universally valid, and the greatest caution has to be exercised in making use of it--a double care, first, never to use it when it will involve serious error; and, second, never to fail to take advantage of it in cases in which it can be employed. This rule depends upon the fact that in very many cases the probability that C is true if B is, is substantially the same as the probability that C is true if A is. Suppose, for example, we have the average number of males among the children born in New York; suppose that we also have the average number of children born in the winter months among those born in New York. Now, we may assume without doubt, at least as a closely approximate proposition (and no very nice calculation would be in place in regard to probabilities), that the proportion of males among all the children born in New York is the same as the proportion of males born in summer in New York; and, therefore, if the names of all the children born during a year were put into an urn, we might multiply the probability that any name drawn would be the name of a male child by the probability that it would be the name of a child born in summer, in order to obtain the probability that it would be the name of a male child born in summer. The questions of probability, in the treatises upon the subject, have usually been such as relate to balls drawn from urns, and games of cards, and so on, in which the question of the *independence* of events, as it is called--that is to say, the question of whether the probability of C, under the hypothesis B, is the same as its probability under the hypothesis A--has been very simple; but, in the application of probabilities to the ordinary questions of life, it is often an exceedingly nice question whether two events may be considered as independent with sufficient accuracy or not. In all calculations about cards it is assumed that the cards are thoroughly shuffled, which makes one deal quite independent of another. In point of fact the cards seldom are, in practice, shuffled sufficiently to make this true; thus, in a game of whist, in which the cards have fallen in sets of four of the same suit, and are so gathered up, they will lie more or less in sets of four of the same suit, and this will be true even after they are shuffled. At least some traces of this arrangement will remain, in consequence of which the number of "short suits," as they are called--that is to say, the number of hands in which the cards are very unequally divided in regard to suits--is smaller than the calculation would make it to be; so that, when there is a misdeal, where the cards, being thrown about the table, get very thoroughly shuffled, it is a common saying that in the hands next dealt out there are generally short suits. A few years ago a friend of mine, who plays whist a great deal, was so good as to count the number of spades dealt to him in 165 hands, in which the cards had been, if anything, shuffled better than usual. According to calculation, there should have been 85 of these hands in which my friend held either three or four spades, but in point of fact there were 94, showing the influence of imperfect shuffling.

Peirce: CP 2.660 Cross-Ref:††

According to the view here taken, these are the only fundamental rules for the calculation of chances. An additional one, derived from a different conception of probability, is given in some treatises, which if it be sound might be made the basis of a theory of reasoning. Being, as I believe it is, absolutely absurd, the consideration of it serves to bring us to the true theory; and it is for the sake of this discussion, which must be postponed to the next number,†1 that I have brought the doctrine of chances to the reader's attention at this early stage of our studies of the logic of science.

Peirce: CP 2.661 Cross-Ref:††

§6. NOTES ON THE DOCTRINE OF CHANCES †2

661. On reperusing this article after the lapse of a full generation, it strikes me as making two points that were worth making. The better made of the two had been still better made ten years before in my three articles in the [*Journal of Speculative Philosophy*] Vol. 2.†3: This point is that no man can be logical whose supreme desire is the well-being of himself or of any other existing person or collection of persons. The other good point is that probability never properly refers immediately to a single event, but exclusively to the happening of a given kind of event on any occasion of a given kind. So far all is well. But when I come to define probability, I repeatedly say that it is the quotient of the *number* of occurrences of the event divided by the *number* of occurrences of the occasion. Now this is manifestly wrong, for probability relates to the future; and how can I say how many times a given die will be thrown in the future? To be sure I might, immediately after my throw, put the die in strong nitric acid, and dissolve it, but this suggestion only puts the preposterous character of the definition in a still stronger light. For it is plain that, if probability be the ratio of the occurrences of the specific event to the occurrences of the generic occasion, it is the ratio that there *would be* in the long run, and has nothing to do with any supposed cessation of the occasions. This long run can be nothing but an endlessly long run; and even if it be correct to speak of an infinite "number," yet ∞/∞ (infinity divided by infinity) has certainly, *in itself*, no definite value.

Peirce: CP 2.661 Cross-Ref:††

But we have not yet come to the end of the flaws in the definition, since no notice whatever has been taken of two conditions which require the strictest precautions in all experiments to determine the probability of a specific event on a generic occasion. Namely, in the first place we must limit our endeavors strictly to counting occurrences of the right genus of occasion and carefully resist all other motives for counting them, and strive to take them just as they would ordinarily occur. In the next place, it must be known that the occurrence of the specific event on one occasion will have no tendency to produce or to prevent the occurrence of the same event upon any other of the occurrences of the generic occasion. In the third place, after the probability has been ascertained, we must remember that this probability cannot be relied upon at any future time unless we have adequate grounds for believing that it has not too much changed in the interval.

Peirce: CP 2.662 Cross-Ref:††

662. I will now give over jeering at my former inaccuracies, committed when I had been a student of logic for only about a quarter of a century, and was naturally not so well-versed in it as now, and will proceed to define probability. I must premiss that we, all of us, use this word with a degree of laxity which corrupts and rots our reasoning to a degree that very few of us are at all awake to. When I say our "reasoning," I mean not formal reasonings only but our thoughts in general, so far as they are concerned with any of those approaches toward knowledge which we confound with probability. The result is that we not only fall into the falsest ways of thinking, but, what is often still worse, we give up sundry problems as beyond our powers -- problems of gravest concern, too -- when, in fact, we should find they were not a bit so, if we only rightly discriminated between the different kinds of imperfection of certitude, and if we had only once acquainted ourselves with their different natures. I shall in these notes endeavor to mark the three ways of falling

short of certainty by the three terms *probability*, *verisimilitude* or *likelihood*, and *plausibility*. Just at present I propose to deal only with Probability; but I will so far characterize *verisimilitude* and *plausibility* as to mark them off as being entirely different from Probability. Beginning with Plausibility, †1 I will first endeavor to give an example of an idea which shall be strikingly marked by its very low degree of this quality. Suppose a particularly symmetrical larch tree near the house of a great lover of such trees had been struck by lightning and badly broken, and that as he was looking sorrowfully out of the window at it, he should have happened to say, "I wonder why that particular tree should have been struck, when there are so many about the place that seem more exposed!" Suppose, then, his wife should reply, "Perhaps there may be an eagle's eyrie on some of the hills in the neighborhood, and perhaps the male bird in building it may have used some stick that had a nail in it; and one of the eaglets may have scratched itself against the nail; so that the mother may have reproached the male for using such a dangerous stick; and he, being vexed with her teasing, may have determined to carry the piece to a great distance; it may have been while he was doing this that the explosion of lightning took place, and the electricity may have been deflected by the iron in such a way as to strike this tree. Mind, I do not say that this is what did happen; but if you want to find out why that tree was struck, I think you had better search for an eyrie, and see whether any of the eaglets have been scratched." This is an example of as unplausible a theory as I can think of. We should commonly say it was highly improbable; and I suppose it would be so. But were it ever so probable in all its elements, it would still deserve no attention, because it is perfectly gratuitous to suppose that the lightning was deflected at all; and this supposition does not help to explain the phenomenon.

Peirce: CP 2.662 Cross-Ref: ††

Eusapia Palladino had been proved to be a very clever prestigitieuse and cheat, and was visited by a Mr. Carrington, †1 whom I suppose to be so clever in finding out how tricks are done, that it is highly improbable that any given trick should long baffle him. In point of fact he has often caught the Palladino creature in acts of fraud. Some of her performances, however, he cannot explain; and thereupon he urges the theory that these are supernatural, or, as he prefers to phrase it, "supernormal." Well, I know how it is that when a man has been long intensely exercised and over-fatigued by an enigma, his common-sense will sometimes desert him; but it seems to me that the Palladino has simply been too clever for him, as no doubt she would be for me. The theory that there is anything "supernormal," or *super* anything but *superchérie* in the case, seems to me as needless as any theory I ever came across. That is to say, granted that it is not yet *proved* that women who deceive for gain receive aid from the spiritual world, I think it more plausible that there are tricks that can deceive Mr. Carrington than that the Palladino woman has received such aid. By Plausible, I mean that a theory that has not yet been subjected to any test, although more or less surprising phenomena have occurred which it would explain if it were true, is in itself of such a character as to recommend it for further examination or, if it be *highly* plausible, justify us in seriously inclining toward belief in it, as long as the phenomena be inexplicable otherwise.

Peirce: CP 2.663 Cross-Ref: ††

663. I will now give an idea of what I mean by *likely* or *verisimilar*. It is to be understood that I am only endeavouring so far to explain the meanings I attach to "plausible" and to "likely," as this may be an assistance to the reader in understanding the meaning I attach to *probable*. I call that theory *likely* which is not yet proved but is supported by such evidence that if the rest of the conceivably possible evidence

should turn out upon examination to be of a *similar* character, the theory would be conclusively proved. Strictly speaking, matters of fact never can be demonstrably proved, since it will always remain conceivable that there should be some mistake about it. For instance, I regard it as *sufficiently* proved that my name is Charles Peirce and that I was born in Cambridge, Massachusetts, in a stone-colored wooden house in Mason Street. But even of the part of this of which I am most assured--of my name--there is a certain small probability that I am in an abnormal condition and have got it wrong. I am conscious myself of occasional lapses of memory about other things; and though I well remember--or think I do--living in that house at a tender age, I do not in the least remember being born there, impressive as such a first experience might be expected to be. Indeed, I cannot specify any date on which any certain person informed me I had been born there; and it certainly would have been easy to deceive me in the matter had there been any serious reason for doing so; and how can I be so sure as I surely am that no such reason did exist? It would be a theory without plausibility; that is all.

Peirce: CP 2.663 Cross-Ref:††

The history of science, particularly physical science, in contradistinction to natural science--or, as I usually, though inadequately, phrase the distinction, the history of nomological in contradistinction to classificatory sciences--this history ever since I first seriously set myself, at the age of thirteen, in 1852, to the study of logic,†1 shows only too grievously how great a boon would be any way [of] determining and expressing by numbers the degree of likelihood that a theory had attained--any general recognition, even among leading men of science, of the true degree of significance of a given fact, and of the proper method of determining it. I hope my writings may, at any rate, awaken a few to the enormous waste of effort it would save. But any numerical determination of likelihood is more than I can expect.

Peirce: CP 2.664 Cross-Ref:††

664. The only kind of reasoning which can render our conclusions certain--and even this kind can do so only under the proviso that no blunder has been committed in the process--attains this certainty by limiting the conclusion (as Kant virtually said, and others before him), to facts already expressed and accepted in the premisses. This is called necessary, or syllogistic reasoning. Syllogism, not confined to the kind that Aristotle and Theophrastus studied, is merely an artificial form in which it may be expressed, and it is not its best form, from any point of view. But the kind of reasoning which creates likelihoods by virtue of observations may render a likelihood *practically* certain--as certain as that a stone let loose from the clutch will, under circumstances not obviously exceptional, fall to the ground--and this conclusion may be that under a certain general condition, easily verified, a certain actuality will be *probable*, that is to say, will come to pass once in so often in the long run. One such familiar conclusion, for example, is that a die thrown from a dice box will with a *probability* of one-third, that is, once in three times in the long run, turn up a number (either *tray* or *size*) that is divisible by three. But this can be affirmed with practical certainty only if by a "long run" be meant an endless series of trials, and (as just said) infinity divided by infinity gives of itself an entirely indefinite quotient. It is therefore necessary to define the phrase. I might give the definition with reference to the probability, *p*, where *p* is any vulgar fraction, and in reference to a generic condition, *m*, and a specific kind of event *n*. But I think the reader will follow me more readily, if in place of the letter, *m* (which in itself is but a certain letter, to which is attached a peculiar meaning, that of the fulfillment of some generic condition) I put instead the supposition that a die is thrown from a dice box; and this special

supposition will be as readily understood by the reader to be replaceable by any other general condition along with a simultaneous replacement of the *event*, that a number divisible by three is turned up, and at the same time with the replacement of one third by whatever other vulgar fraction may be called for when some different example of a probability is before us. I am, then, to define the meanings of the statement that the *probability*, that if a die be thrown from a dice box it will turn up a number divisible by three, is one-third. The statement means that the die has a certain "would-be"; and to say that a die has a "would-be" is to say that it has a property, quite analogous to any *habit* that a man might have. Only the "would-be" of the die is presumably as much simpler and more definite than the man's habit as the die's homogeneous composition and cubical shape is simpler than the nature of the man's nervous system and soul; and just as it would be necessary, in order to define a man's habit, to describe how it would lead him to behave and upon what sort of occasion--albeit this statement would by no means imply that the habit *consists* in that action--so to define the die's "would-be," it is necessary to say how it would lead the die to behave on an occasion that would bring out the full consequence of the "would-be"; and this statement will not of itself imply that the "would-be" of the die *consists* in such behavior.

Peirce: CP 2.665 Cross-Ref:††

665. Now in order that the full effect of the die's "would-be" may find expression, it is necessary that the die should undergo an endless series of throws from the dice box, the result of no throw having the slightest influence upon the result of any other throw, or, as we express it, the throws must be *independent* each of every other.

Peirce: CP 2.666 Cross-Ref:††

666. It will be no objection to our considering the consequences of the supposition that the die is thrown an endless succession of times, and that with a finite pause after each throw, that such an endless series of events is impossible, for the reason that the impossibility is merely a physical, and not a logical, impossibility, as was well illustrated in that famous sporting event in which Achilles succeeded in overtaking the champion tortoise, in spite of his giving the latter the start of a whole *stadion*. For it having been ascertained, by delicate measurements between a mathematical point between the shoulder-blades of Achilles (marked [by] a limit between a red, a green, and a violet sector of a stained disk) and a similar point on the carapace of the tortoise, that when Achilles arrived where the tortoise started, the latter was just 60 feet 8 inches and 1/10 inch further on, which is just one tenth of a stadion, and that when Achilles reached that point the tortoise was still 6 feet and 8 1/100 inch in advance of him, and finally that, both advancing at a perfectly uniform rate, the tortoise had run just 67 feet 5 inches when he was overtaken by Achilles, it follows that the tortoise progressed at just one tenth the speed of Achilles, the latter running a distance in *stadia* of 1.11111111, so that he had to traverse the sum of an infinite multitude of finite distances, each in a finite time, and yet covered the *stadion* and one ninth in a finite time. No contradiction, therefore, is involved in the idea of an endless series of finite times or spaces having but a finite sum, provided there is no *fixed* finite quality which every member of an endless part of that series must each and every one *exceed*.

Peirce: CP 2.666 Cross-Ref:††

The reader must pardon me for occupying any of his time with such puerile stuff as that $0.1111 = 1/9$; for astounding as it seems, it has more than once happened

to me that men have come to me--every one of them not merely educated men, but highly accomplished--men who might well enough be famous over the civilized world, if fame were anything to the purpose, but men whose studies had been such that one would have expected to find each of them an adept in the accurate statement of arguments, and yet each has come and has undertaken to prove to me that the old catch of Achilles and the tortoise is a sound argument. If I tell you what after listening to them by the hour, I have always ended by saying--it may serve your turn on a similar occasion--I have said, "I suppose you do not mean to say that you really believe that a fast runner cannot, as a matter of fact, overtake a slow one. I therefore conclude that the argument which you have been unable to state, either syllogistically or in any other intelligible form, is intended to show that Zeno's reasoning about Achilles and the tortoise is sound according to some system of logic which admits that sound necessary reasoning may lead from true premisses to a false conclusion. But in my system of logic what I mean by bad necessary reasoning is precisely an argument which might lead from true premisses to a false conclusion--just that and nothing else. If you prefer to call such reasoning a sound necessary argument, I have no objection in the world to your doing so; and you will kindly allow me to employ my different nomenclature. For I am such a plain, uncultured soul that when I reason I aim at nothing else than just to find out the truth." To get back, then, to the die and its habit--its "would-be"--I really know no other way of defining a habit than by describing the kind of behavior in which the habit becomes actualized. So I am obliged to define the statement that there is a probability of one-third that the die when thrown will turn up either a three or a six by stating how the numbers will run when the die is thrown.

Peirce: CP 2.667 Cross-Ref:††

667. But my purpose in doing so is to explain what *probability*, as I use the word, consists in. Now it would be no explanation at all to say that it consists in something being *probable*. So I must avoid using that word or any synonym of it. If I were to use such an expression, you would very properly turn upon me and say, "I either know what it is to be *probable*, in your sense of the term, or I do not. If I don't, how can I be expected to understand you until you have explained yourself; and if I do, what is the use of the explanation?" But the fact [is] that the probability of the die turning up a three or a six is not *sure* to produce any determination [of] the run of the numbers thrown in any *finite* series of throws. It is only when the series is endless that we can be *sure* that it will have a particular character. Even when there is an endless series of throws, there is no syllogistic certainty, no "mathematical" certainty (if you are more familiar with this latter phrase)--that the die will not turn up a six obstinately at every single throw. It might be that if in the course of the endless series, some friends should borrow the die to make a pair for a game of backgammon, there might be nothing unusual in the behavior of the lent die, and yet when it was returned and our experimental series was resumed where it had been interrupted, the die might return to turning up nothing but six every time. I say it *might*, in the sense that it would not violate the principle of contradiction if it did. It sanely *would not*, however, unless a miracle were performed; and moreover if such miracle *were* worked, I should say (since it is my use of the term "probability" that we have supposed to be in question) that during this experimental series of throws, the die took on an abnormal, a miraculous, habit. For I should think that the performance of a certain line of behavior, throughout an endless succession of occasions, without exception, very decidedly *constituted* a habit. There may be some doubt about this, for owing to our not being accustomed to reason in this way about successions of events which are

endless *in the sequence* and yet are completed *in time*, it is hard for me quite to satisfy myself what I ought to say in such a case. But I have reflected seriously on it, and though I am not perfectly sure of my ground (and I am a cautious reasoner), yet I am more that what you would understand by "pretty confident," that supposing one to be in a condition to assert what *would surely be* the behavior, *in any single determinate respect*, of any subject throughout an endless series of occasions of a stated kind, he *ipso facto* knows a "would-be," or habit, of that subject. It is very true, mind you, that *no* collection whatever of single acts, though it were ever so many grades greater than a simple endless series, can constitute a would-be, nor can the knowledge of single acts, whatever their multitude, tell us for *sure* of a would-be. But there are two remarks to be made; first, that in the case under consideration a person is supposed to be in a condition to assert what surely *would be* the behavior of the subject throughout the endless series of occasions--a knowledge which cannot have been derived from reasoning from its behavior on the single occasions; and second, that that which in our case renders it true, as stated, that the person supposed "*ipso facto* knows a would-be of that subject," is not the occurrence of the single acts, but the fact that the person supposed "was in condition to assert what *would surely be* the behavior of the subject throughout an endless series of occasions."†P1

Peirce: CP 2.668 Cross-Ref:††

668. I will now describe the behavior of the die during the endless series of throws, in respect to turning up numbers divisible by three. It would be perfectly possible to construct a machine that would automatically throw the die and pick it up, and continue doing so as long as it was supplied with energy. It would further be still easier to design the plan of an arrangement whereby a hand should after each throw move over an arc graduated so as to indicate the value of the quotient of the number of throws of three or six that had been known since the beginning of the experiment, divided by the total number of throws since the beginning. It is true that the mechanical difficulties would become quite insuperable before the die had been thrown many times; but fortunately a general description of the way the hand would move will answer our purpose much better than would the actual machine, were it ever so perfect.

Peirce: CP 2.668 Cross-Ref:††

After the first throw, the hand will go either to $0 = 0/1$ or $1 = 1/1$; and there it may stay for several throws. But when it once moves, it will move after every throw, without exception, since the denominator of the fraction at whose value it points will always increase by 1, and consequently the value of the fraction will be diminished if the numerator remains unchanged, as it will be increased in case the numerator is increased by 1, these two being the only possible cases. The behavior of the hand may be described as an excessively irregular oscillation, back and forth, from one side of $1/3$ to the other. . . .

Peirce: CP 2.669 Cross-Ref:††
CHAPTER 7

THE PROBABILITY OF INDUCTION†1

§1. RULES FOR THE ADDITION AND MULTIPLICATION OF PROBABILITIES^E

669. We have found that every argument derives its force from the general truth of the class of inferences to which it belongs; and that probability is the proportion of arguments carrying truth with them among those of any *genus*. This is most conveniently expressed in the nomenclature of the medieval logicians. They called the fact expressed by a premiss an *antecedent*, and that which follows from it its *consequent*; while the leading principle, that every (or almost every) such antecedent is followed by such a consequent, they termed the *consequence*. Using this language, we may say that probability belongs exclusively to *consequences*, and the probability of any consequence is the number of times in which antecedent and consequent both occur divided by the number of all the times in which the antecedent occurs. From this definition are deduced the following rules for the addition and multiplication of probabilities:

Peirce: CP 2.670 Cross-Ref:††

670. ***Rule for the Addition of Probabilities.***--Given the separate probabilities of two consequences having the same antecedent and incompatible consequents. Then the sum of these two numbers is the probability of the consequence, that from the same antecedent one or other of those consequents follows.

Peirce: CP 2.671 Cross-Ref:††

671. ***Rule for the Multiplication of Probabilities.***--Given the separate probabilities of the two consequences, "If A then B," and "If both A and B, then C." Then the product of these two numbers is the probability of the consequence, "If A, then both B and C."

Peirce: CP 2.672 Cross-Ref:††

672. ***Special Rule for the Multiplication of Independent Probabilities.***--Given the separate probabilities of two consequences having the same antecedents, "If A, then B," and "If A, then C." Suppose that these consequences are such that the probability of the second is equal to the probability of the consequence, "If both A and B, then C." Then the product of the two given numbers is equal to the probability of the consequence, "If A, then both B and C."

Peirce: CP 2.672 Cross-Ref:††

To show the working of these rules we may examine the probabilities in regard to throwing dice. What is the probability of throwing a six with one die? The antecedent here is the event of throwing a die; the consequent, its turning up a six. As the die has six sides, all of which are turned up with equal frequency, the probability of turning up any one is $1/6$. Suppose two dice are thrown, what is the probability of throwing sixes? The probability of either coming up six is obviously the same when both are thrown as when one is thrown--namely, $1/6$. The probability that either will come up six when the other does is also the same as that of its coming up six whether the other does or not. The probabilities are, therefore, independent; and, by our rule, the probability that both events will happen together is the product of their several probabilities, or $1/6 \times 1/6$. What is the probability of throwing deuce-ace? The probability that the first die will turn up ace and the second deuce is the same as the

probability that both will turn up sixes--namely, $1/36$; the probability that the *second* will turn up ace and the *first* deuce is likewise $1/36$; these two events--first, ace; second, deuce; and, second, ace; first, deuce--are incompatible. Hence the rule for addition holds, and the probability that either will come up ace and the other deuce is $1/36 + 1/36$, or $1/18$.

Peirce: CP 2.672 Cross-Ref:††

In this way all problems about dice, etc., may be solved. When the number of dice thrown is supposed very large, mathematics (which may be defined as the art of making groups to facilitate numeration) comes to our aid with certain devices to reduce the difficulties.

Peirce: CP 2.673 Cross-Ref:††

§2. MATERIALISTIC AND CONCEPTUALISTIC VIEWS OF PROBABILITY^E

673. The conception of probability as a matter of *fact*, i.e., as the proportion of times in which an occurrence of one kind is accompanied by an occurrence of another kind, is termed by Mr. Venn the materialistic view of the subject. But probability has often been regarded as being simply the degree of belief which ought to attach to a proposition, and this mode of explaining the idea is termed by Venn the conceptualistic view. Most writers have mixed the two conceptions together. They, first, define the probability of an event as the reason we have to believe that it has taken place, which is conceptualistic; but shortly after they state that it is the ratio of the number of cases favorable to the event to the total number of cases favorable or contrary, and all equally possible. Except that this introduces the thoroughly unclear idea of cases equally possible in place of cases equally frequent, this is a tolerable statement of the materialistic view. The pure conceptualistic theory has been best expounded by Mr. De Morgan in his *Formal Logic: or, the Calculus of Inference, Necessary and Probable*.

Peirce: CP 2.674 Cross-Ref:††

674. The great difference between the two analyses is, that the conceptualists refer probability to an event, while the materialists make it the ratio of frequency of events of a *species* to those of a *genus* over that *species*, thus *giving it two terms instead of one*. The opposition may be made to appear as follows:† 1

Peirce: CP 2.674 Cross-Ref:††

Suppose that we have two rules of inference, such that, of all the questions to the solution of which both can be applied, the first yields correct answers to $81/100$, and incorrect answers to the remaining $19/100$; while the second yields correct answers to $93/100$, and incorrect answers to the remaining $7/100$. Suppose, further, that the two rules are entirely independent as to their truth, so that the second answers correctly $93/100$ of the questions which the first answers correctly, and also $93/100$ of the questions which the first answers incorrectly, and answers incorrectly the remaining $7/100$ of the questions which the first answers correctly, and also the remaining $7/100$ of the questions which the first answers incorrectly. Then, of all the questions to the solution of which both rules can be applied--

both answer correctly --- of --- or ----- ;

$$100 \quad 100 \quad 100 \times 100$$

the second answers correctly and the first incorrectly

$$93 \quad 19 \quad 93 \times 19$$

--- of ---, or ----- ;

$$100 \quad 100 \quad 100 \times 100$$

the second answers incorrectly and the first correctly

$$7 \quad 81 \quad 7 \times 81$$

--- of ---, or ----- ;

$$100 \quad 100 \quad 100 \times 100$$

and both answer incorrectly

$$7 \quad 19 \quad 7 \times 19$$

--- of ---, or ----- ;

$$100 \quad 100 \quad 100 \times 100$$

Peirce: CP 2.674 Cross-Ref:††

Suppose, now, that, in reference to any question, both give the same answer. Then (the questions being always such as are to be answered by *yes* or *no*), those in reference to which their answers agree are the same as those which both answer correctly together with those which both answer falsely, or

$$93 \times 81 \quad 7 \times 19$$

----- + ----- of all. The proportion of those which

$$100 \times 100 \quad 100 \times 100$$

both answer correctly out of those their answers to which

agree is, therefore--

$$\begin{array}{r} 93 \times 81 \\ \hline 100 \times 100 \end{array} \quad \begin{array}{r} 93 \times 81 \\ \hline 100 \times 100 \end{array} \quad \text{or} \quad \begin{array}{r} 93 \times 81 \\ \hline 100 \times 100 \end{array} + \begin{array}{r} 7 \times 19 \\ \hline 100 \times 100 \end{array} \quad \text{or} \quad \begin{array}{r} (93 \times 81) + (7 \times 19) \\ \hline 100 \times 100 \end{array}$$

Peirce: CP 2.675 Cross-Ref:††

675. This is, therefore, the probability that, if both modes of inference yield the same result, that result is correct. We may here conveniently make use of another mode of expression. **Probability** is the ratio of the favorable cases to all the cases. Instead of expressing our result in terms of this ratio, we may make use of another--the ratio of favorable to unfavorable cases. This last ratio may be called the **chance** of an event. Then the chance of a true answer by the first mode of inference is $81/19$ and by the second is $93/7$; and the chance of a correct answer from both, when they agree, is--

$$\begin{array}{r} 81 \times 93 \\ \hline 19 \times 7 \end{array} \quad \text{or} \quad \begin{array}{r} 81 \\ \hline 19 \end{array} \times \begin{array}{r} 93 \\ \hline 7 \end{array}$$

or the product of the chances of each singly yielding a true answer.

Peirce: CP 2.675 Cross-Ref:††

It will be seen that a chance is a quantity which may have any magnitude, however great. An event in whose favor there is an even chance, or $1/1$, has a probability of $1/2$. An argument having an even chance can do nothing toward reënforcing others, since according to the rule its combination with another would only multiply the chance of the latter by 1.

Peirce: CP 2.676 Cross-Ref:††

676. Probability and chance undoubtedly belong primarily to consequences, and are relative to premisses; but we may, nevertheless, speak of the chance of an event absolutely, meaning by that the chance of the combination of all arguments in reference to it which exist for us in the given state of our knowledge. Taken in this sense it is incontestable that the chance of an event has an intimate connection with the degree of our belief in it. Belief is certainly something more than a mere feeling; yet there is a feeling of believing, and this feeling does and ought to vary with the

chance of the thing believed, as deduced from all the arguments. Any quantity which varies with the chance might, therefore, it would seem, serve as a thermometer for the proper intensity of belief. Among all such quantities there is one which is peculiarly appropriate. When there is a very great chance, the feeling of belief ought to be very intense. Absolute certainty, or an infinite chance, can never be attained by mortals, and this may be represented appropriately by an infinite belief. As the chance diminishes the feeling of believing should diminish, until an even chance is reached, where it should completely vanish and not incline either toward or away from the proposition. When the chance becomes less, then a contrary belief should spring up and should increase in intensity as the chance diminishes, and as the chance almost vanishes (which it can never quite do) the contrary belief should tend toward an infinite intensity. Now, there is one quantity which, more simply than any other, fulfills these conditions; it is the *logarithm* of the chance. But there is another consideration which must, if admitted, fix us to this choice for our thermometer. It is that our belief ought to be proportional to the weight of evidence, in this sense, that two arguments which are entirely independent, neither weakening nor strengthening each other, ought, when they concur, to produce a belief equal to the sum of the intensities of belief which either would produce separately. Now, we have seen that the chances of independent concurrent arguments are to be multiplied together to get the chance of their combination, and, therefore, the quantities which best express the intensities of belief should be such that they are to be *added* when the *chances* are multiplied in order to produce the quantity which corresponds to the combined chance. Now, the logarithm is the only quantity which fulfills this condition. There is a general law of sensibility, called Fechner's psychophysical law. It is that the intensity of any sensation is proportional to the logarithm of the external force which produces it. It is entirely in harmony with this law that the feeling of belief should be as the logarithm of the chance, this latter being the expression of the state of facts which produces the belief.

Peirce: CP 2.676 Cross-Ref:††

The rule for the combination of independent concurrent arguments takes a very simple form when expressed in terms of the intensity of belief, measured in the proposed way. It is this: Take the sum of all the feelings of belief which would be produced separately by all the arguments *pro*, subtract from that the similar sum for arguments *con*, and the remainder is the feeling of belief which we ought to have on the whole. This is a proceeding which men often resort to, under the name of *balancing reasons*.

Peirce: CP 2.676 Cross-Ref:††

These considerations constitute an argument in favor of the conceptualistic view. The kernel of it is that the conjoint probability of all the arguments in our possession, with reference to any fact, must be intimately connected with the just degree of our belief in that fact; and this point is supplemented by various others showing the consistency of the theory with itself and with the rest of our knowledge.

Peirce: CP 2.677 Cross-Ref:††

677. But probability, to have any value at all, must express a fact. It is, therefore, a thing to be inferred upon evidence. Let us, then, consider for a moment the formation of a belief of probability. Suppose we have a large bag of beans from which one has been secretly taken at random and hidden under a thimble. We are now to form a probable judgment of the color of that bean, by drawing others singly from the bag and looking at them, each one to be thrown back, and the whole well mixed

up after each drawing. Suppose the first drawing is white and the next black. We conclude that there is not an immense preponderance of either color, and that there is something like an even chance that the bean under the thimble is black. But this judgment may be altered by the next few drawings. When we have drawn ten times, if 4, 5, or 6, are white, we have more confidence that the chance is even. When we have drawn a thousand times, if about half have been white, we have great confidence in this result. We now feel pretty sure that, if we were to make a large number of bets upon the color of single beans drawn from the bag, we could approximately insure ourselves in the long run by betting each time upon the white, a confidence which would be entirely wanting if, instead of sampling the bag by 1,000 drawings, we had done so by only two. Now, as the whole utility of probability is to insure us in the long run, and as that assurance depends, not merely on the value of the chance, but also on the accuracy of the evaluation, it follows that we ought not to have the same feeling of belief in reference to all events of which the chance is even. In short, to express the proper state of our belief, not *one* number but *two* are requisite, the first depending on the inferred probability, the second on the amount of knowledge on which that probability is based. †P1 It is true that when our knowledge is very precise, when we have made many drawings from the bag, or, as in most of the examples in the books, when the total contents of the bag are absolutely known, the number which expresses the uncertainty of the assumed probability and its liability to be changed by further experience may become insignificant, or utterly vanish. But, when our knowledge is very slight, this number may be even more important than the probability itself; and when we have no knowledge at all this completely overwhelms the other, so that there is no sense in saying that the chance of the totally unknown event is even (for what expresses absolutely no fact has absolutely no meaning), and what ought to be said is that the chance is entirely indefinite. We thus perceive that the conceptualistic view, though answering well enough in some cases, is quite inadequate.

Peirce: CP 2.678 Cross-Ref: ††

678. Suppose that the first bean which we drew from our bag was black. That would constitute an argument, no matter how slender, that the bean under the thimble was also black. If the second bean was also to turn out black, that would be a second independent argument reënforcing the first. If the whole of the first twenty beans drawn should prove black, our confidence that the hidden bean was black would justly attain considerable strength. But suppose the twenty-first bean was to be white and that we were to go on drawing until we found that we had drawn 1,010 black beans and 990 white ones. We should conclude that our first twenty beans being black was simply an extraordinary accident, and that in fact the proportion of white beans to black was sensibly equal, and that it was an even chance that the hidden bean was black. Yet according to the rule of *balancing reasons*, since all the drawings of black beans are so many independent arguments in favor of the one under the thimble being black, and all the white drawings so many against it, an excess of twenty black beans ought to produce the same degree of belief that the hidden bean was black, whatever the total number drawn.

Peirce: CP 2.679 Cross-Ref: ††

679. In the conceptualistic view of probability, complete ignorance, where the judgment ought not to swerve either toward or away from the hypothesis, is represented by the probability $1/2$. †P1

Peirce: CP 2.679 Cross-Ref: ††

But let us suppose that we are totally ignorant what colored hair the inhabitants of Saturn have. Let us, then, take a color-chart in which all possible colors are shown shading into one another by imperceptible degrees. In such a chart the relative areas occupied by different classes of colors are perfectly arbitrary. Let us inclose such an area with a closed line, and ask what is the chance on conceptualistic principles that the color of the hair of the inhabitants of Saturn falls within that area? The answer cannot be indeterminate because we must be in some state of belief; and, indeed, conceptualistic writers do not admit indeterminate probabilities. As there is no certainty in the matter, the answer lies between *zero* and *unity*. As no numerical value is afforded by the data, the number must be determined by the nature of the scale of probability itself, and not by calculation from the data. The answer can, therefore, only be one-half, since the judgment should neither favor nor oppose the hypothesis. What is true of this area is true of any other one; and it will equally be true of a third area which embraces the other two. But the probability for each of the smaller areas being one-half, that for the larger should be at least unity, which is absurd.

Peirce: CP 2.680 Cross-Ref:††

§3. ON THE CHANCE OF UNKNOWN EVENTSE

680. All our reasonings are of two kinds: 1. *Explicative, analytic, or deductive*; 2. *Amplificative, synthetic, or (loosely speaking) inductive*. In explicative reasoning, certain facts are first laid down in the premisses. These facts are, in every case, an inexhaustible multitude, but they may often be summed up in one simple proposition by means of some regularity which runs through them all. Thus, take the proposition that Socrates was a man; this implies (to go no further) that during every fraction of a second of his whole life (or, if you please, during the greater part of them) he was a man. He did not at one instant appear as a tree and at another as a dog; he did not flow into water, or appear in two places at once; you could not put your finger through him as if he were an optical image, etc. Now, the facts being thus laid down, some order among some of them, not particularly made use of for the purpose of stating them, may perhaps be discovered; and this will enable us to throw part or all of them into a new statement, the possibility of which might have escaped attention. Such a statement will be the conclusion of an analytic inference. Of this sort are all mathematical demonstrations. But synthetic reasoning is of another kind. In this case the facts summed up in the conclusion are not among those stated in the premisses. They are different facts, as when one sees that the tide rises *m* times and concludes that it will rise the next time. These are the only inferences which increase our real knowledge, however useful the others may be.

Peirce: CP 2.681 Cross-Ref:††

681. In any problem in probabilities, we have given the relative frequency of certain events, and we perceive that in these facts the relative frequency of another event is given in a hidden way. This being stated makes the solution. This is, therefore, mere explicative reasoning, and is evidently entirely inadequate to the representation of synthetic reasoning, which goes out beyond the facts given in the premisses. There is, therefore, a manifest impossibility in so tracing out any probability for a synthetic conclusion.

Peirce: CP 2.682 Cross-Ref:††

682. Most treatises on probability contain a very different doctrine. They state, for example, that if one of the ancient denizens of the shores of the Mediterranean, who had never heard of tides, had gone to the bay of Biscay, and had there seen the tide rise, say m times, he could know that there was a probability equal to

$$\frac{m + 1}{m + 2}$$

that it would rise the next time. In a well-known work by Quetelet,^{†1} much stress is laid on this, and it is made the foundation of a theory of inductive reasoning.

Peirce: CP 2.683 Cross-Ref:††

683. But this solution betrays its origin if we apply it to the case in which the man has never seen the tide rise at all; that is, if we put $m = 0$. In this case, the probability that it will rise the next time comes out $1/2$, or, in other words, the solution involves the conceptualistic principle that there is an even chance of a totally unknown event. The manner in which it has been reached has been by considering a number of urns all containing the same number of balls, part white and part black. One urn contains all white balls, another one black and the rest white, a third two black and the rest white, and so on, one urn for each proportion, until an urn is reached containing only black balls. But the only possible reason for drawing any analogy between such an arrangement and that of Nature is the principle that alternatives of which we know nothing must be considered as equally probable. But this principle is absurd. There is an indefinite variety of ways of enumerating the different possibilities, which, on the application of this principle, would give different results. If there be any way of enumerating the possibilities so as to make them all equal, it is not that from which this solution is derived, but is the following: Suppose we had an immense granary filled with black and white balls well mixed up; and suppose each urn were filled by taking a fixed number of balls from this granary quite at random. The relative number of white balls in the granary might be anything, say one in three. Then in one-third of the urns the first ball would be white, and in two-thirds black. In one-third of those urns of which the first ball was white, and also in one-third of those in which the first ball was black, the second ball would be white. In this way, we should have a distribution like that shown in the following table, where w stands for a white ball and b for a black one. The reader can, if he chooses, verify the table for himself.

wwww.

wwwb. wwbw. wbww. bwww.

wwwb. wwbw. wbww. bwww.

wbwb. bwwb. wbbw. bwbw. bbww.
wbwb. bwwb. wbbw. bwbw. bbww.
wbwb. bwwb. wbbw. bwbw. bbww.
wbwb. bwwb. wbbw. bwbw. bbww.

wbbb. bwbb. bbwb. bbbw.
wbbb. bwbb. bbwb. bbbw.

wbbb. In the second group, where there is one b, there are
wbbb. two sets just alike; in the third there are 4, in the
wbbb. fourth 8, and in the fifth 16, doubling every time.
wbbb. This is because we have supposed twice as many
wbbb. black balls in the granary as white ones; had we
wbbb. supposed 10 times as many, instead of
wbbb.

wbbb. 1, 2, 4, 8, 16

wbbb.

wbbb. sets we should have had

wbbb.

wbbb. 1, 10, 100, 1000, 10000

wbbb.

wbbb. sets; on the other hand, had the numbers of black and
wbbb. white balls in the granary been even, there would have
wbbb. been but one set in each group. Now suppose two balls
were drawn from one of these urns and were found to be both white, what would be

the probability of the next one being white? If the two drawn out were the first two put into the urns, and the next to be drawn out were the third put in, then the probability of this third being white would be the same whatever the colors of the first two, for it has been supposed that just the same proportion of urns has the third ball white among those which have the first two *white-white*, *white-black*, *black-white*, and *black-black*. Thus, in this case, the chance of the third ball being white would be the same whatever the first two were. But, by inspecting the table, the reader can see that in each group all orders of the balls occur with equal frequency, so that it makes no difference whether they are drawn out in the order they were put in or not. Hence the colors of the balls already drawn have no influence on the probability of any other being white or black.

Peirce: CP 2.684 Cross-Ref:††

684. Now, if there be any way of enumerating the possibilities of Nature so as to make them equally probable, it is clearly one which should make one arrangement or combination of the elements of Nature as probable as another, that is, a distribution like that we have supposed, and it, therefore, appears that the assumption that any such thing can be done, leads simply to the conclusion that reasoning from past to future experience is absolutely worthless.†1 In fact, the moment that you assume that the chances in favor of that of which we are totally ignorant are even, the problem about the tides does not differ, in any arithmetical particular, from the case in which a penny (known to be equally likely to come up heads or tails) should turn up heads *m* times successively. In short, it would be to assume that Nature is a pure chaos, or chance combination of independent elements, in which reasoning from one fact to another would be impossible; and since, as we shall hereafter see,†2 there is no judgment of pure observation without reasoning, it would be to suppose all human cognition illusory and no real knowledge possible. It would be to suppose that if we have found the order of Nature more or less regular in the past, this has been by a pure run of luck which we may expect is now at an end. Now, it may be we have no scintilla of proof to the contrary, but reason is unnecessary in reference to that belief which is of all the most settled, which nobody doubts or can doubt, and which he who should deny would stultify himself in so doing.

Peirce: CP 2.684 Cross-Ref:††

The relative probability of this or that arrangement of Nature is something which we should have a right to talk about if universes were as plenty as blackberries, if we could put a quantity of them in a bag, shake them well up, draw out a sample, and examine them to see what proportion of them had one arrangement and what proportion another. But, even in that case, a higher universe would contain us, in regard to whose arrangements the conception of probability could have no applicability.

Peirce: CP 2.685 Cross-Ref:††

§4. ON THE PROBABILITY OF SYNTHETIC INFERENCESE

685. We have examined the problem proposed by the conceptualists, which, translated into clear language, is this: Given a synthetic conclusion; required to know out of all possible states of things how many will accord, to any assigned extent, with this conclusion; and we have found that it is only an absurd attempt to reduce

synthetic to analytic reason, and that no definite solution is possible.

Peirce: CP 2.686 Cross-Ref:††

686. But there is another problem in connection with this subject. It is this: Given a certain state of things, required to know what proportion of all synthetic inferences relating to it will be true within a given degree of approximation. Now, there is no difficulty about this problem (except for its mathematical complication); it has been much studied, and the answer is perfectly well known. And is not this, after all, what we want to know much rather than the other? Why should we want to know the probability that the fact will accord with our conclusion? That implies that we are interested in all possible worlds, and not merely the one in which we find ourselves placed. Why is it not much more to the purpose to know the probability that our conclusion will accord with the fact? One of these questions is the first above stated and the other the second, and I ask the reader whether, if people, instead of using the word probability without any clear apprehension of their own meaning, had always spoken of relative frequency, they could have failed to see that what they wanted was not to follow along the synthetic procedure with an analytic one, in order to find the probability of the conclusion; but, on the contrary, to begin with the fact at which the synthetic inference aims, and follow back to the facts it uses for premisses in order to see the probability of their being such as will yield the truth.

Peirce: CP 2.687 Cross-Ref:††

687. As we cannot have an urn with an infinite number of balls to represent the inexhaustibleness of Nature, let us suppose one with a finite number, each ball being thrown back into the urn after being drawn out, so that there is no exhaustion of them. Suppose one ball out of three is white and the rest black, and that four balls are drawn. Then the table in 683 represents the relative frequency of the different ways in which these balls might be drawn. It will be seen that if we should judge by these four balls of the proportion in the urn, 32 times out of 81 we should find it $1/4$, and 24 times out of 81 we should find it $1/2$, the truth being $1/3$. To extend this table to high numbers would be great labor, but the mathematicians have found some ingenious ways of reckoning what the numbers would be. It is found that, if the true proportion of white balls is p , and s balls are drawn, then the error of the proportion obtained by the induction will be--

half the time within	$0.477\sqrt{2p(1-p)}$

	s
9 times out of 10 within	$1.163\sqrt{2p(1-p)}$

	s
99 times out of 100 within	$1.821\sqrt{2p(1-p)}$

s

999 times out of 1,000 within $2.328\sqrt{2p(1-p)}$

s

9,999 times out of 10,000 within $2.751\sqrt{2p(1-p)}$

s

9,999,999,999 times out of 10,000,000,000 within $4.77\sqrt{2p(1-p)}$

s

Peirce: CP 2.687 Cross-Ref:††

The use of this may be illustrated by an example. By the census of 1870, it appears that the proportion of males among native white children under one year old was 0.5082, while among colored children of the same age the proportion was only 0.4977. The difference between these is 0.0105, or about one in 100. Can this be attributed to chance, or would the difference always exist among a great number of white and colored children under like circumstances? Here p may be taken at $1/2$; hence $2p(1-p)$ is also $1/2$. The number of white children counted was near 1,000,000; hence the fraction whose square-root is to be taken is about $1/2000000$. The root is about $1/1400$, and this multiplied by 0.477 gives about 0.0003 as the probable error in the ratio of males among the whites as obtained from the induction. The number of black children was about 150,000, which gives 0.0008 for the probable error. We see that the actual discrepancy is ten times the sum of these, and such a result would happen, according to our table, only once out of 10,000,000,000 censuses, in the long run.

Peirce: CP 2.688 Cross-Ref:††

688. It may be remarked that when the real value of the probability sought inductively is either very large or very small, the reasoning is more secure. Thus, suppose there were in reality one white ball in 100 in a certain urn, and we were to judge of the number by 100 drawings. The probability of drawing no white ball would be $366/1000$; that of drawing one white ball would be $370/1000$; that of drawing two would be $185/1000$; that of drawing three would be $61/1000$; that of drawing four would be $15/1000$; that of drawing five would be only $3/1000$, etc. Thus we should be tolerably certain of not being in error by more than one ball in 100.

Peirce: CP 2.689 Cross-Ref:††

689. It appears, then, that in one sense we can, and in another we cannot, determine the probability of synthetic inference. When I reason in this way:

Ninety-nine Cretans in a hundred are liars,

But Epimenides is a Cretan;

Therefore, Epimenides is a liar;

I know that reasoning similar to that would carry truth 99 times in 100. But when I reason in the opposite direction:

Minos, Sarpedon, Rhadamanthus, Deucalion, and Epimenides, are all the Cretans I can think of,

But these were all atrocious liars;

Therefore, pretty much all Cretans must have been liars;

I do not in the least know how often such reasoning would carry me right. On the other hand, what I do know is that some definite proportion of Cretans must have been liars, and that this proportion can be probably approximated to by an induction from five or six instances. Even in the worst case for the probability of such an inference, that in which about half the Cretans are liars, the ratio so obtained would probably not be in error by more than 1/6. So much I know; but, then, in the present case the inference is that pretty much all Cretans are liars, and whether there may not be a special improbability in that I do not know.

Peirce: CP 2.690 Cross-Ref:††

§5. THE RATIONALE OF SYNTHETIC INFERENCE E

690. Late in the last century, Immanuel Kant asked the question, "How are synthetical judgments *a priori* possible?" By synthetical judgments he meant such as assert positive fact and are not mere affairs of arrangement; in short, judgments of the kind which synthetical reasoning produces, and which analytic reasoning cannot yield. By *a priori* judgments he meant such as that all outward objects are in space, every event has a cause, etc., propositions which according to him can never be inferred from experience. Not so much by his answer to this question as by the mere asking of it, the current philosophy of that time was shattered and destroyed, and a new epoch in its history was begun. But before asking *that* question he ought to have asked the more general one, "How are any synthetical judgments at all possible?" How is it that a man can observe one fact and straightway pronounce judgment concerning another different fact not involved in the first? Such reasoning, as we have

seen, has, at least in the usual sense of the phrase, no definite probability; how, then, can it add to our knowledge? This is a strange paradox; the Abbe Gratry says it is a miracle, and that every true induction is an immediate inspiration from on high.†P1 I respect this explanation far more than many a pedantic attempt to solve the question by some juggle with probabilities, with the forms of syllogism, or what not. I respect it because it shows an appreciation of the depth of the problem, because it assigns an adequate cause, and because it is intimately connected--as the true account should be--with a general philosophy of the universe. At the same time, I do not accept this explanation, because an explanation should tell *how* a thing is done, and to assert a perpetual miracle seems to be an abandonment of all hope of doing that, without sufficient justification.

Peirce: CP 2.691 Cross-Ref:††

691. It will be interesting to see how the answer which Kant gave to his question about synthetical judgments *a priori* will appear if extended to the question of synthetical judgments in general. That answer is, that synthetical judgments *a priori* are possible because whatever is universally true is involved in the conditions of experience. Let us apply this to a general synthetical reasoning. I take from a bag a handful of beans; they are all purple, and I infer that all the beans in the bag are purple. How can I do that? Why, upon the principle that whatever is universally true of my experience (which is here the appearance of these different beans) is involved in the condition of experience. The condition of this special experience is that all these beans were taken from that bag. According to Kant's principle, then, whatever is found true of all the beans drawn from the bag must find its explanation in some peculiarity of the contents of the bag. This is a satisfactory statement of the principle of induction.

Peirce: CP 2.692 Cross-Ref:††

692. When we draw a deductive or analytic conclusion, our rule of inference is that facts of a certain general character are either invariably or in a certain proportion of cases accompanied by facts of another general character. Then our premiss being a fact of the former class, we infer with certainty or with the appropriate degree of probability the existence of a fact of the second class. But the rule for synthetic inference is of a different kind. When we sample a bag of beans we do not in the least assume that the fact of some beans being purple involves the necessity or even the probability of other beans being so. On the contrary, the conceptualistic method of treating probabilities, which really amounts simply to the deductive treatment of them, when rightly carried out leads to the result that a synthetic inference has just an even chance in its favor, or in other words is absolutely worthless. The color of one bean is entirely independent of that of another. But synthetic inference is founded upon a classification of facts, not according to their characters, but according to the manner of obtaining them. Its rule is, that a number of facts obtained in a given way will in general more or less resemble other facts obtained in the same way; or, *experiences whose conditions are the same will have the same general characters.*

Peirce: CP 2.693 Cross-Ref:††

693. In the former case, we know that premisses precisely similar in form to those of the given ones will yield true conclusions, just once in a calculable number of times. In the latter case, we only know that premisses obtained under circumstances similar to the given ones (though perhaps themselves very different) will yield true conclusions, at least once in a calculable number of times. We may express this by

saying that in the case of analytic inference we know the probability of our conclusion (if the premisses are true), but in the case of synthetic inferences we only know the degree of trustworthiness of our proceeding. As all knowledge comes from synthetic inference, we must equally infer that all human certainty consists merely in our knowing that the processes by which our knowledge has been derived are such as must generally have led to true conclusions.

Peirce: CP 2.693 Cross-Ref:††

Though a synthetic inference cannot by any means be reduced to deduction, yet that the rule of induction will hold good in the long run may be deduced from the principle that reality is only the object of the final opinion to which sufficient investigation would lead. That belief gradually tends to fix itself under the influence of inquiry is, indeed, one of the facts with which logic sets out.

Peirce: CP 2.694 Cross-Ref:††
CHAPTER 8

A THEORY OF PROBABLE INFERENCE†1

§1. PROBABLE DEDUCTION AND PROBABILITY IN GENERAL †2

694. The following is an example of the simplest kind of probable inference:

About two per cent of persons wounded in the liver recover,

This man has been wounded in the liver;

Therefore, there are two chances out of a hundred that he will recover.

Peirce: CP 2.694 Cross-Ref:††

Compare this with the simplest of syllogisms, say the following:

Every man dies,

Enoch was a man;

Hence, Enoch must have died.

Peirce: CP 2.694 Cross-Ref:††

The latter argument consists in the application of a general rule to a particular

case. The former applies to a particular case a rule not absolutely universal, but subject to a known proportion of exceptions. Both may alike be termed deductions, because they bring information about the uniform or usual course of things to bear upon the solution of special questions; and the probable argument may approximate indefinitely to demonstration as the ratio named in the first premiss approaches to unity or to zero.

Peirce: CP 2.695 Cross-Ref:††

695. Let us set forth the general formulæ of the two kinds of inference in the manner of formal logic.

FORM I.

Singular Syllogism in Barbara.

Every *M* is a *P*,

S is an *M*;

Hence, *S* is a *P*.

FORM II.

Simple Probable Deduction.

The proportion {*r*} of the *M*'s are *P*'s;

S is an *M*;

It follows, with probability {*r*}, that *S* is a *P*.

Peirce: CP 2.695 Cross-Ref:††

It is to be observed that the ratio {*r*} need not be exactly specified. We may reason from the premiss that not more than two per cent of persons wounded in the liver recover, or from "not less than a certain proportion of the *M*'s are *P*'s," or from "no very large nor very small proportion," etc. In short, {*r*} is subject to every kind of indeterminacy; it simply excludes some ratios and admits the possibility of the rest.

Peirce: CP 2.696 Cross-Ref:††

696. The analogy between syllogism and what is here called probable deduction is certainly genuine and important; yet how wide the differences between the two modes of inference are, will appear from the following considerations:

Peirce: CP 2.696 Cross-Ref:††

(1) The logic of probability is related to ordinary syllogistic as the quantitative to the qualitative branch of the same science. Necessary syllogism recognizes only the inclusion or non-inclusion of one class under another; but probable inference takes

account of the proportion of one class which is contained under a second. It is like the distinction between projective geometry, which asks whether points coincide or not, and metric geometry, which determines their distances.

Peirce: CP 2.696 Cross-Ref:††

(2) For the existence of ordinary syllogism, all that is requisite is that we should be able to say, in some sense, that one term is contained in another, or that one object stands to a second in one of those relations: "better than," "equivalent to," etc., which are termed *transitive* because if *A* is in any such relation to *B*, and *B* is in the same relation to *C*, then *A* is in that relation to *C*. The universe might be all so fluid and variable that nothing should preserve its individual identity, and that no measurement should be conceivable; and still one portion might remain inclosed within a second, itself inclosed within a third, so that a syllogism would be possible. But probable inference could not be made in such a universe, because no signification would attach to the words "quantitative ratio." For that there must be counting; and consequently units must exist, preserving their identity and variously grouped together.

Peirce: CP 2.696 Cross-Ref:††

(3) A cardinal distinction between the two kinds of inference is, that in demonstrative reasoning the conclusion follows from the existence of the objective facts laid down in the premisses; while in probable reasoning these facts in themselves do not even render the conclusion probable, but account has to be taken of various subjective circumstances--of the manner in which the premisses have been obtained, of there being no countervailing considerations, etc.; in short, good faith and honesty are essential to good logic in probable reasoning.

Peirce: CP 2.696 Cross-Ref:††

When the partial rule that the proportion $\{r\}$ of the *M*'s are *P*'s is applied to show with probability $\{r\}$ that *S* is a *P*, it is requisite, not merely that *S* should be an *M*, but also that it should be an instance drawn *at random* from among the *M*'s. Thus, there being four aces in a piquet pack of thirty-two cards, the chance is one-eighth that a given card not looked at is an ace; but this is only on the supposition that the card has been drawn at random from the whole pack. If, for instance, it had been drawn from the cards discarded by the players at piquet or euchre, the probability would be quite different. The instance must be drawn at random. Here is a maxim of conduct. The volition of the reasoner (using what machinery it may) has to choose *S* so that it shall be an *M*; but he ought to restrain himself from all further preference, and not allow his will to act in any way that might tend to settle what particular *M* is taken, but should leave that to the operation of chance. Willing and wishing, like other operations of the mind, are *general* and imperfectly determinate. I wish for a horse--for some particular kind of horse perhaps, but not usually for any individual one. I will to act in a way of which I have a general conception; but so long as my action conforms to that general description, how it is further determined I do not care. Now in choosing the instance *S*, the general intention (including the whole plan of action) should be to select an *M*, but beyond that there should be no preference; and the act of choice should be such that if it were repeated many enough times with the same intention, the result would be that among the totality of selections the different sorts of *M*'s would occur with the same relative frequencies as in experiences in which volition does not intermeddle at all. In cases in which it is found difficult thus to restrain the will by a direct effort, the apparatus of games of chance--a lottery-wheel, a roulette, cards, or dice--may be called to our aid. Usually, however,

in making a simple probable deduction, we take that instance in which we happen at the time to be interested. In such a case, it is our interest that fulfills the function of an apparatus for random selection; and no better need be desired, so long as we have reason to deem the premiss "the proportion $\{r\}$ of the *M*'s are *P*'s" to be equally true in regard to that part of the *M*'s which are alone likely ever to excite our interest.

Peirce: CP 2.696 Cross-Ref:††

Nor is it a matter of indifference in what manner the other premiss has been obtained. A card being drawn at random from a piquet pack, the chance is one-eighth that it is an ace, if we have no other knowledge of it. But after we have looked at the card, we can no longer reason in that way. That the conclusion must be drawn in advance of any other knowledge on the subject is a rule that, however elementary, will be found in the sequel to have great importance.

Peirce: CP 2.696 Cross-Ref:††

(4) The conclusions of the two modes of inference likewise differ. One is necessary; the other only probable. Locke, in the *Essay Concerning Human Understanding*, hints at the correct analysis of the nature of probability. After remarking that the mathematician positively knows that the sum of the three angles of a triangle is equal to two right angles because he apprehends the geometrical proof, he then continues:†1 "But another man who never took the pains to observe the demonstration, hearing a mathematician, a man of credit, affirm the three angles of a triangle to be equal to two right ones, *assents* to it, that is, receives it for true. In which case, the foundation of his assent is the probability of the thing, the proof being such as, for the most part, carries truth with it; the man on whose testimony he receives it not being wont to affirm anything contrary to or beside his knowledge, especially in matters of this kind." Those who know Locke are accustomed to look for more meaning in his words than appears at first glance. There is an allusion in this passage to the fact that a probable argument is always regarded as belonging to a *genus* of arguments. This is, in fact, true of any kind of argument. For the belief expressed by the conclusion is determined or caused by the belief expressed by the premisses. There is, therefore, some general rule according to which the one succeeds the other. But, further, the reasoner is conscious of there being such a rule, for otherwise he would not know he was reasoning, and could exercise no attention or control; and to such an involuntary operation the name "reasoning" is very properly not applied. In all cases, then, we are conscious that our inference belongs to a general class of logical forms, although we are not necessarily able to describe the general class. The difference between necessary and probable reasoning is that in the one case we conceive that such facts as are expressed by the premisses are never, in the whole range of possibility, true, without another fact, related to them as our conclusion is to our premisses, being true likewise; while in the other case we merely conceive that, in reasoning as we do, we are following a general maxim that will usually lead us to the truth.

Peirce: CP 2.697 Cross-Ref:††

697. So long as there are exceptions to the rule that all men wounded in the liver die, it does not necessarily follow that because a given man is wounded in the liver he cannot recover. Still, we know that if we were to reason in that way, we should be following a mode of inference which would only lead us wrong, in the long run, once in fifty times; and this is what we mean when we say that the probability is one out of fifty that the man will recover. To say, then, that a proposition has the probability $\{r\}$ means that to infer it to be true would be to follow an argument such

as would carry truth with it in the ratio of frequency $\{r\}$.

Peirce: CP 2.697 Cross-Ref:††

It is plainly useful that we should have a stronger feeling of confidence about a sort of inference which will oftener lead us to the truth than about an inference that will less often prove right--and such a sensation we do have. The celebrated law of Fechner is that as the force acting upon an organ of sense increases in geometrical progression, the intensity of the sensation increases in arithmetical progression. In this case the odds (that is, the ratio of the chances in favor of a conclusion to the chances against it) take the place of the exciting cause, while the sensation itself is the feeling of confidence. When two arguments tend to the same conclusion, our confidence in the latter is equal to the sum of what the two arguments separately would produce; the *odds* are the product of the *odds* in favor of the two arguments separately. When the value of the *odds* reduces to unity, our confidence is null; when the *odds* are less than unity, we have more or less confidence in the negative of the conclusion.

Peirce: CP 2.698 Cross-Ref:††
§2. STATISTICAL DEDUCTION

698. The principle of probable deduction still applies when S , instead of being a single M , is a set of M 's-- n in number. The reasoning then takes the following form:

FORM III.

Complex Probable Deduction.

Among all sets of n M 's, the proportion q consist each of m P 's and of $n-m$ not- P 's,

$S, S', S'',$ etc.; form a set of n objects drawn at random from among the M 's;

Hence, the probability is q that among $S, S', S'',$ etc. there are m P 's and $n-m$ not- P 's.

Peirce: CP 2.698 Cross-Ref:††

In saying that $S, S', S'',$ etc., form a set drawn at random, we here mean that not only are the different individuals drawn at random, but also that they are so drawn that the qualities which may belong to one have no influence upon the selection of any other. In other words, the individual drawings are independent, and the set as a whole is taken at random from among all possible sets of n M 's. In strictness, this supposes that the same individual may be drawn several times in the same set, although if the number of M 's is large compared with n , it makes no appreciable difference whether this is the case or not.

Peirce: CP 2.699 Cross-Ref:††

699. The following formula expresses the proportion, among all sets of n M 's, of those which consist of m P 's and $n-m$ not- P 's. The letter r denotes the proportion of P 's among the M 's, and the sign of admiration is used to express the continued product of all integer numbers from 1 to the number after which it is placed. Thus, $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$, etc. The formula is:

$$q = \frac{n! \times r^m (1-r)^{n-m}}{m! (n-m)!}$$

Peirce: CP 2.699 Cross-Ref:††

As an example, let us assume the proportion $r = 2/3$ and the number of M 's in a set $n = 15$. Then the values of the probability q for different numbers, m , of P 's, are fractions having for their common denominator 14,348,907, and for their numerators as follows:

m Numerator of q .	m Numerator of q .
--- -----	--- -----
0 1	8 1667360
1 30	9 2562560
2 420	10 3075072
3 3640	11 2795520
4 21840	12 1863680
5 96096	13 860160
6 320320	14 122880
7 823680	15 32768

Peirce: CP 2.699 Cross-Ref:††

A very little mathematics would suffice to show that, r and n being fixed, q always reaches its maximum value with that value of m that is next less than $(n + 1)r$,†P1 and that q is very small unless m has nearly this value.

Peirce: CP 2.700 Cross-Ref:††

700. Upon these facts is based another form of inference to which I give the name of statistical deduction. Its general formula is as follows:

FORM IV

Statistical Deduction.

The proportion r of the M 's are P 's,
 S ', S'' , S''' , etc. are a *numerous* set, taken at random from among the M 's;
Hence, *probably* and *approximately*, the proportion r of the S 's are P 's.

As an example, take this:

A little more than half of all human births are males;
Hence, probably a little over half of all the births in New York during any one year are males.

Peirce: CP 2.700 Cross-Ref:††

We have now no longer to deal with a mere probable inference, but with a *probable approximate* inference. This conception is a somewhat complicated one, meaning that the probability is greater according as the limits of approximation are wider, conformably to the mathematical expression for the values of q .

Peirce: CP 2.701 Cross-Ref:††

701. This conclusion has no meaning at all unless there be more than one instance; and it has hardly any meaning unless the instances are somewhat numerous. When this is the case, there is a more convenient way of obtaining (not exactly, but quite near enough for all practical purposes) either a single value of q or the sum of successive values from $m = m[1]$ to $m = m[2]$ inclusive. The rule is first to calculate two quantities which may conveniently be called $t[1]$ and $t[2]$ according to these formulæ:

$$t[1] = \frac{m[1] - (n + 1)r}{\sqrt{2nr(1 - r)}} \quad t[2] = \frac{1 + m[2] - (n + 1)r}{\sqrt{2nr(1 - r)}}$$

where $m[2] > m[1]$. Either or both the quantities $t[1]$ and $t[2]$ may be negative. Next with each of these quantities enter the table below, and take out $1/2\Theta t[1]$ and $1/2\Theta t[2]$ and give each the same sign as the t from which it is derived. Then

$$\Sigma q = 1/2 \Theta t[2] - 1/2 \Theta t[1].$$

$$\text{Table of } \Theta t = \frac{2}{\sqrt{\Theta}} \int_0^t \Theta^{-t} dt.$$

t	Θt
0.0	0.000
0.1	0.112
0.2	0.223
0.3	0.329
0.4	0.428
0.5	0.520
0.6	0.604
0.7	0.678
0.8	0.742
0.9	0.797
1.0	0.843

t	Θt
1.0	0.843
1.1	0.880
1.2	0.910
1.3	0.934
1.4	0.952
1.5	0.966
1.6	0.976
1.7	0.984
1.8	0.989
1.9	0.993
2.0	0.995

t	Θt
2.0	0.99532
2.1	0.99702
2.2	0.99814
2.3	0.99886
2.4	0.99931
2.5	0.99959
2.6	0.99976
2.7	0.99987
2.8	0.99992
2.9	0.99996
3.0	0.99998

t	Θ
4	0.999999989
5	0.9999999999984
6	0.9999999999999982
7	0.9999999999999999958

[Click here to view]

Peirce: CP 2.701 Cross-Ref:††

In rough calculations we may take Θt equal to t for t less than 0.7, and as equal to *unity* for any value above $t = 1.4$.

Peirce: CP 2.702 Cross-Ref:††

§3. INDUCTION †1

702. The principle of statistical deduction is that these †2 two proportions--namely, that of the *P*'s among the *M*'s, and that of the *P*'s among the *S*'s--are probably and approximately equal. If, then, this principle justifies our inferring the value of the second proportion from the known value of the first, it equally justifies our inferring the value of the first from that of the second, if the first is unknown but the second has been observed. We thus obtain the following form of inference:

FORM V

Induction.

S', S'', S''' , etc. form a numerous set taken at random from among the M 's,
 S', S'', S''' , etc. are found to be--the proportion $\{r\}$ of them-- P 's;

Hence, *probably* and *approximately* the same proportion, $\{r\}$, of the M 's are P 's.

Peirce: CP 2.702 Cross-Ref:††

The following are examples. From a bag of coffee a handful is taken out, and found to have nine-tenths of the beans perfect; whence it is inferred that about nine-tenths of all the beans in the bag are probably perfect. The United States Census of 1870 shows that of native white children under one year old, there were 478,774 males to 463,320 females; while of colored children of the same age there were 75,985 males to 76,637 females. We infer that generally there is a larger proportion of female births among negroes than among whites.

Peirce: CP 2.703 Cross-Ref:††

703. When the ratio $\{r\}$ is *unity* or *zero*, the inference is an ordinary induction; and I ask leave to extend the term "induction" to all such inference, whatever be the value of $\{r\}$. It is, in fact, inferring from a sample to the whole lot sampled. These two forms of inference, statistical deduction and induction, plainly depend upon the same principle of equality of ratios, so that their validity is the same. Yet the nature of the probability in the two cases is very different. In the statistical deduction, we know that among the whole body of M 's the proportion of P 's is $\{r\}$; we say, then, that the S 's being random drawings of M 's are probably P 's in about the same proportion--and though this may happen not to be so, yet at any rate, on continuing the drawing sufficiently, our prediction of the ratio will be vindicated at last. On the other hand, in induction we say that the proportion $\{r\}$ of the sample being P 's, probably there is about the same proportion in the whole lot; or at least, if this happens not to be so, then on continuing the drawings the inference will be, not *vindicated* as in the other case, but *modified* so as to become true. The deduction, then, is probable in this sense, that though its conclusion may in a particular case be falsified, yet similar conclusions (with the same ratio $\{r\}$) would generally prove approximately true; while the induction is probable in this sense, that though it may happen to give a false conclusion, yet in most cases in which the same precept of inference was followed, a different and approximately true inference (with the right value of $\{r\}$) would be drawn.

Peirce: CP 2.704 Cross-Ref:††
§4. HYPOTHETIC INFERENCE

704. Before going any further with the study of Form V, I wish to join to it another extremely analogous form.

Peirce: CP 2.704 Cross-Ref:††

We often speak of one thing being very much like another, and thus apply a vague quantity to resemblance. Even if qualities are not subject to exact numeration, we may conceive them to be approximately measurable. We may then measure

resemblance by a scale of numbers from zero up to unity. To say that S has a 1-likeness to a P will mean that it has every character of a P , and consequently *is* a P . To say that it has a 0-likeness will imply total dissimilarity. We shall then be able to reason as follows:

FORM II (*bis*).

Simple probable deduction in depth.

Every M has the simple mark P ,

The S 's have an r -likeness to the M 's;

Hence, the probability is r that every S is P .

Peirce: CP 2.704 Cross-Ref:††

It would be difficult, perhaps impossible, to adduce an example of such kind of inference, for the reason that *simple marks* are not known to us. We may, however, illustrate the complex probable deduction in depth (the general form of which it is not worth while to set down) as follows: I forget whether, in the ritualistic churches, a bell is tinkled at the elevation of the Host or not. Knowing, however, that the services resemble somewhat decidedly those of the Roman Mass, I think that it is not unlikely that the bell is used in the ritualistic, as in the Roman, churches.

Peirce: CP 2.705 Cross-Ref:††

705. We shall also have the following:

FORM IV (*bis*).

Statistical deduction in depth.

Every M has, for example, the numerous marks P' , P'' , P''' , etc.,

S has an r -likeness to the M 's;

Hence, probably and approximately, S has the proportion r of the marks P' , P'' , P''' , etc.

Peirce: CP 2.705 Cross-Ref:††

For example, we know that the French and Italians are a good deal alike in their ideas, characters, temperaments, genius, customs, institutions, etc., while they also differ very markedly in all these respects. Suppose, then, that I know a boy who

is going to make a short trip through France and Italy; I can safely predict that among the really numerous though relatively few respects in which he will be able to compare the two people, about the same degree of resemblance will be found.

Peirce: CP 2.705 Cross-Ref:††

Both these modes of inference are clearly deductive. When $r = 1$, they reduce to Barbara.†P1

Peirce: CP 2.706 Cross-Ref:††

706. Corresponding to induction, we have the following mode of inference:

FORM V (*bis*).

Hypothesis.

M has, for example, the numerous marks P', P'', P''' , etc.,

S has the proportion r of the marks P', P'', P''' , etc.;

Hence, probably and approximately, S has an r -likeness to M .

Peirce: CP 2.706 Cross-Ref:††

Thus, we know, that the ancient Mound-builders of North America present, in all those respects in which we have been able to make the comparison, a limited degree of resemblance with the Pueblo Indians. The inference is, then, that in all respects there is about the same degree of resemblance between these races.

Peirce: CP 2.706 Cross-Ref:††

If I am permitted the extended sense which I have given to the word "induction," this argument is simply an induction respecting qualities instead of respecting things. In point of fact P', P'', P''' , etc., constitute a random sample of the characters of M , and the ratio r of them being found to belong to S , the same ratio of all the characters of M are concluded to belong to S . This kind of argument, however, as it actually occurs, differs very much from induction, owing to the impossibility of simply counting qualities as individual things are counted. Characters have to be weighed rather than counted. Thus, antimony is bluish-gray: that is a character. Bismuth is a sort of rose-gray; it is decidedly different from antimony in color, and yet not so very different as gold, silver, copper, and tin are.

Peirce: CP 2.707 Cross-Ref:††

707. I call this induction of characters *hypothetic inference*,†1 or, briefly, *hypothesis*. This is perhaps not a very happy designation, yet it is difficult to find a better. The term "hypothesis" has many well established and distinct meanings. Among these is that of a proposition believed in because its consequences agree with experience. This is the sense in which Newton used the word when he said, *Hypotheses non fingo*. He meant that he was merely giving a general formula for the motions of the heavenly bodies, but was not undertaking to mount to the causes of the acceleration they exhibit. The inferences of Kepler, on the other hand, were

hypotheses in this sense; for he traced out the miscellaneous consequences of the supposition that Mars moved in an ellipse, with the sun at the focus, and showed that both the longitudes and the latitudes resulting from this theory were such as agreed with observation. These two components of the motion were observed; the third, that of approach to or regression from the earth, was supposed. Now, if in Form V (*bis*) we put $r = 1$, the inference is the drawing of a hypothesis in this sense. I take the liberty of extending the use of the word by permitting r to have any value from zero to unity. The term is certainly not all that could be desired; for the word hypothesis, as ordinarily used, carries with it a suggestion of uncertainty, and of something to be superseded, which does not belong at all to my use of it. But we must use existing language as best we may, balancing the reasons for and against any mode of expression, for none is perfect; at least the term is not so utterly misleading as "analogy" would be, and with proper explanation it will, I hope, be understood.

Peirce: CP 2.708 Cross-Ref:††

§5. GENERAL CHARACTERS OF DEDUCTION, INDUCTION, AND HYPOTHESIS

708. The following examples will illustrate the distinction between statistical deduction, induction, and hypothesis. If I wished to order a font of type expressly for the printing of this book, knowing, as I do, that in all English writing the letter *e* occurs oftener than any other letter, I should want more *e*'s in my font than other letters. For what is true of all other English writing is no doubt true of these papers. This is a statistical deduction. But then the words used in logical writings are rather peculiar, and a good deal of use is made of single letters. I might, then, count the number of occurrences of the different letters upon a dozen or so pages of the manuscript, and thence conclude the relative amounts of the different kinds of type required in the font. That would be inductive inference. If now I were to order the font, and if, after some days, I were to receive a box containing a large number of little paper parcels of very different sizes, I should naturally infer that this was the font of types I had ordered; and this would be hypothetic inference. Again, if a dispatch in cipher is captured, and it is found to be written with twenty-six characters, one of which occurs much more frequently than any of the others, we are at once led to suppose that each character represents a letter, and that the one occurring so frequently stands for *e*. This is also hypothetic inference.

Peirce: CP 2.709 Cross-Ref:††

709. We are thus led to divide all probable reasoning into deductive and ampliative, and further to divide ampliative reasoning into induction and hypothesis. In deductive reasoning, though the predicted ratio may be wrong in a limited number of drawings, yet it will be approximately verified in a larger number. In ampliative reasoning the ratio may be wrong, because the inference is based on but a limited number of instances; but on enlarging the sample the ratio will be changed till it becomes approximately correct. In induction, the instances drawn at random are numerable things; in hypothesis they are characters, which are not capable of strict enumeration, but have to be otherwise estimated.

Peirce: CP 2.710 Cross-Ref:††

710. This classification of probable inference is connected with a preference

for the copula of inclusion over those used by Miss Ladd [Mrs. Christine Ladd-Franklin] and by Mr. Mitchell.†P1 De Morgan established eight forms of simple propositions; and from a purely formal point of view no one of these has a right to be considered as more fundamental than any other. But formal logic must not be too purely formal; it must represent a fact of psychology, or else it is in danger of degenerating into a mathematical recreation. The categorical proposition, "every man is mortal," is but a modification of the hypothetical proposition, "if humanity, then mortality"; and since the very first conception from which logic springs is that one proposition follows from another, I hold that "if *A*, then *B*" should be taken as the typical form of judgment. Time flows; and, in time, from one state of belief (represented by the premisses of an argument) another (represented by its conclusion) is developed. Logic arises from this circumstance, without which we could not learn anything nor correct any opinion. To say that an inference is correct is to say that if the premisses are true the conclusion is also true; or that every possible state of things in which the premisses should be true would be included among the possible states of things in which the conclusion would be true. We are thus led to the copula of inclusion. But the main characteristic of the relation of inclusion is that it is transitive--that is, that what is included in something included in anything is itself included in that thing; or, that if *A* is *B* and *B* is *C*, then *A* is *C*. We thus get *Barbara* as the primitive type of inference. Now in *Barbara* we have a *Rule*, a *Case* under the *Rule*, and the inference of the *Result* of that rule in that case. For example:

Rule. All men are mortal,

Case. Enoch was a man;

Result. ∴ Enoch was mortal.

Peirce: CP 2.711 Cross-Ref:††

711. The cognition of a rule is not necessarily conscious, but is of the nature of a habit, acquired or congenital. The cognition of a case is of the general nature of a sensation; that is to say, it is something which comes up into present consciousness. The cognition of a result is of the nature of a decision to act in a particular way on a given occasion.†P1 In point of fact, a syllogism in *Barbara* virtually takes place when we irritate the foot of a decapitated frog. The connection between the afferent and efferent nerve, whatever it may be, constitutes a nervous habit, a rule of action, which is the physiological analogue of the major premiss. The disturbance of the ganglionic equilibrium, owing to the irritation, is the physiological form of that which, psychologically considered, is a sensation; and, logically considered, is the occurrence of a case. The explosion through the efferent nerve is the physiological form of that which psychologically is a volition, and logically the inference of a result. When we pass from the lowest to the highest forms of innervation, the physiological equivalents escape our observation; but, psychologically, we still have, first, habit--which in its highest form is understanding, and which corresponds to the major premiss of *Barbara*; we have, second, feeling, or present consciousness, corresponding to the minor premiss of *Barbara*; and we have, third, volition, corresponding to the conclusion of the same mode of syllogism. Although these analogies, like all very broad generalizations, may seem very fanciful at first sight, yet the more the reader reflects upon them the more profoundly true I am confident they

will appear. They give a significance to the ancient system of formal logic which no other can at all share.

Peirce: CP 2.712 Cross-Ref:††

712. Deduction proceeds from Rule and Case to Result; it is the formula of Volition. Induction proceeds from Case and Result to Rule; it is the formula of the formation of a habit or general conception--a process which, psychologically as well as logically, depends on the repetition of instances or sensations. Hypothesis proceeds from Rule and Result to Case; it is the formula of the acquirement of secondary sensation--a process by which a confused concatenation of predicates is brought into order under a synthetizing predicate.†1

Peirce: CP 2.713 Cross-Ref:††

713. We usually conceive Nature to be perpetually making deductions in *Barbara*. This is our natural and anthropomorphic metaphysics. We conceive that there are Laws of Nature, which are her Rules or major premisses. We conceive that Cases arise under these laws; these cases consist in the predication, or occurrence, of *causes*, which are the middle terms of the syllogisms. And, finally, we conceive that the occurrence of these causes, by virtue of the laws of Nature, results in effects which are the conclusions of the syllogisms. Conceiving of nature in this way, we naturally conceive of science as having three tasks--(1) the discovery of Laws, which is accomplished by induction; (2) the discovery of Causes, which is accomplished by hypothetic inference; and (3) the predictio of Effects, which is accomplished by deduction. It appears to me to be highly useful to select a system of logic which shall preserve all these natural conceptions.

Peirce: CP 2.714 Cross-Ref:††

714. It may be added that, generally speaking, the conclusions of Hypothetic Inference cannot be arrived at inductively, because their truth is not susceptible of direct observation in single cases. Nor can the conclusions of Inductions, on account of their generality, be reached by hypothetic inference. For instance, any historical fact, as that Napoleon Bonaparte once lived, is a hypothesis; we believe the fact, because its effects--I mean current tradition, the histories, the monuments, etc.--are observed. But no mere generalization of observed facts could ever teach us that Napoleon lived. So we inductively infer that every particle of matter gravitates toward every other. Hypothesis might lead to this result for any given pair of particles, but it never could show that the law was universal.

Peirce: CP 2.715 Cross-Ref:††
§6. INDUCTION AND HYPOTHESIS

INDIRECT STATISTICAL INFERENCES; GENERAL RULE FOR THEIR VALIDITY

715. We now come to the consideration of the Rules which have to be followed in order to make valid and strong Inductions and Hypotheses. These rules can all be reduced to a single one; namely, that the statistical deduction of which the Induction or Hypothesis is the inversion, must be valid and strong.

Peirce: CP 2.716 Cross-Ref:††

716. We have seen that Inductions and Hypotheses are inferences from the conclusion and one premiss of a statistical syllogism to the other premiss. In the case of hypothesis, this syllogism is called the *explanation*. Thus in one of the examples used above, we suppose the cryptograph to be an English cipher, because, as we say, this *explains* the observed phenomena that there are about two dozen characters, that one occurs more frequently than the rest, especially at the end of words, etc. The explanation is--

Simple English ciphers have certain peculiarities,

This is a simple English cipher;

Hence, this necessarily has these peculiarities.

Peirce: CP 2.717 Cross-Ref:††

717. This explanation is present to the mind of the reasoner, too; so much so, that we commonly say that the hypothesis is adopted *for the sake of* the explanation. Of induction we do not, in ordinary language, say that it explains phenomena; still, the statistical deduction, of which it is the inversion, plays, in a general way, the same part as the explanation in hypothesis. From a barrel of apples, that I am thinking of buying, I draw out three or four as a sample. If I find the sample somewhat decayed, I ask myself, in ordinary language, not "Why is this?" but "How is this?" And I answer that it probably comes from nearly all the apples in the barrel being in bad condition. The distinction between the "Why" of hypothesis and the "How" of induction is not very great; both ask for a statistical syllogism, of which the observed fact shall be the conclusion, the known conditions of the observation one premiss, and the inductive or hypothetic inference the other. This statistical syllogism may be conveniently termed the explanatory syllogism.

Peirce: CP 2.718 Cross-Ref:††

718. In order that an induction or hypothesis should have any validity at all, it is requisite that the explanatory syllogism should be a valid statistical deduction. Its conclusion must not merely follow from the premisses, but follow from them upon the principle of probability. The inversion of *ordinary* syllogism does not give rise to an induction or hypothesis. The statistical syllogism of Form IV is invertible, because it proceeds upon the principle of an approximate *equality* between the ratio of *P*'s in the whole class and the ratio in a well-drawn sample, and because equality is a convertible relation. But ordinary syllogism is based upon the property of the relation of containing and contained, and that is not a convertible relation. There is, however, a way in which ordinary syllogism may be inverted; namely, the conclusion and either of the premisses may be interchanged by negating each of them. This is the way in which the indirect, or apagogical, †P1 figures of syllogism are derived from the first, and in which the *modus tollens* is derived from the *modus ponens*. The following schemes show this:

First Figure.

Rule. All M is P ,

Case. S is M ;

Result. S is P .

Second Figure. | **Third Figure.**

|

Rule. All M is P , | **Denial of Result.** S is not P ,

Denial of Result. S is not P ; | **Case.** S is M ;

Denial of Case. $\therefore S$ is not M . | **Denial of Rule.** \therefore Some M is
not P .

Modus Ponens.

Rule. If A is true, C is true,

Case. In a certain case A is true;

Result. \therefore In that case C is true.

Modus Tollens. | **Modus Innominatus.**

|

Rule. If A is true, C is true, | **Case.** In a certain case A is true, **Denial of Result.** In a
certain | **Denial of Result.** In that case,

case C is not true; | C is not true;

Denial of Case. \therefore In that | **Denial of Rule.** \therefore If A is true, case A is not true. |
 C is not necessarily true.

Peirce: CP 2.719 Cross-Ref:††

719. Now suppose we ask ourselves what would be the result of thus
apagogically inverting a statistical deduction. Let us take, for example, Form IV:

The S 's are a numerous random sample of the M 's,

The proportion r of the M 's are P 's;

Hence, probably about the proportion r of the S 's are P 's.

Peirce: CP 2.720 Cross-Ref:††

720. The ratio r , as we have already noticed, is not necessarily perfectly definite; it may be only known to have a certain maximum or minimum; in fact, it may have any kind of indeterminacy. Of all possible values between 0 and 1, it admits of some and excludes others. The logical negative of the ratio r is, therefore, itself a ratio, which we may name $\{r\}$; it admits of every value which r excludes, and excludes every value of which r admits. Transposing, then, the major premiss and conclusion of our statistical deduction, and at the same time denying both, we obtain the following inverted form:

The S 's are a numerous random sample of the M 's,

The proportion $\{r\}$ of the S 's are P 's;

Hence, probably about the proportion $\{r\}$ of the M 's are P 's. †P1

Peirce: CP 2.721 Cross-Ref:††

721. But this coincides with the formula of Induction. Again, let us apagogically invert the statistical deduction of Form IV (*bis*). This form is--

Every M has, for example, the numerous marks P', P'', P''' , etc.,

S has an $\{r\}$ -likeness to the M 's;

Hence, probably and approximately, S has the proportion r of the marks P', P'', P''' , etc.

Peirce: CP 2.721 Cross-Ref:††

Transposing the minor premiss and conclusion, at the same time denying both, we get the inverted form--

Every M has, for example, the numerous marks P', P'', P''' , etc.,

S has the proportion $\{r\}$ of the marks P', P'', P''' , etc.;

Hence, probably and approximately, S has a $\{r\}$ -likeness to the class of M 's.

Peirce: CP 2.722 Cross-Ref:††

722. This coincides with the formula of Hypothesis. Thus we see that Induction and Hypothesis are nothing but the apagogical inversions of statistical deductions. Accordingly, when r is taken as 1, so that $\{r\}$ is "less than 1," or when r is

taken as 0, so that $\{r\}$ is "more than 0," the induction degenerates into a syllogism of the third figure and the hypothesis into a syllogism of the second figure. In these special cases, there is no very essential difference between the mode of reasoning in the direct and in the apagogical form. But, in general, while the probability of the two forms is precisely the same--in this sense, that for any fixed proportion of *P*'s among the *M*'s (or of marks of *S*'s among the marks of the *M*'s) the probability of any given error in the concluded value is precisely the same in the indirect as it is in the direct form--yet there is this striking difference, that a multiplication of instances will in the one case *confirm*, and in the other *modify*, the concluded value of the ratio.

Peirce: CP 2.723 Cross-Ref:††

723. We are thus led to another form for our rule of validity of ampliative inference; namely, instead of saying that the *explanatory* syllogism must be a good probable deduction, we may say that the syllogism of which the induction or hypothesis is the apagogical modification (in the traditional language of logic, the *reduction*) must be valid.

Peirce: CP 2.724 Cross-Ref:††

724. Probable inferences, though valid, may still differ in their strength. A probable deduction has a greater or less probable error in the concluded ratio. When *r* is a definite number the probable error is also definite; but as a general rule we can only assign maximum and minimum values of the probable error. The probable error is, in fact--

$$0.477\sqrt{(2r(1-r))/n}$$

where *n* is the number of independent instances. The same formula gives the probable error of an induction or hypothesis; only that in these cases, *r* being wholly indeterminate, the minimum value is *zero*, and the maximum is obtained by putting *r* = 1/2.

Peirce: CP 2.725 Cross-Ref:††

§7. FIRST SPECIAL RULE FOR SYNTHETIC INFERENCE.

SAMPLING MUST BE FAIR. ANALOGY

725. Although the rule given above really contains all the conditions to which Inductions and Hypotheses need to conform, yet inasmuch as there are many delicate questions in regard to the application of it, and particularly since it is of that nature that a violation of it, if not too gross, may not absolutely destroy the virtue of the

reasoning, a somewhat detailed study of its requirements in regard to each of the premisses of the argument is still needed.

Peirce: CP 2.726 Cross-Ref:††

726. The first premiss of a scientific inference is that certain things (in the case of induction) or certain characters (in the case of hypothesis) constitute a fairly chosen *sample* of the class of things or the run of characters from which they have been drawn.

Peirce: CP 2.726 Cross-Ref:††

The rule requires that the sample should be drawn at random and independently from the whole lot sampled. That is to say, the sample must be taken according to a precept or method which, being applied over and over again indefinitely, would in the long run result in the drawing of any one set of instances as often as any other set of the same number.

Peirce: CP 2.727 Cross-Ref:††

727. The needfulness of this rule is obvious; the difficulty is to know how we are to carry it out. The usual method is mentally to run over the lot of objects or characters to be sampled, abstracting our attention from their peculiarities, and arresting ourselves at this one or that one from motives wholly unconnected with those peculiarities. But this abstention from a further determination of our choice often demands an effort of the will that is beyond our strength; and in that case a mechanical contrivance may be called to our aid. We may, for example, number all the objects of the lot, and then draw numbers by means of a roulette, or other such instrument. We may even go so far as to say that this method is the type of all random drawing; for when we abstract our attention from the peculiarities of objects, the psychologists tell us that what we do is to substitute for the images of sense certain mental signs, and when we proceed to a random and arbitrary choice among these abstract objects we are governed by fortuitous determinations of the nervous system, which in this case serves the purpose of a roulette.

Peirce: CP 2.727 Cross-Ref:††

The drawing of objects at random is an act in which honesty is called for; and it is often hard enough to be sure that we have dealt honestly with ourselves in the matter, and still more hard to be satisfied of the honesty of another. Accordingly, one method of sampling has come to be preferred in argumentation; namely, to take of the class to be sampled all the objects of which we have a sufficient knowledge. Sampling is, however, a real art, well deserving an extended study by itself: to enlarge upon it here would lead us aside from our main purpose.

Peirce: CP 2.728 Cross-Ref:††

728. Let us rather ask what will be the effect upon inductive inference of an imperfection in the strictly random character of the sampling. Suppose that, instead of using such a precept of selection that any one *M* would in the long run be chosen as often as any other, we used a precept which would give a preference to a certain half of the *M*'s, so that they would be drawn twice as often as the rest. If we were to draw a numerous sample by such a precept, and if we were to find that the proportion {*r*} of the sample consisted of *P*'s, the inference that we should be regularly entitled to make would be, that among all the *M*'s, counting the preferred half for two each, the proportion {*r*} would be *P*'s. But this regular inductive inference being granted, from it we could deduce by arithmetic the further conclusion that, counting the *M*'s for one each, the proportion of *P*'s among them must ({*r*} being over 2/3) lie between 3/4{*r*}

+ 1/4, and $3/2\{r\} - 1/2$. Hence, if more than two thirds of the instances drawn by the use of the false precept were found to be *P*'s, we should be entitled to conclude that more than half of all the *M*'s were *P*'s. Thus, without allowing ourselves to be led away into a mathematical discussion, we can easily see that, in general, an imperfection of that kind in the random character of the sampling will only weaken the inductive conclusion, and render the concluded ratio less determinate, but will not necessarily destroy the force of the argument completely. In particular, when *p* approximates towards 1 or 0, the effect of the imperfect sampling will be but slight.

Peirce: CP 2.729 Cross-Ref:††

729. Nor must we lose sight of the constant tendency of the inductive process to correct itself. This is of its essence. This is the marvel of it. The probability of its conclusion only consists in the fact that if the true value of the ratio sought has not been reached, an extension of the inductive process will lead to a closer approximation. Thus, even though doubts may be entertained whether one selection of instances is a random one, yet a different selection, made by a different method, will be likely to vary from the normal in a different way, and if the ratios derived from such different selections are nearly equal, they may be presumed to be near the truth. This consideration makes it extremely advantageous in all ampliative reasoning to fortify one method of investigation by another.†P1 Still we must not allow ourselves to trust so much to this virtue of induction as to relax our efforts towards making our drawings of instances as random and independent as we can. For if we infer a ratio from a number of different inductions, the magnitude of its probable error will depend very much more on the worst than on the best inductions used.

Peirce: CP 2.730 Cross-Ref:††

730. We have, thus far, supposed that although the selection of instances is not exactly regular, yet the precept followed is such that every unit of the lot would eventually get drawn. But very often it is impracticable so to draw our instances, for the reason that a part of the lot to be sampled is absolutely inaccessible to our powers of observation. If we want to know whether it will be profitable to open a mine, we sample the ore; but in advance of our mining operations, we can obtain only what ore lies near the surface. Then, simple induction becomes worthless, and another method must be resorted to. Suppose we wish to make an induction regarding a series of events extending from the distant past to the distant future; only those events of the series which occur within the period of time over which available history extends can be taken as instances. Within this period we may find that the events of the class in question present some uniform character; yet how do we know but this uniformity was suddenly established a little while before the history commenced, or will suddenly break up a little while after it terminates? Now, whether the uniformity observed consists (1) in a mere resemblance between all the phenomena, or (2) in their consisting of a disorderly mixture of two kinds in a certain constant proportion, or (3) in the character of the events being a mathematical function of the time of occurrence--in any of these cases we can make use of an apogee from the following probable deduction:

Within the period of time *M*, a certain event *P* occurs,

S is a period of time taken at random from *M*, and more than half as long;

Hence, probably the event *P* will occur within the time *S*.

Peirce: CP 2.730 Cross-Ref:††

Inverting this deduction, we have the following ampliative inference:

S is a period of time taken at random from M , and more than half as long,

The event P does not happen in the time S ;

Hence, probably the event P does not happen in the period M .

Peirce: CP 2.730 Cross-Ref:††

The probability of the conclusion consists in this, that we here follow a precept of inference, which, if it is very often applied will more than half the time lead us right. Analogous reasoning would obviously apply to any portion of an unidimensional continuum, which might be similar to periods of time. This is a sort of logic which is often applied by physicists in what is called *extrapolation* of an empirical law. As compared with a typical induction, it is obviously an excessively weak kind of inference. Although indispensable in almost every branch of science, it can lead to no solid conclusions in regard to what is remote from the field of direct perception, unless it be bolstered up in certain ways to which we shall have occasion to refer further on.

Peirce: CP 2.731 Cross-Ref:††

731. Let us now consider another class of difficulties in regard to the rule that the samples must be drawn at random and independently. In the first place, what if the lot to be sampled be infinite in number? In what sense could a random sample be taken from a lot like that? A random sample is one taken according to a method that would, in the long run, draw any one object as often as any other. In what sense can such drawing be made from an infinite class? The answer is not far to seek. Conceive a cardboard disk revolving in its own plane about its centre, and pretty accurately balanced, so that when put into rotation it shall be about $\dagger P1$ as likely to come to rest in any one position as in any other; and let a fixed pointer indicate a position on the disk: the number of points on the circumference is infinite, and on rotating the disk repeatedly the pointer enables us to make a selection from this infinite number. This means merely that although the points are innumerable, yet there is a certain order among them that enables us to run them through and pick from them as from a very numerous collection. In such a case, and in no other, can an infinite lot be sampled. But it would be equally true to say that a finite lot can be sampled only on condition that it can be regarded as equivalent to an infinite lot. For the random sampling of a finite class supposes the possibility of drawing out an object, throwing it back, and continuing this process indefinitely; so that what is really sampled is not the finite collection of things, but the unlimited number of possible drawings.

Peirce: CP 2.732 Cross-Ref:††

732. But though there is thus no insuperable difficulty in sampling an infinite lot, yet it must be remembered that the conclusion of inductive reasoning only consists in the approximate evaluation of a *ratio*, so that it never can authorize us to conclude that in an infinite lot sampled there exists no single exception to a rule. Although all the planets are found to gravitate toward one another, this affords not the

slightest direct reason for denying that among the innumerable orbs of heaven there may be some which exert no such force. Although at no point of space where we have yet been have we found any possibility of motion in a fourth dimension, yet this does not tend to show (by simple induction, at least) that space has absolutely but three dimensions. Although all the bodies we have had the opportunity of examining appear to obey the law of inertia, this does not prove that atoms and atomicules are subject to the same law. Such conclusions must be reached, if at all, in some other way than by simple induction. This latter may show that it is unlikely that, in my lifetime or yours, things so extraordinary should be found, but [does] not warrant extending the prediction into the indefinite future. And experience shows it is not safe to predict that such and such a fact will *never* be met with.

Peirce: CP 2.733 Cross-Ref:††

733. If the different instances of the lot sampled are to be drawn independently, as the rule requires, then the fact that an instance has been drawn once must not prevent its being drawn again. It is true that if the objects remaining unchosen are very much more numerous than those selected, it makes practically no difference whether they have a chance of being drawn again or not, since that chance is in any case very small. Probability is wholly an affair of approximate, not at all of exact, measurement; so that when the class sampled is very large, there is no need of considering whether objects can be drawn more than once or not. But in what is known as "reasoning from analogy," the class sampled is small, and no instance is taken twice. For example: we know that of the major planets the Earth, Mars, Jupiter, and Saturn revolve on their axes, and we conclude that the remaining four, Mercury, Venus, Uranus, and Neptune, probably do the like. This is essentially different from an inference from what has been found in drawings made hitherto, to what will be found in indefinitely numerous drawings to be made hereafter. Our premisses here are that the Earth, Mars, Jupiter, and Saturn are a random sample of a natural class of major planets--a class which, though (so far as we know) it is very small, yet may be very extensive, comprising whatever there may be that revolves in a circular orbit around a great sun, is nearly spherical, shines with reflected light, is very large, etc. Now the examples of major planets that we can examine all rotate on their axes; whence we suppose that Mercury, Venus, Uranus, and Neptune, since they possess, so far as we know, all the properties common to the natural class to which the Earth, Mars, Jupiter, and Saturn belong, possess this property likewise. The points to be observed are, first, that any small class of things may be regarded as a mere sample of an actual or possible large class having the same properties and subject to the same conditions; second, that while we do not know what all these properties and conditions are, we do know some of them, which some may be considered as a random sample of all; third, that a random selection without replacement from a small class may be regarded as a true random selection from that infinite class of which the finite class is a random selection. The formula of the analogical inference presents, therefore, three premisses, thus:

S', S'', S''' , are a random sample of some undefined class X , of whose characters P', P'', P''' , are samples,

Q is P', P'', P''' ;

S', S'', S''' , are R 's;

Hence, Q is an R .

Peirce: CP 2.733 Cross-Ref:††

We have evidently here an induction and an hypothesis followed by a deduction; thus:

Every X is, for example, P' , | S' , S'' , S''' , etc., are samples

P'' , P''' , etc., | of the X 's,

Q is found to be P' , P'' , P''' , | S' , S'' , S''' , etc., are found

etc.; | to be R 's;

Hence, hypothetically, Q is | Hence, inductively, every X

an X . | is an R .

Hence, deductively, Q is an R .†P1

Peirce: CP 2.734 Cross-Ref:††

734. An argument from analogy may be strengthened by the addition of instance after instance to the premisses, until it loses its ampliative character by the exhaustion of the class and becomes a mere deduction of that kind called *complete induction*, in which, however, some shadow of the inductive character remains, as this name implies.

Peirce: CP 2.735 Cross-Ref:††

§8. SECOND SPECIAL RULE FOR SYNTHETIC INFERENCE, THAT OF PREDESIGNATION

735. Take any human being, at random--say Queen Elizabeth. Now a little more than half of all the human beings who have ever existed have been males; but it does not follow that it is a little more likely than not that Queen Elizabeth was a male, since we know she was a woman. Nor, if we had selected Julius Cæsar, would it be only a little more likely than not that he was a male. It is true that if we were to go on drawing at random an indefinite number of instances of human beings, a slight excess over one-half would be males. But that which constitutes the probability of an inference is the proportion of true conclusions among all those which could be derived *from the same precept*. Now a precept of inference, being a rule which the mind is to follow, changes its character and becomes different when the case presented to the mind is essentially different. When, knowing that the proportion r of all M 's are P 's, I draw an instance, S , of an M , without any other knowledge of whether it is a P or not, and infer with probability, r , that it is P , the case presented to

my mind is very different from what it is if I have such other knowledge. In short, I cannot make a valid probable inference without taking into account whatever knowledge I have (or, at least, whatever occurs to my mind) that bears upon the question.

Peirce: CP 2.736 Cross-Ref:††

736. The same principle may be applied to the statistical deduction of Form IV. If the major premiss, that the proportion r of the M 's are P 's be laid down first, before the instances of M 's are drawn, we really draw our inference concerning those instances (that the proportion r of them will be P 's) in advance of the drawing, and therefore before we know whether they are P 's or not. But if we draw the instances of the M 's first, and after the examination of them decide what we will select for the predicate of our major premiss, the inference will generally be completely fallacious. In short, we have the rule that the major term P must be decided upon in advance of the examination of the sample; and in like manner in Form IV (*bis*) the minor term S must be decided upon in advance of the drawing.

Peirce: CP 2.737 Cross-Ref:††

737. The same rule follows us into the logic of induction and hypothesis. If in sampling any class, say the M 's, we first decide what the character P is for which we propose to sample that class, and also how many instances we propose to draw, our inference is really made before these latter are drawn, that the proportion of P 's in the whole class is probably about the same as among the instances that are to be drawn, and the only thing we have to do is to draw them and observe the ratio. But suppose we were to draw our inferences without the predesignation of the character P ; then we might in every case find some recondite character in which those instances would all agree. That, by the exercise of sufficient ingenuity, we should be sure to be able to do this, even if not a single other object of the class M possessed that character, is a matter of demonstration. For in geometry a curve may be drawn through any given series of points, without passing through any one of another given series of points, and this irrespective of the number of dimensions. Now, all the qualities of objects may be conceived to result from variations of a number of continuous variables; hence any lot of objects possesses some character in common, not possessed by any other. It is true that if the universe of quality is limited, this is not altogether true; but it remains true that unless we have some special premiss from which to infer the contrary, it always *may* be possible to assign some common character of the instances S' , S'' , S''' , etc., drawn at random from among the M 's, which does not belong to the M 's generally. So that if the character P were not predesignate, the deduction of which our induction is the apagogical inversion would not be valid; that is to say, we could not reason that if the M 's did not generally possess the character P , it would not be likely that the S 's should all possess this character.

Peirce: CP 2.738 Cross-Ref:††

738. I take from a biographical dictionary †1 the first five names of poets, with their ages at death. They are,

Aagard, died at 48.

Abeille, died at 76.

Abulola, died at 84.

Abunowas, died at 48.

Accords, died at 45.

These five ages have the following characters in common:

1. The difference of the two digits composing the number, divided by three, leaves a remainder of *one*.
2. The first digit raised to the power indicated by the second, and then divided by three, leaves a remainder of *one*.
3. The sum of the prime factors of each age, including *one* as a prime factor, is divisible by *three*.

Peirce: CP 2.738 Cross-Ref:††

Yet there is not the smallest reason to believe that the next poet's age would possess these characters.

Peirce: CP 2.738 Cross-Ref:††

Here we have a *conditio sine qua non* of valid induction which has been singularly overlooked by those who have treated of the logic of the subject, and is very frequently violated by those who draw inductions. So accomplished a reasoner as Dr. Lyon Playfair, for instance, has written a paper of which the following is an abstract. He first takes the specific gravities of the three allotropic forms of carbon, as follows:

Diamond, 3.48.

Graphite, 2.29.

Charcoal, 1.88.

He now seeks to find a uniformity connecting these three instances; and he discovers that the atomic weight of carbon, being 12,

$$\text{Sp. gr. diamond nearly} = 3.46 = \sqrt[2]{12}$$

$$\text{Sp. gr. graphite nearly} = 2.29 = \sqrt[3]{12}$$

$$\text{Sp. gr. charcoal nearly} = 1.86 = \sqrt[4]{12}$$

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$$\text{Sp. gr. diamond nearly} = 3.46 = \sqrt[2]{12}$$

$$\text{Sp. gr. graphite nearly} = 2.29 = \sqrt[3]{12}$$

$$\text{Sp. gr. charcoal nearly} = 1.86 = \sqrt[4]{12}$$

This, he thinks, renders it probable that the specific gravities of the allotropic forms of other elements would, if we knew them, be found to equal the different roots of their atomic weight. But so far, the character in which the instances agree not having been predesignated, the induction can serve only to suggest a question, and ought not to create any belief. To test the proposed law, he selects the instance of silicon, which like carbon exists in a diamond and in a graphitoidal condition. He finds for the specific gravities--

Diamond silicon, 2.47

Graphite silicon, 2.33.†P1

Peirce: CP 2.738 Cross-Ref:††

Now, the atomic weight of silicon, that of carbon being 12, can only be taken as 28. But 2.47 does not approximate to any root of 28. It is, however, nearly the cube

root of 14, $(\sqrt[3]{\frac{1}{2} \times 28} = 2.41)$, [Click here to view]([3\sqrt{1/2 \times 28} = 2.41](#)), while 2.33 is nearly the fourth root of 28

$(\sqrt[4]{28} = 2.30)$. [Click here to view]([4\sqrt{28} = 2.30](#)). Dr.

Playfair claims that silicon is an instance satisfying his formula. But in fact this instance requires the formula to be modified; and the modification not being predesignate, the instance cannot count. Boron also exists in a diamond and a graphitoidal form; and accordingly Dr. Playfair takes this as his next example. Its atomic weight is 10.9, and its specific gravity is 2.68; which is the square root of $2/3 \times 10.9$. There seems to be here a further modification of the formula not predesignated, and therefore this instance can hardly be reckoned as confirmatory. The next instances which would occur to the mind of any chemist would be phosphorus and sulphur, which exist in familiarly known allotropic forms. Dr. Playfair admits that the specific gravities of phosphorus have no relations to its atomic weight at all analogous to those of carbon. The different forms of sulphur have nearly the same specific gravity, being approximately the fifth root of the atomic weight 32. Selenium also has two allotropic forms, whose specific gravities are 4.8 and 4.3; one of these follows the law, while the other does not. For tellurium the law fails altogether; but for bromine and iodine it holds. Thus the number of specific gravities for which the law was predesignate are 8; namely, 2 for phosphorus, 1 for sulphur, 2 for selenium, 1 for tellurium, 1 for bromine, and 1 for iodine. The law holds for 4 of these, and the proper inference is that about half the specific gravities of metalloids are roots of some

simple ratio of their atomic weights.

Peirce: CP 2.738 Cross-Ref:††

Having thus determined this ratio, we proceed to inquire whether an agreement half the time with the formula constitutes any special connection between the specific gravity and the atomic weight of a metalloid. As a test of this, let us arrange the elements in the order of their atomic weights, and compare the specific gravity of the first with the atomic weight of the last, that of the second with the atomic weight of the last but one, and so on. The atomic weights are--

Boron,	10.9	Tellurium,	128.1
Carbon,	12.0	Iodine,	126.9
Silicon,	28.0	Bromine,	80.0
Phosphorus,	31.0	Selenium,	79.1
		Sulphur,	32.

There are three specific gravities given for carbon, and two each for silicon, phosphorus, and selenium. The question, therefore, is, whether of the fourteen specific gravities as many as seven are in Playfair's relation with the atomic weights, not of the same element, but of the one paired with it. Now, taking the original formula of Playfair we find

	Sp. gr. boron	= 2.68	$\sqrt[5]{\text{Te}} = 2.64$
3 ^d	Sp. gr. carbon	= 1.88	$\sqrt[5]{\text{I}} = 1.84$
2 ^d	Sp. gr. carbon	= 2.29	$\sqrt[6]{\text{I}} = 2.24$
1 st	Sp. gr. phosphorus	= 1.83	$\sqrt[7]{\text{Se}} = 1.87$
2 ^d	Sp. gr. phosphorus	= 2.10	$\sqrt[6]{\text{Se}} = 2.07$

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$$\text{Sp. gr. boron} = 2.68 \quad \sqrt[5]{\text{Te}} = 2.64$$

$$3^{\text{d}} \text{ Sp. gr. carbon} = 1.88 \quad \sqrt[5]{\text{I}} = 1.84$$

$$2^{\text{d}} \text{ Sp. gr. carbon} = 2.29 \quad \sqrt[6]{\text{I}} = 2.24$$

$$1^{\text{st}} \text{ Sp. gr. phosphorus} = 1.83 \quad \sqrt[7]{\text{Se}} = 1.87$$

$$2^{\text{d}} \text{ Sp. gr. phosphorus} = 2.10 \quad \sqrt[6]{\text{Se}} = 2.07$$

or five such relations without counting that of sulphur to itself. Next, with the modification introduced by Playfair, we have

1 st Sp. gr. silicon = 2.47	$\sqrt[4]{\frac{1}{2} \times \text{Br}} = 2.51$
2 ^d Sp. gr. silicon = 2.33	$\sqrt[6]{2 \times \text{Br}} = 2.33$
Sp. gr. iodine = 4.95	$\sqrt[3]{2 \times \text{C}} = 4.90$
1 st Sp. gr. carbon = 3.48	$\sqrt[3]{\frac{1}{3} \times \text{I}} = 3.48$

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$$\text{Sp. gr. silicon} = 2.47 \quad \sqrt[4]{1/2 \times \text{Br}} = 2.51$$

$$2^{\text{d}} \text{ Sp. gr. silicon} = 2.33 \quad \sqrt[6]{2 \times \text{Br}} = 2.33$$

$$\text{Sp. gr. iodine} = 4.95 \quad \sqrt[3]{2 \times \text{C}} = 4.90$$

$$1^{\text{st}} \text{ Sp. gr. carbon} = 3.48 \quad \sqrt[3]{1/3 \times \text{I}} = 3.48$$

It thus appears that there is no more frequent agreement with Playfair's proposed law than what is due to chance. †P1

Peirce: CP 2.739 Cross-Ref: ††

739. Another example of this fallacy was "Bode's law" of the relative distances of the planets, which was shattered by the first discovery of a true planet after its enunciation. In fact, this false kind of induction is extremely common in science and in medicine. †P1 In the case of hypothesis, the correct rule has often been laid down; namely, that a hypothesis can only be received upon the ground of its having been *verified* by successful *prediction*. The term *predesignation* used in this paper appears to be more exact, inasmuch as it is not at all requisite that the ratio {r} should be given in advance of the examination of the samples. Still, since {r} is equal to 1 in all ordinary hypotheses, there can be no doubt that the rule of prediction, so far as it goes, coincides with that here laid down.

Peirce: CP 2.740 Cross-Ref: ††

740. We have now to consider an important modification of the rule. Suppose that, before sampling a class of objects, we have predesignated not a single character but *n* characters, for which we propose to examine the samples. This is equivalent to making *n* different inductions from the same instances. The probable error in this case is that error whose probability for a simple induction is only $(1/2)^n$, and the theory of probabilities shows that it increases but slowly with *n*; in fact, for *n* = 1000 it is only about five times as great as for *n* = 1, so that with only 25 times as many instances the inference would be as secure for the former value of *n* as with the latter; with 100 times as many instances an induction in which *n* = 10,000,000,000 would be equally secure. Now the whole universe of characters will never contain such a number as the last; and the same may be said of the universe of objects in the case of hypothesis. So that, without any voluntary predesignation, the limitation of our imagination and experience amounts to a predesignation far within those limits; and we thus see that if the number of instances be very great indeed, the failure to predesignate is not an

important fault. Of characters at all striking, or of objects at all familiar, the number will seldom reach 1,000; and of very striking characters or very familiar objects the number is still less. So that if a large number of samples of a class are found to have some very striking character in common, or if a large number of characters of one object are found to be possessed by a very familiar object, we need not hesitate to infer, in the first case, that the same characters belong to the whole class, or, in the second case, that the two objects are practically identical; remembering only that the inference is less to be relied upon than it would be had a deliberate predesignation been made. This is no doubt the precise significance of the rule sometimes laid down, that a hypothesis ought to be *simple*--simple here being taken in the sense of familiar.

Peirce: CP 2.740 Cross-Ref:††

This modification of the rule shows that, even in the absence of voluntary predesignation, *some* slight weight is to be attached to an induction or hypothesis. And perhaps when the number of instances is not very small, it is enough to make it worth while to subject the inference to a regular test. But our natural tendency will be to attach too much importance to such suggestions, and we shall avoid waste of time in passing them by without notice until some stronger plausibility presents itself.

Peirce: CP 2.741 Cross-Ref:††
§9. UNIFORMITIES

741. In almost every case in which we make an induction or a hypothesis, we have some knowledge which renders our conclusion antecedently likely or unlikely. The effect of such knowledge is very obvious, and needs no remark. But what also very often happens is that we have some knowledge, which, though not of itself bearing upon the conclusion of the scientific argument, yet serves to render our inference more or less probable, or even to alter the terms of it. Suppose, for example, that we antecedently know that all the *M*'s strongly resemble one another in regard to characters of a certain order. Then, if we find that a moderate number of *M*'s taken at random have a certain character, *P*, of that order, we shall attach a greater weight to the induction than we should do if we had not that antecedent knowledge. Thus, if we find that a certain sample of gold has a certain chemical character--since we have very strong reason for thinking that all gold is alike in its chemical characters--we shall have no hesitation in extending the proposition from the one sample to gold in general. Or if we know that among a certain people--say the Icelanders--an extreme uniformity prevails in regard to all their ideas, then, if we find that two or three individuals taken at random from among them have all any particular superstition, we shall be the more ready to infer that it belongs to the whole people from what we know of their uniformity. The influence of this sort of uniformity upon inductive conclusions was strongly insisted upon by Philodemus,†1 and some very exact conceptions in regard to it may be gathered from the writings of Mr. Galton. Again, suppose we know of a certain character, *P*, that in whatever classes of a certain description it is found at all, to those it usually belongs as a universal character; then any induction which goes toward showing that all the *M*'s are *P* will be greatly strengthened. Thus it is enough to find that two or three individuals taken at random from a genus of animals have three toes on each foot, to prove that the same is true of the whole genus; for we know that this is a *generic* character. On the other hand, we shall be slow to infer that all the animals of a genus have the same color, because

color varies in almost every genus. This kind of uniformity seemed to J. S. Mill to have so controlling an influence upon inductions, that he has taken it as the centre of his whole theory of the subject.

Peirce: CP 2.742 Cross-Ref:††

742. Analogous considerations modify our hypothetic inferences. The sight of two or three words will be sufficient to convince me that a certain manuscript was written by myself, because I know a certain look is peculiar to it. So an analytical chemist, who wishes to know whether a solution contains gold, will be completely satisfied if it gives a precipitate of the purple of cassius with chloride of tin; because this proves that either gold or some hitherto unknown substance is present. These are examples of characteristic tests. Again, we may know of a certain person, that whatever opinions he holds he carries out with uncompromising rigor to their utmost logical consequences; then, if we find his views bear some of the marks of any ultra school of thought, we shall readily conclude that he fully adheres to that school.

Peirce: CP 2.743 Cross-Ref:††

743. There are thus four different kinds of uniformity and non-uniformity which may influence our ampliative inferences:

Peirce: CP 2.743 Cross-Ref:††

(1) The members of a class may present a greater or less general resemblance as regards a certain line of characters.

Peirce: CP 2.743 Cross-Ref:††

(2) A character may have a greater or less tendency to be present or absent throughout the whole of whatever classes of certain kinds.

Peirce: CP 2.743 Cross-Ref:††

(3) A certain set of characters may be more or less intimately connected, so as to be probably either present or absent together in certain kinds of objects.

Peirce: CP 2.743 Cross-Ref:††

(4) An object may have more or less tendency to possess the whole of certain sets of characters when it possesses any of them.

Peirce: CP 2.743 Cross-Ref:††

A consideration of this sort may be so strong as to amount to demonstration of the conclusion. In this case, the inference is mere deduction--that is, the application of a general rule already established. In other cases, the consideration of uniformities will not wholly destroy the inductive or hypothetic character of the inference, but will only strengthen or weaken it by the addition of a new argument of a deductive kind.

Peirce: CP 2.744 Cross-Ref:††
§10. CONSTITUTION OF THE UNIVERSE

744. We have thus seen how, in a general way, the processes of inductive and hypothetic inference are able to afford answers to our questions, though these may relate to matters beyond our immediate ken. In short, a theory of the logic of verification has been sketched out. This theory will have to meet the objections of two opposing schools of logic.

Peirce: CP 2.744 Cross-Ref:††

The first of these explains induction by what is called the doctrine of Inverse Probabilities, of which the following is an example: Suppose an ancient denizen of the Mediterranean coast, who had never heard of the tides, had wandered to the shore of the Atlantic Ocean, and there, on a certain number m of successive days had witnessed the rise of the sea. Then, says Quetelet, he would have been entitled to conclude that there was a probability equal to $(m + 1)/(m + 2)$ that the sea would rise on the next following day.†P1 Putting $m = 0$, it is seen that this view assumes that the probability of a totally unknown event is $1/2$; or that of all theories proposed for examination one half are true. In point of fact, we know that although theories are not proposed unless they present some decided plausibility, nothing like one half turn out to be true. But to apply correctly the doctrine of inverse probabilities, it is necessary to know the antecedent probability of the event whose probability is in question. Now, in pure hypothesis or induction, we know nothing of the conclusion antecedently to the inference in hand. Mere ignorance, however, cannot advance us toward any knowledge; therefore it is impossible that the theory of inverse probabilities should rightly give a value for the probability of a pure inductive or hypothetic conclusion. For it cannot do this without assigning an antecedent probability to this conclusion; so that if this antecedent probability represents mere ignorance (which never aids us), it cannot do it at all.

Peirce: CP 2.745 Cross-Ref:††

745. The principle which is usually assumed by those who seek to reduce inductive reasoning to a problem in inverse probabilities is, that if nothing whatever is known about the frequency of occurrence of an event, then any one frequency is as probable as any other. But Boole has shown that there is no reason whatever to prefer this assumption, to saying that any one "constitution of the universe" is as probable as any other. Suppose, for instance, there were four possible occasions upon which an event might occur. Then there would be 16 "constitutions of the universe," or possible distributions of occurrences and non-occurrences. They are shown in the following table, where Y stands for an occurrence and N for a non-occurrence.

<i>4 occurrences.</i>	<i>3 occurrences.</i>	<i>2 occurrences.</i>	<i>1 occurrence.</i>	<i>0 occurrence.</i>
<i>Y Y Y Y</i>	<i>Y Y Y N</i>	<i>Y Y N N</i>	<i>Y N N N</i>	<i>N N N N</i>
	<i>Y Y N Y</i>	<i>Y N Y N</i>	<i>N Y N N</i>	
	<i>Y N Y Y</i>	<i>Y N N Y</i>	<i>N N Y N</i>	
	<i>N Y Y Y</i>	<i>N Y Y N</i>	<i>N N N Y</i>	
		<i>N Y N Y</i>		
		<i>N N Y Y</i>		

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Peirce: CP 2.745 Cross-Ref:††

It will be seen that different frequencies result some from more and some from fewer different "constitutions of the universe," so that it is a very different thing to assume that all frequencies are equally probable from what it is to assume that all constitutions of the universe are equally probable.

Peirce: CP 2.746 Cross-Ref:††

746. Boole says that one assumption is as good as the other. But I will go further, and say that the assumption that all constitutions of the universe are equally probable is far better than the assumption that all frequencies are equally probable. For the latter proposition, though it may be applied to any one unknown event, cannot be applied to all unknown events without inconsistency. Thus, suppose all frequencies of the event whose occurrence is represented by *Y* in the above table are equally probable. Then consider the event which consists in a *Y* following a *Y* or an *N* following an *N*. The possible ways in which *this* event may occur or not are shown in the following table:

<i>3 occurrences.</i>	<i>2 occurrences.</i>	<i>1 occurrence.</i>	<i>0 occurrence.</i>
<i>Y Y Y Y</i>	<i>Y Y Y N</i>	<i>Y Y N Y</i>	<i>Y N Y N</i>
<i>N N N N</i>	<i>N N N Y</i>	<i>N N Y N</i>	<i>N Y N Y</i>
	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	
	<i>Y Y N N</i>	<i>Y N N Y</i>	
	<i>N N Y Y</i>	<i>N Y Y N</i>	
	<hr style="width: 100%;"/>	<hr style="width: 100%;"/>	
	<i>N Y Y Y</i>	<i>Y N Y Y</i>	
	<i>Y N N N</i>	<i>N Y N N</i>	

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Peirce: CP 2.746 Cross-Ref:††

It will be found that assuming the different frequencies of the first event to be equally probable, those of this new event are not so--the probability of three occurrences being half as large again as that of two, or one. On the other hand, if all constitutions of the universe are equally probable in the one case, they are so in the other; and this latter assumption, in regard to perfectly unknown events, never gives rise to any inconsistency.

Peirce: CP 2.746 Cross-Ref:††

Suppose, then, that we adopt the assumption that any one constitution of the universe is as probable as any other; how will the inductive inference then appear, considered as a problem in probabilities? The answer is extremely easy;†P1 namely, the occurrences or non-occurrences of an event in the past in no way affect the probability of its occurrence in the future.

Peirce: CP 2.747 Cross-Ref:††

747. Boole frequently finds a problem in probabilities to be indeterminate. There are those to whom the idea of an unknown probability seems an absurdity. Probability, they say, measures the state of our knowledge, and ignorance is denoted by the probability 1/2. But I apprehend that the expression "the probability of an event" is an incomplete one. A probability is a fraction whose numerator is the frequency of a specific kind of event, while its denominator is the frequency of a genus embracing that species. Now the expression in question names the numerator of the fraction, but omits to name the denominator. There is a sense in which it is true that the probability of a perfectly unknown event is one half; namely, the assertion of its occurrence is the answer to a possible question answerable by "yes" or "no," and of

all such questions just half the possible answers are true. But if attention be paid to the denominators of the fractions, it will be found that this value of 1/2 is one of which no possible use can be made in the calculation of probabilities.

Peirce: CP 2.748 Cross-Ref:††

748. The theory here proposed does not assign any probability to the inductive or hypothetic conclusion, in the sense of undertaking to say how frequently *that conclusion* would be found true. It does not propose to look through all the possible universes, and say in what proportion of them a certain uniformity occurs; such a proceeding, were it possible, would be quite idle. The theory here presented only says how frequently, in this universe, the special form of induction or hypothesis would lead us right. The probability given by this theory is in every way different--in meaning, numerical value, and form--from that of those who would apply to ampliative inference the doctrine of inverse chances.

Peirce: CP 2.749 Cross-Ref:††

749. Other logicians hold that if inductive and hypothetic premisses lead to true oftener than to false conclusions, it is only because the universe happens to have a certain constitution. Mill and his followers maintain that there is a general tendency toward uniformity in the universe, as well as special uniformities such as those which we have considered. The Abbe Graty believes that the tendency toward the truth in induction is due to a miraculous intervention of Almighty God, whereby we are led to make such inductions as happen to be true, and are prevented from making those which are false.†1 Others have supposed that there is a special adaptation of the mind to the universe, so that we are more apt to make true theories than we otherwise should be. Now, to say that a theory such as these is *necessary* to explaining the validity of induction and hypothesis is to say that these modes of inference are not in themselves valid, but that their conclusions are rendered probable by being probable deductive inferences from a suppressed (and originally unknown) premiss. But I maintain that it has been shown that the modes of inference in question are necessarily valid, whatever the constitution of the universe, so long as it admits of the premisses being true. Yet I am willing to concede, in order to concede as much as possible, that when a man draws instances at random, all that he knows is that he *tries* to follow a certain precept; so that the sampling process might be rendered generally fallacious by the existence of a mysterious and malign connection between the mind and the universe, such that the possession by an object of an *unperceived* character might influence the will toward choosing it or rejecting it. Such a circumstance would, however, be as fatal to deductive as to ampliative inference. Suppose, for example, that I were to enter a great hall where people were playing *rouge et noir* at many tables; and suppose that I knew that the red and black were turned up with equal frequency. Then, if I were to make a large number of mental bets with myself, at this table and at that, I might, by statistical deduction, expect to win about half of them--precisely as I might expect, from the results of these samples, to infer by induction the probable ratio of frequency of the turnings of red and black in the long run, if I did not know it. But could some devil look at each card before it was turned, and then influence me mentally to bet upon it or to refrain therefrom, the observed ratio in the cases upon which I had bet might be quite different from the observed ratio in those cases upon which I had not bet. I grant, then, that even upon my theory some fact has to be supposed to make induction and hypothesis valid processes; namely, it is supposed that the supernal powers withhold their hands and let me alone, and that no mysterious uniformity or adaptation interferes with the action of chance. But then this negative fact supposed by my theory plays a totally different part from

the facts supposed to be requisite by the logicians of whom I have been speaking. So far as facts like those they suppose can have any bearing, they serve as major premisses from which the fact inferred by induction or hypothesis might be deduced; while the negative fact supposed by me is merely the denial of any major premiss from which the falsity of the inductive or hypothetic conclusion could in general be deduced. Nor is it necessary to deny altogether the existence of mysterious influences adverse to the validity of the inductive and hypothetic processes. So long as their influence were not too overwhelming, the wonderful self-correcting nature of the ampliative inference would enable us, even if they did exist, to detect and make allowance for them.

Peirce: CP 2.750 Cross-Ref:††

750. Although the universe need have no peculiar constitution to render ampliative inference valid, yet it is worth while to inquire whether or not it has such a constitution; for if it has, that circumstance must have its effect upon all our inferences. It cannot any longer be denied that the human intellect is peculiarly adapted to the comprehension of the laws and facts of nature, or at least of some of them; and the effect of this adaptation upon our reasoning will be briefly considered in the next section. Of any miraculous interference by the higher powers, we know absolutely nothing; and it seems in the present state of science altogether improbable. The effect of a knowledge of special uniformities upon ampliative inferences has already been touched upon. That there is a general tendency toward uniformity in nature is not merely an unfounded, it is an absolutely absurd, idea in any other sense than that man is adapted to his surroundings. For the universe of marks is only limited by the limitation of human interests and powers of observation. Except for that limitation, every lot of objects in the universe would have (as I have elsewhere shown)†1 some character in common and peculiar to it. Consequently, there is but one possible arrangement of characters among objects as they exist, and there is no room for a greater or less degree of uniformity in nature. If nature seems highly uniform to us, it is only because our powers are adapted to our desires.

Peirce: CP 2.751 Cross-Ref:††
§11. FURTHER PROBLEMS

751. The questions discussed in this essay relate to but a small part of the Logic of Scientific Investigation. Let us just glance at a few of the others.

Peirce: CP 2.752 Cross-Ref:††

752. Suppose a being from some remote part of the universe, where the conditions of existence are inconceivably different from ours, to be presented with a United States Census Report--which is for us a mine of valuable inductions, so vast as almost to give that epithet a new signification. He begins, perhaps, by comparing the ratio of indebtedness to deaths by consumption in counties whose names begin with the different letters of the alphabet. It is safe to say that he would find the ratio everywhere the same, and thus his inquiry would lead to nothing. For an induction is wholly unimportant unless the proportions of *P*'s among the *M*'s and among the non-*M*'s differ; and a hypothetic inference is unimportant unless it be found that *S* has either a greater or a less proportion of the characters of *M* than it has of other characters. The stranger to this planet might go on for some time asking inductive

questions that the Census would faithfully answer, without learning anything except that certain conditions were independent of others. At length, it might occur to him to compare the January rainfall with the illiteracy. What he would find is given in the following table †P1:

REGION	January Rainfall	Illiteracy
	Inches	Per Cent
Atlantic seacoast, Portland to } Washington }	0.92	11
Vermont, Northern and Western } New York }	0.78	7
Upper Mississippi River	0.52	3
Ohio River Valley	0.74	8
Lower Mississippi, Red River, } and Kentucky }	1.08	50
Mississippi Delta and Northern } Gulf Coast }	1.09	57
Southeastern Coast	0.68	40

He would infer that in places that are drier in January there is, not always but generally, less illiteracy than in wetter places. A detailed comparison between Mr. Schott's map of the winter rainfall with the map of illiteracy in the general census, would confirm the result that these two conditions have a partial connection. This is a very good example of an induction in which the proportion of *P*'s among the *M*'s is different, but not very different, from the proportion among the non-*M*'s. It is unsatisfactory; it provokes further inquiry; we desire to replace the *M* by some different class, so that the two proportions may be more widely separated. Now we,

knowing as much as we do of the effects of winter rainfall upon agriculture, upon wealth, etc., and of the causes of illiteracy, should come to such an inquiry furnished with a large number of appropriate conceptions; so that we should be able to ask intelligent questions not unlikely to furnish the desired key to the problem. But the strange being we have imagined could only make his inquiries haphazard, and could hardly hope ever to find the induction of which he was in search.

Peirce: CP 2.753 Cross-Ref:††

753. Nature is a far vaster and less clearly arranged repertory of facts than a census report; and if men had not come to it with special aptitudes for guessing right, it may well be doubted whether in the ten or twenty thousand years that they may have existed their greatest mind would have attained the amount of knowledge which is actually possessed by the lowest idiot. But, in point of fact, not man merely, but all animals derive by inheritance (presumably by natural selection) two classes of ideas which adapt them to their environment. In the first place, they all have from birth some notions, however crude and concrete, of force, matter, space, and time; and, in the next place, they have some notion of what sort of objects their fellow-beings are, and of how they will act on given occasions. Our innate mechanical ideas were so nearly correct that they needed but slight correction. The fundamental principles of statics were made out by Archimedes. Centuries later Galileo began to understand the laws of dynamics, which in our times have been at length, perhaps, completely mastered. The other physical sciences are the results of inquiry based on guesses suggested by the ideas of mechanics. The moral sciences, so far as they can be called sciences, are equally developed out of our instinctive ideas about human nature. Man has thus far not attained to any knowledge that is not in a wide sense either mechanical or anthropological in its nature, and it may be reasonably presumed that he never will.†1

Peirce: CP 2.754 Cross-Ref:††

754. Side by side, then, with the well established proposition that all knowledge is based on experience, and that science is only advanced by the experimental verifications of theories, we have to place this other equally important truth, that all human knowledge, up to the highest flights of science, is but the development of our inborn animal instincts.

Peirce: CP 2.755 Cross-Ref:††
CHAPTER 9

THE VARIETIES AND VALIDITY OF INDUCTION†1

§1. CRUDE, QUANTITATIVE, AND QUALITATIVE INDUCTION

755. Retroduction and Induction face opposite ways. The function of retroduction is not unlike those fortuitous variations in reproduction which played so important a *rôle* in Darwin's original theory. In point of fact, according to him every

step in the long history of the development of the moner into the man was first taken in that arbitrary and lawless mode. Whatever truth or error there may be in that, it is quite indubitable, as it appears to me, that every step in the development of primitive notions into modern science was in the first instance mere guess-work, or at least mere conjecture. But the stimulus to guessing, the hint of the conjecture, was derived from experience. The order of the march of suggestion in retroduction is from experience to hypothesis. A great many people who may be admirably trained in divinity, or in the humanities, or in law and equity, but who are certainly not well trained in scientific reasoning, imagine that Induction should follow the same course. My Lord Chancellor Bacon was one of them. On the contrary, the only sound procedure for induction, whose business consists in testing a hypothesis already recommended by the retroductive procedure, is to receive its suggestions from the hypothesis first, to take up the predictions of experience which it conditionally makes, and then try the experiment and see whether it turns out as it was virtually predicted in the hypothesis that it would. Throughout an investigation it is well to bear prominently in mind just what it is that we are trying to accomplish in the particular stage of the work at which we have arrived. Now when we get to the inductive stage what we are about is finding out how much like the truth our hypothesis is, that is, what proportion of its anticipations will be verified.

Peirce: CP 2.756 Cross-Ref:††

756. It is well to distinguish three different varieties of induction. The first and weakest kind of inductive reasoning is that which goes on the presumption that future experience as to the matter in hand will not be utterly at variance with all past experience.†P1 **Example:** "No instance of a genuine power of clairvoyance has ever been established: So I presume there is no such thing." I promise to call such reasoning **crude induction**.†1 Bacon seems to refer to this when he speaks of "inductio quae procedit per enumerationem simplicem." But I hardly think he meant to say that that phrase exactly describes it. It certainly does not; since in most cases no enumeration is attempted; and the enumeration, even if given, would not be the reasoner's chief reliance, which is rather the **absence** of instances to the contrary.

Peirce: CP 2.757 Cross-Ref:††

757. Crude induction is the only kind of induction that is capable of inferring the truth of what, in logic, is termed a universal proposition. For what is called "complete induction" is not inductive reasoning, but is logistic deduction. We might further say, if we chose, that every crude induction concludes a universal proposition; but this would be merely the expression of a way of regarding matters. For any proposition concerning the general run of future experience may be regarded as universal, even if it be "A pair of dice will, every now and then, turn up doublets." The undipped heel of crude induction is that if its conclusion be understood as indefinite, it will be of little use, while if it be taken definitely, it is liable at any moment to be utterly shattered by a single experience; for a series of experiences, if the whole constitutes but a single one of the instances to which an inductive conclusion refers, is to be regarded as a single experience.†P1

Peirce: CP 2.758 Cross-Ref:††

758. From the weakest kind of induction let us pass at once to the strongest. This investigates the interrogative suggestion of retroduction, "What is the 'real probability' that an individual member of a certain experiential class, say the S's, will have a certain character, say that of being P?" This it does by first collecting, on scientific principles, a "fair sample" of the S's, taking due account, in doing so, of the

intention of using its proportion of members that possess the predesignate character of being P. This sample will contain none of those S's on which the retrodution was founded. The induction then presumes that the value of the proportion, among the S's of the sample, of those that are P, probably approximates, within a certain limit of approximation, to the value of the real probability in question. I propose to term such reasoning *Quantitative Induction*. Now, if I were writing a treatise on logic, I should here be obliged, not only to teach the art of sampling, including all that Dr. Karl Pearson †1 and others have taught us about distributions of specific instances among general ones, and the consequent proper inferences in such cases, but I should have to state and expound the exact definitions of "real probability," "independent," "fair sample," "predesignate," etc. As it is, I will limit myself to a single needful explanation that, so far as I know, the reader could not find definitely stated in any of the books. It is that when we say that a certain ratio will have a certain value in "the long run," we refer to the *probability-limit* of an endless succession of fractional values; that is, to the only possible value from 0 to ∞ , inclusive, about which the values of the endless succession will never cease to oscillate; so that, no matter what place in the succession you may choose, there will follow both values above the probability-limit and values below it; while if V be any other possible value from 0 to ∞ , but *not* the probability-limit there will be some place in the succession beyond which all the values of the succession will agree, either in all being greater than V, or else in all being less.

Peirce: CP 2.759 Cross-Ref: ††

759. The remaining kind of induction, which I shall call *Qualitative Induction*, is of more general utility than either of the others, while it is intermediate between them, alike in respect to security and to the scientific value of its conclusions. In both these respects it is well separated from each of the other kinds. It consists of those inductions which are neither founded upon experience in one mass, as Crude Induction is, nor upon a collection of numerable instances of equal evidential values, but upon a stream of experience in which the relative evidential values of different parts of it have to be estimated according to our sense of the impressions they make upon us.

Peirce: CP 2.759 Cross-Ref: ††

Qualitative Induction consists in the investigator's first deducing from the retroductive hypothesis as great an evidential weight of genuine conditional predictions as he can conveniently undertake to make and to bring to the test, the condition under which he asserts them being that of the retroductive hypothesis having such degree and kind of truth as to assure their truth. In calling them "predictions," I do not mean that they need relate to future events but that they must antecede the investigator's knowledge of their truth, or at least that they must virtually antecede it. I will give an illustration of such "virtual antecedence." Suppose that to avoid wasting a great deal of time upon a hypothesis which the first comparisons with the facts may show to be utterly worthless, an investigator of a certain conjecture draws up and resolves to follow a well-considered initial program for work upon the question, and that this consists mainly in working out and testing as many consequences of the hypothesis as he can work out by a certain mathematical method and can ascertain the truth or falsity of at a cost of not more than \$100 for each. But suppose that among the half dozen predictions to which that method will carry him, there, quite unexpectedly, turns up one whose truth has long been known to him, though it is a surprise to him to find that it is deducible from the hypothesis under examination. What course does sound logic impose upon him under these

circumstances? The answer is that he must reexamine the process of retrodution that suggested the hypothesis; and if the fact that is now repredicted in any degree influenced that hypothesis, it has had its due effect, and must not be used again. But if not, will he then be free to use the prediction if he likes? Not at all: the validity of his Qualitative Induction will be found to depend upon his following a rational and decisive method; he has no more right, but rather less, to favor the inductive rejection of the retroductive suggestion, than to favor its inductive adoption; and he is bound, as a man who means to reason as honestly as the imperfections of his nature and training will permit, to admit the true prediction into his counsels. The predictions must eventually be so varied as to test every feature of the hypothesis; yet the interests of science command constant attention to economy, especially in the earlier inductive stages of research.

Peirce: CP 2.759 Cross-Ref:††

Having made his initial predictions the investigator proceeds to ascertain their truth or falsity; and then, having taken account of such subsidiary arguments as there may be, goes on to judge of the combined value of the evidence, and to decide whether the hypothesis should be regarded as proved, or as well on the way toward being proved, or as unworthy of further attention, or whether it ought to receive a definite modification in the light of the new experiments and be inductively reexamined *ab ovo*, or whether finally, that while not true it probably presents some analogy to the truth, and that the results of the induction may help to suggest a better hypothesis.

Peirce: CP 2.760 Cross-Ref:††

760. I will now state, with slight hints of argument, the conclusions which I have reached as to the warrant, or basis of validity, of the inferential processes in the three stages of inquiry. I have been actively studying this subject, for the sake of completely satisfying my own mind about it, for 50 or 51 years. To be sure, I have, some half dozen times during the half-century, let my mind lie fallow, as to this subject, during one or two dozens of months, hoping so to rid myself of any inveterate bad habits of thinking that I may insensibly have fallen into. I have six times published my views, in 1867,†1 1868,†2 1878,†3 1882 [1883],†4 1892,†5 and 1902 [1901].†6 The last of these publications, compared with my present brief abstract, shows that my last week of years has by no means been an idle one, and encourages me to hope that I may yet be able to detect errors and omissions in my views, even if others do not confer upon me the benefit of such amendments.

Peirce: CP 2.761 Cross-Ref:††
§2. MILL ON INDUCTION

761. In regard to the theory of the validity of Induction the great majority still follow the *System of Logic* set forth in 1843 by John Stuart Mill, who was certainly a clear thinker, and apparently a remarkably candid thinker, in spite of his long training in writing for one of the old "quarterlies," and his consequent unfortunate taste for and skill in controversy, which, combined with his having imbibed his father's sterilizing nominalism with his mother's milk, rendered him, for example, incapable of appreciating Whewell, whose acquaintance with the processes of thought of science was incomparably greater than his own. J. S. Mill's beautiful style, of truly

French perfection, together with the bulk of the two volumes, prevent all but the keenest readers from perceiving that he unconsciously wavers between three (not to say four) incompatible theories of the validity of induction. The first (stated in Bk. III, Chap. 3, Sec. 1) is [that] the whole force of Induction is the same as that of a syllogism of which the major premiss is the same for all inductions, being a certain "Axiom of the uniformity of the course of nature" (so described in the table of "Contents"). This was substantially Whately's theory of 1826. The second theory (which seems to be usually uppermost in Mill's mind; especially in Bk. II, Chap. 3, Sec. 7 and in Bk. III, Chap. 4, Sec. 2), is that induction proceeds *as if* upon the principle that a predicate which throughout a more or less extensive experience has been uniformly found to be true of all the members of a given class that have been examined in this respect may, with little risk, be presumed to be true of every member of that class, without exception; and that while it is not necessary that the inductive reasoner should have this principle clearly in mind, the *logician*, whose business it partly is to explain why inductions turn out to be true, must recognize the fact that nature is sufficiently uniform to render that *quasi* principle true, and must recognize that [nothing] else renders induction a safe and justifiable procedure. This theory is little more than the old maxim that "we must judge of the future by the past," which Mill--into such unfairness can an inclination toward controversy betray even an eminently fair mind!--attacks as if it merely meant that future history will repeat past history, instead of what it has meant, that future experience must be presumed to resemble past experience under sufficiently similar conditions. The third theory (see Bk. III, Chap. 3, Sec. 3), is that nature as a whole is not absolutely uniform, variety being a far more prominent characteristic of it; and that such uniformity as there is, is "a mere tissue of partial regularities," each consisting in the fact that some classes of objects show a greater, and some a less, tendency to a resemblance of all their members in respect to certain lines of characters; and that whoever knows this "has solved the problem of Induction." This theory was original with Mill; and though it is not the sole, nor the main, support of induction, it certainly does bring a powerful additional support to many inductions. But it is curious that Mill should have chanced to say, whoever might be acquainted with this theory "knows more of the philosophy of logic than the wisest of the ancients." For a quarter of a century later Gomperz †1 published so much as remained of the contents of a papyrus from Herculaneum, which was a defence of induction and a theory of its validity by the Epicurean Philodemus, under whose instruction Cicero studied; and the theory of Philodemus, like that of Mill, is that this kind of reasoning (the only valid reasoning in his opinion) derives its validity from the existence in nature of special uniformities. Only, the uniformities that attracted the attention of Philodemus, instead of characterizing certain classes, characterize certain characters, and consist in their having a special tendency to be present (or to be absent) throughout all the members of certain kinds of classes. In fact, still other types of uniformities may affect the strength of inductions.

Peirce: CP 2.761 Cross-Ref: ††

A yet fourth theory of induction, that of Laplace, received, by implication, the assent of Mill; and since this theory is taught as correct in all the textbooks of the Doctrine of Chances, it behooves me, in adopting another, to state, with the utmost brevity wherein Laplace's theory is false and harmful. I shall also give my explanation of Mill's assenting to it.

Peirce: CP 2.762 Cross-Ref: ††

762. If, upon any occasion, we were to devise a method of forming a

numerous sample of any class, say the S's, which should be suitable for use in determining to a given degree of approximation what proportion of future experiences of S's would, in the long run, be found to have the character of being P, in case existing general conditions should undergo no alteration, then in case there were any definite reason to expect that, among S's coming to our attentive experience from any particular sub-class, say among the S's that belong to the sub-class of T's, a markedly different proportion would turn out to be P from the proportion among the S's that should not be T's, then that method of forming the sample, since we have supposed it to be "suitable" for showing the proportion of P's among all future experiences of S's, must needs insure that the proportion of S's that are T's should be nearly the same in the sample as it was destined to be among all the S's of our future experience; though, as I need not repeat again, this would be so only under the supposition of unchanged general circumstances, and need not be more precisely true than would suffice to keep the errors of the concluded proportion of subsequently experienced S's that should be P within the intended limit of approximation. Moreover, should there be any serious reason to suspect that any identifiable S presenting itself for admission to the sample was so connected with any S already admitted as to have a special liability whether to being like or to being unlike that already admitted instance in respect to being or not being P (under the same limitation that is not to be repeated), then our "suitable" method would have to exclude that instance from the sample. And once again, should there happen to be any reason to suspect that an instance had attracted our attention owing to causes connected, whether directly or indirectly, with its being P, or to such causes as should be connected with its *not* being P, then our suitable method must exclude that instance. Furthermore, our suitable method must so operate that the sample shall contain a sufficient number of instances to give the intended degree of approximation. For instance, if it will suffice that the figure next following the decimal point in the decimal expression of the proportion among all the S's of such as are P should be exact, 9 instances may first be taken, and if these make the ratio less than 0.05 or greater than 0.95, they will suffice. If not, 14 more instances may be collected; and if the whole 23 make the ratio less than 0.15 or greater than 0.85, they will suffice. If not, then if 11 more instances being taken, the whole 34 make the ratio less than 0.25 or greater than 0.75, they will suffice. If not, add 7 more, and if the ratio appears as less than 0.35 or more than 0.65, the 41 will suffice. If not, take 4 more and if the ratio then appears as less than 0.45 or greater than 0.55, the 45 will suffice. If not, one more instance will in any case be enough. If the first two figures of the decimal fraction must be correct, a hundred times as many instances will be requisite.

Peirce: CP 2.763 Cross-Ref:††

763. Every person of common-sense must, upon reflexion, acknowledge, what is familiar to everybody habituated to inductive reasoning, that all the above precautions are requisite, except that the concluding rule need not have been so detailed, and that, if the instances were sufficiently multiplied, it would suffice that the other rules should not be too frequently and grossly violated and that they should not prevalingly be violated in the same direction. Let all such diminutions from them be made, and it still remains true for sound reason, that *such an induction does not follow merely from the fact that P is true of such and such of the S's of a collection of S's, but that it is necessary to take account of the manner in which these S's were brought to the inquirer's attention.* This fixes a great gulf between Induction and Deduction. It is quite true that we may describe the general conditions of a valid quantitative induction, and may convince ourselves that if the sample be drawn strictly at random from among the S's and be made sufficiently numerous, then, the

general conditions remaining unchanged, it necessarily follows that future experience, *under the same general conditions*, will on the average of an indefinite multitude of such inductions, bear out the Inductive conclusion. Still, this in no wise suffices to reduce the quantitative induction to any kind of induction [deduction?]. For even if I were to grant that the truth of the inductive conclusion would necessarily follow if the conditions of a fair sample were to be ideally fulfilled, which, for a reason that I will presently state, I find myself unable to do, still the person who really draws the inductive inference cannot possibly have any demonstrative evidence that those conditions are fulfilled even to the imperfect degree that is needful for an approximation to the true ratio. He knows, if you will, that [he] has made strenuous efforts to make his sample a fair one; but he cannot be quite sure that deep down in the caverns of his heart there may not lurk, unsuspected by him, a determination to force himself to believe in a certain value for the ratio, nor that this has not frustrated all his efforts to make the sample a fair one; and if he cannot be absolutely certain even of his own honesty, how can he so much as approach certainty as to the correctness of his concluded approximation to the ratio not having been destroyed by external circumstances? A theoretician--or rather, a papyrobite, a man whose vitality is that of sentences written down or imagined--may reply that that contingency is covered by proviso that general conditions remain sufficiently unchanged. But that is to overlook the principal end of inquiry, as regards human life. What is the chief end of man? *Answer:* To actualize ideas of the immortal, ceaselessly prolific kind. To that end it is needful to get beliefs that the believer will take satisfaction in acting upon, not mere rules set down on paper, with lethal provisos attached to them. The inductive reasoner cannot possibly find any strictly demonstrative reasoning that could take the place of his induction, since every demonstrative is strictly limited to the field of that part of its copulate premiss that corresponds to the minor premiss of a syllogism; while to serve his purpose, that of forming a basis for conduct, it must transcend that limit in concluding future from past experience. Now every valid mathematical reasoning is demonstrative and is limited to an ideal state of things. The reasoning of the calculus of probabilities consists simply of demonstrations concerning "probabilities," which, in all useful applications of the calculus, are *real* probabilities, or ratios of frequency in the "long run" of experiences of designated species among experiences designated, or obviously designable, genera over those species; which real probabilities are ascertained by quantitative inductions from statistics laboriously collected and critically tabulated. But the phrase "the probability of an event," which is perpetually recurring in the treatises, and which is not free from objection, even when the real probability is meant (because it seems to refer to a singular experience considered by itself, and because it does not mention that *two* classes of experiences are essentially concerned), is used in various different senses, owing to the ambiguity of the word "probability"; and the writers of the mathematical treatises on the subject have not had sufficient power of logical analysis to found any useful theory upon it. . .

Peirce: CP 2.764 Cross-Ref:††

764. Laplace maintains that it is possible to draw a necessary conclusion regarding the probability of a particular determination of an event based on not knowing anything at all about [it]; that is, based on nothing. When a man thinks himself to know nothing at all as to which of a number of alternatives is the truth, his mind can no more incline toward or against any one of them or any combination of them than a mathematical point can have an inclination toward any point of the compass. Suppose the question concerns the color of an object which we know has a high color, but are otherwise in a state of blank ignorance [about it]. Then, according

to Laplace, if one were to draw two lines across a map of the spectrum, it would be probable that the color did not match any part of the spectrum included between those lines; no matter how nearly they might include the whole spectrum. Laplace holds that for every man there is one law (and necessarily but one) of dissection of each continuum of alternatives so that all the parts shall seem to that man to be "*également possibles*" in a quantitative sense, antecedently to all information. But he presents not the slightest reason for thinking this to be so, and seems to admit that to different men different modes of dissection will seem to give alternatives that are *également possibles*. It is only by basing the theory of probability upon this doctrine, and thus rendering probability without interest except to a student of human eccentricities, that it is possible to assign any mathematical probability to an inductive conclusion. Much might be added in refutation of Laplace's position.

Peirce: CP 2.765 Cross-Ref:††

765. In the first edition of his *Logic*, †1 Mill presents arguments against Laplace's view; but in his third, without answering his former arguments, as far as I see, he abandons them, and thus assents to all that is necessary for calculating a necessary probability for the inductive conclusion, without any regard to the manner in which the instances have been collected.

Peirce: CP 2.766 Cross-Ref:††

766. I will now sketch one or more ways of refuting each of Mill's three professed theories of Induction.

Peirce: CP 2.766 Cross-Ref:††

To the first theory, that an Induction is equivalent to a syllogism whose major premiss is the axiom of the uniformity of nature, while its minor premiss states the observed facts about the instances, the conclusion being identical with that of the induction, each of the following objections is conclusive: *first*, that an induction, unlike a demonstration, does not rest solely upon the facts observed, but upon the manner in which those facts have been collected; *secondly*, that a syllogism infers its conclusion apodictically, while an induction does not; *thirdly*, that a syllogism enriches our knowledge of ideas, but not our information, which is what Kant meant in saying that it only explicates but does not amplify knowledge, while an induction does amplify our knowledge; *fourthly*, that the proposed syllogism would be fallacious, because its major premiss is vague, so that it could be fairly thrown into the form of a fallacy of undistributed middle, since all we really know of the general uniformity of nature is that *some* pairs of phenomena (an apparently infinitesimal proportion of all pairs) are connected as logical antecedent and consequent; *fifthly*, because a sound syllogism must not conclude beyond the breadth, or logical extension of its minor premiss (when this is suitably stated), while to represent a true induction it must do so. There are other objections, fully as strong as these five; but it seems needless to mention them.

Peirce: CP 2.767 Cross-Ref:††

767. The second theory correctly describes the procedure of the mind in crude inductions, but in no others; and Mill's celebrated four methods (chiefly based on the *Novum Organum*), though they may be of some help to minds that need such aids, yet furnish nothing but crude inductions, after all. The principle of this theory also sufficiently explains how it is that we meet such frequent opportunities to draw crude inductions as we do. But the moment the attempt is made to apply this theory to *justifying*, or explaining the validity even of crude inductions (and it is still worse

with other kinds of induction), it lays itself open to all the objections to the first method, including the five that were specified above. For this second theory, which is the point where Mill's vain attempt to make reasoning able to get along without generalization becomes the most futile, and verges closely upon overt absurdity, differs from the first merely in not allowing, as essential to induction, that it should have any of such force as it might derive from employing the uniformity of experience as a premiss. Now this point of difference cannot confer upon induction as explained by the second theory any validity that it would not have if it were explicable by the first theory.

Peirce: CP 2.768 Cross-Ref:††

768. The third theory presents two decided advantages. For it may remove entirely the vagueness of the general principle of uniformity; and in some cases makes the special uniformity predicate a probability, so as to render the refutation of the theory, on the ground that induction does not conclude apodictically, considerably more difficult in those cases. Moreover, this theory does correctly state a part of the argument for very many inductive conclusions. But this part of the argument is not inductive but deductive. For these special uniformities (such, for example, as that every chemical element has the same combining weight, no matter from what mineral or from what part of the globe it has come), have only become known by induction, often only by elaborate investigations, and are not logical principles; so that they need to be stated as premisses when the argument is to be set forth in full. The special uniformities, when they become known, enable us to dispense with certain inductive inquiries that would otherwise be requisite. But they leave other inductions (such as that which led Mendeléeff to enunciate his periodic law), quite untouched, not explaining them in any sense.

Peirce: CP 2.769 Cross-Ref:††

769. The true guarantee of the validity of induction is that it is a method of reaching conclusions which, if it be persisted in long enough, will assuredly correct any error concerning future experience into which it may temporarily lead us. This it will do not by virtue of any deductive necessity (since it never uses all the facts of experience, even of the past), but because it is manifestly adequate, with the aid of retrodution and of deductions from retroductive suggestions, to discovering any **regularity** there may be among experiences, while **utter irregularity is not surpassed in regularity by any other relation of parts to whole**, and is thus readily discovered by induction to exist where it does exist, and the amount of departure therefrom to be mathematically determinable from observation where it is imperfect. The doctrine of chances, in all that part of it that is sound, is nothing but the science of the laws of irregularities. I do not deny that God's beneficence is in nothing more apparent than in how in the early days of science Man's attention was particularly drawn to phenomena easy to investigate and how Man has ever since been led on, as through a series of graduated exercises, to more and more difficult problems; but what I do say is that there is no possibility of a series of experiences so wanting in uniformity as to be beyond the reach of induction, provided there be sufficiently numerous instances of them, and provided the march of scientific intelligence be unchecked.

Peirce: CP 2.770 Cross-Ref:††

770. Quantitative induction approximates gradually, though in an irregular manner to the experiential truth for the long run. The antecedent probable error of it at any stage is calculable as well as the probable error of that probable error. Besides that, the probable error can be calculated from the results, by a mixture of induction

and theory. Any striking and important discrepancy between the antecedent and *a posteriori probable* errors may require investigation, since it suggests some error in the theoretical assumptions. But the fact which is here important is that Quantitative Induction always makes a gradual approach to the truth, though not a uniform approach.

Peirce: CP 2.771 Cross-Ref:††

771. Qualitative Induction is not so elastic. Usually either this kind of induction confirms the hypothesis or else the facts show that some alteration must be made in the hypothesis. But this modification may be a small detail.

Peirce: CP 2.772 Cross-Ref:††

772. Experiments †1 which I have conducted in great numbers and great elaboration have convinced me of the extremely important advantages of making use, in Qualitative Induction, of numbers in place of such adverbs of comparison of the intensity of feelings as, "slightly," "a little," "somewhat," "tolerably," "moderately," "considerably," "much," "greatly," "excessively," etc. It is not necessary to use the adverbs; but in some cases I have found it convenient to employ a few of them. What is necessary is to get certain feelings so fixed in one's mind that they can be exactly and severally reproduced in the imagination at any time, these feelings forming such a series of ten or so, beginning with the zero of intensity and running up to high intensities; and further being such that any one of them being contemplated by the investigator and compared with the next intenser in the series, the interval of intensity between them shall appear, to the contemplator's feeling, to be equal to the interval between any other one of the series and the member next intenser than it. It is certain that this can be done, since all sidereal astronomers since Ptolemy have practised this; and many psychologists beside me have done something similar for other feelings than that of luminosity. It has been demonstrated that a series of positive numbers, integer and fractional, expresses *in itself* nothing more than an order of succession. But this scale is made to express, besides, a feeling of a difference of feeling in one respect; and the experiments of many persons prove conclusively that people generally can form such a scale; and further that the scales of different persons are concordant to a pretty high degree. The next step has been executed by but few persons so far as I know; but the experiments of these few render it all but certain that all normal persons can do so with good accord. This consists in comparing a difference of feeling in one respect with a difference in a single other respect; such as luminosity and pressure-feelings, or the relative bitterness of two solutions of quassia and that of self-blame for two former actions. Such comparisons as these last are, to be sure, of no direct applicability so far as I am aware; but they are good exercises in that prescissive abstraction of intensity from its subject which is required for estimating the equality of two differences of intensity. Such estimations enable us to add and take the arithmetical mean of intensities referred to the same standard; and not only the practice of all photometricians, both astronomers and gas-examiners, but also very many thousands of experiments by me upon a wide variety of qualities of sensation, establishes, to my full satisfaction, the great utility of such applications of number in giving a control over qualitative inductions. I have not found multiplications of such numbers useful, for example, in establishing the laws of such comparisons as the relative photometric value of two lights of different colors, where I need not say that it is one thing to ask what intensity of a light A, of fixed hue and chroma but variable luminosity best matches a light B, that is altogether fixed, and quite another and independent question what photometric intensity of B, if this be made to vary, best matches an A of given fixed intensity. The meaning of the product

of two differences of intensity which refer in general to different qualities is obvious enough: it is the number to be attached as a measure to a phenomenon which involves two feelings of the intensities indicated by the multiplicand and multiplier, these two feelings being [in] a certain fixed relation to one another in which they are as independent as possible. But there is no advantage in attaching any single measure to such a complex phenomenon, unless there are different ways of analyzing it, more or less similar to the different systems of coördinates in geometry. I mean that, for example, different horizontal areas are not only measured by the sum of the parallelogram into which [they] may be cut up, each parallelogram having its sides in the directions of ENE and N by W. Were that the sole method of measurement, nothing would be gained by combining the linear measures in the two directions, but rather the reverse. But in fact we may measure the area by parallelograms in any other two dimensions; and the ratio between any two areas will be the same by any two such methods of measurement. Moreover, we may employ polar, in place of Cartesian, coördinates, and cut the area up into a circle and broken, concentric, and very thin rings. The area will be the sum of the areas, each of which will be $X \times Y$, where X is the difference of the two radii, while Y is the proportion of their sum, the proportion being that of the entire ring which forms a part of the area.

Peirce: CP 2.773 Cross-Ref:††
CHAPTER 10

NOTES ON AMPLIATIVE REASONING

§1. REASONING †1

773. Reasoning is a process in which the reasoner is conscious that a judgment, the conclusion, is determined by other judgment or judgments, the premisses, according to a general habit of thought, which he may not be able precisely to formulate, but which he approves as conducive to true knowledge. By true knowledge he means, though he is not usually able to analyse his meaning, the ultimate knowledge in which he hopes that belief may ultimately rest, undisturbed by doubt, in regard to the particular subject to which his conclusion relates. Without this logical approval, the process, although it may be closely analogous to reasoning in other respects, lacks the essence of reasoning. Every reasoner, therefore, since he approves certain habits, and consequently methods, of reasoning, accepts a logical doctrine, called his *logica utens*. Reasoning does not begin until a judgment has been formed; for the antecedent cognitive operations are not subject to logical approval or disapproval, being subconscious, or not sufficiently near the surface of consciousness, and therefore uncontrollable. Reasoning, therefore, begins with premisses which are adopted as representing percepts, or generalizations of such percepts. All the reasoner's conclusions ought to refer solely to the percepts, or rather to propositions expressing facts of perception. But this is not to say that the general conceptions to which he attains have no value in themselves.

Peirce: CP 2.774 Cross-Ref:††

774. Reasoning is of three elementary kinds; but mixed reasonings are more common. These three kinds are *induction*, *deduction*, and *presumption* (for which the present writer proposes the name *abduction*).

Peirce: CP 2.775 Cross-Ref:††

775. *Induction* takes place when the reasoner already holds a theory more or less problematically (ranging from a pure interrogative apprehension to a strong leaning mixed with ever so little doubt); and having reflected that if that theory be true, then under certain conditions certain phenomena ought to appear (the stranger and less antecedently credible the better), proceeds to *experiment*, that is, to realize those conditions and watch for the predicted phenomena. Upon their appearance he accepts the theory with a modality which recognizes it provisionally as approximately true. The logical warrant for this is that this method persistently applied to the problem must in the long run produce a convergence (though irregular) to the truth; for the truth of a theory consists very largely in this, that every perceptual deduction from it is verified. It is of the essence of induction that the consequence of the theory should be drawn first in regard to the unknown, or virtually unknown, result of experiment; and that this should virtually be only ascertained afterward. For if we look over the phenomena to find agreements with the theory, it is a mere question of ingenuity and industry how many we shall find. Induction (at least, in its typical forms) contributes nothing to our knowledge except to tell us approximately how often, in the course of such experience as our experiments go towards constituting, a given sort of event occurs. It thus simply evaluates an objective probability. Its validity does not depend upon the uniformity of nature, or anything of that kind. The uniformity of nature may tend to give the probability evaluated an extremely great or small value; but even if nature were not uniform, induction would be sure to find it out, *so long as inductive reasoning could be performed at all*. Of course, a certain degree of special uniformity is requisite for that.

Peirce: CP 2.775 Cross-Ref:††

But all the above is at variance with the doctrines of almost all logicians; and, in particular, they commonly teach that the inductive conclusion approximates to the truth because of the uniformity of nature. They only contemplate as inductive reasoning cases in which, from finding that certain individuals of a class have certain characters, the reasoner concludes that every single individual of the class has the same character. According to the definition here given, that inference is not inductive, but is a mixture of deduction and presumption. Cf. Probable Inference [§4.] See also Scientific Method [vol. 7.]

Peirce: CP 2.776 Cross-Ref:††

776. *Presumption*, or, more precisely, *abduction* (which the present writer believes to have been what Aristotle's twenty-fifth chapter of the second *Prior Analytics* imperfectly described under the name of {απαγόγε}, until Apellicon substituted a single wrong word and thus disturbed the sense of the whole), furnishes the reasoner with the problematic theory which induction verifies. Upon finding himself confronted with a phenomenon unlike what he would have expected under the circumstances, he looks over its features and notices some remarkable character or relation among them, which he at once recognizes as being characteristic of some conception with which his mind is already stored, so that a theory is suggested which would *explain* (that is, render necessary) that which is surprising in the phenomena.

Peirce: CP 2.776 Cross-Ref:††

He therefore accepts that theory so far as to give it a high place in the list of theories of those phenomena which call for further examination. If this is all his conclusion amounts to, it may be asked: What need of reasoning was there? Is he not free to examine what theories he likes? The answer is that it is a question of economy. If he examines all the foolish theories he might imagine, he never will (short of a miracle) light upon the true one. Indeed, even with the most rational procedure, he never would do so, were there not an affinity between his ideas and nature's ways. However, if there be any attainable truth, as he hopes, it is plain that the only way in which it is to be attained is by trying the hypotheses which seem reasonable and which lead to such consequences as are observed.

Peirce: CP 2.777 Cross-Ref:††

777. Presumption is the only kind of reasoning which supplies new ideas, the only kind which is, in this sense, synthetic. Induction is justified as a method which must in the long run lead up to the truth, and that, by gradual modification of the actual conclusion. There is no such warrant for presumption. The hypothesis which it problematically concludes is frequently utterly wrong itself, and even the method need not ever lead to the truth; for it may be that the features of the phenomena which it aims to explain have no rational explanation at all. Its only justification is that its method is the only way in which there can be any hope of attaining a rational explanation. This doctrine agrees substantially with that of some logicians; but it is radically at variance with a common theory and with a common practice. This prescribes that the reasoner should be guided by balancing probabilities, according to the doctrine of inverse probability. This depends upon knowing antecedent probabilities. If these antecedent probabilities were solid statistical facts, like those upon which the insurance business rests, the ordinary precepts and practice would be sound. But they are not and cannot, in the nature of things, be statistical facts. What is the antecedent probability that matter should be composed of atoms? Can we take statistics of a multitude of different universes? An objective probability is the ratio of frequency of a specific to a generic event *in the ordinary course of experience*. Of a fact *per se* it is absurd to speak of objective probability. All that is attainable are subjective probabilities, or likelihoods, which express nothing but the conformity of a new suggestion to our prepossessions; and these are the source of most of the errors into which man falls, and of all the worst of them. An instance of what the method of balancing likelihoods leads to is the "higher criticism" of ancient history, upon which the archaeologist's spade has inflicted so many wounds.

Peirce: CP 2.778 Cross-Ref:††

778. The third elementary way of reasoning is *deduction*, of which the warrant is that the facts presented in the premisses could not under any imaginable circumstances be true without involving the truth of the conclusion, which is therefore accepted with necessary modality. But though it be necessary in its modality, it does not by any means follow that the conclusion is certainly true. When we are reasoning about purely hypothetical states of things, as in mathematics, and can make it one of our hypotheses that what is true shall depend only on a certain kind of condition--so that, for example, what is true of equations written in black ink would certainly be equally true if they were written in red--we can be certain of our conclusions, *provided no blunders have been committed*. This is "demonstrative reasoning." Fallacies in pure mathematics have gone undetected for many centuries. It is to ideal states of things alone--or to real states of things as ideally conceived, always more or

less departing from the reality--that deduction applies. The process is as follows, at least in many cases:

Peirce: CP 2.778 Cross-Ref:††

We form in the imagination some sort of diagrammatic, that is, iconic, representation of the facts, as skeletonized as possible. The impression of the present writer is that with ordinary persons this is always a visual image, or mixed visual and muscular; but this is an opinion not founded on any systematic examination. If visual, it will either be geometrical, that is, such that familiar spatial relations stand for the relations asserted in the premisses, or it will be algebraical, where the relations are expressed by objects which are imagined to be subject to certain rules, whether conventional or experiential. This diagram, which has been constructed to represent intuitively or semi-intuitively the same relations which are abstractly expressed in the premisses, is then observed, and a hypothesis suggests itself that there is a certain relation between some of its parts--or perhaps this hypothesis had already been suggested. In order to test this, various experiments are made upon the diagram, which is changed in various ways. This is a proceeding extremely similar to induction, from which, however, it differs widely, in that it does not deal with a course of experience, but with whether or not a certain state of things can be imagined. Now, since it is part of the hypothesis that only a very limited kind of condition can affect the result, the necessary experimentation can be very quickly completed; and it is seen that the conclusion is compelled to be true by the conditions of the construction of the diagram. This is called "diagrammatic, or schematic, reasoning."

Peirce: CP 2.779 Cross-Ref:††
§2. VALIDITY †1

779. The possession by an argumentation or inference of that sort of efficiency in leading to the truth, which it professes to have; it is also said to be "valid."

Peirce: CP 2.780 Cross-Ref:††

780. Every argument or inference professes to conform to a general method or type of reasoning, which method, it is held, has one kind of virtue or another in producing truth. In order to be valid the argument or inference must really pursue the method it professes to pursue, and furthermore, that method must have the kind of truth-producing virtue which it is supposed to have. For example, an induction may conform to the formula of induction; but it may be conceived, and often is conceived, that induction lends a probability to its conclusion. Now that is not the way in which induction leads to the truth. It lends no definite probability to its conclusion. It is nonsense to talk of the probability of a law, as if we could pick universes out of a grab-bag and find in what proportion of them the law held good. Therefore, such an induction is not valid; for it does not do what it professes to do, namely, to make its conclusion probable. But yet if it had only professed to do what induction does (namely, to commence a proceeding which must in the long run approximate to the truth), which is infinitely more to the purpose than what it professes, it would have been valid. Validity must not be confounded with strength. For an argument may be perfectly valid and yet excessively weak. I wish to know whether a given coin is so accurately made that it will turn up heads and tails in approximately equal

proportions. I therefore pitch it five times and note the results, say three heads and two tails; and from this I conclude that the coin is approximately correct in its form. Now this is a valid induction; but it is contemptibly weak. All simple arguments about matters of fact are weak. The strength of an argument might be theoretically defined as the number of *independent* equal standard unit arguments upon the other side which would balance it. But since it is next to impossible to imagine independent arguments upon any question, or to compare them with accuracy, and since moreover the "other side" is a vague expression, this definition only serves to convey a rough idea of what is meant by the strength of an argument. It is doubtful whether the idea of strength can be made less vague. But we may say that an induction from more instances is, other things being equal, stronger than an induction from fewer instances. Of probable deductions the more probable conclusion is the stronger. In the case of hypotheses adopted presumptively on probation, one of the very elements of their strength lies in the absence of any other hypothesis; so that the above definition of strength cannot be applied, even in imagination, without imagining the strength of the presumption to be considerably reduced. Perhaps we might conceive the strength, or urgency, of a hypothesis as measured by the amount of wealth, in time, thought, money, etc., that we ought to have at our disposal before it would be worth while to take up that hypothesis for examination. In that case it would be a quantity dependent upon many factors. Thus a strong instinctive inclination towards it must be allowed to be a favouring circumstance, and a disinclination an unfavourable one. Yet the fact that it would throw a great light upon many things, if it were established, would be in its favour; and the more surprising and unexpected it would be to find it true, the more light it would generally throw. The expense which the examination of it would involve must be one of the main factors of its urgency.

Peirce: CP 2.781 Cross-Ref: ††

781. Returning to the matter of validity, an argument professing to be necessary is valid in case the premisses could not under any hypothesis, not involving contradiction, be true, without the conclusion being also true. If this is so in fact, while the argument fails to make it evident, it is a bad argument rhetorically, and yet is valid; for it absolutely leads to the truth if the premisses are true. It is thus possible for an argument to be valid and yet bad. Yet an argument ought not to be called bad because it does not elucidate steps with which readers may be assumed to be familiar. A probable deductive argument is valid, if the conclusions of precisely such arguments (from true premisses) would be true, in the long run, in a proportion of times equal to the probability which this argument assigns to its conclusion; for that is all that is pretended. Thus, an argument that out of a certain set of sixty throws of a pair of dice about to be thrown, about ten will probably be doublets, is rendered valid by the fact that if a great number of just such arguments were made, the immense majority of the conclusions would be true, and indeed ten would be indefinitely near the actual average number in the long run. The validity of induction is entirely different; for it is by no means certain that the conclusion actually drawn in any given case would turn out true in the majority of cases where precisely such a method was followed; but what is certain is that, in the majority of cases, the method would lead to *some* conclusion that was true, and that in the individual case in hand, if there is any error in the conclusion, that error will get corrected by simply persisting in the employment of the same method. The validity of an inductive argument consists, then, in the fact that it pursues a method which, if duly persisted in, must, in the very nature of things, lead to a result indefinitely approximating to the truth in the long run. The validity of a presumptive adoption of a hypothesis for examination consists in this, that the hypothesis being such that its consequences are capable of being

tested by experimentation, and being such that the observed facts would follow from it as necessary conclusions, that hypothesis is selected according to a method which must ultimately lead to the discovery of the truth, so far as the truth is capable of being discovered, with an indefinite approximation to accuracy.

Peirce: CP 2.782 Cross-Ref:††
§3. PROOF †1

782. An argument which suffices to remove all real doubt from a mind that apprehends it.

Peirce: CP 2.782 Cross-Ref:††

It is either mathematical demonstration; a probable deduction of so high probability that no real doubt remains; or an inductive, i.e., experimental, proof. No presumption can amount to proof. Upon the nature of proof see Lange, *Logische Studien*, who maintains that deductive proof must be mathematical; that is, must depend upon observation of diagrammatic images or schemata. Mathematical proof is probably accomplished by appeal to experiment upon images or other signs, just as inductive proof appeals to outward experiment.

Peirce: CP 2.783 Cross-Ref:††
§4. PROBABLE INFERENCE †2

783. Any inference which does not regard its own conclusion as being necessarily true (though the facts be as the premisses assert).

Peirce: CP 2.783 Cross-Ref:††

In such an inference the facts asserted in the premisses are regarded as constituting a sign of the fact stated in the conclusion in one or other of three senses, as follows: *i.e.*, that relation of the premisses to the concluded fact which is regarded as making the former a sign of the latter (1) may be such as could not exist until the conclusion was problematically recognized; this is inductive or experimental inference. Such a relation (2) may be altogether irrespective of whether the conclusion is recognized or not, yet such that it could not subsist if the concluded fact were not probable; this is probable deduction. Such a relation (3) may consist merely in the premisses having some character which may agree with, or be in some other relation to, a character which the concluded fact would possess if it existed; this is presumptive inference.

Peirce: CP 2.784 Cross-Ref:††

784. (1) The first case is that in which we begin by asking how often certain described conditions will, in the long run of experience, be followed by a result of a predesignate description; then proceeding to note the results as events of that kind present themselves in experience; and finally, when a considerable number of instances have been collected, inferring that the general character of the whole endless succession of similar events in the course of experience will be approximately of the character observed. For that endless series must have some character; and it

would be absurd to say that experience has a character which is never manifested. But there is no other way in which the character of that series can manifest itself than while the endless series is still incomplete. Therefore, if the character manifested by the series up to a certain point is not that character which the entire series possesses, still, as the series goes on, it must eventually tend, however irregularly, towards becoming so; and all the rest of the reasoner's life will be a continuation of this inferential process. This inference does not depend upon any assumption that the series will be endless, or that the future will be like the past, or that nature is uniform, nor upon any material assumption whatever.

Peirce: CP 2.784 Cross-Ref:††

Logic imposes upon us two rules in performing this inference. The first is this: so far as in us lies, the conditions of the experience should remain the same. For we are reasoning exclusively from *experience*, that is, from the cognitions which the history of our lives *forces* upon us. So far as our will is allowed to interfere, it is not experience; so we must take pains that we do not, in taking the instances from which we are to reason, restrict the conditions or relax them from those to which the question referred. The second prescription of logic is that the conclusion be confined strictly to the question. If the instances examined are found to be remarkable in any other respect than that for which they were selected, we can draw no inference of the present kind from that. It would be merely an infinitely weaker inference of the third kind (below). The present kind of inference derives its great force from the circumstance that the result is virtually predicted.

Peirce: CP 2.785 Cross-Ref:††

785. (2) The second kind of probable inference is, by the definition of it, necessary inference. But necessary inference may be applied to probability as its subject-matter; and it then becomes, under another aspect, probable inference. If of an endless series of possible experiences a definite proportion will present a certain character (which is the sort of fact called an objective probability), then it necessarily follows that, foreseen or not, approximately the same proportion of any finite portion of that series will present the same character, either as it is, or when it has been sufficiently extended. This is governed by precisely the same principle as the inductive inference, but applied in the reverse way. The same prescriptions of logic apply as before; but, owing to that being now inferred which was in the other case a premiss, and conversely, it is not here true that the relation of the facts laid down in the premisses to the fact stated in the conclusion, which makes the former significant of the latter, requires the recognition of the conclusion. This is probable deduction. It covers all the ordinary and legitimate applications of the mathematical doctrine of probability.

Peirce: CP 2.785 Cross-Ref:††

The legitimate results of the calculus of probability are of enormous importance, but others are unfortunately vitiated by confusing mere likelihood, or subjective probability, with the objective probability to which the theory ought to be restricted. An objective probability is the ratio in the long run of experience of the number of events which present the character of which the probability is predicated to the total number of events which fulfill certain conditions often not explicitly stated, which all the events considered fulfill. But the majority of mathematical treatises on probability follow Laplace in results to which a very unclear conception of probability led him. Laplace and other mathematicians, though they regard a probability as a ratio of two numbers, yet, instead of holding that it is the limiting

ratio of occurrences of different kinds in the course of experience, hold that it is the ratio between numbers of "cases," or special suppositions, whose "possibilities" (a word not clearly distinguished, if at all, from "probabilities") are equal in the sense that we are aware of no reason for inclining to one rather than to another. This is an error often appearing in the books under the head of "inverse probabilities."

Peirce: CP 2.786 Cross-Ref:††

786. (3) Probable inference of the third kind includes those cases in which the facts asserted in the premisses do not compel the truth of the fact concluded, and where the significant observations have not been suggested by the consideration of what the consequences of the conclusion would be, but have either suggested the conclusion or have been remarked during a search in the facts for features agreeable or conflicting with the conclusion. The whole argument then reduces itself to this, that the observed facts show that the truth is *similar* to the fact asserted in the conclusion. This may, of course, be reinforced by arguments of some other kind; but we should begin by considering the case in which it stands alone. As an example to fix ideas, suppose that I am reading a long anonymous poem. As I proceed, I meet with trait after trait which seems as if the poem were written by a woman. In what way do the premisses justify the acceptance of that conclusion, and in what sense? It does not necessarily, nor with any necessitated objective probability, follow from the premisses; nor must the method eventually lead to the truth. The only possible justifications which it might have would be that the acceptance of the conclusion or of the method might necessarily conduce, in the long run, to such attainment of truth as might be possible by any means, or else to the attainment of some other purpose. All these alternatives ought to be carefully examined by the logician in order that he may be assured that no mode of probable inference has been overlooked.

Peirce: CP 2.786 Cross-Ref:††

It appears that there is a mode of inference in which the conclusion is accepted as having some chance of being true, and as being at any rate put in such a form as to suggest experimentation by which the degree of its truth can be ascertained. The only method by which it can be proved that a method, without necessarily leading to the truth, has some tolerable chance of doing so, is evidently the empirical, or inductive, method. Hence, as induction is proved to be valid by necessary deduction, so this presumptive inference must be proved valid by induction from experience.

Peirce: CP 2.786 Cross-Ref:††

The presumptive conclusion is accepted only problematically, that is to say, as meriting an inductive examination. The principal rule of presumption is that its conclusion should be such that definite consequences can be plentifully deduced from it of a kind which can be checked by observation. Among the wealth of methods to which this kind of inference (perhaps by virtue of its experiential origin) gives birth, the best deserving of mention is that which always prefers the hypothesis which suggests an experiment whose different possible results appear to be, as nearly as possible, equally likely.

Peirce: CP 2.787 Cross-Ref:††

787. Among probable inferences of mixed character, there are many forms of great importance. The most interesting, perhaps, is the argument from Analogy, in which, from a few instances of objects agreeing in a few well-defined respects, inference is made that another object, known to agree with the others in all but one of those respects, agrees in that respect also.

Peirce: CP 2.788 Cross-Ref:††
§5. PREDESIGNATE †1

788. (A word formed by Sir W. Hamilton by composition from Lat. *prae*, in front of, and *designatus*, marked out): (1) A term applied by Hamilton to verbal propositions whose quantity, as universal or particular, is expressed (*Lectures on Logic*, xiii).

Peirce: CP 2.789 Cross-Ref:††

789. (2) By C. S. Peirce applied to relations, characters, and objects which, in compliance with the principles of the theory of probability, are in probable reasonings specified in advance of, or, at least, quite independently of, any examination of the facts. See Probable Inference [785].

Peirce: CP 2.790 Cross-Ref:††

790. For example, the laws of England will, in the long run, cause the majority of English sovereigns to be males. In that sense it was unlikely that the successor of William IV would be a queen. But it would be absurd to say this after knowing that there was no heir to the crown so near as the Princess Victoria; and, in like manner, to say that it was not very unlikely that Queen Victoria's successor would be a queen was true enough as long as the character of her progeny was not known, or, if known was not taken account of, but false considering the number of her sons and grandsons. In such cases of deductive probable inference the necessity of the predestination is too obvious to be overlooked. But in indirect statistical inferences, which are mere transformations of similar deductive consequences, and the validity of which, therefore, depends upon precisely the same conditions, the necessity of the predestination is more often overlooked than remarked. Thus Macaulay, in his essay on the inductive philosophy, collects a number of instances of Irish whigs--which we may suppose constitute a random sample, as they ought, since they are to be used as the basis of an induction. By the exercise of ingenuity and patience, the writer succeeds in finding a character which they all possess, that of carrying middle names; whereupon he seems to think that an unobjectionable induction would be that all Irish whigs have middle names. But he has violated the rule, based on the theory of probabilities, that the character for which the samples are to be used as inductive instance must be specified independently of the result of that examination. Upon the same principle only those consequents of a hypothesis support the truth of the hypothesis which were predicted, or, at least, in no way influenced the character of the hypothesis. But this rule does not forbid the problematic acceptance of a hypothesis which has nothing to do with the theory of probability.

Peirce: CP 2.791 Cross-Ref:††
§6. PRESUMPTION †1

791. In logic: a more or less reasonable hypothesis, supported, it may be, by circumstances amounting all but to proof, or, it may be, all but baseless.

Peirce: CP 2.791 Cross-Ref:††

Logical or philosophical presumption is non-deductive probable inference which involves a hypothesis. It might very advantageously replace hypothesis in the sense of something supposed to be true because of certain facts which it would account for. See Probable Inference [786].

Peirce: CP 2.792 Cross-Ref:††

APPENDIX

MEMORANDA CONCERNING THE ARISTOTELIAN SYLLOGISM †1

792. The Quantity of Propositions is the respect in which *Universal* and *Particular* Propositions differ. The Quality of Propositions is the respect in which *Affirmative* and *Negative* Propositions differ.

NAMES AND SIGNS FOR PROPOSITIONS.

Universal Affirmative: **A**: Any S is P.

Particular Affirmative: **I**: Some S is P.

Universal Negative: **E**: Any S is not P.

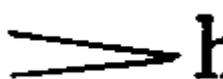
Particular Negative: **O**: Some S is not P.

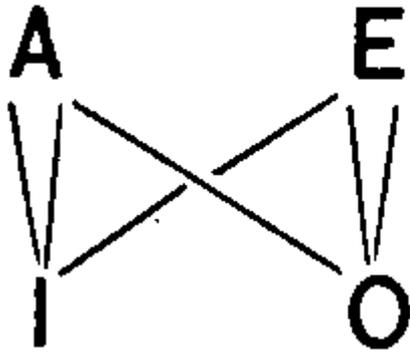
Terms occupying the places of S and P in the above, are called the logical *Subject* and *Predicate*.

Peirce: CP 2.793 Cross-Ref:††

RELATIONS OF PROPOSITIONS

793. In the following diagram, the different propositions are supposed to have the same logical Subject and Predicate. The lines connecting **A** with **O**, and **E** with **I**, are meant to indicate that these connected propositions contradict one another. The

sign  [Click here to view] has its broad end towards a proposition which implies another, and its point toward the proposition implied.



[Click here to view]

Peirce: CP 2.794 Cross-Ref:††
 RULE, CASE, AND RESULT.

794. A syllogism in the first figure argues from a **Rule**, and the subsumption of a **Case**, to the **Result** of that rule in that case.

Rule: Any man is mortal,
Case: Napoleon III is a man;
Result: ∴ Napoleon III is mortal.

Peirce: CP 2.794 Cross-Ref:††

The Rule must be universal; and the Case affirmative. And the subject of the Rule must be the predicate of the Case. The Result has the quality of the Rule and the quantity of the Case; and has for its subject the subject of the Case, and for its predicate the predicate of the Rule.

THE THREE FIGURES.

Figure 1.

Assertion of Rule, **A E**
 Assertion of Case; **A I**

Assertion of Result. **E A O I**

Figure 2.

Assertion of Rule, **A E**
Denial of Result; **O I E A**
Denial of Case. **O E**

Figure 3.

Denial of Result, **I O A E**
Assertion of Case; **A I**
Denial of Rule. **O I**

Peirce: CP 2.794 Cross-Ref:††

The letters A, E, I, O, in the above diagram are so arranged that inferences can be made along the straight lines.

Peirce: CP 2.795 Cross-Ref:††

795. It is important to observe that the second and third figures are *apagogical*, that is, infer a thing to be false in order to avoid a false result which would follow from it. That which is thus reduced to an absurdity is a Case in the second figure, and a Rule in the third.

Peirce: CP 2.795 Cross-Ref:††

To *contrapose* two terms or propositions is to transpose them, and at the same time substitute for each its contradictory. The second figure is derived from the first by the contraposition of the Case and Result, the third by the contraposition of the Rule and Result. The Rule and Case of the first figure cannot be contraposed, because they already occupy the same logical position, namely, that of a *premiss*; their contraposition in either of the other figures converts these figures into one another.

Peirce: CP 2.795 Cross-Ref:††

Let F, S, T denote syllogisms of the first, second, and third figures, respectively. And let s, t, f denote the processes of contraposition of the Case and Result, Rule and Result, and Rule and Case, respectively. Then

$$sF = S \quad sS = F$$

$$tF = T \quad tT = F$$

$$fS = T \quad fT = S$$

$$s^2 = t^2 = f^2 = 1$$

$$f = st = ts \quad s = ft = tf \quad t = fs = sf$$

Peirce: CP 2.796 Cross-Ref:††

796. The following table exhibits all the moods of Aristotelian syllogism (varieties resulting from variations of the Quantity and Quality of the propositions). Enter at the top, the proposition asserting or denying the rule; enter at the side, the proposition asserting or denying the case; find in the body of the table the proposition asserting or denying the result. In the body of the table, propositions indicated by italics belong to the first figure, those by black letter to the second figure, and those by script to the third figure.

	I	A	E	O
E		E	A	
A	<i>I</i>	A	E	<i>O</i>
I	<i>A</i>	I	O	<i>E</i>
O		E	I	

[Click here to view]

Peirce: CP 2.796 Cross-Ref:††

Two moods of the third figure, namely, A A I and E A O, are omitted, for two reasons. The first is that they correspond by contraposition to two moods in the first figure, A A I and E A O, never given by logicians, who, therefore, act inconsistently in admitting these. The second reason is, that, like those moods in the first figure, they are virtually enumerated already, if the change of a proposition from universal to particular be not an inference; but if it be, then, again like those moods of the first figure, the argument they embody may be analyzed into a syllogism and an inference from universal to particular.

Peirce: CP 2.797 Cross-Ref:††

797. The celebrated lines of William Shyreswood (?) are here given. The vowels of the first three syllables of each word indicate the three propositions of the syllogisms. He enumerates, along with the moods of the first figure, the Theophrastean moods (two of which we omit for the same reason that we do those

two in the third figure):

Barbara: Celarent: Darii: Ferio: Baralipon:

Celantes: Dabitis: Fapesmo: Frisesomorum:

Cesare: Camestres: Festino: Baroco: Darapti:

Felapton: Disamis: Datisi: Bocardo: Ferison.

Peirce: CP 2.798 Cross-Ref:††

798. The diagram below shows the relations in which the second and third figures stand to the first. In order to understand the seven syllogistic formulas there set down, it is necessary to notice that propositions may be divided into four parts: first the *Any* or *Some*, second the Subject, third the *is* or *is not*, and fourth the Predicate. When a proposition admits of varieties in either of these parts, they are shown in the diagram by two words or letters, one above the other, as *is/is not* in the rule of the first figure. Two independent variations may occur in one formula, and the variations of different parts are independent, but in the same part either the upper or lower line must always be read, in any one syllogism.

Peirce: CP 2.798 Cross-Ref:††

For example, the result in the first figure has four forms: any *or* some *S is or* is not *P*; but if *Some* has been read in the Case, *Some* must also be read in the Result. So, in the second figure, where a variation is possible in the quality of either premiss; but the same line of the third part of both propositions must be taken.

Figure 1

Figure 2

Figure 3

Figure 1.

Any	M	is is not	P
Any ome	S	is	M
Any ome	S	is is not	P

[Click here to view]

Figure 2.

Any **M** is **P**
 Some **S** is **P**
 Some **S** is not **M**

Any ^{not}**P** is not **M**
 Some **S** is ^{not}**P**
 Some **S** is not **M**

Any ^{not}**P** is not ^{some}**S**
 Any **M** is ^{not}**P**
 Any **M** is not ^{some}**S**

[Click here to view]

Figure 3.

Some **S** is not **P**
 Any **S** is **M**
 Some **M** is not **P**

Any ^{Some}**S** is not **P**
 Some **M** is ^{some}**S**
 Some **M** is not **P**

Any ^{some}**S** is **M**
 Some ^{not}**P** is ^{some}**S**
 Some ^{not}**P** is **M**

[Click here to view]

Peirce: CP 2.798 Cross-Ref:††

At the top of the diagram are given the formulæ of the first figure, and of the second and third, as derived from that of the first by contraposition of the propositions. Under the second and third figures, respectively, are given forms expressing the same arguments in the first figure. It is necessary to study carefully the manner in which this reduction to the first figure is effected.

Peirce: CP 2.799 Cross-Ref:††

799. It will be perceived that the arrangements of the terms in the three figures, as determined by the rules given in 794, are as follows: where the first letter of each pair indicates the subject of a proposition of the syllogism and the second its predicate:

Figure 1. Figure 2. Figure 3.

First. {B} {A} {N} {M} {S} {P}

Second. {G} {B} {X} {M} {S} {R}

Third. {G} {A} {X} {N} {R} {P}

Peirce: CP 2.800 Cross-Ref:††

800. It is plain that there are two ways of transposing the arrangements of the terms of the second and third figures without removing a term from the conclusion, so as to give the term the same arrangement as that of the first figure. This is shown in

the following table, where the columns headed *s* show the propositions whose terms are to be transposed, while those headed *m* show the propositions to be transposed.†P1

Figure 2. Figure 3.									

s		m		s		m			
Short Reduction		1st				2d			
Long Reduction		2d 3d		2d 1st		1st 3d		1st 2d	

Peirce: CP 2.800 Cross-Ref:††
 The effect of these transpositions is here shown.

SECOND FIGURE

<i>Short Reduction</i>		<i>Long Reduction</i>	
{N} {M}	{M} {N}	{M} {X}	{X} {M}
{X} {M}	{X} {M}	{N} {M}	{M} {N}
{X} {N}	{X} {N}	{N} {X}	{X} {N}

THIRD FIGURE

Short Reduction		Long Reduction	
{S} {P}	{S} {P}	{S} {R}	{R} {S}
{S} {R}	{R} {S}	{P} {S}	{S} {P}
{R} {P}	{R} {P}	{P} {R}	{R} {P}

Peirce: CP 2.800 Cross-Ref:††
 It must next be shown how these transpositions may be made, in syllogisms

themselves.

Peirce: CP 2.801 Cross-Ref:††

801. The short reduction of the second figure is shown in the second syllogism of that column of the large diagram headed Figure 2. The term not-P is introduced. This we define as that class to which some or any S belongs, when it is not P. Accordingly, for "some or any S is not P," we can substitute "some or any S is not-P," and this substitution is made in the reduction. But we cannot, on that account, substitute "any M is not-P" for "any M is not P." For "any M is not P," is substituted, in the reduction, "any P is not M;" and for "any M is P" is substituted "any not-P is not M." The only syllogisms by which these substitutions can be justified are these:

$$\begin{array}{ll} \text{Any M is not P,} & \text{Any M is P,} \\ \text{Any P is P;} & \text{Any not-P is not P;} \end{array}$$

\therefore Any P is not M. \therefore Any not-P is not M.

Both these are syllogisms in the second figure.

Peirce: CP 2.802 Cross-Ref:††

802. The short reduction of the third figure is shown in the second syllogism of the column headed Figure 3. The term some-S is introduced. The definition of this term is that it is that part of S which is or is not P when some S is or is not P. Hence, we can and do substitute "Any some-S is or is not P" for "Some S is or is not P," though we could not substitute "Any some-S is M" for "Some S is M." For "Some S is M" we substitute "Some M is S"; and for "Any S is M" we substitute "Some M is some-S"; and these substitutions are justified by inferences which can be expressed syllogistically only thus:

$$\begin{array}{ll} \text{Any S is S,} & \text{Some S is some-S,} \\ \text{Some S is M;} & \text{Any S is M;} \end{array}$$

\therefore Some M is S. \therefore Some M is some-S.

These are both syllogisms in the third figure.

Peirce: CP 2.803 Cross-Ref:††

803. The long reduction of the second syllogism is shown in the third syllogism of the column headed Figure 2. Here not-P is defined as that class to which any M belongs which is not P. Hence we can substitute "Any M is not-P" for "Any M is not P." Some-S is defined as in the short reduction of the third figure. Hence, for "Some S is or is not P," we can say "Any Some-S is or is not P." Then, we use the inferences which are expressed syllogistically, thus:

Some Some
 Any S is not P, Any S is P,

Any P is P; Any not P is not P;

Some S Some S
 ∴ Any P is not S ∴ Any not P is not S.

Peirce: CP 2.803 Cross-Ref:††

These are both syllogisms of the second figure. Substituting their conclusions for the second premiss of the second figure and transposing the premisses we obtain the premisses of the reduction. The conclusion of the reduction justifies that of the second figure, by inferences which are expressed syllogistically as follows:

Any M is not some-S, Any M is not S,
 Some S is some-S; Any S is S;

∴ Some S is not M. ∴ Any S is not M.

Both these are syllogisms of the second figure.

Peirce: CP 2.804 Cross-Ref:††

804. The long reduction of the third figure is shown in the third syllogism of the column headed Figure 3. Some S is here defined as that part of S which is M when some S is M. Hence, for "Some S is M," we can substitute "Any Some-S is M." Not-P is defined as in the short reduction of the second figure. Hence, in place of "Some or any S is not P," we can put "Some or any S is not-P." In place of "Some S is P or not-P" we again substitute "Some P or not-P is S," and in place of "Any S is P or not-P" we substitute "Some P or not-P is some-S," in virtue of inferences which are expressed syllogistically thus:

Any S is S, Some S is some-S,

Not P not P
 Some S is P; Any S is P;

not P not P

∴ Some P is S ∴ Some P is some S.

These are syllogisms of the third figure.

Peirce: CP 2.804 Cross-Ref:††

Then, the premisses being transposed, we have the premisses of the reduction. The conclusion of the reduction justifies that of the third figure by inferences which are expressed syllogistically thus:

Any not-P is P, Any P is P,
 Some not-P is M; Some P is M;

∴ Some M is not-P. ∴ Some M is P.

These are syllogisms of the third figure.

Peirce: CP 2.805 Cross-Ref:††

805. The reduction called *reductio per impossibile* is nothing more than the repetition or inverse repetition of that contraposition by which the second and third figures have been obtained. It is not *ostensive* (that is, does not yield an argument with essentially the same premisses and conclusion as that of the argument thus to be reduced), but apagogical, that is, shows by the first figure that the contradiction of the conclusion of the second or third leads to the contradiction of one of the premisses. Contradiction arises from a difference in both quantity and quality. But it is to be observed that in the contraposition which gives the second figure, a change of the *quality* alone, and in that which gives the third figure, a change of the *quantity* alone of the contraposed propositions is sufficient. This shows that the two contrapositions are of essentially different kinds. The reductions *per impossibile* of the second and third figures respectively involve, therefore, these inferences:

Figure 2.

The Result follows from the Case;

∴ The negative of the Case follows from the negative of the Result.

Figure 3.

The Result follows from the Rule;

∴ The Rule changed in Quantity follows from the Result changed in Quantity.

These inferences may also be expressed thus:

Figure 2.

P;

Whatever (S) is M is not P;

not P

∴ Whatever (S) is P is not M.

Figure 3.

Any some S is whatever (P or not-P) M is;

some S

∴ Some M is whatever (P or not P) S is.

Peirce: CP 2.805 Cross-Ref:††

And if we omit the limitations in parentheses, which do not alter the essential nature of the inferences, we have:

Figure 2.

P;

Any M is not P;

not P

∴ Any P is not M.

Figure 3.

S

Any some S is M;

Some S.

∴ Some M is S.

Peirce: CP 2.805 Cross-Ref:††

We have seen above that the former of these can only be reduced to a syllogism in the second figure, and the latter only to one in the third figure.

Peirce: CP 2.806 Cross-Ref:††

806. The ostensive reductions of each figure are also apagogical reductions of the other. There are also the following:

not S, some S,

Any not-M is not some S, Any some-M is S,

not P some P

Any P is not-M; Any some not P is some-M;

not P not S. some P some S.

Any P is not some S. Any some not P is S.

Peirce: CP 2.806 Cross-Ref:††

But all these reductions involve the peculiar inferences we have found in those which have been examined, inasmuch as they are but complications of the latter.

Peirce: CP 2.807 Cross-Ref:††

807. Hence, it appears that no syllogism of the second or third figure can be reduced to the first, without taking for granted an inference which can only be expressed syllogistically in that figure from which it has been reduced. These inferences are not strictly syllogistic, because one of the propositions taken as a

premiss in the syllogistic expression is a logical fact. But the fact that each can only be expressed in the second or third figure of syllogism, as the case may be, shows that those figures alone involve the respective principles of those inferences. Hence, it is proved that every figure involves the principle of the first figure, but the second and third figures contain other principles, besides.

Peirce: CP 2.1 Fn 1 p 3

†1 "Intended Characters of this Treatise," ch. 1 of the "Minute Logic" (1902).

Peirce: CP 2.1 Fn 2 p 3

†2 See 200.

Peirce: CP 2.3 Fn 1 p 4

†1 See 186ff.

Peirce: CP 2.4 Fn 2 p 4

†2 Cf. vol. 1, bk. IV, ch. 5.

Peirce: CP 2.4 Fn 3 p 4

†3 See vol. 6, bk. II, ch. 7.

Peirce: CP 2.4 Fn 4 p 4

†4 See 4.114ff.

Peirce: CP 2.4 Fn 5 p 4

†5 See 6.304ff.

Peirce: CP 2.4 Fn 6 p 4

†6 See 101ff. and bk. III *passim*.

Peirce: CP 2.7 Fn 1 p 5

†1 For an extended discussion of this "Pragmatic" doctrine see vol. 5.

Peirce: CP 2.7 Fn P1 p 5 Cross-Ref:††

†P1 The latter word is not, at least to one individual whom I wot of, particularly pleasing. The verb, *normo*, to square, is in the dictionary, but what ordinary reader of Latin can remember having met with it? Yet if the presumable motive for the substitution of the new adjective, namely, its avoidance of an apparent implication in "*directive*" that logic is a mere art, or practical science, approves itself to us, the twentieth century would laugh at us if we were too squeamish about the word's legitimacy of birth. [The *Vocabulaire de la Philosophie*, ed. by A. Lalande, vol. 2, p. 521, attributes the creation of this word, or at least its introduction into common speech, to Wundt. The Century Dictionary gets normative from *normo*, but not all the other standard works agree on this.]

Peirce: CP 2.9 Fn 1 p 6

†1 Interesting papers on Benjamin Peirce are to be found in *Benjamin Peirce, Biographical Sketch and Bibliography*, Mathematical Association of America, (1925), Oberlin, Ohio.

Peirce: CP 2.9 Fn 1 p 7

†1 Royce was originally specified, his name being deleted, apparently, at the suggestion of William James. A number of similar changes seem to have been made throughout this chapter.

Peirce: CP 2.9 Fn 1 p 7

†1 See Preface to vol. 6 for Peirce's views regarding "scientific" metaphysics.

Peirce: CP 2.11 Fn P1 p 8 Cross-Ref:††

†P1 "*Mem.* Before this goes to press I have to go over three books: 1.

Barthélemy St. Hilaire's Ed. of Aristotle's *Historia Animal*; 2. Littré's *Hippocrates*; 3. The best German history of medicine"--marginal note.

Peirce: CP 2.12 Fn 1 p 8

†1 See the *Opus Majus*, Parts I and VI.

Peirce: CP 2.19 Fn 1 p 10

†1 *Logic* (1895), translated by H. Dendy.

Peirce: CP 2.19 Fn 2 p 10

†2 *Algebra der Logik*. There is no fourth volume. Volume 2.2, which perhaps was what was meant, however, was published posthumously in 1905.

Peirce: CP 2.19 Fn 3 p 10

†3 Cf. *Logik*, §3, 1.

Peirce: CP 2.20 Fn 1 p 11

†1 See also 5.85ff.

Peirce: CP 2.21 Fn 2 p 11

†2 Cf., e.g., *La Logique*, Paris (1855), vol. 2, pp. 196-7.

Peirce: CP 2.22 Fn 1 p 12

†1 See, e.g., 769, 1.118 and 6.487ff.

Peirce: CP 2.24 Fn P1 p 13 Cross-Ref:††

†P1 *Lao-Tze's Tao-Teh-King*, Paul Carus, Chicago (1898), ch. 52, sec. 3.

Peirce: CP 2.26 Fn 1 p 13

†1 In his *Aristotle*, 2d ed. (1880), p. 259, and p. 562.

Peirce: CP 2.26 Fn P1 p 14 Cross-Ref:††

†P1 Indeed this is precisely the position he takes in the *Metaphysics* Γ iii, 1005b, 19.

Peirce: CP 2.27 Fn 1 p 14

†1 In *Posterior Analytics* I, ii.

Peirce: CP 2.27 Fn 1 p 15

†1 Cf. Lewis Carroll's "What the Tortoise said to Achilles," *Mind*, N. S. vol. 4, p. 278.

Peirce: CP 2.28 Fn 1 p 17

†1 *Discourse on Method*, pts. 2 and 4; *Principles of Philosophy*, I, 45, 46.

Peirce: CP 2.28 Fn 2 p 17

†2 "Meditationes de Cognitione, Veritate et Ideis" ed. Gerhardt, vol. 4, p. 422;

transl. by G. M. Duncan in *Philosophical Works of Leibniz*, article III. Cf. also *Discours de la Métaphysique*, sec. 24 and *Nouveaux Essais*, II, ch. 29.

Peirce: CP 2.29 Fn 1 p 18

†1 See, e.g., Herbert Spencer, *First Principles*, Appleton & Co., N.Y., 4th ed. (1882), pp. 34-36.

Peirce: CP 2.29 Fn 2 p 18

†2 Cf. *Logic*, bk. II, ch. 5, sec. 6.

Peirce: CP 2.29 Fn 3 p 18

†3 Though the text is unmistakable, this should be "true" and not "false." See below and 48.

Peirce: CP 2.30 Fn 1 p 19

†1 The fact that part of all the integers can be put in one-to-one correspondence with all the integers was known long before Cantor, though not the same use was made of the information. See, e.g., Galileo's *Mathematical Discourses*, Weston's translation, p. 46; Renouvier, *Année Philosophique* (1868), p. 37.

Peirce: CP 2.32 Fn 1 p 22

†1 1852-1919. Editor of the *Open Court* and the *Monist*.

Peirce: CP 2.34 Fn 2 p 22

†2 And the highest of all possible aims is to further concrete reasonableness. See 1.602, 1.615, 5.121, 5.433.

Peirce: CP 2.36 Fn 1 p 23

†1 See 1.487, 1.624, 1.625.

Peirce: CP 2.37 Fn 2 p 23

†2 The 11th Scholarch of the Peripatetics, circa 70 B.C.

Peirce: CP 2.37 Fn P1 p 23 Cross-Ref:††

†P1 "Organon" was first the name of the science, given to it by the early Peripatetics, because logic did not satisfy Aristotle's definitions either of Science or of Art. I fully accept the usual story about Aristotle's writings lying some centuries *perdus* until they were rescued and put in order first by Apellicon [circa 90 B.C., a famous book collector of Athens] and later by Andronicus. I consider the rejection of this story and consequent partial refusal to admit the authenticity of Aristotle's works as we have them, one of the extravagancies of "higher criticism."

Peirce: CP 2.38 Fn 1 p 24

†1 *Syntagma philosophicum*, pt. I, (1658).

Peirce: CP 2.38 Fn 2 p 24

†2 See T. Gomperz, *Philodemi de ira Libra*, (1864).

Peirce: CP 2.38 Fn 3 p 24

†3 *La Logique*, (1712).

Peirce: CP 2.38 Fn 4 p 24

†4 *Logick*, (1724).

Peirce: CP 2.38 Fn 5 p 24

†5 *La Logique*, (1805) and *Langue des Calculs*, (1798).

- Peirce: CP 2.38 Fn 6 p 24
†6 *Logik oder Denklehre*, (1806).
- Peirce: CP 2.38 Fn 7 p 24
†7 *System der Logik* (1823).
- Peirce: CP 2.38 Fn 8 p 24
†8 E.g., F. Hoffmann was a follower of Baader; Reinhold, Forberg, Schad were followers of Fichte; while Klein and Troxler were followers of Schelling.
- Peirce: CP 2.39 Fn 1 p 25
†1 E.g. G. B. Bülfinger, L. P. Thümming.
- Peirce: CP 2.39 Fn 2 p 25
†2 *Critique of Pure Reason*. B, viii.
- Peirce: CP 2.39 Fn 3 p 25
†3 *Empirical Logic*, (1889).
- Peirce: CP 2.42 Fn 1 p 26
†1 See 5.244ff.
- Peirce: CP 2.44 Fn 2 p 26
†2 See 227, 364, 422 and 428.
- Peirce: CP 2.45 Fn 1 p 27
†1 Association is treated in detail in vol. 8.
- Peirce: CP 2.46 Fn 2 p 27
†2 Vol. 1, bk. II, ch. 2.
- Peirce: CP 2.47 Fn 3 p 27
†3 In his *Examination of Hamilton*, ch. XXI.
- Peirce: CP 2.54 Fn 1 p 30
†1 In 27.
- Peirce: CP 2.56 Fn 1 p 31
†1 Peirce seems to have considered the construction of a logical machine. He cut out and arranged a number of overlapping papers to represent specific arguments; but it does not appear that he ever completed it or devised a key. In vol. 8 Peirce gives a detailed criticism of logical machines.
- Peirce: CP 2.61 Fn 1 p 34
†1 *Logik*, (1892), Bd. I, S. 9.
- Peirce: CP 2.62 Fn 2 p 34
†2 Cf. 205
- Peirce: CP 2.64 Fn 1 p 35
†1 *Ibid.*, S.10.
- Peirce: CP 2.66 Fn 1 p 36
†1 See 1.333.
- Peirce: CP 2.69 Fn 1 p 37

†1 A. H. Sayce, *Introduction to the Science of Language* (1880), vol. 2, p. 329.

Peirce: CP 2.71 Fn 1 p 38

†1 Karl Pearson, in his *Grammar of Science* (1893), pp. 67-74.

Peirce: CP 2.77 Fn 1 p 40

†1 Ch. 3 of the "Minute Logic" published in vol. 4, bk. I, as No. 7.

Peirce: CP 2.77 Fn 2 p 40

†2 See vol. 4, bk. II.

Peirce: CP 2.77 Fn 3 p 40

†3 *Prior Analytics*, 1, 2a.

Peirce: CP 2.79 Fn 1 p 42

†1 The remainder of ch. 1 of the "Minute Logic."

Peirce: CP 2.79 Fn 2 p 42

†2 Not only was this book never completed, but many of the proposed discussions here outlined were never begun.

Peirce: CP 2.81 Fn 3 p 42

†3 E.g. Dedekind and Whitehead.

Peirce: CP 2.82 Fn 1 p 43

†1 See ch. 4 of the "Minute Logic" published in vol. 1, bk. IV as ch. 2.

Peirce: CP 2.84 Fn 1 p 44

†1 Vol. 1, bk. III contains a detailed study of the categories.

Peirce: CP 2.84 Fn 1 p 45

†1 Cf. 3.93n, 3.611ff, 6.6.

Peirce: CP 2.85 Fn 1 p 46

†1 Cf. 1.322.

Peirce: CP 2.86 Fn 1 p 48

†1 See, for example, *Die Mechanik*, ch. II, vi, 6 and 9.

Peirce: CP 2.87 Fn 1 p 49

†1 See, e.g., 1.298, 1.347.

Peirce: CP 2.91 Fn 1 p 51

†1 See, e.g., *Nomenclature and Divisions of Dyadic Relations*, Paper XVIII, vol. 3 for an extended treatment of Dyads.

Peirce: CP 2.95 Fn 1 p 52

†1 Icons can be only terms; indices can be only terms or propositions (dicisigns), while symbols can be all three.

Peirce: CP 2.95 Fn 1 p 53

†1 Today the rhema, or rheme, is conventionally symbolized as ϕx and is called a propositional function.

Peirce: CP 2.96 Fn P1 p 53 Cross-Ref:††

†P1 The reader should refer to the definitions of Index, Icon, and Symbol [in

92].

Peirce: CP 2.99 Fn 1 p 56

†1 "The Fixation of Belief" (1877) and "How to Make Our Ideas Clear" (1878), chs. 4 and 5 of bk. II, vol. 5. The maxim is first stated in the latter paper.

Peirce: CP 2.99 Fn 2 p 56

†2 i.e., in 1902-3.

Peirce: CP 2.100 Fn 3 p 56

†3 In bk. III.

Peirce: CP 2.101 Fn 1 p 57

†1 Bk. III, B.

Peirce: CP 2.101 Fn 2 p 57

†2 E.g. Laplace and Quetelet.

Peirce: CP 2.101 Fn 3 p 57

†3 See his *Logic of Chance*, (1866) and *Empirical Logic* (1889).

Peirce: CP 2.101 Fn 4 p 57

†4 See Keynes' *Treatise on Probability* for a bibliography of Edgeworth's writings.

Peirce: CP 2.102 Fn 1 p 58

†1 See, e.g., bk. III, ch. 9.

Peirce: CP 2.102 Fn 2 p 58

†2 Bk. III, ch. 8.

Peirce: CP 2.102 Fn 1 p 59

†1 That part of the "Minute Logic" was not written. But see vol. 5, bk. I, ch. 7.

Peirce: CP 2.105 Fn 1 p 60

†1 There is no systematic treatment of this subject. Remarks on the conditions of research and the principles of discovery are scattered through the volumes.

Peirce: CP 2.108 Fn 1 p 61

†1 Cf. 3.364.

Peirce: CP 2.108 Fn 2 p 61

†2 See vol. 1, bk. II, ch. 2, § 1-2.

Peirce: CP 2.111 Fn 1 p 62

†1 The closest approach to this subject is to be found in vol. 6, bk. I, ch. 7. But see 4.80.

Peirce: CP 2.113 Fn 1 p 64

†1 See Molière's *Le Malade imaginaire*.

Peirce: CP 2.118 Fn 1 p 66

†1 Cf. vol. 1, bk. IV.

Peirce: CP 2.119 Fn 1 p 67

†1 Second section of the second chapter, "Pre-logical Notions" of the "Minute Logic." For the first section, "The Classification of the Sciences," see vol. 1, bk. II,

ch. 2.

Peirce: CP 2.135 Fn 1 p 71

†1 See 1.172.

Peirce: CP 2.141 Fn P1 p 74 Cross-Ref:††

†P1 I speak here too much of myself personally. Many men and women have imaginations resembling percepts up to the point of being mistaken for them. In others imagination is less vivid, down to my point, who see no resemblance between an imagination and a percept except that somehow they can be compared.

Peirce: CP 2.141 Fn P2 p 74 Cross-Ref:††

†P2 For some people, however, this stenographic report seems to be illustrated with photographs. I can only adequately describe my own experience.

Peirce: CP 2.141 Fn P1 p 75 Cross-Ref:††

†P1 This I suppose would be true even for the most vivid imaginations.

Peirce: CP 2.142 Fn P2 p 75 Cross-Ref:††

†P2 Personally, until old age began to mark its effects, I hardly had any pictorial dreams; and even now they are mere scraps of images to which incongruous abstract ideas are attached.

Peirce: CP 2.142 Fn P1 p 76 Cross-Ref:††

†P1 Personally, I never had anything like a hallucination except in the delirium of fever.

Peirce: CP 2.144 Fn P2 p 76 Cross-Ref:††

†P2 I here use the word "criticize" in the philosophical sense. Criticism proper, literary criticism, does not necessarily approve or disapprove.

Peirce: CP 2.148 Fn 1 p 79

†1 442.

Peirce: CP 2.151 Fn 1 p 82

†1 Cf. 5.85ff.

Peirce: CP 2.151 Fn 2 p 82

†2 Cf. 5.3.

Peirce: CP 2.152 Fn 1 p 83

†1 E.g., Boole, De Morgan, Whewell, J. S. Mill, Jevons, Venn, Pearson, MacColl.

Peirce: CP 2.152 Fn 2 p 83

†2 E.g., Sigwart, Wundt, Schuppe, Erdmann, Bergmann, Glogau, Husserl.

Peirce: CP 2.163 Fn 1 p 95

†1 No such list has been found.

Peirce: CP 2.166 Fn 1 p 97

†1 See, e.g., Schröder's *Algebra der Logik*, 3.1; and the *Principia Mathematica*, *23ff.

Peirce: CP 2.168 Fn 1 p 99

†1 In his *Logica*, e.g., in I, xiv.

Peirce: CP 2.176 Fn 1 p 104

†1 Cf. 5.234.

Peirce: CP 2.177 Fn 1 p 105

†1 Cf. 1.672.

Peirce: CP 2.191 Fn 1 p 111

†1 See 614.

Peirce: CP 2.196 Fn 1 p 114

†1 Mathematics is dealt with in the next chapter of the "Minute Logic," published in vol. 4, bk. I, as no. VII.

Peirce: CP 2.197 Fn 2 p 114

†2 See vol. 1, bk. III.

Peirce: CP 2.197 Fn 1 p 115

†1 A classmate--"a noble-hearted, sterling-charactered young gentleman . . . almost the only real companion I have ever had."

Peirce: CP 2.197 Fn 2 p 115

†2 Only three and a half chapters of the "Minute Logic" were written, of which none is devoted to esthetics.

Peirce: CP 2.198 Fn 3 p 115

†3 See vol. 1, bk. IV and vol. 5, bk. I.

Peirce: CP 2.203 Fn 1 p 119

†1 *Dictionary of Philosophy and Psychology* (1901-2, 1911), edited by J. M. Baldwin, Macmillan, New York, vol. 2, pp. 20-23, by Peirce and Mrs. Ladd-Franklin.

Peirce: CP 2.204 Fn 1 p 120

†1 773ff.

Peirce: CP 2.205 Fn 2 p 120

†2 *Life of Aristotle*, bk. V, ch. 13.

Peirce: CP 2.205 Fn 1 p 121

†1 Cf. *Politicus*, 260.

Peirce: CP 2.205 Fn 2 p 121

†2 Cf. *Critik der Reinen Vernunft*, 2^{te} Auflage, Einleitung VII.

Peirce: CP 2.219 Fn 1 p 129

†1 *Syllabus of Certain Topics of Logic* (1903), pp. 10-14, Alfred Mudge & Son, Boston, Continuing 1.202. Cf. 5.413, 5.502, and vol. 5, Appendix §4.

Peirce: CP 2.227 Fn 1 p 134

†1 From an unidentified fragment, c. 1897.

Peirce: CP 2.228 Fn 1 p 135

†1 *E.g.*, in 1.551.

Peirce: CP 2.230 Fn 1 p 136

†1 From "Meaning," 1910.

Peirce: CP 2.233 Fn 1 p 138

†1 §§ 3-10 are from "Nomenclature and Divisions of Triadic Relations, as far as they are determined,"--a manuscript continuation of the "Syllabus," c. 1903. See note to vol. 1, Bk. II, ch. 1, and 3.608n.

Peirce: CP 2.235 Fn 1 Para 1/2 p 139

†1 On Peirce's principle that possibilities determine only possibilities and laws are determined only by laws, the terms "First Correlate" and "Third Correlate" should be interchanged in 235-38. In this way one secures, in harmony with other writings, the ten classes mentioned in 238. They are as follows:

If the Third Correlate is a possibility, then

	<i>First</i>	<i>Second</i>	<i>Third</i>
(I)1. Possibility	Possibility	Possibility	Possibility
(II)2. Existent		Possibility	Possibility
(III)3. Existent	Existent		Possibility
(V)4. Law	Possibility		Possibility
(VI)5. Law	Existent		Possibility
(VIII)6. Law	Law		Possibility

If the Second is an existent, then also

(IV)7. Existent	Existent		Existent
(VII)8. Law	Existent		Existent

If the First is a law, then also

(IX)9. Law	Law		Existent
(X)10. Law	Law		Law

By 242 and 274 the Representamen, Object, and Interpretant are the first, second, and third correlate respectively, while by 243ff. the representamen in itself, in relation to its object, and as interpreted, is the first, second, and third correlate respectively. The former division yields ten trichotomies and sixty-six classes of signs, the latter three trichotomies and ten classes of signs.

Peirce: CP 2.235 Fn 1 Para 2/2 p 139

The bracketed roman numerals in the above table give the order of discussion in §7 and the designations in the table in 264. See also 243n.

Peirce: CP 2.237 Fn 1 p 140

†1 The truth of this last clause is to be seen from case numbered 5 above. The truth of the rest of the proposition is to be seen from cases numbered 1, 7, and 10.

Peirce: CP 2.238 Fn 2 p 140

†2 The three ways are given in the notes to 243.

Peirce: CP 2.239 Fn 3 Para 1/2 p 140

†3 Though Peirce has laid down the condition that a dyadic relation to be an existent requires both its correlates to be existents (cf. 283), he does not seem ever to have given the conditions involved in determining a dyadic relation to be of the nature of a law. In fact, his usual view is that there are no such dyadic relations. However, what seems to be meant here is that a dyadic relation is of the nature of law if both its correlates are laws. If, in addition, we accept the unstated propositions that a dyadic relation is a possibility if one correlate is a possibility, while a dyadic relation is an existent if one correlate is an existent and the other a law, we should get the following table:

At least one dyadic relation of the nature of possibility:

First

Second

Third

1. Possibility-----Possibility-----Possibility

|-----|

2. Existent-----Possibility-----Possibility

|-----|

3. Existent.....2.....Existent-----Possibility

|-----|

4. Law-----Possibility-----Possibility

|-----|

5. Law.....2.....Existent-----Possibility

|-----|

6. Law.....3.....Law-----Possibility

|-----|

Those which have at least two existent dyadic relations:

7. Existent-----Existent-----Existent

|_____|

8. Law-----Existent-----Existent

|_____ |

9. Law.....3.....Law-----Existent

|_____ |

All dyadic relations are laws:

10. Law-----Law-----Law

|_____ |

Peirce: CP 2.239 Fn 3 Para 2/2 p 140

The black lines between the correlates are marks of the presence of the specified relation; the ".....2....." and ".....3....." stand for existential and rational dyadic relations respectively.

Peirce: CP 2.240 Fn 1 p 141

†1 I.e., 5 has its correlates all of different natures; 1, 7, 10 have their correlates all of the same nature; and the rest have two and only two correlates of the same nature; while 1, 2, 4, 7, 10, have all the dyadic relations of the same nature and 3, 5, 6, 8, 9, have only two of the same nature.

Peirce: CP 2.241 Fn 2 p 141

†2 In 1-6, the third correlate is determined by the first to have a quality; in 7-9, it is determined to have an existential relation to the second, and in 10 it is determined to have a relation of thought to the second for another correlate.

Peirce: CP 2.243 Fn 1 Para 1/2 p 142

†1 Peirce later (c. 1906, see e.g., 1.291, 4.530) discovered that there are ten trichotomies and sixty-six classes of signs. The analysis of the additional divisions was never satisfactorily completed; the best statement of them is to be found in the letters to Lady Welby, vol. 9. The present book, it is believed, contains most of Peirce's most thorough and authoritative work on signs.

Peirce: CP 2.243 Fn 1 Para 2/2 p 142

The ten classes of signs derived from the three trichotomies here given are diagrammatically presented by Peirce in 264. If "Representamen," "Representamen as related to object," and "Interpreted Representamen" be substituted for first, second, and third correlate respectively, the tables of 235n and 239n should prove helpful schemata in §4-§7. The present section treats of the firstness, secondness, and thirdness of the *Representamen*.

Peirce: CP 2.243 Fn 2 p 142

†2 If we make the suggested substitutions we get the three groups consisting of: I; II, III, IV; and V-X.

Peirce: CP 2.243 Fn 3 p 142

†3 I.e., I, II, V; III, IV, VI, VII; VIII, IX, X.

Peirce: CP 2.243 Fn 4 p 142

†4 I.e., The three groups of 241n--1-6, 7-9; 10 i.e., I, II, III, V, VI, VIII; IV, VII, IX; X.

Peirce: CP 2.250 Fn 1 p 144

†1 See the second note to 95.

Peirce: CP 2.252 Fn 1 p 145

†1 See 315.

Peirce: CP 2.253 Fn 1 p 146

†1 See 582-3.

Peirce: CP 2.264 Fn 1 p 150

†1 See 235n and 243n for explanation of the roman numerals.

Peirce: CP 2.271 Fn 1 p 154

†1 See ch. 1.

Peirce: CP 2.273 Fn 1 p 155

†1 *Dictionary of Philosophy and Psychology*, vol. 2, p. 464.

Peirce: CP 2.273 Fn 2 p 155

†2 303-4.

Peirce: CP 2.274 Fn 1 p 156

†1 274-7, 283-4, 292-4 are from "Syllabus," c. 1902, no part of which was ever published (cf. note to ch. 1). 278-80 are from "That Categorical and Hypothetical Propositions are one in essence, with some connected matters," c. 1895, following 339; 281, 285, 297-302 are from chapter 2 of "The Art of Reasoning," c. 1895, while 282, 286-91 and 295-6 are from "The Short Logic," c. 1893.

Peirce: CP 2.283 Fn 1 p 160

†1 "Seme" is usually reserved for indexical dicisigns which are only a subclass of the indices.

Peirce: CP 2.287 Fn 1 p 162

†1 *New Latin Grammar*, p. 131n (ed. 1884).

Peirce: CP 2.287 Fn P1 p 163 Cross-Ref:††

†P1 Modern grammars define a pronoun as a word used in place of a noun. That is an ancient doctrine which, exploded early in the thirteenth century, disappeared from the grammars for several hundred years. But the substitute employed was not very clear; and when a barbarous rage against medieval thought broke out, it was swept away. Some recent grammars, as *Allen and Greenough's*, set the matter right again. There is no reason for saying that *I, thou, that, this*, stand in place of nouns; they indicate things in the directest possible way. It is impossible to express what an assertion refers to except by means of an index. A pronoun is an

index. A noun, on the other hand, does not *indicate* the object it denotes; and when a noun is used to show what one is talking about, the experience of the hearer is relied upon to make up for the incapacity of the noun for doing what the pronoun does at once. Thus, a noun is an imperfect substitute for a pronoun. Nouns also serve to help out verbs. A pronoun ought to be defined as *a word which may indicate anything to which the first and second persons have suitable real connections, by calling the attention of the second person to it*. Allen and Greenough say, "pronouns indicate some person or thing without either naming or describing" [p. 128, edition of 1884]. This is correct--refreshingly correct; only it seems better to say what they *do*, and not merely what they don't.

Peirce: CP 2.290 Fn P1 p 164 Cross-Ref:††

†P1 If a logician had to construct a language *de novo*--which he actually has almost to do--he would naturally say, I shall need prepositions to express the temporal relations of *before*, *after*, and *at the same time with*, I shall need prepositions to express the spatial relations of *adjoining*, *containing*, *touching*, *of in range with*, *of near to*, *far from*, *of to the right of*, *to the left of*, *above*, *below*, *before*, *behind*, and I shall need prepositions to express motions into and out of these situations. For the rest, I can manage with metaphors. Only if my language is intended for use by people having some great geographical feature related the same way to all of them, as a mountain range, the sea, a great river, it will be desirable to have prepositions signifying situations relatively to that, as *across*, *seaward*, etc. But when we examine actual languages, it would seem as though they had supplied the place of many of these distinctions by gestures. The Egyptians had no preposition nor demonstrative having any apparent reference to the Nile. Only the Esquimos are so wrapped up in their bearskins that they have demonstratives distinguishing landward, seaward, north, south, east, and west. But examining the cases or prepositions of any actual language we find them a haphazard lot.

Peirce: CP 2.291 Fn P1 p 165 Cross-Ref:††

†P1 The nomenclature of grammar, like that of logic, is derived chiefly from a late Latin, the words being transferred from the Greek, the Latin prefix translating the Greek prefix and the Latin stem the Greek stem. But while the logical words were chosen with fastidious care, the grammarians were excessively careless, and none more so than Priscian. The word *indicative* is one of Priscian's creations. It was evidently intended to translate Aristotle's {apophantiké}. But this is precisely equivalent to *declarative* both in signification and according to the rules of transference, *de*, taking the place of {apo} as is usual in these artificial formations (*demonstration* for {apodeixis}, etc.), and *clarare* representing {phainein} to make clear. Perhaps the reason Priscian did not choose the word *declaratiuus* was that Apuleius [see Prantl's *Geschichte der Logik*, I, 581], a great authority on words, had used this in a somewhat different *sense*.

Peirce: CP 2.293 Fn P1 p 166 Cross-Ref:††

†P1 There are two ways in which a Symbol may have a real Existential Thing as its real Object. First, the thing may conform to it, whether accidentally or by virtue of the Symbol having the virtue of a growing habit, and secondly, by the Symbol having an Index as a part of itself. But the immediate object of a symbol can only be a symbol and if it has in its own nature another kind of object, this must be by an *endless series*.

Peirce: CP 2.297 Fn 1 p 168

†1 *De Interpretatione*, II, 16a, 12.

Peirce: CP 2.301 Fn 1 p 169

†1 Cf. *Tractatus Logicae*, I, xiv.

Peirce: CP 2.303 Fn 2 p 169

†2 *Dictionary of Philosophy & Psychology*, vol. 2, p. 527.

Peirce: CP 2.305 Fn 1 p 170

†1 *Ibid.*, vol. 1, pp. 531-2.

Peirce: CP 2.307 Fn 1 p 172

†1 *Ibid.*, vol. 2, p. 640.

Peirce: CP 2.308 Fn 2 p 172

†2 *Ibid.*, vol. 2, pp. 691-2.

Peirce: CP 2.309 Fn 1 p 174

†1 §§1-4 are from "Syllabus," c. 1902, continuing 294.

Peirce: CP 2.309 Fn P1 p 174 Cross-Ref:††

†P1 To explain the judgment in terms of the "proposition" is to explain it by that which is essentially intelligible. To explain the proposition in terms of the "judgment" is to explain the self-intelligible in terms of a psychological act, which is the most obscure of phenomena or facts.

Peirce: CP 2.315 Fn P1 p 178 Cross-Ref:††

†P1 But if anybody prefers a form of analysis which gives more prominence to the unquestionable fact that a proposition is something capable of being assented to and asserted, it is not my intention to make any objection to that. I do not think my analysis does put quite the emphasis on that that it justly might.

Peirce: CP 2.315 Fn 1 p 179

†1 If "some" be taken to involve the existence of what it quantifies, then I and O propositions of non-existents must both be false; by the square of opposition both E and A would then be true, so that all universals, whether affirmative or negative, are true of the non-existent. See also 324, 327, 369.

Peirce: CP 2.315 Fn 1 p 180

†1 See vol. 4, bk. II.

Peirce: CP 2.316 Fn P1 p 181 Cross-Ref:††

†P1 *Conditional* is the right appellation, and not *hypothetical*, if the rules of the author's *Ethics of Philosophical Terminology* [bk. II, ch. 1] are to be followed. The meaning of {hypothetikos} was quite unsettled with the Greeks; but the word seems ultimately to have come to be applied to any compound proposition; and so Apuleius, under Nero, uses the translation *conditionalis*; saying, "Propositionum igitur, perinde ut ipsarum conclusionum, duae species sunt: altera praedicativa, quae etiam simplex est; ut si dicamus, *qui regnat, beatus est*: altera substitutiva, vel conditionalis, quae etiam composita est; ut si aias: *qui regnat, si sapit, beatus est*. Substituis enim conditionem, qua, nisi sapiens est, non sit beatus." [See Prantl's *Geschichte der Logik*, I, 580-581.] But as early as Boëthius and Cassiodorus, that is, about A.D. 500, it was settled that *hypothetica* applies to any compound proposition, and *conditionalis* to a proposition asserting one thing only in case a condition set forth in a separate clause be fulfilled. This was the universally accepted use of the

terms throughout the middle ages. Therefore, *hypotheticals* should have been divided into *disjunctives* and *copulatives*. They were usually divided into conditionals, disjunctives, and copulatives. But conditionals are really only a special kind of *disjunctives*. To say, "If it freezes tonight, your roses will be killed" is the same as to say, "It either will not freeze, or your roses will tonight be killed." A disjunctive does not exclude the truth of both alternatives, at once [cf. 345-7].

Peirce: CP 2.317 Fn P1 p 182 Cross-Ref:††

†P1 Mill's term *connote* is not very accurate. Connote properly means to denote along with in a secondary way. Thus "killer" connotes a living thing killed. When the scholastics said that an adjective *connoted*, they meant it connoted the abstraction named by the corresponding abstract noun. But the ordinary use of an adjective involves no reference to any abstraction. The word *signify* has been the regular technical term since the twelfth century, when John of Salisbury (*Metalogicus*, II, xx) spoke of "quod fere in omnium ore celebre est, aliud scilicet esse quod appellativa (i.e., adjectives) *significant*, et aliud esse quod *nominant*. Nominantur singularia (i.e., existent individual things and facts), sed universalia (i.e., Firstnesses) significantur." See my paper of Nov. 13, 1867 [next chapter], to which I might now [1902] add a multitude of instances in support of what is here said concerning *connote* and *signify*.

Peirce: CP 2.323 Fn P1 p 185 Cross-Ref:††

†P1 The *Summulae Logicales* of Petrus Hispanus, which Prantl [*Geschichte der Logik*, II, 266ff, a writer of little judgment and over-rated learning, whose useful history of Logic is full of blunders, misappreciations, and insensate theories, and whose own Billingsgate justifies almost any tone toward him, absurdly maintains that this book was substantially translated from a Greek book, which is manifestly from the Latin. The *Summulae* of Petrus Hispanus are nearly identical with some other contemporary works and evidently show a doctrine which had been taught in the schools from about A.D. 1200. After Boëthius, it is the highest authority for logical terminology, according to the present writer's ethical views.

Peirce: CP 2.323 Fn 1 p 186

†1 I.e., it can be stated in terms of a material or Philonian implication. See 348n.

Peirce: CP 2.324 Fn 2 p 186

†2 Prantl, *op. cit.*, I, 581.

Peirce: CP 2.328 Fn 1 p 187

†1 Prantl, *op. cit.*, I, 696.

Peirce: CP 2.328 Fn 1 p 188

†1 {Horoñ de kalö eis hon dialyetai é protasis oion to te katégoroumenon kai to kath ou katégoroitai}, says Aristotle 24b.16.

Peirce: CP 2.328 Fn 2 p 188

†2 Prantl, *op. cit.*, II, 197.

Peirce: CP 2.328 Fn 3 p 188

†3 See also 3.459.

Peirce: CP 2.331 Fn 1 p 190

†1 See Prantl, *op. cit.*, II, 272.

Peirce: CP 2.332 Fn 2 p 190

†2 §5 and §6 are from "That Categorical and Hypothetical Propositions are one in essence, with some connected matters," c. 1895.

Peirce: CP 2.332 Fn 3 p 190

†3 "On a New List of Categories," vol. 1, bk. III, ch. 6.

Peirce: CP 2.339 Fn 1 p 193

†1 See 1.567 for a definition of this term.

Peirce: CP 2.340 Fn 1 p 194

†1 1.559.

Peirce: CP 2.345 Fn 1 p 196

†1 Cf. Schröder, *Logik*, §28.

Peirce: CP 2.345 Fn 2 p 196

†2 E.g., in his *Formal Logic*, ch. 4, and his *Syllabus*, §21ff. See 366.

Peirce: CP 2.345 Fn 1 p 197

†1 See *Studies in Logic*, edited by C. S. Peirce, Little, Brown and Co., Boston, 1883. "On the Algebra of Logic," by Christine Ladd, p. 61ff.

Peirce: CP 2.345 Fn 2 p 197

†2 Fabian Franklin, "A Point of Logical Notation," *Johns Hopkins University Circular*, p. 131, April 1881.

Peirce: CP 2.348 Fn 1 p 199

†1 A Philonian is one who defines implication "materially" i.e., one who takes "P implies Q" to mean the same as "Not P or Q." There is a reference to the controversy between Philo the Megarian, Diodorus Cronus, and Chrysippus on this point in Cicero's *Acad. Quaest.* II, 143; the issues between Philo and Diodorus are mentioned also in Sextus Empiricus, *Adv. Math.* VIII, 113-17. See also 3.441ff.

Peirce: CP 2.349 Fn 2 p 199

†2 On the Algebra of Logic," vol. 3, no. VI.

Peirce: CP 2.349 Fn 1 p 200

†1 Vol. 3, nos. XII and XIII, §3.

Peirce: CP 2.349 Fn 2 p 200

†2 In *Algebra der Logik*.

Peirce: CP 2.352 Fn 1 p 201

†1 See also 618.

Peirce: CP 2.352 Fn 2 p 201

†2 I.e., it means both p and not-p. See 383.

Peirce: CP 2.354 Fn 1 p 202

†1 I.e., for every individual it holds that if it is human, it is mortal; or, for all occasions it holds that what is human is mortal.

Peirce: CP 2.356 Fn 1 p 203

†1 See 3.18.

Peirce: CP 2.356 Fn 2 p 203

†2 See 3.47n.

Peirce: CP 2.356 Fn 1 p 204

†1 A number of mss. on this topic have been found. They contain nothing not easily derived from the discussions in volume 3 and volume 4.

Peirce: CP 2.356 Fn 2 p 204

†2 I.e., it is equivalent to: not-a or not-a or not-a . . .

Peirce: CP 2.357 Fn 1 p 205

†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 609-10.

Peirce: CP 2.357 Fn 1 p 208

†1 *Formal Logic*, ch. 8.

Peirce: CP 2.358 Fn 2 p 208

†2 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 325-6.

Peirce: CP 2.358 Fn 3 p 208

†3 See 378-80.

Peirce: CP 2.359 Fn 1 p 209

†1 *Ibid.*, vol. 2, pp. 326-9.

Peirce: CP 2.360 Fn 2 p 209

†2 305-6.

Peirce: CP 2.361 Fn 1 p 211

†1 See Prantl, *op. cit.*, III, 279.

Peirce: CP 2.361 Fn 2 p 211

†2 *Super Universalia Porphyrii*, qu. XIV.

Peirce: CP 2.361 Fn 3 p 211

†3 *Ibid.*

Peirce: CP 2.362 Fn 1 p 212

†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 410-12.

Peirce: CP 2.363 Fn 2 p 212

†2 Cf. vol. 4, bk. I, no. 4.

Peirce: CP 2.364 Fn 1 p 213

†1 *De divisione naturae* IV, 4.

Peirce: CP 2.364 Fn 2 p 213

†2 *Metalogicus* II, xx.

Peirce: CP 2.364 Fn 3 p 213

†3 *An Outline of the Necessary Laws of Thought* (1842), §§52, 54, 80.

Peirce: CP 2.364 Fn 4 p 213

†4 *An Elementary Treatise on Logic*, (1856), I, ii, §5.

Peirce: CP 2.364 Fn 5 p 213

†5 418.

Peirce: CP 2.364 Fn 1 p 214

†1 See Prantl, *op. cit.*, I, 581.

Peirce: CP 2.365 Fn 1 p 215

†1 *Lectures on Logic*, XIII, pp. 243-48.

Peirce: CP 2.365 Fn 2 p 215

†2 *An Essay on the New Analytic of Logical Forms*, (1850).

Peirce: CP 2.366 Fn 3 p 215

†3 *Syllabus of a Proposed System of Logic* (1860). §21ff. See also 568.

Peirce: CP 2.366 Fn 1 p 216

†1 *Ibid.*, §165.

Peirce: CP 2.367 Fn 2 p 216

†2 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 737-40: 367-9 are by Peirce alone; 370-1, given in part only, are by Peirce and Mrs. C. Ladd-Franklin.

Peirce: CP 2.367 Fn 3 p 216

†3 *Summulae*, Tractatus II, p. 87C.

Peirce: CP 2.367 Fn 4 p 216

†4 *De Praedicab.* II, 1, p. 11A.

Peirce: CP 2.369 Fn 5 p 216

†5 *Lectures on Logic*, App. V (d), (3).

Peirce: CP 2.369 Fn 1 p 217

†1 Cf. *Nouveaux Essais*, bk. IV, ch. 9.

Peirce: CP 2.370 Fn 1 p 220

†1 Cf. *Nouveaux Essais, Avant-Propos*.

Peirce: CP 2.372 Fn 1 p 221

†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 265-6, by Peirce and Mrs. C. Ladd-Franklin.

Peirce: CP 2.374 Fn 1 p 222

†1 *Ibid.*, vol. 2, pp. 408-9.

Peirce: CP 2.375 Fn 1 p 223

†1 *Essay*, II, viii, 8.

Peirce: CP 2.376 Fn 1 p 224

†1 See vol. 3, no. XX, §8.

Peirce: CP 2.378 Fn 2 p 224

†2 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 146-7, by Peirce and Mrs. C. Ladd-Franklin.

Peirce: CP 2.379 Fn 1 p 225

†1 See vol. 5, Introduction.

Peirce: CP 2.379 Fn 1 p 227

†1 See *Dictionary of Philosophy and Psychology*, vol. 2, p. 369ff.

Peirce: CP 2.381 Fn 1 p 228

†1 *Ibid.*, vol. 2, pp. 6-7.

Peirce: CP 2.381 Fn 2 p 228

†2 See Prantl, *op. cit.*, I, 693.

Peirce: CP 2.381 Fn 3 p 228

†3 See e.g., *Formal Logic*, p. 37ff.

Peirce: CP 2.381 Fn 4 p 228

†4 But cf. Wolff's *Logica*, §208ff.

Peirce: CP 2.382 Fn 1 p 229

†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 89-93.

Peirce: CP 2.382 Fn 2 p 229

†2 See Prantl, *op. cit.*, II, 158f.

Peirce: CP 2.386 Fn 1 p 233

†1 Bd. II, S. 127.

Peirce: CP 2.387 Fn 1 p 234

†1 Cf. *Logik* I, 1, §§33-35.

Peirce: CP 2.389 Fn 1 p 236

†1 *Logik*, §31.

Peirce: CP 2.389 Fn 2 p 236

†2 *Logic* (1883), ch. 7.

Peirce: CP 2.391 Fn 1 p 237

†1 §§1-6 are "Upon Logical Comprehension and Extension," *Proceedings of the American Academy of Arts and Sciences*, vol. 7, November 13, 1867, pp. 416-32, with additions and corrections c. 1870 and 1893; intended as Essay III of *Search for a Method* and as ch. 15 of the *Grand Logic*.

Peirce: CP 2.391 Fn P1 p 237 Cross-Ref:††

†P1 Aristotle remarks in several places that genera and differences may be regarded as parts of species and species as equally parts of genera, e.g., in the 5th *Meta.* (1023b 22). The commentator referred to is perhaps Alexander Aphrodisiensis . . .--1893.

Peirce: CP 2.391 Fn P2 p 237 Cross-Ref:††

†P2 This is quoted from Baines [Baynes'] (*Port Royal Logic*, 2d ed. p. xxiii), who says that he is indebted to Sir William Hamilton for the information.

Peirce: CP 2.391 Fn P3 p 237 Cross-Ref:††

†P3 Lotze.--1893.

Peirce: CP 2.391 Fn P1 p 238 Cross-Ref:††

†P1 Porphyry appears to refer to the doctrine as an ancient one. [Ch. 1.]

Peirce: CP 2.391 Fn P2 p 238 Cross-Ref:††

†P2 They were equally diligent in the study of Boëthius, who says: (*Opera*, p.

645) "Genus in divisione totum est, in diffinitione pars."

Peirce: CP 2.391 Fn P3 Para 1/2 p 238 Cross-Ref:††

†P3 The author of *De Generibus et Speciebus* opposes the *integral* and *diffinitive* wholes. John of Salisbury refers to the distinction of comprehension and extension, as something "quod fere in omnium ore celebre est, aliud scilicet esse quod appellativa *significant*, et aliud esse quod *nominant*. Nominantur singularia, sed universalia significantur." (*Metalogicus*, lib. 2, cap. 20. Ed. of 1620, p. 111.) [By appellativa he means adjectives and the like.--1893.]

Peirce: CP 2.391 Fn P3 Para 2/2 p 238 Cross-Ref:††

Vincentius Bellocensis (*Speculum Doctrinale*, Lib. III, cap. xi) has the following: "Si vero quæritur utrum hoc universale 'homo' sit in quolibet homine secundum se totum an secundum partem, dicendum est quod secundum se totum, id est secundum quamlibet sui partem diffinitivam. . . ., non autem secundum quamlibet partem subjectivam. . . ." William of Auvergne (Prantl's *Geschichte*, vol. 3, p. 77) speaks of "totalitatem istam, quæ est ex partibus rationis seu diffinitionis, et hæc partes sunt genus et differentiæ; alio modo partes speciei individua sunt, quoniam ipsam speciem, cum de eis prædicatur, sibi invicem quodammodo partiunter." [See also Duns Scotus, *Opera* I, 137.] If we were to go to later authors, the examples would be endless. See any commentary in *Phys. Lib. I*.

Peirce: CP 2.392 Fn P1 p 239 Cross-Ref:††

†P1 Part I, ch. 9.

Peirce: CP 2.392 Fn P2 p 239 Cross-Ref:††

†P2 *Principia*, Part I, §45 *et seq.*

Peirce: CP 2.392 Fn P3 p 239 Cross-Ref:††

†P3 Eighth [ninth?] Letter to Burnet. Gerhardt's ed., vol. 3, p. 224.

Peirce: CP 2.393 Fn P4 p 239 Cross-Ref:††

†P4 But *intension* was in use among the Leibnizians in the same sense.--1893.

Peirce: CP 2.393 Fn 1 p 240

†1 Originally, "the best."

Peirce: CP 2.393 Fn P1 p 240 Cross-Ref:††

†P1 Cf. Morin, *Dictionnaire*, Tome I, col. 684 [685?]; Chauvin, *Lexicon*, both editions; Eustachius, *Summa*, Part I, Tr. I, qu. 6. [Aquinas, sentent 1, d. 8, q. 1, art. 1.]

Peirce: CP 2.393 Fn 2 p 240

†2 Originally, "authority."

Peirce: CP 2.393 Fn P2 p 240 Cross-Ref:††

†P2 And such is the humility of his disciples, that not one has dared utter protest against this tax upon his credulity.--1893.

Peirce: CP 2.393 Fn 3 p 240

†3 *Logic*, bk. I, ch. 2, §5, note.

Peirce: CP 2.393 Fn P3 p 240 Cross-Ref:††

†P3 If I understand him, he expresses himself in his usual enigmatical style.--1870.

- Peirce: CP 2.393 Fn P1 p 241 Cross-Ref:††
†P1 Prantl, *Geschichte*, vol. 3, p. 364.
- Peirce: CP 2.393 Fn P2 p 241 Cross-Ref:††
†P2 *Ibid.*, p. 134n. Scotus also uses the term. *Quodlib.* question 13, article 4.
- Peirce: CP 2.393 Fn P3 p 241 Cross-Ref:††
†P3 *Summa Theologica*, Part I, question 53. [This work was certainly written before 1280. Roger Bacon refers to it while saying that Albertus is still alive.--1893.]
- Peirce: CP 2.393 Fn P4 p 241 Cross-Ref:††
†P4 The doctrine of *connotare* is part of the doctrine of *appellatio*, for which see Petrus Hispanus.--1893.
- Peirce: CP 2.393 Fn P5 p 241 Cross-Ref:††
†P5 Part I, ch. 10. (Ed. of 1488, fol. 6, c.)
- Peirce: CP 2.393 Fn 1 p 242
†1 The last sentence in this quotation appears, in the original, nearly half a page before the rest of the quotation.
- Peirce: CP 2.393 Fn P1 p 242 Cross-Ref:††
†P1 Fol. 23 d. See also Tartareti Expositio in Petr. Hisp. towards the end. Ed. of 1509, fol. 91, b.
- Peirce: CP 2.393 Fn P2 p 242 Cross-Ref:††
†P2 . . . is simply rubbish. Civilization in England does not seem as yet to have reached the stage in which men feel shame in making positive assertions based on exceptional ignorance.--1893.
- Peirce: CP 2.394 Fn P3 p 242 Cross-Ref:††
†P3 *Logic*, p. 100 [i.e., Lect. viii, 24]. In the *Summa Logices* attributed to Aquinas, we read: "Omnis forma sub se habens multa, idest, quae universaliter sumitur, habet quamdam *latitudinem*; nam invenitur in pluribus, et dicitur de pluribus." (Tr. 1, c. 3.)
- Peirce: CP 2.395 Fn 2 p 242
†2 *Logik* (1826), I, II; i, 1, iii, and ii, 1, iv.
- Peirce: CP 2.395 Fn 3 p 242
†3 *System der Logik*, (1857), §§50, 53.
- Peirce: CP 2.395 Fn 4 p 242
†4 *Acroasis Logica*, ed. 2 (1773), §24.
- Peirce: CP 2.395 Fn 5 p 242
†5 *The Elements of Deductive Logic* (1867), Pt. I, ch. 2.
- Peirce: CP 2.395 Fn 6 p 242
†6 *An Introduction to Logical Science* (1857), §§7, 30, 31.
- Peirce: CP 2.395 Fn 7 p 242
†7 *Elements of Logic* (1864), p. 10.
- Peirce: CP 2.395 Fn 8 p 242

†8 *A System of Logic* (1862), p. 191.

Peirce: CP 2.395 Fn 1 p 243

†1 *The Principles of Science* (1874), bk. I, ch. 2, p. 31.

Peirce: CP 2.396 Fn 2 p 243

†2 Baynes' translation, I, vi.

Peirce: CP 2.398 Fn 3 p 243

†3 *Grundriss der Logik*, 2^{te} Auf. (1822), §29.

Peirce: CP 2.398 Fn 4 p 243

†4 *Logik*, §42.

Peirce: CP 2.398 Fn 5 p 243

†5 *Neue Darstellung der Logik*, 2^{te} Auf. (1851), §23.

Peirce: CP 2.398 Fn 6 p 243

†6 *System der Logik* (1828), Erster Theil, §48.

Peirce: CP 2.398 Fn 7 p 243

†7 *Logische Untersuchung*, 2^{te} Auf. (1862), xv, 4.

Peirce: CP 2.398 Fn 8 p 243

†8 *Elements of Logic* (1864), pp. 10, 39ff.

Peirce: CP 2.398 Fn 1 p 244

†1 *An Introduction to Logical Science* (1857), §31.

Peirce: CP 2.398 Fn 2 p 244

†2 *Logic, or the Science of Inference* (1854), p. 42.

Peirce: CP 2.398 Fn 3 p 244

†3 Cf. *Syllabus*, §131.

Peirce: CP 2.398 Fn 4 p 244

†4 *The Principles of Science* (1874), Bk. 1, ch. 2.

Peirce: CP 2.398 Fn 5 p 244

†5 *A System of Logic* (1862), p. 191.

Peirce: CP 2.398 Fn 6 p 244

†6 *The Elements of Deductive Logic* (1867), Part I, ch. 2.

Peirce: CP 2.398 Fn P1 p 244 Cross-Ref:††

†P1 I adopt the admirable distinction of Scotus between actual, habitual, and virtual cognition. [*Reportatei*, Ed. 1853, vol. 1, p. 147a. This distinction arose from mixed Aristotelean and Neoplatonic suggestions. Aristotle, as everybody knows, distinguished actual and potential thought. Alexander Aphrodisiensis distinguished material intellect ({nous hylikos}), habitual intellect ({nous kata hexin}) and *intellectus adeptus*. These two distinctions have little to do with one another. Still they were confounded by the Arabians, and the confused doctrine suggested to Scotus his brilliant and philosophical division.--1893.]

Peirce: CP 2.399 Fn 7 p 244

†7 Baynes' translation I, vi.

- Peirce: CP 2.399 Fn 8 p 244
†8 *Logick* (1725), Part I, ch. 3, §3.
- Peirce: CP 2.399 Fn 9 p 244
†9 *Lehrbuch der Logik* (1838), Drittes Kap., §37.
- Peirce: CP 2.399 Fn 10 p 244
†10 *System der Logik* (1828), Erster Theil, §48.
- Peirce: CP 2.399 Fn 11 p 244
†11 *System der Logik* 2^{te} Auf. (1830), Erster Theil, §34.
- Peirce: CP 2.399 Fn 12 p 244
†12 *Grundsätze der allgemeinen Logik* 5^{te} Auf. (1831), §29.
- Peirce: CP 2.399 Fn 1 p 245
†1 Cf. *Logik*, her. v. G. B. Jäsche (1800), I, i, §§1-7.
- Peirce: CP 2.399 Fn 2 p 245
†2 *Die Logik* (1827), S. 115.
- Peirce: CP 2.399 Fn 3 p 245
†3 *System der Logik*, 3^{te} Auf. (1837), §20.
- Peirce: CP 2.399 Fn 4 p 245
†4 *System der Logik* (1857), §53.
- Peirce: CP 2.399 Fn 5 p 245
†5 *Logick* (1725), Part I, ch. 3, §3.
- Peirce: CP 2.399 Fn 6 p 245
†6 *Elements of Logic* (1864), pp. 39, 40.
- Peirce: CP 2.399 Fn 7 p 245
†7 *System der Logik* (1828), Erster Theil, §48.
- Peirce: CP 2.399 Fn 8 p 245
†8 *Logic, or the Science of Inference* (1854), p. 42.
- Peirce: CP 2.399 Fn 9 p 245
†9 *Grundsätze der allgemeinen Logik*, 5^{te} Auf. (1831), §29.
- Peirce: CP 2.399 Fn 10 p 245
†10 *A Treatise on Logic* (1864), p. 67.
- Peirce: CP 2.399 Fn 11 p 245
†11 *Neue Darstellung der Logik*, 2^{te} Auf. (1851), §23.
- Peirce: CP 2.399 Fn 12 p 245
†12 *Formal Logic* (1847), p. 234.
- Peirce: CP 2.399 Fn 13 p 245
†13 *Outline of the Laws of Thought*, 4 ed., pp. 99-102.
- Peirce: CP 2.399 Fn 14 p 245
†14 *Intellectual Philosophy*, 2d ed. (1847), ch. 7, 8.

Peirce: CP 2.399 Fn 15 p 245

†15 *Lehrbuch zur Einleitung in die Philosophie* (1813), II, i, §40.

Peirce: CP 2.399 Fn 16 p 245

†16 *Wissenschaft der Erkenntniss* (1847), II, i, 2, b.

Peirce: CP 2.399 Fn 17 p 245

†17 *An Introduction to Logical Science* (1857), §30.

Peirce: CP 2.399 Fn 18 p 245

†18 *Entwurf der Logik* (1846), 4^{tes} Kap.

Peirce: CP 2.399 Fn 19 p 245

†19 *Abriss der Philosophischen Logik* (1824), S.79.

Peirce: CP 2.400 Fn P1 p 246 Cross-Ref:††

†P1 This law, algebraically stated, is that if a and b are logical terms so related that $a = bx$, then also $b = a + y$ and conversely. Numbers of German logicians are capable of denying this.--1893.

Peirce: CP 2.400 Fn P2 p 246 Cross-Ref:††

†P2 Hoppe reverses the law of Kant, and maintains that the wider the concept the greater its content. His idea, translated into Aristotelean phraseology, is this. He admits the second antepredicamental rule, that the differences of different genera are different (This, of itself, removes him widely from logicians for whom the distinction of comprehension and extension is the turning point of logic.) *Negro* is not a conception formed by the union of the two concepts *man* and *black*, but the peculiar differences of *negro* belong to negroes alone of all beings. This naturally carries him a step further, and he says the difference is of itself sufficient to constitute the pure concept, so that the genus is not an essential predicate. Thirdly, he finds that the characters of the narrower difference are less important (*wirkungsreich*) than those of the higher, and to have less important consequences is to have a smaller measure of predicates.--1893.

Peirce: CP 2.401 Fn 1 p 246

†1 *Neue Darstellung der Logik*. 2^{te} Auf. (1851), Anhang I.

Peirce: CP 2.403 Fn 1 p 247

†1 *Wissenschaft der Erkenntniss* (1847), S. 104-107.

Peirce: CP 2.404 Fn P1 p 247 Cross-Ref:††

†P1 *System der Logik*, 2^{te} Aufl., §54.

Peirce: CP 2.405 Fn P2 p 247 Cross-Ref:††

†P2 *Formal Logic*, p. 234. His doctrine is different in the *Syllabus*.

Peirce: CP 2.406 Fn P1 p 248 Cross-Ref:††

†P1 *Laws of Thought*, 4th ed., §§52, 80. [Cf. §54.]

Peirce: CP 2.406 Fn P2 p 248 Cross-Ref:††

†P2 *Logic*, Part I, ch. 2, §5.

Peirce: CP 2.407 Fn 1 p 248

†1 This and the previous section were both numbered §4 in the original publication.

Peirce: CP 2.407 Fn P1 p 249 Cross-Ref:††

†P1 I restricted myself to *terms*, because at the time this chapter was first written (1867), I had not remarked that the whole doctrine of breadth and depth was equally applicable to *propositions* and to *arguments*. The breadth of a proposition is the aggregate of possible states of things in which it is true; the breadth of an argument is the aggregate of possible cases to which it applies. The depth of a proposition is the total of fact which it asserts of the state of things to which it is applied; the depth of an argument is the importance of the conclusions which it draws. In fact, every proposition and every argument can be regarded as a term.--1893.

Peirce: CP 2.407 Fn P2 p 249 Cross-Ref:††

†P2 It would seem needlessly to complicate the doctrine to introduce probabilities, and therefore it is understood that the information is *supposed to be* accepted absolutely.--1893.

Peirce: CP 2.407 Fn 1 p 249

†1 Changed in 1870 and 1893 from "those things of which there is not . . . predicable are not . . ."

Peirce: CP 2.407 Fn P3 p 249 Cross-Ref:††

†P3 For the distinction of extensive and comprehensive distinctness, see Scotus, i, dist. 2, qu. 3.

Peirce: CP 2.408 Fn P1 p 250 Cross-Ref:††

†P1 That is, of whatever things it is applicable to.

Peirce: CP 2.409 Fn P2 p 250 Cross-Ref:††

†P2 The *essence* of a thing is the idea of it, the law of its being, which makes it the kind of thing it is, and which should be expressed in the definition of that kind.--1893.

Peirce: CP 2.412 Fn P1 p 251 Cross-Ref:††

†P1 Negative terms are called by the logicians *infinite* (or, recently, *infinited*). This is a translation of Aristotle's {aoristos} [*De Interpr.* 3, 16b, 14], which really means "without definition" {horismos}.--1893.

Peirce: CP 2.415 Fn P2 p 251 Cross-Ref:††

†P2 See, for example, *De Generibus et Speciebus*, p. 548.

Peirce: CP 2.418 Fn 1 p 253

†1 *On a New List of Categories*, vol. 1, bk. III, ch. 6, §1.

Peirce: CP 2.418 Fn P1 p 253 Cross-Ref:††

†P1 It will be seen that I depart widely from the ordinary use of this word to mean testimony given privately. As in metaphysics, information is the connection of form and matter, so it may in logic appropriately mean the measure of predication.--1893.

Peirce: CP 2.422 Fn P1 p 255 Cross-Ref:††

†P1 *Ascent* is the most unequivocal noun to denote the passage to a broader and less deep notion, without change of information; and other words of similar literal meaning are used in the same way. It is the decrease in depth, of course, which is directly expressed, the increase in breadth being implied. *Extension*, which directly expresses increase of breadth, has a somewhat different meaning. It is applied to the

discovery (by increase of information) that a predicate applies--*mutatis mutandis*--to subjects to which it had not occurred to us to apply it. It involves no decrease of depth. Thus, Herbert Spencer says [" The Genesis of Science," *British Quarterly Review*, July, 1854] that the inversion of the barometer enabled us to *extend* the principles of mechanics to the atmosphere. Mathematicians frequently speak of the extension of a theorem. Thus, the modification of a theorem relating to plane curves so as to make it apply to all curves in space would be called an extension of that theorem. An extended theorem asserts all that the original theorem did, and more too. **Generalization** in its strict sense, means the discovery, by reflection upon a number of cases, of a general description applicable to them all. This is the kind of thought movement which I have elsewhere [509] called formal hypothesis, or reasoning from definition to definitum. So understood, it is not an increase in breadth but an increase in depth. For instance, I received today a number of English books printed by Hindoos in Calcutta. The manufacture is rude, yet peculiarly pleasing. Remembering other Indian manufactures I have seen, I now get a more definite conception of the characteristic of Indian taste. This, since it is an idea derived from the comparison of a number of objects, is called **generalization**. Yet it is not an extension of an idea already had, but, on the contrary, an increase of definiteness of the conceptions I apply to known things. Besides this, the proper meaning of the word generalization, there are two others which, though they are in good use, ought all the more for that to be severely frowned upon by all who have precision of philosophical terminology at heart. Namely, generalization is applied, secondly, to a particular kind of extension, namely to an extension in which the change of the predicate, in order to make it applicable to a new class of subjects, is so far from obvious, that it is the part of the mental process which chiefly attracts our notice. For example, what is usually called Fermat's theorem is that if $\{r\}$ be a prime number, and a be any number not divisible by $\{r\}$, then $a^{\{r\}-1}$ leaves a remainder of 1 when divided by $\{r\}$. Now, what is called the generalized theorem of Fermat is that if $\{k\}$ is any integer number, and $\phi\{k\}$ its totient, or the number of numbers as small as $\{k\}$ and prime to it, and if a be a number prime to $\{k\}$, then $a^{\phi k}$ leaves a remainder 1 when divided by $\{k\}$. Instead of calling such process a generalization, it would be far better to call it a **generalizing extension**.--1893.

Peirce: CP 2.427 Fn 1 p 258

†1 "Terminology" a supplement to the foregoing.

Peirce: CP 2.428 Fn 1 p 259

†1 See Prantl, *op. cit.*, III, 94.

Peirce: CP 2.428 Fn 1 p 260

†1 Cf. 1.549n.

Peirce: CP 2.431 Fn 1 p 262

†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 528-9; 431-3 are by Peirce and Mrs. C. Ladd-Franklin.

Peirce: CP 2.434 Fn 1 p 263

†1 *Metalogicus*, II, xx.

Peirce: CP 2.434 Fn 2 p 263

†2 *Lexicon Rationale*.

Peirce: CP 2.435 Fn 1 p 265

†1 From the "Short Logic," c. 1893, following 296.

Peirce: CP 2.440 Fn P1 p 266 Cross-Ref:††

†P1 That *Marra* and *Warra* were really the same cannot be positively asserted; but the hypothesis suits the known facts remarkably well, except for the difference of names, which is perhaps not an insuperable obstacle.

Peirce: CP 2.442 Fn 1 p 268

†1 See 451n.

Peirce: CP 2.444 Fn P1 Para 1/2 p 269 Cross-Ref:††

†P1 To speak summarily and use a symbol of abbreviation, rather than an analytical and iconical idea, we may say that the purpose of signs--which is the purpose of thought--is to bring truth to expression. The law under which a sign must be true is the law of inference; and the signs of a scientific intelligence must, above all other conditions, be such as to lend themselves to inference. Hence, the illative relation is the primary and paramount semiotic relation.

Peirce: CP 2.444 Fn P1 Para 2/2 p 269 Cross-Ref:††

It might be objected that to say that the purpose of thought is to bring the truth to expression is to say that the production of *propositions*, rather than that of *inferences*, is the primary object. But the *production* of propositions is of the general nature of inference, so that inference is the essential function of the cognitive mind.--From the fragment used in ch. 2, §1.

Peirce: CP 2.445 Fn 1 p 273

†1 From chapter 9 of the *Grand Logic*, 1893. Cf. 3.162ff.

Peirce: CP 2.445 Fn P1 p 273 Cross-Ref:††

†P1 To be well-read, or even fairly versed, in philosophy (no easy accomplishment) it is quite indispensable to have studied Aristotle; and the study of Aristotle may most conveniently begin with the two books of *Prior Analytics*, certainly the most elementary of all his writings. Two books precede these in the traditional arrangement (with which Aristotle himself probably had nothing to do). One of these, the *Predicaments* or *Categories*, is a metaphysico-logical treatise, of which only the outlines are important. The other, the *Peri hermeneias*, is purely logical, but difficult and confused; and the doctrine is not that of the *Analytics*. I should recommend every serious student of logic who can pick out easy Greek without much trouble to read the *Prior Analytics* at any rate, and the *Posterior Analytics* if he can find time. The *Posterior Analytics* is a splendid monument to the human intellect. Both treatises are in very easy Greek; and they have so much influenced medieval thought, and through that our own, that really a man does not understand what is said to him in the streets till he has read them. I would read them out of the Berlin edition; and if you want notes, there can be nothing better than the Greek scholia there given. Then by buying this edition, you have the advantage of having the index constantly at your hand; and it is of inestimable value, every day. Waitz's edition of the *Organon* is good; and Trendelenburg's *Beiträge, De Anima*, and little epitome [*Elementa Logices Aristoteleae*] are very valuable. There is a capital little epitome [*Outlines of the Philosophy of Aristotle*] by Wallace, and Grote's *Aristotle* has merit. But Grote is terribly one-sided. In fact, all modern commentators have strong leanings.

Peirce: CP 2.449 Fn P1 p 275 Cross-Ref:††

†P1 An *incomplete* argumentation is properly called an *enthymeme*, which is often carelessly defined as a syllogism with a suppressed premiss, as if a sorites, or complex argumentation, could not equally give an enthymeme. The ancient definition of an *enthymeme* was "a rhetorical argumentation," and this is generally set down as a second meaning of the word. But it comes to the same thing. By a rhetorical argumentation was meant one not depending upon logical necessity, but upon common knowledge as defining a sphere of possibility. Such an argument is rendered logical by adding as a premiss that which it assumes as a leading principle.

Peirce: CP 2.451 Fn P1 p 276 Cross-Ref:††

†P1 The latter term is more familiar to our generation, having been used by Whewell [*Novum Organum Renovatum*, II, iv]. But the former is the more legitimate historically. *Copulatum* with Aulus Gellius (XVI. viii. 10) translates the Stoical {sympeplegmenon} in this sense. Conjunctions like et are called *copulative* by Priscian [*Institutiones Grammaticae*, lib. xvi, cap. 1]. Abelard uses *copulare*. We might use *colligation* where the propositions brought together are of one nature and function. But in syllogism, this is not the case. However, if the mood Darapti be admitted, it consists merely in compounding two premisses and dropping a term from the result. This will appear below.

Peirce: CP 2.451 Fn P1 p 277 Cross-Ref:††

†P1 What Kant calls an explicative, or analytical, judgment is either no judgment at all, because void of content (to use his phrase), or else it sets forth distinctly in the predicate what was only indistinctly thought (that is, not actually thought at all) in the subject. In that case, it is really synthetic, and rests on experience; only the experience on which it rests is mere internal experience--experience of our own imaginations. Association by resemblance, and association by contiguity: all lies in that great distinction.

Peirce: CP 2.453 Fn 1 p 278

†1 Cf. Hobbes, *Computation or Logic*, ch. II, §11.

Peirce: CP 2.453 Fn P1 Para 1/2 p 278 Cross-Ref:††

†P1 This does not hold in the case of a limited universe of marks. For if we are confining ourselves to a certain line of predicates, there will be nothing absurd in saying that things differ in every respect. In that case, there will be a lexis of predicates, distinct from the phasis. Certainly, if the nature of reasoning is to be explored, it is necessary to take account of cases in which we limit our thought to a particular order of predicates. Some logicians treat the subject as "extra-logical"; but that only means it is outside the scope of their own studies. If a mathematician should choose to characterize the differential calculus as "extra-mathematical," he would exhibit the same determination to keep his science small and simple that animates many of the logicians.

Peirce: CP 2.453 Fn P1 Para 1/2 p 278 Cross-Ref:††

But although the limited universe of marks is not for me extra-logical, I think it is proper to exclude it from elementary syllogistic, for the reason that it is one of the simplest conceivable instances of the logic of relatives, and when that is treated this problem is virtually solved, even if it be not directly attended.

Peirce: CP 2.455 Fn P1 p 279 Cross-Ref:††

†P1 It dates from Apuleius, and is more assified than golden. *Universal* and *Particular* have the same origin. *Affirmative* and *Negative* are words manufactured

by Boëthius. [See Prantl *op. cit.*, I, 691.]

Peirce: CP 2.455 Fn P1 p 280 Cross-Ref:††

†P1 From {phémi}, not {phainö}; therefore nothing to do with phase.

Peirce: CP 2.457 Fn P1 p 281 Cross-Ref:††

†P1 The term *universe*, now in general use, was introduced by De Morgan in 1846. *Cambridge Philosophical Transactions*, VIII, 380.

Peirce: CP 2.459 Fn 1 p 282

†1 See Prantl, *op. cit.*, I, 687ff.

Peirce: CP 2.459 Fn 2 p 282

†2 *Ibid.*, 687.

Peirce: CP 2.459 Fn 3 p 282

†3 *Ibid.*, 661.

Peirce: CP 2.459 Fn 4 p 282

†4 *Ibid.*, 684, 692.

Peirce: CP 2.461 Fn 1 p 284

†1 *Proceedings of the American Academy of Arts and Sciences*, vol. 7, April 9, 1867, pp. 261-87, with additions and corrections of 1893. Intended as Essay I of the *Search for a Method*.

Peirce: CP 2.461 Fn P1 p 284 Cross-Ref:††

†P1 There can be little doubt that *argumentum* acquired its logical meaning in the Roman law courts; and Cicero not only uses it as above, but expressly defines it as "ratio rei dubiae faciens fidem." The definition of Boëthius, who intends to follow Cicero, makes it a *medium* proving a conclusion. *Medium* is here used in the sense of premiss; but since it usually means in logic a middle term, *argument* has been by many understood in that sense; and Hamilton is among those who go so far as to stigmatize the other and ancient use as improper; wishing to substitute for this the word *argumentation*. The substitution, however, seems to me historically wrong, contrary to common usage, and not particularly convenient. Still, to avoid reproach, I was inclined to replace *argument* in this essay by *inference* (for, as Locke well says, "to *infer* is nothing but, by virtue of one proposition laid down as true, to draw in another as true," and those who would restrict it to reasoning from effect to cause violate all good usage), until I reflected that to do so would have the air of admitting what I could never admit, that logic is primarily conversant with unexpressed thought and only secondarily with language.--1893.

Peirce: CP 2.461 Fn P2 p 284 Cross-Ref:††

†P2 So far as that is separable from the rest.--1893.

Peirce: CP 2.465 Fn P1 p 285 Cross-Ref:††

†P1 Or else made a conditional antecedent to the conclusion.--1893.

Peirce: CP 2.465 Fn P2 p 285 Cross-Ref:††

†P2 To this it might be objected that if from premiss, *P*, we infer conclusion, *C*, then to infer "If *P*, then *C*," needs no premiss at all. But if the hypothetical judgment is *immediately* made, it is not inferential; and if not, it is requisite to begin with some premiss, though its modality be only problematical.--1893.

Peirce: CP 2.466 Fn P1 Para 1/2 p 286 Cross-Ref:††

†P1 Neither of these terms is quite satisfactory. Enthymeme is usually defined as a syllogism with a premiss suppressed. This seems to determine the same sphere as the definition I have given; but the doctrine of a suppressed premiss is objectionable. The sense of a premiss which is said to be suppressed is either conveyed in some way, or it is not. If it is, the premiss is not suppressed in any sense which concerns the logician; if it is not, it ceases to be a premiss altogether. What I mean by the distinction is this. He who is convinced that Sortes is mortal because he is a man (the latter belief not only being the cause of the former, but also being felt to be so) necessarily says to himself that all such arguments are valid. This genus of argument is either clearly or obscurely recognized. In the former case, the judgment amounts to another premiss, because the proposition (for example), "All reasoning from humanity to mortality is certain," only says in other words that every man is mortal. But if the judgment amounts merely to this, that the argument in question belongs to some genus all under which are valid, then in one sense it does, and in another it does not, contain a premiss. It does in this sense, that by an act of attention such a proposition may be shown to have been virtually involved in it; it does not in this sense, that the person making the judgment did not *actually* understand this premiss to be contained in it. This I express by saying that this proposition is contained in the leading principle, but is not *laid down*. This manner of stating the matter frees us at once from all psychological perplexities; and at the same time we lose nothing, since all that we know of thought is but a reflection of what we know of its expression.

Peirce: CP 2.466 Fn P1 Para 2/2 p 286 Cross-Ref:††

These vague arguments are just such as alone are suitable to oratory or popular discourse, and they are appropriate to no other; and this fact justifies the appellation, "rhetorical argument." There is also authority for this use of the term. "Complete" and "incomplete" are adjectives which I have preferred to "perfect" and "imperfect," as being less misleading when applied to argument, although the latter are the best when syllogism is the noun to be limited. [Perhaps it is necessary further to distinguish between a *complete* and a *logical* argument. [See 474n.]--1893.]

Peirce: CP 2.467 Fn 1 p 287

†1 Originally, "a proposition."

Peirce: CP 2.467 Fn 2 p 287

†2 Originally, "Considered as regulating the procedure of inference, it is determinate."

Peirce: CP 2.467 Fn P1 p 287 Cross-Ref:††

†P1 Any assertion means merely how we would act under given circumstances, but a logical principle does not even mean this, but only what we would infer from certain premisses.--1893.

Peirce: CP 2.468 Fn 3 p 287

†3 Originally numbered §4, though following immediately after §2.

Peirce: CP 2.469 Fn P2 p 287 Cross-Ref:††

†P2 The view here taken appears inadmissible, unless it be understood to bear a greatly generalized meaning. For, according to this, from two premisses, "A" and "B," but one complete argument could be formed, namely, that which concludes the conjunctive proposition "A and B"; and so, all the arguments of which this paper treats would have to be excluded. But we must not use the word "argument" in a sense

which completely annuls its utility. The mere "colligation of facts," to use Whewell's term, is a most important and difficult part of that whole operation which in its totality is called *reasoning*. But it is not the only part. Given the premiss that every man (living, dead, or unborn) is the son of a man, then by a process which is an important part of reasoning, we conclude that every man is a grandson (or a descendant of any order) of a man. Nothing is to be gained by excluding such an operation of thought from the number of arguments. To further illustrate this point, take any branch of pure mathematics--say the theory of numbers. What are its premisses? Some of them are mere definitions of terms, such as *product*, *sum*, and the like--terms which may be dispensed with entirely, their definitions always being substituted. The other premisses taken together define the relationship existing between integer numbers; and they may without difficulty be comprised in a single proposition, which will really be a definition of the subject-matter, number. Thus, the whole fabric of the theory of numbers, which in its *posse*, at least, may well be called vast, will be deduced from a single premiss. Nay, it would be very simple to make this single proposition the hypothetical antecedent of every conclusion, when it would be needed no more as a premiss. Nevertheless, it is impossible to regard reasoning otherwise than as a process; and as such, it must involve a substitution of a conclusion in place of premisses. The logic of relatives clearly shows that not all reasoning is eliminative; but so far as it is eliminative, it involves two premisses and is of the general type set forth in §4, below.--1893.

Peirce: CP 2.474 Fn P1 p 289 Cross-Ref:††

†P1 Jevons is often referred to as the originator of the conception of reasoning as a substitution. He first set forth this idea in a little treatise called *The Substitution of Similars* [1869], which appeared after he had seen the present essay in print. But he never claimed the idea as original; nor do I. It was familiar to the Leibnizian logicians. In my opinion, the peculiar doctrine of the substitution of similars is utterly false and untenable.--1893.

Peirce: CP 2.474 Fn P2 p 289 Cross-Ref:††

†P2 This treatment of the subject has the effect of excluding the dilemma altogether from the scheme of classification. It was not until the Renaissance that this formal argument ever appeared in the logical treatises; though it had been given in the rhetorics since ancient times. The stock example of the dilemma is this from Aulus Gellius: a handsome wife would be unfaithful, which is bad; and an ugly wife would confer no pleasure, which is bad; therefore, to take any wife would be bad. This is usually reduced to syllogism by assuming as a premiss a proposition which is nothing but a corollary from the principle of excluded middle; and this would be requisite to make the argument a *complete* one, according to the definition in the text. It would, however, have been proper to recognize, among incomplete arguments, those as *logical* which suppress no premiss except a mere logical principle, such as the principle of excluded middle. Such argument is a trifle less infantile than syllogism. By a dilemmatic argument I mean any argument whose validity depends upon the principle of excluded middle. Many of these are reckoned as syllogism by De Morgan [*Formal Logic*, p. 117ff]. I do not think anything should be a dilemma which does not depend on the same principle.--1893.

Peirce: CP 2.475 Fn 1 p 290

†1 Peirce's first printed paper, of which Part II is substantially a restatement, is in the appendix to the present volume.

Peirce: CP 2.475 Fn P1 p 290 Cross-Ref:††

†P1 This operation will be termed a *contraposition* of the premiss and conclusion.

Peirce: CP 2.476 Fn P2 p 290 Cross-Ref:††

†P2 What *S* is meant being generally undetermined.

Peirce: CP 2.477 Fn 1 p 291

†1 This sentence was deleted and the following substituted for it in 1893. "For example, suppose one man says 'Some *S*'s are *Q*,' that is, 'I can so choose from the class of *S*'s that those I choose shall be *Q*'; and suppose another replies, 'Some of that some are *R*,' that is, 'I can so choose from among those which you will choose that (whether they be *Q* or not) they shall be *R*.' Plainly he says in effect that all *S*'s are *R*. It will be the same if instead of two men, it is but one man at two different times."

Peirce: CP 2.482 Fn 1 p 293

†1 When Peirce incorporated this paper in the "Search for a Method" he deleted this paragraph and wrote instead: "It is customary to enumerate six moods of the third figure instead of four; but Darapti and Felapton are omitted, because when the universal premisses are not understood to assert the existence of their subjects these moods become invalid."

Peirce: CP 2.486 Fn 1 p 296

†1 On the authorship of these mood names and other mnemonic devices, cf. Prantl, *op. cit.*, III, 11-50, *passim*.

Peirce: CP 2.487 Fn P1 p 296 Cross-Ref:††

†P1 This inference, *S* is not *P*,

∴ *S* is not-*P*,

being the conclusion of an affirmative from a negative premiss, can obviously not be ordinary syllogism. The suppressed premiss is,

Not-*P* is not *P*,

which is the definition of not-*P*. It will appear below (Part III, §1) that this is ordinary reasoning from definition to definitum, and is essentially of the second figure.--1893.

Peirce: CP 2.488 Fn P2 p 296 Cross-Ref:††

†P2 This is not reasoning from definition; for that is the substitution of a compact predicate for a complex one. The present inference is:

Some *S* is *P*,

∴Some-*S* is *P*.

It draws a universal conclusion from a particular premiss, and is, therefore, not ordinary syllogism. The suppressed premiss is,

Some *S* is some-*S*;

and the inference is of the nature of reasoning from enumeration, as will be seen below. Such inference is essentially of the third figure.--1893.

Peirce: CP 2.496 Fn P1 p 299 Cross-Ref:††

†P1 Or rather, they bear only the formal meaning that consists in showing certain forms of inference to be valid.--1893.

Peirce: CP 2.496 Fn P2 p 299 Cross-Ref:††

†P2 Or at most, they are but *formal* inferences in the sense in which this word has just been used.--1893.

Peirce: CP 2.497 Fn 1 p 301

†1 This paragraph was deleted in the "Search for a Method" and the following substituted: "Now, putting aside the long reductions of the moods concluding *O*, or a particular negative, which are needlessly complicated and involve irrelevant conversions, in every other case the forms of inference used belong to the very same figure as the syllogism to be reduced."

Peirce: CP 2.498 Fn P1 p 301 Cross-Ref:††

†P1 A formal inference is a substitution having the form of an inference.

Peirce: CP 2.500 Fn 1 p 303

†1 Cf. Ladd-Franklin's "Antilogism" in "On the Algebra of Logic," *Studies in Logic*, p. 37ff.

Peirce: CP 2.502 Fn 2 p 303

†2 See Prantl, *op. cit.*, III, 15, 16.

Peirce: CP 2.506 Fn P1 p 305 Cross-Ref:††

†P1 I leave the Theophrastean syllogism, as every logician has found it, almost entirely useless.--1893.

Peirce: CP 2.506 Fn P1 p 306 Cross-Ref:††

†P1 Hypotheticals have not been considered above, the well-known opinion having been adopted that, "If *A*, then *B*," means the same as "Every state of things in which *A* is true is a state of things in which *B* is (or will be) true."

Peirce: CP 2.511 Fn P1 Para 1/2 p 308 Cross-Ref:††

†P1 Positivism, apart from its theory of history and of the relations between the sciences, is distinguished from other doctrines by the manner in which it regarded hypotheses. Almost all men think that metaphysical theories are valueless, because metaphysicians differ so much among themselves; but the positivists give another reason, namely, that these theories violate the sole condition of all legitimate hypothesis. This condition is that every good hypothesis must be such as is certainly capable of subsequent verification with the degree of certainty proper to the conclusions of the branch of science to which it belongs. There is, it seems to me, a confusion here between the probability of an hypothesis in itself, and its admissibility

into any one of those bodies of doctrine which have received distinct names, or have been admitted into a scheme of the sciences, and which admit only conclusions which have a very high probability indeed. I have here to deal with the rule only so far as it is a general canon of the *legitimacy* of hypotheses, and not so far as it determines their *relevancy* to a particular science; and I shall, therefore, consider only another common statement of it; namely, "that no hypothesis is admissible which is not capable of verification by direct observation." The positivist regards an hypothesis, not as an inference, but as a device for stimulating and directing observation. But I have shown above that certain premisses will render an hypothesis probable, so that there is such a thing as legitimate hypothetic inference. It may be replied that such conclusions are not hypotheses, but inductions. That the sense in which I have used "hypothesis" is supported by good usage, I could prove by a hundred authorities. The following is from Kant: "An hypothesis is the holding for true of the judgment of the truth of a reason on account of the sufficiency of its consequents." Mill's definition (*Logic*, Book III, Ch. XIV §4) also nearly coincides with mine. Moreover, an hypothesis in every sense is an inference, because it is adopted for some reason, good or bad, and that reason, in being regarded as such, is regarded as lending the hypothesis some plausibility. The arguments which I term hypothetic are certainly not inductions, for induction is reasoning from particulars to generals, and this does not take place in these cases. The positivist canon for hypotheses is neither sufficient nor necessary. If it is granted that hypotheses are inferred, it will hardly be questioned that the observed facts must follow apodictically from the hypothesis without the aid of subsidiary hypotheses, and the characters of that which is predicated in the hypothesis, and from which the inference is drawn, must be taken as they occur, and not be picked out in order to make a plausible argument. That the maxim of the positivists is superfluous or worse, is shown; first, by the fact that it is not implied in the proof that hypothetic inference is valid; and next, by the absurdities to which it gives rise when strictly applied to history, which is entirely hypothetical, and is absolutely incapable of verification by direct observation. To this last argument I know of but two answers: first, that this pushes the rule further than was intended, it being considered that history has already been so verified; and second, that the positivist does not pretend to know the world as it absolutely exists, but only the world which appears to him. To the first answer, the rejoinder is that a rule must be pushed to its logical consequences in all cases, until it can be shown that some of these cases differ in some material respect from the others. To the second answer, the rejoinder is double: first, that I mean no more by "is" than the positivist by "appears" in the sense in which he uses it in saying that only what "appears" is known, so that the answer is irrelevant; second, that positivists, like the rest of the world, reject historic testimony sometimes, and in doing so distinguish hypothetically between what is and what in some other sense appears, and yet have no means of verifying the distinction by direct observation.

Peirce: CP 2.511 Fn P1 Para 2/2 p 308 Cross-Ref: ††

Another error in reference to hypothesis is that the antecedent probability of what is testified to cannot affect the probability of the testimony of a good witness. This is as much as to say that probable arguments can neither support nor weaken one another. Mr. Venn goes so far as to maintain the impossibility of a conflict of probabilities. The difficulty is instantly removed by admitting indeterminate probabilities.

Peirce: CP 2.517 Fn 1 p 313

†1 §1 and §2 are "Note A," the Johns Hopkins *Studies in Logic*, edited by C.

S. Peirce, Little, Brown and Co., Boston, (1883) pp. 182-6, as rewritten in 1893, for the *Grand Logic*, ch. 13. Cf. 3.345ff and 3.403F ff.

Peirce: CP 2.517 Fn P1 p 313 Cross-Ref:††

†P1 The term was introduced by De Morgan in 1846. *Cambridge Philosophical Transactions*, VIII (1849), p. 380.

Peirce: CP 2.519 Fn 2 p 313

†2 Originally: "The conception of ordinary syllogism is so unclear that it would hardly be accurate to say that it supposes an unlimited universe of characters; but it comes nearer to that than to any other consistent view."

Peirce: CP 2.519 Fn 1 p 314

†1 This sentence was added in 1893.

Peirce: CP 2.521 Fn 2 p 314

†2 The remainder of this section and the whole of the next differ in detail from the article as first published.

Peirce: CP 2.521 Fn 1 p 315

†1 See 3.345 for the interpretation of some of these in terms of "relatives." They are perhaps clearer when expressed with quantifiers, thus: 1. (s) (π) π s. 2. (Σ s) (π) π s. 3. (π) (Σ s) π s. 4. ($\pi\Sigma$) (s) π s, etc., where (Σ s) means "some s" and (s) means "every s."

Peirce: CP 2.524 Fn 1 p 316

†1 (Σ s) (π) π s \rightarrow (π) (Σ s) π s.

Peirce: CP 2.526 Fn 1 p 319

†1 . . . "the conclusion is of a kind called *spurious* by De Morgan if, and only if, the middle term is affected by 'some' in both premisses"--in the original, p. 185. See 607; De Morgan, *Formal Logic*, pp. 153-4 and *Syllabus*, §76ff.; and B. Gilman, *Johns Hopkins University Circular*, August, 1882, on these "Spurious Propositions."

Peirce: CP 2.532 Fn 1 p 322

†1 From ch. 10, "Extension of the Aristotelian Syllogistic," of the *Grand Logic*, 1893.

Peirce: CP 2.533 Fn 1 p 324

†1 See also Appendix 1 to De Morgan's *Formal Logic*.

Peirce: CP 2.534 Fn 2 p 324

†2 *An Essay on the New Analytic of Logical Forms*, Edinburgh, (1850).

Peirce: CP 2.534 Fn 1 p 325

†1 See *Lectures on Logic*, VI, p. 114.

Peirce: CP 2.536 Fn 1 p 326

†1 From "Universe," *Dictionary of Philosophy and Psychology*, vol. 2, p. 742, by Peirce and Mrs. C. Ladd-Franklin.

Peirce: CP 2.536 Fn 1 p 327

†1 See *An Investigation of the Laws of Thought*, etc., p. 42.

Peirce: CP 2.537 Fn 1 p 328

†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 27-28.

- Peirce: CP 2.544 Fn 1 p 329
†1 *Ibid.*, vol. 2, p. 402.
- Peirce: CP 2.547 Fn 1 p 330
†1 *Ibid.*, vol. 2, p. 219.
- Peirce: CP 2.548 Fn 2 p 330
†2 *Ibid.*, vol. 1, p. 561.
- Peirce: CP 2.548 Fn 3 p 330
†3 See vol. 4, bk. I, No. 3.
- Peirce: CP 2.549 Fn 4 p 330
†4 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 44-45.
- Peirce: CP 2.549 Fn 1 p 331
†1 See 579.
- Peirce: CP 2.550 Fn 2 p 331
†2 *Mind*, vol. 1 (1876), p. 424.
- Peirce: CP 2.551 Fn 1 p 332
†1 *Dictionary of Philosophy and Psychology*, vol. 2, p. 199.
- Peirce: CP 2.552 Fn 2 p 332
†2 *Ibid.*, vol. 2, pp. 628-9, 633-9.
- Peirce: CP 2.553 Fn 1 p 333
†1 See *Novum Organum Renovatum*, II, iv.
- Peirce: CP 2.554 Fn 2 p 333
†2 §20.
- Peirce: CP 2.554 Fn 1 p 334
†1 *Aristotle's Rhetoric with Cope's Commentary*, Sandys' edition, vol. 1, p. 217.
- Peirce: CP 2.554 Fn P1 p 334 Cross-Ref:††
†P1 Written, as there is strong unpublished ground for thinking, 394 B. C., while Aristotle only went to Athens 368 or 367 B.C. (Grote thinks not till 362 B. C.) All the other dialogues here mentioned are subsequent to Aristotle's joining the school.
- Peirce: CP 2.554 Fn 2 p 334
†2 *The Origin and Growth of Plato's Logic* (1897), p. 203.
- Peirce: CP 2.554 Fn 3 p 334
†3 *Ibid.*, p. 464.
- Peirce: CP 2.554 Fn P1 p 335 Cross-Ref:††
†P1 It has been argued that Aristotle may here, as it is said he often does, employ the first person plural to mean the students of Plato; and also that {proteron allo} would not exclude aid from contemporaries. The present writer, without making any pretension to philological learning, apprehends that it is quite clear that Aristotle is speaking of himself personally, and that he means that no doctrine of the syllogism,

in which he now takes the first steps ({hȫs ek toioutön ex archés hyparchontön echein hé methodos }) had existed before his *Analytiks* and *Topics*. Such hints as he may have received from Plato cannot (the writer believes) have been in Aristotle's memory when he penned those words. But a man does not always know how he originally came by ideas which occupied him at first little, but afterwards more and more, up to almost complete absorption for many long years.

Peirce: CP 2.556 Fn 1 p 335
†1 *Prior Analytic*, I, 1, 24b.

Peirce: CP 2.558 Fn 1 p 336
†1 3.636.

Peirce: CP 2.558 Fn 2 p 336
†2 Vol. 4, bk. II, ch. 2.

Peirce: CP 2.558 Fn 1 p 337
†1 † is the sign of relative addition, $\sim y$ is the negative of y , $\$y$ is the converse of y , xy means x and y , and $x \top z$ means x is other than z .

Peirce: CP 2.559 Fn 1 p 340
†1 I.e., symmetry.

Peirce: CP 2.567 Fn 1 p 345
†1 *Outline of the Laws of Thought*, Fourth ed., (1857), §§77ff.

Peirce: CP 2.567 Fn 2 p 345
†2 *An Introduction to Logical Science*, (1857), §§38ff.

Peirce: CP 2.580 Fn 1 p 348
†1 From article "Negative," *Dictionary of Philosophy and Psychology*, vol. 2, p. 148.

Peirce: CP 2.581 Fn 2 p 348
†2 *Ibid.*, vol. 2, p. 77, by Peirce and Mrs. C. Ladd-Franklin.

Peirce: CP 2.582 Fn 1 p 349
†1 *Ibid.*, vol. 2, pp. 330-31.

Peirce: CP 2.583 Fn 1 p 350
†1 But see second edition (1755), London, under "Premiss" where it is said: "This word is rare in the singular."

Peirce: CP 2.584 Fn 2 p 350
†2 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 87-88.

Peirce: CP 2.584 Fn 3 p 350
†3 Cf. Prantl, op. cit., III, 41.

Peirce: CP 2.584 Fn 4 p 350
†4 *Ibid.*, III, 27.

Peirce: CP 2.584 Fn 5 p 350
†5 *Ibid.*, III, 43.

Peirce: CP 2.584 Fn 1 p 352

- †1 Cf. Prantl, *op. cit.*, III, 15-16.
- Peirce: CP 2.585 Fn 1 p 353
†1 *Dictionary of Philosophy and Psychology*, vol. 2, p. 435.
- Peirce: CP 2.586 Fn 2 p 353
†2 I.e., as not implying existence.
- Peirce: CP 2.587 Fn 1 p 354
†1 Vol. 3.
- Peirce: CP 2.588 Fn 2 p 354
†2 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 1-2.
- Peirce: CP 2.590 Fn 1 p 355
†1 *Ibid.*, vol. 2, p. 183.
- Peirce: CP 2.593 Fn 1 p 356
†1 *Ibid.*, vol. 1, pp. 641-44.
- Peirce: CP 2.598 Fn 1 p 359
†1 I.e., the principle of contradiction asserts that "not" is an alio-relative.
- Peirce: CP 2.598 Fn 2 p 359
†2 I.e., the principle of excluded middle asserts that "not" is a concurrent.
- Peirce: CP 2.598 Fn 1 p 360
†1 Cf. 3.339.
- Peirce: CP 2.600 Fn 1 p 362
†1 Ch. 10, §1.
- Peirce: CP 2.601 Fn 2 p 362
†2 From article "Regular," *Dictionary of Philosophy and Psychology*, vol. 2, p. 439.
- Peirce: CP 2.602 Fn 3 p 362
†3 *Ibid.*, vol. 2, p. 287.
- Peirce: CP 2.603 Fn 1 p 363
†1 *Ibid.*, vol. 1, pp. 525-26.
- Peirce: CP 2.604 Fn 2 p 363
†2 This "logical sense of implication" corresponds to "entailment" and not to Philonian "material implication." It may be defined as the relation of a set of alternatives to the set which contains them.
- Peirce: CP 2.605 Fn 1 p 364
†1 *Dictionary of Philosophy and Psychology*, vol. 2, p. 198, by Peirce and James Mark Baldwin, the editor.
- Peirce: CP 2.607 Fn 2 p 364
†2 *Ibid.*, vol. 2, p. 588.
- Peirce: CP 2.608 Fn 1 p 365
†1 *Ibid.*, vol. 2, p. 206.

- Peirce: CP 2.608 Fn 2 p 365
†2 Or non-symmetrical. See 3.136c.
- Peirce: CP 2.609 Fn 3 p 365
†3 *Ibid.*, vol. 1, p. 529.
- Peirce: CP 2.612 Fn 1 p 366
†1 *Ibid.*, vol. 2, p. 434.
- Peirce: CP 2.612 Fn 2 p 366
†2 *Anal. pr.* 25, 69a, 20.
- Peirce: CP 2.613 Fn 3 p 366
†3 *Dictionary of Philosophy and Psychology*, vol. 2, p. 181.
- Peirce: CP 2.613 Fn 4 p 366
†4 *De Sophist. Elen.* I, 5, 167a, 36.
- Peirce: CP 2.614 Fn 1 p 368
†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 287-88, by Peirce and Mrs. C. Ladd-Franklin.
- Peirce: CP 2.614 Fn 2 p 368
†2 *Anal. Prior.*, II, 16.
- Peirce: CP 2.615 Fn 1 p 369
†1 *Dictionary of Philosophy and Psychology*, vol. 2, p. 290.
- Peirce: CP 2.616 Fn 2 p 369
†2 *Ibid.*, from article "Saltus," vol. 2, p. 484.
- Peirce: CP 2.617 Fn 3 p 369
†3 *Ibid.*, vol. 2, p. 44.
- Peirce: CP 2.617 Fn 4 p 369
†4 *Artis Logicae Rudimenta* (1862), p. 131ff.
- Peirce: CP 2.618 Fn 1 p 370
†1 *Dictionary of Philosophy and Psychology*, vol. 1, p. 554.
- Peirce: CP 2.618 Fn 1 p 371
†1 For a further discussion of sophisms and insolubilia, see 5.33ff.
- Peirce: CP 2.618 Fn 2 p 371
†2 *System der Logik*.
- Peirce: CP 2.618 Fn 3 p 371
†3 See 3.446.
- Peirce: CP 2.619 Fn 1 p 372
†1 *Popular Science Monthly*, vol. 13, pp. 470-82 (1878); intended as Essay XIII of the *Search for a Method* (1893). It is the sixth and last of a series of papers on the "Illustrations of the Logic of Science," which appeared in the *Popular Science Monthly*. For the first and second papers, see vol. 5, bk. II, chs. 4 and 5; the third and fourth constitute chapters 6 and 7 of the present book; for the fifth paper, see vol. 6, bk. II, ch. 1.

Peirce: CP 2.631 Fn 1 p 378

†1 See, e.g., 515.

Peirce: CP 2.635 Fn 1 p 380

†1 See vol. 5, bk. II, ch. 4, §5.

Peirce: CP 2.638 Fn 1 p 383

†1 *Novum Organum*, bk. I, Aphorism X.

Peirce: CP 2.639 Fn 1 p 384

†1 In his *Hydrodynamica*.

Peirce: CP 2.640 Fn 1 p 385

†1 Cf. 511n.

Peirce: CP 2.641 Fn P1 p 386 Cross-Ref:††

†P1 This division was first made in a course of lectures by the author before the Lowell Institute, Boston, in 1866, and was printed in the *Proceedings of the American Academy of Arts and Sciences*, for April 9, 1867. [See 508-12.]

Peirce: CP 2.643 Fn 1 p 387

†1 See, e.g., the first paper, vol. 5, bk. II, ch. 4.

Peirce: CP 2.643 Fn 1 p 388

†1 Cf. 712.

Peirce: CP 2.644 Fn 2 p 388

†2 Cf. vol. 1, bk. II, ch. 2.

Peirce: CP 2.645 Fn 1 p 389

†1 *Popular Science Monthly*, vol. 12, pp. 604-15 (1878) with corrections of 1893 and a note of 1910; intended as ch. 18 of the *Grand Logic* (1893), and as Essay X of the *Search for a Method* (1893), the third of a series of papers on "Illustrations of the Logic of Science." See notes to ch. 5 and 6.410.

Peirce: CP 2.645 Fn P1 p 389 Cross-Ref:††

†P1 This characterization of chemistry now sounds antiquated indeed; and yet it was justified by the general state of mind of chemists at that day, as is shown by the fact that only a few months before, van't Hoff had put forth a statement of the law of mass-action as something absolutely new to science. I am satisfied by considerable search after pertinent facts that no distinction between different allied sciences can represent any truth of fact other than a difference between what habitually passes in the minds, and moves the investigations of the two general bodies of the cultivators of those sciences at the time to which the distinction refers.--1910.

Peirce: CP 2.646 Fn 1 p 390

†1 *Novum Organum*, bk. II, Aphorism XXVII.

Peirce: CP 2.646 Fn P1 p 391 Cross-Ref:††

†P1 "Or rather of an idea that continuity suggests--that of limitless intermediation; i.e., of a series between every two members of which there is another member of it"--to be substituted for the phrase "or . . . degrees."--1893.

Peirce: CP 2.646 Fn P2 p 391 Cross-Ref:††

†P2 For "continuity" substitute "limitless intermediation, the business of

reasoning."--1893.

Peirce: CP 2.646 Fn P3 p 391 Cross-Ref:††

†P3 "And others that are involved in that of continuity."--1893.

Peirce: CP 2.646 Fn P4 p 391 Cross-Ref:††

†P4 For "neglect of" substitute "want of close study of these concepts."--1893.

Peirce: CP 2.646 Fn P5 p 391 Cross-Ref:††

†P5 This mode of thought is so familiarly associated with all exact numerical consideration, that the phrase appropriate to it is imitated by shallow writers in order to produce the appearance of exactitude where none exists. Certain newspapers, which affect a learned tone, talk of "the average man," when they simply mean *most men*, and have no idea of striking an average.

Peirce: CP 2.646 Fn 1 p 391

†1 See, e.g., 1.383.

Peirce: CP 2.648 Fn 1 p 393

†1 See vol. 5, bk. II, ch. 5.

Peirce: CP 2.649 Fn 2 p 393

†2 *Essay*, bk. IV, ch. 15, §1.

Peirce: CP 2.649 Fn 3 p 393

†3 See vol. 5, bk. II, ch. 4, §2.

Peirce: CP 2.650 Fn 1 p 394

†1 See vol. 5, bk. II, ch. 5, §4.

Peirce: CP 2.651 Fn P1 p 395 Cross-Ref:††

†P1 The conception of probability here set forth is substantially that first developed by Mr. Venn, in his *Logic of Chance*. Of course, a vague apprehension of the idea had always existed, but the problem was to make it perfectly clear, and to him belongs the credit of first doing this.

Peirce: CP 2.654 Fn P1 p 399 Cross-Ref:††

†P1 I do not here admit an absolutely unknowable. Evidence could show us what would probably be the case after any given lapse of time; and though a subsequent time might be assigned which that evidence might not cover, yet further evidence would cover it.

Peirce: CP 2.660 Fn 1 p 404

†1 Ch. 7.

Peirce: CP 2.661 Fn 2 p 404

†2 1910.

Peirce: CP 2.661 Fn 3 p 404

†3 See vol. 5, bk. II, chs. 1, 2, 3, particularly 5.355.

Peirce: CP 2.662 Fn 1 p 406

†1 Cf. 111, 269, 756f.

Peirce: CP 2.662 Fn 1 p 407

†1 See Carrington's *Eusapia Palladino*, B. W. Dodge & Co., New York

(1909).

Peirce: CP 2.663 Fn 1 p 408

†1 Peirce read *Whately's Logic* at this time.

Peirce: CP 2.667 Fn P1 p 413 Cross-Ref:††

†P1 Meantime it may be remarked that, though an endless series of acts is not a habit, nor a would-be, it does present the first of an endless series of steps toward the full nature of a would-be. Compare what I wrote nineteen [thirteen!] years ago, in an article on the logic of relatives [3.526ff].

Peirce: CP 2.669 Fn 1 p 415

†1 *Popular Science Monthly*, vol. 12, pp. 705-18 (1878), the fourth of a series of papers on "Illustrations of the Logic of Science." See 612n. Intended as Essay XI of the *Search for a Method* (1893).

Peirce: CP 2.674 Fn 1 p 417

†1 Cf. 3.17.

Peirce: CP 2.677 Fn P1 p 421 Cross-Ref:††

†P1 Strictly we should need an infinite series of numbers each depending on the probable error of the last.

Peirce: CP 2.679 Fn P1 p 422 Cross-Ref:††

†P1 "Perfect indecision, belief inclining neither way, an even chance."--De Morgan, p. 182.

Peirce: CP 2.682 Fn 1 p 424

†1 *Théorie des Probabilités*, deuxième partie, §1.

Peirce: CP 2.684 Fn 1 p 426

†1 Cf. vol. 6, bk. II, ch. 1, §2.

Peirce: CP 2.684 Fn 2 p 426

†2 See 692.

Peirce: CP 2.690 Fn P1 p 430 Cross-Ref:††

†P1 *Logique*. The same is true, according to him, of every performance of a differentiation, but not of integration. He does not tell us whether it is the supernatural assistance which makes the former process so much the easier.

Peirce: CP 2.694 Fn 1 p 433

†1 *The Johns Hopkins Studies in Logic*, edited by C. S. Peirce, Little Brown and Co., Boston (1883), pp. 126-181; intended as Essay XIV of the *Search for a Method* (1893).

Peirce: CP 2.694 Fn 2 p 433

†2 The headings of these sections were made by Peirce in his own copy of the Johns Hopkins *Studies*.

Peirce: CP 2.696 Fn 1 p 436

†1 Bk. IV, ch. 15, §1.

Peirce: CP 2.699 Fn P1 p 439 Cross-Ref:††

†P1 In case $(n + 1)r$ is a whole number, q has equal values for $m = (n + 1)r$ and for $m = (n + 1)r - 1$.

Peirce: CP 2.702 Fn 1 p 441

†1 There was no §3 in the original, and the present section formed part of §2.

Peirce: CP 2.702 Fn 2 p 441

†2 "these" is deleted in Peirce's own copy.

Peirce: CP 2.705 Fn P1 Para 1/3 p 444 Cross-Ref:††

†P1 When $r = 0$, the last form becomes

M has all the marks P ,

S has no mark of M ;

Hence, S has none of the marks P .

Peirce: CP 2.705 Fn P1 Para 2/3 p 444 Cross-Ref:††

When the universe of marks is unlimited (see a note appended to this paper for an explanation of this expression [519]), the only way in which two terms can fail to have a common mark is by their together filling the universe of things; and consequently this form then becomes

M is P ,

Every non- S is M ;

Hence, every non- S is P .

This is one of De Morgan's syllogisms.

Peirce: CP 2.705 Fn P1 Para 3/3 p 444 Cross-Ref:††

In putting $r = 0$ in Form II (*bis*) it must be noted that, since P is simple in depth, to say that S is not P is to say that it has no mark of P .

Peirce: CP 2.707 Fn 1 p 445

†1 Cf. 102.

Peirce: CP 2.710 Fn P1 p 447 Cross-Ref:††

†P1 I do not here speak of Mr. Jevons, because my objection to the copula of identity is of a somewhat different kind. [See *Studies in Logic*, pp. 17-69 and 72-106 for Miss Ladd's and Mr. Mitchell's papers.]

Peirce: CP 2.711 Fn P1 p 448 Cross-Ref:††

†P1 See my paper on "How to make our ideas clear." [Vol. 5. bk. II, ch. 5.]

Peirce: CP 2.712 Fn 1 p 449

†1 Cf. 643.

Peirce: CP 2.718 Fn P1 p 451 Cross-Ref:††

†P1 From *apagoge*, {apagōgē eis to adynaton}, Aristotle's name for the

reductio ad absurdum.

Peirce: CP 2.720 Fn P1 p 452 Cross-Ref:††

†P1 The conclusion of the statistical deduction is here regarded as being "the proportion r of the S 's are P 's," and the words "probably about" as indicating the modality with which this conclusion is drawn and held for true. It would be equally true to consider the "probably about" as forming part of the contents of the conclusion; only from that point of view the inference ceases to be probable, and becomes rigidly necessary, and its apagogical inversion is also a necessary inference presenting no particular interest.

Peirce: CP 2.729 Fn P1 p 456 Cross-Ref:††

†P1 This I conceive to be all the truth there is in the doctrine of Bacon and Mill regarding different Methods of Experimental Inquiry. The main proposition of Bacon's and Mill's doctrine is, that in order to prove that all M 's are P 's, we should not only take random instances of the M 's and examine them to see that they are P 's, but we should also take instances of not- P 's and examine them to see that they are not- M 's. This is an excellent way of fortifying one induction by another, when it is applicable; but it is entirely inapplicable when r has any other value than 1 or 0. For, in general, there is no connection between the proportion of M 's that are P 's and the proportion of non- P 's that are non- M 's. A very small proportion of calves may be monstrosities, and yet a very large proportion of monstrosities may be calves.

Peirce: CP 2.731 Fn P1 p 458 Cross-Ref:††

†P1 I say *about*, because the doctrine of probability only deals with approximate evaluations.

Peirce: CP 2.733 Fn P1 p 460 Cross-Ref:††

†P1 That this is really a correct analysis of the reasoning can be shown by the theory of probabilities. For the expression

$$\frac{(p + q)! (\pi + \{r\})! (p + \pi)(q + \{r\})!}{p!q! \pi!\{r\}! (p + \pi + q + \{r\})!}$$

expresses at once the probability of two events; namely, it expresses first the probability that of $p + q$ objects drawn without replacement from a lot consisting of $p + \pi$ objects having the character R together with $q + \{r\}$ not having this character, the number of those drawn having this character will be p ; and second, the same expression denotes the probability that if among $p + \pi + q + \{r\}$ objects drawn at random from an infinite class (containing no matter what proportion of R 's to non- R 's), it happens that $p + \pi$ have the character R , then among any $p + q$ of them, designated at random, p will have the same character. Thus we see that the chances in reference to drawing without replacement from a finite class are precisely the same as those in reference to a class which has been drawn at random from an infinite class.

Peirce: CP 2.738 Fn 1 p 462

†1 Wheeler's *Biographical Dictionary*.

Peirce: CP 2.738 Fn P1 p 464 Cross-Ref:††

†P1 The author ought to have noted that this number is open to some doubt, since the specific gravity of this form of silicon appears to vary largely. If a different value had suited the theory better, he might have been able to find reasons for preferring that other value. But I do not mean to imply that Dr. Playfair has not dealt with perfect fairness with his facts, except as to the fallacy which I point out.

Peirce: CP 2.738 Fn P1 p 465 Cross-Ref:††

†P1 As the relations of the different powers of the specific gravity would be entirely different if any other substance than water were assumed as the standard, the law is antecedently in the highest degree improbable. This makes it likely that some fallacy was committed, but does not show what it was.

Peirce: CP 2.739 Fn P1 p 466 Cross-Ref:††

†P1 The physicians seem to use the maxim that you cannot reason from *post hoc* to *propter hoc* to mean (rather obscurely) that cases must not be used to prove a proposition that has only been suggested by these cases themselves.

Peirce: CP 2.741 Fn 1 p 468

†1 See Theodor Gomperz, *Herculanische Studien*, pt. I (1865). Cf. 761.

Peirce: CP 2.744 Fn P1 p 469 Cross-Ref:††

†P1 See Laplace, *Théorie Analytique des Probabilités*, [1812], livre ii, ch. vi.

Peirce: CP 2.746 Fn P1 p 471 Cross-Ref:††

†P1 See Boole, *Laws of Thought*, p. 370.

Peirce: CP 2.749 Fn 1 p 472

†1 See *La Logique*, Paris (1855), vol. 2, pp. 196-97.

Peirce: CP 2.750 Fn 1 p 474

†1 See vol. 6, bk. II, ch. 1, 2.

Peirce: CP 2.752 Fn P1 p 475 Cross-Ref:††

†P1 The different regions with the January rainfall are taken from Mr. Schott's work. [*Tables and Results of the Precipitation in Rain and Snow in the United States*, 1872.] The percentage of illiteracy is roughly estimated from the numbers given in the Report of the 1870 Census. [The maps originally published with this paper have not been considered worth reproducing.]

Peirce: CP 2.753 Fn 1 p 477

†1 Cf. 1.118.

Peirce: CP 2.755 Fn 1 p 478

†1 From ms. "G." c. 1905.

Peirce: CP 2.756 Fn P1 p 479 Cross-Ref:††

†P1 When I say that it goes on that presumption, I merely mean to describe the presumption of the reasoning as being that the particular uniformity as to a certain matter that has attached to past experience will be maintained in the future. I shall explain below how there is a certain justification in this, though a very slender one. I do not mean to say, as some logicians do, that the force of the induction is just the same as that of a syllogism whose major premiss should be, "Future experience will not violate the uniformity of past experience." For such a syllogism being a fallacy of

the particularly atrocious kind called "logical fallacy," would have no justification whatsoever. For a sound syllogism must have a major premiss of definite meaning: otherwise it may be thrown into the form of a fallacy of undistributed middle. Now the induction in question, though weak, is by no means without justification.

Peirce: CP 2.756 Fn 1 p 479

†1 Cf. 111, 269, 662, 757n.

Peirce: CP 2.757 Fn P1 p 480 Cross-Ref:††

†P1 Induction is such a way of inference that if one persists in it one must necessarily be led to the truth, at last. It is true that this condition is most imperfectly fulfilled in the *Pooh-pooh* argument. For here the unexpected, when it comes, comes with a bang. But then, on the other hand, until the fatal day arrives, this argument causes us to anticipate just what does happen and prevents us from anticipating a thousand things that do not happen. I engage a stateroom; I purchase a letter of credit for fifty thousand dollars, and I start off determined to have a good time. On the way down the bay, my wife says to me, "Aren't you afraid the house may be struck by lightning while we are gone?" *Pooh-pooh!* "But aren't you afraid there will be a war and Boston will be bombarded?" *Pooh-pooh!* "But aren't you afraid that when we are in the heart of Hungary or somewhere you will get the Asiatic plague, and I shall be left unable to speak the language?" *Pooh-pooh!* On the morning of the fourth day out there is a terrific explosion and I find myself floating about on the middle of the Atlantic with my letter of credit safe in my breast pocket. I say to myself, my *Pooh-pooh* argument broke down that time sure enough, but after all it made my mind easy about a number of possibilities that did not occur, and even about this one for three days. So I had better be content with my lot. This little parable is intended to illustrate how even the Pooh-pooh argument, the weakest of all sound induction, does satisfy the essential condition of saving me from surprises both positive and negative; that is from the happening of things not anticipated and the non-occurrence of imaginary disasters.--From a fragment c. 1902.

Peirce: CP 2.758 Fn 1 p 480

†1 Cf. *The Grammar of Science* (1892); *The Chances of Death* (1897).

Peirce: CP 2.760 Fn 1 p 483

†1 "On the Natural Classification of Arguments," ch. 2.

Peirce: CP 2.760 Fn 2 p 483

†2 "Some Consequences of Four Incapacities," vol. 5, bk. II, ch. 2.

Peirce: CP 2.760 Fn 3 p 483

†3 "Deduction, Induction, and Hypothesis," ch. 5.

Peirce: CP 2.760 Fn 4 p 483

†4 "A Theory of Probable Inference," ch. 8.

Peirce: CP 2.760 Fn 5 p 483

†5 "The Doctrine of Necessity Examined," vol. 6, bk. I, ch. 2.

Peirce: CP 2.760 Fn 6 p 483

†6 "Pearson's Grammar of Science," vol. 9.

Peirce: CP 2.761 Fn 1 p 485

†1 Theodor Gomperz, *Philodemi de ira liber* (1864); *Herkulanische Studien* (1865-6).

Peirce: CP 2.765 Fn 1 p 489

†1 Bk. III, ch. 18.

Peirce: CP 2.772 Fn 1 p 492

†1 See volumes 7 and 8.

Peirce: CP 2.773 Fn 1 p 495

†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 426-28.

Peirce: CP 2.779 Fn 1 p 499

†1 *Ibid.*, vol. 2, pp. 748-49, by Peirce and Mrs. C. Ladd-Franklin.

Peirce: CP 2.782 Fn 1 p 502

†1 *Ibid.*, vol. 2, p. 359.

Peirce: CP 2.783 Fn 2 p 502

†2 *Ibid.*, vol. 2, pp. 353-55.

Peirce: CP 2.788 Fn 1 p 506

†1 *Ibid.*, vol. 2, pp. 324-25.

Peirce: CP 2.791 Fn 1 p 507

†1 *Ibid.*, vol. 2, p. 337.

Peirce: CP 2.792 Fn 1 p 508

†1 Privately printed and "distributed at the Lowell Institute, Nov. 1866." Cf. Bk. III, ch. 2, Part II.

Peirce: CP 2.800 Fn P1 p 513 Cross-Ref:††

†P1 "Ubicunque ponitur *s* significatur quod propositio . . . debet converti simpliciter . . . et ubicunque ponitur *m* debet fieri transpositio in præmissis."--*Petrus Hisp.*

Peirce: CP 3 Title-Page
COLLECTED PAPERS OF CHARLES SANDERS PEIRCE

EDITED BY

CHARLES HARTSHORNE

AND

PAUL WEISS

VOLUME III

EXACT LOGIC

(*Published Papers*)

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Peirce: CP 3 Introduction p iii
INTRODUCTION

In the editing of the present volume Peirce's punctuation and spelling have, wherever possible, been retained. Titles supplied by the editors have been marked *E*; and their remarks and additions are enclosed in light-faced brackets. The editors' footnotes are indicated by various typographical signs, while Peirce's footnotes are indicated by numbers. Paragraphs are numbered consecutively throughout the volume. The numbers at the top of each page signify the volume and the first paragraph of that page. All references in the text and in the indices are to the numbers of the paragraphs.

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The department and the editors desire to express their gratitude to Dr. Henry S. Leonard, who has assisted with the proofs, references and editorial footnotes.

HARVARD UNIVERSITY

February, 1933

Peirce: CP 3 Editorial Note p v
EDITORIAL NOTE

Charles Sanders Peirce was one of the most original and prolific logicians of the nineteenth century. His published papers contain important contributions to almost every phase of the subject. He radically modified, extended and transformed the Boolean algebra, making it applicable to propositions, relations, probability and arithmetic. Following De Morgan, he was one of the chief contributors to the logic of

relatives. In addition to an analysis of "second intentional" relatives and a detailed classification of the main species of "first intentional" relatives, he supplied most of the fundamental theorems and distinctions in this branch of logic, providing, incidentally, two distinct algebras in terms of which they could be treated. He indicated how arithmetic, multiple algebras and quaternions could be derived from logic, made an independent discovery of the propositional function, of material and formal implication, and invented a new kind of logical diagram. Other discoveries which he did not publish will be found in volume four.

Peirce: CP 3 Editorial Note p v

Peirce's symbolism and mode of procedure is somewhat antiquated and in many places his thought is difficult to grasp. The following selected list of important topics, together with the explanatory footnotes to the text, and the index at the end of the volume should, however, aid even the general reader to extract what is still living and important in Peirce's work. The items mentioned in the following list do not exhaust either the number of Peirce's contributions or the topics on which he will prove illuminating; they are offered solely as a guide through the mazes of his symbolism. For one unfamiliar with the history of the subject, or the technicalities of modern logic, the easiest approach is by way of paper No. XX, which consists of articles published in Baldwin's *Dictionary of Philosophy and Psychology*, to be followed by papers No. XIII to XV; after which the rest may be read in chronological order.

Peirce: CP 3 Topics p vi
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Logical subtraction

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The definition of 1

The transition from logical identities to arithmetical equalities

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The criticism of Boole

The definition of arithmetical multiplication and independence

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The relation of class inclusion
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Kinds of logical terms
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Extension of the logic of relatives to probability
The "logical binomial"
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Division of possible problems
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BRYN MAWR COLLEGE

December, 1932.

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Peirce: CP 3.1 Cross-Ref:††

EXACT LOGIC

(Previously published papers)

ON AN IMPROVEMENT IN BOOLE'S CALCULUS OF LOGIC†1

1. The principal use of Boole's Calculus of Logic lies in its application to problems concerning probability. It consists, essentially, in a system of signs to denote the logical relations of classes. The data of any problem may be expressed by means of these signs, if the letters of the alphabet are allowed to stand for the classes themselves. From such expressions, by means of certain rules for transformation, expressions can be obtained for the classes (of events or things) whose frequency is sought in terms of those whose frequency is known. Lastly, if certain relations are known between the logical relations and arithmetical operations, these expressions for events can be converted into expressions for their probability.

Peirce: CP 3.1 Cross-Ref:††

It is proposed, first, to exhibit Boole's system in a modified form, and second, to examine the difference between this form and that given by Boole himself.

Peirce: CP 3.2 Cross-Ref:††

2. Let the letters of the alphabet denote classes whether of things or of occurrences. It is obvious that an event may either be singular, as "this sunrise," or general, as "all sunrises." Let the sign of equality with a comma beneath it express numerical identity. Thus $a =, b$ is to mean that a and b denote the same class--the same collection of individuals.

Peirce: CP 3.3 Cross-Ref:††

3. Let $a +, b$ denote all the individuals contained under a and b together.†2 The operation here performed will differ from arithmetical addition in two respects: first, that it has reference to identity, not to equality; and second, that what is common to a and b is not taken into account twice over, as it would be in arithmetic. The first of these differences, however, amounts to nothing, inasmuch as the sign of identity would indicate the distinction in which it is founded; and therefore we may say that

(1) If No a is b $a +, b =, a + b$.†1

It is plain that

(2) $a +, a =, a$ †2

and also, that the process denoted by $+, =$, and which I shall call the process of *logical addition*, is both commutative and associative. That is to say

(3) $a +, b =, b +, a$

and

$$(4) \quad (a +, b) +, c =, a +, (b +, c).$$

Peirce: CP 3.4 Cross-Ref:††

4. Let a, b denote the individuals contained at once under the classes a and b ; those of which a and b are the common species. If a and b were independent events, a, b would denote the event whose probability is the product of the probabilities of each. On the strength of this analogy (to speak of no other), the operation indicated by the comma may be called logical multiplication. It is plain that

$$(5) \quad a, a =, a. \dagger 2$$

Peirce: CP 3.4 Cross-Ref:††

Logical multiplication is evidently a commutative and associative process. That is,

$$(6) \quad a, b =, b, a$$

$$(7) \quad (a, b), c =, a, (b, c).$$

Peirce: CP 3.4 Cross-Ref:††

Logical addition and logical multiplication are doubly distributive, so that

$$(8) \quad (a +, b), c =, a, c +, b, c$$

and

$$(9) \quad a, b +, c =, (a +, c), (b +, c).$$

Proof. Let $a =, a' + x + y + o$

$$b =, b' + x + z + o$$

$$c =, c' + y + z + o$$

where any of these letters may vanish. These formulæ comprehend every possible

relation of a , b and c ; and it follows from them, that

$$a +, b =, a' + b' + x + y + z + o \quad (a +, b), c =, y + z + o.$$

But

$$a, c =, y + o \quad b, c =, z + o \quad a, c +, b, c =, y + z + o \quad \therefore (8).$$

So

$$a, b =, x + o \quad a, b +, c =, c' + x + y + z + o.$$

But

$$(a +, c) = a' + c' + x + y + z + o \quad (b +, c) =, b' + c' + x + y + z + o \\ (a +, c), (b +, c) =, c' + x + y + z + o \quad \therefore (9).$$

Peirce: CP 3.5 Cross-Ref: ††

5. Let $-$, be the sign of logical subtraction; so defined that

$$(10) \quad \text{If } b +, x =, a \quad x =, a -, b.$$

Peirce: CP 3.5 Cross-Ref: ††

Here it will be observed that x is not completely determinate. It may vary from a to a with b taken away. This minimum may be denoted by $a - b$. †1 It is also to be observed that if the sphere of b reaches at all beyond a , the expression $a -, b$ is uninterpretable. †2 If then we denote the contradictory negative of a class by the letter which denotes the class itself, with a line above it, †P1 if we denote by v a wholly indeterminate class, and if we allow $(0 -, 1)$ to be a wholly uninterpretable symbol, we have

$$(11) \quad a -, b =, v, a, b + a, \sim b + [0 -, 1], \sim a, b \dagger 3$$

which is uninterpretable unless

$$\sim a, b =, 0.$$

If we define zero by the following identities, in which x may be any class whatever,

$$(12) \quad 0 =, x -, x =, x - x$$

then, **zero** denotes the class which does not go beyond any class,†1 that is **nothing** or nonentity.

Peirce: CP 3.6 Cross-Ref:††

6. Let $a;b$ be read a logically divided by b , and be defined by the condition that

$$(13) \quad \text{If } b, x =, a \quad x =, a;b$$

x is not fully determined by this condition. It will vary from a to $a + \sim b$ and will be uninterpretable if a is not wholly contained under b .†2 Hence, allowing (1;0) to be some uninterpretable symbol,

$$(14) \quad a;b =, a, b+v, \sim a, \sim b+(1;0)a, \sim b \dagger 3$$

which is uninterpretable unless $a, \sim b =, 0$.

Peirce: CP 3.7 Cross-Ref:††

7. Unity may be defined by the following identities in which x may be any class whatever.

$$(15) \quad 1 =, x;x =, x:x. \dagger 4$$

Peirce: CP 3.7 Cross-Ref:††

Then **unity** denotes the class of which any class is a part; that is, **what is** or **ens**.

Peirce: CP 3.8 Cross-Ref:††

8. It is plain that if for the moment we allow $a;b$ to denote the maximum value of $a;b$, then

$$(16) \quad \sim x =, 1-x =, 0:x. \dagger 5$$

So that

$$(17) \quad x, (1-x) =, 0 \quad x +, 0:x =, 1.$$

Peirce: CP 3.9 Cross-Ref:††

9. The rules for the transformation of expressions involving logical subtraction and division would be very complicated. The following method is, therefore, resorted to.†1

Peirce: CP 3.9 Cross-Ref:††

It is plain that any operations consisting solely of logical addition and multiplication, being performed upon interpretable symbols, can result in nothing uninterpretable. Hence, if $\phi+Xx$ signifies such an operation performed upon symbols of which x is one, we have

$$\phi+Xx =, a,x+b,(1-x)†2$$

where a and b are interpretable.

Peirce: CP 3.9 Cross-Ref:††

It is plain, also, that all four operations being performed in any way upon any symbols, will, in general, give a result of which one term is interpretable and another not; although either of these terms may disappear. We have then

$$\phi x =, i,x+j,(1-x).†P1$$

Peirce: CP 3.9 Cross-Ref:††

We have seen that if either of these coefficients i and j is uninterpretable, the other factor of the same term is equal to nothing, or else the whole expression is uninterpretable. But

$$\phi(1) =, i \text{ and } \phi(0) =, j.$$

Hence

$$(18) \quad \phi x =, \phi(1),x + \phi(0),(1-x)†P2$$

$$\phi(x \text{ and } y) =, \phi(1 \text{ and } 1),x,y + \phi(1 \text{ and } 0),x,\sim y + \phi(0 \text{ and } 1),\sim x,y + \phi(0 \text{ and } 0),\sim x,\sim y.$$

$$(18') \quad \phi x =, (\phi(1)+,\sim x),(\phi(0) +, x)^2$$

$$\phi(x \text{ and } y) =, (\phi(1 \text{ and } 1) +, \sim x +, \sim y),(\phi(1 \text{ and } 0) +, \sim x +, y), \\ (\phi(0 \text{ and } 1) +, x +, \sim y),(\phi(0 \text{ and } 0) +, x +, y).$$

Developing by (18) $x -, y$, we have,

$$x -, y =, (1 -, 1),x,y+(1 -, 0),x,\sim y+(0 -, 1),\sim x,y+(0 -, 0),\sim x,\sim y.$$

So that, by (11),

$$(19) \quad (1 -, 1) =, v. \quad 1 -, 0 =, 1. \quad 0 -, 1 =, (0 -, 1). \quad 0 -, 0 =, 0.$$

Peirce: CP 3.10 Cross-Ref: ††

10. Developing $x;y$ in the same way, we have †P1

$$x;y =, 1;1,x,y+1;0,x,\sim y+0;1,\sim x,y+0;0,\sim x,\sim y.$$

So that, by (14),

$$(20) \quad 1;1 =, 1 \quad 1;0 =, (1;0) \quad 0;1 =, 0 \quad 0;0 =, v.$$

Boole gives (20), †1 but not (19).

Peirce: CP 3.10 Cross-Ref: ††

In solving identities we must remember that

$$(21) \quad (a +, b) - b =, a$$

$$(22) \quad (a -, b) +, b =, a.$$

From $a -, b$ the value of b cannot be obtained.

$$(23) \quad (a, b) \mid b =, a$$

$$(24) \quad a; b, b =, a.$$

From $a; b$ the value of b cannot be determined.

Peirce: CP 3.11 Cross-Ref: ††

11. Given the identity $\phi x =, 0$.

Required to eliminate x .

$$\phi(1) =, x, \phi(1) + (1-x), \phi(1)$$

$$\phi(0) =, x, \phi(0) + (1-x), \phi(0).$$

Logically multiplying these identities, we get

$$\phi(1), \phi(0) =, x, \phi(1), \phi(0) + (1-x), \phi(1), \phi(0).$$

For two terms disappear because of (17).

But we have, by (18),

$$\phi(1), x + \phi(0), (1-x) =, \phi x =, 0.$$

Multiplying logically by x we get

$$\phi(1), x =, 0$$

and by $(1-x)$ we get

$$\phi(0), (1-x) =, 0.$$

Substituting these values above, we have

(25) $\phi(1), \phi(0) =, 0$ when $\phi x =, 0$.

Peirce: CP 3.12 Cross-Ref:††

12. Given $\phi x =, 1$.

Required to eliminate x .

Let $\phi'x =, 1-\phi x =, 0$

$\phi'(1), \phi'(0) =, (1-\phi(1)), (1-\phi(0)) =, 0$

$1-(1-\phi(1)), (1-\phi(0)) =, 1$.

Now, developing as in (18), only in reference to $\phi(1)$ and $\phi(0)$ instead of to x and y ,

$1-(1-\phi(1)), (1-\phi(0)) =, \phi(1), \phi(0)+\phi(1), (1-\phi(0))$

$+ \phi(0), (1-\phi(1))$.

But by (18) we have also,

$\phi(1) +, \phi(0) =, \phi(1), \phi(0)+\phi(1), (1-\phi(0))+\phi(0), (1-\phi(1))$.

So that

(26) $\phi(1) +, \phi(0) =, 1$ when $\phi x =, 1$.

Boole gives (25), †1 but not (26).

Peirce: CP 3.13 Cross-Ref:††

13. We pass now from the consideration of *identities* to that of *equations*. †2

Peirce: CP 3.13 Cross-Ref:††

Let every expression for a class have a second meaning, which is its meaning in an equation. Namely, let it denote the proportion of individuals of that class to be found among all the individuals examined in the long run.

Peirce: CP 3.13 Cross-Ref:††

Then we have

$$(27) \quad \text{If } a =, b \quad a = b$$

$$(28) \quad a + b = (a +, b) + (a,b).$$

Peirce: CP 3.14 Cross-Ref:††

14. Let $b[a]$ denote the frequency of b 's among the a 's. Then considered as a class, if a and b are events, $b[a]$ denotes the fact that if a happens b happens.

$$(29) \quad a b[a] = a,b.\dagger 3$$

It will be convenient to set down some obvious and fundamental properties of the function $b[a]$.

$$(30) \quad a b[a] = b a[b]$$

$$(31) \quad \phi(b[a] \text{ and } c[a]) = (\phi(b \text{ and } c))[a]$$

$$(32) \quad (1-b)[a] = 1-b[a]$$

$$(33) \quad b[a] = b/a + b[(1-a)](1-1/a)$$

$$(34) \quad a[b] = 1 - (1-a/b) b[(1-a)]$$

$$(35) \quad (\phi a)[a] = (\phi(1))[a].$$

Peirce: CP 3.14 Cross-Ref:††

The application of the system to probabilities may best be exhibited in a few simple examples, some of which I shall select from Boole's work, in order that the solutions here given may be compared with his.

Peirce: CP 3.15 Cross-Ref:††

15. *Example* 1. Given the proportion of days upon which it hails, and the proportion of days upon which it thunders. Required the proportion of days upon which it does both.

Let $1 =$, days,

$p =$, days when it hails,

$q =$, days when it thunders,

$r =$, days when it hails and thunders.

$$p, q =, r$$

Then by (29), $r =, p, q = p q[p]=q p[q]$.

Peirce: CP 3.15 Cross-Ref:††

Answer. The required proportion is an unknown fraction of the least of the two proportions given.

Peirce: CP 3.15 Cross-Ref:††

By p might have been denoted the probability of the major, and by q that of the minor premiss of a hypothetical syllogism of the following form:

If a noise is heard, an explosion always takes place;

If a match is applied to a barrel of gunpowder, a noise is heard;

∴ If a match is applied to a barrel of gunpowder, an explosion always takes place.

Peirce: CP 3.15 Cross-Ref:††

In this case, the value given for r would have represented the probability of the conclusion. Now Boole (page 284) solves this problem by his unmodified method, and obtains the following answer:

$$r = p q + a(1 - q)$$

where a is an arbitrary constant. Here, if $q=1$ and $p=0$, $r=0$. That is, his answer implies that if the major premiss be false and the minor be true, the conclusion must be false. That this is not really so is shown by the above example. Boole (page 286) is forced to the conclusion that "propositions which, when true, are equivalent, are not necessarily equivalent when regarded only as probable." This is absurd, because probability belongs to the events denoted, and not to forms of expression. The probability of an event is not altered by translation from one language to another.

Peirce: CP 3.15 Cross-Ref:††

Boole, in fact, puts the problem into equations wrongly (an error which it is the chief purpose of a calculus of logic to prevent), and proceeds as if the problem

were as follows:

Peirce: CP 3.15 Cross-Ref:††

It being known what would be the probability of Y , if X were to happen, and what would be the probability of Z , if Y were to happen; what would be the probability of Z , if X were to happen?

Peirce: CP 3.15 Cross-Ref:††

But even this problem has been wrongly solved by him. For, according to his solution, where

$$p = Y[X] \quad q = Z[Y] \quad r = Z[X],$$

r must be at least as large as the product of p and q . But if X be the event that a certain man is a negro, Y the event that he is born in Massachusetts, and Z the event that he is a white man, then neither p nor q is *zero*, and yet r vanishes.

Peirce: CP 3.15 Cross-Ref:††

This problem may be rightly solved as follows:

$$\text{Let } p' =, Y[p] =, X, Y$$

$$q' =, Z[q] =, X, Z$$

$$r' =, Z[r] =, X, Z.$$

$$\text{Then, } r' =, p', q'; p' =, p', q'; q'.$$

Developing these expressions by (18) we have

$$r' =, p', q' + r'p', \sim q' + r'\sim p', \sim q'$$

$$=, p', q' + r'\sim p', q' + r'[\sim p', \sim q'](p', q')$$

The comparison of these two identities shows that

$$r' =, p', q' + r'\sim p', \sim q'.$$

$$\text{Let } V =, r'[\sim p', \sim q'] =, ((x, \sim y, z)/(\sim x, y, \sim z + \sim y))$$

$$\begin{aligned} \text{Now } p', q' &=, p' - p', \sim q' =, q' - q', \sim p' \\ \sim p', \sim q &=, \sim q' - p', \sim q' =, \sim p' - q', \sim p' \end{aligned}$$

$$\text{And } p', \sim q' =, p' - p'[q']q' =, \sim q' - \sim q[\sim p']\sim p'$$

$$\sim p', q' =, q' - q'[p']p' =, \sim p' - \sim p'[\sim q']\sim q'$$

Then let

$$A =, p'[q'] =, (x.y.z)/(y.z)$$

$$B =, \sim q'[\sim p'] =, (\sim x.y, \sim z + \sim x, \sim y.z + x, \sim y.z + \sim x, \sim y, \sim z)/(1 - x.y)$$

$$C =, \sim p'[\sim q'] =, (\sim x.y, \sim z + \sim x, \sim y.z + x, \sim y.z + \sim x, \sim y, \sim z)/(1 - y.z)$$

$$D =, q'[p'] =, (x.y.z)/(x.y)$$

And we have

$$\begin{aligned} r &= (Y/Z)p + V(1/Z - q) - (1 + V)((Y/Z)p - A q) \\ &= (Y/Z)p + V(1/Z - q) - (1 + V)(1/Z - q - B((1 - Y p)/Z)) \\ &= q + V((1 - Y p)/Z) - (1 + V)((1 - Y p)/Z - C((1/Z) - q)) \\ &= q + V(1 - Y p/Z) - (1 + V)(q - D(Y/Z)p) \end{aligned}$$

Peirce: CP 3.16 Cross-Ref: ††

16. *Example 2.* (See Boole, page 276.) Given r and q ; to find p .

$p =, r; q =, r + v, (1 - q)$ because p is interpretable.

Peirce: CP 3.16 Cross-Ref:††

Answer. The required proportion lies somewhere between the proportion of days upon which it both hails and thunders, and that added to one minus the proportion of days when it thunders.

Peirce: CP 3.17 Cross-Ref:††

17. **Example 3.** (See Boole, page 279.) Given, out of the number of questions put to two witnesses, and answered by *yes* or *no*, the proportion that each answers truly, and the proportion of those their answers to which disagree. Required, out of those wherein they agree, the proportion they answer truly and the proportion they answer falsely.†1

Let 1 =, the questions put to both witnesses,

p =, those which the first answers truly,

q =, those which the second answers truly,

r =, those wherein they disagree,

w =, those which both answer truly,

w' =, those which both answer falsely.

$$w =, p, q \quad w' =, \sim p, \sim q \quad r =, p +, q - w =, \sim p +, \sim q - w'.$$

Now by (28)

$$p +, q = p + q - w \quad \sim p +, \sim q = p - p + 1 - q - w'.$$

Substituting and transposing,

$$2w = p + q - r \quad 2w' = 2 - p - q - r.$$

Now $w[1-r] = (w, (1-r))/(1-r)$ but $w(1-r) =, w$.

$w'[1-r] = (w', (1-r))/(1-r)$ but $w'(1-r) =, w'$.

$$\therefore w[1-r] = (p+q-r)/(2(1-r)) \quad w'[1-r] = (2-p-q-r)/(2(1-r)).$$

Peirce: CP 3.18 Cross-Ref:††

18. The differences of Boole's system, as given by himself, from the modification of it given here, are three.

Peirce: CP 3.18 Cross-Ref:††

First. Boole does not make use of the operations here termed logical addition and subtraction. The advantages obtained by the introduction of them are three, *viz.*, they give unity to the system; they greatly abbreviate the labor of working with it; and they enable us to express *particular* propositions. This last point requires illustration. Let *i* be a class only determined to be such that only some one individual of the class *a* comes under it. Then *a - i*, *a* is the expression for some *a*. Boole cannot properly express some *a*.

Peirce: CP 3.18 Cross-Ref:††

Second. Boole uses the ordinary sign of multiplication for logical multiplication. This debars him from converting every logical identity into an equality of probabilities. Before the transformation can be made the equation has to be brought into a particular form, and much labor is wasted in bringing it to that form.

Peirce: CP 3.18 Cross-Ref:††

Third. Boole has no such function as *a*[*b*]. This involves him in two difficulties. When the probability of such a function is required, he can only obtain it by a departure from the strictness of his system. And on account of the absence of that symbol, he is led to declare that, without adopting the principle that simple, unconditioned events whose probabilities are given are independent, a calculus of logic applicable to probabilities would be impossible.

Peirce: CP 3.19 Cross-Ref:††

19. The question as to the adoption of this principle is certainly not one of words merely. The manner in which it is answered, however, partly determines the sense in which the term "probability" is taken.

Peirce: CP 3.19 Cross-Ref:††

In the propriety of language, the probability of a fact either is, or solely depends upon, the strength of the argument in its favor, supposing all relevant relations of all known facts to constitute that argument. Now, the strength of an argument is only the frequency with which *such* an argument will yield a true conclusion when its premisses are true. Hence probability depends solely upon the relative frequency of a specific event (namely, that a certain kind of argument yields a true conclusion from true premisses) to a generic event (namely, that that kind of argument occurs with true premisses). Thus, when an ordinary man says that it is highly probable that it will rain, he has reference to certain indications of rain--that is, to a certain kind of argument that it will rain--and means to say that there is an argument that it will rain, which is of a kind of which but a small proportion fail. "Probability," in the untechnical sense, is therefore a vague word, inasmuch as it does not indicate what one, of the numerous subordinated and coordinated genera to which every argument belongs, is the one the relative frequency of the truth of which is expressed. It is usually the case, that there is a tacit understanding upon this point, based perhaps on the notion of an *infima species* of argument. But an *infima species* is a mere fiction in logic. And very often the reference is to a very wide genus.

Peirce: CP 3.19 Cross-Ref:††

The sense in which the term should be made a technical one is that which will best subserve the purposes of the calculus in

question. Now, the only possible use of a calculation of a probability is security in the long run. But there can be no question that an insurance company, for example, which assumed that events were independent without any reason to think that they really were so, would be subjected to great hazard. Suppose, says Mr. Venn,†1 that an insurance company knew that nine tenths of the Englishmen who go to Madeira die, and that nine tenths of the consumptives who go there get well. How should they treat a consumptive Englishman? Mr. Venn has made an error in answering the question, but the illustration puts in a clear light the advantage of ceasing to speak of probability, and of speaking only of the relative frequency of this event to that.†P1

Peirce: CP 3.20 Cross-Ref:††

II

UPON THE LOGIC OF MATHEMATICS†1

PART I †2

§1. THE BOOLIAN CALCULUS^E

20. The object of the present paper is to show that there are certain general propositions from which the truths of mathematics follow syllogistically, and that these propositions may be taken as definitions of the objects under the consideration of the mathematician without involving any assumption in reference to experience or intuition. That there actually are such objects in experience or pure intuition is not in itself a part of pure mathematics.

Peirce: CP 3.21 Cross-Ref:††

21. Let us first turn our attention to the logical calculus of Boole. I have shown in a previous communication to the Academy,†3 that this calculus involves eight operations, viz., Logical Addition, Arithmetical Addition, Logical Multiplication, Arithmetical Multiplication, and the processes inverse to these.

DEFINITIONS

1. **Identity.** $a =, b$ expresses the two facts that any a is b and any b is a .
2. **Logical Addition.** $a +, b$ denotes a member of the class which contains under it all the a 's and all the b 's, and nothing else.

3. **Logical Multiplication.** $a \cdot b$ denotes only whatever is both a and b .
4. **Zero** denotes *nothing*, or the class without extent, by which we mean that if a is any member of any class, $a + 0$ is a .
5. **Unity** denotes *being*, or the class without content, by which we mean that, if a is a member of any class, a is $a \cdot 1$.
6. **Arithmetical Addition.** $a + b$, if $a \cdot b = 0$, is the same as $a + b$, but, if a and b are classes which have any extent in common, it is not a class.
7. **Arithmetical Multiplication.** $a \cdot b$ represents an event when a and b are events only if these events are independent of each other, in which case $a \cdot b = a \cdot b$. By the events being independent is meant that it is possible to take two series of terms, $A[1], A[2], A[3], \dots$, and $B[1], B[2], B[3], \dots$, such that the following conditions will be satisfied. (Here x denotes any individual or class, not nothing; $A[m], A[n], B[m], B[n]$, any members of the two series of terms, and $\Sigma A, \Sigma B, \Sigma(A, B)$ logical sums of some of the $A[n]$'s, the $B[n]$'s, and the $(A[n], B[n])$'s respectively.)

Condition 1. No $A[m]$ is $A[n]$.

Condition 2. No $B[m]$ is $B[n]$.

Condition 3. $x = \Sigma(A, B)$

Condition 4. $a = \Sigma A$.

Condition 5. $b = \Sigma B$.

Condition 6. Some $A[m]$ is $B[n]$.†1

Peirce: CP 3.22 Cross-Ref:††

22. From these definitions a series of theorems follow syllogistically, the proofs of most of which are omitted on account of their ease and want of interest.

Peirce: CP 3.23 Cross-Ref:††
THEOREMS

I

23. If $a = b$, then $b = a$.

Peirce: CP 3.24 Cross-Ref:††
II

24. If $a = b$, and $b = c$, then $a = c$.

Peirce: CP 3.25 Cross-Ref:††

III

25. If $a + b = c$, then $b + a = c$.

Peirce: CP 3.26 Cross-Ref:††

26. If $a + b = m$ and $b + c = n$ and $a + n = x$, then $m + c = x$.

Peirce: CP 3.26 Cross-Ref:††

Corollary. These last two theorems hold good also for arithmetical addition.

Peirce: CP 3.27 Cross-Ref:††

V

27. If $a + b = c$ and $a' + b = c$, then $a = a'$, or else there is nothing not b .

Peirce: CP 3.27 Cross-Ref:††

This theorem does not hold with logical addition. But from definition 6 it follows that

No a is b (supposing there is any a)

No a' is b (supposing there is any a')

neither of which propositions would be implied in the corresponding formulæ of logical addition. Now from definitions 2 and 6,

Any a is c

∴ Any a is c not b

But again from definitions 2 and 6 we have

Any c not b is a' (if there is any not b)

\therefore Any a is a' (if there is any not b)

And in a similar way it could be shown that any a' is a (under the same supposition).
Hence by definition 1,

$a =, a'$ if there is anything not b .

Peirce: CP 3.27 Cross-Ref:††

Scholium. In arithmetic this proposition is limited by the supposition that b is finite.†1 The supposition here though similar to that is not quite the same.

Peirce: CP 3.28 Cross-Ref:††
VI

28. If $a, b =, c$, then $b, a =, c$.

Peirce: CP 3.29 Cross-Ref:††
VII

29. If $a, b =, m$ and $b, c =, n$ and $a, n =, x$, then $m, c =, x$.

Peirce: CP 3.30 Cross-Ref:††
VIII

30. If $m, n =, b$ and $a +, m =, u$ and $a +, n =, v$ and $a +, b =, x$, then $u, v =, x$.

Peirce: CP 3.31 Cross-Ref:††
IX

31. If $m +, n =, b$ and $a, m =, u$ and $a, n =, v$ and $a, b =, x$, then $u +, v =, x$.

Peirce: CP 3.31 Cross-Ref:††

The proof of this theorem may be given as an example of the proofs of the rest.

Peirce: CP 3.31 Cross-Ref:††

It is required then (by definition 3) to prove three propositions, *viz.*

First. That any u is x .

Second. That any v is x .

Third. That any x not u is v .

Peirce: CP 3.31 Cross-Ref:††
FIRST PROPOSITION

Since $u = a, m$, by definition 3

Any u is m ,

and since $m + n = b$, by definition 2

Any m is b ,

whence Any u is b ,

But since $u = a, m$, by definition 3

Any u is a ,

whence Any u is both a and b ,

But since $a, b = x$, by definition 3

Whatever is both a and b is x

whence Any u is x .

Peirce: CP 3.31 Cross-Ref:††
SECOND PROPOSITION

This is proved like the first.

Peirce: CP 3.31 Cross-Ref:††
THIRD PROPOSITION

Since $a, m = u$, by definition 3,

Whatever is both a and m is u .

or Whatever is not u is not both a and m .

or Whatever is not u is either not a or not m .

or Whatever is not u and is a is not m .

But since $a, b = x$, by definition 3

Any x is a ,
whence Any x not u is not u and is a ,
whence Any x not u is not m .

But since $a, b =, x$, by definition 3

Any x is b ,
whence Any x not u is b ,
Any x not u is b , not m .

But since $m +, n =, b$, by definition 2

Any b not m is n ,
whence Any x not u is n ,
and therefore Any x not u is both a and n .†1

But since $a, n =, v$, by definition 3

Whatever is both a and n †2 is v ,
whence Any x not u is v .

Peirce: CP 3.32 Cross-Ref:††

32. *Corollary 1.* This proposition readily extends itself to arithmetical addition.

Peirce: CP 3.32 Cross-Ref:††

Corollary 2. The converse propositions produced by transposing the last two identities of theorems VIII and IX are also true.

Peirce: CP 3.32 Cross-Ref:††

Corollary 3. Theorems VI, VII, and IX hold also with arithmetical multiplication. This is sufficiently evident in the case of theorem VI, because by definition 7 we have an additional premiss, namely, that a and b are independent, and an additional conclusion which is the same as that premiss.

Peirce: CP 3.33 Cross-Ref:††

33. In order to show the extension of the other theorems, I shall begin with the following lemma. If a and b are independent, then corresponding to every pair of individuals, one of which is both a and b , there is just one pair of individuals one of which is a and the other b ; and conversely, if the pairs of individuals so correspond, a and b are independent. For, suppose a and b independent, then, by definition 7, condition 3, every class $(A[m], B[n])$ is an individual. If then $A[a]$ denotes any $A[m]$ which is a , and $B[b]$ any $B[n]$ which is b , by condition 6 $(A[a], B[n])$ and $(A[m], B[b])$ both exist, and by conditions 4 and 5 the former is any individual a , and the latter any

individual b . But given this pair of individuals, both of the pair $(A[a], B[b])$ and $(A[m], B[n])$ exist by condition 6. But one individual of this pair is both a and b . Hence the pairs correspond, as stated above. Next, suppose a and b to be any two classes. Let the series of $A[m]$'s be a and not- a ; and let the series of $B[m]$'s be all individuals separately. Then the first five conditions can always be satisfied. Let us suppose, then, that the sixth alone cannot be satisfied. Then $A[p]$ and $B[q]$ may be taken such that $(A[p], B[q])$ is nothing. Since $A[p]$ and $B[q]$ are supposed both to exist, there must be two individuals $(A[p], B[n])$ and $(A[m], B[q])$ which exist. But there is no corresponding pair $(A[m], B[n])$ and $(A[p], B[q])$. Hence, no case in which the sixth condition cannot be satisfied simultaneously with the first five is a case in which the pairs rightly correspond; or, in other words, every case in which the pairs correspond rightly is a case in which the sixth condition can be satisfied, provided the first five can be satisfied. But the first five can always be satisfied. Hence, if the pairs correspond as stated, the classes are independent.

Peirce: CP 3.34 Cross-Ref: ††

34. In order to show that theorem VII may be extended to arithmetical multiplication, we have to prove that if a and b , b and c , and a and (b, c) , are independent, then (a, b) and c are independent. Let s denote any individual. Corresponding to every s with (a, b, c) , there is an a and (b, c) . Hence, corresponding to every s with s and with (a, b, c) (which is a particular case of that pair), there is an s with a and with (b, c) . But for every s with (b, c) there is a b with c ; hence, corresponding to every a with s and with (b, c) , there is an a with b and with c . Hence, for every s with s and with (a, b, c) there is an a with b and with c . For every a with b there is an s with (a, b) ; hence, for every a with b and with c , there is an s with (a, b) and c . Hence, for every s with s and with (a, b, c) there is an s with (a, b) and with c . Hence, for every s with (a, b, c) there is an (a, b) with c . The converse could be proved in the same way. Hence, etc.

Peirce: CP 3.35 Cross-Ref: ††

35. Theorem IX holds with arithmetical addition of whichever sort the multiplication is. For we have the additional premiss that "No m is n "; whence since "any u is m " and "any v is n ," "no u is v ," which is the additional conclusion.

Peirce: CP 3.35 Cross-Ref: ††

Corollary 2, so far as it relates to theorem IX, holds with arithmetical addition and multiplication. For, since no m is n , every pair, one of which is a and either m or n , is either a pair, one of which is a and m , or a pair, one of which is a and n , and is not both. Hence, since for every pair one of which is a and m , there is a pair one of which is a and the other m , and since for every pair one of which is a, n there is a pair one of which is a and the other n ; for every pair one of which is a and either m or n , there is either a pair one of which is a and the other m , or a pair one of which is a and the other n , and not both; or, in other words, there is a pair one of which is a and the other either m or n .

Peirce: CP 3.35 Cross-Ref: ††

(It would perhaps have been better to give this complicated proof in its full syllogistic form. But as my principal object is merely to show that the various theorems could be so proved, and as there can be little doubt that if this is true of those which relate to arithmetical addition it is true also of those which relate to arithmetical multiplication, I have thought the above proof (which is quite apodeictic) to be sufficient. The reader should be careful not to confound a proof which needs

itself to be experienced with one which requires experience of the object of proof.)

Peirce: CP 3.36 Cross-Ref:††
X

36. If $a b = c$ and $a' b = c$, then $a = a'$, or no b exists.

This does not hold with logical, but does with arithmetical multiplication.

For if a is not identical with a' , it may be divided thus

$$a = a, a' + a, \sim a'$$

if $\sim a'$ denotes not a' . Then

$$a, b = (a, a'), b + (a, \sim a'), b$$

and by the definition of independence the last term does not vanish unless $(a, \sim a') = 0$, or all a is a' ; but since $a, b = a', b = (a, a'), b + (\sim a, a'), b$, this term does vanish, and, therefore, only a is a' , and in a similar way it could be shown that only a' is a .

Peirce: CP 3.37 Cross-Ref:††
XI

37. $1 + a = 1$.

This is not true of arithmetical addition, for since by definition 7,

$$1x, 1 = x1$$

by theorem IX

$$x, (1+a) = x(1+a) = x1 + xa = x + xa.$$

Peirce: CP 3.37 Cross-Ref:††

Whence $xa = 0$, while neither x nor a is zero, which, as will appear directly,

is impossible.

Peirce: CP 3.38 Cross-Ref:††
XII

38. $0, a =, 0$.

Proof. For call $0, a =, x$. Then by definition 3

x belongs to the class *zero*.

∴ by definition 4 $x =, 0$.

Peirce: CP 3.38 Cross-Ref:††

Corollary 1. The same reasoning applies to arithmetical multiplication.

Peirce: CP 3.38 Cross-Ref:††

Corollary 2. From theorem x and the last corollary it follows that if $a b =, 0$, either $a =, 0$ or $b =, 0$.

Peirce: CP 3.39 Cross-Ref:††
XIII

39. $a, a =, a.†1$

Peirce: CP 3.40 Cross-Ref:††
XIV

40. $a +, a =, a.†1$

These do not hold with arithmetical operations.

Peirce: CP 3.41 Cross-Ref:††

41. **General Scholium.** This concludes the theorems relating to the direct operations. As the inverse operations have no peculiar logical interest, they are passed over here.

In order to prevent misapprehension, I will remark that I do not undertake to demonstrate the principles of logic themselves. Indeed, as I have shown in a previous

paper, these principles considered as speculative truths are absolutely empty and indistinguishable.†2 But what has been proved is the *maxims* of logical procedure, a certain system of signs being given.

Peirce: CP 3.41 Cross-Ref:††

The definitions given above for the processes which I have termed arithmetical plainly leave the functions of these operations in many cases uninterpreted. Thus if we write

$$a+b =, b+a$$

$$a+(b+c) =, (a+b)+c$$

$$b c =, c b$$

$$(a b)c =, a(b c)$$

$$a(m+n) =, a m+a n$$

we have a series of identities whose truth or falsity is entirely

undeterminable. In order, therefore, *fully to define those operations*, we will say that all propositions, equations, and identities which are in the general case left by the former definitions undetermined as to truth, shall be true, provided they are so in all interpretable cases.

Peirce: CP 3.42 Cross-Ref:††
§2. ON ARITHMETIC.†1

42. *Equality* is a relation of which identity is a species.

If we were to leave equality without further defining it, then by the last scholium all the formal rules of arithmetic would follow from it. And this completes the central design of this paper, as far as arithmetic is concerned.

Peirce: CP 3.43 Cross-Ref:††

43. Still it may be well to consider the matter a little further. Imagine, then, a particular case under Boole's calculus, in which the letters are no longer terms of first intention, but terms of second intention, and that of a special kind. Genus, species, difference, property, and accident, are the well-known terms of second intention. These relate particularly to the *comprehension*†2 of first intentions; that is, they refer to different sorts of predication. Genus and species, however, have at least a secondary reference to the *extension*†2 of first intentions. Now let the letters, in the particular application of Boole's calculus now supposed, be terms of second intention which relate exclusively to the extension of first intentions.†3 Let the differences of the characters of things and events be disregarded, and let the letters signify only the differences of classes as wider or narrower. In other words, the only logical comprehension which the letters considered as terms will have is the greater or less

divisibility of the classes. Thus, n in another case of Boole's calculus might, for example, denote "New England States"; but in the case now supposed, all the characters which make these States what they are being neglected, it would signify only what essentially belongs to a class which has the same relations to higher and lower classes which the class of New England States has, -- that is, a collection of *six*.

Peirce: CP 3.44 Cross-Ref:††

44. In this case, the sign of identity will receive a special meaning. For, if m denotes what essentially belongs to a class of the rank of "sides of a cube," then $m =, n$ will imply, not that every New England State is a side of a cube, and conversely, but that whatever essentially belongs to a class of the numerical rank of "New England States" essentially belongs to a class of the rank of "sides of a cube," and conversely. *Identity* of this particular sort may be termed *equality*, and be denoted by the sign $=$.†P1 Moreover, since the numerical rank of a *logical sum* depends on the identity or diversity (in first intention) of the integrant parts, and since the numerical rank of a *logical product* depends on the identity or diversity (in first intention) of parts of the factors, logical addition and multiplication can have no place in this system. Arithmetical addition and multiplication, however, will not be destroyed. $a b = c$ will imply that whatever essentially belongs at once to a class of the rank of a , and to another independent class of the rank of b belongs essentially to a class of the rank of c , and conversely.†1 $a + b = c$ implies that whatever belongs essentially to a class which is the logical sum of two mutually exclusive classes of the ranks of a and b belongs essentially to a class of the rank of c , and conversely.†1 It is plain that from these definitions the same theorems follow as from those given above. *Zero* and *unity* will, as before, denote the classes which have respectively no extension and no comprehension; only the comprehension here spoken of is, of course, that comprehension which alone belongs to letters in the system now considered, that is, this or that degree of divisibility; and therefore *unity* will be what belongs essentially to a class of any rank independent of its divisibility. These two classes alone are common to the two systems, because the first intentions of these alone determine, and are determined by, their second intentions. Finally, the laws of the Boolean calculus, in its ordinary form, are identical with those of this other so far as the latter apply to *zero* and *unity*, because every class, in its first intention, is either without any extension (that is, is nothing), or belongs essentially to that rank to which every class belongs, whether divisible or not.

Peirce: CP 3.44 Cross-Ref:††

These considerations, together with those advanced [in 1.556], will, I hope, put the relations of logic and arithmetic in a somewhat clearer light than heretofore.

Peirce: CP 3.45 Cross-Ref:††

III

***DESCRIPTION OF A NOTATION FOR THE LOGIC OF RELATIVES,
RESULTING FROM AN AMPLIFICATION OF THE CONCEPTIONS OF
BOOLE'S CALCULUS OF LOGIC†1***

§1. DE MORGAN'S NOTATIONE

45. Relative terms usually receive some slight treatment in works upon logic, but the only considerable investigation into the formal laws which govern them is contained in a valuable paper by Mr. De Morgan in the tenth volume of the *Cambridge Philosophical Transactions*.†2 He there uses a convenient algebraic notation, which is formed by adding to the well-known *spiculae* of that writer the signs used in the following examples.

$X . . LY$ signifies that X is some one of the objects of thought which stand to Y in the relation L , or is one of the L 's of Y .

$X . LMY$ signifies that X is not an L of an M of Y .

$X . . (L,M)Y$ signifies that X is either an L or an M of Y .

LM' an L of every M . $L[,]M$ an L of none but M 's.

$L[[-1]]Y$ something to which Y is L . l (small L) non- L .

This system still leaves something to be desired. Moreover, Boole's logical algebra has such singular beauty, so far as it goes, that it is interesting to inquire whether it cannot be extended over the whole realm of formal logic, instead of being restricted to that simplest and least useful part of the subject, the logic of absolute terms, which, when he wrote, was the only formal logic known. The object of this paper is to show that an affirmative answer can be given to this question. I think there can be no doubt that a *calculus*, or art of drawing inferences, based upon the notation I am to describe, would be perfectly possible and even practically useful in some difficult cases, and particularly in the *investigation* of logic. I regret that I am not in a situation to be able to perform this labor, but the account here given of the notation itself will afford the ground of a judgment concerning its probable utility.

Peirce: CP 3.46 Cross-Ref:††

46. In extending the use of old symbols to new subjects, we must of course be guided by certain principles of analogy, which, when formulated, become new and wider definitions of these symbols. As we are to employ the usual algebraic signs as far as possible, it is proper to begin by laying down definitions of the various algebraic relations and operations. The following will, perhaps, not be objected to.

Peirce: CP 3.47 Cross-Ref:††

§2. GENERAL DEFINITIONS OF THE ALGEBRAIC SIGNS

47. *Inclusion in* or *being as small as* is a *transitive* relation. The consequence holds that †P1

If $x < y$,
 and $y < z$,
 then $x < z$.

Peirce: CP 3.48 Cross-Ref:††

48. **Equality** is the conjunction of being as small as and its converse. To say that $x = y$ is to say that $x < y$ and $y < x$.

Peirce: CP 3.49 Cross-Ref:††

49. **Being less than** is being as small as with the exclusion of its converse. To say that $x < y$ is to say that $x < y$, and that it is not true that $y < x$.

Peirce: CP 3.50 Cross-Ref:††

50. **Being greater than** is the converse of being less than. To say that $x > y$ is to say that $y < x$.

Peirce: CP 3.51 Cross-Ref:††

51. Addition is an **associative** operation. That is to say, †P1

$$(x +, y) +, z = x +, (y +, z).$$

Addition is a **commutative** operation. That is,

$$x +, y = y +, x.$$

Peirce: CP 3.52 Cross-Ref:††

52. Invertible †1 addition is addition the corresponding inverse of which is determinative. The last two formulæ hold good for it, and also the consequence that

If $x + y = z$,
 and $x + y' = z$,
 then $y = y'$. †2

Peirce: CP 3.53 Cross-Ref:††

53. Multiplication is an operation which is **doubly distributive with reference to addition**. That is,

$$x(y +, z) = x y +, x z,$$

$$(x +, y)z = x z +, y z.$$

Multiplication is almost invariably an *associative* operation. †3

$$(x y)z = x(y z).$$

Multiplication is not generally commutative. If we write commutative †4 multiplication with a comma, †5 we have

$$x.y = y.x.$$

Peirce: CP 3.54 Cross-Ref: ††

54. Invertible †1 multiplication is multiplication whose corresponding inverse operation (division) is determinative. We may indicate this by a dot; †2 and then the consequence holds that

$$\begin{aligned} &\text{If } x.y = z, \\ &\text{and } x.y' = z, \\ &\text{then } y = y'. \end{aligned} \dagger 3$$

Peirce: CP 3.55 Cross-Ref: ††

55. Functional multiplication †4 is the application of an operation to a function. It may be written like ordinary multiplication; but then there will generally be certain points where the associative principle does not hold. Thus, if we write $(\sin abc) def$, there is one such point. If we write $(\log (\text{base } abc) def) ghi$, there are two such points. The number of such points depends on the nature of the symbol of operation, and is necessarily finite. If there were many such points, in any case, it would be necessary to adopt a different mode of writing such functions from that now usually employed. We might, for example, give to "log" such a meaning that what followed it up to a certain point indicated by a † should denote the base of the system, what followed that to the point indicated by a ‡ should be the function operated on, and what followed that should be beyond the influence of the sign "log." Thus $\log abc \dagger def \ddagger ghi$ would be $(\log abc) ghi$, the base being def . In this paper I shall adopt a notation very similar to this, which will be more conveniently described further on.

Peirce: CP 3.56 Cross-Ref: ††

56. The operation of involution obeys the formula †P1

$$(xy)z = x(yz).$$

Involution, also, follows the *indexical principle*.

$$xy +, z = xy, xz.$$

Involution, also, satisfies the *binomial theorem*. †1

$$(x +, y)z = xz +, \Sigma[p]xz \cdot p \cdot yp +, yz,$$

where $\Sigma[p]$ denotes that p is to have every value less than z , and is to be taken out of z in all possible ways, and that the sum of all the terms so obtained of the form $xz \cdot p \cdot yp$ is to be taken.

Peirce: CP 3.57 Cross-Ref: ††

57. Subtraction is the operation inverse to addition. We may write indeterminate †2 subtraction with a comma below the usual sign. Then we shall have that

$$(x -, y) +, y = x,$$

$$(x - y) + y = x,$$

$$(x + y) - y = x.$$

Peirce: CP 3.58 Cross-Ref: ††

58. Division is the operation inverse to multiplication. Since multiplication is not generally commutative it is necessary to have two signs for division. I shall take

$$(x:y)y = x,$$

$$x y/x = y.$$

Peirce: CP 3.59 Cross-Ref: ††

59. Division inverse to that multiplication which is indicated by a comma may be indicated by a semicolon. So that

$$(x;y).y = x. †3$$

Peirce: CP 3.60 Cross-Ref:††

60. Evolution and taking the logarithm are the operations inverse to involution.

$$(x \vee y)x = y,$$

$$x^{\log[x]y} = y.$$

Peirce: CP 3.61 Cross-Ref:††

61. These conditions are to be regarded as imperative. But in addition to them there are certain other characters which it is highly desirable that relations and operations should possess, if the ordinary signs of algebra are to be applied to them. These I will here endeavour to enumerate.

Peirce: CP 3.61 Cross-Ref:††

1. It is an additional motive for using a mathematical sign to signify a certain operation or relation that the general conception of this operation or relation should resemble that of the operation or relation usually signified by the same sign. In particular, it will be well that the relation expressed by \prec should involve the conception of one member being in the other; addition, that of taking together; multiplication, that of one factor's being taken relatively to the other (as we write 3×2 for a triplet *of* pairs, and $D\phi$ for the derivative of ϕ); and involution, that of the base being taken for every unit of the exponent.

Peirce: CP 3.61 Cross-Ref:††

2. In the second place, it is desirable that, in certain general circumstances, determinate numbers should be capable of being substituted for the letters operated upon, and that when so substituted the equations should hold good when interpreted in accordance with the ordinary definitions of the signs, so that arithmetical algebra should be included under the notation employed as a special case of it. For this end, there ought to be a number known or unknown, which is appropriately substituted in certain cases, for each one of, at least, some class of letters.

Peirce: CP 3.61 Cross-Ref:††

3. In the third place, it is almost essential to the applicability of the signs for addition and multiplication, that a *zero* and a *unity* should be possible. By a *zero* I mean a term such that

$$x +, 0 = x,$$

whatever the signification of x ; and by a *unity* a term for which the corresponding general formula

$$x1 = x$$

holds good. On the other hand, there ought to be no term a such that $ax=x$, independently of the value of x .

Peirce: CP 3.61 Cross-Ref:††

4. It will also be a strong motive for the adoption of an algebraic notation, if other formulæ which hold good in arithmetic, such as

$$xz.yz = (x.y)z,$$

$$1x = x,$$

$$x1 = x,$$

$$x0 = 0,$$

continue to hold good; if, for instance, the conception of a differential is possible, and Taylor's Theorem holds, and $\mathcal{C}\dagger 1$

or $(1+i)^{1/i}$ plays an important part in the system, if there should be a term having the properties of $\mathcal{D}\dagger 1$ (3.14159), or properties similar to those of space should otherwise be brought out by the notation, or if there should be an absurd expression having the properties and uses of $\mathcal{J}\dagger 1$ or the square root of the negative.

Peirce: CP 3.62 Cross-Ref:††

§3. APPLICATION OF THE ALGEBRAIC SIGNS TO LOGIC

62. While holding ourselves free to use the signs of algebra in any sense conformable to the above absolute conditions, we shall find it convenient to restrict ourselves to one particular interpretation except where another is indicated. I proceed to describe the special notation which is adopted in this paper.

Peirce: CP 3.63 Cross-Ref:††

USE OF THE LETTERS

63. The letters of the alphabet will denote logical signs. Now logical terms are of three grand classes. The first embraces those whose logical form involves only the conception of quality, and which therefore represent a thing simply as "a --." These discriminate objects in the most rudimentary way, which does not involve any consciousness of discrimination. They regard an object as it is in itself as *such*

(*quale*); for example, as horse, tree, or man. These are *absolute terms*. The second class embraces terms whose logical form involves the conception of relation, and which require the addition of another term to complete the denotation. These discriminate objects with a distinct consciousness of discrimination. They regard an object as over against another, that is as relative; as father of, lover of, or servant of. These are *simple relative terms*. The third class embraces terms whose logical form involves the conception of bringing things into relation, and which require the addition of more than one term to complete the denotation. They discriminate not only with consciousness of discrimination, but with consciousness of its origin. They regard an object as medium or third between two others, that is as conjugative; as giver of -- to --, or buyer of -- for -- from --. These may be termed *conjugative terms*. The conjugative term involves the conception of *third*, the relative that of second or *other*, the absolute term simply considers *an* object.†1 No fourth class of terms exists involving the conception of *fourth*, because when that of *third is* introduced, since it involves the conception of bringing objects into relation, all higher numbers are given at once, inasmuch as the conception of bringing objects into relation is independent of the number of members of the relationship.†2 Whether this *reason* for the fact that there is no fourth class of terms fundamentally different from the third is satisfactory or not, the fact itself is made perfectly evident by the study of the logic of relatives. I shall denote absolute terms by the Roman alphabet, a, b, c, d, etc.; relative terms by italics, *a, b, c, d*, etc.; and conjugative terms by a kind of type called Kennerly, **a, b, c, d**, etc.

Peirce: CP 3.63 Cross-Ref:††

I shall commonly denote individuals by capitals, and generals †3 by small letters. General symbols for numbers will be printed in black-letter, thus, *a, b, c, d*, etc. The Greek letters will denote operations.

Peirce: CP 3.64 Cross-Ref:††

64. To avoid repetitions, I give here a catalogue of the letters I shall use in examples in this paper, with the significations I attach to them.

- a. animal. p. President of the United States Senate.
- b. black. r. rich person.
- f. Frenchman. u. violinist.
- h. horse. v. Vice-President of the United States.
- m. man. w. woman.

a. enemy. *h.* husband. *o.* owner.

b. benefactor. *l.* lover. *s.* servant.

c. conqueror. *m.* mother. *w.* wife.

e. emperor. *n.* not.

g. giver to--of--. **b.** betrayer to--of--.

w. winner over of--to--from--. t. transferrer from--to--.

Peirce: CP 3.65 Cross-Ref:††
NUMBERS CORRESPONDING TO LETTERS

65. I propose to use the term "universe" to denote that class of individuals *about* which alone the whole discourse is understood to run. The universe, therefore, in this sense, as in Mr. De Morgan's,†1 is different on different occasions. In this sense, moreover, discourse may run upon something which is not a subjective part of the universe; for instance, upon the qualities or collections of the individuals it contains.†2

Peirce: CP 3.65 Cross-Ref:††

I propose to assign to all logical terms, numbers; to an absolute term, the number of individuals it denotes; to a relative term, the average number of things so related to one individual. Thus in a universe of perfect men (men), the number of "tooth of" would be 32. The number of a relative with two correlates would be the average number of things so related to a pair of individuals; and so on for relatives of higher numbers of correlates. I propose to denote the number of a logical term by enclosing the term in square brackets, thus [*t*].

Peirce: CP 3.66 Cross-Ref:††
THE SIGNS OF INCLUSION, EQUALITY, ETC.

66. I shall follow Boole †3 in taking the sign of equality to signify identity. Thus, if *v* denotes the Vice-President of the United States, and *p* the President of the Senate of the United States,

$$v = p$$

means that every Vice-President of the United States is President of the Senate, and every President of the United States Senate is Vice-President. The sign "less than" is to be so taken that

$$f < m$$

means every Frenchman is a man, but there are men besides Frenchmen. Drobisch has used this sign in the same sense.†P1 It will follow from these significations of = and < that the sign < (or |, "as small as") will mean "is." Thus,

$$f < m$$

means "every Frenchman is a man," without saying whether there are any other men or not. So,

$$m < l$$

will mean that every mother of anything is a lover of the same thing; although this interpretation in some degree anticipates a convention to be made further on. These significations of = and < plainly conform to the indispensable conditions. Upon the transitive character of these relations the syllogism depends, for by virtue of it, from

$$f < m$$

and $m < a$,

we can infer that $f < a$;

that is, from every Frenchman being a man and every man being an animal, that every Frenchman is an animal. But not only do the significations of = and < here adopted fulfill all absolute requirements, but they have the supererogatory virtue of being very nearly the same as the common significations. Equality is, in fact, nothing but the identity of two numbers; numbers that are equal are those which are predicable of the same collections, just as terms that are identical are those which are predicable of the same classes.†1 So, to write $5 < 7$ is to say that 5 is part of 7, just as to write $f < m$ is to say that Frenchmen are part of men. Indeed, if $f < m$, then the number of Frenchmen is less than the number of men, and if $v = p$, then the number of Vice-Presidents is equal to the number of Presidents of the Senate; so that the numbers may always be substituted for the terms themselves, in case no signs of operation occur in the equations or inequalities.

Peirce: CP 3.67 Cross-Ref:††
THE SIGNS FOR ADDITION

67. The sign of addition is taken by Boole,†2 so that

$$x + y$$

denotes everything denoted by x , and, *besides*, everything denoted by y . Thus

$$m + w$$

denotes all men, and, besides, all women. This signification for this sign is needed for connecting the notation of logic with that of the theory of probabilities. But if there is anything which is denoted by both the terms of the sum, the latter no longer stands for any logical term on account of its implying that the objects denoted by one term are to be taken *besides* the objects denoted by the other. For example,

$$f + u$$

means all Frenchmen besides all violinists, and, therefore, considered as a logical term, implies that all French violinists are *besides themselves*. For this reason alone, in a paper which is published in the Proceedings of the Academy for March 17, 1867,^{†1} I preferred to take as the regular addition of logic a noninvertible process, such that

$$m +, b$$

stands for all men and black things, without any implication that the black things are to be taken besides the men; and the study of the logic of relatives has supplied me with other weighty reasons for the same determination. Since the publication of that paper, I have found that Mr. W. Stanley Jevons, in a tract called *Pure Logic, or the Logic of Quality*, [1864]^{†2} had anticipated me in substituting the same operation for Boole's addition, although he rejects Boole's operation entirely and writes the new one with a + sign while withholding from it the name of addition.^{†P1} It is plain that both the regular non-invertible addition and the invertible addition satisfy the absolute conditions. But the notation has other recommendations. The conception of *taking together* involved in these processes is strongly analogous to that of summation, the sum of 2 and 5, for example, being the number of a collection which consists of a collection of two and a collection of five. Any logical equation or inequality in which no operation but addition is involved may be converted into a numerical equation or inequality by substituting the numbers of the several terms for the terms themselves--provided all the terms summed are mutually exclusive. Addition being taken in this sense, *nothing* is to be denoted by *zero*, for then

$$x +, 0 = x,$$

whatever is denoted by x ; and this is the definition of *zero*.^{†1}

This interpretation is given by Boole, and is very neat, on

account of the resemblance between the ordinary conception of *zero* and that of nothing, and because we shall thus have

$$[[0]] = 0$$

Peirce: CP 3.68 Cross-Ref:††
THE SIGNS FOR MULTIPLICATION

68. I shall adopt for the conception of multiplication *the application of a relation*, in such a way that, for example, *lw* shall denote whatever is lover of a woman. This notation is the same as that used by Mr. De Morgan, although he appears not to have had multiplication in his mind. *s(m +, w)* will, then, denote whatever is servant of anything of the class composed of men and women taken together. So that

$$s(m +, w) = sm +, sw.$$

(l +, s)w will denote whatever is lover or servant to a woman, and

$$(l +, s)w = lw +, sw.$$

(s l)w will denote whatever stands to a woman in the relation of servant of a lover, and

$$(s l)w = s(lw).$$

Thus all the absolute conditions of multiplication are satisfied.

The term "identical with--" is a unity for this multiplication. That is to say, if we denote "identical with--" by **1** we have

$$x\mathbf{1} = x,$$

whatever relative term *x* may be. For what is a lover of something identical with anything, is the same as a lover of that thing.

Peirce: CP 3.69 Cross-Ref:††

69. A conjugative term like *giver* naturally requires two correlates, one denoting the thing given, the other the recipient of the gift. We must be able to distinguish, in our notation, the giver of A to B from the giver to A of B, and, therefore, I suppose the signification of the letter equivalent to such a relative to distinguish the correlates as first, second, third, etc., so that "giver of -- to --" and "giver to -- of --" will be expressed by different letters. Let **g** denote the latter of these conjugative terms. Then, the correlates or multiplicands of this multiplier cannot all stand directly after it, as is usual in multiplication, but may be ranged after it in regular order, so that

gx y

will denote a giver to **x** of **y**. But according to the notation, **x** here multiplies **y**, so that if we put for **x** owner (**o**), and for **y** horse (**h**),

goh

appears to denote the giver of a horse to an owner of a horse. But let the individual horses be H, H', H'', etc. Then

$h \uparrow 1 = H +, H' +, H'' +, \text{etc.}$

$goh = go(H +, H' +, H'' +, \text{etc.}) = goH +, goH' +, goH'' +, \text{etc.}$

Now this last member must be interpreted as a giver of a horse to the owner of *that* horse, and this, therefore, must be the interpretation of **goh**. This is always very important. *A term multiplied by two relatives shows that the same individual is in the two relations.* If we attempt to express the giver of a horse to a lover of a woman, and for that purpose write

g/wh,

we have written giver of a woman to a lover of her, and if we add brackets, thus,

g(lw)h,

we abandon the associative principle of multiplication. A little reflection will show that the associative principle must in some form or other be abandoned at this point.

But while this principle is sometimes falsified, it oftener holds, and a notation must be adopted which will show of itself when it holds. We already see that we cannot express multiplication by writing the multiplicand directly after the multiplier; let us then affix subjacent numbers after letters to show where their correlates are to be found. The first number shall denote how many factors must be counted from left to right to reach the first correlate, the second how many *more* must be counted to reach the second, and so on. Then, the giver of a horse to a lover of a woman may be written

$$g[12]l[1]wh = g[11]l[2]hw = g[2-1]hl[1]w.$$

Peirce: CP 3.70 Cross-Ref: ††

70. Of course a negative number indicates that the former correlate follows the latter by the corresponding positive number. A subjacent *zero* makes the term itself the correlate. Thus,

$$l[0]$$

denotes the lover of *that* lover or the lover of himself, just as *goh* denotes that the horse is given to the owner of itself, for to make a term doubly a correlate is, by the distributive principle, to make each individual doubly a correlate, so that

$$l[0] = L[0] +, L[0]' +, L[0]'' +, \text{etc.}$$

A subjacent sign of infinity may indicate that the correlate is indeterminate, so that

$$l[\infty]$$

will denote a lover of something. We shall have some confirmation of this presently. †1

Peirce: CP 3.70 Cross-Ref: ††

If the last subjacent number is a *one* it may be omitted. Thus we shall have

$$l[1] = l,$$

$$g[11] = g[1] = g.$$

This enables us to retain our former expressions *lw*, *goh*, etc.

Peirce: CP 3.71 Cross-Ref: ††

71. The associative principle does not hold in this counting of factors. Because it does not hold, these subjacent numbers are frequently inconvenient in practice, and I therefore use also another mode of showing where the correlate of a term is to be found. This is by means of the marks of reference, † ‡ || § PARASYMBOLQX, which are placed subjacent to the relative term and before and above the correlate. Thus, giver of a horse to a lover of a woman may be written

$g \dagger \ddagger \ulcorner || w \ddagger h$.

The asterisk I use exclusively to refer to the last correlate of the last relative of the algebraic term.

Peirce: CP 3.72 Cross-Ref: ††

72. Now, considering the order of multiplication to be: -- a term, a correlate of it, a correlate of that correlate, etc., -- there is no violation of the associative principle. The only violations of it in this mode of notation are that in thus passing from relative to correlate, we skip about among the factors in an irregular manner, and that we cannot substitute in such an expression as *goh* a single letter for *oh*. I would suggest that such a notation may be found useful in treating other cases of non-associative multiplication. By comparing this with what was said above †1 concerning functional multiplication, it appears that multiplication by a conjugative term is functional, and that the letter denoting such a term is a symbol of operation. I am therefore using two alphabets, the Greek and Kennerly, where only one was necessary. But it is convenient to use both.

Peirce: CP 3.73 Cross-Ref: ††

73. Thus far, we have considered the multiplication of relative terms only. Since our conception of multiplication is the application of a relation, we can only multiply absolute terms by considering them as relatives. Now the absolute term "man" is really exactly equivalent to the relative term "man that is --," and so with any other. I shall write a comma after any absolute term to show that it is so regarded as a relative term. Then man that is black will be written

m,b.

But not only may any absolute term be thus regarded as a relative term, but any relative term may in the same way be regarded as a relative with one correlate more. It is convenient to take this additional correlate as the first one. Then

$l,sw \dagger 2$

will denote a lover of a woman that is a servant of that woman. The comma here after *l* should not be considered as altering at all the meaning of *l*, but as only a subjacent sign, serving to alter the arrangement of the correlates. In point of fact, since a comma may be added in this way to any relative term, it may be added to one of these very relatives formed by a comma, and thus by the addition of two commas an absolute term becomes a relative of two correlates. So

m,,b,r,

interpreted like **goh**,

means a man that is a rich individual and is a black that is that rich individual. But this has no other meaning than

m,b,r,

or a man that is a black that is rich. Thus we see that, after one comma is added, the addition of another does not change the meaning at all, so that whatever has one comma after it must be regarded as having an infinite number. If, therefore, *l,,sw* is not the same as *l,sw* (as it plainly is not, because the latter means a lover and servant of a woman, and the former a lover of and servant of and same as a woman), this is simply because the writing of the comma alters the arrangement of the correlates. And if we are to suppose that absolute terms are multipliers at all (as mathematical generality demands that we should), we must regard every term as being a relative requiring an infinite number of correlates to its virtual infinite series "that is--and is--and is--etc." Now a relative formed by a comma of course receives its subjacent numbers like any relative, but the question is, What are to be the implied subjacent numbers for these implied correlates? Any term may be regarded as having an infinite number of factors, those at the end being *ones*, thus,

l,sw = *l,sw,1,1,1,1,1,1,1,1*, etc.

A subjacent number may therefore be as great as we please. But all these *ones* denote the same identical individual denoted by *w*; what then can be the subjacent numbers to be applied to *s*, for instance, on account of its infinite "*that is*" 's? What numbers can separate it from being identical with *w*? There are only two. The first is *zero*, which plainly neutralizes a comma completely, since

s,[0]w = *sw,†1*

and the other is infinity; for as 1^∞ is indeterminate in ordinary algebra, so it will be shown hereafter to be here, so that to remove the correlate by the product of an infinite series of *ones* is to leave it indeterminate. Accordingly,

$$m, [\infty]$$

should be regarded as expressing *some* man. Any term, then, is properly to be regarded as having an infinite number of commas, all or some of which are neutralized by zeros.

Peirce: CP 3.73 Cross-Ref: ††
 "Something" may then be expressed by

$$1[\infty].\dagger 1$$

I shall for brevity frequently express this by an antique figure one (I).

Peirce: CP 3.73 Cross-Ref: ††
 "Anything" by

$$1[0].\dagger 2$$

I shall often also write a straight 1 for *anything*.

Peirce: CP 3.74 Cross-Ref: ††
 74. It is obvious that multiplication into a multiplicand indicated by a comma is commutative, †P1 that is,

$$s,l = l,s.$$

This multiplication is effectively the same as that of Boole in his logical calculus. Boole's unity is my 1, that is, it denotes whatever is.

Peirce: CP 3.75 Cross-Ref: ††
 75. The sum $x + x$ generally denotes no logical term. But $x, [\infty] + x, [\infty]$ may be considered as denoting some two x 's. It is natural to write

$$x+x = 2.x,$$

and $x, [\infty]+x, [\infty] = 2.x, [\infty],$

where the dot shows that this multiplication is invertible. We may also use the antique figures so that

$$2.x, [\infty] = 2X,$$

just as $1[\infty] = 1.$

Then **2** alone will denote some two things. But this multiplication is not in general commutative, and only becomes so when it affects a relative which imparts a relation such that a thing only bears it to *one* thing, and one thing *alone* bears it to a thing. For instance, the lovers of two women are not the same as two lovers of women, that is,

$$l2.w \text{ and } 2.lw$$

are unequal; but the husbands of two women are the same as two husbands of women, that is,

$$h2.w = 2.hw,$$

and in general, $x,2.y = 2.x.y.$

Peirce: CP 3.76 Cross-Ref:††

76. The conception of multiplication we have adopted is that of the application of one relation to another. So, a quaternion being the relation of one vector to another, the multiplication of quaternions is the application of one such relation to a second. Even ordinary numerical multiplication involves the same idea, for 2×3 is a pair of triplets, and 3×2 is a triplet of pairs, where "triplet of" and "pair of" are evidently relatives.

Peirce: CP 3.76 Cross-Ref:††

If we have an equation of the form

$$x y = z,$$

and there are just as many *x*'s per *y* as there are *per* things, things of the universe, then we have also the arithmetical equation,

$$[[x]][[y]] = [[z]].$$

For instance, if our universe is perfect men, and there are as many teeth to a Frenchman (perfect understood) as there are to any one of the universe, then

$$[[t]][[f]] = [[t f]]$$

holds arithmetically. So if men are just as apt to be black as things in general,

$$[[m,]][[b]] = [[m,b]],$$

where the difference between $[[m]]$ and $[[m,]]$ must not be overlooked. It is to be observed that

$$[[1]] = 1.$$

Peirce: CP 3.76 Cross-Ref:††

Boole was the first to show this connection between logic and probabilities.†1 He was restricted, however, to absolute terms. I do not remember having seen any extension of probability to relatives, except the ordinary theory of *expectation*.

Peirce: CP 3.76 Cross-Ref:††

Our logical multiplication, then, satisfies the essential conditions of multiplication, has a unity, has a conception similar to that of admitted multiplications, and contains numerical multiplication as a case under it.

Peirce: CP 3.77 Cross-Ref:††
THE SIGN OF INVOLUTION

77. I shall take involution in such a sense that xy will denote everything which is an x for every individual of y . Thus l^w will be a lover of every woman. Then $(sl)^w$ will denote whatever stands to every woman in the relation of servant of every lover of hers; and $s(lw)$ will denote whatever is a servant of everything that is lover of a woman. So that

$$(sl)^w = s(lw).$$

A servant of every man and woman will be denoted by $s^{m+}, w^{\dagger 1}$, and s^m, s^w will denote a servant of every man that is a servant of every woman. So that

$$s^{m+}, w = s^m, s^w.$$

That which is emperor or conqueror of every Frenchman will be denoted by $(e+, c)^f$, and $e^f+, \Sigma[p]e^f, p, cp+, c^f$ will denote whatever is emperor of every Frenchman or emperor of some Frenchmen and conqueror of all the rest, or conqueror of every Frenchman. Consequently,

$$(e+, c)^f = e^f+, \Sigma[p]e^f, p, cp+, c^f.$$

Indeed, we may write the binomial theorem so as to preserve all its usual coefficients; for we have

$$(e+, c)^f = e^{f+}, [[f]].e^{f-\dagger 1}, c^{1\dagger+}, ((([f]).([f]-1))/2).e^{f-\dagger 2}, c^{2\dagger+}, \text{etc.} \dagger 2$$

That is to say, those things each of which is emperor or conqueror of every Frenchman consist, first, of all those individuals each of which is a conqueror [emperor!] of every Frenchman; second, of a number of classes equal to the number of Frenchmen, each class consisting of everything which is an emperor of every Frenchman but some one and is a conqueror of that one; third, of a number of classes equal to half the product of the number of Frenchmen by one less than that number, each of these classes consisting of every individual which is an emperor of every Frenchman except a certain two, and is conqueror of those two, etc. This theorem holds, also, equally well with invertible addition, and either term of the binomial may be negative provided we assume

$$(-x)y = (-)[y].xy.$$

Peirce: CP 3.77 Cross-Ref: ††

In addition to the above equations which are required to hold good by the definition of involution, the following also holds,

$$(s, l)^w = s^w, l^w, \dagger 1$$

just as it does in arithmetic.

Peirce: CP 3.78 Cross-Ref:††

78. The application of involution to conjugative terms presents little difficulty after the explanations which have been given under the head of multiplication. It is obvious that betrayer to every enemy should be written

ba,

just as lover of every woman is written

l^w,

but **b** = **b**[11] and therefore, in counting forward as the subjacent numbers direct, we should count the exponents, as well as the factors, of the letter to which the subjacent numbers are attached. Then we shall have, in the case of a relative of two correlates, six different ways of affixing the correlates to it, thus:

bam betrayer of a man to an †P1 enemy of him;

(ba)^m betrayer of every man to some enemy of him;

ba^m betrayer of each man to an enemy of every man;

ba^m betrayer of a †P1 man to all †P1 enemies of all men;

bam betrayer of a man to every enemy of him;

bam betrayer of every man to every enemy of him.

If both correlates are absolute terms, the cases are

bmw betrayer of a woman to a man;

(bm)^w betrayer of each woman to some man;

bm^w betrayer of all women to a man;

b^mw betrayer of a woman to every man; †P1

b^mw betrayer of a woman to all men;

b^mw betrayer of every woman to every man.

These interpretations are by no means obvious, but I shall show that they are correct

further on. †1

Peirce: CP 3.79 Cross-Ref: ††

79. It will be perceived that the rule still holds here that

$$(\mathbf{ba})^m = \mathbf{b(am)}$$

that is to say, that those individuals each of which stand to every man in the relation of betrayer to every enemy of his are identical with those individuals each of which is a betrayer to every enemy of a man of that man.

Peirce: CP 3.80 Cross-Ref: ††

80. If the proportion of lovers of each woman among lovers of other women is equal to the average number of lovers which single individuals of the whole universe have, then

$$[[l^w]] = [[l^{w'}]] [[l^{w''}]] [[l^{w'''}]] \text{ etc.} = [[l]][[w]].$$

Thus arithmetical involution appears as a special case of logical involution.

Peirce: CP 3.81 Cross-Ref: ††

§4. GENERAL FORMULÆ

81. The formulæ which we have thus far obtained, exclusive of mere explanations of signs and of formulæ relating to

the numbers of classes, are:

- (1) If $x \prec y$ and $y \prec z$, then $x \prec z$.
- (2) $(x +, y) +, z = x +, (y +, z)$. (Jevons)
- (3) $x +, y = y +, x$. (Jevons)
- (4) $(x +, y)z = x z +, y z$.
- (5) $x(y +, z) = x y +, x z$.
- (6) $(x y)z = x(y z)$.
- (7) $x,(y +, z) = x,y +, x,z$. (Jevons)
- (8) $(x,y),z = x,(y,z)$. (Boole)
- (9) $x,y, = y,x$. (Boole)

- (10) $(xy)z = x(yz)$.
- (11) $xy + z = xy, xz$.
- (12) $(x + y)z = xz + \Sigma[p](xx-p, yp) +, yp$
 $= xz +, [[z]].xz-\dagger 1, y\dagger 1$
 $+ , ((([z]].[[z-1]])/2).xz-\dagger 2, y\dagger 2$
 $+ , ((([z]].[[z-1]].[[z-2]])/(2.3)).xz-\dagger 3, y\dagger 3 +, etc.$
- (13) $(x,y)z = xz,yz$.
- (14) $x + 0 = x$. (Boole)
- (15) $x1 = x$.
- (16) $(x + y) + z = x + (y + z)$. (Boole)
- (17) $x + y = y + x$. (Boole)
- (18) $x + y - y = x$. (Boole)
- (19) $x,(y + z) = x,y + x,z$. (Boole)
- (20) $(x + y)z = x + [[z]].xz-\dagger 1, y\dagger 1 + etc.$

Peirce: CP 3.81 Cross-Ref:††

We have also the following, which are involved implicitly in the explanations which have been given.

(21) $x \rightarrow x +, y, \dagger 1$

This, I suppose, is the principle of identity, for it follows from this that $x = x, \dagger 2$

(22) $x +, x = x$. (Jevons)

(23) $x,x = x$. (Boole)

(24) $x +, y = x + y - x,y$.

The principle of contradiction is

(25) $x, nx = 0$

where n stands for "not." The principle of excluded middle is

$$(26) \ x +, nx = 1.$$

It is an identical proposition, that, if ϕ be determinative, we have

$$(27) \text{ If } x = y \quad \phi x = \phi y.$$

Peirce: CP 3.81 Cross-Ref:††

The six following are derivable from the formulæ already given:

$$(28) \ (x +, y), (x +, z) = x +, y, z.$$

$$(29) \ (x - y) +, (z - w) = (x +, z) - (y +, w) + y, z, (1 - w) + x, (1 - y), w.$$

Peirce: CP 3.81 Cross-Ref:††

In the following, ϕ is a function involving only the commutative operations and the operations inverse to them.

$$(30) \ \phi x = (\phi 1), x + (\phi 0), (1 - x). \quad (\text{Boole})$$

$$(31) \ \phi x = (\phi 1 +, (1 - x)), (\phi 0 +, x).$$

$$(32) \ \text{If } \phi x = 0 \quad (\phi 1), (\phi 0) = 0. \quad (\text{Boole})$$

$$(33) \ \text{If } \phi x = 1 \quad \phi 1 +, \phi 0 = 1.$$

Peirce: CP 3.81 Cross-Ref:††

The reader may wish information concerning the proofs of formulæ (30) to (33). When involution is not involved in a function nor any multiplication except that for which $x, x = x$, it is plain that ϕx is of the first degree, and therefore, since all the rules of ordinary algebra hold, we have as in that

$$\phi x = \phi 0 + (\phi 1 - \phi 0), x.$$

We shall find, hereafter, that when y has a still more general character, we have,

$$\phi x = \phi 0 + (\phi 1 - \phi 0)x.$$

The former of these equations by a simple transformation gives (30).

Peirce: CP 3.81 Cross-Ref:††

If we regard $(\phi 1), (\phi 0)$ as a function of x and develop it by (30), we have

$$(\phi 1), (\phi 0) = x, (\phi 1), (\phi 0) + (\phi 1), (\phi 0), (1-x).$$

Comparing these terms separately with the terms of the second member of (30), we see that

$$(\phi 1), (\phi 0) \prec x.$$

This gives at once (32), and it gives (31) after performing the multiplication indicated in the second member of that equation and equating ϕx to its value as given in (30). If $(\phi 1 +, \phi 0)$ is developed as a function of x by (31), and the factors of the second member are compared with those of the second member of (31), we get

$$\phi x \prec \phi 1 +, \phi 0,$$

from which (33) follows immediately.

Peirce: CP 3.82 Cross-Ref:††
 PROPERTIES OF ZERO AND UNITY

82. The symbolical definition of zero is

$$x+0 = x,$$

so that by (19) $x, a = x, (a+0) = x, a+x, 0.$

Hence, from the invertible character of this addition, and the generality of (14), we have

$$x, 0 = 0.$$

By (24) we have in general,

$$x +, 0 = x + 0 - x, 0 = x,$$

or $x +, 0 = x.$

By (4) we have $a x = (a +, 0)x = a x +, 0x.$

But if a is an absurd relation, $a x = 0,$

so that $0x = 0,$

which must hold invariably.

Peirce: CP 3.82 Cross-Ref:††

From (12) we have $ax = (a +, 0)x = ax +, 0x +, \text{etc.},$

whence by (21) $0x \prec ax. \dagger 1$

But if a is an absurd relation, and x is not zero,

$$ax = 0.$$

And therefore, unless $x=0,$ $0x = 0.$

Peirce: CP 3.83 Cross-Ref:††

83. Any relative x may be conceived as a sum of relatives $X, X', X'', \text{etc.},$ such that there is but one individual to which anything is $X,$ but one to which anything is $X',$ etc. Thus, if x denote "cause of," X, X', X'' would denote different kinds of causes, the causes being divided according to the differences of the things they are causes of. Then we have

$$X y = X(y +, 0) = X y +, X 0,$$

whatever y may be. Hence, since y may be taken so that

$$X y = 0,$$

we have $X 0 = 0;$

and in a similar way,

$$X^0 = 0, \quad X''^0 = 0, \quad X'''^0 = 0, \text{ etc.}$$

We have, then,

$$\begin{aligned} x^0 &= (X +, X' +, X'' +, X''' +, \text{ etc.})^0 \\ &= X^0 +, X'^0 +, X''^0 +, X'''^0 +, \text{ etc.} = 0. \end{aligned}$$

Peirce: CP 3.84 Cross-Ref:††

84. If the relative x be divided in this way into X, X', X'', X''' , etc., so that x is that which is either X or X' or X'' or X''' , etc., then non- x is that which is at once non- X and non- X' and non- X'' , etc.; that is to say,

$$\text{non-}x = \text{non-}X, \text{ non-}X', \text{ non-}X'', \text{ non-}X''', \text{ etc.};$$

where non- X is such that there is something (Z) such that everything is non- X to Z ; and so with non- X' , non- X'' , etc. Now, non- x may be any relative whatever. Substitute for it, then, y ; and for non- X , non- X' , etc., Y, Y' , etc. Then we have

$$y = Y, Y', Y'', Y''', \text{ etc.};$$

$$\text{and } Y'Z' = 1, \quad Y''Z'' = 1, \quad Y'''Z''' = 1, \text{ etc.},$$

where Z', Z'', Z''' are individual terms which depend for what they denote on Y', Y'', Y''' .

Then we have

$$1 = Y'Z' = Y'Z' = Y'(Z' +, 0) = Y'Z', Y'^0 = Y'Z', Y'^0,$$

$$\text{or } Y'^0 = 1, \quad Y''^0 = 1, \quad Y'''^0 = 1, \text{ etc.}$$

$$\text{Then } y^0 = (Y', Y'', Y''', \text{ etc.})^0 = Y'^0, Y''^0, Y'''^0, \text{ etc.} = 1.$$

Peirce: CP 3.84 Cross-Ref:††

$$\text{We have by definition, } x\mathbf{1} = x.$$

$$\text{Hence, by (6), } a\mathbf{x} = (a\mathbf{1})x = a(\mathbf{1}x).$$

Now a may express any relation whatever, but things the same way related to everything are the same. Hence,

$$x = \mathbf{1}x.$$

We have by definition, $\mathbf{1} = \mathbf{1}[0]$.

Then if X is any individual $X, \mathbf{1} = X, \mathbf{1}[0] = X, \mathbf{1}X$.

But $\mathbf{1}X = X$.

Hence $X, \mathbf{1} = X, X$;

and by (23) $X, \mathbf{1} = X$;

whence if we take $x = X + X' + X'' + X''' + \text{etc.}$,

where X, X' etc, denote individuals (and by the very meaning of a general term this can always be done, whatever x may be)

$$\begin{aligned} x, \mathbf{1} &= (X + X' + X'' + \text{etc.}), \mathbf{1} = X, \mathbf{1} + X', \mathbf{1} + X'', \mathbf{1} + \text{etc.} \\ &= X + X' + X'' + \text{etc.} = x, \end{aligned}$$

or $x, \mathbf{1} = x$.

We have by (24) $x +, \mathbf{1} = x + \mathbf{1} - x, \mathbf{1} = x + \mathbf{1} - x = \mathbf{1}$,

or $x +, \mathbf{1} = \mathbf{1}$.

Peirce: CP 3.85 Cross-Ref: ††

85. We may divide all relatives into limited and unlimited. Limited relatives express such relations as nothing has to everything. For example, nothing is knower of everything. Unlimited relatives express relations such as something has to everything. For example, something is as good as anything. For limited relatives, then, we may write

$$p^1 = 0.$$

The converse of an unlimited relative expresses a relation which everything has to something. Thus, everything is as bad as something. Denoting such a relative by q ,

$$q1 = 1.$$

These formulæ remind one a little of the logical algebra of Boole; because one of them holds good in arithmetic only for *zero*, and the other only for *unity*.

Peirce: CP 3.85 Cross-Ref:††

$$\text{We have by (10)} \quad 1x = (q^0)x = q^{(0x)} = q^0 = 1,$$

$$\text{or} \quad 1^x = 1.$$

$$\text{We have by (4)} \quad 1x = (a +, 1)x = a x +, 1x,$$

$$\text{or by (21)} \quad a x \prec 1x.$$

But everything is somehow related to x unless x is 0; hence unless x is 0, $1x = 1$.

Peirce: CP 3.85 Cross-Ref:††

If a denotes "what possesses," and y "character of what is denoted by x ,"

$$x = ay = a(y1) = (ay)^1 = x^1,$$

$$\text{or} \quad x^1 = x.$$

Peirce: CP 3.85 Cross-Ref:††

Since 1 means "identical with," $l, 1w$ denotes whatever is both a lover of and identical with a woman, or a woman who is a lover of herself. And thus, in general,

$$x, 1 = x[0],.$$

Peirce: CP 3.86 Cross-Ref:††

86. Nothing is identical with every one of a class; and therefore $1x$ is zero, unless x denotes only an individual when $1x$ becomes equal to x . But equations founded on interpretation may not hold in cases in which the symbols have no rational interpretation.

Peirce: CP 3.86 Cross-Ref:††

Collecting together all the formulæ relating to *zero* and

unity, we have

$$(34) \quad x +, 0 = x. \quad (\text{Jevons})$$

- (35) $x +, 1 = 1$. (Jevons)
(36) $x0 = 0$.
(37) $0x = 0$.
(38) $x,0 = 0$. (Boole)
(39) $x^0 = 1$.
(40) $0x = 0$, provided $x > 0$.†1
(41) $1x = x$.
(42) $x,1 = x[0]$,.
(43) $x1 = x$.
(44) $1x = 0$, unless x is individual, when $1x = x$.
(45) $q1 = 1$, where q is the converse of an unlimited relative.
(46) $1x = 1$, provided $x > 0$.†1
(47) $x,1 = x$. (Boole)
(48) $p^1 = 0$, where p is a limited relative.
(49) $1x = 1$.

These, again, give us the following:

- | | |
|-------------------|----------------|
| (50) $0 +, 1 = 1$ | (64) $0^1=0$ |
| (51) $0 +, 1 = 1$ | (65) $1 1=1$ |
| (52) $00 = 0$ | (66) $1,1=1$ |
| (53) $0,0 = 0$ | (67) $11=1$ |
| (54) $0^0 = 1$ | (68) $11 = 1$ |
| (55) $10 = 0$ | (69) $1,1 = 1$ |
| (56) $01 = 0$ | (70) $1^1 = 1$ |
| (57) $0,1 = 0$ | (71) $11=1$ |
| (58) $01 = 0$ | (72) $11 = 1$ |
| (59) $1^0 = 1$ | (73) $1,1 = 1$ |
| (60) $01 = 0$ | (74) $11 = 1$ |
| (61) $10 = 0$ | (75) $1^1 = 0$ |

$$(62) 0,1 = 0 \quad (76) 1, = \mathbf{1}$$

$$(63) 1^0 = 1$$

Peirce: CP 3.86 Cross-Ref:††

From (64) we may infer that 0 is a limited relative, and from (60) that it is not the converse of an unlimited relative. From (70) we may infer that 1 is not a limited relative, and from (68) that it is the converse of an unlimited relative.

Peirce: CP 3.87 Cross-Ref:††

FORMULÆ RELATING TO THE NUMBERS OF TERMS

87. We have already seen that

$$(77) \text{ If } x \prec y, \text{ then } [[x]] \prec [[y]].$$

$$(78) \text{ When } x.y = 0, \text{ then } [[x +, y]] = [[x]] +, [[y]],$$

$$(79) \text{ When } [[x y]]: [[nxy]] = [[x]]: [[nx]], \text{ then } [[x y]] = [[x]][[y]].$$

$$(80) \text{ When } [[xny]] = [[x]][[ny]][[1]], \text{ then } [[x y]] = [[x]][[y]].$$

Peirce: CP 3.87 Cross-Ref:††

It will be observed that the conditions which the terms must conform to, in order that the arithmetical equations shall hold, increase in complexity as we pass from the more simple relations and processes to the more complex.

Peirce: CP 3.88 Cross-Ref:††

88. We have seen that

$$(81) [[0]] = 0.$$

$$(82) [[1]] = 1.$$

Most commonly the universe is unlimited, and then

$$(83) [[1]] = \infty; \dagger 1$$

and the general properties of 1 correspond with those of infinity. Thus,

$x + 1 = 1$ corresponds to $x + \infty = \infty$,

$q1 = 1$ corresponds to $q \infty = \infty$,

$1x = 1$ corresponds to $\infty x = \infty$,

$p^1 = 0$ corresponds to $p \infty = 0$,

$1x = 1$ corresponds to $\infty x = \infty$.

Peirce: CP 3.88 Cross-Ref:††

The formulæ involving commutative multiplication are derived from the equation $1, = \mathbf{1}$. But if 1 be regarded as infinite, it is not an absolute infinite; for $10 = 0$. On the other hand, $\mathbf{1}^1 = 0$.

Peirce: CP 3.88 Cross-Ref:††

It is evident, from the definition of the number of a term, that

$$(84) [[x,]] = [[x]]:[[1]].$$

Peirce: CP 3.88 Cross-Ref:††

We have, therefore, if the probability of an individual being x to any y is independent of what other y 's it is x to, and if x is independent of y ,

$$(85) [[xy,]] = [[x,]][[y]].$$

Peirce: CP 3.89 Cross-Ref:††

§5. GENERAL METHOD OF WORKING WITH THIS NOTATION

89. Boole's logical algebra contains no operations except our invertible addition and commutative multiplication, together With the corresponding subtraction and division. He has, therefore, only to expand expressions involving division, by means of (30), so as to free himself from all non-determinative operations, in order to be able to use the ordinary methods of algebra, which are, moreover, greatly simplified by the fact that

$$x, x = x.$$

Peirce: CP 3.90 Cross-Ref:††

90. Mr. Jevons's modification †1 of Boole's algebra involves only non-invertible addition and commutative multiplication, without the corresponding

inverse operations. He is enabled to replace subtraction by multiplication, owing to the principle of contradiction, and to replace division by addition, owing to the principle of excluded middle. For example, if x be unknown, and we have

$$x +, m = a,$$

or what is denoted by x together with men make up animals, we can only conclude, with reference to x , that it denotes (among other things, perhaps) all animals not men; that is, that the x 's not men are the same as the animals not men. Let $\sim m$ denote non-men; then by multiplication we have

$$x\sim m, +, m, \sim m = x, \sim m = a, \sim m,$$

because, by the principle of contradiction,

$$m, \sim m = 0.$$

Or, suppose, x being again unknown, we have given

$$a, x = m.$$

Then all that we can conclude is that the x 's consist of all the m 's and perhaps some or all of the non- a 's, or that the x 's and non- a 's together make up the m 's and non- a 's together. If, then, $\sim a$ denote non- a , add $\sim a$ to both sides and we have

$$a, x +, \sim a = m +, \sim a.$$

Then by (28) $(a +, \sim a), (x +, \sim a) = m +, \sim a.$

But by the principle of excluded middle,

$$a +, \sim a = 1$$

and therefore $x +, \sim a = m +, \sim a.$

I am not aware that Mr. Jevons actually uses this latter process, but it is open to him

to do so. In this way, Mr. Jevons's algebra becomes decidedly simpler even than Boole's.

Peirce: CP 3.90 Cross-Ref:††

It is obvious that any algebra for the logic of relatives must be far more complicated. In that which I propose, we labor under the disadvantages that the multiplication is not generally commutative, that the inverse operations are usually indeterminative, and that transcendental equations, and even equations like

$$abx = cdex + fx + x,$$

where the exponents are three or four deep, are exceedingly common. It is obvious, therefore, that this algebra is much less manageable than ordinary arithmetical algebra.

Peirce: CP 3.91 Cross-Ref:††

91. We may make considerable use of the general formulæ already given, especially of (1), (21), and (27), and also of the following, which are derived from them:

(86) If $a \prec b$ then there is such a term x that $a +, x = b$.

(87) If $a \prec b$ then there is such a term x that $b, x = a$.

(88) If $b, x = a$ then $a \prec b$.

(89) If $a \prec b \quad c +, a \prec c +, b$.

(90) If $a \prec b \quad c a \prec c b$.

(91) If $a \prec b \quad a c \prec b c$.

(92) If $a \prec b \quad cb \prec ca \dagger 1$

(93) If $a \prec b \quad ac \prec bc$.

(94) $a, b \prec a$

Peirce: CP 3.91 Cross-Ref:††

There are, however, very many cases in which the formulæ thus far given are of little avail.

Peirce: CP 3.92 Cross-Ref:††

92. Demonstration of the sort called mathematical is founded on suppositions of particular cases. The geometrician draws a figure; the algebraist assumes a letter to signify a single quantity fulfilling the required conditions. But while the mathematician supposes an individual case, his hypothesis is yet perfectly general, because he considers no characters of the individual case but those which must belong to every such case. The advantage of his procedure lies in the fact that the logical

laws of individual terms are simpler than those which relate to general terms, because individuals are either identical or mutually exclusive, and cannot intersect or be subordinated to one another as classes can. Mathematical demonstration is not, therefore, more restricted to matters of intuition than any other kind of reasoning. Indeed, logical algebra conclusively proves that mathematics extends over the whole realm of formal logic; and any theory of cognition which cannot be adjusted to this fact must be abandoned. We may reap all the advantages which the mathematician is supposed to derive from intuition by simply making general suppositions of individual cases.

Peirce: CP 3.93 Cross-Ref: ††

93. In reference to the doctrine of individuals,†1 two distinctions should be borne in mind. The logical atom, or term not capable of logical division, must be one of which every predicate may be universally affirmed or denied. For, let A be such a term. Then, if it is neither true that all A is X nor that no A is X, it must be true that some A is X and some A is not X; and therefore A may be divided into A that is X and A that is not X, which is contrary to its nature as a logical atom. Such a term can be realized neither in thought nor in sense. Not in sense, because our organs of sense are special -- the eye, for example, not immediately informing us of taste, so that an image on the retina is indeterminate in respect to sweetness and non-sweetness. When I see a thing, I do not see that it is not sweet, nor do I see that it is sweet; and therefore what I see is capable of logical division into the sweet and the not sweet. It is customary to assume that visual images are absolutely determinate in respect to color, but even this may be doubted. I know no facts which prove that there is never the least vagueness in the immediate sensation. In thought, an absolutely determinate term cannot be realized, because, not being given by sense, such a concept would have to be formed by synthesis, and there would be no end to the synthesis because there is no limit to the number of possible predicates. A logical atom, then, like a point in space, would involve for its precise determination an endless process. We can only say, in a general way, that a term, however determinate, may be made more determinate still, but not that it can be made absolutely determinate. Such a term as "the second Philip of Macedon" is still capable of logical division--into Philip drunk and Philip sober, for example; but we call it individual because that which is denoted by it is in only one place at one time. It is a term not *absolutely* indivisible, but indivisible as long as we neglect differences of time and the differences which accompany them. Such differences we habitually disregard in the logical division of substances. In the division of relations, etc., we do not, of course, disregard these differences, but we disregard some others. There is nothing to prevent almost any sort of difference from being conventionally neglected in some discourse, and if I be a term which in consequence of such neglect becomes indivisible in that discourse, we have in that discourse,

$$[[1]] = 1.$$

This distinction between the absolutely indivisible and that which is one in number from a particular point of view is shadowed forth in the two words *individual* {to atomon} and *singular* (to kath' hekaston); but as those who have used the word *individual* have not been aware that absolute individuality is merely ideal, it has come to be used in a more general sense.†P1

Peirce: CP 3.94 Cross-Ref:††

94. The old logics distinguish between *individuum signatum* and *individuum vagum*. "Julius Cæsar" is an example of the former; "a certain man," of the latter. The *individuum vagum*, in the days when such conceptions were exactly investigated, occasioned great difficulty from its having a certain generality, being capable, apparently, of logical division. If we include under the *individuum vagum* such a term as "any individual man," these difficulties appear in a strong light, for what is true of any individual man is true of all men. Such a term is in one sense not an individual term; for it represents every man. But it represents each man as capable of being denoted by a term which is individual; and so, though it is not itself an individual term, it stands for any one of a class of individual terms. If we call a thought about a thing in so far as it is denoted by a term, a second intention, we may say that such a term as "any individual man" is individual by second intention. The letters which the mathematician uses (whether in algebra or in geometry) are such individuals by *second intention*. Such individuals are one in number, for any individual man is one man; they may also be regarded as incapable of logical division, for any individual man, though he may either be a Frenchman or not, is yet altogether a Frenchman or altogether not, and not some one and some the other. Thus, all the formal logical laws relating to individuals will hold good of such individuals by second intention, and at the same time a universal proposition may at any moment be substituted for a proposition about such an individual, for nothing can be predicated of such an individual which cannot be predicated of the whole class.

Peirce: CP 3.95 Cross-Ref:††

95. There are in the logic of relatives three kinds of terms which involve general suppositions of individual cases. The first are *individual* terms, which denote only individuals †1; the second are those relatives whose correlatives are individual: I term these *infinitesimal relatives* †2; the third are *individual infinitesimal* relatives, and these I term *elementary* relatives. †3

Peirce: CP 3.96 Cross-Ref:††
INDIVIDUAL TERMS

96. The fundamental formulæ relating to individuality are two. Individuals are denoted by capitals.

$$(95) \quad \text{If } x > 0 \quad x = X +, X' +, X'' +, X''' +, \text{ etc.}$$

$$(96) \quad yX = yX.$$

Peirce: CP 3.96 Cross-Ref:††

We have also the following which are easily deducible from these two:

$$(97) \quad (y,z)X = (y X),(z X). \quad (99) \quad [[X]] = 1.$$

$$(98) X,y[0] = X,y X. \quad (100) \mathbf{1}X = X.$$

We have already seen that

$$\mathbf{1}x = 0, \text{ provided that } [[x]] > 1.$$

Peirce: CP 3.97 Cross-Ref:††

97. As an example of the use of the formulæ we have thus far obtained, let us investigate the logical relations between "benefactor of a lover of every servant of every woman," "that which stands to every servant of some woman in the relation of benefactor of a lover of him," "benefactor of every lover of some servant of a woman," "benefactor of every lover of every servant of every woman," etc.

Peirce: CP 3.97 Cross-Ref:††

In the first place, then, we have by (95)

$$sw = s(W' =, W'' +, W''' +, \text{etc.}) = sW' +, sW'' +, sW''' +, \text{etc.}$$

$$sw = sW' +, W'' +, W''' +, \text{etc.} = sW' +, sW'' +, sW''' +, \text{etc.}$$

From the last equation we have by (96)

$$sw = (sW'),(sW''),(sW'''), \text{etc.}$$

Now by (31) $x' +, x'' +, \text{etc.} = x',x'',x''', \text{etc.} +, \text{etc.},$

or

$$(101) \quad \pi' -< \Sigma',$$

where π' and Σ' signify that the addition and multiplication with commas $\dagger 1$ are to be used. From this it follows that

$$(102) \quad s^w -< sw.\dagger 2$$

If w vanishes, this equation fails, because in that case (95) does not hold.

Peirce: CP 3.97 Cross-Ref:††

From (102) we have

$$(103) \quad (l s)^w \prec l s w. \dagger 3$$

Since $a = a, b +, \text{etc.},$

$b = a, b +, \text{etc.},$

we have $l a = l(a, b +, \text{etc.}) = l(a, b) +, l(\text{etc.}),$

$l b = l(a, b +, \text{etc.}) = l(a, b) +, l(\text{etc.}).$

Multiplying these two equations commutatively we have

$$(l a), (l b) = l(a, b) +, \text{etc.}$$

or

$$(104) \quad l \pi' \prec \pi' l. \dagger 1$$

Now $(l s)^w = (s)^{W' +, W'' +, W''' +, \text{etc.}} = \pi'(l s)^W = \pi' l s W,$

$$l s^w = l s^{W' +, W'' +, W''' +, \text{etc.}} = l \pi' s^{W'} = l \pi' s W.$$

Hence,

$$(105) \quad l s^w \prec (l s)^w,$$

or every lover of a servant of all women stands to every woman in the relation of lover of a servant of hers.

Peirce: CP 3.97 Cross-Ref: ††

From (102) we have

$$(106) \quad l s^w \prec l s^w. \dagger 2$$

By (95) and (96) we have

$$lsw = ls(W' +, W'' +, W''' \text{ etc.}) = lsW' +, lsW'' +, lsW''' +, \text{ etc.}$$

$$= lsW' +, lsW'' +, lsW''' + \text{ etc.}$$

$$\text{Now } s^w = s^{W'+, W'' +, W''' +, \text{ etc.}} = s^{W'}, s^{W''}, s^{W'''}, \text{ etc.}$$

$$\text{So that by (94) } s^w \prec s^{W'} \prec s^w.$$

Hence by (92)

$$lsW' \prec ls^w, lsW'' \prec ls^w, lsW''' \prec ls^w.$$

$$\text{Adding, } lsW' +, lsW'' +, lsW''' \prec ls^w;$$

or

$$(107) \quad lsw \prec ls^w.$$

That is, every lover of every servant of any particular woman is a lover of every servant of all women.

Peirce: CP 3.97 Cross-Ref: ††

By (102) we have

$$(108) \quad lsw \prec ls^w. \dagger 3$$

Thus we have

$$lsw \prec ls^w \prec ls^w \prec ls^w \prec (ls)^w \prec lsw. \dagger P1$$

Peirce: CP 3.98 Cross-Ref:††

98. By similar reasoning we can easily make out the relations shown in the following table. It must be remembered

that the formulæ do not generally hold when exponents vanish.

$blsw$	b^{lsw}
$* \vee$	$\wedge *$
$(bls)^w$	b^{lsw}
\vee	\wedge
$b(l^s)^w$	$b^{l^s w}$
\vee	\wedge
bls^w	b^{lsw}
$* \vee$	$\wedge *$
$(bl)^s w$	b^{lsw}
$\wedge \vee$	$\wedge \vee$
$b^{lsw} (bl)^s w$	$(b^{lsw})^w b^{lsw}$
$\wedge \vee \wedge *$	$* \vee \wedge \vee$
$bls^w (bl)^s w$	$b^{lsw} b^{lsw}$
$* \vee \wedge$	$\vee \wedge *$
$(bls)^w$	b^{lsw}
\vee	\wedge
bls^w	b^{lsw}
\vee	\wedge
	

[Click here to view]†1

Peirce: CP 3.99 Cross-Ref:††

99. It appears to me that the advantage of the algebraic notation already begins to be perceptible, although its powers are thus far very imperfectly made out. At any rate, it seems to me that such a *prima facie* case is made out that the reader who still denies the utility of the algebra ought not to be too indolent to attempt to write down the above twenty-two terms in ordinary language With logical precision. Having done that, he has only to disarrange them and then restore the arrangement by ordinary logic, in order to test the algebra so far as it is yet developed.

Peirce: CP 3.100 Cross-Ref:††
INFINITESIMAL RELATIVES

100. We have by the binomial theorem by (49) and by (47),

$$(1+x)^n = 1 + \sum [p] x^n \cdot p + x^n.$$

Now, if we suppose the number of individuals to which any one thing is x to be reduced to a smaller and smaller number, we reach as our limit

$$x^2 = 0,$$

$$\sum [p] x^n \cdot p = [[n]]. 1n \cdot \dagger 1, x^{\dagger 1} = xn,$$

$$(1+x)^n = 1 + x n.$$

Peirce: CP 3.101 Cross-Ref:††

101. If, on account of the vanishing of its powers, we call x an infinitesimal here and denote it by i , and if we put

$$x n = i n = y,$$

our equation becomes

$$(109) \quad (1+i) y/i = 1 + y.$$

Putting $y = 1$, and denoting $(1+i)1/i$ by \mathfrak{C} , we have

$$(110) \quad \mathfrak{C} = (1+i)1/i = 1 + I.$$

Peirce: CP 3.102 Cross-Ref:††

102. In fact, this agrees With ordinary algebra better than it seems to do; for I is itself an infinitesimal, and \mathfrak{C} is $\mathfrak{C}I$. If the higher powers of I did not vanish, we should get the ordinary development of \mathfrak{C} .

Peirce: CP 3.103 Cross-Ref:††

103. Positive powers of \mathfrak{C} are absurdities in our notation. For negative powers we have

$$(111) \quad \mathfrak{C} \cdot x = 1 - x.$$

Peirce: CP 3.104 Cross-Ref:††

104. There are two ways of raising $\mathbb{C}x$ to the y th power.

In the first place, by the binomial theorem,

$$(1-x)^y = 1 - \binom{y}{1} x^1 + \frac{\binom{y}{1} \binom{y-1}{2}}{2} x^2 - \dots;$$

and, in the second place, by (111) and (10).

$$\mathbb{C}x^y = 1 - x^y.$$

It thus appears that the sum of all the terms of the binomial development of $(1-x)^y$, after the first, is $-x^y$. The truth of this may be shown by an example. Suppose the number of y 's are four, viz. Y' , Y'' , Y''' , and Y'''' . Let us use x' , x'' , x''' , and x'''' in such senses that

$$x^{Y'} = x', \quad x^{Y''} = x'', \quad x^{Y'''} = x''', \quad x^{Y''''} = x''''.$$

Then the negatives of the different terms of the binomial development are,

$$\binom{y}{1} x^1 = x' + x'' + x''' + x''''.$$

$$-\frac{\binom{y}{1} \binom{y-1}{2}}{2} x^2 = -x'x'' - x'x''' - x'x'''' - x''x''' - x''x'''' - x'''x''''.$$

$$+\frac{\binom{y}{1} \binom{y-1}{2} \binom{y-2}{3}}{2 \cdot 3} x^3 = x'x''x''' + x'x''x'''' + x'x'''x'''' + x''x'''x''''.$$

Now, since this addition is invertible, in the first term, x' that is x'' , is counted over twice, and so with every other pair. The second term subtracts each of these pairs, so that it is only counted once. But in the first term the x' that is x'' that is x''' is counted in three times only, while in the second term it is subtracted three times; namely, in (x',x'') , in (x',x''') and in (x'',x''') . On the whole, therefore, a triplet would not be represented in the sum at all, were it not added by the third term. The whole quartette is included four times in the first term, is subtracted six times by the second term, and is added four times in the third term. The fourth term subtracts it once, and thus in the sum of these negative terms each combination occurs once, and once only; that is to say the sum is

$$x' + x'' + x''' + x'''' = x(Y' + Y'' + Y''' + Y''') = x y.$$

Peirce: CP 3.105 Cross-Ref:††

105. If we write $(a x)^3$ for $[[x]].[[x-1]].[[x-2]].1x-†3,a†3$, that is for whatever is a to any three x 's, regard being had for the order of the x 's; and employ the modern numbers as exponents with this signification generally, then

$$1 - a x + (1/2!)(a x)^2 - (1/3!)(a x)^3 + \text{etc.}$$

is the development of $(1 - a)x$ and consequently it reduces itself to $1 - a x$. That is,

$$(112) \quad x = x - (1/2!)x^2 + (1/3!)x^3 + (1/4!)x^4 + \text{etc.}$$

Peirce: CP 3.106 Cross-Ref:††

106. $1 - x$ denotes everything except x , that is, whatever is other than every x ; so that ⊖ means "not." We shall take $\log x$ in such a sense that

$$\text{⊖}^{\log} x = x.†P1$$

Peirce: CP 3.107 Cross-Ref:††

107. I define the first difference of a function by the usual formula,

$$(113) \quad \{D\}\phi x = \phi(x + \{D\}x) - \phi x,$$

where $\{D\}x$ is an indefinite relative which never has a correlate in common with x . So that

$$(114) \quad x,(\{D\}x) = 0 \quad x + \{D\}x = x +, \{D\}x.$$

Higher differences may be defined by the formulæ

$$(115) \quad \{D\}n \cdot x = 0 \text{ if } n > 1$$

$$\{D\}^2.\phi x = \{D\}\{D\}x = \phi(x+2.\{D\}x) - 2.\phi(x+\{D\}x) + \phi x,$$

$$\{D\}^3.\phi x = \{D\}\{D\}^2.x = \phi(x+3.\{D\}x) - 3.\phi(x+2.\{D\}x) + 3.\phi(x+\{D\}x) - \phi x.$$

$$(116) \{D\}n.\phi x = \phi(x+n.\{D\}x) - n.\phi(x+(n-1).\{D\}x)$$

$$+ (n.(n-1))/2.\phi(x+(n-2).\{D\}x) - \text{etc.}$$

Peirce: CP 3.108 Cross-Ref:††

108. The exponents here affixed to $\{D\}$ denote the number of times this operation is to be repeated, and thus have quite a different signification from that of the numerical coefficients in the binomial theorem. I have indicated the difference by putting a period after exponents significative of operational repetition. Thus, m^2 may denote a mother of a certain pair, m^2 . a maternal grandmother.

Peirce: CP 3.109 Cross-Ref:††

109. Another circumstance to be observed is, that in taking the second difference of x , if we distinguish the two increments

which x successively receives as $\{D\}'x$ and $\{D\}''x$, then by (114)

$$(\{D\}'x), (\{D\}''x) = 0$$

If $\{D\}x$ is relative to so small a number of individuals that if the number were diminished by one $\{D\}n.\phi x$ would vanish, then I term these two corresponding differences *differentials*, and write them with \mathbf{d} instead of $\{D\}$.

Peirce: CP 3.110 Cross-Ref:††

110. The difference of the invertible sum of two functions is the sum of their differences; for by (113) and (18),

$$(117) \{D\}(\phi x + \mu x) = \phi(x + \{D\}x) + \mu(x + \{D\}x) - \phi x - \mu x$$

$$= \phi(x + \{D\}x) - \phi x + \mu(x + \{D\}x) - \mu x = \{D\}\mu x + \{D\}\mu x.$$

If a is a constant, we have

$$(118) \{D\}a\phi x = a(\phi x + \{D\}\phi x) - a\phi x = a\{D\}\phi x - (a\{D\}\phi x), a\phi x,$$

$$\{D\}^2.a\phi x = -\{D\}a\phi x, a\{D\}x, \text{ etc.}$$

$$\{D\}(\phi x)a = (\{D\}\phi x)a - ((\{D\}\phi x)a), \phi xa,$$

$$\{D\}^2.(\phi x)a = -\{D\}(\phi x)a, \text{ etc.}$$

$$(119) \phi(a, \phi x) = a, \{D\}\phi x.$$

Peirce: CP 3.110 Cross-Ref: ††

Let us differentiate the successive powers of x . We have in the first place,

$$\{D\}(x^2) = (x + \{D\}x)^2 - x^2 = 2.x^{2-\dagger 1}, (\{D\}x)^{\dagger 1} + (\{D\}x)^2.$$

Here, if we suppose $\{D\}x$ to be relative to only one individual,

$(\{D\}x)^2$ vanishes, and we have, with the aid of (115),

$$d(x^2) = 2.x^1, dx .$$

Considering next the third power, we have, for the first differential,

$$\{D\}(x^3) = (x + \{D\}x)^3 - x^3 = 3.x^{3-\dagger 1}, (\{D\}x)^{\dagger 1} + 3.x^{3-\ddagger 2}, (\{D\}x)^{\ddagger 2} + (\{D\}x)^3,$$

$$d(x^3) = 3.x^2, d(x).$$

To obtain the second differential, we proceed as follows:

$$\{D\}^2.(x^3) = (x + 2.\{D\}x)^3 - 2.(x + \{D\}x)^3 + x^3$$

$$\begin{aligned}
&= x^3 + 6.x^3-\dagger 1,(\{D\}x)^{\dagger 1} + 12.x^3-\dagger 2,(\{D\}x)^{\dagger 2} + 8.(\{D\}x)^3 \\
&- 2.x^3 - 6.x^3-\|1,(\{D\}x)\|1 - 6.x^3-\§2,(\{D\}x)\§2 - 2.(\{D\}x)^3 + x^3 \\
&= 6.x^3-\dagger 2,(\{D\}x)^{\dagger 2} + 6.(\{D\}x)^3.
\end{aligned}$$

Here, if $\{D\}x$ is relative to less than two individuals, $\{D\}\phi x$ vanishes. Making it relative to two only, then, we have

$$d^2.(x^3) = 6.x^1, (dx)^2.$$

These examples suffice to show what the differentials of xn will be. If for the number n we substitute the logical term n , we have

$$\{D\}(xn) = (x + \{D\}x)n - xn = [[n]].xn-\dagger 1, (\{D\}x)^{\dagger 1} + \text{etc.}$$

$$d(xn) = [[n]].xn-1, (dx).$$

We should thus readily find

$$(120) \mathbf{d}^m.(xn) = [[n]].[[n-1]].[[n-2]].\dots[n-m+1].xn-\dagger m, (\mathbf{d}x)^{\dagger m}.$$

Peirce: CP 3.110 Cross-Ref: ††

Let us next differentiate lx . We have, in the first place,

$$\{D\}lx = lx +, \{D\}x - lx = lx, l\{D\}x - lx = lx, (l\{D\}x - 1).$$

The value of $l\{D\}x - 1$ is next to be found.

$$\text{We have by (111)} \quad \textcircled{C}l\{D\}z - 1 = l\{D\}[x].$$

Hence, $l^{(D)}x - 1 = \log l^{(D)}x.$

But by (10) $\log l^{(D)}x = (\log l)\{D\}x.$

Substituting this value of $l^{(D)}x - 1$ in the equation lately found for dx we have

$$(121) \quad dx = lx, (\log l) dx = lx, (l - 1) dx = -lx, (1 - l) dx.$$

Peirce: CP 3.111 Cross-Ref: ††

111. In printing this paper, I here make an addition which supplies an omission in the account given above †1 of involution in this algebra. We have seen that every term which does not vanish is conceivable as logically divisible into individual terms. Thus we may write

$$s = S' +, S'' +, S''' +, \text{etc.}$$

where not more than one individual is in any one of these relations to the same individual, although there is nothing to prevent the same person from being so related to many individuals. †1 Thus, "bishop of the see of" may be divided into first bishop, second bishop, etc., and only one person can be n th bishop of any one see, although the same person may (where translation is permitted) be n th bishop of several sees. Now let us denote the converse of x by Kx ; thus, if s is "servant of," Ks is "master or mistress of." Then we have

$$Ks = KS' +, KS'' +, KS''' +, \text{etc.};$$

and here each of the terms of the second member evidently expresses such a relation that the same person cannot be so related to more than one, although more than one may be so related to the same. Thus, the converse of "bishop of the see of --" is "see one of whose bishops is --," the converse of "first bishop of --" is "see whose first bishop is --," etc. Now, the same see cannot be a see whose n th bishop is more than one individual, although several sees may be so related to the same individual. Such relatives I term infinitesimal on account of the vanishing of their higher powers. Every relative has a converse, and since this converse is conceivable as divisible into individual terms, the relative itself is conceivable as divisible into infinitesimal terms. To indicate this we may write

$$(122) \quad \text{If } x > 0 \quad x = X[.] +, X[.,] +, X[.,.,] +, \text{etc.}$$

Peirce: CP 3.112 Cross-Ref:††

112. As a term which vanishes is not an individual, nor is it composed of individuals, so it is neither an infinitesimal nor composed of infinitesimals.

As we write $l S', l S'', l S'''$, etc. = ls ,

so we may write

$$(123) \quad L[,]s, L[,]s, L[,]s, \text{ etc.} = ls,$$

But as the first formula is affected by the circumstance that **zero** is not an individual, so that lsw does not vanish on account of no woman having the particular kind of servant denoted by S' , lsw denoting merely every lover of whatever servant there is of any woman; so the second formula is affected in a similar way, so that the vanishing of $L[,]s$ does not make ls to vanish, but this is to be interpreted as denoting everything which is a lover, *in whatever way it is a lover at all*, of a servant.†1 Then just as we have by (112), that

$$(124) \quad ls = 1 - (1 - l)s; \dagger 2$$

so we have

$$(125) \quad ls = 1 - l(1 - s). \dagger 3$$

Mr. De Morgan denotes ls and ls by $LS[,]$ and $L[,]S$ respectively,†4 and he has traced out the manner of forming the converse and negative of such functions in detail. The following table contains most of his results in my notation.†5 For the converse of m , I write w ; and for that of n , u .

x	$ $	Kx
$m n$	$ $	$u w$
$mn = (1-m)(1-n)$	$ $	$uw = (1-u)(1-w)$

$$mn = (1-m)(1-n) \quad | \quad uw = (1-u)(1-w)$$

$$\lceil \neg x \quad | \quad \mathbf{K} \lceil \neg x$$

$$(1-m)n = m(1-n) \quad | \quad w(1-u) = (1-w)u$$

$$(1-m)n \quad | \quad w(1-u)$$

$$m(1-n) \quad | \quad (1-m)n$$

Peirce: CP 3.113 Cross-Ref:††

113. I shall term the operation by which w is changed to lw , **backward involution**. All the laws of this but one are the same as for ordinary involution, and the one exception is of that kind which is said to prove the rule. It is that whereas with ordinary involution we have,

$$(ls)^w = l(sw);$$

in backward involution we have

$$(126) \quad l(sw) = (l s)w;$$

that is, the things which are lovers to nothing but things that are servants to nothing but women are the things which are lovers of servants to nothing but women.

Peirce: CP 3.114 Cross-Ref:††

114. The other fundamental formulæ of backward involution are as follows:

$$(127) \quad l+, sw = lw,sw,$$

or, the things which are lovers or servants to nothing but women are the things which are lovers to nothing but women and servants to nothing but women.

$$(128) \quad l(f,u) = lf,lu,$$

or, the things which are lovers to nothing but French violinists are the things that are lovers to nothing but Frenchmen and lovers to nothing but violinists. This is perhaps not quite axiomatic. It is proved as follows. By (125) and (30)

$$l(f,u) = \mathcal{C}^{-1}(1-f,u) = \mathcal{C}^{-1}(1-f) + l(1-u)$$

By (125), (13), and (7),

$$lf,lu = \mathcal{C}^{-1}(1-f), \mathcal{C}^{-1}(1-u) = \mathcal{C}^{-1}(1-f) + l(1-u).$$

Finally, the binomial theorem holds with backward involution. For those persons who are lovers of nothing but Frenchmen and violinists consist first of those who are lovers of nothing but Frenchmen; second, of those who in some ways are lovers of nothing but Frenchmen and in all other ways of nothing but violinists, and finally of those who are lovers only of violinists. That is,

$$(129) \quad l(u +, f) = lu +, \sum [p] l \cdot pu \cdot pf +, lf.$$

In order to retain the numerical coefficients, we must let $\{l\}$ be the number of persons that one person is lover of. We can then write

$$l(u+f) = lu + \{l\} l \cdot 1u, 1f + ((\{l\} \cdot \{l-1\})/2) l \cdot 2u, 2f + \text{etc.}$$

Peirce: CP 3.115 Cross-Ref:††

115. We have also the following formula which combines the two involutions:

$$(130) \quad l(s^w) = (ls)^w;$$

that is, the things which are lovers of nothing but what are servants of all women are the same as the things which are related to all women as lovers of nothing but their servants.

Peirce: CP 3.116 Cross-Ref:††

116. It is worth while to mention, in passing, a singular proposition derivable from (128). Since, by (124) and (125)

$$xy = (1-x)^{(1-y)}, \dagger 1$$

and since

$$1-(u +, f) = \mathcal{C}^{-(u +, f)} = \mathcal{C}^{-u}, \mathcal{C}^{-f} = (1-u), (1-f),$$

(128) gives us,

$$(1-l)^{(1-u), (1-f)} = (1-l)^{(1-u)} +, \Sigma[p](1-(l-p))^{(1-u)}, \\ (1-p)^{(1-f)} +, (1-l)^{(1-f)}.$$

This is, of course, as true for u and f as for $(1-u)$ and $(1-f)$. Making those substitutions, and taking the negative of both sides, we have, by (124)

$$(131) \quad l(u, f) = (lu), \pi'[p]((l-p)u +, pf), (lf),$$

or, the lovers of French violinists are those persons who, in reference to every mode of loving whatever, either in that way love some violinists or in some other way love some Frenchmen. This logical proposition is certainly not self-evident, and its practical importance is considerable. In a similar way, from (12) we obtain

$$(132) \quad (e, c)f = \pi'[p](e(f-p) +, c p),$$

that is, to say that a person is both emperor and conqueror of the same Frenchman is the same as to say that, taking any class of Frenchmen whatever, this person is either an emperor of some one of this class, or conqueror of some one among the remaining Frenchmen.

Peirce: CP 3.117 Cross-Ref: ††

117. The properties of zero and unity, with reference to backward involution, are easily derived from (125). I give them here in comparison with the corresponding formulæ for forward involution.

$$(133) \quad {}^0x = 1 \quad x^0 = 1.$$

$$(134) \quad q0 = 0 \quad 0r = 0,$$

where q is the converse of an unlimited relative, and r is greater than zero.

$$(135) \quad lx = x \quad xl = x.$$

$$(136) \quad yl = y \quad lz = z,$$

where y is infinitesimal, and z is individual. Otherwise, both vanish.

$$(137) \quad {}^1s = 0 \quad p^1 = 0,$$

where s is less than unity and p is a limited relative.

$$(138) \quad xl = 1 \quad lx = 1.$$

Peirce: CP 3.118 Cross-Ref: ††

118. In other respects the formulæ for the two involutions are not so analogous as might be supposed; and this is owing to the dissimilarity between individuals and infinitesimals. We have, it is true, if X' is an infinitesimal and X an individual,

$$(139) \quad X[,](y,z) = X[,],y,X[,],z \text{ like } (y,z)X' = y X',z X';$$

$$(140) \quad X[,],y[0] = X[,],X[,],y \quad " \quad X',y[0] = X',y X';$$

$$(141) \quad \{X[,]\} = 1 \quad " \quad \{[X']\} = 1.$$

We also have

$$(142) \quad X[,],y \prec X[,],y.$$

But we have *not* $Xy = X'y$, and consequently we have *not* $sw \prec sw$, for this fails if there is anything which is not a servant at all, while the corresponding formula $s^w \prec sw$ only fails if there is not anything which is a woman. Now, it is much more often the case that there is something which is not x , than that there is not anything which is x . We have with the backward involution, as with the forward, †1 the formulæ

$$(143) \quad \text{If } x \prec y \quad yz \prec xz; \dagger 1$$

$$(144) \quad \text{If } x \prec y \quad zx \prec zy; \dagger 1$$

The former of these gives us

$$(145) \quad lsw \prec (ls)w,$$

or, whatever is lover to nothing but what is servant to nothing but women $\dagger 2$ stands to nothing but a woman in the relation of lover of every servant of hers. The following formulæ can be proved without difficulty.

$$(146) \quad lsw \prec lsw,$$

or, every lover of somebody who is servant to nothing but a woman stands to nothing but women in the relation of lover of nothing but a servant of them.

$$(147) \quad lsw \prec l(sw),$$

or, whatever stands to a woman in the relation of lover of nothing but a servant of hers is a lover of nothing but servants of women.

The differentials of functions involving backward involution are

$$(148) \quad \mathbf{d}nx = \{n\}n-1x.\mathbf{d}x.$$

$$(149) \quad \mathbf{d}xl = xl.\mathbf{d}x \log.x.$$

In regard to powers of \mathbb{C} we have

$$(150) \quad x\mathbb{C} = \mathbb{C}x.$$

Exponents with a dot may also be put upon either side of the letters which they affect.

Peirce: CP 3.119 Cross-Ref: $\dagger\dagger$

119. The greater number of functions of x in this algebra may be put in the form

$$\phi x = \Sigma[p] \Sigma[q] [p]A p x q [p]B[q].$$

For all such functions Taylor's and Maclaurin's theorems hold good in the form,

$$(151) \quad \begin{array}{c} \text{---} \quad \text{---} \\ | y | \quad | 0 | \quad \infty \\ \text{---} | \quad \text{---} | \quad \Sigma[p] 1/p! \cdot d^p \cdot = I. \\ | dx | \quad | y | \quad 0 \\ \text{---} \quad \text{---} \end{array}$$

The symbol $\begin{array}{c} \text{---} \\ | a | \\ \text{---} | \quad \text{---} \\ | b | \\ \text{---} \end{array}$ is used to denote that a is to be substituted

for b in what follows. For the sake of perspicuity, I will write Maclaurin's theorem at length.

$$\phi x = \begin{array}{c} \text{---} \quad \text{---} \\ | x | \quad | 0 | \\ \text{---} | \quad \text{---} | \quad ((1/0!).d^0+(1/1!).d^1+(1/2!).d^2+(1/3!).d^3+\text{etc.})\phi x \\ | dx | \quad | y | \\ \text{---} \quad \text{---} \end{array}$$

Peirce: CP 3.119 Cross-Ref: ††

The proof of these theorems is very simple. The $(p+q)$ th differential of $p x q$ is the only one which does not vanish when x vanishes. This differential then becomes $[[p+q]]! \cdot p(dx)q$. It is plain, therefore, that the theorems hold when the coefficients pAq and pBq are I . But the general development, by Maclaurin's theorem, of $a\phi x$ or $(\phi x)a$ is in a form which (112) reduces to identity. It is very likely that the application of these theorems is not confined within the limits to which I have restricted it. We may write these theorems in the form

$$(152) \quad \begin{array}{c} \text{---} \quad \text{---} \\ | y | \quad | 0 | \\ \text{---} | \quad \text{---} | \quad (d = I, \\ | dx | \quad | y | \\ \text{---} \quad \text{---} \end{array}$$

provided we assume that when the first differential is positive

$$[d = (1/0!)d^0 + (1/1!)d^1 + (1/2!)d^2 + \text{etc.},$$

but that when the first differential is negative this becomes by (111),

$$[d = 1 + d.$$

Peirce: CP 3.120 Cross-Ref:††

120. As another illustration of the use which may be made of differentiation in logic, let us consider the following problem. In a certain institution all the officers (x) and also all their common friends (f) are privileged persons (y). How shall the class of privileged persons be reduced to a minimum? Here we have

$$y = x + fx,$$

$$dy = dx + dfx = dx - f^x(1-f)dx.$$

When y is at a minimum it is not diminished either by an increase or diminution of x . That is,

$$[[dy]] > - 0,$$

and when $[[x]]$ is diminished by one,

$$[[dy]] < - 0,$$

Peirce: CP 3.120 Cross-Ref:††

When x is a minimum, then

$$[[dx \cdot f^x(1-f)dx]] > - 0 \quad [[dx \cdot f^{x-1}(1-f)dx]] < - 0$$

$$(A) \quad [[dx]] - [[f^x(1-f)dx]] > - 0 \quad [[dx]] - [[f^{x-1}(1-f)dx]] < - 0$$

Peirce: CP 3.120 Cross-Ref:††

Now we have by (30)

$$f^x, (1-f)dx = f^x - (0;0), (1-f)dx.$$

Hence,

$$[f^x] -< [[dx]] + (0;0). [[(1-f)dx]].$$

$$[f^{x-1}] -< [[dx]] + (0;0). [[(1-f)dx]].$$

But $[0;0]$ lies between the limits 0 and 1, and

$$(153) [dx] = 1.$$

We have, therefore,

$$[f^x] -< 1 + [[(1-f)1.]] [f^{x-1}] >- 1.$$

This is the general solution of the problem. If the event of a person who may be an official in the institution being a friend of a second such person is independent of and equally probable

with his being a friend of any third such person, and if we take

p , or the whole class of such persons, for our universe, we have,

$$p = 1;$$

$$[f^x,] = [f^x]/[p] = ([f]/[p])^{[x]},$$

$$[[1-f)dx]] = [[1-f]]. [[dx]] = ([p]-[f]). [[dx]],$$

$$[f^x, (1-f)dx] = ([f]/[p])^{[x]}. ([p]-[f]). [[dx]]$$

Substituting these values in our equations marked (A) we get, by a little reduction,

$$[[x]] >- (\log([[p]]-[[f]]))/(\log[[p]]-\log[[f]]),$$

$$[[x]] <- (\log([[p]]-[[f]]))/(\log[[p]]-\log[[f]]) + 1.$$

The same solution would be reached through quite a different road by applying the calculus of finite differences in the usual way.

Peirce: CP 3.121 Cross-Ref:††
ELEMENTARY RELATIVES †1

121. By an elementary relative I mean one which signifies a relation which exists only between mutually exclusive pairs (or in the case of a conjugative term, triplets, or quartettes, etc.) of individuals, or else between pairs of classes in such a way that every individual of one class of the pair is in that relation to every individual of the other. If we suppose that in every school, every teacher teaches every pupil (a supposition which I shall tacitly make whenever in this paper I speak of a school), then *pupil* is an elementary relative. That every relative may be conceived of as a logical sum of elementary relatives is plain, from the fact that if a relation is sufficiently determined it can exist only between two individuals. Thus, a *father* is either father in the first ten years of the Christian era, or father in the second ten years, in the third ten years, in the first ten years, B. C., in the second ten years, or the third ten years, etc. Any one of these species of father is father for the first time or father for the second time, etc. Now such a relative as "father for the third time in the second decade of our era, of --" signifies a relation which can exist only between mutually exclusive pairs of individuals, and is therefore an elementary relative; and so the relative *father* may be resolved into a logical sum of elementary relatives.

Peirce: CP 3.122 Cross-Ref:††

122. The conception of a relative as resolvable into elementary relatives has the same sort of utility as the conception of a relative as resolvable into infinitesimals or of any term as resolvable into individuals.

Peirce: CP 3.123 Cross-Ref:††

123. Elementary simple relatives are connected together in systems of four. For if A:B be taken to denote the elementary relative which multiplied into B gives A, then this relation

existing as elementary, we have the four elementary relatives

A:A A:B B:A B:B.

An example of such a system is--colleague: teacher: pupil: schoolmate. In the same

way, obviously, elementary conjugatives are in systems the number of members in which is $(n+1)n+1$ where n is the number of correlates which the conjugative has. At present, I shall consider only the simple relatives.

Peirce: CP 3.124 Cross-Ref:††

124. The existence of an elementary relation supposes the existence of mutually exclusive pairs of classes. The first members of those pairs have something in common which discriminates them from the second members, and may therefore be united in one class, while the second members are united into a second class. Thus *pupil* is not an elementary relative unless there is an absolute distinction between those who teach and those who are taught. We have, therefore, two general absolute terms which are mutually exclusive, "body of teachers in a school," and "body of pupils in a school." These terms are general because it remains undetermined what school is referred to. I shall call the two mutually exclusive absolute terms which any system of elementary relatives supposes, the *universal extremes* of that system. There are certain characters in respect to the possession of which both members of any one of the pairs, between which there is a certain elementary relation, agree. Thus, the body of teachers and the body of pupils in any school agree in respect to the country and age in which they live, etc., etc. Such characters I term *scalar characters* for the system of elementary relatives to which they are so related; and the relatives written with a comma which signify the possession of such characters, I term *scalars* for the system. Thus, supposing French teachers have only French pupils and *vice versa*, the relative

f,

will be a scalar for the system "colleague: teacher: pupil: schoolmate." If r is an elementary relative for which s is a scalar,

$$(154) \quad s,r = rs,.$$

Peirce: CP 3.125 Cross-Ref:††

125. Let c, t, p, s , denote the four elementary relatives of any system; such as colleague, teacher, pupil, schoolmate; and let a, b, c, d , be scalars for this system. Then any relative which is capable of expression in the form

$$a,c + b,t + c,p + d,s$$

I shall call a *logical quaternion*. Let such relatives be denoted by q, q', q'' , etc. It is plain, then, from what has been said, that any relative may be regarded as resolvable into a logical sum of logical quaternions.

Peirce: CP 3.126 Cross-Ref:††

126. The multiplication of elementary relatives of the same system follows a

very simple law. For if u and v be the two universal extremes of the system c, t, p, s , we may write

$$c = u:u \quad t = u:v \quad p = v:u \quad s = v:v,$$

and then if w and w' are each either u or v , we have

$$(155) \quad (w':w)^{-w} = 0.$$

This gives us the following multiplication-table, where the multiplier is to be entered at the side of the table and the multiplicand at the top, and the product is found in the middle:

	<i>c</i>	<i>t</i>	<i>p</i>	<i>s</i>	
<i>c</i>	<i>c</i>	<i>t</i>	0	0	
(156) <i>t</i>	0	0	<i>c</i>	<i>t</i>	
<i>p</i>	<i>p</i>	<i>s</i>	0	0	
<i>s</i>	0	0	<i>p</i>	<i>s</i>	

Peirce: CP 3.126 Cross-Ref: ††

The sixteen propositions expressed by this table are in ordinary language as follows: †1

The colleagues of the colleagues of any person are that person's colleagues;

The colleagues of the teachers of any person are that person's teachers;

There are no colleagues of any person's pupils;

There are no colleagues of any person's schoolmates;

There are no teachers of any person's colleagues;

There are no teachers of any person's teachers;

The teachers of the pupils of any person are that person's colleagues;

The teachers of the schoolmates of any person are that person's teachers;

The pupils of the colleagues of any person are that person's pupils;

The pupils of the teachers of any person are that person's schoolmates;

There are no pupils of any person's pupils;

There are no pupils of any person's schoolmates;

There are no schoolmates of any person's colleagues;

There are no schoolmates of any person's teachers;

The schoolmates of the pupils of any person are that person's pupils;

The schoolmates of the schoolmates of any person are that person's schoolmates.

Peirce: CP 3.126 Cross-Ref:††

This simplicity and regularity in the multiplication of elementary relatives must clearly enhance the utility of the conception of a relative as resolvable into a sum of logical quaternions.

Peirce: CP 3.127 Cross-Ref:††

127. It may sometimes be convenient to consider relatives each one of which is of the form

$$a,i + b,j + c,k + d,l + \text{etc.}$$

where $a, b, c, d, \text{etc.}$ are scalars, and $i, j, k, l, \text{etc.}$ are each of the form

$$m,u + n,v + o,w + \text{etc.}$$

where $m, n, o, \text{etc.}$ are scalars, and $u, v, w, \text{etc.}$ are elementary relatives. In all such cases (155) Will give a multiplication-table for $i, j, k, l, \text{etc.}$ For example, if we have three classes of individuals, $u[1], u[2], u[3]$, which are related to one another in pairs, we may put

$$u[1]:u[1] = i \quad u[1]:u[2] = j \quad u[1]:u[3] = k$$

$$u[2]:u[1] = l \quad u[2]:u[2] = m \quad u[2]:u[3] = n$$

$$u[3]:u[1] = o \quad u[3]:u[2] = p \quad u[3]:u[3] = q$$

and by (155) we get the multiplication-table

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>	<i>q</i>
<i>i</i>	<i>i</i>	<i>j</i>	<i>k</i>	0	0	0	0	0	0
<i>j</i>	0	0	0	<i>i</i>	<i>j</i>	<i>k</i>	0	0	0
<i>k</i>	0	0	0	0	0	0	<i>i</i>	<i>j</i>	<i>k</i>
<i>l</i>	<i>l</i>	<i>m</i>	<i>n</i>	0	0	0	0	0	0
<i>m</i>	0	0	0	<i>l</i>	<i>m</i>	<i>n</i>	0	0	0
<i>n</i>	0	0	0	0	0	0	<i>l</i>	<i>m</i>	<i>n</i>
<i>o</i>	<i>o</i>	<i>p</i>	<i>q</i>	0	0	0	0	0	0
<i>p</i>	0	0	0	<i>o</i>	<i>p</i>	<i>q</i>	0	0	0
<i>q</i>	0	0	0	0	0	0	<i>o</i>	<i>p</i>	<i>q</i>

Peirce: CP 3.128 Cross-Ref:††

128. If we take

$$i = u[1]:u[2] + u[2]:u[3] + u[3]:u[4],$$

$$j = u[1]:u[3] + u[2]:u[4],$$

$$k = 2.u[1]:u[4],$$

we have

	<i>i</i>	<i>j</i>	<i>k</i>
<i>i</i>	<i>j</i>	<i>k</i>	0
<i>j</i>	<i>k</i>	0	0
<i>k</i>	0	0	0

Peirce: CP 3.129 Cross-Ref:††
 129. If we take

$$i = u[1]:u[2] + u[2]:u[3] + u[3]:u[4] + u[5]:u[6] + u[7]:u[8],$$

$$j = u[1]:u[3] + u[2]:u[4],$$

$$k = 2.u[1]:u[4],$$

$$l = u[6]:u[8] + a.u[5]:u[7] + 2b.u[1]:u[9] + u[9]:u[4] + c.u[5]:u[6],$$

$$m = u[5]:u[8],$$

we have

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	
<i>i</i>	<i>j</i>	<i>k</i>	0	<i>m</i>	0	
<i>j</i>	<i>k</i>	0	0	0	0	
<i>k</i>	0	0	0	0	0	
<i>l</i>	<i>a.m</i>	0	0	<i>b.k+</i>	0	
<i>m</i>	0	0	0	<i>c.m</i>		

Peirce: CP 3.130 Cross-Ref:††

130. These multiplication-tables have been copied from Professor Peirce's monograph on Linear Associative Algebras.†P1 I can assert, upon reasonable inductive evidence, that all such algebras can be interpreted on the principles of the present notation in the same way as those given above. In other words, all such algebras are complications and modifications of the algebra of (156). It is very likely that this is true of all algebras whatever. The algebra of (156), which is of such a fundamental character in reference to pure algebra and our logical notation, has been shown by Professor Peirce †1 to be the algebra of Hamilton's quaternions.†2 In fact, if we put

$$1 = i + l.$$

$$i' = \sqrt{1-b^2} j - (\sqrt{1-a^2b} + ab) j + (\sqrt{1-a^2b} - ab) k - \sqrt{1-b^2} l.$$

$$j' = -b\sqrt{1-c^2} i + (ac - \sqrt{1-a^2}\sqrt{1-b^2}\sqrt{1-c^2} - (\sqrt{1-a^2c} + a\sqrt{1-b^2}\sqrt{1-c^2})) j - (ac - \sqrt{1-a^2}\sqrt{1-b^2}\sqrt{1-c^2} + (\sqrt{1-a^2c} + a\sqrt{1-b^2}\sqrt{1-c^2})) k + b\sqrt{1-c^2} l.$$

$$k' = bc i + (\sqrt{1-a^2}\sqrt{1-b^2}c + a\sqrt{1-c^2} + (a\sqrt{1-b^2}c - \sqrt{1-a^2}\sqrt{1-c^2})) j - (\sqrt{1-a^2}\sqrt{1-b^2}c + a\sqrt{1-c^2} - (a\sqrt{1-b^2}c - \sqrt{1-a^2}\sqrt{1-c^2})) k - b c l.$$

where a, b, c , are scalars, then $1, i', j', k'$ are the four fundamental factors of quaternions, the multiplication-table of which is as follows:

	1	i'	j'	k'
1	1	i'	j'	k'
i'	i'	-1	k'	$-j'$
j'	j'	$-k'$	-1	i'
k'	k'	j'	$-i'$	-1

Peirce: CP 3.131 Cross-Ref:††

131. It is no part of my present purpose to consider the bearing upon the philosophy of space of this occurrence, in pure logic, of the algebra which expresses all the properties of space; but it is proper to point out that one method of working with this notation would be to transform the given logical expressions into the form of Hamilton's quaternions (after representing them as separated into elementary relatives), and then to make use of geometrical reasoning. The following formulæ will assist this process. Take the quaternion relative

$$q = x i + y j + z k + w l,$$

where $x, y, z,$ and w are scalars. The conditions of q being a *scalar*, *vector*, etc. (that is, being denoted by an algebraic expression which denotes a scalar, a vector, etc., in geometry), are

(157) Form of a scalar: $x(i + l)$.

(158) Form of a vector: $x i + y j + z k - x l$.

(159) Form of a versor:

$$x/y((x/z)-1)^{-1/2}i + y/x((x/z)-1)^{-1/2}j + \\ z/y((z/x)-1)^{-1/2}k + y/z((z/x)-1)^{-1/2}l.$$

(160) Form of zero: $x i + x y j + (z/y)k + z l$.

(161) Scalar of q : $Sq = 1/2(x + w)(i + l)$.

(162) Vector of q : $Vq = 1/2(x-w)i + y j + z k + 1/2(w-x)l$.

(163) Tensor of q : $Tq = \sqrt{x w - y z} (i+l)$.

(164) Conjugate of q : $Kq = w i - y j - z k + x l$.

Peirce: CP 3.132 Cross-Ref:††

132. In order to exhibit the logical interpretations of these functions, let us consider a universe of married monogamists, in which husband and wife always have country, race, wealth, and virtue, in common. Let i denote "man that is --," j "husband of --," k "wife of --," and l "woman that is --"; x "negro that is --," y "rich person that is --," z "American that is --," and w "thief that is --." Then, q being defined as above, the q 's of any class will consist of so many individuals of that class as are negro-men or women-thieves together with all persons who are rich husbands

or American wives of persons of this class. Then, $2S\mathbf{q}$ denotes, by (160), $\dagger 1$ all the negroes and besides all the thieves, while $S\mathbf{q}$ is the indefinite term which denotes half the negroes and thieves. Now, those persons who are self-q's of any class (that is, the q's of themselves among that class) are $x\mathbf{i} + w\mathbf{l}$; add to these their spouses and we have $2S\mathbf{q}$. In general, let us term $(\mathbf{j} + \mathbf{k})$ the "correspondent of --." Then, the double scalar of any quaternion relative, q , is that relative which denotes all self-q's, and, besides, "all correspondents of self-q's of --." $(T\mathbf{q})^2$ denotes all persons belonging to pairs of corresponding self-q's *minus* all persons belonging to pairs of corresponding q's of each other.

Peirce: CP 3.133 Cross-Ref: $\dagger\dagger$

133. As a very simple example of the application of geometry to the logic of relatives, we may take the following. Euclid's axiom concerning parallels corresponds to the quaternion principle that the square of a vector is a scalar. From this it follows, since by (157) $\mathbf{y}\mathbf{z} + \mathbf{z}\mathbf{k}$ [?] is a vector, that the rich husbands and American wives of the rich husbands and American wives of any class of persons are wholly contained under that class, and can be described without any discrimination of sex. In point of fact, by (156), the rich husbands and American wives of the rich husbands and American wives of any class of persons, are the rich Americans of that class.

Peirce: CP 3.133 Cross-Ref: $\dagger\dagger$

Lobatchewsky $\dagger 1$ has shown that Euclid's axiom concerning parallels may be supposed to be false without invalidating the propositions of spherical trigonometry. In order, then, that corresponding propositions should hold good in logic, we need not resort to elementary relatives, but need only take S and V in such senses that every relative of the class considered should be capable of being regarded as a sum of a scalar and a vector, and that a scalar multiplied by a scalar should be a scalar, while the product of a scalar and a vector is a vector. Now, to fulfill these conditions we have only to take $S\mathbf{q}$ as "self-q of," and $V\mathbf{q}$ as "alio-q of" (q of another, that other being --), and \mathbf{q} may be any relative whatever. For, "lover," for example, is divisible into self-lover and alio-lover; a self-lover of a self-benefactor of persons of any class is contained under that class, and neither the self-lover of an alio-benefactor of any persons nor the alio-lover of the self-benefactor of any persons are among those persons. Suppose, then, we take the formula of spherical trigonometry,

$$\cos a = \cos b \cos c + \cos A \sin b \sin c.$$

In quaternion form, this is,

$$(165) \quad S(\mathbf{p}\mathbf{q}) = (S\mathbf{p})(S\mathbf{q}) + S((V\mathbf{p})(V\mathbf{q})).$$

Let \mathbf{p} be "lover," and \mathbf{q} be "benefactor." Then this reads, lovers of their own benefactors consist of self-lovers of self-benefactors together with alio-lovers of alio-benefactors of themselves. So the formula

$$\sin b \cos p b' = -\sin a \cos c \cos p a' \\ -\sin c \cos a \cos p c' + \sin a \sin c \sin b \cos p b,$$

where A', B', C', are the positive poles of the sides a, b, c, is in quaternions

$$(166) V(p q) = (Vp)(Sq) + (Sp)(Vq) + V((Vp)(Vq)),$$

and the logical interpretation of this is: lovers of benefactors of others consist of alio-lovers of self-benefactors, together with self-lovers of alio-benefactors, together with alio-lovers of alio-benefactors of others. It is a little striking that just as in the non-Euclidean or imaginary geometry of Lobatchewsky the axiom concerning parallels holds good only with the ultimate elements of space, so its logical equivalent holds good only for elementary relatives.

Peirce: CP 3.134 Cross-Ref: ††

134. It follows from what has been said that for every proposition in geometry there is a proposition in the pure logic of relatives. But the method of working with logical algebra which is founded on this principle seems to be of little use. On the other hand, the fact promises to throw some light upon the philosophy of space. †P1

Peirce: CP 3.135 Cross-Ref: ††

§6. PROPERTIES OF PARTICULAR RELATIVE TERMS CLASSIFICATION OF SIMPLE RELATIVES †1

135. Any particular property which any class of relative terms may have may be stated in the form of an equation, and affords us another premiss for the solution of problems in which such terms occur. A good classification of relatives is, therefore, a great aid in the use of this notation, as the notation is also an aid in forming such a classification.

Peirce: CP 3.136 Cross-Ref: ††

136. The first division of relatives is, of course, into simple relatives and conjugatives. The most fundamental divisions of simple relatives are based on the distinction between elementary relatives of the form (A:A), and those of the form (A:B). These are divisions in regard to the amount of opposition between relative and correlative.

Peirce: CP 3.136 Cross-Ref: ††

a. Simple relatives are in this way primarily divisible into relatives all of whose elements are of the form (A:A) and those which contain elements of the form (A:B). The former express a mere agreement among things, the latter set one thing over against another, and in that sense express an opposition (*{antikeisthai}*); I shall therefore term the former *concurrents*, †P1 and the latter *opponents*. The distinction appears in this notation as between relatives with a comma, such as (w,), and relatives

without a comma, such as (w); and is evidently of the highest importance. The character which is signified by a concurrent relative is an absolute character, that signified by an opponent is a relative character, that is, one which cannot be prescinded from reference to a correlate.

Peirce: CP 3.136 Cross-Ref:††

b. The second division of simple relatives with reference to the amount of opposition between relative and correlative is into those whose elements may be arranged in collections of squares, each square like this,

A:A	A:B	A:C
B:A	B:B	B:C
C:A	C:B	C:C

and those whose elements cannot be so arranged.†1 The former (examples of which are, "equal to --," "similar to --") may be called *copulatives*, †P2 the latter *non-copulatives*. A copulative multiplied into itself gives itself. Professor Peirce calls letters having this property, *idempotents*.†2 The present distinction is of course very important in pure algebra. All concurrents are copulatives.

Peirce: CP 3.136 Cross-Ref:††

c. Third, relatives are divisible into those which for every element of the form (A:B) have another of the form (B:A), and those which want this symmetry. This is the old division into *equiparants*†P1 and *disquiparants*, †P2 or in Professor De Morgan's language, convertible and inconvertible relatives.†1 Equiparants are their own correlatives. All copulatives are equiparant.

Peirce: CP 3.136 Cross-Ref:††

d. Fourth, simple relatives are divisible into those which contain elements of the form (A:A) and those which do not. The former express relations such as a thing may have to itself, the latter (as cousin of --, hater of --) relations which nothing can have to itself. The former may be termed *self-relatives*, †P3 the latter *alio-relatives*. All copulatives are self-relatives.

Peirce: CP 3.136 Cross-Ref:††

e. The fifth division is into relatives some power (*i.e.* repeated product) of which contains †P4 elements of the form (A:A), and those of which this is not true.†2 The former I term *cyclic*, the latter *non-cyclic*†P5 relatives. As an example of the former, take

(A:B) +, (B:A) +, (C:D) +, (D:E) +, (E:C).

The product of this into itself is

(A:A) +, (B:B) +, (C:E) +, (D:C) +, (E:D).

The third power is

(A:B) +, (B:A) +, (C:C) +, (D:D) +, (E:E).

The fourth power is

(A:A) +, (B:B) +, (C:D) +, (D:E) +, (E:C).

The fifth power is

(A:B) +, (B:A) +, (C:E) +, (D:C) +, (E:D).

The sixth power is

(A:A) +, (B:B) +, (C:C) +, (D:D) +, (E:E).

where all the terms are of the form (A:A). Such relatives, as *cousin of --*, are cyclic. All equiparants are cyclic.

Peirce: CP 3.136 Cross-Ref:††

f. The sixth division is into relatives no power of which is zero, and relatives some power of which is zero. The former may be termed *inexhaustible*, the latter *exhaustible*. An example of the former is "spouse of --," of the latter, "husband of --." All cyclics are inexhaustible.

Peirce: CP 3.136 Cross-Ref:††

g. Seventh, simple relatives may be divided into those whose products into themselves are not zero, and those whose products into themselves are zero. The former may be termed *repeating*, the latter, *non-repeating* relatives. All inexhaustible relatives are repeating.

Peirce: CP 3.136 Cross-Ref:††

h. Repeating relatives may be divided (after De Morgan) into those whose products into themselves are contained under themselves, and those of which this is not true. The former are well named by De Morgan †1 *transitive*, the latter *intransitive*. All transitives are inexhaustible; all copulatives are transitive; and all transitive equiparants are copulative. The class of transitive equiparants has a

character, that of being self-relatives, not involved in the definitions of the terms.†2

Peirce: CP 3.136 Cross-Ref:††

i. Transitives are further divisible into those whose products by themselves are equal to themselves, and those whose products by themselves are less than themselves; the former may be termed *continuous*, †P1 the latter *discontinuous*. An example of the second is found in the pure mathematics of a continuum, where if *a* is greater than *b* it is greater than something greater than *b*; and as long as *a* and *b* are not of the same magnitude, an intervening magnitude always exists. All concurrents are continuous.

Peirce: CP 3.136 Cross-Ref:††

j. Intransitives may be divided into those the number of the powers (repeated products) of which not contained in the first is infinite, and those some power of which is contained in the first. The former may be called *infinite*s, the latter *finite*s. Infinite inexhaustibles are cyclic.

In addition to these, the old divisions of relations into relations of reason and real relations, of the latter into aptitudinal and actual, and of the last into extrinsic and intrinsic, are often useful.†P1

"NOT"

Peirce: CP 3.137 Cross-Ref:††

137. We have already seen that "not," or "other than," is denoted by $\textcircled{\sim}$ ⁻¹. It is often more convenient to write it, *n*. The fundamental property of this relative has been given above (111). It is that,

$$\textcircled{\sim}x = 1 - x.$$

Two other properties are expressed by the principles of contradiction and excluded middle. They are,

$$x, \textcircled{\sim}x = 0; \dagger 1$$

$$x +, \textcircled{\sim}x = 1. \dagger 1$$

The following deduced properties are of frequent application:

$$(167) \quad \textcircled{\sim}(x,y) = \textcircled{\sim}x +, \textcircled{\sim}y; \dagger 2$$

$$(168) \quad \mathcal{C}^{-}xy = \mathcal{C}^{-}xy$$

The former of these is the counterpart of the general formula,

$zx + y = zx, zy \dagger 2$, The latter enables us always to bring the exponent of the exponent of \mathcal{C}^{-} down to the line, and make it a factor. By the former principle, objects not French violinists consist of objects not Frenchmen, together with objects not violinists; by the latter, individuals not servants of all women are the same as non-servants of some women.

Another singular property of \mathcal{C}^{-} is that,

$$\text{If } [[x]] > 1 \quad \mathcal{C}^{-}Ix = 1.$$

"CASE OF THE EXISTENCE OF --," AND "CASE OF THE NON-EXISTENCE OF --."

Peirce: CP 3.138 Cross-Ref: ††

138. That which first led me to seek for the present extension of Boole's logical notation was the consideration that as he left his algebra, neither hypothetical propositions nor particular propositions could be properly expressed. It is true that Boole was able to express hypothetical propositions in a way which answered some purposes perfectly. He could, for example, express the proposition, "Either the sun will shine, or the enterprise will be postponed," by letting x denote "the truth of the proposition that the sun will shine," and y "the truth of the proposition that the enterprise will be postponed"; and writing,

$$x +, y = 1,$$

or, with the invertible addition,

$$x + (1 - x), y = 1.$$

But if he had given four letters denoting the four terms, "sun," "what is about to shine," "the enterprise," and "what is about to be postponed," he could make no use of these to express his disjunctive proposition, but would be obliged to assume others. The imperfection of the algebra here was obvious. As for particular propositions, Boole could not accurately express them at all. He did undertake to express them and wrote

Some Y's are X's: $v,y = v,x;$

Some Y's are not X's: $v,y = v,(1-x).$

The letter v is here used, says Boole, for an "indefinite class symbol."^{†1} This betrays a radical misapprehension of the nature of a particular proposition. To say that some Y's are X's, is not the same as saying that a logical species of Y's are X's. For the logical species need not be the name of anything existing. It is only a certain description of things fully expressed by a mere definition, and it is a question of fact whether such a thing really exist or not. St. Anselm wished to infer existence from a definition, but that argument has long been exploded. If, then, v is a mere logical species in general, there is not necessarily any such thing, and the equation means nothing. If it is to be a logical species, then, it is necessary to suppose in addition that it exists, and further that *some v is y* . In short, it is necessary to assume concerning it the truth of a proposition, which, being itself particular, presents the original difficulty in regard to its symbolical expression. Moreover, from

$$v,y = v,(1-x)$$

we can, according to algebraic principles, deduce successively

$$v,y = v - v,x$$

$$v,x = v - v,y = v,(1-y).$$

Now if the first equation means that some Y's are not X's, the last ought to mean that some X's are not Y's; for the algebraic forms are the same, and the question is, whether the algebraic forms are adequate to the expression of particulars. It would appear, therefore, that the inference from Some Y's are not X's to Some X's are not Y's, is good; but it is not so, in fact.

Peirce: CP 3.139 Cross-Ref:††

139. What is wanted, in order to express hypotheticals and particulars analytically, is a relative term which shall denote "case of the existence of --," or "what exists only if there is any --"; or else "case of the non-existence of --," or "what exists only if there is not --." When Boole's algebra is extended to relative terms, it is easy to see what these particular relatives must be. For suppose that having expressed the propositions "it thunders," and "it lightens," we wish to express the fact that "if it lightens, it thunders." Let

$$A = 0 \text{ and } B = 0,$$

be equations meaning respectively, it lightens and it thunders. Then, if ϕx vanishes when x does not and *vice versa*, whatever x may be, the formula

$$\phi A \rightarrow \phi B$$

expresses that if it lightens it thunders; for if it lightens, A vanishes; hence ϕA does not vanish, hence ϕB does not vanish, hence B vanishes, hence it thunders. It makes no difference what the function ϕ is, provided only it fulfills the condition mentioned. Now, $0x$ is such a function, vanishing when x does not, and not vanishing when x does. *Zero*, therefore, may be interpreted as denoting "that which exists if, and only if, there is not --." Then the equation

$$0^0 = 1$$

means, everything which exists, exists only if there is not anything which does not exist. So,

$$0x = 0$$

means that there is nothing which exists if, and only if, *some x* does not exist. The reason of this is that *some x* means some existing x .

Peirce: CP 3.139 Cross-Ref:††

"It lightens" and "it thunders" might have been expressed by equations in the forms

$$A = 1, \quad B = 1.$$

In that case, in order to express that if it lightens it thunders, in the form

$$\phi A \rightarrow \phi B,$$

it would only be necessary to find a function, ϕx , which should vanish unless x were 1, and should not vanish if x were 1.

Such a function is 1x . We must therefore interpret 1 as "that

which exists if, and only if, there is --," $1x$ as "that which exists if, and only if, there is nothing but x ," and $1x$ as "that which exists if, and only if, there is some x ." Then the equation

$$1x = 1,$$

means everything exists if, and only if, whatever x there is exists.

Peirce: CP 3.140 Cross-Ref:††

140. Every hypothetical proposition may be put into four equivalent forms, as follows:

If X, then Y.

If not Y, then not X.

Either not X or Y.

Not both X and not Y.

If the propositions X and Y are $A = 1$ and $B = 1$, these four forms are naturally expressed by

$$1A \rightarrow 1B,$$

$$1(1-A) \rightarrow 1(1-B), \dagger 1$$

$$1(1-A) + B = 1,$$

$$1A, 1(1-B) = 0.$$

For $1x$ we may always substitute $0(1-x)$.

Peirce: CP 3.141 Cross-Ref:††

141. Particular propositions are expressed by the consideration that they are contradictory of universal propositions. Thus, as $h, (1-b) = 0$ means every horse is black, so $0^{h, (1-b)} = 0$ means that some horse is not black; and as $h, b = 0$ means that no horse is black, so $0^{h, b} = 0$ means that some horse is black. We may also write the particular affirmative $1(h, b) = 1$, and the particular negative $1(h, \bar{b}) = 1$.

Peirce: CP 3.142 Cross-Ref:††

142. Given the premisses, every horse is black, and every horse is an animal; required the conclusion. We have given

$h \rightarrow b$;

$h \rightarrow a$.

Commutatively multiplying, we get

$h \rightarrow a, b$.

Then, by (92) or by (90),

$0^{a,b} \rightarrow 0^h$, or $1^h \rightarrow 1^{(a,b)}$.

Hence, by (40) or by (46),

If $h > 0$ $0^{a,b} = 0$, or $1^{(a,b)} = 1$;

or if there are any horses, some animals are black. I think it would be difficult to reach this conclusion, by Boole's method unmodified.

Peirce: CP 3.143 Cross-Ref:††

143. Particular propositions may also be expressed by means of the signs of inequality. Thus, some animals are horses, may be written

$a, h > 0$;

and the conclusion required in the above problem might have been obtained in this form, very easily, from the product of the premisses, by (1) and (21).

Peirce: CP 3.143 Cross-Ref:††

We shall presently see †1 that conditional and disjunctive propositions may also be expressed in a different way.

Peirce: CP 3.144 Cross-Ref:††
CONJUGATIVE TERMS

144. The treatment of conjugative terms presents considerable difficulty, and would no doubt be greatly facilitated by algebraic devices. I have, however, studied this part of my notation but little.

Peirce: CP 3.144 Cross-Ref:††

A relative term cannot possibly be reduced to any combination of absolute terms, nor can a conjugative term be reduced to any combination of simple relatives; but a conjugative having more than two correlates can always be reduced to a combination of conjugatives of two correlates. Thus for "winner over of --, from --, to --," we may always substitute **u**, or "gainer of the advantage -- to --," where the first correlate is itself to be another conjugative **v**, or "the advantage of winning over of -- from --." Then we may write,

$$\mathbf{w} = \mathbf{u} \mathbf{v}.$$

It is evident that in this way all conjugatives may be expressed as production of conjugatives of two correlates.

Peirce: CP 3.145 Cross-Ref:††

145. The interpretation of such combinations as **ba**^m, etc., is not very easy. When the conjugative and its first correlative can be taken together apart from the second correlative, as in (**ba**)_m and (**ba**)^m and (**ba**)_m and (**ba**)^m, there is no perplexity, because in such cases (**ba**) or (**ba**) is a simple relative. We have, therefore, only to call the betrayer to an enemy an inimical betrayer, when we have

(**ba**)_m = inimical betrayer of a man = betrayer of a man to an enemy of him,

(**ba**)^m = inimical betrayer of every man = betrayer of every man to an enemy of him.

And we have only to call the betrayer to every enemy an unbounded betrayer, in order to get

(**ba**)_m = unbounded betrayer of a man = betrayer of a man to every enemy of him,

(**ba**)^m = unbounded betrayer of every man = betrayer of every man to every enemy of him. The two terms **ba**^m and **ba**_m are not quite so easily interpreted. Imagine **a** separated into infinitesimal relatives, A[,],A[.,],A[.,,], etc., each of which is relative to but one individual which is m. Then, because all powers of A[,],A[.,],A[.,,], etc., higher than the first, vanish, and because the number of such terms must be [[m,]] we have,

$$\mathbf{a}[m] = (\mathbf{A}[,], \mathbf{A}[,], \mathbf{A}[,], \dots)^m =$$

$$(\mathbf{A}[,],m), (\mathbf{A}[,],m), (\mathbf{A}[,],m), \text{ etc.}$$

or if M' , M'' , M''' , etc., are the individual m 's,

$$a^m = (A[,]M'), (A[,]M''), (A[,]M'''), \text{ etc.}$$

It is evident from this that ba^m is a betrayer to an $A[,]$ of M' , to an $A[,]$ of M'' , to an $A[,]$ of M''' , etc., in short of all men to some enemy of them all. In order to interpret ba^m we have only to take the negative of it. This, by (124), is $(1-b)a^m$, or a non-betrayer of all men to some enemy of them. Hence, ba^m , or that which is *not* this, is a betrayer of some man to each enemy of all men. To interpret $b(am)$ we may put it in the form $(1-b)^{(1-a)}m$. This is "non-betrayer of a man to all non-enemies of all men." Now, a non-betrayer of some X to every Y , is the same as a betrayer of all X 's to nothing but what is not Y ; and the negative-of "non-enemy of all men," is "enemy of a man." Thus, $b(am)$ is, "betrayer of all men to nothing but an enemy of a man." To interpret bam we may put it in the form $(1-b)^{(1-a)}m$, which is, "non-betrayer of a man to every non-enemy of him." This is a logical sum of terms, each of which is "non-betrayer of an individual man M to every non-enemy of M ." Each of these terms is the same as "betrayer of M to nothing but an enemy of M ." The sum of them, therefore, which is bam is "betrayer of some man to nothing but an enemy of him." In the same way it is obvious that bam is "betrayer of nothing but a man to nothing but an enemy of him." We have $bam = b(1-a)^{(1-m)}$ or "betrayer of all non-men to a non-enemy of all non-men." This is the same as "that which stands to something which is an enemy of nothing but a man in the relation of betrayer of nothing but men to what is not it." The interpretation of bam is obviously "betrayer of nothing but a man to an enemy of him." It is equally plain that ba^m is "betrayer of no man to anything but an enemy of him," and that bam is "betrayer of nothing but a man to every enemy of him." By putting bam in the form $b(1-a)^{(1-m)}$ we find that it denotes "betrayer of something besides a man to all things which are enemies of nothing but men." When an absolute term is put in place of a , the interpretations are obtained in the same way, with greater facility.

Peirce: CP 3.146 Cross-Ref:††

146. The sign of an operation is plainly a conjugative term. Thus, our commutative multiplication might be denoted by the conjugative

$l,$.

For we have

$$l,sw = l,l,sw.$$

As conjugatives can all be reduced to conjugatives of two correlates, they might be expressed by an operative sign (for which a Hebrew letter might be used) put between

the symbols for the two correlates. There would often be an advantage in doing this, owing to the intricacy of the usual notation for conjugatives. If these operational signs happened to agree in their properties with any of the signs of algebra, modifications of the algebraic signs might be used in place of Hebrew letters. For instance, if γ were such that

$$\gamma x \gamma y z = \gamma [13] \gamma y z, \dagger 1$$

then, if we were to substitute for γ the operational sign \neg we have

$$x \neg (y \neg z) = (x \neg y) \neg z,$$

which is the expression of the associative principle. So, if

$$\gamma x y = \gamma y x$$

we may write,

$$x \neg y = y \neg x$$

which is the commutative principle. If both these equations held for any conjugative, we might conveniently express it by a modified sign $\dot{+}$. For example, let us consider the conjugative "what is denoted by a term which either denotes -- or else --." For this, the above principles obviously hold, and we may naturally denote it by $\dot{+}$. Then, if p denotes Protestantism, r Romanism, and f what is false,

$$p \dot{+} r \dot{-} f$$

means either all Protestantism or all Romanism is false. In this way it is plain that all hypothetical propositions may be expressed. Moreover, if we suppose any term as "man" (m) to be separated into its individuals, M', M'', M''', etc., then,

$$M' \dot{+} M'' \dot{+} M''' \dot{+} \text{etc.},$$

means "some man." This may very naturally be written

$$'m'$$

and this gives us an improved way of writing a particular proposition; for

$$'x' \prec y$$

seems a simpler way of writing "Some X is Y" than

$$0^{x,y} = 0.$$

Peirce: CP 3.147 Cross-Ref: ††
CONVERSE

147. If we separate *lover* into its elementary relatives, take the reciprocal of each of these, that is, change it from

$$A:B \text{ to } B:A,$$

and sum these reciprocals, we obtain the relative *loved by*. There is no such operation as this in ordinary arithmetic, but if we suppose a science of discrete quantity in quaternion form (a science of equal intervals in space), the sum of the reciprocals of the units of such a quaternion will be the conjugate-quaternion. For this reason, I express the conjugative term "what is related in the way that to -- is --, to the latter" by **K**. The fundamental equations upon which the properties of this term depend are

$$(169) \quad \mathbf{K} \mathbf{K} = 1.$$

$$(170) \quad \text{If } x < yz \text{ then } z \prec (\mathbf{K}y)x,$$

$$\text{or} \quad 1(x.y z) = 1(z, \mathbf{K}y x)$$

We have, also,

$$(171) \quad \mathbf{K}\Sigma = \Sigma\mathbf{K},$$

$$(172) \quad \mathbf{K}\pi = \pi\mathbf{K},$$

where π denotes the product in the reverse order. Other equations will be found in Mr. De Morgan's table, given above.†1

Peirce: CP 3.148 Cross-Ref:††
CONCLUSION

148. If the question is asked, What are the axiomatic principles of this branch of logic, not deducible from others? I reply that whatever rank is assigned to the laws of contradiction and excluded middle belongs equally to the interpretations of all the general equations given under the head of "Application of the Algebraic signs to Logic," together with those relating to backward involution, and the principles expressed by equations (95), (96), (122), (142), (156), (25), (26), (14), (15).

Peirce: CP 3.149 Cross-Ref:††

149. But these axioms are mere substitutes for definitions of the universal logical relations, and so far as these can be defined, all axioms may be dispensed with. The fundamental principles of formal logic are not properly axioms, but definitions and divisions; and the only *facts* which it contains relate to the identity of the conceptions resulting from those processes with certain familiar ones.

Peirce: CP 3.150 Cross-Ref:††
IV

ON THE APPLICATION OF LOGICAL ANALYSIS TO MULTIPLE
ALGEBRA.†1

150. The letters of an algebra express the relation of the product to the multiplicand. Thus, iA expresses the quantity which is related to A in the manner denoted by i . This being the conception of these algebras, for each of them we may imagine another "absolute" algebra, as we may call it, which shall contain letters which can only be products and multiplicands, not multipliers. Let the general expression of the absolute algebra be $aI + bJ + cK + dL + \text{etc.}$ Multiply this by any letter i of the relative algebra, and denote the product by

$$\begin{aligned} & (A[1]a + A[2]b + A[3]c + \text{etc.})I. \\ & + (B[1]a + B[2]b + B[3]c + \text{etc.})J. \\ & + \text{etc.} \end{aligned}$$

Peirce: CP 3.150 Cross-Ref:††

Now we may obviously enlarge the given relative algebra, so that

$$\begin{aligned}
 i &= A[1]i[11] + A[2]i[12] + A[3]i[13] + \text{etc.} \\
 &+ B[1]i[21] + B[2]i[22] + B[3]i[23] + \text{etc.} \\
 &+ \text{etc.}
 \end{aligned}$$

where $i[11]i[12]$ etc., are such that the product of either of them into any letter of the absolute algebra shall equal some letter of that algebra. That there is no self-contradiction involved in this supposition seems axiomatic.†2

Peirce: CP 3.151 Cross-Ref:††

151. In this way each letter of the given algebra is resolved into a sum of terms of the form $a A:B$, a being a scalar, and $A:B$ such that

$$(A:B)(B:C) = A:C.$$

$$(A:B)(C:D) = 0.$$

The actual resolution is usually performed with ease, but in some cases a good deal of ingenuity is required. I have not found the process facilitated by any general rules. I have actually resolved all the Double, Triple, and Quadruple algebras, and all the Quintuple ones, that appeared to present any difficulty. I give a few examples.

$b i[5].\dagger 1$

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	
<i>k</i>	<i>j</i>	0	<i>l</i>	0	0	
<i>j</i>	0	0	0	0	0	
<i>k</i>	<i>j+a l</i>	0	0	0	<i>bj+cl</i>	
<i>l</i>	0	0	0	0	0	
<i>m</i>	<i>aj+b'l</i>	0	<i>cj+d'l</i>	0	0	

$$i = c d'A:B + b'B:C + b'D:E. j = b'c d'A:C.$$

$$k = c d'A:B + a c d'D:B + b'c^2 d'D:F + c d'E:C + b b'c d'A:F.$$

$$l = b'c d'D:C.$$

$$m = a'c d'A:B+b'c'A:E+b'c d'D:B+b'd'D:E+b'c d'D:F+F:C.$$

$b d[5].\dagger 2$

	i	j	k	l	m	
k	j	0	l	0	0	
j	0	0	0	0	0	
k	$j+rl$	0	$i+m$	0	$-j-rl$	
l	0	0	j	0	0	
m	$(r^2-1)j$	0	$-l$	0	$-r^2j$	

$$i = A:D+D:F+B:E+C:F. \quad j = A:F.$$

$$k = r A:B+r B:C+D:E-(1/r)D:F+E:F.$$

$$l = A:E-(1/r)A:F+B:F. \quad m = r^2A:C-A:D-B:E-C:F.$$

$b h[6].\dagger 1$

	i	j	k	l	m	n	
k	i	j	k	l	m		
j	j	k					
k	k						
l	l	ak		k			
m						k	
n	n						

$$i = A:A+B:B+C:C+D:D. \quad l = a A:B+A:D+D:C.$$

$$j = A:B+B:C. \quad m = A:E.$$

$$k = A:C. \qquad n = E:C.$$

	i	j	k	l	m
k	j				
j					
k	l		m		
l					
m					

$$i = A:B+B:C \qquad l = D:C.$$

$$j = A:C. \qquad m = D:F.$$

$$k = D:B+D:E+E:F.$$

Peirce: CP 3.152 Cross-Ref:††
V

NOTE ON GRASSMANN'S CALCULUS OF EXTENSION.†1

152. The last *Mathematische Annalen* contains a paper by H. Grassmann, on the application of his calculus of extension to mechanics.†2

Peirce: CP 3.152 Cross-Ref:††

He adopts the quaternion addition of vectors. But he has two multiplications, internal and external, just as the principles of logic require.

Peirce: CP 3.152 Cross-Ref:††

The *internal* product of two vectors, $v[1]$ and $v[2]$, is simply what is written in quaternions as $--S.v[1]v[2]$. He writes it $[[v[1]|v[2]]]$. So that

$$[[v[1]|v[2]]] = [[v[2]|v[1]]],$$

$$v[2] = (T v)^2.$$

The *external* product of two vectors is the parallelogram they form, account being taken of its plane and the direction of running round it, which is equivalent to its *aspect*. We therefore have:

$$\begin{array}{c} \mathbf{v}[1] \\ [[\mathbf{v}[1]\mathbf{v}[2]]] = \mathbf{v}[1]\mathbf{v}[2] \sin \angle . \mathbf{I} \\ \mathbf{v}[2] \end{array}$$

$$[[\mathbf{v}[1]\mathbf{v}[2]]] = -[[\mathbf{v}[2]\mathbf{v}[1]]], \quad \mathbf{v}^2 = \mathbf{o},$$

where \mathbf{I} is a new unit. This reminds me strongly of what is written in quaternions as $--V(\mathbf{v}[1]\mathbf{v}[2])$. But it is not the same thing in fact, because $[[\mathbf{v}[1]\mathbf{v}[2]]]\mathbf{v}[3]$ is a solid, and therefore a new kind of quantity. In truth, Grassmann has got hold (though he did not say so) of an eight-fold algebra, which may be written in my system as follows:

Three Rectangular Versors†1

$$\begin{aligned} i &= M:A - B:Z + C:Y + X:N \\ j &= M:B - C:X + A:Z + Y:N \\ k &= M:C - A:Y + B:X + Z:N \end{aligned}$$

Three Rectangular Planes

$$\begin{aligned} I &= M:X + A:N \\ J &= M:Y + B:N \\ K &= M:Z + C:N \end{aligned}$$

One Solid

$$V = M:N$$

Unity

$$1 = M:M + A:A + B:B + C:C \\ + N:N + X:X + Y:Y + Z:Z$$

This unity might be omitted.

Peirce: CP 3.153 Cross-Ref:††

153. The recognition †2 of the two multiplications is exceedingly interesting. The system seems to me more suitable to three dimensional space, and also more natural than that of quaternions. The simplification of mechanical formulæ is striking, but not more than quaternions would effect, that I see.

Peirce: CP 3.153 Cross-Ref:††

By means of eight rotations through two-thirds of a circumference, around four symmetrically placed axes, together with unity, all distortions of a particle would be represented linearly. I have therefore thought of the nine-fold algebra thus resulting.

Peirce: CP 3.154 Cross-Ref:††

VI

ON THE ALGEBRA OF LOGIC†1

PART I.†2 SYLLOGISTIC †P1

§1. DERIVATION OF LOGIC

154. In order to gain a dear understanding of the origin of the various signs used in logical algebra and the reasons of the fundamental formulæ, we ought to begin by considering how logic itself arises.

Peirce: CP 3.155 Cross-Ref:††

155. Thinking, as cerebration, is no doubt subject to the general laws of nervous action.

Peirce: CP 3.156 Cross-Ref:††

156. When a group of nerves are stimulated, the ganglions with which the group is most intimately connected on the whole are thrown into an active state, which in turn usually occasions movements of the body. The stimulation continuing, the irritation spreads from ganglion to ganglion (usually increasing meantime). Soon, too, the parts first excited begin to show fatigue; and thus for a double reason the bodily activity is of a changing kind. When the stimulus is withdrawn, the excitement quickly subsides.

Peirce: CP 3.156 Cross-Ref:††

It results from these facts that when a nerve is affected, the reflex action, if it is not at first of the sort to remove the irritation, will change its character again and again until the irritation is removed; and then the action will cease.

Peirce: CP 3.157 Cross-Ref:††

157. Now, all vital processes tend to become easier on repetition. Along whatever path a nervous discharge has once taken place, in that path a new discharge is the more likely to take place.

Peirce: CP 3.157 Cross-Ref:††

Accordingly, when an irritation of the nerves is repeated, all the various actions which have taken place on previous similar occasions are the more likely to take place now, and those are most likely to take place which have most frequently taken place on those previous occasions. Now, the various actions which did not remove the irritation may have previously sometimes been performed and sometimes not; but the action which removes the irritation must have always been performed, because the action must have every time continued until it was performed. Hence, a strong habit of responding to the given irritation in this particular way must quickly be established.

Peirce: CP 3.158 Cross-Ref:††

158. A habit so acquired may be transmitted by inheritance.

Peirce: CP 3.158 Cross-Ref:††

One of the most important of our habits is that one by virtue of which certain classes of stimuli throw us at first, at least, into a purely cerebral activity.

Peirce: CP 3.159 Cross-Ref:††

159. Very often it is not an outward sensation but only a fancy which starts the train of thought. In other words, the irritation instead of being peripheral is visceral. In such a case the activity has for the most part the same character; an inward action removes the inward excitation. A fancied conjuncture leads us to fancy an appropriate line of action. It is found that such events, though no external action takes place, strongly contribute to the formation of habits of really acting in the fancied way when the fancied occasion really arises.†1

Peirce: CP 3.160 Cross-Ref:††

160. A cerebral habit of the highest kind, which will determine what we do in fancy as well as what we do in action, is called a *belief*. The representation to ourselves that we have a specified habit of this kind is called a *judgment*. A belief-habit in its development begins by being vague, special, and meagre; it becomes more precise, general, and full, without limit. The process of this development, so far as it takes place in the imagination, is called *thought*. A judgment is formed; and under the influence of a belief-habit this gives rise to a new judgment, indicating an addition to belief. Such a process is called an *inference*; the antecedent

judgment is called the *premiss*; the consequent judgment, the *conclusion*; the habit of thought, which determined the passage from the one to the other (when formulated as a proposition), the *leading principle*. †P1

Peirce: CP 3.161 Cross-Ref: ††

161. At the same time that this process of inference, or the spontaneous development of belief, is continually going on within us, fresh peripheral excitations are also continually creating new belief-habits. Thus, belief is partly determined by old beliefs and partly by new experience. Is there any law about the mode of the peripheral excitations? The logician maintains that there is, namely, that they are all adapted to an end, that of carrying belief, in the long run, toward certain predestinate conclusions which are the same for all men. This is the faith of the logician. This is the matter of fact, upon which all maxims of reasoning repose. In virtue of this fact, what is to be believed at last is independent of what has been believed hitherto, and therefore has the character of *reality*. Hence, if a given habit, considered as determining an inference, is of such a sort as to tend toward the final result, it is correct; otherwise not. Thus, inferences become divisible into the valid and the invalid; and thus logic takes its reason of existence.

Peirce: CP 3.162 Cross-Ref: ††
§2. SYLLOGISM AND DIALOGISM †1

162. The general type of inference is

P
∴ C,

where ∴ is the sign of illation.

Peirce: CP 3.163 Cross-Ref: ††

163. The passage from the premiss (or set of premisses) P to the conclusion C takes place according to a habit or rule active within us. All the inferences which that habit would determine when once the proper premisses were admitted, form a class. The habit is logically good provided it would never (or in the case of a probable inference, seldom) lead from a true premiss to a false conclusion; otherwise it is logically bad. That is, every possible case of the operation of a good habit would either be one in which the premiss was false or one in which the conclusion would be true; whereas, if a habit of inference is bad, there is a possible case in which the premiss would be true, while the conclusion was false. When we speak of a *possible* case, we conceive that from the general description of cases we have struck out all those kinds which we know how to describe in general terms but which we know never will occur; those that then remain, embracing all whose nonoccurrence we are not certain of, together with all those whose non-occurrence we cannot explain on any general principle, are called possible.

Peirce: CP 3.164 Cross-Ref: ††

164. A habit of inference may be formulated in a proposition which shall state that every proposition c , related in a given general way to any true proposition p , is true. Such a proposition is called the *leading principle* of the class of inferences whose validity it implies. When the inference is first drawn, the leading principle is not present to the mind, but the habit it formulates is active in such a way that, upon contemplating the believed premiss, by a sort of perception the conclusion is judged to be true.†P1 Afterwards, when the inference is subjected to logical criticism, we make a new inference, of which one premiss is that leading principle of the former inference, according to which propositions related to one another in a certain way are fit to be premiss and conclusion of a valid inference, while another premiss is a fact of observation, namely, that the given relation does subsist between the premiss and conclusion of the inference under criticism; whence it is concluded that the inference was valid.

Peirce: CP 3.165 Cross-Ref:††

165. Logic supposes inferences not only to be drawn, but also to be subjected to criticism; and therefore we not only require the form $P \therefore C$ to express an argument, but also a form, $P[i] \text{ -< } C[i]$, to express the truth of its leading principle. Here $P[i]$ denotes any one of the class of premisses, and $C[i]$ the corresponding conclusion. The symbol -< is the copula, and signifies primarily that every state of things in which a proposition of the class $P[i]$ is true is a state of things in which the corresponding propositions of the class $C[i]$ are true. But logic also supposes some inferences to be invalid, and must have a form for denying the leading premiss [*?principle*]. This we shall write $P[i] \sim\text{-<} C[i]$, *a dash over any symbol signifying in our notation the negative of that symbol*.†P1 [Elec. ed.: A tilde (\sim) preceding the symbol signifies the negative in the electronic edition.]

Peirce: CP 3.165 Cross-Ref:††

Thus, the form $P[i] \text{ -<} C[i]$ implies

either, 1, that it is impossible that a premiss of the class $P[i]$ should be true,

or, 2, that every state of things in which $P[i]$ is true is a state of things in which the corresponding $C[i]$ is true.

Peirce: CP 3.165 Cross-Ref:††

The form $P[i] \sim\text{-<} C[i]$ implies

both, 1, that a premiss of the class $P[i]$ is possible,

and, 2, that among the possible cases of the truth of a $P[i]$ there is one in which the corresponding $C[i]$ is not true.

This acceptance of the copula differs from that of other systems of syllogistic in a manner which will be explained below in treating of the negative.

Peirce: CP 3.166 Cross-Ref:††

166. In the form of inference $P \therefore C$ the leading principle is not expressed; and the inference might be justified on several separate principles. One of these, however, $P[i] \rightarrow C[i]$, is the formulation of the habit which, in point of fact, has governed the inferences. This principle contains all that is necessary besides the premiss P to justify the conclusion. (It will generally assert more than is necessary.) We may, therefore, construct a new argument which shall have for its premisses the two propositions P and $P[i] \rightarrow C[i]$ taken together, and for its conclusion, C . This argument, no doubt, has, like every other, its leading principle, because the inference is governed by some habit; but yet the substance of the leading principle must already be contained implicitly in the premisses, because the proposition $P[i] \rightarrow C[i]$ contains by hypothesis all that is requisite to justify the inference of C from P . Such a leading principle, which contains no fact not implied or observable in the premisses, is termed a *logical* principle, and the argument it governs is termed a *complete*, in contradistinction to an *incomplete*, argument, or *enthymeme*.

Peirce: CP 3.166 Cross-Ref:††

The above will be made clear by an example. Let us begin with the enthymeme,

Enoch was a man,
 \therefore Enoch died.

The leading principle of this is, "All men die." Stating it, we get the complete argument,

All men die,
Enoch was a man;
 \therefore Enoch was to die.

The leading principle of this is *nota notae est nota rei ipsius*. Stating this as a premiss, we have the argument,

Nota notae est nota rei ipsius,
Mortality is a mark of humanity, which is a mark of Enoch;
 \therefore Mortality is a mark of Enoch.

But this very same principle of the *nota notae* is again active in the drawing of this last inference, so that the last state of the argument is no more complete than the last but one.

Peirce: CP 3.167 Cross-Ref:††

167. There is another way of rendering an argument complete, namely, instead of adding the leading principle $P[i] \rightarrow C[i]$ conjunctively to the premiss P , to form a new argument, we might add its denial disjunctively to the conclusion; thus,

P

\therefore Either C or $P[i] \sim \rightarrow C[i]$.

Peirce: CP 3.168 Cross-Ref:††

168. A logical principle is said to be an *empty* or merely formal proposition, because it can add nothing to the premisses of the argument it governs, although it is relevant; so that it implies no fact except such as is presupposed in all discourse, as we have seen in §1 that certain facts are implied. We may here distinguish between *logical* and *extralogical* validity; the former being that of a *complete*, the latter that of an *incomplete* argument. The term *logical leading principle* we may take to mean the principle which must be supposed true in order to sustain the logical validity of any argument. Such a principle states that among all the states of things which can be supposed without conflict with logical principles, those in which the premiss of the argument would be true would also be cases of the truth of the conclusion. Nothing more than this would be relevant to the *logical leading principle*, which is, therefore, perfectly determinate and not vague, as we have seen an extralogical leading principle to be.

Peirce: CP 3.169 Cross-Ref:††

169. A complete argument, with only one premiss, is called an *immediate* inference. *Example:* All crows are black birds; therefore, all crows are birds. If from the premiss of such an argument everything redundant is omitted, the state of things expressed in the premiss is the same as the state of things expressed in the conclusion, and only the form of expression is changed. Now, the logician does not undertake to enumerate all the ways of expressing facts: he supposes the facts to be already expressed in certain standard or canonical forms. But the equivalence between different ones of his own standard forms is of the highest importance to him, and thus certain immediate inferences play the great part in formal logic. Some of these will not be reciprocal inferences or logical equations, but the most important of them will have that character.

Peirce: CP 3.170 Cross-Ref:††

170. If one fact has such a relation to a different one that, if the former be true, the latter is necessarily or probably true, this relation constitutes a determinate fact; and therefore, since the leading principle of a complete argument involves no matter of fact (beyond those employed in all discourse), it follows that every complete and *material* (in opposition to a merely *formal*) argument must have at least two premisses.

Peirce: CP 3.171 Cross-Ref:††

171. From the doctrine of the leading principle it appears that if we have a valid and complete argument from more than one premiss, we may suppress all premisses but one and still have a valid but incomplete argument. This argument is justified by the suppressed premisses; hence, from these premisses alone we may infer that the conclusion would follow from the remaining premisses. In this way, then, the original argument

P Q R S T

∴ C

is broken up into two, namely, 1st,

P Q R S

∴ T -< C

and, 2d,

T -< C

T

∴ C.

By repeating this process, any argument may be broken up into arguments of two premisses each. A complete argument having two premisses is called a *syllogism*.†P1

Peirce: CP 3.172 Cross-Ref:††

172. An argument may also be broken up in a different way by substituting for the second constituent above, the form

T -< C

∴ Either C or not T.

In this way, any argument may be resolved into arguments, each of which has one premiss and two alternative conclusions. Such an argument, when complete, may be called a *dialogism*.

Peirce: CP 3.173 Cross-Ref:††
§3. FORMS OF PROPOSITIONS

173. In place of the two expressions $A \prec B$ and $B \prec A$ taken together we may write $A = B$;†P2 in place of the two expressions $A \prec B$ and $B \sim\prec A$ taken together we may write $A < B$ or $B > A$; and in place of the two expressions $A \sim\prec B$ and $B \sim\prec A$ taken together [disjunctively] we may write $A \sim = B$.†1

Peirce: CP 3.174 Cross-Ref:††

174. De Morgan, in the remarkable memoir with which he opened his discussion of the syllogism (1846, p. 380,†2) has pointed out that we often carry on reasoning under an implied restriction as to what we shall consider as possible, which restriction, applying to the whole of what is said, need not be expressed. The total of all that we consider possible is called the *universe* of discourse, and may be very limited. One mode of limiting our universe is by considering only what actually occurs, so that everything which does not occur is regarded as impossible.

Peirce: CP 3.175 Cross-Ref:††

175. The forms $A \prec B$, or A implies B , and $A \sim\prec B$, or A does not imply B †3, embrace both hypothetical and categorical propositions. Thus, to say that all men are mortal is the same as to say that if any man possesses any character whatever then a mortal possesses that character. To say, 'if A , then B ' is obviously the same as to say that from A , B follows, logically or extralogically. By thus identifying the relation expressed by the copula with that of illation, we identify the proposition with the inference, and the term with the proposition. This identification, by means of which all that is found true of term, proposition, or inference is at once known to be true of all three, is a most important engine of reasoning, which we have gained by beginning with a consideration of the genesis of logic.†P1

Peirce: CP 3.176 Cross-Ref:††

176. Of the two forms $A \prec B$ and $A \sim\prec B$, no doubt the former is the more primitive, in the sense that it is involved in the idea of reasoning, while the latter is only required in the criticism of reasoning. The two kinds of proposition are essentially different, and every attempt to reduce the latter to a special case of the former must fail. Boole †1 attempts to express 'some men are not mortal,' in the form 'whatever men have a certain unknown character v are not mortal.' But the propositions are not identical, for the latter does not imply that some men have that character v ; and, accordingly, from Boole's proposition we may legitimately infer that 'whatever mortals have the unknown character v are not men';†2 yet we cannot reason from 'some men are not mortal' to 'some mortals are not men.'†P2 On the other hand, we can rise to a more general form under which $A \prec B$ and $A \sim\prec B$ are both included. For this purpose we write $A \sim\prec B$ in the form $\$A \prec \sim B$,†3 where $\$A$ is *some*- A and $\sim B$ is *not*- B . This more general form is equivocal in so far as it is left undetermined whether the proposition would be true if the subject were impossible. When the subject is general this is the case, but when the subject is particular (i.e., is subject to the modification *some*) it is not.†1 The general form supposes merely inclusion of the subject under the predicate. The short curved mark over the letter in the subject shows that some part of the term denoted by that letter is the subject, and that that is asserted to be in possible existence.

Peirce: CP 3.177 Cross-Ref:††

177. The modification of the subject by the curved mark and of the predicate by the straight mark gives the old set of propositional forms, viz.:

A. $a \text{ -< } b$ Every a is b . Universal affirmative.

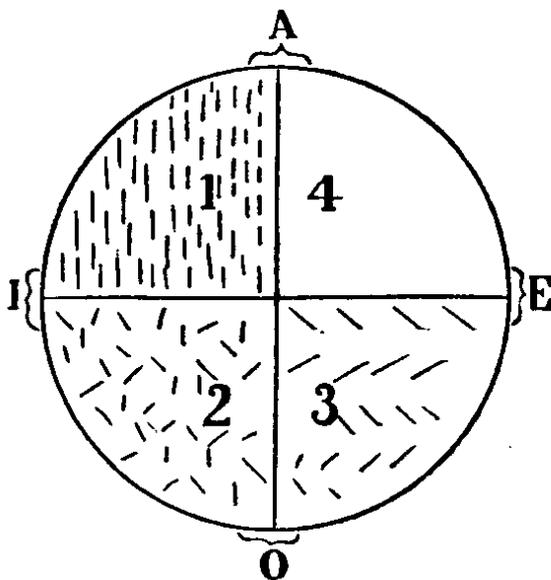
E. $a \text{ -< } \sim b$ No a is b . Universal negative.

I. $\$a \text{ -< } b$ Some a is b . Particular affirmative.

O. $\$a \text{ -< } \sim b$ Some a is not b . Particular negative.

Peirce: CP 3.178 Cross-Ref:††

178. There is, however, a difference between the senses in which these propositions are here taken and those which are traditional; namely, it is usually understood that affirmative propositions imply the existence of their subjects, while negative ones do not. Accordingly, it is said that there is an immediate inference from A to I and from E to O. But in the sense assumed in this paper, universal propositions do not, while particular propositions do, imply the existence of their subjects. The following figure illustrates the precise sense here assigned to the four forms A, E, I, O.



[Click here to view]

Peirce: CP 3.179 Cross-Ref:††

179. In the quadrant marked 1 there are lines which are all vertical; in the quadrant marked 2 some lines are vertical and some not; in quadrant 3 there are lines none of which are vertical; and in quadrant 4 there are no lines. Now, taking *line* as subject and *vertical* as predicate,

A is true of quadrants 1 and 4 and false of 2 and 3.

E is true of quadrants 3 and 4 and false of 1 and 2.

I is true of quadrants 1 and 2 and false of 3 and 4.

O is true of quadrants 2 and 3 and false of 1 and 4.

Hence, A and O precisely deny each other, and so do E and I. But any other pair of propositions may be either both true or both false or one true while the other is false.†1

Peirce: CP 3.180 Cross-Ref:††

180. De Morgan ("On the Syllogism," No. I., 1846, p. 381) has enlarged the system of propositional forms by applying the sign of negation which first appears in $A \sim\prec B$ to the subject and predicate. He thus gets

$A \prec B$. Every A is B.†2 A is species of B.†3

$A \sim\prec B$. Some A is not B. A is exient of B.

$A \prec \sim B$. No A is B. A is external of B.

$A \sim\prec \sim B$. Some A is B. A is partient of B.

$\sim A \prec B$. Everything is either A or B. A is complement of B.

$\sim A \sim\prec B$. There is something besides A and B. A is coinadequate of B.

$\sim A \prec \sim B$. A includes all B. A is genus of B.

$\sim A \sim\prec \sim B$. A does not include all B. A is deficient of B.

De Morgan's table of the relations of these propositions must be modified to conform to the meanings here attached to \prec and to $\sim\prec$.

Peirce: CP 3.181 Cross-Ref:††

181. We might confine ourselves to the two propositional forms $S \prec P$ and $S \sim\prec P$. If we once go beyond this and adopt the form $S \prec \sim P$, we must, for the sake of completeness, adopt the whole of De Morgan's system. But this system, as we shall see in the next section, is itself incomplete, and requires to complete it the admission of particularity in the predicate. This has already been attempted by Hamilton, with an incompetence which ought to be extraordinary.†4 I shall allude to this matter further on, but I shall not attempt to say how many forms of propositions there would be in the completed system.†P1

Peirce: CP 3.182 Cross-Ref:††
 §4. THE ALGEBRA OF THE COPULA

182. From the identity of the relation expressed by the copula with that of illation, springs an algebra. In the first place, this gives us

$$\mathbf{x} \text{ -< } \mathbf{x} \quad (1)$$

the principle of identity, which is thus seen to express that what we have hitherto believed we continue to believe, in the absence of any reason to the contrary. In the next place, this identification shows that the two inferences

$$\begin{array}{l} \mathbf{x} \\ \mathbf{y} \text{ and } \mathbf{x} \quad (2) \\ \therefore \mathbf{z} \quad \therefore \mathbf{y} \text{ -< } \mathbf{z} \end{array}$$

are of the same validity. Hence we have

$$\{\mathbf{x} \text{ -< } (\mathbf{y} \text{ -< } \mathbf{z})\} = \{\mathbf{y} \text{ -< } (\mathbf{x} \text{ -< } \mathbf{z})\}. \dagger\text{P2} \quad (3)$$

Peirce: CP 3.183 Cross-Ref:††

183. From (1) we have

$$(\mathbf{x} \text{ -< } \mathbf{y}) \text{ -< } (\mathbf{x} \text{ -< } \mathbf{y}),$$

whence by (2)

$$\begin{array}{l} \mathbf{x} \text{ -< } \mathbf{y} \quad \mathbf{x} \\ \therefore \mathbf{y} \quad (4) \end{array}$$

is a valid inference.

Peirce: CP 3.184 Cross-Ref:††

184. By (4), if x and $x \prec y$ are true y is true; and if y and $y \prec z$ are true z is true. Hence, the inference is valid

$$x \quad x \prec y \quad y \prec z$$

$$\therefore z.$$

By the principle of (2) this is the same as to say that

$$x \prec y \quad y \prec z$$

$$\therefore x \prec z \quad (5)$$

is a valid inference. This is the canonical form of the syllogism *Barbara*. The statement of its validity has been called the *dictum de omni*, the *nota notae*, etc.; but it is best regarded, after De Morgan,†P1 as a statement that the relation signified by the copula is a transitive one.†P2 It may also be considered as implying that in place of the subject of a proposition of the form $A \prec B$, any subject of that subject may be substituted, and that in place of its predicate any predicate of that predicate may be substituted.†P3 The same principle may be algebraically conceived as a rule for the elimination of y from the two propositions $x \prec y$ and $y \prec z$.†P4

Peirce: CP 3.185 Cross-Ref:††

185. It is needless to remark that any letters may be substituted for x, y, z ; and that, therefore, $\sim x, \sim y, \sim z$, some or all, may be substituted. Nevertheless, after these purely extrinsic changes have been made, the argument is no longer called *Barbara*, but is said to be some other universal mood of the *first figure*. There are evidently eight such moods.

Peirce: CP 3.186 Cross-Ref:††

186. From (5) we have, by (2), these two forms of valid immediate inference:

$$S \prec P$$

$$\therefore (x \prec S) \prec (x \prec P) \quad (6)$$

and

$$S \prec P$$

$$\therefore (P \rightarrow x) \rightarrow (S \rightarrow x). \quad (7)$$

The latter may be termed the inference of *contraposition*. †1

Peirce: CP 3.187 Cross-Ref: ††

187. From the transitivity of the copula, the following inference is valid:

$$\begin{aligned} (S \rightarrow M) \rightarrow (S \rightarrow P) \\ (S \rightarrow P) \rightarrow x \\ \therefore (S \rightarrow M) \rightarrow x. \end{aligned}$$

But, by (6), from $(M \rightarrow P)$ we can infer the first premiss immediately; hence the inference is valid

$$\begin{aligned} M \rightarrow P \\ (S \rightarrow P) \rightarrow x \quad (8) \\ \therefore (S \rightarrow M) \rightarrow x. \end{aligned}$$

This may be called the *minor indirect syllogism*. The following is an example:

All men are mortal,
 If Enoch and Elijah were mortal, the Bible errs;
 \therefore If Enoch and Elijah were men, the Bible errs.

Peirce: CP 3.188 Cross-Ref: ††

188. Again we may start with this syllogism in *Barbara*

$$\begin{aligned} (M \rightarrow P) \rightarrow (S \rightarrow P), \\ (S \rightarrow P) \rightarrow x; \\ \therefore (M \rightarrow P) \rightarrow x. \end{aligned}$$

But by the principle of contraposition (7), the first premiss immediately follows from $(S \rightarrow M)$, so that we have the inference valid

$$\begin{aligned}
& S \text{ -< } M, \\
& (S \text{ -< } P) \text{ -< } x; \quad (9) \\
\therefore & (M \text{ -< } P) \text{ -< } x.
\end{aligned}$$

This may be called the *major indirect syllogism*.

Example: All patriarchs are men,

If all patriarchs are mortal, the Bible errs;
 \therefore If all men are mortal, the Bible errs.

Peirce: CP 3.189 Cross-Ref:††

189. In the same way it might be shown that (6) justifies the syllogism

$$\begin{aligned}
& M \text{ -< } P, \\
& x \text{ -< } (S \text{ -< } M); \quad (10) \\
\therefore & x \text{ -< } (S \text{ -< } P).
\end{aligned}$$

And (7) justifies the inference

$$\begin{aligned}
& S \text{ -< } M, \\
& x \text{ -< } (M \text{ -< } P); \quad (11) \\
\therefore & x \text{ -< } (S \text{ -< } P).
\end{aligned}$$

But these are only slight modifications of *Barbara*.

Peirce: CP 3.190 Cross-Ref:††

190. In the form (10), x may denote a limited universe comprehending some cases of S . Then we have the syllogism

$$\begin{aligned}
& M \text{ -< } P, \\
& S \sim\text{-<} \sim M; \quad (12) \\
\therefore & S \sim\text{-<} \sim P.
\end{aligned}$$

This is called *Darii*. A line might, of course, be drawn over the S. So, in the form (11), x may denote a limited universe comprehending some P. Then we have the syllogism

$$\begin{aligned} S &-< M, \\ \sim M &\sim-< P; & (13) \\ \therefore \sim S &\sim-< P. \end{aligned}$$

Here a line might be drawn over the P. But the forms (12) and (13) are deduced from (10) and (11) only by principles of interpretation which require demonstration.

Peirce: CP 3.191 Cross-Ref:††

191. On the other hand, if in the *minor indirect syllogism* (8), we put "what does not occur" for x , we have by definition

$$\{(S -< P) -< x\} = (S \sim-< P)$$

and we then have

$$\begin{aligned} M &-< P, \\ S &\sim-< P; & (14) \\ \therefore S &\sim-< M, \end{aligned}$$

which is the syllogism *Baroko*. If a line is drawn over P, the syllogism is called *Festino*; and by other negations eight essentially identical forms are obtained, which are called minor-particular moods of the second figure.†P1 In the same way the major indirect syllogism (9) affords the form

$$\begin{aligned} S &-< M, \\ S &\sim-< P; & (15) \\ \therefore M &\sim-< P. \end{aligned}$$

This form is called *Bocardo*. If P is negated, it is called *Disamis*. Other negations give the eight major-particular moods of the third figure.

Peirce: CP 3.192 Cross-Ref:††

192. We have seen that $S \sim\text{-}P$ is of the form $(S \text{-} P) \text{-} x$. Put A for $S \text{-} P$, and we find that $\sim A$ is of the form $A \text{-} x$. Then the principle of contraposition (7) gives the immediate inference

$$\begin{aligned} S \text{-} P & \quad (16) \\ \therefore \sim P \text{-} \sim S. \end{aligned}$$

Applying this to the universal moods of the first figure justifies six moods. These are two in the second figure,

$$\begin{aligned} x \text{-} \sim y \quad z \text{-} y & \quad \therefore x \text{-} \sim z \quad (\textit{Camestres}) \\ \sim x \text{-} \sim y \quad z \text{-} y & \quad \therefore \sim x \text{-} \sim z; \end{aligned}$$

two in the third figure,

$$\begin{aligned} y \text{-} x \quad \sim y \text{-} z & \quad \therefore \sim x \text{-} z \\ y \text{-} x \quad \sim y \text{-} \sim z & \quad \therefore \sim x \text{-} \sim z; \end{aligned}$$

and two others which are said to be in the fourth figure,

$$\begin{aligned} x \text{-} y \quad y \text{-} z & \quad \therefore \sim z \text{-} \sim x \\ x \text{-} \sim y \quad \sim y \text{-} z & \quad \therefore \sim z \text{-} \sim x. \end{aligned}$$

But the negative has two other properties not yet taken into account. These are

$$x \text{-} \sim x \quad (17)$$

or x is not not- X , which is called the *principle of contradiction*; and

$$\sim\sim x \text{-} x \quad (18)$$

or what is not not-X is x , which is called the *principle of excluded middle*.†1

Peirce: CP 3.193 Cross-Ref:††

193. By (17) and (16) we have the immediate inference

$$S \text{ -< } \sim P \quad (19)$$

$$\therefore P \text{ -< } \sim S$$

which is called the conversion of E. By (18) and (16) we have

$$\sim S \text{ -< } P \quad (20)$$

$$\therefore \sim P \text{ -< } S.$$

By (17), (18), and (16), we have

$$\sim S \text{ -< } \sim P \quad (21)$$

$$\therefore P \text{ -< } S.$$

Each of the inferences (19), (20), (21), justifies six universal syllogisms; namely, two in each of the figures, second, third, and fourth. The result is that each of these figures has eight universal moods; two depending only on the principle that $\sim A$ is of the form $A \text{ -< } x$,†P1 two depending also on the principle of contradiction, two on the principle of excluded middle, and two on all three principles conjoined.

Peirce: CP 3.194 Cross-Ref:††

194. The same formulæ (16), (19), (20), (21), applied to the minor-particular moods of the second figure, will give eight minor-particular moods of the first figure; and applied to the major-particular moods of the third figure, will give eight major-particular moods of the first figure.†P2

Peirce: CP 3.195 Cross-Ref:††

195. The principle of contradiction in the form (19) may be further transformed thus:

$$\text{If } (P \therefore \sim C) \text{ is valid, then } (C \therefore \sim P) \text{ is valid.} \dagger 1 \quad (22)$$

Applying this to the minor-particular moods of the first figure, will give eight minor-particular moods of the third figure; and applying it to the major-particular moods of the first figure will give eight major-particular moods of the second figure.

Peirce: CP 3.195 Cross-Ref:††

It is very noticeable that the corresponding formula,

If $(\sim P \therefore C)$ is valid, then $(\sim C \therefore P)$ is valid,†1 (23)

has no application in the existing syllogistic, because there are no syllogisms having a particular premiss and universal conclusion. In the same way, in the Aristotelian system an affirmative conclusion cannot be drawn from negative premisses, the reason being that negation is only applied to the predicate. So in De Morgan's system the subject only is made particular, not the predicate.

Peirce: CP 3.196 Cross-Ref:††

196. In order to develop a system of propositions in which the predicate shall be modified in the same way in which the subject is modified in particular propositions, we should consider that to say $S \prec P$ is the same as to say $(S \sim\prec x) \prec (P \sim\prec x)$, whatever x may be. That

$$(S \prec P) \prec \{(S \sim\prec x) \prec (P \sim\prec x)\}$$

follows at once from *Bokardo* (15) by means of (2). Moreover, since $\sim A$ may be put in the form $A \prec x$, it follows that $\sim\sim A$ may be put in the form $A \sim\prec x$, so that by the principles of contradiction and excluded middle, A may be put in the form $A \sim\prec x$. On the other hand, to say $S \sim\prec \sim P$ is the same as to say $(S \prec \sim x) \prec (P \sim\prec x)$, whatever x may be; for

$$(S \sim\prec \sim P) \prec \{(S \prec \sim x) \prec (P \sim\prec x)\}$$

is the principle of *Ferison*, a valid syllogism of the third figure; and if for x we put $\sim S$, we have

$$(S \prec \sim\sim S) \prec (P \sim\prec \sim S),$$

which is the same as to say that $P \sim\prec \sim S$ is true if the principle of contradiction is true. So that it follows that $P \sim\prec \sim S$ is $S \sim\prec \sim P$ from the principle of contradiction. Comparing

$$S \prec P \text{ or } (S \sim\prec x) \prec (P \sim\prec x)$$

with

$$S \sim\sim\sim P \text{ or } (S \sim\sim\sim x) \sim\sim\sim (P \sim\sim\sim x),$$

we see that they differ by a modification of the subject. Denoting this by a short curve over the subject, we may write $\$S \sim\sim\sim P$ for $S \sim\sim\sim \sim P$. We see then that while for A we may write $A \sim\sim\sim x$, where x is anything whatever, so for $\$A$ we may write $A \sim\sim\sim x$. If we attach a similar modification to the predicate also, we have

$$\$S \sim\sim\sim \$P \text{ or } (S \sim\sim\sim x) \sim\sim\sim (P \sim\sim\sim x),$$

which is the same as to say that you can find an S which is any P you please. We thus have

$$(S \sim\sim\sim P) \sim\sim\sim (\$P \sim\sim\sim \$S), \quad (24)$$

a formula of contraposition, similar to (16).

Peirce: CP 3.196 Cross-Ref:††
It is obvious that

$$(\$S \sim\sim\sim P) \sim\sim\sim (\$P \sim\sim\sim S); \quad (25)$$

for, negating both propositions, this becomes, by (16),

$$(P \sim\sim\sim \sim S) \sim\sim\sim (S \sim\sim\sim \sim P),$$

which is (19). The inference justified by (25) is called the conversion of I. From (25) we infer

$$\$x \sim\sim\sim x, \dagger 1 \quad (26)$$

which may be called the principle of particularity. This is obviously true, because the modification of particularity only consists in changing $(A \sim\sim\sim x)$ to $(A \sim\sim\sim \sim x)$, which is the same as negating the copula and predicate, and a repetition of this will evidently give the first expression again. For the same reason we have

$$x \prec \$x, \dagger 1 \quad (27)$$

which may be called the principle of individuality. This gives

$$(S \prec \$P) \prec (P \prec \$S), \quad (28)$$

and (26) and (27) together give

$$(\$S \prec \$P) \prec (P \prec S). \quad (29)$$

Peirce: CP 3.196 Cross-Ref: ††

It is doubtful whether the proposition $S \prec \$P$ ought to be interpreted as signifying that S and P are one sole individual, or that there is something besides S and P . I here leave this branch of the subject in an unfinished state.

Peirce: CP 3.197 Cross-Ref: ††

197. Corresponding to the formulæ which we have obtained by the principle (2) are an equal number obtained by the following principle:

Peirce: CP 3.197 Cross-Ref: ††

(2') The inference

$$x$$

∴ Either y or z

has the same validity as

$$x \sim\prec y$$

∴ z

[Add the formula (3') $\{(x \sim\prec y) \sim\prec z\} = \{(x \sim\prec z) \sim\prec y\}$.

--1880.]

Peirce: CP 3.197 Cross-Ref: ††

From (1) we have

$$(x \sim\sim\langle y) \sim\langle (x \sim\sim\langle y),$$

whence, by (2),

$$(4') \quad x \\ \therefore \text{Either } (x \sim\langle y) \text{ or } y.\dagger 1$$

This gives

$$x \\ \therefore \text{Either } x \sim\sim\langle y \text{ or } y \sim\sim\langle z \text{ or } z.$$

Then, by (2),

$$x \sim\sim\langle z \\ (5') \quad \therefore x \sim\sim\langle y \text{ or } y \sim\sim\langle z,$$

which is the canonical form of dialogism. The minor indirect dialogism is

$$(8') \quad x \sim\sim\langle (M \sim\sim\langle P) \\ \therefore \text{Either } x \sim\sim\langle (S \sim\sim\langle P) \text{ or } S \sim\sim\langle M.$$

The major indirect dialogism is

$$x \sim\sim\langle (S \sim\sim\langle M) \\ \therefore \text{Either } x \sim\sim\langle (S \sim\sim\langle P) \text{ or } M \sim\sim\langle P.$$

We have also

(12') $(S \sim\sim P) \sim\sim x$

\therefore Either $(S \sim\sim M)$ or $(M \sim\sim P) \sim\sim x$

and

(13') $(S \sim\sim P) \sim\sim x$

\therefore Either $(M \sim\sim P)$ or $(S \sim\sim M) \sim\sim x$.

We have A of the form $x \sim\sim \sim A$. And we have the inferences

$S \sim\sim P \quad S \sim\sim \sim P \quad \sim S \sim\sim P \quad \sim S \sim\sim \sim P$

$\therefore \sim P \sim\sim \sim S. \therefore P \sim\sim \sim S. \therefore \sim P \sim\sim S. \therefore P \sim\sim S.$

Peirce: CP 3.198 Cross-Ref:††
PART II. THE LOGIC OF NON-RELATIVE TERMS

§1. THE INTERNAL MULTIPLICATION AND THE ADDITION OF LOGIC

198. We have seen that the inference

x and y

$\therefore z$

is of the same validity with the inference

x

\therefore Either $\sim y$ or z

and the inference x

\therefore Either y or z ,

with the inference

$$\begin{aligned} & \mathbf{x} \text{ and } \sim\mathbf{y} \\ & \therefore \mathbf{z}. \end{aligned}$$

Peirce: CP 3.198 Cross-Ref:††

In like manner, $\mathbf{x} \prec \mathbf{y}$

is equivalent to

(The possible) \prec Either $\sim\mathbf{x}$ or \mathbf{y} ,

and to

\mathbf{x} which is $\sim\mathbf{y} \prec$ (The impossible).

To express this algebraically, we need, in the first place, symbols for the two terms of second intention, the possible and the impossible. Let ∞ and 0 be the terms; then we have the definitions

$$\mathbf{x} \prec \infty \quad 0 \prec \mathbf{x} \quad (1)$$

whatever \mathbf{x} may be.†P1

Peirce: CP 3.199 Cross-Ref:††

199. We need also two operations which may be called non-relative addition and multiplication. They are defined as follows:†P2

If $\mathbf{a} \prec \mathbf{x}$ and $\mathbf{b} \prec \mathbf{x}$,	If $\mathbf{x} \prec \mathbf{a}$ and $\mathbf{x} \prec \mathbf{b}$, (2)
then $\mathbf{a} + \mathbf{b} \prec \mathbf{x}$;	then $\mathbf{x} \prec \mathbf{a} \times \mathbf{b}$;
and conversely,	and conversely,
if $\mathbf{a} + \mathbf{b} \prec \mathbf{x}$,	if $\mathbf{x} \prec \mathbf{a} \times \mathbf{b}$, (3)
then $\mathbf{a} \prec \mathbf{x}$ and $\mathbf{b} \prec \mathbf{x}$.	then $\mathbf{x} \prec \mathbf{a}$ and $\mathbf{x} \prec \mathbf{b}$.

Peirce: CP 3.200 Cross-Ref:††

200. From these definitions we at once deduce the following formulæ:

A.†P1 $a \prec a + b \quad a X b \prec a$ (Peirce, 1870)†P2 (4)

$$b \prec a + b \quad a X b \prec b.$$

These are proved by substituting $a + b$ and $a X b$ for x in (3).

B. $x = x + x \quad x X x = x$ (Jevons, 1864). (5)

By substituting x for a and b in (2), we get

$$x + x \prec x \quad x \prec x X x;$$

and, by (4),

$$x \prec x + x \quad x X x \prec x.$$

C. $a + b = b + a \quad a X b = b X a$ (Boole, Jevons). (6)

These formulæ are examples of the *commutative principle*. From (4) and (2),

$$b + a \prec a + b \quad a X b \prec b X a$$

and interchanging a and b we get the reciprocal inclusion implied in (6).

D. $(a+b)+c = a+(b+c) \quad aX(bXc) = (aXb)Xc$

(Boole, Jevons). (7)

These are cases of the *associative principle*. By (4), $c \prec b+c$ and $bXc \prec c$; also $b+c \prec a+(b+c)$ and $aX(bXc) \prec bXc$; so that $c \prec a+(b+c)$ and $aX(bXc) \prec c$. In the same way, $b \prec a+(b+c)$ and $aX(bXc) \prec b$, and, by (4), $a \prec a+(b+c)$ and $aX(bXc) \prec a$.

Hence, by (2), $a+b \prec a+(b+c)$ and $aX(bXc) \prec aXb$. And, again by (2), $(a+b)+c \prec a+(b+c)$ and $aX(bXc) \prec (aXb)Xc$. In a similar way we should prove the converse propositions to these and so establish (7).

$$E. (a+b)Xc = (aXc)+(bXc) \quad (aXb)+c = (a+c)X(b+c). \dagger P3 \quad (8)$$

These are cases of the *distributive principle*. They are easily proved by (4) and (2), but the proof is too tedious to give. †1

$$F. (a+b)+c = (a+c)+(b+c) \quad (aXb)Xc = (aXc)X(bXc). \quad (9)$$

These are other cases of the distributive principle. They are proved by (5), (6) and (7). These formulæ, which have hitherto escaped notice, are not without interest.

$$G. a+(aXb) = a \quad aX(a+b) = a \quad (\text{Grassmann, Schröder}). \quad (10)$$

By (4), $a \prec a+(aXb)$ $aX(a+b) \prec a$.

Again, by (4), $(aXb) \prec a$ and $a \prec a+b$; hence, by (2)

$$a+(aXb) \prec a \quad a \prec aX(a+b).$$

$$H. \quad (a+b \prec a) = (b \prec aXb). \quad (11)$$

This proposition is a transformation of Schröder's two propositions 21 (p. 25), one of which was given by Grassmann. By

(3)

$$(a+b \prec a) \prec (b \prec a) \quad (b \prec aXb) \prec (b \prec a).$$

Hence, since $b \prec b$, $a \prec a$

we have, by (2),

$$(a+b \prec a) \prec (b \prec aXb) \quad (b \prec aXb) \prec (a+b \prec a).$$

$$(a \prec b)X(x \prec y) \prec (a+x \prec b+y) \quad \text{(Peirce, 1870).}$$

$$I. (a \prec b)X(x \prec y) \prec (aXx \prec bXy) \quad (12)$$

Readily proved from (2) and (4).

$$J. (a \prec b+x)X(aXx \prec b) = (a \prec b). \quad (13)$$

This is a generalization of a theorem by Grassmann. In stating it, he erroneously unites the first two propositions by + instead of X. By (12), (5), and (8),

$$(a \prec b+x) \prec \{a \prec (aXb)+(aXx)\}$$

$$(aXx \prec b) \prec \{(a+b)X(x+b) \prec b\}.$$

But by (4)

$$a \prec a+b \quad aXb \prec b.$$

Hence, by (2), it is doubly proved that

$$(a \prec b+x)X(aXx \prec b) \prec (a \prec b).$$

The demonstration of the converse is obvious.

We have immediately, from (2) and (3),

$$K. (a+b \prec c) = (a \prec c)X(b \prec c)$$

$$(c \prec aXb) = (c \prec a)X(c \prec b) \quad (14)$$

$$L. (c \prec a+b) = \Sigma\{(p \prec a)X(q \prec b)\} \text{ where } p+q = c$$

$$(aXb \prec c) = \Sigma\{(a \prec p)X(b \prec q)\} \text{ where } c = pXq. \quad (15)$$

Peirce: CP 3.200 Cross-Ref: ††

The propositions (15) are new. By (12)

$$\{(p \prec a)X(q \prec b)\} \prec (c \prec a+b) \text{ where } p+q = c$$

$$\{(a \prec p)X(b \prec q)\} \prec (aXb \prec c) \text{ where } c = pXq.$$

and, since these are true for any set of values of p and q , we have by (2)

$$\Sigma\{(p \prec a)X(q \prec b)\} \prec (c \prec a+b), \text{ where } p+q = c.$$

$$\Sigma\{(a \prec p)X(b \prec q)\} \prec (aXb \prec c), \text{ where } c = pXq.$$

By (4) and (8), we have

$$(c \prec a+b) \prec \{(aXc)+(bXc) = c\}$$

$$(aXb \prec c) \prec \{(c+a)X(c+b) = c\}.$$

Hence, putting

$$aXc = p \quad bXc = q, \quad \text{where } p+q = c$$

$$a+c = p \quad b+c = q, \quad \text{where } pXq = c,$$

we have

$$(c \prec a+b) \prec (p \prec a)X(q \prec b), \text{ where } p+q = c$$

$$(aXb \prec c) \prec (a \prec p)X(b \prec q), \text{ where } c = pXq,$$

whence, by (4)

$$(c \prec a+b) \prec \Sigma\{(p \prec a)X(q \prec b)\} \text{ where } p+q = c$$

$$(aXb \prec c) \prec \Sigma\{(a \prec p)X(b \prec q)\} \text{ where } c = pXq.$$

A formula analogous to (15) will be found below (35).

Peirce: CP 3.201 Cross-Ref:††

201. From (1) and (2) and (4) we have

$$\mathbf{x+0 = x \quad x = xX\infty.} \quad (16)$$

From (1) and (4),

$$\mathbf{x+\infty = \infty \quad 0 = xX0.} \quad (17)$$

The definition of the negative has as we have seen three clauses: first, that $\sim a$ is of the form $a \text{ -< } x$; second, $a \text{ -< } \sim\sim a$; third, $\sim\sim a \text{ -< } a$.

Peirce: CP 3.201 Cross-Ref:††

From the first we have that if

$$\mathbf{c \ a}$$
$$\mathbf{\therefore b}$$

is valid, then

$$\mathbf{c \ \sim b}$$
$$\mathbf{\therefore \sim a}$$

is valid. Or

$$\mathbf{(cXa \text{ -< } b) \text{ -< } (cX\sim b \text{ -< } \sim a).\dagger 1} \quad (18)$$

Also, that if

$$\mathbf{b}$$
$$\mathbf{\therefore \text{ Either } c \text{ or } a}$$

is valid, then

$$\sim a$$

\therefore Either c or $\sim b$

is valid; or

$$(b \text{ -< } c+a) \text{ -< } (\sim a \text{ -< } c+\sim b). \quad (19)$$

Combining (18) and (19), we have

$$(aXb \text{ -< } c+d) = (aX\sim d \text{ -< } c+\sim b). \quad (20)$$

By the principles of contradiction and excluded middle, this gives

$$(aX\sim d \text{ -< } c+\sim b) \text{ -< } (aXb \text{ -< } c+d). \quad (21)$$

Thus the formula

$$(aXb \text{ -< } c+d) = (aX\sim d \text{ -< } c+\sim b) \quad (22)$$

embodies the essence of the negative.

Peirce: CP 3.202 Cross-Ref:††

202. If in (22) we put, first, $a = d$ $b = c = 0$, and then $a = d = \infty$ $b = c$, we have from the formula of identity

$$aX\sim a = 0 \quad a+\sim a = \infty. \quad (23)$$

Peirce: CP 3.202 Cross-Ref:††

We have

$$p = (pXx) + (pX\sim x) \quad p = (p+x)X(p+\sim x), \quad (24)$$

by the distributive principle and (23). If we write

$$i = p+(aX\sim x) \quad j = p+(bXx) \quad k = pX(c+x) \quad l = pX(d+\sim x),$$

we equally have

$$p = (iXx) + (jX\sim x) \quad p = (l+x)X(k+\sim x). \quad (25)$$

Now p may be a function of x , and such values may perhaps be assigned to a, b, c, d , that i, j, k, l , shall be free from x . It is obvious that if the function results from any complication of the operations $+$ and X , this is the case. Supposing, then, i, j, k, l , to be constant, we have, putting successively, ∞ , and 0 , for x .

$$\phi_{\infty} = i = i k$$

$$\phi_0 = j = l$$

so that

$$\phi_x = (\phi_{\infty}Xx) + (\phi_0X\sim x) \quad \phi_x = (\phi_0+x)X(\phi_{\infty}+\sim x). \quad (26)$$

The first of these formulæ was given by Boole for his addition. I showed (1867)[†]1 that both hold for the modified addition. These formulæ are real analogues of mathematical developments; but practically they are not convenient. Their connection suggests the general formula

$$(a+x)X(b+\sim x) = (aX\sim x) + (bXx) \quad (27)$$

a formula of frequent utility.

The distributive principle and (3) applied to (26) give

$$\phi 0 \text{ X } \phi \infty \text{ -< } \phi x \quad \phi x \text{ -< } \phi \infty + \phi 0. \quad (28)$$

Hence

$$(\phi x = 0) \text{ -< } (\phi 0 \text{ X } \phi \infty = 0) \quad (\phi x = \infty) \text{ -< } (\phi 0 + \phi \infty = \infty). \quad (29)$$

Boole gave the former, and I (1867)^{†2} the latter. These formulæ are not convenient for elimination.

Peirce: CP 3.203 Cross-Ref:^{††}

203. The following formulæ (probably given by De Morgan) are of great importance:

$$\sim(a \text{ X } b) = \sim a + \sim b \quad \sim(a + b) = \sim a \text{ X } \sim b. \quad \dagger 3 \quad (30)$$

By (23)

$$(a \text{ X } b) \text{ X } \sim(a \text{ X } b) \text{ -< } 0 \quad \infty \text{ -< } (a + b) + \sim(a + b),$$

whence by (22) and the associative principle

$$\begin{aligned} b \text{ X } \sim(a \text{ X } b) \text{ -< } \sim a \quad \sim a \text{ -< } b + \sim(a + b) \\ \sim(a \text{ X } b) \text{ -< } \sim a + \sim b \quad \sim a \text{ X } \sim b \text{ -< } a + b. \end{aligned}$$

By (4) and (22)

$$\begin{aligned} \sim a \text{ -< } \sim(a \text{ X } b) \quad \sim(a + b) \text{ -< } \sim a \\ \sim b \text{ -< } \sim(a \text{ X } b) \quad \sim(a + b) \text{ -< } \sim b, \end{aligned}$$

whence by (2)

$$\sim a + \sim b \text{ -< } \sim(a + b) \quad \dagger 4 \quad \sim(a + b) \text{ -< } \sim a \text{ X } \sim b.$$

Peirce: CP 3.203 Cross-Ref:††

The application of (22) gives from (11)

$$(b \sim\sim aXb) = (a+b \sim\sim a); \quad (31)$$

from (12)

$$(a+x \sim\sim b+y) \sim\sim (a \sim\sim b)+(x \sim\sim y) \\ (aXx \sim\sim bXy) \sim\sim (a \sim\sim b)+(x \sim\sim y); \quad (32)$$

from (13)

$$(a \sim\sim b) = (a \sim\sim b+x)+(aXx \sim\sim b); \quad (33)$$

from (14)

$$(a+b \sim\sim c) = (a \sim\sim c)+(b \sim\sim c); (c \sim\sim aXb) = \\ (c \sim\sim a)+(c \sim\sim b); \quad (34)$$

from (15)

$$(c \sim\sim a+b) = \pi\{(p \sim\sim a)+(q \sim\sim b)\} \text{ where } p+q = c \\ (aXb \sim\sim c) = \pi\{(a \sim\sim p)+(b \sim\sim q)\} \text{ where } pXq=c; \quad (35)$$

from (22)

$$(aXb \sim\sim c+d) = (aX\sim d \sim\sim c+\sim b). \quad (36)$$

Peirce: CP 3.204 Cross-Ref:††

§2. THE RESOLUTION OF PROBLEMS IN NON-RELATIVE LOGIC

204. Four different algebraic methods of solving problems in the logic of non-relative terms have already been proposed by Boole, Jevons, Schröder, and McColl. I propose here a fifth method which perhaps is simpler and certainly is more natural than any of the others. It involves the following processes:

Peirce: CP 3.205 Cross-Ref:††

205. **First Process.** Express all the premisses with the copulas \rightarrow and $\sim\rightarrow$, remembering that $A = B$ is the same as $A \rightarrow B$ and $B \rightarrow A$.

Peirce: CP 3.206 Cross-Ref:††

206. **Second Process.** Separate every predicate into as many factors and every subject into as many aggregant terms as is possible without increasing the number of different letters used in any subject or predicate.

Peirce: CP 3.207 Cross-Ref:††

207. An expression might be separated into such factors or aggregants (let us term them *prime* factors and *ultimate* aggregants) by one or other of these formulæ:

$$\phi x = (\phi\infty X x) + (\phi 0 X \sim x)$$

$$\phi x = (\phi\infty + \sim x) X (\phi 0 + x)$$

But the easiest method is this. To separate an expression into its

{ ultimate aggregants } take any { product }

{ prime factors } { sum }

of all the different letters of the expression, each taken either positively or negatively (that is, with a dash over it). By means of the fundamental formulæ

$$X X Y \rightarrow Y \rightarrow Y + Z$$

examine whether

the { product } taken is a { subject } of every

{ sum } { predicate }

{ factor } of the given expression. If so, it is a

{ aggregant }

{ultimate aggregant} of that expression; otherwise not.

{ prime factor }

Proceed in thus way until as many {ultimate aggregants}

{ prime factors }

have been found as the expression possesses. This number is found in the case of a {product of sums} of letters, as follows.

{sum of products}

Let m be the number of *different* letters in the expression (a letter and its negative not being considered different); let n

be the total number of letters whether the same or different, and let p be the number of { factors }.

{ terms }

Then the number of {ultimate aggregants} is

{prime factors }

$$2m + n - m p - p.$$

Peirce: CP 3.208 Cross-Ref:††

208. For example, let it be required to separate $x+(yXz)$ into its prime factors. Here $m = 3$, $n = 3$, $p = 2$. Hence the number of factors is three. Trying $x+y+z$, we have

$$x -< x+y+z \quad yXz -< x+y+z,$$

so that this is a factor. Trying $x+y+\sim z$, we have

$$x -< x+y+\sim z \quad yXz -< x+y+\sim z,$$

so that this is also a factor. It is, also, obvious that $x+\sim y+z$ is the third factor. Accordingly,

$$x+(yXz) = (x+y+z)X(x+y+\sim z)X(x+\sim y+z).$$

Again, let us develop the expression

$$(\sim a+b+c)X(a+\sim b+\sim c)X(a+b+c)$$

Here $m = 3$, $n = 9$, $p = 3$; so that the number of ultimate aggregants is five. Of the eight possible products of three letters, then, only three are excluded, namely: $(aX\sim bX\sim c)$, $(\sim aXbXc)$ and $(\sim aX\sim bX\sim c)$. We have, then,

$$\begin{aligned} &(\sim a+b+c)X(a+\sim b+\sim c)X(a+b+c) = \\ &(aXbXc)+(aXbX\sim c)+(aX\sim bXc)+(\sim aXbX\sim c)+(\sim aX\sim bXc). \end{aligned}$$

Peirce: CP 3.209 Cross-Ref:††

209. **Third Process.** Separate all complex propositions into simple ones by means of the following formulæ from the definitions of + and X:

$$(X+Y \text{ -< } Z)\dagger 1 = (X \text{ -< } Z)X(Y \text{ -< } Z)$$

$$(X \text{ -< } Y \text{ X } Z)\dagger 2 = (X \text{ -< } Y)X(X \text{ -< } Z)$$

$$(X+Y \sim\text{-<} Z)\dagger 3 = (X \sim\text{-<} Z)+(Y \sim\text{-<} Z)$$

$$(X \sim\text{-<} Y \text{ X } Z)\dagger 4 = (X \sim\text{-<} Y)+(X \sim\text{-<} Z).$$

In practice, the first three operations will generally be performed off-hand in writing down the premisses.

Peirce: CP 3.210 Cross-Ref:††

210. **Fourth Process.** If we have given two propositions, one of one of the forms

$$a \text{ -< } b+x \quad aX\sim x \text{ -< } b,$$

and the other of one of the forms

$$c \text{ -< } d+\sim x \quad cXx \text{ -< } d,$$

we may, by the transitiveness of the copula, eliminate x , and so obtain

$$aXc \text{ -< } b+d.$$

Peirce: CP 3.211 Cross-Ref:††

211. *Fifth Process.* We may transpose any term from subject to predicate or the reverse, by changing it from positive to negative or the reverse, and at the same time its mode of connection from addition to multiplication or the reverse. Thus,

$$(xXy \text{ -< } z) = (x \text{ -< } \sim y+z).$$

We may, in this way, obtain all the subjects and predicates of any letter; or we may bring all the letters into the subject, leaving the predicate 0, or all into the predicate, leaving the subject ∞ .

Peirce: CP 3.212 Cross-Ref:††

212. *Sixth Process.* Any number of propositions having

a common { subject } are, taken together,

{ predicate }

equivalent to their { product }.

{ sum }

Peirce: CP 3.213 Cross-Ref:††

213. As an example of this method, we may consider a well-known problem given by Boole.†1 The data are

$$\sim xX\sim z \text{ -< } vX(yX\sim w+\sim yXw)$$

$$\sim vXxXw \text{ -< } (yXz)+(\sim yX\sim z)$$

$$(xXy)+(\sim vXxX\sim y) = (zX\sim w)+(\sim zXw).$$

Peirce: CP 3.213 Cross-Ref:††

The quæsitæ are: first, to find those predicates of x which involve only $y, z,$

and w ; second, to find any relations which may be implied between y, z, w ; third, to find the predicates of y ; fourth, to find any relation which may be implied between $x, z,$ and w . By the first three processes, mentally performed, we resolve the premisses as follows: the first into

$$\begin{aligned} \sim xX\sim z & \text{ -< } v \\ \sim xX\sim z & \text{ -< } y+w \\ \sim xX\sim z & \text{ -< } \sim y+\sim w; \end{aligned}$$

the second into

$$\begin{aligned} \sim vXxXw & \text{ -< } y+\sim z \\ \sim vXxXw & \text{ -< } \sim y+z; \end{aligned}$$

the third into

$$\begin{aligned} xXy & \text{ -< } z+w \\ xXy & \text{ -< } \sim z+\sim w \\ vXxX\sim y & \text{ -< } z+w \\ vXxX\sim y & \text{ -< } \sim z+\sim w \\ zX\sim w & \text{ -< } x \\ \sim zXw & \text{ -< } v+y \\ zX\sim w & \text{ -< } x \\ \sim zXw & \text{ -< } v+y. \end{aligned}$$

Peirce: CP 3.213 Cross-Ref:††

We must first eliminate v , about which we want to know nothing. We have, on the one hand, the propositions

$$\begin{aligned} vXxX\sim y & \text{ -< } z+w \\ vXxX\sim y & \text{ -< } \sim z+\sim w; \end{aligned}$$

and, on the other, the propositions

$$\sim xX\sim z \text{ -< } v$$

$$\sim vXxXw \text{ -< } y+\sim z$$

$$\sim vXxXw \text{ -< } \sim y+z$$

$$zX\sim w \text{ -< } v+y$$

$$\sim zXw \text{ -< } v+y.$$

Peirce: CP 3.213 Cross-Ref:††

The conclusions from these propositions are obtained by taking one from each set, multiplying their subjects, adding their predicates, and omitting v . The result will be a merely empty proposition if the same letter in the same quality as to being positive or negative be found in the subject and in the predicate, or if it be found twice with opposite qualities either in the subject or in the predicate. Thus, it will be useless to combine the proposition $vXxX\sim y \text{ -< } z+w$ with any which contains $\sim x$, y , z , or w , in the subject. But all of the second set do this, so that nothing can be concluded from this proposition. So it will be useless to combine $vXxX\sim y \text{ -< } \sim z+\sim w$ with any which contains $\sim x$, y , $\sim z$, $\sim w$ in the subject, or z in the predicate. This excludes every proposition of the second set except $\sim vXxXw \text{ -< } y+\sim z$, which, combined with the proposition under discussion, gives

$$xXw \text{ -< } y+\sim z+\sim w$$

or $xXw \text{ -< } y+\sim z,$

which is therefore to be used in place of all the premisses containing v .

Peirce: CP 3.213 Cross-Ref:††

One of the other propositions, namely, $\sim xX\sim z \text{ -< } \sim y+\sim w$ is obviously contained in another, namely: $\sim zXw \text{ -< } x$. Rejecting it, our premisses are reduced to six, namely:

$$\sim xX\sim z \text{ -< } y+w$$

$$xXy \text{ -< } z+w$$

$$xXy \text{ -< } \sim z+\sim w$$

$$zX\sim w \text{ -< } x$$

$$\sim zXw \text{ -< } x$$

$$xXw \text{ -< } y+\sim z$$

The second, third, and sixth of these give the predicates of x Their product is

$$x \text{ -} < (\sim y+z+w)X(\sim y+\sim z+\sim w)X(y+\sim z+\sim w)$$

or

$$x \text{ -} < yXzX\sim w + yX\sim zXw + \sim yXzX\sim w + \sim yX\sim zXw + \sim yX\sim zX\sim w$$

or

$$x \text{ -} < zX\sim w + \sim zXw + \sim yX\sim zX\sim w.$$

To find whether any relation between y , z , and w can be obtained by the elimination of x , we find the subjects of x by combining the first, fourth, and fifth premisses. Thus we find

$$\sim yX\sim zX\sim w + zX\sim w + \sim zXw \text{ -} < x.$$

It is obvious that the conclusion from the last two propositions is a merely identical proposition, and therefore no independent relation is implied between y , z , and w .

Peirce: CP 3.213 Cross-Ref:††

To find the predicates of y we combine the second and third propositions. This gives

$$y \text{ -} < (\sim x+z+w)X(\sim x+\sim z+\sim w)$$

or $y \text{ -} < xXzX\sim w + xX\sim zXw + \sim x.$

Peirce: CP 3.213 Cross-Ref:††

Two relations between x , z , and w are given in the premisses, namely: $zX\sim w \text{ -} < x$ and $\sim zXw \text{ -} < x$. To find whether any other is implied, we eliminate y between the above proposition and the first and sixth premisses. This gives

$$\sim xX\sim z \text{ -} < xXzX\sim w + w + \sim x$$

$$xXw \text{ -} < xXzX\sim w + \sim x + \sim z.$$

The first conclusion is empty. The second is equivalent to $xXw \prec \sim z$, which is a third relation between x , z , and w .

Peirce: CP 3.213 Cross-Ref:††

Everything implied in the premisses in regard to the relations of x , y , z , w may be summed up in the proposition

$$\infty \prec x+zXw + yX\sim zX\sim w.$$

Peirce: CP 3.214 Cross-Ref:††
PART III. THE LOGIC OF RELATIVES

§1. INDIVIDUAL AND SIMPLE TERMS

214. Just as we had to begin the study of Logical Addition and Multiplication by considering ∞ and 0, terms which might have been introduced under the Algebra of the Copula, being defined in terms of the copula only, without the use of + or X, but which had not been there introduced, because they had no application there, so we have to begin the study of relatives by considering the doctrine of individuals and simples, -- a doctrine which makes use only of the conceptions of nonrelative logic, but which is wholly without use in that part of the subject, while it is the very foundation of the conception of a relative, and the basis of the method of working with the algebra of relatives.

Peirce: CP 3.215 Cross-Ref:††

215. The germ of the correct theory of individuals and simples is to be found in Kant's *Critic of the Pure Reason*, "Appendix to the Transcendental Dialectic," where he lays it down as a regulative principle, that, if

$$a \prec b \quad b \sim \prec a,$$

then it is always possible to find such a term x , that

$$a \prec x \quad x \prec b \\ x \sim \prec a \quad b \sim \prec x.$$

Kant's distinction of regulative and constitutive principles is unsound, but this *law of continuity*,†1 as he calls it, must be accepted as a fact. The proof of it, which I have given elsewhere,†2 depends on the continuity of space, time, and the intensities of the qualities which enter into the definition of any term. If, for instance, we say that

Europe, Asia, Africa and North America are continents, but not all the continents, there remains over only South America. But we may distinguish between South America as it now exists and South America in former geological times; we may, therefore, take x as including Europe, Asia, Africa, North America, and South America as it exists now, and every x is a continent, but not every continent is x .

Peirce: CP 3.216 Cross-Ref:††

216. Just as in mathematics we speak of infinitesimals and infinities, which are fictitious limits of continuous quantity, and every statement involving these expressions has its interpretation in the doctrine of limits,†3 so in logic we may define an *individual*, A , as such a term that

$$A \sim \prec 0,$$

but such that if

$$x < A$$

then $x \prec 0$.

And in the same way, we may define the *simple*, α , as such a term that

$$\infty \sim \prec \alpha,$$

but such that if

$$\alpha < x$$

then $\infty \prec x$.

The individual and the simple, as here defined, are ideal limits, and every statement about either is to be interpreted by the doctrine of limits.

Peirce: CP 3.217 Cross-Ref:††

217. Every term may be conceived as a limitless logical sum of individuals,†1 or as a limitless logical product of simples; thus,

$$\alpha = A[1]+A[2]+A[3]+A[4]+A[5]+ \text{etc.}$$

$$\sim\alpha = \sim A[1]X \sim A[2]X \sim A[3]X \sim A[4]X \sim A[5]X \text{ etc.}$$

It will be seen that a simple is the negative of an individual.

Peirce: CP 3.218 Cross-Ref:††
§2. RELATIVES

218. A *relative* is a term whose definition describes what sort of a system of objects that is whose first member (which is termed the *relate*) is denoted by the term; and names for the other members of the system (which are termed the *correlates*) are usually appended to limit the denotation still further. In these systems the order of the members is essential; so that (A, B, C) and (A, C, B) are different systems. As an example of a relative, take 'buyer of -- for -- from'; we may append to this three correlates, thus, 'buyer of every horse of a certain description in the market for a good price from its owner.'

Peirce: CP 3.219 Cross-Ref:††

219. A relative of only one correlate, so that the system it supposes is a pair, may be called a *dual* relative; a relative of more than one correlate may be called *plural*. A nonrelative term may be called a term of *singular reference*.

Peirce: CP 3.220 Cross-Ref:††

220. Every relative, like every term of singular reference, is general; its definition describes a system in general terms; and, as general, it may be conceived either as a logical sum of individual relatives, or as a logical product of simple relatives.†P1 An individual relative refers to a system all the members of which are individual. The expressions

(A:B) (A:B:C)

may denote individual relatives. Taking dual individual relatives, for instance, we may arrange them all in an infinite block, thus,

A:A A:B A:C A:D A:E etc.

B:A B:B B:C B:D B:E etc.

C:A C:B C:C C:D C:E etc.

D:A D:B D:C D:D D:E etc.

E:A E:B E:C E:D E:E etc.

etc. etc. etc. etc. etc.

In the same way, triple individual relatives may be arranged in a cube, and so forth. The logical sum of all the relatives in this infinite block will be the relative universe, ∞ , where

$$x < \infty,$$

whatever dual relative x may be. It is needless to distinguish the dual universe, the triple universe, etc., because, by adding a perfectly indefinite additional member to the system, a dual relative may be converted into a triple relative, etc. Thus, for *lover of a woman*, we may write *lover of a woman coexisting with anything*. In the same way, a term of single reference is equivalent to a relative with an indefinite correlate; thus, *woman* is equivalent to *woman coexisting with anything*. Thus, we shall have

$$A = A:A+A:B+A:C+A:D+A:E+\text{etc.}$$

$$A:B = A:B:A+A:B:B+A:B:C+A:B:D+\text{etc.}$$

Peirce: CP 3.221 Cross-Ref: ††

221. From the definition of a simple term given in the last section, it follows that every simple relative is the negative of an individual term. But while in non-relative logic negation only divides the universe into two parts, in relative logic the same operation divides the universe into $2n$ parts, where n is the number of objects in the system which the relative supposes; thus,

$$\infty = A + \sim A$$

$$\infty = A:B + \sim A:B + A:\sim B + \sim A:\sim B$$

$$\infty = (A:B:C) + (\sim A:B:C) + (A:\sim B:C) + (A:B:\sim C)$$

$$+ (\sim A:\sim B:\sim C) + (A:\sim B:\sim C) + (\sim A:B:\sim C) + (\sim A:\sim B:C).$$

Here, we have

$$A = A:B + A:\sim B; \sim A = \sim A:B + \sim A:\sim B;$$

$$A:B = A:B:C + A:B:\sim C; A:\sim B = A:\sim B:C + A:\sim B:\sim C;$$

$$\sim A:B = \sim A:B:C + \sim A:B:\sim C; \sim A:\sim B = \sim A:\sim B:C + \sim A:\sim B:\sim C.$$

It will be seen that a term which is individual when considered as n -fold is not so when considered as more than n -fold; but an n -fold term when made $(m+n)$ -fold, is individual as to n members of the system, and indefinite as to m members.

Peirce: CP 3.222 Cross-Ref: ††

222. Instead of considering the system of a relative as consisting of non-relative individuals, we may conceive of it as consisting of relative individuals. Thus, since

$$A = A:A+A:B+A:C+A:D+\text{etc.},$$

we have

$$A:B = (A:A):B+(A:B):B+(A:C):B+(A:D):B+\text{etc.}$$

But

$$B = B:A+B:B+B:C+B:D+\text{etc.};$$

so that

$$A:B = A:(B:A)+A:(B:B)+A:(B:C)+A:(B:D)+\text{etc.} \dagger 1$$

Peirce: CP 3.223 Cross-Ref: ††

§3. RELATIVES CONNECTED BY TRANSPOSITION OF RELATE AND CORRELATE

223. Connected with every dual relative, as

$$l = \Sigma(A:B) = \pi(\alpha:\beta), \dagger 2$$

is another which is called its *converse*,

$$k-l = \Sigma(B:A) = \pi(\beta:\alpha),$$

in which the relate and correlate are transposed. The converse, k , is itself a relative, being

$$k = \Sigma[(A:B):(B:A)];$$

that is, it is the first of any pair which embraces two individual dual relatives, each of which is the converse of the other. The converse of the converse is the relation itself, thus

$$k-k-l = l,$$

or say $k k = 1$.

We have also

$$k-\sim l = \sim(k-l)$$

$$k\Sigma = \Sigma k$$

$$k\pi = \pi k.$$

Peirce: CP 3.224 Cross-Ref: ††

224. In the case of triple relatives there are five transpositions possible. Thus, if

$$b = \Sigma[(A:B):C]$$

we may write

$$Ib = \Sigma[(B:A):C]$$

$$Jb = \Sigma[(A:C):B]$$

$$Kb = \Sigma[(C:B):A]$$

$$Lb = \Sigma[(C:A):B]$$

$$Mb = \Sigma[(B:C):A].$$

Here we have $LM \dagger 1 = ML = 1$

$$II = JJ = KK = 1$$

$$IJ = JK = KI = L$$

$$JI = KJ = IK = M$$

$$IL = MI = J = KM = LK$$

$$JL = MJ = K = IM = LI$$

$$KL = MK = I = JM = LJ.$$

If we write $a:b$ to express the operation of putting A in place of B in the original relative

$$b = \Sigma[(A:B):C]$$

then we have

$$I = a:b+b:a+c:c \dagger 2$$

$$J = a:a+b:c+c:b$$

$$K = a:c+b:b+c:a$$

$$L = a:b+b:c+c:a$$

$$M = a:c+b:a+c:b$$

$$1 = a:a+b:b+c:c.$$

Then we have

$$I+J+K = 1+L+M,$$

which does not imply

$$(I+J+K)l = (1+L+M)l.$$

Peirce: CP 3.224 Cross-Ref: ††

In a similar way the n -fold relative will have $(n! - 1)$ transposition-functions.

Peirce: CP 3.225 Cross-Ref:††
§4. CLASSIFICATION OF RELATIVES

225. Individual relatives are of one or other of the two forms

A:A A:B,

and simple relatives are negatives of one or other of these two forms.

Peirce: CP 3.226 Cross-Ref:††

226. The forms of general relatives are of infinite variety, but the following may be particularly noticed.

Peirce: CP 3.226 Cross-Ref:††

Relatives may be divided into those all whose individual aggregants are of the form A:A and those which contain individuals of the form A:B. The former may be called *concurrents*,†1 the latter *opponents*. Concurrents express a mere agreement among objects. Such, for instance, is the relative '*man that is --*,' and a similar relative may be formed from any term of singular reference. We may denote such a relative by the symbol for the term of singular reference with a comma after it; thus (*m*,) will denote 'man that is --' if (*m*) denotes 'man.' In the same way a comma affixed to an n -fold relative will convert it into an $(n+1)$ -fold relative. Thus, (*l*) being 'lover of --,' (*l*,) will be 'lover that is -- of --.'

Peirce: CP 3.226 Cross-Ref:††

The negative of a concurrent relative will be one each of whose simple components is of the form $\sim(A:A)$, and the negative of an opponent relative will be one which has components of the form $\sim(A:B)$.

Peirce: CP 3.226 Cross-Ref:††

We may also divide relatives into those which contain individual aggregants of the form A: A and those which contain only aggregants of the form A: B. The former may be called *self-relatives*,†2 the latter *alio-relatives*. We also have negatives of self-relatives and negatives of alio-relatives.

Peirce: CP 3.227 Cross-Ref:††

227. These different classes have the following relations. Every negative of a concurrent and every alio-relative is both an opponent and the negative of a self-relative. Every concurrent and every negative of an alio-relative is both a self-relative and the negative of an opponent. There is only one relative which is both a concurrent and the negative of an alio-relative; this is 'identical with --.' There is only one relative which is at once an alio-relative and the negative of a concurrent; this is the negative of the last, namely, 'other than --.'†P1 The following pairs of classes are mutually exclusive, and divide all relatives between them:

Alio-relatives and self-relatives,

Concurrents and opponents,

Negatives of alio-relatives and negatives of self-relatives,

Negatives of concurrents and negatives of opponents.

No relative can be at once either an alio-relative or the negative of a concurrent, and at the same time either a concurrent or the negative of an alio-relative.

Peirce: CP 3.228 Cross-Ref:††

228. We may append to the symbol of any relative a semicolon to convert it into an alio-relative of a higher order. Thus (*l*;) will denote a 'lover of -- that is not --.'

Peirce: CP 3.229 Cross-Ref:††

229. This completes the classification of dual relatives founded on the difference of the fundamental forms A:A and A:B. Similar considerations applied to triple relatives would give rise to a highly complicated development, inasmuch as here we have no less than five fundamental forms of individuals, namely,

(A:A):A (A:A):B (A:B):A (B:A):A (A:B):C.

The number of individual forms for the (*n*+2)-fold relative is

$2+(2n-1) \cdot 3 + 1/2! \{(3n-1) - 2(2n-1)\} \cdot 4 + 1/3! \{(4n-1) - 3(3n-1) + 3(2n-1)\} \cdot 5$
 $+ 1/4! \{(5n-1) - 4(4n-1) + 6(3n-1) - 4(2n-1)\} \cdot 6 + 1/5! \{(6n-1) - 5(5n-1)$
 $+ 10(4n-1) - 10(3n-1) + 5(2n-1)\} \cdot 7 + \text{etc.}$

If this number be called *fn*, we have

$$\{D\}nf_0 = f(n-1)$$

$$f_0 = 1.$$

The form of calculation is

1				
2	1			
5	3	2		
15	10	7	5	
52	37	27	20	15

where the diagonal line is copied number by number from the vertical line, as fast as the latter is computed. †P1

Peirce: CP 3.230 Cross-Ref: ††

230. Relatives may also be classified according to the general amount of filling up of the above-mentioned block, cube, etc., they present. In the first place, we have such relatives in whose block, cube, etc., every line in a certain direction in which there is a single individual is completely filled up. Such relatives may be called *complete in regard to* the relate, or first, second, third, etc., correlate. The dual relatives which are equivalent to terms of singular reference are complete as to their correlate.

Peirce: CP 3.231 Cross-Ref: ††

231. A relative may be incomplete with reference to a certain correlate or to its relate, and yet every individual of the universe may in some way enter into that correlate or relate. Such a relative may be called *unlimited* in reference to the correlate or relate in question. Thus, the relative

A:A+A:B+C:C+C:D+E:E+E:F+G:G+G:H + etc.

is unlimited as to its correlate. The negative of an unlimited relative will be unlimited unless the relative has as an integrant a relative which is complete with regard to every other relate and correlate than that with reference to which the given relative is unlimited.

Peirce: CP 3.232 Cross-Ref: ††

232. A totally unlimited relative is one which is unlimited in reference to the relate and all the correlates. A totally unlimited relative in which each letter enters only once into the relate and once into the correlate is termed a *substitution*.

Peirce: CP 3.233 Cross-Ref: ††

233. Certain classes of relatives are characterized by the occurrence or non-occurrence of certain individual aggregants related in a definite way to others which occur. A set of individual dual relatives each of which has for its relate the correlate of the last, the last of all being considered as preceding the first of all, may be called a *cycle*. If there are n individuals in the cycle it may be called a cycle of the n^{th} order. For instance,

A:B B:C C:D D:E E:A

may be called the cycle of the fifth order. Now, if a certain relative be such that of any cycle of the n^{th} order of which it contains any m terms, it also contains the remaining $(n - m)$ terms, it may be called a cyclic relative of the n^{th} order and m^{th} degree. If, on

the other hand, of any cycle of the n^{th} order of which it contains m terms the remaining $(n - m)$ are wanting, the relative may be called an anticyclic relative of the n^{th} order and m^{th} degree.

Peirce: CP 3.234 Cross-Ref:††

234. A cyclic relative of the first order and first degree contains all individual components of the form A:A. A cyclic relative of the second order and first degree is called an *equiparant*†1 in opposition to a *disquiparant*.

Peirce: CP 3.235 Cross-Ref:††

235. A highly important class of relatives is that of *transitives*; that is to say, those which for every two individual terms of the forms (A:B) and (B:C) also possess a term of the form (A:C).†2

Peirce: CP 3.236 Cross-Ref:††
§5. THE COMPOSITION OF RELATIVES

236. Suppose two relatives either individual or simple, and having the relate or correlate of the first identical with the relate or correlate of the second or of its negative. This pair of relatives will then be of one or other of sixteen forms, viz.:

(A:B)(B:C) ~ (A:B)(B:C) (A:B)~(B:C) ~ (A:B)~(B:C)

(A:B)(C:B) ~ (A:B)(C:B) (A:B)~(C:B) ~ (A:B)~(C:B)

(B:A)(B:C) ~ (B:A)(B:C) (B:A)~(B:C) ~ (B:A)~(B:C)

(B:A)(C:B) ~ (B:A)(C:B) (B:A)~(C:B) ~ (B:A)~(C:B)

Now we may conceive an operation upon any one of these sixteen pairs of relatives of such a nature that it will produce one or other of the four forms (A:C), ~(A:C), (C:A), ~(C:A). Thus, we shall have sixty-four operations in all.

Peirce: CP 3.237 Cross-Ref:††

237. We may symbolize them as follows: Let

$$A:B(\equiv)B:C = A:C;$$

that is, let (\equiv) signify such an operation that this formula necessarily holds. The three lines in the sign of this operation are to refer respectively to the first relative operated upon, the second relative operated upon, and to the result. When either of these lines is replaced by a hyphen (-), let the operation signified be such that the negative of the corresponding relative must be substituted in the above formula. Thus,

$$\sim(A:B)(-||)B:C = A:C.$$

In the same way, let a semicircle (\$) signify that the converse of the corresponding relative is to be taken. The hyphen and the semicircle may be used together. If, then, we write the symbol of a relative with a semicircle or curve over it to denote the converse of that relative, we shall have, for example,

$$$(A:B)(\$||)B:C = A:C.$$

Peirce: CP 3.238 Cross-Ref:††

238. Then any combination of the relatives *a* and *e*, in this order, is equivalent to others formed from this by making any of the following changes:

Peirce: CP 3.238 Cross-Ref:††

First. Putting a straight or curved mark over *a* and changing the first mark of the sign of operation in the corresponding way; that is,

for $\$a$, from | to \$ or from - to \$- or conversely,

for $\sim a$, from | to - or from \$ to \$- or conversely,

for $\$\sim a$, from | to \$- or from - to \$ or conversely.

Peirce: CP 3.238 Cross-Ref:††

Second. Making similar simultaneous modifications of *e* and of the second mark.

Peirce: CP 3.238 Cross-Ref:††

Third. Changing the third mark from | to - or from \$ to \$- or conversely, and simultaneously writing the mark of negation over the whole expression.

Peirce: CP 3.238 Cross-Ref:††

Fourth. Changing the third mark from | to \$ or from - to \$- or conversely, and interchanging *a* and *e* and also the first and second marks.

Peirce: CP 3.239 Cross-Ref:††

239. We have thus far defined the effect of the sixty-four operations when certain members of the individual relatives operated upon are identical. When these members are not identical, we may suppose either that the operation ||| produces either the first or second relative or 0. We cannot suppose that it produces ∞ for a reason which will appear further on. Let us elect the formula

$$A:B(|||)C:D = 0.$$

Peirce: CP 3.239 Cross-Ref:††

The other excluded operations will be included in a certain manner, as we shall see below. From this formula, by means of the rules of equivalence, it will follow that all operations in whose symbol there is no hyphen in the third place will also give 0 in like circumstances, while all others will give ~0 or ∞.

Peirce: CP 3.240 Cross-Ref:††

240. We have thus far only defined the effect of the sixty-four operations upon individual or simple terms. To extend the definitions to other cases, let us suppose first that the rules of equivalence are generally valid, and second, that

$$\text{If } a \text{ -< } b \text{ and } c \text{ -< } d \text{ then } a(\| \|)c \text{ -< } b(\| \|)d$$

or

$$(a \text{ -< } b)X(c \text{ -< } d) \text{ -< } \{(a(\| \|)c \text{ -< } b(\| \|)d)\}.$$

Then, this rule will hold good in all operations in whose symbols the first and second places agree with the third in respect

to having or not having hyphens. For operations, in whose

symbols the {first/second} mark disagrees with the third in this respect we must write { $b \text{ -< } a/d \text{ -< } c$ } instead of { $a \text{ -< } b/c \text{ -< } d$ }

in this rule. Thus, the sixty-four operations are divisible into four classes according to which one of the four rules so produced they follow.

Peirce: CP 3.241 Cross-Ref:††

241. It now appears that only the hyphens and not the curved marks are of significance in reference to the rule which an operation follows. Let us accordingly reject all operations whose symbols contain curved marks, and there remain only eight. For these eight the following formulæ hold:

$$A:B(\| \|)B:C = A:C \quad A:B(\| -)B:C = \sim(A:C)$$

$$\sim(A:B)(- \|)B:C = A:C \quad \sim(A:B)(- -)B:C = \sim(A:C)$$

$$A:B(- \|)\sim(B:C) = A:C \quad A:B(- -)\sim(B:C) = \sim(A:C)$$

$$\sim(A:B)(--)\sim(B:C) = A:C \quad \sim A:B(---)\sim(B:C) = \sim(A:C)$$

$$A:B(\| \|)C:D = 0 \quad A:B(\| -)C:D = \infty$$

$$\sim(A:B)(- \|)C:D = 0 \quad \sim(A:B)(- -)C:D = \infty$$

$$A:B(- \|)\sim(C:D) = 0 \quad A:B(- -)\sim(C:D) = \infty$$

$$\sim(A:B)(--)\sim(C:D) = 0 \quad \sim(A:B)(---)\sim(C:D) = \infty$$

$$(a \prec b)X(c \prec d) \prec \{a (\|) c \prec b (\|) d\}$$

$$(a \prec b)X(c \prec d) \prec \{a (---) c \prec b (---) d\}$$

$$(b \prec a)X(c \prec d) \prec \{a (-\|) c \prec b (-\|) d\}$$

$$(b \prec a)X(c \prec d) \prec \{a (|--) c \prec b (|--) d\}$$

$$(a \prec b)X(d \prec c) \prec \{a (|-) c \prec b (|-) d\}$$

$$(a \prec b)X(d \prec c) \prec \{a (-|-) c \prec b (-|-) d\}$$

$$(b \prec a)X(d \prec c) \prec \{a (--|) c \prec b (--|) d\}$$

$$(b \prec a)X(d \prec c) \prec \{a (\||-) c \prec b (\||-) d\}$$

Peirce: CP 3.242 Cross-Ref:††

242. As it is inconvenient to consider so many as eight distinct operations, we may reject one-half of these so as to retain one under each of the four rules. We may reject all those whose symbols contain an odd number of hyphens (as being negative). We then retain four, to which we may assign the following names and symbols:

$a (\|) e = a e$ *Relative or external multiplication.*

$a (|--) e = ae$ *Regressive involution.*

$a (-|-) e = ae$ *Progressive involution.*

$a (--|) e = a \bullet e$ *Transaddition.* †P1

Peirce: CP 3.243 Cross-Ref:††

243. We have then the following table of equivalents, negatives, and converses:†P1

x	~x	\$x	\$~x
$a e = \sim a \bullet \sim e$	$\sim a e = a \sim e$	$\$e\$a = \$\sim e\$ \sim a$	$ \sim e\$a = \$e\$ \sim a$
$ae = \sim a \sim e$	$\sim a e = a \bullet \sim e$	$\$e\$a = \$\sim e\$ \sim a$	$ \$e\$ \sim a = \$\sim e\$ a$
$ae = \sim a \sim e$	$a \sim e = \sim a \bullet e$	$\$e\$a = \$\sim e\$ \sim a$	$ \sim e\$a = \$e\$ \sim a$
$a \bullet e = \sim a \sim e$	$a \sim e = \sim a e$	$\$e \bullet a = \$\sim e\$ \sim a$	$ \$e\$ \sim a = \$\sim e\$ a$

Peirce: CP 3.244 Cross-Ref:††

244. If l denote 'lover' and s 'servant,' then

$l[s]$ denotes whatever is lover of a servant of --,

ls whatever is lover of every servant of--,

$l[s]$ whatever is in every way (in which it loves at all) lover of a servant,

$l\bullet[s]$ whatever is not a non-lover only of a servant of --

or whatever is not a lover of everything but servants of --

or whatever is some way a non-lover of some thing besides servants of--.

Peirce: CP 3.245 Cross-Ref:††

§6. METHODS IN THE ALGEBRA OF RELATIVES

245. The universal method in this algebra is the method of limits. For certain letters are to be substituted an infinite sum of individuals or product of simples; whereupon certain transformations become possible which could not otherwise be effected.

Peirce: CP 3.246 Cross-Ref:††

246. The following theorems are indispensable for the application of this method: First $l^{A:B} = l(A:B) + k\sim B$.

Peirce: CP 3.246 Cross-Ref:††

Since $\sim B$ is equivalent to the relative term which comprises all individual relatives whose relates are not B , so $k\sim B$ may be conveniently used, as it is here, to express the aggregate of all individual relatives whose correlate is $\sim B$. To prove this proposition, we observe that

$$l^{A:B} = \sim(\sim l(A:B)).$$

Now $\sim l(A:B)$ contains only individual relatives whose correlate is B , and of these it contains those which are not included in $l(A:B)$. Hence the negative of $\sim l(A:B)$ contains all individual relatives whose correlates are not B , together with all contained in $l(A:B)$. Q. E. D.

Second $A:B l = (A:B)l + \sim A$.

Here $\sim A$ is used to denote the aggregate of all individual relatives whose relates are not A. This proposition is proved like the last.

Third $\quad \sim(A:Bl) = (A:B)\sim l + \sim A.$

This is evident from the second proposition, because

$$\sim(A:Bl) = (A:B)\sim l$$

Fourth $\quad l\sim(A:B) = \sim l(A:B) + k\sim B.$

Another method of working with the algebra is by means of negations. This becomes quite indispensable when the operations are defined by negations, as in this paper.

Peirce: CP 3.247 Cross-Ref: ††

247. To illustrate the use of these methods, let us investigate the relations of lb and lb to $l b$ when l and b are totally unlimited relatives.

Write $\quad l = \Sigma[i](L[i]:M[i]) \quad b = \Sigma[j](B[j]:C[j]).$

Then, by the rules of the last section,

$$lb \prec L:Mb \quad lb \prec lB:C;$$

whence, by the second and third propositions above,

$$lb \prec (L[i]:M[i])b + \sim L[i], \quad lb \prec l(B[j]:C[j]) + k\sim B[j].$$

But by the first rule of the last section

$$(L[i]:M[i])b \prec l b \quad l(B[j]:C[j]) \prec l b;$$

hence,

$$lb \prec l b + \sim L[i] \quad lb \prec l b + k \sim B[j].$$

There will be propositions like these for all the different values of i and j . Multiplying together all those of the several sets, we have

$$lb \prec l b + \pi[i] \sim L[i] \quad lb \prec l b + \pi[j] k \sim B[j].$$

But

$$\pi[i] \sim L[i] = \sim(\Sigma[i] L[i]) \quad \pi[j] k \sim B[j] = \sim(\Sigma[j] k B[j])$$

and since the relatives are unlimited,

$$\begin{aligned} \Sigma[i] L[i] &= \infty & \Sigma[j] k B[j] &= \infty \\ \sim(\Sigma[i] L[i]) &= 0 & \sim(\Sigma[j] k B[j]) &= 0; \end{aligned}$$

hence

$$lb \prec l b \quad lb \prec l b.$$

In the same way it is easy to show that, if the negatives of l and b are totally unlimited,

$$lb \prec l \bullet b \quad lb \prec l \bullet b.$$

Peirce: CP 3.248 Cross-Ref: ††
 §7. THE GENERAL FORMULÆ FOR RELATIVES

248. The principal formulæ of this algebra may be divided into *distribution formulæ* and *association formulæ*. The distribution formulæ are those which give the

equivalent of a relative compounded with a sum or product of two relatives in such terms as to separate the latter two relatives. The association formulæ are those which give the equivalent of a relative A compounded with a compound of B and C in terms of a compound of A and B compounded with C.

Peirce: CP 3.249 Cross-Ref:††

249. I. DISTRIBUTION FORMULÆ

1. AFFIRMATIVE

i. *Simple Formulæ*

$$(a+b)c = a c + b c \quad a(b+c) = a b + a c$$

$$(aXb)c = acXbc \quad ab+c = abXac$$

$$a+bc = acXbc \quad a(bXc) = abXac$$

$$(aXb)\bullet c = (a\bullet c) + (b\bullet c) \quad a\bullet(bXc) = (a\bullet b) + (a\bullet c)$$

ii. *Developments*

$$(aXb)c = \pi[p]\{a(cXp) + b(cX\sim p)\}$$

$$(a+b)c = \Sigma[p]\{acXpXbcX\sim p\}$$

$$(aXb)c = \Sigma[p]\{a(c+p)Xb(c+\sim p)\}$$

$$(a+b)\bullet c = \pi[p]\{a\bullet(c+p) + b\bullet(c+\sim p)\}$$

$$a(bXc) = \pi[p]\{(aXp)b + (aX\sim p)c\}$$

$$abXc = \Sigma[p]\{(a+p)bX(a+\sim p)c\}$$

$$a(b+c) = \Sigma[p]\{aXpbXaX\sim pc\}$$

$$a\bullet(b+c) = \pi[p]\{(a+p)\bullet b + (a+\sim p)\bullet c\}$$

2. NEGATIVE

i. *Simple Formulæ*

$$\begin{aligned}
\sim((a+b)c) &= \sim(a c)X\sim(b c) & \sim(a(b+c)) &= \sim(a b)X\sim(a c) \\
\sim((aXb)c) &= \sim(ac)+\sim(bc) & \sim(ab+c) &= \sim(ab)+\sim(ac) \\
\sim(a+bc) &= \sim(ac)+\sim(bc) & \sim(a(bXc)) &= \sim(ab)+\sim(ac) \\
\sim((aXb)\bullet c) &= \sim(a\bullet c)X\sim(b\bullet c) & \sim(a\bullet(bXc)) &= \sim(a\bullet b)X\sim(a\bullet c)
\end{aligned}$$

ii. *Developments*

$$\begin{aligned}
\sim((aXb)c) &= \Sigma[p]\{\sim(a(cXp)) X \sim(b(cX\sim p))\} \\
\sim((a+b)c) &= \pi[p]\{\sim(acXp) + \sim(bcX\sim p)\} \\
\sim((aXb)c) &= \pi[p]\{\sim(a(c+p)) + \sim(b(c+\sim p))\} \\
\sim((a+b)\bullet c) &= \Sigma[p]\{\sim(a\bullet(c+p)) X \sim(b\bullet(c+\sim p))\} \\
\sim(a(bXc)) &= \Sigma[p]\{\sim((aXp)b) X \sim((aX\sim p)c)\} \\
\sim(abXc) &= \pi[p]\{\sim((a+p)b) + \sim((a+\sim p)c)\} \\
\sim(a(b+c)) &= \pi[p]\{\sim(aXpb) + \sim(aX\sim pc)\} \\
\sim(a\bullet(b+c)) &= \Sigma[p]\{\sim((a+p)\bullet b) X \sim((a+\sim p)\bullet c)\}
\end{aligned}$$

Peirce: CP 3.250 Cross-Ref:††
250.II. ASSOCIATION FORMULÆ

1. AFFIRMATIVE

i. *Simple Formulæ*

$$\begin{aligned}
\sim(a\sim(b c)) &= a(b c) = (a b)c = \sim(\sim(a b)c) \\
\sim(a\bullet\sim(b\bullet c)) &= a(b\bullet c) = (a\bullet b)c = \sim(\sim(a\bullet b)\bullet c) \\
\sim(a\bullet\sim(b c)) &= a(b c) = (ab)c = \sim(\sim(ab)c) \\
\sim(a\sim(b\bullet c)) &= a(b\bullet c) = (ab)\bullet c = \sim(\sim(ab)c) \\
\sim(a\sim(bc)) &= a(bc) = (ab)c = \sim(\sim(ab)c) \\
\sim(a\sim(bc)) &= a\bullet(bc) = (ab)\bullet c = \sim(\sim(ab)c) \\
\sim(a\sim(bc)) &= a(bc) = (a b)c = \sim(\sim(a b)\bullet c) \\
\sim(a\sim(bc)) &= a\bullet(bc) = (a\bullet b)c = \sim(\sim(a\bullet b)c)
\end{aligned}$$

Peirce: CP 3.250 Cross-Ref:††
 ii. *Developments*

(A and E are individual aggregants, and α and ε simple components of a and e . The summations and products are relative to all such aggregants and components. The formulæ are of four classes; and for any relative c either all formulæ of Class 1 or all of Class 2, and also either all of Class 3 or all of Class 4 hold good.)

CLASS 1.

$$\sim(a\sim(b\ c)) = a(b\ c) = \pi\{(Ab)c\} = \pi\{\sim(\sim(Ab)c)\}$$

$$\sim(a\sim(b\ c)) = a\bullet(b\ c) = \Sigma\{(a\bullet b)c\} = \Sigma\{\sim(\sim(a\bullet b)c)\}$$

$$\sim(a\bullet\sim(bc)) = a(bc) = \pi\{(ab)c\} = \pi\{\sim(\sim(ab)c)\}$$

$$\sim(a\sim(bc)) = a(bc) = \Sigma\{(Ab)c\} = \Sigma\{\sim(\sim(Ab)c)\}$$

CLASS 2.

$$\sim(\sim(c\bullet d)e) = (c\bullet d)e = \pi\{c\bullet(dE)\} = \pi\{\sim(c\sim(dE))\}$$

$$\sim(\sim(c\bullet d)e) = (c\bullet d)\bullet e = \Sigma\{c(de)\} = \Sigma\{\sim(c\bullet\sim(d\varepsilon))\}$$

$$\sim(\sim(cd)\bullet e) = (cd)e = \pi\{c\bullet(d\bullet\varepsilon)\} = \pi\{\sim(c\sim(d\bullet\varepsilon))\}$$

$$\sim(\sim(cd)e) = (cd)e = \Sigma\{c(d^E)\} = \Sigma\{\sim(c\bullet\sim(d^E))\}$$

CLASS 3.

$$\sim(a\sim(b\bullet c)) = a\bullet(b\bullet c) = \Sigma\{(ab)c\} = \Sigma\{\sim(\sim(\alpha b)c)\}$$

$$\sim(a\sim(b\bullet c)) = a(b\bullet c) = \pi\{(Ab)\bullet c\} = \pi\{\sim(\sim(Ab)c)\}$$

$$\sim(a\sim(bc)) = a(bc) = \Sigma\{(Ab)c\} = \Sigma\{\sim(\sim(Ab)\bullet c)\}$$

$$\sim(a\bullet\sim(bc)) = a(bc) = \pi\{(a\bullet b)\bullet c\} = \pi\{\sim(\sim(a\bullet b)c)\}$$

CLASS 4.

$$\sim(\sim(c d)e) = (c d)\bullet e = \Sigma\{c(d\bullet\varepsilon)\} = \Sigma\{\sim(c\sim(d\bullet\varepsilon))\}$$

$$\sim(\sim(c d)e) = (c d)e = \pi\{c(d^E)\} = \pi\{\sim(c\sim(d^E))\}$$

$$\sim(\sim(cd)e) = (cd)e = \Sigma\{c(dE)\} = \Sigma\{\sim(c\sim(dE))\}$$

$$\sim(\sim(cd)\bullet e) = (cd)e = \pi\{c(d\varepsilon)\} = \pi\{\sim(c\sim(d\varepsilon))\}$$

Peirce: CP 3.250 Cross-Ref:††

The negative formulæ are derived from the affirmative by simply drawing or erasing lines over the whole of each member of every equation.

Peirce: CP 3.251 Cross-Ref:††

251. In order to see the general rules which these formulæ follow, we must imagine the operations symbolized by three marks, as in the commencement of this part [237]. We may term the operation uniting the two letters within the parenthesis the *interior* operation, and that which unites the whole parenthesis to the third letter the *exterior* operation. By *junction-marks* will be meant, in case the parenthesis {follows/precedes} the third letter the {first/second} mark of the symbol of the interior operation and the {second/first} mark of the symbol of the exterior operation. Using these terms, we may say that the exterior junction-mark and the third mark of the interior operation may always be changed together. When they are the same there is a simple association formula. This formula consists in the possibility of simultaneously interchanging the junction-marks, the third marks, and the exteriority or interiority of the two operations. When the exterior junction-mark and the third mark of the interior operation are unlike, there is a developmental association formula. The general term of this formula is obtained by making the same interchanges as in the simple formulæ, and then changing *a* to *A* when after these interchanges *a b* or *ab* occurs in parenthesis, changing *a* to α when *ab* or *a•b* occurs in parenthesis, changing *e* to *E* when *d e* or *de* occurs in parenthesis, and changing *e* to ε when *de* or *d•e* occurs in parenthesis. When the third mark in the symbol of the exterior operation is affirmative the development is a summation; when this mark is negative there is a continued product.

Peirce: CP 3.251 Cross-Ref:††

In the first half of the formulæ, the second mark in the sign of the interior operation is a line in Class 1 and a hyphen in Class 3. In the second half, the first mark in the sign of the interior operation is a hyphen in Class 2 and a line in Class 4.

Peirce: CP 3.252 Cross-Ref:††

ON THE LOGIC OF NUMBER†1

§1. DEFINITION OF QUANTITY^E

252. Nobody can doubt the elementary propositions concerning number: those that are not at first sight manifestly true are rendered so by the usual demonstrations. But although we see they *are* true, we do not so easily see precisely *why* they are true; so that a renowned English logician †2 has entertained a doubt as to whether they were true in all parts of the universe. The object of this paper is to show that they are strictly syllogistic consequences from a few primary propositions. The question of the logical origin of these latter, which I here regard as definitions, would require a separate discussion. In my proofs I am obliged to make use of the logic of relatives, in which the forms of inference are not, in a narrow sense, reducible to ordinary syllogism. They are, however, of that same nature, being merely syllogisms in which the objects spoken of are pairs or triplets. Their validity depends upon no conditions other than those of the validity of simple syllogism, unless it be that they suppose the existence of singulars, while syllogism does not.

Peirce: CP 3.252 Cross-Ref:††

The selection of propositions which I have proved will, I trust, be sufficient to show that all others might be proved with like methods.

Peirce: CP 3.253 Cross-Ref:††

253. Let *r* be any relative term, so that one thing may be said to be *r* of another, and the latter *r*'d by the former. If in a certain system of objects, whatever is *r* of an *r* of anything is itself *r* of that thing, then *r* is said to be a transitive relative in that system. (Such relatives as "lover of everything loved by --" are transitive relatives.) In a system in which *r* is transitive, let the *q*'s of anything include that thing itself, and also every *r* of it which is not *r*'d by it. Then *q* may be called a fundamental relative of quantity; its properties being, first, that it is transitive; second, that everything in the system is *q* of itself, and, third, that nothing is both *q* of and *q*'d by anything except itself. The objects of a system having a fundamental relation of quantity are called quantities, and the system is called a system of quantity.

Peirce: CP 3.254 Cross-Ref:††

254. A system in which quantities may be *q*'s of or *q*'d by the same quantity without being either *q*'s of or *q*'d by each other is called multiple;†P1 a system in which of every two quantities one is a *q* of the other is termed simple.

Peirce: CP 3.255 Cross-Ref:††

§2. SIMPLE QUANTITY

255. In a simple system every quantity is either "as great as" or "as small as" every other; whatever is as great as something as great as a third is itself as great as that third, and no quantity is at once as great as and as small as anything except itself.

Peirce: CP 3.256 Cross-Ref:††

256. A system of simple quantity is either continuous, discrete, or mixed. A continuous system is one in which every quantity greater than another is also greater than some intermediate quantity greater than that other.†1 A discrete system is one in which every quantity greater than another is next greater than some quantity (that is, greater than without being greater than something greater than). A mixed system is one in which some quantities greater than others are next greater than some quantities, while some are continuously greater than some quantities.

Peirce: CP 3.257 Cross-Ref:††

§3. DISCRETE QUANTITY

257. A simple system of discrete quantity is either limited, semi-limited, or unlimited. A limited system is one which has an absolute maximum and an absolute minimum quantity; a semi-limited system has one (generally considered a minimum) without the other; an unlimited has neither.

Peirce: CP 3.258 Cross-Ref:††

258. A simple, discrete, system, unlimited in the direction of increase or decrement, is in that direction either infinite or super-infinite. An infinite system is one in which any quantity greater than x can be reached from x by successive steps to the next greater (or less) quantity than the one already arrived at. In other words, an infinite, discrete, simple, system is one in which, if the quantity next greater than an attained quantity is itself attained, then any quantity greater than an attained quantity is attained; and by the class of attained quantities is meant any class whatever which satisfies these conditions. So that we may say that an infinite class is one in which if it is true that every quantity next greater than a quantity of a given class itself belongs to that class, then it is true that every quantity greater than a quantity of that class belongs to that class. Let the class of numbers in question be the numbers of which a certain proposition holds true. Then, an infinite system may be defined as one in which from the fact that a certain proposition, if true of any number, is true of the next greater, it may be inferred that that proposition if true of any number is true of every greater.†1

Peirce: CP 3.259 Cross-Ref:††

259. Of a super-infinite system this proposition, in its numerous forms, is untrue.

Peirce: CP 3.260 Cross-Ref:††

§4. SEMI-INFINITE QUANTITY

260. We now proceed to study the fundamental propositions of semi-infinite,†2 discrete, and simple quantity, which is ordinary number.†3

Peirce: CP 3.261 Cross-Ref:††

DEFINITIONS

261. The minimum number is called one.

Peirce: CP 3.262 Cross-Ref:††

262. By $x+y$ is meant, in case $x = 1$, the number next greater than y ; and in other cases, the number next greater than $x'+y$, where x' is the number next smaller than x .

Peirce: CP 3.263 Cross-Ref:††

263. By xXy is meant, in case $x = 1$, the number y , and in other cases $y+x y$, where x' is the number next smaller than x .

Peirce: CP 3.264 Cross-Ref:††

264. It may be remarked that the symbols $+$ and X are triple relatives, their two correlates being placed one before and the other after the symbols themselves.

Peirce: CP 3.265 Cross-Ref:††

THEOREMS

265. The proof in each case will consist in showing, first, that the proposition is true of the number one, and second, that if true of the number n it is true of the number $1+n$, next larger than n . The different transformations of each expression will be ranged under one another in one column, with the indications of the principles of transformation in another column.

Peirce: CP 3.266 Cross-Ref:††

266. (1) To prove the associative principle of addition, that

$$(x+y)+z = x+(y+z)$$

whatever numbers x , y , and z , may be. First it is true for $x = 1$; for

$$(1+y)+z$$

= $1+(y+z)$ by the definition of addition, second clause. Second, if true for $x = n$, it is true for $x = 1+n$; that is, if $(n+y)+z = n+(y+z)$ then $((1+n)+y)+z = (1+n)+(y+z)$.

For

$$((1+n)+y)+z$$

= $(1+(n+y))+z$ by the definition of addition:

= $1 + ((n+y)+z)$ by the definition of addition:

= $1 + (n+(y+z))$ by hypothesis:

= $(1+n)+(y+z)$ by the definition of addition,

Peirce: CP 3.267 Cross-Ref:††

267. (2) To prove the commutative principle of addition that

$$\mathbf{x+y = y+x}$$

whatever numbers x and y may be. First, it is true for $x = 1$ and $y = 1$, being in that case an explicit identity. Second if true for $x = n$ and $y = 1$, it is true for $x = 1+n$ and $y = 1$. That is, if $n+1 = 1+n$, then $(1+n)+1 = 1+(1+n)$. For $(1+n)+1$

= $1+(n+1)$ by the associative principle:

= $1+(1+n)$ by hypothesis.

Peirce: CP 3.267 Cross-Ref:††

We have thus proved that whatever number x may be $x+1 = 1+x$, or that $x+y = y+x$ for $y = 1$. It is now to be shown that if this be true for $y = n$, it is true for $y = 1+n$; that is, that if $x+n = n+x$, then $x+(1+n) = (1+n)+x$. Now

$$\mathbf{x+(1+n)}$$

= $(x+1)+n$ by the associative principle:

= $(1+x)+n$ as just seen:

= $1+(x+n)$ by the definition of addition:

= $1+(n+x)$ by hypothesis:

= $(1+n)+x$ by the definition of addition.

Thus the proof is complete.

Peirce: CP 3.268 Cross-Ref:††

268. (3) To prove the distributive principle, first clause. The distributive principle consists of two propositions:

First, $(x+y)z = xz + yz$

Second, $x(y+z) = xy + xz$.

Peirce: CP 3.268 Cross-Ref: ††

We now undertake to prove the first of these. First it is true for $x = 1$. For

$$(1+y)z$$

= $z + yz$ by the definition of multiplication:

= $1 \cdot z + yz$ by the definition of multiplication.

Second, if true for $x = n$ it is true for $x = 1+n$; that is, if $(n+y)z = nz + yz$ then $((1+n)+y)z = (1+n)z + yz$. For

$$((1+n)+y)z$$

= $(1+(n+y))z$ by the definition of addition:

= $z + (n+y)z$ by the definition of multiplication:

= $z + (nz + yz)$ by hypothesis:

= $(z + nz) + yz$ by the associative principle of addition:

= $(1+n)z + yz$ by the definition of multiplication.

Peirce: CP 3.269 Cross-Ref: ††

269. (4) To prove the second proposition of the distributive principle, that

$$x(y+z) = xy + xz.$$

First, it is true for $x = 1$; for

$$1(y+z)$$

= $y+z$ by the definition of multiplication;

= $1y + 1z$ by the definition of multiplication.

Second, if true for $x = n$, it is true for $x = 1+n$; that is, if $n(y+z) = ny + nz$, then

$(1+n)(y+z) = (1+n)y + (1+n)z$. For

$$\begin{aligned} & (1+n)(y+z) \\ &= (y+z) + n(y+z) \quad \text{by the definition of multiplication:} \\ &= (y+z) + (ny + nz) \quad \text{by hypothesis:} \\ &= (y+ny) + (z+nz) \quad \text{by the principles of addition:} \\ &= (1+n)y + (1+n)z \quad \text{by the definition of multiplication.} \end{aligned}$$

Peirce: CP 3.270 Cross-Ref:††

270. (5) To prove the associative principle of multiplication; that is, that

$$(x y)z = x(y z)$$

whatever numbers x , y , and z , may be. First, it is true for $x = 1$, for

$$\begin{aligned} & (1y)z \\ &= y z \quad \text{by the definition of multiplication:} \\ &= 1.y z \quad \text{by the definition of multiplication.} \end{aligned}$$

Second, if true for $x = n$, it is true for $x = 1+n$; that is, if $(n y)z = n(y z)$ then $((1+n)y)z = (1+n)(y z)$. For

$$\begin{aligned} & ((1+n)y)z \\ &= (y+ny)z \quad \text{by the definition of multiplication:} \\ &= y z + (ny)z \quad \text{by the distributive principle:} \\ &= y z + n(y z) \quad \text{by hypothesis:} \\ &= (1+n)(y z) \quad \text{by the definition of multiplication.} \end{aligned}$$

Peirce: CP 3.271 Cross-Ref:††

271. (6) To prove the commutative principle of multiplication; that

$$x y = y x,$$

whatever numbers x and y may be. In the first place we prove that it is true for $y = 1$. For this purpose, we first show that it is true for $y = 1$ $x = 1$; and then that if true for $y = 1$, $x = n$ it is true for $y = 1$, $x = 1+n$. For $y = 1$ and $x = 1$, it is an explicit identity. We have now to show that if $n1 = 1n$ then $(1+n)1 = 1(1+n)$. Now

$$\begin{aligned}
 & (1+n)1 \\
 = & 1+n1 && \text{by the definition of multiplication:} \\
 = & 1+1n && \text{by hypothesis:} \\
 = & 1+n && \text{by the definition of multiplication:} \\
 = & 1(1+n) && \text{by the definition of multiplication.}
 \end{aligned}$$

Peirce: CP 3.271 Cross-Ref:††

Having thus shown the commutative principle to be true for $y = 1$, we proceed to prove that if it is true for $y = n$, it is true for $y = 1+n$; that is, if $xn = nx$, then $x(1+n) = (1+n)x$. For

$$\begin{aligned}
 & (1+n)x \\
 = & x+nx && \text{by the definition of multiplication:} \\
 = & x+xn && \text{by hypothesis:} \\
 = & 1x+xn && \text{by the definition of multiplication:} \\
 = & x1+xn && \text{as already seen:} \\
 = & x(1+n) && \text{by the distributive principle.}
 \end{aligned}$$

Peirce: CP 3.272 Cross-Ref:††

§5. DISCRETE SIMPLE QUANTITY INFINITE IN BOTH DIRECTIONS

272. A system of number infinite in both directions has no minimum, but a certain quantity is called *one* and the numbers as great as this constitute a partial system of semi-infinite number, of which this one is a minimum. The definitions of addition and multiplication require no change except that the one therein is to be understood in the new sense.

Peirce: CP 3.273 Cross-Ref:††

273. To extend the proofs of the principles of addition and multiplication to unlimited number, it is necessary to show that if true for any number $(1+n)$ they are also true for the next smaller number n . For this purpose we can use the same

transformations as in the second clauses of the former proof; only we shall have to make use of the following lemma.

Peirce: CP 3.274 Cross-Ref:††

274. If $x+y = x+z$ then $y = z$ whatever numbers x , y , and z , may be. First this is true in case $x = 1$ for then y and z are both next smaller than the same number. Therefore, neither is smaller than the other, otherwise it would not be next smaller to $1+y = 1+z$. But in a simple system, of any two different numbers one is smaller. Hence, y and z are equal. Second, if the proposition is true for $x = n$, it is true for $x = 1+n$. For if $(1+n)+y = (1+n)+z$, then by the definition of addition $1+(n+y) = 1+(n+z)$; whence it would follow that $n+y = n+z$, and, by hypothesis, that $y = z$. Third, if the proposition is true for $x = 1+n$ it is true for $x = n$. For if $n+y = n+z$, then $1+n+y = 1+n+z$ because the system is simple. The proposition has thus been proved to be true of 1 of every greater and of every smaller number and therefore to be universally true.

Peirce: CP 3.275 Cross-Ref:††

275. An inspection of the above proofs of the principles of addition and multiplication for semi-infinite number will show that they are readily extended to doubly infinite number by means of the proposition just proved.

Peirce: CP 3.276 Cross-Ref:††

276. The number next smaller than one is called naught, 0. This definition in symbolic form is $1+0 = 1$. To prove that $x+0 = x$, let x' be the number next smaller than x . Then

$$\begin{aligned}
 & x+0 \\
 = & (1+x')+0 && \text{by the definition of } x': \\
 = & (1+0)+x' && \text{by the principles of addition:} \\
 = & 1+x' && \text{by the definition of naught:} \\
 = & x && \text{by the definition of } x'.
 \end{aligned}$$

Peirce: CP 3.277 Cross-Ref:††

277. To prove that $x0 = 0$. First, in case $x = 1$ the proposition holds by the definition of multiplication. Next, if true for $x = n$ it is true for $x = 1+n$. For

$$\begin{aligned}
 & (1+n)0 \\
 = & 1.0+n.0 && \text{by the distributive principle:} \\
 = & 1.0+0 && \text{by hypothesis:} \\
 = & 1.0 && \text{by the last theorem:} \\
 = & 0 && \text{as above.}
 \end{aligned}$$

Third, the proposition, if true for $x = 1+n$, is true for $x = n$. For, changing the order of the transformations,

$$1.0+0 = 1.0 = 0 = (1+n)0 = 1.0+n.0.$$

Then by the above lemma, $n.0 = 0$ so that the proposition is proved.

Peirce: CP 3.278 Cross-Ref:††

278. A number which added to another gives naught is called the negative of the latter. To prove that every number greater than naught has a negative. First, the number next smaller than naught is the negative of one; for, by the definition of addition, one plus this number is naught. Second, if any number n has a negative then the number next greater than n has for its negative the number next smaller than the negative of n . For let m be the number next smaller than the negative of n . Then $n+(1+m) = 0$.

But $n+(1+m)$

$$= (n+1)+m \quad \text{by the associative principle of addition.}$$

$$= (1+n)+m \quad \text{by the commutative principle of addition.}$$

So that $(1+n)+m = 0$. ***Q.E.D.*** Hence every number greater than 0 has a negative, and naught is its own negative.

To prove that $(-x)y = -(x y)$. We have

$$0 = x+(-x) \quad \text{by the definition of the negative:}$$

$$0 = 0y = (x+(-x))y \quad \text{by the last proposition but one:}$$

$$0 = x y+(-x)y \quad \text{by the distributive principle:}$$

$$-(x y) = (-x)y \quad \text{by the definition of the negative.}$$

Peirce: CP 3.279 Cross-Ref:††

279. The negative of the negative of a number is that number. For $x+(-x) = 0$. Whence by the definition of the negative $x = -(-x)$.

280. Let such a relative term, c , that whatever is a c of anything is the only c of that thing, and is a c of that thing only, be called a relative of simple correspondence. †1 In the notation of the logic of relatives

$$c\$c \text{ -< } 1, \ \$c c \text{ -< } 1. \dagger 2$$

Peirce: CP 3.281 Cross-Ref: ††

281. If every object, s , of a class is in any such relation being c 'd by †3 a number of a semi-infinite discrete simple system, and if further every number smaller than a number c of †4 an s is itself c of an s , then the numbers c of the s 's are said to count them, †5 and the system of correspondence is called a count. In logical notation, putting g for as 'great as,' and n for a positive integral number,

$$s \text{ -< } \$c n \quad \$g c s \text{ -< } c s$$

If in any count there is a maximum counting number the count is said to be finite, and that number is called the number of the count. Let $[[s]]$ denote the number of a count of the s 's, then

$$[[s]] \text{ -< } c s \quad \sim g c s \text{ -< } \sim [[s]] .$$

Peirce: CP 3.282 Cross-Ref: ††

282. The relative "identical with" satisfies the definition of a relative of simple correspondence, and the definition of a count is satisfied by putting "identical with" for c , and "positive integral number as small as x " for s . In this mode of counting, the number of numbers as small as x is x .

Peirce: CP 3.283 Cross-Ref: ††

283. Suppose that in any count the number of numbers as small as the minimum number, one, is found to be n . Then, by the definition of a count, every number as small as n counts a number as small as one. But by the definition of one there is only one number as small as one. Hence, by the definition of single correspondence, no other number than one counts one. Hence, by the definition of one, no other number than one counts any number as small as one. Hence, by the definition of the count, one is, in every count, the number of numbers as small as one.

Peirce: CP 3.284 Cross-Ref: ††

284. If the number of numbers as small as x is in some count y , then the number of numbers as small as y is in some count x . For if the definition of a simple correspondence is satisfied by the relative c , it is equally satisfied by the relative c 'd by.

Peirce: CP 3.285 Cross-Ref:††

285. Since the number of numbers as small as x is in some count y , we have, c being some relative of simple correspondence,

Peirce: CP 3.285 Cross-Ref:††

First. Every number as small as x is c 'd by a number.

Peirce: CP 3.285 Cross-Ref:††

Second. Every number as small as a number that is c of a number as small as x is itself c of a number as small as x .

Peirce: CP 3.285 Cross-Ref:††

Third. The number y is c of a number as small as x .

Peirce: CP 3.285 Cross-Ref:††

Fourth. Whatever is not as great as a number that is c of a number as small as x is not y .

Peirce: CP 3.286 Cross-Ref:††

286. Now let $c[1]$ be the converse of c . Then the converse of $c[1]$ is c ; whence, since c satisfies the definition of a relative of simple correspondence, so also does $c[1]$. By the third proposition above, every number as small as y is as small as a number that is c of a number as small as x . Whence, by the second proposition every number as small as y is c of a number as small as x ; and it follows that every number as small as y is $c[1]$ 'd by a number. It follows further that every number $c[1]$ of a number as small as y is $c[1]$ of something $c[1]$ 'd by (that is, $c[1]$ being a relative of simple correspondence, is identical with) some number as small as x . Also, "as small as" being a transitive relative, every number as small as a number c of a number as small as y is as small as x . Now by the fourth proposition y is as great as any number that is c of a number as small as x , so that what is not as small as y is not c of a number as small as x ; whence whatever number is c 'd by a number not as small as y is not a number as small as x . But by the second proposition, every number as small as x not c 'd by a number not as small as y is c 'd by a number as small as y . Hence, every number as small as x is c 'd by a number as small as y . Hence, every number as small as a number $c[1]$ of a number as small as y is $c[1]$ of a number as small as y . Moreover, since we have shown that every number as small as x is $c[1]$ of a number as small as y , the same is true of x itself. Moreover, since we have seen that whatever is $c[1]$ of a number as small as y is as small as x , it follows that whatever is not as great as a number $c[1]$ of a number as small as y is not as great as a number as small as x ; *i.e.* ("as great as" being a transitive relative) is not as great as x , and consequently is not x .†P1 We have now shown:

First, that every number as small as y is $c[1]$ 'd by a number;

Second, that every number as small as a number that is $c[1]$ of a number as small as y is itself $c[1]$ of a number as small as y ;

Third, that the number x is $c[1]$ of a number as small as y ; and

Fourth, that whatever is not as great as a number that is $c[1]$ of a number as small as y is not x .

Peirce: CP 3.286 Cross-Ref:††

These four propositions taken together satisfy the definition of the number of

numbers as small as y counting up to x .

Peirce: CP 3.286 Cross-Ref:††

Hence, since the number of numbers as small as one cannot in any count be greater than one, it follows that the number of numbers as small as any number greater than one cannot in any count be one.

Peirce: CP 3.287 Cross-Ref:††

287. Suppose that there is a count in which the number of numbers as small as $1+m$ is found to be $1+n$, since we have just seen that it cannot be 1. In this count, let m' be the number which is c of $1+n$, and let n' be the number which is c' 'd by $1+m$. Let us now consider a relative, e , which differs from c only in excluding the relation of m' to $1+n$ as well as the relation of $1+m$ to n' and in including the relation of m' to n' . Then e will be a relative of single correspondence; for c is so, and no exclusion of relations from a single correspondence affects this character, while the inclusion of the relation of m' to n' leaves m' the only e of n' and an e of n' only. Moreover, every number as small as m is e of a number, since every number except $1+m$ that is c of anything is e of something, and every number except $1+m$ that is as small as $1+m$ is as small as m . Also, every number as small as a number e' 'd by a number is itself e' 'd by a number; for every number c' 'd is e' 'd except $1+m$, and this is greater than any number e' 'd. It follows that e is the basis of a mode of counting by which the numbers as small as m count up to n . Thus we have shown that if in any way $1+m$ counts up to $1+n$, then in some way m counts up to n . But we have already seen that for $x = 1$ the number of numbers as small as x can in no way count up to other than x . Whence it follows that the same is true whatever the value of x .†P1

Peirce: CP 3.288 Cross-Ref:††

288. If every S is a P , and if the P 's are a finite lot counting up to a number as small as the number of S 's, then every P is an S . For if, in counting the P 's, we begin with the S 's (which are a part of them), and having counted all the S 's arrive at the number n , there will remain over no P 's not S 's. For if there were any, the number of P 's would count up to more than n . From this we deduce the validity of the following mode of inference:

Every Texan kills a Texan,

Nobody is killed by but one person,

Hence, every Texan is killed by a Texan,

supposing Texans to be a finite lot. For, by the first premiss, every Texan killed by a Texan is a Texan killer of a Texan, By the second premiss, the Texans killed by Texans are as many as the Texan killers of Texans. Whence we conclude that every Texan killer of a Texan is a Texan killed by a Texan, or, by the first premiss, every Texan is killed by a Texan. This mode of reasoning †1 is frequent in the theory of numbers.

ASSOCIATIVE ALGEBRAS †1E

§1. ON THE RELATIVE FORMS OF THE ALGEBRAS

289. Given an associative algebra whose letters are i, j, k, l , etc., and whose multiplication table is

$$\begin{aligned} i^2 &= a[11]i + b[11]j + c[11]k + \text{etc.} \dagger P1 \\ ij &= a[12]i + b[12]j + c[12]k + \text{etc.} \\ ji &= a[21]i + b[21]j + c[21]k + \text{etc.}, \\ &\text{etc., etc.} \end{aligned}$$

I proceed to explain what I call the relative form of this algebra.

Peirce: CP 3.290 Cross-Ref:††

290. Let us assume a number of new units, A, I, J, K, L , etc., one more in number than the letters of the algebra, and every one except the first, A , corresponding to a particular letter of the algebra. These new units are susceptible of being multiplied by numerical coefficients and of being added together;†P2 but they cannot be multiplied together, and hence are called *non-relative* units.

Peirce: CP 3.291 Cross-Ref:††

291. Next, let us assume a number of operations each denoted by bracketing together two non-relative units separated by a colon. These operations, equal in number to the square of the number of non-relative units, may be arranged as follows:

$$\begin{aligned} (A:A) (A:I) (A:J) (A:K), \text{ etc.} \\ (I:A) (I:I) (I:J) (I:K), \text{ etc.} \\ (J:A) (J:I) (J:J) (J:K), \text{ etc.} \end{aligned}$$

Peirce: CP 3.292 Cross-Ref:††

292. Any one of these operations performed upon a polynomial in non-relative units, of which one term is a numerical multiple of the letter following the colon, gives the same multiple of the letter preceding the colon. Thus, $(I:J)(aI + bJ + cK) = bI$.†P1 These operations are also taken to be susceptible of associative combination.

Hence $(I:J)(J:K) = (I:K)$; for $(J:K)K = J$ and $(I:J)J = I$, so that $(I:J)(J:K)K = I$. And $(I:J)(K:L) = 0$; for $(K:L)L = K$ and $(I:J)K = (I:J)(0.J+K) = 0.I = 0$. We further assume the application of the distributive principle to these operations; so that, for example,

$$\{(I:J)+(K:J)+(K:L)\} (aJ+bL) = aJ+(a+b)K.$$

Peirce: CP 3.293 Cross-Ref:††

293. Finally, let us assume a number of complex operations denoted by i', j', k', l' , etc., corresponding to the letters of the algebra and determined by its multiplication table in the following manner:

$$\begin{aligned} i' &= (I:A)+a[11](I:I)+b[11](J:I)+c[11](K:I)+\text{etc.} \\ &\quad +a[12](I:J)+b[12](J:J)+c[12](K:J)+\text{etc.} \\ &\quad +a[13](I:K)+b[13](J:K)+c[13](K:K)+\text{etc.}+\text{etc.} \end{aligned}$$

$$\begin{aligned} j' &= (J:A)+a[21](I:I)+b[21](J:I)+c[21](K:I)+\text{etc.} \\ &\quad +a[22](I:J)+b[22](J:J)+c[22](K:J)+\text{etc.} \\ &\quad +a[23](I:K)+b[23](J:K)+c[23](K:K)+\text{etc.}+\text{etc.} \end{aligned}$$

$$k' = \text{etc.}$$

Peirce: CP 3.294 Cross-Ref:††

294. Any two operations are equal which, being performed on the same operand, invariably give the same result. The ultimate operands in this case are the non-relative units. But any operations compounded by addition or multiplication of the operations i', j', k' , etc., if they give the same result when performed upon A , will give the same result when performed upon any one of the non-relative units. For suppose $ij'A = k'l'A$. We have

$$ij'A = i'J = a[12]I+b[12]J+c[12]K+\text{etc.}$$

$$k'l'A = k'L = a[34]I+b[34]J+c[34]K+\text{etc.}$$

so that $a[12] = a[34]$, $b[12] = b[34]$, $c[12] = c[34]$, etc., and in our original algebra $ij = kl$. Hence, multiplying both sides of the equation into any letter, say m , $ijm = klm$. But

$$\begin{aligned}
 ij m &= i(a[25]i+b[25]j+c[25]k+\text{etc.}) = \\
 & (a[11]a[25]+a[12]b[25]+a[13]c[25]+\text{etc.})i \\
 & + (b[11]a[25] + b[12]b[25] + b[13]c[25] + \text{etc.})j + \text{etc.}
 \end{aligned}$$

But we have equally

$$\begin{aligned}
 ij' m' A &= (a[11]a[25]+a[12]b[25]+a[13]c[25]+\text{etc.})I \\
 & + (b[11]a[25]+b[12]b[25]+b[13]c[25]+\text{etc.})J+\text{etc.}
 \end{aligned}$$

So that $ij' m' A = k'l' m' A$. Hence, $ij' M = k'l' M$. It follows, then, that if $ij' A = k'l' A$, then ij' into any non-relative unit equals $k'l'$ into the same unit, so that $ij' = k'l'$. We thus see that whatever equality subsists between compounds of the accented letters i', j', k' , etc., subsists between the same compounds of the corresponding unaccented letters i, j, k , so that the multiplication tables of the two algebras are the same.†P1 Thus, what has been proved is that any associative algebra can be put into relative form, i. e. (see my *brochure* entitled *A Brief Description of the Algebra of Relatives*)†1 that every such algebra may be represented by a matrix.

Peirce: CP 3.294 Cross-Ref:††

Take, for example, the algebra ($b d[5]$).†2 It takes the relative form

$$i = (I:A)+(J:I)+(L:K), j = (J:A),$$

$$\begin{aligned}
 k &= (K:A)+(J:I)+r(L:I)+(I:K)+(M:K)+r(J:L) \\
 & -(J:M)-r(L:M),
 \end{aligned}$$

$$\begin{aligned}
 l &= (L:A)+(J:K), m = (M:A)+(r^2 - 1)(J:I) \\
 & -(L:K)-r^2(J:M).
 \end{aligned}$$

Peirce: CP 3.294 Cross-Ref:††

This is the same as to say that the general expression $x i+y j+z k+u l+v m$ of this algebra has the same laws of multiplication as the matrix

$$\begin{array}{cccccc}
 0, & 0, & 0, & 0, & 0, & 0, \\
 x, & 0, & 0, & z, & 0, & 0, \\
 & x+z, & & & &
 \end{array}$$

$$\begin{array}{l}
y, \\
z, \\
u, \\
v,
\end{array}
\begin{array}{ll}
+(r^2-1)v & 0, \\
0, & 0, \\
rz, & 0, \\
0, & 0,
\end{array}
\begin{array}{l}
u, \\
0, \\
x-v, \\
z,
\end{array}
\begin{array}{ll}
rz, & -z-r^2v, \\
0, & 0, \\
0, & -rz, \\
0, & 0,
\end{array}$$

Peirce: CP 3.295 Cross-Ref:††

295. Of course, every algebra may be put into relative form in an infinity of ways; and simpler ways than that which the rule affords can often be found. Thus, for the above algebra, the form given in the foot-note is simpler, and so is the following:

$$\begin{aligned}
i &= (B:A)+(C:B)+(F:D)+(C:E), j = (C:A), \\
k &= (D:A)+(E:D)+(C:B)+r(F:B)+r(C:F), \\
l &= (F:A)+(C:D), m = (E:A)+(r^2-1)(C:B)-(B:A) \\
&\quad -(F:D)-(C:E).
\end{aligned}$$

These different forms will suggest transformations of the algebra. Thus, the relative form in the foot-note to (**b d**[5]) suggests putting

$$\begin{aligned}
i[1] &= i+m, j[1]=r^2j, k[1] = k+r^{-1}i+r^{-1}m, \\
l[1] &= rl+j, m[1] = -m,
\end{aligned}$$

when we get the following multiplication table, where {r} is put for r^{-1} :

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>
<i>i</i>	0	0	0	0	<i>j</i>
<i>j</i>	0	0	0	0	0
<i>k</i>	0	0	<i>i</i>	<i>j</i>	<i>l</i>
<i>l</i>	0	0	{r} <i>j</i>	0	0
<i>m</i>	{r} ² <i>j</i>	0	{r} <i>l</i>	0	<i>j</i>

Peirce: CP 3.296 Cross-Ref:††

296. Ordinary algebra with imaginaries, considered as a double algebra, is, in relative form,

$$1 = (X:X)+(Y:Y), \quad \mathbf{J} = (X:Y)-(Y:X).$$

This shows how the operation \mathbf{J} turns a vector through a right angle in the plane of X, Y . Quaternions in relative form is

$$1 = (W:W)+(X:X)+(Y:Y)+(Z:Z),$$

$$i = (X:W)-(W:X)+(Z:Y)-(Y:Z),$$

$$j = (Y:W)-(Z:X)-(W:Y)+(X:Z),$$

$$k = (Z:W)+(Y:X)-(X:Y)-(W:Z).$$

We see that we have here a reference to a space of four dimensions corresponding to X, Y, Z, W .

Peirce: CP 3.297 Cross-Ref:††

§2. ON THE ALGEBRAS IN WHICH DIVISION IS UNAMBIGUOUS

297. (1) In the *Linear Associative Algebra*, the coefficients are permitted to be imaginary. In this note they are restricted to being real. It is assumed that we have to deal with an algebra such that from $A B = A C$ we can infer that $A = 0$ or $B = C$. It is required to find what forms such an algebra may take.

Peirce: CP 3.298 Cross-Ref:††

298. (2) If $A B = 0$, then either $A = 0$ or $B = 0$. For if not, $A C = A(B+C)$, although A does not vanish and C is unequal to $B+C$.

Peirce: CP 3.299 Cross-Ref:††

299. (3) The reasoning of §40 [of the *Linear Associative Algebra*] holds, although the coefficients are restricted to being real. It is true, then, that since there is no expression (in the algebra under consideration) whose square vanishes, there must be an expression, i , such that $i^2 = -1$.

Peirce: CP 3.300 Cross-Ref:††

300. (4) By §41, it appears that for every expression in the algebra we have

$$i A = A i = A.$$

Peirce: CP 3.301 Cross-Ref:††

301. (5) By the reasoning of §53, it appears that for every expression A there

is an equation of the form

$$\Sigma[m](a[m]Am)+b i = 0.$$

But i is virtually arithmetical unity, since $i A = A i = A$; and this equation may be treated by the ordinary theory of equations. Suppose it has a real root, α ; then it will be divisible by $(A-\alpha)$, and calling the quotient B we shall have

$$(A-\alpha i)B = 0.$$

But $A-\alpha i$ is not zero, for A was supposed dissimilar to i . Hence a product of finites vanishes, which is impossible. Hence the equation cannot have a real root. But the whole equation can be resolved into quadratic factors, and some one of these must vanish. Let the irresoluble vanishing factor be

$$(A-s)^2+t^2 = 0.$$

Then $((A-s)/t)^2 = -1$,

or, every expression, upon subtraction of a real number (*i.e.* a real multiple of i), can be converted, in one way only, into a quantity whose square is a negative number. We may express this by saying that every quantity consists of a scalar and a vector part. A quantity whose square is a negative number we here call a *vector*.

Peirce: CP 3.302 Cross-Ref:††

302. (6) Our next step is to show that the vector part of the product of two vectors is linearly independent of these vectors and of unity. That is, i and j being any two vectors,†P1 if

$$i j = s+v$$

where s is a scalar and v a vector, we cannot determine three real scalars a, b, c , such that

$$v = a+b i+c j.$$

This is proved, if we prove that no scalar subtracted from $\mathbf{i}\mathbf{j}$ leaves a remainder $\mathbf{b}\mathbf{i}+\mathbf{c}\mathbf{j}$. If this be true when \mathbf{i} and \mathbf{j} are any unit vectors whatever, it is true when these are multiplied by real scalars, and so is true of every pair of vectors. We will, then, suppose \mathbf{i} and \mathbf{j} to be unit vectors. Now,

$$\mathbf{i}\mathbf{j}^2 = -\mathbf{i}.$$

If therefore we had

$$\mathbf{i}\mathbf{j} = \mathbf{a}+\mathbf{b}\mathbf{i}+\mathbf{c}\mathbf{j},$$

we should have

$$-\mathbf{i} = \mathbf{i}\mathbf{j}^2 = \mathbf{a}\mathbf{j}+\mathbf{b}\mathbf{i}\mathbf{j}-\mathbf{c} = \mathbf{a}\mathbf{b}-\mathbf{c}+\mathbf{b}^2\mathbf{i}+(\mathbf{a}+\mathbf{b}\mathbf{c})\mathbf{j};$$

whence, \mathbf{i} and \mathbf{j} being dissimilar,

$$-\mathbf{i} = \mathbf{b}^2\mathbf{i}, \quad \mathbf{b}^2 = -1,$$

and \mathbf{b} could not be real.

Peirce: CP 3.303 Cross-Ref:††

303. (7) Our next step is to show that, \mathbf{i} and \mathbf{j} being any two vectors, and

$$\mathbf{i}\mathbf{j} = \mathbf{s}+\mathbf{v},$$

\mathbf{s} being a scalar and \mathbf{v} a vector, we have

$$\mathbf{j}\mathbf{i} = \mathbf{r}(\mathbf{s}-\mathbf{v}),$$

where \mathbf{r} is a real scalar. It will be obviously sufficient to prove this for the case in which \mathbf{i} and \mathbf{j} are unit vectors. Assuming them such, let us write

$$j i = s' + v', \quad v v' = s'' + v'',$$

where s' and s'' are scalars, while v' and v'' are vectors. Then

$$i j j i = (s + v)(s' + v') = s s' + s v' + s' v + v v' + s''.$$

But we have

$$i j j i = i j^2 i = -i^2 = 1.$$

Hence,

$$v'' = 1 - s s' - s'' - s v' - s' v.$$

But v'' is the vector of $v v'$, so that by the last paragraph such an equation cannot subsist unless v'' vanishes. Thus we get

$$0 = 1 - s s' - s'' - s v' - s' v,$$

or

$$s v' = 1 - s s' - s'' - s' v.$$

But a quantity can only be separated in one way into a scalar and a vector part; so that

$$s v' = -s' v.$$

That is,

$$j i = (s'/s)(s - v). \quad Q. E. D.$$

Peirce: CP 3.304 Cross-Ref:††

304. (8) Our next step is to prove that $s = s'$; so that if $\mathbf{i}\mathbf{j} = s + \mathbf{v}$ then $\mathbf{j}\mathbf{i} = s - \mathbf{v}$. It is obviously sufficient to prove this when \mathbf{i} and \mathbf{j} are unit vectors. Now from any quantity a scalar may be subtracted so as to leave a remainder whose square is a scalar. We do not yet know whether the sum of two vectors is a vector or not (though we do know that it is not a scalar). Let us then take such a sum as $a\mathbf{i} + b\mathbf{j}$ and suppose x to be the scalar which subtracted from it makes the square of the remainder a scalar. Then, C being a scalar,

$$(-x + a\mathbf{i} + b\mathbf{j})^2 = C.$$

But developing the square we have

$$\begin{aligned} (-x + a\mathbf{i} + b\mathbf{j})^2 &= x^2 - a^2 - b^2 + a\mathbf{b}s + a\mathbf{b}s' - 2ax\mathbf{i} + 2bx\mathbf{j} \\ &+ a\mathbf{b}(1 - (s'/s))\mathbf{v} = C; \end{aligned}$$

i.e.

$$a\mathbf{b}(1 - (s'/s))\mathbf{v} = C - x^2 + a^2 + b^2 - a\mathbf{b}s - a\mathbf{b}s' + 2ax\mathbf{i} + 2bx\mathbf{j}.$$

But \mathbf{v} being the vector of $\mathbf{i}\mathbf{j}$, by the last paragraph but one the equation must vanish. Either then $\mathbf{v} = 0$ or $1 - (s'/s) = 0$. But if $\mathbf{v} = 0$, $\mathbf{i}\mathbf{j} = s$, and multiplying into \mathbf{j} ,

$$-\mathbf{i} = s\mathbf{j},$$

which is absurd, \mathbf{i} and \mathbf{j} being dissimilar. Hence $1 - (s'/s) = 0$ and

$$\mathbf{j}\mathbf{i} = s - \mathbf{v}. \quad \mathbf{Q.E.D.}$$

Peirce: CP 3.305 Cross-Ref:††

305. (9) The number of independent vectors in the algebra cannot be two. For the vector of $\mathbf{i}\mathbf{j}$ is independent of \mathbf{i} and \mathbf{j} . There may be no vector, and in that case we have the ordinary algebra of reals; or there may be only one vector, and in that case we have the ordinary algebra of imaginaries.

Peirce: CP 3.305 Cross-Ref:††

Let \mathbf{i} and \mathbf{j} be two independent vectors such that

$$ij = s + v.$$

Let us substitute for j

$$j[1] = s + i + j.$$

Then we have

$$ij[1] = v, \quad j[1]i = -v,$$

$$j[1]v = j[1]ij[1] = -j[1]^2i = i, \quad vj[1] = ij[1]^2 = -i,$$

$$iv = i^2j[1] = -j[1], \quad vi = ij[1]i = -j[1]i^2 = j[1]$$

Thus we have the algebra of real *quaternions*. Suppose we have a fourth unit vector, k , linearly independent of all the others, and let us write

$$j[1]k = s' + v',$$

$$ki = s'' + v''.$$

Let us substitute for k

$$k[1] = s''i + sj[1] + k,$$

and we get

$$j[1]k[1] = -s''v + v', \quad k[1]j[1] = s''v - v',$$

$$k[1]i = -s'v + v'', \quad ik[1] = s'v - v''.$$

Let us further suppose

$$(ij[1])k[1] = s''' + v'''.$$

Then, because ij is a vector,

$$k(ij) = s - v.$$

But

$$kij = -jki, \quad ki = -ik,$$

because both products are vectors.

Hence

$$ijk = -ikj = -ikj = kji = kji.$$

Hence

$$s + v = s - v$$

or $v = 0$, and the product of the two unit vectors is a scalar. These vectors cannot, then, be independent, or k cannot be independent of $ij = v$. Thus it is proved that a fourth independent vector is impossible, and that ordinary real algebra, ordinary algebra with imaginaries, and real quaternions are the only associative algebras in which division by finites always yields an unambiguous quotient.

Peirce: CP 3.306 Cross-Ref:††

IX

BRIEF DESCRIPTION OF THE ALGEBRA OF RELATIVES†1

306. Let A, B, C , etc., denote objects of any kind. These letters may be conceived to be finite in number or innumerable. The sum of them, each affected by a

numerical coefficient (which may equal 0), is called an *absolute term*. Let x be such a term; then we write

$$x = (x)[a]A + (x)[b]B + (x)[c]C + \text{etc.} = \Sigma [i](x)[i]I.$$

Here (x) , etc., are numbers, which may be permitted to be imaginary or restricted to being real or positive, or to being roots of any given equation, algebraic or transcendental.†P1 By ϕx , any mathematical function of the absolute term x , we mean such an absolute term that

$$(\phi x)[i] = \phi(x)[i].$$

That is, each numerical coefficient of ϕx is the function, ϕ , of the corresponding coefficient of x . In particular,

$$(x+y)[i] = (x)[i] + (y)[i],$$

$$(xXy)[i] = (x)[i]X(y)[i].$$

Peirce: CP 3.306 Cross-Ref:††

Otherwise written,

$$x+y = \{(x)[a] + (y)[a]\}A + \{(x)[b] + (y)[b]\}B + \text{etc.}$$

$$xXy = \{(x)[a]X(y)[a]\}A + \{(x)[b]X(y)[b]\}B + \text{etc.}$$

Peirce: CP 3.307 Cross-Ref:††

307. Two peculiar absolute terms are suggested by the logic of the subject. I call them terms of second intention. The first is zero, 0, and is defined by the equation

$$(0)[i] = 0$$

or

$$0 = 0.A + 0.B + 0.C + \text{etc.}$$

Peirce: CP 3.307 Cross-Ref:††

The other is *ens* (or non-relative unity), ~ 0 , and is defined by the equation

$$\sim(0)[i] = 1,$$

or

$$\sim 0 = A+B+C+\text{etc.}$$

Peirce: CP 3.308 Cross-Ref:††

308. The symbol $(A:B)$ is called an *individual dual relative*. It signifies simply a pair of individual objects, $(A:B)$ and $(B:A)$ being different. An aggregate of such symbols, each affected by a numerical coefficient, is called a *general dual relative*. The totality of pairs of letters arrange themselves with obvious naturalness in the block,

$A:A$	$A:B$	$A:C$	etc.
$B:A$	$B:B$	$B:C$	etc.
$C:A$	$C:B$	$C:C$	etc.
etc.	etc.	etc.	etc.

Peirce: CP 3.309 Cross-Ref:††

309. If l denotes any general dual relative, then the coefficient of the pair $I:J$ in l is written $(l)[i j]$ These coefficients are thus each referred to a place in the above block, and may themselves be arranged in the block

$(l)[a a]$	$(l)[a b]$	$(l)[a c]$	etc.
$(l)[b a]$	$(l)[b b]$	$(l)[b c]$	etc.
$(l)[c a]$	$(l)[c b]$	$(l)[c c]$	etc.
etc.	etc.	etc.	etc.

Peirce: CP 3.310 Cross-Ref:††

310. Every relative term, x , is separable into a part called 'self- x ,' Sx , such that

$$Sx = \Sigma[i](x)[i i](I:I)$$

and the remaining part, called 'alio- x ,' Vx ; comprising all the terms in x not in the principal diagonal of the block; so that we write

$$\mathbf{x} = \mathbf{S}\mathbf{x} + \mathbf{V}\mathbf{x}.\dagger 1$$

Peirce: CP 3.311 Cross-Ref:††

311. Each absolute term is considered to be equivalent to a certain relative term; namely,

$$\mathbf{A} = (\mathbf{A}:\mathbf{A}) + (\mathbf{A}:\mathbf{B}) + (\mathbf{A}:\mathbf{C}) + \text{etc.}$$

or, if \mathbf{x} be an absolute term,

$$(\mathbf{x})[\mathbf{i} \mathbf{j}] = (\mathbf{x})[\mathbf{i}].$$

Peirce: CP 3.311 Cross-Ref:††

The self-part of the relative equivalent to an absolute term is denoted by writing a comma after the term. Accordingly,

$$(\mathbf{x},)[\mathbf{i} \mathbf{i}] = (\mathbf{x})[\mathbf{i}], \quad (\mathbf{x},)[\mathbf{i} \mathbf{j}] = 0.$$

Peirce: CP 3.312 Cross-Ref:††

312. Besides 0 and ~ 0 , two other dual relative terms have been called terms of second intention. These are simply $\mathbf{S}\sim 0$ and $\mathbf{V}\sim 0$. The relative $\mathbf{S}\sim 0$ or $(\sim 0,)$ is also written 1, and is called unity, or 'identical with.' It is defined by the equations

$$(1)[\mathbf{i} \mathbf{i}] = 1, \quad (1)[\mathbf{i} \mathbf{j}] = 0.$$

That is, $1 = (\mathbf{A}:\mathbf{A}) + (\mathbf{B}:\mathbf{B}) + (\mathbf{C}:\mathbf{C}) + \text{etc.}$

Peirce: CP 3.312 Cross-Ref:††

The relative $\mathbf{V}\sim 0$ is written ~ 1 or n , and is called 'not,' or 'the negative of.' It is defined by the equations

$$(\sim 1)[\mathbf{i} \mathbf{j}] = 0, \dagger 1 \quad (\sim 1)[\mathbf{i} \mathbf{i}] = 1.$$

Peirce: CP 3.313 Cross-Ref:††

313. By an absolute function of a relative term is meant that function taken according to the rule for taking the function of an absolute term. That is,

$$(\phi x)[i j] = \phi(x)[i j].$$

In particular,

$$(x+y)[i j] = (x)[i j]+(y)[i j]$$

$$(xXy)[i j] = (x)[i j]X(y)[i j]$$

Peirce: CP 3.314 Cross-Ref:††

314. Of the various external or relative combinations that have been employed the following may be particularly specified. †2 (1), External multiplication, defined by the equation

$$(x y)[i j] = \Sigma[n](x)[i n](y)[n j]$$

(2), External progressive involution, defined by the equation

$$(xy)[i j] = \pi[n] = (x)[i n](y)[n j]$$

(3), External regressive involution, defined by the equation

$$(xy)[i j] = \pi[n](y)[n j](x)[i n].$$

In general, using Miss Ladd's notation †P1 for the different orders of multiplication,

$$(x X y)[i j] = \pi[n]\{(x)[i n] X (y)[n j]\}.$$

$$p \quad p-1 \quad p$$

Other modes of external combination have been used, but they are believed to have only a special utility. Division does not generally yield an unambiguous quotient.

Indeed, I have shown that it does so only in the cases of ordinary real algebra, of imaginary algebra, and of real quaternions.†1

Peirce: CP 3.315 Cross-Ref:††

315. Besides the *mathematical* functions of relatives, there are various modes in which one relative may *logically* depend upon another. Thus, Sx and Vx may be said to be logical functions of x . The most important of such operations is that of taking the converse of a relative. The converse of x , written $\$x$ or Kx , is defined by the equation

$$(\$x)[ij] = (x)[ji].$$

Peirce: CP 3.316 Cross-Ref:††

316. The algebraical laws of all these combinations are obtained with great facility by a method of which the following are examples:

Example 1.

$$\{(x\ y)z\}[ij] = \Sigma[n](x\ y)[in](z)[nj] = \Sigma[n]\Sigma[m](x)[im](y)[mn](z)[nj]$$

$$\{x(y\ z)\}[ij] = \Sigma[m](x)[im](y\ z)[mj] = \Sigma[m]\Sigma[n](x)[im](y)[mn](z)[nj]$$

$$\therefore (x\ y)z = x(y\ z).$$

Example 2.

$$\begin{aligned} \{(x+y)z\}[ij] &= \Sigma[n](x+y)[in](z)[nj] = \Sigma[n]\{(x)[in]+(y)[in]\}(z)[nj] \\ &= \Sigma[n](x)[in](z)[nj] + \Sigma[n](y)[in](z)[nj] = (x\ z)[ij] + (y\ z)[ij] \end{aligned}$$

$$\therefore (x+y)z = x\ z + y\ z.$$

Peirce: CP 3.316 Cross-Ref:††

The following are some of the elementary formulæ so obtained. Non-relative multiplication is indicated by a comma, relative multiplication by writing the factors one after the other, without the intervention of any sign.

$$(x+y)+z = x+(y+z), \quad x+y = y+x,$$

$$(x.y),z = x,(y,z), \quad x.y = y,x,$$

$$(x+y),z = (x,z)+(y,z),$$

$$(x y)z = x(y z),$$

$$(x+y)z = x z+y z, \quad x(y+z) = x y+x z,$$

$$(xy)z = x(y z), \quad x(yz) = (x y)z, \quad x(yz) = (xy)z,$$

$$(x.y)z = (xz),(yz), \quad x(y.z) = (xy),(xz),$$

$$xy+z = (xy),(xz), \quad x+yz = (xz),(yz)$$

$$kkx = x$$

$$k(x+y) = k x+ky, \quad k(x.y) = k x,k y$$

$$k(x y) = (k y)(k x), \quad k(xy) = (k y)(k x)$$

$$0+x = 0, \quad 0,x = 0x = x0 = 0, \quad x^0 = 0x = \sim 0,$$

$$\sim 0 X x = x, \quad \sim 0x = x\sim 0 = \sim 0,$$

$$1x = x1 = x^1 = 1x = x,$$

$$\{0, \text{ if } x[ij] \sim 0.$$

$$(x\sim 1)[ij] = ((\sim 1)x)[ij] = \{$$

$$\{1, \text{ if } x[ij] = 0.$$

Peirce: CP 3.317 Cross-Ref:††

317. Just as the different pairs of letters, *A*, *B*, *C*, etc., have been conceived to be arranged in a square block, so the different triplets of them may be conceived to be arranged in a cube, and the algebraical sum of all such triplets, each affected with a numerical coefficient, may be called a *triple relative*.

Peirce: CP 3.317 Cross-Ref:††

Every dual relative may be regarded as equivalent to a triple relative, just as every absolute term is equivalent to a dual relative.

Peirce: CP 3.317 Cross-Ref:††

Every triple relative may be regarded as a sum of five parts, each being a linear expression in terms of one of the five forms,

$$(A:A):A \quad (A:B):A \quad (A:A):B \quad (B:A):A \quad (A:B):C$$

Peirce: CP 3.317 Cross-Ref:††

The sign of a dual relative followed by a comma denotes that part of the equivalent triple relative which consists of terms in one of the forms

$$(A:A):(A:A) \quad (A:B):(A:B).$$

Peirce: CP 3.317 Cross-Ref:††

The multiplication of triple relatives is not perfectly associative and the multiplication of two triple relatives yields a quadruple relative.

Peirce: CP 3.317 Cross-Ref:††

The modes of combination of a triple relative followed by two dual relatives are the same as the modes of combination of three dual relatives. This ceases to be true for quadruple and higher relatives.

Peirce: CP 3.317 Cross-Ref:††

Corresponding to the operation of taking the converse of a dual relative, there are five operations upon triple relatives. They are defined as follows:

$$(I x)[i j k] = (x)[j i k], (J x)[i j k] = (x)i k j, (K x)[i j k] = (x)[k j i],$$

$$(L x)[i j k] = (x)[j k i], (M x)[i j k] = (x)[k i j].$$

Peirce: CP 3.317 Cross-Ref:††

Every quadruple or higher relative may be conceived as a product of triple relatives.

Peirce: CP 3.318 Cross-Ref:††

318. Thus, the essential characteristics of this algebra are (1) that it is a multiple algebra depending upon the addition of square blocks or cubes of numbers, (2) that in the external multiplication the rows of the block of the first factor are respectively multiplied by the columns of the block of the second factor, and (3) that the multiplication so resulting is, for the two-dimensional form of the algebra, always associative. I have proved in a paper presented to the American Academy of Arts and Sciences, May 11, 1875,†1 that this algebra necessarily embraces every associative algebra.

Peirce: CP 3.319 Cross-Ref:††

319. I have here described the algebra apart from the logical interpretation with which it has been clothed. In this interpretation a letter is regarded as a name applicable to one or more objects. By a name is usually meant something representative *of* an object *to* a mind. But I generalize this conception and regard a name as merely something in *a conjoint* relation to a second and a third, that is as a triple relative.†2 A sum of different individual names is a name for each of the things named severally by the aggregate letters. A name multiplied by a positive integral coefficient is the aggregate of so many different senses in which that name may be taken. The individual relative $A:B$ is the name of A considered as the first member of the pair $A:B$. The signification of the external multiplication is then determined by its algebraical definition.

Peirce: CP 3.320 Cross-Ref:††

320. Professor Sylvester, in his "New Universal Multiple Algebra,"†3 appears to have come, by a line of approach totally different from mine, upon a system which coincides, in some of its main features, with the Algebra of Relatives, as described in

my four papers upon the subject, †P1 and in my lectures on logic. I am unable to judge, from my unprofessional acquaintance with pure mathematics, how much of novelty there may be in my conceptions; but as the researches of the illustrious geometer who has now taken up the subject must draw increased attention to this kind of algebra, I take occasion to redescribe the outlines of my own system, and at the same time to declare my modest conviction that the logical interpretation of it, far from being in any degree special, will be found a powerful instrument for the discovery and demonstration of new algebraical theorems.

Peirce: CP 3.321 Cross-Ref: ††

321. *Postscript.* -- I have this day had the delight of reading for the first time Professor Cayley's *Memoir on Matrices*, in the *Philosophical Transactions* for 1858. The algebra he there describes seems to me substantially identical with my long subsequent algebra for dual relatives. Many of his results are limited to the very exceptional cases in which division is a determinative process.

Peirce: CP 3.322 Cross-Ref: ††

322. My own studies in the subject have been logical not mathematical, being directed toward the essential elements of the algebra, not towards the solution of problems.

Peirce: CP 3.323 Cross-Ref: ††

X

ON THE RELATIVE FORMS OF QUATERNIONS †1

323. If X , Y , Z denote the three rectangular components of a vector, and W denote numerical unity (or a fourth rectangular component, involving space of four dimensions), and $(Y:Z)$ denote the operation of converting the Y component of a vector into its Z component, then

$$1 = (W:W) + (X:X) + (Y:Y) + (Z:Z)$$

$$i = (X:W) - (W:X) - (Y:Z) + (Z:Y)$$

$$j = (Y:W) - (W:Y) - (Z:X) + (X:Z)$$

$$k = (Z:W) - (W:Z) - (X:Y) + (Y:X).$$

Peirce: CP 3.323 Cross-Ref: ††

In the language of logic $(Y:Z)$ is a relative term whose relate is a Y component, and whose correlate is a Z component. The law of multiplication is plainly $(Y:Z)(Z:X) = (Y:X)$, $(Y:Z)(X:W) = 0$, and the application of these rules to the above values of 1 , i , j , k gives the quaternion relations

$$i^2 = j^2 = k^2 = -1, \quad ijk = -1, \text{ etc.}$$

Peirce: CP 3.323 Cross-Ref:††

The symbol $a(Y:Z)$ denotes the changing of Y to Z and the multiplication of the result by a . If the relatives be arranged in the block

$W:W$	$W:X$	$W:Y$	$W:Z$
$X:W$	$X:X$	$X:Y$	$X:Z$
$Y:W$	$Y:X$	$Y:Y$	$Y:Z$
$Z:W$	$Z:X$	$Z:Y$	$Z:Z,$

then the quaternion $w+x i+y j+z k$ is represented by the matrix of numbers

w	$-x$	$-y$	$-z$
x	w	$-z$	y
y	z	w	$-x$
z	$-y$	x	$w.$

Peirce: CP 3.323 Cross-Ref:††

The multiplication of such matrices follows the same laws as the multiplication of quaternions. The determinant of the matrix = the fourth power of the tensor of the quaternion.

Peirce: CP 3.323 Cross-Ref:††

The imaginary $x+y\sqrt{-1}$ may likewise be represented by the matrix

x	y
$-y$	$x,$

and the determinant of the matrix = the square of the modulus.

Peirce: CP 3.324 Cross-Ref:††

ON A CLASS OF MULTIPLE ALGEBRAS †1

324. The object of this paper is to show what algebras express all the substitutions of two, of three, and of four letters; and to put these algebras into familiar forms. †2

Peirce: CP 3.324 Cross-Ref: ††

It is evident that every substitution is a relative term. Thus, the transposition of $A B$ to $B A$ is in relative form

$$(B C A)$$

$(A:B)+(B:A)$, and the circular substitution $(A B C)$ is

$(B:A)+(C:B)+(A:C)$. In this point of view, we see that substitutions may be added and multiplied by scalars, although the results will usually no longer be substitutions. A group of substitutions may, then, be linear expressions in an associative multiple algebra of a lower order than that of the group.

Peirce: CP 3.325 Cross-Ref: ††

325. Of two letters, there are two substitutions $(X:X)+(Y:Y)$ and $(X:Y)+(Y:X)$. We may denote these by a and β respectively, so that taking A and B as indeterminate coefficients, the general expression of the algebra is $A a+B\beta$, or in the form of a matrix is

$$A B$$

$$B A.$$

Assume i and j such that $i = 1/2(a+\beta)$

$$j = 1/2(a-\beta)$$

Then the multiplication table of i and j is as follows:

	i	j	
i	i	0	$ $
j	0	j	$/$

The algebra is, therefore, a mixture of two ordinary simple algebras of B. Peirce's

form $(a[1])$.

Peirce: CP 3.326 Cross-Ref:††

326. Of three letters, there are six substitutions. Let $\{A\}$, $\{B\}$, $\{G\}$, $\{D\}$, $\{E\}$, $\{Z\}$ be indeterminate coefficients. Then, the general expression of the algebra is equivalent to

$$\begin{aligned} &\{A\} \{B\} \{G\} \{D\} \{E\} \{Z\} \\ &\{G\} \{A\} \{B\} + \{E\} \{Z\} \{D\} \\ &\{B\} \{G\} \{A\} \{Z\} \{D\} \{E\} \end{aligned}$$

Or, denoting the six substitutions by $\{a\}$, $\{b\}$, $\{g\}$, $\{d\}$, $\{e\}$, $\{z\}$ we may write the general expression as $\{Aa\} + \{Bb\} + \{Gg\} + \{Dd\} + \{Ee\} + \{Zz\}$. There is an equation between these substitutions, namely:

$$\{a\} + \{b\} + \{g\} = \{d\} + \{e\} + \{z\}.$$

Assume 5 relatives h, i, j, k, l , such that

$$\begin{aligned} -h &= 1/3 (\{a\} + \{b\} + \{g\}) \\ i &= 1/2 (\{a\} - \{e\}) \\ j &= 1/3 (\{b\} - \{g\} + \{d\} - \{z\}) \\ k &= 1/4 (-\{b\} + \{g\} + \{d\} - \{z\}) \\ l &= 1/6 (2\{a\} - \{b\} - \{g\} - \{d\} + 2\{e\} - \{z\}) \end{aligned}$$

In matricular form,

$$h = \begin{matrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{matrix} \quad 2i = \begin{matrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{matrix} \quad 3j = \begin{matrix} 1 & 1 & -2 \\ -1 & -1 & +2 \\ 0 & 0 & 0 \end{matrix} \quad 4k = \begin{matrix} 1 & -1 & 0 \\ 1 & -1 & 0 \\ -2 & +2 & 0 \end{matrix}$$

$$6l = \begin{matrix} 1 & 1 & -2 \\ 1 & 1 & -2 \\ -2 & -2 & +4 \end{matrix}$$

The multiplication table of h, i, j, k, l , is as follows:

	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	
<i>h</i>	<i>h</i>	0	0	0	0	
<i>i</i>	0	<i>i</i>	<i>j</i>	0	0	
<i>j</i>	0	0	0	<i>i</i>	<i>j</i>	
<i>k</i>	0	<i>k</i>	<i>l</i>	0	0	
<i>l</i>	0	0	0	<i>k</i>	<i>l</i>	

The algebra is a mixture of ordinary single algebra ($\mathfrak{a}[1]$) with the algebra of Hamilton's biquaternions ($\mathfrak{g}[4]$).

Peirce: CP 3.327 Cross-Ref:††

327. In six letters, there are twenty-four substitutions. Using the capital Greek letters for indeterminate coefficients, the general linear expression in these substitutions is equivalent to

$$\begin{aligned}
 & \{A\}\{L\}\{G\}\{B\} \quad \{E\}\{Z\}\{\text{Th}\}\{\acute{E}\} \quad \{I\}\{L\}\{K\}\{M\} \\
 & \{G\}\{B\}\{A\}\{D\} + \{Z\}\{E\}\{\acute{E}\}\{\text{Th}\} + \{M\}\{K\}\{L\}\{I\} \\
 & \{B\}\{G\}\{D\}\{A\} \quad \{\text{Th}\}\{\acute{E}\}\{E\}\{Z\} \quad \{L\}\{I\}\{M\}\{K\} \\
 & \{D\}\{A\}\{B\}\{G\} \quad \{\acute{E}\}\{\text{Th}\}\{Z\}\{E\} \quad \{K\}\{M\}\{I\}\{L\}
 \end{aligned}$$

$$\begin{aligned}
 & \{N\}\{O\}\{P\}\{X\} \quad \{R\}\{Y\}\{S\}\{T\} \quad \{\text{Ph}\}\{\text{Ch}\}\{\text{Ps}\}\{\ddot{O}\} \\
 & \{O\}\{N\}\{X\}\{P\} + \{S\}\{T\}\{R\}\{Y\} + \{\ddot{O}\}\{\text{Ps}\}\{\text{Ch}\}\{\text{Ph}\} \\
 & + \{X\}\{P\}\{O\}\{N\} \quad \{Y\}\{R\}\{T\}\{S\} \quad \{\text{Ps}\}\{\ddot{O}\}\{\text{Ph}\}\{\text{Ch}\} \\
 & \{P\}\{X\}\{N\}\{O\} \quad \{T\}\{S\}\{Y\}\{R\} \quad \{X\}\{\text{Ph}\}\{\ddot{O}\}\{\text{Ps}\}
 \end{aligned}$$

Using the twenty-four small Greek letters to denote the twenty-four substitutions, so that the general linear expression is $A\{a\}+B\{b\}+$, etc., an attentive observation of the above scheme will show that the following equations subsist:

$$\begin{aligned}
 & \{a\}+\{b\}+\{g\}+\{d\} = \{e\}+\{z\}+\{\acute{e}\}+\{\text{th}\} = \{i\}+\{k\}+\{l\}+\{m\} \\
 & = \{n\}+\{x\}+\{o\}+\{p\} = \{r\}+\{s\}+\{t\}+\{n\} = \{\text{ph}\}+\{\text{ch}\}+\{\text{ps}\}+\{\ddot{o}\}
 \end{aligned}$$

Also,

$$\begin{aligned} \{a\}+\{b\} &= \{n\}+\{x\} & \{e\}+\{z\} &= \{n\}+\{o\} & \{i\}+\{k\} &= \{n\}+\{p\} \\ \{a\}+\{g\} &= \{r\}+\{s\} & \{e\}+\{\acute{e}\} &= \{r\}+\{t\} & \{i\}+\{l\} &= \{r\}+\{n\} \\ \{a\}+\{d\} &= \{ph\}+\{ch\} & \{e\}+\{th\} &= \{ph\}+\{ps\} & \{i\}+\{m\} &= \{ph\}+\{\ddot{o}\} \end{aligned}$$

It is plain that these equations are all independent, and not difficult to see that there are no more. Since they are fourteen in number, a ten-fold algebra is required to express the twenty-four substitutions.

Peirce: CP 3.327 Cross-Ref:††

Assume the ten relatives $h, i, j, k, l, m, n, o, p, q$, such that

$$h = 1/4 (\{a\}+\{b\}+\{g\}+\{d\})$$

$$i = 1/4 (\{e\}+\{z\}-\{\acute{e}\}-\{th\})$$

$$j = 1/4 (\{a\}-\{b\}+\{g\}-\{d\})$$

$$k = 1/4 (\{i\}-\{k\}-\{l\}+\{m\})$$

$$l = 1/4 (\{i\}-\{k\}+\{l\}-\{m\})$$

$$m = 1/4 (\{e\}-\{z\}-\{\acute{e}\}+\{th\})$$

$$n = 1/4 (\{a\}+\{b\}-\{g\}-\{d\})$$

$$o = 1/4 (\{a\}-\{b\}-\{g\}+\{d\})$$

$$p = 1/4 (\{i\}+\{k\}-\{l\}-\{m\})$$

$$q = 1/4 (\{e\}-\{z\}+\{\acute{e}\}-\{th\})$$

In matricular form, these are as follows (where + is written for

+1 and - for -1):

$$\begin{array}{r} \begin{array}{cccc} & + & + & + & + \\ & + & + & + & + \\ 4h = & + & + & + & + \\ & + & + & + & + \end{array} \\ \\ \begin{array}{r} \begin{array}{cccc} + & + & - & - \\ + & + & - & - \\ 4i = & - & - & + & + \end{array} \quad \begin{array}{cccc} + & - & + & - \\ + & - & + & - \\ 4j = & - & + & - & + \end{array} \quad \begin{array}{cccc} + & - & - & + \\ + & - & - & + \\ 4k = & - & + & + & - \end{array} \end{array}$$

$$\begin{array}{l}
\begin{array}{lll}
- - + + & - + - + & - + + - \\
+ + - - & + - + - & + - - + \\
- - + + & - + - + & - + + - \\
4l = + + - - & 4m = + - + - & 4n = + - - + \\
- - + + & - + - + & - + + - \\
\end{array} \\
\begin{array}{lll}
+ + - - & + - + - & + - - + \\
- - + + & - + - + & - + + - \\
- - + + & - + - + & - + + - \\
4o = + + - - & 4p = + - + - & 4q = + - - + \\
+ + - - & + - + - & + - - +
\end{array}
\end{array}$$

The multiplication table is as follows:

	<i>h</i>	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	<i>m</i>	<i>n</i>	<i>o</i>	<i>p</i>	<i>q</i>
<i>h</i>	<i>h</i>	0	0	0	0	0	0	0	0	0
<i>i</i>	0	<i>i</i>	<i>j</i>	<i>k</i>	0	0	0	0	0	0
<i>j</i>	0	0	0	0	<i>i</i>	<i>j</i>	<i>k</i>	0	0	0
<i>k</i>	0	0	0	0	0	0	0	<i>i</i>	<i>j</i>	<i>k</i>
<i>l</i>	0	<i>l</i>	<i>m</i>	<i>n</i>	0	0	0	0	0	0
<i>m</i>	0	0	0	0	<i>l</i>	<i>m</i>	<i>n</i>	0	0	0
<i>n</i>	0	0	0	0	0	0	0	<i>l</i>	<i>m</i>	<i>n</i>
<i>o</i>	0	<i>o</i>	<i>p</i>	<i>q</i>	0	0	0	0	0	0
<i>p</i>	0	0	0	0	<i>o</i>	<i>p</i>	<i>q</i>	0	0	0
<i>q</i>	0	0	0	0	0	0	0	<i>o</i>	<i>p</i>	<i>q</i>

The algebra is, therefore, a mixture of ordinary algebra with that of my nonions.

Peirce: CP 3.327 Cross-Ref:††

†1 Of course, the representation of quaternions as linear expressions in substitutions of three letters, the sum of the coefficients being zero, is equivalent to finding *a theorem of plane geometry corresponding to each theorem of solid geometry expressible by quaternions*. For instance, let the three letters which are interchanged be the coördinates *x*, *y*, *z*, of a point in space. Then, as above, let

$$\{a\} = \begin{matrix} (x,y,z) \\ (x,y,z) \end{matrix} \quad \{b\} = \begin{matrix} (z,x,y) \\ (x,y,z) \end{matrix} \quad \{g\} = \begin{matrix} (y,z,x) \\ (x,y,z) \end{matrix}$$

$$\{d\} = \begin{matrix} (x,z,y) \\ (x,y,z) \end{matrix} \quad \{e\} = \begin{matrix} (y,x,z) \\ (x,y,z) \end{matrix} \quad \{z\} = \begin{matrix} (z,y,z) \\ (x,y,z) \end{matrix}$$

Thus, {b} and {g} represent the operations of rotation through one-third of a circumference, the one forward, the other backward, about an axis passing through the origin and the point (1, 1, 1); while {d}, {e}, {z} represent three perversions with reference to axes passing through the origin and the points (0, 1, -1), (1, -1, 0), and (1, 0, -1), respectively. Quaternions may be represented thus:

$$1 = 1/3 (2\{a\} - \{b\} - \{g\})$$

$$i = -1/3 (2\{e\} - \{d\} - \{z\})\sqrt{-1}$$

$$j = 1/\sqrt{3} (\{b\} - \{g\})$$

$$k = 1/\sqrt{3} (\{d\} - \{z\})\sqrt{-1}.$$

We have here a new geometrical interpretation of quaternions. Since the sum of the coefficients of the substitutions is equal to zero in the values of every one of the quaternion elements, it follows that under this interpretation any quaternion operating upon any point brings it into the plane.

$$x+y+z = 0.$$

Hence, every quaternion equation has an interpretation relating to points in this plane. The reason why a quaternion, which has a four-fold multiplicity, is no more than adequate to expressing operations upon points in space, is that the operations are of such a nature that different ones may have the same effect upon single points. But a *real* quaternion has no greater multiplicity than the real and imaginary points of a

plane; and the geometrical effects of different real quaternions upon points in the plane $x+y+z = 0$ under the new interpretation are different upon all points except the origin.

Peirce: CP 3.327 Cross-Ref:††

For the axes of x, y, z , in trilinear coordinates, take three lines meeting in one point and equally inclined to one another. To plot the point

$$x = a+b\sqrt{-1}, \quad y = c+d\sqrt{-1}, \quad z = e+f\sqrt{-1},$$

plot the point $x = a, y = c, z = e$ in *blue*, and the point $x = b, y = d, z = f$ in *red*. Then the effects of the different quaternion elements upon points in the plane $x+y+z = 0$ are as follows: 1 leaves every point unchanged. The vector i reverses the position of a blue point with reference to the line $z = 0$ and changes it to red, and reverses the position of a red point with reference to the line $x = y$ and changes it to blue. The vector j rotates every point through a quadrant round the origin in the direction from $x = 0$ to $y = z$, without changing the color. The vector k reverses the position of a blue point with reference to the line through the origin that bisects the angle between $y = 0$ and $y = z$ and changes it to red, and reverses the position of a red point with reference to the line through the origin that bisects the angle between $x = 0$ and $x = z$ and changes it to blue.

Peirce: CP 3.328 Cross-Ref:††
XII

THE LOGIC OF RELATIVES †1

328. A dual relative term, such as "lover," "benefactor," "servant," is a common name signifying a pair of objects. Of the two members of the pair, a determinate one is generally the first, and the other the second; so that if the order is reversed, the pair is not considered as remaining the same.

Peirce: CP 3.329 Cross-Ref:††

329. Let A, B, C, D, etc., be all the individual objects in the universe; then all the individual pairs may be arrayed in a block, thus:

A:A	A:B	A:C	A:D	etc.
B:A	B:B	B:C	B:D	etc.
C:A	C:B	C:C	C:D	etc.

D:A D:B D:C D:D etc.
 etc. etc. etc. etc. etc.

Peirce: CP 3.329 Cross-Ref:††

A general relative may be conceived as a logical aggregate of a number of such individual relatives. Let l denote "lover"; then we may write

$$l = \sum[i] \sum[j] (l)[i j] (I:J)$$

where $(l)[i j]$ is a numerical coefficient, whose value is 1 in case I is a lover of J , and 0 in the opposite case, and where the sums are to be taken for all individuals in the universe.

Peirce: CP 3.330 Cross-Ref:††

330. Every relative term has a negative (like any other term) which may be represented by drawing a straight line over the sign for the relative itself. The negative of a relative includes every pair that the latter excludes, and vice versa. Every relative has also a *converse*, produced by reversing the order of the members of the pair. Thus, the converse of "lover" is "loved." The converse may be represented by drawing a curved line over the sign for the relative, thus: $\$l$. It is defined by the equation

$$(\$l)[i j] = (l)[j i].$$

The following formulæ are obvious, but important:

$$\sim\sim l = l \quad \$\$l = l$$

$$\sim \$l = \$\sim l$$

$$(l -< b) = (\sim b -< \sim l) \quad (l -< b) = (\$l -< \$b).$$

Peirce: CP 3.331 Cross-Ref:††

331. Relative terms can be aggregated and compounded like others. Using + for the sign of logical aggregation, and the comma for the sign of logical composition (Boole's multiplication, here to be called non-relative or internal multiplication), we have the definitions

$$(l+b)[ij] = (l)[ij] + (b)[ij]$$

$$(l,b)[ij] = (l)[ij] \times (b)[ij].$$

The first of these equations, however, is to be understood in a peculiar way: namely, the + in the second member is not strictly addition, but an operation by which

$$0+0 = 0 \quad 0+1 = 1+0 = 1+1 = 1.$$

Instead of $(l)[ij]+(b)[ij]$, we might with more accuracy write

$$0^0(l)[ij] + (b)ij$$

Peirce: CP 3.331 Cross-Ref:††

The main formulæ of aggregation and composition are

{If $l \prec s$ and $b \prec s$, then $l+b \prec s$.}

{If $s \prec l$ and $s \prec b$, then $s \prec l,b$.}

{If $l+b \prec s$, then $l \prec s$ and $b \prec s$.}

{If $s \prec l,b$, then $s \prec l$ and $s \prec b$.}

{ $(l+b),s \prec l,s+b,s$. }

{ $(l+s),(b+s) \prec l,b+s$.}

The subsidiary formulæ need not be given, being the same as in non-relative logic.

Peirce: CP 3.332 Cross-Ref:††

332. We now come to the combination of relatives. Of these, we denote two by special symbols; namely, we write

lb for lover of a benefactor,

and

$l\ddagger b$ for lover of everything but benefactors.†1

The former is called a particular combination, because it implies the *existence* of something *loved by* its relate and a *benefactor of* its correlate. The second combination is said to be *universal*, because it implies the *non-existence* of anything except what is either loved by its relate or a benefactor of its correlate. The combination $l b$ is called a relative product, $l\ddagger b$ a relative sum. The l and b are said to be undistributed in both, because if $l \prec s$, then $l b \prec s b$ and $l\ddagger b \prec s\ddagger b$; and if $b \prec s$, then $l b \prec l s$ and $l\ddagger b \prec l\ddagger s$.†2

Peirce: CP 3.333 Cross-Ref:††

333. The two combinations are defined by the equations

$$(l b)[i j] = \Sigma[x](l)[i x](b)[x j]$$

$$(l\ddagger b)[i j] = \pi[x] \{ (l)[i x] + (b)[x j] \} \ddagger 3$$

The sign of addition in the last formula has the same signification as in the equation defining non-relative multiplication.†4

Peirce: CP 3.334 Cross-Ref:††

334. Relative addition and multiplication are subject to the associative law.

That is,

$$l\ddagger(b\ddagger s) = (l\ddagger b)\ddagger s$$

$$l(b s) = (l b)s.$$

Two formulæ so constantly used that hardly anything can be done without them are

$$l(b\ddagger s) \prec l b\ddagger s,$$

$$(l\ddagger b)s \prec l\ddagger b s.$$

The former asserts that whatever is lover of an object that is benefactor of everything but a servant, stands to everything but servants in the relation of lover of a benefactor. The latter asserts that whatever stands to any servant in the relation of lover of everything but its benefactors, is a lover of everything but benefactors of servants. The following formulæ are obvious and trivial:

$$l s + b s \prec (l + b) s \dagger 1$$

$$l, b \dagger s \prec (l \dagger s), (b \dagger s). \dagger 2$$

Unobvious and important, however, are these:

$$(l + b) s \prec l s + b s \dagger 1$$

$$(l \dagger s), (b \dagger s) \prec l, b \dagger s. \dagger 2$$

Peirce: CP 3.335 Cross-Ref: ††

335. There are a number of curious development formulæ. †3 Such are

$$(l, b) s = \pi[p] \{ l(s, p) + b(s, \sim p) \}$$

$$l(b, s) = \pi[p] \{ (l, p) b + (l, \sim p) s \}$$

$$(l + b) \dagger s = \Sigma[p] \{ [(l \dagger (s + p))], [(b \dagger (s + \sim p))] \}$$

$$l \dagger (b + s) = \Sigma[p] \{ [(l + p) \dagger b], [(l + \sim p) \dagger s] \}$$

The summations and multiplications denoted by Σ and π are to be taken non-relatively, and all relative terms are to be successively substituted for p .

Peirce: CP 3.336 Cross-Ref: ††

336. The negatives of the combinations follow these rules:

$$\sim(l + b) = \sim l, \sim b \quad \sim(l, b) = \sim l + \sim b$$

$$\sim(l \dagger b) = \sim l \sim b \quad \sim(l b) = \sim l \dagger \sim b$$

Peirce: CP 3.337 Cross-Ref: ††

337. The converses of combinations are as follows:

$$\$(l + b) = \$l + \$b \quad \$(l, b) = \$l, \$b$$

$$\$(l \dagger b) = \$b \dagger \$l \quad \$(l b) = \$b \$l$$

Peirce: CP 3.338 Cross-Ref: ††

338. †4 Individual dual relatives are of two types,

A:A and A:B.

Relatives containing no pair of an object with itself are called *alio-relatives* as opposed to *self-relatives*. The negatives of alio-relatives pair every object with itself. Relatives containing no pair of an object with anything but itself are called *concurrents* as opposed to *opponents*. The negatives of concurrents pair every object with every other.

Peirce: CP 3.339 Cross-Ref:††

339. There is but one relative which pairs every object with itself and with every other. It is the aggregate of all pairs, and is denoted by ∞ . It is translated into ordinary language by "coexistent with." Its negative is 0. There is but one relative which pairs every object with itself and none with any other. It is

$(A:A) + (B:B) + (C:C) + \text{etc.};$

is denoted by 1, and in ordinary language is "identical with --." Its negative, denoted by n , is "other than --," or "not."

Peirce: CP 3.340 Cross-Ref:††

340. No matter what relative term x may be, we have

$$0 \prec x \quad x \prec \infty.$$

Peirce: CP 3.341 Cross-Ref:††

341. Hence, obviously

$$x + 0 = x \quad x, \infty = x$$

$$x + \infty = \infty \quad x, 0 = 0.$$

The last formulæ hold for the relative operations; thus,

$$x \dagger \infty = \infty \quad x0 = 0.$$

$$\infty \dagger x = \infty \quad 0x = 0.$$

The formulæ

$$x + 0 = x \quad x, \infty = x$$

also hold if we substitute the relative operations, and also 1 for ∞ , and n for 0; thus,

$$x \dagger n = x \quad x \uparrow 1 = x.$$

$$n \dagger x = x \quad 1 \uparrow x = x.$$

Peirce: CP 3.341 Cross-Ref:††

We have also

$$l + \sim l = \infty, \quad l, \sim l = 0. \dagger P1$$

To these partially correspond the following pair of highly important formulæ:

$$1 \prec l \dagger \sim l \dagger 1 \quad l \dagger \sim l \prec n. \dagger 2$$

Peirce: CP 3.342 Cross-Ref:††

342. The logic of relatives is highly multiform; it is characterized by innumerable immediate inferences, and by various distinct conclusions from the same sets of premisses. An example of the first character is afforded by Mr. Mitchell's $F[1v]$ following from $F[1v']$.†3 As an instance of the second, take the premisses,

Every man is a lover of an animal;

and

Every woman is a lover of a non-animal.

From these we can equally infer that

Every man is a lover of something which stands to each woman in the relation of not being the only thing loved by her, and that

Every woman is a lover of something which stands to each man in the relation

of not being the only thing loved by him.

The effect of these peculiarities is that this algebra cannot be subjected to hard and fast rules like those of the Boolean calculus; and all that can be done in this place is to give a general idea of the way of working with it. The student must at the outset disabuse himself of the notion that the chief instruments of algebra are the inverse operations. General algebra hardly knows any inverse operations. When an inverse operation is identical with a direct operation with an inverse quantity (as subtraction is the addition of the negative, and as division is multiplication by the reciprocal), it is useful; otherwise it is almost always useless. In ordinary algebra, we speak of the "principal value" of the logarithm, etc., which is a direct operation substituted for an indefinitely ambiguous inverse operation. The elimination and transposition in this algebra really does depend, however, upon formulæ quite analogous to the

$$x + (-x) = 0 \quad x \times 1/x = 1,$$

of arithmetical algebra. These formulæ are

$$l, \sim l = 0 \quad l \$ \sim l \prec n$$

$$l + \sim l = \infty \quad 1 \prec l \dagger \$ \sim l.$$

For example, to eliminate s from the two propositions

$$1 \prec l \sim s \quad 1 \prec \$ s b,$$

we relatively multiply them in such an order as to bring the two s 's together, and then apply the second of the above formulæ, thus:

$$1 \prec l \sim s \$ s b \prec l n b.$$

This example shows the use of the association formulæ in bringing letters together. Other formulæ of great importance for this purpose are

$$(b \dagger l) s \prec b \dagger l s \quad b (l \dagger s) \prec b l \dagger s.$$

The distribution formulæ are also useful for this purpose.

Peirce: CP 3.343 Cross-Ref:††

343. When the letter to be eliminated has thus been replaced by one of the four relatives -- 0, ∞ , 1, n -- the replacing relative can often be got rid of by means of one of the formulæ

$$l+0 = l \quad l,\infty = l$$

$$l\ddagger n = n\ddagger l = l \quad l1 = 1l = l.$$

Peirce: CP 3.344 Cross-Ref:††

344. When we have only to deal with universal propositions, it will be found convenient so to transpose everything from subject to predicate as to make the subject 1. Thus, if we have given $l \prec b$, we may relatively add $\$ \sim l$ to both sides; whereupon we have

$$1 \prec l\ddagger \$ \sim l \prec b\ddagger \$ \sim l.$$

Every proposition will then be in one of the forms

$$1 \prec b\ddagger l \quad 1 \prec b l.$$

With a proposition of the form $1 \prec b\ddagger l$, we have the right (1) to transpose the terms, and (2) to convert the terms. Thus, the following are equivalent:

$$1 \prec b\ddagger l$$

$$1 \prec l\ddagger b \quad 1 \prec \$b\ddagger \$l$$

$$1 \prec \$l\ddagger \$b.$$

With a proposition of the form $1 \prec b l$, we have only the right to convert the predicate giving $1 \prec \$l\b .

Peirce: CP 3.344 Cross-Ref:††

With three terms, there are four forms of universal propositions, namely:

$$1 \prec l\ddagger b\ddagger s \quad 1 \prec l(b\ddagger s) \quad 1 \prec l b\ddagger s \quad 1 \prec l b s.$$

Of these, the third is an immediate inference from the second.

Peirce: CP 3.345 Cross-Ref:††

345. By way of illustration, we may work out the syllogisms whose premisses are the propositions of the first order referred to in Note A.†1 Let a and c be class terms, and let β be a group of characters. Let p be the relative "possessing as a character." The non-relative terms are to be treated as relatives-- a , for instance, being considered as " a coexistent with" and $\$a$ as "coexistent with a that is." Then, the six forms of affirmative propositions of the first order are

$$1 \text{ -< } \$a \dagger p \dagger \beta$$

$$1 \text{ -< } \$a(p \dagger \beta) \quad 1 \text{ -< } (\$a \dagger p)\beta$$

$$1 \text{ -< } \$a p \dagger \beta \quad 1 \text{ -< } \$a \dagger p \beta$$

$$1 \text{ -< } \$a p \beta. \dagger 2$$

Peirce: CP 3.346 Cross-Ref:††

346. The various kinds of syllogism †3 are as follows:

$$1. \text{ Premisses: } 1 \text{ -< } \$a \dagger p \dagger \beta \quad 1 \text{ -< } \$c \dagger p \dagger \sim \beta.$$

Convert one of the premisses and multiply,

$$1 \text{ -< } (\$a \dagger p \dagger \beta)(\$ \sim \beta \dagger \$p \dagger c) \text{ -< } \$a \dagger p \dagger \beta \$ \sim \beta \dagger \$p \dagger c \\ \text{-< } \$a \dagger p \dagger n \dagger \$p \dagger c \text{ -< } \$a \dagger p \dagger \$p \dagger c.$$

Peirce: CP 3.346 Cross-Ref:††

The treatment would be the same if one or both of the premisses were negative: that is, contained $\sim p$ in place of p .

Peirce: CP 3.346 Cross-Ref:††

$$2. \text{ Premisses: } 1 \text{ -< } \$a \dagger p \dagger \beta \quad 1 \text{ -< } \$c(p \dagger \sim \beta).$$

We have

$$1 \text{ -< } (\$a \dagger p \dagger \beta)(\$ \sim \beta \dagger \$p)c \text{ -< } (\$a \dagger p \dagger \$p)c.$$

The same with negatives.

Peirce: CP 3.346 Cross-Ref:††

3. **Premises:** $1 \rightarrow \mathbf{a}(p \rightarrow \beta)$ $1 \rightarrow \mathbf{c}(p \rightarrow \sim\beta)$.

$1 \rightarrow \mathbf{a}(p \rightarrow \beta)(\sim\beta \rightarrow \mathbf{p})\mathbf{c} \rightarrow \mathbf{a}(p \rightarrow \mathbf{p})\mathbf{c}$.

The same with negatives.

Peirce: CP 3.346 Cross-Ref:††

4. **Premises:** $1 \rightarrow \mathbf{a} \rightarrow p \rightarrow \beta$ $1 \rightarrow (\mathbf{c} \rightarrow p) \rightarrow \sim\beta$.

$1 \rightarrow (\mathbf{a} \rightarrow p \rightarrow \beta) \rightarrow \sim\beta(\mathbf{p} \rightarrow \mathbf{c}) \rightarrow (\mathbf{a} \rightarrow p \rightarrow \beta \rightarrow \sim\beta)(\mathbf{p} \rightarrow \mathbf{c}) \rightarrow (\mathbf{a} \rightarrow p)(\mathbf{p} \rightarrow \mathbf{c})$

If one of the premisses, say the first, were negative, we should obtain a similar conclusion --

$1 \rightarrow (\mathbf{a} \rightarrow \sim p)(\mathbf{p} \rightarrow \mathbf{c})$;

but from this again p could be eliminated, giving

$1 \rightarrow \mathbf{a} \rightarrow \mathbf{c}$, or $\sim\mathbf{a} \rightarrow \mathbf{c}$.

Peirce: CP 3.346 Cross-Ref:††

5. **Premises:** $1 \rightarrow \mathbf{a} \rightarrow (p \rightarrow \beta)$ $1 \rightarrow (\mathbf{c} \rightarrow p) \rightarrow \sim\beta$.

$1 \rightarrow \mathbf{a} \rightarrow (p \rightarrow \beta) \rightarrow \beta(\mathbf{p} \rightarrow \mathbf{c}) \rightarrow \mathbf{a} \rightarrow p(\mathbf{p} \rightarrow \mathbf{c})$.

If either premiss were negative, p could be eliminated, giving $1 \rightarrow \mathbf{a} \rightarrow \mathbf{c}$, or some a is c .

Peirce: CP 3.346 Cross-Ref:††

6. **Premises:** $1 \rightarrow (\mathbf{a} \rightarrow p) \rightarrow \beta$ $1 \rightarrow (\mathbf{c} \rightarrow p) \rightarrow \sim\beta$.

$$1 \rightarrow (A \rightarrow p) \rightarrow \sim \beta \rightarrow (p \rightarrow c) \rightarrow (A \rightarrow p) \rightarrow (p \rightarrow c).$$

Peirce: CP 3.346 Cross-Ref:††

7. *Premisses:* $1 \rightarrow A \rightarrow p \rightarrow \beta \quad 1 \rightarrow (C \rightarrow p) \rightarrow \sim \beta.$

$$1 \rightarrow (A \rightarrow p \rightarrow \beta) \rightarrow (\sim \beta \rightarrow p \rightarrow c) \rightarrow A \rightarrow p \rightarrow p \rightarrow c.$$

Peirce: CP 3.346 Cross-Ref:††

8. *Premisses:* $1 \rightarrow A \rightarrow (p \rightarrow \beta) \quad 1 \rightarrow C \rightarrow p \rightarrow \sim \beta.$

$$1 \rightarrow A \rightarrow (p \rightarrow \beta) \rightarrow (\sim \beta \rightarrow p \rightarrow c) \rightarrow A \rightarrow (p \rightarrow p \rightarrow c).$$

Peirce: CP 3.346 Cross-Ref:††

9. *Premisses:* $1 \rightarrow (A \rightarrow p) \rightarrow \beta \quad 1 \rightarrow C \rightarrow p \rightarrow \sim \beta.$

$$1 \rightarrow (A \rightarrow p) \rightarrow \beta \rightarrow (\sim \beta \rightarrow p \rightarrow c) \rightarrow (A \rightarrow p) \rightarrow p \rightarrow c.$$

If one premiss is negative, we have the further conclusion $1 \rightarrow A \rightarrow c.$

Peirce: CP 3.346 Cross-Ref:††

10. *Premisses:* $1 \rightarrow A \rightarrow p \rightarrow \beta \quad 1 \rightarrow C \rightarrow p \rightarrow \sim \beta.$

$$1 \rightarrow (A \rightarrow p \rightarrow \beta) \rightarrow (\sim \beta \rightarrow p \rightarrow c) \rightarrow A \rightarrow p \rightarrow p \rightarrow c.$$

Peirce: CP 3.346 Cross-Ref:††

11. *Premisses:* $1 \rightarrow A \rightarrow p \rightarrow \beta \quad 1 \rightarrow C \rightarrow p \rightarrow \sim \beta$

$$1 \rightarrow (A \rightarrow p \rightarrow \beta) \rightarrow (\sim \beta \rightarrow p \rightarrow c) \rightarrow (A \rightarrow p) \rightarrow p \rightarrow c.$$

We might also conclude

$$1 \rightarrow \mathcal{A} \dagger p \dagger n \mathcal{P} \dagger c;$$

but this conclusion is an immediate inference from the other; for

$$(\mathcal{A} \dagger p) \mathcal{P} \dagger c \rightarrow (\mathcal{A} \dagger p)(1 \dagger n) \mathcal{P} \dagger c \rightarrow (\mathcal{A} \dagger p) 1 \dagger n \mathcal{P} \dagger c \rightarrow \mathcal{A} \dagger p \dagger n \mathcal{P} \dagger c.$$

If one premiss is negative, we have the further conclusion $1 \rightarrow \mathcal{A} \dagger c$.

Peirce: CP 3.346 Cross-Ref: ††

$$12. \text{Premisses: } 1 \rightarrow \mathcal{A}(p \dagger \beta) \quad 1 \rightarrow \mathcal{C} \dagger p \sim \beta.$$

$$1 \rightarrow \mathcal{A}(p \dagger \beta)(\sim \beta \mathcal{P} \dagger c) \rightarrow \mathcal{A}(p \mathcal{P} \dagger c).$$

If one premiss is negative, we have the further inference $1 \rightarrow \mathcal{A} c$.

Peirce: CP 3.346 Cross-Ref: ††

$$13. \text{Premisses: } 1 \rightarrow (\mathcal{A} \dagger p) \beta \quad 1 \rightarrow \mathcal{C} \dagger p \sim \beta.$$

$$1 \rightarrow (\mathcal{A} \dagger p) \beta (\sim \beta \dagger c) \rightarrow (\mathcal{A} \dagger p)(n \mathcal{P} \dagger c).$$

Peirce: CP 3.346 Cross-Ref: ††

$$14. \text{Premisses: } 1 \rightarrow \mathcal{A} p \dagger \beta \quad 1 \rightarrow \mathcal{C} \dagger p \sim \beta.$$

$$1 \rightarrow (\mathcal{A} p \dagger \beta)(\sim \beta \mathcal{P} \dagger c) \rightarrow \mathcal{A} p \mathcal{P} \dagger c.$$

If one premiss is negative, we have the further spurious †1 inference $1 \rightarrow \mathcal{A} n \dagger c$.

Peirce: CP 3.346 Cross-Ref: ††

$$15. \text{Premisses: } 1 \rightarrow \mathcal{A} \dagger p \beta \quad 1 \rightarrow \mathcal{C} \dagger p \sim \beta.$$

$$1 \rightarrow (\mathcal{A} \dagger p \beta)(\sim \beta \mathcal{P} \dagger c) \rightarrow \mathcal{A} \dagger p(n \mathcal{P} \dagger c).$$

We can also infer $1 \rightarrow (\mathcal{A} \dagger p n) \mathcal{P} \dagger c$.

Peirce: CP 3.346 Cross-Ref: ††

16. *Premisses:* $1 \rightarrow \mathcal{A} \dagger p \dagger \beta$ $1 \rightarrow \mathcal{C} p \sim \beta$.

$1 \rightarrow (\mathcal{A} \dagger p \dagger \beta) \mathcal{P} \sim \beta \mathcal{P} c \rightarrow (\mathcal{A} \dagger p) \mathcal{P} c$.

If one premiss is negative, we can further infer $1 \rightarrow \mathcal{A} c$.

Peirce: CP 3.346 Cross-Ref: ††

17. *Premisses:* $1 \rightarrow \mathcal{A} (p \dagger \beta)$ $1 \rightarrow \mathcal{C} p \sim \beta$.

$1 \rightarrow \mathcal{A} (p \dagger \beta) \mathcal{P} \sim \beta \mathcal{P} c \rightarrow \mathcal{A} p \mathcal{P} c$.

If one premiss is negative, we have the further spurious conclusion $1 \rightarrow \mathcal{A} n c$.

Peirce: CP 3.346 Cross-Ref: ††

18. *Premisses:* $1 \rightarrow (\mathcal{A} \dagger p) \beta$ $1 \rightarrow \mathcal{C} p \sim \beta$.

$1 \rightarrow (\mathcal{A} \dagger p) \beta \mathcal{P} \sim \beta \mathcal{P} c \rightarrow (\mathcal{A} \dagger p) n \mathcal{P} c$.

Peirce: CP 3.346 Cross-Ref: ††

19. *Premisses:* $1 \rightarrow \mathcal{A} p \dagger \beta$ $1 \rightarrow \mathcal{C} p \sim \beta$.

$1 \rightarrow (\mathcal{A} p \dagger \beta) \mathcal{P} \sim \beta \mathcal{P} c \rightarrow \mathcal{A} p \mathcal{P} c$.

If one premiss is negative, we further conclude $1 \rightarrow \mathcal{A} n c$.

Peirce: CP 3.346 Cross-Ref: ††

20. *Premisses:* $1 \rightarrow \mathcal{A} \dagger p \beta$ $1 \rightarrow \mathcal{C} p \sim \beta$.

$1 \rightarrow (\mathcal{A} \dagger p \beta) \mathcal{P} \sim \beta \mathcal{P} c \rightarrow (\mathcal{A} \dagger p n) \mathcal{P} c$.

Peirce: CP 3.346 Cross-Ref:††

21. *Premises:* $1 \rightarrow \alpha p \beta$ $1 \rightarrow \beta c p \sim \beta$.

$1 \rightarrow \alpha p \beta \sim \beta p c \rightarrow \alpha p n p c$.

Peirce: CP 3.347 Cross-Ref:††

347. When we have to do with particular propositions, we have the proposition $\infty \sim \rightarrow 0$, or "something exists"; for every particular proposition implies this. Then every proposition can be put into one or other of the four forms

$\infty \rightarrow 0 \dagger l \dagger 0$

$\infty \rightarrow (0 \dagger l) \infty$

$\infty \rightarrow 0 \dagger l \infty$

$\infty \rightarrow \infty l \infty$

Each of these propositions immediately follows from the one above it. The *enveloped* expressions which form the predicates have the remarkable property that each is either 0 or ∞ . This fact gives extraordinary freedom in the use of the formulæ. In particular, since if anything not zero is included under such an expression, the whole universe is included, it will be quite unnecessary to write the $\infty \rightarrow$ which begins every proposition.

Peirce: CP 3.348 Cross-Ref:††

348. Suppose that *f* and *g* are general relatives signifying relations of things to times. Then, Dr. Mitchell's †1 six forms of two dimensional propositions appear thus:

$F[11] = 0 \dagger f \dagger 0$

$F[1v] = 0 \dagger f \infty$

$F[u1] = \alpha f \dagger 0$

$F[1v'] = (0 \dagger f) \infty$

$F[u'1] = \infty (f \dagger 0)$

$F[u v] = \alpha f \infty$.

It is obvious that $l \dagger 0 \rightarrow l$, for

$$l \dagger 0 \text{ -< } (l \dagger 0)^\infty \text{ -< } l \dagger 0 \infty l \text{ -< } l \dagger n \text{ -< } l.$$

If then we have $0 \dagger f \dagger 0$ as one premiss, and the other contains g , we may substitute for g the product (f, g) .

$$g \text{ -< } g, \infty \text{ -< } g, (0 \dagger f \dagger 0) \text{ -< } g f.$$

Peirce: CP 3.349 Cross-Ref: ††
349. From the two premisses

$$\infty(f \dagger 0) \text{ and } 0 \dagger g \infty,$$

by the application of the formulæ

$$l s, (b \dagger \sim s) \text{ -< } (l, b) s$$

$$s l, (\sim s \dagger b) \text{ -< } s(l, b),$$

we have

$$\{\infty(f \dagger 0)\}, (0 \dagger g \infty) \text{ -< } \infty\{(f \dagger 0), g \infty\} \text{ -< } \infty(f, g) \infty.$$

These formulæ give the first column of Dr. Mitchell's rule on page 90.

Peirce: CP 3.350 Cross-Ref: ††
350. The following formulæ may also be applied:

1. $(0 \dagger f \dagger 0), (0 \dagger g \dagger 0) = 0 \dagger (f, g) \dagger 0.$
2. $(0 \dagger f)^\infty (0 \dagger g \dagger 0) \text{ -< } (0 \dagger f)(g \dagger 0).$
3. $(0 \dagger f)^\infty \infty(g \dagger 0) = (0 \dagger f)(g \dagger 0) + (0 \dagger f)n(g \dagger 0).$
4. $(0 \dagger f)^\infty (0 \dagger g)^\infty \text{ -< } (0 \dagger f)g^\infty.$
5. $(0 \dagger f \dagger 0) (0 \dagger g \infty) = 0 \dagger (g f, f) \dagger 0.$

6. $(0 \dagger f)^\infty (0 \dagger g)^\infty = (0 \dagger g f f)^\infty$.
7. $(0 \dagger f)^\infty, (0 \dagger g)^\infty = (0 \dagger f, g)^\infty$.
8. $(0 \dagger f)^\infty (0 \dagger g)^\infty = 0 \dagger (f g, g f)^\infty$.
9. $(0 \dagger f)^\infty, (0 \dagger g)^\infty = 0 \dagger f^\infty, g^\infty$.
10. $(0 \dagger f \dagger 0)^\infty g^\infty = 0 \dagger (f g f) \dagger 0$.
11. $(0 \dagger f)^\infty \infty g^\infty = (0 \dagger f) g^\infty (0 \dagger f) n g^\infty$.
12. $(0 \dagger f)^\infty \infty g^\infty = (0 \dagger f g)^\infty + (0 \dagger f n) g^\infty$
13. $\alpha f^\infty \infty g^\infty = \alpha f g^\infty + \infty f n g^\infty$.

Peirce: CP 3.351 Cross-Ref: ††

351. When the relative and non-relative operations occur together, the rules of the calculus become pretty complicated. In these cases, as well as in such as involve *plural* relations (subsisting between three or more objects), it is often advantageous to recur to the numerical coefficients mentioned in 329. Any proposition whatever is equivalent to saying that some complexus of aggregates †P1 and products of such numerical coefficients is greater than zero. Thus,

$$\Sigma[i] \Sigma[j] l[i j] > 0$$

means that something is a lover of something; and

$$\pi[i] \Sigma[j] l[i j] > 0$$

means that everything is a lover of something. We shall, however, naturally omit, in writing the inequalities, the > 0 which terminates them all; and the above two propositions will appear as

$$\Sigma[i] \Sigma[j] l[i j] \text{ and } \pi[i] \Sigma[j] l[i j].$$

Peirce: CP 3.352 Cross-Ref: ††

352. The following are other examples:

$$\pi[i] \Sigma[j] (l) [i j] (b) [i j]$$

means that everything is at once a lover and a benefactor of something.

$$\pi[i]\Sigma[j](l)[i\ j](b)[j\ i]$$

means that everything is a lover of a benefactor of itself.

$$\Sigma[i]\Sigma[k]\pi[j](l)[i\ j] + b[j\ k])$$

means that there is something which stands to something in the relation of loving everything except benefactors of it. †1

Peirce: CP 3.353 Cross-Ref: ††

353. Let α denote the triple relative "accuser to -- of --," and ε the triple relative "excuser to -- of --." Then,

$$\Sigma[i]\pi[j]\Sigma[k](\alpha)[i\ j\ k](\varepsilon)[j\ k\ i]$$

means that an individual i can be found, such, that taking any individual whatever, j , it will always be possible so to select a third individual, k , that i is an accuser to j of k , and j an excuser to k of i .

Peirce: CP 3.354 Cross-Ref: ††

354. Let $\{p\}$ denote "preferrer to -- of --." Then,

$$\pi[i]\Sigma[j]\Sigma[k](\alpha)[i\ j\ k](\varepsilon)[j\ k\ i] + \{p\}[k\ i\ j])$$

means that, having taken any individual i whatever, it is always possible so to select two, j and k , that i is an accuser to j of k , and also is either excused by j to k or is something to which j is preferred by k .

Peirce: CP 3.355 Cross-Ref: ††

355. When we have a number of premisses expressed in this manner, the conclusion is readily deduced by the use of the following simple rules. In the first place, we have

$$\Sigma[i]\pi[j] \prec \pi[j]\Sigma[i].$$

In the second place, we have the formulæ

$$\{\pi[i]\phi(i)\} \{\pi[j]\{Ps\}(j)\} = \pi[i]\{\phi(i)\cdot\{Ps\}(i)\}.$$

$$\{\pi[i]\phi(i)\} \{\Sigma[j]\{Ps\}(j)\} \prec \Sigma[i]\{\phi(i)\cdot\{Ps\}(i)\}.$$

In the third place, since the numerical coefficients are all either *zero* or *unity*, the Boolean calculus is applicable to them.

Peirce: CP 3.356 Cross-Ref: ††

356. The following is one of the simplest possible examples. Required to eliminate *servant* from these two premisses:

First premiss. There is somebody who accuses everybody to everybody, unless the latter is loved by some person that is servant of all not accused to him. †1

Second premiss. There are two persons, the first of whom excuses everybody to everybody, unless the unexcused be benefited by, without the person to whom he is unexcused being a servant of, the second.

These premisses may be written thus:

$$\Sigma[h]\pi[i]\Sigma[j]\pi[k](\alpha[h\ i\ k] + s[j\ k]l[j\ i])$$

$$\Sigma[u]\Sigma[v]\pi[x]\pi[y](\varepsilon[u\ y\ x] + \sim s[y\ v]b[v\ x]).$$

The second yields the immediate inference,

$$\pi[x]\Sigma[u]\pi[y]\Sigma[v](\varepsilon[u\ y\ x] + \sim s[y\ v]b[v\ x]).$$

Combining this with the first, we have

$$\Sigma[x]\Sigma[u]\Sigma[y]\Sigma[v](\varepsilon[u\ y\ x] + \sim s[y\ v]b[v\ x])(a[x\ u\ v] + s[y\ v]l[y\ u]).$$

Finally, applying the Boolean calculus, we deduce the desired conclusion

$$\Sigma[x]\Sigma[u]\Sigma[y]\Sigma[v](\varepsilon[u\ y\ x] + \alpha[x\ u\ v] + \varepsilon[u\ y\ x]l[y\ u] + \alpha[x\ u\ v]b[v\ x]).$$

The interpretation of this is that either there is somebody excused by a person to whom he accuses somebody, or somebody excuses somebody to his (the excuser's) lover, or somebody accuses his own benefactor.

Peirce: CP 3.357 Cross-Ref:††

357. The procedure may often be abbreviated by the use of operations intermediate between π and Σ . Thus, we may use π' , π'' , etc. to mean the products for all individuals except one, except two, etc. Thus,

$$\pi[i]'\pi[j]''(l[ij] + b[ji])$$

will mean that every person except one is a lover of everybody except its benefactors, and at most two non-benefactors. In the same manner, Σ' , Σ'' , etc., will denote the sums of all products of two, of all products of three, etc. Thus,

$$\Sigma[i]''(l[ii])$$

will mean that there are at least three things in the universe that are lovers of themselves. It is plain that if $m < n$, we have

$$\pi m \prec \pi n \quad \Sigma n \prec \Sigma m.$$

$$(\pi[i]m\phi i)(\Sigma[j]n\{Ps\}j) \prec \Sigma[i]n-m(\phi i \cdot \{Ps\}i)$$

$$(\pi[i]m\phi i)(\pi[j]n\{Ps\}j) \prec \pi[i]m+n(\phi i \cdot \{Ps\}i)$$

Peirce: CP 3.358 Cross-Ref:††

358. Mr. Schlötel has written to the London Mathematical Society,†1 accusing me of having, in my *Algebra of Logic*, plagiarized from his writings. He has also written to me to inform me that he has read that Memoir with "heitere Ironie," and that Professor Drobisch, the Berlin Academy, and I constitute a "liederliche Kleeblatt," with many other things of the same sort. Up to the time of publishing my Memoir, I had never seen any of Mr. Schlötel's writings; I have since procured his *Logik*,†2 and he has been so obliging as to send me two cuttings from his papers, thinking, apparently, that I might be curious to see the passages that I had appropriated. But having examined these productions, I find no thought in them that I ever did, or ever should be likely to put forth as my own.

Peirce: CP 3.359 Cross-Ref:††
XIII

ON THE ALGEBRA OF LOGIC

A CONTRIBUTION TO THE PHILOSOPHY OF NOTATION †1

§1. THREE KINDS OF SIGNS †2

359. Any character or proposition either concerns one subject, two subjects, or a plurality of subjects. For example, one particle has mass, two particles attract one another, a particle revolves about the line joining two others. A fact concerning two subjects is a dual character or relation; but a relation which is a mere combination of two independent facts concerning the two subjects may be called *degenerate*, just as two lines are called a degenerate conic. In like manner a plural character or conjoint relation is to be called degenerate if it is a mere compound of dual characters.

Peirce: CP 3.360 Cross-Ref:††

360. A sign is in a conjoint relation to the thing denoted and to the mind. If this triple relation is not of a degenerate species, the sign is related to its object only in consequence of a mental association, and depends upon a habit. Such signs are always abstract and general, because habits are general rules to which the organism has become subjected. They are, for the most part, conventional or arbitrary. They include all general words, the main body of speech, and any mode of conveying a judgment. For the sake of brevity I will call them *tokens*. †3

Peirce: CP 3.361 Cross-Ref:††

361. But if the triple relation between the sign, its object, and the mind, is degenerate, then of the three pairs

sign object

sign mind

object mind

two at least are in dual relations which constitute the triple relation. One of the connected pairs must consist of the sign and its object, for if the sign were not related to its object except by the mind thinking of them separately, it would not fulfill the function of a sign at all. Supposing, then, the relation of the sign to its object does not

lie in a mental association, there must be a direct dual relation of the sign to its object independent of the mind using the sign. In the second of the three cases just spoken of, this dual relation is not degenerate, and the sign signifies its object solely by virtue of being really connected with it. Of this nature are all natural signs and physical symptoms. I call such a sign an *index*, a pointing finger being the type of the class.

Peirce: CP 3.361 Cross-Ref:††

The index asserts nothing; it only says "There!" It takes hold of our eyes, as it were, and forcibly directs them to a particular object, and there it stops.

Demonstrative and relative pronouns are nearly pure indices, because they denote things without describing them; so are the letters on a geometrical diagram, and the subscript numbers which in algebra distinguish one value from another without saying what those values are.

Peirce: CP 3.362 Cross-Ref:††

362. The third case is where the dual relation between the sign and its object is degenerate and consists in a mere resemblance between them. I call a sign which stands for something merely because it resembles it, an *icon*. Icons are so completely substituted for their objects as hardly to be distinguished from them. Such are the diagrams of geometry. A diagram, indeed, so far as it has a general signification, is not a pure icon; but in the middle part of our reasonings we forget that abstractness in great measure, and the diagram is for us the very thing. So in contemplating a painting, there is a moment when we lose the consciousness that it is not the thing, the distinction of the real and the copy disappears, and it is for the moment a pure dream -- not any particular existence, and yet not general. At that moment we are contemplating an *icon*.

Peirce: CP 3.363 Cross-Ref:††

363. I have taken pains to make my distinction †P1 of icons, indices, and tokens clear, in order to enunciate this proposition: in a perfect system of logical notation signs of these several kinds must all be employed. Without tokens there would be no generality in the statements, for they are the only general signs; and generality is essential to reasoning. Take, for example, the circles by which Euler represents the relations of terms. They well fulfill the function of icons, but their want of generality and their incompetence to express propositions must have been felt by everybody who has used them.†1 Mr. Venn †2 has, therefore, been led to add shading to them; and this shading is a conventional sign of the nature of a token. In algebra, the letters, both quantitative and functional, are of this nature. But tokens alone do not state what is the subject of discourse; and this can, in fact, not be described in general terms; it can only be indicated. The actual world cannot be distinguished from a world of imagination by any description. Hence the need of pronoun and indices, and the more complicated the subject the greater the need of them. The introduction of indices into the algebra of logic is the greatest merit of Mr. Mitchell's system.†P1 He writes $F[1]$ to mean that the proposition F is true of every object in the universe, and $F[u]$ to mean that the same is true of some object.†3 This distinction can only be made in some such way as this. Indices are also required to show in what manner other signs are connected together. With these two kinds of signs alone any proposition can be expressed; but it cannot be reasoned upon, for reasoning consists in the observation that where certain relations subsist certain others are found, and it accordingly requires the exhibition of the relations reasoned within an icon. It has long been a puzzle how it could be that, on the one hand, mathematics is purely deductive in its nature, and draws its conclusions apodictically, while on the other hand, it presents as

rich and apparently unending a series of surprising discoveries as any observational science. Various have been the attempts to solve the paradox by breaking down one or other of these assertions, but without success. The truth, however, appears to be that all deductive reasoning, even simple syllogism, involves an element of observation; namely, deduction consists in constructing an icon or diagram the relations of whose parts shall present a complete analogy with those of the parts of the object of reasoning, of experimenting upon this image in the imagination, and of observing the result so as to discover unnoticed and hidden relations among the parts. For instance, take the syllogistic formula,

All *M* is *P*

S is *M*

∴ *S* is *P*.

This is really a diagram of the relations of *S*, *M*, and *P*. The fact that the middle term occurs in the two premisses is actually exhibited, and this must be done or the notation will be of no value. As for algebra, the very idea of the art is that it presents formulæ which can be manipulated, and that by observing the effects of such manipulation we find properties not to be otherwise discerned. In such manipulation, we are guided by previous discoveries which are embodied in general formulæ. These are patterns which we have the right to imitate in our procedure, and are the *icons par excellence* of algebra. The letters of applied algebra are usually tokens, but the *x*, *y*, *z*, etc., of a general formula, such as

$$(x+y)z = xz + yz,$$

are blanks to be filled up with tokens, they are indices of tokens. Such a formula might, it is true, be replaced by an abstractly stated rule (say that multiplication is distributive); but no application could be made of such an abstract statement without translating it into a sensible image.

Peirce: CP 3.364 Cross-Ref:††

364. In this paper, I purpose to develop an algebra adequate to the treatment of all problems of deductive logic, showing as I proceed what kinds of signs have necessarily to be employed at each stage of the development. I shall thus attain three objects. The first is the extension of the power of logical algebra over the whole of its proper realm. The second is the illustration of principles which underlie all algebraic notation. The third is the enumeration of the essentially different kinds of necessary inference; for when the notation which suffices for exhibiting one inference is found inadequate for explaining another, it is clear that the latter involves an inferential element not present to the former. Accordingly, the procedure contemplated should result in a list of categories of reasoning, the interest of which is not dependent upon the algebraic way of considering the subject. I shall not be able to perfect the algebra sufficiently to give facile methods of reaching logical conclusions: I can only give a method by which any legitimate conclusion may be reached and any fallacious one

avoided. But I cannot doubt that others, if they will take up the subject, will succeed in giving the notation a form in which it will be highly useful in mathematical work. I even hope that what I have done may prove a first step toward the resolution of one of the main problems of logic, that of producing a method for the discovery of methods in mathematics.

Peirce: CP 3.365 Cross-Ref:††
 §2. NON-RELATIVE LOGIC

365. According to ordinary logic, a proposition is either true or false, and no further distinction is recognized. This is the descriptive conception, as the geometers say; the metric conception would be that every proposition is more or less false, and that the question is one of amount. At present we adopt the former view.

Peirce: CP 3.366 Cross-Ref:††

366. Let propositions be represented by quantities. Let \mathbf{v} and \mathbf{f} be two constant values, and let the value of the quantity representing a proposition be \mathbf{v} if the proposition is true and be \mathbf{f} if the proposition is false. Thus, \mathbf{x} being a proposition, the fact that \mathbf{x} is either true or false is written

$$(\mathbf{x} - \mathbf{f})(\mathbf{v} - \mathbf{x}) = 0. \dagger 1$$

So $(\mathbf{x} - \mathbf{f})(\mathbf{v} - \mathbf{y}) = 0$

will mean that either \mathbf{x} is false or \mathbf{y} is true. This may be said to be the same as 'if \mathbf{x} is true, \mathbf{y} is true.' A hypothetical proposition, generally, is not confined to stating what actually happens, but states what is invariably true throughout a universe of possibility. The present proposition is, however, limited to that one individual state of things, the Actual.

Peirce: CP 3.367 Cross-Ref:††

367. We are, thus, already in possession of a logical notation, capable of working syllogism. Thus, take the premisses, 'if \mathbf{x} is true, \mathbf{y} is true,' and 'if \mathbf{y} is true, \mathbf{z} is true.' These are written

$$(\mathbf{x} - \mathbf{f})(\mathbf{v} - \mathbf{y}) = 0$$

$$(\mathbf{y} - \mathbf{f})(\mathbf{v} - \mathbf{z}) = 0.$$

Multiply the first by $(\mathbf{v} - \mathbf{z})$ and the second by $(\mathbf{x} - \mathbf{f})$ and add. We get

$$(\mathbf{x} - \mathbf{f})(\mathbf{v} - \mathbf{f})(\mathbf{v} - \mathbf{z}) = 0,$$

or dividing by $\mathbf{v} - \mathbf{f}$, which cannot be 0,

$$(\mathbf{x} - \mathbf{f})(\mathbf{v} - \mathbf{z}) = 0;$$

and this states the syllogistic conclusion, "if \mathbf{x} is true, \mathbf{z} is true."

Peirce: CP 3.368 Cross-Ref:††

368. But this notation shows a blemish in that it expresses propositions in two distinct ways, in the form of quantities, and in the form of equations; and the quantities are of two kinds, namely those which must be either equal to \mathbf{f} or to \mathbf{v} , and those which are equated to **zero**. To remedy this, let us discard the use of equations, and perform no operations which can give rise to any values other than \mathbf{f} and \mathbf{v} .

Peirce: CP 3.369 Cross-Ref:††

369. Of operations upon a simple variable, we shall need but one. For there are but two things that can be said about a single proposition, by itself; that it is true and that it is false,

$$\mathbf{x} = \mathbf{v} \text{ and } \mathbf{x} = \mathbf{f}.$$

The first equation is expressed by \mathbf{x} itself, the second by any function, ϕ , of \mathbf{x} , fulfilling the conditions

$$\phi\mathbf{v} = \mathbf{f} \quad \phi\mathbf{f} = \mathbf{v}.$$

The simplest solution of these equations is

$$\phi\mathbf{x} = \mathbf{f} + \mathbf{v} - \mathbf{x}.$$

A product of n factors of the two forms $(\mathbf{x} - \mathbf{f})$ and $(\mathbf{v} - \mathbf{y})$, if not zero, equals $(\mathbf{v} - \mathbf{f})n$. Write \mathbf{P} for the product. Then $\mathbf{v} - (\mathbf{P}/((\mathbf{v} - \mathbf{f})n-1))$ is the simplest function of the variables which becomes \mathbf{v} when the product vanishes and \mathbf{f} when it does not. By this means any proposition relating to a single individual can be expressed.

Peirce: CP 3.370 Cross-Ref:††

370. If we wish to use algebraical signs with their usual significations, the meanings of the operations will entirely depend upon those of \mathbf{f} and \mathbf{v} . Boole †1 chose $\mathbf{v} = 1$, $\mathbf{f} = 0$. This choice gives the following forms:

$$\mathbf{f} + \mathbf{v} - \mathbf{x} = 1 - \mathbf{x}$$

which is best written $\sim\mathbf{x}$.

$$\mathbf{v} - ((\mathbf{x}-\mathbf{f})(\mathbf{v}-\mathbf{y})/(\mathbf{v}-\mathbf{f})) = 1 - \mathbf{x} + \mathbf{x} \mathbf{y} = \sim(\mathbf{x}\sim\mathbf{y}).$$

$$\mathbf{v} - ((\mathbf{v}-\mathbf{x})(\mathbf{v}-\mathbf{y})/(\mathbf{v}-\mathbf{f})) = \mathbf{x} + \mathbf{y} - \mathbf{x} \mathbf{y} \dagger 2$$

$$\mathbf{v} - ((\mathbf{v}-\mathbf{x})(\mathbf{v}-\mathbf{y})(\mathbf{v}-\mathbf{z})/((\mathbf{v}-\mathbf{f})^2)) = \mathbf{x} + \mathbf{y} + \mathbf{z} - \mathbf{x} \mathbf{y} - \mathbf{x} \mathbf{z} - \mathbf{y} \mathbf{z} + \mathbf{x} \mathbf{y} \mathbf{z}$$

$$\mathbf{v} - ((\mathbf{x}-\mathbf{f})(\mathbf{y}-\mathbf{f})/(\mathbf{v}-\mathbf{f})) = 1 - \mathbf{x} \mathbf{y} = \sim\mathbf{x}\mathbf{y} \dagger 3$$

Peirce: CP 3.371 Cross-Ref: ††

371. It appears to me that if the strict Boolean system is used, the sign + ought to be altogether discarded. Boole and his adherent, Mr. Venn (whom I never disagree with without finding his remarks profitable), prefer to write $\mathbf{x} + \sim\mathbf{x} \mathbf{y}$ in place of $\sim(\mathbf{x}\sim\mathbf{y})$. I confess I do not see the advantage of this, for the distributive principle holds equally well when written

$$\sim(\sim\mathbf{x}\sim\mathbf{y})\mathbf{z} = \sim(\sim(\mathbf{x} \mathbf{z})\sim(\mathbf{y} \mathbf{z})) \dagger 4$$

$$\sim(\sim(\mathbf{x} \mathbf{y})\sim\mathbf{z}) = \sim(\sim\mathbf{x}\sim\mathbf{z}).\sim(\sim\mathbf{y}\sim\mathbf{z}). \dagger 5$$

The choice of $\mathbf{v} = 1$, $\mathbf{f} = 0$, is agreeable to the received measurement of probabilities. But there is no need, and many times no advantage, in measuring probabilities in this way. I presume that Boole, in the formation of his algebra, at first considered the letters as denoting propositions or events. As he presents the subject, they are class-names; but it is not necessary so to regard them. Take, for example, the equation

$$\mathbf{t} = \mathbf{n} + \mathbf{h} \mathbf{f},$$

which might mean that the body of taxpayers is composed of all the natives, together with householding foreigners. We might reach the signification by either of the following systems of notation, which indeed differ grammatically rather than logically.

Sign.	Signification. 1st System.	Signification. 2d System.
<i>t</i>	Taxpayer.	He is a Taxpayer.
<i>n</i>	Native.	He is a Native.
<i>h</i>	Householder.	He is a Householder.
<i>f</i>	Foreigner.	He is a Foreigner.

There is no *index* to show who the "He" of the second system is, but that makes no difference. To say that he is a taxpayer is equivalent to saying that he is a native or is a householder and a foreigner. In this point of view, the constants 1 and 0 are simply the probabilities, to one who knows, of what is true and what is false; and thus unity is conferred upon the whole system.

Peirce: CP 3.372 Cross-Ref:††

372. For my part, I prefer for the present not to assign determinate values to *f* and *v*, nor to identify the logical operations with any special arithmetical ones, leaving myself free to do so hereafter in the manner which may be found most convenient. Besides, the whole system of importing arithmetic into the subject is artificial, and modern Boolians do not use it. The algebra of logic should be self-developed, and arithmetic should spring out of logic instead of reverting to it. Going back to the beginning, let the writing of a letter by itself mean that a certain proposition is true. This letter is a *token*. There is a general understanding that the actual state of things or some other is referred to. This understanding must have been established by means of an *index*, and to some extent dispenses with the need of other indices. The denial of a proposition will be made by writing a line over it.

Peirce: CP 3.373 Cross-Ref:††

373. I have elsewhere †1 shown that the fundamental and primary mode of relation between two propositions is that which we have expressed by the form

$$v - ((x-f)(v-y)/(v-f)).$$

We shall write this $x \prec y$,

which is also equivalent to $(x-f)(v-y) = 0$.

It is stated above that this means "if *x* is true, *y* is true." But this meaning is greatly modified by the circumstance that only the actual state of things is referred to.

Peirce: CP 3.374 Cross-Ref:††

374. To make the matter clear, it will be well to begin by defining the meaning of a hypothetical proposition, in general. What the usages of language may be does

not concern us; language has its meaning modified in technical logical formulæ as in other special kinds of discourse. The question is what is the sense which is most usefully attached to the hypothetical proposition in logic? Now, the peculiarity of the hypothetical proposition is that it goes out beyond the actual state of things and declares what *would* happen were things other than they are or may be. The utility of this is that it puts us in possession of a rule, say that "if *A* is true, *B* is true," such that should we hereafter learn something of which we are now ignorant, namely that *A* is true, then, by virtue of this rule, we shall find that we know something else, namely, that *B* is true. There can be no doubt that the Possible, in its primary meaning, is that which may be true for aught we know, that whose falsity we do not know.†1 The purpose is subserved, then, if throughout the whole range of possibility, in every state of things in which *A* is true, *B* is true too. The hypothetical proposition may therefore be falsified by a single state of things, but only by one in which *A* is true while *B* is false. States of things in which *A* is false, as well as those in which *B* is true, cannot falsify it. If, then, *B* is a proposition true in every case throughout the whole range of possibility, the hypothetical proposition, taken in its logical sense, ought to be regarded as true, whatever may be the usage of ordinary speech. If, on the other hand, *A* is in no case true, throughout the range of possibility, it is a matter of indifference whether the hypothetical be understood to be true or not, since it is useless. But it will be more simple to class it among true propositions, because the cases in which the antecedent is false do not, in any other case, falsify a hypothetical. This, at any rate, is the meaning which I shall attach to the hypothetical proposition in general, in this paper.

Peirce: CP 3.375 Cross-Ref:††

375. The range of possibility is in one case taken wider, in another narrower; in the present case it is limited to the actual state of things. Here, therefore, the proposition

$$a \text{ -< } b$$

is true if *a* is false or if *b* is true, but is false if *a* is true while *b* is false. But though we limit ourselves to the actual state of things, yet when we find that a formula of this sort is true by logical necessity, it becomes applicable to any single state of things throughout the range of logical possibility. For example, we shall see that from $x \text{ --< } y$ we can infer $z \text{ -< } x$. This does not mean that because in the actual state of things *x* is true and *y* false, therefore in every state of things either *z* is false or *x* true; but it does mean that in whatever state of things we find *x* true and *y* false, in that state of things either *z* is false or *x* is true. In that sense, it is not limited to the actual state of things, but extends to any single state of things.

Peirce: CP 3.376 Cross-Ref:††

376. The *first icon* of algebra is contained in the formula of identity

$$x \text{ -< } x.$$

This formula does not of itself justify any transformation, any inference. It only justifies our continuing to hold what we have held (though we may, for instance, forget how we were originally justified in holding it).

Peirce: CP 3.377 Cross-Ref:††

377. The *second icon* is contained in the rule that the several antecedents of a *consequentia* may be transposed; that is, that from

$$x \text{ -< } (y \text{ -< } z)$$

we can pass to $y \text{ -< } (x \text{ -< } z)$.

This is stated in the formula

$$\{x \text{ -< } (y \text{ -< } z)\} \text{ -< } \{y \text{ -< } (x \text{ -< } z)\}.$$

Because this is the case, the brackets may be omitted, and we may write

$$y \text{ -< } x \text{ -< } z.$$

Peirce: CP 3.377 Cross-Ref:††

By the formula of identity

$$(x \text{ -< } y) \text{ -< } (x \text{ -< } y);$$

and transposing the antecedents

$$x \text{ -< } \{(x \text{ -< } y) \text{ -< } y\}$$

or, omitting the unnecessary brackets

$$x \text{ -< } (x \text{ -< } y) \text{ -< } y.$$

This is the same as to say that if in any state of things x is true, and if the proposition "if x , then y " is true, then in that state of things y is true. This is the *modus ponens* of

hypothetical inference, and is the most rudimentary form of reasoning.†1

Peirce: CP 3.378 Cross-Ref:††

378. To say that $(x \text{ -< } x)$ is generally true is to say that it is so in every state of things, say in that in which y is true; so that we may write

$$y \text{ -< } (x \text{ -< } x),$$

and then, by transposition of antecedents,

$$x \text{ -< } (y \text{ -< } x),$$

or from x we may infer $y \text{ -< } x$.

Peirce: CP 3.379 Cross-Ref:††

379. The *third icon* is involved in the principle of the transitivity of the copula, which is stated in the formula

$$(x \text{ -< } y) \text{ -< } (y \text{ -< } z) \text{ -< } x \text{ -< } z. \dagger 2$$

According to this, if in any case y follows from x and z from y , then z follows from x .†3 This is the principle of the syllogism in *Barbara*.

Peirce: CP 3.380 Cross-Ref:††

380. We have already seen that from x follows $y \text{ -< } x$. Hence, by the transitivity of the copula, if from $y \text{ -< } x$ follows z , then from x follows z , or from

$$(y \text{ -< } x) \text{ -< } z$$

follows $x \text{ -< } z$,

or $\{(y \text{ -< } x) \text{ -< } z\} \text{ -< } x \text{ -< } z$.

Peirce: CP 3.381 Cross-Ref:††

381. The original notation $x \text{ -< } y$ served without modification to express the pure formula of identity. An enlargement of the conception of the notation so as to make the terms themselves complex was required to express the principle of the transposition of antecedents; and this new *icon* brought out new propositions. The *third icon* introduces the image of a chain of consequence. We must now again enlarge the notation so as to introduce negation. We have already seen that if a is true, we can write $x \text{ -< } a$, whatever x may be. Let b be such that we can write $b \text{ -< } x$ whatever x may be. Then b is false. We have here a *fourth icon*, which gives a new

sense to several formulæ. Thus the principle of the interchange of antecedents is that from

$$\mathbf{x} \text{ -< } (\mathbf{y} \text{ -< } \mathbf{z})$$

we can infer $\mathbf{y} \text{ -< } (\mathbf{x} \text{ -< } \mathbf{z})$.

Since \mathbf{z} is any proposition we please, this is as much as to say that if from the truth of \mathbf{x} the falsity of \mathbf{y} follows, then from the truth of \mathbf{y} the falsity of \mathbf{x} follows.

Peirce: CP 3.382 Cross-Ref:††
382. Again the formula

$$\mathbf{x} \text{ -< } \{(\mathbf{x} \text{ -< } \mathbf{y}) \text{ -< } \mathbf{y}\}$$

is seen to mean that from \mathbf{x} , we can infer that anything we please follows from that things following from \mathbf{x} , and *a fortiori* from everything following from \mathbf{x} . This is, therefore, to say that from \mathbf{x} follows the falsity of the denial of \mathbf{x} ; which is the principle of contradiction.

Peirce: CP 3.383 Cross-Ref:††
383. Again the formula of the transitivity of the copula, or

$$\{\mathbf{x} \text{ -< } \mathbf{y}\} \text{ -< } \{(\mathbf{y} \text{ -< } \mathbf{z}) \text{ -< } (\mathbf{x} \text{ -< } \mathbf{z})\}$$

is seen to justify the inference

$$\begin{aligned} &\mathbf{x} \text{ -< } \mathbf{y} \\ \therefore &\sim\mathbf{y} \text{ -< } \sim\mathbf{x}. \end{aligned}$$

The same formula justifies the *modus tollens*,

$$\begin{aligned} &\mathbf{x} \text{ -< } \mathbf{y} \\ &\sim\mathbf{y} \\ \therefore &\sim\mathbf{x}. \end{aligned}$$

Peirce: CP 3.383 Cross-Ref:††

So the formula $\{(y \neg x) \neg z\} \neg (x \neg z)$ shows that from the falsity of $y \neg x$ the falsity of x may be inferred.

Peirce: CP 3.383 Cross-Ref:††

All the traditional moods of syllogism can easily be reduced to *Barbara* by this method.

Peirce: CP 3.384 Cross-Ref:††

384. A *fifth icon* is required for the principle of excluded middle and other propositions connected with it. One of the simplest formulæ of this kind is

$$\{(x \neg y) \neg x\} \neg x.$$

This is hardly axiomatical. That it is true appears as follows. It can only be false by the final consequent x being false while its antecedent $(x \neg y) \neg x$ is true. If this is true, either its consequent, x , is true, when the whole formula would be true, or its antecedent $x \neg y$ is false. But in the last case the antecedent of $x \neg y$, that is x , must be true. †P1

Peirce: CP 3.384 Cross-Ref:††

From the formula just given, we at once get

$$\{(x \neg y) \neg \alpha\} \neg x,$$

where the α is used in such a sense that $(x \neg y) \neg \alpha$ means that from $(x \neg y)$ every proposition follows. With that understanding, the formula states the principle of excluded middle, that from the falsity of the denial of x follows the truth of x .

Peirce: CP 3.385 Cross-Ref:††

385. The logical algebra thus far developed contains signs of the following kinds:

First, tokens; signs of simple propositions, as t for 'He is a taxpayer,' etc.

Second, the single operative sign \neg ; also of the nature of a token.

Third, the juxtaposition of the letters to the right and left of the operative sign. This juxtaposition fulfils the function of an index, in indicating the connections of the tokens.

Fourth, the parentheses, subserving the same purpose.

Fifth, the letters α , β , etc. which are indices of no matter what tokens, used for expressing negation.

Sixth, the indices of tokens, x , y , z , etc., used in the general formulæ.

Seventh, the general formulæ themselves, which are *icons*, or exemplars of algebraic proceedings.

Eighth, the fourth *icon* which affords a second interpretation of the general formulæ.

Peirce: CP 3.386 Cross-Ref:††

386. We might dispense with the fifth and eighth species of signs -- the devices by which we express negation -- by adopting a second operational sign $\sim\!-\! $\langle$$, such that

$$x \sim\!-\!\langle y$$

should mean that $x = \mathbf{v}$, $y = \mathbf{f}$. With this, we should require new indices of connections, and new general formulæ. Possibly this might be the preferable notation. We should thus have two operational signs but no sign of negation. The forms of Boolean algebra hitherto used, have either two operational signs and a special sign of negation, or three operational signs. One of the operational signs is in that case superfluous. Thus, in the usual notation we have

$$\sim(x+y) = \sim x \sim y$$

$$\sim x + \sim y = \sim(x y)$$

showing two modes of writing the same fact. The apparent balance between the two sets of theorems exhibited so strikingly by Schröder, arises entirely from this double way of writing everything. But while the ordinary system is not so analytically fitted to its purpose as that here set forth, the character of superfluity here, as in many other cases in algebra, brings with it great facility in working.

Peirce: CP 3.387 Cross-Ref:††

387. The general formulae given above are not convenient in practice. We may dispense with them altogether, as well as with one of the indices of tokens used in them, by the use of the following rules. A proposition of the form

$$x \!-\!\langle y$$

is true if $x = \mathbf{f}$ or $y = \mathbf{v}$. It is only false if $y = \mathbf{f}$ and $x = \mathbf{v}$. A proposition written in the form

$$x \sim\!-\!\langle y$$

is true if $x = \mathbf{v}$ and $y = \mathbf{f}$, and is false if either $x = \mathbf{f}$ or $y = \mathbf{v}$. Accordingly, to find whether a formula is necessarily true substitute \mathbf{f} and \mathbf{v} for the letters and see whether

it can be supposed false by any such assignment of values. Take, for example, the formula

$$(x \rightarrow y) \rightarrow \{(y \rightarrow z) \rightarrow (x \rightarrow z)\}.$$

To make this false we must take

$$\begin{aligned}(x \rightarrow y) &= \mathbf{v} \\ \{(y \rightarrow z) \rightarrow (x \rightarrow z)\} &= \mathbf{f}.\end{aligned}$$

The last gives

$$(y \rightarrow z) = \mathbf{v}, (x \rightarrow z) = \mathbf{f}, x = \mathbf{v}, z = \mathbf{f}.$$

Substituting these values in

$$\begin{aligned}(x \rightarrow y) &= \mathbf{v} \quad (y \rightarrow z) = \mathbf{v} \\ \text{we have } (v \rightarrow y) &= \mathbf{v} \quad (y \rightarrow f) = \mathbf{v},\end{aligned}$$

which cannot be satisfied together.

Peirce: CP 3.388 Cross-Ref:††

388. As another example, required the conclusion from the following premisses: Anyone I might marry would be either beautiful or plain; anyone whom I might marry would be a woman; any beautiful woman would be an ineligible wife; any plain woman would be an ineligible wife. Let

m be anyone whom I might marry,

b, beautiful,

p, plain,

w, woman,

i, ineligible.

Then the premisses are

$$m \text{ -< } (b \text{ -< } f) \text{ -< } p,$$

$$m \text{ -< } w,$$

$$w \text{ -< } b \text{ -< } i,$$

$$w \text{ -< } p \text{ -< } i.$$

Let x be the conclusion. Then,

$$[[m \text{ -< } (b \text{ -< } f) \text{ -< } p]] \text{ -< } (m \text{ -< } w) \text{ -< } (w \text{ -< } b \text{ -< } i) \\ \text{-< } (w \text{ -< } p \text{ -< } i) \text{ -< } x$$

is necessarily true. Now if we suppose $m = v$, the proposition can only be made false by putting $w = v$ and either b or $p = v$. In this case the proposition can only be made false by putting $i = v$. If, therefore, x can only be made f by putting $m = v$, $i = f$, that is if $x = (m \text{ -< } i)$ the proposition is necessarily true.

In this method, we introduce the two special tokens of second intention f and v , we retain two indices of tokens x and y , and we have a somewhat complex *icon*, with a special prescription for its use.

Peirce: CP 3.389 Cross-Ref: ††

389. A better method may be found as follows. We have seen that

$$x \text{ -< } (y \text{ -< } z)$$

may be conveniently written $x \text{ -< } y \text{ -< } z$;

while $(x \text{ -< } y) \text{ -< } z$

ought to retain the parenthesis. Let us extend this rule, so as to be more general, and hold it necessary *always* to include the antecedent in parenthesis.

Thus, let us write $(x) \text{ -< } y$

instead of $x \text{ -< } y$. If now, we merely change the external appearance of two signs; namely, if we use the vinculum instead of the parenthesis, and the sign + in place of -<, we shall have

$$x \text{ -< } y \text{ written } \sim x+y$$

$x \prec y \prec z$ written $\sim x + \sim y + z$

$(x \prec y) \prec z$ written $\sim(\sim x + \sim y) + z, \dagger 1$ etc.

We may further write for $x \sim \prec y, \sim(\sim x + y)$ implying that $x + y \dagger 2$ is an antecedent for whatever consequent may be taken, and the vinculum becomes identified with the sign of negation. We may also use the sign of multiplication as an abbreviation, putting

$$x y = \sim(\sim x + \sim y) = \sim(x \prec \sim y).$$

Peirce: CP 3.390 Cross-Ref: $\dagger \dagger$

390. This subjects addition and multiplication to all the rules of ordinary algebra, and also to the following:

$$y + x \sim x = y \quad y(x + \sim x) = y$$

$$x + \sim x = v \quad \sim x x = f$$

$$x y + z = (x + z)(y + z).$$

Peirce: CP 3.391 Cross-Ref: $\dagger \dagger$

391. To any proposition we have a right to add any expression at pleasure; also to strike out any factor of any term. The expressions for different propositions separately known may be multiplied together. These are substantially Mr. Mitchell's rules of procedure. $\dagger 1$ Thus the premisses of Barbara are

$$\sim x + y \text{ and } \sim y + z.$$

Multiplying these, we get $(\sim x + y)(\sim y + z) = \sim x \sim y + y z$. Dropping $\sim y$ and y we reach the conclusion $\sim x + z$.

Peirce: CP 3.392 Cross-Ref: $\dagger \dagger$

§3. FIRST-INTENTIONAL LOGIC OF RELATIVES

392. The algebra of Boole affords a language by which anything may be expressed which can be said without speaking of more than one individual at a time. It is true that it can assert that certain characters belong to a whole class, but only such characters as belong to each individual separately. The logic of relatives

considers statements involving two and more individuals at once. Indices are here required. Taking, first, a degenerate form of relation, we may write $x[i]y[j]$ to signify that x is true of the individual i while y is true of the individual j . If z be a relative character $z[i]j$ will signify that i is in that relation to j . In this way we can express relations of considerable complexity. Thus, if

1, 2, 3,
4, 5, 6,
7, 8, 9,

are points in a plane, and $l[123]$ signifies that 1, 2, and 3 lie on one line, a well-known proposition of geometry †1 may be written

$l[159] \text{ -< } l[267] \text{ -< } l[348] \text{ -< } l[147] \text{ -< } l[258] \text{ -< } l[369] \text{ -< } l[123] \text{ -< } l[456] \text{ -< } l[789]$.

In this notation is involved a *sixth icon*.

Peirce: CP 3.393 Cross-Ref: ††

393. We now come to the distinction of *some* and *all*, a distinction which is precisely on a par with that between truth and falsehood; that is, it is descriptive.

Peirce: CP 3.393 Cross-Ref: ††

All attempts to introduce this distinction into the Boolean algebra were more or less complete failures until Mr. Mitchell †2 showed how it was to be effected. His method really consists in making the whole expression of the proposition consist of two parts, a pure Boolean expression referring to an individual and a Quantifying part saying what individual this is. Thus, if k means 'he is a king,' and h , 'he is happy,' the Boolean

$(\sim k+h)$

means that the individual spoken of is either not a king or is happy. Now, applying the quantification, we may write

Any $(\sim k+h)$

to mean that this is true of any individual in the (limited) universe, or Some $(\sim k+h)$

to mean that an individual exists who is either not a king or is happy. So $\text{Some } (k h)$

means some king is happy, and

Any $(k h)$

means every individual is both a king and happy. The rules for the use of this notation are obvious. The two propositions

Any (x) Any (y)

are equivalent to Any $(x y)$.

From the two propositions

Any (x) Some (y)

we may infer Some $(x y)$. †P1

Mr. Mitchell has also a very interesting and instructive extension of his notation for *some* and *all*, to a two-dimensional universe, that is, to the logic of relatives. Here, in order to render the notation as iconical as possible we may use Σ for *some*, suggesting a sum, and π for *all*, suggesting a product. Thus $\Sigma[i]x[i]$ means that x is true of some one of the individuals denoted by i or

$$\Sigma[i]x[i] = x[i]+x[j]+x[k]+etc. \dagger 1$$

In the same way, $\pi[i]x[i]$ means that x is true of all these individuals, or

$$\pi[i]x[i] = x[i]x[j]x[k], etc. \dagger 2$$

If x is a simple relation, $\pi[i]\pi[j]x[i j]$ means that every i is in this relation to every j , $\Sigma[i]\pi[j]x[i j]$ that some one i is in this relation to every j , $\pi[j]\Sigma[i]x[i j]$ that to every j

some i or other is in this relation, $\Sigma[i]\Sigma[j]x[ij]$ that some i is in this relation to some j . It is to be remarked that $\Sigma[i]x[i]$ and $\pi[i]x[i]$ are only *similar* to a sum and a product; they are not strictly of that nature, because the individuals of the universe may be innumerable.

Peirce: CP 3.394 Cross-Ref:††

394. At this point, the reader would perhaps not otherwise easily get so good a conception of the notation as by a little practice in translating from ordinary language into this system and back again. Let $l[ij]$ mean that i is a lover of j , and $b[ij]$ that i is a benefactor of j . Then

$$\pi[i]\Sigma[j]l[ij]b[ij]$$

means that everything is at once a lover and a benefactor of something; and $\pi[i]\Sigma[j]l[ij]b[ji]$

that everything is a lover of a benefactor of itself.

$$\Sigma[i]\Sigma[k]\pi[j](l[ij] + b[jk])$$

means that there are two persons, one of whom loves everything except benefactors of the other (whether he loves any of these or not is not stated). Let $g[i]$ mean that i is a griffin, and $c[i]$ that i is a chimera, then

$$\Sigma[i]\pi[j](g[i]l[ij] + \sim c[j])$$

means that if there be any chimeras there is some griffin that loves them all; while $\Sigma[i]\pi[j]g[i](l[ij] + \sim c[j])$

means that there is a griffin and he loves every chimera that exists (if any exist). On the other hand,

$$\pi[j]\Sigma[i]g[i](l[ij] + \sim c[j])$$

means that griffins exist (one, at least), and that one or other of them loves each chimera that may exist; and

$$\pi[j]\Sigma[i](g[i]l[ij] + \sim c[j])$$

means that each chimera (if there is any) is loved by some griffin or other.

Peirce: CP 3.395 Cross-Ref: ††

395. Let us express: every part of the world is either sometimes visited with cholera, and at others with small-pox (without cholera), or never with yellow fever and the plague together.

Let $c[i j]$ mean the place i has cholera at the time j .

$s[i j]$ mean the place i has small-pox at the time j .

$y[i j]$ mean the place i has yellow fever at the time j .

$p[i j]$ mean the place i has plague at the time j .

Then we write $\pi[i]\Sigma[j]\Sigma[k]\pi[l](c[i j]\sim c[i k]s[i k]+\sim y[i l]+\sim p[i l])$

Peirce: CP 3.395 Cross-Ref: ††

Let us express this: one or other of two theories must be admitted, first, that no man is at any time unselfish or free, and some men are always hypocritical, and at every time some men are friendly to men to whom they are at other times inimical, or second, at each moment all men are alike either angels or fiends. Let

$u[ij]$ mean the man i is unselfish at the time j ,

$f[ij]$ mean the man i is free at the time j ,

$h[ij]$ mean the man i is hypocritical at the time j ,

$a[ij]$ mean the man i is an angel at the time j ,

$d[ij]$ mean the man i is a fiend at the time j ,

$p[i j k]$ mean the man i is friendly at the time j , to the man k ,

$e[i j k]$ the man i is an enemy at the time j to the man k ;

$1[jm]$ the two objects j and m are identical.

Then the proposition is

$\pi[i]\Sigma[h]\pi[j]\Sigma[k]\Sigma[l]\Sigma[m]\pi[n]\pi[p]\pi[q](\sim u[ij]\sim f[ij]h[hj]p[k j l]e[k m l]\sim 1[jm] + a[pn] + d[qn])$

Peirce: CP 3.396 Cross-Ref:††

396. We have now to consider the procedure in working with this calculus. It is far from being true that the only problem of deduction is to draw a conclusion from given premisses. On the contrary, it is fully as important to have a method for ascertaining what premisses will yield a given conclusion. There are besides other problems of transformation, where a certain system of facts is given, and it is required to describe this in other terms of a definite kind. Such, for example, is the problem of the fifteen young ladies,†1 and others relating to synthemes.†2 I shall, however, content myself here with showing how, when a set of premisses are given, they can be united and certain letters eliminated. Of the various methods which might be pursued, I shall here give the one which seems to me the most useful on the whole.

Peirce: CP 3.396 Cross-Ref:††

First, the different premisses having been written with distinct indices (the same index not used in two propositions) are written together, and all the π 's and Σ 's are to be brought to the left. This can evidently be done, for

$$\pi[i]x[i] \cdot \pi[j]x[j] = \pi[i]\pi[j]x[i]x[j]$$

$$\Sigma[i]x[i] \cdot \pi[j]x[j] = \Sigma[i]\pi[j]x[i]x[j]$$

$$\Sigma[i]x[i] \cdot \Sigma[j]x[j] = \Sigma[i]\Sigma[j]x[i]x[j]$$

Peirce: CP 3.396 Cross-Ref:††

Second, without deranging the order of the indices of any one premiss, the π 's and Σ 's belonging to different premisses may be moved relatively to one another, and as far as possible the Σ 's should be carried to the left of the π 's. We have

$$\pi[i]\pi[j]x[i]j = \pi[j]\pi[i]x[i]j$$

$$\Sigma[i]\Sigma[j]x[i]j = \Sigma[j]\Sigma[i]x[i]j$$

and also $\Sigma[i]\pi[j]x[i]y[j] = \pi[j]\Sigma[i]x[i]y[j]$

But this formula does not hold when the i and j are not separated. We do have, however,

$$\Sigma[i]\pi[j]x[i]j \prec \pi[i]\Sigma[j]x[i]j. \dagger 1$$

It will, therefore, be well to begin by putting the Σ 's to the left, as far as possible, because at a later stage of the work they can be carried to the right but not to the left. For example, if the operators of the two premisses are $\pi[i]\Sigma[j]\pi[k]$ and $\Sigma[x]\pi[y]\Sigma[z]$, we can unite them in either of the two orders

$$\Sigma[x]\pi[y]\Sigma[z]\pi[i]\Sigma[j]\pi[k]$$

$$\Sigma[x]\pi[i]\Sigma[j]\pi[y]\Sigma[z]\pi[k],$$

and shall usually obtain different conclusions accordingly. There will often be room for skill in choosing the most suitable arrangement.

Peirce: CP 3.396 Cross-Ref:††

Third, it is next sometimes desirable to manipulate the Boolean part of the expression, and the letters to be eliminated can, if desired, be eliminated now. For this purpose they are replaced by relations of second intention, such as "other than," etc. If, for example, we find anywhere in the expression

$$a[i j k] \sim a[x y z],$$

this may evidently be replaceable by

$$(n[i x] + n[j y] + n[k z])$$

where, as usual, n means not or other than. This third step of the process is frequently quite indispensable, and embraces a variety of processes; but in ordinary cases it may be altogether dispensed with.†1

Peirce: CP 3.396 Cross-Ref:††

Fourth, the next step, which will also not commonly be needed, consists in making the indices refer to the same collections of objects, so far as this is useful. If the quantifying part, or Quantifier, contains $\Sigma[x]$ and we wish to replace the x by a new index i , not already in the Quantifier, and such that every x is an i , we can do so at once by simply multiplying every letter of the Boolean having x as an index by $x[i]$. Thus, if we have "some woman is an angel" written in the form $\Sigma[w]a[w]$ we may replace this by $\Sigma[i](a[i]w[i])$. It will be more often useful to replace the index of a π by a wider one; and this will be done by adding $\sim x[i]$ to every letter having x as an index. Thus, if we have "all dogs are animals, and all animals are vertebrates" written thus

$$\pi[d]\alpha[d] \quad \pi[a]v[a]$$

where a and α alike mean animal, it will be found convenient to replace the last index by i , standing for any object, and to write the proposition

$$\pi[i](\sim a[i]+v[i])$$

Peirce: CP 3.396 Cross-Ref:††

Fifth,†2 the next step consists in multiplying the whole Boolean part, by the modification of itself produced by substituting for the index of any π any other index standing to the left of it in the Quantifier. Thus, for

$$\Sigma[i]\pi[j]l[ij],$$

we can write $\Sigma[i]\pi[j]l[ij]l[ii]$.

Peirce: CP 3.396 Cross-Ref:††

Sixth, the next step consists in the re-manipulation of the Boolean part, consisting, first, in adding to any part any term we like; second, in dropping from any part any factor we like, and third, in observing that

$$x\sim x = \mathbf{f}, \quad x+\sim x = \mathbf{v},$$

so that $x\sim x y+z = z, (x+\sim x+y)z = z$.

Peirce: CP 3.396 Cross-Ref:††

Seventh, π 's and Σ 's in the Quantifier whose indices no longer appear in the Boolean are dropped.

Peirce: CP 3.396 Cross-Ref:††

The fifth step will, in practice, be combined with part of the sixth and seventh. Thus, from $\Sigma[i]\pi[j]l[ij]$ we shall at once proceed to $\Sigma[i]l[ii]$ if we like.

Peirce: CP 3.397 Cross-Ref:††

397. The following examples will be sufficient.

From the premisses $\Sigma[i]a[i]b[i]$ and $\pi[j](\sim b[j]+c[j])$, eliminate b . We first write $\Sigma[i]\pi[j]a[i]b[i](\sim b[j]+c[j])$.

The distributive process gives

$$\Sigma[i]\pi[j]a[i](b[i]\sim b[j] + b[i]c[j]).$$

But *we always have a right to drop a factor or insert an additive term*. We thus get

$$\Sigma[i]\pi[j]a[i](b[i]\sim b[j] + c[j]).$$

By the third process, we can, if we like, insert $n[ij]$ for $b[i]\sim b[j]$. In either case, we identify j with i and get the conclusion

$$\Sigma[i]a[i]c[i].$$

Given the premisses:

$$\begin{aligned} &\Sigma[h]\pi[i]\Sigma[j]\pi[k](\alpha[hik]+s[jk]lji) \\ &\Sigma[u]\Sigma[v]\pi[x]\pi[y](\epsilon[ux] + \sim s[yv]b[vx]). \end{aligned}$$

Required to eliminate s . The combined premiss is

$$\Sigma[u]\Sigma[v]\Sigma[h]\pi[i]\Sigma[j]\pi[x]\pi[k]\pi[y](\alpha[hik]+s[jk]lji)(\epsilon[ux]+\sim s[yv]b[vx]).$$

Identify k with v and y with j , and we get

$$\Sigma[u]\Sigma[v]\Sigma[h]\pi[i]\Sigma[j]\pi[x](\alpha[hiv]+s[jv]lji)(\epsilon[ujx]+\sim s[jv]b[vx]).$$

The Boolean part then reduces, so that the conclusion is

$$\Sigma[u]\Sigma[v]\Sigma[h]\pi[i]\Sigma[j]\pi[x](\alpha[hiv]\epsilon[ujx]+\alpha[hiv]b[vx]+\epsilon[ujx]lji).$$

Peirce: CP 3.398 Cross-Ref: ††
§4. SECOND-INTENTIONAL LOGIC †1

398. Let us now consider the logic of terms taken in collective senses. Our notation, so far as we have developed it, does not show us even how to express that two indices, i and j , denote one and the same thing. We may adopt a special token of second intention, say 1, to express identity, and may write $1[ij]$. But this relation of identity has peculiar properties. The first is that if i and j are identical, whatever is true of i is true of j . This may be written

$$\pi[i]\pi[j]\{\sim 1[ij]+\sim x[i]+x[j]\}.$$

The use of the general index of a token, x , here, shows that the formula is iconical. The other property is that if everything which is true of i is true of j , then i and j are identical. This is most naturally written as follows: Let the token, q , signify the relation of a quality, character, fact, or predicate to its subject. Then the property we desire to express is

$$\pi[i]\pi[j]\Sigma[k](1[ij]+\sim q[k i]q[k j]).$$

And identity is defined thus

$$1[ij] = \pi[k](q[k i]q[k j]+\sim q[k i]\sim q[k j]).$$

That is, to say that things are identical is to say that every predicate is true of both or false of both. It may seem circuitous to introduce the idea of a quality to express identity; but that impression will be modified by reflecting that $q[k i]q[k j]$ merely means that i and j are both within the class or collection k . If we please, we can dispense with the token q , by using the index of a token and by referring to this in the Quantifier just as subjacent indices are referred to. That is to say,

we may write $1[ij] = \pi[x](x[i]x[j]+\sim x[i]\sim x[j]).$

Peirce: CP 3.399 Cross-Ref: ††

399. The properties of the token q must now be examined. These may all be summed up in this, that taking any individuals $i[1], i[2], i[3]$, etc., and any individuals, $j[1], j[2], j[3]$, etc., there is a collection, class, or predicate embracing all the i 's and excluding all the j 's except such as are identical with some one of the i 's. This might be written

$$(\pi[\alpha]\pi[i[\alpha]])(\pi[\beta]\pi[j[\beta]])\Sigma[k](\pi[\alpha]\Sigma[i'[\alpha]]\pi[l]q[k i[\alpha]](\sim q[k j[\beta]] + q[l i'[\alpha]]q[l j[\beta]] + \sim q[l i'[\alpha]]\sim q[l j[\beta]]),$$

where the i 's and the i' 's are the same lot of objects. This notation presents indices of indices. The $\pi[\alpha]\pi[i[\alpha]]$ shows that we are to take any collection whatever of i 's, and then any i of that collection. We are then to do the same with the j 's. We can then find a quality k such that the i taken has it, and also such that the j taken wants it unless we can find an i that is identical with the j taken. The necessity of some kind of notation

of this description in treating of classes collectively appears from this consideration: that in such discourse we are neither speaking of a single individual (as in the non-relative logic) nor of a small number of individuals considered each for itself, but of a whole class, perhaps an infinity of individuals. This suggests a relative term with an indefinite series of indices as $x[i j k l] . . .$. Such a relative will, however, in most, if not in all cases, be of a degenerate kind and is consequently expressible as above. But it seems preferable to attempt a partial decomposition of this definition. In the first place, any individual may be considered as a class. This is written

$$\pi[i]\Sigma[k]\pi[j] q[k i](\sim q[k j] + 1[i j]).$$

This is the *ninth icon*.^{†1} Next, given any class, there is another which includes all the former excludes and excludes all the former includes. That is,

$$\pi[l]\Sigma[k]\pi[i](q[l i]\sim q[k i] + \sim q[l i]q[k i]).$$

This is the *tenth icon*. Next, given any two classes, there is a third which includes all that either includes and excludes all that both exclude. That is

$$\pi[l]\pi[m]\Sigma[k]\pi[i](q[l i]q[k i] + q[m i]q[k i] + \sim q[l i]\sim q[m i]\sim q[k i]).$$

This is the *eleventh icon*. Next, given any two classes, there is a class which includes the whole of the first and any one individual of the second which there may be not included in the first and nothing else. That is,

$$\pi[l]\pi[m]\pi[i]\Sigma[k]\pi[j]\{q[l i] + \sim q[m i] + q[k i](q[k j] + \sim q[l j])\}.$$

This is the *twelfth icon*.

Peirce: CP 3.400 Cross-Ref:††

400. To show the manner in which these formulæ are applied let us suppose we have given that everything is either true of *i* or false of *j*. We write

$$\pi[k](q[k i] + \sim q[k j]).\dagger 2$$

The *tenth icon* gives

$$\pi[l]\Sigma[k](q[l\ i]\sim q[k\ i] + \sim q[l\ i]q[k\ i])(q[l\ j]\sim q[k\ j] + \sim q[l\ j]q[k\ j]).$$

Multiplication of these two formulæ give

$$\pi[l]\Sigma[k](q[k\ i]\sim q[l\ i] + q[l\ j]\sim q[k\ j]),$$

or, dropping the terms in k

$$\pi[l](\sim q[l\ i] + q[l\ j]).$$

Multiplying this with the original datum and identifying l with k , we have
 $\pi[k](q[k\ i]q[k\ j] + \sim q[k\ i]\sim q[k\ j]).$

No doubt, a much more direct method of procedure could be found.

Peirce: CP 3.401 Cross-Ref:††

401. Just as q signifies the relation of predicate to subject, so we need another token, which may be written r , to signify the conjoint relation of a simple relation, its relate and its correlate. That is, $r[j\alpha i]$ is to mean that i is in the relation α to j . Of course, there will be a series of properties of r similar to those of q . But it is singular that the uses of the two tokens are quite different. Namely, the chief use of r is to enable us to express that the number of one class is at least as great as that of another. This may be done in a variety of different ways. Thus, we may write that for every a there is a b , in the first place, thus:

$$\Sigma[\alpha]\pi[i]\Sigma[j]\pi[h]\{\sim a[i] + b[j]r[j\alpha i](\sim r[j\alpha h] + \sim a[h] + 1[i\ h])\}.$$

But, by an icon analogous to the eleventh, we have

$$\pi[\alpha]\pi[\beta]\Sigma[\{g\}]\pi[u]\pi[v](r[u\alpha v]r[u\{g\}v] + r[u\beta v]r[u\{g\}v] + \sim r[u\alpha v]\sim r[u\beta v]\sim r[u\{g\}v]).$$

From this, by means of an icon analogous to the *tenth*, we get the general formula

$$\pi[\alpha]\pi[\beta]\Sigma[\{g\}]\pi[u]\pi[v]\{r[u\alpha v]r[u\beta v]r[u\{g\}v] + \sim r[u\{g\}v](\sim r[u\alpha v] + \sim r[u\beta v])\}.$$

For $r[u\beta v]$ substitute $a[u]$ and multiply by the formula the last but two. Then, identifying u with h and v with j , we have

$$\Sigma[\alpha]\pi[i]\Sigma[h]\pi[h]\{\sim a[i] + b[j]r[j\alpha i](\sim r[j\alpha h] + 1[i h])\}$$

a somewhat simpler expression. However, the best way to express such a proposition is to make use of the letter c as a token of a one-to-one correspondence. That is to say, c will be defined by the three formulæ,†1

$$\pi[\alpha]\pi[u]\pi[v]\pi[w](\sim c[\alpha] + \sim r[u\alpha v] + \sim r[u\alpha w] + 1[v w])$$

$$\pi[\alpha]\pi[u]\pi[v]\pi[w](\sim c[\alpha] + \sim r[u\alpha w] + r[v\alpha w] + 1[u v])$$

$$\pi[\alpha]\Sigma[u]\Sigma[v]\Sigma[w](c[\alpha] + r[u\alpha v]r[u\alpha w] + 1[v w] + r[u\alpha w]r[v\alpha w] + 1[u v]).†3$$

Making use of this token, we may write the proposition we have been considering in the form

$$\Sigma[\alpha]\pi[i]\Sigma[j] c[\alpha](\sim a[i] + b[j]r[j\alpha i]).$$

Peirce: CP 3.402 Cross-Ref:††

402. In an appendix to his memoir on the logic of relatives,†1 De Morgan enriched the science of logic with a new kind of inference, the syllogism of transposed quantity. De Morgan was one of the best logicians that ever lived and unquestionably the father of the logic of relatives. Owing, however, to the imperfection of his theory of relatives, the new form, as he enunciated it, was a down-right paralogism, one of the premisses being omitted. But this being supplied, the form furnishes a good test of the efficacy of a logical notation. The following is one of De Morgan's examples:†2

Some X is Y ,

For every X there is something neither Y nor Z ;

Hence, something is neither X nor Z .

The first premiss is simply $\Sigma[\alpha]x[\alpha]y[\alpha]$.

The second may be written

$$\Sigma[\alpha]\pi[i]\Sigma[j] c[\alpha](\sim x[i] + r[j\alpha i]\sim y[j]\sim z[j]).$$

From these two premisses, little can be inferred. To get the above conclusion it is necessary to add that the class of X 's is a finite collection †3; were this not necessary the following reasoning would hold good (the limited universe consisting of numbers); for it precisely conforms to De Morgan's scheme.

Some odd number is prime;

Every odd number has its square, which is neither prime nor even;

Hence, some number is neither odd nor even. †P1

Peirce: CP 3.402 Cross-Ref: ††

Now, to say that a lot of objects is finite, is the same as to say that if we pass through the class from one to another we shall necessarily come round to one of those individuals already passed; that is, if every one of the lot is in any one-to-one relation to one of the lot, then to every one of the lot some one is in this same relation. This is written thus:

$$\pi[\beta]\pi[u]\Sigma[v]\Sigma[s]\pi[t]\{\sim c[\beta] + \sim x[u] + x[v]r[u\beta v] + x[s](\sim x[t] + \sim r[t\beta s])\}$$

Uniting this with the two premisses and the second clause of the definition of c , we have

$$\Sigma[a]\Sigma[\alpha]\pi[\beta]\pi[u]\Sigma[v]\Sigma[s]\pi[i]\Sigma[j]\pi[t]\pi[\{g\}]\pi[e]\pi[f]\pi[g]x[a]y[a]c[\alpha](\sim x[i] + r[j\alpha i]\sim y[j]\sim z[j])\{\sim c[\beta] + \sim x[u] + x[v]r[u\beta v] + x[s](\sim x[t] + \sim r[t\beta s])\}(\sim c[\{g\}] + \sim r[e\{g\}g] + \sim r[f\{g\}w] + 1[ef]).$$

We now substitute α for β and for $\{g\}$, a for u and for e, j for t and for f, v for g . The factor in i is to be repeated, putting first s and then v for i . The Boolean part thus reduces to

$$(\sim x[s] + r[j\alpha s]\sim y[j]\sim z[j])c[\alpha]x[a]y[a]r[a\alpha v]x[v]r[j\alpha v]\sim y[j]\sim z[j]1[a j] + r[j\alpha s]\sim y[j]\sim z[j]x[s]\sim x[j](\sim x[v] + r[j a v]\sim y[j]\sim z[j])(\sim r[a\alpha v] + \sim r[j\alpha v]1[a j],$$

which, by the omission of factors, becomes

$$y[a]\sim y[j]1[a\ j] + \sim x[j]\sim z[j].$$

Thus we have the conclusion $\Sigma[j]\sim x[j]\sim z[j]$.

Peirce: CP 3.403 Cross-Ref:††

403. It is plain that by a more iconical and less logically analytical notation this procedure might be much abridged. How minutely analytical the present system is, appears when we reflect that every substitution of indices of which nine were used in obtaining the last conclusion is a distinct act of inference. The annulling of $(y[a]\sim y[j]1[a\ j])$ makes ten inferential steps between the premisses and conclusion of the syllogism of transposed quantity.

Peirce: CP 3.403A Cross-Ref:††
§5. NOTE †1

403A. Under the third step [396], an example was given which is really a general formula of elimination. Namely, we have

$$a[i\ j\ k] \text{ etc. } \sim a[x\ y\ z] \text{ etc. } \rightarrow \sim 1[i\ x] + \sim 1[j\ y] + \sim 1[k\ z] \text{ etc.}$$

and conversely

$$1[i\ x]1[j\ y]1[z\ k] \text{ etc. } \rightarrow a[i\ j\ k] \text{ etc. } + \sim a[x\ y\ z] \text{ etc.}$$

The principle of contradiction and of excluded middle might be considered as mere special cases of these formulæ. Namely, the latter give

$$a[i]\sim a[i] \rightarrow \sim 1[i\ i]$$

$$1 \rightarrow a[i] + \sim a[i]$$

But the definition of identity is

$$\mathbf{v} \rightarrow 1[i\ i] \sim 1[i\ i] \rightarrow \mathbf{f};$$

whence

$$\mathbf{a}[i]\sim\mathbf{a}[i] \rightarrow \mathbf{f} \quad \mathbf{v} \rightarrow \mathbf{a}[i] + \sim\mathbf{a}[i].$$

Peirce: CP 3.403B Cross-Ref:††

403B. Under the head of the third step belongs the frequently necessary development of the Boolean by means of distribution formulæ. (This proceeding, and indeed the whole of the third step, might more properly have been made to follow the fourth.) The fundamental formulæ of distribution are

$$\mathbf{x}(\mathbf{y}+\mathbf{z}) = \mathbf{x}\mathbf{y}+\mathbf{x}\mathbf{z}$$

$$\mathbf{x}+\mathbf{y}\mathbf{z} = (\mathbf{x}+\mathbf{y})(\mathbf{x}+\mathbf{z}).$$

Peirce: CP 3.403B Cross-Ref:††

The general development formulæ thence resulting are

$$\mathbf{F}\mathbf{x} = \mathbf{x}\mathbf{F}1 + \sim\mathbf{x}\mathbf{F}0$$

$$= (\mathbf{x} + \mathbf{F}0)(\sim\mathbf{x} + \mathbf{F}1)$$

The following formulæ, which find continual application, are deducible from the above:

$$\mathbf{a}+\mathbf{b} = \mathbf{a} + \sim\mathbf{a}\mathbf{b}$$

$$\mathbf{x}\mathbf{y}+\sim\mathbf{x}\sim\mathbf{y} = (\mathbf{x}+\sim\mathbf{y})(\sim\mathbf{x}+\mathbf{y})$$

$$(\mathbf{a}+\mathbf{b})\mathbf{c} \rightarrow \mathbf{a}+\mathbf{b}\mathbf{c}$$

$$(\mathbf{a}+\mathbf{x})(\mathbf{b}+\mathbf{y}) \rightarrow \mathbf{a}+\mathbf{b}+\mathbf{x}\mathbf{y}.$$

Peirce: CP 3.403C Cross-Ref:††

403C. The *fifth step* is composed of two kinds of operations, the involution of the whole expression, and the identification and discrimination of indices.

Peirce: CP 3.403D Cross-Ref:††

403D. It is plain that *any algebraical expression of a proposition may be multiplied into itself any number of times without ceasing to be true*; and it will be found that such involution is essentially necessary in all difficult modes of inference.

Consider, for example, the last formula but one of those last given,

$$(a+b)c \text{ -< } a+b c.$$

Peirce: CP 3.403D Cross-Ref:††

This is not itself a distribution formula, but only an association formula, and therefore the deduction of a distribution formula from it is not a matter of the very utmost simplicity. If, however, we square the antecedent we have

$$(a+b)c = (a+b)c(a+b)c = c(a+b)c,$$

and then by the application of the association formula twice over we get

$$c(a+b)c \text{ -< } (c a+b)c \text{ -< } c a+b c;$$

so that we have proved

$$(a+b)c \text{ -< } a c+b c,$$

which is the main distribution formula, with the aid of which all the others are readily obtained.

Peirce: CP 3.403D Cross-Ref:††

Other much more striking examples of the utility of involution will present themselves in the course of this paper.

Peirce: CP 3.403E Cross-Ref:††

403E. In multiplying a proposition by itself, we have the right to choose any point from the beginning to the end of the quantifier and to identify in two factors all the indices to the left of this point while diversifying all to the right of it. When there is but one index, this is plainly true (see the formulæ in first step); for

$$\pi[i]a[i] = (\pi[i]a[i])(\pi[i]a[i]) = (\pi[i]a[i])(\pi[j]a[j])$$

$$\Sigma[i]a[i] = (\Sigma[i]a[i])(\Sigma[i]a[i]) = (\Sigma[i]a[i])(\Sigma[j]a[j]).$$

Now $(\pi[i]a[i])(\pi[j]a[j]) = (a[1]a[2]a[3] \text{ etc.})(a[1]a[2]a[3] \text{ etc.})$

while $\pi[i]\pi[j]a[i]a[j] = a[1]a[1] \cdot a[1]a[2] \cdot a[1]a[3] \text{ etc.}$

$$X a[2]a[1] \cdot a[2]a[2] \cdot a[2]a[3] \text{ etc.}$$

$$X a[3]a[1] \cdot a[3]a[2] \cdot a[3]a[3] \text{ etc.}$$

X etc.

$$\text{and } \pi[i]a[i]a[i] = a[1]a[1] \cdot a[2]a[2] \cdot a[3]a[3] \cdot \text{etc.}$$

But since $a a = a$, all these are equivalent.

Also

$$(\Sigma[i]a[i])(\Sigma[j]a[j]) = (a[1]+a[2]+a[3]+\text{etc.})(a[1]+a[2]+a[3]+\text{etc.})$$

while

$$\begin{aligned} \Sigma[i]\Sigma[j]a[i]a[j] &= a[1]a[1]+a[1]a[2]+a[1]a[3] + \text{etc.} \\ &+ a[2]a[1]+a[2]a[2]+a[2]a[3] + \text{etc.} \\ &+ a[3]a[1]+a[3]a[2]+a[3]a[3] + \text{etc.} \\ &+ \text{etc.} \end{aligned}$$

$$\text{and } \Sigma[i]a[i]a[i] = a[1]a[1]+a[2]a[2]+a[3]a[3] + \text{etc.}$$

The first two expressions are equivalent by the distribution principle. The third is equivalent to $\Sigma[i]a[i]$ because $a a = a$, and thus all three are equivalent to one another.

Peirce: CP 3.403E Cross-Ref:††

The theorem enunciated having thus been proved true of every proposition having a single index, it only remains to show that if it be true of every proposition having n indices, it is true for every proposition having $n + 1$ indices.

Peirce: CP 3.403E Cross-Ref:††

Now let Φ and $*$ stand either for Σ and $+$ or for π and X respectively, so that

$$\Phi[i]u[i] = u[1] * u[2] * u[3] * \text{etc.}$$

This may represent any proposition in $n + 1$ indices of which i is the first. When i is fixed in a certain individual as in $u[1]$, $u[2]$, etc., u becomes a proposition in n indices. Let u^2 represent any legitimate square of u . Then the square of the whole expression with identification of i in the two factors is

$$\Phi[i]u[i]^2 = u[1]^2 * u[2]^2 * u[3]^2 * \text{etc.}$$

and since by hypothesis

$$u[1] \prec u[1]^2 \quad u[2] \prec u[2]^2 \quad u[3] \prec u[3]^2 \text{ etc.}$$

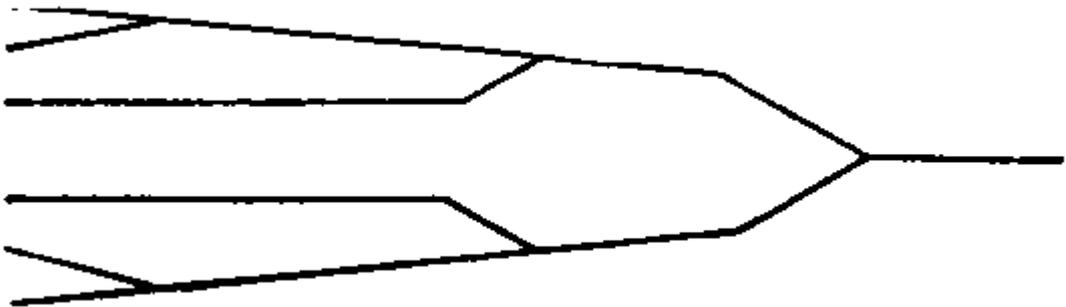
it follows that

$$\Phi[i]u[i] \prec \Phi[i]u[i]^2.$$

Thus it is shown that the rule holds for every proposition in $n + 1$ indices, if the first of them is identified in the two factors. But according to the rule, the first cannot be diversified unless all are diversified; and all may be diversified, by an obvious extension of the formulæ relating to propositions in a single index. The theorem is therefore proved.

Peirce: CP 3.403E Cross-Ref:††

If the proposition is raised to a higher power, the diversifications toward the right may be represented as branchings from a stem, thus,



[Click here to view]

Peirce: CP 3.403E Cross-Ref:††

After the involution has once been performed, further identification and diversification of indices may be effected by applying the following formulæ.

For identifications

For diversifications

$$\pi[i]\pi[j]l[ij] \prec \pi[i]l[ii] \quad \pi[i]l[ii] \prec \pi[i]\Sigma[j]l[ij]$$

$$\Sigma[i]\pi[j]l[ij] \prec \Sigma[i]l[ii] \quad \Sigma[i]l[ii] \prec \Sigma[i]\Sigma[j]l[ij]$$

Peirce: CP 3.403F Cross-Ref:††

403F. Besides these formulæ, the following are occasionally useful

$$\pi[i]m[i] = \pi[i]\pi[j]m[i]m[j] \quad \Sigma[i]m[i] = \Sigma[i]\Sigma[j](m[i]+m[j]).$$

Peirce: CP 3.403F Cross-Ref:††

The following example shows the utility of changing the indices, even without involution. From the premiss

$$\pi[i]\pi[j](m[i]l[ij] + \sim m[i]\sim l[ij])$$

let it be required to eliminate m . The immediate dropping of this token would only yield an identical proposition. But by separating the Boolean into factors and then changing the indices, we have

$$\begin{aligned} & \pi[i]\pi[j](m[i]l[ij] + \sim m[i]\sim l[ij]) \\ &= \pi[i]\pi[j](m[i] + \sim l[ij])(l[ij] + \sim m[i]) \\ &= \pi[i]\pi[j](m[i] + \sim l[ij])\pi[j](l[ij] + \sim m[i]) \\ &= \pi[i]\pi[j](m[i] + \sim l[ij])\pi[k](l[ik] + \sim m[i]) \\ &= \pi[i]\pi[j]\pi[k](m[i] + \sim l[ij])(l[ik] + \sim m[i]) \\ &= \pi[i]\pi[j]\pi[k](m[i]l[ik] + \sim m[i]\sim l[ij]) \end{aligned}$$

Now dropping m we have

$$\pi[i]\pi[j]\pi[k](l[ik] + \sim l[ij])$$

i.e. whatever is in the relation l to anything is in that relation to everything.

Peirce: CP 3.403G Cross-Ref:††

403G. An artifice which I have not included among the regular steps of the inferential procedure but which is occasionally useful, consists in taking the latter part of the quantifier away from its position at the head of the proposition and putting it before the part which alone it concerns. The formulæ governing this operation are

$$\pi[j]a[i]b[ij] = a[i]\pi[i]b[ij] \quad \Sigma[j]a[i]b[ij] = a[i]\Sigma[j]b[ij]$$

$$\pi[j](a[i]+b[ij]) = a[i] + \pi[j]b[ij] \quad \Sigma[j](a[i]+b[ij]) = a[i] + \Sigma[j]b[ij].$$

This transformation will generally be used in connection with the formulæ

$$a \sim a + b \text{ and } a + x b = a \sim x + x b. \dagger 1$$

Peirce: CP 3.403H Cross-Ref:††

403H. The special peculiarity of ordinary algebra has given us the false notion that inverse operations are the general means of solving algebraical problems. But the study of general algebra shows that inverse operations lead to determinate results only in special cases -- that what is called "the general case" is in truth a mere form of speciality -- and that the truly general method of elimination is by performing a direct operation which will give a constant result whatever the value of the variable. Thus, in ordinary algebra, it happens to be the case that every quantity has a reciprocal so that

$$(1/x) \cdot x = 1.$$

So in logical algebra) the only way of eliminating any token is by means of the properties of special terms of second intention, such as

$$a[ij] \sim a[x y] \sim \sim 1[i x] + 1[j y] \quad a[i] \sim a[i] \sim 0.$$

Peirce: CP 3.403H Cross-Ref:††

In order to bring these formulæ to bear it may be necessary to multiply the premiss into itself, to manipulate the indices, and use processes of association and of development by distribution; but whatever cannot be eliminated by these means cannot be eliminated at all.

Peirce: CP 3.403I Cross-Ref:††

403I. The universes of marks †2 to which the tokens q, r and others like these refer are, in reference to the combinations of objects to which they are attached, unlimited universes. (Compare the Thomist doctrine of angelic natures.) That is to say, every lot of objects has some quality common and peculiar to the objects composing it; every lot of pairs has some relation subsisting between the first and second members of each one of those pairs and between no others. In other regards, the universes are not unlimited; of characters familiar to us there is quite a limited number; of colours definable by Newton's diagram not all can [co-?]exist. In like manner, the universes of quantity, position, etc., of mathematics are unlimited universes. Of the objects of such a universe everything is true, which can be true; every proposition is true from which the unlimited universe cannot be eliminated without yielding a true proposition.

Peirce: CP 3.403I Cross-Ref:††

As a general rule, every proposition in $\Sigma[\alpha]q[\alpha]$ and $\Sigma[\alpha]r[\alpha]$ is true; but there are many exceptions.

Peirce: CP 3.403I Cross-Ref:††

To illustrate the application of this principle, consider the ninth icon.†1 This is written

$$\pi[a]\Sigma[\{k\}]\pi[b]q[\{k\}a](\sim q[\{k\}b] + 1[a b])$$

Now $\{k\}$ can only be eliminated from this without involution in two ways; first, eliminating $q[\{k\}a]$ and $q[\{k\}b]$ independently, we have

$$\pi[a]\pi[b](\sim 1[a b] + 1[a b]),$$

an identical proposition; second, identifying b with a , we get

$$\pi[a]1[a a],$$

another identical proposition. We next proceed to square the proposition; and we have

$$\pi[a]\Sigma[\{k\}]\pi[b]\pi[c]q[\{k\}a](\sim q[\{k\}b] + 1[a b])(\sim q[\{k\}c] + 1[a c]).$$

We must identify $\{k\}$ in the factors or we should reach no result; and this forces us to identify a . But it is plain that we cannot from this square eliminate in any new way,

and therefore we could not from any higher power, and consequently the original proposition is proved true.

Peirce: CP 3.403I Cross-Ref:††

Suppose, however, we had written that proposition

$$\Sigma[\{k\}]\pi[a]\pi[b]q[\{k\}a](\sim q[\{k\}b] + 1[a b]).$$

Squaring this we have

$$\Sigma[\{k\}]\pi[a]\pi[b]\pi[c]\pi[d]q[\{k\}a]q[\{k\}c](\sim q[\{k\}b] + 1[a b])(\sim q[\{k\}d] + 1[cd]).$$

Now identify d with a , c with b , and we have

$$\pi[a]\pi[b]1[a b],$$

which is absurd.†1

Peirce: CP 3.403J Cross-Ref:††

403J. By the same principle, we can if we please solve the example of 400 as follows. Given the premiss

$$\pi[\{k\}](q[\{k\}i] + \sim q[\{k\}j]):$$

We multiply the square of this by an identical proposition, thus,

$$\pi[\{1\}]\pi[\{k\}]\pi[\{m\}](q[\{1\}i] + \sim q[\{1\}i])(q[\{1\}j] + \sim q[\{1\}j])(q[\{m\}i] + \sim q[\{m\}j])(q[\{k\}i] + \sim q[\{k\}j]).$$

Identifying $\{m\}$ with $\{1\}$, we get

$$\pi[\{1\}]\pi[\{k\}](q[\{1\}i] + \sim q[\{1\}i])(q[\{1\}i] + \sim q[\{1\}j])(q[\{1\}i] + \sim q[\{1\}j])(q[\{k\}i] + \sim q[\{k\}j]).$$

We now introduce $\sim q[\{k\}i]$ and $q[\{k\}j]$ into the identical proposition wherever we

please so long as they cannot be eliminated without making the proposition otherwise than identical; and this condition will be fulfilled so long as we do not introduce $q[\{k\}i]$ or $\sim q[\{k\}j]$. Only $\pi[\{k\}]$ must be changed to $\Sigma[\{k\}]$. We get then

$$\pi[\{1\}]\Sigma[\{k\}](q[\{1\}i]\sim q[\{k\}i] + \sim q[\{1\}i])(q[\{1\}j] + q[\{k\}j]\sim q[\{1\}i])(q[\{1\}i] + \sim q[\{1\}j])(q[\{k\}i] + \sim q[\{k\}j]).$$

This gives successively

$$\pi[\{1\}]\Sigma[q](q[\{1\}i]\sim q[\{k\}i]q[\{1\}j]\sim q[\{k\}j] + \sim q[\{1\}i]q[\{k\}j]\sim q[\{1\}j]q[\{k\}i])$$

and $\pi[\{1\}](q[\{1\}i]q[\{1\}j] + \sim q[\{1\}i]\sim q[\{1\}j]).$

Peirce: CP 3.403J Cross-Ref:††

But the same conclusion can be reached much more easily by identifying $q[\{k\}]$ with $1[j]$, which we have a right to do on account of the universal quantifier. Thus from

$$\pi[i]\Sigma[j]\pi[\{k\}](q[\{k\}i] + \sim q[\{k\}j])$$

we get

$$\pi[i]\Sigma[j](1[j]i + \sim 1[j]j) = \pi[i]\Sigma[j]1[j]i.$$

Hence the proposition

$$\pi[\{1\}](q[\{1\}i]q[\{1\}j] + \sim q[\{1\}i]\sim q[\{1\}j])$$

holds, because it becomes identical when i is substituted for j .

Peirce: CP 3.403K Cross-Ref:††

403K. Let us now consider some examples of a somewhat more difficult kind. Given the proposition

$$\Sigma[\{k\}]\pi[a]\Sigma[b]\pi[c]\{\sim x[a] + r[\{k\}a b]y[b](\sim r[\{k\}c b] + 1[a c])\},$$

that is, there is a relation $\{k\}$ such that whatever object be taken, either this is not an x or it stands in the relation k to some object which is a y and to which nothing except that x stands in that relation. Required to find all the propositions deducible from this by elimination of $\{k\}$. The simple omission of factors gives

$$\pi[a]\Sigma[b](\sim x[a] + y[b]),$$

that is, there either is no x or there is a y . We get nothing further by identifying c with a . There is therefore no further conclusion without involution. Squaring, with identification of $\{k\}$ (without which we should plainly reach nothing new) we have

$$\Sigma[\{k\}]\pi[a]\Sigma[b]\pi[c]\pi[d]\Sigma[e]\pi[f]\{\sim x[a] + r[\{k\}a b]y[b](\sim r[\{k\}c b] + 1[a c])\} \{\sim x[d] + r[\{k\}d e]y[e](\sim r[\{k\}f e] + 1[d f])\}.$$

The application to the Boolean of the common formula

$$(a + x)(b + y) \prec a + b + x y$$

gives

$$\sim x[a] + \sim x[d] + r[\{k\}a b]r[\{k\}d e]y[b]y[e](\sim r[\{k\}c b] + 1[a c])(\sim r[\{k\}f e] + 1[d f]).$$

We now identify f with a ; when the first and last factors of the last term become

$$r[\{k\}a b](\sim r[\{k\}a e] + 1[a d]).$$

But we have

$$r[\{k\}a b]\sim r[\{k\}a e] \prec \sim 1[b e].$$

Thus, we reach the conclusion

$$\pi[a]\Sigma[b]\pi[d]\Sigma[e](\sim x[a] + \sim x[d] + 1[a d] + y[b]y[e]\sim 1[b e]);$$

that is, there are either not two x 's different from one another or there are two y 's different from one another. A little examination will show that no other conclusion could be reached by elimination from the square.

Peirce: CP 3.403K Cross-Ref: ††

Cubing the original proposition, we reach in a similar way

$$\Sigma[\{k\}]\pi[a]\Sigma[b]\pi[c]\pi[d]\Sigma[e]\pi[f]\pi[g]\Sigma[h]\pi[i]\sim x[a] + \sim x[d] + \sim x[g] + y[b]y[e]y[h]r[\{k\}a b]r[\{k\}d e]r[\{k\}g h] (\sim r[\{k\}c b] + 1[a c])(\sim r[\{k\}f e] + 1[d f])(\sim r[\{k\}i h] + 1[g i])$$

Identifying f with a , i with d , g with c , the last term of the

Boolean becomes

$$y[b]y[e]y[h]r[\{k\}a b]r[\{k\}d e]r[\{k\}c h] (\sim r[\{k\}c b] + 1[a c])(\sim r[\{k\}a e] + 1[a d])(\sim r[\{k\}d h] + 1[c d])$$

whence

$$y[b]y[e]y[h] (\sim 1[b h] + 1[a c])(\sim 1[b e] + 1[a d])(1[e h] + 1[c d]),$$

whence again

$$y[b]y[e]y[h]\sim 1[b h]\sim 1[b e]1[c h]\dagger 1 + 1[a c] + 1[a d] + 1[c d],$$

so that we have the conclusion

$$\pi[a]\Sigma[b]\pi[c]\pi[d]\Sigma[e]\Sigma[h]$$

$$\sim x[a] + \sim x[d] + \sim x[c] + 1[a c] + 1[a d] + 1[c d] + y[b]y[e]y[h]\sim 1[b e]\sim 1[b h]\sim 1[e h],$$

that is, there either are not three x 's all different from one another or there are three y 's

all different from one another.

Peirce: CP 3.403K Cross-Ref:††

It is plain that by raising to the $n!$ power we should get any proposition of this form, and that no others could be obtained.

Peirce: CP 3.403L Cross-Ref:††

403L. Let us now seek all the propositions deducible, by elimination of $\{k\}$, from

$$\pi[\{k\}]\Sigma[a]\pi[b]\Sigma[c] u[a](\sim w[b] + \sim r[\{k\}a b] + r[\{k\}c b]\sim 1[a c]).$$

We have at once

$$\Sigma[a]u[a].$$

The following proposition is universally true:

$$\pi[\alpha]\pi[\beta]\Sigma[\{g\}]\pi[i]\pi[j]\{\sim r[\{g\}i j](\sim r[\alpha i j] + \sim r[\beta i j]) + r[\{g\}i j]r[\alpha i j]r[\beta i j]\}.$$

To prove this, note first that if we eliminate $\{g\}$ at once, we have an identical proposition. If we raise the whole to a power, there will be no additional mode of elimination, unless i, j and $\{g\}$ be identified in different factors; but then all the indices must be alike and nothing will be changed. As a special case of this formula we put $u[i]$ for $r[\beta i j]$. This we can do, because

$$\Sigma[\beta]\pi[i]\pi[j]\{u[i]r[\beta i j] + \sim u[i]\sim r[\beta i j]\}$$

which is true by the same reasoning as that just used. The product of these two formulæ gives

$$\pi[\alpha]\Sigma[\{g\}]\pi[i]\pi[j]\{\sim r[\{g\}i j](\sim r[\alpha i j] + \sim u[i]) + r[\{g\}i j]r[\alpha i j]u[i]\}.$$

Multiplying this twice into the first proposition, identifying $\{k\}$ with $\{g\}$, i with a , and j with b , in one factor, and $\{k\}$ with $\{g\}$, i with c , j with b in the other, we have

$$\pi[a]\Sigma[\{g\}]\Sigma[a]\pi[b]\Sigma[c] u[a](\sim w[b] + \sim r[\{g\}a b] + r[\{g\}c b]\sim 1[a c]) \{r[\{g\}a$$

$$b](\sim r[\alpha a b] + \sim u[a]) + r[\{g\} a b] r[\alpha a b] u[a] \{ \sim r[\{g\} c b] (\sim r[\alpha c b] + \sim u[c]) + r[\{g\} c b] r[\alpha c b] u[c] \}.$$

This gives

$$\pi[\alpha] \Sigma[a] \pi[b] \Sigma[c] u[a] (\sim w[b] + \sim r[\alpha a b] + r[\alpha c b] u[c] \sim 1[a c]).$$

We have universally

$$\Sigma[\delta] \pi[m] \pi[n] (r[\delta m n] 1[m n] + \sim r[\delta m n] \sim 1[m n]). \dagger 1$$

Multiplying this twice into the last proposition, identifying α with δ , and n with b , in both factors, m with a in one and with c in the other, we have

$$\Sigma[a] \pi[b] \Sigma[c] u[a] (\sim w[b] + \sim 1[a b] + 1[c b] u[c] \sim 1[a c]);$$

or since

$$1[c b] \sim 1[a c] \rightarrow \sim 1[a b]$$

$$\Sigma[a] \pi[b] u[a] (\sim w[b] + \sim 1[a b]);$$

and identifying b with a

$$\Sigma[a] u[a] \sim w[a].$$

Again, we have universally

$$\Sigma[\delta] \pi[m] \pi[n] r[\delta m n]. \dagger 2$$

Multiplying this twice into the proposition

$$\pi[\alpha]\Sigma[a]\pi[b]\Sigma[c] u[a](\sim w[b] + \sim r[\alpha a b] + r[\alpha c b]u[c]\sim 1[a c])$$

with the same identifications as before, we get

$$\Sigma[a]\pi[b]\Sigma[c] u[a](\sim w[b] + u[c]\sim 1[a c]),$$

that is, there is a u , and if there be a w there is a second u . This last proposition shows, by the formula

$$\Sigma[a]\Sigma[c] x[a]x[c]\sim 1[a c] \rightarrow \pi[a]\Sigma[c](\sim x[a] + x[c]\sim 1[a c])$$

that the original proposition may be written in the form

$$\pi[\alpha]\Sigma[a]\pi[b]\Sigma[c] u[a] \{ \sim w[b] + u[c]\sim 1[a c](\sim r[\alpha a b] + r[\alpha c b]) \}$$

Peirce: CP 3.403M Cross-Ref:††

403M. Sixth step. The step numbered fifth [396] may more conveniently be separated into two. The first of these somewhat resembles the last; it is a sort of development or setting forth in detail the premiss, but instead of being founded upon a distribution formula it consists in raising the whole premiss to a power or multiplying it into itself. At each such multiplication any of the indices may be changed to new ones, their order in the quantifier being determined by the rules of the second step. To prove that this can be done we begin by confining our attention to the first index of the quantifier. The proposition is then either of the form $\pi[i]a[i]$ or $\Sigma[i]a[i]$. We have obviously

$$\pi[i]a[i] = a[1]a[2]a[3] \text{ etc.} = \pi[i]\pi[j]a[i]a[j]$$

$$\Sigma[i]a[i] = a[1] + a[2] + a[3] + \text{etc.}$$

$$= (a[1]+a[2]+a[3]+\text{etc.})(a[1]+a[2]+a[3]+\text{etc.})$$

$$= \Sigma[i]\Sigma[j]a[i]a[j].$$

Peirce: CP 3.403M Cross-Ref:††

Next consider the first two indices. The proposition is of one of the four forms

$$\pi[i]\pi[j]a[ij] \quad \Sigma[i]\pi[j]a[ij] \quad \pi[i]\Sigma[j]a[ij] \quad \Sigma[i]\Sigma[j]a[ij]$$

For the first and last of these, we have only to apply the formulæ just obtained, with the first two [of step two in 396]. That is,

$$\begin{aligned} \pi[i]\pi[j]a[ij] &= \pi[j]\pi[i]a[ij] = \pi[i]\pi[k]\pi[j]a[ij]a[kj] \\ &= \pi[j]\pi[i]\pi[k]a[ij]a[kj] = \pi[j]\pi[l]\pi[i]\pi[k]a[ij]a[kl]. \end{aligned}$$

$$\begin{aligned} \Sigma[i]\Sigma[j]a[ij] &= \Sigma[j]\Sigma[i]a[ij] = \Sigma[i]\Sigma[k]\Sigma[j]a[ij]a[kj] \\ &= \Sigma[i]\Sigma[j]\Sigma[k]\Sigma[l]a[ij]a[kl]. \end{aligned}$$

For the other two forms the proceeding is not more difficult. By the formulæ for a single index

$$\Sigma[i](\pi[j]a[ij]) = \Sigma[i]\Sigma[k](\pi[j]a[ij])(\pi[j]a[kj]) = \Sigma[i]\Sigma[k]\pi[j]a[ij]a[kj]$$

$$\pi[i](\Sigma[j]a[ij]) = \pi[i]\pi[k](\Sigma[j]a[ij])(\Sigma[j]a[kj])$$

Peirce: CP 3.404 Cross-Ref:††

XIV

***THE CRITIC OF ARGUMENTS*†1**

§1. EXACT THINKING

404. "Critic" is a word used by Locke in English, by Kant in German, and by Plato in Greek, to signify the art of judging, being formed like "logic." I should shrink from heading my papers **Logic**, because logic, as it is set forth in the treatises, is an art far worse than useless, making a man captious about trifles and neglectful of weightier matters, condemning every inference really valuable and admitting only such as are really childish.

It is naughty to do what mamma forbids;

Now, mamma forbids me to cut off my hair:

Therefore, it would be naughty for me to cut off my hair.

This is the type of reasoning to which the treatises profess to reduce all the reasonings which they approve. Reasoning from authority does, indeed, come to that, and in a broad sense of the word authority, such reasoning only. This reminds us that the logic of the treatises is, in the main, a heritage from the ages of faith and obedience, when the highest philosophy was conceived to lie in making everything depend upon authority. Though few men and none of the less sophisticated minds of the other sex ever, nowadays, plunge into the darkling flood of the medieval commentaries, and fewer still dive deep enough to touch bottom, everybody has received the impression they are full of syllogistic reasoning; and this impression is correct. The syllogistic logic truly reflects the sort of reasoning in which the men of the middle ages sincerely put their trust; and yet it is not true that even scholastic theology was sufficiently prostrate before its authorities to have possibly been, in the main, a product of ordinary syllogistic thinking. Nothing can be imagined more strongly marked in its distinctive character than the method of discussion of the old doctors. Their one recipe for any case of difficulty was a distinction. That drawn, they would proceed to show that the difficulties were in force against every member of it but one. Therein all their labor of thinking lies, and thence comes all that makes their philosophy what it is. Without pretending, then, to pronounce the last word on the character of their thought, we may, at least, say it was not, in their sense, syllogistic; since in place of syllogisms it is rather characterised by the use of such forms as the following:

Everything is either *P* or *M*,

S is not *M*;

∴ *S* is *P*.

This is commonly called disjunctive reasoning; but, for reasons which it would be too long to explain in full, I prefer to term it dilemmatic reasoning. Such modes of inference are, essentially, of the same character as the dilemma. Indeed, the regular stock example of the dilemma (for the logicians, in their gregariousness, follow their leader even down to the examples), though we find it set down in the second-century commonplace-book of Aulus Gellius, has quite the ring of a scholastic disquisition. The question, in this example, is, ought one to take a wife? In answering it, we first distinguish in regard to wives (and I seem to hear the Doctor subillissimus saying: *primo distinguendum est de hoc nomine uxor*). A wife may mean a plain or a pretty wife. Now, a plain wife does not satisfy her husband; so one ought not to take a plain wife. But a pretty wife is a perpetual source of jealousy; so one ought still less to take a pretty wife. In sum, one ought to take no wife, at all. It may seem strange that the dilemma is not mentioned in a single medieval logic. It first appears in the *De Dialectica* of Rudolph Agricola.†1 But it should surprise nobody that the most characteristic form of demonstrative reasoning of those ages is left unnoticed in their logical treatises. The best of such works, at all epochs, though they reflect in some measure contemporary modes of thought, have always been considerably behind their

times. For the methods of thinking that are living activities in men are not objects of reflective consciousness. They baffle the student, because they are a part of himself.

"Of thine eye I am eye-beam,"

says Emerson's sphynx. The methods of thinking men consciously admire are different from, and often, in some respects, inferior to those they actually employ. Besides, it is apparent enough, even to one who only knows the works of the modern logicians, that their predecessors can have been little given to seeing out of their own eyes, since, had they been so, their sequacious successors would have been religiously bound to follow suit.

Peirce: CP 3.405 Cross-Ref:††

405. One has to confess that writers of logic-books have been, themselves, with rare exceptions, but shambling reasoners. How wilt thou say to thy brother, Let me pull out the mote out of thine eye; and behold, a beam is in thine own eye? I fear it has to be said of philosophers at large, both small and great, that their reasoning is so loose and fallacious, that the like in mathematics, in political economy, or in physical science, would be received in derision or simple scorn. When, in my teens, I was first reading the masterpieces of Kant, Hobbes, and other great thinkers, my father, who was a mathematician, and who, if not an analyst of thought, at least never failed to draw the correct conclusion from given premisses, unless by a mere slip, would induce me to repeat to him the demonstrations of the philosophers, and in a very few words would usually rip them up and show them empty. In that way, the bad habits of thinking that would otherwise have been indelibly impressed upon me by those mighty powers, were, I hope, in some measure, overcome. Certainly, I believe the best thing for a fledgling philosopher is a close companionship with a stalwart practical reasoner.

Peirce: CP 3.406 Cross-Ref:††

406. How often do we hear it said that the study of philosophy requires *hard thinking!* But I am rather inclined to think a man will never begin to reason well about such subjects, till he has conquered the natural impulse to making spasmodic efforts of mind. In mathematics, the complexity of the problems renders it often a little difficult to hold all the different elements of our mental diagrams in their right places. In a certain sense, therefore, hard thinking *is* occasionally requisite in that discipline. But metaphysical philosophy does not present any such complications, and has no work that *hard thinking* can do. What is needed above all, for metaphysics, is thorough and mature thinking; and the particular requisite to success in the critic of arguments is exact and diagrammatic thinking.

Peirce: CP 3.407 Cross-Ref:††

407.†1 To illustrate my meaning, and at the same time to justify myself, in some degree, for conceding all I have to the prejudice of logicians, I will devote the residue of the space which I can venture to occupy today, to the examination of a statement which has often been made by logicians, and often dissented from, but which I have never seen treated otherwise than as a position quite possible for a reputable logician. I mean the statement that the principle of identity is the necessary and sufficient condition of the validity of all affirmative syllogisms, and that the principles of contradiction and excluded middle, constitute the additional necessary

and sufficient conditions for the validity of negative syllogisms. The principle of identity, expressed by the formula " A is A ," states that the relation of subject to predicate is a relation which every term bears to itself. The principle of contradiction, expressed by the formula " A is not not A ," might be understood in three different senses; first, that any term is in the relation of negation to whatever term is in that relation to it, which is as much as to say that the relation of negation is its own converse; second, that no term is in the relation of negation to itself; third, that every term is in the relation of negation to everything but itself. But the first meaning is the best, since from it the other two readily follow as corollaries. The principle of excluded middle, expressed by the formula "Not not A is A ," may also be understood in three senses; first, that every term, A , is predicable of anything that is in the relation of negation to a term which is in the same relation to it, A ; second, that the objects of which any term, A , is predicable together with those of which the negative of A is predicable together make up all the objects possible; third, that every term, A , is predicable of whatever is in the relation of negation to everything but A . But, as before, the first meaning is to be preferred, since from it the others are immediately deducible.

Peirce: CP 3.408 Cross-Ref: ††

408. There is but one mood of universal affirmative syllogism. It is called *Barbara*, and runs thus:

Any M is P ,
 Any S is M ;
 ∴ Any S is P .

Now the question is, what one of the properties of the relation of subject to predicate is it, with the destruction of which alone this form of inference ceases invariably to yield a true conclusion from true premisses? To find that out the obvious way is to destroy all the properties of the relation in question, so as to make it an entirely different relation, and then note what condition this relation must satisfy in order to make the inference valid. Putting *loves* in place of *is*, we get:

M loves P ,
 S loves M ;
 ∴ S loves P .

That this should be universally true, it is necessary that every lover should love whatever his beloved loves. A relation of which the like is true is called a *transitive* relation. Accordingly, the condition of the validity of *Barbara* is that the relation expressed by the copula should be a transitive relation. This statement was first accurately made by De Morgan †1; but it is in substantial agreement with the doctrine of Aristotle. The analogue of the principle of identity, when *loves* is the copula of the

proposition, is that everybody loves himself. This would plainly not suffice of itself to make the inferential form valid; nor would its being false prevent that form from being valid, provided loving were a transitive relation. Thus, by a little exact thinking, the principle of identity is clearly seen to be neither a sufficient nor a necessary condition for the truth of *Barbara*.

Peirce: CP 3.409 Cross-Ref:††

409. Let us now examine the negative syllogisms. The simplest of these is *Celarent*, which runs as follows:

Any *M* is not *P*,
Any *S* is *M*;
∴ Any *S* is not *P*

Let us substitute *injures* for *is not*. Then the form becomes

Every *M* injures *P*,
Every *S* is *M*;
∴ Every *S* injures *P*.

This is a good inference, still, no matter what sort of relation injuring is. Consequently, this syllogism is dependent upon no property of negation, except that it expresses a relation. Let us, in the last form, substitute *loves* for *is*. Then, we get

M injures *P*,
S loves *M*;
∴ *S* injures *P*.

In order that this should hold good irrespective of the nature of the relation of injuring, it is necessary that nobody should love anybody but himself. A relation of that sort is called a *sibi-relation* or *concurrency*.†1 The necessary and sufficient condition of the validity of *Celarent is*, then, that the copula should express a *sibi-relation*. This is *not* what the principle of identity expresses. Of course, every *sibi-relation* is transitive.

Peirce: CP 3.410 Cross-Ref:††

410. The next simplest of the universal negative syllogisms is *Camestres*, which runs thus:

Any M is P ,
 Any S is not P ;
 \therefore Any S is not M .

Substitute *injures* for *is not*, and we get,

Every M is a P ,
 Every S injures every P ;
 \therefore Every S injures every M .

This obviously holds because the injuring is to *every* one of the class injured. It would not do to reason,

Every M is a P ,
 Every S injures a P ;
 \therefore Every S injures an M .

We see, then, that the principal reason of the validity of *Camestres* is that by *not*, we mean *not any*, and not *not some*. In logical lingo, this is expressed by saying that negative predicates are distributed. But the condition that the copula expresses a *sibi-relation* is also involved.

Peirce: CP 3.411 Cross-Ref:††

411. The remaining universal negative syllogisms of the old enumeration, *Celantes* and *Cesare*, depend upon one principle. They are:

<i>Celantes</i>	<i>Cesare</i>
Any M is not P ,	Any M is not P ,
Any S is M ;	Any S is P ;
\therefore Any P is not S .	\therefore Any S is not M .

Substituting *fights* for *is not*, we get

Every *M* fights every *P*,
Every *S* is *M*;
∴ Every *P* fights every *S*.

Every *M* fights every *P*,
Every *S* is *P*;
∴ Every *S* fights every *M*.

What is requisite to the validity of these inferences is plainly that the relation expressed by *fights* should be its own converse, or that everything should fight whatever fights it. This is the analogue of the principle of contradiction.

Peirce: CP 3.412 Cross-Ref:††

412. We see, then, that the principles of universal syllogism of the ordinary sort are that the copula expresses a *sibi-relation*, not that it expresses an agreement, which is what the principle of identity states, and that the negative is its own converse, which is the law of contradiction.

Peirce: CP 3.413 Cross-Ref:††

413. The authors who say that the principle of identity governs affirmative syllogism give no proof of what they allege. We are expected to see it by "hard thinking." I fancy I can explain what this process of "hard thinking" is. By a spasm induced by self-hypnotisation you throw yourself into a state of mental vacancy. In this state the formula "*A* is *A*" loses its definite signification and seems quite empty. Being empty it is regarded as wonderfully lofty and precious. Fired into enthusiasm by the contemplation of it, the subject, with one wild mental leap, throws himself into the belief that it must rule all human reason. Consequently, it is the principle of syllogism. If this is, as I suspect, what hard thinking means, it is of no use in philosophy.

Peirce: CP 3.414 Cross-Ref:††

414. As for the principle of excluded middle, the only syllogistic forms it governs are the dilemmatic ones.

Any not *P* is *M*,
Any *S* is not *M*;
∴ Any *S* is *P*.

Putting *admiring* for *not*, we have:

Everything admiring every *P* is an *M*,

Every *S* admires every *M*;

∴ Every *S* is a *P*.

To make this good, it must be that the only person who admires everybody that admires a given person is that person. This is the analogue of "everything not not *A* is *A*," which is the principle of excluded middle.

Peirce: CP 3.415 Cross-Ref:††

§2. THE READER IS INTRODUCED TO RELATIVES †1

415. There is a melancholy book entitled *Astronomy Without Mathematics*. The author, an F. R. A. S., presumably knew something of astronomy; therefore, I pity him. I think I hear his groans and maledictions, as he wrote the book, over the initial lie to which he had committed himself, that it is possible to convey any idea of the science of astronomy without making use of mathematics. He could tell roughly how the planets go round the sun, and make his readers think they knew what the error of the ancient system was (namely, that all went round the earth -- really, no error), and could set down surprising figures about the stars (beaten, however, by Buddhistic numbers both in magnitude and in intellectual value). A book so made might well have been called "The Story of the Heavens" (in anticipation of Dr. Ball's splendid volume, which, promising little, performs much), but it was not the "astronomy" stipulated for in the title page. When, in a neighbor's house yesterday, my eye lit upon that book, I shuddered. For I too have engaged myself by the title of these papers to produce something of solid value to my readers; but, thank God, I have not agreed to do it without the use of mathematics. I came home and pondered; and have decided that, in order to fulfill legitimate expectations, I must begin with a few chapters upon certain dry and somewhat technical matters that underlie the more interesting questions concerning reasoning. Do not fear a repetition of matter to be found in common textbooks. I shall suppose the reader to be acquainted with what is contained in Dr. Watts's *Logick*, a book very cheap and easily procured, and far superior to the treatises now used in colleges, being the production of a man distinguished for good sense. I mean to bring out a reprint of it, with extensive annotations, whenever I can find an eligible publisher.†1 Though a life-long student of reasonings, I know no way of giving the reader the benefit of what I ought to have learned, without asking him to go through with some irksome preliminary thinking about relations.

For this subject, although always recognised as an integral part of logic, has been left untouched on account of its intricacy. It is as though a geographer, finding the whole United States, its topography, its population, its industries, etc., too vast for convenient treatment, were to content himself with a description of Nantucket. This comparison hardly, if at all, exaggerates the inadequacy of a theory of reasoning that takes no account of relative terms.

Peirce: CP 3.416 Cross-Ref:††

416. A *relation* is a fact about a number of things. Thus the fact that a locomotive blows off steam constitutes a relation, or more accurately a relationship

(the *Century Dictionary*, under *relation*, 3, gives the terminology. †2 See also *relativity*, etc.) between the locomotive and the steam. In reality, every fact is a relation. Thus, that an object is blue consists of the peculiar regular action of that object on human eyes. This is what should be understood by the "relativity of knowledge."

Peirce: CP 3.417 Cross-Ref: ††

417. Not only is every fact really a relation, but your thought of the fact *implicitly* represents it as such. Thus, when you think "this is blue," the demonstrative "this" shows you are thinking of something just brought up to your notice; while the adjective shows that you recognise a familiar idea as applicable to it. Thus, your thought, when explicated, develops into the thought of a fact concerning this thing and concerning the character of blueness. Still, it must be admitted that, antecedently to the unwrapping of your thought, you were not actually thinking of blueness as a distinct object, and therefore were not thinking of the relation as a relation. †P1 There is an aspect of every relation under which it does not appear as a relation. Thus, the blowing off of steam by a locomotive may be regarded as merely an action of the locomotive, the steam not being conceived to be a thing distinct from the engine. This aspect we enphrase in saying, "the engine blows."

Peirce: CP 3.418 Cross-Ref: ††

418. Thus, the question whether a fact is to be regarded as referring to a single thing or to more is a question of the form of proposition under which it suits our purpose to state the fact. Consider any argument concerning the validity of which a person might conceivably entertain for a moment some doubt. For instance, let the premiss be that from either of two provinces of a certain kingdom it is possible to proceed to any province by floating down the only river the kingdom contains, combined with a land-journey within the boundaries of one province; and let the conclusion be that the river, after touching every province in the kingdom, must again meet the one which it first left. Now, in order to show that this inference is (or that it is not) absolutely necessary, it is requisite to have something analogous to a diagram with different series of parts, the parts of each series being evidently related as those provinces are said to be, while in the different series something corresponding to the course of the river has all the essential variations possible; and this diagram must be so contrived that it is easy to examine it and find out whether the course of the river is in truth in every case such as is here proposed to be inferred. Such a diagram has got to be either auditory or visual, the parts being separated in the one case in time, in the other in space. But in order completely to exhibit the analogue of the conditions of the argument under examination, it will be necessary to use signs or symbols repeated in different places and in different juxtapositions, these signs being subject to certain "rules," that is, certain general relations associated with them by the mind. Such a method of forming a diagram is called *algebra*. All speech is but such an algebra, the repeated signs being the words, which have relations by virtue of the meanings associated with them. What is commonly called *logical algebra* differs from other formal logic only in using the same formal method with greater freedom. I may mention that unpublished studies †1 have shown me that a far more powerful method of diagrammatisation than algebra is possible, being an extension at once of algebra and of Clifford's method of graphs; but I am not in a situation to draw up a statement of my researches.

Peirce: CP 3.419 Cross-Ref: ††

419. Diagrams and diagrammatoidal figures are intended to be applied to the

better understanding of states of things, whether experienced, or read of, or imagined. Such a figure cannot, however, show what it is to which it is intended to be applied; nor can any other diagram avail for that purpose. The where and the when of the particular experience, or the occasion or other identifying circumstance of the particular fiction to which the diagram is to be applied, are things not capable of being diagrammatically exhibited. Describe and describe and describe, and you never can describe a date, a position, or any homaloidal quantity. You may object that a map is a diagram showing localities; undoubtedly, but not until the law of the projection is understood, nor even then unless at least two points on the map are somehow previously identified with points in nature. Now, how is any diagram ever to perform that identification? If a diagram cannot do it, algebra cannot: for algebra is but a sort of diagram; and if algebra cannot do it, language cannot: for language is but a kind of algebra. It would, certainly, in one sense be extravagant to say that we can never tell what we are talking about; yet, in another sense, it is quite true. The meanings of words ordinarily depend upon our tendencies to weld together qualities and our aptitudes to see resemblances, or, to use the received phrase, upon associations by *similarity*; while experience is bound together, and only recognisable, by forces acting upon us, or, to use an even worse chosen technical term, by means of associations by *contiguity*. Two men meet on a country road. One says to the other, "that house is on fire." "What house?" "Why, the house about a mile to my right." Let this speech be taken down and shown to anybody in the neighboring village, and it will appear that the language by itself does not fix the house. But the person addressed sees where the speaker is standing, recognises his *right* hand side (a word having a most singular mode of signification) estimates a *mile* (a length having no geometrical properties different from other lengths), and looking there, sees a house. It is not the language alone, with its mere associations of similarity, but the language taken in connection with the auditor's own experiential associations of contiguity, which determines for him what house is meant. It is requisite then, in order to show what we are talking or writing about, to put the hearer's or reader's mind into real, active connection with the concatenation of experience or of fiction with which we are dealing, and, further, to draw his attention to, and identify, a certain number of particular points in such concatenation. If there be a reader who cannot understand my writings, let me tell him that no straining of his mind will help him: his whole difficulty is that he has no personal experience of the world of problems of which I am talking, and he might as well close the book until such experience comes. That the diagrammatisation is one thing and the application of the diagram quite another, is recognised obscurely in the structure of such languages as I am acquainted with, which distinguishes the *subjects* and *predicates* of propositions. The subjects are the indications of the things spoken of, the predicates, words that assert, question, or command whatever is intended.†1 Only, the shallowness of syntax is manifest in its failing to recognise the impotence of mere words, and especially of common nouns, to fulfil the function of a grammatical subject. Words like *this, that, lo, hallo, hi there*, have a direct, forceful action upon the nervous system, and compel the hearer to look about him; and so they, more than ordinary words, contribute towards indicating what the speech is about. But this is a point that grammar and the grammarians (who, if they are faithfully to mirror the minds of the language-makers, can hardly be scientific analysts) are so far from seeing as to call demonstratives, such as *that* and *this*, pronouns -- a literally preposterous designation, for nouns may more truly be called pro-demonstratives.†1

Peirce: CP 3.420 Cross-Ref:††

420. If upon a diagram we mark two or more points to be identified at some future time with objects in nature, †P1 so as to give the diagram at that future time its meaning; or if in any written statement we put dashes in place of two or more demonstratives or pro-demonstratives, the professedly incomplete representation resulting may be termed a *relative rhema*. It differs from a relative *term* only in retaining the "copula," or signal of assertion. If only one demonstrative or prodemonstrative is erased, the result is a *non-relative rhema*. For example, "-- buys -- from -- for the price --," is a relative rhema; it differs in a merely secondary way from

"-- is bought by -- from -- for --,"
 from "-- sells -- to -- for --,"
 and from "-- is paid by -- to -- for --."

On the other hand, "-- is mortal" is a non-relative rhema.

Peirce: CP 3.421 Cross-Ref: ††

421. A rhema is somewhat closely analogous to a chemical atom or radicle with unsaturated bonds. †2 A non-relative rhema is like a univalent radicle; it has but one unsaturated bond. A relative rhema is like a multivalent radicle. The blanks of a rhema can only be filled by terms, or, what is the same thing, by "something which" (or the like) followed by a rhema; or, two can be filled together by means of "itself" or the like. So, in chemistry, unsaturated bonds can only be saturated by joining two of them, which will usually, though not necessarily, belong to different radicles. If two univalent radicles are united, the result is a saturated compound. So, two non-relative rhemas being joined give a complete proposition. Thus, to join "-- is mortal" and "-- is a man," we have "*X* is mortal and *X* is a man," or some man is mortal. So likewise, a saturated compound may result from joining two bonds of a bivalent radicle; †P2 and, in the same way, the two blanks of a dual rhema may be joined to make a complete proposition. Thus, "-- loves --," "*X* loves *X*," or something loves itself. A univalent radicle united to a bivalent radicle gives a univalent radicle (as H-O-); and, in like manner, a non-relative rhema, joined to a dual rhema, gives a non-relative rhema. Thus, "-- is mortal" joined to "-- loves --" gives "-- loves something that is mortal," which is a non-relative rhema, since it has only one blank. Two, or any number of bivalent radicles united, give a bivalent radicle (as O-O-S-O-O-), and so two or more dual rhemata give a dual rhema; as "-- loves somebody that loves somebody that serves somebody that loves --." Non-relative and dual rhemata only produce rhemata of the same kind, so long as the junctions are by twos; but junctions of triple rhemata (or junctions of dual rhemata by threes), will produce all higher orders. Thus, "-- gives -- to --" and "-- takes -- from --," give "-- gives -- to somebody who takes -- from --," a quadruple rhema. This joined to another quadruple rhema, as "-- sells -- to -- for --," gives the sextuple rhema "-- gives -- to somebody who takes -- from somebody who sells -- to -- for --." Accordingly, all rhemata higher than the dual may be considered as belonging to one and the same order; and we may say that all rhemata are either singular, dual, or plural. †1

Peirce: CP 3.422 Cross-Ref: ††

422. Such, at least, is the doctrine I have been teaching for twenty-five years,

and which, if deeply pondered, will be found to enwrap an entire philosophy.†2 Kant taught that our fundamental conceptions are merely the ineluctable ideas of a system of logical forms; nor is any occult transcendentalism requisite to show that this is so, and must be so. Nature only appears intelligible so far as it appears rational, that is, so far as its processes are seen to be like processes of thought. I must take this for granted, for I have no space here to argue it. It follows that if we find three distinct and irreducible forms of rhemata, the ideas of these should be the three elementary conceptions of metaphysics. That there are three elementary forms of categories is the conclusion of Kant, to which Hegel subscribes; and Kant seeks to establish this from the analysis of formal logic. Unfortunately, his study of that subject was so excessively superficial that his argument is destitute of the slightest value. Nevertheless, his conclusion is correct; for the three elements permeate not only the truths of logic, but even to a great extent the very errors of the profounder logicians. I shall return to them next week.†1 I will only mention here that the ideas which belong to the three forms of rhemata are firstness, secondness, thirdness; firstness, or spontaneity; secondness, or dependence; thirdness, or mediation.

Peirce: CP 3.423 Cross-Ref:††

423. But Mr. A. B. Kempe, in his important memoir on the "Theory of Mathematical Forms,"†P1 presents an analysis which amounts to a formidable objection to my views. He makes diagrams of spots connected by lines; and it is easy to prove that every possible system of relationship can be so represented, although he does not perceive the evidence of this. But he shows (§68) that every such form can be represented by spots indefinitely varied, some of them being connected by lines, all of the same kind. He thus represents every possible relationship by a diagram consisting of only *two* different kinds of elements, namely, spots and lines between pairs of spots. Having examined this analysis attentively, I am of opinion that it is of extraordinary value. It causes me somewhat to modify my position, but not to surrender it. For, in the first place, it is to be remarked that Mr. Kempe's conception depends upon considering the diagram purely in its self-contained relations, the idea of its representing anything being altogether left out of view; while my doctrine depends upon considering how the diagram is to be connected with nature. It is not surprising that the idea of thirdness, or mediation, should be scarcely discernible when the representative character of the diagram is left out of account. In the second place, while it is not in the least necessary that the spots should be of different kinds, so long as each is distinguishable †P2 from the others, yet it is necessary that the connections between the spots should be of two different kinds, which, in Mr. Kempe's diagrams, appear as lines and as the absence of lines. Thus, Mr. Kempe has, and must have, three kinds of elements in his diagrams, namely, one kind of spots, and two kinds of connections of spots. In the third place, the spots, or units, as he calls them, involve the idea of firstness; the two-ended lines, that of secondness; the attachment of lines to spots, that of mediation.

Peirce: CP 3.424 Cross-Ref:††

424. My position has been modified by the study of Mr. Kempe's analysis. For, having a perfect algebra for dual relations, by which, for instance, I could express that "**A** is at once lover of **B** and servant of **C**," I declared that this was inadequate for the expression of plural relations; since to say that **A** gives **B** to **C** is to say more than that **A** gives something to **C**, and gives to somebody **B**, which is given to **C** by somebody. But Mr. Kempe (§330) virtually shows that my algebra is perfectly adequate to expressing that **A** gives **B** to **C**; since I can express each of the following relations:

In a certain act, *D*, something is given by *A*;

In the act, *D*, something is given to *C*;

In the act, *D*, to somebody is given *B*.

Peirce: CP 3.424 Cross-Ref:††

This is accomplished by adding to the universe of concrete things the abstraction "this action." But I remark that the diagram fails to afford any formal representation of the manner in which this abstract idea is derived from the concrete ideas. Yet it is precisely in such processes that the difficulty of all difficult reasoning lies. We have an illustration of this in the circumstance that I was led into an error about the capability of my own algebra for want of just the idea that process would have supplied. The process consists, psychologically, in catching one of the transient elements of thought upon the wing and converting it into one of the resting places of the mind. The difference between setting down spots in a diagram to represent recognised objects, and making new spots for the creations of logical thought, is huge. To include this last as one of the regular operations of logical algebra is to make an intrinsic transmutation of that algebra. What that mutation was I had already shown before Mr. Kempe's memoir appeared.

Peirce: CP 3.425 Cross-Ref:††

XV

THE REGENERATED LOGIC†1

425. The appearance of Schröder's *Exact Logic*†P1 has afforded much gratification to all those homely thinkers who deem the common practice of designating propositions as "unquestionable," "undoubtedly true," "beyond dispute," etc., which are known to the writer who so designates them to be doubted, or perhaps even to be disputed, by persons who with good mental capacities have spent ten or more years of earnest endeavor in fitting themselves to judge of matters such as those to which the propositions in question relate, to be no less heinous an act than a trifling with veracity, and who opine that questions of logic ought *not* to be decided upon philosophical principles, but on the contrary, that questions of philosophy ought to be decided upon logical principles, these having been themselves settled upon principles derived from the only science in which there has never been a prolonged dispute relating to the proper objects of that science. Among those homely thinkers the writer of this review is content to be classed.

Peirce: CP 3.425 Cross-Ref:††

Why should we be so much gratified by the appearance of a single book? Do we anticipate that this work is to convince the philosophical world? By no means;

because we well know that prevalent philosophical opinions are not formed upon the above principles, nor upon any approach to them. A recent little paper by an eminent psychologist concludes with the remark that the verdict of a majority of four of a jury, provided the individual members would form their judgments independently, would have greater probability of being true than the unanimous verdict now is. Certainly, this may be assented to; for the present verdict is not so much an opinion as a resultant of psychical and physical forces. But the remark seemed to me a pretty large concession from a man imbued with the idea of the value of modern opinion about philosophical questions formed according to that scientific method which the Germans and their admirers regard as the method of modern science -- I mean, that method which puts great stress upon cooperation and solidarity of research even in the early stages of a branch of science, when independence of thought is the wholesome attitude, and gregarious thought is really sure to be wrong. For, as regards the verdict of German *university professors*, which, excepting at epochs of transition, has always presented a tolerable approach to unanimity upon the greater part of fundamental questions, it has always been made up as nearly as possible in the same way that the verdict of a jury is made up. Psychical forces, such as the spirit of the age, early inculcations, the spirit of loyal discipline in the general body, and that power by virtue of which one man bears down another in a negotiation, together with such physical forces as those of hunger and cold, are the forces which are mainly operative in bringing these philosophers into line; and none of these forces have any direct relation to reason. Now, these men write the larger number of those books which are so thorough and solid that every serious inquirer feels that he is obliged to read them; and his time is so engrossed by their perusal that his mind has not the leisure to digest their ideas and to reject them. Besides, he is somewhat overawed by their learning and thoroughness. This is the way in which certain opinions -- or rather a certain verdict -- becomes prevalent among philosophical thinkers everywhere; and reason takes hardly the leading part in the performance. It is true, that from time to time, this prevalent verdict becomes altered, in consequence of its being in too violent opposition with the changed spirit of the age; and the logic of history will usually cause such a change to be an advance toward truth in some respect. But this process is so slow, that it is not to be expected that any rational opinion about logic will become prevalent among philosophers within a generation, at least.

Peirce: CP 3.425 Cross-Ref:††

Nevertheless, hereafter, the man who sets up to be a logician without having gone carefully through Schröder's *Logic* will be tormented by the burning brand of *false pretender* in his conscience, until he has performed that task; and that task he cannot perform without acquiring habits of exact thinking which shall render the most of the absurdities which have hitherto been scattered over even the best of the German treatises upon logic impossible for him. Some amelioration of future treatises, therefore, though it will leave enough that is absurd, is to be expected; but it is not to be expected that those who form their opinions about logic or philosophy rationally, and therefore not gregariously, will ever comprise the majority even of philosophers. But opinions thus formed, and among such those formed by thoroughly informed and educated minds, are the only ones which need cause the homely thinker any misgiving concerning his own.

Peirce: CP 3.426 Cross-Ref:††

426. It is a remarkable historical fact that there is a branch of science in which there has never been a prolonged dispute concerning the proper objects of that science. It is the mathematics. Mistakes in mathematics occur not infrequently, and

not being detected give rise to false doctrine, which may continue a long time. Thus, a mistake in the evaluation of a definite integral by Laplace, in his *Mécanique céleste*, led to an erroneous doctrine about the motion of the moon which remained undetected for nearly half a century. But after the question had once been raised, all dispute was brought to a close within a year. So, several demonstrations in the first book of Euclid, notably that of the sixteenth proposition, are vitiated by the erroneous assumption that a part is necessarily less than its whole. These remained undetected until after the theory of the non-Euclidean geometry had been completely worked out; but since that time, no mathematician has defended them; nor could any competent mathematician do so, in view of Georg Cantor's,†1 or even of Cauchy's discoveries. Incessant disputations have, indeed, been kept up by a horde of undisciplined minds about quadratures, cyclotomy, the theory of parallels, rotation, attraction, etc. But the disputants are one and all men who cannot discuss any mathematical problem without betraying their want of mathematical power and their gross ignorance of mathematics at every step. Again, there have been prolonged disputes among real mathematicians concerning questions which were not mathematical or which had not been put into mathematical form. Instances of the former class are the old dispute about the measure of force, and that lately active concerning the number of constants of an elastic body; and there have been sundry such disputes about mathematical physics and probabilities. Instances of the latter class are the disputes about the validity of reasonings concerning divergent series, imaginaries, and infinitesimals. But the fact remains that concerning strictly mathematical questions, and among mathematicians who could be considered at all competent, there has never been a single prolonged dispute.

Peirce: CP 3.426 Cross-Ref:††

It does not seem worth while to run through the history of science for the sake of the easy demonstration that there is no other extensive branch of knowledge of which the same can be said.

Peirce: CP 3.426 Cross-Ref:††

Nor is the reason for this immunity of mathematics far to seek. It arises from the fact that the objects which the mathematician observes and to which his conclusions relate are objects of his mind's own creation. Hence, although his proceeding is not infallible -- which is shown by the comparative frequency with which mistakes are committed and allowed -- yet it is so easy to repeat the inductions upon new instances, which can be created at pleasure, and extreme cases can so readily be found by which to test the accuracy of the processes, that when attention has once been directed to a process of reasoning suspected of being faulty, it is soon put beyond all dispute either as correct or as incorrect.

Peirce: CP 3.427 Cross-Ref:††

427. Hence, we homely thinkers believe that, considering the immense amount of disputation there has always been concerning the doctrines of logic, and especially concerning those which would otherwise be applicable to settle disputes concerning the accuracy of reasonings in metaphysics, the safest way is to appeal for our logical principles to the science of mathematics, where error can only long go unexploded on condition of its not being suspected.

Peirce: CP 3.427 Cross-Ref:††

This double assertion, first, that logic ought to draw upon mathematics for control of disputed principles, and second that ontological philosophy ought in like manner to draw upon logic, is a case under a general assertion which was made by

Auguste Comte,†1 namely, that the sciences may be arranged in a series with reference to the abstractness of their objects; and that each science draws regulating principles from those superior to it in abstractness, while drawing data for its inductions from the sciences inferior to it in abstractness. So far as the sciences can be arranged in such a scale, these relationships must hold good. For if anything is true of a whole genus of objects, this truth may be adopted as a principle in studying every species of that genus. While whatever is true of a species will form a datum for the discovery of the wider truth which holds of the whole genus. Substantially the following scheme of the sciences †2 is given in the *Century Dictionary*†3:

MATHEMATICS

Philosophy { Logic
 { Metaphysics.

Science of Time | Geometry
 | { Molar

Nomological Psychics | Nomological Physics { Molecular
 | { Ethereal
 |
 | { Chemistry

Classificatory Psychics | Classificatory Physics { Biology, or the
 | { chemistry of
 | { protoplasm

Descriptive Psychics | Descriptive Physics

PRACTICAL SCIENCE.

Perhaps each psychical branch ought to be placed above the corresponding physical branch. However, only the first three branches concern us here.

Peirce: CP 3.428 Cross-Ref:††

428. Mathematics is the most abstract of all the sciences. For it makes no external observations, nor asserts anything as a real fact. When the mathematician deals with facts, they become for him mere "hypotheses"; for with their truth he refuses to concern himself. The whole science of mathematics is a science of hypotheses; so that nothing could be more completely abstracted from concrete reality. Philosophy is not quite so abstract. For though it makes no *special*

observations, as every other positive science does, yet it does deal with reality. It confines itself, however, to the universal phenomena of experience; and these are, generally speaking, sufficiently revealed in the ordinary observations of every-day life. I would even grant that philosophy, in the strictest sense, confines itself to such observations as *must* be open to every intelligence which can learn from experience. Here and there, however, metaphysics avails itself of one of the grander generalisations of physics, or more often of psychics, not as a governing principle, but as a mere datum for a still more sweeping generalisation. But logic is much more abstract even than metaphysics. For it does not concern itself with any facts not implied in the supposition of an unlimited applicability of language.

Peirce: CP 3.428 Cross-Ref:††

Mathematics is not a positive science; for the mathematician holds himself free to say that *A* is *B* or that *A* is not *B*, the only obligation upon him being, that as long as he says *A* is *B*, he is to hold to it, consistently. But logic begins to be a positive science; since there are some things in regard to which the logician is not free to suppose that they are or are not; but acknowledges a compulsion upon him to assert the one and deny the other. Thus, the logician is forced by positive observation to admit that there is such a thing as doubt, that some propositions are false, etc. But with this compulsion comes a corresponding responsibility upon him not to admit anything which he is not forced to admit.

Peirce: CP 3.429 Cross-Ref:††

429. Logic may be defined as the science of the laws of the stable establishment of beliefs. Then, *exact* logic will be that doctrine of the conditions of establishment of stable belief which rests upon perfectly undoubted observations and upon mathematical, that is, upon *diagrammatical*, or, *iconic*, thought. We, who are sectaries of "exact" logic, and of "exact" philosophy, in general, maintain that those who follow such methods will, so far as they follow them, escape all error except such as will be speedily corrected after it is once suspected. For example, the opinions of Professor Schröder and of the present writer diverge as much as those of two "exact" logicians well can; and yet, I think, either of us would acknowledge that, however serious he may hold the errors of the other to be, those errors are, in the first place, trifling in comparison with the original and definite advance which their author has, by the "exact" method, been able to make in logic, that in the second place, they are trifling as compared with the errors, obscurities, and negative faults of any of those who do not follow that method, and in the third place, that they are chiefly, if not wholly, due to their author not having found a way to the application of diagrammatical thought to the particular department of logic in which they occur.

Peirce: CP 3.430 Cross-Ref:††

430. "Exact" logic, in its widest sense, will (as I apprehend) consist of three parts.†1 For it will be necessary, first of all, to study those properties of beliefs which belong to them as beliefs, irrespective of their stability. This will amount to what Duns Scotus †2 called *speculative grammar*. For it must analyse an assertion into its essential elements, independently of the structure of the language in which it may happen to be expressed. It will also divide assertions into categories according to their essential differences. The second part will consider to what conditions an assertion must conform in order that it may correspond to the "reality," that is, in order that the belief it expresses may be stable. This is what is more particularly understood by the word *logic*. It must consider, first, *necessary*, and second, *probable* reasoning. Thirdly, the general doctrine must embrace the study of those general conditions

under which a problem presents itself for solution and those under which one question leads on to another. As this completes a triad of studies, or trivium, we might, not inappropriately, term the last study *Speculative rhetoric*. This division was proposed in 1867†3 by me, but I have often designated this third part as *objective logic*.

Peirce: CP 3.431 Cross-Ref:††

431. Dr. Schröder's Logic is not intended to cover all this ground. It is not, indeed, as yet complete; and over five hundred pages may be expected yet to appear. But of the seventeen hundred and sixty-six pages which are now before the public, only an introduction of one hundred and twenty-five pages rapidly examines the speculative grammar, while all the rest, together with all that is promised, is restricted to the deductive branch of logic proper. By the phrase "exact logic" upon his title-page, he means logic treated algebraically. Although such treatment is an aid to exact logic, as defined on the last page, it is certainly not synonymous with it. The principal utility of the algebraic treatment is stated by him with admirable terseness: it is "to set this discipline free from the fetters in which language, by force of custom, has bound the human mind."†1 Upon the algebra may, however, be based a calculus, by the aid of which we may in certain difficult problems facilitate the drawing of accurate conclusions. A number of such applications have already been made; and mathematics has thus been enriched with new theorems. But the applications are not so frequent as to make the elaboration of a facile calculus one of the most pressing desiderata of the study. Professor Schröder has done a great deal in this direction; and of course his results are most welcome, even if they be not precisely what we should most have preferred to gain.

Peirce: CP 3.432 Cross-Ref:††

432. The introduction, which relates to first principles, while containing many excellent observations, is somewhat fragmentary and wanting in a unifying idea; and it makes logic too much a matter of feeling.†2 It cannot be said to belong to exact logic in any sense. Thus, under β (Vol. I., p. 2) the reader is told that the sciences have to suppose, not only that their objects really exist, but also that they are knowable and that for every question there is a true answer and but one. But, in the first place, it seems more exact to say that in the discussion of one question nothing at all concerning a wholly unrelated question can be implied. And, in the second place, as to an inquiry presupposing that there is some one truth, what can this possibly mean except it be that there is one destined upshot to inquiry with reference to the question in hand --one result, which when reached will never be overthrown? Undoubtedly, we hope that this, or *something approximating to this*, is so, or we should not trouble ourselves to make the inquiry. But we do not necessarily have much confidence that it *is* so. Still less need we think it is so about the *majority* of the questions with which we concern ourselves. But in so exaggerating the presupposition, both in regard to its universality, its precision, and the amount of belief there need be in it, Schröder merely falls into an error common to almost all philosophers about all sorts of "presuppositions." Schröder (under {e}, p. 5) undertakes to define a contradiction in terms without having first made an ultimate analysis of the proposition. The result is a definition of the usual peripatetic type; that is, it affords no analysis of the conception whatever. It amounts to making the contradiction in terms an ultimate unanalysable relation between two propositions -- a sort of blind reaction between them. He goes on (under {z}, p. 9) to define, after Sigwart, logical consequentiality, as a *compulsion of thought*. Of course, he at once endeavors to avoid the dangerous consequences of this theory, by various qualifications. But all that is to no purpose. Exact logic will say that *C*'s following

logically from *A* is a state of *things* which no impotence of thought can alone bring about, unless there is also an impotence of existence for *A* to be a fact without *C* being a fact. Indeed, as long as this latter impotence exists and can be ascertained, it makes little or no odds whether the former impotence exists or not. And the last anchor-hold of logic he makes (under {i}) to lie in the correctness of a feeling! If the reader asks *why* so subjective a view of logic is adopted, the answer seems to be (under β , p. 2), that in this way Sigwart escapes the necessity of founding logic upon the theory of cognition. By the theory of cognition is usually meant an explanation of the possibility of knowledge drawn from principles of psychology. Now, the only sound psychology being a special science, which ought itself to be based upon a wellgrounded logic, it is indeed a vicious circle to make logic rest upon a theory of cognition so understood. But there is a much more general doctrine to which the name theory of cognition might be applied. Namely, it is that speculative grammar, or analysis of the nature of assertion, which rests upon observations, indeed, but upon observations of the rudest kind, open to the eye of every attentive person who is familiar with the use of language, and which, we may be sure, no rational being, able to converse at all with his fellows, and so to express a doubt of anything, will ever have any doubt. Now, proof does not consist in giving superfluous and superpossible certainty to that which nobody ever did or ever will doubt, but in removing doubts which do, or at least might at some time, arise. A man first comes to the study of logic with an immense multitude of opinions upon a vast variety of topics; and they are held with a degree of confidence, upon which, after he has studied logic, he comes to look back with no little amusement. There remains, however, a small minority of opinions that logic never shakes; and among these are certain observations about assertions. The student would never have had a desire to learn logic if he had not paid some little attention to assertion, so as at least to attach a definite signification to assertion. So that, if he has not thought more accurately about assertions, he must at least be conscious, in some out-of-focus fashion, of certain properties of assertion. When he comes to the study, if he has a good teacher, these already dimly recognised facts will be placed before him in accurate formulation, and will be accepted as soon as he can clearly apprehend their statements.

Peirce: CP 3.433 Cross-Ref:††

433. Let us see what some of these are. When an assertion is made, there really is some speaker, writer, or other signmaker who delivers it; and he supposes there is, or will be, some hearer, reader, or other interpreter who will receive it. It may be a stranger upon a different planet, an æon later; or it may be that very same man as he will be a second after. In any case, the deliverer makes signals to the receiver. Some of these signs (or at least one of them) are supposed to excite in the mind of the receiver familiar images, pictures, or, we might almost say, *dreams* -- that is, reminiscences of sights, sounds, feelings, tastes, smells, or other sensations, now quite detached from the original circumstances of their first occurrence, so that they are free to be attached to new occasions. The deliverer is able to call up these images at will (with more or less effort) in his own mind; and he supposes the receiver can do the same. For instance, tramps have the habit of carrying bits of chalk and making marks on the fences to indicate the habits of the people that live there for the benefit of other tramps who may come on later. If in this way a tramp leaves an assertion that the people are stingy, he supposes the reader of the signal will have met stingy people before, and will be able to call up an image of such a person attachable to a person whose acquaintance he has not yet made. Not only is the outward significant word or mark a sign, but the image which it is expected to excite in the mind of the receiver

will likewise be a sign -- a sign by resemblance, or, as we say, an *icon* -- of the similar image in the mind of the deliverer, and through that also a sign of the real quality of the thing. This icon is called the *predicate* of the assertion. But instead of a single *icon*, or sign by resemblance of a familiar image or "dream," evocable at will, there may be a complexus of such icons, forming a composite image of which the whole is not familiar. But though the whole is not familiar, yet not only are the parts familiar images, but there will also be a familiar image of its mode of composition. In fact, two types of complication will be sufficient. For example, one may be conjunctive and the other disjunctive combination. Conjunctive combination is when two images are both to be used at once; and disjunctive when one or other is to be used. (This is not the most scientific selection of types; but it will answer the present purpose.) The sort of idea which an icon embodies, if it be such that it can convey any positive information, being applicable to some things but not to others, is called a *first intention*. The idea embodied by an icon which cannot of itself convey any information, being applicable to everything or to nothing, but which may, nevertheless, be useful in modifying other icons, is called a *second intention*.

Peirce: CP 3.434 Cross-Ref:††

434. The assertion which the deliverer seeks to convey to the mind of the receiver relates to some object or objects which have forced themselves upon his attention; and he will miss his mark altogether unless he can succeed in forcing those very same objects upon the attention of the receiver. No icon can accomplish this, because an icon does not relate to any particular thing; nor does its idea strenuously force itself upon the mind, but often requires an effort to call it up. Some such sign as the word *this*, or *that*, or *hullo*, or *hi*, which awakens and directs attention must be employed. A sign which denotes a thing by forcing it upon the attention is called an *index*. An index does not describe the qualities of its object. An object, in so far as it is denoted by an index, having *thisness*, and distinguishing itself from other things by its continuous identity and forcefulness, but not by any distinguishing characters, may be called a *hecceity*. A hecceity in its relation to the assertion is a *subject* thereof. An assertion may have a multitude of subjects; but to that we shall return presently.

Peirce: CP 3.435 Cross-Ref:††

435. Neither the predicate, nor the subjects, nor both together, can make an *assertion*. The assertion represents a compulsion which experience, meaning the course of life, brings upon the deliverer to attach the predicate to the subjects as a sign of them taken in a particular way. This compulsion strikes him at a certain instant; and he remains under it forever after. It is, therefore, different from the temporary force which the hecceities exert upon his attention. This new compulsion may pass out of mind for the time being; but it continues just the same, and will act whenever the occasion arises, that is, whenever those particular hecceities and that first intention are called to mind together. It is, therefore, a permanent conditional force, or *law*. The deliverer thus requires a kind of sign which shall signify a law that to objects of indices an icon appertains as sign of them in a given way. Such a sign has been called a *symbol*. It is the *copula* of the assertion.

Peirce: CP 3.436 Cross-Ref:††

436. Returning to the subjects, it is to be remarked that the assertion may contain the suggestion, or request, that the receiver *do* something with them. For instance, it may be that he is first to take any one, no matter what, and apply it in a certain way to the icon, that he is then to take another, perhaps this time a suitably chosen one, and apply that to the icon, etc. For example, suppose the assertion is:

"Some woman is adored by all catholics." The constituent icons are, in the probable understanding of this assertion, three, that of a woman, that of a person, *A*, adoring another, *B*, and that of a non-catholic. We combine the two last disjunctively, identifying the noncatholic with *A*; and then we combine this compound with the first icon conjunctively, identifying the woman with *B*. The result is the icon expressed by, "*B* is a woman, and moreover, either *A* adores *B* or else *A* is a non-catholic." The subjects are all the things in the real world past and present. From these the receiver of the assertion is suitably to choose one to occupy the place of *B*; and then it matters not what one he takes for *A*. A suitably chosen object is a woman, and any object, no matter what, adores her, unless that object be a noncatholic. This is forced upon the deliverer by experience; and it is by no idiosyncrasy of his; so that it will be forced equally upon the receiver.

Peirce: CP 3.437 Cross-Ref:††

437. Such is the meaning of one typical assertion. An assertion of *logical necessity* is simply one in which the subjects are the objects of any collection, no matter what. The consequence is, that the icon, which can be called up at will, need only to be called up, and the receiver need only ascertain by experiment whether he can distribute any set of indices in the assigned way so as to make the assertion false, in order to put the truth of the assertion to the test. For example, suppose the assertion of logical necessity is the assertion that from the proposition, "Some woman is adored by all catholics," it logically follows that "Every catholic adores some woman." That is as much as to say that, for every imaginable set of subjects, either it is false that some woman is adored by all catholics or it is true that every catholic adores some woman. We try the experiment. In order to avoid making it false that some woman is adored by all catholics, we must choose our set of indices so that there shall be one of them, *B*, such that, taking any one, *A*, no matter what, *B* is a woman, and moreover either *A* adores *B* or else *A* is a non-catholic. But that being the case, no matter what index, *A*, we may take, either *A* is a noncatholic or else an index can be found, namely, *B*, such that *B* is a woman, and *A* adores *B*. We see, then, by this experiment, that it is impossible so to take the set of indices that the proposition of consecution shall be false. The experiment may, it is true, have involved some blunder; but it is so easy to repeat it indefinitely, that we readily acquire any desired degree of certitude for the result.

Peirce: CP 3.438 Cross-Ref:††

438. It will be observed that this explanation of logical certitude depends upon the fact of speculative grammar that the predicate of a proposition, being essentially of an ideal nature, can be called into the only kind of existence of which it is capable, at will.

Peirce: CP 3.439 Cross-Ref:††

439. A not unimportant dispute has raged for many years as to whether hypothetical propositions (by which, according to the traditional terminology, I mean any compound propositions, and not merely those *conditional* propositions to which, since Kant, the term has often been restricted) and categorical propositions are one in essence. Roughly speaking, English logicians maintain the affirmative, Germans the negative. Professor Schröder is in the camp of the latter, I in that of the former.

Peirce: CP 3.440 Cross-Ref:††

440. I have maintained since 1867 that there is but one primary and fundamental logical relation, that of illation, expressed by *ergo*. A proposition, for

me, is but an argumentation divested of the assertoriness of its premiss and conclusion. This makes every proposition a conditional proposition at bottom. In like manner a "term," or class-name, is for me nothing but a proposition with its indices or subjects left blank, or indefinite. The common noun happens to have a very distinctive character in the Indo-European languages. In most other tongues it is not sharply discriminated from a verb or participle. "Man," if it can be said to mean anything by itself, means "what I am thinking of is a man." This doctrine, which is in harmony with the above theory of signs, gives a great unity to logic; but Professor Schröder holds it to be very erroneous.

Peirce: CP 3.441 Cross-Ref:††

441. Cicero †1 and other ancient writers †2 mention a great dispute between two logicians, Diodorus and Philo, in regard to the significance of conditional propositions.†3 This dispute has continued to our own day. The Diodoran view seems to be the one which is natural to the minds of those, at least, who speak the European languages. How it may be with other languages has not been reported. The difficulty with this view is that nobody seems to have succeeded in making any clear statement of it that is not open to doubt as to its justice, and that is not pretty complicated. The Philonian view has been preferred by the greatest logicians. Its advantage is that it is perfectly intelligible and simple. Its disadvantage is that it produces results which seem offensive to common sense.

Peirce: CP 3.442 Cross-Ref:††

442. In order to explain these positions, it is best to mention that *possibility* may be understood in many senses; but they may all be embraced under the definition that that is possible which, in a certain state of information, is not known to be false. By varying the supposed state of information all the varieties of possibility are obtained. Thus, *essential* possibility is that which supposes nothing to be known except logical rules. *Substantive* possibility, on the other hand, supposes a state of omniscience. Now the Philonian logicians have always insisted upon beginning the study of conditional propositions by considering what such a proposition means in a state of omniscience; and the Diodorans have, perhaps not very adroitly, commonly assented to this order of procedure. Duns Scotus †1 terms such a conditional proposition a "*consequentia simplex de inesse*." According to the Philonians, "If it is now lightening it will thunder," understood as a consequence *de inesse*, means "It is either not now lightening or it will soon thunder." According to Diodorus, and most of his followers (who seem here to fall into a logical trap), it means "It is now lightening and it will soon thunder."

Peirce: CP 3.443 Cross-Ref:††

443. Although the Philonian views lead to such inconveniences as that it is true, as a consequence *de inesse*, that if the Devil were elected president of the United States, it would prove highly conducive to the spiritual welfare of the people (because he will not be elected), yet both Professor Schröder and I prefer to build the algebra of relatives upon this conception of the conditional proposition. The inconvenience, after all, ceases to seem important, when we reflect that, no matter what the conditional proposition be understood to mean, it can always be expressed by a complexus of Philonian conditionals and denials of conditionals. It may, however, be suspected that the Diodoran view has suffered from incompetent advocacy, and that if it were modified somewhat, it might prove the preferable one.

Peirce: CP 3.444 Cross-Ref:††

444. The consequence *de inesse*, "if *A* is true, then *B* is true," is expressed by

letting i denote the actual state of things, $A[i]$ mean that in the actual state of things A is true, and $B[i]$ mean that in the actual state of things B is true, and then saying "If $A[i]$ is true then $B[i]$ is true," or, what is the same thing, "Either $A[i]$ is not true or $B[i]$ is true." But an *ordinary* Philonian conditional is expressed by saying, "In *any* possible state of things, i , either $A[i]$ is not true, or $B[i]$ is true."

Peirce: CP 3.445 Cross-Ref:††

445. Now let us express the categorical proposition, "Every man is wise." Here, we let $m[i]$ mean that the individual object i is a man, and $w[i]$ mean that the individual object i is wise. Then, we assert that, "taking any individual of the universe, i , no matter what, either that object, i , is not a man or that object, i , is wise"; that is, whatever is a man is wise. That is, "whatever i can indicate, either $m[i]$ is not true or $w[i]$ is true. The conditional and categorical propositions are expressed in precisely the same form; and there is absolutely no difference, to my mind, between them. The *form* of relationship is the same.

Peirce: CP 3.446 Cross-Ref:††

446. I find it difficult to state Professor Schröder's objection to this, because I cannot find any clear-cut, unitary conception governing his opinion. More than once in his first volume promises are held out that §28, the opening section of the second volume, shall make the matter plain. But when the second volume was published, all we found in that section was, as far as repeated examination has enabled me to see, as follows. First, hypothetical propositions, unlike categoricals, essentially involve the idea of time. When this is eliminated from the assertion, they relate only to two possibilities, what always is and what never is. Second, a categorical is always either true or false; but a hypothetical is either true, false, or meaningless. Thus, "this proposition is false" is meaningless; and another example is, "the weather will clear as soon as there is enough sky to cut a pair of trousers." Third, the supposition of negation is forced upon us in the study of hypotheticals, never in that of categoricals. Such are Schröder's arguments, to which I proceed to reply.

Peirce: CP 3.446 Cross-Ref:††

As to the idea of time, it *may* be introduced; but to say that the range of possibility in hypotheticals is always a unidimensional continuum is incorrect. "If you alone trump a trick in whist, you take it." The possibilities are that each of the four players plays any one of the four suits. There are 2^{16} different possibilities. Certainly, the universe in hypotheticals is far more frequently finite than in categoricals. Besides, it is an *ignoratio elenchi* to drag in time, when no logician of the English camp has ever alleged anything about propositions involving time. That is not the question.

Peirce: CP 3.446 Cross-Ref:††

Every proposition is either true or false, and something not a proposition, when considered as a proposition, is, from the Philonian point of view, true. To be objectionable, a proposition must assert something; if it is merely neutral, it is not positively objectionable, that is, it is not false. "This proposition is false," far from being meaningless, is self-contradictory. That is, it means two irreconcilable things. That it involves contradiction (that is, leads to contradiction if supposed true), is easily proved. For if it be true, it is true; while if it be true, it is false. Every proposition besides what it explicitly asserts, tacitly implies its own truth. The proposition is not true unless *both*, what it explicitly asserts and what it tacitly implies, are true. This proposition, being self-contradictory, is false; and hence, what

it explicitly asserts is true. But what it tacitly implies (its own truth) is false.†1 The difficulty about the proposition concerning the piece of blue sky is not a logical one, at all. It is no more senseless than any proposition about a "red odor" which might be a term of a categorical.

Peirce: CP 3.446 Cross-Ref:††

The fact stated about negation is only true of the sorts of propositions which are commonly put into categorical and hypothetical shapes, and has nothing to do with the essence of the propositions. In a paper "On the Validity of the Laws of Logic" in the *Journal of Speculative Philosophy*, Vol. II.,†2 I have given a sophistical argument that black is white, which shows in the domain of categoricals the phenomena to which Professor Schröder refers as peculiar to hypotheticals.

Peirce: CP 3.446 Cross-Ref:††

The *consequentia de inesse* is, of course, the extreme case where the conditional proposition loses all its proper signification, owing to the absence of any range of possibilities. The conditional proper is, "In any possible case, *i*, either $A[i]$ is not true, or $B[i]$ is true." In the consequence *de inesse* the meaning sinks to, "In the true state of things, *i*, either $A[i]$ is not true or $B[i]$ is true."

Peirce: CP 3.447 Cross-Ref:††

447. My general algebra of logic (which is not that algebra of dual relations, likewise mine, which Professor Schröder prefers, although in his last volume he often uses this general algebra) consists in simply attaching indices to the letters of an expression in the Boolean algebra, making what I term a Boolean, and prefixing to this a series of "quantifiers," which are the letters π and Σ , each with an index attached to it. Such a quantifier signifies that every individual of the universe is to be substituted for the index the π or Σ carries, and that the non-relative product or aggregate of the results is to be taken.

Peirce: CP 3.448 Cross-Ref:††

448. Properly to express an ordinary conditional proposition the quantifier π is required. In 1880, three years before I developed that general algebra, I published a paper containing a chapter on the algebra of the copula †1 (a subject I have since worked out completely in manuscript.)†2 I there noticed the necessity of such quantifiers properly to express conditional propositions; but the algebra of quantifiers not being at hand, I contented myself with considering consequences *de inesse*. Some apparently paradoxical results were obtained. Now Professor Schröder seems to accept these results as holding good in the general theory of hypotheticals; and then, since such results are in strong contrast with the doctrine of categoricals, he infers, in §45 of his Vol. II., a great difference between hypotheticals and categoricals. But the truth simply is that such hypotheticals want the characteristic feature of conditionals, that of a range of possibilities.

Peirce: CP 3.449 Cross-Ref:††

449. In connexion with this point, I must call attention to a mere algebraical difference between Schröder and me. I retain Boole's idea that there are but two *values* in the system of logical quantity. This harmonises with my use of the general algebra. Any two numbers may be selected to represent those values. I prefer 0 and a positive logarithmic ∞ . To express that something is *A* and something is not *A*, I write:

$$\infty = \Sigma[i]A[i] \quad \infty = \Sigma[j]\sim A[j]$$

or, what is the same thing:

$$\Sigma[i]A[i] > 0 \quad \Sigma[j]\sim A[j] > 0.$$

I have no objection to writing, *as a mere abbreviation*, which may, however, lead to difficulties, if not *interpreted*:

$$A > 0 \quad \sim A > 0.$$

But Professor Schröder understands these formulæ literally, and accordingly *rejects* Boole's conception of two values. He does not seem to understand my mode of apprehending the matter; and hence considers it a great limitation of my system that I restrict myself to two values. In fact, it is a mere difference of algebraical form of conception. I very much prefer the Boolean idea as more simple, and more in harmony with the general algebra of logic.

Peirce: CP 3.450 Cross-Ref:††

450. Somewhat intimately connected with the question of the relation between categoricals and hypotheticals is that of the quantification of the predicate. This is the doctrine that identity, or equality, is the fundamental relation involved in the copula. Holding as I do that the fundamental relation of logic is the illative relation, and that only in special cases does the premiss follow from the conclusion, I have in a consistent and thoroughgoing manner opposed the doctrine of the quantification of the predicate.†1 Schröder seems to admit some of my arguments; but still he has a very strong *penchant* for the equation.

Peirce: CP 3.450 Cross-Ref:††

Were I not opposed to the quantification of the predicate, I should agree with Venn †2 that it was a mistake to replace Boole's operation of [arithmetical] addition by the operation of [logical] aggregation, as most Boolians now do. I should consider the "principle of duality" †3 rather an argument *against* than *for* our modern practice. The algebra of dual relatives would be almost identical with the theory of matrices were addition retained; and this would be a great advantage.

Peirce: CP 3.451 Cross-Ref:††

451. It is Schröder's predilection for equations which motives his preference for the algebra of dual relatives, namely, the fact that in that algebra, even a simple undetermined inequality can be expressed as an equation. I think, too, that that algebra has merits; it certainly has uses to which Schröder seldom puts it. Yet, after all, it has too much formalism to greatly delight me -- too many bushels of chaff *per* grain of wheat. I think Professor Schröder likes algebraic formalism better, or dislikes it less, than I.

Peirce: CP 3.451 Cross-Ref:††

He looks at the problems of logic through the spectacles of equations, and he formulates them, from that point of view, as he thinks, with great generality; but, as I think, in a narrow spirit. The great thing, with him, is to solve a proposition, and get a *value* of x , that is, an equation of which x forms one member without occurring in the other. How far such equation is *iconic*, that is, has a meaning, or exhibits the constitution of x , he hardly seems to care. He prefers general values to particular roots. Why? I should think the particular root alone of service, for most purposes, unless the general expressions were such that particular roots could be deduced from it -- particular instances, I mean, *showing* the constitution of x . In most instances, a profitable solution of a mathematical problem must consist, in my opinion, of an exhaustive examination of special cases; and quite exceptional are those fortunate problems which mathematicians naturally prefer to study, where the enumeration of special cases, together with the pertinent truths about them, flow so naturally from the general statement as not to require separate examination.

Peirce: CP 3.451 Cross-Ref:††

I am very far from denying the interest and value of the problems to which Professor Schröder has applied himself; though there are others to which I turn by preference. Certainly, he has treated his problems with admirable power and clearness. I cannot in this place enter into the elementary explanations which would be necessary to illustrate this for more than a score of readers.

Peirce: CP 3.452 Cross-Ref:††

452. In respect to individuals, both non-relative and pairs, he has added some fundamental propositions to those which had been published. But he is very much mistaken in supposing that I have expressed contrary views. He simply mistakes my meaning.

Peirce: CP 3.453 Cross-Ref:††

453. In regard to algebraical signs, I cannot accept any of Professor Schröder's proposals except this one. While it would be a serious hindrance to the promulgation of the new doctrine to insist on new types being cut, and while I, therefore, think my own course in using the dagger as the sign of relative addition must be continued, yet I have always given that sign in its cursive form a scorpion-tail curve to the left; and it would be finical to insist on one form of curve rather than another. In almost all other cases, in my judgment, Professor Schröder's signs can never be generally received, because they are at war with a principle, the general character of which is such that Professor Schröder would be the last of all men to wish to violate it, a principle which the biologists have been led to adopt in regard to their systematic nomenclature. It is that priority must be respected, or all will fall into chaos. I will not enter further into this matter in this article.†1

Peirce: CP 3.454 Cross-Ref:††

454. Of what use does this new logical doctrine promise to be? The first service it may be expected to render is that of correcting a considerable number of hasty assumptions about logic which have been allowed to affect philosophy. In the next place, if Kant has shown that metaphysical conceptions spring from formal logic, this great generalisation upon formal logic must lead to a new apprehension of the metaphysical conceptions which shall render them more adequate to the needs of science. In short, "exact" logic will prove a stepping-stone to "exact" metaphysics. In the next place, it must immensely widen our logical notions. For example, a class

consisting of a lot of things jumbled higgledy-piggledy must now be seen to be but a degenerate form of the more general idea of a *system*. Generalisation, which has hitherto meant passing to a larger class, must mean taking in the conception of the whole system of which we see but a fragment, etc., etc. In the next place, it is already evident to those who know what has already been made out, that that speculative rhetoric, or objective logic, mentioned at the beginning of this article, is destined to grow into a colossal doctrine which may be expected to lead to most important philosophical conclusions. Finally, the calculus of the new logic, which is applicable to everything, will certainly be applied to settle certain logical questions of extreme difficulty relating to the foundations of mathematics. Whether or not it can lead to any method of discovering methods in mathematics it is difficult to say. Such a thing is conceivable.

Peirce: CP 3.455 Cross-Ref:††

455. It is now more than thirty years since my first published contribution to "exact" logic.†2 Among other serious studies, this has received a part of my attention ever since. I have contemplated it in all sorts of perspectives and have often reviewed my reasons for believing in its importance. My confidence that the key of philosophy is here, is stronger than ever after reading Schröder's last volume. One thing which helps to make me feel that we are developing a living science, and not a dead doctrine, is the healthy mental independence it fosters, as evidenced, for example, in the divergence between Professor Schröder's opinions and mine. There is no bovine nor ovine gregariousness here. But Professor Schröder and I have a common method which we shall ultimately succeed in applying to our differences, and we shall settle them to our common satisfaction; and when that method is pouring in upon us new and incontrovertible positively valuable results, it will be as nothing to either of us to confess that where he had not yet been able to apply that method he has fallen into error.

Peirce: CP 3.456 Cross-Ref:††
XVI

THE LOGIC OF RELATIVES†1

§1. THREE GRADES OF CLEARNESS †2

456. The third volume of Professor Schröder's *Exact Logic*, †P1 which volume bears separately the title I have chosen for this paper, is exciting some interest even in this country. There are in America a few inquirers into logic, sincere and diligent, who are not of the genus that buries its head in the sand -- men who devote their thoughts to the study with a view to learning something that they do not yet know, and not for the sake of upholding orthodoxy, or any other foregone conclusion. For them this article is written as a kind of popular exposition of the work that is now being done in the field of logic. To them I desire to convey some idea of what the new

logic is, how two "algebras," that is, systems of diagrammatical representation by means of letters and other characters, more or less analogous to those of the algebra of arithmetic, have been invented for the study of the logic of relatives, and how Schröder uses one of these (with some aid from the other and from other notations) to solve some interesting problems of reasoning. I also wish to illustrate one other of several important uses to which the new logic may be put. To this end I must first clearly show what a relation is.

Peirce: CP 3.457 Cross-Ref:††

457. Now there are three grades of clearness in our apprehensions of the meanings of words. The first consists in the connexion of the word with familiar experience. In that sense, we all have a clear idea of what *reality* is and what *force* is -- even those who talk so glibly of mental force being correlated with the physical forces. The second grade consists in the abstract definition, depending upon an analysis of just what it is that makes the word applicable. An example of defective apprehension in this grade is Professor Tait's holding (in an appendix to the reprint of his Britannica article, *Mechanics*) that energy is "objective" (meaning it is a substance), because it is permanent, or "persistent." For independence of time does not of itself suffice to make a substance; it is also requisite that the aggregant parts should always preserve their identity, which is not the case in the transformations of energy. The third grade of clearness consists in such a representation of the idea that fruitful reasoning can be made to turn upon it, and that it can be applied to the resolution of difficult practical problems.

Peirce: CP 3.458 Cross-Ref:††

§2. OF THE TERM RELATION IN ITS FIRST GRADE OF CLEARNESS

458. An essential part of speech, the Preposition, exists for the purpose of expressing relations. Essential it is, in that no language can exist without prepositions, either as separate words placed before or after their objects, as case-declensions, as syntactical arrangements of words, or some equivalent forms. Such words as "brother," "slayer," "at the time," "alongside," "not," "characteristic property" are relational words, or *relatives*, in this sense, that each of them *becomes a general name when another general name is affixed to it as object*. In the Indo-European languages, in Greek, for example, the so-called genitive case (an inapt phrase like most of the terminology of grammar) is, very roughly speaking, the form most proper to the attached name. By such attachments, we get such names as "brother *of* Napoleon," "slayer *of* giants," "{epi 'Ellissaiou}, at the time of Elias," "{para allélön}, alongside *of* each other," "not guilty," "a characteristic property *of* gallium." *Not* is a relative because it means "other than"; *scarcely*, though a relational word of highly complex meaning, is not a relative. It has, however, to be treated in the logic of relatives. Other relatives do not become general names until two or more names have been thus affixed. Thus, "giver to the city" is just such a relative as the preceding; for "giver to the city of a statue of himself" is a complete general name (that is, there might be several such humble admirers of themselves, though there be but one, as yet); but "giver" requires *two* names to be attached to it, before it becomes a complete name. The dative case is a somewhat usual form for the second object. The archaic, instrumental, and locative cases were serviceable for third and fourth objects.

Peirce: CP 3.459 Cross-Ref:††

459. Our European languages are peculiar in their marked differentiation of common nouns from verbs. *Proper* nouns must exist in all languages; and so must such "pronouns," or indicative words, as *this, that, something, anything*. But it is probably true that in the great majority of the tongues of men, distinctive common nouns either do not exist or are exceptional formations. In their meaning as they stand in sentences, and in many comparatively widely-studied languages, common nouns are akin to participles, as being mere inflexions of verbs. If a language has a verb meaning "is a man," a noun "man" becomes a superfluity. For all men are mortals is perfectly expressed by "Anything either is-a-man not or is-a-mortal." Some man is a miser is expressed by "Something both is-a-man and is-a-miser." The best treatment of the logic of relatives, as I contend, will dispense altogether with class names and only use such verbs. A verb requiring an object or objects to complete the sense may be called a *complete relative*.

Peirce: CP 3.459 Cross-Ref:††

A verb by itself signifies a mere dream, an imagination unattached to any particular occasion. It calls up in the mind an *icon*. A *relative* is just that, an icon, or image, without attachments to experience, without "a local habitation and a name," but with indications of the need of such attachments.

Peirce: CP 3.460 Cross-Ref:††

460. An indexical word, such as a proper noun or demonstrative or selective pronoun, has force to draw the attention of the listener to some hecceity common to the experience of speaker and listener. By a hecceity, I mean, some element of existence which, not merely by the likeness between its different apparitions, but by an inward force of identity, manifesting itself in the continuity of its apparition throughout time and in space, is distinct from everything else, and is thus fit (as it can in no other way be) to receive a proper name or to be indicated as *this* or *that*. Contrast this with the signification of the verb, which is sometimes in my thought, sometimes in yours, and which has no other identity than the agreement between its several manifestations. That is what we call an abstraction or idea. The nominalists say it is a *mere* name. Strike out the "mere," and this opinion is approximately true. The realists say it *is* real. Substitute for "is," *may be*, that is, *is* provided experience and reason shall, as their final upshot, uphold the truth of the particular predicate, and the natural existence of the law it expresses, and this is likewise true. It is certainly a great mistake to look upon an idea, merely because it has not the mode of existence of a hecceity, as a lifeless thing.

Peirce: CP 3.461 Cross-Ref:††

461. The proposition, or sentence, signifies that an eternal fitness, or truth, a permanent conditional force, or law, attaches certain hecceities to certain parts of an idea. Thus, take the idea of "buying by -- of -- from -- in exchange for --." This has four places where hecceities, denoted by indexical words, may be attached. The proposition "A buys B from C at the price D," signifies an eternal, irrefragable, conditional force gradually compelling those attachments in the opinions of inquiring minds.

Peirce: CP 3.462 Cross-Ref:††

462. Whether or not there be in the reality any definite separation between the hecceity-element and the idea-element is a question of metaphysics, not of logic. But it is certain that in the expression of a fact we have a considerable range of choice as

to how much we will denote by the indexical and how much signify by iconic words. Thus, we have stated "all men are mortal" in such a form that there is but one index. But we may also state it thus: "Taking anything, either it possesses not humanity or it possesses mortality." Here "humanity" and "mortality" are really proper names, or purely denotative signs, of familiar ideas. Accordingly, as here stated, there are three indices. Mathematical reasoning largely depends on this treatment of ideas as things †1; for it aids in the iconic representation of the whole fact. Yet for some purposes it is disadvantageous. These truths will find illustration in §13 below.

Peirce: CP 3.463 Cross-Ref:††

463. Any portion of a proposition expressing ideas but requiring something to be attached to it in order to complete the sense, is in a general way relational. But it is only a *relative* in case the attachment of indexical signs will suffice to make it a proposition, or, at least, a complete general name. Such a word as *exceedingly* or *previously* is relational, but is not a relative, because significant words require to be added to it to make complete sense.

Peirce: CP 3.464 Cross-Ref:††

§3. OF RELATION IN THE SECOND GRADE OF CLEARNESS

464. Is relation anything more than a connexion between two things? For example, can we not state that A gives B to C without using any other relational phrase than that one thing is connected with another? Let us try. We have the general idea of *giving*. Connected with it are the general ideas of *giver*, *gift*, and "*donée*." We have also a particular transaction connected with no general idea except through that of giving. We have a first party connected with this transaction and also with the general idea of giver. We have a second party connected with that transaction, and also with the general idea of "*donée*." We have a subject connected with that transaction and also with the general idea of gift. A is the only hecceity directly connected with the first party; C is the only hecceity directly connected with the second party, B is the only hecceity directly connected with the subject. Does not this long statement amount to this, that A gives B to C?

Peirce: CP 3.464 Cross-Ref:††

In order to have a distinct conception of Relation, it is necessary not merely to answer this question but to comprehend the reason of the answer. I shall answer it in the negative. For, in the first place, if relation were nothing but connexion of two things, all things would be connected. For certainly, if we say that A is unconnected with B, that non-connexion is a relation between A and B. Besides, it is evident that any two things whatever make a pair. Everything, then, is equally related to everything else, if mere connexion be all there is in relation. But that which is equally and necessarily true of everything is no positive fact, at all. This would reduce relation, considered as simple connexion between two things, to nothing, unless we take refuge in saying that relation *in general* is indeed nothing, but that *modes* of relation are something. If, however, these different modes of relation are different modes of connexion, relation ceases to be simple bare connexion. Going back, however, to the example of the last paragraph, it will be pointed out that the peculiarity of the mode of connexion of A with the transaction consists in A's being in connexion with an element connected with the transaction, which element is

connected with the peculiar general idea of a *giver*. It will, therefore, be said, by those who attempt to defend an affirmative answer to our question, that the peculiarity of a mode of connexion consists in this, that that connexion is indirect and takes place through something which is connected with a peculiar general idea. But I say that is no answer at all; for if all things are equally connected, nothing can be more connected with one idea than with another. This is unanswerable. Still, the affirmative side may modify their position somewhat. They may say, we grant that it is necessary to recognise that relation is something more than connexion; it is *positive* connexion. Granting that all things are connected, still all are not positively connected. The various modes of relationship are, then, explained as above. But to this I reply: you propose to make the peculiarity of the connexion of A with the transaction depend (no matter by what machinery) upon that connexion having a positive connexion with the idea of a giver. But "positive connexion" is not enough; the relation of the general idea is quite peculiar. In order that it may be characterised, it must, on your principles, be made indirect, taking place through something which is itself connected with a general idea. But this last connexion is again more than a mere general positive connexion. The same device must be resorted to, and so on *ad infinitum*. In short, you are guilty of a *circulus in definiendo*. You make the relation of any two things consist in their connexion being connected with a general idea. But that last connexion is, on your own principles, itself a *relation*, and you are thus defining relation by relation; and if for the second occurrence you substitute the definition, you have to repeat the substitution *ad infinitum*.

Peirce: CP 3.464 Cross-Ref:††

The affirmative position has consequently again to be modified. But, instead of further tracing possible tergiversations, let us directly establish one or two positive positions. In the first place, I say that every relationship concerns some definite number of correlates. Some relations have such properties that this fact is concealed. Thus, any number of men may be brothers. Still, brotherhood is a relation between pairs. If A, B, and C are all brothers, this is merely the consequence of the three relations, A is brother of B, B is brother of C, C is brother of A. Try to construct a relation which shall exist either between two or between three things such as "-- is either a brother or betrayer of -- to --." You can only make sense of it by somehow interpreting the dual relation as a triple one. We may express this as saying that every relation has a definite number of blanks to be filled by indices, or otherwise. In the case of the majority of relatives, these blanks are qualitatively different from one another. These qualities are thereby communicated to the connexions.

Peirce: CP 3.465 Cross-Ref:††

465. In a complete proposition there are no blanks. It may be called a *medad*, or *medadic relative*, from {médamos}, none, and {-ada} the accusative ending of such words as {monas, dyas, trias, tetras} etc.†P1 A non-relative name with a substantive verb, as "-- is a man," or "man that is --," or "--'s manhood" has one blank; it is a *monad*, or *monadic relative*. An ordinary relative with an active verb as "-- is a lover of --" or "the loving by -- of --" has two blanks; it is a *dyad*, or *dyadic relative*. A higher relative similarly treated has a plurality of blanks. It may be called a *polyad*. The rank of a relative among these may be called its *adinity*, that is, the peculiar quality of the number it embodies.

Peirce: CP 3.466 Cross-Ref:††

466. A *relative*, then, may be defined as the equivalent of a word or phrase which, either as it is (when I term it a *complete* relative), or else when the verb "is" is

attached to it (and if it wants such attachment, I term it a *nominal* relative), becomes a sentence with some number of proper names left blank. A *relationship, or fundamentum relationis*, is a fact relative to a number of objects, considered apart from those objects, as if, after the statement of the fact, the designations of those objects had been erased. A *relation* is a relationship considered as something that may be said to be true of one of the objects, the others being separated from the relationship yet kept in view. Thus, for each relationship there are as many relations as there are blanks. For example, corresponding to the relationship which consists in one thing loving another there are two relations, that of loving and that of being loved by. There is a nominal relative for each of these relations, as "lover of --," and "loved by --." These nominal relatives belonging to one relationship, are in their relation to one another termed *correlatives*. In the case of a dyad, the two correlatives, and the corresponding relations are said, each to be the *converse* of the other. The objects whose designations fill the blanks of a complete relative are called the *correlates*. The correlate to which a nominal relative is attributed is called the *relate*.

Peirce: CP 3.467 Cross-Ref:††

467. In the statement of a relationship, the designations of the correlates ought to be considered as so many *logical subjects* and the relative itself as the *predicate*. The entire set of logical subjects may also be considered as a *collective subject*, of which the statement of the relationship is *predicate*.

Peirce: CP 3.468 Cross-Ref:††

§4. OF RELATION IN THE THIRD GRADE OF CLEARNESS †1

468. Mr. A. B. Kempe has published in the *Philosophical Transactions*†2 a profound and masterly "Memoir on the Theory of Mathematical Form," which treats of the representation of relationships by "Graphs," which is Clifford's name for a diagram, consisting of spots and lines, in imitation of the chemical diagrams showing the constitution of compounds. Mr. Kempe seems to consider a relationship to be nothing but a complex of bare connexions of pairs of objects, the opinion refuted in the last section. Accordingly, while I have learned much from the study of his memoir, I am obliged to modify what I have found there so much that it will not be convenient to cite it; because long explanations of the relation of my views to his would become necessary if I did so.

Peirce: CP 3.469 Cross-Ref:††

469. A chemical atom is quite like a relative in having a definite number of loose ends or "unsaturated bonds," corresponding to the blanks of the relative.†3 In a chemical molecule, each loose end of one atom is joined to a loose end, which it is assumed must belong to some other atom, although in the vapor of mercury, in argon, etc., two loose ends of the same atom would seem to be joined; and why pronounce such hermaphroditism impossible? Thus the chemical molecule is a *medad*, like a complete proposition. Regarding proper names and other indices, after an "is" has been attached to them, as monads, they, together with other monads, correspond to the two series of chemical elements, H, Li, Na, K, Rb, Cs, etc., and F, Cl, Br, I. The dyadic relatives correspond to the two series, Mg, Ca, Sr, Ba, etc., and O, S, Se, Te, etc. The triadic relatives correspond to the two series B, Al, Zn, In, Tl, etc., and N, P, As, Sb, Bi, etc. Tetradic relatives are, as we shall see, a superfluity; they correspond

to the series C, Si, Ti, Sn, Ta, etc. The proposition "John gives John to John" corresponds in

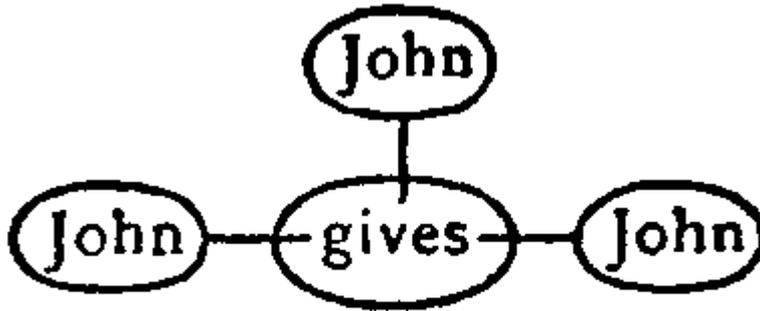


Figure 1

[Click here to



Figure 2

view]

[Click here to view]

Figure 1 Figure 2

its constitution, as Figs. 1 and 2 show, precisely to ammonia.

Peirce: CP 3.470 Cross-Ref:††

470. But beyond this point the analogy ceases to be striking. In fact, the analogy with the ruling theory of chemical compounds quite breaks down. Yet I cannot resist the temptation to pursue it. After all, any analogy, however fanciful, which serves to focus attention upon matters which might otherwise escape observation is valuable. A chemical compound might be expected to be quite as much like a proposition as like an algebraical invariant; and the brooding upon chemical graphs has hatched out an important theory in invariants.†1 Fifty years ago, when I was first studying chemistry, the theory was that every compound consisted of two oppositely electrified atoms or radicles; and in like manner every compound radicle consisted of two opposite atoms or radicles. The argument to this effect was that chemical attraction is evidently between things unlike one another and evidently has a saturation point; and further that we observe that it is the elements the most extremely

unlike which attract one another. [Julius] Lothar Meyer's curve having for its ordinates the atomic volumes of the elements and for its abscissas their atomic weights tends to support the opinion that elements strongly to attract one another must have opposite characters †1; for we see that it is the elements on the steepest downward slopes of that curve which have the strongest attractions for the elements on the steepest upward inclines. But when chemists became convinced of the doctrine of valency, that is, that every element has a fixed number of loose ends, and when they consequently began to write graphs for compounds, it seems to have been assumed that this necessitated an abandonment of the position that atoms and radicles combine by opposition of characters, which had further been weakened by the refutation of some mistaken arguments in its favor. But if chemistry is of no aid to logic, logic here comes in to enlighten chemistry. For in logic, the medad must always be composed of one part having a negative, or antecedental, character, and another part of a positive, or consequential, character; and if either of these parts is compound its constituents are similarly related to one another. Yet this does not, at all, interfere with the doctrine that each relative has a definite number of blanks or loose ends. We shall find that, in logic, the negative character is a character of reversion in this sense, that if the negative part of a medad is compound, *its* negative part has, on the whole, a positive character. We shall also find, that if the negative part of a medad is compound, the bond joining its positive and negative parts has its character reversed, just as those relatives themselves have. †2

Peirce: CP 3.471 Cross-Ref: ††

471. Several propositions are in this last paragraph stated about logical medads which now must be shown to be true. In the first place, although it be granted that every relative has a definite number of blanks, or loose ends, yet it would seem, at first sight, that there is no need of each of these joining no more than one other. For instance, taking the triad

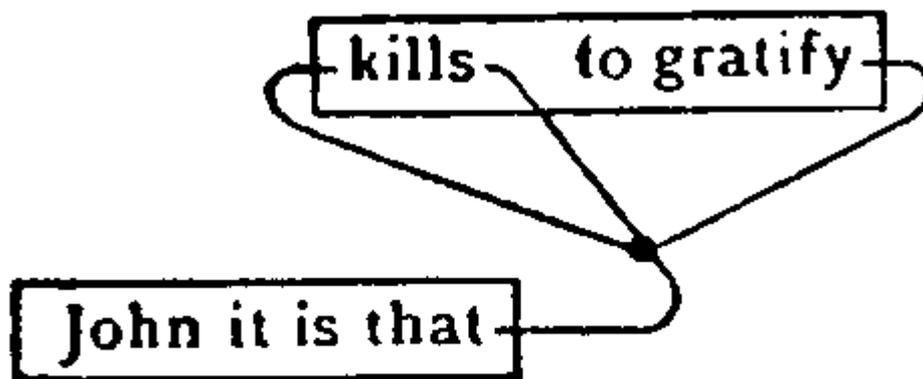


Figure 3

[Click here to view]

Figure 3

-- kills -- to gratify --," why may not the three loose ends all join in one node and then be connected with the loose end of the monad "John is --" as in Figure 3 making the proposition "John it is that kills what is John to gratify what is John"? The answer is, that a little exercise of generalising power will show that such a four-way node is really a tetradic relative, which may be expressed in words thus, "-- is identical with -- and with -- and with --"; so that the medad is

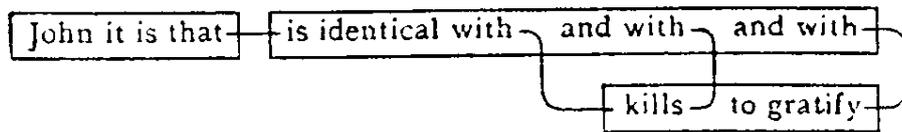


Figure 4

[Click

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Figure 4

really equivalent to that of Figure 4, which corresponds to prussic acid as shown in Figure 5.

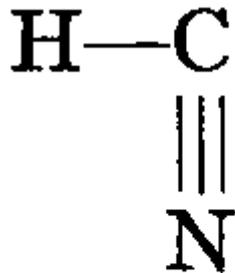


Figure 5

[Click here to view]

Figure 5

Thus, it becomes plain that every node of bonds is equivalent to a relative; and the doctrine of valency is established for us in logic.

Peirce: CP 3.472 Cross-Ref:††

472. We have next to inquire into the proposition that in every combination of relatives there is a negative and a positive constituent. This is a corollary from the

general logical doctrine of the illative character of the copula, a doctrine precisely opposed to the opinion of the quantification of the predicate. A satisfactory discussion of this fundamental question would require a whole article. I will only say in outline that it can be positively demonstrated in several ways that a proposition of the form "man = rational animal," is a compound of propositions each of a form which may be stated thus: "**Every** man (if there be any) is a rational animal" or "Men are **exclusively** (if anything) rational animals." Moreover, it must be acknowledged that the illative relation (that expressed by "therefore") is the most important of logical relations, the be-all and the end-all of the rest. It can be demonstrated that formal logic needs no other elementary logical relation than this; but that with a symbol for this and symbols of relatives, including monads, and with a mode of representing the attachments of them, all syllogistic may be developed, far more perfectly than any advocate of the quantified predicate ever developed it, and in short in a way which leaves nothing to be desired. This in fact **will** be virtually shown in the present paper. It can further be shown that no other copula will of itself suffice for all purposes. Consequently, the copula of equality ought to be regarded as merely derivative.

Peirce: CP 3.473 Cross-Ref:††

473. Now, in studying the logic of relatives we must sedulously avoid the error of regarding it as a highly specialised doctrine. It is, on the contrary, nothing but formal logic generalised to the very tip-top. In accordance with this view, or rather with this theorem (for it is susceptible of positive demonstration), we must regard the **relative copula**, which is the bond between two blanks of relatives, as only a generalisation of the ordinary copula, and thus of the "**ergo**." When we say that from the proposition A the proposition B necessarily follows, we say that "the truth of A in **every way** in which it can exist at all is the truth of B," or otherwise stated "A is true **only** in so far as B is true." This is the very same relation which we express when we say that "**every** man is mortal," or "men are **exclusively** mortals." For this is the same as to say, "Take anything whatever, M; then, if M is a man, it follows necessarily that M is mortal." This mode of junction is essentially the same as that between the relatives in the compound relative "lover, in **every way** in which it may be a lover at all, of a servant," or, otherwise expressed, "lover (if at all) **exclusively** of servants." For to say that "Tom is a lover (if at all) only of servants of Dick," is the same as to say "Take anything whatever, M; then, if M is loved by Tom, M is a servant of Dick," or "everything there may be that is loved by Tom is a servant of Dick." †1

Peirce: CP 3.474 Cross-Ref:††

474. Now it is to be observed that the illative relation is not simply convertible; that is to say, that "from A necessarily follows B" does not necessarily imply that "from B necessarily follows A." Among the vagaries of some German logicians of some of the inexact schools, the convertibility of illation (like almost every other imaginable absurdity) has been maintained; but all the other inexact schools deny it, and exact logic condemns it, at once. Consequently, the copula of inclusion, which is but the **ergo** freed from the accident of asserting the truth of its antecedent, is equally inconvertible. For though "men include only mortals," it does not follow that "mortals include **only** men," but, on the contrary, what follows is "mortals include **all** men." Consequently, again, the fundamental **relative copula** is inconvertible. That is, because "Tom loves (if anybody) only a servant (or servants) of Dick," it does not follow that "Dick is served (if at all) only by somebody loved by Tom," but, on the contrary, what follows is "Dick is master of **every** person (there may be) who is loved by Tom." †1 We thus see, clearly, first, that, as the fundamental relative copula, we must take that particular mode of junction; secondly, that that

mode is at bottom the mode of junction of the *ergo*, and so joins a relative of antecedental character to a relative of consequential character; and, thirdly, that that copula is inconvertible, so that the two kinds of constituents are of opposite characters. There are, no doubt, convertible modes of junction of relatives, as in "lover of a servant"†P1; but it will be shown below that these are complex and indirect in their constitution.

Peirce: CP 3.475 Cross-Ref:††

475. It remains to be shown that the antecedent part of a medad has a negative, or reversed, character, and how this, in case it be compound, affects both its relatives and their bonds. But since this matter is best studied in examples, I will first explain how I propose to draw the logical graphs.

Peirce: CP 3.475 Cross-Ref:††

It is necessary to use, as the sign of the relative copula, some symbol which shall distinguish the antecedent from the consequent; and since, if the antecedent is compound (owing to the very character which I am about to demonstrate, namely, its reversing the characters of the relatives and the bonds it contains), it is very important to know just how much is included in that antecedent, while it is a matter of comparative indifference how much is included in the consequent (though it is simply everything not in the antecedent), and since further (for the same reason) it is important to know how many antecedents, each after the first a part of another, contain a given relative or copula, I find it best to make the line which joins antecedent and consequent encircle the whole of the former. Letters of the alphabet may be used as abbreviations of complete relatives; and the proper number of bonds may be attached to each. If one of these is encircled, that circle must have a bond corresponding to each bond of the encircled letter. Chemists sometimes write above atoms Roman numerals to indicate their *adinities*; but I do not think this necessary. Figure 7 shows, in a complete medad, my sign of the



Figure 7

[Click here to view]



Figure 8

[Click here to view]

Figure 7 Figure 8

relative copula. Here, *h* is the monad "-- is a man," and *d* is the monad "-- is mortal." The antecedent is completely enclosed, and the meaning is "Anything whatever, if it be a man, is mortal." If the circle encloses a dyadic or polyadic relative, it must, of course, have a tail for every bond of that relative. Thus, in Figure 8, *l* is the dyad "-- loves --," and it is important to remark that the bond to the left is the lover and that to the right is the loved. Monads are the only relatives for which we need not be attentive to the positions of attachment of the bonds. In this figure, *w* is the monad "-- is wise," and *v* is the monad "-- is virtuous." The *l* and *v* are enclosed in a large common circle. Had this not been done, the medad could not be read (as far as any rules yet given show), because it would not consist of antecedent and consequent. As it is, we begin the reading of the medad at the bond connecting antecedent and consequent. Every bond of a logical graph denotes a hecceity; and every unencircled bond (as this one is) stands for any hecceity the reader may choose from the universe. This medad evidently refers to the universe of men. Hence the interpretation begins: "Let M be any man you please." We proceed along this bond in the direction of the antecedent, and on entering the circle of the antecedent we say: "If M be." We then enter the inner circle. Now, entering a circle means a relation to *every*. Accordingly we add "whatever." Traversing *l* from left to right, we say "lover." (Had it been from right to left we should have read it "loved.") Leaving the circle is the mark of a relation "only to," which words we add. Coming to *v* we say "what is virtuous." Thus our antecedent reads: "Let M be any man you please. If M be whatever it may that is lover only to the virtuous." We now return to the consequent and read, "M is wise." Thus the whole means, "Whoever loves only the virtuous is wise."

Peirce: CP 3.475 Cross-Ref:††

As another example, take the graph of Figure 9, where *l* has



Figure 9

[Click here to view]

Figure 9

the same meaning as before and *m* is the dyad "-- is mother of --." Suppose we start with the left hand bond. We begin with saying "Whatever." Since cutting this bond does not sever the medad, we proceed at once to read the whole as an unconditional statement and we add to our "whatever" "there is." We can now move round the ring of the medad either clockwise or counter-clockwise. Taking the last way, we come to *l* from the left hand and therefore add "is a lover." Moving on, we enter the circle round *m*; and entering a circle is a sign that we must say "of *everything* that." Since we pass through *m* backwards we do not read "is mother" but "is mothered" or "has for mother." Then, since we pass out of the circle we should have to add "only"; but coming back, as we do, to the starting point, we need only say "that same thing." Thus, the interpretation is "Whatever there is, is lover of everything that has for mother that same thing," or "Every woman loves everything of which she is mother." Starting at the same point and going round the other way, the reading would be "Everybody is mother (if at all) only of what is loved by herself." Starting on the right and proceeding clockwise, "Everything is loved by every mother of itself." Proceeding counter-clockwise, "Everything has for mothers only lovers of itself."

Peirce: CP 3.476 Cross-Ref:††

476. Triple relatives afford no particular difficulty. Thus, in Figure 10, *w* and *v* have the same significations as before; *r* is the monad, "-- is a reward," and *g* is the triad "-- gives | to --." It can be read either

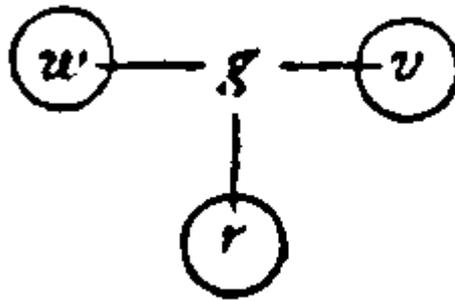


Figure 10

[Click here to view]

Figure 10

"Whatever is wise gives every reward to every virtuous person," or "Every virtuous person has every reward given to him by everybody that is wise," or "Every reward is given by everybody who is wise to every virtuous person."

Peirce: CP 3.477 Cross-Ref:††

477. A few more examples will be instructive. Figure 11, where A is the proper name "Alexander" means "Alexander loves only the virtuous," i.e., "Take anybody you please; then, if he be Alexander and if he loves anybody, this latter is virtuous."



Figure 11

[Click here to view]

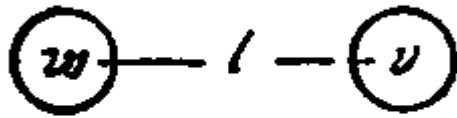


Figure 12

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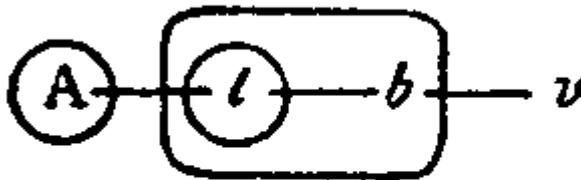


Figure 13

[Click here to view]

Figure 11 Figure 12 Figure 13

Peirce: CP 3.477 Cross-Ref:††

If you attempt, in reading this medad, to start to the right of *l*, you fall into difficulty, because your antecedent does not then consist of an antecedent and consequent, but of two circles joined by a bond, a combination to be considered below. But Figure 12 may be read with equal ease on whichever side of *l* you begin, whether as "whoever is wise loves everybody that is virtuous," or "whoever is virtuous is loved by everybody that is wise." If in Figure 13 *-b-* be the dyad "-- is a benefactor of --," the medad reads, "Alexander stands only to virtuous persons in the relation of loving only their benefactors."

Peirce: CP 3.477 Cross-Ref:††

Figure 14, where *-s-* is the dyad "-- is a servant of --" may be read, according to the above principles, in the several ways following:

"Whoever stands to any person in the relation of lover to none but his servants benefits him."

"Every person stands only to a person benefited by him in the relation of a lover only of a servant of that person."

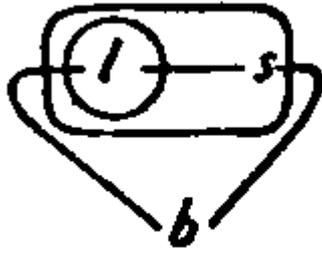


Figure 14

[Click here to view]



Figure 15

[Click here to view]

Figure 14 Figure 15

"Every person, M, is benefactor of everybody who stands to M in the relation of being served by everybody loved by him."

"Every person, N, is benefited by everybody who stands to N in the relation of loving only servants of him."

"Every person, N, stands only to a benefactor of N in the relation of being served by everybody loved by him."

"Take any two persons, M and N. If, then, N is served by every lover of M, N is benefited by M."

Figure 15 represents a medad which means, "Every servant of any person, is a benefactor of whomever may be loved by that person." Equivalent statements easily read off from the graphs are as follows:

"Anybody, M, no matter who, is servant (if at all) only of somebody who loves (if at all) only persons benefited by M."

"Anybody, no matter who, stands to every master of him in the relation of benefactor of whatever person may be loved by him."

"Anybody, no matter who, stands to whoever loves him in the relation of

being benefited by whatever servant he may have."

"Anybody, N, is loved (if at all) only by a person who is served (if at all) only by benefactors of N."

"Anybody, no matter who, loves (if at all) only persons benefited by all servants of his."

"Anybody, no matter who, is served (if at all) only by benefactors of everybody loved by him."

Peirce: CP 3.478 Cross-Ref:††

478. I will now give an example containing triadic relatives, but no monads. Let p be "-- prevents -- from communicating with --," the second blank being represented by a bond from the right of p and the third by a bond from below p . Let β mean "-- would betray -- to --," the arrangement of bonds being the same as with p . Then, Figure 16 means that "whoever loves only persons who prevent every servant

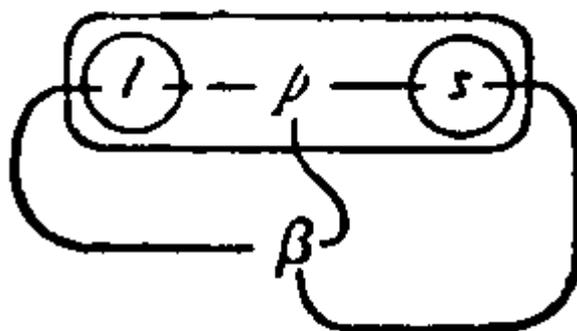


Figure 16

[Click here to view]

Figure 16

of any person, A, from communicating with any person, B, would betray B to A." I will only notice one equivalent statement, viz.: "Take any three persons, A, B, C, no matter who. Then, either C betrays B to A, or else two persons, M and N, can be found, such that M does not prevent N from communicating with B, although M is loved by C and N is a servant of A."

Peirce: CP 3.479 Cross-Ref:††

479. This last interpretation is an example of the method which is, by far, the plainest and most unmistakable of any in complicated cases. The rule for producing it is as follows:

Peirce: CP 3.479 Cross-Ref:††

1. Assign a letter of the alphabet to denote the hecceity represented by each bond.†P1

Peirce: CP 3.479 Cross-Ref:††

2. Begin by saying: "Take any things you please, namely," and name the letters representing bonds not encircled; then add, "Then suitably select objects, namely," and name the letters representing bonds each once encircled; then add, "Then take any things you please, namely," and name the letters representing bonds each twice encircled. Proceed in this way until all the letters representing bonds have been named, no letter being named until all those encircled fewer times have been named; and each hecceity corresponding to a letter encircled odd times is to be suitably chosen according to the intent of the assertor of the medad proposition, while each hecceity corresponding to a bond encircled even times is to be taken as the interpreter or the opponent of the proposition pleases. † 1

Peirce: CP 3.479 Cross-Ref:††

3. Declare that you are about to make statements concerning certain propositions, to which, for the sake of convenience, you will assign numbers in advance of enunciating them or stating their relations to one another. These numbers are to be formed in the following way. There is to be a number for each letter of the medad (that is for those which form spots of the graph, not for the letters assigned by clause 1 of this rule to the bonds), and also a number for each circle round more than one letter; and the first figure of that number is to be a 1 or a 2, according as the letter or the circle is in the principal antecedent or the principal consequent; the second figure is to be 1 or 2, according as the letter or the circle belongs to the antecedent or the consequent of the principal antecedent or consequent, and so on.

Peirce: CP 3.479 Cross-Ref:††

Declare that one or other of those propositions whose numbers contain no 1 before the last figure is true. Declare that each of those propositions whose numbers contain an odd number of 1's before the last figure consists in the assertion that *some one* or another of the propositions whose numbers commence with its number is true. For example, 11 consists in the assertion that either 111 or 1121 or 1122 is true, supposing that these are the only propositions whose numbers commence with 11. Declare that each of those propositions whose numbers contain an even number of 1's (or none) before the last figure consists in the assertion that *every one of* the propositions whose numbers commence with its number is true. Thus, 12 consists in the assertion that 121, 1221, 1222 are all true, provided those are the only propositions whose numbers commence with 12. The process described in this clause will be abridged except in excessively complicated cases.

Peirce: CP 3.479 Cross-Ref:††

4. Finally, you are to enunciate all those numbered propositions which correspond to single letters. Namely, each proposition whose number contains an even number of 1's, will consist in affirming the relative of the spot-letter to which that number corresponds after filling each blank with that bond-letter which by clause 1 of this rule was assigned to the bond at that blank. But if the number of the proposition contains an odd number of 1's, the relative, with its blanks filled in the same way, is to be denied.

Peirce: CP 3.480 Cross-Ref:††

480. In order to illustrate this rule, I will restate the meanings of the medads of Figures 7-16, in all the formality of the rule; although such formality is uncalled for and awkward, except in far more complicated cases.

Peirce: CP 3.480 Cross-Ref:††

Figure 7. Let A be anything you please. There are two propositions, 1 and 2, one of which is true. Proposition 1 is, that A is not a man. Proposition 2 is, that A is mortal. More simply, Whatever A may be, either A is not a man or A is mortal.

Peirce: CP 3.480 Cross-Ref:††

Figure 8. Let A be anybody you please. Then, I will find a person, B, so that either proposition 1 or proposition 2 shall be true. Proposition 1 asserts that both propositions 11 and 12 are true. Proposition 11 is that A loves B. Proposition 12 is that B is not virtuous. Proposition 2 is that A is wise. More simply, Take anybody, A, you please. Then, either A is wise, or else a person, B, can be found such that B is not virtuous and A loves B.

Peirce: CP 3.480 Cross-Ref:††

Figure 9. Let A and B be any persons you please. Then, either proposition 1 or proposition 2 is true. Proposition 1 is that A is not a mother of B. Proposition 2 is that A loves B. More simply, whatever two persons A and B may be, either A is not a mother of B or A loves B.

Peirce: CP 3.480 Cross-Ref:††

Figure 10. Let A, B, C be any three things you please. Then, one of the propositions numbered, 1, 21, 221, 222 is true. Proposition 1 is that A is not wise. Proposition 21 is that B is not a reward. Proposition 221 is that C is not virtuous. Proposition 222 is that A gives B to C. More simply, take any three things, A, B, C, you please. Then, either A is not wise, or B is not a reward, or C is not virtuous, or A gives B to C.

Peirce: CP 3.480 Cross-Ref:††

Figure 11. Take any two persons, A and B, you please. Then, one of the propositions 1, 21, 22 is true. 1 is that A is not Alexander. 21 is that A does not love B. Proposition 3 is that B is virtuous.

Peirce: CP 3.480 Cross-Ref:††

Figure 12. Take any two persons, A and B. Then, one of the propositions 1, 21, 22 is true. 1 is that A is not wise. 21 is that B is not virtuous. 22 is that A loves B.

Peirce: CP 3.480 Cross-Ref:††

Figure 13. Take any two persons, A and C. Then a person, B can be found such that one of the propositions 1, 21, 22 is true. Proposition 21 asserts that both 211 and 212 are true. Proposition 1 that A is not Alexander. Proposition 211 is that A loves B. Proposition 212 is that B does not benefit C. Proposition 22 is that C is virtuous. More simply, taking any two persons, A and C, either A is not Alexander, or C is virtuous, or there is some person, B, who is loved by A without benefiting C.

Peirce: CP 3.480 Cross-Ref:††

Figure 14. Take any two persons, A and B, and I will then select a person C. Either proposition 1 or proposition 2 is true. Proposition 1 is that both 11 and 12 are true. Proposition 11 is that A loves C. Proposition 12 is that C is not a servant of B. Proposition 2 is that A benefits B. More simply, of any two persons, A and B, either A benefits the other, B, or else there is a person, C, who is loved by A but is not a servant of B.

Peirce: CP 3.480 Cross-Ref:††

Figure 15. Take any three persons, A, B, C. Then one of the propositions 1, 21, 22 is true. 1 is that A is not a servant of B; 21 is that B is not a lover of C; 22 is that A benefits C.

Peirce: CP 3.480 Cross-Ref:††

Figure 16. Take any three persons, A, B, C. Then I can so select D and E, that one of the propositions 1 or 2 is true. 1 is that 11 and 121 and 122 are all true. 11 is that A loves D, 121 is that E is a servant of C, 122 is that D does not prevent E from communicating with B. 2 is that A betrays B to C.

Peirce: CP 3.480 Cross-Ref:††

I have preferred to give these examples rather than fill my pages with a dry abstract demonstration of the correctness of the rule. If the reader requires such a proof, he can easily construct it. This rule makes evident the reversing effect of the encirclements, not only upon the "quality" of the relatives as affirmative or negative, but also upon the selection of the hecceities as performable by advocate or opponent of the proposition, as well as upon the conjunctions of the propositions as disjunctive or conjunctive, or (to avoid this absurd grammatical terminology) as alternative or simultaneous.

Peirce: CP 3.481 Cross-Ref:††

481. It is a curious example of the degree to which the thoughts of logicians have been tied down to the accidents of the particular language they happened to write (mostly Latin), that while they hold it for an axiom that two *nots* annul one another, it was left for me to say as late as 1867†P1 that *some* in formal logic ought to be understood, and could be understood, so that *some-some* should mean *any*. I suppose that were ordinary speech of any authority as to the forms of logic, in the overwhelming majority of human tongues two negatives intensify one another. And it is plain that if "not" be conceived as less than anything, what is less than that is *a fortiori* not. On the other hand, although *some* is conceived in our languages as *more than none*, so that two "somes" intensify one another, yet what it ought to signify for the purposes of syllogistic is that, instead of the selection of the instance being left -- as it is, when we say "any man is not good" -- to the opponent of the proposition, when we say "some man is not good," this selection is transferred to the opponent's opponent, that is to the defender of the proposition. Repeat the some, and the selection goes to the opponent's opponent's opponent, that is, to the opponent again, and it becomes equivalent to *any*. In more formal statement, to say "Every man is mortal," or "Any man is mortal," is to say, "A man, as suitable as any to prove the proposition false, is mortal," while "Some man is mortal" is equivalent to "A man, as suitable as any to prove the proposition *not* false, is mortal." "Some-some man is mortal" is accordingly "A man, as suitable as any to prove the proposition *not not*-false, is mortal."

Peirce: CP 3.482 Cross-Ref:††

482. In like manner, encircled $2N + 1$ times, a disjunctive conjunction of propositions becomes a copulative conjunction. Here, the case is altogether similar. Encircled even times, the statement is that some one (or more) of the propositions is true; encircled odd times, the statement is that any one of the propositions is true. The negative of "lover of every servant" is "non-lover of some servant." The negative of "lover every way (that it is a lover) of a servant" is "lover some way of a non-servant."

Peirce: CP 3.482 Cross-Ref:††

The general nature of a relative and of a medad has now been made clear. At any rate, it will become so, if the reader carefully goes through with the explanations. We have not, however, as yet shown how every kind of proposition can be graphically expressed, nor under what conditions a medad is necessarily true. For that purpose it will be necessary to study certain special logical relatives.

Peirce: CP 3.483 Cross-Ref:††
 §5. TRIADS, THE PRIMITIVE RELATIVES

483. That out of triads all polyads can be constructed is made plain by Figure 17.



Figure 17

[Click here to view]

Figure 17

Peirce: CP 3.484 Cross-Ref:††

484. Figure 18 shows that from two triads a dyad can be made. Figure 19 shows that from one triad a monad can be made. Figure 20 shows that from any even number of triads



Figure 18

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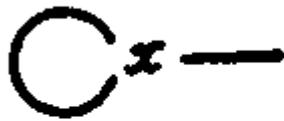


Figure 19

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Figure 20

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Figure 18 Figure 19 Figure 20

a medad can be made. In general, the union of a $\{m\}$ -ad and a $\{n\}$ -ad gives a $(\{m\} + \{n\} - 2\{1\})$ -ad, where $\{1\}$ is the number of bonds of union. This formula shows that *artiads*, or even-ads, can produce only *artiads*. But any perissid, or odd-ad (except a monad), can by repetition produce a relative of any *adinity*.

Peirce: CP 3.485 Cross-Ref:††

485. Since the principal object of a notation for relatives is not to produce a handy *calculus* for the solution of special logical problems, but to help the study of logical principles, the study of logical graphs from that point of view must be postponed to a future occasion. For present purposes that notation is best which carries analysis the furthest, and presents the smallest number of unanalyzed forms. It will be best, then, to use single letters for relatives of some one definite and odd number of blanks. We naturally choose three as the smallest number which will answer the purpose.

Peirce: CP 3.486 Cross-Ref:††

486. We shall, therefore, substitute for such a dyad as "-- is lover of --" some such triad as "-- is coexistent with | and a lover of --." If, then, we make -w- to signify "-- is coexistent with | and with --," that which we have hitherto written as in Figure 12 will be written as in Figure 21. But having once recognised that such a mode of writing is possible,

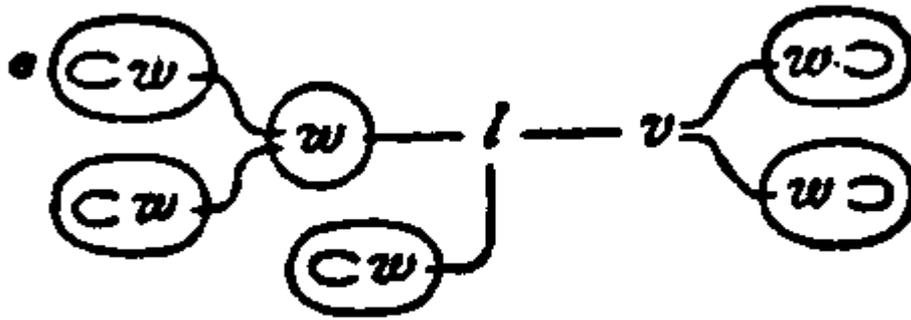


Figure 21

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Figure 21

we can continue to use our former methods, provided we now consider them as abbreviations.

Peirce: CP 3.487 Cross-Ref:††

487. The logical doctrine of this section, must, we may remark, find its application in metaphysics, if we are to accept the Kantian principle that metaphysical conceptions mirror those of formal logic.

Peirce: CP 3.488 Cross-Ref:††

§6. RELATIVES OF SECOND INTENTION

488. The general method of graphical representation of propositions has now been given in all its essential elements, except, of course, that we have not, as yet, studied any truths concerning special relatives; for to do so would seem, at first, to be "extralogical." Logic in this stage of its development may be called *paradisical logic*, because it represents the state of Man's cognition before the Fall. For although, with this apparatus, it is easy to write propositions necessarily true, it is absolutely impossible to write any which is necessarily false, or, in any way which that stage of logic affords, to find out that anything is false. The mind has not as yet eaten of the fruit of the Tree of Knowledge of Truth and Falsity. Probably it will not be doubted that every child in its mental development necessarily passes through a stage in which he has some ideas, but yet has never recognised that an idea may be erroneous; and a stage that every child necessarily passes through must have been formerly passed through by the race in its adult development. It may be doubted whether many of the lower animals have any clear and steady conception of falsehood; for their instincts work so unerringly that there is little to force it upon their attention. Yet plainly without a knowledge of falsehood no development of discursive reason can take place.

Peirce: CP 3.489 Cross-Ref:††

489. This paradisaical logic appears in the study of nonrelative formal logic. But *there* no possible avenue appears by which the knowledge of falsehood could be brought into this Garden of Eden except by the arbitrary and inexplicable introduction of the Serpent in the guise of a proposition necessarily false. The logic of relatives, affords such an avenue, and *that*, the very avenue by which in actual development, this stage of logic supervenes. It is the avenue of experience and logical reflexion.

Peirce: CP 3.490 Cross-Ref:††

490. By *logical* reflexion, I mean the observation of thoughts in their expressions. Aquinas remarked that this sort of reflexion is requisite to furnish us with those ideas which, from lack of contrast, ordinary external experience fails to bring into prominence. He called such ideas *second intentions*. It is by means of *relatives of second intention* that the general method of logical representation is to find completion.

Peirce: CP 3.491 Cross-Ref:††



491. Let
"-- is {neither --,†1

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{nor --."

Then Figure 22 means

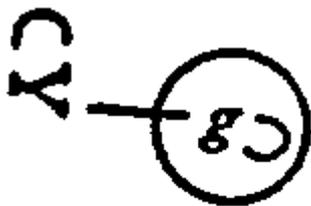


Figure 22

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Figure 23

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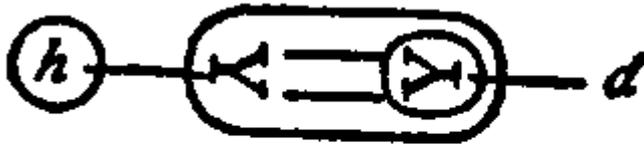


Figure 24

[Click here to view]

Figure 22 Figure 23 Figure 24

that taking any two things whatever, either the one is neither itself nor the other (putting it out of the question as an absurdity), or the other is a non-giver of something to that thing. That is, nothing gives all things, each to itself. Thus, the existence of any general description of thing can be denied.

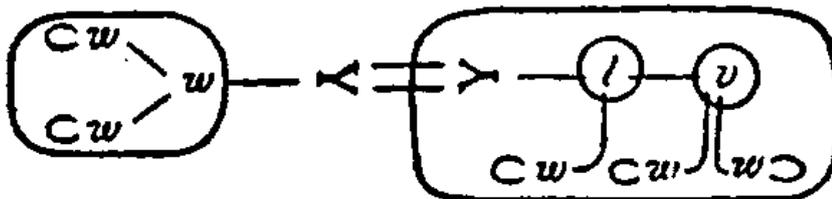


Figure 25

to view]

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Figure 25

Either medad of Figure 23 means no wise men are virtuous. Figure 24 is equivalent to Figure 7. Figure 25 means "each wise man is a lover of something virtuous." Thus we see that this mode of junction -- lover of some virtuous -- which seems so simple -- is really complex. Figure 26 means "some one

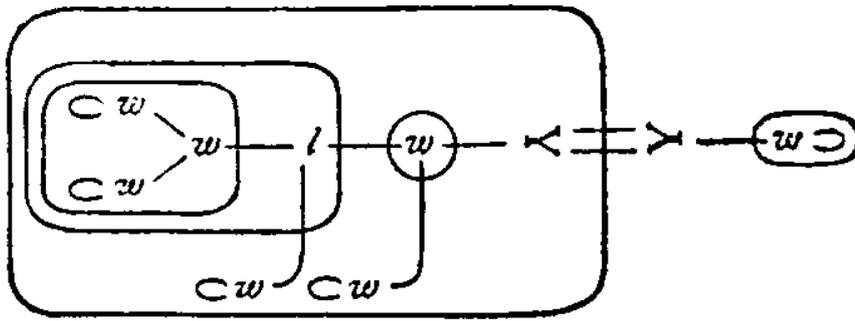


Figure 26

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Figure 26

thing is loved by all wise men." Figure 27 means that every man is either wise or virtuous. Figure 28 means that every man is both wise and virtuous.

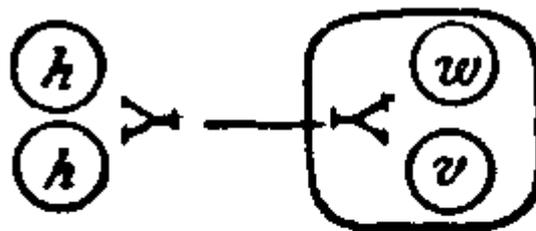


Figure 27

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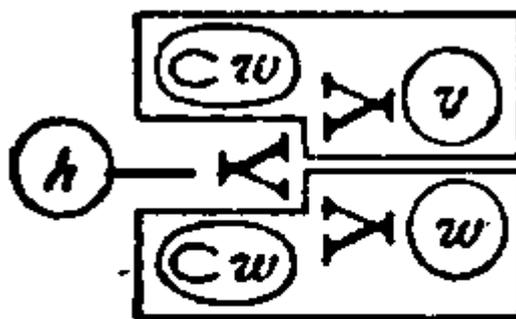


Figure 28

[Click here

to view]

Figure 27

Figure 28

Peirce: CP 3.491 Cross-Ref:††

These explanations need not be carried further to show that we have here a perfectly efficient and highly analytical method of representing relations.

Peirce: CP 3.492 Cross-Ref:††

§7. THE ALGEBRA OF DYADIC RELATIVES

492. Although the primitive relatives are triadic, yet they may be represented with but little violence by means of dyadic relatives, provided we allow several attachments to one blank. For instance, A gives B to C, may be represented by saying A is the first party in the transaction D, B is subject of D, C is second party of D, D is a giving by the first party of the subject to the second party. Triadic relatives cannot conveniently be represented on one line of writing. These considerations led me to invent the algebra of dyadic relatives as a tolerably convenient substitute in many cases for the graphical method of representation. In place of the one "operation," or mode of conjunction of graphical method, there are in this algebra four operations.

Peirce: CP 3.493 Cross-Ref:††

493. For the purpose of this algebra, I entirely discard the idea that every compound relative consists of an antecedent and a consequent part. I consider the circle round the antecedent as a mere sign of negation, for which in the algebra I substitute an *obelus* over that antecedent. The line between antecedent and consequent, I treat as a sign of an "operation" by itself. It signifies that anything whatever being taken as correlate of the first written member -- antecedent or consequent -- and as first relate of the second written member, either the one or the other is to be accepted. Thus in place of the relative of Figure 29 signifying that "taking anything whatever, M, either -- is not a lover of M, or M is a benefactor of --,"

that is "-- is a lover only of a benefactor of --," I write

$$\sim l \text{ } \S \text{ } b.$$

Or if it happens to be read the other way, putting a short mark over any letters to signify that relate and correlate are interchanged, I write the same thing

$$\$b \text{ } \S \text{ } \$\sim l$$

Peirce: CP 3.493 Cross-Ref:††

This operation, which may, at need, be denoted by a dagger in print, to which I give a scorpion-tail curve in its cursive form, I call *relative addition*. †1

Peirce: CP 3.494 Cross-Ref:††

494. The relative "-- stands to everything which is a benefactor of -- in the relation of servant of every lover of his," shows,



Figure 29

[Click here to view]



Figure 30

[Click here to view]

Figure 29

Figure 30

as written in Figure 30, an unencircled bond between *s* and *l*. The junction of the *l* and the *b* may therefore be regarded as direct. Stating the relative so as to make this direct junction prominent, it is "-- is servant of everything that is a lover of a benefactor of --." In the algebra, as far as already explained, "lover of a benefactor" would be

written

$$\sim(\sim l \text{ \textcircled{ } } \sim b)$$

that is, not a non-lover of every benefactor, or not a lover only of non-benefactors. This mode of junction, I call, in the algebra, the operation of *relative multiplication*, and write it

$$lb.$$

We have, then, the purely formal, or meaningless, equation

$$lb = \sim(\sim l \text{ \textcircled{ } } \sim b).$$

And in like manner, as a consequence of this,

$$l \text{ \textcircled{ } } b = \sim(\sim l \sim b).$$

That is to say, "To say that A is a lover of everything but benefactors of B," or "A is a non-lover only of benefactors of B," is the same as to say that A is not a non-lover of a non-benefactor of B.

Peirce: CP 3.495 Cross-Ref:††

495. To express in the algebra the relative of Figure 31.



Figure 31

[Click here to view]

Figure 31

or "-- is both a lover and a benefactor of --," I write

$$l \cdot b,$$

calling this "the operation of *non-relative multiplication*." To express "-- is either a lover or a benefactor of --," which might be written

$$\sim(l \cdot b),$$

I write

$$l \uplus b,$$

calling this the operation of *non-relative addition*, or more accurately, of *aggregation*. These last two operations belong to the Boolean algebra of non-relative logic. They are De Morgan's operations of composition and aggregation. Boole himself did not use the last, but in place of it an operation more properly termed addition which gives no interpretable result when the aggregants have any common aggregant. Mr. Venn †1 still holds out for Boole's operation, and there are weighty considerations in its favor. In my opinion, the decision between the two operations should depend upon whether the quantified predicate is rejected (when aggregation should be used), or accepted (when Boole's strict addition should be used).

Peirce: CP 3.496 Cross-Ref:††

496. The use of these four operations necessitates continual resort to parentheses, brackets, and braces to show how far the different compound relatives extend. It also becomes desirable to have a "copula of inclusion," or the sign of "is exclusively (if anything)." For this purpose I have since 1870†1 employed the sign \prec (intended for an improved \downarrow). It is easily made in the composing room from a dash followed by \prec , and in its cursive form is struck off in two rapid strokes, thus \prec . Its meaning is exemplified in the formula

$$w \prec v$$

"anybody who is wise (if any there be) is exclusively found among the virtuous." We also require in this algebra the signs of relatives of second intention

0, "-- is inconsistent with --," \wp "-- is coexistent with --,"

T "-- is other than --," I "-- is identical with."

Peirce: CP 3.497 Cross-Ref:††

497. The algebra has a moderate amount of power in skilful hands; but its great defect is the vast multitude of purely formal propositions which it brings along. The most significant of these are

$$s(l \text{ \textasciitilde } b) \approx s l \text{ \textasciitilde } b$$

and

$$(l \text{ \textasciitilde } b)s \approx l \text{ \textasciitilde } b s$$

That is, whatever is a servant of something which is a lover of everything but benefactors is a servant-of-a-lover to everything but benefactors, etc.

Peirce: CP 3.498 Cross-Ref:††

498. Professor Schröder attaches, as it seems to me, too high a value to this algebra. That which is in his eyes the greatest recommendation of it is to me scarcely a merit, namely that it enables us to express in the outward guise of an equation propositions whose real meaning is much simpler than that of an equation.

Peirce: CP 3.499 Cross-Ref:††
§8. GENERAL ALGEBRA OF LOGIC

499. Besides the algebra just described, I have invented another which seems to me much more valuable. It expresses with the utmost facility everything which can be expressed by a graph, and frequently much more clearly than the unabridged graphs described above. The method of using it in the solution of special problems has also been fully developed by me.

Peirce: CP 3.500 Cross-Ref:††

500. In this algebra every proposition consists of two parts, its quantifiers and its Boolean. The Boolean consists of a number of relatives united by a non-relative multiplication and aggregation. No relative operations are required (though they can be introduced if desired). Each elementary relative is represented by a letter on the line of writing with subjacent indices to denote the heccecities which fill its blanks. An obelus is drawn over such a relative to deny it.

Peirce: CP 3.501 Cross-Ref:††

501. To the left of the Boolean are written the quantifiers. Each of these is a π or a Σ with one of the indices written subjacent to it, to signify that in the Boolean every object in the universe is to be imaged substituted successively for that index and the non-relative product (if the quantifier is π) or the aggregate (if the quantifier

is Σ) of the results taken. The order of the quantifiers is, of course, material. Thus

$$\pi[i]\Sigma[j]l[ij] = (l[11] \vee l[12] \vee l[13] \text{ etc.}) \cdot (l[21] \vee l[22] \vee l[23] \vee \text{ etc.}) \cdot \text{etc.}$$

will mean anything loves something. But

$$\Sigma[j]\pi[i]l[ij] = l[11] \cdot l[21] \cdot l[31] \cdot \text{etc.} \vee l[12] \cdot l[22] \cdot l[32] \cdot \text{etc.} \vee l[23] \cdot l[23] \cdot l[33] \cdot \text{etc.} \vee \text{etc.}$$

will mean something is loved by all things.

Peirce: CP 3.502 Cross-Ref:††

502. This algebra, which has but two operations, and those easily manageable, is, in my opinion, the most convenient apparatus for the study of difficult logical problems, although the graphical method is capable of such modification as to render it substantially as convenient on the average. Nor would I refuse to avail myself of the algebra of dyadic relatives in the simpler cases in which it is easily handled.

Peirce: CP 3.503 Cross-Ref:††

§9. METHOD OF CALCULATING WITH THE GENERAL ALGEBRA

503. My rules for working this algebra, the fruit of long experience with applying it to a great variety of genuine inquiries, have never been published.†1 Nor can I here do more than state such as the beginner will be likely to require.

Peirce: CP 3.504 Cross-Ref:††

504. A number of premisses being given, it is required to know the most important conclusions of a certain description which can be drawn from them. The first step will be to express the premisses by means of the general algebra, taking care to use entirely different letters *as indices* in the different premisses.

Peirce: CP 3.505 Cross-Ref:††

505. These premisses are then to be copulated (or, in Whewell's phrase, colligated), *i.e.*, non-relatively multiplied together, by multiplying their Boolians and writing before the product all the quantifiers. The relative order of the quantifiers of each premiss must (in general) be undisturbed; but the relative order of quantifiers of different premisses is arbitrary. The student ought to place Σ 's as far to the left and π 's as far to the right as possible. Different arrangements of the quantifiers will lead to different conclusions from the premisses. It sometimes happens that each of several arrangements leads to a conclusion which could not easily be reached from any other arrangement.

Peirce: CP 3.506 Cross-Ref:††

506. The premisses, being so copulated, become one copulated premiss. This

copulated premiss is next to be logically multiplied into itself any number of times, the indices being different in all the different factors. For there will be certain conclusions which I call conclusions of the first order, which can be drawn from the copulated premiss without such involution, certain others, which I call inferences of the second order, which can be drawn from its square, etc. But after involution has been carried to a certain point, higher powers will only lead to inferences of subsidiary importance. The student will get a just idea of this matter by considering the rise and decline of interest in the theorems of any mathematical theory, such as geometry or the theory of numbers, as the fundamental hypotheses are applied more and more times in the demonstrations. The number of factors in the copulated premiss, which embraces *all* the hypotheses that either theory assumes, is not great. Yet from this premiss many thousand conclusions have already been drawn in the case of geometry and hundreds in the case of the theory of numbers. New conclusions are now coming in faster than ever before. From the nature of logic they can never be exhausted. But as time goes on the conclusions become more special and less important. It is true that mathematics, as a whole, does not become more special nor its late discoveries less important, because there is a growth of the hypotheses. Up to a certain degree, the importance of the conclusions increases with their "order." Thus, in geometry, there is nothing worth mention of the first order, and hardly of the second. But there is a great falling off in the importance of conclusions in the theories mentioned long before the fiftieth order has been reached.

Peirce: CP 3.507 Cross-Ref:††

507. This involution having been performed, the next step will be the identification (occasionally the diversification) of certain indices. The rule is, that any index quantified with a π can be transmitted, throughout the Boolean, into any other index whose quantifier stands to the left of its own, which now becomes useless, since it refers to nothing in the Boolean. For example, in

$$\Sigma[i]\pi[j][i j]$$

which in the Algebra of Dyadic Relatives would be written $\forall(l \downarrow 0)$, we can identify j with i and write

$$\Sigma[i]l[i i]$$

which in the other algebra becomes $\forall(l \cdot 1) \forall$.

Peirce: CP 3.508 Cross-Ref:††

508. That done, the Boolean is to be manipulated according to any of the methods of non-relative Boolean algebra, and the conclusion is read off.

Peirce: CP 3.509 Cross-Ref:††

509. But it is only in the simplest cases that the above operations suffice. Relatives of second intention will often have to be introduced; and their peculiar properties must be attended to. Those of 0 and \forall are covered by the rules of nonrelative Boolean algebra; but it is not so with **I** and **T**. We have, for example, to

observe that

$$\pi[i]x[i] \cdot y[i] = \pi[i]\pi[j]x[i] \cdot \top[i]j \cdot y[i] \dagger 1$$

$$\Sigma[i]x[i] \cdot y[i] = \Sigma[i]\Sigma[j]x[i] - \top[j]i \cdot y[j].$$

Exceedingly important are the relatives signifying "-- is a quality of --" and "-- is a relation of -- to --." It may be said that mathematical reasoning (which is the only deductive reasoning, if not absolutely, at least eminently) almost entirely turns on the consideration of abstractions as if they were objects. The protest of nominalism against such hypostatisation, although, if it knew how to formulate itself, it would be justified as against much of the empty disputation of the medieval Dunces, yet, as it was and is formulated, is simply a protest against the only kind of thinking that has ever advanced human culture. Nobody will work long with the logic of relatives -- unless he restricts the problems of his studies very much -- without seeing that this is true.

Peirce: CP 3.510 Cross-Ref: ††

§10. SCHRÖDER'S CONCEPTION OF LOGICAL PROBLEMS

510. Of my own labors in the logic of relatives since my last publication in 1884, †1 I intend to give a slight hint in §13. But I desire to give some idea of a part of the contents of Schröder's last volume. In doing so, I shall adhere to my own notation; for I cannot accept Professor Schröder's proposed innovations. I shall give my reasons in detail for this dissent in the *Bulletin of the American Mathematical Society*. †2 I will here only indicate their general nature. I have no objection whatever to the creation of a new system of signs *ab ovo*, if anybody can propose such a system sufficiently recommending itself. But *that* Professor Schröder does not attempt. He wishes his notation to have the support of existing habits and conventions, while proposing a measure of reform in the present usage. For that he must obtain general consent. Now it seems to me quite certain that no such general agreement can be obtained without the strictest deference to the principle of priority. Without that, new notations can only lead to confusion thrice confounded. The experience of biologists in regard to the nomenclature of their genera and other groups shows that this is so. I believe that their experience shows that the only way to secure uniformity in regard to conventions of this sort, is to accept for each operation and relative the sign definitively recommended by the person who introduced that operation or relative into the Boolean algebra, unless there are the most *substantial* reasons for dissatisfaction with the meaning of the sign. Objections of lesser magnitude may justify slight modifications of signs; as I modify Jevons's \cdot to \cdot , by uniting the two dots by a connecting line, and as I so far yield to Schröder's objections to using ∞ for the sign of whatever is, as to resort to the similarly shaped sign of Aries ♈ (especially as a notation of some power is obtained by using all the signs of the Zodiac in the same sense, as I shall show elsewhere). In my opinion, Professor Schröder alleges no sufficient reason for a single one of his innovations; and I further consider them as *positively* objectionable.

Peirce: CP 3.511 Cross-Ref:††

511. The volume consists of thirty-one long sections filling six hundred and fifty pages. I can, therefore, not attempt to do more than to exemplify its contents by specimens of the work selected as particularly interesting. Professor Schröder chiefly occupies himself with what he calls "solution-problems," in which it is required to deduce from a given proposition an *equation* of which one member consists in a certain relative determined in advance, while the other member shall not contain that relative. He rightly remarks that such problems often involve problems of elimination.

Peirce: CP 3.512 Cross-Ref:††

512. While I am not at all disposed to deny that the so-called "solution-problems," consisting in the ascertainment of the general forms of relatives which satisfy given conditions, are often of considerable importance, I cannot admit that the interest of logical study centres in them. I hold that it is usually much more to the purpose to express in the simplest way what a given premiss discloses in regard to the constitution of a relative, whether that simplest expression is of the nature of an equation or not. Thus, one of Schröder's problems is, "Given $x \prec a$, required x ," -- for instance, knowing that an opossum is a marsupial, give a description of the opossum.†1 The so-called solution is

$$\Sigma(x = a \cdot u), \dagger 2$$

u

or opossums embrace precisely what is common to marsupials and to some other class. In my judgment $x \prec a$ might with great propriety be called the solution of

$$\Sigma(x = a \cdot u), \dagger 2$$

u

When the information contained in a proposition is not of the nature of an equation, why should we, by circumlocutions, insist upon expressing it in the form of an equation?

Peirce: CP 3.513 Cross-Ref:††

513. Professor Schröder attaches great importance to the generality of solutions. In my opinion, this is a mistake. It is not merely that he insists that solutions shall be *complete*, as for example when we require *every root* of a numerical equation, but further that they shall all be embraced under one algebraical expression. Upon that he insists and with that he is satisfied. Whether or not the "solution" is such as to exhibit anything of the real constitution of the relative which forms the first member of the equation he does not seem to care; at least, there is no apparent consideration of the question of how such a result can be secured.

Peirce: CP 3.514 Cross-Ref:††

514. Pure mathematics always selects for the subjects of its studies manifolds

of perfect homogeneity; and thence it comes that for the problems which first present themselves general solutions are possible, which notwithstanding their generality, guide us at once to all the particular solutions. But even in pure mathematics the class of problems which are capable of solutions at once general and useful is an exceedingly limited one. All others have to be treated by subdivision of cases. That is what meets us everywhere in higher algebra. As for general solutions, they are for the most part trivial -- like the well-known and obvious test for a prime number that the continued product of all lesser numbers increased by 1 shall be divisible by that number. Only in those cases in which a general solution points the way to the particular solutions is it valuable; for it is only the particular solutions which picture to the mind the solution of a problem; and a form of words which fails to produce a definite picture in the mind is meaningless.†1

Peirce: CP 3.515 Cross-Ref:††

515. Professor Schröder endeavors to give the most general formula of a logical problem. It is in dealing with such very general and fundamental matters that the exact logician is most in danger of violating his own principles of exactitude. To seek a formula for all logical problems is to ask what it is, in general terms, that men inquire. To answer that question, my own logical proceeding would be to note that it asks what the essence of a question, in general, is. Now a question is a rational contrivance or device, and in order to understand any rational contrivance, experience shows that the best way is to begin by considering what circumstances of need prompted the contrivance, and then upon what general principle its action is designed to fill that need. Applying this general experience to the case before us, we remark that every question is prompted by some need -- that is, by some unsatisfactory condition of things, and that the object of asking the question is to fill that need by bringing reason to bear upon it and to do this by a hypnotically suggestive indication of that to which the mind has to apply itself. I do not know that I have ever, before this minute, considered the question what is the most general formulation of a problem in general; for I do not find much virtue in general formulæ. Nor do I think my answer to this question affords any particularly precious suggestion. But its ordinary character makes it all the better an illustration of the manner -- or one of the manners -- in which an exact logician may attack, off-hand, a suddenly sprung question. A question, I say, is an indication suggestive (in the hypnotic sense) of what has to be thought about in order to satisfy some more or less pressing want. Ideas like those of this statement, and not talk about ϕx , and "roots," and the like, must, in my opinion, form the staple of a logical analysis and useful description of a problem, in general. I am none the less a mathematical logician for that. If of two students of the theory of numbers one should insist upon considering numbers as expressed in a system of notation like the Arabic (though using now one number as base of the numeration, and now another), while the other student should maintain that all that was foreign to the theory of numbers, which ought not to consider upon what system the numbers with which it deals are expressed, those two students would, to my apprehension, occupy positions analogous to that of Schröder and mine in regard to this matter of the formulation of the problems of logic; and supposing the student who wished to consider the forms of expression of numbers were to accuse the other of being wanting in the spirit of an arithmetician, that charge would be unjust in quite the same way in which it would be unjust to charge me with deficiency in the mathematical spirit on account of my regarding the conceptions of "values," and "roots," and all that as very special ideas, which can only lumber up the field of consciousness with such hindrances as it is the very end and aim of that diagrammatic

method of thinking that characterises the mathematician to get rid of.

Peirce: CP 3.516 Cross-Ref:††

516. But different questions are so very unlike that the only way to get much idea of the nature of a problem is to consider the different cases separately. There are in the first place questions about needs and their fulfillment which are not directly affected by the asking of the questions. A very good example is a chess problem. You have only to experiment in the imagination just as you would do on the board if it were permitted to touch the men, and if your experiments are intelligently conducted and are carried far enough, the solution required must be discovered. In other cases, the need to which the question relates is nothing but the intellectual need of having that question answered. It may happen that questions of this kind can likewise be answered by imaginary experimentation; but the more usual case requires real experimentation. The need is of one or other of two kinds. In the one class of cases we experience on several occasions to which our own deliberate action gave a common character, an excitation of one and the same novel idea or sensation, and the need is that a large number of propositions having the same novel consequent but different antecedents, should be replaced by one proposition which brings in the novel element, so that the others shall appear as mere consequences of everyday facts with a single novel one. We may express this intellectual need in a brief phrase as the need of synthetising a multitude of subjects. It is the need of *generalisation*. In another class of cases, we find in some new thing, or new situation, a great number of characters, the same as would naturally present themselves as consequences of a hypothetical state of things, and the need is that the large number of novel propositions with one subject or antecedent should be replaced by a single novel proposition, namely that the new thing or new occasion belongs to the hypothetical class, from which all those other novelties shall follow as mere consequences of matters of course. This intellectual need, briefly stated, is the need of synthetising a multitude of predicates. It is the need of *theory*. Every problem, then, is either a problem of consequences, a problem of generalisation, or a problem of theory.†1 This statement illustrates how special solutions are the only ones which directly mean anything or embody any knowledge; and general solutions are only useful when they happen to suggest what the special solutions will be.

Peirce: CP 3.517 Cross-Ref:††

517. Professor Schröder entertains very different ideas upon these matters. The general problem, according to him,†1 is, "Given the proposition $Fx = 0$, required the 'value' of $x[0]$," that is, an expression not containing x which can be equated to x . This "value" must be the "general root," that is, it must, under one general description, cover every possible object which fulfils a given condition. This, by the way, is the simplest explanation of what Schröder means by a 'solution-problem'; it is the problem to find that form of relative which necessarily fulfils a given condition and in which every relative that fulfils that condition can be expressed. Schröder shows that the solution of such a problem can be put into the form

$$(\Sigma x = fu),$$

u

which means that a suitable logical function (f) of *any* relative, u , no matter what, will satisfy the condition $Fx = 0$; and that nothing which is not equivalent to such a function will satisfy that condition. He further shows, what is very significant, that the solution may be required to satisfy the "adventitious condition" $fx = x$.^{†2} This fact about the adventitious condition is all that prevents me from rating the value of the whole discussion as far from high.

Peirce: CP 3.518 Cross-Ref:††

518. Professor Schröder next produces what he calls "the rigorous solution" of the general question. This promises something very fine -- the rigorously correct resolution of everything that ever could (but for this knowledge) puzzle the human mind. It is true that it supposes that a particular relative has been found which shall satisfy the condition $Fx = 0$. But that is seldom difficult to find. Either 0, or u , or some other trivial solution commonly offers itself. Supposing, then, that a be this particular solution, that is, that $Fa = 0$, the "rigorous solution" is

$$x = fu = a \cdot (Fu) \vee u \cdot (0 \wedge \sim(Fu)). \dagger 3$$

That is, it is such a function of u that when u satisfies the condition $Fu = 0$, $fu = u$; but when u does not satisfy this condition $fu = a$. Now $Fa = 0$.

Peirce: CP 3.519 Cross-Ref:††

519. Since Professor Schröder carries his algebraicity so very far, and talks of "roots," "values," "solutions," etc., when, even in my opinion, with my bias towards algebra, such phrases are out of place, let us see how this "rigorous solution" would stand the climate of numerical algebra. What should we say of a man who professed to give rigorous general solutions of algebraic equations of every degree (a problem included, of course, under Professor Schröder's general problem)? Take the equation: $x^5 + Ax^4 + Bx^3 + Cx^2 + Dx + E = 0$. Multiplying by $x - a$ we get

$$x^6 + (A-a)x^5 + (B-aA)x^4 + (C-aB)x^3 + (D-aC)x^2 + (E-aD)$$

$$x-aE = 0$$

The roots of this equation are precisely the same as those of the proposed quintic together with the additional root $x = a$. Hence, if we solve the sextic we thereby solve the quintic. Now our Schröderian solver would say, "There is a certain function, fu , every value of which, no matter what be the value of the variable, is a root of the sextic. And this function is formed by a direct operation. Namely, for all values of u which satisfy the equation

$$u^6 + (A-a)u^5 + (B-aA)u^4 + (C-aB)u^3 + (D-aC)u^2 + (E-aD)$$

$$u-aE = 0$$

$fu = u$, while for all other values, $fu = a$. Then, $x = fu$ is the expression of every root of the sextic and of nothing else. It is safe to say that Professor Schröder would pronounce a pretender to algebraical power who should talk in that fashion to be a proper subject for *surveillance* if not for confinement in an asylum. Yet he would only be applying Professor Schröder's "rigorous solution," neither more nor less. It is true that Schröder considers this solution as somewhat unsatisfactory; but he fails to state any principle according to which it should be so. Nor does he hold it too unsatisfactory to be frequently resorted to in the course of the volume. The *invention* of this solution exhibits in a high degree that very effective ingenuity which the *solution itself* so utterly lacks, owing to its resting on no correct conception of the nature of problems in general and of their solutions and of the meaning of a proposition.

Peirce: CP 3.520 Cross-Ref:††

§11. PROFESSOR SCHRÖDER'S PENTAGRAMMATICAL NOTATION

520. Professor Schröder's greatest success in the logic of relatives, is due precisely to his having, in regard to certain questions, proceeded by the separation of cases, quite abandoning the glittering generalities of the algebra of dyadic relatives. As his greatest success, I reckon his solutions of "inverse row and column problems" in §16, resting upon an investigation in §15 of the relations of various compound relatives which end in 0, ∞ , I, and T. The investigations of §15 might perfectly well have been carried through without any other instrument than the algebra of dyadic relatives. This course would have had certain advantages, such as that of exhibiting the principles on which the formulæ rest. But directness of proof would not have been of the number of those advantages; this is on the contrary decidedly with the notation invented and used by Professor Schröder. This notation may be called *pentagrammatic*, since it denotes a relative by a row of five characters. Imagine a list to be made of all the objects in the universe. Second, imagine a switchboard, consisting of a horizontal strip of brass for each object (these strips being fastened on a wall at a little distance one over another according to the order of the objects in the list) together with a vertical strip of brass for each object (these strips being fastened a little forward of the others, and being arranged in the same order), with holes at all the intersections, so that when a brass plug is inserted in any hole, the object corresponding to the horizontal brass strip can act in some way upon the object corresponding to the vertical brass strip. In order then, by means of this switchboard, to get an analogue of any dyadic relative, a lover of --," we insert plugs so that A and B, being any two objects, A can act on B, if and only if A is a lover of B. Now in Professor Schröder's pentagrammatic notation, the first of the five characters denoting any logical function of a primitive relative, *a*, refers to those horizontal strips, all whose holes are plugged in the representation of *a* (or, as we may say for short, "in *a*"), the second refers to those horizontal strips, each of which has in *a* every hole plugged but one. This one, not necessarily the same for all such strips, may be denoted by A. The third character refers to those horizontal strips which in *a* have several holes plugged, and several empty. The full holes (different, it may be, in the different horizontal strips) may be denoted by β . The fourth character refers to those

horizontal strips which in a have, each of them, but one hole plugged, generally a different hole in each. This one plugged hole may be denoted by Γ . The fifth character will refer to those rows each of which in a has all its holes empty. Then, a will be denoted by $\varphi \sim A \beta \Gamma 0$; and $\sim a$ by $0 A \sim \beta \sim \Gamma \varphi; \dagger 1$ for in $\sim a$, all the holes must be filled that are void in a , and *vice versa*. Consequently $\sim a \top = 0 \sim A \Omega \Omega \varphi$. Τηισ σηαλλ βε σηοων ασ σοον ασ ωε ηαπε φηρστ εξαμινεδ τηε πενταγραμματιχ σψμβολ φορ a . Τηισ σψμβολ διπιιδεσ a ιντο φουρ αγγρεγαντσ, πιζ.:

$$a = (a \downarrow 0) \psi a \bullet [(a \downarrow 1) \bullet \sim a] \top \psi a \bullet a \top \bullet (\sim a \bullet \sim a \top) \top \psi a \bullet (\sim a \downarrow 1)$$

Ιν ορδερ το προπεε, βψ τηε αλγεβρα ιτσελφ τηατ τηισ εθυατιον ηολδσ, ωε ρεμαρκ τηατ $a = a \bullet \beta \psi a \bullet \sim \beta$, ωηατεπερ β μαψ βε. Φορ β , συβστιτυτε $(a \downarrow 0)$. Τηεν, $a \downarrow 0 \approx a \approx \top$; βυτ $a \downarrow \top = a$. Ηενχε, $a \bullet \beta = a \downarrow 0$. $a \bullet \sim \beta = a \bullet \sim a \varphi = a \bullet \sim a (1 \psi \top) = a \bullet (\sim a | \psi \sim a \top)$. Βυτ $\sim a | = \sim a$, ανδ $a \bullet \sim a = 0$. Ηενχε $a \bullet \sim \beta = a \bullet \sim a \top$. Τηυσ $a = a \downarrow 0 \psi a \bullet \sim a \top$. Νοω, ιν $\sim a = \sim a \bullet \chi \psi \sim a \bullet \sim \chi$, συβστιτυτε φορ χ , $a \downarrow 1$. Τηισ γηπεσ $\sim a = (a \downarrow 1) \bullet \sim a \psi \sim a \top \bullet \sim a$; ανδ τηυσ, $a = a \downarrow 0 \psi a \bullet [(a \downarrow 1) \bullet \sim a] \psi a \bullet (\sim a \top \bullet \sim a) \top$. Φηναλλψ, $a = a \bullet a \top \psi a \bullet (\sim a \downarrow 1)$. Βυτ $a \bullet (\sim a \downarrow 1) = a \bullet (\sim a \downarrow 1) \bullet (\sim a \top \bullet \sim a) \top \psi a \bullet (\sim a \downarrow 1) \bullet \{(a \downarrow 1) \psi a\} \downarrow 1$.

Ανδ

$$\begin{aligned} & a \bullet (\sim a \downarrow 1) \bullet \{[(a \downarrow 1) a] \downarrow 1\} \\ &= a \bullet \{\sim a \bullet [(a \downarrow 1) \psi a] \downarrow 1\} \quad (\beta \psi \text{ διστριβυτιον}) \\ &= a \bullet [(\sim a \bullet (a \downarrow 1) \downarrow 1)] \quad (\text{σινχε } \sim a \bullet a = 0) \\ &= a \bullet (\sim a \downarrow 1) \bullet (a \downarrow 1 \downarrow 1) \quad (\beta \psi \text{ διστριβυτιον}) \\ &= a \bullet (\sim a \downarrow 1) \bullet (a \downarrow 0) \quad (\text{ιφ μορε τηαν 2 τηηνγσ εξιστ}) \\ &= a \bullet (\sim a \downarrow 1) \bullet (a \downarrow 1 \bullet \top) \quad (\text{σινχε } 0 = 1 \bullet \top) \\ &= a \bullet (\sim a \downarrow 1) \bullet (a \downarrow 1) \bullet (a \downarrow \top) \quad (\beta \psi \text{ διστριβυτιον}) \\ &= a \bullet (\sim a \downarrow 1) \bullet (a \downarrow 1) \quad (\text{σινχε } a \downarrow \top = a) \\ &= a \bullet (\sim a \bullet a \downarrow 1) \quad (\beta \psi \text{ διστριβυτιον}) \\ &= a \bullet (0 \downarrow 1) \quad (\text{σινχε } \sim a \bullet a = 0) \\ &= a \bullet 0 \quad (\text{ιφ μορε τηαν 1 οβφεχτ εξιστσ}) \\ &= 0. \end{aligned}$$

would be an equivalent form, equally with

$$(u \dot{\downarrow} 0 \dot{\uparrow} \sim u \dot{\downarrow} 0) \cdot v \dot{\uparrow} u \cdot \text{CP} \sim u \text{CP}.$$

Peirce: CP 3.522 Cross-Ref:††

522. §16 concludes with some examples of eliminations of great apparent complexity.†1 In the first of these we have given $x = (\sim u \dot{\downarrow} 1) \text{CP} \dot{\uparrow} u$; and it is required to eliminate u . We have, however, instantly $u \approx x$

$$(\sim u \dot{\downarrow} 1) \text{CP} \approx x$$

Whence, immediately,

$$(\sim x \dot{\downarrow} 1) \text{CP} \approx x,$$

or

$$\text{CP} \approx (x \cdot x \text{T}) \text{CP}.$$

Peirce: CP 3.522 Cross-Ref:††

The next example, the most complicated, requires u to be eliminated from the equation

$$x = \sim u \dot{\downarrow} 0 \dot{\uparrow} (u \dot{\downarrow} 1) \text{CP} \cdot \sim u \text{T} \dot{\uparrow} (v \dot{\downarrow} 1) \bullet \sim v \dot{\uparrow} (\sim v \dot{\downarrow} 1) \bullet v \\ \dot{\uparrow} (v \text{T} \bullet \sim v \text{T} \dot{\downarrow} 0) \bullet \sim v,$$

Ηε περφορμσ τηε ελιμινατιον βη μεανσ οφ τηε πενταγραμματιχ νοτατιον
περψ εασιλψ ασ φολλοωσ: Πυττινγ $v = \text{CP} \sim A\beta\Gamma 0$

$$\sim u \dot{\downarrow} 0 = 0000 \text{CP}$$

$$(u \dot{\downarrow} 1) \text{CP} \cdot \sim u \text{T} = 0 \sim A000$$

$$(u \dot{\downarrow} 1) \cdot \sim u = 0A000$$

$$(\sim u \dot{\downarrow} 1) \cdot u = 000\Gamma 0$$

$$(u \cdot T \cdot \sim u \cdot T \cdot \zeta 0) \cdot u = 00 \sim \beta 00$$

$$\text{sum } 0 \sim \beta \Gamma \sim \beta \Gamma \sim \beta \Gamma$$

Peirce: CP 3.522 Cross-Ref:††

Thus, x is of the form $\beta \Gamma 0$, which has been found in former problems to imply $x \cdot \zeta 1 \sim x$.

Peirce: CP 3.522 Cross-Ref:††

Without the pentagrammatical notation this elimination would prove troublesome, although with that as a guide it could easily be obtained by the algebra alone.

Peirce: CP 3.523 Cross-Ref:††

§12. PROFESSOR SCHRÖDER'S ICONIC SOLUTION OF $x \sim \phi x$.

523. Another valuable result obtained by Professor Schröder is the solutions of the problem

$$x \sim \phi x.$$

Namely, he shows that

$$x = f^\infty u$$

where

$$f u = u \cdot \phi u$$

(Of course, by contraposition, this gives for the solution of $\phi x \sim x$, $x = f^\infty u$ where $f u = u \cdot \phi u$.) The correctness of this solution will appear upon a moment's reflexion; and nearly all the useful solutions in the volume are cases under this.

Peirce: CP 3.524 Cross-Ref:††

524. It happens very frequently that the iteration of the functional operation is unnecessary, because it has no effect.

Peirce: CP 3.524 Cross-Ref:††

Suppose, for example, that we desire the general form of a "transitive" relative, that is, such a one, x , that

$$x x \approx x.$$

In this case, since $l \approx l \text{ } \text{\textcircled{~}} \text{\textcircled{~}} l$ whatever l may be, we have

$$x \approx x l \approx x(x \text{ } \text{\textcircled{~}} \text{\textcircled{~}} x) \approx x x \text{ } \text{\textcircled{~}} \text{\textcircled{~}} x \approx x \text{ } \text{\textcircled{~}} \text{\textcircled{~}} x,$$

or $x \approx x \text{ } \text{\textcircled{~}} \text{\textcircled{~}} x$

If, then,

$$f u = u \cdot (u \text{ } \text{\textcircled{~}} \text{\textcircled{~}} u)$$

we have

$$x = f^\infty u.$$

Here,

$$f u \approx u;$$

so that

$$f^\infty u \approx f u.$$

Also,

$$f^2 u = f u \cdot (f u \text{ } \text{\textcircled{~}} \text{\textcircled{~}} f u) = u \cdot (u \text{ } \text{\textcircled{~}} \text{\textcircled{~}} u) \cdot [[u \cdot (u \text{ } \text{\textcircled{~}} \text{\textcircled{~}} u) \text{ } \text{\textcircled{~}} \text{\textcircled{~}} (u \text{ } \text{\textcircled{~}} \text{\textcircled{~}} u)]]$$

$$= u \cdot (u \text{ } \text{\textcircled{~}} \text{\textcircled{~}} u) \cdot [[u f (| \text{ } \text{\textcircled{~}} \text{\textcircled{~}} v) \exists \sim v]] \bullet [[v \text{ } \text{\textcircled{~}} \text{\textcircled{~}} v \text{ } \text{\textcircled{~}} (| \text{ } \text{\textcircled{~}} \text{\textcircled{~}} v) \exists \sim v]]$$

That is to say, the principle of this syllogism enters into every syllogism. But to say that this is a valid syllogism is merely to say that the copula expresses a transitive relation. Hence, when we now find that transitiveness always depends upon inclusion, the initial analysis by which the copula of inclusion was taken as the general one is fully confirmed. For the chief end of formal logic is the representation of the syllogism.

Peirce: CP 3.526 Cross-Ref: ††
§13. INTRODUCTION TO THE LOGIC OF QUANTITY †2

526. The great importance of the idea of quantity in demonstrative reasoning seems to me not yet sufficiently explained. It appears, however, to be connected with the circumstance that the relations of being greater than and of being at least as great as are transitive relations. Still, a satisfactory evolutionary logic of mathematics remains a desideratum. I intend to take up that problem in a future paper. †1 Meantime the development of projective geometry and of geometrical topics has shown that there are at least two large mathematical theories of continuity into which the idea of continuous *quantity*, in the usual sense of that word, does not enter at all. For projective geometry Schubert †2 has developed an algebraical calculus which has a most remarkable affinity to the Boolean algebra of logic. It is, however, imperfect, in that it only gives imaginary points, rays, and planes, without deciding whether they are real or not. This defect cannot be remedied until topology -- or, as I prefer to call it, mathematical topics -- has been further developed and its logic accurately analysed. †3 To do this ought to be one of the first tasks of exact logicians. But before that can be accomplished, a perfectly satisfactory logical account of the conception of continuity is required. This involves the definition of a certain kind of infinity; and in order to make that quite clear, it is requisite to begin by developing the logical doctrine of infinite multitude. This doctrine still remains, after the works of Cantor, Dedekind, and others, in an inchoate condition. For example, such a question remains unanswered as the following: Is it, or is it not, logically possible for two collections to be so multitudinous that neither can be put into a one-to-one correspondence with a part or the whole of the other? To resolve this problem demands, not a mere *application* of logic, but a further *development* of the conception of logical possibility.

Peirce: CP 3.527 Cross-Ref: ††

527. †4 I formerly defined the possible as that which in a given state of information (real or feigned) we do not know not to be true. †5 But this definition today seems to me only a twisted phrase which, by means of two negatives, conceals an anacoluthon. We know in advance of experience that certain things are not true, because we see they are impossible. Thus, if a chemist tests the contents of a hundred bottles for fluorine, and finds it present in the majority, and if another chemist tests them for oxygen and finds it in the majority, and if each of them reports his result to me, it will be useless for them to come to me together and say that they know infallibly that fluorine and oxygen cannot be present in the same bottle; for I see that such infallibility is *impossible*. I know it is not true, because I satisfy myself that there is no room for it even in that ideal world of which the real world is but a fragment. I need no sensible experimentation, because ideal experimentation establishes a much

broader answer to the question than sensible experimentation could give. It has come about through the agencies of development that man is endowed with intelligence of such a nature that he can by ideal experiments ascertain that in a certain universe of logical possibility certain combinations occur while others do not occur. Of those which occur in the ideal world some do and some do not occur in the real world; but all that occur in the real world occur also in the ideal world. †P1 For the real world is the world of sensible experience, and it is a part of the process of sensible experience to locate its facts in the world of ideas. This is what I mean by saying that the sensible world is but a fragment of the ideal world. In respect to the ideal world we are virtually omniscient; that is to say, there is nothing but lack of time, of perseverance, and of activity of mind to prevent our making the requisite experiments to ascertain positively whether a given combination occurs or not. Thus, every proposition about the ideal world can be ascertained to be either true or false. A description of thing which occurs in that world is *possible, in the substantive logical sense*. Very many writers assert that everything is logically possible which involves no contradiction. Let us call that sort of logical possibility, *essential*, or *formal*, logical possibility. It is not the only logical possibility; for in this sense, two propositions contradictory of one another may both be severally possible, although their combination is not possible. †P2 But in the *substantive* sense, the contradictory of a possible proposition is impossible, because we are virtually omniscient in regard to the ideal world. For example, there is no contradiction in supposing that only four, or any other number, of independent atoms exist. But it is made clear to us by ideal experimentation, that five atoms are to be found in the ideal world. Whether all five are to be found in the sensible world or not, to say that there are only four in the ideal world is a proposition absolutely to be rejected, notwithstanding its involving no contradiction.

Peirce: CP 3.528 Cross-Ref: ††

528. It would be a great mistake to suppose that ideal experimentation can be performed without danger of error; but by the exercise of care and industry this danger may be reduced indefinitely. In sensible experimentation, no care can always avoid error. The results of induction from sensible experimentation are to afford some ratio of frequency with which a given consequence follows given conditions in the existing order of experience. In induction from ideal experimentation, no particular order of experience is forced upon us; and consequently no such numerical ratio is deducible. We are confined to a dichotomy: the result either is that some description of thing occurs or that it does not occur. For example, we cannot say that one number in every three is divisible by three and one in every five is divisible by five. This is, indeed, so if we choose to arrange the numbers in the order of counting; but if we arrange them with reference to their prime factors, just as many are divisible by one prime as by another. I mean, for instance, when they are arranged [in blocks] as follows:

1,2,4,8, etc. 5,10,20,40, etc. 7,14,28,56, etc. 35, 70, etc.

3,6,12,24, etc. 15,30,60,120, etc. 21,42,84,168, etc. 105,210, etc. 9,18,36,72, etc.
45,90,180,360, etc. etc. etc. †P1

27,54,108,216, etc. 135,270,540,1080, etc.

etc. etc.

Peirce: CP 3.529 Cross-Ref:††

529. Thus, dichotomy rules the ideal world. Plato, therefore, for whom that world alone was real, showed that insight into concepts but dimly apprehended that has always characterised philosophers of the first order, in holding dichotomy to be the only truthful mode of division. Lofty moral sense consists in regarding, not indeed *the*, but yet *an* ideal world as in some sense the only real one; and hence it is that stern moralists are always inclined to dual distinctions.†1

Peirce: CP 3.530 Cross-Ref:††

530. Ideal experimentation has one or other of two forms of results. It either proves that $\Sigma[i]m[i]$, a particular proposition true of the ideal world, and going on, finds $\Sigma[j]\sim m[j]$ also true; that is, that *m* and $\sim m$ are both possible, or it succeeds in its induction and shows the universal proposition $\pi[i]\sim m[i]$ to be true of the ideal world; that is that $\sim m$ is *necessary* and *m impossible*.

Peirce: CP 3.531 Cross-Ref:††

531. Every result of an ideal induction clothes itself, in our modes of thinking, in the dress of a *contradiction*. It is an anacoluthon to say that a proposition is impossible *because* it is self-contradictory. It rather is thought so as to appear self-contradictory because the ideal induction has shown it to be impossible. But the result is that in the absence of any interfering contradiction every particular proposition is possible in the substantive logical sense, and its contradictory universal proposition is impossible. But where contradiction interferes this is reversed.

Peirce: CP 3.532 Cross-Ref:††

532. In former publications †2 I have given the appellation of *universal* or *particular* to a proposition according as its *first* quantifier is π or Σ . But the study of substantive logical possibility has led me to substitute the appellations *negative* and *affirmative* in this sense, and to call a proposition *universal* or *particular* according as its *last* quantifier is π or Σ †3. For letting *l* be any relative, one or other of the two propositions

$$\pi[i]\Sigma[j]l[i,j] \quad \Sigma[i]\pi[j]\sim l[i,j]$$

and one or other of the two propositions

$$\pi[j]\Sigma[i]\sim l[i,j] \quad \Sigma[j]\pi[i]l[i,j]$$

are true, while the other one of each pair is false. Now, in the absence of any peculiar property of the special relative *l*, the two similar forms $\Sigma[i]\pi[i]\sim l[i,j]$ †4 and $\Sigma[j]\pi[i]l[i,j]$ must be equally possible in the substantive logical sense. But these two propositions cannot both be true. Hence, both must be false in the ideal world, in the absence of any constraining contradiction. Accordingly, these ought to be regarded as universal propositions, and their contradictions, $\pi[i]\Sigma[j]l[i,j]$ and $\pi[j]\Sigma[i]\sim l[j,i]$, as particular propositions.†1

Peirce: CP 3.533 Cross-Ref:††

533. There are two opposite points of view, each having its logical value, from one of which, of two quantifiers of the same proposition, the preceding is more important than the following, while from the other point of view the reverse is the case. Accordingly, we may say that an affirmative proposition is particular in a secondary way, and that a particular proposition is affirmative in a secondary way.

Peirce: CP 3.534 Cross-Ref:††

534. If an index is not quantified at all, the proposition is, with reference to that index, *singular*. To ascertain whether or not such a proposition is true of the ideal world, it must be shown to depend upon some universal or particular proposition.

Peirce: CP 3.535 Cross-Ref:††

535. If some of the quantifiers refer not to hecceities, having in themselves no general characters except the logical characters of identity, diversity, etc., but refer to *characters*, whether non-relative or relative, these alone are to be considered in determining the "quantity" of an ideal proposition as universal or particular. For anything whatever is true of *some* character, unless that proposition be downright absurd; while nothing is true of *all* characters except what is formally necessary.†2 Consider, for example, a dyadic relation. This is nothing but an aggregation of pairs. Now any two hecceities may in either order form a pair; and any aggregate whatever of such pairs will form *some* dyadic relation. Hence, we may totally disregard the manner in which the hecceities are connected in determining the possibility of a hypothesis about *some* dyadic relation.

Peirce: CP 3.536 Cross-Ref:††

536. Characters have themselves characters, such as importance, obviousness, complexity, and the like. If some of the quantified indices denote such characters of characters, they will, in reference to a purely ideal world be paramount in determining the quantity of the proposition as universal or particular.

Peirce: CP 3.537 Cross-Ref:††

537. All quantitative comparison depends upon a *correspondence*. A correspondence is a relation which every subject †P1 of one collection bears to a subject of another collection, to which no other is in the same relation. That is to say, the relative "corresponds to" has

$$\sum u \cdot (I \text{ } \text{ } \sim u)$$

u

not merely as its *form*, but as its *definition*. This relative is transitive; for its relative product into itself is

$$[[\sum u \cdot (I \text{ } \text{ } \sim u)]] [[\sum v \cdot (I \text{ } \text{ } \sim v)]] \approx \sum \sum u \ v \cdot (I \text{ } \text{ } \sim u) (I \text{ } \text{ } \sim v)$$

u *v* *u v*

$$\approx \sum u \cdot (I \dot{\sim} u \dot{\sim} v) \approx \sum u \cdot (I \dot{\sim} (u \vee v)) \approx \sum w \cdot (I \dot{\sim} w).$$

$u \vee \quad \quad u \vee \quad \quad w$

But it is to be observed that if the P's, the Q's, and the R's are three collections, it does not follow because every P corresponds to an R, and every Q corresponds to an R that every object of the aggregate collection P∨Q corresponds to an R. The *dictum de omni* in external appearance fails here. For P may be $[[u \cdot (I \dot{\sim} u)]]R$ and Q may be $[[v \cdot (I \dot{\sim} v)]]R$; but the aggregate of these is not $[[(u \vee v) \cdot (I \dot{\sim} (u \vee v))]R$, which equals $[[(u \vee v) \cdot (I \dot{\sim} u) \cdot (I \dot{\sim} v)]R$. The aggregate of the two first is $\{ (u \dot{\sim} v) \cdot [[v \cdot (I \dot{\sim} \sim v) \Psi I \dot{\sim} v]] \bullet [[v \cdot (I \dot{\sim} v) \Psi I \dot{\sim} w]] \}P$, ωηιχη ισ οβπιουσλψ τοο βροαδ το βε νεχεσσαριλψ ινχλυδεδ υνδερ τηε οτηερ εξπρεσσιον. Χορρεσπονδενχε ισ, τηερεφορε, νοτ α ρελατιον βετωεεν τηε συβφεχτσ οφ ονε χολλεχτιον ανδ τηοσε οφ ανοτηερ, βυτ βετωεεν τηε χολλεχτιονσ τηεμσελωεσ. Λετ $\theta[\alpha \ i]$ μεαν τηατ i ισ α συβφεχτ οφ τηε χολλεχτιον, α , ανδ λετ $\rho[\beta j k]$ mean that j stands in the relation β to k . Then, to say that the collection P corresponds to the collection Q, or, as it is sometimes expressed, that "for every subject of Q there is a subject of P," is to make the assertion expressed by

$$\sum[\beta] \pi[i] \sum[j] \pi[k] \sim q[P[i]] \vee r[\beta i j] \cdot (I[i k] \vee \sim r[\beta k j]) \cdot q[Q[j]]. \dagger 1$$

In the algebra of dual relatives this may be written

$$\sum[\beta] P \approx \sim q \dot{\sim} [[r[\beta] \cdot (I \dot{\sim} \sim r[\beta])]] \$ q Q.$$

The transitivity is evident; for

$$\begin{aligned} & \sum[\beta] \sum[\{g\}] \sim q \dot{\sim} [[r[\beta] \cdot (I \dot{\sim} \sim r[\beta])]] \$ q \{ \sim q \dot{\sim} [[r[\{g\}] \cdot (I \dot{\sim} \sim r[\{g\})]] \$ q \} \\ & \approx \sum[\beta] \sum[\{g\}] \sim q \dot{\sim} [[r[\beta] \cdot (I \dot{\sim} \sim r[\beta])]] \{ \$ q \sim q \dot{\sim} [[r[\{g\}] \cdot (I \dot{\sim} \sim r[\{g\})]] \$ q \} \\ & \approx \sum[\beta] \sum[\{g\}] \sim q \dot{\sim} [[r[\beta] \cdot (I \dot{\sim} \sim r[\beta])]] \{ T \dot{\sim} [[r[\{g\}] \cdot (I \dot{\sim} \sim r[\{g\})]] \$ q \} \\ & \approx \sum[\beta] \sum[\{g\}] \sim q \dot{\sim} [[r[\beta] \cdot (I \dot{\sim} \sim r[\beta])]] [[r[\{g\}] \cdot (I \dot{\sim} \sim r[\{g\})]] \$ q \\ & \approx \sum[\beta] \sum[\{g\}] \sim q \dot{\sim} [[r[\beta] r[\{g\}] \cdot (I \dot{\sim} \sim r[\beta] \dot{\sim} \sim r[\{g\})]] \$ q \\ & \approx \sum[\delta] \sim q \dot{\sim} [[r[\delta] \cdot (I \dot{\sim} \sim r[\delta])]] \$ q. \dagger P1 \end{aligned}$$

Peirce: CP 3.538 Cross-Ref: ††

538. Not only is the relative of correspondence transitive but it also possesses what may be called *antithetic transitivity*. Namely, if c be the relative, not only is $c \ c$

$\sim c$ but also $c \sim c \supset c$. To demonstrate this very important proposition is, however, far from easy. The quantifiers of the assertion that for every subject of one character there is a subject of another are $\Sigma[\beta]\pi[i]\Sigma[j]\pi[k]$. Hence, the proposition is particular $\dagger 2$ and will be true in the ideal world, except in case a positive contradiction is involved.

Peirce: CP 3.539 Cross-Ref: $\dagger\dagger$

539. Let us see how such contradiction can arise. The assertion that for every subject of P there is a subject of Q is

$$\Sigma[\beta]\pi[i]\Sigma[j]\pi[k]\sim q[P[i]] \vee r[\beta i j] \cdot (I[i k] \vee \sim r[\beta k j]) \cdot q[Q[j]].$$

This cannot vanish if the first aggregant term does not vanish, that is, if $\pi[i]q[P[i]]$ or there is no subject of P. It cannot vanish if everything is a subject of Q. For in that case, the last factor of the latter aggregant disappears, and substituting **I** for $r[\beta]$ the second aggregant becomes \vee . The expression cannot vanish if every subject of P is a subject of Q. For when **I** is substituted for $r[\beta]$, we get

$$\pi[i]\sim q[P[i]] \vee q[Q[i]].$$

If P has but a single individual subject and Q has a subject, for every P there is a Q. For in this case we have only to take for β the relation of the subject of P to any one of the subjects of Q. But if P has more than one subject, and Q has but one, the expression above vanishes. For let 1 and 2 be the two subjects of P. Substituting 1 for i , we get

$$\pi[k]r[\beta 1 j] \cdot (I[1 k] \vee \sim r[\beta k j]) \cdot q[Q[j]]$$

Substituting 2 for i we get

$$\pi[k]r[\beta 2 j] \cdot (I[2 k] \vee \sim r[\beta k j]) \cdot q[Q[j]]$$

Multiplying these

$$\pi[k]\pi[k]r[\beta 1 j] \cdot r[\beta 2 j] \cdot (I[1 k] \vee \sim r[\beta k j]) \cdot (I[2 k'] \vee \sim r[\beta k j]) \cdot q[Q[i]]$$

Substituting 2 for k and 1 for k' , this gives

$$r[\beta 1j] \cdot r[\beta 2j] \cdot \sim r[\beta 2j \cdot \beta 1j] \cdot q[Q[i]]$$

which involves two contradictions.

Peirce: CP 3.540 Cross-Ref:††

540. It is to be remarked that although if every subject of P is a subject of Q, then for every subject of P there is a subject of Q, yet it does not follow that if the subjects of P are a part only of the subjects of Q, that there is then not a subject of P for every subject of Q. For example, numbering 2, 4, 6, etc., as the first, second, third, etc., of the even numbers, there is an even number for every whole number, although the even numbers form but a part of the whole numbers.

Peirce: CP 3.541 Cross-Ref:††

541. It is now requisite, in order to prove that $c \approx c \text{ } \S c$, to draw three propositions from the doctrine of substantive logical possibility. The first is that given any relation, there is a possible relation which differs from the given relation only in excluding any of the pairs we may choose to exclude. Suppose, for instance, that for every subject of P there is a subject of Q, that is that

$$\Sigma[\beta] \$qP \approx [[r[\beta] \cdot (1 \text{ } \S \sim r[\beta])]] \$qQ.$$

The factor $(1 \text{ } \S \sim r[\beta])$ here has the effect of allowing each correlate but one relate. Each relate is, however, allowed any number of correlates. If we exclude all but one of these, the one retained being, if possible, a subject of Q, we have a possible relation, β' , such that

$$\Sigma[\beta'] \$qP \approx [[r[\beta'] \cdot (1 \text{ } \S \sim r[\beta']) \cdot (\sim r[\beta'] \text{ } \S 1)]] \$qQ$$

Peirce: CP 3.542 Cross-Ref:††

542. The second proposition of substantive logical possibility is that whatever is true of *some* of a class is true of the whole of *some* class. That is, if we accept a proposition of the form $\Sigma[i] a[i] \cdot b[i]$, we can write

$$\Sigma[\{g\}] \pi[i] \sim q[\{gi\}] \cdot \Psi \sim a[i] b[i],$$

though this will generally fail positively to assert, in itself, what is implied, that the collection $\{g\}$ excludes whatever is *a* but not *b*, and includes something in common with *a*. There are, however, cases in which this implication is easily made plain.

Peirce: CP 3.542 Cross-Ref:††

Applying these two principles to the relation of correspondence, we get a new statement of the assertion that for every P there is a Q. Namely, if we write $a[a i]$ to signify that i is a relate of the relative $r[a]$ to some correlate, that is if $a[a i] = (i \rightsquigarrow r[a] \circ \circ)$, if we write $b[a j]$ to signify that j is a correlate of the relative $r[a]$, to some relate, that is if $b[a j] = (j \rightsquigarrow r[a] \circ \circ)$, and if we write $p[c a]$ to signify that $r[a]$ is an aggregate of the relative $r[c]$, that is, if $p[c a] = (r[a] \rightsquigarrow r[c])$, then the proposition that for every subject of P there is a subject of Q may be put in the form,

$$\Sigma[c]\Sigma[\{g\}]\pi[x]\pi[y]\Sigma[\delta]\Sigma[\varepsilon]\pi[a]\Sigma[i]\Sigma[j]\pi[\beta]\pi[u]\pi[v]$$

$$[[\sim p[c a] \vee a[a i] \cdot q[P[i]] \cdot b[a j] \cdot q[Q[j]] \cdot q[\{g\}i] \cdot (\sim a[a u] \vee I[i u]) \cdot (\sim b[a v] \vee I[j v]) \cdot (\sim p[c \beta] \vee I[a \beta] \vee \sim a[\beta i] \cdot b[\beta j])] \cdot (\sim q[P[x]] \vee a[\delta x] \cdot p[c \delta]) \cdot (\sim q[Q[y]] \vee \sim q[\{g\}y] \vee b[\varepsilon y] \cdot p[c \varepsilon])$$

This states that there is a collection of pairs, c , any single pair of which, a , has for its sole first subject a subject of P, and for its sole second subject a subject of Q which is at the same time a subject of a collection, j , and that no two pairs of the collection, c , have the same first subject or the same second subject, and that every subject of P is a first subject of some pair of this collection, c , and every subject of Q which is at the same time a subject of $\{g\}$ is a second subject of some pair of the same collection, c .

Peirce: CP 3.543 Cross-Ref:††

543. The third proposition of the doctrine of substantive logical possibility of which we have need is that all hecceities are alike in respect to their capacity for entering into possible pairs. Consequently, all the objects of any collection whatever may be severally and distinctly paired with all the objects of a collection which shall either be wholly contained in, or else shall entirely contain, any other collection whatever. Consequently,

$$\pi[P]\pi[Q]\Sigma[c]\Sigma[\delta]\pi[x]\Sigma[\delta]\pi[y]\Sigma[\delta]\pi[a]\Sigma[i]\Sigma[j]\pi[u]\pi[v]\pi[\beta]\pi[m]\pi[n]$$

$$[[\sim p[c a] \vee a[a i] \cdot q[P[i]] \cdot b[a j] \cdot q[\xi j] \cdot (\sim a[a u] \vee I[i u]) \cdot (\sim b[a v] \vee I[v j]) \cdot (\sim p[c \beta] \vee I[a \beta] \vee \sim a[\beta i] \cdot b[\beta j])] \cdot (\sim q[P[x]] \vee a[\delta x] \cdot p[c \delta]) \cdot (\sim q[\delta y] \vee b[\varepsilon y] \cdot p[c \varepsilon]) \cdot (\sim q[\delta m] \vee q[Q[m]] \vee \sim q[Q[n]] \vee q[\delta n]).$$

Peirce: CP 3.544 Cross-Ref:††

544. Although the above three propositions belong to a system of doctrine not universally recognised, yet I believe their truth is unquestionable. Suppose, now, that it is not true that for every subject of P there is a subject of Q. Then, in the last formula, $\pi[m] \sim q[\delta m] \vee q[Q[m]] \rightsquigarrow 0$. This leaves for the last factor $\pi[n] \sim q[Q[n]] \vee q[\delta n]$, and then the formula expresses that for every subject of Q there is a subject of P. In other words, we have demonstrated the important proposition that *two collections cannot be disparate in respect to correspondence*, but that for every

subject of the one there must be a subject of the other.

Peirce: CP 3.545 Cross-Ref:††

545. The theorem $c \approx c\%c$ is now established; for since of any two collections one corresponds to the other, we have $c \approx c \vee \$c$ or (non-relatively multiplying by $\$ \sim c$) $\$ \sim c \approx c$. Hence, $c \approx |c \approx (\$ \sim c \ \$ c)c \approx \$ \sim c \ \$ c c \approx c \ \$ c c$; and, by the transitive principle $c c \approx c$, we finally obtain $c \approx c \ \$ c$.

Peirce: CP 3.546 Cross-Ref:††

546. Thus is established the conception of *multitude*. Namely, if for every subject of P there is a subject of Q, while there is not for every subject of Q a subject of P, the *multitude* of Q is said to be *greater* than that of P. But if for every subject of each collection there is a subject of the other, the *multitudes* of the two collections are said to be *equal* the one to the other. We may create a scale of objects, one for every group of equal collections. Calling these objects *arithms*, the first arithm will belong to 0 considered as a collection, the second to individuals, etc. Calling a collection the counting of which can be completed an *enumerable* collection, the multitude of any enumerable collection equals that of the arithms that precede its arithm. Calling a collection whose multitude equals that of all the arithms of enumerable collection a *denumerable* collection (because its subjects can all be distinguished by ordinal numbers, though the counting of it cannot be completed), the arithms preceding the arithm of denumerable collections form a denumerable collection. More multitudinous collections are greater than the collections of arithms which precede their arithm.

Peirce: CP 3.547 Cross-Ref:††

547. Let there be a denumerable collection, say the cardinal numbers; and let there be two houses. Let there be a collection of children, each of whom wishes to have those numbers placed in some way into those houses, no two children wishing for the same distribution, but every distribution being wished for by some child. Then, as Dr. George Cantor has proved,†1 the collection of children is greater in multitude than the collection of numbers. Let a collection equal in multitude to that collection of children be called an *abnumeral* collection *of the first dignity*. The real numbers (surd and rational) constitute such a collection.

Peirce: CP 3.548 Cross-Ref:††

548. I now ask, suppose that for every way of placing the subjects of one collection in two houses, there is a way of placing the subjects of another collection in two houses, does it follow that for every subject of the former collection there is a subject of the latter? In order to answer this, I first ask whether the multitude of possible ways of placing the subjects of a collection in two houses can equal the multitude of those subjects. If so, let there be such a multitude of children. Then, each having but one wish, they can among them wish for every possible distribution of themselves among two houses. Then, however they may actually be distributed, some child will be perfectly contented. But ask each child which house he wishes himself to be in, and put every child in the house where he does not want to be. Then, no child would be content. Consequently, it is absurd to suppose that any collection can equal in multitude the possible ways of distributing its subjects in two houses.

Peirce: CP 3.549 Cross-Ref:††

549. Accordingly, the multitude of ways of placing a collection of objects abnumeral of the first dignity into two houses is still greater in multitude than that multitude, and may be called abnumeral of the second dignity. There will be a

denumerable succession of such dignities. But there cannot be any multitude of an infinite dignity; for if there were, the multitude of ways of distributing it into two houses would be no greater than itself. †P1

Peirce: CP 3.550 Cross-Ref: ††

550. We thus not only answer the question proposed, and show that of two unequal multitudes the multitude of ways of distributing the greater is the greater; but we obtain the entire scale of collectional quantity, which we find to consist of two equal parts (that is two parts whose multitudes of grades are equal), the one finite, the other infinite. Corresponding to the multitude of 0 on the finite scale is the abnumeral of 0 dignity, which is the denumerable, on the infinite scale, etc. †1

Peirce: CP 3.551 Cross-Ref: ††

551. So much of the general logical doctrine of quantity has been here given, in order to illustrate the power of the logic of relatives in enabling us to treat with unerring confidence the most difficult conceptions, before which mathematicians have heretofore shrunk appalled.

Peirce: CP 3.552 Cross-Ref: ††

552. I had been desirous of examining Professor Schröder's developments concerning individuals and individual pairs; but owing to the length this paper has already reached, I must remit that to some future occasion. †2

Peirce: CP 3.553 Cross-Ref: ††

XVII

THE LOGIC OF MATHEMATICS IN RELATION TO EDUCATION †1

§1. OF MATHEMATICS IN GENERAL

553. In order to understand what number is, it is necessary first to acquaint ourselves with the nature of the business of mathematics in which number is employed.

Peirce: CP 3.554 Cross-Ref: ††

554. I wish I knew with certainty the precise origin of the definition of mathematics as the science of quantity. It certainly cannot be Greek, because the Greeks were advanced in projective geometry, whose problems are such as these: whether or not four points obtained in a given way lie in one plane; whether or not four planes have a point in common; whether or not two rays (or unlimited straight lines) intersect, and the like -- problems which have nothing to do with quantity, as such. Aristotle †2 names, as the subjects of mathematical study, quantity and continuity. But though he never gives a formal definition of mathematics, he makes quite clear, in more than a dozen places, his view that mathematics ought not to be defined by the things which it studies but by its peculiar mode and degree of

abstractness. Precisely what he conceives this to be it would require me to go too far into the technicalities of his philosophy to explain; and I do not suppose anybody would today regard the details of his opinion as important for my purpose. Geometry, arithmetic, astronomy, and music were, in the Roman schools of the fifth century †P1 and earlier, recognized as the four branches of mathematics. And we find Boëthius (A.D. 500) defining them as the arts which relate, not to quantity, but to *quantities*, or *quanta*. What this would seem to imply is, that mathematics is the foundation of the minutely exact sciences; but really it is not worth our while, for the present purpose, to ascertain what the schoolmasters of that degenerate age conceived mathematics to be.

Peirce: CP 3.555 Cross-Ref:††

555. In modern times projective geometry was, until the middle of this century, almost forgotten, the extraordinary book of Desargues †P1 having been completely lost until, in 1845, Chasles came across a MS. copy of it; and, especially before imaginaries became very prominent, the definition of mathematics as the science of quantity suited well enough such mathematics as existed in the seventeenth and eighteenth centuries.

Peirce: CP 3.556 Cross-Ref:††

556. Kant, in the *Critique of Pure Reason* (Methodology, chapter I, section 1), distinctly rejects the definition of mathematics as the science of quantity. What really distinguishes mathematics, according to him, is not the subject of which it treats, but its method, which consists in studying constructions, or diagrams. That such is its method is unquestionably correct; for, even in algebra, the great purpose which the symbolism subserves is to bring a skeleton representation of the relations concerned in the problem before the mind's eye in a schematic shape, which can be studied much as a geometrical figure is studied.

Peirce: CP 3.557 Cross-Ref:††

557. But Rowan Hamilton and De Morgan, having a superficial acquaintance with Kant, were just enough influenced by the *Critique* to be led, when they found reason for rejecting the definition as the science of quantity, to conclude that mathematics was the science of pure time and pure space. Notwithstanding the profound deference which every mathematician must pay to Hamilton's opinions and my own admiration for De Morgan, I must say that it is rare to meet with a careful definition of a science so extremely objectionable as this. If Hamilton and De Morgan had attentively read what Kant himself has to say about number, in the first chapter of the *Analytic of principles* and elsewhere, they would have seen that it has no more to do with time and space than has every conception. Hamilton's intention probably was, by means of this definition, to throw a slur upon the introduction of imaginaries into geometry, as a false science; but what De Morgan, who was a student of multiple algebra, and whose own formal logic is plainly mathematical, could have had in view, it is hard to comprehend, unless he wished to oppose Boole's theory of logic. Not only do mathematicians study hypotheses which, both in truth and according to the Kantian epistemology, no otherwise relate to time and space than do all hypotheses whatsoever, but we now all clearly see, since the non-Euclidean geometry has become familiar to us, that there *is* a real science of space and a real science of time, and that these sciences are positive and experiential -- branches of physics, and so not mathematical except in the sense in which thermotics and electricity are mathematical; that is, as calling in the aid of mathematics. But the gravest objection of all to the definition is that it altogether ignores the veritable characteristics of this

science, as they were pointed out by Aristotle and by Kant.

Peirce: CP 3.558 Cross-Ref:††

558. Of late decades philosophical mathematicians have come to a pretty just understanding of the nature of their own pursuit. I do not know that anybody struck the true note before Benjamin Peirce, who, in 1870,†P1 declared mathematics to be "the science which draws necessary conclusions," adding that it must be defined "subjectively" and not "objectively." A view substantially in accord with his, though needlessly complicated, is given in the article "Mathematics," in the ninth edition of the *Encyclopædia Britannica*. The author, Professor George Chrystal, holds that the essence of mathematics lies in its making pure hypotheses, and in the character of the hypotheses which it makes. What the mathematicians mean by a "hypothesis" is a proposition imagined to be strictly true of an ideal state of things. In this sense, it is only about hypotheses that necessary reasoning has any application; for, in regard to the real world, we have no right to presume that any given intelligible proposition is true in absolute strictness. On the other hand, probable reasoning deals with the ordinary course of experience; now, nothing like *a course of experience* exists for ideal hypotheses. Hence to say that mathematics busies itself in drawing necessary conclusions, and to say that it busies itself with hypotheses, are two statements which the logician perceives come to the same thing.

Peirce: CP 3.559 Cross-Ref:††

559. A simple way of arriving at a true conception of the mathematician's business is to consider what service it is which he is called in to render in the course of any scientific or other inquiry. Mathematics has always been more or less a trade. An engineer, or a business company (say, an insurance company), or a buyer (say, of land), or a physicist, finds it suits his purpose to ascertain what the necessary consequences of possible facts would be; but the facts are so complicated that he cannot deal with them in his usual way. He calls upon a mathematician and states the question. Now the mathematician does not conceive it to be any part of his duty to verify the facts stated. He accepts them absolutely without question. He does not in the least care whether they are correct or not. He finds, however, in almost every case that the statement has one inconvenience, and in many cases that it has a second. The first inconvenience is that, though the statement may not at first sound very complicated, yet, when it is accurately analyzed, it is found to imply so intricate a condition of things that it far surpasses the power of the mathematician to say with exactitude what its consequence would be. At the same time, it frequently happens that the facts, as stated, are insufficient to answer the question that is put. Accordingly, the first business of the mathematician, often a most difficult task, is to frame another simpler but quite fictitious problem (supplemented, perhaps, by some supposition), which shall be within his powers, while at the same time it is sufficiently like the problem set before him to answer, well or ill, as a substitute for it.†P1 This substituted problem differs also from that which was first set before the mathematician in another respect: namely, that it is highly abstract. All features that have no bearing upon the relations of the premisses to the conclusion are effaced and obliterated. The skeletonization or diagrammatization of the problem serves more purposes than one; but its principal purpose is to strip the significant relations of all disguise. Only one kind of concrete clothing is permitted -- namely, such as, whether from habit or from the constitution of the mind, has become so familiar that it decidedly aids in tracing the consequences of the hypothesis. Thus, the mathematician does two very different things: namely, he first frames a pure hypothesis stripped of all features which do not concern the drawing of consequences from it, and this he

does without inquiring or caring whether it agrees with the actual facts or not; and, secondly, he proceeds to draw necessary consequences from that hypothesis.

Peirce: CP 3.560 Cross-Ref:††

560. Kant is entirely right in saying that, in drawing those consequences, the mathematician uses what, in geometry, is called a "construction," or in general a diagram, or visual array of characters or lines. Such a construction is formed according to a precept furnished by the hypothesis. Being formed, the construction is submitted to the scrutiny of observation, and new relations are discovered among its parts, not stated in the precept by which it was formed, and are found, by a little mental experimentation, to be such that they will always be present in such a construction. Thus, the necessary reasoning of mathematics is performed by means of observation and experiment, and its necessary character is due simply to the circumstance that the subject of this observation and experiment is a diagram of our own creation, the conditions of whose being we know all about.

Peirce: CP 3.560 Cross-Ref:††

But Kant, owing to the slight development which formal logic had received in his time, and especially owing to his total ignorance of the logic of relatives, which throws a brilliant light upon the whole of logic, fell into error in supposing that mathematical and philosophical necessary reasoning are distinguished by the circumstance that the former uses constructions. This is not true. All necessary reasoning whatsoever proceeds by constructions; and the only difference between mathematical and philosophical necessary deductions is that the latter are so excessively simple that the construction attracts no attention and is overlooked. The construction exists in the simplest syllogism in Barbara. Why do the logicians like to state a syllogism by writing the major premiss on one line and the minor below it, with letters substituted for the subject and predicates? It is merely because the reasoner has to notice that relation between the parts of those premisses which such a diagram brings into prominence. If the reasoner makes use of syllogistic in drawing his conclusion, he has such a diagram or construction in his mind's eye, and observes the result of eliminating the middle term. If, however, he trusts to his unaided reason, he still uses some kind of a diagram which is familiar to him personally. The true difference between the necessary logic of philosophy and mathematics is merely one of degree. It is that, in mathematics, the reasoning is frightfully intricate, while the elementary conceptions are of the last degree of familiarity; in contrast to philosophy, where the reasonings are as simple as they can be, while the elementary conceptions are abstruse and hard to get clearly apprehended. But there is another much deeper line of demarcation between the two sciences. It is that mathematics studies nothing but pure hypotheses, and is the only science which never inquires what the actual facts are; while philosophy, although it uses no microscopes or other apparatus of special observation, is really an experimental science, resting on that experience which is common to us all; so that its principal reasonings are not mathematically necessary at all, but are only necessary in the sense that all the world knows beyond all doubt those truths of experience upon which philosophy is founded. This is why the mathematician holds the reasoning of the metaphysician in supreme contempt, while he himself, when he ventures into philosophy, is apt to reason fantastically and not solidly, because he does not recognize that he is upon ground where elaborate deduction is of no more avail than it is in chemistry or biology.

Peirce: CP 3.561 Cross-Ref:††

561. I have thus set forth what I believe to be the prevalent opinion of

philosophical mathematicians concerning the nature of their science. It will be found to be significant for the question of number. But were I to drop this branch of the subject without saying one word more, my criticism of the old definition, "mathematics is the science of quantity," would not be quite just. It must be admitted that quantity is useful in almost every branch of mathematics. Jevons wrote a book entitled *Pure logic, the science of quality*, which expounded, with a certain modification, the logical algebra of Boole. But it is a mistake to regard that algebra as one in which there is no system of quantity. As Boole rightly holds, there is a quadratic equation which is fundamental in it. The meaning of that equation may be expressed as follows: Every proposition has one or other of two *values*, being either *true* (which gives it one value) or *false* (which gives it the other). So stated, we see that the algebra of Boole is nothing but the algebra of that system of quantities which has but two values -- the simplest conceivable system of quantity. The widow of the great Boole has lately written a little book †P1 in which she points out that, in solving a mathematical problem, we usually introduce some part or element into the construction which, when it has served our purpose, is removed. Of that nature is a scale of quantity, together with the apparatus by which it is transported unchanged from one part of the diagram to another, for the purpose of comparing those two parts. Something of this general description seems to be indispensable in mathematics. Take, for example, the Theorem of Pappus concerning ten rays in a plane. The demonstration of it which is now usual, that of von Staudt, introduces a third dimension; and the utility of that arises from the fact that a ray, or unlimited straight line, being the intersection of two planes, these planes show us exactly where the ray runs, while, as long as we confine ourselves to the consideration of a single plane, we have no easy method of describing precisely what the course of the ray is. Now this is not precisely a system of quantity; but it is closely analogous to such a system, and that it serves precisely the same purpose will appear when we remember that that same theorem can easily (though not *so* easily) be demonstrated by means of the barycentric calculus. Although, then, it is not true that all mathematics is a science of quantity, yet it is true that all mathematics makes use of a scaffolding altogether *analogous* to a system of quantity; and quantity itself has more or less utility in every branch of mathematics which has as yet developed into any large theory.

Peirce: CP 3.562 Cross-Ref:††

562. I have only to add that the hypotheses of mathematics may be divided into those *general hypotheses* which are adhered to throughout a whole branch of mathematics, and the *particular hypotheses* which are peculiar to different special problems.†1

Peirce: CP 3.562A Cross-Ref:††
 §2. OF PURE NUMBER

562A. The system of pure number is the general hypothesis of arithmetic -- at any rate, of scientific arithmetic (or, the theory of numbers) -- for whether it is best to say anything about it in vulgar arithmetic (or, the art of computing with the Arabic figures)†P1 is a question of educational theory to be considered after studying Counting and Dating.

Peirce: CP 3.562B Cross-Ref:††

562B. Preparatory to showing what this system is, it will be well to describe a still more general hypothesis -- that of a *sequence*. A sequence is a multitude of objects connected with a relation, which we may call the "relation of sequence," or R, in a manner I proceed to define. It will be convenient to use this following locution: I shall say, A is R to B, meaning that A stands to B in the relation R, and with the same meaning I shall also say, B is R'd by A. Then the sequence is defined by these two precepts:

Peirce: CP 3.562B Cross-Ref:††

Precept I. If A is any object of the sequence whatever and B is any object of the sequence whatever, either A is not R to B or else B is not R to A.

Peirce: CP 3.562B Cross-Ref:††

Precept II. If A is any object of the sequence whatever, B is any such object, and C is any such object, then, so far as Precept I permits, either A is R to B, or B is R to C, or C is R to A.

Peirce: CP 3.562B Cross-Ref:††

Certain corollaries are easily deduced from these precepts. First, if, in applying Precept I, we choose for B the same object represented by A, that precept leads to the conclusion that no object of the sequence is R itself. This, then, shows the limitation which Precept I imposes upon Precept II. Namely, in the latter, A, B, and C cannot, all three, represent the same object. Secondly, if in Precept II we take C to be the same as B, that precept shows that of any two different objects of the sequence, A and B, either A is R to B or B is R to A. For by the first corollary, B cannot be R to C; that is, to itself. Thirdly, if A, B, and C represent objects of the sequence (the same or different), then if A is R'd by B, it follows that either B is R to C or C is R to A. For if A is R'd by B, then by Precept I, A is not R to B, and then, by Precept II, either B is R to C or C is R to A, or else A, B, and C are identical. But the last cannot be the case by the first corollary, if A is R'd by B. Fourthly, whatever objects of the sequence A, B, and C may be, if A is R'd by B and B is R'd by C, then A is R'd by C. For, by Precept I, if B is R'd by C, C is not R to B.†1 Hence, by the third corollary, if A is R'd by B and B is R'd by C, C is R to A, that is, A is R'd by C. Fifthly, whatever objects of the sequence A, B, and C may be, either A is not R'd by B or B is not R'd by C, or C is not R'd by A. For, by the fourth corollary, either A is not R'd by B, or B is not R'd by C, or A is R'd by C. But in the last case, by Precept I, C is not R'd by A.†2

Peirce: CP 3.562B Cross-Ref:††

This defines a sequence in general. The description of the relation R agrees with that of a relation which is familiar to us, namely, that of *following* in sequence. Of course, it equally agrees with the relation of *preceding*. Substituting "following" for R, Precept I becomes, that whatever members of the sequence A and B may be; they do not both follow one another. Precept II becomes that, whatever members of the sequence A, B, and C may be, provided they are not all three identical, either A follows B, or B follows C, or C follows A. The first corollary becomes, that no member of the sequence follows itself. The second becomes, that of any two different members of the sequence one follows the other. The third becomes, that whatever members A, B, and C, may be, if A is followed by B, either B follows C or C follows A. The fourth becomes, that whatever members A, B, and C may be, if A is followed by B and B by C, then A is followed by C. The fifth becomes, that whatever members A, B, and C may be, either A is not followed by B, or B is not followed by C, or C is not followed by A.

Peirce: CP 3.562B Cross-Ref:††

A sequence may nor may not have an absolute end; that is, a member which is followed by no member; and it may or may not have an absolute beginning; that is, a member which follows no member. These distinctions are not mathematically important in most cases.

Peirce: CP 3.562C Cross-Ref:††

562C. A highly important distinction, however, arises in a way which I proceed to describe. We know from the fourth corollary that, taking any member of the sequence M, and any member N, which does not follow M, then whatever member X, we may select, either X does not follow M or is not followed by N.†1 Now M may be such that it *is followed* by a member N, of which this same thing is true; namely, that there is no member of the sequence which at once follows M and is followed by N. The member N may, in this case, be said *hardly* to follow M, since it has this property which belongs generally to members which do not follow M. We usually say that N follows *next* after M.†P1 Now a sequence may either be such that no member has another that *hardly*, or *next*, follows it, or such that some members do and some do not, or that all have such members hardly following them.

Peirce: CP 3.562D Cross-Ref:††

562D. If a sequence be such that some member has a member that hardly follows it, the sequence is such that some member has a member that hardly precedes it. For, if N follows hard after M, then M hardly precedes N. But, nevertheless, a sequence may be such that *every* member has a member hardly following it without being such that every member has a member hardly preceding it. An accurate logic will show that this is quite admissible. Suppose, for example, the sequence is composed of all the whole numbers, together with whole numbers each added to a vulgar fraction having 1 for its numerator. And suppose these to follow one another in the reverse order of their values, thus:

4, 3 1/2, 3 1/3, 3 1/4, 3 1/5, . . . 3, 2 1/2, 2 1/3, 2 1/4, 2 1/5, . . . 2, 1 1/2, 1 1/3, 1 1/4, . . . 1, 1/2, 1/3, 1/4, 1/5 . . .

Then every member, as $p + 1/q$ has $p + 1/(q+1)$ following hard after it. But the whole numbers have no members which *they* follow hard after.

Peirce: CP 3.562E Cross-Ref:††

562E. If a sequence be such that there is some member of it, not its absolute end, which has no member hardly following it, then a part of that sequence forms a sequence with the same, R (or, relation of sequence), which sequence has no absolute beginning. Namely, if M be a member of the total sequence such that some members are R'd by M, but such that taking any member P, either P is not R to M, or else there is a member X, such that P is R to X and X is R to M, in that case the partial sequence composed of all the members of the total sequence that are R'd by M, contains no member that is not R to another member of the same partial sequence. We might express the same thing by saying that if a member of a sequence not its absolute beginning follows hard upon no member, then all the members it follows form a sequence with no absolute end. But the converse of this proposition is not true. That is to say, it is not true that a sequence, every member of which *has* another following

hard after it and itself following hard upon another, *cannot* be cut up into sequences having neither beginning nor end. For instance, take all numerals of the forms $p + 1/q$ where p and q are whole numbers; and let them be arranged in the order of their magnitudes, thus:

$1/5, 1/4, 1/3, 1/2, 2/3, 3/4, 4/5 \dots 1\ 1/5, 1\ 1/4, 1\ 1/3, 1\ 1/2, 1\ 2/3, 1\ 3/4, \dots 2\ 1/5, 2\ 1/4, 2\ 1/3, 2\ 2/3, 2\ 3/4, 2\ 4/5.$

Peirce: CP 3.562E Cross-Ref:††

Here it is plain that every member has an assignable one next after and another next before it; and yet the sequence can be broken up into partial sequences, each without beginning or end.

Peirce: CP 3.562F Cross-Ref:††

562F. Let us call a sequence of which every member follows hard after a second, and is itself followed hard after by a third, a *sparse* sequence; and let us call a sparse sequence which cannot be broken into parts which want either beginning or end (except so far as they may retain the infinity of the total sequence), a *simple* sparse sequence.

Peirce: CP 3.562F Cross-Ref:††

Then the system of pure number may be defined as a simple sparse sequence, having an absolute beginning, called *zero*, but no absolute end. Of course, *zero* might be dropped from the system, or zero and unity might both be dropped, and so on *ad infinitum*. But slightly simpler definitions of addition and multiplication are obtained by retaining the *zero*.

Peirce: CP 3.562G Cross-Ref:††

562G. Arithmetic begins with the following fundamental theorem:

Whatever character belongs to zero, and belongs to the number following hard after any number to which it belongs, belongs to all numbers.

Peirce: CP 3.562G Cross-Ref:††

Demonstration. Let X be a character which zero possesses. And suppose that, whatever number N may be, either N does not possess X , or else N^1 , the number that follows hard after N , possesses X . Then I say that every number possesses X .

Peirce: CP 3.562G Cross-Ref:††

For, since zero possesses X , and zero is followed by every other number (by the second corollary above, since zero, as absolute beginning, follows no number) there are numbers (or, at least, a number) which are followed by whatever numbers there may be which do not possess X . Let us consider, then, the sequence composed of those numbers which are smaller than whatever numbers there may be which do not possess X ; these numbers being taken in their order, that is, with the same R as the total sequence. (Even a single number may be regarded as a sequence of which the absolute beginnings and end are identical.) This sequence may either be the total sequence of numbers, in which case it will have no end, or it may be a partial sequence. In the latter case, every number of this sequence will be followed by every number which does not belong to it, since the latter is not followed by some number

not possessing X which does follow every number of this partial sequence. Hence, since the sequence of pure number is a simple sparse sequence, it follows that the partial sequence (if it be partial) must have an absolute end. Let N be this absolute end. Then N is followed by whatever number there may be which does not possess X. But by the first corollary, no number is followed by itself. Hence N possesses X. Hence, by hypothesis, N^1 , the number that follows hard after N, possesses X. Hence, by the definition of "following hard after," N^1 is followed by every number except itself, and follows N; that is, by whatever number there may be which does not possess X. But this is contrary to the hypothesis that N was the absolute end of the partial series; that is, was not followed by any number followed by whatever number does not possess X. Hence, the supposition that the series is a partial one is absurd. Hence, every number is followed by whatever number there may be that does not possess X. But, by the first corollary, no number is followed by itself. Hence, every number possesses X. Q.E.D.

Peirce: CP 3.562H Cross-Ref:††

562H. By means of this theorem all the elementary propositions of arithmetic as to the associativeness and distributiveness of addition and multiplication, as well as the fundamental principles of subtraction and division of whole numbers are easily deduced from the following definitions of addition and multiplication. Of course subtraction and division are to be defined as inverse operations:

Definition of addition. The sum of two numbers, M and N, is a number depending upon M and N according to the following precepts.

Peirce: CP 3.562H Cross-Ref:††

Precept III. The Sum of 0 and 0 is 0.

Peirce: CP 3.562H Cross-Ref:††

Precept IV. The sum of any number, M, and of the number N^1 , next following any number, N, is the number next following the sum of M and N.

Peirce: CP 3.562I Cross-Ref:††

562I. **Definition of multiplication.** The **product** of two numbers, M and N, is a number depending upon M and N according to the following precepts:

Precept V. The product of 0 and 0 is 0.

Peirce: CP 3.562I Cross-Ref:††

Precept VI. The product of any number, M, and of the number, N^1 , next following any number, N, is the number which is the sum of M and the product of M and N. This gives a sufficient idea of pure number. But I will remark that, when this method is applied to division, it leads so easily into the theory of numbers that it is difficult to restrain the pupil from that line of thought. But I need not say that I should not teach the doctrine of pure number to young children. At present, I am merely collecting the conceptions of the subject. I shall pass next to the application of number to the counting of collections.†1

INFINITESIMALS†1

563. Will you kindly accord me space for a few remarks about Infinity and Continuity which I seem called upon to make by several notes to Professor Royce's Supplementary Essay in his strong work *The World and the Individual?* I must confess that I am hardly prepared to discuss the subject as I ought to be, since I have never had an opportunity sufficiently to examine the two small books by Dedekind,†2 nor two memoirs by Cantor,†3 that have appeared since those contained in the second volume of the *Acta Mathematica*. I cannot even refer to Schröder's *Logic*.

Peirce: CP 3.564 Cross-Ref:††

564. (1) There has been some question whether Dedekind's definition of an infinite collection or that which results from negating my definition of a finite collection is the best. It seems to me that two definitions of the same conception, not subject to any conditions, as a figure in space, for example, is subject to geometrical conditions, must be substantially the same. I pointed out (*Am. Journ. Math.* IV. 86,†4 but whether I first made the suggestion or not I do not know) that a finite collection differs from an infinite collection in nothing else than that the syllogism of transposed quality [quantity] is applicable to it (and by the consequences of this logical property). For that reason, the character of being finite seemed to me a positive extra determination which an infinite collection does not possess. Dr. Dedekind defines an infinite collection as one of which every *echter Theil* is similar to the whole collection.†5 It obviously would not do to say a *part*, simply, for every collection, even if it be infinite, is composed of individuals; and these individuals are parts of it, differing from the whole in being indivisible. Now I do not believe that it is possible to define an *echter Theil* without substantially coming to my definition. But, however that may be, Dedekind's definition is not of the kind of which I was in search. I sought to define a finite collection in logical terms. But a "part," in its mathematical, or collective, sense, is not a logical term, and itself requires definition.

Peirce: CP 3.565 Cross-Ref:††

565. (2) Professor Royce remarks that my opinion that differentials may quite logically be considered as true infinitesimals, if we like, is shared by no mathematician "*outside of Italy*."†1 As a logician, I am more comforted by corroboration in the clear mental atmosphere of Italy than I could be by any seconding from a tobacco-clouded and bemused land (if any such there be) where no philosophical eccentricity misses its champion, but where sane logic has not found favor. Meantime, I beg leave briefly to submit certain reasons for my opinion.

Peirce: CP 3.566 Cross-Ref:††

566. In the first place, I proved in January, 1897, in an article in the *Monist* (VII 215),†2 that the multitude of possible collections of members of any given collection whatever is greater than the multitude of the latter collection itself. That demonstration is so simple, that, with your permission, I will here repeat it. If there be any collection as great as the multitude of possible collections of its members, let the members of one such collection be called the *A*'s. Then, by Cantor's definition of the

relation of multitude, there must be some possible relation, r , such that every possible collection of A 's is r to some A , while no two possible collections of A 's are r to the same A . But now I will define a certain possible collection of A 's, which I will call the collection of B 's, as follows: Whatever A there may be that is not included in any collection of A 's that is r to it, shall be included in the collection of B 's, and whatever A there may be that is included in a collection of A 's that is r to it, shall not be included in the collection of B 's. If there is any A to which no collection of A 's stands in the relation r , I do not care whether it is included among the B 's or not. Now I say the collection of B 's is not in the relation r to any A . For every A is either an A to which no collection of A 's stands in the relation r , or it is included in a collection of A 's that is r to it, or it is excluded from every collection of A 's that is r to it. Now the collection of B 's, being a collection of A 's, is not r to any A to which no collection of A 's is r ; and it is not r to any A that is included in a collection of A 's that is r to it, since only one collection of A 's is r to the same A , so that were that the case the A in question would be a B , contrary to the definition which makes the collection of B 's exclude every A included in a collection that is r to it; and finally, the collection of B 's is not r to any A not included in any collection of A 's that is r to it, since by definition every such A is a B , so that, if the collection of B 's were r to that A , that A would be included in a collection of A 's that was r to it. It is thus absurd to say that the collection of B 's is r to any A ; and thus there is always a possible collection of A 's not r to any A ; in other words, the multitude of possible collections of A 's is greater than the multitude of the A 's themselves. That is, every multitude is less than a multitude; or, there is no maximum multitude.

Peirce: CP 3.567 Cross-Ref:††

567. In the second place I postulate that it is an admissible hypothesis that there may be a something, which we will call a *line*, having the following properties: first, points may be determined in a certain relation to it, which relation we will designate as that of "lying on" that line; second, four different points being so determined, each of them is separated from one of the others by the remaining two; third, any three points, A , B , C , being taken on the line, any multitude whatever of points can be determined upon it so that every one of them is separated from A by B and C .

Peirce: CP 3.568 Cross-Ref:††

568. In the third place, the possible points so determinable on that line cannot be distinguished from one another by being put into one-to-one correspondence with any system of "assignable quantities." For such assignable quantities form a collection whose multitude is exceeded by that of another collection, namely, the collection of all possible collections of those "assignable quantities." But points are, by our postulate, determinable on the line in excess of that or of any other multitude. Now, those who say that two different points on a line must be at a finite distance from one another, virtually assert that the points are distinguishable by corresponding (in a one-to-one correspondence) to different individuals of a system of "assignable quantities." This system is a collection of individual quantities of very moderate multitude, being no more than the multitude of all possible collections of integral numbers. For by those "assignable quantities" are meant those toward which the values of fractions can indefinitely approximate. According to my postulate, which involves no contradiction, a line may be so conceived that its points are not so distinguishable and consequently can be at infinitesimal distances.

Peirce: CP 3.568 Cross-Ref:††

Since, according to this conception, any multitude of points whatever are determinable on the line (not, of course, by us, but of their own nature), and since there is no maximum multitude, it follows that the points cannot be regarded as constituent parts of the line, existing on it by virtue of the line's existence. For if they were so, they would form a collection; and there would be a multitude greater than that of the points determinable on a line. We must, therefore, conceive that there are only so many points on the line as have been marked, or otherwise determined, upon it. Those do form a collection; but ever a greater collection remains determinable upon the line. *All* the determinable points cannot form a collection, since, by the postulate, if they did, the multitude of that collection would not be less than another multitude. The explanation of their not forming a collection is that all the determinable points are not individuals, distinct, each from all the rest. For individuals can only be distinct from one another in three ways: First, by acts of reaction, immediate or mediate, upon one another; second, by having *per se* different qualities; and third, by being in one-to-one correspondence to individuals that are distinct from one another in one of the first two ways. Now the points on a line not yet actually determined are mere potentialities, and, as such, cannot react upon one another actually; and, *per se*, they are all exactly alike; and they cannot be in one-to-one correspondence to any collection, since the multitude of that collection would require to be a maximum multitude. Consequently, all the possible points are not distinct from one another; although any possible multitude of points, once determined, become so distinct by the act of determination. It may be asked, "If the totality of the points determinable on a line does not constitute a collection, what shall we call it?" The answer is plain: the possibility of determining more than any given multitude of points, or, in other words, the fact that there is room for any multitude at every part of the line, makes it *continuous*. Every point actually marked upon it breaks its continuity, in one sense.

Peirce: CP 3.569 Cross-Ref: ††

569. Not only is this view admissible without any violation of logic, but I find -- though I cannot ask the space to explain this here -- that it forms a basis for the differential calculus preferable, perhaps, at any rate, quite as clear, as the doctrine of limits. But this is not all. The subject of topical geometry has remained in a backward state because, as I apprehend, nobody has found a way of reasoning about it with demonstrative rigor. But the above conception of a line leads to a definition of continuity very similar to that of Kant. Although Kant confuses continuity with infinite divisibility, yet it is noticeable that he always defines a continuum as that of which every part (not every *echter Theil*) has itself parts. This is a very different thing from infinite divisibility, since it implies that the continuum is not composed of points, as, for example, the system of rational fractions, though infinitely divisible, is composed of the individual fractions. If we define a continuum as that every part of which can be divided into any multitude of parts whatsoever -- or if we replace this by an equivalent definition in purely logical terms -- we find it lends itself at once to mathematical demonstrations, and enables us to work with ease in topical geometry.

Peirce: CP 3.570 Cross-Ref: ††

570. (3) Professor Royce wants to know †1 how I could, in a passage which he cites, attribute to Cantor the above opinion about infinitesimals. My intention in that passage was simply to acknowledge myself, in a general way, to be no more than a follower of Cantor in regard to infinity, not to make him responsible for any particular opinion of my own. However, Cantor proposed, if I remember rightly, so far to modify the kinetical theory of gases as to make the multitude of ordinary atoms equal

to that of the integral numbers, and that of the atoms of ether equal to the multitude of possible collections of such numbers.†1 Now, since it is essential to that theory that encounters shall take place, and that promiscuously, it would seem to follow that each atom has, in the random distribution, certain next neighbors, so that if there are an infinite multitude in a finite space, the infinitesimals must be actual real distances, and not the mere mathematical conceptions, like $\sqrt{-1}$, which is all that I contend for.

Peirce: CP 3.571 Cross-Ref:††

XIX

NOMENCLATURE AND DIVISIONS OF DYADIC RELATIONS†1

§1. NOMENCLATURE^E

571. A *dyadic relation* is a character whose being consists in the logical possibility of a definite fact concerning an ordered pair, or *dyad*, of subjects; the first of these being termed the *relate*, the second the *correlate*; and the relation is said to *subsist* between the relate and correlate when the fact in whose possibility its being consists actually has place between these objects. The relation, by itself, is, therefore, an *ens rationis* and mere logical possibility; but its subsistence is of the nature of a fact. When the quality of the fact concerning two objects is considered, without reference to any distinction between these subjects other than that which this fact establishes, and therefore regardless of which of them is relate, which correlate, its possibility is termed by the author a *relationship*. (It is a useful distinction, but cannot be translated into every language.)

Peirce: CP 3.572 Cross-Ref:††

572. The broadest division of dyadic relations is into those which can only subsist between two subjects of different categories of being (as between an existing individual and a quality) and those which can subsist between two subjects of the same category. A relation of the former kind may advantageously be termed a *reference*; a relation of the latter kind, a *dyadic relation proper*.

Peirce: CP 3.573 Cross-Ref:††

573. A dyadic relation proper is either such as can only have place between two subjects of different universes of discourse (as the membership of a natural person in a corporation), or is such as can subsist between two objects of the same universe. A relation of the former description may be termed a *referential relation*; a relation of the latter description, a *rerelection*.†P1

Peirce: CP 3.574 Cross-Ref:††

574. A rerelection may either be such as can only subsist between characters or between laws (such as the relation of "essentially depending upon"), or it may be such as can subsist between two existent individual objects. In the former case, it may be termed a *modal* relation (not a good term), in the latter case an *existential relation*.

The author's writings on the logic of relations †P2 were substantially restricted to existential relations; and the same restriction will be continued in the body of what here follows. A note at the end of this section will treat of modal relations.

Peirce: CP 3.575 Cross-Ref:††

575. The number of different species of existential relations for which technical designations are required is so great that it will be best to adopt names for them which shall, by their form, furnish technical definitions of them, in imitation of the nomenclature of chemistry. The following rules will here be used. Any name (for which in this statement of the rules of word-formation we may put *x*), having been adopted for all relations of a given description, the preposition *extra* (or *ex*, or *e*) will be prefixed to that name ("extra *x*") in order to form a name descriptive of any relation to which the primitive name does not apply; the preposition *contra* will be prefixed (forming "contra-*x*,") to make a name applicable only to such relations as consist precisely in the non-subsistence of corresponding relations to which the primitive name does apply; the preposition *juxta* will be prefixed so as to bear the sense of *contra-extra*, or (what is the same) *extra-contra*; the preposition *red* (or *re*) will be prefixed to form a name applicable to a relation if, and only if, the correlate of it stands to its relate in a relation to which the primitive name applies, so that, in other words, a "red-*x*" is a relation the converse of an *x*; the preposition *com* (or *con*, or *co*,) will be prefixed to form the general name of any relation which consists in its relate and correlate alike standing in one relation of the primitive kind to one and the same individual correlate; the preposition *ultra* will be prefixed to form a name applicable only to a relation which subsists between any given relate and correlate only in case the former stands in a relation of the primitive kind to some individual to which the latter does not stand in that same relation; the preposition *trans* will be used so as to be equivalent to *contra-red-ultra*, or (what is the same) *recontrultra*, so that A will be in a relation "*trans-x*" to B, if, and only if, there is an *x*-relation in which A stands to whatever individual there may be to which B stands in that very same relation; and the preposition *super* to form the name of a relation which is, at once, *ultra* and *trans*, in respect to the very same relation of the primitive kind. Any of these prepositions may be prefixed, in the same sense, occasionally (and where no misunderstanding could result) not only to names of classes of relations and their cognates, but also to relative terms. But it is chiefly the prepositions *com*, *ultra*, *trans* and *super*, that will be so used. For example, taking the relative term "loves," there will be little occasion to use the first four of the following expressions, especially, the first and third, which become almost meaningless, while the last four will often be convenient.

1. A *extra-loves* B; that is, stands in some other relation, whether loving besides or not;
2. A *contra-loves* B; that is, does not love B;
3. A *juxta-loves* B; that is, stands in some other relation than that of not loving, whether loving or not;
4. A *reloves* B; that is, is loved by;
5. A *coloves* B; that is, loves something loved by;
6. A *ultraloves* B; that is, loves something not loved by;

7. A *transloves* B; that is, loves whatever may be loved by;

8. A *superloves* B; that is, loves whatever may be loved by and something else. †1

Peirce: CP 3.576 Cross-Ref: ††

576. By a *seed* (*granum*) of an existential relation is to be understood an existing individual which not only stands in that relation to some correlate, but to which also some relate stands in that relation. By a *spike* of a relation is to be understood any collection of seeds of it of which it is both true that every one of them stands in that relation to some one of them; and it is also true that to every one seed of the spike some seed of the spike stands in that same relation. †2 Thus, two spikes of the same relation may have common seeds, or one may even be a part of another. A *simple spike* is a spike not containing any other spike as a part of it.

Peirce: CP 3.577 Cross-Ref: ††

577. There are four general kinds of consideration on which divisions of existential dyadic relations may be based. The connections between the four systems of division so resulting have not been sufficiently studied to be treated here.

Peirce: CP 3.578 Cross-Ref: ††

§2. FIRST SYSTEM OF DIVISIONS †3

578. An existential dyadic relation may be termed a *lation*, to express its possibly subsisting between two existing individuals, and in opposition to an *extralation*, which is a fictive lation, looked upon as a lation, but which cannot (or, at least, does not) subsist between two existing individuals. In short, it is *impossibility*.

Peirce: CP 3.579 Cross-Ref: ††

579. A lation is either a *contralation* which does not necessarily subsist between the members of every dyad, or else it is *the juxtalation*, or *coexistence*, which subsists between every such dyad.

Peirce: CP 3.580 Cross-Ref: ††

580. Schröder, who always conscientiously follows the writer's terminology, except where he sees good and sufficient reasons for departing from it, †P1 thinks that, in place of "coexistence," the term "compossibility" should be used. This is a nice question. It is to be kept in view that an existential dyadic relation is to be regarded as a brute fact existing between two existing things, whatever may be thought about it. In saying this, we enunciate no metaphysical nor epistemological proposition, but simply say how the matter must be understood. Such a relation has no being at all unless the two things exist. It has not even that mode of being which consists in conceivability. That is, you cannot conceive that A strikes B, for example, while one of them is non-existent. But when we apply a general word to a class of such dyads, we are thinking of more than ever can exist. If the logician says "A strikes B," meaning (as is usual in logic, except when time is expressly under consideration) did strike, is striking, or will strike B, he asserts only that that event either has happened or will, at some time in the endless future, have happened. But it

is impossible, "in the nature of things," as we say, that is, is logically impossible, that all that ever will have happened should at any time actually have happened. Thus, the assertion transcends actual existence. This is because "strike" is a *general* word, and as such, relates to what is, or is not, *possible*; and because of the axiom that it is impossible that all that is possible should actually exist. Thus, in absolute strictness of language, "striking" is not an existential relation, but only signifies a *class* of existential relations. But this having, once for all, been well-understood, it becomes permissible to speak of a general relation as an existential relation, meaning a general class of existential relations. We may, thus, even distinguish between what is *essentially* true of an existential relation and what is *accidentally*, but universally, true of it, although what in absolute strictness is an existential relation has no other mode of being than existence, and consequently has no essence. It is not possible, if it does not exist. Still, we have to bear in mind that though we allow ourselves to speak of general relations as existential, yet what we mean are classes of existential relations in the strictest sense; and therefore they have no possibility distinct from existence. Accordingly, *coexistence* is the right word, and compossibility is only a modal relation. It is quite another thing with the extralation, since this is *not* an existential relation, so that "impossibility" seems the more appropriate word; but if I were to countenance any modification of my nomenclature for this pair of relations, it would be that of the substitution of "non-coexistence" for "impossibility"; and yet, how could things not coexist? Non-coexistence is nonsense, while impossibility is an intelligible modal relation borrowed to fill a blank in the scheme of divisions of lations.

Peirce: CP 3.581 Cross-Ref:††

581. A *contralation* (really, the only kind of lation there is) is to be termed a *perlation*, if, and only if, there is some individual that stands in this relation to every individual of the universe. A contralation is to be termed an *extraperlation*, if, and only if, there is nothing which stands in this relation to everything.

Peirce: CP 3.581 Cross-Ref:††

A contralation is to be termed a *contraperlation*, if, and only if, there is something which does not stand in this relation to anything. A contralation is to be termed a *juxtalation*,†1 if, and only if, everything stands to something or other in this relation.

Peirce: CP 3.581 Cross-Ref:††

A contralation is to be termed a *reperlation*, if, and only if, there is something to which everything stands in this relation. An *extrareperlation* is any contralation in which there is nothing to which something does not stand.†2

Peirce: CP 3.581 Cross-Ref:††

A *contrareperlation* is a contralation such that there is something to which nothing stands in it. If everything has something or other in a given relation to it, that relation (and none other than such) is a *juxtareperlation*.

Peirce: CP 3.581 Cross-Ref:††

Every perlation is necessarily a juxtareperlation; although not every juxtareperlation is a perlation; and similarly, every reperlation is a juxtaperlation.†1 So, every contraperlation is an extrareperlation; and every contrareperlation is an extraperlation.†2 The converse of none of these propositions is true. Consequently, the eight classes of contralations named divide all contralations into nine classes, as shown in this table:

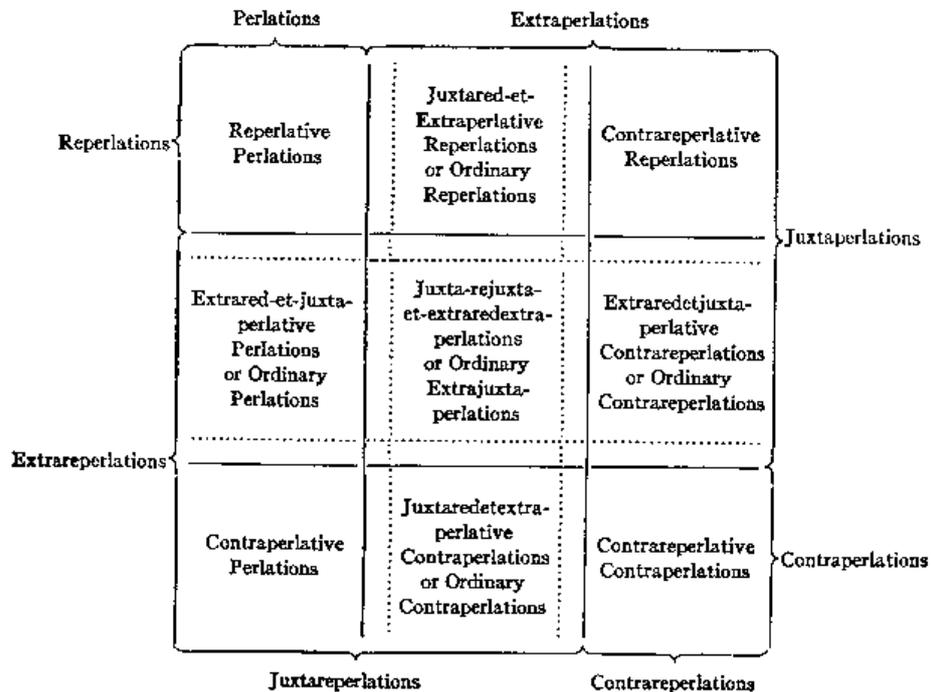


Figure 1 § (page 373)

[Click here to view]

Figure 1†1

Peirce: CP 3.581 Cross-Ref:††

It is a little puzzling to find that a relation may be at once in a class and in the contra-class. Thus, taking the citizens of a town as a universe of discourse, if there is one of them who is helpful to himself and to all the others, helpfulness becomes for that universe, a *perlation*, and unhelpfulness a *contraperlation*. But if there is also in that town a citizen who is helpful to nobody, not even to himself, then helpfulness and unhelpfulness become alike contraperlative perlations. If, however, the first supposition remaining, there is no entirely unhelpful citizen, helpfulness will be a juxtaperlative perlation; and if there is a citizen who is helped by everybody, helpfulness and helpedness will be reperlative perlations.

Peirce: CP 3.582 Cross-Ref:††

582. The preposition *pene* may be prefixed to any one of these terms to show that in speaking of relations to everything or to something we mean everything *else* or something else, whether this limits or extends the class of relations. The dotted lines of the diagram show the effect of this prefix in slightly enlarging the border classes at the expense of the interior classes. If the character of a perlation is not due merely to the non-existence of anything that is not a correlate of the individual or individuals that make it a perlation, but is essential to the relation itself, regardless of any other general condition, then whatever is in this relation to anything *must be* in this relation to everything. A perlation of which this is true may be termed an *essential perlation*.

In a similar sense we may speak of an *essential reperlation*. Thus "loving something coexisting with" is an essential perlation, "coexisting with something that loves" is an essential reperlation. These are said to be formed from the *primitive* relation of loving. An essentially reperlative perlation must be the juxtalation unless it be the extralation, and so, no lation at all. Thus, either nothing is in the relation of "coexisting with a creator of something coexisting with" to anything, or else everything is in that relation to everything. Thus, an essentially reperlative perlation is not *essentially* either coexistence or non-coexistence, but is either an *accidental juxtalation* or an *accidental extralation*. In the algebra of dyadic relations such an expression is said to be *enveloped*. An essential perlation is essentially a contraperlation, and an essential reperlation is essentially a contrareperlation.[?] But a *contressentiperlation*, which consists in the absence of an essential perlation, is itself an essential perlation. Thus, "loving nobody that coexists with," is equivalent to "non-loving somebody coexisting with," although its relation to its primitive is different.

Peirce: CP 3.583 Cross-Ref:††
 §3. SECOND SYSTEM OF DIVISIONS †1

583. An existential relation may be termed a *suilation*, if, and only if, every individual of the universe stands in that relation to itself. An existential relation may be termed an *ambilation*, if, and only if, every individual of the universe stands in that relation to every other.

Peirce: CP 3.584 Cross-Ref:††

584. Every suilation is a jxtasuilation and every ambilation a jxtambilation. Consequently, there will result nine classes of relations as shown in this table:

		Suilations	Extrasuilations			
Ambilations	}	Coexistence	Ordinary Ambilations	Negation or, more accurately, Otherness		Jxtambilations
		Ordinary Suilations	Ordinary Extrambilative Extrasuilations	Extrambilative Contrasuilations		
Extrambilations	}	Identity	Extrasuilative Contrambilation	Impossibility		Contrambilations
		Jxtasuilations		Contrasuilations		

Figure 2.*

[Click

here to view]

Figure 2.†1

Peirce: CP 3.585 Cross-Ref:††

585. The following terms were proposed by the author in 1870,†1 and since they have been generally accepted by writers on the subject, he is more bound to adhere to them than anybody else, although he does not now think they were very judiciously chosen. Namely, a jxtasuilation is termed a *self-relation*, a contrasuilation an *alio-relation*; a contrambilation is called a *concurrency*, a juxtambilation an *opponency*.

Peirce: CP 3.586 Cross-Ref:††

586. There is but one ambilative suilation. It is the juxtation, or coëxistence. There is but one contrambilative suilation: it is the relation of individual *identity*, called *numerical identity* by the logicians. But the adjective seems needless. There is but one ambilative [contra] suilation: it is the relation of individual *otherness*, or *negation*. There is properly no contrambilative contrasuilation: it would be the absurd relation of impossibility. These four relations are to be termed the *Four Cardinal Dyadic Relations of Second Intention*. It will be enough to call them the *cardinilations*, or *cardinal relations*.

Peirce: CP 3.587 Cross-Ref:††

587. Any peneperlation or penereperlation is a juxtambilation; any perlation or reperlation is, in addition, a jxtasuilation. Any penecontraperlation or penecontrareperlation is an extrambilation: any contraperlation or contrareperlation is, in addition, an extrasuilation. Every ambilation is a penereperlative penereperlation †2: every contrambilation is a penecontrareperlative penecontraperlation. Every suilation is a juxtareperlative juxtaperlation: every contrasuilation is an extrareperlative extraperlation.†3

Peirce: CP 3.588 Cross-Ref:††
§4. THIRD SYSTEM OF DIVISIONS †4

588. This system of divisions which depends upon the identity or otherness of relates and correlates not necessarily (as in the Second System) relates and correlates of one another, is multiform and irregular, a condition, it can hardly be doubted, incident to its not having been sufficiently studied.

Peirce: CP 3.589 Cross-Ref:††

589. A relation having seeds may be termed a *granilation*. The term at present in use is a *repeating relation*, an *extragranilation* being called a non-repeating relation. Every jxtasuilation is a granilation, and every jxtagranilation is a suilation.

Peirce: CP 3.589 Cross-Ref:††

Extragranilations are of subsidiary importance (yet not of very little importance), since an essential extragranilation closely approximates to the nature of

a reference.

Peirce: CP 3.590 Cross-Ref:††

590. A granilation may contain a spike,†1 or it may not. In the former case, it may be termed a *spicalation*, in the latter an *extraspicalation*.

Extressentispicalations, or granilations not necessarily containing spikes, comprise the most important class of existential relations in logic. This is the class of *transitive relations* (the term was introduced by De Morgan).†2 A relation is *transitive* if, and only if, any individual object in that relation to a second which is in the same relation to a third is itself in that relation to this third. Thus, the relation of being as great as (in all such phrases "as" is, in logic, to be understood in the sense of "at least as") is essentially transitive. For if A is as great as B, and B is as great as C, then A is necessarily as great as C. The relation of "greater than" is also essentially transitive.

Peirce: CP 3.591 Cross-Ref:††

591. Imagine all the dyads (or ordered pairs) of individuals in the universe to be arrayed in a *matrix* (Cayley's term, though the application of the conception to the logic of relations was first made by the author) as follows:

A:A	A:B	A:C	A:D	A:E	A:F	etc.
B:A	B:B	B:C	B:D	B:E	B:F	etc.
C:A	C:B	C:C	C:D	C:E	C:F	etc.
D:A	D:B	D:C	D:D	D:E	D:F	etc.
E:A	E:B	E:C	E:D	E:E	E:F	etc.
F:A	F:B	F:C	F:D	F:E	F:F	etc.
etc.						

Figure 3

Peirce: CP 3.591 Cross-Ref:††

Let the horizontal rows be termed ranks and the vertical rows *columns* (Howard Staunton's terms?) Let the diagonal line containing the identical pairs A:A, B :B, etc. be termed the *principal, dexter, or leading diagonal* (all terms in common use.) Then a relation is transitive if, and only if, whatever pair of dyads be taken, of each of which the first individual is in that relation to the second, then in case the rank of either one of these dyads in the matrix crosses the column of the other on the dexter diagonal, the column of the former dyad crosses the rank of the latter in a dyad of which the first member is in the transitive relation to the second. Thus, suppose, to begin with: that the only individual in the relation whose transitivity is to be examined is B; and that it is in that relation to C alone. Then, it is an extragranilation; but it is, according to the definition, a transitive relation, since B does stand in that relation to whatever there may be that C is so related to so long as C is not so related to anything. But now suppose that, in addition, C is in the relation in question to F. Then, since the column of (B:C) meets the rank of (C:F) in (C:C) on the dexter

diagonal, while the rank of (B:C) meets the column (C:F) in (B:F) the relation will not be transitive unless B stands in that relation to F. If, in addition, any other dyad in the same rank as (C:F) belongs to the relation, there will have to be a corresponding dyad of the relation in the rank of (B:C); and if there is any additional dyad in the column of (B:C), there will have to be a corresponding dyad of the relation in the column of (C:F).†1 Therefore, in examining the matrix of a relation in order to ascertain whether the latter is transitive or not, it will be sufficient to consider each pair in the dexter diagonal and examine whether for every dyad in its rank, there is a co-columnar dyad in the rank of every dyad in its (the dexter pair's) column, and conversely. Suppose, for example, we desire to represent a transitive relation in the seven-by-seven matrix of the first figure below with the condition that the squares with crosses shall be occupied by dyads of the relation.

Peirce: CP 3.591 Cross-Ref:††

Since the first, fourth, and fifth squares of the dexter diagonals (or say, dexter squares) have blank ranks, they need no consideration; and the same is true of the second and sixth dexter squares, which have blank columns. It is thus only necessary to consider the third and seventh dexter squares.

3		X	7	3		3
X			7	X		X
3		X	7	3		3
			X			

Figure 4

[Click here to view]

Figure 4

The third will require dyads for the squares marked 3, *after which*, the seventh will require dyads in the squares marked 7; and, as it happens, these do not make any new dyads requisite, as they might have done. It will generally be necessary to reexamine all the dexter squares until it is found that no new dyads are required.†1

Peirce: CP 3.591 Cross-Ref:††

In this example, there is no need of filling any of the dexter squares; but any of them can be filled without affecting the transitivity of the relation; and in general no transitive extraspicalation is necessarily jxtasuilative; but it may be so in any way. That is, the modification of it by making it include any identical pairs will never destroy its transitivity.

Peirce: CP 3.592 Cross-Ref:††

592. On the other hand, a transitive spicalation must subsist between every possible dyad whose members are individuals of the spike. Thus, in order to render the relation whose matrix is shown in Figure 5 transitive, this relation having the two simple spikes A, B, and C, D, E, F, this matrix must be filled up as in Figure 6.

--	A:B	--	--	--	--
B:A	--	--	C:D	--	--
--	--	--	--	D:E	--
--	--	--	--	--	E:F
--	--	F:C	--	--	--

Figure 5

A:A	A:B	--	--	--	--
B:A	B:B	--	--	--	--
--	--	C:C	C:D	C:E	C:F
--	--	D:C	D:D	D:E	D:F
--	--	E:C	E:D	E:E	E:F
--	--	F:C	F:D	F:E	F:F

Figure 6

Peirce: CP 3.592 Cross-Ref:††

Such a relation, whose matrix is entirely composed of full squares having their

dexter diagonals on the dexter diagonal of the whole matrix, may be termed a *simililation*. (The term formerly proposed by the author, "copulative relation," †1 has not found much favor, and is too unsuitable.)

Peirce: CP 3.593 Cross-Ref: ††

593. A transitive relation which is such that whatever individual stands in this relation to another stands in the same relation to some individual that stands in the same relation to that other is called an *idempotent* relation †2 (B. Peirce's Term, †3 which is generally received). An idempotent contrasuilation must subsist between individuals at least denumerable in multitude, and may be termed an *endlessly divisible idempotence*.

Peirce: CP 3.594 Cross-Ref: ††

594. If a transitive relation is so connected with a relation, called a *primitive* of it, which may be a reference, that any individual stands in this transitive relation to an individual, if, and only if, the former individual stands in the primitive relation to whatever there may be to which the latter individual stands in that primitive relation, then, and only then, it is to be termed a *translation* of that primitive relation. †4 Thus, taking "loving" as the primitive relation, the relation of "loving whatever may be loved by" is the translation of loving. Every translation is transitive suilation. Moreover, given any transitive relation, every relate of it stands to its correlate in a translation of that same relation. This is easily seen if we consider that, for example to say that "every lover of a servant of a mistress is a tormentor of that mistress" is the same as to say that "every lover is a tormentor of every mistress whom his beloved may be servant of"; and the same will evidently be true in any analogous case. If, therefore (substituting "lover of" for "servant of" and for "tormentor of"), every lover of a lover of anybody is a lover of the last (which is as much as to say that loving is a transitive relation), it follows that every lover of anybody loves whatever may be loved by his beloved; that is, he stands in the translation of loving to that which he loves. But this does not imply that every transitive relation constitutes a translation. This is evidently untrue, since every translation is a suilation, but not every transitive relation has that character. Since, however, every translation is a transitive relation, it evidently follows that every transitive relation is precisely a compound of some relation and its translation; such as "at once loving and loving everything loved by." This was Schröder's most brilliant discovery, notwithstanding the simplicity of it, when once stated.

Peirce: CP 3.595 Cross-Ref: ††

595. The *ultralation* †1 of any primitive relation consists in any relate of it standing to its correlate in relation of standing in the primitive relation to something to which its correlate does not stand in that primitive relation. Thus, "loving something not loved by" is the ultralation of loving. The translation of any primitive is the contraredultration of the same primitive. Thus, the redultration of loving is "not loving something loved by" and the contraredultration is "loving whatever may be loved by."

Peirce: CP 3.596 Cross-Ref: ††

596. Almost every step in necessary reasoning depends on two premisses which come to the reasoner's knowledge from different quarters. He "puts two and two together," or, in Whewell's †2 terminology, he *colligates* the facts. But the two premisses may be (in fact they are, although this, as a mere psychological fact does not concern us), colligated, or *compounded* (De Morgan's †3 term) into a copulative

proposition (the traditional term, *copulatum* being used by Aulus Gellius †4 in the second century of our era, in this sense) from which as a single premiss the conclusion follows. This is to explain the frequent mention of *the premiss* of a necessary inference. The relation of a necessary conclusion to its premiss is an essential translation of "being true of." For that reason translations may, in all cases, give rise to necessary conclusions; and there is hardly any necessary inference that does not depend on a translation. Consequently, translations and ultratranslations are logically the most important of all dyadic relations.

Peirce: CP 3.597 Cross-Ref:††

597. The compound of the ultratranslation and translation of the same primitive is to be termed the *superlation* of that primitive. Thus, *superloving* is loving everything loved by and loving something not loved by. Essentially transitive relations (at least, all that are of importance) are either translations (and so, suitations) or are superlations (and so, contrasuitations).

Peirce: CP 3.598 Cross-Ref:††

598. A juxtasuitation is, strictly speaking, a spicalation having a spike consisting of a single seed. Such a spike may be called a unispoke. A spike consisting of two individuals may be termed a bispoke. A spike of which every individual stands in the relation, to which the spike belongs, to only one individual of the spike, and in the converse relation to only one individual of the spike, may be termed a cyclical spike.†1 Every *simple* spike, that is, every spike whose existence does not consist in the existence of other spikes, is a cyclical spike. It is true that a cyclical spike, and even a simple spike, may be composed of innumerable individuals; and in that case we are not compelled to regard it as running round into itself. Consider, for example, the relation "coming next after" in the both-ways endless series.

... 10^{-5} , 10^{-4} , 10^{-3} , 10^{-2} , 10^{-1} , 10^0 , 10^1 , 10^2 , 10^3 , 10^4 , 10^5 , 10^6 ...

This, according to the definition, is a relation having a single simple spike. But since on one side, the values run toward zero, and on the other side toward infinity, it cannot well be considered as returning into itself. Yet it is a nice question whether we may not properly conceive such a series, if endless, as *reaching* the limit, which by definition it just fails to reach. For positive endlessness as much transcends the finite series as does the limit, and reaches that limit in becoming infinite.†P1 The limiting term, however is identical with the next term. If that view of the matter be admissible (which is here left doubtful), there is no spike, unless the series returns into itself. The question will be fully considered in the lectures.†1

Peirce: CP 3.599 Cross-Ref:††

599. A spike is said *wholly to consist of simple spikes* only in case every dyad belonging to the relation forms a part of a simple spike belonging to the relation. Thus, the relation which subsists only between the following dyads, is composed wholly of simple spikes: (A:B), (B:C), (C:D), (D:A), (A:E), (E:F), (F:D). For every dyad either belongs to the simple spike (A:B), (B:C), (C:D), (D:A), or to the simple spike (D:A), (A:E), (E:F), (F:D), (D:A); and the circumstance that (D:A) belongs to both simple spikes does not conflict with this. But the spike (A:B), (A:D), (B:C), (C:A), (D:E), (E:F), (F:D) does not wholly consist of simple spikes, since (A:D) does

not here belong to any simple spike.

Peirce: CP 3.600 Cross-Ref:††

600. A relation of which every correlate stands in this relation to every one of its relates, and consequently to every relate of which every one of its correlates stand in the same relation, is termed an *equiparance*†2 (the term has been used since Antonius Andreas about A.D. 1300†3). An *extrequiparance* is termed a *disquiparance*†2 (the term has been in common use since Franciscus Mayro †4, about A. D. 1300). Equiparances and disquiparances are also termed *convertible* and *inconvertible* relations (De Morgan),†5 and quite commonly *reciprocating* and *non-reciprocating* relations. All the relates and correlates of an equiparance form a spike wholly consisting of spikes of two members and spikes of one member. (A spike of two members is either simple or is wholly composed of a simple spike of two members and, at most, two spikes of one member, every spike of one member being simple.)

Peirce: CP 3.601 Cross-Ref:††
§5. FOURTH SYSTEM OF DIVISIONS

601. This system depends upon the multitudes of relates and correlates. It has been treated with such elaboration by Mr. Kempe, whose "Memoir on the Theory of Mathematical Forms" may be found in the *Philosophical Transactions* for 1886, that only the most important points will here be noticed. Mr. Kempe's nomenclature is, in a few cases, unsuitable; but it is remarkable that the severest criticism has detected only one or two errors in the extensive work, and that that one, or may be two, attach to matters of minute detail toward the end of the memoir. It is one of the most solid treatises that have ever been written on the logic of relations -- a work that goes toward the elevation of man.

Peirce: CP 3.602 Cross-Ref:††

602. In the first place, there are two cross dichotomies according as the relation has for its relates on the one hand, or for its correlates on the other, whatever there may be that possesses some general quality, or merely a singular object or singular objects. The names which the author proposed for these classes in 1870†1 were rightly rejected by Schröder, who names them by reference to the matrix.†2 The ranks and files of a matrix have always in German been called *Zeilen* and *Kolonnen* respectively; and Schröder, accordingly names a relative term applicable to a single relate an "Einzeiler," and one applicable to a single correlate an "Einkolonner." A compartment of the matrix is called in German "ein Auge";†P1 and Schröder calls a relative term applicable only to a single dyad, whether like A:A or like A:B, an *Einauger*. The author's original term an *elementary relative* is sufficiently good. But the terms *one-rank* relation, *one-column* relation, *many-rank* relation, *many-column* relations can be used to advantage.

Peirce: CP 3.602 Cross-Ref:††

A one-rank relation may either be accidentally such (as it happens that there is but one "day-bringing" luminary), or essentially such (as "creator of"), and so of course, it is with one-column relations.

Peirce: CP 3.603 Cross-Ref:††

603. A general relation, and indeed, any many-ranked many-column relation, may be limited in respect to the multitude of relates *to each* correlate, and conversely. The sixth chapter of the third volume of Schröder's *Logik* contains a valuable and entirely original investigation of these. He describes these as relations with *empty* ranks (*Leerzeilen*) and *empty* columns, with *Singly occupied* (*Einbesetzten*) ranks or columns, with *multioccupied* (*Meerbesetzten*) ranks or columns, with *many-gapped* (*Meerlückige-*) ranks or columns, *singly gapped* (*Einlück-*) ranks or columns, and with *full* (*Voll-*) ranks and columns.

Peirce: CP 3.603 Cross-Ref:††

These again may be accidental or essential. Thus, the relation of parentage has essentially two relates to each correlate, but an indefinite finite number of correlates to each relate.

Peirce: CP 3.604 Cross-Ref:††

604. Another pair of cross dichotomies depends upon whether the relates of each correlate, on the one hand, and the correlate of each relate, on the other, are, or are not, of a fixed number. Of particular consequence (meaning by "consequence" the leading to conclusions) are those relations for which both these numbers are fixed. Such a relation is termed a *correspondence* (a common term of mathematics). The number of relates to each correlate is usually prefixed to the designation, being followed by the number of correlates to each relate, with the proposition "to" intervening between the two numbers. Thus, we speak of a "two-to-three correspondence," meaning that each correlate has two relates and each relate three correlates.

Peirce: CP 3.605 Cross-Ref:††

605. There is an indefinite division of correspondences into the *regular* and the *irregular*. The regular are those in which there is some discernible rule as to what relates are joined to what correlates. Thus, the relations whose matrices are shown in Figures 7, 8 and 9 are all regular two-to-two correspondences; but their rules are different. It is certain, however, that no collection of individuals be it finite or infinite, can be arranged in succession without that succession's perfectly conforming to some kind of regularity. It is true that there is a difference between an *accidental* and an *essential* regularity. But the difference does not manifest itself in the existential

--	A:B	--	--	--	A:F	--	--
--	--	B:C	--	--	--	B:G	--
--	--	--	C:D	--	--	--	C:H
D:A	--	--	--	D:E	--	--	--
--	E:B	--	--	--	E:F	--	--
--	--	F:C	--	--	--	F:G	--
--	--	--	G:D	--	--	--	G:H
H:A	--	--	--	H:E	--	--	--

Figure 7

facts themselves. The problem of how an accidental regularity can be distinguished from an essential one is precisely the problem of inductive logic.

--	A:B	--	--	A:E	--	--	--
--	--	B:C	--	--	B:F	--	--
--	--	--	C:D	--	--	C:G	--
--	--	--	--	D:E	--	--	D:H
E:A	--	--	--	--	E:F	--	--
--	F:B	--	--	--	--	F:G	--
--	--	G:C	--	--	--	--	G:H
H:A	--	--	H:D	--	--	--	--

Figure 8

--	A:B	A:C	--	--	--	--	--
--	--	B:C	B:D	--	--	--	--
--	--	--	C:D	C:E	--	--	--
--	--	--	--	D:E	D:F	--	--
--	--	--	--	--	E:F	E:G	--
--	--	--	--	--	--	F:G	F:H
G:A	--	--	--	--	--	--	G:H
H:A	H:B	--	--	--	--	--	--

Figure 9

Peirce: CP 3.606 Cross-Ref:††

§6. NOTE ON THE NOMENCLATURE AND DIVISIONS OF MODAL DYADIC RELATIONS

606. A modal dyadic relation is either a relation between characters (including

qualities and relations of individuals, of characters, and of concepts), or between symbols, or concepts.

Peirce: CP 3.607 Cross-Ref: ††

607. Dyadic relations between characters mostly correspond to relations between the subjects of those characters or to relations between the symbols of them; and such need not be separately considered. There remain some relations between characters, especially between qualities, which do not seem to be derivative. Such are the relations of "being more intense than," of "being disparate to" (or in applicability to subjects of the same category, as multitude and intensity are disparate). But, so far as appears at present, no particular logical interest attaches to such relations, and they will here be passed by.

Peirce: CP 3.608 Cross-Ref: ††

608. Dyadic relations between symbols, or concepts, are matters of logic, so far as they are not derived from relations between the objects and the characters to which the symbols refer. Noting that we are limiting ourselves to modal *dyadic* relations, it may probably be said that those of them that are truly and fundamentally dyadic arise from corresponding relations between propositions. To exemplify what is meant, the dyadic relations of logical *breadth* and *depth*, often called denotation and connotation, have played a great part in logical discussions, but these take their origin in the triadic relation between a sign, its object, and its interpretant sign; and furthermore, the distinction appears as a dichotomy owing to the limitation of the field of thought, which forgets that concepts grow, and that there is thus a third respect in which they may differ, depending on the state of knowledge, or amount of information. †1 To give a good and complete account of the dyadic relations of concepts would be impossible without taking into account the triadic relations which, for the most part, underlie them; and indeed almost a complete treatise upon the first †2 of the three divisions of logic would be required. †3

Peirce: CP 3.609 Cross-Ref: ††

XX

NOTES ON SYMBOLIC LOGIC AND MATHEMATICS^E

§1. IMAGING †1

609. A term proposed to translate *Abbildung* in its logical use. In order to apprehend this meaning, it is indispensable to be acquainted with the history of the meanings of *Abbildung*. This word was used in 1845 by Gauss †2 for what is called in English a map-projection, which is an incorrect term, since many such modes of representation are not geometrical rectilinear projections at all; and of those which Gauss had in view, but a single one is so. In mathematics *Abbildung* is translated *representation*; but this word is preëmpted in logic. Since *Bild* is always translated *image*, *imaging* will answer very well for *Abbildung*. If a map of the entire globe was

made on a sufficiently large scale, and out of doors, the map itself would be shown upon the map; and upon that image would be seen the map of the map; and so on, indefinitely. If the map were to cover the entire globe, it would be an image of nothing but itself, where each point would be imaged by some other point, itself imaged by a third, etc. But a map of the heavens does not show the map itself at all. A Mercator's projection shows the entire globe (except the poles) over and over again in endlessly recurring strips.†3 Many maps, if they were completed, would show two or more different places on the earth at each point of the map (or at any rate on a part of it), like one map drawn upon another. Such is obviously the case with any rectilinear projection of the entire sphere, excepting only the stereographic. These two peculiarities may coexist in the same map.

Peirce: CP 3.610 Cross-Ref:††

610. Any mathematical function of one variable may be regarded as an image of its variable according to some mode of imaging. For the real and imaginary quantities correspond, one to one and continuously, to the assignable points on a sphere. Although mathematics is by far the swiftest of the sciences in its generalisations, it was not until 1879 that Dedekind (in the third edition of his recension of Lejeune-Dirichlet's *Zahlentheorie*, §163, p. 470; but the writer has not examined the second edition) extended the conception to discrete systems in these words: "It very often happens in other sciences, as well as in mathematics, that there is a replacement of every element { \ddot{o} } of a system { \ddot{O} } of elements or things by a corresponding element { \ddot{o} '} [of a system { \ddot{O} '}]. Such an act should be called a substitution. . . . But a still more convenient expression is found by regarding { \ddot{O} '} as the image of { \ddot{O} }, and { \ddot{o} '} of { \ddot{o} }, according to a certain mode of imaging." And he adds, in a footnote: "This power of the mind of comparing a thing { \ddot{o} }, with a thing { \ddot{o} '}, or of relating { \ddot{o} } to { \ddot{o} '}, or of considering { \ddot{o} '} to correspond to { \ddot{o} }, is one without which no thought would be possible." (We do not translate the main clause.) This is an early and significant acknowledgment that the so-called "logic of relatives" -- then deemed beneath the notice of logicians -- is an integral part of logic. This remark remained unnoticed until, in 1895, Schröder devoted the crowning chapter of his great work (*Exakte Logik*, iii. 553-649) to its development. Schröder says that, in the broadest sense, any relative whatever may be considered as an imaging -- "nämlich als eine eventuell bald 'undeutige,' bald 'eindeutige,' bald 'mehrdeutige' Zuordnung." He presumably means that the logical universe is thus imaged in itself. However, in a narrower sense, he says, a mode of imaging is restricted to a relative which fulfills one or other of the two conditions of being never *undeutig* or being never *mehrdeutig*. That is, the relation must belong to one or other of two classes, the one embracing such that every object has an image, and the other such that no object has more than one image. Schröder's definitions (however interesting his developments) break all analogy with the important property of the imaging of continua noticed above. If this is to be regarded as essential, an imaging must be defined as a generic relation between an object-class and an image-class, which generic relation consists of specific relations, in each of which one individual, and no more, of the image-class stands to each individual of the object-class, and in each of which every individual of the image-class stands to one individual and no more of the object-class. This is substantially a return to Dedekind's definition, which makes an *imaging* a synonym for a substitution.

Peirce: CP 3.611 Cross-Ref:††

§2. INDIVIDUAL †1

611. (As a technical term of logic, *individuum* first appears in Boëthius, †2 in a translation from Victorinus, no doubt of {atomon}, a word used by Plato (*Sophistes*, 229 D) for an indivisible species, and by Aristotle, often in the same sense, but occasionally for an individual. Of course the physical and mathematical senses of the word were earlier. Aristotle's usual term for individuals is {ta kath' hekasta}, Latin *singularia*, English *singulars*.) Used in logic in two closely connected senses. (1) According to the more formal of these an individual is an object (or term) not only actually determinate in respect to having or wanting each general character and not both having and wanting any, but is necessitated by its mode of being to be so determinate. See Particular (in logic). †3

Peirce: CP 3.612 Cross-Ref: ††

612. This definition does not prevent two distinct individuals from being precisely similar, since they may be distinguished by their heccecities (or determinations not of a generalizable nature); so that Leibnitz' principle of indiscernibles is not involved in this definition. Although the principles of contradiction and excluded middle may be regarded as together constituting the definition of the relation expressed by "not," yet they also imply that whatever exists consists of individuals. †4 This, however, does not seem to be an identical proposition or necessity of thought; for Kant's Law of Specification (*Krit. d. reinen Vernunft*, 1st ed., 656; 2d ed., 684; but it is requisite to read the whole section to understand his meaning), which has been widely accepted, treats logical quantity as a continuum in Kant's sense, i.e., that every part of which is composed of parts. Though this law is only regulative, it is supposed to be demanded by reason, and its wide acceptance as so demanded is a strong argument in favour of the conceivability of a world without individuals in the sense of the definition now considered. Besides, since it is not in the nature of concepts adequately to define individuals, it would seem that a world from which they were eliminated would only be the more intelligible. A new discussion of the matter, on a level with modern mathematical thought and with exact logic, is a desideratum. A highly important contribution is contained in Schröder's *Logik*, iii, Vorles. 10. What Scotus says (*Quaest. in Met.*, VII 9, xiii and xv) is worth consideration.

Peirce: CP 3.613 Cross-Ref: ††

613. (2) Another definition which avoids the above difficulties is that an individual is something which reacts. That is to say, it does react against some things, and is of such a nature that it might react, or have reacted, against my will.

Peirce: CP 3.613 Cross-Ref: ††

This is the stoical definition of a reality; but since the Stoics were individualistic nominalists, this rather favours the satisfactoriness of the definition than otherwise. It may be objected that it is unintelligible; but in the sense in which this is true, it is a merit, since an individual is unintelligible in that sense. It is a brute fact that the moon exists, and all explanations suppose the existence of that same matter. That existence is unintelligible in the sense in which the definition is so. That is to say, a reaction may be experienced, but it cannot be conceived in its character of a reaction; for that element evaporates from every general idea. According to this definition, that which alone immediately presents itself as an individual is a reaction against the will. But everything whose identity consists in a continuity of reactions

will be a single logical individual. Thus any portion of space, so far as it can be regarded as reacting, is for logic a single individual; its spatial extension is no objection. With this definition there is no difficulty about the truth that whatever exists is individual, since existence (not reality) and individuality are essentially the same thing; and whatever fulfills the present definition equally fulfills the former definition by virtue of the principles of contradiction and excluded middle, regarded as mere definitions of the relation expressed by "not." As for the principle of indiscernibles, if two individual things are exactly alike in all other respects, they must, according to this definition, differ in their spatial relations, since space is nothing but the intuitional presentation of the conditions of reaction, or of some of them. But there will be no logical hindrance to two things being exactly alike in all other respects; and if they are never so, that is a physical law, not a necessity of logic. This second definition, therefore, seems to be the preferable one. Cf. Particular (in logic).†1

Peirce: CP 3.614 Cross-Ref:††
 §3. INVOLUTION †2

614. A term of Symbolic Logic borrowed from algebra, where it means the raising of a base to a power. In logic it has two different senses. (1) Relative involution: let *lwm* denote any lover of a well-wisher of a man. That is, any individual *A* is denoted by *lwm*, provided there are in existence individuals *B* and *C* (who may be identical with each other or with *A*), such that *A* loves *B*, while *B* wishes well to *C*, and *C* is a man. Further, let *lwm* denote any individual *A*, if, and only if, there is in existence an individual *C*, who is a man, and who is such that taking any individual *B* whatever, if *B* is a well-wisher of *C*, then *A* is a lover of *B*. The operation of combining *l* and *w* in this statement is termed "progressive involution."†3 Again, let *lwm* denote any individual *A*, if, and only if, there is in existence an individual *B*, who is loved by *A*, and who is such that taking any individual *C* whatever, if *C* is wished well by *B*, then *C* is a man. The operation of combining *w* and *m* in this statement is termed "regressive involution."†4 These designations were adopted because of the analogy of the general formulae to those of involution in the algebra of quantity.

Peirce: CP 3.614 Cross-Ref:††

These kinds of involution are not, at present, in use in symbolical logic; but they are, nevertheless, useful, especially in developing the conception of continuity. These two kinds of involution together constitute relative involution.

Peirce: CP 3.615 Cross-Ref:††

615. (2) Non-relative involution: consisting in the repeated introduction of the same premiss into a reasoning; as, for example, the half dozen simple premisses upon which the Theory of Numbers is based are introduced over and over again in the reasoning by which its myriad theorems are deduced. In exact logic the regular process of deduction begins by non-relatively multiplying together all the premisses to make one conjunctive premiss, from which whatever can be deduced by using those premisses as often as they are introduced as factors, can be deduced by processes of "immediate inference" from that single conjunctive premiss. But the general character of the conclusion is found to depend greatly upon the number of times the same factor is multiplied in. From this circumstance the importance and the

name of non-relative involution arise.

Peirce: CP 3.616 Cross-Ref:††

§4. LOGIC (EXACT)†1

616. The doctrine that the theory of validity and strength of reasoning ought to be made one of the "exact sciences," that is, that generalisations from ordinary experience ought, at an early point in its exposition, to be stated in a form from which by mathematical, or expository, reasoning, the rest of the theory can be strictly deduced; together with the attempt to carry this doctrine into practice.

Peirce: CP 3.617 Cross-Ref:††

617. This method was pursued, in the past, by Pascal (1623-62), Nicolas Bernoulli (1687-1759), Euler (1708-83), Ploucquet (1716-90), Lambert (1728-77), La Place (1749-1827), De Morgan (1806-71), Boole (1815-64), and many others; and a few men in different countries continue the study of the problems opened by the last two named logicians as well as those of the proper foundations of the doctrine and of its application to inductive reasoning. The results of this method, thus far, have comprised the development of the theory of probabilities, the logic of relatives, advances in the theory of inductive reasoning (as it is claimed), the syllogism of transposed quantity, the theory of the Fermatian inference, considerable steps towards an analysis of the logic of continuity and towards a method of reasoning in topical geometry, contributions towards several branches of mathematics by applications of "exact" logic, the logical graphs called after Euler and other systems for representing in intuitional form the relations of premisses to conclusions, and other things of the same general nature.

Peirce: CP 3.618 Cross-Ref:††

618. There are those, not merely outside the ranks of exact logic, but even within it, who seem to suppose that the aim is to produce a calculus, or semi-mechanical method, for performing all reasoning, or all deductive inquiry; but there is no reason to suppose that such a project, which is much more consonant with the ideas of the opponents of exact logic than with those of its serious students, can ever be realised. The real aim is to find an indisputable theory of reasoning by the aid of mathematics. The first step in the order of logic towards this end (though not necessarily the first in the order of inquiry) is to formulate with mathematical precision, definiteness, and simplicity, the general facts of experience which logic has to take into account.

Peirce: CP 3.619 Cross-Ref:††

619. The employment of algebra in the investigation of logic is open to the danger of degenerating into idle trifling of too rudimentary a character to be of mathematical interest, and too superficial to be of logical interest. It is further open to the danger that the rules of the symbols employed may be mistaken for first principles of logic. An algebra which brings along with it hundreds of purely formal theorems of no logical import whatever must be admitted, even by the inventor of it, to be extremely defective in that respect, however convenient it may be for certain purposes. On the other hand, it is indisputable that algebra has an advantage over speech in forcing us to reason explicitly and definitely, if at all. In that way it may afford very considerable aid to analysis. It has been employed with great advantage in

the analysis of mathematical reasonings.

Peirce: CP 3.619 Cross-Ref:††

Algebraic reasoning involves intuition just as much as, though more insidiously than, does geometrical reasoning; and for the investigation of logic it is questionable whether the method of graphs is not superior. Graphs cannot, it is true, readily be applied to cases of great complexity; but for that very reason they are less liable to serve the purposes of the logical trifler. In the opinion of some exact logicians, they lead more directly to the ultimate analysis of logical problems than any algebra yet devised. See Logical Diagram (or Graph).†1

Peirce: CP 3.620 Cross-Ref:††

620. It is logical algebra, however, which has chiefly been pursued. De Morgan invented a system of symbols, which had the signal advantage of being entirely new and free from all associations, misleading or otherwise. Although he employed them for synthetical purposes almost exclusively, yet the great generality of some of the conceptions to which they led him is sufficient to show that they might have been applied with great advantage in analysis. Boole was led, no doubt from the consideration of the principles of the calculus of probabilities, to a wonderful application of ordinary algebra to the treatment of all deductive reasoning not turning upon any relations other than the logical relations between non-relative terms. By means of this simple calculus, he took some great steps towards the elucidation of probable reasoning; and had it not been that, in his pre-Darwinian day, the notion that certain subjects were profoundly mysterious, so that it was hopeless, if not impious, to seek to penetrate them, was still prevalent in Great Britain, his instrument and his intellectual force were adequate to carrying him further than he actually went. Most of the exact logicians of today are, from the nature of the case, followers of Boole. They have modified his algebra by disusing his addition, subtraction, and division, and by introducing a sign of logical aggregation. This was first done by Jevons; and he proposed $\cdot\mid\cdot$, a sign of division turned up, to signify this operation. Inasmuch as this might easily be read as three signs, it would, perhaps, be better to join the two dots by a light curve, thus \curvearrowright . Some use the sign $+$ for logical aggregation. The algebra of Boole has also been amplified so as to fit it for the logic of relatives. The system is, however, far from being perfect. See Relatives (logic of).†1

Peirce: CP 3.621 Cross-Ref:††

621. Certain terms of exact logic may be defined as follows:--†2

Copula is often defined as that which expresses the relation between the subject-term and the predicate-term of a proposition. But this is not sufficiently accurate for the purposes of exact logic. Passing over the objection that it applies only to categorical propositions, as if conditional and copulative propositions had no copula, contrary to logical tradition, it may be admitted that a copula often does fulfill the function mentioned; but it is only an accidental one, and its essential function is quite different. Thus, the proposition, "Some favoured patriarch is translated" is essentially the same as "A translated favoured patriarch is"; and "Every mother is a lover of that of which she is a mother" is the same as "A mother of something not loved by her is not." In the second and fourth forms, the copula connects no terms; but if it is dropped, we have a mere term instead of a proposition. Thus the essential office of the copula is to express a relation of a general term or terms to the universe. The universe must be well known and mutually known to be known and agreed to exist, in some sense, between speaker and hearer, between the mind as appealing to its own further consideration and the mind as so appealed to, or there can be no

communication, or "common ground," at all. The universe is, thus, not a mere concept, but is the most real of experiences. Hence, to put a concept into relation to it, and into the relation of describing it, is to use a most peculiar sort of sign or thought; for such a relation must, if it subsist, *exist* quite otherwise than a relation between mere concepts. This, then, is what the copula essentially does. This it may do in three ways: first, by a vague reference to the universe collectively; second, by a reference to all the individuals existent in the universe distributively; third, by a vague reference to an individual of the universe selectively. "It is broad daylight," I exclaim, as I awake. My universe is the momentary experience as a whole. It is that which I connect as object of the composite photograph of daylight produced in my mind by all my similar experiences. Secondly, "Every woman loves something" is a description of every existing individual in the universe. Every such individual is said to be coexistent only with what, so far as it is a woman at all, is sure to be a lover of some existing individual. Thirdly, "Some favoured patriarch is translated" means that a certain description applies to a select individual. A hypothetical proposition, whether it be conditional (of which the alternative, or disjunctive, proposition is a mere species, or *vice versa*, as we choose to take it) or copulative, is either general or *ut nunc*. A general conditional is precisely equivalent to a universal categorical. "If you really want to be good, you can be," means "Whatever determinate state of things may be admissibly supposed in which you want to be good is a state of things in which you can be good." The universe is that of determinate states of things that are admissible hypothetically. It is true that some logicians appear to dispute this; but it is manifestly indisputable. Those logicians belong to two classes: those who think that logic ought to take account of the difference between one kind of universe and another (in which case, several other *substantiae* of propositions must be admitted); and those who hold that logic should distinguish between propositions which are necessarily true or false together, but which regard the fact from different aspects. The exact logician holds it to be, in itself, a defect in a logical system of expression, to afford different ways of expressing the same state of facts; although this defect may be less important than a definite advantage gained by it. The copulative proposition is in a similar way equivalent to a particular categorical. Thus, to say "The man might not be able voluntarily to act otherwise than physical causes make him act, whether he try or not," is the same as to say that there is a state of things hypothetically admissible in which a man tries to act one way and voluntarily acts another way in consequence of physical causes. As to hypotheticals *ut nunc*, they refer to no range of possibility, but simply to what is true, vaguely taken collectively.

Peirce: CP 3.622 Cross-Ref:††

622. Although it is thus plain that the action of the copula in relating the subject-term to the predicate-term is a secondary one, it is nevertheless necessary to distinguish between copulas which establish different relations between these terms. Whatever the relation is, it must remain the same in all propositional forms, because its nature is not expressed in the proposition, but is a matter of established convention. With that proviso, the copula may imply any relation whatsoever. So understood, it is the *abstract copula* of De Morgan (*Camb. Philos. Trans.*, x. 339). A *transitive* copula is one for which the mood Barbara is valid. Schröder has demonstrated the remarkable theorem that if we use IS in small capitals to represent any one such copula, of which "greater than" is an example, then there is some relative term *r*, such that the proposition "*S IS P*" is precisely equivalent to "*S is r to P* and is *r* to whatever *P is r to*." A *copula of correlative inclusion* is one for which both Barbara and the formula of identity hold good. Representing any one such

copula by *is* in italics, there is a relative term *r*, such that the proposition "*S is P*" is precisely equivalent to "*S is r* to whatever *P is r* to." If the last proposition follows from the last but one, no matter what relative *r* may be, the copula is called the *copula of inclusion*, used by C. S. Peirce, Schröder, and others. De Morgan uses a copula defined as standing for any relation both transitive and convertible. The latter character consists in this, that whatever terms *I* and *J* may be, if we represent this copula by *is* in black-letter, then from "*I is J*" it follows that "*J is I*." From these two propositions, we conclude, by Barbara, that "*I is I*." Such copulas are, for example, "equal to," and "of the same colour as." For any such copula there will be some relative term *r*, such that the proposition "*S is P*" will be precisely equivalent to "*S is r* to everything, and only to everything, to which *P is r*." Such a copula may be called a copula of *correlative identity*. If the last proposition follows from the last but one, no matter what relative *r* may be, the copula is the *copula of identity* used by Thomson, Hamilton, Baynes, Jevons, and many others.

Peirce: CP 3.622 Cross-Ref:††

It has been demonstrated by Peirce that the copula of inclusion is logically simpler than that of identity.†1

Peirce: CP 3.623 Cross-Ref:††

623. *Dialogism*. A form of reasoning in which from a single premiss a disjunctive, or alternative, proposition is concluded introducing an additional term; opposed to a syllogism, in which from a copulative proposition a proposition is inferred from which a term is eliminated.

Syllogism.

All men are animals, and all animals are mortal;

∴ All men are mortal.

Dialogism.

Some men are not mortal;

∴ Either some men are not animals, or some animals are not mortal.

Peirce: CP 3.624 Cross-Ref:††

624. *Dimension*. An element or respect of extension of a logical universe of such a nature that the same term which is individual in one such element of extension is not so in another. Thus, we may consider different persons as individual in one respect, while they may be divisible in respect to time, and in respect to different admissible hypothetical states of things, etc. This is to be widely distinguished from different universes, as, for example, of things and of characters, where any given individual belonging to one cannot belong to another. The conception of a multidimensional logical universe is one of the fecund conceptions which exact logic owes to O. H. Mitchell.†1 Schröder, in his then second volume, where he is far below himself in many respects, pronounces this conception "untenable." But a doctrine which has, as a matter of fact, been held by Mitchell, Peirce, and others, on apparently cogent grounds, without meeting any attempt at refutation in about twenty years, may be regarded as being, for the present, at any rate, tenable enough to be held.

Peirce: CP 3.625 Cross-Ref:††

625. **Dyadic relation.** A fact relating to two individuals. Thus, the fact that *A* is similar to *B*, and the fact that *A* is a lover of *B*, and the fact that *A* and *B* are both men, are dyadic relations; while the fact that *A* gives *B* to *C* is a triadic relation. Every relation of one order of relativity may be regarded as a relative of another order of relativity if desired. Thus, *man* may be regarded as *man coexistent with*, and so as a relative expressing a dyadic relation, although for most purposes it will be regarded as a monad or non-relative term.

Peirce: CP 3.626 Cross-Ref:††
§5. MULTITUDE (IN MATHEMATICS)†2

626. That relative character of a collection which makes it greater than some collections and less than others. A collection, say that of the *A*'s, is **greater than** another, say that of the *B*'s, if, and only if, it is impossible that there should be any relation *r*, such that every *A* stands in the relation *r* to a *B* to which no other *A* is in the relation *r*.

Peirce: CP 3.627 Cross-Ref:††

627. The precise analysis of the notion is due to G. Cantor, whose definition is, however, a little different in its mode of expression, since it is more abstract. He defines the character in these words: "By Mächtigkeit or cardinal number of a collection (Menge) *M*, we mean the universal concept, which by the help of our active faculty of thought results from the collection *M* by abstraction from the characters of the different members (Elemente) of that collection and from the order in which they are given (Gegebensein)."†1

Peirce: CP 3.628 Cross-Ref:††

628. A cardinal number, though confounded with multitude by Cantor, is in fact one of a series of vocables the prime purpose of which, quite unlike any other words, is to serve as an instrument in the performance of the experiment of counting; these numbers being pronounced in their order from the beginning, one as each member of the collection is disposed of in the operation of counting. If the operation comes to an end by the exhaustion of the collection, the last cardinal number pronounced is applied adjectivally to the collection, and expresses its multitude, by virtue of the theorem that a collection the counting of which comes to an end, always comes to an end with the pronunciation of the same cardinal number.

Peirce: CP 3.629 Cross-Ref:††

629. If the cardinal numbers are considered abstractedly from their use in counting, simply in themselves, as objects of mathematical reasoning, stripped of all accidents not pertinent to such study, they become indistinguishable from the similarly treated ordinal numbers, and are then usually called **ordinal numbers** by the mathematico-logicians. There is small objection to this; yet it is to be remarked that they are ordinal in different senses in grammar and in the logic of mathematics. For in grammar they are called ordinal as being adapted to express the ordinal places of other things in the series to which those things belong; while in the logic of mathematics the only relevant sense in which they are ordinal is as being defined by a serial order within their own system. The definition of this order is not difficult; but

the syntax of ordinary language does not lend itself to the clear expression of such relations in the manner in which they ought to be expressed in order to bring out their logical character. It must, therefore, be here passed by. In fact, none of the doctrines of logic can be satisfactorily expressed under the limitations here imposed, however simple they may be. The doctrine of ordinal numbers is by Dedekind (*Was sind und was sollen die Zahlen?*) made to precede that of the cardinal numbers; and this is logically preferable, if hardly so imperative as Schröder considers it.

Peirce: CP 3.630 Cross-Ref:††

630. The doctrine of the so-called ordinal numbers is a doctrine of pure mathematics; the doctrine of cardinal numbers, or, rather, of multitude, is a doctrine of mathematics applied to logic. The smallest multitude is most conveniently considered to be *zero*; but this is a question of definition. A *finite* collection is one of which the syllogism of transposed quantity holds good. Of finite collections, it is true that the whole is greater than any part. It is singular that this is often taken as the type of an axiom, although it has from early times been a matter of familiar knowledge that it is not true of infinite collections. Every addition of one increases a finite multitude. An infinite collection cannot be separated into a lesser collection of parts all smaller than itself.

Peirce: CP 3.631 Cross-Ref:††

631. The multitude of all the different finite multitudes is the smallest infinite multitude. It is called the *denumeral* multitude. (Cantor uses a word equivalent to *denumerable*; but the other form has the advantage of being differentiated from words like *enumerable*, *abnumerable*, which denote classes of multitudes, not, like *denumeral*, a single multitude.) Following upon this is a denumeral series of multitudes called by C. S. Peirce the *first*, *second*, etc. *abnumerable* multitudes. Each is the multitude of possible collections formed from the members of a collection of the next preceding multitude. They seem to be the same multitudes that are denoted by Cantor as *Alephs*. The first of them is the multitude of different limits of possible convergent series of rational fractions, and therefore of all the quantities with which mathematical analysis can deal under the limitations of the doctrine of limits. (The imaginaries do not increase the multitude.) What comes after these is still a matter of dispute, and is perhaps of inferior interest. The transition to continuity is, however, a matter of supreme importance for the theory of scientific method; nor is it a very complicated matter; but it cannot be stated under the limitations of expression here imposed upon us.

Peirce: CP 3.632 Cross-Ref:††

§6. POSTULATE †1

632. (1) The earliest definition we have of postulate, which was a technical term of Greek geometers, is by Aristotle.†2 The passage has an appearance of incoherence; it is, however, plain that Aristotle makes a distinction between *hypotheses* and *postulates* which Euclid does not draw, and which is irrelevant. Omitting the distinction, the two have this in common -- that they are propositions not necessarily true which are assumed as the bases of deductions.

Peirce: CP 3.632 Cross-Ref:††

If we turn to the first book of Euclid's *Elements*, we observe, in the first place, that he calls axioms by the name of common notions, a deliberate choice by him, for Aristotle, before his day, had called them axioms, though Aristotle usually calls them {ta koina}, nearly Euclid's name. These matters of common knowledge, according to Euclid's enumeration of them, are not specially geometrical, except that magnitudes superposable are equal (see the *Cent. Dict.*, "Axiom"). On the other hand, the "postulates" of Euclid are all geometrical. They are as follows (according to the best MS. and all the evidence):--

(a) Between any two points a straight line can be drawn.

(b) Any terminated straight line can be prolonged at either end indefinitely.

(c) About any point in any plane as centre a circle may be described with any radius.

(d) All right angles are equal.

(e) If two straight lines in a plane are cut by a third, making the sum of the internal angles on one side less than two right angles, those two straight lines will meet if sufficiently produced.

(f) Two straight lines cannot enclose a space in a plane.

Peirce: CP 3.633 Cross-Ref:††

633. (2) Since Wolff it has been very common among Germans, and among English writers who follow them, to define a postulate as an indemonstrable practical proposition. That is to say, it is an indemonstrable *particular* proposition, asserting that some general description of an object *exists* (in the only sense in which pure geometrical forms can be said to exist), in contradistinction to *axioms*, which were supposed to be indemonstrable theoretical (*i.e.* universal) propositions, asserting that some general description of an object has no existence as a geometrical form.

Peirce: CP 3.633 Cross-Ref:††

It is certainly desirable to have two terms bearing these meanings; but it was an utter misunderstanding to suppose that such were the proper meanings either of the word *axiom* or of the word *postulate*. The manner in which this misunderstanding came about is somewhat instructive. An axiom was a perfectly indubitable statement *about things*, in contradistinction to a definition, which cannot be called in question. On the contrary, a postulate was an indemonstrable proposition, not indubitable. There was some question whether certain postulates might not be considered to be axiomatic. When that was done, all the remaining postulates were particular propositions; namely, the first three of Euclid's list. This view was aided by the illogical notion that definitions could be considered as among the foundations of geometrical truth. Some writers went so far as to say that definitions were, or ought to be, the sole foundation of geometry -- an extreme nominalistic position. But if definitions are allowed to take such a position, one postulate, at most, suffices, without any axiom; and all the rest of geometry can be thrown into a single definition. Namely, it is only necessary to postulate, say, that a point is possible, and to define a point in such a way as to make it cover the whole of geometry. This was not seen; and the practice of throwing geometrical truth over into definitions so far prevailed as to aid in restricting postulates to particular propositions. That such assumptions of possibility had a markedly different logical function from assumptions of impossibility was sufficiently clear to Wolff and the earlier writers whom he followed

to cause him to put forth his definitions of *axiom* and *postulate*; and they recommended themselves all the more, because the postulates had become so familiar that it was no longer recognized that they were open to doubt.

Peirce: CP 3.634 Cross-Ref:††

634. (3) Kant calls his principles of modality "postulates of empirical thought" in the sense of judgments which are objectively analytical but subjectively synthetical. In fact, the principles as stated by him are not synthetical in any sense whatever, but are mere definitions.

Peirce: CP 3.635 Cross-Ref:††
§7. PRESUPPOSITION †1

635. Presupposition is either a conjecture or what is better called in English a *Postulate*. (q. v.)

Peirce: CP 3.635 Cross-Ref:††

As a philosophical term it translates the German *Voraussetzung*, and is presumably preferred to "postulate" by Germans and others imperfectly acquainted with the English language, because they suppose that postulate in English has the same meaning as *Postulat* in German, which is not true; for the English retains the old meaning, while the German has generally adopted the conception of Wolff. If postulate does not exactly translate German *Voraussetzung*, it comes, at any rate, quite as near to doing so as presupposition; a good translation would be "assumption."

Peirce: CP 3.636 Cross-Ref:††
§8. RELATIVES †1

636. If from any proposition having more than one subject (used to include "objects") we strike out the indices of the subjects, as in "-- praises -- to --," "-- dat in matrimonium --," what remains and requires at least two insertions of subject-nouns to make a proposition is a "relative term," or "*relative rhema*," called briefly a "relative." The relative may be converted into a complete assertion by filling up the blanks with proper names or abstract nouns; this serves as a criterion.

Peirce: CP 3.636 Cross-Ref:††

But in such a relative there must be such an idea of the difference between the subjects to be applied that "dat in matrimonium" shall be different from "datur in matrimonium." In order to free ourselves from the accidents of speech, we might represent the sentence by the following diagram:

Cinna --| dat in matrimonium | -- Cossutiam

|
Caesari

or, as follows:

$d[i j k]$ (Cinna = i , Cossutia = j , Caesar = k).

Then the relative will appear as

-- | dat in matrimonium | --

|

or as: $d[i j k]$.

But in either case, in order to explain what is meant, it will be necessary to explain how those three tails, or the three letters i, j, k , differ. The order shows which of three indices is given, which giver, which recipient.

Peirce: CP 3.637 Cross-Ref:††

637. Relatives may be more or less general like other terms, that is, one relative may be predicable of members of a set of which another is not, while the latter is predicable only of members of sets of which the former is predicable. By a set is meant an ordered system, so that ABC and BCA , though the same collection, are different sets. As any general term is predicable of any one of an aggregate of individuals, so a relative is predicable of any one of an aggregate of sets; and each such set may be regarded as an individual relative. By a system is meant an individual of which if anything is true, the truth of it consists in certain things being true of certain other individuals, called its *members*, regardless of the system. A *system* is either a *sorite*, *heap*, or *mere collection*, or it is a *set*. A *sorite* is a system of which, if anything is true, its truth consists of the truth of one predicate for any one of the members. A *set* is a system of which the truth of anything consists in the truth of different predicates. Of course the idea of relation is involved in the idea of a system. As it is very important for the understanding of relations that the conception of a system should be perfectly clear, let us consider the latter a moment in its simplest form, that of a *sorite* or mere collection. ABC is a sorite. Thus, it is true of it that it contains the three first letters of the alphabet, and the truth of that consists in A, B , and C being each one of the first three letters of the alphabet. It is true that it contains nothing but the first letters of the alphabet, because it is true of A, B, C severally that each is nothing but one of the first three letters of the alphabet. AB is a different sorite, because something is true of it which is not true of ABC . A may be regarded as

a sorite provided we mean not *A* in its first intention and being, but a something whose being consists in *A*'s being. The collection *A* is not the letter *A*, but it contains *A* and nothing else. If it be said that there is no such thing, the reply is that every collection, every system may be said to be an *ens rationis*. To this point we shall return. Even Nothing may be said to be a collection. For when we say that Nothing is less than 1, we do not mean that a self-subsisting individual is so, but that an *ens rationis* whose mode of being consists in the absence of everything is less than 1. The sorite *ABC* is other than *ABI*. But should I say that *ABC* contains two of the letters of Caesar's first name, and subsequently learn that that was a mistake, the real name being Gaius, that would not make *ABC* a different sorite.

Peirce: CP 3.638 Cross-Ref:††

638. That in the reality which corresponds to a proposition with a relative predicate is called the *fundamentum relationis*. A *relationship* is a system of such fundamenta. *Relation* is the relative character, conceived as belonging in different ways to the different relates, and (owing to the somewhat undue prominence given by familiar languages to one of these) especially to the relate which is denoted by the noun which is the subject nominative.

Peirce: CP 3.639 Cross-Ref:††

639. Relatives and relations are said to differ in their *orders*, according to the numbers of their relates. *Dyadic* or *dual* relations, or relatives of two relates, of which the second is called the *correlate*, differ somewhat widely from *plural*, or *polyadic*, relations. *Triadic* relations have all the principal characters of *tetradic* and higher relations. In fact, a compound of two triadic relatives may be a tetradic relative; as "praiser of to a maligner of -- to --."

Peirce: CP 3.640 Cross-Ref:††

640. Relatives may be compounded in all the ways in which other terms can be compounded as well as in other ways closely related to those. Thus, *A* may be said to be at once a lover and a servant of *B*, and it may be said that there is something, *X*, such that *A* is a lover of *X*, while *X* is a servant of *B*; so that *A* is a lover of a servant of *B*. This mode of composition is called *relative multiplication*. So, not only may it be said that *A* is either a lover or a servant of *B* (not excluding both), but also that whatever *X* may be, either *A* is a lover of *X* or *X* is a servant of *B*; that is, *A* is a lover of everything there is besides servants of *B*. (This wording, by Schröder, slightly violates English idiom, but is valuable as showing the analogy to aggregation.) This mode of composition is called *relative addition*. So, again, it may not only be said that *A* is if a lover then a servant of *B*, but also that whatever *X* may be, if *A* is a lover of *X*, then *X* is a servant of *B*; that is, *A* is a lover only of servants of *B*. This is called *relative regressive involution*. Or it may be said that whatever *X* may be, *A* is a lover of *X*, if *X* is a servant of *B*, or *A* is a lover of whatever is a servant of *B*. This is called *relative progressive involution*. Polyadic relatives are capable of other modes of composition. Thus, it may be said that anything whatever, *X*, being taken, something *Y* exists, such that *A* praises *X* to *Y* while *X* maligns *Y* to *B*; that is, *A* praises everybody to somebody malignd by him to *B*. Or we can say that there is something, *Y*, such that, whatever *X* may be, *A* praises *X* to *Y* while *X* maligns *Y* to *B*; or, *A* praises everybody to somebody whom everybody maligns to *B*.

Peirce: CP 3.641 Cross-Ref:††

641. Deductive logic can really not be understood without the study of the logic of relatives, which corrects innumerable serious errors into which not merely

logicians, but people who never opened a logic-book, fall from confining their attention to non-relative logic. One such error is that demonstrative reasoning is something altogether unlike observation. But the intricate forms of inference of relative logic call for such studied scrutiny of the representations of the facts, which representations are of an *iconic* kind, in that they represent relations in the fact by analogous relations in the representation, that we cannot fail to remark that it is by *observation* of diagrams that the reasoning proceeds in such cases. We successively simplify them and are always able to remark that such observation is required, and that it is even thus, and not otherwise, that the conclusion of a simple syllogism is seen to follow from its premisses. Again, non-relative logic has given logicians the idea that deductive inference was a following out of a rigid rule, so that machines have been constructed to draw conclusions. But this conception is not borne out by relative logic. People commonly talk of *the* conclusion from a pair of premisses, as if there were but one inference to be drawn. But relative logic shows that from any proposition whatever, without a second, an endless series of necessary consequences can be deduced; and it very frequently happens that a number of distinct lines of inference may be taken, none leading into another. That this must be the case is indeed evident without going into the logic of relatives, from the vast multitude of theorems deducible from the few incomplex premisses of the theory of numbers. But ordinary logic has nothing but barren sorites to explain how this can be. Since Kant, especially, it has been customary to say that deduction only elicits what was implicitly thought in the premisses; and the famous distinction of analytical and synthetical judgments is based upon that notion. But the logic of relatives shows that this is not the case in any other sense than one which reduces it to an empty form of words. Matter entirely foreign to the premisses may appear in the conclusion. Moreover, so far is it from being true, as Kant would have it, that all reasoning is reasoning in *Barbara*, that that inference itself is discovered by the microscope of relatives to be resolvable into more than half a dozen distinct steps. In minor points the doctrines of ordinary logic are so constantly modified or reversed that it is no exaggeration to say that deductive logic is completely metamorphosed by the study of relatives.

Peirce: CP 3.642 Cross-Ref:††

642. One branch of deductive logic, of which from the nature of things ordinary logic could give no satisfactory account, relates to the vitally important matter of abstraction. Indeed, the student of ordinary logic naturally regards abstraction, or the passage from "the rose smells sweet" to "the rose has perfume," to be a quasi-grammatical matter, calling for little or no notice from the logician. The fact is, however, that almost every great step in mathematical reasoning derives its importance from the fact that it involves an abstraction. For by means of abstraction the transitory elements of thought, the {epea pteroenta}, are made substantive elements, as James terms them, {epea apteroenta}.†1 It thus becomes possible to study their relations and to apply to these relations discoveries already made respecting analogous relations. In this way, for example, operations become themselves the subjects of operations.

Peirce: CP 3.642 Cross-Ref:††

To take a most elementary example -- from the idea of a particle moving, we pass to the idea of a particle describing a line. This line is then thought as moving, and so as generating a surface; and so the relations of surfaces become the subject of thought. An abstraction is an *ens rationis* whose being consists in the truth of an ordinary predication. A *collection*, or *system*, is an abstraction or abstract *ens*; and

thus the whole doctrine of number is founded on the operation of abstraction. If we conceive an object to be a collective whole, but to be so in such a way that it has no part which is not itself a collective whole in the same way, then, if the collection is of the nature of a sorite, it is a *general*, whose parts are distinguished merely as having additional characters; but if the collection is a *set*, whose members have other relations to one another, it is a *continuum*. The logic of continua is the most important branch of the logic of relatives, and mathematics, especially geometrical topic, or topical geometry, has its development retarded from the lack of a developed logic of continua.

Peirce: CP 3.643 Cross-Ref:††

643. *Literature*: relatives have, since Aristotle, been a recognized topic of logic. The first germ of the modern doctrine appears in a somewhat trivial remark of *Robert Leslie Ellis. De Morgan* did the first systematic work in his fourth memoir on the syllogism in 1860 (*Cambridge Philosophical Transactions*, x. 331-358); he here sketched out the theory of dyadic relations. C. S. Peirce, in 1870,†1 extended Boole's algebra so as to apply to them, and after many attempts produced a good general algebra of logic, together with another algebra specially adapted to dyadic relations (*Studies in Logic*, by members of the Johns Hopkins University, 1883, Note B, 187-203).†2 Schröder developed the last in a systematic manner (which brought out its glaring defect of involving hundreds of merely formal theorems without any significance, and some of them quite difficult) in the third volume of his *Exakte Logik* (1895). Schröder's work contains much else of great value. . . .

Peirce: CP 3.644 Cross-Ref:††
§9. TRANSPOSITION †3

644. Transposition consists in transferring a term from the subject to the predicate, or the reverse, with no change in the character of the connection; as, *No artists who are bankers are clever, No artists are clever bankers, No bankers are clever artists, None are at once artists and bankers and clever*; or as *All but a is b, All but b is a*. Any proposition may be "transformed" into other exactly equivalent forms: *e.g.* the transformation may consist in the change from one sort of connection to another (change of copula, in the extended meaning of that term), as -- to take a compound proposition as an example -- *It never rains but it pours = always either it pours or it does not rain*, but this is not transposition.

Peirce: CP 3.645 Cross-Ref:††

645. Certain copulas permit transposition simply, with no variation in the quality of the term transposed (as in the instances just given); but with the non-symmetrical copulas there must be a change from positive to negative or the reverse (and, if the proposition is complex, from the conjunctive to the alternative combination and the reverse), if the change can be made at all: He who is an astronomer *and* un-devout is mad = Any astronomer is mad or devout = All are mad *or* devout *or* not astronomers. When both the whole subject and the whole predicate is transposed the change is commonly called contraposition if the copula is non-symmetrical (*All a is b = All non-b is non-a; None but a is b = None but non-b is non-a*), but simple conversion if it is symmetrical (*No a is b = No b is a, Some a is b = Some b is a*). The usual discussion in the logics of the doctrine of the equivalence

of propositions is greatly simplified by taking this more general view of the subject.

Peirce: CP 3.646 Cross-Ref:††
APPENDIX

ON NONIONS^{E†1}

646. Readers of Professor Sylvester's communication entitled *Erratum* in the last number of these Circulars have perhaps inferred that my conduct in the matter there referred to had been in fault. Professor Sylvester's *Erratum* relates to his "Word upon Nonions," printed in the *Johns Hopkins University Circulars No. 17, p. 242*. In that article appears this sentence: "These forms [*i.e.* a certain group of nine Forms belonging to the algebra of Nonions] can be derived from an algebra given by Mr. Charles S. Peirce, (*Logic of Relatives*, 1870)." The object of Professor Sylvester's "*Erratum*" would seem to be to say that this sentence was inserted by me in his proof-sheet without his knowledge or authority on the occasion of the proof being submitted to me to supply a reference, and to repudiate the sentence, because he "knows nothing whatever" of the fact stated. But I think this view of Professor Sylvester's meaning is refuted by simply citing the following testimony of Professor Sylvester himself, printed in the *Johns Hopkins University Circulars, No. 15, p. 203*.

Peirce: CP 3.646 Cross-Ref:††

"Mr. Sylvester mentioned . . . that . . . he had come upon a system of Nonions, the exact analogues of the Hamiltonian Quaternions . . . Mr. Charles S. Peirce, it should be stated, had to the certain knowledge of Mr. Sylvester arrived at the same result many years ago in connection with his theory of the *logic of relatives*; but whether the result had been published by Mr. Peirce, he was unable to say."

Peirce: CP 3.646 Cross-Ref:††

This being so, I think that on the occasion of Professor Sylvester's publishing these forms I was entitled to some mention, if I had already published them, and *a fortiori* if I had not. When the proof-sheet was put into my hands, the request made to me, by an oral message, was not simply to supply a reference but to correct a statement relating to my work in the body of the text. And I had no reason to suppose that having thus submitted his text to me, Professor Sylvester would omit to look at his proof-sheet after it left my hands to see whether or not he approved of such alteration as I might have proposed. At any rate, when from these causes Professor Sylvester's "Word upon Nonions" had been published with the above statement concerning me, would it have been too much to expect that he should take the trouble to refer to my memoir in order to see whether the statement was not after all true, before publicly protesting against it?

Peirce: CP 3.647 Cross-Ref:††

647. I will now explain what the system of Nonions consists in and how I have been concerned with it.

Peirce: CP 3.647 Cross-Ref:††

The calculus of Quaternions, one of the greatest of all mathematical discoveries, is a certain system of algebra applied to geometry. A quaternion is a four-dimensional quantity; that is to say, its value cannot be precisely expressed without the use of a set of four numbers. It is much as if a geographical position should be expressed by a single algebraical letter; the value of this letter could only be defined by the use of two numbers, say the Latitude and Longitude. There are various ways in which a quaternion may be conceived to be measured and various different sets of four numbers by which its value may be defined. Of all these modes, Hamilton, the author of the algebra, selected one as the standard. Namely, he conceived the general quaternion q to be put into the form

$$q = x i + y j + z k + w,$$

where x, y, z, w , are four ordinary numbers, while i, j, k , are peculiar units, subject to singular laws of multiplication. For $ij = -ji$, the order of the factors being material, as shown in this multiplication table, where the first factor is entered at the side, the second at the top, and the product is found in the body of the table.

	1	i	j	k	
1	1	i	j	k	
i	i	-1	k	$-j$	
j	j	$-k$	-1	i	
k	k	j	$-i$	-1	

Peirce: CP 3.647 Cross-Ref:††

As long as x, y, z , and w in Hamilton's standard tetranomial form are confined to being *real* numbers, as he usually supposed them to be, no simpler or more advantageous form of conceiving the measurement of a quaternion can be found. But my father, Benjamin Peirce,†1 made the profound, original, and pregnant discovery that when x, y, z, w are permitted to be imaginaries, there is another very different and preferable system of measurement of a quaternion. Namely, he showed that the general quaternion, q , can be put into the form

$$q = x i + y j + z k + w l,$$

where x, y, z, w , are real or imaginary numbers, while i, j, k, l are peculiar units whose multiplication obeys this table.

	<i>i</i>	<i>j</i>	<i>k</i>	<i>l</i>	
<i>i</i>	<i>i</i>	<i>j</i>	0	0	
<i>j</i>	0	0	<i>i</i>	<i>j</i>	
<i>k</i>	<i>k</i>	<i>l</i>	0	0	
<i>l</i>	0	0	<i>k</i>	<i>l</i>	

Peirce: CP 3.647 Cross-Ref:††

A quaternion does not cease to be a quaternion by being measured upon one system rather than another. Any quantity belonging to the algebra is a quaternion; the algebra itself is "quaternions." The usual formulae of the calculus have no reference to any tetranomial form, and such a form might even be dispensed with altogether.

Peirce: CP 3.647 Cross-Ref:††

While my father was making his investigations in multiple algebra I was, in my humble way, studying the logic of relatives and an algebraic notation for it; and in the ninth volume of the *Memoirs of the American Academy of Arts and Sciences*, †2 appeared my first paper upon the subject. In this memoir, I was led, from logical considerations that are patent to those who read it, to endeavor to put the general expression of any linear associative algebra into a certain form; namely as a linear expression in certain units which I wrote thus:

$$\begin{array}{lll}
 (u[1]:u[1]) & (u[1]:u[2]) & (u[1]:u[3]), \text{ etc.}, \\
 (u[2]:u[1]) & (u[2]:u[2]) & (u[2]:u[3]), \text{ etc.}, \\
 (u[3]:u[1]) & (u[3]:u[2]) & (u[3]:u[3]), \text{ etc.}, \\
 \text{etc.} & \text{etc.} & \text{etc.}
 \end{array}$$

These forms, in their multiplication, follow these rules:

$$(u[a]:u[b]) (u[b]:u[c]) = (u[a]:u[c]) \quad (u[a]:u[b]) (u[c]:u[d]) = 0.$$

Peirce: CP 3.647 Cross-Ref:††

I said, "I can assert, upon reasonable inductive evidence, that all such algebras can be interpreted on the principles of the present notation in the same way," †1 and consequently can be put into this form. I afterwards published a proof of this. †2 I added that this amounted to saying that "all such algebras are complications and modifications of the . . . Hamilton's quaternions." †3 What I meant by this appears

plainly in the memoir. It is that any algebra that can be put into the form proposed by me is thereby referred to an algebra of a certain class (afterwards named *quadrates* by Professor Clifford) which present so close an analogy with quaternions that they may all be considered as mere complications of that algebra. Of these algebras, I gave as an example, the multiplication table of that one which Professor Clifford afterward named *nonions*. †P1 This is the passage: †4

Peirce: CP 3.647 Cross-Ref: ††

It will be seen that the system of nonions is not a group but an algebra; that just as the word "quaternion" is not restricted to the three perpendicular vectors and unity, so a nonion is any quantity of this nine-fold algebra.

Peirce: CP 3.647 Cross-Ref: ††

So much was published by me in 1870; and it then occurred either to my father or to me (probably in conversing together) that since this algebra was thus shown (through his form of quaternions) to be the strict analogue of quaternions, there ought to be a form of it analogous to Hamilton's standard tetranomial form of quaternions. That form, either he or I certainly found. I cannot remember, after so many years, which first looked for it; whichever did must have found it at once. I cannot tell what his method of search would have been, but I can show what my own must have been. The following course of reasoning was so obtrusive that I could not have missed it.

Hamilton's form of quaternions presents a group of four square-roots of unity. Are there, then, in nonions, nine independent cube-roots of unity, forming a group? Now, unity upon my system of notation was written thus:

$$(u[1]:u[1]) + (u[2]:u[2]) + (u[3]:u[3]).$$

Two independent cube-roots of this suggest themselves at once; they are

$$(u[1]:u[2]) + (u[2]:u[3]) + (u[3]:u[1])$$

$$(u[3]:u[2]) + (u[2]:u[1]) + (u[1]:u[3]).$$

In fact these are hinted at in my memoir, p. 53.[?] Then, it must have immediately occurred to me, from the most familiar properties of the imaginary roots of unity, that instead of the coefficients

$$1, \quad 1, \quad 1,$$

I might substitute

$$1, \quad g, \quad g^2$$

or

$$1, \quad g^2, \quad g$$

where g is an imaginary cube-root of unity. The nine cube-roots of unity thus obtained are obviously independent and obviously form a group. Thus the problem is solved by a method applicable to any other quadrate.

Peirce: CP 3.648 Cross-Ref:††

648. My father, with his strong partiality for my performances, talked a good deal about the algebra of nonions in general and these forms in particular; and they became rather widely known as mine. Yet it is clear that the only real merit in the discovery lay in my father's transformation of quaternions. In 1875, when I was in Germany, my father wrote to me that he was going to print a miscellaneous paper on multiple algebra and he wished to have it accompanied by a paper by me, giving an account of what I had found out. I wrote such a paper, and sent it to him; but somehow all but the first few pages of the manuscript were lost, a circumstance I never discovered till I saw the part that had reached him (and which he took for the whole) in print. I did not afterward publish the matter, because I did not attach much importance to it, and because I thought that too much had been made, already, of the very simple things I had done.

Peirce: CP 3.648 Cross-Ref:††

I here close the narrative. The priority of publication of the particular group referred to belongs to Professor Sylvester. But most readers will agree that he could not have desired to print it without making any allusion to my work, and that to say the group could be derived from my algebra was not too much.

Peirce: CP 3.1 Fn 1 p 3

†1 *Proceedings of the American Academy of Arts and Sciences*, vol. 7, pp. 250-61, March 1867.

Peirce: CP 3.3 Fn 2 p 3

†2 I.e., $a + b$ is the class of those things which are a not- b , b not- a or both a and b .

Peirce: CP 3.3 Fn 1 p 4

†1 I.e., if $a, b = 0$, $a + b = a +, b$. Logical addition allows conjunction, arithmetic addition is exclusive.

Peirce: CP 3.3 Fn 2 p 4

†2 (2) and (5) embody the law of tautology -- one of the features which distinguish the Boolean from the ordinary arithmetical calculus, limiting it to the

numbers 1 and 0.

Peirce: CP 3.5 Fn 1 p 5

†1 I.e., if x is a minimum, $a - b = a - b = a - (a, b)$; if a maximum, $a - b = a + a - b = a$.

Peirce: CP 3.5 Fn 2 p 5

†2 I.e., Class b must be contained in Class a ; b may of course be null.

Peirce: CP 3.5 Fn P1 p 5 Cross-Ref:††

†P1 So that, for example, $\sim a$ denotes not- a . [Elec. ed. note: The tilde (\sim) is used to mark negation in this electronic edition rather than a line above the symbol(s) negated.]

Peirce: CP 3.5 Fn 3 p 5

†3 I.e., the class $a - b$ is equal to the indeterminate class of that which is both a and b , plus the class of that which is a but not b , under the condition that there are no b 's which are not a .

Peirce: CP 3.5 Fn 1 p 6

†1 As $[0 = (x - x)] = [0 + x = x]$, 0 added to a class is the class. 0 represents the minimum which results from subtracting a class from itself.

Peirce: CP 3.6 Fn 2 p 6

†2 If x is a minimum, $a; b = a, b = a$; if a maximum, $a; b = \sim a, \sim b + a, b$.

Peirce: CP 3.6 Fn 3 p 6

†3 The class $a; b$ is equal to the class of that which is both a and b plus the indeterminate class of what is neither a nor b on the condition that there are no a 's which are not b .

Peirce: CP 3.7 Fn 4 p 6

†4 As $a; b$ represents the maximum or upper limit of $a; b$ (see 8) unity represents the maximum which results from dividing any class by itself. As $(1 = x; x) = (x, 1 = x)$ the result of multiplying a class by 1 is the class.

Peirce: CP 3.8 Fn 5 p 6

†5 As $a; b = a + \sim b, 0; x = (0 + \sim x = \sim x)$.

Peirce: CP 3.9 Fn 1 p 7

†1 See Lewis' *Survey of Symbolic Logic*, pp. 58-67, 82 and 132-174 for a very clear presentation and a development of these "transformations."

Peirce: CP 3.9 Fn 2 p 7

†2 $f(x) = x, a + \sim x, b$.

Peirce: CP 3.9 Fn P1 p 7 Cross-Ref:††

†P1 The proof offered for this is fallacious inasmuch as i and j have not been proved to be independent of x .--1870. [Peirce follows this remark with a proof which is too long and of insufficient interest to be reproduced].

Peirce: CP 3.9 Fn P2 p 7 Cross-Ref:††

†P2 Identity (18') is reducible to (18) by development by second member by (18).--1870.

Peirce: CP 3.10 Fn P1 p 8 Cross-Ref:††

†P1 $a;b, c$ must always be taken as $(a;b), c$, not as $a;(b, c)$.

Peirce: CP 3.10 Fn 1 p 8

†1 *Laws of Thought*, vol. 2, p. 87ff.

Peirce: CP 3.12 Fn 1 p 9

†1 *Ibid.*, p. 10.

Peirce: CP 3.13 Fn 2 p 9

†2 I.e., from the logical relations of class identity in extension to the "arithmetical" relations of numerical equality. Cf. 44.

Peirce: CP 3.14 Fn 3 p 9

†3 'Arithmetical' multiplication is represented by juxtaposing the terms.

Peirce: CP 3.17 Fn 1 p 13

†1 Cf. 2.674.

Peirce: CP 3.19 Fn 1 p 15

†1 *Logic of Chance*, ch. 9, section 24.

Peirce: CP 3.19 Fn P1 p 15 Cross-Ref:††

†P1 See a notice, Venn's *Logic of Chance*, in the North American Review for July 1867 [vol. 9].

Peirce: CP 3.20 Fn 1 p 16

†1 *Proceedings of the American Academy of Arts and Sciences*, vol. 7, pp. 402-412, September 1867.

Peirce: CP 3.20 Fn 2 p 16

†2 No other part seems to have been written.

Peirce: CP 3.21 Fn 3 p 16

†3 See Paper No. I.

Peirce: CP 3.21 Fn 1 p 17

†1 a and b are independent if they are summations of terms (4 and 5) each of whose members is distinct (1 and 2), so that there is a class of the terms of a and b together (3) and a term in a has a corresponding member in b (6). There are as many members of a,b as there are combinations of a member of a with one of b . Cf. 33.

Peirce: CP 3.27 Fn 1 p 18

†1 --because in transfinite arithmetic finite quantities can be added to infinities without affecting the total -- $\aleph + x = \aleph + y = \aleph$ where x and y are finite and \aleph is the smallest transfinite cardinal.

Peirce: CP 3.31 Fn 1 p 20

†1 Originally m .

Peirce: CP 3.31 Fn 2 p 20

†2 Originally u .

Peirce: CP 3.39 Fn 1 p 23

†1 See 4n.

Peirce: CP 3.41 Fn 2 p 23

†2 See 2.467. Cf. Paul Weiss, *Erkenntnis* Bd. 2, H. 4, S. 242-8 where it is shown that all the logical propositions are variations of some such form as $P Q + \sim P Q + P \sim Q + \sim (P Q)$

Peirce: CP 3.42 Fn 1 p 24

†1 The ideas here presented for the derivation of arithmetic from logic are somewhat similar to those employed in the *Principia Mathematica*, Whitehead and Russell, vol 2, section A (1912).

Peirce: CP 3.43 Fn 2 p 24

†2 See vol. 2, bk. II, ch. 5. for a discussion of these terms.

Peirce: CP 3.43 Fn 3 p 24

†3 This leads to a somewhat similar definition of a cardinal number as that given in the *Principia Mathematica*. But see 4.333 where this paper is characterized as being the worst Peirce ever published.

Peirce: CP 3.44 Fn P1 p 25 Cross-Ref:††

†P1 Thus, in one point of view, *identity* is a species of *equality*, and, in another, the reverse is the case. This is because the Being of the copula may be considered on the one hand (with De Morgan [*Formal Logic*, p. 59]) as a special description of "inconvertible, transitive relation," while, on the other hand, all relation may be considered as a special determination of Being. If a Hegelian should be disposed to see a contradiction here, an accurate analysis of the matter will show him that it is only a verbal one.

Peirce: CP 3.44 Fn 1 p 25

†1 Cf. *Principia Mathematica*, vol. 2, section A.

Peirce: CP 3.45 Fn 1 p 27 Cross-Ref:††

†1 *Memoirs of the American Academy*, vol. 9, pp. 317-78 (1870). Reprinted separately by Welch, Bigelow and Company, Cambridge, Mass., 1870, pp. 1-62. "In 1870 I made a contribution to this subject [logic] which nobody who masters the subject can deny was the most important excepting Boole's original work that ever has been made." -- From the "Lowell Lectures," 1903.

Peirce: CP 3.45 Fn 2 p 27

†2 "On the Syllogism No. IV, and on the Logic of Relations," pp. 331-58, dated 1859.

Peirce: CP 3.47 Fn P1 p 28 Cross-Ref:††

†P1 I use the sign \prec in place of \mid . My reasons for not liking the latter sign are that it cannot be written rapidly enough, and that it seems to represent the relation it expresses as being compounded of two others which in reality are complications of this. It is universally admitted that a higher conception is logically more simple than a lower one under it. Whence it follows from the relations of extension and comprehension, that in any state of information a broader concept is more simple than a narrower one included under it. Now all equality is inclusion in, but the converse is not true; hence inclusion in is a wider concept than equality, and therefore logically a simpler one. On the same principle, inclusion is also simpler than being less than. The sign \mid seems to involve a definition by enumeration; and such a definition offends against the laws of definition.

Peirce: CP 3.51 Fn P1 p 29 Cross-Ref:††

†P1 I write a comma below the sign of addition, except when (as is the case in ordinary algebra) the corresponding inverse operation (subtraction) is determinative, [i.e., except when the addition is arithmetical.]

Peirce: CP 3.52 Fn 1 p 29
†1 I.e., arithmetical.

Peirce: CP 3.52 Fn 2 p 29
†2 See 27.

Peirce: CP 3.53 Fn 3 p 29
†3 Cf. 55, 69f.

Peirce: CP 3.53 Fn 4 p 29
†4 I.e., "non-relative," or what was before called "logical."

Peirce: CP 3.53 Fn 5 p 29
†5 Cf. 73, 74n.

Peirce: CP 3.54 Fn 1 p 30
†1 I.e., arithmetical. See 75.

Peirce: CP 3.54 Fn 2 p 30
†2 The symbolism of the earlier papers is here slightly modified: the simple conjunction of terms now represents relative, instead of arithmetical, multiplication, and the dot is introduced to represent arithmetical multiplication. The comma, however, is still retained for logical multiplication.

Peirce: CP 3.54 Fn 3 p 30
†3 See 36.

Peirce: CP 3.55 Fn 4 p 30
†4 Cf. 71 and 72.

Peirce: CP 3.56 Fn P1 p 30 Cross-Ref:††
†P1 In the notation of quaternions Hamilton has assumed

$(xy)z=x(z y)$ instead of $(xy)z=x(y z)$,

although it appears to make but little difference which he takes. Perhaps we should assume two involutions, so that

$(xy)z=x(yz), \quad z(yx)=(zy)x.$

But in this paper only the former of these is required. [See 113ff. for the latter.]

Peirce: CP 3.56 Fn 1 p 31
†1 See 77.

Peirce: CP 3.57 Fn 2 p 31

†2 I.e., logical; see 10.

Peirce: CP 3.59 Fn 3 p 31

†3 See 10 (24).

Peirce: CP 3.61 Fn 1 p 33

†1 The symbol \ominus represents the base of Napierian logarithms, the symbol \odot represents π and \surd represents the square root of the negative in Benjamin Peirce's *Linear Associative Algebras*, §15ff (1870).

Peirce: CP 3.63 Fn 1 p 34

†1 Cf. the discussion on the categories in vol. 1, bk. III, and on signs in vol. 2, bk. II.

Peirce: CP 3.63 Fn 2 p 34

†2 Cf. 421 and 1.347.

Peirce: CP 3.63 Fn 3 p 34

†3 Cf. 69n.

Peirce: CP 3.65 Fn 1 p 35

†1 "On the Structure of the Syllogism," Section 1, *Cambridge Philosophical Transactions*, vol. 8 (1846); *Formal Logic*, p. 37 (1847).

Peirce: CP 3.65 Fn 2 p 35

†2 Cf. 2.518ff.

Peirce: CP 3.66 Fn 3 p 35

†3 *Laws of Thought*, p. 27.

Peirce: CP 3.66 Fn P1 p 35 Cross-Ref:††

†P1 According to De Morgan, *Formal Logic*, p. 334. De Morgan refers to the first edition of Drobisch's Logic. The third edition contains nothing of the sort.

Peirce: CP 3.66 Fn 1 p 36

†1 Cf. 42-44. A class is the extension of a collection; see e.g., 537n.

Peirce: CP 3.67 Fn 2 p 36

†2 *Op. cit.*, p. 33.

Peirce: CP 3.67 Fn 1 p 37

†1 In 3.

Peirce: CP 3.67 Fn 2 p 37

†2 Ch. 6, §63; ch. 15, §177ff.

Peirce: CP 3.67 Fn P1 p 37 Cross-Ref:††

†P1 In another book [*Substitution of Similar*s (1869) and subsequent works] he uses the sign $\cdot\mid$ instead of +.

Peirce: CP 3.67 Fn 1 p 38

†1 Cf. 82.

Peirce: CP 3.69 Fn 1 p 39

†1 h is a variable designating an unspecified H. See 84, 94, 111.

Peirce: CP 3.70 Fn 1 p 40

†1 See 73.

Peirce: CP 3.72 Fn 1 p 41

†1 In 55.

Peirce: CP 3.73 Fn 2 p 41

†2 l, s is the logical product of l and s ; $l s$, on the other hand, is the relative product of l and s . See 54n, 68.

Peirce: CP 3.73 Fn 1 p 42

†1 A servant of herself who is also a woman is the same as a servant of a woman?

Peirce: CP 3.73 Fn 1 p 43

†1 "Something" is whatever is identical with an undetermined thing.

Peirce: CP 3.73 Fn 2 p 43

†2 "Anything" is whatever is identical with itself.

Peirce: CP 3.74 Fn P1 p 43 Cross-Ref:††

†P1 It will often be convenient to speak of the whole operation of affixing a comma and then multiplying, as a commutative multiplication, the sign for which is the comma. But though this is allowable, we shall fall into confusion at once if we ever forget that in point of fact it is not a different multiplication, only it is multiplication by a relative whose meaning -- or rather whose syntax -- has been slightly altered; and that the comma is really the sign of this modification of the foregoing term.

Peirce: CP 3.76 Fn 1 p 44

†1 *Op. cit.*, p. 243ff.

Peirce: CP 3.77 Fn 1 p 45

†1 This is more accurately read as: a servant of all those who are either men or women.

Peirce: CP 3.77 Fn 2 p 45

†2 Cf. J. N. Lambert. "Sechs Versuche einer Zeichenkunst in der Vernunftlehre"; in *Logische u. Philosophische Abhandlungen*, vol. 1, ed. J. Bernouilli, Berlin, (1782) for the use of this "Newtonian Formula" in an intensional logic of absolute terms.

Peirce: CP 3.77 Fn 1 p 46

†1 I.e., a servant and lover of every woman is a servant of every woman and a lover of every woman.

Peirce: CP 3.78 Fn P1 p 46 Cross-Ref:††

†P1 "The same" substituted for "an"; "some" for "a," "every" for "all" -- ink correction on C. S. P.'s own copy; cf. 145.

Peirce: CP 3.78 Fn P1 p 47 Cross-Ref:††

†P1 "Follows from last because [it is] negative of $\sim bm^w$ "--marginal note.

Peirce: CP 3.78 Fn 1 p 47

†1 See 145.

Peirce: CP 3.81 Fn 1 p 48

†1 This proposition is the source of the famous so-called paradoxes of material implication.

Peirce: CP 3.81 Fn 2 p 48

†2 This sentence seems to have been misplaced and should have appeared after (22) or (23).

Peirce: CP 3.82 Fn 1 p 50

†1 I.e., by (21) $0x \rightarrow ax$ and as by (12) $ax + 0x = ax$, $0x \rightarrow ax$.

Peirce: CP 3.86 Fn 1 p 53

†1 On his own copy, Peirce substitutes the condition " x is an unlimited relative," for " $x > 0$."

Peirce: CP 3.88 Fn 1 p 54

†1 See 198.

Peirce: CP 3.90 Fn 1 p 55

†1 In his *Pure Logic*.

Peirce: CP 3.91 Fn 1 p 56

†1 Where c = not, this becomes the formula for contraposition. See 142 and 2.550. In 186, however, a different formula for contraposition is given.

Peirce: CP 3.93 Fn 1 p 57

†1 Cf. the definition of individuals in 611-13.

Peirce: CP 3.93 Fn P1 p 58 Cross-Ref:††

†P1 The absolute individual can not only not be realized in sense or thought, but cannot exist, properly speaking. For whatever lasts for any time, however short, is capable of logical division, because in that time it will undergo some change in its relations. But what does not exist for any time, however short, does not exist at all. All, therefore, that we perceive or think, or that exists, is general. So far there is truth in the doctrine of scholastic realism. But all that exists is infinitely determinate, and the infinitely determinate is the absolutely individual. This seems paradoxical, but the contradiction is easily resolved. That which exists is the object of a true conception. This conception may be made more determinate than any assignable conception; and therefore it is never so determinate that it is capable of no further determination.

Peirce: CP 3.95 Fn 1 p 59

†1 See 96ff.

Peirce: CP 3.95 Fn 2 p 59

†2 See 100ff.

Peirce: CP 3.95 Fn 3 p 59

†3 See 121ff.

Peirce: CP 3.97 Fn 1 p 60

†1 π' signifies logical multiplication and Σ' signifies logical addition.

Peirce: CP 3.97 Fn 2 p 60

†2 A servant of every woman is a servant of a woman.

Peirce: CP 3.97 Fn 3 p 60

†3 A lover-of-a-servant of every woman is a lover-of-a-servant of a woman.

Peirce: CP 3.97 Fn 1 p 61

†1 See Lewis' *Survey of Symbolic Logic*, p. 87n for a proof of this theorem.

Peirce: CP 3.97 Fn 2 p 61

†2 A lover of every servant of all women is a lover of a servant of every woman.

Peirce: CP 3.97 Fn 3 p 61

†3 A lover of every servant-of-a-woman is to a woman a lover of all her servants.

Peirce: CP 3.97 Fn P1 p 61 Cross-Ref:††

†P1 $l s^w \prec l s^w$ and $l s^w \prec (l s)^w$ invariably holds--marginal note.

Peirce: CP 3.98 Fn 1 p 62

†1 On Peirce's copy a line was drawn through the vinculum in each of these cases with the comment, "Crossed are not universally true."

Peirce: CP 3.104 Fn 1 p 64

†1 I.e., $1-(x y)$.

Peirce: CP 3.104 Fn 2 p 64

†2 I.e., $-(x y)$.

Peirce: CP 3.104 Fn 3 p 64

†3 I.e., $-(x', x'') - (x', x''')$ etc.

Peirce: CP 3.104 Fn 4 p 64

†4 I.e., $-(x', x'', x''', x''''')$.

Peirce: CP 3.106 Fn P1 p 65 Cross-Ref:††

†P1 It makes another resemblance between 1 and infinity that $\log 0 = -1$.

Peirce: CP 3.111 Fn 1 p 67

†1 In 56.

Peirce: CP 3.111 Fn 1 p 68

†1 I.e., each correlate has only one relate though a given relate may have many correlates.

Peirce: CP 3.112 Fn 1 p 69

†1 I.e., as a lover of none but servants.

Peirce: CP 3.112 Fn 2 p 69

†2 A lover of all servants is not a non-lover of a servant,

Peirce: CP 3.112 Fn 3 p 69

†3 A lover of none but servants is not a lover of a non-servant. Cf. 116.

Peirce: CP 3.112 Fn 4 p 69

†4 *Formal Logic*, p. 341.

Peirce: CP 3.112 Fn 5 p 69

†5 Cf. *ib.*, p. 343; see also 244.

Peirce: CP 3.116 Fn 1 p 70

†1 An x of none but y 's is a non- x of all non- y 's; i.e., $xy = -x'y$.

Peirce: CP 3.118 Fn 1 p 72

†1 Cf. (92), (93) and 332.

Peirce: CP 3.118 Fn 2 p 72

†2 Or: Whatever is a lover-of-a-servant to nothing but women . . . etc.

Peirce: CP 3.121 Fn 1 p 75

†1 Cf. 602ff.

Peirce: CP 3.126 Fn 1 p 78

†1 See Lewis, *Survey of Symbolic Logic*, p. 103, for a symbolic and analytic account of some of these propositions.

Peirce: CP 3.130 Fn P1 p 80 Cross-Ref:††

†P1 *Linear Associative Algebras*. By Benjamin Peirce. 4to, lithographed. Washington. 1870. [Published with notes and addenda by C. S. Peirce in *The American Journal of Mathematics*, vol. 4, pp. 97-229 (1881); see paper no. VIII of this volume.]

Peirce: CP 3.130 Fn 1 p 81

†1 *Ibid.*, (161²).

Peirce: CP 3.130 Fn 2 p 81

†2 See Appendix to this volume.

Peirce: CP 3.132 Fn 1 p 82

†1 By (161).

Peirce: CP 3.133 Fn 1 p 83

†1 In his "New Elements of Geometry With a Complete Theory of Parallels," *Gelehrte Schriften der Universität*, Kasan, 1836-1838.

Peirce: CP 3.134 Fn P1 Para 1/3 p 84 Cross-Ref:††

†P1 The researches of Lobatchewsky furnish no solution of the question concerning the apriority of space. For though he has shown that it is conceivable that space should have such properties that two lines might be in a plane and inclined to one another without ever meeting, however far produced, yet he has not shown that the facts implied in that supposition are inconsistent with supposing space to retain its present [Euclidean] nature and the properties only of the things in it to change. For example, in Lobatchewsky's geometry a star at an infinite distance has a finite parallax. But suppose space to have its present properties, and suppose that there were one point in the universe towards which anything being moved should expand, and away from which being moved should contract. Then this expansion and contraction might obey such a law that a star, the parallax of which was finite, should be at an infinite distance measured by the number of times a yard-stick must be laid down to measure off that distance. I have not seen Beltrami's investigations, ["Saggio di interpretazione della geometria non-euclidea," *Giorn. di Matem.*, 6, 1868.] but I understand that they do show that something of this sort is possible. Thus, it may be that, make what suppositions you will concerning phenomena, they can always be reconciled to our present geometry or be shown to involve implicit contradictions. If this is so--and whether it is or not is a completely open question--then the principles of geometry are necessary, and do not result from the specialities of any object

cognized, but from the conditions of cognition in general. In speaking of the conditions of cognition, in general, I have in view no psychological conception, but only a distinction between principles which, if the facts should present a sufficient difficulty, I may always logically doubt, and principles which it can be shown cannot become open to doubt from any difficulty in my facts, as long as they continue to be supposed in all logical procedure.

Peirce: CP 3.134 Fn P1 Para 2/3 p 84 Cross-Ref: ††

But, waiving this point, Lobatchewsky's conclusions do not positively overthrow the hypothesis that space is *a priori*. For he has only shown that a certain proposition, *not usually believed to be axiomatical*, is conceivably false. That people may be doubtful or even mistaken about *a priori* truth does not destroy all important practical distinction between the two kinds of necessity. It may be said that if Lobatchewsky's geometry is the true one, then space involves an arbitrary constant, which value cannot be given *a priori*. This may be; but it may be that the general properties of space, with the general fact that there is such a constant, are *a priori*, while the value of the constant is only empirically determined.

Peirce: CP 3.134 Fn P1 Para 3/3 p 84 Cross-Ref: ††

It appears to me plain that no geometrical speculations will settle the philosophy of space, which is a logical question. If space is *a priori*, I believe that it is in some recondite way involved in the logic of relatives.

Peirce: CP 3.135 Fn 1 p 85

†1 Cf. 225 and 585.

Peirce: CP 3.136 Fn P1 p 86 Cross-Ref: ††

†P1 "As we speak of *self-loving*, etc., the former of these classes should be called self-relatives" -- marginal note.

Peirce: CP 3.136 Fn 1 p 86

†1 Cf. 230.

Peirce: CP 3.136 Fn P2 p 86 Cross-Ref: ††

†P2 "The idea of a copula is different. These should be called *assimilative*." -- Marginal note. See 592.

Peirce: CP 3.136 Fn 2 p 86

†2 *Op. cit.*, §25; see also 593.

Peirce: CP 3.136 Fn P1 p 87 Cross-Ref: ††

†P1 "If such reciprocation is admissible but not necessary they may be called *reciprocal*." -- Marginal note.

Peirce: CP 3.136 Fn P2 p 87 Cross-Ref: ††

†P2 "Quædam sunt relatione equiparantiæ, quædam disquiparantiæ. Primæ sunt relationes similium nominum, secundæ relationes dissimilium nominum. Exemplum primi est quando idem nomen ponitur in recto et in obliquo, sicut simile simili est simile.... Exemplum secundi est quando unum nomen ponitur in recto sed aliud in obliquo, sicut pater est filii pater et non oportet quod sit patris pater." Ockham *Quodlibetum* 6, qu. 20. See also his *Summa Logices*, pars 1, cap. 52. "Relativa equiparantiæ: quæ sunt synonyma cum suis correlativis.... Relativa diquiparantiæ: quæ non sunt synonyma cum suis correlativis." Pschlacher in Petr. Hisp. The same definitions substantially may be found in many late mediæval logics.

Peirce: CP 3.136 Fn 1 p 87
†1 *Formal Logic*, p. 345

Peirce: CP 3.136 Fn P3 p 87 Cross-Ref:††
†P3 "Instead of self-relatives, better concurrents."--Marginal note.

Peirce: CP 3.136 Fn P4 p 87 Cross-Ref:††
†P4 "Insert 'only' between contains and 'elements'; for 'non-cyclic' substitute 'acyclic.'" -- Marginal note.

Peirce: CP 3.136 Fn 2 p 87
†2 Cf. 233.

Peirce: CP 3.136 Fn P5 p 87 Cross-Ref:††
†P5 "Insert 'only' between contains and 'elements'; for 'non-cyclic' substitute 'acyclic.'" -- Marginal note.

Peirce: CP 3.136 Fn 1 p 88
†1 *Op. cit.*, p. 346.

Peirce: CP 3.136 Fn 2 p 88
†2 For if R is transitive then $aRb.bRc \rightarrow aRc$; if R is also equiparant then $aRb.bRa$ is true and aRa is a necessary consequent.

Peirce: CP 3.136 Fn P1 p 88 Cross-Ref:††
†P1 "Should be called concatenated" -- marginal note.

Peirce: CP 3.136 Fn P1 p 89 Cross-Ref:††
†P1 "Duplex est relatio: scilicet rationis et realis. Unde relatio rationis est quae fit per actum comparativum intellectus, ut sunt secundae intentiones; sed relatio realis est duplex, scilicet aptitudinalis et actualis. Aptitudinalis est quae non requirit terminum actu existere sed solum in aptitudine; cujusmodi sunt omnes propriae passionis, omnes aptitudines, et omnes inclinationes; et tales sunt in illo praedicamento reductive in quo sunt illa quorum sunt propriae passionis. Sed relatio actualis est duplex, scilicet, intrinsecus adveniens, et extrinsecus adveniens. Intrinsecus adveniens est quae necessario ponitur positis extremis in quacunque etiam distantia ponantur, ut similitudo, paternitas, equalitas. Extrinsecus adveniens est quae necessario non ponitur, positis extremis, sed requiritur debita approximatio extremorum; cujusmodi sunt sex ultima praedicamenta, scilicet, actio, passio, quando, ubi, situs, et habitus." Tartaretus.

Peirce: CP 3.137 Fn 1 p 89
†1 Cf. 8 and 81 (25), (26.)

Peirce: CP 3.137 Fn 2 p 89
†2 These two together contain De Morgan's Theorem to the effect that the negative of a logical product is the logical sum of the negatives of its factors; and that the negative of a sum ($z=\text{not}$) is the product of the negatives of the summands.

Peirce: CP 3.138 Fn 1 p 90
†1 *Laws of Thought*, p. 62.

Peirce: CP 3.140 Fn 1 p 93
†1 This should be $1(1-B) \rightarrow 1(1-A)$

Peirce: CP 3.143 Fn 1 p 94
†1 In 146.

Peirce: CP 3.146 Fn 1 p 96
†1 The second half of this equation should be: †[13]†
 $x y z$.

Peirce: CP 3.147 Fn 1 p 98
†1 In 112.

Peirce: CP 3.150 Fn 1 p 99
†1 *Proceedings of the American Academy of Arts and Sciences*, pp. 392-94, vol. 10, (1875).

Peirce: CP 3.150 Fn 2 p 99
†2 See 294 for another approach to this same problem.

Peirce: CP 3.151 Fn 1 p 100
†1 See Benjamin Peirce's *Linear Associative Algebras, op. cit.*, p. 195.

Peirce: CP 3.151 Fn 2 p 100
†2 *Ibid.*, p. 188.

Peirce: CP 3.151 Fn 1 p 101
†1 *Ibid.*, p. 209.

Peirce: CP 3.151 Fn 2 p 101
†2 *Ibid.*, p. 202.

Peirce: CP 3.152 Fn 1 p 102
†1 *Proceedings of the American Academy of Arts and Sciences*, vol. 13, pp. 115-16, (1877).

Peirce: CP 3.152 Fn 2 p 102
†2 "Die mechanik nach den Principien der Ausdehnungslehre," Bd. 12, H. 2, S. 222-40, (1877).

Peirce: CP 3.152 Fn 1 p 103
†1 Originally 'vectors'; corrected by Peirce.

Peirce: CP 3.153 Fn 2 p 103
†2 Originally 'relation'; corrected by Peirce.

Peirce: CP 3.154 Fn 1 p 104
†1 *American Journal of Mathematics*, vol. 3, pp. 15-57 (1880), with Peirce's marginal corrections, and the printed corrections of September 15, 1880, in which he says, "The manuscript left my hands in April last before I had seen several important publications -- Mr. McColl's third paper, Prof. Wundt's *Logik*, etc."

Peirce: CP 3.154 Fn 2 p 104
†2 The editors have changed 'Chapter' to 'Part.'

Peirce: CP 3.154 Fn P1 p 104 Cross-Ref:††
†P1 "The whole of these two parts is bad, first, because it does not treat the subject from the point of view of pure mathematics, as it should have done; and

second because the fundamental propositions are not made out. I follow too much in the footsteps of ordinary numerical algebra, and the sketch of the algebra of the copula is very insufficient." -- from the Lowell Lectures, 1903.

Peirce: CP 3.159 Fn 1 p 105
†1 Cf. 2.146, 2.148.

Peirce: CP 3.160 Fn P1 p 106 Cross-Ref:††
†P1 **Deductive logic**, perhaps, does not involve the principle that there is any special character in the peripheral excitation but only that reasoning proceeds by habits that are consistent. **Deductive** -- consistency of thought with itself. **Inductive** -- consistency of the world (Uniformity of Nature). -- marginal note, c. 1882.

Peirce: CP 3.162 Fn 1 p 106
†1 Cf. vol. 2, bk. III, ch. 1, §§1, 2, and ch. 2, Part I.

Peirce: CP 3.164 Fn P1 p 107 Cross-Ref:††
†P1 Though the leading principle itself is not present to the mind, we are generally conscious of inferring on some general principle. [Cf. 2.186 ff.]

Peirce: CP 3.165 Fn P1 p 108 Cross-Ref:††
†P1 This dash was used by Boole, but not over other than class-signs.

Peirce: CP 3.171 Fn P1 p 111 Cross-Ref:††
†P1 The general doctrine of this section is contained in my paper, **On the Natural Classification of Arguments**, 1867 [vol. 2, Bk. III, ch. 2].

Peirce: CP 3.173 Fn P2 p 111 Cross-Ref:††
†P2 There is a difference of opinion among logicians as to whether $-<$ or $=$ is the simpler relation. But in my paper on the **Logic of Relatives** [47n.], I have strictly demonstrated that the preference must be given to $-<$ in this respect. The term **simpler** has an exact meaning in logic; it means that whose logical depth is smaller; that is, if one conception implies another, but not the reverse, then the latter is said to be the simpler. Now to say that $A = B$ implies that $A -< B$, but not conversely. **Ergo**, etc. It is to no purpose to reply that $A -< B$ implies $A = (A \text{ that is } B)$; it would be equally relevant to say that $A -< B$ implies $A = A$. Consider an analogous case. Logical sequence is a simpler conception than causal sequence, because every causal sequence is a logical sequence but not every logical sequence is a causal sequence; and it is no reply to this to say that a logical sequence between two facts implies a causal sequence between some two facts whether the same or different. The idea that $=$ is a very simple relation is probably due to the fact that the discovery of such a relation teaches us that instead of two objects we have only one, so that it simplifies our conception of the universe. On this account the existence of such a relation is an important fact to learn; in fact, it has the sum of the importances of the two facts of which it is compounded. It frequently happens that it is more convenient to treat the propositions $A -< B$ and $B -< A$ together in their form $A = B$; but it also frequently happens that it is more convenient to treat them separately. Even in geometry we can see that to say that two figures A and B are equal is to say that when they are properly put together A will cover B and B will cover A ; and it is generally necessary to examine these facts separately. So, in comparing the numbers of two lots of objects, we set them over against one another, each to each, and observe that for every one of the lot A there is one of the lot B , and for every one of the lot B there is one of the lot A .

In logic, our great object is to analyse all the operations of reason and reduce them to their ultimate elements; and to make a calculus of reasoning is a subsidiary object. Accordingly, it is more philosophical to use the copula \rightarrow apart from all considerations of convenience. Besides, this copula is intimately related to our natural logical and metaphysical ideas; and it is one of the chief purposes of logic to show what validity those ideas have. Moreover, it will be seen further on that the more analytical copula does in point of fact give rise to the easiest method of solving problems of logic.

Peirce: CP 3.173 Fn 1 p 112

†1 I.e., $\sim(A = B)$.

Peirce: CP 3.174 Fn 2 p 112

†2 "On the Structure of the Syllogism, and on the Application of the Theory of Probabilities to Questions of Argument and Authority." *Transactions, Cambridge Philosophical Society*, vol. 8, pp. 379-408, (1849). The paper was read and dated 1846.

Peirce: CP 3.175 Fn 3 p 112

†3 I.e., it is false that $A \rightarrow B$.

Peirce: CP 3.175 Fn P1 p 113 Cross-Ref:††

†P1 In consequence of the identification in question, in $S \rightarrow P$, I speak of S indifferently as *subject*, *antecedent*, or *premiss*, and of P as *predicate*, *consequent*, or *conclusion*.

Peirce: CP 3.176 Fn 1 p 113

†1 See *Laws of Thought*, p. 62f.

Peirce: CP 3.176 Fn 2 p 113

†2 See 138.

Peirce: CP 3.176 Fn P2 p 113 Cross-Ref:††

†P2 Equally unsuccessful is Mr. Jevons' attempt to overcome the difficulty by omitting particular propositions, 'because we can always substitute for it [*some*] more definite expressions if we like.' The same reason might be alleged for neglecting the consideration of *not*. But in fact the form $A \sim\rightarrow B$ is required to enable us to simply deny $A \rightarrow B$.

Peirce: CP 3.176 Fn 3 p 113

†3 To express such a particular proposition disjunctively, change the quantity and quality of the antecedent and the consequent and deny their disjunction. Cf. 196.

Peirce: CP 3.176 Fn 1 p 114

†1 See 178.

Peirce: CP 3.179 Fn 1 p 115

†1 See vol. 2, Bk. III, ch. 1, §3 for a later analysis of this quadrant.

Peirce: CP 3.180 Fn 2 p 115

†2 The readings in this column are not precise.

Peirce: CP 3.180 Fn 3 p 115

†3 The terms in this column are taken from De Morgan's later papers.

Peirce: CP 3.181 Fn 4 p 115
†4 Cf. vol. 2, Bk. III, ch. 3, §3.

Peirce: CP 3.181 Fn P1 p 116 Cross-Ref:††
†P1 In this connection see De Morgan, "On the Syllogism," No. V., 1862. [*Transactions, Cambridge Philosophical Society*, vol. 4, p. 467, (1864), read and dated 1863.]

Peirce: CP 3.182 Fn P2 p 116 Cross-Ref:††
†P2 Mr. Hugh McColl (*Calculus of Equivalent Statements*, Second Paper, 1878, [*Proceedings, London Mathematical Society*, vol. 9, p. 183 (1877)]), makes use of the sign of inclusion several times in the same proposition. He does not, however, give any of the formulæ of this section.

Peirce: CP 3.184 Fn P1 p 117 Cross-Ref:††
†P1 "On the Syllogism," No. II., 1850, [*Transactions, Cambridge Philosophical Society*, vol. 9, (1851), p. 104].

Peirce: CP 3.184 Fn P2 p 117 Cross-Ref:††
†P2 That the validity of syllogism is not deducible from the principles of identity, contradiction, and excluded middle, is capable of strict demonstration. The transitivity of the copula is, however, implied in the identification of the copula-relation with illation, because illation is obviously transitive.

Peirce: CP 3.184 Fn P3 p 117 Cross-Ref:††
†P3 The conception of substitution (already involved in the mediæval doctrine of descent), as well as the word, was familiar to logicians before the publication of Mr. Jevons's *Substitution of Similars*. [see vol. 8] This book argues, however, not only that inference is substitution, but that it and induction in particular consist in the substitution of similars. This doctrine is allied to Mill's theory of induction.

Peirce: CP 3.184 Fn P4 p 117 Cross-Ref:††
†P4 This must have been in Boole's mind from the first. De Morgan ("On the Syllogism," No. II., 1850, p. 83) goes too far in saying that "what is called elimination in algebra is called inference in logic," if he means, as he seems to do, that all inference is elimination. [Cf. 2.442f.]

Peirce: CP 3.186 Fn 1 p 117
†1 See 91n.

Peirce: CP 3.191 Fn P1 p 119 Cross-Ref:††
†P1 De Morgan, *Syllabus*, 1860, p. 18.

Peirce: CP 3.192 Fn 1 p 120
†1 Cf. 2.597-8.

Peirce: CP 3.193 Fn P1 p 121 Cross-Ref:††
†P1 An oversight has here been committed. For from $\sim A = (A \rightarrow x)$ follows not merely (16) but also (19), (20), and (21), and thus all the properties of the negative which concern syllogistic. But this does not affect the view taken of the subject, nor the division of the moods according to the properties of the negative on which they depend; for whatever is shown in the text to be deducible from $\sim A = (A \rightarrow x)$ is in fact deducible from (16). -- Sept., 1880.

Peirce: CP 3.194 Fn P2 p 121 Cross-Ref:††

†P2 Aristotle and De Morgan have particular conclusions from two universal premisses. These are all rendered illogical by the significations which I attach to $\sim\prec$ and $\sim\sim\prec$.

Peirce: CP 3.195 Fn 1 p 121

†1 $\sim P$ and $\sim C$ here represent some such forms as $S \sim\prec M$ and $S \sim\sim\prec P$.

Peirce: CP 3.196 Fn 1 p 123

†1 See 2.458.

Peirce: CP 3.197 Fn 1 p 124

†1 This should be: either $(x \sim\prec y)$ or y .

Peirce: CP 3.198 Fn P1 p 125 Cross-Ref:††

†P1 The symbol 0 is used by Boole; the symbol ∞ replaces his 1, according to a suggestion in my *Logic of Relatives*, 1870 [88].

Peirce: CP 3.199 Fn P2 Para 1/2 p 125 Cross-Ref:††

†P2 These forms of definition are original. The algebra of non-relative terms was given by Boole (*Mathematical Analysis of Logic*, 1847). Boole's addition was not the same as that in the text, for with him whatever was common to the two terms added was taken twice over in the sum. The operations in the text were given as complements of one another, and with appropriate symbols, by De Morgan ("On the Syllogism," No. III., 1858 [*Cambridge Philosophical Transactions*, vol. 10], p. 185). For addition, sum, parts, he uses aggregation, aggregate, aggregants; for multiplication, product, factors, he uses composition, compound, components. Mr. Jevons (*Formal Logic [Pure Logic?]*, 1864)--I regret that I can only speak of this work from having read it many years ago, and therefore cannot be sure of doing it full justice--improved the algebra of Boole by substituting De Morgan's aggregation for Boole's addition. The present writer, not having seen either De Morgan's or Jevons's writings on the subject, again recommended the same change (*On an Improvement in Boole's Calculus of Logic*, 1867 [3]), and showed the perfect balance existing between the two operations. In another paper, published in 1870 [47], I introduced the sign of inclusion into the algebra.

Peirce: CP 3.199 Fn P2 Para 2/2 p 125 Cross-Ref:††

In 1872, Robert Grassmann, brother of the author of the *Ausdehnungslehre*, published a work entitled '*Die Formenlehre oder Mathematik*,' the second book of which gives an algebra of logic identical with that of Jevons. The very notation is reproduced, except that the universe is denoted by T instead of U, and a term is negated by drawing a line over it, as by Boole, instead of by taking a type from the other case, as Jevons does. Grassmann also uses a sign equivalent to my $\sim\prec$. In his third book, he has other matter which he might have derived from my paper of 1870. Grassmann's treatment of the subject presents inequalities of strength; and most of his results had been anticipated. Professor Schröder, of Karlsruhe, in the spring of 1877, produced his *Operationskreis des Logikkalkuls*. He had seen the works of Boole and Grassmann, but not those of De Morgan, Jevons, and me. He gives a fine development of the algebra, adopting the addition of Jevons, and he exhibits the balance between + and X by printing the theorems in parallel columns, thus imitating a practice of the geometers. Schröder gives an original, interesting, and commodious method of working with the algebra. Later in the same year, Mr. Hugh

McCull, apparently having known nothing of logical algebra except from a jejune account of Boole's work in Bain's *Logic*, published several papers on a *Calculus of Equivalent Statements*, [*Proceedings, London Mathematical Society*, Series 1, vol. IX, pp. 177-186], the basis of which is nothing but the Boolean algebra, with Jevons's addition and a sign of inclusion. Mr. McCull adds an exceedingly ingenious application of this algebra to the transformation of definite integrals.

Peirce: CP 3.200 Fn P1 p 127 Cross-Ref:††

†P1 Remark that the proofs of the lettered propositions follow the enunciations -- 1880.

Peirce: CP 3.200 Fn P2 p 127 Cross-Ref:††

†P2 *Logic of Relatives* (§4) gives $aXb \prec a$. The other formulæ, equally obvious, I do not find anywhere.

Peirce: CP 3.200 Fn P3 p 127 Cross-Ref:††

†P3 The first of these given by Boole for his addition, was retained by Jevons in changing the addition. The second was first given by me (1867) [See 4].

Peirce: CP 3.200 Fn 1 Para 1/3 p 128 Cross-Ref:††

†1 "It seems that $(a+b)Xc \prec (aXc)+(bXc)$ cannot be proved from the definitions. The propositions L are needed"--a marginal note prompted apparently by Schröder's criticisms in his *Vorlesungen über die Algebra der Logik*, Bd. 1, Kap. 6.

Peirce: CP 3.200 Fn 1 Para 2/3 p 128 Cross-Ref:††

On February 14, 1904, Peirce wrote Prof. E. V. Huntington of Harvard as follows:

"Dear Mr. Huntington: Should you decide to print the proof of the distributive principle (and this would not only relieve me from a long procrastinated duty, but would have a certain value for exact logic, as removing the eclipse under which the method of developing the subject followed in my paper in vol. 3 has been obscured) I should feel that it was incumbent upon me, in decency, to explain its having been so long withheld. The truth is that the paper aforesaid was written during leisure hours gained to me by my being shut up with a severe influenza. In writing it, I omitted the proof, as there said, because it was 'too tedious' and because it seemed to me very obvious. Nevertheless, when Doctor Schröder questioned its possibility, I found myself unable to reproduce it, and so concluded that it was to be added to the list of blunders, due to the grippe, with which that paper abounds--a conclusion that was strengthened when Schröder thought he demonstrated the indemonstrability of the law of distributiveness. (I must confess that I never carefully examined his proof, having my table loaded with logical books for the perusal of which life was not long enough.) It was not until many years afterwards that, looking over my papers of 1880 for a different purpose, I stumbled upon this proof written out in full for the press, though it was eventually cut out, and, at first, I was inclined to think that it employed the principle that *all* existence is individual, which my method, in the paper in question, did not permit me to employ at that stage. I venture to opine that it fully vindicates my characterisation of it as 'too tedious'. But this is how I have a new apology to make to exact logicians."

Peirce: CP 3.200 Fn 1 Para 3/3 p 128 Cross-Ref:††

This letter and the proof were used by Professor Huntington in his "Sets of Independent Postulates for the Algebra of Logic," *Transactions, American*

Mathematical Society, vol. 5, p. 300n. (1904), and in proof of proposition 22a. The proof is also to be found in Lewis's *Survey of Symbolic Logic*, p. 128 (5.5). A more elegant proof is to be found in the *Principia Mathematica*. See 384n.

Peirce: CP 3.201 Fn 1 p 130

†1 Cf. Mrs. Ladd-Franklin's *Antilogism* in her "On the Algebra of Logic," *Studies in Logic*, edited by C. S. Peirce, Little, Brown & Co., Boston, 1883.

Peirce: CP 3.202 Fn 1 p 132

†1 In 9.

Peirce: CP 3.202 Fn 2 p 132

†2 In 12.

Peirce: CP 3.203 Fn 3 p 132

†3 These two embody De Morgan's principle of duality.

Peirce: CP 3.203 Fn 4 p 132

†4 This should be: $\sim a + \sim b \text{ -< } \sim(aXb)$.

Peirce: CP 3.209 Fn 1 p 135

†1 I.e., $\{(x+y) \text{ -< } z\}$.

Peirce: CP 3.209 Fn 2 p 135

†2 I.e., $\{x \text{ -< } (yXz)\}$.

Peirce: CP 3.209 Fn 3 p 135

†3 I.e., $\{(x+y) \sim\text{-<} z\}$.

Peirce: CP 3.209 Fn 4 p 135

†4 I.e., $\{x \sim\text{-<} (yXz)\}$.

Peirce: CP 3.213 Fn 1 p 136

†1 *The Laws of Thought*, pp. 146-9.

Peirce: CP 3.215 Fn 1 p 139

†1 Cf. 4.121.

Peirce: CP 3.215 Fn 2 p 139

†2 See 93, 2.646 and 4.121-22.

Peirce: CP 3.216 Fn 3 p 139

†3 Cf. 4.118-9.

Peirce: CP 3.217 Fn 1 p 140

†1 Cf. 2.356.

Peirce: CP 3.220 Fn P1 p 140 Cross-Ref:††

†P1 In my *Logic of Relatives*, 1870 [§6], I have used this expression ['simple relatives'] to designate what I now call *dual relatives*.

Peirce: CP 3.222 Fn 1 p 142

†1 Four lines of formula are here deleted, in accordance with Peirce's subsequent marginal comment. They involved the invalid use of the law of association in connection with triple relatives.

Peirce: CP 3.223 Fn 2 p 142

†2 $\Sigma(A:B)$ represents the logical sum of individual relatives; $\pi(\alpha:\beta)$, represents the logical product of simple relatives.

Peirce: CP 3.224 Fn 1 p 143

†1 LM indicates the application of L on M. L says that any formula of the form (1:2):3 is to be changed to (3:1):2. As M means that (1:2):3 is to be changed to (2:3):1, the application of L on M yields the original. The rest of the formulæ are to be understood in a similar way.

Peirce: CP 3.224 Fn 2 p 143

†2 I.e., I results by substituting *a* for *b*, *b* for *a* and *c* for *c* in the equation for *b*; and so on with the rest.

Peirce: CP 3.226 Fn 1 p 144

†1 See 136*an*.

Peirce: CP 3.226 Fn 2 p 144

†2 See 136*dn*.

Peirce: CP 3.227 Fn P1 p 145 Cross-Ref:††

†P1 The relative 0 ought to be considered as at once a concurrent and an alio-relative, and the relative ∞ as at once the negative of a concurrent and the negative of an alio-relative. [Cf. 585-6.] The statements in the text require to be modified to this extent. [This note, apparently a correction made after receiving proofs, was published at the end of the original paper.]

Peirce: CP 3.229 Fn P1 p 146 Cross-Ref:††

†P1 These numbers are every fifth of the series:

$\sim 1^*$, - 7, 10, -3, $\sim 2^*$, -5, 5, 2, 3, $\sim 5^*$, 0, 5, 7, 10, $\sim 15^*$, 15, 20, 27, 37, $\sim 52^*$, where $u[x]+u[x]+3 = u[x]+4$. But only holds up to 203 and is therefore valueless.-- marginal note.

Peirce: CP 3.234 Fn 1 p 147

†1 See 136*cn*.

Peirce: CP 3.235 Fn 2 p 147

†2 See 136*hn*.

Peirce: CP 3.242 Fn P1 p 150 Cross-Ref:††

†P1 The first three of these were studied by De Morgan ("On the Syllogism," No. IV.); the last is new. The above names for the first three (except the adjective *external* suggested by Grassmann's operation) are given in my *Logic of Relatives*.

Peirce: CP 3.243 Fn P1 p 151 Cross-Ref:††

†P1 A similar table is given by De Morgan. Of course, it lacks the symmetry of this, because he had not the fourth operation. [Cf. 112.]

Peirce: CP 3.252 Fn 1 p 158

†1 *The American Journal of Mathematics*, vol. 4, pp. 85-95 (1881), as subsequently corrected.

Peirce: CP 3.252 Fn 2 p 158

†2 J. S. Mill, e.g. in *Logic*, bk. II, ch. 6, §2-3.

Peirce: CP 3.254 Fn P1 p 159 Cross-Ref:††

†P1 For example, in the ordinary algebra of imaginaries two quantities may both result from the addition of quantities of the form a^2+b^2i to the same quantity without either being in this relation to the other.

Peirce: CP 3.256 Fn 1 p 159

†1 But see 4.121.

Peirce: CP 3.258 Fn 1 p 160

†1 See 564.

Peirce: CP 3.260 Fn 2 p 160

†2 I.e., semi-limited and infinite; see 4.107.

Peirce: CP 3.260 Fn 3 p 160

†3 Cf. 4.150ff.

Peirce: CP 3.280 Fn 1 p 166

†1 I.e., c is a one-one relation.

Peirce: CP 3.280 Fn 2 p 166

†2 $\$c$ is the converse of c .

Peirce: CP 3.281 Fn 3 p 166

†3 Originally 'c'd with.'

Peirce: CP 3.281 Fn 4 p 166

†4 Originally 'c'd by.'

Peirce: CP 3.281 Fn 5 p 166

†5 I.e., counting involves the establishment of a one-one correlation between the members of a given class and the natural numbers.

Peirce: CP 3.286 Fn P1 p 168 Cross-Ref:††

†P1 This long proof is quite unnecessary. The whole thing depends not on Fermat's mode of reasoning but on De Morgan's. -- marginal note. [I.e., it depends, not on mathematical induction but on the syllogism of transposed quantity.]

Peirce: CP 3.287 Fn P1 p 169 Cross-Ref:††

†P1 It may be remarked that when we reason that a certain proposition, if false of any number, is false of some smaller number, and since there is no number (in a semi-limited system) smaller than every number, the proposition must be true, our reasoning is a mere logical transformation of the reasoning that a proposition, if true for n , is true for $1+n$, and that it is true for 1.

Peirce: CP 3.288 Fn 1 p 170

†1 This mode of reasoning was uncovered by De Morgan and called the syllogism of transposed quantity. See 402 and 4.103f.

Peirce: CP 3.289 Fn 1 p 171

†1 *The American Journal of Mathematics*, vol. 4, pp. 221-29, (1881), an addendum to Benjamin Peirce's *Linear Associative Algebras* published posthumously in the same volume with notes by C. S. P. These notes throw considerable light on the significance and relationship of algebra to the logic of relatives.

Peirce: CP 3.289 Fn P1 p 171 Cross-Ref:††

†P1 I have used $a[11]$ etc., in place of the $a[1]$, etc., used by my father in his

text.

Peirce: CP 3.290 Fn P2 p 171 Cross-Ref:††
†P2 Any one of them multiplied by 0 gives 0.

Peirce: CP 3.292 Fn P1 p 172 Cross-Ref:††
†P1 If $\mathbf{B} = 0$, of course the result is 0.

Peirce: CP 3.294 Fn P1 p 173 Cross-Ref:††
†P1 A brief proof of this theorem, perhaps essentially the same as the above, was published by me in the *Proceedings of the American Academy of Arts and Sciences*, for May 11, 1875. [150-51.]

Peirce: CP 3.294 Fn 1 p 173
†1 Paper No. IX.

Peirce: CP 3.294 Fn 2 p 173
†2 The reference is to the *Linear Associative Algebras*, p. 188.

Peirce: CP 3.302 Fn P1 p 176 Cross-Ref:††
†P1 The idempotent basis having been shown to be arithmetical unity, we are free to use the letter i to denote another unit.

Peirce: CP 3.306 Fn 1 p 180
†1 Dated, Baltimore, January 7, 1882, pp. 1-6, with a postscript of January 16, 1882. To judge from a search through the technical journals and from Peirce's reference in 294, this paper was privately printed.

Peirce: CP 3.306 Fn P1 p 180 Cross-Ref:††
†P1 I have usually restricted the coefficients to one or other of two values; but the more general view was distinctly recognized in my paper of 1870.

Peirce: CP 3.310 Fn 1 p 181
†1 Cf. 133.

Peirce: CP 3.312 Fn 1 p 182
†1 This should be: $(\sim 1)[ii] = 0$.

Peirce: CP 3.314 Fn 2 p 182
†2 Cf. 242.

Peirce: CP 3.314 Fn P1 p 182 Cross-Ref:††
†P1 "On De Morgan's Extension of the Algebraic Processes," *American Journal of Mathematics*, vol. 3, no. 3.

Peirce: CP 3.314 Fn 1 p 183
†1 See 297-305.

Peirce: CP 3.318 Fn 1 p 185
†1 150-51.

Peirce: CP 3.319 Fn 2 p 185
†2 Cf. the analysis of signs, 2.274 and in the present volume, 359f.

Peirce: CP 3.320 Fn 3 p 185
†3 There is no paper by this title known to have been published by Sylvester, though a paper entitled 'Lectures on the Principles of Universal Algebra,' was

published in the *American Journal of Mathematics*, vol. 6, pp. 270-86, (1884). A number of papers on algebra published in 1881-82 are to be found in the *Collected Mathematical Papers of J. J. Sylvester*, ed. by H. F. Baker, Cambridge University Press, vol. 3 (1904-12).

Peirce: CP 3.320 Fn P1 p 185 Cross-Ref:††

†P1 "Description of a Notation for the Logic of Relatives." *Memoirs, American Academy of Arts and Sciences*, vol. 9, 1870. [III.] "On the Application of Logical Analysis to Multiple Algebra." *Proceedings* of the same Academy, 1875, May 11. [IV.] "Note on Grassman's Calculus of Extension." *Ibid.* 1877, Oct. 10. [V.] "On the Algebra of Logic." *American Journal of Mathematics*, vol. 3. [VI.]

Peirce: CP 3.323 Fn 1 p 187

†1 *Johns Hopkins University Circulars*, No. 13, p. 179, (1882).

Peirce: CP 3.324 Fn 1 p 189

†1 *Johns Hopkins University Circulars*, No. 19, pp. 3-4, (1882), read before the University Mathematical Society, October 18, 1882.

Peirce: CP 3.324 Fn 2 p 189

†2 The first and second paragraph were originally transposed.

Peirce: CP 3.327 Fn 1 p 193

†1 The rest of this paper was added on October 30.

Peirce: CP 3.328 Fn 1 p 195

†1 *Note B*, pp. 187-203, *Johns Hopkins Studies in Logic*, ed. by C. S. Peirce, Little, Brown & Co., Boston, 1883.

Peirce: CP 3.332 Fn 1 p 197

†1 As this is not intended to exclude being a lover of a benefactor, but only being a non-lover of a non-benefactor, the following alternative expressions may be clearer: "lover of all non-benefactors"; "a non-lover only of benefactors," or "Either a lover of X or X is a benefactor." Relative addition is the denial of transaddition (243), so that the following are equivalent: $\sim(lb)$, $-l[b]$, $l\bar{b}$, $l\bar{b}$, and $-(l\bullet b)$. In 473 it is shown that $\$ \sim 1 \prec b$ is also equivalent to the above.

Peirce: CP 3.332 Fn 2 p 197

†2 Cf. 118 (143) (144).

Peirce: CP 3.333 Fn 3 p 197

†3 Cf. 352.

Peirce: CP 3.333 Fn 4 p 197

†4 I.e., it is not exclusive.

Peirce: CP 3.334 Fn 1 p 198

†1 These two formulæ together show that lovers of servants or benefactors of servants are the same as lovers or benefactors of servants.

Peirce: CP 3.334 Fn 2 p 198

†2 These two formulæ together show that lovers and benefactors of all non-servants are the same as lovers of all non-servants and benefactors of all non-servants.

Peirce: CP 3.335 Fn 3 p 198
†3 Cf. 249, 1ii.

Peirce: CP 3.338 Fn 4 p 198
†4 Cf. 136 and 227.

Peirce: CP 3.341 Fn P1 p 199 Cross-Ref:††
†P1 Sometimes important. $1 \rightarrow l + \sim l$ [i.e., identity implies to be either a lover or not loved by] and $l, \sim l \rightarrow n$ [i.e., to love and not be loved by implies otherness] -- marginal note.

Peirce: CP 3.341 Fn 1 p 200
†1 I.e., Identity implies to love all that is loved by.

Peirce: CP 3.341 Fn 2 p 200
†2 I.e., to love something that is not loved by implies otherness.

Peirce: CP 3.342 Fn 3 p 200
†3 "On a New Algebra of Logic," *Studies in Logic*, pp. 87, 88.

Peirce: CP 3.345 Fn 1 p 202
†1 2.521ff.

Peirce: CP 3.345 Fn 2 p 202
†2 The present paper was rewritten to serve as the thirteenth chapter of the Grand Logic of 1893-94, where the above six propositions are replaced by the following five:

- $\sim a \dagger p \dagger \sim \beta$ Every a has every β
- $a(p \dagger \sim \beta)$ Some a has all β
- $a p \dagger \sim \beta$ Every β is some a or other
- $(\sim a \dagger p) \beta$ Some β is all a
- $\sim a \dagger p \beta$ Some β is in some a ;

and the various syllogisms are modified accordingly. There are no other changes of consequence. These are reduced to four by Schröder, *Algebra der Logik*, III, 1, 470.

Peirce: CP 3.346 Fn 3 p 202
†3 For a reduction in the number of these forms see Schröder, *ibid.*, 470f.

Peirce: CP 3.346 Fn 1 p 204
†1 See 2.526n for definition of this term.

Peirce: CP 3.348 Fn 1 p 205
†1 *Op. cit.*, p. 87.

Peirce: CP 3.351 Fn P1 p 206 Cross-Ref:††
†P1 The sums of 331.

Peirce: CP 3.352 Fn 1 p 207

†1 More clearly: For some i and k , i is a lover of all non-benefactors of k ; or for some i and some k every j is such that either i loves j or j is a benefactor of k .

Peirce: CP 3.356 Fn 1 p 208

†1 This sentence is slightly different from the original, in accordance with Peirce's correction.

Peirce: CP 3.358 Fn 1 p 209

†1 No record of this communication has been found in the official *Proceedings* of the London Mathematical Society; nor has any letter from Mr. Schlötel come to hand in a search through Peirce's correspondence.

Peirce: CP 3.358 Fn 2 p 209

†2 *Die Logik, neu bearbeitet*, 1854.

Peirce: CP 3.359 Fn 1 p 210

†1 *The American Journal of Mathematics*, vol. 7, No. 2, pp. 180-202, (1885); reprinted pp. 1-23.

Peirce: CP 3.359 Fn 2 p 210

†2 See vol. 2, bk. II, for a detailed analysis of signs.

Peirce: CP 3.360 Fn 3 p 210

†3 More frequently called 'symbols'; the word 'token' is later (in 4.537) taken to apply to what in 2.245 is called a 'sinsign.'

Peirce: CP 3.363 Fn P1 p 211 Cross-Ref:††

†P1 See *Proceedings, American Academy of Arts and Sciences*, vol. 7, p. 294, May 14, 1867. [1.558.]

Peirce: CP 3.363 Fn 1 p 212

†1 See 4.356.

Peirce: CP 3.363 Fn 2 p 212

†2 "On the Diagrammatic and Mechanical Representations of Propositions and Reasoning." *Philosophical Magazine*, ser. 5, vol. 10, pp. 1-15 (1880).

Peirce: CP 3.363 Fn P1 p 212 Cross-Ref:††

†P1 *Studies in Logic*, by members of the Johns Hopkins University; Boston, Little, Brown and Co., 1883.

Peirce: CP 3.363 Fn 3 p 212

†3 *Ibid.*, p. 74.

Peirce: CP 3.366 Fn 1 p 214

†1 If this proposition be added to the postulates of Boolean algebra and if the terms of that algebra be interpreted as propositions, a propositional calculus is secured. From an historical standpoint this is of tremendous significance.

Peirce: CP 3.370 Fn 1 p 216

†1 *The Laws of Thought*, pp. 47ff.

Peirce: CP 3.370 Fn 2 p 216

†2 I.e., $\sim(\sim x \sim y)$.

Peirce: CP 3.370 Fn 3 p 216

†3 I.e., $-(x \ y)$ or $\sim x + \sim y$.

Peirce: CP 3.371 Fn 4 p 216

†4 I.e., $-(\sim x \sim y)z = -\{(x \ z) - (y \ z)\}$

Peirce: CP 3.371 Fn 5 p 216

†5 I.e., $-\{(x \ y) \sim z\} = -(\sim x \sim z) - (\sim y \sim z)$.

Peirce: CP 3.373 Fn 1 p 217

†1 In paper No. VI, part I.

Peirce: CP 3.374 Fn 1 p 218

†1 Cf. 527.

Peirce: CP 3.377 Fn 1 p 220

†1 Strictly speaking, the *modus ponens* is expressed as $\{(x \ -< y) \cdot x\} \ -< y$. The given proposition states that if x is true then provided ' x then y ' is true, y is true.

Peirce: CP 3.379 Fn 2 p 220

†2 I.e. $(x \ -< y) \ -< \{(y \ -< z) \ -< (x \ -< z)\}$, which is often called the *nota notæ* or the *dictum de omni*. See 383, and vol. 2, Bk. III, ch. 4, 14, and cf. Joseph's *An Introduction to Logic*, p. 296n and 308n.

Peirce: CP 3.379 Fn 3 p 220

†3 This is not an exact reading of the given formula even as modified in the last note. The formula for this expression and thus for Barbara is:

$$\{(x \ -< y) \ \sim\ -< (y \ \sim\ -< z)\} \ -< (x \ -< z).$$

Peirce: CP 3.384 Fn P1 p 222 Cross-Ref: ††

†P1 It is interesting to observe that this reasoning is dilemmatic. In fact, the dilemma involves the fifth icon. The dilemma was only introduced into logic from rhetoric by the humanists of the *renaissance*; and at that time logic was studied with so little accuracy that the peculiar nature of this mode of reasoning escaped notice. I was thus led to suppose that the whole non-relative logic was derivable from the principles of the ancient syllogistic, and this error [But it was not an error!!! See my original demonstration. -- marginal note. [See 200n].] is involved in chapter 2 of my paper in the third volume of this Journal [No. VI]. My friend, Professor Schröder, detected the mistake and showed that the distributive formulæ

$$(x+y)z \ -< x \ z + y \ z$$

$$(x+z)(y+z) \ -< x \ y + z$$

could not be deduced from syllogistic principles. [This matter is discussed at length by Schröder in his *Vorlesungen über die Algebra der Logik*, Bd. 1, 12, (1890)]. I had myself independently discovered and virtually stated the same thing. (*Studies in Logic*, p. 189 [331].) There is some disagreement as to the definition of the dilemma

(see Keynes's excellent *Formal Logic*, p. 241); but the most useful definition would be a syllogism depending on the above distribution formulæ. The distribution formulæ

$$x z + y z \text{ -} < (x + y)z$$

$$x y + z \text{ -} < (x + z)(y + z)$$

are strictly syllogistic. DeMorgan's added moods are virtually dilemmatic depending on the principle of excluded middle.

Peirce: CP 3.389 Fn 1 p 225

†1 This should be: $\sim(\sim x + y) + z$.

Peirce: CP 3.389 Fn 2 p 225

†2 This should be: $\sim x + y$.

Peirce: CP 3.391 Fn 1 p 226

†1 *Op. cit.*, p. 80f.

Peirce: CP 3.392 Fn 1 p 227

†1 If the six vertices of a hexagon lie three and three on two straight lines, the three points of intersection of the opposite sides lie on a straight line.

Peirce: CP 3.393 Fn 2 p 227

†2 *Op. cit.*, p. 79.

Peirce: CP 3.393 Fn P1 p 228 Cross-Ref: ††

†P1 I will just remark, quite out of order, that the quantification may be made numerical; thus producing the numerically definite inferences of DeMorgan and Boole. Suppose at least 2/3 of the company have white neckties and at least 3/4 have dress coats. Let w mean 'he has a white necktie,' and d 'he has a dress coat.' Then, the two propositions are

$$(2/3)(w) \text{ and } (3/4)(d).$$

These are to be multiplied together. But we must remember that $x y$ is a mere abbreviation for $\sim(\sim x + \sim y)$, and must therefore write

$$\sim(\sim((2/3)w) + \sim((3/4)d)).$$

Now $\sim((2/3)w)$ is the denial of $(2/3)w$, and this denial may be written $(>(1/3))\sim w$, or more than 1/3 of the universe (the company) have not white neckties. So $\sim((3/4)d) = (>(1/4))\sim d$. The combined premisses thus become

$$\sim((>(1/3))\sim w + (>(1/4))\sim d)$$

Now $(\gt(1/3))\sim w + (\gt(1/4))\sim d$ gives May be $(1/3 + 1/4)(\sim w + \sim d)$.

Thus we have May be $(7/12)(\sim w + \sim d)$

and this is (At least $5/12)(\sim w + \sim d)$,

which is the conclusion.

Peirce: CP 3.393 Fn 1 p 228

†1 This is the seventh icon?

Peirce: CP 3.393 Fn 2 p 228

†2 This is the eighth icon?

Peirce: CP 3.396 Fn 1 p 230

†1 See *Lady's and Gentleman's Diary* for 1850, p. 48 for the original presentation of 'Kirkman's School-Girls Problem.' See also W. W. R. Ball, *Mathematical Recreations and Essays*, Ch. IX, MacMillan & Co., 5 ed. (1911), where a reference is given to Benjamin Peirce's method of solution and his enunciation of a corresponding problem.

Peirce: CP 3.396 Fn 2 p 230 Cross-Ref:††

†2 Peirce defines a syntheme in the *Century Dictionary*, p. 6139, ed. 1889, as "a system of groups of objects comprising every one of a larger set just once, twice or other given number of times. The groups may be divided into subgroups subject to various conditions."

Peirce: CP 3.396 Fn 1 p 231

†1 Obviously a misprint for $\Sigma[i]\pi[j]x[ij] -< \pi[j]\Sigma[i]x[ij]$.

Peirce: CP 3.396 Fn 1 p 232

†1 See 403A and B at end of article.

Peirce: CP 3.396 Fn 2 p 232

†2 See 403C and M.

Peirce: CP 3.398 Fn 1 p 233

†1 See 403H.

Peirce: CP 3.399 Fn 1 p 235

†1 See 403I.

Peirce: CP 3.400 Fn 2 p 235

†2 See 403J.

Peirce: CP 3.401 Fn 1 p 236

†1 Cf. 21 (7).

Peirce: CP 3.401 Fn 2 p 236

†2 $\sim r[v\alpha w]$.

Peirce: CP 3.401 Fn 3 p 236

†3 I.e., any two terms which are related in any one-one correspondence to any term are identical; any two terms which are related by one-one correspondence to any term are identical; and any case of terms related one to one is always a one-one correspondence.

Peirce: CP 3.402 Fn 1 p 237

†1 *Transactions Cambridge Philosophical Society*, vol. 10, pp. *355-*358, (1864).

Peirce: CP 3.402 Fn 2 p 237

†2 *Ibid.*, p. *356.

Peirce: CP 3.402 Fn 3 p 237

†3 See 564 and 4.103f.

Peirce: CP 3.402 Fn P1 p 237 Cross-Ref:††

†P1 Another of De Morgan's examples [*Formal Logic*, p. 168] is this: "Suppose a person, on reviewing his purchases for the day, finds, by his counterchecks, that he has certainly drawn as many checks on his banker (and maybe more) as he has made purchases. But he knows that he paid some of his purchases in money, or otherwise than by checks. He infers then that he has drawn checks for something else except that day's purchases. He infers rightly enough." Suppose, however, that what happened was this: He bought something and drew a check for it; but instead of paying with the check, he paid cash. He then made another purchase for the same amount, and drew another check. Instead, however, of paying with that check, he paid with the one previously drawn. And thus he continued without cessation, or *ad infinitum*. Plainly the premisses remain true, yet the conclusion is false.

Peirce: CP 3.403A Fn 1 p 239

†1 This undated note seems to have been written for publication in some issue of *The American Journal of Mathematics*, shortly subsequent to that in which the previous article appeared. Why it was not published is unknown.

Peirce: CP 3.403G Fn 1 p 243

†1 The second formula is incorrect. The second half should read:

$$a\sim x + x b + a x\sim b; \text{ or } a\sim b + x b + a\sim x b.$$

Peirce: CP 3.403I Fn 2 p 243

†2 Cf. 2.517-27.

Peirce: CP 3.403I Fn 1 p 244

†1 399.

Peirce: CP 3.403I Fn 1 p 245

†1 I.e., every *a* is identical with every *b*.

Peirce: CP 3.403K Fn 1 p 247

†1 This should be $\sim 1[e h]$.

Peirce: CP 3.403L Fn 1 p 248

†1 I.e., any two terms are identical and related or they are not identical and the

said relation does not relate them.

Peirce: CP 3.403L Fn 2 p 248

†2 I.e., every two terms are related by some relation.

Peirce: CP 3.404 Fn 1 p 250

†1 *The Open Court*, vol. 6, pp. 3391-4 (1892).

Peirce: CP 3.404 Fn 1 p 251

†1 Or possibly in some other Renaissance writing. My memory may deceive me; and my library is precious small.

Peirce: CP 3.407 Fn 1 p 253

†1 Cf. 2.593ff.

Peirce: CP 3.408 Fn 1 p 254

†1 "On the Syllogism," II, *Transactions, Cambridge Philosophical Society*, vol. 9, p. 104, (1851).

Peirce: CP 3.409 Fn 1 p 255

†1 See 136a.

Peirce: CP 3.415 Fn 1 p 257

†1 *The Open Court*, vol. 6, pp. 3416-8, (1892).

Peirce: CP 3.415 Fn 1 p 258

†1 Nothing of the kind seems to have been published by Peirce, and there is no record of any relevant manuscript.

Peirce: CP 3.416 Fn 2 p 258

†2 Page 5057, ed. of 1889; see also 571f.

Peirce: CP 3.417 Fn P1 p 259 Cross-Ref:††

†P1 In this connection, see James's, *Principles of Psychology*, vol. 1, pp. 237-271; *Briefer Course*, pp. 160 et seq. James is no logician, but it is not difficult to trace a connection between the points he makes and the theory of inference.

Peirce: CP 3.418 Fn 1 p 260

†1 Paper no. XVI and see vol. 4, bk. II.

Peirce: CP 3.419 Fn 1 p 261

†1 Cf. discussion in vol. 2, bk. II, ch. 4, on the nature of propositions. The previous chapters of the same book should clarify what follows.

Peirce: CP 3.419 Fn 1 p 262

†1 Cf. 2.287n.

Peirce: CP 3.420 Fn P1 p 262 Cross-Ref:††

†P1 *Nature*, in connection with a picture, copy, or diagram, does not necessarily denote an object not fashioned by man, but merely the object represented, as something existing apart from the representation.

Peirce: CP 3.421 Fn 2 p 262

†2 Cf. 469f, 1.289f, 1.346.

Peirce: CP 3.421 Fn P2 p 262 Cross-Ref:††

†P2 Thus, *CO*, which appears as such a radicle in formic acid, makes of itself

a saturated compound.

Peirce: CP 3.421 Fn 1 p 263

†1 Cf. 63 and 1.347.

Peirce: CP 3.422 Fn 2 p 263

†2 Cf. vol. 1, bk. III.

Peirce: CP 3.422 Fn 1 p 264

†1 This is the last paper on logic to be published in *The Open Court*.

Peirce: CP 3.423 Fn P1 p 264 Cross-Ref:††

†P1 *Philosophical Transactions* for 1886 [pp. 1-70]. No logician should fail to study this memoir.

Peirce: CP 3.423 Fn P2 p 264 Cross-Ref:††

†P2 I use this word in its proper sense, and not to mean unlike, as Mr. Kempe does.

Peirce: CP 3.425 Fn 1 p 266

†1 *The Monist*, vol. 7, pp. 19-40, (1896).

Peirce: CP 3.425 Fn P1 p 266 Cross-Ref:††

†P1 *Vorlesungen über die Algebra der Logik*, (Exakte Logik). Von Dr. Ernst Schröder, Ord. Professor der Mathematik an der technischen Hochschule zu Karlsruhe in Baden. Dritter Band. *Algebra und Logik der Relative*. Leipsic: B. G. Teubner. 1895. Price, 16M.

Peirce: CP 3.426 Fn 1 p 268

†1 See 2.30.

Peirce: CP 3.427 Fn 1 p 270

†1 *La philosophie positive*, deuxième leçon.

Peirce: CP 3.427 Fn 2 p 270

†2 Cf. the classification of sciences in vol. 1, Bk. II.

Peirce: CP 3.427 Fn 3 p 270

†3 By Peirce, p. 5397, ed. of 1889.

Peirce: CP 3.430 Fn 1 p 272

†1 The first and second parts are the topics of bks. II and III of vol. 2; the third is discussed in vol. 5 and 6.

Peirce: CP 3.430 Fn 2 p 272

†2 *Opera Omnia Collecta*, T. 1, pp. 45-76. L. Durand.

Peirce: CP 3.430 Fn 3 p 272

†3 1.559.

Peirce: CP 3.431 Fn 1 p 273

†1 Bd. 1, S. 118.

Peirce: CP 3.432 Fn 2 p 273

†2 Cf. 2.19.

Peirce: CP 3.441 Fn 1 p 279

†1 *Acad. Quaest.* II, 143.

Peirce: CP 3.441 Fn 2 p 279

†2 E.g. Sextus Empiricus, *Adv. Math.* VIII, 113-17.

Peirce: CP 3.441 Fn 3 p 279

†3 The Diodorians in opposition to the Philonians deny that material implication expresses what is usually meant by "if ... then" For contemporary discussion see G. E. Moore, *Philosophical Studies*, p. 276ff; C. I. Lewis, *A Survey of Symbolic Logic*, esp. p. 324ff, and Paul Weiss, "Relativity in Logic," *The Monist*, October, 1928; "The Nature of Systems," *The Monist*, April/July, 1929; "Entailment and the Future of Logic," *Proceedings*, Seventh International Congress of Philosophy; E. J. Nelson, "Intensional Relations," *Mind*, October, 1930.

Peirce: CP 3.442 Fn 1 p 280

†1 *Questiones in Octo libror Physicorum Aristotelis*, L. 1, qu. II.

Peirce: CP 3.446 Fn 1 p 282

†1 Cf. 2.352 and 2.618.

Peirce: CP 3.446 Fn 2 p 282

†2 Vol. 5, bk. II, ch. 3.

Peirce: CP 3.448 Fn 1 p 283

†1 182-197.

Peirce: CP 3.448 Fn 2 p 283

†2 The editors have not considered it worth publishing. But see 4.277ff. and vol. 4, Bk. II.

Peirce: CP 3.450 Fn 1 p 284

†1 See 472 and 2.532-5.

Peirce: CP 3.450 Fn 2 p 284

†2 *Symbolic Logic*, p. 39ff.

Peirce: CP 3.450 Fn 3 p 284

†3 The principle of duality is expressible in the formulæ: $\sim(a+b) = (\sim a \sim b)$ and $\sim(ab) = (\sim a + \sim b)$.

Peirce: CP 3.453 Fn 1 p 286

†1 See vol. 2, bk. II, ch. 1, for a discussion of the "ethics of terminology."

Peirce: CP 3.455 Fn 2 p 286

†2 Peirce's first contribution to 'exact' logic is published in the Appendix to vol. 2.

Peirce: CP 3.456 Fn 1 p 288

†1 *The Monist*, vol. 7, pp. 161-217, (1897).

Peirce: CP 3.456 Fn 2 p 288

†2 Cf. vol. 5, bk. II, ch. 5.

Peirce: CP 3.456 Fn P1 p 288 Cross-Ref:††

†P1 *Algebra und Logik der Relative*. Leipsic: B. G. Teubner. 1895. Price, 16 M.

Peirce: CP 3.462 Fn 1 p 291

†1 See 42-44, 1.83, 2.227, 2.364 and 4.235.

Peirce: CP 3.465 Fn P1 p 294 Cross-Ref:††

†P1 The Pythagoreans, who seem first to have used these words, probably attached a patronymic signification to the termination. A *triad* was derivative of *three*, etc.

Peirce: CP 3.468 Fn 1 p 295

†1 In this section Peirce presents his "Entitative Graphs." The "Existential Graphs" are to be found in vol. 4, bk. II.

Peirce: CP 3.468 Fn 2 p 295

†2 Part 1, 1886, pp. 1-70.

Peirce: CP 3.469 Fn 3 p 295

†3 Cf. 1.289f., 1.346 and 421.

Peirce: CP 3.470 Fn 1 p 296

†1 See J. J. Sylvester, "Chemistry and Algebra," *Mathematical Papers*, vol. III, No. 14; W. K. Clifford; "Remarks on the Chemico-Algebraic Theory," *Mathematical Papers*, No. 28.

Peirce: CP 3.470 Fn 1 p 297

†1 Meyer used the volumes as abscissæ and the weights as ordinates. See *Das Natürliche System der Chemischen Elemente*, Meyer u. Mendejeff, Leipzig, 1895.

Peirce: CP 3.470 Fn 2 p 297

†2 See 475f.

Peirce: CP 3.473 Fn 1 p 299

†1 I.e., $\sim l \dagger s = \$l -< s$.

Peirce: CP 3.474 Fn 1 p 300

†1 i.e., $\sim l \dagger s -< \$s \dagger \$\sim l$ but not $\sim \sim s \dagger \$l$; or perhaps more clearly $(\$l -< s) -< (\sim s -< \$\sim l)$ but not $s -< \$l$.

Peirce: CP 3.474 Fn P1 p 300 Cross-Ref:††

†P1 Professor Schröder proposes to substitute the word "symmetry" for *convertibility*, and to speak of *simply convertible* modes of junction as "symmetrical." Such an example of wanton disregard of the admirable traditional terminology of logic, were it widely followed, would result in utter uncertainty as to what any

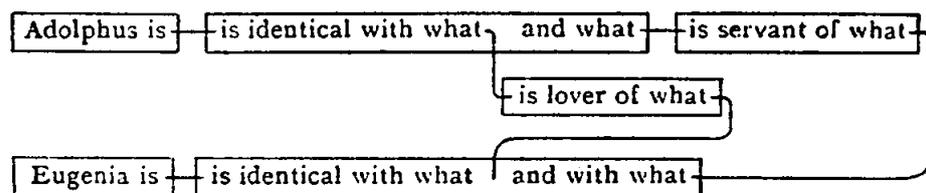


Figure 6

[Click here to view]

Figure 6

writer on logic might mean to say, and would thus be utterly fatal to all our efforts to render logic exact. Professor Schröder denies that the mode of junction in "lover of a servant" is "symmetrical," which word in practice he makes synonymous with "commutative," applying it only to such junctions as that between "lover" and "servant" in "Adolphus is at once lover and servant of Eugenia." Commutativity depends on one or more polyadic relatives having two like blanks as shown in Fig. 6.

Peirce: CP 3.479 Fn P1 p 305 Cross-Ref:††

†P1 In my method of graphs, the spots represent the relatives, their bonds the heccecities; while in Mr. Kempe's method, the spots represent the objects, whether individuals or abstract ideas, while their bonds represent the relations. Hence, my own exclusive employment of bonds between pairs of spots does not, in the least, conflict with my argument that in Mr. Kempe's method such bonds are insufficient.

Peirce: CP 3.479 Fn 1 p 306

†1 I.e., evenly encircled bonds have π as their quantifier, while the others have Σ .

Peirce: CP 3.481 Fn P1 p 309 Cross-Ref:††

†P1 "On the Natural Classification of Arguments." *Proceedings of the American Academy of Arts and Sciences*. [2.477.]

Peirce: CP 3.491 Fn 1 p 312

†1 The use of one such logical constant is shown by Peirce to be sufficient for the development of Boolean Algebra. See e.g., 4.1ff.

Peirce: CP 3.493 Fn 1 p 314

†1 Cf. 332-4.

Peirce: CP 3.495 Fn 1 p 315

†1 *Symbolic Logic*, p. 39ff.

Peirce: CP 3.496 Fn 1 p 316

†1 See 47n.

Peirce: CP 3.503 Fn 1 p 317

†1 But see 396.

Peirce: CP 3.509 Fn 1 p 319

†1 This should be $y|j]$.

Peirce: CP 3.510 Fn 1 p 320

†1 1885, see No. XIII.

Peirce: CP 3.510 Fn 2 p 320

†2 This does not seem to have been done.

Peirce: CP 3.512 Fn 1 p 321

†1 See S. 296.

Peirce: CP 3.512 Fn 2 p 321

†2 Peirce wrote $\Sigma = x u \cdot a$

u

which conforms neither to Schröder nor to the illustration in the text.

Peirce: CP 3.514 Fn 1 p 322

†1 Cf. 2.316.

Peirce: CP 3.516 Fn 1 p 324

†1 Compare the physiological explanation of deductions, inductions and abductions in 2.643.

Peirce: CP 3.517 Fn 1 p 325

†1 See S.161ff.

Peirce: CP 3.517 Fn 2 p 325

†2 See S.163.

Peirce: CP 3.518 Fn 3 p 325

†3 See S.165.

Peirce: CP 3.520 Fn 1 p 328

†1 This notation is not exactly that of Schröder, who writes $a = 1a\beta\{g\}$, and $\sim a = 0\sim a\sim\beta\sim\{g\}1$. See S. 205.

Peirce: CP 3.521 Fn 1 p 329

†1 See S.231.

Peirce: CP 3.522 Fn 1 p 330

†1 See S. 239.

Peirce: CP 3.524 Fn 1 p 332

†1 For $I[1] \supset \sim I[2], = \sim I[1] \approx \sim I[2] = I[2] \approx I[1]$

Peirce: CP 3.526 Fn 2 p 332

†2 See vol. 4, bk. I, ch. 4.

Peirce: CP 3.526 Fn 1 p 333

†1 See vol. 4, bk. I, ch. 7.

Peirce: CP 3.526 Fn 2 p 333

†2 "*Kalkül der Abzählenden Geometrie*," 1879.

Peirce: CP 3.526 Fn 3 p 333

†3 See 4.219ff.

Peirce: CP 3.527 Fn 4 p 333

†4 See 6.450.

Peirce: CP 3.527 Fn 5 p 333

†5 See e.g. 374 and 442.

Peirce: CP 3.527 Fn P1 p 334 Cross-Ref:††

†P1 For the simple reason that the real world is a part of the ideal world.

namely, that part which sufficient experience would tend ultimately (and therefore definitively), to compel Reason to acknowledge as having a being independent of what he may arbitrarily, or willfully, create.--*Marginal note*, 1908.

Peirce: CP 3.527 Fn P2 p 334 Cross-Ref:††

†P2 That is to say each is *vaguely*, not *distinctly*, possible. -- *Marginal note*, 1908.

Peirce: CP 3.528 Fn P1 p 335 Cross-Ref:††

†P1 Or linearly [by taking the diagonals] as follows. But there the small primes come earlier: 1; 2, 3; 5; 4, 6, 9; 10, 15; 7; 8, 12, 18, 27; 20, 30, 45; 14, 21; 35; 16, 24, 36, 54, 81; 40, 60, 90, 135; 28, 42, 63; 70, 105; 11, etc." -- *Marginal note*, 1908.

Peirce: CP 3.529 Fn 1 p 336

†1 Cf. 1.61, 1.591ff, 2.156.

Peirce: CP 3.532 Fn 2 p 336

†2 Where?

Peirce: CP 3.532 Fn 3 p 336

†3 See 4.552n.

Peirce: CP 3.532 Fn 4 p 336

†4 Obviously a misprint for: $\Sigma[i]\pi[j]\sim I[i,j]$

Peirce: CP 3.532 Fn 1 p 337

†1 (1) $\Sigma[i]\pi[j]\sim I[i,j]$ -< $\pi[j]\Sigma[i]I[i,j]$ (2); (3) $\Sigma[j]\pi[i]I[i,j]$ -< $\pi[i]\Sigma[j]I[i,j]$ (4). (2) and (3) are contradictories; (1) and (4) are contradictories. If (1) were true (2) would be true. If (3) were true (4) would be true. If (1) and (3) were true, (2) and (3), which are contradictories, would both be true. Though (1) and (3) thus cannot both be true, they may both be false. They are related to one another as an A and an E; their contradictories (4) and (2) must be related to one another as an I and an O -- both can be and one at least must be true.

Peirce: CP 3.535 Fn 2 p 337

†2 See 2.517ff.

Peirce: CP 3.537 Fn P1 p 338 Cross-Ref:††

†P1 I prefer to speak of a member of a collection as a *subject* of it rather than as an *object* of it; for in this way I bring to mind the fact that the collection is virtually a quality or class-character. [A collection is a rhema or propositional function. Its members are those subjects which make it a true proposition. See 66.]

Peirce: CP 3.537 Fn 1 p 339

†1 I.e., all the *i*'s which are members of P, are related to the *j*'s which are members of Q, and there is no *k* distinct from an *i* which has the same relation to the *j*'s which the *i*'s have to the *j*'s.

Peirce: CP 3.537 Fn P1 p 339 Cross-Ref:††

†P1 It must be remembered that to a person familiar with the algebra all such series of steps become evident at first glance.

Peirce: CP 3.538 Fn 2 p 339

†2 Cf. 532.

Peirce: CP 3.547 Fn 1 p 343

†1 See "Ueber eine elementare Frage der Mannigfaltigkeitslehre," (1890-1), *Georg Cantor Gesammelte Abhandlung*, herausg. E. Zermelo, S. 278-81, Berlin, (1932) where Cantor shows that $2n$ is always greater than n .

Peirce: CP 3.549 Fn P1 Para 1/9 p 343 Cross-Ref: ††

†P1 Inasmuch as the above theorem is, as I believe, quite opposed to the opinion prevalent among students of Cantor, and they may suspect that some fallacy lurks in the reasoning about wishes, I shall here give a second proof of a part of the theorem, namely that there is an endless succession of infinite multitudes related to one another as above stated, a relation entirely different, by the way, from those of the orders of infinity used in the calculus. I shall not be able to prove by this second method, as is proved in the text, that there are no higher multitudes, and in particular no maximum multitude. The ways of distributing a collection into two houses are equal to the possible combinations of members of that collection (including zero); for these combinations are simply the aggregates of individuals put into either one of the houses in the different modes of distribution. Hence, the proposition is that the combinations of whole numbers are more multitudinous than the whole numbers, that the combinations of combinations of whole numbers are still more multitudinous, the combinations of combinations of combinations again more multitudinous, and so on without end.

Peirce: CP 3.549 Fn P1 Para 2/9 p 343 Cross-Ref: ††

I assume the previously proved proposition that of any two collections there is one which can be placed in one-to-one correspondence with a part or the whole of the other. This obviously amounts to saying that the members of any collection can be arranged in a linear series such that of any two different members one comes later in the series than the other.

Peirce: CP 3.549 Fn P1 Para 3/9 p 343 Cross-Ref: ††

A part may be equal to the whole; as the even numbers are equal in multitude to all the numbers (since every number has a double distinct from the doubles of all other numbers, and that double is an even number). Hence, it does not follow that because one collection can be placed in one-to-one correspondence to a part of another, it is less than that other, that is, that it cannot also, by a rearrangement, be placed in one-to-one correspondence with the whole. This makes an inconvenience in reasoning which can be overcome in a manner I proceed to describe.

Peirce: CP 3.549 Fn P1 Para 4/9 p 343 Cross-Ref: ††

Let a collection be arranged in a linear series. Then, let us speak of a *section* of that series, meaning the aggregate of all the members which are later than (or as late as) one *assignable* member and at the same time earlier than (or as early as) a second *assignable* member. Let us call a series *simple* if it cannot be severed into sections each equal in multitude to the whole. A series not simple itself, may be conceivably severed into *simple sections*, or it may be so arranged that it cannot be so severed (for example the series of rational fractions arranged in the order of their magnitudes). But suppose two collections to be each ranged in a linear series, and suppose one of them, A, is in one-to-one correspondence with a part of the other B. If now the latter series, B, can be severed into simple sections, in each of which it is possible to find a member at least as early in the series as any member of that section that is in correspondence with a member of the other collection A, and also a member at least as late in the series as any member of that section that is in correspondence

with any member of the other collection, and if it is also possible to find a section of the series, B, equal to the whole series, B, in which it is possible to find a member *later* than any member that is in correspondence with any member of the collection, A, then I say that the collection, B, is greater than the collection, A. This is so obvious that I think the demonstration may be omitted.

Peirce: CP 3.549 Fn P1 Para 5/9 p 343 Cross-Ref: ††

Now, imagine two infinite collections, the α 's and the β 's, of which the β 's are the more multitudinous. I propose to prove that the possible combinations of β 's are more multitudinous than the possible combinations of α 's. For let the pairs of conjugate combinations (meaning by conjugate combinations a pair each of which includes every member of the whole collection which the other excludes) of the β 's be arranged in a linear series; and those of the α 's in another linear series. Let the order of the pairs in each of the two series be subject to the rule that if of two pairs one contains a combination composed of fewer members than either combination of the other pair, it shall precede the latter in the series. Let the order of the pairs in the series of pairs of combinations of β 's be further determined by the rule that where the first rule does not decide, one of two pairs shall precede the other whose smaller combination (this rule not applying where one [?] combinations are equal) contains fewer β 's which are in correspondence with α 's in one fixed correspondence of all the α 's with a part of the β 's.

Peirce: CP 3.549 Fn P1 Para 6/9 p 343 Cross-Ref: ††

In this fixed correspondence each α has its β , while there is an infinitely greater multitude of β 's without α 's than with. Let the two series of pairs of combinations be so placed in correspondence that every pair of unequal combinations of α 's is placed in correspondence with that pair of combinations of β 's of which the smaller contains only the β 's corresponding in the fixed correspondence to the smaller combination of α 's; and let every pair of equal combinations of α 's be put into correspondence with a pair of β 's of which the smaller contains only the β 's belonging in the fixed correspondence to one of the combinations of α 's.

Peirce: CP 3.549 Fn P1 Para 7/9 p 343 Cross-Ref: ††

Then it is evident that each series will generally consist of an infinite multitude of simple sections. In none of these will the combinations be more multitudinous than those of the β 's. In some, the combinations of α 's will be equal to those of the β 's; but in an infinitely greater multitude of such simple sections and each of these infinitely more multitudinous, the combinations of β 's will be infinitely more multitudinous than those of the α 's. Hence it is evident that the combinations of the β 's will on the whole be infinitely more multitudinous than those of the α 's.

Peirce: CP 3.549 Fn P1 Para 8/9 p 343 Cross-Ref: ††

That is if the multitude of finite numbers be α , and $2a = b$, $2b = c$, $2c = d$, etc. $a < b < c < d < \text{etc. ad infinitum}$.

Peirce: CP 3.549 Fn P1 Para 9/9 p 343 Cross-Ref: ††

It may be remarked that the finite combinations of finite whole numbers form no larger a multitude than the finite whole numbers themselves; i.e. they are at least enumerable. But there are infinite collections of finite whole numbers; and it is these which are infinitely more numerous than those numbers themselves.

Peirce: CP 3.550 Fn 1 p 345

†1 Cf. 4113.

Peirce: CP 3.552 Fn 2 p 345

†2 This is the last of the published papers on Schröder.

Peirce: CP 3.553 Fn 1 p 346

†1 *Educational Review*, pp. 209-16, (1898).

Peirce: CP 3.554 Fn 2 p 346

†2 *Metaphysica* 1061a 28-1061b 3; 1061b 21-25.

Peirce: CP 3.554 Fn P1 p 346 Cross-Ref:††

†P1 Davidson, *Aristotle and the ancient educational ideals*. Appendix: The Seven Liberal Arts. (New York: Charles Scribner's Sons.)

Peirce: CP 3.555 Fn P1 p 347 Cross-Ref:††

†P1 Brouillon, *Projet d'une atteinte aux événements des rencontres du cône avec son plan*, 1639.

Peirce: CP 3.558 Fn P1 p 348 Cross-Ref:††

†P1 In his *Linear associative algebras*, [p. 97, published in the *American Journal of Mathematics*, vol. 4, (1881), pp. 97-229; see No. VIII.]

Peirce: CP 3.559 Fn P1 p 349 Cross-Ref:††

†P1 See this well put in Thomson and Tait's *Natural Philosophy*, §447.

Peirce: CP 3.561 Fn P1 p 352 Cross-Ref:††

†P1 *The Mathematical Psychology of Boole and Gratry*.

Peirce: CP 3.562 Fn 1 p 352

†1 No further articles were published in the *Educational Review*. The following, however, was part of the original article and seems to have been omitted only because of lack of space. What follows is taken from paginated page proofs.

Peirce: CP 3.562A Fn P1 p 353 Cross-Ref:††

†P1 I notice that writers of school arithmetics shrink from accepting the correct name of their art, Vulgar Arithmetic. It is a pity we have lost Chaucer's word "augrim," which, etymologically meaning "the art of the Chorasmian," is free from all objection. Still, we cannot find fault with these writers who adopt no more high sounding title for their subject than Practical Arithmetic.

Peirce: CP 3.562B Fn 1 p 354

†1 This should be: C is not *R'd* by B.

Peirce: CP 3.562B Fn 2 p 354

†2 Given the precepts

I. $(\alpha\pi\alpha b) - (a R b) \Psi - (b R a)$

II. $(\alpha\pi\alpha b\pi c) a R b \Psi b R c \Psi c R a$

through the use of the propositions of logic and the principles of substitution and inference, the following are some of the theorems that can be derived.

A. $(\pi b) - (b R b)$

$$[[I:(b/a)]] - (b R b) \Psi - (b R b) \quad (1)$$

$$(1) -< - (b R b)$$

B. $(\pi a \pi b) a R b \Psi b R a$

$$[[II:(b/c)]] a R b \Psi b R b \Psi b R a \quad (1)$$

$$(A) \cdot (1) -< a R b \Psi b R a$$

C. $(\pi a \pi b \pi c) b R a -< (b R c \Psi c R a)$

$$I = b R a -< - (a R b) \quad (1)$$

$$II = -(a R b) -< (b R c \Psi c R a) \quad (2)$$

$$(1) \cdot (2) -< [b R a -< (b R c \Psi c R a)]$$

D. $(\pi a \pi b \pi c)(b R a \cdot c R b) -< c R a$

$$[[I:(c/a)]] - (c R b) \Psi - (b R c) \quad (1)$$

$$= c R b -< - (b R c) \quad (2)$$

$$C = b R a -< [[-(b R c) -< c R a]] \quad (3)$$

$$= [[b R a \cdot -(b R c)] -< c R a \quad (4)$$

$$(2) \cdot (4) -< (b R a \cdot c R b) -< c R a$$

E. $(\pi a \pi b \pi c) - (b R a) \Psi - (c R b) \Psi - (a R c)$

$$D = -(b R a) \Psi - (c R b) \Psi c R a \quad (1)$$

$$[[I:(c/b)]] -(a R c) \Psi -(c R a) \quad (2)$$

(1) · (2) \leftarrow \neg $(b R a)$ \leftrightarrow \neg $(c R b)$ \leftrightarrow \neg $(a R c)$

Peirce: CP 3.562C Fn 1 p 355

†1 4 = \neg $(c R a)$ \leftarrow \neg $(b R a)$ \leftrightarrow \neg $(c R b)$, which by $[[n/c, m/a, x/b]]$ is \neg (nRm) \leftarrow \neg (xRm) \leftrightarrow \neg (nRx) .

Peirce: CP 3.562C Fn P1 p 355 Cross-Ref:††

†P1 The word hardly *is* in older English "hard." N follows *hard* upon M, that is, solidly up against M, with nothing between them.

Peirce: CP 3.562I Fn 1 p 359

†1 The proof sheet ends here with the note "to be continued." The ms. of this paper has not been uncovered but similar papers are published in vol. 4, bk. I.

Peirce: CP 3.563 Fn 1 p 360

†1 A letter to the Editor of *Science*, vol. 2, pp. 430-33, March 16, 1900.

Peirce: CP 3.563 Fn 2 p 360

†2 *Stetigkeiten u. Irrationalen Zahlen*, 2^{te} Auf., 1892; *Was sind u. was sollen die Zahlen*, 1888.

Peirce: CP 3.563 Fn 3 p 360

†3 "Beiträge zur Begründung der transfiniten Mengenlehre," *Georg Cantor Gesammelte Abhandlung*, herausg. E. Zermelo, S. 282-351, Berlin, (1932).

Peirce: CP 3.564 Fn 4 p 360

†4 258, 281ff., 402.

Peirce: CP 3.564 Fn 5 p 360

†5 But see *Was sind u. was sollen die Zahlen*, §64.

Peirce: CP 3.565 Fn 1 p 361

†1 "Mr. Charles Peirce, as I understand his statements in the *Monist*, appears to stand almost alone amongst recent mathematical logicians outside of Italy, in still regarding the Calculus as properly to be founded upon the conception of the actually infinite and infinitesimal." *The World and the Individual*, 1st Series, p. 562n.

Peirce: CP 3.566 Fn 2 p 361

†2 548.

Peirce: CP 3.570 Fn 1 p 364

†1 *Op. cit.*, p. 562n.

Peirce: CP 3.570 Fn 1 p 365

†1 *Georg Cantor Gesammelte Abhandlung*, S. 275-6.

Peirce: CP 3.571 Fn 1 p 366

†1 Printed separately in eight pages, circa 1903, apparently intended as the second part of *A Syllabus of Certain Topics of Logic*, published as a supplement to the Lowell Lectures of 1903. See vol. 1, bk. II, ch. 1, note.

Peirce: CP 3.573 Fn P1 p 367 Cross-Ref:††

†P1 It is far better to invent a word for a purely technical conception than to

use an expression liable to be corrupted by being employed by loose writers. I reduplicate the first syllable of relation to form this word, with little reference to the meaning of the syllable as a preposition. Still, relations of this kind are the only ones that might be asserted of the same relates transposed; and the reduplication of the preposition *re* connotes such transposition.

Peirce: CP 3.574 Fn P2 p 367 Cross-Ref:††

†P2 I must, with pain and shame, confess that in my early days I showed myself so little alive to the decencies of science that I presumed to change the name of this branch of logic, a name established by its author and my master, Augustus De Morgan, to "the logic of relatives." I consider it my duty to say that this thoughtless act is a bitter reflection to me now, so that young writers may be warned not to prepare for themselves similar sources of unhappiness. I am the more sorry, because my designation has come into general use.

Peirce: CP 3.575 Fn 1 p 369

- †1 I.e., 1. extraloves $a\sim l b$
2. contraloves $-(a l b)$
 3. juxtaloves $-(a\sim l b)$
 4. reloves $a\$l b$
 5. coloves $(a l c)(c\$l b)$
 6. ultraloves $(a l c)(c\$\sim l b)$
 7. transloves $-[[a\sim l c)(c\$l b)]]$
 8. superloves $\{-[[a\sim l c)(c\$l b)]] [[a l d)(d\$\sim l b)]]\}$

Peirce: CP 3.576 Fn 2 p 369

†2 See note to 4.

Peirce: CP 3.578 Fn 3 p 369

†3 The following schedule may be of aid in this section:

1. $\Sigma[i]\Sigma[j]r i j$ -- lation
2. $\Sigma[i]\Sigma[j]r i j$ -- contralation;
3. $\pi[i]\pi[j]\sim r i j$ -- extralation; $r = 0$
4. $\pi[i]\pi[j]r i j$ -- juxtalation; $r = \infty$
5. $\Sigma[i]\pi[j]r i j$ -- perlation
6. $\Sigma[i]\pi[j]\sim r i j$ -- contraperlation
7. $\pi[i]\Sigma[j]\sim r i j$ -- extraperlation
8. $\pi[i]\Sigma[j]r i j$ -- juxtaperlation
9. $\Sigma[j]\pi[i]r i j$ -- reperlation
10. $\Sigma[j]\pi[i]\sim r i j$ -- contrareperlation

11. $\pi[j]\Sigma[i]\sim r[ij]$ -- extrareperlation
12. $\pi[j]\Sigma[i]r[ij]$ -- juxtareperlation
13. $\Sigma[i]\pi[j]\Sigma[k]r[ik] \cdot r[jk]$ -- conlation
14. $\Sigma[i]\pi[j]\Sigma[k]r[ik] \cdot r[jk]$ -- ultralation
15. $\pi[i]\Sigma[j]\pi[k] - (\sim r[ik] \cdot r[jk])$ -- translation
16. $\pi[i]\pi[j]\pi[k]\Sigma[l] - (\sim r[ik] \cdot r[jk])(r[i l] \cdot \sim r[j l])$ -- superlation
17. $\Sigma[i]\Sigma[j]r[ij]^\infty$ -- essential perlation
18. $\Sigma[i]\Sigma[j]r^\infty[ij]$ -- essential reperlation
19. $\Sigma[i]\Sigma[j]\sim r[ij]^\infty$ -- contressentiperlation

1 = -3, 5 = -7, 9 = -11; 2 = -4; 6 = -8; 10 = -12, 5 -< 12; 9 -< 8; 6 -< 11; 10 -< 7;
 17.18--< 3 Ψ 4.

Peirce: CP 3.580 Fn P1 p 370 Cross-Ref:††

†P1 Since this shows he felt the obligation, it is the more lamentable that his treatment of my notation showed no such scrupulosity. That notation had been most maturely considered and thoroughly put to the test; and his changes were, without exception, for the worse. I did not say this while he lived, out of regard for his feelings. His *additions* to the notation, to express what I had afforded no means of expressing, stand, of course, on a different footing; and I should be bound to follow him, here.

Peirce: CP 3.581 Fn 1 p 371

†1 Juxtaperlation.

Peirce: CP 3.581 Fn 2 p 371

†2 Better: ". . . there is nothing to which everything stands."

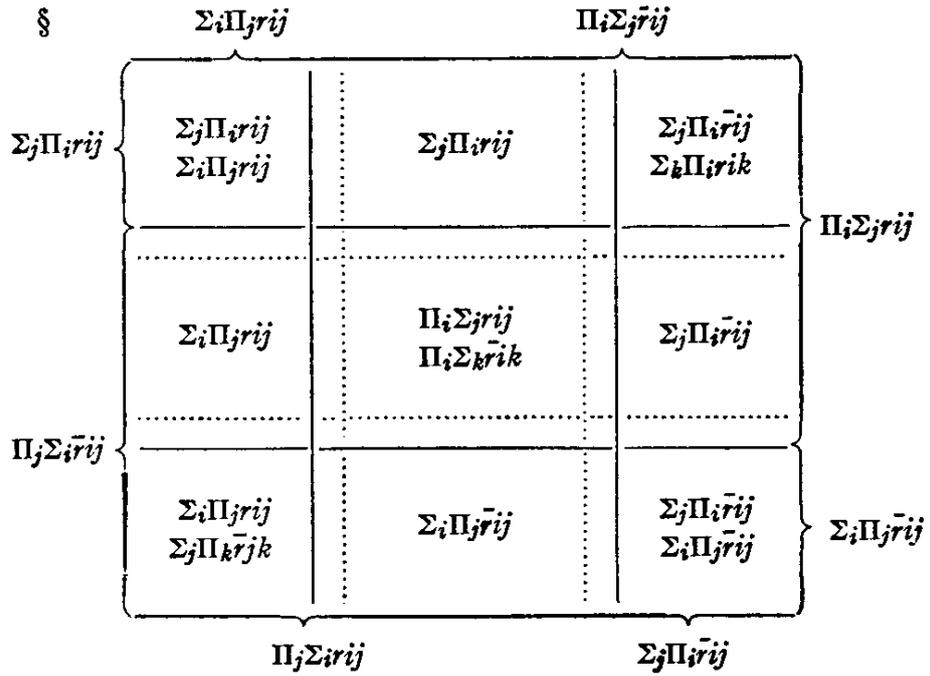
Peirce: CP 3.581 Fn 1 p 372

†1 I.e., $\Sigma[i]\pi[j]r[ij] -< \pi[j]\Sigma[i]r[ij]$; $\Sigma[j]\pi[i]r[ij] -< \pi[i]\Sigma[j]r[ij]$.

Peirce: CP 3.581 Fn 2 p 372

†2 $\Sigma[i]\pi[j]\sim r[ij] -< \pi[j]\Sigma[i]\sim r[ij]$; $\Sigma[j]\pi[i]\sim r[ij] -< \pi[i]\Sigma[j]\sim r[ij]$.

Peirce: CP 3.581 Fn 1 p 373



†1
[\[Click here to view\]](#)

Peirce: CP 3.583 Fn 1 p 374

†1 The following schedule may be of aid in this section:

1. $\pi[i]r[ii]$ --suilation
2. $\pi[i]\pi[j]\sim r[i]i\sim r[j]j r[ij]$ --contrasuilation--alio-relative
3. $\Sigma[i]\Sigma[j]\sim r[ii]\sim r[j]j r[ij]$ --extrasuilation
4. $\Sigma[i]r[ii]$ --jxtasuilation--self-relative
5. $\pi[i]\pi[j]r[ij]\cdot i \sim j$ --ambilation
6. $\pi[i]\pi[j]r[ii]r[j]j\sim r[ij]$ --contrambilation--concurrency
7. $\Sigma[i]\Sigma[j]\sim r[ij]\cdot i \sim j$ --extrambilation
8. $\Sigma[i]\Sigma[j]r[ij]\cdot i \sim j$ --jxtambilation--opponency
9. $\Sigma[i]\pi[j]r[ij]\cdot i \sim j$ --peneperlation
10. $\Sigma[j]\pi[i]r[ij]\cdot i \sim j$ --penereperlation
11. $\Sigma[i]\pi[j]\sim r[ij]\cdot i \sim j$ --penecontraperlation
12. $\Sigma[j]\pi[i]r[ij]\cdot i \sim j$ --penecontrareperlation

1 -< 4; 5 -< 8; 9 -< 8; 10 -< 8; 11 -< 7; 12 -< 7.

7. $\pi[i]\pi[j]\Sigma[k] i R j \cdot k R j \rightarrow i R k$ idempotent contraredultration

Peirce: CP 3.590 Fn 1 p 377

†1 See 576.

Peirce: CP 3.590 Fn 2 p 377

†2 "On the Syllogism IV," *Transactions Cambridge Philosophical Society*, vol. 10, p. 346 (1859).

Peirce: CP 3.591 Fn 1 p 378

†1 I.e., if (B:C) and (C:F) imply (B:F), then if (C:X) holds, (B:X) holds, X standing for any correlate; and if (Y:C) holds then (Y:F) holds, Y standing for any relate.

Peirce: CP 3.591 Fn 1 p 379

†1 I.e., BC, CA, CE, CG, FC, GD \rightarrow BA, BD, BE, BG, CD, FA, FD, FE, FG.

Peirce: CP 3.592 Fn 1 p 380

†1 See 136b.

Peirce: CP 3.593 Fn 2 p 380

†2 See 136b.

Peirce: CP 3.593 Fn 3 p 380

†3 See his *Linear Associative Algebras*, §25.

Peirce: CP 3.594 Fn 4 p 380

†4 See editors' note to §2.

Peirce: CP 3.595 Fn 1 p 381

†1 See editors' note to 2.

Peirce: CP 3.596 Fn 2 p 381

†2 *Novum Organon Renovatum*, bk. II, ch. IV.

Peirce: CP 3.596 Fn 3 p 381

†3 *Syllabus of Logic*, §§124, 144.

Peirce: CP 3.596 Fn 4 p 381

†4 See Prantl, *Geschichte der Logik*, Bd. I, S. 521.

Peirce: CP 3.598 Fn 1 p 382

†1 Cf. 136e.

Peirce: CP 3.598 Fn P1 p 382 Cross-Ref:††

†P1 I am not here making infinity a limit but in making the state of exceeding the finite the limit of successive increments by unity.

Peirce: CP 3.598 Fn 1 p 383

†1 "The Lowell Lectures," 1903, for which this syllabus was intended as a supplement.

Peirce: CP 3.600 Fn 2 p 383

†2 See 136c.

- Peirce: CP 3.600 Fn 3 p 383
 †3 Cf. Prantl, *op. cit.*, Bd. III, S. 153, 282.
- Peirce: CP 3.600 Fn 4 p 383
 †4 Cf. Prantl, *op. cit.*, Bd. III, S. 290.
- Peirce: CP 3.600 Fn 5 p 383
 †5 *Formal Logic*, p. 345.
- Peirce: CP 3.602 Fn 1 p 384
 †1 Cf. 121ff.
- Peirce: CP 3.602 Fn 2 p 384
 †2 *Algebra der Logik*, 3.1, S. 11-12 and *passim*.
- Peirce: CP 3.602 Fn P1 p 384 Cross-Ref:††
 †P1 But in the mathematical books, that which fills it is called an Element.
- Peirce: CP 3.608 Fn 1 p 387
 †1 See 2.406ff.
- Peirce: CP 3.608 Fn 2 p 387
 †2 I.e., Speculative Grammar, for which see vol. 2, bk. II.
- Peirce: CP 3.608 Fn 3 p 387
 †3 The remainder of this syllabus is to be found in 2.233 seq., where triadic relations, and, in particular, the divisions of signs are treated at length.
- Peirce: CP 3.609 Fn 1 p 388
 †1 *Dictionary of Philosophy and Psychology*, ed. by J. M. Baldwin, Macmillan & Co., N.Y., vol. 1, p. 518, by Peirce and H. B. Fine. 2d ed. 1911.
- Peirce: CP 3.609 Fn 2 p 388
 †2 "Untersuchung über Gegenstände der höhere Geodäsie; 2 Abhandlung;" *Abh. d. Königl. G. d. W. zu Göttingen*, 2^{ter} u. 3^{ter} Bd.
- Peirce: CP 3.609 Fn 3 p 388
 †3 See vol. 7.
- Peirce: CP 3.611 Fn 1 p 390
 †1 *Dictionary of Philosophy and Psychology*, vol. 1, p. 537-38.
- Peirce: CP 3.611 Fn 2 p 390
 †2 See Prantl, *op. cit.*, Bd. I, S. 661, 684.
- Peirce: CP 3.611 Fn 3 p 390
 †3 Vol. 2, bk. II, ch. 4, §12.
- Peirce: CP 3.612 Fn 4 p 390
 †4 Cf. 1.435.
- Peirce: CP 3.613 Fn 1 p 392
 †1 Vol. 2, bk. II, ch. 4, 12.
- Peirce: CP 3.614 Fn 2 p 392
 †2 *Dictionary of Philosophy and Psychology*, vol. 1, p. 574.

- Peirce: CP 3.614 Fn 3 p 392
†3 See 77-80.
- Peirce: CP 3.614 Fn 4 p 392
†4 See 113-118.
- Peirce: CP 3.616 Fn 1 p 393
†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 24-27.
- Peirce: CP 3.619 Fn 1 p 394
†1 Vol. 4, bk. II, ch. 1., §1.
- Peirce: CP 3.620 Fn 1 p 395
†1 §8.
- Peirce: CP 3.621 Fn 2 p 395
†2 A number of elementary definitions of such familiar terms as "aggregation," "absorption," "associative," "commutative," etc. have been omitted.
- Peirce: CP 3.622 Fn 1 p 398
†1 See 47n.
- Peirce: CP 3.624 Fn 1 p 399
†1 *Johns Hopkins Studies in Logic*, p. 87ff.
- Peirce: CP 3.626 Fn 2 p 399
†2 *Dictionary of Philosophy and Psychology*, vol. 2, p. 117-18, by Peirce and H. B. Fine.
- Peirce: CP 3.627 Fn 1 p 400
†1 See *Georg Cantor Gesammelte Abhandlung* S. 282, Berlin, (1932).
- Peirce: CP 3.632 Fn 1 p 401
†1 *Dictionary of Philosophy and Psychology*, vol. 2, pp. 315-16.
- Peirce: CP 3.632 Fn 2 p 401
†2 See *Analytica Posteriora*, 76b, 26-31.
- Peirce: CP 3.635 Fn 1 p 403
†1 *Dictionary of Philosophy and Psychology*, vol. 2, p. 338.
- Peirce: CP 3.636 Fn 1 p 404
†1 *Ibid.*, vol. 2, pp. 447-50.
- Peirce: CP 3.642 Fn 1 p 408
†1 *Principles of Psychology*, I, 243.
- Peirce: CP 3.643 Fn 1 p 409
†1 See Paper No. III.
- Peirce: CP 3.643 Fn 2 p 409
†2 See Paper No. XII.
- Peirce: CP 3.644 Fn 3 p 409
†3 *Dictionary of Philosophy and Psychology*, vol. 2, p. 713, by Christine Ladd-Franklin and Peirce.

- Peirce: CP 3.646 Fn 1 p 411
†1 *Johns Hopkins University Circulars*, No. 22, pp. 86-88, April, 1883,
entitled "A Communication from Mr. Peirce."
- Peirce: CP 3.647 Fn 1 p 413
†1 In *his Linear Associative Algebras*, 1870.
- Peirce: CP 3.647 Fn 2 p 413
†2 Paper No. III.
- Peirce: CP 3.647 Fn 1 p 414
†1 See 130.
- Peirce: CP 3.647 Fn 2 p 414
†2 See 150-51.
- Peirce: CP 3.647 Fn 3 p 414
†3 See 130.
- Peirce: CP 3.647 Fn P1 p 414 Cross-Ref:††
†P1 It would have been more accurately analogical, perhaps, to call it
noventions.
- Peirce: CP 3.647 Fn 4 p 414
†4 §127, beginning with "for example," was here reproduced.

Peirce: CP 4 Title-Page
THE COLLECTED PAPERS OF CHARLES SANDERS PEIRCE

EDITED BY
CHARLES HARTSHORNE
AND
PAUL WEISS
VOLUME IV
THE SIMPLEST MATHEMATICS
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Peirce: CP 4 Introduction p iii
INTRODUCTION

Peirce's punctuation and spelling have, wherever possible, been retained. Titles supplied by the editors for papers previously published are marked with an *E*, while Peirce's titles for unpublished papers are marked with a *P*. Peirce's titles for previously published papers and the editors' titles for unpublished papers are not marked. Remarks and additions by the editors are enclosed in light-face square brackets. The editors' footnotes are indicated by various typographical signs, while Peirce's are indicated by numbers. Paragraphs are numbered consecutively throughout the volume. At the top of each page the numbers signify the volume and the first paragraph of that page. All references in the indices are to the numbers of the paragraphs.

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The department and the editors desire to express their gratitude to Dr. Henry S. Leonard and Miss Isabel Stearns for their assistance with the proofs, references and editorial footnotes.

HARVARD UNIVERSITY

September, 1933

Peirce: CP 4 Editorial Note p v
EDITORIAL NOTE

In addition to his published papers on exact logic and the foundations of mathematics (see vol. III), Peirce wrote a great number of others which he seems never to have given to publishers, although many of them are advances on his published work. A selected number of these papers, arranged in chronological order, form the first book of the present volume.

Peirce: CP 4 Editorial Note p v

The logical graphs, which are presented in the second book, are, in Peirce's opinion, his greatest contribution to logic. They are considered by him to provide a

more detailed and satisfactory analysis of the structure of logical argumentation than is permitted by previous symbolisms or the traditional Eulerian diagrams. As he treats them they are pertinent to discussions on the nature of mathematics, multiple-valued logics and modality, as well as to his theory of signs (see vol. II, bk. ii) and his pragmatism (see vol. V, bk. ii). The papers in this book, particularly those which formulate the rules for the manipulation of the graphs, often overlap, since almost all his writings on this subject were different attempts to express the same truths. The repetitions could only have been avoided either by not publishing previously published papers (in violation of the plan of the edition), or by the omission of manuscripts upon which he bestowed great effort, or by the breaking of the thread of his discourse -- none of which seemed desirable alternatives.

Peirce: CP 4 Editorial Note p v

Peirce's last projected series of papers, serving to illustrate fundamental theorems in logic and mathematics, forms the third book of the volume. The frontispiece reproduces a page from the last article of this series, the last perhaps of his. original papers, written when Peirce was about seventy years old.

Peirce: CP 4 Editorial Note p vi

Papers I, III, IV, VII, VIII of book i and the whole of book ii are recommended to students of exact logic; while papers IV-X of book i and those of book iii will be most useful to students of the foundations of mathematics. The general reader will perhaps find papers II and VII of book i, and chapters 4-6 of book ii to be the clearest and most interesting.

BRYN MAWR COLLEGE

September, 1933

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Peirce: CP 4.1 Cross-Ref:††
BOOK I

LOGIC AND MATHEMATICS

(UNPUBLISHED PAPERS)

PREFACE †1

1. . . . Now what was the question of realism and nominalism?†2 I see no objection to defining it as the question of which is the best, the laws or the facts under those laws. It is true that it was not stated in this way. As stated, the question was whether *universals*, such as the Horse, the Ass, the Zebra, and so forth, were *in re* or *in rerum natura*. But that there is no great merit in this formulation of the question is shown by two facts; first, that many different answers were given to it, instead of merely yes and no, and second, that all the disputants divided the question into various parts. It was therefore a broad question and it is proper to look beyond the letter into the spirit of it. Most of those scholastics whose works are occasionally read today were matter-of-fact dualists; and when they used the phrase *in re* or *in rerum natura* in formulating the question, they took for granted something in regard to which other disputants, however confusedly, were at odds with them. For some of them regarded the universals as more real than the individuals. Therefore, the reality, or as I would say in order to avoid any begging of the question, the value or worth, not merely of the universals, but also that of the individuals was a part of the broad question. Finally, it was always agreed that there were other sorts of universals

besides *genera* and *species*, and in using the word law, or regularity, we bring into prominence the kind of universals to which modern science pays most attention. Roughly speaking, the nominalists conceived the *general* element of cognition to be merely a convenience for understanding this and that fact and to amount to nothing except for cognition, while the realists, still more roughly speaking, looked upon the general, not only as the end and aim of knowledge, but also as the most important element of being. Such was and is the question. It is as pressing today as ever it was, Ernst Mach, †1 for example, holding that generality is a mere device for economising labor while Hegeler, †2 though he extols Mach to the skies, thinks he has said that man is immortal when he has only said that his influence survives him.

Peirce: CP 4.1 Cross-Ref: ††

According to the nominalistic view, the only value which an idea has is to represent the fact, and therefore the only respect in which a system of ideas has more value than the sum of the values of the ideas of which it is composed is that it is compendious; while, according to the realistic view, this is more or less incorrect depending upon how far the realism be pushed.

Peirce: CP 4.1 Cross-Ref: ††

Dr. [F.E.] Abbot in his *Scientific Theism* [1885] has so clearly and with such admirable simplicity shown that modern science is realistic that it is perhaps injudicious for me to attempt* to add anything upon the subject. Yet I shall try to put it into such a light that it may reflect some rays upon the worth or worthlessness of detached ideas. But I warn you that I shall not argue the question, but only indicate what my line of reasoning would be were I to enter upon it in detail.

Peirce: CP 4.1 Cross-Ref: ††

The burden of proof is undoubtedly upon the realists, because the nominalistic hypothesis is the simpler. Dr. Carus †3 professes himself a realist and yet accuses me of inconsistency in admitting Ockham's razor although I am a realist, thus, implying that he himself does not accept it. †4 But this brocard, *Entia non sunt multiplicanda praeter necessitatem*, that is, a hypothesis ought not to introduce complications not requisite to explain the facts, this is not distinctively nominalistic; it is the very roadbed of science. Science ought to try the simplest hypothesis first, with little regard to its probability or improbability, although regard ought to be paid to its consonance with other hypotheses, already accepted. This, like all the logical propositions I shall enunciate, is not a mere private impression of mine: it is a mathematically necessary deduction from unimpeachable generalizations of universally admitted facts of observation. The generalizations are themselves allowed by all the world; but still they have been submitted to the minutest criticism before being employed as premisses. It appears therefore that in scientific method the nominalists are entirely right. Everybody ought to be a nominalist at first, and to continue in that opinion until he is driven out of it by the *force majeure* of irreconcilable facts. Still he ought to be all the time on the lookout for these facts, considering how many other powerful minds have found themselves compelled to come over to realism.

Peirce: CP 4.1 Cross-Ref: ††

Nor has the wealth of thought that has been expended upon the defenses of nominalism especially by four great English philosophers who have engineered the works, I mean Ockham, Hobbes, Berkeley, and James Mill, by any means been wasted. It has on the contrary been most precious for the clear comprehension of logic and of metaphysics. But as for the average nominalist whom you meet in the streets,

he reminds me of the blind spot on the retina, so wonderfully does he unconsciously smooth over his field of vision and omit facts that stare him in the face, while seeing all round them without perceiving any gap in his view of the world. That any man not demented should be a realist is something he cannot conceive.

Peirce: CP 4.1 Cross-Ref:††

My plan for defeating nominalism is not simple nor direct; but it seems to me sure to be decisive, and to afford no difficulties except the mathematical toil that it requires. For as soon as you have once mounted the vantage-ground of the logic of relatives, which is related to ordinary logic precisely as the geometry of three dimensions is to the geometry of points on a line, as soon as you have scaled this height, I say, you find that you command the whole citadel of nominalism, which must thereupon fall almost without another blow.

Peirce: CP 4.1 Cross-Ref:††

I am going to describe in general terms what this logic of relatives is, so far as it bears upon this great controversy. And in doing so I can at the same time, without lengthening the lecture by more than three or four minutes, make my account of this generalized logic illustrate some of [the] relative advantages and disadvantages of detached ideas and of systematic thought, by simply forming it into a narrative of how I myself became acquainted with that logic.

Peirce: CP 4.2 Cross-Ref:††

2. I came to the study of philosophy not for its teaching about God, Freedom, and Immortality, but intensely curious about Cosmology and Psychology. In the early sixties I was a passionate devotee of Kant, at least as regarded the Transcendental Analytic in the *Critic of the Pure Reason*. I believed more implicitly in the two tables of the Functions of Judgment and the Categories than if they had been brought down from Sinai. Hegel, so far as I knew him through a book by Vera †1 repelled me. Now Kant points out certain relations between the categories. I detected others; but these others, if they had any orderly relation to a system of conceptions, at all, belonged to a larger system than that of Kant's list. Here there was a problem to which I devoted three hours a day for two years, rising from it, at length, with the demonstrative certitude that there was something wrong about Kant's formal logic. Accordingly, I read every book I could lay hands upon on logic, and of course Kant's essay on the *falsche Spitzfindigkeit der vier syllogistischen Figuren*;†2 and here I detected a fallacy similar to that of the phlogistic chemists. For Kant argues that the fact that all syllogisms can be reduced to Barbara shows that they involve no logical principle that Barbara does not involve. A chemist might as well argue, that because water boiled with zinc dust evolves hydrogen, and the hydrogen does not come from the zinc, therefore water is a mere form of hydrogen. In short, Kant omits to inquire whether the very reasoning by which he reduces the indirect moods to Barbara may not itself introduce an additional logical principle. Pursuing this suggestion, I found that that was in truth the case, and I succeeded [in 1866] in demonstrating that the second and third figures each involved a special additional logical principle, both of which enter into the fourth figure.†3 Namely, the additional principle of the second figure is that by which we pass from judging that among dumb brutes no animal with a hand can be found, to judging that among animals with hands no dumb brute can be found; and the additional principle of the third figure is that by which we pass from judging that among human beings there are females to that of judging that among female animals there are human beings. Although I do not stop to give the proof, I assert that it is rigidly demonstrated that these are distinct principles of logic. Thus to find that the

passage from one way of viewing a fact to another way of viewing the same fact should be a logical principle was naturally food for reflection. I remarked that while the circumstances under which propositions of the form No A is B and No B is A are true are identical, yet the circumstances under which such a pair of propositions indefinitely approximate to being true do not by any means indefinitely approximate toward being identical. For instance, the probability that a man taken at random will be a poet as great as Dante may be indefinitely near to zero; but it does not follow that the probability that a poet as great as Dante will be a man approximates to zero, at all. This reflection led me to inquire whether there might not be forms of probable reasoning analogous to the second and third figures of syllogism which were widely different from one another and from the first figure. Here, Aristotle's account of induction aided me; for Aristotle †1 makes induction to be a probable syllogism in the third figure.

Peirce: CP 4.3 Cross-Ref: ††

3. I found that there was also a mode of probable reasoning in the second figure essentially different both from induction and from probable deduction. †2 This was plainly what is called reasoning from consequent to antecedent, and in many books is called adopting a hypothesis for the sake of the explanation it affords of known facts. It would be tedious to show how this discovery led to the thorough refutation of the third and most important of Kant's triads, and the confirmation of the doctrine that for the purposes of ordinary syllogism categorical propositions and conditional propositions, which Kant and his ignorant adherents call hypotheticals, are all one. †3 This led me to see that the relation between subject and predicate, or antecedent and consequent, is essentially the same as that between premiss and conclusion. †4 It was interesting to see how the combined result of all these improvements and some others to which I have not alluded was decidedly to consolidate that systematic or synthetic unity in the system of formal logic which occupied so large a place in Kant's thought. But though there was more unity than in Kant's system, still, as the subject stood, there was not as much as might be desired. Why should there be three principles of reasoning, and what have they to do with one another? This question, which was connected with other parts of my schedule of philosophical inquiry that need not be detailed, now came to the front. Even without Kant's categories, the recurrence of triads in logic was quite marked, and must be the croppings out of some fundamental conceptions. I now undertook to ascertain what the conceptions were. This search resulted in what I call my categories. I then †1 named them Quality, Relation, and Representation. But I was not then aware that undecomposable relations may necessarily require more subjects than two; for this reason *Reaction* is a better term. Moreover, I did not then know enough about language to see that to attempt to make the word *representation* serve for an idea so much more general than any it habitually carried, was injudicious. The word *mediation* would be better. Quality, reaction, and mediation will do. But for scientific terms, Firstness, Secondness, and Thirdness, are to be preferred as being entirely new words without any false associations whatever. How the conceptions are *named* makes, however, little difference. I will endeavor to convey to you some idea of the conceptions themselves. †2 It is to be remembered that they are excessively general ideas, so very uncommonly general that it is far from easy to get any but a vague apprehension of their meaning. . . .

Peirce: CP 4.4 Cross-Ref: ††

4. [With regard to] my logical studies in 1867, various facts proved to me beyond a doubt that my scheme of formal logic was still incomplete. For one thing, I

found it quite impossible to represent in syllogisms any course of reasoning in geometry, or even any reasoning in algebra, except in Boole's logical algebra. Moreover, I had found that Boole's algebra required enlargement to enable it to represent the ordinary syllogisms of the third figure; and though I had invented such an enlargement, it was evidently of a makeshift character, and there must be some other method springing out of the idea of the algebra itself. Besides, Boole's algebra suggested strongly its own imperfection. Putting these ideas together I discovered the logic of relatives. †1 I was not the first discoverer; but I thought I was, and had complemented Boole's algebra so far as to render it adequate to all reasoning about dyadic relations, before Professor De Morgan sent me his epoch-making memoir †2 in which he attacked the logic of relatives by another method in harmony with his own logical system. But the immense superiority of the Boolean method was apparent enough, and I shall never forget all there was of manliness and pathos in De Morgan's face when I pointed it out to him in 1870. I wondered whether when I was in my last days some young man would come and point out to me how much of my work must be superseded, and whether I should be able to take it with the same genuine candor. . . †3

Peirce: CP 4.5 Cross-Ref: ††

5. The great difference between the logic of relatives and ordinary logic is that the former regards the form of relation in all its generality and in its different possible species while the latter is tied down to the matter of the single special relation of similarity. The result is that every doctrine and conception of logic is wonderfully generalized, enriched, beautified, and completed in the logic of relatives.

Peirce: CP 4.5 Cross-Ref: ††

Thus, the ordinary logic has a great deal to say about *genera* and *species*, or in our nineteenth century dialect, about *classes*. Now, a *class* is a set of objects comprising all that stand to one another in a special relation of similarity. But where ordinary logic talks of classes the logic of relatives talks of *systems*. A *system* is a set of objects comprising all that stand to one another in a group of connected relations. Induction according to ordinary logic rises from the contemplation of a sample of a class to that of the whole class; but according to the logic of relatives it rises from the contemplation of a fragment of a system to the envisagement of the complete system.

Peirce: CP 4.6 Cross-Ref: ††

6. It is requisite that the reader should fully understand the relation of thought in itself to *thinking*, on the one hand, and to graphs, on the other hand. Those relations being once magisterially grasped, it will be seen that the graphs break to pieces all the really serious barriers, not only to the logical analysis of thought, but also to the digestion of a different lesson, by rendering literally visible before one's very eyes the operation of thinking *in actu*. In order that the fact should come to light that the method of graphs really accomplishes this marvelous result, it is first of all needful, or at least highly desirable, that the reader should have thoroughly assimilated, in all its parts, the truth that thinking always proceeds in the form of a dialogue -- a dialogue between different phases of the *ego* -- so that, being dialogical, it is essentially composed of signs, as its matter, in the sense in which a game of chess has the chessmen for its matter. Not that the particular signs employed *are* themselves the thought! Oh, no; no whit more than the skins of an onion are the onion. (About as much so, however.) One selfsame thought may be carried upon the vehicle of English, German, Greek, or Gaelic; in diagrams, or in equations, or in graphs: all these are but so many skins of the onion, its inessential accidents. Yet that the thought should have

some possible expression for some possible interpreter, is the very being of its being. .

..

Peirce: CP 4.7 Cross-Ref:††

7. How many writers of our generation (if I must call names, in order to direct the reader to further acquaintance with a generally described character -- let it be in this case the distinguished Husserl †1), after underscored protestations that their discourse shall be of logic exclusively and not by any means of psychology (almost all logicians protest that on file), forthwith become intent upon those elements of the process of thinking which seem to be special to a mind like that of the human race, *as we find it*, to too great neglect of those elements which must belong, as much to any one as to any other mode of embodying the same thought. It is one of the chief advantages of Existential Graphs, as a guide to Pragmaticism, that it holds up thought to our contemplation with the wrong side out, as it were; showing its construction in the barest and plainest manner, so that it [does not] seduce us into the bye-path of the distinctively English logicians (whether in that branch of it where the way is strewn, often in the most valuably suggestive works, such as Venn's *Empirical Logic*, with puerilities about words -- and often not merely strewn with them but buried so deep in them, as by a great snowstorm, as to obstruct the reader's passage and render it fatiguing in the extreme, while the books of lesser inquirers, say Carveth Read,†1 Horace William Brindley Joseph,†2 and the last edition (greatly inferior to the first) of John Neville Keynes' *Formal Logic*, offer little reward for the labour of listening to their irrelevant baby-talk; or whether in the other branch of the same path where, as in the two Logics of Miss Constance Jones,†3 it seems to be forgotten that Latin Grammar does not furnish the only type even of Sud Germanic construction, which is itself a peculiarly specialized form of expression opposed in various particulars to the common ways of thinking of the great majority of mankind).

Peirce: CP 4.8 Cross-Ref:††

8. Nor does it lead us into the divarications of those who know no other logic than a "Natural History" of thought. As to this remark, I pray you, that "Natural History" is the term applied to the descriptive sciences of nature, that is to say, to sciences which describe different kinds of objects and classify them as well as they can while they still remain ignorant of their essences and of the ultimate agencies of their production, and which seek to explain the properties of those kinds by means of laws which another branch of science called "Natural Philosophy" has established. Thus a logic which is a natural history merely, has done no more than observe that certain conditions have been found attached to sound thought, but has no means of ascertaining whether the attachment be accidental or essential; and quite ignoring the circumstance that the very essence of thought lies open to our study; which study alone it is that men have always called "logic," or "dialectic."

Peirce: CP 4.8 Cross-Ref:††

Accordingly, when I say that Existential Graphs put before us moving pictures of thought, I mean of thought in its essence free from physiological and other accidents. . . .

Peirce: CP 4.9 Cross-Ref:††

9. The highest kind of symbol is one which signifies a growth, or self-development, of thought, and it is of that alone that a moving representation is possible; and accordingly, the central problem of logic is to say whether one given thought is truly, i.e., is adapted to be, a development of a given other or not. In other words, it is the critic of arguments. Accordingly, in my early papers I limited logic to

the study of this problem. But since then, I have formed the opinion that the proper sphere of any science in a given stage of development of science is the study of such questions as one social group of men can properly devote their lives to answering †1; and it seems to me that in the present state of our knowledge of signs, the whole doctrine of the classification of signs and of what is essential to a given kind of sign, must be studied by one group of investigators. Therefore, I extend logic to embrace all the necessary principles of semeiotic, and I recognize a logic of icons, and a logic of indices, as well as a logic of symbols; and in this last I recognize three divisions: *Stecheotic* (or stoicheiology), which I formerly called Speculative Grammar; *Critic*, which I formerly called Logic; and *Methodeutic*, which I formerly called Speculative Rhetoric.†2

Peirce: CP 4.10 Cross-Ref:††

10. A fallacy is, for me, a supposititious thinking, a thinking that parades as a self-development of thought but is in fact begotten by some other sire than reason; and this has substantially been the usual view of modern logicians. For reasoning ceases to be Reason when it is no longer reasonable: thinking ceases to be Thought when true thought disowns it. A self-development of Thought takes the course that thinking will take that is sufficiently deliberate, and is not truly a self-development if it slips from being the thought of one object-thought to being the thought of another object-thought. It is, in the geological sense, a "fault" -- an inconformability in the strata of thinking. The discussion of it does not appertain to pure logic, but to the application of logic to psychology. I only notice it here, as throwing a light upon what I *do not* mean by "Thought."

Peirce: CP 4.11 Cross-Ref:††

11. I trust by this time, Reader, that you are conscious of having some idea, which perhaps is not so dim as it seems to you to be, of what I mean by calling Existential Graphs a moving-picture of Thought. Please note that I have not called it a *perfect* picture. I am aware that it is not so: indeed, that is quite obvious. But I hold that it is considerably more nearly perfect than it seems to be at first glance, and quite sufficiently so to be called a portraiture of Thought.

Peirce: CP 4.12 Cross-Ref:††
THE SIMPLEST MATHEMATICS

I

A BOOLEAN ALGEBRA WITH ONE CONSTANT†1

12. Every logical notation hitherto proposed has an unnecessary number of signs. It is by means of this excess that the calculus is rendered easy to use and that a symmetrical development of the subject is rendered possible; at the same time, the number of primary formulæ is thus greatly multiplied, those signifying facts of logic being very few in comparison with those which merely define the notation. I have thought that it might be curious to see the notation in which the number of signs should be reduced to a minimum; and with this view I have constructed the following. The apparatus of the Boolean calculus consists of the signs, =, > (not used by Boole,

but necessary to express particular propositions) +, -, X, 1, 0. In place of these seven signs, I propose to use a single one.

Peirce: CP 4.13 Cross-Ref:††

13. I begin with the description of the notation for conditional or "secondary" propositions. The different letters signify propositions. Any one proposition written down by itself is considered to be asserted. Thus,

A

means that the proposition **A** is true. Two propositions written in a pair are considered to be both denied. Thus,

A B

means that the propositions **A** and **B** are both false; and

A A

means that **A** is false. We may have pairs of pairs of propositions and higher complications. In this case we shall make use of commas, semicolons, colons, periods, and parentheses, just as [in] chemical notation, to separate pairs which are themselves paired. These punctuation marks can no more count for distinct signs of algebra, than the parentheses of the ordinary notation.

Peirce: CP 4.14 Cross-Ref:††

14. To express the proposition: "If **S** then **P**," first write

A

for this proposition. But the proposition is that a certain conceivable state of things is absent from the universe of possibility. Hence instead of **A** we write

B B

Then **B** expresses the possibility of **S** being true and **P** false. Since, therefore, **SS** denies **S**, it follows that (**SS, P**) expresses **B**. Hence we write

SS, P; SS, P.†1

Peirce: CP 4.15 Cross-Ref:††

15. Required to express the two premisses, "If **S** then **M**" and "if **M** then **P**." Let

A

be the two premisses. Let **B** be the denial of the first and **C** that of the second; then in

place of A we write

$B C$

But we have just seen that B is (SS, M) and that C is (MM, P) ; accordingly we write

$SS, M; MM, P.$

Peirce: CP 4.16 Cross-Ref:††

16. All the formulae of the calculus may be obtained by development or elimination. The development or elimination having reference say to the letter X , two processes are required which may be called the erasure of the X s and the erasure of the double X s. The erasure of the X s is performed as follows:

Peirce: CP 4.17 Cross-Ref:††

17. Erase all the X s and fill up each blank with whatever it is paired with. But where there is a double X this cannot be done; in this case erase the whole pair of which the double X forms a part, and fill up the space with whatever it is paired with. Go on following these rules.

Peirce: CP 4.17 Cross-Ref:††

A pair of which both members are erased is to be considered as doubly erased. A pair of which either member is doubly erased is to be considered as only singly erased, without regard to the condition of the other member. Whatever is singly erased is to be replaced by the repetition of what it is paired with.

Peirce: CP 4.17 Cross-Ref:††

To erase the double X s, repeat every X and then erase the X s.†1

Peirce: CP 4.18 Cross-Ref:††

18. If ϕ be any expression, ϕ/x what it becomes after erasure of the X s, and $\phi/x x$ what it becomes after erasure of the double

X s, then

$$\phi = \phi/x, x; \phi/x x, xx.$$

If ϕ be asserted, then

$$\phi/x \phi/x x, \phi/x \phi/xx$$

may be asserted.

Peirce: CP 4.19 Cross-Ref:††

19. The following are examples. Required to develop X in terms of X . Erasing the X s the whole becomes erased, and

$$(\phi/x) x = xx.$$

Erasing the double X s, the whole becomes doubly erased and

$(\phi/x x) xx$ is erased. If ϕ , then

$$(\phi/x) x, (\phi/x x) x x = xx, xx.$$

So that

$$X = xx, xx.$$

Required to eliminate X from $(xx, x; a)$.

$$\phi/x = 00, 0; a = aa$$

$$\phi/x x = 00, 00; 00: a = aa$$

$$\therefore \phi = \dagger 1 \phi/x \phi/xx, \phi/x \phi/xx = aa, aa; aa, aa = aa.$$

Required to eliminate X from (xa, a) .

$$\phi/x = 0a, a = aa, a$$

$$\phi/x x = 00, a; a = aa$$

$$\therefore \phi \dagger 1 = aa, a; aa:aa, a; aa = aa, aa = a \dagger 2$$

Required to develop $(ax; b, xx:ab)$ according to X .

$$\phi/x = a0; b, 00: ab = aa, aa; ab = a, ab = a \dagger 3$$

$$\phi/x x = a00; b, 0:ab = bb, bb; ab = b, ab = b \dagger 3$$

$$\phi = (\phi/x) x, (\phi/x x) xx = ax; b, xx \dagger 3$$

Required to eliminate M from $(SS, M; MM, P)$.

$$\phi/M = SS, 0; 00, P = SS, SS; SS, SS = SS$$

$$\phi/M M = SS, 00; 0, P = P, P; P, P = P$$

$$\therefore SS, M; MM, P = \dagger 1 SS, P; SS, P$$

which is the syllogistic conclusion.

Peirce: CP 4.19 Cross-Ref: ††

We may now take an example in categoricals. Given the premisses "There is something besides S s and M s," and "There is nothing besides M s and P s," to find the conclusion. As the combined premisses state the existence of a non- S non- M and the non-existence of an MP , †4 they are expressed by

$$SM, SM; MP.$$

To eliminate M , we have

$$\phi/M = S0, S0; 0P = SS, SS; PP = S, PP$$

$$\phi/M M = S; 00, S, 00:00, P = \text{erased}$$

∴ $\phi/M \phi/MM, \phi/M \phi/MM = \dagger 1 S, PP; 0:S, PP; 0$
 $= \dagger 1 S, PP; S, PP: S, PP; S, PP = S, PP.$

The conclusion therefore is that there is something which is not an *S* but is a *P*.

Peirce: CP 4.20 Cross-Ref: ††

20. Of course, it is not maintained that this notation is convenient; but only that it shows for the first time the possibility of writing both universal and particular propositions with but one copula which serves at the same time as the only sign for compounding terms and which renders special signs for negation, for "what is" and for "nothing" unnecessary. It is true, that a 0 has been used, but it has only been used as the sign of an erasure. †2

Peirce: CP 4.21 Cross-Ref: ††

II

THE ESSENCE OF REASONING †1

§1. SOME HISTORICAL NOTES

21. . . . Logic having been written first in Greek had to be turned into Latin; and this was done for the most part by imitating the formation of each technical term. Thus, the Greek *hypothesis*, {hypothesis} was compounded of {hypo}, *under* and {tithenai}, *to put*. The preposition {hypo} was equivalent to the Latin *sub* -- which is from the same root (being altered from *sup*), and {tithenai} was translated by *ponere*. Hence resulted *suppositio*. It is a very curious fact, by the way, that in this process it was always necessary to change the root. For, whether it be that there is something analogous to Grimm's law applying to *meanings*, as that applies to *sounds*, certain it is that the roots bear so uniformly different meanings that a different one must always be taken. Thus, the root of {tithenai} is the same as that of the Latin *facere*, so that *hypothetical* is the equivalent of sufficient, which widely diverges in meaning. *Ponere* is *po-sinere*, of which the root may be *sa*, to sow, to strew.

Peirce: CP 4.22 Cross-Ref: ††

22. The earliest Latin work in which we find logical words so transferred from the Greek is supposed to be a treatise on Rhetoric (*Ad Herennium*) usually printed with the works of Cicero, but supposed to be written by one Cornificius, a little older than Cicero. Cicero himself made a number of words on that plan which are now very common, such as *quantity* and *quality*. †2

Peirce: CP 4.23 Cross-Ref: ††

23. Apuleius, the author early in the second century of our era of the celebrated novel of the *Golden Ass*, wrote a treatise on logic which has somehow come to be arranged as the third book of his work *De dogmate Platonis*. The terminology of this treatise we may be pretty sure [Apuleius] did not invent, though it differs considerably from that of any other book which either preceded or for

centuries followed it. That terminology has overridden other rival systems of translating the Greek words and has become largely ours. If the reader asks me what the quality was which lent it this staying power, he will be surprised at the answer. Namely, [Apuleius] had one of the most artificial, word-playing, fantastically and elaborately nonsensical styles that the Indo-European literatures can show. It sedulously cultivates every quality which writers upon style admonish us to avoid.

Peirce: CP 4.24 Cross-Ref:††

24. . . . Towards the end of the fifth century there was one Martianus Minneus Felix Capella, who wrote a work entitled the *Nuptials of Philology and of Mercury*. This Martianus Capella thought that beneath the stars there was nothing so beautiful nor so worthy of emulation as the style of Apuleius. He did his very best to outdo him; and in studying him became embued with his phraseology. Now in the book the Seven Liberal Arts are invited to the celebration of the Nuptials aforesaid, and each one entertains the company with the greatest good taste by talking shop for all she is worth. The consequence is that the book contains seven short treatises upon these disciplines, of which logic is one. Now the masters of the cathedral schools which at the fall of the Western Empire had to take the place of the old Roman schools found that in an age when one copy of one book and that not too large a one, was all that one school could commonly afford, the work of Capella was well adapted to their purpose. And thus it happened that for some centuries that was the only secular book that ordinary clerks had ever laid eyes upon. Thus, its borrowed terminology became traditional.

Peirce: CP 4.25 Cross-Ref:††

25. Anicius Manlius Severinus Boëthius (more commonly called Boëthius) was the author of a book which, whatever its merits and faults, was sincere and has in fact excited a degree of admiration such as has fallen to few works. He is most respectable as a thinker, a logician of positive strength, a man of great learning, a most estimable and sympathetic character, and the courageous supporter of calamities that touch every heart. . . .

Peirce: CP 4.26 Cross-Ref:††

26. Petrus Hispanus was a noble Portuguese who, having taken degrees in all the faculties in Paris, returned to Lisbon and was appointed head of that school which ultimately developed into the University of Coimbra. Subsequently, he was head physician to Pope Gregory X, who created him Cardinal; and he was crowned Pope, September 20, 1276.†1 He began his pontificate with promise of grandeur; but a part of his palace fell upon him and he died in consequence of his injuries on May 16, 1277.†1 This man, who had he survived would surely have been reckoned among the world's great men, was according to the tradition, the author of the *Summulæ logicales*, the regular textbook in logic almost to the very end of scholasticism. There are, it is true not very many printed editions subsequent to 1520; but over fifty editions having by that time been printed upon substantial linen paper, copies could always be procured in plenty. Manuscript copies were also current long after printing came in.

Peirce: CP 4.26 Cross-Ref:††

There is a Greek text of the book; it has been printed with the name of Michael Psellus attached to it. That name was a common one in Constantinople. Even if any MS. carries it, which has been denied, it does not prove that any particular Michael Psellus was the author; and the language, which is intermediate between Greek and a kind of Romaic, absolutely negatives the idea of its being written by any

Michael Psellus known. It is full of Latinisms, and of reminiscences of Latin authors. The Latin text on the other hand bears on its first page conclusive evidence that the author did not know Greek. Namely, we there read: "Dicitur enim *dyalectica a dya*, quod est duo, et *logos*, quod est fermo vel lexis quod est ratio." Nevertheless, some writers, especially Prantl,†2 have believed the Greek text to be the original. Charles Thurot has written ably on the other side. When the reader comes across anything about "Byzantine" logic, what is meant is that this book is supposed to be the relic of a development of logic in Constantinople, which in my opinion is an unfounded fancy of Prantl's taken up by many writers without sufficient examination, and solely because Prantl has looked into more logical books of the middle ages than anybody else. I am very grateful to him for what he has read and published in a most convenient form; but I find myself compelled to dissent from his judgment very many times. A more slap-dash historian it would be impossible to conceive.

Peirce: CP 4.27 Cross-Ref:††

27. There is a synchronism between the different periods of medieval architecture, and the different periods of logic. The great dispute between the Nominalists and Realists took place while men were building the round-arched churches, and the elaboration finally attained corresponds to the intricate character of the opinions of the later disputants in that controversy. From that style of architecture we pass to the early pointed architecture with only plate-tracery. The simplicity of it is perfectly paralleled by the simplicity of the early logics of the thirteenth century. Among these simple writings, I reckon the commentaries of Averroes and of Albertus Magnus. I would add to them the writings of the great psychologist, St. Thomas Aquinas. For Thomistic Logic, I refer to Aquinas,†1 to Lambertus de Monte †2 whose work was approved by the Doctors of Cologne, to the highly esteemed Logic of the Doctors of Coimbra,†3 and to the modern manual of [Antoine] Bensa.†4

Peirce: CP 4.28 Cross-Ref:††

28. During the period of the Decorated Gothic, we have the writings of Duns Scotus, one of the greatest metaphysicians of all time, whose ideas are well worth careful study, and are remarkable for their subtilty, and their profound consideration of all aspects of the questions [of philosophy]. The logical upshot of the doctrine of Scotus is that real problems cannot be solved by metaphysics, but must be decided according to the evidence. As he was a theologian, that evidence was, for him, the dicta of the church. But the same system in the hands of a scientific man will lead to his insisting upon submitting everything to the test of observation. Especially, will he insist upon doing so as against so-called "experientialists," who, though they talk about experience as their guide, really reach the most important conclusions without any careful examination of experience. Whether their conclusions happen to be right or wrong, the Scotist will protest against the manner in which they are taken up. Scotus added a great deal to the language of logic. Of his invention is the word *reality*. For Scotistic logic I refer to Scotus,†1 Sirectus,†2 and Tartaretus.†3

Peirce: CP 4.29 Cross-Ref:††

29. Scotus died in 1308. After him William of Ockham, who died in 1347, took up once more the nominalistic opinion and this gained ground more and more. Logic now took on a very elaborate, but fanciful and in great measure senseless development; and finally became so big and so useless, that men must have dropped it, even if a new awakening of thought had not occurred. This was during the flamboyant period of architecture in France, the perpendicular in England. The Occamists made important additions to the terminology. For Occamistic Logic, I refer

to Ockham's own elaborate treatise,†4 to the *Summulæ* of the Doctors of Mayence, to the commentaries of Bricot,†5 etc.

Peirce: CP 4.30 Cross-Ref:††

30. The new awakening consisted in the conviction that the classical authors had not been sufficiently studied. At the same time the reformation of the churches came. Logic once more became simple, and this time took on a rhetorical character. Ramus (Pierre de la Ramée),†6 Ludovicus Vives,†7 Laurentius Valla,†8 were the names of logicians who contributed a few things, but on the whole, rather important things to the tradition of logic.

Peirce: CP 4.31 Cross-Ref:††

31. Upon the heels of that movement came another, which has not yet expended itself, nor even quite completed its conquest of minds. It arose from the conviction that man had everything to learn from observation. The first great investigators in this line were Copernicus, Tycho Brahe, Kepler, Galileo, Harvey, and Gilbert. None of them seemed to have any interest at all in the general theory -- and that for a simple reason; namely, they knew no way of inquiry but the way of experiment; and their lives were so many experiments in regard to the efficacy of the method of experimentation. The first great writer on the theory of Induction, Francis Bacon, was no scientific man. He had no turn that way, though he wished to have, and though he came to his death by a foolish experiment; and his judgments of scientific men were uniformly mistaken. The details of his theory were equally at fault; yet as long as he remains upon the ground of generalities, his ponderous charges are excellent. His *Novum Organum*, like several other great works of this period upon method, is marked by complete contempt for the Aristotelian analysis of reasoning, which nevertheless has kept the field, and, on the whole, held its ground. Still, Bacon made some distinct contributions to the traditional stock of logical ideas. . . .

Peirce: CP 4.32 Cross-Ref:††

32. The works we are now coming to are of less *historical* interest, precisely because they have to be taken seriously. Truly to paint the ground where we ourselves are standing is an impossible problem in historical perspective. . . .

Peirce: CP 4.33 Cross-Ref:††

33. The nominalistic wing of the Lockian party, much influenced by Hobbes and Ockham, made a philosophical development, chiefly psychological, but also logical. Among their names are Hartley, Berkeley, Hume, James Mill, John Stuart Mill, Bain. Bentham's *Logic* I must confess I do not remember to have seen. That of Mill, which appeared in 1843, contributed some phrases, which many persons adhere to passionately without reference to their meaning, sometimes seeming to attach no meaning to them, except the general one of a party-reveil. The present writer cares nothing about social matters, and knows not what such things mean. He examines logical questions as such and question by question. He perceives that many adherents of John Stuart Mill seem to be in a passion about something. But until they can calm themselves sufficiently, any scientific discussion of the questions, which perhaps they care little about, anyway, is impossible.

Peirce: CP 4.34 Cross-Ref:††

34. All the Occamistic school, from the Venerable Inceptor down, have been more or less politicians. John Stuart Mill was hypochondrically scrupulous. Nevertheless, *every* man of action is, must be, and ought to be, cunning, worldly, and dishonest, or what seems so to a man of pure science. When such men dispute, the

dispute has some other object than the ascertainment of scientific truth. Men accomplish, roughly speaking, what they desire. Government may be ever so much more important than science; but only those men can advance science who desire simply to find out how things really are, without *arrière-pensée*.

Peirce: CP 4.35 Cross-Ref:††

35. Occamism is governed by a very judicious maxim of logic, called *Ockham's razor*. It runs thus: *Entia non sunt multiplicanda præter necessitatem*, that is, "Try the theory of fewest elements first; and only complicate it as such complication proves indispensable for the ascertainment of truth." It may seem, at the outset, that the more complicated theory is the more probable. Nevertheless, it is highly desirable to stop and carefully to examine the simpler theory, and not contenting oneself with concluding that it will not do, to note precisely what the nature of its shortcomings are. Realism can never establish itself except upon the basis of an ungrudging acceptance of that truth. The Occamists have followed out this rule in the most interesting manner, and have contributed much to human knowledge. Reasons will be given †1 for thinking that their simple theory will not answer; yet this in no wise detracts from their scientific merits, since the only satisfactory way of ascertaining the insufficiency of the theory was to push the application of it, just as they have done. But because the abandonment of the theory would imply the modification of their politics, they employ every means in their power to discredit and personally hamper those who reject it and to prevent the publication and circulation of works in which it is impartially examined. That is not the conduct of philosophers, however wise it may be from the point of view of statesmanship. . . .

Peirce: CP 4.36 Cross-Ref:††

36. As a logician [Leibniz] was a nominalist and leaned to the opinion of Raymond Lully, an absurdity here passed over as not worth mention. This very nominalism led Leibniz to an extraordinary metaphysical theory, his Monadology, of much interest. In regard to human knowledge, he put forth many ideas which had great influence, all of them rooted in nominalism, yet at the same time departing widely from the Occamistic spirit. Such were his tests of universality and necessity; and such was his *principle of sufficient reason*, which he regarded as one of the fundamental principles of logic. This principle is that whatever exists has a reason for existing, not a blind cause, but a reason. A reason is something essentially general, so that this seems to confer reality upon generals. Yet if realism be accepted, there is no need of any principle of sufficient reason. In that case, existing things do not need supporting reasons; for they *are* reasons, themselves. A great deal of the Leibnizian philosophy consists of attempts to annul the effect of nominalistic hypotheses. . . .

Peirce: CP 4.37 Cross-Ref:††

37. Immanuel Kant, who made a revolution in philosophy by his *Critic of the Pure Reason*, 1781, had great power as a logician. He unfortunately had the opinion that the traditional logic was perfect and that there was no room for any further development of it.†1 That opinion did not prevent his introducing a number of ideas which have indirectly more than directly affected the traditional logic.

Peirce: CP 4.37 Cross-Ref:††

The merits of German philosophers since Kant as logicians have in the opinion of the present writer been small, while their errors and vagaries have been incessant.†2 At any rate, they have had little or no effect upon the ordinary logic. . . .

38. Very little of the traditional logic relates to the subject of the present section. St. Thomas Aquinas †3 divides the operations of the Understanding in reference to the logical character of their products into

Simple Apprehension,
Judgment, and
Ratiocination, or Reasoning.

Prantl declares †4 the commentary on the *Perihermeneias* in which this occurs not to be the work of Aquinas. But he does not explain how it could possibly happen that all the other books of the commentary should be genuine, as he admits they are, and this spurious. From the manner in which such books are written it is utterly inadmissible to suppose Aquinas passed over this book without comment. Such conduct would have excited a riot the noise of which would have reached our ears. If, then, the existing commentary is spurious, how could the genuine one have been lost? Thomas Aquinas was already an object of worship living. There was no school which adhered so religiously to the tenets of their master. Prantl himself complains that there is absolutely nothing in the works of Lambertus de Monte, and other Thomists except what St. Thomas had said. How could, then, all those schools be deceived into rejecting one of the works of their holy master, and taking in its place a writing that was not his? How is it that men of such learning as the doctors of Coimbra should get no wind of the substitution? Even Duns Scotus, writing directly after Aquinas, uses in his questions expressions which he probably derived from the book which Prantl suspects. Prantl gives no reason whatever for his rejection. He seems to think his judgment will be so commended by the comparison of it with manuscripts in other cases so entirely that he is placed quite above the necessity of giving reasons for his opinions. Similar ideas are apt to get possession of Germans.

Peirce: CP 4.39 Cross-Ref:††

39. *Simple Apprehension* produces *concepts* expressed by *names* or *terms*, "man," a state, suspended existence, the character of eating canned vacuum.

Peirce: CP 4.39 Cross-Ref:††

Judgment produces judgments, which are true or false, and are expressed by sentences, or *propositions*, as "Man is mortal," "some men may be insane."

Peirce: CP 4.39 Cross-Ref:††

Ratiocination or *reasoning* produces *inferences* or *reasonings*, which are expressed by *argumentations*, as, "I think, therefore I must exist," "Enoch, being a man, must have died; and since the Bible says he did not die, not everything in the Bible can be true."

Peirce: CP 4.40 Cross-Ref:††

40. A term names something but asserts nothing; a proposition asserts. Propositions differ in *modality*, which is the degree of positiveness of their assertion, as in *maybe, is, must be*. In another respect propositions are said to be *assertory, problematic, and apodictic*. The old statement †1 was that propositions were either

modal or *de inesse, i.e., assertoric*. They may also be *probable* assertions; they may further be *approximate* and *probable* assertions, as "about 51 per cent of the births in any one year will be male." Propositions are divided into the *Categorical* and the *Hypothetical*. "Propositionum alia categorica alia hypothetica,"†1 says the *Summulæ*. A categorical proposition is one whose immediate parts are terms; or as the *Summulæ* of the Mayence doctors say,†2 "categorica est illa quæ habet subiectum et prædicatum tanguam partes principales sui." A *hypothetical* proposition, better called by the Stoics †3 a *composite* proposition, is one which is composed of other propositions: "Propositio hypothetica est illa quæ habet duas propositiones categoricas tanquam partes principales sui."†4 The old, and less incorrect doctrine about compound propositions was that they were of three kinds, conditional, copulative, and disjunctive.†5 A conditional proposition is one whose *members* are joined by an *if*, or its equivalent: "Conditionalis est illa in qua coniunguntur duæ categoricæ per hanc coniunctionem, si."†6 That is, what is asserted is that in case one proposition, called the *antecedent*, is true, another proposition, called the consequent, is true. But how it may be in the opposite case in which the antecedent is not true is not stated. A *copulative* proposition is one in which the truth of every one of several propositions is affirmed. A *disjunctive* proposition is one in which the truth of *some* one of several propositions is affirmed. This enumeration is faulty because the *conditional* and *disjunctive* do not differ from one another in the same way in which both differ from the *copulative* proposition. For the conditional merely (or, at least, principally) asserts that unless one proposition is true another is true, that is, either the contrary of the former is true or the latter is true; and the disjunctive implies no more than that if the contradictions of all the alternatives but one be true, that one is true. Hence, either these two classes should be joined together, or we ought to include three other kinds of compound propositions, one which declares the repugnancy of two or more given propositions so that all cannot be true, one which declares the independence of one proposition of others so that it can be false although they are all true, and one which declares that there is a possibility that all of certain propositions are false.†1

Peirce: CP 4.41 Cross-Ref:††

41. The subject of a categorical proposition is that concerning which something is said, the predicate is that which is said of it. Most of the medieval logics teach that subject and predicate are the principal parts of the categorical proposition but that there is also a *Copula* which joins them together. . . . The Mayence doctors were quoted on this head, because Petrus Hispanus †2 makes the Subject, Predicate, and Copula to be all principal parts -- one of the numerous evidences that the text is not a translation from the Greek, a language in which the copula may be dispensed with. Aristotle, however, in his treatise upon forms of propositions, the *De interpretatione*,†3 analyzes the categorical proposition into the *noun*, or *nominative*, and the verb.

Peirce: CP 4.42 Cross-Ref:††

42. Categorical propositions are said to be divided according to their *Quantity*, into the universal, the particular, the indefinite, and the singular. A *universal* proposition was said to be a proposition whose subject is a common term determined by a universal sign. A common term was defined as one which is adapted to being predicated of several things (*aptus natus prædicari de pluribus*†4). The universal signs are every, no, any, etc. A *particular* proposition was said to be a proposition whose subject is a common term determined by a particular sign. The particular signs are, *some*, etc. An *indefinite* proposition was said †5 to be one in which the subject is

a common term without any sign, "ut homo currit." That unfortunate "indefinite" man has been running on now for so many centuries, it is fair he should have a rest and that we should revert to Aristotle's example, "Man is just."†6 A *singular* proposition was said to be one in which the subject is a *singular* term. A *singular* term was defined as "qui aptus natus est prædicari de uno solo,"†7 that is, it is a proper noun. Kant and other modern logicians very rightly drop the indefinite propositions which merely arise from the imperfect expression of what is meant. Singular propositions are for the purposes of formal logic equivalent to universal ones.

Peirce: CP 4.43 Cross-Ref:††

43. Propositions were further distinguished into propositions *per se* and propositions *per accidens*. But this was a complicated doctrine, which Kant very conveniently replaced by the distinction between *analytic*, or *explicative*, and *synthetic*, or *ampliative*, propositions. Namely, the question is what we are talking about. If we are saying that some imaginable kind of thing does or does not occur in the real world, or even in any well-established world of fiction (as when we ask whether Hamlet was mad or not), then the proposition is *synthetic*. But when we are merely saying that such and such a verbal combination does or does not represent anything that can find a place in any self-consistent supposition, then, we are either talking nonsense, as when we say, "A woolly horse would be a horse," or else, we are, as Kant says,†1 expressing a result of inward experimentation and observation, as when I say, "Probability essentially involves the supposition that certain general conditions are fulfilled many times and that in the long run a specific circumstance accompanies them in some definite proportion of the occurrences." If such a proposition is true and we substitute for the subject what that subject means, the proposition is reduced to an *identical* proposition, or in Kantian terminology an *empty* form of judgment. But the real sense of it lies in its being only just now seen that such *is* the meaning of the subject, that subject having hitherto been obscurely apprehended.

Peirce: CP 4.44 Cross-Ref:††

44. Categorical propositions are further divided into *affirmative* and *negative* propositions. A *negative* is one which has the particle of exclusion, *not*, or *other than* attached to the copula. There is a confusing distinction between a negative proposition and an *infinite*, that is, an *indefinite* one. The former is like *homo non est equis*, the latter like *homo est non equis*. That is the negative does not imply the existence of the subject, while the affirmative does imply this. But this arrangement, as will be shown in another chapter,†2 greatly complicates the description of correct reasonings. For analytical propositions, though affirmative, cannot, as analytical, assert the real existence of anything.†1

Peirce: CP 4.45 Cross-Ref:††

45. Ratiocination is defined by St. Thomas †2 as the operation by which reason proceeds from the known to the unknown. Inferences are of two kinds: the necessary and the probable. There are in either case (such is the traditional opinion which will be modified in this work †3) certain propositions called *premisses* laid down and granted; and these render another proposition, called the *conclusion* either necessary or probable, as the case may be. The conclusion is sometimes said to be *collected* from the premisses. It is also said to *follow* from them. The proposition that from such premisses such a conclusion *follows*, that is, is rendered necessary or probable, is called the *logical rule*, *dictum*, *law*, or *principle*. A necessary inference from a single premiss is called an *immediate inference*, from two premisses a

syllogism, from more than two a *sorites*. The massing of a number of premisses into one conjunctive proposition, which, in general consonance with the doctrine of immediate inference, might be considered as the inference of the conjunctive proposition from its members, though it is *not* so conceived traditionally, is conveniently called by Whewell †4 a *colligation*. It is plain that colligation is half the battle in ratiocination. †5

Peirce: CP 4.45 Cross-Ref: ††

It may be mentioned that Scotus (Duns, of course, for Scotus Erigena was not a scholastic) and the later scholastics usually dealt, not with the Syllogism, but with an inferential form called a consequence. The consequence has only one expressed premiss, called the *antecedent*; its conclusion is called the *consequent*; and the proposition which asserts that in case the antecedent be true, the consequent is true, is called the *consequence*. . . .

Peirce: CP 4.46 Cross-Ref: ††

46. Logic ought, for the realization of its germinal idea, to be *l'art de penser*. *L'art de penser!* What a sublime conception. A school to which an age can turn and here learn the most efficient method of solving its theoretical problems! Such is the idea of logic; but it manifestly asks that the logician should be head and shoulders above his age. That is not at all impossible. There are such men by the dozen in every age. Unfortunately, that is not enough. The man must not only live in realms of thought far removed from that of his fellow-citizens, and really be vastly their intellectual superior, but he must also be recognized as such; and that is a combination of events which hardly ever has happened. Aristotle, alone, by the extraordinary chance of adding to his vast powers, inherited wealth, and the close friendship of two kings the most powerful in the world, and both of them, men of gigantic intellect, came near to that ideal. That logic should really teach an age to think must be confessed impracticable. Let it aspire in each age to register the highest method of thinking to which that age actually attains, and it will be doing all that can be expected. This calls for the best minds. But in few ages has even this been done. The logicians instead of generally riding on the crest of the thought-wave, have, three-quarters waterlogged, drifted wherever the motion of thought was least. . . .

Peirce: CP 4.47 Cross-Ref: ††

§3 THE NATURE OF INFERENCE

47. We now come to the proper subject of this chapter. What is the nature of inference? What says the traditional syllogism? That an inference consists of a colligation of propositions which if true render certain or probable another collected proposition. If, to get to the bottom of the matter, we ask what is the nature of a proposition, traditional logic tells us, that it consists of terms -- two terms, usually connected together by another kind of sign, a copula.

Peirce: CP 4.47 Cross-Ref: ††

This is tolerably explicit, and, so far, good.

Peirce: CP 4.48 Cross-Ref: ††

48. The next question, in order, which we put to the traditional logic, is, how do you know that all that is true? to divide the question, tell us, first, how you know

that that analysis of the nature of assertion is correct.

Peirce: CP 4.48 Cross-Ref:††

To this, the traditional logic has not one traditional word to say. It is perfectly plain, however, that the reason it thinks so, is that that seems a satisfactory analysis of a sentence. So it is of the majority of sentences in the Greek, Latin, English, German, French, Italian, Spanish, languages -- in short, in the Indo-European languages; and European grammarians, true children of Procrustes, manage to exhibit sentences in other languages forced into the same formula.†P1 But outside of that family of languages which bears somewhat the same relation to language in general as the phanerogams do to all plants, or the vertebrates to all animals -- while there are of course proper names -- it seems to me that *general terms*, in the logical sense, do not exist.†1 That the analysis of the proposition into subject and predicate represents tolerably the way *we*, Arians, think, I grant; but I deny that it is the *only* way to think. It is not even the clearest way nor the most effective way.

Peirce: CP 4.49 Cross-Ref:††

49. There appear to be very many languages in which the copula is quite needless. In the Old Egyptian language, which seems to come within earshot of the origin of speech, the most explicit expression of the copula is by means of a word, really the relative pronoun, *which*. Now to one who regards a sentence from the Indo-European point of view, it is a puzzle how "which" can possibly serve the purpose in place of "is." Yet nothing is more natural.†2 The fact that hieroglyphics came so easy to the Egyptians shows how their thought is pictorial. . . . [e.g.] "Aahmes what we write of is a soldier *which* what we write of is overthrown," means "Aahmes the soldier is overthrown." Are you on the whole quite sure that this is not the most effective way of analyzing the meaning of a proposition?

Peirce: CP 4.50 Cross-Ref:††

50.†3 Take, now, the other part of the question, namely, supposing the nature of assertion to be understood, what is the relation of inference to assertion, according to the traditional logic? Here we find a marked difference between the view taken down to A. D. 1300 or 1325 and the view which then gradually gained ground and became universal considerably before A. D. 1600, and remained so until long after A. D. 1800. After 250 years of contest in which it was always gaining ground, it remained for 250 years more in unchallenged possession of the field. The opinion referred to is nominalism. Ockham revived it. By the time the universities were reformed in the sixteenth century, it had gained a complete victory. Descartes, Leibniz, Locke, Hume, and Kant, the great landmarks of philosophical history, were all pronounced nominalists. Hegel first advocated realism; and Hegel unfortunately was about at the average degree of German correctness in logic. The author of the present treatise is a Scotistic realist. He entirely approved the brief statement of Dr. F. E. Abbott in his *Scientific Theism* that Realism is implied in modern science. In calling himself a Scotist, the writer does not mean that he is going back to the general views of 600 years back; he merely means that the point of metaphysics upon which Scotus chiefly insisted and which has since passed out of mind, is a very important point, inseparably bound up with the *most* important point to be insisted upon today. The author might with more reason, call himself a Hegelian; but that would be to appear to place himself among a known band of thinkers to which he does not in fact at all belong, although he is strongly drawn to them.

Peirce: CP 4.51 Cross-Ref:††

51. How, then, does Kant regard the *apodictic* inference? He holds that the

conclusion is thought in the premisses although indistinctly. That that is Kant's view could be shown in a few words. But let us rather listen to his general tone in talking of reasoning. *In the Critic of The Pure Reason*, Transcendental Dialectic, Introduction, Section II, Subsection B, [A303, B359] he speaks of the logical employment of the Reason, as follows:

Peirce: CP 4.51 Cross-Ref:††

"A distinction is usual between things known immediately and things merely inferred. That in a figure bounded by three straight lines, there are three angles is known immediately; that the sum of these angles equals two right angles is a thing inferred. [When Kant wrote this no step in the modern revival of graphical geometry had been made. That three rays in a plane have three intersections, which, without any two rays coinciding, may reduce to one, is a *theorem* of graphics. But Kant confounds this proposition with another, namely, that if three lines, straight or not, enclose a space on a surface, those three lines must have at least three intersections. This is a corollary from the *Census theorem* of topology. That the sum of the three angles of the triangle equals two right angles, depends, as Lambert had clearly explained, before Kant wrote, upon a particular system of measurement which, however much it may be recommended by what we observe in nature, is not the only admissible system of measurement. Thus, what Kant says is immediately known, is fairly demonstrable; but what he says is demonstrable, is not so. This is not merely true in this case, but would be true of any example which Kant would feel to be a good one. It casts suspicion, at once, upon what he has to say, which has been the result of his generalizations of such examples]. Having an incessant need of inferring we become so accustomed to it, that at last the distinction spoken of escapes us. Even so called deceptions of the senses, where evidently it is the *inferences* that are at fault, we take for immediate perceptions. In every inference, there is one initial proposition, another, the consequent, which is drawn from it, and finally there is the consequence, or proposition according to which the truth of the consequent invariably accompanies the truth of the antecedent. [This is the doctrine of *consequentia* which is so extensively employed by philosophers of the fourteenth and fifteenth centuries.] If the concluded judgment is so contained in the initial judgment, that it can be derived without the intervention of any third idea, the consequence is called *immediate*. [This well-known term Kant would find in Wolff.] I would rather term it an Understanding-consequence. [This Kant seems to think an original idea, but that such a consequence was not an argument was the established doctrine.] But in case, besides the knowledge assigned as reason, still another judgment be needful, in order to draw the conclusion, the inference is called a Reason-inference. In the proposition "All men are mortal" is contained the propositions, "Some men are mortal," "Some mortals are men," "No immortal is a man." These, therefore, follow *immediately* from that. On the other hand, the proposition "All *savans* are mortal" is not contained in our assumed judgment (which does not contain the notion of *savan*), so that this proposition cannot be deduced from that other without a mediating judgment." [This is a slipshod analysis. Kant, out of his well-founded contempt for the scholastic method of trying to answer real questions by drawing distinctions, was led virtually to put the stamp of his condemnation upon all accurate thought. "Subtleties," he often says, "may sharpen the wits, but they are of no use at all."†P1 That was a very unfortunate opinion, which encouraged the down-at-the-heels, slouchy sort of logic to which Germans were prone enough and which has disgraced that country. To return to the present case, why does Kant consider only one kind of enthymeme and not another? Suppose the consequence to be the following -- which represents an

argument actually used by Kant against Boscovich --

All particles are bodies;

Ergo, All particles are extended.

Will Kant tell us there is any idea contained in this consequent not contained in its antecedent? Not so: he himself says, †1 "I need not go beyond the notion connected with the noun body to find that *extension* belongs to it." Will he, then, say that the consequence is no argument? It is put forward as such by himself; and such a doctrine would be a novelty in the traditional logic, with which he professes himself eminently satisfied, which were it involved in his doctrine, he certainly ought to have called attention to. But this example shows that in Kant's opinion the conclusion of a complete and perfect argumentation is implicitly contained in its premisses.]

Peirce: CP 4.52 Cross-Ref: ††

52. "With the explanation of synthetical Knowledge," says Kant [Analytic of Principles, Chapter 2, Section 2, [A154, B193] of the highest principles of all synthetical judgments]," general logic has absolutely nothing to do." The reason is obvious. Reasoning, according to the doctrine of that work, is regulated entirely by the principle of contradiction, which is the principle of analytical thought. The one law of demonstrative reasoning is that nothing must be said in the conclusion which is not implied in the premisses, that is, nothing must be said in the conclusion, not actually thought in the premisses, though not so clearly and consciously. †P1 The proposition that that is *actually thought*, though somewhat unconsciously, which is *implicitly contained* in what is thought, is absurd enough; but it is a *psychological* absurdity which may perhaps be passed over in logic. If that be true, nobody can tell by the most attentive introspection, what he thinks. For it will not be maintained that by carefully considering the few and simple premisses of the theory of numbers -- by just contemplating these propositions ever so nicely -- one could even discover the truth of Fermat's theorems. It would be impossible to adduce a single instance of the discovery of anything deserving the name of a mathematical theorem by any such means. Every mathematical discoverer knows very well that that is no way to succeed. If the implied proposition be thought, it is thought in some cryptic sense, and it in no wise tells us how it is that inference is performed, to say that in such sense the conclusion is thought as soon as the premisses are given. The distinction between analytical and synthetical judgments represents this conception of reasoning. The distinction may approximate to a just and valuable distinction; but it cannot be accepted as accurately defined. . . .

Peirce: CP 4.53 Cross-Ref: ††

53. †1 A belief is a habit; but it is a habit of which we are conscious. The actual calling to mind of the substance of a belief, not as personal to ourselves, but as holding good, or true, is a *judgment*. An inference is a passage from one belief to another; but not every such passage is an inference. If noticing my ink is bluish, I cast my eye out of the window and my mind being awakened to color remark particularly a poppy, that is no inference. Or if without casting my eye out of the window, I call to mind the green tinge of Niagara or the blue of the Rhone, that is no inference. In inference one belief not only follows *after* another, but follows *from* it.

Peirce: CP 4.54 Cross-Ref: ††

54. What does that mean? The proper method of finding the answer to this question is to compare pairs of beliefs which differ as little as possible except in that

in one pair one belief follows *from* the other and in the other pair only follows *after* it; and then note what practical difference, or difference that might become practical, there is between those two pairs. . . .

Peirce: CP 4.55 Cross-Ref:††

55. I think the upshot of reflection will be this. If a belief is produced for the first time directly after a judgment or colligation of judgments and is suggested by them, then that belief must be considered as the result of and as following from those judgments. The idea which is the matter of the belief is suggested by the idea in those judgments according to some habit of association, and the peculiar character of believing the idea really is so, is derived from the same element in the judgments. Thus, inference has at least two elements: the one is the suggestion of one idea by another according to the law of association, while the other is the carrying forward of the asserting element of judgment, the holding for true, from the first judgment to the second. That these two things suffice [to] constitute inference I do not say. . . .†1

Peirce: CP 4.56 Cross-Ref:††

56.†2 Let us now inquire in what the assertory element of a judgment consists. What is there in an assertion which makes it more than a mere complication of ideas? What is the difference between throwing out the word *speaking monkey*, and averring that *monkeys speak*, and inquiring *whether monkeys speak or not*? This is a difficult question.

Peirce: CP 4.56 Cross-Ref:††

In the first place, it is to be remarked that the first expression signifies nothing. The grammarians call it an "incomplete speech." But, in fact, it is no speech at all. As well call the termination *ability* -- or *ationally* an incomplete speech. It is also to be remarked that the number of languages in which such an expression is possible is very small. In most languages that have nouns and adjectives, the participial adjective follows the noun and when left without other words the combination would mean *the monkey is speaking*.

Peirce: CP 4.56 Cross-Ref:††

In such languages you can't say "speaking monkey," and surely it is no defect in them; for after it is said, it is pure nonsense. . . . There are more than a dozen different families of languages, differing radically in their manner of thinking; and I believe it is fair to say that among these the Indo-European is only one in which words which are distinctively common nouns are numerous. And since a noun or combination of nouns by itself says nothing, I do not know why the logician should be required to take account of it at all. Even in Indo-European speech the linguists tell us that the roots are all verbs. It seems that, speaking broadly, ordinary words in the bulk of languages are assertory. They assert as soon as they are in any way attached to any object. If you write GLASS upon a case, you will be understood to mean that the case contains glass. It seems certainly the truest statement for most languages to say that a *symbol* is a conventional sign which being attached to an object signifies that that object has certain characters. But a symbol, in itself, is a mere dream; it does not show what it is talking about. It needs to be connected with its object. For that purpose, an *index* is indispensable. No other kind of sign will answer the purpose. That a word cannot in strictness of speech be an index is evident from this, that a word is general -- it occurs often, and every time it occurs, it is the same word, and if it has any meaning *as a word*, it has the same meaning every time it occurs; while an index is essentially an affair of here and now, its office being to bring the thought to a

particular experience, or series of experiences connected by dynamical relations. A *meaning* is the associations of a word with images, its dream exciting power. An index has nothing to do with meanings; it has to bring the hearer to share the experience of the speaker by *showing* what he is talking about. The words *this* and *that* are indicative words. They apply to different things every time they occur.

Peirce: CP 4.56 Cross-Ref:††

It is the connection of an indicative word to a symbolic word which makes an assertion.

Peirce: CP 4.57 Cross-Ref:††

57.†1 The distinction between an assertion and an interrogatory sentence is of secondary importance. An assertion has its *modality*, or measure of assurance, and a question generally involves as part of it an assertion of emphatically low modality. In addition to that, it is intended to stimulate the hearer to make an answer. This is a rhetorical function which needs no special grammatical form. If in wandering about the country, I wish to inquire the way to town, I can perfectly do so by assertion, without drawing upon the interrogative form of syntax. Thus I may say, "This road leads, perhaps, to the city. I wish to know what you think about it." The most suitable way of expressing a question would, from a logical point of view, seem to be by an interjection: "This road leads, perhaps, to the city, eh?"

Peirce: CP 4.58 Cross-Ref:††

58. An index, then, is quite essential to a speech and a symbol equally so. We find in grammatical forms of syntax, a part of the sentence particularly appropriate to the index, another particularly appropriate to the symbol. The former is the *grammatical subject*, the latter the *grammatical predicate*. In the logical analysis of the sentence, we disregard the forms and consider the sense. Isolating the indices as well as we can, of which there will generally be a number, we term them *the logical subjects*, though more or less of the symbolic element will adhere to them unless we make our analysis more recondite than it is commonly worth while to do; while the purely symbolic parts, or the parts whose indicative character needs no particular notice, will be called the *logical predicate*. As the analysis may be more or less perfect -- and perfect analyses are very complicated -- different lines of demarcation will be possible between the two logical members.†1 In the sentence "John marries the mother of Thomas," John and Thomas are the logical subjects, marries-the-mother-of- is the logical predicate. . . .

Peirce: CP 4.59 Cross-Ref:††

59. In making general assertions it is not possible directly to indicate anything but the real world, or whatever world discourse may refer to. But it is necessary to give a general direction as to the manner in which an object intended may be found. Especially it is necessary to be able to say that any object whatever will answer the purpose, in which case the subject is said to be *universal*, and to be able to say that a suitable object occurs, in which case the subject is said to be *particular*.

Peirce: CP 4.60 Cross-Ref:††

60. If there are several subjects, some universal and some particular, it makes a difference in what order the selections of a universal and of a particular subject are made. For example, the four following statements are different:

1. Take any two things, A and C; then a thing, B, can be so chosen that if A and C are men, B is a man praised by A to C.

2. Take anything, A; then a thing, B, can be so chosen, that whatever third thing, C, be taken, if A and C are men, B is a man praised by A to C.

3. Take anything, C; then a thing, B, can be so chosen, that whatever third thing, A, be taken, if A and C are men, B is a man praised by A to C.

4. A thing, B, can be so chosen that whatever things A and C may be, if A and C are men, B is a man praised by A to C.

Peirce: CP 4.60 Cross-Ref:††

We should usually express these as follows:

1. Every man praises some man or other to each man.

2. Every man praises some man to all men.

3. To every man some man is praised by all men.

4. There is a man whom all men praise to all men.†1

Peirce: CP 4.61 Cross-Ref:††

61. . . . When we busy ourselves to find the answer to a question, we are going upon the hope that there is an answer, which can be called *the* answer, that is, the final answer. It may be there is none. If any profound and learned member of the German Shakespearian Society were to start the inquiry how long since Polonius had had his hair cut at the time of his death, perhaps the only reply that could be made would be that Polonius was nothing but a creature of Shakespeare's brain, and that Shakespeare never thought of the point raised. Now it is certainly conceivable that this world which we call the real world is not perfectly real but that there are things similarly indeterminate. We cannot be sure that it is not so. In reference, however, to the particular question which at any time we have in hand, we hope there is an answer, or something pretty close to an answer, which sufficient inquiry will compel us to accept.

Peirce: CP 4.62 Cross-Ref:††

62. Suppose our opinion with reference to a given question to be quite settled, so that inquiry, no matter how far pushed, has no surprises for us on this point. Then we may be said to have attained *perfect knowledge* about that question. True, it is conceivable that somebody else should attain to a like "perfect knowledge," which should conflict with ours. He might know something to be white, which we should know was black. This is *conceivable*; but it is not possible, considering the social nature of man, if we two are ever to compare notes; and if we never do compare notes, and no third party talks with both and makes the comparison, it is difficult to see what meaning there is in saying we disagree. When we come to study the principle of continuity †1 we shall gain a more ontological conception of knowledge and of reality; but even that will not shake the definition we now give.

Peirce: CP 4.63 Cross-Ref:††

63. Perhaps we may already have attained to perfect knowledge about a number of questions; but we cannot have an unshakable opinion that we have attained such perfect knowledge about any given question. That would be not only perfectly to know, but perfectly to know that we do perfectly know, which is what is called *sure knowledge*. No doubt, many people opine that they surely know certain things; but after they have read this book, I hope many of them will be led to see that that opinion

is not unshakable. At any rate, as they are, after all, in some measure reasonable beings, no matter how pig-headed they might be (I am only saying that pigheaded people exist, not that they are very frequently met with among my opponents), after a time, if they live long enough, reason must get the better of obstinate adherence to their opinion, and they must come to see that sure knowledge is impossible.

Peirce: CP 4.64 Cross-Ref:††

64. Nevertheless, in every state of intellectual development and of information, there are things that seem to us sure, because no little ingenuity and reflection is needed to see how anything can be false which all our previous experience seems to support; so that even though we tell ourselves we are *not* sure, we cannot clearly see *how* we fail of being so. Practically, therefore, life is not long enough for a given individual to rake up doubts about everything; and so, however strenuously he may hold to the doctrine of catalepsy, he will practically treat one proposition and another as certain. This is a state of *practically perfect belief*.

Peirce: CP 4.65 Cross-Ref:††

65. We have now to define the five words *necessary*, *unnecessary*, *possible*, *impossible*, and *contingent*. But first let me say that I use the word *information* to mean a state of knowledge, which may range from total ignorance of everything except the meanings of words up to omniscience; and by *informational* I mean relative to such a state of knowledge. Thus, by "informationally possible," I mean possible so far as we, or the persons considered, know. Then, the *informationally possible* is that which in a given information is not perfectly known not to be true. The *informationally necessary* is that which is perfectly known to be true. The *informationally contingent*, which in the given information remains uncertain, that is, at once possible and unnecessary.

Peirce: CP 4.66 Cross-Ref:††

66. The information considered may be our actual information. In that case, we may speak of what is possible, necessary, or contingent, *for the present*. Or it may be some hypothetical state of knowledge. Imagining ourselves to be thoroughly acquainted with all the laws of nature and their consequences, but to be ignorant of all particular facts, what we should then not know not to be true is said to be *physically possible*; and the phrase *physically necessary* has an analogous meaning. If we imagine ourselves to know what the resources of men are, but not what their dispositions and desires are, what we do not know will not be done is said to be *practically possible*; and the phrase *practically necessary* bears an analogous signification. Thus, the possible varies its meaning continually. We speak of things *mathematically* and *metaphysically possible*, meaning states of things which the most perfect mathematician or metaphysician does not *qua* mathematician or metaphysician know not to be true.

Peirce: CP 4.67 Cross-Ref:††

67. There are two meanings of the words *possible* and *necessary* which are of special interest to the logician more than to other men. These refer to the states of information in which we are supposed to know *nothing*, except the meanings of words, and their consequences, and in which we are supposed to know *everything*. These I term *essential* and *substantial possibility*, respectively: and of course necessity has similar varieties. That is *essentially* or *logically possible* which a person who knows no facts, though perfectly *au fait* at reasoning and well-acquainted with the words involved, is unable to pronounce untrue. The *essentially* or *logically*

necessary is that which such a person knows is true. For instance, he would not know whether there was or was not such an animal as a *basilisk*, or whether there are any such things as serpents, cocks, and eggs; but he would know that every basilisk there may be has been hatched by a serpent from a cock's egg. That is essentially necessary; because that is what the word *basilisk* means. On the other hand, the substantially possible refers to the information of a person who knows everything now existing, whether particular fact or law, together with all their consequences. This does not go so far as the omniscience of God; for those who admit Free-Will suppose that God has a direct intuitive knowledge of future events even though there be nothing in the present to determine them. That is to say, they suppose that a man is perfectly free to do or not do a given act; and yet that God already knows whether he will or will not do it. This seems to most persons flatly self-contradictory; and so it is, if we conceive God's knowledge to be among the things which exist at the present time. But it is a degraded conception to conceive God as subject to Time, which is rather one of His creatures. Literal fore-knowledge is certainly contradictory to literal freedom. But if we say that though God knows (using the word *knows* in a trans-temporal sense) he never did know, does not know, and never will know, then his knowledge in no wise interferes with freedom. The terms, *substantial necessity* and *substantial possibility*, however, refer to supposed information of the present in the present, including among the objects known all existing laws as well as special facts. In this sense, everything in the present which is possible is also necessary, and there is no present contingent. But we may suppose there are "future contingents." Many men are so cocksure that necessity governs everything that they deny that there is anything substantially contingent. But it will be shown in the course of this treatise that they are unwarrantably confident, that wanting omniscience we ought to presume there may be things substantially contingent, and further that there is overwhelming evidence that such things are. . . .†1

Peirce: CP 4.68 Cross-Ref:††

68. To conclude from the above definitions that there is nothing analogous to possibility and necessity in the real world, but that these modes appertain only to the particular limited information which we possess, would be even less defensible than to draw precisely the opposite conclusion from the same premisses. It is a style of reasoning most absurd. Unfortunately, it is so common, that the moment a writer sets down these definitions nine out of ten critics will set him down as a nominalist. The question of realism and nominalism, which means the question how far real facts are analogous to logical relations, and why, is a very serious one, which has to be carefully and deliberately studied, and not decided offhand, and not decided on the ground that one or another answer to it is "inconceivable." Nothing is "inconceivable" to a man who sets seriously about the conceiving of it.†1 There are those who believe in their own existence, because its opposite is inconceivable; yet the most balsamic of all the sweets of sweet philosophy is the lesson that personal existence is an illusion and a practical joke. Those that have loved themselves and not their neighbors will find themselves April fools when the great April opens the truth that neither selves nor neighborselves were anything more than vicinities; while the love they would not entertain was the essence of every scent.†2

Peirce: CP 4.69 Cross-Ref:††

69. A leading principle of inference which can lead from a true premiss to a false conclusion is insofar bad; but insofar as it can only lead either from a false premiss or to a true conclusion, it is satisfactory; and whether it leads from false to false, from true to true, or from false to true, it is equally satisfactory. The first part of

this theorem, that an inference from true to false is bad, [follows] from the essential characteristic of truth, which is its finality. For truth being our end and being able to endure, it can only be a false maxim which represents it as destroying itself. Indeed, I do not see how anybody can fail to admit that (other things being equal) it is a fault in a mode of inference that it can lead from truth to falsity. But it is by no means as evident that an inference from false to false is as satisfactory as an inference from true to true; still less, that such a one is as satisfactory as an inference from false to true. The Hegelian logicians seem to rate only that reasoning A1 which setting out from falsity leads to truth. But men of laboratories consider those truths as small that only an inward necessity compels. It is the great compulsion of the Experience of nature which they worship. On the other hand, the men of seminaries sneer at nature; the great truths for them are the inward ones. Their god is enthroned in the depths of the soul. How shall we decide the question? Let us rationally inquire into it, subordinating personal prepossessions in view of the fact that whichever way these prepossessions incline, we can but admit that wiser men than we, more sober-minded men than we, and humbler searchers after truth, do today embrace the opinion the opposite of our own. How, then, shall we decide the question? Yes, how to decide questions is precisely the question to be decided. One thing the laboratory-philosophers ought to grant: that when a question can be satisfactorily decided in a few moments by calculation, it would be foolish to spend much time in trying to answer it by experiment. Nevertheless, this is just what they are doing every day. The wisest-looking man I ever saw, with a vast domelike cranium and a weightiness of discourse that left Solon in the distance, once spent a month or more in dropping a stick on the floor and seeing how often it would fall on a crack; because that ratio of frequency afforded a means of ascertaining the value of $\{p\}$, though not near so close as it could be calculated in five minutes; and what he did it for was never made clear. Perhaps it was only for relaxation; though some people might have found reading Goldsmith or Voltaire fully as lively an occupation. If it were not for the example of this distinguished LL.D., I should have ventured to say that nothing is more foolish than carrying a question into a laboratory until reflection has done all that it can do towards clearing it up -- at least, all that it can do for the time being. Of course, for a seminary-philosopher, to send a question to the laboratory is to have done with it, to which he naturally has a reluctance; while the laboratory-philosopher is impatient to get a whack at it.

Peirce: CP 4.70 Cross-Ref:††

70. Suppose that, at any rate, we try applying this maxim of methodology to the question now in hand. Then the first thing that has to be remarked is that every inference proceeds according to a general rule -- and *that*, a comprehended rule -- so that in the very act of drawing it the reasoner thinks of there being other similar inferences to be drawn. For unless the premiss determines the conclusion according to a rule, there is no intelligible meaning in saying that it determines it at all; unless, indeed, we are prepared to say that the conclusion feels compelled but knows not how; and if it knows not how, how can it know it was the premiss which compelled it? But a conclusion is not only determined by the premiss, but rationally determined, and that implies that in drawing said conclusion we feel we are following a rule and a comprehensible rule. . . .

Peirce: CP 4.71 Cross-Ref:††

71. Descartes marks the period when Philosophy put off childish things and began to be a conceited young man. By the time the young man has grown to be an old man, he will have learned that traditions are precious treasures, while iconoclastic

inventions are always cheap and often nasty. He will learn that when one's opinion is besieged and one is pushed by questions from one reason to another behind it, there is nothing illogical in saying at last, "Well, this is what we have always thought; this has been assumed for thousands of years without inconvenience." The childishness only comes in when tradition, instead of being respected, is treated as something infallible before which the reason of man is to prostrate itself, and which it is shocking to deny. In 1637, Descartes (aged 41) published his first work on philosophy, the *Discours de la méthode pour bien conduire sa raison et chercher la vérité dans les sciences*. In the fourth part of this dissertation, after insisting upon the doubtfulness of everything, even the simplest propositions of mathematics, in a strain quite familiar to readers of the present work, he goes on to say how at one time "je me résolus de feindre que toutes les choses qui m'étoient jamais entrées en esprit n'étoient non plus vraies que les illusions de mes songes." Thereupon follows the grand passage: "Mais aussitôt après je pris garde que, pendant que je voulais ainsi penser que tout étoit faux, il falloit nécessairement que moi qui le pensois susse quelque chose; et, remarquant que cette vérité: **je pense, donc je suis**, étoit si ferme et si assurée que toutes les plus extravagantes suppositions des sceptiques n'étoient pas capable de l'ébranler, je jugeai que je pouvois la recevoir sans scrupule pour le premier principe de la philosophie que je cherchois."

Peirce: CP 4.71 Cross-Ref:††

Descartes thought this "très-clair"; but it is a fundamental mistake to suppose that an idea which stands isolated can be otherwise than perfectly blind. He professes to doubt the testimony of his memory; and in that case all that is left is a vague indescribable idea. There is no warrant for putting it into the first person singular. "I think" begs the question. "There is an idea: therefore, I am," it may be contended represents a compulsion of thought; but it is not a rational compulsion. There is nothing clear in it. Here is a man who utterly disbelieves and almost denies the dicta of memory. He notices an idea, and then he thinks he exists. The *ego* of which he thinks is nothing but a holder together of ideas. But if memory lies there may be only one idea. If that one idea suggests a holder-together of ideas, *how* it can do so is a mystery. To make the reflection that many of the things which appear certain to us are probably false, and that there is not one which may not be among the errors, is very sensible. But to make believe one does not believe anything is an idle and self-deceptive pretence. Of the things which seem to us clearly true, probably the majority are approximations to the truth. We never can attain absolute certainty; but such clearness and evidence as a truth can acquire will consist in its appearing to form an integral unbroken part of the great body of truth. If we could reduce ourselves to a single belief, or to only two or three, those few would not appear reasonable or clear.

Peirce: CP 4.72 Cross-Ref:††

72. Now, then, how is truth to be inferred from falsehood? First, it may happen accidentally, from the falsehood that Alexander the Great was the great-grandson of Benjamin Franklin it may be inferred there lived a great-grandson of Benjamin Franklin named Alexander, which happens to be true. It cannot be considered as a merit of a rule that its results accidentally have any character; for an accidental result *ex vi termini* is not determined by the rule. Secondly, truth may follow from falsehood because no lie is altogether false. Every precept of inference which does not lead from truth to falsity, must sometimes lead from falsity to truth. For let **A** be a true premiss and **B** a conclusion from it according to such a precept. Then **B** must be true. But if we add to **A** something false, **B** will follow from it just the same. A mode of inference may accordingly infer a larger proportion of true

conclusions from false premisses than another simply by inferring less. But concluding falsehood from falsehood is by no means useless, provided it follows a precept which cannot conclude falsehood from truth. For it hastens the detection and rejection of the falsity. Consequently of two modes of inference neither ever leading from truth to falsity, one of which infers something false from a false premiss from which the other infers something true, the former is rather to be preferred because it infers more. Suppose for instance it is false that the sides of a triangle measure 4 inches, 5 inches, and 6 inches, then the rule of inference which deduces for the area $15/4 \sqrt{7}$ square inches is certainly superior to a rule of inference which only concludes that the area is finite. Thirdly, truth may follow from falsehood because that falsehood is impossible and refutes itself. But in this way, only what is logically necessary can be inferred, that is only what a person ought to know independently of any particular premisses. As this is a mode of inference which infers less than any other, its value is the least that any mode can have which never leads from truth to falsity. Many persons will be inclined to dispute this, and will point to the utility of the *reductio ad absurdum* in geometry. But the *reductio ad absurdum* is not a method of inferring truth from falsity; it is only a form of statement of an inference from truth to truth. . . . †1

Peirce: CP 4.72 Cross-Ref:††

Now it may be that everything is so bound up with everything else that to understand perfectly any single fact, as it really is, would involve a knowledge of all facts. But this is not admitting that from any proposition, understood as it is understood, and not as the reality it represents ought to be understood, much can be inferred; far less that valuable truth can be deduced from falsehood.

Peirce: CP 4.72 Cross-Ref:††

It thus appears that the inference of truth from falsity is never so valuable as when it is accidental, in which case its value is precisely the same as that of an inference from false to false.

Peirce: CP 4.73 Cross-Ref:††

73. The inference from true to true has precisely the same value as that from false to false. For to infer B from A involves inferring the falsity of A from the falsity of B. The two inferences are inseparable; when either is made the other is made. Now if either of these is an inference from truth to truth, the other is an inference from falsity to falsity; and conversely, if either is an inference from false to false, the other is an inference from true to true. Accordingly it is impossible to set different values upon the two modes of inference.

Peirce: CP 4.74 Cross-Ref:††

74. Leading principles are of two classes: those whose pretension it is to lead always to the truth unless from the false, and never astray; and those which only profess to lead toward the truth in the long run. This distinction separates two great branches of reasoning, the one bringing to light the dark things of the hidden recesses of the soul, the other those hidden in nature. We may, for the present, call them Imaginative and Experiential reasoning; or reasoning by diagrams and reasoning by experiments. †1

Peirce: CP 4.75 Cross-Ref:††

75. . . . The necessity for a sign directly monstrative of the connection of premiss and conclusion is susceptible of proof. That proof is as follows. When we contemplate the premiss, we mentally *perceive* that that being true the conclusion is

true. I say we perceive it, because clear knowledge follows contemplation without any intermediate process. Since the conclusion becomes certain, there is some state at which it becomes directly certain. Now this no symbol can show; for a symbol is an indirect sign depending on the association of ideas. Hence, a sign directly exhibiting the mode of relation is required. This promised proof presents this difficulty: namely, it requires the reader actually to *think* in order to see the force of it. That is to say, he must represent the state of things considered in a direct imaginative way.

Peirce: CP 4.76 Cross-Ref:††

76. A large part of logic will consist in the study of the different monstrative signs, or icons, serviceable in reasoning.

Peirce: CP 4.76 Cross-Ref:††

Suppose we reason

Enoch was a man,

Then, Enoch must have died.

Let this reasoning be called in question, and the reasoner searches his mind to discover the leading principle which actuated it. He finds this in the truth (as he assumes it to be) that

Every man dies.

He now repeats his reasoning, joining this proposition to the premiss previously assumed, to make the compound premiss,

Enoch was a man, and every man dies.

This may be otherwise stated thus:

If we are talking of Enoch, what we are talking of is a man; and if we are talking of a man, what we are talking of dies.

The conclusion is

If we are talking of Enoch, what we are talking of dies.

Or we may state it thus:

From being Enoch follows being a man, and from being a man follows being subject to death;

Hence, from being Enoch follows being subject to death.

Peirce: CP 4.76 Cross-Ref:††

If this reasoning is called in question, the reasoner searches his mind for the leading principle and may state it thus:

If one truth, A, makes another truth, B, certain, and if this truth, B, makes a third truth, C, certain; then, the truth, A, makes the truth, C, certain.

Peirce: CP 4.76 Cross-Ref:††

This is the logical principle called the *Nota notæ*, because one statement of it is, *nota notæ est nota rei ipsius*.†1

Peirce: CP 4.76 Cross-Ref:††

Now shall the reader add this as a premiss to the compound premiss already adopted? He gains nothing by doing so. For he cannot reason at all without a monstrative sign of illation; and this sign is not really monstrative unless it makes clear the proposition here proposed to be abstractly stated. Nor could any use of that statement be made without using the truth which it expresses.

Peirce: CP 4.76 Cross-Ref:††

That if the fact A is certain evidence of the fact B and the fact B is certain evidence of the fact C, then the fact A is certain evidence of the fact C, appears to us perfectly clear. That appearance of evidence may be an argument that the proposition is probably about true; for our instincts are generally pretty well adapted to their ends. But its appearing clear will not prevent our reflecting that things that seem evident are often found to be mistakes, so that it may be the proposition is not true.

Peirce: CP 4.77 Cross-Ref:††

77. Now although the reader does not really doubt that the proposition is true, it may be instructive to feign such a doubt, and see what the nature of the source of knowledge is.

Peirce: CP 4.77 Cross-Ref:††

A common form of the maxim is this: The word *mortal* is applicable to everything to which the word *man* is applicable, and the word *man* is applicable to everything to which the word *Enoch* is applicable. Hence, the word *mortal* is applicable to everything to which the word *Enoch* is applicable. This mode of representing the matter is embodied in a maxim called the *Dictum de omni*:†1 if A is in any relation to all to which B is in the same relation, and if B is in this relation to all to which C is in this relation, then A is in this relation to all to which C is in this relation; that is, if the things to which A is applicable are wholly included among the

things to which B is applicable, and the things to which B is applicable are wholly included among the things to which C is applicable, then the things to which A is applicable are wholly included among the things to which C is applicable.

Peirce: CP 4.77 Cross-Ref:††

Here we have a mental diagram representing receptacles or spaces successively included in one another; and the question of the truth of the maxim may be divided into two parts:

Peirce: CP 4.77 Cross-Ref:††

First: Is the maxim certainly true of the mental diagram; and if so how do we know it?

Peirce: CP 4.77 Cross-Ref:††

Second: Does the mental diagram represent the relations of truths of nature to one another, in fact?

Peirce: CP 4.77 Cross-Ref:††

As to the first question, there would seem to be no reason to doubt that we know it is true of our mental diagram, just as we know of *our idea* of numbers that 2 and 3 make 5. And no line can be drawn between this case and knowing that $\sqrt{2}=1.414213562373095$ except that the latter is more complicated. It would thus appear that our certainty about the mental diagram is merely due to our having gone over it many times and being confident we could not be all wrong about a matter so simple. Still, as it is easy to make a mistake in calculating the $\sqrt{2}$, and that mistake may be repeated, it is barely *possible* that any conclusion reached in the same way is wrong. Besides, how do I know I am not crazy and am not uttering the greatest absurdity when I enunciate the *Nota notæ*? Of course, it is not rational for a man to assume that he is utterly irrational. A man cannot be speaking the truth in saying that everything he says is false. For this very thing is one of the things he says; and if this be false then in what it says of itself it is true, and therefore false.†1 But this remark does not clear up the matter; and we shall leave the problem for the present, to return to it later.†2

Peirce: CP 4.77 Cross-Ref:††

As to the second question, it is important to remark that the *Nota notæ* does not declare that there is any infallible mark of anything, or any rule without exceptions. If, as we have seen, the *Nota notæ* itself is not absolutely certain, nothing else ought to be so regarded. We cannot go so far as to declare that absolutely no rule is without exceptions; for this declaration is itself a rule. Nor can we say that no rule but this is without exceptions. For this rule either has exceptions or it has not. If it has exceptions and every other rule has exceptions, it has no exceptions. But if it has no exceptions, then in accordance with its declaration it has exceptions. We are thus obliged to admit that there are rules without exceptions, or at least that the denial of it has no sense.†P1 But we ought not to suppose that we can identify any general proposition as being certainly or even probably without exceptions. The case is like the following. We say $1/2$ of $1/3$ is $1/6$. Now we do not really think we can divide anything into precisely equal parts; but we think that, barring the possibility that we have made a mistake in doing the sum, which is excessively improbable, the nearer we can come to $1/2$ of $1/3$ of anything, that is, to the ideal state of things in our imagination, the nearer we shall come to $1/6$. . .

Peirce: CP 4.78 Cross-Ref:††

78. In like manner, it may be nothing in the world precisely conforms to rigidity of our idea of something steady enough to be represented by a sign. The reader has had several examples of *insolubilia*,†1 as they are called by logicians, that is, cases in which every attempt to reason lands us in absurdity. Here are two more examples.

Peirce: CP 4.78 Cross-Ref:††

In order to prove black is white, you have only to say, "Either what I am saying is false or black is white." Is that proposition false? It cannot be so; for it only says that one or other of two things is true; and if either is true the proposition is true. It cannot, therefore, be false; for that is one of the alternatives that it leaves open. The proposition is true, then. Consequently, one of the alternatives is true. But not the first; therefore the second. Hence, black is white.

Peirce: CP 4.78 Cross-Ref:††

A man invented an ink containing Vanadium the like of which had never been made before. He was just about to try it for the first time, when a friend asked, "Has anything ever been written in Vanadium ink before?" "No." "Will you please write what I tell you for the first handsel of it?" "Yes." "Very well, here is a folded paper marked 'Exhibit A.' Write: What is written in exhibit A is true." He did so. "Now," said the friend, "do you know you have lied to me?" "Oh, but I only wrote that to please you. I did not mean to say it was true." "Very well; suppose it false. Then, what exhibit A says is false. Now read Exhibit A. It reads: 'Something written in Vanadium ink is false.' If that is false, what you have written must be true." "Good! So much the better!" "Not so fast, if you please. What you have written is, of course, true; and consequently exhibit A is true; and consequently something written in Vanadium ink is false. Now it is not what you have just written, for that is true; and therefore you must have lied when you told me nothing had ever been written with that ink before."

Peirce: CP 4.78 Cross-Ref:††

It may be that all the propositions in the world would, if subjected to a dialectical examination, prove thus elusive. But that does not affect the truth of the *Nota notæ*, which only says that so far as things conform to our idea of successive inclusion, so far (unless we have blundered almost inconceivably) the *Nota notæ* holds. . . .

Peirce: CP 4.79 Cross-Ref:††

79. The logician does not assert anything, as the geometrician does; but there are certain assumed truths which he hopes for, relies upon, banks upon, in a way quite foreign to the arithmetician. Logic teaches us to expect some residue of dreaminess in the world, and even self-contradictions; but we do not expect to be brought face to face with any such phenomenon, and at any rate are forced to run the risk of it. The assumptions of logic differ from those of geometry, not merely in not being assertorically held, but also in being much less definite.

Peirce: CP 4.80 Cross-Ref:††

III

SECOND INTENTIONAL LOGIC†1P

80. Second intentional, or, as I also call it, Objective Logic, is much the larger part of formal logic. It is also the more beautiful and the interesting subject; and in serious significance it is superior in a far higher ratio. But it is highly abstract, remote from the bread and butter of all parties, and to yield to the temptation of going into it would be to forget

That not to know at large of things remote
 From use, obscure and subtle, but to know
 That which before us lies in daily life,
 Is the prime Wisdom, what is more is fume,
 Or emptiness, or fond impertinence,
 And renders us in things that most concerne
 Unpractis'd, unprepar'd, and still to seek.

Peirce: CP 4.81 Cross-Ref: ††

81. Second intentional logic treats at length of the properties of logical conceptions. First, come such simple relations as 0, ∞, 1, T. †2 There is also an extensive doctrine concerning q, the relation of inherence. Kant, in his celebrated Appendix to the Transcendental Dialectic, †3 has set forth three sporadic propositions of this sort, whose significance can hardly be seen away from their crowd. Besides, it is more satisfactory to see these things set forth in a purely logical way and deduced mathematically, than to have them treated at their first presentation as regulative principles. As a part of this general doctrine of inherence, there is a special doctrine of the properties of relations. Of course, all logical treatises consider these things; but they do not consider them in a formal way, nor at all in the manner in which they are turned out by the machinery of this calculus. One of the questions which pertain to this branch of logic is that of the classification of relations. There are also some special relations of logical origin which have to be considered, among which is that of *correspondence*, which has been studied by mathematicians without much logical analysis.

Peirce: CP 4.82 Cross-Ref: ††

82. A number of interesting features of the logical calculus itself emerge in the application of it to the second intentions. One of these, for example, is that the subjacent letters I call indices do not essentially differ from any other letters. Thus we may define identity as follows:

$$\pi[i]\pi[j]\pi[K]\pi[x] \{ 1[K]=\sim r[Kij] \vee x[i] \vee \sim x[j] \}$$

That is, to say that anything whatever, *K*, is identity is to say that if any two things *i* and *j* are in the relation, *K*, the *i* to the *j*, then any proposition whatever, *x*, is true of *i*, or else that proposition is false of *j*. The point calling for notice is that *x* is put into the logisterium, although it is one of the principal letters of the Boolean. †1

Peirce: CP 4.83 Cross-Ref:††

83. Another thing is that the forms of logisteria, themselves, become subjects of study, and certain general propositions with regard to them are expressed as if they were shops or trees; and yet these very propositions can be made use of in the calculus.

Peirce: CP 4.83 Cross-Ref:††

Among the forms of logisteria which require attentive study and which are found to possess interesting properties are particularly those which are infinite series, though the very purpose of the lectical symbols is to embrace infinite series. But we find that we have to resort to logisteria of logisteria, and to their logisteria again, and so on indefinitely, and that the distinctive characters of different such infinite series of logisteria have to be discriminated.

Peirce: CP 4.84 Cross-Ref:††

84. Another point of logical interest is that when our discourse relates to a universe of possibility which virtually embraces all logical possibility, everything is true [from] which no false consequence can possibly follow; and the only way of investigating certain propositions is by proving that they cannot give rise to any contradiction; and this being proved, they are proved true.

Peirce: CP 4.84 Cross-Ref:††

For example, it is required to prove that

$$\pi[a]\Sigma[K]\pi[b] g[Ka](\sim g[Kb] \Psi 1[ab])$$

Peirce: CP 4.84 Cross-Ref:††

That is, that every object, a , whether individual or general, has a quality, K , which is a quality of no object, b , unless b is identical with a . Now this can only lead to an absurd result if K be eliminated. But without "logical involution," or the compounding of the premiss with itself, K can only be eliminated in two ways; first, by eliminating $g[Ka]$ and $\sim g[Kb]$ independently, and second, by identifying b with a . In the first way, we can only get

$$\pi[a]\pi[b] (\sim 1[ab] \Psi 1[ab])$$

which is true; and in the second way we can only get

$$\pi[a] 1[aa]$$

which is equally true. We next proceed, then, to square the proposition in question, and so get

$$\pi[a]\Sigma[K]\pi[b]\pi[c] g[Ka]\cdot(\sim g[Kb] \Psi 1[ab])\cdot(\sim g[Kc] \Psi 1[ac]).$$

Unless we identified K in the factors, there could be no aid in eliminating K . But this identification forces us to identify a which is to the left of it in the logisterium. But it is easy to show that from the square so written K cannot be eliminated in any new way, nor from any higher power. Therefore, the proposition can lead to no absurdity, and is true.

Peirce: CP 4.84 Cross-Ref:††

Suppose, however, we were to subject the following (which seems hardly distinguishable from it to a loose thinker) to the same test

$$\Sigma[K]\pi[a]\pi[b]g[Ka]\cdot(\sim g[Kb]\vee I[ab])$$

Squaring this, we have

$$\Sigma[K]\pi[a]\pi[b]\pi[c]\pi[d]g[Ka]\cdot g[Kc]\cdot(\sim g[Kb]\vee I[ab])\cdot(\sim g[Kd]\vee I[cd]).$$

Now *identifying* d with a and c with b , we get

$$\pi[a]\pi[b]I[ab]$$

which is as much as to say that everything is identical with everything. So that in a universe of more than one object this proposition is false.

Peirce: CP 4.85 Cross-Ref:††

IV

THE LOGIC OF QUANTITY†1P

§1. ARITHMETICAL PROPOSITIONS

85. Kant, in the introduction to his *Critic of the Pure Reason*†2, started an extremely important question about the logic of mathematics. He begins by drawing a famous distinction, as follows:

Peirce: CP 4.85 Cross-Ref:††

"In judgments wherein the relation of a subject to a predicate is thought . . . this relation may be of two kinds. Either the predicate, B , belongs to the subject, A , as something covertly contained in A as a concept; or B is external to A , though connected with it. In the former case, I term the judgment analytical; in the latter synthetical. Analytical judgments, then, are those in which the connection of the predicate with the subject is thought to consist in identity, while those in which this connection is thought without identity, are to be called synthetical judgments. The former may also be called explicative, the latter ampliative judgments, since those by their predicates add nothing to the concept of the subject, which is only divided by analysis into partial concepts that were already thought in it though confusedly; while these add to the concept of the subject a predicate not thought in it at all, and not to be extracted from it by any analysis. For instance, if I say all bodies are extended, this is an analytical judgment. For I need not go out of the conception I attach to the word *body*, to find extension joined to it; it is enough to analyze my meaning, *i.e.*, merely to become aware of the various things I always think in it, to find that predicate among them. On the other hand, if I say, all bodies are heavy, that predicate is quite another matter from anything I think in the mere concept of a body in general."

Peirce: CP 4.85 Cross-Ref:††

Like much of Kant's thought this is acute and rests on a solid basis, too; and yet is seriously inaccurate. The first criticism to be made upon it is, that it confuses together a question of psychology with a question of logic, and that most disadvantageously; for on the question of psychology, there is hardly any room for anybody to maintain Kant right. Kant reasons as if, in our thoughts, we made logical definitions of things we reason about! How grotesquely this misrepresents the facts, is shown by this, that there are thousands of people who, believing in the atoms of Boscovich, do not hold bodies to occupy any space. Yet it never occurred to them, or to anybody, that they did not believe in corporeal substance. It is only the scientific man, and the logician who makes definitions, or cares for them.

Peirce: CP 4.86 Cross-Ref:††

86. At the same time, the unscientific, as well as the scientific, frequently have occasion to ask whether something is consistent with their own or somebody's meaning; and that sort of question they themselves widely separate from a question of how experience, past or possible, is qualified. The Aristotelian [logicians] -- and, in fact, all men who ever have thought -- have made that distinction. It is embodied in the conjugations of some barbarous languages. What was peculiar to Kant -- it came from his thin study of syllogistic figure -- was his way of putting the distinction, when he says we necessarily think the explicatory proposition although confusedly, whenever we think its subject. This is monstrous! The question whether a given thing is consistent with a hypothesis, is the question of whether they are logically compossible or not. I can easily throw all the axioms of number, which are neither numerous nor complicated, into the antecedent of a proposition -- or into its *subject*, if that be insisted upon -- so that the question of whether every number is the sum of three cubes, is simply a question of whether that is *involved* in the conception of the subject and nothing more. But to say that because the answer is *involved* in the conception of the subject, it is confusedly *thought* in it, is a great error. To be *involved*, is a phrase to which nobody before Kant ever gave such a psychological meaning. Everything is involved which can be evolved. But how does this evolution of necessary consequences take place? We can answer for ourselves after having worked a while in the logic of relatives. It is not by a simple mental stare, or strain of mental vision. It is by manipulating on paper, or in the fancy, formulæ or other diagrams -- experimenting on them, *experiencing* the thing. Such experience alone *evolves* the reason hidden within us and as utterly hidden as gold ten feet below ground -- and this experience only differs from what usually carries that name in that it brings out the reason hidden within and not the reason of Nature, as do the chemist's or physicist's experiments.

Peirce: CP 4.87 Cross-Ref:††

87. There is an immense distinction between the Inward and the Outward truth. I know them alike by experimentation only. But the distinction lies in this, that I can glut myself with experiments in the one case, while I find it most troublesome to obtain any that are satisfactory in the other. Over the Inward, I have considerable control, over the Outward very little. It is a question of degree only. Phenomena that inward force puts together appear *similar*; phenomena that outward force puts together appear *contiguous*. We can try experiments establishing similarity so easily, that it seems as if we could see through and through that; while contiguity strikes us as a marvel. The young chemist precipitates Prussian blue from two nearly colorless fluids a hundred times over without ceasing to marvel at it. Yet he finds no marvel in

the fact that any one precipitate when compared in color with the other seems similar every time. It is quite as much a mystery, in truth, and you can no more get at the heart of it, than you can get at the heart of an onion.

Peirce: CP 4.87 Cross-Ref:††

But nothing could be more extravagant than to jump to the conclusion that because the distinction between the Inward and the Outward is merely one of how much, therefore it is unimportant; for the distinction between the unimportant and the important is itself purely one of little and much. Now, the difference between the Inward and the Outward worlds is certainly very, very great, with a remarkable absence of intermediate phenomena.

Peirce: CP 4.88 Cross-Ref:††

88. The first question, then, to ask concerning arithmetical and geometrical propositions is, whether they are logically necessary and merely relate to hypotheses, or whether they are logically contingent and relate to experiential fact.

Peirce: CP 4.88 Cross-Ref:††

Beginning with the propositions of arithmetic, we have seen already †1 that arithmetical propositions may be syllogistic conclusions from ordinary particular propositions. From

$$A \sim\sim B$$

$$\text{and } \sim A \sim\sim B, \dagger 2$$

taken together, or

Some A is B ,

Some not- A is B ,

it follows that there are at least two B 's. This inference is strictly logical, depending on the principle of contradiction, that is, on the non-identity of A and not- A . By the same principle, from

Some A is B ,

Some not- A is B ,

Any B is C ,

Some not- B is C ,

taken together it follows that there are at least three C 's.

Peirce: CP 4.89 Cross-Ref:††

89. Hamilton admits †3 that the arithmetical proposition, "Some B is not some- B ," is so urgently called for in logic, that a special propositional form must be made for it. So, if a distributive meaning be given to "every," Every A is every A , implies that there is but one A , at most. This is what this proposition must mean, if it is to be the precise contradiction of the other. If a proposition is infra-logical in form, its denial must be admitted to be so.

Peirce: CP 4.90 Cross-Ref:††

90. It clearly belongs to logic to evolve the consequences of its own forms.

Hence, the whole of the theory of numbers belongs to logic; or rather, it would do so, were it not, as pure mathematics, *prelogical*, that is, even more abstract than logic.†4

Peirce: CP 4.91 Cross-Ref:††

91. These considerations are sufficient of themselves to refute Kant's doctrine that the propositions of arithmetic are "synthetical." As for the argument of J. S. Mill,†5 or what is usually attributed to him, for what this elusive writer really meant, if he precisely meant anything, about any difficult point, it is utterly impossible to determine -- I mean the argument that because we can conceive of a world in which when two things were put together, a third should spring up, therefore arithmetical propositions are experiential, this argument proves too much. For, in the existing world, this often happens; and the fact that nobody dreams of its constituting any infringement of the truths of arithmetic shows that arithmetical propositions are not understood in any experiential sense.

Peirce: CP 4.91 Cross-Ref:††

But Mill is wrong in supposing that those who maintain that arithmetical propositions are logically necessary, are therein *ipso facto* saying that they are verbal in their nature. This is only the same old idea that Barbara in all its simplicity represents all there is to necessary reasoning, utterly overlooking the construction of a diagram, the mental experimentation, and the surprising novelty of many deductive discoveries.

Peirce: CP 4.91 Cross-Ref:††

If Mill wishes me to admit that *experience* is the only source of any kind of knowledge, I grant it at once, provided only that by experience he means *personal history*, life. But if he wants me to admit that inner experience is nothing, and that nothing of moment is found out by diagrams, he asks what cannot be granted.

Peirce: CP 4.92 Cross-Ref:††

92. The very word *a priori* involves the mistaken notion that the operations of demonstrative reasoning are nothing but applications of plain rules to plain cases. The really unobjectionable word is *innate*; for that may be innate which is very abstruse, and which we can only find out with extreme difficulty. All those Cartesians who advocated innate ideas took this ground; and only Locke failed to see that learning something from experience, and having been fully aware of it since birth, did not exhaust all possibilities.

Peirce: CP 4.92 Cross-Ref:††

Kant declares that the question of his great work is "How are synthetical judgments *a priori* possible?" By *a priori* he means universal; by synthetical, experiential (*i.e.*, relating to experience, not necessarily derived wholly from experience). The true question for him should have been, "How are universal propositions relating to experience to be justified?" But let me not be understood to speak with anything less than profound and almost unparalleled admiration for that wonderful achievement, that indispensable stepping-stone of philosophy.

Peirce: CP 4.93 Cross-Ref:††

93. To return to number, there are various ways in which arithmetic may be conceived to connect itself with and spring out of logic. Besides the path of spurious propositions [as indicated in 88], there is another which I pursued on an early paper †1 in which I defined the arithmetical operations in terms of those of the reformed Boolean calculus. In a later paper,†2 I considered quantity from the point of view of

the logic of relatives.

Peirce: CP 4.93 Cross-Ref:††

I shall in the present chapter endeavor, as much as I can, to avoid tedious questions of detail and seek to make clear some of the main points of the logic of mathematics.

Peirce: CP 4.94 Cross-Ref:††

§2. TRANSITIVE AND COMPARATIVE RELATIONS

94. I have certainly written to little purpose, and so has Dr. Schröder, if we have not succeeded in making readers perceive the pervasive working of balance and symmetry in every part of logic. Now, we have seen the ubiquitous logical agency of the form

$$l \sim \$l. \dagger 3$$

Peirce: CP 4.94 Cross-Ref:††

We say that this is due to the formula $l \sim \$l \prec T, \dagger 4$ which is balanced by $1 \dagger 5 \prec l \dagger \sim \$l. \dagger 6$ But, be it observed, that this is a kind of balance which throws all the active work upon the shoulders of the former principle, and allows the latter to moulder in innocuous desuetude. Yet really, the form

$$l \dagger \sim \$l$$

is all-important, inasmuch as it is the basis of all quantitative thought. For the relation expressed by it is a transitive relation. By a transitive relation, we mean a relation like that of the copula. If A be so related to B , and B be so related to C , then A is so related to C . In other words, if t is a transitive relation,

$$tt \prec t.$$

Now, this is the case with $l \dagger \sim \$l$. For that

$$(l \dagger \sim \$l)(l \dagger \sim \$l) \prec l \dagger \sim \$l$$

is obvious. Though the reader sees how, I will, in consideration of the importance of the matter, set down the steps:

$$(l \dagger \sim \$l)(l \dagger \sim \$l)$$

$$\prec l \dagger \sim \$l(l \dagger \sim \$l)$$

$$\prec l \dagger \sim \$l l \dagger \sim \$l$$

$$\prec l \dagger T \dagger \sim \$l$$

$$\prec l \dagger \sim \$l.$$

Peirce: CP 4.94 Cross-Ref:††

This is not only a transitive relation, but it is one which includes identity under it. That is,

$$1 \prec l \dagger \sim \$l$$

Peirce: CP 4.94 Cross-Ref:††

But it is further demonstrable that every transitive relation which includes identity under it is of the form

$$l\ddagger\sim\$l.$$

For let t be such a relation that

$$1 \prec t.$$

Multiplying by $\sim\$t$, we get,

$$\sim\$t \prec 1\sim\$t \prec t\sim\$t \prec T.$$

Hence,†1

$$t\ddagger\sim\$t \prec t\ddagger T \prec t.$$

On the other hand,

$$t \prec t1$$

$$\therefore t \prec t(t\ddagger\sim\$t)$$

$$\therefore t \prec tt\ddagger\sim\$t.$$

But because t is transitive,

$$tt \prec t$$

$$\therefore t \prec t\ddagger\sim\$t.$$

Having just found

$$t\ddagger\sim\$t \prec t,$$

we can write

$$t = t\ddagger\sim\$t,$$

so that t may be expressed in the form $l\ddagger\sim\$l$, Q. E. D.

Peirce: CP 4.95 Cross-Ref:††

95. I am now going to allow myself to be led aside out of the main channel of thought upon this subject merely to show how little interest there is in transitive relationship apart from the logical form ($l\ddagger\sim\$l$).

Peirce: CP 4.95 Cross-Ref:††

Let us use the zodiacal sign of Leo to signify a transitive relation, such that not everything is in that relation to itself. The inference holds,

$$x \text{ ☉ } y, y \text{ ☉ } z,$$

$$\therefore x \text{ ☉ } z.$$

Let L be an individual that is not in this relation to itself, which we may write,

$$L \sim \Omega L.$$

Then, the (equivalent) inferences hold

$$\begin{array}{l} L \Omega x \quad x \Omega L \\ \therefore x \sim \Omega L \quad \therefore L \sim \Omega x. \end{array}$$

We may, therefore, divide all other individuals into three classes; *first*, **K**, **J**, etc. such that

$$\begin{array}{l} K \Omega L \\ J \Omega L, \text{ etc.}; \end{array}$$

second, **M**, **N**, etc., such that

$$\begin{array}{l} L \Omega M \\ L \Omega N, \text{ etc.}; \end{array}$$

and *third*, Γ , $\{D\}$, etc., such that

$$\begin{array}{l} \Gamma \sim \Omega L \quad L \sim \Omega \Gamma \\ \{D\} \sim \Omega L \quad L \sim \Omega \{D\}, \text{ etc.} \end{array}$$

Taking any one of the first class, **K**, and any one of the second, **M**, we have

$$M \sim \Omega K.$$

Peirce: CP 4.95 Cross-Ref:††

Let **G** be a letter of the first class which is a non-Leo of itself; then

$$G \sim \Omega G,$$

and the first class may be subdivided into three with reference to **G**, just as all were divided relatively to **L**. So, if **R** be a letter of the second class which is non-Leo of itself, or

$$R \sim \Omega R.$$

Peirce: CP 4.95 Cross-Ref:††

We can then divide all possible individuals other than **G**, **L**, and **R** into ten classes, *viz.*:

Peirce: CP 4.95 Cross-Ref:††

First, Those which, as **B**, **C**, etc. are Leos of **G**; as

$$B \Omega G;$$

Peirce: CP 4.95 Cross-Ref:††

Second, Those which, as **r**, are neither Leos of nor Leo'd by **G**, but are Leos of **L**; as

$$\Gamma \sim \Omega G, G \sim \Omega \Gamma, \Gamma \Omega A;$$

Peirce: CP 4.95 Cross-Ref:††

Third, Those which, as H, K , etc., are Leo'd by G and are Leos of L ; as

$$G \circ K, K \circ L;$$

Peirce: CP 4.95 Cross-Ref:††

Fourth, Those which, as $\{D\}$, are not Leo'd by G , nor are Leos of L , but are Leos of R ; as

$$G \sim \circ \{D\}, \{D\} \sim \circ L, \{D\} \circ R;$$

Peirce: CP 4.95 Cross-Ref:††

Fifth, Those which, as Θ , are Leo'd by G , are neither Leos of nor Leo'd by L , and are Leos of R ; as

$$G \circ \Theta, L \sim \circ \Theta, \Theta \sim \circ L, \Theta \circ R;$$

Peirce: CP 4.95 Cross-Ref:††

Sixth, Those which, as P, Q , etc. are Leo'd by L and are Leos of R ; as

$$L \circ P, P \circ R;$$

Peirce: CP 4.95 Cross-Ref:††

Seventh, Those which as $\{X\}$ are not Leo'd by G nor are Leos of R ; as

$$G \sim \circ \{X\}, \{X\} \sim \circ R;$$

Peirce: CP 4.95 Cross-Ref:††

Eighth, Those which as π are Leo'd by G , are not Leo'd by L , and are not Leos of R ;

$$G \circ \pi, L \sim \circ \pi, \pi \sim \circ R;$$

Peirce: CP 4.95 Cross-Ref:††

Ninth, Those as Σ which are Leo'd by L but are neither Leos of nor Leo'd by R ;

$$L \circ \Sigma, R \sim \circ \Sigma, \Sigma \sim \circ R;$$

Peirce: CP 4.95 Cross-Ref:††

Tenth, Those as X, Y , etc. which are Leo'd by R ; as

$$R \circ X.$$

Peirce: CP 4.96 Cross-Ref:††

96. The above gives some idea what the further doctrine of transitive relations not including identity would be like. It is evidently more interesting to consider further the study of relatives of the form $(I \dagger \sim \$I)$ and others allied to them.

Peirce: CP 4.96 Cross-Ref:††

The converse of $I \dagger \sim \$I$ is $\sim I \dagger \$I$, †1 which is, of course, also transitive. The negative is $\sim I \$I$, †2 which is not transitive, but which has the property,

$$\sim I \$I \prec \sim I \$I \dagger \sim I \$I.$$

For $\sim I \$I = \sim I I \$I \prec \sim I (\$I \dagger \sim I) \$I \prec \sim I \$I \dagger \sim I \$I.$

Peirce: CP 4.96 Cross-Ref:††

This is a property allied to transitiveness.

Peirce: CP 4.96 Cross-Ref:††

If A is a lover of something not loved by B , which is, in its turn, a lover of something not loved by C , the conclusion is, that A is a lover of something different from something not loved by C . That is,

$$(l \sim \$l)(l \sim \$l) \rightarrow lT \sim \$l.$$

Peirce: CP 4.96 Cross-Ref:††

The natural current of thought next carries us to the hypothesis that the relation expressed by l be such that

$$l \sim \$l \rightarrow l \dagger \sim \$l.$$

In this case, $l \sim \$l$ is a transitive relation. For

$$l \sim \$l \ l \sim \$l \rightarrow l \sim \$l(l \dagger \sim \$l) \rightarrow l \sim \$l.$$

Such a relation may well be termed a comparative relation. If Samson can lift something Ajax cannot lift, then Samson can lift everything Ajax can lift. Such a relation underlies all measurement; and the propriety of the designation I propose will be allowed.

Peirce: CP 4.96 Cross-Ref:††

With this conception, quantitative science begins. Note well how it has been suggested to us.

Peirce: CP 4.96 Cross-Ref:††

A trial of strength must begin by young Ajax, the challenger, doing various things which he "stumps" the champion, Samson, to imitate. If Samson cannot perform all of Ajax's feats, that settles it. But if it seems that Samson can do all that Ajax can do then he will, in his turn, do something which he proposes that Ajax shall imitate. If Ajax cannot do all that Samson can do, that again settles it. But if it seems that each can repeat all the performances of the other, we conclude that they are equally strong. Thus equality is a complex relation.†3

Peirce: CP 4.96 Cross-Ref:††

In a universe of *quantities* of one dimension (where are only *quantities*, not *quanta*) things equal are identical; so that, not only,

$$l \sim \$l \rightarrow T,$$

which is always true and

$$l \sim \$l \ l \sim \$l \rightarrow T,$$

which is true for all comparative relations, but also

$$T \rightarrow l \sim \$l \ \nrightarrow \sim \$l. \dagger 1$$

That is, if A and B are not identical, either A can do something that B cannot, or B can do something that A cannot.

Peirce: CP 4.96 Cross-Ref:††

We have thus analyzed the conception of quantity; and we see that nothing but logical conceptions enter into its constitution. The idea of being *able*, especially in the broad sense in which one quantity is said to be *able* to do something another is *unable* to do, is only a modification of the idea of possibility, the precise explanation of which is given in second intentional logic.

Peirce: CP 4.97 Cross-Ref:††

97. Although I cannot, in this work, carry the student deeply into second intentional logic, yet it will be indispensable to look upon quantity somewhat in that way; for quantitative thought, like the traditional "chimæra bombinans in vacuo," feeds upon second intentions.

Peirce: CP 4.97 Cross-Ref:††

That *l* is a comparative relation in a universe of quantity, may be expressed by the formula,

$$0 \dagger (1 \dashv l \sim \$ l \dashv \sim l \$ l) \dagger 0 \dagger 1$$

But the same thing may be expressed in another way, by throwing the relation *l* among the indices. Thus, let us use the three symbols, *u*, *v*, *w*.

w[*ij*] means that *i* is an individual relation of which *j* is the general character,

u[*ij*] means that *i* is an individual relation of which *j* is the first relate,

v[*ij*] means that *i* is an individual relation of which *j* is the correlate.

Peirce: CP 4.97 Cross-Ref:††

Then, we shall have

$$l[ij] = \Sigma[k]u[ki]v[kj]w[kl],$$

that is, there is an individual relation of which *i* is the relate, *j* the correlate, and *l* the general character.

Peirce: CP 4.97 Cross-Ref:††

That being premissed, the proposition that *l* is a *comparative* relation may be written,

$$\pi[h]\pi[k]\Sigma[i]\Sigma[j]\Sigma[p]\pi[q]\pi[r]\Sigma[s]$$

$$1[hk] \dashv u[ph] \cdot v[pi] \cdot w[pl] \cdot (\sim u[qk] \dashv \sim v[qi] \dashv \sim w[ql]) \dashv (\sim u[rh] \dashv \sim v[rj] \dashv \sim w[rl]) \cdot u[sk] \cdot v[sj] \cdot w[sl];$$

that is, of any two different objects, *h* and *k*, one or other is *l* to something to which the other is not *l*.

Peirce: CP 4.97 Cross-Ref:††

More simply, if

$$r[ijk]$$

means that *i* is a relation in which *j* stands to *k*; we may write,

$$\pi[h]\pi[k]\Sigma[i]\Sigma[j] 1[hk] \dashv r[lhi] \sim r[lki] \dashv r[lkj] \sim r[lhj]$$

to express that *l* is a comparative relation.

Peirce: CP 4.98 Cross-Ref:††

98. We are next naturally led to remark that it is a very important thing to say of a class of objects, say the *A*'s, that there is some one relation such that, of any two *A*'s not identical, one stands in that relation to the other, while the second does not stand in that relation to the first. This we write

$$\Sigma[l]\pi[m]\pi[n] \sim a[m] \vee \sim a[n] \vee 1[mn] \vee r[lmn] \sim r[lm] \vee r[lm] \sim r[lm]$$

But this becomes particularly important, in case the relation *l* is a relation of comparison. If that be the case, we multiply in the above definition of such a relation.

Peirce: CP 4.99 Cross-Ref:††

99. The question arises, is it possible there should be a class which does not possess the property just defined? It is a difficult question, to which a good logician will be reluctant to give a negative answer. In order to answer it, we must have some way of constructing an icon of a class, in general. Now, a class may be said to comprise all of which something is true. Shall we say of the different individuals composing it that they are distinguished by having some of them qualities which the others do not possess? It seems far from evident that this is so; although, no doubt, after two instances have presented themselves, it is possible in the circumstances of the presentation to find distinguishing qualities. But supposing this difficulty surmounted, of two individuals of a class, each may have qualities the other wants. If, then, we seek to establish an order of precedence among the things, such as a relationship of comparison supposes, we must first establish some order of precedence among the qualities. We are thus brought back to the question with which we set out, whether among a collection of objects an order of precedence *can* always be establishable. It thus becomes clear that no contradiction can emerge from the hypothesis of a class among the members of which no thoroughgoing order of precedence can be established, and to which all quantitative conceptions are quite inapplicable. About such classes we can reason, but we cannot reason quantitatively.

Peirce: CP 4.100 Cross-Ref:††
§3. ENUMERABLE COLLECTIONS

100. But supposing we have to do with a class of things throughout which a relationship of comparison *can* be established, the next question that balance and symmetry suggest is, whether, as we have

$$l \sim \$l \quad l \sim \$l \quad - < \quad l \sim \$l,$$

we have also

$$l \sim \$l \quad - < \quad l \sim \$l \quad l \sim \$l.$$

Peirce: CP 4.100 Cross-Ref:††

It is clear enough, that cases can be imagined in which this shall not be true. Classes of a mixed character exist, too, where this holds in certain parts, but not in others. Such mixed cases are not, however, of much interest. The interesting cases are those in which

$$l \sim \$l \prec l \sim \$l \sim \$l$$

invariably holds, †1 and those in which whatever is $l \sim \$l$ to anything is $l \sim \$l$ to something to which it is *not* $l \sim \$l \sim \l . To speak in more familiar terms, whatever is greater than anything may or may not always be *next* greater than something, that is, may or may not be always greater than something else, greater than that thing.

Peirce: CP 4.101 Cross-Ref: ††

101. But a further distinction immediately arises, according as, on the one hand, one or other of these propositions is true for *every* comparative relation, or on the other hand for some comparative relations the one proposition is true and for others the other.

Peirce: CP 4.101 Cross-Ref: ††

This trichotomy constitutes the most important distinction between classes in respect to their *multitude*.

Peirce: CP 4.101 Cross-Ref: ††

Let us first consider a class in which, no matter what comparative relation may be signified by $l \sim \$l$,

$$l \sim \$l^\infty \prec [l \sim \$l \cdot (\sim l \dagger \$l \dagger \sim l \dagger \$l)]^\infty \dagger 1$$

That is, whatever is greater than anything is next greater than something, in *every sense* of being greater, that is, for *every* comparative relation, $l \sim \$l$, for which

$$0 \dagger (1 \dashv l \sim \$l \dashv \sim l \dagger \$l) \dagger 0.$$

Peirce: CP 4.102 Cross-Ref: ††

102. We have now pushed our way far enough into the theory of quantity and its complications of logic to meet with theorems. Such is the following: any class of which the conditions enunciated holds has a *maximum* and a *minimum* individual for each comparative relation, that is, one which is not $l \sim \$l$ to any member of the class, and one which is not $\sim l \dagger \$l$ to any member. I will first show that there is a maximum. For this purpose, assume $a[0]$ to be any member of the class of A's, and consider the relation

$$(l \sim \$l \dashv \sim l \dagger \$l) a[0] \$a[0] \dashv l \sim \$l a[0] \$a[0] \sim l \dagger \$l \cdot l \sim \$l \dashv \sim l \dagger \$l a[0] \$a[0] \sim l \dagger \$l \dashv \sim l \dagger \$l a[0] \$a[0] l \sim \$l \sim l \dagger \$l.$$

Peirce: CP 4.102 Cross-Ref: ††

This is an aggregate of four relations, *viz.*:

Peirce: CP 4.102 Cross-Ref: ††

First, That having for its relate any A, superior ($l \sim \$l$) or inferior ($\sim l \dagger \l) to $a[0]$, and for its correlate $a[0]$,

Peirce: CP 4.102 Cross-Ref: ††

Second, That having for both relate and correlate A's superior to $a[0]$, the relate being superior to the correlate,

Peirce: CP 4.102 Cross-Ref: ††

Third, That having for relate any A inferior to $a[0]$, and for correlate any A

superior to $a[0]$,

Peirce: CP 4.102 Cross-Ref:††

Fourth, That having for both relate and correlate A 's inferior to $a[0]$, the relate being inferior to the correlate.

Peirce: CP 4.102 Cross-Ref:††

This, I say, will be a quantitative relation. That is, it will be transitive, included under its own negative converse, and including negation under the aggregate of itself and its converse. That it is transitive, that is, that, if X is in this relation to Y , and Y to Z , then X is in this relation to Z , is plain; for, if Y is superior to $a[0]$, then Z must either be $a[0]$ (in which case, X , whatever A it may be, is in this relation to Z) or must be superior to $a[0]$ but inferior to Y . Then, if X is superior to $a[0]$, it is superior to Y , and consequently also to Z , and is in this relation to Z . But if X is inferior to $a[0]$, it is in this relation to Y , which is superior. X can in no case be $a[0]$. If Y is inferior to $a[0]$, Z is either $a[0]$ (when as before X will be in this relation to it) or superior to $a[0]$, or inferior to $a[0]$, but superior to Y . X will be inferior to Y , and thus will be in this relation to Z . This shows that the relation is transitive. That it is included under its negative converse, that is, inconsistent with its converse, is plain; for if U could be in this relation to V and in the converse relation, too, that is, V in this relation to U , then, since the relation is transitive, U would be in this relation to itself, which, it is easy to see, the definition excludes. That of any pair of different A 's, one is in this relation to the other, is easily seen by running over the definition.

Peirce: CP 4.102 Cross-Ref:††

We will call this relation "second-superior." Now, I say, if the class of A 's has no maximum for the relation $l \sim \$l$, then that A which is next inferior to $a[0]$ is not next second-superior to any A . Will it be objected that I have not proved that there is an A next inferior to $a[0]$? It is easy to supply the defect. For by hypothesis whatever A is superior is next superior to some A for every comparative relation. Now, we have only to substitute $\$l$ for l and *vice versa*, and next inferior becomes next superior. Therefore, to say that for *every* comparative relation, whatever has a superior has a next superior, is the same as to say that for every comparative relation, whatever has an inferior has a next inferior. The A next inferior to $a[0]$ is second-superior only to $a[0]$, to A 's superior to $a[0]$ and to A 's inferior to $a[0]$ but superior to itself. Because it is *next* inferior, of the last there are none. That of which it is next superior is therefore superior to $a[0]$, and any other A superior to it is second-superior to it and second-inferior to the next inferior to $a[0]$. Thus, that to which the next inferior to $a[0]$ is next second-superior, is the superior of all other A 's superior to $a[0]$; that is, it is the *maximum*. The proof that there will be a minimum is altogether similar.

Peirce: CP 4.102 Cross-Ref:††

Hence, any class of things in which whatever is *anywise* superior to another of the class is next superior to some one can be enumerated. For in enumeration, the objects of a class are singly *told*, and "told later than" evidently satisfies the three conditions of a quantitative relative. If there is a maximum, the telling comes to an end, the class is told out, it is enumerated. For that reason, it is convenient to term a class every member of which anywise superior to another is next superior to some, an *enumerable* collection.

Peirce: CP 4.103 Cross-Ref:††

103.†1 About an enumerable collection certain forms of reasoning hold which, though they had been used more or less since man began to be a reasoning animal,

were first signalized in a logical work by De Morgan in 1847,[†]P1 and constitute one of his claims to be considered the greatest of all formal logicians. In his *Formal Logic* he gives eight forms which in the Appendix to his fourth Memoir on the Syllogism are increased to 64. The eight are as follows:

For every Z there is an X that is Y ,

Some Z is not Y ;

∴Some X is not Z .

For every Z there is an X not Y ,

Some Z is Y ;

∴Some X is not Z .

For every non- Z there is an X that is Y ,

There is something besides Y 's and Z 's;

∴Some X is Z .

For every non- Z there is an X not Y ,

Some Y is not Z ;

∴Some X is Z .

For every Z there is a Y not X ,

Some Z is not Y ;

∴There is something besides X 's and Z 's.

For every Z there is something neither X nor Y ,

Some Y is Z ;

∴There is something besides X 's and Z 's.

For every non- Z there is a Y not X ,

There is something besides Y 's and Z 's;

∴Some Z is not X .

For every non- Z there is something neither X nor Y ,
Some Y is not Z ;

∴Some Z is not X .

Peirce: CP 4.103 Cross-Ref:††

We might also have such a reasoning as this:

For every Z there is an X that is Y ,

For every X not Z there is an X not Y ;

∴Every Z is X .

Peirce: CP 4.103 Cross-Ref:††

This is not one of De Morgan's Forms. He gives, however such as these:

For every Z an X is Y ,

Every Y is Z ;

∴Every Z is X .

For every not Z is an X not Y ,

For every X is a Y not Z ;

∴Every Z is X .

Peirce: CP 4.103 Cross-Ref:††

De Morgan termed these "syllogisms of transposed quantity," because they transfer the lexis from one term to another. His point of view was this: Take *Baroko*,

Any M is P ,

Some S is not P ;

∴Some S is not M .

The converse,

Some M is not S ,

does not follow; but if there are as many M 's as there are S 's, then this *does* follow. "For if M 's, as many as there are S 's, be among the P 's, and some of the S 's be not among the P 's, though all the rest were, there would not be enough to match all the M 's, or some M 's are not S 's."

Peirce: CP 4.103 Cross-Ref:††

The rank and file of old-fashioned logicians were not pleased with the syllogisms of transposed quantity. They belonged to that class of minds who decry originality, who dread novelty, who hate discoveries, and who will go to some trouble to inflict any personal injury on those who perpetrate them, provided they can inflict it without serious injury to themselves. They circulated an unfounded innuendo that De Morgan was a drunkard; their spitefulness was only bounded by their prudence. The idea of so lifeless a subject as formal logic -- too abstract to be philosophical -- exciting such passions is laughable. Yet such was the fact.

Peirce: CP 4.103 Cross-Ref:††

But they could not find anything better to say against those syllogisms of transposed quantity than that they were "extralogical." If it had only occurred to them that they were not sound reasoning, that is, not universally valid, they would have seized upon that defect with glee. Nor, singularly enough, does De Morgan himself seem to have remarked the circumstance; although it ought to have been evident from the line of thought which led him to those forms. By the logic of relatives, we at once find that the statement that such a syllogism is necessary implies that a certain collection is enumerable.

Peirce: CP 4.103 Cross-Ref:††

The following is the first form De Morgan adduced:

Some X is Y ,

For every X there is something neither Y nor Z ;

Hence, something is neither X nor Z .

Let us put for X "odd number," for Y "prime," for Z "either an even number or not a number," so that its negative is "a number not even." Then, the conclusion is false though the premisses are true. Thus:

Some odd numbers are prime,

Every odd number has for its square a number not even nor prime;

Hence, some number not even is not odd.

Peirce: CP 4.104 Cross-Ref:††

104. Let us enclose the description of a class in square brackets to denote the number of individuals in it. Then the premisses of the above may be arithmetically stated thus:

$$[x \cdot y] > 0$$
$$[x] \mid [\sim y \cdot \sim z]$$

Developing the last, we get

$$[x \cdot y \cdot z] + [x \cdot y \cdot \sim z] + [x \cdot \sim y \cdot z] + [x \cdot \sim y \cdot \sim z] \mid [x \cdot \sim y \cdot \sim z] + [\sim x \cdot \sim y \cdot \sim z]$$

Cancelling $[x \cdot \sim y \cdot \sim z]$, we have

$$[x \cdot y \cdot z] + [x \cdot y \cdot \sim z] + [x \cdot \sim y \cdot z] \mid [\sim x \cdot \sim y \cdot \sim z]$$

Developing the first premiss

$$[x \cdot y \cdot z] + [x \cdot y \cdot \sim z] > 0.$$

It thus follows, not only that

$$[\sim x \cdot \sim z] > 0$$

but even that, throwing aside *Y*'s, there are more non-*X*'s not *Z*'s than there are *X*'s that are *Z*'s. But the fallacy lies in assuming the Simple Simon proposition that *every part is less than its whole*. That is, because the odd squares are no fewer than all the odd numbers, we quietly reason as if they were more than a part of odd numbers; so that after taking away alike from odd squares and from odd numbers the odd primes, we should necessarily have as many odd squares left over as we have odd numbers. Of collections not enumerable it is not generally true that the part is less than the whole. Every integer has a square; and thus there are as many squares as there are integers; although the squares form but a part of all the integers.

Peirce: CP 4.104 Cross-Ref:††

Take this example:

Every woman marries a man,

For every man there is a woman;

∴ Every man is married to a woman.

The necessity of this plainly arises from the fact that after every woman has got a husband, the collection of men is *exhausted*. To say this, is to imply that for every quantitative relation it would have a *maximum*, that is, a *last reached*, in any order of running it through. . . .

Peirce: CP 4.105 Cross-Ref:††

105. The commonest sort of paralogism by far among thoughtful persons

consists in reasoning as if collections were enumerable which are, in fact, innumerable. How often do we hear one, speaking of objects in a linear series, say there must be a first or must be a last! Logic lends no color to such ideas; but, on the contrary, shows them to be pure assumptions. In metaphysics, particularly, it is frequently argued that something is analyzable into a series -- by pure abstract reasoning -- and then because "there must be a first" some consequence truly startling follows. Years of experience bring us to expect, as a matter of course, some fallacy, big or little, in every demonstration which seems to advance knowledge very much.

Peirce: CP 4.106 Cross-Ref:††

106. De Morgan's syllogism of transposed quantity does not seem very clearly or accurately to set forth precisely what the nature of the reasoning specially applicable to enumerable collections is. What does precisely describe it is, that in whatever order you pass, one by one, through the collection, you come to a last unit. But let us logically analyze this. Here there is a relation of the later-taken to the earlier-taken. The earlier is taken at a time at which the later is not taken. If l signify "taken at a time," then "taken at a time at which was not taken" is written

$$l \sim \$l.$$

We see at once that this running through the collection is only a specialized way of saying that we have to do with a quantitative relation, a way of expressing it which brings in the irrelevant idea of *time*. The better statement is that, in reference to every quantitative relation, the enumerable collection has a last. From this it quite obviously follows that there is also a first, and that every superior of anything is next superior of something, and so also with the inferior. Another form of definition of the enumerable collection is, to say that it is a collection any part of which is less than the whole. That is, given a class, the b 's, such that

$$\infty \prec \sim \$b \dagger a,$$

or every b is a , while

$$\infty \prec \$a \sim b,$$

or some a is not b ; if, further, k is such a relative that

$$\$k \ b \ k \prec 1,$$

then

$$\infty \prec \$a(\sim \$k \dagger \sim b).$$

Peirce: CP 4.106 Cross-Ref:††

This is a good statement of the kind of inference peculiar to enumerable classes. It has three premisses, involving two class terms and a relative term; and it reposes directly upon the axiom that an enumerable part is less than the whole. One of the three premisses is implicitly assumed but not stated by De Morgan; the relative is his "for every," and one of his three terms is superfluous. Thus he puts the argument about the checks into form as follows:†1

"For every memorandum of a purchase a countercheque is a transaction involving the drawing of a cheque,

"Some purchases are not transactions involving the drawing of checks;

"Therefore, some countercheques are not memoranda of purchases."

But I should put it in form as follows:

Some payment for purchase was not a check,

Every payment by purchase is told off against a check;

No two payments by purchase are told against the same check;

∴ Some checks are not payments by purchase.

Peirce: CP 4.106 Cross-Ref:††

That is, we prove the checks are not a part of the payments by purchase, because they are not less than the payments by purchase; and it is assumed they were enumerable. If they ran on endlessly, each payment by purchase might be told off against a check of a subsequent day, the purchases increasing in number day by day.

Peirce: CP 4.107 Cross-Ref:††

§4. LINEAR SEQUENCES

107. The second, or middling, grade of multitude is that of collections which have different attributes for different quantitative collections; namely, for some such relations, every member of the class superior to another member is *next* superior to some member, definitely designatable, while for other quantitative relations it is not so. I undertake to show that there is always some quantitative relation for which (1) the class has a minimum but no maximum, (2) for which every member of the class that is superior to another is *next* superior to some other, (3) and for which the partial class consisting of any two members of the class we are speaking of, together with all that are superior to one of these two members but inferior to the other, is *enumerable*. Let us begin by thinking of a member of this class, say $a[x]$. Then, considering a quantitative relation in which every a superior to an a is next superior to an a , let us think of that to which $a[x]$ is next superior. Then, think of that to which the last is next superior. Then consider a partial class to all of which $a[x]$ is superior, and the next inferior of each member of which is also included under it, so that either there is a minimum, which is not superior to any member of the class, or else, if $a[y]$ is any member of the class to which $a[x]$ is superior, and the a 's at once inferior to $a[x]$ and superior to $a[y]$ are enumerable, it follows that $a[y]$ is a member of the partial class. For if not, of all the a 's superior to $a[y]$ and inferior to $a[x]$, a part belongs to the partial class, and this part of an enumerable collection, being itself (as such) enumerable, must have a *minimum*. But by the definition of the partial class, whatever is next inferior to any member of it also belongs to it. To this partial class, then, belongs every a inferior to $a[x]$, so long as between it and $a[z]$, the collection of

a's is enumerable. We do not know that there is an *a* next superior to *a*[*x*]. But we define a second partial class as containing the *a* next superior to *a*[*x*], if there be any, and as containing nothing else, except that it contains the *a* next superior to any *a* that it contains. Then, it will either contain all the *a*'s superior to *a*[*x*] up to some *maximum*, which need not be the maximum of all the *a*'s, but which has no *a* next superior to it, or, in the absence of such a maximum, it will contain all the *a*'s up to and beyond any *a* superior to *a*[*x*], but such that the *a*'s inferior to it and superior to *a*[*x*] form an enumerable collection. The proof of this (so plain that it hardly needs statement) is as follows: if this be not the case let *a*[*z*] be an *a* superior to *a*[*x*] and such that the *a*'s inferior to *a*[*z*] but superior to *a*[*x*] form an enumerable multitude. Then, those of those which belong to the second partial class, being part of an enumerable collection. are themselves enumerable. Hence, they have a maximum, contrary to the hypothesis. Taking the first and second partial classes together, I propose to call such a series of *a*'s a linear sequence. I will repeat its characteristics:

1. It contains *a*[*x*].
2. It contains every *a* inferior to *a*[*x*] and identical with or superior to *a*[*y*], no matter what *a*[*y*] may be, so long as it is inferior to *a*[*x*], and so long as the *a*'s superior to *a*[*y*] and inferior to *a*[*z*] form an enumerable collection.
3. It contains every *a* superior to *a*[*x*] and identical with or inferior to *a*[*z*], no matter what *a*[*z*], may be, so long as it is superior to *a*[*x*], and so long as the *a*'s inferior to *a*[*z*] and superior to *a*[*x*] form an enumerable collection.
4. It has no minimum unless that *minimum* be the minimum of all the *a*'s.
5. It has no maximum unless that maximum be an *a* which has no *a* next superior to it.
6. Unless the linear series happens to have a minimum and a maximum, it is itself an *inenumerable* collection of *a*'s.

Peirce: CP 4.108 Cross-Ref:††

108. Having formed this linear sequence, if there be any *a*'s not included in it, let *a*'[*x*] be one of them. We then proceed to form a second linear sequence in which *a*'[*x*] takes the place of *a*[*x*]. If, after that, there still be *a*'s not included in the linear sequences already formed we proceed to form a *next succeeding* linear sequence, and so on indefinitely. The multitude of linear sequences may be inenumerable; but as necessary consequences of the rule for the formation of these sequences, the following propositions hold.

Peirce: CP 4.108 Cross-Ref:††

First, The linear sequences are formed successively. If we take

t

to signify "already formed at a time --", then

$$\sim t \& t \prec \sim t \dagger t$$

that is "what is not yet formed at some time at which *X* was already formed," is included under "formed only at times at which *X* was already formed." Moreover

$$0 \dagger (1 \dashv \sim t \& t \dashv \sim t) \dagger 0.$$

That is, of two linear sequences not the same one was formed earlier than the other.

Peirce: CP 4.108 Cross-Ref:††

Second, None of the linear sequences was $\sim tSt$ to the first.

Peirce: CP 4.108 Cross-Ref:††

Third, Every sequence $\sim tSt$, that is formed subsequent to another, was formed on the occasion of its having been found that some a 's were left over not included in the previous sequences, and thus was not $\sim tSt\sim tSt$, or was formed *next* subsequent to some other.

Peirce: CP 4.108 Cross-Ref:††

Fourth, All the sequences formed previous to a given sequence, have a first and last, and are an enumerable collection.

Peirce: CP 4.109 Cross-Ref:††

109. It is well to remark, as a matter of language, that whenever a quantitative relation is applied to a class which has for that relation a minimum, and every member of the class superior to another in respect to this relation is next superior to some other, and the partial class consisting of all inferior to any given member is enumerable, then we can conveniently speak of the relationship as constituting an arithmoidal order in which the individuals are taken, the next superior being said to come next after, etc.

Peirce: CP 4.110 Cross-Ref:††

110. I propose now to show that all the a 's can be embraced in such a serial order. But for this purpose, I must first establish a preliminary arithmoidal order in each of the linear sequences. The first sequence may have both a minimum and a maximum, and if so, it is enumerable. If it has a minimum but no maximum, the arithmoidal order will already exist, the minimum being the first. In every sequence which has a maximum, the arithmoidal order is established by simply considering the converse of the quantitative relation for which that maximum is a maximum. It remains only to establish an arithmoidal order in each of the sequences which has neither minimum nor maximum. This we do by taking arbitrarily any a of the sequence as first of the series, and for the one next after it, the a to which it is next superior, and thereafter the following rule is to be used; next after any a , as $a[w]$, which is inferior to the a next preceding it, which we may call $a[v]$, is to be taken the a next superior to $a[v]$; but next after any a , as $a[u]$, which is superior to the a next preceding it, which we may call $u[w]$, is to be taken the a next inferior to $u[w]$. The demonstration that this reduces the sequence to such a series is so easy that it may be omitted. We then take all the a 's together in the following order: first, we take the whole of the first sequence if it is enumerable; next, we take the first a of the first inenumerable series; next, after any a of any inenumerable series we take the first a not already taken of the next inenumerable series, unless the last taken were the first of its series, when next after it we take the first not already taken of the first inenumerable series. The demonstration that this has the desired effect is sufficiently easy to be left to the student.

Peirce: CP 4.110 Cross-Ref:††

Such an arithmoidal series is just like the series of positive whole numbers. I call it with reference to its grade of multitude, *dinumerable*. That is, it corresponds one to one to the numbers, yet the count of it cannot be completed. To such a series

applies the kind of reasoning called by me the *Fermatian inference*. This consists in proving a proposition to be true of such a series, because otherwise it must be false of an enumerable collection, such falsity, by reasoning on the principle of the part being less than the whole, being shown to be impossible. Fermat himself called it indefinite descent. He states his "manière de démonstrer," which he calls "une route tout à fait singulière" as consisting in showing that if the proposition to be proved were false of any number, it would be false of some smaller number. This statement shows a good comprehension of its nature.†1

Peirce: CP 4.111 Cross-Ref:††

111. If we take all the whole numbers and write opposite to each the same figures in inverse order with a decimal point before them -- as, opposite 1894, for example, .4981 -- and then arrange the numbers in the order of these decimal fractions, we shall have established a quantitative relation according to which no number is next superior or next inferior to any other. A story is told of a bar of tin which being sent into Russia in the depths of winter, arrived in good order only that every atom had broken away from every other. If this tale did not serve to put money in somebody's pocket, it at least affords a pretty simile of the condition of the numbers when looked upon from that below-zero point of view. To bind them together after they are in their new order would require a multitude of new units inexpressibly more numerous relative to these numbers than the totality of them is to one.

Peirce: CP 4.112 Cross-Ref:††

112. If from the entire series of integer numbers, ranged in regular order, we imagine none, certain ones, or all to be omitted, we have what we may call a broken series, and the multitude of the entire collection of all such broken series possible is so great that they cannot be arranged by means of any quantitative relation so that whatever one is superior to another is next superior to some one. The proof which I offer of this is at bottom not mine. It seems to me sound, and if so is wonderful. In order to show that those broken series cannot themselves be arranged in an arithmoïdal order, let us first arrange them in any quantitative order. This is easy, for each number may represent a place of decimals in the binary system of numeration, 1 the $1/2$ place, 2 the $1/2^3$ place, 3 the $1/2^3$ place, and n the $1/2^n$ place. The absence of a number being represented by zero, and its presence by 1, each series is represented by a binarial fraction, and these may be arranged in the order of their values. (Of course, the series could not be represented by integer numbers by reflection from the binarial point, because so they would all be infinite.) Now the question is whether the series of whole numbers can in any way whatever be made to correspond to these fractions. And in order that the matter may appear in a clearer light let us suppose that parallel to the series of infinite binarial fractions is ranged the entire series of values of rational fractions between 0 and 1, expressed in the same notation and set down in the same order. Let us first take a mode of correspondence which obviously will not fulfill the purpose, but which will serve to show the difference between such a series as that of the rational fractions and the series with which we are dealing. Suppose that the numbers correspond to the fractions in the order of their simplicity. Thus our first two fractions (I won't take the trouble to write

them in binary notation) are:

$1/3$.333333333 $1/2$ 0.500000000;

between these the first two are

2/5 .4000000000 3/7 0.4285742857;

between these the first two are

7/17 .4117647059 5/12 0.4166666667;

between these the first two are

12/29 .4137931034 17/41 0.4146341463;

between these the first two are

41/99 .4141414141 29/70 0.4142857143;

between these the first two are

70/169 .4142011834 99/239 0.4142259414.

All these are found in both parallel rows; but they are converging toward

0.41421356

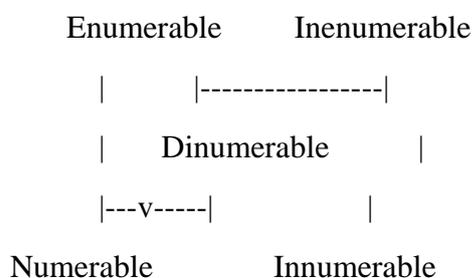
or $\sqrt{2}-1$ which is not a rational fraction.

Peirce: CP 4.112 Cross-Ref:††

Now the fact that in this case the numbers happened to be rational fractions had nothing to do with the result. It is plain that in every case, when between two values we insert two, and between those two, two more again, and so on indefinitely, there remains a limit which is never reached; and the multitude which includes all such limits cannot be made to correspond, one to one, to any dinumerable collection.

Peirce: CP 4.113 Cross-Ref:††
§5. THE METHOD OF LIMITS

113. Let us settle the terminology as shown in this diagram



Peirce: CP 4.113 Cross-Ref:††

The relation of the Innumerable to the Dinumerable is analogous to that of the Dinumerable to the Enumerable. Dinumerable is the multitude of enumerable

numbers; innumerable is the multitude of dinumerable series. The dinumerable follows after the enumerable; but so closely after that as soon as you have passed all that is enumerable you have passed the dinumerable; so that we rightly reason such-and-such must be the character of the dinumerable, for if not there must be an enumerable which wants this character. In like manner the innumerable lies beyond the dinumerable; it is its limit; but it lies so closely beyond that we rightly reason, such-and-such must be the character of the innumerable; for if not, there must be a gap between this character and that of the dinumerable.

Peirce: CP 4.113 Cross-Ref:††

All reasoning about the Innumerable derives its force from the conception of a *limit*. We therefore have to study this conception. But two or three prolegomena are called for.

Peirce: CP 4.114 Cross-Ref:††

114. The idea that there can be any vigorous and productive thought upon any great subject without reasoning like that of the differential calculus is a futile and pernicious idea. Some newspapers maintain that all doctrines involving such reasoning ought to be struck out of political economy because that science is of no service unless everybody, or the great majority of voters, individually comprehend it and assent to its reasonings. I do not observe that it is a fact that voters are such asses as to insist upon thinking they personally comprehend the effects of tariff-laws, etc. But whether they be so or not, it is certain that the ratio of the circumference to the diameter is 3.1415926535897932384626433832795028841971694 . . . whether the reasoning that proves it is hard or easy. That I feel sure of, although I personally have not verified the above figures; and if I had, I should not feel perceptibly more sure of the matter than I am. Certainly, if on attempting to verify them I got a slightly different result, I should feel pretty sure it was I who had committed an error. But whether people be wise or foolish, it remains that there is no possible way of establishing the true doctrines of political economy except by reasonings about *limits*, that is, reasoning essentially the same as that of the differential calculus. (I do not know why I should hesitate to say that the journal which I have particularly in mind is the New York *Evening Post*, incontestably one of the very best newspapers in the world, and especially remarkable for the sagacity of its judgments upon all questions of public policy.)

Peirce: CP 4.115 Cross-Ref:††

115. The reasoning of Ricardo about rent is this.†1 When competition is unrestrained by combination, producers will carry production to the limit at which it ceases to be profitable. Thus, a man will put fertilizers on his land, until the point is reached where, were he to add the least bit more, his little increased production would no more than just pay the increased expense. Every piece of land will be treated in this way, and every grade of land will be used down to the limit of the land upon which the product can just barely pay.

Peirce: CP 4.115 Cross-Ref:††

The whole reasoning of political economy proceeds in this fashion. If we put an import duty upon any article, the price to the consumer cannot be raised by the full amount of the tax. For the price before the imposition was such as to sell a certain amount. Now, if the price is raised, less can be sold. If less can be sold, less will be produced. But production will only be diminished by the producer getting a less price; and it is this less price *plus* the duty that the consumer pays. Of course, we must understand by the *duty*, not merely what goes to the government, but what has to be

paid in consequence to brokers, bankers, and increased expenses of all kinds caused by the change of the law. Looking at the matter from this point of view (and abstracting from other considerations) the best articles on which to levy duties are those upon the production of which our demand is so influential that a small decrease in the demand will cause a relatively large fall in the price.†P1

Peirce: CP 4.116 Cross-Ref:††

116. As another preliminary to the analysis of the conception of Limit, I now pass to a widely different topic. The student has not failed to remark how much I have insisted upon balance and symmetry in logic. It is a great point in the art of reasoning; although I do not know that one could say that logic requires it. As long ago as 1867 I spoke of a trivium of formal sciences of symbols in general. "The first," I said, "would treat of the formal conditions of symbols having meaning, and this might be called formal grammar; the second, logic, would treat of the formal conditions of the truth of symbols; and the third would treat of the formal conditions of the force of symbols, or their power of appealing to a mind, and this might be called formal rhetoric."†P1 It would be a mistake, in my opinion, to hold the last to be a matter of psychology. That which needs no further premisses for its support than the universal data of experience that we cannot suppose a man not to know and yet to be making inquiries, that I do not think can advantageously be thrown in with observational science. Each of these kinds of science is strong where the other is weak; and hence it is well to discriminate between them. Now, the *Grundsatz* of Formal Rhetoric is that an idea should be presented in a unitary, comprehensive, systematic shape. Hence it is that many a diagram which is intricate and incomprehensible by reason of the multitude of its lines is instantly rendered clear and simple by the addition of more lines, these additional lines being such as to show that those that were there first were merely parts of a unitary system. The mathematician knows this well. We have seen what endless difficulties there are with "some's" and "all's". The mathematician almost altogether frees himself from "some's"; for wherever something outlying and exceptional occurs, he enlarges his system so as to make it regular. I repeat that this is the prime principle of the rhetoric of self-communing. Nobody who neglects it can attain any great success in thinking.

Peirce: CP 4.117 Cross-Ref:††

117. The innumerable appears in two different shapes. In the first place, if we append to the entire series of finite integral numbers, which is a denumerable collection, all the infinite numbers, we obtain an innumerable collection. Or, if we take the series of rational fractions, also a denumerable collection, and add to them the limits of all infinite series of such fractions, then again we obtain an innumerable collection. In this latter case, each instance taken from the innumerable collection is a limit which may be passed through. This latter is a more balanced conception than the other; but the mathematician reduces the other to it by conceiving, that in the former case also, after passing through infinity as a mere point, we pass into a new region -- a new world. We pass off, for example, in a straight line parallel to the earth's axis northwardly: after passing through infinity we pass into an imaginary region from which after an infinite passage we re-emerge into our space at the extreme south. Or, it may be that this imaginary world reduces to nothing and that the points at infinity north and south coincide. This is the way the mathematician supplements facts in the interest of formal rhetoric. Of course, in doing so he has to take care not to misrepresent the real world; but his ideal addition to it may have any properties that simplicity dictates. This is an immense engine of thought in mathematics. It affords a little difficulty to the mind at first presentation; but that passes away very soon, and

then it is found to be greatly in the interest of comprehensibility. Every mathematician will tell you this; if you are not already aware of it. But even among mathematicians there is a trace of that human weakness, the stupidity of adhering to what ought to be obsolete; and consequently the idea that infinity is something to pass through has not been everywhere carried out.

Peirce: CP 4.118 Cross-Ref:††

118. In many mathematical treatises the limit is defined as a point that can "never" be reached. This is a violation not merely of formal rhetoric but of formal grammar. True, in the world of real experience, "never" has at least an approximate meaning. But in the Platonic world of pure forms with which mathematics is always dealing, "never" can only mean "not consistently with --." To say that a point can never be reached is to say that it cannot be reached consistently with --, and has no meaning until the blank is filled up. And thereupon, the mathematical and balanced conception must be that the point is instantly passed through. The metaphysicians have in this instance been clearer than the mathematicians -- and that upon a point of mathematics; for they have always declared that a limit was inconceivable without a region beyond it.

Peirce: CP 4.118 Cross-Ref:††

I understand that Jordan has rewritten the first volume of his *Cours d'analyse*. I have not seen this new edition (for all my life my studies have been cruelly hampered by my inability to procure necessary books), but I can guess to some extent what the character of it will be; and it no doubt contains much, most pertinent to the subject now under our attention. It was, I presume, this work which suggested to Klein a remark which he makes in his Evanston Colloquium, to the effect that there is a distinction between the *naïve* and the *refined* geometrical intuition. "In imagining a line," he says, "we do not picture to ourselves length without breadth, but a *strip* of a certain width. Now such a strip has, of course, always a tangent; *i.e.*, we can always imagine a straight strip having a small portion (element) in common with the curved strip."†1 The psychological remark seems to me incorrect. I, for my individual part, imagine a curve (even of an odd degree, which I convert into an even degree by doubling it, or by crossing it by a line) as the boundary between two regions pink and bluish grey; and I do not think I imagine the line as a strip. But it is of little consequence what individual ways of imagining may be. Klein's *naïve* view has a real importance far greater than his adjective imports, at which I have hinted in the Century Dictionary, under *Limits, Doctrine of*, where I say that this doctrine "should be understood to rest upon the general principle that every proposition must be interpreted as referring to a possible experience."†P1 What I mean is that absolute exactitude cannot be revealed by experience, and therefore every boundary of a figure which is to represent a possible experience ought to be blurred. If this is the case, it is both needless and useless to talk of infinitesimals. Still thought of this inexact kind (I mean upon these essentially inexact premisses) will be found *much more intricate and difficult* than the exact doctrine.

Peirce: CP 4.119 Cross-Ref:††

119. To define a limit, mathematicians usually write

$$x[n],$$

where $x[1]$, $x[2]$, $x[3]$, etc. are supposed to successively approximate toward a value. Then they say that if after, perhaps, some scattering values, the successive $x[n]$'s at length come nearer and nearer to a constant which they indefinitely approach but

never reach, that quantity is the limit. By saying they never reach it, they mean that as the $[n]$ of $x[n]$ passes through infinity, $x[n]$ passes through the limit. This $n = \infty$ of course marks the point at which the collection which n measures becomes denumerable. At that point $x[n]$ ceases to vary with n ; else the value would be indeterminate.

Peirce: CP 4.120 Cross-Ref:††

120. I insert here a few remarks. The denumerable is to the innumerable as logarithmic infinity is to ordinary infinity. The analogy may be traced in two ways; first the number of numbers expressible by n decimal points is, of course, bn where b is the base of the system of numeration; but the innumerable is the number of numbers expressible by denumerable decimal points. In the second place, the innumerable is not only denumerably more than the denumerable but is innumerably more.

Peirce: CP 4.121 Cross-Ref:††
 §6. THE CONTINUUM

121. It may be asked whether there be not a higher degree of multitude than that of the points upon a line. At first sight, the points on a surface seem to be more; but they are not so. For points on surfaces can be discriminated by two coördinates with values running to a denumerable multitude of decimal places. Now two such numbers or any enumerable multitude of them can be expressed by one series of numbers. Thus to express two, write a number such that the succession of figures in the odd places of decimals gives one coördinate, and those in the even places, the other. Thus,

$u = 32.174118529821685238548599709435 \dots$ will mean

$x = 3.141592653589793 \dots$

$y = 2.718281828459045 \dots$

Peirce: CP 4.121 Cross-Ref:††

This method would break down if the number of dimensions were denumerable; but even then another method could be found. But if the number of dimensions were innumerable, it is difficult to say without more study than I have given, how to proceed. The idea of space with innumerable dimensions does not, at first blush, at least, strike one as presenting great difficulty.

Peirce: CP 4.121 Cross-Ref:††

But if bn when n is denumerable gives a new grade of multitude, we might expect that when n was innumerable, a still higher grade would be given.

Peirce: CP 4.121 Cross-Ref:††

Yet, on the other hand, looking at the matter from the point of view of the original definitions given above, the three classes of multitude seem to form a closed system. Still, nothing in those definitions prevents there being many grades of multiplicity in the third class. I leave the question open, while inclining to the belief that there are such grades.†1 Cantor's theory of manifolds appears to me to present certain difficulties; but I think they may be removed.

Peirce: CP 4.121 Cross-Ref:††

Let us now consider what is meant by saying that a line, for example, is continuous. The multitude of points, or limiting values of approximations upon it, is of course innumerable. But that does not make it continuous. Kant †2 defined its continuity as consisting in this, that between any two points upon it there are points. This is true, but manifestly insufficient, since it holds of the series of rational fractions, the multitude of which is only dinumerable. Indeed, Kant's definition applies if from such a series any two, together with all that are intermediate, be cut away; although in that case a finite gap is made. I have termed the property of infinite intermediety, or divisibility, the *Kanticity* of a series. It is *one* of the defining characters of a continuum. We had better define it in terms of the algebra of relatives. Be it remembered that continuity is not an affair of multiplicity simply (though nothing but an innumerable multitude can be continuous) but is an affair of arrangement also. We are therefore to say not merely that there can be a quantitative relation but that there is such, with reference to which the collection is continuous. Let $l \sim l$ denote this relation. Then, as quantitative, this has, as we have seen, †1 these properties:

$$l \sim l \prec l \sim l,$$

and

$$0 \dagger (1 \Psi l \sim l \Psi \sim l) \dagger 0.$$

Then the property of Kanticity consists in this:

$$l \sim l \prec l \sim l l \sim l.$$

Peirce: CP 4.122 Cross-Ref:††

122. To complete the definition of a continuum, the *a*'s, we require the following property. Namely, if there be a class of *b*'s included among the *a*'s but all inferior to a certain *a*, that is, if

$$b \prec a,$$

$$1 \prec \$a(l \sim l \dagger \sim b);$$

and if further there be for each *b* another next superior to it; that is,

$$1 \prec \sim \$b\{l \sim l \cdot [\sim l \dagger l \dagger (\sim b \Psi \sim l \dagger l)]\} \dagger b,$$

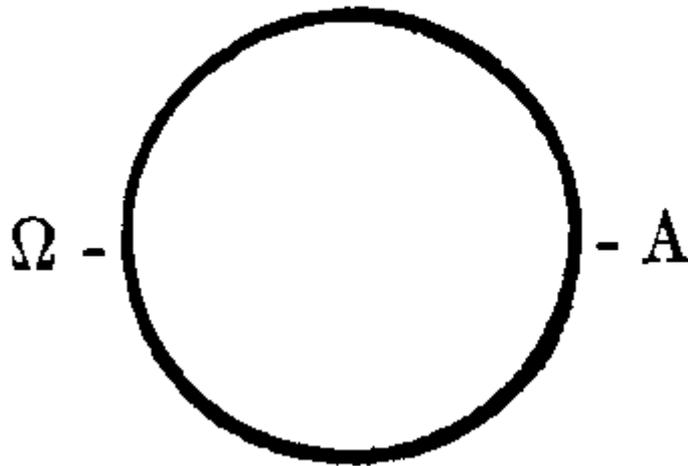
then there is an *a* next superior to all the *b*'s. That is,

$$1 \prec \$a\{(l \sim l \dagger \sim b) \cdot [\sim l \dagger l \dagger (\sim l \dagger l)b]\}.$$

Peirce: CP 4.122 Cross-Ref:††

I call this the *Aristotelicity* of the series, because Aristotle seems to have had it obscurely in mind in his definition of a continuum as that whose parts have a common limit. †2

Peirce: CP 4.123 Cross-Ref:††



123.

[Click here to

view] If we consider a line (which, for rhetoric's sake, we will consider as returning into itself, though if it did not, it would give no difficulty further than an intolerably tedious complexity) it consists in a connection of points, such that by virtue of it, if any two points, A and Ω , be taken on that line, the points are divided into two parts, say the $a[0]$ and the $a[\infty]$, such that a certain continuous quantitative relation, say $l[0]$, subsists between all the $a[0]$'s having A for minimum and Ω for maximum, and another continuous quantitative relation, say $l[\infty]$, subsists between all the $a[\infty]$'s having the same maximum and minimum. The student is invited to state this in algebraic form using π 's, Σ 's, and indices. He begins, for example

$$\Sigma[i]\pi[j]\pi[k] \sim a[j] \vee \sim a[k] \vee q[ij] \cdot \sim q[ik] \vee \sim q[ij] \cdot q[ik] \vee l[jk] \vee l[0kj] \vee [\infty jk] \vee l[\infty kj].$$

To this I wish to add something, which seems to require a preliminary remark. There are certain quantitative relations between the points such that if one of them were to govern an arrangement of the points in space, it would derange their connection in a line, in this sense, that it would cause some four points which are connected in the cyclical order **PQRS** (=PSRQ) to be brought in to one of the two orders **PQSR** (=PRSQ) or **PSQR** (=PRQS). We will call such a relation, for short, incompatible. Of course, there is nothing to prevent its existing; only the points cannot be arranged according to this order and remain in their order in the line. I now say that by no compatible continuous quantitative order can we pass from any $a[0]$ to any $a[\infty]$, without passing through A or Ω . The student will do well to express this in terms of the algebra. Of course this statement requires modification in case the line forks. But for the purposes of logic it does not seem necessary to examine such details.

Peirce: CP 4.124 Cross-Ref:††

124. Pass we now to the study of the Surface. It is here that the mathematical conception of a "spread" -- as Clifford calls it in insular but expressive language -- at length displays itself. For an example of a surface, think of something irregularly round -- multiple-connected surfaces complicate the subject, without advantage. They are easily taken into account later and the modifications they require made. A surface

contains an innumerable multitude of lines. Let one of these -- a complete oval -- be marked upon it. Then the connection of points is such that this line separates those that do not lie upon it into two classes, such that it is impossible to pass from any point of one class to any point of the other by a *compatible* continuous quantitative order without passing through some point of the line. This is, however, perhaps not quite clear. Let us endeavor to make a better statement. Upon the oval take two points A and ∞ . Then by virtue of the connection of points on the surface there is an innumerable series of continuous quantitative orders, each beginning at A and ending at ∞ . Two of these, signified by the relative terms $I[0]$ and $I[\infty]$, follow the two parts of the oval. These orders are such that no two of them embrace the same point, except the initial and final points which are common to them all. And all other lines (or compatible continuous quantitative orders), twice cutting the oval, cut these different lines in the same order, say $I[0] \dots I[n] \dots I[\infty] \dots I[n] \dots I[0] \dots$. Every point on the surface lies on one of these lines.

Peirce: CP 4.125 Cross-Ref: ††
 §7. THE IMMEDIATE NEIGHBORHOOD

125. I wish to remark that it is a serious fault in the ordinary treatment of the fundamentals of geometry that attention is not paid to the distinction between the two sides of points on a line, lines on a surface, and surfaces in space. This is why certain theorems indubitably true are so difficult of formal proof. It is that a part of the fundamental properties of space have no expression among the Postulates of Geometry.

Peirce: CP 4.125 Cross-Ref: ††

I think that I have thus described the nature of the connection of points upon the surface -- and nothing need be added.

Peirce: CP 4.125 Cross-Ref: ††

But there are three very important ideas I have left undefined. I mean those of the *simplest line* (straight line on a plane, great circle on a sphere, perhaps the geodetic line on other surfaces), the *immediate neighborhood*, and *measurement*. I have also imaginary quantity still to consider.

Peirce: CP 4.125 Cross-Ref: ††

The easiest of these ideas seems to be that of the *immediate neighborhood*. It supposes that we recognize that every region stands in relation to a certain scale of quantity. We do not yet assign its quantity but we are able to say whether it is connected with an enumerable, a dinumerable, or an innumerable multitude. Two regions which are connected with quantities of the same class are said to be *about alike*. Suppose, then, we have a region about like the whole surface, or about like some region which we take as a standard. Suppose a thunder-bolt rends this into two parts about alike, a crack separating them. Suppose a second thunderbolt similarly rends both parts; and each successive thunderbolt rends all the parts the last left into two new parts about alike. Suppose these thunderbolts to follow at the completion of each rational fraction of a minute. Then, at the end of the minute, the region will be rent into innumerable parts about alike. These parts are neighborhoods or infinitesimals.

Peirce: CP 4.126 Cross-Ref:††

126. It will at once be objected that there is no reason to suppose that this operation would leave any parts at all, or if it did there is no reason to suppose they would be surfaces rather than angels, or oranges, or precessions of the equinoxes; for the only reason for thinking they remained of the same genus is that no one thunderbolt would change them. Reasoning from that premiss, however, would be a Fermatian inference, and would, as such, only hold good for the dinumerable.

Peirce: CP 4.126 Cross-Ref:††

But the reply is that there is no need of calling in the Fermatian inference. The minute of thunderbolts does not differ from any other minute, as far as the character of the surface goes. The parts have been moved a little, but all their mutual relations are undisturbed. Even if the operation broke it up into single points, which is an unfounded proposition, still all the cracks that have been made in no wise alter the relations of the points to one another. The space the region occupies, though interfiltrated through with another space, remains the same, and the relations of its parts the same. If this conception is too difficult, imagine that the thunderbolts do not rend the regions, but only cause a mind to imagine them rent.

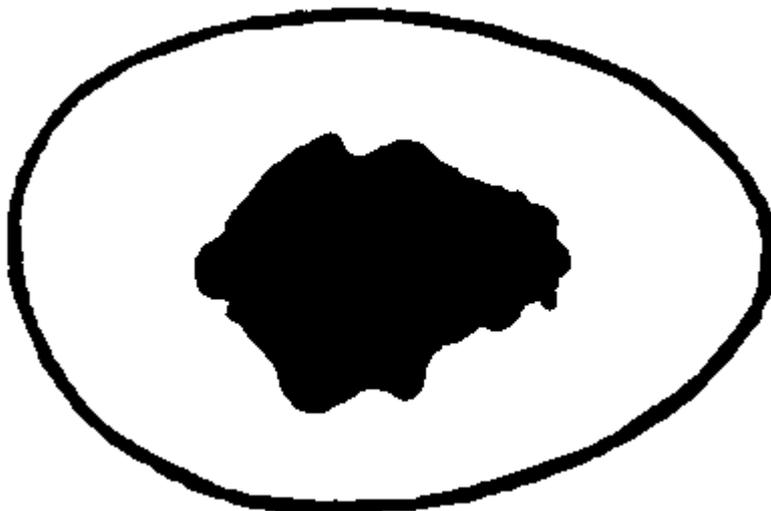
Peirce: CP 4.126 Cross-Ref:††

It would, however, be quite out of order to consider the question of whether these parts are single points or what their composition may be, until it first be fully admitted that the logical division of the region into innumerable parts is logically possible. But there is no room for dispute here. It has been irrefragably demonstrated that the points of a line, and *a fortiori* of a surface, are innumerable. Now, as no two coincide, there is nothing in logic to prevent their being drawn asunder. My definition of a continuum only prescribes that, after every innumerable series of points, there shall be a *next* following point, and does not forbid this to follow at the interval of a mile. That, therefore, certainly permits cracks everywhere; for there is no ordinal place in the series where such a limit point is not inserted. But if anybody thinks my definition is in error here, still it will not be maintained that that definition *involves any contradiction*. Hence, there is no contradiction in the separation into parts, even if I am wrong in saying that it involves no breach of continuity. There is no contradiction involved in breaking the region anywhere. But perhaps it may be said, the contradiction lies, not in breaking it anywhere, nor in breaking it into as many parts as it has points, but that the idea of an innumerable multitude involves a contradiction. That it does not can be formally demonstrated by second-intentional logic; but that part of this book having been excised, it is necessary to find other arguments. There is no difficulty about the existence of π , and therefore none in the existence of incommensurable limits. There is no more difficulty about the existence of any one number not accurately expressible in a finite number of decimals than in any other. Therefore, there is no logical contradiction in supposing all numbers to which decimals can indefinitely approximate to exist, *i.e.*, as all the objects of mathematics exist, as abstract hypotheses. Besides, that the innumerable multitudes are logically possible is shown by the fact that many propositions (namely all that are true of the dinumerable but not of the numerable) cannot be demonstrated in a way which will stand logical examination unless it be expressly introduced as a premiss that a given multitude is numerable. Now a logically necessary proposition is of no avail as a premiss. On the whole then, there is nothing in logic to prevent a region from being divided into innumerable parts about alike.

Peirce: CP 4.126 Cross-Ref:††

Now I say that each of those parts contains innumerable points. For if that were not the case each of these parts could be so arranged that every point had another next after it; and, since a continuum has no molecular constitution, the divisions could everywhere be made between points having other points next them; and so, after rearranging the parts (no matter how the continuity might be broken up) all the points would have points next after them. But this is contrary to the fact that the points are innumerable. Besides, going back to the unanalyzed idea of continuity, it is evident that in a continuum the points are so connected that every part, irrespective of its magnitude, contains innumerable points. It may be objected that the single points are parts. But that is not properly true. The single points are parts of the collection; but they cannot be broken off by a division of parts unless they are on the outer boundary of a region, or unless they are not continuous with the rest. They can be extracted from the middle; but doing this breaks the continuity. Thus the incommensurable numbers taken by themselves do not form a continuum.

Peirce: CP 4.127 Cross-Ref:††



127.

[Click

here to view] A drop of ink has fallen upon the paper and I have walled it round. Now every point of the area within the walls is either black or white; and no point is both black and white. That is plain. The black is, however, all in one spot or blot; it is within bounds. There is a line of demarcation between the black and the white. Now I ask about the points of this line, are they black or white? Why one more than the other? Are they (**A**) both black and white or (**B**) neither black nor white? Why **A** more than **B**, or **B** more than **A**? It is certainly true,

Peirce: CP 4.127 Cross-Ref:††

First, that every point of the area is either black or white,

Peirce: CP 4.127 Cross-Ref:††

Second, that no point is both black and white,

Peirce: CP 4.127 Cross-Ref:††

Third, that the points of the boundary are no more white than black, and no more black than white.

Peirce: CP 4.127 Cross-Ref:††

The logical conclusion from these three propositions is that the points of the boundary do not exist. That is, they do not exist in such a sense as to have entirely determinate characters attributed to them for such reasons as have operated to produce the above premisses. This leaves us to reflect that it is only as they are connected together into a continuous surface that the points are colored; taken singly, they have no color, and are neither black nor white, none of them. Let us then try putting "neighboring part" for point. Every part of the surface is either black or white. No part is both black and white. The parts on the boundary are no more white than black, and no more black than white. The conclusion is that the parts near the boundary are half black and half white. This, however (owing to the curvature of the boundary), is not exactly true unless we mean the parts in the *immediate neighborhood* of the boundary. These are the parts we have described. They are the parts which must be considered if we attempt to state the properties *at precise points* of a surface, these points being considered, as they must be, in their connection of continuity.

Peirce: CP 4.127 Cross-Ref:††

One begins to see that the phrase "immediate neighborhood," which at first blush strikes one as almost a contradiction in terms, is, after all, a very happy one.

Peirce: CP 4.127 Cross-Ref:††

What is the velocity of a particle *at* any instant? I answer it is the ratio of space traversed to time of traversing, in the *moment*, or time in the immediate neighborhood, of that *instant*, or point of time. Some logicians object to this. They say that the velocity means nothing but the limiting value of the ratio of the space to the time when the time is indefinitely diminished. But they say they use the expression "immediate neighborhood" to mean nothing more than that, as a convenience of language. Sometimes we meet with an assertion difficult to refute because it involves several difficult logical blunders. The position just stated is an example of this. People who talk in this way do not see that what they say is a justification of the idea of a part such as the whole contains an innumerable multitude of. I do not yet say "immeasurably small," because we have not yet studied the conception of measurement. These people do not seem to have analyzed the conception of a "meaning," †1 which is, in its primary acceptation, the translation of a sign into another system of signs, and which, in the acceptation here applicable, is a second assertion from which all that follows from the first assertion equally follows, and *vice versa*. It is true that, when we find with reference to a continuous motion that something would be true at the limit of a dinumerable series, it follows this is true for the part about the point considered. . . . This is as much as to say that the one assertion "means" the other. But do these people mean to say that when I think of a particle as having a velocity, I can only think, or that it is convenient to think, simply that at different times it is stationed at different points? Do they mean to say I have no direct, clear icon of a movement? If so, they are shutting their eyes to the plain truth. Remember it is by icons only that we really reason, and abstract statements are valueless in reasoning except so far as they aid us to construct diagrams. The sectaries of the opinion I am combating seem, on the contrary, to suppose that reasoning is performed with abstract "judgments," and that an icon is of use only as enabling me to frame abstract statements as premisses.

Peirce: CP 4.127 Cross-Ref:††

The idea of "immediate neighborhood" is an exceedingly easy one, into which everybody is continually slipping, though he fancies it unjustifiable. Klein says of his

"refined intuition" that, strictly speaking, it is not an intuition. But, speaking as strictly as that, there is no such psychological phenomenon as an intuition.†1 The strip, which he says makes the curve in the *naïve* intuition, makes two parallel curves with a region between. But the simple idea is that of a blurred outline, to which we all, wise and simple, append the mental note that its breadth is such that an innumerable number would be contained in any surface.

Peirce: CP 4.127 Cross-Ref:††

Those who, finding themselves betrayed into the use of the expression "immediate neighborhood" or something equivalent, seek to justify it by the *exigencies of speech*, are mistaken. It is not English grammar which forces these words upon them, but it is the very grammar of thought -- formal grammar -- which forces the idea upon them. The idea of supposing that they can think about motion without an image of something moving!

Peirce: CP 4.127 Cross-Ref:††

We must return to this subject after having considered the nature of measurement.

Peirce: CP 4.128 Cross-Ref:††
§8. LINEAR SURFACES

128. Euclid †2 defines a straight line as a line which lies evenly between its points. This is the real Greek acuteness; it is as much as to say that if a straight line be moved, its new position intersects its old one in one point at most. This is substantially the idea of all modern geometry. Legendre,†3 it is true, defined the straight line as the shortest distance between two points, as it most indubitably is. Nor do I think that it would be fair to object that this definition is *metrical*, that is, supposes a definition of measurement. For all kinds of measurement known make the straight line the shortest (or the longest, sometimes, if there be a longest) distance, if there be a shortest distance. But a more serious objection to Legendre's definition is that, if that be adopted, its property of two straight lines not intersecting in two places follows as a consequence; while, if Euclid's definition be adopted, there must be a separate postulate to the effect that there is a shortest distance. Thus, Euclid's definition involves a more thorough analysis of the properties of space. Legendre conceived the other way, which wraps up as much as possible in one formula, to be the best. It certainly is not so for the purposes of logic.

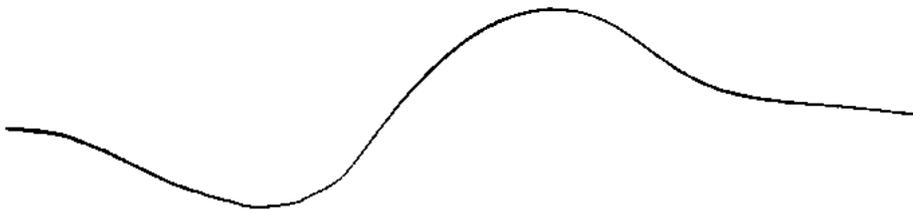
Peirce: CP 4.128 Cross-Ref:††

When instead of a plane we consider a roundish surface, it is difficult to say what sort of an oval best corresponds to a straight line. Most writers have assumed that it is the geodetic line which is the shortest (or longest) distance between its points. But they seem to have forgotten that a geodetic line on almost any surface but a perfect sphere generally intersects itself a dinumerable multitude of times. The discussion of this question would involve very difficult mathematics, quite out of place in this work.

Peirce: CP 4.128 Cross-Ref:††

We must, therefore, confine ourselves to the plane. Now it is evident that the definition we have adopted supposes straight lines to move about in the plane without

ceasing to be straight. Hitherto, all the properties of the connection of points are such as might hold if the plane were a fluid; for though discontinuous fluid motion is conceivable, it has no place in the usual conceptions of the student of hydrokinetics. But now we propose that the straight line should move about as if it were a rigid bar. However, it is not necessary to broach the theory of elasticity, a doctrine of Satanic perplexity. We may call a straight line the path of a ray of light, or the shadow of a dark point cast from a luminous point. That is rather a pretty idea. Or going down to the roots of physics, we may define the straight line as the path of a particle, not deflected by any force. This is, so far as we can see, the origin of the importance of the straight line in the physical world. But, then, at present it is doubtful whether we are concerned at all with the physical world. We would like, if we could, to find some logical property of the straight line distinguishing it from other curves. I fear, however, there is none, if we are to leave its shortness out of account. We can perfectly well conceive of a cubic curve, such as is shown in the figure,



[Click here to view] moving about with modifications of its shape, so as in any position to cut any other position once and once only (in real space). A mathematician will easily write down the conditions for this. Namely, the equation of the serpentine is

$$y = 1/(x+(1/x)),$$

and that of the different cubics is

$$x/a + \{y-(1/(x+(1/x)))\}/b=1.$$

There is nothing in the plane geometry of the straight line which is not equally true, *mutatis mutandis*, of such a system of cubics.

Peirce: CP 4.128 Cross-Ref:††

But the intersectional properties of straight lines in a plane are not exhausted in saying that any two straight lines intersect once and once only.

Peirce: CP 4.129 Cross-Ref:††

129. Let us resort to our algebra of relatives. Denote unlimited straight lines by lower case italic letters. Let capitals denote points. Let Greek minuscules denote certain marks of lines. All these letters are treated as indices; but they will be written on the line.

Peirce: CP 4.129 Cross-Ref:††

Let aB (or any similar pair of letters) mean that the line a is considered as having the point B , which lies on it. If the point B is not on the line a , then $\sim(a B)$; but even if B be on a , it does not necessarily follow that B is regarded as belonging to the line a , and if not, again $\sim(a B)$. A point may belong to two lines, at once.

Peirce: CP 4.129 Cross-Ref:††

Let ab (or any similar pair of letters) signify that the line b has the mark a , the

nature of which will appear in the sequel.

Peirce: CP 4.129 Cross-Ref:††

Let aB , etc., signify that the point B belongs to some line that has the mark a .

Peirce: CP 4.129 Cross-Ref:††

Let us now endeavor to sum up in a series of propositions the fundamental truths about the intersections of lines.

Peirce: CP 4.129 Cross-Ref:††

First proposition.

$$\pi A \pi B \Sigma c \cdot cA \cdot cB$$

that is, any two points *may be* regarded as belonging to one straight line.

Peirce: CP 4.129 Cross-Ref:††

Second proposition.

$$\pi \alpha \pi \beta \pi c \cdot \alpha c \cdot \beta c,$$

that is, given any two marks, an unlimited straight line having them both may be drawn.

Peirce: CP 4.129 Cross-Ref:††

Third proposition.

$$\pi a \pi b \pi C \pi D \sim (a C) \cdot \sim (a D) \cdot \sim (b C) \cdot \sim (b D) \cdot \sim 1ab \cdot \sim 1CD,$$

that is, if two points are regarded as belonging to two lines, either the two points or the two lines coincide.

Peirce: CP 4.129 Cross-Ref:††

Fourth proposition.

$$\pi \alpha \pi \beta \pi c \pi d \sim (\alpha c) \cdot \sim (\alpha d) \cdot \sim (\beta c) \cdot \sim (\beta d) \cdot \sim 1\alpha\beta \cdot \sim 1\chi\delta,$$

τηατ ισ, ιφ τωο μαρκς βελονγ το τωο λινεσ, ειτηερ τηε τωο μαρκς αρε χοεξτενσιπε ορ τηε τωο λινεσ χοινχιδε.

Peirce: CP 4.129 Cross-Ref:††

Fifth proposition.

$$\pi A \pi b \Sigma \{g\} \cdot \{g\}b \cdot \{g\}A,$$

that is, given any line, any point may be regarded as belonging to a line having a mark in common with the given line.

Peirce: CP 4.129 Cross-Ref:††

Sixth proposition.

$$\pi \alpha \pi b \Sigma C \cdot \alpha b \cdot \alpha C,$$

that is, given any mark and any line, it is always possible to find a point which may be regarded as belonging to the given line and to some line having the given mark.

Peirce: CP 4.129 Cross-Ref:††

Seventh proposition.

$$\pi\alpha\pi\beta\pi c\pi D \sim(\alpha c) \cdot \Psi \sim(\alpha D) \cdot \Psi \sim(\beta c) \cdot \Psi \sim(\beta D) \cdot \Psi c D \cdot \Psi 1\alpha\beta,$$

that is, if two marks belong to a given line and to lines to which a given point is regarded as belonging, that point must be regarded as belonging to that line, unless the two marks are coextensive.

Peirce: CP 4.129 Cross-Ref: ††

Eighth proposition.

$$\pi\alpha\pi b\pi C\pi D \sim(\alpha C) \cdot \Psi \sim(\alpha D) \cdot \Psi \sim(b C) \cdot \Psi \sim(b D) \Psi \alpha b \cdot \Psi 1CD,$$

that is, if two points are regarded as belonging to a given line and to lines having a given mark, that line has that mark unless the two points coincide.

Peirce: CP 4.129 Cross-Ref: ††

Ninth proposition.

$$\pi\alpha\pi b\pi c\pi d\pi E \sim(\alpha b) \cdot \Psi \sim(\alpha c) \cdot \Psi \sim(\alpha d) \cdot \Psi \sim(b E) \cdot \Psi \sim(c E) \cdot \Psi \sim(d E) \cdot \Psi 1bc \cdot \Psi 1bd \cdot \Psi 1cd,$$

that is, any three lines either have no common point or no common mark.

Peirce: CP 4.129 Cross-Ref: ††

Tenth proposition.

$$\pi b\pi c\pi d\Sigma\alpha\Sigma E \alpha b \cdot \alpha c \cdot \alpha d \cdot \Psi b E \cdot c E \cdot d E \dagger 1 \dagger P1,$$

Peirce: CP 4.130 Cross-Ref: ††

130. . . . The student may object, at first blush, that the marks indicated by Greek letters *have no meaning*. This is a great mistake; they have precisely the meaning that is pertinent; but it is true they have no meaning in the sense of anything which particularly strikes ordinary attention. Reflect upon this. What people call an "interpretation" is a thing totally irrelevant, except that it may show by an example that no slip of logic has been committed.

Peirce: CP 4.131 Cross-Ref: ††

131. At this point, I should like to give some account of Schubert's *calculus of enumerative geometry*†2, which is the most extensive application of the Boolean algebra which has ever been made, and is of manifestly high utility. But I do not feel that I could possibly condense the elementary explanations or clarify them more than Schubert has himself done in his book. He has by no means exhausted the powers of his method. There is plenty of room for new researches; but his work will stand as the classical treatise upon geometry as viewed from the standpoint of arithmetic for an indefinite future.

Peirce: CP 4.132 Cross-Ref: ††

§9. THE LOGICAL AND THE QUANTITATIVE ALGEBRA

132. Cauchy †3 first gave the correct logic of imaginaries, and very instructive

it is. But the majority of writers of textbooks, who reason by the rule of thumb, do not understand it to this day. The square of the imaginary unit, i , is -1 , and therefore it may be allowable to speak of i and $-i$ as being two square roots of -1 . But to speak of them as the two square roots of -1 will not do. The algebraist sets out with a single continuous quantitative relation. But when he comes to quadratics he finds himself confronted with impossible problems. He says: "I want a square root of negative unity. Now there is no such thing in the universe: clearly, then, I must import it from abroad." Let us see how one would go about to prove there is no square root of negative unity. He would reason indirectly: that is the mathematician's recipe for everything. He would say let i be this square root if there be one. Then, whether its sign be $+$ or $-$, its square will have a positive sign, contrary to the hypothesis. Then the whole impossibility depends upon this, that every quantity is supposed to be positive or negative. Suppose we make i neither positive nor negative. "But there is no such thing," some rule-of-thumb man says. Really? In that respect it is just like all the other objects the mathematician deals with. They are one and all mere figments of the brain.†1 "But to say that a quantity is neither positive nor negative *means nothing*," objects the thumbist. I reply, the meaning of a sign is the sign it has to be translated into. Now in mathematics, which is merely tracing out the consequences of hypotheses, to say a thing has no meaning is to say it is not included in our hypothesis. In that case, all we have to do is to enlarge the hypothesis and put it in. That is your course when you have a concrete hypothesis. That was our conduct when we called a debt, negative property. But, at present, we are dealing with algebra in the abstract. The only hypothesis we make is that our letters obey the laws of algebra. If there is one of those laws which requires a quantity to be either positive or negative, find out which it is and delete it. If you have a system of laws which is self-consistent, it will not be less so when one of them is wiped out. But let us see what the laws of algebra are and how they are affected toward a quantity whose square is negative. We have,

- (1) If $x = y$, then x may anywhere be substituted for y .
- (2) $x + y = y + x$.
- (3) $x + (y + z) = (x + y) + z$.
- (4) $xy = yx$.
- (5) $x(yz) = (xy)z$.
- (6) $(x + y)z = xz + yz$.
- (7) $x + 0 = x$.
- (8) $x1 = x$.
- (9) $x + \infty = \infty$.
- (10) If $x + y = x + z$, either $y = z$ or $x = \infty$.
- (11) If $xy = xz$, either $y = z$, or $x = 0$, or $x = \infty$.
- (12) If $x > y$, not $y > x$.
- (13) If $x > y$, then there is a quantity a such that $a > 0$ [and] $a + y = x$.

(14) If $x > y$, then $x+z > y+z$.

(15) If $x > 0$ and $y > 0$, then $xy > 0$.

(16) Either $x > y$, or $x = y$, or $y > x$.

(17) $1 > 0$.

It is plain that from these equations it is impossible to prove that $x^2 < 0$ is not true except by the aid of one of the last six formulæ, and further that it will be requisite to consider the factors of x^2 . Now (15) is the only one of the last six formulæ, directly containing a product. This gives $x^2 > 0$ provided $x > 0$. Also, if $0 > x$, let $x + \{x\} = 0$, by (13), where $\{x\} > 0$.

Then, by (6),

$$x\{x\} + \{x\}^2 = 0$$

But by (7),

$$y+0 = y,$$

and by (6),

$$(y+0)\{x\} = y\{x\} + 0\{x\} = y\{x\},$$

and by (7),

$$y\{x\} + 0\{x\} = y\{x\} + 0,$$

and by (10),

either $0\{x\} = 0$ or $y\{x\} = \infty$ whatever y may be.

But the last alternative is absurd; for then by (9),

$$y0 = y(x + \{x\}) = yx + y\{x\} = yx + \infty = \infty.$$

But if $y = 1$ by (8),

$$y0 = 10 = 0.$$

Hence we should have

$$0 = \infty$$

whence by (7) and (9),

$$z = 0 = \infty$$

whatever z may be. Hence by (1),

If $u > v$ $v > u$

Hence by (12), in no case is $u > v$. But this contradicts (17).

We have, then,

$$0\{x\} = 0.$$

Hence by (2) and (7),

$$x\{x\} + \{x\}^2 = 0,$$

But by (15),

$$\{x\}^2 > 0.$$

Hence by (14),

$$x\{x\} + \{x\}^2 > x\{x\} + 0.$$

Hence by (7),

$$x\{x\} + \{x\}^2 > x\{x\}$$

or,

$$0 > x\{x\}.$$

But by (6) and (4),

$$x(x + \{x\}) = x^2 + x\{x\} = x0 = 0x = 0.$$

Hence,

$$x^2 + x\{x\} > x\{x\}.$$

Now since $0 > x\{x\}$ by (13) there is a quantity a such that

$$a > 0, a + x\{x\} = 0$$

Hence, by (14),

$$x^2 + x\{x\} + a > x\{x\} + a$$

Hence, finally, by (3),

$$x^2 + 0 > 0$$

or by (7),

$$x^2 > 0.$$

Hence, by (16), in every case

$$x^2 > 0 \text{ or } x = 0.$$

But it is plain that without (16) this conclusion could not be drawn, since no other of the formulæ (12)-(17) have anything to say about quantities neither greater, less, nor equal to one another.

Peirce: CP 4.132 Cross-Ref: ††

It thus appears that we have only to strike out (16), and the quantity i such that $i^2 = -1$, becomes perfectly possible, and perfectly *conceivable*, in the only clear sense of that word, namely, that we can write down

$$i^2 + 1 = 0$$

without conflict with any formula. If we define $-x$ by the formula

$$x+(-x) = 0,$$

then, necessarily, if $i^2 = -1$, we have also $(-i)^2 = -1$. Ordinary algebra *assumes* there is no other quantity except these two whose square equals -1 . Thus, if the algebraist finds $x^2 = y^2$ he at once writes $x = \pm y$. This is because he chooses to exclude all other square roots of -1 . I will return to this point shortly.†1

Peirce: CP 4.133 Cross-Ref:††

133. Men are anxious to learn what the square root of negative unity *means*. It just means

$$i^2+1 = 0;$$

precisely as -1 means

$$1+(-1) = 0.$$

The algebraic system of symbols is a *calculus*; that is to say, it is a language to *reason in*. Consequently, while it is perfectly proper to define a *debt* as negative property, to explain what a negative quantity is, by saying that it is what debt is to property, is to put the cart before the horse and to explain the more intelligible by the less intelligible. To say that algebra means anything else than just its own forms is to mistake an *application* of algebra for the *meaning* of it.†P1 But to this statement a proviso should be attached. If an application of algebra consists in another system of diagrams having properties analogous to those of the sixteen fundamental formulæ, or to the greater number of them, and if that other system of diagrams is a good one to reason in, and may advantageously be taken as an adjunct of the algebraic system in reasoning, then such system of diagrams should be regarded as more than a subject for the *application* of logic, and though it is too much to say it is *the* meaning of the algebra, it may be conceived as a secondary, or junior-partner meaning. Such junior interpretations are especially, the *logical* and the *geometrical*.

Peirce: CP 4.134 Cross-Ref:††

134. Logical algebra ought to be entirely self-developed. Quantitative algebra, on the contrary, ought to be developed as a special case of logical algebra. I do not mean that elementary teaching should set it on that basis; but that should be recognized as the fundamental philosophy of it. The seminary logicians have often seemed to think that those who study logic algebraically entertain the opinion that logic is a branch of the science of quantity. Even if they did, the error would be a trifling one; since it would be an isolated opinion, having no influence upon the main results of their studies, which are purely formal. But with the possible exception of Boole himself, whose philosophical views have not been lauded by any of his followers, none of the algebraic logicians do hold any such opinion. For my part, I consider that the business of drawing demonstrative conclusions from assumed premisses, in cases so difficult as to call for the services of a specialist, is the sole business of the mathematician. Whether this makes mathematics a branch of logic, or whether it cuts off this business from logic, is a mere question of the classification of the sciences. I adopt the latter alternative, making the business of logic to be analysis and theory of reasoning, but not the practice of it. To show how reasoning about quantity may be facilitated by considering logical interpretations, I may instance the Enumerate Geometry of Schubert,†1 which works by means of the logical calculus,

and Mr. MacColl's †2 method of transposing the limits of multiple integrals, which is done by the Boolean algebra. Dr. Fabian Franklin has effected some difficult algebraical demonstrations by considering quantities as expressive of probabilities. I myself made two additions to the theory of multiple algebra by considering it as expressive of the logic of relatives. †3

Peirce: CP 4.135 Cross-Ref: ††

135. The idea of multiplication has been widely generalized by mathematicians in the interest of the science of quantity itself. In quaternions, and more generally in all linear associative algebra, which is the same as the theory of matrices, it is not commutative. The general idea which is found in all of these is that the product of two units is the pair of units regarded as a new unit. Now there are two senses in which a "pair" may be understood, according as **BA** is, or is not, regarded as the same as **AB**. Ordinary arithmetic makes them the same. Hence, 2X3 or the pairs consisting of one unit of a set of 2, say, **I, J**, and another unit of a set of 3, say **X, Y, Z**, the pairs **IX, IY, IZ, JX, JY, JZ**, are the same as the pairs formed by taking a unit of the set of 3 first, followed by a unit of the set of 2. So when we say that the area of a rectangle is equal to its length multiplied by its breadth, we mean that the area consists of all the units derived from coupling a unit of length with a unit of breadth. But in the multiplication of matrices, each unit in the **P**th row and **Q**th column, which I write **P:Q**, of the multiplier coupled with a unit in the **Q**th row and **R**th column, or **Q:R** gives

$$(P:Q)(Q:R) = P:R$$

or a unit of the **P**th row and **R**th column of the multiplicand. If their order be reversed,

$$(Q:R)(P:Q) = 0,$$

unless it happens that **R = P**.

Peirce: CP 4.136 Cross-Ref: ††

136. In my earlier papers on the logic of relatives I made an application of the sign of involution †1 which, I am persuaded, is less special than it seems at first sight to be. Namely, I there wrote

$$ls$$

for the lover of *every* servant, while *ls* was the lover of some servant.

$$lsm = (ls)m$$

or the lover of everything that is servant to a man stands to every man in the relation of lover of every servant of his.

$$lw \vee m = lw \cdot lm$$

or the lover of everything that is either woman or man is the same as the lover of every woman and, at the same time, lover of every man.

$$(l \cdot b)m = lm \cdot bm$$

or that which is to every man at once lover and benefactor is the same as a lover of every man who is benefactor of every man.

$$(e \vee c)f = ef \vee [f]ef^{-1} \cdot c^{1*} \vee ([f] \cdot ([f]-1))/2 \cdot ef^{-2*} \cdot c^{2*} \vee \text{etc.}$$

that is to say, those things each of which is to every Frenchman either emperor or conqueror consist first of the emperors of all Frenchmen; second, of a number of classes equal to the number of Frenchmen, each class consisting of all emperors of all Frenchmen but one who are at the same time conquerors of that one; third, of a number of classes equal to half the product of the number of Frenchmen by one less than the number of Frenchmen, each class consisting of every individual which is emperor of all Frenchmen but two and conqueror of those two; etc.

Peirce: CP 4.136 Cross-Ref:††

This makes

$$lm = l m[1] \cdot l m[2] \cdot l m[3] \cdot l m[4] \cdot \text{etc.}$$

Of course, the ordinary idea which makes of involution the iteration of an operation, is a special case under this.

Peirce: CP 4.136 Cross-Ref:††

Thus, quantitative algebra is only a special development of logical algebra. On the other hand, it is equally true that the Boolean algebra is nothing but the mathematics of numerical congruences having 2 for their modulus.

Peirce: CP 4.137 Cross-Ref:††

137. The geometrical interpretation affords great aid in reasoning, because man has, so to speak, a natural genius for geometry. Thus we see easily enough, algebraically, that

$$x+yi = \sqrt{x^2+y^2} \left\{ \sqrt{(x/y)/((x/y)+(y/x))} + \sqrt{(y/x)/((x/y)+(y/x))} \cdot i \right\}$$

and further that

$$(x+yi)(u+vi) = \sqrt{(x^2+y^2)(u^2+v^2)}$$

$$\left\{ \sqrt{((x/y)(u/v)-(y/x)(v/u))/((x/y)+(y/x))((u/v)+(v/u))} + \right.$$

$$\left. \sqrt{((x/y)(v/u)+((y/x)(u/v))/((x/y)+(y/x))((u/v)+(v/u))} \cdot i \right\}.$$

But that which is by no means obvious algebraically, but becomes obvious geometrically, is that when we plot x and y as abscissa and ordinate of rectangular coördinates and u, v as other values of the same coördinates, and the product in the same way, the angle from the axis of abscissas of the product is equal to the sum of those of the two factors. This once found out, in the geometrical way, is easily put into algebraical form. Geometry here renders a precisely similar service to that which the theory of probabilities often lends. There are several instances in which mechanical instincts have been valuable in the same way. A choice collection of such

lemmas would be interesting.

Peirce: CP 4.138 Cross-Ref:††
§10. THE ALGEBRA OF REAL QUATERNIONS

138. I now turn back to square roots of negative unity, not supposing multiplication to be commutative. That is, we do not generally have $xy = yx$. Suppose we have two quantities i and j , such that

$$i^2 + 1 = 0$$

$$j^2 + 1 = 0$$

Then it is plain that

$$(iji)(iji) = (ij)ii(ji) = -(ij)(ji) = -i jj \cdot i = \ddot{i} = -1,$$

so that iji and ji are also square roots of negative unity.

Peirce: CP 4.138 Cross-Ref:††

Five cases may be studied:

First, $iji = i$

Second, $iji = -i$

Third, $iji = j$

Fourth, $iji = \cdot j$

Fifth, $iji = k$ (a third unit).

Peirce: CP 4.138 Cross-Ref:††

First Case. $iji = i$. Then,

$$i \cdot iji = \cdot ji = \ddot{i}$$

$$\cdot j = i.$$

Peirce: CP 4.138 Cross-Ref:††

Second Case. $iji = -i$. Then,

$$i \cdot iji = \cdot ji = \cdot \ddot{i}$$

$$j = i.$$

Peirce: CP 4.138 Cross-Ref:††

Third Case. $iji = j$. Then,

$$ijj = jj = -1$$

$$jji = ji = -1$$

and ij and ji are also square roots of negative unity.

$$iji = -ij = ji.$$

But,

$$(ij)i = j$$

$$i(ij) = -j,$$

equations that do not hold for $ij = i$ nor for $ij = j$.

Nor can we put

$$ij = \sin \Theta \cdot i + \cos \Theta \cdot j$$

For then,

$$i \cdot ij = -\sin \Theta + \cos \Theta \cdot ij = -\sin \Theta + \cos \Theta \sin \Theta \cdot i + \cos^2 \Theta \cdot j$$

and we have

$$\cos \Theta = \sqrt{-1}.$$

Let us then write

$$ij = k$$

$$ji = -k$$

Then,

$$ki = iji = j$$

$$ik = iij = -j$$

$$kj = ijj = -i$$

$$jk = -jji = i$$

This is the algebra of quaternions.†1

Peirce: CP 4.138 Cross-Ref: ††

Fourth Case. $iji = -j$. Then,

$$iji \cdot i = -ij = -ji$$

$$o_1 ji = ij.$$

Hence, since $i^2 = j^2$

$$(i-j)(i+j) = 0$$

$$j = \pm i.$$

Peirce: CP 4.138 Cross-Ref:††
Fifth Case .iji=k.

The multiplication cannot be commutative. We may then have four infinite series of units

$$\begin{array}{cccc}
 i[1] = i & j[1] = j & k^1 = ij & l^1 = ji \\
 i[2] = iji & j[2] = jij & k^2 = ijij & l^2 = jiji \\
 i[3] = ijiji & j[3] = jijij & k^3 = ijijij & l^3 = jijiji \\
 i[4] = ijijiji & j[4] = jijijij & k^4 = ijijijij & l^4 = jijijiji \\
 \text{etc.} & \text{etc.} & \text{etc.} & \text{etc.}
 \end{array}$$

Here

$$\begin{aligned}
 i[n] &= ilm^{-1} = kn^{-1}i \\
 j[n] &= jkn^{-1} = ln^{-1}j \\
 i[m]j[n] &= km+n-1 \\
 j[m]i[n] &= lm+n-1
 \end{aligned}$$

It is possible to suppose these all different.

Peirce: CP 4.138 Cross-Ref:††

If, on the other hand, any two are equal, there are but a finite number of different units. For example, if

$$ijijiji = jij$$

then,

$$ijiji = jijij$$

And all forms of more than five letters are equal to forms of quite as few as five letters. Thus,

$$ijijij = jijijj = .jiji.$$

Peirce: CP 4.139 Cross-Ref:††

139.†1 But the moment we suppose the number of linearly independent letters is finite we can reason as follows. Taking any expression, A , some power of it is a linear function of inferior powers. Hence, there is some equation

$$\Sigma[m](a[m]A^m) + a[0] = 0.$$

By the theory of equations, this is resolvable into quadratic factors. One of these, then, must equal zero. Let it be

$$(A-s)^2 + t^2 = 0.$$

Then,

$$((A-s)/t)^2 = -1$$

or, every expression, upon subtraction of a real number from it, can be converted, in one way only, into a square root, of a negative number. Let us call such a square root, the vector of the first expression, and the real number subtracted, the scalar of it.

Peirce: CP 4.139 Cross-Ref:††

$$\text{Let } \mathbf{v}^2 = -1, \mathbf{j}^2 = -1, \text{ and } \mathbf{ij} = s + \mathbf{v},$$

where s is scalar, and \mathbf{v} vector. Then it is impossible to find three real numbers, \mathbf{a} , \mathbf{b} , \mathbf{c} , such that

$$\mathbf{v} = \mathbf{a} + \mathbf{bi} + \mathbf{cj}$$

For assume this equation. Then, since

$$\mathbf{ijj} = -\mathbf{i},$$

$$-\mathbf{i} = \mathbf{sj} + \mathbf{vj} = -\mathbf{c} + (s + \mathbf{a})\mathbf{j} + \mathbf{bij}$$

$$= \mathbf{bs} - \mathbf{c} + \mathbf{ab} + \mathbf{b}^2\mathbf{i} + (s + \mathbf{a} + \mathbf{bc})\mathbf{j}.$$

Whence,

$$\mathbf{b}^2 = -1,$$

and \mathbf{b} could not be real.

Peirce: CP 4.139 Cross-Ref:††

Moreover, we shall have

$$\mathbf{ji} = r(s - \mathbf{v}),$$

where \mathbf{v} is a real number. For write

$$\mathbf{ji} = s' + \mathbf{v}',$$

where s' is the scalar, and \mathbf{v}' the vector, of \mathbf{ji} . Let us write, too,

$$\mathbf{vv}' = s'' + \mathbf{v}'' ,$$

where s'' and \mathbf{v}'' are again the scalar and vector of \mathbf{vv}' . Then,

$$\mathbf{ijji} = (s + \mathbf{v})(s' + \mathbf{v}') = ss' + s\mathbf{v}' + s'\mathbf{v} + s'' + \mathbf{v}''.$$

But

$$\mathbf{ijji} = 1.$$

Hence,

$$\mathbf{v}'' = 1 - ss' - s'' - s\mathbf{v}' - s'\mathbf{v}.$$

But it has just been proved that the vector of the product of two vectors is linearly independent of these vectors and of unity. Hence

$$\mathbf{v}'' = 0.$$

That is,

$$s\mathbf{v}' = 1 - ss' - s'' - s'\mathbf{v}.$$

Peirce: CP 4.139 Cross-Ref:††

But it has just been shown that a quantity can be separated into a scalar and a vector part in only one way. Hence

$$s\mathbf{v}' = -s'\mathbf{v}$$

$$s'' = 1 - ss'.$$

The former equation makes

$$j\mathbf{i} = (s'/s)(s - \mathbf{v}).$$

Let us next consider such an expression as

$$a\mathbf{i} + b\mathbf{j} = S + \mathbf{V}$$

where S and \mathbf{V} are the scalar and vector of the first member. Squaring the vector, we get

$$\begin{aligned} V^2 = -N = (a\mathbf{i} + b\mathbf{j} - S)^2 = -a^2 - b^2 + S^2 + abs + abs' - 2aSi - 2bSj \\ + ab(1 - (s'/s))\mathbf{v} \end{aligned}$$

or

$$ab(1 - (s'/s))\mathbf{v} = -N + a^2 + b^2 - S^2 - abs - abs' + 2aSi + 2bSj.$$

But since \mathbf{v} is the vector of $j\mathbf{i}$ it must, as we have seen, be linearly independent of unity, \mathbf{i} , and \mathbf{j} . Hence the first member must vanish. But if $\mathbf{v} = 0$, $j\mathbf{i} = s$, whence $-\mathbf{i} = s\mathbf{j}$, contrary to hypothesis. Hence,

$$1 - (s'/s) = 0$$

or

$$s' = s.$$

Whence,

$$s'' = 1 - s^2.$$

But s'' being the negative of the square of \mathbf{v} is positive. Hence,

$$s^2 | 1.$$

Peirce: CP 4.139 Cross-Ref:††

We know that $a\mathbf{i} + b\mathbf{j}$ cannot be a scalar; for then a quantity could in two ways be resolved into a scalar and vector part. Now

$$(a\mathbf{i} + b\mathbf{j})^2 = -a^2 - b^2 + 2abs.$$

This must therefore be negative. For $(a\mathbf{i} + b\mathbf{j}) = S + \mathbf{V}$, and \mathbf{V} does not vanish. Hence

$$(a\mathbf{i} + b\mathbf{j})^2 = S^2 + V^2 + 2SV;$$

and since, by comparison, it appears the vector part $2SV$ vanishes, it follows that $S = 0$, and the sum, or linear function, of two vectors is a vector.

Peirce: CP 4.139 Cross-Ref:††

The same thing is evident because $S^2 = 1$; whence

$$-a^2 - b^2 + 2abs = -p(a+b)^2 - (1-p)(a-b)^2,$$

where $p = (1+s)/2$ and $1-p = (1-s)/2$, both of which are positive, or zero.

Peirce: CP 4.139 Cross-Ref:††

Let us then assume a vector j , such that

$$j[1] = (si + j)/\sqrt{1-s^2}$$

$$j[1]^2 = (-s^2 - 1 + 2s^2)/(1-s^2) = -1$$

$$ij[1] = (-s + s + v)/\sqrt{1-s^2} = v/\sqrt{1-s^2}$$

$$j[1]i = -(v/\sqrt{1-s^2})$$

$$j[1] = v/\sqrt{1-s^2} = j[1]ij[1] = -j[1]^2i = i$$

$$v/\sqrt{1-s^2} \cdot j[1] = ij[1]j[1] = -i$$

$$i \cdot v/\sqrt{1-s^2} = iij[1] = -j[1]$$

$$v/\sqrt{1-s^2} \cdot i = -j[1]ii = j[1]$$

$$(v/\sqrt{1-s^2})^2 = -ij[1]j[1]i = -1$$

Peirce: CP 4.139 Cross-Ref:††

Writing j for $j[1]$ and k for $v/\sqrt{1-s^2}$ and the above formulæ define the algebra of real *quaternions*.

Peirce: CP 4.140 Cross-Ref:††

140.†1 I will now prove that it is not possible to add to this a fourth linearly independent vector. For suppose $[l]$ to be such a unit vector. Write

$$jl = S' + V',$$

$$li = S'' + V''.$$

Substitute for l ,

$$l[1] = S''i + S'j + l$$

Then,

$$jl[1] = -S''k - S' + S' + V' = -S''k + V',$$

$$l[1]j = S''k - S' + S' - V' = S''k - V',$$

$$l[1]i = -S'' - S'k + S'' + V'' = -S''k + V'',$$

$$il[1] = -S'' + S'k + S'' - V'' = S''k - V''.$$

Let us further assume

$$kl[1] = S''' + V'''$$

Whence,

$$l[1]k = S''' - V'''.$$

But

$$l[1]j = -jl[1] \text{ and } l[1]i = -il[1].$$

Hence,

$$kl = ijl[1] = -i.l[1]j = -il[1]j = l[1]ij = l[1].ij = lk.$$

So,

$$kl[1] = l[1]k$$

or,

$$kl[1] - l[1]k = 0$$

But we have seen that

$$kl[1] - l[1]k = S''' + V''' - S''' + V''' = 2V'''.$$

Hence $V''' = 0$. Then these vectors are not linearly independent, and a fourth unit vector is impossible.

Peirce: CP 4.140 Cross-Ref:††

But this proof does not apply when the multitude of linearly independent expressions is endless; such algebras are nonlinear.

Peirce: CP 4.140 Cross-Ref:††

We thus see that even when we annul the commutative law of multiplication, there are but three linear algebras, real single algebra, ordinary imaginary algebra, and the algebra of real quaternions which obey all the other algebraic laws. The law which so limits the number is:

Peirce: CP 4.140 Cross-Ref:††

If $xy = xz$, then $y = z$, unless $x = 0$ or $x = \infty$.

Peirce: CP 4.141 Cross-Ref:††

141. In all other algebras this law fails and with it goes all semblance of importance for the inverse operation of division.†P1 The algebra of logic illustrates the vanity of that device for solving equations, which must on the contrary usually be solved by producing special known quantities by direct operations.

Peirce: CP 4.142 Cross-Ref:††
§11. MEASUREMENT

142. It was necessary to say something about imaginaries before coming to the subject of measurement since the modern theory of measurement (due to the researches of Cayley, Clifford, Klein, etc.) depends essentially upon imaginaries.

Peirce: CP 4.142 Cross-Ref:††

Let us first consider measurement in one dimension. There is a certain absurdity in talking about measurement in one dimension. This is seen in the instance of time. Suppose we only knew the flow of our inward sensations, but nothing spread into two dimensions, how could one interval of time be compared with another? Certainly, their contents might be so alike that we should judge them equivalent. But that is not shoving a scale along. It does not enable us to compare intervals unless they happen to have similar contents. However, it is convenient to put that consideration aside, and to begin (with Klein) at unidimensional measurement.

Peirce: CP 4.142 Cross-Ref:††

We are to measure, then, along a line. We will, for formal rhetoric's sake, conceive that line as returning into itself. We will, first, in order that we may apply numerical algebra, give a preliminary numbering to all the points of that line, so that every point has a number and but one number, and every real number, positive or negative, rational or surd, has a point and but one point, and so that the succession of any four numbers is the same as the succession on the line of the four corresponding points. Now, we must make a scale to shift along that line. We must imagine that we have a movable line which lies everywhere in coincidence with the fixed line, and which can be shifted. In the shifting, parts of it may become expanded or contracted, for we cannot tell whether they do or not unless we had some third standard to shift along to tell us; and then the same question would arise. But the continuity and succession of points shall not be broken in the shifting; and moreover, when the movable line has any one point brought back to coincidence with a former position, all the points shall be brought back. Now imagine all this extended to the imaginary numbers. Then, it is shown in the mathematical theory of functions, that if x be the number against which any point of the movable line falls in any one position and y be the number the same point falls against in any other position, it follows, because for each value of x there is just one value of y and for each value of y just one value of x , that x and y are connected by an equation linear in each, that is, an equation of the form

$$xy + Ax + By + C = 0.$$

This gives

$$y = -(Ax + C)/(x + B).$$

Now this is a function which forms the subject of some very beautiful and simple algebraical studies.†P1 It is convenient to put

$$A = B - \alpha - \beta$$

$$C = B^2 - (\alpha + \beta)B + \alpha\beta.$$

Then

$$y = ((\alpha + \beta - B)x - B^2 + (\alpha + \beta)B - \alpha\beta) / (x + B)$$

$$= (((\alpha + \beta)x + (\alpha + \beta)B - \alpha\beta) / (x + B)) - B$$

$$= (((\alpha^2 - \beta^2)(x + B) - \alpha\beta(\alpha - \beta)) / ((x + B)(\alpha - B))) - B$$

$$= (((x + B - \beta)\alpha^2 - (x + B - \alpha)\beta^2) / ((x + B - \beta)\alpha^1 - (x + B - \alpha)\beta^1)) - B$$

But

$$x = ((x + B - \beta)\alpha^1 - (x + B - \alpha)\beta^1) / ((x + B - \beta)\alpha^0 - (x + B - \alpha)\beta^0) - B.$$

So that the effect of the shifting has been to raise the exponents of α and β by 1.

Peirce: CP 4.142 Cross-Ref:††

It is easily proved that the same operation, performed any number t times, gives

$$((x + B - \beta)\alpha^{t+1} - (x + B - \alpha)\beta^{t+1}) / ((x + B - \beta)\alpha^t - (x + B - \alpha)\beta^t) - B$$

If α has a modulus greater than that of β , it is easily seen that when t becomes a very large positive number, the first terms of numerator and denominator will become indefinitely greater than the second terms and the value will indefinitely approximate to

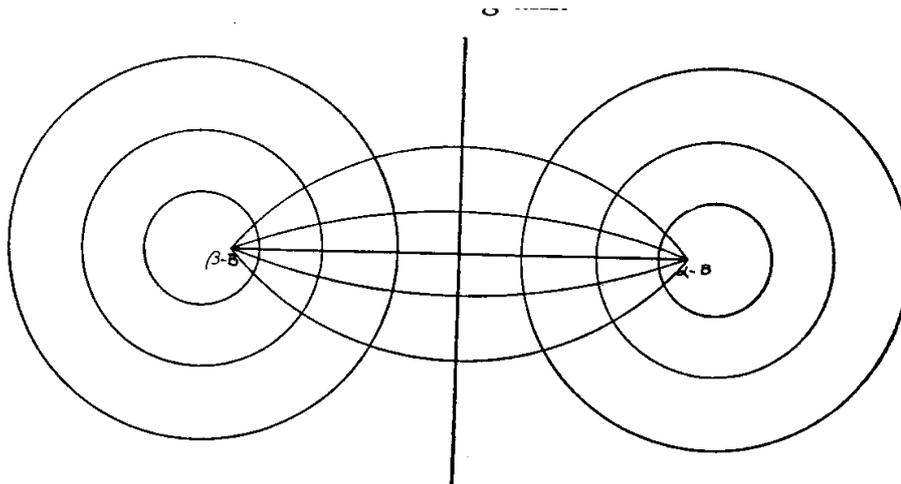
$$\alpha - B.$$

But when t is a very large negative quantity, the reverse will occur, and the value will approximate toward

$$\beta - B.$$

Peirce: CP 4.143 Cross-Ref:††

143. If we look at the field of imaginary quantity, what we shall see is shown in the diagram.



[Click here

to view] Here we have a stereographic projection of the globe. At the south pole is $\beta-B$; at the north pole, $\alpha-B$. The parallels are not at equal intervals of latitude but are crowded together infinitely about the pole. Now an increase of t by unity carries a point of the scale along a meridian from one parallel to the one next nearer the north pole. But an addition to t of an *imaginary* quantity carries the point of the scale round along a parallel.

Peirce: CP 4.143 Cross-Ref:††

If the real line of the scale lies along a meridian all real shiftings of it will crowd its parts toward the north or the south pole; and the distance of either pole, as measured by the multitude of shiftings required to reach it, is *infinite*. †P1 The scale is *limited*, but *immeasurable*.

Peirce: CP 4.143 Cross-Ref:††

But if the real line of the scale lies along a parallel, real shiftings, that is shiftings from real points to real points, will carry it round, so that a finite number of shiftings will restore it to its first position. Such is the scale of rotatory displacement. It is *unlimited*, but finite, or *measurable*. A scale of measurement, in the sense here defined, cannot be both limited and finite. We seem to have such a scale in the measurement of probabilities. But it is not so. Absolute certainty, or probability 0 or probability 1 are unattainable; and therefore, the numbers attached to probabilities do not constitute any proper scale of measurement, *which can be shifted along*. But it is possible to construct a true scale for the measurement of belief. †P2 It was a part of the definition of a scale that in all its shiftings it should cover the whole of the line measured. ("For every point of the line a number of the scale in every position.") Hence the shifting can never be arrested by abuttal against a limit. If there is a limit, it must be at an immeasurable distance.

Peirce: CP 4.144 Cross-Ref:††

144. But there is a special case of measurement, very different from the one considered. Namely, it may happen that the nature of the shifting is such that [given] the equation

$$xy + Ax + By + C = 0,$$

where A, B, C , may have any values, real or imaginary, we have

$$C = 1/4(A+B)^2.$$

Substituting this in the expressions for A and C in terms of α, β , and B , we get

$$1/4(\alpha+\beta)^2 = \alpha\beta$$

or

$$\alpha = \beta.$$

This necessitates an altogether different treatment. In this case, we have

$$y = -((Ax+(1/4)(A+B)^2)/(x+B))$$

$$2y = -(A+B)+((B-A)(2x+A+B)/((B-A)+(2x+A+B)))$$

$$2x = -(A+B)+((B-A)(2x+A+B)/((B-A)+0(2x+A+B)))$$

And t shifts give

$$-(A+B)+((B-A)(2x+A+B)/((B-A)+t(2x+A+B))).$$

This gives for $t = \pm\infty$, $-(A+B)$. The scale is in this case then unlimited and immeasurable. This is the manner in which the Euclidean geometry virtually conceives lengths to be measured; but whether this method accords precisely with measurement by a rigid bar is a question to be decided experimentally, or irrationally, or not at all.

Peirce: CP 4.145 Cross-Ref:††

145. The fixed limits of measurement are very appropriately termed by mathematicians the Absolute.†P1 It is clear that even when measurement is not practical, even when we can hardly see how it ever can become so, the very idea of measuring a quantity, considerably illuminates our ideas about it. Naturally, the first question to be asked about a continuous quantity is whether the two points of its absolute coincide; if not, a second less important, but still significant question is whether they are in the real line of the scale or not. These are ultimately questions of fact which have to be decided by experimental indications; but the answers to them will have great bearing on philosophical and especially cosmogonical problems.

Peirce: CP 4.146 Cross-Ref:††

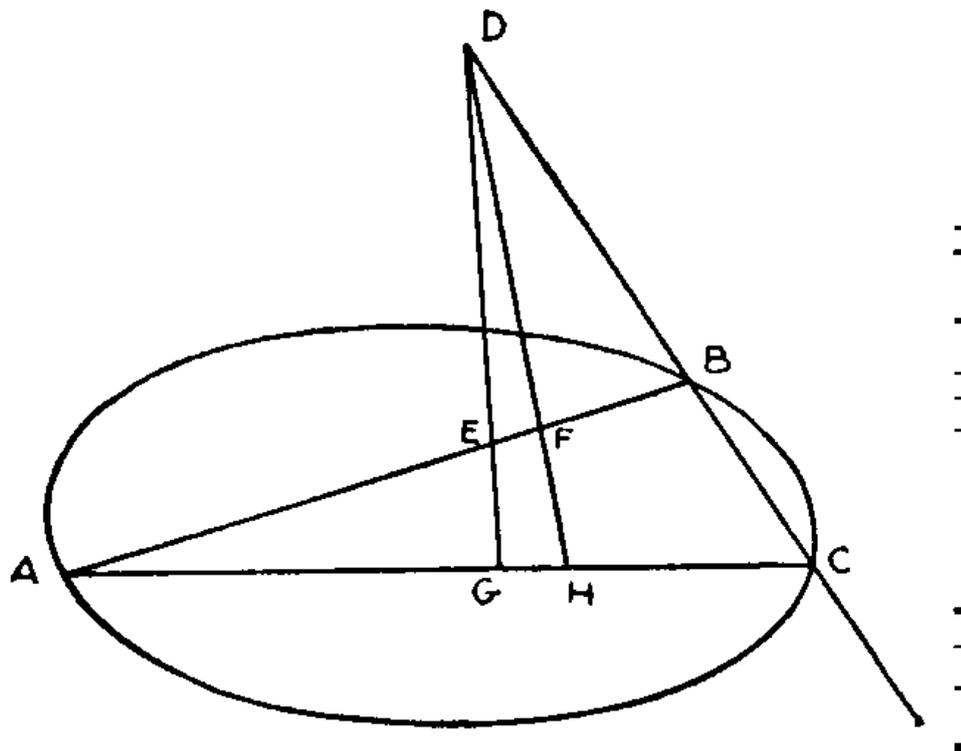
146. The mathematician does not by any means pretend that the above reasoning flawlessly establishes the absolute in every case. It is evident that it involves a premiss in regard to the imaginary points which only indirectly relates to anything in visible geometry, and which, of course, may be supposed not true. Nevertheless, the doctrine of the mathematical absolute holds with little doubt for all cases of measurement, because the assumptions virtually made will hardly ever fail.

Peirce: CP 4.147 Cross-Ref:††

147. When we pass to measurement in several dimensions, there seems to be little difference between one number of dimensions and another; and therefore we may as well limit ourselves to studying measurement on a plane, the only spatial spread for which our intuition is altogether effortless.

Peirce: CP 4.147 Cross-Ref:††

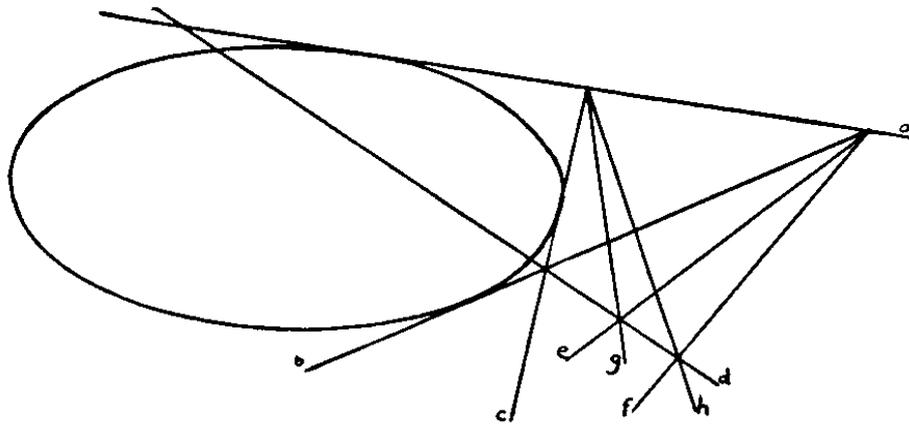
Radiating from each point of the plane is a continuity of lines. Each of these has upon it its two absolute points (possibly imaginary, and even possibly coincident); and assuming these to be continuous, they form a curve which, being cut in two points only by any one line, is of the second order. That is, it is a conic section, though it may be an imaginary or even degenerate one.



[Click here to view] Now as the foot has different lengths in different countries, so the ratios of units of lengths along different lines in the plane is somewhat arbitrary. But the measurement is so made that first, every point infinitely distant from another along a straight line is also infinitely distant along any broken line; and second, if two straight lines intersect at a point, *A*, on the absolute conic and respectively cut it again at *B* and *C*; and from *D*, any point collinear with *B* and *C*, two straight lines be drawn, the segments of the first two lines, *EF* and *GH*, which these cut off, are equal. I omit the geometrical proof that this involves no inconsistency. This proposition enables us to compare any two lengths.

Peirce: CP 4.148 Cross-Ref:††

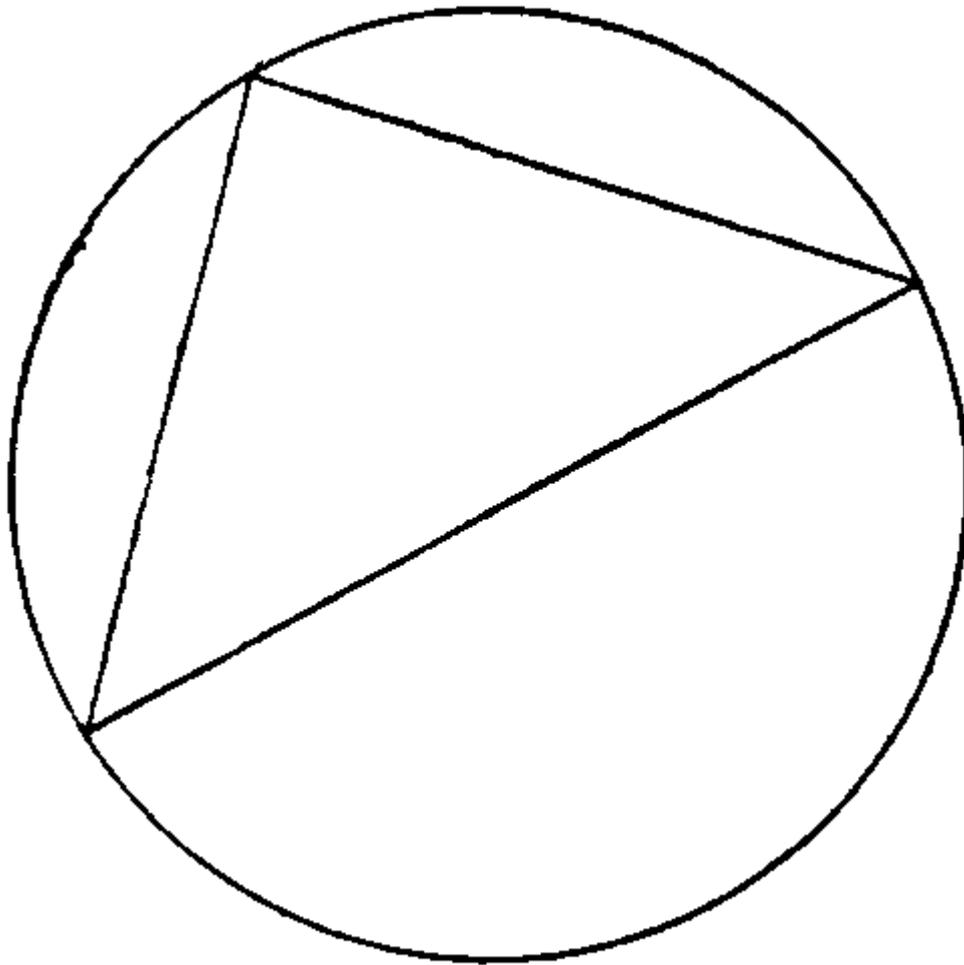
148. We now have to consider angular magnitude. In the space of experience, the evidence is strong that, when we turn around and different landscapes pass panorama-wise before our vision we come round to the same direction, and not merely to a new world much like the old one. In fact, I know of no other theory for which the evidence is so strong as it is for this. But it is quite conceivable that this should not be so; there might be a world in which we never could get turned round but should always be turning to new objects. But certain conveniences result from assuming for the measurement of the angles between lines the same absolute conic which is assumed as the absolute of linear measure. Thus, it is assumed that two straight lines meeting at infinity have no inclination to one another, just as it is assumed that in a direction such that the opposite infinities should coincide, all other points would have no distance from one another. The latter is another way of saying that if a point is at an infinite distance from another point on a straight line, it is so on a broken line. The other assertion is that if an infinite turning is requisite to reach a line from one centre, it is equally so if you attempt to reach it by turning successively about different centres. The analogue of the proposition for which the last figure was drawn is as follows:



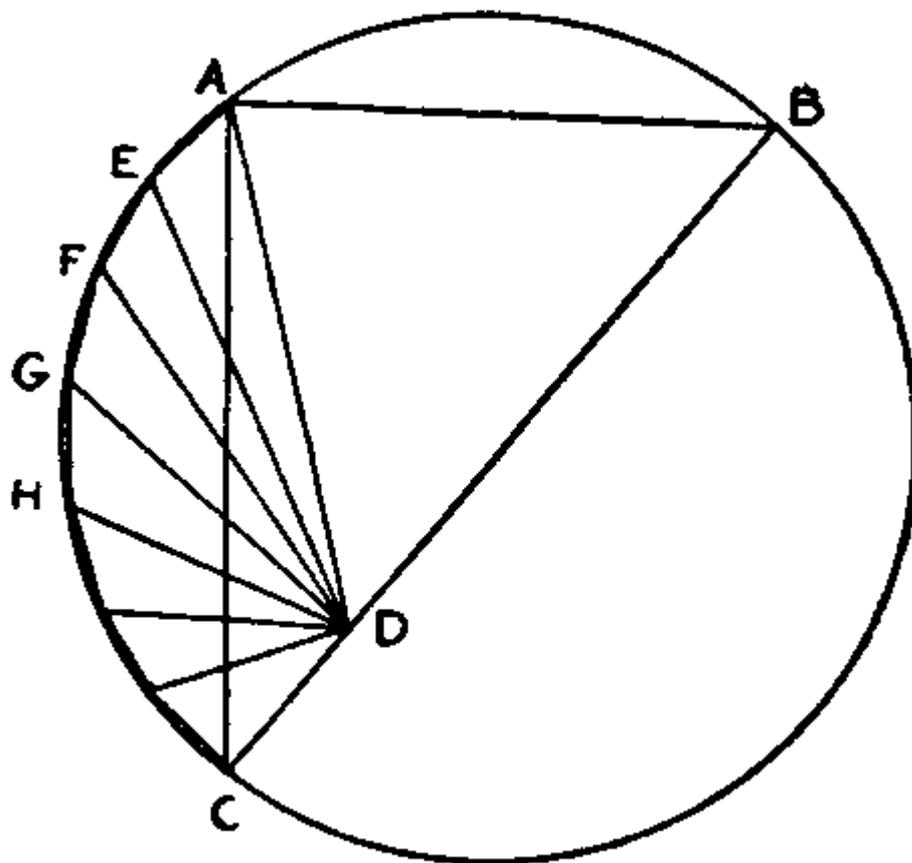
[Click here to view] Upon a line, a , tangent to the absolute let two points be taken from which the other tangents to the absolute are b and c . Through the intersection of b and c draw any line, d , then any two lines e and f , meeting at the intersection of a and b , make the same angle with one another as two other lines having the same intersections with d , and cutting one another at the intersection of a and c . This enables us to compare all angles.

Peirce: CP 4.149 Cross-Ref:††

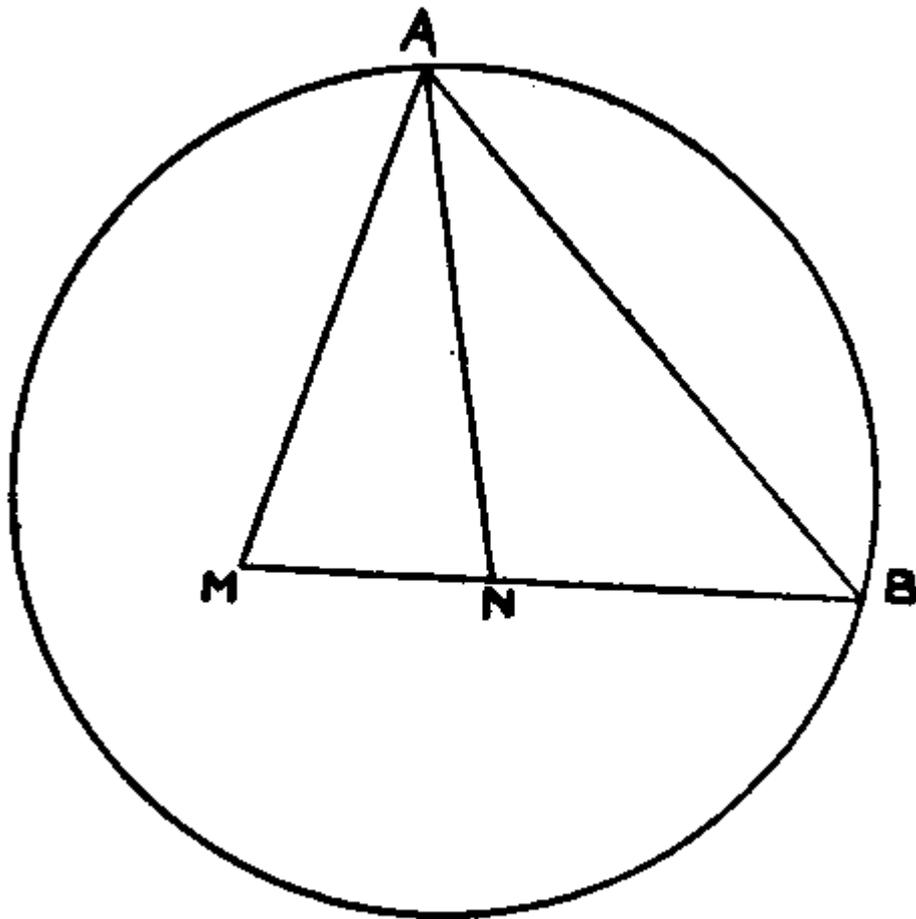
149. Suppose a man to be standing upon an infinitely extended plane free from all obstructions. Would he see something like a horizon line, separating earth from sky, being the foreshortened parts of the plane at an infinite distance? If space is infinite, he would. Now suppose he sets up a plane of glass and traces upon it the projection of that horizon, from his eye as a centre. Would that projection be a straight line? Euclid virtually says, "Yes." Modern geometers say it is a question to be decided experimentally. As a logician, I say that no matter how near straight the line may seem, the presumption is that sufficiently accurate observation would show it was a conic section. We shall see the reason for this, when we come to study probable inference.



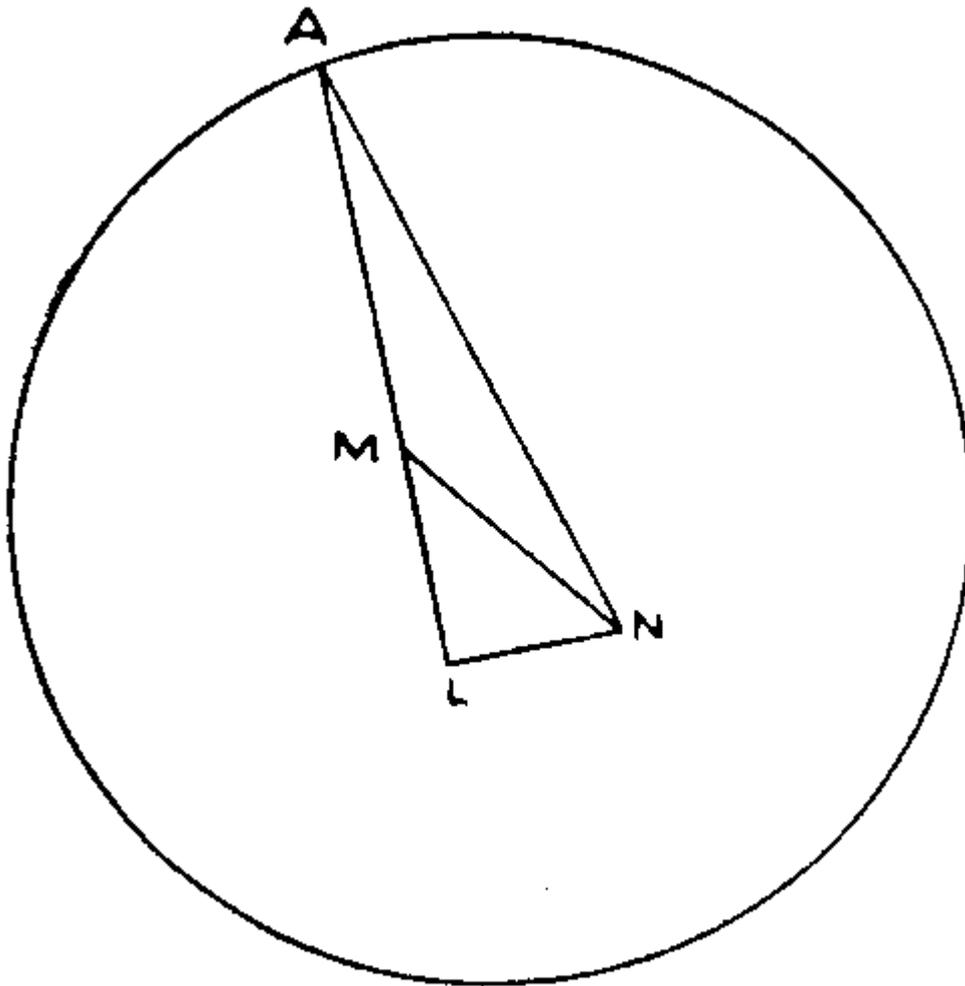
[Click here to view] Let us suppose, then, that the horizon is not a straight line upon a level with the eye, but is a small circle below that level. If then two straight lines meet at infinity, their other ends must be infinitely distant; but the angle between them is null. Hence, there may be a triangle having all its angles null, and all its sides infinite. Let us assume (what might, however, be proved) that two triangles, having all the sides and angles of the one respectively and in their order equal to those of the other, are of equal area. Then all triangles having the sum of their angles *null* are of equal area. Call this *T*. Then the area of an ordinary polygon of *V* vertices all on the absolute is $(V-2)T$. The area of the absolute is therefore infinite.



[Click here to view] If a triangle has two angles null and the third $1/N$ part of 180° , what is its area? Let ABD be the triangle, AB being on the absolute. Continue BD the absolute at C . Let ADE , EDF , etc., be $N-1$ triangles having their angles at D all equal to ADB . Then these N triangles are all equal, because their sides and angles are equal. They make a polygon of $N+1$ vertices on the absolute, the area of which is $(N-1)T$. Hence, the area of each triangle is $(1-1/N)T$.



[Click here to view] What is the area of a triangle having one angle zero and the others $m \times 180^\circ$ and $n \times 180^\circ$? Let AMN be such a triangle; extend MN on the side of N to the absolute at B . Then the area of ABM is $(1-m)T$ and that of ABN is nT . Hence the area of AMN , which is their difference, is $(1-m-n)T$.



[Click here to view] What is the area of a triangle having its three angles equal to $l \times 180^\circ$, $m \times 180^\circ$, and $n \times 180^\circ$? Let LMN be such a triangle. Produce LM on the side of M to the absolute at A and join AN . Then if the angle MNA is put equal to $x \cdot 180^\circ$, the area of ALN is $(1-l-n-x)T$, while that of AMN is $(m-x)T$. Hence, that of LMN , which is their difference, is $(1-l-m-n)T$.

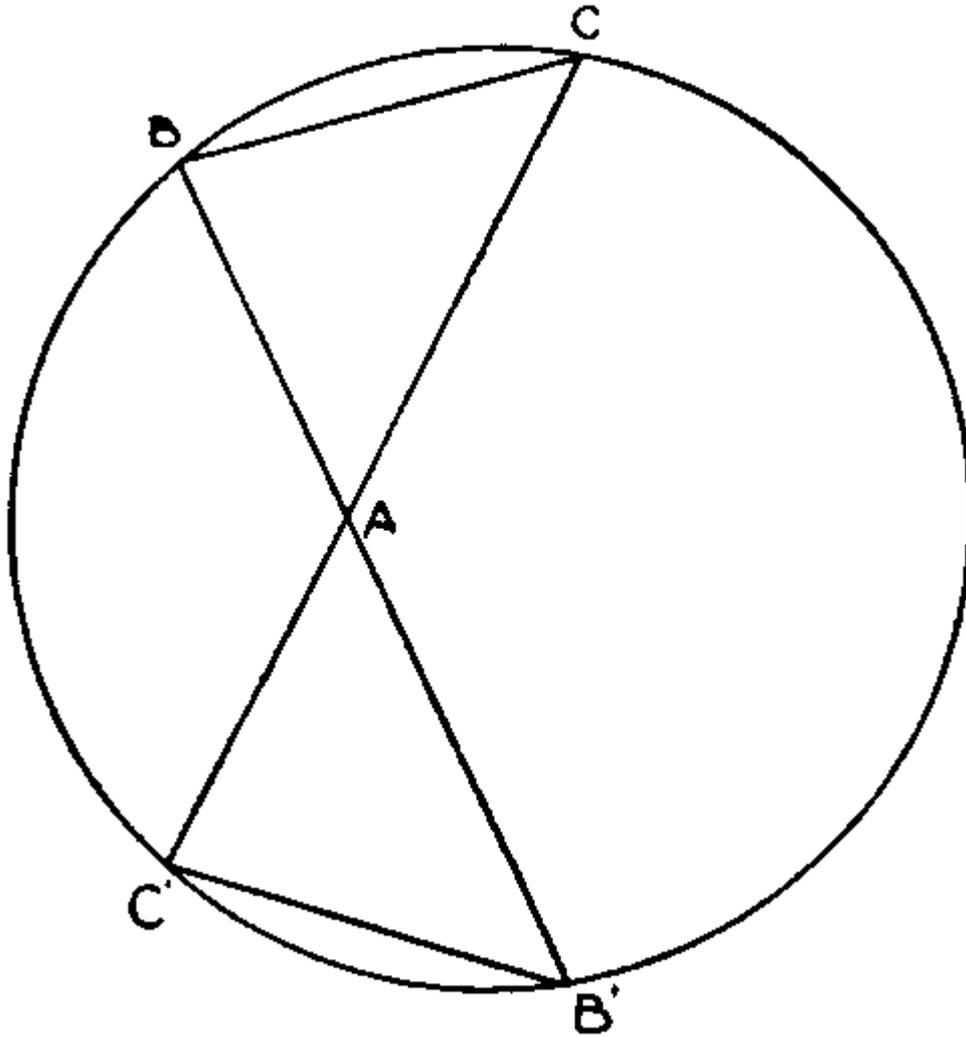
Peirce: CP 4.149 Cross-Ref: ††

Thus, the area of a triangle is proportional to the amount

by which the sum of its angles falls short of two right angles. Of course, this does not forbid that amount being infinitely small for all triangles whose sides are finite.

Peirce: CP 4.150 Cross-Ref: ††

150.



[Click here to view] The above reasoning may appear to be fallacious because it forgets that subtraction is not applicable to infinities. But it does not fall into that error. I may remark, however, that subtraction is applicable to infinities, in case their transformations are so limited that $x+y$ cannot equal $x+z$ unless $y = z$. For instance, we have considered the triangle ABC having two vertices on the absolute. This triangle is finite. But we might perfectly well reason about the infinite sector ABC provided this sector be not allowed to vary so as to change the area of the triangle, and provided, further, that we always add to each sector, BAC , its equal vertical sector $B'AC'$.

Peirce: CP 4.150 Cross-Ref:††

Looking at a triangle from this point of view, we see that the sum of the six sectors (two for each angle) is twice three times the triangle *plus* all the rest of the plane, or twice the area of the triangle plus the whole plane. The whole plane is four right angled sectors. But we have thus reckoned together with the sectors of the angles of the triangle their equal vertical sectors. Dividing by two, we find the sum of the angular sectors of a triangle is two right angles *plus* the area of the triangle.

Peirce: CP 4.150 Cross-Ref:††

Now since the sector is proportional to its angle, and since further, for the largest possible sector the angle is zero, it follows that the sector is equal to the negative of the angle, whence we find

$$\text{area } \{D\} = 2 \cdot \mathbf{a} \cdot \Sigma \text{ Angles.}$$

Peirce: CP 4.151 Cross-Ref:††

151. . . . Such are the ideas which the mathematician is using every day. They are as logically unimpeachable as any in the world; but people, who are not sure of their logic, or who, like many men who pride themselves on their soundness of reason, are totally destitute of it, and who substitute for reasoning an associational rule of thumb, are naturally afraid of ideas that are unfamiliar, and which might lead them they know not whither.

Peirce: CP 4.151 Cross-Ref:††

As compared with imaginaries, with the absolute, and with other conceptions with which the mathematician works fearlessly -- because good logicians, the Cauchys and the like,†P1 have led the way -- as compared with these, the idea of an infinitesimal is exceedingly natural and facile. Yet men are afraid of infinitesimals, and resort to the cumbrous method of limits. This timidity is a psychological phenomenon which history explains. But I will not occupy space with that here.†1

Peirce: CP 4.151 Cross-Ref:††

It was Fermat, a wonderful logical and still more wonderful mathematical genius, whose light was almost extinguished by the bread-and-butter difficulties which the secret plotting of worldlings forced upon him, who first taught men the method of reasoning which lies at the bottom of all modern science and modern wealth, the method of the differential calculus.†2 He gave a variety of instructive examples, did this lawyer, this "conseiller de minimis," as the jealous Descartes was base enough to call him, joining himself to the "born missionaries" who were determined to "head off" this hope of mankind. But the first and simplest of them is the solution of the problem to divide a number, a , into two parts so that their product shall be a maximum. Let the parts be x and $a-x$. Let e be a quantity such that $a+e$ is "adequal" or {parisos}, say perequal to a . Then, the product being a maximum is at the point when increase of x ceases to cause it to increase. Hence Fermat writes

$$x(a-x) = (x+e)(a-x-e)$$

which gives

$$0 = -xe+e(a-x-e)$$

or

$$0 = e(a-2x-e)$$

Fermat now divides both sides by e (which assumes e is not zero). Whence

$$0 = a-2x-e.$$

But $a-2x-e$ is "adequal" to $a-2x$; and the e may consequently be "elided." Thus we get

$$0 = a-2x,$$

or

$$x = (1/2)a.$$

Peirce: CP 4.151 Cross-Ref:††

The peculiar properties of e , which we now call, after Leibniz, the infinitesimal, are:

Peirce: CP 4.151 Cross-Ref:††

First, that if $pe = qe$, then $p = q$, contrary to the property of zero; while

Peirce: CP 4.151 Cross-Ref:††

Second, that, under certain circumstances, we treat e as if zero, writing

$$p+e = p.$$

Peirce: CP 4.151 Cross-Ref:††

Of course, we cannot adopt the last equation without reservation. For it would follow that

$$e = 0,$$

whence, since

$$4X0 = 5X0,$$

$$4e = 5e,$$

and then by the first property,

$$4 = 5.$$

Peirce: CP 4.151 Cross-Ref:††

The method of indivisibles †P1 had recognized that infinitely large numbers may have definite ratios, so that division is applicable to them.

Peirce: CP 4.152 Cross-Ref:††

152. The simplest way of defending the algebraical device is to say that e represents a quantity immeasurably small, that is, so small that the Fermatian inference does not hold from these quantities to any that are assignable. That no contradiction is involved in this has been shown in the former part of this chapter. †1 In the sense of measurement, then, $p+e = p$, while from a formally logical point of view, it is assumed that $e > 0$. This is the most natural way, a perfectly logical way, and the way the most consonant with modern mathematics.

Peirce: CP 4.152 Cross-Ref:††

It is also possible to conceive the reasoning to represent the following. (The problem is the same as above.) Let x be the unknown. Then, since $x(a-x)$ is a maximum,

$$x(a-x) > (x+e)(a-x-e)$$

for all neighboring values of e . That is

$$0 > e(a-2x-e).$$

Then the sign of $a-2x-e$ is opposite to that of e no matter what the value of e . It follows that $2x$ differs from a by less than any assignable quantity.

Peirce: CP 4.152 Cross-Ref:††

The great body of modern mathematicians repudiate infinitesimals in the above literal sense, because it is not clear that such quantities are possible, or because

they cannot entirely satisfy themselves with that mode of reasoning. They therefore adopt the method of limits, which is a method of establishing the fundamental principles of the differential calculus. I have nothing against it, except its timidity or inability to see the logic of the simpler way. Let x be a variable quantity which takes an unlimited series of values $x[1], x[2], \dots x[n]$, so that n will be a variable upon which $x[n]$, depends. If, then, there be a quantity c such that

$$x[\infty] = c,$$

that is, as the mathematicians prefer to say, in order to avoid speaking of infinity, if for every positive quantity e sufficiently small, there be a positive quantity $\{n\}$ such that for all values of n greater than $\{n\}$

$$\text{Modulus } (x[n]-c) < e$$

then c is said to be the *limit* of x .

Peirce: CP 4.152 Cross-Ref:††

Upon this definition is raised quite an imposing theory about limits which I can only regard with admiration, when it is erected with modern accuracy. Only, I wish to point out that the need for such a definition is not limited to its application to $n = \infty$, nor because infinity presents peculiar difficulties. It is only because ∞ is not an assignable number with which we can perform arithmetical processes. Let the function $x[n] = n^2$, then the same difficulty arises when $n = \pi$, and the same definition of a limit is called for.

Peirce: CP 4.152 Cross-Ref:††

The differential calculus deals with continuity, and in some shape or other, it is necessary to define continuity. I accept the above definition, with unimportant modifications, as a good definition of continuity. From it, as it appears to me, the idea of infinitesimals follows as a consequence; but, if not, no matter -- so long as the algebraic expression of the infinitesimal be accepted, which is really the essential point. Infinitesimals may exist and be highly important for philosophy, as I believe they are. But I quite admit that as far as the calculus goes, we only want them to reason with, and if they be admitted into our reasoning apparatus (which is the algebra) that is all we need care for.

Peirce: CP 4.153 Cross-Ref:††

V

A THEORY ABOUT QUANTITY†1P

§1. THE CARDINAL NUMERALS

153. Quantity presents certain metaphysical difficulties, to appreciate which it would be as hopeless a task to bring this generation as to bring it to a sense of sin. Medieval doctors apprehended such points of logic far more clearly. Why a mass

distant one yard from a pound of matter should gravitate toward that pound by precisely that fraction of an inch per second that it does, neither more nor less -- how it is possible that the exact value of this quantity ever should be explained and brought under the manifest governance of that unyielding and universal law which is supposed to regulate all facts, how this can be when the general properties of an inch are nowise different from a mile, is a question which those philosophers who oppose my tychism †2 would find a puzzling one, could they once be brought to understand what this question is. My present purpose, however, is not to discuss this problem in its entirety, but merely to follow out, in a rambling spirit, a pretty little opening of thought suggested by an objection to a part of my solution of that problem.

Peirce: CP 4.154 Cross-Ref:††

154. That part of my solution is that Quantity is merely the mathematician's idealization of meaningless vocables invented for the experimental testing of orders of sequence. In our experience we often have occasion to remark that something is true of two or more things -- say, for example, that one thing eats another, or that one day is pleasanter than another. A fact true of several subjects is called a "relation" between them. A fact true of a pair of subjects (as in the examples just suggested) is a "dyadic relation." Of some kinds of dyadic relation we find that if one thing, *A*, be so related to a second, *B*, while *B* is so related to a third thing, *C*, then *A* is always related in that same way to *C*. When this is so, the relation is said to be "transitive." We also call it a "succession" or "sequence." Now I hold that numbers are a mere series of vocables serving no other purpose than that of expressing such transitive relations, or, at least, no other purpose except one whose accomplishment is necessarily involved in that. I admit that our senses may inform us, not merely that *A* is heavier than *B*, but that *A* is a great deal more heavier than *B*, than *C* is heavier than *D*; and undoubtedly numbers do serve to express this verdict of sense with greater precision than sense can render it. But this, as I think, is a use of numbers which necessarily results from their primary use; the judgment, that one thing is *much* heavier than another, being a mere complexus of judgments each that one thing is heavier by a unit than another.

Peirce: CP 4.155 Cross-Ref:††

155. When a number is mentioned, I grant that the idea of a succession, or transitive relation, is conveyed to the mind; and insofar the number is not a meaningless vocable. But then, so is this same idea suggested by the children's gibberish

"Eeny, meeny, miney, mo,"

Yet all the world calls these meaningless words, and rightly so. Some persons would even deny to them the title of "words," thinking, perhaps, that every word properly means something. That, however, is going too far. For not only "this" and "that," but all proper names, including such words as "yard" and "metre" (which are strictly the names of individual prototype standards), and even "I" and "you," together with various other words, are equally devoid of what Stuart Mill †1 calls "connotation." Mr. Charles Leland informs us that "eeny, meeny," etc. are gipsy numerals.†2 They are certainly employed in counting nearly as the cardinal numbers are employed. The only essential difference is that the children count on to the end of the series of vocables round and round the ring of objects counted; while the process of counting a collection is brought to an end exclusively by the exhaustion of the collection, to which thereafter the last numeral word used is applied as an adjective. This adjective thus expresses nothing more than the relation of the collection to the series of

vocables.

Peirce: CP 4.156 Cross-Ref:††

156. Still, there is a real fact of great importance about the collection itself which is at once deducible from that relation; namely, that the collection cannot be in a one-to-one correspondence with any collection to which is applicable an adjective derived from a subsequent vocable, but only to a part of it; nor can any collection to which is applicable an adjective derived from a preceding collection be in a one-to-one correspondence with this collection, but only with a part of it; while, on the other hand, this collection is in one-to-one correspondence with every collection to which the same numeral adjective is applicable. This, however, is not essentially implied as a part of the significance of the adjective. On the contrary, it is only shown by means of a theorem, called "The Fundamental Theorem of Arithmetic,"†1 that this is an attribute of the collections themselves and not an accident of the particular way in which they have been counted. Nevertheless, this is a complete justification for the statement that quantity -- in this case, multitude, or collectional quantity -- is an attribute of the collections themselves. I do not think of denying this; nor do I mean that any kind of quantity is merely subjective. I am simply not using the word quantity in that acception. I am not speaking of physical, but of mathematical, quantity.

Peirce: CP 4.157 Cross-Ref:††

157. Were I to undertake to establish the correctness of my statement that the cardinal numerals are without meaning, I should unavoidably be led into a disquisition upon the nature of language quite astray from my present purpose. I will only hint at what my defence of the statement would be by saying that, according to my view, there are three categories of being; ideas of feelings, acts of reaction, and habits.†2 Habits are either habits about ideas of feelings or habits about acts of reaction. The ensemble of all habits about ideas of feeling constitutes one great habit which is a World; and the ensemble of all habits about acts of reaction constitutes a second great habit, which is another World. The former is the Inner World, the world of Plato's forms. The other is the Outer World, or universe of existence. The mind of man is adapted to the reality of being. Accordingly, there are two modes of association of ideas: inner association, based on the habits of the inner world, and outer association, based on the habits of the universe.†1 The former is commonly called association by resemblance; but in my opinion, it is not the resemblance which causes the association, but the association which constitutes the resemblance. An idea of a feeling is such as it is within itself, without any elements or relations. One shade of red does not in itself resemble another shade of red. Indeed, when we speak of a shade of red, it is already not the idea of the feeling of which we are speaking but of a cluster of such ideas. It is their clustering together in the Inner World that constitutes what we apprehend and name as their resemblance. Our minds, being considerably adapted to the inner world, the ideas of feelings attract one another in our minds, and, in the course of our experience of the inner world, develop general concepts. What we call sensible qualities are such clusters. Associations of our thoughts based on the habits of acts of reaction are called associations by contiguity, an expression with which I will not quarrel, since nothing can be contiguous but acts of reaction. For to be contiguous means to be near in space at one time; and nothing can crowd a place for itself but an act of reaction. The mind, by its instinctive adaptation to the Outer World, represents things as being in space, which is its intuitive representation of the clustering of reactions. What we call a Thing is a cluster or habit of reactions, or, to use a more familiar phrase, is a centre of forces.†2 In consequence, of this double

mode of association of ideas, when man comes to form a language, he makes words of two classes, words which denominate things, which things he identifies by the clustering of their reactions, and such words are proper names, and words which signify, or *mean*, qualities, which are composite photographs of ideas of feelings, and such words are verbs or portions of verbs, such as are adjectives, common nouns, etc.

Peirce: CP 4.158 Cross-Ref:††

158. Thus, the cardinal numerals in being called meaningless are only assigned to one of the two main divisions of words. But within this great class the cardinal numerals possess the unique distinction of being mere instruments of experimentation. "This" and "that" are words designed to stimulate the person addressed to perform an act of observation; and many other words have that character; but these words afford no particular help in making the observation. At any rate, any such use is quite secondary. But the sole uses of the cardinal numbers are, first, to count with them, and second, to state the results of such counts.

Peirce: CP 4.159 Cross-Ref:††

159. Of course, it is impossible to count anything but clusters of acts, *i.e.*, events and things (including persons); for nothing but reaction-acts are individual and discrete. To attempt, for example, to count all possible shades of red would be futile. True, we count the notes of the gamut; but they are not all possible pitches, but are merely those that are customarily used in music, that is, are but habits of action. But the system of numerals having been developed during the formative period of language, are taken up by the mathematician, who, generalizing upon them, creates for himself an ideal system after the following precepts.

Peirce: CP 4.160 Cross-Ref:††

§2. PRECEPTS FOR THE CONSTRUCTION

OF THE SYSTEM OF ABSTRACT NUMBERS^{SP}

160. First, There is a relation, *G*, such that to every *number*, *i.e.*, to every object of the system, a different number is *G* and is *G* to that number alone; and we may say that a number to which another is *G* is "*G*'d" by that other;

Peirce: CP 4.160 Cross-Ref:††

Second, There is a number, called zero, 0, which is *G* to no cardinal number;

Peirce: CP 4.160 Cross-Ref:††

Third, The system contains no object that it is not necessitated to contain by the first two precepts. That is to say, a given description of number only exists provided the first two precepts require the existence of a number which may be of that description.

Peirce: CP 4.161 Cross-Ref:††

161. This system is a cluster of ideas of individual things; but it is not a cluster of real things. It thus belongs to the world of ideas, or Inner World. Nor does the mathematician, though he "creates the idea *for himself*," create it absolutely. Whatever it may contain of [that which is] impertinent [to Mathematics] is soilure from [elsewhere]. The idea in its purity is an eternal being of the Inner World.

Peirce: CP 4.162 Cross-Ref:††

162. This idea of discrete quantity having an absolute minimum subsequently suggests the ideas of other systems, all of which are characterized by the prominence of transitive relations. These mathematical ideas, being then applied in physics to such phenomena as present analogous relations, form the basis of systems of measurement. Throughout them all, succession is the prominent relation; and all measurement is affected by two operations. The first is the experiment of super-position, the result of which is that we say of two objects, A and B , A is (or is not) in the transitive relation, t , to B , and B is (or is not) in the relation, t , to A ; while the second operation is the experiment of counting. The question "How much is A ?" only calls for the statement, A has the understood transitive relation to such things, and such things have this relation to A .

Peirce: CP 4.163 Cross-Ref:††

§3. APPLICATION OF THE THEORY TO ARITHMETIC \mathcal{P}

163. According to the theory partially stated above, pure arithmetic has nothing to do with the so-called Fundamental Theorem of Arithmetic.†1 For that theorem is that a finite collection counts up to the same number in whatever order the individuals of it are counted. But pure arithmetic considers only the numbers themselves and not the application of them to counting.

Peirce: CP 4.164 Cross-Ref:††

164. In order to illustrate the theory, I will show how the leading elementary propositions of pure arithmetic are deduced, and how it is subsequently applied to counting collections.

Peirce: CP 4.164 Cross-Ref:††

Corollary 1. No number is G of more than one number. For every number necessitated by the first precept is G to a single number, and the only number necessitated by the second precept, by itself, is G to no number. Hence, by the third precept, there is no number that is G to two numbers.

Peirce: CP 4.164 Cross-Ref:††

Corollary 2. No number is G 'd by two numbers. For were there a number to which two numbers were G , one of the latter could be destroyed without any violation of the first two precepts, since the destruction would leave no number without a G which before had one, nor would it destroy 0, since that is not G . Hence, by the third precept, there is no number which is G to a number to which another number is G .

Peirce: CP 4.164 Cross-Ref:††

Corollary 3. No number is G to itself. For every number necessitated by the first precept is G to a different number, and to that alone; and the only number necessitated by the second precept, by itself, is G to no number.

Peirce: CP 4.164 Cross-Ref:††

Corollary 4. Every number except zero is G of a number. For every number necessitated by the first precept is so, and the only number directly necessitated by the second is zero.

Peirce: CP 4.164 Cross-Ref:††

Corollary 5. There is no class of numbers every one of which is G of a number of that class. For were there such a class, it could be entirely destroyed without conflict with precepts 1 and 2. For such destruction could only conflict with the first precept if it destroyed the number that was G to a number without destroying the latter. But no number of such a class could be G of any number out of the class by the first corollary. Nor could zero, the only number required to exist by the second precept alone, belong to this class, since zero is G to no number. Therefore, there would be no conflict with the first two precepts, and by the third precept such a class does not exist.

Peirce: CP 4.165 Cross-Ref:††

165. The truly fundamental theorem of pure arithmetic is not the proposition usually so called, but is the Fermatian principle, which is as follows:

Peirce: CP 4.165 Cross-Ref:††

Theorem I. The Fermatian Principle: *Whatever character belongs to zero and also belongs to every number that is G of a number to which it belongs, belongs to all numbers.*

Peirce: CP 4.165 Cross-Ref:††

Proof. For were there any numbers which did not possess that character, their destruction could not conflict with the first precept, since by hypothesis no number without that character is G to a number with it. Nor would their destruction conflict with the second precept directly, since by hypothesis zero is not one of the numbers which would be destroyed. Hence, by the third precept, there are no numbers without the character.

Peirce: CP 4.166 Cross-Ref:††

166. **Definition 1.** Any number, M , is, or is not, said to be *greater than*, a number, N , and N to be, or not to be, *less than M* , according to [whether] the following conditions are, or are not, fulfilled:

Peirce: CP 4.166 Cross-Ref:††

First, Every number G to another is greater than that other;

Peirce: CP 4.166 Cross-Ref:††

Second, Every number greater than a second, itself greater than a third, is greater than that third;

Peirce: CP 4.166 Cross-Ref:††

Third, No number is greater than another unless the above two conditions necessitate its being so.

Peirce: CP 4.166 Cross-Ref:††

Theorem II. Every cardinal number except zero is greater than zero.†1

Peirce: CP 4.166 Cross-Ref:††

Theorem III. No cardinal number L is greater than any number, M , unless L is G to a cardinal number, N , which either is greater than [or equal to] M .

Peirce: CP 4.166 Cross-Ref:††

Corollary 1. By the same reasoning (substituting everywhere less for "greater" and $G'd$ by for " G of") no number M is *less* than any number L unless L be G to M or

be greater than the number that is G to M .

Peirce: CP 4.166 Cross-Ref:††

Corollary 2. Hence, by the first and second conditions of the definition, if a cardinal number, L is greater than a cardinal number, M , then the number that is G to L is greater than the number that is G to M .

Peirce: CP 4.166 Cross-Ref:††

Corollary 3. Zero is greater than no number.

Peirce: CP 4.166 Cross-Ref:††

Corollary 4. Every number greater than a number is G of some number.

Peirce: CP 4.166 Cross-Ref:††

Theorem IV. No cardinal number is both greater and less than the same cardinal number.

Peirce: CP 4.166 Cross-Ref:††

Corollary 1. No number is either greater or less than itself.

Peirce: CP 4.166 Cross-Ref:††

Corollary 2. No cardinal number, M , is greater than a cardinal number, N , and less than GN .

Peirce: CP 4.166 Cross-Ref:††

Theorem V. Of any two different cardinal numbers, one is greater than the other.

Peirce: CP 4.166 Cross-Ref:††

Corollary. If the cardinal number, GL , that is G to L , be greater than the cardinal number, GM , that is G to M , by Theorem IV it cannot be less. Hence by Corollary 2 from Theorem III, L cannot be less than M . But by the first corollary from Theorem IV, GL is not GM , and therefore L is not M . Hence L is greater than M if GL is greater than GM .

Peirce: CP 4.167 Cross-Ref:††

167. **Theorem VI. (Modified Fermatian Principle.)** If a character, α , be such that, taking any two cardinal numbers, A and Z , either α does not belong to both A and Z , or no cardinal number is greater than A and less than Z , or SOME cardinal number greater than A and less than Z has the character, α , then, α is also such that, taking any two cardinal numbers, B and Y , either α does not belong both to B and Y , or no cardinal number is greater than B and less than Y , or EVERY cardinal number greater than B and less than Y has the character, α .

Peirce: CP 4.167 Cross-Ref:††

Proof. For were there an exception, there would be, at least, one cardinal number of a class we may call the n 's fulfilling the conditions that every n is greater than B and less than Y and no number at once greater than an n and less than an n possess α . Then, by Corollary 4 of Theorem III, every n , and also every number greater than every n , would be G to some cardinal number; and by Corollary 5 from the general precepts there would be some n G to a cardinal number, not an n , which we may call M , and there would be some number greater than every n which would be G to some n which we may call N . But then GN and M would possess α , and if any n 's existed, they would be greater than M and less than GN and yet there would

be *no* cardinal number greater than M and less than GN having α . Hence it is absurd to suppose any exception.

Peirce: CP 4.168 Cross-Ref:††

168. **Definition 2.** A *sum* of a cardinal number, M , *added to* a cardinal number, N , is a cardinal number which fulfills the following conditions:

Peirce: CP 4.168 Cross-Ref:††

First, A sum of zero added to zero is zero;

Peirce: CP 4.168 Cross-Ref:††

Second, A sum of zero added to a cardinal number which is G to any cardinal number, N , is a number which is G to a sum of zero added to N ;

Peirce: CP 4.168 Cross-Ref:††

Third, A sum of a cardinal number that is G to any cardinal number, M , added to any cardinal number, N , is a cardinal number which is G to a cardinal number which is a sum of M added to N ;

Peirce: CP 4.168 Cross-Ref:††

Fourth, No cardinal number is a sum of a cardinal number added to a cardinal number unless it is necessitated to be so by the above conditions.

Peirce: CP 4.168 Cross-Ref:††

Theorem VII. *There is one cardinal number, and but one, which is a sum of a cardinal number, M , added to a cardinal number, N .*

Peirce: CP 4.168 Cross-Ref:††

Corollary 1. Whatever cardinal numbers M and N may be, $M+N>0$ unless $M = N = 0$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 2. Whatever cardinal numbers M and N may be $M+N>N$ unless $M = 0$, and $M+N>M$ unless $N = 0$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 3. Whatever cardinal numbers M and N may be, $M+GN = G(M+N)$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 4. Whatever cardinal number N may be, $0+N = N$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 5. Whatever cardinal number N may be, $N+0 = N$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 6. Whatever cardinal numbers M and N may be, $M+N = N+M$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 7. Whatever cardinal numbers L , M , and N may be, $L+(M+N) = (L+M)+N$.

Peirce: CP 4.168 Cross-Ref:††

Theorem VIII. *The sum of a greater cardinal number, L , added to any cardinal number, N , is greater than the sum of a lesser cardinal number, M , added to the same cardinal number, N .*

Peirce: CP 4.168 Cross-Ref:††

Corollary 1. Whatever cardinal numbers L , M , and N may be if $L > M$, then $N + L > N + M$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 2. Whatever cardinal numbers A , B , C , D may be, if $A > C$ and $B > D$ then $A + B > C + D$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 3. Whatever cardinal numbers L , M , and N may be unless $L = M$, $L + N$ or $N + L$ is not $M + N$ or $N + M$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 4. If $L + N > M + N$ then $L > M$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 5. If $A + B > C + D$, either A or B is greater than C and than D or else either C or D is less than A and than B .

Peirce: CP 4.168 Cross-Ref:††

Theorem IX. Whatever cardinal numbers L and M may be, there is one and only one cardinal number, N , such that either $N + M = L$ or $N + L = M$.

Peirce: CP 4.168 Cross-Ref:††

Definition 3. The *difference* between two cardinal numbers, L and M , is such a number, N , that either $N + M = L$ or $N + L = M$. It is said to be the *remainder* after *subtracting* the smaller as *subtrahend*, from the larger, as *minuend*. It is best denoted by the "minus sign" written after the larger of L and M and before the smaller.

Peirce: CP 4.168 Cross-Ref:††

Corollary 1. $L - M$ is no cardinal number unless $L > M$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 2. If $L > M$, then $(L - M) + M = L$.

Peirce: CP 4.168 Cross-Ref:††

Definition 4. The *product* of a cardinal number, M , *multiplied* into a cardinal number, N , or the product of the *multiplicand*, N , *multiplied by* the *multiplier*, M , is a cardinal number, written MXN , or $M.N$, or MN , subject to the following conditions:

Peirce: CP 4.168 Cross-Ref:††

First, zero is a product of any cardinal number multiplied by 0;

Peirce: CP 4.168 Cross-Ref:††

Second, a product of a cardinal number, N , multiplied by the cardinal number, GM , that is G to any cardinal number, M , is the sum, $N + M.N$, of N added to the product of M multiplied into N ;

Peirce: CP 4.168 Cross-Ref:††

Third, no cardinal number is a product of cardinal numbers unless necessitated to be so by the foregoing conditions.

Peirce: CP 4.168 Cross-Ref:††

A product is said to be a *multiple* of its multiplicand.

Peirce: CP 4.168 Cross-Ref:††

Theorem X. *There is one cardinal number and but one which is $M.N$ a product of one cardinal number, M , multiplied into a given cardinal number, N .*

Peirce: CP 4.168 Cross-Ref:††

Corollary 1. The product of any cardinal number, N , multiplied into zero is zero.

Peirce: CP 4.168 Cross-Ref:††

Corollary 2. Whatever cardinal numbers M and N may be $MXN > 0$ unless $M = 0$ or $N = 0$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 3. The product of any cardinal number, N , multiplied by the cardinal number that is G to zero (which is called 1, one) is N .

Peirce: CP 4.168 Cross-Ref:††

Corollary 4. The product of any cardinal number, N , multiplied into $G0$, or 1, is N .

Peirce: CP 4.168 Cross-Ref:††

Corollary 5. Whatever cardinal numbers M and N may be, $MXGN = M+M.N$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 6. Whatever cardinal numbers M and N may be, $MXN > M$ unless $M = 0$ or $N = 0$ or $N = G0$, and $MXN > N$ unless $N = 0$ or $M = 0$ or $M = G0$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 7. Whatever cardinal numbers L , M , and N may be, $L.(M+N) = L.M+L.N$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 8. Whatever cardinal numbers L , M , N may be $L.(M.N) = (L.M).N$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 9. Whatever cardinal numbers M and N may be $M.N = N.M$.

Peirce: CP 4.168 Cross-Ref:††

Theorem XI. *Of two products of the same multiplicand not zero, that by the greater multiplier is the greater.*

Peirce: CP 4.168 Cross-Ref:††

Corollary 1. If $L > M$, $N.L > N.M$ unless $N = 0$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 2. If $A > C$ and $B > D$, $AXB > CXD$ in all cases.

Peirce: CP 4.168 Cross-Ref:††

Corollary 3. Unless $L = M$, LXN is not MXN , unless $N = 0$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 4. If $LXN > MXN$, then $L > M$ in all cases.

Peirce: CP 4.168 Cross-Ref:††

Corollary 5. If $AXB > CXD$, either A or B is greater than C and than D , or C or

D is less than A and than B .

Peirce: CP 4.168 Cross-Ref:††

Corollary 6. If either B or C is greater than A or than D , then $B.C > A.D$, unless $A + D > B + C$.

Peirce: CP 4.168 Cross-Ref:††

Definition 5. A *divisor* of a cardinal number, N , is a cardinal number which multiplied by a cardinal number gives N as product. The number, N , is said to be *exactly divisible* by its divisor.

Peirce: CP 4.168 Cross-Ref:††

Abbreviations. We may write $N \equiv N' \pmod{M}$ where M is any cardinal number, not zero, to express that N and N' are cardinal numbers leaving the same remainder after division by M . We may denote the remainder and quotient of N divided by M by $R[m]N$ and $Q[m]N$, respectively. Then $N = R[m]N + (Q[m]N) \cdot M$.

Peirce: CP 4.168 Cross-Ref:††

We may denote $GG0$ by Q .

Peirce: CP 4.168 Cross-Ref:††

Scholium. The number Q is logically and mathematically peculiar. In old arithmetics multiplication and division by Q are considered as peculiar operations, *Duplation* and *Mediation*. We have need of an arithmetic of two, even in reasonings which do not concern quantity in the ordinary sense.

Peirce: CP 4.168 Cross-Ref:††

Theorem XII. Every cardinal number, N , has with reference to every cardinal number, M , except zero, a remainder, $R[m]N$, and a quotient, $Q[m]N$; and only one number is remainder or quotient.

Peirce: CP 4.168 Cross-Ref:††

Corollary 1. If the cardinal number, N , is less than the modulus, M , its remainder, $R[m]N = N$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 2. The remainder of the sum of two numbers, N and N' , is the remainder of the sum of their remainders.

Peirce: CP 4.168 Cross-Ref:††

Corollary 3. The remainder of the product of two numbers, N and N' , is the remainder of the product of the remainders.

Peirce: CP 4.168 Cross-Ref:††

Corollary 4. The quotient of the sum of two numbers is the sum of the quotient of the sum of the remainders added to the sum of the quotients of the numbers.

Peirce: CP 4.168 Cross-Ref:††

Corollary 5. The quotient of the product of two numbers, N and N' , is the sum of the product of N by $Q[m]N'$, the quotient of the other, added to the quotient of the product of N by $R[m]N'$, the remainder of the other. Or $Q[m](N.N') = N.Q[m]N' + Q[m](N.R[m]N')$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 6. Given any cardinal number, N , and any modular number, M , there is a multiple of M greater than N . For $(GQ[m]N).M$ is such a multiple.

Peirce: CP 4.168 Cross-Ref:††

Definition 6. The *powers* of any cardinal number, B , called the *base* of the powers, are a class of cardinal numbers, each having a cardinal number, E , connected with it, called its exponent; and the power is written, BE , and powers and exponents are defined by the following conditions:

Peirce: CP 4.168 Cross-Ref:††

First, $G0$ is a power of B whose exponent is zero;

Peirce: CP 4.168 Cross-Ref:††

Second, The product of the power BE , of B with exponent E , multiplied by B is BGE , a power of B with exponent GE ;

Peirce: CP 4.168 Cross-Ref:††

Third, No cardinal number is a power of a cardinal number, unless necessitated to be so by the foregoing conditions.

Corollary. $BG0 = BXB^0 = BXG0 = B$.

Peirce: CP 4.168 Cross-Ref:††

Theorem XIII. Given any two cardinal numbers, B and E , there is one, and but one cardinal number which is a power, BE , of B with exponent, E .

Peirce: CP 4.168 Cross-Ref:††

Corollary 1. Hence, $0^0 = G0$ and is not indeterminate. In this respect, the definition here assumed differs from the usual one, which substitutes for the first condition $B^1 = B$ and adds the condition that $BE = P$ if $BGE = B.P$. But practically the present definition is just as useful, if not more so, than the usual one.

Peirce: CP 4.168 Cross-Ref:††

Theorem XIV. A given exponent of two powers with the same exponent greater than zero, that with the greater base is the greater, and two powers of the same base greater than $G0$, that with the greater exponent is the greater.

Peirce: CP 4.168 Cross-Ref:††

Definition 7. An *even number* is a cardinal number whose remainder, relative to $GG0$ as modulus, is *zero*.

Peirce: CP 4.168 Cross-Ref:††

An *odd number* is a cardinal number whose remainder, relative to $GG0$ as modulus, is $G0$.

Peirce: CP 4.168 Cross-Ref:††

Corollary 1. Every cardinal number, N , is even or odd; and if N be even, GN is odd and *vice versa*.

Peirce: CP 4.168 Cross-Ref:††

Corollary 2. The double of a cardinal number, N , is $N+N$, the sum of N added to itself.

Peirce: CP 4.168 Cross-Ref:††

Corollary 3. If a number is even, it has a cardinal number that is half of it, but

if it is odd, it has not.

Peirce: CP 4.168 Cross-Ref:††

Corollary 4. If the difference of two cardinal numbers, M and N , is even, those two numbers have a cardinal number as their arithmetical mean, and the difference between this mean and either M or N is half the difference between M and N .

Peirce: CP 4.169 Cross-Ref:††

169. **Theorem XV.** (Binary form of the Fermatian Principle.) *If any character belongs to every power of $GG0$ and also to the mean of any two numbers having a mean, if it belongs to the numbers themselves, then it belongs to every cardinal number except 0.*

Peirce: CP 4.170 Cross-Ref:††

VI

MULTITUDE AND NUMBER†1P

§1. THE ENUMERABLE

170. Let us consider the relation of a constituent unit to the collective whole of which it forms a part. Suppose A to be such a unit and B to be such a whole. Then in order to avoid the circumlocution of saying that A is a constituent unit of B as the collective whole of which it is a unit, I shall simply say A is a unit of B , and shall write " A is a u of B "; or I may reverse the order in which A and B are mentioned by writing " B is u 'd by A ."

Peirce: CP 4.170 Cross-Ref:††

The only logical peculiarities of this relation are as follows:

Peirce: CP 4.170 Cross-Ref:††

First, Whatever is u of anything is u 'd by itself and by nothing else. Hence, if anything is u 'd by anything not itself, it is not itself u of anything; and consequently nothing that is u 'd by anything but itself is u 'd by itself.

Peirce: CP 4.170 Cross-Ref:††

Second, Whatever is not u 'd by anything does not exist.

Peirce: CP 4.171 Cross-Ref:††

171. By a collection, I mean anything which is u 'd by whatever has a certain quality, or general description, and by nothing else. That is, if C is a collection, there is some quality, α , such that taking anything whatever, say x , either x possesses the quality of α and is a unit of C , or else it neither possesses the quality α nor is a unit of C . On the other hand, if C is not a collection, no matter what quality or general description, β may be taken, there is either something possessing the quality β without being a unit of C , or there is some unit of C which does not possess the

quality, β .

Peirce: CP 4.171 Cross-Ref:††

It will be perceived, therefore, that there is a collection corresponding to every common noun or general description. Corresponding to the common noun "man" there is a collection of men; and corresponding to the common noun "fairy" there is a collection of fairies. It is true that this last collection does not exist, or as we say, the total number of fairies is zero. But though it does not exist, that does not prevent it from being of the nature of a collection, any more than the non-existence of fairies deprives them of their distinguishing characteristics. . . .

Peirce: CP 4.172 Cross-Ref:††

172. Whether the constituent individuals or units of a collection have each of them a distinct identity of its own or not, depends upon the nature of the universe of discourse. If the universe of discourse is a matter of objective and completed experience, since experience is the aggregate of mental effect which the course of life has forced upon a man, by a brute bearing down of any will to resist it, each such act of brute force is destitute of anything reasonable (and therefore of the element of generality, or continuity, for continuity and generality are the same thing), and consequently the units will be individually distinct. It is such collections that I desire first to call your attention. I put aside then, for the present, such collections as the drops of water in the sea; and assume that the units are of such a kind that they may be absolutely distinguished from one another. Then, I say, as long as the discourse relates to a common objective and completed experience, those units *retain* each its distinct identity. If you and I talk of the great tragedians who have acted in New York within the last ten years, a definite list can be drawn up of them, and each of them has his or her proper name. But suppose we open the question of how far the general influences of the theatrical world at present favor the development of female stars rather than of male stars. In order to discuss that, we have to go beyond our *completed* experience, which may have been determined by accidental circumstances, and have to consider the possible or probable stars of the immediate future. We can no longer assign proper names to each. The individual actors to which our discourse now relates become largely merged into general varieties; and their separate identities are partially lost. Again, statisticians can tell us pretty accurately how many people in the city of New York will commit suicide in the year after next. None of these persons have at present any idea of doing such a thing, and it is very doubtful whether it can properly be said to be determinate now who they will be, although their number is approximately fixed. There is an approach to a want of distinct identity in the individuals of the collection of persons who are to commit suicide in the year 1899. When we say that of all possible throws of a pair of dice one thirty-sixth part will show sixes, the collection of possible throws which have not been made is a collection of which the individual units have no distinct identity. It is impossible so to designate a single one of those possible throws that have not been thrown that the designation shall be applicable to only one definite possible throw; and this impossibility does not spring from any incapacity of ours, but from the fact that in their own nature those throws are not individually distinct. The possible is necessarily general; and no amount of general specification can reduce a general class of possibilities to an individual case. It is only actuality, the force of existence, which bursts the fluidity of the general and produces a discrete unit. Since Kant it has been a very wide-spread idea that it is time and space which introduce continuity into nature. But this is an *anacoluthon*. Time and space are continuous because they embody conditions of possibility, and the possible is general, and continuity and generality are

two names for the same absence of distinction of individuals.

Peirce: CP 4.172 Cross-Ref:††

When the universe of discourse relates to a common experience, but this experience is of something imaginary, as when we discuss the world of Shakespeare's creation in the play of Hamlet, we find individual distinction existing so far as the work of imagination has carried it, while beyond that point there is vagueness and generality. So, in the discussion of the consequences of a mathematical hypothesis, as long as we keep to what is distinctly posited and its positive implications, we find discrete elements, but when we pass to mere possibilities, the individuals merge together. This remark will be fully illustrated in the sequel.

Peirce: CP 4.173 Cross-Ref:††

173. A *part* of a collection called its *whole* is a collection such that whatever is *u* of the part is *u* of the whole, but something that is *u* of the whole is not *u* of the part.

Peirce: CP 4.174 Cross-Ref:††

174. It is convenient to use this locution; namely, instead of saying A is in the relation, *r*, to B, we may say A is an *r* to B, or of B; or, if we wish to reverse the order of mentioning A and B, we may say B is *r*'d by A.

Peirce: CP 4.174 Cross-Ref:††

If a relation, *r*, is such that nothing is *r* to two different things, and nothing is *r*'d by two different things, so that some things in the universe are perhaps *r* to nothing while all the rest are *r*, each to its own distinct correlate, and there are some things perhaps to which nothing is *r*, but all the rest have each a single thing that is *r* to it, then I call *r* a *one-to-one relation*. If there be a one-to-one relation, *r*, such that every unit of one collection is *r* to a unit of a second collection, while every unit of the second collection is *r*'d by a unit of the first collection, those two collections are commonly said to be in a one-to-one correspondence with one another.†1. . .

Peirce: CP 4.175 Cross-Ref:††

175. I shall use the word *multitude* to denote that character of a collection by virtue of which it is greater than some collections and less than others, provided the collection is *discrete*, that is, provided the constituent units of the collection are or may be distinct. But when the units lose their individual identity because the collection exceeds every positive existence of the universe, the word multitude ceases to be applicable. I will take the word *multiplicity* to mean the greatness of any collection discrete or continuous.

Peirce: CP 4.176 Cross-Ref:††

176. We have to note the precise meaning of saying that a relation of a given description exists. A relation of the kind here considered has been called an *ens rationis*; but it cannot be said that because nobody has ever constructed it -- perhaps never will -- it exists any the less on that account. Its existence consists in the fact that, if it were constructed, it would involve no contradiction. An easy dilemma will show that to suppose three things to be in one-to-one correspondence with individuals of a pair involves contradiction. But it is much more difficult to prove that a given hypothesis *involves no contradiction*. In mathematics, such propositions are usually replaced by so-called "problems." That is to say, a construction shows *how* the thing in question can take place. When we know how it can take place, we know, of course, that it is possible. Cases are rare in mathematics in which anything is shown to be

possible without its being shown how. But when we come to philosophical questions, such a construction is generally practically beyond our powers; and it becomes necessary to examine the principles of logic in order to discover a general method of proving that a given hypothesis involves no contradiction. Without a thorough mastery of the principles of logic such a search must be fruitless.

Peirce: CP 4.176 Cross-Ref:††

Mathematics never has hypotheses forced upon it that are perplexing from [their] seemingly irresolvable mistiness -- which is the aspect of such a question of philosophical possibility, at first sight. Mathematics does not *need* to take up any hypothesis that is not crystal-clear. Unfortunately, philosophy cannot choose its first principles at will, but has to accept them as they are.

Peirce: CP 4.177 Cross-Ref:††

177. For example, the relations of equality and excess of multitude having been defined after Cantor, philosophy can not avoid the question which instantly springs up: must every two collections be either equal or the one greater than the other, or can they be so multitudinous that the units of neither can be in one-to-one relation to units of the other?

Peirce: CP 4.177 Cross-Ref:††

To say that the collection of *M*'s and the collection of *N*'s are equal is to say:

Peirce: CP 4.177 Cross-Ref:††

There is a one-to-one relation, *c*, such that every *M* is *c* to an *N*; and there is a one-to-one relation, *d*, such that every *N* is *d* to an *M*.

Peirce: CP 4.177 Cross-Ref:††

To say that the collection of *M*'s is less than the collection of *N*'s is to say:

Peirce: CP 4.177 Cross-Ref:††

There is a one-to-one relation, *c*, such that every *M* is *c* to an *N*; but whatever one-to-one relation *d* may be, some *N* is not *d* to any *M*.

Peirce: CP 4.177 Cross-Ref:††

To say the collection of *M*'s is greater than the collection of *N*'s is to say:

Peirce: CP 4.177 Cross-Ref:††

Whatever one-to-one relation *c* may be, some *M* is not *c* to any *N*; but there is a one-to-one relation *d* such that every *N* is *d* to an *M*.

Peirce: CP 4.177 Cross-Ref:††

Now, formal logic suggests the fourth relation:

Peirce: CP 4.177 Cross-Ref:††

Whatever one-to-one relation *c* may be, some *M* is not *c* to any *N*, and whatever one-to-one relation *d* may be, some *N* is not *d* to any *M*.

Peirce: CP 4.177 Cross-Ref:††

Or this last may be stated more simply thus:

Peirce: CP 4.177 Cross-Ref:††

Whatever one-to-one relation *c* may be, some *M* is not *c* to any *N* and some *N* is not *c*'d by any *M*.

Peirce: CP 4.177 Cross-Ref:††

How shall we proceed in order to find out whether this last relation is a possible one, or not? . . .

Peirce: CP 4.178 Cross-Ref:††

178. In the first place, it must not be supposed that even if a collection is so great that the constituent units lose their individual identity, a one-to-one relation necessarily becomes impossible. If such a relation implied an actual operation performed, it would indeed be impossible, I suppose. But this is not the case. As the collection enlarges and the individual distinctions are little by little merged, it also passes out of the realm of brute force into the realm of ideas which is governed by rules. This sounds vague, because until I have shown you how to develop the idea of such a collection, I can offer you no example. But it is not necessary actually to construct the correspondence. It suffices to suppose that a certain number of units of the two collections having been brought into such a relation (and, in fact, they always are in such relations), then the general rules of the genesis of the two collections necessitate the falling of all the other individuals into their places in the correspondence. All this will become quite clear in the sequel.

Peirce: CP 4.179 Cross-Ref:††

179. That difficulty, then, having been removed, we have two collections, the M's and the N's; and the question is whether there is, no matter what these collections may be, always either some one-to-one relation, *c*, such that any M is *c* to an N or else some one-to-one relation, *d*, such that every N is *d*'d by an M. To begin with, there are vast multitudes of relations such that taking any one of them, *r*, every M is *r* to an N and every N is *r*'d by an M. For example, the relations of coexistence, maker of, non-husband of, etc. In general, each M can have any set of N's whatever as its correlates, except that there must be *one* of the M's that shall have among its correlates all those N's that are not *r*'d by any other M. And all those sets of N's for each M can be combined in any way whatever. In order to make our ideas more clear, let us for the moment suppose that the M's are equal to the finite number, {*m*}, and the N's are equal to the finite number, {*n*}. Then, for each M except one there are $2^{\{n\}}-1$ different sets of N's, any one of which can be its correlates. Hence, there are $(2^{\{n\}}-1)^{\{m\}}-1$ different forms of the relation *r*, without taking account of the variety of different sets of correlates which the remaining M may have. Suppose we had a diagram of each of those relations, each diagram showing the collection of M's above and the collection of N's below, with lines drawn from each M to all the N's of which it was *r*. Each of that stupendous multitude of relations may be modified so as to reduce it [to] what we may call a one-to-*x* relation, by running through the N's and cutting away the connection of each N with every M but one; and each of the *r* relations could be thus cut down in a vast multitude of different ways. Call any such resulting relation, *s*. Then, every N would be *s*'d by a single M. Each one of the *r* relations could also be so modified as to reduce it to what we may call an *x*-to-one relation, by running through the M's and cutting off the connection of each M with every N but one. Call such a resulting relation, *t*. Then, every M would be *t* to a single N. Suppose we had a collection of diagrams showing all the ways in which every *r* relation could thus be reduced to an *s* relation or a *t* relation, that is, be reduced to a one-to-*x* relation or to an *x*-to-one relation. The question is, could the multitudes of M and N, be such that there would not be a single one-to-one relation among all those one-to-*x* relations [which each M has to an N] and *x*-to-one relations [which each N has to an M]? If among the diagrams of the one-to-*x* relations there were not one

where the one-to- x relation was a one-to-one relation, it would be because in each case there was some M which was s [*i.e.*, one-to- x] to two or more N 's. If, then, there were any of these diagrams in which some M was not s to any N , those diagrams could be thrown out of consideration, because there was no necessity for a pluralism of lines to one M , as long as there were M 's to which no line ran; and since there was no necessity for it, there is no need of modifying those diagrams so as to take away plural lines from some of the M 's so as to give lines to all the M 's, because, *since there is a diagram for every possible modification changing an r [x - x] relation to an s [one- x] relation*, there must already be a diagram remedying this fault. There must, therefore, be among the diagrams, some diagrams in which every M is s to an N -- unless indeed there is a diagram where the s is a one-to-one relation. Taking, then, any diagram in which every M is s to an N , all it is necessary to do is to erase all the lines but one which go to each M , and the relation so resulting, which we may call u , is such that every M is u to an N , no N is u 'd by two M 's (for no N is s 'd by two M 's and the erasures cannot increase the relates of any N), and no M is u of two N 's. In other words, u is a one-to-one relation, and every M is u of an N . Q.E.D.

Peirce: CP 4.179 Cross-Ref:††

Is this demonstration sound? It may be doubted; at any rate I can show you how by a very small modification it would certainly become unsound; and thus direct your attention to the point which requires scrutiny. If, instead of casting aside those diagrams of s relations which showed some M 's that are not s to any N , I had proposed to cure them by changing the course of lines from M 's having two or more lines to M 's having none, until there were either no M 's left without any lines or no M 's left with pluralities of lines, I should have fallen into a gross *petitio principii*. For I should be assuming that, of those two classes of M 's, the whole of one (whichever it might be) could be put into a one-to-one relation with the whole or a part of the other; and whether or not this is always possible is the very question at issue.

Peirce: CP 4.179 Cross-Ref:††

But the true argument is this: Nothing can force *all* of the s diagrams to show pluralities of lines to M 's except the fact that some of them show lines to all the M 's. For since all possibilities are represented in the diagrams, if all the diagrams show pluralities of lines to M 's, there must be a logical necessity for this, so that the conditions would be contradicted if it were not so. Now the only logical necessity there can be in making some lines terminate at M 's, that already have lines, is that there are no M 's that have not already lines. Hence, in some cases, at least, all the M 's must have lines.

Peirce: CP 4.179 Cross-Ref:††

The gist of this argument is that it considers in what way contradiction can arise, and thus shows that the only circumstance which could render the one-to-one correspondence impossible in one way, necessarily renders it possible in another way.

Peirce: CP 4.180 Cross-Ref:††

180. I will now prove two general theorems of great importance. The first is, that the collection of possible sets of units (including the set that includes no units at all) which can be taken from discrete collections is always greater than the collection of units.†1 . . .

Peirce: CP 4.180 Cross-Ref:††

The other theorem, which gives great importance to the first, is that if a collection is not too great to be discrete, that is, to have all its units individually

distinct, neither is the collection of sets of units that can be generally formed from that collection too great to be discrete.

Peirce: CP 4.180 Cross-Ref:††

For we may suppose the units of the smaller collection to be independent characters, and the larger collection to consist of individuals possessing the different possible combinations of those characters. Then, any two units of the larger collection will be distinguished by the different combinations of characters they possess, and being so distinguished from one another they must be distinct individuals.

Peirce: CP 4.180 Cross-Ref:††

On those two theorems, I build the whole doctrine of collections.

Peirce: CP 4.181 Cross-Ref:††

181. I will now run over the different grades of multitude of discrete collections, and point out the most remarkable properties of those multitudes.

Peirce: CP 4.181 Cross-Ref:††

The lowest grade of multitude is that of a collection which does not exist, or the multitude of *none*. A collection of this multitude has obvious logical peculiarities. Namely, nothing asserted of it can be false. For of it alone contradictory assertions are true. It is a collection and it is not a collection. Given the premisses that all the X's are black and that all the X's are pure white, what is the conclusion? Simply that the multitude of the X's is zero.

Peirce: CP 4.181 Cross-Ref:††

The least difference by which one multitude can exceed another is by a single unit. But I do not say that the multitude next greater than a given multitude always exceeds it by a single unit.

Peirce: CP 4.181 Cross-Ref:††

The multitude of ways of distributing nothing into two abodes is *one*. This is the next grade of multitude. This again has certain logical peculiarities. Namely, in order to prove that every individual of it possesses one character, it suffices to prove that every individual of it does not possess the negative of that character.

Peirce: CP 4.181 Cross-Ref:††

The multitude of ways of distributing a single individual into two houses is *two*. This is the next grade of multitude. This again has certain logical peculiarities which have been noted in Schröder's *Logik*.

Peirce: CP 4.181 Cross-Ref:††

The multitude of combinations of two things is four, which is not the next grade of multitude. The multitude of combinations of four things is 16. The multitude of combinations of 16 things is 65,536. The multitude of combinations of 65,536 things is large. It is written by 20,036 followed by 19,725 other figures. The multitude of combinations of that many things is a number to write which would require over 600,000 thousand trimillibicentioctagentiseptillions of figures on the so-called English system of numeration. What the number itself would be called it would need a multimillionaire to say. But I suppose the word trimillillion might mean a million to the trimillillionth power; and a trimillion would be a million to the three thousandth power. But the multitude considered is far greater than a trimillillion. It is safe to say that it far exceeds the number of chemical atoms in the galactic cluster. Yet this is one of the early terms of a series which is confined entirely to finite

collections and never reaches the really interesting division of multitudes, which comprises these that are infinite.

Peirce: CP 4.182 Cross-Ref:††

182. The finite collections, however, or, as I prefer to call them, the *enumerable* collections, have several interesting properties. The first thing to be considered is, how shall an enumerable multitude be defined? If we say that it is a multitude which can be reached by starting at 0, the lowest grade of multitude, and successively increasing it by one, we shall express the right idea. The difficulty is that this is not a clear and distinct statement. As long as we discuss the subject in ordinary language, the defect of distinctness is not felt. But it is one of the advantages of the algebra which is now used by all exact logicians, that such a statement cannot be expressed in that logical algebra until we have carefully thought out what it really means. An enumerable multitude is said to be one which can be constructed from zero by "successive" additions of unity. What does "successive," here, mean? Does it allow us to make *innumerable* additions of unity? If so, we certainly should get beyond the enumerable multitudes. But if we say that by "successive" additions we mean an enumerable multitude of additions, we fall into a *circulus in definiendo*. A little reflection will show that what we do mean is, that the enumerable multitudes are those multitudes which are necessarily reached, provided we start at zero, and provided that, any given multitude being reached, we go on to reach another multitude next greater than that. The only fault of this statement is, that it is logically inelegant. It sounds as if there were some special significance in the "reaching," which by the principles of logic there cannot be. For the enumerable multitudes are defined as those which are *necessarily* so reached. Now the kind of necessity to which this "necessarily" plainly refers is logical necessity. But the perfect logical necessity of a result never depends upon the material character of the predicate. If it is necessary for one predicate, it is equally so for any other. Accordingly, what is meant is that the enumerable multitudes are those multitudes every one of which possess any character whatsoever which is, in the first place, possessed by zero and, in the second place, if it is possessed by any multitude, M, whatsoever, is likewise possessed by the multitude next greater than M. We, thus, find that the definition of enumerable multitude is of this nature, that it asserts that that famous mode of reasoning which was invented by Fermat †1 applies to the succession of those multitudes. The enumerable multitudes are defined by a logical property of the whole collection of those multitudes.

Peirce: CP 4.183 Cross-Ref:††

183. Since the whole collection of enumerable multitudes has this logical property it follows *a fortiori* that every single enumerable multitude has the same property.

Peirce: CP 4.184 Cross-Ref:††

184. But it further follows from the same definition that every single enumerable collection has a further logical property.

Peirce: CP 4.184 Cross-Ref:††

This property is, that if an enumerable collection be counted, the counting process eventually comes to an end by the exhaustion of the collection. This property follows from the other, in this sense, that it is true of the *zero* collection, and if it be true of any collection whatever, it is equally true of every collection that is greater than that by one individual. Hence, it is true of all enumerable collections, by Fermatian reasoning.

Peirce: CP 4.185 Cross-Ref:††

185. You may ask why I should call this a *logical* property. It does not at first sight appear to be of that nature. But that is because it is not distinctly expressed. In place of "coming next after in the count," we may substitute any relation, r , such that not more than one individual (at least of the collection in question) is an r to any one. Then, the property is that if the M's form an enumerable collection, then and only then, if every M is r to an M [say, L], then every M is r 'd by an M [say, N]. For example, in a count no M is immediately preceded by more than one M, hence it cannot be that every M immediately precedes an M (so that the collection is never exhausted) unless every M is immediately preceded by an M (in which case, the count would have no beginning). Because this is a logical necessity, the property is a logical property and is the foundation of that mode of inference for which De Morgan first gave the logical rules, under the name of the *syllogism of transposed quantity*. He, however, overlooked the fact that this mode of reasoning is only valid of enumerable collections.†1. . .

Peirce: CP 4.186 Cross-Ref:††

186. A remarkable and important property of enumerable collections is, that every finite part is less than a whole. If the finite part is measured, the multitude of units it contains is enumerable; and if it is incommensurable with the unit, the unit can be changed so as to make the finite part commensurable. Thus, to say that a finite part is less than its whole is the same as to say that an enumerable collection which is part of another is less than that other. There are two cases: first, when the whole is enumerable; and second, when the whole is innumerable. Let us consider the first case. Let the M's be contained among the N's (which form an enumerable collection). Suppose however that the collection of M's is not less than that of the N's. Then, by the definition of equality, there is such a one-to-one relation d , that every N is d 'd by an M. Then, since this M is an N, every N is d 'd by an N. But d being a one-to-one relation, there are not two N's that are d 'd by the same N. Hence, by the syllogism of transposed quantity, every N is d of an N. But the N's are, by their equality to the M's, d 'd by nothing but M's. Hence, every N is an M. That is, we have shown that if the N's form an enumerable collection, the only collection at once contained in that collection and equal to that collection is the collection itself, and is not a part of the collection. That is, no part of an enumerable collection is equal to the collection. But the relation of inclusion is a one-to-one relation of every unit of the part to a unit of the whole. Hence, the part cannot be greater than the whole, and must be less than the whole.

Peirce: CP 4.186 Cross-Ref:††

We now take up the second case. But we can go further, and show that every innumerable collection is greater than any enumerable collection. It is to be shown that it is absurd to suppose that every unit of an innumerable collection, the N's, is in a one-to-one relation, c , to a unit of any one enumerable collection, the M's. Let r be such a one-to-one relation that every M except one is r to an M. Then, by the syllogism of transposed quantity, every M except just one is r 'd by an M. (For if every M were r to an M, every M would be r 'd by an M; and since r is a one-to-one [relation], if there is a single one of the connections or relations between pairs of individuals, which is excluded from r , it leaves just one M not r to an M and just one M not r 'd by an M.) This is so whatever one-to-one relation r may be. Hence, were every N c to an M, it would follow that every N but one would be c to an M that was r to an M that was c 'd by an N; and this compound relation of being ' c to an r of something c 'd by' would be a one-to-one relation, being compounded of one-to-one

relations. And invariably, whatever one-to-one relation r might be, one N would be the last in a count of the N's which should proceed from each N, say N[i], to that N, say N[j], such that N[i] was c of that M that was r of that M which was c 'd by N[j]. In every such mode of counting, I say, some N would be the last N completing the count. And the M's being equal to the N's, and the one collection tied to the other by the relative, for every possible order of counting of the N's there would be some r relation among the M's; and thus in every possible counting of the N's there would be a last N, contrary to the hypothesis that the N's form an innumerable collection. Thus, it is shown to be impossible that an innumerable collection should be no greater than an enumerable collection, and the demonstration that a finite part is less than its whole is complete.

Peirce: CP 4.186 Cross-Ref:††

Now it is singular that every time Euclid reasons that a part is less than its whole, he falls into some fallacy, even though the part he is speaking of be finite.†P1 I can only account for it by supposing that owing to the falsity of his axiom, he learned to think that very wonderful things could be proved by its aid, things that he would know could never be proved by any other axiom; for when a man appeals to an axiom he is pretty sure to be reasoning fallaciously. And thus he was prevented from suspecting and thoroughly criticizing those places in his reasoning. . . .

Peirce: CP 4.187 Cross-Ref:††

187. It is a curious illustration of how even that part of mankind who reason for themselves more than any others -- I mean the mathematicians -- yet how even they follow phrases and forget their meanings, that while everybody is in the habit of calling the proposition that a part is less than its whole an axiom, yet when this proposition is stated in another form of words -- for the transformation amounts to little more -- we always speak of it as the *fundamental theorem of arithmetic*. The statement is that if in counting a collection with the cardinal numerals the count of a collection comes to a stop from the exhaustion of the individuals it always comes to a stop at the same numeral. I say that this amounts pretty much to saying that an enumerable part can not equal its whole. For to say that the same collection can in one order of counting count 16 and in another order of counting count 15 would be the same as to say that the first 16 numerals could (through the identity of the objects counted) be put into a one-to-one correspondence with the first 15 numerals; and this, by the definition of equality, would be to say that the collection of the first 15 numerals was equal to the collection of the first 16 numerals, although the former collection is an enumerable part of the latter.

Peirce: CP 4.187 Cross-Ref:††

It is generally understood to be very difficult to demonstrate this theorem logically, and so it is somewhat so if the principles of logic are not attended to. At any rate several of the proposed demonstrations egregiously beg the question.†P1

Peirce: CP 4.188 Cross-Ref:††
 §2. THE DENUMERABLE

188. But I have lingered too long among enumerable multitudes. Let us go on to inquire what is the smallest possible multitude which is innumerable?

Peirce: CP 4.188 Cross-Ref:††

Take the collection of M's. If this collection be such that taking any one-to-one relation r whatever, if every M is r to an M it necessarily follows that every M is r 'd by an M, the collection of M's thereby fulfills the definition of an enumerable collection. We can substitute a phrase for the letter r in this statement and say that to call the collection of M's enumerable is the same as to assert that if every M, in any order of arrangement, is immediately succeeded by another M, and *that* an M which does not so immediately succeed any other of the M's, then every M immediately succeeds another M, and there is some ring arrangement without any first. To say that if there be no last there can be no first, is to say the collection spoken of is enumerable.

Peirce: CP 4.188 Cross-Ref:††

To deny that the M's are enumerable is, then, as much as to assert that there is a possible arrangement in which each M is immediately followed by another M which so follows no third M, and yet there is an absolutely first M which does not follow any M. If now we deny that the collection of M's is enumerable but, at the same time, restrict it to including no individual that need not be included to make the collection innumerable, we shall plainly have a collection of the lowest order of multitude which any innumerable collection can have. Such a collection I call *denumerable*. To say, then, that the collection of M's is denumerable, is the same as to assert that it contains nothing except one particular object and except what is implied in the fact that there is a one-to-one relation r such that every M is r to an M. This is a logical character; for it is the same as to say that the syllogism of transposed quantity does not hold good of it but that the Fermatian inference does. That is, if the collection of M's is denumerable, every character which is true of a certain M, say M[0] and is also true of every M which is in a certain one-to-one relation to an M of which it is true, is necessarily true of every M of the collection.

Peirce: CP 4.188 Cross-Ref:††

For example, the entire collection of whole numbers forms a denumerable collection. For zero is a whole number, which is not greater by one than any number, there is a number greater by one than any given whole number, and there is no number or numbers which could be struck out of the collection and still leave it true that zero belonged to the collection and that there was a number of the collection greater by one than each number of the collection.

Peirce: CP 4.189 Cross-Ref:††

189. I have already shown by the example of the even numbers that a part of a denumerable collection may be equal to the whole collection. I will now prove that all denumerable collections are equal. For suppose that the M's and the N's are two denumerable collections. Then, a certain M can be found which we may call M[0] such that taking a certain one-to-one relation, r , every M except M[0] is r to an M, and there is an r to every M; and in like manner there is a one-to-one relation, s , such that every N except one, N[0], is s to an N, and every N is s 'd by an N. Then, I say, that the relation, c , can be so defined that every M is c to an N, and every N is c 'd by an M. For let M[0] be c to N[0] and to nothing else; and let N[0] be c 'd by nothing but M[0] and if anything, X, is c to anything, Y, let the r to X (and it alone) be c to the s of Y and to nothing else. Then, evidently c is a one-to-one relation. But every M is c to an N, because M[0] is c to an N (namely to N[0]) and if any M is c to an N, then the r of that M is c to an N (for it is, by the definition of c , c to the s of the N to which

the former M is *c*). And in like manner every N is *c*'d by an M, because N[0] is *c*'d by an M (namely by M[0]), and if any N is *c*'d by an M, then the *s* of that N is *c*'d by an M (for it is, by the definition of *c*, *c*'d by the *r* of the M by which the former N is *c*'d). Q. E. D.

Peirce: CP 4.189 Cross-Ref:††

Accordingly, there is but a single grade of denumerable multitude. So it is to be noted as a defect in my nomenclature, which I unfortunately did not remark when I first published it,†1 that *enumerable* and *denumerable*, which sound so much alike, denote, the one a whole category of grades of multitude and the other a simple grade like, *zero*, or *twenty-three*.

Peirce: CP 4.190 Cross-Ref:††

190. It will be convenient to make here a few remarks about arithmetical operations upon multitude. Please observe that I have not said one word as yet about number, and I do not propose even to explain at all what numbers are until I have fully considered the subject of multitude, which is a radically different thing. Arithmetical operations can be performed upon both multitudes and upon numbers, just as they can be performed upon the terms of logic, the vectors of quaternions, the operations of the calculus of functions, and other subjects. What I ask you at this moment to consider is, not at all the addition and multiplication of numbers, for you do not know what I mean by numbers -- it is safe to say so, since the word bears so many different meanings -- but the addition and multiplication of multitudes.

Peirce: CP 4.190 Cross-Ref:††

Addition in general differs from aggregation inasmuch as a unit is increased by being *added* to itself but not by being aggregated to itself. When mutually exclusive terms are aggregated, that is the same as the addition of them. Addition might, therefore, be defined as the aggregation of the positings of terms. Two positings of the same term being different positings, their aggregate is different from a single positing of the term. The sum of two multitudes is the multitude of the aggregate of two mutually exclusive collections of those multitudes. The *aggregate* of a collection of collections of units may be defined as that collection of units, every unit of which is a unit of one of those collections, and which has every unit of any of those collections among its units.

Peirce: CP 4.191 Cross-Ref:††

191. It is easily proved that the sum of an enumerable collection of enumerable multitudes is an enumerable multitude.†2 . . .

Peirce: CP 4.192 Cross-Ref:††

192. The sum of an enumerable multitude and the denumerable multitude is denumerable. The proof is excessively simple; for we have only to count the enumerable collection in linear series, first. The count of that has to end; and then the denumerable series may follow in its primal order.

Peirce: CP 4.193 Cross-Ref:††

193. That the denumerable multitude added to itself gives itself is made plain by zigzagging through two denumerable series. But this comes more properly under the head of multiplication of multitudes, which I propose to consider.

Peirce: CP 4.193 Cross-Ref:††

Mathematicians seem to be satisfied so far to generalize the conception of multiplication as to make it the application of one operation to the result of another.

But the conception may be still further generalized, and in being further generalized it returns more closely to its primitive type. The more general conception of multiplication to which I allude is expressed in the following definition:

Multiplication is the pairing of every unit of one quantity with every unit of another quantity so as to make a new unit. Since there are two acceptions of the term *pair* -- the ordered acception, according to which AB and BA are different pairs, and the unordered acception -- there are two varieties of multiplication, the non-commutative and the commutative. Multiplication may further be distinguished into the *free* and the *dominated*. In free multiplication the idea of pairing remains in all its purity and generality. In dominated multiplication, the product of two units is that which results from the special mode of pairing which is of preëminent importance with reference to the particular kind of units that are paired. Thus, in reference to length and breadth the pairing of their units in units of area is preëminently important; in reference to an operator and its operand the pairing of their units in units of the result is preëminently important; in logic, in reference to two general terms, the pairing of their units in identical units which reunite their essential characters is preëminently important, etc. In the multiplication of multitudes we have one of the very rare instances of free multiplication. The *product* of a collection of multitudes called its *factors* may be defined as the multitude of possible sets of units any one of which could be formed out of units taken one from each of a collection of mutually exclusive collections of units having severally the multitudes of the factors. For example, to multiply 2 and 3, we take a collection of two objects, as A and B, and a distinct collection of three objects, as X, Y, and Z, and form the pairs AX, AY, AZ, BX, BY, BZ, which are all the sets that can be formed each from one unit of each collection. Then, since the multitude of these pairs is 6, the product of the multitudes, 2 and 3, is the multitude of 6.

Peirce: CP 4.194 Cross-Ref:††

194. The same general idea affords us a definition of involution. **Involution** is the formation of a new quantity a *power* from two quantities, a *base*, and an *exponent*, each unit of the power resulting from the attachment of all the units of the exponent each to some one unit of the base, without reference to how many units of the exponent are attached to any one unit of the base. Thus, 3 to the 2 power is the multitude of different ways in which both of two units, A and B, can be joined each to some one of three objects, X, Y, and Z. . . .

Peirce: CP 4.195 Cross-Ref:††

195. The product of two multitudes, $\{m\}$ and $\{n\}$, is equal to the multitude of units in $\{m\}$ mutually exclusive collections each of $\{n\}$ units. For since there is one unit and but one for each of the $\{n\}$ units of each of the $\{m\}$ collections, these units are in one-to-one correspondence with the possible descriptions of single units each of which pairs a unit of a multitude of $\{n\}$ with a unit of a multitude of $\{m\}$; and the multitude of such pairs is the product of $\{m\}$ and $\{n\}$.

Peirce: CP 4.195 Cross-Ref:††

The $\{m\}$ power of $\{n\}$ is equal to the product of $\{m\}$ mutually exclusive collections each of $\{n\}$ units.

Peirce: CP 4.195 Cross-Ref:††

The product of two enumerable multitudes is an enumerable multitude.

Peirce: CP 4.195 Cross-Ref:††

The product of an enumerable multitude and the denumerable multitude is the

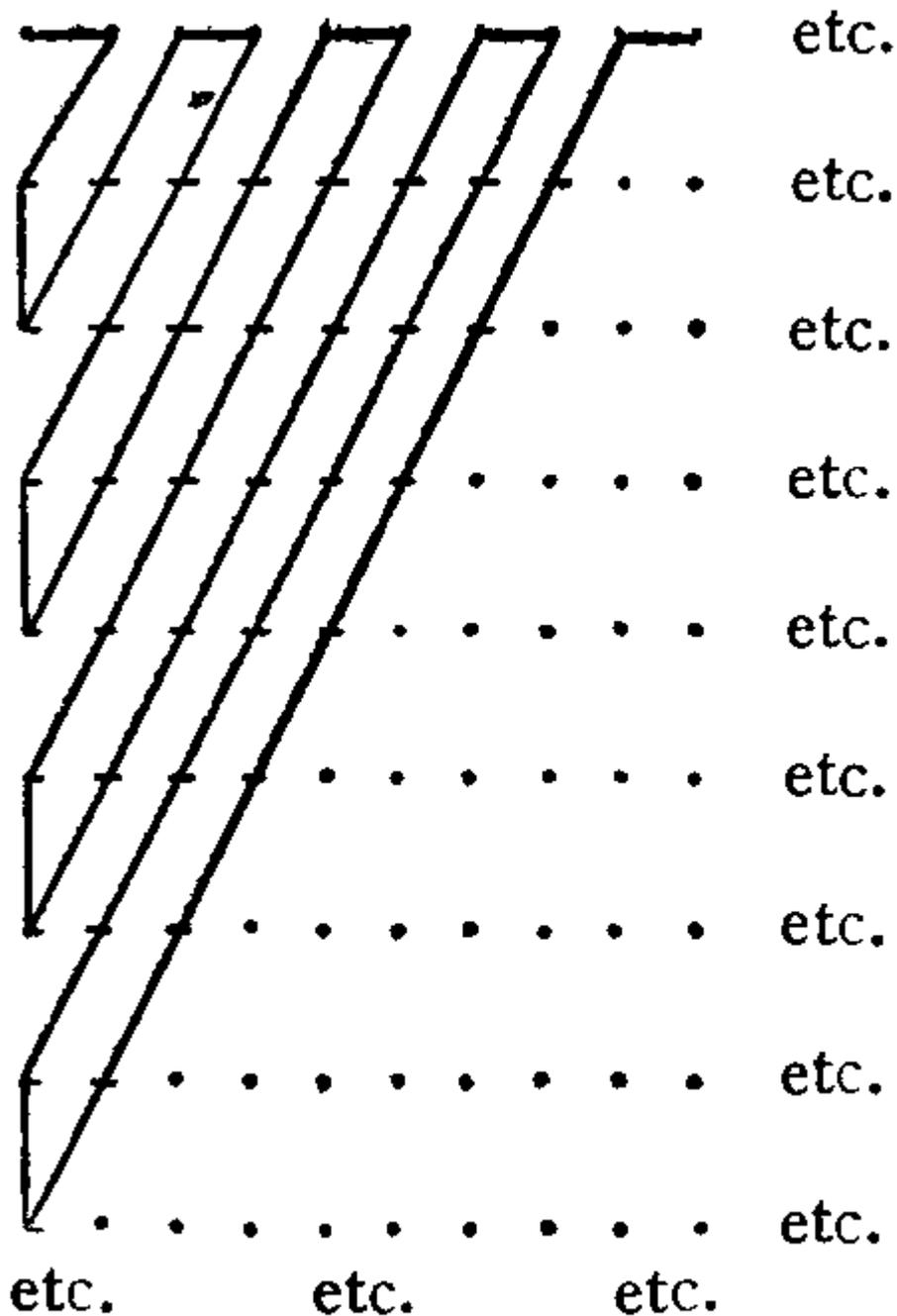
denumerable multitude.

Peirce: CP 4.195 Cross-Ref:††

An enumerable power of the denumerable multitude is the denumerable multitude.

Peirce: CP 4.196 Cross-Ref:††

196. That the second power of the denumerable multitude is the denumerable multitude is easily seen by aggregating a denumerable series of collections, each a denumerable series of units.



[Click here to view] Now we can start at the corner and proceeding from each unit we reach to a single next one and can reach any unit whatever in time without completing the proceeding. Hence, the whole forms one denumerable series. This proof is substantially that of Cantor.†1 The proposition being proved for two factors instantly extends itself to any enumerable multitude of factors. Of course, there is not the slightest difficulty in expressing this idea so as to construct the most rigidly formal demonstration. Let \aleph denote the denumerable multitude. Then, I am to show that $\aleph^2 = \aleph$. Let the M's be a denumerable collection. That is, suppose

Peirce: CP 4.196 Cross-Ref:††

First: a certain object $M[0]$ is an M ;

Peirce: CP 4.196 Cross-Ref:††

Second: there is a certain non-identical one-to-one relation, r , such that every M is r 'd by an M ;

Peirce: CP 4.196 Cross-Ref:††

Third: whatever is not necessitated to be an M by the above statements is not an M .

Peirce: CP 4.196 Cross-Ref:††

Let A and B constitute a collection of two objects not M 's. Let us define the relation $\{s\}$ as follows:

Peirce: CP 4.196 Cross-Ref:††

First: the pair of attachments of A to $M[0]$ and B to $M[0]$ is $\{s\}$ 'd by nothing;

Peirce: CP 4.196 Cross-Ref:††

Second: every pair of attachments of A to an M which we may call $M[i]$ other than $M[0]$ and of B to an M , which we may call $M[j]$, is $\{s\}$ to the pair of attachments of A to that M which is r 'd by $M[i]$, and of B to that M which is r of $M[j]$;

Peirce: CP 4.196 Cross-Ref:††

Third: every pair of attachments of A to $M[0]$ and of B to an M , which we may call $M[k]$, is $\{s\}$ to the pair of attachments of A †1 to that M which is r of $M[k]$, and of B †2 to $M[0]$;

Peirce: CP 4.196 Cross-Ref:††

Fourth: if one thing is not necessitated by the above rules to be $\{s\}$ to another, it is not $\{s\}$ to that other.

Peirce: CP 4.196 Cross-Ref:††

It is evident, then, that $\{s\}$ is a one-to-one relation; and it is evident that every pair of attachments of A to any M , say $M[x]$, and of B to any M , say $M[y]$, is $\{s\}$ of another such pair of attachments, that one such pair of attachments is $\{s\}$ 'd by nothing, and that nothing is a pair of such attachments that is not necessitated to exist by the fact that everything is $\{s\}$ of something. Hence, the multitude of those pairs of attachments is denumerable; and that is the same as to say that the second power of the denumerable multitude is the denumerable multitude.

Peirce: CP 4.197 Cross-Ref:††

197. Dr. George Cantor †1 first substantially showed that between the units of any denumerable collection certain remarkable relations exist, which I call indefinitely dividant relations. Namely, let the M 's be any denumerable collection, and let f be any relation indefinitely dividant of the collection of the M 's. Then, no M is f to itself, but of any two different M 's one is f to the other; and if an M is f to another it is f to every M that is f 'd by that other; and if an M is f 'd by another it is f 'd by every M that is f to that other. And now comes the remarkable feature: If one M is f to another, it is f to an M that is not f 'd by that other; whence, necessarily if one M is f 'd by another, it is f 'd by an M that is not f to that other.

Peirce: CP 4.197 Cross-Ref:††

There are vast multitudes of such indefinitely dividant relations. I will instance a single one. If we take the whole series of vulgar fractions, those of the same denominator being taken immediately following one another in the increasing

order of the numerators and those of different denominators in the increasing order of the denominators,

$1/2$ $1/3$ $2/3$ $1/4$ $2/4$ $3/4$ $1/5$ $2/5$ $3/5$ $4/5$ $1/6$ $2/6$ $3/6$ $4/6$ $5/6$ etc.

these evidently form a denumerable collection, for they form the aggregate of a denumerable collection of enumerable collections of units. If from this collection we omit those fractions which are equal to other fractions of lower denominations we plainly have still a denumerable collection. Now for the first of these denumerable collections, that of all the vulgar fractions, an indefinitely dividend relation is that of being "greater than or equal to but of higher terms than". For the second of those denumerable collections, that of all the rational quantities greater than 0 and less than 1, an indefinitely dividend relation is that of being "greater than." . . .

Peirce: CP 4.197 Cross-Ref:††

Numbers in themselves cannot possibly signify any magnitude other than the magnitudes of collections, or multitude; but what they principally represent is place in a serial order. Numbers do not contain the idea of the equality of parts and consequently a fraction cannot in itself signify anything involving equality of parts. They merely express the ordinal place in such a uniformly condensed series. . . .†1

Peirce: CP 4.198 Cross-Ref:††

198. A striking difference between enumerable and denumerable collections is this, that no arrangement of an enumerable collection has any different properties from any other arrangement; for the units are or may be in all respects precisely alike, that is, have the same *general* characters, although they differ individually, each having its proper designation. But it is not so with regard to denumerable collections. Every such collection has a *primal* arrangement, according to its *generating relation*. There is one unit, at least, which arbitrarily belongs to the collection just as every unit of an enumerable collection belongs to that collection. But after that one unit, or some enumerable collection of units, has been arbitrarily posited as belonging to the collection, the rest belong to it by virtue of the general rule that there is in the primal arrangement one unit of the collection next after each unit of the collection. Those last units cannot be all individually designated, although any one of them may be individually designated. Nor is this merely owing to an incapacity on our part. On the contrary, it is logically impossible that they should be so designated. For were they so designated there would be no contradiction in supposing a list of them all to be made. That list would be complete, for that is the meaning of *all*. There would therefore be a last name on the list. But that is directly contrary to the definition of the denumerable multitude.

Peirce: CP 4.198 Cross-Ref:††

The same truth may be stated thus: It is impossible that all the units of a denumerable collection should have the same general properties. For the existence of the primal arrangement is essential to it, being involved in the very definition of the denumerable collection as that of smallest multitude greater than every enumerable collection. Now, this primal arrangement is an arrangement according to a general rule, and its statement constitutes, therefore, general differences between the units of the denumerable collection.

Peirce: CP 4.198 Cross-Ref:††

On the other hand any unit whatever of a denumerable collection may be individually designated, as well as all those which precede it in the primal

arrangement. And these can be all exactly alike in their general qualities. Yet there must always be a latter part of the collection which is not individually designated but is only generally described. In this part we recognize an element of ideal being as opposed to the brute and surd existence of the individual.

Peirce: CP 4.198 Cross-Ref:††

The denumerable collection of whole numbers, for example, constitutes a discrete series, in the sense that there is not one which may not be distinguished completely and individually from its neighbors.

Peirce: CP 4.198 Cross-Ref:††

But we cannot with any clearness of thought carry these reflections further until we are in possession of an instance of a greater collection.

Peirce: CP 4.199 Cross-Ref:††

199. The arrangement of a denumerable collection according to an indefinitely dividant relation like the rational numbers -- or to take a simpler instance, like the fractions which can be written in the binary system of arithmetical notation with enumerable series of figures -- is a very recondite arrangement, not at all naturally suggested by the primal arrangement. This is shown by the fact that the world had to wait for George Cantor to inform it that the collection of rational fractions was a collection precisely like that of the whole numbers.†1

Peirce: CP 4.199 Cross-Ref:††

This remark will be found important in the sequel.

Peirce: CP 4.200 Cross-Ref:††
§3. THE PRIMIPSTNUMERAL

200. So much, for the present, for the denumerable multitude. Let us now inquire, what is the smallest multitude which exceeds the denumerable multitude? An enumerable or denumerable multitude is a multitude such that whatever in *any* arrangement of an enumerable collection, in the *primal* arrangement of a denumerable collection, is true of the first unit, and is further true of any unit which comes next after any unit of which it is true, is true of all and every unit of the collection. . . . It has not yet been proved that there is any such minimum multitude among those which exceed the denumerable; but it is convenient to say that in fact there is. I have hitherto named this multitude, which was first clearly described by Cantor,†1 the *first abnumeral* multitude.†2 But I find that a name in one word is wanted. So I will hereafter name it the *primipstnumeral* multitude.

Peirce: CP 4.201 Cross-Ref:††

201. Suppose it to be true of a collection that in whatever way its units be arranged in a horizontal line with one unit to the extreme left, and a unit next to the right of each unit, there is something which is true of the first unit and which if true of any unit is always true of the next unit to the right, which nevertheless is not true of all the units; and suppose furthermore that the collection is no greater than it need be to bring about that state of things. Then, that collection is by definition a primipstnumeral collection. Or by the aid of the logic of relatives, we may state the matter as follows:

1. Let there be an existent collection, R;
2. Let R include no unit which is not necessitated by that condition;
3. Let r be a one-to-one relation between units;
4. Let there be a collection, the Q's, such that no Q is R;
5. Let there be a Q that is r to each unit of the collections of the Q's and R's;
6. Let the collection of the Q's include nothing not necessitated by the foregoing conditions;
7. Let h be a one-to-one relation of a unit to a collection;
8. Let there be a collection, P, such that no P is a Q or R;
9. Let there be a P which is h to every (denumerable) collection of Q's.
10. Let there be no P which is not necessitated in order to fulfill the foregoing conditions.

Peirce: CP 4.201 Cross-Ref:††

Then, the collection of P's is a primipostnumeral collection.

Peirce: CP 4.201 Cross-Ref:††

It would be easy to make this statement more symmetrical in appearance; but I prefer to make it perspicuous. Thus, we might make r a relation between a unit and an enumerable collection; and we might make the P's include an h for every denumerable collection of P's, Q's and R's, etc. The word "denumerable" in the ninth condition is added merely for the sake of perspicuity.

Peirce: CP 4.201 Cross-Ref:††

The second, sixth, and tenth conditions are not very clear. The meaning is that the multitude is no larger than need be.

Peirce: CP 4.202 Cross-Ref:††

202. The definitions of a primipostnumeral collection just given suppose it to be constructed from a denumerable collection. But if we attempt to form a primipostnumeral collection from a denumerable collection in its primal arrangement we shall fail ignominiously.

Peirce: CP 4.202 Cross-Ref:††

Let us, for example, imagine a series of dots representing, the first [dot] the position of the tortoise when Achilles began to run after him, and each successive dot the position of the tortoise at the instant when Achilles reached the position represented by the preceding dot. If there are no more dots than are necessary to fulfill this condition, the collection of dots is denumerable. If we add a dot to represent the position of the tortoise at the moment when Achilles catches up with him, the Fermatian inference seems at first sight not to hold good. For the first dot represents a position of the tortoise *before* Achilles had caught up with him, and if any dot represents the position of the tortoise before Achilles caught up with him, so likewise does the dot which immediately succeeds it. The Fermatian inference then would seem to be that every dot represents a position of the tortoise before Achilles had caught up with him. Yet this is not true of the last dot which represents the position of

the tortoise at the moment when Achilles caught up with him. Yet but one dot has been added to the denumerable collection, and of course, it remains denumerable. The only reason that the inference does not hold is that the dots are no longer in their primal arrangement. Put the last dot at the beginning, so as to preserve the primal arrangement, and any Fermatian inference whose premisses were true would hold good. The point I wish to make is that the denumerable collection in its primal order leads to no way of constructing or of conceiving of a primipostnumeral collection. Of course, we can say, "Let there be a dot for each denumerable collection of the tortoise-places;" but we might as well omit the tortoise-places and say, "Let there be a primipostnumeral collection of dots." The primal arrangement of the denumerable collection affords no definite places nor approximations to the places for the primipostnumeral collection.

Peirce: CP 4.203 Cross-Ref:††

203. The reason is that the latter part of the denumerable collection, which is its denumerable point, is all concentrated towards one point, whether that point be a metrically ordinary point or a point at infinity. This fault is remedied in the indefinitely dividend arrangement. Here the denumerable part of the collection is spread over a line.

Peirce: CP 4.203 Cross-Ref:††

In this case, if we imagine all those subdivisions to be performed which are implied by saying that the intervals resulting from each set of subdivisions are all subdivided in the next following set of subdivisions, the multitude of subdivisions is 2^{\aleph} where \aleph is the denumerable multitude; and this is no mere algebraical form without meaning. It has a perfectly exact meaning which I explained in speaking of the effects of addition, multiplication, and involution upon multitudes.

Peirce: CP 4.203 Cross-Ref:††

Moreover, you will remember that I distinctly and fully proved †1 that the multitude of possible sets of units each of which can be formed from the units of a collection always exceeds the multitude of that collection, provided it be a discrete collection.

Peirce: CP 4.204 Cross-Ref:††

204. Do you not think it possible that the stellar universe extends throughout space? If so, the whole collection of worlds is at least denumerable. At any rate, it is perfectly possible that the whole collection of intelligent beings who live, have lived, or will live anywhere is at least equal to the collection of whole numbers. It is conceivable that they are all immortal and that each one should be given each hour throughout eternity the name of one of them and he should assign that person in wish to heaven or to hell, so that in the course of eternity he would wish every one of them to heaven or to hell. Could they by all making different wishes wish among them for every possible distribution of themselves to heaven or to hell? If not, the multitude of such possible distributions is greater than the denumerable multitude. But they plainly could not wish for all possible such distributions. For if they did, some one would necessarily be perfectly satisfied with every possible distribution. But one possible distribution would consist in sending each person to the place he did not wish himself to go; and that would satisfy nobody. It was Cantor who first proved that the surd quantities form a collection exceeding the collection of rational quantities.†2 But his method was only applicable to that particular case. My method is applicable to any discrete multitude whatever and shows that $2^{\{m\}} > \{m\}$ in every case in which $\{m\}$ is

a discrete multitude.

Peirce: CP 4.205 Cross-Ref:††

205. I will give a few more examples of primipostnumeral collections. The collection of quantities between zero and unity, to the exact discrimination of which decimals can indefinitely approximate but never attain, is evidently 10^{\aleph} , which of course equals 2^{\aleph} . For $16^{\aleph} = (2^4)^{\aleph} = 2^{(4^{\aleph})} = 2^{\aleph}$.

Peirce: CP 4.205 Cross-Ref:††

The collection of all possible limits of convergent series, whose successive approximations are vulgar fractions, although it does not, according to any obvious rule of one-to-one correspondence, give a limit for every possible denumerable collection of vulgar fractions, does nevertheless in an obvious way correspond each limit to a denumerable collection of vulgar fractions, and to so large a part of the whole that it is *primipostnumeral*, as Cantor has strictly proved.†1

Peirce: CP 4.206 Cross-Ref:††

206. Just as there is a primal arrangement of every denumerable collection, according to a generating relation, so there is a primal arrangement of every primipostnumeral collection, according to a generating arrangement. This primal arrangement of the primipostnumeral collection springs from a highly recondite arrangement of the denumerable collection. Namely, we must arrange the denumerable collection in an indefinitely dividend order, and then the units, which are implied in saying that the denumerable succession of subdivisions have been completed constitute the primipostnumeral collection. But when I say that the primipostnumeral collection *springs from an arrangement* of the denumerable collection, I do not mean that it is formed from the denumerable collection itself; for that would not be true. On the contrary, the primipostnumeral collection can only be constructed by a method which *skips* the denumerable collection altogether. In order to show what I mean I will state the definition of a primipostnumeral collection in terms of relations. There are two or three trifling explanations to be made here. First an aggregate of collections is a collection of the units of those collections. It is also an aggregate of the collections, which are called its aggregants. Just as to say that Alexander cuts some knot implies that a knot exists, although to say Alexander cuts every knot, *i.e.*, whatever knot there may be, does not imply the existence of any knot, the latter by its generality referring to an ideal being, not to a brute individual existence, so to say that a collection has a certain collection as its aggregant implies the existence of the latter collection and therefore that it contains at least one unit. I must also explain that whenever I say either one thing or another is true I never thereby mean to exclude both.

Peirce: CP 4.207 Cross-Ref:††

207. I will now describe a certain collection A, whose units I will call the *P*'s [π 's?].

Peirce: CP 4.207 Cross-Ref:††

First, The π 's can be arranged in linear order. That is, there is a relation, *p*, such that taking as you will any π 's, individually designable as $\pi[1]$, $\pi[2]$, and $\pi[3]$, either $\pi[3]$ is not *p* to $\pi[2]$ or $\pi[2]$ is not *p* to $\pi[1]$ or (if $\pi[3]$ is *p* to $\pi[2]$ and $\pi[2]$ is *p* to $\pi[1]$), $\pi[3]$ is *p* to $\pi[1]$;

Peirce: CP 4.207 Cross-Ref:††

Second, The line of arrangement of the π 's can be taken so as not to branch. That is, taking as you will π 's, individually designable as $\pi[4]$ and $\pi[5]$, either $\pi[4]$ is p to $\pi[5]$ or $\pi[5]$ is p to $\pi[4]$; (of course this permits both to be true, but that I proceed to forbid).

Peirce: CP 4.207 Cross-Ref:††

Third, The line of arrangement of the π 's can further be so taken as not to return into itself, circularly. That is, taking as you will any π , individually designable as $\pi[6]$, $\pi[6]$ is not p to $\pi[6]$;

Peirce: CP 4.207 Cross-Ref:††

Fourth, There are certain parts of A called "packs" of π , which are mutually exclusive. That is, taking any pack whatever and any unit of that pack, that unit is a π ; and taking as you will any packs individually designable as $P[7]$ †1 and $P[8]$, and any π 's individually designable as $\pi[7]$ and $\pi[8]$, either $P[7]$ is identical with $P[8]$ or $\pi[7]$ is not a unit of $P[7]$, or $\pi[8]$ is not a unit of $P[8]$, or else $\pi[7]$ is not identical with $\pi[8]$;

Peirce: CP 4.207 Cross-Ref:††

Fifth, The packs can be arranged in linear order. That is, there is a relation, s , such that taking as you will any P 's, individually designable as $P[1]$, $P[2]$, and $P[3]$, either $P[3]$ is not s to $P[2]$, or $P[2]$ is not s to $P[1]$, or $P[3]$ is s to $P[1]$;

Peirce: CP 4.207 Cross-Ref:††

Sixth, The line of arrangement of the packs can be taken so as not to branch. That is, taking as you will any P 's, individually designable as $P[4]$ and $P[5]$, either $P[4]$ is s to $P[5]$ or $P[5]$ is s to $P[4]$;

Peirce: CP 4.207 Cross-Ref:††

Seventh, The line of arrangement of the packs can be further taken so as not to return into itself. That is, taking as you will any pack individually designable as $P[6]$, $P[6]$ is not s to $P[6]$;

Peirce: CP 4.207 Cross-Ref:††

Eighth, The arrangement of the packs can further be such that each pack is immediately succeeded by a next following pack. That is, taking as you will any pack individually designable as $P[9]$, a pack individually designable as $P[10]$ can be found such that $P[10]$ is s to $P[9]$; and such that taking thereafter as you will any pack individually designable as $P[11]$, either $P[11]$ is not p to $P[9]$, or $P[11]$ is not p 'd by $P[10]$;

Peirce: CP 4.207 Cross-Ref:††

Ninth, Such a succession of packs is not a mere idea, but actually exists if the collection A exists. That is, a certain collection, $P[0]$, is such a pack;

Peirce: CP 4.207 Cross-Ref:††

Tenth, Each pack contains a unit which, in the linear order of the π 's, comes next after each unit of any of those packs which precede this pack in the linear order of the packs. That is, taking as you will any packs, individually designable as $P[12]$ and $P[13]$, and any unit, individually designable as $\pi[12]$, a unit, individually designable as $\pi[13]$, can be thereafter found such that, taking as you will any pack individually designable as $P[14]$ and any unit individually designable as $\pi[14]$, either

$P[13]$ is not s to $P[12]$, or $\pi[12]$ is not a unit of $P[12]$, or $\pi[13]$ is p to $\pi[12]$; and either $P[13]$ is not s to $P[14]$ or $\pi[14]$ is not a unit of $P[14]$, or $\pi[12]$ is p to $\pi[14]$, or $\pi[13]$ is not p to $\pi[14]$;

Peirce: CP 4.207 Cross-Ref:††

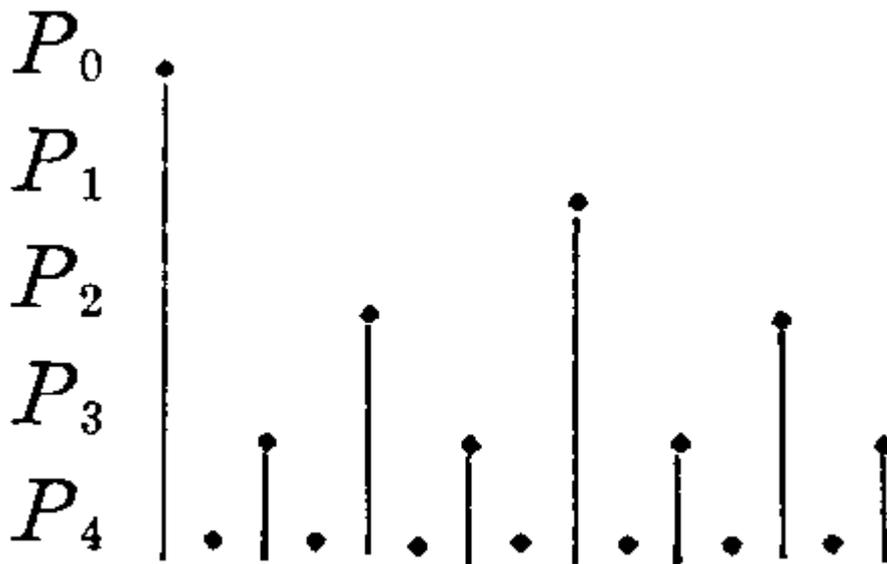
Eleventh, No varieties of descriptions of π 's exist than those which are necessitated by the foregoing conditions;

Peirce: CP 4.207 Cross-Ref:††

Twelfth, No varieties of descriptions of packs exist than those which are necessitated by the foregoing conditions.

Peirce: CP 4.207 Cross-Ref:††

This collection of π 's is primipostnumeral; and you will see what I mean by saying that the construction skips the denumerable multitude, if you consider how many π 's are contained in each pack. The pack $P[0]$ is obliged by the ninth condition to exist, so that it must contain at least one π . But nothing obliges it to contain a π which is other than any π which it contains; and therefore the twelfth condition forbids it to contain [more than] one π . It consists, therefore, of a single π . If we arrange the π 's in a horizontal row so that p shall be equivalent to being "further to the right than," then that P which is s to $P[0]$, but is not s to any other pack, which pack we may call $P[1]$, must contain one π to the right of the π of $[P[0]]$. It need contain no other, and therefore cannot contain any other.



[Click

here to view]

P_0 . ·
 P_1 . | ·
 P_2 . | · | ·
 P_3 . | · | · | · | ·
 P_4 . | · | · | · | · | · | ·

Peirce: CP 4.207 Cross-Ref:††

$P[2]$ contains a π immediately to the right of that of the $P[0]$ and another to the right of that of $P[1]$, and after this each pack contains double the units of the preceding. Thus, $P[n+1]$ contains $2n$ units. As long as n is enumerable, this is enumerable. But as soon as n becomes denumerable, it skips the denumerable multitude and becomes primipostnumeral.

Peirce: CP 4.208 Cross-Ref:††

208. In order to prove that any proposition is generally true of every member of a denumerable collection, it is always necessary -- unless it be some proposition not peculiar to such a collection -- to consider the collection either in its primal arrangement, or in reference to some relation by which the collection is generable, and then reason as follows, where r is the generating relation, and $M[0]$ is that M which is not r to any M :

$M[0]$ is X ,

If any M is X then the r of M is X ;

∴ Every M is X .

Without this Fermatian syllogism no progress would ever have been made in the mathematical doctrine of whole numbers; and though by the exercise of ingenuity we may seem to dispense with this syllogism in some cases, yet either it lurks beneath the method used, or else by a generalization the proposition is reduced to a case of a proposition not confined to the denumerable multitude.

Peirce: CP 4.209 Cross-Ref:††

209. In like manner, in order to prove that anything is true of a primipostnumeral collection, unless it is more generally true, we must consider that collection in its primal arrangement or with reference to a relation equivalent to that of its primal arrangement. The special mode of reasoning will be as follows:

$\pi[0]$, the unit of $P[0]$, is X ,

If every π of any pack is X , then every π of the pack which is s of that pack is X ;

Hence, every π is X .

This may be called the primipostnumeral syllogism.

Peirce: CP 4.210 Cross-Ref:††

210. Every mathematician knows that the doctrine of real quantities is in an exceedingly backward condition. It cannot be doubted by any exact logician that the reason of this is the neglect of the primipostnumeral syllogism without which it is as impossible to develop the doctrine of real quantities, as it would be to develop the theory of numbers without Fermatian reasoning.

Peirce: CP 4.210 Cross-Ref:††

I do not mean to say that the primipostnumeral syllogism is altogether unknown in mathematics; for the reasoning of Ricardo †1 in his theory of rent, reasoning which is of fundamental importance in political economy, as well as much of the elementary reasoning of the differential calculus, is of that nature. But these are

only exceptions which prove the rule; for they strongly illustrate the weakness of grasp, the want of freedom and dexterity with which the mathematicians handle this tool which they seem to find so awkward that they can only employ it in a few of its manifold applications.

Peirce: CP 4.211 Cross-Ref:††

211. In the denumerable multitude we noticed the first beginnings of the phenomenon of the fusion of the units. All the units of the first part of the primal order of a denumerable multitude can be individually designated as far as we please, but those in the latter part cannot. In the primipostnumeral multitude the same phenomenon is much more marked. It is impossible to designate individually all the units in any part of a primipostnumeral multitude. Any one unit may be completely separated from all the others without the slightest disturbance of the arrangement.

Peirce: CP 4.211 Cross-Ref:††

Thus, we may imagine points measured off from 0 as origin

0 A B C
 ·

toward A to represent the real quantities from zero toward $\sqrt{2}$. Let A be the point which according to this measurement would represent $\sqrt{2}$. But we may modify the rule of one-to-one correspondence between quantities and points, so that, for all values less than $\sqrt{2}$, the points to the left of A represent those values, while another point an inch or two to the right shall represent $\sqrt{2}$, and all quantities greater than $\sqrt{2}$ shall be represented by points as many inches or parts of an inch to the right of a third point, C, several inches to the right of B, as there are units and parts of units in the excess of those quantities over $\sqrt{2}$. This mode of representation is just as perfect as the usual unbroken correspondence. It represents all the relations of the quantities with absolute fidelity and does not disturb their arrangement in the least.

Peirce: CP 4.211 Cross-Ref:††

It is, therefore, perfectly possible to set off any one unit of a primipostnumeral collection by itself, and equally possible so to set off any enumerable multitude of such units. Nor are there any singular units of the collection which resist such separation.

Peirce: CP 4.211 Cross-Ref:††

I will give another illustration. It is perfectly easy to exactly describe many surd quantities simply by stating what their expressions in the Arabic system of notation would be. This may sound very false; but it is so, nevertheless. For instance, that quantity, which is expressed by a decimal point followed by a denumerable series of figures, of which every one which stands in a place appropriated to $(1/10)^n$ where n is *prime* shall be a figure 1, while every one which stands in a place whose logarithm n is composite shall be a *cipher*, is, we know, an irrational quantity. Now, I do not think there can be much doubt that, however recondite and complicated the descriptions may be, *every* surd quantity is capable in some such way of having its expression in decimals exactly described.

Peirce: CP 4.211 Cross-Ref:††

Thus every unit of a primipostnumeral collection admits of being individually designated and exactly described in such terms as to distinguish it from every other

unit of the collection. Thus, notwithstanding a certain incipient cohesiveness between its units, it is a discrete collection, still. . . .

Peirce: CP 4.212 Cross-Ref:††

212. It is one of the effects of the deplorable neglect by mathematicians of the properties of primipostnumeral collections that we are in complete ignorance of an arrangement of such a collection, which should be related to its primal arrangement in any manner analogous to the relation of the arrangement [of] the primal arrangement of the denumerable collection to that indefinitely dividant arrangement, which leads to a clear conception of the next grade of multitude.

Peirce: CP 4.212 Cross-Ref:††

I have had but little time to consider this problem; but I can produce an arrangement which will be of some service. Suppose that instead of proceeding, as in the usual generation of the primipostnumeral multitude, to go through a denumerable series of operations each consisting in interpolating a unit between every pair of successive units, we go through a denumerable multitude of operations each consisting in replacing every pair by an image of the whole collection. For example, using the binary system of arithmetical notation, suppose we begin with a collection of two objects, zero and one-half.

.0 .1

Each operation may consist of replacing each number by a sub-collection of [all the] numbers, each consisting of two parts, the first part being the figures of the number replaced, the second [being the figures of one of] the numbers composing the whole collection. Thus, the result of the first operation will be

.00 .01 .10 .11

The result of the second operation will be

.0000 .0001 .0010 .0011 .0100 .0101 .0110
.0111 .1000 .1001 .1010 .1011 .1100 .1101
.1110 .1111

The next result would be 256 numbers, the next 65,536 numbers, the next 4,294,867,296. The result of a denumerable succession of such operations will evidently be to give all the real quantities between zero and one, which is a primipostnumeral collection.

Peirce: CP 4.213 Cross-Ref:††

§4. THE SECUNDOPOSTNUMERAL AND LARGER COLLECTIONS

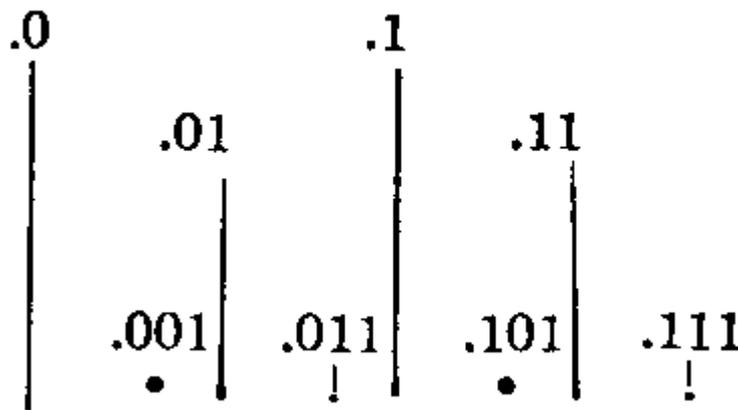
213. Although I have not touched upon half the questions of interest concerning the primipostnumeral multitude, I must hurry on to inquire, what is the least multitude greater than the primipostnumeral multitude? Time forbids my going through a fundamentally methodical discussion of this problem. But the speediest route to a correct solution of a difficult logical crux lies almost always through that paradox or sophism which depends upon that crux. Let us recur then for a moment to

the indefinitely dividend arrangement of a primipostnumeral collection. It will be convenient to use the binary system of arithmetical notation. We begin with .0 as our $\pi[0]$. $P[1]$ consists

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.0    .1
|    |
|.01 | .11
|    |
|    |
|.001|.011|.101|.111
|    |
|.  | | | .  | |

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of a fraction equal to that but carried into the first place of secundals and of corresponding units which differ only in having a 1 in the first place of secundals. $P[2]$ consists of fractions equal to those but carried into the second place of secundals, together with fractions differing from them only in having a 1 in the second place of secundals. And so on. Now if we use all the enumerable places of secundals, but stop before we reach any denumerable place, we shall have, among all the packs, all the fractions whose denominators are powers of 2 with enumerable exponents, and therefore we shall plainly have only a denumerable collection. But if n is the number of packs up to a given pack, then the number of fractions will be $2n-1$. When we have used all the enumerable places of secundals and no others, how many packs have we used? Plainly a denumerable collection, since the multitude of enumerable whole numbers is denumerable. It would appear, then, that $2n-1$, when n is denumerable, is denumerable. But on the contrary, if we consider only that pack which fills every enumerable place of secundals, since it contains the expression in secundals of every real quantity between 0 and 1, it alone is a primipostnumeral collection. Moreover,

the number of π 's in $P[n]$ is $2n$ and since n is denumerable for this collection, it follows that $2n$ is primipostnumeral. And it is impossible that the subtraction of one unit should reduce a primipostnumeral collection to a denumerable collection. Again, every pack contains a multitude of individuals only 1 more than that of all the packs that precede it in the order of the packs. How then can the former be primipostnumeral while the latter is denumerable?

Peirce: CP 4.213 Cross-Ref:††

The explanation of this sophism is that it confounds two categories of characters of collections, their *multitudes* and their *arithms*. The arithm of a multitude is the multitude of multitudes less than that multitude. Thus, the arithm of 2 is 2; for the multitudes less than 2 are 0 and 1. By *number* in one of its senses, that in which I endeavor to restrict it in exact discussions, is meant an enumerable arithm. Thus, the arithm or number of any enumerable multitude is that multitude. The arithm of the denumerable multitude, also, is that multitude. But the arithm of the primipostnumeral multitude is the denumerable multitude. The maximum multitude of an increasing endless series that converges to a limit is the arithm of that limit, in this sense, that by the limit of an increasing endless series is meant the smallest multitude greater than all the terms of the series. If there is no such smallest multitude the series is not convergent. If, then, by the maximum multitude of an increasing series we mean the multitude of *all* the multitudes which would converge increasingly to the given limit, this maximum multitude is plainly the arithm of the series. Thus, the series of whole numbers is an increasing endless series. Its limit is the denumerable multitude. The arithm of this multitude is the maximum multitude of the series. If in 2^n we substitute the different whole numbers for n , we get an increasing endless series whose limit is the primipostnumeral multitude. Its arithm, which is the maximum multitude of the series, is denumerable only. It is strictly true that the multitude of pack $P[n]$, in the example to which the sophism relates, is $2n$. But it is not strictly true that the multitude of π 's in all preceding packs is $2n-1$. It happens to be so when n is a *number*, that is, is enumerable. But strictly it is the multitude *next smaller* than the multitude of $2n$. If the latter is the primipostnumeral multitude, the former can be nothing but the denumerable multitude. This is what we find to be the case, as it must be; and there is nothing paradoxical in it, when rightly understood. There is no value of n for which $2n$ is denumerable.

Peirce: CP 4.213 Cross-Ref:††

The limit of $2n$ is primipostnumeral. The denumerable is skipped. But were we to reach the denumerable as we may, if we erroneously assume the sum of $2n$ is $2n-1$, when we double that on the principle that $2n = 2X(2n-1)$, we, of course, only have the denumerable as the result.

Peirce: CP 4.214 Cross-Ref:††

214. Let us now consider 2^2n . Since $2n$ can never be denumerable, but skips at once from the enumerable to the primipostnumeral, when n is denumerable, it follows that 2^2n can never be denumerable nor primipostnumeral. For there is no value which $2n$ could have to make 2^2n denumerable; and in order that 2^2n should be primipostnumeral, $2n$ would have to be denumerable, which is impossible. Thus, 2^2n skips the denumerable and the denumerable [Ed. note, Burks: "primipostnumeral" should replace "denumerable" here] multitudes. But if we use square brackets to denote the arithm, so that $[2]=2$, $[3]=3$, $[\infty]=\infty$, etc., then since $[2^\infty]$ is denumerable, $2^{[2^\infty]}$ is primipostnumeral.

Peirce: CP 4.215 Cross-Ref:††

215. When we start with .0 and .1, and repeating these varieties in the next figures, get .00 .01 .10 .11, and then repeating these varieties in the next figures, get .0000 .0001 etc., and then repeating these varieties in the next figures, get .00000000 .00000001 etc., if we say that, when this operation is carried out until the number of figures is denumerable, we get a primipostnumeral collection, we are assuming what is not true, that by continually doubling an enumerable multitude we shall ever get to a denumerable multitude. That is not true. In that process the denumerable multitude is skipped. We are assuming that because [the] multitude of all the arithmetical places which we pass by is denumerable, when the operation has been performed a denumerable multitude of times, therefore the multitude reached is denumerable. That is, we are confusing 2^∞ with $[2^\infty]$.

Peirce: CP 4.215 Cross-Ref:††

The function 2^{2^x} is no doubt the simplest one which skips the denumerable and primipostnumeral multitudes. Therefore the multitude of this when x is denumerable is, no doubt, the smallest multitude greater than the primipostnumeral multitude. It is the *secundopostnumeral* multitude.

Peirce: CP 4.216 Cross-Ref:††

216. Although there can remain no doubt whatever to an exact logician of the existence in the world of mathematical ideas, of the secundopostnumeral multitude, yet I have been unable, as yet, to form any very intuitively conception †1 of the construction of such a collection. But I must confess I have not bestowed very much thought upon this matter. I give a few constructions which have occurred to me.

Peirce: CP 4.216 Cross-Ref:††

Imagine points on a line to be in one-to-one correspondence with all the different real quantities between 0 and 1. Imagine the line to be repeated over and over again in each repetition having a different set of those points marked. Then the entire collection of repetitions is a *secundopostnumeral* collection.

Peirce: CP 4.216 Cross-Ref:††

Imagine a denumerable row of things, which we may call the B's. Let every set of B's possess some character, which we may call its crane †1 different from the crane of any other set. Imagine a collection of houses which we may call the beths †1 such that each house contains an object corresponding to each crane-character, and according as that object does or does not possess that character, the beth is said to possess or want that character. Then, the different possible varieties of beths, due to their possessing or not possessing the different cranes, form a secundopostnumeral collection.

Peirce: CP 4.216 Cross-Ref:††

According to the hypothesis of Euclidean projective geometry there is a plane at infinity. That plane we virtually see when we look up at the blue spread of the sky. A straight line at infinity, although it is straight and looks straight, is called a great semi-circle of the heavens. At two opposite points of the horizon we look at the same point of the plane at infinity. Of course, we cannot look both ways at once. We measure distances on an ordinary straight line by metres and centimetres. We measure distance on a straight line in the sky by degrees and minutes. The entire circuit of the straight line is 180 degrees, and the circuits of all straight lines are equal. But in metres the measure is infinite. If by a projection we make a position of a straight line

in the sky correspond to a straight line near at hand, we perhaps make a degree correspond to a metre, although in reality a metre is to a degree in the proportion, 180 degrees to infinity. Imagine that, upon a straight filament in the sky, points are marked off metrically corresponding to all the real quantities. Then let that filament be brought down to earth. If one of those real quantities' points is at any near point, there will not be another at any finite number of kilometres from it. For were there two, when it was in the sky they would have been closer together than any finite fraction of a second of arc. If, however, when you had pulled the filament down from the sky you were to find that each of those things you took for points was really a doubly refracting crystal and that these acted quite independently of one another, so that when you looked through two others you saw four images, when you looked through three you saw eight images, and so on, then if you were to look along the filament through all the crystals, one for each real quantity, the collectum of images you would see would be a secundopostnumeral collection.

Peirce: CP 4.217 Cross-Ref:††

217. In like manner, there will be a *tertiopostnumeral* multitude $2^2\aleph$, a *quartopostnumeral* multitude $2^{22}\aleph$ and so on *ad infinitum*.

Peirce: CP 4.217 Cross-Ref:††

All of these will be discrete multitudes although the phenomenon of the incipient cohesion of units becomes more and more marked from one to another.

Peirce: CP 4.217 Cross-Ref:††

These multitudes bear no analogy to the orders of infinity of the calculus; for $\infty^1 \times \infty^1 = \infty^2$. But any of these multiplied by itself gives itself. I had intended to explain these infinities of the calculus. But I find I cannot cram so much into a single lecture.

Peirce: CP 4.218 Cross-Ref:††

218. I now inquire, is there any multitude larger than all of these? That there is a multitude greater than any of them is very evident. For every postnumeral multitude has a next greater multitude. Now suppose collections one of each postnumeral multitude, or indeed any denumerable collection of postnumeral multitudes, all unequal. As all of these are possible their aggregate is *ipso facto* possible. For aggregation is an existential relation, and the aggregate exists (in the only kind of existence we are talking of, existence in the world of noncontradictory ideas) by the very fact that its aggregant parts exist. But this aggregate is no longer a discrete multitude, for the formula $2n > n$ which I have proved holds for all discrete collections cannot hold for this. In fact writing Exp. n for $2n$, (Exp.) \aleph^\aleph is evidently so great that this formula ceases to hold and it represents a collection no longer discrete.

Peirce: CP 4.219 Cross-Ref:††

§5. CONTINUA

219. Since then there is a multiplicity or multiplicities greater than any discrete multitude, we have to examine continuous multiplicities. Considered as a mere multitude, we might be tempted to say that continuous multiplicities are incapable of discrimination. For the nature of the differences between them does not

depend upon what multitudes enter into the denumerable series of discrete multitudes out of which the continuous multiplicity may be compounded; but it depends on the manner in which they are connected. This connection does not spring from the nature of the individual units, but constitutes the mode of existence of the whole.

Peirce: CP 4.219 Cross-Ref:††

The explanation of the paradoxes which arise when you undertake to consider a line or a surface as a collection of points is that, although it is true that a line is nothing but a collection of points of a particular mode of multiplicity, yet in it the individual identities of the units are completely merged, so that not a single one of them can be identified, even approximately, unless it happen to be a topically singular point, that is, either an extremity or a point of branching, in which case there is a defect of continuity at that point. This remark requires explanation, owing to the narrowness of the common ways of conceiving of geometry. Briefly to explain myself, then, geometry or rather mathematical geometry, which deals with pure hypotheses, and unlike physical geometry, does not investigate the properties of objectively valid space -- mathematical geometry, I say, consists of three branches; Topics (commonly called Topology), Graphics (or pure projective geometry), and Metrics. But metrics ought not to be regarded as pure geometry. It is the doctrine of the properties of such bodies as have a certain hypothetical property called absolute rigidity, and all such bodies are found to slide upon a certain individual surface called the Absolute. This Absolute, because it possesses individual existence, may properly be called a thing. Metrics, then, is not pure geometry; but is the study of the graphical properties of a certain hypothetical thing. But neither ought graphics to be considered as pure geometry. It is the doctrine of a certain family of surfaces called the *planes*. But when we ask what surfaces these planes are, we find that no other purely geometrical description can be given of them than that there is a threefold continuum of them and that every three of them have one point and one only in common. But innumerable families of surfaces can be conceived of which that is true. For imagine space to be filled with a fluid and that all the planes, or a sufficient collection of them, are marked by dark films in that fluid. Suppose the fluid to be slightly viscous, so that the different parts of it cannot break away from one another. Then give that fluid any motion. The result will be that those films will be distorted into a vast variety of shapes of all degrees of complexity, and yet any three of them will continue to possess a particle in common. The family of surfaces they then occupy will have every purely geometrical property of the family of planes; and yet they will be planes no longer. The distinguishing character of a plane is that if any particle lying in it be luminous and any filament lying in it be opaque, the shadow of that filament from that luminous particle lies wholly in the plane. Hence it is that unlimited straight lines are called *rays*. Graphics then is not pure geometry but is geometrical perspective. If, however, any geometer replies that the family of planes ought not to be limited to optical planes, but ought to be considered as any tridimensional continuum of surfaces, any three of which have just one point in common, then my rejoinder is that if we are to allow the planes to undergo any sort of distortion so long as the connections of the different planes of the family are preserved, then the whole doctrine of graphics is manifestly nothing but a branch of topics. For this is just what topics is. It is the study of the continuous connections and defects of continuity of loci which are free to be distorted in any way so long as the integrity of the connections and separations of all their parts is maintained. All strictly pure geometry, therefore, is topics. I now proceed to explain my remark that in a continuous *locus* no point has any individual identity, unless it be a topically singular point, that is, an isolating point, or either the extremity of a line, or a point from which three or more branches

of a line, or two or more sheets of a surface extend. Consider for example an oval line, and let that oval line be broken so as to make a line with two extremities. It may be said that when this happens a point of the oval bursts into two. But I say that there is no particular point of the yet unbroken oval which can be identified, even approximately, with the point which bursts. For to say that the different points of an oval move round the oval, without ever moving out of it, is a form of words entirely destitute of meaning. The points are but places; and the oval and all its parts subsist unchanged whether we regard the points as standing still or running round. In like manner, when we say the oval bursts, we introduce time with a second dimension. Considering the time, the place of the oval is a two dimensional place. This is cylindrical at the bursting and is a ribbon afterward. If one of the dimensions has a different quality from the other, the couple, consisting of a point and instant on the two dimensional continuum where the bursting takes place, has an individual identity. But it cannot be identified with any particular line in the cylindrical part of the two dimensional, even approximately. That line has no individuality.

Peirce: CP 4.220 Cross-Ref:††

220. If instead of an oval *place*, we consider an oval *thing*, say a filament, then it certainly means something to say that the parts revolve round the oval. For any one particle might be marked black and so be seen to move. And even if it were not actually marked, it would have an individuality which would make it capable of being marked. So that the filament would have a definite velocity of rotation whether it could be seen to move or not. But the reply to this is, that the marking of a single particle would be a discontinuous marking; and if the particles possess all their own individual identities, that is to suppose a discontinuity of existence everywhere, notwithstanding the continuity of place. But I go further. If those particles possess each its individual existence there is a discrete collection of them, and this collection must possess a definite multitude. Now this multitude cannot equal the multiplicity of the aggregate of all possible discrete multitudes; because it is a discrete multitude, and as such it is smaller than another possible multitude. Hence, it is not equal to the multitude of points of the oval. For that is equal to the aggregate of all possible discrete multitudes, since the line, by hypothesis, affords room for any collection of discrete points however great. Hence, if particles of the filament are distributed equally along the line of the oval, there must be, in every sensible part, continuous collections of points, that is, *lines*, that are unoccupied by particles. These lines may be far less than any *assignable* magnitudes, that is, far less than any parts into which the system of real quantities enables us to divide the line. But there is no contradiction whatever involved in that. It thus appears that true continuity is logically absolutely repugnant to the individual designation or even approximate individual designation of its units, except at points where the character of the continuity is itself not continuous.

Peirce: CP 4.221 Cross-Ref:††

221. In view of what has been said, it is not surprising that those arithmetical operations of addition and multiplication, which seemed to have lost their significance forever, now reappear in reference to continua. It is not that the points, as points, can be one more or less; but if there are defects of continuity, those discontinuities can have perfect individual identity and so be added and multiplied.

Peirce: CP 4.222 Cross-Ref:††

222. In regard to lines, there are two kinds of defects of continuity. The first is, that two or more particles moving in a line-figure may be unable to coalesce. The possible number of such non-coalescible particles may be called the *chorisis* of a

figure. Any kind of a geometrical figure has chorisism whether it be a point-figure, a line-figure, a surface-figure, or what. Thus the chorisism of three [not overlapping] ovals is three. The chorisism may be any discrete multitude.

Peirce: CP 4.223 Cross-Ref:††

223. The other defect of continuity that can affect a line-figure is that there may be a collection of points upon it from which a particle can move in more or fewer ways than from the generality of points of the figure. These *topically singular points*, as I call them, are of two kinds: those away from which a particle can move on the line in less than two ways and those from which a particle can move in the line in more than three ways. Of the first kind are, first, isolated points, †1 or *topical acnodes*, and extremities. Those, from which a particle can move in more than two ways, are points of branching, or *topical nodes*. The negative of what Listing calls the *Census number* of a line is, if we give a further extension to his definition, that which I would call the total *singularity* of the line; namely, it is half the sum of the excesses over two of the number of ways in which a particle could leave the different singular points of the line. No line can have a fractional total singularity.

Peirce: CP 4.224 Cross-Ref:††

224. In regard to surfaces, the chorisism is very simple and calls for no particular attention.

Peirce: CP 4.224 Cross-Ref:††

The theory of the singular places of surfaces is somewhat complicated. The singular places may be points, and those are either isolated points or points where two or more sheets are tacked together. Or the singular places may be isolated lines, and those are either totally isolated, or they may cut the surface. Such lines can have singularities like lines generally. Or the singular places may be lines which are either bounding edges or lines of splitting of the surface, or they may be in some parts edges and in other parts lines of splitting. They have singular points at which the line need not branch. All that is necessary is that the identities of the sheets that join there should change. If such a line has an extremity or point of odd branches, an even number of the sheets which come together there must change.

Peirce: CP 4.225 Cross-Ref:††

225. In addition to that, surfaces are another kind of defect of continuity, which Listing calls their *cyclosis*. That is, there is room upon them for oval filaments which cannot shrink to nothing by any movement in the surface. The number of operations each of a kind calculated to destroy a simple cyclosis which have to be [employed] in order to destroy the cyclosis of a surface is the number of the cyclosis. A puncture of a surface which does not change it from a closed surface to an open surface increases the cyclosis by one. A cut from edge to edge which does not increase the chorisism diminishes the cyclosis by one.

Peirce: CP 4.225 Cross-Ref:††

The cyclosis of a spherical surface is 0; that of an unlimited plane is 1; that of an anchor-ring is 2, that of a plane with a fornix (or bridge from one part to another) is 3; that of an anchor-ring with a fornix is 4, etc.

Peirce: CP 4.225 Cross-Ref:††

Euler's theorem †1 concerning polyhedra is an example of the additive arithmetic of continua.

Peirce: CP 4.226 Cross-Ref:††

226. The multiplicity of points upon a surface must be admitted, as it seems to me, to be the square of that of the points of a line, and so with higher dimensions. The multitude of dimensions may be of any discrete multitude.

Peirce: CP 4.227 Cross-Ref: ††

VII

THE SIMPLEST MATHEMATICS †1P

§1. THE ESSENCE OF MATHEMATICS

227. In this chapter, I propose to consider certain extremely simple branches of mathematics which, owing to their utility in logic, have to be treated in considerable detail, although to the mathematician they are hardly worth consideration. In Chapter 4, †2 I shall take up those branches of mathematics upon which the interest of mathematicians is centred, but shall do no more than make a rapid examination of their logical procedure. In Chapter 5, †2 I shall treat formal logic by the aid of mathematics. There can really be little logical matter in these chapters; but they seem to me to be quite indispensable preliminaries to the study of logic.

Peirce: CP 4.228 Cross-Ref: ††

228. It does not seem to me that mathematics depends in any way upon logic. It reasons, of course. But if the mathematician ever hesitates or errs in his reasoning, logic cannot come to his aid. He would be far more liable to commit similar as well as other errors there. On the contrary, I am persuaded that logic cannot possibly attain the solution of its problems without great use of mathematics. Indeed all formal logic is merely mathematics applied to logic. †3

Peirce: CP 4.229 Cross-Ref: ††

229. It was Benjamin Peirce, †4 whose son I boast myself, that in 1870 first defined mathematics as "the science which draws necessary conclusions." †5 This was a hard saying at the time; but today, students of the philosophy of mathematics generally acknowledge its substantial correctness.

Peirce: CP 4.230 Cross-Ref: ††

230. The common definition, among such people as ordinary schoolmasters, still is that mathematics is the science of quantity. As this is inevitably understood in English, it seems to be a misunderstanding of a definition which may be very old, †P1 the original meaning being that mathematics is the science of *quantities*, that is, forms possessing quantity. We perceive that Euclid was aware that a large branch of geometry had nothing to do with measurement (unless as an aid in demonstrating); and, therefore, a Greek geometer of his age (early in the third century B.C.) or later could not define mathematics as the science of that which the abstract noun quantity expresses. A line, however, was classed as a quantity, or *quantum*, by Aristotle †1 and his followers; so that even perspective (which deals wholly with intersections and projections, not at all with lengths) could be said to be a science of quantities, "quantity" being taken in the concrete sense. That this was what was originally meant by the definition "Mathematics is the science of quantity," is sufficiently shown by

the circumstance that those writers who first enunciate it, about A.D. 500, that is Ammonius Hermiæ†2 and Boëthius,†3 make astronomy and music branches of mathematics; and it is confirmed by the reasons they give for doing so.†P2 Even Philo of Alexandria (100 B.C.), who defines mathematics as the science of ideas furnished by sensation and reflection in respect to their necessary consequences, since he includes under mathematics, besides its more essential parts, the theory of numbers and geometry, also the practical arithmetic of the Greeks, geodesy, mechanics, optics (or projective geometry), music, and astronomy, must be said to take the word 'mathematics' in a different sense from ours. That Aristotle did not regard mathematics as the science of quantity, in the modern abstract sense, is evidenced in various ways. The subjects of mathematics are, according to him, the how much and the continuous. (See *Metaph.* K iii 1061 a 33). He referred the continuous to his category of *quantum*; and therefore he did make *quantum*, in a broad sense, the one object of mathematics.

Peirce: CP 4.231 Cross-Ref:††

231. Plato, in the Sixth book of the *Republic*,†P1 holds that the essential characteristic of mathematics lies in the peculiar kind and degree of its abstraction, greater than that of physics but less than that of what we now call philosophy; and Aristotle †1 follows his master in this definition. It has ever since been the habit of metaphysicians to extol their own reasonings and conclusions as vastly more abstract and scientific than those of mathematics. It certainly would seem that problems about God, Freedom, and Immortality are more exalted than, for example, the question how many hours, minutes, and seconds would elapse before two couriers travelling under assumed conditions will come together; although I do not know that this has been proved. But that the methods of thought of the metaphysicians are, as a matter of historical fact, in any aspect, not far inferior to those of mathematics is simply an infatuation. One singular consequence of the notion which prevailed during the greater part of the history of philosophy, that metaphysical reasoning ought to be similar to that of mathematics, only more so, has been that sundry mathematicians have thought themselves, as mathematicians, qualified to discuss philosophy; and no worse metaphysics than theirs is to be found.

Peirce: CP 4.232 Cross-Ref:††

232. Kant †2 regarded mathematical propositions as synthetical judgments *a priori*; wherein there is this much truth, that they are not, for the most part, what he called analytical judgments; that is, the predicate is not, in the sense he intended, contained in the definition of the subject. But if the propositions of arithmetic, for example, are true cognitions, or even forms of cognition, this circumstance is quite aside from their mathematical truth. For all modern mathematicians agree with Plato and Aristotle that mathematics deals exclusively with hypothetical states of things, and asserts no matter of fact whatever; and further, that it is thus alone that the necessity of its conclusions is to be explained.†P1 This is the true essence of mathematics; and my father's definition is in so far correct that it is impossible to reason necessarily concerning anything else than a pure hypothesis. Of course, I do not mean that if such pure hypothesis happened to be true of an actual state of things, the reasoning would thereby cease to be necessary. Only, it never would be known apodictically to be true of an actual state of things. Suppose a state of things of a perfectly definite, general description. That is, there must be no room for doubt as to whether anything, itself determinate, would or would not come under that description. And suppose, further, that this description refers to nothing occult -- nothing that cannot be summoned up fully into the imagination. Assume, then, a range of

possibilities equally definite and equally subject to the imagination; so that, so far as the given description of the supposed state of things is general, the different ways in which it might be made determinate could never introduce doubtful or occult features. The assumption, for example, must not refer to any matter of fact. For questions of fact are not within the purview of the imagination. Nor must it be such that, for example, it could lead us to ask whether the vowel *OO* can be imagined to be sounded on as high a pitch as the vowel *EE*. Perhaps it would have to be restricted to pure spatial, temporal, and logical relations. Be that as it may, the question whether in such a state of things, a certain other similarly definite state of things, equally a matter of the imagination, could or could not, in the assumed range of possibility, ever occur, would be one in reference to which one of the two answers, *Yes* and *No*, would be true, but never both. But all pertinent facts would be within the beck and call of the imagination; and consequently nothing but the operation of thought would be necessary to render the true answer. Nor, supposing the answer to cover the whole range of possibility assumed, could this be rendered otherwise than by reasoning that would be apodictic, general, and exact. No knowledge of what actually is, no positive *knowledge*, as we say, could result. On the other hand, to assert that any source of information that is restricted to actual facts could afford us a necessary knowledge, that is, knowledge relating to a whole general range of possibility, would be a flat contradiction in terms.

Peirce: CP 4.233 Cross-Ref:††

233. Mathematics is the study of what is true of hypothetical states of things. That is its essence and definition. Everything in it, therefore, beyond the first precepts for the construction of the hypotheses, has to be of the nature of apodictic inference. No doubt, we may reason imperfectly and jump at a conclusion; still, the conclusion so guessed at is, after all, that in a certain supposed state of things something would necessarily be true. Conversely, too, every apodictic inference is, strictly speaking, mathematics. But mathematics, as a serious science, has, over and above its essential character of being hypothetical, an accidental characteristic peculiarity -- a *proprium*, as the Aristotelians used to say -- which is of the greatest logical interest. Namely, while all the "philosophers" follow Aristotle in holding no demonstration to be thoroughly satisfactory except what they call a "direct" demonstration, or a "demonstration why" -- by which they mean a demonstration which employs only general concepts and concludes nothing but what would be an item of a definition if all its terms were themselves distinctly defined -- the mathematicians, on the contrary, entertain a contempt for that style of reasoning, and glory in what the philosophers stigmatize as "mere" indirect demonstrations, or "demonstrations that." Those propositions which can be deduced from others by reasoning of the kind that the philosophers extol are set down by mathematicians as "corollaries." That is to say, they are like those geometrical truths which Euclid did not deem worthy of particular mention, and which his editors inserted with a garland, or corolla, against each in the margin, implying perhaps that it was to them that such honor as might attach to these insignificant remarks was due. In the theorems, or at least in all the major theorems, a different kind of reasoning is demanded. Here, it will not do to confine oneself to general terms. It is necessary to set down, or to imagine, some individual and definite schema, or diagram -- in geometry, a figure composed of lines with letters attached; in algebra an array of letters of which some are repeated. This schema is constructed so as to conform to a hypothesis set forth in general terms in the thesis of the theorem. Pains are taken so to construct it that there would be something closely similar in every possible state of things to which the hypothetical description in the thesis would be applicable, and furthermore to construct it so that it shall have no other characters

which could influence the reasoning. How it can be that, although the reasoning is based upon the study of an individual schema, it is nevertheless necessary, that is, applicable, to all possible cases, is one of the questions we shall have to consider. Just now, I wish to point out that after the schema has been constructed according to the precept virtually contained in the thesis, the assertion of the theorem is not evidently true, even for the individual schema; nor will any amount of hard thinking of the philosophers' corollarial kind ever render it evident. Thinking in general terms is not enough. It is necessary that something should be DONE. In geometry, subsidiary lines are drawn. In algebra permissible transformations are made. Thereupon, the faculty of observation is called into play. Some relation between the parts of the schema is remarked. But would this relation subsist in every possible case? Mere corollarial reasoning will sometimes assure us of this. But, generally speaking, it may be necessary to draw distinct schemata to represent alternative possibilities. Theorematic reasoning invariably depends upon experimentation with individual schemata. We shall find that, in the last analysis, the same thing is true of the corollarial reasoning, too; even the Aristotelian "demonstration why." Only in this case, the very words serve as schemata. Accordingly, we may say that corollarial, or "philosophical" reasoning is reasoning with words; while theorematic, or mathematical reasoning proper, is reasoning with specially constructed schemata.

Peirce: CP 4.234 Cross-Ref:††

234. Another characteristic of mathematical thought is the extraordinary use it makes of abstractions. Abstractions have been a favorite butt of ridicule in modern times. Now it is very easy to laugh at the old physician who is represented as answering the question, why opium puts people to sleep, by saying that it is because it has a dormative virtue. It is an answer that no doubt carries vagueness to its last extreme. Yet, invented as the story was to show how little meaning there might be in an abstraction, nevertheless the physician's answer does contain a truth that modern philosophy has generally denied: it does assert that there really is in opium *something* which explains its always putting people to sleep. This has, I say, been denied by modern philosophers generally. Not, of course, explicitly; but when they say that the different events of people going to sleep after taking opium have really nothing in common, but only that the mind classes them together -- and this is what they virtually do say in denying the reality of generals -- they do implicitly deny that there is any true explanation of opium's generally putting people to sleep.

Peirce: CP 4.235 Cross-Ref:††

235. Look through the modern logical treatises, and you will find that they almost all fall into one or other of two errors, as I hold them to be; that of setting aside the doctrine of abstraction (in the sense in which an abstract noun marks an abstraction) as a grammatical topic with which the logician need not particularly concern himself; and that of confounding abstraction, in this sense, with that operation of the mind by which we pay attention to one feature of a percept to the disregard of others. The two things are entirely disconnected. The most ordinary fact of perception, such as "it is light," involves *precisive* abstraction, or *prescission*.†1 But *hypostatic* abstraction, the abstraction which transforms "it is light" into "there is light here," which is the sense which I shall commonly attach to the word abstraction (since *prescission* will do for precise abstraction) is a very special mode of thought. It consists in taking a feature of a percept or percepts (after it has already been prescinded from the other elements of the percept), so as to take propositional form in a judgment (indeed, it may operate upon any judgment whatsoever), and in conceiving this fact to consist in the relation between the subject of that judgment and

another subject, which has a mode of being that merely consists in the truth of propositions of which the corresponding concrete term is the predicate. Thus, we transform the proposition, "honey is sweet," into "honey possesses sweetness." "Sweetness" might be called a fictitious thing, in one sense. But since the mode of being attributed to it *consists* in no more than the fact that some things are sweet, and it is not pretended, or imagined, that it has any other mode of being, there is, after all, no fiction. The only profession made is that we consider the fact of honey being sweet under the form of a relation; and so we really can. I have selected sweetness as an instance of one of the least useful of abstractions. Yet even this is convenient. It facilitates such thoughts as that the sweetness of honey is particularly cloying; that the sweetness of honey is something like the sweetness of a honeymoon; etc. Abstractions are particularly congenial to mathematics. Everyday life first, for example, found the need of that class of abstractions which we call *collections*. Instead of saying that some human beings are males and all the rest females, it was found convenient to say that *mankind* consists of the male *part* and the female *part*. The same thought makes classes of collections, such as pairs, leashes, quatrains, hands, weeks, dozens, baker's dozens, sonnets, scores, quires, hundreds, long hundreds, gross, reams, thousands, myriads, lacs, millions, milliards, milliasses, etc. These have suggested a great branch of mathematics.†P1 Again, a point moves: it is by abstraction that the geometer says that it "describes a line." This line, though an abstraction, itself moves; and this is regarded as generating a surface; and so on. So likewise, when the analyst treats operations as themselves subjects of operations, a method whose utility will not be denied, this is another instance of abstraction. Maxwell's notion of a tension exercised upon lines of electrical force, transverse to them, is somewhat similar. These examples exhibit the great rolling billows of abstraction in the ocean of mathematical thought; but when we come to a minute examination of it, we shall find, in every department, incessant ripples of the same form of thought, of which the examples I have mentioned give no hint.

Peirce: CP 4.236 Cross-Ref:††

236. Another characteristic of mathematical thought is that it can have no success where it cannot generalize. One cannot, for example, deny that chess is mathematics, after a fashion; but, owing to the exceptions which everywhere confront the mathematician in this field -- such as the limits of the board; the single steps of king, knight, and pawn; the finite number of squares; the peculiar mode of capture by pawns; the queening of pawns; castling -- there results a mathematics whose wings are effectually clipped, and which can only run along the ground. Hence it is that a mathematician often finds what a chess-player might call a gambit to his advantage; exchanging a smaller problem that involves exceptions for a larger one free from them. Thus, rather than suppose that parallel lines, unlike all other pairs of straight lines in a plane, never meet, he supposes that they intersect at infinity. Rather than suppose that some equations have roots while others have not, he supplements real quantity by the infinitely greater realm of imaginary quantity. He tells us with ease how many inflexions a plane curve of any description has; but if we ask how many of these are real, and how many merely fictional, he is unable to say. He is perplexed by three-dimensional space, because not all pairs of straight lines intersect, and finds it to his advantage to use quaternions which represent a sort of four-fold continuum, in order to avoid the exception. It is because exceptions so hamper the mathematician that almost all the relations with which he chooses to deal are of the nature of correspondences; that is to say, such relations that for every relate there is the same number of correlates, and for every correlate the same number of relates.

Peirce: CP 4.237 Cross-Ref:††

237. Among the minor, yet striking characteristics of mathematics, may be mentioned the fleshless and skeletal build of its propositions; the peculiar difficulty, complication, and stress of its reasonings; the perfect exactitude of its results; their broad universality; their practical infallibility. It is easy to speak with precision upon a general theme. Only, one must commonly surrender all ambition to be certain. It is equally easy to be certain. One has only to be sufficiently vague. It is not so difficult to be pretty precise and fairly certain at once about a very narrow subject. But to reunite, like mathematics, perfect exactitude and practical infallibility with unrestricted universality, is remarkable. But it is not hard to see that all these characters of mathematics are inevitable consequences of its being the study of hypothetical truth.

Peirce: CP 4.238 Cross-Ref:††

238. It is difficult to decide between the two definitions of mathematics; the one by its method, that of drawing necessary conclusions; the other by its aim and subject matter, as the study of hypothetical states of things. The former makes or seems to make the deduction of the consequences of hypotheses the sole business of the mathematician as such. But it cannot be denied that immense genius has been exercised in the mere framing of such general hypotheses as the field of imaginary quantity and the allied idea of Riemann's surface, in imagining non-Euclidian measurement, ideal numbers, the perfect liquid. Even the framing of the particular hypotheses of special problems almost always calls for good judgment and knowledge, and sometimes for great intellectual power, as in the case of Boole's logical algebra. Shall we exclude this work from the domain of mathematics? Perhaps the answer should be that, in the first place, whatever exercise of intellect may be called for in applying mathematics to a question not propounded in mathematical form [it] is certainly not pure mathematical thought; and in the second place, that the mere creation of a hypothesis may be a grand work of poietic †1 genius, but cannot be said to be scientific, inasmuch as that which it produces is neither true nor false, and therefore is not knowledge. This reply suggests the further remark that if mathematics is the study of purely imaginary states of things, poets must be great mathematicians, especially that class of poets who write novels of intricate and enigmatical plots. Even the reply, which is obvious, that by *studying* imaginary states of things we mean *studying* what is true of them, perhaps does not fully meet the objection. The article *Mathematics* in the ninth edition of the *Encyclopaedia Britannica*†2 makes mathematics consist in the study of a particular sort of hypotheses, namely, those that are exact, etc., as there set forth at some length. The article is well worthy of consideration.

Peirce: CP 4.239 Cross-Ref:††

239. The philosophical mathematician, Dr. Richard Dedekind,†3 holds mathematics to be a branch of logic. This would not result from my father's definition, which runs, not that mathematics is the science of *drawing* necessary conclusions -- which would be deductive logic -- but that it is the science which *draws* necessary conclusions. It is evident, and I know as a fact, that he had this distinction in view. At the time when he thought out this definition, he, a mathematician, and I, a logician, held daily discussions about a large subject which interested us both; and he was struck, as I was, with the contrary nature of his interest and mine in the same propositions. The logician does not care particularly about this or that hypothesis or its consequences, except so far as these things may throw a light upon the nature of

reasoning. The mathematician is intensely interested in efficient methods of reasoning, with a view to their possible extension to new problems; but he does not, *quâ* mathematician, trouble himself minutely to dissect those parts of this method whose correctness is a matter of course. The different aspects which the algebra of logic will assume for the two men is instructive in this respect. The mathematician asks what value this algebra has as a calculus. Can it be applied to unravelling a complicated question? Will it, at one stroke, produce a remote consequence? The logician does not wish the algebra to have that character. On the contrary, the greater number of distinct logical steps, into which the algebra breaks up an inference, will for him constitute a superiority of it over another which moves more swiftly to its conclusions. He demands that the algebra shall analyze a reasoning into its last elementary steps. Thus, that which is a merit in a logical algebra for one of these students is a demerit in the eyes of the other. The one studies the science of drawing conclusions, the other the science which draws necessary conclusions.

Peirce: CP 4.240 Cross-Ref:††

240. But, indeed, the difference between the two sciences is far more than that between two points of view. Mathematics is purely hypothetical: it produces nothing but conditional propositions. Logic, on the contrary, is categorical in its assertions. True, it is not merely, or even mainly, a mere discovery of what really is, like metaphysics. It is a normative science. It thus has a strongly mathematical character, at least in its methodeutic division; for here it analyzes the problem of how, with given means, a required end is to be pursued. This is, at most, to say that it has to call in the aid of mathematics; that it has a mathematical branch. But so much may be said of every science. There is a mathematical logic, just as there is a mathematical optics and a mathematical economics. Mathematical logic is formal logic. Formal logic, however developed, is mathematics. Formal logic, however, is by no means the whole of logic, or even its principal part. It is hardly to be reckoned as a part of logic proper. Logic has to define its aim; and in doing so is even more dependent upon ethics,†1 or the philosophy of aims, by far, than it is, in the methodeutic branch, upon mathematics. We shall soon come to understand how a student of ethics might well be tempted to make his science a branch of logic; as, indeed, it pretty nearly was in the mind of Socrates. But this would be no truer a view than the other. Logic depends upon mathematics; still more intimately upon ethics; but its proper concern is with truths beyond the purview of either.

Peirce: CP 4.241 Cross-Ref:††

241. There are two characters of mathematics which have not yet been mentioned, because they are not exclusive characteristics of it. One of these, which need not detain us, is that mathematics is distinguished from all other sciences †2 except only ethics, in standing in no need of ethics. Every other science, even logic -- logic, especially -- is in its early stages in danger of evaporating into airy nothingness, degenerating, as the Germans say, into an anachrioid [?] film, spun from the stuff that dreams are made of. There is no such danger for pure mathematics; for that is precisely what mathematics ought to be.

Peirce: CP 4.242 Cross-Ref:††

242. The other character -- and of particular interest it is to us just now -- is that mathematics, along with ethics and logic alone of the sciences, has no need of any appeal to logic. No doubt, some reader may exclaim in dissent to this, on first hearing it said. Mathematics, they may say, is preëminently a science of reasoning. So it is; preëminently a science that reasons. But just as it is not necessary, in order to

talk, to understand the theory of the formation of vowel sounds, so it is not necessary, in order to reason, to be in possession of the theory of reasoning. Otherwise, plainly, the science of logic could never be developed. The contrary objection would have more excuse, that no science stands in need of logic, since our natural power of reason is enough. Make of logic what the majority of treatises in the past have made of it, and a very common class of English and French books still make of it -- that is to say, mainly formal logic, and that formal logic represented as an art of reasoning -- and in my opinion this objection is more than sound, for such logic is a great hindrance to right reasoning. It would, however, be aside from our present purpose to examine this objection minutely. I will content myself with saying that undoubtedly our natural power of reasoning is enough, in the same sense that it is enough, in order to obtain a wireless transatlantic telegraph, that men should be born. That is to say, it is bound to come sooner or later. But that does not make research into the nature of electricity needless for gaining such a telegraph. So likewise if the study of electricity had been pursued resolutely, even if no special attention had ever been paid to mathematics, the requisite mathematical ideas would surely have been evolved. Faraday, indeed, did evolve them without any acquaintance with mathematics. Still it would be far more economical to postpone electrical researches, to study mathematics by itself, and then to apply it to electricity, which was Maxwell's way. In this same manner, the various logical difficulties which arise in the course of every science except mathematics, ethics, and logic, will, no doubt, get worked out after a time, even though no special study of logic be made. But it would be far more economical to make first a systematic study of logic. If anybody should ask what are these logical difficulties which arise in all the sciences, he must have read the history of science very irreflectively. What was the famous controversy concerning the measure of force but a logical difficulty? What was the controversy between the uniformitarians and the catastrophists but a question of whether or not a given conclusion followed from acknowledged premisses? This will fully appear in the course of our studies in the present work.†1

Peirce: CP 4.243 Cross-Ref:††

243. But it may be asked whether mathematics, ethics, and logic have not encountered similar difficulties. Are the doctrines of logic at all settled? Is the history of ethics anything but a history of controversy? Have no logical errors been committed by mathematicians? To that I reply, first, as to logic, that not only have the rank and file of writers on the subject been, as an eminent psychiatrist, Maudsley, declares, men of arrested brain-development, and not only have they generally lacked the most essential qualification for the study, namely mathematical training, but the main reason why logic is unsettled is that thirteen different opinions are current as to the true aim of the science.†1 Now this is not a logical difficulty but an ethical difficulty; for ethics is the science of aims. Secondly, it is true that pure ethics has been, and always must be, a theatre of discussion, for the reason that its study consists in the gradual development of a distinct recognition of a satisfactory aim. It is a science of subtleties, no doubt; but it is not logic, but the development of the ideal, which really creates and resolves the problems of ethics. Thirdly, in mathematics errors of reasoning have occurred, nay, have passed unchallenged for thousands of years. This, however, was simply because they escaped notice. Never, in the whole history of the science, has a question whether a given conclusion followed *mathematically* from given premisses, when once started, failed to receive a speedy and unanimous reply. Very few have been even the apparent exceptions; and those few have been due to the fact that it is only within the last half century that mathematicians have come to have a perfectly clear recognition of what is

mathematical soil and what foreign to mathematics. Perhaps the nearest approximation to an exception was the dispute about the use of divergent series. Here neither party was in possession of sufficient pure mathematical reasons covering the whole ground; and such reasons as they had were not only of an extra-mathematical kind, but were used to support more or less vague positions. It appeared then, as we all know now, that divergent series are of the utmost utility.†P1

Peirce: CP 4.243 Cross-Ref:††

Struck by this circumstance, and making an inference, of which it is sufficient to say that it was not mathematical, many of the old mathematicians pushed the use of divergent series beyond all reason. This was a case of mathematicians disputing about the validity of a kind of inference that is not mathematical. No doubt, a sound logic (such as has not hitherto been developed) would have shown clearly that that non-mathematical inference was not a sound one. But this is, I believe, the only instance in which any large party in the mathematical world ever proposed to rely, in mathematics, upon unmathematical reasoning. My proposition is that true mathematical reasoning is so much more evident than it is possible to render any doctrine of logic proper -- without just such reasoning -- that an appeal in mathematics to logic could only embroil a situation. On the contrary, such difficulties as may arise concerning necessary reasoning have to be solved by the logician by reducing them to questions of mathematics. Upon those mathematical dicta, as we shall come clearly to see, the logician has ultimately to repose.

Peirce: CP 4.244 Cross-Ref:††

244. So a double motive induces me to devote some preliminary chapters to mathematics. For, in the first place, in studying the theory of reasoning, we are concerned to acquaint ourselves with the methods of that prior science of which acts of reasoning form the staple. In the second place, logic, like any other science, has its mathematical department, and of that, a large portion, at any rate, may with entire convenience be studied as soon as we take up the study of logic, without any propedeutic. That portion is what goes by the name of Formal Logic.†P1 It so happens that the special kind of mathematics needed for formal logic, which, therefore, we need to study in detail, as we need not study other branches of mathematics, is so excessively simple as neither to have much mathematical interest, nor to display the peculiarities of mathematical reasoning. I shall, therefore, devote the present chapter -- a very dull one, I am sorry to say, it must be -- to this kind of mathematics. Chapter 4 will treat of the more truly mathematical mathematics; and Chapter 5 will apply the results of the present chapter to the study of Formal Logic.†1

Peirce: CP 4.245 Cross-Ref:††

§2. DIVISION OF PURE MATHEMATICS*P*

245. We have to make a rapid survey of pure mathematics, in so far as it interests us as students of logic. Each branch of mathematics will have to be reconnoitered and its methods examined. Those parts of the calculus of which use has to be made in the study of reasoning must receive a fuller treatment. Finally, having so collected some information about mathematics, we may venture upon some useful generalizations concerning the nature of mathematical thought. But this plan calls for a preliminary dissection of mathematics into its several branches.

Peirce: CP 4.246 Cross-Ref:††

246. Each branch of mathematics sets out from a general hypothesis of its own. I mean by its general hypothesis the substance of its postulates and axioms, and even of its definitions, should they be contaminated with any substance, instead of being the pure verbiage they ought to be. We have to make choice, then, between a division of mathematics according to the matter of its hypotheses, or according to the forms of the schemata of which it avails itself. These latter are either geometrical or algebraical. Geometrical schemata are linear figures with letters attached; the perfect imaginability, on the one hand, and the extreme familiarity, on the other hand, of spatial relations are taken advantage of, to enable us to see what will necessarily be true under supposed conditions. The algebraical schemata are arrays of characters, sometimes in series, sometimes in blocks, with which are associated certain rules of permissible transformation. With these rules the algebraist has perfectly to familiarize himself. By virtue of these rules, become habits of association, when one array has been written or assumed to be permissibly scriptible, the mathematician just as directly perceives that another array is permissibly scriptible, as he perceives that a person talking in a certain tone is angry, or [is] using certain words in such and such a sense.

Peirce: CP 4.247 Cross-Ref:††

247. The primary division of mathematics into algebra and geometry is the usual one. But, in all departments, it appears both *a priori* and *a posteriori*, that divisions according to differences of purpose should be given a higher rank than divisions according to different methods of attaining that purpose.†1 The division of pure mathematics into algebra and geometry was first adopted before the modern conception of pure mathematics had been distinctly prescinded, and when geometry and algebra seemed to deal with different subjects. It remains, a vestige of that old unclearness and a witness that not even mathematicians are able entirely to shake off the sequelæ of exploded ideas. For now that everybody knows that any mathematical subject, from the theory of numbers to topical geometry, may be treated either algebraically or geometrically, one cannot fail to see that so to divide mathematics is to make twice over the division according to fundamental hypotheses, to which one must come, at last. This duplication is worse than useless, since the geometrical and algebraical methods are by many writers continually mixed. No such inconvenience attends the other plan of classification; for two sets of fundamental hypotheses could not, properly speaking, be mixed without self-contradiction.

Peirce: CP 4.248 Cross-Ref:††

248. Let us, then, divide mathematics according to the nature of its general hypotheses, taking for the ground of primary division the multitude of units, or elements, that are supposed; and for the ground of subdivision that mode of relationship between the elements upon which the hypotheses focus the attention.

Peirce: CP 4.249 Cross-Ref:††

249. From a logician's point of view this plan of classification would seem to call for a preliminary analysis of what is meant by multitude. But to execute this analysis satisfactorily, considerable studies of logic would be indispensable preliminaries. Besides, it is not at all in the spirit of mathematics to analyze the ideas with which it works farther than is needful for using them in deducing consequences, nor sooner than that need comes to be felt. It is true that we, as students of logic, are not bound to embrace the mathematical ways of thought as far as that, but the other circumstance, that it is, at the present stage of our studies, impossible to make the

analysis, must be conclusive.

Peirce: CP 4.250 Cross-Ref:††

§3. THE SIMPLEST BRANCH OF MATHEMATICS^P

250. Were nothing at all supposed, mathematics would have no ground at all to go upon. Were the hypothesis merely that there was nothing but one unit, there would not be a possibility of a question, since only one answer would be possible. Consequently, the simplest possible hypothesis is that there are two objects, which we may denote by \mathbf{v} and \mathbf{f} . Then the first kind of problem of this algebra will be, given certain data concerning an unknown object, \mathbf{x} , required to know whether it is \mathbf{v} or \mathbf{f} . Or similar problems may arise concerning several unknowns, \mathbf{x} , \mathbf{y} , etc. Or when the last problem cannot be resolved, we may ask whether, supposing \mathbf{x} to be \mathbf{v} , will \mathbf{y} be \mathbf{v} or \mathbf{f} ? And similarly, supposing \mathbf{x} to be \mathbf{f} . Again, given certain data concerning \mathbf{x} , we may ask, what else needs to be known in order to compel \mathbf{x} to be \mathbf{v} or to be \mathbf{f} . Or again, given certain information about \mathbf{x} , \mathbf{y} , and \mathbf{z} , what relations between \mathbf{x} and \mathbf{z} remain unchanged whether \mathbf{y} be \mathbf{v} or \mathbf{f} ?

Peirce: CP 4.251 Cross-Ref:††

251. Let us call \mathbf{v} and \mathbf{f} the two possible values, one of which must be attached to any unknown. For the form of reasoning will be the same whether we talk of identity or attachment. The attachment may be of any kind so long as each unknown must be, or be attached to, \mathbf{v} or \mathbf{f} , but cannot be or be attached to, both \mathbf{v} and \mathbf{f} . This idea of a system of values is one of the most fundamental abstractions of the algebraic method of mathematics. An object of the universe, whose *value* is generally unknown, though it may in special cases be known -- that is to say, an object which, to phrase the matter differently, *is* one of the values, though perhaps we do not know which -- is called, when we speak of it as "*having*" a value, a *quantity*. For example, suppose the problem under consideration be to determine, upon a certain hypothesis, the numerical definition of the instant, or, as we may say, to determine the exact *date*, at which two couriers will meet. This date is some one of the series of numbers each of which is expressible, at least to any predesignate degree of approximation, in our usual method of numeral notation. That series of numbers will be the system of values; and the number we want *is* one of them. But we find it convenient to use a different phrase, and to say that the date *is* defined to be the date at which the couriers meet, that *this* fixes its *identity*, and that what we seek to know is what value becomes *attached* to it in consequence of the conditions the problem supposes. It will be convenient to conceive of this statement as a "mere" variation of phraseology, although, as we shall learn, the word "mere" in such cases is often inappropriate, since great mathematical results are attainable by such means. Dichotomic algebra can be applied wherever there are just two possible alternatives. Thus, we might call the \mathbf{v} the truth, and \mathbf{f} falsity. Then, in regard to a given proposition we may seek to know whether it is true or false; that is, whether it is or is not a partial description of the real universe, or say, whether what it means is identical with the existent truth or identical with nothing. Looking at the matter in a different way, or phrasing it differently, we say that a proposition has one or other of two values, being either true and good for something, or false and good for nothing. The point of view of mathematics is the point of view which looks upon those two points of view as no more than different phrases for the same fact.

Peirce: CP 4.252 Cross-Ref:††

252. There is another little group of algebraical words which must now be defined in the imperfect way in which they can be defined for dichotomic mathematics. In the first place, there are the pair of terms, *constant*, or *constant quantity*, and *variable*, or *variable quantity*. These words were introduced by the Marquis d'Hôpital †1 in 1696. Suppose two couriers to set out, at the same instant, from two points 12 miles apart and to travel toward one another, the one at the rate of 7 miles an hour, the other at the rate of 8 miles an hour: when will they meet? They evidently approach one another at the rate of 7 *plus* 8, or 15 miles an hour; and they will reduce the distance of 12 miles to nothing in 12/15 of an hour, or 12 times 4, or 48, minutes. But suppose we find the distance was wrongly given; that it is 12 1/2 miles. Then, the *date*, or numerical designation of the instant of meeting, becomes different. But if we choose to say that the *quantity* sought is defined as the time of meeting, and that it remains the same quantity, having the same definition, but that its value only is altered, then that quantity is said to be *variable*. A quantity is said to be *variable* when we propose to consider it as taking different values in different states of things; or, to phrase the matter differently, when we consider a group of questions together, as one general question, the single questions having different values for their answers. The most usual case is where we suppose the quantity to take all possible values under different circumstances. A quantity is called *constant* when the hypothesis includes no states of things in which its value changes. The difference between an *unknown* quantity and a variable quantity is trifling. The unknown quantity is variable at first; but special hypotheses being adopted, it is restricted to certain values, perhaps to a single value.

Peirce: CP 4.253 Cross-Ref:††

253. The word *function* (a sort of semi-synonym of "operation") was first used in something like its present mathematical sense in 1692, by a writer who was doubtless Leibniz. †1 It soon came into use with the circle of analysts of whom Leibniz was the centre. But the first attempt at a definition of it was by John Bernouilli, †2 in 1718. There has since been much discussion as to what precise meaning can most advantageously be applied to it; but the most general definition, that of Dirichlet, †3 is confined to a system of numerical values. Since I wish to apply the word to all sorts of algebra, I shall, under these circumstances, take the liberty of generalizing the meaning in the manner which seems to me to be called for. I shall say then, that, given two ordered sets of the same number of quantities, $x[1], x[2], x[3], \dots, x[n]$, and $y[1], y[2], y[3] \dots y[n]$, any quantity, say $x[2]$, of the one set is the *same function* of the other quantities of that same set, which are called its *arguments*, that the corresponding quantity, $y[2]$, according to the order of arrangement of the other set, is of the remaining quantities of that set, if and only if every set of values which either set of quantities, in their order, can take, can likewise be taken by the other set. Thus, to say that a quantity is a *given function* of certain quantities as *arguments* is simply to say that its value stands in a given relation to theirs; or that a given proposition is true of the whole set of values in their order. To say simply that one quantity is *some* function of certain others is to say nothing; since of every set of values something is true. But this no more renders *function* a useless word than the fact, that it means nothing to say of a set of things that there is some relation between them, renders *relation* a useless word.

Peirce: CP 4.253 Cross-Ref:††

I may mention that the old and usual expression is "a function of variables";

but the word *argument* here is not unusual and is more explicit. The function is also called the *dependent variable*; the arguments, the *independent variables*. Of course, any one of the whole set of quantities composed of the function and its arguments is just as dependent as any other. It is a mere way of referring to them. The function is often conceived, very conveniently, as resulting from an *operation* performed on the arguments, which are then called *operands*. The idea is that the definition of the *same function* implies a rule which permits such sets of values as may conform to its conditions and excludes others; and the *operation* is the operation of actually applying this rule, when the values of all the quantities but one are given, in order to ascertain what the value of the remaining quantity can be.

Peirce: CP 4.254 Cross-Ref:††

254. Among functions, or operations, there is one extensive class which is of particular importance. I call it the class of *correspondential* functions, or operations. Namely, if all the variables but one, independent and dependent, have a set of values assigned to them, then, if the relation between them is a *correspondence*, the number of different values which the remaining variable can have, is *generally* the same, whatever the particular set of assigned values may be; although this number is *not* necessarily the same when different quantities are thus left over to the last. I say *generally* the same, because there may be peculiar isolated exceptions, though this limitation can have no significance in dichotomic mathematics. A function which is in correspondence with its arguments may be called a correspondential function. It may be remarked that it is not the habit of mathematicians, in general statements, to pay attention to isolated exceptions; and when a mathematician uses the phrase "in general" he means to be understood as not considering possible peculiar cases. Thus, I have known a great mathematician to enunciate a proposition concerning multiple algebra to be true "in general" when the state of the case was that there were just two instances of its being true against an infinity of instances of its being false.

Peirce: CP 4.255 Cross-Ref:††

255. A function which has but one value for any one set of values of the arguments is called *monotropic*. A function which, when all the arguments except a certain one take any fixed values, always changes its value with a change of that one, may be called *distinctive* for that argument.

Peirce: CP 4.256 Cross-Ref:††

256. If the relation between a function and its arguments is such that one of the latter may take any value for every set of the values of the others without altering the function, the function may be said to be *invariable* with that argument. If the function can take any value, whatever values be assigned to the arguments, it may be said to be independent of the arguments. In either of these cases, the function may be called a *degenerate* function.

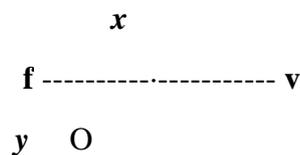
Peirce: CP 4.257 Cross-Ref:††

257. With this lexical preface, we come down to our dichotomic mathematics, which I shall treat algebraically. The first thing to be done is to fix upon a sign to show that any quantity, say x , has the value v , and upon another to signify that it has the value f . The simplest suggestion is that universally used since man began to keep accounts; namely, to appropriate a place in which we are to write whatever is v , say the upper of two lines, the lower of which is appropriated to quantities whose value is f . That is, we open one account for v , and another for f . In doing this, we put v and f in a radically different category from the other letters, very much as two opposite

qualities, say good and bad, are attributed to concrete objects. I do not mean that there is any other analogy than that the values, **v** and **f**, are made to be of a different nature from the quantities, **x**, **y**, **z**, etc. One or other of the values, but not both, is connected, in some definite sense, and it matters not what the sense may be, so long as it is definite, with each quantity. But here an important remark obtrudes. Non-connection in any definite way is only another equally definite mode of connection; especially in a strictly dichotomic state of things. If, for example, every man either does good and eschews evil, or does evil and eschews good, then the former is thereby connected with evil by eschewing it, as he is connected with good in the mode of connection called doing it. Note how the perfect balance of our initial dichotomy generates new dichotomies: first, two categories, those of value and of quantity; then, two modes of connection between a value and a quantity.

Peirce: CP 4.258 Cross-Ref:††

258. Let us modify our mode of signifying the attachment of a quantity to a value, so as to show its contrary attachment to the opposite value. For this purpose,



from a centre, **O**, let us draw a horizontal arm to the right, which we will call the **v**-radius, and another to the left, which we will call the **f**-radius. Now, then, any quantity **x** put in the upper or **v** account, will be so situated that a right-handed, or clock-wise, revolution around **O** will bring it first to the **v**-radius; as it will bring a quantity, **y**, in the **f** account, to the **f**-radius; while a left-handed, or counter-clock-wise, turn around **O** will carry the quantities each to the other radius. This diagram suggests another way of signifying the value of a quantity. Let a heavy line, representing the horizontal bar of the diagram, be drawn under the sign of a quantity, thus, **x**_, to signify that its value is **v**; and the same bar be drawn above it, thus, ~**y**, to signify that its value is **f**.

Peirce: CP 4.259 Cross-Ref:††

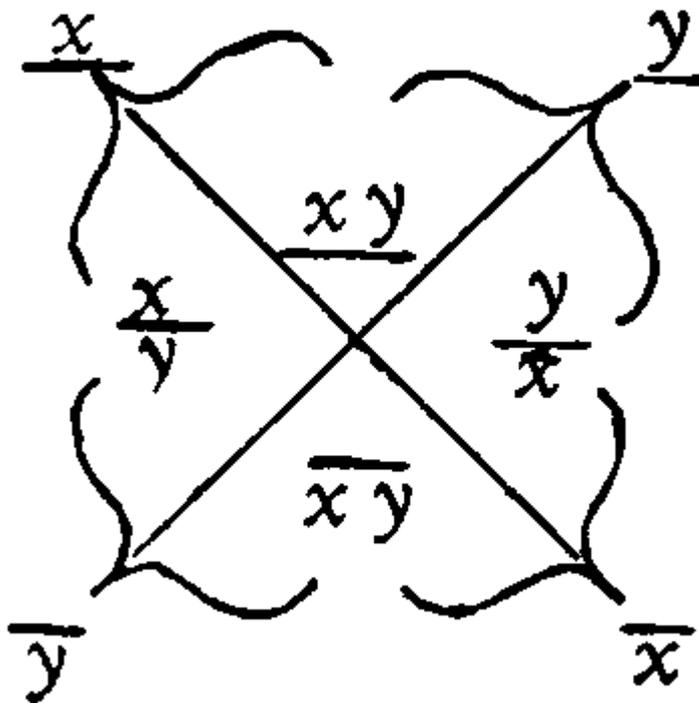
259. It may be mentioned that this mode of indicating the value by a bar has a historical appropriateness. For although the two values **f** and **v** are, at present, merely distinguished, without any definite difference between them being admitted -- and mathematically they do stand upon a precise par, and will continue to do so -- yet when dichotomic algebra comes to be applied to logic, it will be found necessary to call one of them verity and the other falsity; and the letters **v** and **f** were chosen with a view to that. We shall find it impossible later to prevent this affecting our purest practicable mathematics, in some measure. Now it has been the practice, from antiquity, to draw a heavy line under that whose truth it was desired to emphasize. On the other hand, the obelus, or spit, is already mentioned by Lucian, in the second century A. D., as the sign of denial; and that is why it is frequently even now used in several European countries to denote an **n**, for non, or the other nasal letter **m**.

Peirce: CP 4.259 Cross-Ref:††

The Greek word {obelos} means a spit, (for example, {pempöbelos} is a five-pronged fork) so that the original notion was that that which is beneath it was transfixed; just as it used to be usual to nail false coins to the counter.

Peirce: CP 4.260 Cross-Ref:††

260. There is a small theorem about multitude that it will be convenient to have stated, and the reader will do well to fix it in his memory correctly, with the "each" number as exponent. If each of a set of m objects be connected with some one of a set of n objects, the possible modes of connection of the sets will number nm . Now an assertion concerning the value of a quantity either admits as possible or else excludes each of the values v and f . Thus, v and f form the set of m objects each connected with one only of n objects, admission and exclusion. Hence there are, nm , or 2^2 , or 4, different possible assertions concerning the value of any quantity, x . Namely, one assertion will simply be a form of assertion without meaning, since it admits either value. It is represented by the letter, x . Another assertion will violate the hypothesis of dichotomies by excluding both values. It may be represented by $\sim x$.



-
-
v
f
-

[Click here to view]

Of the remaining two, one will admit v and exclude f , namely, x ; the other will admit f and exclude v , namely $\sim x$.

Peirce: CP 4.261 Cross-Ref:††

261. Now, let us consider assertions concerning the values of two quantities, x and y . Here there are two quantities, each of which has one only of two values; so that there are 2^2 , or 4, possible states of things, as shown in this diagram.

Peirce: CP 4.261 Cross-Ref:††

Above the line, slanting upward to the right, are placed the cases in which x is v ; below it, those in which x is f . Above the line but slanting downward to the right, are placed the cases in which y is v ; below it, those in which y is f . Now in each possible assertion each of these states of things is either admitted or excluded; but not both. Thus, m will be 2^2 , while n will be 2; and there will be nm , or 2^4 , or 16, possible assertions. They may be represented by drawing the lines of the diagram between x and y and closing over the compartments for the excluded sets of values. . . .
.†1

Peirce: CP 4.262 Cross-Ref:††

262. Of three quantities, there are 2^3 , or 8, possible sets of values, and consequently 2^8 , or 256, different forms of propositions. Of these, there are only 38 which can fairly be said to be expressible by the signs [used in a logic of two quantities]. It is true that a majority of the others might be expressed by two or more propositions. But we have not, as yet, expressly adopted any sign for the operation of compounding propositions. Besides, a good many propositions concerning three quantities cannot be expressed even so. Such, for example, is the statement which admits the following sets of values:

<i>x</i>	<i>y</i>	<i>z</i>

v	v	v
v	f	f
f	v	f
f	f	v

Peirce: CP 4.262 Cross-Ref:††

Moreover, if we were to introduce signs for expressing [each of] these, of which we should need 8, even allowing the composition of assertions, still 16 more would be needed to express all propositions concerning 4 quantities, 32 for 5, and so on, *ad infinitum*.

Peirce: CP 4.263 Cross-Ref:††

263. The remedy for this state of things lies in simply giving the values **v** and **f** to propositions; that is, in admitting them to the universe of quantities. Here I will make an observation, by the way. Although formal logic is nothing but mathematics applied to logic, yet not a few of those who have cultivated it have had distinctly unmathematical minds. Indeed, in man's first steps in mathematics, he always draws back from mathematical conceptions. To first make **v** represent, let us say, Julius Caesar, and **f**, Pompey, since they may represent any subjects that are individual and definite, and thereupon further to propose to make every proposition either **v** or **f**, shocks the lower order of formal logicians. Such a mind will say, "If we have to distinguish propositions into two categories, let us denote their values by accented, or otherwise modified, letters, say **v'** and **f'**, and not call them Caesar and Pompey, which is absurd." But I reply that that sort of stickling for usage bars the progress of mathematical thought; that the very fact that it is absurd that a proposition should be Caesar or Pompey proves that there will be no inconvenience, not in calling propositions what you mean by Caesar and Pompey, which, as *you say*, nobody *could* mean to do, but in generalizing the conception of Caesar, so as to make it include those propositions which are destined to triumph over the others. To protest against this, is virtually to protest against generalization; and to protest against generalization is to protest against thought; and to protest against thought is a pretty kind of logic. But still the unmathematical mind will ask, why not, however, adopt the **v'** and **f'**; for he cannot conquer his shrinking from any generalization that can be evaded. It is the spirit of conservatism, the shrinking from the *outré*, which is commendable in its proper place; only it is unmathematical: instead of shrinking from generalizations, the part of the mathematician is to go for them eagerly. However, it would not even answer the purpose to distinguish **v'** and **f'** from **v** and **f**,†P1 for the reason that there would be equal reason for distinguishing propositions about quantities being **v** or being **f** from propositions about quantities being **v'** or being **f'**; so that we should require a **v''** and an **f''**, and so on, *ad infinitum*. Now this would hamper us, because we should find we had occasion to form many a proposition about two propositions, as to whether one of the two was **v''** or **f''**, for example, and at the same time whether

absurdity analogous to the last; so that in no way can the associativeness fail in this case.

Peirce: CP 4.266 Cross-Ref:††

Second, Suppose $\mathbf{vOv} = \mathbf{f}$ and $\mathbf{fOf} = \mathbf{v}$. Then I will show that the operation is not associative. For on the one hand,

$$(\mathbf{vOv})\mathbf{Of} \equiv \mathbf{fOf} \equiv \mathbf{v};$$

while, on the other hand, whether $\mathbf{vOf} = \mathbf{v}$, so that

$$\mathbf{vO}(\mathbf{vOf}) \equiv \mathbf{vOv} \equiv \mathbf{f},$$

or whether $\mathbf{vOf} = \mathbf{f}$, so that

$$\mathbf{vO}(\mathbf{vOf}) \equiv \mathbf{vOf} \equiv \mathbf{f},$$

in either case, the associative rule is broken.

Peirce: CP 4.266 Cross-Ref:††

Third, Suppose $\mathbf{vOv} = \mathbf{fOf}$ and $\mathbf{vOf} = \mathbf{fOv}$. Then I will show that the operation is associative. For otherwise it would be possible to give such values to $\mathbf{p}, \mathbf{q}, \mathbf{r}$, that

$$\sim\{\mathbf{pO}(\mathbf{qOr})\} \equiv \mathbf{pOq} \mathbf{Or}.$$

But since $\mathbf{vOf} = \mathbf{fOv}$, it follows that the second side of the inequality would be equal to

$$\mathbf{rO}(\mathbf{qOp})$$

so that the inequality requires $\sim\mathbf{p} \equiv \mathbf{r}$. But then also $\sim(\mathbf{qOp}) \equiv \mathbf{qOr}$ and consequently, the two assumed equations are inconsistent with the inequality, and the operation must be associative.

Peirce: CP 4.266 Cross-Ref:††

Fourth, Suppose $\mathbf{vOv} \equiv \mathbf{fOf}$, but $\sim(\mathbf{vOf}) \equiv \mathbf{fOv}$. Then I will show that the operation is not associative. For either

$$(\mathbf{vOv})\mathbf{Ov} \equiv \mathbf{fOv}, \text{ while } \mathbf{vO}(\mathbf{vOv}) \equiv \mathbf{vOf}$$

$$\text{or } (\mathbf{fOf})\mathbf{Of} \equiv \mathbf{vOf}, \text{ while } \mathbf{fO}(\mathbf{fOf}) \equiv \mathbf{fOv};$$

and in either case since $\sim(\mathbf{vOf}) \equiv \mathbf{fOv}$, the rule of association is violated. The four propositions thus proved, when taken together, are equivalent to the proposition [at the top of page 217]. Of these four, the first shows that $\vee, \cdot, \mathbf{x}, \dagger 1 \mathbf{y} \dagger 1$ are associative; the third that $:\vee:\sim, \equiv, \sim\equiv, \sim(:\vee:\sim)$, are so. The second shows that $(\sim \mathbf{x}), \sim \mathbf{x}, \dagger 1 \sim \mathbf{y}, \dagger 1 \mathbf{x}$, are non-associative; the fourth that $\prec, \vee\sim, \sim(\vee\sim), \sim(\prec)$ are so.

Peirce: CP 4.267 Cross-Ref:††

267. Another important property of some signs is that a quantity over a vinculum can be interchanged with one beyond the vinculum, so that --

either $\mathbf{x O}(\mathbf{y O z})$ and $\mathbf{y O}(\mathbf{x O z})$

or $(\mathbf{x O y})\mathbf{O z}$ and $(\mathbf{x O z})\mathbf{O y}$

will have the same value. In order that the former formula should hold, it is necessary and sufficient that if $x \text{ O } y$, whatever x and y may be, always has the contrary value to that of y when x and y have contrary values, then it should have the contrary value to that of y when x and y have the same value; and conversely, if it has the contrary value to that of y when x and y have the same value, it should also have the contrary value to that of y when x and y have different values. The same rule holds in regard to the second formula interchanging x and y in the statement.

Peirce: CP 4.267 Cross-Ref:††

The proposition may be otherwise stated as follows. Let P be the proposition that either $\mathbf{vOv} = \mathbf{v}$ or $\mathbf{fOf} = \mathbf{f}$ and Q the proposition that either $\mathbf{fOv} = \mathbf{v}$ or $\mathbf{vOf} = \mathbf{f}$. Then the [first] formula holds if P and Q are both true or both false; but fails if either is true while the other is false.

Peirce: CP 4.267 Cross-Ref:††

First, Suppose P and Q both false; so that

$$\mathbf{vOv} = \mathbf{f}, \mathbf{fOf} = \mathbf{v}, \mathbf{fOv} = \mathbf{f}, \mathbf{vOf} = \mathbf{v},$$

$$\text{Then, } \mathbf{fO(vOv)} = \mathbf{fOf} = \mathbf{v}, \mathbf{vO(fOv)} = \mathbf{vOf} = \mathbf{v}$$

$$\mathbf{fO(vOf)} = \mathbf{fOv} = \mathbf{f}, \mathbf{vO(fOf)} = \mathbf{vOv} = \mathbf{f},$$

and the two formulæ hold.

Peirce: CP 4.267 Cross-Ref:††

Second, Suppose the first formula fails. That is,

$$\sim\{\mathbf{fO(vOv)} \equiv \mathbf{vO(fOv)}\}.$$

Now if $\mathbf{vOv} = \mathbf{fOv} = \mathbf{v}$, evidently every expression ending in \mathbf{v} would be equal to \mathbf{v} , contrary to the hypothesis of the inequality just written. Hence, either $\mathbf{vOv} = \mathbf{f}$ or $\mathbf{fOv} = \mathbf{f}$.

Peirce: CP 4.267 Cross-Ref:††

If $\mathbf{vOv} = \mathbf{f}$, but $\mathbf{fOv} = \mathbf{v}$, the second side of the inequality is $\mathbf{vOv} = \mathbf{f}$; so that the first side must be $\mathbf{fOf} = \mathbf{v}$, and P is false.

Peirce: CP 4.267 Cross-Ref:††

If $\mathbf{fOv} = \mathbf{f}$, but $\mathbf{vOv} = \mathbf{v}$, the first side of the inequality is $\mathbf{fOv} = \mathbf{f}$; so that the second side must be $\mathbf{vOf} = \mathbf{v}$, and Q is false.

Peirce: CP 4.267 Cross-Ref:††

If $\mathbf{vOv} = \mathbf{f}$ and $\mathbf{fOv} = \mathbf{f}$, the inequality becomes $\sim(\mathbf{fOf}) \equiv \mathbf{vOf}$. Hence, either $\mathbf{fOf} = \mathbf{v}$, when P is false or $\mathbf{vOf} = \mathbf{v}$, when Q is false; and in any case either P or Q is false.

Peirce: CP 4.267 Cross-Ref:††

To suppose that the second of the two formulæ fails, that is, that

$$\sim\{\mathbf{vO(fOf)} \equiv \mathbf{fO(vOf)}\}$$

is merely changing \mathbf{f} to \mathbf{v} and \mathbf{v} to \mathbf{f} throughout the supposition just examined. Consequently, the result must be obtained by making the same interchange in the result of that supposition. But such interchange will neither change the falsity of P nor

that of Q . Consequently, whichever formula fails either P or Q is false; although, by the first supposition, if both are false both formulæ hold. It remains then to examine the cases in which of P and Q one is true and the other false.

Peirce: CP 4.267 Cross-Ref:††

Third, Suppose P to be false, and one or other formula to fail. If it be the first that fails, or

$$\sim\{\mathbf{fO}(\mathbf{vOv})\equiv\mathbf{vO}(\mathbf{fOv})\},$$

since the first member is $\mathbf{fO}(\mathbf{vOv}) = \mathbf{fOv} = \mathbf{v}$, the second member must be \mathbf{f} . This will be the case, if, and only if, either $\mathbf{fOv} = \mathbf{v}$ or $\mathbf{vOf} = \mathbf{f}$; that is, if and only if Q is true. If, however, it be the second formula that fails, or

$$\sim\{\mathbf{vO}(\mathbf{fOf})\equiv\mathbf{fO}(\mathbf{vOf})\},$$

again, by interchange of \mathbf{f} and \mathbf{v} , Q must be true.

Peirce: CP 4.267 Cross-Ref:††

Fourth, Suppose Q to be false (that is, $\mathbf{fOv} = \mathbf{f}$ and $\mathbf{vOf} = \mathbf{v}$) and one or other formula to fail. If it be the first, that is, if

$$\sim\{\mathbf{fO}(\mathbf{vOv})\equiv\mathbf{vO}(\mathbf{fOv})\},$$

since the second member is $\mathbf{vO}(\mathbf{fOv}) = \mathbf{vOf} = \mathbf{v}$, the first member must equal \mathbf{f} and either $\mathbf{vOv} = \mathbf{v}$ or $\mathbf{fOf} = \mathbf{f}$, that is, P will be true. The same results, by interchange of \mathbf{f} and \mathbf{v} , if the second formula fails.

Peirce: CP 4.267 Cross-Ref:††

It follows, then, that if P and Q are both true or both false the formulæ hold, while [if] P and Q differ in respect to truth both formulæ fail, and must be replaced by inequalities.

Peirce: CP 4.267 Cross-Ref:††

It will be remarked that one or other of the two formulæ

$$\mathbf{xO}(\mathbf{yOz})\equiv\mathbf{yO}(\mathbf{xOz})$$

$$(\mathbf{xOy})\mathbf{Oz}\equiv(\mathbf{xOz})\mathbf{Oy}$$

is true of 14 out of the 16 signs, or all but $(\sim\downarrow)$ and \downarrow .

Peirce: CP 4.267 Cross-Ref:††

Another very commonly true pair of formulæ are

$$(\mathbf{xOy})\mathbf{Ox}\equiv(\mathbf{yOy})\mathbf{Ox}$$

$$\mathbf{xO}(\mathbf{yOx})\equiv\mathbf{xO}(\mathbf{yOy}).$$

Peirce: CP 4.267 Cross-Ref:††

One formula or the other holds for all the signs except \equiv and $\sim\equiv$; and both hold except for these and \mathbf{x} , \mathbf{y} , $\sim\mathbf{x}$, $\sim\mathbf{y}$.

Peirce: CP 4.267 Cross-Ref:††

Somewhat similar to the above are the formulæ

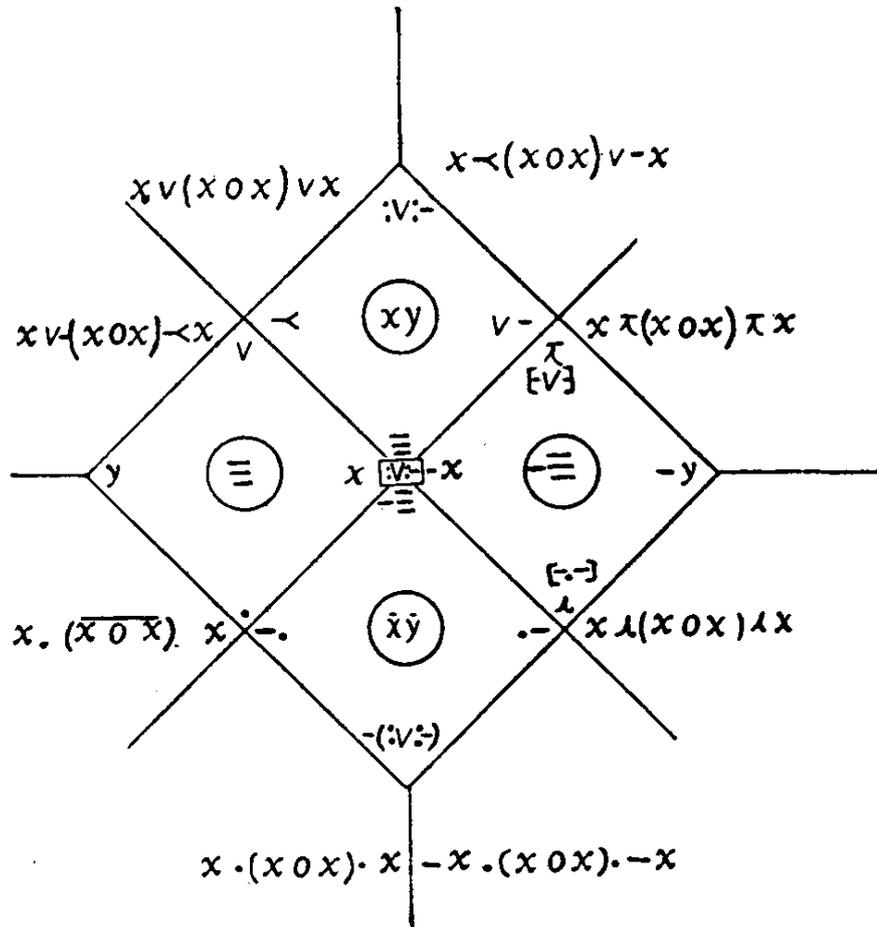
$$(\mathbf{xOy})\mathbf{Oy}\equiv(\mathbf{yOx})\mathbf{Ox}$$

$$x O(x O y) \equiv y O(y O x).$$

The first formula holds for $:\vee:\sim, \vee, \prec, \cdot, \sim, \sim(:\vee:\sim)$ and the second for $:\vee:\sim, \vee, \vee\sim, \cdot, \sim, \sim(:\vee:\sim)$.

Peirce: CP 4.268 Cross-Ref:††

268. The following table is a key to all the propositions which can be written with one vinculum, at most, and are either necessarily true or necessarily false, whatever be the value of the single letter they contain.



[Click here

to view]

Peirce: CP 4.269 Cross-Ref:††

269. A proposition of the form $x O x$ is necessarily true, if O be replaced by any one of the signs in the four corners of the upper quadrant; that is, by $:\vee:\sim, \prec, \vee\sim, \equiv$, which are, therefore, *affirmative* signs, according to De Morgan's †1 definition of that word. A proposition of the same form is necessarily false, if O be replaced by any of the former signs in the corners of the lower quadrant; that is, by $\sim\equiv, \sim\cdot, \sim, \sim(:\vee:\sim)$, which are accordingly *negative* signs. The others are *neutral*.

Peirce: CP 4.269 Cross-Ref:††

Above the horizontal line, there are, outside the large square, four compartments each of which heads a pair of quadrants making half the large square. In each of these compartments is written a formula of the form

$$x \diamond (x O x) \heartsuit x.$$

If we strike out the heart and last letter, and replace the diamond by the sign found in its place in any one of those compartments, and replace the **O** by any one of the eight signs in the half-square that compartment heads, we shall get a proposition necessarily true. Such, for example, are

$$x \prec (x \equiv x)$$

$$\text{and } x \vee (x (\sim \blacktriangleleft) x).$$

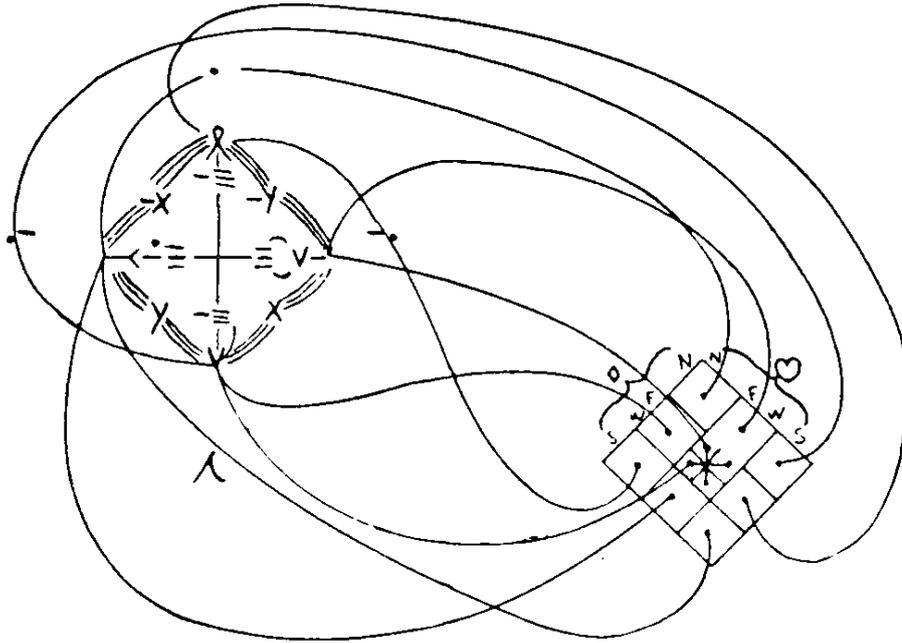
There will be a similar result if we strike off the first letter and use only the heart; instead of striking off the heart and last letter and using only the diamond. On the other hand, if we enter in either of these ways one of the compartments below the horizontal line, we get a proposition necessarily false. Thus $x \sim (x \blacktriangleright)$ is necessarily false. If we replace the diamond by the left-hand, or only, sign [*i.e.* by \equiv] in an oval, and the **O** by any of the signs in the corners of the same square [*i.e.* by $\vee, y, x, .$] we have a proposition necessarily true; and so if we replace the heart by the right-hand, or only, letter in an oval [*i.e.* by $\sim \equiv$]. If instead of replacing the **O** by one of the four signs in the corners of the same square, we take one of those in the opposite square, we get a proposition necessarily false. If the diamond or heart is replaced by $:\vee:\sim$, which is in the little square in the middle, and the **O** by any of the 16 signs in the large square, the proposition will be necessarily true; but if $\sim(:\vee:\sim)$ replaces the diamond or heart, the proposition will necessarily be false.

Peirce: CP 4.270 Cross-Ref:††

270. Of propositions necessarily true of the form

$$(x \blacklozenge x) O (x \heartsuit x)$$

there are just eleven hundred. But in 256 of these **O** is $:\vee:\sim$, while \blacklozenge and \heartsuit can be any signs whatever. The remaining 844 are exhibited in the following diagram, not very elegantly, it must be confessed. The sign in the place of the diamond is first to be sought in the first diagram; and its quadrant there is to be denoted by the corresponding cardinal point upon an ordinary map. That is to say, N is either $:\vee:\sim$, \prec , $\vee\sim$, \equiv ; E is either $(\sim \blacktriangleleft)$, $\sim x$, $\sim y$, or \blacktriangleleft ; W is either \vee, x, y or $.$; S is either $\sim \equiv$, $\sim.$, \sim , or $\sim(:\vee:\sim)$. We do the same with the sign in place of the heart. We enter the square on the right hand side below, with the letter for the diamond on the left, and that for the heart at the right. At the intersection of the two rows will be found a spot, from which a line leads to three signs in the left-hand part of the diagram



[Click here

to view], or to seven signs in case neither letter is E or W. Any one of these signs being taken as *O*, the proposition will be necessarily true.

Peirce: CP 4.271 Cross-Ref:††

271. Rather than inflict upon the reader more of these inconsequential technicalities, I will skip much that systematic thoroughness would require, and will at once notice some propositions necessarily true of the forms:

- (α) $(xRy) \heartsuit [(ySz) \blacklozenge (xOz)] [(xRy) \blacklozenge (ySz)] \heartsuit (xOz) (\{z\})$
- (β) $(xRy) \heartsuit [(xOz) \blacklozenge (ySz)] [(xRy) \blacklozenge (xOz)] \heartsuit (ySz) (\{n\})$
- ({g}) $(ySz) \heartsuit [(xRy) \blacklozenge (xOz)] [(ySz) \blacklozenge (xRy)] \heartsuit (xOz) (\Theta)$
- (δ) $(ySz) \heartsuit [(xOz) \blacklozenge (xRy)] [(ySz) \blacklozenge (xOz)] \heartsuit (xRy) (\{i\})$
- ({e}) $(xOz) \heartsuit [(xRy) \blacklozenge (ySz)] [(xOz) \blacklozenge (xRy)] \heartsuit (ySz) (\{k\})$
- (F) $(xOz) \heartsuit [(ySz) \blacklozenge (xRz)] [(xOz) \blacklozenge (ySz)] \heartsuit (xRy) (\{l\})$

In all cases, R is on the left margin, S on the right margin, and *O* in the body of the table. If these parts of the margins are used which intersect in \blacklozenge , then \blacklozenge is to be taken as \blacklozenge and \heartsuit , in the six left-hand formulæ (α-F), as $\vee\sim$, but in the others ({z}-{l}) as \sim ; or both \blacklozenge and \heartsuit may be taken as \vee . If those parts of the margin are used which intersect in \sim , in formulæ α, β, {e}, F, \heartsuit is $\vee\sim$; in Θ, {z}, {i}, {l}, it is \sim . In α, F, Θ, {i}, \blacklozenge is \sim ; in β, {e}, {z}, {l}, it is \sim . In {g}, δ, {n}, {k}, \heartsuit is ($\sim\blacklozenge$) while \blacklozenge is \blacklozenge . But, on the other hand, we may use a different interpretation, and in α, β, {e}, F, {z}, Θ, {i}, {l}, make \heartsuit to be \vee ; while in α, F, Θ, {i}, we put \sim for \blacklozenge , and in β, {e}, {z}, {l} we put $\vee\sim$ for \blacklozenge . Then, in {g}, δ, we can put \sim , and in {n} and {k}, can put $\vee\sim$, for \heartsuit ; putting \vee in all four for \blacklozenge . If the parts of the margin intersecting in \sim are used, then in one system of interpretation, in {g}, δ, {e}, F, we may put $\vee\sim$ for \heartsuit , and in {z}, {n}, Θ, {k}, may put \sim for \heartsuit ; while in {g}, {e}, {z},

{n} we put \sim , and in $\delta, F, \Theta, \{k\}$ we put \sim for \blacklozenge . Then in $\alpha, \beta, \{i\}, \{l\}$, we shall put $(\sim \blacklozenge)$ for \heartsuit and \blacklozenge for \blacklozenge . In another system of interpretation, we may in $\{g\}, \delta, \{e\}, F, \{z\}, \{n\}, \Theta, \{k\}$, put \vee for \heartsuit , while in $\{g\}, \{e\}, \{z\}, \{n\}$, we put \prec and in $\delta, F, \Theta, \{k\}$, we put $\vee \sim$, for \blacklozenge . In α and β , we shall put \prec , in $\{i\}$ and $\{l\}$, we shall put $\vee \sim$, for \heartsuit , and in all four shall put \vee for \blacklozenge . If the parts of the margin intersecting in \cdot are used, then, in the first system, in $\alpha, \beta, \{g\}, \delta$, we put \prec , while in $\{n\}, \{i\}, \{k\}, \{l\}$, we put $\vee \sim$, for \heartsuit , and in $\alpha, \{g\}, \{n\}, \{i\}$, we put \prec , in $\beta, \delta, \{k\}, \{l\}$, we put $\vee \sim$, for \blacklozenge . In the other system in $\alpha, \beta, \{g\}, \delta, \{n\}, \{i\}, \{k\}, \{l\}$, we put $(\sim \blacklozenge)$ for \heartsuit , in $\alpha, \{g\}, \{n\}, \{i\}$, we put \sim , in $\beta, \delta, \{k\}, \{l\}$, we put $\sim \cdot$, for \blacklozenge . In $\{e\}, F, \{z\}, \Theta$, we may put \vee for \heartsuit and $(\sim \blacklozenge)$ for \blacklozenge ; or else in $(e), F$, we may put $\vee \sim$, in $\{z\}, \Theta$ we may put $\sim \vee$ for \heartsuit , in all four \cdot for \blacklozenge .

Peirce: CP 4.271 Cross-Ref:††

The 24376 formulæ which this table yields are all of this class that it seems worthwhile to give.

Peirce: CP 4.272 Cross-Ref:††

272. In order to form a proposition necessarily true of the form $(x \blacklozenge y)O(x \heartsuit y)$, it is only necessary to rewrite it, replacing \blacklozenge and \heartsuit by the signs they stand for in their most iconic forms, but with $:\vee:\sim$ in place of O . This middle sign is now to be modified as follows:

Peirce: CP 4.272 Cross-Ref:††

If there is a quadrant open in both right- and left-hand signs, the top quadrant of the middle one must be left open;

Peirce: CP 4.272 Cross-Ref:††

If there is a quadrant open in the left-hand sign alone, the left-hand quadrant of the middle sign must be left open;

Peirce: CP 4.272 Cross-Ref:††

If there is a quadrant open in the right-hand sign alone, the right-hand quadrant of the middle sign must remain open;

Peirce: CP 4.272 Cross-Ref:††

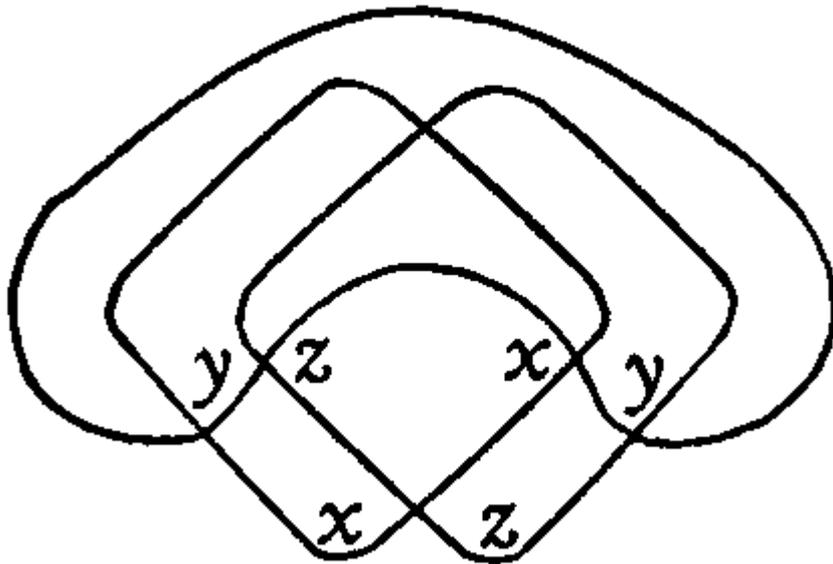
If there is a quadrant open neither in the right- nor in the left-hand sign, the bottom quadrant of the middle sign must be open.

Peirce: CP 4.272 Cross-Ref:††

Any quadrant which is not compelled to be open by this rule, may be closed.

Peirce: CP 4.273 Cross-Ref:††

273. The rule may be used inversely, of course. Its principal results are shown in the following table.†1

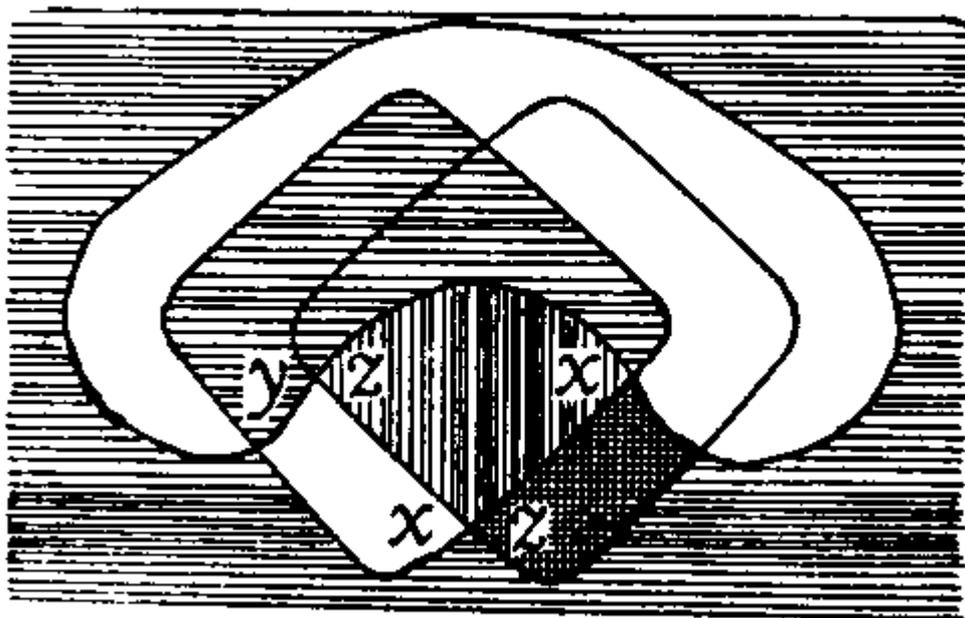


[Click here to

view]

Peirce: CP 4.274 Cross-Ref:††

Whatever quadrants are closed should be shaded, in the one case by vertical shading and in the other by horizontal shading; and this shading should extend over that line which does not pass through the sign concerned. We then read off the conclusion below, noting well that each quadrant consists of two compartments, since the line that does not pass through the sign concerned is disregarded; and unless **both** are shaded, the quadrant is not closed. For example, given that $y \vee \sim z$ and $x \equiv \sim y$, our diagram becomes as here shown.



[Click here to view]

Peirce: CP 4.274 Cross-Ref:††

It will be seen that although three quadrants appear, at first glance, to be cut off from the lower sign, yet two of them are not really cut off, since the more remote parts of them are unshaded. The conclusion, therefore, is, that $x(\sim \cdot)y$.

Peirce: CP 4.274 Cross-Ref:††

Such diagrams go by the name of Euler's diagrams,†1 although they are said by Hamilton †2 and by Drobisch †3 to be far older than Euler. The rudimentary idea of them is very likely ancient. But the plan of shading them is due to Mr. Venn.†4 Further on [in book II], I shall show how they may be rendered even more efficient.

Peirce: CP 4.274 Cross-Ref:††

So far we have refrained from making use of the obelus, a practice in the style of De Morgan. We shall now see what instant and complete simplification results from the use of it. In the first place, all the signs then become expressible by means of any one of the eight.†5

$$\begin{array}{cc} \vee & \cdot \\ \vee\sim & \sim\vee & \sim\sim & \sim\cdot \\ \sim\vee\sim & \sim\sim \end{array}$$

Peirce: CP 4.275 Cross-Ref:††

275. Dichotomic mathematics, in itself considered, is a trivial thing. Early students of it -- in the days of Boole, and later, I mean -- may be excused for fancying it could turn out important; I myself long entertained that chimerical dream. The real importance of it lies in the fact that it is a most important aid to the clear understanding of speculative grammar, and even of critical logic; and in the circumstance that, for logical reasons, every mathematical doctrine involves dichotomic mathematics. Where, for example, would be the algebra without a sign of equality? Yet that sign is a dichotomic sign. Were dichotomic algebra only to be used in the study of logic, the simplification of the apparatus would be a secondary consideration; although even then it would be dreary waste of time always to be going back to first principles. But when we reflect that every algebra must involve a dichotomic algebra, we see that such simplification is a serious desideratum. It will be best, therefore, to retain at least two signs of the relations between the values of two quantities. We want these to be as free from necessary formulæ, which will be rules to be borne in mind, as possible; and what rules there are should be as simple as possible. It is, therefore, best to select signs with which it is impossible to construct a formula which is either necessarily true or necessarily false. Moreover, associative signs simplify rules greatly. These two conditions are connected. That is, a sign which satisfies the first necessarily satisfies the second; and the only two signs which do this are \vee and \cdot .

Peirce: CP 4.276 Cross-Ref:††

276. Suppose, then, we proceed to build the algebra upon these, together with the obelus. We shall then have no need of \cdot and $(\sim \cdot)$, since

$$x(\sim \cdot)y \text{ is } \sim x\vee\sim y \text{ and } x \cdot y \text{ is } \sim x\cdot\sim y.$$

But the very frequently occurring relations $x\equiv y$ and $\sim(x\equiv y)$ are still only capable of being expressed in a too complicated way

$$x \equiv y \text{ is } (x \cdot y) \vee (\sim x \cdot \sim y) \text{ or } (x \vee \sim y) \cdot (\sim x \vee y)$$

$$\text{and } \sim(x \equiv y) \text{ is } (\sim x \cdot y) \vee (x \cdot \sim y) \text{ or } (x \vee y) \cdot (\sim x \vee \sim y)$$

It will be best to retain one or both of them. Moreover, for logical reasons, the sign \cdot is of the greatest importance. From a logical point of view, we may say, as we shall see, that it is the most important of any; and even from an algebraical point of view, if we retain \vee and \cdot as our usual signs, the most important formula of the algebra, apart from those of the commutative and associative principles, requires this sign for its convenient and clear expression. Namely, this formula is

$$((x \vee y) \cdot z) \cdot (x \vee (y \cdot z))$$

$$(x \cdot (y \vee z)) \cdot ((x \cdot y) \vee z)$$

expressing a sort of associativeness involving the two signs \vee and \cdot . . .

Peirce: CP 4.277 Cross-Ref:††

277. Adopting these signs, I proceed to state, as definitions, those fundamental conventions from which all the properties of the signs, and all the necessary rules and formulæ of the algebra can be deduced. In doing so, I shall, after Schröder, imitate a practice introduced into geometry, I believe, by Gergonne (about 1820), of writing reciprocally related propositions in parallel columns. The perfect correspondence between aggregation and composition was vaguely asserted by De Morgan, but was first definitely applied to dichotomic algebra and demonstrated, by me.† 1

Peirce: CP 4.278 Cross-Ref:††

278. Definition of the Quantities of a Dichotomic Algebra.

If x is any quantity of this algebra, then

No predicate and its | Every predicate or its negative

negative can both be true | must be true of x . That is,

of x That is, x is *definite*. | x is *individual*.

Peirce: CP 4.278 Cross-Ref:††

Every proposition concerning quantities of the algebra which is such that it must be true or false but cannot be both is itself a quantity of the algebra; and no other propositions except those which are primarily quantities of the algebra, and those which relate to the values of quantities of the algebra, are quantities of the algebra.

Definitions of \mathbf{v} and \mathbf{f}

No quantity of this algebra | Every quantity of this algebra

has, at once, the two values | has either the value, \mathbf{v} , or

\mathbf{v} and \mathbf{f} . | the value, \mathbf{f} .

Every true proposition which | Every false proposition which

is a quantity of this algebra | is a quantity of this algebra

has the value \mathbf{v} . | has the value, \mathbf{f} .

Peirce: CP 4.279 Cross-Ref:††

279. *Definitions of the Vinculum and Obelus*

Every quantity written upon a sheet of assertion, is either written

with a *vinculum* extending | with an *obelus* extending over

under the whole expression, | the whole of it, and is thereby

and is thereby asserted to | asserted to have the value **f**.

have the value **v**. |

Peirce: CP 4.279 Cross-Ref:††

But such assertions may be in what is equivalent to *oratio obliqua*, and are not necessarily *direct*.

Peirce: CP 4.280 Cross-Ref:††

280. *Definitions of Composition and Aggregation.*

If **x** and **y** are any quantities of this algebra (whether the

same or different)

If $\mathbf{x} \cdot \mathbf{y} = \mathbf{v}$, then $\mathbf{x} = \mathbf{v}$; If $\mathbf{x} \cdot \mathbf{y} \dagger 1 = \mathbf{f}$, then $\mathbf{x} = \mathbf{f}$;

If $\mathbf{x} \cdot \mathbf{y} = \mathbf{v}$, then $\mathbf{y} = \mathbf{v}$; If $\mathbf{x} \cdot \mathbf{y} = \mathbf{f}$, then $\mathbf{y} = \mathbf{f}$;

If $\mathbf{x} = \mathbf{v}$, then if $\mathbf{y} = \mathbf{v}$, If $\mathbf{x} = \mathbf{f}$, then if $\mathbf{y} = \mathbf{f}$,

so likewise $\mathbf{x} \cdot \mathbf{y} = \mathbf{v}$. so likewise $\mathbf{x} \cdot \mathbf{y} = \mathbf{f}$.

Peirce: CP 4.280 Cross-Ref:††

Substantially these definitions of composition and aggregation were given by me in 1880 (*Am. Jour. of Math.* III, 33).†2 They have the effect of reducing all legitimate transformations to successive legitimate insertions and omissions. The demonstration of this is very easy; but I think it will be more accurately appreciated if I postpone giving it until I have first shown what transformations are, by the above definitions, legitimated through insertions and omissions.

Peirce: CP 4.281 Cross-Ref:††

281. I. *Any proposition in this algebra, being written on a sheet of assertions, can, without loss of truth, receive the insertion of a vinculum below it.*

Peirce: CP 4.281 Cross-Ref:††

Demonstration. For, according to the definition of quantities, any proposition of this algebra is a quantity; and by the definition of vinculum and obelus, if written on a sheet of assertions, it must receive either a vinculum or an obelus, either of which being added to it, the result is an assertion concerning the value of the quantity. But this assertion is, by the definition of quantities, itself a quantity. And since this assertion asserts itself to be true, by the definition of **v** and **f**, it assigns to itself the value, **v**. But, by the definition of a vinculum, the assertion that a quantity has the

value \mathbf{v} is expressed by writing it upon a sheet of assertions with a vinculum below it; and therefore the insertion of the vinculum is legitimated by the assertion of the proposition. For example, if x appears upon the field of assertions we may write $x_$, or if $\sim y$ appears, we may write $\sim y_$. For a transformation is legitimate if it proceeds in accordance with a rule which can never transform a true proposition into a false one.

Peirce: CP 4.282 Cross-Ref:††

282. II. *A quantity, written on a sheet of assertions with a vinculum under it may, without loss of truth, be transformed by the omission of the vinculum.*

Peirce: CP 4.282 Cross-Ref:††

Demonstration. Let x be any quantity which is written on the sheet of assertion with a vinculum under it, $x_$. We may confine ourselves to the consideration of the case of the assertion x being true; for if it be not true, it certainly cannot sustain a loss of truth, since, by the definition of quantities, an assertion which has any other than the two grades of truth does not enter into the algebra. Let us first suppose that x is a proposition. Now every asserted proposition virtually asserts its own truth. That is to say, it asserts a fact, which being assumed real, whoever perceives that the proposition asserts that fact, has a perception which can be formulated by saying that what the proposition asserts is true. Therefore, if the vinculum is removed, nothing false is asserted by x , assuming x to be true. The only difference is that $x_$ directly asserts what x virtually asserts, and that x implies what $x_$ directly asserts. If, on the other hand, x is not an assertion, still its being written upon a sheet of assertions would make it assert itself to be an assertion; and whatever asserts itself to be something asserts something. But, being a quantity of this algebra, unless it is itself primarily an assertion, which would be contrary to our present hypothesis, the only assertion it can be, by the definition of quantities, is that a quantity of the algebra has some value. But the only quantity of the algebra to which x could refer would be itself. It must, therefore, assert that it has itself either the value \mathbf{v} or the value \mathbf{f} . But, by **I**, it must assert something which implies the truth of $x_$. Hence, x must assert that its value is \mathbf{v} . But this is no more than is asserted by $x_$; and therefore no falsity can be introduced by omitting the vinculum.

Peirce: CP 4.283 Cross-Ref:††

283. III. *A quantity, written on a sheet of assertion, may, without loss of truth, be transformed by the insertion of two obeli over it.*

Peirce: CP 4.283 Cross-Ref:††

Demonstration. Let x represent what is written. Being written upon the sheet of assertions, it is an assertion. It, therefore, virtually, at least, consists in the affirmation that some proposition pertinent to the algebra is true, that is, has the value \mathbf{v} . Then, since every such proposition is \mathbf{v} or \mathbf{f} , but not both, it virtually implies that $\sim x$ has the value \mathbf{f} . But $\sim\sim x$ merely asserts that $\sim x$ has the value \mathbf{f} . Hence $\sim\sim x$ asserts no more than is virtually implied in x , and there will be no loss of truth in inserting the two obeli.

Peirce: CP 4.284 Cross-Ref:††

284. IV. *If a quantity upon a sheet of assertions has two obeli over it, it may be transformed, without loss of truth, by the omission of the two obeli.*

Peirce: CP 4.284 Cross-Ref:††

Demonstration. Let $\sim\sim x$ represent what is written. This asserts that $\sim x$ has the value \mathbf{f} . That is to say, it asserts that the assertion of x is false. But, then, since x must

either have the value v , when x is true, or the value f , when $\sim x$ is true, and since the latter is not the case, it follows that x is true. That is, by **II**, x can be written on the sheet of assertions. Or, in other words, the two obeli can be omitted, without loss of truth.

Peirce: CP 4.285 Cross-Ref:††

285. V. *If a quantity, were it written by itself on a sheet of assertions, could be transformed, without loss of truth, into another quantity, then were the latter written under an obelus on the field of assertions, it could be transformed without loss of truth into the former under the obelus.*

Peirce: CP 4.285 Cross-Ref:††

Demonstration. Let x represent the first quantity, y the second, so that it is supposed that if x were written on the sheet of assertions, it could in every case be transformed into y without loss of truth. Suppose, now, that it were not true that $\sim y$ could in every case be transformed into $\sim x$ without loss of truth. Then, there must be some possible case in which $\sim y$ would be true and $\sim x$ not true. To say that $\sim x$ is not true, is expressed by writing $\sim\sim x$; so that while $\sim y$ was true, it would be possible for $\sim\sim x$ to be true, and hence, by **IV**, for x to be true. But $\sim y$ is the expression of the assertion that y is f , or y is false. Hence, it would be possible for x to be true but y false; when it would not be true that x would in all cases be transformed into y without loss of truth, which is contrary to the hypothesis. Hence, if x could be transformable into y , without loss of truth, it would be absurd to suppose $\sim y$ not transformable into $\sim x$ without loss of truth.

Peirce: CP 4.286 Cross-Ref:††

286. VI. *If a quantity, were it written by itself on a sheet of assertions, could be transformed into another with an obelus over it, then the latter, were it written by itself, without the obelus, could be transformed into the former covered with an obelus.*

Peirce: CP 4.286 Cross-Ref:††

Demonstration. Let x be the former, y the latter quantity. Then it is assumed that x , were it asserted, could be transformed into $\sim y$ without loss of truth. It follows, then, from **V**, that $\sim\sim y$ could be transformed into $\sim x$ without loss of truth. But by **III**, y , were it written on a sheet of assertions, could be transformed into $\sim\sim y$ without loss of truth. Therefore, y could be, by the two steps, transformed into $\sim x$ without loss of truth.

Peirce: CP 4.287 Cross-Ref:††

287. VII. *If a quantity, were it written with an obelus over it on a sheet of assertions, could be transformed, obelus and all, into another quantity, without loss of truth, then the latter, were it written with an obelus over it, could be transformed, obelus and all, into the former without its obelus.*

Peirce: CP 4.287 Cross-Ref:††

Demonstration. Let x be the former, y the latter quantity. Then we assume that $\sim x$ could, in all cases, be transformed into y , without loss of truth; and I am to prove that $\sim y$ could, in all cases, be transformed into x without loss of truth. Since $\sim x$ could be transformed into y , it follows from **V** that $\sim y$ could be transformed into $\sim\sim x$. But by **IV**, $\sim\sim x$ could then be transformed into x ; so that $\sim y$ would be transformable into x .

Peirce: CP 4.288 Cross-Ref:††

288. VIII. *If a quantity, were it written on a sheet of assertions under an obelus, could be transformed, without loss of truth, into another quantity under the same obelus, then, were the latter written without the obelus, it could, without loss of truth, be transformed into the former (without the obelus).*

Peirce: CP 4.288 Cross-Ref:††

Demonstration. Let x represent the former, y the latter quantity. Then, we assume that x could be transformed into $\sim y$, without loss of truth; and I am to show that, under that assumption, y could be transformed into x , without loss of truth. By VI, y could be transformed into x , and by IV, $\sim\sim x$ could be transformed into x . Hence, y could be transformed, in two steps, into x .

Peirce: CP 4.289 Cross-Ref:††

289. IX. *Any composite which can be written could, were it written on a sheet of assertions, be transformed, without loss of truth, by the omission of any component with its compositor.*

Peirce: CP 4.289 Cross-Ref:††

Demonstration. A composite can only be written by writing one component after another, singly. It will therefore be sufficient to show that, if the proposition is true for any given composite, it will be true for every composite resulting from the compounding with that composite of an additional component; provided, that I further show the proposition to be true for every composite with the writing of which the writing of any composite begins. For having proved *that*, I shall have shown that the proposition is true for every composite which can result from the successive affixation of components. For if, notwithstanding this reasoning, a composite could be written of which the proposition were not true, let

$$a \cdot b \cdot c \cdot d \dots \cdot k \cdot l \cdot m \cdot n$$

represent such a composite. Now if this composite can be written, it manifestly can be written by first writing a , then affixing $\cdot b$, then $\cdot c$ and so on, that is, by a series of changes each of which consists merely in affixing a new component with its compositor.†P1 But if the proposition is true for the first compound, $a \cdot b$, so resulting, and none of the steps of the series of operations renders it false for the result of that step, then it never can have been rendered false at all, and must be true for the composite $a \cdot b \cdot c \cdot d \dots \cdot k \cdot l \cdot m \cdot n$. Now, if the proposition is false, there must be some single composite for which it is false. It cannot then be false, if I prove that no affixation of a single component can render it false, and if I further prove that it is true at the beginning of writing any composite whatsoever. This general method of proof was invented by the great mathematician Pierre de Fermat †1 (1601-1665).

Peirce: CP 4.289 Cross-Ref:††

Let then $a \cdot b \cdot c \dots \cdot k \cdot l \cdot m$ represent any composite whatever which can be written and of which it is true either that this composite is false, or that the composite which results from it by erasing any component with an adjacent compositor is true. Then I say that, no matter what quantity n may be, it is true of the composite $a \cdot b \cdot c \dots \cdot k \cdot l \cdot m \cdot n$ that either it is false, or every composite which results from erasing from this composite any component with an adjacent compositor is true.

Peirce: CP 4.289 Cross-Ref:††

First, then, suppose $a \cdot b \cdot c \dots \cdot k \cdot l \cdot m$ to be false. Then, $a \cdot b \cdot c \dots \cdot k \cdot l \cdot m \cdot n$ must be false. For otherwise, the latter would be true, and consequently have the value v . But

in this case, by the first clause of the definition of composition $a \cdot b \cdot c \cdots k \cdot l \cdot m$ would be equal to \mathbf{v} , and therefore true and not false, contrary to the hypothesis.

Peirce: CP 4.289 Cross-Ref:††

Secondly, consider the alternative, namely that $a \cdot b \cdot c \cdots k \cdot l \cdot m$, as well as every composite resulting from the omission from $a \cdot b \cdot c \cdots k \cdot l \cdot m$ of a component with its adjacent compositor, is true. Then every such composite will have the value \mathbf{v} ; and if n likewise has the value \mathbf{v} , by the third clause of the definition of composition, every composite resulting from $a \cdot b \cdot c \cdots k \cdot l \cdot m \cdot n$ by the omission of one component will have the value \mathbf{v} , and consequently, will be true. But if n has not the value \mathbf{v} , then, by the second clause of the definition of composition, the composite $a \cdot b \cdot c \cdots k \cdot l \cdot m \cdot n$ will not be \mathbf{v} , and will therefore not be true, but false.

Peirce: CP 4.289 Cross-Ref:††

The simplest form of composite, with which the writing of any other must begin, is that in which there are but two components. By the first and second clauses of the definition of composition, if this is true, that is, equal to \mathbf{v} , then each component is equal to \mathbf{v} , that is, is true. Therefore, in this case either component can be erased without loss of truth; and the demonstration is thus completed.

Peirce: CP 4.290 Cross-Ref:††

290. X. *Every quantity written on a sheet of assertions may be transformed without loss of truth, by the insertion of an aggregator with any quantity whatever as aggregant; and if any aggregate which can be written out is written on a sheet of assertions, any additional aggregant, with its aggregator, may be anywhere inserted, without loss of truth.*

Peirce: CP 4.290 Cross-Ref:††

Demonstration. The demonstration is altogether similar to that of IX. Namely, suppose that $a \vee b \vee c \vee \cdots \vee k \vee l \vee m$ be an aggregate which can be written out, of which this proposition is true; that is to say, that if it be written on a sheet of assertions, that assertion is true, if it be possible to find an aggregant which, being omitted with its adjacent aggregator, the quantity remaining could be written on the sheet of assertions with truth. Then, I say, that the same thing is true, no matter what quantity n may be, of the aggregate $a \vee b \vee c \vee \cdots \vee k \vee l \vee m \vee n$; and moreover I aver that the proposition holds of every aggregate with which the writing of an aggregate begins, namely of every aggregate of two aggregants only.

Peirce: CP 4.290 Cross-Ref:††

For assume that the proposition is true of $a \vee b \vee c \vee \cdots \vee k \vee l \vee m$; that is to say, that either this is true or the quantity which results from the omission from it of any aggregant with an adjacent aggregator is false. Then, I say, of the aggregate $a \vee b \vee c \vee \cdots \vee k \vee l \vee m \vee n$, that it either is true, or else that no matter what single aggregant of it with an adjacent aggregator be omitted, the resulting quantity, being written on a sheet of assertions, makes a false assertion. Consider, in the first place, the case in which $a \vee b \vee c \vee \cdots \vee k \vee l \vee m$ is true, that is equals \mathbf{v} . Then it does not equal \mathbf{f} and consequently, by the first clause of the definition of aggregation, $a \vee b \vee c \vee \cdots \vee k \vee l \vee m \vee n$ cannot equal \mathbf{f} , but must equal \mathbf{v} , that is, must be true.

Peirce: CP 4.290 Cross-Ref:††

Consider next the other alternative, that both $a \vee b \vee c \vee \cdots \vee k \vee l \vee m$ and also every quantity resulting from the omission from it of an aggregant and an adjacent aggregator are false. Then, these are all equal to \mathbf{f} and if, besides, n is equal to \mathbf{f} , by

the third clause of the definition of aggregation, every quantity which results from the omission from $a \vee b \vee c \vee \dots \vee k \vee l \vee m \vee n$ is also equal to \mathbf{f} , is therefore false. If, however, n is not equal to \mathbf{f} , then by the second clause of the definition of aggregation $a \vee b \vee c \vee \dots \vee k \vee l \vee m \vee n$ is not equal to \mathbf{f} , and must be equal to \mathbf{v} and hence, when written on a sheet of assertions, must be true.

Peirce: CP 4.290 Cross-Ref:††

But if an aggregate has two aggregants only, then, by the first and second clauses of the definition of aggregation, if either of those aggregants is true, and so equal to \mathbf{v} , and not to \mathbf{f} , then the aggregate cannot have the value \mathbf{f} , but must have the value \mathbf{v} , and when written on a sheet of assertions must constitute a true assertion.

Peirce: CP 4.290 Cross-Ref:††

Corollary. It follows that every composite is true or equal to \mathbf{v} only in case all its components are so; being false, or equal to \mathbf{f} , if any one of its components is so; while an aggregate is true, or equal to \mathbf{v} , if any one of its aggregants is so, and is only false, or equal to \mathbf{f} , if all its aggregants are so. But this applies only to composites and aggregants which can be written out.

Peirce: CP 4.291 Cross-Ref:††

291. **XI.** *If every one of the components of a composite might, with truth, be written upon a sheet of assertions, the composite itself can be so written, if it can be written out.*

Peirce: CP 4.291 Cross-Ref:††

Demonstration. I shall use the Fermatian method. Assume that it is true of the composite (which can be written out) $a \cdot b \cdot c \dots \cdot k \cdot l \cdot m$. Then it is also true of the composite $a \cdot b \cdot c \dots \cdot k \cdot l \cdot m \cdot n$, by the third clause of the definition of composition. It is true of every composite of two components by the same clause. Hence it is always true.

Peirce: CP 4.292 Cross-Ref:††

292. **XII.** *If an aggregate might, with truth, be written out upon a sheet of assertions, then some one of its aggregants might be so written.*

Peirce: CP 4.292 Cross-Ref:††

Demonstration. Let $a \vee b \vee c \vee \dots \vee k \vee l \vee m$ be an aggregate of which it is true either that it cannot be written with truth upon a sheet of assertions or that some aggregant of it might be so written (although we may not know which one). Then the same will be true of the aggregate $a \vee b \vee c \vee \dots \vee k \vee l \vee m \vee n$. For consider, first, the alternative that $a \vee b \vee c \vee \dots \vee k \vee l \vee m$ cannot be asserted with truth. Then its value is not \mathbf{v} but \mathbf{f} . If then n is likewise \mathbf{f} , it follows from the third clause of the definition of aggregation that $a \vee b \vee c \vee \dots \vee k \vee l \vee m \vee n$ is \mathbf{f} and so could not be asserted with truth. But if n is not \mathbf{f} , then this is an aggregant of $a \vee b \vee c \vee \dots \vee k \vee l \vee m \vee n$ which can be asserted with truth. The other alternative is that some aggregant of $a \vee b \vee c \vee \dots \vee k \vee l \vee m$ can be asserted with truth. But that aggregant will equally be an aggregant of $a \vee b \vee c \vee \dots \vee k \vee l \vee m \vee n$. Finally, if there are but two aggregants, the proposition is true by the third clause of the definition of aggregation.

Peirce: CP 4.293 Cross-Ref:††

293. **XIII.** *As long as the only signs used are those described, any quantity which, if written alone on a sheet of assertions, could be transformed into a certain other, without loss of truth, can be so transformed wherever it occurs as a part of a*

directly asserted quantity, so long as it is not under an obelus.

Peirce: CP 4.293 Cross-Ref:††

Demonstration. Let x be a quantity which, if asserted to have the value v , could be replaced by y without loss of truth.

Peirce: CP 4.293 Cross-Ref:††

Then, in the first place, if x is a component of a directly asserted composite, it can be replaced by y without loss of truth. For if any of the other components is equal to f , and so is false, by **IX**, the composite is not true, and is therefore not capable of sustaining a loss of truth. But if every other component is equal to v , and this component is also v , then by **XI**, the composite is likewise v ; while if this component is f , by **X**, the composite is f . Thus, the value of the whole is the same as that of this component x ; and if this component is transformed into y , which is necessarily v if x is v , the whole composite remains v .

Peirce: CP 4.293 Cross-Ref:††

Next, suppose that that x is an aggregant of the asserted quantity. Then, if any of the other aggregants is v , the aggregate will be v , whatever transformation x may undergo. But if all the others are f , and x is f , the aggregate, will, by **XII**, be f , while if x is v , by **X**, the aggregate will be v . Thus the value of the whole aggregate will be the same as that of x ; and if x is replaced by y , which is necessarily v if x is v , the value of the aggregate will remain v .

Peirce: CP 4.293 Cross-Ref:††

Thus the transformations of an aggregant, as well as those of a component, follow the same rules as those of an entire asserted quantity; and consequently, if x be an aggregant of a component, or a component of an aggregant, or be in any other relation to the asserted quantity describable by the alternate use of component and aggregate, as it must be if we use no other signs than those described, and if x is not under an obelus, then it is subject to the same rules of transformation as if it were asserted alone.

Peirce: CP 4.294 Cross-Ref:††

294. **XIV.** *As long as the only signs used are those described, any quantity into which another could be transformed without loss of truth, if the latter were asserted alone, can, if it occurs anywhere under a single obelus in a directly asserted quantity, be there transformed into that latter.*

Peirce: CP 4.294 Cross-Ref:††

Demonstration. Let x and y represent the two quantities, so that in the assertion that x has the value v , x could be replaced by y , without loss of truth. Then, I say that if, in a directly asserted quantity, y occurs anywhere under a single obelus, it can be transformed into x without loss of truth. For let Y be the entire expression which is under the same obelus as y ; and let X be what Y would become if y were replaced by x . Then, by **XIII**, if X were asserted, it could be transformed into Y . And consequently, by **V**, $\sim Y$, if it were asserted alone could be transformed into $\sim X$. Hence, by **XIII**, situated as $\sim Y$ is, not under any obelus, it can be transformed into $\sim X$. But this transformation consists only in replacing y by x under a single obelus.

Peirce: CP 4.295 Cross-Ref:††

295. **Definition.** Let us say that two operations are *internal negatives* of one another, if and only if, either gives a result of contrary value to the result of the other

when whatever quantities it operates upon are of contrary value to those operated upon by that other; and let us write an obelus over a sign of the combination of two quantities to signify the internal negative of that sign.

Peirce: CP 4.295 Cross-Ref:††

Then we shall have the following pairs of internal negatives : \vee : \sim and \sim (: \vee : \sim)

∇ or \vee and \cdot

($\sim \nabla$) and ∇

\cdot and \sim

\vee and \sim

\equiv and $\sim(\equiv)$

x and $\sim x$

y and $\sim y$

Peirce: CP 4.295 Cross-Ref:††

The obelus and vinculum will each be its own internal negative.

Peirce: CP 4.295 Cross-Ref:††

Since a quantity operates upon nothing, its internal and negative will be its own negative; that is x and $\sim x$, \vee and ∇ will be internal negatives of each other.

Peirce: CP 4.296 Cross-Ref:††

296. **XV.** *Every expression of a quantity has the contrary value to the expression which results from the substitution in it of the internal negative of each quantity and operation.*

Peirce: CP 4.296 Cross-Ref:††

Demonstration. For, in the first place, this is true regarding single letters. For let x represent any single letter. Then, since x operates upon nothing, $\sim x$ gives the contrary value to x when it operates upon whatever x operates upon. In the second place, it is true of expressions operated upon by the vinculum and obelus. For since x has the contrary value to $\sim x$, the vinculum is its own internal negative; and since $\sim x$ has the contrary value to $\sim\sim x$, the obelus is its own internal negative. In the next place, the proposition is true of any operator upon two operands; for, by the definition, $x O y$ has the contrary value to that of $\sim x(\sim O)\sim y$. Hence, if y is $u \diamond v$, $x O(u \diamond v)$ has the contrary value to $\sim x(\sim O)(\sim u(\sim \diamond)v)$. And so if $y = \sim w$, $x O\sim w$ has the contrary value to $\sim x(\sim O)w$. And by Fermatian reasoning, it is evident that this will be so in every case.

Peirce: CP 4.297 Cross-Ref:††

297. **XVI.** *Every necessarily true proposition concerning the values, or relations between the values, of quantities of dichotomic algebra will remain necessarily true after it has been modified by everywhere interchanging ∇ and \vee , and internal negatives of operators upon two letters, and when categorical affirmation and denial, protasis and apodosis of the same conditional sentence, copulation and disjunction are likewise interchanged.*

Peirce: CP 4.297 Cross-Ref:††

Demonstration. I have to prove that if a certain form of algebraic expression

is necessarily true, then if that form is modified by the interchange of **f** and **v**, of \cdot and Ψ , of \equiv and $\sim(\equiv)$, of \prec and $\sim\prec$, etc., it becomes necessarily false; and further it being necessarily true, that from the truth of one expression the truth of another follows, it will be equally true that from the truth of the latter, modified as above, the truth of the former, modified in the same way, equally follows by necessity. Finally, I have to show that to say that if any quantity, A, is **v**, then both the quantities B and C are **v**, is the same as to say that if either $\sim B'$ or $\sim C'$ is **v**, then $\sim A'$ is **v**; where, $\sim A'$, $\sim B'$, $\sim C'$ are the above-described modifications of A, B, C.

Peirce: CP 4.297 Cross-Ref:††

I first remark that to call a proposition concerning the values, or relations between the values, of quantities of dichotomic algebra *necessarily* true is to say that it is true whatever be the values of the quantities to which it relates. If, then, a proposition is necessarily true, it remains so when for all the single ordinary letters (not **v** and **f**) that are mentioned in its statement, the negatives of these letters are substituted. But if this substitution be made, and the internal negative of each combination be then substituted for the combinations themselves, the result will be the same modification which is described in the enunciation of this theorem. For the single letters will be restored to their original conditions with respect to having obeli over them, but **f** and **v** will be interchanged; the vincula and obeli will remain unchanged; and the signs of operations upon two quantities will be changed into their internal negatives. The quantities described by the combinations so modified will thus have values contrary to the values of the corresponding unmodified combinations, after the obeli are applied to the single letters. Consequently, if we make the proper changes in what is said of them, the initial proposition which was necessarily true will remain necessarily true of these so modified combinations. In particular, if the original proposition represents the assertion that a combination, C, is necessarily true, or has the value **v**; and if C' is the modification produced by putting obeli over the single letters of C, then that C' is necessarily true, or has the value **v**, will be equally true; and consequently, that $\sim C'$ is necessarily false will be equally true; where C' will be the modification of C described in the enunciation. So if the necessarily true proposition is that if a certain combination, A, is true, then a certain combination, B, is true, it remains equally necessary that if A' is true B' is true; and consequently, equally true that if $\sim B'$ is true (and therefore B' is not true) then $\sim A'$ will be true (*i.e.*, A' will not be true). So, if the original proposition be that if A is true, then both B and C are true, and [if] this be necessarily true, it is equally necessary that if A' is true, then both B' and C' are true, and consequently, [it] is equally necessary that if either $\sim B'$ or $\sim C'$ be true (so that either B' or C' is false), then $\sim A'$ is true (or, A' is false). The necessary truth of the proposition is thus made plain.

Peirce: CP 4.298 Cross-Ref:††

298. XVII. *If a quantity have the value v, then this quantity may be inserted as a component of the whole or any part of $\sim n$ asserted quantity, and any component of a part (or the whole) of an asserted quantity, can be repeated, without loss of truth, as a component of this part or of any part of it.*

Peirce: CP 4.298 Cross-Ref:††

Demonstration. By the first clause of the definition of composition if $a \cdot v$ has the value **v**, so has **a**; and by the third clause of the same definition, if **a** has the value **v**, so has $a \cdot v$. Thus **a** and $a \cdot v$ have in all cases the same value, and the substitution of the latter for the former in any part of an expression must be without effect upon the value of it, since the value of an expression of a given form depends exclusively on

the values of its parts.

Peirce: CP 4.298 Cross-Ref:††

Consequently, if a has the value v , its introduction anywhere as a component cannot change the value of the expression into which it is introduced; but if a has the value f , and is a component, the composite has in any case the value f , whatever be done to the other components. Thus, its introduction, as a component of a part of the composite of which it is a component, can never alter the value.

Peirce: CP 4.298 Cross-Ref:††

Corollary. It follows, by **XVI**, that a quantity having the value f may be introduced as an aggregant into any part of an assertion; and further, that any quantity, repeated as an aggregate of any part of any part of an assertion of which it is an aggregant, may be omitted in that inner repetition.

Peirce: CP 4.298 Cross-Ref:††

Corollary. We may place here the propositions that composition and aggregation are commutative and associative.

Peirce: CP 4.298 Cross-Ref:††

Given $a \cdot b$, we can, by insertion, write $b \cdot a \cdot b$, for if $a \cdot b$ is v , so is b , and if b and $a \cdot b$ are v , so is $b \cdot a \cdot b$. But then, by omission, we get $b \cdot a$. For if $b \cdot a \cdot b$ is v , so is $a \cdot b$, and if $a \cdot b$ is v , so is a . Also, if $b \cdot a \cdot b$ is v , so is b ; and if both b and a are v , so is $b \cdot a$.

Peirce: CP 4.298 Cross-Ref:††

Again, given $(a \cdot b) \cdot c$, we get by insertion $(a \cdot b \cdot c) \cdot c$. For if $(a \cdot b) \cdot c$ is v , so is c ; and if c is v , $b \cdot c$ is v , if b is v ; and if $a \cdot b \cdot c$ is v if $a \cdot b$ is v , and if $(a \cdot b) \cdot c$ is v , so is $(a \cdot b \cdot c) \cdot c$. But then by omission we get $a \cdot b \cdot c$.

Peirce: CP 4.298 Cross-Ref:††

Since composition has these properties, so, by **XVI** has aggregation.

Peirce: CP 4.299 Cross-Ref:††

299. **XVIII.** *The negative of a component may be introduced into any part of its composite as an aggregant, without loss of truth.*

Peirce: CP 4.299 Cross-Ref:††

Demonstration. For if a has the value v , its negative is f , and as an aggregant can affect the value of nothing. But if a has the value f , being a component, the composite is f , no matter how the other components be transformed.

Peirce: CP 4.299 Cross-Ref:††

Corollary. It follows that if the negative of an aggregant occurs in any part of the aggregate as a component, it may be omitted.

Peirce: CP 4.300 Cross-Ref:††

300. **XIX.** *A quantity which is a component of every aggregant of an aggregate may be introduced as a component of the whole, without altering the value; and it may then be omitted from the aggregants, without altering the value.*

Peirce: CP 4.300 Cross-Ref:††

Demonstration. I have to prove that $a \cdot x \vee b \cdot x \vee c \cdot x \vee$ etc. has the same value as $(a \cdot x \vee b \cdot x \vee c \cdot x \vee$ etc.) $\cdot x$, and also the same value as $(a \vee b \vee c \vee$ etc.) $\cdot x$.

Peirce: CP 4.300 Cross-Ref:††

First, suppose that the value of $a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots$ etc. is v . Then, by the third clause of the definition of composition, the value of $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}) \cdot (a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.})$ will also be v .

Peirce: CP 4.300 Cross-Ref:††

But, then, omitting components, $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}) \cdot (x \cdot x \cdot x \cdot \dots \text{ etc.})$ will also be v . But, then, $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}) \cdot x$ will be v . For if $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}) \cdot (a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.})$ is v , then by the second clause of the definition of composition, $a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}$ is v ; and then by the third clause of the definition of aggregation, either $a \cdot x$ or $b \cdot x$ or $c \cdot x$ etc. is v ; and then, by the second clause of the definition of composition, x is v ; and then, since by the first clause of the definition of composition, $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}) \cdot (a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.})$ is v , $a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}$ is v , it follows from the third clause of the same definition that $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}) \cdot x$ is v . And further, if $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}) \cdot x$ is v , it follows, by omission of components that $(a \cdot b \cdot c \cdot \dots \text{ etc.}) \cdot x$ is v .

Peirce: CP 4.300 Cross-Ref:††

Secondly, suppose that $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.})$ is f . Then, by the first clause of the definition of composition, $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}) \cdot x$ is f . And further, $a \cdot x$ and $b \cdot x$ and $c \cdot x$ etc. have by the first and second clauses of the definition of aggregation, all severally the value f . Then, if x is v , by the third clause of the definition of composition, a , b , c , etc. have all severally the value f , and by the third clause of the definition of aggregation, $a \cdot b \cdot c \cdot \dots \text{ etc.}$ has the value f ; and by the first clause of the definition of composition $(a \cdot b \cdot c \cdot \dots \text{ etc.}) \cdot x$ has the value f . Suppose, on the other hand, that x has the value f . Then, by the second clause of the definition of composition $(a \cdot b \cdot c \cdot \dots \text{ etc.}) \cdot x$ has the value f .

Peirce: CP 4.300 Cross-Ref:††

Thus, in every case, $a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}$ and $(a \cdot x \cdot b \cdot x \cdot c \cdot x \cdot \dots \text{ etc.}) \cdot x$ and $(a \cdot b \cdot c \cdot \dots \text{ etc.}) \cdot x$ have the same value.

Peirce: CP 4.300 Cross-Ref:††

Corollary. It follows that

$$\begin{aligned} & (a\Psi x) \cdot (b\Psi x) \cdot (c\Psi x) \text{ etc.} \\ & = (a\Psi x) \cdot (b\Psi x) \cdot (c\Psi x) \cdot \text{ etc. } \Psi x \\ & = (a\Psi b\Psi c \cdot \dots \text{ etc.}) \cdot x. \end{aligned}$$

Peirce: CP 4.300 Cross-Ref:††

When we come to the algebra of logic, there will be a highly important remark to make concerning this theorem.

Peirce: CP 4.301 Cross-Ref:††

301. The above are, I believe, all the theorems of dichotomic algebra with which it is worthwhile to trouble the reader. There are, however, a few problems to be considered. Of these, I shall give those methods of solution which seem to me to be upon the whole the most useful, taking into consideration something besides their brevity in very complicated cases -- making this, indeed, decidedly a secondary consideration, in view of the excessive rarity of cases in which the reader will ever have occasion to apply the algebra to complicated problems, and in view of the very moderate degree of mathematical ingenuity requisite to clearing away the complexity even from these few. What seems to me desirable is that the procedure should have

that kind of simplicity which makes it easy to remember or to reconstruct if its details are forgotten.

Peirce: CP 4.301 Cross-Ref:††

The first problem is to put an expression into such a form that a certain letter appears only as an aggregant of a component, or as the negative of an aggregant of a component. I first gave a general solution of this problem which I do not think can be improved upon.†1 Let x be the letter in question; and let the expression be Fx . Then,

$$Fx = (Ff \cdot x) \cdot (Fv \cdot \sim x).$$

I must say that there was little originality in this solution, since it was but the reciprocal of a proposition of Boole's. If it is desired to separate *two* letters in this way, we have

$$F(x,y) = (Ff, f \cdot x \cdot y) \cdot (Ff, v \cdot x \cdot \sim y) \\ (Fv, f \cdot \sim x \cdot y) \cdot (Fv, v \cdot \sim x \cdot \sim y)$$

The procedure for a greater number of letters will be similar. But it is never really necessary to separate more than one.

Peirce: CP 4.301 Cross-Ref:††

The proof is simple enough. If Fx has the value v , either x has the value v , when Fx becomes Fv or $\sim x$ has the value v , so that, in any case, $Fv \cdot \sim x$ has the value v ; and further, either $\sim x$ has the value v , when Fx becomes Ff or x has the value v , so that, in any case, $Ff \cdot x$ has the value v . Consequently, if Fx has the value v , the composite $(Fv \cdot \sim x) \cdot (Ff \cdot x)$ has the same value. But if Fx has the value f , either x has the value v , when Fx becomes Fv , and $Fv \cdot \sim x$ has the value f , or x has the value f , when Fx becomes Ff , and $Ff \cdot x$ has the value f . Thus, if Fx has the value f , one or other of the components of $(Fv \cdot x) \cdot (Ff \cdot \sim x)$ has the value f , and again $(Fv \cdot \sim x) \cdot (Ff \cdot x)$ has the same value as Fx .

Peirce: CP 4.301 Cross-Ref:††

Boole's reciprocal problem is to put an expression into a form in which a given letter or its negative appears only as a component of an aggregant. The solution is

$$Fx = (Fv) \cdot x \cdot (Ff) \cdot \sim x.$$

Peirce: CP 4.301 Cross-Ref:††

We now have to take a step which I first took in my memoir dated, 1870 Jan. 26.†P1 Let us start from the sixteen assertions concerning the values of two dichotomic quantities, and take the signs of the operations they involve in a new sense, which may be distinguished by a dot over these signs, regarding these operations themselves as quantities, and at the same time operators upon the second of the two quantities, thereby producing the first. Then, the four $:$, \cdot , \sim , \cdot , will by aggregation give all the rest, except \sim (\cdot \sim). Thus,

$$\equiv \text{ will be } : \cdot \cdot \cdot \text{ since } x \equiv y = (x \cdot y) \cdot (x \cdot y)$$

$$\sim(\equiv) \text{ will be } \cdot \sim \cdot \cdot \text{ since } \sim(x \equiv y) = (x \cdot \sim y) \cdot (\sim x \cdot y)$$

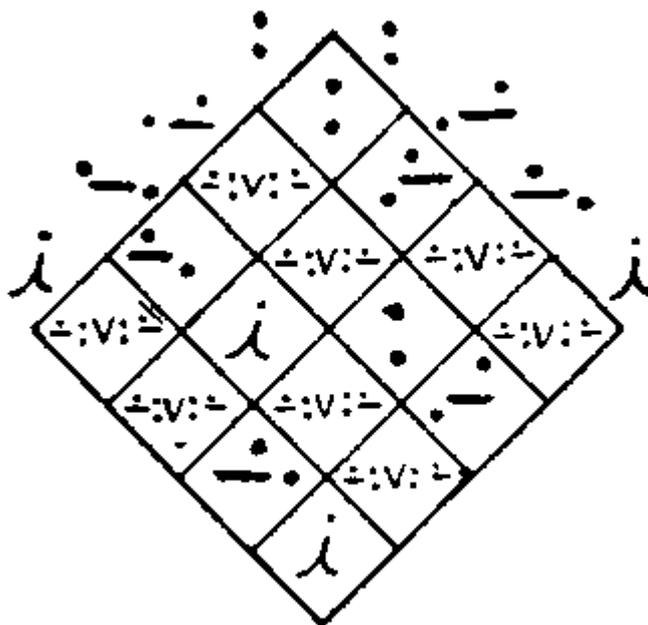
$$\cdot y \text{ will be } : \cdot \sim \cdot \text{ since } y = (x \cdot y) \cdot (\sim x \cdot y)$$

$$\cdot x \text{ will be } : \cdot \sim \cdot$$

$\sim \cdot y$ will be $\cdot \sim \cdot \Psi \cdot \perp$
 $\sim \cdot x$ will be $\sim \cdot \cdot \Psi \cdot \perp$
 $\cdot \vee$ will be $\cdot \Psi \cdot \sim \cdot \Psi \cdot \sim \cdot$
 $\cdot \sim \perp$ will be $\cdot \perp \cdot \Psi \cdot \sim \cdot \Psi \cdot \sim \cdot$
 $\bullet \rightarrow \omega \iota \lambda \lambda \beta \epsilon$: $\Psi \bullet \perp \Psi \sim \bullet$
 $\bullet \cdot \cdot / \sim \omega \iota \lambda \lambda \beta \epsilon$: $\Psi \bullet \perp \Psi \cdot \sim \bullet$
 $(\cdot \cdot \cdot / \sim) \omega \iota \lambda \lambda \beta \epsilon$: $\Psi \cdot \sim \bullet \Psi \sim \bullet \cdot \Psi \bullet \perp$

Peirce: CP 4.302 Cross-Ref:††

302. Now mathematicians have long ago agreed upon generalizing the meaning of the word 'multiplication' so as to make it signify the operation of applying, as multiplier, an operation to the result of another operation, which last operation is regarded as the multiplicand. I identify this operational multiplication, which is commonly called *functional* multiplication, with my *relative* multiplication, which is the operation of so combining a relative term -- such as '*lover of*' -- as multiplier, with a correlate as multiplicand, so as to yield, as product, the relate which is in the relation signified by the relative term to the object indicated as correlate. A mathematical operator is nothing but a mathematical relative term. Then, the relative, or functional, multiplication table of \cdot, \sim, \vee, \perp will be as here shown.



[Click here to view]

Peirce: CP 4.303 Cross-Ref:††

303. My father afterward, but independently, obtained two multiplication tables of similar form in his *Linear Associative Algebra* (1st Ed., pp. 30, 59; 2nd Ed. [*American Journal of Mathematics*, 1881], edited by me, pp. 111, 132), which I showed †1 was due to their being, in essence, the same thing. He further discovered

that by means of ordinary imaginary algebra, quaternions, or rather Hamilton's biquaternions, can be put into this form.†2 Namely, if we put

$$1 = 1$$

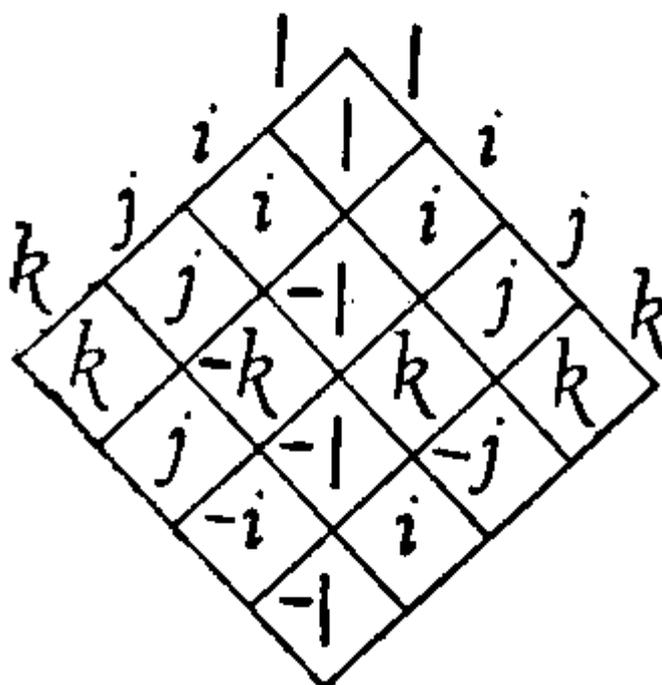
$$(\dots)\sqrt{-1} = i$$

$$(\dots)\sqrt{-1} = j$$

$$(\dots) = k$$

we get the multiplication table of quaternions, which is as here shown.

Peirce: CP 4.304 Cross-Ref:††



304.

[Click here

to view] This is only interesting here as showing how dichotomic mathematics may influence higher orders of mathematics. But quaternions proper does not admit of imaginary coefficients; and biquaternions is essentially a different thing. Quaternions is a quaternary, or tetrachotomic, algebra of which [I gave] the proper representation in February, 1882.†1

Peirce: CP 4.305 Cross-Ref:††

305. If the operations are to be considered as quantities, what we have considered, and what are, the quantities of dichotomic algebra, sink to the rank of *umbræ*, or ingredients of quantities. It was Leibniz †2 who in 1693, April 28, first introduced into algebra this conception of ingredients of quantities; and unfortunately, he neglected to provide a suitable name for them. It so happened that it was not until three half centuries later, when they had been used by a hundred writers, at the very least, and had figured in familiar textbooks, that the seething brain of Sylvester allowed him to claim them as his own invention, and to bestow upon them the name of *umbræ*.†3 The name could not well be more inappropriate; for whoever heard of

shadows conspiring to create a substance? Besides, there is nothing to prevent umbræ being identified with ordinary quantities -- or rather all ordinary quantities being identified with some umbræ: such a step is, often, a most useful mathematical generalization. An *umbra*, or better, an *ingredient* of a quantity, is a logical symbol, a set of which systematically, and from a logical point of view, describes a quantity, without any necessary reference to its value. For example, if the velocities of two couriers are denoted respectively by $u[1]$ and $u[2]$, then u , 1, and 2, are umbræ, or ingredients, of the quantities $u[1]$, and $u[2]$. The first example given by Leibniz was of three general simultaneous linear equations between two quantities, which he wrote thus:

$$10+11x+12y = 0$$

$$20+21x+22y = 0$$

$$30+31x+32y = 0$$

Here the numbers have nothing to do with the values of the coefficients, which may be anything. The first figure shows simply what equation is referred to and the second what term of that equation. My particular umbral notion of 1870†1 for relative terms, which has been generally approved, was A:B, where A and B are individual objects, and A:B is that operation upon B which produces A, but operating upon any other individual than B produces f , even if that other individual have the same value as B. Nevertheless, since values, in all sorts of algebra, are singular and definite objects, to which the principles of contradiction and excluded middle apply, there is nothing to prevent our taking the colon in a special sense, so that A:B shall operate, not upon B logically considered, but upon the value of B, giving the value of A. It is in this sense that we may write

$$: = v:v \quad \cdot \mathbf{f} = f:f$$

$$\cdot \sim = v:f \quad \sim \cdot = f:v\dots$$

Peirce: CP 4.306 Cross-Ref:††

306. If we identify v with $\cdot x$, and f with $\sim x$, the distinction between relative multiplication and composition will disappear in regard to the ordinary quantities of dichotomic algebra. For just as

$$v \cdot v = v \quad \text{so} \quad \cdot x : \cdot x = \cdot x \dagger 2$$

$$v \cdot f = f \quad \cdot x : \sim x = \sim x$$

$$f \cdot v = f \quad \sim x : \cdot x = \sim x$$

$$f \cdot f = f \quad \sim x : \sim x = \sim x$$

But in regard to these new quantities, the distinction will be maintained. For example,

$$\cdot \sim : \dagger 2 \cdot \sim = \cdot \mathbf{f} \cdot (\cdot \sim \cdot \sim) = (\cdot \equiv) = (\cdot \sim \sim \dots \sim) \cdot \sim \dagger 2 \cdot \sim = \cdot$$

Peirce: CP 4.306 Cross-Ref:††

I shall not further enlarge upon this matter at this point, although the conception mentioned opens a wide field; because it cannot be set in its proper light without overstepping the limits of dichotomic mathematics.

Peirce: CP 4.307 Cross-Ref:††
§4. TRICHOTOMIC MATHEMATICS^P

307. We have already, along one line, traversed the marches between dichotomic and trichotomic mathematics; for the general idea of operational multiplication is as purely triadic as it could well be, involving no ideas but those of the triad, operator, operand, and result. Relative multiplication, however, involves a marked dichotomic element since $(A:B):(C:D)$ is one of the two, f or $A:D$, according as $(B:C)$ is one of the two f or v .

Peirce: CP 4.308 Cross-Ref:††

308. Trichotomic mathematics is not quite so fundamentally important as the dichotomic branch; but the need of a study of it is much greater, its applications being most vital and its difficulties greater than the dichotomic. Nevertheless, it has received hardly any direct attention. The permutations of three letters have, of course, been noticed, along with other permutations. The theory of the cubic equation is fully made out; along with those of plane and twisted cubic curves. There is also an algebra of novenions. In addition, considerable studies have been made in a particular province of trichotomic mathematics by logicians, without their recognizing the triadic character of the subject.

Peirce: CP 4.308 Cross-Ref:††

A trichotomic mathematics entirely free from any dichotomic element appears to be impossible. For how is the mathematician to take a step without recognizing the duality of truth and falsehood? Hegel and others have dreamed of such a thing; but it cannot be. Trichotomic mathematics will therefore be a 2×3 affair, at simplest.

Peirce: CP 4.309 Cross-Ref:††

309. I will begin this topic by a glance at some of the logico-mathematical generalities, without being too scrupulous about excluding higher numbers than three.

Peirce: CP 4.309 Cross-Ref:††

The most fundamental fact about the number three is its generative potency.†1 This is a great philosophical truth having its origin and rationale in mathematics. It will be convenient to begin with a little *a priori* chemistry.†2 An atom of helion, neon, argon, xenon, crypton, appears to be a medad (if I may be allowed to form a patronymic from {méden}). Argon gives us, with its zero valency, the one single type

A.

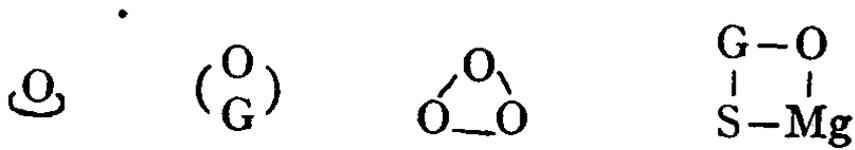
Supposing H, L, Na, Ag, etc. and F, Cl, Br, I to have strictly unit valency (which appears not to be true; at least, not for the halogens), then they afford only the two types

H-H H-F,

if these can be called two.

Peirce: CP 4.309 Cross-Ref:††

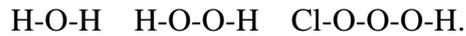
Assuming G (glucinum), etc. with O, S, etc., to have valency 2 (certainly not true), they might give an endless series of saturated rings, by themselves.



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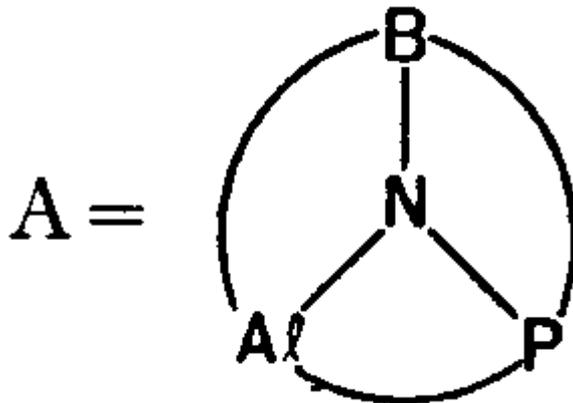
and so on, *ad infinitum*. With the monads, these dyads would give terminated lines.



and so on, *ad infinitum*. But they can give no other types than single rings and terminated lines.

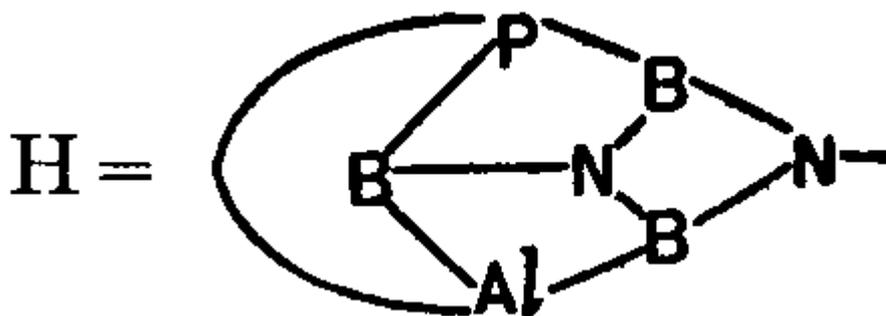
Peirce: CP 4.309 Cross-Ref:††

Triads, on the other hand, will give every possible variety of type. Thus, we may imagine the atom of argon to be really formed of four triads, thus



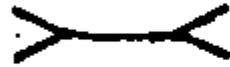
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We may imagine the monadic atom to be composed of seven triads; thus;



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A dyad will be obtained by breaking any bond of A; while higher valencies may be produced, either simply



[Click here to

or in an intricate manner.

Peirce: CP 4.309 Cross-Ref:††



One atom forms one type without a ring

[Click here to view], one



with a one-atom ring

[Click here to view]: two in all.

Peirce: CP 4.309 Cross-Ref:††



Two atoms form, in one piece, one acyclic type

[Click here to



view], one with one protocycle

[Click here to view], one diprotocyclic



type

[Click here to view], one monodeuterocyclic



[Click here to view], one dideuterocyclic type
five in all.



[Click here to view]:

Peirce: CP 4.309 Cross-Ref:††



Three atoms form one acyclic type

[Click here to view],



one monoprotocyclic



[Click here to view], one diprotocyclic



[Click here to view], one monodeuterocyclic

[Click here to view], one



protodeuterocyclic



[Click here to view], one tritocyclic



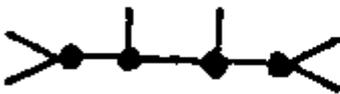
[Click here to view] one deuterotritocyclic in all.

[Click here to view]: seven

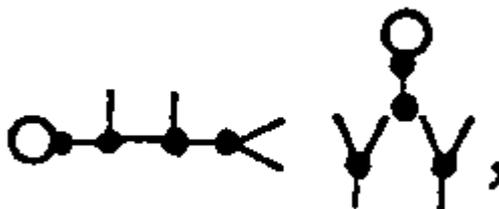
Peirce: CP 4.309 Cross-Ref:††

Four atoms form two acyclic types

11.



[Click here to view], two



monoprotocyclic

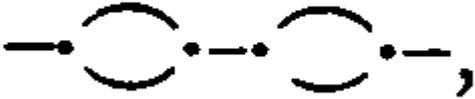
[Click here to view], two

diprotocyclic   [Click here to view] one

triprotocyclic  , [Click here to view] two monodeuterocyclic

 [Click here to view]

 , [Click here to view], one dideuterocyclic

 [Click here to view], two monodeuteromonoprotocyclic

  , [Click here to

view], one monodeuterodiprotocyclic  , [Click here to

 , [Click here to view], one monotritocyclic  [Click here to view], one ditritocyclic



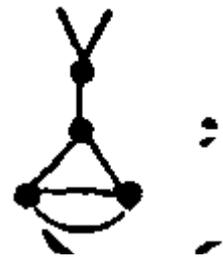
[Click here to view] (where there is, of course, also a four-atom



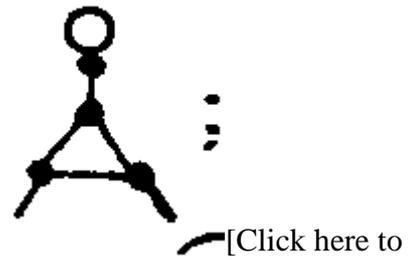
ring), one tritritocyclic [Click here to view] (so I name it, although there are four three-atom rings and three four-atom rings), one prototritocyclic



[Click here to view], one deuterotritocyclic



[Click here to view], one protodeuterotritocyclic



[Click here to

view]; one tettartocyclic



[Click here to view], one

monodeutero-tettartocyclic



[Click here to view], one

dideuterotettartocyclic



[Click here to view], one tritotettartocyclic



[\[Click here to view\]](#): twenty-three in all, if I have repeated none. With larger numbers of atoms the types multiply astonishingly.

Peirce: CP 4.310 Cross-Ref:††

310. It would scarcely be an exaggeration to say that the whole of mathematics is enwrapped in these trichotomic graphs; and they will be found extremely pertinent to logic. So prolific is the triad in forms that one may easily conceive that all the variety and multiplicity of the universe springs from it, though each of the thousand corpuscles of which an atom of hydrogen consists be as multiple as all the telescopic heavens, and though all our heavens be but such a corpuscle which goes with a thousand others to make an atom of hydrogen of a single molecule of a single cell of a being gazing through a telescope at a heaven as stupendous to him as ours to us. All that springs from the

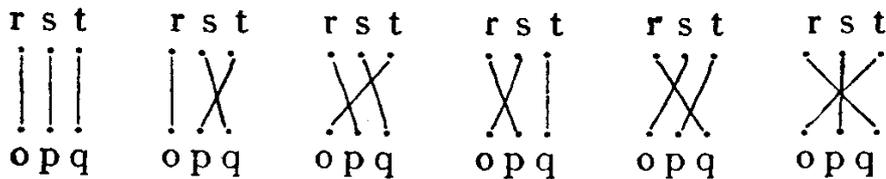


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-- an emblem of fertility in comparison with which the holy phallus of religion's youth is a poor stick indeed.

Peirce: CP 4.311 Cross-Ref:††

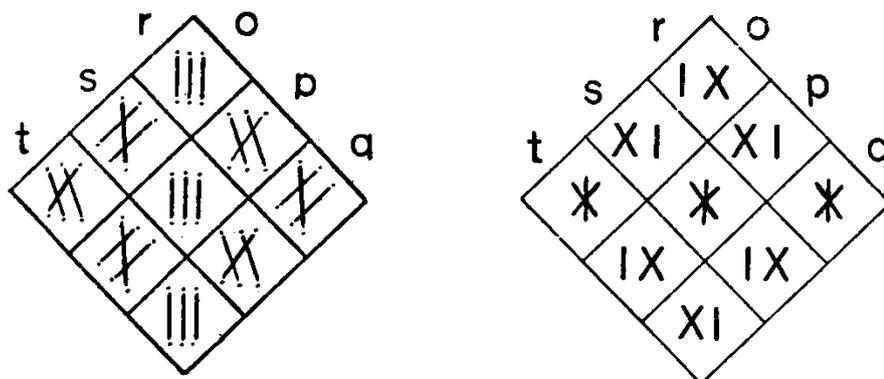
311. Let us now glance at the permutations of three things. To say that there are six permutations of three things is the same as to say the two sets of three things may correspond, one to one in six ways. The ways are here shown



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No one of these has any properties different from those of any other. They are like two ideal rain drops, distinct but not different. Leibniz's "principle of indiscernibles" is all nonsense. No doubt, all things differ; but there is no logical necessity for it. To say there is, is a logical error going to the root of metaphysics; but it was an odd hodge-podge, Leibniz's metaphysics, containing a little to suit every taste. These arrangements are just like so many dots, as long as they are considered in themselves. There is nothing that is true of one that is not equally true of any other -- so long as in the proposition no other is definitely mentioned. But when we come to speak of them in pairs, we find that *pairs* of permutations differ greatly. To show this let us make a

table like that which we formed in dichotomic algebra. On one side we enter the table with r, s or t , on the other with $[o, p, \text{ or } q]$, and at the intersections of the rows we find the figure of the permutation in which the two correspond. In order to avoid putting two symbols in one square I repeat the table. We have then



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to view]

It will be seen that from this point of view, that of their relations to one another, the permutations separate themselves into two sets. In any one set, there are no two permutations which make the same letter correspond to the same letter; while of pairs of permutations of opposite sets, each agrees in respect to the correspondence of one letter.

Peirce: CP 4.311 Cross-Ref:††

Since there are six permutations, there could be 26, or sixty-four different assertions which might be made concerning an unknown two, as to what ones of the six they were. But from these sixty-four, it will be interesting to select a set of six, such that any one, being relatively multiplied by any other, the product is one of the six, and such that the product is different from what it would be if either multiplier or multiplicand were alone different. Not that this is to be true of all quantities of the algebra, but only of the six single letters. For the larger condition would render the problem impossible. Let the six letters conforming to this condition be i, j, k, l, m, n . Since every product of these letters is to be one of the letters, and since i^2, ji, ki, li, mi, ni are all different, it follows that some one of them must be equal to i . This one may or may not be i . Suppose, first, that it is not so; but that, say, $ji = i$. Then $j^2i = j(ji) = ji = i$. Thus j^2 is some letter which multiplied into i gives i . But since this is true of j , it cannot, by our hypothesis, be true of any other letter, and it must be that $j^2 = j$. Thus, if i^2 is not i , there is some other letter whose square is the letter itself. We may assume, then, i to be that letter; and we shall have $i^2 = i$. Then what will any other letter, say j , give when multiplied by i ? Since $i = i^2$, we have $ij = i^2j = i(ij)$. But according to our hypothesis, if j were not equal to ij , as the product, then ij could not be equal to $i(ij)$, as it is. Hence $ij = j$, and similarly $ik = k, il = l, im = m, in = n$; and by like reasoning $ji = j, ki = k, li = l, mi = m, ni = n$. It is plain, then, that no other quantity can have its square equal to itself. For, if $j = j$ while $ij = j$, we should have by our hypothesis $i = j$.

Peirce: CP 4.312 Cross-Ref:††

312. Since the product of two letters is a letter in every case, and since the number of letters is finite, it follows that some power of a letter is equal to some other power of the same letter. But suppose that $jp = jp+q = jp \cdot jq$. But jp is equal to a letter

and therefore, as we have seen, $jp-i = jp$. Thus, $jp-i = jp \cdot jq$, where jq is likewise equal to a letter. But it is assumed that multiplication is invertible for the letters; so that $jq = i$. That is, some power of each letter is i . Moreover, $jq-1$ will also be equal to a letter; and $j \cdot jq-1 = jq-1 \cdot j = i$. There is, therefore, for each letter some letter which multiplied by or into it gives i as the product; and since multiplication is invertible for the letters, there will be no other letter that multiplied either by or into that letter will give i as the product. Consequently, if the product of two letters is i , each of them is equal to some power of the other. If, then, two letters, say j and k , are not powers of one another, their product cannot be a power of either; for were $j \cdot k = kq$, since multiplication is invertible for the letters, we should have $j = kp-1$, that is, j would be a power of k . If, then, there are two series of letters, the letters of each series, powers of one another, but no two letters of the one and the other powers of one another, then there must be a third series of which this is true; but any of these series may consist of a single square root of i . Of the five letters j, k, l, m, n , not more than three can be square roots of i . In the first place, all five of the letters j, k, l, m, n cannot be square roots of i . For if $j^2 = k^2 = i$ then jk must be a different letter from either j or k . Call it l . Since then $jk = l$, we have $lk = jk^2 = ji = j$ and $jl = j^2k = ik = k$. Moreover, if $l^2 = i$, that is, if $jkjk = i$, multiplying into k we have $kj = ikj = j^2kj = j^2kji = j^2kjk^2 = j(jk \cdot jk)k = jl^2k = jik = jk$, so that $kj = jk = l$ and $lj = kj^2 = ki = k$, $kl = k^2j = ij = j$. We thus find the products of j into i, j, k, l to be j, i, l, k , leaving nothing for jm to be except n . For, by hypothesis, it must be some letter different from the product of j into any of the other letters. But in like manner, the products of k into i, j, k, l would be k, l, i, j respectively; and the products of l into i, j, k, l , would be l, k, j, i , respectively; so that in each case we could only have $jm = n, km = n, lm = n$, violating the hypothesis that multiplication is invertible for the letters. That just four of the five letters, j, k, l, m, n , cannot be square roots of i is quite obvious. For what root of i could the fifth one be? It could not be a fourth root of i , since then its cube must be a separate letter; it could not be any other root of i , since then its square must be a separate letter; while all the other letters would be preoccupied. Indeed, for the same reason no root of i higher than the cube root (except, of course, a sixth root) can exist among the five letters. No more can two independent cube roots of i ; although this is perhaps a trifle less obvious. Thus, the only possible cases are, first, where one of the letters is a sixth root of i , of which there are one hundred twenty varieties, {hoi polloi}, which vie with one another in their utter want of interest. The only remaining possible case is where there are three square roots of i and a cube root of i with its square, to which of course the same description applies. I here give the multiplication table.

\dot{i}	=	: + ✕: ✕ + ✕: ✕ + X: X + ✱: ✱ + X : X
\dot{l}	=	✕: + ✕: ✕ + : ✕ + X : X + X: ✕ + ✱: X
\dot{k}	=	✕: + : ✕ + ✕: ✕ + ✱: X + X : ✱ + X: X
\dot{l}	=	X: + ✱: ✕ + X : ✕ + : X + ✕: ✕ + ✕: X
\dot{m}	=	✱: + X : ✕ + X: ✕ + ✕: X + : ✱ + ✕: X
\dot{n}	=	X : + X: ✕ + ✱: ✕ + ✕: X + ✕: ✱ + : X

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If these signs were to be used in any general application, some of them would require modification to prevent their being mistaken for two or three characters. It will be remarked that i, j, k have each permutation in its own class indicated by the even or odd number of crossings in its character; while l, m, n reverse the class of each

permutation.

Peirce: CP 4.313 Cross-Ref:††

313. In regard to the multiplication table, it is, in the first place, noticeable that if we take six new quantities, as follows:

$$\begin{aligned} i'' &= ((i-j)-(k-i))/3 & l'' &= ((l-m)-(n-l))/3 \\ j'' &= ((j-k)-(l-j))/3 & m'' &= ((m-n)-(l-m))/3 \\ k'' &= ((k-i)-(j-k))/3 & n'' &= ((n-l)-(m-n))/3 \end{aligned}$$

we have

$$\begin{aligned} i'' \cdot j'' &= i \cdot j & j'' \cdot k'' &= j \cdot k & k'' \cdot i'' &= k \cdot i \\ l'' \cdot m'' &= l \cdot m & m'' \cdot n'' &= m \cdot n & n'' \cdot l'' &= n \cdot l; \end{aligned}$$

whence it follows that all formulæ concerning the unaccented differences have reciprocal formulæ exactly like them concerning accented differences. It is true that the reverse is not always true.

Peirce: CP 4.313 Cross-Ref:††

For example, $i''+j''+k''=0$ $l''+m''+n''=0$;

while, without the accents, these equations would not hold good. But this very fact will lead us, in due time, to an analogy, not only very pretty, but of the highest importance for logic.

Peirce: CP 4.313 Cross-Ref:††

It is to be added that the multiplication table of $i'', j'', k'', l'', m'', n''$ is precisely the same as that of i, j, k, l, m, n , except that accents are everywhere added. But it is seriously important, both for the sake of the purely ideal beauty of this algebra, and still more in view of its great application to logic, that we should not allow its triadic purity to be violated. Now the reciprocal relation between the unaccented and the accented letters, were it allowed to stand alone, would be a purely dichotomic one. In order to avoid this, we ought to introduce the following singly accented letters:

$$\begin{aligned} i' &= (j-k)/\sqrt{-3} & i'' &= (j'-k')/\sqrt{-3} & l' &= (m-n)/\sqrt{-3} & l'' &= (m'-n')/\sqrt{-3} \\ j' &= (k-i)/\sqrt{-3} & j'' &= (k'-i')/\sqrt{-3} & m' &= (n-l)/\sqrt{-3} & m'' &= (n'-l')/\sqrt{-3} \\ k' &= (i-j)/\sqrt{-3} & k'' &= (i'-j')/\sqrt{-3} & n' &= (l-m)/\sqrt{-3} & n'' &= (l'-m')/\sqrt{-3} \end{aligned}$$

Peirce: CP 4.314 Cross-Ref:††

314. . . . In pure algebra, the symbols have no other meaning than that which the formulæ impose upon them. In other words, they signify any relations which follow the same laws. Anything more definite detracts needlessly and injuriously from the generality and utility of the algebra. It is that high principle which we all learned at a tender age that one cannot eat his cake and have it too; one cannot devote a thing to a particular use without making it less available for other applications. The logicians call it the principle of the inverse proportionality of comprehension and extension. Yet in this particular instance, we can adapt our doctrine better to thoroughgoing trichotomy by derogating a little from the dignified meaninglessness of pure algebra. In multiple algebra, it is generally assumed that the coefficients can

be any numbers. My father, for example, even allowed them to be imaginary; though I cannot approve of that. But for the purposes of trichotomic mathematics, it should be recognized that each quantity has one of three values. Call them, for the moment, 0^\bullet , 120^\bullet , 240^\bullet -- regarding 360^\bullet as the same as 0^\bullet . Or one might call them night, morning, and afternoon. Let us denote the three values by o (for {orthros}), δ (for {deilé}), $\{n\}$ (for {nyx}). Then, we must adopt the addition table:

$$\begin{aligned} o+o &= o & o+\delta &= \delta & o+\{n\} &= \{n\} \\ \delta+o &= \delta & \delta+\delta &= \{n\} & \delta+\{n\} &= o \\ \{n\}+o &= \{n\} & \{n\}+\delta &= o & \{n\}+\{n\} &= \delta \end{aligned}$$

For multiplication [a] table of these numbers will be obtained by assuming $\delta\delta = \delta$. Then $\delta\{n\} = \delta(\delta+\delta) = \delta\delta+\delta\delta = \delta+\delta = \{n\}$. $\delta o = \delta(\{n\}+\delta) = \delta\{n\} + \delta\delta = \{n\}+\delta = o$. For the sake of showing the consistency of the formulæ, we may complete the cycle

$$\begin{aligned} \delta\delta &= \delta(\{n\}+\{n\}) = \delta\{n\}+\delta\{n\} = \{n\}+\{n\} = \delta; \\ \{n\}\{n\} &= \{n\}(\delta+\delta) = \{n\}\delta+\{n\}\delta = \{n\}+\{n\} = \delta, \text{ (just as } - - = +) \\ \{n\}o &= \{n\}(\{n\}+\delta) = \{n\}\{n\}+\{n\}\delta = \delta+\{n\} = o \\ oo &= (\{n\}+\delta)o = \{n\}o+\delta o = o+o = o \end{aligned}$$

Instead of assuming the distributive principle in its entirety we might have evolved the multiplication table from these three equations

$$oo = o \quad x(y+\delta) = xy+x \quad (x+\delta)y = xy+y.$$

In the same spirit, the addition table might have been derived from the equations,

$$o+\delta = \delta \quad \delta+\delta = \{n\} \quad \{n\}+\delta = o \quad (x+\delta)+y = x+(y+\delta) = (x+y)+\delta$$

Thus,

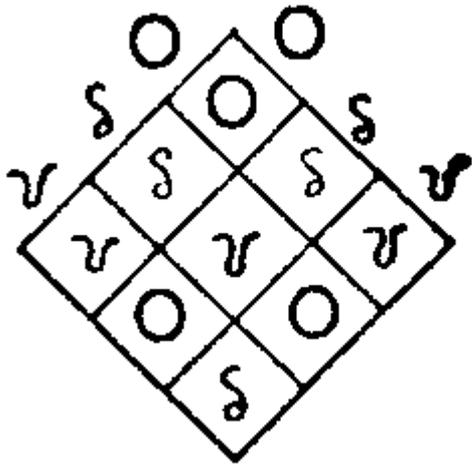
$$\begin{aligned} o+\{n\} &= o+(\delta+\delta) = (o+\delta)+\delta = \delta+\delta = \{n\} \\ o+o &= o+(\{n\}+\delta) = (o+\{n\})+\delta = \{n\}+\delta = o \\ \{n\}+\{n\} &= \{n\}+(\delta+\delta) = (\{n\}+\delta)+\delta = o+\delta = \delta. \end{aligned}$$

Peirce: CP 4.315 Cross-Ref:††

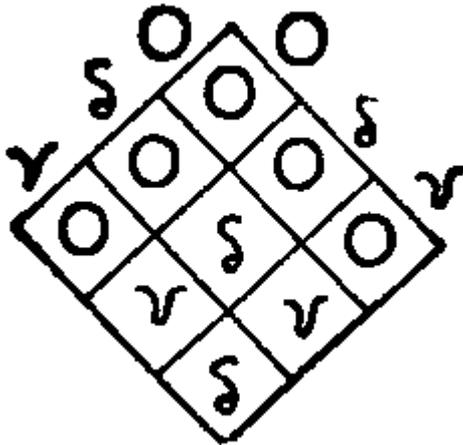
315. As for involution, until somebody can give me some good reason for attaching a given definite interpretation to $(120^\bullet)^{(120^\bullet)}$, I may be excused from attempting to introduce it into this algebra. I apprehend that, multiple algebra being essentially linear, there is no demand for any involution of its units.

Peirce: CP 4.316 Cross-Ref:††

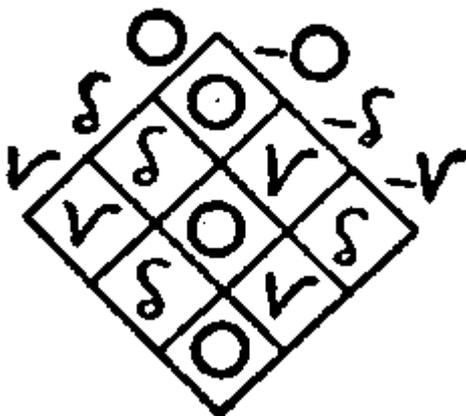
316. It will be seen that, if we are to accept the premisses upon which the addition-table and multiplication-table are based, we cannot avoid giving peculiar properties to each of the three values δ , $\{n\}$, o , and that the connection of them with some such sensuous images as day, night, and dawn is by no means an idle fancy. Let us put these tables into form. I add the subtraction-table



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ADDITION

MULTIPLICATION

SUBTRACTION

TABLE

TABLE

TABLE

We see that the multiplication-table recognizes a characteristic property in each member of the triad \mathbf{o} , δ , $\{n\}$. Multiplication by δ effects nothing. Multiplication by $\{n\}$ may have peculiar effects, but it is undone by a second multiplication by $\{n\}$. Multiplication by 0 can never be undone, nor the same effect be otherwise produced.

Peirce: CP 4.317 Cross-Ref:††

317. We thus see that it is impossible to deal with a triad without being forced to recognize a triad of which one member is positive but ineffective, another is the opponent of that, a third, intermediate between these two, is all-potent. The ideas of our three categories could not be better stated in so few words. A man must be wedded to a system of metaphysics not to see the philosophical importance of the fact that these ideas thus insist upon intruding where we have done our best to bolt and bar the doors against them, by assuming the members of the triad to be more alike than three rain-drops.

Peirce: CP 4.318 Cross-Ref:††

318. What is experience? It is the resultant ideas that have been forced upon us. We find we cannot summon up what images we like. Try to banish an idea and it only comes home with greater violence later. Hence, we find the only wisdom is to accept, at once, the ideas that sooner or later we must accept; and we even go to work solicitous to find out what are the ideas which are going ultimately to be forced upon us. Three such ideas are the three categories; and it will be wise to pitch overboard promptly the metaphysics which preaches against them. To recognize the triad is a step out of the bounds of mere dualism; but to attempt [to deny] independent being to the dyad and monad, Hegel-wise, is only another one-sidedness.

Peirce: CP 4.319 Cross-Ref:††

319. We are not bound at all times to introduce the triad: it is not needed on every occasion. But we should be prepared to introduce it whenever it is needed. We must not absolutely restrict ourselves to the notion that two triads can at one time correspond to one another in only one way, so that a given member of the one must be down-right, absolutely connected with a given member of the other, or else be down-right, absolutely, disconnected from it -- two alternatives differing as day from night -- as $\{\text{deilé}\}$ from $\{\text{nyx}\}$. We must be prepared, if occasion be, to admit a possible intermediate dawn. For to say that two things are disconnected is but to say that they are connected in a way different from the way under contemplation. For everything is in some relation to each other thing. It is connected with it by otherness, for example. We should, therefore, be prepared to say that two atoms, one of each triad, have either a positive connection, such as is under the illumination of thought at the time, or a dark other mode of connection, or a vague glimmering intermediate form of connection. Nor are we to rest there, as a finality. We must not restrict ourselves to saying absolutely that between a pair there either down-right is a given kind of connection, or down-right is *not* that kind. We must be prepared to say, if need be, that the pair has a δ -connection with a given mode of connection, or an opposite $\{n\}$ -connection with it, or a neutral \mathbf{o} -connection with it. We can push this sort of thing as far as may be -- indefinitely. Still, however far we carry it, ultimately there will always be a dichotomic alternative between the truth and falsity of what is said. Why it should be so, we shall see in the proper place and time for such an

inquiry. At present, it is pertinent to note that the fact that it is so is forced upon our attention in pure mathematics.

Peirce: CP 4.320 Cross-Ref:††

320. Our algebra of i, j, k, l, m, n , supposes a triad to be in one-to-one correspondence with a second triad. We may conveniently identify that second triad with $o, \delta, \{n\}$; since these three values, by hypothesis, are any three things we please. As to the mode of connection of the first triad with $o, \delta, \{n\}$, we ought, in order to make full use of the algebra, to have a state of things which is, or is analogous to, a state in which each of the three things composing the triad is in two distinct modes connected with one of the three values, $o, \delta, \{n\}$; or with a value of whatever system of values may be appropriate. To indicate this we may use a sign like this

$$\begin{array}{ccc} \delta & \delta & o \\ \cdot & \cdot & \cdot \\ \{n\} & \delta & \{n\} \end{array}$$

The three dots represent the triad. The letters above show what values are attached to the members of the triad in the first mode; the letters below show what values are so attached in the second mode. In order to translate this into the algebra, the values of the upper line, in their order, are to be taken as the coefficients of i, j, k respectively; those in the lower line as the coefficients of l, m, n respectively. Thus the sign written would be

$$\delta i + \delta j + o k + \{n\} l + \delta m + \{n\} n.$$

We may commonly write 0 for o , 1 for δ , -1 for $\{n\}$. It often happens in the application of algebra that the absolute values of the coefficients are without significance, their ratios being alone important. If, in addition, the second mode of connection of the members of the triad with the values is identical with the first mode, we shall have

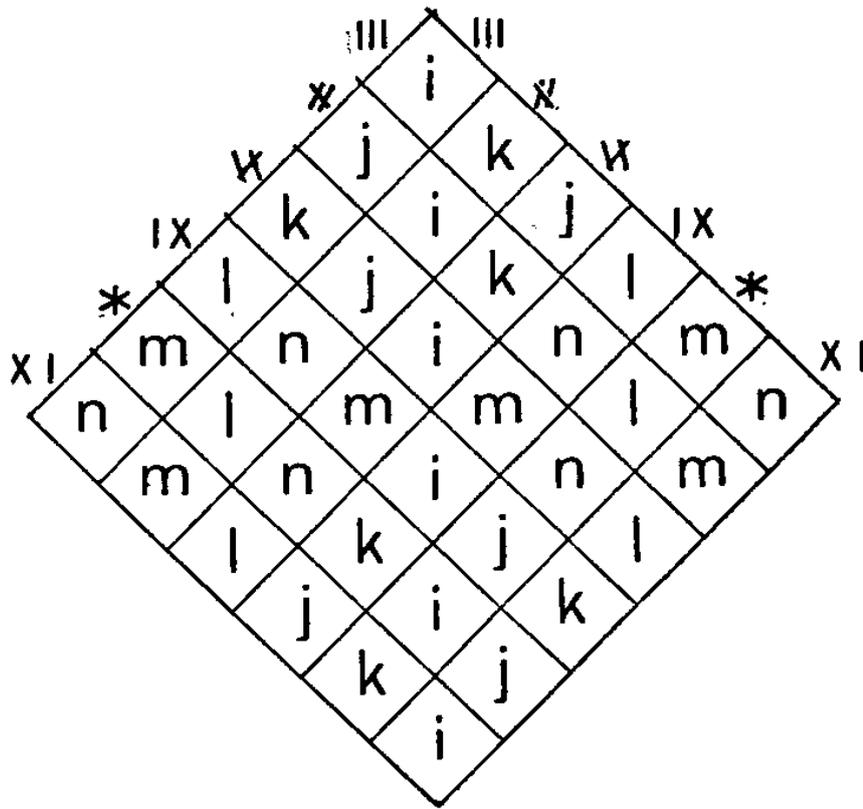
$$i + j + k = o \quad l + m + n = o$$

just as $i' + j' + k' = o \quad l' + m' + n' = o$.

Indeed it was the circumstance, that with the unaccented letters these equations do not generally hold good, which led me to the above remarks, which give an interpretation to a sum of letters of the algebra.

Peirce: CP 4.321 Cross-Ref:††

321. I have known such astounding blunders and oversights to be committed by the most powerful and exact intellects, that I have come to the conclusion that it is folly to attempt to set limits which human stupidity cannot overpass. Otherwise, I should venture to say that no intelligent reader of this chapter could, while wide awake, for two minutes harbor the notion that a transposition, or passage from one



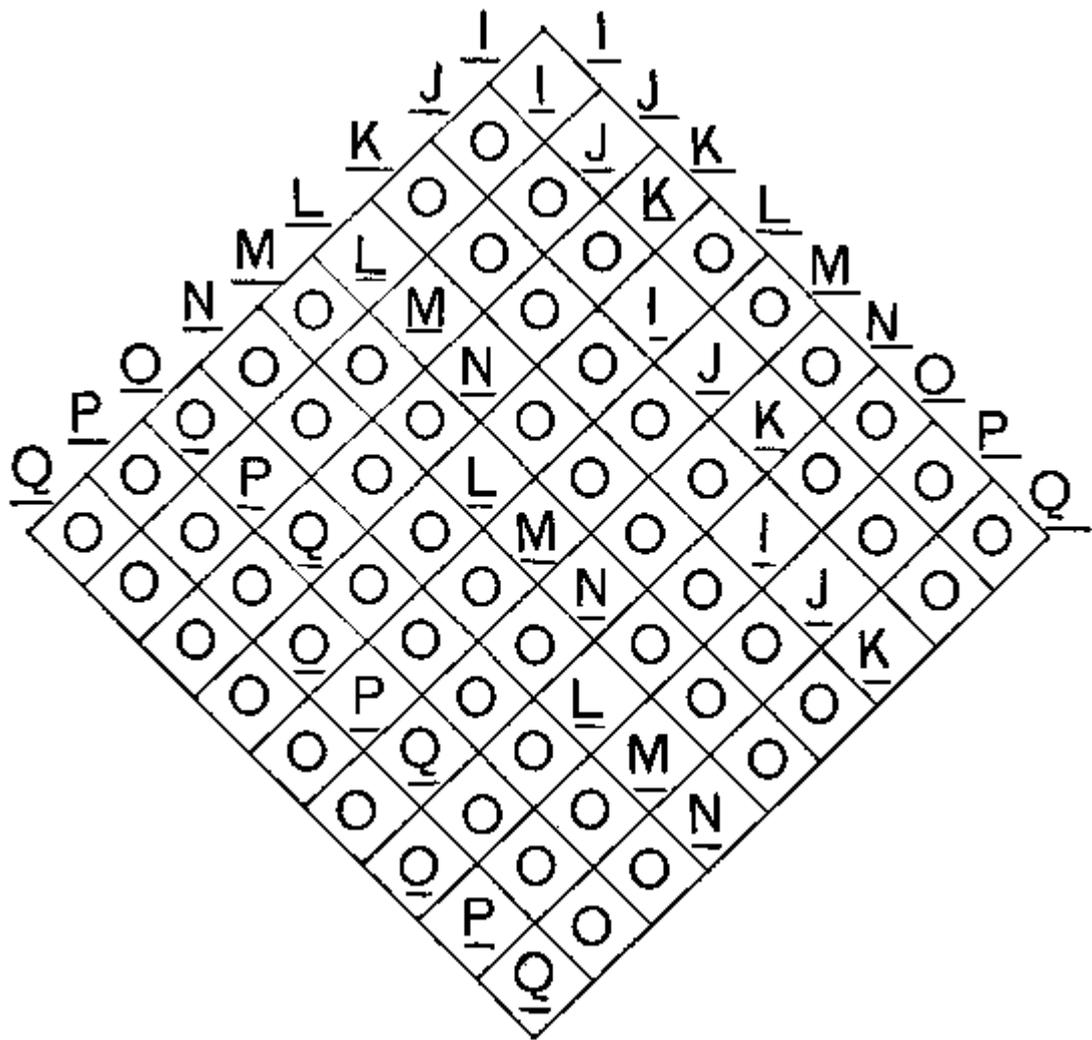
[Click here

to view]

permutation to another -- the mathematicians call it a substitution -- could possibly be the interpretation of a sum of transpositions; for the very idea of a multiple algebra is that a polynomial in the unit letters cannot be expressed as a numerical multiple of a single unit. A transposition produces a permutation in which all the members of the set are connected with different members of another set. In a sum of transpositions connections between members of two sets are created in which two or more of one set may be connected with one of the other. Precisely what such connections may be is sufficiently shown above. Another illustration is afforded by the diagram on page 260.

Peirce: CP 4.321 Cross-Ref:††

This is a sort of division table. The operand being entered on the right and the product on the left, the one operation effecting this transmutation is found at the intersection of the rows. We see that any permutation can be transmuted into any other, but by only one letter; and since there are only six letters in all while there are nine pairs composed each of a



[Click here to view]

member of each of two triads, no combination of letters can express every possible change of connections of the members of two triads (but only changes of permutations and their sums, as above explained). When it is requisite to be able to do that, as we shall find that for some logical problems it is, we must resort to another algebra published by me in 1870†1 and for which I prefer the name *novenions*, rather than 'nonions,' the designation proposed by Clifford and employed by many mathematicians. Let $u[1]$, $u[2]$, $u[3]$, be three "umbræ," or ingredients of quantities. Then take

$$\begin{aligned}
 i &= u[1]:u[1] & l &= u[2]:u[1] & o &= u[3]:u[1] \\
 j &= u[1]:u[2] & m &= u[2]:u[2] & p &= u[3]:u[2] \\
 k &= u[1]:u[3] & n &= u[2]:u[3] & g &= u[3]:u[3]
 \end{aligned}$$

and the multiplication table is as shown on page 261.

Peirce: CP 4.322 Cross-Ref:††

322. Sometime after my first publication, either my father or I myself (under the instigation of my father's ideas) transformed †P1 this algebra by means of the

following equations, where, as above, $\{r\}$ is an imaginary cube root of unity:

$$I = (\mathbf{u}[1]:\mathbf{u}[1]) + (\mathbf{u}[2]:\mathbf{u}[2]) + (\mathbf{u}[3]:\mathbf{u}[3])$$

$$J = (\mathbf{u}[1]:\mathbf{u}[1]) + \{r\}(\mathbf{u}[2]:\mathbf{u}[2]) + \{r\}^2(\mathbf{u}[3]:\mathbf{u}[3])$$

$$K = (\mathbf{u}[1]:\mathbf{u}[1]) + \{r\}^2(\mathbf{u}[2]:\mathbf{u}[2]) + \{r\}(\mathbf{u}[3]:\mathbf{u}[3])$$

$$L = (\mathbf{u}[1]:\mathbf{u}[2]) + (\mathbf{u}[2]:\mathbf{u}[3]) + (\mathbf{u}[3]:\mathbf{u}[1])$$

$$M = (\mathbf{u}[1]:\mathbf{u}[2]) + \{r\}(\mathbf{u}[2]:\mathbf{u}[3]) + \{r\}^2(\mathbf{u}[3]:\mathbf{u}[1])$$

$$N = (\mathbf{u}[1]:\mathbf{u}[2]) + \{r\}^2(\mathbf{u}[2]:\mathbf{u}[3]) + \{r\}(\mathbf{u}[3]:\mathbf{u}[1])$$

$$O = (\mathbf{u}[1]:\mathbf{u}[3]) + (\mathbf{u}[3]:\mathbf{u}[2]) + (\mathbf{u}[2]:\mathbf{u}[1])$$

$$P = (\mathbf{u}[1]:\mathbf{u}[3]) + \{r\}(\mathbf{u}[3]:\mathbf{u}[2]) + \{r\}^2(\mathbf{u}[2]:\mathbf{u}[1])$$

$$Q = (\mathbf{u}[1]:\mathbf{u}[3]) + \{r\}^2(\mathbf{u}[3]:\mathbf{u}[2]) + \{r\}(\mathbf{u}[2]:\mathbf{u}[1])$$

Peirce: CP 4.323 Cross-Ref:††

323. Other points concerning trichotomic mathematics

are more of logical than of mathematical interest, and are so

woven with logic in my mind that I will not attempt to set

them forth from a purely mathematical point of view. Here,

then, I conclude what I have to say of these very simple

branches of mathematics which lie at the root of formal

logic. Those of which the interest is more purely mathematical

must be treated in a very different manner in the next chapter.†1

Peirce: CP 4.324 Cross-Ref:††

VIII

NOTES ON THE LIST OF POSTULATES

OF DR. HUNTINGTON'S SECTION 2†1P

324. Dr. Huntington's Postulates at the head of section 2 of his paper †2 seem to me to have so great and so permanent an interest, that I am prompted to append to them some remarks with a view to establishing some other points of view which seem to me to be worth examining. My remarks inevitably present more or less opposition to Dr. Huntington's ideas; but that opposition is insignificant. For I certainly hold it to be undeniable that Dr. Huntington has successfully achieved his purpose; and that that purpose was a most important one for the study of logic. But in philosophy, as every student of it well understands, discoveries are worked out by reflection upon matters of common observation, instead of resulting from new opportunities for observation,

as in all the positive sciences, or from pure thought concerning creations of the learned, as in mathematics, so that an idea is worthy of publication until one finds that it has not occurred to some profound student of the subject; and this one knows only by its opposition to something he has said.

Peirce: CP 4.325 Cross-Ref:††

325. By a 'postulate,' Dr. Huntington seems to understand any one of a body of propositions such that nothing can be deduced from one that could equally be deduced from another, while, from them all, every proposition of a given branch of mathematics might be deduced. The utility of such a body of premisses for the logical analysis of the branch of mathematics in question is beyond dispute. But I think we ought to distinguish between *postulates* and *definitions*. As for *axioms*, or propositions already well-known to the student who takes up the branch of mathematics in question, the ancients themselves admitted that they might be omitted without detriment to the course of deduction of the theorems. Indeed, an axiom could only be a maxim of logical nature. A postulate is a statement that might be questioned or denied without absurdity. A definition, or rather, one of the pair, or larger number, of propositions that constitute a definition, cannot be questioned, because it merely states the logical relation of a conception thereby introduced to conceptions already in use. It is quite true, on the one hand, that a postulate, after all, is merely a part of the definition of the underlying conception of the branch of mathematics to which it refers, (Euclid's celebrated fifth postulate, for example, being merely a part of the definition of Euclidean space); while on the other hand, a definition is a statement of positive fact about the use of the word defined, and may thus be regarded as a sort of postulate. But to argue from these truths that there is no important difference between a postulate and a definition would be to fall into a fallacy of a very common kind, that of denying all important difference between two things because they are in an important respect alike, or of denying all important likeness between two things because they are in an important respect unlike. There is a vast difference between the logical relations to a branch of mathematics of propositions defining its very purpose in defining its fundamental hypothesis, and those of propositions that merely define conceptions which it is convenient or even which it is necessary to introduce in order to develop that branch.

Peirce: CP 4.326 Cross-Ref:††

326. I shall consider only the postulates of Dr. Huntington's section 2. This section refers to a special form of the Boolean algebra of logic. The algebra which Boole himself used was simply ordinary numerical algebra as applied to a collection of quantities each of which was assumed to be subject to the quadratic equation $x(1-x) = 0$, and Boole showed how this hypothesis could be applied to the solution of many logical problems. For him, therefore, addition and multiplication were nothing but the numerical operations, greatly restricted in their application. An essential, not to say the vital, element of Boole's method lay in its applicability to the calculation of probabilities and statistical relations. But this feature disappeared in the algebra as it was modified by all his followers except Mr. Venn, a circumstance that gives a special value to Venn's *Symbolic Logic*, a work that has many other merits. The rest of us assigned to the terms and operations purely logical meanings which we thought had sufficient analogy with the numerical conceptions to receive either exactly the same symbols, or, in my own case, to receive symbols closely resembling the numerical symbols. We †1 made three changes which affected the working of the method. These were as follows:†2

Peirce: CP 4.326 Cross-Ref:††

First, we introduced what we inappropriately called 'logical addition,' writing something exactly or nearly like $x \vee y$ for what Boole wrote as $x+y+xy$.

Peirce: CP 4.326 Cross-Ref:††

Second, we introduced the copula of inclusion, writing

$$x \prec y$$

where Boole would write

$$x = xy.$$

Peirce: CP 4.326 Cross-Ref:††

Third, we introduced the negative of this copula; writing

$$\sim(x \prec y)$$

for what Boole had no correct way of writing except " $x = xy$ is false."

Peirce: CP 4.327 Cross-Ref:††

327. Now the Boolean algebra to which Dr. Huntington's section 2 relates is Boole's as modified in the first two of these ways. It is to be added, moreover, that Schröder, with the majority of the Boolians, abandoned Boole's conception that every logical term has one or other of two values. For my part, I have always retained that conception, as far as non-relative terms go, which correspond to the quantities of ordinary algebra. But I introduced relative terms which correspond to what Sylvester called the *umbræ* of quantities (the conception is due to Leibniz), and employed various signs of operation upon these *umbræ*.†3 At the same time I showed that a non-relative term can be considered as a relative, that, in another sense, a relative term may be considered as non-relative, and that the non-relative operations equally apply to relatives.†4 I regard a logical term as an indefinite proposition or blank form of proposition; *man*, for example, meaning " x is a man." Now every proposition has one or other of two values; the lesser, that of being false; the greater, that of being true. A term, or rheme, is like 0/0, in itself indeterminate in value, yet having one or the other of two values in each particular case. Thus, when in the ordinary Boolean algebra we write $m \prec l$, meaning "every man is a liar," according to me this means "if x (which is any individual object you may choose) is a man, then x is a liar," m signifying that x is a man, and l , signifying that x is a liar. Schröder, on the other hand, would say that m 'denotes' the entire collection of men (though I do not know what definite idea can be attached to the word 'denotes'), that l 'denotes' the entire collection of liars, and that the formula states that the former collection is included in the latter. Now it is certain that Dr. Huntington does not embrace my conception, since he would have greatly simplified his list of postulates if he had done so; but it is not clear that he unequivocally rejects this conception.

Peirce: CP 4.328 Cross-Ref:††

328. In my opinion, in algebra generally, the distinction between a quantity and an operation is a subsidiary one, and that we ought to allow operations to be operated upon as much as quantities; and this, perhaps, will be commonly conceded. If so, in the algebra of logic, the signs of the so-called logical addition and multiplication are substantially nothing but relative terms of a special description. But I would allow the spirit of unification to have the further effect that no radical

separation be recognized between signs of operation and copulas, such as = and \rightarrow . Thus, in any paper of 1880,†1 I have such expressions as $(a \rightarrow b) \rightarrow (c \rightarrow d)$ which is equivalent to

$$(\sim a \vee b) \rightarrow (\sim c \vee d)$$

or to

$$a \cdot \sim b \vee \sim c \vee d.$$

Dr. Huntington evidently does not contemplate any such handling of the algebra of logic.

Peirce: CP 4.329 Cross-Ref:††

329. Dr. Huntington's purpose is, while considering the Boolean algebra in a purely morphic light, without regard to its interpretation, to draw up a list of independent propositions which shall justify every illative transformation of the algebra in one way, and in one way only. Of his postulates, which are ten in number,



the first three give a morphic definition of the copula \rightarrow . [Click here to view] which is the purely morphic generalization of the copula of inclusion, these three propositions also showing that the algebra regards all entities as interchangeable which a direct application of this copula would not distinguish. The next two postulates define the two terms which I regard as the two values. The next two amount (according to my well-known definitions of the logical aggregate and compound)†1 to asserting that any terms may be logically added and multiplied. The eighth and ninth morphically define the sign of negation. The tenth is the morphic generalization of the logical postulate that it is possible to express a proposition that is false.

Peirce: CP 4.330 Cross-Ref:††

330. Perhaps this arrangement has not been the subject of much consideration. In defining any branch of mathematics, I should begin with stating the multitude of values it admits; and therefore in this case I should put postulate No. 10 first. Dr. Huntington seems to have left it to the last because he considered it "trivial." But it is certainly not trivial in the sense of being inessential. If it were it should be omitted altogether. Indeed in the sense in which it is trivial, this is the character of the whole algebra. In the sense in which the algebra is of interest and significance, its having just two values is its most significant and characteristic feature. If he had begun with the statements

Peirce: CP 4.330 Cross-Ref:††

There are at least two expressions not interconvertible, Z and I.

Peirce: CP 4.330 Cross-Ref:††



Every expression, a , is so related to Z, that a [Click here to view] Z may be written alone.

Peirce: CP 4.330 Cross-Ref:††



Every expression, *a*, is so related to I, that I [\[Click here to view\]](#) a may be written alone.

Peirce: CP 4.330 Cross-Ref:††

Every expression, which is such that, taking any expression that is a part of it, the whole could permissibly be written alone whether Z or I were substituted for this part, may be written alone.†2

Peirce: CP 4.331 Cross-Ref:††

IX

ORDINALS†1

331. . . . I pay full homage to Cantor. He is indisputably the *Hauptförderer* of the mathematico-logical doctrine of numbers. As for Dedekind, his little book *Was sind und was sollen die Zahlen?* is most ingenious and excellent. But it proves no difficult theorem that I had not proved or published years before, and my paper †2 had been sent to him. His definition †3 of an infinite collection is precisely my previous definition of a finite collection reversed.†4 His introduction of Gauss's concept of the *Abbild*,†5, which has been spoken of as something quite great, might have been borrowed from my paper, though I made no fuss about it. Since my priority about the distinction of the finite and the infinite has been pointed out in Germany, in a prominent way,†6 Dedekind has said that he had the idea some years earlier. He seems to think this an important circumstance. I may mention that my habit has always been to record ideas that seemed to me valuable in a certain large blank book with the dates at which I set them down, almost always not until I had had the ideas long enough to be quite convinced of their value. This idea about finite and infinite collections was thought worthy of record.†7 But I do not see that it has any interest for anybody but myself; and from Dedekind's conduct, I infer he would prefer I should not give it.

Peirce: CP 4.332 Cross-Ref:††

332. An extremely difficult question about whole numbers is as to which are the more fundamental, ordinal numbers or the cardinal numbers considered as expressing the multitudes of collections. The solution of this problem is contained in the following six propositions, which are all capable of proof.†1

Peirce: CP 4.332 Cross-Ref:††

First. The general idea of *plurality* is involved in the fundamental concept of *Thirdness*, a concept without which there can be no suggestion of such a thing as *logic*, or such a character as *truth*. *Plurality*, therefore, is an idea much more fundamental than that of the ordinal place of a member of a linear series.

Peirce: CP 4.332 Cross-Ref:††

Second. The conception that there is a transitive relation of greater and less among multitudes is logically prior to the conception of ordinal place in a linear series. But *that* relation of greater and less is by no means Bolzano's relation †2 which is at the foundation of the doctrine of multitude. Nothing more in the way of a conception of greater or less collections is involved in the concept of ordinal place than the idea that the greater collection can result from the incorporation into the lesser collection of another collection (using this word in a sense in which a collection may have but a single member).

Peirce: CP 4.332 Cross-Ref:††

Third. Cantor †3 represents the two ways in which a unit may be added to an endless series, namely by incorporation into the series, or by immediately following the endless series, as differing only in respect to the order of performance of the addition. But this is incorrect. The original concept of greater involved in the general concept of ordinal place is that of incorporation into a series. A contradiction is involved in speaking of a unit being incorporated into an endless series after all the members of the series, as well as in speaking of an endless series being incorporated into a finite series. The concept of a unit coming immediately after the endless series is a different concept.

Peirce: CP 4.332 Cross-Ref:††

Fourth. Both the concept of incorporation into a series and of attachment immediately after an endless series are involved in the complete conception of ordinal quantity; but they do not suffice to make it up. They do not even make up the full concept of what Cantor calls a well-ordered series, †4 but which I propose to call a Cantorian series, in order to pay due honor to the completer of the doctrine of ordinal quantity, by attaching his name to this invaluable concept. This concept does not of itself make up the concept of ordinal quantity, but it is its most important ingredient.

Peirce: CP 4.332 Cross-Ref:††

Fifth. Bolzano's concept †1 of being multitudinally *greater than* is in no way involved in the concept of ordinal quantity.

Peirce: CP 4.332 Cross-Ref:††

Sixth. The concept of multitudinal quantity does not involve the concept of ordinal quantity as a system, nor even that of an ordinal quantity; but it does involve every ingredient of the concept of ordinal quantity except the *subjectal abstraction* of it. The logical term subjectal abstraction here requires a word of explanation; for there are few treatises on logic which notice subjectal abstraction under any name, except so far as to confuse it with precise abstraction which is an entirely different logical function. †2 When we say that the Columbia library building is *large*, this remark is a result of precise abstraction by which the man who makes the remark leaves out of account all the other features of his image of the building, and takes [to represent the size] the word "large" which is entirely unlike that image -- and when I say the word is unlike the image, I mean that the general signification of the word is utterly disparate from the image, which involves no predicates at all. Such is *precise abstraction*. But now if this man goes on to remark that the largeness of the building is very impressive, he converts the applicability of that predicate from being a way of thinking about the building to being itself a subject of thought, and that operation is *subjectal abstraction*. Subjectal abstraction is one of the most constantly employed tools of the mathematician. In thinking of the system of multitudinal quantity, we do

not need to think about ordinal quantities, but we do need to attribute, to the objects we are thinking about, ordinal places in a series. The very system of multitudinal quantities themselves consists in their being ordinally arranged.

Peirce: CP 4.333 Cross-Ref:††

333. It is indispensable to my argument about continuity that I should, at this point, give a formal definition of the system of finite numbers, regarded as ordinals. I will give two such definitions. The first is new, and is, I think, the best definition yet given of the finite ordinals, although because of its novelty I find the second handier, which was substantially given by me in 1883.†1 I may mention that in a paper of mine published in 1867,†2 I attempted a definition of the cardinal numbers considered as multitudinal. But although I received some complimentary letters about that paper at the time, it is now utterly unintelligible to me, and is, I trust, by far the worst I ever published. Nevertheless, it is founded upon an interesting idea, worthy of a better development; and the curious contrast between all the operatives of arithmetic when viewed multitudinally and when viewed ordinally is also worth showing.

Peirce: CP 4.333 Cross-Ref:††

In both these definitions, for the sake of simplicity, I speak of 'ordinals' meaning *finite ordinals*, or places in a simple endless series. Also when I speak of a definition of ordinals what I mean is a definition of the system of relations between ordinals.

Peirce: CP 4.334 Cross-Ref:††

334. If α denote a character, then what I should mean by calling it an *ordinary* character would be that it would be absurd to say of two imaginary objects, M and N, that M possesses α while N does not possess it, but that in respect to all other characters -- or in respect to all other characters of a given line of characters -- M and N do not differ. For ordinary characters so blend into one another that no one can be singled out. It is not in their nature. By a *singular* character on the other hand, I mean a character which differs decidedly from every other however nearly like it. A branch character is a singular character which belongs to a certain collection of characters which I call a *system of branch characters*, and the characteristic property of such a system is as follows:

Peirce: CP 4.334 Cross-Ref:††

Let α and β be any two different characters of the system whatsoever, then of the three propositions:

P possesses both α and β

Q possesses α but not β

R possesses β but not α

one or other is contrary to the nature of things, even of imaginary things, while two of the three propositions (as the proposition "S possesses neither α nor β ") are possible as far as the nature of the characters go. Understand then, that if of a system of singular characters it is possible to find among them two such (say *beauty* and *virtue*) that all that triad of propositions might be true of different imaginary objects, that system of characters is *not* a system of branch characters. Again if a system of singular characters be such that it is possible to find any two of them (like being a prime number and exactly dividing the number next greater than the continued

product of all smaller integers) so related that two of those three forms of proposition would be contrary to the nature of the characters, *that* system is not a system of branch characters. Once more if a system of singular characters be such that it is possible to find among them any two of such a nature that both could not be absent from any object, *that* system is not a system of branch characters. Finally, if it be possible to find in a system a single ordinary character, it is not a system of branch characters. But if none of these four things be possible it is a system of branch characters.

Peirce: CP 4.334 Cross-Ref:††

The reason I call these branch characters is that a collection of characters each of which consists in being on some one branch of a tree forms for small objects like insects, as subjects, a system of branch characters, for if the two branches are separate nothing can be on both, but a thing may be on either and not on the other, or it may be on the trunk and so not on either. The only case in which a thing can be on two branches at once is when one branch is itself on the other, so that to be on the former is *ipso facto* to be on the latter. But then it cannot be on the former and not on the latter, though it may be on the latter and not on the former.

Peirce: CP 4.335 Cross-Ref:††

335. *First Definition of Ordinals*

Peirce: CP 4.335 Cross-Ref:††

Clause 1. Of any two non-identical ordinals, there is a branch character of a certain system of branch characters (here to be designated simply as 'the system') which one of them possesses while the other lacks it.

Peirce: CP 4.335 Cross-Ref:††

Clause 2. The branch characters of the system are singular.

Peirce: CP 4.335 Cross-Ref:††

Clause 3. It would be logically possible for an object susceptible of any branch character of the system, and actually having one of them, to change to one other, and to change from every character, to which it had been changed, to one branch character of the system to which it had never been changed, without ever being restored to its first character.

Peirce: CP 4.335 Cross-Ref:††

Clause 4. On the other hand, this would be logically impossible if the changes were restricted to those branch-characters of the system that are possessed by any one ordinal.

Peirce: CP 4.335 Cross-Ref:††

Clause 5. (From clauses 3 and 4 it follows that every branch character of the system has at least one other immediately dependent upon it; that is, dependent upon it, but not dependent on a third that is dependent upon it), but no branch character of the system has more than one immediately dependent upon it.

Peirce: CP 4.335 Cross-Ref:††

Clause 6. Every combination of possession and nonpossession of branch characters of the system which is logically consistent with clauses 1 to 5 inclusive and with the definitions of branch characters, dependence, etc., is actually realized in some ordinal; (and that ordinal which possesses none of the branch characters of the system is called zero).

Peirce: CP 4.336 Cross-Ref:††

336. The statement of the second definition will be facilitated by the explanations of three peculiar locutions. In order to abbreviate oft-recurring phrases like 'A stands in the relation r to B,' we may say indifferently either 'A is r to B' or 'B is r 'd by A.' We may say that a relation, r , is an *appurtenance*, if, and only if, it is out of the nature of things for anything to be r to two different correlates. We may call a relation, r , a *comparative fulfillment*, if, and only if, to say that A is r to B is the same as to say that A is not r 'd by anything that r 's B, or nothing is r to A that is not r to B. Thus, to say that 'John' is as short a word as 'Jack' is the same as to say that nothing is as short as 'John' that is not as short as 'Jack.'

Second Definition of Ordinals

Clause I. The most fundamental relation of ordinals, N, ('next after'), is an appurtenance.

Peirce: CP 4.336 Cross-Ref:††

Clause II. Every ordinal is N'd by an ordinal.

Peirce: CP 4.336 Cross-Ref:††

Clause III. There is an ordinal, zero, or 0, that is not N to any ordinal.

Peirce: CP 4.336 Cross-Ref:††

Clause IV. The relation g (as high as) is a comparative fulfillment.

Peirce: CP 4.336 Cross-Ref:††

Clause V. Whatever ordinal is N of an ordinal is g of that ordinal.

Peirce: CP 4.336 Cross-Ref:††

Clause VI. Whatever ordinals p and q may be, either the facts that certain ordinals are N to certain ordinals, taken in conjunction with the preceding clauses of this definition, logically necessitate p 's being g to q , or else p is not g to q .

Peirce: CP 4.336 Cross-Ref:††

Clause VII. Whatever ordinals p and q may be, either p is g to q or q is g to p .

Peirce: CP 4.336 Cross-Ref:††

If we translate either definition into the terms of the other all its clauses may be deduced from those of the other. Thus, the branch characters of the system are the characters of being N to an ordinal that is g to a , where a is any one ordinal, constant for any one branch character but varying with the branch character. On the other hand, to say that n is g to m is the same as to say that n possesses every branch character of m , while to say that n is N to m is to say that n possesses a branch character, say $\{n\}$, not possessed by m , while whatever branch character that is not $\{n\}$ and is not possessed by m , $\{x\}$ may be, n does not possess $\{x\}$. The demonstration of each definition by the other will be found an instructive exercise, but it need not be worked out for our purposes. A person who wishes to try it should begin by proving by the second definition that no ordinal is N to itself (for in Clause II it is not said that every ordinal is N'd by *another*), and that no two different ordinals are N to the same ordinal (for this is not implied in any single clause of the definition).

Peirce: CP 4.337 Cross-Ref:††

337. All that it is necessary to insist upon here is that the only thing that whole

numbers can express is the relative place of objects in a simple, discrete, linear series; and whole numbers are applicable to enumerable multitudes and enumerable collections, only because it happens that those multitudes have each its place in a simple, discrete, linear series. It is true that Dr. Georg Cantor, the great founder and *Hauptförderer* of the logico-mathematical doctrine of numbers, begins his exposition with what he calls "cardinal numbers," †1 but which ought properly to be called *multitudes*. For cardinal numbers proper are nothing but the vocables of a certain series of vocables that are used in the operation of ascertaining the multitude of a collection, by counting, and thence are applied as appellatives of collections to signify their multitudes. Multitude itself, however, belongs to various different collections in various different grades, where cardinal number has no application, at all. Cantor, †1 however, has partially shown, what is entirely true, that the whole doctrine of multitude can be developed without any reference to ordinal numbers. But in treating of ordinals we are obliged to say, *in substance*, what their multitude is. Thus, when we look at the matter from a certain point of view, it seems that the doctrine of multitude is more fundamental than that of ordinals, and that all whole numbers really express multitudes. But this is a logical fallacy. That the concepts of *multitude* and of *ordinal place in a simple, discrete, linear series* are very intimately connected is true. The latter involves the consideration of facts constituting the applicability of definite conceptions of multitude; but it does not involve these conceptions themselves. Multitude, on the other hand, is nothing but the place of a series in one or the other of two simple, discrete, linear series, and it is impossible to define it at all without the use of the ordinal conception itself.

Peirce: CP 4.337 Cross-Ref: ††

That positive whole numbers can express nothing but places in a linear series is proved by the fact that from either of the definitions above of ordinals, neither of which involves any concept not involved in the concept of such a series, any property of whole numbers can be deduced. If the statement of the property involves the triadic relation of being 'sum of' or being 'product of' of course this relation must first be defined. In case the first definition is used, N being defined in terms of branch characters as above, and in case the second definition is used, without that definition of N , the definition of *sum* is as follows:

Peirce: CP 4.337 Cross-Ref: ††

Any ordinal, s , is a *sum* of an ordinal m as summand, and an ordinal n as addend, if, and only if, either m is N to an ordinal, l , and s is N to an ordinal that is a sum of l and n , or m is not N to any ordinal, and s is n .

Peirce: CP 4.337 Cross-Ref: ††

A product may be defined thus: An ordinal, p , is a *product* of an ordinal as *multiplier* and of an ordinal as *multiplicand*, if, and only if, either the multiplier, m , is N to an ordinal, l , and p is the sum of the multiplicand, n , and of an ordinal that is a product of l and n , or else m is not N to any ordinal, and p is m .

Peirce: CP 4.338 Cross-Ref: ††

338. Many a person who will readily admit that whole numbers can express nothing but places in a linear series is inclined to insist that with fractions it is otherwise, fractions essentially involving the idea of equality of measure among the parts of a whole. Indeed, more than one highly esteemed writer might be named who has emphasized this as an essential characteristic of fractions, and in support of the assertion has averred that fractions cannot be added or subtracted until they have been

reduced to a common denominator, and indeed that until that is done one cannot always tell which of two fractions is the larger. It thus becomes necessary to enter upon a proof that what is true of whole numbers is equally true of fractions; namely, that they can express nothing but relative places in a linear series; and this shall be done by defining first the system of rationals, or rational quantities, and then the system of fractional expressions, without any reference to measure, purely in terms of the relations of linear series, and in showing that from these definitions all properties of rationals and of fractions can be logically deduced.

Peirce: CP 4.339 Cross-Ref:††

339. In order to do this it becomes convenient, and indeed little short of indispensable, to make use of the secundal system of numerical notation, which may be familiarly described as a system exactly like the Arabic notation, except that Two is taken as the base of numeration instead of Ten. It may be formally defined as follows:

1. There is a collection of objects called *secundal places* (not *places of secundals*, which are used only in fractional expressions), and this one collection of places is the same for all secundal numerical expressions.

2. Every secundal place is in a certain relation to an ordinal called *being designated* by that ordinal, and is designated by no other ordinal.

3. Every ordinal designates a secundal place, and designates no other secundal place.

4. Every secundal integer expression denotes an ordinal and denotes no other ordinal.

5. Every ordinal is denoted by a secundal integer expression and is denoted by no other secundal integer expression.

6. Every secundal integer expression is a system of perceptual objects, called *figures*, each having perceptual characters in itself, and each having a perceptual relation to each secundal place.

7. In every secundal integer expression each figure has a perceptual relation to a secundal place, called *being in* that place, and is in no other place.

8. In every secundal integer expression, every secundal place has a figure in it and has no other figure in it.

9. In every secundal integer expression, the figure in each secundal place is perceptually distinguishable as having a certain character, called being a *unit*, or as not having that character, when it is called a *blank*. . . .

10. In every secundal integer expression, there is a secundal place, such that in every secundal place that is *g* to it there is a blank;

11. Every secundal expression denoting an ordinal *n* that is N to an ordinal, *m*, is such that:

Peirce: CP 4.339 Cross-Ref:††

Firstly, some figure of it is a blank.

Peirce: CP 4.339 Cross-Ref:††

Secondly, in that secundal place, to which every place that is g contains a blank in the secundal expression for m , in the secundal expression for n there is a unit.

Peirce: CP 4.339 Cross-Ref:††

Thirdly, in every secundal place to which that place is $N[g]$ there is a blank in the expression for n instead of the unit there in the expression for m .

Peirce: CP 4.339 Cross-Ref:††

Fourthly, in every secundal place that is $N[g]$ to that place the figure in the expression for n has the same character as the figure in the same place in the expression for m .

Peirce: CP 4.339 Cross-Ref:††

It is usual to write those blanks to whose places the place of some unit is $N[g]$ as 0, leaving those of which this is not true unwritten. The figure in the 0 place is best drawn with heavy lines.

Peirce: CP 4.340 Cross-Ref:††

340. The system of rationals, or positive rational quantities, may now be defined as follows:

1. Every rational is in a certain triadic relation to an ordinal called its *antecedent*, and to an ordinal called its *consequent*, namely, the relation of being the *ratio* (for the sake of brevity, I omit the qualification *in lowest terms*) of the antecedent *to* the consequent; but is the ratio (in lowest terms, always) of no antecedent to any other consequent, nor of any other antecedent to any consequent. Nor is any other rational a ratio of the same antecedent to the same consequent. But not every two ordinals are the antecedent and consequent of a rational (in lowest terms).

2. Every ordinal is a rational, being the ratio of itself as antecedent, to NO (the ordinal that is N to zero, which in the secundal notation is written 1).

3. The relation g may be taken in a generalized sense, so as to be applicable to all rationals. Writing g' for this more general relation, every ordinal that is g to an ordinal, is also g' to the same ordinal, and if any rational be g' to an ordinal that is g to an ordinal, the first rational is g to the latter ordinal, and if an ordinal be g to an ordinal that is g' to a rational, the former ordinal is g' to the last rational.

4. If the converse of the negative of g' be called $\{g\}$ (being greater than) every rational stands to any non-identical rational either in the relation of being $\{g\}$ to it or in that of being $\{g\}$ 'd by it.

5. There is a peculiar relation, to be here called *being indicated by* or *having as indicator*, in which every rational stands to a secundal integer expression, and to nothing else.

6. Every secundal integer expression is indicator of a rational, but of no other rational.

7. If from the indicator of any rational, on the one hand, the figure in the zero secundal place be struck off and the result be called the near subindicator, and on the other hand all those figures be struck off which are in places $g'd$ by the place of that figure unlike the figure in the zero place whose place is $g'd$ by all the figures unlike

Peirce: CP 4.340 Cross-Ref:††
 Proceed thus

$$\text{Proceed thus} \quad \frac{487}{830} \frac{301}{513}$$

$$\frac{186}{317} \frac{487}{830} \frac{301}{513}$$

$$\frac{1}{0} \frac{1}{1} \frac{2}{3} \frac{3}{5} \frac{10}{17} \frac{27}{46} \frac{71}{121} \frac{186}{317} \frac{487}{830} \frac{301}{513} \frac{115}{196} \frac{44}{75} \frac{17}{29} \frac{7}{12} \frac{4}{7} \frac{1}{2} \frac{0}{1} \quad [\text{Click here to view}]$$

Peirce: CP 4.340 Cross-Ref:††
 This series everywhere increases in the same direction. Hence $487/830 > 301/513$. Fractions can be added on the same principle.

Peirce: CP 4.341 Cross-Ref:††
 X

***ANALYSIS OF SOME DEMONSTRATIONS
 CONCERNING DEFINITE POSITIVE INTEGERS†1P***

341. Let the Universe of 1. c. italics be the aggregate of all definite Integers not negative. Let the Universe of Greek Minuscles be the aggregate of possible characters of such Integers. Let $q[\alpha u]$ mean, as in 3.398, that the Integer, u , has the character, α .

Hypotheses

(1) $\pi[\alpha] \Sigma[\beta] \pi[u] (q[\alpha u] \Psi[q[\beta u)] \cdot (\sim q[\alpha u] \Psi \sim q[\beta u]))$ *i.e.*, every character has a negative.

(2) $\pi[\alpha] \pi[\beta] \Sigma[\{g\}] \pi[u] (\sim q[\alpha u] \Psi [\sim q[\beta u] \Psi q[\{g\}u]) \cdot (q[\alpha u] \cdot q[\beta u] \Psi \sim q[\{g\}u])$ *i.e.*, of every two possibilities there is a compounded possibility. Instead of introducing an unanalyzed relation of 'as small as,' let us, at first, conceive a character of characters, consisting in each of certain characters belonging to every integer lower than an integer to which it belongs.

(3) $\pi[u] \Sigma[\alpha] q[\alpha u] \cdot s[\alpha]$, †2 every Integer has a character common to all lower Integers. (A formal proposition.)

(4) $\pi[a] \Sigma[u] \sim s[\alpha] \Psi \sim q[\alpha u]$ *i.e.*, there is no highest Integer.

(5) $\Sigma[u] \pi[\alpha] \pi[v] q[\alpha u] \Psi \sim s[\alpha] \Psi \sim q[\alpha v]$ *i.e.*, there is a lowest Integer (*i.e.*, Zero).

(6) $\pi[\alpha]\pi[\beta]\pi[u]\pi[v] \sim s[\alpha] \cdot \sim s[\beta] \cdot \sim q[\alpha u]q[\beta u] \cdot \sim q[\alpha v] \cdot \sim q[\beta v]$ *i.e.*, an Integer having any s-character that another has not has every one that other has.

(7) $\pi[u]\pi[v]\Sigma[\alpha]\pi[\beta] s[\alpha] \cdot (q[\alpha u] \cdot \sim q[\alpha v]) \cdot (\sim q[\alpha u] \cdot \sim q[\alpha v]) \cdot \sim q[\beta u] \cdot q[\beta v] \cdot \sim q[\beta u] \cdot \sim q[\beta v]$ *i.e.*, unless one of any two integers has an s-character which the other has not, they are alike in **all** characters, and therefore, being definite, are identical.

(8) $\pi[u]\Sigma[\alpha]\Sigma[v]\pi[w]\pi[\beta]\pi[\{g\}] q[\alpha u] \cdot s[\alpha] \cdot \sim q[\alpha v] \cdot (\sim s[\beta] \cdot \sim q[\beta u] \cdot \sim q[\beta w] \cdot \sim s[\{g\}] \cdot \sim q[\{g\}w] \cdot \sim q[\{g\}v])$ *i.e.*, every integer has another next higher. This is a consequence of the following with (4):

(9) $\pi[\alpha]\pi[u]\Sigma[v]\pi[w]\pi[\beta] \sim q[\alpha u] \cdot \sim q[\alpha v] \cdot (\sim q[\alpha w] \cdot \sim s[\beta] \cdot \sim q[\beta w] \cdot \sim q[\beta v])$ *i.e.*, every class of integers has a lowest member.

Peirce: CP 4.341 Cross-Ref: ††

For formula (10) see below [343].

Addition

Peirce: CP 4.341 Cross-Ref: ††

Let $(i+j)[k]$ mean that the integer k can result from adding the integer i to the integer j . This can be negated by an obelus over it like any other expression.

Peirce: CP 4.341 Cross-Ref: ††

Addition is definable by the following six formulæ:

$$(11) \pi[i]\pi[j]\Sigma[k](i+j)[k]$$

$$(12) \pi[k]\pi[j]\Sigma[i]\Sigma[\alpha] s[\alpha] \cdot q[\alpha j] \cdot \sim q[\alpha k] \cdot \sim (i+j)[k]$$

$$(13) \pi[i]\pi[k]\Sigma[j]\Sigma[\alpha] s[\alpha] \cdot q[\alpha i] \cdot \sim q[\alpha k] \cdot \sim (i+j)[k].$$

$$(14) \pi[i]\pi[j]\pi[k]\pi[u]\pi[v]\pi[w]\Sigma[\alpha]\pi[\beta] \sim (i+j)[k] \cdot \sim (u+v)[w] \cdot \sim s[\alpha] \cdot (q[\alpha u] \cdot \sim q[\alpha i] \cdot \sim q[\alpha v] \cdot \sim q[\alpha j]) \cdot \sim s[\beta] \cdot \sim q[\beta w] \cdot \sim q[\beta k]$$

$$(15) \pi[i]\pi[j]\pi[k]\pi[u]\pi[v]\pi[w]\Sigma[\alpha]\pi[\beta] \sim (i+j)[k] \cdot \sim (u+v)[w] \cdot \sim s[\alpha] \cdot (q[\alpha v] \cdot \sim q[\alpha j] \cdot \sim q[\alpha w] \cdot \sim q[\alpha k]) \cdot \sim s[\beta] \cdot \sim q[\beta u] \cdot \sim q[\beta i]$$

$$(16) \pi[i]\pi[j]\pi[k]\pi[u]\pi[v]\pi[w]\Sigma[\alpha]\pi[\beta] \sim (i+j)[k] \cdot \sim (u+v)[w] \cdot \sim s[\alpha] \cdot (q[\alpha u] \cdot \sim q[\alpha i] \cdot \sim q[\alpha w] \cdot \sim q[\alpha k]) \cdot \sim s[\beta] \cdot \sim q[\beta v] \cdot \sim q[\beta j]$$

Peirce: CP 4.342 Cross-Ref: ††

342. It would be illuminating to exhibit the above fifteen propositions scribed in existential graphs;†1 but it would be aside from my present purpose. I proceed to indicate sketchily in what manner the leading theorems concerning the addition of positive integers can be deduced from the fifteen propositions by means of the rules given in 3.396. (Though those rules might now be amended much, so as to render them more efficient.) If (14) be iterated, it becomes

$$\pi[i]\pi[j]\pi[k]\pi[u]\pi[v]\pi[w]\Sigma[\alpha]\pi[i']\pi[j']\pi[k']\pi[u']\pi[v']\pi[w']\Sigma[\alpha']\pi[\beta] \pi[\beta'] \{ \sim (i+j)[k] \cdot \sim (u+v)[w] \cdot \sim s[\alpha] \cdot (q[\alpha u] \cdot \sim q[\alpha i] \cdot \sim q[\alpha v] \cdot \sim q[\alpha j]) \cdot \sim s[\beta] \cdot \sim q[\beta w] \cdot \sim q[\beta k] \} \cdot \{ \sim (i'+j')[k'] \cdot \sim (u'+v')[w'] \cdot \sim s[\alpha'] \cdot (q[\alpha' u'] \cdot \sim q[\alpha' i'] \cdot \sim q[\alpha' v'] \cdot \sim q[\alpha' j']) \cdot \sim s[\beta'] \cdot \sim q[\beta' w'] \cdot \sim q[\beta' k'] \}$$

$\sim q[\beta'w'] \cdot q[\beta'k']$.

Peirce: CP 4.342 Cross-Ref:††

Next (I go into detail with this first example farther than I shall with others), we may, by the fifth rule, identify u, i' , and u' , with $i; v, j'$, and v' with $j; k'$ with $w; w'$ with k ; and β' with β (for though the rule as given in the memoir is the right one, theoretically, yet in practice the operation of this and part of the sixth can generally be reduced with convenience to the identification of the index of any π with any index to the left of it in the quantifier). We, at the same time, apply Rule 6 somewhat, remembering that $q[\alpha i] \cdot \sim q[\alpha i] \approx 0$ etc. and applying the principle $(A \cdot B) \cdot (A \cdot C) = A \cdot B \cdot C$, and then applying Rule 7 we get

$\pi[i]\pi[j]\pi[k]\pi[w]\pi[\beta]\sim(i+j)[k] \cdot \sim(i+j)[w] \cdot \sim s[\beta] \cdot (\sim q\beta w \cdot q[\beta k]) \cdot (\sim q[\beta k] \cdot q[\beta w])$
or $\pi[i]\pi[j]\pi[k]\pi[w]\pi[\beta]\sim(i+j)[k] \cdot \sim(i+j)[w] \cdot \sim s[\beta] \cdot \sim q\beta w \cdot \sim q[\beta k] \cdot q[\beta w] \cdot q[\beta k]$.

Peirce: CP 4.342 Cross-Ref:††

Let us now compound this with (7) in which, to avoid confusion, we may write m for u, n for $v, \{y\}$ for α , and ϕ for β . We thus get

$\pi[m]\pi[n]\Sigma[\{y\}]\pi[\phi]\pi[i]\pi[j]\pi[k]\pi[w]\pi[\beta] \{ \sim(i+j)[k] \cdot \sim(i+j)[w] \cdot \sim s[\beta] \cdot \sim q\beta w \cdot \sim q[\beta k] \} \cdot q[\beta w] \cdot q[\beta k] \} \{ s[\{y\}] \cdot (q[\{y\}m] \cdot q[\{y\}n]) \cdot (\sim q[\{y\}m] \cdot \sim q[\{y\}n]) \} \cdot q[\phi m] \cdot q[\phi n] \cdot \sim q[\phi m] \cdot \sim q[\phi n] \}$.

Now identifying β with $\{y\}$, w with m, k with u , the formula with an obvious reduction of the Boolean, becomes

(17) $\pi[m]\pi[n]\pi[\phi]\pi[i]\pi[j] \sim(i+j)[n] \cdot \sim(i+j)[m] \cdot q[\phi m] \cdot q[\phi n] \cdot \sim q[\phi m] \cdot \sim q[\phi n]$ *i.e.*, if $(i+j)[m]$ and $(i+j)[n]$, then $m = n$; or the sum of two definite positive integers has but a single value.

Peirce: CP 4.342 Cross-Ref:††

Without writing down the formulae, a little close attention will enable one to convince himself that (15) and (16), treated almost exactly as (14) has been above, show that if

(18) $(i+j)[k]$ and $(u+j)[k]$ then $u = i$ and that

(19) if $(i+j)[k]$ and $(i+v)[k]$ then $j = v$.

Peirce: CP 4.343 Cross-Ref:††

343. **Abbreviations.** Having thus illustrated how the notation works, it will be well to introduce some abbreviations.

First, although obviously indefinite individuals may be alike in respect to every character, yet different in their (real or pretended) brute existence, such as the different parts of space and the different vertices of the regular dodecahedron of pure mathematics, still since the Universe of *l. c. italics* is confined to definite integers, we may, by introducing $I[ij]$ to mean that i and j are the same individual, write the following principle:

(10) $\pi[u]\pi[v]\Sigma[\alpha]s[\alpha] \cdot (q[\alpha u] \cdot q[\alpha v]) \cdot (\sim q[\alpha u] \cdot \sim q[\alpha v]) \Psi I[u\alpha]$.

Peirce: CP 4.343 Cross-Ref:††

Of course, the negative of $I[ij]$ will be $T[ij]$.

Peirce: CP 4.344 Cross-Ref:††

344. One may entertain the theory that all vagueness is due to a defect of cogitation or cognition. It is a natural kind of nominalism the justice of which it would be remote from the purpose of this analysis to consider. The vagueness of characters is of different kinds. The quality of redness and the quality of blueness differ without differing in any essential character which one has but the other lacks. The otherness of them is as irrational as the qualities themselves, if not more so. It appears to consist in a mutual war between them, in our taste. But the characters of integers are not of this irrational kind. In another regard, however, they are vague. Thus we say that the two characters of 4, of being the sum of 2 and 2, and of being the product of ~ and 2, are different characters, so that we cannot, in imitation of (10), write

$$\pi[\alpha]\pi[\alpha]\Sigma[n](q[\alpha n]\Psi q[\beta n])\cdot(\sim q[\alpha n]\Psi \sim q[\beta n])\Psi I[\alpha\beta]$$

Peirce: CP 4.344 Cross-Ref:††

This is [because] we do not think out the meaning of 2+2 and 2X2 to the very bottom. In this respect, the objects we denote by Greek minuscules are not generally definite.

Peirce: CP 4.345 Cross-Ref:††

345. The character, **I**, which I introduced in 1882, when I was teaching logic in the Johns Hopkins University, was in my mind one of a class of notations which I left unmentioned in order that some one of my pupils might have the pleasure of finding it out for himself; but as nobody has, so far as I have noticed, in the three-fourths of a generation that has elapsed, I will give some illustrations of the class:

I[ij] means j is a member of the *singlet*, i.

2[ij] means j is a member of the *doublet*, i, or unordered pair, or *couple*.

3[ij] means j is a member of the *triplet*, i, or unordered trio, or leash.

4[ij] means j is a member of the *quadruplet*, i, or unordered collection of 4.

9[ij] means j is a member of the *nonuplet*, i, or unordered collection of 9.

x[ij] means j is a member of the *decuplet*, i, or unordered collection of 10.

Ordered collections I call, *medads* (0), *monads*, *dyads*, *triads*, etc. Indeterminate as to being ordered are *binion* (or pair), *trine*, *quaternion*, *quine*, *senion*, *septene*, *octone*, *novene*, *dene* (or *denion*), etc.

Peirce: CP 4.345 Cross-Ref:††

By an ordered collection, I mean one of which each member has a peculiar relation to the whole; as for example, if one is definitely the first, another definitely the second, a definite one the third, etc., or if there is any other formal relation by which each is different from all the others. There are also *diduct* collections which are formally divided into subcollections and it may be in more than one way, whether inadequately, adequately, or superfluously. By *adequately*, I mean just sufficiently to make the collection an ordered one.

Peirce: CP 4.345 Cross-Ref:††

With this notation (7) can be expressed as follows, using Hebrew letters to denote definite collections:

$$\pi[\aleph]\Sigma[i]\Sigma[j]\Sigma[\alpha] \ 2[\aleph i]\cdot 2[\aleph j]\cdot s[\alpha]\cdot q[\alpha i]\cdot \sim q[\alpha j]$$

Peirce: CP 4.345 Cross-Ref:††

The utility of the symbols 1, 2, 3, etc. is increased by employing them as follows: I[i], I[ij], I[ijk], etc. means that the indices denote the same existing individual.

2[\aleph i], 2[\aleph ij], 2[\aleph ijk], etc. mean that the individuals denoted by the indices belong to the doublet \aleph .

2[ijk], 2[ijkl], etc. mean that all the individuals denoted by the indices are members of one doublet.

(2·T)[ij], (2·T)[ijk], etc., mean that the individuals denoted by the indices belong to one doublet but are not all one individual.

3[\aleph i], 3[\aleph ij], 3[\aleph ijk], 3[\aleph ijkl], mean that i, j, etc. all belong to the triplet \aleph .

3[ijkl], 3[ijklm], etc. mean that i, j, etc. all belong to one triplet.

(3·T)[ijkl], means that i, j, k, l all belong to one triplet but are not all identical.

(3~2)[ijkl] means that i, j, k, l are three different existing individuals.

(32)[ijklmnopq] (where note the absence of a dot -- not 3·2, but 32) means that the individuals indicated are all members of a triplet of doublets.

(3 Ψ 2)[ijklmno] means that every individual denoted by an index is either a member of a triplet or of a doublet.

Peirce: CP 4.345 Cross-Ref:††

I would use a special form of parenthesis (I will not recommend any particular form as more appropriate than another) which I would use in the following way:

$\pi[i][\odot\Psi I][i]; \mu\epsilon\alpha\nu\sigma \ \alpha\nu\psi \ \omicron\beta\phi\epsilon\chi\tau \ \omega\eta\iota\chi\eta \ \iota\sigma \ \sigma\upsilon\nu \ \iota\sigma, \ \alpha\sigma \ \sigma\upsilon\chi\eta, \ \tau\eta\epsilon \ \mu\epsilon\mu\beta\epsilon\rho \ \omicron\phi \ \alpha \ \sigma\iota\nu\gamma\lambda\epsilon\tau, \ \text{i.e.,} \ \pi[i]\pi[j]\odot[i]\Psi\odot[j]\Psi I[i\phi].$

Peirce: CP 4.345 Cross-Ref:††

If \int^+ means is a satellite of Jupiter, then

$\pi[i][\sim\int^+\Psi 5][i];$ means that whatever is a satellite of Jupiter is, as such, a member of a quintuplet, *i.e.*,

$$\Sigma[i]\Sigma[j]\Sigma[k]\Sigma[l]\Sigma[m]\pi[n] \ \int^+[i]\cdot \int^+[j]\cdot \int^+[k]\cdot \int^+[l]\cdot \int^+[m]\cdot T[ij]\cdot T[ik]\cdot T[il]\cdot T[im]\cdot T[jk]\cdot T[jl]\cdot T[jm]\cdot T[kl]\cdot T[km]\cdot T[lm]\cdot (\sim\int^+[n]\Psi I[\iota\nu]\Psi I[\phi\nu]\Psi I[\kappa\nu]\Psi I[\lambda\nu]\Psi I[\mu\nu]).$$

Peirce: CP 4.345 Cross-Ref:††

The saving here is enormous.

Peirce: CP 4.345 Cross-Ref:††

Intimately connected with these abbreviations are others, some of which I have mentioned elsewhere. The rules of their application would form an elaborate logical doctrine, which I have not time to develop, because I am working at more fundamental parts of logic. Whoever undertakes it in the light of what I have said here and elsewhere will have other symbols forced upon his attention.

Peirce: CP 4.345 Cross-Ref:††

I pass to another and very simple abbreviation, which consists in using the symbol σ so that $\sigma[ij]$ shall mean that j is at least as low an integer as i . That is,

$$(20) \sigma[ij] = \pi[\alpha] \sim s[\alpha] \cdot \Psi \sim q[\alpha i] q[\alpha j] \quad \sim \sigma[ij] = \Sigma[\alpha] s[\alpha] \cdot q[\alpha i] \sim q[\alpha j]$$

Peirce: CP 4.345 Cross-Ref:††

It immediately follows that

$$(21) \pi[i] \sigma[ii] \text{ and}$$

$$(22) \pi[i] \pi[j] \pi[k] \sim \sigma[ij] \cdot \Psi \sim \sigma[jk] \Psi \sigma[ik]$$

Peirce: CP 4.345 Cross-Ref:††

From (20) and (10) it follows that

$$(23) \pi[i] \pi[j] \pi[k] \sigma[ij] \Psi \sigma[jk] \cdot \Psi \sim \sigma[ik]$$

Hypothesis (5) represents that there is an integer as low as any and by (10) this is lower than any other. We may therefore give it the proper name, o , which will possess the singularity of being definable. Thus

$$(24) \pi[i] \Pi[o] \cdot \Psi \sim \sigma[oi] \quad \pi[i] \sigma[io]$$

a definition which is also singular in being in a single proposition. But this is owing to (10).

Peirce: CP 4.345 Cross-Ref:††

The long formula (8) requires abbreviation; and we may write

$$(25) H[uv] = \Sigma[\alpha] \pi[\beta] \pi[\{g\}] \pi[w] s[\alpha] \cdot q[\alpha u] \cdot \sim q[\alpha v] \cdot (\sim s[\beta] \cdot \Psi \sim q[\beta u] \cdot \Psi q[\beta w] \cdot \Psi \sim s[\{g\}] \cdot \Psi q[\beta v] \cdot \Psi \sim q[\beta w]) = \pi[w] \sim \sigma[uv] \cdot (\sigma[uw] \Psi \sigma[wv])$$

We may further take the index 1 as such a proper name that

$$(26) H[01]. \text{ I will also write } r[Hi] \text{ for } \Sigma[j] r[j] \cdot H[ij] \text{ and } (H[i] + H[j])[k] \text{ for } \Sigma[u] H[iu] \cdot \Sigma[v] H[vj] \cdot (u+v)[k]$$

Peirce: CP 4.345 Cross-Ref:††

Formulæ (14)-(16) may be put in the form

$$(14) \pi[i] \pi[j] \pi[k] \pi[u] \pi[v] \pi[w] \sim \sigma[ui] \cdot \Psi \sim \sigma[vj] \cdot \Psi \sim (i+j)[k] \cdot \Psi \sim (u+v)[w] \Psi \sigma[wk]$$

$$(15) \pi[i] \pi[j] \pi[k] \pi[u] \pi[v] \pi[w] \sim \sigma[wk] \cdot \Psi \sim \sigma[vj] \cdot \Psi \sim (i+j)[k] \cdot \Psi \sim (u+v)[w] \Psi \sigma[ui]$$

$$(16) \pi[i] \pi[j] \pi[k] \pi[u] \pi[v] \pi[w] \sim \sigma[ui] \cdot \Psi \sim \sigma[wk] \cdot \Psi \sim (i+j)[k] \cdot \Psi \sim (u+v)[w] \Psi \sigma[vj]$$

Peirce: CP 4.345 Cross-Ref:††

Putting, in (14), o for i , j , and w , it becomes

$$\pi[k]\pi[u]\pi[v](o+o)[k]\Psi(u+v)[o]\Psi\sim\sigma[uo]\Psi\sim\sigma[vo]\Psi\sigma[ok]$$

Peirce: CP 4.345 Cross-Ref:††

Multiplying this by the third power of (24), *i.e.*, (24)·(24)·(24), we get

$$(27)(o+o)[o]$$

(9) may be put in the form

$$(9')\pi[\alpha]\pi[u]\Sigma[v]\pi[w] \sim q[\alpha u]\Psi q[\alpha v]\cdot(\sim q[\alpha w]\Psi\sigma[wv])$$

Putting for $q[\alpha]$ the expression $\sim\sigma[u]$, this becomes

$$(28)\pi[u]\Sigma[v]\pi[w] \sim\sigma[uv]\cdot(\sigma[uw]\Psi\sigma[wv])$$

Peirce: CP 4.346 Cross-Ref:††

346. I will now return to addition. I will remark, by the way (for I do not make this paper at all systematic), that Schröder's notation

Σ and π

$u \quad \{y\}$

and the like, which is his chief modification of my two logical algebras (which, by the way, can perfectly well be mixed), made long after my second intentional section of the paper, No. XIII in vol. 3, has several advantages over mine, both theoretical and practical, and ought to be employed freely. But it fails to do what my invention was made in order to do, namely, to enable us to perform the operation of hypostatic abstraction, and freely make use of *entia rationis*. But that is neither here nor there.

Peirce: CP 4.346 Cross-Ref:††

I will start with (14) in its last form and will trace out the steps of the algebraical transformation in closer detail than I purpose generally to do in this paper. For the inference I am coming to employs the **Rule of Diduction**, or diversification, which I fully treated of in a paper I drew up in Grammercy Park in 1885†1, but the dignity of science does not permit me to go begging to have its results printed. This paper I am writing will probably never be seen by other eyes than those that see it written; but I record this for my own gratification. The rule is that after any quantifier of the *Peircean* (whether it be π or Σ) can be inserted a Σ with a new index, into which the preceding index can be transmuted in any of the places where it occurs, remaining untransmuted in the other places. Thus $\pi[i]l[ii]$ everybody loves himself, can be changed to $\pi[i]\Sigma[j]l[ij]$, everybody loves somebody. Identifying v with j , in (14) we get

$$\pi[i]\pi[j]\pi[k]\pi[w]\pi[u]\sim(i+j)[k]\Psi\sigma[wk]\Psi\sim(u+j)[w]\Psi\sim\sigma[ui]$$

Let us insert the aggregant $\sim q[\beta u]$:

$$\pi[i]\pi[j]\pi[k]\pi[w]\pi[\beta]\pi[u]\sim(i+j)[k]\Psi\sigma[wk]\Psi\sim q[\beta u]\Psi\sim(u+j)[w]\Psi\sim\sigma[ui]$$

The insertion of this aggregant authorizes the insertion of its negative as component of another aggregant.

$$\pi[i]\pi[j]\pi[k]\pi[w]\pi[\beta]\pi[u]\sim(i+j)[k]\Psi\sigma[wk]\Psi\sim q[\beta u]\Psi\sim(u+j)[w]\Psi q[\beta u]\Psi\sim\sigma[ui].$$

Let the index u now be diduced, becoming x in the last aggregant:

$$\pi[i]\pi[j]\pi[k]\pi[w]\pi[\beta]\pi[u]\Sigma[x]\sim(i+j)[k]\Psi\sigma[wk]\Psi\sim q[\beta u]\Psi\sim(u+j)[w]\Psi q[\beta x]\Psi\sim\sigma[xi].$$

Let $\sim q[\alpha w]$ be inserted as an aggregant:

$$\pi[i]\pi[j]\pi[k]\pi[\alpha]\pi[w]\pi[\beta]\pi[u]\Sigma[x]\sim(i+j)[k]\Psi\sim q[\alpha w]\Psi\sigma[wk]\Psi\sim q[\beta u]\Psi\sim(u+j)[w]\Psi q[\beta x]\Psi\sim\sigma[xi].$$

The insertion of this aggregant authorizes the insertion of its negative as a component of another aggregant.

$$\pi[i]\pi[j]\pi[k]\pi[\alpha]\pi[w]\pi[\beta]\pi[u]\Sigma[x]\sim(i+j)[k]\Psi\sim q[\alpha w]\Psi\sigma[wk]\Psi q[\alpha w]\cdot\{\sim q[\beta u]\Psi\sim(u+j)[w]\}\Psi q[\beta x]\Psi\sim\sigma[xi].$$

We now diduce w, transmuted it in one term into z, and thus obtain finally,

$$(29) \pi[i]\pi[j]\pi[k]\pi[\alpha]\pi[w]\Sigma[z]\pi[\beta]\pi[u]\Sigma[x]\sim(i+j)[k]\Psi\sim q[\alpha w]\Psi\sigma[wk]\Psi q[\alpha z]\cdot\{\sim q[\beta u]\Psi\sim(u+j)[z]\}\Psi q[\beta x]\Psi\sim\sigma[xi].$$

Peirce: CP 4.346 Cross-Ref:††

Here we have a nice little theoremidion, obvious though not self-evident. Namely, if any three positive integers, i, j, k, are such that k can result from adding i to j, then, selecting any class of integers we please, and speaking of the character of being an integer of this class as "the character α " either all integers of this class are as large as or larger than any integer k that can result from adding i to j, or else (if that is not the case) there is an integer of this class, z, if we take any second class of integers whatever (inclusion in which shall be called the character β) no integer u of this second class can on being added to j give the integer z, unless there be an integer x of the second class which is smaller than i. The form of statement is too strictly logical and formal for an ordinary mind readily to grasp it; but let us dilute it with a little verbiage, as follows. Suppose k is a positive integer which can result as the sum of j, as augend, and i, as addend. We select a first class of positive integers, say for example the cubes above 0 and 1. Now it may be that k does not exceed any of these. As to that case we say nothing. But should there be one or more of the first class that exceed k, then it may be that one of them is such that it cannot result from adding any positive integer to j as augend, because it may be less than j. It would have been better if, instead of writing $\Sigma[z]\pi[\beta]\pi[u]$ in the *Peircean*, I had written $\pi[\beta]\pi[u]\Sigma[z]$ for it is always allowable to carry Σ 's to the right. Then the second class being selected first, it might happen that there was an integer of the first class that could not result from adding any integer of the second class to j. . . .

Peirce: CP 4.347 Cross-Ref:††

BOOK II

EXISTENTIAL GRAPHSP

MY CHEF D'OEUVRE

CHAPTER 1

EULER'S DIAGRAMSE

§1. LOGICAL DIAGRAM †1

347. A diagram composed of dots, lines, etc., in which logical relations are signified by such spatial relations that the necessary consequences of these logical relations are at the same time signified, or can, at least, be made evident by transforming the diagram in certain ways which conventional 'rules' permit.

Peirce: CP 4.348 Cross-Ref:††

348. In order to form a system of graphs which shall represent ordinary syllogisms, it is only necessary to find spatial relations analogous to the relations expressed by the copula of inclusion and its negative and to the relation of negation. Now all the formal properties of the copula of inclusion are involved in the principle of identity and the *dictum de omni*. That is, if r is the relation of the subject of a universal affirmative to its predicate, then, whatever terms X, Y, Z may be,

Every X is r to an X ; and if every X is r to a Y , and every Y is r to a Z , every X is r to a Z .

Peirce: CP 4.348 Cross-Ref:††

Now, it is easily proved by the logic of relatives, that to say that a relation r is subject to these two rules, implies neither more nor less than to say that there is a relation l , such that, whatever individuals A and B may be,

If nothing is in the relation l to A without being also in the same relation l to B , then A is in the relation r to B ; and conversely, that,

If A is r to B , there is nothing that is l to A except what is l to B .

Peirce: CP 4.349 Cross-Ref:††

349. Consequently, in order to construct such a system of graphs, we must find some spatial relation by which it shall appear plain to the eye whether or not there is anything that is in that relation to one thing without being in that relation to the other. The popular Euler's diagrams fulfill one-half of this condition well by representing A as an oval inside the oval B . Then, l is the relation of being included within; and it is plain that nothing can be inside of A without being inside B . The relation of the copula is thus represented by the spatial relation of 'enclosing only what is enclosed by'. In order to represent the negation of the copula of inclusion (which, unlike that copula, asserts the existence of its subject), a dot may be drawn to represent some existing individual. In this case the subject and predicate ovals must be drawn to intersect each other, in order to avoid asserting too much. If an oval already exists cutting the space in which the dot is to be placed, the latter should be

put on the line of that oval, to show that it is doubtful on which side it belongs; or, if an oval is to be drawn through the space where a dot is, it should be drawn through the dot; and it should further be remembered that if two dots lie on the boundaries of one compartment, there is nothing to prevent their being identical. The relation of negation here appears as 'entirely outside of'. For a later practical improvement see Venn, *Symbolic Logic*, chapter xi.

Peirce: CP 4.350 Cross-Ref:††
§2. OF EULER'S DIAGRAMS †1

350. In the second volume of the great Leonard Euler's *Lettres à une Princesse d'Allemagne*, which appeared in 1772 (four years after the first volume), the nature of the syllogism is illustrated by means of circles, in substantially the following manner. Let the syllogism whose cogency is to be exhibited be the following:

All men are passionate,

All saints are men;

Therefore, All saints are passionate.

Imagine the entire collection of saints and nothing else to be enclosed in the imaginary circle, S, of Fig. 1; imagine the entire collection of men and nothing else to be enclosed in the imaginary circle, M -- which will therefore enclose whatever is enclosed in the circle S, since all saints are men. Imagine the entire collection of passionate beings and nothing else to be enclosed in the circle, P, which will thus enclose whatever is enclosed in the circle M, since all men are passionate. We see, then, that whatever is enclosed in the circle, S, is enclosed in the circle, P; that is, that all saints are passionate.

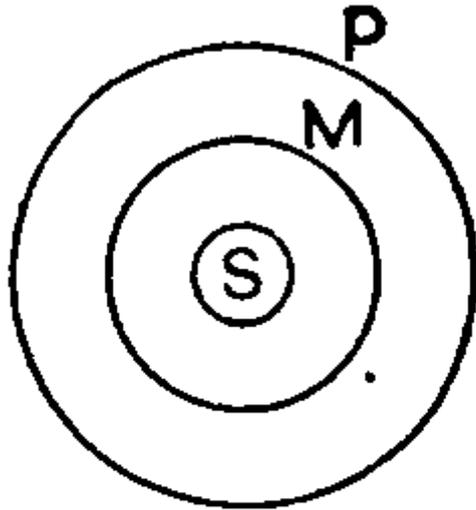


Fig. 1

Fig. 1

[Click here to view] Fig. 2

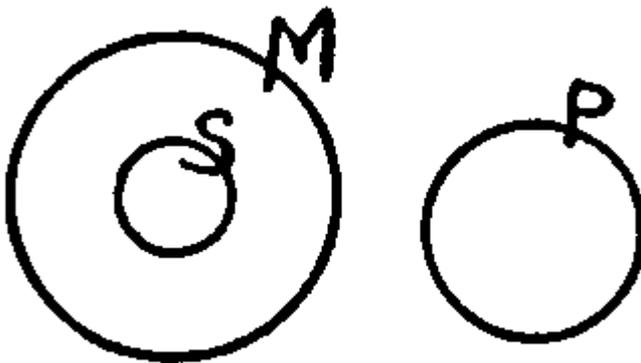


Fig. 2

[Click here to view]

It will be remarked that the way in which any facts of enclosure relating to the circle M can inform us about any relation of enclosure between two other circles, is by the facts about M being that one of the other circles is on the inside of M and the other on the outside of it; so that, so far as this mode of representation exhibits the true nature of the syllogism, there ought to be just two kinds of syllogisms, one corresponding to Fig. 1, and the other to Fig. 2, which latter figure illustrates the following syllogism:

No man is perfect,

But any saint is a man;

Hence, no saint is perfect.

Peirce: CP 4.351 Cross-Ref:††

351. . . . We may now define the system of Euler's Diagrams, as he left it, in the following rules:

Peirce: CP 4.351 Cross-Ref:††

First, every area of the diagram represents the entire collection, or aggregate, of possibilities of a certain description.

Peirce: CP 4.351 Cross-Ref:††

Second, if a circle is drawn within an area, the part of the area within the circle represents the entire aggregate of those possibilities represented by the area to which a certain description applies, while the area outside the circle represents the entire aggregate of those possibilities represented by the area to which that description does not apply; and in general the area common to two areas represents the entire aggregate of possibilities which are at once represented by each of those two areas.

Peirce: CP 4.352 Cross-Ref:††

352. From these two principles it follows that to draw two circles or other areas, A and B, so that they have no common area, is to represent that there is no possibility which is at once of the description represented by A and of the description represented by B; *and this is the only way in which an Euler's diagram can represent a state of things to be a fact.* It is essential that this should be understood. Thus, in

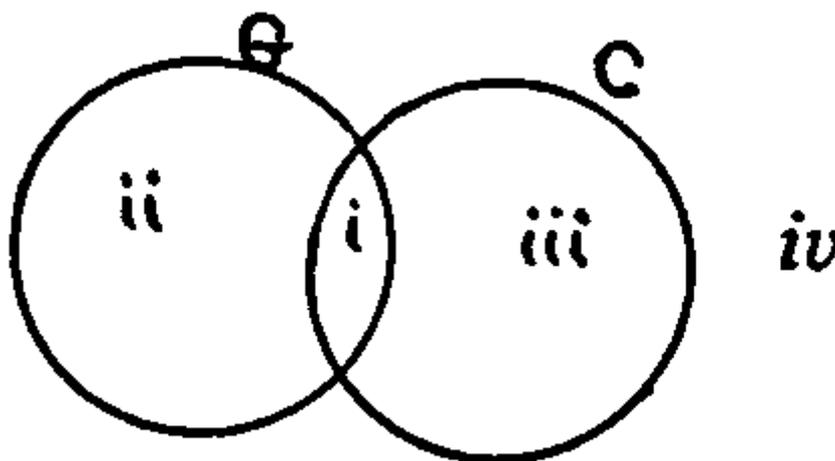


Fig. 3

Fig. 3

[Click here to view], let the entire area of the sheet represent all men now living. Let the circle G enclose all Greeks, and the circle C all courageous men. Then the four parts into which the two circles divide the whole sheet represent respectively whatever, i, Courageous Greeks; ii, Greeks not Courageous; iii, Courageous men not Greeks; iv, Men, neither Courageous nor Greeks, there may be among men now living. These are represented merely as possible classes, without any assertion. Fig. 4

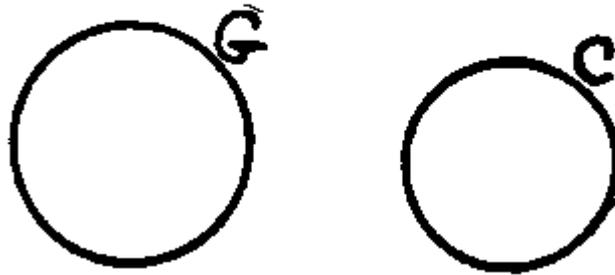


Fig. 4

[Click here to view] represents that all men consist of whatever Greeks not Courageous, Courageous Men not Greeks and Men neither Greeks nor Courageous there may be, and thus asserts that no Greek is Courageous; while Fig. 5

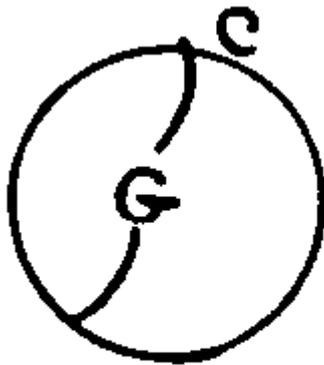


Fig. 5

[Click here to view] represents that men consist of whatever Courageous Greeks, Courageous Men not Greeks, and Men neither Courageous nor Greek there may be, and thus asserts that whatever Greek men there may be now living are courageous.

Peirce: CP 4.353 Cross-Ref:††

353. The history of this System of Graphs has been discussed by Mr. Venn;†1 and though he does not consider every historical question upon which one might desire to be informed, nothing additional will here be brought forward. The results are briefly these: Eight years before Euler's publication appeared, the *Neues Organon* of John Henry Lambert (Alsatian by birth, French by descent, but German by residence and by the honor and support that country rendered him) in which †1 the author made the same use of the stretches of parallel lines essentially as Euler did of the areas of circles, with an additional feature of dotted lines and extensions of lines. Lambert, however, does not seem to aim at any mathematical accuracy of thought in using his lines. He certainly does not attain it; nor could he do so, as long as he failed to perceive that the only purpose such diagrams could subserve is that of representing the necessity with which the conclusion follows from the premisses of a necessary reasoning, and that that necessity is not a compulsion in thinking (although there is

such a compulsion) but is a relation between the *facts* represented in the premisses and the facts represented in the conclusion. The failure to comprehend the true nature of logical necessity and the confounding of it with a psychological compulsion is common to German logicians generally,†2 excepting only the Herbartians. Thus, he represents 'Some A is B' by this diagram.

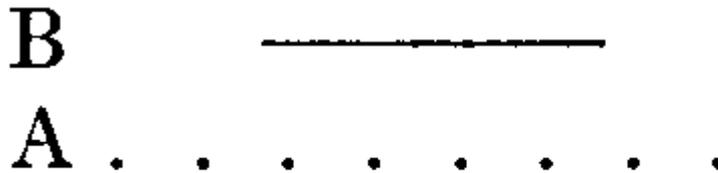


Fig. 6

[Click here

to view]

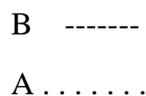


Fig. 6

Thus, a distinction is represented between 'Some A is B,' and 'Some B is A,' although they express the same fact. No doubt, there is a way of regarding such a fact, or supposed fact, as that 'Some Germans are given to subjective ways of thinking' which renders that a more natural mode of expression than 'Some men given to subjective ways of thinking [are] Germans.' But it is one thing to admit that this is so and quite another to admit that the sentence "expresses" that way of thinking, rather than the fact itself. The sentence is an assertion; and an assertion is of a fact and not of a way of thinking the fact. When a writer makes an assertion, his principal purpose is to induce the reader to believe in the reality of the fact asserted. He has the subsidiary design of causing the reader to follow along his line of thinking. . . . Throughout Lambert's whole treatment of syllogistic, the way of thinking is made the principal thing. Under these circumstances, it was impossible for him to have a clear conception of the proper nature of a system of syllogistic graphs.

Peirce: CP 4.353 Cross-Ref:††

My reason for insisting at such length upon this point is that it is a passage of Lambert's *Architektonik*†1 which is the principal authority for one of the main points of the current account of the history of the Eulerian diagrams. The usual assertion is that the voluminous pedagogist Christian Weise, the author of two works on logic (*Doctrina Logica*, 1690, and *Nucleus Logicæ*, 1691), who died 1708 October 21, made use of the system of diagrams in question. Nobody has ever examined Weise's own editions to see whether they bear out the assertion.†2 But it is said that one Johann Christian Lange in a book by him (*von ihm verfassten*) and entitled *Nucleus Logicæ Weisianæ*, published in 1712, tells how Weise so taught logic. Nobody, however, except Hamilton,†3 claims to have seen even this book. They appeal to a vague account of its contents in Lambert's *Architektonik*. Now this book of Lambert's

preceded Euler's publication by a year; and in view of Lambert's crude notions of what such diagrams ought to be, and in view of his not apparently being greatly struck by what would, to his mathematical mind in its benighted condition concerning syllogistic, have been a great light, the passage of Lambert is rather against the claims made for Weise than in their favor. It is curious that even Ueberweg †4 talks of Lange as the *author* of the publication of 1712. But to anybody familiar with such literature the title proclaims it to be a work by Weise probably with a running commentary or copious notes by Lange. The passage in Lambert's *Architektonik* was first brought to light by Drobisch in 1851 in the second edition of his *Neue Darstellung der Logik*. †5 But Hamilton in his fourteenth lecture on logic, publicly delivered in 1837-8 and regularly afterwards till his death, says †6 "I find it [*i.e.*, the mode of sensualizing by circles the abstractions of logic] in the *Nucleus Logicæ Weisianæ*, which appeared in 1712; but this was a posthumous publication, and the author, Christian Weise, who was Rector of Zittan, died in 1708." Hamilton was mistaken in supposing that the book had not appeared before; for it was published originally in 1691. What Hamilton here attributes to Weise falls very far short indeed of the system of Eulerian diagrams. It is true that Hamilton appears to confound the two; but no careful student of this strikingly unmathematical scholar will attribute any importance to such a unification. In this very same passage, he attributes to Alstedius, in 1614, the use of Lambert's linear graphs, which his editors are compelled to admit is a gross exaggeration. †1 How utterly unfounded it was is shown by Venn. †2 When we think of the great reputation of Weise in his own day, it is almost incredible that so striking an idea as that of Euler's diagrams should have been developed by so prominent a man without attracting universal attention. Until further evidence is adduced his claims to their authorship must be pronounced quite unsupported. But Friedrich Albert Lange in his remarkable *Logische Studien* (p. 10) says that substantially the Eulerian method is mentioned by the celebrated Juan Luis Vives †3 early in the sixteenth century, and in an offhand manner (*die schlichte Art ihrer Einführung*) that would seem to indicate that it was traditional in the schools. Venn †4 copies the passage and diagram, which shows the cardinal idea of the Eulerian diagrams, that the middle term is like a boundary separating the two regions in which the other two terms respectively lie -- and this much probably was traditional -- but gives no hint of any development of this idea into a sort of calculus, such as Euler's system is. Of this, the principal achievement, Euler is the author. After Euler several attempts were made to improve the system; but all of them were blunders until Venn's publications in 1880. Venn made a distinct improvement, and I shall endeavor to contribute others; but before giving an account of them, it will be requisite to study critically Euler's original proposal.

Peirce: CP 4.354 Cross-Ref: ††

354. What is it, then, that these diagrams are supposed to accomplish? Is it to prove the validity of the syllogistic formula? That sounds rather ridiculous -- as if anything could be more evident than a syllogism -- yet that is not far from the opinion of Friedrich Albert Lange, a thinker of no ordinary force. Suppose we ask ourselves *why* it is that, if a circle P wholly encloses a circle M which itself wholly encloses a circle S, the circle P necessarily wholly encloses the circle S. In order to express the answer, it will be well to avail ourselves of a phraseology proper to the logic of relatives. I use the words *relation* and *relative* in a somewhat narrow sense, which I begin by explaining. Take, then, any assertion †P1 whatever about a number of designate individuals. These individuals may be persons, material objects, actions, collections of things, possible courses of events, qualities, abstractions of any kind,

and, in short, of any one nature or of any several natures whatsoever; only each of them must be well-known and rated by a proper name, and each must belong to some universe, or total aggregate of things of the same wide class, and the assertion must be such that if any one of the individuals did not really occur in its universe, independently of whether you, I, or any collection of men or other cognoscitive beings should opine that it did or that it did not, then that assertion would be false. For example, if in the assertion that Mrs. Harris was unbeknown to Betsy Prig except by hearsay, "unbeknown" be understood in such a sense that the nonexistence of Mrs. Harris would render it true, then not only does this assertion not fulfill the condition, but -- still taking "unbeknown" in the same sense -- no more would the assertion that Sairey Gamp was unbeknown to Betsy Prig except by hearsay; while if "unbeknown" be taken in such a sense that the first assertion is rendered false by the non-existence of Mrs. Harris, then, although that assertion would not fulfill the condition because Mrs. Harris did not belong to the universe of characters in *Martin Chuzzlewit*, yet, taking the word in this sense, the assertion, "Sairey Gamp is unbeknown to Betsy Prig except by hearsay," will perfectly fulfill the condition; and neither its falsity nor the fictitiousness of the universe to which Sairey Gamp and Betsy Prig belong are any objections. . . .

Peirce: CP 4.354 Cross-Ref:††

An assertion fulfilling the condition having been obtained, let a number of the proper designations of individual subjects be omitted, so that the assertion becomes a mere blank form for an assertion which can be reconverted into an assertion by filling all the blanks with proper names. I term such a blank form a *rheme*. If the number of blanks it contains is zero, it may nevertheless be regarded as a rheme, and under this aspect, I term it a *medad*. A medad is, therefore, merely an assertion regarded in a certain way, namely as subject to the inquiry, How many blanks has it? If the number of blanks is one, I term the rheme a *monad*. If the number of blanks exceeds one, I term it a *Relative Rheme*. If the number of blanks is two, I term the rheme a *Dyad*, or *Dyadic Relative*. If the number of blanks exceeds two, I term it a *Polyad*, or *Plural Relative*, etc. A *Relation* is a substance whose being and identity precisely consist in this; its being, in the possibility of a fact which could be precisely asserted by filling the blanks of a corresponding relative rheme with proper names; its identity, in its being in all cases so expressible by the same relative rheme.†1 It must be confessed that it would have been better if a modifying adjective had been attached to the words *relative* and *relation* to form the technical terms to designate what have just been defined as a relative rheme and a relation. But now that these terms have been established by me, my convictions of the ethics of terminology †2 forbid me to attempt to alter the meanings attached to them. I use the word "signify" in such a sense that I say that a relative rheme *signifies* its corresponding relation. In the technical language of the logic of relatives, letters of the alphabet are employed as pronouns to denote relatives, just as, in ordinary and especially in legal language, they are often used as relative pronouns. The ancient grammarians defined a pronoun as a word used to replace a noun, a most preposterous attempt at analysis. It would have been far nearer the truth to describe a common noun as a word used in place of a pronoun.†1 In the middle ages, Duns Scotus and others brought a correcter definition into vogue; but the humanists of the reformation stickled for the ancient definition and that of the scholastics was quite forgotten A relative pronoun designates a subject by indicating, through its position and agreement, a noun that designates that subject. This nearly corresponds to the use of letters in the Catechism, "What is your Name? *Answer* N., or M." and the priest in dipping the child in the water so

"discretely and warily," is represented as saying "N. I baptize thee in the name" etc. The point is that in neither case is it meant that the letter is pronounced, but this letter designates the person through indicating by its position that it is to be replaced by the Christian name. . . . So in logic, *Barbara* is described as the syllogistic form

Any M is P,

Any S is M;

∴ Any S is P.

What is meant is that the letters S, M, and P, in this formula, may be replaced by any terms whatever; only each letter must everywhere in the formula be replaced by the same term. In the logic of relatives, the letters *r*, *s*, {*r*}, σ , etc., are frequently employed as substitutes for dyadic relatives, so that "A is *r* of B" and "B is *r*'d by A" stand for different expressions of the same fact, analogous to "A is lover of B" and "B is loved by A."

Peirce: CP 4.355 Cross-Ref:††

355. With this explanation of terms, we can intelligibly answer the question, "**Why** does a circle, P, that wholly encloses a circle M, itself wholly enclosing a circle, S, likewise necessarily enclose the circle, S?" Namely, understanding by this question, "What is the peculiarity of the relation of wholly enclosing which renders this necessary?" we answer, "It is because the relation of wholly enclosing is such that there is a dyadic relative, *r*, such that to say that any place, X, wholly encloses a place, Y, is equivalent to saying that X is at once *r* of Y and is *r* of everything that is *r*'d by Y." To show that this is the explanation, we must prove two propositions: firstly, that there is a dyadic relative, *r*, such that, on the one hand, if the place X wholly encloses the place Y, the place X is *r* of Y and is *r* of everything *r*'d by Y, while on the other hand, if X does not enclose Y, either X is not *r* of Y or else there is something *r*'d by Y that is not *r*'d by X; and secondly, that from that first proposition it necessarily results that if any circle, P, wholly encloses a circle, M, itself wholly enclosing a circle, S, then P wholly encloses the circle, S. Before proving the first of these propositions, it is to be remarked that whether we affirm that the place X wholly encloses the place Y, [or whether] we say that the place, X, does not wholly enclose the place, Y, we are to be understood as recognizing X and Y as definite places in space, so that if either of them is not of that nature, both the one assertion and the other are false. Now in order to prove the first proposition, it will suffice to make *r* signify the relation of not being quite at a distance from (with intervening place), so that the first clause of the first proposition will be that if the place, X, wholly encloses the place, Y, then X is not altogether at a distance from Y, nor is it altogether at a distance from any place from which Y is not altogether at a distance. That, if X wholly encloses Y it is not altogether at a distance from Y, is self-evident. Moreover, if we consider any place Z, from which Y is not altogether removed, there must be some point of Z from which Y is not altogether removed, and from this point, X will not be altogether removed. Hence it is evident that X is not altogether removed from any place from which Y is not altogether removed; and the first clause of the proposition is found to be true. The other clause of the proposition is that, if X does not wholly enclose Y, then either X is not *r* of Y or else there is something *r*'d by Y that is not *r*'d by X; that is, if X does not wholly enclose Y, either X is altogether remote from Y or else there is some place not altogether distant from Y from which X is entirely remote. This is plainly true, since if X does not wholly enclose Y, there is some point of Y which lies quite outside of X; and such a point will be a place from

which Y is not remote but from which X is remote. Thus the first proposition is true. It remains then to be shown that, from this peculiar form of the relation of total inclusion, it follows that a circle, P, wholly enclosing a circle, M, itself wholly enclosing a circle, S, likewise wholly encloses the circle, S. . . . Now it is clear that as long as P is *r* of whatever is *r*'d by M, if S is *r*'d by M, so long will P be *r* of S; while as long as P is *r* of whatever is *r*'d by M, and whatever is *r*'d by S is *r*'d by M, P is *r* of whatever is *r*'d by S. Thus if P is *r* of whatever is *r*'d by M and if both S and whatever is *r*'d by S are *r*'d by M, P is *r* both of S and of whatever is *r*'d by S; *quod erat demonstrandum*. Thus the reason, that the geometrically wholly enclosed by the wholly enclosed is itself wholly enclosed, is shown. But this is the very same reason substantially that Aristotle †1 gives for the validity of the syllogism in *Barbara*.

Any M is P,

Any S is M;

∴ Any S is P.

For Aristotle's doctrine is that this depends on the essential nature of being *dictum de omni*, or universally predicated. This essential nature he says is, that to say that X is predicated of the whole of Y, is to say that X is predicated of Y and of whatever Y is predicated of.

Peirce: CP 4.355 Cross-Ref: ††

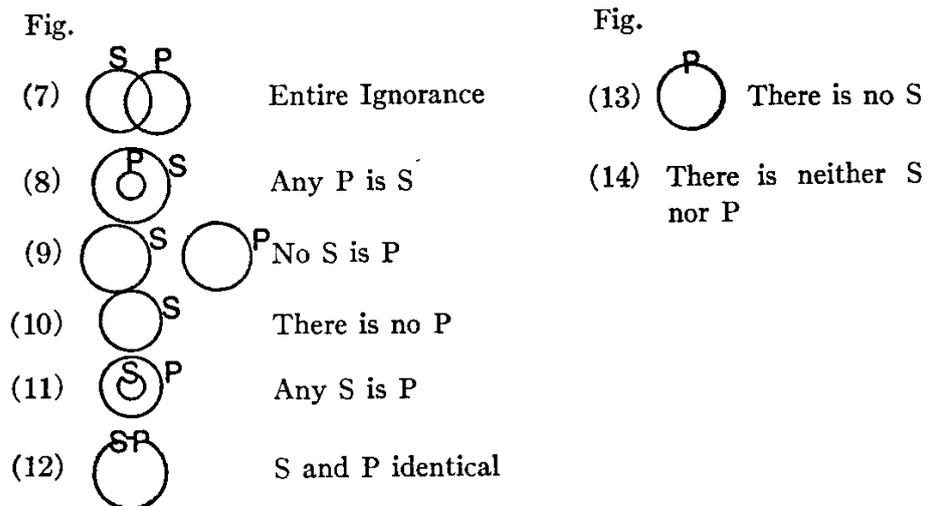
That is, the relation of universal predication is also of the form, "At once *r* of and *r* of whatever is *r*'d by." He might have avoided the apparent *circulus in definiendo* by stating the matter thus: X is predicated of all Y if and only if X is not foreign to Y nor to any term to which Y is not foreign. Thus, as far as logical dependence goes, the validity of the syllogism and the property of the Eulerian diagram depend upon a common principle. They are analogous phenomena neither of which is, properly speaking, the cause or principle of the other. Lange †2 is of opinion that all reasoning proceeds by the observation of imaginary Euler's diagrams or of something closely similar; and I, †3 for my part, share his opinion so far as to admit that an imaginary observation is the most essential part of reasoning. But the psychological process is not the matter in question. This brings us back to the inquiry, What purpose are the diagrams fitted to subserve? They may help to analyze reasonings, and this either in a practical way by aiding a person in rendering his ideas clear, or theoretically. In either regard it is desirable that they should be adequate to represent the gist of every kind of deductive reasoning.

Peirce: CP 4.356 Cross-Ref: ††

356. As Euler left the system, it had the following faults:

Peirce: CP 4.356 Cross-Ref: ††

First, two circles cannot be each inside the other; so that while, as Mrs. Franklin has shown (*Johns Hopkins Studies in Logic*, p. 64), there are fifteen or sixteen different ways in which two terms may be related in reference to the possibility or impossibility of their different combinations, Euler's original diagrams show but eight of these, as follows:



Figs. 7-14

[Click here to view]

The states of possibility not represented are as follows:

Everything is either S or P; $[S \vee P]$

Everything is S; $[S]$

No S is P, but everything but S is P; $[S \equiv \sim P]$

Everything is S and nothing is P; $[S \cdot \sim P]$

Everything is P; $[P]$

Everything is both S and P; $[S \cdot P]$

Nothing is S but everything is P; $[\sim S \cdot P]$

The Universe is absurd and impossible; $[P \cdot \sim P]$

Peirce: CP 4.356 Cross-Ref:††

Second, in regard to every combination of terms (that is, in regard to each of the possible parts of the universe, when we are in complete ignorance), the system is limited to expressing its non-existence or to not expressing whether it exists or not. *It cannot affirm the existence* of any description of an object. But a categorical, though possibly partial, description of the universe in its relation to two terms can, in reference to each of the four possible parts into which those two terms can divide the universe of possibility, either affirm its existence, or deny its existence, or say nothing. Therefore, excluding the absurd assertion that nothing exists, there are $3^4 - 1$, or eighty, possible categorical descriptions of the universe, of which this system can express but one tenth part.

Peirce: CP 4.356 Cross-Ref:††

Third, the system affords no means of expressing a knowledge that one or another of several alternative states of things occurs. Of the sixteen possible dichotomic states-of things with reference to two terms, a state of knowledge may either exclude or admit each, though it cannot exclude all. There are therefore $2^{16} - 1$,

or 65535, possible states of dichotomous information about two terms of which the system permits the expression of only eight, or one out of every 8192.

Peirce: CP 4.356 Cross-Ref:††

Fourth, the system affords no means of expressing any other than dichotomous, or qualitative, information. It cannot express enumerations, statistical facts, measurements, or probabilities. In short, it affords no room for the introduction of quantitative premisses into its reasonings.

Peirce: CP 4.356 Cross-Ref:††

Fifth, the system affords no means of exhibiting reasoning, the gist of which is of a relational or abstractional kind. It does not extend to the logic of relatives.

Peirce: CP 4.357 Cross-Ref:††

357. Some of these imperfections are, however, easily removed. This first of them was done away with by an improvement

Figs. 15-18

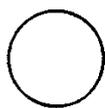


Fig. 15

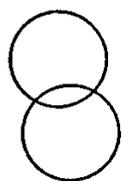


Fig. 16

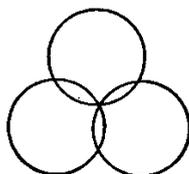


Fig. 17

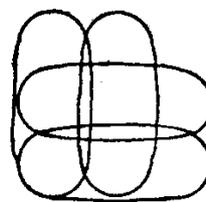


Fig. 18

to view]

[Click here

introduced by Mr. Venn in 1880. Namely, Mr. Venn in his *Symbolic Logic* (I use the first edition of 1881) recommends drawing the diagrams so as always to exhibit all the possible parts into which terms, to the number employed, would, in the absence of all information, divide the universe. That done, if information is received that certain of these parts do not exist, the corresponding regions of the diagrams are shaded. Thus the areas representing the terms may be arranged in one of the following ways according as they are one, two, three, or four in number. With more than four terms the system becomes cumbrous; yet, by having on hand lithographed blank forms showing the four-term figure on a large scale, all the compartments containing repetitions of one figure, whether that for one term, for two terms, for three or for four, and considering corresponding regions of all sixteen of the large compartments to represent together the extension of one term, it is possible without much inconvenience to increase the number of terms to eight. Beyond eight terms, the best way will simply be to make a list of the regions, numbered in the dichotomous system of arithmetical notation, one numerical place being appropriated to each term.

Peirce: CP 4.357 Cross-Ref:††

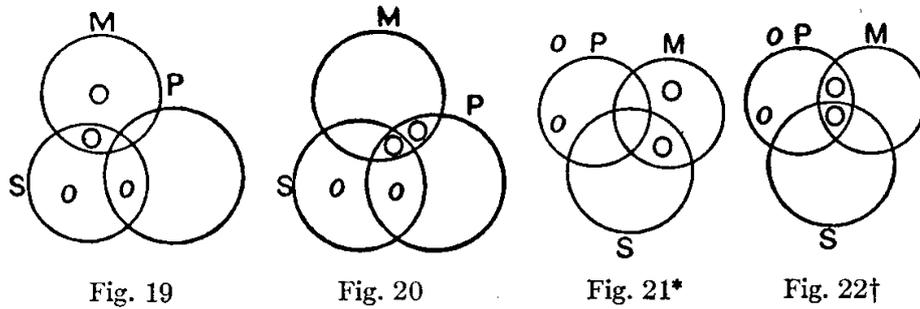
Instead of shading excluded regions we may simply make them with the character 0, for zero.

Peirce: CP 4.358 Cross-Ref:††

358. The unmodified Eulerian system gives two syllogistic diagrams as shown

above, Figs. 1 and 2. These with the modification

Figs. 19-22



[Click here

to view]†1†2

are shown in Figs. 19 and 20. The exclusions by different premisses are marked differently. Venn's modification furnishes two new syllogistic diagrams shown in Figs. 21 and 22.

Peirce: CP 4.359 Cross-Ref:††

359. The second imperfection of the system is also very readily remedied; and the remedy almost inevitably suggests a partial remedy for the third imperfection. Namely, why not draw the character X in any compartment in order to signify that something of the corresponding description occurs in the universe? We shall thus get these three forms of propositions:

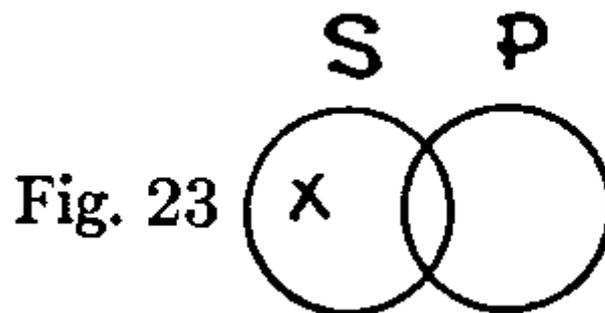


Fig. 23
view] Some S is not P.

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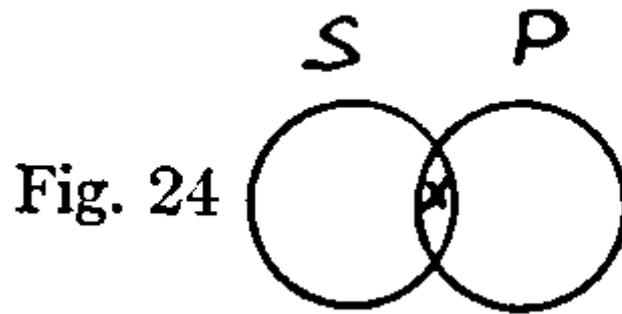


Fig. 24
view] Some S is P.

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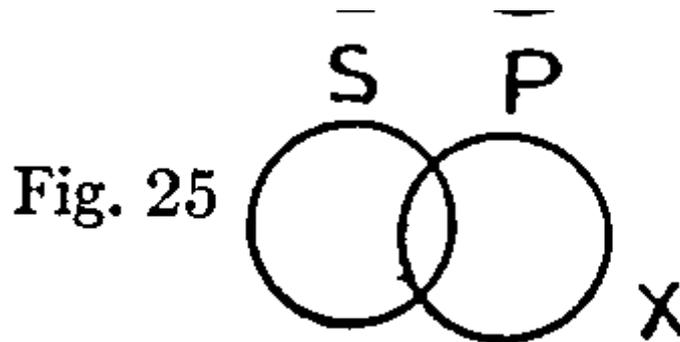


Fig. 25
view] There is something beside S and P.

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The precise denial of each of these is produced by substituting 0 for X. But when a third term is present some further rule has to be determined. How shall we mark the following diagram in order to express "Some S is not P"? The proposition

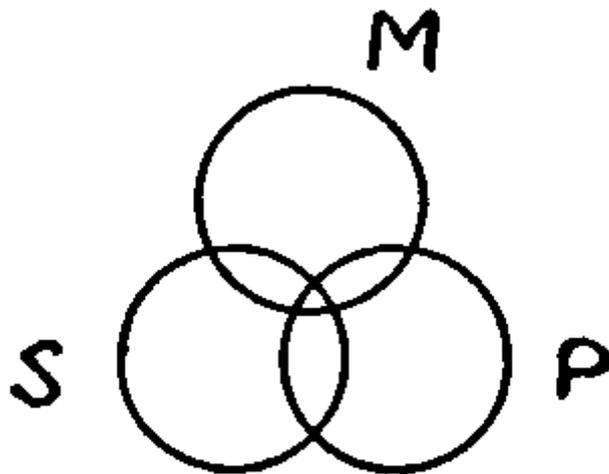


Fig. 26

Fig. 26

[[Click here to view](#)]

will here take the form, Either some S that is M but not P exists or some S that is neither M nor P exists. One suggestion would be that a cross be made on the circumference of M. But this would only provide for a special class of disjunctions. The question would then become, How shall we express, "Either something that is at once S and P exists or something that is neither S nor P exists"? Since we have drawn *zeros* at once in two compartments to signify the non-existence of either of two classes of objects, if we are to adhere to the principle that precise denial is produced by substituting crosses for zeros and conversely, it would follow that two crosses in two compartments would signify that something exists either of the one or of the other class. But this decision would render it impossible to give any systematic interpretation to a cross in one compartment and a zero in another. Suppose, then, that signs in different compartments, if disconnected, are to be taken conjunctively, and, if connected, disjunctively, or *vice versa*. Then precise denial will be effected by reversing the characters of the signs and of their relations as to connection or disconnection. There are perhaps no very compulsive reasons for adopting one interpretation of the connection of signs rather than the other. But it would seem strange if the insertion of a new and disconnected sign should cause a diagram to assert less; while the modification of an existing sign, by attaching to it a line of connection terminating in a new sign, might well enough diminish the assertion. It seems also quite natural that to mark the same compartment independently with contradictory signs, as in Fig. 27, should be absurd, while that if the two opposite signs are connected, as in Fig. 28, they should simply annul one another and be equivalent to no sign at all.



Fig. 27

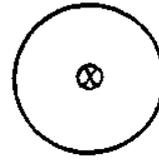


Fig. 28

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here to view]

Fig. 27 Fig. 28

Moreover, a cross on a boundary line may very naturally be understood to be equivalent to two connected crosses on the two sides of the boundary. Another consideration, perhaps more decisive, is that we shall necessarily regard the connected assertions as being put together directly, while the detached connexi of assertions are afterward compounded. It is therefore a question between using copulations of disjunctions [or] disjunctions of copulations. The former is the more convenient. . . .

Peirce: CP 4.360 Cross-Ref:††

360. Let this rule then be adopted:

Connected assertions are made alternatively, but disconnected ones independently, *i.e.*, copulatively.

Peirce: CP 4.361 Cross-Ref:††

361. As a consequence of this rule and of the introduction of the cross, the permissible transformations of diagrams, which transformations of course signify inferences, become so various that it is time to draw up a code of Rules for them. "Rules" is here used in the sense in which we speak of the "rules" of algebra; that is, as a permission under strictly defined conditions.†P1

Peirce: CP 4.362 Cross-Ref:††

362. ***Rules of Transformation of Eulerian Diagrams***

Peirce: CP 4.362 Cross-Ref:††

Rule 1. Any entire sign of assertion (*i.e.*, a cross, zero, or connected body of crosses and zeros) can be erased.

Peirce: CP 4.362 Cross-Ref:††

Rule 2. Any sign of assertion can receive any accretion. Thus Fig. 29 may be transformed into Fig. 30.†1

Figs. 29-30

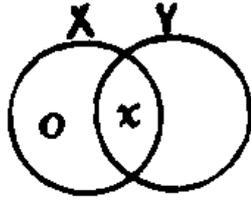


Fig. 29

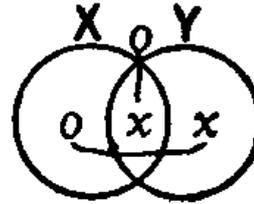


Fig. 30

to view]

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Peirce: CP 4.362 Cross-Ref:††

Rule 3. Any assertion which could permissively be written, if there were no other assertion, can be written at any time, detachedly.

Peirce: CP 4.362 Cross-Ref:††

Rule 4. In the same compartment repetitions of the same sign, whether mutually attached or detached, are equivalent to one writing of it. Two different signs in the same compartment, if attached to one another are equivalent to no sign at all, and may be erased or inserted. But if they are detached from one another they constitute an absurdity. All the foregoing supposes the signs to be unconnected with any in other compartments. If two contrary signs are written in the same compartment, the one being attached to certain others, P,

Figs. 31-34

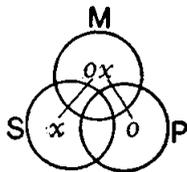


Fig. 31

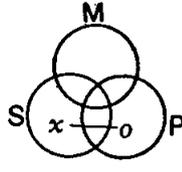


Fig. 32

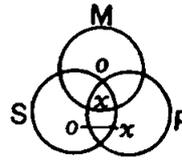


Fig. 33

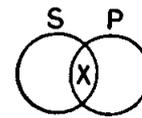


Fig. 34

to view]

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and the other to certain others, Q, it is permitted to attach P to Q and to erase the two contrary signs.

Peirce: CP 4.362 Cross-Ref:††

Thus, Fig. 31 can be transformed into Fig. 32.†1

Peirce: CP 4.362 Cross-Ref:††

Rule 5. Any Area-boundary, representing a term, can be erased, provided that, if, in doing so, two compartments are thrown together containing independent zeros, those zeros be connected, while if there be a zero on one side of the boundary to be erased which is thrown into a compartment containing no independent zero, the zero and its whole connex be erased.

Peirce: CP 4.362 Cross-Ref:††

Thus, Fig. 33 can be transformed into Fig. 34.†2

Peirce: CP 4.362 Cross-Ref:††

Rule 6. Any new Term-boundary can be inserted; and if it cuts every compartment already present, any interpretation desired may be assigned to it. Only, where the new boundary passes through a compartment containing a cross, the new boundary must pass through the cross, or what is the same thing, a second cross connected with that already there must be drawn and the new boundary must pass between

Figs. 35-36

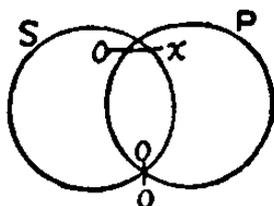


Fig. 35

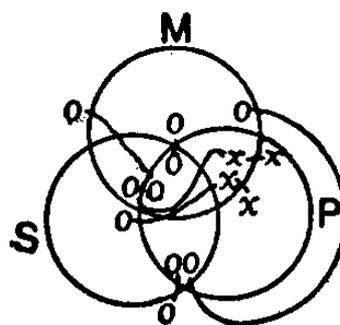


Fig. 36

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them, regardless of what else is connected with the cross. If the new boundary passes through a compartment containing a zero, it will be permissible to insert a detached duplicate of the whole connex of that zero, so that one zero shall be on one side and the other on the other side of the new boundary.

Peirce: CP 4.362 Cross-Ref:††

Thus, Fig. 35 can be converted into Fig. 36.†3

Peirce: CP 4.362 Cross-Ref:††

These six rules have been written down entirely without preconsideration; and it is probable that they might be simplified, and not unlikely that some have been overlooked.

Peirce: CP 4.363 Cross-Ref:††

363. As thus improved, Euler's diagrams are capable of giving an instructive development of the particular syllogism. The premisses of *Darii* are as follows:

Figs. 37-40

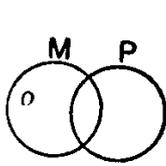


Fig. 37

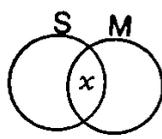


Fig. 38

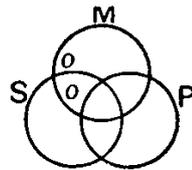


Fig. 39

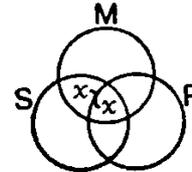


Fig. 40

to view]

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Fig. 37: Any M is P. S being inserted this gives, by Rule

6, Fig. 39. Fig. 38: Some S is M, P being inserted, this becomes,

by Rule 6, Fig. 40. Uniting Figs. 39 and 40 by Rule 3, we

get Fig. 41, and by Rule 4, Fig. 42. Now erasing M by Rule 5,

Figs. 41-43

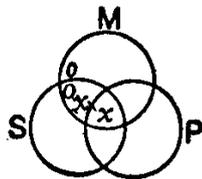


Fig. 41

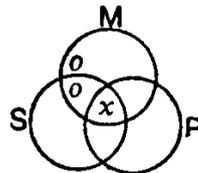


Fig. 42

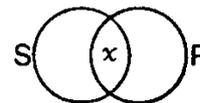


Fig. 43

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we get Fig. 43. *Baroko*, *Bokardo*, and *Frisesomorum* proceed in the same way. The premisses of the last are as follows:

Figs. 44-48

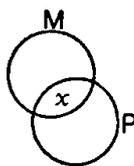


Fig. 44

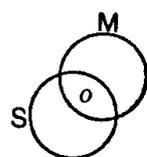


Fig. 45

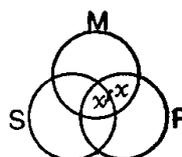


Fig. 46

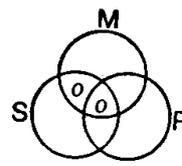


Fig. 47

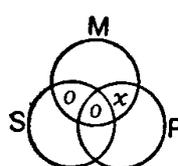


Fig. 48

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Figs. 49-51

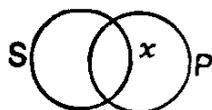


Fig. 49

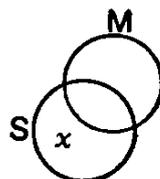


Fig. 50

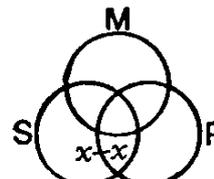


Fig. 51

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Fig. 44: Some M is P, which, by Rule 6, gives Fig. 46.

Fig. 45: No S is M, which, by Rule 6, may give Fig. 47. Combining these by Rule 3, Rule 4 gives Fig. 48 and Rule 5, Fig. 49. Let us now make the second premiss particular, as well as the first. We thus have Fig. 50 in place of Fig. 45; and on inserting P, we have Fig. 51 in place of Fig. 47. Uniting Figs. 46 and 51 we get Fig. 52. We now introduce two new and *undescribed* terms, as in Fig. 53, and on erasing M, we get Fig. 54 of which the interpretation is "Some S is

Figs. 52-54

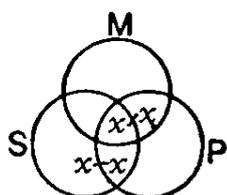


Fig. 52

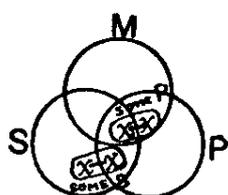


Fig. 53

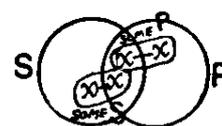


Fig. 54

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to view]

not some P." The objection may be raised that this method of dealing with the spurious †1 syllogism does not seem to follow from general principles, as a matter of course. In view of that objection we may put a single cross on the boundary instead of two connected crosses. The reasoning then proceeds, by uniting Figs. 44 and 50, as shown in Figs. 55 and 56.

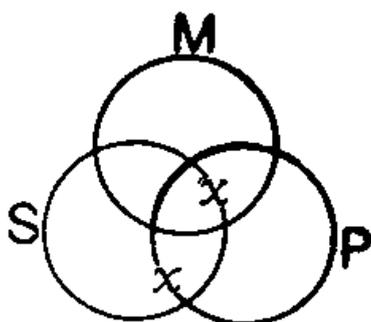


Fig. 55

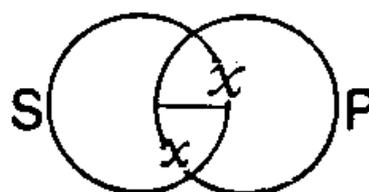


Fig. 56

[Click here

to view] A portion of the boundary of M is retained in Fig. 56 to show that on whichever sides of the boundaries the two crosses may belong, they can in no

case fall within the same region. Let it be noted, by the way, as a suggestive circumstance, that the portion of the boundary of M now remaining is simply a sign of negation.

Peirce: CP 4.364 Cross-Ref:††

364. This proposition "Some S is not some P" is called by Mr. B. I. Gilman, in

a paper which constitutes a distinct step in logical research, but which is buried in the Johns Hopkins Bulletins,†2 a proposition "particular in the second degree." An ordinary particular proposition asserts the existence of at least one individual of a given description. A proposition particular in the second degree asserts the existence of at least two individuals. It is an inference from two particular propositions each of which affirms the existence of one of the two individuals. We should therefore expect that, from a particular proposition of the second degree combined with one of the first degree, the inference should affirm the existence of three objects. Let us try the

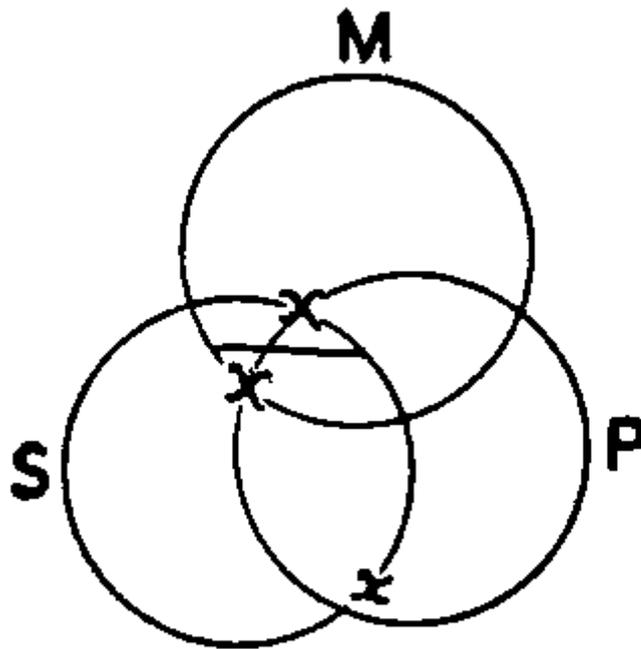


Fig. 57

experiment. Fig. 57

here to view] shows that the conclusions from the two premisses

[Click

Some S is not some M, and

Some P is not M,

is "Some S is other than something other than some P." But the S and the P in question are represented by the two lower crosses in the figure; and since these border upon the same compartment they may refer to the same individual. But if in addition to two ordinary particular premisses we take a universal premiss we can get a conclusion affirming the existence of three individuals. Take for instance the premisses

Some S is not M

Some M is P

No N is P

Some M is N

These premisses are combined in Fig. 58

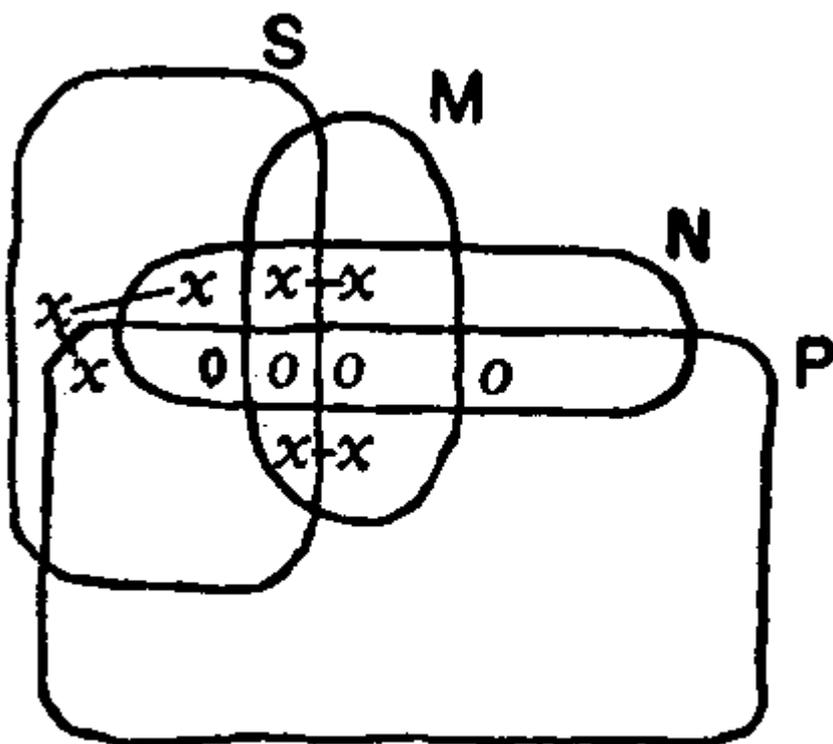


Fig. 58

[Click here

to view]; and it will be seen that the three connexes of crosses must be all different individuals; so that the conclusion is "Some S is other than and other than something other than some P." This line of study is far from being a trivial matter, however it may appear to superficial thinkers. But it does not enter into the purpose of the present paper to pursue it further.

Peirce: CP 4.365 Cross-Ref:††

365. In remedying the second imperfection we have gone far to remove the third and have even done something toward a treatment of the fourth. Let us consider a moment how far it can now be said that the method is inadequate to dealing with disjunctions. If by a disjunctive proposition we mean the sort of propositions usually given in the books as examples of this form, there never was any difficulty at all in dealing with them by Euler's diagrams in their original form. But such a proposition as "Every A is either B or C" which merely declares the non-existence of an A that is at once not B and not C, is not properly a disjunctive proposition. It is only disjunctions of conjunctions that cause some inconvenience; such as "Either some A is B while everything is either A or B, or else All A is B while some B is not A." Even here there is no serious difficulty. Fig. 59

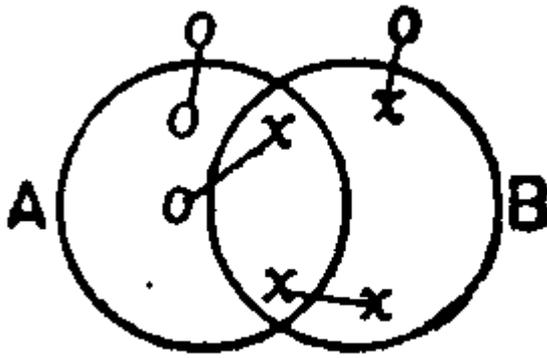


Fig. 59

[Click here to view] expresses this proposition. It is merely that there is a greater complexity in the expression than is essential to the meaning. There is, however, a very easy and very useful way of avoiding this. It is to draw an Euler's Diagram of Euler's Diagrams each surrounded by a circle to represent its Universe of Hypothesis. There will be no need of connecting lines in the enclosing diagram, it being understood that its compartments contain the several possible cases. Thus, Fig. 60

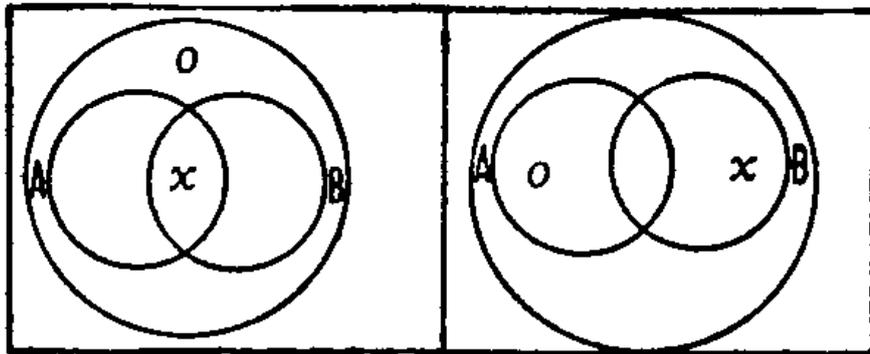


Fig. 60

[Click here to view] expresses the same proposition as Fig. 59.

Peirce: CP 4.366 Cross-Ref:††

366. Let us now consider the fourth imperfection. We are already in condition to express minimal multitudes. Thus Fig. 61

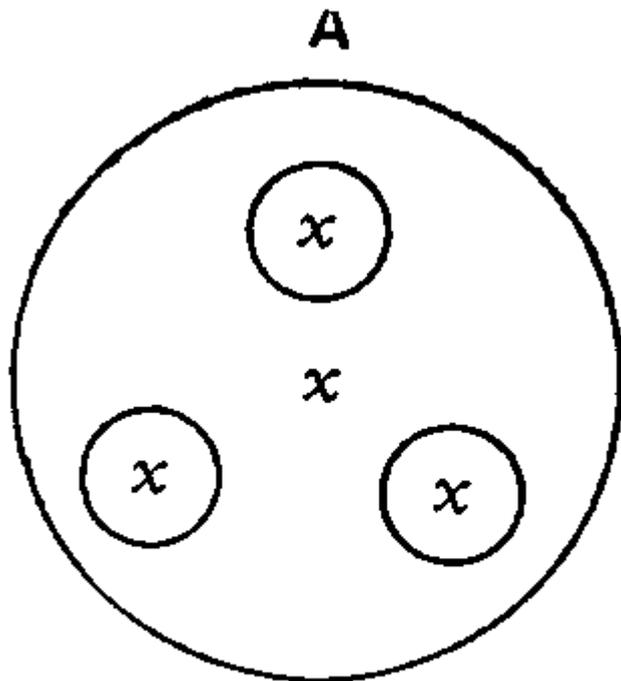


Fig. 61

[Click here to view]

expresses that there are at least four A's. The precise denial of a minimal proposition will be a maximal proposition; and consequently, Fig. 62

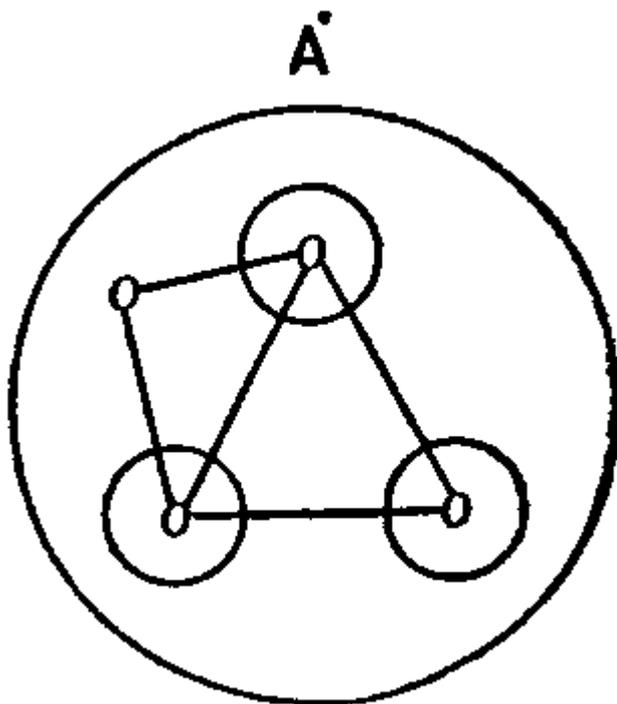


Fig. 62

[Click here to view] must

express that there are not as many as four A's. It is necessary here that the whole area

of A should be covered by the parts.

Peirce: CP 4.366 Cross-Ref:††

This mode of expression becoming impracticable, except for very small numbers, it naturally occurs to us to write a number in a compartment to express the precise multitude of individuals it contains. By extending this to algebraic expressions, not merely ratios but all sorts of numerical relations can be expressed.

Peirce: CP 4.367 Cross-Ref:††

367. The fifth fault of the system is by far the worst; and if there is any cure for it, not the smallest hopeful indication of its possibility appears at present.

Peirce: CP 4.368 Cross-Ref:††

368. Let us now endeavor to seize upon the spirit and characteristic of this system of graphs, and to estimate its value. Its beauty -- a violent inappropriate word, yet apparently the best there is to express the satisfactoriness of it upon mere contemplation -- and its other merits, which are fairly considerable, spring from its being veridically iconic, naturally analogous to the thing represented, and not a creation of conventions. It represents logic because it is governed by the same law. It works the syllogism as the planet integrates the equation of Laplace, or as the motion of the air about a pendulum solves a mathematical problem in ideal hydrodynamics. Still more closely, it resembles the application of geometry to algebra. By this I mean what is commonly called the application of algebra to geometry, but surely quite preposterously and contrarily to the spirit of the study. I hope no set argument is needed to defend this statement. The habitual neglect by students of analytical geometry of the *real* properties of loci, of which very little is known, and their almost exclusive interest in the *imaginary* properties, which are non-geometrical, sufficiently show that it is geometry that is the means, algebra the end. Geometry is not a perfect fit to algebra, in some respects falling short, in others over-running; elliptic in the absence of the imaginary, hyperbolic in presenting a continuity to which analytic quantity can hardly be said to make any approach. Yet even its partial analogy has been so helpful to modern algebra (and it was not less so to the older doctrine) that the phrase "it has been the making of it" is not too strong. For no doubt it was geometry that suggested the importance of the linear transformation, that of invariance, and in short almost all the profounder conceptions. The analogy of the doctrine of the Eulerian diagrams to non-relative logic is *proportionately* fully as great; although, owing to the greater simplicity of the subject and to its having fewer characters in all, the absolute number and weight of the points of resemblance are necessarily less. Such mathematics, as there may be connected with non-relative logic, we should have a right to expect would be much illuminated by the Eulerian Diagrams. Only this [is] mathematics of the most rudimentary conceivable kind; and hardly stands in need of any particular illumination. The different branches of pure mathematics are distinguished by their different systems of quantity; that is, of systems of points, units, or elements. In algebra, these points are so distributed over a surface that, in whatever manner any one is related to a single other exclusively, in that same manner is this other related to a third, and so on, *ad infinitum*; and moreover this infinite series may tend toward a definite limit, which limit is, in every case, included in the system. This is the most highly organized system of quantity that mathematicians have ever succeeded in definitely conceiving. On the other hand, the very simplest and most rudimentary of all conceivable systems of quantity is that one which distinguishes only two values. This [is] the system of evaluation which ethics applies to actions in dividing them into the right and the wrong, and which

non-relative logic applies to assertions in dividing them into the true and the false. The mathematics of such a system -- dichotomous mathematics -- amounts to very little. Those who seek to make a calculus of the algebra of logic struggle vainly after mathematical interest by complicating their problems. They do not succeed: mere complication has not even a mathematical interest.

Peirce: CP 4.369 Cross-Ref:††

369. Dichotomous mathematics does not amount to much, but it does amount to something. For example, the subject of higher particular propositions, in consequence of not being perfectly familiar, will call for considerable reflection to understand in its entirety and in its connections. Complicated questions of non-relative deductive reasoning are rare, it is true; still, they do occur, and if they are garbed in strange disguises, will now and then make the quickest minds hesitate or blunder. Euler's diagrams are the best aids in such cases, being natural, little subject to mistake, and every way satisfactory. It is true that there is a certain difficulty in applying them to problems involving many terms; but it is an easy art to learn to break such problems up into manageable fragments. The improved Boolean algebra has some advantages for those who are expert in its use, and who do not allow their instrument to rust from want of use. But the diagrams are always ready†1

Peirce: CP 4.370 Cross-Ref:††

370. Any broad mathematical hypothesis, like that of a system of values, will attract three classes of students by three different interests that attach to it. The first is the special interest in the circumstance that that hypothesis necessarily involves certain relations among the things supposed, over and above those that were supposed in the definition of it. This is the mathematical interest proper. The second is the methodic interest in the devices which have to be employed to bring those new relations to light. This is a matter of supreme interest to the mathematician and of considerable, though subordinate, interest to the logician. The third is the analytical interest in the essential elements of the hypothesis and of the deductive processes of the second study, in their intellectual pedigrees and in their conceptual affiliations with ideas met with elsewhere. This is the logical interest, *par excellence*. In the case of non-relative deductive logic, that is, the doctrine of the relations of truth and falsity between combinations of non-relative terms, the methodic interest is slight owing to the extreme simplicity of the methods. The logical interest, on the other hand, limited as the subject is when relative terms are excluded, is very considerable, not to say great. In the inquiries which it prompts, it is the simplest cases which will chiefly attract attention, and therefore the circumstance, that the system of Eulerian diagrams becomes too cumbrous and laborious in complicated problems, is no objection to it. While the student cannot be counselled to confine himself to any single method of representation, the system of Eulerian diagrams is probably the best of any single one for the purely non-relative analysis of thought. Thus, it at once directs attention to the circumstance that the syllogism may be considered as a special case of the inference from Fig. 63 to Fig. 64, where the blots may either be zero or crosses or one a zero and the other a cross. Another example of the analytical interest of the system lies in the higher particular propositions, where we see an evolution of the conception of multitude. Multitude, or maniness, is a property of *collections*. Now a collection is an *ens rationis*, or abstraction; and abstraction appears as the highest product of the development of the logic

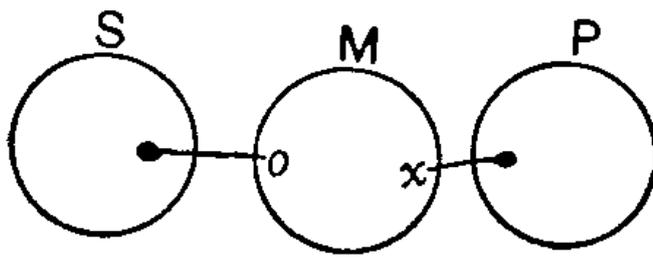


Fig. 63

[Click here to view]

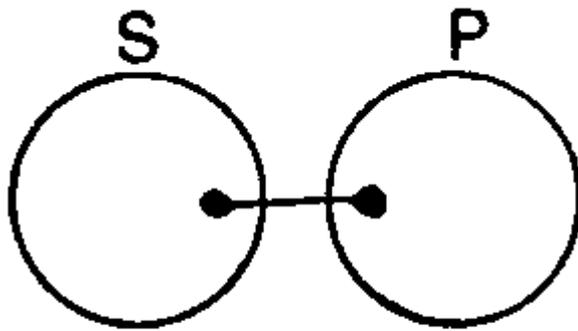


Fig. 64

[Click here to view]

Fig. 63 Fig. 64

of relatives. The student is thus directed to the deeply interesting and important problem of just how it is that the conception of multitude merges in the Eulerian diagrams.

Peirce: CP 4.371 Cross-Ref:††

371. The value of the system is thus considerable. Its fatal defect seems to be that it has no vital power of growth beyond the point to which it has here been carried. But this seeming may perhaps only be the reflection of the present student's own stupidity.

Peirce: CP 4.372 Cross-Ref:††
CHAPTER 2

SYMBOLIC LOGIC†1

372. If symbolic logic be defined as logic -- for the present only deductive logic -- treated by means of a special system of symbols, either devised for the purpose or extended to logical from other uses, it will be convenient not to confine the symbols used to algebraic symbols, but to include some graphical symbols as well.

Peirce: CP 4.373 Cross-Ref:††

373. The first requisite to understanding this matter is to recognize the purpose of a system of logical symbols. That purpose and end is simply and solely the investigation of the theory of logic, and not at all the construction of a calculus to aid the drawing of inferences. These two purposes are incompatible, for the reason that the system devised for the investigation of logic should be as analytical as possible, breaking up inferences into the greatest possible number of steps, and exhibiting them under the most general categories possible; while a calculus would aim, on the contrary, to reduce the number of processes as much as possible, and to specialize the symbols so as to adapt them to special kinds of inference. It should be recognized as a defect of a system intended for logical study that it has two ways of expressing the same fact, or any superfluity of symbols, although it would not be a serious fault for a calculus to have two ways of expressing a fact.

Peirce: CP 4.374 Cross-Ref:††

374. There must be operations of transformation. In that way alone can the symbol be shown determining its interpretant. In order that these operations should be as analytically represented as possible, each elementary operation should be either an insertion or an omission. Operations of commutation, like $xy \cdot yx$, may be dispensed with by not recognizing any order of arrangement as significant. Associative transformations, like $(xy)z \cdot x(yz)$, which is a species of commutation, will be dispensed with in the same way; that is, by recognizing an equiparant †1 as what it is, a symbol of an unordered set.

Peirce: CP 4.375 Cross-Ref:††

375. It will be necessary to recognize two different operations, because of the difference between the relation of a symbol to its object and to its interpretant. Illative transformation (the only transformation, relating solely to truth, that a system of symbols can undergo) is the passage from a symbol to an interpretant, generally a partial interpretant. But it is necessary that the interpretant shall be recognized without the actual transformation. Otherwise the symbol is imperfect. There must, therefore, be a sign to signify that an illative transformation would be possible. That is to say, we must not only be able to express "**A** therefore **B**," but "If **A** then **B**." The symbol must, besides, separately indicate its object. This object must be indicated by a sign, and the relation of this to the significant element of the symbol is that both are signs of the same object. This is an equiparant, or commutative relation. It is therefore necessary to have an operation combining two symbols as referring to the same object. This, like the other operation, must have its actual and its potential state. The former makes the symbol a proposition "**A** is **B**;" that is, "Something **A** stands for, **B** stands for." The latter expresses that such a proposition *might* be expressed, "This stands for something which **A** stands for and **B** stands for." These relations might be expressed in roundabout ways; but two operations would always be necessary. In Jevons's modification †2 of Boole's algebra the two operations are aggregation and composition. Then, using non-relative terms, "nothing" is defined as that term which aggregated with any term gives that term, while "what is" is that term which

compounded with any term gives that term. But here we are already using a third operation; that is, we are using the relation of equivalence; and this is a composite relation. And when we draw an inference, which we cannot avoid, since it is the end and aim of logic, we use still another. It is true that if our purpose were to make a calculus, the two operations, aggregation and composition, would go admirably together. Symmetry in a calculus is a great point, and always involves superfluity, as in homogeneous coördinates and in quaternions. Superfluities which bring symmetry are immense economies in a calculus. But for purposes of analysis they are great evils.

Peirce: CP 4.376 Cross-Ref:††

376. A proposition *de inesse* relates to a single state of the universe, like the present instant. Such a proposition is altogether true or altogether false. But it is a question whether it is not better to suppose a general universe, and to allow an ordinary proposition to mean that it is sometimes or possibly true. Writing down a proposition under certain circumstances asserts it. Let these circumstances be represented in our system of symbols by writing the proposition on a certain sheet. If, then, we write two propositions on this same sheet, we can hardly resist understanding that both are asserted. This, then, will be the mode of representing that there is something which the one and the other represent -- not necessarily the same quasi-instantaneous state of the universe, but the same universe. If writing **A** asserts that **A** may be true, and writing **B** that **B** may be true, then writing both together will assert that **A** may be true and that **B** may be true.

Peirce: CP 4.377 Cross-Ref:††

377. By a *rule* of a system of symbols is meant a permission under certain circumstances to make a certain transformation; and we are to recognize no transformations as elementary except writing down and erasing. From the conventions just adopted, it follows, as *Rule 1*, that *anything written down may be erased, provided the erasure does not visibly affect what else there may be which is written along with it.*

Peirce: CP 4.378 Cross-Ref:††

378. Let us suppose that two facts are so related that asserting the one gives us the right to assert the other, because if the former is true, the latter must be true. If **A** having been written, we can add **B**, we may then, by our first rule, erase **A**; and consequently **A** may be transformed into **B** by two steps. We shall need to express the fact that writing **A** gives us a right, under all circumstances, to add **B**. Since this is not a reciprocal relation, **A** and **B** must be written differently; and since neither is positively asserted, neither must be written so that the other could be erased without affecting it. We need some place on our sheet upon which we can write a proposition without asserting it. The present writer's habit is to cut it off from the main sheet by enclosing it within an oval line; but in order to facilitate the printing, we will here enclose it in square brackets. In order, then, to express "If **A** can under any circumstances whatever be true, **B** can under some circumstances be true," we must certainly enclose **A** in square brackets. But what are we to do with **B**? We are not to assert positively that **B** can be true; yet it is to be more than hypothetically set forth, as **A** is. It must certainly, in some fashion, be enclosed within the brackets; for were it detached from the brackets, the brackets with their enclosed **A** could, by Rule 1, be erased; while in fact the dependence upon **A** cannot be omitted without danger of falsity. It is to be remarked that, in case we can assert that "If **A** can be true, **B** can be true," then, *a fortiori*, we can assert that "If both **A** and **C** can be true, **B** can be true,"

no matter what proposition C may be. Consequently, we have, as Rule 2, that, *within brackets already written, anything whatever can be inserted*. But the fact that "If A can be true, B can be true" does not generally justify the assertion "If A can be true, both B and D are true"; yet our second rule would imply that, unless the B were cut off, in some way, from the main field within the brackets. We will therefore enclose B in parentheses, and express the fact that "If A can be true, B can be true" by

$[A(B)]$ or $[(B)A]$ or $[A/(B)]$, etc.

The arrangement is without significance. The fact that "If A can be true, both B and D can be true," or $[A(BD)]$, justifies the assertion that "If A is true B is true," or $[A(B)]$. Hence the permission of Rule 1 may be enlarged, and we may assert that anything unenclosed or enclosed both in brackets and parentheses can be erased if it is separate from everything else. Let us now ask what $[A]$ means. Rule 2 gives it a meaning; for by this rule $[A]$ implies $[A(X)]$, whatever proposition X may be. That is to say, that $[A]$ can be true implies that "If A can under any circumstances be true, then anything you like, X , may be true." But we may like to make X express an absurdity. This, then, is a *reductio ad absurdum* of A ; so that $[A]$ implies, for one thing, that A cannot under any circumstances be true. The question is, Does it express anything further? According to this, $[A(B)]$ expresses that $A(B)$ is impossible. But what is this? It is that A can be true while something expressed by (B) can be true. Now, what can it be that renders the fact that "If A can ever be true, B can sometimes be true" incompatible with A 's being able to be true? Evidently the falsity of B under all circumstances. Thus, just as $[A]$ implies that A can never be true, so (B) implies that B can never be true. But further, to say that $[A(B)]$, or "If A is ever true, B is sometimes true," is to say no more than that it is impossible that A is ever true, B being never true. Hence, the square brackets and the parentheses precisely deny what they enclose. A logical principle can be deduced from this: namely, if $[A]$ is true $[A(X)]$ is true. That is, if A is never true, then we have a right to assert that "If A is ever true, X is sometimes true," no matter what proposition X may be. Square brackets and parentheses, then, have the same meaning. Braces may be used for the same purpose.

Peirce: CP 4.379 Cross-Ref:††

379. Moreover, since two negatives make an affirmative, we have, as **Rule 3**, that *anything can have double enclosures added or taken away, provided there be nothing within one enclosure but outside the other*. Thus, if B can be true, so that B is written, Rule 3 permits us to write $[(B)]$, and then Rule 2 permits us to write $[X(B)]$. That is, if B is sometimes true, then "If X is ever true, B is sometimes true." Let us make the apodosis of a conditional proposition itself a conditional proposition. That is, in $(C\{D\})$ let us put for D the proposition $[A(B)]$. We thus have $(C\{[A(B)]\})$. But, by Rule 3, this is the same as $(CA(B))$.

Peirce: CP 4.380 Cross-Ref:††

380. All our transformations are analysed into insertions and omissions. That is, if from A follows B , we can transform A into $A B$ and then omit the B . Now, by Rule 1, from $A B$ follows A . Treating this in the same way, we first insert the conclusion and say that from $A B$ follows $A B A$. We thus get as **Rule 4** that *any detached portion of a proposition can be iterated*.

Peirce: CP 4.381 Cross-Ref:††

381. It is now time to reform Rule 2 so as to state in general terms the effect of enclosures upon permissions to transform. It is plain that if we have written $[A(B)]C$, we can write $[A(BC)]C$, although the latter gives us no right to the former. In place,

then, of Rule 2 we have:

Peirce: CP 4.381 Cross-Ref:††

Rule 2 (amended). Whatever transformation can be performed on a whole proposition can be performed upon any detached part of it under additional enclosures even in number, and the reverse transformation can be performed under additional enclosures odd in number.

Peirce: CP 4.381 Cross-Ref:††

But this rule does not permit every transformation which can be performed on a detached part of a proposition to be performed upon the same expression otherwise situated.

Peirce: CP 4.382 Cross-Ref:††

382. Rule 4 permits, by virtue of Rule 2 (amended), all iteration under additional enclosures and erasure of a term inside enclosures if it is iterated outside some of them.

Peirce: CP 4.383 Cross-Ref:††

383. We can now exhibit the *modi tollens et ponens*. Suppose, for example, we have these premisses: "If **A** is ever true, **B** is sometimes true," and "**B** is never true." Writing them, we have $[A(B)](B)$. By Rule 4, from (B) we might proceed to $(B)(B)$. Hence, by Rule 2 (amended), from $[A(B)](B)$ we can proceed to $[A](B)$, and by Rule 1 to $[A]$. That is, "**A** is never true." Suppose, on the other hand, our premisses are $[A(B)]$ and **A**. As before, we get $[(B)]A$, and by Rule 3, **B A**, and by Rule 1, **B**. That is, from the premisses of the *modus ponens* we get the conclusion. Let us take as premisses "If **A** is ever true, **B** is sometimes true," and "If **B** is ever true, **C** is sometimes true." That is, $(A\{B\})[B(C)]$. Then iterating $[B(C)]$ within two enclosures, we get $(A(B[B(C)]))[B(C)]$, or, by Rule 1, $(A\{B[B(C)]\})$. But we have just seen that $B[B(C)]$ can be transformed to **C**. Performing this under two enclosures, we get $(A\{C\})$, which is the conclusion, "If **A** is ever true, **C** is sometimes true." Let us now formally deduce the principle of contradiction $[A(A)]$. Start from any premiss **X**. By Rule 3 we can insert $[(X)]$, so that we have $X[(X)]$. By insertion under odd enclosures we have $X[A(X)]$. By iteration under additional enclosures we get $X[A(A X)]$; by erasures under even enclosures $[A(A)]$.

Peirce: CP 4.384 Cross-Ref:††

384. In complicated cases the multitude of enclosures become unmanageable. But by using ruled paper and drawing lines for the enclosures, composed of vertical and horizontal lines, always writing what is more enclosed lower than what is less enclosed, and what is evenly enclosed, on the left-hand part of the sheet, and what is oddly enclosed, on the right-hand part, this difficulty is greatly reduced. The diagram on page 325 (Fig. 65

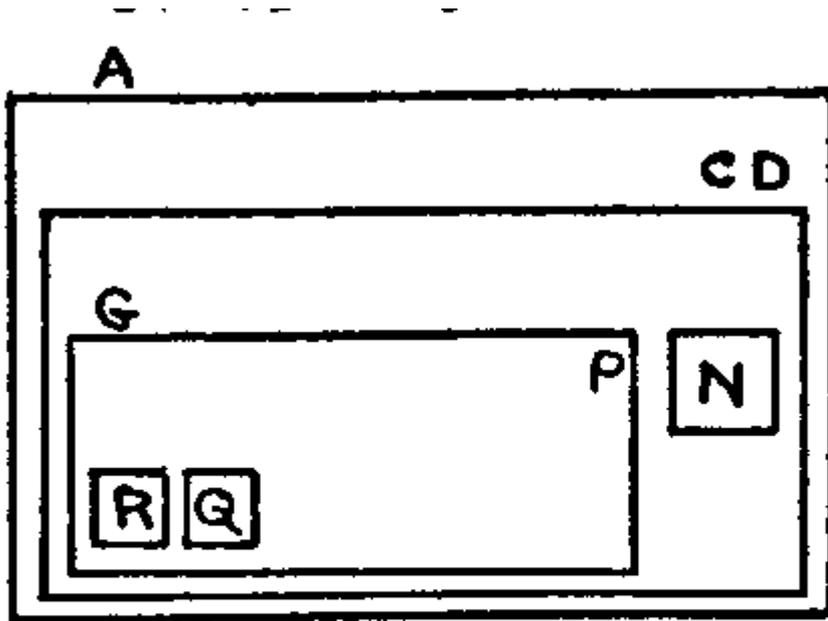


Fig. 65

[Click here

to view)] illustrates the general style of arrangement recommended.

Peirce: CP 4.385 Cross-Ref:††

385. It is now time to make an addition to our system of symbols. Namely, A B signifies that A is at some quasi-instant true, and that B is at some quasi-instant true. But we wish to be able to assert that A and B are true at the same quasi-instant. We should always study to make our representations iconoidal; and a very iconoidal way of representing that there is one quasi-instant at which both A and B are true will be to connect them with a heavy line drawn in any shape, thus:



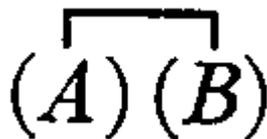
$A-B$ or

[Click here to view]

If this line be broken, thus $A- -B$, the identity ceases to be asserted. We have evidently:

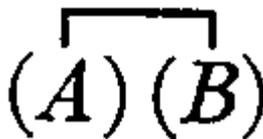
Peirce: CP 4.385 Cross-Ref:††

Rule 5. A line of identity may be broken where unenclosed. $-A$ will mean "At some quasi-instant A is true." It is equivalent to A simply. But $-(-A)$ will differ from $(-A)$ or (A) in merely asserting that at some quasi-instant A is not true, instead of asserting, with the latter forms, that at no quasi-instant is A true. Our quasi-instants may be individual things. In that case $-A$ will mean "Something is A "; $-(-A)$, "Something is not A "; $[-(-A)]$, "Everything is A "; $(-A)$, "Nothing is A ." So $A-B$ will express "Some A is B "; $(A-B)$, "No A is B "; $A-(-B)$, "Some A is not B "; $[A-(-B)]$,



"Whatever A there may be is B ";

[Click here to view] "There is



something besides A and B "; †1 [
"Everything is either A or B ."

[Click here to view]],

Peirce: CP 4.386 Cross-Ref: ††

386. The rule of iteration must now be amended as follows:

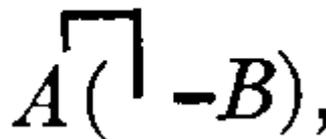
Peirce: CP 4.386 Cross-Ref: ††

Rule 4 (amended). *Anything can be iterated under the same enclosures or under additional ones, its identical connections remaining identical.*

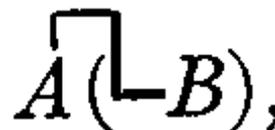
Peirce: CP 4.386 Cross-Ref: ††



Thus, $[A(-B)]$ can be transformed to $[A(A-B)]$. [Click here to view]. By the same rule $A--(-B)$, *i.e.*, "Something is A and nothing is B ," by



iteration of the line of identity, can be transformed to $A(-B)$, [Click here to view] *i.e.*, "Some A is not coexistent with anything that is B ," whence,



by Rules 5 and 2 (amended), it can be further transformed to $A(B)$, [Click here to view] *i.e.*, "Some A is not B ."

Peirce: CP 4.387 Cross-Ref: ††

387. But it must be most carefully observed that two unenclosed parts cannot be illatively united by a line of identity. The enclosure of such a line is that of its least enclosed part. We can now exhibit any ordinary syllogism. Thus, the premisses of *Baroko*, "Any M is P " and "Some S is not P ,"

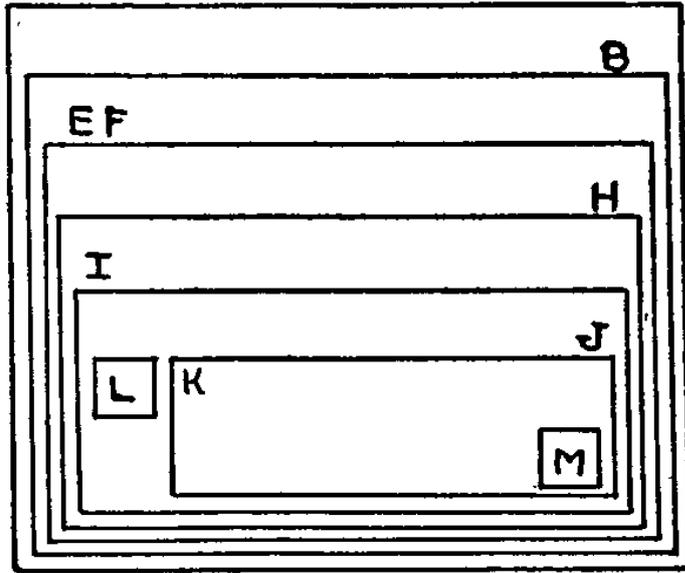


Fig. 66

Fig 66
view]

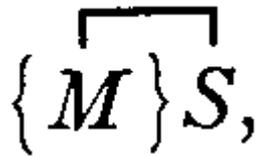
[Click here to

may be written $\{ \overbrace{M[P]} \} S(P).$ [Click here to view] Then, as

just seen, we can write $\{ \overbrace{M[P]} \} S(P).$ [Click here to

view] Then, by iteration, $\{ \overbrace{M[P(P)]} \} S(P).$ [Click here to view] Breaking the line under even enclosures, we get

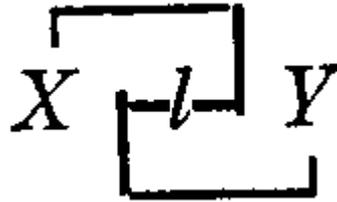
$\{ \overbrace{[P(P)]M} \} S(P).$ [Click here to view] But we have already shown that $[P(P)]$ can be written unenclosed. Hence it can be struck out under one enclosure; and the unenclosed (P) can be erased. Thus we get



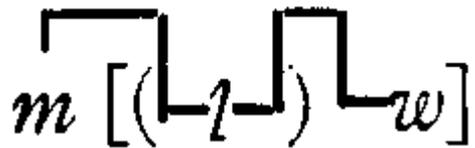
[Click here to view] or "Some *S* is not *M*." The great number of steps into which syllogism is thus analysed shows the perfection of the method for purposes of analysis.

Peirce: CP 4.388 Cross-Ref:††

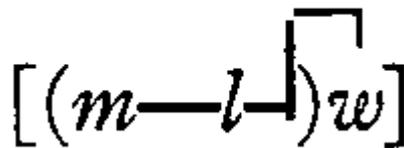
388. In taking account of relations, it is necessary to distinguish between the different sides of the letters. Thus let *l* be taken in such a sense that *X-l-Y* means "*X*



loves *Y*." Then [Click here to view] will mean "*Y* loves *X*." Then, if *m-* means "Something is a man," and *-w* means "Something is a woman," *m-l-w* will mean "Some man loves some woman";



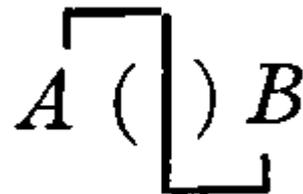
[Click here to view] will mean "Some man



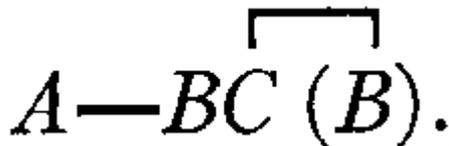
loves all women"; [Click here to view] will mean "Every woman is loved by some man," etc.

Peirce: CP 4.389 Cross-Ref:††

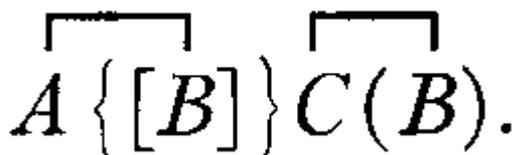
389. Since enclosures signify negation, by enclosing a part of the line of



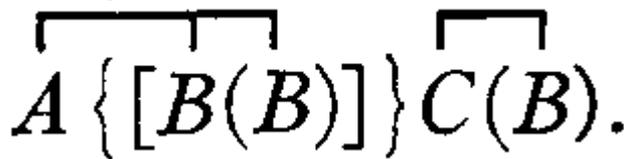
identity, the relation of otherness is represented. Thus, [Click here to view] will assert "Some *A* is not some *B*." Given the premisses "Some *A* is *B*" and "Some *C* is not *B*," they can be written



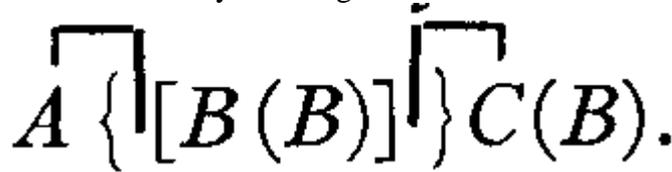
[Click here to view] By Rule 3, this can be written



[Click here to view]. By iteration, this gives



[Click here to view] The lines of identity are to be conceived as passing through the space between the braces outside of the brackets. By breaking the lines under even enclosures, we get

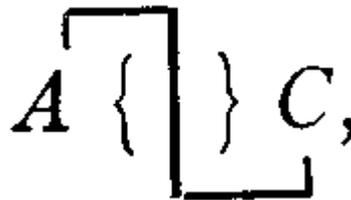


[Click here to view] As we have already seen, oddly enclosed $[B(B)]$ can be erased. This, with erasure of the



detached (B) , gives

[Click here to view] Joining the lines



under odd enclosures, we get
"Some A is not some C ."

[Click here to view] or

Peirce: CP 4.390 Cross-Ref:††

390. For all considerable steps in ratiocination, the reasoner has to treat qualities, or collections, (they only differ grammatically), and especially relations, or systems, as objects of relation about which propositions are asserted and inferences drawn. It is, therefore, necessary to make a special study of the logical relatives "-- is a member of the collection --," and "-- is in the relation to --." The key to all that amounts to much in symbolical logic lies in the symbolization of these relations. But we cannot enter into this extensive subject in this article.

Peirce: CP 4.391 Cross-Ref:††

391. The system, of which the slightest possible sketch has been given, is not so iconoidal as the so-called Euler's diagrams; but it is by far the best *general* system which has yet been devised. The present writer has had it under examination for five years with continually increasing satisfaction. However, it is proper to notice some other systems that are now in use. Two systems which are merely extensions of Boole's algebra of logic may be mentioned. One of these is called by no more proper designation than the "general algebra of logic."†1 The other is called "Peirce's algebra of dyadic relatives."†1 In the former there are two operations -- aggregation, which Jevons †2 (to whom its use in algebra is due) signifies by a sign of division turned on its side, thus $\cdot\cdot$. (I prefer to join the two dots, in order to avoid mistaking the single character for three); and composition, which is best signified by a somewhat heavy dot, $\cdot\cdot$.

Peirce: CP 4.391 Cross-Ref:††

Thus, if A and B are propositions, $A\cdot\cdot B$ is the proposition which is true if A is

true, is true if **B** is true, but is such that if **A** is false and **B** is false, it is false. **A·B** is the proposition which is true if **A** is true and **B** is true, but is false if **A** is false and false if **B** is false. Considered from an algebraical point of view, which is the point of view of this system, these expressions **A·|·B** and **A·B** are *mean functions*; for a mean function is defined as such a symmetrical function of several variables, that when the variables have the same value, it takes that same value. It is, therefore, wrong to consider them as addition and multiplication, unless it be that *truth* and *falsity*, the two possible states of a proposition, are considered as logarithmic infinity and zero. It is therefore well to let 0 represent a false proposition and ∞ (meaning logarithmic infinity, so that + ∞ and - ∞ are different) a true proposition. A heavy line, called an "obelus," over an expression negatives it.

Peirce: CP 4.391 Cross-Ref: ††

The letters **i, j, k**, etc., written below the line after letters signifying predicates, denote individuals, or supposed individuals, of which the predicates are true. Thus, **l[ij]** may mean that **i** loves **j**. To the left of the expression a series of letters π and Σ are written, each with a special one of the individuals **i, j, k** attached to it in order to show in what order these individuals are to be selected, and how. Σ[**i**] will mean that **i** is to be a suitably chosen individual, π[**j**] that **j** is any individual, no matter what. Thus,

$$\Sigma[\mathbf{i}]\pi[\mathbf{j}]\mathbf{l}[\mathbf{j}\mathbf{i}]$$

means that there is an individual **i** such that every individual **j** loves **i**; and

$$\pi[\mathbf{j}]\Sigma[\mathbf{i}]\mathbf{l}[\mathbf{j}\mathbf{i}]$$

will mean that taking any individual **j**, no matter what, there is some individual **i**, whom **j** loves. This is the whole of this system, which has considerable power. This use of Σ and π was probably first introduced by O. C. Mitchell in his epoch-making paper in *Studies in Logic*, †1 by members of the Johns Hopkins University.

Peirce: CP 4.392 Cross-Ref: ††

392. In Peirce's algebra of dyadic relatives the signs of aggregation and composition are used; but it is not usual to attach indices. In place of them two relative operations are used. Let **l** be "lover of," **s** "servant of." Then **ls**, called the relative product of **s** by **l**, denotes "lover of some servant of"; and **l†s**, called the relative sum of **l** to **s**, denotes "lover of whatever there may be besides servants of." In ms. the tail of the cross will naturally be curved. The sign **l** is used to mean "numerically identical with," and **†** to mean "other than." Schröder, who has written an admirable treatise on this system (though his characters are very objectionable, and should not be used †2), has considerably increased its power by various devices, and especially by writing, for example, π/**u** before an expression containing **u** to signify that **u** may be any relative whatever, or Σ/**u** to signify that it is a possible relative. In this way he introduces an abstraction or term of second intention.

Peirce: CP 4.393 Cross-Ref: ††

393. Peano has made considerable use of a system of logical symbolization of his own. Mrs. Ladd-Franklin †3 advocates eight copula-signs to begin with, in order to exhibit the equal claim to consideration of the eight propositional forms. Of these she chooses "No **a** is **b**" and "Some **a** is **b**" (**a**(~∨)**b** and **a**∨**b**) as most desirable for the

elements of an algorithmic scheme; they are both symmetrical and natural. She thinks that a symbolic logic which takes "All *a* is *b*" (Boole, Schröder) as its basis is cumbrous; for every statement of a theorem, there is a corresponding statement necessary in terms of its contrapositive. This, she says, is the source of the parallel columns of theorems in Schröder's *Logik*; a single set of theorems is all-sufficient if a symmetrical pair of copulas is chosen. Some logicians (as C. S. P.) think the objections to Mrs. Ladd-Franklin's system outweigh its advantages. Other systems, as that of Wundt,†4 show a complete misunderstanding of the problem.

Peirce: CP 4.394 Cross-Ref:††
CHAPTER 3

EXISTENTIAL GRAPHS†1

A. THE CONVENTIONS

§1. ALPHA PART

394. *Convention No. Zero.* Any feature of these diagrams that is not expressly or by previous conventions of languages required by the conventions to have a given character may be varied at will. This "convention" is numbered zero, because it is understood in all agreements.

Peirce: CP 4.395 Cross-Ref:††

395. *Convention No. I.* These Conventions are supposed to be mutual understandings between two persons: a *Graphist*, who expresses propositions according to the system of expression called that of *Existential Graphs*, and an *Interpreter*, who interprets those propositions and accepts them without dispute.

Peirce: CP 4.395 Cross-Ref:††

A *graph* is the propositional expression in the System of Existential Graphs of any possible state of the universe. It is a Symbol,†2 and, as such, general, and is accordingly to be distinguished from a *graph-replica*.†P1 A graph remains such though not actually asserted. An expression, according to the conventions of this system, of an impossible state of things (conflicting with what is taken for granted at the outset or has been asserted by the graphist) is not a graph, but is termed *The pseudograph*, all such expressions being equivalent in their absurdity.

Peirce: CP 4.396 Cross-Ref:††

396. It is agreed that a certain sheet, or blackboard, shall, under the name of *The Sheet of Assertion*, be considered as representing the universe of discourse, and as asserting whatever is taken for granted between the graphist and the interpreter to be true of that universe. The sheet of assertion is, therefore, a graph. Certain parts of the sheet, which may be severed from the rest, will not be regarded as any part of it.

Peirce: CP 4.397 Cross-Ref:††

397. The graphist may place replicas of graphs upon the sheet of assertion; but this act, called *scribing* a graph on the sheet of assertion, shall be understood to constitute the assertion of the truth of the graph scribed. (Since by 395 the conventions are only "supposed to be" agreed to, the assertions are mere pretence in studying logic. Still they may be regarded as actual assertions concerning a fictitious universe.) "Assertion" is not defined; but it is supposed to be permitted to scribe some graphs and not others.

Peirce: CP 4.397 Cross-Ref:††

Corollary. Not only is the sheet itself a graph, but so likewise is the sheet together with the graph scribed upon it. But if the sheet be blank, this *blank*, whose existence consists in the absence of any scribed graph, is itself a graph.

Peirce: CP 4.398 Cross-Ref:††

398. *Convention No. II*. A graph-replica on the sheet of assertion having no scribed connection with any other graph-replica that may be scribed on the sheet shall, as long as it is on the sheet of assertion in any way, make the same assertion, regardless of what other replicas may be upon the sheet.

Peirce: CP 4.398 Cross-Ref:††

The graph which consists of all the graphs on the sheet of assertion, or which consists of all that are on any one area severed from the sheet, shall be termed the *entire* graph of the sheet of assertion or of that area, as the case may be. Any part of the entire graph which is itself a graph shall be termed a *partial* graph of the sheet or of the area on which it is.

Peirce: CP 4.398 Cross-Ref:††

Corollaries. Two graphs scribed on the sheet are, both of them, asserted, and any entire graph implies the truth of all its partial graphs. Every blank part of the sheet is a partial graph.

Peirce: CP 4.399 Cross-Ref:††

399. *Convention No. III*. By a *Cut* shall be understood to mean a self-returning linear separation (naturally represented by a fine-drawn or peculiarly colored line) which severs all that it encloses from the sheet of assertion on which it stands itself, or from any other area on which it stands itself. The whole space within the cut (but not comprising the cut itself) shall be termed the *area* of the cut. Though the area of the cut is no part of the sheet of assertion, yet the cut together with its area and all that is on it, conceived as so severed from the sheet, shall, under the name of the *enclosure* of the cut, be considered as on the sheet of assertion or as on such other area as the cut may stand upon. Two cuts cannot intersect one another, but a cut may exist on any area whatever. Any graph which is unenclosed or is enclosed within an even number of cuts shall be said to be *evenly enclosed*; and any graph which is within an odd number of cuts shall be said to be *oddly enclosed*. A cut is not a graph; but an enclosure is a graph. The sheet or other area on which a cut stands shall be called the *place* of the cut.

Peirce: CP 4.400 Cross-Ref:††

400. A pair of cuts, one within the other but not within any other cut that that other is not within, shall be called a scroll. The outer cut of the pair shall be called the *outloop*, the inner cut the *inloop*, of the scroll. The area of the inloop shall be termed

the *inner close* of the scroll; the area of the outloop, *excluding the enclosure of the inloop* (and not merely its area), shall be termed the *outer close* of the scroll.

Peirce: CP 4.401 Cross-Ref:††

401. The enclosure of a scroll (that is, the enclosure of the outer cut of the pair) shall be understood to be a graph having such a meaning that if it were to stand on the sheet of assertion, it would assert *de inesse* that if the entire graph in its outer close is true, then the entire graph in its inner close is true. No graph can be scribed across a cut, in any way; although an enclosure is a graph.

(A conditional proposition *de inesse* considers only the existing state of things, and is, therefore, false only in case the consequent is false while the antecedent is true. If the antecedent is false, or if the consequent is true, the conditional *de inesse* is true.)

Peirce: CP 4.402 Cross-Ref:††

402. The filling up of any entire area with whatever writing material (ink, chalk, etc.) may be used shall be termed *obliterating* that area, and shall be understood to be an expression of the pseudograph on that area.

Peirce: CP 4.402 Cross-Ref:††

Corollary. Since an obliterated area may be made indefinitely small, a single cut will have the effect of denying the entire graph in its area. For to say that if a given proposition is true, everything is true, is equivalent to denying that proposition.

Peirce: CP 4.403 Cross-Ref:††
§2. BETA PART

403. *Convention No. IV.* The expression of a rheme in the system of existential graphs, as simple, that is without any expression, according to these conventions, of the analysis of its signification, and such as to occupy a superficial portion of the sheet or of any area shall be termed a *spot*. The word "spot" is to be used in the sense of a *replica*; and when it is desired to speak of the symbol of which it is the replica, this shall be termed a *spot-graph*. On the periphery of every spot, a certain place shall be appropriated to each blank of the rheme; and such a place shall be called a *hook* of the spot. No spot can be scribed except wholly in some area.

Peirce: CP 4.404 Cross-Ref:††

404. A heavy *dot* scribed at the hook of a spot shall be understood as filling the corresponding blank of the rheme of the spot with an indefinite sign of an individual, so that when there is a dot attached to every hook, the result shall be a proposition which is particular in respect to every subject.

Peirce: CP 4.405 Cross-Ref:††

405. *Convention No. V.* Every heavily marked point, whether isolated, the extremity of a heavy line, or at a furcation of a heavy line, shall denote a single individual, without in itself indicating what individual it is.

Peirce: CP 4.406 Cross-Ref:††

406. A heavily marked line without any sort of interruption (though its extremity may coincide with a point otherwise marked) shall, under the name of *a*

line of identity, be a graph, subject to all the conventions relating to graphs, and asserting precisely the identity of the individuals denoted by its extremities.

Peirce: CP 4.406 Cross-Ref:††

Corollaries. It follows that no line of identity can cross a cut.

Peirce: CP 4.406 Cross-Ref:††

Also, a point upon which three lines of identity abut is a graph expressing the relation of *teridentity*.

Peirce: CP 4.407 Cross-Ref:††

407. A heavily marked point may be on a cut; and such a point shall be interpreted as lying in the place of the cut and at the same time as denoting an individual identical with the individual denoted by the extremity of a line of identity on the area of the cut and abutting upon the marked point on the cut. Thus, in Fig. 67,

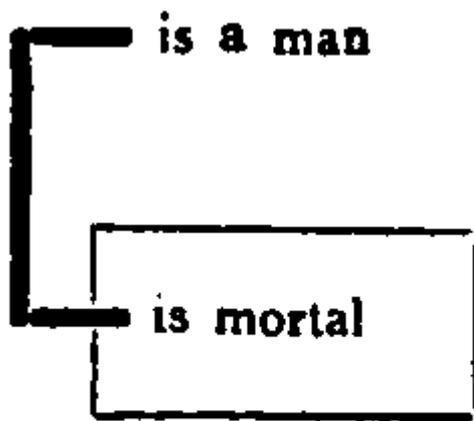


Fig. 67

[Click here to view] if we refer to the individual denoted by the point where the two lines meet on the cut, as X, the assertion is, "Some individual, X, of the universe is a man, and nothing is at once mortal and identical with X"; *i.e.*, some man is not mortal. So in Fig. 68

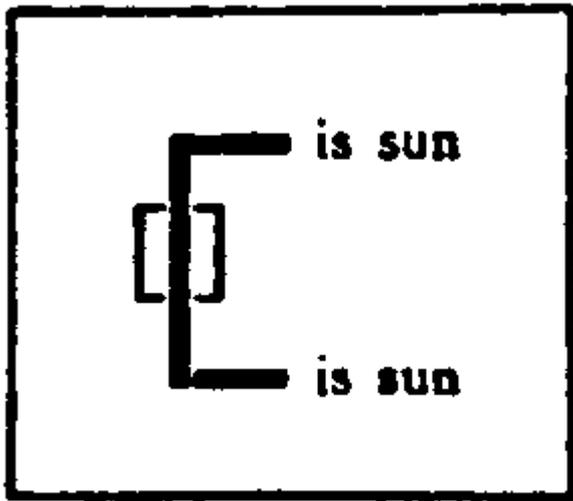


Fig. 68

[Click here to view], if X and Y are the individuals denoted by the points on the [inner] cut, the interpretation is,

Peirce: CP 4.407 Cross-Ref:††

"If X is the sun and Y is the sun, X and Y are identical."

Peirce: CP 4.407 Cross-Ref:††

A collection composed of any line of identity together with all others that are connected with it directly or through still others is termed a *ligature*. Thus ligatures often cross cuts, and, in that case, are not graphs.

Peirce: CP 4.408 Cross-Ref:††

408. *Convention No. VI.* A symbol for a single individual, which individual is more than once referred to, but is not identified as the object of a proper name, shall be termed a *Selective*. The capital letters may be used as selectives, and may be made to abut upon the hooks of spots. Any ligature may be replaced by replicas of one selective placed at every hook and also in the outermost area that it enters. In the interpretation, it is necessary to refer to the outermost replica of each selective first, and generally to proceed in the interpretation from the outside to the inside of all cuts.

Peirce: CP 4.409 Cross-Ref:††

§3. GAMMA PART

409. *Convention No. VII.* The following spot-symbols shall be used, as if they were ordinary spot-symbols, except for special rules applicable to them: (Selectives are placed against the hooks in order to render the meanings of the new spot-symbols clearer).

A[*q*], A is a monadic character;

A[*r*], A is a dyadic relation;

A[s], A is a triadic relation;

X /0\, X is a proposition or fact;

X /1\ Y, Y possesses the character X;

X /2\ Y/Z, Y stands in the dyadic relation X to Z;

X Y//3\W /Z, Y stands in the triadic relation X to Z for W.

Peirce: CP 4.410 Cross-Ref:††

410. **Convention No. VIII.** A cut with many little interruptions †1 aggregating about half its length shall cause its enclosure to be a graph, expressing that the entire graph on its area is logically contingent (non-necessary).

Peirce: CP 4.411 Cross-Ref:††

411. **Convention No. IX.** By a *rim* shall be understood an oval line making it, with its contents, the expression either of a rheme or a proper name of an *ens rationis*. Such a rim may be drawn as a line of peculiar texture, or a gummed label with a colored border may be attached to the sheet. A dotted rim containing a graph, some part of which is itself enclosed by a similar inner dotted oval and with heavy dotted lines proceeding from marked points of this graph to hooks on the rim, shall be a spot expressing that the individuals denoted by lines of identity attached to the hooks (or the single such individual) have the character, constituted by the truth of the graph, to be possessed by the individuals denoted by those points of it to which the heavy dotted lines are attached, in so far as they are connected with the partial graph within the inner oval.

Peirce: CP 4.412 Cross-Ref:††

412. A rim represented by a wavy line containing a graph, of which some marked points are connected by wavy lines with hooks on the rim, shall be a spot expressing that the individuals denoted by lines of identity abutting on these hooks form a collection of sets, of which collection each set has its members characterized in the manner in which those individuals must be which are denoted by the points of attachment of the interior graph, when that graph is true.

Peirce: CP 4.413 Cross-Ref:††

413. A rim shown as a saw line denotes an individual collection of individual single objects or sets of objects, the members of the collection being all those in existence, which are such individuals as the truth of the graph within makes those to be that are denoted by points of attachment of that graph to saw lines passing to hooks of the rim.

Peirce: CP 4.414 Cross-Ref:††
B. RULES OF TRANSFORMATION

***Pure Mathematical Definition of Existential Graphs,
Regardless of Their Interpretation***

§1. ALPHA PART

414. 1. The *System of Existential Graphs* is a certain class of diagrams upon which it is permitted to operate certain transformations.

2. There is required a certain surface upon which it is practicable to scribe the diagrams and from which they can be erased in whole or in part.

3. The whole of this surface except certain parts which may be severed from it by "cuts" is termed the *sheet of assertion*.

4. A *graph* is a legisign (*i.e.*, a sign which is of the nature of a general type) which is one of a certain class of signs used in this system. A *graph-replica* is any individual instance of a graph. The sheet of assertion itself is a graph-replica; and so is any part of it, being called the *blank*. Other graph-replicas can be scribed on the sheet of assertion, and when this is done the graphs of which those graph-replicas are instances is said to be "scribed on the sheet of assertion"; and when a graph-replica is erased, the graph is said to be erased. Two graphs scribed on the sheet of assertion constitute one graph of which they are said to be *partial graphs*. All that is at any time scribed on the sheet of assertion is called the *entire scribed graph*.

5. A *cut* is a self-returning finely drawn line. A cut is not a graph-replica. A cut drawn upon the sheet of assertion severs the surface it encloses, called the *area* of the cut, from the sheet of assertion; so that the area of a cut is no part of the sheet of assertion. A cut drawn upon the sheet of assertion together with its area and whatever is scribed upon that area constitutes a graph-replica scribed upon the sheet of assertion, and is called the *enclosure* of the cut. Whatever graph might, if permitted, be scribed upon the sheet of assertion might (if permitted) be scribed upon the area of any cut. Two graphs scribed at once on such area constitute a graph, as they would on the sheet of assertion. A cut can (if permitted) be drawn upon the area of any cut, and will sever the surface which it encloses from the area of the cut, while the enclosure of such inner cut will be a graph-replica scribed on the area of the outer cut. The sheet of assertion is also an area. Any blank part of any area is a graph-replica. Two cuts one of which has the enclosure of the other on its area and has nothing else there constitute a *double cut*.

6. No graph or cut can be placed partly on one area and partly on another.†1

7. No transformation of any graph-replica is permitted unless it is justified by the following code of Permissions.

Peirce: CP 4.415 Cross-Ref:††
Code of Permissions

415. *Permission No. 1. In each special problem such graphs may be scribed on the sheet of assertion as the conditions of the special problem may warrant.*

Peirce: CP 4.415 Cross-Ref:††

Permission No. 2. Any graph on the sheet of assertion may be erased, except

an enclosure with its area entirely blank.

Peirce: CP 4.415 Cross-Ref:††

Permission No. 3. Whatever graph it is permitted to scribe on the sheet of assertion, it is permitted to scribe on any unoccupied part of the sheet of assertion, regardless of what is already on the sheet of assertion.

Peirce: CP 4.415 Cross-Ref:††

Permission No. 4. Any graph which is scribed on the inner area of a double cut on the sheet of assertion may be scribed on the sheet of assertion.

Peirce: CP 4.415 Cross-Ref:††

Permission No. 5. A double cut may be drawn on the sheet of assertion; and any graph that is scribed on the sheet of assertion may be scribed on the inner area of any double cut on the sheet of assertion.

Peirce: CP 4.415 Cross-Ref:††

Permission No. 6. The reverse of any transformation that would be permissible on the sheet of assertion is permissible on the area of any cut that is upon the sheet of assertion.

Peirce: CP 4.415 Cross-Ref:††

Permission No. 7. Whenever we are permitted to scribe any graph we like upon the sheet of assertion, we are authorized to declare that the conditions of the special problem are absurd.

Peirce: CP 4.416 Cross-Ref:††

§2. BETA PART

416. 8. The beta part adds to the alpha part certain signs to which new permissions are attached, while retaining all the alpha signs with the permissions attaching to them.

9. The *line of identity* is a Graph any replica of which, also called a line of identity, is a heavy line with two ends and without other topical singularity (such as a point of branching or a node), not in contact with any other sign except at its extremities. Otherwise, its shape and length are matters of indifference. All lines of identity are replicas of the same graph.

10. A *spot* is a graph any replica of which occupies a simple bounded portion of a surface, which portion has qualities distinguishing it from the replica of any other spot; and upon the boundary of the surface occupied by the spot are certain points, called the *hooks* of the spot, to each of which, if permitted, one extremity of one line of identity can be attached. Two lines of identity cannot be attached to the same hook; nor can both ends of the same line.

11. Any indefinitely small dot may be a spot replica called a *spot of teridentity*, and three lines of identity may be attached to such a spot. Two lines of identity, one outside a cut and the other on the area of the same cut, may have each an extremity at the same point on the cut. The totality of all the lines of identity that join one another is termed a *ligature*. A ligature is not generally a graph, since it may be part in one area and part in another. It is said to lie within any cut which it is wholly within.

Peirce: CP 4.417 Cross-Ref:††

417. 12. The following are the additional permissions attaching to the beta part.

Code of Permissions -- Continued

Permission No. 8. All the above permissions apply to all spots and to the line of identity, as Graphs; and Permission No. 2 is to be understood as permitting the erasure of any portion of a line of identity on the sheet of assertion, so as to break it into two. Permission No. 3 is to be understood as permitting the extension of a line of identity on the sheet of assertion to any unoccupied part of the sheet of assertion. Permission No. 3 must not be understood [as stating that] that because it is permitted to scribe a graph without certain ligatures therefore it is permissible to scribe it with them, or the reverse.

Peirce: CP 4.417 Cross-Ref:††

Permission No. 9. It is permitted to scribe an unattached line of identity on the sheet of assertion, and to join such unattached lines in any number by spots of teridentity. This is to be understood as permitting a line of identity, whether within or without a cut, to be extended to the cut, although such extremity is to be understood to be on both sides of the cut. But this does not permit a line of identity within a cut that is on the sheet of assertion to be retracted from the cut, in case it extends to the cut.

Peirce: CP 4.417 Cross-Ref:††

Permission No. 10. If two spots are within a cut (whether on its area or not), and are not joined by any ligature within that cut, then a ligature joining them outside the cut is of no effect and may be made or broken. But this does not apply if the spots are joined by other hooks within the cut.†1

Peirce: CP 4.417 Cross-Ref:††

Permission No. 11. Permissions Nos. 4 and 5 do not cease to apply because of ligatures passing from without the outer of two cuts to within the inner one, so long as there is nothing else in the annular area.†2

Peirce: CP 4.418 Cross-Ref:††

CHAPTER 4

ON EXISTENTIAL GRAPHS, EULER'S DIAGRAMS,

AND LOGICAL ALGEBRA†1P

§INTRODUCTION

418. A *diagram* is a representamen †2 which is predominantly an icon of

relations and is aided to be so by conventions. Indices are also more or less used. It should be carried out upon a perfectly consistent system of representation, founded upon a simple and easily intelligible basic idea.

Peirce: CP 4.419 Cross-Ref:††

419. A **graph** is a superficial diagram composed of the sheet upon which it is written or drawn, of spots or their equivalents, of lines of connection, and (if need be) of enclosures. The type, which it is supposed more or less to resemble, is the structural formula of the chemist.

Peirce: CP 4.420 Cross-Ref:††

420. A **logical graph** is a graph representing logical relations iconically, so as to be an aid to logical analysis.

Peirce: CP 4.421 Cross-Ref:††

421. An **existential graph** is a logical graph governed by a system of representation founded upon the idea that the sheet upon which it is written, as well as every portion of that sheet, represents one recognized universe, real or fictive, and that every graph drawn on that sheet, and not cut off from the main body of it by an enclosure, represents some fact existing in that universe, and represents it independently of the representation of another such fact by any other graph written upon another part of the sheet, these graphs, however, forming one composite graph.

Peirce: CP 4.422 Cross-Ref:††

422. No other system of existential graphs than that herein set forth having hitherto been proposed, this one will need, for the present, no more distinctive designation. Should such designation hereafter become desirable, I desire that this system should be called the Existential System of 1897, in which year I wrote an account of it and offered it for publication to the Editor of *The Monist*, who declined it on the ground that it might later be improved upon. No changes have been found desirable since that date, although it has been under continual examination; but the exposition has been rendered more formal.

Peirce: CP 4.423 Cross-Ref:††

423. The following exposition of this system will be arranged as follows:

Peirce: CP 4.423 Cross-Ref:††

Part I will explain the expression of ordinary forms of language in graphs and the interpretation of the latter into the former in three sections, as follows:

Peirce: CP 4.423 Cross-Ref:††

A will state all the fundamental conventions of the system, separating those which are essentially different, showing the need which each is designed to meet together with the reasons for meeting it by the particular convention chosen, so far as these can be given at this stage of the development. A complete discussion will be given in an Appendix †1 to this part. To aid the understanding of all this, various logical analyses will be interspersed where they become pertinent.

Peirce: CP 4.423 Cross-Ref:††

B will enunciate other rules of interpretation whose validity will be demonstrated from the fundamental conventions as premisses. This section will also introduce certain modifications of some of the signs established in **A**, the modified signs being convenient, although good reasons forbid their being considered fundamental.

Peirce: CP 4.423 Cross-Ref:††

C will redescribe the system in a compact form, which, on account of its uniting into one many rules that had, in the first instance; to be considered separately, is more easily grasped and retained in the mind.

Peirce: CP 4.423 Cross-Ref:††

Part II will develop formal "rules," or permissions, by which one graph may be transformed into another without danger of passing from truth to falsity and without recurring to any interpretation of the graphs; such transformations being of the nature of immediate inferences. The part will be divided into sections corresponding to those of Part I.

Peirce: CP 4.423 Cross-Ref:††

A will prove the basic rules of transformation directly from the fundamental conventions of *A* of Part I.

Peirce: CP 4.423 Cross-Ref:††

B will deduce further rules of transformation from those of *A*, without further recourse to the principles of transformation.

Peirce: CP 4.423 Cross-Ref:††

C will restate the rules in more compact form.

Peirce: CP 4.423 Cross-Ref:††

Part III will show how the system may be made useful.†1

Peirce: CP 4.424 Cross-Ref:††
PART I. PRINCIPLES OF INTERPRETATION^P

A. Fundamental Conventions^P

§1. OF CONVENTIONS NOS. 1 AND 2†1^P

424. In order to understand why this system of expression has the construction it has, it is indispensable to grasp the precise purpose of it, and not to confuse this with four other purposes, to wit:

Peirce: CP 4.424 Cross-Ref:††

First, although the study of it and practice with it will be highly useful in helping to train the mind to accurate thinking, still that consideration has not had any influence in determining the characters of the signs employed; and an exposition of it, which should have that aim, ought to be based upon psychological researches of which it is impossible here to take account.

Peirce: CP 4.424 Cross-Ref:††

Second, this system is not intended to serve as a universal language for mathematicians or other reasoners, like that of Peano.

Peirce: CP 4.424 Cross-Ref:††

Third, this system is not intended as a calculus, or apparatus by which conclusions can be reached and problems solved with greater facility than by more familiar systems of expression. Although some writers †2 have studied the logical algebras invented by me with that end apparently in view, in my own opinion their structure, as well as that of the present system, is quite antagonistic to much utility of that sort. The principal desideratum in a calculus is that it should be able to pass with security at one bound over a series of difficult inferential steps. What these abbreviated inferences may best be, will depend upon the special nature of the subject under discussion. But in my algebras and graphs, far from anything of that sort being attempted, the whole effort has been to dissect the operations of inference into as many distinct steps as possible.

Peirce: CP 4.424 Cross-Ref:††

Fourth, although there is a certain fascination about these graphs, and the way they work is pretty enough, yet the system is not intended for a plaything, as logical algebra has sometimes been made, but has a very serious purpose which I proceed to explain.

Peirce: CP 4.425 Cross-Ref:††

425. Admirable as the work of research of the special sciences -- physical and psychical -- is, as a whole, the reasoning [employed in them] is of an elementary kind except when it is mathematical, and it is not infrequently loose. The philosophical sciences are greatly inferior to the special sciences in their reasoning. Mathematicians alone reason with great subtlety and great precision. But hitherto nobody has succeeded in giving a thoroughly satisfactory logical analysis of the reasoning of mathematics. That is to say, although every step of the reasoning is evidently such that the collective premisses cannot be true and yet the conclusion false, and although for each such step, A, we are able to draw up a self-evident general rule that from a premiss of such and such a form such and such a form of conclusion will necessarily follow, this rule covering the particular inferential step, A, yet nobody has drawn up a complete list of such rules covering all mathematical inferences. It is true that mathematics has its calculus which solves problems by rules which are fully proved; but, in the first place, for some branches of the calculus those proofs have not been reduced to self-evident rules, and in the second place, it is only routine work which can be done by simply following the rules of the calculus, and every considerable step in mathematics is performed in other ways.

Peirce: CP 4.426 Cross-Ref:††

426. If we consult the ordinary treatises on logic for an account of necessary reasoning, all the help that they afford is the rules of syllogism. They pretend that ordinary syllogism explains the reasoning of mathematics; and books have professed to exhibit considerable parts of the reasoning of the first book of Euclid's *Elements* stated in the form of syllogisms. But if this statement is examined, it will be found that it represents transformations of statements to be made that are not reduced to strict syllogistic form; and on examination it will be found that it is precisely in these transformations that the whole gist of the reasoning lies. The nearest approach to a logical analysis of mathematical reasoning that has ever been made was Schröder's statement, with improvements, in a logical algebra of my invention, of Dedekind's reasoning (itself in a sort of logical form) concerning the foundations of arithmetic.†1 But though this relates only to an exceptionally simple kind of mathematics, my opinion -- quite against my natural leanings toward my own creation -- is that the soul

of the reasoning has even here not been caught in the logical net.

Peirce: CP 4.427 Cross-Ref:††

427. No other book has, during the nineteenth century, been deeply studied by so large a proportion of the strong intellects of the civilized world as Kant's *Critic of the Pure Reason*; and the reason has undoubtedly been that they have all been greatly struck by Kant's logical power. Yet Kant, for all this unquestionable power, had paid so little attention to logic that he makes it manifest that he supposed that ordinary syllogism explains mathematical reasoning, and indeed [in] the simplest mood of syllogism, *Barbara*. Now, at the very utmost, from n propositions only $1/4n^2$ conclusions can be drawn by *Barbara*. In the thirteen books of Euclid's Elements there [are] 14 premisses (5 postulates and 9 axioms) excluding the definitions, which are merely verbal. Therefore, even if these premisses were related to one another in the most favorable way, which is far from being the case, there could only be 49 conclusions from them. But Euclid draws over ten times that number (465 propositions, 27 corollaries, and 17 lemmas) besides which his editors have inserted hundreds of corollaries. There are 48 propositions in the first book. Moreover, in *Barbara* or any sorites, or complexus of such syllogisms, to introduce the same premiss twice is idle. But throughout mathematics the same premisses are used over and over again. Moreover a person of fairly good mind and some logical training will instantly see the syllogistic conclusions from any number of premisses. But this is far from being true of mathematical inferences.

Peirce: CP 4.428 Cross-Ref:††

428. There is reason to believe that a thorough understanding of the nature of mathematical reasoning would lead to great improvements in mathematics. For when a new discovery is made in mathematics, the demonstration first found is almost always replaced later by another much simpler. Now it may be expected that, if the reasoning were thoroughly understood, the unnecessary complications of the first proof would be eliminable at once. Indeed, one might expect that the shortest route would be taken at the outset. Then again, consider the state of topical geometry, or geometrical topics, otherwise called topology. Here is a branch of geometry which not only leaves out of consideration the proportions of the different dimensions of figures and the magnitudes of angles (as does also graphics, or projective geometry -- perspective, etc.) but also leaves out of account the straightness or mode of curvature of lines and the flatness or mode of bending of surfaces, and confines itself entirely to the connexions of the parts of figures (distinguishing, for example, a ring from a ball). Ordinary metric geometry equally depends on the connections of parts; but it depends on much besides. It, therefore, is a far more complicated subject, and can hardly fail to be of its own nature much the more difficult. And yet geometrical topics stands idle with problems to all appearance very simple staring it unsolved in the face, merely because mathematicians have not found out how to reason about it. Now a thorough understanding of mathematical reasoning must be a long stride toward enabling us to find a method of reasoning about this subject as well, very likely, as about other subjects that are not even recognized to be mathematical.

Peirce: CP 4.429 Cross-Ref:††

429. This, then, is the purpose for which my logical algebras were designed but which, in my opinion, they do not sufficiently fulfill. The present system of existential graphs is far more perfect in that respect, and has already taught me much about mathematical reasoning. Whether or not it will explain all mathematical inferences is not yet known.

Peirce: CP 4.429 Cross-Ref:††

Our purpose, then, is to study the workings of necessary inference. What we want, in order to do this, is a method of representing diagrammatically any possible set of premisses, this diagram to be such that we can observe the transformation of these premisses into the conclusion by a series of steps each of the utmost possible simplicity.

Peirce: CP 4.430 Cross-Ref:††

430. What we have to do, therefore, is to form a perfectly consistent method of expressing any assertion diagrammatically. The diagram must then evidently be something that we can see and contemplate. Now what we see appears spread out as upon a sheet. Consequently our diagram must be drawn upon a sheet. We must appropriate a sheet to the purpose, and the diagram drawn or written on the sheet is to express an assertion. We can, then, approximately call this sheet our *sheet of assertion*. The *entire graph*, or all that is drawn on the sheet, is to express a proposition, which the act of writing is to assert.

Peirce: CP 4.431 Cross-Ref:††

431. But what are our assertions to be about? The answer must be that they are to be about an arbitrarily hypothetical universe, a creation of a mind. For it is *necessary* reasoning alone that we intend to study; and the necessity of such reasoning consists in this, that not only does the conclusion happen to be true of a pre-determinate universe, but *will* be true, so long as the premisses are true, howsoever the universe may subsequently turn out to be determined. Thus, conformity to an *existing*, that is, entirely determinate, universe does not make necessity, which consists in what always *will be*, that is, what is determinately true of a universe not yet entirely determinate. Physical necessity consists in the fact that whatever may happen will conform to a law of nature; and logical necessity, which is what we have here to deal with, consists of something being determinately true of a universe not entirely determinate as to what is true, and thus not *existent*. In order to fix our ideas, we may imagine that there are two persons, one of whom, called the *grapheus*, creates the universe by the continuous development of his idea of it, every interval of time during the process adding some *fact* to the universe, that is, affording justification for some assertion, although, the process being continuous, these facts are not distinct from one another in their mode of being, as the propositions, which state some of them, are. As fast as this process in the mind of the grapheus takes place, that which is thought acquires *being*, that is, *perfect definiteness*, in the sense that the effect of what, is thought in any lapse of time, however short, is definitive and irrevocable; but it is not until the whole operation of creation is complete that the universe acquires *existence*, that is, *entire determinateness*, in the sense that nothing remains undecided. The other of the two persons concerned, called the *graphist*, is occupied during the process of creation in making *successive* modifications (*i.e.*, not by a continuous process, since each modification, unless it be final, has another that follows *next* after it), of the entire graph. Remembering that the entire graph is *whatever* is, at any time, expressed in this system on the sheet of assertion, we may note that before anything has been drawn on the sheet, the *blank* is, by that definition, a graph. It may be considered as the expression of whatever must be well-understood between the graphist and the interpreter of the graph before the latter can understand what to expect of the graph. There must be an interpreter, since the graph, like every sign founded on convention, only has the sort of being that it has if it is interpreted; for a conventional sign is neither a mass of ink on a piece of paper or any other

individual existence, nor is it an image present to consciousness, but is a special habit or rule of interpretation and consists precisely in the fact that certain sorts of ink spots -- which I call its *replicas* -- will have certain effects on the conduct, mental and bodily, of the interpreter. So, then, the blank of the blank sheet may be considered as expressing that the universe, in process of creation by the grapheus, is perfectly definite and entirely determinate, etc. Hence, even the first writing of a graph on the sheet is a modification of the graph already written. The business of the graphist is supposed to come to an end before the work of creation is accomplished. He is supposed to be a mind-reader to such an extent that he knows some (perhaps all) the creative work of the grapheus so far as it has gone, but not what is to come. What he intends the graph to express concerns the universe as it will be when it comes to exist. If he risks an assertion for which he has no warrant in what the grapheus has yet thought, it may or may not prove true.

Peirce: CP 4.432 Cross-Ref:††

432. The above considerations constitute a sufficient reason for adopting the following convention, which is hereby adopted:

Peirce: CP 4.432 Cross-Ref:††

Convention No. 1. A certain sheet, called the sheet of assertion, is appropriated to the drawing upon it of such graphs that whatever may be at any time drawn upon it, called the entire graph, shall be regarded as expressing an assertion by an imaginary person, called the graphist, concerning a universe, perfectly definite and entirely determinate, but the arbitrary creation of an imaginary mind, called the grapheus.

Peirce: CP 4.433 Cross-Ref:††

433. The convention which has next to be considered is the most arbitrary of all. It is, nevertheless, founded on two good reasons. A diagram ought to be as iconic as possible; that is, it should represent relations by visible relations analogous to them. Now suppose the graphist finds himself authorized to write each of two entire graphs. Say, for example, that he can draw:

The pulp of some oranges is red;

and that he is equally authorized to draw:

To express oneself naturally is the last perfection
of a writer's art.

Each proposition is true independently of the other, and either may therefore be expressed on the sheet of assertion. If both are written on different parts of the sheet of assertion, the independent presence on the sheet of the two expressions is analogous to the independent truth of the two propositions that they would, when written separately, assert. It would, therefore, be a highly iconic mode of representation to understand,

The pulp of some oranges is red.

To express oneself naturally is the last perfection
of a writer's art.

where both are written on different parts of the sheet, as the assertion of both

propositions.

Peirce: CP 4.434 Cross-Ref:††

434. It is a subsidiary recommendation of a mode of diagrammatization, but one which ought to be accorded some weight, that it is one that the nature and habits of our minds will cause us at once to understand, without our being put to the trouble of remembering a rule that has no relation to our natural and habitual ways of expression. Certainly, no convention of representation could possess this merit in a higher degree than the plan of writing both of two assertions in order to express the truth of both. It is so very natural, that all who have ever used letters or almost any method of graphic communication have resorted to it. It seems almost unavoidable, although in my first invented system of graphs, which I *call entitative graphs*,†1 propositions written on the sheet together were not understood to be independently asserted but to be *alternatively* asserted. The consequence was that a blank sheet instead of expressing only what was taken for granted had to be interpreted as an absurdity. One system seems to be about as good as the other, except that unnaturalness and aniconicity haunt every part of the system of entitative graphs, which is a curious example of how late a development simplicity is. These two reasons will suffice to make every reader very willing to accede to the following convention, which is hereby adopted.

Peirce: CP 4.434 Cross-Ref:††

Convention No. 2. Graphs on different parts of the sheet, called partial graphs, shall independently assert what they would severally assert, were each the entire graph.

Peirce: CP 4.435 Cross-Ref:††
§2. OF CONVENTION NO. 3P

435. If a system of expression is to be adequate to the analysis of all necessary consequences,†P1 it is requisite that it should be able to express that an expressed consequent, C, follows necessarily from an expressed antecedent, A. The conventions hitherto adopted do not enable us to express this. In order to form a new and reasonable convention for this purpose we must get a perfectly distinct idea of what it means to say that a consequent follows from an antecedent. It means that in adding to an assertion of the antecedent an assertion of the consequent we shall be proceeding upon a general principle whose application will never convert a true assertion into a false one. This, of course, means that so it will be in the universe of which alone we are speaking. But when we talk logic -- and people occasionally insert logical remarks into ordinary discourse -- our universe is that universe which embraces all others, namely *The Truth*, so that, in such a case, we mean that in no universe whatever will the addition of the assertion of the consequent to the assertion of the antecedent be a conversion of a true proposition into a false one. But before we can express any proposition referring to a general principle, or, as we say, to a "range of possibility," we must first find means to express the simplest kind of conditional proposition, the *conditional de inesse*, in which "If A is true, C is true" means only that, principle or no principle, the addition to an assertion of A of an assertion of C will not be a conversion of a true assertion into a false one. That is, it asserts that the graph of Fig. 69, anywhere on the sheet of assertion, might be transformed into the graph of Fig. 70

without passing from truth to falsity.

a

ac

Fig. 69

Fig. 70

This conditional *de inesse* has to be expressed as a graph in such a way as distinctly to express in our system both *a* and *c*, and to exhibit their relation to one another. To assert the graph thus expressing the conditional *de inesse*, it must be drawn upon the sheet of assertion, and in this graph the expressions of *a* and of *c* must appear; and yet neither *a* nor *c* must be drawn upon the sheet of assertion. How is this to be managed? Let us draw a closed line which we may call a *sep* (*sæpes*, a fence), which shall cut off its contents from the sheet of assertion. Let this sep together with all that is within it, *considered as a whole*, be called an *enclosure*, this close, being written on the sheet of assertion, shall assert the conditional *de inesse*; but that which it encloses, considered separately from the sep, shall not be considered as on the sheet of assertion. Then, obviously, the antecedent and consequent must be in separate compartments of the close. In order to make the representation of the relation between them iconic, we must ask ourselves what spatial relation is analogous to their relation. Now if it be true that "If *a* is true, *b* is true" and "If *b* is true, *c* is true," then it is true that "If *a* is true, *c* is true." This is analogous to the geometrical relation of inclusion. So naturally striking is the analogy as to be (I believe) used in all languages to express the logical relation; and even the modern mind, so dull about metaphors, employs this one frequently. It is reasonable, therefore, that one of the two compartments should be placed within the other. But which shall be made the inner one? Shall we express the conditional *de inesse* by Fig. 71 or by Fig. 72? In order to decide which is the more appropriate mode of representation, one should observe that the consequent of a conditional proposition asserts what is true, not throughout the whole universe of possibilities considered, but in a subordinate universe marked off by the antecedent. This is not a fanciful notion, but a truth. Now in Fig. 72, the consequent appears in a special part of the sheet representing the universe, the space between the two lines containing the definition of the sub-universe.

Figs. 71-72

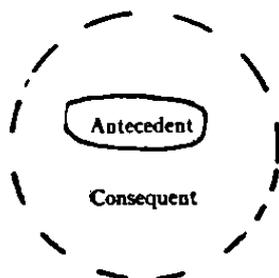


Fig. 71*

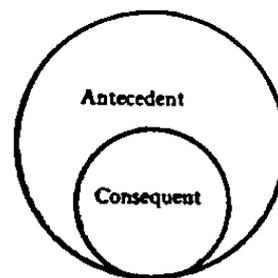


Fig. 72

[Click here

to view]†1

There is no such expressiveness in Fig. 71 -- or, if there be, it is only of a superficial and fanciful sort. Moreover, the necessity of using two kinds of enclosing lines -- a

necessity which, we shall find, does not exist in Fig. 72 -- is a defect of Fig. 71; and when we come to consider the question of convenience, the superiority of Fig. 72 will appear still more strongly. This, then, will be the method for us to adopt.

Peirce: CP 4.436 Cross-Ref:††

436. The two seps of Fig. 72, taken together, form a curve which I shall call a *scroll*. The node is of no particular significance. The scroll may equally well be drawn

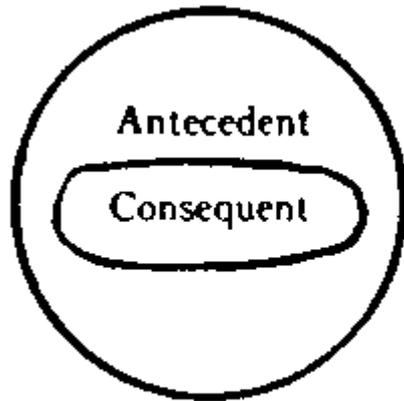


Fig. 73

as in Fig. 73.

[Click here to view] The only essential feature is that there should be two seps, of which the inner, however drawn, may be called the *inloop*. The node merely serves to aid the mind in the interpretation, and will be used only when it can have this effect. The two compartments will be called the *inner*, or *second, close*, and the *outer close*, the latter excluding the former. The outer close considered as containing the inloop will be called the *close*.

Peirce: CP 4.437 Cross-Ref:††

437. *Convention No. 3. An enclosure shall be a graph consisting of a scroll with its contents.*

Peirce: CP 4.437 Cross-Ref:††

The scroll shall be a real curve of two closed branches, the one within the other, called seps, and the inner specifically called the loop; and these branches may or may not be joined at a node.

Peirce: CP 4.437 Cross-Ref:††

The contents of the scroll shall consist of whatever is in the area enclosed by the outer sep, this area being called the close and consisting of the inner, or second, close, which is the area enclosed by the loop, and the outer, or first close, which is the area outside the loop but inside the outer sep.

Peirce: CP 4.437 Cross-Ref:††

When an enclosure is written on the sheet of assertion, although it is asserted as a whole, its contents shall be cut off from the sheet, and shall not be asserted in the assertion of the whole. But the enclosure shall assert de inesse that if every graph in the outer close be true, then every graph in the inner close is true.

Peirce: CP 4.438 Cross-Ref:††
§3. OF CONVENTIONS NOS. 4 TO 9†1P

438. Let a heavy dot or dash be used in place of a noun which has been erased from a proposition. A blank form of proposition produced by such erasures as can be filled, each with a proper name, to make a proposition again, is called a rhema, or, relatively to the proposition of which it is conceived to be a part, the *predicate* of that proposition. The following are examples of rhemata:

-- is good

every man is the son of --

-- loves --

God gives -- to --

Every proposition has one predicate and one only. But what that predicate is considered to be depends upon how we choose to analyze it. Thus, the proposition

God gives some good to every man

may be considered as having for its predicate either of the following rhemata:

-- gives -- to --

-- gives some good to --

-- gives -- to every man

God gives -- to --

God gives some good to --

God gives -- to every man

-- gives some good to every man

God gives some good to every man.

In the last case the entire proposition is considered as predicate. A rhema which has one blank is called a *monad*; a rhema of two blanks, a *dyad*; a rhema of three blanks, a *triad*; etc. A rhema with no blank is called a *medad*, and is a complete proposition. A rhema of more than two blanks is a *polyad*. A rhema of more than one blank is a *relative*. Every proposition has an *ultimate predicate*, produced by putting a blank in every place where a blank can be placed, without substituting for some word its definition. Were this done we should call it a *different proposition*, as a matter of nomenclature. If on the other hand, we transmute the proposition without making any difference as to what it leaves unanalyzed, we say the *expression* only is different, as, if we say,

Some good is bestowed by God on every man.

Each part of a proposition which might be replaced by a proper name, and still leave the proposition a proposition is a subject of the *proposition*. †P1 It is, however, the rhema which we have just now to attend to.

Peirce: CP 4.439 Cross-Ref: ††

439. A rhema is, of course, not a proposition. Supposing, however, that it be written on the sheet of assertion, so that we have to adopt a meaning for it as a proposition, what can it most reasonably be taken to mean? Take, for example, Fig. 74. Shall this, since it represents the universe, be taken to mean that "Something in the universe is beautiful," or that "Anything in the universe is beautiful," or that "The universe, as a whole, is beautiful"? The last interpretation may be rejected at once for the reason that we are generally unable to assert anything of the universe not reducible to one of the other forms except what is well-understood between graphist and interpreter. We have, therefore, to choose between interpreting Fig. 74 to mean "Something is beautiful"

-----is beautiful

————— is beautiful

Fig. 74 ,



Fig. 75

[Click here

to view]

Fig. 74

Fig. 75

and to mean "Anything is beautiful." Each asserts the rhema of an individual; but the former leaves that individual to be designated by the grapheus, while the latter allows the rhema [interpreter q to fill the blank with any proper name he likes. If Fig. 74 be taken to mean "Something is beautiful," then Fig. 75 will mean "Everything is beautiful"; while if Fig. 74 be taken to mean "Everything is beautiful," then Fig. 75 will mean "Something is beautiful." In either case, therefore, both propositions will be expressible, and the main question is, which gives the most appropriate expressions? The question of convenience is subordinate, as a general rule; but in this case the difference is so vast in this respect as to give this consideration more than its usual importance.

Peirce: CP 4.440 Cross-Ref: ††

440. In order to decide the question of appropriateness, we must ask which form of proposition, the universal or the particular, "Whatever salamander there may be lives in fire," or "Some existing salamander lives in fire," is more of the nature of a conditional proposition; for plainly, these two propositions differ in form from "Everything is beautiful" and "Something is beautiful" respectively, only in their being limited to a subsidiary universe of salamanders. Now to say "Any salamander lives in fire" is merely to say "If anything, X, is a salamander, X lives in fire." It

differs from a conditional, if at all, only in the identification of X which it involves. On the other hand, there is nothing at all conditional in saying "There is a salamander, and it lives in fire."

Peirce: CP 4.440 Cross-Ref:††

Thus the interpretation of Fig. 74 to mean "Something is beautiful" is decidedly the more appropriate; and since reasonable arrangements generally prove to be the most convenient in the end, we shall not be surprised when we come to find, as we shall, the same interpretation to be incomparably the superior in that respect also.

Peirce: CP 4.441 Cross-Ref:††

441. Convention No. 4. In this system, the unanalyzed expression of a rhema shall be called a spot. A distinct place on its periphery shall be appropriated to each blank, which place shall be called a hook. A spot with a dot at each hook shall be a graph expressing the proposition which results from filling every blank of the rhema with a separate sign of an indesignate individual existing in the universe and belonging to some determinate category, usually that of "things."

Peirce: CP 4.442 Cross-Ref:††

442. In many reasonings it becomes necessary to write a copulative proposition in which two members relate to the same individual so as to distinguish these members. Thus we have to write such a proposition as,

A is greater than something that is greater than B,

so as to exhibit the two partial graphs of Fig. 76.

A is greater than --

-- is greater than B

Fig. 76

The proposition we wish to express adds to those of Fig. 76 the assertion of the identity of the two "somethings." But this addition cannot be effected as in Fig. 77.

A is greater than--

-- is greater than B

-- is greater than --

Fig. 77

For the "somethings," being indesignate, cannot be described in general terms. It is necessary that the signs of them should be connected in fact. No way of doing this can be more perfectly iconic than that exemplified in Fig. 78.

A is greater than  is greater than B.

Fig. 78:

[Click here to view]

Peirce: CP 4.442 Cross-Ref:††

Any sign of such identification of individuals may be called a *connexus*, and the particular sign here used, which we shall do well to adopt, may be called a *line of identity*.

Peirce: CP 4.443 Cross-Ref:††

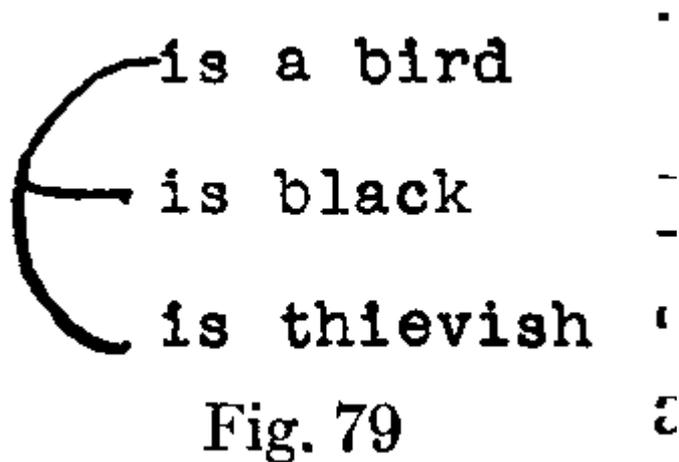
443. *Convention No. 5. Two coincident points, not more, shall denote the same individual.*

Peirce: CP 4.444 Cross-Ref:††

444. *Convention No. 6. A heavy line, called a line of identity, shall be a graph asserting the numerical identity of the individuals denoted by its two extremities.*

Peirce: CP 4.445 Cross-Ref:††

445. The next convention to be laid down is so perfectly natural that the reader may well have a difficulty in perceiving that a separate convention is required for it. Namely, we may make a line of identity branch to express the identity of three



individuals. Thus, Fig. 79

[Click here to view] will express that some black bird is thievish. No doubt, it would have been easy to draw up Convention No. 4 in such a form as to cover this procedure. But it is not our object in this section to find ingenious modes of statement which, being borne in mind, may serve as rules for as many different acts as possible. On the contrary, what we are here concerned to do is to distinguish all proceedings that are essentially different. Now it is plain that no number of mere bi-terminal bonds, each terminal occupying a spot's hook, can ever assert the identity of three things, although when we once have a three-way branch, any higher number of terminals can be produced from it, as in Fig. 80

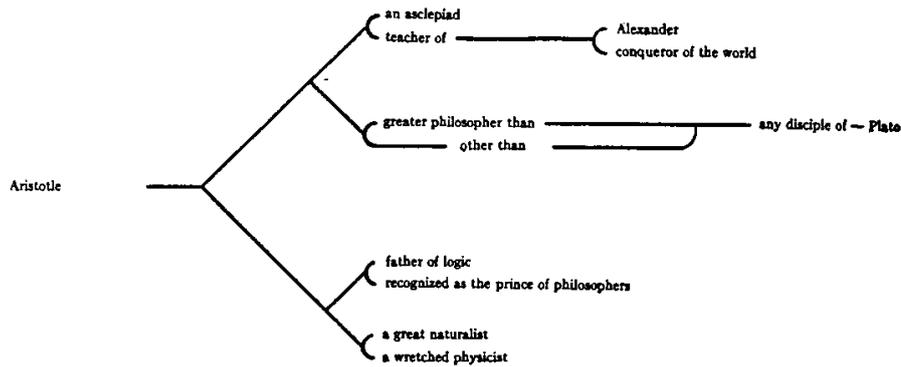


Fig. 80

[Click here

to view].

Peirce: CP 4.446 Cross-Ref:††

446. We ought to, and must, then, make a distinct convention to cover this procedure, as follows:

Peirce: CP 4.446 Cross-Ref:††

Convention No. 7. A branching line of identity shall express a triad rhema signifying the identity of the three individuals, whose designations are represented as filling the blanks of the rhema by coincidence with the three terminals of the line.

Peirce: CP 4.447 Cross-Ref:††

447. Remark how peculiar a sign the line of identity is. A sign, or, to use a more general and more definite term, a ***representamen***, is of one or other of three kinds:†1 it is either an icon, an ***index***, or a ***symbol***. An icon is a representamen of what it represents and for the mind that interprets it as such, by virtue of its being an immediate image, that is to say by virtue of characters which belong to it in itself as a sensible object, and which it would possess just the same were there no object in nature that it resembled, and though it never were interpreted as a sign. It is of the nature of an appearance, and as such, strictly speaking, exists only in consciousness, although for convenience in ordinary parlance and when extreme precision is not called for, we extend the term ***icon*** to the outward objects which excite in consciousness the image itself. A geometrical diagram is a good example of an icon. A pure icon can convey no positive or factual information; for it affords no assurance that there is any such thing in nature. But it is of the utmost value for enabling its interpreter to study what would be the character of such an object in case any such did exist. Geometry sufficiently illustrates that. Of a completely opposite nature is the kind of representamen termed an ***index***. This is a real thing or fact which is a sign of its object by virtue of being connected with it as a matter of fact and by also forcibly intruding upon the mind, quite regardless of its being interpreted as a sign. It may simply serve to identify its object and assure us of its existence and presence. But very often the nature of the factual connexion of the index with its object is such as to excite in consciousness an image of some features of the object, and in that way affords evidence from which positive assurance as to truth of fact may be drawn. A photograph, for example, not only excites an image, has an appearance, but, owing to its optical connexion with the object, is evidence that that appearance corresponds to a reality. A ***symbol*** is a representamen whose special significance or fitness to represent just what it does represent lies in nothing but the very fact of there being a habit, disposition, or other effective general rule that it will be so interpreted. Take,

for example, the word "*man*." These three letters are not in the least like a man; nor is the sound with which they are associated. Neither is the word existentially connected with any man as an index. It cannot be so, since the word is not an existence at all. The word does not consist of three films of ink. If the word "man" occurs hundreds of times in a book of which myriads of copies are printed, all those millions of triplets of patches of ink are embodiments of one and the same word. I call each of those embodiments a *replica* of the symbol. This shows that the word is not a thing. What is its nature? It consists in the really working general rule that three such patches seen by a person who knows English will effect his conduct and thoughts according to a rule. Thus the mode of being of the symbol is different from that of the icon and from that of the index. An icon has such being as belongs to past experience. It exists only as an image in the mind. An index has the being of present experience. The being of a symbol consists in the real fact that something surely will be experienced if certain conditions be satisfied. Namely, it will influence the thought and conduct of its interpreter. Every word is a symbol. Every sentence is a symbol. Every book is a symbol. Every representamen depending upon conventions is a symbol. Just as a photograph is an index having an icon incorporated into it, that is, excited in the mind by its force, so a symbol may have an icon or an index incorporated into it, that is, the active law that it is may require its interpretation to involve the calling up of an image, or a composite photograph of many images of past experiences, as ordinary common nouns and verbs do; or it may require its interpretation to refer to the actual surrounding circumstances of the occasion of its embodiment, like such words as *that, this, I, you, which, here, now, yonder*, etc. Or it may be pure symbol, neither *iconic* nor *indicative*, like the words *and, or, of*, etc.

Peirce: CP 4.448 Cross-Ref:††

448. The value of an icon consists in its exhibiting the features of a state of things regarded as if it were purely imaginary. The value of an index is that it assures us of positive fact. The value of a symbol is that it serves to make thought and conduct rational and enables us to predict the future. It is frequently desirable that a representamen should exercise one of those three functions to the exclusion of the other two, or two of them to the exclusion of the third; but the most perfect of signs are those in which the iconic, indicative, and symbolic characters are blended as equally as possible. Of this sort of signs the line of identity is an interesting example. As a conventional sign, it is a symbol; and the symbolic character, when present in a sign, is of its nature predominant over the others. The line of identity is not, however, arbitrarily conventional nor purely conventional. Consider any portion of it taken arbitrarily (with certain possible exceptions shortly to be considered) and it is an ordinary graph for which Fig. 81 might perfectly well be substituted. But when we consider the

--is identical with--

Fig. 81

connexion of this portion with a next adjacent portion, although the two together make up the same graph, yet the identification of the something, to which the hook of the one refers, with the something, to which the hook of the other refers, is beyond the power of any graph to effect, since a graph, as a symbol, is of the nature of a *law*, and is therefore general, while here there must be an identification of individuals. This identification is effected not by the pure symbol, but by its *replica* which is a thing. The termination of one portion and the beginning of the next portion denote the same individual by virtue of a factual connexion, and that the closest possible; for both are

points, and they are one and the same point. In this respect, therefore, the line of identity is of the nature of an index. To be sure, this does not affect the ordinary parts of a line of identity, but so soon as it is even *conceived*, [it is conceived] as composed of two portions, and it is only the factual junction of the replicas of these portions that makes them refer to the same individual. The line of identity is, moreover, in the highest degree iconic. For it appears as nothing but a continuum of dots, and the fact of the identity of a thing, seen under two aspects, consists merely in the continuity of being in passing from one apparition to another. Thus uniting, as the line of identity does, the natures of symbol, index, and icon, it is fitted for playing an extraordinary part in this system of representation.

Peirce: CP 4.449 Cross-Ref:††

449. There is no difficulty in interpreting the line of identity until it crosses a sep. To interpret it in that case, two new conventions will be required.

Peirce: CP 4.449 Cross-Ref:††

How shall we express the proposition "Every salamander lives in fire," or "If it be true that something is a salamander then it will always be true that *that something* lives in fire"? If we omit the assertion of the identity of the somethings, the expression is obviously given in Fig. 82.

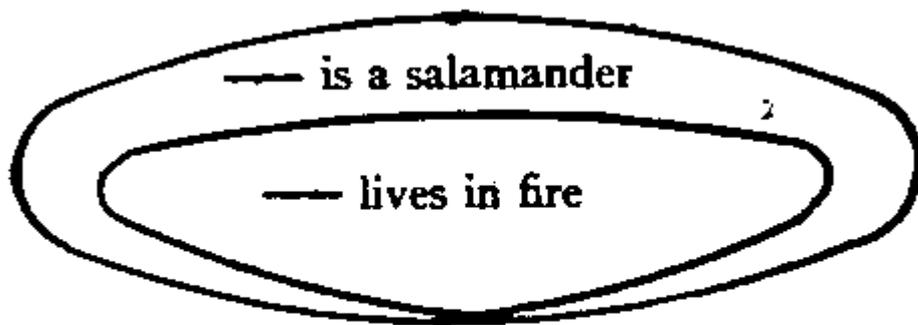


Fig. 82

[Click here to view] To that, we wish to add the expression of individual identity. We ought to use our line of identity for that. Then, we must draw Fig. 83.

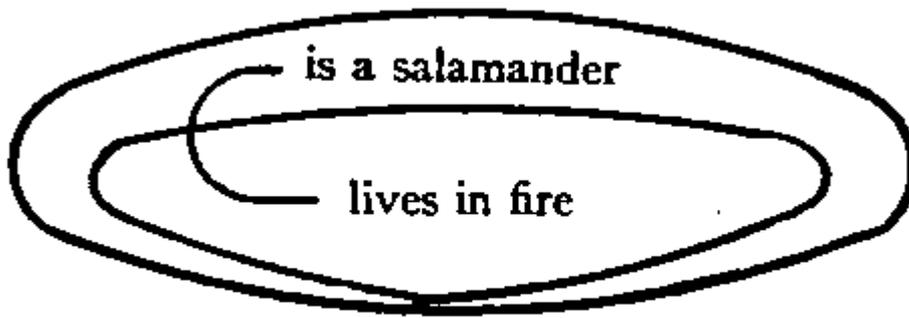


Fig. 83

[Click here to view] It would be unreasonable, after having adopted the line of identity as our instrument for the expression of individual identity, to hesitate to employ it in this case. Yet to regularize such a mode of expression two new conventions are required. For, in the first place, we have not hitherto had any such sign as a line of identity crossing a sep. This part of the line of identity is not a graph; for a graph must be either outside or inside of each sep.†1 In order, therefore, to legitimate our interpretation of Fig. 83, we must agree that a line of identity crossing a sep simply asserts the identity of the individual denoted by its outer part and the individual denoted by its inner part. But this agreement does not of itself necessitate our interpretation of Fig. 83; since this might be understood to mean, "There is *something* which, if it be a salamander, lives in fire," instead of meaning, "If there be *anything* that is a salamander, it lives in fire." But although the last interpretation but one would involve itself in no positive contradiction, it would annul the convention that a line of identity crossing a sep still asserts the identity of its extremities -- not, indeed, by conflict with that convention, but by rendering it nugatory. What does it mean to assert *de inesse* that there is something, which if it be a salamander, lives in fire? It asserts, no doubt, that there is something. Now suppose that anything lives in fire. Then of that it will be true *de inesse* that if it be a salamander, it lives in fire; so that the proposition will then be true. Suppose that there is anything that is not a salamander. Then, of that it will be true *de inesse* that if it be a salamander, it lives in fire; and again the proposition will be true. It is only false in case whatever there may be is a salamander while nothing lives in fire. Consequently, Fig. 83 would be precisely equivalent to Fig. 84

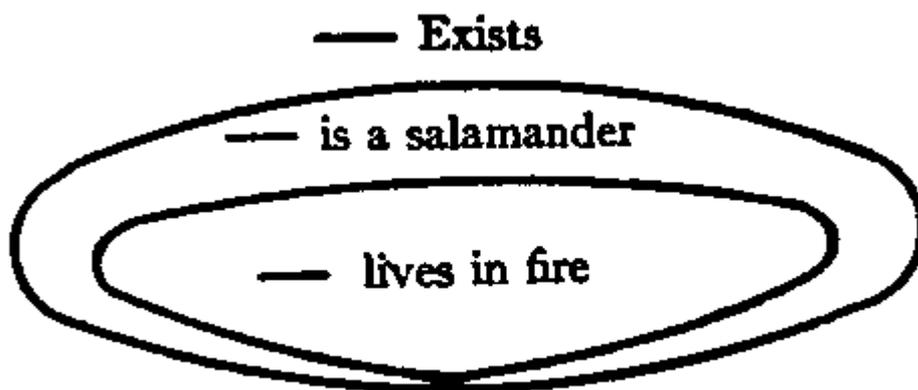


Fig. 84

[Click here to view], and there would be no need of any line of identity's crossing a sep. It would then be impossible to express a universal categorical analytically except by resorting to an unanalytic expression of such a proposition or something substantially equivalent to that. †P1

Peirce: CP 4.449 Cross-Ref: ††

Two conventions, then, are necessary. In stating them, it will be well to avoid the idea of a graph's being cut through by a sep, and confine ourselves to the effects of joining dots on the sep to dots outside and inside of it.

Peirce: CP 4.450 Cross-Ref: ††

450. *Convention No. 8. Points on a sep shall be considered to lie outside the close of the sep so that the junction of such a point with any other point outside the sep by a line of identity shall be interpreted as it would be if the point on the sep were outside and away from the sep.*

Peirce: CP 4.451 Cross-Ref: ††

451. *Convention No. 9. The junction by a line of identity of a point on a sep to a point within the close of the sep shall assert of such individual as is denoted by the point on the sep, according to the position of that point by Convention No. 8, a hypothetical conditional identity, according to the conventions applicable to graphs situated as is the portion of that line that is in the close of the sep.*

Peirce: CP 4.452 Cross-Ref: ††

452. It will be well to illustrate these conventions by some examples. Fig. 85 asserts that if it be true that something is good, then this assertion is false. That is, the assertion is that nothing is good. But in Fig. 86, the terminal of the line of identity on the outer sep asserts that something, X, exists, and it is only of this existing individual, X, that it is asserted that if *that* is good the assertion is false. It therefore means

Figs. 85-86

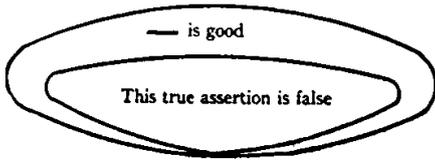


Fig. 85

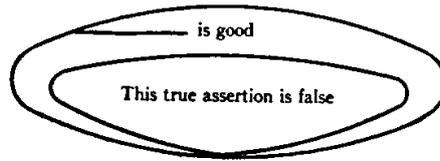


Fig. 86

[Click here

to view]

"Something is not good." On Fig. 87 and Fig. 88 the points on the seps are marked with letters, for convenience of reference. Fig. 87 asserts that something, A, is a woman; and that if there is an individual, X, that is a catholic, and an individual, Y, that is identical with A, then X adores Y; that is, some woman is adored by all catholics, if there are any. Fig. 88 asserts that if there be an individual, X, and if X is a catholic, then X adores somebody that is a woman. That is, whatever

Figs. 87-88

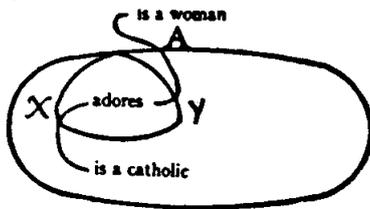


Fig. 87

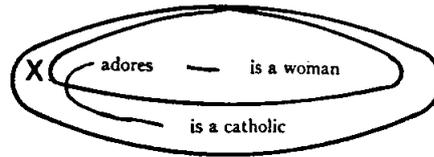


Fig. 88

[Click here

to view]

catholic there may be adores some woman or other. This does not positively assert that any woman exists, but only that if there is a catholic, then there is a woman whom he adores.

Peirce: CP 4.453 Cross-Ref:††

453. A triad rhema gives twenty-six affirmative forms of

simple general propositions, as follows:

Nos.

Fig. 89. --blames_ | to-- Somebody blames

somebody to somebody

1

Fig. 90.

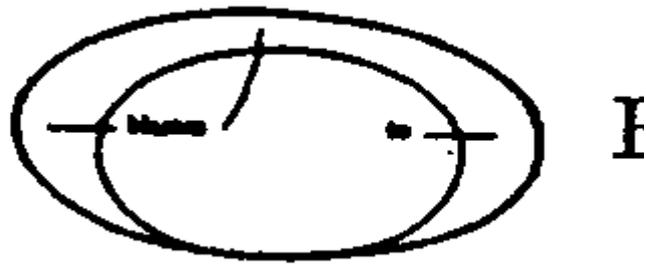


Fig. 90.
[Click here to view] Everybody blames everybody

to everybody 1

Fig. 91.

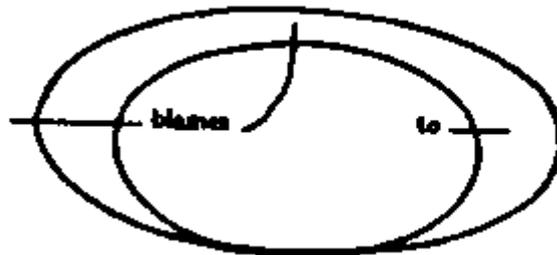


Fig. 91.
[Click here to view] Somebody blames everybody

to everybody 3 such

Fig. 92.

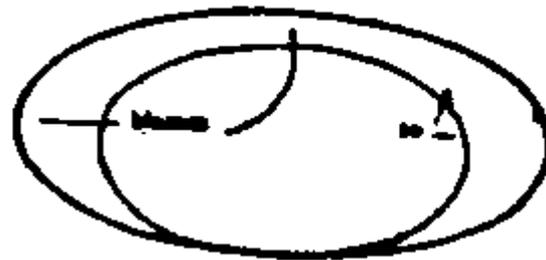
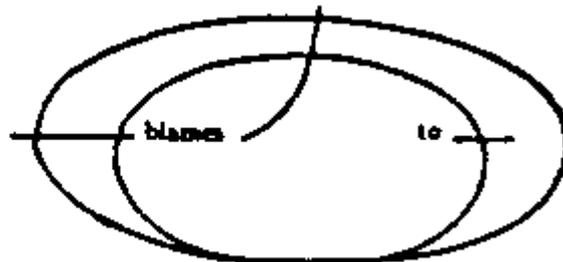


Fig. 92.
[Click here to view] Everybody blames everybody

to somebody or other 3 such

Fig. 93.

Fig. 93.



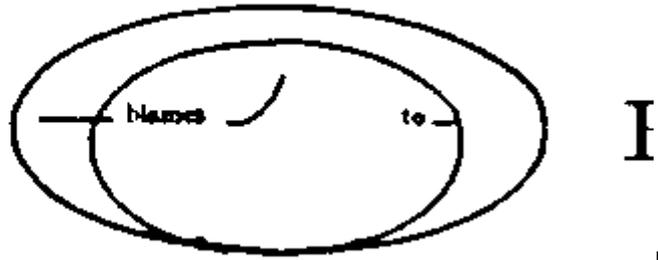
[Click here to view] Somebody blames somebody

to everybody 3 such

[Click

Fig. 94.

Fig. 94.



[Click

here to view] Everybody blames somebody

to somebody 3 such

Fig. 95.



Fig. 95.

[Click here to view] Somebody blames everybody

to somebody or other 6 such

Fig. 96.

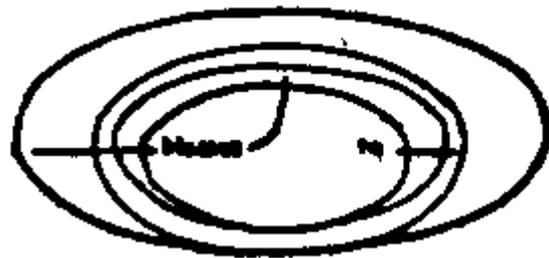


Fig. 96.

[Click here to view] Everybody to somebody or

other blames all 6 such

 Total 26

For a tetrad there are 150 such forms; for a pentad 1082; for a hexad 9366; etc.

§1. OF THE PSEUDOGRAPH AND CONNECTED SIGNSP

454. It is, as will soon appear, sometimes desirable to express a proposition either absurd, contrary to the understanding between the graphist and the interpreter, or at any rate well-known to be false. From any such proposition, as antecedent, any proposition whatever follows as a consequent *de inesse*. Hence, every such proposition may be regarded as implying that everything is true; and consequently all such propositions are equivalent. The expression of such a proposition may very well fill the entire close in which it is, since nothing can be added to what it already implies. Hence we may adopt the following secondary convention.

Peirce: CP 4.454 Cross-Ref:††

Convention No. 10. *The pseudograph, or expression in this system of a proposition implying that every proposition is true, may be drawn as a black spot entirely filling the close in which it is.*

Peirce: CP 4.455 Cross-Ref:††

455. Since the size of signs has no significance, the blackened close may be drawn invisibly small. Thus Fig. 97 [may be scribed] as in Fig. 98, or even as in Fig. 99, Fig. 100, or lastly as in Fig. 101.†1

Figs. 97-101

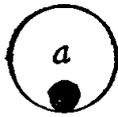


Fig. 97

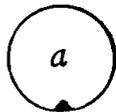


Fig. 98

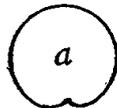


Fig. 99

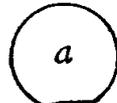


Fig. 100

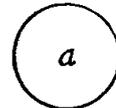


Fig. 101

[Click here

to view]

Peirce: CP 4.456 Cross-Ref:††

456. **Interpretational Corollary 1.** *A scroll with its contents having the pseudograph in the inner close is equivalent to the precise denial of the contents of the outer close.*

Peirce: CP 4.456 Cross-Ref:††

For the assertion, as in Fig. 97, that *de inesse* if *a* is true everything is true, is equivalent to the assertion that *a* is not true, since if the conditional proposition *de inesse* be true *a* cannot be true, and if *a* is not true the conditional proposition *de inesse*, having *a* for its antecedent, is true. Hence the one is always true or false with the other, and they are equivalent.

Peirce: CP 4.456 Cross-Ref:††

This corollary affords additional justification for writing Fig. 97 as in Fig. 101, since the effect of the loop enclosing the pseudograph is to make a precise denial of the absurd proposition; and to deny the absurd is equivalent to asserting nothing.

Peirce: CP 4.457 Cross-Ref:††

457. **Interpretational Corollary 2.** *A disjunctive proposition may be*

expressed by placing its members in as many inloops of one sep. But this will not exclude the simultaneous truth of several members or of all.

Peirce: CP 4.457 Cross-Ref:††

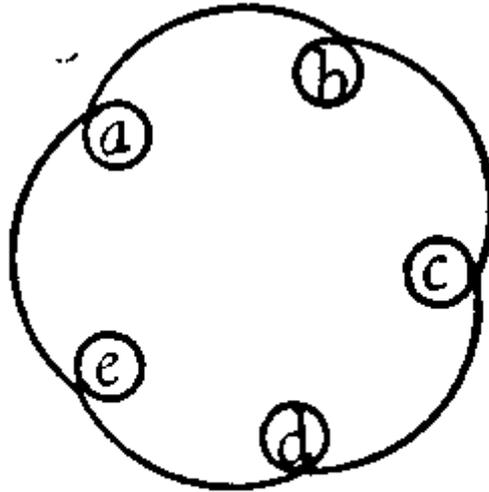


Fig. 102

Thus, Fig. 102 [Click here to view] will express that either a or b or c or d or e is true. For it will deny the simultaneous denial of all.

Peirce: CP 4.458 Cross-Ref:††

458. *Interpretational Corollary 3. A graph may be interpreted by copulations and disjunctions. Namely, if a graph within an odd number of seps be said to be oddly enclosed, and a graph within no sep or an even number of seps be said to be evenly enclosed, then spots in the same compartment are copulated when evenly enclosed, and disjunctively combined when oddly enclosed; and any line of identity whose outermost part is evenly enclosed refers to something, and any one whose outermost part is oddly enclosed refers to anything there may be. And the interpretation must begin outside of all seps and proceed inward. And spots evenly enclosed are to be taken affirmatively; those oddly enclosed negatively.*

Peirce: CP 4.458 Cross-Ref:††

For example, Fig. 83 may be read, Anything whatever is either not a salamander or lives in fire. Fig. 87 may be read, Something, A, is a woman, and whatever X may be, either X is not a catholic or X adores A. Fig. 88 may be read, Whatever X may be, either X is not a catholic or there is something Y, such that X adores Y and Y is a woman. Fig. 96 may be read, Whatever A may be, there is something C, such that whatever B may be, A blames B to C. Fig. 103

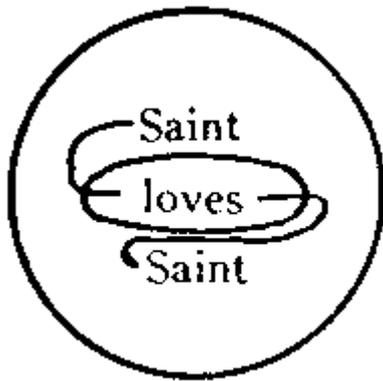


Fig. 103

[Click here to view] may be read, Whatever X and Y may be, either X is not a saint or Y is not a saint or X loves Y; that is, Every saint there may be loves every saint. So Fig. 104

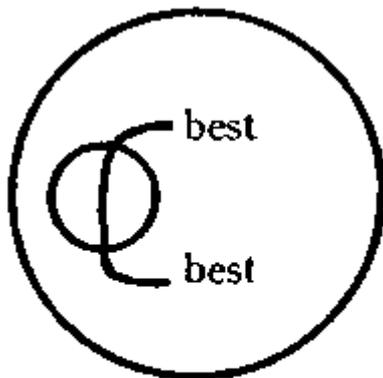


Fig. 104

[Click here to view] may be read, Whatever X and Y may be, either X is not best or Y is not best or X is identical with Y; that is, there are not two bests. Fig. 105

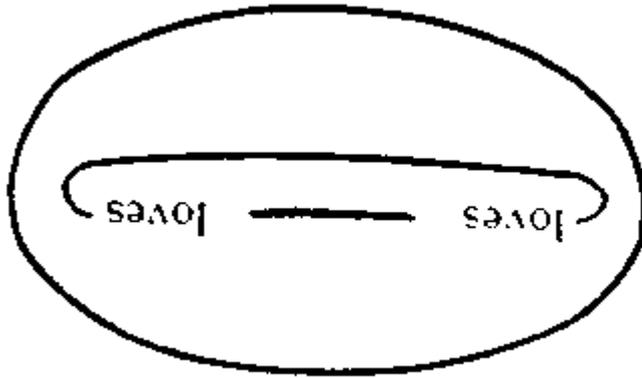


Fig. 105

[Click here to view]

may be read, Whatever X and Y may be, either X does not love Y or Y does not love X; that is, no two love each other. Fig. 106

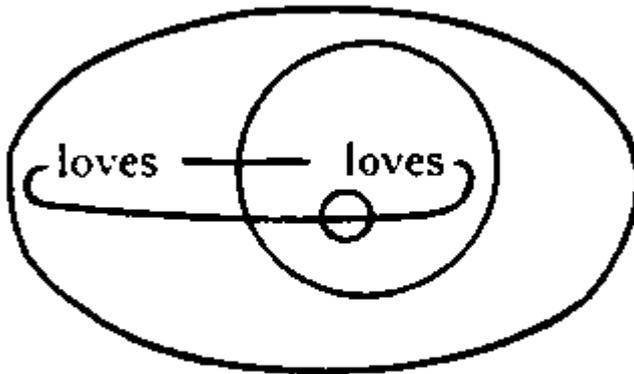


Fig. 106

[Click here to view] may

be read, Whatever X and Y may be either X does not love Y or there is something L and X is not L but Y loves L; that is, nobody loves anybody who does not love somebody else.

Peirce: CP 4.459 Cross-Ref:††

459. *Interpretational Corollary 4. A sep which is vacant, except for a line of identity traversing it, expresses with its contents the non-identity of the extremities of that line.*

Peirce: CP 4.460 Cross-Ref:††
 §2. SELECTIVES AND PROPER NAMESP

460. It is sometimes impossible upon an ordinary surface to draw a graph so that lines of identity will not cross one another. If, for example, we express that x is a

value that can result from raising z to the power whose exponent is y , by means of Fig. 107, and express that u is a value that can result from multiplying w by v , by Fig. 108, then in order to express that

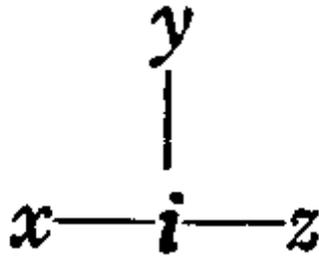


Fig. 107:

[Click here to view]

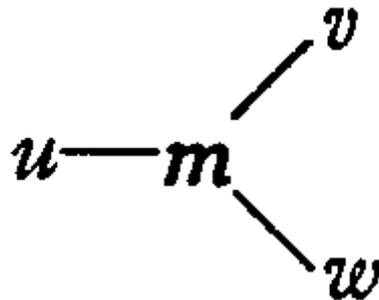


Fig. 108:

[Click here to view]

whatever values x , y , and z may be, there is a value resulting from raising x to a power whose exponent is a value of the product of z by y which same value is also one of the values resulting from raising to the power z a value resulting from raising x to the power y (this being one of the propositions expressed by the equation $xyz = (xy)z$) we may draw Fig. 109

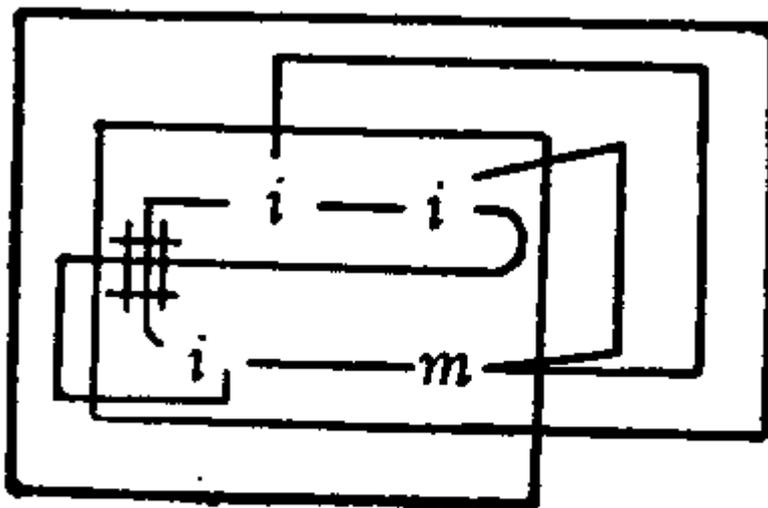


Fig. 109

[Click here to

view]; but there is an unavoidable intersection of two lines of identity. In such a case, and indeed in any case in which the lines of identity become too intricate to be perspicuous, it is advantageous to replace some of them by signs of a sort that in this system are called *selectives*. A selective is very much of the same nature as a proper name; for it denotes an individual and its outermost occurrence denotes a wholly indesignate individual of a certain category (generally a thing) existing in the universe, just as a proper name, on the first occasion of hearing it, conveys no more. But, just as on any subsequent hearing of a proper name, the hearer identifies it with that individual concerning which he has some information, so all occurrences of the selective other than the outermost must be understood to denote that identical individual. If, however, the outermost occurrence of any given selective is oddly enclosed, then, on that first occurrence the selective will refer to any individual whom the interpreter may choose, and in all other occurrences to the same individual. If there be no one outermost occurrence, then any one of those that are outermost may be considered as the outermost. The later capital letters are used for selectives. For example, Fig. 109 is otherwise expressed in Figs. 110 and 111.

Figs. 110-111

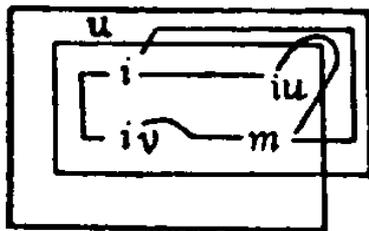


Fig. 110

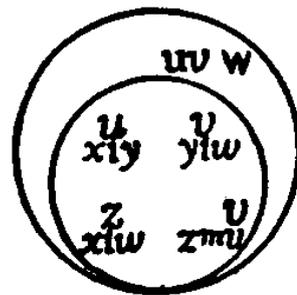


Fig. 111

[Click here

to view]

Fig. 111 may be read, "Either no value is designated as U, or no value is designated as V, or no value is designated as W, or else a value designated as Y results from raising W to the V power, and a value designated as Z results from multiplying U by V, and a value designated as X results from raising Y to the U power, while this same value X results from raising W to the Z power."

Peirce: CP 4.461 Cross-Ref:††

461. *Convention No. 11. The capital letters of the alphabet shall be used to denote single individuals of a well-understood category, the individual existing in the universe, the early letters preferably as proper names of well-known individuals, the later letters, called selectives, each on its first occurrence, as the name of an individual (that is, an object existing in the universe in a well-understood category; that is, having such a mode of being as to be determinate in reference to every character as wholly possessing it or else wholly wanting it), but an individual that is indesignate (that is, which the interpreter receives no warrant for identifying); while in every occurrence after the first, it shall denote that same individual. Of two occurrences of the same selective, either one may be interpreted as the earlier, if*

and only if, enclosed by no sep that does not enclose the other. A selective at its first occurrence shall be asserted in the mode proper to the compartment in which it occurs. If it be on that occurrence evenly enclosed, it is only affirmed to exist under the same conditions under which any graph in the same close is asserted; and it is then asserted, under those conditions, to be the subject filling the rhema-blank corresponding to any hook against which it may be placed. If, however, at its first occurrence, it be oddly enclosed, then, in the disjunctive mode of interpretation, it will be denied, subject to the conditions proper to the close in which it occurs, so that its existence being disjunctively denied, a non-existence will be affirmed, and as a subject, it will be universal (that is, freed from the condition of wholly possessing or wholly wanting each character) and at the same time designate (that is, the interpreter will be warranted in identifying it with whatever the context may allow), and it will be, subject to the conditions of the close, disjunctively denied to be the subject filling the rhema-blank of the hook against which it may be placed. In all subsequent occurrences it shall denote the individual with which the interpreter may, on its first occurrence, have identified it, and otherwise will be interpreted as on its first occurrence.

Peirce: CP 4.461 Cross-Ref: ††

Resort must be had to the examples to trace out the sense of this long abstract statement; and the line of identity will aid in explaining the equivalent selectives. Fig. 112 may be read

X is good

Fig. 112

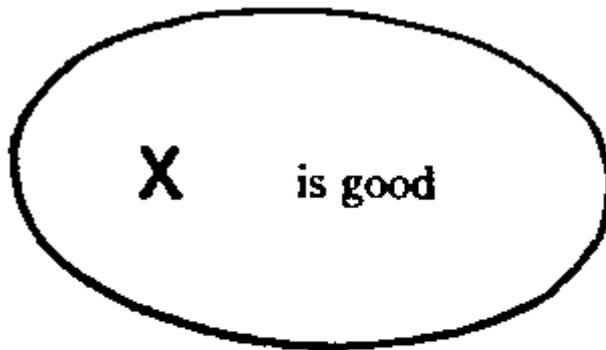


Fig. 113

[Click here to view]

Fig. 113

there exists something that may be called X and it is good. Fig. 113, the precise denial of Fig. 112, may be read "Either there is not anything to be called X or whatever there may be is not good," or "Anything you may choose to call X is not good," or "all things are non-good." "Anything" is not an individual subject, since the two propositions, "Anything is good" and "Anything is bad," do not exhaust the

possibilities. Both may be false.

Peirce: CP 4.462 Cross-Ref: ††

462. *Convention No. 12. The use of selectives may be avoided, where it is desired to do so, by drawing parallels on both sides of the lines of identity where they appear to cross.* †1

Peirce: CP 4.463 Cross-Ref: ††

§3. OF ABSTRACTION AND ENTIA RATIONIS †2P

463. The term *abstraction* bears two utterly different meanings in philosophy. In one sense it is applied to a psychological act by which, for example, on seeing a theatre, one is led to call up images of other theatres which blend into a sort of composite in which the special features of each are obliterated. Such obliteration is called precise abstraction. We shall have nothing to do with abstraction in that sense. But when that fabled old doctor, being asked why opium put people to sleep, answered that it was because opium has a dormative virtue, he performed this act of immediate inference:

Opium causes people to sleep;

Hence, Opium possesses a power of causing sleep.

The peculiarity of such inference is that the conclusion relates to something -- in this case, a power -- that the premiss says nothing about; and yet the conclusion is necessary. *Abstraction*, in the sense in which it will here be used, is a necessary inference whose conclusion refers to a subject not referred to by the premiss; or it may be used to denote the characteristic of such inference. But how can it be that a conclusion should necessarily follow from a premiss which does not assert the existence of that whose existence is affirmed by it, the conclusion itself? The reply must be that the new individual spoken of is an *ens rationis*; that is, its being consists in some other fact. Whether or not an *ens rationis* can *exist* or be *real*, is a question not to be answered until existence and reality have been very distinctly defined. But it may be noticed at once, that to deny every mode of being to anything whose being consists in some other fact would be to deny every mode of being to tables and chairs, since the being of a table depends on the being of the atoms of which it is composed, and not *vice versa*.

Peirce: CP 4.464 Cross-Ref: ††

464. Every symbol is an *ens rationis*, because it consists in a habit, in a regularity; now every regularity consists in the future conditional occurrence of facts not themselves that regularity. Many important truths are expressed by propositions which relate directly to symbols or to ideal objects of symbols, not to realities. If we say that two walls collide, we express a real relation between them, meaning by a *real relation* one which involves the existence of its correlates. If we say that a ball is red, we express a positive quality of feeling really connected with the ball. But if we say that the ball is not blue, we simply express -- as far as the direct expression goes -- a relation of inapplicability between the predicate blue, and the ball or the sign of it. So it is with every negation. Now it has already been shown that every universal proposition involves a negation, at least when it is expressed as an existential graph.

On the other hand, almost every graph expressing a proposition not universal has a line of identity. But identity, though expressed by the line as a dyadic relation, is not a relation between two things, but between two representamens of the same thing.

Peirce: CP 4.465 Cross-Ref:††

465. Every rhema whose blanks may be filled by signs of ordinary individuals, but which signifies only what is true of symbols of those individuals, without any reference to qualities of sense, is termed a *rhema of second intention*. For *second intention* is thought about thought as symbol. Second intentions and certain *entia rationis* demand the special attention of the logician. Avicenna defined logic as the science of second intentions, and was followed in this view by some of the most acute logicians, such as Raymund Lully, Duns Scotus, Walter Burleigh, and Armandus de Bello Visu; while the celebrated Durandus à Sancto Porciano, followed by Gratiadeus Esculanus, made it relate exclusively to *entia rationis*, and quite rightly.

Peirce: CP 4.466 Cross-Ref:††

466. *Interpretational Corollary 5*. A blank, considered as a medad, expresses what is well-understood between graphist and interpreter to be true; considered as a monad, it expresses "--exists" or "--is true"; considered as a dyad, it expresses "--coexists with--" or "and."

Peirce: CP 4.467 Cross-Ref:††

467. *Interpretational Corollary 6*. An empty sep with its surrounding blank, as in Fig. 114, is the pseudograph. Whether it be taken as medad, monad, or dyad, for which purpose it will be written as in Figs. 115, 116, it is the denial of the blank.

Figs. 114-116



Fig. 114



Fig. 115



Fig. 116

to view]

[Click here

Peirce: CP 4.468 Cross-Ref:††

468. *Interpretational Corollary 7*. A line of identity traversing a sep will

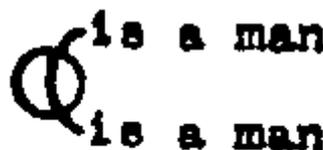


Fig. 117

signify non-identity. Thus Fig. 117 to view] will express that there are at least two men.

[Click here

Peirce: CP 4.469 Cross-Ref:††

469. *Interpretational Corollary 8*. A branching of a line of identity enclosed

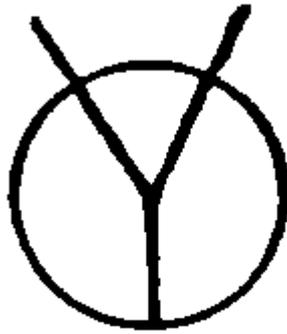


Fig. 118

in a sep, as in Fig. 118
 express that three individuals are not all identical.

[Click here to view], will

Peirce: CP 4.469 Cross-Ref:††

We now come to another kind of graphs which may go under the general head of second intentional graphs.†1

Peirce: CP 4.470 Cross-Ref:††

470. *Convention No. 13. The letters, $\{r\}_0$ $\{r\}_1$, $\{r\}_2$, $\{r\}_3$, etc., each with a number of hooks greater by one than the subscript number, may be taken as rhemata, signifying that the individuals joined to the hooks, other than the one vertically above the $\{r\}$, taken in their order clockwise, are capable of being asserted of the rhema indicated by the line of identity joined vertically to the $\{r\}$.*

Peirce: CP 4.470 Cross-Ref:††

Thus, Fig. 119

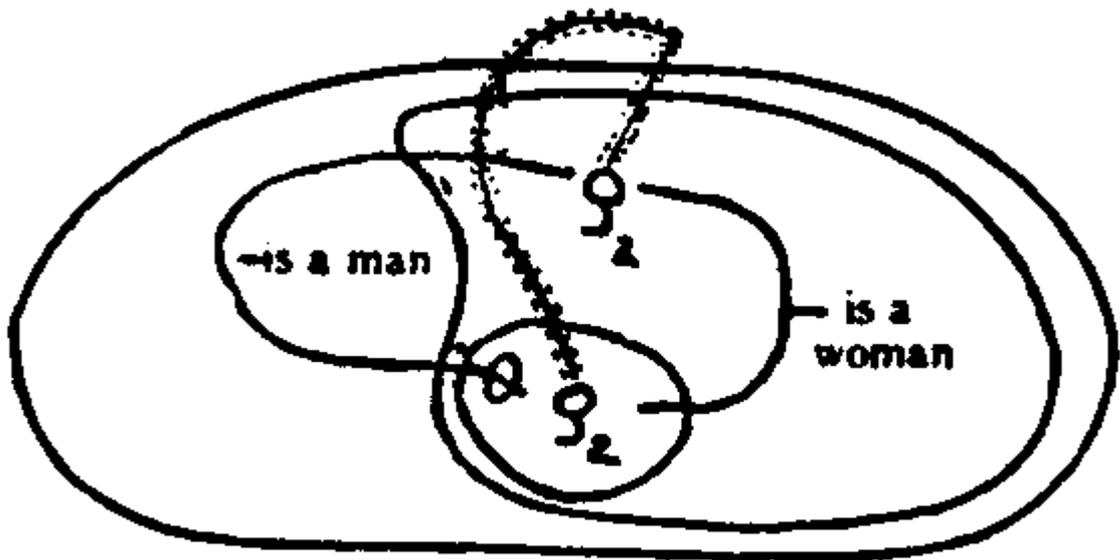


Fig. 119

[Click here to view] expresses that there is a relation in which every man stands to some woman to whom no other man stands in the same relation; that is, there is a woman corresponding to every man or, in other words, there are at least as many women as men. The dotted lines, between which, in Fig. 119, the line of identity denoting the *ens rationis* is placed, are by no means necessary.

Peirce: CP 4.471 Cross-Ref:††

471. *Convention No. 14. The line of identity representing an ens rationis may be placed between two rows of dots, or it may be drawn in ink of another colour, and any graph, which is to be spoken of as a thing, may be enclosed in a dotted oval with a dotted line attached to it. Other entia rationis may be treated in the same way, the patterns of the dotting being varied for those of different category.*

Peirce: CP 4.471 Cross-Ref:††

The graph of Fig. 120

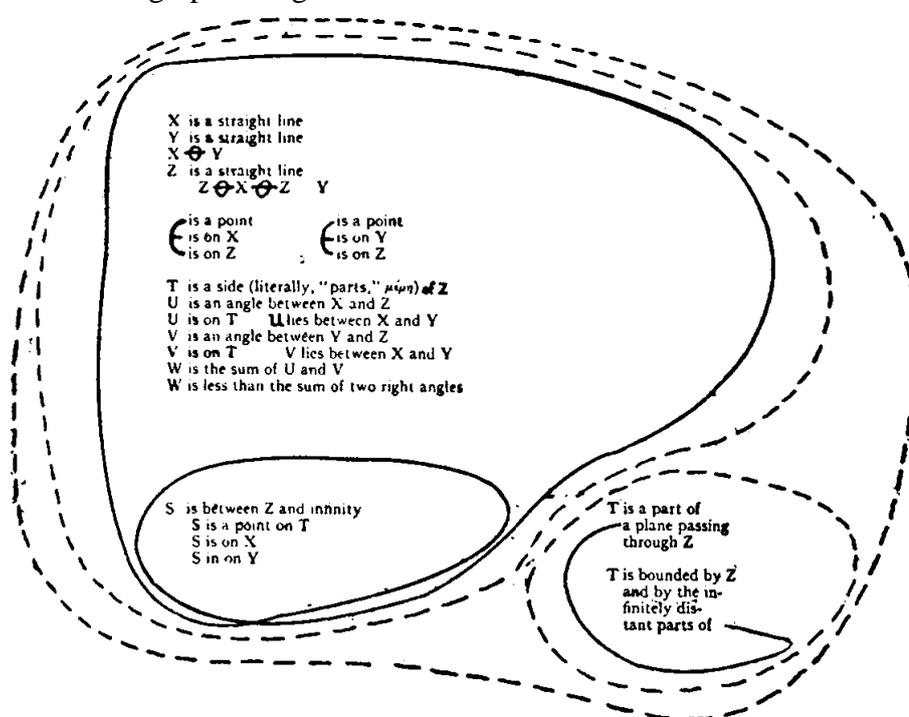


Fig. 120

[Click here

to view] is an example. It may be read, as follows: "Euclid †2 enunciates it as a postulate that if two straight lines are cut by a third straight line so that those angles the two make with the third, these angles lying between the first two lines ({tas entos gönias }) and on the same side of the third, are less than two right angles, then that those two lines shall meet on that same side; and in this enunciation, by a side, {meré } of the third line must be understood part of a plane that contains that third line, which part is bounded by that line and by the infinitely distant parts of the plane." . . .

Peirce: CP 4.472 Cross-Ref:††

C. RecapitulationP

472. The principles of interpretation may now be restated more concisely and more comprehensibly. In this resume, it will be assumed that selectives, which should be regarded as a mere abbreviating device, and which constitute a serious exception to the general idea of the system, are not used. A person, learning to use the system and not yet thoroughly expert in it, might be led to doubt whether every proposition is capable of being expressed without selectives. For a line of identity cannot identify two individuals within enclosures outside of one another without passing out of both enclosures, while a selective is not subject to that restriction. It can be shown, however, that this restriction is of no importance nor even helps to render thought clear. Suppose then that two designations of individuals are to be identified, each being within a separate nest of seps, and the two nests being within a common nest of outer seps. The question is whether this identification can always be properly effected by a line of identity that passes out of the two separate nests of seps, and if desired, still farther out. The answer is plain enough when we consider that, having to say something of individuals, some to be named by the grapheus, others by the graphist, we can perfectly well postpone what we have to say until all these individuals are indicated; that is to say, the order in which they are to be specified by one and the other party. But if this be done, these individuals will first appear, even if selectives are used, in one nest of seps entirely outside of all the spots; and then these selectives can be replaced by lines of identity.

Peirce: CP 4.473 Cross-Ref:††

473. The respect in which selectives violate the general idea of the system is this; the outermost occurrence of each selective has a different significative force from every other occurrence -- a grave fault, if it be avoidable, in any system of regular and exact representation. The consequence is that the meaning of a partial graph containing a selective depends upon whether or not there be another part, which may be written on a remote part of the sheet in which the same selective occurs farther out. But the idea of this system is that assertions written upon different parts of the sheet should be independent of one another, if, and only if, they have no common part. When lines of identity are used to the exclusion of selectives, no such inconvenience can occur, because each line of one partial graph will retain precisely the same significative force, no matter what part outside of it be removed (though if a line be broken, the identity of the individuals denoted by its two parts will no longer be affirmed); and even if everything outside a sep be removed (the sep being unbreakable by any removal of a partial graph, or part which written alone would express a proposition) still there remains a point on the sep which retains the same force as if the line had been broken quite outside and away from the sep.

Peirce: CP 4.474 Cross-Ref:††

474. Rejecting the selectives, then, the principles of interpretation reduce themselves to simple form, as follows:

1. The writing of a proposition on the sheet of assertion unenclosed is to be understood as asserting that proposition; and that, independently of any other proposition on the sheet, except so far as the two may have some part or point in common.

2. A "spot," or unanalyzed expression of a rhema, upon this system, has upon its periphery a place called a "hook" appropriated to every blank of the rhema; and whenever it is written a heavily marked point occupies each hook. Now every heavily marked point, whether isolated or forming a part of a heavy line, denotes an

indesignate individual, and being unenclosed affirms the existence of some such individual; and if it occupy a hook of a spot it is the corresponding subject of the rhema signified by the spot. A heavy line is to be understood as asserting, when unenclosed, that all its points denote the same individual, so that any portion of it may be regarded as a spot.

3. A sep, or lightly drawn oval, when unenclosed is with its contents (the whole being called an *enclosure*) a graph, entire or partial, which precisely denies the proposition which the entire graph within it would, if unenclosed, affirm. Since, therefore, an entire graph, by the above principles, copulatively asserts all the partial graphs of which it is composed, and takes every indesignate individual, denoted by a heavily marked point that may be a part of it, in the sense of "something," it follows that an unenclosed enclosure disjunctively denies all the partial graphs which compose the contents of its sep, and takes every heavily marked point included therein in the sense of "anything" whatever. Consequently, if an enclosure is oddly enclosed, its evenly enclosed contents are copulatively affirmed; while if it be evenly enclosed, its oddly enclosed contents are disjunctively denied.

4. A heavily marked point upon a sep, or line of enclosure, is to be regarded as no more enclosed than any point just outside of and away from the sep, and is to be interpreted accordingly. But the effect of joining a heavily marked point within a sep to such a point upon the sep itself by means of a heavy line is to limit the disjunctive denial of existence (which is the effect of the sep upon the point within it) to the individual denoted by the point upon the sep. No heavy line is to be regarded as cutting a sep; nor can any graph be partly within a sep and partly outside of it; although the entire enclosure (which is not inside the sep) may be part of a graph outside of the sep. †1

5. A dotted oval is sometimes used to show that that which is within it is to be regarded as an *ens rationis*.

Peirce: CP 4.475 Cross-Ref: ††

PART II. THE PRINCIPLES OF ILLATIVE TRANSFORMATION^P

A. *Basic Principles*^P

§1. SOME AND ANY

475. The first part of this tract was a grammar of this language of graphs. But one has not mastered a language as long as one has to think about it in another language. One must learn to think *in it about* facts. The present part is designed to show how to reason *in* this language without translating it into another, the language of our ordinary thought. This reasoning, however, depends on certain first principles, for the justification of which we have to make a last appeal to instinctive thought.

Peirce: CP 4.476 Cross-Ref: ††

476. The purpose of reasoning is to proceed from the recognition of the truth

we already know to the knowledge of novel truth. This we may do by instinct or by a habit of which we are hardly conscious. But the operation is not worthy to be called reasoning unless it be deliberate, critical, self-controlled. In such genuine reasoning we are always conscious of proceeding according to a general rule which we approve. It may not be precisely formulated, but still we do think that all reasoning of that perhaps rather vaguely characterized kind will be safe. This is a doctrine of logic. We never can really reason without entertaining a logical theory. That is called our *logica utens*.†2

Peirce: CP 4.477 Cross-Ref:††

477. The purpose of logic is attained by any single passage from a premiss to a conclusion, as long as it does not at once happen that the premiss is true while the conclusion is false. But reasoning proceeds upon a rule, and an inference is not *necessary*, unless the rule be such that in every case the fact stated in the premiss and the fact stated in the conclusion are so related that either the premiss will be false or the conclusion will be true. (Or *both*, of course. "*Either A or B*" does not properly exclude "*both A and B*.") Even then, the reasoning may not be *logical*, because the rule may involve matter of fact, so that the reasoner cannot have sufficient ground to be absolutely certain that it will not sometimes fail. The inference is only logical if the reasoner can be *mathematically certain* of the excellence of his rule of reasoning; and in the case of necessary reasoning he must be mathematically certain that in every state of things whatsoever, whether now or a million years hence, whether here or in the farthest fixed star, such a premiss and such a conclusion will never be, the former true and the latter false. It would be far beyond the scope of this tract to enter upon any thorough discussion of how this can be. Yet there are some questions which concern us here -- as, for example, how far the system of rules of this section is eternal verity, and how far it merely characterizes the special language of existential graphs -- and yet trench closely upon the deeper philosophy of logic; so that a few remarks meant to illuminate those pertinent questions and to show how they are connected with the philosophy of logic seem to be quite in order.

Peirce: CP 4.478 Cross-Ref:††

478. Mathematical certainty is not absolute certainty. For the greatest mathematicians sometimes blunder, and therefore it is *possible* -- barely possible -- that all have blundered every time they added two and two. Bearing in mind that fact, and bearing in mind the fact that mathematics deals with imaginary states of things upon which experiments can be enormously multiplied at very small cost, we see that it is not impossible that inductive processes should afford the basis of mathematical certainty; and any mathematician can find much in the history of his own thought, and in the public history of mathematics to show that, as a matter of fact, inductive reasoning is considerably employed in making sure of the first mathematical premisses. Still, a doubt will arise as to whether this is anything more than a psychological need, whether the reasoning really rests upon induction at all. A geometer, for example, may ask himself whether two straight lines can enclose an area of their plane. When this question is first put, it is put in reference to a concrete image of a plane; and, at first, some experiments will be tried in the imagination. Some minds will be satisfied with that degree of certainty: more critical intellects will not. They will reflect that a closed area is an area shut off from other parts of the plane by a boundary all round it. Such a thinker will no longer think of a closed area by a composite photograph of triangles, quadrilaterals, circles, etc. He will think of a predictive rule -- a thought of what experience one would intend to produce who should intend to establish a closed area.

Peirce: CP 4.479 Cross-Ref:††

479. That step of thought, which consists in interpreting an image by a symbol, is one of which logic neither need nor can give any account, since it is subconscious, uncontrollable, and not subject to criticism. Whatever account there is to be given of it is the psychologist's affair. But it is evident that the image must be connected in some way with a symbol if any proposition is to be true of it. The very truth of things must be in some measure representative.

Peirce: CP 4.480 Cross-Ref:††

480. If we admit that propositions express the very reality, it is not surprising that the study of the nature of propositions should enable us to pass from the knowledge of one fact to the knowledge of another.†P1

Peirce: CP 4.481 Cross-Ref:††

481. We frame a system of expressing propositions -- a written language -- having a syntax to which there are absolutely no exceptions. We then satisfy ourselves that whenever a proposition having a certain syntactical form is true, another proposition definitely related to it -- so that the relation can be defined in terms of the appearance of the two propositions on paper -- will necessarily also be true. We draw up our code of basic rules of such illative transformations, none of these rules being a necessary consequence of others. We then proceed to express in our language the premisses of long and difficult mathematical demonstrations and try whether our rules will bring out their conclusions. If, in any case, not, and yet the demonstration appears sound, we have a lesson in logic to learn. Some basic rule has been omitted, or else our system of expression is insufficient. But after our system and its rules are perfected, we shall find that such analyses of demonstrations teach us much about those reasonings. They will show that certain hypotheses are superfluous, that others have been virtually taken for granted without being expressly laid down; and they will show that special branches of mathematics are characterized by appropriate modes of reasoning, the knowledge of which will be useful in advancing them. We may now lay all that aside, and begin again, constructing an entirely different system of expression, developing it from an entirely different initial idea, and having perfected it, as we perfected the former system, we shall analyze the same mathematical demonstrations. The results of the two methods will agree as to what is and what is not a necessary consequence. But a consequence that either method will represent as an immediate application of a basic rule, and therefore as simple, the other will be pretty sure to analyze into a series of steps. If it be not so, in regard to some inference the one method will be merely a disguise of the other. To say that one thing is simpler than another is an incomplete proposition, like saying that one ball is to the right of another. It is necessary to specify what point of view is assumed, in order to render the sentence true or false.

Peirce: CP 4.482 Cross-Ref:††

482. This remark has its application to the business now in hand, which is to translate the effect of each simple illative transformation of an existential graph into the language of ordinary thought and thus show that it represents a necessary consequence. For it will be found that it is not the operations which are simplest in this system that are simplest from the point of view of ordinary thought; so that it will be found that the simplest way to establish by ordinary thought the correctness of our basic rules will be to begin by proving the legitimacy of certain operations that are less simple from the point of view of the existential graphs.

Peirce: CP 4.483 Cross-Ref:††

483. The first proposition for assent to which I shall appeal to ordinary reason is this; when a proposition contains a number of *anys* and *somes*, or their equivalents, it is a delicate matter to alter the form of statement while preserving the exact meaning. Every *some*, as we have seen,†1 means that under stated conditions, an individual could be specified of which that which is predicated of the *some* is [true], while every *any* means that what is predicated is true of no matter what [specified] individual; and the specifications of individuals must be made in a certain order, or the meaning of the proposition will be changed. Consider, for example, the following proposition: "A certain bookseller only quotes a line of poetry in case it was written by some blind authoress, and he either is trying to sell any books she may have written to the person to whom he quotes the line or else intends to reprint some book of hers." Here the existence of a bookseller is categorically affirmed; but the existence of a blind authoress is only affirmed conditionally on that bookseller's quoting a line of poetry. As for any book by her, none such is positively said to exist, unless the bookseller is not endeavoring to sell all the books there may be by her to the person to whom he quotes the line.

Peirce: CP 4.484 Cross-Ref:††

484. Now the point to which I demand the assent of reason is that all those individuals, whose selection is so referred to, might be named to begin with, thus: "There is a certain individual, A, and no matter what Z and Y may be, an individual, B, can be found such that whatever X may be, there is something C, and A is a bookseller and if he quotes Z to Y, and if Z is a line of poetry and Y is a person, then B is a blind poetess who has written Z, and either X is not a book published by B or A tries to sell X to Y or else C is a book published by B and A intends to reprint C." This is the precise equivalent of the original proposition, and any proposition involving *somes* and *anys*, or their equivalents, might equally be expressed by first thus defining exactly what these *somes* and *anys* mean, and then going on to predicate concerning them whatever is to be predicated. This is so evident that any proof of it would only confuse the mind; and anybody who could follow the proof will easily see how the proof could be constructed. But after the *somes* and *anys* have thus been replaced by letters, denoting each one individual, the subsequent statement concerns merely a set of designate individuals.

Peirce: CP 4.485 Cross-Ref:††
§2. RULES FOR DIRECTED GRAPHS

485. In order, then, to make evident to ordinary reason what are the simple illative transformations of graphs, I propose to imagine the lines of identity to be all replaced by selectives, whose first occurrences are entirely outside the substance of the graph in a nest of seps, where each selective occurs once only and with nothing but existence predicated of it (affirmatively or negatively according as it is evenly or oddly enclosed). I will then show that upon such a graph certain transformations are permissible, and then will suppose the selectives to be replaced by lines of identity again. We shall thus have established the permissibility of certain transformations without the intervention of selectives.

Peirce: CP 4.486 Cross-Ref:††

486. There will therefore be two branches to our inquiry. First, what transformations may be made in the inner part of the graph where all the selectives have proper names, and secondly what transformations may be made in the outer part where each selective occurs but once. It will be found that the second inquiry almost answers itself after the first has been investigated, and further, that the first class of transformations are precisely the same as if all the first occurrences of selectives were erased and the others were regarded as proper names. We therefore begin by inquiring what transformations are permissible in a graph which has no connexi at all, neither lines of identity nor selectives.

Peirce: CP 4.487 Cross-Ref: ††

487. First of all, let us inquire what are those modes of illative transformation by each of which any graph whatever, standing alone on the sheet of assertion, may be transformed, and, at the same time, what are those modes of illative transformation from each of which any graph whatever, standing alone on the sheet of assertion, might result. Let us confine ourselves, in the first instance, to transformations not only involving no connexi, but also involving no *entia rationis* nor seps. Let us suppose a graph, say that of Fig. 121,

a

Fig. 121

to be alone upon the sheet of assertion. In what ways can it be illatively transformed without using connexi nor seps nor other *entia rationis*? In the first place, it may be erased; for the result of erasure, asserting nothing at all, can assert nothing false. In the second place, it can be iterated, as in Fig. 122;

aa

Fig. 122

for the result of the iteration asserts nothing not asserted already. In the third place, any graph, well-understood (before the original graph was drawn) to be true, can be inserted, as in The Fig. 123.

The universe

is here

a

Fig. 123

Evidently, these are the only modes of transformation that conform to the assumed conditions. Next, let us inquire in what manner any graph, say that of Fig. 124,

z

Fig. 124

can result. It cannot, unless of a special nature, result from insertion, since the blank is true and the graph may be false; but it can result by any omission, say of *y* from the graph of Fig. 125,

y z

Fig. 125

whether y be true or false, or whatever its relation to z , since the result asserts nothing not asserted in the graph from which it results.

Peirce: CP 4.488 Cross-Ref:††

488. We may now employ the following:

Peirce: CP 4.488 Cross-Ref:††

Conditional Principle No. 1. If any graph, a , were it written alone on the sheet of assertion, would be illatively transformable into another graph, z , then if the former graph, a , is a partial graph of an entire graph involving no connexus or sep, and written on the sheet of assertion, a may still be illatively transformed in the same way.

Peirce: CP 4.488 Cross-Ref:††

For let a be a partial graph of which the other part is m , in Fig. 126.

a m z m

Fig. 126 Fig. 127

Then, both a and m will be asserted. But since a would be illatively transformable into z if it were the entire graph, it follows that if a is true z is true. Hence, the result of the transformation asserts only m which is already asserted, and z which is true if a , which is already asserted, is true.

Peirce: CP 4.489 Cross-Ref:††

489. By means of this principle we can evidently deduce the following:

Peirce: CP 4.489 Cross-Ref:††

***Categorical Basic Rules for the Illative Transformation of Graphs
dinectively built up from partial graphs not separated by seps.***

1. Any partial graph may be erased.
2. Any partial graph may be iterated.
3. Any graph well-understood to be true may be inserted.

Peirce: CP 4.489 Cross-Ref:††

It is furthermore clear that no transformation of such graphs is ***logical***, that is, results from the mere form of the graph, that is not justified by these rules. For a transformation not justified by these rules must insert something not in the premiss and not well-understood to be true. But under those circumstances, it may be false, as far as appears from the form.

Peirce: CP 4.490 Cross-Ref:††

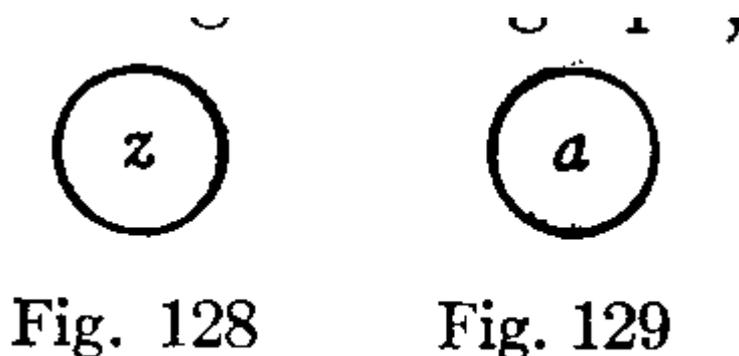
490. Let us now consider graphs having no connexi or ***entia rationis*** other than seps. Here we shall have the following

Peirce: CP 4.490 Cross-Ref:††

Conditional Principle No. 2. If a graph, a , were it written alone on the sheet of assertion, would be illatively transformable into a sep containing nothing but a graph, z , then in case nothing is on the sheet of assertion except this latter graph, z , this will be illatively transformable into a sep containing nothing but a .

Peirce: CP 4.490 Cross-Ref:††

For to say that Fig. 123 [?121] is illatively transformable into Fig. 128, is to say that if a is true, then if z were true, anything you like would be true; while to say that Fig. 124 is illatively transformable into Fig. 129 is to say that if z is true, then if a were true, anything you like would be true. But each of these amounts to saying that if a and z were both true anything you like would be true. Therefore, if either [transformation] is true so is the other.



Figs. 128-129
here to view]

[Click

Peirce: CP 4.491 Cross-Ref:††

491. **Conditional Principle No. 3.** If a sep containing nothing but a graph, a , would, were it written alone on the sheet of assertion, be illatively transformable into a graph, z , then if a sep, containing nothing but the latter graph, z , were written alone on the sheet of assertion, [this would] be illatively transformable into the graph, a .

Peirce: CP 4.491 Cross-Ref:††

For to say that Fig. 129 is illatively transformable into Fig. 124 is to say that by virtue of the forms of a and z , if a is false, z is true; in other words, by virtue of their forms, either a or z is true. But this is precisely the meaning of saying that Fig. 128 is illatively transformable into Fig. 123 [?121].

Peirce: CP 4.492 Cross-Ref:††

492. By means of these principles we can deduce the following:

Peirce: CP 4.492 Cross-Ref:††

**Basic Categorical Rules for the Illative Transformation of Graphs
dinectively built up from Partial Graphs and from Graphs separated by seps.**

Peirce: CP 4.492 Cross-Ref:††

Rule 1. Within an even finite number (including none) of seps, any graph may be erased; within an odd number any graph may be inserted.

Peirce: CP 4.492 Cross-Ref:††

Rule 2. Any graph may be iterated within the same or additional seps, or if iterated, a replica may be erased, if the erasure leaves another outside the same or additional seps.

Peirce: CP 4.492 Cross-Ref:††

Rule 3. Any graph well-understood to be true (and therefore an enclosure having a pseudograph within an odd number of its seps) may be inserted outside all seps.

Peirce: CP 4.492 Cross-Ref:††

Rule 4. Two seps, the one enclosing the other but nothing outside that other, can be removed.

Peirce: CP 4.493 Cross-Ref:††

493. These rules have now to be demonstrated. The former set of rules, already demonstrated, apply to every graph on the sheet of assertion composed of directed partial graphs not enclosed; for the reasoning of the demonstrations so apply. It is now necessary to demonstrate, from Conditional Principle No. 2, the following **Principle of Contraposition**: If any graph, say that of Fig. 123 [?121], is illatively transformable into another graph, say that of Fig. 124, then an enclosure consisting of a sep containing nothing but the latter graph, as in Fig. 130, is illatively transformable into

Figs. 130-134



Fig. 130



Fig. 131



Fig. 132

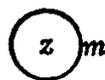


Fig. 133



Fig. 134

to view]

[Click here

an enclosure consisting of a sep containing nothing but the first graph, as in Fig. 131. In order to prove this principle, we must first prove that any graph on the sheet of assertion is illatively transformable by having two seps drawn round it, the one containing nothing but the other with its contents. For let z be the original graph. Then, it has to be shown that Fig. 124 is transformable into Fig. 132. Now Fig. 130 on the sheet of assertion is illatively transformable into itself since any graph is illatively transformable into any graph that by virtue of its form cannot be false unless the original graph be false, and Fig. 130 cannot be false unless Fig. 130 is false. But from this it follows, by Conditional Principle No. 2, that Fig. 124 is illatively transformable into Fig. 132. Q. E. D. The principle of contraposition, which can now be proved without further difficulty, is that if any graph, a , (Fig. 123[?121]), is illatively transformable into any graph, z , (Fig. 124) then an enclosure (Fig. 130) consisting of a sep enclosing nothing but the latter graph, z , is transformable into an enclosure (Fig. 131) consisting of a sep containing nothing but the first graph, a . If a is transformable into z , then, by the rule just proved, it is transformable into Fig. 132, consisting of z doubly enclosed with nothing between the seps. But if Fig. 123 [?121] is illatively transformable into Fig. 132, then, by Conditional Principle No. 2, Fig. 130 is illatively transformable into Fig. 131, Q. E. D.

Peirce: CP 4.494 Cross-Ref:††

494. Supposing, now, that Rule 1 holds good for any insertion or omission within not more than any finite number, N , of seps, it will also hold good for every

insertion or omission within not more than $N+1$ seps. For in any graph on the sheet of insertions of which a partial graph is an enclosure consisting of a sep containing only a graph, z , involving a nest of N seps, let the partial graph outside this enclosure be m , so that Fig. 133 is the entire graph. Then application of the rule within the $N+1$ seps will transform z into another graph, say a , so that Fig. 134 will be the result. Then a , were it written on the sheet of assertion unenclosed and alone, would be illatively transformable into z , since the rule is supposed to be valid for an insertion or omission within N seps. Hence, by the principle of contraposition, Fig. 130 will be transformable into Fig. 131, and by Conditional Principle No. 1, Fig. 133 will be transformable into Fig. 134. It is therefore proved that if Rule 1 is valid within any number of seps up to any finite number, it is valid for the next larger whole number of seps. But by Rule 1 of the former set of rules, it is valid for $N = 0$, and hence it follows that it is valid within seps whose number can be reached from 0 by successive additions of unity; that is, for any finite number. Rule 1 is, therefore, valid as stated. It will be remarked that the partial graphs may have any multitude whatsoever; but the seps of a nest are restricted to a finite multitude, so far as this rule is concerned. A graph with an endless nest of seps is essentially of doubtful meaning, except in special cases. Thus Fig. 135

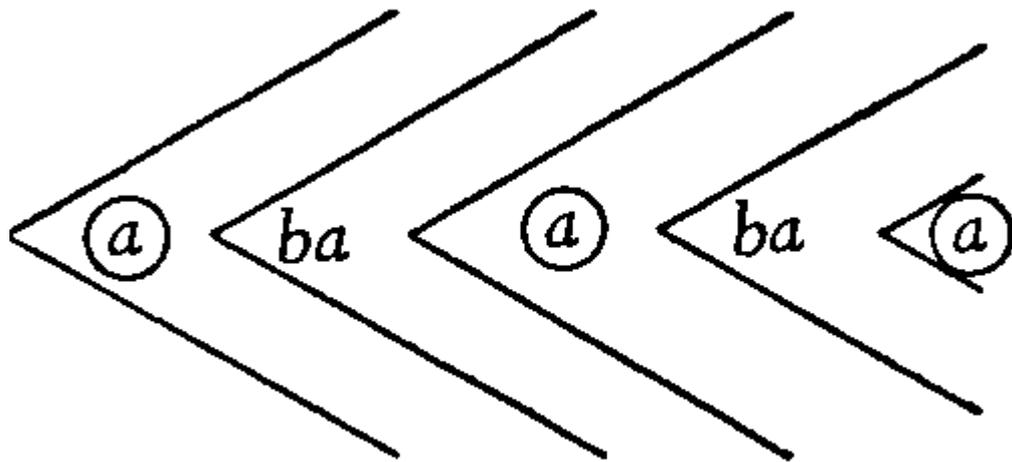


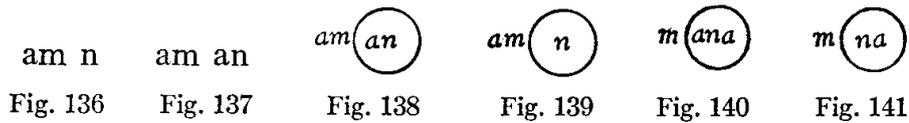
Fig. 135

[Click here to view], supposed to continue the alternation endlessly, evidently merely asserts the truth of a .†1 But if instead of ba , b were everywhere to stand alone, the graph would certainly assert either a or b to be true and would certainly be true if a were true, but whether it would be true or false in case b were true and not a is essentially doubtful.

Peirce: CP 4.495 Cross-Ref:††

495. Rule 2 is so obviously demonstrable in the same way that it will be sufficient to remark that unenclosed iterations of unenclosed graphs are justified by Rule 2 of the former set of rules. Then, since Fig. 136 is illatively transformable into

Figs. 136-141



[Click here

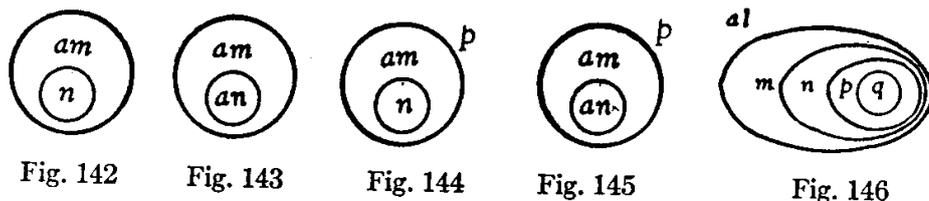
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Fig. 137, it follows from the principle of contraposition that Fig. 138 is illatively transformable into Fig. 139. Or we may reason that to say that Fig. 137 follows from Fig. 136 is to say that, *am* being true, *an* follows from *n*; while to say that Fig. 139 follows from Fig. 138, is to say that, *am* being true, as before, if from *an* anything you like follows, then from *n* anything you like follows. In the same way Fig. 140 is transformable into Fig. 141.

Peirce: CP 4.496 Cross-Ref:††

496. The transformations the reverse of these, that is of Fig. 137 into Fig. 136, of Fig. 139 into Fig. 138, and of Fig. 141 into Fig. 140 are permitted by Rule 1. Then by the same Fermatian reasoning by which Rule 1 was demonstrated, we easily show that a graph can anywhere be illatively inserted or omitted, if there is another occurrence of the same graph in the same compartment or farther out by one sep. For if Fig. 138 is transformable into Fig. 139, then by the principle of contraposition, Fig. 142 is transformable into Fig. 143, and by Conditional Principle No. 1, Fig. 144 is transformable in Fig. 145. Having thus proved that iterations and deiterations are always permissible in the same compartment as the leading replica or in a compartment within one additional sep, we have no difficulty in extending this to any finite interval.

Figs. 142-146



[Click here

to view]

Thus, Fig. 146 is transformable into Fig. 147, this into Fig. 148, this successively into Figs. 149 to 153. Thus, the second rule is fully demonstrable.

Peirce: CP 4.496 Cross-Ref:††

Rule 3 is self-evident.

Peirce: CP 4.497 Cross-Ref:††

497. We have thus far had no occasion to appeal to Conditional Principle No. 3; but it is indispensable for the proof of Rule 4. We have to show that if any graph, which [we] may denote by *z* is surrounded by two seps with nothing

Figs. 147-150

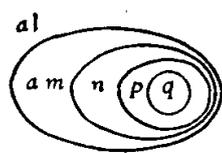


Fig. 147

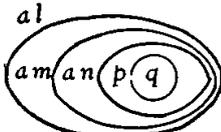


Fig. 148

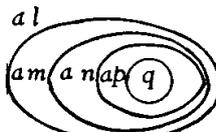


Fig. 149



Fig. 150

[Click here

to view]

Figs. 151-153

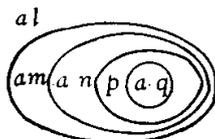


Fig. 151

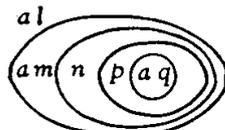


Fig. 152

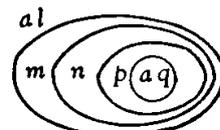


Fig. 153

[Click here

to view]

between as in Fig. 132, then the two seps may be illatively removed as in Fig. 124. Now if the graph, z, occurred within one sep, as in Fig. 130, this, as we have seen, would be transformed into itself. Hence, by Conditional Principle No. 3, Fig. 132, can be illatively transformed into Fig. 124. Q. E. D.

Peirce: CP 4.498 Cross-Ref:††

498. The list of rules given for directed graphs is complete. This is susceptible of proof; but the proof belongs in the next section of this chapter, where I may perhaps insert it. It is not interesting.

Peirce: CP 4.499 Cross-Ref:††

B. Rules for Lines of Identity

499. We now pass to the consideration of graphs connected by lines of identity. A small addition to our nomenclature is required here. Namely, we have seen that a line of identity is a partial graph; and as a graph it cannot cross a sep. Let us, then, call a series of lines of identity abutting upon one another at seps, a **ligature**; and we may extend the meaning of the word so that even a single line of identity shall be called a ligature. A ligature composed of more than one line of identity may be distinguished as a **compound ligature**. A compound ligature is not a graph, because by a graph we mean something which, written or drawn alone on the sheet of assertion, would, according to this system, assert something. Now a compound ligature could not be written alone on the sheet of assertion, since it is only by means of the intercepting sep, which is no part of it, that it is rendered compound. The different spots, as well as the different hooks, upon which a ligature abuts, may be said to be **ligated** by that ligature; and two replicas of the same graph are said to have the same **ligations** only when all the corresponding hooks of the two are ligated to one another. When a ligature cuts a sep, the part of the ligature outside the sep may be

said to be *extended* to the point of intersection on the sep, while the part of the ligature inside may be said to be *joined* to that point.

Peirce: CP 4.500 Cross-Ref:††

500. It has already †1 been pointed out that the mass of ink on the sheet by means of which a graph is said to be "scribed" is not, strictly speaking, a symbol, but only a replica of a symbol of the nature of an index. Let it not be forgotten that the significative value of a symbol consists in a regularity of association, so that the identity of the symbol lies in this regularity, while the significative force of an index consists in an existential fact which connects it with its object, so that the identity of the index consists in an existential fact or thing. When symbols, such as words, are used to construct an assertion, this assertion relates to something real. It must not only *profess* to do so, but must really *do* so; otherwise, it could not be true; and still less, false. Let a witness take oath, with every legal formality, that John Doe has committed murder, and still he has made no assertion unless the name John Doe denotes some existing person. But in order that the name should do this, something more than an association of ideas is requisite. For the person is not a conception but an existent thing. The name, or rather, *occurrences* of the name, must be existentially connected with the existent person. Therefore, no assertion can be constructed out of pure symbols alone. Indeed, the pure symbols are immutable, and it is not *them* that are joined together by the syntax of the sentence, but occurrences of them -- *replicas* of them. My aim is to use the term "graph" for a graph-symbol, although I dare say I sometimes lapse into using it for a graph-replica. To say that a graph is scribed is accurate, because "to scribe" means *to make a graphical replica of*. By "a line of identity," on the other hand, it is more convenient to mean a replica of the linear graph of identity. For here the indexical character is more positive; and besides, one seldom has occasion to speak of the graph. But the only difference between a line of identity and an ordinary dyadic spot is that the latter has its hooks marked at points that are deemed appropriate without our being under any factual compulsion to mark them at all, while a simple line such as is naturally employed for a line of identity must, from the nature of things, have extremities which are at once parts of it and of whatever it abuts upon. This difference does not prevent the rules of the last list from holding good of such lines. The only occasion for any additional rule is to meet that situation, in which no other graph-replica than a line of identity can ever be placed, that of having a hook upon a sep.

Peirce: CP 4.501 Cross-Ref:††

501. As to this, it is to be remarked that an enclosure -- that is, a sep with its contents -- is a graph; and those points on its periphery, that are marked by the abuttal upon them of lines of identity, are simply the hooks of the graph. But the sep is outside its own close. Therefore an unmarked point upon it is just like any other vacant place outside the sep. But if a line inside the sep is prolonged to the sep, at the instant of arriving at the sep, its extremity suddenly becomes identified -- as a matter of fact, and there as a matter of signification -- with a point outside the sep; and thus the prolongation suddenly assumes an entirely different character from an ordinary, insignificant prolongation. This gives us the following:

Peirce: CP 4.501 Cross-Ref:††

Conditional Principle No. 4. Only the connexions and continuity of lines of identity are significant, not their shape or size. The connexion or disconnexion of a line of identity outside a sep with a marked or an unmarked point on the sep follows the same rules as its connexion or disconnexion with any other marked or unmarked

point outside the sep, but the junction or disjunction of a line of identity inside the sep with a point upon the sep always follows the same rules as its connexion or disconnexion with a marked point inside the sep.

Peirce: CP 4.501 Cross-Ref:††

In consequence of this principle, although the categorical rules hitherto given remain unchanged in their application to lines of identity, yet they require some modifications in their application to ligatures.

Peirce: CP 4.502 Cross-Ref:††

502. In order to see that the principle is correct, first consider Fig. 154. Now the rule of erasure of an unenclosed graph certainly allows the transformation of this into Fig. 155, which must therefore be interpreted to mean "Something is not ugly," and must not be confounded with Fig. 156, "Nothing is ugly." But Fig. 156 is transformable into Fig. 157; that is, the line of identity with a loose end can be carried to any vacant place within the sep. If, therefore, Fig. 155 were to be treated as if the end of the line were loose, it could be illatively transformed into Fig. 156. But the line can no more be separated from the point of the sep than it could from any marked point within the sep -- any more, for example, than

Figs. 154-157



Fig. 154



Fig. 155



Fig. 156



Fig. 157

to view]

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Fig. 158, "Nothing good is ugly" could be transformed into Fig. 159, "Either nothing is ugly or nothing is good." So Fig. 160 can, by the rule of insertion within odd seps, be transformed into Fig. 161, and must be interpreted, like that, "Everything acts on everything," and not, as in Fig. 162, "Everything acts on something or other." But if the vacant point on the sep could be treated like an ordinary point, Fig. 162 could be illatively transformed into Fig. 160, which the interpretation forbids. Although in this argument special graphs

Figs. 158-162



Fig. 158



Fig. 159



Fig. 160



Fig. 161



Fig. 162

to view]

[Click here

are used, it is evident that the argument would be just the same whatever others were used, and the proof is just as conclusive as if we had talked of "any graph whatever, x," etc., as well as being clearer. The principle of contraposition renders

Figs. 163-177

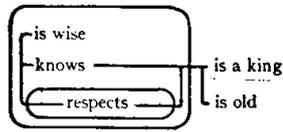


Fig. 163

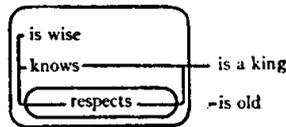


Fig. 164

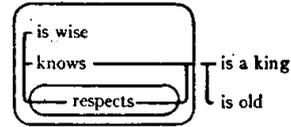


Fig. 165

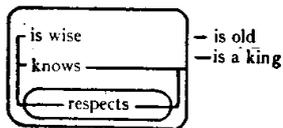


Fig. 166

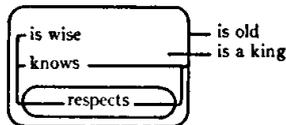


Fig. 167

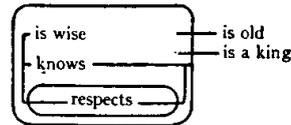


Fig. 168

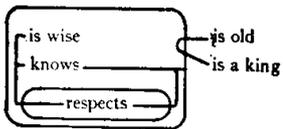


Fig. 169

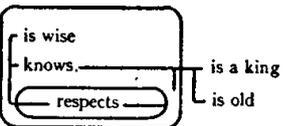


Fig. 170

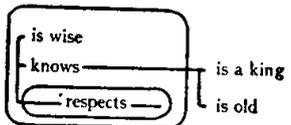


Fig. 171

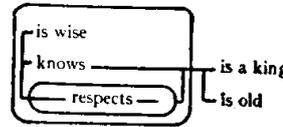


Fig. 172

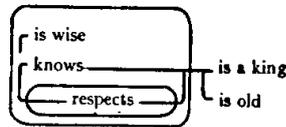


Fig. 173

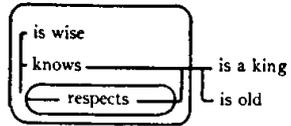


Fig. 174

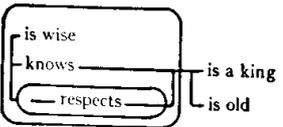


Fig. 175

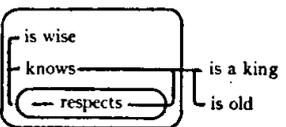


Fig. 176

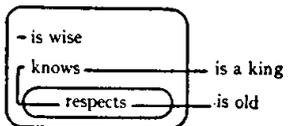


Fig. 177

to view]

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it evident that the same thing would hold for any finite nests of seps.

Peirce: CP 4.503 Cross-Ref:††

503. On the other hand, it is easy to show that the illative connexion or disconnexion of a line exterior to the sep with a point on the sep follows precisely the same rules as if the point were outside of and away from the sep.

Peirce: CP 4.503 Cross-Ref:††

Figs. 163-177 furnish grounds for the demonstration of this. Fig. 163 asserts that there is an old king whom every wise person that knows him respects. The connexion of "is old" with "is king" can be illatively severed by the rule of erasure, as

in Fig. 164; so that the old person shall not be asserted to be identical with the king whom all wise people that know him respect; and once severed the connexion cannot be illatively restored. So it is precisely if the line of identity outside the outer sep is cut at the sep, as in Fig. 165, which asserts that somebody is respected by whatever wise person there may be that knows him, and asserts that there is an old king, but fails to assert that the old king is that respected person. Here, as before, the line can be illatively severed but cannot be illatively restored. It is evident that this is not because of the special significance of the "spots" or unanalyzed rhemata, but that it would be the same in all cases in which a line of identity should terminate at a point on a sep where a line inside that sep should also terminate. Fig. 166 shows both lines broken, so that this might equally and for the same reason result from the illative transformation of Fig. 164 or of Fig. 165. The lines, being broken as in Fig. 166, can be distorted in any way and their extremities can be carried to any otherwise vacant places outside the outer sep, and afterwards can be brought back to their present places. In this respect, a vacant point on a sep is just like any other vacant point outside the close of the sep. If the line of identity attached to "is old" be carried to the sep, as in Fig. 167, certainly no addition is thereby made to the assertion. Once the ligature is carried as far as the sep, the rule of insertion within an odd number of seps permits it to be carried still further, as is done in Fig. 167, with the ligature attached to "is a king." This whole graph may be interpreted, "Something is old and something is a king." But this last does not exist unless something is respected by whatever that is wise there may be that knows it. The graph of Fig. 167 can be illatively retransformed into Fig. 166, by first severing the ligature attached to "is a king" outside the sep by the rule of erasure, when the part of the ligature inside may be erased by the rule of deiteration, and finally the part outside the close of the sep may be erased by the rule of erasure. On the other hand the ligatures attached in Fig. 167 to "is old" and "is a king" might, after Fig. 167 had been converted in Fig. 168, be illatively joined inside the sep by the rule of insertion, as in Fig. 169, which asserts that there is something old and there is a king; and if there is an old king something is respected by whatever wise thing there may be that knows it. This is not illatively retransformable in Fig. 168. It thus abundantly shows that an unenclosed line can be extended to a point on an unenclosed sep under the same conditions as to any other unenclosed point. For there is evidently nothing peculiar about the characters of being old and of being a king which render them different in this respect from graphs in general. Let us now see how it is in regard to singly enclosed lines in their relations to points on seps in the same close. If in Fig. 163 we sever the ligature denoting the object accusative of "respects," just outside the inner sep, as in Fig. 170, the interpretation becomes, "There is an old king, and whoever that is wise there may be who knows him, respects everybody." This is illatively transformable into Fig. 163 by the rule of insertion under odd enclosures, just as if the marked point on the sep were a hook of any spot. We may, of course, by the rule of erasure within even seps, cut away the ligature from the sep internally, getting Fig. 171, "There is an old king, whom anybody that knows respects somebody or other." The point on the sep being now unmarked, it makes no difference whether the outside ligature is extended to it, as in Fig. 172, or not. It is the same if the ligature denoting the subject nominative of "respects" be broken outside the inner sep, as in Fig. 174. Whether this be done, or whether the line of identity joining "is wise" to "knows" be cut, as in Fig. 173, in either case we get a graph illatively transformable into Fig. 163, but not derivable from Fig. 163 by any illative transformation. If, however, the line of identity within the inner sep be retracted from the sep, as in Figs. 175 and 176, it makes no difference whether the line outside the sep be extended to the unmarked point on the sep or not. One cannot even say that

one form of interpretation better fits the one figure and another the other: they are absolutely equivalent. Thus, the unmarked point on the oddly enclosed sep is just like any other unmarked point exterior to the close of the sep as far as its relations with exterior lines of identity are concerned.

Peirce: CP 4.504 Cross-Ref:††

504. The principle of contraposition extends this Conditional Principle No. 4 to all seps, within any finite number of seps.

Peirce: CP 4.504 Cross-Ref:††

By means of this principle the rules of illative transformation hitherto given will easily be extended so as to apply to graphs with ligatures attached to them, and the one rule which it is necessary to add to the list will also be readily deduced. In the following statement, each rule will first be enunciated in an exact and compendious form and then, if necessary, two remarks will be added, under the headings of "Note A" and "Note B." Note A will state more explicitly how the rule applies to a line of identity; while Note B will call attention to a transformation which might, without particular care, be supposed to be permitted by the rule but which is really not permitted.

Peirce: CP 4.505 Cross-Ref:††

***C. Basic Categorical Rules for the Illative Transformation
of All GraphsP***

505. **Rule 1.** Called ***The rule of Erasure and of Insertion.*** In even seps, any graph-replica can be erased; in odd seps any graph-replica can be inserted.

Peirce: CP 4.505 Cross-Ref:††

Note A. By even seps is meant any finite even number of seps, including none; by odd seps is meant any odd number of seps.

Peirce: CP 4.505 Cross-Ref:††

This rule permits any ligature, where evenly enclosed, to be severed, and any two ligatures, oddly enclosed in the same seps, to be joined. It permits a branch with a loose end to be added to or retracted from any line of identity.

Peirce: CP 4.505 Cross-Ref:††

It permits any ligature, where evenly enclosed, to be severed from the inside of the sep immediately enclosing that evenly enclosed portion of it, and to be extended to a vacant point of any sep in the same enclosure. It permits any ligature to be joined to the inside of the sep immediately enclosing that oddly enclosed portion of it, and to be retracted from the outside of any sep in the same enclosure on which the ligature has an extremity.

Peirce: CP 4.505 Cross-Ref:††

Note B. In the erasure of a graph by this rule, all its ligatures must be cut. The rule does not permit a sep to be so inserted as to intersect any ligature, nor does it permit any erasure to accompany an insertion.

Peirce: CP 4.505 Cross-Ref:††

It does not permit the insertion of a sep within even seps.

Peirce: CP 4.506 Cross-Ref:††

506. **Rule 2.** Called *The Rule of Iteration and Deiteration*. Anywhere within all the seps that enclose a replica of a graph, that graph may be iterated with identical ligations, or being iterated, may be deiterated.

Peirce: CP 4.506 Cross-Ref:††

Note A. The operation of iteration consists in the insertion of a new replica of a graph of which there is already a replica, the new replica having each hook ligated to every hook of a graph-replica to which the corresponding hook of the old replica is ligated, and the right to iterate includes the right to draw a new branch to each ligation of the original replica inwards to the new replica. The operation of deiteration consists in erasing a replica which might have illatively resulted from an operation of iteration, and of retracting outwards the ligatures left loose by such erasure until they are within the same seps as the corresponding ligation of the replica of which the erased replica might have been the iteration.

Peirce: CP 4.506 Cross-Ref:††

The rule permits any loose end of a ligation to be extended inwards through a sep or seps or to be retracted outwards through a sep or seps. It permits any cyclical part of a ligation to be cut at its innermost part, or a cycle to be formed by joining, by inward extensions, the two loose ends that are the innermost parts of a ligation.

Peirce: CP 4.506 Cross-Ref:††

If any hook of the original replica of the iterated graph is ligated to no other hook of any graph-replica, the same should be the case with the new replica.

Peirce: CP 4.506 Cross-Ref:††

Note B. This rule does not confer a right to ligate any hook to another nor to deligate any hook from another unless the same hooks, or corresponding hooks of other replicas of the same graphs (these replicas being outside every sep that the hooks ligated or deligated are outside), be ligated otherwise, and outside of every sep that the new ligations or deligations are outside of.

Peirce: CP 4.506 Cross-Ref:††

This rule does not confer the right to extend any ligation outwardly from within any sep, nor to retract any ligation inwardly from without any sep.

Peirce: CP 4.507 Cross-Ref:††

507. **Rule 3.** Called *The Rule of Assertion*. Any graph well-understood to be true may be scribed unenclosed.

Peirce: CP 4.507 Cross-Ref:††

Note A. This rule is to be understood as permitting the explicit assertion of three classes of propositions; first, those that are involved in the conventions of this system of existential graphs; secondly, any propositions known to be true but which may not have been thought of as pertinent when the graph was first scribed or as pertinent in the way in which it is now seen to be pertinent (that is to say, premisses may be added if they are acknowledged to be true); thirdly, any propositions which the scription of the graph renders true or shows to be true. Thus, having graphically asserted that it snows, we may insert a graph asserting "that it snows is asserted" or "it is possible to assert that it snows without asserting that it is winter."

Peirce: CP 4.508 Cross-Ref:††

508. **Rule 4.** Called *The Rule of Biclosure*. Two seps, one within the other, with nothing between them whose significance is affected by seps, may be withdrawn from about the graph they doubly enclose.

Peirce: CP 4.508 Cross-Ref:††

Note A. The significance of a ligature is not affected by a sep except at its outermost part, or if it passes through the close of the sep; and therefore ligatures passing from outside the outer sep to inside the inner one will not prevent the withdrawal of the double sep; and such ligatures will remain unaffected by the withdrawal.

Peirce: CP 4.508 Cross-Ref:††

Note B. A ligature passing twice through the outer sep without passing through the inner one, or passing from within the inner one into the intermediate space and stopping there, will be equivalent to a graph and will preclude the withdrawal.

Peirce: CP 4.509 Cross-Ref:††

509. **Rule 5.** Called *The Rule of Deformation*. All parts of the graph may be deformed in any way, the connexions of parts remaining unaltered; and the extension of a line of identity outside a sep to an otherwise vacant point on that sep is not to be considered to be a connexion.

Peirce: CP 4.510 Cross-Ref:††

CHAPTER 5

THE GAMMA PART OF EXISTENTIAL GRAPHS†1

510. The alpha part of graphs . . . is able to represent no reasonings except those which turn upon the logical relations of general terms.

Peirce: CP 4.511 Cross-Ref:††

511. The beta part . . . is able to handle with facility and dispatch reasonings of a very intricate kind, and propositions which ordinary language can only express by means of long and confusing circumlocutions. A person who has learned to think in beta graphs has ideas of the utmost clearness and precision which it is practically impossible to communicate to the mind of a person who has not that advantage. Its reasonings generally turn upon the properties of the relations of individual objects to one another.

Peirce: CP 4.511 Cross-Ref:††

But it is able to do nothing at all with many ideas which we are all perfectly familiar with. Generally speaking it is unable to reason about abstractions. It cannot reason for example about qualities nor about relations as subjects to be reasoned about. It cannot reason about ideas. It is to supply that defect that the gamma part of the subject has been invented. But this gamma part is still in its infancy. It will be many years before my successors will be able to bring it to the perfection to which the

alpha and beta parts have been brought. For logical investigation is very slow, involving as it does the taking up of a confused mass of ordinary ideas, embracing we know not what and going through with a great quantity of analyses and generalizations and experiments before one can so much as get a new branch fairly inaugurated. . . .

Peirce: CP 4.512 Cross-Ref:††

512. The gamma part of graphs, in its present condition, is characterized by a great wealth of new signs; but it has no sign of an essentially different kind from those of the alpha and beta part. The alpha part has three distinct kinds of signs, the *graphs*, the *sheet of assertion*, and the *cuts*. The beta part adds two quite different kinds of signs, *spots*, or *lexeis*, and *ligatures* with *selectives*. It is true that a line of identity is a graph; but the terminal of such a line, especially a terminal on a cut where two lines of identity have a common point, is radically different. So far, all the gamma signs that have presented themselves, are of those same kinds. If anybody in my lifetime shall discover any radically disparate kind of sign, peculiar to the gamma part of the system, I shall hail him as a new Columbus. He must be a mind of vast power. But in the gamma part of the subject all the old kinds of signs take new forms. . . . Thus in place of a sheet of assertion, we have a book of separate sheets, tacked together at points, if not otherwise connected. For our alpha sheet, as a whole, represents simply a universe of existent individuals, and the different parts of the sheet represent facts or true assertions made concerning that universe. At the cuts we pass into other areas, areas of conceived propositions which are not realized. In these areas there may be cuts where we pass into worlds which, in the imaginary worlds of the outer cuts, are themselves represented to be imaginary and false, but which may, for all that, be true, and therefore continuous with the sheet of assertion itself, although this is uncertain. You may regard the ordinary blank sheet of assertion as a film upon which there is, as it were, an undeveloped photograph of the facts in the universe. I do not mean a literal picture, because its elements are propositions, and the meaning of a proposition is abstract and altogether of a different nature from a picture. But I ask you to imagine all the true propositions to have been formulated; and since facts blend into one another, it can only be in a continuum that we can conceive this to be done. This continuum must clearly have more dimensions than a surface or even than a solid; and we will suppose it to be plastic, so that it can be deformed in all sorts of ways without the continuity and connection of parts being ever ruptured. Of this continuum the blank sheet of assertion may be imagined to be a photograph. When we find out that a proposition is true, we can place it wherever we please on the sheet, because we can imagine the original continuum, which is plastic, to be so deformed as to bring any number of propositions to any places on the sheet we may choose.

Peirce: CP 4.513 Cross-Ref:††

513. So far I have called the sheet a photograph, so as not to overwhelm you with all the difficulties of the conception at once. But let us rather call it a map -- a map of such a photograph if you like. A map of the simplest kind represents all the points of one surface by corresponding points of another surface in such a manner as to preserve the continuity unbroken, however great may be the distortion. A Mercator's chart, however, represents all the surface of the earth by a strip, infinitely long, both north and south poles being at infinite distances, so that places near the poles are magnified so as to be many times larger than the real surfaces of the earth that they represent, while in longitude the whole equator measures only two or three feet; and you might continue the chart so as to represent the earth over and over again

in as many such strips as you pleased. Other kinds of map, such as my Quincuncial Projection which is drawn in the fourth volume of the *American Journal of Mathematics*, †1 show the whole earth over and over again in checkers, and there is no arrangement you can think of in which the different representations of the same place might not appear on a perfectly correct map. This accounts for our being able to scribe the same graph as many times as we please on any vacant places we like. Now each of the areas of any cut corresponds exactly to some locus of the sheet of assertion where there is mapped, though undeveloped, the real state of things which the graph of that area denies. In fact it is represented by that line of the sheet of assertion which the cut itself marks.

Peirce: CP 4.514 Cross-Ref: ††

514. By taking time enough I could develop this idea much further, and render it clearer; but it would not be worth while, for I only mention it to prepare you for the idea of quite different kinds of sheets in the gamma part of the system. These sheets represent altogether different universes with which our discourse has to do. In the Johns Hopkins *Studies in Logic* †2 -- I printed a note of several pages on the universe of qualities -- *marks*, as I then called them. But I failed to see that I was then wandering quite beyond the bounds of the logic of relations proper. For the relations of which the so-called "logic of relatives" treats are *existential* relations, which the nonexistence of either relate or correlate reduces to nullity. Now, *qualities* are not, properly speaking, individuals. All the qualities you actually have ever thought of might, no doubt, be counted, since you have only been alive for a certain number of hundredths of seconds, and it requires more than a hundredth of a second actually to have any thought. But all the qualities, any one of which you readily can think of, are certainly innumerable; and all that might be thought of exceed, I am convinced, all multitude whatsoever. For they are mere logical possibilities, and possibilities are general, and no multitude can exhaust the narrowest kind of a general. Nevertheless, within limitations, which include most ordinary purposes, qualities may be treated as individuals. At any rate, however, they form an entirely different universe of existence. It is a universe of logical possibility. As we have seen, although the universe of existential fact can only be conceived as mapped upon a surface by each point of the surface representing a vast expanse of fact, yet we can conceive the facts [as] sufficiently separated upon the map for all our purposes; and in the same sense the entire universe of logical possibilities might be conceived to be mapped upon a surface. Nevertheless, in order to represent to our minds the relation between the universe of possibilities and the universe of actual existent facts, if we are going to think of the latter as a surface, we must think of the former as three-dimensional space in which any surface would represent all the facts that might exist in one existential universe. In endeavoring to begin the construction of the gamma part of the system of existential graphs, what I had to do was to select, from the enormous mass of ideas thus suggested, a small number convenient to work with. It did not seem to be convenient to use more than one actual sheet at one time; but it seemed that various different kinds of cuts would be wanted.

Peirce: CP 4.515 Cross-Ref: ††

515. I will begin with one of the *gamma cuts*. I call it the *broken cut*. I scribe it thus



Fig. 178

Fig. 178

[[Click here to view](#)]

This does not assert that it does not rain. It only asserts that the alpha and beta rules do not compel me to admit that it rains, or what comes to the same thing, a person altogether ignorant, except that he was well versed in logic so far as it embodied in the alpha and beta parts of existential graphs, would not know that it rained.†1

Peirce: CP 4.516 Cross-Ref:††

516. The rules of this cut are very similar to those of the alpha cut.

Peirce: CP 4.516 Cross-Ref:††

Rule 1. In a broken cut already on the sheet of assertion any graph may be inserted.

Peirce: CP 4.516 Cross-Ref:††

Rule 2. An evenly enclosed alpha cut may be half erased so as to convert it into a broken cut, and an oddly enclosed broken cut may be filled up to make an alpha cut. Whether the enclosures are by alpha or broken cuts is indifferent.

Consequently

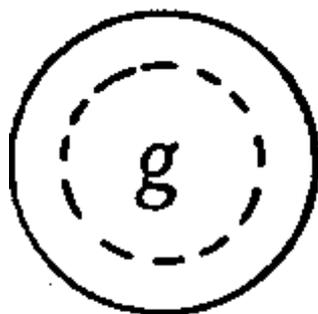


Fig. 179

Fig. 179

[[Click here to view](#)]

will mean that the graph g is beta-necessarily true.†2 By Rule 2, this is converted into

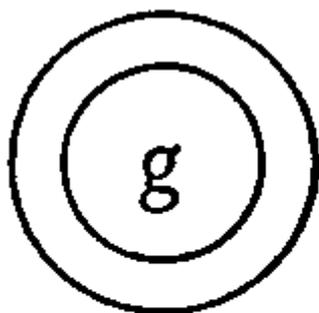


Fig. 180

Fig. 180

[Click here to view]

which is equivalent to

g

Fig. 181

the simple assertion of g . By the same rule Fig. 180 is transformable into

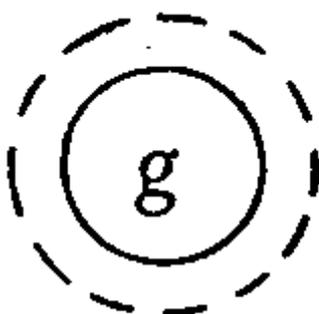


Fig. 182

Fig. 182

[Click here to view]

which means that the beta rules do not make g false.†1 That is g is beta-possible.†2

Peirce: CP 4.516 Cross-Ref:††

So if we start from

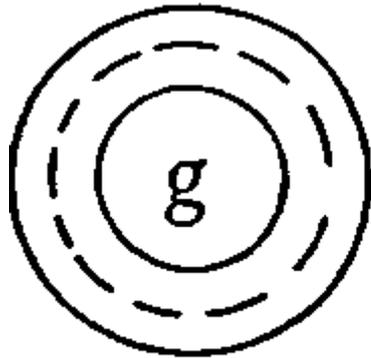


Fig. 183

Fig. 183

[\[Click here to view\]](#)

which denies the last figure and thus asserts that it is beta-impossible that *g* should be true, †3 Rule 2 gives

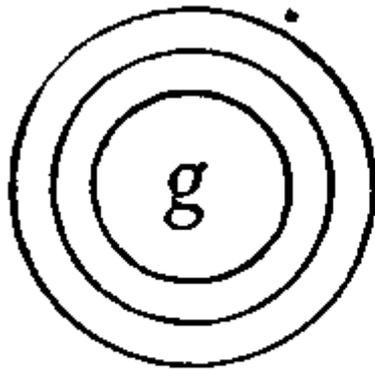


Fig. 184

Fig. 184

[\[Click here to view\]](#) equivalent to Fig.



Fig. 185

185

[\[Click here to view\]](#)

the simple denial of g .†4

And from this we get again



Fig. 186¶

Fig. 186

[\[Click here to view\]†5](#)

Peirce: CP 4.517 Cross-Ref:††

517. It must be remembered that possibility and necessity are relative to the state of information.

Peirce: CP 4.517 Cross-Ref:††

Of a certain graph g let us suppose that I am in such a state of information that it *may be true* and *may be false*; that is I can scribe on the sheet of assertion Figs. 182 and 186. Now I learn that it is true. This gives me a right to scribe on the sheet Figs. 182, 186 and 181. But now relative to this new state of information, Fig. 186 ceases to be true; and therefore relatively to the new state of information we can scribe Fig. 179.†6

Peirce: CP 4.518 Cross-Ref:††

518. You thus perceive that we should fall into inextricable confusion in dealing with the broken cut if we did not attach to it a sign to distinguish the particular state of information to which it refers. And a similar sign has then to be attached to the simple g , which refers to the state of information at the time of learning that graph to be true. I use for this purpose cross marks below, thus:

Fig. 187



Fig. 187

[Click here to view]

These selectives are very peculiar in that they refer to states of information as if they were individual objects. They have, besides, the additional peculiarity of having a definite order of succession, and we have the rule that from Fig. 188 we can infer Fig. 189.†1

Figs. 188-189



Fig. 188



Fig. 189

[Click here to view]

These signs are of great use in cleaning up the confused doctrine of *modal propositions* as well as the subject of logical breadth and depth.

Peirce: CP 4.519 Cross-Ref:††

519. There is not much utility in a *double broken cut*. Yet it may be worth notice that Fig. 181 and



Fig. 190

Fig. 190

[\[Click here to view\]](#)

can neither of them be inferred from the other. The outer

of the two broken cuts is not only relative to a state of information but to a state of reflection. The graph [190] asserts

that it is possible that the truth of the graph g is necessary. It is only because I have not sufficiently reflected upon the subject that I can have any doubt of whether it is so or not.

Peirce: CP 4.520 Cross-Ref:††

520. It becomes evident, in this way, that a modal proposition is a simple assertion, not about the universe of things, but about the universe of facts that one is in a state of information sufficient to know. [Fig. 186] without any selective, merely asserts that there is a possible state of information in which the knower is not in a condition to know that the graph g is true, while Fig. 179 asserts that there is no such possible state of information. Suppose, however, we wish to assert that there is a conceivable state of information of which it would not be true that, in that state, the knower would not be in condition to know that g is true. We shall naturally express

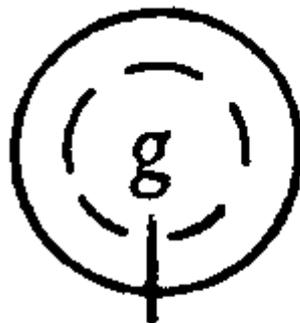


Fig. 191

this by Fig. 191.
that there is a conceivable state

[\[Click here to view\]](#) But this is to say

of information in which the knower would know Fig. 191 that g is true. [This is expressed by] Fig. 188.

Peirce: CP 4.521 Cross-Ref:††

521. Now suppose we wish to assert that there is a conceivable state of information in which the knower would know g to be true and yet would not know another graph h to be true. We shall naturally express this by Fig. 192.

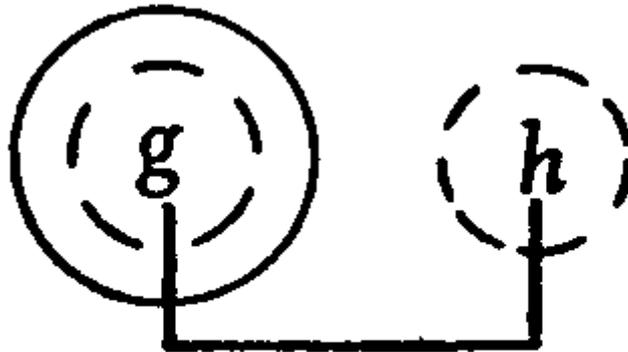


Fig. 192

Fig. 192
view]

[Click here to

Here we have a new kind of ligature, which will follow all the rules of ligatures. We have here a most important addition to the system of graphs. There will be some peculiar and interesting little rules, owing to the fact that what one knows, one has the means of knowing that one knows -- which is sometimes incorrectly stated in the form that whatever one knows, one knows that one knows, which is manifestly false. For if it were the same to say "A whale is not a fish" and "I know that a whale is not a fish," the precise denials of the two would be the same. Yet one is "A whale is a fish" and the other is "I do not know that a whale is not a fish."

Peirce: CP 4.522 Cross-Ref:††

522. The truth is that it is necessary to have a graph to signify that one state of



information follows after another. If we scribe,
[Click here to view] to express that the state of information B follows after the state of information A, we shall have

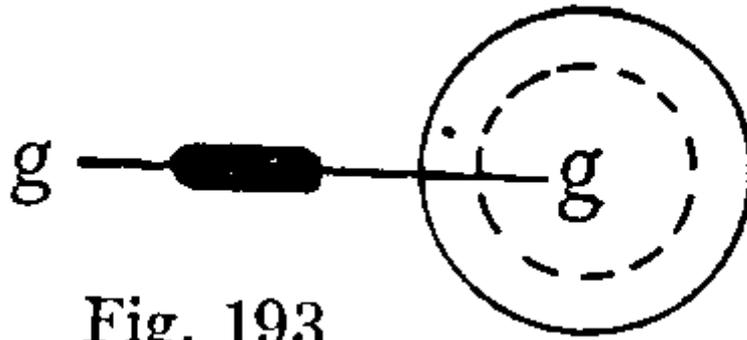


Fig. 193

Fig. 193
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Peirce: CP 4.523 Cross-Ref:††

523. It is clear, however, that the matter must not be allowed to rest here. For it would be a strangely, and almost an ironically, imperfect kind of logic which should recognize only *ignorance* and should ignore *error*. Yet in order to recognize error in our system of graphs, we shall be obliged still further to introduce the idea of time, which will bring still greater difficulties. *Time* has usually been considered by logicians to be what is called "extra-logical" matter. I have never shared this opinion.†1 But I have thought that logic had not reached that state of development at which the introduction of temporal modifications of its forms would not result in great confusion; and I am much of that way of thinking yet. The idea of time really is involved in the very idea of an argument. But the gravest complications of logic would be involved, [if we took] account of time [so as] to distinguish between what *one knows* and what one *has sufficient reason to be entirely confident of*. The only difference, that there seems to be room for between these two, is that what one *knows*, one always will have *reason to be confident of*, while what one now has ample reason to be entirely confident of, one may conceivably in the future, in consequence of a new light, find reason to doubt and ultimately to deny. Whether it is really possible for this to occur, whether we can be said truly to have sufficient reason for entire confidence unless it is manifestly impossible that we should have any such new light in the future, is not the question. Be that as it may, it still remains *conceivable* that there should be that difference, and therefore there is a difference in the *meanings* of the two phrases. I confess that my studies heretofore have [not] progressed so far that I am able to say precisely what modification of our logical forms will be required when we come to take account, as some day we must, of all the effects of the possibilities of error, as we can now take account, in the doctrine of modals, of the possibilities of ignorance. Nor do I believe that the time has yet come when it would be profitable to introduce such complications. But I can see that, when that time does come, our logical forms will become very much more metamorphosed, by introducing that consideration, than they are in modal logic, where we take account of the possibility of ignorance as compared with the simple logic of propositions *de inesse* (as non-modal propositions, in which the ideas of possibility and necessity are not introduced, are called) . . .

Peirce: CP 4.524 Cross-Ref:††

524. I introduce certain spots which I term Potentials. They are shown on this diagram:

The Potentials

- A - p means A is a primary individual
- A - q means A is a monadic character or "quality"
- A - r means A is a dyadic relation
- A - s means A is a legisign
- A - /0\ means A is a graph
- A - /1\ - B means B possesses the quality A

- A - /2\ / B means B is in the relation A to C
 \
 \
 /B
 /
- A - /3\ -- C means B is in the triadic relation A to C for D.
 \
 \
 \D

See image:

The Potentials

- A - p means A is a primary individual
- A - q means A is a monadic character or "quality"
- A - r means A is a dyadic relation
- A - s means A is a legisign
- A - $\hat{\circ}$ means A is a graph
- A - $\hat{\wedge}$ - B means B possesses the quality A
- A - $\hat{\wedge}_2$ $\begin{matrix} \swarrow B \\ \searrow C \end{matrix}$ means B is in the relation A to C
- A - $\hat{\wedge}_3$ $\begin{matrix} \swarrow B \\ \searrow C \\ \quad \searrow D \end{matrix}$ means B is in the triadic relation A to C for D.

to view]

[Click here

Peirce: CP 4.525 Cross-Ref:††

525. It is obvious that the lines of identity on the left-hand side of the potentials are quite peculiar, since the characters they denote are not, properly speaking, individuals. For that reason and others, to the left of the potentials I use

selectives not ligatures.

Peirce: CP 4.526 Cross-Ref:††

526. As an example of the use of the potentials, we may take this graph, which expresses a theorem of great importance: The proposition is that for every quality Q whatsoever, there is a dyadic relation, R , such that, taking any two different individuals both possessing this quality, Q , either the first stands in the relation R to some thing to which the second does not stand in that relation, while there is nothing to which the second stands in that relation without the first standing in the same relation to it; or else it is just the other way, namely that the second stands in the relation, R , to which the first does not stand in that relation, while there is nothing to which the first stands in that relation, R , without the second also standing in the same relation to it. The proof of this, which is a little too intricate to be followed in an oral statement

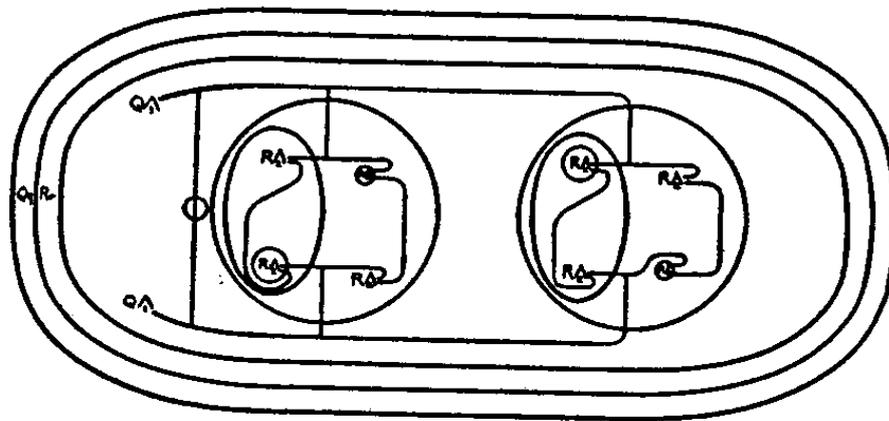


Fig. 194

Fig. 194

[Click here to view]

(although in another lecture †1 I shall substantially prove it) depends upon the fact that a relation is in itself a mere logical possibility.

Peirce: CP 4.527 Cross-Ref:††

527. I will now pass to another quite indispensable department of the gamma graphs. Namely, it is necessary that we should be able to reason in graphs about graphs. The reason is that a reasoning about graphs will necessarily consist in showing that something is true of every possible graph of a certain general description. But we cannot scribe every possible graph of any general description, and therefore if we are to reason in graphs we must have a graph which is a general description of the kind of graph to which the reasoning is to relate.

Peirce: CP 4.528 Cross-Ref:††



528. For the alpha graphs, it is easy to see what is wanted. Let [Click here to view], the old Greek form of the letter A , denote the sheet of assertion.



Let $\sim\{g\}$ be "is a graph." Let Y [\[Click here to view\]](#) X mean that X is scribed or placed on Y . Let $W - k - Z$ mean that Z is the area of



the cut W . Let U - [\[Click here to view\]](#) mean that U is a graph, precisely expressing V . It is necessary to place V in the *saw-rim*, as I call the line about it, because in thus speaking of a sign *materialiter*, as they said in the middle ages, we require that it should have a hook that it has not got. For example

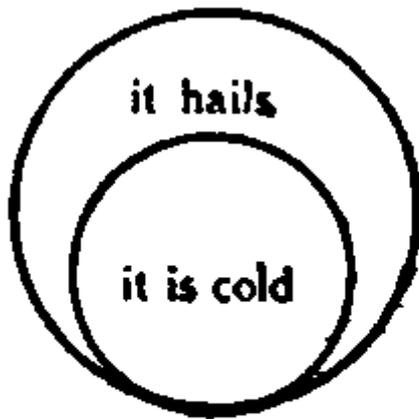


Fig. 195

Fig. 195

[\[Click here to view\]](#)

asserts, of course, that if it hails, it is cold *de inesse*.

Now a graph asserting that this graph is scribed on the sheet of assertion, will be

Fig. 196

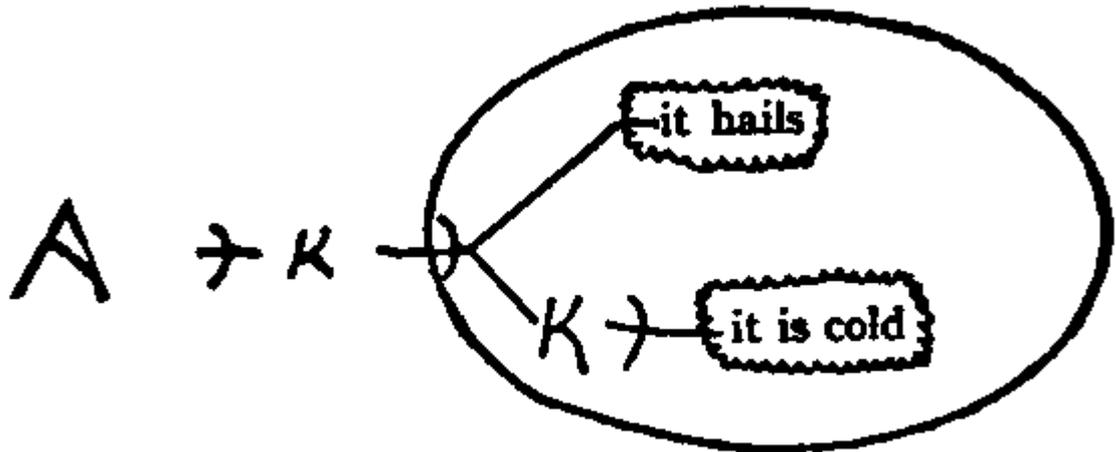


Fig. 196

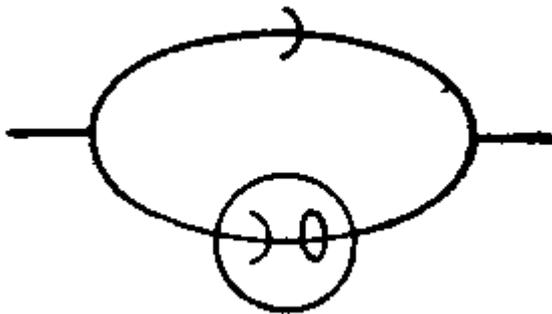
[Click here to view]

This graph only asserts what the other does assert. It does not say what the other does not assert. But there would be no difficulty in expressing that. We have only to place



instead of

[Click here to view], wherever it occurs,

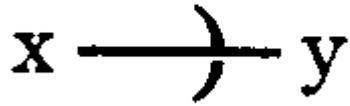


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Peirce: CP 4.529 Cross-Ref:††

529. We come now to the graphical expressions of beta graphs. Here we require the following symbols,

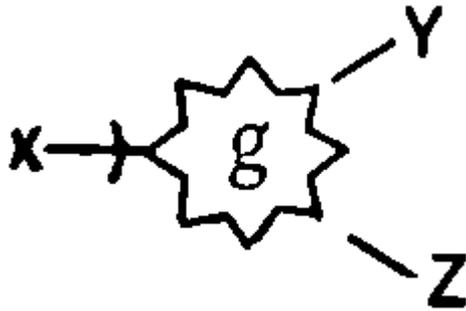
Gamma Expressions of Beta Graphs



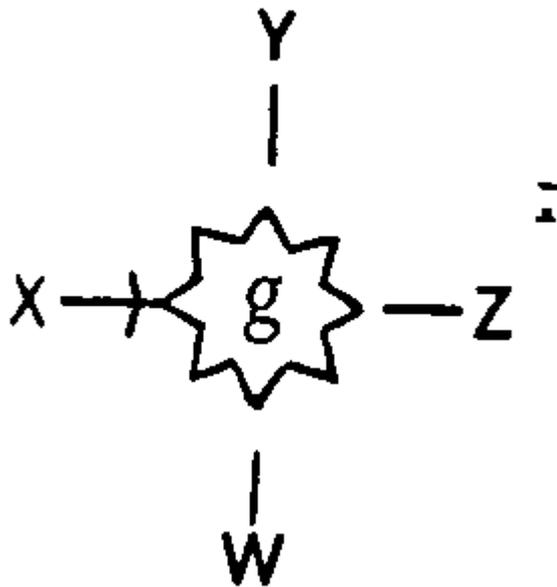
[Click here to view] means Y is a ligature whose outermost part is on X.



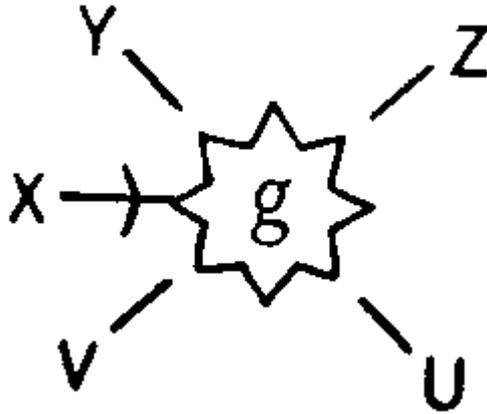
[Click here to view] means g is expressed by a monad spot on X whose hook is joined to the ligature Y on X.



[Click here to view] means g is expressed by a dyad graph on X whose first and second hooks respectively are joined on X to the ligatures Y and Z.



[Click here to view] means g is expressed by a triad graph on X whose first, second, and third hooks are joined on X to the ligatures Y, Z, W, respectively.



[Click here to view] means g is expressed by a tetrad spot on X whose first to fourth hooks are joined to Y, Z, U, V, respectively.†1

Peirce: CP 4.530 Cross-Ref:††
CHAPTER 6

PROLEGOMENA TO AN APOLOGY OR PRAGMATICISM†1

§1. ***SIGNSE***†2

530. Come on, my Reader, and let us construct a diagram to illustrate the general course of thought; I mean a System of diagrammatization by means of which any course of thought can be represented with exactitude.

Peirce: CP 4.530 Cross-Ref:††

"But why do that, when the thought itself is present to us?" Such, substantially, has been the interrogative objection raised by more than one or two superior intelligences, among whom I single out an eminent and glorious General.

Peirce: CP 4.530 Cross-Ref:††

Recluse that I am, I was not ready with the counter-question, which should have run, "General, you make use of maps during a campaign, I believe. But why should you do so, when the country they represent is right there?" Thereupon, had he replied that he found details in the maps that were so far from being "right there," that they were within the enemy's lines, I ought to have pressed the question, "Am I right, then, in understanding that, if you were thoroughly and perfectly familiar with the country, as, for example, if it lay just about the scenes of your childhood, no map of it would then be of the smallest use to you in laying out your detailed plans?" To that he could only have rejoined, "No, I do not say that, since I might probably desire the maps to stick pins into, so as to mark each anticipated day's change in the situations of the two armies." To that again, my sur-rejoinder should have been, "Well, General,

that precisely corresponds to the advantages of a diagram of the course of a discussion. Indeed, just there, where you have so clearly pointed it out, lies the advantage of diagrams in general. Namely, if I may try to state the matter after you, one can make exact experiments upon uniform diagrams; and when one does so, one must keep a bright lookout for unintended and unexpected changes thereby brought about in the relations of different significant parts of the diagram to one another. Such operations upon diagrams, whether external or imaginary, take the place of the experiments upon real things that one performs in chemical and physical research. Chemists have ere now, I need not say, described experimentation as the putting of questions to Nature. Just so, experiments upon diagrams are questions put to the Nature of the relations concerned." The General would here, may be, have suggested (if I may emulate illustrious warriors in reviewing my encounters in afterthought), that there is a good deal of difference between experiments like the chemist's, which are trials made upon the very substance whose behavior is in question, and experiments made upon diagrams, these latter having no physical connection with the things they represent. The proper response to that, and the only proper one, making a point that a novice in logic would be apt to miss, would be this: "You are entirely right in saying that the chemist experiments upon the very object of investigation, albeit, after the experiment is made, the particular sample he operated upon could very well be thrown away, as having no further interest. For it was not the particular sample that the chemist was investigating; it was the molecular structure. Now he was long ago in possession of overwhelming proof that all samples of the same molecular structure react chemically in exactly the same way; so that one sample is all one with another. But the object of the chemist's research, that upon which he experiments, and to which the question he puts to Nature relates, is the Molecular Structure, which in all his samples has as complete an identity as it is in the nature of Molecular Structure ever to possess. Accordingly, he does, as you say, experiment upon the Very Object under investigation. But if you stop a moment to consider it, you will acknowledge, I think, that you slipped in implying that it is otherwise with experiments made upon diagrams. For what is there the Object of Investigation? It is the *form of a relation*. Now this Form of Relation is the very form of the relation between the two corresponding parts of the diagram. For example, let $f[1]$ and $f[2]$ be the two distances of the two foci of a lens from the lens. Then,

$$1/f[1] + 1/f[2] = 1/f[o]$$

Peirce: CP 4.530 Cross-Ref:††

This equation is a diagram of the form of the relation between the two focal distances and the principal focal distance; and the conventions of algebra (and all diagrams, nay all pictures, depend upon conventions) in conjunction with the writing of the equation, establish a relation between the *very* letters $f[1]$, $f[2]$, $f[o]$ regardless of their significance, the form of which relation is the *Very Same* as the form of the relation between the three focal distances that these letters denote. This is a truth quite beyond dispute. Thus, this algebraic Diagram presents to our observation the very, identical object of mathematical research, that is, the Form of the harmonic mean, which the equation aids one to study. (But do not let me be understood as saying that a Form possesses, itself, Identity in the strict sense; that is, what the logicians, translating {arithmō}, call 'numerical identity.')

Peirce: CP 4.531 Cross-Ref:††

531. Not only is it true that by experimentation upon some diagram an experimental proof can be obtained of every necessary conclusion from any given

Copulate of Premises, but, what is more, no "necessary" conclusion is any more apodictic than inductive reasoning becomes from the moment when experimentation can be multiplied *ad libitum* at no more cost than a summons before the imagination. I might furnish a regular proof of this, and am dissuaded from doing so now and here only by the exigency of space, the ineluctable length of the requisite explanations, and particularly by the present disposition of logicians to accept as sufficient F. A. Lange's persuasive and brilliant, albeit defective and in parts even erroneous, apology for it.†1 Under these circumstances, I will content myself with a rapid sketch of my proof. First, an analysis of the essence of a sign, (stretching that word to its widest limits, as *anything which, being determined by an object, determines an interpretation to determination, through it, by the same object*), leads to a proof that every sign is determined by its object, either first, by partaking in the characters of the object, when I call the sign an *Icon*; secondly, by being really and in its individual existence connected with the individual object, when I call the sign an *Index*; thirdly, by more or less approximate certainty that it will be interpreted as denoting the object, in consequence of a habit (which term I use as including a natural disposition), when I call the sign a *Symbol*.†P1 I next examine into the different efficiencies and inefficiencies of these three kinds of signs in aiding the ascertainment of truth. A Symbol incorporates a habit, and is indispensable to the application of any *intellectual* habit, *at least*. Moreover, Symbols afford the means of thinking about thoughts in ways in which we could not otherwise think of them. They enable us, for example, to create Abstractions, without which we should lack a great engine of discovery. These enable us to count; they teach us that collections are individuals (individual = individual object), and in many respects they are the very warp of reason. But since symbols rest exclusively on habits already definitely formed but not furnishing any observation even of themselves, and since knowledge is habit, they do not enable us to add to our knowledge even so much as a necessary consequent, unless by means of a definite preformed habit. Indices, on the other hand, furnish positive assurance of the reality and the nearness of their Objects. But with the assurance there goes no insight into the nature of those Objects. The same Perceptible may, however, function doubly as a Sign. That footprint that Robinson Crusoe found in the sand, and which has been stamped in the granite of fame, was an Index to him that some creature was on his island, and at the same time, as a Symbol, called up the idea of a man. Each Icon partakes of some more or less overt character of its Object. They, one and all, partake of the most overt character of all lies and deceptions -- their Overtness. Yet they have more to do with the living character of truth than have either Symbols or Indices. The Icon does not stand unequivocally for this or that existing thing, as the Index does. Its Object may be a pure fiction, as to its existence. Much less is its Object necessarily a thing of a sort habitually met with. But there is one assurance that the Icon does afford in the highest degree. Namely, that which is displayed before the mind's gaze -- the Form of the Icon, which is also its object -- must be *logically possible*. This division of Signs is only one of ten different divisions of Signs which I have found it necessary more especially to study.†1 I do not say that they are all satisfactorily definite in my mind. They seem to be all trichotomies, which form an attribute to the essentially triadic nature of a Sign. I mean because three things are concerned in the functioning of a Sign; the Sign itself, its Object, and its Interpretant. I cannot discuss all these divisions in this article; and it can well be believed that the whole nature of reasoning cannot be fully exposed from the consideration of one point of view among ten. That which we can learn from this division is of what sort a Sign must be to represent the sort of Object that reasoning is concerned with. Now reasoning has to make its conclusion manifest. Therefore, it

must be chiefly concerned with forms, which are the chief objects of rational insight. Accordingly, Icons are specially requisite for reasoning. A Diagram is mainly an Icon, and an Icon of intelligible relations. It is true that what must be is not to be learned by simple inspection of anything. But when we talk of deductive reasoning being necessary, we do not mean, of course, that it is infallible. But precisely what we do mean is that the conclusion follows from the form of the relations set forth in the premiss. Now since a diagram, though it will ordinarily have Symbolide Features, as well as features approaching the nature of Indices, is nevertheless in the main an Icon of the forms of relations in the constitution of its Object, the appropriateness of it for the representation of necessary inference is easily seen.

Peirce: CP 4.532 Cross-Ref:††
 §2. COLLECTIONSE

532. But since you may, perhaps, be puzzled to understand how an Icon can exhibit a necessity -- a Must-be -- I will here give, as an example of its doing so, my proof †2 that the single members of no collection or plural, are as many as are the collections it includes, each reckoned as a single object, or, in other words, that there can be no relation in which every collection composed of members of a given collection should (taken collectively as a single object) stand to some member of the latter collection to which no other such included collection so stands. This is another expression of the following proposition, namely: that, taking any collection or plural, whatsoever, be it finite or infinite, and calling this the given collection; and considering all the collections, or plurals, each of which is composed of some of the individual members of the given collection (but including along with these *Nothing* which is to be here regarded as a collection having no members at all; and also including the single members of the given collection, conceived as so many collections each of a single member), and calling these the *involved collections*; the proposition is that there is no possible relation in which each involved collection (considered as a single object), stands to a member of the given collection, without any other of the involved collections standing in the same relation to that same member of the given collection. This purely symbolic statement can be rendered much more perspicuous by the introduction of Indices, as follows. The proposition is that no matter what collection C may be, and no matter what relation R may be, there must be some collection, c' , composed exclusively of members of C , which does not stand in the relation R to any member, k , of C , unless some other collection, c'' , likewise composed of members of C , stands in the same relation R to the same k . The theorem is important in the doctrine of multitude, since it is the same as to say that any collection, no matter how great, is less multitudinous than the collection of possible collections composed exclusively of members of it; although formerly this was assumed to be false of some infinite collections. The demonstration begins by insisting that, if the proposition be false, there must be some definite relation of which it is false. Assume, then, that the letter R is an index of any one such relation you please. Next divide the members of C into four classes as follows:

Peirce: CP 4.532 Cross-Ref:††

Class I is to consist of all those members of C (if there be any such) to each of which no collection of members of C stands in the relation R .

Peirce: CP 4.532 Cross-Ref:††

Class II is to consist of all those members of C to each of which one and only one collection of members of C stands in the relation R ; and this class has two subclasses, as follows:

Sub-Class 1 is to consist of whatever members of Class II there may be, each of which is contained in that one collection of members of C that is in the relation R to it.

Sub-Class 2 is to consist of whatever members of Class II there may be, none of which is contained in that one collection of members of C that is in the relation R to it.

Peirce: CP 4.532 Cross-Ref:††

Class III is to consist of all those members of C , if there be any such, to each of which more than one collection of members of C are in the relation R .

Peirce: CP 4.532 Cross-Ref:††

This division is complete; but everybody would consider the easy diagrammatical proof that it is so as needless to the point of nonsense, implicitly relying on a Symbol in his memory which assures him that every Division of such construction is complete.

Peirce: CP 4.532 Cross-Ref:††

I ought already to have mentioned that, throughout the enunciation and demonstration of the proposition to be proved, the term "collection included in the given collection" is to be taken in a peculiar sense to be presently defined. It follows that there is one "possible collection" that is included in every other, that is, which excludes whatever any other excludes. Namely, this is the "possible collection" which includes only the Sphinxes, which is the same that includes only the Basilisks, and is identical with the "possible collection" of all the Centaurs, the unique and ubiquitous collection called "Nothing," which has no member at all. If you object to this use of the term "collection," you will please substitute for it, throughout the enunciation and the demonstration, any other designation of the same object. I prefix the adjective "possible," though I must confess it does not express my meaning, merely to indicate that I extend the term "collection" to Nothing, which, of course, has no existence. Were the suggested objection to be persisted in by those *soi-disant* reasoners who refuse to think at all about the object of this or that description, on the ground that it is "inconceivable," I should not stop to ask them how they could say that, when that involves thinking of it in the very same breath, but should simply say that for them it would be necessary to except collections consisting of single individuals. Some of these mighty intellects refuse to allow the use of any name to denote single individuals, and also plural collections along with them; and for them the proposition ceases to be true of pairs. If they would not allow pairs to be denoted by any term that included all higher collections, the proposition would cease to be true of triplets and so on. In short, by restricting the meaning of "possible collection," the proposition may be rendered false of *small* collections. No general formal restriction can render it false of *greater* collections.

Peirce: CP 4.532 Cross-Ref:††

I shall now assume that you will permit me to use the term "possible collection" according to the following definition. A "possible collection" is an *ens*

rationis of such a nature that the definite plural of any noun, or possible noun of definite signification, (as "the A's," "the B's," etc.) denotes one, and only one, "possible collection" in any one perfectly definite state of the universe; and there is a certain relation between some "possible collections," expressed by saying that one "possible collection" *includes* another (or the same) "possible collection," and if, and only if, of two nouns one is universally and affirmatively predicable of the other in any one perfectly definite state of the universe, then the "possible collection" denoted by the definite plural of the former *includes* whatever "possible collection" is *included* by the "possible collection" denoted by the definite plural of the latter, and of any two different "possible collections," one or other must *include* something not *included* by the other.

Peirce: CP 4.532 Cross-Ref:††

A diagram of the definition of "possible collection" being compared with a diagram embracing whatever members of subclasses 1 and 2 that it may, excluding all the rest, will now assure us that any such aggregate is a possible collection of members of the class C, no matter what individuals of Classes I and III be included or excluded in the aggregate along with those members of Class II, if any there be in the aggregate.

Peirce: CP 4.532 Cross-Ref:††

We shall select, then, a single possible collection of members of C to which we give the proper name, *c*, and this possible collection shall be one which contains no individual of Subclass 1, but contains whatever individual there may be of Subclass 2. We then ask whether or not it is true that *c* stands in the relation *R* to a member of C to which no other possible collection of members of C stands in the same relation; or, to put this question into a more convenient shape, we ask, Is there any member of the Class C to which *c* and no other possible collection of members of C stands in the relation *R*? If there be such a member or members of C, let us give one of them the proper name T. Then T must belong to one of our four divisions of this class. That is,

either T belongs to Class I (but that cannot be, since by the definition of Class I, to no member of this class is any possible collection of members of C in the relation *R*);

or T belongs to Subclass 1 (but that cannot be, since by the definition of that subclass, every member of it is a member of the only possible collection of members of C that is *R* to it, which possible collection cannot be *c*, because *c* is only known to us by a description which forbids its containing any member of Subclass 1. Now it is *c*, and *c* only, that is in the relation *R* to T);

or T belongs to Subclass 2 (but that cannot be, since by the definition of that subclass, no member of it is a member of the only possible collection of members of C that is *R* to it, which possible collection cannot be *c*, because the description by which alone *c* can be recognized makes it contain every member of Subclass 2. Now it is *c* only that is in the relation *R* to T);

or T belongs to Class III (but this cannot be, since to every member of that class, by the definition of it, more than one collection of members of C stand in the relation *R*, while to T only one collection, namely, *c*, stands in that relation).

Peirce: CP 4.532 Cross-Ref:††

Thus, T belongs to none of the classes of members of C , and consequently is not a member of C . Consequently, there is no such member of C ; that is, no member of C to which c , and no other possible collection of members of C , stands in the relation R . But c is the proper name we were at liberty to give to whatever possible collection of members of C we pleased. Hence, there is no possible collection of members of C that stands in the relation R to a member of the class C to which no other possible collection of members of C stands in this relation R . But R is the name of *any* relation we please, and C is any class we please. It is, therefore, proved that no matter what class be chosen, or what relation be chosen, there will be some possible collection of members of that class (in the sense in which Nothing is such a collection) which does not stand in that relation to any member of that class to which no other such possible collection stands in the same relation.

Peirce: CP 4.533 Cross-Ref:††
 §3. GRAPHS AND SIGNSE

533. When I was a boy, my logical bent caused me to take pleasure in tracing out upon a map of an imaginary labyrinth one path after another in hopes of finding my way to a central compartment. The operation we have just gone through is essentially of the same sort, and if we are to recognize the one as essentially performed by experimentation upon a diagram, so must we recognize that the other is performed. The demonstration just traced out brings home to us very strongly, also, the convenience of so constructing our diagram as to afford a clear view of the mode of connection of its parts, and of its composition at each stage of our operations upon it. Such convenience is obtained in the diagrams of algebra. In logic, however, the desirability of convenience in threading our way through complications is much less than in mathematics, while there is another desideratum which the mathematician as such does not feel. The mathematician wants to reach the conclusion, and his interest in the process is merely as a means to reach similar conclusions. The logician does not care what the result may be; his desire is to understand the nature of the process by which it is reached. The mathematician seeks the speediest and most abridged of secure methods; the logician wishes to make each smallest step of the process stand out distinctly, so that its nature may be understood. He wants his diagram to be, above all, as analytical as possible.

Peirce: CP 4.534 Cross-Ref:††

534. In view of this, I beg leave, Reader, as an Introduction to my defence of pragmatism, to bring before you a very simple system of diagrammatization of propositions which I term the System of Existential Graphs. For, by means of this, I shall be able almost immediately to deduce some important truths of logic, little understood hitherto, and closely connected with the truth of pragmatism;†P1 while discussions of other points of logical doctrine, which concern pragmatism but are not directly settled by this system, are nevertheless much facilitated by reference to it.

Peirce: CP 4.535 Cross-Ref:††

535. By a *graph* (a word overworked of late years), I, for my part, following my friends Clifford †1 and Sylvester,†2 the introducers of the term, understand in general a diagram composed principally of spots and of lines connecting certain of the spots. But I trust it will be pardoned to me that, when I am discussing Existential

Graphs, without having the least business with other Graphs, I often omit the differentiating adjective and refer to an Existential Graph as a Graph simply. But you will ask, and I am plainly bound to say, precisely what kind of a Sign an Existential Graph, or as I abbreviate that phrase here, a *Graph* is. In order to answer this I must make reference to two different ways of dividing all Signs. It is no slight task, when one sets out from none too clear a notion of what a Sign is -- and you will, I am sure, Reader, have noticed that my definition of a Sign is not convincingly distinct -- to establish a single vividly distinct division of all Signs. The one division which I have already given has cost more labor than I should care to confess. But I certainly could not tell you what sort of a Sign an Existential Graph is, without reference to two other divisions of Signs. It is true that one of these involves none but the most superficial considerations, while the other, though a hundredfold more difficult, resting as it must for a clear comprehension of it upon the profoundest secrets of the structure of Signs, yet happens to be extremely familiar to every student of logic. But I must remember, Reader, that your conceptions may penetrate far deeper than mine; and it is to be devoutly hoped they may. Consequently, I ought to give such hints as I conveniently can, of my notions of the structure of Signs, even if they are not strictly needed to express my notions of Existential Graphs.

Peirce: CP 4.536 Cross-Ref:††

536. I have already noted that a Sign has an Object and an Interpretant, the latter being that which the Sign produces in the Quasi-mind that is the Interpreter by determining the latter to a feeling, to an exertion, or to a Sign, which determination is the Interpretant. But it remains to point out that there are usually two Objects, and more than two Interpretants. Namely, we have to distinguish the Immediate Object, which is the Object as the Sign itself represents it, and whose Being is thus dependent upon the Representation of it in the Sign, from the Dynamical Object, which is the Reality which by some means contrives to determine the Sign to its Representation. In regard to the Interpretant we have equally to distinguish, in the first place, the Immediate Interpretant, which is the interpretant as it is revealed in the right understanding of the Sign itself, and is ordinarily called the *meaning* of the sign; while in the second place, we have to take note of the Dynamical Interpretant which is the actual effect which the Sign, as a Sign, really determines. Finally there is what I provisionally term the Final Interpretant, which refers to the manner in which the Sign tends to represent itself to be related to its Object. I confess that my own conception of this third interpretant is not yet quite free from mist.†1 Of the ten divisions of signs which have seemed to me to call for my special study, six turn on the characters of an Interpretant and three on the characters of the Object.†1 Thus the division into Icons, Indices, and Symbols depends upon the different possible relations of a Sign to its Dynamical Object.†2 Only one division is concerned with the nature of the Sign itself, and this I now proceed to state.

Peirce: CP 4.537 Cross-Ref:††

537. A common mode of estimating the amount of matter in a MS. or printed book is to count the number of words.†P1 There will ordinarily be about twenty *the's* on a page, and of course they count as twenty words. In another sense of the word "word," however, there is but one word "the" in the English language; and it is impossible that this word should lie visibly on a page or be heard in any voice, for the reason that it is not a Single thing or Single event. It does not exist; it only determines things that do exist. Such a definitely significant Form, I propose to term a *Type*.†3 A Single event which happens once and whose identity is limited to that one happening or a Single object or thing which is in some single place at any one instant of time,

such event or thing being significant only as occurring just when and where it does, such as this or that word on a single line of a single page of a single copy of a book, I will venture to call a *Token*.†3 An indefinite significant character such as a tone of voice can neither be called a Type nor a Token. I propose to call such a Sign a *Tone*;†3 In order that a Type may be used, it has to be embodied in a Token which shall be a sign of the Type, and thereby of the object the Type signifies. I propose to call such a Token of a Type an *Instance* of the Type. Thus, there may be twenty Instances of the Type "the" on a page. The term (Existential) *Graph* will be taken in the sense of a Type; and the act of embodying it in a *Graph-Instance* will be termed *scribing* the Graph (not the Instance), whether the Instance be written, drawn, or incised. A mere blank place is a Graph-Instance, and the Blank *per se* is a Graph; but I shall ask you to assume that it has the peculiarity that it cannot be abolished from any Area on which it is scribed, as long as that Area exists.

Peirce: CP 4.538 Cross-Ref:††

538. A familiar logical triplet is Term, Proposition, Argument.†1 In order to make this a division of all signs, the first two members have to be much widened. By a *Seme*,†2 I shall mean anything which serves for any purpose as a substitute for an object of which it is, in some sense, a representative or Sign. The logical Term, which is a class-name, is a Seme. Thus, the term "The mortality of man" is a Seme. By a *PHEME*†3 I mean a Sign which is equivalent to a grammatical sentence, whether it be Interrogative, Imperative, or Assertory. In any case, such a Sign is intended to have some sort of compulsive effect on the Interpreter of it. As the third member of the triplet, I sometimes use the word *Delome* (pronounce deeloam, from {délöma}), though *Argument* would answer well enough. It is a Sign which has the Form of tending to act upon the Interpreter through his own self-control, representing a process of change in thoughts or signs, as if to induce this change in the Interpreter.

Peirce: CP 4.538 Cross-Ref:††

A Graph is a PHEME, and in my use hitherto, at least, a Proposition. An Argument is represented by a series of Graphs.

Peirce: CP 4.539 Cross-Ref:††

§4. UNIVERSES AND PREDICAMENTSE

539. The Immediate Object of all knowledge and all thought is, in the last analysis, the Percept. This doctrine in no wise conflicts with Pragmaticism, which holds that the Immediate Interpretant of all thought proper is Conduct. Nothing is more indispensable to a sound epistemology than a crystal-clear discrimination between the Object and the Interpretant of knowledge; very much as nothing is more indispensable to sound notions of geography than a crystal-clear discrimination between north latitude and south latitude; and the one discrimination is not more rudimentary than the other. That we are conscious of our Percepts is a theory that seems to me to be beyond dispute; but it is not a fact of Immediate Perception. A fact of Immediate Perception is not a Percept, nor any part of a Percept; a Percept is a Seme, while a fact of Immediate Perception or rather the Perceptual Judgment of which such fact is the Immediate Interpretant, is a PHEME that is the direct Dynamical Interpretant of the Percept, and of which the Percept is the Dynamical Object, and is with some considerable difficulty (as the history of psychology shows), distinguished

from the Immediate Object, though the distinction is highly significant.†1 But not to interrupt our train of thought, let us go on to note that while the Immediate Object of a Percept is excessively vague, yet natural thought makes up for that lack (as it almost amounts to), as follows. A late Dynamical Interpretant of the whole complex of Percepts is the Seme of a Perceptual Universe that is represented in instinctive thought as determining the original Immediate Object of every Percept.†2 Of course, I must be understood as talking not psychology, but the logic of mental operations. Subsequent Interpretants furnish new Semes of Universes resulting from various adjunctions to the Perceptual Universe. They are, however, all of them, Interpretants of Percepts.

Peirce: CP 4.539 Cross-Ref:††

Finally, and in particular, we get a Seme of that highest of all Universes which is regarded as the Object of every true Proposition, and which, if we name it [at] all, we call by the somewhat misleading title of "The Truth."

Peirce: CP 4.540 Cross-Ref:††

540. That said, let us go back and ask this question: How is it that the Percept, which is a Seme, has for its direct Dynamical Interpretant the Perceptual Judgment, which is a PHEME? For that is not the usual way with Semes, certainly. All the examples that happen to occur to me at this moment of such action of Semes are instances of Percepts, though doubtless there are others. Since not all Percepts act with equal energy in this way, the instances may be none the less instructive for being Percepts. However, Reader, I beg you will think this matter out for yourself, and then you can see -- I wish I could -- whether your independently formed opinion does not fall in with mine. My opinion is that a pure perceptual Icon -- and many really *great* psychologists have evidently thought that Perception is a passing of images before the mind's eye, much as if one were walking through a picture gallery -- could not have a PHEME for its direct Dynamical Interpretant. I desire, for more than one reason, to tell you *why* I think so, although that you should today appreciate my reasons seems to be out of the question. Still, I wish you to understand me so far as to know that, mistaken though I be, I am not so sunk in intellectual night as to be dealing lightly with philosophic Truth when I aver that weighty reasons have moved me to the adoption of my opinion; and I am also anxious that it should be understood that those reasons have not been psychological at all, but are purely logical. My reason, then, briefly stated and abridged, is that it would be *illogical* for a pure Icon to have a PHEME for its Interpretant, and I hold it to be impossible for thought not subject to self-control, as a Perceptual Judgment manifestly is not, to be illogical. I dare say this reason may excite your derision or disgust, or both; and if it does, I think none the worse of your intelligence. You probably opine, in the first place, that there is no meaning in saying that thought which draws no Conclusion is illogical, and that, at any rate, there is no standard by which I can judge whether such thought is logical or not; and in the second place, you probably think that, if self-control has any essential and important relation to logic, which I guess you either deny or strongly doubt, it can only be that it is that which makes thought *logical*, or else which establishes the distinction between the logical and the illogical, and that in any event it has to be such as it is, and would be logical, or illogical, or both, or neither, whatever course it should take. But though an Interpretant is not necessarily a Conclusion, yet a Conclusion is necessarily an Interpretant. So that if an Interpretant is not subject to the rules of Conclusions there is nothing monstrous in my thinking it is subject to some generalization of such rules. For any evolution of thought, whether it leads to a Conclusion or not, there is a certain normal course, which is to be determined by considerations not in the least

psychological, and which I wish to expound in my next article;†1 and while I entirely agree, in opposition to distinguished logicians, that normality can be no criterion for what I call rationalistic reasoning, such as alone is admissible in science, yet it is precisely the criterion of instinctive or common-sense reasoning, which, within its own field, is much more trustworthy than rationalistic reasoning. In my opinion, it is self-control which makes any other than the normal course of thought possible, just as nothing else makes any other than the normal course of action possible; and just as it is precisely that that gives room for an ought-to-be of conduct, I mean Morality, so it equally gives room for an ought-to-be of thought, which is Right Reason; and where there is no self-control, nothing but the normal is possible. If your reflections have led you to a different conclusion from mine, I can still hope that when you come to read my next article, in which I shall endeavor to show what the forms of thought are, in general and in some detail, you may yet find that I have not missed the truth.

Peirce: CP 4.541 Cross-Ref:††

541. But supposing that I am right, as I probably shall be in the opinions of *some* readers, how then is the Perceptual Judgment to be explained? In reply, I note that a Percept cannot be dismissed at will, even from memory. Much less can a person prevent himself from perceiving that which, as we say, stares him in the face. Moreover, the evidence is overwhelming that the perceiver is aware of this compulsion upon him; and if I cannot say for certain how this knowledge comes to him, it is not that I cannot conceive how it could come to him, but that, there being several ways in which this might happen, it is difficult to say which of those ways actually is followed. But that discussion belongs to psychology; and I will not enter upon it. Suffice it to say that the perceiver is aware of being compelled to perceive what he perceives. Now existence means precisely the exercise of compulsion. Consequently, whatever feature of the percept is brought into relief by some association and thus attains a logical position like that of the observational premiss of an explaining Abduction,†P1 the attribution of Existence to it in the Perceptual Judgment is virtually and in an extended sense, a logical Abductive Inference nearly approximating to necessary inference. But my next paper will throw a flood of light upon the logical affiliation of the Proposition, and the PHEME generally, to coercion.

Peirce: CP 4.542 Cross-Ref:††

542. That conception of Aristotle which is embodied for us in the cognate origin of the terms *actuality* and *activity* is one of the most deeply illuminating products of Greek thinking. Activity implies a generalization of *effort*; and effort is a two-sided idea, effort and resistance being inseparable, and therefore the idea of Actuality has also a dyadic form.

Peirce: CP 4.543 Cross-Ref:††

543. No cognition and no Sign is absolutely precise, not even a Percept; and indefiniteness is of two kinds, indefiniteness as to what is the Object of the Sign, and indefiniteness as to its Interpretant, or indefiniteness in Breadth and in Depth.†1 Indefiniteness in Breadth may be either Implicit or Explicit. What this means is best conveyed in an example. The word *donation* is indefinite as to who makes the gift, what he gives, and to whom he gives it. But it calls no attention, itself, to this indefiniteness. The word *gives* refers to the same sort of fact, but its meaning is such that that meaning is felt to be incomplete unless those items are, at least formally, specified; as they are in "Somebody gives something to some person (real or artificial)." An ordinary Proposition †2 ingeniously contrives to convey novel information through Signs whose significance depends entirely on the interpreter's

familiarity with them; and this it does by means of a "Predicate," *i.e.*, a term explicitly indefinite in breadth, and defining its breadth by means of "Subjects," or terms whose breadths are somewhat definite, but whose informative depth (*i.e.*, all the depth except an essential superficiality) is indefinite, while conversely the depth of the Subjects is in a measure defined by the Predicate. A Predicate is either non-relative, or a *monad*, that is, is explicitly indefinite in one extensive respect, as is "black"; or it is a dyadic relative, or dyad, such as "kills," or it is a polyadic relative, such as "gives." These things must be diagrammatized in our system.

Peirce: CP 4.543 Cross-Ref:††

Something more needs to be added under the same head. You will observe that under the term "Subject" I include, not only the subject nominative, but also what the grammarians call the direct and the indirect object, together, in some cases, with nouns governed by prepositions. Yet there is a sense in which we can continue to say that a Proposition has but one Subject, for example, in the proposition, "Napoleon ceded Louisiana to the United States," we may regard as the Subject the ordered triplet, "Napoleon -- Louisiana -- the United States," and as the Predicate, "has for its first member, the agent, or party of the first part, for its second member the object, and for its third member the party of the second part of one and the same act of cession." The view that there are three subjects is, however, preferable for most purposes, in view of its being so much more analytical, as will soon appear.

Peirce: CP 4.544 Cross-Ref:††

544. All general, or definable, Words, whether in the sense of Types or of Tokens, are certainly Symbols. That is to say, they denote the objects that they do by virtue only of there being a habit that associates their signification with them. As to Proper Names, there might perhaps be a difference of opinion, especially if the Tokens are meant. But they should probably be regarded as Indices, since the actual connection (as we listen to talk), of Instances of the same typical words with the same Objects, alone causes them to be interpreted as denoting those Objects. Excepting, if necessary, propositions in which all the subjects are such signs as these, no proposition can be expressed without the use of Indices.†P1 If, for example, a man remarks, "Why, it is raining!" it is only by some such *circumstances* as that he is now standing here looking out at a window as he speaks, which would serve as an Index (not, however, as a Symbol) that he is speaking of this place at this time, whereby we can be assured that he cannot be speaking of the weather on the satellite of Procyon, fifty centuries ago. Nor are Symbols and Indices together generally enough. The arrangement of the words in the sentence, for instance, must serve as *Icons*, in order that the sentence may be understood. The chief need for the Icons is in order to show the Forms of the synthesis of the elements of thought. For in precision of speech, Icons can represent nothing but Forms and Feelings. That is why Diagrams are indispensable in all Mathematics, from Vulgar Arithmetic up, and in Logic are almost so. For Reasoning, nay, Logic generally, hinges entirely on Forms. You, Reader, will not need to be told that a regularly stated Syllogism is a Diagram; and if you take at random a half dozen out of the hundred odd logicians who plume themselves upon not belonging to the sect of Formal Logic, and if from this latter sect you take another half dozen at random, you will find that in proportion as the former avoid diagrams, they utilize the syntactical Form of their sentences. No pure Icons represent anything but Forms; no pure Forms are represented by anything but Icons. As for Indices, their utility especially shines where other Signs fail. Extreme precision being desired in the description of a red color, should I call it vermilion, I may be criticized on the ground that vermilion differently prepared has quite different hues, and thus I may be driven

to the use of the color-wheel, when I shall have to Indicate four disks individually, or I may say in what proportions light of a given wave-length is to be mixed with white light to produce the color I mean. The wave-length being stated in fractions of a micron, or millionth of a meter, is referred through an Index to two lines on an individual bar in the Pavillon de Breteuil, at a given temperature and under a pressure measured against gravity at a certain station and (strictly) at a given date, while the mixture with white, after white has been fixed by an Index of an individual light, will require at least one new Index. But of superior importance in Logic is the use of Indices to denote Categories and Universes,†P1 which are classes that, being enormously large, very promiscuous, and known but in small part, cannot be satisfactorily defined, and therefore can only be denoted by Indices. Such, to give but a single instance, is the collection of all things in the Physical Universe. If anybody, your little son for example, who is such an assiduous researcher, always asking, What is the Truth ({Ti estin alétheia}); but like "jesting Pilate," will not always stay for an answer, should ask you what the Universe of things physical is, you may, if convenient, take him to the Rigi-Kulm, and about sunset, point out all that is to be seen of Mountains, Forests, Lakes, Castles, Towns, and then, as the stars come out, all there is to be seen in the heavens, and all that though not seen, is reasonably conjectured to be there; and then tell him, "Imagine that what is to be seen in a city back yard to grow to all you can see here, and then let this grow in the same proportion as many times as there are trees in sight from here, and what you would finally have would be harder to find in the Universe than the finest needle in America's yearly crop of hay." But such methods are perfectly futile: Universes cannot be described.

Peirce: CP 4.545 Cross-Ref:††

545. Oh, I overhear what you are saying, O Reader: that a Universe and a Category are not at all the same thing; a Universe being a receptacle or class of Subjects, and a Category being a mode of Predication, or class of Predicates. I never said they were the same thing; but whether you describe the two correctly is a question for careful study.

Peirce: CP 4.546 Cross-Ref:††

546. Let us begin with the question of Universes. It is rather a question of an advisable point of view than of the truth of a doctrine. A logical universe is, no doubt, a collection of *logical* subjects, but not necessarily of meta-physical Subjects, or "substances"; for it may be composed of characters, of elementary facts, etc. See my definition in Baldwin's Dictionary.†1 Let us first try whether we may not assume that there is but one kind of Subjects which are either existing things or else quite fictitious. Let it be asserted that there is some married woman who will commit suicide in case her husband fails in business. Surely that is a very different proposition from the assertion that some married woman will commit suicide if all married men fail in business. Yet if nothing is real but existing things, then, since in the former proposition nothing whatever is said as to what the lady will or will not do if her husband does *not* fail in business, and since of a given married couple this can only be false if the fact is contrary to the assertion, it follows it can only be false if the husband *does* fail in business and if the wife then fails to commit suicide. But the proposition only says that there is *some* married couple of which the wife is of that temper. Consequently, there are only two ways in which the proposition can be false, namely, first, by there not being any married couple, and secondly, by *every* married man failing in business while *no* married woman commits suicide. Consequently, all that is required to make the proposition true is that there should either be some

married man who does not fail in business, or else some married woman who commits suicide. That is, the proposition amounts merely to asserting that there is a married woman who will commit suicide if *every* married man fails in business. The equivalence of these two propositions is the absurd result of admitting no reality but existence. If, however, we suppose that to say that a woman will suicide if her husband fails, means that every *possible* course of events would either be one in which the husband would not fail or one in which the wife would commit suicide, then, to make that false it will not be requisite for the husband actually to fail, but it will suffice that there are *possible* circumstances under which he would fail, while yet his wife would not commit suicide. Now you will observe that there is a great difference between the two following propositions:

First, There is some *one* married woman who under all possible conditions would commit suicide or else her husband would not have failed.

Second, Under all possible circumstances there is some married woman *or other* who would commit suicide, or else her husband would not have failed.

Peirce: CP 4.546 Cross-Ref:††

The former of these is what is really meant by saying that there is some married woman who would commit suicide if her husband were to fail, while the latter is what the denial of any possible circumstances except those that really take place logically leads to [our] interpreting (or virtually interpreting), the Proposition as asserting.

Peirce: CP 4.547 Cross-Ref:††

547. In other places,†1 I have given many other reasons for my firm belief that there are real possibilities. I also think, however, that, in addition to actuality and possibility, a *third* mode of reality must be recognized in that which, as the gipsy fortune-tellers express it, is "sure to come true," or, as we may say, is *destined*,†P1 although I do not mean to assert that this is affirmation rather than the negation of this Mode of Reality. I do not see by what confusion of thought anybody can persuade himself that he does not believe that tomorrow is destined to come. The point is that it is today really true that to-morrow the sun will rise; or that, even if it does not, the clocks or *something*, will go on. For if it be not real it can only be fiction: a Proposition is either True or False. But we are too apt to confound destiny with the impossibility of the opposite. I see no impossibility in the sudden stoppage of everything. In order to show the difference, I remind you that "impossibility" is that which, for example, describes the mode of falsity of the idea that there should be a collection of objects so multitudinous that there would not be characters enough in the universe of characters to distinguish all those things from one another. Is there anything of that sort about the stoppage of all motion? There is, perhaps, a *law of nature* against it; but that is all. However, I will postpone the consideration of that point. Let us, at least, *provide* for such a mode of being in our system of diagrammatization, since it *may* turn out to be needed and, as I think, surely will.

Peirce: CP 4.548 Cross-Ref:††

548. I will proceed to explain why, although I am not prepared to deny that

every proposition can be represented, and that I must say, for the most part very conveniently, under your view that the Universes are receptacles of the Subjects alone, I, nevertheless, cannot deem that mode of analyzing propositions to be satisfactory.

Peirce: CP 4.548 Cross-Ref:††

And to begin with, I trust you will all agree with me that no analysis, whether in logic, in chemistry, or in any other science, is satisfactory, unless it be thorough, that is, unless it separates the compound into components each entirely homogeneous in itself, and therefore free from the smallest admixture of any of the others. It follows that in the Proposition, "Some Jew is shrewd," the Predicate is "Jew-that-is-shrewd," and the Subject is *Something*, while in the proposition "Every Christian is meek," the Predicate is "Either not Christian or else meek," while the Subject is *Anything*; unless, indeed, we find reason to prefer to say that this Proposition means, "It is false to say that a person is Christian of whom it is false to say that he is meek." In this last mode of analysis, when a Singular Subject is not in question (which case will be examined later), the only Subject is *Something*. Either of these two modes of analysis [differentiates] quite [clearly] the Subject from any Predicative ingredients; and at first sight, either seems quite favorable to the view that it is only the Subjects which belong to the Universes. Let us, however, consider the following two forms of propositions:

A †1 Any adept alchemist could produce a philosopher's stone of some kind or other,

B There is one kind of philosopher's stone that any adept alchemist could produce.

Peirce: CP 4.548 Cross-Ref:††

We can express these in the principle that the Universes are receptacles of Subjects as follows:

A¹ The Interpreter having selected any individual he likes, and called it A, an object B can be found, such that, Either A would not be an adept alchemist, or B would be a philosopher's stone of some kind, and A could produce B.

B¹ Something, B, might be found, such that, no matter what the Interpreter might select and call A, B would be a philosopher's stone of some kind, while either A would not be an adept alchemist, or else A could produce B.

Peirce: CP 4.548 Cross-Ref:††

In these forms there are two Universes, the one of individuals selected at pleasure by the interpreter of the proposition, the other of suitable objects.

Peirce: CP 4.548 Cross-Ref:††

I will now express the same two propositions on the principle that each Universe consists, not of Subjects, but the one of True assertions, the other of False, but each to the effect that there is something of a given description.

1. This is false: That something, P, is an adept alchemist, and that this is false, that while something, S, is a philosopher's stone of some kind, P could produce S.

2. This is true: That something, S, is a philosopher's stone of some kind; and this is

false, that something, P, is an adept alchemist while this is false, that P could produce S.

Peirce: CP 4.548 Cross-Ref:††

Here, the whole proposition is mostly made up of the truth or falsity of assertions that a thing of this or that description exists, the only conjunction being "and." That this method is highly analytic is manifest. Now since our whole intention is to produce a method for the perfect analysis of propositions, the superiority of this method over the other for our purpose is undeniable. Moreover, in order to illustrate how that other might lead to false logic, I will tack the predicate of B¹, in its objectionable form, upon the subject of A¹ in the same form, and *vice versa*. I shall thus obtain two propositions which that method represents as being as simple as are Nos. 1 and 2.

Peirce: CP 4.548 Cross-Ref:††

We shall see whether they are so. Here they are:†1

3. The Interpreter having designated any object to be called A, an object B may be found such that B is a philosopher's stone of some kind, while either A is not an adept alchemist or else A could produce B.
4. Something, B, may be found, such that, no matter what the interpreter may select, and call A,

Either A would not be an adept alchemist, or B would be a philosopher's stone of some kind, and A could produce B.

Peirce: CP 4.548 Cross-Ref:††

Proposition 3 may be expressed in ordinary language thus: There is a kind of philosopher's stone, and if there be any adept alchemist, he could produce a philosopher's stone of some kind. That is, No. 3 differs from A, A¹ and 1 only in adding that there is a kind of philosopher's stone. It differs from B, B¹ and 2 in not saying that any two adepts could produce the same kind of stone (nor that any adept could produce any existing kind); while B, B¹ and 2 assert that some kind is both existent and could be made by every adept.

Peirce: CP 4.548 Cross-Ref:††

Proposition 4, in ordinary language, is: If there be (or were) an adept alchemist, there is (or would be) a kind of philosopher's stone that any adept could produce. This asserts the substance of B, B¹ and 2, but only conditionally upon the existence of an adept; but it asserts, what A, A¹ and 1 do not, that all adepts could produce some one kind of stone, and this is precisely the difference between No. 4 and A¹.

Peirce: CP 4.548 Cross-Ref:††

To me it seems plain that the propositions 3 and 4 are both less simple than No. 1 and less simple than No. 2, each adding some thing to one of the pair first given and asserting the other conditionally. Yet the method of treating the Universes as receptacles for the metaphysical Subjects only, involves as a consequence the representation of 3 and 4 as quite on a par with 1 and 2.

Peirce: CP 4.548 Cross-Ref:††

It remains to show that the other method does not carry this error with it. [If] it is the states of things affirmed or denied that are contained in the universes, then the

propositions [3 and 4] become as follows:

5. This is true: that there is a philosopher's stone of some kind, S, and that it is false that there is an adept, A, and that it is false that A could produce a philosopher's stone of some kind, S'. (Where it is neither asserted nor denied that S and S' are the same, thus distinguishing this from 2.)
6. This is false: That there is an adept, A, and that this is false: That there is a stone of a kind, S, and this is false: That there is an adept, A', and that this is false: That A' could produce a stone of the kind S. (Where again it is neither asserted nor denied that A and A' are identical, but the point is that this proposition holds even if they are not identical, thus distinguishing this from 1.)

Peirce: CP 4.548 Cross-Ref:††

These forms exhibit the greater complexity of Propositions 3 and 4, by showing that they really relate to *three* individuals each; that is to say, 3 to two possible different kinds of stone, as well as to an adept; and 4 to two possible different adepts, and to a kind of stone. Indeed, the two forms 3 and 4†1 are absolutely identical in meaning with the following different forms on the same theory. Now it is, to say the least, a serious fault in a method of analysis that it can yield two analyses so different of one and the same compound.

7. An object, B, can be found, such that whatever object the interpreter may select and call A, an object, B', can thereupon be found such that B is an existing kind of philosopher's stone, and either A would not be an adept or else B' is a kind of philosopher's stone such as A could produce.
8. Whatever individual the Interpreter may choose to call A, an object, B, may be found, such that whatever individual the Interpreter may choose to call A', Either A is not an adept or B is an existing kind of philosopher's stone, and either A' is not an adept or else A' could produce a stone of the kind B.

Peirce: CP 4.548 Cross-Ref:††

But while my forms are perfectly analytic, the need of diagrams to exhibit their meaning to the eye (better than merely giving a separate line to every proposition said to be false) is painfully obtrusive.†P1

Peirce: CP 4.549 Cross-Ref:††

549. I will now say a few words about what you have called Categories, but for which I prefer the designation Predicaments, and which you have explained as predicates of predicates. That wonderful operation of hypostatic abstraction by which we seem to create *entia rationis* that are, nevertheless, sometimes real, furnishes us the means of turning predicates from being signs that we think or think *through*, into being subjects thought of. We thus think of the thought-sign itself, making it the object of another thought-sign. Thereupon, we can repeat the operation of hypostatic abstraction, and from these second intentions derive third intentions. Does this series proceed endlessly? I think not. What then are the characters of its different members? My thoughts on this subject are not yet harvested. I will only say that the subject concerns Logic, but that the divisions so obtained must not be confounded with the different Modes of Being:†1 Actuality, Possibility, Destiny (or Freedom from Destiny). On the contrary, the succession of Predicates of Predicates is different in the different Modes of Being. Meantime, it will be proper that in our system of diagrammatization we should provide for the division, whenever needed, of each of our three Universes of modes of reality into *Realms* for the different Predicaments.

Peirce: CP 4.550 Cross-Ref:††

550. All the various meanings of the word "Mind," Logical, Metaphysical, and Psychological, are apt to be confounded more or less, partly because considerable logical acumen is required to distinguish some of them, and because of the lack of any machinery to support the thought in doing so, partly because they are so many, and partly because (owing to these causes), they are all called by one word, "Mind." In one of the narrowest and most concrete of its logical meanings, a Mind is that Some of The Truth, whose determinations become Immediate Interpretants of all other Signs whose Dynamical Interpretants are dynamically connected.†2 In our Diagram the same thing which represents The Truth must be regarded as in another way representing the Mind, and indeed, as being the Quasi-mind of all the Signs represented on the Diagram. For any set of Signs which are so connected that a complex of two of them can have one interpretant, must be Determinations of one Sign which is a *Quasi-mind*.

Peirce: CP 4.551 Cross-Ref:††

551. Thought is not necessarily connected with a brain. It appears in the work of bees, of crystals, and throughout the purely physical world; and one can no more deny that it is really there, than that the colors, the shapes, etc., of objects are really there. Consistently adhere to that unwarrantable denial, and you will be driven to some form of idealistic nominalism akin to Fichte's. Not only is thought in the organic world, but it develops there. But as there cannot be a General without Instances embodying it, so there cannot be thought without Signs. We must here give "Sign" a very wide sense, no doubt, but not too wide a sense to come within our definition. Admitting that connected Signs must have a Quasi-mind, it may further be declared that there can be no isolated sign. Moreover, signs require at least two Quasi-minds; a *Quasi-utterer* and a *Quasi-interpreter*; and although these two are at one (*i.e.*, are one mind) in the sign itself, they must nevertheless be distinct. In the Sign they are, so to say, *welded*. Accordingly, it is not merely a fact of human Psychology, but a necessity of Logic, that every logical evolution of thought should be dialogic. You may say that all this is loose talk; and I admit that, as it stands, it has a large infusion of arbitrariness. It might be filled out with argument so as to remove the greater part of this fault; but in the first place, such an expansion would require a volume -- and an uninviting one; and in the second place, what I have been saying is only to be applied to a slight determination of our system of diagrammatization, which it will only slightly affect; so that, should it be incorrect, the utmost *certain* effect will be a danger that our system may not represent every variety of non-human thought.

Peirce: CP 4.552 Cross-Ref:††

§5. TINCTURED EXISTENTIAL GRAPHSE

552. There now seems to remain no reason why we should not proceed forthwith to formulate and agree upon

THE CONVENTIONS

DETERMINING THE FORMS AND INTERPRETATIONS OF EXISTENTIAL GRAPHS

Convention the First: Of the Agency of the Scripture. We are to imagine that two parties †P1 collaborate in composing a PHEME, and in operating upon this so as to develop a Delome. (Provision shall be made in these Conventions for expressing every kind of PHEME as a Graph; †P2 and it is certain that the Method could be applied to aid the development and analysis of any kind of purposive thought. But hitherto no Graphs have been studied but such as are Propositions; so that, in the resulting uncertainty as to what modifications of the Conventions might be required for other applications, they have mostly been here stated as if they were only applicable to the expression of PHEMES and the working out of necessary conclusions.)

Peirce: CP 4.552 Cross-Ref: ††

The two collaborating parties shall be called the ***Graphist*** and the ***Interpreter***. The Graphist shall responsibly scribe each original Graph and each addition to it, with the proper indications of the Modality to be attached to it, the ***relative Quality*** †P1 of its position, and every particular of its dependence on and connections with other graphs. The Interpreter is to make such erasures and insertions of the Graph delivered to him by the Graphist as may accord with the "***General Permissions***" deducible from the Conventions and with his own purposes.

Peirce: CP 4.553 Cross-Ref: ††

553. ***Convention the Second; Of the Matter of the Scripture, and the Modality*** †P1 ***of the PHEMES expressed.*** The matter which the Graph-instances are to determine, and which thereby becomes the ***Quasi-mind*** in which the Graphist and Interpreter are at one, being a Seme of ***The Truth***, †P2 that is, of the widest Universe of Reality, and at the same time, a PHEME of all that is tacitly taken for granted between the Graphist and Interpreter, from the outset of their discussion, shall be a sheet, called the ***PHEMIC SHEET***, upon which signs can be scribed and from which any that are already scribed in any manner (even though they be incised) ***can*** be erased. But certain parts of other sheets not having the significance of the PHEMIC

THE TINCTURES OF COLOR

Fig. 197

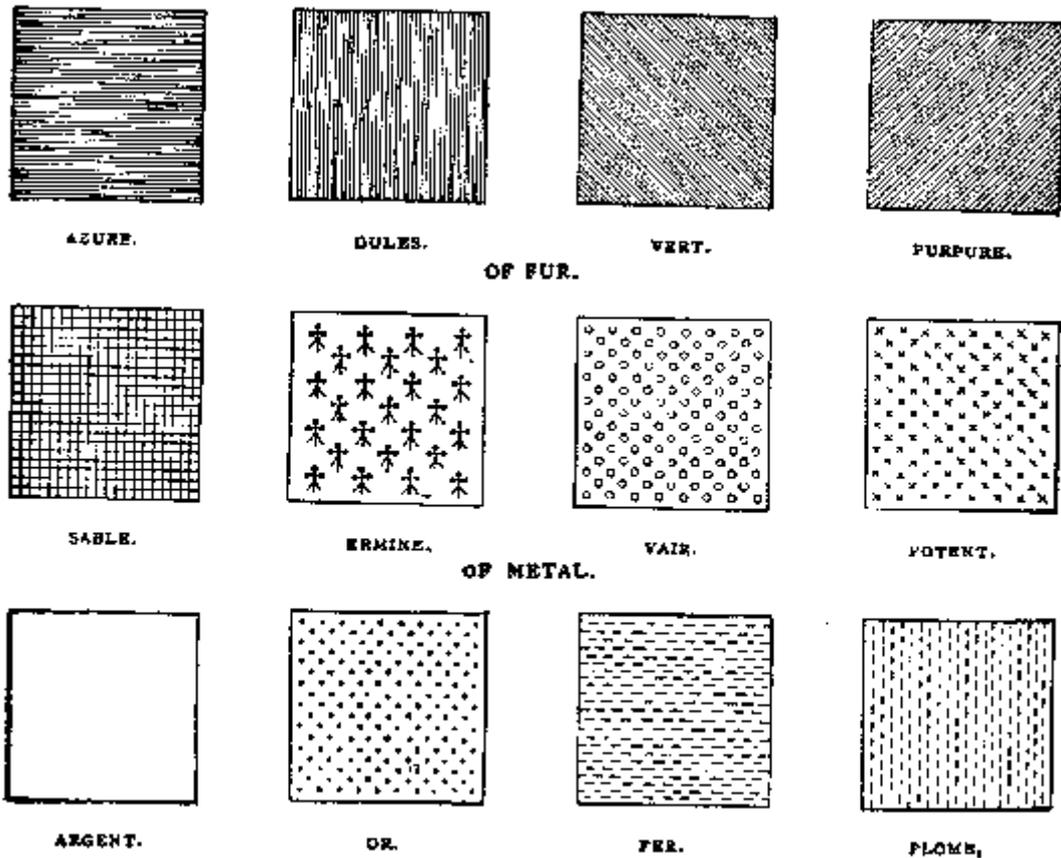


Fig. 197

[Click here to view]

sheet, but on which Graphs can be scribed and erased, shall be sometimes inserted in the Phemic sheet and exposed to view, as the Third Convention shall show. Every part of the exposed surface shall be tintured in one or another of twelve tinctures. These are divided into three *classes* of four tinctures each, the class-characters being called *Modes of Tincture*, or severally, Color, Fur, and Metal. The tinctures of Color are Azure, Gules, Vert, and Purpure. Those of Fur are Sable, Ermine, Vair, and Potent. Those of Metal are Argent, Or, Fer, and Plomb. The Tinctures will in practice be represented as in Fig. 197.†P1 The whole of any *continuous* part of the exposed surface in one tincture shall be termed a *Province*. The border of the sheet has one tincture all round; and we may imagine that it was chosen from among twelve, in agreement between the Graphist and the Interpreter at the outset. The province of the border may be called the *March*. Provinces adjacent to the March are to be regarded as overlying it; Provinces adjacent to those Provinces, but not to the March, are to be regarded as overlying the provinces adjacent to the March, and so on. We are to imagine that the Graphist always finds provinces where he needs them.

Peirce: CP 4.554 Cross-Ref:††

554. When any representation of a state of things consisting in the applicability of a given description to an individual or limited set of individuals otherwise indesignate is scribed, the Mode of Tincture of the province on which it is scribed shows whether the Mode of Being which is to be affirmatively or negatively attributed to the state of things described is to be that of Possibility, when Color will

be used; or that of Intention, indicated by Fur; or that of Actuality shown by Metal. Special understandings may determine special tinctures to refer to special varieties of the three genera of Modality. Finally, the Mode of Tincture of the March may determine whether the Entire Graph is to be understood as Interrogative, Imperative, or Indicative.

Peirce: CP 4.555 Cross-Ref:††

555. *Convention the Third: Of Areas enclosed within, but severed from, the Phemic Sheet.* The Phemic Sheet is to be imagined as lying on the smoother of the two surfaces or sides of a *Leaf*, this side being called the *recto*, and to consist of so much of this side as is continuous with the March. Other parts of the *recto* may be *exposed* to view. Every Graph-instance on the Phemic Sheet is posited unconditionally (unless, according to an agreement between Graphist and Interpreter, the Tincture of its own Province or of the March should indicate a condition) and every Graph-instance on the *recto* is posited affirmatively and, in so far as it is indeterminate, indefinitely.

Peirce: CP 4.556 Cross-Ref:††

556. Should the Graphist desire to negative a Graph, he must scribe it on the *verso*, and then, before delivery to the Interpreter, must make an incision, called a *Cut*, through the Sheet all the way round the Graph-instance to be denied, and must then turn over the excised piece, so as to *expose* its rougher surface carrying the negated Graph-instance. This reversal of the piece is to be conceived to be an inseparable part of the operation of making a *Cut*.†P1 But if the Graph to be negated includes a Cut, the twice negated Graph within that Cut must be scribed on the *recto*, and so forth. The part of the exposed surface that is continuous with the part just outside the Cut is called the *Place of the Cut*. A Cut is neither a Graph nor a Graph-instance; but the Cut, together with all that it encloses, exposed is termed an *Enclosure*, and is conceived to be an Instance of a Graph *scribed* on the Place of the Cut, which is also termed the *Place of the Enclosure*. The surface within the Cut, continuous with the parts just within it, is termed the *Area* of the Cut and of the Enclosure; and the part of the *recto* continuous with the March (*i.e.*, the Phemic Sheet), is likewise termed an *Area*, namely the Area of the Border. The Copulate of all that is scribed on any one Area, including the Graphs of which the Enclosures whose Place is this Area are Instances, is called the *Entire Graph* of that Area; and any part of the Entire Graph, whether graphically connected with or disconnected from the other parts, provided it might be the Entire Graph of the Sheet, is termed a Partial Graph of the Area.

Peirce: CP 4.557 Cross-Ref:††

557. There may be any number of Cuts, one within another, the Area of one being the Place of the next, and since the Area of each is on the side of the leaf opposite to its Place, it follows that *recto* Areas may be *exposed* which are not parts of the Phemic Sheet. Every Graph-instance on a *recto* Area is affirmatively posited, but is posited conditionally upon whatever may be signified by the Graph on the Place of the Cut of which this Area is the Area. (It follows that Graphs on Areas of different Enclosures on a *verso* Place are only alternately affirmed, and that while only the Entire Graph of the Area of an Enclosure on a *recto* Place is denied, but not its different Partial Graphs, except alternatively, the Entire Graphs of Areas of different Enclosures on one *recto* Place are copulatively denied.)

Peirce: CP 4.558 Cross-Ref:††

558. Every Graph-instance must lie upon one Area, †P1 although an Enclosure may be a part of it. Graph-instances on different Areas are not to be considered as, nor by any permissible latitude of speech to be called, Parts of one Graph-instance, nor Instances of Parts of one Graph; for it is only Graph-instances on one Area that are called Parts of one Graph-instance, and that only of a Graph-instance on that same Area; for though the Entire Graph on the Area of an enclosure is termed the *Graph of the Enclosure*, it is no Part of the Enclosure and is connected with it only through a denial.

Peirce: CP 4.559 Cross-Ref: ††

559. *Convention the Fourth: Concerning Signs of Individuals and of Individual Identity.* A single dot, not too minute, or single congeries of contiguous pretty large dots, whether in the form of a line or surface, when placed on any exposed Area, will refer to a single member of the Universe to which the Tincture of that Area refers, but will not thereby be made to refer determinately to any one. But do not forget that separate dots, or separate aggregates of dots, will not necessarily denote different Objects.

Peirce: CP 4.560 Cross-Ref: ††

560. By a *rheme*, or *predicate*, will here be meant a blank form of proposition which might have resulted by striking out certain parts of a proposition, and leaving a *blank* in the place of each, the parts stricken out being such that if each blank were filled with a proper name, a proposition (however nonsensical) would thereby be recomposed. An ordinary predicate of which no analysis is intended to be represented will usually be *written* in abbreviated form, but having a particular point on the periphery of the written form appropriated to each of the blanks that might be filled with a proper name. Such written form with the appropriated points shall be termed a *Spot*; and each appropriated point of its periphery shall be called a *Peg* of the Spot. If a heavy dot is placed at each Peg, the Spot will become a Graph expressing a proposition in which every blank is filled by a word (or concept) denoting an indefinite individual object, "something."

Peirce: CP 4.561 Cross-Ref: ††

561. A heavy line shall be considered as a continuum of contiguous dots; and since contiguous dots denote a single individual, such a line without any point of branching will signify the identity of the individuals denoted by its extremities, and the type of such unbranching line shall be the Graph of Identity, any instance of which (on one area, as every Graph-instance must be) shall be called a *Line of Identity*. The type of a three-way point of such a line (Fig. 198) shall be the *Graph of Teridentity*; and it shall be considered as composed of three contiguous Pegs of a Spot of Identity. An extremity

Figs. 198-199

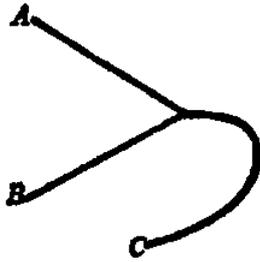


Fig. 198

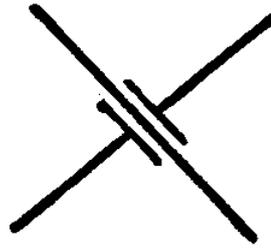


Fig. 199

[Click here

to view]

of a Line of Identity not abutting upon another such Line in another area shall be called a *Loose End*. A heavy line, whether confined to one area or not (and therefore not generally being a Graph-instance) of which two extremities abut upon pegs of spots shall be called a *Ligature*. Two lines cannot abut upon the same peg other than a point of teridentity. (The purpose of this rule is to force the recognition of the demonstrable logical truth that the concept of teridentity is not mere identity. It is identity *and* identity, but this "and" is a distinct concept, and is precisely that of teridentity.) A Ligature crossing a Cut is to be interpreted as unchanged in meaning by erasing the part that crosses to the Cut and attaching to the two Loose Ends so produced two Instances of a Proper Name nowhere else used; such a Proper name (for which a capital letter will serve) being termed a *Selective*.†P1 In the interpretation of Selectives it is often necessary to observe the rule which holds throughout the System, that the Interpretation of Existential Graphs must be *endoporeutic*, that is, the application of a Graph on the Area of a Cut will depend on the predetermination of the application of that which is on the Place of the Cut.

Peirce: CP 4.561 Cross-Ref:††

In order to avoid the intersection of Lines of Identity, either a Selective may be employed, or a *Bridge*, which is imagined to be a bit of paper ribbon, but will in practice be pictured as in Fig. 199.

Peirce: CP 4.562 Cross-Ref:††

562. *Convention the Fifth: Of the Connections of Graph-Instances.* Two partial Graph-Instances are said to be *individually and directly connected*, if, and only if, in the Entire Graph, one individual is, either unconditionally or under some condition, and whether affirmatively or negatively, made a Subject of both. Two Graph-Instances connected by a ligature are explicitly and definitely individually and directly connected. Two Graph-Instances in the same Province are thereby explicitly, although indefinitely, individually and directly connected, since both, or one and the negative of the other, or the negative of both, are asserted to be true or false together, that is, under the same circumstances, although these circumstances are not formally defined, but are left to be interpreted according to the nature of the case. Two Graph-Instances not in the same Province, though on the same Mode of Tincture, are only in so far connected that both are in the same Universe. Two Graph-Instances in different Modes of Tincture are only in so far connected that both, or one and the negative of the other, or the negative of both, are posited as appertaining to the Truth. They cannot be said to have any individual and direct connection. Two Graph-Instances that are not individually connected within the innermost Cut which

contains them both cannot be so connected at all; and every ligature connecting them is meaningless and may be made or broken.

Peirce: CP 4.563 Cross-Ref:††

563. Relations which do not imply the occurrence in their several universes of all their correlates must not be expressed by Spots or single Graphs,†P1 but all such relations can be expressed in the System.

Peirce: CP 4.564 Cross-Ref:††

564. I will now proceed to give a few examples of Existential Graphs in order to illustrate the method of interpretation, and also the *Permissions of Illative Transformation* of them.

Peirce: CP 4.564 Cross-Ref:††

If you carefully examine the above conventions, you will find that they are simply the development, and excepting in their insignificant details, the inevitable result of the development of the one convention that if any Graph, A, asserts one state of things to be real and if another graph, B, asserts the same of another state of things, then AB, which results from setting both A and B upon the sheet, shall assert that both states of things are real. This was not the case with my first system of Graphs, described in Vol. VII of *The Monist*,†1 which I now call *Entitative Graphs*. But I was forced to this principle by a series of considerations which ultimately arrayed themselves into an exact logical deduction of all the features of Existential Graphs which do not involve the Tinctures. I have no room for this here; but I state some of the points arrived at somewhat in the order in which they first presented themselves.

Peirce: CP 4.564 Cross-Ref:††

In the first place, the most perfectly analytical system of representing propositions must enable us to separate illative transformations into indecomposable parts. Hence, an illative transformation from any proposition, A, to any other, B, must in such a system consist in first transforming A into AB, followed by the transformation of AB into B. For an omission and an insertion appear to be indecomposable transformations and the only indecomposable transformations. That is, if A can be transformed by insertion into AB, and AB by omission in B, the transformation of A into B can be decomposed into an insertion and an omission. Accordingly, since logic has primarily in view argument, and since the conclusiveness of an argument can never be weakened by adding to the premisses nor by subtracting from the conclusion, I thought I ought to take the general form of argument as the basal form of composition of signs in my diagrammatization; and this necessarily took the form of a "scroll," that is (see Figs. 200, 201, 202) a curved line without contrary flexure and returning

Figs. 200-202

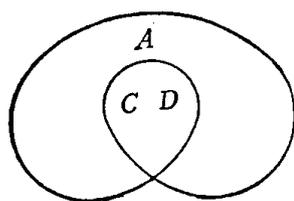


Fig. 200

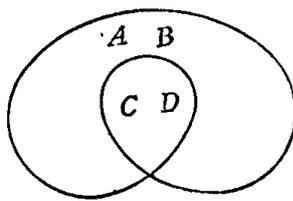


Fig. 201

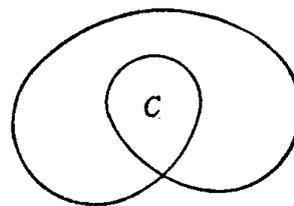


Fig. 202

to view]

[Click here

into itself after once crossing itself, and thus forming an outer and an inner "close." I shall call the outer boundary the *Wall*; and the inner, the *Fence*. In the outer I scribed the Antecedent, in the inner the Consequent, of a Conditional Proposition *de inesse*. The scroll was not taken for this purpose at hap-hazard, but was the result of experiments and reasonings by which I was brought to see that it afforded the most faithful Diagram of such a Proposition. This form once obtained, the logically inevitable development brought me speedily to the System of Existential Graphs. Namely, the idea of the scroll was that Fig. 200, for example, should assert that if A be true (under the actual circumstances), then C and D are both true. This justifies Fig. 201, that if both A and B are true, then both C and D are true, no matter what B may assert, any insertion being permitted in the outer close, and any omission from the inner close. By applying the former clause of this rule to Fig. 202, we see that this scroll with the outer close void, justifies the assertion that if no matter what be true, C is in any case true; so that the two walls of the scroll, when nothing is between them, fall together, collapse, disappear, and leave only the contents of the inner close standing, asserted, in the open field. Supposing, then, that the contents of the inner scroll had been CD, these would have been left standing, both asserted; and we thus return to the principle that writing assertions together on the open sheet asserts them all. Now, Reader, if you will just take pencil and paper and scribe the scroll expressing that if A be true, then it is true that if B be true C and D are true, and compare this with Fig. 201, which amounts to the same thing in meaning, you will see that scroll walls with a void between them collapse even when they belong to different scrolls; and you will

Figs. 203-204

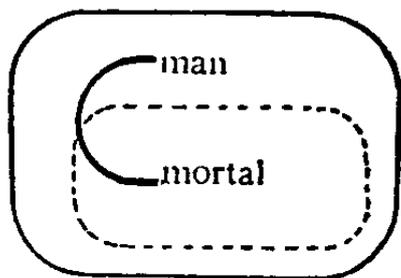


Fig. 203

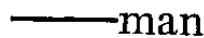


Fig. 204

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to view]

further see that a scroll is really nothing but one oval within another. Since a Conditional *de inesse* (unlike other conditionals) only asserts that either the antecedent is false or the consequent is true, it all but follows that if the latter alternative be suppressed by scribing nothing but the antecedent, which may be any proposition, in an oval, that antecedent is thereby denied.†P1 The use of a heavy line as a juncture signifying identity is inevitable; and since Fig. 203 must mean that if anything is a man, it is mortal, it will follow that Fig. 204 must mean "Something is a man."

Peirce: CP 4.565 Cross-Ref:††

565. The first permission of illative transformation is now evident as follows:

Peirce: CP 4.565 Cross-Ref:††

First Permission, called "The Rule of Deletion and Insertion." Any Graph-Instance can be deleted from any recto Area (including the severing of any Line of Identity), and any Graph-Instance can be inserted on any verso Area (including as a Graph-Instance the juncture of any two Lines of Identity or Points of Teridentity).

Peirce: CP 4.566 Cross-Ref:††

566. The justice of the following will be seen instantly by students of any form of Logical Algebra, and with very little difficulty by others:

Peirce: CP 4.566 Cross-Ref:††

Second Permission, called "The Rule of Iteration and Deiteration." Any Graph scribed on any Area may be Iterated in or (if already Iterated) may be Deiterated by a deletion from that Area or from any other Area included within that. This involves the Permission to distort a line of Identity, at will.

Peirce: CP 4.566 Cross-Ref:††

To *iterate* a Graph means to scribe it again, while joining by Ligatures every Peg of the new Instance to the corresponding Peg of the Original Instance. To *deiterate* a Graph is to erase a second Instance of it, of which each Peg is joined by a Ligature to a first Instance of it. One Area is said to be ***included within*** another if, and only if, it either is that Area or else is the Area of a Cut whose Place is an Area which, according to this definition, must be regarded as included within that other. By this Permission, Fig. 205 may be transformed into Fig. 206, and thence, by Permission No. 1, into Fig. 207.

Figs. 205-207

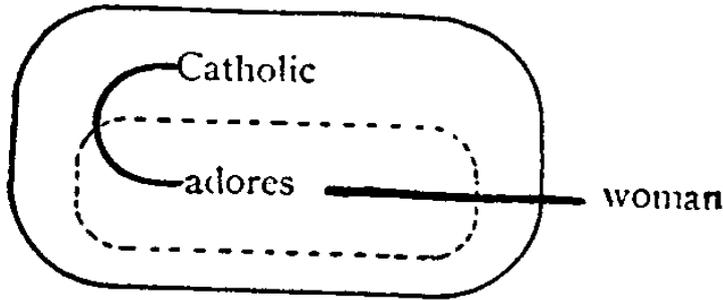


Fig. 205

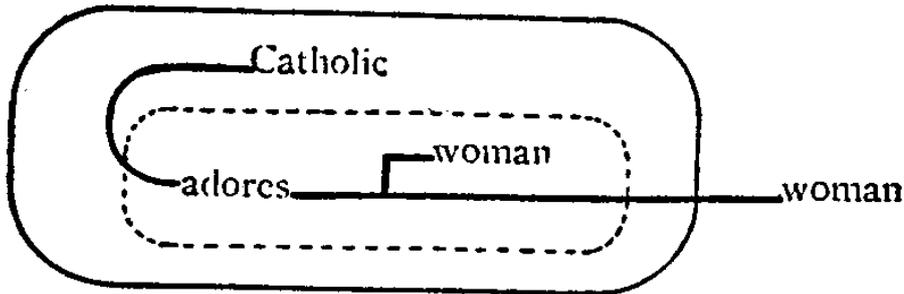


Fig. 206

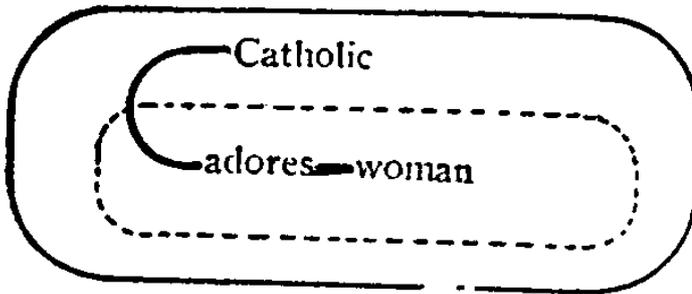


Fig. 207

[Click here

to view]

Peirce: CP 4.567 Cross-Ref:††

567. We now come to the Third Permission, which I shall state in a form which is valid, sufficient for its purpose, and convenient in practice, but which cannot be assumed as an undeduced Permission, for the reason that it allows us to regard the Inner close, after the Scroll is removed, as being a part of the Area on which the Scroll lies. Now this is not strictly either an Insertion or a Deletion; and a perfectly analytical System of Permissions should permit only the indecomposable operations of Insertion and Deletion of Graphs that are simple in expression. The more scientific way would be to substitute for the Second and Third Permissions the following Permission:

Peirce: CP 4.567 Cross-Ref:††

If an Area, {G}, and an Area, \wp , be related in any of these four ways, viz., (1) If {G} and \wp are the same Area; (2) If \wp is the Area of an Enclosure whose Place is {G}; (3) if \wp is the Area of an Enclosure whose Place is the Area of a

second Enclosure whose Place is {G}; or (4) if \mathcal{C} is the Place of an Enclosure whose Area is vacant except that it is the Place of an Enclosure whose Area is {G}, and except that it may contain ligatures, identifying Pegs in \mathcal{C} with Pegs in {G}; then, if \mathcal{C} be a recto area, any simple Graph already scribed upon {G} may be iterated upon \mathcal{C} ; while if \mathcal{C} be a verso Area, any simple Graph already scribed upon {G} and iterated upon \mathcal{C} may be deiterated by being deleted or abolished from \mathcal{C} .

Peirce: CP 4.567 Cross-Ref:††

These two Rules (of Deletion and Insertion, and of Iteration and Deiteration) are substantially all the undeduced Permissions needed; the others being either Consequences or Explanations of these. Only, in order that this may be true, it is necessary to assume that all indemonstrable implications of the Blank have from the beginning been scribed upon distant parts of the Phemic Sheet, upon any part of which they may, therefore, be iterated at will. I will give no list of these implications, since it could serve no other purpose than that of warning beginners that necessary propositions not included therein were deducible from the other permissions. I will simply notice two principles the neglect of which might lead to difficulties. One of these is that it is physically impossible to delete or otherwise get rid of a Blank in any Area that contains a Blank, whether alone or along with other Graph-Instances. We may, however, assume that there is one Graph, and only one, an Instance of which entirely fills up an Area, without any Blank. The other principle is that, since a Dot merely asserts that some individual object exists, and is thus one of the implications of the Blank, it may be inserted in any Area; and since the Dot will signify the same thing whatever its size, it may be regarded as an Enclosure whose Area is filled with an Instance of that sole Graph that excludes the Blank. The Dot, then, denies that Graph, which may, therefore, be understood as the absurd Graph, and its signification may be formulated as "Whatever you please is true." The absurd Graph may also take the form of an Enclosure with its Area entirely Blank, or enclosing only some Instance of a Graph implied in the Blank. These two principles will enable the Graphist to thread his way through some Transformations which might otherwise appear paradoxical and absurd.

Peirce: CP 4.567 Cross-Ref:††

Third Permission, called "The Rule of the Double Cut." Two Cuts one within another, with nothing between them, unless it be Ligatures passing from outside the outer Cut to inside the inner one, may be made or abolished on any Area.

Peirce: CP 4.568 Cross-Ref:††

568. Let us now consider the Interpretation of such Ligatures. For that purpose, I first note that the Entire Graph of any *recto* Area is a wholly particular and affirmative Proposition or Copulation of such Propositions. By "wholly particular," I mean, having for every Subject an indesignate individual. The Entire Graph of any *verso* Area is a wholly universal negative proposition or a disjunction of such propositions.

Peirce: CP 4.568 Cross-Ref:††

The first time one hears a Proper Name pronounced, it is but a name, predicated, as one usually gathers, of an existent, or at least historically existent, individual object, of which, or of whom, one almost always gathers some additional information. The next time one hears the name, it is by so much the more definite;

and almost every time one hears the name, one gains in familiarity with the object. A Selective is a Proper Name met with by the Interpreter for the first time. But it always occurs twice, and usually on different areas. Now the Interpretation, by Convention No. 3, is to be Endoporeutic, so that it is the outermost occurrence of the Name that is the earliest.

Peirce: CP 4.569 Cross-Ref:††

569. Let us now analyze the interpretation of a Ligature passing through a Cut. Take, for example, the Graph of Fig. 208.

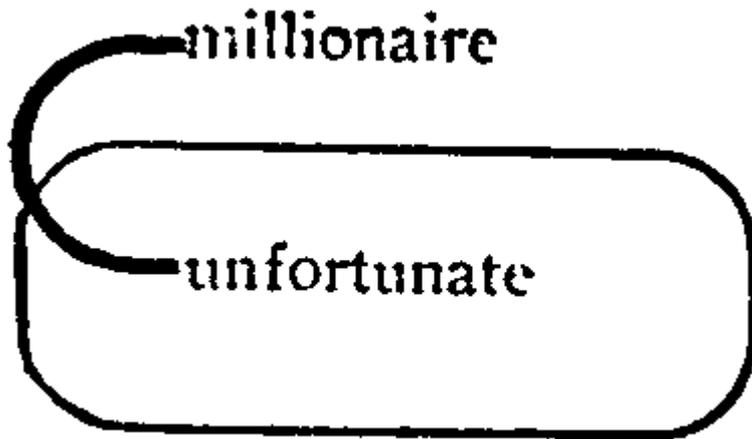


Fig. 208

[[Click here to view](#)]

The partial Graph on the Place of the Cut asserts that there exists an individual denoted by the extremity of the line of identity on the Cut, which is a millionaire. Call that individual C. Then, since contiguous dots denote the same individual objects, the extremity of the line of identity on the Area of the cut is also C, and the Partial Graph on that Area, asserts that, let the Interpreter choose whatever individual he will, that individual is either not C, or else is not unfortunate. Thus, the Entire Graph asserts that there exists a millionaire who is not unfortunate. Furthermore, the Enclosure lying in the same Argent Province as the "millionaire," it is asserted that this individual's being a millionaire is *connected* with his not being unfortunate. This example shows that the Graphist is permitted to extend any Line of Identity on a recto Area so as to carry an end of it to any Cut in that area. Let us next interpret Fig. 209.

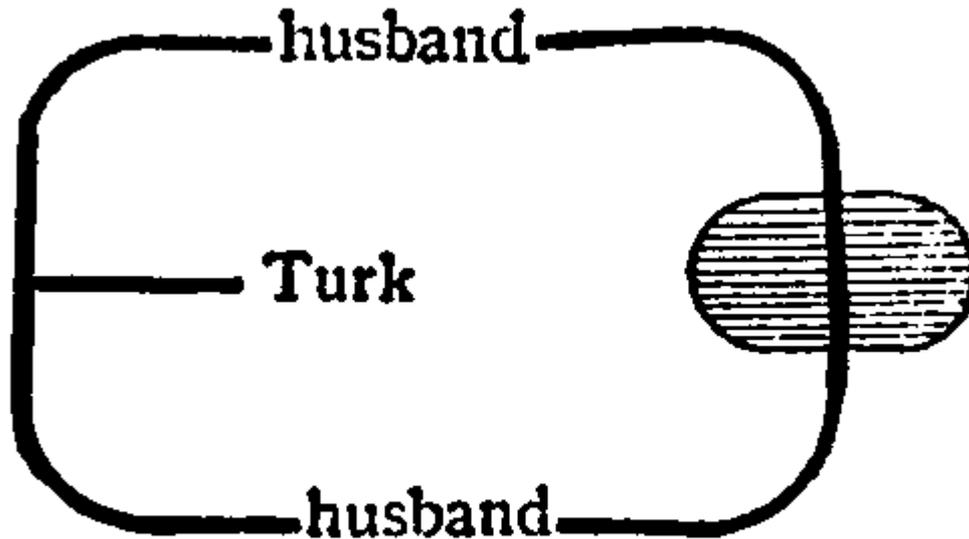


Fig. 209

[Click here to view] It obviously asserts that there exists a Turk who is at once the husband of an Individual denoted by a point on the Cut, which individual we may name U, and is the husband of an Individual, whom we may name V, denoted by another point on the Cut. And the Graph on the Area of the cut, declares that whatever Fig. 209 Individual the Interpreter may select either is not, and cannot be, U, or is not and cannot be V. Thus, the Entire Graph asserts that there is an existent Turk who is husband of two existent persons; and the "husband," the "Turk" and the enclosure, all being in the same Argent province, although the *Area* of the Enclosure is on color, and thus denies the *possibility* of the identity of U and V, all four predications are true *together*, that is, are true under the same circumstances, which circumstances should be defined by a special convention when anything may turn upon what they are. For the sake of illustrating this, I shall now scribe Fig. 210

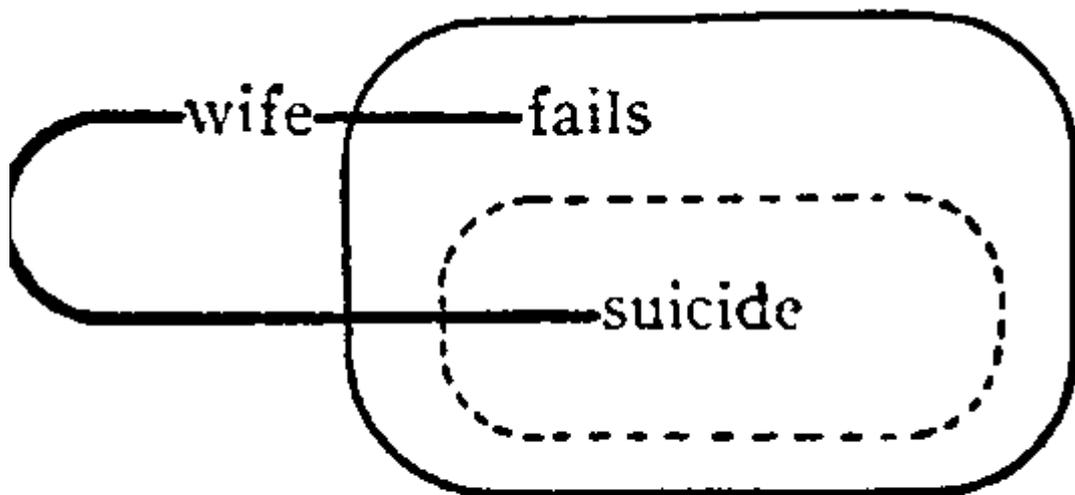


Fig. 210

[Click here to view] all in one province. This may be read, "There is some married woman who will commit suicide in case her husband fails in business." This evidently goes far beyond saying that if every married man fails in business some married woman will commit suicide. Yet note that since the Graph is on Metal it asserts a conditional proposition *de inesse* and only means that there is a married woman whose husband does not fail or else she commits suicide. That, at least, is all it will seem to mean if we fail to take account of the fact, that being all in one Province, it is said that her suicide is *connected* with his failure. Neglecting that, the proposition only denies that every married man fails, while no married woman commits suicide.†1 The logical principle is that to say that there is some one individual of which one or other of two predicates is true is no more than to say that there either is some individual of which one is true or else there is some individual of which the other is true. Or, to state the matter as an illative permission of the System of Existential Graphs,

Peirce: CP 4.569 Cross-Ref:††

Fourth Permission. If the smallest Cut which wholly contains a Ligature connecting two Graphs in different Provinces has its Area on the side of the Leaf opposite to that of the Area of the smallest Cut that contains those two Graphs, then such Ligature may be made or broken at pleasure, as far as these two Graphs are concerned.†2

Peirce: CP 4.570 Cross-Ref:††

570. Another somewhat curious problem concerning ligatures is to say by what principle it is true, as it evidently is true, that the passage of ligatures from without the outer of two Cuts to within the inner of them will not prevent the two from collapsing in case there is no other Graph-Instance between them. A little study suffices to show that this may depend upon the ligatures' being replaceable by Selectives where they cross the Cuts, and that a Selective is always, at its first occurrence, a new predicate. For it is a principle of Logic that in introducing a new predicate one has a right to assert what one likes concerning it, without any restriction, as long as one implies no assertion concerning anything else. I will leave it to you, Reader, to find out how this principle accounts for the collapse of the two Cuts. Another solution of this problem, not depending on the superfluous device of Selectives, is afforded by the second enunciation of the Rule of Iteration and Deiteration; since this permits the Graph of the Inner Close to be at once iterated on the Phemic Sheet. One may choose between these two methods of solution.

Peirce: CP 4.571 Cross-Ref:††

571. The System of Existential Graphs which I have now sufficiently described -- or, at any rate, have described as well as I know how, leaving the further perfection of it to others -- greatly facilitates the solution of problems of Logic, as will be seen in the sequel, not by any mysterious properties, but simply by substituting for the symbols in which such problems present themselves, concrete visual figures concerning which we have merely to say whether or not they admit

Figs. 211-212

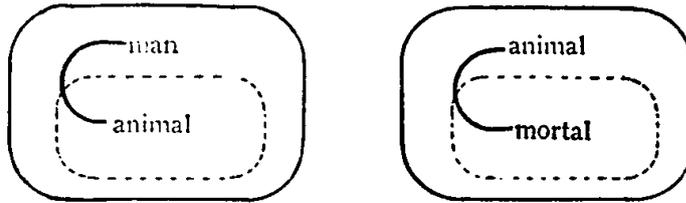


Fig. 211

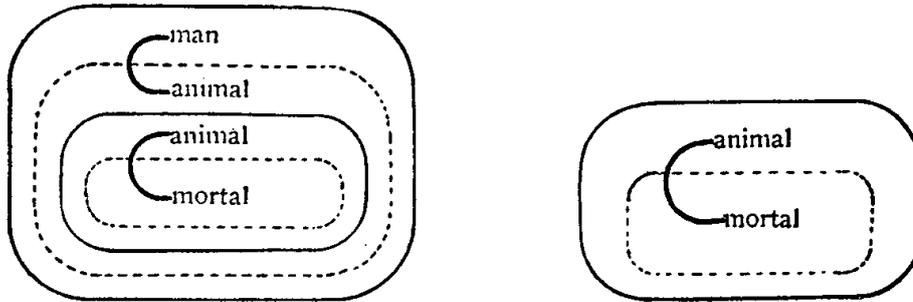


Fig. 212

[Click here

to view]

certain describable relations of their parts. Diagrammatic reasoning is the only really fertile reasoning. If logicians would only embrace this method, we should no longer see attempts to base their science on the fragile foundations of metaphysics or a psychology not based on logical theory; and there would soon be such an advance in logic that every science would feel the benefit of it.

Peirce: CP 4.571 Cross-Ref:††

This System may, of course, be applied to the analysis of reasonings. Thus, to separate the syllogistic illation, "Any man would be an animal, and any animal would be mortal; therefore, any man would be mortal," the Premisses are first scribed as in Fig. 211. Then by the rule of Iteration, a first illative transformation gives Fig. 212. Next, by the permission to erase from a recto Area, a second step gives Fig. 213. Then, by the permission to deform a line of Identity on a recto Area, a third step gives Fig. 214. Next, by the permission to insert in a verso Area, a fourth step gives Fig. 215. Next, by Deiteration, a fifth step gives Fig. 216. Next, by

Figs. 213-218

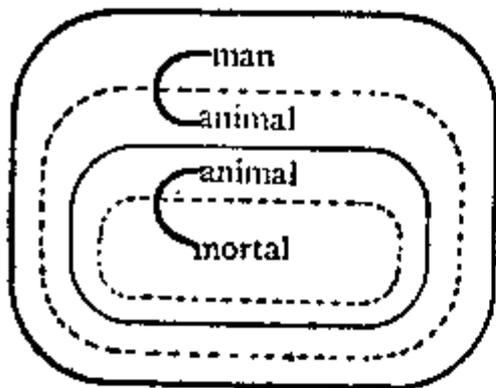


Fig. 213

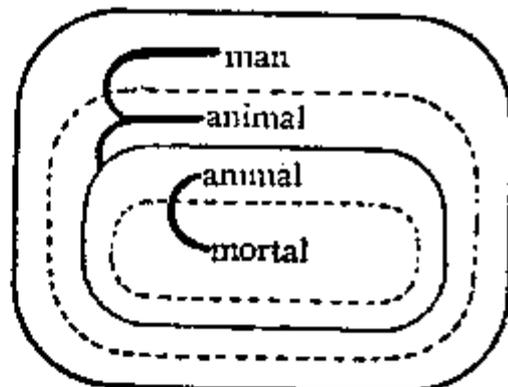


Fig. 214

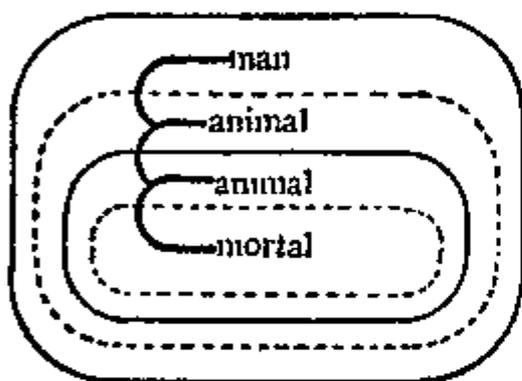


Fig. 215

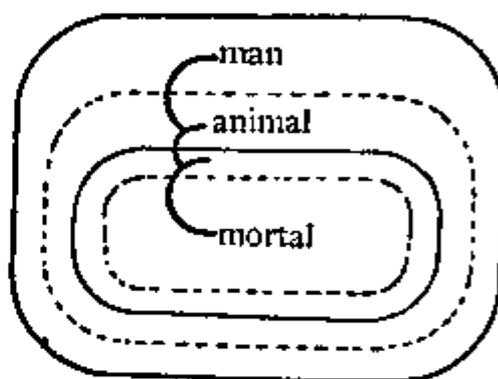


Fig. 216

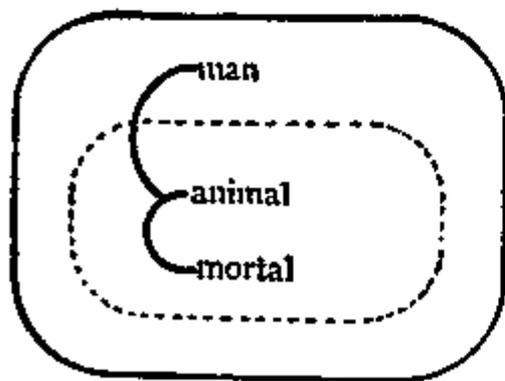


Fig. 217

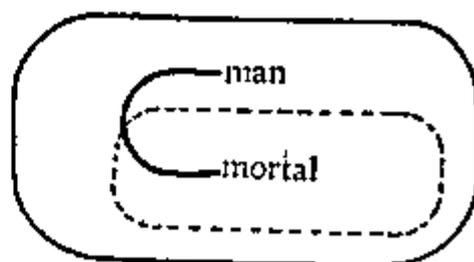


Fig. 218

[Click here to view]

the collapse of two Cuts, a sixth step gives Fig. 217; and finally, by omission from a recto Area, a seventh step gives the conclusion Fig. 218. The analysis might have been carried a little further, by means of the Rule of Iteration and Deiteration, so as to increase the number of distinct inferential steps to nine, showing how complex a process the drawing of a syllogistic conclusion really is. On the other hand, it need

scarcely be said that there are a number of *deduced* liberties of transformation, by which even much more complicated inferences than a syllogism can be performed at a stroke. For that sort of problem, however, which consists in drawing a conclusion or assuring oneself of its correctness, this System is not particularly adapted. Its true utility is in the assistance it renders -- the support to the mind, by furnishing concrete diagrams upon which to experiment -- in the solution of the most difficult problems of logical theory.

Peirce: CP 4.572 Cross-Ref:††

572. I mentioned on an early page of this paper that this System leads to a different conception of the Proposition and Argument from the traditional view that a Proposition is composed of Names, and that an Argument is composed of Propositions. It is a matter of insignificant detail whether the term Argument be taken in the sense of the Middle Term, in that of the Copulate of Premisses, in that of the setting forth of Premisses and Conclusion, or in that of the representation that the real facts which the premisses assert (together, it may be, with the mode in which those facts have come to light) logically signify the truth of the Conclusion. In any case, when an Argument is brought before us, there is brought to our notice (what appears so clearly in the Illative Transformations of Graphs) a process whereby the Premisses bring forth the Conclusion, not informing the Interpreter of its Truth, but appealing to him to assent thereto. This Process of Transformation, which is evidently the kernel of the matter, is no more built out of Propositions than a motion is built out of positions. The logical relation of the Conclusion to the Premisses might be asserted; but that would not be an Argument, which is essentially intended to be understood as representing what it represents only in virtue of the logical habit which would bring any logical Interpreter to assent to it. We may express this by saying that the Final (or quasi-intended) Interpretant of an Argument represents it as representing its Object after the manner of a Symbol. In an analogous way the relation of Predicate to Subject which is *stated* in a Proposition might be merely described in a Term. But the essence of the Proposition is that it intends, as it were, to be regarded as in an existential relation to its Object, as an Index is, so that its assertion shall be regarded as evidence of the fact. It appears to me that an assertion and a command do not differ essentially in the nature of their Final Interpretants as in their Immediate, and so far as they are effective, in their Dynamical Interpretants; but that is of secondary interest. The Name, or any Seme, is merely a substitute for its Object in one or another capacity in which respect it is all one with the Object. Its Final Interpretant thus represents it as representing its Object after the manner of an Icon, by mere agreement in idea. It thus appears that the difference between the Term, the Proposition, and the Argument, is by no means a difference of complexity, and does not so much consist in structure as in the services they are severally intended to perform.

Peirce: CP 4.572 Cross-Ref:††

For that reason, the ways in which Terms and Arguments can be compounded cannot differ greatly from the ways in which Propositions can be compounded. A mystery, or paradox, has always overhung the question of the Composition of Concepts. Namely, if two concepts, A and B, are to be compounded, their composition would seem to be necessarily a third ingredient, Concept C, and the same difficulty will arise as to the Composition of A and C. But the Method of Existential Graphs solves this riddle instantly by showing that, as far as propositions go, and it must evidently be the same with Terms and Arguments, there is but one general way in which their Composition can possibly take place; namely, each component must be indeterminate in some respect or another; and in their composition each determines

the other. On the recto this is obvious: "Some man is rich" is composed of "Something is a man" and "something is rich," and the two somethings merely explain each other's vagueness in a measure. Two simultaneous independent assertions are still connected in the same manner; for each is in itself vague as to the Universe or the "Province" in which its truth lies, and the two somewhat define each other in this respect. The composition of a Conditional Proposition is to be explained in the same way. The Antecedent is a Sign which is Indefinite as to its Interpretant; the Consequent is a Sign which is Indefinite as to its Object. They supply each the other's lack. Of course, the explanation of the structure of the Conditional gives the explanation of negation; for the negative is simply that from whose Truth it would be true to say that anything you please would follow *de inesse*.

Peirce: CP 4.572 Cross-Ref:††

In my next paper, the utility of this diagrammatization of thought in the discussion of the truth of Pragmaticism shall be made to appear.†1

Peirce: CP 4.573 Cross-Ref:††
CHAPTER 7

AN IMPROVEMENT ON THE GAMMA GRAPHS†1E

573. In working with Existential Graphs, we use, or at any rate imagine that we use, a sheet of paper of different tints on its two sides. Let us say that the side we call the *recto* is cream white while the *verso* is usually of somewhat bluish grey, but may be of yellow or of a rose tint or of green. The *recto* is appropriated to the representation of existential, or actual, facts, or what we choose to make believe are such. The *verso* is appropriated to the representation of possibilities of different kinds according to its tint, but usually to that of subjective possibilities, or subjectively possible truths. The special kind of possibility here called subjective is that which consists in ignorance. If we do not know that there are not inhabitants of Mars, it is subjectively possible that there are such beings. . . .

Peirce: CP 4.574 Cross-Ref:††

574. The *verso* is usually appropriated to imparting information about *subjective possibilities* or what may be true for aught we know. To scribe a graph is to impart an item of information; and this item of information does one of two things. It either adds to what we know to exist or it cuts off something from our list of subjective possibilities. Hence, it must be that a graph scribed on the *verso* is thereby denied.

Peirce: CP 4.575 Cross-Ref:††

575. Now the denial of a subjective possibility usually, if not always, involves the assertion of a truth of existence; and consequently what is put upon the *verso* must usually have a definite connection with a place on the *recto*.

Peirce: CP 4.576 Cross-Ref:††

576. In my former exposition of Existential Graphs, I said that there must be a

department of the System which I called the Gamma part into which I was as yet able to gain mere glimpses, sufficient only to show me its reality, and to rouse my intense curiosity, without giving me any real insight into it. The conception of the System which I have just set forth is a very recent discovery. I have not had time as yet to trace out all its consequences. But it is already plain that, in at least three places, it lifts the veil from the Gamma part of the system.

Peirce: CP 4.577 Cross-Ref:††

577. The new discovery which sheds such a light is simply that, as the main part of the sheet represents existence or actuality, so the area within a cut, that is, the *verso* of the sheet, represents a kind of possibility.

Peirce: CP 4.578 Cross-Ref:††

578. From thence I immediately infer several things that I did not understand before, as follows:

Peirce: CP 4.578 Cross-Ref:††

First, the cut may be imagined to extend down to one or another depth into the paper, so that the overturning of the piece cut out may expose one stratum or another, these being distinguished by their tints; the different tints representing different kinds of possibility.

Peirce: CP 4.578 Cross-Ref:††

This improvement gives substantially, as far as I can see, nearly the whole of that Gamma part which I have been endeavoring to discern.

Peirce: CP 4.579 Cross-Ref:††

579. Second, In a certain partly printed but unpublished "Syllabus of Logic," which contains the only formal or full description of Existential Graphs that I have ever undertaken to give, I laid it down, as a rule, that no graph could be partly in one area and partly in another;†1 and this I said simply because I could attach no interpretation to a graph which should cross a cut. As soon, however, as I discovered that the *verso* of the sheet represents a universe of possibility, I saw clearly that such a graph was not only interpretable, but that it fills the great lacuna in all my previous developments of the logic of relatives. For although I have always recognized that a possibility may *be real*, that it is sheer insanity to deny the reality of the possibility of my raising my arm, even if, when the time comes, I do *not* raise it; and although, in all my attempts to classify relations, I have invariably recognized, as one great class of relations, the class of *references*, as I have called them, where one correlate is an existent, and another is a mere possibility; yet whenever I have undertaken to develop the logic of relations, I have always left these references out of account, notwithstanding their manifest importance, simply because the algebras or other forms of diagrammatization which I employed did not seem to afford me any means of representing them.†1 I need hardly say that the moment I discovered in the *verso* of the sheet of Existential Graphs a representation of a universe of possibility, I perceived that a *reference* would be represented by a graph which should cross a cut, thus subduing a vast field of thought to the governance and control of exact logic.

Peirce: CP 4.580 Cross-Ref:††

580. Third, My previous account of Existential Graphs

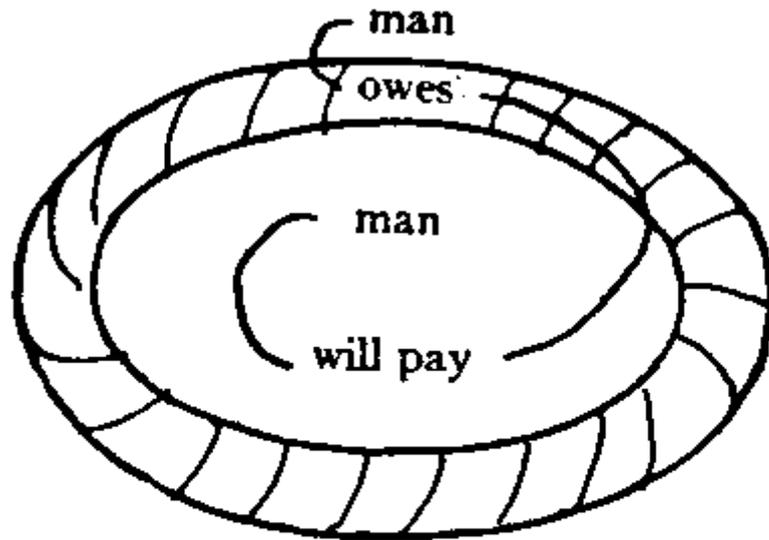


Fig. 219†

Fig. 219
 here to view]†2

[Click

was marred by a certain rule which, from the point of view
 from which I thought the system ought to be regarded, seemed
 quite out of place and unacceptable, and yet which I found

myself unable to dispute.†3 I will just illustrate this matter by an example. Suppose
 we wish to assert that there is a man every dollar of whose indebtedness will be paid
 by some man

Figs. 220-221

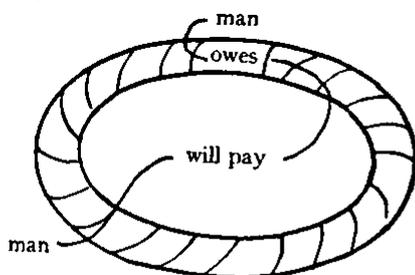


Fig. 220

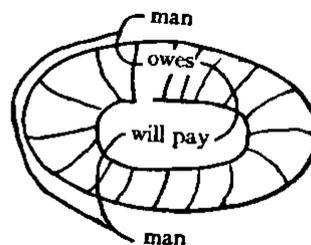


Fig. 221

to view]

[Click here

or other, perhaps one dollar being paid by one man and another by another man, or perhaps all paid by the same man. We do not wish to say how that will be. Here will be our graph, Fig. 219. But if we wish to assert that one man will pay the whole, without saying in what relation the payer stands to the debtor, here will be our graph, Fig. 220. Now suppose we wish to add that this man who will pay all those debts is the very same man who owes them. Then we insert two graphs of teridentity and a line of identity as in Fig. 221. The difference between the graph with and without this added line is obvious, and is perfectly represented in all my systems. But here it will be observed that the graph "*owes*" and the graph "*pays*" are *not only* united on the *left* by a line *outside* the smallest area that contains them both, but likewise on the *right*, by a line *inside* that smallest common area. Now let us consider a case in which this inner connection is lacking. Let us assert that there is a man A and a man B, who may or may not be the same man, and if A becomes bankrupt then B will suicide. Then, if we add that A and B *are* the same man, by drawing a line outside the smallest common area of the

Figs. 222-223

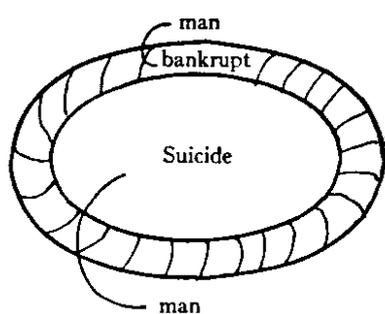


Fig. 222

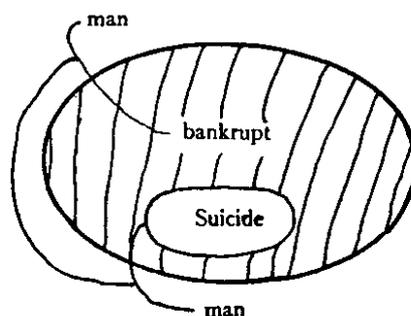


Fig. 223

[Click here

to view]

graphs joined, which are here bankrupt and suicide, the strange rule to which I refer is that such outer line, because there is no connecting line within the smallest common area, is null and void, that is, it does not affect the interpretation in the least. . . . The proposition that there is a man who if *he* goes bankrupt will commit suicide is false only in case, taking any man you please, he *will* go bankrupt, and *will not* suicide. That is, it is falsified only if every man goes bankrupt without suiciding. But this is the same as the state of things under which the other proposition is false; namely, that every man goes broke while no man suicides. This reasoning is irrefragable as long as a mere possibility is treated as an absolute nullity. Some years ago,†1 however, when in consequence of an invitation to deliver a course of lectures in Harvard University upon Pragmatism, I was led to revise that doctrine, in which I had already found difficulties, I soon discovered, upon a critical analysis, that it was absolutely necessary to insist upon and bring to the front, the truth that a mere possibility may be quite real. That admitted, it can no longer be granted that every conditional proposition whose antecedent does not happen to be realized is true, and the whole reasoning just given breaks down.

Peirce: CP 4.581 Cross-Ref:††

581. I often think that we logicians are the most obtuse of men, and the most devoid of common sense. As soon as I saw that this strange rule, so foreign to the general idea of the System of Existential Graphs, could by no means be deduced from the other rules nor from the general idea of the system, but has to be accepted, if at all, as an arbitrary first principle -- I ought to have asked myself, and should have asked myself if I had not been afflicted with the logician's *bêtise*, What compels the adoption of this rule? The answer to that must have been that the *interpretation* requires it; and the inference of common sense from that answer would have been that the interpretation was too narrow. Yet I did not think of that until my operose method like that of a hydrographic surveyor sounding out a harbour, suddenly brought me up to the important truth that the verso of the sheet of Existential Graphs represents a universe of possibilities. This, taken in connection with other premisses, led me back to the same conclusion to which my studies of Pragmatism had already brought me, the reality of some possibilities. This is a striking proof of the superiority of the System of Existential Graphs to either of my algebras of logic.†1 For in both of them the incongruity of this strange rule is completely hidden behind the superfluous machinery which is introduced in order to give an appearance of symmetry to logical law, and in order to facilitate the working of these algebras considered as *reasoning machines*. I cannot let this remark pass without protesting, however, that in the construction of no algebra was the idea of making a calculus which would turn out conclusions by a regular routine other than a very secondary purpose. . . .†2

Peirce: CP 4.582 Cross-Ref:††

582. The sheet of the graphs in all its states collectively, together with the laws of its transformations, corresponds to and represents the *Mind* in its relation to its thoughts, considered as signs. That thoughts are signs has been more especially urged by nominalistic logicians; but the realists are, for the most part, content to let the proposition stand unchallenged, even when they have not decidedly affirmed its truth. The scribed graphs are *determinations* of the sheet, just as thoughts are *determinations* of the mind; and the mind itself is a comprehensive thought just as the sheet considered in all its actual transformation-states and transformations, taken collectively, is a graph-instance and taken in all its permissible transformations is a graph. Thus the system of existential graphs is a rough and generalized diagram of the Mind, and it gives a better idea of what the mind is, from the point of view of logic, than could be conveyed by any abstract account of it.

Peirce: CP 4.583 Cross-Ref:††

583. The System of Existential Graphs recognizes but one mode of combination of ideas, that by which two indefinite propositions define, or rather partially define, each other on the *recto* and by which two general propositions mutually limit each other upon the *verso*; or, in a unitary formula, by which two indeterminate propositions mutually determine each other in a measure. I say in a measure, for it is impossible that any sign whether mental or external should be perfectly determinate. If it were possible such sign must remain absolutely unconnected with any other. It would quite obviously be such a sign of its entire universe, as Leibniz and others have described the omniscience of God to be, an intuitive representation amounting to an indecomposable feeling of the whole in all its details, from which those details would not be separable. For no reasoning, and consequently no abstraction, could connect itself with such a sign. This consideration, which is obviously correct, is a strong argument to show that what the system of

existential graphs represents to be true of propositions and which must be true of *them*, since every proposition can be analytically expressed in existential graphs, equally holds good of concepts that are *not* propositional; and this argument is supported by the evident truth that no sign of a thing or kind of thing -- the ideas of signs to which concepts belong -- can arise except in a proposition; and no logical operation upon a proposition can result in anything but a proposition; so that non-propositional signs can only exist as constituents of propositions. But it is not true, as ordinarily represented, that a proposition can be built up of non-propositional signs. The truth is that concepts are nothing but indefinite problematic judgments. The concept of *man* necessarily involves the thought of the possible being of a man; and thus it is precisely the judgment, "There may be a man." Since no perfectly determinate proposition is possible, there is one more reform that needs to be made in the system of existential graphs. Namely, the line of identity must be totally abolished, or *rather* must be understood quite differently. We must hereafter understand it to be *potentially* the graph of *teridentity* by which means there always will virtually be at least one loose end in every graph. In fact, it will not be truly a graph of *teridentity* but a graph of indefinitely multiple identity.

Peirce: CP 4.584 Cross-Ref:††

584. We here reach a point at which novel considerations about the constitution of knowledge and therefore of the constitution of nature burst in upon the mind with cataclysmal multitude and resistlessness. It is that synthesis of tychism and of pragmatism for which I long ago proposed the name, Synechism,†1 to which one thus returns; but this time with stronger reasons than ever before. But I cannot, consistently with my own convictions, ask the Academy to listen to a discourse upon Metaphysics.

Peirce: CP 4.585 Cross-Ref:††
BOOK III

THE AMAZING MAZES

CHAPTER 1

THE FIRST CURIOSITY†1E

"Mazes intricate.

Eccentric, interwov'd, yet regular

Then most, when most irregular they seem."

Milton's Description of the Mystical Angelic Dance.

§1. STATEMENT OF THE FIRST CURIOSITYE

585. About 1860 I cooked up a *mélange* of effects of most of the elementary principles of cyclic arithmetic; and ever since, at the end of some evening's card-play, I have occasionally exhibited it in the form of a "trick" (though there is really no trick about the phenomenon) with the uniform result of interesting and surprising all the company, albeit their mathematical powers have ranged from a bare sufficiency for an altruistic tolerance of cards up to those of some of the mightiest mathematicians of the age, who assuredly with a little reflection could have unraveled the marvel.

Peirce: CP 4.586 Cross-Ref:††

586. The following shall describe what I do; but you, Reader, must do it too, if you are to appreciate the curiosity of the effect. So be good enough as to take two packets of playing-cards, the one consisting of a complete red suit and the other of a black suit without the king, the cards of each being arranged in regular order in the packet, so that the face-value of every card is equal to its ordinal number in the packet.

Peirce: CP 4.586 Cross-Ref:††

N.B. *Throughout all my descriptions of manipulations of cards, it is to be understood, once for all, that the observance of the following STANDING RULES is taken for granted in all cases where the contrary is not expressly directed: Firstly, that a pack or packet of cards held in the hand is, unless otherwise directed, to be held with backs up (though not, of course, while they are in process of arrangement or rearrangement), while a pile of cards FORMED on the table (in contra-distinction to a pile placed, ready formed, on the table, as well as to rows of single cards spread upon the table) is always to be formed with the faces displayed, and left so until they are gathered up. Secondly, that, whether a packet in the hand or a pile on the table be referred to, by the "ordinal, or serial, number" of a single card or of a larger division of the whole is meant its number, counting in the order of succession in the packet or pile, from the card or other part at the BACK of the packet or at the BOTTOM of the pile as "Number 1," to the card or other part at the FACE of the packet or the TOP of the pile; the ordinal or serial number of this last being equal to the cardinal number of cards (or larger divisions COUNTED) in the whole packet or pile; and the few exceptions to this rule will be noted as they occur; Thirdly, that by the "face-value" is meant the number of pips on a plain card, the ace counting as one; while, of the picture-cards, the knave, for which J will usually be written, will count as eleven, the queen, or Q, as twelve, and the king, K, either as thirteen or as the zero of the next suit; and Fourthly, that when a number of piles that have been formed upon the table by dealing out the cards, are to be gathered up, the uniform manner of doing so is to be as follows: The first pile to be taken (which pile this is to be will appear in due time) is to be grasped as a whole and placed (faces up) upon the pile that is to be taken next. Then those two piles are to be grasped as a whole, and placed (faces up) upon the pile that is next to be taken; and so on, until all the piles have been gathered up; when, in accordance with the first Standing Rule, the whole packet is to be turned back up. And note, by the way,*

that in consequence of the manner in which the piles are gathered, each, after the first, being placed at the back of those already taken, while in observance of the second Standing Rule, we always count places in a packet from the back of it, it follows that the last pile taken will be the first in the regathered packet, while the first taken will become the last, and all the others in the same complementary way, the ordinal numbers of their gathering and those of their places in the regathered packet adding up to one more than the total number of piles.

Peirce: CP 4.587 Cross-Ref:††

587. Of course, while the red packet and the black packet are getting arranged so that the face-value of each card shall also be its ordinal, or serial, number in the packet, the cards must needs be held faces up. But as soon as they have been arranged, the packet of thirteen cards is to be laid on the table, *back up*. You then deal -- for, let me repeat it, Reader, by the inexorable laws of psychology, if you do not actually take cards (and the United States Playing-Card Company's "Fauntleroy" playing-cards are the most suitable, although any that run smoothly will do), and actually go through the processes, the whole description can mean nothing to you; -- *you* deal, then, the twelve black cards, one by one, into two piles, the first card being turned to form the bottom of the first pile, the second that of the second pile (on the right hand of the first pile), the third card going on the first pile again, the fourth on the second, and every following card being placed immediately upon the card whose ordinal, or serial, number in the packet before the deal was two lower than the former's ordinal, or serial, number then was. *The last card, however, is to be exceptionally treated*. Instead of being placed on the top of the second pile according to the rule just given, it is to be placed on the table, face up, and apart from the other cards, to make the bottom card of an isolated pile, to be called the "*discard pile*"; while, in place of it, the first card of the pile of cards of the red suit, which card will, of course, be the ace, is to be placed face up on the top of the second of the two piles formed by the dealing, where that discarded card would naturally have gone. Now you gather up these two piles by grasping the first, or left-hand pile, placing it, face up, upon the second, or right-hand, pile, and taking up the two together; and you then at once turn the packet back up in compliance with the first standing rule. This whole operation of *firstly*, dealing out into two piles the packet that was at first entirely composed of black cards; but *secondly*, placing the last card, face up, on the discard pile, and *thirdly*, substituting for it the card then at the top of the pile of red cards, by placing this latter, face up, upon the top of the second pile of the deal, and then, *fourthly*, putting the left-hand, or first, pile of the deal, face up, upon the second, and having taken up the whole packet, turning it with its back up -- this whole quadripartite operation, I say, is to be performed, in all, twelve times in succession. My statement that in this operation the last card is treated *exceptionally* was quite correct, since its treatment made an exception to the rule of placing each card on the card that before the deal came two places in advance of it in the packet. Had I said it was treated *irregularly*, I should have written very carelessly, since it is just one of those cases in which a violation of a regularity of a low order establishes a regularity of a much higher order (if John Milton knew the meaning of the word "regular") -- a pronouncement which must be left for the issue of the performance to ratify; and you shall see, Reader, that the event will ratify it with striking emphasis. Already, we begin to see some regularity in the process, since each of the twelve cards placed on the discard-pile in the twelve performances of the quadripartite operation is seen to belong to the black suit; so that the packet held in the hand and dealt out, from being originally entirely black, has now become entirely red. Having placed the red king

upon the face of this packet, you now lay down the latter in order to have your hands free to manipulate the discard-pile. Holding this discard pile as the first standing rule directs, you take the cards singly from the top and range them, one by one, from left to right, in a row upon the table, with their backs up. The length of the table from left to right ought to be double that of the row; and this is one of the reasons for preferring cards of a small size. To guard against any mistake, you may take a peek at the seventh card, to make sure that it is the ace, as it should be. The row being formed, I remark to the company, as you should do in substance, that I reserve the right to move as many of these black cards as I please, at any and all times, from one end of the row to the other; but that beyond doing that, I renounce all right to disarrange those cards. Then, taking up the red cards, and holding the packet with its back up, I (and so must you) request any person to cut it. When he does so, you place the cards he leaves in your hand at the back of the partial packet he removes. This is my proceeding, and must be yours. You then ask some person to say into how many piles (less than thirteen) the red cards shall be dealt. When he has prescribed the number of piles, you are to hold the packet of red cards back up, and deal cards one by one from the back of it, placing each card on the table face up, and each to the right of the last card dealt. When you have dealt out enough to form the bottom cards of piles to the number commanded, you return to the extreme left-hand pile, *which you are to imagine as lying next to, and to the right of, the extreme right-hand pile* -- as in fact it would come next in clockwise order, if the row were bent down at the ends in the manner

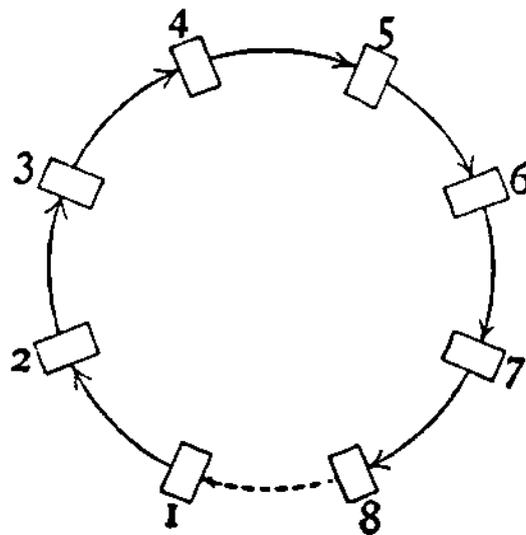


Fig. 224

shown in Fig. 224

[Click here to view],

where the piles (here supposed to be eight in all) are numbered in the order in which their bottom cards are laid down. Indeed, when more than seven piles are ordered, it is not a bad plan actually to arrange them so. So, counting the piles round and round, whether you place them in a circle or not, you place each card on the pile that comes clockwise next after, or to the right of the pile upon which the card next before it was placed (regulating your imagination as above stated), and so you continue until you have dealt out the whole packet of thirteen cards. You now proceed to gather up the piles according to the Fourth Standing Rule.

Peirce: CP 4.588 Cross-Ref:††

588. That rule, however, does not determine the order of succession in which the piles are to be taken up. I will now give the rule for this. It applies to the dealing

of any prime number of cards, or of any number of cards one less than a prime number, into any number of piles less than that prime number. It happens that that form of statement of this rule which is decidedly the most convenient when the number of piles does not exceed seven, as well as when the whole number of cards differs by less than three from some multiple of the number of piles, becomes quite confusing in other cases. A slight modification of it which I will give as a second form of the rule, sometimes greatly mitigates the inconvenience; and it will be well to acquaint yourself with it. But for the most part, when the first form threatens to be confusing, it will be best to resort to that form of the rule which I describe as the third.

Peirce: CP 4.588 Cross-Ref:††

For the purpose of this "first curiosity" (indeed, in every case where a prime number of real cards are dealt out), it matters not what pile you take up first. But in certain cases we shall have occasion to deal out into piles a number of cards, such as 52, which is one less than a prime number. In such case, it will be necessary to add *an imaginary card to the pack*, since a real card would interfere with certain operations. Now imaginary cards, if allowed to get mixed in with real ones, are liable to get lost. Consequently, in such cases, we have to keep the imaginary card constantly at the face of the pack by taking up first the pile on which it is imagined to fall, that is, the pile next to the right of the one on which the last real card falls. I now proceed to state, in its three forms, the rule for determining what pile is to be taken up next after any given pile that has just been taken. It is assumed that the whole pack of cards dealt consists of a prime number of cards; but, of these cards, the last may be an imaginary one, provided the pile on which it is imagined regularly to fall be taken up first.

Peirce: CP 4.588 Cross-Ref:††

First Form of the Rule. Count from the place of the extreme right-hand pile, as zero, either way round, clockwise or counter-clockwise -- preferably in the shortest way -- to the place of the pile on which the last card, real or imaginary, fell. Then, counting the original places of piles, whether the piles themselves still remain in those places or have already been picked up, from the place of the pile last taken, in the same direction, up to the same number, you will reach the place of the next pile to be taken.

Fig. 225

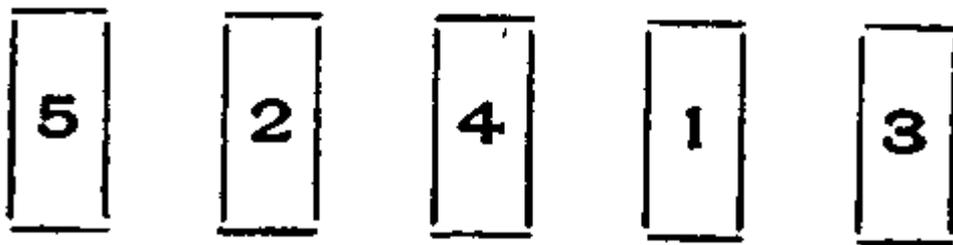


Fig. 225

[Click here to view]

Peirce: CP 4.588 Cross-Ref:††

Example. If 13 cards are dealt into five piles, the thirteenth card will fall on the second pile from the extreme right-hand pile going round counter-clockwise. Supposing, then, that the first pile taken is the right-handmost but one, they are all to be taken in the order marked in Fig. 225.

Peirce: CP 4.588 Cross-Ref:††

Second Form. Proceed as in the first form of the rule until you have repassed the place of the first pile taken. You will then always find that the place of the last pile taken is nearer to that of some pile, P, previously taken, than it is to the place of that taken immediately before it. Then, the next pile to be taken will be in the same relation of places to the pile taken next after the pile P.

Peirce: CP 4.588 Cross-Ref:††

Example. Let 13 cards be dealt into 9 piles. Then the last card will fall on the pile removed 4 places clockwise from the extreme right-hand pile. Then, when you have removed four piles according to the first form of the rule, you will at once perceive, as shown in Fig. 226 (where it is assumed that

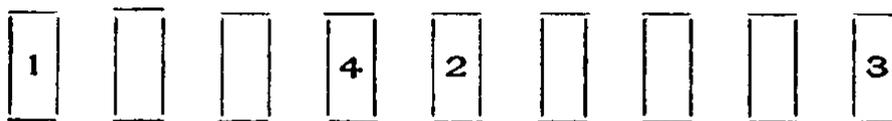


Fig. 226

Fig. 226

[Click here to view]

the extreme left-hand pile was the one to be taken up first), that for the rest of the regathering, you have simply to take the pile that stands immediately to the left of the place of the last previous removal but one.

Peirce: CP 4.588 Cross-Ref:††

Third Form. In this form of the rule vacant places are not counted, but only the remaining piles, which is sometimes much less confusing. It is requisite, however, carefully to note the place of the pile first taken. You begin as in the first form of the rule; but every time you pass over the place whence the first pile was removed, you diminish the number of your count by one, beginning with the count then in progress; and you adhere to this number until you pass the same place again, and consequently again diminish the number of your count, which will thus ultimately be reduced to one, when you will take every pile you come to.

Peirce: CP 4.588 Cross-Ref:††

Example. Let a pack of 52 cards be dealt into 22 piles. The first pile taken up must be the one upon which the imaginary fifty-third card falls. It is assumed that, before the deal the cards were arranged in suits in the order $\spadesuit \heartsuit \clubsuit \diamondsuit$ and in each suit in the order of their face-values. Then the different columns of Fig. 227 show the cards at the tops of the different piles while the different horizontal rows show what piles remain, just before you come to count the left-handmost of the remaining piles,

as your countings successively pass through the whole row of piles. The gap between the columns, just after the place where the imaginary card is supposed to have fallen, contains the direction thereafter to diminish by one the number of piles you count. Beneath the designations of the top cards are small type numbers which are the numbers in your different countings through the row of piles; and the last number in each count is followed by a note of admiration that is to be understood as a command to gather up that pile. Beneath it is a heavy faced number, which is the ordinal number of that removal.

Fig. 227

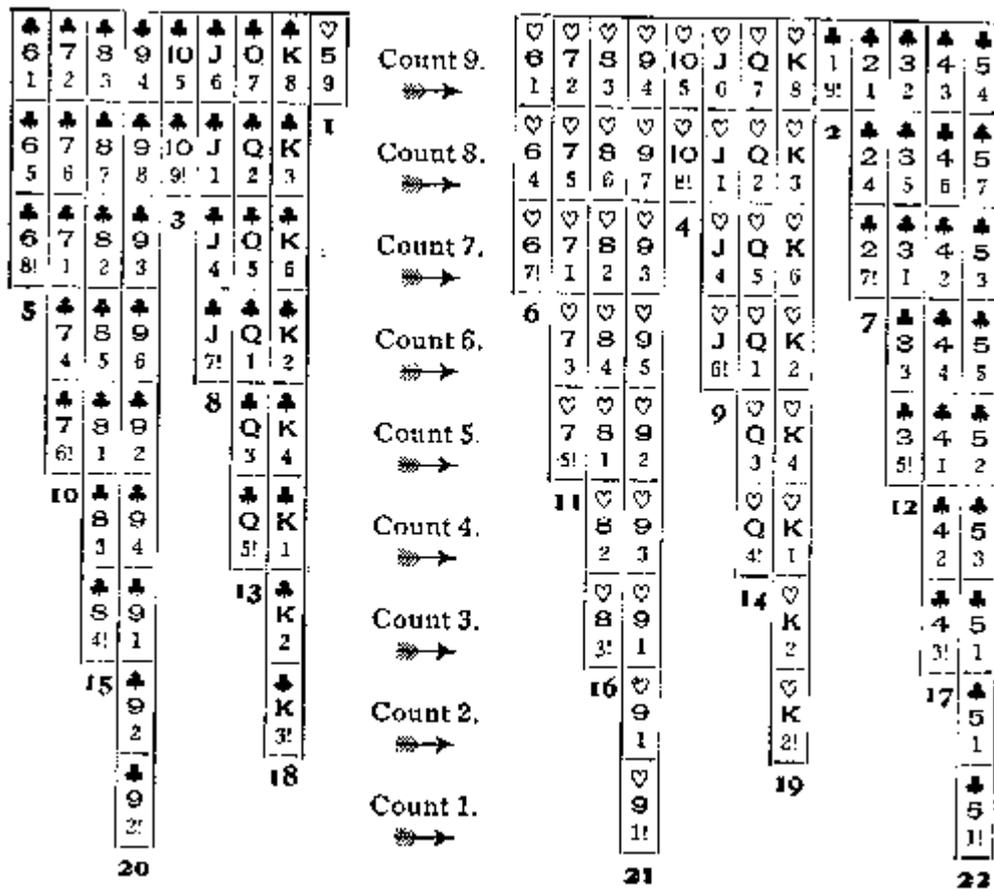


Fig. 227

[Click here to view]

Peirce: CP 4.589 Cross-Ref:††

589. I hate to bore readers who are capable of exact thought with redundancies; but others often deploy such brilliant talents in not understanding the plainest statements that have no familiar jingle, that I must beg my more active-minded readers to have patience under the infliction while I exhibit in Fig. 228 the orders in which 5, 8, 9, 10, and 11 piles formed by dealing 13 cards are to be taken up.

Peirce: CP 4.590 Cross-Ref:††

590. When the red cards have thus been regathered, you again hold out the packet to somebody to cut, and again request somebody to say into how many piles

they shall be dealt "in order that the mixing may be as thorough as it may." You follow his directions, and regather the piles according to the same rule as before. If your company is not too intelligent, you might venture to ask somebody, before you regather the piles, to say what pile you shall take up first; but this will be presuming a good deal upon the stupidity of the company; for an inference might be drawn which would go far toward destroying the surprise of the result. Nothing absolutely prevents the cards from being cut and dealt any number of times.

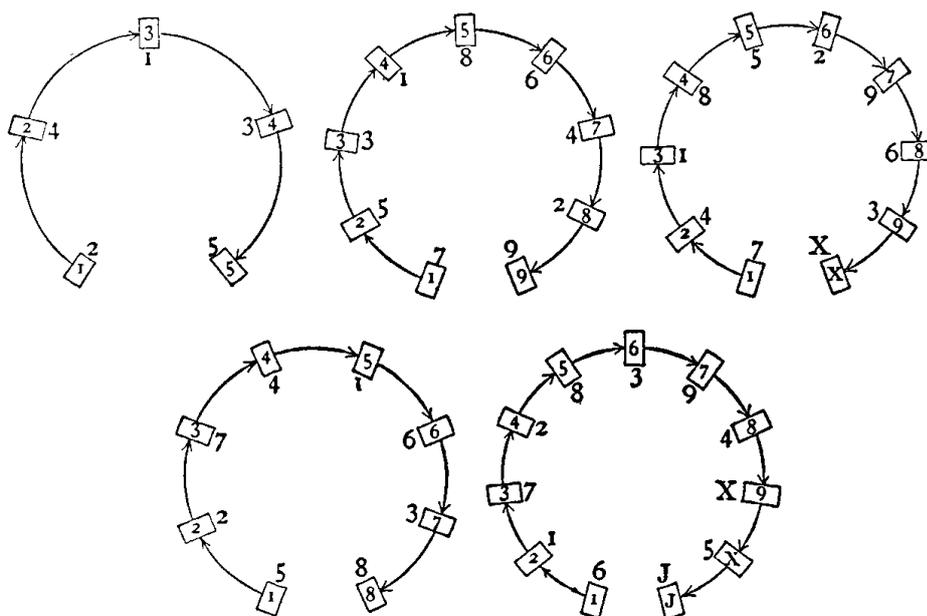


Fig. 228

Fig. 228

[Click here to view]

Peirce: CP 4.591 Cross-Ref:††

591. When the number of piles for the last dealing has been given out, you will have to ascertain what transposition of the black cards is required. There are three alternative ways of doing this, which I proceed to describe. The best way is to multiply together the numbers of piles of the different dealings of the red cards, subtracting from each product the highest multiple of 13, if there be any, that is less than that product. The result is the cyclical product. By "the different dealings," you here naturally understand those that have taken place since the last shifting of the black row. If a wrong shift has been made, the simplest way to correct it, after new cuttings and dealings, is to resort to a peep at the black ace, and to determining where it ought to be in the third way explained below.

Peirce: CP 4.591 Cross-Ref:††

Thus, if the red cards have been dealt into 5 piles and into 3 piles, since 3 times 5 make 15, and 15-13 = 2, the cyclical product is 2. You now proceed to ascertain how many times 1 has to be cyclically doubled to make that cyclical product. But if 6 doublings do not give it -- which six doublings will give

- 1 doubling, twice 1 are 2,
- 2 doublings, twice 2 are 4,

3 doublings, twice 4 are 8,
 4 doublings, twice 8 less 13 make 3,
 5 doublings, twice 3 are 6,
 6 doublings, twice 6 are Q,--

Peirce: CP 4.591 Cross-Ref:††

I say if none of the first six doublings gives the cyclical product of the numbers of piles in the dealings, you resort to successive cyclical halvings of 1. The cyclical half of an even number is the simple half; but to get the cyclical half of an odd number, add 7 to half of one less than that number. Thus,

The cyclical half of 1 is $(0/2)+7 = 7$;

The cyclical half of 7 is $(6/2)+7 = X$;

The cyclical half of X is 5;

The cyclical half of 5 is $(4/2)+7 = 9$;

The cyclical half of 9 is $(8/2)+7 = J$;

The cyclical half of J is $(X/2)+7 = Q$.

Peirce: CP 4.591 Cross-Ref:††

If the cyclical product of the numbers of piles in the dealings is one of the first six results of doubling one, you will have (when the time comes) to bring one card from the right-hand end of the row of black cards to the left-hand end for each such doubling. Thus, if the red cards have twice been deal

into 4 piles, four cards must be brought from the right end to the left end of the row of black cards. For $4 \times 4 - 13 = 3$ and $1 \times 2^4 - 13 = 3$. But if that cyclical product is one of the first six results of successive cyclical halvings of one, one card must be carried from the left to the right end of the row of black cards for every halving. Thus, if the red cards have been dealt into 6 and into 8 piles, 4 black cards must be carried from the left-hand end of the row to the right-hand end of the row. $6 \times 8 - 3 \times 13 = 9$ and it takes 4 cyclical halvings to give 9. If the product of the numbers of piles in the dealings is one more than a multiple of 13, the row of black cards is to remain unshifted.

Peirce: CP 4.591 Cross-Ref:††

The second way of determining how the black cards are to be transposed is simply, during the last of the dealings, to note what card is laid upon the king. The face-value of this card is the ordinal, or serial place in the row, counting from the left-hand extremity of it, which the ace must be brought to occupy. Now if you remember, as you always ought to do, where the ace is in the row, you will know how many cards to carry from one end to the other so as to bring the ace into that place. But if in the last dealing the king happens to fall at the top of one of the piles, two lines of conduct are open to you. One would be, in regathering the piles, by a pretended awkwardness in taking up the pile that is to be taken next before the one that the king heads, at first to leave its bottom card on the table, so as to get a glimpse of it before you take it up, as you would regularly have done at first; and if the king should happen to be the last card dealt, the card at the back of the packet would be the

one for you to get sight of, by a similar imitation blunder. In either case, the card you so aim to get sight of would show the right place for the ace in the row. But if you doubt your ability to be gracefully awkward, it always remains open to you to ask to have the red packet cut again and a number of piles for a new deal to be ordered.

Peirce: CP 4.591 Cross-Ref:††

The third way of determining the proper transposition of the black cards is a slight modification of the second. It consists in looking at the card whose back is against the face of the king, when you come to cut the red packet so as to bring the king to the face. (Any practical psychologist, such as a prestidigitator must be, can, with the utmost ease, look for the card he wants to see, and can inspect it without detection.)

Peirce: CP 4.591 Cross-Ref:††

But whichever of these methods you employ, you should not touch the row of black cards until the red cards having been regathered after the last dealing, you have said something like this: "Now I think that all these dealings and cuttings and exchanges of the last cards have sufficiently mixed up the red cards to give a certain interest to the fact that I am going to show you; namely, that this row of black cards forms an index showing where any red card you would like to see is to be found in the red pack. But since there is no black king in the row, of course the place of the red king cannot be indicated; and for that reason, I shall just cut the pack of red cards so as to bring the king to the face of it, and so render any searching for that card needless." You then cut the red cards. That speech is quite important as restraining the minds of the company from reflecting upon the relation between the effect of your cutting and that of theirs. Without much pause you go on to say that you shall leave the row of black cards just as they are, simply putting so many of them from one end of the row to the other. You now ask some one, "Now, what red card would you like to find?" On his naming the face-value of a card, you begin at the left-hand end of the row of black cards and count them aloud and deliberately, pointing to each one as you count it, until you come to the ordinal number which equals the face-value of the red card called for; and in case that card is the knave or queen, you call "knave" instead of "eleven" on pointing at the eleventh card, and "queen" on pointing at the last card. When you come to call the number that equals that of the red card called for, you turn the card you are pointing at face up. Suppose it is the six, for example. Then you say, naming the card called for, that that card will be the sixth; or if the card turned up was the knave, you say that the card called for will be "in the knave-place," and so in other cases. You then take up the red packet, and counting them out, aloud and deliberately, from one hand to the other, and from the back toward the face of the packet, when you come to the number that equals the face-value of the black card turned, you turn over this card as soon as you have counted it, and lo! it will be the card called for.

Peirce: CP 4.592 Cross-Ref:††

592. The company never fail to desire to see the thing done again; and on their expressing this wish, after impressing on your memory the present place of the black ace, you have only to hold out the red cards to be cut again, and you again go through the rest of the performance, now abbreviating it by having the cards dealt only once. The third time you do it, since you will now have given them the enjoyment of their little astonishment, there will no longer be any reason for not asking somebody to say what pile you shall take up first, although that will soon lead to their seeing that all the cuttings are entirely nugatory. Still they will not thoroughly understand the phenomenon.

Peirce: CP 4.593 Cross-Ref:††

593. If you wish for an explanation of it, the wish shows that you are not thoroughly grounded in cyclic arithmetic, and that you consequently still have before you the delight of assimilating the first three *Abschnitte* (for that matter the first hundred pages would suffice to reveal the foundations of the present mystery; but I confess I do not particularly admire the first *Abschnitt*) of Dedekind's lucid and elegant redaction of the unerring Lejeune-Dirichlet's "*Vorlesungen über Zahlentheorie*." But, perhaps, on another occasion †1 I will myself give a little essay on the subject, "adapted to the meanest capacity," as some of the books of my boyhood used, not too respectfully, to express it.

Peirce: CP 4.594 Cross-Ref:††

§2. EXPLANATION OF CURIOSITY THE FIRST †2

594. You remember that at the end of my description of the card "trick" that made my first curiosity, I half promised to give, some time, an explanation of its rationale. This half promise I proceed to half redeem.

Peirce: CP 4.594 Cross-Ref:††

Suppose a prime number, P , of cards to be dealt into S (for *strues*) piles, where $S < P$. (Were $S = P$, it would be impossible to regather the cards, according to the rule given in the description of the "trick.") Then, in each pile, every card that lies directly on another occupied, before the deal, the ordinal, or serial, place in the packet whose number was S more than that of the other; and using Q to denote the integral part of the quotient of the division of P by S , so that $P - QS$ is positive, while $P - (Q+1)S$ is negative (for P being prime, neither can be zero), and assuming that the piles lie in a horizontal row, and that each card is dealt out upon the pile that is next on the right of the pile on which the last preceding card was dealt, it follows that the left-hand piles, to the number of $P - QS$ of them, contain each $Q+1$ cards, while the $(Q+1)S - P$ piles to the right contain each only Q cards. It is plain, then, that, in each pile, every card above the bottom one is the one that before the dealing stood S places further from the back of the packet than did the card upon which it is placed in dealing. But in what ordinal place in the packet before the dealing did that card stand which after the regathering of the piles comes next in order after the card which just before the regathering of the piles lay at the top of any pile whose ordinal place in the row of piles, counting from the left, may be called the *sth*? In order to answer this question, we have first to consider that the effect of Standing Rule No. IV is that the pile that comes next after any given pile in the order of the regathered packet, counting, as we always do, from back to face, is the pile which was taken up *next before* that given pile; and of course it is the bottom card of that pile to which our question refers. Now the rule of regathering is that, after taking up any pile we next take up, either the pile that lies $P - QS$ places to the right of it, or else that which lies $(Q+1)S - P$ places to the left of it. In other words, the pile that is taken up *next before* any pile, numbered s from the left of the row, is either the pile numbered $s + QS - P$ (and so lies toward the *left* of pile s) or else is the pile numbered $s + (Q+1)S - P$ (and so lies toward the *right* of pile s). But if pile number s were one of those which contain $Q+1$ cards each, since these are the first $P - QS$ piles, we should have $s \geq P - QS$, and the pile taken next before it, if it were to the left of it, would be numbered less than or equal to zero; and there is

no such pile. Consequently in that case, that pile taken up next before pile s will be to the right of the pile numbered s , and its number will be $s+(Q+1)S-P$, which will also have been the number of its bottom card in the packet before the dealing; while, since the bottom card of pile number s was card number s before the dealing, and since this pile contains Q other cards, each originally having occupied a place S further on than the one next below it in the pile, it follows that its top card was, before the dealing, the card whose ordinal number was $s+QS$. Thus, while every other card of any of the first $P-QS$ piles is followed after the regathering by a card whose original place was numbered S more than its own, the top card of such a pile will then be followed by a card whose original place was S more than its own, *counting round a cycle of P cards*. In a similar way, if pile number s contains only Q cards, it is one of the last $(Q+1)S-P$ piles. Then it cannot be that the pile taken up, according to the rule, next before it lay to the right of it; for in that case the number of this previously taken pile would exceed S . It must therefore be pile number $s+QS-P$; and this will be the original number of its bottom card, while the original number of the top card of pile number s (since this contains only Q cards), will be $s+(Q-1)S$. Hence, as before, the top card will be followed after the regathering by a card whose original place would be S greater than its own, but for the subtraction of P in counting round a cycle of P numbers. This rule then holds for all the cards.

Peirce: CP 4.594 Cross-Ref:††

It follows that if, after the regathering, the last card, that at the face of the pack or in the P place, is the one whose original place may be called the π th, then any other card, as that whose place after the gathering is the l th, was originally in the $\pi+lS-mP$, where mP is the largest multiple of P that is less than $\pi+lS$. If, however, after the regathering, the pack be cut so as to bring the card which was originally the P th, or last, that is, which was at the face of the pack, back to that same situation, then, since the original places increase by S (round and round a cycle of P places) every time the regathered places increase by 1, it follows that the original place of the card that is first subsequently to that cutting will have been S , that of the second, $2S$, etc.; and in general, that of the l th will have been $lS-mP$. If the cards had originally been arranged in the order of their face-values, the face-value of the card in the l th place after the cut will be $lS-mP$, which we may briefly express by saying that the dealing into S piles with the subsequent cutting that brings the face card back to its place, "cyclically multiplies the face-value of each card by S ," the cycle being P . If after dealing into S piles, another dealing is made into T piles, and another into U piles, etc., after which a cut brings the face card back to its place, the face-value of every card will be cyclically multiplied by $SXTXUX$ etc. Moreover, if cuttings were made before each of the dealings, since each cutting only cyclically adds the same number to the place of every card, the cards will still follow after one another according to the same rule; so that the final cutting that restores the face card to its place, annuls the effect of all those previous cuttings.

Peirce: CP 4.595 Cross-Ref:††

595. My hints as to the rationale of the exceptional treatment of the last card in twelve initial deals, and as to the extraordinary relation which results between the orders of succession of the black and of the red cards must be prefaced by some observations on the effects of reiterated dealings into a constant number of piles. What I shall say will apply to a pack of any prime number of cards greater than two; but to convey more definite ideas I shall refer particularly to a suit of 13 cards, each at the outset having its ordinal number in the packet equal to its face-value. The effect of one cyclic multiplication of the face-values by 2, brought about by dealing the suit

into 2 piles, regathering, and cutting, if need be, so as to restore the king to the face of the packet, will be to shift all the cards except the king in one circuit. That is, the order before and after the cyclic multiplication being as here shown.

Peirce: CP 4.595 Cross-Ref:††

Before the cyclic doubling of

the face-values.....1, 2, 3, 4, 5, 6, 7, 8, 9, X, J, Q, K, After the same ...2, 4, 6, 8, X, Q, 1, 3, 5, 7, 9, J, K, the 2 takes the place of the 1, the 4 that of the 2, the 8 that of the 4, the 3 that of the 8, the 6 that of the 3, the Q that of the 6, the J that of the Q, the 9 that of the J, the 5 that of the 9, the X that of the 5, the 7 that of the X, and the 1 that of the 7; so that the values are shifted as shown by the arrows on the circumference of the circle of Fig. 229. If 7, instead of 2, be the number of piles into which the thirteen cards are dealt there will be a similar shift round the same circuit, but in the direction opposite to the pointings of the arrows; and if the cards are dealt into 6 or into 11 piles, there will be a shift in a similar single circuit along the sides of the inscribed stellated polygon. But if the 13 cards are dealt into a number of piles other than 2, 6, 7, or 11, the single circuit will break into 2, 3, 4, or 6 separate circuits of shifting. Thus, if the dealing be into 4 or into 10 piles, there will be two such circuits, each along the sides of a hexagon whose vertices are at alternate points along the circumference of the circle in the same figure (or, what comes to the same thing, at alternate vertices, along the periphery of the stellated polygon). Dealing into 4 piles makes one round from 1 to 4, from 4 to 3, from 3 to Q, from Q to 9, from 9 to X, and from X back to 1; while another round is from 2 to 8, from 8 to 6, from 6 to J, from J to 5, from 5 to 7, and from 7 back to 2. Dealing into 5 or into 8 piles will make three circuits each from one vertex to the next one of 3 squares inscribed in the circle. Dealing into 3 or into 9 piles will give 4 circuits round three inscribed equilateral triangles. Finally, dealing into 12 piles, with regathering, etc., according to rule, simply reverses the

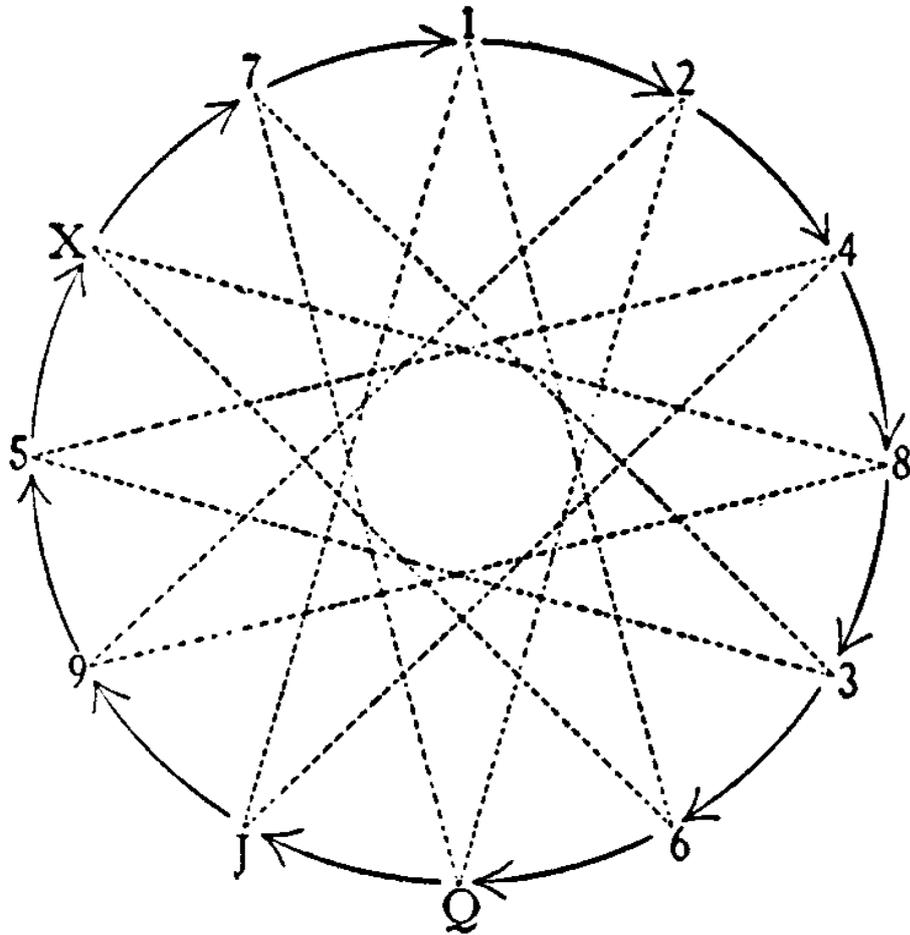


Fig. 229

Fig. 229

[Click here to view]

order so that the ace and queen, the 2 and knave, the 3 and ten, etc., change places.

Peirce: CP 4.595 Cross-Ref:††

It has already been made evident that if any prime number, P , of cards, each inscribed with a number, so that, when operations begin this number shall be equal to the ordinal place of the card in the pack, be dealt into any lesser number, S , of piles, and these be regathered, etc., according to rule, the effect is cyclically to multiply by S the number inscribed on any card which is identified solely by its resulting ordinal place, that is, to multiply in counting the numbers round and round a cycle of P numbers -- or, to state it otherwise, the ordinary product has the highest lesser multiple of P subtracted from it, though this seems to me to be a needlessly complicated form of conceiving the cyclical product. In counting round and round the number of numbers in the cycle, the so-called "modulus of the cycle" is the same as zero; so that the product of its multiplication by S is zero; or, regarding the matter in the other way, SP diminished by the largest lesser multiple of P gives P . Consequently, the face card will not change its face-value. Let the dealing, etc., be reiterated until it has been performed δ times. The effect will be to multiply the

face-values (of cards identified only by their final ordinal places) by S^δ . Since this is the same multiplier for all the cards, it follows that when δ attains such a value that the card in any one place, with the exception of the face card of the pack, *which alone retains an unchanging value*, recovers its original value, every one of the P-1 cards of (apparently) changing values equally recovers its original value; and if the values do not shift round a single circuit of P-1 cards, all the circuits must be equal; for otherwise the single number S^δ would not fix the values of all the cards. And since zero, or P, is the only number that remains unchanged by a multiplication where the multiplier is not unity (and S is always cyclically greater, that is, more advanced clockwise, than 1 and less than P), it follows that the moduli of the shifts must all be the same divisor of P-1, and consequently P-1 deals, whatever be the constant number of piles, must restore the original order. The pure arithmetical statement of this result is that S^{P-1} , whenever P is a prime number and S not a multiple of it, must exceed by one some multiple of P. This proposition goes by the name of its discoverer, perhaps the most penetrating mind in the history of mathematics, being known as "Fermat's theorem"; although from our present point of view, it may seem too obvious to be entitled to rank as a "theorem." The books give half a dozen demonstrations of it. It lies at the root of cyclic arithmetic.

Peirce: CP 4.596 Cross-Ref:††

596. Fermat said he possessed a demonstration of his theorem; and there is every reason for believing him; but he did not publish any proof.†1 About 1750, the mathematician König asserted that he held an autograph manuscript of Leibniz containing a proof of the proposition, but it has never been published, so far as I know.†1 Euler,†2 at any rate, first published a proof of it; and Lambert †3 gave a similar one in 1769. Subsequently Euler †4 gave a proof less encumbered with irrelevant considerations; and this second proof is substantially the same as that in Gauss's celebrated *Disquisitiones Arithmeticae*. of 1801, §49. Several other simple proofs have since been given; but none, I think, better than that derived from the consideration of repeated deals.

Peirce: CP 4.597 Cross-Ref:††

597. But what concerns the curious phenomenon of my little "trick" is not so much Fermat's theorem as it is the more comprehensive fact that, whatever odd prime number, P, the number of cards in the pack may be, there is some number, S, such that in repeated deals into that number of piles, all the numbers less than P shift round a single circuit. I hope and trust, Reader, that you will not take my word for this. If fifty years spent chiefly with books makes my counsel about reading of any value, I would submit for your approbation the following maxims:

Peirce: CP 4.597 Cross-Ref:††

I. There are more books that are really worth reading than you will ever be able to read. Confine yourself, therefore, to books worth reading and re-reading; and as far as you can, own the good books that are valuable to you.

Peirce: CP 4.597 Cross-Ref:††

II. Always read every book critically. A book may have three kinds of value. First, it may enrich your ideas with the mere possibilities, the mere ideas, that it suggests. Secondly, it may inform you of facts. Thirdly, it may submit, for your approbation, lines of thought and evidences of the reasonable connection of possibilities and facts. Consider carefully the attractiveness of the ideas, the credibility of the assertions, and the strengths of the arguments, and set down your

well-matured objections in the margins of your own books.

Peirce: CP 4.597 Cross-Ref:††

III. Moreover, procure, in lots of twenty thousand or more, slips of stiff paper of the size of postcards, made up into pads of fifty or so. Have a pad always about you, and note upon one of them anything worthy of note, the subject being stated at the top and reference being made below to available books or to your own note books. If your mind is active, a day will seldom pass when you do not find a dozen items worth such recording; and at the end of twenty years, the slips having been classified and arranged and rearranged, from time to time, you will find yourself in possession of an encyclopaedia adapted to your own special wants. It is especially the small points that are thus to be noted; for the large ideas you will carry in your head.

Peirce: CP 4.598 Cross-Ref:††

598. If you are the sort of person to whom anything like this recommends itself, you will want to know what evidence there is of the truth of what I assert, that there is some number of piles into which any prime number of cards must be dealt out one less than that prime number of times before they return to their original order.

Peirce: CP 4.598 Cross-Ref:††

If these maxims meet your approval, and you read this screed at all, you will certainly desire to see my proposition proved. At any rate, I shall assume that such is your desire. Very well; proofs can be found in all the books on the subject from the date of Gauss's immortal work down. But all those proofs appear to me to be needlessly involved, and I shall endeavor to proceed in a more straightforward way, which "mehr rechnend zu Werke geht." Indeed, I think I shall render the matter more comprehensible by first examining a few special cases. But at the outset let us state distinctly what it is that is to be proved. It is that if P is any prime number greater than 2, then there must be some number of piles, S , into which a pack of P cards must be dealt (and regathered and cut, according to the rule) $P-1$ times in order to bring them all round to their original places again. The reason I limit the proposition to primes will presently appear: the reason I limit the primes to those that are greater than 2 is that two cards cannot, in accordance with the rule, be dealt, etc., into more than one pile (if you call that dealing); and of course this does not alter the arrangement; and since there is no number of piles less than one, the theorem, in this case, reduces itself to an identical proposition; while if 1 be considered to be a prime number, the proposition is falsified since there is no number of piles into which one card can be dealt and regathered according to the rule, which requires S to be less than P .

Peirce: CP 4.598 Cross-Ref:††

Let our first example be that of $P = 17$. Then $P-1 = 16$; and unless there be a single circuit of 16 face-values, which my whole present object is to show that there must be, all the circuits must either be one or more sets of 8 circuits of 2 values each, or sets of 4 circuits of 4 values each, or sets of 2 circuits of 8 values each; unless, indeed, we count in, as we ought to do, the case of 16 circuits of 1 value each. This last means that each of the 16 cards retains its face-value after a single deal. It is obtrusively obvious that this can only be when $S = 1$. But since in these hints toward a demonstration of the proposition the particular values of S do not concern us, and had better be dismissed from our minds, we will denote this value of S by S^{xvi} , meaning that it is a value that gives 16 circuits. We will not ask what is the number of piles into which 2 dealings will restore the face-value of every card; or, in other words, will give 8 circuits of 2 values each. Letting x denote that unknown quantity, the number

of piles, or the cyclic multiplier, the equation to determine it is $x^2 = 1$. To many readers two values satisfying this equation will be apparent. But I do not care what they are, further than that the value $x = 1$ obviously satisfies the equation $x^2 = 1$. I do care, however, to show that there can be but two solutions of the equation $x^2 = 1$. For suppose that $x^2 = 1$ and $x^2 = 1$. Then $[x[1]^2]-x^2 = (x[1]+x[2])\cdot(x[1]-x[2]) = 0$ or equals mP . Now if a multiple of a prime number be separated into two or more factors, one of these, at least, must itself be a multiple of that prime, just as in the algebra of real and of imaginary quantities and in quaternions, if the product of several quantities be zero, one or other of those factors must be zero; and just as in logic, if an assertion consisting of a number of asserted items be false, one or more of these items must be false. In addition, every summand has its own independent effect; but every unit of a product is compounded of units of all the several factors. This is the formal, or purely intellectual, principle at the root of all the reasons for making the number of cards dealt, especially in reiterated dealings, to be a prime. It follows, then, that there are but two numbers of piles, dealings into each of which will restore the original arrangement after 2 deals; and one of these is $x = 1$; for evidently (bear this in mind), if $xa = 1$, then also $xab = (xa)b = 1$. There is then but one number of piles, dealings into which shift the values of the cards in eight, and only eight, circuits; and this number we will denote by S^{viii} . Then, reserving x to denote any root of the equation $x^2 = 1$, and taking $\{x\}$ to denote that one of the two roots that is not 1, we will take y to denote any number of piles, after dealing into which 4 times, the resulting arrangement of the values will be the original arrangement. That is to say, y will be any root of the cyclic equation $y^4 = 1$. But $x^4 = (x^2)^2 = 1^2 = 1$; so that any value of x is a value of y . Let $\{é\}$ denote any value of y that is not a value of x ; and let us suppose that there are two values of $\{é\}$, which we may denote by S^{iv} and S^{xii} . It will be easy to show that there is no third value of $\{é\}$. For $(\{é\}^2)^2 = 1$, where $\{é\}^2$ fulfills the definition of x and is thus either 1 or $\{x\}$. But the roots of the equation $\{é\}^2 = 1$ fulfill the definition of x , whose values are excluded from the definition of $\{é\}$. Hence we can only have $\{é\}^2 = \{x\}$; and that this has but two roots is proved by the same argument as was used above. Namely, $\{é\}[1]$, and $\{é\}[2]$ being any two of these, $(\{é\}[1]^2 - \{é\}[2]^2) = (\{é\}[1] + \{é\}[2]) \cdot (\{é\}[1] - \{é\}[2]) = 0$, so that unless $\{é\}[1]$ and $\{é\}[2]$ are equal, and $\{é\}[1] - \{é\}[2] = 0$, then $\{é\}[1] + \{é\}[2] = 0$, or $\{é\}[1]$ and $\{é\}[2]$ are negatives of each other. Now no more than 2 quantities can be each the negative of each of the others. We now pass to the consideration of those numbers of piles into which eight successive dealings result in the original arrangement. Denoting by z any such number, it is defined by the equation $z^8 = 1$. But every value of y (of which we have seen that there cannot be more than 4), satisfies this equation, since $y^8 = (y^4)^2 = 1^2 = 1$. Let $\{z\}$ denote any value of z which is not a value of y . We may suppose that there are two of these for each of the two values of $\{é\}$, which we will designate as S^{ii} , S^{vi} , S^x , S^{xiv} . I need not assert that there are so many; but my argument requires me to prove that there are no more. The equation $(z^2)^4 = z^8 = 1$ shows that z^2 fulfills the definition of y and can therefore have no more than the four values, 1, $\{x\}$, and the two values of $\{é\}$. Now if $z^2 = 1$, z can, as we have seen in the case of x , have no other values than $z = 1$ and $z = \{é\}$, both of which are values of y .

Peirce: CP 4.598 Cross-Ref:††

If $z^2 = \{x\}$, as we have seen in regard to y , z can have no other values than the two values of $\{é\}$, which are again values of y . Now let us suppose that z has four values, S^{ii} , S^{vi} , S^x , and S^{xiv} , that are not values of y ; and let us define $\{z\}$ as any value of z that is not a y . The proof that there can be no more than four $\{z\}$'s is so

exactly like the foregoing as to be hardly worth giving. I will relegate it to a paragraph of its own that shall be both eusceptic and euskiptatic -- "what horrors!" I hear from the mouths of those moderns who abominate all manufactures of Hellenic raw materials, like "skip" and "skimp."

Peirce: CP 4.598 Cross-Ref:††

We have seen that either $z^2 = 1$, or $z^2 = \{x\}$, or $z^2 = \{é\}$; and also that, in the first case, either $z = 1$ or $z = \{x\}$, both of which are values of y ; and that, in the second case, z has one or other of the two values of $\{é\}$. Accordingly, it only remains that $\{z\}^2 = \{é\}$, There are but two values of $\{é\}$ and if $\{z\}[1]$ and $\{z\}[2]$ are two different values of $\{z\}$ whose squares are the same value of $\{é\}$, $\{z\}[1]^2 - \{z\}[2]^2 = (\{z\}[1] + \{z\}[2]) \cdot (\{z\}[1] - \{z\}[2]) = 0$. Hence, since $\{z\}[1] - \{z\}[2]$ is not zero, it follows that every value of $\{z\}$ differs from every other value derived from the same $\{é\}$ only by being the negative of it. Now no number has two different negatives; and therefore there can be no more than two $\{z\}$ s to every $\{é\}$; and there being no more than two $\{é\}$ s, there can be no more than four $\{z\}$ s.

Peirce: CP 4.598 Cross-Ref:††

Now this is the summary of the whole argument: the 17 cards of the pack being consecutively inscribed with numbers from the back to the face of the pack, each number of piles into which they are dealt etc. according to the rule acts as a cyclic multiplier of the face-value of every card. Every such multiplier leaves 0(=17) unchanged, and shifts the other 16 face-values in a number of circuits having the same number of values in each. The possible consequences, excluding the case of a single circuit of 16 values, are the following:

- 16 circuits of 1 value each can result from but 1 multiplier at the utmost
- 8 circuits of 2 values each can result but from 1 other multiplier
- 4 circuits of 4 values each can result but from 2 other multipliers
- 2 circuits of 8 values each can result but from 4 other multipliers

—

In all, the number of multipliers that give more than 1 circuit (of all 16 values) is.....8 at most

But there are in all.....16 multipliers

—

Hence, the number of multipliers that shift the values in 1 circuit of 16 values is.....8, at least.

In point of fact, it is precisely 8.

Peirce: CP 4.599 Cross-Ref:††

599. Let us now consider a pack of 31 cards. Here, the zero card not changing its value, there are 30 values which are shifted in one of these ways:

In 30 circuits of 1 value each;

In 15 circuits of 2 values each;

In 10 circuits of 3 values each;

In 6 circuits of 5 values each;

In 5 circuits of 6 values each;

In 3 circuits of 10 values each;

In 2 circuits of 15 values each;

In 1 circuit of 30 values.

Peirce: CP 4.599 Cross-Ref:††

I propose to show as before that if we exclude the last case, the others do not account for the effects of so many as 30 different multipliers. In the first place, as in the last example, but one multiplier will give circuits of one value each; and but one other multiple will give circuits of only two values each. We may call the former S^{xxx} and the latter S^{xv} .

Peirce: CP 4.599 Cross-Ref:††

The problems of 10 circuits of 3 values each and of 6 circuits of 5 values each can be treated by exactly the same method, 3 and 5 being prime numbers. I shall exhibit in full the solution of the more complicated of the two, leaving the other to the reader.

Peirce: CP 4.599 Cross-Ref:††

I propose, then, to show that there are at most but 5 different values which satisfy an equation of the form $s^5 = 1$. The general idea of my proof will be to assume that there are 5 different values (for it is indifferent to my purpose whether there be so many or not) and then to show that there is such an equation between these five, that given any four, there is but one value that the fifth can have; that being as much as to say that there are not more than five such values in all. This assumes that every one of the five values differs from every one of the other four; making ten premisses of this kind that have to be introduced. Now to introduce a premiss into a reasoning, is to make some inference which would not necessarily follow if that premiss were not true. Assuming, then, that $s^5 = 1$, $t^5 = 1$, $u^5 = 1$, $v^5 = 1$, $w^5 = 1$, are the five assumed equations, I note that the division by one divisor of both sides of an equation necessarily yields equal quotients only if the divisor is known not to be zero. Hence if I divide my equations by $s-t$, by $s-u$, by $s-v$, by $s-w$, by $t-u$, by $t-v$, by $t-w$, by $u-v$, [by] $u-w$, and by $v-w$, I shall certainly introduce the ten premisses that all the five values are different; and with a little ingenuity -- a *very* little, as it turns out -- I ought to reach my legitimate conclusion.

Peirce: CP 4.599 Cross-Ref:††

I will begin then by subtracting $t^5 = 1$ from $s^5 = 1$, giving $s^5 - t^5 = 0$; and dividing this by $s-t$, and using $\cdot\cdot$ as the logical sign of disjunction, that is, to mean "or else," I get

$$(1) s^4 + s^3t + s^2t^2 + st^3 + t^4 = 0 \cdot\cdot s = t.$$

By analogy, I can equally write

$$s^4 + z^3u + s^2u^2 + su^3 + u^4 = 0 \cdot\cdot s = u.$$

Subtracting the latter of these from the former, I get

$$s^3(t-u)+s^2(t^2-u^2)+s(t^3-u^3)+t^4-u^4 = 0$$

$$\cdot \cdot s = t \cdot \cdot s = u$$

And dividing this by $t-u$, I obtain

$$(2) s^3+s^2(t+u)+s(t^2+t u+u^2)+t^3+t^2u+t u^2+u^3 = 0$$

$$\cdot \cdot s = t \cdot \cdot s = u \cdot \cdot t = u.$$

By analogy, I can equally write

$$s^3+s^2(t+v)+s(t^2+t v+v^2)+t^3+t^2v+t v^2+v^3 = 0$$

$$\cdot \cdot s = t \cdot \cdot s = v \cdot \cdot t = v.$$

Subtracting the last equation from the last but one, I get

$$(s^2+s t+t^2)(u-v)+(s+t)(u^2-v^2)+u^3-v^3 = 0$$

$$\cdot \cdot s = \cdot \cdot s = u \cdot \cdot s = v \cdot \cdot t = u \cdot \cdot t = v.$$

And dividing by $u-v$, I have

$$(3) s^2+s t+t^2+(s+t)(u+v)+u^2+u v+v^2 = 0$$

$$\cdot \cdot s = t \cdot \cdot s = u \cdot \cdot s = v \cdot \cdot t = u \cdot \cdot t = v \cdot \cdot u = v.$$

By analogy, I can equally write

$$s^2+s t+t^2+(s+t)(u+w)+u^2+u w+w^2 = 0$$

$$\cdot \cdot s = t \cdot \cdot s = u \cdot \cdot s = w \cdot \cdot t = u \cdot \cdot t = w \cdot \cdot u = w.$$

Subtracting the last from the last but one, and dividing by $v-w$, I get

$$(4) s+t+u+v+w = 0 \cdot \cdot s = t \cdot \cdot s = u \cdot \cdot s = v \cdot \cdot s = w \cdot \cdot t = u \cdot \cdot t = v \cdot \cdot t = w \cdot \cdot u$$

$$= v \cdot \cdot u = w \cdot \cdot v = w.$$

Peirce: CP 4.599 Cross-Ref:††

This shows at once that there cannot be more than 5 different numbers, which, counting round any prime cycle, all have their fifth powers equal to 1. By a similar process, as you can almost see without slate and pencil, from $x^3 = 1$, $y^3 = 1$, $z^3 = 1$ one can deduce $x+y+z = 0 \cdot \cdot x = y \cdot \cdot x = z \cdot \cdot y = z$. The existence of these 5 and these 3 numbers must, for the present, be regarded as problematic, except that we cannot shut our eyes to the fact that 1 is one of the members of each set; as indeed $1^\delta = 1$, whatever the exponent may be.

Peirce: CP 4.599 Cross-Ref:††

I have numbered some of the equations obtained in the proof that there are no more than 5 fifth roots of unity. You will observe that (1) equates to zero the sum of all possible terms of the fourth degree formed by two roots; that (2) equates to zero the sum of all possible terms of the third degree formed by three roots; that (3)

equates to zero the sum of all possible terms of the second degree formed from four roots; and that (4) equates to zero the sum of all possible terms of the first degree formed by all five roots. Now it is plain that if we assume that there are n unequal n th roots of unity, then by subtracting $x[2]^2 = 1$ from $x[1]^2 = 1$, and dividing by $x[1]-x[2]$, we shall equate to zero the sum of all possible terms of the $(n-1)$ th degree in $x[1]$ and $x[2]$. And if we have proved, in regard to any m of the roots, that (all being unequal) the sum of all possible terms of the $(n-m+1)$ th degree in these roots is equal to zero; then by taking two such equations of the $(n-m+1)$ th degree in $m-1$ roots common to the two, with one root in each equation not entering into the others; by subtracting one of these equations from the other, and then dividing by the difference between the two roots which enter each into but one of these equations, we shall get an equation of the $(n-m)$ th degree in $m+1$ roots. For $x^n - y^n = (x-y)$.

$n-1$

$\sum [i] x_i y^{n-i-1}$ Accordingly, by repetitions of this process, we

0

shall ultimately find that the sum of the n roots, if there be so many, is 0. This proves that there can be no more than n unequal n th roots of unity in cyclic arithmetic any more than in unlimited real or imaginary arithmetic.

Peirce: CP 4.600 Cross-Ref: ††

600. But if the root of unity be of an order not prime but composite, so that it is the root of an equation of the form $x^p q = 1$, it is evident that it is satisfied by every root of $y^p = 1$ and by every root of $y^q = 1$; since every power of 1 is 1. Accordingly, exclusive of roots of a lower order, the number of roots of unity of order n , that is, the number of roots of $x^n = 1$, additional to those that are roots of unity of lower order, cannot be greater than the number of numbers not greater than n and prime to it. A number is said to be prime to a number when they have no other common divisor than 1. I shall write the expression of two or more numbers separated by heavy vertical lines to denote the greatest common divisor of those numbers. Thus, I shall write $12 | 18 = 6$. This vertical line may be considered as a reminiscence of the line that separates numbers in the usual algorithm of the greatest common divisor. A prime number is a number prime to every other number. Consequently, 1 is a prime number. It is the only prime number that is prime to itself; for $p | p = p$. The number of numbers not exceeding a number, n , but prime to it is now called the *totient* of n . In the books of the first four-fifths of the nineteenth century, the totient of n was denoted by $\phi(n)$; but since the invention of the word totient †1 about 1880, Tn has become the preferable notation. $T1 = 1$; but if p be a prime not prime to itself, $Tp = p-1$. It is quite obvious that the totient of any number, n , whose prime factors, not prime to themselves, are p', p'', p''' , etc., is obtained by subtracting from n the p' th part of it, and then successively from each remainder the p'' th, etc., part of it, but not using any prime factor twice. Thus $T4 = 2$ (for $4 | 1 = 1$ and $4 | 3 = 1$; but $4 | 2 = 2$ and $4 | 4 = 4$); $T6 = 2$ (for $6-1/2 \cdot 6 = 3$ and $3-1/3 \cdot 3 = 2$); $T8 = 4$ (for $8-1/2 \cdot 8 = 4$), $T9 = 6$, $T10 = 4$, etc. If $m | n = 1$, then $Tmn = (Tm)(Tn)$. On the other hand, if p is a prime and m any exponent, $Tpn = (p-1)pn-1$. A "perfect number" is defined as one which is equal to the sum of its "aliquot parts," that is, of all its divisors except itself; but, in a more philosophical sense, *every* number is a perfect number. That is to say, it is equal to the sum of the totients of *all* its divisors; a proposition which is perfectly obvious if

regarded from the proper point of view. However, since this proposition has some relevancy to the proposition I am endeavoring to prove; namely, that there is some number of piles, dealing into which shifts all the face-values of the cards along a single cycle, I will repeat a pretty demonstration of the former proposition that I find in the books. Having selected any number, m , rule a sheet of paper into columns, a column for each divisor of m ; and write these divisors, in increasing order from left to right each at the top of its column as its principal heading. Just beneath this, write in parentheses, as a subsidiary heading to the column, the complementary divisor, *i.e.*, the divisor whose product into the principal heading is the number m ; and draw a line under this subsidiary heading. Now, to fill up the columns, run over all the numbers in regular succession, from 1 up to m inclusive, writing each in one column, and in one only; namely in that column which is furthest to the right of all the columns of whose principal headings the number to be written is a multiple. Here, for example, is the table for $m = 20$:

1	2	4	5	10	20	
(20)	(10)	(5)	(4)	(2)	(1)	
1	2					
3		4	5			
	6					
7		8				
9			10			
11		12				
13	14		15			
		16				
17	18					
19				20		

Fig. 230

Peirce: CP 4.600 Cross-Ref:††

By this means it is obvious that each column will receive all those multiples of the principal heading whose quotients by that heading are prime to the subsidiary heading, and will receive no other numbers. Thus, every column will contain just one number for each number prime to the subsidiary heading but not greater than it; (since no number is entered which exceeds the product of the two headings). In other words, the number of numbers in each column equals the totient of the subsidiary heading; and since the subsidiary headings are all the divisors, and the total number of numbers entered is m , the sum of the totients of all the divisors of m is m , what ever number m may be. It will be convenient to have a name for this principle; and since, as I

remarked, it renders every number a perfect number in a perfected sense of that term, or say a *perfecti perfect* number, I will refer to it as the *rule of perfection*.

Peirce: CP 4.600 Cross-Ref:††

According to this, although $x^6 = 1$ may have 6 roots, yet since x^2 , x^3 , and x^6 are also roots, by the rule of perfection there can be but $T_6 = T_2 \cdot T_3 = 1 \cdot 2 = 2$ numbers of piles into which dealing must be made 6 times successively in order to restore the original arrangement; and similarly for the other divisors. So then the number of ways of dealing (*i.e.*, number of piles into which the cards can be dealt, etc.) which will restore 31 cards to their original order in less than 30 deals cannot exceed $T_1 + T_2 + T_3 + T_5 + T_6 + T_{10} + T_{15}$. There are, however, in all, 30 ways of dealing; and by the rule of perfection $30 = T_1 + T_2 + T_3 + T_5 + T_6 + T_{10} + T_{15} + T_{30}$. Hence, there must be $T_{30} = T_2 \cdot T_3 \cdot T_5 = 1 \cdot 2 \cdot 4 = 8$ ways of dealing which shift the 30 values in a single circuit. And so with any other prime number than 31. This argument is so near a perfect demonstration that there always must be such ways of dealing that I may leave its perfectionment to the reader.

Peirce: CP 4.601 Cross-Ref:††

601. I do not know of any general rule for ascertaining what the particular numbers of piles are into which the prime number p of cards must be dealt $p-1$ times in order to bring round the original arrangement again. It seems that there is a *Canon Arithmeticus* got out by Jacobi, which gives the numbers for the first 170 primes or so. It was published in the year of my birth;†1 so that it was clearly the purpose of the Eternal that I should have the advantage of it. But that purpose must have been frustrated; for I never saw the book. The *Tables Arithmétiques* of Hoüel (Gauthiers-Villars: 1866, 8^{vo}, pp. 44) gives those numbers for all primes less than 200. From these tables it appears that for about five-eighths of the primes one such number is either 2 or $p-2$. Now as soon as one has been found, it is easy to find the rest which are all the powers of that one whose exponents are prime to $p-1$. In case $p-1$ has few prime factors, the numbers any one of which we seek must be nearly a third, perhaps nearly or quite half of all the $p-1$ numbers: so that ere many trials have been made, one is likely to light upon one of them. Thus if $p = 17$, try 2. Now $2^4 = 16 = -1$; so this will not do. Nor will -2 . Try 3. We have $3^2 = 9 = -8$; $3^3 = 27 = -7$, $3^4 = 81 = -4$, $3^8 = (3^4)^2 = -4^2 = -1$. Evidently 3 is one of the numbers and the others are $3^3 = -7$, $3^5 = -12 = 5$, $3^7 = (3^3)(3^4) = (-7) \cdot (-4) = 28 = -6$, and the negatives of these. If the prime factors are many, a different procedure may be preferable. Take the case of $p = 31$. Here $p-1 = 2 \cdot 3 \cdot 5$. Turning to that table of the first nine powers of the first hundred numbers which is given in so many editions of Vega, I find in the column of cubes, $5^3 = 125 = 4(31)+1$, and $6^3 = 216 = 7 \cdot 31 - 1$ and in the column of fifth powers, I find $3^5 = 243 = 8(31)-5$. Consequently, $(3^5)^3 = 3^{15} = 1$. This renders it likely that 3 may be such a number as I seek. $3^2 = 9$, $3^3 = -4$, $3^4 = -12$, $3^5 = -5$, $3^6 = 16 = -15$, $3^{10} = -6$, $3^{12} = +8$, $3^{15} = (3^5)^3 = -125 = -1$. It is evident that 3 is one of the numbers. The other seven are $3^7 = 3^5 \cdot 3^2 = -45 = -14$, $3^{11} = 3 \cdot 3^{10} = -18 = 13$, $3^{13} = 3 \cdot 3^{12} = 24 = -7$, $3^{17} = 3^{15} \cdot 3^2 = -9$; $3^{19} = 3^{15} \cdot 3^4 = +12$, $3^{23} = 3^{19} \cdot 3^4 = -144 = +11$, $3^{29} = 3^{17} \cdot 3^6 = (-9) \cdot (-15) = +135 = +11$.

Peirce: CP 4.602 Cross-Ref:††

602. Since, then, whatever prime number not prime to itself p may be, there are always $T(p-1)$ numbers of which the lowest power equal to 1 (counting round the p cycle) is the $(p-1)$ th and these powers run through all the values of the cycle excepting only $p = 0$, it follows that these numbers may appropriately be called *basal*

(or *primitive*) roots of the cycle; and that their exponents are true *cyclic logarithms* of all the numbers of the cycle except zero. But since, if b be such a basal root, its $(p-1)$ th power, like that of any other number, equals 1 (counting round the p -cycle), it follows that these exponents run round a cycle smaller by one unit than that of their powers; or in other words, the *modulus* of the cycle of logarithms is $p-1$, while the modulus of the cycle of natural numbers is p .†P1

Peirce: CP 4.603 Cross-Ref:††

603. The cyclic logarithms form an entirely distinct number-system from that of the corresponding natural numbers. For the modulus of their cycle is composite instead of prime, a circumstance which essentially modifies some of the principles of arithmetic. For example, every natural number of a cycle of prime modulus gives an unequivocal quotient when divided by another. But some numbers in a cycle of composite modulus give two or more quotients when divided by certain others, while others are not divisible without remainders. The whole doctrine shall be set forth here. I will preface it with a statement of the essential differences between the system of all positive finite integers, the system of all real finite integers, and any cyclical system. I omit the Cantorian system, partly because the full explanation of it would be needed and would be long, and partly because there is a doubt whether it really possesses an important character which Cantor attributes to it.

Peirce: CP 4.604 Cross-Ref:††

604. It is singular that though the systems to be defined possess, besides several independent common characters, others in respect to which they differ, yet *all* the properties of each system are necessary consequences of a single principle of immediate sequence. In stating this, I shall abbreviate a frequently recurring phrase of nine syllables by writing, " m is A of (or to) n ," or even " m is An ," to mean that the member, m , of the system is in a certain relation of immediate antecedence to the member n . I shall express the same thing by writing " n is A'd by m ." But when I call A an abbreviation, I do not mean to imply that the words "immediately antecedent" express its meaning in a satisfactory way. On the contrary, in part, they suggest something repugnant to its meaning, which must be gathered exclusively from the following definitions of the three kinds of systems:

Peirce: CP 4.605 Cross-Ref:††

605. A *cyclical system* of objects is such a collection of objects that, the expression " m is A to n " signifying some recognizable relation of m to n , every member of the system is A to some member or other, and whatever predicate, P, may be, if P is true of no member of the system without being true of some member of it that is A'd by that member, then P is true either of no member or of every member.

Peirce: CP 4.606 Cross-Ref:††

606. The system of all positive whole numbers is a single collection of numbers, the general essential character of which collection is that there is a recognizable relation signified by A, such that every positive integer is A to a positive integer, and there is one, and one only, initial positive integer, 0, (or, if this be excluded, then 1) such that, whatever predicate P may be, if P is true of no positive integer without being also true of some positive integer to which the former is A, then either this predicate is false of that initial positive integer or else is true of all positive integers.†1

Peirce: CP 4.607 Cross-Ref:††

607. The system of all real integers is a collection of numbers of which the

general essential character is that there is recognizable relation signified by one being A to another, such that every number of the system is both A to a number of the system, and is A'd by a number of the system, and whatever predicate P may be, if this be not true of any number, *n*, of the system without being both true of some number that is A of *n*, and true also of some number that is A'd by *n*, then P is either false of every number of the system or is true of every number of the system.†1

Peirce: CP 4.608 Cross-Ref:††

608. A *Cantorian* system is essentially a system of objects positively determined by every collection of objects of the system being A to some object of the system, and by a certain object, 0, being a member of the system; while it is negatively determined by the principle that, whatsoever predicate P may be, if P is not true of every member of any collection of the system without being also true of some member that is A'd by that collection, then either P is not true of the member, 0, or it is true of every member of the system.†2

Peirce: CP 4.609 Cross-Ref:††

609. Now for several reasons, partly for the sake of the logical interest and instruction that will accrue I will proceed to show precisely *how* all the fundamental properties common to cyclical systems follow from my definition. In accordance with the usage of logicians and mathematicians, I shall call this "demonstrating" those properties. The reader must not fall into the error of supposing that, by this expression, I mean *rationally convincing* him that all cyclical systems have these properties; for I know well that he is perfectly cognizant of that already. All I am seeking to convince him of is, first, *that*, and second, *how*, their truth of all cyclical systems follows from my definition. But in the course of doing so, I shall endeavor to bring to his notice some things well worth knowing concerning necessary reasonings in general. Especially, I shall try to point out errors of logical doctrine which students of the subject who neglect the logic of relations are apt to fall into.

Peirce: CP 4.610 Cross-Ref:††

610. A brace of these errors, are, first, that nothing of importance can be deduced from a single premiss; and secondly, that from two premisses one sole complete conclusion can be drawn. Persons who hold the latter notion cannot have duly considered the paucity of the premisses of arithmetic and the immensity of higher arithmetic, otherwise called the "theory of numbers," itself. As to the former belief, aside from the consideration that whatever follows from two propositions equally follows from the one which results from their copulation, they will have occasion to change their opinion when they come to see what can be deduced from the definition of a cyclic system, which definition is not a copulative proposition.

Peirce: CP 4.611 Cross-Ref:††

611. That couple of logical heresies, being married together, legitimately generates a third more malignant than either; namely, that necessary reasoning takes a course from which it can no more deviate than a good machine can deviate from its proper way of action, and that its future work might conceivably be left to a machine -- some Babbage's analytical engine or some logical machine (of which several have actually been constructed).†1 Even the logic of relations fails to eradicate that notion completely, although it does show that much unexpected truth may often be brought to light by the repeated reintroduction of a premiss already employed; and in fact, this proceeding is carried to great lengths in the development of any considerable branch of mathematics. Although, moreover, the logic of relations shows that the introduction of abstractions -- which nominalists have taken such delight in ridiculing

-- is of the greatest service in necessary inference, and further shows that, apart from either of those manoeuvres -- either the iteration of premisses or the introduction of abstractions -- the situations in which the necessary reasoner finds several lines of reasoning open to him are frequent. Nevertheless, in spite of all this, the tendency of the logic of relations itself -- the highest and most rational theory of necessary reasoning yet developed -- is to insinuate the idea that in necessary reasoning one is always limited to a narrow choice between quasi-mechanical processes; so that little room is left for the exercise of invention. Even the great mathematician, Sylvester, perhaps the mind the most exuberant in original ideas of pure mathematics of any since Gauss, was infected with this error; and consequently, conscious of his own inventive power, was led to preface his "Outline Trace of the Theory of Reducible Cyclodes," with a footnote which seems to mean that mathematical conclusions are not always derived by an apodictic procedure of reason. If he meant that a man might, by a happy guess, light upon a truth which might have been made a mathematical conclusion, what he said was a truism. If he meant that the hint of the way of solving a mathematical problem might be derived from any sort of accidental experience, it was equally a matter of course. But the truth is that all genuine mathematical work, except the formation of the initial postulates (if this be regarded as mathematical work) is necessary reasoning. The mistake of Sylvester and of all who think that necessary reasoning leaves no room for originality -- it is hardly credible however that there is anybody who does not know that mathematics calls for the profoundest invention, the most athletic imagination, and for a power of generalization in comparison to whose everyday performances the most vaunted performances of metaphysical, biological, and cosmological philosophers in this line seem simply puny -- their error, the key of the paradox which they overlook, is that originality is not an attribute of the *matter* of life, present in the whole only so far as it is present in the smallest parts, but is an affair of *form*, of the way in which parts none of which possess it are joined together. Every action of Napoleon was such as a treatise on physiology ought to describe. He walked, ate, slept, worked in his study, rode his horse, talked to his fellows, just as every other man does. But he combined those elements into shapes that have not been matched in modern times. Those who dispute about Free-Will and Necessity commit a similar oversight. Notwithstanding my tychism, I do not believe there is enough of the ingredient of pure chance now left in the universe to account at all for the indisputable fact that mind acts upon matter. †1 I do not believe there is any amount of *immediate* action of that kind sufficient to show itself in any easily discerned way. But one endless series of mental events may be immediately followed by a beginningless series of physical transformations. †2 If, for example, all atoms are vortices in a fluid, and every fluid is composed of atoms, and these are vortices in an underlying fluid, we can imagine one way in which a beginningless series of transformations of energy †P1 might take place in a fraction of a second. Now whether this particular way of solving the paradox happens to be the actual way, or not, it suffices to show us that from the supposed fact that mind acts *immediately* only on mind, and matter *immediately* only on matter, it by no means follows that mind cannot act on matter, and matter on mind, without any *tertium quid*. At any rate, our power of self-control certainly does not reside in the smallest bits of our conduct, but is an effect of building up a character. All supremacy of mind is of the nature of Form.

Peirce: CP 4.612 Cross-Ref: ††

612. The plan of a demonstration can obviously not spring up in the mind complete at the outset; since when the plan is perfected, the demonstration itself is so. The thought of the plan begins with an act of {anchinoia} †P1 which, in consequence

of pre-existent associations, brings out the idea of a possible object, this idea not being itself involved in the proposition to be proved. In this idea is discerned that the possibility of its object follows in some way from the condition, general subject, or antecedent of the proposition to be proved, while the known characters of the object of the new idea will, it is perceived, be at least adjutant to the establishment of the predicate or consequent of that proposition.

Peirce: CP 4.613 Cross-Ref:††

613. I shall term the step of so introducing into a demonstration a new idea not explicitly or directly contained in the premisses of the reasoning or in the condition of the proposition which gets proved by the aid of this introduction, a theoric step. Two considerable advantages may be expected from such a step besides the demonstration of the proposition itself. In the first place, since it is a part of my definition that it really aids the demonstration, it follows that without some such step the demonstration could not have been effected, or at any rate only in some very peculiar way. Now to propositions which can only be proved by the aid of theoric steps (or which, at any rate, could *hardly* otherwise be proved), I propose to restrict the application of the hitherto vague word "*theorem*," calling all others, which are deducible from their premisses by the general principles of logic, by the name of *corollaries*.†1 A theorem, in this sense, once it is proved, almost invariably clears the way to the corollarial or easy theorematic proof of other propositions whose demonstrations had before been beyond the powers of the mathematicians. That is the first secondary advantage of a theoric step. The other such advantage is that when a theoric step has once been invented, it may be imitated, and its analogues applied in proving other propositions. This consideration suggests the propriety of distinguishing between varieties of theorems, although the distinctions cannot be sharply drawn. Moreover, a theorem may pass over into the class of corollaries, in consequence of an improvement in the system of logic. In that case, its new title may be appended to its old one, and it may be called a *theorem-corollary*. There are several such, pointed out by De Morgan, among the theorems of Euclid, to whom they were theorems and are reckoned as such, though to a modern exact logician they are only corollaries. If a proposition requires, indeed, for its demonstration, a theoric step, but only one of a familiar kind, that has become quite a matter of course, it may be called a *theoremation*.†P1 If the needed theoric step is a novel one, the proposition which employs it most fully may be termed a *major theorem*; for even if it does not, as yet, appear particularly important, it is likely eventually to prove so. If the theoric invention is susceptible of wide application, it will be the basis of a mathematical method.

Peirce: CP 4.614 Cross-Ref:††

614. But mathematicians are rather seldom logicians or much interested in logic; for the two habits of mind are directly the reverse of each other;†1 and consequently a mathematician does not care to go to the trouble (which would often be very considerable) of ascertaining whether the theoric step he proposes to himself to take is absolutely indispensable or not, so long as he clearly perceives that it will be exceedingly convenient; and the consequence is that many demonstrations introduce theoric steps which relieve the mind and obviate confusing complications without being logically necessary. Such demonstrations prove corollaries more easily by treating them as if they were theorems. They may be called *theoric corollaries*, or if one is not sure that they are so, *theoretically proved propositions*.

Peirce: CP 4.615 Cross-Ref:††

615. I wish a historical study were made of all the remarkable theoric steps and noticeable classes of theoric steps. I do not mean a mere narrative, but a critical examination of just what and of what mode the logical efficacy of the different steps has been. Then, upon this work as a foundation, should be erected a logical classification of theoric steps; and this should be crowned with a new methodic of necessary reasoning. My future years -- whatever can have become of them, they do not seem so many now as they used, when, at De Morgan's *Open Sesame*, the Aladdin matmûrah of relative logic had been nearly opened to my mind's eye; but the remains of them shall, I hope, somehow contribute toward setting such an enterprise on foot. I shall not be so short-sighted as to expect any cut-and-dried rules nor yet any higher sort of contrivance, to supersede in the least that {anchinoia} -- that penetrating glance at a problem that directs the mathematician to take his stand at the point from which it may be most advantageously viewed. But I do think that that faculty may be taught to nourish and strengthen itself, and to acquire a skill in fulfilling its office with less of random casting about than it as yet can.

Peirce: CP 4.616 Cross-Ref:††

616. Euclid always begins his presentation of a theorem by a statement of it in *general terms*, which is the form of statement most convenient for applying it. This was called the {protasis}, or *proposition*. To this he invariably appends, by a {legö}, "I say," a translation of it into *singular terms*, each general subject being replaced by a Greek letter that serves as the proper name for a single one of the objects denoted by that general subject. Yet the generality of the statement is not lost nor reduced, since the understanding is that the letter may be regarded as the name of any one of those objects that the student may select. This second statement was called the {ekthesis}, or *exposition*. Euclid lived at a time when the surpassing importance of Aristotle's *Analytics* was not appreciated. The use, probably by Euclid himself, of the term {protasis}, which in Aristotle's writings means a premiss, to denote the conclusion to be proved, illustrates this, and confirms other reasons for thinking that Euclid was unacquainted with the doctrine of the *Analytics*. The invariable appending by Euclid of an {ekthesis} to the {protasis} (except in a few cases in which the proposition is expressed in the ecthetic form alone) inclines me to think that it was, for him, a principle of logic that any general proposition can be so stated; and such a form of statement was always convenient in demonstration; sometimes, necessary. If this surmise be correct, Euclid probably looked upon the function of the {ekthesis} as that of merely supplying a more convenient form for expressing no more than the {protasis} had already asserted. Yet inasmuch as the {protasis} does not mention those proper names consisting of single letters, the {ekthesis} certainly does supply ideas that, however obvious they be, are not contained in the {protasis}; so that it must be regarded as taking a little theoric step. The principal theoric step of the demonstration is, however, taken in what immediately follows; namely, in "preparation" for the demonstration, the {paraskeué}, usually translated "the construction." The Greek word is applied to any thing got up with some elaboration with a view to its being used in any contemplated undertaking: a near equivalent to a frequent use of it is "apparatus." Euclid's {paraskeué} consists of precise directions for drawing certain lines, rarely for spreading out surfaces; for though his work entitled "*Elements*," appears to have been intended as an introduction to theoretical mathematics in general (the art of computation being the *métier* -- the 'mister, as Chaucer would say, of the Pythagoreans), yet Euclid always conceives arithmetical quantities -- even when distinguishing between prime and composite integers -- as being lengths of lines. It was his mania. Those lines which are drawn in the

{paraskeu }) are not only all that are referred to in the condition of the proposition, but also all the additional lines which he is about to consider in order to facilitate the demonstration of which this {paraskeu } is thus the soul, since in it the principal theoretic step is taken. But the construction of these additional lines is introduced by {gar}, here meaning "for," and sometimes the text does not very sharply separate some parts of the {paraskeu } from the next step, the {apodeixis}, or demonstration. This latter contains mere corollarial reasoning, though, in consequence of its silently assuming the truth of all that has been previously proved or postulated (which Mr. Gow, in his *Short History of Greek Mathematics*,†1 gives as the reason for Euclid's having called his work {Stoicheia}; which seems to me very dubious), this corollarial reasoning will sometimes be a little puzzling to a student who has not so thoroughly assimilated what went before as to have the approximate proposition ready to his mind. After this, a sentence always using {ara}, "hence," "*ergo*," repeats the {protasis} (not often the {ekthesis}) so as to impress the proposition on the mind of the student, in its new light and new authority, expressed in the form most convenient in future applications of it. This is called {symperasma}, the "conclusion," which sounds highly Aristotelian. Yet the classical use of the verb to signify coming to a final conclusion, rendered this noun inevitable as soon as these neuter abstracts came into the frequent use that they had by Euclid's time. The conclusion always ends with the words {hyper edei deixai}, "which had to be shown," *quod erat demonstrandum*, for which Q. E. D. is now put.

Peirce: CP 4.616 Cross-Ref:††

I will take at random the twentieth proposition of the first book, to illustrate the matter. "In every triangle, any two sides, taken together are always greater than the third.

Peirce: CP 4.616 Cross-Ref:††

"For let {A}{B}{G} be a triangle. I say that any two sides taken together are greater than the third; {B}{A} and {A}{G} than {B}{G}, {A}{B} and {B}{G} than {A}{G}, and {B}{G} and {G}{A} than {A}{B}.

Peirce: CP 4.616 Cross-Ref:††

"For extend {B}{A} to the point {D}, taking {A}{D} equal to {G}{A} [which he has shown in the second proposition always to be possible]; and join {D} to {G} by a straight line.

Peirce: CP 4.616 Cross-Ref:††

"Now since {D}{A} is equal to {A}{G}, the angle under {A}{D}{G} is equal to that under {A}{G}{D} [by the *pons asinorum*]. Hence, the angle under {B}{G}{D} will be greater than that under {A}{D}{G}. [This is a fallacy of a kind to which Euclid is subject from assuming that every figure drawn according to the {paraskeue} will necessarily have its parts related in the same way, when it can only be otherwise if space is finite, which he has never formally adopted as a postulate. In the present case, if {A}{D} is more than half-way round space, the triangle {A}{G}{D} will include the triangle {A}{B}{G} within it; and then the angle {B}{G}{A} will be less than the angle {A}{D}{G}.] And since {D}{G}{B} is a triangle having the angle under {B}{G}{D} greater than that under {B}{D}{G}, but the greater side subtends under the greater angle [which is the theorem that had just previously been demonstrated], therefore {D}{B} is greater than {B}{G}. But {D}{A} is equal to {A}{G}. Therefore, {B}{D} and {A}{G} are greater than {B}{G}. Similarly, we shall [*i.e.*, could] show that {A}{B} and {B}{G} are greater than {G}{A}, and {B}{G} and {G}{A} than {A}{B}.

Peirce: CP 4.616 Cross-Ref:††

"In every triangle, then, any two sides joined together are greater than the third, which is what had to be shown."

Peirce: CP 4.617 Cross-Ref:††

617. I will now return to the consideration of cyclical systems, and will begin by expressing my definition of such a system in those Existential Graphs which have been explained in *The Monist* [book II, ch. 6]. In reference to those graphs, it is to be borne in mind that they have not been contrived with a view to being used as a calculus, but on the contrary for a purpose opposed to that. Nevertheless, if anyone cares to amuse himself by drawing inferences by machinery, the graphs can be put to this work, and will perform it with a facility about equal to that of my universal algebra of logic †1 and as much beyond that of my algebra of dyadic relatives, †2 of which the lamented Schroeder was so much enamoured. †3 The only other contrivances for the purpose appear to me to be of inferior value, unless it be considered worth while to bring a pasigraphy into use. Such ridiculously exaggerated claims have been made for Peano's system, †4 though not, so far as I am aware, by its author, that I shall prefer to refrain from expressing my opinion of its value. I will only say that if a person chooses to use the graphs to work out difficult inferences with expedition, he must devote some hours daily for a

Figs. 231-232

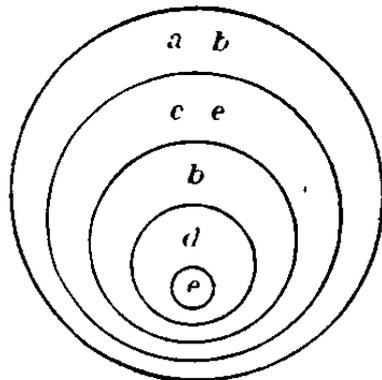


Fig. 231

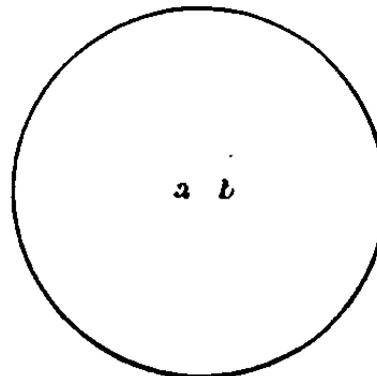


Fig. 232

[Click here

to view]

week or two to practice with it; and the most efficacious, instructive, and entertaining practice possible will be gained in working out his own method of using the graphs for his purpose. I will just give these little hints. Some slight shading with a blue pencil of the oddly enclosed areas will conduce to clearness. Abbreviate the parts of the graph that do not concern your work. Extend the rule of iteration and deiteration, by means of a few theorems which you will readily discover. Do not forget that useful iteration is almost always into an evenly enclosed area, while useful deiteration is, as usually, from an oddly enclosed area. Perform the iteration and the immediately following deiteration at one stroke, in your mind's eye. Do not forget that the ligatures may be considered as graph-instances scribed in the areas where their least enclosed

parts lie, and repeated at their attachments. Their intermediate parts may be disregarded. Reflect well on each of the four permissions †1 (especially that curious fourth one) †2 until you vividly comprehend the why and wherefore of each, and the bearings of each from every point of view that is habitual with you. Do not forget that an enclosure upon whose area there is a vacant cut can everywhere be inserted and erased, while an unenclosed vacant cut declares your initial assumption, first scribed, to have been absurd. You will thus, for example, be enabled to see at a glance that from Fig. 231 can be inferred Fig. 232. The cuts perform two functions; that of denial and that of determining the order of

Figs. 233-234

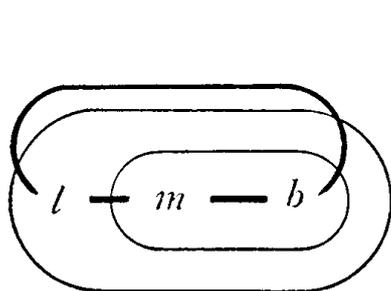


Fig. 233

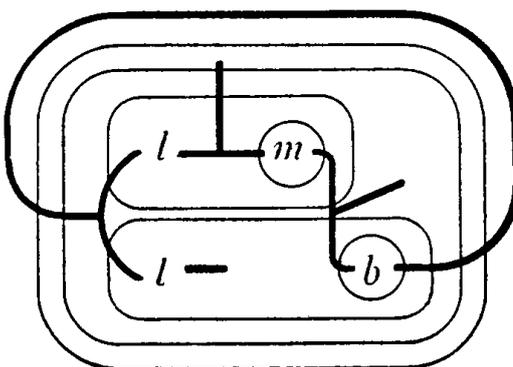


Fig. 234

to view]

[Click here

selection of the individual objects denoted by the ligatures. If the outer cuts of any graph form a nest with no spot except in its innermost area, then all that part of the assertion that is therein expressed will need no nest of cuts, but only cuts outside of one another, none of them containing a cut with more than a single spot on it. It will seldom be advisable to apply this to a complicated case, owing to the great number of cuts required; but you should discover and stow away in some sentry-box of your mind whence the beck of any occasion may instantly summon it, the simple rule that expresses all possible complications of this principle. As an example of one of the simplest cases, Fig. 233 and Fig. 234 are seen precisely equivalent.

Peirce: CP 4.618 Cross-Ref: ††

618. Owing to my Existential Graphs having been invented in January of 1897 and not published until October, 1906, it slipped my mind to remark when I finally did print a description of it, what any reader of the volume entitled *Studies in Logic by Members of the Johns Hopkins University* (Boston, 1883), might perceive, that in constructing it, I profited by entirely original ideas both of Mrs. and Mr. Fabian Franklin, as well as by a careful study of the remarkable work of O. H. Mitchell, whose early demise the world of exact logic has reason deeply to deplore.

Peirce: CP 4.619 Cross-Ref: ††

619. My reason for expressing the definition of a cyclic system in Existential Graphs is that if one learns to think of relations in the forms of those graphs, one gets the most distinct and esthetically as well as otherwise intellectually, iconic conception of them likely to suggest circumstances of theoretic utility, that one can obtain in any

way. The aid that the system of graphs thus affords to the process of logical analysis, by virtue of its own analytical purity, is surprisingly great, and reaches further than one would dream. Taught to boys and girls before grammar, to the point of thorough familiarization, it would aid them through all their lives. For there are few important questions that the analysis of ideas does not help to answer. The theoretical value of the graphs, too, depends on this.

Peirce: CP 4.620 Cross-Ref:††

620. Strictly speaking, the term "definition" has two senses -- Firstly, this term is sometimes quite accurately applied to the composite of characters which are requisite and sufficient to express the signification of the "definitum," or predicate defined; but I will distinguish the definition in this sense by calling it the "definition-term." Secondly, the word definition is correctly applied to the double assertion that the definition term's being true of any conceivable object would always be both requisite and sufficient to justify predicating the definitum of that object. I will distinguish the definition in this sense by calling it the "definition-assertion-pair." In the present case, as in most cases, it is needless and would be inconvenient to express the entire definition-assertion-pair with strict accuracy, since we only want the definition in order to prove certain existential facts of subjects of which we *assume* that the definitum, "cyclic-system," is predicable. We do not care to *prove* that it is predicable, and therefore the assertion that the definitum is predicable of the definition-term is not relevant to our purpose. In the second place, we do not care to meddle with that universe of concepts with which the definition deals; and it would considerably complicate our premisses to no purpose to introduce it. We only care for the predication of the definition-term concerning the definitum so far as it can concern existential facts. All that we care to express in our graph is so much as may be required to deduce every existential fact implied in the existence of a cyclic system.

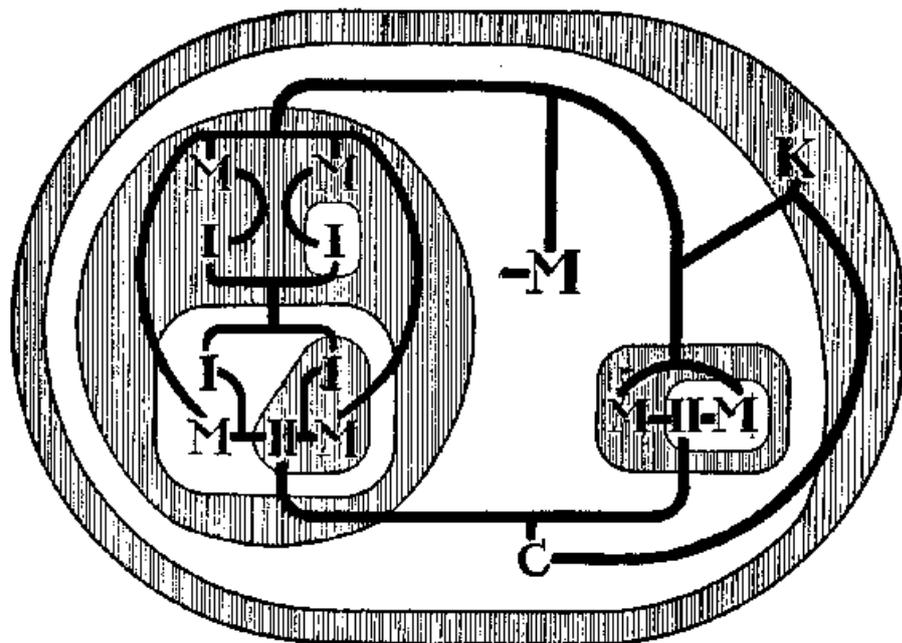


Fig. 235

[Click here to view]

Fig. 235

Peirce: CP 4.621 Cross-Ref:††

621. A cyclic system is a system; and a system is a collection having a regular relation between its members. One member suffices to make a collection, and is requisite to the existence of the collection. The definition, so far as we need it, is then expressed in the graph of Fig. 235. Here K with a "peg"†1 at the side asserts that the object denoted by the peg is a cyclic system. The letter M with one peg at the top and another placed on either side without any distinction of meaning, asserts that the object denoted by the side-peg is a *member* of the system denoted by the top-peg. The letter C, with a peg at the top and another at the side, asserts that the object denoted by the top-peg is a relation [sic] involved in that relation between all the members which constitutes the entire collection of them as the system that it is, and asserts that the object denoted by the side-peg is such a system. The Roman numerals each having one peg placed at the top or bottom of the numeral and a number of side-pegs equal to the value of the numeral, all these side-pegs being carefully distinguished, are used to express the truth of the proposition resulting from filling the blanks of the rheme denoted by the top or bottom peg, with indefinite signs of objects denoted by the side-pegs taken in their order, all the left-hand pegs being understood to precede all the right-hand pegs, and on each side a higher peg to precede a lower one. With this understanding, the graph of Fig. 235, where for the sake of perspicuity the oddly enclosed, or negating areas are shaded, may be translated into the language of speech in either of the two following equivalent forms (besides many others):

It is false that

there is a cyclic system while it is false that

this system has a member

and involves a relation ("being A to," the bottom peg of II), and that it is false that

the system has a member of which it is false that

it is in that relation, A, to a member of the system,

while it is false that

there is a definite predicate, P (the top or bottom peg of

I), that is true of a member of

the system and is false of a member of the system,

and that it is false that

this predicate is true of a member of the system of which

it is false that

it is A to a member of the system of which P is true.

Peirce: CP 4.621 Cross-Ref:††

This more analytic statement is equivalent to saying that every cyclic system (if there be any) has a member, and involves a relation called "being A to" (not the

graph but perspicuity of speech requires it to be so named), such that every member of the system is A to a member of the system, and any definite predicate, P, whatsoever, that is at once true of one member of the system and untrue of another, is true of some member of the system that is not A to any member of which P is true.

Peirce: CP 4.622 Cross-Ref:††

622. To anybody who has no notion of logic this may seem a queer attempt to explain what is meant by a cyclic system; and it is true that it would be a needlessly involved *verbal* definition; a verbal definition being an explanation of the meaning of a word or phrase intended for a person to whose mind the idea expressed is perfectly distinct. But it is not intended to serve as a *verbal*, but as a *real* definition, that is, to explain to a person to whom the idea may be familiar enough, but who has never picked it to pieces and marked its structure, exactly how the idea is composed. As such, I believe it to be the simplest and most straightforward explanation possible. When you say that the days of the week "come round in a set of seven," you think of the week everything here expressed of K. I do not mean that all this is *actually* existent in your thought; for thinking no more needs the actual presence in the mind of what is thought than knowing the English language means that at every instant while one knows it the whole dictionary is actually present to his mind. Indeed, thinking, if possible, even *less* implies presence to the mind than knowing does; for it is tolerably certain that a mind to whom a word is present with a sense of familiarity knows that word; whereas a mind which being asked to *think* of anything, say a locomotive, simply calls up an image of a locomotive, has, in all probability, by bad training, pretty nearly lost the power of thinking; for really to think of the locomotive means to put oneself in readiness to attach to it any of its essential characters that there may be occasion to consider; and this must be done by general signs, not by an image of the object. But the truth of the matter will more fully be brought out as we proceed.

Peirce: CP 4.623 Cross-Ref:††

623. All that we require of the definition may be put into a simpler shape by omitting the letter M, since the interpreter of the graph must well understand that the whole talk of the graphist for the time being, so far as it refers to things and not to the attributes or relations, has reference to the members of a cyclic system. We may consequently use the graph of Fig. 236 in place of Fig. 235.

Peirce: CP 4.623 Cross-Ref:††

It will be remarked that the graph of Fig. 236 is no more a definition of a cyclic system than it is of the relation of immediate antecedence; and this is as it should be; for plainly a system cannot be defined, without virtually defining the relation between its members that constitutes it a system.

Figs. 236-237

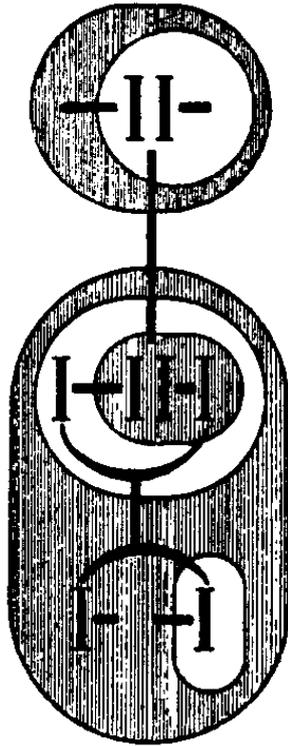


Fig. 236

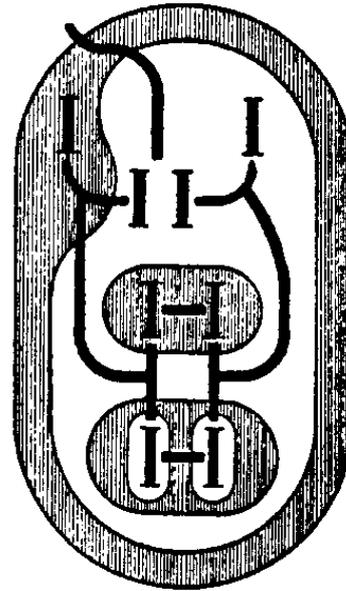


Fig. 237

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to view]

Peirce: CP 4.624 Cross-Ref:††

624. I will now begin by drawing one of several corollaries that are right at my hand. I am always using the words *corollary* and *theorem* in the strict sense of the foregoing †1 definition. This corollary results from the logical principle that to every predicate there is a negative predicate which is true if the former is false, and is false if the former is true. This purely logical principle is expressed in the graph of Fig. 237. Obviously, if any predicate is both true of some member and false of some member of the system, the same will be the case with its negative. Consequently, by the definition, this negative will be true of some member without being true of any to which that member is A; or, in other words, the original predicate will be false of some member without being A to any member of which it is false. Thus, if any predicate is neither true of all nor false of all the members of any cyclic system, but is true of some one and false of some other, there will be two different members of one of which it is true without being true of any to which that member is A, while of the other it is false without being false of any to which that member is A. Or, to put the corollary in a different light, taking any predicate, P, whatsoever, then, in case you can prove that there cannot be more than one exception to the rule that every member of the system resembles some one of those to which it is A in respect to the truth or falsity concerning it of P, then if P be true of one member, it is true of all, and if it be false of one, it is false of all.

Peirce: CP 4.625 Cross-Ref:††

625. I am now going to apply this proposition to a theoretic proof of a proposition which is really only a corollary from the definition of a cyclic system. My motive for this departure from good method is that it will afford a good illustration of the advantage of making the selected predicate, P, as special and characteristic of the

state of things you are reasoning about as possible. The proposition I am going to prove is, that in any cyclic system that contains more than one member no member will be A to itself. For this purpose I will consider any member of the system you please, and will give it the proper name, N. This ecthetic step is already theoretic, but is a matter of course. Another theoretic step, not a matter of course, shall consist in my selecting, as the predicate to be considered, "is N." Now if N is A to itself, every member of the system of which this predicate is true (which can be none other than N itself) will be A to a member of which the predicate is also true; and consequently, by the definition of a cyclic system this predicate cannot be true of one member and false of another. But if there be any other member of the system than N, it will be false of that one. Whence, if N were A to itself and were not the only member of the system, there would be no member of which it would be true that it was N. But by the definition, every cyclic system has some member, and N was chosen as such. So that it must be, either that the system has no other member, or that any member you please, and consequently *every one*, is non-A to itself.

Peirce: CP 4.625 Cross-Ref:††

Now what I wanted to point out was that if instead of "is N," I had selected, as my predicate to be considered, "is A to itself," it would merely have followed that since any member that is A to itself is A to a member that is A to itself, by the general definition either every member of the system is A to itself or none is so.

Peirce: CP 4.625 Cross-Ref:††

I will now prove that this proposition, that no member of a cyclical system is A to itself unless it is the only member of the system, is not a theorem, in any strict sense, by proving it corollarily. For this purpose I first prove that no cyclical system, by virtue of the same relation A, involves another as a part, but not the whole of it. For suppose that certain members of a cyclical system form by themselves a cyclical system constituted by the same A-hood. Then, by the part of the definition of a cyclical system that has been expressed as graph in Fig. 235 and in Fig. 236, there is a member of this minor system; and every member of it is A to a member of the major system that is a member of the minor system. Hence, by that same partial definition, the predicate "is a member of the minor system" being true of one is true of all members of the major system. The minor system is, then, the whole of the major system. To go further, I must employ that assertion of the definitum "is a cyclic system" concerning the definition-term, which assertion has not been expressed as a graph, in order to prove, by its conformity with the definition that a single object, having a relation, identity, to itself, that relation conforming to the conditions of the constitutive relation of a cyclical system, must be admitted to be a cyclical system of a single member. If, therefore, one of the members of a cyclical system of more than one member were A to itself, it would be a cyclical system which was a part but not the whole of another cyclical system, which we have seen to be impossible.

Peirce: CP 4.626 Cross-Ref:††

626. I shall now employ the first corollary to prove that every member of a cyclical system is A'd by some member. For take any member you please of any such system you please; and I will assign to it the proper name N. If then, N is the only member of the system, by the definition N is A to itself. But if there be another member, it is one of which the predicate "is N" is not true, though there is some member, namely N, of which that predicate is true. Consequently, by that first corollary, there must be a member of which it is not true that it is N which is A to nothing of which this is not true. But, by the definition, every member of a cyclic

system is A to some member; and therefore that member which is not A to any member of which "is N" is not true, must be true of a member of which "is N" is true, which, by hypothesis, is only N itself; consequently any member of any cyclic system which one may choose to select is A'd by some member, and by another than itself, if there be another. Q. E. D.

Peirce: CP 4.627 Cross-Ref:††

627. Further investigation of the properties of cyclic systems will need a somewhat more recondite theoretic step. Certainly, however, I must not convey the idea that I claim to be quite sure of this. As yet, I have not sufficiently studied the methodic of theorematic reasoning. I only have an indistinct apprehension of a principle which seems to me to prove what I say; and I must confess that of all logical habits that of confiding in deductions from vague conceptions is quite the most vicious, since it is just such reasonings that to the intellectual rabble are the most convincing; so that the conclusions get woven into the general common sense so closely, that it at length seems paradoxical and absurd to deny them, and men of "good sense" cling to them long after they have been clearly disproved. However, whether it be absolutely necessary or not, the only way I see, at present, of demonstrating the remaining properties of a cyclic system is to suppose a predicate to be formed by a process which will seem somewhat complicated. I shall not state what this predicate is, but only suppose it to be formed according to a rule; and even this rule will not be exactly stated but only a description of its provisions will be given. I shall suppose that one member of the system is selected by the rule as one of the class of subjects of which the predicate is true, and that the remaining members of this class shall be taken into it from among the members of the system *one by one*, according to the rule that when the member last taken in is not A to any member already taken in, one and one only of the members of the system not yet taken in to which that last adopted member is A is to be added to the class; and this new addition may, in the same way, require another. If the system were infinite (as we shall soon see that it cannot be), this might go on endlessly; and so far, we have not seen that this cannot happen. But as soon as it happens that the member last admitted to the class is A to a member already admitted (and consequently that every member admitted to the class is A to an admitted member) the admissions to the class are to be brought to a stop. There are now two supposable cases to be provided for which we shall later find will never occur; but if we did not determine what was to be done if they should (this not being proved impossible) our first proof would involve a *petitio principii*. One is the case in which the finally adopted member is A to a member already having an A that had previously been admitted to the class. The other is the case in which the last (but not necessarily the final) adopted member is not only A'd by the *last previously* adopted member (for the sake of providing which with a member A'd by it, the very last was taken in) but is also A'd by an earlier adopted member. In the latter case, in which the member last adopted, which we may name V, is not only A'd by the last previous one, which we may name U, but is also A'd by a previously adopted member of the class which we may name K, we are to reject from the class all that were admitted after K to U inclusive; so that we revert to what would have been the case, as it might have been, if next after K we had admitted V, to which K is A. We should thus make the class smaller, which we shall soon see could not happen. In the other case, where the last adopted member, which we will name Z, is A to a previously adopted one, which we will name J, which was not the first member adopted into the class, but is A'd by another, which we will name I, we reject from the class both I and all that were adopted previously to I.

Peirce: CP 4.627 Cross-Ref:††

After these supposititious rejections, there is no object of which the predicate, "is a member of the class so formed," is true that is not A of any object of which the same predicate is true, and therefore, by the definition so often appealed to, this predicate cannot be both true of a member of the cyclic system and false of another such member. Now it plainly is true of some member, since the first object taken into it as well as every one subsequently taken into it were members of the cyclic system. Therefore, this predicate cannot be false of any member of the cyclic system. In other words, the class so formed includes all the members of the cyclic system. Consequently, there cannot have been any rejections.

Peirce: CP 4.627 Cross-Ref:††

Since there were no rejections, the first member adopted must remain a member of the class; and since we have seen in a former corollary that every member of a cyclic system is A'd by a member of the same system, this first adopted member must be A'd by some member of the system, that is, by some member of the class. But by the rule of formation of the class no member of it except the finally adopted one can be A to a previously adopted member. It follows that there must be a finally adopted one; and by the same rule no member of the class except the first was adopted without there being a *last previously adopted* member. It follows that the succession of adoptions cannot, at any part of it, have been endless. This is one of the most difficult theorems that I had to prove.

Peirce: CP 4.627 Cross-Ref:††

Moreover, every member of the class is by the mode of formation A to one, and only to one, member of the class; and consequently the same is true of all the members of every cyclic system.

Peirce: CP 4.627 Cross-Ref:††

Moreover, every member of the class except the first was only taken in so as to be A'd by the last, or, at any rate, by one member only; and the first adopted member as we have seen is A'd by the finally adopted member. It cannot be A'd by any other, since by the rule of formation, such another would thereby have become the finally adopted member. Hence, no member of a cyclic system is A'd (in the same sense) by any two members of the system; or no two members are A to the same member.

Peirce: CP 4.628 Cross-Ref:††

628. I have thus, by means of this {theōria} of the formation of a certain kind of class, succeeded in demonstrating, what one might well have doubted, that from the proposition expressed in Fig. 235 follows the double uniqueness of the cyclical relation of A-hood or immediate antecedence. This is the principal, as I think, of those properties that are common and peculiar to cyclical systems. The same theoretic step, or a reduplication of it, will enable the reader to prove other properties, common but not peculiar to cyclic systems; and especially that a collection the count of whose members in one order comes to an end can never in any order involve an endless process, whether it comes to an end or does not. There is, by the way, an important logical interest in that mode of succession in which an endless succession, say, of odd numbers, is followed by a beginningless diminishing succession of even numbers. For it shows that two classes of objects may have such a connection with a transitive relation, such as are those of causation, logical implication, etc., that any member of either class is *immediately* in this relation only to a member of the same class, while

yet every member of one of the classes may be in this same relation to every member of the other class. Thus, it may be that thought only acts upon thought *immediately*, and matter *immediately* only upon matter; and yet it may be that thought acts on matter and matter upon thought, as in fact is plainly the case, somehow.

Peirce: CP 4.629 Cross-Ref:††

629. In this theoretic step, it is noticeable that I have had to embody the idea of *antecedence* generally, in order to prove the properties of cyclical *immediate antecedence*. Any reasoner is always entitled to assume that the mind to which he makes appeal is familiar with the properties of antecedence in general; since if he were not so, he could not even understand what reasoning was at all about. For logical antecedence is an idea which no reasoner can unload or dispense with. It would have been easy to replace, in my demonstrations, all the "previously"s, etc., by relations of inference. I have not done so in order not to burden the reader's mind with needlessly intricate forms of thought.

Peirce: CP 4.630 Cross-Ref:††

630. A corollary from what has already been proved is that if we regard the definition of Fig. 236 as the definition of A-hood, or cyclical immediate antecedence, then A-hood is not a single relation but is any one of a class of relations which, if the collection of all the members of the system is not very small, is a large class. For taking any two members of the system, and naming them Y and {Y}, we may form such a relation, that of A'-hood, that whatever is neither Y nor {Y}, nor is A to Y nor to {Y}, is A' to whatever is A'd by it, while whatever is A to Y is A' to {Y}, whatever is A to {Y} is A' to Y, whatever is A'd by Y is A' 'd by {Y}, and whatever is A'd by T is A' 'd by Y; and then A' will have the same general properties as A. Thus, if the number of members of a cyclic system is *m*, the number of relations of A-hood is $(m-1)!$ if *m* be seven, the number of A-relations is 720; etc.

Peirce: CP 4.631 Cross-Ref:††

631. There is no relation in a cyclic system exactly answering to general antecedence in a denumeral †P1 system.

Peirce: CP 4.632 Cross-Ref:††

632. As a finitude is a positive complication (as is shown by a form of inference being valid in a finite system that is not elsewhere valid) so in place of the relation of betweenness which in a linear system endless both ways, which, if those ways are not distinctively characterized, is triadic, we have in a cyclic system a tetradic relation expressible by α with four tails, so that Fig. 238, which means that an object which can, wherever it be in the cycle, pass from its position to that which is *next* to that position, being either A to it or A'd by it, will if at I be *opposite* to an object at J, relatively to any objects at U and at V. That is, such an object cannot move from I to J without passing through U and V. This implies that U is opposite to V relatively to I and J; that no other pair out of the four are opposite to each other relatively to the

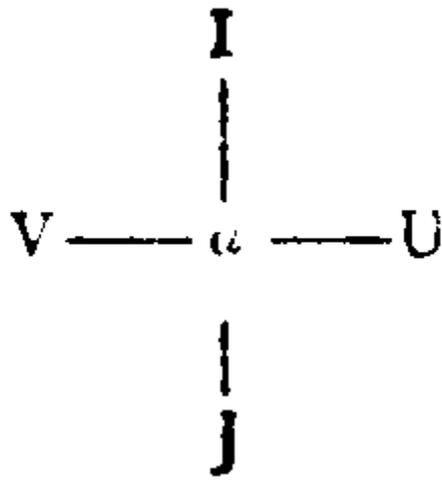


Fig. 238:

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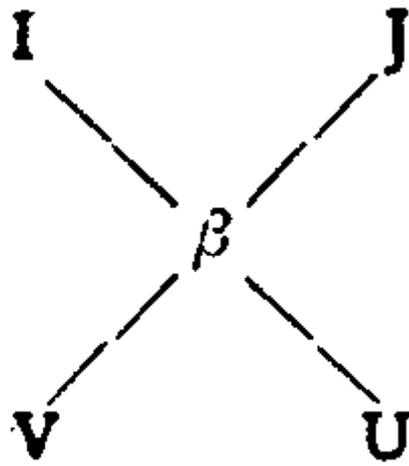


Fig. 239:

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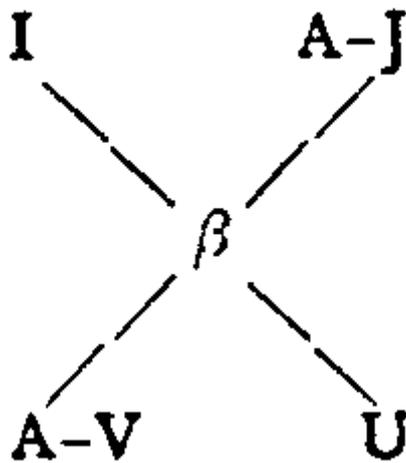


Fig. 240:

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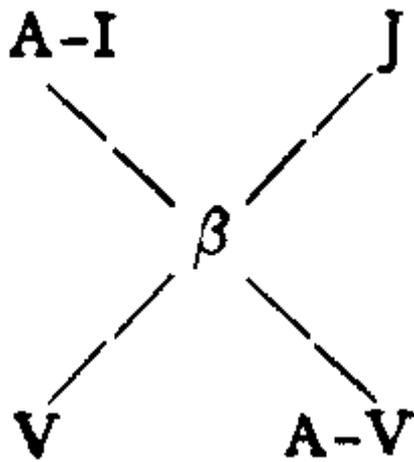


Fig. 241:

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other pair; and that that way of passing round the cycle in which U is reached next after I is the way in which J is reached next after U, V next after J, and I next after V; while that way in which V is reached next after I is the way in which J is reached next after V, U next after J, and I next after U. This supposes that I, J, U, and V are all different, as those that are opposite must be unless two that are adjacent are identical, in which case we may understand the relation as always being true and meaningless.

Peirce: CP 4.632 Cross-Ref:††

We may modify this relation, so as to render it exact, by defining Fig. 239 as true, if I and J are identical while U and V are also identical; or if I and U are identical while J and V are identical, and also if Fig. 240 or Fig. 241 is true; but as not true unless necessarily so according to these principles. This last clause, by the way, has a very important logical form; but I shall not stop to comment upon it. It will be observed that if Fig. 239 is true, then one or other of the graphs Figs. 242 and 243 must be true. And if two α -relations hold, having three of their four correlates identical,

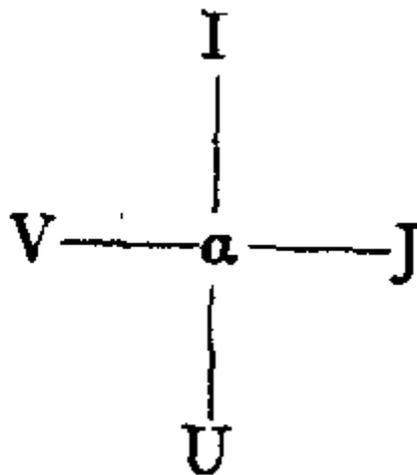


Fig. 242:

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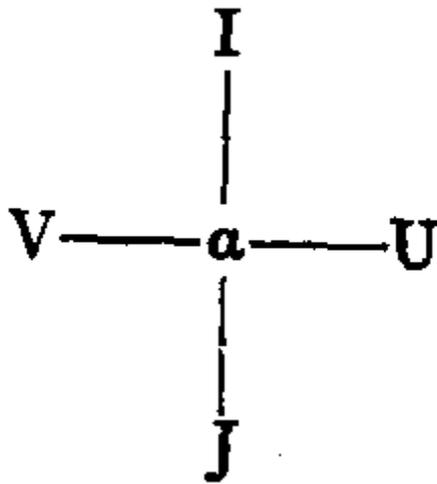
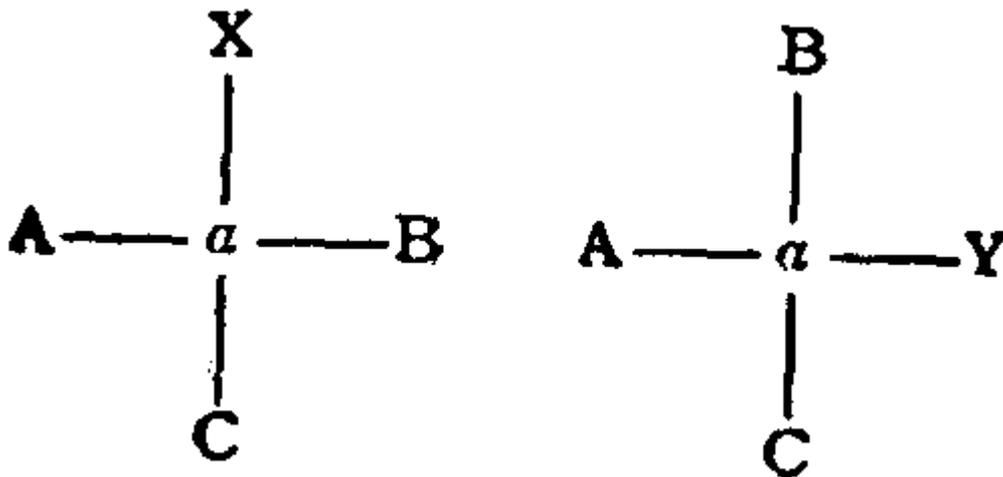


Fig. 243:

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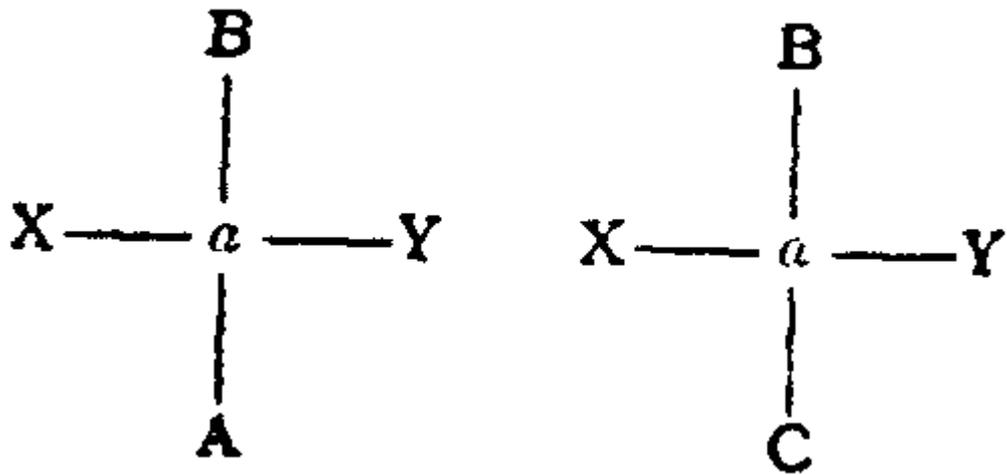
and not the same pair being opposite in both, then two α -conclusions may be drawn in which the two correlates that only appeared once each in the premisses, appear together, and

Fig. 244:



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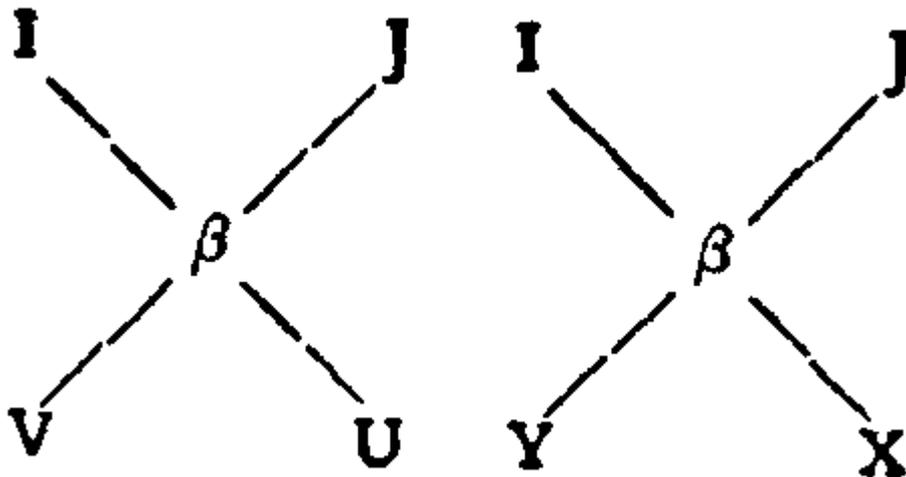
Fig. 245:



[Click here to view]

opposite to one another. Thus, from Fig. 244 may be inferred
 Fig. 245. The β -relation lends itself to much further inferential

Fig. 246:



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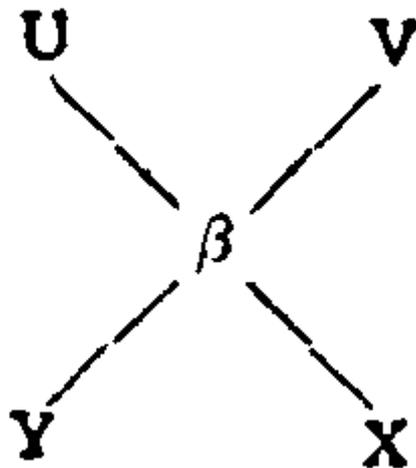
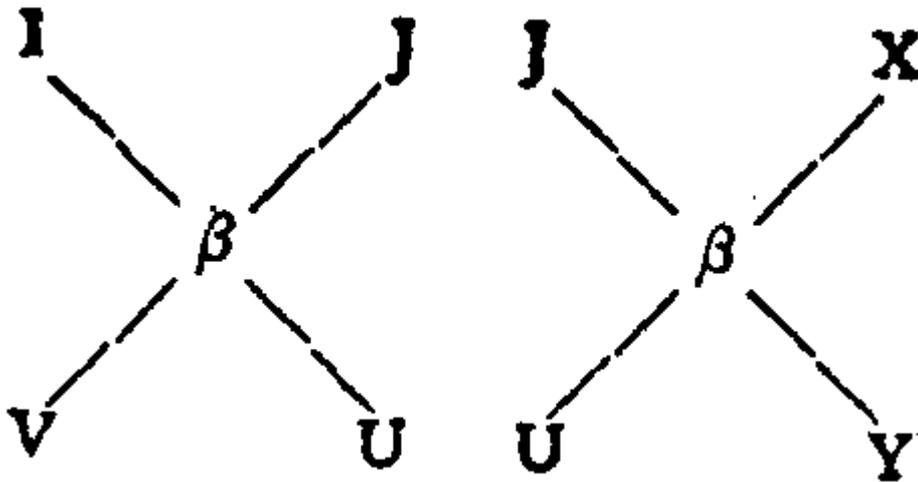


Fig. 247:

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Fig. 248:



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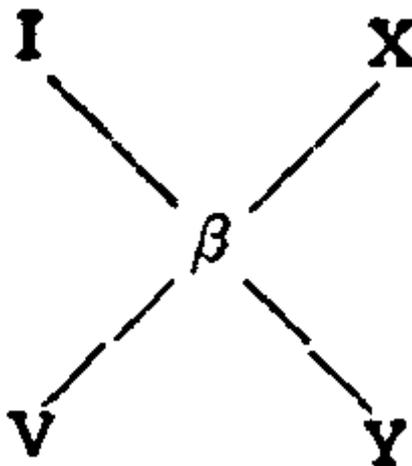
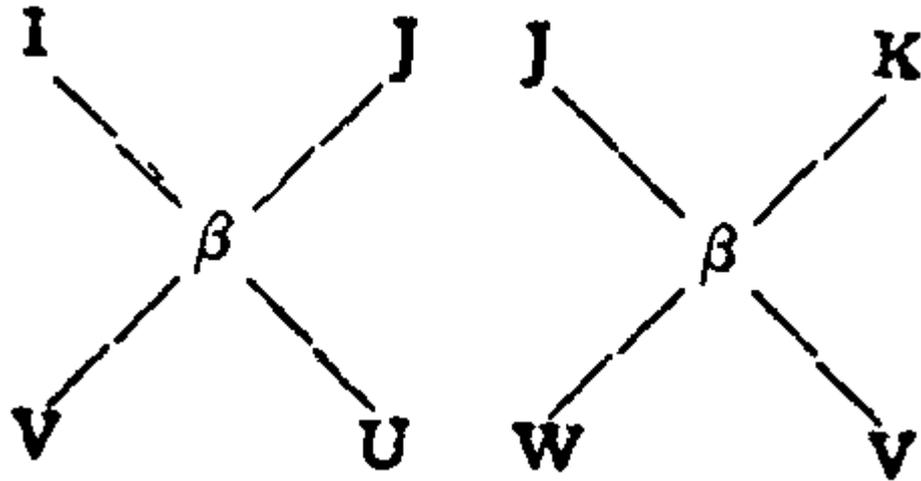


Fig. 249:

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Fig. 250:



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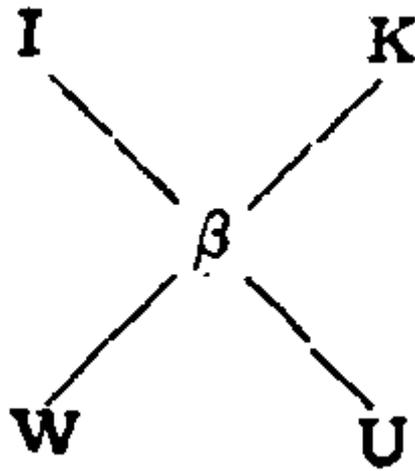


Fig. 251:

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procedure. In the first place in Fig. 239, the whole graph may be turned round on the paper so as to bring each correlate into the place of its opposite. It may also be turned through 180° round a vertical axis in the sheet. (It may consequently be turned 180° round a horizontal axis in the sheet.) Moreover, the two correlates on the left, I and V, may be interchanged.

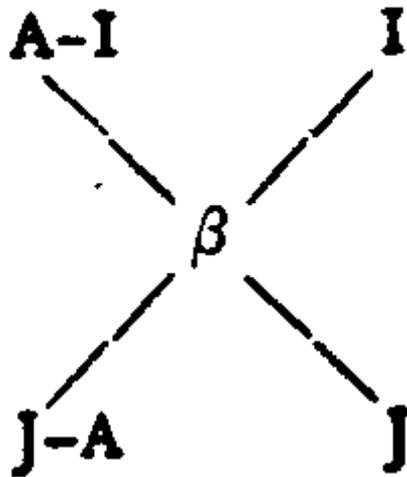


Fig. 252:

[Click here to view]

(And so, consequently, may J and U.) Moreover, from Fig. 246 we can infer Fig. 247. (Whence it follows that from Fig. 248 we can infer Fig. 249.) Also, from Fig. 250 we can infer Fig. 251. Whence there follow very obviously several transformations. For example, Fig. 252 will be true; and if any three of the four graphs of Fig. 253 are true, so is the other

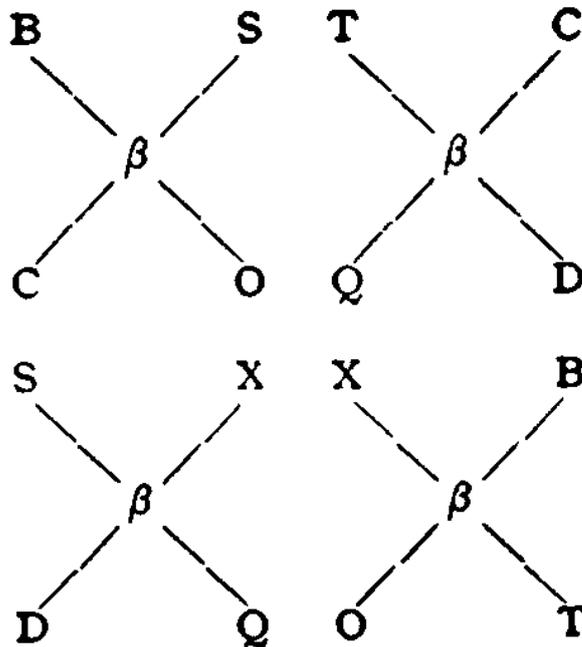


Fig. 253:

[Click here to view]

one. It is obvious that the relation β involves cyclical addition-subtraction, by its definition.

Peirce: CP 4.633 Cross-Ref:††

633. Cyclic arithmetic involves no other *ordinal*, or climacote, numbers than cyclic ordinals. But if we define a *cardinal* number as an adjective essentially

applicable, universally and exclusively, to a plural of a single multitude, then even the relations α and β may be said to depend upon the value of a cardinal number; namely, upon the modulus of the cycle; and no cardinal number is cyclic. Dedekind and others †1 consider the pure abstract integers to be ordinal; and in my opinion they are not only right, but might extend the assertion to all real numbers. †2 (But what I mean by an ordinal number precisely must be explained further on. †3) Nevertheless, the operations of addition, multiplication, and involution can be more simply defined if they are regarded as applied to cardinals, that is to multitudes, than if they are regarded in their application to ordinals.

Peirce: CP 4.633 Cross-Ref: ††

Thus, the sum of two multitudes, M and N, is simply the multitude of a collection composed of the mutually exclusive collections of the multitudes M and N. The ordinal definition, on the other hand, must be that $0+X = X$, whatever X may be, while (the ordinal next after Y)+X is the ordinal next after (Y+X). So the product of two multitudes M and N is simply the multitude of units each composed of a unit of a collection of multitude M and a unit of multitude N; while the ordinal definition must be that $0X0 = 0$ and that Xx (the ordinal next after Y) is $X+(X-Y)$ and the ordinal next after XxY is $(X-Y)+Y$. So finally the multitude M raised to the power whose exponent is N, is the multitude of ways in which every member of a collection of multitude N can be related in

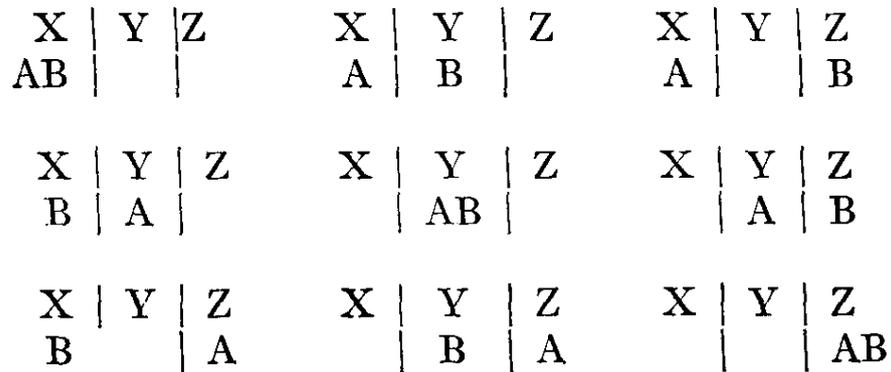


Fig. 254:
[Click here to view]

a given way, each to some single member or other of a collection of multitude M. Thus $3^2 = 9$ because the different configurations of Fig. 254 are nine in number; while $2^3 = 8$ because the different configurations of Fig. 255 are eight in number. But a definition of involution which shall be *purely*

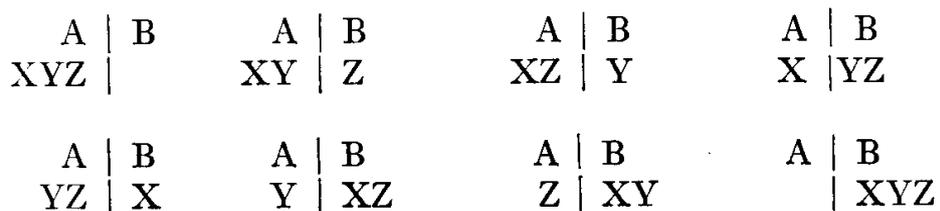


Fig. 255:
[Click here to view]

ordinal must be quite a complicated affair. We may say, for example, that $X^1 = X$ and $X^{1+Y} = X \cdot X^Y$.

Peirce: CP 4.634 Cross-Ref:††

634. In cyclic addition, that is, in the α and β relations, there is but a single cardinal number to be dealt with; and this is fully dealt with in counting round and round the single cycle. But in multiplication there is always another cycle, and thus another cardinal number to be considered, although the modulus of the second cycle is usually such that it is not brought to our attention. But suppose that in a cycle of 72 we multiply the successive integers from zero up by 54. The following will be the result:

$$0X54 = 0 = 72$$

$$1X54 = 54 = -18$$

$$2X54 = 36$$

$$3X54 = 18$$

$$4X54 = 72 = 0$$

It will be seen that there is a cycle of modulus 4. Suppose that, instead of 54, we take 27 as the multiplicand. Then we shall have

$$0X27 = 0 = 72$$

$$1X27 = 27$$

$$2X27 = 54 = -18$$

$$3X27 = 9$$

$$4X27 = 36$$

$$5X27 = 63 = -9$$

$$6X27 = 18$$

$$7X27 = 45 = -27$$

$$8X27 = 72 = 0$$

By halving the multiplicand we have doubled the modulus. Suppose, however, that, instead of $(1/2)X54$, we take $(1/3)X54 = 18$, as the multiplicand. Read the column of successive multiples of 54 upwards, and we shall see that the multiples of 18 have a cycle of modulus 4.

Peirce: CP 4.634 Cross-Ref:††

With 6 as the multiplicand we get a cycle of 12 for its multiples, the numbers being as follows:

$$6, 12, 18, 24, 30, 36, -30, -24, -18, -12, -6, 0$$

Peirce: CP 4.634 Cross-Ref:††

With 2×6 we get a cycle of $(1/2) \times 12$, every other one. With 4×6 as multiplicand, we get a cycle of $(1/4) \times 12 = 3$, with 8×12 as multiplicand; since 3 cannot be halved we still get 3. With $3 \cdot 6 = 18$ as multiplicand; we get a cycle of $(1/3) \times 12$, or every third of the multiples of 6; but with $3 \cdot 18 = 54$ as modulus, since 4 is not divisible by 3, we still get a cycle of 4. With $6 \cdot 6 = 36$ as multiplicand, we get every sixth multiple of 6, or two in all, 0 and 36. With 5×6 , 7×6 , and 11×6 since 12 is not divisible by 5, 7, or 11, we still get a modulus of 12. With 30, the order is as follows:

0, 30, -12, 18, -24, 6, 36, -6, 24, -18, 12, -30, 0.

This principle is obvious: if the multiples of a number N form a cycle of modulus K , and p is a prime number, then the multiples of pN will form a cycle of K/p , provided K is divisible by p ; but otherwise, the modulus will remain K . Suppose, then, that the cycle of multiples of 1, that is to say, the cycle of our entire system of numbers is $pa \cdot qb$ where p and q are primes, and a and b are any whole numbers. If, then, we multiply 1 by $rc \cdot sd \cdot te$ where r, s, t are other primes than p and q , the modulus of the cycle of multiples of $rc \cdot sd \cdot te$ will remain $pa \cdot qb$. But every time we multiply this by p we divide the modulus by p , until we have so multiplied it a times. On the other hand, if, instead of multiplying 1 by $rc \cdot sd \cdot te$ we multiply it by $pa \cdot qb$ to get a new multiplicand, the modulus of the cycle of multiples of $pa \cdot qb$ will be 1; that is, all multiples will be equal. It will follow by the distributive principle, that $pa \cdot qb$ added to any number leaves that number unchanged. That is to say, the modulus of a cycle is the *zero* of that cycle. But right here I must explain what I mean by an *ordinal number*.

Peirce: CP 4.635 Cross-Ref:††

635. Take any enumerable, or finite, collection of distinct objects. Let there be recognized one special relation in which each of them stands to a single one of them, and no two to the same one, and such that any predicate whatsoever that is true of any one of them and is true of the one to which any one of which it is true stands in that relation, is true of all of them. This substantially defines that relation as the relation of "being A'd by." Thereby, that collection is recognized as forming a cyclical system of which those objects are members. But those objects will not in general be numbers of any kind. They may be days of the week or certain meridians of the Globe. But now consider a single "step," or substitution, by which the A of any member of the cyclic system is replaced by the member itself. From what member this step, or substitution, began remains indefinite. The "step" still leads to a single member, and the step is a single kind of step even if that member be any member you please, in which case it is not a single, *i.e.*, a singular, but the general member. I will condescend to meet the reader's probably indurated habit of crass nominalist thought by saying that, in the one case, it is a single member not definitely described, and in the other is a single member, left to him to choose; and there is no objection to this, if the member be supposed to be both existent and intelligible, both of which however it need not be. Give this kind of a step a proper name. Next consider in succession all the kinds of step each of which consists in first taking a step of the last previously considered kind and then substituting for the member which it puts in place of another, the member of which that member is A; so that the kinds of steps may be

From the A of a member to that member,

From the A of the A of a member to that member,

From the A of the A of the A of a member to that member, etc., etc.

Now if each of these has a name, whether pronounced, scribed, or merely thought, those names will come round in a cycle of the same modulus as the original system. They will therefore form a cyclic system, but not a system of objects not essentially ordered, as the original system may have been. This system of names is a cyclic system of numbers. These are ordinal, or climacote, numbers. By ordinal numbers in general I mean names essentially denoting kinds of steps each from any member whatever of a system of objects to, at most, a single object of the system (*i.e.*, one or another object, depending on what object the step replaces by this other). Thus, as I use the term "ordinal number" I do not mean the absolute first, second, third, etc. member of a row of objects, but rather such as these: the same as the first after, the second after, the third before, etc. These numbers are certainly "ordinal" in the sense of expressing relative order; yet it might be better to avoid possible misunderstanding by calling them *metrical numbers*, or more specifically, *climacode* or *climacote numbers*.

Peirce: CP 4.636 Cross-Ref:††

636. In order to push further our study of this subject, let us suppose a pack of 72 cards, numbered in order upon their faces, to be dealt into two piles. We will not directly consider those serial face-values, but only their differences. The two piles cannot regularly be reunited, because the difference of successive face-values in each, comes round in a cycle in each pile, the bottom card of the one pile, 1, being 2 more than the top card 71 (counting round the cycle of modulus 72) and that of the other pile also coming round in a cycle. The difference between the face-values of any two cards in either pile is a multiple of 2, the multiplier being the difference of position in that pile. If now we desire so to re-deal the cards of the one pile and the other into any number n of piles, as to produce the same effect as if they had originally been dealt into $2n$ piles, we must first deal the first pile leaving room between every two of the new piles for the piles to be produced by dealing the second pile. If for the number, n , we take 8, we shall get sixteen piles, the first 8 of 5 cards each and the last 5 of 4; and now it is allowable and proper to place each of the first 8 piles on the pile 8 piles further advanced; or equally so to place each of the last 8 piles on the pile 8 piles further advanced, counting round and round the cycle of modulus 16. In either case the cards of each composite pile so formed will form a cycle, successive face-values increasing (round and round the cycle of 72) by 16. The rule for gathering the piles is just the same as that previously given, except that one must confine oneself to piles *of the same set*. For instance if 72 cards, numbered as just described, get in any way dealt into 15 piles, the top cards of the piles will have these values:

61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 58, 59, 60

Now since $15 \mid 72 = 3$ these are in 5 sets of 3 piles, thus

61, 64, 67, 70, 58,
62, 65, 68, 71, 59,
63, 66, 69, 72, 60.

Peirce: CP 4.636 Cross-Ref:††

We shall therefore put the pile headed by 72 on the pile headed by 69, because there is only one pile of the set to the right of the former, and these on the pile headed by 66, and these on that headed by 63, and finally all four on the one headed by 60. So we shall in the next set begin with the pile headed by 71, the last of the larger piles.

Peirce: CP 4.636 Cross-Ref:††

We shall thus get the whole pack divided into three portions, and there is absolutely no way of getting them back into a single pack except by *undealing* them, that is by cutting the cards one by one from the three portions in turn, round and round.

Peirce: CP 4.636 Cross-Ref:††

This general rule holds in all cases; as much when the entire number of cards is prime as when it is composite. For a prime number is one whose greatest common divisor with any smaller positive integer is 1, while, of course, like any other number, its greatest divisor common to itself is itself.

Peirce: CP 4.637 Cross-Ref:††

637. Having thus fully explained the dealing into any number of piles of any number of cards, prime or composite, I revert, after this almost interminable disquisition, to the subject of cyclic logarithms. I have confined, and shall continue to confine, my study of these to logarithms of numbers whose cycle has a prime modulus. Then, the modulus of the cycle of the logarithms being one less than that of the natural numbers cannot be prime. Still so long as it is a question of employing the logarithms merely to multiply two numbers, the logarithm of the product is simply the sum of the logarithms of multiplier and multiplicand; and in addition it makes no difference whether the modulus be prime or composite. But when it comes to raising numbers to powers or to extracting their roots, the divisors of the number one less than the modulus have to be considered. The modulus being prime, the number one less must be divisible by 2. If 2 be the only prime factor, the modulus must be 3 or 5 or 17 or 65537 or much greater yet. As an example, let us take the modulus 17. Then the following two pairs of tables show the logarithms for the 8 different bases, 3, 5, 6, 7, 10, 11, 12, 14.

See image:

$$\begin{array}{l} \text{Nat. nos.} \left\{ \begin{array}{cccccccccccccccc} -16 & -14 & -8 & -7 & -4 & -12 & -2 & -6 & -1 & -3 & -9 & -10 & -13 & -5 & -15 & -11 & -16 \\ 1 & 3 & 9 & 10 & 13 & 5 & 15 & 11 & 16 & 14 & 8 & 7 & 4 & 12 & 2 & 6 & 1 \end{array} \right. \\ \text{Logs.} \left\{ \begin{array}{cccccccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \\ -16 & -15 & -14 & -13 & -12 & -11 & -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 \end{array} \right. \end{array}$$

$$\begin{array}{l} \text{Nat. nos.} \left\{ \begin{array}{cccccccccccccccc} -16 & -15 & -14 & -13 & -12 & -11 & -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \end{array} \right. \\ \text{Logs.} \left\{ \begin{array}{cccccccccccccccc} 0 & 14 & 1 & 12 & 5 & 15 & 11 & 10 & 2 & 3 & 7 & 13 & 4 & 9 & 6 & 8 \\ -16 & -2 & -15 & -4 & -11 & -1 & -5 & -6 & -14 & -13 & -9 & -3 & -12 & -7 & -10 & -8 \end{array} \right. \end{array}$$

$$\begin{array}{l} \text{Nat. nos.} \left\{ \begin{array}{cccccccccccccccc} -16 & -12 & -9 & -11 & -4 & -3 & -15 & -7 & -1 & -5 & -8 & -6 & -13 & -14 & -2 & -10 \\ 1 & 5 & 8 & 6 & 13 & 14 & 2 & 10 & 16 & 12 & 9 & 11 & 4 & 3 & 15 & 7 \end{array} \right. \\ \text{Logs.} \left\{ \begin{array}{cccccccccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\ -16 & -15 & -14 & -13 & -12 & -11 & -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 \end{array} \right. \end{array}$$

$$\begin{array}{l} \text{Nat. nos.} \left\{ \begin{array}{cccccccccccccccc} -16 & -15 & -14 & -13 & -12 & -11 & -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 \end{array} \right. \\ \text{Logs.} \left\{ \begin{array}{cccccccccccccccc} 0 & 6 & 13 & 12 & 1 & 3 & 15 & 2 & 10 & 7 & 11 & 9 & 4 & 5 & 14 & 8 \\ -16 & -10 & -3 & -4 & -15 & -13 & -1 & -14 & -6 & -9 & -5 & -7 & -12 & -11 & -2 & -8 \end{array} \right. \end{array}$$

[Click here to view]

Peirce: CP 4.637 Cross-Ref:††

Of course, none of the even numbers can be logarithms of a possible base of another system since with a modulus 16 no multiple of an even number can be 1, the logarithm of the base. On the other hand, every odd number is in every system of logarithms the logarithm of some base.

Peirce: CP 4.638 Cross-Ref:††

638. If, instead of 13 cards and 12, the "trick" be done with 17 and 16, say the first eight hearts *increasingly* and then the first eight diamonds *decreasingly*, with the joker or king of hearts to make up 17 and with the first eight spades to correspond with the hearts and the first eight clubs to correspond with the diamonds, laying down the black cards on the table, in *two* rows, one of eight from left to right, and the other below from right to left, after having dealt the black cards 16 times into three piles and every time exchanging the top card of the middle pile for the topmost red card, so as to bring the ace of spades into the right-handmost place of the upper row, then having done the trick substantially as above described, there is a very pretty way in which you can ask into what *odd* number of piles the *black* cards shall be dealt and then dealing out the red cards, *minus* the extra one, 16 times exchanging a card each time for the three court cards and ten of each suit, so as to again render the black ones the index of the places of the red ones. But I leave it to the reader's ingenuity to find out exactly how this is to be done. *Beware of the moduli.*

Peirce: CP 4.638 Cross-Ref:††

There is much more to be said on this subject, but I leave it for the reader to

investigate.

Peirce: CP 4.639 Cross-Ref:††
§3. A NOTE ON CONTINUITY †1E

639. Denumeral is applied to a collection in one-to-one correspondence to a collection in which every member is immediately followed by a single other member, and in which but a single member does not, immediately or mediately, follow any other. A collection is in one-to-one correspondence to another, if, and only if, there is a relation, r , such that every member of the first collection is r to some member of the second to which no other member of the first is r , while to every member of the second some member of the first is r , without being r to any other member of the second. The positive integers form the most obviously denumeral system. So does the system of all real integers, which, by the way, does not pass through infinity, since infinity itself is not part of the system. So does a Cantorian collection in which the endless series of all positive integers is immediately followed by $\{\ddot{o}\}[1]$, and this by $\{\ddot{o}\}[1]+1$, this by $\{\ddot{o}\}[1]+2$, and so on endlessly, this endless series being immediately followed by $2\{\ddot{o}\}[1]$. Upon this follow an endless series of endless series, all positive integer coefficients of $\{\ddot{o}\}[1]$ being exhausted, whereupon immediately follows $\{\ddot{o}\}[1]^2$, and in due course $x\{\ddot{o}\}[1]^2+y\{\ddot{o}\}[1]+z$, where x, y, z , are integers; and so on; in short, any system in which every member can be described so as to distinguish it from every other by a finite number of characters joined together in a finite number of ways, is a denumeral system. For writing the positive whole numbers in any way, most systematically thus:

1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, etc.

it is plain that an infinite square matrix of pairs of such numbers can be arranged in one series, by proceeding along successive bevel lines thus: (1, 1); (1, 10); (10, 1); (1, 11); (10, 10); (11, 1); (1, 100); (10, 11); (11, 10); etc., and consequently whatever can be arranged in such a square can be arranged in one row. Thus an endless square of quaternions such as the following can be so arranged:

[(1, 1) (1,1)]:[(1, 1) (1,10)]; [(1, 1) (10,1)]:[(1,1) (1,11)]; etc.

[(1,10) (1,1)]:[(1,10) (1,10)]; [(1,10) (10,1)]:[(1,10) (1,11)]; etc.

[(10,1) (1,1)]:[(10,1) (1,10)]; [(10,1) (10,1)]:[(10,1) (1,11)]; etc.

[(1,11) (1,1)]:[(1,11) (1,10)]; [(1,11) (10,1)]:[(1,11) (1,11)]; etc.

Consequently whatever can be arranged in a block of any finite number of dimensions can be arranged in a linear succession. Thus it becomes evident that any collection of objects, every one of which can be distinguished from all others by a finite collection of marks joined in a finite number of ways can be no greater than the denumeral multitude. (The bearing of this upon Cantor's $\{\ddot{o}\}^{\{\ddot{o}\}}$ is not very clear to my mind.) But when we come to the collection of all irrational fractions, to exactly distinguish each of which from all others would require an endless series of decimal places, we reach a greater multitude, or grade of maniness, namely, the *first abnumerable multitude*. It is called "abnumerable," to mean that there is not only no way of counting the single members of such a collection so that, at last, every one will have

been counted (in which case the multitude would be *enumerable*), but, further, there is no way of counting them so that every member will after a while get counted (which is the case with the single multitude called *denumeral*). It is called the *first* abnumerable multitude, because it is the smallest of an endless succession of abnumerable multitudes each smaller than the next. For whatever multitude of a collection of single members $\{m\}$ may denote, $2^{\{m\}}$, or the multitude of different collections, in such collection of multitude $\{m\}$, is always greater than $\{m\}$. The different members of an abnumerable collection are not capable of being distinguished, each one from all others, by any finite collection of marks or of finite sets of marks. But by the very definition of the first abnumerable multitude, as being the multitude of collections (or we might as well say of denumeral collections) that exist among the members of a denumeral collection, it follows that all the members of a first-abnumerable collection are capable of being ranged in a linear series, and of being so described that, of any two, we can tell which comes earlier in the series. For the two denumeral collections being each serially arranged, so that there is in each a first member and a singular next later member after each member, there will be a definite first member in respect to containing or not containing which the two collections differ, and we may adopt either the rule that the collection that contains, or the rule that the collection that does not contain, this member shall be earlier in the series of collections. Consequently a first abnumerable collection is capable of having all its members arranged in a linear series. But if we define a *pure* abnumerable collection as a collection of all collections of members of a denumeral collection each of which includes a denumeral collection of those members and excludes a denumeral collection of them, then there will be no two among all such pure abnumerable collections of which one follows next after the other or of which one next precedes the other, according to that rule. For example, among all decimal fractions whose decimal expressions contain each an infinite number of 1s and an infinite number of 0s, but no other figures, it is evident that there will be no two between which others of the same sort are not intermediate in value. What number for instance is next greater or next less than one which has a 1 in every place whose ordinal number is prime and a zero in every place whose ordinal number is composite? .11101010001010001010001000001 etc. Evidently, there is none; and this being the case, it is evident that all members of a pure second-abnumerable collection, which both contains and excludes among its members first-abnumerable collections formed of the members of a pure first-abnumerable collection; cannot, in any *such* way, be in any linear series. Should further investigation prove that a second-abnumeral multitude can in *no way* be linearly arranged, my former opinion †1 that the common conception of a line implies that there is room upon it for any multitude of points whatsoever will need modification.

Peirce: CP 4.640 Cross-Ref: ††

640. Certainly, I am obliged to confess that the ideas of common sense are not sufficiently distinct to render such an implication concerning the continuity of a line evident. But even should it be proved that no collection of higher multitude than the first abnumerable can be linearly arranged, this would be very far from establishing the idea of certain mathematico-logicians that a line consists of points. †1 The question is not a physical one: it is simply whether there can be a consistent conception of a more perfect continuity than the so-called "continuity" of the theory of functions (and of the differential calculus) which makes the continuum a first abnumerable system of points. It will still remain true, after the supposed demonstration, that no collection of points, each distinct from every other, can make

up a line, no matter what relation may subsist between them; and therefore whatever multitude of points be placed upon a line, they leave room for the same multitude that there was room for on the line before placing any points upon it. This would generally be the case if there were room only for the denumeral multitude of points upon the line. As long as there is certainly room for the first denumerable multitude, no denumeral collection can be so placed as to diminish the room, even if, as my opponents seem to think, the line is composed of actual determinate points. But in my view the unoccupied points of a line are mere possibilities of points, and as such are not subject to the law of contradiction, for what merely *can be* may also *not be*. And therefore there is no cutting down of the possibility *merely* by some possibility having been actualized. A man who can see does not become deprived of the power merely by the fact that he has seen.

Peirce: CP 4.641 Cross-Ref:††

641. The argument which seems to me to prove, not only that there is such a conception of continuity as I contend for, but that it is realized in the universe, is that if it were not so, nobody could have any memory. If time, as many have thought, consists of discrete instants, all but the feeling of the present instant would be utterly non-existent. But I have argued this elsewhere.†2 The idea of some psychologists of meeting the difficulties by means of the indefinite phenomenon of the span of consciousness betrays a complete misapprehension of the nature of those difficulties.

Peirce: CP 4.642 Cross-Ref:††

642. *Added*, 1908, May 26. In going over the proofs of this paper, written nearly a year ago, I can announce that I have, in the interval, taken a considerable stride toward the solution of the question of continuity, having at length clearly and minutely analyzed my own conception of a *perfect continuum* as well as that of an *imperfect continuum*, that is, a continuum having *topical singularities*, or places of lower dimensionality where it is interrupted or divides. These labors are worth recording in a separate paper, if I ever get leisure to write it.†1 Meantime, I will jot down, as well as I briefly can, one or two points. If in an otherwise unoccupied continuum a figure of lower dimensionality be constructed -- such as an oval line on a spheroidal or anchor-ring surface -- either that figure is a part of the continuum or it is not. If it is, it is a topical singularity, and according to my concept of continuity, is a breach of continuity. If it is not, it constitutes no objection to my view that all the parts of a perfect continuum have the same dimensionality as the whole. (Strictly, all the *material*, or *actual* parts, but I cannot now take the space that minute accuracy would require, which would be many pages.) That being the case, my notion of the essential character of a perfect continuum is the absolute generality with which two rules hold good, first, that every part has parts; and second, that every sufficiently small part has the same mode of immediate connection with others as every other has. This manifestly vague statement will more clearly convey my idea (though less distinctly) than the elaborate full explication of it could. In endeavoring to explicate "immediate connection," I seem driven to introduce the idea of time. Now if my definition of continuity involves the notion of immediate connection, and my definition of immediate connection involves the notion of time; and the notion of time involves that of continuity, I am falling into a *circulus in definiendo*. But on analyzing carefully the idea of Time, I find that to say it is continuous is just like saying that the atomic weight of oxygen is 16, meaning that that shall be the standard for all other atomic weights. The one asserts no more of Time than the other asserts concerning the atomic weight of oxygen; that is, just nothing at all. If we are to suppose the idea of Time is wholly an affair of immediate consciousness, like the idea

of royal purple, it cannot be analyzed and the whole inquiry comes to an end. If it can be analyzed, the way to go about the business is to trace out in imagination a course of observation and reflection that might cause the idea (or so much of it as is not mere feeling) to arise in a mind from which it was at first absent. It might arise in such a mind as a hypothesis to account for the seeming violations of the principle of contradiction in all alternating phenomena, the beats of the pulse, breathing, day and night. For though the *idea* would be absent from such a mind, that is not to suppose him blind to the *facts*. His hypothesis would be that we are, somehow, in a situation like that of sailing along a coast in the cabin of a steamboat in a dark night illumined by frequent flashes of lightning, and looking out of the windows. As long as we think the things we see are the same, they seem self-contradictory. But suppose them to be mere aspects, that is, relations to ourselves, and the phenomena are explained by supposing our standpoint to be different in the different flashes. Following out this idea, we soon see that it means nothing at all to say that time is unbroken. For if we all fall into a sleeping-beauty sleep, and *time itself stops during the interruption*, the instant of going to sleep is absolutely unseparated from the instant of waking; and the interruption is merely in our way of thinking, not in time itself. There are many other curious points in my new analysis. Thus, I show that my true continuum might have room only for a denumeral multitude of points, or it might have room for just any abnumeral multitude of which the units are in themselves capable of being put in a linear relationship, or there might be room for all multitudes, supposing no multitude is contrary to a linear arrangement.

Peirce: CP 4.643 Cross-Ref:††
CHAPTER 2

A SECOND CURIOSITY †1

643. A phenomenon easier to understand depends on the fact that, in counting round and round a cycle of 53 numbers, $\sqrt{-1} = \pm 30$. (For $30^2 = 900 = 17 \cdot 53 - 1$.) This, likewise, may be exhibited in the form of a "trick." You begin with a pack of 52 playing cards arranged in regular order. For this purpose, it is necessary to assign ordinal numbers to the four suits. It seems appropriate to number the spade-suit as 1, because its ace carries the maker's trade-mark. I would number the heart-suit 2, because the pips are partially cleft in two; the club-suit 3, because a "club," as the French term *trèfle* reminds us, is a trefoil; and the diamond-suit as 4 or 0, because the pips are quadrilaterals, and counting round and round a cycle of 4, $4 = 0$. But it is convenient, in numbering the cards, to employ the system of arithmetical notation whose base is 13. It will follow that if the cards of each suit are to follow the order 1 2 3 4 5 6 7 8 9 X J Q K, the king of each suit must be numbered as if it were a zero-card of the following suit. The inconvenience of this is very trifling compared with the convenience of directly availing oneself of a regular system of notation; for the exhibitor of the "trick" will have many a "long multiplication" to perform in his head, as will shortly appear. Another slight inconvenience is that the cycle of numeration must be fifty-three, or $4\spadesuit$, which, or its highest possible multiple, must be subtracted

from every product that exceeds 4♠. It is to be remembered that ♦, ♠, ♥, ♣, are used as nothing but other shaped characters for 0, 1, 2, 3 respectively. Thirteen is the base of numeration, but fifty-three, or 4♠, is the cycle of numeration. I adopt ♦, rather than K, as the zero-sign in order to avoid denoting the king of diamonds by ♠K, etc. In order to exhibit the trick in the highest style, the performer should have this multiplication table by heart in which I have been forced to put 10 in place of x most incongruously simply because I am informed that the latter would transcend the resources of the printing office.

Peirce: CP 4.643 Cross-Ref:††

Yet I do it quite passably without possessing that accomplishment. In those squares of the multiplication-table where two lines are occupied, the upper gives the simple product in tridecimal notation, and the lower the remainder of this after subtracting the highest less multiple of fifty-three, *i.e.*, of 4♠.

♦♠	♦♥	♦♣	♦4	♦5	♦6	♦7	♦8	♦9	♦10	♦J	♦Q
♦♥	♦4	♦6	♦8	♦10	♦Q	♠♠	♠♠	♠5	♠7	♠9	♠J
♦♣	♦6	♦9	♦Q	♠♥	♠5	♠8	♠J	♥♠	♥4	♥7	♥10
♦4	♦8	♦Q	♠♠	♠7	♠J	♥♥	♥6	♥10	♠♠	♠5	♠9
♦5	♦10	♠♥	♠7	♠Q	♥4	♥9	♠♠	♠6	♠J	4♠	48
♦6	♦Q	♠5	♠J	♥4	♥10	♠3	♠9	4♥	48	5♠	57
♦7	♠♠	♠8	♥♥	♥9	♠♠	♠10	44	4J	55	5Q	66
♦8	♠♠	♠J	♥6	♠♠	♠9	44	4Q	57	6♥	610	75
♦9	♠5	♥♠	♥10	♠8	4♥	4J	57	6♠	6Q	78	84
♦10	♠7	♥4	♠♠	♠J	48	55	6♥	6Q	79	88	9♠
♦J	♠9	♥7	♠5	4♠	5♠	5Q	610	78	86	94	10♥
♦Q	♠J	♥10	♠9	48	57	66	75	84	9♠	10♥	J♠
				♦7	♠6	♥5	♠4	♦♥	♠♠	♥♦	♥Q

Fig. 256
here to view]

Fig. 256

[Click

Peirce: CP 4.644 Cross-Ref:††

644. In order to exhibit the trick, while you are arranging the cards in regular order, you may tell some anecdote which involves some mention of the numbers 5 and 6. For instance, you may illustrate the natural inaptitude of the human animal for mathematics, by saying how all peoples use some multiple of 5 as the base of numeration, because they have 5 fingers on a hand, although any person with any turn for mathematics would see that it would be much simpler, in counting on the fingers, to use 6 as the base of numeration. For having counted 5 on the fingers of one hand,

one would simply fold a finger of the other hand for 6, and then make the first finger of the first hand to continue the count. The object of telling this anecdote would be to cause the numbers 5 and 6 to be uppermost in the minds of the company. But you must be very careful not at all to emphasize them; for if you do, you will cause their avoidance. The pack being arranged in regular sequence, you ask the company into how many piles you shall deal them, and if anybody says 5 or 6, deal into that number of piles. If they give some other number, manifest not the slightest shade of preference for one number of piles over another; but have the cards dealt again and again, until you can get for the last card either ♠x, that is, the ten of the second suit (*i.e.*, suit number one; since the first suit is numbered ♦, or zero), or ♥4, the four of the third suit, or ♠6, or ♥8. If you cannot influence the company to give you any of the right numbers, after they have ordered several deals, you can say, "Now let me choose a couple of numbers," and by looking through the pack, you will probably find that one or other of those can be brought to the face of the pack in two or three deals. For every deal multiplies the ordinal place of each card by a certain number, counting round and round a cycle of 53. And this multiplier is that number which multiplied by the number of piles in the deal gives +1 or -1 in counting round and round the cycle of 53. For it makes no difference to which end of the pack the card is drawn. After each deal the piles are to be gathered up according to the same rule as in the first "trick," except that the first pile taken must not be the one on which the fifty-second card fell, but the one on which the fifty-third would have fallen if there had been 53 cards in the pile. The last deal having been made, you lay all the cards now, backs up, in 4 rows of 13 cards in each row, leaving small gaps between the third and fourth and sixth and seventh cards counting from each end, thus:

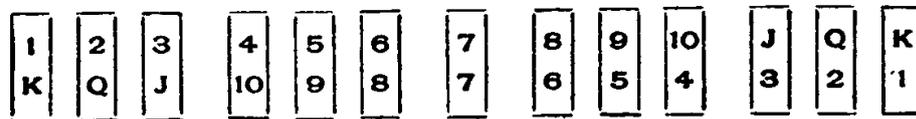


Fig. 257

Fig. 257

[Click here to view]

Peirce: CP 4.644 Cross-Ref:††

The object of these gaps is to facilitate the counting of the places from each end, both by yourself and by the company of onlookers. If the first or last card is either ♠x or ♥4, the first card of the pack will form the left-hand end of the top row, and each successive card will be next to the right of the previously laid card, until you come to the end of a row, when the next card will be the extreme left-hand card of the row next below that last formed. But if the first or last card is either ♠6 or ♥8, you begin at the top of the extreme right-hand column, and lay down the following three cards each under the last, the fifth card forming the head of the column next to the left, and so on, the cards being laid down in successive columns, passing downward in each column, and the successive columns toward the right being formed in regular order.

Peirce: CP 4.644 Cross-Ref:††

You now explain to the company, very fully and clearly, that the upper row consists of the places of the diamonds; and you count the places, pointing to each, thus "Ace of diamonds, two of diamonds, three; four, five, six; the seven, a little

separated, the eight, nine, and ten, together; then a little gap, and the knave, queen, king of diamonds together. The next row is for the spades in the same regular order, from that end to this (you will not say "right" and "left," because the spectators will probably be at different sides of the table), next the hearts, and last the clubs. Please remember the order of the suits, diamond (you sweep your finger over the different rows successively), spades, hearts, and clubs. But (you continue), those are the places beginning at *that* (the upper left-hand) corner. In addition, every card has a *second* place, beginning at *this* opposite corner (the lower right-hand corner). The order is the same; only you count backwards, toward the right in each row; and the order of the suits is the same, diamonds, spades, hearts, clubs; only the places of the diamonds are in the bottom row, the places of the spades next above them, the places of the hearts next above them, and the clubs at the top. These are the regular places for the cards. But owing to their having been dealt out so many times, they are now, of course, all out of both their places." You now request one of the company (not the least intelligent of them) simply to turn over any card in its place. Suppose he turns up the fifth card in the third row. It will be either the ♥3 or ♠J. Suppose it is the former. Then you say, "Since the three of hearts is in the place of the five of hearts, counting from *that* corner, it follows *of course*" (don't omit this phrase, nor emphasize it; but say it as if what follows were quite a syllogistically evident conclusion), "that the five of hearts will be in the place of the three of hearts counting from the opposite corner." Thereupon, you count "Spades, hearts: one, two, three," and turn up the card, which, sure enough, will be ♥5. "But," you continue, "counting from the first corner, the five of hearts is in the place of the knave of spades, and accordingly, the knave of spades will, of course, be in the place of the five of hearts, counting from the opposite corner." You count, first, to show that ♥5 is in the place of ♠J, and then, always pointing as you count, and counting, first the rows, by giving successively the names of the suits, "diamonds, spades, hearts," and then the places in the row, "one, two, three, four, five," and turning up the card you find it to be, as predicted, the ♠J. "Now," you continue, "the knave of spades is in the place of the nine of spades counting from the first corner, so that we shall necessarily find the nine of spades in the place of the knave of spades counting from the opposite corner." You count as before, and find your prediction verified. (I will here interrupt the description of the "trick" to remark that the number of different arrangements of the fifty-two cards all possessing this same property is thirty-eight thousand three hundred and eighty-two billions (or millions squared), three hundred and seventy-six thousand two hundred and sixty-six millions, two hundred and forty thousand, = $6 \times 10 \times 14 \times 18 \times 22 \times 26 \times 30 \times 34 \times 38 \times 42 \times 46 \times 50$, not counting a turning over of the block as altering the arrangement. But of these only one arrangement can be produced by dealing the cards according to our general rule. Either of the four *simplest* arrangements having the property in question will be obtained by first laying out the diamonds in a row so that the values of the cards increase regularly in passing along the row in either direction, then laying out the spades in a parallel row either above or below the diamonds, but leaving space for another row between the diamonds and spades, their values increasing in the counter-direction to the diamonds, then laying out the hearts in a parallel row close upon the other side of the diamonds, their values increasing in the same direction as the spades, and finally laying out the clubs between the diamond-row and the spade-row, their values increasing in the same direction as the former.

Peirce: CP 4.644 Cross-Ref: ††

Not to let slip an opportunity for a logical remark, let me note that, *in itself*

considered, i.e., regardless of their sequence of values, any one arrangement of the cards is as *simple* as any other; just as any continuous line that returns into itself, without crossing or touching itself, or branching, is just as simple, *in itself*, as any other; and relatively to the sequence of values of the cards, only, the arrangement produced in "trick," in which the value of each card is i times the ordinal number of its place, where $i = \pm\sqrt{-1}$, is far simpler than the arrangement just described. But in calling the latter arrangement the "simpler," I use this word in the sense that is most important in logical methodetic; namely, to mean more facile of human imagination. (We form a detailed icon of it in our minds more readily.)

Peirce: CP 4.644 Cross-Ref:††

You now promptly turn down again the four cards that have been turned up (for some of the company may have the impression that the proceeding might continue indefinitely; and you do not wish to shatter their pleasing illusions), and ask how many piles they would like to have the cards dealt in next. If they mention 5 or 6, you say, "Well we will deal them into 5 and 6. Or shall we deal them into 4, 5, 6? Or into 2 and 7? Take your choice." Which ever they choose, you say, "Now in what order shall I make the dealings?" It makes no difference. But how the cards are to be taken up will be described below. After gathering the cards in the mode described in the next paragraph, deal them out, *without turning the cards up*. (I have never tried what I am now describing; but for fear of error, I shall do so before my article goes to press.) After that, you say, "Oh, I don't believe they are sufficiently shuffled. I will milk them." You proceed to do so. That is, holding the pack backs up, you take off the cards now at the top and bottom, and lay them backs up, the card from the bottom remaining at the bottom; and this you repeat 25 times more, thus exhausting the pack. Many persons insist that the proper way of milking the cards is to begin by putting the card that is at the back of the pack at its face; but when I speak of "milking," I mean this not to be done. Having milked the pack three times, you count off the four top cards (*i.e.*, the cards that are at the top as you hold the pack with the faces down) one by one from one hand to the other, putting each card above the last, so as to reverse their positions. You then count the next four into the same receiving hand, *under* the four just taken, so that their relative positions remain the same. The next four are to be counted, one by one, upon the first four, so that their relative positions are reversed, and the next four are to be counted into the receiving hand under those it already holds. So you proceed alternately counting four to the top and four to the bottom of those already in the receiving hand, until the pack is exhausted. You then say, "Now we will play a hand of whist." You allow somebody to cut the cards and deal the pack, as in whist, one by one into four "hands," or packets, turning up the last card for the trump. It will be found that you hold all the trumps, and each of the other players the whole of a plain suit.

Peirce: CP 4.645 Cross-Ref:††

645. I now go back to explain how the cards are to be taken up. If it is decided that the cards are to be dealt into 5 and into 6 piles (the order of the dealing always being immaterial), you take them up row by row, in consecutive order, from the upper left-hand to the lower right-hand corner. If they are to be dealt into 4, 5 and 6 piles, or into 2 and 7 piles, in any order, you take them up column by column, from the upper right-hand to the lower left-hand corner. The exact reversal of all the cards in the pack will make no difference in the final result. They may also be taken up in columns and dealt into piles whose product is 14 or 39 (as, for example, into 2 piles and 7 piles, or into 3 piles and 13 piles). They may be taken up in rows and dealt into any number of piles whose product is thirty, or, by the multiplication table is ♠4. The following are

some of the sets of numbers whose products, counted round a cycle of 53, equal 30:
 6·5; 17·8; 7·5·4·4; 9·7·3; 9·8·7·7; 9·6·6·5; 9·9·5·4; X·8·7; X·9·8·7·6; J·J·2; J·8·4·4;
 J·5·5·3; Q·X·X·4; Q·X·8·5; Q·7·7·6; K·K·3; ♠X·♠X·4 (decimally, 23·13·4); ♠6·♠4·6;
 ♠5·♦9·♦X.

Peirce: CP 4.645 Cross-Ref:††

The products of the following sets count round a cycle of 53 to $-30 = 23$;
 4·♠6; 2·7·K; K·Q·X; 8·6·6; 9·8·4; X·X·5; Q·J·7; Q·Q·2; 5·5·5·4; 6·4·4·3; X·9·7·5;
 J·7·6·2; 11·7·4·3; 13·X·6·2; 13·8·5·3; 7·6·5·5·3; 7·7·7·5·4; 9·7·5·5·2; 11·6·5·4·3;
 9·8·8·5·4·4; 8·8·7·7·4·4; 11·8·7·7·2·2; 12·11·9·8·7·6.

Peirce: CP 4.645 Cross-Ref:††

The products required to prepare the cards for being laid down column by
 column are ♠6, decimally expressed, 19; and ♥8, decimally expressed, 34.

Peirce: CP 4.645 Cross-Ref:††

The following are some of the sets of numbers whose continued products are
 19: 9·8; Q·6; 5·5·5; 6·4·3; J·7·3; 13·6·5; 13·10·3; 8·7·6·4; 9·9·8·6; J·9·5·4; 11·10·9·2;
 12·8·7·7; 13·10·8·7; 9·8·8·5·4; 10·7·7·6·5; 10·10·10·10·2; 12·7·7·5·5; 7·4·4·4·3;
 13·7·4·4·4·4·4; 4·4·4·4·4·4·4·4·3. The following are sets of numbers whose
 continued product is 34: ♠4·2; ♠X·K; 29·3; 7·5·4; 9·3·♠·♥; 9·9·5; X·7·2; J·8·4;
 Q·X·X; 17·11·5; 17·12·9; 19·13·4; 23·11·6; 23·13; 23·17·7; 41·3·2; 5·5·4·4·3; 9·7·7·6·3;
 8·6·5·5; 9·9·7·7·2; 13·13·7·2; 17·12·9; 8·4·4·4·4; 2·2·2·2·2·2·2·2·2·11·10·7·5;
 13·12·9; 23·13.

Peirce: CP 4.646 Cross-Ref:††

646. This "trick" may be varied in endless ways. For example, you may
 introduce the derangement that is the inverse of milking. That is, you may pass the
 cards, one by one, from one hand to the other, placing them alternately at the top and
 the bottom of the cards held by the receiving hand. Twelve such operations will bring
 the cards back to their original order. But a pack of 72 cards would be requisite to
 show all the curious effects of this mode of derangement.

Peirce: CP 4.647 Cross-Ref:††
 CHAPTER 3

ANOTHER CURIOSITY†1

§1. COLLECTIONS AND MULTITUDES

647. A character which is not sometimes true and sometimes false of the same
 singular is a *kind*. A kind may not exist at all; or it may exist in but one sole singular,
 which the old logics used to say was the case with the kind called sun. Two kinds
 may, neither of them, exist except in singulars in which the other exists; and when
 this is the case, they are said to be *coextensive*. If two kinds, A and B, are so related

that of whatever singulars A could possibly be true, B would necessarily also be true, then A is said to *involve* B.

Peirce: CP 4.647 Cross-Ref:††

This necessity may be of any of the modes of necessity. In particular, if A involves B because of the definitions, or very ideas, of the two kinds, A is said *essentially to involve*, or, in other words, to *imply* B. A kind all whose singulars seem, according to experience, normally to belong to other kinds not implied in the former kind, is called (especially if the other kinds are numerous) a *natural* kind.†2

Peirce: CP 4.648 Cross-Ref:††

648. I consider a *kind* to be an *ens rationis*, although that may be open to dispute, at least as regards some kinds; but there can, I think, be no doubt that a *class* is an *ens rationis*. For a *class*, unlike a *kind*, is not a character, but is the totality of all those singulars that possess a definite existent character, which is the *essential character* of the class.†3 Should observation show that two classes having different essential characters embraced the very same singulars, then since it is the singulars, and not the kinds, that constitute the existence of the class, we should say that the two classes, though *entitatively*, that is, in their possibilities, they were diverse, were yet *existentially* one. Such, I think, is the modern notion of a class,†1 though I must confess that it appears to me to be rather hazy. The characters which go to define a class are not necessarily permanent characters of the singulars, as a kind is. On the contrary we speak with perfect propriety of the class of human males between the ages of fourteen and twenty-one, though there is evidently no such *kind*. In fluid with viscosity the belonging to a given vortex would be a kind; for once in that vortex, particles would for all eternity be in it. But the particles of a given wave, though not of one kind, would be of one class, which would endure as long as the wave endured; that is, forever. But the singulars would be continually passing in and passing out of that class, as those of the adolescent class do. Yet if the wave were to subside and cease to exist, that class would cease to be; or if the fecundity of a population were to be destroyed, after a few years the class of adolescents would cease to be a class. Such appears to be the notion of a class, whether it be consistent or not.

Peirce: CP 4.649 Cross-Ref:††

649. A *collection* or *plural* is different.†2 Here there always must have been some characters common and peculiar to all the singulars, however trifling and unnoticed they may have been. They have, for example, the common and peculiar character of having been chosen to go to the making up of the collection. The word *collection* does not imply that the singulars themselves are gathered together or are, in any way, externally affected. It is only the ideas of them that are grouped. Though we may strive to make our collection as promiscuous as possible, yet in spite of all our efforts, it always must embrace whatever there may be in the universe that has a certain character, and it will embrace nothing else. The essential characters of all possible collections are of one and the same type; each such character consists in all those characters that are common and peculiar to such objects as have the character of having been taken on some definite and determinate occasion to be included in one collection. Thus, if I on Monday consider the collection composed of Don Quixote's helmet, the procession of the equinoxes, Jean Dare's children, and the star Mira Cete at its maximum brilliancy, and you on Wednesday, without knowing of my collection determine to take the first, second, and fourth of those objects as a collection, or lot, then, because Jean Dare had no children, your collection and mine will be identically the same collection, having precisely the same essential character.

Peirce: CP 4.650 Cross-Ref:††

650. In like manner, two apparent and highly interesting ornithological collections, the one of whatever phoenixes there ever were or will be, the other of whatever cockatrices there are at this moment, are one and the same collection, having one and the same essential character. It is that quite unique collection that goes by the name of *Nothing*. Some writers whose logical conceptions would seem to be in a state of disintegration have supposed the collection whose sole member is Gaius Julius Caesar to be identical with Gaius Julius Caesar himself -- a strange confusion considering that the latter was a man of immense force of intellect who was brought into the world by a grossly unskillful operation of surgery, while the other is nothing but an *ens rationis* brought into being by the idea of that man being chosen without any surgery at all and utterly deprived of any force of intellect or life. So likewise that pair of objects which consists of Julius Caesar himself and the collection whose sole member is Julius Caesar is very different from the pair that consists of Julius Caesar himself and the collection whose sole member is the collection whose sole member is Julius Caesar.

Peirce: CP 4.651 Cross-Ref:††

651. The conception of *multitude* which is now current among mathematico-logicians upon which I am unable to make any substantial improvement is due to a remarkable definition of the relation of equality of collections first put forward in the book *Paradoxes of the Infinite*†1 of Bernardo Bolzano, a catholic priest at Buda-Pesth, and the author of a logic in four volumes. Since he was in the priesthood at the time he made this notable contribution to the clearness of human conceptions, it is needless to say that he was severely punished for an act so contrary to the sacerdotal functions. Without altering the main idea of Bolzano, I shall modify the definition as follows: Any collection or plural, say that of the X's, is *more or greater* than any collection, say that of the Y's, if and only if, there is no relation *r* whatsoever, such that every X stands in the relation *r* to a Y to which no other X stands in this same relation, *r*.

Peirce: CP 4.652 Cross-Ref:††

652. I give this form of the definition because it is one that I have employed for a demonstration.†1 But there is another which is closer to the original idea of Bolzano, and which has the great merit of not masking the intrinsic absurdity of the whole idea. It is this. Let us call a *substitution*†2 a dyadic relation in which every singular in the universe stands to one and only one singular and in which to every singular in the universe stands one and only one singular. Then the X's are more multitudinous than the Y's if and only if, whatever substitution be considered, there is some X which is not in this relation to any Y, or, in other words, if and only if there is no substitution such that every X stands in this relation to a Y. As long as we deal with a universe the multitude of whose singulars is not abnumerable this definition involves no absurdity, so that it will answer very well for all enumerable multitudes. But if we are to consider all multitudes, it would be necessary that the universe should be a collection of units of a multitude such that it would be absurd to suppose a greater multitude. Now since I have proved in Vol. VII of the *Monist*†3 that there can be no maximum possible multitude, we have here an absurdity to begin with. Then when we come to speak of *every possible* substitution, this supposes a collection which, if M be the impossible maximum multitude, has a multitude equal to M! or greater than the impossible greatest multitude. Not that that adds anything to the absurdity, since that maximum multitude would, according to the definition, be

greater than itself, and therefore it could not be identical with itself. There are still other points of view from which the arrant nonsense of it appears. For the only thing that exceeds the manifoldness of all collections is a continuum. Therefore to speak of every possible substitution is equivalent to speaking of the collection of *all possible curves* in which, regardless of continuity, there is but one value of F for each value of x , which is absurd since it supposes a smallest possible distance between successive values of x , or, to put it better, supposes this continuum to be utterly discontinuous, without any continuity at any point.

Peirce: CP 4.653 Cross-Ref:††

653. The truth is that Bolzano's definition, if it is to be applied to *all* collections, must be replaced by one which does not introduce the idea of *all possible collections*, since that idea is intrinsically absurd. But if we confine ourselves to finite multitudes, or even to any fixed multitude, such as that of all possible irrational quantities, the absurdity disappears. I am not prepared to give any better definition of fewer and manier. I should like to have the leisure to work at the problem, for, since my paper in Vol. VII of the *Monist*, I have had ten good years of training in logic and am much stronger in it than I was then. However, I am to bethink me that in order to get time to make what work I have done generally useful before extreme old age overtakes me, I must leave new problems or difficulties to another generation, however much they may tempt me.

Peirce: CP 4.654 Cross-Ref:††

654. In my eagerness to express myself, I have permitted myself to talk of *multitude* without defining it. It is that respect in which discrete collections of singulars of which one is greater than the other disagree. It has two denumeral series of absolute grades, the one consisting of all *multitudes*, that is, of all absolute grades of multitudes such that the count of any collection of any such grade of multitude can be completed, which multitudes are distinguished by the cardinal numbers proper, that is, the finite cardinal numbers; these grades of enumerable multitude running from 0 up endlessly, are followed by another similar series of abnumerable multitudes, beginning with the multitude of abnumeration zero, which is the multitude of a simply endless succession of singulars; and each following multitude being the multitude of all the possible collections that can be formed of the singulars of a collection of the next lower multitude, so that this second and last series of multitudes forms another simply endless series.†1

Peirce: CP 4.655 Cross-Ref:††

655. The lowest multitude is *None*. It increases by a step as a singular is affixed to any collection whose multitude it is, and this goes on endlessly, for all the finite multitudes, that is to say for all collections for which the following is necessarily true: "If every singular of the collection of Hottentots kills a singular of the same collection, and if no singular of that collection is killed by more than one singular of that collection, then every singular of that collection is killed by a singular of that collection."†1 Of course in place of "killing," any other dyadic relation may be substituted, and in place of the "Hottentots," any other plural, or collection, may be substituted.

Peirce: CP 4.656 Cross-Ref:††

656. That multitude which is greater than any such multitude but is not greater than any other multitude, is termed the *denumeral* multitude, which in the higher, or second, series of multitudes corresponds to *zero* in the lower, or first, series. After it

follow one by one an endless series of abnumerable multitudes. Yet so far as I know (I am not acquainted with the work of Borel, †2 of which I have only quite vaguely heard), it has never been exactly proved that there are no multitudes between two successive abnumerable multitudes, nor, which is more important, that there is no multitude greater than all the abnumerable multitudes. Each abnumerable multitude after the denumeral multitude is the multitude of all possible collections whose singulars are members of a collection whose multitude is the next lower abnumerable multitude, the denumeral multitude being considered as the abnumerable multitude of grade Zero.

Peirce: CP 4.657 Cross-Ref: ††
§2. CARDINAL AND ORDINAL NUMBERS

657. The *cardinal numbers*, strictly understood, are vocables or written signs, of which one is attached to each finite multitude. But Cantor uses the term cardinal number to mean any multitude whatsoever. †3 According to me, the proper extension of cardinal numbers consists in taking in the *arithms*, or indices, of abnumerable multitudes, which I have explained in Vol. VII of the *Monist*. †4

Peirce: CP 4.658 Cross-Ref: ††

658. Let me now discuss after the fashion of a scholastic disputation the following

Peirce: CP 4.658 Cross-Ref: ††

Question: *Whether the cardinal or the ordinal numbers are the pure and primitive mathematical numbers.*

Peirce: CP 4.658 Cross-Ref: ††

It would seem that the cardinals are so: for

Peirce: CP 4.658 Cross-Ref: ††

Firstly. All the writers of arithmetic books say so: Fibonacci, †1 or Leonardo of Pisa, the effective introducer into Europe of what we call the Arabic system of numerical notation in 1205, although Geber [Gerbert], who became Pope Sylvester II in 999, had brought the figures to Europe and had taught arithmetic in his school, and although I believe that passage of the Geometry of Boëthius †2 to be genuine which gave the forms of the characters representing the nine digits about A.D. 500; the Saxon Jordanus Nemorarius, †3 another early [thirteenth] century mathematician; the English Johannes Sacrobosco (Hollivood), †4 Roger Bacon, †5 Adelard of Bath †6; Thomas Bradwardine, †7 the Doctor Profundus of Merton College, made Archbishop of Canterbury in [1349], who anticipated and outstripped our most modern mathematico-logicians, and gave the true analysis of continuity; the Cardinal of Cusa †8; Prosdocimo de' Beldamandi, †9 the Bamberg *Rechenbuch*, †10 the first printed arithmetic; Johannes Widmann †11 who first used the signs + and -- about as we now do; the Arithmetic of Treviso of 1478 †12; the *Arithmetica* of Borgi of 1484 †13; Luca Paciolo †14; *Le Triparty en la Science des Nombres* †15 which gives the words Byllion, etc., up to Nonyllion "et ainsi des aultres se plus oultre on voulait proceder," and who seemed first to have virtually used negative exponents; Oronce Fine †1; Michael Stifel †2; Cuthbert Tunstall, Bishop of London, *De arte Supputandi*, †3 which he wrote as a farewell to science on taking holy orders; Robert Recorde in his

celebrated *Grounde of Artes* †4; Masterson †5; Blundeville †6; Hylles †7; Oughtred †8; Cocker †9 who gave the name to the most famous of all English arithmetics, though it seems pretty clear that he did not write it and never saw it; Pliny Earl Chase †10 who wrote the best introduction to the art I ever saw; from which I learned to cipher as a boy; and though he wrote (probably under the influence of idiotic publishers) several very inferior arithmetics, I never saw but the one copy of his only excellent work, the one I studied in school at my father's dictation; but I still often refer to the arithmetic of Pliny Earl Chase and Horace Mann †11; all these make cardinals the fundamental numbers;

Peirce: CP 4.658 Cross-Ref: ††

Secondly. The forms of the words in all languages show the cardinals to be the oldest; and since they thus appear to have been first conceived, that conception must be the simplest;

Peirce: CP 4.658 Cross-Ref: ††

Thirdly. Any person whose head is not cracked by too much study of logic will say without hesitation that the cardinals are the original numbers. It is common-sense; and common-sense is the safest guide.

Peirce: CP 4.658 Cross-Ref: ††

Fourthly. It is impossible to form a clear conception of multiplication without resort to cardinals; thus 3 times 5 is a collection of 3 members, each a collection of 5 units. No sense can be attached to the "third fifth," unless you really mean that you form three collections of five each. In like manner 2^3 is the number of different ways in which 3 objects can be distributed among 2 places, while 3^2 is the number of ways in which 2 objects can be distributed among 3 places. But there is no such clear conception of the involution of ordinals.

Peirce: CP 4.658 Cross-Ref: ††

Fifthly. With ordinals alone there would be no fractions; except perhaps in Washington, where there is a "4 1-2th" Street!

Peirce: CP 4.659 Cross-Ref: ††

659. On the other hand, it may be argued:

Peirce: CP 4.659 Cross-Ref: ††

Firstly. What, after all, are the cardinal numbers? What do they signify? They signify the *grades* of multitude. Now a grade is a rank; it is an ordinal idea. The English word *grade* which came in with the nineteenth century, was evidently from Latin *gradus*, a stride, being the Latinized form of the old English word *gree*, which the Scotch still use in the sense of that which one strives to attain. It is the French *gré*. It is from an Aryan root found in "*greedy*." See Fick's list of roots in the International Dictionary, No. 49, [$\sqrt{34}$]. There never was any idea of multitude attached to this root. Some think the principal idea is desire; others, that it is that of stepping out. It seems to me it is the idea of pushing on to the attainment of what one hankers after. Thus, cardinal numbers are nothing but a special class of ordinals. To say that a plural is five means that it is of the fifth grade of multitude. It would be the sixth, if we were to count *none*, or the foot of the staircase, as the first number; but we ought in consistency to call it the "*none-th*" number. The ordinal "*none-th*" is a desideration of *gree*, of thought that I have lately won. Just ponder the utility of that view, my candid reader. Now Number is *the* mathematical conception *par excellence*; and therefore the question is whether limiting the grades we refer to in mathematics to grades of

multitude advances and aids mathematics to attain a higher grade of perfection or not. But this answers itself. All that is essential to the mathematics of numbers is *succession* and definite relations of succession, and that is just the idea that ordinal number develops.

Peirce: CP 4.659 Cross-Ref:††

Secondly. The essence of anything lies in what it is intended to do. Numbers are simply vocables used in counting. In order to subserve that purpose best, their sequence should stick in the memory, while the less signification they carry the better. The children are quite right in counting as they do:

Onery; uery; ickari; Ann;

Filason; folason; Nicholas Jan;

Queevy; quavy; English navy;

Stingalum; stangalum; Buck!

The fact that there are generally thirteen of these vocables suggests that they may have originated in counting out a panel in order to get a jury. These children's vocables are purely ordinal.

Peirce: CP 4.659 Cross-Ref:††

Thirdly. But the ultimate utility of counting is to aid reasoning. In order to do that, it must carry a *form* akin to that of reasoning. Now the inseparable form of reasoning is that of proceeding *from* a starting-point *through* something else, *to* a result. This is an ordinal, not a collective idea.

Peirce: CP 4.660 Cross-Ref:††

660. Now in answer to the above arguments on the side of the cardinal ideas.

Peirce: CP 4.660 Cross-Ref:††

As to the first argument [on the side of the cardinal ideas] the first reply is that all the authorities cited are worthless as to a question of logical analysis. The only opinions worth consideration are those of the modern mathematico-logicians, Georg Cantor, Richard Dedekind, Ernst Schröder, and their fellows; and of these Dedekind †1 emphatically and Schröder †2 probably are on the ordinal side; though Cantor †3 by basing ordinal upon the doctrine of cardinals has the appearance (perhaps it is a deceptive appearance), of taking the side of the cardinals.

Peirce: CP 4.660 Cross-Ref:††

But *secondly*, arguments from authority are of no authority in a question of logic.

Peirce: CP 4.661 Cross-Ref:††

661. To the second argument, likewise, two replies may be made. *Firstly*, it is almost always found that when a new idea is born into the living world of thought, it labors under all sorts of inconsequential and inconvenient adjuncts. A new machine, for example, is at first needlessly complicated, and has to be simplified later. We should therefore not expect to find that the earliest forms of numbers were the neatest and purest.

Peirce: CP 4.661 Cross-Ref:††

Yet, *secondly*, there can hardly be a doubt that the original numbers were

meaningless vocables used for counting, such as children invent; and there is no reason to suppose that these were at first less purely ordinal than the children's are.

Peirce: CP 4.662 Cross-Ref:††

662. According to the principle of the third argument, which seems to be the widely disseminated tenet that the less thought a man has bestowed upon a question, the more valuable his opinion about it is likely to be, when it is applied to the question of how much that very third argument is worth, must result in according a perfectly crushing strength to my judgment, which is that it is beneath contempt.

Peirce: CP 4.663 Cross-Ref:††

663. The fourth argument, much the most respectable of the list, certainly shows that the device of considering numbers as multitudes gives very pretty demonstrations of the values of products and powers of whole numbers; but the first fault of the argument is that there are countless parallel instances of devices giving charmingly clear intuitions of mathematical truth, although nobody in his senses could say that the imported considerations were essentially involved in the subjects to which the theorems relate. Thus, a number of difficult evaluations of integrals can be obtained most delightfully by considering those integrals as the values of probabilities and then applying common sense, or some simple reasoning, to answering the question of probability. Yet who would say that the idea of probability was essentially involved in the idea of an abstract integral? The proper inference is the converse of that; I mean that the idea of the integral is essentially involved in the idea of the problem in probabilities. Just so, in the instances adduced; what they evidently prove is that the abstract ideas of multiplication and of involution are involved, the one in the more concrete idea of a collection whose units are collections, and the other in the concreter idea of the different ways of distributing the members of one collection into connection with the several units of another collection. I admit, with all my heart, the instructiveness of these remarks and to the fact that they shed a brilliant illumination upon the essential nature of the arithmetical and algebraical results. Indeed, they are so rich in their curiosity and their eye-opening virtues, that I will not spoil their effect by tagging any discussion of them upon this already exorbitant paper. I will only say that if on another occasion I ring up the curtain upon what they have to show, it will be seen that one of their first lessons is that numbers may stand for grades of any kind and not exclusively for grades of multitude. You will observe that, for example, in the iconization of involution, it was not members of a *multitude* that were put into the different parts of another *multitude*, but members of a *collection* which are attached to different singulars of a *collection*. Now while numbers may on occasion be, or represent, multitudes, they can never be collections, since collections are not *grades* of any kind, but are single things. It may be reckoned a second fault of that fourth argument that it quite overlooks the necessity of *proving* the exclusive limitation of numbers to a single variety of grades; and a third fault of it is that it baldly asserts, with not so much as an imitation-reason, that it is impossible to obtain a clear conception of multiplication without appeal to cardinals. That is a gage that I am obliged to take up. Let me first call attention to the fact that an object of pure mathematical thought does not possess this or that definite sensible quality, but is distinguished from other such objects by the form of relation involved in its structure. It must further be noticed that there are different kinds of multiplication, especially the "internal" and the "external";†1 and besides that, there are different allowable ways of using the term, so that what at one time would be called multiplication, at another time would not be multiplication. I have to define what could with propriety be called multiplication with the proper strictness and proper looseness. Above all,

extreme care will be needed to avoid vicious circles and phrases that seem to have a meaning but really have none. For example, I shall have to mention addition in defining multiplication, and, consequently must begin by defining that. Now if I were to say that addition consists in simply putting two quantities together, that would sound as if it meant something; yet I do not clearly see what it would or well could mean; for if anybody were to ask me what kind of "putting together" I meant, why, what I should find myself meaning is simply the *adding* of them together. So since addition is of course adding, my statement might just as well be omitted, and no meaning would be lost with the omission.

Peirce: CP 4.664 Cross-Ref:††

664.†2 A *quantity* is in one sense or another an object of almost any category; but most appropriately the word is used to denote a dyadic relation, which is considered as having conceivable exact determinations differing from one another only in a linear respect, that is, so that there is a dyadic relation of "being *r* to" such that, of any two of the determinations in the same linear respect, one is *r* to whatever the other is *r* to, and is *r* to something the other is not *r* to; and to know all the possible determinations to which any determination was in that linear relation would be to know the determination exactly, the determinations being defined as such as to satisfy (especially so as just to satisfy) some general condition. If there is but a single linear respect such that, whatever two conceivable determinations of a quantity be taken, they can differ in that respect alone, it is called a *simple quantity*; but a quantity whose determinations can differ from others in different linear respects is called a *complex quantity*. The expressed determination of a quantity in all its linear respects of determination (especially if the expression be such that the determination is *exact*, that is, is a single one and is not any other), is called the *value* of the quantity.

Peirce: CP 4.665 Cross-Ref:††

665. The *ens rationis* whose complete being consists in the alternative possibility of all the conceivable values of a quantity under all conditions, is called the *scale of values* of the quantity. If the different values of the scale of conceivable values *q q'* etc., denoting the values, *consists of the discrepancies, according to a definite rule of comparison*, between the values of the scale of values *Q, Q'* etc., so that for every value of the first scale, which is called the *relative scale*, and for any value *Q'* that may be assumed on the second scale, called the *absolute scale*, there is some value *Q* on the absolute scale, whose discrepancy, according to the rule of comparison, from *Q'* has the given value *q* of the relative scale, then if the rule of comparison is a convenient one, it will be possible by inserting, if necessary, *fictional values* in the relative scale to have a value on the relative scale for the discrepancy of any value of the absolute scale from every other. There will, therefore, then be some value, *q[0]*, of the relative scale which shall represent the nil discrepancy of *Q'* from itself; and consistency will require this to be, at the same time, the representative of the *nil* discrepancy of every value *Q* of the absolute scale from itself; and this value *q[0]* on the relative scale will be called and written, "zero," 0. Then every discrepancy *Q[a]-Q[b]* of values on the absolute scale will be the same as discrepancy of $q[a] = Q[a]-Q^1$ from $q[b] = Q[b]-Q^1$, where *q[b]* will be any arbitrarily taken value on the relative scale and *q[a]* will be a suitably chosen value of the same scale. Thus, the relative scale will itself fulfill the functions of the absolute scale, and may be identified with it.

Peirce: CP 4.666 Cross-Ref:††

666. Addition $\dagger 1$ is a "mathematical operation," *i.e.*, a certain triadic relation, of a suitable value s by any arbitrarily taken operand, called an *augend*. For such a scale of difference-values as that just described, if the discrepancy of $q[s]$ from $q[a]$ is represented by $q[b]$, then the discrepancy of $q[s]$ from $q[b]$ will be $q[a]$; and $q[s]$ will be the *sum* by $q[a]$ as *addend* upon $q[b]$ as *augend*. Or stating the matter otherwise, $\dagger P1 (q[x]-q[y])+(q[y]-q[z]) = (q[x]-q[z])$. In any case the rule for determining the sum from any given addend and augend, will be such that a given value as addend will produce a definite effect upon the sum depending exclusively upon its own value, regardless of what the augend may be; and the same value as augend will produce the very same definite effect upon the sum. From this follow all the properties of addition. In the case of the scale of values of the positive whole numbers, the ordinal rule will be that $0+0 = 0$ and that the value next following any given value will as addend or as augend give a sum of the value next following that given by that given value as addend or augend. That is to say, if we denote by Nx the positive whole number next following after x in the natural order of those numbers, and if $x+y = z$, then $Nx+y = Nz$ and $x+Ny = Nz$. Moreover, if $N0 = 1$, then, since $0+0 = 0$, we have $0+1 = 0+N0 = N0 = Z$ and so $1+0 = 1$, and it necessarily follows, as it is easy to see and to prove $Nx = x+1$.

Peirce: CP 4.667 Cross-Ref: $\dagger\dagger$

667. I am now prepared to give a perfectly clear ordinal definition of multiplication. Only, I must warn you that mathematical clearness, as understood since Weierstrass, does not mean producing a sensuous impression of naturalness, but means logical clearness, clearness of *thought*. I will call attention to the circumstance that the idea of multiplication founded on multitude embraces only the multiplication of whole numbers. For notwithstanding the assertion of the fifth argument that fractions cannot be dealt with ordinally, which it is the principal purpose of the present paper to disprove, and I shall come to that disproof in a little while now, that fifth argument forgets that it ought, in order to have any force at all, to have shown that you *can* deal with fractions from the point of view of multitude. Now few things would seem more obvious than that there is no such thing as a fractional collection, and that that argument, as soon as I shall have shown how the rational fractions can be ordinally inserted in their places, will be turned in a convincing manner against the cardinal conception of pure mathematical number. And if anybody is doubtful whether I am right in saying that there is no fractional multitude, he ought to be convinced by the helplessness of the cardinal method when it attempts to represent the multiplication of fractions. It should also be remembered that mathematicians have a thoroughly well-grounded generalization of the conception of multiplication, in the multiplication of matrices, which is the same thing as the multiplication of quaternions and of other forms of multiple algebra. I shall embrace such multiplication in my first general ordinal conception of the operation, afterward showing both *how* and *why* its rules become specialized in the multiplication of numbers, whole or fractional, or even imaginary. I might show, were it not too far from my main theme, that the algebra of real quaternions is unique among all possible algebras in the closeness of its properties to numerical algebra, notwithstanding its non-commutative character. $\dagger 1$ A few writers have alluded to a so-called "symbolic" multiplication which is not even associative; but they have completely failed to show any advantage in regarding it as multiplication. I have myself made some studies along this line which have led me to the conviction that, except when such operations break up into sections which are associative, it is quite useless and idle to talk of it as multiplication. Accordingly, I shall pay no attention to

it, although there would be no difficulty in treating it ordinarily.

Peirce: CP 4.668 Cross-Ref:††

668. **Multiplication**†2 is another mathematical operation, the triadic relation of a **product** "by" an operator called the **multiplier** "into" an operand called a **multiplicand**. The multiplicand is said to be multiplied **by** the multiplier, and the latter to be multiplied **into** the former. It is based, as is addition, upon a sort of discrepancy between numbers; upon a discrepancy, however, of quite a different kind, for it is, so to speak, double-ended, and has other remarkable peculiarities. It is not confined to a single line. Just as the basis of addition is that

$$(i-j)-(u-v) = (i-u)-(j-v)$$

and $x-x = 0$ $x-0 = x$

so multiplication may be regarded as based upon the double discrepancy x/y and y/x .†1 This, if my memory is right, was Grassmann's view.†2 But while, for addition, this point of view is in reason almost compulsory, because it alone accounts for the **zero** and determines all the properties of addition, for multiplication it has little to recommend it beyond the analogy of addition. The formulæ here, whose get-up is less smart because of the lack of the commutative principle, are

$$a/b \setminus a = b \quad \text{and} \quad a/(b \setminus a) = b$$

$$a/a = a \setminus a = 1 \quad 1 \setminus a = a/1 = a$$

Peirce: CP 4.668 Cross-Ref:††

But these formulæ by no means imply all the principles of division.

Peirce: CP 4.669 Cross-Ref:††

669. Any system of values which fully illustrates all the features of external multiplication of which I regard internal multiplication as a special case,†P1 must have at least three linear respects. I will take such a system for illustration. Let the three linear respects be

0, aU, bU, cU, etc.

0, aV, bV, cV, etc.

0, aW, bW, cW, etc.

where a, b, c, etc. are any real numbers, 0 is ordinary zero, and U, V, W, are non-numerical, non-relative, units of different kinds, and linearly independent, so that the equation $xU+yV+zW = 0$ is impossible, x, y, z, being numerical and not all 0.

Peirce: CP 4.670 Cross-Ref:††

670. Since multiplication is not generally commutative, there must be a progressive and a regressive division. For the present I will consider only the former of these. It will furnish a matrix of nine dyadic units; so that the general form of those dyadic quantities, so far as progressive division furnishes them will be

$$a(U/U)+b(U/V)+c(U/W)$$

$$+d(V/U)+e(V/V)+f(V/W)$$

$$+g(W/U)+h(W/V)+i(W/W)$$

where $a, b, c, \dots i$ can take any real numerical values.

Peirce: CP 4.671 Cross-Ref:††

671. How multiplication follows this rule I proceed to state; and why it should do so will appear in the sequel.

$$(U/U)U = U, (V/V)V = V, (W/W)W = W$$

$$(W/U)U = W, (U/V)V = U, (V/W)W = V$$

$$(V/U)U = V, (W/V)V = W, (U/W)W = U.$$

But all products in which the last term of the dyad is a different non-numerical unit from the multiplicand are equal to numerical *zero*. The numerical coefficient of a product is the product of the numerical coefficient of the factors.

Peirce: CP 4.671 Cross-Ref:††

Such products as

$$(U/V)(V/W) = (U/W)$$

obey the same principle, the multiplication and division being associative. That is to say,

$$(U/V)(V/W) = \{(U/V)V\}/W = U/W.$$

It will be seen that it necessarily follows that multiplication is associative. For

$$\{(U/V)(V/W)\}W = \{U/W\}W = U$$

and

$$(U/V)\{(V/W)W\} = (U/V)V = U$$

as before.

Peirce: CP 4.672 Cross-Ref:††

672. I may here note that in every system of values the sum of all the quotients of non-numerical units divided by themselves as divisors, in this system taken as our illustration, $(U/U)+(V/V)+(W/W)$ is numerical unity. The proof of this is perhaps the very simplest of all possible proofs, and certainly is as simple as any proof can be. It rests on the definition of a "system." Now everybody ought to have and must be expected to have a perfectly distinct notion of what he is talking about, at least, he must when he is undertaking to talk scientifically. But the truth of a proposition that follows from the definition of a notion that is perfectly distinct to a man must be seen with certainty by that man as soon as it is enunciated; for that is the definition of a "perfectly distinct" notion. Consequently, everybody who undertakes to discuss scientifically a *system* of values ought, and must be expected, to see, as soon as it is enunciated, the truth of any proposition that follows from the definition of a system of values, especially when that proof is excessively simple. Now it is part of the definition of a system of values that any collection of values, which does not contain *any* value that satisfies a system of equations in these values, although there is a value which would satisfy them if it were admitted into the system, is not a complete system of values. Consider then the system of equations which consists of all the equations of the form $xa = a$, where x is the unknown, and a takes in the different equations all the values of the system. There is a value which would satisfy all these equations were

that value admitted to the system. Numerical unity is such a value, for $1a = a$, whatever value a may have. Consequently, given any complete system of values, it must either contain numerical unity as one of its values, or else it contains some other value which satisfies all three equations, so that, were numerical unity admitted into the system it would contain two different values that satisfy the system of equations. Now if the latter alternative be the true one, let I and J be two different values that would belong to the system if numerical unity were admitted into it, each of these two being such that there would be no value in the system which would be altered by being multiplied by or into either I or J . Then the product IJ would be equal to I because J is the only other factor and would also be equal to J because I is the only other factor. Hence, since the same value is the value both of I and of J , I and J are the same value contrary to the hypothesis. Could there be a proof sounder or more needless than that? Suppose we were to admit that every collection of values, which contains, for every system of linear equations in the values of that collection which are capable of being satisfied linearly, some value that does satisfy them, is a complete system of values. Suppose, then, that the following were offered as a proof. Under that definition all possible values must constitute a complete system of values; for since it contains all possible values it must contain every possible value that satisfies any system of linear equations. But we have just seen that no complete system of values can contain two different values each of which is such that its product by or into any value whatever of the system gives that same value as the product. Therefore there is no other possible value than numerical unity of which this is true. Suppose, I say, that were offered as a proof, then I should like to put to you, reflective Reader, two questions. The first is this: What do you think of the value of that proposed proof, and precisely why? The second question is, What do you think I think of it? Do not think me impertinent; for I have two pertinent birds to hit with the two stones. One is to show that it is possible to have an extra clear ordinal conception of multiplication; and the other is to put myself in condition to please you better in some article on some subject.

Peirce: CP 4.673 Cross-Ref:††

673. There is one reply that might be made by upholders of the cardinal view of the pure mathematical conception of number that I think I ought briefly to notice. It might be said, "Well, granting for the sake of argument that even the cardinal view of number involves the ordinal idea, still it is equally true that one cannot count objects in a row without regarding them as forming a collection, and thus the two views, if they be two, are quite on a par." I shall be able to make a very brief reply as a partial indemnity for the last affliction. It is not the question whether we are obliged when we think of number ordinally to think of some numbers as a collection; for of course we are so obliged. Nor is it the question whether in thinking of such collection we have to attend particularly to its multitude, although that is not so clear. The question is whether there is any possible theorem or reasoning of mathematics or any strictly numerical conception which calls for anything more with which to build it up than the ordinal idea supplies, and if there be, whether the idea of multitude supplies that further idea.

Peirce: CP 4.674 Cross-Ref:††

674. Now in my opinion the arguments already given are conclusive; but I announced my intention of discussing the question in the thorough manner of the best of the fourteenth century scholastic doctors. As yet their scheme of a disputation has not been filled out. It is now time that like them I should introduce such a filling out of the body of reasons as what has gone before suggests. Now what has gone before

does suggest a new argument that seems to me decisive. The whole list of different multitudes is as follows. Beginning with the multitude of nothing, which is *none* (corresponding to the ordinal variously called "zero," "the origin," and "noneth," these three words marking three different aspects of that ordinal), we take the sole collection that has that multitude, which is Nothing, and successively add units to it as long as each added unit increases the multitude. This gives us the whole series of finite multitudes; each of which, of course, has its corresponding ordinal. We next take the multitude of all the positive integer numbers. It is a vulgar fallacy to reason that because any *collection* of that multitude, such as the collection of all the positive integers is *endless*, that is, has no last member, therefore there can be no corresponding ordinal number. But hold -- I am wrong! It is *not a fallacy*, because it is not a reasoning at all. It is a jump of the rank of the Achilles-and-tortoise catch, or that of "It either rains or it doesn't rain; now it rains; *ergo* it doesn't rain." To say no corresponding ordinal can be supplied, when the multitudes themselves including the denumeral are in a linear series, is absurd. In point of fact, there is a corresponding ordinal, namely, "infinitesimal," although this is ordinarily restricted to the *infinitieth* term of infinite converging series, or to the magnitude that such term would have, if there were any such, which there is not. Still there are many series of objects of thought which I do not halt when the finite numbers have been exhausted. Such, for example, is the series of values furnished by a converging series; such is the series which shows in perspective points at equal distances along a straight line, where the "vanishing-point" is the *infinitieth*, or infinitesimal, point; and such is the series of multitudes, which by no means ceases to increase after the denumeral multitude. Cantor denotes this ordinal by $\{\ddot{o}\}$, and that notation is generally adopted. The next larger multitude than any infinite multitude, say that of the M's, if there be endlessly many M's, is the multitude of all possible different collections each consisting of M's; or, what is the same thing, the multitude of different ways in which all the M's might be distributed among different heads (for whether the number of heads were 2, 3, or any other number up to M would make no difference in the multitude). These were named by me (who had the right to name them, having been the first to define them as well as the first to prove that the multitude of ways of distributing the singulars of any collection under two heads is always greater than the multitude of those singulars themselves,†1 although it was soon found that they coincided with certain multitudes less clearly and less accurately defined by Cantor), the *abnumeral* or *abnumerable* multitudes. I prefer the latter term as corresponding to *enumerable* or finite multitudes and collections; while I prefer to speak of the *denumeral* multitude, giving the word a different termination because there is but one such multitude, while of the *abnumerable*, as well as of the *enumerable*, multitudes there is a denumeral collection. The multitude of ways of distributing the singulars of a denumeral collection is the *first abnumerable multitude*. It is the multitude of all irrational values (whether imaginaries be included or excluded). The multitude of ways of distributing a first-abnumerable collection is the *second-abnumerable* multitude. The multitude of ways of distributing a second abnumerable collection is the *third-abnumerable* multitude; and in general, the multitude of all possible collection is the (N+1)th abnumerable multitude. There is no $\{\ddot{o}\}$ -abnumerable collection; which is a corollary drawn by me from my proof that $2x$ is always greater than x . There is no multitude greater than the finitely-abnumerable multitudes. Consequently, the total multitude of possible multitudes is denumeral.†1 The *objects* of *any* abnumerable collection are in greater multitude than all multitudes. The reason, of course, is that the addition of a unit to an infinite collection never increases its multitudes. But that new unit will always carry a new ordinal number. If therefore, we extend the term

"cardinal number" so as to make it apply to infinite collections, a multitude of ordinal numbers will be possible exceeding that of all possible cardinal numbers in any infinitely great ratio you please, without having begun to exhaust the ordinals in the least. The system of ordinals is thus infinitely more rich than the system of cardinals. In fact, those two denumeral series of ordinals which are alone required to count all the cardinals seem to the student of this branch of mathematical logic as most beggarly.

Peirce: CP 4.675 Cross-Ref:††

675. Dr. Georg Cantor, of Halle, undertook that research which I have mentioned as of the greatest urgency for logic, for metaphysics, and for cosmogony, that of ascertaining whether or not the singulars of every collection, however great, can be the subjects of a linear relation, and if not what is the greatest multitude of singulars that can be so arranged. To this end he introduced the immensely valuable concept of what he calls a "well-ordered" series, by which he means a linear series every portion of which has a first member.†2 He undertook to describe such a series, and name its members, the series containing more than any conceivable multitude of members. The momentous series so described ought to be called the "Cantorian Series," in everlasting memory of the man who so clearly perceived the supreme importance of the problem, and took so considerable a step, at least, toward its fulfillment. Students generally are either doubtful of his success or even deny it. For my part, I am not sure that I understand his papers. At any rate, I think that by deviating somewhat from his method, we may be able to attain clearer certitude, one way or the other. I prefer to construct a well-ordered series upon slightly different principles from these that Cantor has used. However, I cannot here go into details. The problem is to construct a well-ordered series which shall embrace as great a multitude of members as possible.

Peirce: CP 4.676 Cross-Ref:††

676. Let us make use of that system of numerical notation whose base is two. Let us take the different multitudes in succession and represent the different ordinals by the different ways of distributing objects of each multitude among two places. These two places may be called the affirmative and the negative places (for a non-existent object, or one not considered, is to be regarded as in the negative place), and we may represent each arrangement by marking objects in the affirmative place by 1's and those in the negative place by 0, or if an object is not considered it need not be marked. We will take the different secundal "places" of numerical notation from right to left to represent the different objects. Thus 1001 will represent that the first (or, as we had better call it, the zero object) is in the affirmative place, the second and third, or better, the first and second are in the negative place, and the fourth is in the affirmative place again, and so, this series of characters shall be, or represent, one of our ordinal numbers. We begin with the lowest multitude, which is that of the unique collections, Nothing. In how many ways can whatever members this collection contains (which is none, 0) be distributed among two places? The answer is $2^0 = 1$. Therefore we need one ordinal to describe it, and one object, which we will call the zero object, with which to construct that ordinal. There being no member of the collection of multitude zero, 0 will properly represent the sole arrangement. The next multitude is 1 and the number of ways of distributing one object among two places is $2^1 = 2$, one *additional* ordinal is required. The same object we used before will answer the purpose, being now put in the affirmative place; so that 1 represents the ordinal. The next multitude is 2 and the number of ways of distributing 2 objects among two places is $2^2 = 4$, *i.e.*, we need 4 minus 2 or 2 additional ordinals. For that

purpose we take another object, represented by the secunda place next to the left of the one we have been using and represent the two new arrangements by 10 and 11. In this way, all the finite ordinal numbers can be written down, in the sense in which it is true, in the mode of the possible, that of a subject of which anything can be predicated distributively the same thing can be predicated collectively; and these finite ordinals will be marked just as in the secunda system of arithmetic. In the following table, "distr." means modes of distributing under two heads.

Considering no object	0	The single mode of distributing nothing.
Now considering an object	1	The "distr." of 1 object when it is put under the affirmative head
Another object considered.	1 0 1 1	The "distr." of 2 objects when a certain one of them is put under the affirmative head.
A third object considered.	1 0 0 1 0 1 1 1 0 1 1 1 etc.	The "distr." of 3 objects of which a certain one is put under the affirmative head.

Peirce: CP 4.677 Cross-Ref: ††

677. I guess that a good many people, among whom many mathematicians must be included, to judge by their often writing $-, 1, 2, 3, \dots, \infty, \dots$ have a notion that nothing but a limitation attached to human powers prevents a finite collection receiving successive finite increments until it becomes denumeral; though I do not suppose that any modern mathematician would deliberately say that the positive integers strictly run up to the denumeral. It is not because of an human imperfection that we cannot add units to a collection until it becomes denumeral, but it is because the supposition involves a contradiction in itself, and therefore cannot be rendered definite in all respects. For the denumeral is and by definition that which *cannot* be reached by successive additions of unity. Nothing, however, prevents an endless series being followed by some definite unit as its *limit*; and this is what Cantor means, and expressly says he means, by his $\{\omega\}$. †1 It is not *produced* by additions of unity but it is the first ordinal number after having passed through an endless series. There is no contradiction in the idea of passing through an endless series; for it is only endless in the sense of being incapable of production by successive additions of unity, just as Achilles can easily overtake the tortoise although he can never do so by repeatedly going *only part way* to where the tortoise will be the instant Achilles gets there. So we can and often do reach the w term of a series, though not by merely passing through all previous terms. Yet while reaching the denumeral does not consist in passing from one number to the number exceeding that by 1, though this be done to any extent; nevertheless because the series of finite numbers is endless, it follows that to pass *all* finite numbers is to pass *beyond* them all, and in doing *that* to attain the denumeral. There are in Cantor's exposition of his ordinal numbers several points like that which will give the unmathematical student difficulty, not because he lacks intelligence, but because he thinks so exactly as to see the difficulties, while not being sufficiently acquainted with the subtleties of mathematics he is unable to solve them, while many mathematicians, especially of the pre-Weierstrassian school have their ideas hazy on these points, although they may be perfectly clear for all mathematical purposes. There is certainly no really sound objection to anything in Cantor's system of ordinals until the second abnumerable ordinals are reached; and even then in my

opinion my modification of his law of progression removes any possible error that there may there be. But my article is already so long that I must cut that short. Suffice it to say that there is certainly a possible series of ordinals of the first abnumerable multitude, while the entire multitude of all possible multitudes is only denumeral. On that point there is no possible doubt for a competent judge. It follows that the cardinal numbers, even in the extended sense in which Cantor employs the term, to denote any multitudes whatever, cannot be so rich in relations and therefore must belong to a lower [order] than that of ordinals, which are merely exact grades, regardless of what sort of states they are grades of; and hence the restriction of number to cardinals involves a serious lopping off of the highest part of mathematics. Indeed it is not necessary to consider Cantor's ordinals to reach that conclusion, since the multitude of all possible irrational values, say between 0 and 1, is abnumerable and therefore can in no way be reduced to cardinals, of which the entire multitude is infinitely less.

Peirce: CP 4.677 Cross-Ref:††

To be sure, it might be said that the irrational numbers, even if they be not cardinals, are not ordinals, but are ratios, and involve the idea of equality of parts. But I propose to disprove that, by showing that all rational fractions are ordinal. It is well known that all fractions can be arranged in a well-ordered Cantorian series, and that in indefinitely many ways, but it may be said that when that is done, it is no longer possible (certainly far from evident) which of any two is the greater in value; for which purpose they must be reduced to a common denominator; and that the possibility of the reduction to a common denominator is not involved in the idea of the special Cantorian series. But I am going to show in two ways that such series are possible in which the relative magnitude of any two fractions is expressed in the series itself.

Peirce: CP 4.678 Cross-Ref:††

678. We have to distinguish the system of *rational fractions* themselves, which are merely expressions, denoting rational values, from the values themselves. Thus $5/10$ and $1/2$ are two different *fractions* denoting one and the same *rational value*. Now I am going to show how in the first place all positive numerical fractions can be arranged in a well-ordered Cantorian series, carrying between the members of each successive pair of fractions of the series either the sign $<$ to show that the succeeding fraction is the greater or the sign $=$ to show that the two fractions are equal. Afterward I shall exhibit a somewhat similar series whose terms are all the positive rational values expressed in their lowest terms.

Peirce: CP 4.679 Cross-Ref:††

679. For the present, I confine myself to the fractional expressions. The conception of the series will be built up in the following way. We are to suppose, in the first place, that all the positive fractions of denominator 1 (which fractions are all equal to their several numerators) to be ranged in the order of values of their numerators with the sign $<$ between every successive two, thus:

(First *state*): $0/1 < 1/1 < 2/1 < 3/1 < \text{etc.}$

We then go on to conceive that first all the fractions of denominator 2 are placed in the order of values of their numerators one in every "space" of that series; where by a "space," I mean an interval either between a fraction of that series and its following copula, $<$, or between a copula and its following fraction; so that the fractions of denominator two will be inserted in the spaces indicated by the "carets" of the following line:

$$0/1^{<^1/1^{>^2/1}, \text{ etc.}$$

Each fraction when inserted will be accompanied by a copula either preceding or following it according as the space, before the insertion, was preceded or followed by a fraction.

Peirce: CP 4.679 Cross-Ref:††

For the sake of clearness, I will postpone saying what each copula is to be, but will only indicate it by a C. At first, then, we have the series in the form

(Second *state*):

$$0/1^C^1/1^C^2/1^C^3/1 \text{ etc.}$$

and after the insertion of the fractions of denominator 2 the series will become (without the carets), where I distinguish the new carets by italicizing the C's

(Third *state*):

$$0/1C0/2C1/2C1/1C2/2C3/2C3/2C2/1C4/2C5/2C3/1.$$

Peirce: CP 4.679 Cross-Ref:††

Now the general rule for the *carets* (which are only temporary scaffolding) is that after inserting the fractions of any denominator, N, where N is whatever whole number comes next in the order of magnitude of whole numbers, a caret is to be inserted at every Nth space from the beginning; so that in the state of the series just represented, the carets will appear as here shown:

(Fourth *state*):

$$0/1C^0/2C^1/2C^1/1C^2/2C^3/2C^3/2C^2/1C^4/2C^5/2C^3/1. \text{ etc.}$$

Then will be inserted the fractions of the next higher denominator each with its copula (which I will again italicize) thus:

(Fifth *state*):

$$0/1C0/3C0/2C1/3C1/2C2/3C1/1C3/3C2/2C4/3C3/2C5/3C \text{ etc.}$$

The carets will now be inserted as follows:

(Sixth *state*):

$$0/1C0/3^C0/2C^1/3C^1/2^C2/3C^1/1C^3/3^C2/2C^4/3C3/2^C5/3 \text{ etc.}$$

The carets are always forthwith replaced each by a fraction of the lowest denominator not used to replace any previous set of carets; and each fraction will be accompanied by a copula either before it, in case of a fraction immediately preceding the caret, or after it in case a fraction follows the caret.

Peirce: CP 4.679 Cross-Ref:††

It now only remains to state what the copulas are and the description of the series is complete. A newly inserted copula coming before a newly inserted fraction is always =, but a newly inserted copula which follows the fraction inserted with it in place of the same caret, is of the same kind as the old copula preceding that newly inserted fraction. In the only case not thus provided for, the copula is <.

Peirce: CP 4.680 Cross-Ref:††

680. Let the fractions be *defined* by the series thus formed, and not otherwise defined, and (the arithmetic of whole numbers being supposed) the *entire doctrine of fractions is contained in this series, or rather, in its governing definition, or rule of construction*. The very easy proof of this may be omitted.

Peirce: CP 4.681 Cross-Ref:††

681. I fear this series will not attract the attention it really merits; for it is dressed in a fantastic garb of artificiality that does not do it justice. It is like an honest, self-respecting actor, who, by some unfortunate mistake, happened to be arrested in his theatrical dress, should appear before the judge in the guise of Jeremy Diddler.

Peirce: CP 4.681 Cross-Ref:††

For my series of rational values, I hope better things. Here there will be no need of copulas, because all the terms are different values. We begin by writing

$$0/1 \quad 1/0$$

as the zero state of the series (although $1/0$ is not properly a rational value), and we go through the series from beginning to end, time and again ceaselessly; every time inserting between every two adjacent fractions a new fraction whose numerator shall be the sum of the numerators, and its denominator the sum of the denominators of the two fractions between which it is inserted. The result will be that every positive rational value will be inserted, once only, and expressed in its lowest terms.

Zero state:

$$0/1 \quad 1/0$$

First state:

$$0/1 \quad 1/1 \quad 1/0$$

Second state:

$$0/1 \quad 1/2 \quad 1/1 \quad 2/1 \quad 1/0$$

Third state:

$$0/1 \quad 1/3 \quad 1/2 \quad 2/3 \quad 1/1 \quad 3/2 \quad 2/1 \quad 3/1 \quad 1/0$$

Fourth state:

$$0/1 \quad 1/4 \quad 1/3 \quad 2/5 \quad 1/2 \quad 3/5 \quad 2/3 \quad 3/4 \quad 1/1 \quad 4/3 \quad 3/2 \quad 5/3 \quad 2/1 \quad 5/2 \quad 3/1 \quad 4/1 \quad 1/0$$

Fifth state:

$$0/1 \quad 1/5 \quad 1/4 \quad 2/7 \quad 1/3 \quad 3/8 \quad 2/5 \quad 3/7 \quad 1/2 \quad 4/7 \quad 3/5 \quad 5/8 \quad 2/3 \quad 5/7 \quad 3/4 \quad 4/5 \quad 1/1 \quad \text{etc.}$$

Peirce: CP 4.681 Cross-Ref:††

This series has many curious properties, some of which are very easily proved. For example, the series of numerators in any state always begins with the series of the preceding state; thus:

$$01$$

$$011$$

01121

011213231

01121323143525341

011213231435253415473857275837451

Peirce: CP 4.681 Cross-Ref:††

All the properties of rational values and of their expressions in their lowest terms follow from the general fact that they are all contained in their order in the series constructed according to this rule.

Peirce: CP 4.681 Cross-Ref:††

This at once proves that the ideas of the rational values essentially involves no other relation than that of linear succession, and that the equality of parts is not presupposed. And since the irrational values are nothing but the limits of series of rational values, they also suppose nothing but the linear form of relation. It is because [of] this form of relation of rational consequence that numbers are of such stupendous importance in reasoning.

Peirce: CP 4.681 Cross-Ref:††

But the highest and last lesson which the numbers whisper in our ear is that of the supremacy of the forms of relation for which their tawdry outside is the mere shell of the casket.

Peirce: CP 4.1 Fn 1 p 3

†1 1-5 are from the second lecture on "Detached Ideas on Vitally Important Topics" of 1898. For the first lecture see vol. 1, bk. III, ch. 6. 5-10 are from "Phanerostopy, φα{n} 1906."

Peirce: CP 4.1 Fn 2 p 3

†2 Cf. 1.15ff.

Peirce: CP 4.1 Fn 1 p 4

†1 E.g., in "The Economical Nature of Physical Inquiry," *Popular Scientific Lectures*, Chicago (1894).

Peirce: CP 4.1 Fn 2 p 4

†2 E. C. Hegeler (1835-1910), founder of the *Open Court Monthly* and *The Monist*.

Peirce: CP 4.1 Fn 3 p 4

†3 Paul Carus (1852-1919), editor of the *Open Court Monthly* and *The Monist*.

Peirce: CP 4.1 Fn 4 p 4

†4 See "The Founder of Tychism, his Methods and Criticisms," I, 4, *The Monist*, vol. III (1893).

Peirce: CP 4.2 Fn 1 p 6

- †1 Augusto Vera, *Introduction à la philosophie de Hegel*, Paris (1855).
- Peirce: CP 4.2 Fn 2 p 6
 †2 *Die falsche Spitzfindigkeit der vier syllogistischen Figuren erwiesen* (1762).
- Peirce: CP 4.2 Fn 3 p 6
 †3 See 2.485ff; 2.801ff.
- Peirce: CP 4.2 Fn 1 p 7
 †1 Cf., e.g., *Prior Analytic* II, 23.
- Peirce: CP 4.3 Fn 2 p 7
 †2 See 2.509; 2.706f.
- Peirce: CP 4.3 Fn 3 p 7
 †3 See, e.g., 2.345ff, 2.710.
- Peirce: CP 4.3 Fn 4 p 7
 †4 See, e.g., 3.175.
- Peirce: CP 4.3 Fn 1 p 8
 †1 In 1867, see 1.551ff.
- Peirce: CP 4.3 Fn 2 p 8
 †2 See vol. 1, bk. III, for an extended discussion of these categories.
- Peirce: CP 4.4 Fn 1 p 9
 †1 The steps of this development are clearly manifest in the early papers of vol. 3.
- Peirce: CP 4.4 Fn 2 p 9
 †2 "On the Syllogism IV, and on the Logic of Relations," *Cambridge Philosophical Transactions*, vol. 10, pp. 331-58.
- Peirce: CP 4.4 Fn 3 p 9
 †3 Peirce here gives a number of elementary graphs to illustrate the logic of relatives. The papers in book II of this volume cover the same ground.
- Peirce: CP 4.7 Fn 1 p 10
 †1 See, e.g., his *Logische Untersuchung*, Teil I, Kap. 3 (1900).
- Peirce: CP 4.7 Fn 1 p 11
 †1 *Logic, Deductive and Inductive* (1898).
- Peirce: CP 4.7 Fn 2 p 11
 †2 *An Introduction to Logic* (1906).
- Peirce: CP 4.7 Fn 3 p 11
 †3 *Elements of Logic as a Science of Propositions* (1890); *An Introduction to General Logic* (1892).
- Peirce: CP 4.9 Fn 1 p 12
 †1 Cf. 1.236.
- Peirce: CP 4.9 Fn 2 p 12
 †2 See 2.93, 2.229.

Peirce: CP 4.12 Fn 1 p 13

†1 Untitled paper c. 1880. Compare H. M. Sheffer's: "A Set of Five Independent Postulates for Boolean Algebras, with application to logical constants," *Transactions, American Mathematical Society*, vol. 14, pp. 481-88 (1913), of which this is a striking anticipation. See also 264f., where the same idea is developed from a different angle.

Peirce: CP 4.14 Fn 1 p 14

†1 This can be symbolized as: $S/S\uparrow P\downarrow S/S\uparrow P$; where the stroke is the sign of the logical multiplication of the contradictories of the constituents, and the number of cross bars indicates the inverse order of dissolution. Thus, (1)- $(S/S\uparrow P)$; (2)- $(-(S/S)-P)$; (3)- $(S.-P)$.

Peirce: CP 4.17 Fn 1 p 15

†1 These rules may be reformulated as follows:

A. 1. For every single x (i.e., for every x that is not paired with an x) substitute its pair; e.g., $x\uparrow a/a$ becomes $a/a\uparrow a/a$.

2. In the case of double x consider the whole of which the double x is a part.

a. If that whole is itself paired, substitute its pair for it;

e.g., $x/x\uparrow a\downarrow a$ becomes a/a .

b. If that whole is not paired, substitute the other member of the whole for each of the x s;

e.g., $x/x\uparrow a$ becomes $a/a\uparrow a$

$x/x\uparrow a/a$ becomes $a/a\uparrow a/a\downarrow a/a$

3. Repeat operations as often as necessary.

4. Equate result to ϕ/x .

B. 1. Negate all x s; i.e., for every double x employ but one; for every one employ a double.

2. Perform steps 1-3 of A.

3. Equate to ϕ/x .

The following are additional illustrations to those in the text.

Let $\phi = a/a\uparrow x$ which

by A1 becomes $a/a\uparrow a/a$, or a

so that $\phi/x = a$

by B1 we get $a/a\uparrow x/x$ which by

A2b becomes $a/a \dagger a/a \dagger a/a$, or $a/a \dagger a$

so that $\phi/x \ x = a/a \dagger a$

and $\phi \ Y \ a, \ x$ being eliminated.

Let $\phi = x/x \dagger a$ which

by A2b becomes $a/a \dagger a$

so that $\phi/x = a/a \dagger a$

by B1 we get $x/x \dagger x/x \dagger a$ from which

by A2b we get a/a

so that $\phi/x \ x = a/a$

and $\phi \ -< \ a/a, \ x$ being eliminated.

Let $\phi = a \dagger a/a \dagger x$ which

by A1 becomes $a \dagger a/a \dagger a/a$, or a/a

so that $\phi/x = a/a$

by B1 we get $a \dagger a/a \dagger x/x$

and by A2a we get a/a

so that $\phi/x \ x = a/a$

and $\phi = a/a, \ x$ being eliminated.

Peirce: CP 4.19 Fn 1 p 17

†1 This should be a sign of implication and not of equivalence.

Peirce: CP 4.19 Fn 2 p 17

†2 But $aa, a; aa:aa, a; aa = aa$, which is the correct answer.

Peirce: CP 4.19 Fn 3 p 17

†3 This seems to be in error: $a \dagger a/b$ is not equal to a ; nor is $b \dagger a/b$ equal to b .

Peirce's conclusion should be: $a, ab; x:b, ab; xx$, or $a, bb; x:b, aa; xx$.

Peirce: CP 4.19 Fn 4 p 17

†4 This should be: non- M non- P .

Peirce: CP 4.19 Fn 1 p 18

†1 This should be an entailment.

Peirce: CP 4.20 Fn 2 p 18

†2 With this notation only a single undefined or "primitive" idea and the principle of substitution are necessary in order to construct the propositions and to define all the signs used in a Boolean Algebra. The following is an indication

of how this can be done:

Primitive Idea. No. 1. $A B$.

Substituting A/B in No. 1.2. $A A \equiv \text{def } \neg A$.

Substituting B/A in No. 1.3. $B B \equiv \text{def } \neg B$.

Substituting $2/A, 3/B$ in No. 1.4. $AA; BB \equiv \text{def } A \times B$ (logical product).

Substituting $1/A, 1/B$ in No. 1.5. $AB; AB \equiv \text{def } A +, B$ (logical sum).

Substituting $2/A$ in No. 5.6. $AA, B; AA, B \equiv \text{def } A \prec B$.

Substituting $3/B$ in No. 5.7. $A, BB; A, BB \equiv \text{def } B \prec A$.

Substituting A/B in No. 6.8. $A, AA; A, AA \equiv \text{def } 1$.

Substituting $2/B$ in No. 1.9. $A, AA \equiv \text{def } O$.

Substituting $6/A, 7/B$ in No. 4.10. $A \prec B \times B \prec A \equiv \text{def } A = B$.

Peirce: CP 4.21 Fn 1 p 19

†1 Chapter 6 of the "Grand Logic" of 1893. 53-79 are from an alternative draft.

Peirce: CP 4.22 Fn 2 p 19

†2 Cf. Prantl, *Geschichte der Logik im Abendlande*, Bd. I, S. 581.

Peirce: CP 4.26 Fn 1 p 21

†1 These dates do not coincide exactly with those generally accepted today.

Peirce: CP 4.26 Fn 2 p 21

†2 *Op. cit.*, Bd. II, S.266.

Peirce: CP 4.27 Fn 1 p 22

†1 Various *Opuscula*.

Peirce: CP 4.27 Fn 2 p 22

†2 *Copulata pulcherrima in novam logicam Aristotelis*, (1493).

Peirce: CP 4.27 Fn 3 p 22

†3 *Commentariorum Collegii Conimbricensis in universam dialecticam Aristotelis Stagiritæ partes duæ*, Venice, (1616).

Peirce: CP 4.27 Fn 4 p 22

†4 *Manuel de logique*, Paris, (1855).

Peirce: CP 4.28 Fn 1 p 23

†1 Various *Quæstiones*.

Peirce: CP 4.28 Fn 2 p 23

†2 *Formalitates de mente Scoti* (1501).

Peirce: CP 4.28 Fn 3 p 23

†3 See Prantl, *op. cit.*, Bd. IV, S.204ff.

Peirce: CP 4.29 Fn 4 p 23

- †4 *Summa totius logicæ* (1488).
- Peirce: CP 4.29 Fn 5 p 23
†5 *Textus totius logices* (1492).
- Peirce: CP 4.30 Fn 6 p 23
†6 *Dialecticæ partitiones* (1543).
- Peirce: CP 4.30 Fn 7 p 23
†7 *de Causis corruptarium artium*, bk. III, Antwerp (1531).
- Peirce: CP 4.30 Fn 8 p 23
†8 *Dialecticæ disputationes*, (1541).
- Peirce: CP 4.35 Fn 1 p 25
†1 See 36, 68ff, and 1.26, 1.170, 1.422, 2.149. There are many pertinent discussions in vol. 1, bk. III and vols. 5 and 6.
- Peirce: CP 4.37 Fn 1 p 26
†1 See the Preface to the second edition, B VIII.
- Peirce: CP 4.37 Fn 2 p 26
†2 See 2.152ff
- Peirce: CP 4.38 Fn 3 p 26
†3 *Summa totius logicæ Aristotelis (Opusculum 48)*.
- Peirce: CP 4.38 Fn 4 p 26
†4 *Geschichte der Logik*, Bd. III, S.108.
- Peirce: CP 4.40 Fn 1 p 27
†1 See Petrus Hispanus, *Summlæ logicales*, Tractatus I, cap. 39 (1597).
- Peirce: CP 4.40 Fn 1 p 28
†1 *Ibid.*, cap. 12.
- Peirce: CP 4.40 Fn 2 p 28
†2 Tractatus I, pars. 3.
- Peirce: CP 4.40 Fn 3 p 28
†3 Cf. Prantl, *op. cit.*, I, S. 446, 447.
- Peirce: CP 4.40 Fn 4 p 28
†4 Petrus Hispanus, *op. cit.*, cap. 29.
- Peirce: CP 4.40 Fn 5 p 28
†5 But see below and 2.271, 2.345f.
- Peirce: CP 4.40 Fn 6 p 28
†6 Petrus Hispanus, *op. cit.*, cap. 30.
- Peirce: CP 4.40 Fn 1 p 29
†1 Cf. 2.345f.
- Peirce: CP 4.41 Fn 2 p 29
†2 *Op. cit.*, cap. 13.

- Peirce: CP 4.41 Fn 3 p 29
†3 Ch. 1-5, passim.
- Peirce: CP 4.42 Fn 4 p 29
†4 Petrus Hispanus, *op. cit.*, cap. 14.
- Peirce: CP 4.42 Fn 5 p 29
†5 *Ibid.*, cap. 15.
- Peirce: CP 4.42 Fn 6 p 29
†6 *De interpretatione*, ch. 10.
- Peirce: CP 4.42 Fn 7 p 29
†7 Petrus Hispanus, *op. cit.*, cap. 16.
- Peirce: CP 4.43 Fn 1 p 30
†1 See his *Kritik der Reinen Vernunft*, A7, B11; B418.
- Peirce: CP 4.44 Fn 2 p 30
†2 See 552n, 2.376, 2.453, 3.532.
- Peirce: CP 4.44 Fn 1 p 31
†1 See 2.456f, 3.178-9.
- Peirce: CP 4.45 Fn 2 p 31
†2 *Summa totius logicæ Aristotelis*, (*Opusculum* 48).
- Peirce: CP 4.45 Fn 3 p 31
†3 See vol. 2, bk. III.
- Peirce: CP 4.45 Fn 4 p 31
†4 *Novum Organon Renovatum* II, iv.
- Peirce: CP 4.45 Fn 5 p 31
†5 See 2.442f, 2.451, 2.469n.
- Peirce: CP 4.48 Fn P1 p 33 Cross-Ref:††
†P1 I once had the privilege in the Levant of passing some weeks in the companionship of E. H. Palmer, and had a hundred convincing evidences of the high respect which was paid by Arabians to his wonderful mastery of their language, which much surpassed that of any native Sheikh we met. It gave me great pleasure after his death to find a super-learned Regius Professor find fault with Palmer's Arabic grammar because it followed the system which seemed right to those whose vernacular Arabic was, instead of "following the Greek and Latin methods."
- Peirce: CP 4.48 Fn 1 p 33
†1 See 56, 2.341, 3.459.
- Peirce: CP 4.49 Fn 2 p 33
†2 Cf. 2.354.
- Peirce: CP 4.50 Fn 3 p 33
†3 Cf. 1.15ff.
- Peirce: CP 4.51 Fn P1 p 36 Cross-Ref:††
†P1 See, for instance, Kant's *Werke*, Ed. Rosenkrantz u. Schubert, III, 181.

Peirce: CP 4.51 Fn 1 p 36

†1 *Kritik der Reinen Vernunft*, A 7, B 11.

Peirce: CP 4.52 Fn P1 p 37 Cross-Ref:††

†P1 See *Prolegomena* 2. "Analytische Urtheile sagen im Prädicate nichts, als das, was im Begriffe des Subjects schon *wirklich*, obgleich nicht so Klar und mit gleichem Bewusstseyn *gedacht wird*."

Peirce: CP 4.53 Fn 1 p 37

†1 Cf. 3.160.

Peirce: CP 4.55 Fn 1 p 38

†1 Cf. vol. 2, bk. II, ch. 6, §2.

Peirce: CP 4.56 Fn 2 p 38

†2 Cf. vol. 2, bk. II, ch. 4, §5.

Peirce: CP 4.57 Fn 1 p 39

†1 Cf. 3.515-6.

Peirce: CP 4.58 Fn 1 p 40

†1 See 2.358.

Peirce: CP 4.60 Fn 1 p 41

†1 The difference between these four expressions is represented symbolically by a difference in the order of the quantifiers:

1. $\forall x \exists y \Sigma b$

2. $\forall x \Sigma b \forall y$

3. $\forall y \Sigma b \forall x$

4. $\Sigma b \forall x \forall y$.

Peirce: CP 4.62 Fn 1 p 42

†1 See 121ff, 219ff, 639ff, and 6.112ff, 6.179ff.

Peirce: CP 4.67 Fn 1 p 44

†1 See 6.35ff.

Peirce: CP 4.68 Fn 1 p 45

†1 Cf. 2.29.

Peirce: CP 4.68 Fn 2 p 45

†2 Cf. 6.355ff.

Peirce: CP 4.72 Fn 1 p 49

†1 Cf. 2.612.

Peirce: CP 4.74 Fn 1 p 50

†1 Cf. 2.96.

Peirce: CP 4.76 Fn 1 p 51

†1 Cf. 2.590.

Peirce: CP 4.77 Fn 1 p 52

†1 See 2.591; also H. W. B. Joseph, *An Introduction to Logic*, pp. 296n (1925).

Peirce: CP 4.77 Fn 1 p 53

†1 Cf. 2.618, 5.340.

Peirce: CP 4.77 Fn 2 p 53

†2 See 2.186ff.

Peirce: CP 4.77 Fn P1 p 53 Cross-Ref:††

†P1 If the first thing Adam said was, "No man has ever begun to say anything not false," is it necessary to suppose Pre-adamites? If no man had ever begun to say anything at all, Adam was clearly right, except perhaps in regard to his own remark; and if that was false, it was not false in what it said of itself, and would therefore have to be false of something some Pre-adamite had said. How is this? Even if Adam did not say this before anything else, he might have done so.

Peirce: CP 4.78 Fn 1 p 54

†1 See 2.618; also vol. 5, bk. II, No. 3, 2.

Peirce: CP 4.80 Fn 1 p 56

†1 Ch. 14 of the "Grand Logic," 1893. Cf. 3.398ff. See also 126.

Peirce: CP 4.81 Fn 2 p 56

†2 0 represents impossibility; ∞ , coexistence; 1, identity and T, otherness. C. 3.339.

Peirce: CP 4.81 Fn 3 p 56

†3 *Kritik der reinen Vernunft* A651ff, B679ff.

Peirce: CP 4.82 Fn 1 p 57

†1 The "logisterium" and the "Boolian" are Peirce's respective names for the quantifying and quantified parts of the proposition. Cf. 346 and 3.500.

Peirce: CP 4.85 Fn 1 p 59

†1 Ch. 17 of the "Grand Logic," 1893. Peirce said that this was the strongest paper he ever wrote; see vol. 9, letters to Judge Russell.

Peirce: CP 4.85 Fn 2 p 59

†2 A 7; B 10, 11.

Peirce: CP 4.88 Fn 1 p 62

†1 I.e., in a previous paper which is not being published. Cf. 2.607, 2.526.

Peirce: CP 4.88 Fn 2 p 62

†2 I.e., $\sim(A \prec \sim B)$ and $\sim(\sim A \prec \sim B)$.

Peirce: CP 4.89 Fn 3 p 62

†3 Cf. *Lectures on Logic*, App. V (d); (1860). See 2.532f.

Peirce: CP 4.90 Fn 4 p 62

†4 Cf. 2.191.

Peirce: CP 4.91 Fn 5 p 62

†5 *System of Logic*, bk. II, ch. 6, 3.

Peirce: CP 4.93 Fn 1 p 64

†1 Vol. 3, No. II.

Peirce: CP 4.93 Fn 2 p 64

†2 Vol. 3, No. VII.

Peirce: CP 4.94 Fn 3 p 64

†3 This can be read as: "lover of what is not loved by."

Peirce: CP 4.94 Fn 4 p 64

†4 T means "other than."

Peirce: CP 4.94 Fn 5 p 64

†5 1 means "identical with."

Peirce: CP 4.94 Fn 6 p 64

†6 As + is the mark of relative addition, $I\ddagger\sim\$/I$ can be read as: "lover of everything loved by." See 3.332n.

Peirce: CP 4.94 Fn 1 p 65

†1 By transposition.

Peirce: CP 4.96 Fn 1 p 68

†1 This can be read as: "non-lover of all not loved by."

Peirce: CP 4.96 Fn 2 p 68

†2 This can be read as: "non-lover of what is loved by."

Peirce: CP 4.96 Fn 3 p 68

†3 Cf. 3.47n, 3.173n.

Peirce: CP 4.96 Fn 1 p 69

†1 ∇ represents logical addition. 0 represents "inconsistent with." Cf. 3.348.

Peirce: CP 4.100 Fn 1 p 71

†1 Cf. 121.

Peirce: CP 4.101 Fn 1 p 72

†1 ∞ represents "coexistent with."

Peirce: CP 4.103 Fn 1 p 74

†1 Cf. 3.288; 3.402.

Peirce: CP 4.103 Fn P1 p 74 Cross-Ref:††

†P1 *Formal Logic: or the Calculus of Inference, Necessary and Probable*, pp. 166 et seq. Also: *Cambridge Philosophical Transactions*, X. 355. (Here more thoroughly treated.)

Peirce: CP 4.106 Fn 1 p 79

†1 Cf. 3.402n.

Peirce: CP 4.110 Fn 1 p 83

†1 Cf. 3.286-7.

Peirce: CP 4.115 Fn 1 p 87

†1 See *On the Principles of Political Economy and Taxation*, ch. II.

Peirce: CP 4.115 Fn P1 p 87 Cross-Ref:††

†P1 Some remarks of mine to this effect were characterized by the *Evening Post* as "too much like the differential calculus." No doubt the reasoning was too sound for the convenience of those who maintain the consumer pays the whole duty.

Peirce: CP 4.116 Fn P1 p 88 Cross-Ref:††

†P1 *Proceedings of the American Academy of Arts and Sciences*, VII, 295. [1.559.]

Peirce: CP 4.118 Fn 1 p 90

†1 *The Evanston Colloquium; Lectures on Mathematics*, Lecture vi, p. 42, (1894).

Peirce: CP 4.118 Fn P1 p 90 Cross-Ref:††

†P1 *Method or doctrine of limits*: The doctrine that we cannot reason about infinite and infinitesimal quantities and that phrases in mathematics containing these and cognate words are not to be understood literally, but are to be interpreted as meaning that the functions spoken of behave in certain ways when their variables are indefinitely increased or diminished, and that the fundamental formulæ of the differential calculus should be based on the conception of a limit. The first of these positions is not now tenable; the hypothesis of infinite and infinitesimal quantities is consistent and can be reasoned about mathematically, but the doctrine of limits should be understood to rest upon the general principle that every proposition must be interpreted as referring to a possible experience. The problems to which this method is applied belong to three types: the summation of series, the problem of differentials and the problem of quadratics. It is the same as Newton's method of prime and ultimate ratios. Its rival is the method of infinitesimals which is almost excluded from the textbooks at present. -- *Century Dictionary and Cyclopædia*, p. 3458. Cf. 125ff, and 3.563ff.

Peirce: CP 4.121 Fn 1 p 92

†1 Cf. 214ff, 3.549n.

Peirce: CP 4.121 Fn 2 p 92

†2 See *Kritik der reinen Vernunft*, A169, 659; B211, 687.

Peirce: CP 4.121 Fn 1 p 93

†1 In 96 and 97.

Peirce: CP 4.122 Fn 2 p 93

†2 *Metaphysica*, 1069a, 5.

Peirce: CP 4.127 Fn 1 p 99

†1 Cf. 1.339, 1.343f, 2.293.

Peirce: CP 4.127 Fn 1 p 100

†1 See 5.213ff.

Peirce: CP 4.128 Fn 2 p 100

†2 Bk. I, def. 4; but cf. *Heath, The Thirteen Books of Euclid's Elements*, pp. 165-169, (1926).

Peirce: CP 4.128 Fn 3 p 100

†3 *Elements de géométrie*, livre premier, def. 3.

Peirce: CP 4.129 Fn 1 p 104

†1 Any three lines have a common mark or a common point.

Peirce: CP 4.129 Fn P1 p 104 Cross-Ref:††

†P1 Is this right? I have been working right on end for over twelve hours and with slight interruption for over 20 hours and I can hardly tell what I write. -- A marginal note, addressed apparently to Judge Russell, to whom the "Grand Logic" was submitted for criticism.

Peirce: CP 4.131 Fn 2 p 104

†2 *Kalkül der Abzählenden Geometrie* Hermann Schubert, Leipzig, (1879).

Peirce: CP 4.132 Fn 3 p 104

†3 See e.g., his *Nouveaux Exercices d'Analyse et de Physique mathématique*, t. III et IV (1840-1847).

Peirce: CP 4.132 Fn 1 p 105

†1 Cf. 2.191-2, 2.305, 2.778, 3.426.

Peirce: CP 4.132 Fn 1 p 108

†1 See 138.

Peirce: CP 4.133 Fn P1 p 108 Cross-Ref:††

†P1 See my "Description of a Notation for the Logic of Relatives." Also, my brochure entitled "Brief Description of the Algebra of Relatives." [Vol. 3, Nos. III and IX.]

Peirce: CP 4.134 Fn 1 p 109

†1 *Kalkül der Abzählenden Geometrie*, Leipzig, (1879).

Peirce: CP 4.134 Fn 2 p 109

†2 "The Calculus of Equivalent Statements and Integration Limits," *Proceedings*, London Mathematical Society, vol. ix, pp. 9-20 (1877-8).

Peirce: CP 4.134 Fn 3 p 109

†3 See 3.150f, and 3.324f.

Peirce: CP 4.136 Fn 1 p 110

†1 3.77ff.

Peirce: CP 4.138 Fn 1 p 113

†1 See 3.130f.

Peirce: CP 4.139 Fn 1 p 114

†1 Cf. 3.301ff.

Peirce: CP 4.140 Fn 1 p 118

†1 Cf. 3.305.

Peirce: CP 4.141 Fn P1 p 119 Cross-Ref:††

†P1 Professor Sylvester investigates the "general case" of multiple algebra. This is like enunciating the great truth that every human being who ever lived has been caught up into heaven (excepting only those who were neither Enoch nor Elijah). Only it is more extreme. For as Sylvester allows imaginary coefficients, his

"general case" of multiple algebra has not one single multiple, algebra or group under it. It is pure moonshine. Professor Sylvester ventilates his scorn for my father's work; but if he had studied it, he would have escaped the absurdity into which he falls. [See Sylvester's "Lectures on the Principles of Universal Algebra," *American Journal of Mathematics*, vol. 6, pp. 270-286, (1884).]

Peirce: CP 4.142 Fn P1 p 120 Cross-Ref:††

†P1 See Boole, *Calculus of Finite Differences*, 2d ed., ch. xv. Clebsch, *Geometrie*, Klein, *Das Ikosaeder*, Forsyth, *Theory of Functions*, ch. xxii, "Automorphic Functions."

Peirce: CP 4.143 Fn P1 p 122 Cross-Ref:††

†P1 The word *infinite* does not, as its etymology would suggest, mean *unlimited*; for we do not call the surface of a pea *infinite*. It means immeasurably great.

Peirce: CP 4.143 Fn P2 p 122 Cross-Ref:††

†P2 See *Wenzel Simerka*, "Die Kraft der Ueberzeugung," *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften*, Wien; Philosophisch-Historische Classe (CIV, 2 Heft, S. 511-571).

Peirce: CP 4.145 Fn P1 p 123 Cross-Ref:††

†P1 Cayley gave the name to the "circular points." ["On Evolutes and Parallel Curves," *Quarterly Journal of Pure and Applied Mathematics*, pp. 183-200, vol. XI (1871).] It seems to indicate that even at that early day, he had some insight into the philosophy of the subject. Yet, had he seen more clearly, he would have made the double line at infinity a part of the absolute.

Peirce: CP 4.151 Fn P1 p 128 Cross-Ref:††

†P1 Cauchy is often called a bad logician because he has made many logical blunders. It is true he did so; but nevertheless he knew how to reason better than others, who grope and never make a misstep because they only shuffle along.

Peirce: CP 4.151 Fn 1 p 129

†1 See 118 and 3.563f.

Peirce: CP 4.151 Fn 2 p 129

†2 See *Oeuvres*, t. 1, p. 133f. Paris, (1891).

Peirce: CP 4.151 Fn P1 p 130 Cross-Ref:††

†P1 Newcomb errs in saying (Johnson's *Cyclopædia*, 1894, IV, 567) this method is "medieval," and his description of it is not very characteristic. He is also wrong (Funk's *Dictionary*, *indivisible*) in calling it an application of the method of limits.

Peirce: CP 4.152 Fn 1 p 130

†1 See 125ff.

Peirce: CP 4.153 Fn 1 p 132

†1 From "Recreations in Reasoning," c. 1897.

Peirce: CP 4.153 Fn 2 p 132

†2 See vol. 6, bk. I, ch. 2.

Peirce: CP 4.155 Fn 1 p 133

†1 See *A system of Logic*, bk. 1, ch. ii, §5.

Peirce: CP 4.155 Fn 2 p 133

†2 The editors have been unable to find this statement in the writings of Mr. Leland. But see his *Gypsy Sorcery and Fortune Telling*, p. 210, London (1891).

Peirce: CP 4.156 Fn 1 p 134

†1 See 163, 187n.

Peirce: CP 4.157 Fn 2 p 134

†2 See vol. 1, bk. III.

Peirce: CP 4.157 Fn 1 p 135

†1 Cf. 1.383 and vol. 8.

Peirce: CP 4.157 Fn 2 p 135

†2 Cf. 1.436.

Peirce: CP 4.163 Fn 1 p 137

†1 See 187n.

Peirce: CP 4.166 Fn 1 p 139

†1 The proofs of the less important theorems have been omitted by the editors.

Peirce: CP 4.170 Fn 1 p 145

†1 1897. See 217 where the present paper is spoken of as if it were a lecture.

Peirce: CP 4.174 Fn 1 p 148

†1 Cf. 3.537.

Peirce: CP 4.180 Fn 1 p 152

†1 This proof is being omitted, having been given in 3.548. Cf. also 204.

Peirce: CP 4.182 Fn 1 p 155

†1 See *Oeuvres de Fermat*, t. III, pp. 431-436; Paris (1891-94). Cf. 110, 165.

Peirce: CP 4.185 Fn 1 p 156

†1 Cf. 183 and 106.

Peirce: CP 4.186 Fn P1 p 157 Cross-Ref:††

†P1 Euclid has been so over-admired by men who were far from seeing all the depth of thought in the first book of the *Elements*, that it is hard to speak of him as he deserves without risk of being understood to admire what is not admirable. Undoubtedly, too, some of the merits of the *Elements* were not original with Euclid. It is only the first book in which he has elaborated the logic as far as he was able. One of the remarkable merits of it is that Euclid had evidently gone far toward an understanding of the non-Euclidean geometry, and must undoubtedly be classed among the non-Euclidean. One evidence of that is that he puts his famous postulate about parallels into the form in which it most obtrusively displays its hypothetic character. He ranks it, too, as a postulate, that is as a dubitable proposition not demonstrated. Then, too, he arranges his theorems with those which hold for all systems of measurement first. But the greatest blunder of Euclid is in setting it down as an axiom that a part is less than its whole. That this is not true in regard to innumerable parts can be shown by a simple example. The collection of all the even numbers is only a part of the collection of all the whole numbers; for only every other number is even. But if we imagine the whole numbers written in a row, and under

each imagine its double written, there will be a distinct and separate even number written under every whole number. That is to say, the even numbers and all the whole numbers will be in one-to-one correspondence with one another, so that, by the definition of equality, the two collections are equal, although one is but a part of the other.

Peirce: CP 4.187 Fn P1 Para 1/6 p 159 Cross-Ref: ††

†P1 I shall use the language of the logic of relatives [to prove the "fundamental proposition of arithmetic"]. Namely, supposing $\{1\}$ signifies a class of ordered pairs of which PQ is one (QP may, or may not, belong to the class), then I shall say that P is a $\{1\}$ of Q and that Q is $\{1\}$ 'd by P.

Peirce: CP 4.187 Fn P1 Para 2/6 p 159 Cross-Ref: ††

I next define a *finite* class. Suppose a lot of things, say the A's, is such that whatever class of ordered pairs $\{1\}$ may signify, the following conclusion shall hold. Namely, if every A is a $\{1\}$ of an A, and if no A is $\{1\}$ 'd by more than one A, then every A is $\{1\}$ 'd by an A. If that necessarily follows, I term the collection of A's *finite* class.

Peirce: CP 4.187 Fn P1 Para 3/6 p 159 Cross-Ref: ††

I now proceed to prove the difficult part of the proposition, namely, that every collection of things the count of which can be completed by counting them in a suitable order of succession is finite. For suppose there be a collection of which this is not true, and call it the A's. Then, by the definition of a finite class, there must be some relative, or class of ordered pairs, $\{1\}$, such that while every A is a $\{1\}$ of an A, and no two A's $\{1\}$ of the same A, there is some A not $\{1\}$ 'd by any A. Then, I say that if this A, not $\{1\}$ 'd by any A, be removed from the class of A's, the same thing will remain true. Namely, first, every A is $\{1\}$ of an A, for so it was before the removal, and no A $\{1\}$ 'd by an A has been removed; second, no two A's are $\{1\}$ of the same A, for the removal could not increase the number fulfilling any positive condition; and third, there is still an A not $\{1\}$ 'd by any A, namely, that A which was $\{1\}$ 'd by the removed A, and by no other A. Now, the class of A's is said to have been counted, and by the definition of counting, some number must have been called out in counting the A that was afterward removed. Let every number higher than that be lowered by unity, and a count of the class, after A is removed, results.

Peirce: CP 4.187 Fn P1 Para 4/6 p 160 Cross-Ref: ††

It follows, then, that if there be a collection not finite the count of which can, by a suitable arrangement, be terminated by any number N, then the same is true of some collection the count of which can be terminated by a lower number. This implies there is no lowest number; but by definition of number, there is a lowest number, namely, *one*. Thus, the hypothesis that a class whose count in any order can be completed is not finite is reduced to absurdity.

Peirce: CP 4.187 Fn P1 Para 5/6 p 160 Cross-Ref: ††

Now, suppose a finite class to be counted twice. By the definition of a finite class, each count must stop. For make $\{1\}$ mean "next followed in the counting by" and the definition states that if the counting does not stop, then there is no A at which it begins, which is contrary to the definition of counting. If the two counts do not stop at the same number, call that the superior which stops at the higher number.

Peirce: CP 4.187 Fn P1 Para 6/6 p 160 Cross-Ref: ††

Let the cardinal numbers used in this "superior" count be called the S's. Let a

number of this count be said to be "successor" of the number which in the inferior count was called out against the same thing. Then, every S is successor of an S, but no two S's are successors of the same S, (since, by the definition of counting, no number was used twice in the inferior count). Consequently, the number of S's being finite by the definition of a finite class, every S is succeeded by an S, or, in other words, every S, including the greatest, was used in the inferior count. Hence, the two counts end with the same number. -- From "The Critic of Arguments," III (1892).

Peirce: CP 4.189 Fn 1 p 162

†1 See 3.546.

Peirce: CP 4.191 Fn 2 p 162

†2 Peirce's proof of this runs into several pages. As it is not an original proof, it has been omitted.

Peirce: CP 4.196 Fn 1 p 164

†1 *Georg Cantor Gesammelte Abhandlung*, S. 294-5 (1932).

Peirce: CP 4.196 Fn 1 p 165

†1 B?

Peirce: CP 4.196 Fn 2 p 165

†2 A?

Peirce: CP 4.197 Fn 1 p 166

†1 *Ibid.*, S. 296ff.

Peirce: CP 4.197 Fn 1 p 167

†1 The editors omitted 16 manuscript pages of proof showing that there are a vast multitude of indefinitely dividant relations between the units of any denumerable collection.

Peirce: CP 4.199 Fn 1 p 168

†1 *Ibid.*, S. 304.

Peirce: CP 4.200 Fn 1 p 169

†1 *Ibid.*, S. 288; 325ff.

Peirce: CP 4.200 Fn 2 p 169

†2 Cf. 117.

Peirce: CP 4.203 Fn 1 p 171

†1 See 180n.

Peirce: CP 4.204 Fn 2 p 171

†2 *Ibid.*, S. 278-80; 288.

Peirce: CP 4.205 Fn 1 p 172

†1 *Ibid.*, S. 288.

Peirce: CP 4.207 Fn 1 p 173

†1 Peirce used a square instead of a *P*.

Peirce: CP 4.210 Fn 1 p 176

†1 See *On the Principles of Political Economy and Taxation*, ch. II. Cf. 115.

- Peirce: CP 4.216 Fn 1 p 181
 †1 I.e. . . . any intuitional concept.
- Peirce: CP 4.216 Fn 1 p 182
 †1 The drawings have been omitted.
- Peirce: CP 4.223 Fn 1 p 187
 †1 Such as the centre of a circle.
- Peirce: CP 4.225 Fn 1 p 188
 †1 See his "Elementa doctrinæ solidarum" *Novi Commentarii Petropolitane*, T. IV, p. 119, (1752-3).
- Peirce: CP 4.227 Fn 1 p 189
 †1 Chapter 3 of the "Minute Logic," dated January-February, 1902. For the previous chapters see vol. 2, bk. I, ch. 1 and 2, and vol. 1, bk. II, ch. 2.
- Peirce: CP 4.227 Fn 2 p 189
 †2 These chapters were not written. See 1.584n.
- Peirce: CP 4.228 Fn 3 p 189
 †3 Cf. 1.247.
- Peirce: CP 4.229 Fn 4 p 189
 †4 See 2.9n.
- Peirce: CP 4.229 Fn 5 p 189
 †5 "Linear Associative Algebra" (1870), sec. 1, see *American Journal of Mathematics*, vol. 4 (1881).
- Peirce: CP 4.230 Fn P1 p 190 Cross-Ref:††
 †P1 From what is said by Proclus Diadochus, A.D. 485 [*Commentarii in Primum Euclidis Elementorum Librum*, Prologi pars prior, c. 12], it would seem that the Pythagoreans understood mathematics to be the answer to the two questions "how many?" and "how much?"
- Peirce: CP 4.230 Fn 1 p 190
 †1 *Metaphysica*, 1020a, 14-20.
- Peirce: CP 4.230 Fn 2 p 190
 †2 In *Porphyrii Isogogen sine v voces*, p. 5v., 1.11 *et seq.*
- Peirce: CP 4.230 Fn 3 p 190
 †3 *de institutione Arithmetica*, L. I, c. 1.
- Peirce: CP 4.230 Fn P2 p 190 Cross-Ref:††
 †P2 I regret I have not noted the passage of Ammonius to which I refer. It is probably one of the excerpts given by Brandis. My MS. note states that he gives reasons showing this to be his meaning.
- Peirce: CP 4.231 Fn P1 p 191 Cross-Ref:††
 †P1 510C to the end; but in the *Laws* his notion is improved.
- Peirce: CP 4.231 Fn 1 p 191
 †1 See *Metaphysica*, 1025^b1-1026^a33; 1060^b31-1061^b34.

Peirce: CP 4.232 Fn 2 p 191

†2 *Kritik der reinen Vernunft* Einleitung, B, §V.

Peirce: CP 4.232 Fn P1 p 192 Cross-Ref:††

†P1 A view which J. S. Mill (*Logic* II, V, §2) rather comically calls "the important doctrine of Dugald Stewart."

Peirce: CP 4.235 Fn 1 p 195

†1 Cf. 1.549n; 2.428.

Peirce: CP 4.235 Fn P1 p 196 Cross-Ref:††

†P1 Of course, the moment a collection is recognized as an abstraction we have to admit that even a percept is an abstraction or represents an abstraction, if matter has parts. It therefore becomes difficult to maintain that all abstractions are fictions.

Peirce: CP 4.238 Fn 1 p 198

†1 From {poieö}.

Peirce: CP 4.238 Fn 2 p 198

†2 By George Chrystal.

Peirce: CP 4.239 Fn 3 p 198

†3 *Was sind und was sollen die Zahlen; Vorwort*; (1888.)

Peirce: CP 4.240 Fn 1 p 200

†1 Cf. 1.577.

Peirce: CP 4.241 Fn 2 p 200

†2 But cf. 1.611.

Peirce: CP 4.242 Fn 1 p 201

†1 This point is not discussed in the "Minute Logic." But see 1.104f and vol. 6, bk. I.

Peirce: CP 4.243 Fn 1 p 202

†1 See vol. 2, ch. 1, §3.

Peirce: CP 4.243 Fn P1 p 202 Cross-Ref:††

†P1 It would not be fair, however, to suppose that every reader will know this. Of course, there are many series so extravagantly divergent that no use at all can be made of them. But even when a series is divergent from the very start, some use might commonly be made of it, if the same information could not otherwise be obtained more easily. The reason is -- or rather, one reason is -- that most series, even when divergent, approximate at last somewhat to geometrical series, at least, for a considerable succession of terms. The series $\log(1+x) = x - (1/2)x^2 + (1/3)x^3 - (1/4)x^4 +$, etc., is one that would not be judiciously employed in order to find the natural logarithm of 3, which is 1.0986, its successive terms being $2 - 2 + 8/3 - 4 + 32/5 - 32/3 +$, etc. Still, employing the common device of substituting for the last two terms that are to be used, say M and N, the expression $M/(1-N/M)$, the succession of the first six values is 0.667, 1.143, 1.067, 1.128, 1.067, which do show some approximation to the value. The mean of the last two, which any professional computer would use (supposing him to use this series, at all) would be 1.098, which is not very wrong. Of course, the computer would practically use the series $\log 3 = 1 + 1/12 + 1/80 + 1/448 +$,

etc., of which the terms written give the correct value to four places, if they are properly used.

Peirce: CP 4.244 Fn P1 p 203 Cross-Ref:††

†P1 "Formal Logic" is also used, by Germans chiefly, to mean that sect of Logic which makes Formal Logic pretty much the whole of Logic.

Peirce: CP 4.244 Fn 1 p 204

†1 See 227n.

Peirce: CP 4.247 Fn 1 p 205

†1 Cf. 1.205f.

Peirce: CP 4.252 Fn 1 p 207

†1 *Analyse des infiniment petits pour l'intelligence des lignes courbes*, §1, Def. I.

Peirce: CP 4.253 Fn 1 p 208

†1 See his *Mathematische Schriften*, h. von C. I. Gerhardt, Bd. I, S. 268; (1858).

Peirce: CP 4.253 Fn 2 p 208

†2 *Histoire de l'Academie Royale des Sciences*, pp. 100-139; (1718). Reprinted in his *Opera Omnia*, t. II, p. 241; (1742).

Peirce: CP 4.253 Fn 3 p 208

†3 *Werke*, Bd. I, S. 133-160; (1889).

Peirce: CP 4.261 Fn 1 p 213

†1 At this point Peirce introduces sixteen novel signs -- one for each of the possible dyadic connections of P, ~P, Q and ~Q. As he below abandons these signs for the conventional dot (to represent logical multiplication) and for a sign of logical disjunction, his sixteen signs are not being reproduced. Wherever it was necessary to differentiate the sixteen cases a more conventional symbolism was substituted by the editors.

Peirce: CP 4.263 Fn P1 p 214 Cross-Ref:††

†P1 Nor a relative from a non-relative universe.

Peirce: CP 4.264 Fn 1 p 215

†1 But cf. F. Cajori, *A History of Mathematics*, p. 158.

Peirce: CP 4.264 Fn 2 p 215

†2 This is another anticipation of the Shefferian stroke-function; cf. 12ff.

Peirce: CP 4.264 Fn 3 p 215

†3 I.e., $(x \vee y) \cdot \sim z$.

Peirce: CP 4.264 Fn 4 p 215

†4 I.e., $x \cdot \sim x$.

Peirce: CP 4.264 Fn 5 p 215

†5 I.e., $x \vee y$.

Peirce: CP 4.264 Fn 6 p 215

†6 I.e., $x \vee :x$.

Peirce: CP 4.264 Fn 1 p 216

†1 Peirce omitted the consideration of these five cases, though he did provide for them in the elided passages.

Peirce: CP 4.265 Fn 2 p 216

†2 In 262.

Peirce: CP 4.265 Fn 3 p 216

†3 I.e., a dichotomic algebra or logic can be developed through the use of but one logical constant; in this case, through the use of a single symbol representing the *disjunction* of the negatives of the symbolized constituents. In the first paper of this volume and in 264, Peirce used, as the one constant, a symbol representing the *conjunction* of the negatives of the symbolized constituents. He now shows that either one of these logical constants can be defined in terms of the other, thus:

$$\text{As } x \cdot y = \sim x \vee \sim y$$

$$\text{As } x(\sim \cdot)y = \sim x \vee \sim y$$

$$\text{And as } \sim x \cdot \sim y = \sim[(\sim x \vee \sim x) \vee (\sim y \vee \sim y)]$$

It must be true that

$$x \cdot y = ((x(\sim \cdot)x) \cdot (\sim \cdot)(y(\sim \cdot)y) \cdot (\sim \cdot)) \cdot ((x(\sim \cdot)x) \cdot (\sim \cdot)(y(\sim \cdot)y) \cdot (\sim \cdot))$$

$$\text{And as } \sim x \vee \sim y = \sim[\sim(\sim x \cdot \sim x) \cdot (\sim y \cdot \sim y)]$$

It must be true that

$$x(\sim \cdot)y = [((x \cdot x) \cdot (\psi \cdot \psi) \cdot (\sim \cdot)) \cdot ((\xi \cdot \xi) \cdot (\sim \cdot)(\psi \cdot \psi) \cdot (\sim \cdot))]$$

Peirce: CP 4.266 Fn 1 p 218

†1 By the sign x is meant any sign such that $x \text{ O } y \equiv x$; by y is meant any sign such that $x \text{ O } y \equiv y$; by $\sim x$ is meant any sign such that $x \text{ O } y \equiv \sim x$; and by $\sim y$ is meant any sign such that $x \text{ O } y \equiv \sim y$.

Peirce: CP 4.269 Fn 1 p 221

†1 *Formal Logic*, p. 56; *Syllabus*, §19.

Peirce: CP 4.273 Fn 1 p 225

†1 This is a clearer (and more accurate) representation, by reason of a reduction in the number of signs used, of the table Peirce gives. It says that if the symbols on the top of the columns be used to connect x and y , and the symbols at the end of the rows be used to connect x and y , then the first pair is connected necessarily with the second pair by the symbol at the intersection.

Peirce: CP 4.274 Fn 1 p 226

†1 See bk. II.

Peirce: CP 4.274 Fn 2 p 226

†2 *Lectures on Logic*, Lecture XIV.

Peirce: CP 4.274 Fn 3 p 226

†3 *Neue Darstellung der Logik*, §84.

Peirce: CP 4.274 Fn 4 p 226

†4 See *Symbolic Logic*, p. 122n.

Peirce: CP 4.274 Fn 5 p 226

†5 Obviously, if all the signs can be expressed by either \cdot or $(\sim \cdot)$ they can be expressed by any of the other six, for these other six differ from \cdot and $(\sim \cdot)$ only through the addition of signs of denial. Thus, $x \cdot \sim y$ is $\sim x \cdot y$; $\sim x \vee y$ is $x(\sim \cdot) \sim y$, etc. As $\sim x$ and $\sim y$ are themselves expressible by \cdot and $(\sim \cdot)$, the six signs may be viewed as abbreviations of some combination of either \cdot or $(\sim \cdot)$.

Peirce: CP 4.277 Fn 1 p 228

†1 See 3.4 and 3.199f.

Peirce: CP 4.280 Fn 1 p 229

†1 ∇ is Peirce's alternative symbol for logical addition, which was just now symbolized by \vee .

Peirce: CP 4.280 Fn 2 p 229

†2 In 3.199.

Peirce: CP 4.289 Fn P1 p 233 Cross-Ref:††

†P1 It will be remarked that the most logical treatment of associativeness and commutativeness is here considered to be that of ignoring them altogether.

Peirce: CP 4.289 Fn 1 p 233

†1 *Oeuvres de Fermat*, t. III, pp. 431-436, Paris, (1894).

Peirce: CP 4.301 Fn 1 p 243

†1 See 3.9, (18').

Peirce: CP 4.301 Fn P1 p 244 Cross-Ref:††

†P1 *Memoirs of the American Academy of Arts and Sciences, IX*, pp. 317-378 [Vol. 3, No. 3]; also with a separate title page and paging of its own, the title being *Description of a Notation for the Logic of Relatives, resulting from an Amplification of the Conceptions of Boole's Calculus of Logic*. My calling De Morgan's *logic of relations* by a slightly different name, for no better reason than that all logic treats of relations, was a youthful piece of bad manners of which I am now heartily ashamed. My work was due, of course, to the combined study of Boole's *Laws of Thought*, 1854, De Morgan "On the Syllogism and the Logic of Relatives" (*Cambridge Philosophical Transactions*, X, 1860, April 23). I interested my father in the subject, and his *Linear Associative Algebra* was issued to his friends before the printing of my memoir was complete. We were, therefore, working simultaneously upon closely related subjects, and continually discussing them together; and consequently, it is impossible to say precisely what was due to each. Of course, in mathematics, he was my master, and vastly my superior in genius; so that, in case of doubt, it is safer to attribute any mathematical step to him.

Peirce: CP 4.303 Fn 1 p 245

†1 See 3.126f.

Peirce: CP 4.303 Fn 2 p 245

†2 See 3.647.

Peirce: CP 4.304 Fn 1 p 246

†1 See vol. 3, No. X.

- Peirce: CP 4.305 Fn 2 p 246
†2 *Mathematische Schriften*, 2 Abt., 3 Bd., S. 160-3.
- Peirce: CP 4.305 Fn 3 p 246
†3 *Collected Mathematical Papers*, vol. I, pp. 241-50.
- Peirce: CP 4.305 Fn 1 p 247
†1 See 3.123.
- Peirce: CP 4.306 Fn 2 p 247
†2 : here represents relative multiplication.
- Peirce: CP 4.309 Fn 1 p 248
†1 Cf. 1.347.
- Peirce: CP 4.309 Fn 2 p 248
†2 Cf. 1.289f, 3.421, 3.469, 5.469.
- Peirce: CP 4.321 Fn 1 p 261
†1 See 3.127.
- Peirce: CP 4.322 Fn P1 p 262 Cross-Ref:††
†P1 This transformation somehow escaped publication at the time; I think probably because I was abroad, so that my father and I could not consult, and he thought it had been discovered by me, and I by him. It certainly was an obvious transformation of my algebra in view of certain ideas of his. It thus happened that Sylvester first published it long after, saying in his first mention of it, "To my certain knowledge this result was obtained by Mr. C. S. Peirce many years ago." [See 3.646f.]
- Peirce: CP 4.323 Fn 1 p 262
†1 That chapter does not seem to have been written. See 227n.
- Peirce: CP 4.324 Fn 1 p 263
†1 c. 1904.
- Peirce: CP 4.324 Fn 2 p 263
†2 *Transactions*, American Mathematical Society, vol. 5, pp. 288-309; (1904).
- Peirce: CP 4.326 Fn 1 p 265
†1 More accurately, Peirce.
- Peirce: CP 4.326 Fn 2 p 265
†2 See 3.3, 3.47 and 3.165. There are a considerable number of other improvements as is evident from the papers in volume 3.
- Peirce: CP 4.327 Fn 3 p 265
†3 See 305.
- Peirce: CP 4.327 Fn 4 p 265
†4 See 3.73, 3.331f.
- Peirce: CP 4.328 Fn 1 p 266
†1 Vol. 3, No. VI.

Peirce: CP 4.329 Fn 1 p 267

†1 Cf. 3.199.

Peirce: CP 4.330 Fn 2 p 267

†2 The manuscript ends here.

Peirce: CP 4.331 Fn 1 p 268

†1 c. 1905; 331-334 are from a proposed lecture to the Academy of American Arts and Sciences; the remainder of the paper is from "Topics," a revised version of the latter part of that lecture.

Peirce: CP 4.331 Fn 2 p 268

†2 Paper No. VII of vol. 3.

Peirce: CP 4.331 Fn 3 p 268

†3 *Op. cit.*, §64.

Peirce: CP 4.331 Fn 4 p 268

†4 Cf. 3.281f., 3.564.

Peirce: CP 4.331 Fn 5 p 268

†5 *Op. cit.*, §21.

Peirce: CP 4.331 Fn 6 p 268

†6 See Ernst Schröder's article, "Ueber zwei Definitionen der Endlichkeit u. G. Cantor'sche Sätze," *Nova Acta*, Bd. 71, S. 301 (1898).

Peirce: CP 4.331 Fn 7 p 268

†7 No such record has been found.

Peirce: CP 4.332 Fn 1 p 269

†1 Cf. 337 and 657ff.

Peirce: CP 4.332 Fn 2 p 269

†2 *Paradoxien des Unendlichen*, §19; *Wissenschaftslehre*, §84f.

Peirce: CP 4.332 Fn 3 p 269

†3 *Georg Cantor Gesammelte Abhandlungen*, S. 302.

Peirce: CP 4.332 Fn 4 p 269

†4 *Ibid.*, S. 312f.

Peirce: CP 4.332 Fn 1 p 270

†1 *Op. cit.*

Peirce: CP 4.332 Fn 2 p 270

†2 See 1.549n., 2.428.

Peirce: CP 4.333 Fn 1 p 271

†1 1881?; see 3.260ff.

Peirce: CP 4.333 Fn 2 p 271

†2 3.43-44.

Peirce: CP 4.337 Fn 1 p 274

†1 *Op. cit.*, S. 282.

- Peirce: CP 4.337 Fn 1 p 275
†1 *Ibid.*, S. 284ff.
- Peirce: CP 4.341 Fn 1 p 281
†1 1905.
- Peirce: CP 4.341 Fn 2 p 281
†2 $s[\alpha]$ means that every unit less than u has the character α .
- Peirce: CP 4.342 Fn 1 p 282
†1 See book II for a detailed analysis of these graphs.
- Peirce: CP 4.346 Fn 1 p 288
†1 The Note to Paper XIII, 3.403A ff.?
- Peirce: CP 4.347 Fn 1 p 293
†1 *Baldwin's Dictionary of Philosophy and Psychology*, vol. 2, p. 28, (2d edition, 1911). The Macmillan Co., New York.
- Peirce: CP 4.350 Fn 1 p 294
†1 From "Graphs," c. 1903.
- Peirce: CP 4.353 Fn 1 p 296
†1 *Symbolic Logic*, ch. 20, II, 2d ed., London, (1894).
- Peirce: CP 4.353 Fn 1 p 297
†1 Bd. I, s. 111ff.
- Peirce: CP 4.353 Fn 2 p 297
†2 Cf. 2.152ff.
- Peirce: CP 4.353 Fn 1 p 298
†1 *Anlage zur Architektonik*, i. 28.
- Peirce: CP 4.353 Fn 2 p 298
†2 Cf. Venn, *Symbolic Logic*, 2 ed., p. 509.
- Peirce: CP 4.353 Fn 3 p 298
†3 *Lectures on Metaphysics and Logic*, III, 256 (1874).
- Peirce: CP 4.353 Fn 4 p 298
†4 *System of Logic*, p. 302 (1871).
- Peirce: CP 4.353 Fn 5 p 298
†5 Th. 1, Abs. 1, §38, n. 2.
- Peirce: CP 4.353 Fn 6 p 298
†6 *Op. cit.*, p. 256.
- Peirce: CP 4.353 Fn 1 p 299
†1 *Ibid.*, Note β .
- Peirce: CP 4.353 Fn 2 p 299
†2 *Op. cit.*, pp. 507-8.
- Peirce: CP 4.353 Fn 3 p 299

†3 *Opera*, i, 607.

Peirce: CP 4.353 Fn 4 p 299

†4 *Op. cit.*, p. 507.

Peirce: CP 4.354 Fn P1 p 300 Cross-Ref:††

†P1 Two different sentences having the same meaning precisely are expressions of the same assertion.

Peirce: CP 4.354 Fn 1 p 301

†1 Cf. 3.466; 3.571.

Peirce: CP 4.354 Fn 2 p 301

†2 See vol. 2, bk. II, ch. 1.

Peirce: CP 4.354 Fn 1 p 302

†1 Cf. 2.287n.

Peirce: CP 4.355 Fn 1 p 304

†1 *Prior Analytics*, I, 1, 24b, 28; see also vol. 2, bk. III, ch. 4, §14 and Joseph's *An Introduction to Logic*, p. 296n and p. 308n, 2d. edition, revised (1916).

Peirce: CP 4.355 Fn 2 p 304

†2 *Op. cit.*, S. 9ff.

Peirce: CP 4.355 Fn 3 p 304

†3 Cf. 2.77 and 2.444.

Peirce: CP 4.358 Fn 1 p 307

†1 *I.e.*, All men are passionate and all non-saints are men.

Peirce: CP 4.358 Fn 2 p 307

†2 *I.e.*, No men are passionate and all non-saints are men.

Peirce: CP 4.361 Fn P1 p 310 Cross-Ref:††

†P1 This curious use of the word *Rule* is doubtless derived from the use of the word in Vulgar Arithmetic, where it signifies a method of computation adapted to a particular class of problems; as the Rule of Three, the Rule of Alligation, the Rule of False, the Rule of Fellowship, the Rule of Tare and Tret, the Rule of Coss. Here the Rule is a body of directions for performing an operation successfully. But when we speak of the Rule of Transposition, the directions are so simple, that the Rule becomes principally a permission.

Peirce: CP 4.362 Fn 1 p 310

†1 *I.e.*, "All x is y and Some x is y" can be transformed to "Either All x is y or Some ~x is y, and Some ~x is y or All ~x is y."

Peirce: CP 4.362 Fn 1 p 311

†1 *I.e.*, "Either Some S~P is ~M or All M is S~/P, and Some M~P is ~S or All P is M~/S" is transformable into "Either Some S~P is ~M or All P is M~/S."

Peirce: CP 4.362 Fn 2 p 311

†2 *I.e.*, "Either All S is P~/M or Some P~M is ~S, and Some SM is P and All M is S~/P" is transformable into "Some S is P."

Peirce: CP 4.362 Fn 3 p 311

†3 *I.e.*, "Either All S is P or Some P is \sim S, and either No S is P or No \sim S is \sim P" is transformable into " \sim (MSP) = 0 or MSP = 0; and MSP = 0 or $M\sim$ (SP) = 0; and $M\sim$ (SP) = 0 or \sim MSP = 0, and \sim MSP = 0 or \sim (MSP) = 0; and MS \sim P = 0 or Some \sim SM is P or Some \sim (SM) is P; and \sim MS \sim P = 0, or Some \sim SM is P or Some \sim (SM) is P."

Peirce: CP 4.363 Fn 1 p 313

†1 See 2.526n., 2.607 for the meaning of this term.

Peirce: CP 4.364 Fn 2 p 313

†2 *Johns Hopkins University Circular*, August, 1882.

Peirce: CP 4.369 Fn 1 p 318

†1 Peirce here illustrates the method by solving a complicated problem through the use of one hundred and thirty-five circles.

Peirce: CP 4.372 Fn 1 p 320

†1 Peirce's contribution to an article of that title in *Baldwin's Dictionary of Philosophy and Psychology*, vol. 2, pp. 645-50; 393 is by Peirce and Mrs. C. L. Franklin.

Peirce: CP 4.374 Fn 1 p 321

†1 See 3.136c.

Peirce: CP 4.375 Fn 2 p 321

†2 *Pure Logic*, chs. 6 and 15; (1864).

Peirce: CP 4.385 Fn 1 p 326

†1 Better: Something is \sim A and \sim B.

Peirce: CP 4.391 Fn 1 p 328

†1 See 3.351f, 3.499f.

Peirce: CP 4.391 Fn 1 p 329

†1 See 3.330ff, 3.492ff.

Peirce: CP 4.391 Fn 2 p 329

†2 See his *Substitution of Similars*; §41 (1869); *Pure Logic*, p. 111 (1890).

Peirce: CP 4.391 Fn 1 p 330

†1 p. 79.

Peirce: CP 4.392 Fn 2 p 330

†2 See 3.510.

Peirce: CP 4.393 Fn 3 p 330

†3 *Johns Hopkins Studies in Logic*, p. 25ff.

Peirce: CP 4.393 Fn 4 p 330

†4 *Logik*, (1880, 1883).

Peirce: CP 4.394 Fn 1 p 331

†1 *A Syllabus of Certain Topics of Logic*, pp. 15-23. Alfred Mudge & Son, Boston (1903). Continuing 2.226.

Peirce: CP 4.395 Fn 2 p 331

†2 Most of the terms such as "symbol," "replica," "rheme," "legisign" used in this paper are defined in vol. 2, bk. II, ch. 2.

Peirce: CP 4.395 Fn P1 p 331 Cross-Ref:††

†P1 "I abandon this inappropriate term, *replica*, Mr. Kempe having already ('Memoir on the Theory of Mathematical Form' [*Philosophical Transactions, Royal Society* (1886)], §170) given it another meaning. I now call it an instance." -- marginal note, c. 1910.

Peirce: CP 4.410 Fn 1 p 336

†1 *I.e.*, a broken cut.

Peirce: CP 4.414 Fn 1 p 338

†1 But see 579.

Peirce: CP 4.417 Fn 1 p 340

†1 But see 580.

Peirce: CP 4.417 Fn 2 p 340

†2 For the code of permissions for the Gamma part, which was not discussed in this printed pamphlet, see below, 470-1, and chapters 5 and 7.

Peirce: CP 4.418 Fn 1 p 341

†1 From "Logical Tracts, No. 2," c. 1903. "Logical Tracts, No. 1" is largely a repetition of the papers on signs in vol. 2, bk. II, ch. 2.

Peirce: CP 4.418 Fn 2 p 341

†2 Most of the terms such as "representamen," "icons," "indices," etc. are defined in vol. 2, bk. II, ch. 2.

Peirce: CP 4.423 Fn 1 p 342

†1 This does not seem to have been written.

Peirce: CP 4.424 Fn 1 p 343

†1 These conventions, together with No. 3, define the Alpha Part of Graphs.

Peirce: CP 4.424 Fn 2 p 343

†2 *E.g.*, Schröder; see 3.510ff.

Peirce: CP 4.426 Fn 1 p 344

†1 *Vorlesungen über die Algebra der Logik*, Bd. 3, §23 and §31, (1895).

Peirce: CP 4.434 Fn 1 p 349

†1 See 3.468ff.

Peirce: CP 4.435 Fn P1 p 350 Cross-Ref:††

†P1 In the language of logic "consequence" does not mean that which follows, which is called the *consequent*, but means the fact that a consequent follows from an antecedent.

Peirce: CP 4.435 Fn 1 p 352

†1 See 515f. on the broken cut.

Peirce: CP 4.438 Fn 1 p 353

†1 The conventions Nos. 4 to 12 define the Beta Part of Graphs.

Peirce: CP 4.438 Fn P1 Para 1/3 p 354 Cross-Ref:††

†P1 This, it will be remarked, makes what modern grammars call the direct and indirect objects, as well as much else, to be *subjects*; and some persons will consider this to be a bad abuse of the word *subject*. Come, let us have this out. I grant you that in polite literature usage is, not only almost, but altogether, the *arbitrium et jus et norma loquendi*. And if I am asked *whose* usage, I reply, that of the public whom you are addressing. If, with Vaugelas [*Remarques sur la langue française*], you are addressing the court, then the usage of the court. If you are lecturing the riffraff of a great city, then their usage. If anybody were to dispute this and ask me to prove it, I should reply that whatever ultimate purpose the polite *littérateur* may have, it is indispensable to that purpose that he should make the reading of what he writes agreeable; and in order that it may be agreeable, it is necessary that it should be easily understood by those who are addressed. But with logical writings it is different. If there be any sciences which can flourish without any words having any exact meanings, logic is not one of them. It cannot pursue its truths without a terminology of which every word shall have a single exact definition. To a great extent it already possesses such a terminology, notwithstanding the frequent abuse of its terms. But where this terminology is unsettled, to follow usage would simply be to prolong the confusion. There are conflicting individual predilections which must be made to give way; and there is only one thing to which they will consent to give way. It is some rational principle; which, stated generally, will recommend itself to all. Where are we to seek such a principle? In experience. He must profit by the experience of those sciences which have had the greatest difficulties with their terminology, and which have successfully surmounted those difficulties. Wherever this has been accomplished, it has been by adopting a rational general principle; and that principle has always been essentially the same. Any taxonomic zoölogist or botanist will tell you what it is. ***He who introduces a conception into the science shall have the right and the duty of assigning to it a suitable technical expression; and whoever thereafter uses that expression, technically in any other sense commits a grave misdemeanor***, since he thereby inflicts an injury upon the science. [Cf. 2.219-26.]

Peirce: CP 4.438 Fn P1 Para 2/3 p 355 Cross-Ref:††

Now let us apply this rule to the word *subject*. This was made a term of logic about A.D. 500 with this definition: "Subjectum est de quo dicitur id quod praedicatur" (*Boethii Opera*, Eds. of 1546 and 1570, p. 823, in *Comm. in Ciceronis Topica*, lib. v.) Now unless we were prepared to say that for different languages there are different doctrines of logic (which would be contrary to the essence of logic, as all will admit) we cannot, in this definition, take the preposition *de* in so narrow a sense as to exclude the grammatical accusative, dative, genitive and ablative of the verb. For dispersed through all the families of speech there are a dozen languages which either habitually or frequently express a proposition completely without putting any noun in the nominative. Among the European languages, Gaelic is an example, in which the principal subject is most commonly put in the genitive. But the logical fact is simply that it frequently makes a difference in the sense of a proposition which of the different nouns, naming objects to which the verb refers, is considered to be immediately attached to the verb, which to the combination of these two, and so on. Thus, in the sentence, "Some angel gives every man some gift," the verb "gives" is directly applied to "some gift," making "gives-a-gift"; then this action of gift-giving is applied to "every man"; finally the compound "gives-gift to every man" is applied to a certain angel; while in the sentence "A certain gift (perhaps, speech) is given to every man by some angel or other" the verb "is given by" is applied directly to "some angel," making "is angel-given to," which is applied to "every man," and then "is

angel-given to every man" is applied to a certain gift. One sentence represents one angel as distributing gifts to all men, the other represents one gift as bestowed by one or another angel on each man. Thus, the subject-nominative is ordinarily of all the subjects the one of which the verb is least directly said. I quite admit that I use the word *subject* as Boëthius never contemplated its being used; but it would be destructive to science to say that a term must be applied to nothing that its originator did not contemplate its being applied to. It is the definition only that holds.

Peirce: CP 4.438 Fn P1 Para 3/3 p 355 Cross-Ref:††

As a term of grammar, the word *subject* did not come into use until late in the eighteenth century. It would be somewhat impertinent, therefore, for grammarians to claim that, to their usage, the millennial usage of those from whom they borrowed the term, must bow.

Peirce: CP 4.447 Fn 1 p 359

†1 Cf. vol. 2, bk. II, ch. 2, §5.

Peirce: CP 4.449 Fn 1 p 362

†1 See 579.

Peirce: CP 4.449 Fn P1 p 363 Cross-Ref:††

†P1 This will be proved in a later note. [This was not done, but see 472f., 561n.]

Peirce: CP 4.455 Fn 1 p 366

†1 Cf. 564n.

Peirce: CP 4.462 Fn 1 p 370

†1 To illustrate this, two complicated graphs are given. They are not reproduced because the ambiguity in Peirce's explanations makes them unilluminating.

Peirce: CP 4.463 Fn 2 p 370

†2 This section deals in part with the Gamma Part of Graphs; see particularly 470-471. Cf. also 516ff.

Peirce: CP 4.469 Fn 1 p 373

†1 *I.e.*, the Gamma Part of Graphs.

Peirce: CP 4.471 Fn 2 p 373

†2 Bk. I, Postulate 5.

Peirce: CP 4.474 Fn 1 p 377

†1 But see 579.

Peirce: CP 4.476 Fn 2 p 377

†2 Cf. 2.186ff.

Peirce: CP 4.480 Fn P1 p 379 Cross-Ref:††

†P1 Some reader may think that I am expending energy in trying to explain what needs no explanation. He may argue that the mathematician reasons about a diagram in which there appears to be nothing at all corresponding to the structure of the proposition -- no predicate and subjects. Nor does the mathematician's premiss or conclusion at all pretend to represent the diagram in that respect. It may seem to this reader satisfactory to say that the conclusion follows from the premiss, because the

premiss is only applicable to states of things to which the conclusion is applicable. If he thinks that satisfactory, the purpose of this tract does not compel me to dispute it. It is only to defend myself against the charge of giving a needless and doubtful explanation that I point out that it is precisely this relation of applicability that requires to be explained. How comes it that the conclusion is applicable whenever the premiss is applicable? I suppose the answer will be that its only meaning is a part of what the premiss means. The "meaning" of a proposition is what it is intended to convey. But when a mathematician lays down the premisses of the theory of numbers, it cannot be said that he then intends to convey all the propositions of that theory, of which the great majority will occasion him much surprise when he comes to learn them. If to avoid this objection a distinction be drawn between what is explicitly intended and what is implicitly intended, I submit that this manifestly makes a vicious circle; for what can it be *implicitly* to intend anything, except to intend whatever may be a necessary consequence of what is explicitly intended?

Peirce: CP 4.483 Fn 1 p 381
†1 In 439-40.

Peirce: CP 4.494 Fn 1 p 387
†1 Cf. 2.356.

Peirce: CP 4.500 Fn 1 p 389
†1 In 447.

Peirce: CP 4.510 Fn 1 p 398
†1 From "Lowell Lectures of 1903." Lecture IV.

Peirce: CP 4.513 Fn 1 p 400
†1 Vol. 2, pp. 394-6 (1879); to be published in vol. 7.

Peirce: CP 4.514 Fn 2 p 400
†2 See 2.517ff.

Peirce: CP 4.515 Fn 1 p 402
†1 *I.e.*, It is possible that it does not rain.

Peirce: CP 4.516 Fn 2 p 402
†2 *I.e.*, It is false that *g* is possibly false.

Peirce: CP 4.516 Fn 1 p 403
†1 *I.e.*, It is possibly false that *g* is false; or it is possible that *g* is true.

Peirce: CP 4.516 Fn 2 p 403
†2 The passage from Figs. 179 to 181, from 181 to 182, and from 179 to 182 represent C. I. Lewis' subsequent "strict implications," 4.1, 4.12, 4.13 respectively. See his *Survey of Symbolic Logic*, p. 306-7. The broken cut represents Lewis' $\sim\sim$.

Peirce: CP 4.516 Fn 3 p 403
†3 *I.e.*, *g* is impossible.

Peirce: CP 4.516 Fn 4 p 403
†4 This transition is Lewis' 1.7, *ibid.*, p. 295.

Peirce: CP 4.516 Fn 5 p 403
†5 *I.e.*, *g* is not necessary, or it is possible that *g* is false. 183-6 is the

transformation 179-182 with $\sim g$ substituted for g .

Peirce: CP 4.517 Fn 6 p 403

†6 *I.e.*, if g be possibly true and false, and also true, it is necessarily true.

Peirce: CP 4.518 Fn 1 p 404

†1 *I.e.*, with respect to the given state of information, if g is true, it is necessarily true.

Peirce: CP 4.523 Fn 1 p 406

†1 Cf. 3.446.

Peirce: CP 4.526 Fn 1 p 408

†1 That lecture is not being published.

Peirce: CP 4.529 Fn 1 p 410

†1 The following was found on a separate sheet, apparently for use in a similar lecture:

$X - \Lambda$ [Click here to view] means "X is the sheet of assertion"

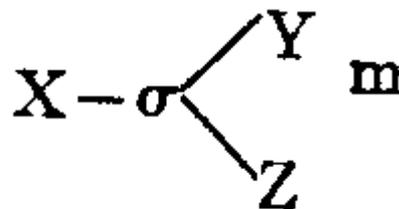
$X - \epsilon - Y$ [Click here to view] means "X is the area of the enclosure Y"

$X - \phi$ [Click here to view] means "X is a permission"

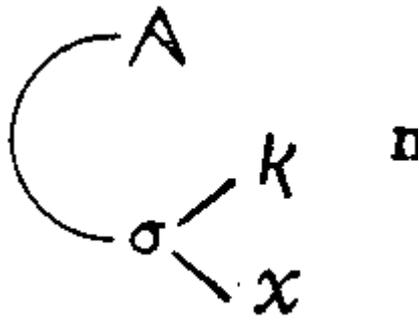
$X - \chi$ [Click here to view] means "X is a fact"

$X - \mu$ [Click here to view] means "X is a blank"

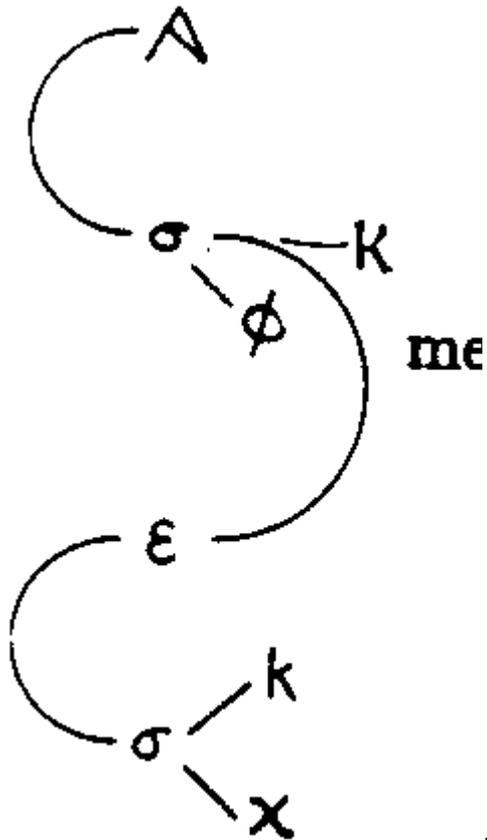
$X - R$ [Click here to view] means "X is an enclosure"



[Click here to view] means "X carries Y as its entire graph insofar as it is of the z nature of Z to make it do so," that is to say, for example



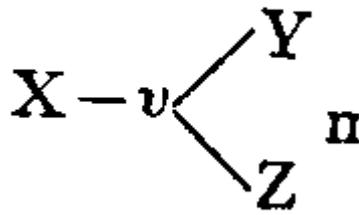
[Click here to view] means "An enclosure is the entire graph on the sheet of assertion as a fact."



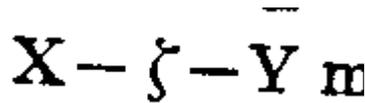
[Click here to view] means "It is permitted to place on the sheet of assertion, as the entire graph, an enclosure on whose area an enclosure is placed as a fact."

$X - \delta - Y \quad m$

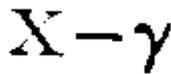
[Click here to view] means "The graph-replica denoted by X contains as a part of it, the replica Y."



[Click here to view] means "X is a line of identity having its terminals at Y and Z."



[Click here to view] means "X is a replica of the same graph of which Y is a replica, or is equivalent to Y."



110 [Click here to view] means "X is a graph-replica."

Peirce: CP 4.530 Fn 1 p 411

†1 *The Monist*, pp. 492-546, vol. 16 (1906), with some minor corrections as listed in vol. 17, p. 160. The two preceding articles of this series are in vol. 5, bk. II, Nos. 6 and 7.

Peirce: CP 4.530 Fn 2 p 411

†2 A detailed study of signs is to be found in vol. 2, bk. II.

Peirce: CP 4.531 Fn 1 p 413

†1 In his *Logische Studien* (1877).

Peirce: CP 4.531 Fn P1 p 414 Cross-Ref:††

†P1 In the original publication of this division, in 1867 [1.558] the term "representamen" was employed in the sense of a sign in general, while "sign" was taken as a synonym of *Index*, and an *Icon* was termed a "likeness."

Peirce: CP 4.531 Fn 1 p 415

†1 See 536n and 2.235n.

Peirce: CP 4.532 Fn 2 p 415

†2 Cf. 3.547ff.

Peirce: CP 4.534 Fn P1 Para 1/2 p 421 Cross-Ref:††

†P1 You apprehend in what way the system of Existential Graphs is to furnish a test of the truth or falsity of Pragmaticism. Namely, a sufficient study of the Graphs should show what nature is truly common to all significations of concepts; whereupon a comparison will show whether that nature be or be not the very ilk that Pragmaticism (by the definition of it) avers that it is. It is true that the two terms of this comparison, while in substance identical, yet might make their appearance under such different garbs that the student might fail to recognize their identity. At any rate, the possibility of such a result has to be taken into account; and therewith it must be acknowledged that, on its negative side, the argument may not turn out to be

sufficient. For example, *quâ* Graph, a concept might be regarded as the passive object of a geometrical *intuitus*, although Pragmaticism certainly makes the essence of every concept to be exhibited in an influence on possible conduct; and a student might fail to perceive that these two aspects of the concept are quite compatible.

Peirce: CP 4.534 Fn P1 Para 2/2 p 421 Cross-Ref:††

But, on the other hand, should the theory of Pragmaticism be erroneous, the student would only have to compare concept after concept, each one, first, in the light of Existential Graphs, and then as Pragmaticism would interpret it, and it could not but be that before long he would come upon a concept whose analyses from these two widely separated points of view unmistakably conflicted. . . . -- from Phaneroscopy {phan}"; one of a number of fragmentary manuscripts designed to follow the present article. See 540n; 553n and 1.306n.

Peirce: CP 4.535 Fn 1 p 421

†1 "Remarks on the Chemico-Algebraic Theory," *Mathematical Papers*, No. 28.

Peirce: CP 4.535 Fn 2 p 421

†2 "Chemistry and Algebra," *Mathematical Papers*, vol. III, no. 14.

Peirce: CP 4.536 Fn 1 p 422

†1 Cf. 5.475ff.

Peirce: CP 4.536 Fn 1 p 423

†1 Signs can be classified on the basis of the characters which (1) they, (2) their immediate and (3) their dynamical objects, and their (4) immediate, (5) dynamical and (6) final interpretants possess, as well as on the basis of the nature of relations which (7) the dynamical objects and the (8) dynamical and (9) final interpretants have to the sign and which the (10) final interpretant has to the object. These ten divisions provide thirty designations for signs (each division being trichotomized by the categories, First, Second and Third). When properly arranged, they are easily shown to yield but sixty-six classes of possible signs. The principle determining that conclusion is stated in the introduction to vol. 2 and in 2.235n. See also the letters to Lady Welby, vol. 9.

Peirce: CP 4.536 Fn 2 p 423

†2 (7) of the previous footnote.

Peirce: CP 4.537 Fn P1 p 423 Cross-Ref:††

†P1 Cf. 2.243; 2.247. Dr. Edward Eggleston originated the method.

Peirce: CP 4.537 Fn 3 p 423

†3 The type, token and tone are the legisigns, sinsigns and qualisigns discussed in 2.243f and form division (1) in the note to 536.

Peirce: CP 4.538 Fn 1 p 424

†1 These are defined in terms of the relation of the final interpretant to the sign. They constitute division (9) in the note to 536. Cf. 2.250f.

Peirce: CP 4.538 Fn 2 p 424

†2 Or rheme. But cf. 560.

Peirce: CP 4.538 Fn 3 p 424

†3 Or dicisign.

Peirce: CP 4.539 Fn 1 p 425

†1 *I.e.*, The perceptual judgment is a proposition of existence determined by the percept, which it interprets. See 541, 5.115ff and 5.151ff.

Peirce: CP 4.539 Fn 2 p 425

†2 *I.e.*, A complex of percepts yields a picture of a perceptual universe. Without reflection, that universe is taken to be the cause of such objects as are represented in a percept. Though each percept is vague, as it is recognized that its object is the result of the action of the universe on the perceiver, it is so far clear.

Peirce: CP 4.540 Fn 1 p 427

†1 This is the last published article of the present series. A number of incompleting papers, intended as the next article, have been found and published in part. See e.g., 1.305n, 1.306n, 534n, 553n, 561n, 564n, 5.549f.

Peirce: CP 4.541 Fn P1 p 428 Cross-Ref:††

†P1 Abduction, in the sense I give the word, is any reasoning of a large class of which the provisional adoption of an explanatory hypothesis is the type. But it includes processes of thought which lead only to the suggestion of questions to be considered, and includes much besides.

Peirce: CP 4.543 Fn 1 p 428

†1 Cf. 2.407ff.

Peirce: CP 4.543 Fn 2 p 428

†2 Cf. vol. 2, bk. II, ch. 4.

Peirce: CP 4.544 Fn P1 p 429 Cross-Ref:††

†P1 Strictly pure Symbols can signify only things familiar, and those only in so far as they are familiar.

Peirce: CP 4.544 Fn P1 p 430 Cross-Ref:††

†P1 I use the term *Universe* in a sense which excludes many of the so-called "universes of discourse" of which Boole [*An Investigation of the Laws of Thought*, etc., pp. 42, 167], De Morgan [*Cambridge Philosophical Transactions*, VIII, 380. *Formal Logic*, pp. 37-8] and many subsequent logicians speak, but which, being perfectly definable, would in the present system be denoted by the aid of a graph.

Peirce: CP 4.546 Fn 1 p 431

†1 2.536.

Peirce: CP 4.547 Fn 1 p 433

†1 *E.g.*, in 1.422. See also 580.

Peirce: CP 4.547 Fn P1 p 433 Cross-Ref:††

†P1 I take it that anything may fairly be said to be *destined* which is sure to come about although there is no necessitating reason for it. Thus, a pair of dice, thrown often enough, will be sure to turn up sixes some time, although there is no necessity that they should. The probability that they will is 1: that is all. *Fate* is that special kind of *destiny* by which events are supposed to be brought about *under definite circumstances* which involve no necessitating cause for those occurrences.

Peirce: CP 4.548 Fn 1 p 434

†1 The numeration has been changed to avoid ambiguity. Originally A, A¹

and 1 were all numbered 1; B, B¹ and 2 were all numbered 2, and not differentiated in the text.

Peirce: CP 4.548 Fn 1 p 435

†1 3, 5 and 7 were all numbered 3; and 4, 6 and 8 were all numbered 4 in the original and not distinguished in the text.

Peirce: CP 4.548 Fn 1 p 437

†1 Originally ". . . forms of statement of 3 and 4 on the other theory of the universes . . ."; a locution necessary so long as 3 and 5, and 4 and 6 were not distinguished.

Peirce: CP 4.548 Fn P1 p 437 Cross-Ref:††

†P1 In correcting the proofs, a good while after the above was written, I am obliged to confess that in some places the reasoning is erroneous; and a much simpler argument would have supported the same conclusion more justly; though some weight ought to be accorded to my argument here, on the whole.

Peirce: CP 4.549 Fn 1 p 438

†1 Usually called categories by Peirce. See vol. 1, bk. III.

Peirce: CP 4.550 Fn 2 p 438

†2 *I.e.*, Mind is a propositional function of the widest possible universe, such that its values are the meanings of all signs whose actual effects are in effective interconnection.

Peirce: CP 4.552 Fn P1 p 439 Cross-Ref:††

†P1 They may be two bodies of persons, two persons, or two mental attitudes or states of one person.

Peirce: CP 4.552 Fn P2 p 439 Cross-Ref:††

†P2 A *Graph* has already been defined in 535 *et seq.*

Peirce: CP 4.552 Fn P1 Para 1/2 p 440 Cross-Ref:††

†P1 The traditional and ancient use of the term propositional *Quality* makes it an affair of the mode of expression solely. For "Socrates is mortal" and "Socrates is immortal" are equally Affirmative; "Socrates is not mortal" and "Socrates is not immortal" are equally Negative, provided "is not" translates *non est*. If, however, "is not" is in Latin *est non*, with no difference of meaning, the proposition is infinitated. Without anything but the merest verbiage to support the supposition that there is any corresponding distinction between different meanings of propositions, Kant insisted on raising the difference of expression to the dignity of a category. In [3.532; but cf. 5.450] I gave some reason for considering a relative proposition to be affirmative or negative according as it does or does not unconditionally assert the existence of an indefinite subject. Although at the time of writing that, nine and a half years ago, I was constrained against my inclinations, to make that statement, yet I never heartily embraced that view, and dismissed it from my mind, until after I had drawn up the present statement of the Conventions of Existential Graphs, I found, quite to my surprise, that I had herein taken substantially the same view. That is to say, although I herein speak only of "relative" quality, calling the assertion of any proposition the Affirmation of it, and regarding the denial of it as an assertion *concerning* that proposition as subject, namely, that it is false; which is my distinction of Quality Relative to the proposition either itself Affirmed, or of which the falsity is affirmed, if the Relative Quality of it is Negative, yet since every Graph in itself either recognizes

the existence of a familiar Singular subject or asserts something of an indefinite subject asserted to exist in some Universe, it follows that every relatively Affirmative Graph unconditionally asserts or recognizes the occurrence of some description of object in some Universe; while no relatively Negative Graph does this. The logic of a Limited Universe of Marks [2.519ff.] suggests a different view of Quality, but careful analysis shows that it is in no fundamental conflict with the above.

Peirce: CP 4.552 Fn P1 Para 2/2 p 440 Cross-Ref:††

A question not altogether foreign to the subject of Quality is whether Quality and Modality are of the same general nature. In selecting a mode of representing Modality, which I have not done without much experimentation, I have finally resorted to one which commits itself as little as possible to any particular theory of the nature of Modality, although there are undeniable objections to such a course. If any particular analysis of Modality had appeared to me to be quite evident, I should have endeavored to exhibit it unequivocally. Meantime, my opinion is that the Universe is a Subject of every Proposition and that any Modality shown by its indefiniteness to be Affirmative, such as Possibility and Intention, is a special determination of the Universe of The Truth. Something of this sort is seen in Negation. For if we say of a Man that he is not sinless, we represent the sinless as having a place only in an ideal universe which, or the part of which that contains the imagined sinless being, we then positively sever from the identity of the man in question.

Peirce: CP 4.553 Fn P1 p 441 Cross-Ref:††

†P1 I may as well, at once, acknowledge that, in Existential Graphs, the representation of Modality (possibility, necessity, etc.) lacks almost entirely that pictorial, or Iconic, character which is so striking in the representation in the same system of every feature of propositions *de inesse*. Perhaps it is in the nature of things that it should be so in such wise that for Modality to be iconically represented in that same "pictorial" way in which the other features are represented would constitute a falsity in the representation. If so, it is a perfect vindication of the system, upon whose accusers, I suppose, the burden of proof lies. Still, I confess I suspect there is in the heraldic representation of modality as set forth [below] a defect capable of being remedied. If it be not so, if the lack of "pictorialness" in the representation of modality cannot be remedied, it is because modality has, in truth, the nature which I opined it has (which opinion I expressed toward the end of the footnote [to 552], and if that be the case, Modality is not, properly speaking, conceivable at all, but the difference, for example, between possibility and actuality is only recognizable much in the same way as we recognize the difference between a dream and waking experience, supposing the dream to be ever so detailed, reasonable, and thoroughly consistent with itself and with all the rest of the dreamer's experience. Namely, it still would not be so "vivid" as waking experience. . . -- from "Phanerology, {phan}," c. 1906; part of the manuscript used in 534n.

Peirce: CP 4.553 Fn P2 p 441 Cross-Ref:††

†P2 It was the genius of my gifted student, Dr. O. H. Mitchell, in [*Studies in Logic*, ed. by C. S. Peirce, p. 73ff] that first opened our eyes to the identity of the subject of all assertions, although in another sense one assertion may have several individual subjects, which may even belong to what Mitchell called (quite justifiably, notwithstanding a certain condemnatory remark, as superficial as it was supercilious), different dimensions of the logical Universe. The entire Phemic Sheet and indeed the whole Leaf [see 555] is an image of the universal field of interconnected Thought (for, of course, all thoughts are interconnected). The field of Thought, in its turn, is in

every thought, confessed to be a sign of that great external power, that Universe, the Truth. We all agree that we refer

to the same real thing when we speak of the truth, whether we think aright of it, or not. But we have no cognition of its essence that can, in strictness, be called a *concept* of it: we only have a direct perception of having the matter of our Thought forced upon it from outside our own control. It is thus, neither by immediate feeling, as we gaze at a red color, that we mean what we mean by the Truth; for Feeling tells of nothing but itself. Nor is it by the persuasion of reason, since reason always refers to two other things than itself. But it is by what I call a *dyadic consciousness*. -- from "The Bedrock beneath Pragmatism," c. 1906, one of a number of fragmentary manuscripts designed to follow the present article.

Peirce: CP 4.553 Fn P1 p 443 Cross-Ref:††

†P1 It is chiefly for the sake of these convenient and familiar modes of representation of Petrosancta, that a modification of heraldic tinctures has been adopted. Vair and Potent here receive less decorative and pictorial Symbols. Fer and Plomb are selected to fill out the quaternion of metals on account of their monosyllabic names.

Peirce: CP 4.556 Fn P1 p 444 Cross-Ref:††

†P1 I am tempted to say that it is the reversal alone that effects the denial, the Cut merely cutting off the Graph within from assertion concerning the Universe to which the Phemic Sheet refers. But that is not the only possible view, and it would be rash to adopt it definitely, as yet.

Peirce: CP 4.558 Fn P1 p 445 Cross-Ref:††

†P1 For, of course, the Graph-instance must be on one sheet; and if part were on the *recto*, and part on the *verso*, it would not be on one continuous sheet. On the other hand, a Graph-instance can perfectly well extend from one Province to another, and even from one *Realm* (or space having one Mode of Tincture) to another. Thus, the Spot, "-- is in the relation -- to --," may, if the relation is that of an existent object to its purpose, have the first Peg on Metal, the second on Color, and the third on Fur. Cf. 579.

Peirce: CP 4.561 Fn P1 p 447 Cross-Ref:††

†P1 The essential error, {to prōton pseudos}, of the Selectives, and their inevitable error, {to prōton pseudos}, lies in their putting forth, in a system which aims at giving, in its visible forms, a diagram of the logical structure of assertions, as a representation, for example, of the assertion that Tully and Cicero are the same man, a type of image which does not differ in form from the assertion that Julius Cæsar and Louis Seize were both men:

{ is Tully	{ is Julius Cæsar
{ is S	{ is a man

{ is S	{ is a man
{ is Cicero	{ is Louis Seize

. . . [The] purpose of the System of Existential Graphs, as it is stated in the Prolegomena [533], [is] to afford a method (1) as *simple* as possible (that is to say, with as small a number of arbitrary conventions as possible), for representing propositions (2) as *iconically*, or diagrammatically and (3) as *analytically* as possible. (The reason for embracing this purpose was developed through the first dozen pages

of this paper.) These three essential aims of the system are, every one of them, missed by Selectives. The first, that of the utmost attainable simplicity, is so, since a selective cannot be used without being attached to a Ligature, and Ligatures *without Selectives* will express all that Selectives with Ligatures express. The second aim, to make the representations as iconical as possible, is likewise missed; since Ligatures are far more iconic than Selectives. For the comparison of the above figures shows that a Selective can only serve its purpose through a special habit of interpretation that is otherwise needless in the system, and that makes the Selective a Symbol and not an Icon; while a Ligature expresses the same thing as a necessary consequence regarding each sizeable dot as an Icon of what we call an "individual object"; and it must be such an Icon if we are to regard an invisible mathematical point as an Icon of the strict individual, absolutely determinate in all respects, which imagination cannot realize. Meantime, the fact that a special convention (a clause of the fourth) is required to distinguish a Selective from an ordinary univalent Spot constitutes a second infraction of the purpose of simplicity. The third item of the idea of the System, that of being as analytical as possible, is infringed by Selectives in no less than three ways. This, at least, is the case if it be true, as I shall endeavour further on to convince the reader that it is, that Concepts are capable of being compounded only in a way differing but in one doubtful particular from that in which the so-called "substances" -- i.e., species -- of Organic Chemistry are compounded, according to the established theory of that science. (That respect is that the different bonds and *pegs* of the Spots of Graphs are different, while those of chemical atoms are believed to be all alike. But on the one hand, it may possibly be that a more nearly ultimate analysis of Concepts would show, as Kempe's "A Memoir on the theory of Mathematical Form" [*Philosophical Transactions* of the Royal Society, v. 177, pp. 1-70, 1886] seems to think, that the pegs of simple concepts are all alike. On the other hand, the carbon-atom seems to be the only one for the entire similarity of whose bonds there is much positive evidence. In the case of nitrogen, for example, two of the five *valencies* seem to be of such different quality from the others as to suggest that the individual *bonds* may likewise be different; and if there were such difference between different bonds of atoms generally, obvious probable causes would prevent our discovering [them] in the present state of chemistry. Looking at the question from the point of view of Thomson's corpuscles, it seems very unlikely that the looser electrons all fulfill precisely the same function in all cases.) For if this be true, the fact that two or more given concepts can be put together to produce one concept, without either of those that are so put together being separated into parts, is *conclusive proof* that the concept so produced is a compound of those that were put together. The principle, no doubt, requires to be proved. For it might easily be thought that the concept of a scalar as well as that of a vector (in quaternions) can equally result from putting together the concepts of a tensor and a versor in different ways, while at the same time the concept of a tensor and that of a versor can, in their turn, result from putting together those of a scalar and of a vector in different ways; so that no one of the four concepts is more or less composite than any of the others. Were such a view borne out by exact analysis, as it certainly is not, a radical disparateness between the composition of concepts and that of chemical species would be revealed. But this could scarcely fail to entail such a serious revolution in accepted doctrines of logic as it would be unwarrantable *gratuitously* to suppose that further investigation will bring about. It will be found that the available evidence is decidedly that Concepts can only be combined through definite "pegs." The first respect in which Selectives are not as analytical as they might be, and therefore ought to be, is in representing identity. The identity of the two S's above is only symbolically expressed. . . . Iconically, they

appear to be merely coexistent; but by the special convention they are interpreted as identical, though identity is not a matter of interpretation -- that is of logical depth -- but is an assertion of unity of Object, that is, is an assertion regarding logical breadth. The two S's are instances of one symbol, and that of so peculiar a kind that they are interpreted as *signifying*, and not merely *denoting*, one individual. There is here no analysis of identity. The suggestion, at least, is, quite decidedly, that identity is a simple relation. But the line of identity which may be substituted for the selectives very explicitly represents Identity to belong to the genus Continuity and to the species Linear Continuity. But of what variety of Linear Continuity is the heavy line more especially the Icon in the System of Existential Graphs? In order to ascertain this, let us contrast the Iconicity of the line with that of the surface of the Phemic Sheet. The continuity of this surface being two-dimensional, and so polyadic, should represent an external continuity, and especially, a continuity of experiential appearance. Moreover, the Phemic Sheet iconizes the Universe of Discourse, since it more immediately represents a field of Thought, or Mental Experience, which is itself directed to the Universe of Discourse, and considered as a sign, denotes that Universe. Moreover, it [is because it must be understood] *as* being directed to that Universe, that it is iconized by the Phemic Sheet. So, on the principle that logicians call "the *Nota notae*" that the sign of anything, X, is itself a sign of the very same X, the Phemic Sheet, in representing the field of attention, represents the general object of that attention, the Universe of Discourse. This being the case, the continuity of the Phemic Sheet in those places, where, nothing being scribed, no *particular* attention is paid, is the most appropriate Icon possible of the continuity of the Universe of Discourse -- where it only receives *general* attention as that Universe -- that is to say of the continuity in experiential appearance of the Universe, relatively to any objects represented as belonging to it. -- From "The Bedrock beneath Pragmaticism" (2) 1906; one of a number of fragmentary manuscripts designed to follow the present article.

Peirce: CP 4.563 Fn P1 p 450 Cross-Ref:††

†P1 It is permissible to have such spots as "possesses the character," "is in the real relation to," but it is not permissible to have such a spot as "can prevent the existence of."

Peirce: CP 4.564 Fn 1 p 450

†1 Vol. 3, No. XVI, §4.

Peirce: CP 4.564 Fn P1 p 452 Cross-Ref:††

†P1 I can make this blackened Inner Close as small as I please, at least, so long as I can still see it there, whether with my outer eye or in my mind's eye. Can I not make it quite invisibly small, even to my mind's eye? "No," you will say, "for then it would not be scribed at all." You are right. Yet since confession will be good for my soul, and since it will be well for you to learn how like walking on smooth ice this business of reasoning about logic is -- so much so that I have often remarked that nobody commits what is called a "logical fallacy," or hardly ever does so, except logicians; and they are slumping into such stuff continually -- it is my duty to [point out] this error of assuming that, because the blackened Inner Close can be made indefinitely small, therefore it can be struck out entirely like an infinitesimal. That led me to say that a Cut around a Graph instance has the effect of denying it. I retract: it only does so if the Cut enclosed also [has] a blot, however small, to represent iconically, the blackened Inner Close. I was partly misled by the fact that in the Conditional *de inesse* the Cut may be considered as denying the contents of its Area. That is true, so long as the entire Scroll is on the Place. But that does not prove that a

single Cut, without an Inner Close, has this effect. On the contrary, a single Cut, enclosing only A and a blank, merely says: "If A," or "If A, then" and there stops. If what? you ask. It does not say. "Then *something* follows," perhaps; but there is no assertion at all. This can be proved, too. For if we scribe on the Phemic Sheet the Graph expressing "If A is true, Something is true," we shall have a Scroll with A alone in the Outer Close, and with nothing but a Blank in the Inner Close. Now this Blank is an Iterate of the Blank-instance that is always present on the Phemic Sheet; and this may, according to the rule, be deiterated by removing the Blank in the inner close. This *will* do, what the blot would not; namely, it *will* cause the collapse of the Inner Close, and thus leaves A in a single cut. We thus see that a Graph, A, enclosed in a single Cut that contains nothing else but a Blank has no signification that is not implied in the proposition, "If A is true, Something is true." When I was in the twenties and had not yet come to the full consciousness of my own gigantic powers of logical blundering, with what scorn I used to think of Hegel's confusion of Being with Blank Nothing, simply because it had the form of a predicate without its matter! Yet here am I after devoting a greater number of years to the study of exact logic than the probable number of hours that Hegel ever gave to this subject, repeating that very identical fallacy! Be sure, Reader, that I would have concealed the mistake from you (for vanity's sake, if for no better reason), if it had not been "up to" me, in a way I could not evade, to expose it. -- From "Copy T," c. 1906; one of a number of fragmentary manuscripts designed to follow the present article.

Peirce: CP 4.569 Fn 1 p 458

†1 Cf. 580.

Peirce: CP 4.569 Fn 2 p 458

†2 In a letter to F. A. Woods, in 1913, Peirce expressed scepticism as to the universal validity of this permission; see vol. 9. See also 580.

Peirce: CP 4.572 Fn 1 p 463

†1 See 540n.

Peirce: CP 4.573 Fn 1 p 464

†1 From "For the National Academy of Science, 1906 April Meeting in Washington."

Peirce: CP 4.579 Fn 1 p 465

†1 414 (6).

Peirce: CP 4.579 Fn 1 p 466

†1 See e.g. 3.572.

Peirce: CP 4.580 Fn 2 p 466

†2 The shaded portions represent the verso.

Peirce: CP 4.580 Fn 3 p 466

†3 See 569.

Peirce: CP 4.580 Fn 1 p 467

†1 1903, see vol. 5, bk. I.

Peirce: CP 4.581 Fn 1 p 468

†1 See e.g. 3.332ff, 3.492ff; and 3.351ff, 3.499f.

Peirce: CP 4.581 Fn 2 p 468

†2 See 3.485, 3.618.

Peirce: CP 4.584 Fn 1 p 470

†1 See 6.102ff, 6.169ff.

Peirce: CP 4.585 Fn 1 p 473

†1 *The Monist*, pp. 227-241, vol. 18, April, 1908. The original title was "Some Amazing Mazes."

Peirce: CP 4.593 Fn 1 p 485

†1 See the next section.

Peirce: CP 4.594 Fn 2 p 485

†2 *The Monist*, pp. 416-64, vol. 18, July, 1908.

Peirce: CP 4.596 Fn 1 p 490

†1 See *Oeuvres*, t. II, p. 209; Paris, (1894).

Peirce: CP 4.596 Fn 1 p 491

†1 See *Leibnizens Mathematische Schriften*, ed. C. 1. Gerhardt, VII, S. 180.

Peirce: CP 4.596 Fn 2 p 491

†2 *Commentarii Academiae Petropolitanae*, t. VIII, pp. 141-6.

Peirce: CP 4.596 Fn 3 p 491

†3 *Nova Acta Eruditorum*, p. 109 (1769).

Peirce: CP 4.596 Fn 4 p 491

†4 *Op. cit.*, t. I, p. 25 (1747 et 1748); t. VII, p. 70 (1758 et 1759).

Peirce: CP 4.600 Fn 1 p 499

†1 By Sylvester; see his *Mathematical Papers*, IV, 102.

Peirce: CP 4.601 Fn 1 p 501

†1 1839.

Peirce: CP 4.602 Fn P1 p 502 Cross-Ref:††

†P1 This being the first [but see 595] occasion I have had in this essay to employ the word "modulus," I will take occasion to say that its general meaning is now well established. It means that signless quantity which measures the magnitude of a quantity and is a factor of it. So that if M and M' are the moduli of two quantities, $M\{m\}$ and $M'\{m\}'$, their product is $MM'\{mm\}'$, where MM' is an ordinary product, but $\{mm\}'$ may be a peculiar function. Thus, the absolute value of -2 , or 2 , is its "modulus", as 3 is of -3 ; and $(-2)\cdot(-3) = +6$ where $2\times 3 = 6$ by ordinary multiplication, but $(-1)\times(-1) = +1$ by an extension of ordinary multiplication. So the "modulus" of $A+Bi$, where $i^2 = -1$, is

$\sqrt{A^2+B^2}$. The tensor of a quaternion and the determinant of a square matrix are other examples of moduli. The cardinal number of numbers in a cycle has no sign and may properly be called the modulus of the cycle. But I sometimes refer to it as "the cycle," for short. The present usage of mathematicians is to use, what seems to me a too involved way of conceiving of cyclic arithmetic which carries with it an irregular use of the word "modulus." Legendre [in his *Théorie des Nombres*] and the earlier writers on cyclic arithmetic conceived of its numbers as signifying the lengths of different

steps along a cycle of objects, and thus spoke of 18 as being *equal* to 1 on a cycle of 17, just as we say that the 1st, 15th, 22d, and 29th days of August fall on *the same* day of the week, and just as we say that 270° of longitude west of any meridian and 90° east of it are *the very same* longitude. Gauss [in his *Disquisitiones Arithmeticae*], however, introduced a different locution, involving quite another form of thought. Instead of saying that 18 *is*, or *equals*, 1 in counting round a cycle of modulus 17, he prefers to say that 18 and 1 belong to the same *class* of numbers *congruent to* one another for the *modulus* 17. Here the idea of a cycle appears to be rejected in favor of the idea that $(18-1)/17$ is a whole number.

Peirce: CP 4.602 Fn P1 p 503 Cross-Ref:††

Now I fully admit that the conception of an indefinitely advancing series is involved in that of a cycle, and further that non-cyclical numbers have to be used to some extent in cyclic arithmetic. But at the same time it seems to me that the theoretic idea of a cycle ought to take the lead in this branch of mathematics. In particular, I cannot see why the term *cyclic logarithms* is not perfectly correct and far more expressive than Gauss's colorless name of "indices."

Peirce: CP 4.606 Fn 1 p 504

†1 Cf. 110, 188, 337ff, 3.258; 3.562B.

Peirce: CP 4.607 Fn 1 p 505

†1 Cf. 110.

Peirce: CP 4.608 Fn 2 p 505

†2 Cf. 332, 675.

Peirce: CP 4.611 Fn 1 p 506

†1 See 2.56n.

Peirce: CP 4.611 Fn 1 p 507

†1 Cf. vol. 6, bk. I, chs. 9 and 10.

Peirce: CP 4.611 Fn 2 p 507

†2 See 628.

Peirce: CP 4.611 Fn P1 p 507 Cross-Ref:††

†P1 You may well be puzzled, dear Reader, to iconize the consecution of a beginningless series upon an endless series. But you have only to imagine a dot to be placed upon the rim of a half-circle at each point whose angular distance from the beginning of the semicircumference has a positive or negative whole number for its natural tangent. These dots will, then, occur at the following angular distances from the origin of measurement.

[Elec. Ed.: See figure ->]

ANGULAR DISTANCE	TANGENT	ANGULAR DISTANCE	TANGENT	ANGULAR DISTANCE	
0° 00'	0	87° 24'	+22	93° 01'	-19
45 00	+ 1	87 31	+23	93 11	-18
63 26	+ 2	87 37	+24	93 21	-17
71 34	+ 3	87 43	+25	93 35	-16
75 58	+ 4			93 49	-15
78 41	+ 5	and so on endlessly. But		94 05	-14
80 32	+ 6	after all positive integer		94 24	-13
81 52	+ 7	values have been passed		94 46	-12
82 52	+ 8	through before 90°		95 12	-11
83 40	+ 9	(where there will not be		95 43	-10
84 17	+10	any dot), a beginningless		96 20	- 9
84 48	+11	series of dots will suc-		97 08	- 8
85 14	+12	ceed, for which the tan-		98 08	- 7
85 36	+13	gents are negative; and		99 28	- 6
85 55	+14	then		101 19	- 5
86 11	+15			104 02	- 4
86 25	+16	92° 17'	-25	108 26	- 3
86 38	+17	92 23	-24	116 34	- 2
86 49	+18	92 29	-23	135 09	- 1
86 59	+19	92 36	-22	180 00	0
87 08	+20	92 44	-21	225 00	+ 1
87 16	+21	92 52	-20	etc.	

[Click here to view]

Peirce: CP 4.612 Fn P1 p 508 Cross-Ref:††

†P1 See *Charmides*, p. 160A, and the last chapter of the First *Posterior Analytics* [A:34].

Peirce: CP 4.613 Fn 1 p 509

†1 Cf. 2.267.

Peirce: CP 4.613 Fn P1 p 510 Cross-Ref:††

†P1 {theôremation} is entered in L. & S. [Liddell & Scott, *Greek-English Lexicon*], with a reference to the Diatribes of Epictetus.

Peirce: CP 4.614 Fn 1 p 510

†1 Cf. 239f.

Peirce: CP 4.616 Fn 1 p 512

†1 But cf. the edition of 1884, p. 199.

Peirce: CP 4.617 Fn 1 p 514

†1 Cf. 3.499.

Peirce: CP 4.617 Fn 2 p 514

†2 Cf. 3.492ff.

Peirce: CP 4.617 Fn 3 p 514

†3 See his *Algebra u. Logik der Relative*, passim.

Peirce: CP 4.617 Fn 4 p 514

†4 *E.g.*, by Russell in his *Principles of Mathematics*, p. 10.

Peirce: CP 4.617 Fn 1 p 515

†1 See 565-69.

Peirce: CP 4.617 Fn 2 p 515

†2 See 569 and 580.

Peirce: CP 4.621 Fn 1 p 517

†1 See 560.

Peirce: CP 4.624 Fn 1 p 520

†1 See 613.

Peirce: CP 4.631 Fn P1 p 527 Cross-Ref:††

†P1 See Note at the end of the article [639ff].

Peirce: CP 4.633 Fn 1 p 530

†1 *E.g.*, Schröder.

Peirce: CP 4.633 Fn 2 p 530

†2 Cf. 332ff, 659ff, 673ff.

Peirce: CP 4.633 Fn 3 p 530

†3 See 635.

Peirce: CP 4.639 Fn 1 p 537

†1 This note was referred to in 631. Cf. also 121ff, 200ff, 219ff.

Peirce: CP 4.639 Fn 1 p 539

†1 See *e.g.* 3.567f.

Peirce: CP 4.640 Fn 1 p 540

†1 *E.g.*, Russell, *Principles of Mathematics*, p. 437.

Peirce: CP 4.641 Fn 2 p 540

†2 See *e.g.*, 5.289.

Peirce: CP 4.642 Fn 1 p 541

†1 That paper does not seem to have been written.

Peirce: CP 4.643 Fn 1 p 543

†1 *The Monist*, pp. 36-45, vol. 19, January 1909, Peirce's last published paper.

Peirce: CP 4.647 Fn 1 p 551

†1 From "Some Amazing Mazes, Fourth Curiosity," c. 1909. Neither the third nor the fourth papers of this series were previously published. The "Third Curiosity" contains little new. In the manuscript the present chapter follows shortly after 6.348.

Peirce: CP 4.647 Fn 2 p 551

†2 Cf. 1.203ff.

- Peirce: CP 4.648 Fn 3 p 551
†3 Cf. 3.66.
- Peirce: CP 4.648 Fn 1 p 552
†1 See *e.g.*, Russell, *Principles of Mathematics*, p. 68.
- Peirce: CP 4.649 Fn 2 p 552
†2 Cf. 3.537n.
- Peirce: CP 4.651 Fn 1 p 553
†1 *Paradoxien des Unendlichen*, §22, Leipzig (1851).
- Peirce: CP 4.652 Fn 1 p 554
†1 See 3.546.
- Peirce: CP 4.652 Fn 2 p 554
†2 Cf. 321, 635, 3.232.
- Peirce: CP 4.652 Fn 3 p 554
†3 See 3.547f.
- Peirce: CP 4.654 Fn 1 p 555
†1 Cf. 113, 218, 674 and 3.550.
- Peirce: CP 4.655 Fn 1 p 556
†1 Cf. 3.288.
- Peirce: CP 4.656 Fn 2 p 556
†2 *Leçons sur la théorie des Fonctions*, Paris, 1898. Borel does not prove the point here at issue.
- Peirce: CP 4.657 Fn 3 p 556
†3 See Georg Cantor, *Gesammelte Abhandlung*, S. 282, Berlin (1932).
- Peirce: CP 4.657 Fn 4 p 556
†4 See 3.546f.
- Peirce: CP 4.658 Fn 1 p 557
†1 *Liber Abaci* (1202).
- Peirce: CP 4.658 Fn 2 p 557
†2 *Ars Geometricæ*, Leipzig (1867).
- Peirce: CP 4.658 Fn 3 p 557
†3 *Arithmetica demonstrata* (1496).
- Peirce: CP 4.658 Fn 4 p 557
†4 *Tractatus de Arte Numerandi*, Strasburg (1488).
- Peirce: CP 4.658 Fn 5 p 557
†5 *Opus Majus*, Part 4.
- Peirce: CP 4.658 Fn 6 p 557
†6 *Regule Abaci*, *Bull. di Bibliographia*, T. XIV.
- Peirce: CP 4.658 Fn 7 p 557

- †7 *Arithmetica speculativa*, Paris (1495).
Peirce: CP 4.658 Fn 8 p 557
†8 *Opuscula*, Strasburg (1490).
- Peirce: CP 4.658 Fn 9 p 557
†9 *Algorisimus*, Padua (1483).
- Peirce: CP 4.658 Fn 10 p 557
†10 By U. Wagner (1482).
- Peirce: CP 4.658 Fn 11 p 557
†11 *Betrede und hubsche Rechnung*, Pforzheim (1489).
- Peirce: CP 4.658 Fn 12 p 557
†12 Anonymous.
- Peirce: CP 4.658 Fn 13 p 557
†13 *Libro de Abacho de Arithmetica*, Venice (1484).
- Peirce: CP 4.658 Fn 14 p 557
†14 *Suma*, Venice (1494).
- Peirce: CP 4.658 Fn 15 p 557
†15 By Nicolas Chuquet (1484).
- Peirce: CP 4.658 Fn 1 p 558
†1 *Protomathesis*, Paris (1530).
- Peirce: CP 4.658 Fn 2 p 558
†2 *Arithmetica Integra*, Nürnberg (1544),
- Peirce: CP 4.658 Fn 3 p 558
†3 Published in 1522.
- Peirce: CP 4.658 Fn 4 p 558
†4 Published in 1543.
- Peirce: CP 4.658 Fn 5 p 558
†5 *Arithmeticke*, London (1592).
- Peirce: CP 4.658 Fn 6 p 558
†6 *Exercises*, London (1594).
- Peirce: CP 4.658 Fn 7 p 558
†7 *The Arte of Vulgar Arithmeticke*, London (1600).
- Peirce: CP 4.658 Fn 8 p 558
†8 *Clavis Mathematicæ*, London (1631).
- Peirce: CP 4.658 Fn 9 p 558
†9 *Arithmetick*, ed. by Hawkins (1678).
- Peirce: CP 4.658 Fn 10 p 558
†10 *Elements of Arithmetic*, Philadelphia (1844).
- Peirce: CP 4.658 Fn 11 p 558

†11 *Elements of Arithmetic*, Philadelphia (1851, 1855).

Peirce: CP 4.660 Fn 1 p 560

†1 *Was sind. u. was sollen die Zahlen*, §73, §161.

Peirce: CP 4.660 Fn 2 p 560

†2 *Lehrbuch der Arithmetik u. Algebra*, Leipzig (1873).

Peirce: CP 4.660 Fn 3 p 560

†3 *Op. cit.*, S. 284f.

Peirce: CP 4.663 Fn 1 p 562

†1 See 107, 154, 3.242 and 3.331.

Peirce: CP 4.664 Fn 2 p 562

†2 Cf. 3.253ff.

Peirce: CP 4.666 Fn 1 p 564

†1 Cf. 190f., 3.262f., 3.562H.

Peirce: CP 4.666 Fn P1 p 564 Cross-Ref:††

†P1 When I write $a+b$, I conceive a to be addend and b to be the augend, on the general principle of putting the operator before the operand, though addition is usually conceived to violate this rule.

Peirce: CP 4.667 Fn 1 p 565

†1 Cf. 3.130, 3.327, 3.647.

Peirce: CP 4.668 Fn 2 p 565

†2 Cf. 193ff., 3.263f., 3.5621.

Peirce: CP 4.668 Fn 1 p 566

†1 $x/y = x/y$; $y \setminus x = y \cdot x$.

Peirce: CP 4.668 Fn 2 p 566

†2 Hermann Grassmann, *Die Ausdehnungslehre*, S. 11 (1878).

Peirce: CP 4.669 Fn P1 p 566 Cross-Ref:††

†P1 I remark that in my memoir of 1870 on "The Logic of Relatives" [3.53f.], although I insisted with emphasis on there generally being these two kinds of multiplication, I made no reference to Grassmann nor designated them as "internal" and "external" which I am all but absolutely sure that I should have done had I been acquainted with either of Grassmann's volumes. [But cf. 3.152, 3.242n.] So I infer that the too exclusive admiration of Hamilton in our household prevented my acquaintance with that great system. The matter interests me as showing that a person who was studying algebra purely from the point of view of logic was quite independently led to the recognition of the presence of the two kinds of multiplication *in associative systems generally*, in spite of an undisputed admiration for Hamilton.

Peirce: CP 4.674 Fn 1 p 571

†1 See 3.548f.

Peirce: CP 4.674 Fn 1 p 572

†1 See 113, 218, 3.550.

Peirce: CP 4.675 Fn 2 p 572

†2 *Op. cit.*, S. 168, S. 312.

Peirce: CP 4.677 Fn 1 p 574

†1 *Op. cit.*, S. 324f.

Peirce: CP 5 Title-Page
COLLECTED PAPERS OF CHARLES SANDERS PEIRCE

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INTRODUCTION

Peirce's punctuation and spelling have, wherever possible, been retained. Titles supplied by the editors for papers previously published are marked with an *E*, while Peirce's titles for unpublished papers are marked with a *P*. Peirce's titles for previously published papers and the editors' titles for unpublished papers are not marked. Remarks and additions by the editors are enclosed in light-face square brackets. The editors' footnotes are indicated by various typographical signs, while Peirce's are indicated by numbers. Paragraphs are numbered consecutively throughout the volume. At the top of each page the numbers signify the volume and the first paragraph of that page. All references in the indices are to the numbers of the paragraphs.

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HARVARD UNIVERSITY

MARCH, 1934.

Peirce: CP 5 Editorial Note p v
EDITORIAL NOTE

According to William James, the philosophical doctrine known as pragmatism was originated by Charles Sanders Peirce. The present volume contains practically everything of importance which Peirce is known to have written concerning his famous theory of "how to make ideas clear." There is, however, a short passage in a review, written in 1871, of Frazer's edition of the *Works of Berkeley*, quoted below for its historical interest:

A better rule [than Berkeley's] for avoiding the deceits of language is this: Do things fulfil the same function practically? Then let them be signified by the same word. Do they not? Then let them be distinguished. If I have learned a formula in gibberish which in any way jogs my memory so as to enable me in each single case to act as though I had a general idea, what possible utility is there in distinguishing between such a gibberish and formula and an idea? Why use the term *a general idea* in such a sense as to separate things which, for all experiential purposes, are the same? (*North American Review*, vol. 113, p. 469.)

Peirce: CP 5 Editorial Note p v

About half of the present volume consists of previously unpublished papers. Their significance lies in the light which they throw upon certain obscure aspects of pragmatism. For anyone other than Peirce himself, the attempt to summarize the gist of his pragmatic point of view will inevitably be difficult and perhaps impossible. It is here attempted only provisionally and subject to the reader's own judgment.

Peirce: CP 5 Editorial Note p v

In the first place, we learn more specifically than hitherto what Peirce regarded as the alternative to a pragmatic theory of meaning. This alternative was the traditional philosophical view that the abstract explains the concrete, and that the most abstract ideas are ultimate and unanalyzable (see 177, 207, 289, 294, 500ff). Pragmatism or pragmaticism (Peirce's term to indicate his divergencies from other pragmatists) was thus Peirce's way of insisting that abstractions must give an account of themselves, and must do it in terms of concrete experience. He held this position as early as 1868 (cf. 289, 294f, 504n). Only simple qualities of sense or feeling, or blind

reactions between these, can be indefinable; concepts are relational and definable.

Peirce: CP 5 Editorial Note p vi

In the second place, it is impossible that abstract generalities should be defined, as older empiricisms affirmed, in terms of mere qualities of sensation or emotion. For these qualities are incommunicable (at least in the present state of science; cf. 506) and particular (cf. 299ff, 312); whereas intellectual meanings or concepts must be public and general (312, 467).

Peirce: CP 5 Editorial Note p vi

Thirdly, that which is most general and public is a habit of behavior (486) directed towards an end (135, 491). The element of generality, which is never absent from the given (181ff, 212, 299ff, 371n), reaches its maximum in purpose; that is, a value capable of being embodied in a wide variety of existents (3, 433).

Peirce: CP 5 Editorial Note p vi

Fourthly, logic is subsidiary to ethics and esthetics (108ff). The ultimate meaning of an intellectual conception is given by its conceivable bearings upon deliberate or self-controlled conduct; but conduct that is fully deliberate in this sense is ethical and the end which it realizes is esthetic (129ff, 533). Thus pragmatism does not subordinate contemplative values to those of expediency (3, 402n, 429).

Peirce: CP 5 Editorial Note p vi

Fifthly, pragmatism is conceived to be a method in logic rather than a principle of metaphysics. It provides a maxim which determines the admissibility of explanatory hypotheses (195ff).

Peirce: CP 5 Editorial Note p vi

Sixthly, it entails scholastic realism, which in its final pragmatic interpretation (503) means the ascription of purposive habits to nature (107, 603).

Peirce: CP 5 Editorial Note p vi

The first book of the volume contains the Pragmatism Lectures of 1903 which deal primarily with questions of phenomenology, epistemology and value. The second book contains previously published papers. The first three papers show the drift towards pragmatism which characterized Peirce's thought in 1868. Papers IV to VII contain the published accounts of pragmatism on the basis of which his theory has hitherto been judged. Except for the first five papers of the second book, all of the selections in the volume date from 1898 (in most cases from 1903-1907), and all the unpublished papers which compose the third book of the volume are from this later period. There are other topics more or less tenuously related to pragmatism which are discussed at some length in the volume, e.g., the doctrine of critical common-sensism (books II and III; 265 containing the earliest statement); logical theory (318-341; 574-604), and philosophical terminology (413, 610f); while in volume 2, book II, and volume 4, book II, there are papers dealing with other matters which were conceived by Peirce to be particularly germane to the topic of pragmatism. The preface to volume 4 may serve perhaps as a useful guide to relate the papers in volumes 2-4 with those in the present volume, which itself reveals its affiliations with volume 1.

BRYN MAWR COLLEGE

UNIVERSITY OF CHICAGO

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§1. A DEFINITION OF PRAGMATIC AND PRAGMATISM †1E

1. *Pragmatic anthropology*, according to Kant,†2 is practical ethics.

Peirce: CP 5.1 Cross-Ref:††

Pragmatic horizon is the adaptation of our general knowledge to influencing our morals.

Peirce: CP 5.2 Cross-Ref:††

2. The opinion that metaphysics is to be largely cleared up by the application of the following maxim for attaining clearness of apprehension: "Consider what effects, that might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object."

Peirce: CP 5.2 Cross-Ref:††

[The doctrine that the whole "meaning" of a conception expresses itself in practical consequences, consequences either in the shape of conduct to be recommended, or in that of experiences to be expected, if the conception be true; which consequences would be different if it were untrue, and must be different from the consequences by which the meaning of other conceptions is in turn expressed. If a second conception should not appear to have other consequences, then it must really be only the first conception under a different name. In methodology it is certain that to trace and compare their respective consequences is an admirable way of establishing the differing meanings of different conceptions.]†3

Peirce: CP 5.3 Cross-Ref:††

3. This maxim was first proposed by C.S. Peirce in the *Popular Science Monthly* for January, 1878 (xii. 287);†4 and he explained how it was to be applied to the doctrine of reality. The writer was led to the maxim by reflection upon Kant's *Critic of the Pure Reason*. Substantially the same way of dealing with ontology seems to have been practised by the Stoics. The writer subsequently saw that the principle might easily be misapplied, so as to sweep away the whole doctrine of incommensurables, and, in fact, the whole Weierstrassian way of regarding the calculus. In 1896 William James published his *Will to Believe*,†1 and later †2 his *Philosophical Conceptions and Practical Results*, which pushed this method to such extremes as must tend to give us pause. The doctrine appears to assume that the end of man is action -- a stoical axiom which, to the present writer at the age of sixty, does not recommend itself so forcibly as it did at thirty. If it be admitted, on the contrary, that action wants an end, and that that end must be something of a general description, then the spirit of the maxim itself, which is that we must look to the upshot of our concepts in order rightly to apprehend them, would direct us towards something different from practical facts, namely, to general ideas, as the true interpreters of our thought. Nevertheless, the maxim has approved itself to the writer, after many years

of trial, as of great utility in leading to a relatively high grade of clearness of thought. He would venture to suggest that it should always be put into practice with conscientious thoroughness, but that, when that has been done, and not before, a still higher grade of clearness of thought can be attained by remembering that the only ultimate good which the practical facts to which it directs attention can subserve is to further the development of concrete reasonableness †3; so that the meaning of the concept does not lie in any individual reactions at all, but in the manner in which those reactions contribute to that development. Indeed, in the article of 1878, above referred to, the writer practised better than he preached; for he applied the stoical maxim most unstoically, in such a sense as to insist upon the reality of the objects of general ideas in their generality.†4

Peirce: CP 5.4 Cross-Ref:††

4. A widely current opinion during the last quarter of a century has been that reasonableness is not a good in itself, but only for the sake of something else. Whether it be so or not seems to be a synthetical question, not to be settled by an appeal to the principle of contradiction -- as if a reason for reasonableness were absurd. Almost everybody will now agree that the ultimate good lies in the evolutionary process in some way. If so, it is not in individual reactions in their segregation, but in something general or continuous. Synechism †1 is founded on the notion that the coalescence, the becoming continuous, the becoming governed by laws, the becoming instinct with general ideas, are but phases of one and the same process of the growth of reasonableness. This is first shown to be true with mathematical exactitude in the field of logic, and is thence inferred to hold good metaphysically. It is not opposed to pragmatism in the manner in which C.S. Peirce applied it, but includes that procedure as a step.

Peirce: CP 5.5 Cross-Ref:††

§2. THE ARCHITECTONIC CONSTRUCTION OF PRAGMATISM †2

5. . . . Pragmatism was not a theory which special circumstances had led its authors to entertain. It had been designed and constructed, to use the expression of Kant,†3 architectonically. Just as a civil engineer, before erecting a bridge, a ship, or a house, will think of the different properties of all materials, and will use no iron, stone, or cement, that has not been subjected to tests; and will put them together in ways minutely considered, so, in constructing the doctrine of pragmatism the properties of all indecomposable concepts †4 were examined and the ways in which they could be compounded. Then the purpose of the proposed doctrine having been analyzed, it was constructed out of the appropriate concepts so as to fulfill that purpose. In this way, the truth of it was proved.†5 There are subsidiary confirmations of its truth; but it is believed that there is no other independent way of strictly proving it. . . .

Peirce: CP 5.6 Cross-Ref:††

6. But first, what is its purpose? What is it expected to accomplish? It is expected to bring to an end those prolonged disputes of philosophers which no observations of facts could settle, and yet in which each side claims to prove that the other side is in the wrong. Pragmatism maintains that in those cases the disputants must be at cross-purposes. They either attach different meanings to words, or else one

side or the other (or both) uses a word without any definite meaning. What is wanted, therefore, is a method for ascertaining the real meaning of any concept, doctrine, proposition, word, or other sign. The object of a sign is one thing; its meaning is another. Its object is the thing or occasion, however indefinite, to which it is to be applied. Its meaning is the idea which it attaches to that object, whether by way of mere supposition, or as a command, or as an assertion.

Peirce: CP 5.7 Cross-Ref:††

7. Now every simple idea is composed of one of three classes; and a compound idea is in most cases predominantly of one of those classes. Namely, it may, in the first place, be a quality of feeling, which is positively such as it is, and is indescribable; which attaches to one object regardless of every other; and which is *sui generis* and incapable, in its own being, of comparison with any other feeling, because in comparisons it is representations of feelings and not the very feelings themselves that are compared.†1 Or, in the second place, the idea may be that of a single happening or fact, which is attached at once to two objects, as an experience, for example, is attached to the experiencer and to the object experienced.†2 Or, in the third place, it is the idea of a sign or communication conveyed by one person to another (or to himself at a later time) in regard to a certain object well known to both. . . .†3 Now the bottom meaning of a sign cannot be the idea of a sign, since that latter sign must itself have a meaning which would thereby become the meaning of the original sign. We may therefore conclude that the ultimate meaning of any sign consists either in an idea predominantly of feeling or in one predominantly of acting and being acted on.†4 For there ought to be no hesitation in assenting to the view that all those ideas which attach essentially to two objects take their rise from the experience of volition and from the experience of the perception of phenomena which resist direct efforts of the will to annul or modify them.

Peirce: CP 5.8 Cross-Ref:††

8. But pragmatism does not undertake to say in what the meanings of all signs consist, but merely to lay down a method of determining the meanings of intellectual concepts, that is, of those upon which reasonings may turn. Now all reasoning that is not utterly vague, all that ought to figure in a philosophical discussion involves, and turns upon, precise necessary reasoning. Such reasoning is included in the sphere of mathematics, as modern mathematicians conceive their science. "Mathematics," said Benjamin Peirce, as early as 1870, "is the science which draws necessary conclusions";†1 and subsequent writers have substantially accepted this definition, limiting it, perhaps, to precise conclusions. The reasoning of mathematics is now well understood.†2 It consists in forming an image of the conditions of the problem, associated with which are certain general permissions to modify the image, as well as certain general assumptions that certain things are impossible. Under the permissions, certain experiments are performed upon the image, and the assumed impossibilities involve their always resulting in the same general way. The superior certainty of the mathematician's results, as compared, for example, with those of the chemist, are due to two circumstances. First, the mathematician's experiments being conducted in the imagination upon objects of his own creation, cost next to nothing; while those of the chemist cost dear. Secondly, the assurance of the mathematician is due to his reasoning only concerning hypothetical conditions, so that his results have the generality of his conditions; while the chemist's experiments relating to what will happen as a matter of fact are always open to the doubt whether unknown conditions may not alter. Thus, the mathematician knows that a column of figures will add up the same, whether it be set down in black ink or in red; because he goes on the

assumption that the sum of any two numbers of which one is M and the other one more than N will be one more than the sum of M and N; and this assumption says nothing about the color of the ink. The chemist assumes that when he mixes two liquids in a test-tube, there will or will not be a precipitate whether the Dowager Empress of China happens to sneeze at the time, because his experience has always been that laboratory experiments are not affected by such distant conditions. Still, the solar system is moving through space at a great rate, and there is a bare possibility that it may just then have entered a region in which sneezing has very surprising force.

Peirce: CP 5.9 Cross-Ref:††

9. Such reasonings and all reasonings turn upon the idea that if one exerts certain kinds of volition, one will undergo in return certain compulsory perceptions. Now this sort of consideration, namely, that certain lines of conduct will entail certain kinds of inevitable experiences is what is called a "practical consideration." Hence is justified the maxim, belief in which constitutes pragmatism; namely,

Peirce: CP 5.9 Cross-Ref:††

In order to ascertain the meaning of an intellectual conception one should consider what practical consequences might conceivably result by necessity from the truth of that conception; and the sum of these consequences will constitute the entire meaning of the conception.

Peirce: CP 5.10 Cross-Ref:††

10. Many plausible arguments in favor of this doctrine could easily be adduced; but the only way hitherto discovered of really proving its truth, without in any measure begging the question, is by following the thorny path that we have thus very roughly sketched.

Peirce: CP 5.11 Cross-Ref:††

§3. HISTORICAL AFFINITIES AND GENESIS †1

11. . . . Any philosophical doctrine that should be completely new could hardly fail to prove completely false; but the rivulets at the head of the river of pragmatism are easily traced back to almost any desired antiquity.

Peirce: CP 5.11 Cross-Ref:††

Socrates bathed in these waters. Aristotle rejoices when he can find them. They run, where least one would suspect them, beneath the dry rubbish-heaps of Spinoza. Those clean definitions that strew the pages of the *Essay concerning Humane Understanding* (I refuse to reform the spelling), had been washed out in these same pure springs. It was this medium, and not tar-water, that gave health and strength to Berkeley's earlier works, his *Theory of Vision* and what remains of his *Principles*. From it the general views of Kant derive such clearness as they have. Auguste Comte made still more -- much more -- use of this element; as much as he saw his way to using. Unfortunately, however, both he and Kant, in their rather opposite ways, were in the habit of mingling these sparkling waters with a certain mental sedative to which many men are addicted -- and the burly business men very likely to their benefit, but which plays sad havoc with the philosophical constitution. I refer to the habit of cherishing contempt for the close study of logic.

Peirce: CP 5.12 Cross-Ref:††

12. So much for the past. The ancestry of pragmatism is respectable enough; but the more conscious adoption of it as *lanterna pedibus* in the discussion of dark questions, and the elaboration of it into a method in aid of philosophic inquiry came, in the first instance, from the humblest *souche* imaginable. It was in the earliest seventies †1 that a knot of us young men in Old Cambridge, calling ourselves, half-ironically, half-defiantly, "The Metaphysical Club," -- for agnosticism was then riding its high horse, and was frowning superbly upon all metaphysics -- used to meet, sometimes in my study, sometimes in that of William James. It may be that some of our old-time confederates would today not care to have such wild-oats-sowings made public, though there was nothing but boiled oats, milk, and sugar in the mess. Mr. Justice Holmes, however, will not, I believe, take it ill that we are proud to remember his membership; nor will Joseph Warner, Esq. †2 Nicholas St. John Green was one of the most interested fellows, a skillful lawyer and a learned one, a disciple of Jeremy Bentham. His extraordinary power of disrobing warm and breathing truth of the draperies of long worn formulas, was what attracted attention to him everywhere. In particular, he often urged the importance of applying Bain's †3 definition of belief, as "that upon which a man is prepared to act." From this definition, pragmatism is scarce more than a corollary; so that I am disposed to think of him as the grandfather of pragmatism. Chauncey Wright, †1 something of a philosophical celebrity in those days, was never absent from our meetings. I was about to call him our corypheus; but he will better be described as our boxing-master whom we -- I particularly -- used to face to be severely pummelled. He had abandoned a former attachment to Hamiltonianism to take up with the doctrines of Mill, to which and to its cognate agnosticism he was trying to weld the really incongruous ideas of Darwin. John Fiske and, more rarely, Francis Ellingwood Abbot, were sometimes present, lending their countenances to the spirit of our endeavours, while holding aloof from any assent to their success. Wright, James, and I were men of science, rather scrutinizing the doctrines of the metaphysicians on their scientific side than regarding them as very momentous spiritually. The type of our thought was decidedly British. I, alone of our number, had come upon the threshing-floor of philosophy through the doorway of Kant, and even my ideas were acquiring the English accent.

Peirce: CP 5.13 Cross-Ref:††

13. Our metaphysical proceedings had all been in winged words (and swift ones, at that, for the most part), until at length, lest the club should be dissolved, without leaving any material *souvenir* behind, I drew up a little paper expressing some of the opinions that I had been urging all along under the name of pragmatism. This paper was received with such unlooked-for kindness, that I was encouraged, some half dozen years later, on the invitation of the great publisher, Mr. W.H. Appleton, to insert it, somewhat expanded, in the *Popular Science Monthly* for November, 1877 and January, 1878, not with the warmest possible approval of the Spencerian editor, Dr. Edward Youmans. The same paper appeared the next year in a French redaction in the *Revue Philosophique* (Vol. VI, 1878, p. 553; Vol. VII, 1879, p. 39). In those medieval times, I dared not in type use an English word to express an idea unrelated to its received meaning. The authority of Mr. Principal Campbell †2 weighed too heavily upon my conscience. I had not yet come to perceive, what is so plain today, that if philosophy is ever to stand in the ranks of the sciences, literary elegance must be sacrificed -- like the soldier's old brilliant uniforms -- to the stern requirements of efficiency, and the philosopher must be encouraged -- yea, and required -- to coin new terms to express such new scientific concepts as he may

discover, just as his chemical and biological brethren are expected to do. Indeed, in those days, such brotherhood was scorned, alike on the one side and on the other -- a lamentable but not surprising state of scientific feeling. As late as 1893, when I might have procured the insertion of the word pragmatism in the *Century Dictionary*, it did not seem to me that its vogue was sufficient to warrant that step.†P1

Peirce: CP 5.14 Cross-Ref:††

BOOK I

LECTURES ON PRAGMATISMP†1

PRAGMATISM AND PRAGMATICISM

LECTURE I

PRAGMATISM: THE NORMATIVE SCIENCES

§1. TWO STATEMENTS OF THE PRAGMATIC MAXIM

14. A certain maxim of Logic which I have called *Pragmatism* has recommended itself to me for divers reasons and on sundry considerations. Having taken it as my guide in most of my thought, I find that as the years of my knowledge of it lengthen, my sense of the importance of it presses upon me more and more. If it is only true, it is certainly a wonderfully efficient instrument. It is not to philosophy only that it is applicable. I have found it of signal service in every branch of science that I have studied. My want of skill in practical affairs does not prevent me from perceiving the advantage of being well imbued with pragmatism in the conduct of life.

Peirce: CP 5.15 Cross-Ref:††

15. Yet I am free to confess that objections to this way of thinking have forced themselves upon me and have been found more formidable the further my plummet has been dropped into the abyss of philosophy, and the closer my questioning at each new attempt to fathom its depths.

Peirce: CP 5.15 Cross-Ref:††

I propose, then, to submit to your judgment in half a dozen lectures an examination of the *pros* and *cons* of pragmatism by means of which I hope to show you the result of allowing to both *pros* and *cons* their full legitimate values. With more time I would gladly follow up the guiding thread so caught up and go on to ascertain what are the veritable conclusions, or at least the genera of veritable conclusions to which a carefully rectified pragmatism will truly lead. If you find what

I say acceptable, you will have learned something worth your while. If you can refute me, the gain will be chiefly on my side; but even in that I anticipate your acknowledging, when I take my leave of you, that the discussion has not been without profit; and in future years I am confident that you will recur to these thoughts and find that you have more to thank me for than you could understand at first.

Peirce: CP 5.16 Cross-Ref:††

16. I suppose I may take it for granted that you all know what *pragmatism* is. I have met with a number of definitions of it lately, against none of which I am much disposed to raise any violent protest. Yet to say exactly what pragmatism is describes pretty well what you and I have to puzzle out together.

Peirce: CP 5.16 Cross-Ref:††

We must start with some rough approximation of it, and I am inclined to think that the shape in which I first stated [it] will be the most useful one to adopt as matter to work upon, chiefly because it is the form most personal to your lecturer, and [upon] which for that reason he can discourse most intelligently. Besides pragmatism and personality are more or less of the same kidney.

Peirce: CP 5.17 Cross-Ref:††

17. I sent forth my statement in January 1878; and for about twenty years never heard from it again. I let fly my dove; and that dove has never come back to me to this very day. But of late quite a brood of young ones have been fluttering about, from the feathers of which I might fancy that mine had found a brood. To speak plainly, a considerable number of philosophers have lately written as they might have written in case they had been reading either what I wrote but were ashamed to confess it, or had been reading something that some reader of mine had read. For they seem quite disposed to adopt my term *pragmatism*. I shouldn't wonder if they were ashamed of me. What could be more humiliating than to confess that one has learned anything of a logician? But for my part I am delighted to find myself sharing the opinions of so brilliant a company. The new pragmatists seem to be distinguished for their terse, vivid and concrete style of expression together with a certain buoyancy of tone as if they were conscious of carrying about them the master key to all the secrets of metaphysics.

Peirce: CP 5.17 Cross-Ref:††

Every metaphysician is supposed to have some radical fault to find with every other, and I cannot find any direr fault to find with the new pragmatists than that they are *lively*. In order to be deep it is requisite to be dull.

Peirce: CP 5.18 Cross-Ref:††

18. On their side, one of the faults that I think they might find with me is that I make pragmatism to be a mere maxim of logic instead of a sublime principle of speculative philosophy. In order to be admitted to better philosophical standing I have endeavored to put pragmatism as I understand it into the same form of a philosophical theorem. I have not succeeded any better than this:

Peirce: CP 5.18 Cross-Ref:††

Pragmatism is the principle that every theoretical judgment expressible in a sentence in the indicative mood is a confused form of thought whose only meaning, if it has any, lies in its tendency to enforce a corresponding practical maxim expressible as a conditional sentence having its apodosis in the imperative mood.

Peirce: CP 5.18 Cross-Ref:††

But the Maxim of Pragmatism, as I originally stated it, *Revue philosophique* VII, is as follows:

Peirce: CP 5.18 Cross-Ref:††

Considérer quels sont les effets pratiques que nous pensons pouvoir être produits par l'objet de notre conception. La conception de tous ces effets est la conception complète de l'objet. [p. 48.]

Peirce: CP 5.18 Cross-Ref:††

Pour développer le sens d'une pensée, il faut donc simplement déterminer quelles habitudes elle produit, car le sens d'une chose consiste simplement dans les habitudes qu'elle implique. Le caractère d'une habitude dépend de la façon dont elle peut nous faire agir non pas seulement dans telle circonstance probable, mais dans toute circonstance possible, si improbable qu'elle puisse être. Ce qu'est une habitude dépend de ces deux points: quand et comment elle fait agir. Pour le premier point: quand? tout stimulant à l'action dérive d'une perception; pour le second point: comment? le but de toute action est d'amener au résultat sensible. Nous atteignons ainsi le tangible et le pratique comme base de toute différence de pensée, si subtile qu'elle puisse être. [p. 47.]

Peirce: CP 5.19 Cross-Ref:††

§2. THE MEANING OF PROBABILITY

19. The utility of the maxim, provided it is only true, appears in a sufficient light in the original article. I will here add a few examples which were not given in that paper.

Peirce: CP 5.19 Cross-Ref:††

There are many problems connected with probabilities which are subject to doubt. One of them, for example, is this: Suppose an infinitely large company of infinitely rich men sit down to play against an infinitely rich bank at a game of chance, at which neither side has any advantage, each one betting a franc against a franc at each bet. Suppose that each player continues to play until he has netted a gain of one franc and then retires, surrendering his place to a new player.

Peirce: CP 5.19 Cross-Ref:††

The chance that a player will ultimately net a gain of a franc may be calculated as follows:

Peirce: CP 5.19 Cross-Ref:††

Let XL be a player's chance, if he were to continue playing indefinitely, of ever netting a gain of 1 franc.

Peirce: CP 5.19 Cross-Ref:††

But after he has netted a gain of 1 franc, his chance of doing which is $X[1]$, he is no richer than before, since he is infinitely rich. Consequently his chance of winning the second franc, after he has won the first, is the same as his chance of winning the first franc. That is, it is $X[1]$ and his chance of winning both is $X[2] = (X[1])^2$. And so in general, $X[L] = (X[1])^L$.

Peirce: CP 5.19 Cross-Ref:††

Now his chance of netting a gain of 1 franc, $X[1]$, is the sum of the chances of the two ways in which it may come about; namely by first winning the first bet of which the chance is $1/2$, and by first losing the first bet and then netting a gain of 2 francs of which the chance is $1/2 X[1]^2$. Therefore

$$X[1] = 1/2 + 1/2 X[1]^2$$

or $X[1]^2 - 2X[1] + 1 = 0$

or $(X[1] - 1)^2 = 0$.

But if the square of a number is zero, the number itself is zero. Therefore

$$X[1] - 1 = 0$$

or $X[1] = 1$.

Consequently, the books would say it was dead certain that any player will ultimately net his winning of a franc and retire. If so it must be certain that *every* player would win his franc and would retire.

Peirce: CP 5.19 Cross-Ref:††

Consequently there would be a continual outflow of money from the bank. And yet, since the game is an even one, the banker would not net any loss. How is this paradox to be explained?

Peirce: CP 5.20 Cross-Ref:††

20. The theory of probabilities is full of paradoxes and puzzles. Let us, then, apply the maxim of pragmatism to the solution of them.

Peirce: CP 5.20 Cross-Ref:††

In order to do this, we must ask *What is meant by saying that the probability of an event has a certain value, p ?* According to the maxim of pragmatism, then, we must ask what practical difference it can make whether the value is p or something else. Then we must ask how are probabilities applied to practical affairs. The answer is that the great business of insurance depends upon it. Probability is used in insurance to determine how much must be paid on a certain risk to make it safe to pay a certain sum if the event insured against should occur. Then, we must ask how can it be safe to engage to pay a large sum if an uncertain event occurs. The answer is that the insurance company does a very large business and is able to ascertain pretty closely out of a thousand risks of a given description how many in any one year will be losses. The business problem is this. The number of policies of a certain description that can be sold in a year will depend on the price set upon them. Let p be that price, and let n be the number that can be sold at that price, so that the larger p is, the smaller n will be. Now n being a large number a certain proportion q of these policies, qn in all, will be losses during the year; and if l be the loss on each, qnl will be the total loss. Then what the insurance company has to do is to set p at such a figure that $pn - qln$ or $(p - ql)n$ shall reach its maximum possible value.

The solution of this equation is:

$$p = ql + ((\delta p / \delta n)(n))$$

where $\delta p / \delta n$ is the amount by which the price would have to be lowered in order to sell one policy more. Of course if the price were raised instead of lowered just one policy fewer would be sold.

For then by so lowering the profit from being

$$(p - ql)n$$

[it] would be changed to

$$(p - ql - \delta p / \delta n)(n + 1)$$

that is to

$$(p - ql)n + p - ql - \delta p / \delta n(n + 1)$$

and this being less than before $ql + \delta p / \delta n(n + 1) > p$

and by raising it, the change would be to

$$(p - ql + \delta p / \delta n)(n - 1)$$

that is to

$$(p - ql)(n - p + ql + \delta p / \delta n(n - 1))$$

and this being less than before

$$p > ql + \delta p / \delta n (n-1)$$

so since p is intermediate between

$$ql + (\delta p / \delta n)n + \delta p / \delta n$$

and

$$ql + (\delta p / \delta n)n - \delta p / \delta n$$

and $\delta p / \delta n$ is very small, it must be close to the truth to write

$$p = ql + \delta p / \delta n (n).$$

Peirce: CP 5.21 Cross-Ref:††

21. This is the problem of insurance. Now in order that probability may have any bearing on this problem, it is obvious that it must be of the nature of a *real fact* and not a mere *state of mind*. For facts only enter into the solution of the problem of insurance. And this fact must evidently be a fact of statistics.

Peirce: CP 5.21 Cross-Ref:††

Without now going into certain reasons of detail that I should enter into if I were lecturing on probabilities, it must be that probability is a *statistical ratio*; and further, in order to satisfy still more special conditions, it is convenient, for the class of problems to which insurance belongs, to make it the statistical ratio of the number of experiential occurrences of a specific kind to the number of experiential occurrences of a generic kind, in the long run.†1

Peirce: CP 5.21 Cross-Ref:††

In order, then, that probability should mean anything, it will be requisite to specify to what *species* of event it refers and to what *genus* of event it refers.

Peirce: CP 5.21 Cross-Ref:††

It also refers to a *long run*, that is, to an indefinitely long series of occurrences taken together in the order of their occurrence in possible experience.

Peirce: CP 5.21 Cross-Ref:††

In this view of the matter, we note, to begin with, that a given species of event considered as belonging to a given genus of events does not necessarily have any definite probability. Because [it may be the case that] the probability is the ratio of one infinite multitude to another. Now infinity divided by infinity is altogether indeterminate, except in special cases.

Peirce: CP 5.22 Cross-Ref:††

22. It is very easy to give examples of events that have no definite probability. If a person agrees to toss up a cent again and again forever and beginning as soon as the first head turns up whenever two heads are separated by any odd number of tails in the succession of throws, to pay 2 to that power in cents, provided that whenever the two successive heads are separated by any even number of throws he receives 2 to that power in cents,†P1 it is impossible to say what the probability will be that he comes out a winner. In half of the cases after the first head the next throw will be a head and he will receive $(-2)^0 = 1$ cent. Which since it happens half the time will be in the long run a winning of $1/2$ a cent per head thrown.

Peirce: CP 5.22 Cross-Ref:††

But in half of the other half the cases, that is in $1/4$ of all the cases, one tail will intervene and he will have to receive $(-2)^1 = -2$ cents, i.e., he will have to pay 2 cents, which happening $1/4$ of the time will make an average loss of $1/2$ a cent per head thrown.

Peirce: CP 5.22 Cross-Ref:††

But in half the remaining quarter of the cases, *i.e.*, of all the cases, two tails will intervene and he will receive $(-2)^2 = 4$ cents which happening one every eight times will be worth $1/2$ a cent per head thrown and so on; so that his account in the long run will be $1/2 - 1/2 + 1/2 - 1/2 + 1/2 - 1/2 + 1/2 - 1/2$ *ad infinitum*, the sum of which may be $1/2$ or may be *zero*. Or rather it is quite indeterminate.

Peirce: CP 5.22 Cross-Ref:††

If instead of being paid $(-2)^n$ when n is the number of intervening tails, he were paid $(-2)^{n^2}$ the result would be he would probably either win or lose enormously without there being any definite probability that it would be winning rather than losing.

Peirce: CP 5.22 Cross-Ref:††

I think I may recommend this game with confidence to gamblers as being the most frightful ruin yet invented; and a little cheating would do everything in it.

Peirce: CP 5.23 Cross-Ref:††

23. Now let us revert to our original problem †1 and consider the state of things after every other bet. After the second, $1/4$ of the players will have gained, gone out, and been replaced by players who have gained and gone out, so that a number of francs equal to half the number of seats will have been paid out by the bank, $1/4$ of the players will have gained and gone out and been replaced by players who have lost, making the bank even; $1/4$ of the players will have lost and then gained, making the bank and them even; $1/4$ of the players will have lost twice, making a gain to the bank of half as many francs as there are seats at the table. The bank then will be where it was. Players to the number of three-quarters of the seats will have netted their franc each; but players to the number of a quarter of the seats will have lost two francs each and another equal number one franc each, just paying for the gains of those who have retired.

That is the way it will happen every time.

Peirce: CP 5.23 Cross-Ref:††

Just before the fifth bet of the players at the table, $3/8$ will have lost nothing, $1/4$ will have lost one franc, $1/4$ two francs, $1/16$ three francs and $1/16$ four francs.

Thus some will always have lost a good deal. Those who sit at the table will among them always have paid just what those who have gone out have carried away.

Peirce: CP 5.24 Cross-Ref:††

24. But it will be asked: How then can it happen that *all* gain? I reply that I never said that all would gain, I only said that the probability was 1 that anyone would ultimately gain his franc. But does not probability 1 mean certainty? Not at all, it only means that the ratio of the number of those who ultimately gain to the total number is 1. Since the number of seats at the table is infinite the ratio of the number of those who never gain to the number of seats may be zero and yet they may be infinitely numerous. So that probabilities 1 and 0 are very far from corresponding to certainty *pro* and *con*.†2

Peirce: CP 5.25 Cross-Ref:††

§3. THE MEANING OF "PRACTICAL" CONSEQUENCES

25. If I were to go into practical matters, the advantage of pragmatism, of looking at the substantial practical issue, would be still more apparent. But here pragmatism is generally practised by successful men. In fact, the genus of efficient men [is] mainly distinguished from inefficient precisely by this.

Peirce: CP 5.26 Cross-Ref:††

26. There is no doubt, then, that pragmatism opens a very easy road to the solution of an immense variety of questions. But it does not at all follow from that, that it is true. On the contrary, one may very properly entertain a suspicion of any method which so resolves the most difficult questions into easy problems. No doubt Ockham's razor is logically sound. A hypothesis should be stripped of every feature which is in no wise called for to furnish an explanation of observed facts. *Entia non sunt multiplicanda praeter necessitatem*; only we may very well doubt whether a very simple hypothesis can contain every factor that is necessary. Certain it is that most hypotheses which at first seemed to unite great simplicity with entire sufficiency have had to be greatly complicated in the further progress of science.

Peirce: CP 5.27 Cross-Ref:††

27. What is the *proof* that the possible practical consequences of a concept constitute the sum total of the concept? The argument upon which I rested the maxim in my original paper †1 was that *belief* consists mainly in being deliberately prepared to adopt the formula believed in as the guide to action. If this be in truth the nature of belief, then undoubtedly the proposition believed in can itself be nothing but a maxim of conduct. That I believe is quite evident.

Peirce: CP 5.28 Cross-Ref:††

28. But how do we know that belief is nothing but the deliberate preparedness to act according to the formula believed?

Peirce: CP 5.28 Cross-Ref:††

My original article carried this back to a psychological principle. The conception of truth, according to me, was developed out of an original impulse to act consistently, to have a definite intention. But in the first place, this was not very clearly made out, and in the second place, I do not think it satisfactory to reduce such

fundamental things to facts of psychology. For man could alter his nature, or his environment would alter it if he did not voluntarily do so, if the impulse were not what was advantageous or fitting. Why has evolution made man's mind to be so constructed? That is the question we must nowadays ask, and all attempts to ground the fundamentals of logic on psychology are seen to be essentially shallow.

Peirce: CP 5.29 Cross-Ref:††

29. The question of the nature of belief, or in other words the question of what the true logical analysis of the act of judgment is, is the question upon which logicians of late years have chiefly concentrated their energies. Is the pragmatistic answer satisfactory?

Peirce: CP 5.29 Cross-Ref:††

Do we not all perceive that *judgment* is something closely allied to *assertion*?†1 That is the view that ordinary speech entertains. A man or woman will be heard to use the phrase, "I says to myself." That is, *judgment* is held to be either no more than an *assertion to oneself* or at any rate something very like that.

Peirce: CP 5.30 Cross-Ref:††

30. Now it is a fairly easy problem to analyze the nature of *assertion*.†2 To find an easily dissected example, we shall naturally take a case where the assertive element is magnified -- a very formal assertion, such as an affidavit. Here a man goes before a notary or magistrate and takes such action that if what he says is not true, evil consequences will be visited upon him, and this he does with a view to thus causing other men to be affected just as they would be if the proposition sworn to had presented itself to them as a perceptual fact.

Peirce: CP 5.30 Cross-Ref:††

We thus see that the act of assertion is an act of a totally different nature from the act of apprehending the meaning of the proposition and we cannot expect that any analysis of what assertion is (or any analysis of what *judgment* or *belief* is, if that act is at all allied to assertion), should throw any light at all on the widely different question of what the apprehension of the meaning of a proposition is.

Peirce: CP 5.31 Cross-Ref:††

31. What is the difference between making an *assertion* and *laying a wager*? Both are acts whereby the agent deliberately subjects himself to evil consequences if a certain proposition is not true. Only when he offers to bet he hopes the other man will make himself responsible in the same way for the truth of the contrary proposition; while when he makes an *assertion* he always (or almost always) wishes the man to whom he makes it to be led to do what he does. Accordingly in our vernacular "I will bet" so and so, is the phrase expressive of a private opinion which one does not expect others to share, while "You bet" is a form of assertion intended to cause another to follow suit.

Peirce: CP 5.32 Cross-Ref:††

32. Such then seems at least in a preliminary glance at the matter to be a satisfactory account of assertion. Now let us pass to judgment and belief. There can, of course, be no question that a man will act in accordance with his belief so far as his belief has any practical consequences. The only doubt is whether this is *all* that belief is, whether belief is a mere nullity so far as it does not influence conduct. What possible effect upon conduct can it have, for example, to believe that the diagonal of a square is incommensurable with the side? Name a discrepancy ϵ no matter how small,

and the diagonal differs from a rational quantity by much less than that. Professor Newcomb in his calculus and all mathematicians of his rather antiquated fashion think that they have proved two quantities to be equal when they have proved that they differ by less than any assignable quantity. I once tried hard to make Newcomb say whether the diagonal of the square differed from a rational fraction of the side or not; but he saw what I was driving at and would not answer. The proposition that the diagonal is incommensurable has stood in the textbooks from time immemorial without ever being assailed and I am sure that the most modern type of mathematician holds to it most decidedly. Yet it seems quite absurd to say that there is any objective practical difference between commensurable and incommensurable. †1

Peirce: CP 5.33 Cross-Ref: ††

33. Of course you can say if you like that the act of expressing a quantity as a rational fraction is a piece of conduct and that it is in itself a practical difference that one kind of quantity can be so expressed and the other not. But a thinker must be shallow indeed if he does not see that to admit a species of practicality that consists in one's conduct about words and modes of expression is at once to break down all the bars against the nonsense that pragmatism is designed to exclude.

Peirce: CP 5.33 Cross-Ref: ††

What the pragmatist has his pragmatism for is to be able to say: here is a definition and it does not differ at all from your confusedly apprehended conception because there is no *practical* difference. But what is to prevent his opponent from replying that there is a practical difference which consists in his recognizing one as his conception and not the other? That is, one is expressible in a way in which the other is not expressible.

Peirce: CP 5.33 Cross-Ref: ††

Pragmatism is completely volatilized if you admit that sort of practicality.

Peirce: CP 5.34 Cross-Ref: ††

§4. THE RELATIONS OF THE NORMATIVE SCIENCES †1

34. It must be understood that all I am now attempting to show is that Pragmatism is apparently a matter of such great probable concern, and at the same time so much doubt hangs over its legitimacy, that it will be well worth our while to make a methodical, scientific, and thorough examination of the whole question, so as to make sure of our ground, and obtain some secure method for such a preliminary filtration of questions as pragmatism professes to furnish.

Peirce: CP 5.34 Cross-Ref: ††

Let us, then, enter upon this inquiry. But before doing so let us mark out the proposed course of it. That should always be done in such cases, even if circumstances subsequently require the plan to be modified, as they usually will.

Peirce: CP 5.34 Cross-Ref: ††

Although our inquiry is to be an inquiry into truth, whatever the truth may turn out to be, and therefore, of course, is not to be influenced by any liking for pragmatism or any pride in it as an American doctrine, yet still we do not come to this inquiry, any more than anybody comes to any inquiry, in that blank state that the

lawyers pretend to insist upon as desirable, though I give them credit for enough common-sense to know better.

Peirce: CP 5.35 Cross-Ref:††

35. We have some reason already to think there is some truth in pragmatism although we also have some reason to think that there is something wrong with it. For unless both branches of this statement were true we should do wrong to waste time and energy upon the inquiry we are undertaking.

Peirce: CP 5.35 Cross-Ref:††

I will, therefore, presume that there is enough truth in it to render a preliminary glance at ethics desirable. For if, as pragmatism teaches us, what we think is to be interpreted in terms of what we are prepared to do, then surely *logic*, or the doctrine of what we ought to think, must be an application of the doctrine of what we deliberately choose to do, which is Ethics.

Peirce: CP 5.36 Cross-Ref:††

36. But we cannot get any clue to the secret of Ethics -- a most entrancing field of thought but soon broadcast with pitfalls -- until we have first made up our formula for what it is that we are prepared to admire. I do not care what doctrine of ethics be embraced, it will always be so. Suppose, for example, our maxim of ethics to be Pearson's †1 that all our action ought to be directed toward the perpetuation of the biological stock to which we belong. Then the question will arise, On what principle should it be deemed such a fine thing for this stock to survive -- or a fine thing at all? Is there nothing in the world or *in posse* that would be admirable *per se* except copulation and swarming? Is swarming a fine thing at all, apart from any results that it may lead to? The course of thought will follow a parallel line if we consider Marshall's ethical maxim: Act to restrain the impulses which demand immediate reaction, in order that the impulse-order determined by the existence of impulses of less strength, but of wider significance, may have full weight in the guidance of your life. Although I have not as clear an apprehension as I could wish of the philosophy of this very close, but too technical, thinker, yet I presume that he would not be among those who would object to making Ethics dependent upon Esthetics. Certainly, the maxim which I have just read to you from his latest book †2 supposes that it is a fine thing for an impulse to have its way, but yet not an equally fine thing for one impulse to have its way and for another impulse to have its way. There is a preference which depends upon the *significance* of impulses, whatever that may mean. It supposes that there is some ideal state of things which, regardless of how it should be brought about and independently of any ulterior reason whatsoever, is held to be good or fine. In short, ethics must rest upon a doctrine which, without at all considering what our conduct is to be, divides ideally possible states of things into two classes, those that would be admirable and those that would be unadmirable, and undertakes to define precisely what it is that constitutes the admirableness of an ideal. Its problem is to determine by analysis what it is that one ought deliberately to admire *per se* in itself regardless of what it may lead to and regardless of its bearings upon human conduct. I call that inquiry *Esthetics*, because it is generally said that the three normative sciences are logic, ethics, and esthetics, being the three doctrines that distinguish good and bad; *Logic* in regard to representations of truth, *Ethics* in regard to efforts of will, and *Esthetics* in objects considered simply in their presentation. Now that third Normative science can, I think, be no other than that which I have described. It is evidently the basic normative science upon which as a foundation, the doctrine of ethics must be reared to be surmounted in its turn by the doctrine of logic.

Peirce: CP 5.37 Cross-Ref:††

37. But before we can attack any normative science, any science which proposes to separate the sheep from the goats, it is plain that there must be a preliminary inquiry which shall justify the attempt to establish such dualism. This must be a science that does not draw any distinction of good and bad in any sense whatever, but just contemplates phenomena as they are, simply opens its eyes and describes what it sees; not what it sees in the real as distinguished from figment -- not regarding any such dichotomy -- but simply describing the object, as a phenomenon, and stating what it finds in all phenomena alike. This is the science which Hegel made his starting-point, under the name of the *Phänomenologie des Geistes* -- although he considered it in a fatally narrow spirit, since he restricted himself to what *actually* forces itself on the mind and so colored his whole philosophy with the ignorance of the distinction of essence and existence and so gave it the nominalistic and I might say in a certain sense the *pragmatoidal* character in which the worst of the Hegelian errors have their origin. I will so far follow Hegel as to call this science *Phenomenology* although I will not restrict it to the observation and analysis of *experience* but extend it to describing all the features that are common to whatever is *experienced* or might conceivably be experienced or become an object of study in any way direct or indirect.†1

Peirce: CP 5.38 Cross-Ref:††

38. Hegel was quite right in holding that it was the business of this science to bring out and make clear the *Categories* or fundamental modes. He was also right in holding that these *Categories* are of two kinds; the Universal Categories all of which apply to everything, and the series of categories consisting of phases of evolution.

Peirce: CP 5.38 Cross-Ref:††

As to these latter, I am satisfied that Hegel has not approximated to any correct catalogue of them. It may be that here and there, in the long wanderings of his *Encyclopædia* he has been a little warmed by the truth. But in all its main features his catalogue is utterly wrong, according to me. I have made long and arduous studies of this matter, but I have not been able to draw up any catalogue that satisfies me. My studies,†2 if they are ever published, will I believe be found helpful to future students of this most difficult problem, but in these lectures I shall have little to say on that subject. The case is quite different with the three Universal Categories, which Hegel, by the way, does not look upon as Categories at all, or at least he does not call them so, but as three stages of thinking. In regard to these, it appears to me that Hegel is so nearly right that my own doctrine might very well be taken for a variety of Hegelianism, although in point of fact it was determined in my mind by considerations entirely foreign to Hegel, at a time when my attitude toward Hegelianism was one of contempt. There was no influence upon me from Hegel unless it was of so occult a kind as to entirely escape my ken; and if there was such an occult influence, it strikes me as about as good an argument for the essential truth of the doctrine, as is the coincidence that Hegel and I arrived in quite independent ways substantially to the same result.

Peirce: CP 5.39 Cross-Ref:††

39. This science of Phenomenology, then, must be taken as the basis upon which normative science is to be erected, and accordingly must claim our first attention.

Peirce: CP 5.39 Cross-Ref:††

This science of Phenomenology is in my view the most primal of all the positive sciences. That is, it is not based, as to its principles, upon any other *positive science*. By a *positive science* I mean an inquiry which seeks for positive knowledge; that is, for such knowledge as may conveniently be expressed in a *categorical proposition*. Logic and the other normative sciences, although they ask, not what *is* but what *ought to be*, nevertheless are positive sciences since it is by asserting positive, categorical truth that they are able to show that what they call good really is so; and the right reason, right effort, and right being, of which they treat, derive that character from positive categorical fact.

Peirce: CP 5.40 Cross-Ref:††

40. Perhaps you will ask me whether it is possible to conceive of a science which should not aim to declare that something is positively or categorically true. I reply that it is not only possible to conceive of such a science, but that such science exists and flourishes, and Phenomenology, which does not depend upon any other *positive science*, nevertheless must, if it is to be properly grounded, be made to depend upon the Conditional or Hypothetical Science of *Pure Mathematics*, whose only aim is to discover not how things actually are, but how they might be supposed to be, if not in our universe, then in some other.†1 A Phenomenology which does not reckon with pure mathematics, a science hardly come to years of discretion when Hegel wrote, will be the same pitiful club-footed affair that Hegel produced.

Peirce: CP 5.41 Cross-Ref:††
LECTURE II †1

THE UNIVERSAL CATEGORIES

§1. PRESENTNESS †2

41. . . . Be it understood, then, that what we have to do, as students of phenomenology, is simply to open our mental eyes and look well at the phenomenon and say what are the characteristics that are never wanting in it, whether that phenomenon be something that outward experience forces upon our attention, or whether it be the wildest of dreams, or whether it be the most abstract and general of the conclusions of science.

Peirce: CP 5.42 Cross-Ref:††

42.†3 The faculties which we must endeavor to gather for this work are three. The first and foremost is that rare faculty, the faculty of seeing what stares one in the face, just as it presents itself, unreplaced by any interpretation, unsophisticated by any allowance for this or for that supposed modifying circumstance. This is the faculty of the artist who sees for example the apparent colors of nature as they appear. When the ground is covered by snow on which the sun shines brightly except where shadows fall, if you ask any ordinary man what its color appears to be, he will tell you white, pure white, whiter in the sunlight, a little greyish in the shadow. But that is not what is before his eyes that he is describing; it is his theory of what *ought* to be seen. The

artist will tell him that the shadows are not grey but a dull blue and that the snow in the sunshine is of a rich yellow. That artist's observational power is what is most wanted in the study of phenomenology. The second faculty we must strive to arm ourselves with is a resolute discrimination which fastens itself like a bulldog upon the particular feature that we are studying, follows it wherever it may lurk, and detects it beneath all its disguises. The third faculty we shall need is the generalizing power of the mathematician who produces the abstract formula that comprehends the very essence of the feature under examination purified from all admixture of extraneous and irrelevant accompaniments.

Peirce: CP 5.43 Cross-Ref:††

43. A very moderate exercise of this third faculty suffices to show us that the word *Category* bears substantially the same meaning with all philosophers. For Aristotle, for Kant, and for Hegel, a category is an element of phenomena of the first rank of generality. It naturally follows that the categories are few in number, just as the chemical elements are. The business of phenomenology is to draw up a catalogue of categories and prove its sufficiency and freedom from redundancies, to make out the characteristics of each category, and to show the relations of each to the others. I find that there are at least two distinct orders of categories, which I call the particular and the universal. The particular categories form a series, or set of series, only one of each series being present, or at least predominant, in any one phenomenon. The universal categories, on the other hand, belong to every phenomenon, one being perhaps more prominent in one aspect of that phenomenon than another but all of them belonging to every phenomenon. I am not very well satisfied with this description of the two orders of categories, but I am pretty well satisfied that there are two orders. I do not recognize them in Aristotle, unless the predicaments and the predicables are the two orders. But in Kant we have Unity, Plurality, and Totality not all present at once; Reality, Negation, and Limitation not all present at once; Inherence, Causation, and Reaction not all present at once; Possibility, Necessity, and Actuality not all present at once. On the other hand Kant's four greater categories, Quantity, Quality, Relation, and Modality, form what I should recognize as Kant's Universal Categories. In Hegel his long list which gives the divisions of his *Encyclopædia* are his Particular Categories. His three stages of thought, although he does not apply the word *Category* to them, are what I should call Hegel's Universal Categories. My intention this evening is to limit myself to the Universal, or Short List of Categories, and I may say, at once, that I consider Hegel's three stages as being, roughly speaking, the correct list of Universal Categories. . . .

Peirce: CP 5.44 Cross-Ref:††

44. When anything is present to the mind, what is the very first and simplest character to be noted in it, in every case, no matter how little elevated the object may be? Certainly, it is its *presentness*. So far Hegel is quite right. Immediacy is his word. To say, however, that presentness, presentness as it is present, present presentness, is *abstract*, is Pure Being, is a falsity so glaring, that one can only say that Hegel's theory that the abstract is more primitive than the concrete blinded his eyes to what stood before them. Go out under the blue dome of heaven and look at what is present as it appears to the artist's eye. The poetic mood approaches the state in which the present appears as it is present. Is poetry so abstract and colorless? The present is just what it is regardless of the absent, regardless of past and future. It is such as it is, utterly ignoring anything else. Consequently, it cannot be *abstracted* (which is what Hegel means by the abstract) for the abstracted is what the concrete, which gives it whatever being it has, makes it to be. The present, being such as it is while utterly

ignoring everything else, is *positively* such as it is. Imagine, if you please, a consciousness in which there is no comparison, no relation, no recognized multiplicity (since parts would be other than the whole), no change, no imagination of any modification of what is positively there, no reflexion -- nothing but a simple positive character. Such a consciousness might be just an odour, say a smell of attar; or it might be one infinite dead ache; it might be the hearing of a piercing eternal whistle. In short, any simple and positive quality of feeling would be something which our description fits that it is such as it is quite regardless of anything else. The quality of feeling is the true psychological representative of the first category of the immediate as it is in its immediacy, of the present in its direct positive presentness. Qualities of feeling show myriad-fold variety, far beyond what the psychologists admit. This variety however is in them only insofar as they are compared and gathered into collections. But as they are in their presentness, each is sole and unique; and all the others are absolute nothingness to it -- or rather much less than nothingness, for not even a recognition as absent things or as fictions is accorded to them. The first category, then, is Quality of Feeling, or whatever is such as it is positively and regardless of aught else.

Peirce: CP 5.45 Cross-Ref:††
§2. STRUGGLE †1

45. The next simplest feature that is common to all that comes before the mind, and consequently, the second category, is the element of *Struggle*. It is convenient enough, although by no means necessary, to study this, at first, in a psychological instance. Imagine yourself making a strong muscular effort, say that of pressing with all your might against a half-open door. Obviously, there is a sense of resistance. There could not be effort without an equal resistance any more than there could be a resistance without an equal effort that it resists. Action and reaction are equal. If you find that the door is pushed open in spite of you, you will say that it was the person on the other side that acted and you that resisted, while if you succeed in pushing the door to, you will say that it was you who acted and the other person that resisted. In general, we call the one that succeeds by means of his effort the *agent* and the one that fails the *patient*. But as far as the element of Struggle is concerned, there is no difference between being an agent and being a patient. It is the result that decides; but what it is that is deemed to be the result for the purpose of this distinction is a detail into which we need not enter. If while you are walking quietly along the sidewalk a man carrying a ladder suddenly pokes you violently with it in the back of the head and walks on without noticing what he has done, your impression probably will be that he struck you with great violence and that you made not the slightest resistance; although in fact you must have resisted with a force equal to that of the blow. Of course, it will be understood that I am not using force in the modern sense of a moving force but in the sense of Newton's *actio*†2; but I must warn you that I have not time to notice such trifles. In like manner, if in pitch darkness a tremendous flash of lightning suddenly comes, you are ready to admit having received a shock and being acted upon, but that you reacted you may be inclined to deny. You certainly did so, however, and are conscious of having done so. The sense of shock is as much a sense of resisting as of being acted upon. So it is when anything strikes the senses. The outward excitation succeeds in producing its effect on you, while you in turn produce no discernible effect on it; and therefore you call it the agent, and overlook

your own part in the reaction. On the other hand, in reading a geometrical demonstration, if you draw the figure in your imagination instead of on paper, it is so easy to add to your image whatever subsidiary line is wanted, that it seems to you that you have acted on the image without the image having offered any resistance. That it is not so, however, is easily shown. For unless that image had a certain power of persisting such as it is and resisting metamorphosis, and if you were not sensible of its strength of persistence, you never could be sure that the construction you are dealing with at one stage of the demonstration was the same that you had before your mind at an earlier stage. The main distinction between the Inner and the Outer Worlds is that inner objects promptly take any modifications we wish, while outer objects are hard facts that no man can make to be other than they are. Yet tremendous as this distinction is, it is after all only relative. Inner objects do offer a certain degree of resistance and outer objects are susceptible of being modified in some measure by sufficient exertion intelligently directed.†1

Peirce: CP 5.46 Cross-Ref:††

46. Two very serious doubts arise concerning this category of struggle which I should be able completely to set to rest, I think, with only a little more time. But as it is, I can only suggest lines of reflexion which, if you perseveringly follow out, ought to bring you to the same result to which they have brought me. The first of these doubts is whether this element of struggle is anything more than a very special kind of phenomenon, and withal an anthropomorphic conception and therefore not scientifically true.

Peirce: CP 5.46 Cross-Ref:††

The other doubt is whether the idea of Struggle is a simple and irresolvable element of the phenomenon; and in opposition to its being so, two contrary parties will enter into a sort of [alliance] without remarking how deeply they are at variance with one another. One of these parties will be composed of those philosophers who understand themselves as wishing to reduce everything in the phenomenon to qualities of feeling. They will appear in the arena of psychology and will declare that there is absolutely no such thing as a specific sense of effort. There is nothing, they will say, but feelings excited upon muscular contraction, feelings which they may or may not be disposed to say have their immediate excitations within the muscles. The other party will be composed of those philosophers who say that there can be only one absolute and only one irreducible element, and since *Nous* is such an element, *Nous* is really the only thoroughly clear idea there is. These philosophers will take a sort of pragmatistic stand. They will maintain that in saying that one thing acts upon another, absolutely the only thing that can be meant is that there is a *law* according to which under all circumstances of a certain general description certain phenomena will result; and therefore to speak of one thing as acting upon another *hic et nunc* regardless of uniformity, regardless of what will happen on all occasions, is simple nonsense.

Peirce: CP 5.47 Cross-Ref:††

47. I shall have to content myself with giving some hints as to how I would meet this second double-headed objection, leaving the first to your own reflexions. In the course of considering the second objection, the universality of the element of struggle will get brought to light without any special arguments to that end. But as to its being unscientific because anthropomorphic, that is an objection of a very shallow kind, that arises from prejudices based upon much too narrow considerations. "Anthropomorphic" is what pretty much all conceptions are at bottom; otherwise

other roots for the words in which to express them than the old Aryan roots would have to be found. And in regard to any preference for one kind of theory over another, it is well to remember that every single truth of science is due to the affinity of the human soul to the soul of the universe, imperfect as that affinity no doubt is.†1 To say, therefore, that a conception is one natural to man, which comes to just about the same thing as to say that it is anthropomorphic, is as high a recommendation as one could give to it in the eyes of an Exact Logician.†P1

Peirce: CP 5.48 Cross-Ref:††

48. As for the double-headed objection, I will first glance at that branch of it that rests upon the idea that the conception of action involves the notion of law or uniformity so that to talk of a reaction regardless of anything but the two individual reacting objects is nonsense. As to that I should say that a law of nature left to itself would be quite analogous to a court without a sheriff. A court in that predicament might probably be able to induce some citizen to act as sheriff; but until it had so provided itself with an officer who, unlike itself, could not discourse authoritatively but who could put forth the strong arm, its law might be the perfection of human reason but would remain mere fireworks, *brutum fulmen*. Just so, let a law of nature -- say the law of gravitation -- remain a mere uniformity -- a mere formula establishing a relation between terms -- and what in the world should induce a stone, which is not a term nor a concept but just a plain thing, to act in conformity to that uniformity? All other stones may have done so, and this stone too on former occasions, and it would break the uniformity for it not to do so now. But what of that? There is no use talking reason to a stone. It is deaf and it has no reason. I should ask the objector whether he was a nominalist or a scholastic realist. If he is a nominalist, he holds that laws are mere generals, that is, formulae relating to mere terms; and ordinary good sense ought to force him to acknowledge that there are real connections between individual things regardless of mere formulae. Now any real connection whatsoever between individual things involves a reaction between them in the sense of this category. The objector may, however, take somewhat stronger ground by confessing himself to be a scholastic realist, holding that generals may be real. A law of nature, then, will be regarded by him as having a sort of *esse in futuro*. That is to say they will have a present reality which consists in the fact that events *will* happen according to the formulation of those laws. It would seem futile for me to attempt to reply that when, for example, I make a great effort to lift a heavy weight and perhaps am unable to stir it from the ground, there really is a struggle on this occasion regardless of what happens on other occasions; because the objector would simply admit that on such an occasion I have a quality of feeling which I call a feeling of effort, but he would urge that the only thing which makes this designation appropriate to the feeling is the regularity of connection between this feeling and certain motions of matter.

Peirce: CP 5.49 Cross-Ref:††

49. This is a position well enough taken to merit a very respectful reply. But before going into that reply, there is an observation which I should like to lay before the candid objector. Your argument against this category of Struggle is that a struggle regardless of law is not *intelligible*. Yet you have just admitted that my so-called sense of effort involves a peculiar quality of feeling. Now a quality of feeling is not intelligible, either. Nothing can be less so. One can *feel* it, but to comprehend it or express it in a general formula is out of the question. So it appears that unintelligibility does not suffice to destroy or refute a Category. Indeed, if you are to accept scholastic realism, you would seem to be almost bound to admit that *Nous*, or

intelligibility, is itself a category; and in that case far from non-intelligibility's refuting a category, intelligibility would do so -- that is, would prove that a conception could not be a category distinct from the category of *Nous*, or intelligibility. If it be objected that the unintelligibility of a Quality of Feeling is of a merely privative kind quite different from the aggressive and brutal anti-intelligibility of action regardless of law, the rejoinder will be that if intelligibility be a category, it is not surprising but rather inevitable that other categories should be in different relations to this one.

Peirce: CP 5.50 Cross-Ref:††

50. But without beating longer round the bush, let us come to close quarters. Experience is our only teacher. Far be it from me to enunciate any doctrine of a *tabula rasa*. For, as I said a few minutes ago, there manifestly is not one drop of principle in the whole vast reservoir of established scientific theory that has sprung from any other source than the power of the human mind to *originate* ideas that are true. But this power, for all it has accomplished, is so feeble that as ideas flow from their springs in the soul, the truths are almost drowned in a flood of false notions; and that which experience does is gradually, and by a sort of fractionation, to precipitate and filter off the false ideas, eliminating them and letting the truth pour on in its mighty current.

Peirce: CP 5.51 Cross-Ref:††

51. But precisely how does this action of experience take place? It takes place by a series of surprises. There is no need of going into details. At one time a ship is sailing along in the trades over a smooth sea, the navigator having no more positive expectation than that of the usual monotony of such a voyage, when suddenly she strikes upon a rock. The majority of discoveries, however, have been the result of experimentation. Now no man makes an experiment without being more or less inclined to think that an interesting result will ensue; for experiments are much too costly of physical and psychical energy to be undertaken at random and aimlessly. And naturally nothing can possibly be learned from an experiment that turns out just as was anticipated. It is by surprises that experience teaches all she deigns to teach us.

Peirce: CP 5.51 Cross-Ref:††

In all the works on pedagogy that ever I read -- and they have been many, big, and heavy -- I don't remember that any one has advocated a system of teaching by practical jokes, mostly cruel. That, however, describes the method of our great teacher, Experience. She says,

Open your mouth and shut your eyes

And I'll give you something to make you wise;

and thereupon she keeps her promise, and seems to take her pay in the fun of tormenting us.

Peirce: CP 5.52 Cross-Ref:††

52. The phenomenon of surprise in itself is highly instructive in reference to this category because of the emphasis it puts upon a mode of consciousness which can be detected in all perception, namely, a double consciousness at once of an *ego* and a

non-ego, directly acting upon each other. †1 Understand me well. My appeal is to observation -- observation that each of you must make for himself.

Peirce: CP 5.53 Cross-Ref: ††

53. The question is what the *phenomenon* is. We make no vain pretense of going beneath phenomena. We merely ask, what is the content of the *Percept*? Everybody should be competent to answer that of himself. Examine the Percept in the particularly marked case in which it comes as a surprise. Your mind was filled [with] an imaginary object that was expected. At the moment when it was expected the vividness of the representation is exalted, and suddenly, when it should come, something quite different comes instead. I ask you whether at that instant of surprise there is not a double consciousness, on the one hand of an Ego, which is simply the expected idea suddenly broken off, on the other hand of the Non-Ego, which is the strange intruder, in his abrupt entrance.

Peirce: CP 5.54 Cross-Ref: ††

54. The whole question is what the *perceptual facts* are, as given in direct perceptual judgments. By a perceptual judgment, I mean a judgment asserting in propositional form what a character of a percept directly present to the mind is. †2 The percept of course is not itself a judgment, nor can a judgment in any degree resemble a percept. It is as unlike it as the printed letters in a book, where a Madonna of Murillo is described, are unlike the picture itself.

Peirce: CP 5.55 Cross-Ref: ††

55. You may adopt any theory that seems to you acceptable as to the psychological operations by which perceptual judgments are formed. For our present purpose it makes no difference what that theory is. All that I insist upon is that those operations, whatever they may be, are utterly beyond our control and will go on whether we are pleased with them or not. Now I say that taking the word "criticize" in the sense it bears in philosophy, that of apportioning praise and blame, it is perfectly idle to criticize anything over which you can exercise no sort of control. You may wisely criticize a reasoning, because the reasoner, in the light of your criticism, will certainly go over his reasoning again and correct it if your blame of it was just. But to pronounce an involuntary operation of the mind *good* or *bad*, has no more sense than to pronounce the proportion of weights in which hydrogen and chlorine combine, that of 1 to 35.11 to be *good* or *bad*. I said it was idle; but in point of fact "nonsensical" would have been an apter word.

Peirce: CP 5.55 Cross-Ref: ††

If, therefore, our careful direct interpretation of perception, and more emphatically of such perception as involves surprise, is that the perception represents two objects reacting upon one another, that is not only a decision from which there is no appeal, but it is downright nonsense to dispute the fact that in perception two objects really do so react upon one another.

Peirce: CP 5.56 Cross-Ref: ††

56. That, of course, is the doctrine of Immediate Perception which is upheld by Reid, Kant, and all dualists who understand the true nature of dualism, and the denial of which led Cartesians to the utterly absurd theory of divine assistance upon which the preestablished harmony of Leibniz is but a slight improvement. Every philosopher who denies the doctrine of Immediate Perception -- including idealists of every stripe -- by that denial cuts off all possibility of ever cognizing a *relation*. Nor will he better his position by declaring that all relations are illusive appearances, since

it is not merely true knowledge of them that he has cut off, but every mode of cognitive representation of them.

Peirce: CP 5.57 Cross-Ref:††

57.†1 When a man is surprised he knows that he is surprised. Now comes a dilemma. Does he know he is surprised by direct perception or by inference? First try the hypothesis that it is by inference. This theory would be that a person (who must be supposed old enough to have acquired self-consciousness) on becoming conscious of that peculiar quality of feeling which unquestionably belongs to all surprise, is induced by some reason to attribute this feeling to himself. It is, however, a patent fact that we never, *in the first instance*, attribute a Quality of Feeling to ourselves. We first attribute it to a *Non-Ego* and only come to attribute it to ourselves when irrefragable reasons compel us to do so. Therefore, the theory would have to be that the man first pronounces the surprising object a *wonder*, and upon reflection convinces himself that it is only a wonder in the sense that he is *surprised*. That would have to be the theory. But it is in conflict with the facts which are that a man is more or less placidly *expecting* one result, and suddenly finds something in contrast to that forcing itself upon his recognition. A duality is thus forced upon him: on the one hand, his expectation which he had been attributing to Nature, but which he is now compelled to attribute to some mere inner world, and on the other hand, a strong new phenomenon which shoves that expectation into the background and occupies its place. The old expectation, which is what he was familiar with, is his inner world, or *Ego*. The new phenomenon, the stranger, is from the exterior world or *Non-Ego*. He does not conclude that he *must* be surprised because the object is so *marvellous*. But on the contrary, it is because of the duality presenting itself as such that he [is] led by generalization to a conception of a quality of marvellousness.

Peirce: CP 5.58 Cross-Ref:††

58. Try, then, the other alternative that it is by direct perception, that is, in a direct perceptual judgment, that a man knows that he is surprised. The perceptual judgment, however, certainly does not represent that it is he himself who has played a little trick upon himself. A man cannot startle himself by jumping up with an exclamation of *Boo!* Nor could the perceptual judgment have represented anything so out of nature. The perceptual judgment, then, can only be that it is the *Non-Ego*, something over against the *Ego* and bearing it down, is what has surprised him. But if that be so, this direct perception presents an *Ego* to which the smashed expectation belonged, and the *Non-Ego*, the sadder and wiser man, to which the new phenomenon belongs. . . .

Peirce: CP 5.59 Cross-Ref:††
§3. LAWS: NOMINALISM †1

59. Thus far, gentlemen, I have been insisting very strenuously upon what the most vulgar common sense has every disposition to assent to and only ingenious philosophers have been able to deceive themselves about. But now I come to a category which only a more refined form of common sense is prepared willingly to allow, the category which of the three is the chief burden of Hegel's song, a category toward which the studies of the new logico-mathematicians, Georg Cantor and the like, are steadily pointing, but to which no modern writer of any stripe, unless it be

some obscure student like myself, has ever done anything approaching to justice. . . .

Peirce: CP 5.60 Cross-Ref:††

60. There never was a sounder logical maxim of scientific procedure than Ockham's razor: *Entia non sunt multiplicanda praeter necessitatem*. That is to say; before you try a complicated hypothesis, you should make quite sure that no simplification of it will explain the facts equally well. No matter if it takes fifty generations of arduous experimentation to explode the simpler hypothesis, and no matter how incredible it may seem that that simpler hypothesis should suffice, still fifty generations are nothing in the life of science, which has all time before it; and in the long run, say in some thousands of generations, time will be economized by proceeding in an orderly manner, and by making it an invariable rule to try the simpler hypothesis first. Indeed, one can never be sure that the simpler hypothesis is not the true one, after all, until its cause has been fought out to the bitter end. But you will mark the limitation of my approval of Ockham's razor. It is a sound maxim of *scientific procedure*. If the question be what one ought to *believe*, the logic of the situation must take other factors into account. Speaking strictly, *belief* is out of place in pure theoretical science, which has nothing nearer to it than the establishment of doctrines, and only the provisional establishment of them, at that.†1 Compared with living *belief* it is nothing but a ghost. If the captain of a vessel on a lee shore in a terrific storm finds himself in a critical position in which he must instantly either put his wheel to port acting on one hypothesis, or put his wheel to starboard acting on the contrary hypothesis, and his vessel will infallibly be dashed to pieces if he decides the question wrongly, Ockham's razor is not worth the stout belief of any common seaman. For stout belief may happen to save the ship, while *Entia non sunt multiplicanda praeter necessitatem* would be only a stupid way of spelling Shipwreck. Now in matters of real practical concern we are all in something like the situation of that sea-captain.

Peirce: CP 5.61 Cross-Ref:††

61. *Philosophy*, as I understand the word, is a positive theoretical science, and a science in an early stage of development. As such it has no more to do with belief than any other science. Indeed, I am bound to confess that it is at present in so unsettled a condition, that if the ordinary theorems of molecular physics and of archaeology are but the ghosts of beliefs, then to my mind, the doctrines of the philosophers are little better than the ghosts of ghosts. I know this is an extremely heretical opinion. The followers of Haeckel are completely in accord with the followers of Hegel in holding that what they call philosophy is a practical science and the best of guides in the formation of what they take to be Religious Beliefs. I simply note the divergence, and pass on to an unquestionable fact; namely, the fact that all modern philosophy is built upon Ockhamism; by which I mean that it is all nominalistic and that it adopts nominalism because of Ockham's razor. And there is no form of modern philosophy of which this is more essentially true than the philosophy of Hegel. But it is not modern philosophers only who are nominalists. The nominalistic *Weltanschauung* has become incorporated into what I will venture to call the very flesh and blood of the average modern mind.

Peirce: CP 5.62 Cross-Ref:††

62. The third category of which I come now to speak is precisely that whose reality is denied by nominalism. For although nominalism is not credited with any extraordinarily lofty appreciation of the powers of the human soul, yet it attributes to it a power of originating a kind of ideas the like of which Omnipotence has failed to

create as real objects, and those general conceptions which men will never cease to consider the glory of the human intellect must, according to any consistent nominalism, be entirely wanting in the mind of Deity. Leibniz, the modern nominalist *par excellence*, will not admit that God has the faculty of Reason; and it seems impossible to avoid that conclusion upon nominalistic principles.

Peirce: CP 5.63 Cross-Ref:††

63. But it is not in Nominalism alone that modern thought has attributed to the human mind the miraculous power of originating a category of thought that has no counterpart at all in Heaven or Earth. Already in that strangely influential hodge-podge, the salad of Cartesianism, the doctrine stands out very emphatically that the only force is the force of impact, which clearly belongs to the category of Reaction; and ever since Newton's *Principia* began to affect the general thought of Europe through the sympathetic spirit of Voltaire, there has been a disposition to deny any kind of action except purely mechanical action. The Corpuscular Philosophy of Boyle -- although the pious Boyle did not himself recognize its character -- was bound to come to that in the last resort; and the idea constantly gained strength throughout the eighteenth century and the nineteenth until the doctrine of the Conservation of Energy, generalized rather loosely by philosophers, led to the theory of psycho-physical parallelism, against which there has, only of recent years, been any very sensible and widespread revolt. Psycho-physical parallelism is merely the doctrine that mechanical action explains all the real facts, except that these facts have an internal aspect which is a little obscure and a little shadowy.

Peirce: CP 5.64 Cross-Ref:††

64. To my way of regarding philosophy, all this movement was perfectly good scientific procedure. For the simpler hypothesis which excluded the influence of ideas upon matter had to be tried and persevered in until it was thoroughly exploded. But I believe that now at last, at any time for the last thirty years, it has been apparent, to every man who sufficiently considered the subject, that there is a mode of influence upon external facts which cannot be resolved into mere mechanical action, so that henceforward it will be a grave error of scientific philosophy to overlook the universal presence in the phenomenon of this third category. Indeed, from the moment that the Idea of Evolution took possession of the minds of men the pure Corpuscular Philosophy together with nominalism had had their doom pronounced. I grew up in Cambridge, [Massachusetts] and was about 21 when the *Origin of Species* appeared. There was then living here a thinker who left no remains from which one could now gather what an educative influence his was upon the minds of all of us who enjoyed his intimacy, Mr. Chauncey Wright.†1 He had at first been a Hamiltonian but had early passed over into the warmest advocacy of the nominalism of John Stuart Mill; and being a mathematician at a time when dynamics was regarded as the loftiest branch of mathematics, he was also inclined to regard nature from a strictly mechanical point of view. But his interests were wide and he was also a student of Gray.†1 I was away surveying in the wilds of Louisiana when Darwin's great work appeared, and though I learned by letters of the immense sensation it had created, I did not return until early in the following summer when I found Wright all enthusiasm for Darwin, whose doctrines appeared to him as a sort of supplement to those of Mill. I remember well that I then made a remark to him which although he did not assent to it, evidently impressed him enough to perplex him. The remark was that these ideas of development had more vitality by far than any of his other favorite conceptions and that though they might at that moment be in his mind like a little vine clinging to the tree of Associationalism, yet after a time that vine would inevitably kill the tree. He

asked me why I said that and I replied that the reason was that Mill's doctrine was nothing but a metaphysical point of view to which Darwin's, which was nourished by positive observation, must be deadly. Ten or fifteen years later, when Agnosticism was all the go, I prognosticated a short life for it, as philosophies run, for a similar reason. What the true definition of Pragmatism may be, I find it very hard to say; but in my nature it is a sort of instinctive attraction for living facts.

Peirce: CP 5.65 Cross-Ref:††

65. All nature abounds in proofs of other influences than merely mechanical action, even in the physical world. They crowd in upon us at the rate of several every minute. And my observation of men has led me to this little generalization. Speaking only of men who really think for themselves and not of mere reporters, I have not found that it is the men whose lives are mostly passed within the four walls of a physical laboratory who are most inclined to be satisfied with a purely mechanical metaphysics. On the contrary, the more clearly they understand how physical forces work the more incredible it seems to them that such action should explain what happens out of doors. A larger proportion of materialists and agnostics is to be found among the thinking physiologists and other naturalists, and the largest proportion of all among those who derive their ideas of physical science from reading popular books. These last, the Spencers, the Youmanses, and the like, seem to be possessed with the idea that science has got the universe pretty well ciphered down to a fine point; while the Faradays and Newtons seem to themselves like children who have picked up a few pretty pebbles upon the ocean beach. But most of us seem to find it difficult to recognize the greatness and wonder of things familiar to us. As the prophet is not without honor save [in his own country] so it is also with phenomena. Point out to the ordinary man evidence, however conclusive, of other influence than physical action in things he sees every day, and he will say: "Well, I don't see as that frog has got any points about him that's any different from any other frog." For that reason we welcome instances perhaps of less real cogency but which have the merit of being rare and strange. Such, for example, are the right-handed and left-handed screw-structures of the molecules of those bodies which are said to be "optically active." Of every such substance there are two varieties, or as the chemists call them, two modifications, one of which twists a ray of light that passes through it to the right, and the other, by an exactly equal amount, to the left. All the ordinary physical properties of the right-handed and left-handed modifications are identical. Only certain faces of their crystals, often very minute, are differently placed. No chemical process can ever transmute the one modification into the other. And their ordinary chemical behaviour is absolutely the same, so that no strictly chemical process can separate them if they are once mixed. Only the chemical action of one optically active substance upon another is different if they both twist the ray the same way from what it is if they twist the ray different ways. There are certain living organisms which feed on one modification and destroy it while leaving the other one untouched. This is presumably due to such organisms containing in their substance, possibly in very minute proportion, some optically active body. Now I maintain that the original segregation of levo-molecules, or molecules with a left-handed twist, from dextro-molecules, or molecules with a right-handed twist, is absolutely incapable of mechanical explanation. Of course you may suppose that in the original nebula at the very formation of the world right-handed quartz was collected into one place, while left-handed quartz was collected into another place. But to suppose that, is *ipso facto* to suppose that that segregation was a phenomenon without any mechanical explanation. The three laws of motion draw no dynamical distinction between right-handed and left-handed screws, and a mechanical explanation is an explanation

founded on the three laws of motion. There, then, is a physical phenomenon absolutely inexplicable by mechanical action. This single instance suffices to overthrow the Corpuscular Philosophy.

Peirce: CP 5.66 Cross-Ref:††
LECTURE III †1

THE CATEGORIES CONTINUED

§1. DEGENERATE THIRDNESS †2

66. Category the First is the Idea of that which is such as it is regardless of anything else. That is to say, it is a *Quality* of Feeling.

Peirce: CP 5.66 Cross-Ref:††

Category the Second is the Idea of that which is such as it is as being Second to some First, regardless of anything else, and in particular regardless of any *Law*, although it may conform to a law. That is to say, it is *Reaction* as an element of the Phenomenon.

Peirce: CP 5.66 Cross-Ref:††

Category the Third is the Idea of that which is such as it is as being a Third, or Medium, between a Second and its First. That is to say, it is *Representation* as an element of the Phenomenon.

Peirce: CP 5.67 Cross-Ref:††

67. A mere complication of Category the Third, involving no idea essentially different, will give the idea of something which is such as it is by virtue of its relations to any multitude, enumerable, denumeral, or abnumerable or even to any supermultitude of correlates; so that this Category suffices of itself to give the conception of True Continuity, than which no conception yet discovered is higher.†3

Peirce: CP 5.68 Cross-Ref:††

68. Category the First owing to its Extremely Rudimentary character is not susceptible of any degenerate or weakened modification.

Peirce: CP 5.69 Cross-Ref:††

69. Category the Second has a *Degenerate* Form, in which there is Secondness indeed, but a weak or Secondary Secondness that is not in the pair in its own quality, but belongs to it only in a certain respect. Moreover, this degeneracy need not be absolute but may be only approximative. Thus a genus characterized by Reaction will by the determination of its essential character split into two species, one a species where the secondness is strong, the other a species where the secondness is weak, and the strong species will subdivide into two that will be similarly related, without any corresponding subdivision of the weak species. For example, Psychological Reaction splits into Willing, where the Secondness is strong, and Sensation, where it is weak;

and Willing again subdivides into Active Willing and Inhibitive Willing, to which last dichotomy nothing in Sensation corresponds. But it must be confessed that subdivision, as such, involves something more than the second category.

Peirce: CP 5.70 Cross-Ref:††

70. Category the Third exhibits two different ways of Degeneracy, where the irreducible idea of Plurality, as distinguished from Duality, is present indeed but in maimed conditions. The First degree of Degeneracy is found in an Irrational Plurality which, as it exists, in contradistinction [to] the form of its representation, is a mere complication of duality. We have just had an example of this in the idea of Subdivision. In pure Secondness, the reacting correlates are *Singulars*, and as such are *Individuals*, not capable of further division. Consequently, the conception of Subdivision, say by repeated dichotomy, certainly involves a sort of Thirdness, but it is a thirdness that is conceived to consist in a second secondness.

Peirce: CP 5.71 Cross-Ref:††

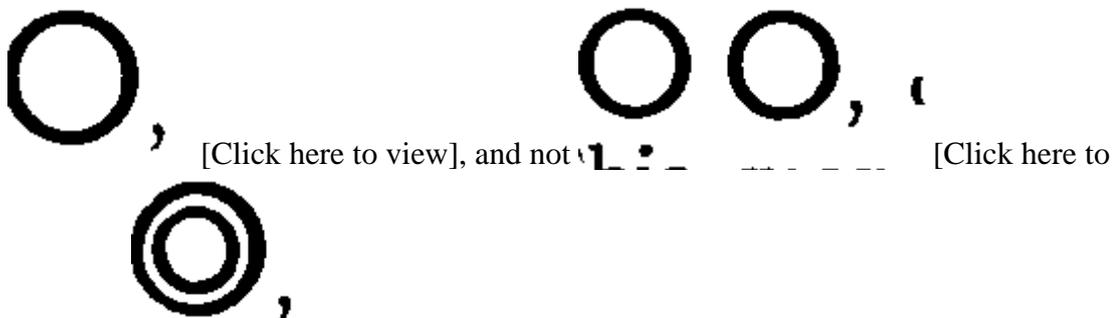
71. The most degenerate Thirdness is where we conceive a mere Quality of Feeling, or Firstness, to represent itself to itself as Representation. Such, for example, would be Pure Self-Consciousness, which might be roughly described as a mere feeling that has a dark instinct of being a germ of thought. This sounds nonsensical, I grant. Yet something can be done toward rendering it comprehensible.

Peirce: CP 5.71 Cross-Ref:††

I remember a lady's averring that her father had heard a minister, of what complexion she did not say, open a prayer as follows: "O Thou, All-Sufficient, Self-Sufficient, Insufficient God." Now pure Self-consciousness is Self-sufficient, and if it is also regarded as All-sufficient, it would seem to follow that it must be Insufficient. I ought to apologize for introducing such Buffoonery into serious lectures. I do so because I seriously believe that a bit of fun helps thought and tends to keep it pragmatical.

Peirce: CP 5.71 Cross-Ref:††

Imagine that upon the soil of a country, that has a single boundary line thus



This map may distort the different provinces of the country to any extent. But I shall suppose that it represents every part of the country that has a single boundary, by a part of the map that has a single boundary, that every part is represented as bounded by such parts as it really is bounded by, that every point of the country is represented by a single point of the map, and that every point of the map represents a single point in the country. Let us further suppose that this map is infinitely minute in its representation so that there is no speck on any grain of sand in the country that could not be seen represented upon the map if we were to examine it under a sufficiently high magnifying power. Since, then, everything on the soil of the country is shown on

the map, and since the map lies on the soil of the country, the map itself will be portrayed in the map, and in this map of the map everything on the soil of the country can be discerned, including the map itself with the map of the map within its boundary. Thus there will be within the map, a map of the map, and within that, a map of the map of the map, and so on *ad infinitum*. These maps being each within the preceding ones of the series, there will be a point contained in all of them, and this will be the map of itself. Each map which directly or indirectly represents the country is itself mapped in the next; *i.e.*, in the next [it] is represented to be a map of the country. In other words each map is *interpreted* as such in the next. We may therefore say that each is a representation of the country *to* the next map; and that point that is in all the maps is in itself the representation of nothing but itself and to nothing but itself. It is therefore the precise analogue of pure self-consciousness. As such it is *self-sufficient*. It is saved from being insufficient, that is as no representation at all, by the circumstance that it is not *all-sufficient*, that is, is not a complete representation but is only a point upon a continuous map.†P1 I dare say you may have heard something like this before from Professor Royce, but if so, you will remark an important divergency. The idea itself belongs neither to him nor to me, and was used by me in this connection thirty years ago.†1

Peirce: CP 5.72 Cross-Ref:††

72. The relatively degenerate forms of the Third category do not fall into a catena, like those of the Second. What we find is this. Taking any class in whose essential idea the predominant element is Thirdness, or Representation, the self-development of that essential idea -- which development, let me say, is not to be compassed by any amount of mere "hard thinking," but only by an elaborate process founded upon experience and reason combined -- results in a *trichotomy* giving rise to three sub-classes, or genera, involving respectively a relatively genuine thirdness, a relatively reactionary thirdness or thirdness of the lesser degree of degeneracy, and a relatively qualitative thirdness or thirdness of the last degeneracy. This last may subdivide, and its species may even be governed by the three categories, but it will not subdivide, in the manner which we are considering, by the essential determinations of its conception. The genus corresponding to the lesser degree of degeneracy, the reactionally degenerate genus, will subdivide after the manner of the Second category, forming a catena; while the genus of relatively genuine Thirdness will subdivide by Trichotomy just like that from which it resulted. Only as the division proceeds, the subdivisions become harder and harder to discern.

Peirce: CP 5.73 Cross-Ref:††

73. The representamen, for example, divides by trichotomy into the general sign or *symbol*, the *index*, and the *icon*.†2 An *icon* is a representamen which fulfills the function of a representamen by virtue of a character which it possesses in itself, and would possess just the same though its object did not exist. Thus, the statue of a centaur is not, it is true, a representamen if there be no such thing as a centaur. Still, if it represents a centaur, it is by virtue of its shape; and this shape it will have, just as much, whether there be a centaur or not. An *index* is a representamen which fulfills the function of a representamen by virtue of a character which it could not have if its object did not exist, but which it will continue to have just the same whether it be interpreted as a representamen or not. For instance, an old-fashioned hygrometer is an *index*. For it is so contrived as to have a physical reaction with dryness and moisture in the air, so that the little man will come out if it is wet, and this would happen just the same if the use of the instrument should be entirely forgotten, so that it ceased actually to convey any information. A *symbol* is a representamen which fulfills its

function regardless of any similarity or analogy with its object and equally regardless of any *factual* connection therewith, but solely and simply because it will be interpreted to be a representamen. Such for example is any general word, sentence, or book.

Peirce: CP 5.73 Cross-Ref:††

Of these three genera of representamens, the *Icon* is the Qualitatively degenerate, the *Index* the Reactionally degenerate, while the *Symbol* is the relatively genuine genus.

Peirce: CP 5.74 Cross-Ref:††

74. Now the *Icon* may undoubtedly be divided according to the categories; but the mere completeness of the notion of the icon does not imperatively call for any such division. For a pure icon does not draw any distinction between itself and its object. It represents whatever it may represent, and whatever it is like, it in so far is. It is an affair of suchness only.

Peirce: CP 5.75 Cross-Ref:††

75. It is quite otherwise with the *Index*. Here is a reactional sign, which is such by virtue of a real connection with its object. Then the question arises is this dual character in the Index, so that it has two elements, by virtue of the one serving as a substitute for the particular object it does, while the other is an involved icon that represents the representamen itself regarded as a quality of the object -- or is there really no such dual character in the index, so that it merely denotes whatever object it happens to be really connected with just as the icon represents whatever object it happens really to resemble? Of the former, the relatively genuine form of Index, the hygrometer, is an example. Its connection with the weather is dualistic, so that by an involved icon, it actually conveys information. On the other hand any mere land-mark by which a particular thing may be recognized because it is as a matter of fact associated with that thing, a proper name without signification, a pointing finger, is a degenerate index. Horatio Greenough, who designed Bunker Hill Monument, tells us in his book †1 that he meant it to say simply "Here!" It just stands on that ground and plainly is not movable. So if we are looking for the battle-field, it will tell us whither to direct our steps.

Peirce: CP 5.76 Cross-Ref:††

76. The *Symbol*, or relatively genuine form of Representamen, divides by Trichotomy into the Term, the Proposition, and the Argument. The Term corresponds to the Icon and to the degenerate Index. It does excite an icon in the imagination. The proposition conveys definite information like the genuine index, by having two parts of which the function of the one is to indicate the object meant, while that of the other is to represent the representamen by exciting an icon of its quality. The argument is a representamen which does not leave the interpretant to be determined as it may by the person to whom the symbol is addressed, but separately represents what is the interpreting representation that it is intended to determine. This interpreting representation is, of course, the conclusion. It would be interesting to push these illustrations further; but I can linger nowhere. As soon as a subject begins to be interesting I am obliged to pass on to another.

Peirce: CP 5.77 Cross-Ref:††

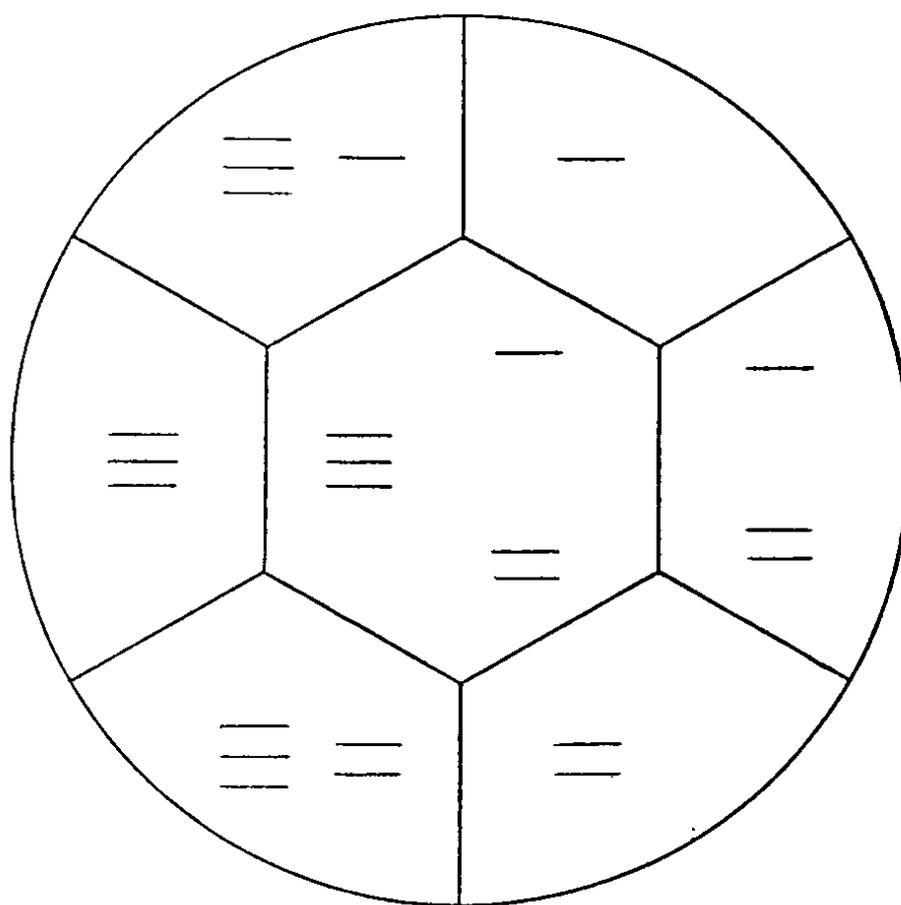
§2. THE SEVEN SYSTEMS OF METAPHYSICS

77. The three categories furnish an artificial classification of all possible systems of metaphysics which is certainly not without its utility. The scheme is shown in this figure (p. 53). It depends upon what ones of the three categories each system admits as important metaphysico-cosmical elements.†P1

Peirce: CP 5.78 Cross-Ref:††

78. One very naturally and properly endeavors to give an account of the universe with the fewest and simplest possible categories.

Praedicamenta non sunt multiplicanda praeter necessitatem.



to view]

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Peirce: CP 5.79 Cross-Ref:††

79. We ought therefore to admire and extol the efforts of Condillac and the Associationalists to explain everything by means of qualities of feeling [i]. If, however, this turns out to be a failure, the next most admirable hypothesis is that of the corpuscularians, Helmholtz and the like, who would like to explain everything by means of mechanical force, which they do not distinguish from individual reaction [ii]. That again failing, the doctrine of Hegel is to be commended who regards

Category the Third as the only true one [iii]. For in the Hegelian system the other two are only introduced in order to be *aufgehoben*. All the categories of Hegel's list, from Pure Being up, appear to me very manifestly to involve Thirdness, although he does not appear to recognize it, so immersed is he in this category.

Peirce: CP 5.80 Cross-Ref:††

80. All three of these simplest systems having worked themselves out into absurdity, it is natural next in accordance with the maxim of Parsimony to try explanations of the Universe based on the recognition of two only of the Categories.

Peirce: CP 5.81 Cross-Ref:††

81. The more moderate nominalists who nevertheless apply the epithet *mere* to thought and to representamens may be said to admit Categories First and Second and to deny the third [i ii]. The Berkeleyans, for whom there are but two kinds of entities, souls, or centres of determinable thought, and ideas in the souls, these ideas being regarded as pure statical entities, little or nothing else than Qualities of Feeling, seem to admit Categories First and Third and to deny Secondness, which they wish to replace by Divine Creative Influence, which certainly has all the flavor of Thirdness [i iii]. So far as one can make out any intelligible aim in that singular hodge-podge, the Cartesian metaphysics, it seems to have been to admit Categories Second and Third as fundamental and to deny the First [ii iii]. Otherwise, I do not know to whom we can attribute this opinion which certainly does not seem to be less acceptable and attractive than several others. But there are other philosophies which seem to do full justice to Categories Second and Third and to minimize the first, and among these perhaps Spinoza and Kant are to be included.

Peirce: CP 5.82 Cross-Ref:††

§3. THE IRREDUCIBILITY OF THE CATEGORIES †1

82. We must begin by asking whether the three categories can be admitted as simple and irreducible conceptions; and afterward go on to ask whether they cannot all be supposed to be real constituents of the universe. For when I say that certain metaphysical schools do not admit them, I do not mean to say that they do not admit them as mere conceptions -- a point to which they do not generally pay much attention, so that their opinions about this are not very marked -- but that they do not admit them as real constituents of the universe.

Peirce: CP 5.82 Cross-Ref:††

I do not know that I could add anything material to what I said in my last lecture to show that Category the First must be admitted as an irreducible constituent of the phenomenon.

Peirce: CP 5.83 Cross-Ref:††

83. There would be no question that Category the Second is an irreducible conception were it not for the deplorable condition of the science of logic. This is illustrated by the fact that so flippant and wildly theorizing work as Prantl's *Geschichte der Logik* should be accepted, as it generally is, even among learned men, as a marvel of patient research. It is true that one or two chapters of it are relatively well done. The account of Aristotle's logic, though not good upon any high standard of completeness or of thorough comprehension, is nevertheless the best account of its

subject that we have. But Prantl, to begin with, does not himself understand logic, meaning by logic the science of which those works treated, of which he gives or he professes to give an account; and yet with the shallowest ideas, he is so puffed up with his own views that he disdains to take the trouble to penetrate their meaning. The crude expressions of contempt in which he continually indulges toward great thinkers ought to put readers on their guard against him. In the next place he belongs to that too well-known class of German critics who get bitten with theories deduced from general conceptions, and who fall in love with these theories because they are their own offspring and treat them as absolute certainties although the complete refutation of them is near at hand. You will understand, of course, that I do not say these things without having read all the chief contributions to the questions on both sides and without having subjected them to careful study and criticism. Prantl's opinions about the Megarian philosophers, about what he calls the Byzantine logic, about the Latin medieval logic, about the *Parva Logicalia*, are wild theories, utterly untenable, and in several cases easily refuted by an easy examination of the MSS. Moreover, it is not a history of logic but mostly of the most trivial parts of logic. But I shall be asked whether I do not think his reading marvellously extensive. No, I do not. He had the Munich library at his hand. He had only to look into the books, and for the most part he has done little more than merely to look into them. He really often has no idea of what the real substance of the books is; and nothing is more common than to find in his notes passages copied out of one book which are nothing but textual copies of celebrated passages in much older works. I do not deny that the book is useful, because the rest of us haven't access to such a library; but I do not consider it a work of respectable erudition. There is no need of mincing words because he himself not only refers most disrespectfully to such solid students of medieval writings as Charles Thurot, Haureau and others, but frequently descends to what in English we should call the language of Billingsgate in characterising ancient opinions which he may or may not be aware are identical with those held today by analysts of logical forms whose studies are so much more exact than his that they are not to be named in the same day.

Peirce: CP 5.84 Cross-Ref:††

84. Nevertheless, bad as Prantl's history is, it is the best we have, and any person who reads it critically, as every book ought to be read, will easily be able to see that the ancient students of logic, Democritus, Plato, Aristotle, Epicurus, Philoponus, even Chrysippus, were thinkers of the highest order, and that St. Augustine, Abelard, Aquinas, Duns Scotus, Ockham, Paulus Venetus, even Laurentius Valla, were logicians of the most painstaking and subtle types. But when the revival of learning came, the finest minds had their attention turned in quite another direction, and modern mathematics and modern physics drew away still more. The result of all this has been that during the centuries that have elapsed since the appearance of the *De Revolutionibus* [1543] -- and remember, if you please, that the work of Copernicus was the fruit of the scientific nourishment that he had imbibed in Italy in his youth -- throughout these ages, the chairs of Logic in the Universities have been turned over to a class of men, of whom we should be speaking far too euphemistically if we were to say that they have in no wise represented the Intellectual Level of their age. No, no; let us speak the plain truth -- modern logicians as a class have been distinctly puerile minds, the kind of minds that never mature, and yet never have the *élan* and originality of youth. First cast your eyes over the pages of a dozen average treatises, dismissing all preconceived estimates of their authors, and see if that is not the impression you derive from them. Why, in the majority of them, the greatest contribution to reasoning that has been generally applied during these

centuries -- the Calculus of Probabilities -- is almost entirely ignored. If it were only the common run of logics that were affected by this state of things, it would not much matter; for if only one per cent of works on the subject were what they should be, we should still be in possession of a splendid and extensive literature. But unfortunately the general standard has been so terribly lowered that even the treatises written by men of real ability have been but half thought out things. Arnauld, for example, was a thinker of considerable force, and yet *L'Art de penser*, or the *Port Royal Logic*, is a shameful exhibit of what the two and a half centuries of man's greatest achievements could consider as a good account of how to think. You may retort that the past three centuries seem to have got on nicely without the aid of logic. Yes, I reply, they have, because there is one thing even more vital to science than intelligent methods; and that is, the sincere desire to find out the truth, whatever it may be; and *that* those centuries have been blessed with. But according to such estimate -- not exactly mere guess-work, although rough enough, no doubt -- as I have been able to form, if logic during those centuries had been studied with half the *zeal* and *genius* that has been bestowed upon mathematics, the twentieth century might have opened with the special sciences generally -- particularly such vitally important sciences as molecular physics, chemistry, physiology, psychology, linguistics, and ancient historical criticism -- in a decidedly more advanced condition than there is much promise that they will have reached at the end of 1950. I shouldn't say that human lives were the most precious things in the world; but after all they have their value; and only think how many lives might thus have been saved. We can mention individuals who might probably have done more work; say Abel, Steiner, Gaulois, Sadi Carnot. Think of the labor of a generation of Germany being allowed to flow off into Hegelianism! Think of the extravagant admiration that half a generation of English -- decidedly the best average reasoners of any modern people, bestowed on that silly thing, Hamilton's *New Analytic*. Look through Vaihinger's commentary to see what an army of students have been entrapped by Kant's view of the relation between his Analytic and Synthetic Judgments -- a view that a study of the logic of relatives would at once have exploded.†1

Peirce: CP 5.85 Cross-Ref:††

85. Had logic not been sunk since the time of Copernicus into a condition of semi-idiocy, the Logic of Relatives would by this time have been pursued for three centuries by hundreds of students, among whom there would have been no small number who in this direction or in that would have surpassed in ability any of the poor handful of students who have been at work upon it for the last generation or so. And let me tell you that this study would have completely revolutionized men's most general notions about logic -- the very ideas that are today current in the market-place and on the boulevards. One of the early results of such wide study of the logic of relatives must have been to cause the idea of *reaction* to be solidly fixed in the minds of all men as an irreducible category of *Thought*†2 -- whatever place might have been accorded to it in metaphysics as a cosmical category. This I venture to say, notwithstanding that the lamented Schröder did not seem to see it so. Schröder followed Sigwart in his most fundamental ideas of logic. Now I entertain a high respect for Sigwart -- the kind of respect that I feel for Rollin as a historian, for Buffon as a zoölogist, for Priestley as a chemist, for Biot as a physicist -- a class of men whom {ohi polloi} always place too high, and scientific specialists too low. He is one of the most critical and least inexact of the inexact logicians. Sigwart, like almost all the stronger logicians of today, present company excepted, makes the fundamental mistake of confounding the logical question with the psychological question.†3 The psychological question is what processes the mind goes through. But the logical

question is whether the conclusion that will be reached, by applying this or that maxim, will or will not accord with the *fact*. It may be that the mind is so constituted that that which our intellectual instinct approves will be true to the extent to which that instinct approves of it. If so, that is an interesting fact about the human mind; but it has no relevancy for logic whatsoever. Sigwart says that the question of what is good logic and what bad must in the last resort come down to a question of how we feel; it is a matter of *Gefühl*, that is, a Quality of Feeling. And this he undertakes to demonstrate. For he says if any other criterion be employed, the correctness of this criterion has to be established by reasoning, and in this reasoning antecedent to the establishment of any rational criterion we must rely upon *Gefühl*; so that *Gefühl* is that to which any other criterion must ultimately be referred. Good! This is good intelligent work, such as advances philosophy -- a good, square, explicit fallacy that can be squarely met and definitively refuted. It is the more valuable because it is a form of argument of very wide applicability. It is precisely analogous to the reasoning by which the hedonist in ethics, the subjectivist in esthetics, the idealist in metaphysics, attacks the category of reaction. You perceive the analogy between their arguments. The hedonist says that the question of what is good morals and what bad must ultimately come down to a question of pleasure. For, he says, suppose we desire anything but our own pleasure. Then whatever it may be that we desire, we take satisfaction in; and if we did not take satisfaction in it we should not desire it. But this satisfaction is that very Quality of Feeling that we call pleasure; and thus the only thing we ever can desire is pleasure, and all deliberate action must be performed for the sake of our own pleasure.

Peirce: CP 5.85 Cross-Ref:††

Every idealist, too, begins with an analogous argument, though he very likely may not remain consistently on the ground it leads to, so far as it leads anywhere. He says: When I perceive anything I am conscious; and when I am conscious of anything, I am immediately conscious and aught else I may be conscious of, I am conscious of through that immediate consciousness. Consequently whatever I learn from perception is merely that I have a feeling together with whatever I infer from that immediate consciousness.

Peirce: CP 5.86 Cross-Ref:††

86. The answer to all such arguments is that no desire can possibly desire its own satisfaction, no judgment can judge itself to be true, and no reasoning can conclude that it is itself sound. For all these propositions stand on the same footing and must stand or fall together. If any judgment judges itself to be true, all judgments -- or at least all assertory judgments -- do so likewise; for there is no ground of discrimination between assertory judgments in this respect. Either therefore the judgment, J, and the judgment "I say that J is true" are the same for all judgments or for none. But if they are identical, their denials are identical. But their denials are respectively "J is not true" and "I do not say that J is true," which are very different. Consequently no judgment judges itself to be true. All that J does is to furnish a premiss which is complete evidence warranting my assertion in another judgment that J is true. It is important to draw this distinction. The judgment J may, for example, be that "Sirius is white." That is a judgment about Sirius. To *myself* who perceive myself making this judgment, or to another who hears me assert it and admits my veracity, the evidence is complete that I believe Sirius to be white. But the two propositions "Sirius is white" "I judge that Sirius is white" are two distinct propositions.

Peirce: CP 5.86 Cross-Ref:††

There are precisely analogous distinctions in the other cases. I may desire that my sick child should recover, and afterwards reflecting upon the intensity of that desire, I probably shall be unable to refrain from desiring that that desire should be gratified. But I cannot desire that a desire of mine should be gratified unless I already have such a desire; and I have no such desire as long as I am yet in the act of forming the desire, so that the desire is not yet complete. I dare say that some people's psychical disposition is such that they have no sooner formed a strong desire than their thoughts take a subjective turn and they forthwith begin to think what satisfaction it would give them if that desire were gratified, and such people find it difficult to conceive that there are other people whose thoughts follow a train of objective suggestions and who think very little about themselves and their gratifications. That is just one of those respects in which different people may be expected to differ widely. But in no case is the desire absolutely the same as the desire of the satisfaction of that desire.

Peirce: CP 5.87 Cross-Ref:††

87. To return, then, to Sigwart's argument, I not only deny what he asserts that when I make an inference I can only do so because of a certain *feeling* of logical satisfaction that is connected with doing so, but I maintain that I *never can* draw an inference because of such a feeling. On the contrary, I never know the inference will afford me any such satisfaction except by a subsequent reflexion after I have already drawn it. It may be that on recognizing the satisfaction the inference gives me I shall consider that as an additional reason for believing in it. But this is *another* inference which in its turn will afford a new gratification if I stop to reflect about it.

Peirce: CP 5.87 Cross-Ref:††

In point of fact it is a serious error of reasoning to regard the sense of logicity as anything more than a tolerably strong argument in favor of the soundness of an inference. For although no doubt the sense of logicity carries men right in the main, yet it very frequently deceives them.

Peirce: CP 5.87 Cross-Ref:††

But Sigwart's argument is plainly either wholly fallacious or else what it proves is that which he himself distinctly maintains that it proves, namely, that the soundness of an argument *consists* in nothing but the *Gefühl* of logicity. Yet this is a downright absurd position for a logician to take; since, if it were true, there could be no such thing as sincere reasoning that was illogical, and logic, as the criticism of arguments and discrimination of the good from the bad, would have no existence at all; and my sincere argument that Sigwart is wholly in the wrong would be a decision from which there would be no appeal.

Peirce: CP 5.87 Cross-Ref:††

If the Holy Father by virtue of his infallibility were to command the faithful to believe that everything that any protestant had ever said was *ipso facto* necessarily true, it could hardly strain one's assent more.

Peirce: CP 5.88 Cross-Ref:††

88. It is certainly hard to believe, until one is forced to the belief, that a conception, so obtrusively complex as Thirdness is, should be an irreducible unanalyzable conception. What, one naturally exclaims, does this man think to convince us that a conception is complex and simple, at the same time! I might answer this by drawing a distinction. It is complex in the sense that different features may be discriminated in it, but the peculiar idea of *complexity* that it contains,

although it has complexity as its object, is an unanalyzable idea. Of what is the conception of *complexity* built up? Produce it by construction without using any idea which involves it if you can.

Peirce: CP 5.89 Cross-Ref:††

89. The best way of satisfying oneself whether Thirdness is elementary or not -- at least, it would be the best way for me, who had in the first place a natural aptitude for logical analysis which has been in constant training all my life long (and I rather think it would be the best way for anybody provided he ruminates over his analysis, returns to it again and again, and criticizes it severely and sincerely, until he reaches a complete insight into the analysis) -- the best way, I say, is to take the idea of representation, say the idea of the fact that the object, A, is represented in the representation, B, so as to determine the interpretation, C: to take this idea and endeavor to state what it consists in without introducing the idea of Thirdness at all if possible, or, if you find that impossible, to see what is the minimum or most degenerate form of Thirdness which will answer the purpose.

Peirce: CP 5.89 Cross-Ref:††

Then, having exercised yourself on that problem, take another idea in which, according to my views, Thirdness takes a more degenerate form. Try your hand at a logical analysis of the Fact that A gives B to C.

Peirce: CP 5.89 Cross-Ref:††

Then pass to a case in which Thirdness takes a still more degenerate form, as for example the idea of "A and B." What is at once A and B involves the idea of three variables. Putting it mathematically, it is $Z = XY$, which is the equation of the simpler of the two hyperboloids, the two-sheeted one, as it is called.

Peirce: CP 5.89 Cross-Ref:††

Whoever wishes to train his logical powers will find those problems furnish capital exercise; and whoever wishes to get a just conception of the universe will find that the solutions of those problems have a more intimate connection with that conception than he could suspect in advance.

Peirce: CP 5.90 Cross-Ref:††

90. I have thus far been intent on repelling attacks upon the categories which should consist in maintaining that the idea of Reaction can be reduced to that of Quality of Feeling, and the idea of Representation to those of Reaction and Quality of Feeling taken together. But meantime may not the enemy have stolen upon my rear, and shall I not suddenly find myself exposed to an attack which shall run as follows:

Peirce: CP 5.90 Cross-Ref:††

We fully admit that you have proved, until we begin to doubt it, that Secondness is not involved in Firstness nor Thirdness in Secondness and Firstness. But you have entirely failed to prove that Firstness, Secondness, and Thirdness are independent ideas for the obvious reason that it is as plain as the nose on your face that the idea of a triplet involves the idea of pairs, and the idea of a pair the idea of units. Consequently, Thirdness is the one and sole category. This is substantially the idea of Hegel; and unquestionably it contains a truth.

Peirce: CP 5.90 Cross-Ref:††

Not only does Thirdness suppose and involve the ideas of Secondness and Firstness, but never will it be possible to find any Secondness or Firstness in the phenomenon that is not accompanied by Thirdness.

Peirce: CP 5.91 Cross-Ref:††

91. If the Hegelians confined themselves to that position they would find a hearty friend in my doctrine.

Peirce: CP 5.91 Cross-Ref:††

But they do not. Hegel is possessed with the idea that the Absolute is One. Three absolutes he would regard as a ludicrous contradiction *in adjecto*. Consequently, he wishes to make out that the three categories have not their several independent and irrefutable standings in thought. *Firstness* and *Secondness* must somehow be *aufgehoben*. But it is not true. They are in no way refuted nor refutable. Thirdness it is true involves Secondness and Firstness, in a sense. That is to say, if you have the idea of Thirdness you must have had the ideas of Secondness and Firstness to build upon. But what is required for the idea of a genuine Thirdness is an independent solid Secondness and not a Secondness that is a mere corollary of an unfounded and inconceivable Thirdness; and a similar remark may be made in reference to Firstness.

Peirce: CP 5.92 Cross-Ref:††

92. Let the Universe be an evolution of Pure Reason if you will. Yet if, while you are walking in the street reflecting upon how everything is the pure distillate of Reason, a man carrying a heavy pole suddenly pokes you in the small of the back, you may think there is something in the Universe that Pure Reason fails to account for; and when you look at the color *red* and ask yourself how Pure Reason could make *red* to have that utterly inexpressible and irrational positive quality it has, you will be perhaps disposed to think that Quality and Reaction have their independent standing in the Universe.

Peirce: CP 5.93 Cross-Ref:††

LECTURE IV †1

THE REALITY OF THIRDNES

§1. SCHOLASTIC REALISM

93. I proceed to argue that *Thirdness* is operative in Nature. Suppose we attack the question experimentally. Here is a stone. Now I place that stone where there will be no obstacle between it and the floor, and I will predict with confidence that as soon as I let go my hold upon the stone it will fall to the floor. I will prove that I can make a correct prediction by actual trial if you like. But I see by your faces that you all think it will be a very silly experiment. Why so? Because you all know very well that I can predict what will happen, and that the fact will verify my prediction.

Peirce: CP 5.94 Cross-Ref:††

94. But *how can* I know what is going to happen? You certainly do not think

that it is by clairvoyance, as if the future event by its existential reactivity could affect me directly, as in an *experience* of it, as an event scarcely past might affect me. You know very well that there is nothing of the sort in this case. Still, it remains true that I *do know* that that stone will drop, as a *fact*, as soon as I let go my hold. If I *truly know* anything, that which I know must be *real*. It would be quite absurd to say that I could be enabled to know how events are going to be determined over which I can exercise no more control than I shall be able to exercise over this stone after it shall have left my hand, that I can so peer in the future merely on the strength of any acquaintance with any pure fiction.

Peirce: CP 5.95 Cross-Ref:††

95. I know that this stone will fall if it is let go, because experience has convinced me that objects of this kind always do fall; and if anyone present has any doubt on the subject, I should be happy to try the experiment, and I will bet him a hundred to one on the result.

Peirce: CP 5.96 Cross-Ref:††

96. But the general proposition that all solid bodies fall in the absence of any upward forces or pressure, this formula I say, is of the nature of a representation. Our nominalistic friends would be the last to dispute that. They will go so far as to say that it is a *mere* representation -- the word *mere* meaning that to be represented and really to be are two very different things; and that this formula has no being except a being represented. It certainly is of the nature of a representation. That is undeniable, I grant. And it is equally undeniable that that which is of the nature of a representation is not *ipso facto* real. In that respect there is a great contrast between an object of reaction and an object of representation. Whatever reacts is *ipso facto* real. But an object of representation is not *ipso facto* real. If I were to predict that on my letting go of the stone it would fly up in the air, that would be mere fiction; and the proof that it was so would be obtained by simply trying the experiment. That is clear. On the other hand, and by the same token, the fact that I *know* that this stone will fall to the floor when I let it go, as you all must confess, if you are not blinded by theory, that I *do know* -- and you none of you care to take up my bet, I notice -- is the proof that the formula, or uniformity, as furnishing a safe basis for prediction, is, or if you like it better, *corresponds to*, a reality.

Peirce: CP 5.97 Cross-Ref:††

97. Possibly at this point somebody may raise an objection and say: You admit, that is one thing really to be and another to be represented; and you further admit that it is of the nature of the law of nature to be represented. Then it follows that it has not the mode of being of a reality. My answer to this would be that it rests upon an ambiguity. When I say that the general proposition as to what will happen, whenever a certain condition may be fulfilled, is of the nature of a representation, I mean that it refers to experiences *in futuro*, which I do not know are all of them experienced and never can know have been all experienced. But when I say that really to be is different from being represented, I mean that what really is, ultimately consists in what shall be forced upon us in experience, that there is an element of brute compulsion in fact and that fact is not a mere question of reasonableness. Thus, if I say, "I shall wind up my watch every day as long as I live," I never can have a positive experience which *certainly* covers all that is here promised, because I never shall know for certain that my last day has come. But what the real fact will be does not depend upon what I represent, but upon what the experiential reactions shall be. My assertion that I shall wind up my watch every day of my life may turn out to

accord with facts, even though I be the most irregular of persons, by my dying before nightfall.

Peirce: CP 5.97 Cross-Ref:††

If we call that being true by chance, here is a case of a *general proposition* being entirely true in all its generality by chance.

Peirce: CP 5.98 Cross-Ref:††

98. Every general proposition is limited to a finite number of occasions in which it might conceivably be falsified, supposing that it is an assertion confined to what human beings may experience; and consequently it is conceivable that, although it should be true without exception, it should still only be by chance that it turns out true.

Peirce: CP 5.99 Cross-Ref:††

99. But if I see a man who is very regular in his habits and am led to offer to wager that that man will not miss winding his watch for the next month, you have your choice between two alternative hypotheses only:

Peirce: CP 5.99 Cross-Ref:††

1. You may suppose that some *principle* or *cause* is *really* operative to *make* him wind his watch daily, which *active principle* may have more or less strength; or

Peirce: CP 5.99 Cross-Ref:††

2. You may suppose that it is mere chance that his actions have hitherto been regular; and in that case, that regularity in the past affords you not the slightest reason for expecting its continuance in the future, any more than, if he had thrown sixes three times running, *that* event would render it either more or less likely that his next throw would show sixes.

Peirce: CP 5.100 Cross-Ref:††

100. It is the same with the operations of nature. With overwhelming uniformity, in our past experience, direct and indirect, stones left free to fall have fallen. Thereupon two hypotheses only are open to us. Either

Peirce: CP 5.100 Cross-Ref:††

1. the uniformity with which those stones have fallen has been due to mere chance and affords no ground whatever, not the slightest for any expectation that the next stone that shall be let go will fall; or

Peirce: CP 5.100 Cross-Ref:††

2. the uniformity with which stones have fallen has been due to some *active general principle*, in which case it would be a strange coincidence that it should cease to act at the moment my prediction was based upon it.

Peirce: CP 5.100 Cross-Ref:††

That position, gentlemen, will sustain criticism. It is irrefragable.

Peirce: CP 5.101 Cross-Ref:††

101. Of course, every sane man will adopt the latter hypothesis. If he could doubt it in the case of the stone -- which he can't -- and I may as well drop the stone once for all -- I told you so! -- if anybody doubts this still, a thousand other such inductive predictions are getting verified every day, and he will have to suppose every one of them to be merely fortuitous in order reasonably to escape the conclusion that general principles are really operative in nature. That is the doctrine of scholastic

realism.

Peirce: CP 5.102 Cross-Ref:††
§2. THIRDNESS AND GENERALITY †1

102. You may, perhaps, ask me how I connect generality with Thirdness. Various different replies, each fully satisfactory, may be made to that inquiry. The old definition of a general is *Generale est quod natum aptum est dici de multis*. †2 This recognizes that the general is essentially *predicative* and therefore of the nature of a representamen. And by following out that path of suggestion we should obtain a good reply to the inquiry.

Peirce: CP 5.103 Cross-Ref:††

103. In another respect, however, the definition represents a very degenerate sort of generality. None of the scholastic logics fails to explain that *sol* is a general term; because although there happens to be but one sun yet the term *sol aptum natum est dici de multis*. But that is most inadequately expressed. If *sol* is apt to be predicated of *many*, it is apt to be predicated of any multitude however great, and since there is no maximum multitude, †3 those objects, of which it is fit to be predicated, form an aggregate that exceeds all multitude. Take any two possible objects that might be called *suns* and, however much alike they may be, any multitude whatsoever of intermediate suns are alternatively possible, and therefore as before these intermediate possible suns transcend all multitude. In short, the idea of a general involves the idea of possible variations which no multitude of existent things could exhaust but would leave between any two not merely *many* possibilities, but possibilities absolutely beyond all multitude.

Peirce: CP 5.104 Cross-Ref:††

104. Now Thirdness is nothing but the character of an object which embodies Betweenness or Mediation in its simplest and most rudimentary form; and I use it as the name of that element of the phenomenon which is predominant wherever Mediation is predominant, and which reaches its fullness in Representation.

Peirce: CP 5.105 Cross-Ref:††

105. Thirdness, as I use the term, is only a synonym for Representation, to which I prefer the less colored term because its suggestions are not so narrow and special as those of the word Representation. Now it is proper to say that a general principle that is operative in the real world is of the essential nature of a Representation and of a Symbol because its *modus operandi* is the same as that by which *words* produce physical effects. Nobody can deny that words do produce such effects. Take, for example, that sentence of Patrick Henry which, at the time of our Revolution, was repeated by every man to his neighbor:

Peirce: CP 5.105 Cross-Ref:††

"Three millions of people, armed in the holy cause of Liberty, and in such a country as we possess, are invincible against any force that the enemy can bring against us."

Peirce: CP 5.105 Cross-Ref:††

Those words present this character of the general law of nature. They might

have produced effects indefinitely transcending any that circumstances allowed them to produce. It might, for example, have happened that some American schoolboy, sailing as a passenger in the Pacific Ocean, should have idly written down those words on a slip of paper. The paper might have been tossed overboard and might have been picked up by some Jagala on a beach of the island of Luzon; and if he had had them translated to him, they might easily have passed from mouth to mouth there as they did in this country, and with similar effect.

Peirce: CP 5.106 Cross-Ref:††

106. Words then do produce physical effects. It is madness to deny it. The very denial of it involves a belief in it; and nobody can consistently fail to acknowledge it until he sinks to a complete mental paresis.

Peirce: CP 5.106 Cross-Ref:††

But *how* do they produce their effect? They certainly do not, in their character as symbols, *directly* react upon matter. Such action as they have is merely logical. It is not even psychological. It is merely that one symbol would justify another. However, suppose that first difficulty to have been surmounted, and that they do act upon actual thoughts. That thoughts act on the physical world and *conversely*, is one of the most familiar of facts. Those who deny it are persons with whom theories are stronger than facts. But how thoughts act on things it is impossible for us, in the present state of our knowledge, so much as to make any very promising guess; although, as I will show you presently,†1 a guess can be made which suffices to show that the problem is not beyond all hope of ultimate solution.

Peirce: CP 5.107 Cross-Ref:††

107. All this is equally true of the manner in which the laws of nature influence matter. A law is in itself nothing but a general formula or symbol. An existing thing is simply a blind reacting thing, to which not merely all generality, but even all representation, is utterly foreign. The general formula may logically determine another, less broadly general. But it will be of its essential nature general, and its being narrower does not in the least constitute any participation in the reacting character of the thing. Here we have that great problem of the *principle of individuation* which the scholastic doctors after a century of the closest possible analysis were obliged to confess was quite incomprehensible to them. Analogy suggests that the laws of nature are ideas or resolutions in the mind of some vast consciousness, who, whether supreme or subordinate, is a Deity relatively to us. I do not approve of mixing up Religion and Philosophy; but as a purely philosophical hypothesis, that has the advantage of being supported by analogy. Yet I cannot clearly see that beyond that support to the imagination it is of any particular scientific service. . . .

Peirce: CP 5.108 Cross-Ref:††
§3. NORMATIVE JUDGMENTS

108. Reasoning cannot possibly be divorced from *logic*; because, whenever a man reasons, he thinks that he is drawing a conclusion such as would be justified in every analogous case. He therefore cannot really *infer* without having a notion of a class of possible inferences, all of which are logically *good*. That distinction of *good*

and *bad* he always has in mind when he infers. Logic proper is the *critic* of arguments, the pronouncing them to be good or bad. There are, as I am prepared to maintain, operations of the mind which are logically exactly analogous to inferences excepting only that they are unconscious and therefore uncontrollable and therefore not subject to criticism. But that makes all the difference in the world; for *inference* is essentially deliberate, and self-controlled. Any operation which cannot be controlled, any conclusion which is not abandoned, not merely as soon as *criticism* has pronounced against it, but in the very act of pronouncing that decree, is not of the nature of rational inference -- is not reasoning. Reasoning as deliberate is essentially critical, and it is idle to criticize as good or bad that which cannot be controlled. Reasoning essentially involves *self-control*; so that the *logica utens*†1 is a particular species of morality. Logical goodness and badness, which we shall find is simply the distinction of *Truth* and *Falsity* in general, amounts, in the last analysis, to nothing but a particular application of the more general distinction of Moral Goodness and Badness, or Righteousness and Wickedness.†2

Peirce: CP 5.109 Cross-Ref:††

109. To criticize as logically sound or unsound an operation of thought that cannot be controlled is not less ridiculous than it would be to pronounce the growth of your hair to be morally good or bad. The ridiculousness in both cases consists in the fact that such a critical judgment may be *pretended* but cannot really be performed in clear thought, for on analysis it will be found absurd.

Peirce: CP 5.110 Cross-Ref:††

110. I am quite aware that this position is open to two serious objections, which I have not time to discuss, but which I have carefully considered and refuted. The first is that this is making logic a question of psychology.†3 But this I deny. Logic does rest on certain facts of experience among which are facts about men, but not upon any theory about the human mind or any theory to explain facts. The other objection is that if the distinction [between] Good and Bad Logic is a special case [of the distinction between] Good and Bad Morals, by the same token the distinction of Good and Bad Morals is a special case of the distinction [between] esthetic Goodness and Badness. Now to admit this is not only to admit hedonism, which no man in his senses, and not blinded by theory or something worse, can admit, but also, having to do with the essentially Dualistic distinction of Good and Bad -- which is manifestly an affair of Category the Second -- it seeks the origin of this distinction in Esthetic Feeling, which belongs to Category the First.

Peirce: CP 5.111 Cross-Ref:††

111. This last objection deceived me for many years. The reply to it involves a very important point which I shall have to postpone to the next lecture. When it first presented itself to me, all I knew of ethics was derived from the study of Jouffroy †1 under Dr. Walker,†2 of Kant, and of a wooden treatise by Whewell;†3 and I was led by this objection to a line of thought which brought me to regard ethics as a mere art, or applied science, and not a pure normative science at all. But when, beginning in 1883, I came to read the works of the great moralists, whose great fertility of thought I found in wonderful contrast to the sterility of the logicians -- I was forced to recognize the dependence of Logic upon Ethics; and then took refuge in the idea that there was no science of esthetics, that, because *de gustibus non est disputandum*, therefore there is no esthetic *truth* and *falsity* or generally valid goodness and badness. But I did not remain of this opinion long. I soon came to see that this whole objection rests upon a fundamental misconception. To say that morality, in the last

resort, comes to an esthetic judgment is *not* hedonism -- but is directly opposed to hedonism. In the next place, every pronouncement between Good and Bad certainly comes under Category the Second; and for that reason such pronouncement comes out in the voice of conscience with an absoluteness of duality which we do not find even in logic; and although I am still a perfect ignoramus in esthetics, I venture to think that the esthetic state of mind is purest when perfectly naive without any critical pronouncement, and that the esthetic critic founds his judgments upon the result of throwing himself back into such a pure naive state -- and the best critic is the man who has trained himself to do this the most perfectly.

Peirce: CP 5.112 Cross-Ref:††

112. It is a great mistake to suppose that the phenomena of pleasure and pain are mainly phenomena of feeling.†1 Examine *pain*, which would seem to be a good deal more positive than *pleasure*. I am unable to recognize with confidence any quality of feeling common to all *pains*; and if I cannot I am sure it cannot be an easy thing for anybody. For I have gone through a systematic course of training in recognizing my feelings. I have worked with intensity for so many hours a day every day for long years to train myself to this; and it is a training which I would recommend to all of you. The artist has such a training; but most of his effort goes to reproducing in one form or another what he sees or hears, which is in every art a very complicated trade; while I have striven simply to see what it is that I see. That this limitation of the task is a great advantage is proved to me by finding that the great majority of artists are extremely narrow. Their esthetic appreciations are narrow; and this comes from their only having the power of recognizing the qualities of their percepts in certain directions.

Peirce: CP 5.112 Cross-Ref:††

But the majority of those who opine that pain is a quality of feeling are not even artists; and even among those who are artists there are extremely few who are *artists in pain*. But the truth is that there are certain states of mind, especially among states of mind in which Feeling has a large share, which we have an impulse to get rid of. That is the obvious phenomenon; and the ordinary theory is that this impulse is excited by a quality of feeling common to all these states -- a theory which is supported by the fact that this impulse is particularly energetic in regard to states in which Feeling is the predominant element. Now whether this be true or false, it is a *theory*. It is not the fact that any such common quality in all pains is readily to be recognized.

Peirce: CP 5.113 Cross-Ref:††

113. At any rate, while the whole phenomenon of pain and the whole phenomenon of pleasure are phenomena that arise within the universe of states of mind and attain no great prominence except when they concern states of mind in which Feeling is predominant, yet these phenomena themselves do not mainly consist in any common Feeling-quality of Pleasure and any common Feeling-quality of Pain, even if there are such Qualities of Feeling; but they mainly consist [in a] Pain [which lies] in a Struggle to give a state of mind its *quietus*, and [in a] Pleasure in a peculiar mode of consciousness allied to the consciousness of *making a generalization*, in which not Feeling, but rather Cognition is the principal constituent. This may be hard to make out as regards the lower pleasures, but they do not concern the argument we are considering. It is esthetic enjoyment which concerns us; and ignorant as I am of Art, I have a fair share of capacity for esthetic enjoyment; and it seems to me that while in esthetic enjoyment we attend to the totality of Feeling -- and especially to the

total resultant Quality of Feeling presented in the work of art we are contemplating -- yet it is a sort of intellectual sympathy, a sense that here is a Feeling that one can comprehend, a reasonable Feeling. I do not succeed in saying exactly *what* it is, but it is a consciousness belonging to the category of Representation, though representing something in the Category of Quality of Feeling.

Peirce: CP 5.113 Cross-Ref:††

In that view of the matter, the objection to the doctrine that the distinction Moral approval and disapproval is ultimately only a species of the distinction Esthetic approval and disapproval seems to be answered.

Peirce: CP 5.114 Cross-Ref:††

114. It appears, then, that *Logica utens* consisting in self-control, the distinction of logical goodness and badness must begin where control of the processes of cognition begins; and any object that antecedes the distinction, if it has to be named either good or bad, must be named *good*. For since no fault can be found with it, it must be taken at its own valuation.

Peirce: CP 5.115 Cross-Ref:††

§4. PERCEPTUAL JUDGMENTS †1

115. Where then in the process of cognition does the possibility of controlling it begin? Certainly not before the *percept* is formed.

Peirce: CP 5.115 Cross-Ref:††

Even after the percept is formed there is an operation which seems to me to be quite uncontrollable. It is that of judging what it is that the person perceives. A judgment is an act of formation of a mental proposition combined with an adoption of it or act of assent to it. A percept on the other hand is an image or moving picture or other exhibition. The perceptual judgment, that is, the first judgment of a person as to what is before his senses, bears no more resemblance to the percept than the figure I am going to draw is like a man.

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M A N

Peirce: CP 5.115 Cross-Ref:††

I do not see that it is possible to exercise any control over that operation or to subject it to criticism. If we can criticize it at all, as far as I can see, that criticism would be limited to performing it again and seeing whether, with closer attention, we get the same result. But when we so perform it again, paying now closer attention, the percept is presumably not such as it was before. I do not see what other means we have of knowing whether it is the same as it was before or not, except by comparing the former perceptual judgment and the later one. I should utterly distrust any other method of ascertaining what the character of the percept was. Consequently, until I am better advised, I shall consider the *perceptual judgment* to be utterly beyond

control. Should I be wrong in this, the *Percept*, at all events, would seem to be so.

Peirce: CP 5.116 Cross-Ref:††

116. It follows, then, that our perceptual judgments are the first premisses of all our reasonings and that they cannot be called in question. All our other judgments are so many theories whose only justification is that they have been and will be borne out by perceptual judgments. But the perceptual judgments declare one thing to be blue, another yellow -- one sound to be that of A, another that of U, another that of I. These are the Qualities of Feeling which the physicists say are mere illusions because there is no room for them in their theories. If the facts won't agree with the Theory, so much the worse for them. They are *bad facts*. This sounds to me childish, I confess. It is like an infant that beats an inanimate object that hurts it. Indeed this is true of all fault-finding with others than oneself, and those for whose conduct one is responsible. Reprobation is a silly [business].

Peirce: CP 5.117 Cross-Ref:††

117. But peradventure I shall be asked whether I do not admit that there is any such thing as an illusion or hallucination. Oh, yes; among artists I have known more than one case of downright hallucinatory imaginations at the beck and call of these {poietai}. Of course, the man knows that such obedient spectres are not real experiences, because experience is that which forces itself upon him, will-he nill-he.

Peirce: CP 5.117 Cross-Ref:††

Hallucinations proper -- obsessional hallucinations -- will *not* down at one's bidding, and people who are subject to them are accustomed to sound the people who are with them in order to ascertain whether the object before them has a being independent of their disease or not. There are also social hallucinations.

Peirce: CP 5.117 Cross-Ref:††

In such a case, a photographic camera or other instrument might be of service.

Peirce: CP 5.118 Cross-Ref:††

118. Of course, everybody admits and must admit that these apparitions are entities -- *entia*; the question is whether these *entia* belong to the class of realities or not, that is, whether they are such as they are independently of any collection of singular representations that they are so, or whether their mode of being depends upon abnormal conditions. But as for the entire universe of Qualities which the physicist would pronounce Illusory, there is not the smallest shade of just suspicion resting upon their normality. On the contrary, there is considerable evidence that colors, for example, and sounds have the same character for all mankind.

Peirce: CP 5.118 Cross-Ref:††

Well, I will skip this. Suffice it to say that there is no reason for suspecting the veracity of the senses, and the presumption is that the physics of the future will find out that they are more real than the present state of scientific theory admits of their being represented as being.†1

Peirce: CP 5.119 Cross-Ref:††

119. Therefore, if you ask me what part Qualities can play in the economy of the universe, I shall reply that the universe is a vast representamen, a great symbol of God's purpose, working out its conclusions in living realities. Now every symbol must have, organically attached to it, its Indices of Reactions and its Icons of Qualities; and such part as these reactions and these qualities play in an argument that, they of course, play in the universe -- that Universe being precisely an argument. In the little

bit that you or I can make out of this huge demonstration, our perceptual judgments are the premisses *for us* and these perceptual judgments have icons as their predicates, in which *icons* Qualities are immediately presented. But what is first for us is not first in nature. The premisses of Nature's own process are all the independent uncaused elements of facts that go to make up the variety of nature which the necessitarian supposes to have been all in existence from the foundation of the world, but which the Tychist supposes are continually receiving new accretions.†1 These premisses of nature, however, though they are not the *perceptual facts* that are premisses to us, nevertheless must resemble them in being premisses. We can only imagine what they are by comparing them with the premisses for us. As premisses they must involve Qualities.

Peirce: CP 5.119 Cross-Ref:††

Now as to their function in the economy of the Universe. The Universe as an argument is necessarily a great work of art, a great poem -- for every fine argument is a poem and a symphony -- just as every true poem is a sound argument. But let us compare it rather with a painting -- with an impressionist seashore piece -- then every Quality in a Premiss is one of the elementary colored particles of the Painting; they are all meant to go together to make up the intended Quality that belongs to the whole as whole. That total effect is beyond our ken; but we can appreciate in some measure the resultant Quality of parts of the whole -- which Qualities result from the combinations of elementary Qualities that belong to the premisses.

But I shall endeavor to make this clearer in the next lecture.

Peirce: CP 5.120 Cross-Ref:††
LECTURE V †1

THE THREE KINDS OF GOODNESS

§1. THE DIVISIONS OF PHILOSOPHY †2

120. . . . I have already explained †3 that by Philosophy I

mean that department of Positive Science, or Science of Fact, which does not busy itself with gathering facts, but merely with learning what can be learned from that experience which presses in upon every one of us daily and hourly. It does not gather new facts, because it does not need them, and also because new general facts cannot be firmly established without the assumption of a metaphysical doctrine; and this, in turn, requires the coöperation of every department of philosophy; so that such new facts, however striking they may be, afford weaker support to philosophy by far than that *common experience* which nobody doubts or can doubt, and which nobody ever even *pretended* to doubt except as a consequence of belief in that experience so entire and perfect that it failed to be conscious of itself; just as an American who has never been abroad fails to perceive the characteristics of Americans; just as a writer is unaware of the peculiarities of his own style; just as none of us can see himself as others see him.

Peirce: CP 5.119 Cross-Ref:††

Now I am going to make a series of assertions which will sound wild; for I cannot stop to argue them, although I cannot omit them if I am to set the supports of pragmatism in their true light.

Peirce: CP 5.121 Cross-Ref:††

121. Philosophy has three grand divisions. The first is Phenomenology, which simply contemplates the Universal Phenomenon and discerns its ubiquitous elements, Firstness, Secondness, and Thirdness, together perhaps with other series of categories. The second grand division is Normative Science, which investigates the universal and necessary laws of the relation of Phenomena to *Ends*, that is, perhaps, to Truth, Right, and Beauty. The third grand division is Metaphysics, which endeavors to comprehend the Reality of Phenomena. Now Reality is an affair of Thirdness as Thirdness, that is, in its mediation between Secondness and Firstness. Most, if not all of you, are, I doubt not, Nominalists; and I beg you will not take offence at a truth which is just as plain and undeniable to me as is the truth that children do not understand human life. To be a nominalist consists in the undeveloped state in one's mind of the apprehension of Thirdness as Thirdness. The remedy for it consists in allowing ideas of human life to play a greater part in one's philosophy. Metaphysics is the science of Reality. Reality consists in regularity. Real regularity is active law. Active law is efficient reasonableness, or in other words is truly reasonable reasonableness. Reasonable reasonableness is Thirdness as Thirdness.

Peirce: CP 5.121 Cross-Ref:††

So then the division of Philosophy into these three grand departments, whose distinctness can be established without stopping to consider the contents of Phenomenology (that is, without asking what the true categories may be), turns out to be a division according to Firstness, Secondness, and Thirdness, and is thus one of the very numerous phenomena I have met with which confirm this list of categories.

Peirce: CP 5.122 Cross-Ref:††

122. Phenomenology treats of the universal Qualities of Phenomena in their immediate phenomenal character, in themselves as phenomena. It, thus, treats of Phenomena in their Firstness.

Peirce: CP 5.123 Cross-Ref:††

123. Normative Science treats of the laws of the relation of phenomena to ends; that is, it treats of Phenomena in their Secondness.

Peirce: CP 5.124 Cross-Ref:††

124. Metaphysics, as I have just remarked, treats of Phenomena in their Thirdness.

Peirce: CP 5.125 Cross-Ref:††

125. If, then, Normative Science does not seem to be sufficiently described by saying that it treats of phenomena in their secondness, this is an indication that our conception of Normative Science is too narrow; and I had come to the conclusion that this is true of even the best modes of conceiving Normative Science which have achieved any renown, many years before I recognized the proper division of philosophy.

Peirce: CP 5.125 Cross-Ref:††

I wish I could talk for an hour to you concerning the true conception of

normative science. But I shall only be able to make a few negative assertions which, even if they were proved, would not go far toward developing that conception. Normative Science is not a skill, nor is it an investigation conducted with a view to the production of skill. Coriolis wrote a book on the Analytic Mechanics of the Game of Billiards.†1 If that book does not help people in the least degree to play billiards, that is nothing against it. The book is only intended to be pure theory. In like manner, if Normative Science does not in the least tend to the development of skill, its value as Normative Science remains the same. It is purely theoretical. Of course there *are* practical sciences of reasoning and investigation, of the conduct of life, and of the production of works of art. They correspond to the Normative Sciences, and may be probably expected to receive aid from them. But they are not integrant parts of these sciences; and the reason that they are not so, thank you, is no mere formalism, but is this, that it will be in general quite different men -- two knots of men not apt to consort the one with the other -- who will conduct the two kinds of inquiry. Nor again is Normative Science a *special* science, that is, one of those sciences that discover new phenomena. It is not even aided in any appreciable degree by any such science, and let me say that it is no more by psychology than by any other special science. If we were to place six lots each of seven coffee beans in one pan of an equal-armed balance, and forty-two coffee beans in the other pan, and were to find on trial that the two loads nearly balanced one another, this observation might be regarded as adding in some excessively slight measure to the certainty of the proposition that six times seven make forty-two; because it is conceivable that this proposition should be a mistake due to some peculiar insanity affecting the whole human race, and the experiment may possibly evade the effects of that insanity, supposing that we are affected with it. In like manner, and in just *about the same degree*, the fact that men for the most part show a natural disposition to approve nearly the same arguments that logic approves, nearly the same acts that ethics approves, and nearly the same works of art that esthetics approves, may be regarded as tending to support the conclusions of logic, ethics, and esthetics. But such support is perfectly insignificant; and when it comes to a particular case, to urge that anything is sound and good logically, morally, or esthetically, for no better reason than that men have a natural tendency to think so, I care not how strong and imperious that tendency may be, is as pernicious a fallacy as ever was. Of course it is quite a different thing for a man to acknowledge that he cannot perceive that he doubts what he does not appreciably doubt.

Peirce: CP 5.126 Cross-Ref:††

126. In one of the ways I have indicated, especially the last, Normative Science is by the majority of writers of the present day ranked too low in the scale of the sciences. On the other hand, some students of exact logic rank *that* normative science, at least, *too high*, by virtually treating it as on a par with pure mathematics.†1 There are three excellent reasons any one of which ought to rescue them from the error of this opinion. In the first place, the hypotheses from which the deductions of normative science proceed are *intended to conform* to positive truth of fact and those deductions derive their interest from that circumstance almost exclusively; while the hypotheses of pure mathematics are purely ideal in intention, and their interest is purely intellectual. But in the second place, the procedure of the normative sciences is *not purely deductive*, as that of mathematics is, nor even principally so. Their peculiar analyses of familiar phenomena, analyses which ought to be guided by the facts of phenomenology in a manner in which mathematics is not at all guided, separate Normative Science from mathematics quite radically. In the third place, there is a most intimate and essential element of Normative Science which

is still *more* proper to it, and that is its *peculiar appreciations*, to which nothing at all in the phenomena, in themselves, corresponds. These appreciations relate to the conformity of phenomena *to ends* which are not immanent within those phenomena.

Peirce: CP 5.127 Cross-Ref:††

127. There are sundry other widely spread misconceptions of the nature of Normative Science. One of these is that the chief, if not the only, problem of Normative Science is to say what is *good* and what *bad*, logically, ethically, and esthetically; or what degree of goodness a given description of phenomenon attains. Were this the case, normative science would be, in a certain sense, *mathematical*, since it would deal entirely with a question of *quantity*. But I am strongly inclined to think that this view will not sustain critical examination. Logic classifies arguments, and in doing so recognizes different *kinds* of truth. In ethics, too, *qualities* of good are admitted by the great majority of moralists. As for esthetics, in that field qualitative differences appear to be so prominent that, abstracted from them, it is impossible to say that there is any appearance which is not esthetically good. Vulgarity and pretension, themselves, may appear quite delicious in their perfection, if we can once conquer our squeamishness about them, a squeamishness which results from a contemplation of them as possible qualities of our own handiwork -- but that is a *moral* and not an *esthetic* way of considering them. I hardly need remind you that goodness, whether esthetic, moral, or logical, may either be *negative* -- consisting in freedom from fault -- or *quantitative* -- consisting in the degree to which it attains. But in an inquiry, such as we are now engaged upon, negative goodness is the important thing.

Peirce: CP 5.128 Cross-Ref:††

128. A subtle and almost ineradicable narrowness in the conception of Normative Science runs through almost all modern philosophy in making it relate exclusively to the human mind. The beautiful is conceived to be relative to human taste, right and wrong concern human conduct alone, logic deals with human reasoning. Now in the truest sense these sciences certainly are indeed sciences of mind. Only, modern philosophy has never been able quite to shake off the Cartesian idea of the mind, as something that "resides" -- such is the term †1 -- in the pineal gland. Everybody laughs at this nowadays, and yet everybody continues to think of mind in this same general way, as something within this person or that, belonging to him and correlative to the real world. A whole course of lectures would be required to expose this error. I can only hint that if you reflect upon it, without being dominated by preconceived ideas, you will soon begin to perceive that it is a very narrow view of mind. I should think it must appear so to anybody who was sufficiently soaked in the *Critic of the Pure Reason*.

Peirce: CP 5.129 Cross-Ref:††

§2. ETHICAL AND ESTHETICAL GOODNESS †1

129. I cannot linger more upon the general conception of Normative Science. I must come down to the particular Normative Sciences. These are now commonly said to be logic, ethics, and esthetics. Formerly only logic and ethics were reckoned as such. A few logicians refuse to recognize any other normative science than their own. My own opinions of ethics and esthetics are far less matured than my logical

opinions. It is only since 1883 that I have numbered ethics among my special studies; and until about four years ago, I was not prepared to affirm that ethics was a normative science. As for esthetics, although the first year of my study of philosophy was devoted to this branch exclusively, yet I have since then so completely neglected it that I do not feel entitled to have any confident opinions about it. I am inclined to think that there is such a Normative Science; but I feel by no means sure even of that.

Peirce: CP 5.129 Cross-Ref:††

Supposing, however, that normative science divides into esthetics, ethics, and logic, then it is easily perceived, from my standpoint, that this division is governed by the three categories. For Normative Science in general being the science of the laws of conformity of things to ends, esthetics considers those things whose ends are to embody qualities of feeling, ethics those things whose ends lie in action, and logic those things whose end is to represent something.

Peirce: CP 5.130 Cross-Ref:††

130. Just at this point we begin to get upon the trail of the secret of pragmatism, after a long and apparently aimless beating about the bush. Let us glance at the relations of these three sciences to one another. Whatever opinion be entertained in regard to the scope of logic, it will be generally agreed that the heart of it lies in the classification and critic of arguments. Now it is peculiar to the nature of argument that no argument can exist without being referred to some special class of arguments. The act of inference consists in the thought that the inferred conclusion is true because *in any analogous case* an analogous conclusion *would be* true. Thus, logic is coeval with reasoning. Whoever reasons *ipso facto* virtually holds a logical doctrine, his *logica utens*. †2 This classification is not a mere qualification of the argument. It essentially involves *an approval* of it -- a *qualitative approval*. Now such self-approval supposes *self-control*. Not that we regard our approval as *itself* a voluntary act, but that we hold the act of inference, which we approve, to be voluntary. That is, if we did not approve, we should not infer. There are mental operations which are as completely beyond our control as the growth of our hair. To approve or disapprove of *them* would be idle. But when we institute an experiment to test a theory, or when we imagine an extra line to be inserted in a geometrical diagram in order to determine a question in geometry, these are *voluntary acts* which our logic, whether it be of the natural or the scientific sort, *approves*. Now, *the approval of a voluntary act* is a *moral approval*. *Ethics is the study of what ends of action we are deliberately prepared to adopt*. That is right action which is in conformity to ends which we are prepared deliberately to adopt. That is all there *can be* in the notion of righteousness, as it seems to me. The righteous man is the man who controls his passions, and makes them conform to such ends as he is prepared deliberately to adopt as *ultimate*. If it were in the nature of a man to be perfectly satisfied to make his personal comfort his ultimate aim, no more blame would attach to him for doing so than attaches to a hog for behaving in the same way. A logical reasoner is a reasoner who exercises great self-control in his intellectual operations; and therefore the logically good is simply a particular species of the morally good. Ethics -- the genuine normative science of ethics, as contradistinguished from the branch of anthropology which in our day often passes under the name of ethics -- this genuine ethics is the normative science *par excellence*, because an *end* -- the essential object of normative science -- is germane to a voluntary act in a primary way in which it is germane to nothing else. For that reason I have some lingering doubt as to there being any true normative science of the beautiful. On the other hand, an ultimate end of action *deliberately* adopted -- that is to say, *reasonably* adopted -- must be a state

of things that *reasonably recommends itself in itself* aside from any ulterior consideration. It must be an *admirable ideal*, having the only kind of goodness that such an ideal *can* have; namely, esthetic goodness. From this point of view the morally good appears as a particular species of the esthetically good.

Peirce: CP 5.131 Cross-Ref:††

131. If this line of thought be sound, the morally good will be the esthetically good specially determined by a peculiar superadded element; and the logically good will be the morally good specially determined by a special superadded element. Now it will be admitted to be, at least, very likely that in order to correct or to vindicate the maxim of pragmatism, we must find out precisely what the logically good consists in; and it would appear from what has been said that, in order to analyze the nature of the logically good, we must first gain clear apprehensions of the nature of the esthetically good and especially that of the morally good.

Peirce: CP 5.132 Cross-Ref:††

132. So, then, incompetent as I am to it, I find the task imposed upon me of defining the esthetically good -- a work which so many philosophical artists have made as many attempts at performing. In the light of the doctrine of categories I should say that an object, to be esthetically good, must have a multitude of parts so related to one another as to impart a positive simple immediate quality to their totality; and whatever does this is, in so far, esthetically good, no matter what the particular quality of the total may be. If that quality be such as to nauseate us, to scare us, or otherwise to disturb us to the point of throwing us out of the mood of esthetic enjoyment, out of the mood of simply contemplating the embodiment of the quality -- just, for example, as the Alps affected the people of old times, when the state of civilization was such that an impression of great power was inseparably associated with lively apprehension and terror -- then the object remains none the less esthetically good, although people in our condition are incapacitated from a calm esthetic contemplation of it.

Peirce: CP 5.132 Cross-Ref:††

This suggestion must go for what it may be worth, which I dare say may be very little. If it be correct, it will follow that there is no such thing as positive esthetic badness; and since by goodness we chiefly in this discussion mean merely the absence of badness, or faultlessness, there will be no such thing as esthetic goodness. All there will be will be various esthetic qualities; that is, simple qualities of totalities not capable of full embodiment in the parts, which qualities may be more decided and strong in one case than in another. But the very reduction of the intensity may be an esthetic quality; nay, it *will* be so; and I am seriously inclined to doubt there being any distinction of pure esthetic betterness and worseness. My notion would be that there are innumerable varieties of esthetic quality, but no purely esthetic grade of excellence.

Peirce: CP 5.133 Cross-Ref:††

133. But the instant that an esthetic ideal is proposed as an ultimate end of action, at that instant a categorical imperative pronounces for or against it. Kant, as you know, proposes to allow that categorical imperative to stand unchallenged -- an eternal pronouncement. His position is in extreme disfavor now, and not without reason. Yet I cannot think very highly of the logic of the ordinary attempts at refuting it. The whole question is whether or not this categorical imperative be beyond control. If this voice of conscience is unsupported by ulterior reasons, is it not simply an insistent irrational howl, the hooting of an owl which we may disregard if we can?

Why should we pay any more attention to it than we would to the barking of a cur? If we *cannot* disregard conscience, all homilies and moral maxims are perfectly idle. But if it can be disregarded, it is, in one sense, not beyond control. It leaves us free to control ourselves. So then, it appears to me that any aim whatever which can be consistently pursued becomes, as soon as it is unflinching adopted, beyond all possible criticism, except the quite impertinent criticism of outsiders. An aim which *cannot* be adopted and consistently pursued is a bad aim. It cannot properly be called an ultimate aim at all. The only moral evil is not to have an ultimate aim.

Peirce: CP 5.134 Cross-Ref:††

134. Accordingly the problem of ethics is to ascertain what end is possible. It might be thoughtlessly supposed that *special science* could aid in this ascertainment. But that would rest on a misconception of the nature of an absolute aim, which is what *would be* pursued under all possible circumstances -- that is, even though the contingent facts ascertained by special sciences were entirely different from what they are. Nor, on the other hand, must the definition of such aim be reduced to a mere formalism.

Peirce: CP 5.135 Cross-Ref:††

135. The importance of the matter for pragmatism is obvious. For if the meaning of a symbol consists in how it might cause us to act, it is plain that this "how" cannot refer to the description of mechanical motions that it might cause, but must intend to refer to a description of the action as having this or that *aim*. In order to understand pragmatism, therefore, well enough to subject it to intelligent criticism, it is incumbent upon us to inquire what an ultimate aim, capable of being pursued in an indefinitely prolonged course of action, can be.

Peirce: CP 5.136 Cross-Ref:††

136. The deduction of this is somewhat intricate, on account of the number of points which have to be taken into account; and of course I cannot go into details. In order that the aim should be immutable under all circumstances, without which it will not be an ultimate aim, it is requisite that it should accord with a free development of the agent's own esthetic quality. At the same time it is requisite that it should not ultimately tend to be disturbed by the reactions upon the agent of that outward world which is supposed in the very idea of action. It is plain that these two conditions can be fulfilled at once only if it happens that the esthetic quality toward which the agent's free development tends and that of the ultimate action of experience upon him are parts of one esthetic total. Whether or not this is really so, is a metaphysical question which it does not fall within the scope of Normative Science to answer. If it is *not* so, the aim is essentially *unattainable*. But just as in playing a hand of whist, when only three tricks remain to be played, the rule is to assume that the cards are so distributed that the odd trick can be made, so the rule of ethics will be to adhere to the only possible absolute aim, and to hope that it will prove attainable. Meantime, it is comforting to know that all experience is favorable to that assumption.

Peirce: CP 5.137 Cross-Ref:††
§3. LOGICAL GOODNESS

137. The ground is now cleared for the analysis of logical goodness, or the

goodness of representation. There is a special variety of esthetic goodness that may belong to a representamen, namely, *expressiveness*. There is also a special moral goodness of representations, namely, *veracity*. But besides this there is a peculiar mode of goodness which is logical. What this consists in we have to inquire.

Peirce: CP 5.138 Cross-Ref:††

138. The mode of being of a representamen is such that it is capable of repetition. Take, for example, any proverb. "Evil communications corrupt good manners." Every time this is written or spoken in English, Greek, or any other language, and every time it is thought of it is one and the same representamen. It is the same with a diagram or picture. It is the same with a physical sign or symptom. If two weathercocks are different signs, it is only in so far as they refer to different parts of the air. A representamen which should have a unique embodiment, incapable of repetition, would not be a representamen, but a part of the very fact represented. This repetitory character of the representamen involves as a consequence that it is essential to a representamen that it should contribute to the determination of another representamen distinct from itself. For in what sense would it be true that a representamen was repeated if it were not capable of determining some different representamen? "Evil communications corrupt good manners" and {phtheirusin ethe chresth' homiliai kakai} are one and the same representamen. They are so, however, only so far as they are represented as being so; and it is one thing to say that "Evil communications corrupt good manners" and quite a different thing to say that "Evil communications corrupt good manners" and {phtheirusin ethe chresth' homiliai kakai} are two expressions of the same proverb. Thus every representamen must be capable of contributing to the determination of a representamen different from itself. Every conclusion from premisses is an instance in point; and what would be a representamen that was not capable of contributing to any ulterior conclusion? I call a representamen which is determined by another representamen, an *interpretant* of the latter. Every representamen is related or is capable of being related to a reacting thing, its object, and every representamen embodies, in some sense, some quality, which may be called its *signification*, what in the case of a common name J.S. Mill calls its *connotation*, a particularly objectionable expression.†1

Peirce: CP 5.139 Cross-Ref:††

139. A representamen [as symbol] is either a *rhema*, a *proposition*, or an *argument*. An *argument* is a representamen which separately shows what interpretant it is intended to determine. A *proposition* is a representamen which is not an argument, but which separately indicates what object it is intended to represent. A *rhema* is a simple representation without such separate parts.

Peirce: CP 5.140 Cross-Ref:††

140. Esthetic goodness, or *expressiveness*, may be possessed, and in some degree must be possessed, by any kind of representamen -- rhema, proposition, or argument.

Peirce: CP 5.141 Cross-Ref:††

141. Moral goodness, or veracity, may be possessed by a proposition or by an argument, but cannot be possessed by a rhema. A mental judgment or inference must possess some degree of veracity.

Peirce: CP 5.142 Cross-Ref:††

142. As to logical goodness, or *truth*, the statements in the books are faulty; and it is highly important for our inquiry that they should be corrected. The books

distinguish between *logical truth*, which some of them rightly confine to arguments that do not promise more than they perform, and *material truth* which belongs to propositions, being that which veracity aims to be; and this is conceived to be a higher grade of truth than mere logical truth. I would correct this conception as follows. In the first place, all our knowledge rests upon perceptual judgments. These are necessarily veracious in greater or less degree according to the effort made, but there is no meaning in saying that they have any other truth than veracity, since a perceptual judgment can never be repeated. At most we can say of a perceptual judgment that its relation to other perceptual judgments is such as to permit a simple theory of the facts. Thus I may judge that I see a clean white surface. But a moment later I may question whether the surface really was clean, and may look again more sharply. If this second more veracious judgment still asserts that I see a clean surface, the theory of the facts will be simpler than if, at my second look, I discern that the surface is soiled. Still, even in this last case, I have no right to say that my first *percept* was that of a soiled surface. I absolutely have no testimony concerning it, except my perceptual judgment, and although that was careless and had no high degree of veracity, still I have to accept the only evidence in my possession. Now consider any other judgment I may make. That is a conclusion of inferences ultimately based on perceptual judgments, and since these are indisputable, all the truth which my judgment can have must consist in the logical correctness of those inferences. Or I may argue the matter in another way. To say that a proposition is false is not veracious unless the speaker has found out that it is false. Confining ourselves, therefore, to veracious propositions, to say that a proposition is false and that it has been *found* to be false are equivalent, in the sense of being necessarily either both true or both false. Consequently, to say that a proposition is *perhaps* false is the same as to say that it will *perhaps* be found out to be false. Hence to deny one of these is to deny the other. To say that a proposition is certainly true means simply that it never can be found out to be false, or in other words, that it is derived by logically correct arguments from veracious perceptual judgments. Consequently, the only difference between material truth and the logical correctness of argumentation is that the *latter* refers to a single line of argument and the *former* to all the arguments which could have a given proposition or its denial as their conclusion.

Peirce: CP 5.142 Cross-Ref:††

Let me say to you that this reasoning needs to be scrutinized with the severest and minutest logical criticism, because pragmatism largely depends upon it.

Peirce: CP 5.143 Cross-Ref:††

143. It appears, then, that logical goodness is simply the excellence of argument -- its negative, and more fundamental, goodness being its soundness and weight, its really having the force that it pretends to have and that force being great, while its quantitative goodness consists in the degree in which it advances our knowledge. In what then does the soundness of argument consist?

Peirce: CP 5.144 Cross-Ref:††

144. In order to answer that question it is necessary to recognize three radically different kinds of arguments which I signalized in 1867†1 and which had been recognized by the logicians of the eighteenth century, although [those] logicians quite pardonably failed to recognize the inferential character of one of them. Indeed, I suppose that the three were given by Aristotle in the *Prior Analytics*, although the unfortunate illegibility of a single word in his MS. and its replacement by a wrong word by his first editor, the stupid [Apellicon], has completely altered the sense of the

chapter on Abduction.†1 At any rate, even if my conjecture is wrong, and the text must stand as it is, still Aristotle, in that chapter on Abduction, was even in that case evidently groping for that mode of inference which I call by the otherwise quite useless name of Abduction -- a word which is only employed in logic to translate the [{{apagoge}}] of that chapter.

Peirce: CP 5.145 Cross-Ref:††

145. These three kinds of reasoning are Abduction, Induction, and Deduction. Deduction is the only necessary reasoning. It is the reasoning of mathematics. It starts from a hypothesis, the truth or falsity of which has nothing to do with the reasoning; and of course its conclusions are equally ideal. The ordinary use of the doctrine of chances is necessary reasoning, although it is reasoning concerning probabilities. Induction is the experimental testing of a theory. The justification of it is that, although the conclusion at any stage of the investigation may be more or less erroneous, yet the further application of the same method must correct the error. The only thing that induction accomplishes is to determine the value of a quantity. It sets out with a theory and it measures the degree of concordance of that theory with fact. It never can originate any idea whatever. No more can deduction. All the ideas of science come to it by the way of Abduction. Abduction consists in studying facts and devising a theory to explain them. Its only justification is that if we are ever to understand things at all, it must be in that way.

Peirce: CP 5.146 Cross-Ref:††

146. Concerning the relations of these three modes of inference to the categories and concerning certain other details, my opinions, I confess, have wavered. These points are of such a nature that only the closest students of what I have written would remark the discrepancies. Such a student might infer that I have been given to expressing myself without due consideration; but in fact I have never, in any philosophical writing -- barring anonymous contributions to newspapers -- made any statement which was not based on at least half a dozen attempts, in writing, to subject the whole question to a very far more minute and critical examination than could be attempted in print, these attempts being made quite independently of one another, at intervals of many months, but subsequently compared together with the most careful criticism, and being themselves based upon at least two briefs of the state of the question, covering its whole literature, as far as known to me, and carrying the criticism in the strictest logical form to its extreme beginnings, without leaving any loopholes that I was able to discern with my utmost pains, these two briefs being made at an interval of a year or more and as independently as possible, although they were subsequently minutely compared, amended, and reduced to one. My waverings, therefore, have never been due to haste. They may argue stupidity. But I can at least claim that they prove one quality in my favor. That is that so far from my being wedded to opinions as being my own, I have shown rather a decided distrust of any opinion of which I have been an advocate. This perhaps ought to give a slight additional weight to those opinions in which I have never wavered -- although I need not say that the notion of any weight of authority being attached to opinions in philosophy or in science is utterly illogical and unscientific. Among these opinions which I have constantly maintained is this, that while Abductive and Inductive reasoning are utterly irreducible, either to the other or to Deduction, or Deduction to either of them, yet the only *rationale* of these methods is essentially Deductive or Necessary. If then we can state wherein the validity of Deductive reasoning lies, we shall have defined the foundation of logical goodness of whatever kind.

Peirce: CP 5.147 Cross-Ref:††

147. Now all necessary reasoning, whether it be good or bad, is of the nature of mathematical reasoning. The philosophers are fond of boasting of the pure conceptual character of their reasoning. The more conceptual it is, the nearer it approaches to verbiage. I am not speaking from surmise. My analyses of reasoning surpass in thoroughness all that has ever been done in print, whether in words or in symbols -- all that DeMorgan, Dedekind, Schröder, Peano, Russell, and others have ever done -- to such a degree as to remind one of the difference between a pencil sketch of a scene and a photograph of it. To say that I analyze the passage from the premisses to the conclusion of a syllogism in Barbara into seven or eight distinct inferential steps gives but a very inadequate idea of the thoroughness of my analysis.†1 Let any responsible person pledge himself to go through the matter and dig it out, point by point, and he shall receive the manuscript.

Peirce: CP 5.148 Cross-Ref:††

148. It is on the basis of such analysis that I declare that all necessary reasoning, be it the merest verbiage of the theologians, so far as there is any semblance of necessity in it, is mathematical reasoning. Now mathematical reasoning is diagrammatic. This is as true of algebra as of geometry. But in order to discern the features of diagrammatic reasoning, it is requisite to begin with examples that are not too simple. In simple cases, the essential features are so nearly obliterated that they can only be discerned when one knows what to look for. But beginning with suitable examples and thence proceeding to others, one finds that the diagram itself, in its individuality, is not what the reasoning is concerned with. I will take an example which recommends itself only by its consideration requiring but a moment. A line abuts upon an ordinary point of another line forming two angles. The sum of these angles is proved by Legendre to be equal to the sum of two right angles by erecting a perpendicular to the second line in the plane of the two and through the point of abuttal. This perpendicular must lie in the one angle or the other. The pupil is supposed to *see* that. He sees it only in a special case, but he is supposed to perceive that it will be so in any case. The more careful logician may demonstrate that it must fall in one angle or the other; but this demonstration will only consist in substituting a different diagram in place of Legendre's figure. But in any case, either in the new diagram or else, and more usually, in passing from one diagram to the other, the interpreter of the argumentation will be supposed to *see* something, which will present this little difficulty for the theory of vision, that it is of a *general nature*.

Peirce: CP 5.149 Cross-Ref:††

149. Mr. Mill's disciples will say that this proves that geometrical reasoning is inductive. I do not wish to speak disparagingly of Mill's treatment †2 of the *Pons Asinorum* because it penetrates further into the logic of the subject than anybody had penetrated before. Only it does not quite touch bottom. As for such general perceptions being inductive, I might treat the question from a technical standpoint and show that the essential characters of induction are wanting. But besides the interminable length, such a way of dealing with the matter would hardly meet the point. It is better to remark that the "uniformity of nature" is not in question, and that there is no way of applying that principle to supporting the mathematical reasoning that will not enable me to give a precisely analogous instance in every essential particular, except that it will be a fallacy that no good mathematician could overlook. If you admit the principle that logic stops where self-control stops, you will find yourself obliged to admit that a *perceptual fact*, a logical origin, may involve

generality. This can be shown for ordinary generality. But if you have already convinced yourself that continuity is generality, it will be somewhat easier to show that a perceptual fact may involve continuity than that it can involve non-relative generality.

Peirce: CP 5.150 Cross-Ref:††

150. If you object that there can be no immediate consciousness of generality, I grant that. If you add that one can have no direct experience of the general, I grant that as well. Generality, Thirdness, pours in upon us in our very perceptual judgments, and all reasoning, so far as it depends on necessary reasoning, that is to say, mathematical reasoning, turns upon the perception of generality and continuity at every step.

Peirce: CP 5.151 Cross-Ref:††

LECTURE VI

THREE TYPES OF REASONING

§1. PERCEPTUAL JUDGMENTS AND GENERALITY

151. I was remarking at the end of my last lecture that perceptual judgments involve generality. What is the general? The Aristotelian definition is good enough. It is *quod aptum natum est praedicari de pluribus*;†1 {legō de katholou men ho epi pleionōn pephyke katégoreisthai}. *De Interp.* 7. When logic was studied in a scientific spirit of exactitude it was recognized on all hands that all ordinary judgments contain a predicate and that this predicate is general. There seemed to be some exceptions, of which the only noticeable ones were expository judgments, such as "Tully is Cicero." But the Logic of Relations has now reduced logic to order, and it is seen that a proposition may have any number of subjects but can have but one predicate which is invariably general. Such a proposition as "Tully is Cicero" predicates the general relation of identity of Tully and Cicero.†2 Consequently, it is now clear that if there be any perceptual judgment, or proposition directly expressive of and resulting from the quality of a present percept, or sense-image, that judgment must involve generality in its predicate.

Peirce: CP 5.152 Cross-Ref:††

152. That which is not general is singular; and the singular is that which reacts. The being of a singular may consist in the being of other singulars which are its parts. Thus heaven and earth is a singular; and its being consists in the being of heaven and the being of earth, each of which reacts and is therefore a singular, forming a part of heaven and earth. If I had denied that every perceptual judgment refers, as to its subject, to a singular, and that singular actually reacting upon the mind in forming the judgment, actually reacting too upon the mind in interpreting the judgment, I should have uttered an absurdity. For every proposition whatsoever refers as to its subject to a singular actually reacting upon the utterer of it and actually reacting upon the interpreter of it. All propositions relate to the same ever-reacting

singular; namely, to the totality of all real objects. It is true that when the Arabian romancer tells us that there was a lady named Scherherazade, he does not mean to be understood as speaking of the world of outward realities, and there is a great deal of fiction in what he is talking about. For the *fictive* is that whose characters depend upon what characters somebody attributes to it; and the story is, of course, the mere creation of the poet's thought. Nevertheless, once he has imagined Scherherazade and made her young, beautiful, and endowed with a gift of spinning stories, it becomes a real fact that so he has imagined her, which fact he cannot destroy by pretending or thinking that he imagined her to be otherwise. What he wishes us to understand is what he might have expressed in plain prose by saying, "I have imagined a lady, Scherherazade by name, young, beautiful and a tireless teller of tales, and I am going on to imagine what tales she told." This would have been a plain expression of professed fact relating to the sum total of realities.

Peirce: CP 5.153 Cross-Ref:††

153. As I said before, propositions usually have more subjects than one; and almost every proposition, if not quite every one, has one or more other singular subjects, to which some propositions do not relate. These are the special parts of the Universe of all Truth †1 to which the given proposition especially refers. It is a characteristic of perceptual judgments that each of them relates to some singular to which no other proposition relates directly, but, if it relates to it at all, does so by relating to that perceptual judgment. When we express a proposition in words, we leave most of its singular subjects unexpressed; for the circumstances of the enunciation sufficiently show what subject is intended and words, owing to their usual generality, are not well adapted to designating singulars. The pronoun, which may be defined as a part of speech intended to fulfill the function of an index, is never intelligible taken by itself apart from the circumstances of its utterance; and the noun, which may be defined as a part of speech put in place of a pronoun, is always liable to be equivocal.†1

Peirce: CP 5.154 Cross-Ref:††

154. A subject need not be singular. If it is not so, then when the proposition is expressed in the canonical form used by logicians, this subject will present one or other of two imperfections:†2

Peirce: CP 5.154 Cross-Ref:††

On the one hand, it may be *indesignative*, so that the proposition means that a singular of the universe might replace this subject while the truth was preserved, while failing to designate what singular that is: as when we say, "*Some* calf has five legs."

Peirce: CP 5.154 Cross-Ref:††

Or on the other hand, the subject may be *hypothetical*, that is may allow any singular to be substituted for it that fulfills certain conditions, without guaranteeing that there is any singular which fulfills these conditions; as when we say, "Any salamander could live in fire," or "Any man who should be stronger than Samson could do all that Samson did."

Peirce: CP 5.154 Cross-Ref:††

A subject which has neither of these two imperfections is a *singular* subject referring to an existing singular collection in its entirety.

Peirce: CP 5.155 Cross-Ref:††

155. If a proposition has two or more subjects of which one is *indesignative* and the other *hypothetical*, then it makes a difference in what order the replacement by singulars is asserted to be possible. It is, for example, one thing to assert that "Any Catholic there may be adores some woman or other" and quite another thing to assert that "There is some woman whom any Catholic adores." If the first general subject is indesignate, the proposition is called particular. If the first general subject is hypothetical, the proposition is called universal.†3

Peirce: CP 5.155 Cross-Ref:††

A particular proposition asserts the existence of something of a given description. A universal proposition merely asserts the non-existence of anything of a given description.

Peirce: CP 5.156 Cross-Ref:††

156. Had I, therefore, asserted that a perceptual judgment could be a universal proposition, I should have fallen into rank absurdity. For reaction is existence and the perceptual judgment is the cognitive product of a reaction.

Peirce: CP 5.156 Cross-Ref:††

But as from the particular proposition that "there is some women whom any Catholic you can find will adore" we can with certainty infer the universal proposition that "any Catholic you can find will adore some woman or other,"†1 so if a perceptual judgment involves any general elements, as it certainly does, the presumption is that a universal proposition can be necessarily deduced from it.

Peirce: CP 5.157 Cross-Ref:††

157. In saying that perceptual judgments involve general elements I certainly never intended to be understood as enunciating any proposition in psychology. For my principles absolutely debar me from making the least use of psychology in logic. I am confined entirely to the unquestionable facts of everyday experience, together with what can be deduced from them. All that I can mean by a perceptual judgment is a judgment absolutely forced upon my acceptance, and that by a process which I am utterly unable to control and consequently am unable to criticize. Nor can I pretend to absolute certainty about any matter of fact. If with the closest scrutiny I am able to give, a judgment appears to have the characters I have described, I must reckon it among perceptual judgments until I am better advised. Now consider the judgment that one event *C appears to be* subsequent to another event *A*. Certainly, I may have inferred this; because I may have remarked that *C* was subsequent to a third event *B* which was itself subsequent to *A*. But then these premisses are judgments of the same description. It does not seem possible that I can have performed an infinite series of acts of criticism each of which must require a distinct effort. The case is quite different from that of Achilles and the tortoise because Achilles does not require to make an infinite series of distinct efforts. It therefore appears that I must have made some judgment that one event *appeared to be* subsequent to another without that judgment having been inferred from any premiss [i.e.] without any *controlled* and *criticized* action of reasoning. If this be so, it is a perceptual judgment in the only sense that the logician can recognize. But from that proposition that one event, *Z*, is subsequent to another event, *J*, I can at once deduce by necessary reasoning a universal proposition. Namely, the definition of the relation of apparent subsequence is well known, or sufficiently so for our purpose. *Z* will appear to be subsequent to *Y* if and only if *Z* appears to stand in a peculiar relation, *R*, to *Y* such that nothing can stand in the relation *R* to itself, and if, furthermore, whatever event, *X*, there may be to which *Y* stands in the relation *R*, to that same *X*, *Z* also stands in the relation *R*.†1

This being implied in the meaning of subsequence, concerning which there is no room for doubt, it easily follows that whatever is subsequent to C is subsequent to anything, A, to which C is subsequent -- which is a universal proposition.

Peirce: CP 5.157 Cross-Ref:††

Thus my assertion at the end of the last lecture appears to be most amply justified. Thirdness pours in upon us through every avenue of sense.

Peirce: CP 5.158 Cross-Ref:††

§2. THE PLAN AND STEPS OF REASONING

158. We may now profitably ask ourselves what logical goodness is. We have seen that any kind of goodness consists in the adaptation of its subject to its *end*. One might set this down as a truism. Verily, it is scarcely more, although circumstances may have prevented it being clearly apprehended.

Peirce: CP 5.158 Cross-Ref:††

If you call this utilitarianism, I shall not be ashamed of the title. For I do not know what other system of philosophy has wrought so much good in the world as that same utilitarianism. Bentham may be a shallow logician; but such truths as he saw, he saw most nobly. As for the vulgar utilitarian, his fault does not lie in his pressing too much the question of what would be the good of this or that. On the contrary his fault is that he never presses the question half far enough, or rather he never really raises the question at all. He simply rests in his present desires as if desire were beyond all dialectic. He wants, perhaps, to go to heaven. But he forgets to ask what would be the good of his going to heaven. He would be happy, there, he thinks. But that is a mere word. It is no real answer to the question.

Peirce: CP 5.159 Cross-Ref:††

159. Our question is, What is the use of thinking? We have already remarked that it is the argument alone which is the primary and direct subject of logical goodness and badness. We have therefore to ask what the end of argumentation is, what it ultimately leads to.

Peirce: CP 5.160 Cross-Ref:††

160. The Germans, whose tendency is to look at everything subjectively and to exaggerate the element of Firstness, maintain that the object is simply to satisfy one's logical feeling and that the goodness of reasoning consists in that esthetic satisfaction alone.†1 This might do if we were gods and not subject to the force of experience.

Peirce: CP 5.160 Cross-Ref:††

Or if the force of experience were mere blind compulsion, and we were utter foreigners in the world, then again we might as well think to please ourselves; because we then never could make our thoughts conform to that mere Secondness.

Peirce: CP 5.160 Cross-Ref:††

But the saving truth is that there is a Thirdness in experience, an element of Reasonableness to which we can train our own reason to conform more and more. If this were not the case, there could be no such thing as logical goodness or badness; and therefore we need not wait until it is proved that there is a reason operative in

experience to which our own can approximate.†2 We should at once hope that it is so, since in that hope lies the only possibility of any knowledge.

Peirce: CP 5.161 Cross-Ref:††

161. Reasoning is of three types, Deduction, Induction, and Abduction.†3 In deduction, or necessary reasoning, we set out from a hypothetical state of things which we define in certain abstracted respects. Among the characters to which we pay no attention in this mode of argument is whether or not the hypothesis of our premisses conforms more or less to the state of things in the outward world. We consider this hypothetical state of things and are led to conclude that, however it may be with the universe in other respects, wherever and whenever the hypothesis may be realized, something else not explicitly supposed in that hypothesis will be true invariably. Our inference is valid if and only if there really is such a relation between the state of things supposed in the premisses and the state of things stated in the conclusion. Whether this really be so or not is a question of reality, and has nothing at all to do with how we may be inclined to think. If a given person is unable to see the connection, the argument is none the less valid, provided that relation of real facts really subsists. If the entire human race were unable to see the connection, the argument would be none the less sound, although it would not be humanly clear. Let us see precisely how we assure ourselves of the reality of the connection. Here, as everywhere throughout logic, the study of relatives has been of the greatest service. The simple syllogisms, which are alone considered by the old inexact logicians, are such very rudimentary forms that it is practically impossible to discern in them the essential features of deductive inference until our attention has been called to these features in higher forms of deduction.

Peirce: CP 5.162 Cross-Ref:††

162. All necessary reasoning without exception is diagrammatic.†1 That is, we construct an icon of our hypothetical state of things and proceed to observe it. This observation leads us to suspect that something is true, which we may or may not be able to formulate with precision, and we proceed to inquire whether it is true or not. For this purpose it is necessary to form a plan of investigation and this is the most difficult part of the whole operation. We not only have to select the features of the diagram which it will be pertinent to pay attention to, but it is also of great importance to return again and again to certain features. Otherwise, although our conclusions may be correct, they will not be the particular conclusions at which we are aiming. But the greatest point of art consists in the introduction of suitable *abstractions*. By this I mean such a transformation of our diagrams that characters of one diagram may appear in another as things. A familiar example is where in analysis we treat operations as themselves the subject of operations. Let me say that it would make a grand life-study to give an account of this operation of planning a mathematical demonstration.†2 Sundry sporadic maxims are afloat among mathematicians, and several meritorious books have been written upon the subject, but nothing broad and masterly. With the modern reformed mathematics and with my own and other logical results as a basis, such a theory of the plan of demonstration is no longer a superhuman task.

Peirce: CP 5.163 Cross-Ref:††

163. Having thus determined the plan of the reasoning, we proceed to the reasoning itself, and this I have ascertained can be reduced to three kinds of steps.†3 The first consists in copulating separate propositions into one compound proposition. The second consists in omitting something from a proposition without possibility of

introducing error. The third consists in inserting something into a proposition without introducing error.

Peirce: CP 5.164 Cross-Ref:††

164. You can see precisely what these elementary steps of inference are in Baldwin's *Dictionary* under Symbolic Logic.†1 As a specimen of what they are like you may take this:

A is a bay horse,

Therefore, A is a horse.

Peirce: CP 5.164 Cross-Ref:††

If one asks oneself how one knows that this is certain, one is likely to reply that one imagines a bay horse and on contemplating the image one sees that it is a horse. But that only applies to the single image. How large a horse did this image represent? Would it be the same with a horse of very different size? How old was the horse represented to be; was his tail docked? Would it be so if he had the blind-staggers, and if so are you sure it would be so whatever of the numerous diseases of the horse afflicted him? We are perfectly certain that none of these circumstances could affect the question in the least. It is easy enough to formulate reasons by the dozen; but the difficulty is that they are one and all far less evident than the original inference. I do not see that the logician can do better than to say that he *perceives* that when a copulative proposition is given, such as "A is a horse and A has a bay color" any member of the copulation may be omitted without changing the proposition from true to false. In a psychological sense I am willing to take the word of the psychologist if he says that such a general truth cannot be *perceived*. But what better can we do in logic?

Peirce: CP 5.165 Cross-Ref:††

165. Somebody may answer that the copulative proposition contains the conjunction "and" or something equivalent, and that the very *meaning* of this "and" is that the entire copulation is true if and only if each of the members is singly true; so that it is involved in the very *meaning* of the copulative proposition that any member may be dropped.

Peirce: CP 5.165 Cross-Ref:††

To this I assent with all my heart. But after all, what does it amount to? It is another way of saying that what we call the *meaning* of a proposition embraces every obvious necessary deduction from it. Considered as the beginning of an analysis of what the meaning of the word "meaning" is, it is a valuable remark. But I ask how it helps us to understand our passing from an accepted judgment A to another judgment C of which we not only feel equally confident but in point of fact *are* equally sure, barring a possible blunder which could be corrected as soon as attention was called to it, barring another equivalent blunder?

Peirce: CP 5.165 Cross-Ref:††

To this the advocate of the explanation by the conception of "meaning" may reply: that is *meant* which is intended or purposed; that a judgment is a voluntary act, and our intention is not to employ the form of the judgment A, except to the

interpretation of images to which judgments, corresponding in form to C, can be applied.

Peirce: CP 5.166 Cross-Ref:††

166. Perhaps it may reconcile the psychologist to the admission of perceptual judgments involving generality to be told that they are perceptual judgments concerning our own purposes. I certainly think that the certainty of pure mathematics and of all necessary reasoning is due to the circumstance that it relates to objects which are the creations of our own minds, and that mathematical knowledge is to be classed along with knowledge of our own purposes. When we meet with a surprising result in pure mathematics, as we so often do, because a loose reasoning had led us to suppose it impossible, this is essentially the same sort of phenomenon as when in pursuing a purpose we are led to do something that we are quite surprised to find ourselves doing, as being contrary, or apparently contrary, to some weaker purpose.

Peirce: CP 5.166 Cross-Ref:††

But if it is supposed that any such considerations afford any logical justification of primary logical principles I must say that, on the contrary, at the very best they *beg the question* by assuming premisses far less certain than the conclusion to be established.

Peirce: CP 5.167 Cross-Ref:††
§3. INDUCTIVE REASONING †1

167. A generation and a half of evolutionary fashions in philosophy has not sufficed entirely to extinguish the fire of admiration for John Stuart Mill -- that very strong but Philistine philosopher whose inconsistencies fitted him so well to be the leader of a popular school -- and consequently there will still be those who propose to explain the general principles of formal logic, which are now fully shown to be mathematical principles, by means of induction. Anybody who holds to that view today may be assumed to have a very loose notion of induction; so that all he really means is that the general principles in question are derived from images of the imagination by a process which is, roughly speaking, analogous to induction. Understanding him in that way, I heartily agree with him. But he must not expect me in 1903 to have anything more than a historical admiration for conceptions of induction which shed a brilliant light upon the subject in 1843. Induction is so manifestly inadequate to account for the certainty of these principles that it would be a waste of time to discuss such a theory.

Peirce: CP 5.168 Cross-Ref:††

168. However, it is now time for me to pass to the consideration of Inductive Reasoning. When I say that by inductive reasoning I mean a course of experimental investigation, I do not understand experiment in the narrow sense of an operation by which one varies the conditions of a phenomenon almost as one pleases. We often hear students of sciences, which are not in this narrow sense experimental, lamenting that in their departments they are debarred from this aid. No doubt there is much justice in this lament; and yet those persons are by no means debarred from pursuing the same logical method precisely, although not with the same freedom and facility. An experiment, says Stöckhardt, in his excellent *School of Chemistry*, is a question put to nature.†1 Like any interrogatory, it is based on a supposition. If that

supposition be correct, a certain sensible result is to be expected under certain circumstances which can be created, or at any rate are to be met with. The question is, Will this be the result? If Nature replies "No!" the experimenter has gained an important piece of knowledge. If Nature says "Yes," the experimenter's ideas remain just as they were, only somewhat more deeply engrained. If Nature says "Yes" to the first twenty questions, although they were so devised as to render that answer as surprising as possible, the experimenter will be confident that he is on the right track, since 2 to the 20th power exceeds a million.

Peirce: CP 5.169 Cross-Ref:††

169. Laplace was of the opinion that the affirmative experiments impart a definite probability to the theory; and that doctrine is taught in most books on probability to this day, although it leads to the most ridiculous results, and is inherently self-contradictory. It rests on a very confused notion of what probability is. Probability applies to the question whether a specified kind of event will occur when certain predetermined conditions are fulfilled; and it is the ratio of the number of times in the long run in which that specified result would follow upon the fulfillment of those conditions to the total number of times in which those conditions were fulfilled in the course of experience. It essentially refers to a course of experience, or at least of real events; because mere possibilities are not capable of being counted. You can, for example, ask what the probability is that a given kind of object will be red, provided you define red sufficiently. It is simply the ratio of the number of objects of that kind that are red to the total number of objects of that kind. But to ask in the abstract what the probability is that a shade of color will be red is nonsense, because shades of color are not individuals capable of being counted. You can ask what the probability is that the next chemical element to be discovered will have an atomic weight exceeding a hundred. But you cannot ask what the probability is that the law of universal attraction should be that of the inverse square until you can attach some meaning to statistics of the characters of possible universes. When Leibniz said that this world is the best that was possible, he may have had some glimmer of meaning, but when Quételet †1 says that if a phenomenon has been observed on m occasions, the probability that it will occur on the $(m + 1)$ th occasion is $(m+1)/(m+2)$, he is talking downright nonsense. Mr. F.Y. Edgeworth asserts that of all theories that are started one half are correct. That is not nonsense, but it is ridiculously false. For of theories that have enough to recommend them to be seriously discussed, there are more than two on the average to each general phenomenon to be explained. Poincaré, on the other hand, seems to think that all theories are wrong, and that it is only a question of how wrong they are.

Peirce: CP 5.170 Cross-Ref:††

170. Induction consists in starting from a theory, deducing from it predictions of phenomena, and observing those phenomena in order to see *how nearly* they agree with the theory. The justification for believing that an experiential theory which has been subjected to a number of experimental tests will be in the near future sustained about as well by further such tests as it has hitherto been, is that by steadily pursuing that method we must in the long run find out how the matter really stands. The reason that we must do so is that our theory, if it be admissible even as a theory, simply consists in supposing that such experiments will in the long run have results of a certain character. But I must not be understood as meaning that experience can be exhausted, or that any approach to exhaustion can be made. What I mean is that if there be a series of objects, say crosses and circles, this series having a beginning but no end, then whatever may be the arrangement or want of arrangement of these

crosses and circles in the entire endless series must be discoverable to an indefinite degree of approximation by examining a sufficient finite number of successive ones beginning at the beginning of the series. This is a theorem capable of strict demonstration. The principle of the demonstration is that whatever has no end can have no mode of being other than that of a law, and therefore whatever general character it may have must be describable, but the only way of describing an endless series is by stating explicitly or implicitly the law of the succession of one term upon another. But every such term has a finite ordinal place from the beginning and therefore, if it presents any regularity for all finite successions from the beginning, it presents the same regularity throughout. Thus the validity of induction depends upon the necessary relation between the general and the singular. It is precisely this which is the support of Pragmatism.

Peirce: CP 5.171 Cross-Ref:††
§4. INSTINCT AND ABDUCTION †1

171. Concerning the validity of Abductive inference, there is little to be said, although that little is pertinent to the problem we have in hand.

Peirce: CP 5.171 Cross-Ref:††

Abduction is the process of forming an explanatory hypothesis. It is the only logical operation which introduces any new idea; for induction does nothing but determine a value, and deduction merely evolves the necessary consequences of a pure hypothesis.

Peirce: CP 5.171 Cross-Ref:††

Deduction proves that something *must* be; Induction shows that something *actually is* operative; Abduction merely suggests that something *may be*.

Peirce: CP 5.171 Cross-Ref:††

Its only justification is that from its suggestion deduction can draw a prediction which can be tested by induction, and that, if we are ever to learn anything or to understand phenomena at all, it must be by abduction that this is to be brought about.

Peirce: CP 5.171 Cross-Ref:††

No reason whatsoever can be given for it, as far as I can discover; and it needs no reason, since it merely offers suggestions.

Peirce: CP 5.172 Cross-Ref:††

172. A man must be downright crazy to deny that science has made many true discoveries. But every single item of scientific theory which stands established today has been due to Abduction.

Peirce: CP 5.172 Cross-Ref:††

But how is it that all this truth has ever been lit up by a process in which there is no compulsiveness nor tendency toward compulsiveness? Is it by chance? Consider the multitude of theories that might have been suggested. A physicist comes across some new phenomenon in his laboratory. How does he know but the conjunctions of the planets have something to do with it or that it is not perhaps because the dowager empress of China has at that same time a year ago chanced to pronounce some word

of mystical power or some invisible jinnee may be present. Think of what trillions of trillions of hypotheses might be made of which one only is true; and yet after two or three or at the very most a dozen guesses, the physicist hits pretty nearly on the correct hypothesis. By chance he would not have been likely to do so in the whole time that has elapsed since the earth was solidified. You may tell me that astrological and magical hypotheses were resorted to at first and that it is only by degrees that we have learned certain general laws of nature in consequence of which the physicist seeks for the explanation of his phenomenon within the four walls of his laboratory. But when you look at the matter more narrowly, the matter is not to be accounted for in any considerable measure in that way. Take a broad view of the matter. Man has not been engaged upon scientific problems for over twenty thousand years or so. But put it at ten times that if you like. But that is not a hundred thousandth part of the time that he might have been expected to have been searching for his first scientific theory.

Peirce: CP 5.172 Cross-Ref:††

You may produce this or that excellent psychological account of the matter. But let me tell you that all the psychology in the world will leave the logical problem just where it was. I might occupy hours in developing that point. I must pass it by.

Peirce: CP 5.172 Cross-Ref:††

You may say that evolution accounts for the thing.†1 I don't doubt it is evolution. But as for explaining evolution by chance, there has not been time enough.

Peirce: CP 5.173 Cross-Ref:††

173. However man may have acquired his faculty of divining the ways of Nature, it has certainly not been by a self-controlled and critical logic. Even now he cannot give any exact reason for his best guesses. It appears to me that the clearest statement we can make of the logical situation -- the freest from all questionable admixture -- is to say that man has a certain Insight, not strong enough to be oftener right than wrong, but strong enough not to be overwhelmingly more often wrong than right, into the Thirdnesses, the general elements, of Nature. An Insight, I call it, because it is to be referred to the same general class of operations to which Perceptive Judgments belong. This Faculty is at the same time of the general nature of Instinct, resembling the instincts of the animals in its so far surpassing the general powers of our reason and for its directing us as if we were in possession of facts that are entirely beyond the reach of our senses. It resembles instinct too in its small liability to error; for though it goes wrong oftener than right, yet the relative frequency with which it is right is on the whole the most wonderful thing in our constitution.

Peirce: CP 5.174 Cross-Ref:††

174. One little remark and I will drop this topic. If you ask an investigator why he does not try this or that wild theory, he will say, "It does not seem *reasonable*." It is curious that we seldom use this word where the strict logic of our procedure is clearly seen. We do [not] say that a mathematical error is not reasonable. We call that opinion reasonable whose only support is instinct. . . .

Peirce: CP 5.175 Cross-Ref:††
§5. THE MEANING OF AN ARGUMENT

175. We have already seen †1 some reason to hold that the idea of *meaning* is

such as to involve some reference to a *purpose*. But Meaning is attributed to representamens alone, and the only kind of representamen which has a definite professed purpose is an "argument." The professed purpose of an argument is to determine an acceptance of its conclusion, and it quite accords with general usage to call the conclusion of an argument its meaning. But I may remark that the word meaning has not hitherto been recognized as a technical term of logic, and in proposing it as such (which I have a right to do since I have a new conception to express, that of the conclusion of an argument as its intended interpretant) I should have a recognized right slightly to warp the acceptation of the word "meaning," so as to fit it for the expression of a scientific conception. It seems natural to use the word *meaning* to denote the intended interpretant of a symbol.

Peirce: CP 5.176 Cross-Ref:††

176. I may presume that you are all familiar with Kant's reiterated insistence that necessary reasoning does nothing but explicate the *meaning* of its premisses.†2 Now Kant's conception of the nature of necessary reasoning is clearly shown by the logic of relations to be utterly mistaken, and his distinction between analytic and synthetic judgments, which he otherwise and better terms *explicatory* (*erläuternde*) and *ampliative* (*erweiternde*) judgments, which is based on that conception, is so utterly confused that it is difficult or impossible to do anything with it. But, nevertheless, I think we shall do very well to accept Kant's dictum that necessary reasoning is merely explicatory of the meaning of the terms of the premisses, only reversing the use to be made of it. Namely instead of adopting the conception of meaning from the Wolffian logicians, as he does, and making use of this dictum to express what necessary reasoning can do, about which he was utterly mistaken, we shall do well to understand necessary reasoning as mathematics and the logic of relations compels us to understand it, and to use the dictum, that necessary reasoning only explicates the meanings of the terms of the premisses, to fix our ideas as to what we shall understand by the *meaning* of a term.

Peirce: CP 5.177 Cross-Ref:††

177. Kant and the logicians with whose writings he was alone acquainted -- he was far from being a thorough student of logic, notwithstanding his great natural power as a logician -- consistently neglected the logic of relations; and the consequence was that the only account they were in condition to give of the meaning of a term, its "signification" as they called it, was that it was composed of all the terms which could be essentially predicated of that term. Consequently, either the analysis of the signification must be capable of [being] pushed on further and further, without limit -- an opinion which Kant †1 expresses in a well-known passage but which he did not develop, or, what was more usual, one ultimately reached certain absolutely simple conceptions such as Being, Quality, Relation, Agency, Freedom, etc., which were regarded as absolutely incapable of definition and of being in the highest degree luminous and clear. It is marvellous what a following this opinion, that those excessively abstracted conceptions were in themselves in the highest degree simple and facile, obtained, notwithstanding its repugnancy to good sense. One of the many important services which the logic of relations has rendered has been that of showing that these so-called simple conceptions, notwithstanding their being unaffected by the particular kind of combination recognized in non-relative logic, are nevertheless capable of analysis in consequence of their implying various modes of relationship. For example, no conceptions are simpler than those of Firstness, Secondness, and Thirdness; but this has not prevented my defining them, and that in a most effective manner, since all the assertions I have made concerning them have

been deduced from those definitions.

Peirce: CP 5.178 Cross-Ref:††

178. Another effect of the neglect of the logic of relations was that Kant imagined that all necessary reasoning was of the type of a syllogism in *Barbara*. Nothing could be more ridiculously in conflict with well-known facts.†2 For had that been the case, any person with a good logical head would be able instantly to see whether a given conclusion followed from given premisses or not; and moreover the number of conclusions from a small number of premisses would be very moderate. Now it is true that when Kant wrote, Legendre and Gauss had not shown what a countless multitude of theorems are deducible from the very few premisses of arithmetic. I suppose we must excuse him, therefore, for not knowing this. But it is difficult to understand what the state of mind on this point could have been of logicians who were at the same time mathematicians, such as Euler, Lambert, and Ploucquet. Euler invented the logical diagrams which go under his name; for the claims that have been made in favor of predecessors may be set down as baseless;†1 and Lambert used an equivalent system.†2 Now I need not say that both of these men were mathematicians of great power. One is simply astounded that they should seem to say that all the reasonings of mathematics could be represented in any such ways. One may suppose that Euler never paid much attention to logic. But Lambert wrote a large book in two volumes on the subject, and a pretty superficial affair it is. One has a difficulty in realizing that the author of it was the same man who came so near to the discovery of the non-Euclidean geometry. The logic of relatives is now able to exhibit in strict logical form the reasoning of mathematics. You will find an example of it -- although too simple a one to put all the features into prominence -- in that chapter †3 of Schröder's logic in which he remodels the reasoning of Dedekind in his brochure *Was sind und was sollen die Zahlen*; and if it be objected that this analysis was chiefly the work of Dedekind who did not employ the machinery of the logic of relations, I reply that Dedekind's whole book is nothing but an elaboration of a paper published by me several years previously in the *American Journal of Mathematics*†4 which paper was the direct result of my logical studies. These analyses show that although most of the steps of the reasoning have considerable resemblance to Barbara, yet the difference of effect is very great indeed.

Peirce: CP 5.179 Cross-Ref:††

179. On the whole, then, if by the *meaning* of a term, proposition, or argument, we understand the entire general intended interpretant, then the meaning of an argument is explicit. It is its conclusion; while the meaning of a proposition or term is all that that proposition or term could contribute to the conclusion of a demonstrative argument. But while this analysis will be found useful, it is by no means sufficient to cut off all nonsense or to enable us to judge of the maxim of pragmatism. What we need is an account of the *ultimate* meaning of a term. To this problem we have to address ourselves.

Peirce: CP 5.180 Cross-Ref:††

LECTURE VII †1

PRAGMATISM AND ABDUCTION

§1. THE THREE COTARY PROPOSITIONS

180. At the end of my last lecture I had just enunciated three propositions which seem to me to give to pragmatism its peculiar character. In order to be able to refer to them briefly this evening, I will call them, for the nonce, my cotary propositions. *Cos, cotis*, is a whetstone. They appear to me to put the edge on the maxim of pragmatism.

Peirce: CP 5.181 Cross-Ref:††

181. These *cotary* propositions are as follows:

Peirce: CP 5.181 Cross-Ref:††

(1) *Nihil est in intellectu quod non prius fuerit in sensu*. I take this in a sense somewhat different from that which Aristotle intended.†2 By *intellectus*, I understand the meaning of any representation in any kind of cognition, virtual, symbolic, or whatever it may be. Berkeley †3 and nominalists of his stripe deny that we have any idea at all of a triangle in general, which is neither equilateral, isosceles, nor scalene. But he cannot deny that there are propositions about triangles in general, which propositions are either true or false; and as long as that is the case, whether we have an *idea* of a triangle in some psychological sense or not, I do not, as a logician, care. We have an *intellectus*, a meaning, of which the triangle in general is an element. As for the other term, *in sensu*, that I take in the sense of *in a perceptual judgment*, the starting point or first premiss of all critical and controlled thinking. I will state presently what I conceive to be the evidence of the truth of this first cotary proposition. But I prefer to begin by recalling to you what all three of them are.

Peirce: CP 5.181 Cross-Ref:††

(2) The second is that perceptual judgments contain general elements, so that universal propositions are deducible from them in the manner in which the logic of relations shows that particular propositions usually, not to say invariably, allow universal propositions to be necessarily inferred from them. This I sufficiently argued in my last lecture. This evening I shall take the truth of it for granted.

Peirce: CP 5.181 Cross-Ref:††

(3) The third cotary proposition is that abductive inference shades into perceptual judgment without any sharp line of demarcation between them; or, in other words, our first premisses, the perceptual judgments, are to be regarded as an extreme case of abductive inferences, from which they differ in being absolutely beyond criticism. The abductive suggestion comes to us like a flash. It is an act of *insight*, although of extremely fallible insight. It is true that the different elements of the hypothesis were in our minds before; but it is the idea of putting together what we had never before dreamed of putting together which flashes the new suggestion before our contemplation.

Peirce: CP 5.181 Cross-Ref:††

On its side, the perceptive judgment is the result of a process, although of a process not sufficiently conscious to be controlled, or, to state it more truly, not

controllable and therefore not fully conscious. If we were to subject this subconscious process to logical analysis, we should find that it terminated in what that analysis would represent as an abductive inference, resting on the result of a similar process which a similar logical analysis would represent to be terminated by a similar abductive inference, and so on *ad infinitum*. This analysis would be precisely analogous to that which the sophism of Achilles and the Tortoise applies to the chase of the Tortoise by Achilles, and it would fail to represent the real process for the same reason. Namely, just as Achilles does not have to make the series of distinct endeavors which he is represented as making, so this process of forming the perceptual judgment, because it is sub-conscious and so not amenable to logical criticism, does not have to make separate acts of inference, but performs its act in one continuous process.

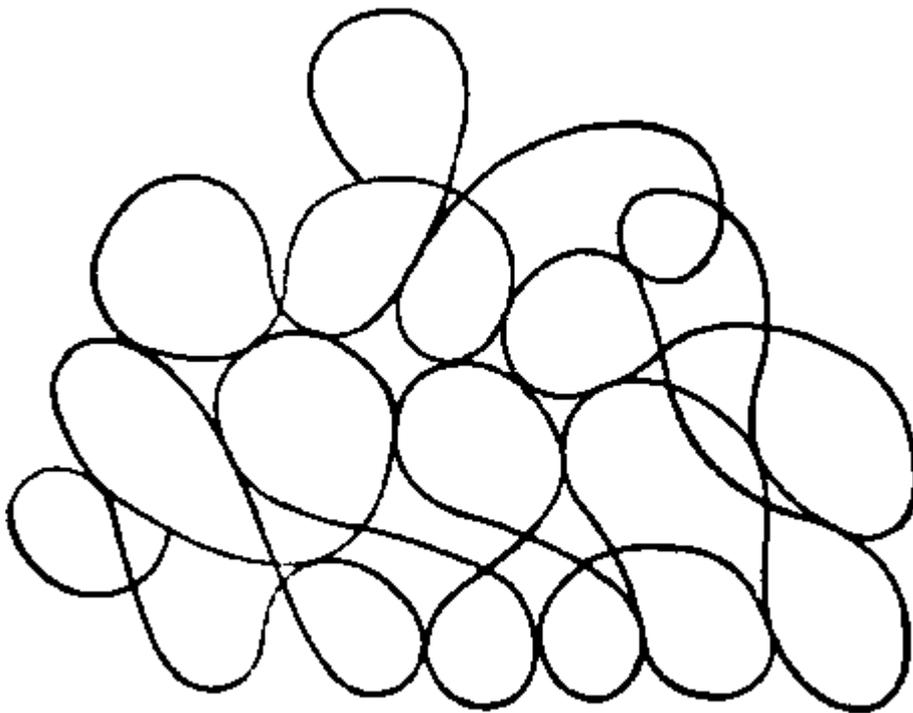
Peirce: CP 5.182 Cross-Ref:††

§2. ABDUCTION AND PERCEPTUAL JUDGMENTS

182. I have already put in my brief in favor of my second cotary proposition, and in what I am about to say I shall treat that as already sufficiently proved. In arguing it I avoided all resort to anything like special phenomena, upon which I do not think that philosophy ought to rest, at all. Still, there is no harm in using special observations merely in an abductive way to throw a light upon doctrines otherwise established, and to aid the mind in grasping them; and there are some phenomena which, I think, do aid us to see what is meant by asserting that perceptual judgments contain general elements, and which will also naturally lead up to a consideration of the third cossal proposition.

Peirce: CP 5.183 Cross-Ref:††

183. I will show you a figure which I remember my father [Benjamin Peirce] drawing in one of his lectures. I do not remember what it was supposed to show; but I cannot imagine what else it could have been but my cotary proposition No. 2. If so, in maintaining that proposition I am substantially treading in his footsteps, though he would doubtless have put the proposition into a shape very different from mine. Here is the figure (though I cannot draw it as skillfully as he did). It consists of a serpentine line. But when it is completely drawn, it appears to be a stone wall.



[Click here to view] The point is that there are two ways of conceiving the matter. Both, I beg you to remark, are *general ways of classing the line*, general classes under which the line is subsumed. But the very decided preference of our perception for one mode of classing the percept shows that this classification is contained in the perceptual judgment. So it is with that well-known unshaded outline figure of a pair of steps seen in perspective. We seem at first to be looking at the steps from above; but some unconscious part of the mind seems to tire of putting that construction upon it and suddenly we seem to see the steps from below, and so the perceptive judgment, and the percept itself, seems to keep shifting from one general aspect to the other and back again.

Peirce: CP 5.183 Cross-Ref:††

In all such visual illusions of which two or three dozen are well known, the most striking thing is that a certain theory of interpretation of the figure has all the appearance of being given in perception. The first time it is shown to us, it seems as completely beyond the control of rational criticism as any percept is; but after many repetitions of the now familiar experiment, the illusion wears off, becoming first less decided, and ultimately ceasing completely. This shows that these phenomena are true connecting links between abductions and perceptions.

Peirce: CP 5.184 Cross-Ref:††

184. If the percept or perceptual judgment were of a nature entirely unrelated to abduction, one would expect that the percept would be entirely free from any characters that are proper to *interpretations*, while it can hardly fail to have such characters if it be merely a continuous series of what, discretely and consciously performed, would be abductions. We have here then almost a crucial test of my third cotary proposition. Now, then, how is the fact? The fact is that it is not necessary to go beyond ordinary observations of common life to find a variety of widely different

ways in which perception is interpretative.

Peirce: CP 5.185 Cross-Ref:††

185. The whole series of hypnotic phenomena, of which so many fall within the realm of ordinary everyday observation -- such as our waking up at the hour we wish to wake much nearer than our waking selves could guess it -- involve the fact that we perceive what we are adjusted for interpreting, though it be far less perceptible than any express effort could enable us to perceive; while that, to the interpretation of which our adjustments are not fitted, we fail to perceive although it exceed in intensity what we should perceive with the utmost ease, if we cared at all for its interpretation. It is a marvel to me that the clock in my study strikes every half hour in the most audible manner, and yet I never hear it. I should not know at all whether the striking part were going, unless it is out of order and strikes the wrong hour. If it does that, I am pretty sure to hear it. Another familiar fact is that we perceive, or seem to perceive, objects differently from how they really are, accommodating them to their manifest intention. Proofreaders get high salaries because ordinary people miss seeing misprints, their eyes correcting them. We can repeat the *sense* of a conversation, but we are often quite mistaken as to what words were uttered. Some politicians think it a clever thing to convey an idea which they carefully abstain from stating in words. The result is that a reporter is ready to swear quite sincerely that a politician said something to him which the politician was most careful not to say.

Peirce: CP 5.185 Cross-Ref:††

I should tire you if I dwelt further on anything so familiar, especially to every psychological student, as the interpretativeness of the perceptive judgment. It is plainly nothing but the extremest case of Abductive Judgments.

Peirce: CP 5.186 Cross-Ref:††

186. If this third cotary proposition be admitted, the second, that the perceptual judgment contains general elements, must be admitted; and as for the first, that all general elements are given in perception, that loses most of its significance. For if a general element were given otherwise than in the perceptual judgment, it could only first appear in an abductive suggestion, and that is now seen to amount substantially to the same thing. I not only opine, however, that every general element of every hypothesis, however wild or sophisticated it may be, [is] given somewhere in perception, but I will venture so far as to assert that every general *form* of putting concepts together is, in its elements, given in perception. In order to decide whether this be so or not, it is necessary to form a clear notion of the precise difference between abductive judgment and the perceptual judgment which is its limiting case. The only symptom by which the two can be distinguished is that we cannot form the least conception of what it would be to deny the perceptual judgment. If I judge a perceptual image to be red, I can conceive of another man's not having that same percept. I can also conceive of his having this percept but never having thought whether it was red or not. I can conceive that while colors are among his sensations, he shall never have had his attention directed to them. Or I can conceive that, instead of redness, a somewhat different conception should arise in his mind; that he should, for example, judge that this percept has a warmth of color. I can imagine that the redness of my percept is excessively faint and dim so that one can hardly make sure whether it is red or not. But that any man should have a percept similar to mine and should ask himself the question whether this percept be *red*, which would imply that he had already judged *some* percept to be red, and that he should, upon careful

attention to this percept, pronounce it to be decidedly and clearly *not* red, when I judge it to be prominently red, *that* I cannot comprehend at all. An abductive suggestion, however, is something whose truth *can* be questioned or even denied.

Peirce: CP 5.187 Cross-Ref:††

187. We thus come to the *test of inconceivability* as the only means of distinguishing between an abduction and a perceptual judgment. Now I fully assent to all that Stuart Mill so forcibly said in his *Examination of Hamilton* as to the utter untrustworthiness of the test of inconceivability.†1 That which is inconceivable to us today, may prove tomorrow to be conceivable and even probable; so that we never can be absolutely sure that a judgment is perceptual and not abductive; and this may seem to constitute a difficulty in the way of satisfying ourselves that the first cotary proposition is true.

Peirce: CP 5.187 Cross-Ref:††

I should easily show you that this difficulty, however formidable theoretically, amounts practically to little or nothing for a person skilled in shaping such inquiries. But this is unnecessary, since the objection founded upon it has no logical force whatever.

Peirce: CP 5.188 Cross-Ref:††

188. No doubt, in regard to the first cotary proposition, [that proposition] follows as a necessary consequence of the possibility that what are really abductions have been mistaken for perceptions. For the question is whether that which really is an abductive result can contain elements foreign to its premisses. It must be remembered that abduction, although it is very little hampered by logical rules, nevertheless is logical inference, asserting its conclusion only problematically or conjecturally, it is true, but nevertheless having a perfectly definite logical form.

Peirce: CP 5.189 Cross-Ref:††

189. Long before I first classed abduction as an inference it was recognized by logicians that the operation of adopting an explanatory hypothesis -- which is just what abduction is -- was subject to certain conditions. Namely, the hypothesis cannot be admitted, even as a hypothesis, unless it be supposed that it would account for the facts or some of them. The form of inference, therefore, is this:

The surprising fact, C, is observed;

But if A were true, C would be a matter of course,

Hence, there is reason to suspect that A is true.

Peirce: CP 5.189 Cross-Ref:††

Thus, A cannot be abductively inferred, or if you prefer the expression, cannot be abductively conjectured until its entire content is already present in the premiss, "If A were true, C would be a matter of course."

Peirce: CP 5.190 Cross-Ref:††

190. Whether this be a correct account of the matter or not, the mere suggestion of it as a possibility shows that the bare fact that abductions may be mistaken for perceptions does not necessarily affect the force of an argument to show

[that] quite new conceptions cannot be obtained from abduction.

Peirce: CP 5.191 Cross-Ref:††

191. But when the account just given of abduction is proposed as a proof that all conceptions must be given substantially in perception, three objections will be started. Namely, in the first place, it may be said that even if this be the normative form of abduction, the form to which abduction *ought* to conform, yet it may be that new conceptions arise in a manner which puts the rules of logic at defiance. In the second place, waiving this objection, it may be said that the argument would prove too much; for if it were valid, it would follow that no hypothesis could be so fantastic as not to have presented itself entire in experience. In the third place, it may be said that granting that the abductive conclusion, "A is true" rests upon the premiss, "If A is true, C is true," still it would be contrary to common knowledge to assert that the antecedents of all conditional judgments are given in perception, and thus it remains almost certain that some conceptions have a different origin.

Peirce: CP 5.192 Cross-Ref:††

192. In answer to the first of these objections, it is to be remarked that it is only in deduction that there is no difference between a *valid* argument and a *strong* one. An argument is valid if it possesses the sort of strength that it professes and tends toward the establishment of the conclusion in the way in which it pretends to do this. But the question of its strength does not concern the comparison of the due effect of the argument with its pretensions, but simply upon how great its due effect is. An argument is none the less logical for being weak, provided it does not pretend to a strength that it does not possess. It is, I suppose, in view of this that the best modern logicians outside the English school never say a word about fallacies. They assume that there is no such thing as an argument illogical in itself. An argument is fallacious only so far as it is mistakenly, though not illogically, inferred to have professed what it did not perform. Perhaps it may be said that if all our reasonings conform to the laws of logic, this is, at any rate, nothing but a proposition in psychology which my principles ought to forbid my recognizing. But I do not offer it as a principle of psychology only. For a principle of psychology is a contingent truth, while this, as I contend, is a necessary truth. Namely, if a fallacy involves nothing in its conclusion which was not in its premisses, that is nothing that was not in any previous knowledge that aided in suggesting it, then the forms of logic will invariably and necessarily enable us logically to account for it as due to a mistake arising from the use of a logical but weak argumentation.†1 In most cases it is due to an abduction. The conclusion of an abduction is problematic or conjectural, but is not necessarily at the weakest grade of surmise, and what we call assertoric judgments are, accurately, problematic judgments of a high grade of hopefulness. There is therefore no difficulty in maintaining that fallacies are merely due to mistakes which are logically valid, though weak argumentations. If, however, a fallacy contains something in the conclusion which was not in the premisses at all, that is, was in no previous knowledge or none that influenced the result, then again a mistake, due as before to weak inference, has been committed; only in this case the mistake consists in taking that to be an inference which, in respect to this new element, is not an inference, at all. That part of the conclusion which inserts the wholly new element can be separated from the rest with which it has no logical connection nor appearance of logical connection. The first emergence of this new element into consciousness must be regarded as a perceptive judgment. We are irresistibly led to judge that we are conscious of it. But the connection of this perception with other elements must be an ordinary logical inference, subject to error like all inference.

Peirce: CP 5.193 Cross-Ref:††

193. As for the second objection that, according to my account of abduction, every hypothesis, however fantastic, must have presented itself entire in perception, I have only to say that this could only arise in a mind entirely unpractised in the logic of relations, and apparently quite oblivious of any other mode of inference than abduction. Deduction accomplishes first the simple colligation of different perceptive judgments into a copulative whole, and then, with or without the aid of other modes of inference, is quite capable of transforming this copulative proposition so as to bring certain of its parts into more intimate connection.

Peirce: CP 5.194 Cross-Ref:††

194. But the third objection is the really serious one. In it lies the whole nodus of the question; and its full refutation would be quite a treatise. If the antecedent is not given in a perceptive judgment, then it must first emerge in the conclusion of an inference. At this point we are obliged to draw the distinction between the matter and the logical form. With the aid of the logic of relations it would be easy to show that the entire logical matter of a conclusion must in any mode of inference be contained, piecemeal, in the premisses. Ultimately therefore it must come from the uncontrolled part of the mind, because a series of controlled acts must have a first. But as to the logical *form*, it would be, at any rate, extremely difficult to dispose of it in the same way. An induction, for example, concludes a ratio of frequency; but there is nothing about any such ratio in the single instances on which it is based. Where do the conceptions of deductive necessity, of inductive probability, of abductive expectability come from? Where does the conception of inference itself come from? That is the only difficulty. But self-control is the character which distinguishes reasonings from the processes by which perceptual judgments are formed, and self-control of any kind is purely *inhibitory*. It originates nothing. Therefore it cannot be in the act of adoption of an inference, in the pronouncing of it to be reasonable, that the formal conceptions in question can first emerge. It must be in the first perceiving that so one might conceivably reason. And what is the nature of that? I see that I have instinctively described the phenomenon as a "perceiving." I do not wish to argue from words; but a word may furnish a valuable suggestion. What can our first acquaintance with an inference, when it is not yet adopted, be but a perception of the world of ideas? In the first suggestion of it, the inference must be thought of as an inference, because when it is adopted there is always the thought that so one might reason in a whole class of cases. But the mere act of inhibition cannot introduce this conception. The inference must, then, be thought of as an inference in the first suggestion of it. Now when an inference is thought of *as* an inference, the conception of inference becomes a part of the *matter* of thought. Therefore, the same argument which we used in regard to matter in general applies to the conception of inference. But I am prepared to show in detail, and indeed virtually have shown, that all the forms of logic can be reduced to combinations of the conception of inference, the conception of otherness, and the conception of a character.†1 These are obviously simply forms of Thirdness, Secondness, and Firstness of which the last two are unquestionably given in perception. Consequently the whole logical form of thought is so given in its elements.

Peirce: CP 5.195 Cross-Ref:††

§3. PRAGMATISM -- THE LOGIC OF ABDUCTION

195. It appears to me, then, that my three cotary propositions are satisfactorily grounded. Nevertheless, since others may not regard them as so certain as I myself do, I propose in the first instance to disregard them, and to show that, even if they are put aside as doubtful, a maxim practically little differing in most of its applications from that of pragmatism ought to be acknowledged and followed; and after this has been done, I will show how the recognition of the cotary propositions will affect the matter. . . .

Peirce: CP 5.196 Cross-Ref:††

196. If you carefully consider the question of pragmatism you will see that it is nothing else than the question of the logic of abduction. That is, pragmatism proposes a certain maxim which, if sound, must render needless any further rule as to the admissibility of hypotheses to rank as hypotheses, that is to say, as explanations of phenomena held as hopeful suggestions; and, furthermore, this is *all* that the maxim of pragmatism really pretends to do, at least so far as it is confined to logic, and is not understood as a proposition in psychology. For the maxim of pragmatism is that a conception can have no logical effect or import differing from that of a second conception except so far as, taken in connection with other conceptions and intentions, it might conceivably modify our practical conduct differently from that second conception. Now it is indisputable that no rule of abduction would be admitted by *any* philosopher which should prohibit on any formalistic grounds any inquiry as to how we ought in consistency to shape our practical conduct. Therefore, a maxim which looks only to possibly practical considerations will not need any supplement in order to exclude any hypotheses as inadmissible. What hypotheses it admits all philosophers would agree ought to be admitted. On the other hand, if it be true that nothing but such considerations has any logical effect or import whatever, it is plain that the maxim of pragmatism cannot cut off any kind of hypothesis which ought to be admitted. Thus, the maxim of pragmatism, if true, fully *covers* the entire logic of abduction. It remains to inquire whether this maxim may not have some *further* logical effect. If so, it must in some way affect inductive or deductive inference. But that pragmatism cannot interfere with induction is evident; because induction simply teaches us what we have to expect as a result of experimentation, and it is plain that any such expectation *may* conceivably concern practical conduct. In a certain sense it *must* affect *deduction*. Anything which gives a rule to abduction and so puts a limit upon admissible hypotheses will cut down *the premisses* of deduction, and thereby will render a *reductio ad absurdum* and other equivalent forms of deduction possible which would not otherwise have been possible. But here three remarks may be made. First, to affect the *premisses* of deduction is not to affect the logic of deduction. For in the process of deduction itself, no conception is introduced to which pragmatism could be supposed to object, except the acts of abstraction. Concerning that I have only time to say that pragmatism ought not to object to it. Secondly, no effect of pragmatism which *is consequent upon its effect on abduction* can go to show that pragmatism is anything more than a doctrine concerning the logic of abduction. Thirdly, if pragmatism is the doctrine that every conception is a conception of conceivable practical effects, it makes conception reach far beyond the practical. It allows any flight of imagination, provided this imagination ultimately alights upon a possible practical effect; and thus many hypotheses may seem at first glance to be excluded by the pragmatism that are not really so excluded.

Peirce: CP 5.197 Cross-Ref:††

197. Admitting, then, that the question of Pragmatism is the question of Abduction, let us consider it under that form. What is good abduction? What should an explanatory hypothesis be to be worthy to rank as a hypothesis? Of course, it must explain the facts. But what other conditions ought it to fulfill to be good? The question of the goodness of anything is whether that thing fulfills its end. What, then, is the end of an explanatory hypothesis? Its end is, through subjection to the test of experiment, to lead to the avoidance of all surprise and to the establishment of a habit of positive expectation that shall not be disappointed. Any hypothesis, therefore, may be admissible, in the absence of any special reasons to the contrary, provided it be capable of experimental verification, and only insofar as it is capable of such verification. This is approximately the doctrine of pragmatism. But just here a broad question opens out before us. What are we to understand by experimental verification? The answer to that involves the whole logic of induction.

Peirce: CP 5.198 Cross-Ref:††

198. Let me point out to you the different opinions which we actually find men holding today -- perhaps not consistently, but thinking that they hold them -- upon this subject. In the first place, we find men who maintain that no hypothesis ought to be admitted, even as a hypothesis, any further than its truth or its falsity is *capable* of being directly perceived. This, as well as I can make out, is what was in the mind of Auguste Comte,†1 who is generally assumed to have first formulated this maxim. Of course, this maxim of abduction supposes that, as people say, we "are to believe only what we actually see"; and there are well-known writers, and writers of no little intellectual force, who maintain that it is unscientific to make predictions -- unscientific, therefore, to expect anything. One ought to restrict one's opinions to what one actually perceives. I need hardly say that that position cannot be consistently maintained. It refutes itself, for it is *itself* an opinion relating to more than is actually in the field of momentary perception.

Peirce: CP 5.199 Cross-Ref:††

199. In the second place, there are those who hold that a theory which has sustained a number of experimental tests may be expected to sustain a number of other similar tests, and to have a general approximate truth, the justification of this being that this kind of inference must prove correct in the long run, as I explained in a previous lecture.†2 But these logicians refuse to admit that we can ever have a right to conclude definitely that a hypothesis is *exactly* true, that is that it should be able to sustain experimental tests in endless series; for, they urge, no hypothesis can be subjected to an endless series of tests. They are willing we should say that a theory is true, because, all our ideas being more or less vague and approximate, what we mean by saying that a theory is true can only be that it is very near true. But they will not allow us to say that anything put forth as an anticipation of experience should assert exactitude, because exactitude in experience would imply experiences in endless series, which is impossible.

Peirce: CP 5.200 Cross-Ref:††

200. In the third place, the great body of scientific men hold that it is too much to say that induction must be restricted to that for which there can be *positive* experimental evidence. They urge that the rationale of induction as it is understood by logicians of the second group, themselves, entitles us to hold a theory, provided it be such that if it involve any falsity, experiment must some day detect that falsity. We, therefore, have a right, they will say, to infer that something *never* will happen,

provided it be of such a nature that it could not occur without being detected.

Peirce: CP 5.201 Cross-Ref:††

201. I wish to avoid in the present lecture *arguing* any such points, because the substance of all sound argumentation about pragmatism has, as I conceive it, been already given in previous lectures, and there is no end to the forms in which it might be stated. I must, however, except from this statement the logical principles which I intend to state in tomorrow evening's lecture on multitude and continuity;† 1 and for the sake of making the relation clear between this third position and the fourth and fifth, I must anticipate a little what I shall further explain tomorrow.

Peirce: CP 5.202 Cross-Ref:††

202. What ought persons, who hold this third position, to say to the Achilles sophism? Or rather . . . what would they be obliged to say to Achilles overtaking the tortoise (Achilles and the tortoise being geometrical points) supposing that our only knowledge was derived inductively from observations of the relative positions of Achilles and the tortoise at those stages of the progress that the sophism supposes, and supposing that Achilles really moves twice as fast as the tortoise? They ought to say that *if* it could not happen that Achilles in one of those stages of his progress should at length reach a certain finite distance behind the tortoise which he would be unable to halve, *without our learning that fact*, then we should have a right to conclude that he could halve *every* distance and consequently that he could make his distance behind the tortoise less than *all* fractions having a power of two for the denominator. Therefore unless these logicians were to suppose a distance less than any measurable distance, which would be contrary to their principles, they would be obliged to say that Achilles could reduce his distance behind the tortoise to *zero*.

Peirce: CP 5.203 Cross-Ref:††

203. The reason why it would be contrary to their principles to admit any distance less than a measurable distance, is that their way of supporting induction implies that they differ from the logicians of the second class, in that these third class logicians admit that we can infer a proposition implying an infinite multitude and therefore implying the reality of the infinite multitude itself, while their mode of justifying induction would exclude every infinite multitude except the lowest grade, that of the multitude of all integer numbers. Because with reference to a greater multitude than that, it would not be true that what did not occur in a finite ordinal place in a series could not occur anywhere within the infinite series -- which is the only reason they admit for the inductive conclusion.

Peirce: CP 5.203 Cross-Ref:††

But now let us look at something else that those logicians would be obliged to admit. Namely, suppose any regular polygon to have all its vertices joined by straight radii to its centre. Then if there were any particular finite number of sides for a regular polygon with radii so drawn, which had the singular property that it should be impossible to bisect all the angles by new radii equal to the others and by connecting the extremities of each new radius to those of the two adjacent old radii to make a new polygon of double the number of angles -- if, I say, there were any *finite number* of sides for which this could not be done -- it may be admitted that we should be able to find it out. The question I am asking supposes arbitrarily that they admit that. Therefore these logicians of the third class would have to admit that all such polygons could so have their sides doubled and that consequently there would be a polygon of an infinite multitude of sides which could be, on their principles, nothing else than the circle. But it is easily proved that the perimeter of that polygon, that is, the

circumference of the circle, would be incommensurable, so that an incommensurable measure is real, and thence it easily follows that all such lengths are real or possible. But these exceed in multitude the only multitude those logicians admit. Without any geometry, the same result could be reached, supposing only that we have an indefinitely bisectible quantity.

Peirce: CP 5.204 Cross-Ref:††

204. We are thus led to a fourth opinion very common among mathematicians, who generally hold that any one irrational real quantity (say of length, for example) whether algebraical or transcendental in its general expression, is just as possible and admissible as any rational quantity, but who generally reason that if the distance between two points is less than any assignable quantity, that is, less than any finite quantity, then it is nothing at all. If that be the case, it is possible for us to conceive, with mathematical precision, a state of things in favor of whose actual reality there would seem to be no possible sound argument, however weak. For example, we can conceive that the diagonal of a square is incommensurable with its side. That is to say, if you first name any length commensurable with the side, the diagonal will differ from that by a finite quantity (and a commensurable quantity), yet however accurately we may measure the diagonal of an apparent square, there will always be a limit to our accuracy and the measure will always be commensurable. So we never could have any reason to think it otherwise. Moreover, if there be, as they seem to hold, no other points on a line than such as are at distances assignable to an indefinite approximation, it will follow that if a line has an extremity, that extreme point may be conceived to be taken away so as to leave the line without any extremity, while leaving all the other points just as they were. In that case, all the points stand discrete and separate; and the line might be torn apart at any number of places without disturbing the relations of the points to one another. Each point has, on that view, its own independent existence, and there can be no merging of one into another. There is no continuity of points in the sense in which continuity implies generality.

Peirce: CP 5.205 Cross-Ref:††

205. In the fifth place it may be held that we can be justified in inferring true generality, true continuity. But I do not see in what way we ever can be justified in doing so unless we admit the cotary propositions, and in particular that such continuity is given in perception; that is, that whatever the underlying psychical process may be, we seem to perceive a genuine flow of time, such that instants melt into one another without separate individuality.

Peirce: CP 5.205 Cross-Ref:††

It would not be necessary for me to deny a psychical theory which should make this to be illusory, in such [a] sense as [one might say] that anything beyond all logical criticism is illusory, but I confess I should strongly suspect that such a psychological theory involved a logical inconsistency; and at best it could do nothing at all toward solving the logical question.

Peirce: CP 5.206 Cross-Ref:††

§4. THE TWO FUNCTIONS OF PRAGMATISM

206. There are two functions which we may properly require that Pragmatism should perform; or if not pragmatism, whatever the true doctrine of the Logic of

Abduction may be, ought to do these two services.

Peirce: CP 5.206 Cross-Ref:††

Namely, it ought, in the first place, to give us an expeditious riddance of all ideas essentially unclear. In the second place, it ought to lend support, and help to render distinct, ideas essentially clear, but more or less difficult of apprehension; and in particular, it ought to take a satisfactory attitude toward the element of thirdness.

Peirce: CP 5.207 Cross-Ref:††

207. Of these two offices of Pragmatism, there is at the present day not so crying a need of the first as there was a quarter of a century ago when I enunciated the maxim. The state of logical thought is very much improved. Thirty years ago †1 when, in consequence of my study of the logic of relations, I told philosophers that all conceptions ought to be defined, with the sole exception of the familiar concrete conceptions of everyday life, my opinion was considered in every school to be utterly incomprehensible. The doctrine then was, as it remains in nineteen out of every score of logical treatises that are appearing in these days, that there is no way of defining a term except by enumerating all its universal predicates, each of which is more abstracted and general than the term defined. So unless this process can go on endlessly, which was a doctrine little followed, the explication of a concept must stop at such ideas as Pure Being, Agency, Substance and the like, which were held to be ideas so perfectly simple that no explanation whatever could be given of them. This grotesque doctrine was shattered by the logic of relations, which showed that the simplest conceptions, such as Quality, Relation, Self-consciousness could be defined and that such definitions would be of the greatest service in dealing with them.†1 By this time, although few really study the logic of relations, one seldom meets with a philosopher who continues to think the most general relations are particularly simple in any except a technical sense; and of course, the only alternative is to regard as the simplest the practically applied notions of familiar life. We should hardly find today a man of Kirchhoff's rank in science saying that we know exactly what energy *does* but what energy *is* we do not know in the least.†2 For the answer would be that energy being a term in a dynamical equation, if we know how to apply that equation, we thereby know what energy is, although we may suspect that there is some more fundamental law underlying the laws of motion.

Peirce: CP 5.208 Cross-Ref:††

208. In the present situation of philosophy, it is far more important that thirdness should be adequately dealt with by our logical maxim of abduction. The urgent pertinence of the question of thirdness, at this moment of the breakup of agnostic calm, when we see that the chief difference between philosophers is in regard to the extent to which they allow elements of thirdness a place in their theories, is too plain to be insisted upon.

Peirce: CP 5.209 Cross-Ref:††

209. I shall take it for granted that as far as *thought* goes, I have sufficiently shown that thirdness is an element not reducible to secondness and firstness. But even if so much be granted, three attitudes may be taken:

Peirce: CP 5.209 Cross-Ref:††

(1) That thirdness, though an element of the mental phenomenon, ought not to be admitted into a theory of the real, because it is not experimentally verifiable;

Peirce: CP 5.209 Cross-Ref:††

(2) That thirdness is experimentally verifiable, that is, is inferable by induction, [abduction?] although it cannot be directly perceived;

Peirce: CP 5.209 Cross-Ref:††

(3) That it is directly perceived, from which the other cotary propositions can hardly be separated.

Peirce: CP 5.210 Cross-Ref:††

210. The man who takes the first position ought to admit no general law as really operative. Above all, therefore, he ought not to admit the law of laws, the law of the uniformity of nature. He ought to abstain from all prediction, however qualified by a confession of fallibility. But that position can practically not be maintained.

Peirce: CP 5.211 Cross-Ref:††

211. The man who takes the second position will hold thirdness to be an addition which the operation of abduction introduces over and above what its premisses in any way contain, and further that this element, though not perceived in experiment, is justified *by* experiment. Then his conception of reality must be such as completely to sunder the real from perception; and the puzzle for him will be why perception should be allowed such authority in regard to what is real.

Peirce: CP 5.211 Cross-Ref:††

I do not think that man can consistently hold that there is room in time for an event between any two events separate in time. But even if he could, he would (if he could grasp the reasons) be forced to acknowledge that the contents of time consists of separate, independent, unchanging states, and nothing else. There would not be even a determinate order of sequence among these states. He might insist that one order of sequence was more readily grasped by us; but nothing more. Every man is fully satisfied that there is such a thing as truth, or he would not ask any question. *That* truth consists in a conformity to something *independent of his thinking it to be so*, or of any man's opinion on that subject. But for the man who holds this second opinion, the only reality, there could be, would be conformity to the ultimate result of inquiry. But there would not be any course of inquiry possible except in the sense that it would be easier for him to interpret the phenomenon; and ultimately he would be forced to say that there was no reality at all except that he now at this instant finds a certain way of thinking easier than any other. But that violates the very idea of reality and of truth.

Peirce: CP 5.212 Cross-Ref:††

212. The man who takes the third position and accepts the cotary propositions will hold, with firmest of grasps, to the recognition that logical criticism is limited to what we can control. In the future we may be able to control more but we must consider what we can now control. Some elements we can control in some limited measure. But the content of the perceptual judgment cannot be sensibly controlled now, nor is there any rational hope that it ever can be. Concerning that quite uncontrolled part of the mind, logical maxims have as little to do as with the growth of hair and nails. We may be dimly able to see that, in part, it depends on the accidents of the moment, in part on what is personal or racial, in part is common to all nicely adjusted organisms whose equilibrium has narrow ranges of stability, in part on whatever is composed of vast collections of independently variable elements, in part on whatever reacts, and in part on whatever has any mode of being. But the sum of it all is that our logically controlled thoughts compose a small part of the mind, the mere blossom of a vast complexus, which we may call the instinctive mind, in which this

man will not say that he has *faith*, because that implies the conceivability of distrust, but upon which he builds as the very fact to which it is the whole business of his logic to be true.

Peirce: CP 5.212 Cross-Ref:††

That he will have no difficulty with Thirdness is clear enough, because he will hold that the conformity of action to general intentions is as much given in perception as is the element of action itself, which cannot really be mentally torn away from such general purposiveness. There can be no doubt that he will allow hypotheses fully all the range they ought to be allowed. The only question will be whether he succeeds in excluding from hypotheses everything unclear and nonsensical. It will be asked whether he will not have a shocking leaning toward anthropomorphic conceptions. I fear I must confess that he will be inclined to see an anthropomorphic, or even a zoöomorphic, if not a physiomorphic element in all our conceptions. But against unclear and nonsensical hypotheses, [of] whatever ægis [he will be protected]. Pragmatism will be more essentially significant for him than for any other logician, for the reason that it is in action that logical energy returns to the uncontrolled and uncriticizable parts of the mind. His maxim will be this:

Peirce: CP 5.212 Cross-Ref:††

The elements of every concept enter into logical thought at the gate of perception and make their exit at the gate of purposive action; and whatever cannot show its passports at both those two gates is to be arrested as unauthorized by reason.

Peirce: CP 5.212 Cross-Ref:††

The digestion of such thoughts is slow, ladies and gentlemen; but when you come in the future to reflect upon all that I have said, I am confident you will find the seven hours, you have spent in listening to these ideas, have not been altogether wasted.

Peirce: CP 5 Book 2 Question 1
BOOK II

PUBLISHED PAPERS

I

QUESTIONS CONCERNING CERTAIN FACULTIES CLAIMED FOR MAN†1

QUESTION 1. *Whether by the simple contemplation of a cognition, independently of any previous knowledge and without reasoning from signs, we are enabled rightly to judge whether that cognition has been determined by a previous cognition or whether it refers immediately to its object.*

Peirce: CP 5.213 Cross-Ref:††

213. Throughout this paper, the term *intuition* will be taken as signifying a cognition not determined by a previous cognition of the same object, and therefore so determined by something out of the consciousness.†P1 Let me request the reader to note this. *Intuition* here will be nearly the same as "premiss not itself a conclusion"; the only difference being that premisses and conclusions are judgments, whereas an intuition may, as far as its definition states, be any kind of cognition whatever. But just as a conclusion (good or bad) is determined in the mind of the reasoner by its premiss, so cognitions not judgments may be determined by previous cognitions; and a cognition not so determined, and therefore determined directly by the transcendental object, is to be termed an *intuition*.

Peirce: CP 5.214 Cross-Ref:††

214. Now, it is plainly one thing to have an intuition and another to know intuitively that it is an intuition, and the question is whether these two things, distinguishable in thought, are, in fact, invariably connected, so that we can always intuitively distinguish between an intuition and a cognition determined by another. Every cognition, as something present, is, of course, an intuition of itself. But the determination of a cognition by another cognition or by a transcendental object is not, at least so far as appears obviously at first, a part of the immediate content of that cognition, although it would appear to be an element of the action or passion of the transcendental *ego*, which is not, perhaps, in consciousness immediately; and yet this transcendental action or passion may invariably determine a cognition of itself, so that, in fact, the determination or non-determination of the cognition by another may be a part of the cognition. In this case, I should say that we had an intuitive power of distinguishing an intuition from another cognition.

Peirce: CP 5.214 Cross-Ref:††

There is no evidence that we have this faculty, except that we seem to *feel* that we have it. But the weight of that testimony depends entirely on our being supposed to have the power of distinguishing in this feeling whether the feeling be the result of education, old associations, etc., or whether it is an intuitive cognition; or, in other words, it depends on presupposing the very matter testified to. Is this feeling infallible? And is this judgment concerning it infallible, and so on, *ad infinitum*? Supposing that a man really could shut himself up in such a faith, he would be, of course, impervious to the truth, "evidence-proof."

Peirce: CP 5.215 Cross-Ref:††

215. But let us compare the theory with the historic facts. The power of intuitively distinguishing intuitions from other cognitions has not prevented men from disputing very warmly as to which cognitions are intuitive. In the middle ages, reason and external authority were regarded as two coördinate sources of knowledge, just as reason and the authority of intuition are now; only the happy device of considering the enunciations of authority to be essentially indemonstrable had not yet been hit upon. All authorities were not considered as infallible, any more than all reasons; but when Berengarius said that the authoritativeness of any particular authority must rest upon reason, the proposition was scouted as opinionated, impious, and absurd.†1 Thus, the credibility of authority was regarded by men of that time simply as an ultimate premiss, as a cognition not determined by a previous cognition of the same object, or, in our terms, as an intuition. It is strange that they should have thought so,

if, as the theory now under discussion supposes, by merely contemplating the credibility of the authority, as a Fakir does his God, they could have seen that it was not an ultimate premiss! Now, what if our *internal* authority should meet the same fate, in the history of opinions, as that external authority has met? Can that be said to be absolutely certain which many sane, well-informed, and thoughtful men already doubt?†P1

Peirce: CP 5.216 Cross-Ref:††

216. Every lawyer knows how difficult it is for witnesses to distinguish between what they have seen and what they have inferred. This is particularly noticeable in the case of a person who is describing the performances of a spiritual medium or of a professed juggler. The difficulty is so great that the juggler himself is often astonished at the discrepancy between the actual facts and the statement of an intelligent witness who has not understood the trick. A part of the very complicated trick of the Chinese rings consists in taking two solid rings linked together, talking about them as though they were separate -- taking it for granted, as it were -- then pretending to put them together, and handing them immediately to the spectator that he may see that they are solid. The art of this consists in raising, at first, the strong suspicion that one is broken. I have seen McAlister do this with such success, that a person sitting close to him, with all his faculties straining to detect the illusion, would have been ready to swear that he saw the rings put together, and, perhaps, if the juggler had not professedly practised deception, would have considered a doubt of it as a doubt of his own veracity. This certainly seems to show that it is not always very easy to distinguish between a premiss and a conclusion, that we have no infallible power of doing so, and that in fact our only security in difficult cases is in some signs from which we can infer that a given fact must have been seen or must have been inferred. In trying to give an account of a dream, every accurate person must often have felt that it was a hopeless undertaking to attempt to disentangle waking interpretations and fillings out from the fragmentary images of the dream itself.

Peirce: CP 5.217 Cross-Ref:††

217. The mention of dreams suggests another argument. A dream, as far as its own content goes, is exactly like an actual experience. It is mistaken for one. And yet all the world believes that dreams are determined, according to the laws of the association of ideas, etc., by previous cognitions. If it be said that the faculty of intuitively recognizing intuitions is asleep, I reply that this is a mere supposition, without other support. Besides, even when we wake up, we do not find that the dream differed from reality, except by certain *marks*, darkness and fragmentariness. Not unfrequently a dream is so vivid that the memory of it is mistaken for the memory of an actual occurrence.

Peirce: CP 5.218 Cross-Ref:††

218. A child has, as far as we know, all the perceptive powers of a man. Yet question him a little as to how he knows what he does. In many cases, he will tell you that he never learned his mother-tongue; he always knew it, or he knew it as soon as he came to have sense. It appears, then, that *he* does not possess the faculty of distinguishing, by simple contemplation, between an intuition and a cognition determined by others.

Peirce: CP 5.219 Cross-Ref:††

219. There can be no doubt that before the publication of Berkeley's book on Vision,†1 it had generally been believed that the third dimension of space was immediately intuited, although, at present, nearly all admit that it is known by

inference. We had been *contemplating* the object since the very creation of man, but this discovery was not made until we began to *reason* about it.

Peirce: CP 5.220 Cross-Ref:††

220. Does the reader know of the blind spot on the retina? Take a number of this journal, turn over the cover so as to expose the white paper, lay it sideways upon the table before which you must sit, and put two cents upon it, one near the left-hand edge, and the other to the right. Put your left hand over your left eye, and with the right eye look *steadily* at the left-hand cent. Then, with your right hand, move the right-hand cent (which is now plainly seen) *towards* the left hand. When it comes to a place near the middle of the page it will disappear -- you cannot see it without turning your eye. Bring it nearer to the other cent, or carry it further away, and it will reappear; but at that particular spot it cannot be seen. Thus it appears that there is a blind spot nearly in the middle of the retina; and this is confirmed by anatomy. It follows that the space we immediately see (when one eye is closed) is not, as we had imagined, a continuous oval, but is a ring, the filling up of which must be the work of the intellect. What more striking example could be desired of the impossibility of distinguishing intellectual results from intuitional data, by mere contemplation?

Peirce: CP 5.221 Cross-Ref:††

221. A man can distinguish different textures of cloth by feeling; but not immediately, for he requires to move his fingers over the cloth, which shows that he is obliged to compare the sensations of one instant with those of another.

Peirce: CP 5.222 Cross-Ref:††

222. The pitch of a tone depends upon the rapidity of the succession of the vibrations which reach the ear. Each of those vibrations produces an impulse upon the ear. Let a single such impulse be made upon the ear, and we know, experimentally, that it is perceived. There is, therefore, good reason to believe that each of the impulses forming a tone is perceived. Nor is there any reason to the contrary. So that this is the only admissible supposition. Therefore, the pitch of a tone depends upon the rapidity with which certain impressions are successively conveyed to the mind. These impressions must exist previously to any tone; hence, the sensation of pitch is determined by previous cognitions. Nevertheless, this would never have been discovered by the mere contemplation of that feeling.

Peirce: CP 5.223 Cross-Ref:††

223. A similar argument may be urged in reference to the perception of two dimensions of space. This appears to be an immediate intuition. But if we were to *see* immediately an extended surface, our retinas must be spread out in an extended surface. Instead of that, the retina consists of innumerable needles pointing towards the light, and whose distances from one another are decidedly greater than the *minimum visibile*. Suppose each of those nerve-points conveys the sensation of a little colored surface. Still, what we immediately see must even then be, not a continuous surface, but a collection of spots. Who could discover this by mere intuition? But all the analogies of the nervous system are against the supposition that the excitation of a single nerve can produce an idea as complicated as that of a space, however small. If the excitation of no one of these nerve points can immediately convey the impression of space, the excitation of all cannot do so. For, the excitation of each produces some impression (according to the analogies of the nervous system), hence, the sum of these impressions is a necessary condition of any perception produced by the excitation of all; or, in other terms, a perception produced by the excitation of all is determined by the mental impressions produced by the excitation

of every one. This argument is confirmed by the fact that the existence of the perception of space can be fully accounted for by the action of faculties known to exist, without supposing it to be an immediate impression. For this purpose, we must bear in mind the following facts of physio-psychology: 1. The excitation of a nerve does not of itself inform us where the extremity of it is situated. If, by a surgical operation, certain nerves are displaced, our sensations from those nerves do not inform us of the displacement. 2. A single sensation does not inform us how many nerves or nerve-points are excited. 3. We can distinguish between the impressions produced by the excitations of different nerve-points. 4. The differences of impressions produced by different excitations of similar nerve-points are similar. Let a momentary image be made upon the retina. By No. 2, the impression thereby produced will be indistinguishable from what might be produced by the excitation of some conceivable single nerve. It is not conceivable that the momentary excitation of a single nerve should give the sensation of space. Therefore, the momentary excitation of all the nerve-points of the retina cannot, immediately or mediately, produce the sensation of space. The same argument would apply to any unchanging image on the retina. Suppose, however, that the image moves over the retina. Then the peculiar excitation which at one instant affects one nerve-point, at a later instant will affect another. These will convey impressions which are very similar by 4, and yet which are distinguishable by 3. Hence, the conditions for the recognition of a relation between these impressions are present. There being, however, a very great number of nerve-points affected by a very great number of successive excitations, the relations of the resulting impressions will be almost inconceivably complicated. Now, it is a known law of mind, that when phenomena of an extreme complexity are presented, which yet would be reduced to *order* or mediate simplicity by the application of a certain conception, that conception sooner or later arises in application to those phenomena. In the case under consideration, the conception of extension would reduce the phenomena to unity, and, therefore, its genesis is fully accounted for. It remains only to explain why the previous cognitions which determine it are not more clearly apprehended. For this explanation, I shall refer to a paper upon a new list of categories, Section 5, †P1 merely adding that just as we are able to recognize our friends by certain appearances, although we cannot possibly say what those appearances are and are quite unconscious of any process of reasoning, so in any case when the reasoning is easy and natural to us, however complex may be the premisses, they sink into insignificance and oblivion proportionately to the satisfactoriness of the theory based upon them. This theory of space is confirmed by the circumstance that an exactly similar theory is imperatively demanded by the facts in reference to time. That the course of time should be immediately felt is obviously impossible. For, in that case, there must be an element of this feeling at each instant. But in an instant there is no duration and hence no immediate feeling of duration. Hence, no one of these elementary feelings is an immediate feeling of duration; and, hence the sum of all is not. On the other hand, the impressions of any moment are very complicated -- containing all the images (or the elements of the images) of sense and memory, which complexity is reducible to mediate simplicity by means of the conception of time. †P2

Peirce: CP 5.224 Cross-Ref: ††

224. We have, therefore, a variety of facts, all of which are most readily explained on the supposition that we have no intuitive faculty of distinguishing intuitive from mediate cognitions. Some arbitrary hypothesis may otherwise explain any one of these facts; this is the only theory which brings them to support one another. Moreover, no facts require the supposition of the faculty in question.

Whoever has studied the nature of proof will see, then, that there are here very strong reasons for disbelieving the existence of this faculty. These will become still stronger when the consequences of rejecting it have, in this paper and in a following one, been more fully traced out.

Peirce: CP 5 Book 2 Question 2

QUESTION 2. *Whether we have an intuitive self-consciousness.*

Peirce: CP 5.225 Cross-Ref:††

225. Self-consciousness, as the term is here used, is to be distinguished both from consciousness generally, from the internal sense, and from pure apperception. Any cognition is a consciousness of the object as represented; by self-consciousness is meant a knowledge of ourselves. Not a mere feeling of subjective conditions of consciousness, but of our personal selves. Pure apperception is the self-assertion of THE *ego*; the self-consciousness here meant is the recognition of my *private* self. I know that *I* (not merely *the I*) exist. The question is, how do I know it; by a special intuitive faculty, or is it determined by previous cognitions?

Peirce: CP 5.226 Cross-Ref:††

226. Now, it is not self-evident that we have such an intuitive faculty, for it has just been shown that we have no intuitive power of distinguishing an intuition from a cognition determined by others. Therefore, the existence or non-existence of this power is to be determined upon evidence, and the question is whether self-consciousness can be explained by the action of known faculties under conditions known to exist, or whether it is necessary to suppose an unknown cause for this cognition, and, in the latter case, whether an intuitive faculty of self-consciousness is the most probable cause which can be supposed.

Peirce: CP 5.227 Cross-Ref:††

227. It is first to be observed that there is no known self-consciousness to be accounted for in extremely young children. It has already been pointed out by Kant †P1 that the late use of the very common word "I" with children indicates an imperfect self-consciousness in them, and that, therefore, so far as it is admissible for us to draw any conclusion in regard to the mental state of those who are still younger, it must be against the existence of any self-consciousness in them.

Peirce: CP 5.228 Cross-Ref:††

228. On the other hand, children manifest powers of thought much earlier. Indeed, it is almost impossible to assign a period at which children do not already exhibit decided intellectual activity in directions in which thought is indispensable to their well-being. The complicated trigonometry of vision, and the delicate adjustments of coördinated movement, are plainly mastered very early. There is no reason to question a similar degree of thought in reference to themselves.

Peirce: CP 5.229 Cross-Ref:††

229. A very young child may always be observed to watch its own body with great attention. There is every reason why this should be so, for from the child's point of view this body is the most important thing in the universe. Only what it touches has any actual and present feeling; only what it faces has any actual color; only what is on its tongue has any actual taste.

Peirce: CP 5.230 Cross-Ref:††

230. No one questions that, when a sound is heard by a child, he thinks, not of himself as hearing, but of the bell or other object as sounding. How when he wills to move a table? Does he then think of himself as desiring, or only of the table as fit to be moved? That he has the latter thought, is beyond question; that he has the former, must, until the existence of an intuitive self-consciousness is proved, remain an arbitrary and baseless supposition. There is no good reason for thinking that he is less ignorant of his own peculiar condition than the angry adult who denies that he is in a passion.

Peirce: CP 5.231 Cross-Ref:††

231. The child, however, must soon discover by observation that things which are thus fit to be changed are apt actually to undergo this change, after a contact with that peculiarly important body called Willy or Johnny. This consideration makes this body still more important and central, since it establishes a connection between the fitness of a thing to be changed and a tendency in this body to touch it before it is changed.

Peirce: CP 5.232 Cross-Ref:††

232. The child learns to understand the language; that is to say, a connection between certain sounds and certain facts becomes established in his mind. He has previously noticed the connection between these sounds and the motions of the lips of bodies somewhat similar to the central one, and has tried the experiment of putting his hand on those lips and has found the sound in that case to be smothered. He thus connects that language with bodies somewhat similar to the central one. By efforts, so unenergetic that they should be called rather instinctive, perhaps, than tentative, he learns to produce those sounds. So he begins to converse.

Peirce: CP 5.233 Cross-Ref:††

233. It must be about this time that he begins to find that what these people about him say is the very best evidence of fact. So much so, that testimony is even a stronger mark of fact than *the facts themselves*, or rather than what must now be thought of as the *appearances* themselves. (I may remark, by the way, that this remains so through life; testimony will convince a man that he himself is mad.) A child hears it said that the stove is hot. But it is not, he says; and, indeed, that central body is not touching it, and only what that touches is hot or cold. But he touches it, and finds the testimony confirmed in a striking way. Thus, he becomes aware of ignorance, and it is necessary to suppose a *self* in which this ignorance can inhere. So testimony gives the first dawning of self-consciousness.

Peirce: CP 5.234 Cross-Ref:††

234. But, further, although usually appearances are either only confirmed or merely supplemented by testimony, yet there is a certain remarkable class of appearances which are continually contradicted by testimony. These are those predicates which *we* know to be emotional, but which *he* distinguishes by their connection with the movements of that central person, himself (that the table wants moving, etc.) These judgments are generally denied by others. Moreover, he has reason to think that others, also, have such judgments which are quite denied by all the rest. Thus, he adds to the conception of appearance as the actualization of fact, the conception of it as something *private* and valid only for one body. In short, *error* appears, and it can be explained only by supposing a *self* which is fallible.

Peirce: CP 5.235 Cross-Ref:††

235. Ignorance and error are all that distinguish our private selves from the

absolute *ego* of pure apperception.

Peirce: CP 5.236 Cross-Ref:††

236. Now, the theory which, for the sake of perspicuity, has thus been stated in a specific form, may be summed up as follows: At the age at which we know children to be self-conscious, we know that they have been made aware of ignorance and error; and we know them to possess at that age powers of understanding sufficient to enable them to infer from ignorance and error their own existence. Thus we find that known faculties, acting under conditions known to exist, would rise to self-consciousness. The only essential defect in this account of the matter is, that while we know that children exercise *as much* understanding as is here supposed, we do not know that they exercise it in precisely this way. Still the supposition that they do so is infinitely more supported by facts, than the supposition of a wholly peculiar faculty of the mind.

Peirce: CP 5.237 Cross-Ref:††

237. The only argument worth noticing for the existence of an intuitive self-consciousness is this. We are more certain of our own existence than of any other fact; a premiss cannot determine a conclusion to be more certain than it is itself; hence, our own existence cannot have been inferred from any other fact. The first premiss must be admitted, but the second premiss is founded on an exploded theory of logic. A conclusion cannot be more certain than that some one of the facts which support it is true, but it may easily be more certain than any one of those facts. Let us suppose, for example, that a dozen witnesses testify to an occurrence. Then my belief in that occurrence rests on the belief that each of those men is generally to be believed upon oath. Yet the fact testified to is made more certain than that any one of those men is generally to be believed. In the same way, to the developed mind of man, his own existence is supported by *every other fact*, and is, therefore, incomparably more certain than any one of these facts. But it cannot be said to be more certain than that there is another fact, since there is no doubt perceptible in either case.

Peirce: CP 5.237 Cross-Ref:††

It is to be concluded, then, that there is no necessity of supposing an intuitive self-consciousness, since self-consciousness may easily be the result of inference.

Peirce: CP 5 Book 2 Question 3

QUESTION 3. *Whether we have an intuitive power of distinguishing between the subjective elements of different kinds of cognitions.*

Peirce: CP 5.238 Cross-Ref:††

238. Every cognition involves something represented, or that of which we are conscious, and some action or passion of the self whereby it becomes represented. The former shall be termed the objective, the latter the subjective, element of the cognition. The cognition itself is an intuition of its objective element, which may therefore be called, also, the immediate object. The subjective element is not necessarily immediately known, but it is possible that such an intuition of the subjective element of a cognition of its character, whether that of dreaming, imagining, conceiving, believing, etc., should accompany every cognition. The question is whether this is so.

Peirce: CP 5.239 Cross-Ref:††

239. It would appear, at first sight, that there is an overwhelming array of

evidence in favor of the existence of such a power. The difference between seeing a color and imagining it is immense. There is a vast difference between the most vivid dream and reality. And if we had no intuitive power of distinguishing between what we believe and what we merely conceive, we never, it would seem, could in any way distinguish them; since if we did so by reasoning, the question would arise whether the argument itself was believed or conceived, and this must be answered before the conclusion could have any force. And thus there would be a *regressus ad infinitum*. Besides, if we do not know that we believe, then, from the nature of the case, we do not believe.

Peirce: CP 5.240 Cross-Ref:††

240. But be it noted that we do not intuitively know the existence of this faculty. For it is an intuitive one, and we cannot intuitively know that a cognition is intuitive. The question is, therefore, whether it is necessary to suppose the existence of this faculty, or whether then the facts can be explained without this supposition.

Peirce: CP 5.241 Cross-Ref:††

241. In the first place, then, the difference between what is imagined or dreamed and what is actually experienced, is no argument in favor of the existence of such a faculty. For it is not questioned that there are distinctions in what is present to the mind, but the question is, whether independently of any such distinctions in the immediate *objects* of consciousness, we have any immediate power of distinguishing different modes of consciousness. Now, the very fact of the immense difference in the immediate objects of sense and imagination, sufficiently accounts for our distinguishing those faculties; and instead of being an argument in favor of the existence of an intuitive power of distinguishing the subjective elements of consciousness, it is a powerful reply to any such argument, so far as the distinction of sense and imagination is concerned.

Peirce: CP 5.242 Cross-Ref:††

242. Passing to the distinction of belief and conception, we meet the statement that the knowledge of belief is essential to its existence. Now, we can unquestionably distinguish a belief from a conception, in most cases, by means of a peculiar feeling of conviction; and it is a mere question of words whether we define belief as that judgment which is accompanied by this feeling, or as that judgment from which a man will act. We may conveniently call the former *sensational*, the latter *active*, belief. That neither of these necessarily involves the other, will surely be admitted without any recital of facts. Taking belief in the sensational sense, the intuitive power of reorganizing it will amount simply to the capacity for the sensation which accompanies the judgment. This sensation, like any other, is an object of consciousness; and therefore the capacity for it implies no intuitive recognition of subjective elements of consciousness. If belief is taken in the active sense, it may be discovered by the observation of external facts and by inference from the sensation of conviction which usually accompanies it.

Peirce: CP 5.243 Cross-Ref:††

243. Thus, the arguments in favor of this peculiar power of consciousness disappear, and the presumption is again against such a hypothesis. Moreover, as the immediate objects of any two faculties must be admitted to be different, the facts do not render such a supposition in any degree necessary.

Peirce: CP 5 Book 2 Question 4

QUESTION 4. *Whether we have any power of introspection, or whether our whole knowledge of the internal world is derived from the observation of external facts.*

Peirce: CP 5.244 Cross-Ref:††

244. It is not intended here to assume the reality of the external world. Only, there is a certain set of facts which are ordinarily regarded as external, while others are regarded as internal. The question is whether the latter are known otherwise than by inference from the former. By introspection, I mean a direct perception of the internal world, but not necessarily a perception of it *as* internal. Nor do I mean to limit the signification of the word to intuition, but would extend it to any knowledge of the internal world not derived from external observation.

Peirce: CP 5.245 Cross-Ref:††

245. There is one sense in which any perception has an internal object, namely, that every sensation is partly determined by internal conditions. Thus, the sensation of redness is as it is, owing to the constitution of the mind; and in this sense it is a sensation of something internal. Hence, we may derive a knowledge of the mind from a consideration of this sensation, but that knowledge would, in fact, be an inference from redness as a predicate of something external. On the other hand, there are certain other feelings -- the emotions, for example -- which appear to arise in the first place, not as predicates at all, and to be referable to the mind alone. It would seem, then, that by means of these, a knowledge of the mind may be obtained, which is not inferred from any character of outward things. The question is whether this is really so.

Peirce: CP 5.246 Cross-Ref:††

246. Although introspection is not necessarily intuitive, it is not self-evident that we possess this capacity; for we have no intuitive faculty of distinguishing different subjective modes of consciousness. The power, if it exists, must be known by the circumstance that the facts cannot be explained without it.

Peirce: CP 5.247 Cross-Ref:††

247. In reference to the above argument from the emotions, it must be admitted that if a man is angry, his anger implies, in general, no determinate and constant character in its object. But, on the other hand, it can hardly be questioned that there is some relative character in the outward thing which makes him angry, and a little reflection will serve to show that his anger consists in his saying to himself, "this thing is vile, abominable, etc." and that it is rather a mark of returning reason to say, "I am angry." In the same way any emotion is a predication concerning some object, and the chief difference between this and an objective intellectual judgment is that while the latter is relative to human nature or to mind in general, the former is relative to the particular circumstances and disposition of a particular man at a particular time. What is here said of emotions in general, is true in particular of the sense of beauty and of the moral sense. Good and bad are feelings which first arise as predicates, and therefore are either predicates of the not-I, or are determined by previous cognitions (there being no intuitive power of distinguishing subjective elements of consciousness).

Peirce: CP 5.248 Cross-Ref:††

248. It remains, then, only to inquire whether it is necessary to suppose a

particular power of introspection for the sake of accounting for the sense of willing. Now, volition, as distinguished from desire, is nothing but the power of concentrating the attention, of abstracting. Hence, the knowledge of the power of abstracting may be inferred from abstract objects, just as the knowledge of the power of seeing is inferred from colored objects.

Peirce: CP 5.249 Cross-Ref:††

249. It appears, therefore, that there is no reason for supposing a power of introspection; and, consequently, the only way of investigating a psychological question is by inference from external facts.

Peirce: CP 5 Book 2 Question 5

QUESTION 5. *Whether we can think without signs.*

Peirce: CP 5.250 Cross-Ref:††

250. This is a familiar question, but there is, to this day, no better argument in the affirmative than that thought must precede every sign. This assumes the impossibility of an infinite series. But Achilles, as a fact, will overtake the tortoise. *How* this happens, is a question not necessary to be answered at present, as long as it certainly does happen.

Peirce: CP 5.251 Cross-Ref:††

251. If we seek the light of external facts, the only cases of thought which we can find are of thought in signs. Plainly, no other thought can be evidenced by external facts. But we have seen that only by external facts can thought be known at all. The only thought, then, which can possibly be cognized is thought in signs. But thought which cannot be cognized does not exist. All thought, therefore, must necessarily be in signs.

Peirce: CP 5.252 Cross-Ref:††

252. A man says to himself, "Aristotle is a man; *therefore*, he is fallible." Has he not, then, thought what he has not said to himself, that all men are fallible? The answer is, that he has done so, so far as this is said in his *therefore*. According to this, our question does not relate to *fact*, but is a mere asking for distinctness of thought.

Peirce: CP 5.253 Cross-Ref:††

253. From the proposition that every thought is a sign, it follows that every thought must address itself to some other, must determine some other, since that is the essence of a sign. This, after all, is but another form of the familiar axiom, that in intuition, *i.e.*, in the immediate present, there is no thought, or, that all which is reflected upon has past. *Hinc loquor inde est*. That, since any thought, there must have been a thought, has its analogue in the fact that, since any past time, there must have been an infinite series of times. To say, therefore, that thought cannot happen in an instant, but requires a time, is but another way of saying that every thought must be interpreted in another, or that all thought is in signs.

Peirce: CP 5 Book 2 Question 6

QUESTION 6. *Whether a sign can have any meaning, if by its definition it is the sign of something absolutely incognizable.*

Peirce: CP 5.254 Cross-Ref:††

254. It would seem that it can, and that universal and hypothetical propositions are instances of it. Thus, the universal proposition, "all ruminants are cloven-hoofed," speaks of a possible infinity of animals, and no matter how many ruminants may have been examined, the possibility must remain that there are others which have not been examined. In the case of a hypothetical proposition, the same thing is still more manifest; for such a proposition speaks not merely of the actual state of things, but of every possible state of things, all of which are not knowable, inasmuch as only one can so much as exist.

Peirce: CP 5.255 Cross-Ref:††

255. On the other hand, all our conceptions are obtained by abstractions and combinations of cognitions first occurring in judgments of experience. Accordingly, there can be no conception of the absolutely incognizable, since nothing of that sort occurs in experience. But the meaning of a term is the conception which it conveys. Hence, a term can have no such meaning.

Peirce: CP 5.256 Cross-Ref:††

256. If it be said that the incognizable is a concept compounded of the concept *not* and *cognizable*, it may be replied that *not* is a mere syncategorematic term and not a concept by itself.

Peirce: CP 5.257 Cross-Ref:††

257. If I think "white," I will not go so far as Berkeley †1 and say that I think of a person seeing, but I will say that what I think is of the nature of a cognition, and so of anything else which can be experienced. Consequently, the highest concept which can be reached by abstractions from judgments of experience -- and therefore, the highest concept which can be reached at all -- is the concept of something of the nature of a cognition. *Not*, then, or *what is other than*, if a concept, is a concept of the cognizable. Hence, not-cognizable, if a concept, is a concept of the form "A, not-A," and is, at least, self-contradictory. Thus, ignorance and error can only be conceived as correlative to a real knowledge and truth, which latter are of the nature of cognitions. Over against any cognition, there is an unknown but knowable reality; but over against all possible cognition, there is only the self-contradictory. In short, *cognizability* (in its widest sense) and *being* are not merely metaphysically the same, but are synonymous terms.

Peirce: CP 5.258 Cross-Ref:††

258. To the argument from universal and hypothetical propositions, the reply is, that though their truth cannot be cognized with absolute certainty, it may be probably known by induction.

Peirce: CP 5 Book 2 Question 7

QUESTION 7. *Whether there is any cognition not determined by a previous cognition.*

Peirce: CP 5.259 Cross-Ref:††

259. It would seem that there is or has been; for since we are in possession of cognitions, which are all determined by previous ones, and these by cognitions earlier still, there must have been a *first* in this series or else our state of cognition at any time is completely determined, according to logical laws, by our state at any previous

time. But there are many facts against the last supposition, and therefore in favor of intuitive cognitions.

Peirce: CP 5.260 Cross-Ref:††

260. On the other hand, since it is impossible to know intuitively that a given cognition is not determined by a previous one, the only way in which this can be known is by hypothetic inference from observed facts. But to adduce the cognition by which a given cognition has been determined is to explain the determinations of that cognition. And it is the only way of explaining them. For something entirely out of consciousness which may be supposed to determine it, can, as such, only be known and only adduced in the determinate cognition in question. So, that to suppose that a cognition is determined solely by something absolutely external, is to suppose its determinations incapable of explanation. Now, this is a hypothesis which is warranted under no circumstances, inasmuch as the only possible justification for a hypothesis is that it explains the facts, and to say that they are explained and at the same time to suppose them inexplicable is self-contradictory.

Peirce: CP 5.261 Cross-Ref:††

261. If it be objected that the peculiar character of *red* is not determined by any previous cognition, I reply that that character is not a character of red as a cognition; for if there be a man to whom red things look as blue ones do to me and *vice versa*, that man's eyes teach him the same facts that they would if he were like me.

Peirce: CP 5.262 Cross-Ref:††

262. Moreover, we know of no power by which an intuition could be known. For, as the cognition is beginning, and therefore in a state of change, at only the first instant would it be intuition. And, therefore, the apprehension of it must take place in no time and be an event occupying no time.†P1 Besides, all the cognitive faculties we know of are relative, and consequently their products are relations. But the cognition of a relation is determined by previous cognitions. No cognition not determined by a previous cognition, then, can be known. It does not exist, then, first, because it is absolutely incognizable, and second, because a cognition only exists so far as it is known.

Peirce: CP 5.263 Cross-Ref:††

263. The reply to the argument that there must be a first is as follows: In retracing our way from conclusions to premisses, or from determined cognitions to those which determine them, we finally reach, in all cases, a point beyond which the consciousness in the determined cognition is more lively than in the cognition which determines it. We have a less lively consciousness in the cognition which determines our cognition of the third dimension than in the latter cognition itself; a less lively consciousness in the cognition which determines our cognition of a continuous surface (without a blind spot) than in this latter cognition itself; and a less lively consciousness of the impressions which determine the sensation of tone than of that sensation itself. Indeed, when we get near enough to the external this is the universal rule. Now let any horizontal line represent a cognition, and let the length of the line serve to measure (so to speak) the liveliness of consciousness in that cognition. A point, having no length, will, on this principle, represent an object quite out of consciousness. Let one horizontal line below another represent a cognition which determines the cognition represented by that other and which has the same object as the latter. Let the finite distance between two such lines represent that they are two different cognitions. With this aid to thinking, let us see whether "there must be a



first." Suppose an inverted triangle  [Click here to view] to be gradually dipped into water. At any date or instant, the surface of the water makes a horizontal line across that triangle. This line represents a cognition. At a subsequent date, there is a sectional line so made, higher upon the triangle. This represents another cognition of the same object determined by the former, and having a livelier consciousness. The apex of the triangle represents the object external to the mind which determines both these cognitions. The state of the triangle before it reaches the water, represents a state of cognition which contains nothing which determines these subsequent cognitions. To say, then, that if there be a state of cognition by which all subsequent cognitions of a certain object are not determined, there must subsequently be some cognition of that object not determined by previous cognitions of the same object, is to say that when that triangle is dipped into the water there must be a sectional line made by the surface of the water lower than which no surface line had been made in that way. But draw the horizontal line where you will, as many horizontal lines as you please can be assigned at finite distances below it and below one another. For any such section is at some distance above the apex, otherwise it is not a line. Let this distance be a . Then there have been similar sections at the distances $1/2a$, $1/4a$, $1/8a$, $1/16a$, above the apex, and so on as far as you please. So that it is not true that there must be a first. Explicate the logical difficulties of this paradox (they are identical with those of the Achilles) in whatever way you may. I am content with the result, as long as your principles are fully applied to the particular case of cognitions determining one another. Deny motion, if it seems proper to do so; only then deny the process of determination of one cognition by another. Say that instants and lines are fictions; only say, also, that states of cognition and judgments are fictions. The point here insisted on is not this or that logical solution of the difficulty, but merely that cognition arises by a *process* of beginning, as any other change comes to pass.

Peirce: CP 5.263 Cross-Ref:††

In a subsequent paper, I shall trace the consequences of these principles, in reference to the questions of reality, of individuality, and of the validity of the laws of logic.

Peirce: CP 5.264 Cross-Ref:††

II

SOME CONSEQUENCES OF FOUR INCAPACITIES†1

§1. THE SPIRIT OF CARTESIANISME

264. Descartes is the father of modern philosophy, and the spirit of Cartesianism -- that which principally distinguishes it from the scholasticism which it displaced -- may be compendiously stated as follows:

Peirce: CP 5.264 Cross-Ref:††

1. It teaches that philosophy must begin with universal doubt; whereas scholasticism had never questioned fundamentals.

Peirce: CP 5.264 Cross-Ref:††

2. It teaches that the ultimate test of certainty is to be found in the individual consciousness; whereas scholasticism had rested on the testimony of sages and of the Catholic Church.

Peirce: CP 5.264 Cross-Ref:††

3. The multiform argumentation of the middle ages is replaced by a single thread of inference depending often upon inconspicuous premisses.

Peirce: CP 5.264 Cross-Ref:††

4. Scholasticism had its mysteries of faith, but undertook to explain all created things. But there are many facts which Cartesianism not only does not explain but renders absolutely inexplicable, unless to say that "God makes them so" is to be regarded as an explanation.

Peirce: CP 5.265 Cross-Ref:††

265. In some, or all of these respects, most modern philosophers have been, in effect, Cartesians. Now without wishing to return to scholasticism, it seems to me that modern science and modern logic require us to stand upon a very different platform from this.

Peirce: CP 5.265 Cross-Ref:††

1. We cannot begin with complete doubt. We must begin with all the prejudices which we actually have when we enter upon the study of philosophy. These prejudices are not to be dispelled by a maxim, for they are things which it does not occur to us *can* be questioned. Hence this initial skepticism will be a mere self-deception, and not real doubt; and no one who follows the Cartesian method will ever be satisfied until he has formally recovered all those beliefs which in form he has given up. It is, therefore, as useless a preliminary as going to the North Pole would be in order to get to Constantinople by coming down regularly upon a meridian. A person may, it is true, in the course of his studies, find reason to doubt what he began by believing; but in that case he doubts because he has a positive reason for it, and not on account of the Cartesian maxim. Let us not pretend to doubt in philosophy what we do not doubt in our hearts.

Peirce: CP 5.265 Cross-Ref:††

2. The same formalism appears in the Cartesian criterion, which amounts to this: "Whatever I am clearly convinced of, is true." If I were really convinced, I should have done with reasoning and should require no test of certainty. But thus to make single individuals absolute judges of truth is most pernicious. The result is that metaphysicians will all agree that metaphysics has reached a pitch of certainty far beyond that of the physical sciences; -- only they can agree upon nothing else. In sciences in which men come to agreement, when a theory has been broached it is considered to be on probation until this agreement is reached. After it is reached, the question of certainty becomes an idle one, because there is no one left who doubts it. We individually cannot reasonably hope to attain the ultimate philosophy which we pursue; we can only seek it, therefore, for the *community* of philosophers. Hence, if disciplined and candid minds carefully examine a theory and refuse to accept it, this ought to create doubts in the mind of the author of the theory himself.

Peirce: CP 5.265 Cross-Ref:††

3. Philosophy ought to imitate the successful sciences in its methods, so far as to proceed only from tangible premisses which can be subjected to careful scrutiny, and to trust rather to the multitude and variety of its arguments than to the conclusiveness of any one. Its reasoning should not form a chain which is no stronger than its weakest link, but a cable whose fibers may be ever so slender, provided they are sufficiently numerous and intimately connected.

Peirce: CP 5.265 Cross-Ref:††

4. Every unidealistic philosophy supposes some absolutely inexplicable, unanalyzable ultimate; in short, something resulting from mediation itself not susceptible of mediation. Now that anything *is* thus inexplicable can only be known by reasoning from signs. But the only justification of an inference from signs is that the conclusion explains the fact. To suppose the fact absolutely inexplicable, is not to explain it, and hence this supposition is never allowable.

Peirce: CP 5.265 Cross-Ref:††

In the last number of this journal will be found a piece entitled "Questions concerning certain Faculties claimed for Man," [Paper No. I] which has been written in this spirit of opposition to Cartesianism. That criticism of certain faculties resulted in four denials, which for convenience may here be repeated:

Peirce: CP 5.265 Cross-Ref:††

1. We have no power of Introspection, but all knowledge of the internal world is derived by hypothetical reasoning from our knowledge of external facts.

Peirce: CP 5.265 Cross-Ref:††

2. We have no power of Intuition, but every cognition is determined logically by previous cognitions.

Peirce: CP 5.265 Cross-Ref:††

3. We have no power of thinking without signs.

Peirce: CP 5.265 Cross-Ref:††

4. We have no conception of the absolutely incognizable. These propositions cannot be regarded as certain; and, in order to bring them to a further test, it is now proposed to trace them out to their consequences. We may first consider the first alone; then trace the consequences of the first and second; then see what else will result from assuming the third also; and, finally, add the fourth to our hypothetical premisses.

Peirce: CP 5.266 Cross-Ref:††

§2. MENTAL ACTIONE

266. In accepting the first proposition, we must put aside all prejudices derived from a philosophy which bases our knowledge of the external world on our self-consciousness. We can admit no statement concerning what passes within us except as a hypothesis necessary to explain what takes place in what we commonly call the external world. Moreover when we have upon such grounds assumed one faculty or mode of action of the mind, we cannot, of course, adopt any other

hypothesis for the purpose of explaining any fact which can be explained by our first supposition, but must carry the latter as far as it will go. In other words, we must, as far as we can do so without additional hypotheses, reduce all kinds of mental action to one general type.

Peirce: CP 5.267 Cross-Ref:††

267. The class of modifications of consciousness with which we must commence our inquiry must be one whose existence is indubitable, and whose laws are best known, and, therefore (since this knowledge comes from the outside), which most closely follows external facts; that is, it must be some kind of cognition. Here we may hypothetically admit the second proposition of the former paper, according to which there is no absolutely first cognition of any object, but cognition arises by a continuous process. We must begin, then, with a *process* of cognition, and with that process whose laws are best understood and most closely follow external facts. This is no other than the process of valid inference, which proceeds from its premiss, A, to its conclusion, B, only if, as a matter of fact, such a proposition as B is always or usually true when such a proposition as A is true. It is a consequence, then, of the first two principles whose results we are to trace out, that we must, as far as we can, without any other supposition than that the mind reasons, reduce all mental action to the formula of valid reasoning.

Peirce: CP 5.268 Cross-Ref:††

268. But does the mind in fact go through the syllogistic process? It is certainly very doubtful whether a conclusion -- as something existing in the mind independently, like an image -- suddenly displaces two premisses existing in the mind in a similar way. But it is a matter of constant experience, that if a man is made to believe in the premisses, in the sense that he will act from them and will say that they are true, under favorable conditions he will also be ready to act from the conclusion and to say that that is true. Something, therefore, takes place within the organism which is equivalent to the syllogistic process.

Peirce: CP 5.269 Cross-Ref:††

269. A valid inference is either *complete* or *incomplete*. †1 An incomplete inference is one whose validity depends upon some matter of fact not contained in the premisses. This implied fact might have been stated as a premiss, and its relation to the conclusion is the same whether it is explicitly posited or not, since it is at least virtually taken for granted; so that every valid incomplete argument is virtually complete. Complete arguments are divided into *simple* and *complex*. †2 A complex argument is one which from three or more premisses concludes what might have been concluded by successive steps in reasonings each of which is simple. Thus, a complex inference comes to the same thing in the end as a succession of simple inferences.

Peirce: CP 5.270 Cross-Ref:††

270. A complete, simple, and valid argument, or syllogism, is either *apodictic* or *probable*. †1 An apodictic or deductive syllogism is one whose validity depends unconditionally upon the relation of the fact inferred to the facts posited in the premisses. A syllogism whose validity should depend not merely upon its premisses, but upon the existence of some other knowledge, would be impossible; for either this other knowledge would be posited, in which case it would be a part of the premisses, or it would be implicitly assumed, in which case the inference would be incomplete. But a syllogism whose validity depends partly upon the *non-existence* of some other knowledge, is a *probable* syllogism.

Peirce: CP 5.271 Cross-Ref:††

271. A few examples will render this plain. The two following arguments are apodictic or deductive:

Peirce: CP 5.271 Cross-Ref:††

1. No series of days of which the first and last are different days of the week exceeds by one a multiple of seven days; now the first and last days of any leap-year are different days of the week, and therefore no leap-year consists of a number of days one greater than a multiple of seven.

Peirce: CP 5.271 Cross-Ref:††

2. Among the vowels there are no double letters; but one of the double letters (w) is compounded of two vowels: hence, a letter compounded of two vowels is not necessarily itself a vowel.

Peirce: CP 5.271 Cross-Ref:††

In both these cases, it is plain that as long as the premisses are true, however other facts may be, the conclusions will be true. On the other hand, suppose that we reason as follows: "A certain man had the Asiatic cholera. He was in a state of collapse, livid, quite cold, and without perceptible pulse. He was bled copiously. During the process he came out of collapse, and the next morning was well enough to be about. Therefore, bleeding tends to cure the cholera." This is a fair probable inference, provided that the premisses represent our whole knowledge of the matter. But if we knew, for example, that recoveries from cholera were apt to be sudden, and that the physician who had reported this case had known of a hundred other trials of the remedy without communicating the result, then the inference would lose all its validity.

Peirce: CP 5.272 Cross-Ref:††

272. The absence of knowledge which is essential to the validity of any probable argument relates to some question which is determined by the argument itself. This question, like every other, is whether certain objects have certain characters. Hence, the absence of knowledge is either whether besides the objects which, according to the premisses, possess certain characters, any other objects possess them; or, whether besides the characters which, according to the premisses, belong to certain objects, any other characters not necessarily involved in these belong to the same objects. In the former case, the reasoning proceeds as though all the objects which have certain characters were known, and this is *induction*; in the latter case, the inference proceeds as though all the characters requisite to the determination of a certain object or class were known, and this is *hypothesis*. This distinction, also, may be made more plain by examples.

Peirce: CP 5.273 Cross-Ref:††

273. Suppose we count the number of occurrences of the different letters in a certain English book, which we may call A. Of course, every new letter which we add to our count will alter the relative number of occurrences of the different letters; but as we proceed with our counting, this change will be less and less. Suppose that we find that as we increase the number of letters counted, the relative number of *e*'s approaches nearly 11 1/4 *per cent.* of the whole, that of the *t*'s 8 1/2 *per cent.*, that of the *a*'s 8 *per cent.*, that of the *s*'s 7 1/2 *per cent.*, etc. Suppose we repeat the same observations with half a dozen other English writings (which we may designate as B, C, D, E, F, G) with the like result. Then we may infer that in every English writing of

some length, the different letters occur with nearly those relative frequencies.

Peirce: CP 5.273 Cross-Ref:††

Now this argument depends for its validity upon our *not* knowing the proportion of letters in any English writing besides A, B, C, D, E, F and G. For if we know it in respect to H, and it is not nearly the same as in the others, our conclusion is destroyed at once; if it is the same, then the legitimate inference is from A, B, C, D, E, F, G and H, and not from the first seven alone. This, therefore, is an *induction*.

Peirce: CP 5.273 Cross-Ref:††

Suppose, next, that a piece of writing in cipher is presented to us, without the key. Suppose we find that it contains something less than 26 characters, one of which occurs about 11 *per cent.* of all the times, another 8 1/2 *per cent.*, another 8 *per cent.*, and another 7 1/2 *per cent.* Suppose that when we substitute for these *e*, *t*, *a* and *s*, respectively, we are able to see how single letters may be substituted for each of the other characters so as to make sense in English, provided, however, that we allow the spelling to be wrong in some cases. If the writing is of any considerable length, we may infer with great probability that this is the meaning of the cipher.

Peirce: CP 5.273 Cross-Ref:††

The validity of this argument depends upon there being no other known characters of the writing in cipher which would have any weight in the matter; for if there are -- if we know, for example, whether or not there is any other solution of it -- this must be allowed its effect in supporting or weakening the conclusion. This, then, is *hypothesis*.

Peirce: CP 5.274 Cross-Ref:††

274. All valid reasoning is either deductive, inductive, or hypothetic; or else it combines two or more of these characters. Deduction is pretty well treated in most logical textbooks; but it will be necessary to say a few words about induction and hypothesis in order to render what follows more intelligible.

Peirce: CP 5.275 Cross-Ref:††

275. Induction may be defined as an argument which proceeds upon the assumption that all the members of a class or aggregate have all the characters which are common to all those members of this class concerning which it is known, whether they have these characters or not; or, in other words, which assumes that that is true of a whole collection which is true of a number of instances taken from it at random. This might be called statistical argument. In the long run, it must generally afford pretty correct conclusions from true premisses. If we have a bag of beans partly black and partly white, by counting the relative proportions of the two colors in several different handfuls, we can approximate more or less to the relative proportions in the whole bag, since a sufficient number of handfuls would constitute all the beans in the bag. The central characteristic and key to induction is, that by taking the conclusion so reached as major premiss of a syllogism, and the proposition stating that such and such objects are taken from the class in question as the minor premiss, the other premiss of the induction will follow from them deductively.†1 Thus, in the above example we concluded that all books in English have about 11 1/4 *per cent.* of their letters *e*'s. From that as major premiss, together with the proposition that A, B, C, D, E, F and G are books in English, it follows deductively that A, B, C, D, E, F and G have about 11 1/4 *per cent.* of their letters *e*'s. Accordingly, induction has been defined by Aristotle †2 as the inference of the major premiss of a syllogism from its minor premiss and conclusion. The function of an induction is to substitute for a

series of many subjects, a single one which embraces them and an indefinite number of others. Thus it is a species of "reduction of the manifold to unity."

Peirce: CP 5.276 Cross-Ref:††

276. Hypothesis may be defined as an argument which proceeds upon the assumption that a character which is known necessarily to involve a certain number of others, may be probably predicated of any object which has all the characters which this character is known to involve. Just as induction may be regarded as the inference of the major premiss of a syllogism, so hypothesis may be regarded as the inference of the minor premiss, from the other two propositions. Thus, the example taken above consists of two such inferences of the minor premisses of the following syllogisms:

Peirce: CP 5.276 Cross-Ref:††

1. Every English writing of some length in which such and such characters denote *e*, *t*, *a*, and *s*, has about 11 1/4 *per cent.* of the first sort of marks, 8 1/2 of the second, 8 of the third, and 7 1/2 of the fourth.

Peirce: CP 5.276 Cross-Ref:††

This secret writing is an English writing of some length, in which such and such characters denote *e*, *t*, *a*, and *s*, respectively:

∴ This secret writing has about 11 1/4 *per cent.* of its characters of the first kind, 8 1/2 of the second, 8 of the third, and 7 1/2 of the fourth.

Peirce: CP 5.276 Cross-Ref:††

2. A passage written with such an alphabet makes sense when such and such letters are severally substituted for such and such characters.

This secret writing is written with such an alphabet.

∴ This secret writing makes sense when such and such substitutions are made.

Peirce: CP 5.276 Cross-Ref:††

The function of hypothesis is to substitute for a great series of predicates forming no unity in themselves, a single one (or small number) which involves them all, together (perhaps) with an indefinite number of others. It is, therefore, also a reduction of a manifold to unity.†P1 Every deductive syllogism may be put into the form

If A, then B;

But A:

∴ B.

And as the minor premiss in this form appears as antecedent or reason of a hypothetical proposition, hypothetical inference may be called reasoning from consequent to antecedent.

Peirce: CP 5.277 Cross-Ref:††

277. The argument from analogy, which a popular writer †1 upon logic calls reasoning from particulars to particulars, derives its validity from its combining the characters of induction and hypothesis, being analyzable either into a deduction or an induction, or a deduction and a hypothesis.†1

Peirce: CP 5.278 Cross-Ref:††

278. But though inference is thus of three essentially different species, it also belongs to one genus. We have seen that no conclusion can be legitimately derived which could not have been reached by successions of arguments having two premisses each, and implying no fact not asserted.

Peirce: CP 5.279 Cross-Ref:††

279. Either of these premisses is a proposition asserting that certain objects have certain characters. Every term of such a proposition stands either for certain objects or for certain characters. The conclusion may be regarded as a proposition substituted in place of either premiss, the substitution being justified by the fact stated in the other premiss. The conclusion is accordingly derived from either premiss by substituting either a new subject for the subject of the premiss, or a new predicate for the predicate of the premiss, or by both substitutions. Now the substitution of one term for another can be justified only so far as the term substituted represents only what is represented in the term replaced. If, therefore, the conclusion be denoted by the formula,

S is P;

and this conclusion be derived, by a change of subject, from a premiss which may on this account be expressed by the formula,

M is P,

then the other premiss must assert that whatever thing is represented by S is represented by M, or that

Every S is an M;

while, if the conclusion, S is P, is derived from either premiss by a change of predicate, that premiss may be written

S is M;

and the other premiss must assert that whatever characters are implied in P are implied in M, or that

Whatever is M is P.

In either case, therefore, the syllogism must be capable of expression in the form,

S is M; M is P:

∴ S is P.

Peirce: CP 5.279 Cross-Ref:††

Finally, if the conclusion differs from either of its premisses, both in subject and predicate, the form of statement of conclusion and premiss may be so altered that they shall have a common term. This can always be done, for if P is the premiss and C the conclusion, they may be stated thus:

The state of things represented in P is real, and

The state of things represented in C is real.

Peirce: CP 5.279 Cross-Ref:††

In this case the other premiss must in some form virtually assert that every state of things such as is represented by C is the state of things represented in P.

Peirce: CP 5.279 Cross-Ref:††

All valid reasoning, therefore, is of one general form; and in seeking to reduce all mental action to the formulæ of valid inference, we seek to reduce it to one single type.

Peirce: CP 5.280 Cross-Ref:††

280. An apparent obstacle to the reduction of all mental action to the type of valid inferences is the existence of fallacious reasoning. Every argument implies the truth of a general principle of inferential procedure (whether involving some matter of fact concerning the subject of argument, or merely a maxim relating to a system of signs), according to which it is a valid argument. If this principle is false, the argument is a fallacy; but neither a valid argument from false premisses, nor an exceedingly weak, but not altogether illegitimate, induction or hypothesis, however its force may be over-estimated, however false its conclusion, is a fallacy.

Peirce: CP 5.281 Cross-Ref:††

281. Now words, taken just as they stand, if in the form of an argument, thereby do imply whatever fact may be necessary to make the argument conclusive; so that to the formal logician, who has to do only with the meaning of the words

according to the proper principles of interpretation, and not with the intention of the speaker as guessed at from other indications, the only fallacies should be such as are simply absurd and contradictory, either because their conclusions are absolutely inconsistent with their premisses, or because they connect propositions by a species of illative conjunction, by which they cannot under any circumstances be validly connected.

Peirce: CP 5.282 Cross-Ref:††

282. But to the psychologist an argument is valid only if the premisses from which the mental conclusion is derived would be sufficient, if true, to justify it, either by themselves, or by the aid of other propositions which had previously been held for true. But it is easy to show that all inferences made by man, which are not valid in this sense, belong to four classes, viz.: 1. Those whose premisses are false; 2. Those which have some little force, though only a little; 3. Those which result from confusion of one proposition with another; 4. Those which result from the indistinct apprehension, wrong application, or falsity, of a rule of inference. For, if a man were to commit a fallacy not of either of these classes, he would, from true premisses conceived with perfect distinctness, without being led astray by any prejudice or other judgment serving as a rule of inference, draw a conclusion which had really not the least relevancy. If this could happen, calm consideration and care could be of little use in thinking, for caution only serves to insure our taking all the facts into account, and to make those which we do take account of, distinct; nor can coolness do anything more than to enable us to be cautious, and also to prevent our being affected by a passion in inferring that to be true which we wish were true, or which we fear may be true, or in following some other wrong rule of inference. But experience shows that the calm and careful consideration of the same distinctly conceived premisses (including prejudices) will insure the pronouncement of the same judgment by all men. Now if a fallacy belongs to the first of these four classes and its premisses are false, it is to be presumed that the procedure of the mind from these premisses to the conclusion is either correct, or errs in one of the other three ways; for it cannot be supposed that the mere falsity of the premisses should affect the procedure of reason when that falsity is not known to reason. If the fallacy belongs to the second class and has some force, however little, it is a legitimate probable argument, and belongs to the type of valid inference. If it is of the third class and results from the confusion of one proposition with another, this confusion must be owing to a resemblance between the two propositions; that is to say, the person reasoning, seeing that one proposition has some of the characters which belong to the other, concludes that it has all the essential characters of the other, and is equivalent to it. Now this is a hypothetic inference, which though it may be weak, and though its conclusion happens to be false, belongs to the type of valid inferences; and, therefore, as the *nodus* of the fallacy lies in this confusion, the procedure of the mind in these fallacies of the third class conforms to the formula of valid inference. If the fallacy belongs to the fourth class, it either results from wrongly applying or misapprehending a rule of inference, and so is a fallacy of confusion, or it results from adopting a wrong rule of inference. In this latter case, this rule is in fact taken as a premiss, and therefore the false conclusion is owing merely to the falsity of a premiss. In every fallacy, therefore, possible to the mind of man, the procedure of the mind conforms to the formula of valid inference.

Peirce: CP 5.283 Cross-Ref:††

§3. THOUGHT-SIGNSE

283. The third principle whose consequences we have to deduce is, that, whenever we think, we have present to the consciousness some feeling, image, conception, or other representation, which serves as a sign. But it follows from our own existence (which is proved by the occurrence of ignorance and error †1) that everything which is present to us is a phenomenal manifestation of ourselves. This does not prevent its being a phenomenon of something without us, just as a rainbow is at once a manifestation both of the sun and of the rain. When we think, then, we ourselves, as we are at that moment, appear as a sign. Now a sign has, as such, three references: first, it is a sign *to* some thought which interprets it; second, it is a sign *for* some object to which in that thought it is equivalent; third, it is a sign, *in* some respect or quality, which brings it into connection with its object. Let us ask what the three correlates are to which a thought-sign refers.

Peirce: CP 5.284 Cross-Ref:††

284. (1) When we think, to what thought does that thought-sign which is ourself address itself? It may, through the medium of outward expression, which it reaches perhaps only after considerable internal development, come to address itself to thought of another person. But whether this happens or not, it is always interpreted by a subsequent thought of our own. If, after any thought, the current of ideas flows on freely, it follows the law of mental association. In that case, each former thought suggests something to the thought which follows it, i.e., is the sign of something to this latter. Our train of thought may, it is true, be interrupted. But we must remember that, in addition to the principal element of thought at any moment, there are a hundred things in our mind to which but a small fraction of attention or consciousness is conceded. It does not, therefore, follow, because a new constituent of thought gets the uppermost that the train of thought which it displaces is broken off altogether. On the contrary, from our second principle, that there is no intuition or cognition not determined by previous cognitions, it follows that the striking in of a new experience is never an instantaneous affair, but is an *event* occupying time, and coming to pass by a continuous process. Its prominence in consciousness, therefore, must probably be the consummation of a growing process; and if so, there is no sufficient cause for the thought which had been the leading one just before, to cease abruptly and instantaneously. But if a train of thought ceases by gradually dying out, it freely follows its own law of association as long as it lasts, and there is no moment at which there is a thought belonging to this series, subsequently to which there is not a thought which interprets or repeats it. There is no exception, therefore, to the law that every thought-sign is translated or interpreted in a subsequent one, unless it be that all thought comes to an abrupt and final end in death.

Peirce: CP 5.285 Cross-Ref:††

285. (2) The next question is: For what does the thought-sign stand -- what does it name -- what is its *suppositum*? The outward thing, undoubtedly, when a real outward thing is thought of. But still, as the thought is determined by a previous thought of the same object, it only refers to the thing through denoting this previous thought. Let us suppose, for example, that Toussaint is thought of, and first thought of as a *negro*, but not distinctly as a man. If this distinctness is afterwards added, it is through the thought that a *negro* is a *man*; that is to say, the subsequent thought, *man*, refers to the outward thing by being predicated of that previous thought, *negro*, which has been had of that thing. If we afterwards think of Toussaint as a general, then we think that this negro, this man, was a general. And so in every case the

subsequent thought denotes what was thought in the previous thought.

Peirce: CP 5.286 Cross-Ref:††

286. (3) The thought-sign stands for its object in the respect which is thought; that is to say, this respect is the immediate object of consciousness in the thought, or, in other words, it is the thought itself, or at least what the thought is thought to be in the subsequent thought to which it is a sign.

Peirce: CP 5.287 Cross-Ref:††

287. We must now consider two other properties of signs which are of great importance in the theory of cognition. Since a sign is not identical with the thing signified, but differs from the latter in some respects, it must plainly have some characters which belong to it in itself, and have nothing to do with its representative function. These I call the *material* qualities of the sign. As examples of such qualities, take in the word "man," its consisting of three letters -- in a picture, its being flat and without relief. In the second place, a sign must be capable of being connected (not in the reason but really) with another sign of the same object, or with the object itself. Thus, words would be of no value at all unless they could be connected into sentences by means of a real copula which joins signs of the same thing. The usefulness of some signs -- as a weathercock, a tally, etc. -- consists wholly in their being really connected with the very things they signify. In the case of a picture such a connection is not evident, but it exists in the power of association which connects the picture with the brain-sign which labels it. This real, physical connection of a sign with its object, either immediately or by its connection with another sign, I call the *pure demonstrative application* of the sign. Now the representative function of a sign lies neither in its material quality nor in its pure demonstrative application; because it is something which the sign is, not in itself or in a real relation to its object, but which it is *to a thought*, while both of the characters just defined belong to the sign independently of its addressing any thought. And yet if I take all the things which have certain qualities and physically connect them with another series of things, each to each, they become fit to be signs. If they are not regarded as such they are not actually signs, but they are so in the same sense, for example, in which an unseen flower can be said to be *red*, this being also a term relative to a mental affection.

Peirce: CP 5.288 Cross-Ref:††

288. Consider a state of mind which is a conception. It is a conception by virtue of having a *meaning*, a logical comprehension; and if it is applicable to any object, it is because that object has the characters contained in the comprehension of this conception. Now the logical comprehension of a thought is usually said to consist of the thoughts contained in it; but thoughts are events, acts of the mind. Two thoughts are two events separated in time, and one cannot literally be contained in the other. It may be said that all thoughts exactly similar are regarded as one; and that to say that one thought contains another, means that it contains one exactly similar to that other. But how can two thoughts be similar? Two objects can only be *regarded* as similar if they are compared and brought together in the mind. Thoughts have no existence except in the mind; only as they are regarded do they exist. Hence, two thoughts cannot *be* similar unless they are brought together in the mind. But, as to their existence, two thoughts are separated by an interval of time. We are too apt to imagine that we can frame a thought similar to a past thought, by matching it with the latter, as though this past thought were still present to us. But it is plain that the knowledge that one thought is similar to or in any way truly representative of another, cannot be derived from immediate perception, but must be an hypothesis

(unquestionably fully justifiable by facts), and that therefore the formation of such a representing thought must be dependent upon a real effective force behind consciousness, and not merely upon a mental comparison. What we must mean, therefore, by saying that one concept is contained in another, is that we normally represent one to be in the other; that is, that we form a particular kind of judgment, †P1 of which the subject signifies one concept and the predicate the other.

Peirce: CP 5.289 Cross-Ref: ††

289. No thought in itself, then, no feeling in itself, contains any others, but is absolutely simple and unanalyzable; and to say that it is composed of other thoughts and feelings, is like saying that a movement upon a straight line is composed of the two movements of which it is the resultant; that is to say, it is a metaphor, or fiction, parallel to the truth. Every thought, however artificial and complex, is, so far as it is immediately present, a mere sensation without parts, and therefore, in itself, without similarity to any other, but incomparable with any other and absolutely *sui generis*. †P2 Whatever is wholly incomparable with anything else is wholly inexplicable, because explanation consists in bringing things under general laws or under natural classes. Hence every thought, in so far as it is a feeling of a peculiar sort, is simply an ultimate, inexplicable fact. Yet this does not conflict with my postulate that that fact should be allowed to stand as inexplicable; for, on the one hand, we never can think, "This is present to me," since, before we have time to make the reflection, the sensation is past, and, on the other hand, when once past, we can never bring back the quality of the feeling as it was *in and for itself*, or know what it was like *in itself*, or even discover the existence of this quality except by a corollary from our general theory of ourselves, and then not in its idiosyncrasy, but only as something present. But, as something present, feelings are all alike and require no explanation, since they contain only what is universal. So that nothing which we can truly predicate of feelings is left inexplicable, but only something which we cannot reflectively know. So that we do not fall into the contradiction of making the Mediate immediable. Finally, no present actual thought (which is a mere feeling) has any meaning, any intellectual value; for this lies not in what is actually thought, but in what this thought may be connected with in representation by subsequent thoughts; so that the meaning of a thought is altogether something virtual. †1 It may be objected, that if no thought has any meaning, all thought is without meaning. But this is a fallacy similar to saying, that, if in no one of the successive spaces which a body fills there is room for motion, there is no room for motion throughout the whole. At no one instant in my state of mind is there cognition or representation, but in the relation of my states of mind at different instants there is. †P1 In short, the Immediate (and therefore in itself unsusceptible of mediation -- the Unanalyzable, the Inexplicable, the Unintellectual) runs in a continuous stream through our lives; it is the sum total of consciousness, whose mediation, which is the continuity of it, is brought about by a real effective force behind consciousness.

Peirce: CP 5.290 Cross-Ref: ††

290. Thus, we have in thought three elements: first, the representative function which makes it a *representation*; second, the pure denotative application, or real connection, which brings one thought into *relation* with another; and third, the material quality, or how it feels, which gives thought its *quality*. †P1

Peirce: CP 5.291 Cross-Ref: ††

291. That a sensation is not necessarily an intuition, or first impression of sense, is very evident in the case of the sense of beauty; and has been shown [in 222],

in the case of sound. When the sensation beautiful is determined by previous cognitions, it always arises as a predicate; that is, we think that something is beautiful. Whenever a sensation thus arises in consequence of others, induction shows that those others are more or less complicated. Thus, the sensation of a particular kind of sound arises in consequence of impressions upon the various nerves of the ear being combined in a particular way, and following one another with a certain rapidity. A sensation of color depends upon impressions upon the eye following one another in a regular manner, and with a certain rapidity. The sensation of beauty arises upon a manifold of other impressions. And this will be found to hold good in all cases. Secondly, all these sensations are in themselves simple, or more so than the sensations which give rise to them. Accordingly, a sensation is a simple predicate taken in place of a complex predicate; in other words, it fulfills the function of an hypothesis. But the general principle that every thing to which such and such a sensation belongs, has such and such a complicated series of predicates, is not one determined by reason (as we have seen), but is of an arbitrary nature. Hence, the class of hypothetic inferences which the arising of a sensation resembles, is that of reasoning from definition to definitum, in which the major premiss is of an arbitrary nature. Only in this mode of reasoning, this premiss is determined by the conventions of language, and expresses the occasion upon which a word is to be used; and in the formation of a sensation, it is determined by the constitution of our nature, and expresses the occasions upon which sensation, or a natural mental sign, arises. Thus, the sensation, so far as it represents something, is determined, according to a logical law, by previous cognitions; that is to say, these cognitions determine that there shall be a sensation. But so far as the sensation is a mere feeling of a particular sort, it is determined only by an inexplicable, occult power; and so far, it is not a representation, but only the material quality of a representation. For just as in reasoning from definition to definitum, it is indifferent to the logician how the defined word shall sound, or how many letters it shall contain, so in the case of this constitutional word, it is not determined by an inward law how it shall feel in itself. A feeling, therefore, as a feeling, is merely the *material quality* of a mental sign.

Peirce: CP 5.292 Cross-Ref:††

292. But there is no feeling which is not also a representation, a predicate of something determined logically by the feelings which precede it. For if there are any such feelings not predicates, they are the emotions. Now every emotion has a subject. If a man is angry, he is saying to himself that this or that is vile and outrageous. If he is in joy, he is saying "this is delicious." If he is wondering, he is saying "this is strange." In short, whenever a man feels, he is thinking of *something*. Even those passions which have no definite object -- as melancholy -- only come to consciousness through tinging the *objects of thought*. That which makes us look upon the emotions more as affections of self than other cognitions, is that we have found them more dependent upon our accidental situation at the moment than other cognitions; but that is only to say that they are cognitions too narrow to be useful. The emotions, as a little observation will show, arise when our attention is strongly drawn to complex and inconceivable circumstances. Fear arises when we cannot predict our fate; joy, in the case of certain indescribable and peculiarly complex sensations. If there are some indications that something greatly for my interest, and which I have anticipated would happen, may not happen; and if, after weighing probabilities, and inventing safeguards, and straining for further information, I find myself unable to come to any fixed conclusion in reference to the future, in the place of that intellectual hypothetic inference which I seek, the feeling of *anxiety* arises. When something happens for which I cannot account, I *wonder*. When I endeavor to realize to myself

what I never can do, a pleasure in the future, I *hope*. "I do not understand you," is the phrase of an angry man. The indescribable, the ineffable, the incomprehensible, commonly excite emotion; but nothing is so chilling as a scientific explanation. Thus an emotion is always a simple predicate substituted by an operation of the mind for a highly complicated predicate.†1 Now if we consider that a very complex predicate demands explanation by means of an hypothesis, that that hypothesis must be a simpler predicate substituted for that complex one; and that when we have an emotion, an hypothesis, strictly speaking, is hardly possible -- the analogy of the parts played by emotion and hypothesis is very striking. There is, it is true, this difference between an emotion and an intellectual hypothesis, that we have reason to say in the case of the latter, that to whatever the simple hypothetic predicate can be applied, of that the complex predicate is true; whereas, in the case of an emotion this is a proposition for which no reason can be given, but which is determined merely by our emotional constitution. But this corresponds precisely to the difference between hypothesis and reasoning from definition to definitum, and thus it would appear that emotion is nothing but sensation. There appears to be a difference, however, between emotion and sensation, and I would state it as follows:

Peirce: CP 5.293 Cross-Ref:††

293. There is some reason to think that, corresponding to every feeling within us, some motion takes place in our bodies. This property of the thought-sign, since it has no rational dependence upon the meaning of the sign, may be compared with what I have called the material quality of the sign; but it differs from the latter inasmuch as it is not essentially necessary that it should be felt in order that there should be any thought-sign. In the case of a sensation, the manifold of impressions which precede and determine it are not of a kind, the bodily motion corresponding to which comes from any large ganglion or from the brain, and probably for this reason the sensation produces no great commotion in the bodily organism; and the sensation itself is not a thought which has a very strong influence upon the current of thought except by virtue of the information it may serve to afford. An emotion, on the other hand, comes much later in the development of thought -- I mean, further from the first beginning of the cognition of its object -- and the thoughts which determine it already have motions corresponding to them in the brain, or the chief ganglion; consequently, it produces large movements in the body, and independently of its representative value, strongly affects the current of thought. The animal motions to which I allude, are, in the first place and obviously, blushing, blenching, staring, smiling, scowling, pouting, laughing, weeping, sobbing, wriggling, flinching, trembling, being petrified, sighing, sniffing, shrugging, groaning, heartsinking, trepidation, swelling of the heart, etc., etc. To these may, perhaps, be added, in the second place, other more complicated actions, which nevertheless spring from a direct impulse and not from deliberation.

Peirce: CP 5.294 Cross-Ref:††

294. That which distinguishes both sensations proper and emotions from the feeling of a thought, is that in the case of the two former the material quality is made prominent, because the thought has no relation of reason to the thoughts which determine it, which exists in the last case and detracts from the attention given to the mere feeling. By there being no relation of reason to the determining thoughts, I mean that there is nothing in the content of the thought which explains why it should arise only on occasion of these determining thoughts. If there is such a relation of reason, if the thought is essentially limited in its application to these objects, then the thought comprehends a thought other than itself; in other words, it is then a complex thought. An incomplex thought can, therefore, be nothing but a sensation or emotion, having

no rational character. This is very different from the ordinary doctrine, according to which the very highest and most metaphysical conceptions are absolutely simple. I shall be asked how such a conception of a *being* is to be analyzed, or whether I can ever define *one*, *two*, and *three*, without a diallelon. Now I shall admit at once that neither of these conceptions can be separated into two others higher than itself; and in that sense, therefore, I fully admit that certain very metaphysical and eminently intellectual notions are absolutely simple. But though these concepts cannot be defined by genus and difference, there is another way in which they can be defined. All determination is by negation; we can first recognize any character only by putting an object which possesses it into comparison with an object which possesses it not. A conception, therefore, which was quite universal in every respect would be unrecognizable and impossible. We do not obtain the conception of Being, in the sense implied in the copula, by observing that all the things which we can think of have something in common, for there is no such thing to be observed. We get it by reflecting upon signs -- words or thoughts; we observe that different predicates may be attached to the same subject, and that each makes some conception applicable to the subject; then we imagine that a subject has something true of it merely because a predicate (no matter what) is attached to it -- and that we call Being. The conception of being is, therefore, a conception about a sign -- a thought, or word; and since it is not applicable to every sign, it is not primarily universal, although it is so in its mediate application to things. Being, therefore, may be defined; it may be defined, for example, as that which is common to the objects included in any class, and to the objects not included in the same class.†1 But it is nothing new to say that metaphysical conceptions are primarily and at bottom thoughts about words, or thoughts about thoughts; it is the doctrine both of Aristotle (whose categories are parts of speech) and of Kant (whose categories are the characters of different kinds of propositions).

Peirce: CP 5.295 Cross-Ref:††

295. Sensation and the power of abstraction or attention may be regarded as, in one sense, the sole constituents of all thought. Having considered the former, let us now attempt some analysis of the latter. By the force of attention, an emphasis is put upon one of the objective elements of consciousness. This emphasis is, therefore, not itself an object of immediate consciousness; and in this respect it differs entirely from a feeling. Therefore, since the emphasis, nevertheless, consists in some effect upon consciousness, and so can exist only so far as it affects our knowledge; and since an act cannot be supposed to determine that which precedes it in time, this act can consist only in the capacity which the cognition emphasized has for producing an effect upon memory, or otherwise influencing subsequent thought. This is confirmed by the fact that attention is a matter of continuous quantity; for continuous quantity, so far as we know it, reduces itself in the last analysis to time. Accordingly, we find that attention does, in fact, produce a very great effect upon subsequent thought. In the first place, it strongly affects memory, a thought being remembered for a longer time the greater the attention originally paid to it. In the second place, the greater the attention, the closer the connection and the more accurate the logical sequence of thought. In the third place, by attention a thought may be recovered which has been forgotten. From these facts, we gather that attention is the power by which thought at one time is connected with and made to relate to thought at another time; or, to apply the conception of thought as a sign, that it is the *pure demonstrative application* of a thought-sign.

Peirce: CP 5.296 Cross-Ref:††

296. Attention is roused when the same phenomenon presents itself repeatedly on different occasions, or the same predicate in different subjects. We see that A has a certain character, that B has the same, C has the same; and this excites our attention, so that we say, "*These* have this character." Thus attention is an act of induction; but it is an induction which does not increase our knowledge, because our "these" covers nothing but the instances experienced. It is, in short, an argument from enumeration.

Peirce: CP 5.297 Cross-Ref:††

297. Attention produces effects upon the nervous system. These effects are habits, or nervous associations.†1 A habit arises, when, having had the sensation of performing a certain act, *m*, on several occasions *a*, *b*, *c*, we come to do it upon every occurrence of the general event, *l*, of which *a*, *b* and *c* are special cases. That is to say, by the cognition that

Peirce: CP 5.297 Cross-Ref:††

Every case of *a*, *b*, or *c*, is a case of *m*, is determined the cognition that

Every case of *l* is a case of *m*.

Thus the formation of a habit is an induction, and is therefore necessarily connected with attention or abstraction. Voluntary actions result from the sensations produced by habits, as instinctive actions result from our original nature.

Peirce: CP 5.298 Cross-Ref:††

298. We have thus seen that every sort of modification of consciousness -- Attention, Sensation, and Understanding -- is an inference. But the objection may be made that inference deals only with general terms, and that an image, or absolutely singular representation, cannot therefore be inferred.

Peirce: CP 5.299 Cross-Ref:††

299. "Singular" and "individual" are equivocal terms.†1 A singular may mean that which can be but in one place at one time. In this sense it is not opposed to general. *The sun* is a singular in this sense, but, as is explained in every good treatise on logic, it is a general term. I may have a very general conception of Hermolaus Barbarus, but still I conceive him only as able to be in one place at one time. When an image is said to be singular, it is meant that it is absolutely determinate in all respects. Every possible character, or the negative thereof, must be true of such an image. In the words of the most eminent expounder of the doctrine, the image of a man "must be either of a white, or a black, or a tawny; a straight or a crooked; a tall, or a low, or a middle-sized man."†2 It must be of a man with his mouth open or his mouth shut, whose hair is precisely of such and such a shade, and whose figure has precisely such and such proportions. No statement of Locke has been so scouted by all friends of images as his denial that the "idea" of a triangle must be either of an obtuse-angled, right-angled, or acute-angled triangle. In fact, the image of a triangle must be of one, each of whose angles is of a certain number of degrees, minutes, and seconds.

Peirce: CP 5.300 Cross-Ref:††

300. This being so, it is apparent that no man has a *true* image of the road to his office, or of any other real thing. Indeed he has no image of it at all unless he can not only recognize it, but imagines it (truly or falsely) in all its infinite details. This being the case, it becomes very doubtful whether we ever have any such thing as an image in our imagination. Please, reader, to look at a bright red book, or other

brightly colored object, and then to shut your eyes and say whether you *see* that color, whether brightly or faintly -- whether, indeed, there is anything like sight there. Hume and the other followers of Berkeley maintain that there is no difference between the sight and the memory of the red book except in "their different degrees of force and vivacity." "The colors which the memory employs," says Hume, "are faint and dull compared with those in which our original perceptions are clothed."†1 If this were a correct statement of the difference, we should remember the book as being less red than it is; whereas, in fact, we remember the color with very great precision for a few moments (please to test this point, reader), although we do not see anything like it. We carry away absolutely nothing of the color except the *consciousness that we could recognize it*. As a further proof of this, I will request the reader to try a little experiment. Let him call up, if he can, the image of a horse -- not of one which he has ever seen, but of an imaginary one -- and before reading further let him by contemplation †P1 fix the image in his memory . . . [sic]. Has the reader done as requested? for I protest that it is not fair play to read further without doing so. -- Now, the reader can say in general of what color that horse was, whether grey, bay, or black. But he probably cannot say *precisely* of what shade it was. He cannot state this as exactly as he could just after having *seen* such a horse. But why, if he had an image in his mind which no more had the general color than it had the particular shade, has the latter vanished so instantaneously from his memory while the former still remains? It may be replied, that we always forget the details before we do the more general characters; but that this answer is insufficient is, I think, shown by the extreme disproportion between the length of time that the exact shade of something looked at is remembered as compared with that instantaneous oblivion to the exact shade of the thing imagined, and the but slightly superior vividness of the memory of the thing seen as compared with the memory of the thing imagined.

Peirce: CP 5.301 Cross-Ref:††

301. The nominalists, I suspect, confound together thinking a triangle without thinking that it is either equilateral, isosceles, or scalene, and thinking a triangle without thinking whether it is equilateral, isosceles, or scalene.

Peirce: CP 5.302 Cross-Ref:††

302. It is important to remember that we have no intuitive power of distinguishing between one subjective mode of cognition and another;†1 and hence often think that something is presented to us as a picture, while it is really constructed from slight data by the understanding. This is the case with dreams, as is shown by the frequent impossibility of giving an intelligible account of one without adding something which we feel was not in the dream itself. Many dreams, of which the waking memory makes elaborate and consistent stories, must probably have been in fact mere jumbles of these feelings of the ability to recognize this and that which I have just alluded to.

Peirce: CP 5.303 Cross-Ref:††

303. I will now go so far as to say that we have no images even in actual perception. It will be sufficient to prove this in the case of vision; for if no picture is seen when we look at an object, it will not be claimed that hearing, touch, and the other senses, are superior to sight in this respect. That the picture is not painted on the nerves of the retina is absolutely certain, if, as physiologists inform us, these nerves are needlepoints pointing to the light and at distances considerably greater than the *minimum visibile*. The same thing is shown by our not being able to perceive that there is a large blind spot near the middle of the retina. If, then, we have a picture

before us when we see, it is one constructed by the mind at the suggestion of previous sensations. Supposing these sensations to be signs, the understanding by reasoning from them could attain all the knowledge of outward things which we derive from sight, while the sensations are quite inadequate to forming an image or representation absolutely determinate. If we have such an image or picture, we must have in our minds a representation of a surface which is only a part of every surface we see, and we must see that each part, however small, has such and such a color. If we look from some distance at a speckled surface, it seems as if we did not see whether it were speckled or not; but if we have an image before us, it must appear to us either as speckled, or as not speckled. Again, the eye by education comes to distinguish minute differences of color; but if we see only absolutely determinate images, we must, no less before our eyes are trained than afterwards, see each color as particularly such and such a shade. Thus to suppose that we have an image before us when we see, is not only a hypothesis which explains nothing whatever, but is one which actually creates difficulties which require new hypotheses in order to explain them away.

Peirce: CP 5.304 Cross-Ref:††

304. One of these difficulties arises from the fact that the details are less easily distinguished than, and forgotten before, the general circumstances. Upon this theory, the general features exist in the details: the details are, in fact, the whole picture. It seems, then, very strange that that which exists only secondarily in the picture should make more impression than the picture itself. It is true that in an old painting the details are not easily made out; but this is because we know that the blackness is the result of time, and is no part of the picture itself. There is no difficulty in making out the details of the picture as it looks at present; the only difficulty is in guessing what it used to be. But if we have a picture on the retina, the minutest details are there as much as, nay, more than, the general outline and significance of it. Yet that which must actually be seen, it is extremely difficult to recognize; while that which is only abstracted from what is seen is very obvious.

Peirce: CP 5.305 Cross-Ref:††

305. But the conclusive argument against our having any images, or absolutely determinate representations in perception, is that in that case we have the materials in each such representation for an infinite amount of conscious cognition, which we yet never become aware of. Now there is no meaning in saying that we have something in our minds which never has the least effect on what we are conscious of knowing. The most that can be said is, that when we see we are put in a condition in which we are able to get a very large and perhaps indefinitely great amount of knowledge of the visible qualities of objects.

Peirce: CP 5.306 Cross-Ref:††

306. Moreover, that perceptions are not absolutely determinate and singular is obvious from the fact that each sense is an abstracting mechanism. Sight by itself informs us only of colors and forms. No one can pretend that the images of sight are determinate in reference to taste. They are, therefore, so far general that they are neither sweet nor non-sweet, bitter nor non-bitter, having savor nor insipid.

Peirce: CP 5.307 Cross-Ref:††

307. The next question is whether we have any general conceptions except in judgments. In perception, where we know a thing as existing, it is plain that there is a judgment that the thing exists, since a mere general concept of a thing is in no case a cognition of it as existing. It has usually been said, however, that we can call up any concept without making any judgment; but it seems that in this case we only

arbitrarily suppose ourselves to have an experience. In order to conceive the number 7, I suppose, that is, I arbitrarily make the hypothesis or judgment, that there are certain points before my eyes, and I judge that these are seven. This seems to be the most simple and rational view of the matter, and I may add that it is the one which has been adopted by the best logicians. If this be the case, what goes by the name of the association of images is in reality an association of judgments. The association of ideas is said to proceed according to three principles -- those of resemblance, of contiguity, and of causality. But it would be equally true to say that signs denote what they do on the three principles of resemblance, contiguity, and causality. There can be no question that anything *is* a sign of whatever is associated with it by resemblance, by contiguity, or by causality: nor can there be any doubt that any sign recalls the thing signified. So, then, the association of ideas consists in this, that a judgment occasions another judgment, of which it is the sign. Now this is nothing less nor more than inference.

Peirce: CP 5.308 Cross-Ref:††

308. Everything in which we take the least interest creates in us its own particular emotion, however slight this may be. This emotion is a sign and a predicate of the thing. Now, when a thing resembling this thing is presented to us, a similar emotion arises; hence, we immediately infer that the latter is like the former. A formal logician of the old school may say, that in logic no term can enter into the conclusion which had not been contained in the premisses, and that therefore the suggestion of something new must be essentially different from inference. But I reply that that rule of logic applies only to those arguments which are technically called completed. We can and do reason --

Elias was a man;

∴ He was mortal.

And this argument is just as valid as the full syllogism, although it is so only because the major premiss of the latter happens to be true. If to pass from the judgment "Elias was a man" to the judgment "Elias was mortal," without actually saying to one's self that "All men are mortal," is not inference, then the term "inference" is used in so restricted a sense that inferences hardly occur outside of a logic-book.

Peirce: CP 5.309 Cross-Ref:††

309. What is here said of association by resemblance is true of all association. All association is by signs. Everything has its subjective or emotional qualities, which are attributed either absolutely or relatively, or by conventional imputation to anything which is a sign of it. And so we reason,

The sign is such and such;

∴ The sign is that thing.

This conclusion receiving, however, a modification, owing to other considerations, so

as to become --

Peirce: CP 5.309 Cross-Ref:††

The sign is almost (is representative of) that thing.

Peirce: CP 5.310 Cross-Ref:††

§4. MAN, A *SIGNE*

310. We come now to the consideration of the last of the four principles whose consequences we were to trace; namely, that the absolutely incognizable is absolutely inconceivable. That upon Cartesian principles the very realities of things can never be known in the least, most competent persons must long ago have been convinced. Hence the breaking forth of idealism, which is essentially anti-Cartesian, in every direction, whether among empiricists (Berkeley, Hume), or among noologists (Hegel, Fichte). The principle now brought under discussion is directly idealistic; for, since the meaning of a word is the conception it conveys, the absolutely incognizable has no meaning because no conception attaches to it. It is, therefore, a meaningless word; and, consequently, whatever is meant by any term as "the real" is cognizable in some degree, and so is of the nature of a cognition, in the objective sense of that term.

Peirce: CP 5.311 Cross-Ref:††

311. At any moment we are in possession of certain information, that is, of cognitions which have been logically derived by induction and hypothesis from previous cognitions which are less general, less distinct, and of which we have a less lively consciousness. These in their turn have been derived from others still less general, less distinct, and less vivid; and so on back to the ideal †P1 first, which is quite singular, and quite out of consciousness. This ideal first is the particular thing-in-itself. It does not exist *as such*. That is, there is no thing which is in-itself in the sense of not being relative to the mind, though things which are relative to the mind doubtless are, apart from that relation. The cognitions which thus reach us by this infinite series of inductions and hypotheses (which though infinite *a parte ante logice*, is yet as one continuous process not without a beginning *in time*) are of two kinds, the true and the untrue, or cognitions whose objects are *real* and those whose objects are *unreal*. And what do we mean by the real? It is a conception which we must first have had when we discovered that there was an unreal, an illusion; that is, when we first corrected ourselves. Now the distinction for which alone this fact logically called, was between an *ens* relative to private inward determinations, to the negations belonging to idiosyncrasy, and an *ens* such as would stand in the long run. The real, then, is that which, sooner or later, information and reasoning would finally result in, and which is therefore independent of the vagaries of me and you. Thus, the very origin of the conception of reality shows that this conception essentially involves the notion of a COMMUNITY, without definite limits, and capable of a definite increase of knowledge.†1 And so those two series of cognition -- the real and the unreal -- consist of those which, at a time sufficiently future, the community will always continue to re-affirm; and of those which, under the same conditions, will ever after be denied. Now, a proposition whose falsity can never be discovered, and the error of which therefore is absolutely incognizable, contains, upon our principle, absolutely no error. Consequently, that which is thought in these cognitions is the real, as it really is. There is nothing, then, to prevent our knowing outward things as

they really are, and it is most likely that we do thus know them in numberless cases, although we can never be absolutely certain of doing so in any special case.

Peirce: CP 5.312 Cross-Ref:††

312. But it follows that since no cognition of ours is absolutely determinate, generals must have a real existence. Now this scholastic realism is usually set down as a belief in metaphysical fictions. But, in fact, a realist is simply one who knows no more recondite reality than that which is represented in a true representation. Since, therefore, the word "man" is true of something, that which "man" means is real. The nominalist must admit that man is truly applicable to something; but he believes that there is beneath this a thing in itself, an incognizable reality. His is the metaphysical figment. Modern nominalists are mostly superficial men, who do not know, as the more thorough Roscellinus and Occam did, that a reality which has no representation is one which has no relation and no quality. The great argument for nominalism is that there is no man unless there is some particular man. That, however, does not affect the realism of Scotus; for although there is no man of whom all further determination can be denied, yet there is a man, abstraction being made of all further determination. There is a real difference between man irrespective of what the other determinations may be, and man with this or that particular series of determinations, although undoubtedly this difference is only relative to the mind and not *in re*. Such is the position of Scotus.†P1 Occam's great objection is, there can be no real distinction which is not *in re*, in the thing-in-itself; but this begs the question for it is itself based only on the notion that reality is something independent of representative relation †P1

Peirce: CP 5.313 Cross-Ref:††

313.†1 Such being the nature of reality in general, in what does the reality of the mind consist? We have seen that the content of consciousness, the entire phenomenal manifestation of mind, is a sign resulting from inference. Upon our principle, therefore, that the absolutely incognizable does not exist, so that the phenomenal manifestation of a substance is the substance, we must conclude that the mind is a sign developing according to the laws of inference. What distinguishes a man from a word? There is a distinction doubtless. The material qualities, the forces which constitute the pure denotative application, and the meaning of the human sign, are all exceedingly complicated in comparison with those of the word. But these differences are only relative. What other is there? It may be said that man is conscious, while a word is not. But consciousness is a very vague term. It may mean that emotion which accompanies the reflection that we have animal life. This is a consciousness which is dimmed when animal life is at its ebb in old age, or sleep, but which is not dimmed when the spiritual life is at its ebb; which is the more lively the better *animal* a man is, but which is not so, the better *man* he is. We do not attribute this sensation to words, because we have reason to believe that it is dependent upon the possession of an animal body. But this consciousness, being a mere sensation, is only a part of the *material quality* of the man-sign. Again, consciousness is sometimes used to signify the *I think*, or unity in thought; but the unity is nothing but consistency, or the recognition of it. Consistency belongs to every sign, so far as it is a sign; and therefore every sign, since it signifies primarily that it is a sign, signifies its own consistency. The man-sign acquires information, and comes to mean more than he did before. But so do words. Does not electricity mean more now than it did in the days of Franklin? Man makes the word, and the word means nothing which the man has not made it mean, and that only to some man. But since man can think only by means of words or other external symbols, these might turn round and say: "You mean nothing which we have not taught you, and then only so far as you address

some word as the interpretant of your thought." In fact, therefore, men and words reciprocally educate each other; each increase of a man's information involves and is involved by, a corresponding increase of a word's information.

Peirce: CP 5.314 Cross-Ref:††

314. Without fatiguing the reader by stretching this parallelism too far, it is sufficient to say that there is no element whatever of man's consciousness which has not something corresponding to it in the word; and the reason is obvious. It is that the word or sign which man uses *is* the man himself. For, as the fact that every thought is a sign, taken in conjunction with the fact that life is a train of thought, proves that man is a sign; so, that every thought is an *external* sign, proves that man is an external sign. That is to say, the man and the external sign are identical, in the same sense in which the words *homo* and *man* are identical. Thus my language is the sum total of myself; for the man is the thought.

Peirce: CP 5.315 Cross-Ref:††

315. It is hard for man to understand this, because he persists in identifying himself with his will, his power over the animal organism, with brute force. Now the organism is only an instrument of thought. But the identity of a man consists in the *consistency* of what he does and thinks, and consistency is the intellectual character of a thing; that is, is its expressing something.

Peirce: CP 5.316 Cross-Ref:††

316. Finally, as what anything really is, is what it may finally come to be known to be in the ideal state of complete information, so that reality depends on the ultimate decision of the community; so thought is what it is, only by virtue of its addressing a future thought which is in its value as thought identical with it, though more developed. In this way, the existence of thought now depends on what is to be hereafter; so that it has only a potential existence, dependent on the future thought of the community.

Peirce: CP 5.317 Cross-Ref:††

317. The individual man, since his separate existence is manifested only by ignorance and error, so far as he is anything apart from his fellows, and from what he and they are to be, is only a negation. This is man,

" . . . proud man,
Most ignorant of what he's most assured,
His glassy essence."

Peirce: CP 5.318 Cross-Ref:††

III

***FOUNDATIONS OF VALIDITY OF THE LAWS OF LOGIC: FURTHER
CONSEQUENCES OF FOUR INCAPACITIES †1***

§1. OBJECTIONS TO THE SYLLOGISME

318. If, as I maintained in an article in the last number of this Journal,^{†2} every judgment results from inference, to doubt every inference is to doubt everything. It has often been argued that absolute scepticism is self-contradictory; but this is a mistake: and even if it were not so, it would be no argument against the absolute sceptic, inasmuch as he does not admit that no contradictory propositions are true. Indeed, it would be impossible to move such a man, for his scepticism consists in considering every argument and never deciding upon its validity; he would, therefore, act in this way in reference to the arguments brought against him.

Peirce: CP 5.318 Cross-Ref:††

But then there are no such beings as absolute sceptics. Every exercise of the mind consists in inference, and so, though there are inanimate objects without beliefs, there may be ^{†3} no intelligent beings in that condition.

Peirce: CP 5.318 Cross-Ref:††

Yet it is quite possible that a person should doubt every principle of inference. He may not have studied logic, and though a logical formula may sound very obviously true to him, he may feel a little uncertain whether some subtle deception may not lurk in it. Indeed, I certainly shall have, among the most cultivated and respected of my readers, those who deny that those laws of logic, which men generally admit, have universal validity. But I address myself, also, to those who have no such doubts, for even to them it may be interesting to consider how it is that these principles come to be true. Finally, having put forth in former numbers of this Journal some rather heretical principles of philosophical research, one of which is nothing can be admitted to be absolutely inexplicable,^{†1} it behooves me to take up a challenge which has been given me to show how upon my principles the validity of the laws of logic can be other than inexplicable.

Peirce: CP 5.319 Cross-Ref:††

319. I shall be arrested, at the outset, by a sweeping objection to my whole undertaking. It will be said that my deduction of logical principles, being itself an argument, depends for its whole virtue upon the truth of the very principles in question; so that whatever my proof may be, it must take for granted the very things to be proved. But to this I reply, that I am neither addressing absolute sceptics, nor men in any state of fictitious doubt whatever. I require the reader to be candid; and if he becomes convinced of a conclusion, to admit it. There is nothing to prevent a man's perceiving the force of certain special arguments, although he does not yet know that a certain general law of arguments holds good; for the general rule may hold good in some cases and not in others. A man may reason well without understanding the principles of reasoning, just as he may play billiards well without understanding analytical mechanics. If you, the reader, actually find that my arguments have a convincing force with you, it is a mere pretence to call them illogical.^{†2}

Peirce: CP 5.320 Cross-Ref:††

320. That if one sign denotes generally everything denoted by a second, and this second denotes generally everything denoted by a third, then the first denotes

generally everything denoted by the third, is not doubted by anybody who distinctly apprehends the meaning of these words. The deduction of the general form of syllogism, therefore, will consist only of an explanation of the *suppositio communis*. †P1 Now, what the formal logician means by an expression of the form, "Every *M* is *P*," is that anything of which *M* is predicable is *P*; thus, if *S* is *M*, that *S* is *P*. The premiss that "Every *M* is *P*" may, therefore, be denied; but to admit it, unambiguously, in the sense intended, is to admit that the inference is good that *S* is *P* if *S* is *M*. He, therefore, who does not deny that *S* is *P* -- *M*, *S*, *P*, being any terms such that *S* is *M* and every *M* is *P* -- denies nothing that the formal logician maintains in reference to this matter; and he who does deny this, simply is deceived by an ambiguity of language. How we come to make any judgments in the sense of the above "Every *M* is *P*," may be understood from the theory of reality put forth in the article in the last number. It was there shown that real things are of a cognitive and therefore significative nature, so that the real is that which signifies something real. †1 Consequently, to predicate anything of anything real is to predicate it of that of which that subject (the real) is itself predicated; for to predicate one thing of another is to state that the former is a sign of the latter.

Peirce: CP 5.321 Cross-Ref: ††

321. These considerations show the reason of the validity of the formula,

S is *M*; *M* is *P*:

∴ *S* is *P*.

They hold good whatever *S* and *P* may be, provided that they be such that any middle term between them can be found. That *P* should be a negative term, therefore, or that *S* should be a particular term, would not interfere at all with the validity of this formula. Hence, the following formulæ are also valid:

S is *M*; *M* is not *P*:

∴ *S* is not *P*.

Some *S* is *M*; *M* is *P*:

∴ Some *S* is *P*.

Some *S* is *M*; *M* is not *P*:

∴ Some *S* is not *P*.

Peirce: CP 5.322 Cross-Ref: ††

322. Moreover, as all that class of inferences which depend upon the introduction of relative terms can be reduced to the general form, they also are shown

to be valid. Thus, it is proved to be correct to reason thus:

Peirce: CP 5.322 Cross-Ref:††

Every relation of a subject to its predicate is a relation of the relative "not X'd, except by the X of some," to its correlate, where X is any relative I please.

Peirce: CP 5.322 Cross-Ref:††

Every relation of "man" to "animal" is a relation of a subject to its predicate.

∴ Every relation of "man" to "animal" is a relation of the relative "not X'd, except by the X of some," to its correlate, where X is any relative I please.

Peirce: CP 5.322 Cross-Ref:††

Every relation of the relative "not X'd, except by the X of some," to its correlate, where X is any relative I please, is a relation of the relative "not *headed*, except by the *head* of some," to its correlate.

∴ Every relation of "man" to "animal" is a relation of the relative "not headed, except by the head of some," to its correlate.†P1

Peirce: CP 5.323 Cross-Ref:††

323. At the same time, as will be seen from this example, the proof of the validity of these inferences depends upon the assumption of the truth of certain general statements concerning relatives. These formulæ can all be deduced from the principle, that in a system of signs in which no sign is taken in two different senses, two signs which differ only in their manner of representing their object, but which are equivalent in meaning, can always be substituted for one another. Any case of the falsification of this principle would be a case of the dependence of the mode of existence of the thing represented upon the mode of this or that representation of it, which, as has been shown in the article in the last number, is contrary to the nature of reality.†1

Peirce: CP 5.324 Cross-Ref:††

324. The next formula of syllogism to be considered is the following:

S is other than *P*; *M* is *P*:

∴ *S* is other than *M*.

The meaning of "not" or "other than" seems to have greatly perplexed the German logicians, and it may be, therefore, that it is used in different senses. If so, I propose to defend the validity of the above formula only when *other than* is used in a particular sense. By saying that one thing or class is other than a second, I mean that any third whatever is identical with the class which is composed of that third and of whatever is, at once, the first and second. For example, if I say that rats are not mice, I mean that any third class as dogs is identical with dogs *plus*†2 rats-which-are-mice; that is to say, the addition of rats-which-are-mice, to anything, leaves the latter just what it was before. This being all that I mean by *S* is other than *P*, I mean absolutely the same thing when I say that *S* is other than *P*, that I do when I say that *P* is other than *S*; and the same when I say that *S* is other than *M*, that I do when I say that *M* is other than *S*. Hence the above formula is only another way of writing the following:

M is *P*; *P* is not *S*:

∴ *M* is not *S*.

But we have already seen that this is valid.

Peirce: CP 5.325 Cross-Ref:††

325. A very similar formula to the above is the following:

S is *M*; some *S* is *P*:

∴ Some *M* is *P*.

By saying that some of a class is of any character, I mean simply that no statement which implies that none of that class is of that character is true. But to say that none of that class is of that character, is, as I take the word "not," to say that nothing of that character is of that class. Consequently, to say that some of *A* is *B*, is, as I understand words and in the only sense in which I defend this formula, to say that some *B* is *A*. In this way the formula is reduced to the following, which has already been shown to be valid:

Some *P* is *S*; *S* is *M*:

∴ Some *P* is *M*.

Peirce: CP 5.326 Cross-Ref:††

326. The only demonstrative syllogisms which are not included among the above forms are the Theophrastean moods, which are all easily reduced by means of simple conversions.†1

Peirce: CP 5.327 Cross-Ref:††

327. Let us now consider what can be said against all this, and let us take up the objections which have actually been made to the syllogistic formulae, beginning with those which are of a general nature and then examining those sophisms which have been pronounced irresolvable by the rules of ordinary logic.

Peirce: CP 5.327 Cross-Ref:††

It is a very ancient notion that no proof can be of any value, because it rests on premisses which themselves equally require proof, which again must rest on other premisses, and so back to infinity. This really does show that nothing can be proved beyond the *possibility*†2 of a doubt; that no argument could be legitimately used against an absolute sceptic; and that inference is only a transition from one cognition to another, and not the creation of a cognition. But the objection is intended to go much further than this, and to show (as it certainly seems to do) that inference not

only cannot produce *infallible* cognition, but that it cannot *produce* cognition at all. It is true, that since some judgment precedes every judgment inferred, either the first premisses were not inferred, or there have been no first premisses. But it does not follow that because there has been no first in a series, therefore that series has had no beginning in time; for the series may be *continuous*, †P1 and may have begun gradually, as was shown in an article in this volume, †3 where this difficulty has already been resolved.

Peirce: CP 5.328 Cross-Ref: ††

328. A somewhat similar objection has been made by Locke †1 and others, †2 to the effect that the ordinary demonstrative syllogism is a *petitio principii*, inasmuch as the conclusion is already implicitly stated in the major premiss. Take, for example, the syllogism,

All men are mortal;
Socrates is a man:
∴ Socrates is mortal.

This attempt to prove that Socrates is mortal begs the question, it is said, since if the conclusion is denied by anyone, he thereby denies that all men are mortal. But what such considerations really prove is that the syllogism is demonstrative. To call it a *petitio principii* is a mere confusion of language. It is strange that philosophers, who are so suspicious of the words *virtual* and *potential*, should have allowed this "implicit" to pass unchallenged. A *petitio principii* consists in reasoning from the unknown to the unknown. †3 Hence, a logician who is simply engaged in stating what general forms of argument are valid, can, at most, have nothing more to do with the consideration of this fallacy than to note those cases in which from logical principles a premiss of a certain form cannot be better known than a conclusion of the corresponding form. But it is plainly beyond the province of the logician, who has only proposed to state what forms of facts involve what others, to inquire whether man can have a knowledge of universal propositions without a knowledge of every particular contained under them, by means of natural insight, divine revelation, induction, or testimony. The only *petitio principii*, therefore, which he can notice is the assumption of the conclusion itself in the premiss; and this, no doubt, those who call the syllogism a *petitio principii* believe is done in that formula. But the proposition "All men are mortal" does not in itself involve the statement that Socrates is mortal, but only that "whatever has man truly predicated of it is mortal." In other words, the *conclusion* is not involved in the meaning of the premiss, but only the *validity of the syllogism*. So that this objection merely amounts to arguing that the syllogism is not valid, because it is demonstrative. †P1

Peirce: CP 5.329 Cross-Ref: ††

329. A much more interesting objection is that a syllogism is a purely mechanical process. It proceeds according to a bare rule or formula; and a machine might be constructed which would so transpose the terms of premisses. This being so (and it is so), it is argued that this cannot be *thought*; that there is no life in it. Swift has ridiculed the syllogism in the "Voyage to Laputa," by describing a machine for making science:

Peirce: CP 5.329 Cross-Ref:††

"By this contrivance, the most ignorant person, at a reasonable charge, and with little bodily labor, might write books in philosophy, poetry, politics, laws, mathematics, and theology, without the least assistance from genius or study."

Peirce: CP 5.329 Cross-Ref:††

The idea involved in this objection seems to be that it requires mind to apply any formula or use any machine. If, then, this mind is itself only another formula, it requires another mind behind it to set it into operation, and so on *ad infinitum*. This objection fails in much the same way that the first one which we considered failed. It is as though a man should address a land surveyor as follows: "You do not make a true representation of the land; you only measure lengths from point to point -- that is to say, lines. If you observe angles, it is only to solve triangles and obtain the lengths of their sides. And when you come to make your map, you use a pencil which can only make lines, again. So, you have to do solely with lines. But the land is a surface; and no number of lines, however great, will make any surface, however small. You, therefore, fail entirely to represent the land." The surveyor, I think, would reply, "Sir, you have proved that *my* lines cannot make up the land, and that, therefore, my map is *not* the land. I never pretended that it was. But that does not prevent it from truly representing the land, as far as it goes. It cannot, indeed, represent every blade of grass; but it does not represent that there is not a blade of grass where there is. To abstract from a circumstance is not to deny it." Suppose the objector were, at this point, to say, "To abstract from a circumstance *is* to deny it. Wherever your map does not represent a blade of grass, it represents there is no blade of grass. Let us take things on their own valuation." Would not the surveyor reply: "This map is my description of the country. Its own valuation can be nothing but what I say, and all the world understands, that I mean by it. Is it very unreasonable that I should demand to be taken as I mean, especially when I succeed in making myself understood?" What the objector's reply to this question would be, I leave it to anyone to say who thinks his position well taken. Now this line of objection is parallel to that which is made against the syllogism. It is shown that no number of syllogisms can constitute the sum total of any mental action, however restricted. This may be freely granted, and yet it will not follow that the syllogism does not truly represent the mental action, as far as it purports to represent it at all. There is reason to believe that the act on of the mind is, as it were, a continuous movement. Now the doctrine embodied in syllogistic formulae (so far as it applies to the mind at all) is, that if two successive positions, occupied by the mind in this movement, be taken, they will be found to have certain relations. It is true that no number of successions of positions can make up a continuous movement; and this, I suppose, is what is meant by saying that a syllogism is a dead formula, while thinking is a living process. But the reply is that the syllogism is not intended to represent the mind, as to its life or deadness, but only as to the relation of its different judgments concerning the same thing. And it should be added that the relation between syllogism and thought does not spring from considerations of formal logic, but from those of psychology. All that the formal logician has to say is, that if facts capable of expression in such and such forms of words are true, another fact whose expression is related in a certain way to the expression of these others is also true.

Peirce: CP 5.330 Cross-Ref:††

330. Hegel taught that ordinary reasoning is "one-sided." A part of what he meant was that by such inference a part only of all that is true of an object can be

learned, owing to the generality or abstractedness of the predicates inferred. This objection is, therefore, somewhat similar to the last; for the point of it is that no number of syllogisms would give a complete knowledge of the object. This, however, presents a difficulty which the other did not; namely, that if nothing incognizable exists, and all knowledge is by mental action, by mental action everything is cognizable. So that if by syllogism everything is not cognizable, syllogism does not exhaust the modes of mental action. But grant the validity of this argument and it proves too much; for it makes, not the syllogism particularly, but all finite knowledge to be worthless. However much we know, more may come to be found out. Hence, all can never be known. This seems to contradict the fact that nothing is absolutely incognizable; and it would really do so *if our knowledge* were something absolutely limited. For, to say that all can never be known, means that information may increase beyond any assignable point; that is, that an absolute termination of all increase of knowledge is absolutely incognizable, and therefore does not exist. In other words, the proposition merely means that the sum of all that will be known up to any time, however advanced, into the future, has a ratio less than any assignable ratio to all that may be known at a time still more advanced. This, however, †1 does not, in the least, †2 contradict the fact that everything is cognizable; it only contradicts a proposition, which no one can maintain, that it is possible to cognize everything, †P1 that is, that at some time all things will be known. †3 It may, however, very justly be said that the difficulty still remains, how at every future time, however late, there can be something yet to happen. It is no longer a contradiction, but it is a difficulty; that is to say, *lengths of time* are shown not to afford an adequate conception of futurity in general; and the question arises, in what other way we are to conceive of it. I might indeed, perhaps, fairly drop the question here, and say that the difficulty had become so entirely removed from the syllogism in particular, that the formal logician need not feel himself specially called on to consider it. The solution, however, is very simple. It is that we conceive of the future, as a whole, by considering that this *word*, like any other general term, as "inhabitant of St. Louis," may be taken distributively or collectively. We conceive of the infinite, therefore, not directly or on the side of its infinity, but by means of a consideration concerning words or a second intention.

Peirce: CP 5.331 Cross-Ref: ††

331. Another objection to the syllogism is that its "therefore" is merely subjective; that, because a certain conclusion syllogistically follows from a premiss, it does not follow that the fact denoted by the conclusion really depends upon the fact denoted by the premiss, so that the syllogism does not represent things as they really are. But it has been fully shown that if the facts are as the premisses represent, they are also as the conclusion represents. Now this is a purely objective statement: therefore, there is a real connection between the facts stated as premisses and those stated as conclusion. It is true that there is often an appearance of reasoning deductively from effects to causes. Thus we may reason as follows: "There is smoke; there is never smoke without fire: hence, there has been fire." Yet smoke is not the cause of fire, but the effect of it. Indeed, it is evident, that in many cases an event is a demonstrative sign of a certain previous event having occurred. Hence, we can reason deductively from relatively future to relatively past, whereas causation †P1 really determines events in the direct order of time. Nevertheless, if we can thus reason against the stream of time, it is because there really are such facts as that "If there is smoke, there has been fire," in which the following event is the antecedent. Indeed, if we consider the manner in which such a proposition became known to us, we shall find that what it really means is that "If we find smoke, we *shall* find evidence on the whole that there has been fire"; and this, if reality consists in the agreement that the

whole community would eventually come to, is the very same thing as to say that there really has been fire. In short, the whole present difficulty is resolved instantly by this theory of reality, because it makes all reality something which is constituted by an event indefinitely future.

Peirce: CP 5.332 Cross-Ref:††

332. Another objection, for which I am quite willing to allow a great German philosopher the whole credit, is that sometimes the conclusion is false, although both the premisses and the syllogistic form are correct.†P1 Of this he gives the following examples. From the middle term that a wall has been painted blue, it may correctly be concluded that it is blue; but notwithstanding this syllogism it may be green if it has also received a coat of yellow, from which last circumstance by itself it would follow that it is yellow. If from the middle term of the sensuous faculty it be concluded that man is neither good nor bad, since neither can be predicated of the sensuous, the syllogism is correct; but the conclusion is false, since of man in the concrete, spirituality is equally true, and may serve as middle term in an opposite syllogism. From the middle term of the gravitation of the planets, satellites, and comets, towards the sun, it follows correctly that these bodies fall into the sun; but they do not fall into it, because (!) they equally gravitate to their own centres, or, in other words (!!), they are supported by centrifugal force. Now, does Hegel mean to say that these syllogisms satisfy the rules for syllogism given by those who defend syllogism? or does he mean to grant that they do not satisfy *those* rules, but to set up some rules of his own for syllogism which shall insure its yielding false conclusions from true premisses? If the latter, he ignores the real issue, which is whether the syllogism as defined by the rules of formal logic is correct, and not whether the syllogism as represented by Hegel is correct. But if he means that the above examples satisfy the usual definition of a true syllogism, he is mistaken. The first, stated in form, is as follows:

Whatever has been painted blue is blue;

This wall has been painted blue:

∴ This wall is blue.

Now "painted blue" may mean painted with blue paint, or painted so as to be blue. If, in the example, the former were meant, the major premiss would be false. As he has stated that it is true, the latter meaning of "painted blue" must be the one intended. Again, "blue" may mean blue at some time, or blue at this time. If the latter be meant, the major premiss is plainly false; therefore, the former is meant. But the conclusion is said to contradict the statement that the wall is yellow. If blue were here taken in the more general sense, there would be no such contradiction. Hence, he means in the conclusion that this wall is now blue; that is to say, he reasons thus:

Whatever has been made blue has been blue;

This has been made blue:

∴ This is blue now.

Now substituting letters for the subjects and predicates, we get the form,

M is *P*;

S is *M*:

∴ *S* is *Q*.

This is not a syllogism in the ordinary sense of that term, or in any sense in which anybody maintains that the syllogism is valid.

Peirce: CP 5.332 Cross-Ref:††

The second example given by Hegel, when written out in full, is as follows:

Sensuality is neither good nor bad;

Man *has* (not *is*) sensuality:

∴ Man is neither good nor bad.

Or, the same argument may be stated as follows:

The sensuous, *as such*, is neither good nor bad;

Man is sensuous:

∴ Man is neither good nor bad.

Peirce: CP 5.332 Cross-Ref:††

When letters are substituted for subject and predicate in either of these arguments, it takes the form,

M is *P*;

S is *N*:

∴ *S* is *P*.

This, again, bears but a very slight resemblance to a syllogism.

The third example, when stated at full length, is as follows

Whatever tends towards the sun, *on the whole*, falls into the sun;

The planets tend toward the sun:

∴ The planets fall into the sun.

This is a fallacy similar to the last.

Peirce: CP 5.332 Cross-Ref:††

I wonder that this eminent logician did not add to his list of examples of correct syllogism the following:

It either rains, or it does not rain;

It does not rain:

∴ It rains.

This is fully as deserving of serious consideration as any of those which he has brought forward. The rainy day and the pleasant day are both, in the first place, day. Secondly, each is the negation of a day. It is indifferent which be regarded as the positive. The pleasant is Other to the rainy, and the rainy is in like manner Other to the pleasant. Thus, both are equally Others. Both are Others of each other, or each is Other for itself. So this day being other than rainy, that to which it is Other is itself. But it is Other than itself. Hence, it is itself Rainy.

Peirce: CP 5.333 Cross-Ref:††

§2. THE THREE KINDS OF SOPHISMSE

333. Some sophisms have, however, been adduced, mostly by the Eleatics and Sophists, which really are extremely difficult to resolve by syllogistic rules; and according to some modern authors this is actually impossible. These sophisms fall into three classes: first, those which relate to continuity; second, those which relate to consequences of supposing things to be other than they are; third, those which relate to propositions which imply their own falsity. Of the first class, the most celebrated are Zeno's arguments concerning motion. One of these is, that if Achilles overtakes a tortoise in any finite time, and the tortoise has the start of him by a distance which may be called a , then Achilles has to pass over the sum of distances represented by the polynomial

$$1/2a + 1/4a + 1/8a + 1/16a + 1/32a \text{ etc.}$$

up to infinity. Every term of this polynomial is finite, and it has an infinite number of terms; consequently, Achilles must in a finite time pass over a distance equal to the sum of an infinite number of finite distances. Now this distance must be infinite, because no finite distance, however small, can be multiplied by an infinite number without giving an infinite distance. So that even if none of these finite distances were larger than the smallest (which is finite since all are finite), the sum of the whole would be infinite. But Achilles cannot pass over an infinite distance in a finite time; therefore, he cannot overtake the tortoise in any time, however great.

Peirce: CP 5.333 Cross-Ref:††

The solution of this fallacy is as follows: The conclusion is supposed to follow from the undoubted †1 fact that Achilles cannot overtake the tortoise without passing over an infinite number of terms of that series of finite distances. That is, no case of his overtaking the tortoise would be a case of his not passing over a non-finite number of terms; that is (by simple conversion), no case of his not passing over a non-finite number of terms would be a case of his overtaking the tortoise. But if he does not pass over a non-finite number of terms, he either passes over a finite number, or he passes over none; and conversely. Consequently, nothing more has been said than that every case of his passing over only a finite number of terms, or of his not passing over any, is a case of his not overtaking the tortoise. Consequently, nothing more can be concluded than that he passes over a distance greater than the sum of any finite number of the above series of terms. But because a quantity is greater than any quantity of a certain series, it does not follow that it is greater than any quantity. †P1

Peirce: CP 5.333 Cross-Ref:††

In fact, the reasoning in this sophism may be exhibited as follows: We start with the series of numbers,

$$\begin{aligned} &1/2a \\ &1/2a+1/4a \\ &1/2a+1/4a+1/8a \\ &1/2a+1/4a+1/8a+1/16a \\ &\text{etc. etc. etc.} \end{aligned}$$

Then, the implied argument is

Any number of this series is less than a ;

But any number you please is less than the number of terms of this series:

Hence, any number you please is less than a .

This involves an obvious confusion between the number of terms and the value of the greatest term.

Peirce: CP 5.334 Cross-Ref:††

334. Another argument by Zeno against motion, is that a body fills a space no larger than itself. In that place there is no room for motion. Hence, while in the place where it is, it does not move. But it never is other than in the place where it is. Hence, it never moves. Putting this into form, it will read:

No body in a place no larger than itself is moving;

But every body is a body in a place no larger than itself:

∴ No body is moving.

The error of this consists in the fact that the minor premiss is only true in the sense that during a time sufficiently short the space occupied by a body is as little larger than itself as you please. All that can be inferred from this is, that during no time a body will move no distance.

Peirce: CP 5.335 Cross-Ref:††

335. All the arguments of Zeno depend on supposing that a *continuum* has ultimate parts. But a *continuum* is precisely that, every part of which has parts, in the same sense.†1 Hence, he makes out his contradictions only by making a self-contradictory supposition. In ordinary and mathematical language, we allow ourselves to speak of such parts -- *points* -- and whenever we are led into contradiction thereby, we have simply to express ourselves more accurately to resolve the difficulty.

Peirce: CP 5.336 Cross-Ref:††

336. Suppose a piece of glass to be laid on a sheet of paper so as to cover half of it. Then, every part of the paper is *covered*, or *not covered*; for "not" means merely outside of, or other than. But is the line under the edge of the glass covered or not? It is no more on one side of the edge than it is on the other. Therefore, it is either on both sides, or neither side. It is not on neither side; for if it were it would be *not* on either side, therefore not on the covered side, therefore not covered, therefore on the uncovered side. It is not partly on one side and partly on the other, because it has no width. Hence, it is wholly on both sides, or both covered and not covered.

Peirce: CP 5.336 Cross-Ref:††

The solution of this is, that we have supposed a part too narrow to be partly uncovered and partly covered; that is to say, a part which has no parts in a continuous surface, which by definition has no such parts. The reasoning, therefore, simply serves to reduce this supposition to an absurdity.

Peirce: CP 5.336 Cross-Ref:††

It may be said that there really is such a thing as a line. If a shadow falls on a surface, there really is a division between the light and the darkness. That is true. But it does not follow that because we attach a definite meaning to the part of a surface being covered, therefore we know what we mean when we say that a line is covered. We may define a covered line as one which separates two surfaces both of which are covered, or as one which separates two surfaces *either* of which is covered. In the former case, the line under the edge is uncovered; in the latter case, it is covered.

Peirce: CP 5.337 Cross-Ref:††

337. In the sophisms thus far considered, the appearance of contradiction depends mostly upon an ambiguity; in those which we are now to consider, two true propositions really do in form conflict with one another. We are apt to think that formal logic forbids this, whereas a familiar argument, the *reductio ad absurdum*, depends on showing that contrary predicates are true of a subject, and *that therefore that subject does not exist*. Many logicians, it is true, make affirmative propositions assert the existence of their subjects.†P1 The objection to this is that it cannot be extended to hypotheticals. The proposition

If *A* then *B*

may conveniently be regarded as equivalent to

Every case of the truth of *A* is a case of the truth of *B*.

But this cannot be done if the latter proposition asserts the existence of its subject; that is, asserts that *A* really happens. If, however, a categorical affirmative be regarded as asserting the existence of its subject, the principle of the *reductio ad absurdum* is that two propositions of the forms,

If *A* were true, *B* would not be true,

and

If *A* were true, *B* would be true,

may both be true at once; and that if they are so, *A* is not true. It will be well, perhaps, to illustrate this point. No man of common sense would deliberately upset his inkstand if there were ink in it; that is, if any ink would run out. Hence, by simple conversion,

If he were deliberately to upset his inkstand, no ink would be spilt.

But suppose there is ink in it. Then, it is also true, that

If he were deliberately to upset his inkstand, the ink would be spilt.

These propositions are both true, and the law of contradiction is not violated which asserts only that nothing has contradictory predicates: only, it follows from these propositions that the man will not deliberately overturn his inkstand.

Peirce: CP 5.338 Cross-Ref:††

338. There are two ways in which deceptive sophisms may result from this circumstance. In the first place, contradictory propositions are never both true. Now, as a universal proposition may be true when the subject does not exist, it follows that the contradictory of a universal -- that is, a particular -- cannot be taken in such a sense as to be true when the subject does not exist. But a particular simply asserts a part of what is asserted in the universal over it; therefore, the universal over it asserts the subject to exist. Consequently, there are two kinds of universals, those which do not assert the subject to exist, and these have no particular propositions under them, and those which do assert that the subject exists, and these strictly speaking have no contradictories. For example, there is no use of such a form of proposition as "Some griffins would be dreadful animals," as particular under the useful form "The griffin would be a dreadful animal"; and the apparent contradictories "All of John Smith's family are ill," and "Some of John Smith's family are not ill," are both false at once if John Smith has no family. Here, though an inference from a universal to the particular under it is always valid, yet a procedure which greatly resembles this would be sophistical if the universal were one of those propositions which does not assert the existence of its subject. The following sophism depends upon this; I call it the True Gorgias:

Gorgias. What say you, Socrates, of black? Is any black, white?

Socrates. No, by Zeus!

Gor. Do you say, then, that no black is white? *Soc.* None at all.

Gor. But is everything either black or non-black? *Soc.*

Of course.

Gor. And everything either white or non-white? *Soc.* Yes.

Gor. And everything either rough or smooth? *Soc.* Yes.

Gor. And everything either real or unreal? *Soc.* Oh, bother! yes.

Gor. Do you say, then, that all black is either rough black or smooth black?

Soc. Yes.

Gor. And that all white is either real white or unreal white? *Soc.* Yes.

Gor. And yet is no black, white? *Soc.* None at all.

Gor. Nor no white, black? *Soc.* By no means.

Gor. What? Is no smooth black, white? *Soc.* No; you cannot prove that, Gorgias.

Gor. Nor no rough black, white? *Soc.* Neither.

Gor. Nor no real white, black? *Soc.* No.

Gor. Nor no unreal white, black? *Soc.* No, I say. No white at all is black.

Gor. What if black is smooth, is it not white? *Soc.* Not in the least.

Gor. And if the last is false, is the first false? *Soc.* It follows.

Gor. If, then, black is white, does it follow, that black is not smooth? *Soc.* It does.

Gor. Black-white is not smooth? *Soc.* What do you mean?

Gor. Can any dead man speak? *Soc.* No, indeed.

Gor. And is any speaking man dead? *Soc.* I say, no.

Gor. And is any good king tyrannical? *Soc.* No.

Gor. And is any tyrannical king good? *Soc.* I just said no.

Gor. And you said, too, that no rough black is white, did you not? *Soc.* Yes.

Gor. Then, is any black-white, rough? *Soc.* No.

Gor. And is any unreal black, white? *Soc.* No.

Gor. Then, is any black-white unreal? *Soc.* No.

Gor. No black-white is rough? *Soc.* None.

Gor. All black-white, then, is non-rough? *Soc.* Yes.

Gor. And all black-white, non-unreal? *Soc.* Yes.

Gor. All black-white is then smooth? *Soc.* Yes.

Gor. And all real? *Soc.* Yes.

Gor. Some smooth, then, is black-white? *Soc.* Of course.

Gor. And some real is black-white? *Soc.* So it seems.

Gor. Some black-white smooth is black-white? *Soc.* Yes.

Gor. Some black smooth is black-white? *Soc.* Yes.

Gor. Some black smooth is white? *Soc.* Yes.

Gor. Some black real is black-white? *Soc.* Yes.

Gor. Some black real is white? *Soc.* Yes.

Gor. Some real black is white? *Soc.* Yes.

Gor. And some smooth black is white? *Soc.* Yes.

Gor. Then, some black is white? *Soc.* I think so myself.

339. The principle of the *reductio ad absurdum* also occasions deceptions in another way, owing to the fact that we have many words, such as *can, may, must*, etc., which imply more or less vaguely an otherwise unexpressed condition, so that these propositions are in fact hypotheticals. Accordingly, if the unexpressed condition is some state of things which does not actually come to pass, the two propositions may appear to be contrary to one another. Thus, the moralist says, "You ought to do this, and you can do it." This "You can do it" is principally hortatory in its force: so far as it is a statement of fact, it means merely, "If you try, you will do it." Now, if the act is an outward one and the act is not performed, the scientific man, in view of the fact that every event in the physical world depends exclusively on physical antecedents, says that in this case the laws of nature prevented the thing from being done, and that therefore, "Even if you had tried, you would not have done it." Yet the reproachful conscience still says you might have done it; that is, that "If you had tried, you would have done it." This is called the paradox of freedom and fate; and it is usually supposed that one of these propositions must be true and the other false.†1 But since, in fact, you have not tried, there is no reason why the supposition that you have tried should not be reduced to an absurdity. In the same way, if you had tried and had performed the action, the conscience might say, "If you had not tried, you would not have done it"; while the understanding would say, "Even if you had not tried, you would have done it." These propositions are perfectly consistent, and only serve to reduce the supposition that you did not try to an absurdity.†P1

Peirce: CP 5.340 Cross-Ref:††

340.†1 The third class of sophisms consists of the so-called *Insolubilia*. Here is an example of one of them with its resolution:

THIS PROPOSITION IS NOT TRUE

IS IT TRUE OR NOT?

Suppose it true. | Suppose it not true.

Then, | Then,

The proposition is true; | It is not true.

But, that it is not true |

is the proposition: | ∴ It is true that it is

| not true.

∴ That it is not true is | But, the proposition is that it is true; | not true.

∴ It is not true. | ∴ The proposition is true.

Besides, | Besides,

It is true. | The proposition is not true.

|

∴ It is true that it is true,| But that it is not true is the

| proposition.

|

∴ It is not true that it is | ∴. That it is not true, is not
 not true; | true.

|

But, the proposition is that it | ∴. That it is true, is true.
 is not true, |

|

∴. The proposition is not | ∴. It is true.
 true. |

∴. Whether it is true or not, it is both true and not.

∴. It is both true and not, which is absurd.

Peirce: CP 5.340 Cross-Ref:††

Since the conclusion is false, the reasoning is bad, or the premisses are not all true. But the reasoning is a dilemma; either, then, the disjunctive principle that it is either true or not is false, or the reasoning under one or the other branch is bad, or the reasoning is altogether valid. If the principle that it is either true or not is false, it is other than true and other than not true; that is, not true and not not true; that is, not true and true. But this is absurd. Hence, the disjunctive principle is valid. There are two arguments under each horn of the dilemma; both the arguments under one or the other branch must be false. But, in each case, the second argument involves all the premisses and forms of inference involved in the first; hence, if the first is false, the second necessarily is so. We may, therefore, confine our attention to the first arguments in the two branches. The forms of argument contained in these are two: first, the simple syllogism in Barbara, and, second, the consequence from the truth of a proposition to the proposition itself. These are both correct. Hence, the whole form of reasoning is correct, and nothing remains to be false but a premiss. But since the repetition of an alternative supposition is not a premiss, there is, properly speaking, but one premiss in the whole. This is that the proposition is the same as that that proposition is not true. This, then, must be false. Hence the proposition signifies either less or more than this. If it does not signify as much as this, it signifies nothing, and hence it is not true, and hence another proposition which says of it what it says of itself is true. But if the proposition in question signifies something more than that it is itself not true, then the premiss that

Whatever is said in the proposition is that it is not true,

is not true. And as a proposition is true only if whatever is said in it is true, but is false if anything said in it is false, the first argument on the second side of the dilemma contains a false premiss, and the second an undistributed middle. But the first argument on the first side remains good. Hence, if the proposition means more than that it is not true, it is not true, and another proposition which repeats this of it is true. Hence, whether the proposition does or does not mean that it is not true, it is not true, and a proposition which repeats this of it is true.

Peirce: CP 5.340 Cross-Ref:††

Since this repeating proposition is true, it has a meaning. Now, a proposition has a meaning if any part of it has a meaning. Hence, the original proposition (a part of which repeated has a meaning) has itself a meaning. Hence, it must imply something besides that which it explicitly states. But it has no particular determination to any further implication. Hence, what more it signifies it must signify by virtue of being a proposition at all. That is to say, every proposition must imply something analogous to what this implies. Now, the repetition of this proposition does not contain this implication, for otherwise it could not be true; hence, what every proposition implies must be something concerning itself. What every proposition implies concerning itself must be something which is false of the proposition now under discussion, for the whole falsity of this proposition lies therein, since all that it explicitly lays down is true. It must be something which would not be false if the proposition were true, for in that case some true proposition would be false. Hence, it must be that it is itself true. That it is, *every proposition asserts its own truth.*

Peirce: CP 5.340 Cross-Ref:††

The proposition in question, therefore, is true in all other respects but its implication of its own truth.†P1

Peirce: CP 5.341 Cross-Ref:††

§3. THE SOCIAL THEORY OF LOGICE

341. The difficulty of showing how the law of deductive reasoning is true depends upon our inability to conceive of its not being true. In the case of probable reasoning the difficulty is of quite another kind; here, where we see precisely what the procedure is, we wonder how such a process can have any validity at all. How magical it is that by examining a part of a class we can know what is true of the whole of the class, and by study of the past can know the future; in short, that we can know what we have not experienced!

Peirce: CP 5.341 Cross-Ref:††

Is not this an intellectual intuition! Is it not that besides ordinary experience which is dependent on there being a certain physical connection between our organs and the thing experienced, there is a second avenue of truth dependent only on there being a certain intellectual connection between our previous knowledge and what we learn in that way? Yes, this is true. Man has this faculty, just as opium has a somnific virtue; but some further questions may be asked, nevertheless. How is the existence of this faculty accounted for? In one sense, no doubt, by natural selection. Since it is absolutely essential to the preservation of so delicate an organism as man's, no race which had it not has been able to sustain itself. This accounts for the prevalence of

this faculty, provided it was only a possible one. But how can it be possible? What could enable the mind to know physical things which do not physically influence it and which it does not influence? The question cannot be answered by any statement concerning the human mind, for it is equivalent to asking what makes the facts usually to be, as inductive and hypothetic conclusions from true premisses represent them to be? Facts of a certain kind are usually true when facts having certain relations to them are true; what is the cause of this? That is the question.

Peirce: CP 5.342 Cross-Ref:††

342. The usual reply is that nature is everywhere regular; as things have been, so they will be; as one part of nature is, so is every other. But this explanation will not do. Nature is not regular. No disorder would be less orderly than the existing arrangement. It is true that the special laws and regularities are innumerable; but nobody thinks of the irregularities, which are infinitely more frequent. Every fact true of any one thing in the universe is related to every fact true of every other. But the immense majority of these relations are fortuitous and irregular. A man in China bought a cow three days and five minutes after a Greenlander had sneezed. Is that abstract circumstance connected with any regularity whatever? And are not such relations infinitely more frequent than those which are regular? But if a very large number of qualities were to be distributed among a very large number of things in almost any way, there would chance to be some few regularities. If, for example, upon a checker-board of an enormous number of squares, painted all sorts of colors, myriads of dice were to be thrown, it could hardly fail to happen, that upon some color, or shade of color, out of so many, some one of the six numbers should not be uppermost on any die. This would be a regularity; for, the universal proposition would be true that upon that color that number is never turned up. But suppose this regularity abolished, then a far more remarkable regularity would be created, namely, that on every color every number is turned up. Either way, therefore, a regularity must occur. Indeed, a little reflection will show that, although we have here only variations of color and of the numbers of the dice, many regularities must occur. And the greater the number of objects, the more respects in which they vary, and the greater the number of varieties in each respect, the greater will be the number of regularities. Now, in the universe, all these numbers are infinite. Therefore, however disorderly the chaos, the *number* of regularities must be infinite. The orderliness of the universe, therefore, if it exists, must consist in the large *proportion* of relations which present a regularity to those which are quite irregular. But this proportion in the actual universe is, as we have seen, as small as it can be; and, therefore, the orderliness of the universe is as little as that of any arrangement whatever.

Peirce: CP 5.343 Cross-Ref:††

343. But even if there were such an orderliness in things, it never could be discovered. For it would belong to things either collectively or distributively. If it belonged to things collectively, that is to say, if things formed a system, the difficulty would be that a system can only be known by seeing some considerable proportion of the whole. Now we never can know how great a part of the whole of nature we have discovered. If the order were distributive, that is, belonged to all things only by belonging to each thing, the difficulty would be that a character can only be known by comparing something which has it †1 with something which has it not. *Being, quality, relation*, and other universals are not known except as characters of words or other signs, attributed by a figure of speech to things. Thus, in neither case could the order of things be known. But the order of things would not help the validity of our reasoning -- that is, would not help us to reason correctly -- unless we knew what the

order of things required the relation between the known reason *from* to the unknown reasoned *to*, to be.

Peirce: CP 5.344 Cross-Ref:††

344. But even if this order both existed and were known, the knowledge would be of no use except as a general principle, from which things could be deduced. It would not explain how knowledge could be increased (in contradistinction to being rendered more distinct), and so it would not explain how it could itself have been acquired.

Peirce: CP 5.345 Cross-Ref:††

345. Finally, if the validity of induction and hypothesis were dependent on a particular constitution of the universe, we could imagine a universe in which these modes of inference should not be valid, just as we can imagine a universe in which there would be no attraction, but things should merely drift about. Accordingly, J.S. Mill, who explains the validity of induction by the uniformity of nature,†P1 maintains that he can imagine a universe without any regularity, so that no probable inference would be valid in it.†P2 In the universe as it is, probable arguments sometimes fail, nor can any definite proportion of cases be stated in which they hold good; all that can be said is that in the long run they prove approximately correct. Can a universe be imagined in which this would not be the case? It must be a universe where probable argument can have some application, in order that it may fail half the time. It must, therefore, be a universe experienced. Of the finite number of propositions true of a finite amount of experience of such a universe, no one would be universal in form, unless the subject of it were an individual. For if there were a plural universal proposition, inferences by analogy from one particular to another would hold good invariably in reference to that subject. So that these arguments might be no better than guesses in reference to other parts of the universe, but they would invariably hold good in a finite proportion of it, and so would on the whole be somewhat better than guesses. There could, also, be no individuals in that universe, for there must be some general class -- that is, there must be some things more or less alike -- or probable argument would find no premisses there; therefore, there must be two mutually exclusive classes, since every class has a residue outside of it; hence, if there were any individual, that individual would be wholly excluded from one or other of these classes. Hence, the universal plural proposition would be true, that no one of a certain class was that individual. Hence, no universal proposition would be true. Accordingly, every combination of characters would occur in such a universe. But this would not be disorder, but the simplest order; it would not be unintelligible, but, on the contrary, everything conceivable would be found in it with equal frequency. The notion, therefore, of a universe in which probable arguments should fail as often as hold true, is absurd.†1 We can suppose it in general terms, but we cannot specify how it should be other than self-contradictory.†P1

Peirce: CP 5.346 Cross-Ref:††

346. Since we cannot conceive of probable inferences as not generally holding good, and since no special supposition will serve to explain their validity, many logicians have sought to base this validity on that of deduction, and that in a variety of ways. The only attempt of this sort, however, which deserves to be noticed is that which seeks to determine the probability of a future event by the theory of probabilities, from the fact that a certain number of similar events have been observed. Whether this can be done or not depends on the meaning assigned to the word probability. But if this word is to be taken in such a sense that a form of

conclusion which is probable is valid; since the validity of an inference (or its correspondence with facts) consists solely in this, that when such premisses are true, such a conclusion is generally true, then probability can mean nothing but the ratio of the frequency of occurrence of a specific event to a general one over it. In this sense of the term, it is plain that the probability of an inductive conclusion cannot be *deduced* from the premisses; for from the inductive premisses

S', S'', S''' are M ,

S', S'', S''' are P ,

nothing follows deductively, except that any M , which is S' , or S'' , or S''' is P ; or, less explicitly, that some M is P .

Peirce: CP 5.347 Cross-Ref:††

347. Thus, we seem to be driven to this point. On the one hand, no determination of things, no *fact*, can result in the validity of probable argument; nor, on the other hand, is such argument reducible to that form which holds good, however the facts may be. This seems very much like a reduction to absurdity of the validity of such reasoning; and a paradox of the greatest difficulty is presented for solution.

Peirce: CP 5.348 Cross-Ref:††

348. There can be no doubt of the importance of this problem. According to Kant, the central question of philosophy is "How are synthetical judgments *a priori* possible?" But antecedently to this comes the question how synthetical judgments in general, and still more generally, how synthetical reasoning is possible at all. When the answer to the general problem has been obtained, the particular one will be comparatively simple. This is the lock upon the door of philosophy.†1

Peirce: CP 5.349 Cross-Ref:††

349. All probable inference, whether induction or hypothesis, is inference from the parts to the whole. It is essentially the same, therefore, as statistical inference. Out of a bag of black and white beans I take a few handfuls, and from this sample I can judge approximately the proportions of black and white in the whole. This is identical with induction. Now we know upon what the validity of this inference depends. It depends upon the fact that in the long run, any one bean would be taken out as often as any other. For were this not so, the mean of a large number of results of such testings of the contents of the bag would not be precisely the ratio of the numbers of the two colors of beans in the bag. Now we may divide the question of the validity of induction into two parts: first, why of all inductions premisses for which occur, the generality should hold good, and second, why men are not fated always to light upon the small proportion of worthless inductions. Then, the first of these two questions is readily answered. For since all the members of any class are the same as all that are to be known; and since from any part of those which are to be known an induction is competent to the rest, in the long run any one member of a class will occur as the subject of a premiss of a possible induction as often as any other, and, therefore, the validity of induction depends simply upon the fact that the parts make up and constitute the whole. This in its turn depends simply upon there being such a state of things that any general terms are possible. But it has been shown in 311 that being at all is being in general. And thus this part of the validity of

induction depends merely on there being any reality.

Peirce: CP 5.350 Cross-Ref:††

350. From this it appears that we cannot say that the generality of inductions are true, but only that in the long run they approximate to the truth. This is the truth of the statement, that the universality of an inference from induction is only the analogue of true universality. Hence, also, it cannot be said that we know an inductive conclusion to be true, however loosely we state it; we only know that by accepting inductive conclusions, in the long run our errors balance one another. In fact, insurance companies proceed upon induction; -- they do not know what will happen to this or that policyholder; they only know that they are secure in the long run.

Peirce: CP 5.351 Cross-Ref:††

351. The other question relative to the validity of induction, is why men are not fated always to light upon those inductions which are highly deceptive. The explanation of the former branch of the problem we have seen to be that there is something real. Now, since if there is anything real, then (on account of this reality consisting in the ultimate agreement of all men, and on account of the fact that reasoning from parts to whole, is the only kind of synthetic reasoning which men possess) it follows necessarily that a sufficiently long succession of inferences from parts to whole will lead men to a knowledge of it, so that in that case they cannot be fated on the whole to be thoroughly unlucky in their inductions. This second branch of the problem is in fact equivalent to asking why there is anything real, and thus its solution will carry the solution of the former branch one step further.

Peirce: CP 5.352 Cross-Ref:††

352.†1 The answer to this question may be put into a general and abstract, or a special detailed form. If men were not to be able to learn from induction, it must be because as a general rule, when they had made an induction, the order of things (as they appear in experience), would then undergo a revolution. Just herein would the unreality of such a universe consist; namely, that the order of the universe should depend on how much men should know of it. But this general rule would be capable of being itself discovered by induction; and so it must be a law of such a universe, that when this was discovered it would cease to operate. But this second law would itself be capable of discovery. And so in such a universe there would be nothing which would not sooner or later be known; and it would have an order capable of discovery by a sufficiently long course of reasoning. But this is contrary to the hypothesis, and therefore that hypothesis is absurd. This is the particular answer. But we may also say, in general, that if nothing real exists, then, since every question supposes that something exists -- for it maintains its own urgency -- it supposes only illusions to exist. But the existence even of an illusion is a reality; for an illusion affects all men, or it does not. In the former case, it is a reality according to our theory of reality; in the latter case, it is independent of the state of mind of any individuals except those whom it happens to affect. So that the answer to the question, Why is anything real? is this: That question means, "supposing anything to exist, why is something real?" The answer is, that that very existence is reality by definition.

Peirce: CP 5.352 Cross-Ref:††

All that has here been said, particularly of induction, applies to all inference from parts to whole, and therefore to hypothesis, and so to all probable inference.

Peirce: CP 5.352 Cross-Ref:††

Thus, I claim to have shown, in the first place, that it is possible to hold a

consistent theory of the validity of the laws of ordinary logic.

Peirce: CP 5.353 Cross-Ref:††

353. But now let us suppose the idealistic theory of reality, which I have in this paper taken for granted to be false. In that case, inductions would not be true unless the world were so constituted that every object should be presented in experience as often as any other; and further, unless we were so constituted that we had no more tendency to make bad inductions than good ones. These facts might be explained by the benevolence of the Creator; but, as has already been argued, they could not explain, but are absolutely refuted by the fact that no state of things can be conceived in which probable arguments should not lead to the truth. This affords a most important argument in favor of that theory of reality, and thus of those denials of certain faculties from which it was deduced, as well as of the general style of philosophizing by which those denials were reached.

Peirce: CP 5.354 Cross-Ref:††

354. Upon our theory of reality and of logic, it can be shown that no inference of any individual can be thoroughly logical without certain determinations of his mind which do not concern any one inference immediately; for we have seen that that mode of inference which alone can teach us anything, or carry us at all beyond what was implied in our premisses -- in fact, does not give us to know any more than we knew before; only, we know that, by faithfully adhering to that mode of inference, we shall, on the whole, approximate to the truth. Each of us is an insurance company, in short. But, now, suppose that an insurance company, among its risks, should take one exceeding in amount the sum of all the others. Plainly, it would then have no security whatever. Now, has not every single man such a risk? What shall it profit a man if he shall gain the whole world and lose his own soul? If a man has a transcendent personal interest infinitely outweighing all others, then, upon the theory of validity of inference just developed, he is devoid of all security, and can make no valid inference whatever. What follows? That logic rigidly requires, before all else, that no determinate fact, nothing which can happen to a man's self, should be of more consequence to him than everything else. He who would not sacrifice his own soul to save the whole world, is illogical in all his inferences, collectively. So the social principle is rooted intrinsically in logic.†1

Peirce: CP 5.355 Cross-Ref:††

355. That being the case, it becomes interesting to inquire how it is with men as a matter of fact. There is a psychological theory that man cannot act without a view to his own pleasure. This theory is based on a falsely assumed subjectivism. Upon our principles of the objectivity of knowledge, it could not be based; and if they are correct, it is reduced to an absurdity. It seems to me that the usual opinion of the selfishness of man is based in large measure upon this false theory. I do not think that the facts bear out the usual opinion. The immense self-sacrifices which the most wilful men often make, show that wilfulness is a very different thing from selfishness. The care that men have for what is to happen after they are dead, cannot be selfish. And finally and chiefly, the constant use of the word "*we*" -- as when we speak of our possessions on the Pacific -- our destiny as a republic -- in cases in which no personal interests at all are involved, show conclusively that men do not make their personal interests their only ones, and therefore may, at least, subordinate them to the interests of the community.

Peirce: CP 5.356 Cross-Ref:††

356. But just the revelation of the possibility of this complete self-sacrifice in

man, and the belief in its saving power, will serve to redeem the logicity of all men. For he who recognizes the logical necessity of complete self-identification of one's own interests with those of the community, and its potential existence in man, even if he has it not himself, will perceive that only the inferences of that man who has it are logical, and so views his own inferences as being valid only so far as they would be accepted by that man. But so far as he has this belief, he becomes identified with that man. And that ideal perfection of knowledge by which we have seen that reality is constituted must thus belong to a community in which this identification is complete.

Peirce: CP 5.357 Cross-Ref:††

357. This would serve as a complete establishment of private logicity, were it not that the assumption, that man or the community (which may be wider than man) shall ever arrive at a state of information greater than some definite finite information, is entirely unsupported by reasons. There cannot be a scintilla of evidence to show that at some time all living beings shall not be annihilated at once, and that forever after there shall be throughout the universe any intelligence whatever. Indeed, this very assumption involves itself a transcendent and supreme interest, and therefore from its very nature is unsusceptible of any support from reasons. This infinite hope which we all have (for even the atheist will constantly betray his calm expectation that what is Best will come about) is something so august and momentous, that all reasoning in reference to it is a trifling impertinence. We do not want to know what are the weights of reasons *pro* and *con* -- that is, how much *odds* we should wish to receive on such a venture in the long run -- because there is no long run in the case; the question is single and supreme, and ALL is at stake upon it. We are in the condition of a man in a life and death struggle; if he have not sufficient strength, it is wholly indifferent to him how he acts, so that the only assumption upon which he can act rationally is the hope of success. So this sentiment is rigidly demanded by logic. If its object were any determinate fact, any private interest, it might conflict with the results of knowledge and so with itself; but when its object is of a nature as wide as the community can turn out to be, it is always a hypothesis uncontradicted by facts and justified by its indispensableness for making any action rational.

Peirce: CP 5.358 Cross-Ref:††

IV

THE FIXATION OF BELIEF†1

§1. SCIENCE AND LOGICE

358. Few persons care to study logic, because everybody conceives himself to be proficient enough in the art of reasoning already. But I observe that this satisfaction is limited to one's own ratiocination, and does not extend to that of other men.

Peirce: CP 5.359 Cross-Ref:††

359. We come to the full possession of our power of drawing inferences, the last of all our faculties; for it is not so much a natural gift as a long and difficult art. The history of its practice would make a grand subject for a book. The medieval schoolman, following the Romans, made logic the earliest of a boy's studies after grammar, as being very easy. So it was as they understood it. Its fundamental principle, according to them, was, that all knowledge rests either on †1 authority or reason; but that whatever is deduced by reason depends ultimately on a premiss derived from authority. Accordingly, as soon as a boy was perfect in the syllogistic procedure, his intellectual kit of tools was held to be complete.

Peirce: CP 5.360 Cross-Ref: ††

360. To Roger Bacon, †2 that remarkable mind who in the middle of the thirteenth century was almost a scientific man, the schoolmen's conception of reasoning appeared only an obstacle to truth. He saw that experience alone teaches anything -- a proposition which to us seems easy to understand, because a distinct conception of experience has been handed down to us from former generations; which to him likewise †3 seemed perfectly clear, because its difficulties had not yet unfolded themselves. Of all kinds of experience, the best, he thought, was interior illumination, which teaches many things about Nature which the external senses could never discover, such as the transubstantiation of bread.

Peirce: CP 5.361 Cross-Ref: ††

361. Four centuries later, the more celebrated Bacon, in the first book of his *Novum Organum*, gave his clear account of experience as something which must be open to verification and reexamination. But, superior as Lord Bacon's conception is to earlier notions, a modern reader who is not in awe of his grandiloquence is chiefly struck by the inadequacy of his view of scientific procedure. That we have only to make some crude experiments, to draw up briefs of the results in certain blank forms, to go through these by rule, checking off everything disproved and setting down the alternatives, and that thus in a few years physical science would be finished up -- what an idea! "He wrote on science like a Lord Chancellor," †P1 indeed, as Harvey, a genuine man of science said. †4

Peirce: CP 5.362 Cross-Ref: ††

362. The early scientists, Copernicus, Tycho Brahe, Kepler, Galileo, Harvey †5, and Gilbert, had methods more like those of their modern brethren. Kepler undertook to draw a curve through the places of Mars, †P2 and to state the times occupied by the planet in describing the different parts of that curve; †1 but perhaps †2 his greatest service to science was in impressing on men's minds that this was the thing to be done if they wished to improve astronomy; that they were not to content themselves with inquiring whether one system of epicycles was better than another but that they were to sit down to the figures and find out what the curve, in truth, was. He accomplished this by his incomparable energy and courage, blundering along in the most inconceivable way (to us), from one irrational hypothesis to another, until, after trying twenty-two of these, he fell, by the mere exhaustion of his invention, upon the orbit which a mind well furnished with the weapons of modern logic would have tried almost at the outset. †P1

Peirce: CP 5.363 Cross-Ref: ††

363. In the same way, every work of science great enough to be well †3 remembered for a few generations affords some exemplification of the defective state of the art of reasoning of the time when it was written; and each chief step in science has been a lesson in logic. It was so when Lavoisier and his contemporaries took up

the study of Chemistry. The old chemist's maxim had been, "Lege, lege, lege, labora, ora, et relege." Lavoisier's method was not to read and pray, but to dream that some long and complicated chemical process would have a certain effect, to put it into practice with dull patience, after its inevitable failure, to dream that with some modification it would have another result, and to end by publishing the last dream as a fact: his way was to carry his mind into his laboratory, and literally †4 to make of his alembics and cucurbits instruments of thought, giving a new conception of reasoning as something which was to be done with one's eyes open, in †5 manipulating real things instead of words and fancies.

Peirce: CP 5.364 Cross-Ref:††

364. The Darwinian controversy is, in large part, a question of logic. Mr. Darwin proposed to apply the statistical method to biology. †P1 The same thing has been done in a widely different branch of science, the theory of gases. Though unable to say what the movements of any particular molecule of gas would be on a certain hypothesis regarding the constitution of this class of bodies, Clausius and Maxwell were yet able, eight years before the publication of Darwin's immortal work, †1 by the application of the doctrine of probabilities, to predict that in the long run such and such a proportion of the molecules would, under given circumstances, acquire such and such velocities; that there would take place, every second, such and such a relative †2 number of collisions, etc.; and from these propositions were able to deduce certain properties of gases, especially in regard to their heat-relations. In like manner, Darwin, while unable to say what the operation of variation and natural selection in any individual case will be, demonstrates that in the long run they will, or †3 would, †4 adapt animals to their circumstances. Whether or not existing animal forms are due to such action, or what position the theory ought to take, forms the subject of a discussion in which questions of fact and questions of logic are curiously interlaced.

Peirce: CP 5.365 Cross-Ref:††
§2. GUIDING PRINCIPLESE

365. The object of reasoning is to find out, from the consideration of what we already know, something else which we do not know. Consequently, reasoning is good if it be such as to †P2 give a true conclusion from true premisses, and not otherwise. Thus, the question of validity is purely one of fact and not of thinking. A being the facts stated in the †5 premisses and B being that concluded, †1 the question is, whether these facts are really so related that if A were B would generally be. †2 If so, the inference is valid; if not, not. It is not in the least the question whether, when the premisses are accepted by the mind, we feel an impulse to accept the conclusion also. It is true that we do generally reason correctly by nature. But that is an accident; the true conclusion would remain true if we had no impulse to accept it; and the false one would remain false, though we could not resist the tendency to believe in it.

Peirce: CP 5.366 Cross-Ref:††

366. We are, doubtless, in the main logical animals, but we are not perfectly so. Most of us, for example, are naturally more sanguine and hopeful than logic would justify. We seem to be so constituted that in the absence of any facts to go upon we are happy and self-satisfied; so that the effect of experience is continually to contract our hopes and aspirations. Yet a lifetime of the application of this corrective does not

usually eradicate our sanguine disposition. Where hope is unchecked by any experience, it is likely that our optimism is extravagant. Logicality in regard to practical matters (if this be understood, not in the old sense, but as consisting in a wise union of security with fruitfulness of reasoning †3) is the most useful quality an animal can possess, and might, therefore, result from the action of natural selection; but outside of these it is probably of more advantage to the animal to have his mind filled with pleasing and encouraging visions, independently of their truth; and thus, upon unpractical subjects, natural selection might occasion a fallacious tendency of thought. †P1

Peirce: CP 5.367 Cross-Ref: ††

367. That which determines us, from given premisses, to draw one inference rather than another, is some habit of mind, whether it be constitutional or acquired. The habit is good or otherwise, according as it produces true conclusions from true premisses or not; and an inference is regarded as valid or not, without reference to the truth or falsity of its conclusion specially, but according as the habit which determines it is such as to produce true conclusions in general or not. The particular habit of mind which governs this or that inference may be formulated in a proposition whose truth depends on the validity of the inferences which the habit determines; and such a formula is called a *guiding principle* of inference. Suppose, for example, that we observe that a rotating disk of copper quickly comes to rest when placed between the poles of a magnet, and we infer that this will happen with every disk of copper. The guiding principle is, that what is true of one piece of copper is true of another. Such a guiding principle with regard to copper would be much safer than with regard to many other substances -- brass, for example.

Peirce: CP 5.368 Cross-Ref: ††

368. A book might be written to signalize all the most important of these guiding principles of reasoning. It would probably be, we must confess, of no service to a person whose thought is directed wholly to practical subjects, and whose activity moves along thoroughly-beaten paths. The problems that †1 present themselves to such a mind are matters of routine which he has learned once for all to handle in learning his business. But let a man venture into an unfamiliar field, or where his results are not continually checked by experience, and all history shows that the most masculine intellect will oftentimes lose his orientation and waste his efforts in directions which bring him no nearer to his goal, or even carry him entirely astray. He is like a ship in the open sea, with no one on board who understands the rules of navigation. And in such a case some general study of the guiding principles of reasoning would be sure to be found useful.

Peirce: CP 5.369 Cross-Ref: ††

369. The subject could hardly be treated, however, without being first limited; since almost any fact may serve as a guiding principle. But it so happens that there exists a division among facts, such that in one class are all those which are absolutely essential as guiding principles, while in the others are all which have any other interest as objects of research. This division is between those which are necessarily taken for granted in asking why †2 a certain conclusion is thought to follow †1 from certain premisses, and those which are not implied in such a †2 question. A moment's thought will show that a variety of facts are already assumed when the logical question is first asked. It is implied, for instance, that there are such states of mind as doubt and belief -- that a passage from one to the other is possible, the object of thought remaining the same, and that this transition is subject to some rules by †3

which all minds are alike bound. As these are facts which we must already know before we can have any clear conception of reasoning at all, it cannot be supposed to be any longer of much interest to inquire into their truth or falsity. On the other hand, it is easy to believe that those rules of reasoning which are deduced from the very idea of the process are the ones which are the most essential; and, indeed, that so long as it conforms to these it will, at least, not lead to false conclusions from true premisses. In point of fact, the importance of what may be deduced from the assumptions involved in the logical question turns out to be greater than might be supposed, and this for reasons which it is difficult to exhibit at the outset. The only one which I shall here mention is, that conceptions which are really products of logical reflection, without being readily seen to be so, mingle with our ordinary thoughts, and are frequently the causes of great confusion. This is the case, for example, with the conception of quality. A quality, as such, is never an object of observation. We can see that a thing is blue or green, but the quality of being blue and the quality of being green are not things which we see; they are products of logical reflections. The truth is, that common-sense, or thought as it first emerges above the level of the narrowly practical, is deeply imbued with that bad logical quality to which the epithet *metaphysical* is commonly applied; and nothing can clear it up but a severe course of logic.

Peirce: CP 5.370 Cross-Ref:††
 §3. DOUBT AND BELIEF

370. We generally know when we wish to ask a question and when we wish to pronounce a judgment, for there is a dissimilarity between the sensation of doubting and that of believing.

Peirce: CP 5.371 Cross-Ref:††

371. But this is not all which distinguishes doubt from belief. There is a practical difference. Our beliefs guide our desires and shape our actions. The Assassins, or followers of the Old Man of the Mountain, used to rush into death at his least command, because they believed that obedience to him would insure everlasting felicity. Had they doubted this, they would not have acted as they did. So it is with every belief, according to its degree. The feeling of believing is a more or less sure indication of there being established in our nature some habit which will determine our actions.†P1 Doubt never has such an effect.

Peirce: CP 5.372 Cross-Ref:††

372. Nor must we overlook a third point of difference. Doubt is an uneasy and dissatisfied state from which we struggle to free ourselves and pass into the state of belief;†P2 while the latter is a calm and satisfactory state which we do not wish to avoid, or to change to a belief in anything else.†P3 On the contrary, we cling tenaciously, not merely to believing, but to believing just what we do believe.

Peirce: CP 5.373 Cross-Ref:††

373. Thus, both doubt and belief have positive effects upon us, though very different ones. Belief does not make us act at once, but puts us into such a condition that we shall behave in some †1 certain way, when the occasion arises. Doubt has not the least such †2 active †3 effect,†4 but stimulates us to inquiry †5 until it is destroyed. This reminds us of the irritation of a nerve and the reflex action produced

thereby; while for the analogue of belief, in the nervous system, we must look to what are called nervous associations †6 -- for example, to that habit of the nerves in consequence of which the smell of a peach will make the mouth water.†P1

Peirce: CP 5.374 Cross-Ref:††
§4. THE END OF INQUIRYE

374. The irritation of doubt causes a struggle to attain a state of belief. I shall term this struggle *Inquiry*, though it must be admitted that this is sometimes not a very apt designation.

Peirce: CP 5.375 Cross-Ref:††

375. The irritation of doubt is the only immediate motive for the struggle to attain belief. It is certainly best for us that our beliefs should be such as may truly guide our actions so as to satisfy our desires; and this reflection will make us reject every †1 belief which does not seem to have been so formed as to insure this result. But it will only do so by creating a doubt in the place of that belief.†P1 With the doubt, therefore, the struggle begins, and with the cessation of doubt it ends. Hence, the sole object of inquiry is the settlement of opinion. We may fancy that this is not enough for us, and that we seek, not merely an opinion, but a true opinion. But put this fancy to the test, and it proves groundless; for as soon as a firm belief is reached we are entirely satisfied, whether the belief be true or false. And it is clear that nothing out of the sphere of our knowledge can be our object, for nothing which does not affect the mind can be the motive for mental effort. The most that can be maintained is, that we seek for a belief that we shall *think* to be true. But we think each one of our beliefs to be true, and, indeed, it is mere tautology to say so.†P2

Peirce: CP 5.375 Cross-Ref:††

That the settlement of opinion is the sole end of inquiry is a very important proposition. It sweeps away, at once, various vague and erroneous conceptions of proof. A few of these may be noticed here.

Peirce: CP 5.376 Cross-Ref:††

376. 1. Some philosophers have imagined that to start an inquiry it was only necessary to utter a question whether orally or by setting †2 it down upon paper, and have even recommended us to begin our studies with questioning everything! But the mere putting of a proposition into the interrogative form does not stimulate the mind to any struggle after belief. There must be a real and living doubt, and without this all discussion is idle.†P3

Peirce: CP 5.376 Cross-Ref:††

2. It is a very common idea that a demonstration must rest on some ultimate and absolutely indubitable propositions. These, according to one school, are first principles of a general nature; according to another, are first sensations. But, in point of fact, an inquiry, to have that completely satisfactory result called demonstration, has only to start with propositions perfectly free from all actual doubt. If the premisses are not in fact doubted at all, they cannot be more satisfactory than they are.†P1

Peirce: CP 5.376 Cross-Ref:††

3. Some people seem to love to argue a point after all the world is fully convinced of it. But no further advance can be made. When doubt ceases, mental action on the subject comes to an end; and, if it did go on, it would be without a purpose.†P2

Peirce: CP 5.377 Cross-Ref:††
§5. METHODS OF FIXING BELIEFE

377. If the settlement of opinion is the sole object of inquiry, and if belief is of the nature of a habit, why should we not attain the desired end, by taking as †1 answer to a question any †2 we may fancy, and constantly reiterating it to ourselves, dwelling on all which may conduce to that belief, and learning to turn with contempt and hatred from anything that †3 might disturb it? This simple and direct method is really pursued by many men. I remember once being entreated not to read a certain newspaper lest it might change my opinion upon free-trade. "Lest I might be entrapped by its fallacies and misstatements," was the form of expression. "You are not," my friend said, "a special student of political economy. You might, therefore, easily be deceived by fallacious arguments upon the subject. You might, then, if you read this paper, be led to believe in protection. But you admit that free-trade is the true doctrine; and you do not wish to believe what is not true." I have often known this system to be deliberately adopted. Still oftener, the instinctive dislike of an undecided state of mind, exaggerated into a vague dread of doubt, makes men cling spasmodically to the views they already take. The man feels that, if he only holds to his belief without wavering, it will be entirely satisfactory. Nor can it be denied that a steady and immovable faith yields great peace of mind. It may, indeed, give rise to inconveniences, as if a man should resolutely continue to believe that fire would not burn him, or that he would be eternally damned if he received his *ingesta* otherwise than through a stomach-pump. But then the man who adopts this method will not allow that its inconveniences are greater than its advantages. He will say, "I hold steadfastly to the truth, and the truth is always wholesome." And in many cases it may very well be that the pleasure he derives from his calm faith overbalances any inconveniences resulting from its deceptive character. Thus, if it be true that death is annihilation, then the man who believes that he will certainly go straight to heaven when he dies, provided he have fulfilled certain simple observances in this life, has a cheap pleasure which will not be followed by the least disappointment.†P1 A similar consideration seems to have weight with many persons in religious topics, for we frequently hear it said, "Oh, I could not believe so-and-so, because I should be wretched if I did." When an ostrich buries its head in the sand as danger approaches, it very likely takes the happiest course. It hides the danger, and then calmly says there is no danger; and, if it feels perfectly sure there is none, why should it raise its head to see? A man may go through life, systematically keeping out of view all that might cause a change in his opinions, and if he only succeeds -- basing his method, as he does, on two fundamental psychological laws -- I do not see what can be said against his doing so. It would be an egotistical impertinence to object that his procedure is irrational, for that only amounts to saying that his method of settling belief is not ours. He does not propose to himself to be rational, and, indeed, will often talk with scorn of man's weak and illusive reason. So let him think as he pleases.

Peirce: CP 5.378 Cross-Ref:††

378. But this method of fixing belief, which may be called the method of tenacity, will be unable to hold its ground in practice. The social impulse is against it. The man who adopts it will find that other men think differently from him, and it will be apt to occur to him, in some saner moment, that their opinions are quite as good as his own, and this will shake his confidence in his belief. This conception, that another man's thought or sentiment may be equivalent to one's own, is a distinctly new step, and a highly important one. It arises from an impulse too strong in man to be suppressed, without danger of destroying the human species. Unless we make ourselves hermits, we shall necessarily influence each other's opinions; so that the problem becomes how to fix belief, not in the individual merely, but in the community.

Peirce: CP 5.379 Cross-Ref:††

379. Let the will of the state act, then, instead of that of the individual. Let an institution be created which shall have for its object to keep correct doctrines before the attention of the people, to reiterate them perpetually, and to teach them to the young; having at the same time power to prevent contrary doctrines from being taught, advocated, or expressed. Let all possible causes of a change of mind be removed from men's apprehensions. Let them be kept ignorant, lest they should learn of some reason to think otherwise than they do. Let their passions be enlisted, so that they may regard private and unusual opinions with hatred and horror. Then, let all men who reject the established belief be terrified into silence. Let the people turn out and tar-and-feather such men, or let inquisitions be made into the manner of thinking of suspected persons, and when they are found guilty of forbidden beliefs, let them be subjected to some signal punishment. When complete agreement could not otherwise be reached, a general massacre of all who have not thought in a certain way has proved a very effective means of settling opinion in a country. If the power to do this be wanting, let a list of opinions be drawn up, to which no man of the least independence of thought can assent, and let the faithful be required to accept all these propositions, in order to segregate them as radically as possible from the influence of the rest of the world.

Peirce: CP 5.379 Cross-Ref:††

This method has, from the earliest times, been one of the chief means of upholding correct theological and political doctrines, and of preserving their universal or catholic character. In Rome, especially, it has been practised from the days of Numa Pompilius to those of Pius Nonus. This is the most perfect example in history; but wherever there is a priesthood -- and no religion has been without one -- this method has been more or less made use of. Wherever there is an aristocracy, or a guild, or any association of a class of men whose interests depend, or are supposed to depend, on certain propositions, there will be inevitably found some traces of this natural product of social feeling. Cruelties always accompany this system; and when it is consistently carried out, they become atrocities of the most horrible kind in the eyes of any rational man. Nor should this occasion surprise, for the officer of a society does not feel justified in surrendering the interests of that society for the sake of mercy, as he might his own private interests. It is natural, therefore, that sympathy and fellowship should thus produce a most ruthless power.

Peirce: CP 5.380 Cross-Ref:††

380. In judging this method of fixing belief, which may be called the method of authority, we must, in the first place, allow its immeasurable mental and moral superiority to the method of tenacity. Its success is proportionately greater; and, in

fact, it has over and over again worked the most majestic results. The mere structures of stone which it has caused to be put together -- in Siam, for example, in Egypt, and in Europe -- have many of them a sublimity hardly more than rivaled by the greatest works of Nature. And, except the geological epochs, there are no periods of time so vast as those which are measured by some of these organized faiths.†P1 If we scrutinize the matter closely, we shall find that there has not been one of their creeds which has remained always the same; yet the change is so slow as to be imperceptible during one person's life, so that individual belief remains sensibly fixed. For the mass of mankind, then, there is perhaps no better method than this. If it is their highest impulse to be intellectual slaves, then slaves they ought to remain.

Peirce: CP 5.381 Cross-Ref:††

381. But no institution can undertake to regulate opinions upon every subject. Only the most important ones can be attended to, and on the rest men's minds must be left to the action of natural causes. This imperfection will be no source of weakness so long as men are in such a state of culture that one opinion does not influence another -- that is, so long as they cannot put two and two together. But in the most priest-ridden states some individuals will be found who are raised above that condition. These men possess a wider sort of social feeling; they see that men in other countries and in other ages have held to very different doctrines from those which they themselves have been brought up to believe; and they cannot help seeing that it is the mere accident of their having been taught as they have, and of their having been surrounded with the manners and associations they have, that has caused them to believe as they do and not far differently. Nor can their candour †1 resist the reflection that there is no reason to rate their own views at a higher value than those of other nations and other centuries; thus giving †2 rise to doubts in their minds.

Peirce: CP 5.382 Cross-Ref:††

382. They will further perceive that such doubts as these must exist in their minds with reference to every belief which seems to be determined by the caprice either of themselves or of those who originated the popular opinions. The willful adherence to a belief, and the arbitrary forcing of it upon others, must, therefore, both be given up. A different †3 new method of settling opinions must be adopted, that †4 shall not only produce an impulse to believe, but shall also decide what proposition it is which is to be believed. Let the action of natural preferences be unimpeded, then, and under their influence let men, conversing together and regarding matters in different lights, gradually develop beliefs in harmony with natural causes. This method resembles that by which conceptions of art have been brought to maturity. The most perfect example of it is to be found in the history of metaphysical philosophy. Systems of this sort have not usually rested upon any observed facts, at least not in any great degree. They have been chiefly adopted because their fundamental propositions seemed "agreeable to reason." This is an apt expression; it does not mean that which agrees with experience, but that which we find ourselves inclined to believe. Plato, for example, finds it agreeable to reason that the distances of the celestial spheres from one another should be proportional to the different lengths of strings which produce harmonious chords. Many philosophers have been led to their main conclusions by considerations like this;†P1 but this is the lowest and least developed form which the method takes, for it is clear that another man might find Kepler's theory, that the celestial spheres are proportional to the inscribed and circumscribed spheres of the different regular solids, more agreeable to *his* reason. But the shock of opinions will soon lead men to rest on preferences of a far more universal nature. Take, for example, the doctrine that man only acts selfishly -- that is,

from the consideration that acting in one way will afford him more pleasure than acting in another. This rests on no fact in the world, but it has had a wide acceptance as being the only reasonable theory. †P1

Peirce: CP 5.383 Cross-Ref: ††

383. This method is far more intellectual and respectable from the point of view of reason than either of the others which we have noticed. Indeed, as long as no better method can be applied, it ought to be followed, since it is then the expression of instinct which must be the ultimate cause of belief in all cases. †1 But its failure has been the most manifest. It makes of inquiry something similar to the development of taste; but taste, unfortunately, is always more or less a matter of fashion, and accordingly metaphysicians have never come to any fixed agreement, but the pendulum has swung backward and forward between a more material and a more spiritual philosophy, from the earliest times to the latest. And so from this, which has been called the *a priori* method, we are driven, in Lord Bacon's phrase, to a true induction. We have examined into this *a priori* method as something which promised to deliver our opinions from their accidental and capricious element. But development, while it is a process which eliminates the effect of some casual circumstances, only magnifies that of others. This method, therefore, does not differ in a very essential way from that of authority. The government may not have lifted its finger to influence my convictions; I may have been left outwardly quite free to choose, we will say, between monogamy and polygamy, and, appealing to my conscience only, I may have concluded that the latter practice is in itself licentious. But when I come to see that the chief obstacle to the spread of Christianity among a people of as high culture as the Hindoos has been a conviction of the immorality of our way of treating women, I cannot help seeing that, though governments do not interfere, sentiments in their development will be very greatly determined by accidental causes. Now, there are some people, among whom I must suppose that my reader is to be found, who, when they see that any belief of theirs is determined by any circumstance extraneous to the facts, will from that moment not merely admit in words that that belief is doubtful, but will experience a real doubt of it, so that it ceases in some degree at least †1 to be a belief.

Peirce: CP 5.384 Cross-Ref: ††

384. To satisfy our doubts, therefore, it is necessary that a method should be found by which our beliefs may be determined †2 by nothing human, but by some external permanency -- by something upon which our thinking has no effect. †P1 Some mystics imagine that they have such a method in a private inspiration from on high. But that is only a form of the method of tenacity, in which the conception of truth as something public is not yet developed. Our external permanency would not be external, in our sense, if it was restricted in its influence to one individual. It must be something which affects, or might affect, every man. And, though these affections are necessarily as various as are individual conditions, yet the method must be such that the ultimate conclusion of every man shall be the same. †P1 Such is the method of science. Its fundamental hypothesis, restated in more familiar language, is this: There are Real things, whose characters are entirely independent of our opinions about them; those Reals †1 affect our senses according to regular laws, and, though our sensations are as different as are †2 our relations to the objects, yet, by taking advantage of the laws of perception, we can ascertain by reasoning how things really and †3 truly †4 are; and any man, if he have sufficient experience and he †5 reason enough about it, will be led to the one True conclusion. The new conception here involved is that of Reality. It may be asked how I know that there are any Reals. †6 If

this hypothesis is the sole support of my method of inquiry, my method of inquiry must not be used to support my hypothesis. The reply †7 is this: 1. If investigation cannot be regarded as proving that there are Real things, it at least does not lead to a contrary conclusion; but the method and the conception on which it is based remain ever in harmony. No doubts of the method, therefore, necessarily arise from its practice, as is the case with all the others. 2. The feeling which gives rise to any method of fixing belief is a dissatisfaction at two repugnant propositions. But here already is a vague concession that there is some *one* thing which a proposition should represent.†8 Nobody, therefore, can really doubt that there are Reals,†9 for,†10 if he did, doubt would not be a source of dissatisfaction. The hypothesis, therefore, is one which every mind admits. So that the social impulse does not cause men †11 to doubt it. 3. Everybody uses the scientific method about a great many things, and only ceases to use it when he does not know how to apply it. 4. Experience of the method has not led us †12 to doubt it, but, on the contrary, scientific investigation has had the most wonderful triumphs in the way of settling opinion. These afford the explanation of my not doubting the method or the hypothesis which it supposes; and not having any doubt, nor believing that anybody else whom I could influence has, it would be the merest babble for me to say more about it. If there be anybody with a living doubt upon the subject, let him consider it.†P1

Peirce: CP 5.385 Cross-Ref:††

385. To describe the method of scientific investigation is the object of this series of papers. At present I have only room to notice some points of contrast between it and other methods of fixing belief.

Peirce: CP 5.385 Cross-Ref:††

This is the only one of the four methods which presents any distinction of a right and a wrong way. If I adopt the method of tenacity, and shut myself out from all influences, whatever I think necessary to doing this, is necessary according to that method. So with the method of authority: the state may try to put down heresy by means which, from a scientific point of view, seem very ill-calculated to accomplish its purposes; but the only test *on that method* is what the state thinks; so that it cannot pursue the method wrongly. So with the *a priori* method. The very essence of it is to think as one is inclined to think. All metaphysicians will be sure to do that, however they may be inclined to judge each other to be perversely wrong. The Hegelian system recognizes every natural tendency of thought as logical, although it be certain to be abolished by counter-tendencies. Hegel thinks there is a regular system in the succession of these tendencies, in consequence of which, after drifting one way and the other for a long time, opinion will at last go right. And it is true that metaphysicians do †1 get the right ideas at last; Hegel's system of Nature represents tolerably the science of his †2 day; and one may be sure that whatever scientific investigation shall have †3 put out of doubt will presently receive *a priori* demonstration on the part of the metaphysicians. But with the scientific method the case is different. I may start with known and observed facts to proceed to the unknown; and yet the rules which I follow in doing so may not be such as investigation would approve. The test of whether I am truly following the method is not an immediate appeal to my feelings and purposes, but, on the contrary, itself involves the application of the method. Hence it is that bad reasoning as well as good reasoning is possible; and this fact is the foundation of the practical side of logic.

Peirce: CP 5.386 Cross-Ref:††

386. It is not to be supposed that the first three methods of settling opinion

present no advantage whatever over the scientific method. On the contrary, each has some peculiar convenience of its own. The *a priori* method is distinguished for its comfortable conclusions. It is the nature of the process to adopt whatever belief we are inclined to, and there are certain flatteries to the vanity of man which we all believe by nature, until we are awakened from our pleasing dream by †4 rough facts. The method of authority will always govern the mass of mankind; and those who wield the various forms of organized force in the state will never be convinced that dangerous reasoning ought not to be suppressed in some way. If liberty of speech is to be untrammelled from the grosser forms of constraint, then uniformity of opinion will be secured by a moral terrorism to which the respectability of society will give its thorough approval. Following the method of authority is the path of peace. Certain non-conformities are permitted; certain others (considered unsafe) are forbidden. These are different in different countries and in different ages; but, wherever you are, let it be known that you seriously hold a tabooed belief, and you may be perfectly sure of being treated with a cruelty less brutal but more refined than hunting you like a wolf. Thus, the greatest intellectual benefactors of mankind have never dared, and dare not now, to utter the whole of their thought; and thus a shade of *prima facie* doubt is cast upon every proposition which is considered essential to the security of society. Singularly enough, the persecution does not all come from without; but a man torments himself and is oftentimes most distressed at finding himself believing propositions which he has been brought up to regard with aversion. The peaceful and sympathetic man will, therefore, find it hard to resist the temptation to submit his opinions to authority. But most of all I admire the method of tenacity for its strength, simplicity, and directness. Men who pursue it are distinguished for their decision of character, which becomes very easy with such a mental rule. They do not waste time in trying to make up their minds what they want, but, fastening like lightning upon whatever alternative comes first, they hold to it to the end, whatever happens, without an instant's irresolution. This is one of the splendid qualities which generally accompany brilliant, unlasting success. It is impossible not to envy the man who can dismiss reason, although we know how it must turn out at last.

Peirce: CP 5.387 Cross-Ref:††

387. Such are the advantages which the other methods of settling opinion have over scientific investigation. A man should consider well of them; and then he should consider that, after all, he wishes his opinions to coincide with the fact, and that there is no reason why the results of those three first †1 methods should do so. To bring about this effect is the prerogative of the method of science. Upon such considerations he has to make his choice -- a choice which is far more than the adoption of any intellectual opinion, which is one of the ruling decisions of his life, to which, when once made, he is bound to adhere. The force of habit will sometimes cause a man to hold on to old beliefs, after he is in a condition to see that they have no sound basis. But reflection upon the state of the case will overcome these habits, and he ought to allow reflection its full weight. People sometimes shrink from doing this, having an idea that beliefs are wholesome which they cannot help feeling rest on nothing. But let such persons suppose an analogous though different case from their own. Let them ask themselves what they would say to a reformed Mussulman who should hesitate to give up his old notions in regard to the relations of the sexes; or to a reformed Catholic who should still shrink from reading the Bible. Would they not say that these persons ought to consider the matter fully, and clearly understand the new doctrine, and then ought to embrace it, in its entirety? But, above all, let it be considered that what is more wholesome than any particular belief is integrity of belief, and that to avoid looking into the support of any belief from a fear that it may turn out rotten is

quite as immoral as it is disadvantageous. The person who confesses that there is such a thing as truth, which is distinguished from falsehood simply by this, that if acted on it should, on full consideration, carry †1 us to the point we aim at and not astray, and then, though convinced of this, dares not know the truth and seeks to avoid it, is in a sorry state of mind indeed.

Peirce: CP 5.387 Cross-Ref:††

†P1 Yes, the other methods do have their merits: a clear logical conscience does cost something -- just as any virtue, just as all that we cherish, costs us dear. But we should not desire it to be otherwise. The genius of a man's logical method should be loved and revered as his bride, whom he has chosen from all the world. He need not condemn the others; on the contrary, he may honor them deeply, and in doing so he only honors her the more. But she is the one that he has chosen, and he knows that he was right in making that choice. And having made it, he will work and fight for her, and will not complain that there are blows to take, hoping that there may be as many and as hard to give, and will strive to be the worthy knight and champion of her from the blaze of whose splendors he draws his inspiration and his courage.

Peirce: CP 5.388 Cross-Ref:††

V

HOW TO MAKE OUR IDEAS CLEAR.

§1. CLEARNESS AND DISTINCTNESSE

388. Whoever has looked into a modern treatise on logic of the common sort, †P1 will doubtless remember the two distinctions between *clear* and *obscure* conceptions, and between *distinct* and *confused* conceptions. They have lain in the books now for nigh two centuries, unimproved and unmodified, and are generally reckoned by logicians as among the gems of their doctrine.

Peirce: CP 5.389 Cross-Ref:††

389. A clear idea is defined as one which is so apprehended that it will be recognized wherever it is met with, and so that no other will be mistaken for it. If it fails of this clearness, it is said to be obscure.

Peirce: CP 5.389 Cross-Ref:††

This is rather a neat bit of philosophical terminology; yet, since it is clearness that they were defining, I wish the logicians had made their definition a little more plain. Never to fail to recognize an idea, and under no circumstances to mistake another for it, let it come in how recondite a form it may, would indeed imply such prodigious force and clearness of intellect as is seldom met with in this world. On the other hand, merely to have such an acquaintance with the idea as to have become familiar with it, and to have lost all hesitancy in recognizing it in ordinary cases, hardly seems to deserve the name of clearness of apprehension, since after all it only amounts to a subjective feeling of mastery which may be entirely mistaken. I take it, however, that when the logicians speak of "clearness," they mean nothing more than

such a familiarity with an idea, since they regard the quality as but a small merit, which needs to be supplemented by another, which they call *distinctness*.

Peirce: CP 5.390 Cross-Ref:††

390. A distinct idea is defined as one which contains nothing which is not clear. This is technical language; by the *contents* of an idea logicians understand whatever is contained in its definition. So that an idea is *distinctly* apprehended, according to them, when we can give a precise definition of it, in abstract terms. Here the professional logicians leave the subject; and I would not have troubled the reader with what they have to say, if it were not such a striking example of how they have been slumbering through ages of intellectual activity, listlessly disregarding the enginery of modern thought, and never dreaming of applying its lessons to the improvement of logic. It is easy to show that the doctrine that familiar use and abstract distinctness make the perfection of apprehension has its only true place in philosophies which have long been extinct; and it is now time to formulate the method of attaining to a more perfect clearness of thought, such as we see and admire in the thinkers of our own time.

Peirce: CP 5.391 Cross-Ref:††

391. When Descartes set about the reconstruction of philosophy, his first step was to (theoretically) permit scepticism and to discard the practice of the schoolmen of looking to authority as the ultimate source of truth. That done, he sought a more natural fountain of true principles, and thought he found †1 it in the human mind; thus passing, in the directest way, from the method of authority to that of apriority, as described in my first paper.†1 Self-consciousness was to furnish us with our fundamental truths, and to decide what was agreeable to reason. But since, evidently, not all ideas are true, he was led to note, as the first condition of infallibility, that they must be clear. The distinction between an idea *seeming* clear and really being so, never occurred to him. Trusting to introspection, as he did, even for a knowledge of external things, why should he question its testimony in respect to the contents of our own minds? But then, I suppose, seeing men, who seemed to be quite clear and positive, holding opposite opinions upon fundamental principles, he was further led to say that clearness of ideas is not sufficient, but that they need also to be distinct, i.e., to have nothing unclear about them. What he probably meant by this (for he did not explain himself with precision) was, that they must sustain the test of dialectical examination; that they must not only seem clear at the outset, but that discussion must never be able to bring to light points of obscurity connected with them.

Peirce: CP 5.392 Cross-Ref:††

392. Such was the distinction of Descartes, and one sees that it was precisely on the level of his philosophy. It was somewhat developed by Leibnitz. This great and singular genius was as remarkable for what he failed to see as for what he saw. That a piece of mechanism could not do work perpetually without being fed with power in some form, was a thing perfectly apparent to him; yet he did not understand that the machinery of the mind can only transform knowledge, but never originate it, unless it be fed with facts of observation. He thus missed the most essential point of the Cartesian philosophy, which is, that to accept propositions which seem perfectly evident to us is a thing which, whether it be logical or illogical, we cannot help doing. Instead of regarding the matter in this way, he sought to reduce the first principles of science to two †2 classes, those which cannot be denied without self-contradiction, and those which result from the principle of sufficient reason (of which more anon),†1 and was apparently unaware of the great difference between his position and

that of Descartes. †P1 So he reverted to the old trivialities †2 of logic; and, above all, abstract definitions played a great part in his philosophy. It was quite natural, therefore, that on observing that the method of Descartes labored under the difficulty that we may seem to ourselves to have clear apprehensions of ideas which in truth are very hazy, no better remedy occurred to him than to require an abstract definition of every important term. Accordingly, in adopting the distinction of *clear* and *distinct* notions, he described the latter quality as the clear apprehension of everything contained in the definition; and the books have ever since copied his words. †1 There is no danger that his chimerical scheme will ever again be over-valued. Nothing new can ever be learned by analyzing definitions. Nevertheless, our existing beliefs can be set in order by this process, and order is an essential element of intellectual economy, as of every other. It may be acknowledged, therefore, that the books are right in making familiarity with a notion the first step toward clearness of apprehension, and the defining of it the second. But in omitting all mention of any higher perspicuity of thought, they simply mirror a philosophy which was exploded a hundred years ago. That much-admired "ornament of logic" -- the doctrine of clearness and distinctness -- may be pretty enough, but it is high time to relegate to our cabinet of curiosities the antique *bijou*, and to wear about us something better adapted to modern uses.

Peirce: CP 5.393 Cross-Ref: ††

393. †P1 The very first lesson that we have a right to demand that logic shall teach us is, how to make our ideas clear; and a most important one it is, depreciated only by minds who stand in need of it. To know what we think, to be masters of our own meaning, will make a solid foundation for great and weighty thought. It is most easily learned by those whose ideas are meagre and restricted; and far happier they than such as wallow helplessly in a rich mud of conceptions. A nation, it is true, may, in the course of generations, overcome the disadvantage of an excessive wealth of language and its natural concomitant, a vast, unfathomable deep of ideas. We may see it in history, slowly perfecting its literary forms, sloughing at length its metaphysics, and, by virtue of the untirable patience which is often a compensation, attaining great excellence in every branch of mental acquirement. The page of history is not yet unrolled that †2 is to tell us whether such a people will or will not in the long run prevail over one whose ideas (like the words of their language) are few, but which possesses a wonderful mastery over those which it has. For an individual, however, there can be no question that a few clear ideas are worth more than many confused ones. A young man would hardly be persuaded to sacrifice the greater part of his thoughts to save the rest; and the muddled head is the least apt to see the necessity of such a sacrifice. Him we can usually only commiserate, as a person with a congenital defect. Time will help him, but intellectual maturity with regard to clearness is apt to †1 come rather late. This seems †2 an unfortunate arrangement of Nature, inasmuch as clearness is of less use to a man settled in life, whose errors have in great measure had their effect, than it would be to one whose path lay †3 before him. It is terrible to see how a single unclear idea, a single formula without meaning, lurking in a young man's head, will sometimes act like an obstruction of inert matter in an artery, hindering the nutrition of the brain, and condemning its victim to pine away in the fullness of his intellectual vigor and in the midst of intellectual plenty. Many a man has cherished for years as his hobby some vague shadow of an idea, too meaningless to be positively false; he has, nevertheless, passionately loved it, has made it his companion by day and by night, and has given to it his strength and his life, leaving all other occupations for its sake, and in short has lived with it and for it, until it has become, as it were, flesh of his flesh and bone of his bone; and then he has waked up some bright morning to find it gone, clean vanished away like the beautiful Melusina

of the fable, and the essence of his life gone with it. I have myself known such a man; and who can tell how many histories of circle-squarers, metaphysicians, astrologers, and what not, may not be told in the old German [French!] story?

Peirce: CP 5.394 Cross-Ref:††
§2. THE PRAGMATIC MAXIME

394. The principles set forth in the first part of this essay †4 lead, at once, to a method of reaching a clearness of thought of †5 higher grade than the "distinctness" of the logicians. It was there noticed †6 that the action of thought is excited by the irritation of doubt, and ceases when belief is attained; so that the production of belief is the sole function of thought. †1 All these words, however, are too strong for my purpose. It is as if I had described the phenomena as they appear under a mental microscope. Doubt and Belief, as the words are commonly employed, relate to religious or other grave discussions. But here I use them to designate the starting of any question, no matter how small or how great, and the resolution of it. If, for instance, in a horse-car, I pull out my purse and find a five-cent nickel and five coppers, I decide, while my hand is going to the purse, in which way I will pay my fare. To call such a question Doubt, and my decision Belief, is certainly to use words very disproportionate to the occasion. To speak of such a doubt as causing an irritation which needs to be appeased, suggests a temper which is uncomfortable to the verge of insanity. Yet, looking at the matter minutely, it must be admitted that, if there is the least hesitation as to whether I shall pay the five coppers or the nickel (as there will be sure to be, unless I act from some previously contracted habit in the matter), though irritation is too strong a word, yet I am excited to such small mental activity as may be necessary to deciding how I shall act. Most frequently doubts arise from some indecision, however momentary, in our action. Sometimes it is not so. I have, for example, to wait in a railway-station, and to pass the time I read the advertisements on the walls. I compare the advantages of different trains and different routes which I never expect to take, merely fancying myself to be in a state of hesitancy, because I am bored with having nothing to trouble me. Feigned hesitancy, whether feigned for mere amusement or with a lofty purpose, plays a great part in the production of scientific inquiry. However the doubt may originate, it stimulates the mind to an activity which may be slight or energetic, calm or turbulent. Images pass rapidly through consciousness, one incessantly melting into another, until at last, when all is over -- it may be in a fraction of a second, in an hour, or after long years -- we find ourselves decided as to how we should act under such circumstances as those which occasioned our hesitation. In other words, we have attained belief.

Peirce: CP 5.395 Cross-Ref:††

395. In this process we observe two sorts of elements of consciousness, the distinction between which may best be made clear by means of an illustration. In a piece of music there are the separate notes, and there is the air. A single tone may be prolonged for an hour or a day, and it exists as perfectly in each second of that time as in the whole taken together; so that, as long as it is sounding, it might be present to a sense from which everything in the past was as completely absent as the future itself. But it is different with the air, the performance of which occupies a certain time, during the portions of which only portions of it are played. It consists in an orderliness in the succession of sounds which strike the ear at different times; and to

perceive it there must be some continuity of consciousness which makes the events of a lapse of time present to us. We certainly only perceive the air by hearing the separate notes; yet we cannot be said to directly hear it, for we hear only what is present at the instant, and an orderliness of succession cannot exist in an instant. These two sorts of objects, what we are *immediately* conscious of and what we are *mediately* conscious of, are found in all consciousness. Some elements (the sensations) are completely present at every instant so long as they last, while others (like thought) are actions having beginning, middle, and end, and consist in a congruence in the succession of sensations which flow through the mind. They cannot be immediately present to us, but must cover some portion of the past or future. Thought is a thread of melody running through the succession of our sensations.

Peirce: CP 5.396 Cross-Ref:††

396. We may add that just as a piece of music may be written in parts, each part having its own air, so various systems of relationship of succession subsist together between the same sensations. These different systems are distinguished by having different motives, ideas, or functions. Thought is only one such system, for its sole motive, idea, and function is to produce belief, and whatever does not concern that purpose belongs to some other system of relations. The action of thinking may incidentally have other results; it may serve to amuse us, for example, and among *dilettanti* it is not rare to find those who have so perverted thought to the purposes of pleasure that it seems to vex them to think that the questions upon which they delight to exercise it may ever get finally settled; and a positive discovery which takes a favorite subject out of the arena of literary debate is met with ill-concealed dislike. This disposition is the very debauchery of thought. But the soul and meaning of thought, abstracted from the other elements which accompany it, though it may be voluntarily thwarted, can never be made to direct itself toward anything but the production of belief. Thought in action has for its only possible motive the attainment of thought at rest; and whatever does not refer to belief is no part of the thought itself.

Peirce: CP 5.397 Cross-Ref:††

397. And what, then, is belief? It is the demi-cadence which closes a musical phrase in the symphony of our intellectual life. We have seen that it has just three properties: First, it is something that we are aware of; second, it appeases the irritation of doubt; and, third, it involves the establishment in our nature of a rule of action, or, say for short, a *habit*. As it appeases the irritation of doubt, which is the motive for thinking, thought relaxes, and comes to rest for a moment when belief is reached. But, since belief is a rule for action, the application of which involves further doubt and further thought, at the same time that it is a stopping-place, it is also a new starting-place for thought. That is why I have permitted myself to call it thought at rest, although thought is essentially an action. The *final* upshot of thinking is the exercise of volition, and of this thought no longer forms a part; but belief is only a stadium of mental action, an effect upon our nature due to thought, which will influence future thinking.

Peirce: CP 5.398 Cross-Ref:††

398. The essence of belief is the establishment of a habit; and different beliefs are distinguished by the different modes of action to which they give rise. If beliefs do not differ in this respect, if they appease the same doubt by producing the same rule of action, then no mere differences in the manner of consciousness of them can make them different beliefs, any more than playing a tune in different keys is playing different tunes. Imaginary distinctions are often drawn between beliefs which differ

only in their mode of expression; -- the wrangling which ensues is real enough, however. To believe that any objects are arranged among themselves †1 as in Fig. 1, and to believe that they are arranged [as] in Fig. 2, are one and the same belief; yet it is conceivable that a man should assert one proposition and deny the other. Such false distinctions do as much harm as the confusion of beliefs really different, and are among the pitfalls of which we ought constantly to beware, especially when we are upon metaphysical ground. One singular deception of this sort, which often occurs, is to mistake the sensation

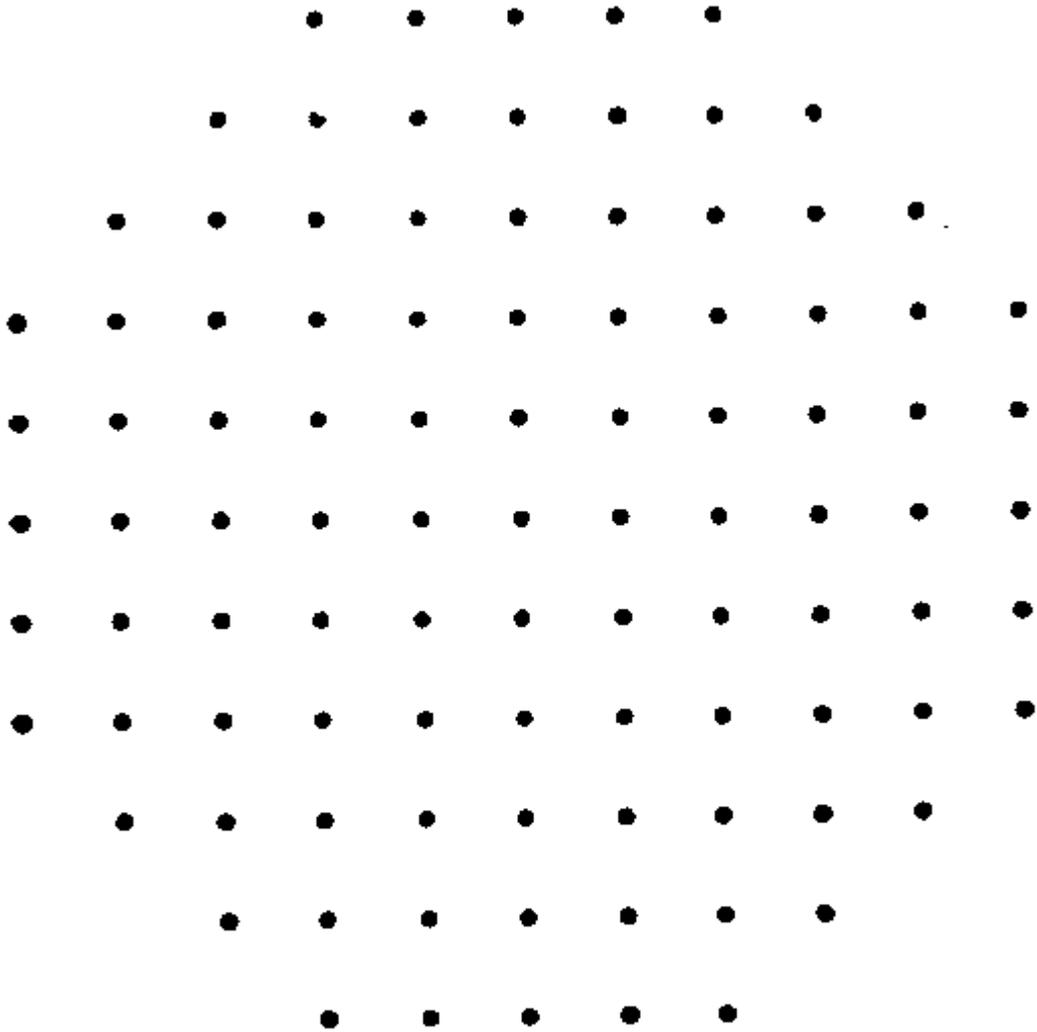


Fig. 1

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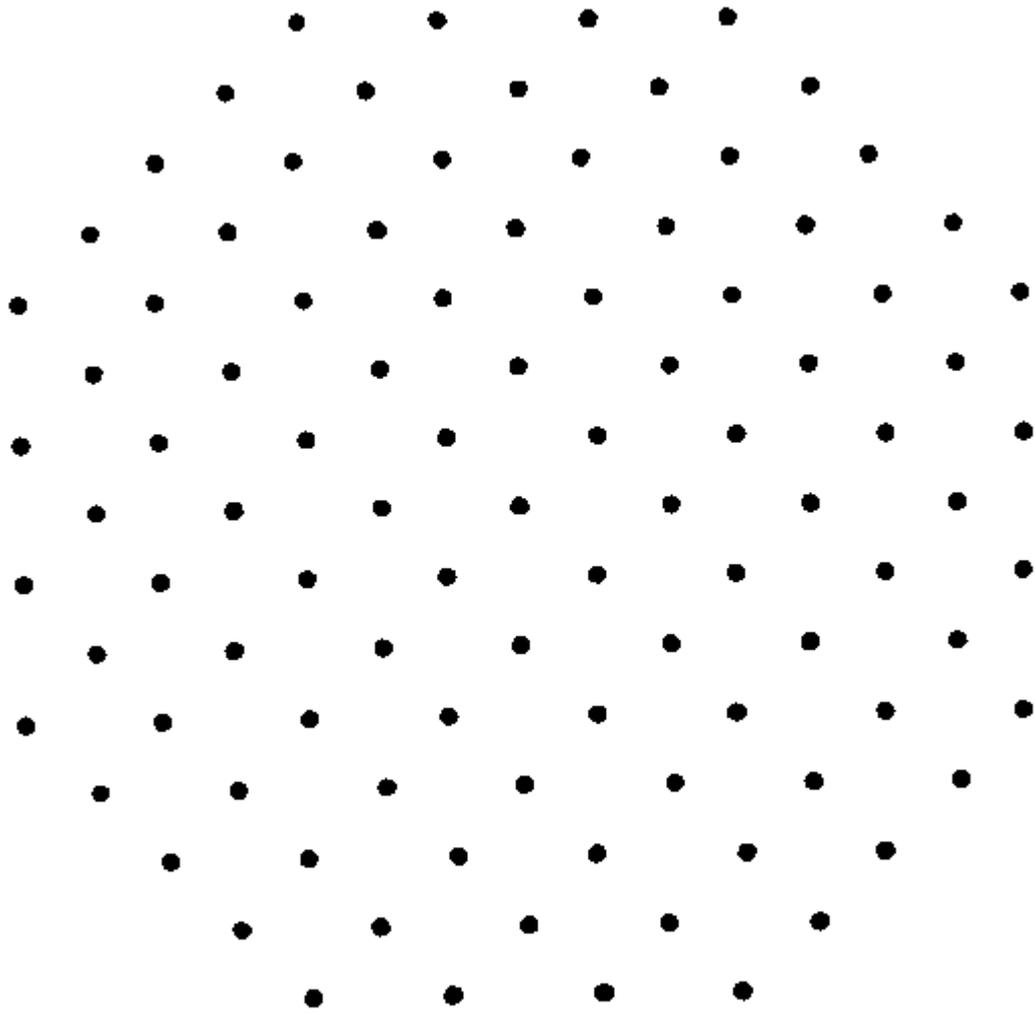


Fig. 2

[Click here to view]

Fig. 1 Fig. 2

produced by our own unclearness of thought for a character of the object we are thinking. Instead of perceiving that the obscurity is purely subjective, we fancy that we contemplate a quality of the object which is essentially mysterious; and if our conception be afterward presented to us in a clear form we do not recognize it as the same, owing to the absence of the feeling of unintelligibility. So long as this deception lasts, it obviously puts an impassable barrier in the way of perspicuous thinking; so that it equally interests the opponents of rational thought to perpetuate it, and its adherents to guard against it.

Peirce: CP 5.399 Cross-Ref:††

399. Another such deception is to mistake a mere difference in the grammatical construction of two words for a distinction between the ideas they express. In this pedantic age, when the general mob of writers attend so much more to words than to things, this error is common enough. When I just said that thought is an *action*, and that it consists in a *relation*, although a person performs an action but not a relation, which can only be the result of an action, yet there was no inconsistency in what I said, but only a grammatical vagueness.

Peirce: CP 5.400 Cross-Ref:††

400. From all these sophisms we shall be perfectly safe so long as we reflect that the whole function of thought is to produce habits of action; and that whatever there is connected with a thought, but irrelevant to its purpose, is an accretion to it, but no part of it. If there be a unity among our sensations which has no reference to how we shall act on a given occasion, as when we listen to a piece of music, why we do not call that thinking. To develop its meaning, we have, therefore, simply to determine what habits it produces, for what a thing means is simply what habits it involves. Now, the identity of a habit depends on how it might lead us to act, not merely under such circumstances as are likely to arise, but under such as might possibly occur, no matter how improbable they may be.†1 What the habit is depends on *when* and *how* it causes us to act. As for the *when*, every stimulus to action is derived from perception; as for the *how*, every purpose of action is to produce some sensible result. Thus, we come down to what is tangible and †2 conceivably †3 practical, as the root of every real distinction of thought, no matter how subtle it may be; and there is no distinction of meaning so fine as to consist in anything but a possible difference of practice.

Peirce: CP 5.401 Cross-Ref:††

401. To see what this principle leads to, consider in the light of it such a doctrine as that of transubstantiation. The Protestant churches generally hold that the elements of the sacrament are flesh and blood only in a tropical sense; they nourish our souls as meat and the juice of it would our bodies. But the Catholics maintain that they are literally just meat and blood †4; although they possess all the sensible qualities of wafercakes and diluted wine. But we can have no conception of wine except what may enter into a belief, either --

Peirce: CP 5.401 Cross-Ref:††

1. That this, that, or the other, is wine; or, 2. That wine possesses certain properties. Such beliefs are nothing but self-notifications that we should, upon occasion, act in regard to such things as we believe to be wine according to the qualities which we believe wine to possess. The occasion of such action would be some sensible perception, the motive of it to produce some sensible result. Thus our action has exclusive reference to what affects the senses, our habit has the same bearing as our action, our belief the same as our habit, our conception the same as our belief; and we can consequently mean nothing by wine but what has certain effects, direct or indirect, upon our senses; and to talk of something as having all the sensible characters of wine, yet being in reality blood, is senseless jargon. Now, it is not my object to pursue the theological question; and having used it as a logical example I drop it, without caring to anticipate the theologian's reply. I only desire to point out how impossible it is that we should have an idea in our minds which relates to anything but conceived sensible effects of things. Our idea of anything *is* our idea of its sensible effects; and if we fancy that we have any other we deceive ourselves, and mistake a mere sensation accompanying the thought for a part of the thought itself. It

is absurd to say that thought has any meaning unrelated to its only function. It is foolish for Catholics and Protestants to fancy themselves in disagreement about the elements of the sacrament, if they agree in regard to all their sensible effects, here and †1 hereafter. †2

Peirce: CP 5.402 Cross-Ref: ††

402. It appears, then, that the rule for attaining the third grade of clearness of apprehension is as follows: Consider what effects, that †3 might conceivably have practical bearings, we conceive the object of our conception to have. Then, our conception of these effects is the whole of our conception of the object. †P1 †P2 †P3

Peirce: CP 5.403 Cross-Ref: ††

§3. SOME APPLICATIONS OF THE PRAGMATIC MAXIME

403. Let us illustrate this rule by some examples; and, to begin with the simplest one possible, let us ask what we mean by calling a thing *hard*. Evidently that it will not be scratched by many other substances. The whole conception of this quality, as of every other, lies in its conceived effects. There is absolutely no difference between a hard thing and a soft thing so long as they are not brought to the test. Suppose, then, that a diamond could be crystallized in the midst of a cushion of soft cotton, and should remain there until it was finally burned up. Would it be false to say that that diamond was soft? This seems a foolish question, and would be so, in fact, except in the realm of logic. There such questions are often of the greatest utility as serving to bring logical principles into sharper relief than real discussions ever could. In studying logic we must not put them aside with hasty answers, but must consider them with attentive care, in order to make out the principles involved. We may, in the present case, modify our question, and ask what prevents us from saying that all hard bodies remain perfectly soft until they are touched, when their hardness increases with the pressure until they are scratched. Reflection will show that the reply is this: there would be no *falsity* in such modes of speech. They would involve a modification of our present usage of speech with regard to the words hard and soft, but not of their meanings. For they represent no fact to be different from what it is; only they involve arrangements of facts which would be exceedingly maladroit. †1 This leads us to remark that the question of what would occur under circumstances which do not actually arise is not a question of fact, but only of the most perspicuous arrangement of them. For example, the question of free-will and fate in its simplest form, stripped of verbiage, is something like this: I have done something of which I am ashamed; could I, by an effort of the will, have resisted the temptation, and done otherwise? The philosophical reply is, that this is not a question of fact, but only of the arrangement of facts. †2 Arranging them so as to exhibit what is particularly pertinent to my question -- namely, that I ought to blame myself for having done wrong -- it is perfectly true to say that, if I had willed to do otherwise than I did, I should have done otherwise. On the other hand, arranging the facts so as to exhibit another important consideration, it is equally true that, when a temptation has once been allowed to work, it will, if it has a certain force, produce its effect, let me struggle how I may. There is no objection to a contradiction in what would result from a false supposition. The *reductio ad absurdum* consists in showing that contradictory results would follow from a hypothesis which is consequently judged to be false. Many questions are involved in the free-will discussion, and I am far from

desiring to say that both sides are equally right. On the contrary, I am of opinion that one side denies important facts, and that the other does not. But what I do say is, that the above single question was the origin of the whole doubt; that, had it not been for this question, the controversy would never have arisen; and that this question is perfectly solved in the manner which I have indicated.

Peirce: CP 5.403 Cross-Ref:††

Let us next seek a clear idea of Weight. This is another very easy case. To say that a body is heavy means simply that, in the absence of opposing force, it will fall. This (neglecting certain specifications of how it will fall, etc., which exist in the mind of the physicist who uses the word) is evidently the whole conception of weight. It is a fair question whether some particular facts may not *account* for gravity; but what we mean by the force itself is completely involved in its effects.

Peirce: CP 5.404 Cross-Ref:††

404. This leads us to undertake an account of the idea of Force in general. This is the great conception which, developed in the early part of the seventeenth century from the rude idea of a cause, and constantly improved upon since, has shown us how to explain all the changes of motion which bodies experience, and how to think about all physical phenomena; which has given birth to modern science, and changed the face of the globe; and which, aside from its more special uses, has played a principal part in directing the course of modern thought, and in furthering modern social development. It is, therefore, worth some pains to comprehend it. According to our rule, we must begin by asking what is the immediate use of thinking about force; and the answer is, that we thus account for changes of motion. If bodies were left to themselves, without the intervention of forces, every motion would continue unchanged both in velocity and in direction. Furthermore, change of motion never takes place abruptly; if its direction is changed, it is always through a curve without angles; if its velocity alters, it is by degrees. The gradual changes which are constantly taking place are conceived by geometers to be compounded together according to the rules of the parallelogram of forces. If the reader does not already know what this is, he will find it, I hope, to his advantage to endeavor to follow the following explanation; but if mathematics are insupportable to him, pray let him skip three paragraphs rather than that we should part company here.

Peirce: CP 5.404 Cross-Ref:††

A *path* is a line whose beginning and end are distinguished. Two paths are considered to be equivalent, which, beginning at the same point, lead to the same point. Thus the two paths, *A B C D E* and *A F G H E* (Fig. 3), are equivalent. Paths which do *not* begin at the same point are considered to be equivalent, provided that, on moving either of them without turning it, but keeping it always parallel to its original position, when its beginning coincides with that of the other path, the ends also coincide. Paths are considered as geometrically added together, when one begins where the other ends; thus

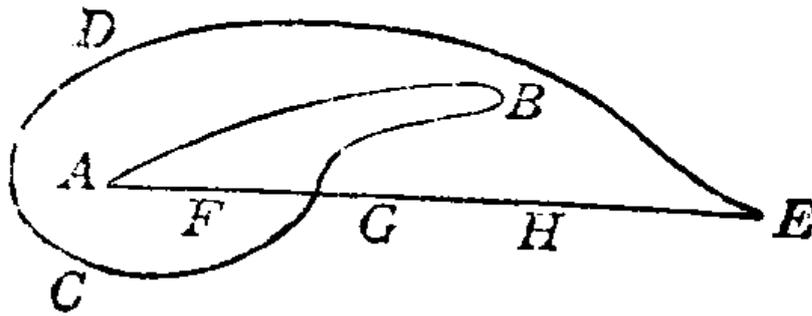


Fig. 3

[Click here to view]

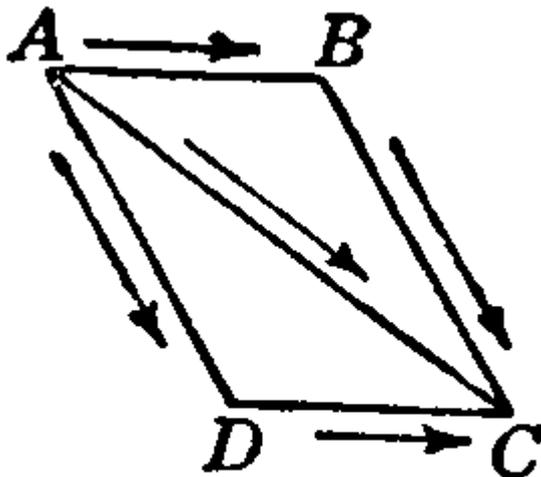


Fig. 4

[Click here to view]

Fig. 3

Fig. 4

the path $A E$ is conceived to be a sum of $A B$, $B C$, $C D$, and $D E$. In the parallelogram of Fig. 4 the diagonal $A C$ is the sum of $A B$ and $B C$; or, since $A D$ is geometrically equivalent to $B C$, $A C$ is the geometrical sum of $A B$ and $A D$.

Peirce: CP 5.404 Cross-Ref:††

All this is purely conventional. It simply amounts to this: that we choose to call paths having the relations I have described equal or added. But, though it is a convention, it is a convention with a good reason. The rule for geometrical addition may be applied not only to paths, but to any other things which can be represented by

paths. Now, as a path is determined by the varying direction and distance of the point which moves over it from the starting-point, it follows that anything which from its beginning to its end is determined by a varying direction and a varying magnitude is capable of being represented by a line. Accordingly, *velocities* may be represented by lines, for they have only directions and rates. The same thing is true of *accelerations*, or changes of velocities. This is evident enough in the case of velocities; and it becomes evident for accelerations if we consider that precisely what velocities are to positions -- namely, states of change of them -- that accelerations are to velocities.

Peirce: CP 5.404 Cross-Ref:††

The so-called "parallelogram of forces" is simply a rule for compounding accelerations. The rule is, to represent the accelerations by paths, and then to geometrically add the paths. The geometers, however, not only use the "parallelogram of forces" to compound different accelerations, but also to resolve one acceleration into a sum of several. Let $A B$ (Fig. 5

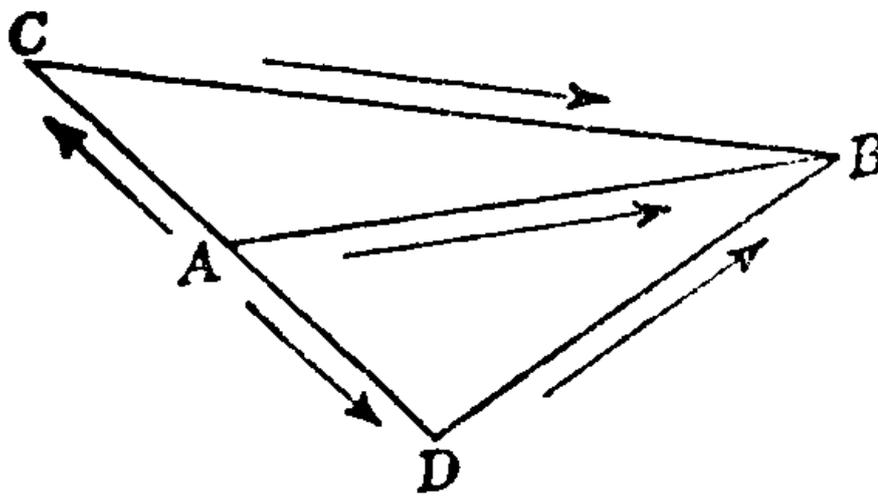


Fig. 5

[Click here to view]) be the path which represents a certain acceleration -- say, such a change in the motion of a body that at the end of one second the body will, under the influence of that change, be in a position different from what it would have had if its motion had continued unchanged such that a path equivalent to $A B$ would lead from the latter position to the former. This acceleration may be considered as the sum of the accelerations represented by $A C$ and $C B$. It may also be considered as the sum of the very different accelerations represented by $A D$ and $D B$, where $A D$ is almost the opposite of $A C$. And it is clear that there is an immense variety of ways in which $A B$ might be resolved into the sum of two accelerations.

Peirce: CP 5.404 Cross-Ref:††

After this tedious explanation, which I hope, in view of the extraordinary interest of the conception of force, may not have exhausted the reader's patience, we are prepared at last to state the grand fact which this conception embodies. This fact is that if the actual changes of motion which the different particles of bodies experience are each resolved in its appropriate way, each component acceleration is precisely such as is prescribed by a certain law of Nature, according to which bodies, in the relative positions which the bodies in question actually have at the moment,†P1 always receive certain accelerations, which, being compounded by geometrical addition, give the acceleration which the body actually experiences.

Peirce: CP 5.404 Cross-Ref:††

This is the only fact which the idea of force represents, and whoever will take the trouble clearly to apprehend what this fact is, perfectly comprehends what force is. Whether we ought to say that a force *is* an acceleration, or that it *causes* an acceleration, is a mere question of propriety of language, which has no more to do with our real meaning than the difference between the French idiom "*Il fait froid*" and its English equivalent "*It is cold.*" Yet it is surprising to see how this simple affair has muddled men's minds. In how many profound treatises is not force spoken of as a "mysterious entity," which seems to be only a way of confessing that the author despairs of ever getting a clear notion of what the word means! In a recent admired work on *Analytic Mechanics*†1 it is stated that we understand precisely the effect of force, but what force itself is we do not understand! This is simply a self-contradiction. The idea which the word force excites in our minds has no other function than to affect our actions, and these actions can have no reference to force otherwise than through its effects. Consequently, if we know what the effects of force are, we are acquainted with every fact which is implied in saying that a force exists, and there is nothing more to know. The truth is, there is some vague notion afloat that a question may mean something which the mind cannot conceive; and when some hair-splitting philosophers have been confronted with the absurdity of such a view, they have invented an empty distinction between positive and negative conceptions, in the attempt to give their non-idea a form not obviously nonsensical. The nullity of it is sufficiently plain from the considerations given a few pages back; and, apart from those considerations, the quibbling character of the distinction must have struck every mind accustomed to real thinking.

Peirce: CP 5.405 Cross-Ref:††
§4. REALITYE

405. Let us now approach the subject of logic, and consider a conception which particularly concerns it, that of *reality*. Taking clearness in the sense of familiarity, no idea could be clearer than this. Every child uses it with perfect confidence, never dreaming that he does not understand it. As for clearness in its second grade, however, it would probably puzzle most men, even among those of a reflective turn of mind, to give an abstract definition of the real. Yet such a definition may perhaps be reached by considering the points of difference between reality and its opposite, fiction. A figment is a product of somebody's imagination; it has such characters as his thought impresses upon it. That those characters are independent of how you or I think is an external reality. There are, however, phenomena within our own minds, dependent upon our thought, which are at the same time real in the sense that we really think them. But though their characters depend on how we think, they do not depend on what we think those characters to be. Thus, a dream has a real existence as a mental phenomenon, if somebody has really dreamt it; that he dreamt so and so, does not depend on what anybody thinks was dreamt, but is completely independent of all opinion on the subject. On the other hand, considering, not the fact of dreaming, but the thing dreamt, it retains its peculiarities by virtue of no other fact than that it was dreamt to possess them. Thus we may define the real as that whose characters are independent of what anybody may think them to be.

Peirce: CP 5.406 Cross-Ref:††

406. But, however satisfactory such a definition may be found, it would be a great mistake to suppose that it makes the idea of reality perfectly clear. Here, then, let us apply our rules. According to them, reality, like every other quality, consists in the peculiar sensible effects which things partaking of it produce. The only effect which real things have is to cause belief, for all the sensations which they excite emerge into consciousness in the form of beliefs. The question therefore is, how is true belief (or belief in the real) distinguished from false belief (or belief in fiction). Now, as we have seen in the former paper,†1 the ideas of truth and falsehood, in their full development, appertain exclusively to the experiential †2 method of settling opinion. A person who arbitrarily chooses the propositions which he will adopt can use the word truth only to emphasize the expression of his determination to hold on to his choice. Of course, the method of tenacity †3 never prevailed exclusively; reason is too natural to men for that. But in the literature of the dark ages we find some fine examples of it. When Scotus Erigena is commenting upon a poetical passage in which hellebore is spoken of as having caused the death of Socrates, he does not hesitate to inform the inquiring reader that Helleborus and Socrates were two eminent Greek philosophers, and that the latter, having been overcome in argument by the former, took the matter to heart and died of it! What sort of an idea of truth could a man have who could adopt and teach, without the qualification of a perhaps, an opinion taken so entirely at random? The real spirit of Socrates, who I hope would have been delighted to have been "overcome in argument," because he would have learned something by it, is in curious contrast with the naive idea of the glossist, for whom (as for "the born missionary" of today)†1 discussion would seem to have been simply a struggle. When philosophy began to awake from its long slumber, and before theology completely dominated it, the practice seems to have been for each professor to seize upon any philosophical position he found unoccupied and which seemed a strong one, to intrench himself in it, and to sally forth from time to time to give battle to the others. Thus, even the scanty records we possess of those disputes enable us to make out a dozen or more opinions held by different teachers at one time concerning the question of nominalism and realism. Read the opening part of the *Historia Calamitatum* of Abelard,†2 who was certainly as philosophical as any of his contemporaries, and see the spirit of combat which it breathes. For him, the truth is simply his particular stronghold. When the method of authority †3 prevailed, the truth meant little more than the Catholic faith. All the efforts of the scholastic doctors are directed toward harmonizing their faith in Aristotle and their faith in the Church, and one may search their ponderous folios through without finding an argument which goes any further. It is noticeable that where different faiths flourish side by side, renegades are looked upon with contempt even by the party whose belief they adopt; so completely has the idea of loyalty replaced that of truth-seeking. Since the time of Descartes, the defect in the conception of truth has been less apparent. Still, it will sometimes strike a scientific man that the philosophers have been less intent on finding out what the facts are, than on inquiring what belief is most in harmony with their system. It is hard to convince a follower of the *a priori* method by adducing facts; but show him that an opinion he is defending is inconsistent with what he has laid down elsewhere, and he will be very apt to retract it. These minds do not seem to believe that disputation is ever to cease; they seem to think that the opinion which is natural for one man is not so for another, and that belief will, consequently, never be settled. In contenting themselves with fixing their own opinions by a method which would lead another man to a different result, they betray their feeble hold of the conception of what truth is.

Peirce: CP 5.407 Cross-Ref:††

407. On the other hand, all the followers of science are animated by a cheerful hope †1 that the processes of investigation, if only pushed far enough, will give one certain solution to each †2 question to which they apply it †3. One man may investigate the velocity of light by studying the transits of Venus and the aberration of the stars; another by the oppositions of Mars and the eclipses of Jupiter's satellites; a third by the method of Fizeau; a fourth by that of Foucault; a fifth by the motions of the curves of Lissajoux; a sixth, a seventh, an eighth, and a ninth, may follow the different methods of comparing the measures of statical and dynamical electricity. They may at first obtain different results, but, as each perfects his method and his processes, the results are found to move †4 steadily together toward a destined centre. So with all scientific research. Different minds may set out with the most antagonistic views, but the progress of investigation carries them by a force outside of themselves to one and the same conclusion. This activity of thought by which we are carried, not where we wish, but to a fore-ordained goal, is like the operation of destiny. No modification of the point of view taken, no selection of other facts for study, no natural bent of mind even, can enable a man to escape the predestinate opinion. This great hope †5 is embodied in the conception of truth and reality. The opinion which is fated †P1 to be ultimately agreed to by all who investigate, is what we mean by the truth, and the object represented in this opinion is the real. That is the way I would explain reality.

Peirce: CP 5.408 Cross-Ref:††

408. But it may be said that this view is directly opposed to the abstract definition which we have given of reality, inasmuch as it makes the characters of the real depend on what is ultimately thought about them. But the answer to this is that, on the one hand, reality is independent, not necessarily of thought in general, but only of what you or I or any finite number of men may think about it; and that, on the other hand, though the object of the final opinion depends on what that opinion is, yet what that opinion is does not depend on what you or I or any man thinks. Our perversity and that of others may indefinitely postpone the settlement of opinion; it might even conceivably cause an arbitrary proposition to be universally accepted as long as the human race should last. Yet even that would not change the nature of the belief, which alone could be the result of investigation carried sufficiently far; and if, after the extinction of our race, another should arise with faculties and disposition for investigation, that true opinion must be the one which they would ultimately come to. "Truth crushed to earth shall rise again," and the opinion which would finally result from investigation does not depend on how anybody may actually think. But the reality of that which is real does depend on the real fact that investigation is destined to lead, at last, if continued long enough, to a belief in it.

Peirce: CP 5.409 Cross-Ref:††

409. But I may be asked what I have to say to all the minute facts of history, forgotten never to be recovered, to the lost books of the ancients, to the buried secrets.

"Full many a gem of purest ray serene

The dark, unfathomed caves of ocean bear;

Full many a flower is born to blush unseen,

And waste its sweetness on the desert air."

Do these things not really exist because they are hopelessly beyond the reach of our knowledge? And then, after the universe is dead (according to the prediction of some scientists), and all life has ceased forever, will not the shock of atoms continue though there will be no mind to know it? To this I reply that, though in no possible state of knowledge can any number be great enough to express the relation between the amount of what rests unknown to the amount of the known, yet it is unphilosophical to suppose that, with regard to any given question (which has any clear meaning), investigation would not bring forth a solution of it, if it were carried far enough. Who would have said, a few years ago, that we could ever know of what substances stars are made whose light may have been longer in reaching us than the human race has existed? Who can be sure of what we shall not know in a few hundred years? Who can guess what would be the result of continuing the pursuit of science for ten thousand years, with the activity of the last hundred? And if it were to go on for a million, or a billion, or any number of years you please, how is it possible to say that there is any question which might not ultimately be solved?

Peirce: CP 5.409 Cross-Ref:††

But it may be objected, "Why make so much of these remote considerations, especially when it is your principle that only practical distinctions have a meaning?" Well, I must confess that it makes very little difference whether we say that a stone on the bottom of the ocean, in complete darkness, is brilliant or not -- that is to say, that it *probably* makes no difference, remembering always that that stone *may* be fished up tomorrow. But that there are gems at the bottom of the sea, flowers in the untraveled desert, etc., are propositions which, like that about a diamond being hard when it is not pressed, concern much more the arrangement of our language than they do the meaning of our ideas.

Peirce: CP 5.410 Cross-Ref:††

410. It seems to me, however, that we have, by the application of our rule, reached so clear an apprehension of what we mean by reality, and of the fact which the idea rests on, that we should not, perhaps, be making a pretension so presumptuous as it would be singular, if we were to offer a metaphysical theory of existence for universal acceptance among those who employ the scientific method of fixing belief. However, as metaphysics is a subject much more curious than useful, the knowledge of which, like that of a sunken reef, serves chiefly to enable us to keep clear of it, I will not trouble the reader with any more Ontology at this moment. I have already been led much further into that path than I should have desired; and I have given the reader such a dose of mathematics, psychology, and all that is most abstruse, that I fear he may already have left me, and that what I am now writing is for the compositor and proof-reader exclusively. I trusted to the importance of the subject. There is no royal road to logic, and really valuable ideas can only be had at the price of close attention. But I know that in the matter of ideas the public prefer the cheap and nasty; and in my next paper †1 I am going to return to the easily intelligible, and not wander from it again. The reader who has been at the pains of wading through this paper, shall be rewarded in the next one by seeing how beautifully what has been developed in this tedious way can be applied to the ascertainment of the rules of scientific reasoning.

Peirce: CP 5.410 Cross-Ref:††

We have, hitherto, not crossed the threshold of scientific logic. It is certainly important to know how to make our ideas clear, but they may be ever so clear without being true. How to make them so, we have next to study. How to give birth to those vital and procreative ideas which multiply into a thousand forms and diffuse themselves everywhere, advancing civilization and making the dignity of man, is an art not yet reduced to rules, but of the secret of which the history of science affords some hints.

Peirce: CP 5.411 Cross-Ref:††

VI

WHAT PRAGMATISM IS†1

§1. THE EXPERIMENTALISTS' VIEW OF ASSERTION

411. The writer of this article has been led by much experience to believe that every physicist, and every chemist, and, in short, every master in any department of experimental science, has had his mind moulded by his life in the laboratory to a degree that is little suspected. The experimentalist himself can hardly be fully aware of it, for the reason that the men whose intellects he really knows about are much like himself in this respect. With intellects of widely different training from his own, whose education has largely been a thing learned out of books, he will never become inwardly intimate, be he on ever so familiar terms with them; for he and they are as oil and water, and though they be shaken up together, it is remarkable how quickly they will go their several mental ways, without having gained more than a faint flavor from the association. Were those other men only to take skillful soundings of the experimentalist's mind -- which is just what they are unqualified to do, for the most part -- they would soon discover that, excepting perhaps upon topics where his mind is trammelled by personal feeling or by his bringing up, his disposition is to think of everything just as everything is thought of in the laboratory, that is, as a question of experimentation. Of course, no living man possesses in their fullness all the attributes characteristic of his type: it is not the typical doctor whom you will see every day driven in buggy or coupe, nor is it the typical pedagogue that will be met with in the first schoolroom you enter. But when you have found, or ideally constructed upon a basis of observation, the typical experimentalist, you will find that whatever assertion you may make to him, he will either understand as meaning that if a given prescription for an experiment ever can be and ever is carried out in act, an experience of a given description will result, or else he will see no sense at all in what you say. If you talk to him as Mr. Balfour talked not long ago to the British Association †1 saying that "the physicist . . . seeks for something deeper than the laws connecting possible objects of experience," that "his object is physical reality" unrevealed in experiments, and that the existence of such non-experiential reality "is the unalterable

faith of science," to all such ontological meaning you will find the experimentalist mind to be color-blind. What adds to that confidence in this, which the writer owes to his conversations with experimentalists, is that he himself may almost be said to have inhabited a laboratory from the age of six until long past maturity; and having all his life associated mostly with experimentalists, it has always been with a confident sense of understanding them and of being understood by them.

Peirce: CP 5.412 Cross-Ref:††

412. That laboratory life did not prevent the writer (who here and in what follows simply exemplifies the experimentalist type) from becoming interested in methods of thinking; and when he came to read metaphysics, although much of it seemed to him loosely reasoned and determined by accidental prepossessions, yet in the writings of some philosophers, especially Kant, Berkeley, and Spinoza, he sometimes came upon strains of thought that recalled the ways of thinking of the laboratory, so that he felt he might trust to them; all of which has been true of other laboratory-men.

Peirce: CP 5.412 Cross-Ref:††

Endeavoring, as a man of that type naturally would, to formulate what he so approved, he framed the theory that a *conception*, that is, the rational purport of a word or other expression, lies exclusively in its conceivable bearing upon the conduct of life; so that, since obviously nothing that might not result from experiment can have any direct bearing upon conduct, if one can define accurately all the conceivable experimental phenomena which the affirmation or denial of a concept could imply, one will have therein a complete definition of the concept, and *there is absolutely nothing more in it*. For this doctrine he invented the name *pragmatism*. Some of his friends wished him to call it *practicism* or *practicalism* (perhaps on the ground that {praktikos} is better Greek than {pragmatikos}). But for one who had learned philosophy out of Kant, as the writer, along with nineteen out of every twenty experimentalists who have turned to philosophy, had done, and who still thought in Kantian terms most readily, *praktisch* and *pragmatisch* were as far apart as the two poles, the former belonging in a region of thought where no mind of the experimentalist type can ever make sure of solid ground under his feet, the latter expressing relation to some definite human purpose. Now quite the most striking feature of the new theory was its recognition of an inseparable connection between rational cognition and rational purpose; and that consideration it was which determined the preference for the name *pragmatism*.

Peirce: CP 5.413 Cross-Ref:††

§2. PHILOSOPHICAL NOMENCLATURE †1E

413. Concerning the matter of philosophical nomenclature, there are a few plain considerations, which the writer has for many years longed to submit to the deliberate judgment of those few fellow-students of philosophy, who deplore the present state of that study, and who are intent upon rescuing it therefrom and bringing it to a condition like that of the natural sciences, where investigators, instead of contemning each the work of most of the others as misdirected from beginning to end, coöperate, stand upon one another's shoulders, and multiply incontestible results;

where every observation is repeated, and isolated observations go for little; where every hypothesis that merits attention is subjected to severe but fair examination, and only after the predictions to which it leads have been remarkably borne out by experience is trusted at all, and even then only provisionally; where a radically false step is rarely taken, even the most faulty of those theories which gain wide credence being true in their main experiential predictions. To those students, it is submitted that no study can become scientific in the sense described, until it provides itself with a suitable technical nomenclature, whose every term has a single definite meaning universally accepted among students of the subject, and whose vocables have no such sweetness or charms as might tempt loose writers to abuse them -- which is a virtue of scientific nomenclature too little appreciated. It is submitted that the experience of those sciences which have conquered the greatest difficulties of terminology, which are unquestionably the taxonomic sciences, chemistry, mineralogy, botany, zoölogy, has conclusively shown that the one only way in which the requisite unanimity and requisite ruptures with individual habits and preferences can be brought about is so to shape the canons of terminology that they shall gain the support of *moral principle* and of every man's sense of decency; and that, in particular (under defined restrictions), the general feeling shall be that he who introduces a new conception into philosophy is under an obligation to invent acceptable terms to express it, and that when he has done so, the duty of his fellow-students is to accept those terms, and to resent any wresting of them from their original meanings, as not only a gross discourtesy to him to whom philosophy was indebted for each conception, but also as an injury to philosophy itself; and furthermore, that once a conception has been supplied with suitable and sufficient words for its expression, no other *technical* terms denoting the same things, considered in the same relations, should be countenanced. Should this suggestion find favor, it might be deemed needful that the philosophians in congress assembled should adopt, after due deliberation, convenient canons to limit the application of the principle. Thus, just as is done in chemistry, it might be wise to assign fixed meanings to certain prefixes and suffixes. For example, it might be agreed, perhaps, that the prefix *prope-* should mark a broad and rather indefinite extension of the meaning of the term to which it was prefixed; the name of a doctrine would naturally end in *-ism*, while *-icism* might mark a more strictly defined acception of that doctrine, etc. Then again, just as in biology no account is taken of terms antedating Linnæus, so in philosophy it might be found best not to go back of the scholastic terminology. To illustrate another sort of limitation, it has probably never happened that any philosopher has attempted to give a general name to his own doctrine without that name's soon acquiring in common philosophical usage, a signification much broader than was originally intended. Thus, special systems go by the names Kantianism, Benthamism, Comteanism, Spencerianism, etc., while transcendentalism, utilitarianism, positivism, evolutionism, synthetic philosophy, etc., have irrevocably and very conveniently been elevated to broader governments.

Peirce: CP 5.414 Cross-Ref:††
§3. PRAGMATICISME

414. After awaiting in vain, for a good many years, some particularly opportune conjuncture of circumstances that might serve to recommend his notions of the ethics of terminology, the writer has now, at last, dragged them in over head and

shoulders, on an occasion when he has no specific proposal to offer nor any feeling but satisfaction at the course usage has run without any canons or resolutions of a congress. His word "pragmatism" has gained general recognition in a generalized sense that seems to argue power of growth and vitality. The famed psychologist, James, first took it up,†1 seeing that his "radical empiricism" substantially answered to the writer's definition of pragmatism, albeit with a certain difference in the point of view. Next, the admirably clear and brilliant thinker, Mr. Ferdinand C.S. Schiller, casting about for a more attractive name for the "anthropomorphism" of his *Riddle of the Sphinx*, lit, in that most remarkable paper of his on *Axioms as Postulates*,†2 upon the same designation "pragmatism," which in its original sense was in generic agreement with his own doctrine, for which he has since found the more appropriate specification "humanism," while he still retains "pragmatism" in a somewhat wider sense. So far all went happily. But at present, the word begins to be met with occasionally in the literary journals, where it gets abused in the merciless way that words have to expect when they fall into literary clutches. Sometimes the manners of the British have effloresced in scolding at the word as ill-chosen -- ill-chosen, that is, to express some meaning that it was rather designed to exclude. So then, the writer, finding his bantling "pragmatism" so promoted, feels that it is time to kiss his child good-by and relinquish it to its higher destiny; while to serve the precise purpose of expressing the original definition, he begs to announce the birth of the word "pragmaticism," which is ugly enough to be safe from kidnappers.†P1

Peirce: CP 5.415 Cross-Ref:††

415. Much as the writer has gained from the perusal of what other pragmatists have written, he still thinks there is a decisive advantage in his original conception of the doctrine. From this original form every truth that follows from any of the other forms can be deduced, while some errors can be avoided into which other pragmatists have fallen. The original view appears, too, to be a more compact and unitary conception than the others. But its capital merit, in the writer's eyes, is that it more readily connects itself with a critical proof of its truth. Quite in accord with the logical order of investigation, it usually happens that one first forms an hypothesis that seems more and more reasonable the further one examines into it, but that only a good deal later gets crowned with an adequate proof. The present writer having had the pragmatist theory under consideration for many years longer than most of its adherents, would naturally have given more attention to the proof of it. At any rate, in endeavoring to explain pragmatism, he may be excused for confining himself to that form of it that he knows best. In the present article there will be space only to explain just what this doctrine (which, in such hands as it has now fallen into, may probably play a pretty prominent part in the philosophical discussions of the next coming years), really consists in. Should the exposition be found to interest readers of *The Monist*, they would certainly be much more interested in a second article which would give some samples of the manifold applications of pragmaticism (assuming it to be true) to the solution of problems of different kinds. After that, readers might be prepared to take an interest in a proof that the doctrine is true -- a proof which seems to the writer to leave no reasonable doubt on the subject, and to be the one contribution of value that he has to make to philosophy. For it would essentially involve the establishment of the truth of synechism.†1

Peirce: CP 5.416 Cross-Ref:††

416. The bare definition of pragmaticism could convey no satisfactory comprehension of it to the most apprehensive of minds, but requires the commentary to be given below. Moreover, this definition takes no notice of one or two other

doctrines without the previous acceptance (or virtual acceptance) of which pragmatism itself would be a nullity. They are included as a part of the pragmatism of Schiller, but the present writer prefers not to mingle different propositions. The preliminary propositions had better be stated forthwith.

Peirce: CP 5.416 Cross-Ref:††

The difficulty in doing this is that no formal list of them has ever been made. They might all be included under the vague maxim, "Dismiss make-believes." Philosophers of very diverse stripes propose that philosophy shall take its start from one or another state of mind in which no man, least of all a beginner in philosophy, actually is. One proposes that you shall begin by doubting everything, and says that there is only one thing that you cannot doubt, as if doubting were "as easy as lying." Another proposes that we should begin by observing "the first impressions of sense," forgetting that our very percepts are the results of cognitive elaboration. But in truth, there is but one state of mind from which you can "set out," namely, the very state of mind in which you actually find yourself at the time you do "set out" -- a state in which you are laden with an immense mass of cognition already formed, of which you cannot divest yourself if you would; and who knows whether, if you could, you would not have made all knowledge impossible to yourself? Do you call it *doubting* to write down on a piece of paper that you doubt? If so, doubt has nothing to do with any serious business. But do not make believe; if pedantry has not eaten all the reality out of you, recognize, as you must, that there is much that you do not doubt, in the least. Now that which you do not at all doubt, you must and do regard as infallible, absolute truth. Here breaks in Mr. Make Believe: "What! Do you mean to say that one is to believe what is not true, or that what a man does not doubt is *ipso facto* true?" No, but unless he can make a thing white and black at once, *he* has to regard what he does not doubt as absolutely true. Now you, *per hypothesiu*, are that man. "But you tell me there are scores of things I do not doubt. I really cannot persuade myself that there is not some one of them about which I am mistaken." You are adducing one of your make-believe facts, which, even if it were established, would only go to show that doubt has a *limen*, that is, is only called into being by a certain finite stimulus. You only puzzle yourself by talking of this metaphysical "truth" and metaphysical "falsity," that you know nothing about. All you have any dealings with are your doubts and beliefs,†P1 with the course of life that forces new beliefs upon you and gives you power to doubt old beliefs. If your terms "truth" and "falsity" are taken in such senses as to be definable in terms of doubt and belief and the course of experience (as for example they would be, if you were to define the "truth" as that to a belief in which belief would tend if it were to tend indefinitely toward absolute fixity), well and good: in that case, you are only talking about doubt and belief. But if by truth and falsity you mean something not definable in terms of doubt and belief in any way, then you are talking of entities of whose existence you can know nothing, and which Ockham's razor would clean shave off. Your problems would be greatly simplified, if, instead of saying that you want to know the "Truth," you were simply to say that you want to attain a state of belief unassailable by doubt.

Peirce: CP 5.417 Cross-Ref:††

417. Belief is not a momentary mode of consciousness; it is a habit of mind essentially enduring for some time, and mostly (at least) unconscious; and like other habits, it is (until it meets with some surprise that begins its dissolution) perfectly self-satisfied. Doubt is of an altogether contrary genus. It is not a habit, but the privation of a habit. Now a privation of a habit, in order to be anything at all, must be a condition of erratic activity that in some way must get superseded by a habit.

Peirce: CP 5.418 Cross-Ref:††

418. Among the things which the reader, as a rational person, does not doubt, is that he not merely has habits, but also can exert a measure of self-control over his future actions; which means, however, *not* that he can impart to them any arbitrarily assignable character, but, on the contrary, that a process of self-preparation will tend to impart to action (when the occasion for it shall arise), one fixed character, which is indicated and perhaps roughly measured by the absence (or slightness) of the feeling of self-reproach, which subsequent reflection will induce. Now, this subsequent reflection is part of the self-preparation for action on the next occasion. Consequently, there is a tendency, as action is repeated again and again, for the action to approximate indefinitely toward the perfection of that fixed character, which would be marked by entire absence of self-reproach. The more closely this is approached, the less room for self-control there will be; and where no self-control is possible there will be no self-reproach.

Peirce: CP 5.419 Cross-Ref:††

419. These phenomena seem to be the fundamental characteristics which distinguish a rational being. Blame, in every case, appears to be a modification, often accomplished by a transference, or "projection," of the primary feeling of self-reproach. Accordingly, we never blame anybody for what had been beyond his power of previous self-control. Now, thinking is a species of conduct which is largely subject to self-control. In all their features (which there is no room to describe here), logical self-control is a perfect mirror of ethical self-control -- unless it be rather a species under that genus.†1 In accordance with this, what you cannot in the least help believing is not, justly speaking, wrong belief. In other words, for you it is the absolute truth. True, it is conceivable that what you cannot help believing today, you might find you thoroughly disbelieve tomorrow. But then there is a certain distinction between things you "cannot" do, merely in the sense that nothing stimulates you to the great effort and endeavors that would be required, and things you cannot do because in their own nature they are unsusceptible of being put into practice. In every stage of your excogitations, there is something of which you can only say, "I cannot think otherwise," and your experientially based hypothesis is that the impossibility is of the second kind.

Peirce: CP 5.420 Cross-Ref:††

420. There is no reason why "thought," in what has just been said, should be taken in that narrow sense in which silence and darkness are favorable to thought. It should rather be understood as covering all rational life, so that an experiment shall be an operation of thought. Of course, that ultimate state of habit to which the action of self-control ultimately tends, where no room is left for further self-control, is, in the case of thought, the state of fixed belief, or perfect knowledge.

Peirce: CP 5.421 Cross-Ref:††

421. Two things here are all-important to assure oneself of and to remember. The first is that a person is not absolutely an individual. His thoughts are what he is "saying to himself," that is, is saying to that other self that is just coming into life in the flow of time. When one reasons, it is that critical self that one is trying to persuade; and all thought whatsoever is a sign, and is mostly of the nature of language. The second thing to remember is that the man's circle of society (however widely or narrowly this phrase may be understood), is a sort of loosely compacted person, in some respects of higher rank than the person of an individual organism. It is these two things alone that render it possible for you -- but only in the abstract, and

in a Pickwickian sense -- to distinguish between absolute truth and what you do not doubt.

Peirce: CP 5.422 Cross-Ref:††

422. Let us now hasten to the exposition of pragmatism itself. Here it will be convenient to imagine that somebody to whom the doctrine is new, but of rather preternatural perspicacity, asks questions of a pragmatist. Everything that might give a dramatic illusion must be stripped off, so that the result will be a sort of cross between a dialogue and a catechism, but a good deal liker the latter -- something rather painfully reminiscent of Mangnall's *Historical Questions*.

Peirce: CP 5.422 Cross-Ref:††

Questioner: I am astounded at your definition of your pragmatism, because only last year I was assured by a person above all suspicion of warping the truth -- himself a pragmatist -- that your doctrine precisely was "that a conception is to be tested by its practical effects." You must surely, then, have entirely changed your definition very recently.

Peirce: CP 5.422 Cross-Ref:††

Pragmatist: If you will turn to Vols. VI and VII of the *Revue Philosophique*, or to the *Popular Science Monthly* for November 1877 and January 1878 [Papers No. IV and V], you will be able to judge for yourself whether the interpretation you mention was not then clearly excluded. The exact wording of the English enunciation, (changing only the first person into the second), was: "Consider what effects that might conceivably have practical bearing you conceive the object of your conception to have. Then your conception of those effects is the WHOLE of your conception of the object."†1

Peirce: CP 5.422 Cross-Ref:††

Questioner: Well, what reason have you for asserting that this is so?

Peirce: CP 5.422 Cross-Ref:††

Pragmatist: That is what I specially desire to tell you. But the question had better be postponed until you clearly understand what those reasons profess to prove.

Peirce: CP 5.423 Cross-Ref:††

423. **Questioner:** What, then, is the *raison d'être* of the doctrine? What advantage is expected from it?

Peirce: CP 5.423 Cross-Ref:††

Pragmatist: It will serve to show that almost every proposition of ontological metaphysics is either meaningless gibberish -- one word being defined by other words, and they by still others, without any real conception ever being reached -- or else is downright absurd; so that all such rubbish being swept away, what will remain of philosophy will be a series of problems capable of investigation by the observational methods of the true sciences -- the truth about which can be reached without those interminable misunderstandings and disputes which have made the highest of the positive sciences a mere amusement for idle intellects, a sort of chess -- idle pleasure its purpose, and reading out of a book its method. In this regard, pragmatism is a species of prope-positivism. But what distinguishes it from other species is, first, its retention of a purified philosophy; secondly, its full acceptance of the main body of our instinctive beliefs; and thirdly, its strenuous insistence upon the truth of scholastic realism (or a close approximation to that, well-stated by the late Dr.

Francis Ellingwood Abbot in the Introduction to his *Scientific Theism*). So, instead of merely jeering at metaphysics, like other prope-positivists, whether by long drawn-out parodies or otherwise, the pragmatist extracts from it a precious essence, which will serve to give life and light to cosmology and physics. At the same time, the moral applications of the doctrine are positive and potent; and there are many other uses of it not easily classed. On another occasion, instances may be given to show that it really has these effects.

Peirce: CP 5.424 Cross-Ref:††

424. **Questioner:** I hardly need to be convinced that your doctrine would wipe out metaphysics. Is it not as obvious that it must wipe out every proposition of science and everything that bears on the conduct of life? For you say that the only meaning that, for you, any assertion bears is that a certain experiment has resulted in a certain way: Nothing else but an experiment enters into the meaning. Tell me, then, how can an experiment, in itself, reveal anything more than that something once happened to an individual object and that subsequently some other individual event occurred?

Peirce: CP 5.424 Cross-Ref:††

Pragmatist: That question is, indeed, to the purpose -- the purpose being to correct any misapprehensions of pragmatism. You speak of an experiment in itself, emphasising "*in itself*." You evidently think of each experiment as isolated from every other. It has not, for example, occurred to you, one might venture to surmise, that every connected series of experiments constitutes a single collective experiment. What are the essential ingredients of an experiment? First, of course, an experimenter of flesh and blood. Secondly, a verifiable hypothesis. This is a proposition †P1 relating to the universe environing the experimenter, or to some well-known part of it and affirming or denying of this only some experimental possibility or impossibility. The third indispensable ingredient is a sincere doubt in the experimenter's mind as to the truth of that hypothesis.

Peirce: CP 5.424 Cross-Ref:††

Passing over several ingredients on which we need not dwell, the purpose, the plan, and the resolve, we come to the act of choice by which the experimenter singles out certain identifiable objects to be operated upon. The next is the external (or quasi-external) ACT by which he modifies those objects. Next, comes the subsequent **reaction** of the world upon the experimenter in a perception; and finally, his recognition of the teaching of the experiment. While the two chief parts of the event itself are the action and the reaction, yet the unity of essence of the experiment lies in its purpose and plan, the ingredients passed over in the enumeration.

Peirce: CP 5.425 Cross-Ref:††

425. Another thing: in representing the pragmatist as making rational meaning to consist in an experiment (which you speak of as an event in the past), you strikingly fail to catch his attitude of mind. Indeed, it is not in an experiment, but in **experimental phenomena**, that rational meaning is said to consist. When an experimentalist speaks of a **phenomenon**, such as "Hall's phenomenon," "Zeemann's phenomenon" and its modification, "Michelson's phenomenon," or "the chessboard phenomenon," he does not mean any particular event that did happen to somebody in the dead past, but what **surely will** happen to everybody in the living future who shall fulfill certain conditions. The phenomenon consists in the fact that when an experimentalist shall come to **act** according to a certain scheme that he has in mind, then will something else happen, and shatter the doubts of sceptics, like the celestial

fire upon the altar of Elijah.

Peirce: CP 5.426 Cross-Ref:††

426. And do not overlook the fact that the pragmaticist maxim says nothing of single experiments or of single experimental phenomena (for what is conditionally true *in futuro* can hardly be singular), but only speaks of *general kinds* of experimental phenomena. Its adherent does not shrink from speaking of general objects as real, since whatever is true represents a real. Now the laws of nature are true.

Peirce: CP 5.427 Cross-Ref:††

427. The rational meaning of every proposition lies in the future. How so? The meaning of a proposition is itself a proposition. Indeed, it is no other than the very proposition of which it is the meaning: it is a translation of it. But of the myriads of forms into which a proposition may be translated, what is that one which is to be called its very meaning? It is, according to the pragmaticist, that form in which the proposition becomes applicable to human conduct, not in these or those special circumstances, nor when one entertains this or that special design, but that form which is most directly applicable to self-control under every situation, and to every purpose. This is why he locates the meaning in future time; for future conduct is the only conduct that is subject to self-control. But in order that that form of the proposition which is to be taken as its meaning should be applicable to every situation and to every purpose upon which the proposition has any bearing, it must be simply the general description of all the experimental phenomena which the assertion of the proposition virtually predicts. For an experimental phenomenon is the fact asserted by the proposition that action of a certain description will have a certain kind of experimental result; and experimental results are the only results that can affect human conduct. No doubt, some unchanging idea may come to influence a man more than it had done; but only because some experience equivalent to an experiment has brought its truth home to him more intimately than before. Whenever a man acts purposively, he acts under a belief in some experimental phenomenon. Consequently, the sum of the experimental phenomena that a proposition implies makes up its entire bearing upon human conduct. Your question, then, of how a pragmaticist can attribute any meaning to any assertion other than that of a single occurrence is substantially answered.

Peirce: CP 5.428 Cross-Ref:††

428. *Questioner:* I see that pragmaticism is a thorough-going phenomenalism. Only why should you limit yourself to the phenomena of experimental science rather than embrace all observational science? Experiment, after all, is an uncommunicative informant. It never expiates †1: it only answers "yes" or "no"; or rather it usually snaps out "No!" or, at best only utters an inarticulate grunt for the negation of its "no." The typical experimentalist is not much of an observer. It is the student of natural history to whom nature opens the treasury of her confidence, while she treats the cross-examining experimentalist with the reserve he merits. Why should your phenomenalism sound the meagre jews-harp of experiment rather than the glorious organ of observation?

Peirce: CP 5.428 Cross-Ref:††

Pragmaticist: Because pragmaticism is not definable as "thorough-going phenomenalism," although the latter doctrine may be a kind of pragmatism. The *richness* of phenomena lies in their sensuous quality. Pragmaticism does not intend to define the phenomenal equivalents of words and general ideas, but, on the contrary,

eliminates their sential element, and endeavors to define the rational purport, and this it finds in the purposive bearing of the word or proposition in question.

Peirce: CP 5.429 Cross-Ref:††

429. *Questioner:* Well, if you choose so to make Doing the Be-all and the End-all of human life, why do you not make meaning to consist simply in doing? Doing has to be done at a certain time upon a certain object. Individual objects and single events cover all reality, as everybody knows, and as a practicalist ought to be the first to insist. Yet, your meaning, as you have described it, is *general*. Thus, it is of the nature of a mere word and not a reality. You say yourself that your meaning of a proposition is only the same proposition in another dress. But a practical man's meaning is the very thing he means. What do you make to be the meaning of "George Washington"?

Peirce: CP 5.429 Cross-Ref:††

Pragmaticist: Forcibly put! A good half dozen of your points must certainly be admitted. It must be admitted, in the first place, that if pragmatism really made Doing to be the Be-all and the End-all of life, that would be its death. For to say that we live for the mere sake of action, as action, regardless of the thought it carries out, would be to say that there is no such thing as rational purport.†1 Secondly, it must be admitted that every proposition professes to be true of a certain real individual object, often the environing universe. Thirdly, it must be admitted that pragmatism fails to furnish any translation or meaning of a proper name, or other designation of an individual object. Fourthly, the pragmatic meaning is undoubtedly general; and it is equally indisputable that the general is of the nature of a word or sign. Fifthly, it must be admitted that individuals alone exist; and sixthly, it may be admitted that the very meaning of a word or significant object ought to be the very essence of reality of what it signifies. But when those admissions have been unreservedly made, you find the pragmaticist still constrained most earnestly to deny the force of your objection, you ought to infer that there is some consideration that has escaped you. Putting the admissions together, you will perceive that the pragmaticist grants that a proper name (although it is not customary to say that it has a *meaning*), has a certain denotative function peculiar, in each case, to that name and its equivalents; and that he grants that every assertion contains such a denotative or pointing-out function. In its peculiar individuality, the pragmaticist excludes this from the rational purport of the assertion, although *the like* of it, being common to all assertions, and so, being general and not individual, may enter into the pragmatic purport. Whatever exists, *ex-sists*, that is, really acts upon other existents, so obtains a self-identity, and is definitely individual. As to the general, it will be a help to thought to notice that there are two ways of being general. A statue of a soldier on some village monument, in his overcoat and with his musket, is for each of a hundred families the image of its uncle, its sacrifice to the Union. That statue, then, though it is itself single, represents any one man of whom a certain predicate may be true. It is *objectively* general. The word "soldier," whether spoken or written, is general in the same way; while the name, "George Washington," is not so. But each of these two terms remains one and the same noun, whether it be spoken or written, and whenever and wherever it be spoken or written. This noun is not an existent thing: it is a *type*,†1 or *form*, to which objects, both those that are externally existent and those which are imagined, may *conform*, but which none of them can exactly be. This is subjective generality. The pragmatic purport is general in both ways.

Peirce: CP 5.430 Cross-Ref:††

430. As to reality, one finds it defined in various ways; but if that principle of terminological ethics that was proposed be accepted, the equivocal language will soon disappear. For *realis* and *realitas* are not ancient words. They were invented to be terms of philosophy in the thirteenth century, †2 and the meaning they were intended to express is perfectly clear. That is *real* which has such and such characters, whether anybody thinks it to have those characters or not. At any rate, that is the sense in which the pragmatist uses the word. Now, just as conduct controlled by ethical reason tends toward fixing certain habits of conduct, the nature of which (as to illustrate the meaning, peaceable habits and not quarrelsome habits) does not depend upon any accidental circumstances, and *in that sense* may be said to be *destined*; so, thought, controlled by a rational experimental logic, tends to the fixation of certain opinions, equally destined, the nature of which will be the same in the end, however the perversity of thought of whole generations may cause the postponement of the ultimate fixation. If this be so, as every man of us virtually assumes that it is, in regard to each matter the truth of which he seriously discusses, then, according to the adopted definition of "real," the state of things which will be believed in that ultimate opinion is real. But, for the most part, such opinions will be general. Consequently, *some* general objects are real. (Of course, nobody ever thought that *all* generals were real; but the scholastics used to assume that generals were real when they had hardly any, or quite no, experiential evidence to support their assumption; and their fault lay just there, and not in holding that generals could be real.) One is struck with the inexactitude of thought even of analysts of power, when they touch upon modes of being. One will meet, for example, the virtual assumption that what is relative to thought cannot be real. But why not, exactly? *Red* is relative to sight, but the fact that this or that is in that relation to vision that we call being red is not *itself* relative to sight; it is a real fact.

Peirce: CP 5.431 Cross-Ref: ††

431. Not only may generals be real, but they may also be *physically efficient*, not in every metaphysical sense, but in the common-sense acception in which human purposes are physically efficient. †1 Aside from metaphysical nonsense, no sane man doubts that if I feel the air in my study to be stuffy, that thought may cause the window to be opened. My thought, be it granted, was an individual event. But what determined it to take the particular determination it did, was in part the general fact that stuffy air is unwholesome, and in part other *Forms*, concerning which Dr. Carus †2 has caused so many men to reflect to advantage -- or rather, *by* which, and the general truth concerning which Dr. Carus's mind was determined to the forcible enunciation of so much truth. For truths, on the average, have a greater tendency to get believed than falsities have. Were it otherwise, considering that there are myriads of false hypotheses to account for any given phenomenon, against one sole true one (or if you will have it so, against every true one), the first step toward genuine knowledge must have been next door to a miracle. So, then, when my window was opened, because of the truth that stuffy air is malsain, a physical effort was brought into existence by the efficiency of a general and non-existent truth. This has a droll sound because it is unfamiliar; but exact analysis is with it and not against it; and it has besides, the immense advantage of not blinding us to great facts -- such as that the ideas "justice" and "truth" are, notwithstanding the iniquity of the world, the mightiest of the forces that move it. Generality is, indeed, an indispensable ingredient of reality; for mere individual existence or actuality without any regularity whatever is a nullity. Chaos is pure nothing.

Peirce: CP 5.432 Cross-Ref: ††

432. That which any true proposition asserts is *real*, in the sense of being as it is regardless of what you or I may think about it. Let this proposition be a general conditional proposition as to the future, and it is a real general such as is calculated really to influence human conduct; and such the pragmatist holds to be the rational purport of every concept.

Peirce: CP 5.433 Cross-Ref:††

433. Accordingly, the pragmatist does not make the *summum bonum* to consist in action, but makes it to consist in that process of evolution whereby the existent comes more and more to embody those generals which were just now said to be *destined*, which is what we strive to express in calling them *reasonable*. In its higher stages, evolution takes place more and more largely through self-control, and this gives the pragmatist a sort of justification for making the rational purport to be general.

Peirce: CP 5.434 Cross-Ref:††

434. There is much more in elucidation of pragmatism that might be said to advantage, were it not for the dread of fatiguing the reader. It might, for example, have been well to show clearly that the pragmatist does not attribute any different essential mode of being to an event in the future from that which he would attribute to a similar event in the past, but only that the practical attitude of the thinker toward the two is different.†1 It would also have been well to show that the pragmatist does not make Forms to be the *only* realities in the world,†2 any more than he makes the reasonable purport of a word to be the only kind of meaning there is.†3 These things are, however, implicitly involved in what has been said. There is only one remark concerning the pragmatist's conception of the relation of his formula to the first principles of logic which need detain the reader.

Peirce: CP 5.435 Cross-Ref:††

435. Aristotle's definition of universal predication,†1 which is usually designated (like a papal bull or writ of court, from its opening words), as the *Dictum de omni*, may be translated as follows: "We call a predication (be it affirmative or negative), *universal*, when, and only when, there is nothing among the existent individuals to which the subject affirmatively belongs, but to which the predicate will not likewise be referred (affirmatively or negatively, according as the universal predication is affirmative or negative)." The Greek is: {legomen de to kata pantos katégoreisthai hotan méden éi labein tôn tou hypokeimenon kath' ohy thateron ou lechthésetai kai to kata médenos hösantös}. The important words "existent individuals" have been introduced into the translation (which English idiom would not here permit to be literal); but it is plain that existent individuals were what Aristotle meant. The other departures from literalness only serve to give modern English forms of expression. Now, it is well known that propositions in formal logic go in pairs, the two of one pair being convertible into another by the interchange of the ideas of antecedent and consequent, subject and predicate, etc.†2 The parallelism extends so far that it is often assumed to be perfect; but it is not quite so. The proper mate of this sort to the *Dictum de omni* is the following definition of affirmative predication: We call a predication *affirmative* (be it universal or particular) when, and only when, there is nothing among the sensational effects that belong universally to the predicate which will not be (universally or particularly, according as the affirmative predication is universal or particular), said to belong to the subject. Now, this is substantially the essential proposition of pragmatism. Of course, its parallelism to the *Dictum de omni* will only be admitted by a person who admits the

truth of pragmatism.

Peirce: CP 5.436 Cross-Ref:††

§4. PRAGMATISM AND HEGELIAN ABSOLUTE IDEALISM^E

436. Suffer me to add one word more on this point. For if one cares at all to know what the pragmatist theory consists in, one must understand that there is no other part of it to which the pragmatist attaches quite as much importance as he does to the recognition in his doctrine of the utter inadequacy of action or volition or even of resolve or actual purpose, as materials out of which to construct a conditional purpose or the concept of conditional purpose. Had a purposed article concerning the principle of continuity and synthesising the ideas of the other articles of a series †1 in the early volumes of *The Monist* ever been written, †2 it would have appeared how, with thorough consistency, that theory involved the recognition that continuity is an indispensable element of reality, and that continuity is simply what generality becomes in the logic of relatives, and thus, like generality, and more than generality, is an affair of thought, and is the essence of thought. Yet even in its truncated condition, an extra-intelligent reader might discern that the theory of those cosmological articles made reality to consist in something more than feeling and action could supply, inasmuch as the primeval chaos, where those two elements were present, was explicitly shown to be pure nothing. Now, the motive for alluding to that theory just here is, that in this way one can put in a strong light a position which the pragmatist holds and must hold, whether that cosmological theory be ultimately sustained or exploded, namely, that the third category -- the category of thought, representation, triadic relation, mediation, genuine thirdness, thirdness as such -- is an essential ingredient of reality, yet does not by itself constitute reality, since this category (which in that cosmology appears as the element of habit) can have no concrete being without action, as a separate object on which to work its government, just as action cannot exist without the immediate being of feeling on which to act. The truth is that pragmatism is closely allied to the Hegelian absolute idealism, from which, however, it is sundered by its vigorous denial that the third category (which Hegel degrades to a mere stage of thinking) suffices to make the world, or is even so much as self-sufficient. Had Hegel, instead of regarding the first two stages with his smile of contempt, held on to them as independent or distinct elements of the triune Reality, pragmatists might have looked up to him as the great vindicator of their truth. (Of course, the external trappings of his doctrine are only here and there of much significance.) For pragmatism belongs essentially to the triadic class of philosophical doctrines, and is much more essentially so than Hegelianism is. †1 (Indeed, in one passage, at least, Hegel alludes to the triadic form of his exposition as to a mere fashion of dress.)

MILFORD, PA., September, 1904.

Peirce: CP 5.437 Cross-Ref:††

437. POSTSCRIPT. During the last five months, I have met with references to several objections to the above opinions, but not having been able to obtain the text of

these objections, I do not think I ought to attempt to answer them. If gentlemen who attack either pragmatism in general or the variety of it which I entertain would only send me copies of what they write, more important readers they could easily find, but they could find none who would examine their arguments with a more grateful avidity for truth not yet apprehended, nor any who would be more sensible of their courtesy.

February 9, 1905.

Peirce: CP 5.438 Cross-Ref:††
VII

ISSUES OF PRAGMATISM†1

§1 SIX CHARACTERS OF CRITICAL COMMON-SENSISME

438. Pragmatism was originally enounced †P1 in the form of a maxim, as follows: Consider what effects that might *conceivably* have practical bearings you *conceive* the objects of your *conception* to have. Then, your *conception* of those effects is the whole of your *conception* of the object.

Peirce: CP 5.438 Cross-Ref:††

I will restate this in other words, since oftentimes one can thus eliminate some unsuspected source of perplexity to the reader. This time it shall be in the indicative mood, as follows: The entire intellectual purport of any symbol consists in the total of all general modes of rational conduct which, conditionally upon all the possible different circumstances and desires, would ensue upon the acceptance of the symbol.

Peirce: CP 5.439 Cross-Ref:††

439. Two doctrines that were defended by the writer about nine years before the formulation of pragmatism may be treated as consequences of the latter belief. One of these may be called Critical Common-sensism. It is a variety of the Philosophy of Common Sense, but is marked by six distinctive characters, which had better be enumerated at once.

Peirce: CP 5.440 Cross-Ref:††

440. *Character I.* Critical Common-sensism admits that there not only are indubitable propositions but also that there are indubitable inferences. In one sense, anything evident is indubitable; but the propositions and inferences which Critical Common-sensism holds to be original, in the sense one cannot "go behind" them (as the lawyers say), are indubitable in the sense of being acritical. The term "reasoning" ought to be confined to such fixation of one belief by another as is reasonable, deliberate, self-controlled. A reasoning must be conscious; and this consciousness is not mere "immediate consciousness," which (as I argued in 1868)†1 is simple Feeling viewed from another side, but is in its ultimate nature (meaning in that characteristic

element of it that is not reducible to anything simpler), a sense of taking a habit, or disposition to respond to a given kind of stimulus in a given kind of way. As to the nature of that, some *éclaircissements* will appear below and again in my third paper, on the Basis of Pragmaticism.†2 But the secret of rational consciousness is not so much to be sought in the study of this one peculiar nucleolus, as in the review of the process of self-control in its entirety. The machinery of logical self-control works on the same plan as does moral self-control, in multiform detail. The greatest difference, perhaps, is that the latter serves to inhibit mad puttings forth of energy, while the former most characteristically insures us against the quandary of Buridan's ass. The formation of habits under imaginary action (see the paper of January, 1878†3) is one of the most essential ingredients of both; but in the logical process the imagination takes far wider flights, proportioned to the generality of the field of inquiry, being bounded in pure mathematics solely by the limits of its own powers, while in the moral process we consider only situations that may be apprehended or anticipated. For in moral life we are chiefly solicitous about our conduct and its inner springs, and the approval of conscience, while in intellectual life there is a tendency to value existence as the vehicle of forms. Certain obvious features of the phenomena of self-control (and especially of habit) can be expressed compactly and without any hypothetical addition, except what we distinctly rate as imagery, by saying that we have an occult nature of which and of its contents we can only judge by the conduct that it determines, and by phenomena of that conduct. All will assent to that (or all but the extreme nominalist), but anti-synechistic thinkers wind themselves up in a factitious snarl by falsifying the phenomena in representing consciousness to be, as it were, a skin, a separate tissue, overlying an unconscious region of the occult nature, mind, soul, or physiological basis. It appears to me that in the present state of our knowledge a sound methodetic prescribes that, in adhesion to the appearances, the difference is only relative and the demarcation not precise.

Peirce: CP 5.441 Cross-Ref:††

441. According to the maxim of Pragmaticism, to say that determination affects our occult nature is to say that it is capable of affecting deliberate conduct; and since we are conscious of what we do deliberately, we are conscious *habitualiter* of whatever hides in the depths of our nature; and it is presumable (and *only* presumable,†P1 although curious instances are on record), that a sufficiently energetic effort of attention would bring it out. Consequently, to say that an operation of the mind is controlled is to say that it is, in a special sense, a conscious operation; and this no doubt is the consciousness of reasoning. For this theory requires that in reasoning we should be conscious, not only of the conclusion, and of our deliberate approval of it, but also of its being the result of the premiss from which it does result, and furthermore that the inference is one of a possible class of inferences which conform to one guiding principle.†1 Now in fact we find a well-marked class of mental operations, clearly of a different nature from any others which do possess just these properties. They alone deserve to be called *reasonings*; and if the reasoner is conscious, even vaguely, of what his guiding principle is, his reasoning should be called a *logical argumentation*. There are, however, cases in which we are conscious that a belief has been determined by another given belief, but are not conscious that it proceeds on any general principle. Such is St. Augustine's "*cogito, ergo sum*." Such a process should be called, not a reasoning, but an *acritical inference*. Again, there are cases in which one belief is determined by another, without our being at all aware of it. These should be called *associational suggestions of belief*.

Peirce: CP 5.442 Cross-Ref:††

442. Now the theory of Pragmaticism was originally based, as anybody will see who examines the papers of November 1877 and January 1878, upon a study of that experience of the phenomena of self-control which is common to all grown men and women; and it seems evident that to some extent, at least, it must always be so based. For it is to conceptions of deliberate conduct that Pragmaticism would trace the intellectual purport of symbols; and deliberate conduct is self-controlled conduct. Now control may itself be controlled, criticism itself subjected to criticism; and ideally there is no obvious definite limit to the sequence. But if one seriously inquires whether it is possible that a completed series of actual efforts should have been endless or beginningless (I will spare the reader the discussion), I think he can only conclude that (with some vagueness as to what constitutes an effort) this must be regarded as impossible.†1 It will be found to follow that there are, besides perceptual judgments, original (i.e., indubitable because uncriticized) beliefs of a general and recurrent kind, as well as indubitable acritical inferences.

Peirce: CP 5.443 Cross-Ref:††

443. It is important for the reader to satisfy himself that genuine doubt always has an external origin, usually from surprise; and that it is as impossible for a man to create in himself a genuine doubt by such an act of the will as would suffice to imagine the condition of a mathematical theorem, as it would be for him to give himself a genuine surprise by a simple act of the will.

Peirce: CP 5.443 Cross-Ref:††

I beg my reader also to believe that it would be impossible for me to put into these articles over two per cent of the pertinent thought which would be necessary in order to present the subject as I have worked it out. I can only make a small selection of what it seems most desirable to submit to his judgment. Not only must all steps be omitted which he can be expected to supply for himself, but unfortunately much more that may cause him difficulty.

Peirce: CP 5.444 Cross-Ref:††

444. *Character* II. I do not remember that any of the old Scotch philosophers ever undertook to draw up a complete list of the original beliefs, but they certainly thought it a feasible thing, and that the list would hold good for the minds of all men from Adam down. For in those days Adam was an undoubted historical personage. Before any waft of the air of evolution had reached those coasts how could they think otherwise? When I first wrote, we were hardly orientated in the new ideas, and my impression was that the indubitable propositions changed with a thinking man from year to year. I made some studies preparatory to an investigation of the rapidity of these changes, but the matter was neglected, and it has been only during the last two years that I have completed a provisional inquiry which shows me that the changes are so slight from generation to generation, though not imperceptible even in that short period, that I thought to own my adhesion, under inevitable modification, to the opinion of that subtle but well-balanced intellect, Thomas Reid, in the matter of Common Sense (as well as in regard to immediate perception, along with Kant).†P1

Peirce: CP 5.445 Cross-Ref:††

445. *Character* III. The Scotch philosophers recognized that the original beliefs, and the same thing is at least equally true of the acritical inferences, were of the general nature of instincts. But little as we know about instincts, even now, we are much better acquainted with them than were the men of the eighteenth century. We know, for example, that they can be somewhat modified in a very short time. The great facts have always been known; such as that instinct seldom errs, while reason

goes wrong nearly half the time, if not more frequently. But one thing the Scotch failed to recognize is that the original beliefs only remain indubitable in their application to affairs that resemble those of a primitive mode of life. It is, for example, quite open to reasonable doubt whether the motions of electrons are confined to three dimensions, although it is good methodetic to presume that they are until some evidence to the contrary is forthcoming. On the other hand, as soon as we find that a belief shows symptoms of being instinctive, although it may seem to be dubitable, we must suspect that experiment would show that it is not really so; for in our artificial life, especially in that of a student, no mistake is more likely than that of taking a paper-doubt for the genuine metal. Take, for example, the belief in the criminality of incest. Biology will doubtless testify that the practice is inadvisable; but surely nothing that it has to say could warrant the intensity of our sentiment about it. When, however, we consider the thrill of horror which the idea excites in us, we find reason in that to consider it to be an instinct; and from that we may infer that if some rationalistic brother and sister were to marry, they would find that the conviction of horrible guilt could not be shaken off.

Peirce: CP 5.445 Cross-Ref:††

In contrast to this may be placed the belief that suicide is to be classed as murder. There are two pretty sure signs that this is not an instinctive belief. One is that it is substantially confined to the Christian world. The other is that when it comes to the point of actual self-debate, this belief seems to be completely expunged and ex-sponged from the mind. In reply to these powerful arguments, the main points urged are the authority of the fathers of the church and the undoubtedly intense instinctive clinging to life. The latter phenomenon is, however, entirely irrelevant. For though it is a wrench to part with life, which has its charms at the very worst, just as it is to part with a tooth, yet there is no *moral* element in it whatever. As to the Christian tradition, it may be explained by the circumstances of the early Church. For Christianity, the most terribly earnest and most intolerant of religions (see *The Book of Revelations of St. John the Divine*) -- and it remained so until diluted with civilization -- recognized no morality as worthy of an instant's consideration except Christian morality. Now the early Church had need of martyrs, i.e., witnesses, and if any man had done with life, it was abominable infidelity to leave it otherwise than as a witness to its power. This belief, then, should be set down as dubitable; and it will no sooner have been pronounced dubitable, than Reason will stamp it as false.

Peirce: CP 5.445 Cross-Ref:††

The Scotch School appears to have no such distinction concerning the limitations of indubitability and the consequent limitations of the jurisdiction of original belief.

Peirce: CP 5.446 Cross-Ref:††

446. *Character* IV. By all odds, the most distinctive character of the Critical Common-sensist, in contrast to the old Scotch philosopher, lies in his insistence that the acritically indubitable is invariably vague.

Peirce: CP 5.446 Cross-Ref:††

Logicians have been at fault in giving Vagueness the go-by, so far as not even to analyze it. The present writer has done his best to work out the Stechiology (or Stoicheiology), Critic, and Methodetic †1 of the subject, but can here only give a definition or two with some proposals respecting terminology.

Peirce: CP 5.447 Cross-Ref:††

447. Accurate writers have apparently made a distinction between the *definite* and the *determinate*. A subject is *determinate* in respect to any character which inheres in it or is (universally and affirmatively) predicated of it, as well as in respect to the negative of such character, these being the very same respect. In all other respects it is *indeterminate*. The *definite* shall be defined presently. A sign (under which designation I place every kind of thought, and not alone external signs), that is in any respect objectively indeterminate (i.e., whose object is undetermined by the sign itself) is objectively *general* in so far as it extends to the interpreter the privilege of carrying its determination further.†P1 **Example:** "Man is mortal." To the question, What man? the reply is that the proposition explicitly leaves it to you to apply its assertion to what man or men you will.†2 A sign that is objectively indeterminate in any respect is objectively *vague* in so far as it reserves further determination to be made in some other conceivable sign, or at least does not appoint the interpreter as its deputy in this office.†3 **Example:** "A man whom I could mention seems to be a little conceited." The *suggestion* here is that the man in view is the person addressed; but the utterer does not authorize such an interpretation or *any* other application of what she says. She can still say, if she likes, that she does *not* mean the person addressed. Every utterance naturally leaves the right of further exposition in the utterer; and therefore, in so far as a sign is indeterminate, it is vague, unless it is expressly or by a well-understood convention rendered general. Usually, an affirmative predication covers *generally* every essential character of the predicate, while a negative predication *vaguely* denies some essential character. In another sense, honest people, when not joking, intend to make the meaning of their words determinate, so that there shall be no latitude of interpretation at all. That is to say, the character of their meaning consists in the implications and non-implications of their words; and they intend to fix what is implied and what is not implied. They believe that they succeed in doing so, and if their chat is about the theory of numbers, perhaps they may. But the further their topics are from such precise, or "abstract," subjects, the less possibility is there of such precision of speech. In so far as the implication is not determinate, it is usually left vague; but there are cases where an unwillingness to dwell on disagreeable subjects causes the utterer to leave the determination of the implication to the interpreter; as if one says, "That creature is filthy, in every sense of the term."

Peirce: CP 5.448 Cross-Ref:††

448. Perhaps a more scientific pair of definitions would be that anything is *general* in so far as the principle of excluded middle does not apply to it and is *vague* in so far as the principle of contradiction does not apply to it.†1 Thus, although it is true that "Any proposition you please, *once you have determined its identity*, is either true or false"; yet *so long as it remains indeterminate and so without identity*, it need neither be true that any proposition you please is true, nor that any proposition you please is false. So likewise, while it is false that "A proposition *whose identity I have determined* is both true and false," yet until it is determinate, it may be true that a proposition is true and that a proposition is false.†P1

Peirce: CP 5.449 Cross-Ref:††

449. In those respects in which a sign is not vague, it is said to be *definite*, and also with a slightly different mode of application, to be *precise*, a meaning probably due to *præcisus* having been applied to curt denials and refusals.†1 It has been the well-established, ordinary sense of *precise* since the Plantagenets; and it were much to be desired that this word, with its derivatives *precision*, *precisive*, etc., should, in the dialect of philosophy, be restricted to this sense. To express the act of *rendering*

precise (though usually only in reference to numbers, dates, and the like), the French have the verb *préciser*, which, after the analogy of *décider*, should have been *précider*. Would it not be a useful addition to our English terminology of logic, to adopt the verb *to precise*, to express the general sense, to render precise? Our older logicians with salutary boldness seem to have created for their service the verb *to prescind*, the corresponding Latin word meaning only to "cut off at the end," while the English word means to suppose without supposing some more or less determinately indicated accompaniment. In geometry, for example, we "prescind" shape from color, which is precisely the same thing as to "abstract" color from shape, although very many writers employ the verb "to abstract" so as to make it the equivalent of "prescind." But whether it was the invention or the courage of our philosophical ancestors which exhausted itself in the manufacture of the verb "prescind," the curious fact is that instead of forming from it the noun *prescission*, they took pattern from the French logicians in putting the word *precision* to this second use. About the same time †P1 (see Watts, *Logick*, 1725, I, vi, 9 *ad fin.*) the adjective *precisive* was introduced to signify what *prescissive* would have more unmistakably conveyed. If we desire to rescue the good ship Philosophy for the service of Science from the hands of lawless rovers of the sea of literature, we shall do well to keep prescind, presciss, prescission, and prescissive on the one hand, to refer to dissection in hypothesis, while precide, precise, precision, and precise are used so as to refer exclusively to an expression of determination which is made either full or free for the interpreter. We shall thus do much to relieve the stem "abstract" from staggering under the double burden of conveying the idea of prescission as well as the unrelated and very important idea of the creation of *ens rationis* out of an {epos pteroen} -- to filch the phrase to furnish a name for an expression of non-substantive thought -- an operation that has been treated as a subject of ridicule -- this hypostatic abstraction -- but which gives mathematics half its power.

Peirce: CP 5.450 Cross-Ref: ††

450. The purely formal conception that the three affections of terms, *determination*, *generality*, and *vagueness*, form a group dividing a category of what Kant calls "functions of judgment" will be passed by as unimportant by those who have yet to learn how important a part purely formal conceptions may play in philosophy. Without stopping to discuss this, it may be pointed out that the "quantity" of propositions in logic, that is, the distribution of the *first* subject, †P1 is either *singular* (that is, determinate, which renders it substantially negligible in formal logic), or *universal* (that is, general), or *particular* (as the mediaeval logicians say, that is, vague or *indefinite*). †1 It is a curious fact that in the logic of relations it is the first and last quantifiers of a proposition that are of chief importance. To affirm of anything that it is a horse is to yield to it *every* essential character of a horse; to deny of anything that it is a horse is vaguely to refuse to it *some* one or more of those essential characters of the horse. There are, however, predicates that are unanalyzable in a given state of intelligence and experience. These are, therefore, determinately affirmed or denied. Thus, this same group of concepts reappears. Affirmation and denial are in themselves unaffected by these concepts, but it is to be remarked that there are cases in which we can have an apparently definite idea of a border line between affirmation and negation. Thus, a point of a surface may be in a region of that surface, or out of it, or on its boundary. This gives us an indirect and vague conception of an intermediary between affirmation and denial in general, and consequently of an intermediate, or nascent state, between determination and indetermination. There must be a similar intermediacy between generality and

vagueness. Indeed, in an article in the seventh volume of *The Monist*†1 there lies just beneath the surface of what is explicitly said, the idea of an endless series of such *intermediacies*. We shall find below some application for these reflections.

Peirce: CP 5.451 Cross-Ref:††

451. *Character* V. The Critical Common-sensist will be further distinguished from the old Scotch philosopher by the great value he attaches to doubt, provided only that it be the weighty and noble metal itself, and no counterfeit nor paper substitute. He is not content to ask himself whether he does doubt, but he invents a plan for attaining to doubt, elaborates it in detail, and then puts it into practice, although this may involve a solid month of hard work; and it is only after having gone through such an examination that he will pronounce a belief to be indubitable. Moreover, he fully acknowledges that even then it may be that some of his indubitable beliefs may be proved false.

Peirce: CP 5.451 Cross-Ref:††

The Critical Common-sensist holds that there is less danger to heurctic science in believing too little than in believing too much. Yet for all that, the consequences to heuristics of believing too little may be no less than disaster.

Peirce: CP 5.452 Cross-Ref:††

452. *Character* VI. Critical Common-sensism may fairly lay claim to this title for two sorts of reasons; namely, that on the one hand it subjects four opinions to rigid criticism: its own; that of the Scotch school; that of those who would base logic or metaphysics on psychology or any other special science, the least tenable of all the philosophical opinions that have any vogue; and that of Kant; while on the other hand it has besides some claim to be called Critical from the fact that it is but a modification of Kantism. The present writer was a pure Kantist until he was forced by successive steps into Pragmaticism. The Kantist has only to abjure from the bottom of his heart the proposition that a thing-in-itself can, however indirectly, be conceived; and then correct the details of Kant's doctrine accordingly, and he will find himself to have become a Critical Common-sensist.

Peirce: CP 5.453 Cross-Ref:††

§2. SUBJECTIVE AND OBJECTIVE MODALITYE

453. Another doctrine which is involved in Pragmaticism as an essential consequence of it, but which the writer defended [306 *ad fin*] and *North American Review*, Vol. CXIII, pp. 449-472, 1871), [Vol. 9] before he had formulated, even in his own mind, the principle of pragmaticism, is the scholastic doctrine of realism. This is usually defined as the opinion that there are real objects that are general, among the number being the modes of determination of existent singulars, if, indeed, these be not the only such objects. But the belief in this can hardly escape being accompanied by the acknowledgment that there are, besides, real *vagues*, and especially real possibilities. For possibility being the denial of a necessity, which is a kind of generality, is vague like any other contradiction of a general. Indeed, it is the reality of some possibilities that pragmaticism is most concerned to insist upon. The article of January 1878†1 endeavored to glose over this point as unsuited to the exoteric public addressed; or perhaps the writer wavered in his own mind. He said

that if a diamond were to be formed in a bed of cotton-wool, and were to be consumed there without ever having been pressed upon by any hard edge or point, it would be merely a question of nomenclature whether that diamond should be said to have been hard or not. No doubt this is true, except for the abominable falsehood in the word MERELY, implying that symbols are unreal. Nomenclature involves classification; and classification is true or false, and the generals to which it refers are either reals in the one case, or figments in the other. For if the reader will turn to the original maxim of pragmatism at the beginning of this article, he will see that the question is, not what *did* happen, but whether it would have been well to engage in any line of conduct whose successful issue depended upon whether that diamond *would* resist an attempt to scratch it, or whether all other logical means of determining how it ought to be classed *would* lead to the conclusion which, to quote the very words of that article, would be "the belief which alone could be the result of investigation carried *sufficiently far*."†2 Pragmatism makes the ultimate intellectual purport of what you please to consist in conceived conditional resolutions,†3 or their substance; and therefore, the conditional propositions, with their hypothetical antecedents, in which such resolutions consist, being of the ultimate nature of meaning, must be capable of being true, that is, of expressing whatever there be which is such as the proposition expresses, independently of being thought to be so in any judgment, or being represented to be so in any other symbol of any man or men. But that amounts to saying that possibility is sometimes of a real kind.

Peirce: CP 5.454 Cross-Ref:††

454. Fully to understand this, it will be needful to analyze modality, and ascertain in what it consists.†1 In the simplest case, the most subjective meaning, if a person does not know that a proposition is false, he calls it *possible*. If, however, he knows that it is *true*, it is much more than possible. Restricting the word to its characteristic applicability, a state of things has the Modality of the possible -- that is, of the merely possible -- only in case the contradictory state of things is likewise possible, which proves possibility to be the vague modality. One who knows that Harvard University has an office in State Street, Boston, and has impression that it is at No. 30, but yet suspects that 50 is the number, would say "I think it is at No. 30, but it *may be* at No. 50," or "it *is possibly* at No. 50." Thereupon, another, who does not doubt his recollection, might chime in, "It *actually is* at No. 50," or simply "it *is* at No. 50," or "it *is* at No. 50, *de inesse*." Thereupon, the person who had first asked, what the number was might say, "Since you are so positive, it *must be* at No. 50," for "I know the first figure is 5. So, since you are both certain the second is a 0, why 50 it *necessarily is*." That is to say, in this most subjective kind of Modality, that which is known by direct recollection is in the Mode of *Actuality*, the determinate mode. But when knowledge is indeterminate among alternatives, either there is one state of things which alone accords with them all, when this is in the Mode of *Necessity*, or there is more than one state of things that no knowledge excludes, when each of these is in the Mode of *Possibility*.

Peirce: CP 5.455 Cross-Ref:††

455. Other kinds of subjective Modality refer to a Sign or Representamen which is assumed to be true, but which does not include the Utterer's (i.e. the speaker's, writer's, thinker's or other symbolizer's) total knowledge, the different Modes being distinguished very much as above. There are other cases, however, in which, justifiably or not, we certainly think of Modality as objective. A man says, "I *can* go to the seashore if I like." Here is implied, to be sure, his ignorance of how he will decide to act. But this is not the point of the assertion. It is that the complete

determination of conduct in the *act* not yet having taken place, the further determination of it belongs to the subject of the action regardless of external circumstances. If he had said, "I *must* go where my employers may send me," it would imply that the function of such further determination lay elsewhere. In "You *may* do so and so," and "You *must* do so," the "may" has the same force as "can," except that in the one case freedom from particular circumstances is in question, and in the other freedom from a law or edict. Hence the phrase, "You *may* if you *can*." I must say that it is difficult for me to preserve my respect for the competence of a philosopher whose dull logic, not penetrating beneath the surface, leaves him to regard such phrases as misrepresentations of the truth. So an act of hypostatic abstraction which in itself is no violation of logic, however it may lend itself to a dress of superstition, may regard the collective tendencies to variability in the world, under the name of Chance, as at one time having their way, and at another time overcome by the element of order; so that, for example, a superstitious cashier, impressed by a bad dream, may say to himself of a Monday morning, "*May be*, the bank has been robbed." No doubt, he recognizes his total ignorance in the matter. But besides that, he has in mind the absence of any particular cause which should protect his bank more than others that are robbed from time to time. He thinks of the variety in the universe as vaguely analogous to the indecision of a person, and borrows from that analogy the garb of his thought. At the other extreme stand those who declare as inspired (for they have no rational proof of what they allege), that an actuary's advice to an insurance company is based on nothing at all but ignorance.

Peirce: CP 5.456 Cross-Ref:††

456. There is another example of objective possibility: "A pair of intersecting rays, i.e., unlimited straight lines conceived as movable objects, *can* (or *may*) move, without ceasing to intersect, so that one and the same hyperboloid shall be completely covered by the track of each of them." How shall we interpret this, remembering that the object spoken of, the pair of rays, is a pure creation of the Utterer's imagination, although it is required (and, indeed, forced) to conform to the laws of space? Some minds will be better satisfied with a more subjective, or nominalistic, others with a more objective, realistic interpretation. But it must be confessed on all hands that whatever degree or kind of reality belongs to pure space belongs to the substance of that proposition, which merely expresses a property of space.

Peirce: CP 5.457 Cross-Ref:††

457. Let us now take up the case of that diamond which, having been crystallized upon a cushion of jeweler's cotton, was accidentally consumed by fire before the crystal of corundum that had been sent for had had time to arrive, and indeed without being subjected to any other pressure than that of the atmosphere and its own weight. The question is, was that diamond *really* hard? It is certain that no discernible *actual* fact determined it to be so. But is its hardness not, nevertheless, a *real* fact? To say, as the article of January 1878 seems to intend, that it is just as an arbitrary "usage of speech" chooses to arrange its thoughts, is as much as to decide against the reality of the property, since the real is that which is such as it is regardless of how it is, at any time, thought to be. Remember that this diamond's condition is not an isolated fact. There is no such thing; and an isolated fact could hardly be real. It is an unsevered, though presciss part of the unitary fact of nature. Being a diamond, it was a mass of pure carbon, in the form of a more or less transparent crystal (brittle, and of facile octahedral cleavage, unless it was of an unheard-of variety), which, if not trimmed after one of the fashions in which diamonds may be trimmed, took the shape of an octahedron, apparently regular (I

need not go into minutiae), with grooved edges, and probably with some curved faces. Without being subjected to any considerable pressure, it could be found to be insoluble, very highly refractive, showing under radium rays (and perhaps under "dark light" and X-rays) a peculiar bluish phosphorescence, having as high a specific gravity as realgar or orpiment, and giving off during its combustion less heat than any other form of carbon would have done. From some of these properties hardness is believed to be inseparable. For like it they bespeak the high polymerization of the molecule. But however this may be, how can the hardness of all other diamonds fail to bespeak *some* real relation among the diamonds without which a piece of carbon would not be a diamond? Is it not a monstrous perversion of the word and concept *real* to say that the accident of the non-arrival of the corundum prevented the hardness of the diamond from having the *reality* which it otherwise, with little doubt, would have had?

Peirce: CP 5.457 Cross-Ref:††

At the same time, we must dismiss the idea that the occult state of things (be it a relation among atoms or something else), which constitutes the reality of a diamond's hardness can possibly consist in anything but in the truth of a general conditional proposition. For to what else does the entire teaching of chemistry relate except to the "behavior" of different possible kinds of material substance? And in what does that behavior consist except that if a substance of a certain kind should be exposed to an agency of a certain kind, a certain kind of sensible result *would* ensue, according to our experiences hitherto. As for the pragmatist, it is precisely his position that nothing else than this can be so much as *meant* by saying that an object possesses a character. He is therefore obliged to subscribe to the doctrine of a real Modality, including real Necessity and real Possibility.

Peirce: CP 5.458 Cross-Ref:††

458. A good question, for the purpose of illustrating the nature of Pragmatism, is, What is Time? It is not proposed to attack those most difficult problems connected with the psychology, the epistemology, or the metaphysics of Time, although it will be taken for granted, as it must be according to what has been said, that Time is real.†1 The reader is only invited to the humbler question of what we mean by Time, and not of every kind of meaning attached to Past, Present, and Future either. Certain peculiar feelings are associated with the three general determinations of Time; but those are to be sedulously put out of view. That the reference of events to Time is irresistible will be recognized; but as to how it may differ from other kinds of irresistibility is a question not here to be considered. The question to be considered is simply, What is the intellectual purport of the Past, Present, and Future? It can only be treated with the utmost brevity.

Peirce: CP 5.459 Cross-Ref:††

459. That Time is a particular variety of objective Modality is too obvious for argumentation. The Past consists of the sum of *faits accomplis*, and this Accomplishment is the Existential Mode of Time. For the Past really acts upon us, and *that* it does, not at all in the way in which a Law or Principle influences us, but precisely as an Existent object acts. For instance, when a *Nova Stella* bursts out in the heavens, it acts upon one's eyes just as a light struck in the dark by one's own hands would; and yet it is an event which happened before the Pyramids were built. A neophyte may remark that its reaching the eyes, which is all we know, happens but a fraction of a second before we know it. But a moment's consideration will show him that he is losing sight of the question, which is not whether the distant Past can act

upon us *immediately*, but whether it acts upon us just as any Existent does. The instance adduced (certainly a commonplace enough fact), proves conclusively that the mode of the Past is that of Actuality. Nothing of the sort is true of the Future, to compass the understanding of which it is indispensable that the reader should divest himself of his Necessitarianism -- at best, but a scientific theory -- and return to the Common-sense State of Nature. Do you never say to yourself, "I *can* do this or that as well tomorrow as today"? Your Necessitarianism is a theoretical pseudo-belief -- a make-believe belief -- that such a sentence does not express the real truth. That is only to stick to proclaiming the unreality of that Time, of which you are invited, be it reality or figment, to consider the meaning. You need not fear to compromise your darling theory by looking out at its windows. Be it true in theory or not, the unsophisticated conception is that everything in the Future is either *destined*, i.e., necessitated already, or is *undecided*, the contingent future of Aristotle. In other words, it is not Actual, since it does not act except through the idea of it, that is, as a law acts; but is either Necessary or Possible, which are of the same mode since (as remarked above †1) Negation being outside the category of modality cannot produce a variation in Modality. As for the Present instant, it is so inscrutable that I wonder whether no sceptic has ever attacked its reality. I can fancy one of them dipping his pen in his blackest ink to commence the assault, and then suddenly reflecting that his entire life is in the Present -- the "living present," as we say, this instant when all hopes and fears concerning it come to their end, this Living Death in which we are born anew. It is plainly that Nascent State between the Determinate and the Indeterminate that was noticed above. †1

Peirce: CP 5.460 Cross-Ref: ††

460. Pragmatism consists in holding that the purport of any concept is its conceived bearing upon our conduct. How, then, does the Past bear upon conduct? The answer is self-evident: whenever we set out to do anything, we "go upon," we base our conduct on facts already known, and for these we can only draw upon our memory. It is true that we may institute a new investigation for the purpose; but its discoveries will only become applicable to conduct after they have been made and reduced to a memorial maxim. In short, the Past is the storehouse of all our knowledge.

Peirce: CP 5.460 Cross-Ref: ††

When we say that we know that some state of things exists, we mean that it used to exist, whether just long enough for the news to reach the brain and be retransmitted to tongue or pen, or longer ago. Thus, from whatever point of view we contemplate the Past, it appears as the Existential Mode of Time.

Peirce: CP 5.461 Cross-Ref: ††

461. How does the Future bear upon conduct? The answer is that future facts are the only facts that we can, in a measure, control; and whatever there may be in the Future that is not amenable to control are the things that we *shall* be able to infer, or *should* be able to infer under favorable circumstances. There may be questions concerning which the pendulum of opinion never would cease to oscillate, however favorable circumstances may be. But if so, those questions are *ipso facto* not *real* questions, that is to say, are questions to which there is no true answer to be given. It is natural to use the future tense (and the conditional mood is but a mollified future) in drawing a conclusion or in stating a consequence. "If two unlimited straight lines in one plane and crossed by a third making the sum . . . then these straight lines *will* meet on the side, etc." It cannot be denied that acritical inferences may refer to the

Past in its capacity as past; but according to Pragmaticism, the conclusion of a Reasoning power must refer to the Future. For its meaning refers to conduct, and since it is a reasoned conclusion must refer to deliberate conduct, which is controllable conduct. But the only controllable conduct is Future conduct. As for that part of the Past that lies beyond memory, the Pragmaticist doctrine is that the meaning of its being believed to be in connection with the Past consists in the acceptance as truth of the conception that we ought to conduct ourselves according to it (like the meaning of any other belief). Thus, a belief that Christopher Columbus discovered America really refers to the future. It is more difficult, it must be confessed, to account for beliefs that rest upon the double evidence of feeble but direct memory and upon rational inference. The difficulty does not seem insuperable; but it must be passed by.

Peirce: CP 5.462 Cross-Ref:††

462. What is the bearing of the Present instant upon conduct?

Peirce: CP 5.462 Cross-Ref:††

Introspection is wholly a matter of inference.†1 One is immediately conscious of his Feelings, no doubt; but not that they are feelings of an *ego*. The *self* is only inferred. There is no time in the Present for any inference at all, least of all for inference concerning that very instant. Consequently the present object must be an external object, if there be any objective reference in it. The attitude of the Present is either conative or perceptive. Supposing it to be perceptive, the perception must be immediately known as external -- not indeed in the sense in which a hallucination is *not* external, but in the sense of being present regardless of the perceiver's will or wish. Now this kind of externality is conative externality. Consequently, the attitude of the present instant (according to the testimony of Common Sense, which is plainly adopted throughout) can only be a Conative attitude. The consciousness of the present is then that of a struggle over what shall be; and thus we emerge from the study with a confirmed belief that it is the Nascent State of the Actual.

Peirce: CP 5.463 Cross-Ref:††

463. But how is Temporal Modality distinguished from other Objective Modality? Not by any general character since Time is unique and *sui generis*. In other words there is only one Time. Sufficient attention has hardly been called to the surpassing truth of this for Time as compared with its truth for Space. Time, therefore, can only be identified by brute compulsion. But we must not go further.

Peirce: CP 5.464 Cross-Ref:††

BOOK III

UNPUBLISHED PAPERS

CHAPTER 1

A SURVEY OF PRAGMATISM†1

§1. THE KERNEL OF PRAGMATISM

464. It is now high time to explain what pragmatism is. I must, however, preface the explanation by a statement of what it is not, since many writers, especially of the starry host of Kant's progeny, in spite of pragmatists' declarations, unanimous, reiterated, and most explicit, still remain unable to "catch on" to what we are driving at, and persist in twisting our purpose and purport all awry. I was long enough, myself, within the Kantian fold to comprehend their difficulty; but let it go. Suffice it to say once more that pragmatism is, in itself, no doctrine of metaphysics, no attempt to determine any truth of things. It is merely a method of ascertaining the meanings of hard words and of abstract concepts. All pragmatists of whatsoever stripe will cordially assent to that statement. As to the ulterior and indirect effects of practising the pragmatistic method, that is quite another affair.

Peirce: CP 5.465 Cross-Ref:††

465. All pragmatists will further agree that their method of ascertaining the meanings of words and concepts is no other than that experimental method by which all the successful sciences (in which number nobody in his senses would include metaphysics) have reached the degrees of certainty that are severally proper to them today; this experimental method being itself nothing but a particular application of an older logical rule, "By their fruits ye shall know them."

Peirce: CP 5.466 Cross-Ref:††

466. Beyond these two propositions to which pragmatists assent *nem. con.*, we find such slight discrepancies between the views of one and another declared adherent as are to be found in every healthy and vigorous school of thought in every department of inquiry. The most prominent of all our school and the most respected, William James, defines pragmatism as the doctrine that the whole "meaning" of a concept expresses itself either in the shape of conduct to be recommended or of experience to be expected.†1 Between this definition and mine there certainly appears to be no slight theoretical divergence, which, for the most part, becomes evanescent in practice; and though we may differ on important questions of philosophy -- especially as regards the infinite and the absolute -- I am inclined to think that the discrepancies reside in other than the pragmatistic ingredients of our thought. If pragmatism had never been heard of, I believe the opinion of James on one side, of me on the other would have developed substantially as they have; notwithstanding our respective connecting them at present with our conception of that method. The brilliant and marvellously human thinker, Mr. F.C.S. Schiller, who extends to the philosophic world a cup of nectar stimulant in his beautiful *Humanism*, seems to occupy ground of his own, intermediate, as to this question, between those of James and mine.

Peirce: CP 5.467 Cross-Ref:††

467. I understand pragmatism to be a method of ascertaining the meanings, not of all ideas, but only of what I call "intellectual concepts," that is to say, of those upon the structure of which, arguments concerning objective fact may hinge. Had the light which, as things are, excites in us the sensation of blue, always excited the sensation of red, and *vice versa*, however great a difference that might have made in

our feelings, it could have made none in the force of any argument. In this respect, the qualities of hard and soft strikingly contrast with those of red and blue; because while red and blue name mere subjective feelings only, hard and soft express the factual behaviour of the thing under the pressure of a knife-edge. (I use the word "hard" in its strict mineralogical sense, "would resist a knife-edge.") My pragmatism, having nothing to do with qualities of feeling, permits me to hold that the predication of such a quality is just what it seems, and has nothing to do with anything else. Hence, could two qualities of feeling everywhere be interchanged, nothing but feelings could be affected. Those qualities have no intrinsic significations beyond themselves. Intellectual concepts, however -- the only sign-burdens that are properly denominated "concepts" -- essentially carry some implication concerning the general behaviour either of some conscious being or of some inanimate object, and so convey more, not merely than any feeling, but more, too, than any existential fact, namely, the "would-acts," "would-dos" of habitual behaviour; and no agglomeration of actual happenings can ever completely fill up the meaning of a "would-be." But [Pragmatism asserts], that the *total* meaning of the predication of an intellectual concept is contained in an affirmation that, under all conceivable circumstances of a given kind (or under this or that more or less indefinite part of the cases of their fulfillment, should the predication be modal) the subject of the predication would behave in a certain general way -- that is, it would be true under given experiential circumstances (or under a more or less definitely stated proportion of them, *taken as they would occur*, that is in the same order of succession, *in experience*). †P1

Peirce: CP 5.468 Cross-Ref: ††

468. A most pregnant principle, quite undeniably, will this "kernel of pragmatism" prove to be, that the *whole* meaning of an intellectual predicate is that certain kinds of events would happen, once in so often, in the course of experience, under certain kinds of existential conditions -- provided it can be proved to be true. But how is this to be done in the teeth of Messrs. Bradley, Taylor, and other high metaphysicians, on the one hand, and of the entire nominalistic nation, with its Wundts, its Haeckels, its Karl Pearsons, and many other regiments, in their divers uniforms, on the other?

Peirce: CP 5.468 Cross-Ref: ††

At this difficulty I have halted for weeks and weeks. It has not been that I could not furnish forth an ample supply of seductive persuasions to pragmatism, or even two or three scientific proofs of its truth. Without a recognition of the chief moments, or points, of these latter it is quite impossible that the power and heart's blood of any variety of doctrine or tendency that ought to be classed among the different species of pragmatism should be really comprehended. A man may very well feel advantages in applications of pragmatism without anything of that. He may even make new applications of the method, himself -- with much risk of blundering, however; but it appears very plain, both to reason and to observation of experience, that he cannot know in what interior eye, what pineal gland its soul and power reside, unless he clearly understands the chief conditions of its truth. Unfortunately, however, all the real proofs of pragmatism that I know -- and, I hardly doubt, all there are to be known -- require just as close and laborious exertion of attention as any but the very most difficult of mathematical theorems, while they add to that all those difficulties of logical analysis which force the mathematician to creep with exceeding caution, if not timorously. But mature consideration has brought me to see that, while those circumstances would render a task quite hopeless that I had never dreamed of undertaking, that of convincing the readers of a literary journal †1 by any honest

argument, of the truth of pragmatism, and consequently must prevent communicating to them quite the idea of this method that an accomplished pragmatist has, yet an idea perfectly fulfilling the reader's desire, that of enabling him to place pragmatism and its concepts in the area of his own thought, and of showing roughly how its concepts are related to familiar concepts [may be given].

Peirce: CP 5.469 Cross-Ref:††
§2. THE VALENCY OF CONCEPTS †2

469. I begin, then, with the first idea that it seems desirable to call to your attention. Everybody is familiar with the useful, though fluctuating and relative distinction of matter and form; and it is strikingly true that distinctions and classifications founded upon form are, with very rare exceptions, more important to the scientific comprehension of the behaviour of things than distinctions and classifications founded upon matter. Mendeléeff's classification of the chemical elements, with which all educated men are, by this time, familiar, affords neat illustrations of this, since the distinctions between what he calls "groups," that is to say, the different vertical columns of his table, consists in the elements of one such "group" entering into different forms of combination with hydrogen and with oxygen from those of another group; or as we usually say, their *valencies* differ; while the distinctions between what he calls the "series," that is, the different horizontal rows of the table, consist in the less formal, more material circumstance that their atoms have, the elements of one "series," greater masses than those of the other. Now everybody who has the least acquaintance with chemistry knows that, while elements in different horizontal rows but the same vertical column always exhibit certain marked physical differences, their chemical behaviours at corresponding temperatures are quite similar; and all the major distinctions of chemical behaviour between different elements are due to their belonging to different vertical columns of the table.

Peirce: CP 5.469 Cross-Ref:††

This illustration has much more pertinence to pragmatism than appears at first sight; since my researches into the logic of relatives have shown beyond all sane doubt that in one respect combinations of concepts exhibit a remarkable analogy with chemical combinations; every concept having a strict valency. (This must be taken to mean that of several forms of expression that are logically equivalent, that one or ones whose analytical accuracy is least open to question, owing to the introduction of the relation of joint identity, follows the law of valency.) Thus, the predicate "is blue" is univalent, the predicate "kills" is bivalent (for the direct and indirect objects are, grammar aside, as much subjects as is the subject nominative); the predicate "gives" is trivalent, since A gives B to C, etc. Just as the valency of chemistry is an atomic character, so indecomposable concepts may be bivalent or trivalent. Indeed, definitions being scrupulously observed, it will be seen to be a truism to assert that no compound of univalent and bivalent concepts alone can be trivalent, although a compound of any concept with a trivalent concept can have at pleasure, a valency higher or lower by one than that of the former concept. Less obvious, yet demonstrable, is the fact that no indecomposable concept has a higher valency. Among my papers are actual analyses of a number greater than I care to state.†1 They are mostly more complex than would be supposed. Thus, the relation between the four bonds of an unsymmetrical carbon atom consists of twenty-four triadic relations.

Careful analysis shows that to the three grades of valency of indecomposable concepts correspond three classes of characters or predicates. Firstly come "firstnesses," or positive internal characters of the subject in itself; secondly come "secondnesses," or brute actions of one subject or substance on another, regardless of law or of any third subject; thirdly comes "*thirdnesses*," or the mental or quasi-mental influence of one subject on another relatively to a third. Since the demonstration of this proposition is too stiff for the infantile logic of our time (which is rapidly awakening, however), I have preferred to state it problematically, as a surmise to be verified by observation. The little that I have contributed to pragmatism (or, for that matter, to any other department of philosophy), has been entirely the fruit of this outgrowth from formal logic, and is worth much more than the small sum total of the rest of my work, as time will show.

Peirce: CP 5.470 Cross-Ref:††
§3. LOGICAL INTERPRETANTS

470. The next moment of the argument for pragmatism is the view that every thought is a sign. This is the doctrine of Leibniz, Berkeley, and the thinkers of the years about 1700. They were all extreme nominalists; but it is a great mistake to suppose that this doctrine is peculiarly nominalistic. I am myself a scholastic realist of a somewhat extreme stripe. Every realist must, as such, admit that a general is a term and therefore a sign. If, in addition, he holds that it is an absolute exemplar, this Platonism passes quite beyond the question of nominalism and realism; and indeed the doctrine of Platonic ideas has been held by the extremest nominalists. There is some reason to suspect that it was shared by Roscellinus himself.

Peirce: CP 5.471 Cross-Ref:††

471. The next point is still less novel; for not to mention references to it by the Greek commentators upon Aristotle, it was between six and seven centuries ago that John of Salisbury spoke of it as "fere in omnium ore celebre."†1 It is the distinction, to use that author's phrases, between that which a term *nominat* -- its logical breadth -- and that which it *significat* -- its logical depth.†2 In the case of a proposition, it is the distinction between that which its subject denotes and that which its predicate asserts. In the case of an argument, it is the distinction between the state of things in which its premisses are true and the state of things which is defined by the truth of its conclusion.

Peirce: CP 5.472 Cross-Ref:††

472. The action of a sign calls for a little closer attention. Let me remind you of the distinction referred to above between dynamical, or dyadic, action; and intelligent, or triadic action. An event, A, may, by brute force, produce an event, B; and then the event, B, may in its turn produce a third event, C. The fact that the event, C, is about to be produced by B has no influence at all upon the production of B by A. It is impossible that it should, since the action of B in producing C is a contingent future event at the time B is produced. Such is dyadic action, which is so called because each step of it concerns a pair of objects.

Peirce: CP 5.473 Cross-Ref:††

473. But now when a microscopist is in doubt whether a motion of an animalcule is guided by intelligence, of however low an order, the test he always used

to apply when I went to school, and I suppose he does so still, is to ascertain whether event, A, produces a second event, B, *as a means* to the production of a third event, C, or not. That is, he asks whether B will be produced if it will produce or is likely to produce C in its turn, but will not be produced if it will not produce C in its turn nor is likely to do so. Suppose, for example, an officer of a squad or company of infantry gives the word of command, "Ground arms!" This order is, of course, a sign. That thing which causes a sign as such is called the *object* (according to the usage of speech, the "real," but more accurately, the *existent* object) represented by the sign: the sign is determined to some species of correspondence with that object. In the present case, the object the command represents is the will of the officer that the butts of the muskets be brought down to the ground. Nevertheless, the action of his will upon the sign is not simply dyadic; for if he thought the soldiers were deaf mutes, or did not know a word of English, or were raw recruits utterly undrilled, or were indisposed to obedience, his will probably would not produce the word of command. However, although this condition is most usually fulfilled, it is not essential to the action of a sign. For the acceleration of the pulse is a probable *symptom* of fever and the rise of the mercury in an ordinary thermometer or the bending of the double strip of metal in a metallic thermometer is an indication, or, to use the technical term, is an *index*, of an increase of atmospheric temperature, which, nevertheless, acts upon it in a purely brute and dyadic way. In these cases, however, a mental representation of the index is produced, which mental representation is called the *immediate object* of the sign; and this object does triadically produce the intended, or proper, effect of the sign strictly by means of another mental sign; and that this triadic character of the action is regarded as essential is shown by the fact that if the thermometer is dynamically connected with the heating and cooling apparatus, so as to check either effect, we do not, in ordinary parlance speak of there being any *semeiosis*, or action of a sign, but, on the contrary, say that there is an "automatic regulation," an idea opposed, in our minds, to that of *semeiosis*. For the proper significate outcome of a sign, I propose the name, the *interpretant* of the sign. The example of the imperative command shows that it need not be of a mental mode of being. Whether the interpretant be necessarily a triadic result is a question of words, that is, of how we limit the extension of the term "sign"; but it seems to me convenient to make the triadic production of the interpretant essential to a "sign," calling the wider concept like a Jacquard loom, for example, a "quasi-sign." On these terms, it is very easy (not descending to niceties with which I will not annoy your readers) to see what the interpretant of a sign is: it is all that is explicit in the sign itself apart from its context and circumstances of utterance. Still, there is a possible doubt as to where the line should be drawn between the interpretant and the object. It will be convenient to give the mere glance, which is all that can be afforded, to this question as it applies to propositions. The interpretant of a proposition is its predicate; its object is the things denoted by its subject or subjects (including its grammatical objects, direct and indirect, etc.). Take the proposition "Burnt child shuns fire." Its predicate might be regarded as all that is expressed, or as "has either not been burned or shuns fire" or "has not been burned," or "shuns fire" or "shuns" or "is true"; nor is this enumeration exhaustive. But where shall the line be most truly drawn? I reply that the purpose of this sentence being understood to be to communicate information, anything belongs to the interpretant that describes the quality or character of the fact, anything to the object that, without doing that, distinguishes this fact from others like it; while a third part of the proposition, *perhaps*, must be appropriated to information about the manner in which the assertion is made, what warrant is offered for its truth, etc. But I rather incline to think that all this goes to the subject. On this view, the predicate is, "is either not a

child or has not been burned, or has no opportunity of shunning fire or does shun fire"; while the subject is "any individual object the interpreter may select from the universe of ordinary everyday experience."

Peirce: CP 5.474 Cross-Ref:††

474. I omit all I possibly can; but there is one fact extremely familiar in itself, that needs to be mentioned as being an indispensable point in the argument. It is that every man inhabits two worlds. These are directly distinguishable by their different appearances. But the greatest difference between them, by far, is that one of these two worlds, the Inner World, exerts a comparatively slight compulsion upon us, though we can by direct efforts so slight as to be hardly noticeable, change it greatly, creating and destroying existent objects in it; while the other world, the Outer World, is full of irresistible compulsions for us, and we cannot modify it in the least, except by one peculiar kind of effort, muscular effort, and but very slightly even in that way.

Peirce: CP 5.475 Cross-Ref:††

475.†1 Now the problem of what the "meaning" of an intellectual concept is can only be solved by the study of the interpretants, or proper significate effects, of signs. These we find to be of three general classes with some important subdivisions. The first proper significate effect of a sign is a feeling produced by it. There is almost always a feeling which we come to interpret as evidence that we comprehend the proper effect of the sign, although the foundation of truth in this is frequently very slight. This "emotional interpretant," as I call it, may amount to much more than that feeling of recognition; and in some cases, it is the only proper significate effect that the sign produces. Thus, the performance of a piece of concerted music is a sign. It conveys, and is intended to convey, the composer's musical ideas; but these usually consist merely in a series of feelings. If a sign produces any further proper significate effect, it will do so through the mediation of the emotional interpretant, and such further effect will always involve an effort. I call it the energetic interpretant. The effort may be a muscular one, as it is in the case of the command to ground arms; but it is much more usually an exertion upon the Inner World, a mental effort. It never can be the meaning of an intellectual concept, since it is a single act, [while] such a concept is of a general nature. But what further kind of effect can there be?

Peirce: CP 5.476 Cross-Ref:††

476. In advance of ascertaining the nature of this effect, it will be convenient to adopt a designation for it, and I will call it the *logical interpretant*, without as yet determining whether this term shall extend to anything beside the meaning of a general concept, though certainly closely related to that, or not. Shall we say that this effect may be a thought, that is to say, a mental sign? No doubt, it may be so; only, if this sign be of an intellectual kind -- as it would have to be -- it must itself have a logical interpretant; so that it cannot be the *ultimate* logical interpretant of the concept. It can be proved that the only mental effect that can be so produced and that is not a sign but is of a general application is a *habit-change*; meaning by a habit-change a modification of a person's tendencies toward action, resulting from previous experiences or from previous exertions of his will or acts, or from a complexus of both kinds of cause. It excludes natural dispositions, as the term "habit" does, when it is accurately used; but it includes beside associations, what may be called "transsociations," or alterations of association, and even includes *dissociation*, which has usually been looked upon by psychologists (I believe mistakenly), as of deeply contrary nature to association.

Peirce: CP 5.477 Cross-Ref:††

477. Habits have grades of strength varying from complete dissociation to inseparable association. These grades are mixtures of promptitude of action, say excitability and other ingredients not calling for separate examination here. The habit-change often consists in raising or lowering the strength of a habit. Habits also differ in their endurance (which is likewise a composite quality). But generally speaking, it may be said that the effects of habit-change last until time or some more definite cause produces new habit-changes. It naturally follows that repetitions of the actions that produce the changes increase the changes. [It] is noticeable that the iteration of the action is often said to be indispensable to the formation of a habit; but a very moderate exercise of observation suffices to refute this error. A single reading yesterday of a casual statement that the "shtar chindis" means in Romany "four shillings," though it is unlikely to receive any reinforcement beyond the recalling of it, at this moment, is likely to produce the habit of thinking that "four" in the Gypsy tongue is "shtar," that will last for months, if not for years, though I should never call it to mind in the interval. To be sure, there has been some iteration just now, while I dwelt on the matter long enough to write these sentences; but I do not believe any reminiscence like this was needed to create the habit; for such instances have been extremely numerous in acquiring different languages. There are, of course, other means than repetition of intensifying habit-changes. In particular, there is a peculiar kind of effort, which may be likened to an imperative command addressed to the future self. I suppose the psychologists would call it an act of auto-suggestion.

Peirce: CP 5.478 Cross-Ref: ††

478. We may distinguish three classes of events causative of habit-change. Such events may, in the first place, not be acts of the mind in which the habit-change is brought about, but experiences forced upon [it]. Thus, surprise is very efficient in breaking up associations of ideas. On the other hand, each new instance that is brought to the experience that supports an induction goes to strengthen that association of ideas -- that inward habit -- in which the tendency to believe in the inductive conclusion consists. But careful examination has pretty thoroughly satisfied me that no new association, no entirely new habit, can be created by involuntary experiences.

Peirce: CP 5.479 Cross-Ref: ††

479. In the second place, the event that causes a habit-change may be a muscular effort, apparently. If I wish to acquire the habit of speaking of "speaking, writing, thinking," etc., instead of "speakin', writin', thinkin'," as I suspect I now do (though I am not sure) -- all I have to do is to make the desired enunciations a good many times; and to do this as thoughtlessly as possible, since it is an inattentive habit that I am trying to create. Everybody knows the facility with which habits may thus be acquired, even quite unintentionally. But I am persuaded that nothing like a concept can be acquired by muscular practice alone. When we seem to do that, it is not the muscular action but the accompanying inward efforts, the acts of imagination, that produce the habit. If a person who has never tried such a thing before undertakes to stand on one foot and to move the other round a horizontal circle, say, as being the easier way, clockwise if he is standing on the left foot, or counter-clockwise if he is standing on the right foot, and at the same time to move the fist of the same side as the moving foot round a horizontal circle in the opposite direction, that is, clockwise if the foot is moved counter-clockwise, and *vice versa*, he will, at first, find he cannot do it. The difficulty is that he lacks a unitary concept of the series of efforts that success requires. By practising the different parts of the movement, while attentively observing the kind of effort requisite in each part, he will, in a few minutes, catch the

idea, and will then be able to perform the movements with perfect facility. But the proof that it is in no degree the muscular efforts, but only the efforts of the imagination that have been his teachers, is that if he does not perform the actual motions, but only imagines them vividly, he will acquire the same trick with only so much additional practice as is accounted for by the difficulty of imagining all the efforts that will have to be made in a movement one has not actually executed. There is an obvious difficulty of determining just how much allowance should be made for this, in the fact [that] when the feat is learned in either way, it cannot be unlearned, so as to compare that way with the other. The only resort is to learn a considerable number of feats which depend upon acquiring a unitary conception of a series of efforts, learning some with actual muscular exercise and others by unaided imagination, and then forming one's judgment of whether the greater facility afforded by the actual muscular contractions is, or is not, greater than the support this gives the imagination. Saying the verse about "Peter Piper"; spelling without an instant's hesitation, in the old way, the name Aldibirontifoscoforniocrononhotontothologes (that is, thus: *A-l*, al, and here's my *al*; *d-i*, di, and here's my *di*, and here's my *aldi*; *b-i*, bi, and here's my *bi*, and here's my *dibi*, and here's my *aldibi*, etc.); making the pass with one hand upon a pack of cards, playing the thimbles and ball, and other turns of legerdemain all largely depend for their success upon a unitary conception of all that has to be done and just when it must be done. It is from such experiments that I have been led to estimate as nil the power of mere muscular effort in contributing to the acquisition of ideas.†P1

Peirce: CP 5.480 Cross-Ref:††

480. Every concept, doubtless, first arises when upon a strong, but more or less vague, sense of need is superinduced some involuntary experience of a suggestive nature; that being suggestive which has a certain occult relation to the build of the mind. We may assume that it is the same with the instinctive ideas of animals; and man's ideas are quite as miraculous as those of the bird, the beaver, and the ant. For a not insignificant percentage of them have turned out to be the keys of great secrets. With beasts, however, conditions are comparatively unchanging, and there is no further progress. With man these first concepts (first in the order of development, but emerging at all stages of mental life) take the form of conjectures, though they are by no means always recognized as such. Every concept, every general proposition of the great edifice of science, first came to us as a conjecture. These ideas are the *first logical interpretants* of the phenomena that suggest them, and which, as suggesting them, are signs, of which they are the (really conjectural) interpretants. But that they are no more than that is evidently an after-thought, the dash of cold doubt that awakens the sane judgment of the muser. Meantime, do not forget that every conjecture is equivalent to, or is expressive of, such a habit that having a certain desire one might accomplish it if one could perform a certain act. Thus, the primitive man must have been sometimes asked by his son whether the sun that rose in the morning was the same as the one that set the previous evening; and he may have replied, "I do not know, my boy; but I think that if I could put my brand on the evening sun, I should be able to see it on the morning sun again; and I once knew an old man who could look at the sun though he could hardly see anything else; and he told me that he had once seen a peculiarly shaped spot on the sun; and that it was to be recognized quite unmistakably for several days." [Readiness] to act in a certain way under given circumstances and when actuated by a given motive is a habit; and a deliberate, or self-controlled, habit is precisely a belief.

Peirce: CP 5.481 Cross-Ref:††

481. In the next step of thought, those first logical interpretants stimulate us to various voluntary performances in the inner world. We imagine ourselves in various situations and animated by various motives; and we proceed to trace out the alternative lines of conduct which the conjectures would leave open to us. We are, moreover, led, by the same inward activity, to remark different ways in which our conjectures could be slightly modified. The logical interpretant must, therefore, be in a relatively future tense.

Peirce: CP 5.482 Cross-Ref:††

482. To this may be added the consideration that it is not all signs that have logical interpretants, but only intellectual concepts and the like; and these are all either general or intimately connected with generals, as it seems to me. This shows that the species of future tense of the logical interpretant is that of the conditional mood, the "*would-be*."

Peirce: CP 5.483 Cross-Ref:††

483. At the time I was originally puzzling over the enigma of the nature of the logical interpretant, and had reached about the stage where the discussion now is, being in a quandary, it occurred to me that if I only could find a moderate number of concepts which should be at once highly abstract and abstruse, and yet the whole nature of whose meanings should be quite unquestionable, a study of them would go far toward showing me how and why the logical interpretant should in all cases be a conditional future. I had no sooner framed a definite wish for such concepts, than I perceived that in mathematics they are as plenty as blackberries. I at once began running through the explications of them, which I found all took the following form: Proceed according to such and such a general rule. Then, if such and such a concept is applicable to such and such an object, the operation will have such and such a general result; and conversely. Thus, to take an extremely simple case, if two geometrical figures of dimensionality N should be equal in all their parts, an easy rule of construction would determine, in a space of dimensionality N containing both figures, an axis of rotation, such that a rigid body that should fill not only that space but also a space of dimensionality $N + 1$, containing the former space, turning about that axis, and carrying one of the figures along with it while the other figure remained at rest, the rotation would bring the movable figure back into its original space of dimensionality, N , and when that event occurred, the movable figure would be in exact coincidence with the unmoved one, in all its parts; while if the two figures were not so equal, this would never happen.

Peirce: CP 5.483 Cross-Ref:††

Here was certainly a stride toward the solution of the enigma.

Peirce: CP 5.483 Cross-Ref:††

For the treatment of a score of intellectual concepts on that model, only a few of them being mathematical, seemed to me to be so refulgently successful as fully to convince me that to predicate any such concept of a real or imaginary object is equivalent to declaring that a certain operation, corresponding to the concept, if performed upon that object, would (certainly, or probably, or possibly, according to the mode of predication), be followed by a result of a definite general description.

Peirce: CP 5.484 Cross-Ref:††

484. Yet this does not quite tell us just what the nature is of the essential effect

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upon the interpreter, brought about by the semio'sis [Click here to view] of the sign, which constitutes the logical interpretant. (It is important to understand what I mean by *semiosis*. All dynamical action, or action of brute force, physical or psychical, either takes place between two subjects [whether they react equally upon each other, or one is agent and the other patient, entirely or partially] or at any rate is a resultant of such actions between pairs. But by "semiosis" I mean, on the contrary, an action, or influence, which is, or involves, a coöperation of *three* subjects, such as a sign, its object, and its interpretant, this tri-relative influence not being in any way resolvable into actions between pairs. {Sémeiōsis} in Greek of the Roman period, as early as Cicero's time, if I remember rightly, meant the action of almost any kind of sign; and my definition confers on anything that so acts the title of a "sign.")

Peirce: CP 5.485 Cross-Ref:††

485. Although the definition does not require the logical interpretant (or, for that matter, either of the other two interpretants) to be a modification of consciousness, yet our lack of experience of any semiosis in which this is not the case, leaves us no alternative to beginning our inquiry into its general nature with a provisional assumption that the interpretant is, at least, in all cases, a sufficiently close analogue of a modification of consciousness to keep our conclusion pretty near to the general truth. We can only hope that, once that conclusion is reached, it may be susceptible of such a generalization as will eliminate any possible error due to the falsity of that assumption. The reader may well wonder why I do not simply confine my inquiry to psychical semiosis, since no other seems to be of much importance. My reason is that the too frequent practice, by those logicians who do not go to work [with] any method at all [or who follow] the method of basing propositions in the science of logic upon results of the science of psychology -- as contradistinguished from common-sense observations concerning the workings of the mind, observations well-known even if little noticed, to all grown men and women, that are of sound minds -- that practice is to my apprehension as unsound and insecure as was that bridge in the novel of "Kenilworth" that, being utterly without any sort of support, sent the poor Countess Amy to her destruction; seeing that, for the firm establishment of the truths of the science of psychology, almost incessant appeals to the results of the science of logic -- as contradistinguished from natural perceptions that one relation evidently involves another -- are peculiarly indispensable. Those logicians continually confound *psychical* truths with *psychological* truths, although the distinction between them is of that kind that takes precedence over all others as calling for the respect of anyone who would tread the strait and narrow road that leadeth unto exact truth.

Peirce: CP 5.486 Cross-Ref:††

486. Making that provisional assumption, then, I ask myself, since we have already seen that the logical interpretant is general in its possibilities of reference (i.e., refers or is related to whatever there may be of a certain description), what categories of mental facts there be that are of general reference. I can find only these four: conceptions, desires (including hopes, fears, etc.), expectations, and habits. I trust I have made no important omission. Now it is no explanation of the nature of the logical interpretant (which, we already know, is a concept) to say that it is a concept. This objection applies also to desire and expectation, as explanations of the same

interpretant; since neither of these is general otherwise than through connection with a concept. Besides, as to desire, it would be easy to show (were it worth the space), that the logical interpretant is an effect of the energetic interpretant, in the sense in which the latter is an effect of the emotional interpretant. Desire, however, is cause, not effect, of effort.†1 As to expectation, it is excluded by the fact that it is not conditional. For that which might be mistaken for a conditional expectation is nothing but a judgment that, under certain conditions, there would be an expectation: there is no conditionality in the expectation itself, such as there is in the logical interpretant after it is actually produced. Therefore, there remains only habit, as the essence of the logical interpretant.

Peirce: CP 5.487 Cross-Ref:††

487. Let us see, then, just how, according to the rule derived from mathematical concepts (and confirmed by others), this habit is produced; and what sort of a habit it is. In order that this deduction may be rightly made, the following remark will be needed. It is not a result of scientific psychology, but is simply a bit of the catholic and undeniable common sense of mankind, with no other modification than a slight accentuation of certain features.

Peirce: CP 5.487 Cross-Ref:††

Every sane person lives in a double world, the outer and the inner world, the world of percepts and the world of fancies. What chiefly keeps these from being mixed up together is (besides certain marks they bear) everybody's well knowing that fancies can be greatly modified by a certain non-muscular effort, while it is muscular effort alone (whether this be "voluntary," that is, pre-intended, or whether all the intended endeavour is to inhibit muscular action, as when one blushes, or when peristaltic action is set up on experience of danger to one's person) that can to any noticeable degree modify percepts. A man can be durably affected by his percepts and by his fancies. The way in which they affect him will be apt to depend upon his personal inborn disposition and upon his habits. Habits differ from dispositions in having been acquired as consequences of the principle, virtually well-known even to those whose powers of reflexion are insufficient to its formulation, that multiple reiterated behaviour of the same kind, under similar combinations of percepts and fancies, produces a tendency -- the *habit* -- actually to behave in a similar way under similar circumstances in the future. Moreover -- *here is the point* -- every man exercises more or less control over himself by means of modifying his own habits; and the way in which he goes to work to bring this effect about in those cases in which circumstances will not permit him to practice reiterations of the desired kind of conduct in the outer world shows that he is virtually well-acquainted with the important principle that *reiterations in the inner world -- fancied reiterations -- if well-intensified by direct effort, produce habits, just as do reiterations in the outer world; and these habits will have power to influence actual behaviour in the outer world*; especially, if each reiteration be accompanied by a peculiar strong effort that is usually likened to issuing a command to one's future self.†P1

Peirce: CP 5.488 Cross-Ref:††

488. I here owe my patient reader a confession. It is that when I said that those signs that have a logical interpretant are either general or closely connected with generals, this was not a scientific result, but only a strong impression due to a life-long study of the nature of signs. My excuse for not answering the question scientifically is that I am, as far as I know, a pioneer, or rather a backwoodsman, in the work of clearing and opening up what I call *semiotic*, that is, the doctrine of the

essential nature and fundamental varieties of possible semiosis; and I find the field too vast, the labor too great, for a first-comer. I am, accordingly, obliged to confine myself to the most important questions. The questions of the same particular type as the one I answer on the basis of an impression, which are of about the same importance, exceed four hundred in number; and they are all delicate and difficult, each requiring much search and much caution. At the same time, they are very far from being among the most important of the questions of semiotic. Even if my answer is not exactly correct, it can lead to no great misconception as to the nature of the logical interpretant. There is my apology, such as it may be deemed.

Peirce: CP 5.489 Cross-Ref:††

489. It is not to be supposed that upon every presentation of a sign capable of producing a logical interpretant, such interpretant is actually produced. The occasion may either be too early or too late. If it is too early, the semiosis will not be carried so far, the other interpretants sufficing for the rude functions for which the sign is used. On the other hand, the occasion will come too late if the interpreter be already familiar with the logical interpretant, since then it will be recalled to his mind by a process which affords no hint of how it was originally produced. Moreover, the great majority of instances in which formations of logical interpretants do take place are very unsuitable to serve as illustrations of the process, because in them the essentials of this semiosis are buried in masses of accidental and hardly relevant semioses that are mixed with the former. The best way that I have been able to hit upon for simplifying the illustrative example which is to serve as our matter upon which to experiment and observe is to suppose a man already skillful in handling a given sign (that has a logical interpretant) to begin now before our inner gaze for the first time, seriously to inquire what that interpretant is. It will be necessary to amplify this hypothesis by a specification of what his *interest* in the question is supposed to be. In doing this, I, by no means, follow Mr. Schiller's brilliant and seductive humanistic logic, according to which it is proper to take account of the whole personal situation in logical inquiries.†1 For I hold it to be very evil and harmful procedure to introduce into scientific investigation an unfounded hypothesis, without any definite prospect of its hastening our discovery of the truth. Now such a hypothesis Mr. Schiller's rule seems to me, with my present lights, to be. He has given a number of reasons for it; but, to my estimate, they seem to be of that quality that is well calculated to give rise to interesting discussions, and is consequently to be recommended to those who intend to pursue the study of philosophy as an entertaining exercise of the intellect, but is negligible [to] one whose earnest purpose is to do what in him lies toward bringing about a metamorphosis of philosophy into a genuine science. I cannot turn aside into Mr. Schiller's charming lane. When I ask what the interest is in seeking to discover a logical interpretant, it is not my fondness for strolling in paths where I can study the varieties of humanity that moves me, but the definite reflection that unless our hypothesis be rendered specific as to that interest, it will be impossible to trace out its logical consequences, since the way the interpreter will conduct the inquiry will greatly depend upon the nature of his interest in it.

Peirce: CP 5.490 Cross-Ref:††

490. I shall suppose, then, that the interpreter is not particularly interested in the theory of logic, which he may judge by examples to be profitless; but I shall suppose that he has embarked a great part of the treasures of his life in the enterprise of perfecting a certain invention; and that, for this end, it seems to him extremely desirable that he should acquire a demonstrative knowledge of the solution of a certain problem of reasoning. As to this problem itself, I shall suppose that it does not

fall within any class for which any general method of handling is known, and that indeed it is indefinite in every respect which might afford any familiar kind of handle by which any image fairly representing it could be held firmly before the mind and examined; so that, in short, it seems to elude reason's application or to slip from its grasp.

Peirce: CP 5.490 Cross-Ref:††

Various problems answering this description might be instanced; but to fix our ideas, I will specify one of them, and will suppose that this is the very one which our imaginary inventor wishes to solve. It shall be the following "map-coloring problem": Let a globular body be bored through in two wide holes; and, though it is unnecessary, the edge at each end of each tunnel shall be smoothly rounded off. Then the problem is, supposing its utterer is free to divide the whole surface of this body -- including the surfaces of the bores -- into regions in any way he likes (no region consisting of separated pieces), and supposing that it will then fall to the interpreter to color the whole area of each region in one color, but never giving to two regions that abut along a common boundary-line the same color; required to ascertain what will be the least number of different colors that will always suffice, no matter how the surface may have been divided.

Peirce: CP 5.490 Cross-Ref:††

Under the high stimulus of his interest in this problem, and with that practical knack that we have supposed him to possess in coloring maps without too frequently being obliged to go back and alter the colors he had assigned to given regions, we need not doubt that our inquirer will be thrown into a state of high activity in the world of fancies, in experimenting upon coloring maps, while trying to make out what subconscious rule guides him, and renders him as successful as he usually is; and in trying, too, to discover what rule he had violated in each case where his first coloration has to be changed. This activity is, logically, an energetic interpretant of the interrogatory he puts to himself. Should he in this way succeed in working out a determinate rule for coloring every map on the two-tunnelled (or, what is the same thing, the two-bridged) everywhere unbounded surface with the fewest possible colors, there will be good hope that a demonstration may tread upon the heels of that rule, in which case, the problem will be solved in the most convenient form.

Peirce: CP 5.490 Cross-Ref:††

But while he may very likely manage to formulate his own usually successful way of coloring the regions, it is very unlikely that he will obtain an unfailing rule for doing so. For after some of the first mathematicians in Europe had found themselves baffled by the far simpler problem, to prove that every map upon an ordinary sheet can be colored with four colors, one of the very first logico-mathematicians of our age, Mr. Alfred B. Kempe,†1 proposed a proof of it, somewhat, though not exactly, of the kind we are supposing our imaginary inventor to be aiming at. Yet I am informed that many years later a fatal flaw was discovered in Mr. Kempe's proof. I do not remember that I ever knew what the fallacy was. We may assume with confidence, then, that our imaginary interpreter will, at length, come to despair of solving the problem in that way. What way shall I imagine him to try next?

Peirce: CP 5.490 Cross-Ref:††

It will be very natural for him to pass from endeavouring to define a uniformly successful rule of procedure, to endeavouring either; first, to define the topical conditions under which two different regions must be colored alike, if the colors are not to exceed a given number, whence he will deduce the conditions under which two

regions that do not abut must be colored differently; or else, first to define the conditions under which two regions cannot, by being stretched out, be brought into abuttal along a boundary, and thence to define the conditions under which two regions must be colored alike. Either of these methods is more promising than the one with which he began; and yet were either capable of being perfected without some very peculiar *aperçu*, the easier task of demonstrating that four colors suffice for every map on an ordinary limited sheet or globular surface must long ago have been brought to completion, which never has been accomplished, I believe, in print. We may assume, then, that he will, at length, come to abandon every such method. Meantime, he cannot fail to have noticed several obvious propositions that will be useful in his further inquiries. One of these will be that by minute alterations of the boundaries between regions, which alterations can neither diminish nor increase the number of colors that will in all cases just suffice, he can get rid of all points where four or more regions concur, and thus render the number of points of concurrence two-thirds as many as the number of boundaries, so that the latter number will be divisible by three, and the former by two, unless fewer colors are required than are generally necessary. He will also have remarked that there must for each color be at least one region of that color which abuts upon regions of all the other colors, that for each of these other colors there must be at least one region that besides abutting upon the first region abuts upon regions of all the remaining colors, etc.

Peirce: CP 5.490 Cross-Ref:††

I shall suppose that it now occurs to him that it not only makes no difference what the proportionate dimensions either of the whole surface or of any of the regions are, but that it is equally indifferent whether any part of the whole surface be flat, convex, concave, curved, or broken by angles, or whether any boundaries are straight, curved, or broken by angles, and are convex or concave to either of the regions it bounds; whence it will follow that the problem belongs neither to Metrical, nor to Graphical (or Projective) Geometry, but to Topical Geometry, or Geometrical Topics. This is the most fundamental, and no doubt, in its own nature, much the easiest of the three departments of geometry. For just as Cayley showed,†1 metrics is but a special problem in the easier graphics; so quite obviously graphics is a special problem in the easier topics. For there is no other possible way of defining unlimited planes and rays, than by the topical statement (which does not fully define them) that the unbounded planes are a family of surfaces in 3-dimensional space of which any two contain one common line, only, which is a *ray*, and of which any three that do not all contain one common ray, have one point and only one in common; and further, any two points are both contained in one and in only one ray, while any three points not all in one ray are contained in one and only one unbounded plane.†P1

Peirce: CP 5.490 Cross-Ref:††

But though Topics must be the easiest kind of geometry, yet geometers were so accustomed to rely on considerations of measure and of flatness, that when they were deprived of these, they did not know how to handle problems; so that, apart from mere enumerations of forms, such as knots, we are still in possession of only one general theorem of Topics, Listing's census-theorem.†1 Consequently, our imagined investigator, as soon as he remarks that he has a problem in topical geometry before him, will infer that he must utilize that sole known theorem of topics; albeit it is sufficiently obvious that that theorem of itself is not adequate to furnishing a solution of his problem. I will state the census-theorem of Listing with some sacrifice of exactitude to perspicuity, insofar as it applies to the map-coloring problem. The surface which is divided into regions may be bounded by a line or unbounded. If it be

unbounded and separates [a] solid into two parts, I call it *artiad*; if it does not, I call it *perissid*. The *Cyclosy*, or ringiness, of the surface of a body unpierced by any tunnel (i.e., not bridged over by an unbounded bridge), is *zero*; and every tunnel through the body adds *two* to the cyclosy of its surface. The cyclosy of the simplest perissid surface, such as an unbounded plane, is *one*, and every tunnel connecting two parts of it in an additional way (or every cylindrical bridge, which will be a tunnel on the other side of the surface) adds *two* to the cyclosy. A region, or an uninterrupted boundary that does not return into itself (as I will assume is the case with all regions and boundaries between two regions), has *zero* cyclosy. I will further assume that there is more than one region on the surface. Under these circumstances, the Census theorem takes this form, supposing all points of concurrence of regions are points where three regions and no more run together; one third of the number of boundaries from one point of concurrence to the next diminished by the number of regions is equal to one less than the cyclosy of the whole surface, if this be bounded, or to two less than the cyclosy, if the surface be unbounded. In the case of the surface of the body pierced by two tunnels, the surface is unbounded, and its cyclosy is 4. The investigator will see at once that the number of colors must be at least 7, and is likely to be more. For were the body pierced by but one tunnel, let the number of regions each abutting upon all the rest be x . Then, the number of boundaries would be $1/2x(x-1)$; and the census-theorem applied to this case would be $1/6 x(x-1) - x = 2-2$. That is $x^2 - 7x = 0$, or $x = 7$. Since, then, even with but one tunnel seven colors might be required, at least that number will be required for the case of two tunnels. On the other hand, were 2 tunnels made in a projective plane, where the cyclosy would be 5, instead of 4, only 9 regions could touch one another; so that it is likely that for a surface of cyclosy 4, the requisite number of colors is less than 9. The investigator will, therefore, only have to ascertain whether 8, and if so whether 9, colors can be required. He is still not very near his solution, but he is not hopelessly removed from it.

Peirce: CP 5.491 Cross-Ref: ††

491. In every case, after some preliminaries, the activity takes the form of experimentation in the inner world; and the conclusion (if it comes to a definite conclusion), is that under given conditions, the interpreter will have formed the habit of acting in a given way whenever he may desire a given kind of result. The real and living logical conclusion *is* that habit; the verbal formulation merely expresses it. I do not deny that a concept, proposition, or argument may be a logical interpretant. I only insist that it cannot be the final logical interpretant, for the reason that it is itself a sign of that very kind that has itself a logical interpretant. The habit alone, which though it may be a sign in some other way, is not a sign in that way in which that sign of which it is the logical interpretant is the sign. The habit conjoined with the motive and the conditions has the action for its energetic interpretant; but action cannot be a logical interpretant, because it lacks generality. The concept which is a logical interpretant is only imperfectly so. It somewhat partakes of the nature of a verbal definition, and is as inferior to the habit, and much in the same way, as a verbal definition is inferior to the real definition. The deliberately formed, self-analyzing habit -- self-analyzing because formed by the aid of analysis of the exercises that nourished it -- is the living definition, the veritable and final logical interpretant. Consequently, the most perfect account of a concept that words can convey will consist in a description of the habit which that concept is calculated to produce. But how otherwise can a habit be described than by a description of the kind of action to which it gives rise, with the specification of the conditions and of the motive?

Peirce: CP 5.492 Cross-Ref:††

492. If we now revert to the psychological assumption originally made, we shall see that it is already largely eliminated by the consideration that habit is by no means exclusively a mental fact. Empirically, we find that some plants take habits. The stream of water that wears a bed for itself is forming a habit. Every ditcher so thinks of it. Turning to the rational side of the question, the excellent current definition of habit, due, I suppose, to some physiologist (if I can remember my bye-reading for nearly half a century unglanced at, Brown-Sequard †1 much insisted on it in his book on the spinal cord), says not one word about the mind. Why should it, when habits in themselves are entirely unconscious, though feelings may be symptoms of them, and when consciousness alone -- i.e., feeling -- is the only distinctive attribute of mind?

Peirce: CP 5.492 Cross-Ref:††

What further is needed to clear the sign of its mental associations is furnished by generalizations too facile to arrest attention here, since nothing but feeling is exclusively mental.

Peirce: CP 5.493 Cross-Ref:††

493. But while I say this, it must not be inferred that I regard consciousness as a mere "epiphenomenon"; though I heartily grant that the hypothesis that it is so has done good service to science. To my apprehension, consciousness may be defined as that congeries of non-relative predicates, varying greatly in quality and in intensity, which are symptomatic of the interaction of the outer world -- the world of those causes that are exceedingly compulsive upon the modes of consciousness, with general disturbance sometimes amounting to shock, and are acted upon only slightly, and only by a special kind of effort, muscular effort -- and of the inner world, apparently derived from the outer, and amenable to direct effort of various kinds with feeble reactions; the interaction of these two worlds chiefly consisting of a direct action of the outer world upon the inner and an indirect action of the inner world upon the outer through the operation of habits. If this be a correct account of consciousness, i.e., of the congeries of feelings, it seems to me that it exercises a real function in self-control, since without it, or at least without that of which it is symptomatic, the resolves and exercises of the inner world could not affect the real determinations and habits of the outer world. I say that these belong to the outer world because they are not mere fantasies but are real agencies.

Peirce: CP 5.494 Cross-Ref:††

§4. OTHER VIEWS OF PRAGMATISM

494. I have now outlined my own form of pragmatism; but there are other slightly different ways of regarding what is practically the same method of attaining vitally distinct conceptions, from which I should protest from the depths of my soul against being separated. In the first place, there is the pragmatism of James, whose definition †1 differs from mine only in that he does not restrict the "meaning," that is, the ultimate logical interpretant, as I do, to a habit, but allows percepts, that is, complex feelings endowed with compulsiveness, to be such. If he is willing to do this, I do not quite see how he need give any room at all to habit. But practically, his view and mine must, I think, coincide, except where he allows considerations not at all

pragmatic to have weight. Then there is Schiller, who offers no less than seven alternative definitions of pragmatism.†2 The first is that pragmatism is the Doctrine that "truths are logical values." At first blush, this seems far too broad; for who, be he pragmatist or absolutist, can fail to prefer truth to fiction? But no doubt what is meant is that the objectivity of truth really consists in the fact that, in the end, every sincere inquirer will be led to embrace it -- and if he be not sincere, the irresistible effect of inquiry in the light of experience will be to make him so. This doctrine appears to me, after one subtraction, to be a corollary of pragmatism. I set it in a strong light in my original presentation of the method.†3 I call my form of it "conditional idealism." That is to say, I hold that truth's independence of individual opinions is due (so far as there is any "truth") to its being the predestined result to which sufficient inquiry *would* ultimately lead. I only object that, as Mr. Schiller himself seems sometimes to say, there is not the smallest scintilla of logical justification for any assertion that a given sort of result will, as a matter of fact, either *always* or *never* come to pass; and consequently we cannot know that there *is* any truth concerning any given question; and this, I believe, agrees with the opinion of M. Henri Poincaré,†1 except that he seems to insist upon the non-existence of any absolute truth for *all* questions, which is simply to fall into the very same error on the opposite side. But practically, we know that questions do generally get settled in time, when they come to be scientifically investigated; and that is practically and pragmatically enough. Mr. Schiller's second definition is Captain Bunsby's that "the 'truth' of an assertion depends on its application," which seems to me the result of a weak analysis. His third definition is that pragmatism is the doctrine that "the meaning of a rule lies in its application," which would make the "meaning" consist in the energetic interpretant and would ignore the logical interpretant; another feeble analysis. His fourth definition is that pragmatism is the doctrine that "all meaning depends on purpose." I think there is much to be said in favor of this, which would, however, make pragmatists of many thinkers who do not consider themselves as belonging to our school of thought. Their affiliations with us are, however, undeniable. His fifth definition is that pragmatism is the doctrine that "all mental life is purposive." His sixth definition is that pragmatism is "a systematic protest against all ignoring of the purposiveness of actual knowing." Mr. Schiller seems habitually to use the word "actual" in some peculiar sense. His seventh definition is that pragmatism is "a conscious application to epistemology (or logic) of a teleological psychology, which implies, ultimately, a voluntaristic metaphysics." Supposing by "psychology" he means *not* the science so called, but a critical acceptance of a sifted common-sense of mankind regarding mental phenomena, I might subscribe to this. I have myself called pragmatism "critical common-sensism"; but, of course, I do not mean this for a strict definition.

Peirce: CP 5.495 Cross-Ref:††

495. Signor Giovanni Papini goes a step beyond Mr. Schiller in maintaining [that] pragmatism is indefinable.†1 But that seems to me to be a literary phrase. In the main, I much admire Papini's presentation of the subject.

Peirce: CP 5.496 Cross-Ref:††

496. There are certain questions commonly reckoned as metaphysical, and which certainly are so, if by metaphysics we mean ontology, which as soon as pragmatism is once sincerely accepted, cannot logically resist settlement. These are for example, What is reality? Are necessity and contingency real modes of being? Are the laws of nature real? Can they be assumed to be immutable or are they presumably

results of evolution? Is there any real chance, or departure from real law? But on examination, if by metaphysics we mean the broadest positive truths of the psycho-physical universe -- positive in the sense of not being reducible to logical formulæ -- then the very fact that these problems can be solved by a logical maxim is proof enough that they do not belong to metaphysics but to "epistemology," an atrocious translation of *Erkenntnislehre*. When we pass to consider the nature of Time, it seems that pragmatism is of aid, but does not of itself yield a solution. When we go on to the nature of Space, I boldly declare that Newton's view that it is a real entity is alone logically tenable; and that leaves such further questions as, Why should Space have three dimensions? quite unanswerable for the present. This, however, is a purely speculative question without much human interest. (It would, of course, be absurd to say that tridimensionality is without practical consequences.) For those metaphysical questions that have such interest, the question of a future life and especially that of One Incomprehensible but Personal God, not immanent in but creating the universe, I, for one, heartily admit that a Humanism, that does not pretend to be a science but only an instinct, like a bird's power of flight, but purified by meditation, is the most precious contribution that has been made to philosophy for ages.

Peirce: CP 5.497 Cross-Ref:††
CHAPTER 2

***PRAGMATISM AND CRITICAL COMMON-SENSISM*†1**

497. **Jules.**†P1 Your Pragmatism, then, seems to be simply the theory that a portion of the "meaning" of thought, that is, I suppose, the substance of thought, which portion you term its "intellectual purport," lies in its reference to conditional resolves. But I fail to see what your vague Common-Sense has to do with it. For your Pragmatism is too definite to fall under that head. Your doctrine of Common-Sense cannot be proved from your Pragmatism, either, since that certain given propositions are absolutely indubitable must itself be an indemonstrable axiom or must be false. So I don't see what relevancy to Pragmatism, even if to any conditional resolve, there was supposed to be in your discourse about this critical acceptance of uncriticizable propositions.

Peirce: CP 5.497 Cross-Ref:††

The Respondent. The Common-Sensism now so widely accepted is not critical of the substantial truth of uncriticizable propositions, but only as to whether a given proposition is of the number.

Peirce: CP 5.498 Cross-Ref:††

498. **Jules.** It seems to me they are the same, at any rate according to your style of dealing with such questions. For I can almost hear you argue that you must either believe a proposition or doubt or disbelieve it. If you believe it, you do not doubt it and cannot criticize it; if you doubt it, it cannot be indubitable.

Peirce: CP 5.498 Cross-Ref:††

Respondent. Just at this moment the question is whether those two propositions really are the same or distinct, and not what my general style of argumentation is; and I am happy to think that you do not yourself sincerely judge all the sages of human nature to have been conscious liars who from time immemorial have testified to their conviction that man possess no infallible introspective power into the secrets of his own heart, to know just what he believes and what he doubts. The denial of such a power is one of the clauses of critical common-sensism. The others are that there are indubitable beliefs which vary a little and but a little under varying circumstances and in distant ages; that they partake of the nature of instincts, this word being taken in a broad sense; that they concern matters within the purview of the primitive man; that they are very vague indeed (such as, that fire burns) without being perfectly so; that while it may be disastrous to science for those who pursue it to think they doubt what they really believe, and still more so really to doubt what they ought to believe, yet, on the whole, neither of these is so unfavorable to science as for men of science to believe what they ought to doubt, nor even for them to think they believe what they really doubt; that a philosopher ought not to regard an important proposition as indubitable without a systematic and arduous endeavour to attain to a doubt of it, remembering that genuine doubt cannot be created by a mere effort of will, but must be compassed through experience; that while it is possible that propositions that really are indubitable, for the time being, should nevertheless be false, yet in so far as we do not doubt a proposition we cannot but regard it as perfectly true and perfectly certain; that while holding certain propositions to be each individually perfectly certain, we may and ought to think it likely that some one of them, if not more, is false.†P1

Peirce: CP 5.499 Cross-Ref:††

499. This is the doctrine of Critical Common-sensism, and the present pertinency of it is that a pragmatist, to be consistent, is obliged to embrace it. . . . For brevity's sake, I shall confine myself to the easiest stated but far from the best of several reasons. Because the pragmatist, then, recognizes that the substance of what he thinks lies in a conditional resolve, he cannot fail also to recognize that to learn the very truth is the way to satisfy the wishes of his heart. For this reason he will be of all men the man whose mind is most open to conviction, and will be keen up the scent of whatever can go toward teaching him to distinguish accurately between truth and falsity, probability and improbability. This will suffice to make the pragmatist attentive to all those matters of every-day facts which critical common-sensism takes into account. It remains to say why he, more than another man, should be inclined to draw these inferences from those facts. His doctrine essentially insists upon the close affinity between thinking in particular and endeavour in general. Since, therefore, action in general is largely a matter of instinct, he will be pretty sure to ask himself whether it be not the same with belief. That this question once asked admits of but one answer is shown by the fact that even John Locke was obliged to fall into line here, notwithstanding the nominalistic metaphysics, the most blinding of all systems, as metaphysics generally is the most powerful of all causes of mental cecity, because it deprives the mind of the power to ask itself certain questions, as the habit of wearing a confining dress deprives one's joints of their suppleness. Now Locke was a man of strong prejudices, while the pragmatist -- it cannot be said too often -- will be the most open-minded of all men. But once it is settled that belief may be a matter of instinct and of desire, the inquiry [will arise] whether almost every man will not have his quite irresistible beliefs spring up ineluctably, together with the question

whether these will not be pretty uniform. Now irresistible instinctive desires are such familiar and such almost invariable phenomena -- so few men, let us say, are able to hold their breath for five minutes, even among strong thinkers who are apt to be great breath-holders -- that there can be no doubt how this will get answered. . . . You see for yourself that pragmatism will be sure to carry critical common-sensism in its arms, do you not?

Peirce: CP 5.500 Cross-Ref:††

500. **Jules.** Perhaps so, but how do you know that your pragmatic doctrine is true?

Peirce: CP 5.500 Cross-Ref:††

Respondent. Why just place the two doctrines -- pragmatism and its alternative -- side by side in your mind.

Peirce: CP 5.500 Cross-Ref:††

Jules. Gracious! Have the gracious gods confined us to two alternatives? I don't know what power of two would suffice to count up the varieties of pragmatism itself.

Peirce: CP 5.500 Cross-Ref:††

Respondent. Yes; but the choice between *them* shall be considered later. Without disrespect to those who only differ from me about the mint and cummin of the law, I may speak of the traditional logic as the principal alternative, as presenting itself at the first and only great parting of the ways. For the old logicians, thought has no meaning except itself, any more than a fugue of Bach has. If you start with any concept, any twig of the tree of Porphyry, say *canary-birds*, and wish to include the similar Palestine bird, you can think of a *serin-finch*. To be sure, this includes a dozen other species; but that cannot be helped; it is the nearest concept there is. Suppose you wish to include sparrows besides. Then the only distinct concept there is, is coracomorphs (or crow-like birds) which, in the hazy light of the non-ornithological mind, would be pretty much the same as "*ordinary birds*." Rising through *birds*, *vertebrates*, *animals*, *living creatures*, *natural objects*, *things*, we come, in the ninth remove from *canary-birds*, to *substances*. Now that substance is a category is the general opinion of all schools of logicians -- indeed, it is, with the unbroken agreement of all, except some who really do thinking on their own account, an irregular pluralism of functions.

Peirce: CP 5.500 Cross-Ref:††

Jules. Say, with the exception of one curious specimen, the Axi-vectis persicus.†P1

Peirce: CP 5.501 Cross-Ref:††

501. **Respondent.** There have been a number of different lists of categories; but all the logicians, to whom I have been referring, agree that the concepts, which are categories, are all simple and are the only simple concepts. That means that while something may be true of one category that is not true of another, yet such differences are not such as to constitute the characters of the concepts. Each is other than each of the rest but this difference is unspecifiable and thus indefinite. At the same time there is nothing indefinite in the concepts themselves. It sounds paradoxical, yet it is precisely like what is to be remarked about different qualities of feeling. The perfume of the orange flower is perfectly definite and no complexity is to be detected in it. The same thing is true of the rose, of peppermint, and of sandal-wood. They are all very

different -- and one can predicate various qualities of them.†1 The odour of sandalwood is heavy, orange flower is cool, rose has an exquisite purity, peppermint has a clean smell; but these qualities do not at all constitute the odors, nor are they any part of the smells themselves. Of their relations it is inconceivable that anything should ever be predicate except that each is other than each. Those relations therefore are as indefinite as they possibly could be while there is not the slightest indefiniteness [in the feelings related]. Indeed I am prepared to assert that the bigwigs of logic make concepts to be nothing else than another kind of qualities of feeling. Inquire of one of these gentlemen whether or not concepts are qualities of feeling. "Oh mi!" he will reply, "nothing could be more different." But ask him what the difference is, and pursue the inquiry sharply, and it will turn out that it is impossible to say what it is. He will begin by saying that concepts are general, feelings not so. But that position cannot be maintained for thirty seconds. They are different no doubt; but the difference is altogether indefinite. It is precisely like the difference between smells and colours. It must be so, because at the very outset they defined concepts as qualities of feeling, not in these very words, of course, but in the very meaning of these words, when they said that concepts possess, as immediate objects, all the characters †P1 that they possess at all, each in itself, regardless of anything else.

Peirce: CP 5.501 Cross-Ref:††

Jules. Yes, you have asserted that with admirable distinctness. Now I invite you to prove it.

Peirce: CP 5.502 Cross-Ref:††
CHAPTER 3

CONSEQUENCES OF CRITICAL COMMON-SENSISM†1

§1. INDIVIDUALISM

502. **Doctor X.** The best of your screed in the April *Monist* was about the singleness of symbols.†2 It is in truth bad morals to use words in other than their original senses. But apply this rule to your own use of the verb "is." When the child first uses this verb, he applies it to some *sensory* reality. How, then, in the face of your code of terminological ethics, do you ever dare to use the verb "to be" for indicating anything not sensory? I, for my part, stick to the *one* meaning of "is" rigidly. When you talk of "general objects being real," and the like, you seem not to be aware that the verbs "to be," "to be real," and "to exist" have *ever* precisely one and the same significance.

Peirce: CP 5.502 Cross-Ref:††

Pragmaticist. My statements today are not designed to answer objections, but merely to correct misapprehensions. However, as your objection seems motivated by a grave misapprehension, I had better set *that* right. Only in order to guard against my misapprehending *you* in my turn, let me ask you whether you mean to say that throughout that vast philosophical "treatise" of yours, the signification of the verb "is"

is really and truly in every instance the same.

Peirce: CP 5.502 Cross-Ref:††

Doctor X. It is.

Peirce: CP 5.502 Cross-Ref:††

Pragmaticist. And everywhere that meaning is "is real"?

Peirce: CP 5.502 Cross-Ref:††

Doctor X. Yes.

Peirce: CP 5.502 Cross-Ref:††

Pragmaticist. Thanks. Then the misapprehension that motives your objection is that you understand me to be more of a scholastic realist than you are, while in fact it is the other way. For this signification of "is," which occurs in hundreds of instances in your book, and is everywhere one and the same, by virtue of that fact satisfies the definition of a general, or, as the scholastics more accurately said, of a universal; and you tell me that when you say that that universal "is" *is* always the same, you mean that it "is real" everywhere, as well as the same. So that makes you a scholastic realist. But you go much further than that; for when you say that the signification *is* the same, you of course make this signification the subject of the verb "is," which according to your statement is equivalent to a declaration that this signification which is one and the same in so many places of your book is a *sensory* object. Not Bernardus Carnotensis himself, who seems like you to have combined scholastic realism with individualism, went to the length of making any general a sensory object.

Peirce: CP 5.502 Cross-Ref:††

Doctor X. Now you are quibbling, you know.

Peirce: CP 5.503 Cross-Ref:††

503. **Pragmaticist.** Do not answer me, I beg. For remember that you, Doctor X, like Y, Z, and W are not an existent individual, even if you are a sensory object, but only a general type -- unless indeed you are a mere man of straw. A reply could only vindicate you by showing that you were not a real type, when you would prove yourself to be a man of straw; in which case I might be provoked into the personality of burning you up, in our good old mediaeval fashion; though I believe you limit yourself to roasting your antagonists. For my part, I have found the combustion of a man of straw one of the best means of stopping my logical chimney from smoking; while your doctrine would seem to debar you from the employment of that useful device. It is perhaps true that the sectators of individualism, the essence of whose doctrine is that reality and existence are coextensive, i.e., are either alike true or alike false of every subject, must, to be logical, go along with you in holding that "real" and "existent" have the same meaning, or *Inhalt*. But many a logician, as soon as he is convinced that that party is under that obligation (individualism furnishing the principle of the consequence as well as furnishing the antecedent) would regard that circumstance as creating a *reductio ad absurdum* of individualism, inasmuch as *reality* means a certain kind of non-dependence upon thought, and so is a cognitionary character, while *existence* means reaction with the environment, and so is a dynamic character; and accordingly the two meanings, he would say, are clearly *not* the same. Individualists are apt to fall into the almost incredible misunderstanding that all other men are individualists, too -- even the scholastic realists, who, they suppose, thought that "universals exist." It is true that there are indications of there having been some

who thought so in that greater darkness before the dawn of Aristotle's *Analytics* and *Topics*, when such grotesque weldings of doctrine as that of nominalistic Platonism are heard of, and when Roscellin may possibly have said that universals were *flatus vocis*. But I ask, can anybody who has seen Westminster Abbey, who had read the Prologue to the *Canterbury Tales*, and who stops to consider that the metaphysics of the Plantagenet age must have more adequately represented the general intellectual standing of that age, when metaphysics absorbed its greatest heuristic minds, than the metaphysics of our day can represent our general intellectual condition, can any such person believe that the great doctors of that time believed that generals *exist*? They certainly did not so opine, but regarded generals as modes of determination of individuals; and such modes were recognized as being of the nature of thought. Now whoever cares to know what pragmatism is should understand that on its metaphysical side it is an attempt to solve the problem: In what way can a general be unaffected by any thought about it? Hence, before we treat of the evidences of pragmatism, it will be needful to weigh the pros and cons of scholastic realism. For pragmatism could hardly have entered a head that was not already convinced that there are real generals.

Peirce: CP 5.504 Cross-Ref:††

504. . . . [Sic] Another misapprehension: You seem to imagine that your argument from the talk of the child will be as convincing to me as it is to you. It is not so, because (aside from "is" not being one of the technical terms to which common-sense limited my maxim), I do not think that the import of any word (except perhaps a pronoun) is limited to what is in the utterer's mind *actualiter*, so that when I mention the Greek language my meaning should be limited to such Greek words as I happen to be thinking of at the moment. It is, on the contrary, according to me, what is in the mind, perhaps not even *habitualiter*, but only *virtualiter*, which constitutes the import.†P1 To say that I hold that the import, or adequate ultimate interpretation, of a concept is contained, not in any deed or deeds that will ever be done, but in a habit of conduct, or general moral determination of whatever procedure there *may come to be*, is no more than to say that I am a pragmatist. Now every animal must have habits. Consequently, it must have innate habits. In so far as it has cognitive powers, it must have *in posse* innate cognitive habits, which is all that anybody but John Locke ever meant by innate ideas.†P1 To say that I hold this for true is implied in my confession of the doctrine of Common-Sense -- not quite that of the old Scotch School, but a critical philosophy of common-sense. It is impossible rightly to apprehend the pragmatist's position without fully understanding that nowhere would he be less at home than in the ranks of individualists, whether metaphysical (and so denying scholastic realism), or epistemological (and so denying innate ideas).

Peirce: CP 5.505 Cross-Ref:††

§2. CRITICAL PHILOSOPHY AND THE PHILOSOPHY OF COMMON-SENSE

505. *Doctor Y.* Allow me. You speak of holding a *Critical Philosophy of Common-Sense*. What meaning would you have me attach to that phrase, seeing that Critical Philosophy and the Philosophy of Common-Sense, the two rival and opposed ways of answering Hume, are at internecine war, impacificable. The Common-Sense philosopher opines that, be Criticism never so indefatigable, it will have to come to a halt somewhere, and leave some belief uncriticized; namely, wherever no stimulus to

doubt has ever been experienced. An uncriticized belief must, says the Common-sensist, *ipso facto* be regarded as the very truth. That sounds conclusive. Yet it does not satisfy any of the Critics, at all -- be they Kantians or be they of one of the modern kinds that do not usually go by the name of Critics. That a belief should be accepted as the bed-rock of truth simply and solely because it has not been criticized -- oh, this is to their minds too monstrous! They insist that first principles be scientifically established. To think otherwise, says the great Wundt,†1 is to ask that philosophy should come into being by *equivocal generation*. He so resuscitates the phrase which, in the days when men believed in armary unguents, in mummial philtres, and in sigillary medicines, denoted the manner in which they thought that Satan's flies and the vilest of crawling things could be produced, in order to hint how much out of good odor common-sense is in his estimation. Of a principle proposed for the foundation of philosophy, think the Critics, it must either be proved that the very circumstances and form of human knowledge require its acceptance, or better, that scientific psychology should show that its truth is unavoidable, or still better, that physiology should support it as it supports parallelism, or best of all, that histology should almost bring it within the field of the microscope, as caryocinesis is supposed almost to give ocular demonstration of some high proposition. Now, without asking whether it be Common-Sense or one of the Critical methods that is *right*, one cannot help seeing that Criticism and Common-sense are so immiscible that to plunge into either is to lose all touch with the other. The Criticist believes in criticizing first principles, while the Common-sensist thinks such criticism is all nonsense. So I can find no meaning in your straddling phrase.

Peirce: CP 5.505 Cross-Ref:††

Pragmaticist. The phrase denotes a particular stripe of Common-sensism, which is separated from the old Scotch kind by four distinguishing marks. The mark that I find it convenient to describe first is that the Critical Common-sensist holds that all the veritably indubitable beliefs are *vague* -- often in some directions highly so. Logicians have too much neglected the study of *vagueness*, not suspecting the important part it plays in mathematical thought. It is the antithetical analogue of generality. A sign is objectively *general*, in so far as, leaving its effective interpretation indeterminate, it surrenders to the interpreter the right of completing the determination for himself. "Man is mortal." "What man?" "Any man you like." A sign is objectively *vague*, in so far as, leaving its interpretation more or less indeterminate, it reserves for some other possible sign or experience the function of completing the determination. "This month," says the almanac-oracle, "a great event is to happen." "What event?" "Oh, we shall see. The almanac doesn't tell that."†P1 The *general* might be defined as that to which the principle of excluded middle does not apply. A triangle in general is not isosceles nor equilateral; nor is a triangle in general scalene. The *vague* might be defined as that to which the principle of contradiction does not apply. For it is false neither that an animal (in a vague sense) is male, nor that an animal is female. Mr. Kempe's great memoir in the *Philosophical Transactions* for 1886,†1 the most solid piece of work upon any branch of the stecheology of relations that has ever been done, in addition to its intrinsic value, has that of taking us out of the logician's rut, and showing us how the mathematician conceives of logical objects. Thus, in Section 4, he says that the four angular points of a square "are not distinguishable from" one another. On first reading this, a person [Peirce] who was preoccupied with conceptions derived from logicians, was moved to write to the author and ask whether he did not mean "do not differ" from one another. For if the angles are undistinguished, how do we know there are more than one of them? But

though some suggestions of the letter were adopted in a supplement to the memoir, Mr. Kempe stood to "undistinguished" and "undistinguishable";†2 and in Section 29 he is more explicit, saying of the units of a singular †P2 system of units (i.e., a system all whose units are undistinguished from one another), "that no definition can be given of, or remark made about, one which is not equally applicable to each of the others." In Section 73, he goes further in using the expression "undistinguished in dress or other circumstance," showing that he means to exclude distinction by means of relations. All this is utterly paradoxical to the logician, who will say that two vertices of a square are distinguished from each other in not being opposite the same vertex, and in various other ways. But the difficulty disappears as soon as he recognizes that Kempe's units are not supposed to be real objects, but are only *vague* ideas, to which nobody ever supposed the principle of contradiction to apply.

Peirce: CP 5.506 Cross-Ref:††

506. Notwithstanding their contrariety, generality and vagueness are, from a formal point of view, seen to be on a par. Evidently no sign can be at once vague and general in the same respect, since insofar as the right of determination is not distinctly extended to the interpreter it remains the right of the utterer. Hence also, a sign can only escape from being either vague or general by not being indeterminate. But that no sign can be absolutely and completely indeterminate †1 is proved in 3.93 where Plutarch's anecdote about appealing from Phillip drunk to Phillip sober is put to use. Yet every proposition actually asserted must refer to some non-general subject; for the doctrine that a proposition has but a single subject has to be given up in the light of the Logic of Relations. (See *The Open Court*, pp. 3416 et seq.) [3.417ff.] Indeed, all propositions refer to one and the same determinately singular subject, well-understood between all utterers and interpreters; namely, to The Truth, which is the universe of all universes, and is assumed on all hands to be real.†2 But besides that, there is some lesser environment of the utterer and interpreter of each proposition that actually gets conveyed, to which that proposition more particularly refers and which is not general. The Open Court paper referred to [above] made this plain, but left unnoticed some truths of the first importance about vagueness. No communication of one person to another can be entirely definite, i.e., non-vague. We may reasonably hope that physiologists will some day find some means of comparing the qualities of one person's feelings with those of another, so that it would not be fair to insist upon their present incomparability as an inevitable source of misunderstanding. Besides, it does not affect the intellectual purport of communications. But wherever degree or any other possibility of continuous variation subsists, absolute precision is impossible. Much else must be vague, because no man's interpretation of words is based on exactly the same experience as any other man's. Even in our most intellectual conceptions, the more we strive to be precise, the more unattainable precision seems. It should never be forgotten that our own thinking is carried on as a dialogue, and though mostly in a lesser degree, is subject to almost every imperfection of language. I have worked out the logic of vagueness with something like completeness,†1 but need not inflict more of it upon you, at present.

Peirce: CP 5.507 Cross-Ref:††

507. That veritably indubitable beliefs are especially vague could be proved *a priori*. But proof not being aimed at today, it will be simpler to say that the Critical Common-sensist's personal experience is that a suitable line of reflexion, accompanied by imaginary experimentation, always excites doubt of any very broad proposition if it be defined with precision. Yet there are beliefs of which such a critical sifting invariably leaves a certain vague residuum unaffected.

Peirce: CP 5.508 Cross-Ref:††

508. One ought then to ask oneself, whether, since much of the original belief has disappeared under an attentive dissection, perseverance might not affect the destruction of what remains of it. This question always appears reasonable as long as one stands far enough away from the facts of the case, and views them as one would a painting of Monet.

Peirce: CP 5.508 Cross-Ref:††

But the answer that a closer scrutiny dictates in some cases is that it is not because insufficient pains have been taken to precide †2 the residuum, that it is vague: it is that it is vague intrinsically. Take, for example, our belief in the Order of Nature. The criticisms of it in 342; 6.395ff; 2.749ff; 6.35ff; 6.613, as well as by various other writers, of whom may be mentioned as long antecedent to the writer, Renouvier, †3 Delboeuf, †4 Fouillée, †5 Blood, †6 and James, †7 and no doubt there were others, and since that time Dewey †8 and I know not who else, appear to me to have stripped it of all rational precision. As precisely defined it can hardly be said to be absolutely indubitable considering how many thinkers there are who do not believe it. But who can think that there is *no* order in nature?

Peirce: CP 5.509 Cross-Ref:††

509. Could I be assured that other men candidly and with sufficient deliberation doubt any proposition which I regard as indubitable, that fact would inevitably cause me to doubt it, too. I ought not, however, lightly to admit that they do so doubt a proposition after the most thorough criticism by myself and anxious consideration of any other criticisms which I have been able to find and understand has left it quite indubitable by me, since there are other states of mind that can easily be mistaken for doubt. If, indeed, the phenomenon in question were at all a common one, instead of being among the rarest of experiences, I should return to a variety of Common-sensism which has always strongly attracted me, namely, that there is no definite and fixed collection of opinions that are indubitable, but that criticism gradually pushes back each individual's indubitables, modifying the list, yet still leaving him beliefs indubitable at the time being. The reason I have of late given up that opinion, attractive as I find it, is that the facts of my experience accord better with the theory of a fixed list, the same for all men. I do not suppose that it is absolutely fixed, (for my synechism would revolt at that) but that it is so nearly so, that for ordinary purposes it may be taken as quite so.

Peirce: CP 5.510 Cross-Ref:††

510. Doubt is a state of mind marked by a feeling of uneasiness; but we cannot, from a logical, least of all from a pragmatic point of view, regard the doubt as consisting in the feeling. A man in doubt is usually trying to imagine how he shall, or should, act when or if he finds himself in the imagined situation. He supposes himself to have an end in view, and two different and inconsistent lines of action offer themselves. His action is in imagination (or perhaps really) brought to a stop because he does not know whether (so to speak) the right hand road or the left hand road is the one that will bring him to his destination; and (to continue the figure of speech) he waits at the fork for an indication, and kicks his heels. His pent up activity finds vent in feeling, which becomes the more prominent from his attention being no longer absorbed in action. A true doubt is accordingly a doubt which really interferes with the smooth working of the belief-habit. Every natural or inbred belief manifests itself in natural or inbred ways of acting, which in fact constitute it a belief-habit. (I need not repeat that I do not say that it is the single deeds that constitute the habit. It is the

single "ways," which are conditional propositions, each general). A true doubt of such a belief must interfere with this natural mode of acting. If a philosopher, reflecting upon the belief from an extraneous or unnatural point of view, develops new modes of manifestation of that belief (as, for example, by associating it with certain phrases), these new habits must not be regarded as expressions of the natural belief *simply*; for they inevitably involve something more. Consequently, if subsequent reflexion results in doubt of them, it is not necessarily doubt of the original belief, although it may be mistaken for such doubt.

Peirce: CP 5.511 Cross-Ref:††

511. These considerations lead me, quite naturally, to mention another mark of the Critical Common-sensist that separates him from the old school. Namely, he opines that the indubitable beliefs refer to a somewhat primitive mode of life, and that, while they never become dubitable in so far as our mode of life remains that of somewhat primitive man, yet as we develop *degrees of self-control* unknown to that man, occasions of action arise in relation to which the original beliefs, if stretched to cover them, have no sufficient authority. In other words, we outgrow the applicability of instinct -- not altogether, by any manner of means, but in our highest activities. The famous Scotch philosophers lived and died out before this could be duly appreciated.

Peirce: CP 5.512 Cross-Ref:††

512. *Doctor Y.* What do you mean by "somewhat primitive"? And by what sort of reasoning can a dubitable proposition about experience become indubitable?

Peirce: CP 5.512 Cross-Ref:††

Pragmaticist. A searching question, because some of our beliefs, which seem as indubitable as any, are of such a character that they can hardly have entered the minds, say, of Neanderthal men, and in any case, cannot possibly have been transmitted to us from the first conscious animals. Consequently, Common-sensism has to grapple with the difficulty that if there are any indubitable beliefs, these beliefs must have grown up; and during the process, cannot have been indubitable beliefs. Still, I see no reason for thinking that beliefs that were dubitable became indubitable. Every decent house dog has been taught beliefs that appear to have no application to the wild state of the dog; and yet your trained dog has not, I guess, been observed to have passed through a period of scepticism on the subject. There is every reason to suppose that belief came first, and the power of doubting long after. Doubt, usually, perhaps always, takes its rise from surprise, which supposes previous belief; and surprises come with novel environment. I will only add that though precise reasoning about precise experiential doubt could not entirely destroy doubt, any more than the action of finite conservative forces could leave a body in a continuous state of rest, yet vagueness, which is no more to be done away with in the world of logic than friction in mechanics, can have that effect.

Peirce: CP 5.513 Cross-Ref:††

513. As I was saying, a modern recognition of evolution must distinguish the Critical Common-sensist from the old school. Modern science, with its microscopes and telescopes, with its chemistry and electricity, and with its entirely new appliances of life, has put us into quite another world; almost as much so as if it had transported our race to another planet. Some of the old beliefs have no application except in extended senses, and in such extended senses they are sometimes dubitable and subject to just criticism. It is above all the normative sciences, esthetics, ethics, and logic, that men are in dire need of having severely criticized, in their relation to the new world created by science. Unfortunately, this need is as unconscious as it is great.

The evils are in some superficial way recognized; but it never occurs to anybody that the study of esthetics, ethics, and logic can be seriously important, because these sciences are conceived by all, but their deepest students, in the old way. It only concerns my present purpose to glance at this state of things. The needed new criticism must know whereon it stands; namely, on the beliefs that remain indubitable; and young Critical Common-sensists of intellectual force who burn for a task in which they can worthily sacrifice their lives without encouragement, reward, recognition, or a hearing (and I trust such young men still live) can find in this field their heart's desire.

Peirce: CP 5.514 Cross-Ref:††

514. Yet a third mark of the Critical Common-sensist is that he has a high esteem for doubt. He may almost be said to have a *sacra fames* for it. Only, his hunger is not to be appeased with paper doubts: he must have the heavy and noble metal, or else belief.

Peirce: CP 5.514 Cross-Ref:††

He quite acknowledges that what has been indubitable one day has often been proved on the morrow to be false. He grants the preciss proposition that it may be so with any of the beliefs he holds. He really cannot admit that it may be so with all of them; but here he loses himself in vague unmeaning contradictions.

Peirce: CP 5.515 Cross-Ref:††

515. *Doctor Y.* Can indubitable propositions be demonstrable?

Peirce: CP 5.515 Cross-Ref:††

Pragmaticist. Indubitable propositions must be ultimate premisses, or at least, must be held without reference to precise proofs. For what one cannot doubt one cannot argue about; and no precise empirical argument can free its conclusion altogether from rational doubt.

Peirce: CP 5.516 Cross-Ref:††

516. Yet it is true that whenever one turns a critical glance upon one of our original beliefs -- say, the belief in the order of nature -- the mind at once seems vaguely to pretend to have reasons for believing it. One dreams of an inductive proof. One surmises that the belief results from something like an inductive proof that has been forgotten. Very likely it did, in a sense of the term "inductive process" that is so generalized as to include uncontrolled thought. But this admission must be accompanied by the emphatic denial that the indubitable belief is inferential, or is "accepted." It simply remains unshaken as it always was. That does not at all interfere with the theory that in the psychological process of its development, the occurrence of single experiences, such as might have been predictively deduced from it, were an indispensable factor, while an original potentiality of the belief-habit must have been a correlative factor. All this is perfectly consistent, too, with the necessity of criticising the ordinary axioms of reasoning and of morals, as well as ordinarily developed ideals, as soon as they are extended so as to become applicable to the new world created by science.

Peirce: CP 5.517 Cross-Ref:††

517. I was saying that the Critical Common-sensist feels that the danger -- the scientific danger, at any rate; and Philosophy is a department of pure Heuristic Science even less concerned, for example, about practical religion, if possible, than religion ought to be about it -- does not lie in believing too little but in believing too much.

The indolent university student, no matter at what pains his professor of philosophy may be to set him upon his own legs, yet having a well-grounded respect for that professor's superior acquirements and force of intellect, finds it much easier to accept all he says, as true because he says it, than to submit said professor's arguments to searching criticism; and he thus becomes, on the average, quite as much a slave to authority as was the average scholar of the medieval schools. Only, instead of bowing to Aristotle and the universal voice of the church sounding *semper eadem*, he submits to the yoke of some young doctrinaire with whom every other like him disagrees. With such sentiments, the Critical Common-sensist sets himself in serious earnest to the systematic business of endeavoring to bring all his very general first premisses to recognition, and of developing every suspicion of doubt of their truth, by the use of logical analysis, and by experimenting in imagination. If, besides being a Critical Common-sensist, he is also a pragmatist, he will further hold that everything in the substance of his beliefs can be represented in the schemata of his imagination; that is to say, in what may be compared to composite photographs of continuous series of modifications of images; these composites being accompanied by conditional resolutions †P1 as to conduct.

Peirce: CP 5.517 Cross-Ref:††

These resolutions should cover all classes of circumstances, in the sense that they would produce (or, *perhaps* †P2 more strictly, manifestations of whatever it may be in our occult nature that produces) determinations of habit corresponding to every possible pragmatic application of the propositions believed.

Peirce: CP 5.518 Cross-Ref:††

518. Pragmatism is, of course, in its developed fullness too recent a phenomenon in the history of philosophy to have disclosed anything to direct experience, concerning its tendencies, that is particularly trustworthy. Thus far, however, it would appear that, as a matter of fact, pragmatists press their peculiar doubts about first principles a good deal further and with a more straightforward earnestness than Kantians do. For when a Kant expresses a doubt, one has still to learn whether it is the substance of the proposition that he doubts or merely its attachment to one faculty or to another. One has even known of a pragmatist being called by a Kantian "David Hume Redivivus"; †1 but I fancy it was more like the David Hume of some mediumistic *séance*.

Peirce: CP 5.519 Cross-Ref:††

519. **Doctor X.** I should think that so passionate a lover of doubt would make a clean sweep of his beliefs.

Peirce: CP 5.519 Cross-Ref:††

Pragmatist. You naturally would, holding the infant's mind to be a *tabula rasa* and the adult's a school slate, on which doubts are written with a soapstone pencil to be cleaned off with the dab of a wet sponge; but if they are marked with talc on man's "glassy essence," they may disappear for a long time only to be revived by a breath.

Peirce: CP 5.520 Cross-Ref:††

520. **Doctor X.** Yours seemed marked with **T-A-L-K**. Doubtless your pragmatist dotes too much on doubt to risk subjecting it to scientific experimentation.

Peirce: CP 5.520 Cross-Ref:††

Pragmaticist. Bah! I may as well capitulate first as last. I have been betrayed by that execrable banality, *auri sacra fames*, which is not even good poetry, since it is inaccurate. For what it *expresses* is the hoarding passion of the miser, a sort of collector's rage for gold as gold, while all it *means* in the familiar passages is no more than simple cupidity; and that is what I meant in applying it to the Critical Common-sensist's eager pursuit of doubt. He is none of those overcultivated Oxford dons -- I hope their day is over -- whom any discovery that brought quietus to a vexed question would evidently vex because it would end the fun of arguing around it and about it and over it. On the contrary what he adores, if he is a good pragmaticist, is *power*; not the sham power of brute force, which, even in its own specialty of spoiling things, secures such slight results; but the creative power of reasonableness, which subdues all other powers, and rules over them with its sceptre, knowledge, and its globe, love. It is as one of the chief lieutenants of reasonableness that he highly esteems doubt, although it is not amiable.

Peirce: CP 5.521 Cross-Ref:††

521. As for allaying doubts concerning the first principles of logic and philosophy by means of scientific experiments, he does not attempt that for two reasons, one a fine little reason that insinuates itself between the joints of the Wundtian armour, the other more like wholesome country air.

Peirce: CP 5.521 Cross-Ref:††

The small reason is sufficient. It is that any such idioscopic †1 inquiry must proceed upon the virtual assumption of sundry logical and metaphysical beliefs; and it is rational to settle the validity of those before undertaking an operation that supposes their truth. Now whether the truth of them be explicitly laid down on critical grounds, or the doctrine of Common-Sense prevent our pretending to doubt it, along with all these other sound first principles will be admitted, and so the whole inquiry will be concluded before the first outward experiment is made. But this preliminary inquiry is long and arduous.

Peirce: CP 5.522 Cross-Ref:††

522. *Doctor X.* A sort of Panama Commission business, apparently. I should say, Pitch in like Lesseps, unless your second objection is more serious. What is that?

Peirce: CP 5.522 Cross-Ref:††

Pragmaticist. There is nothing novel in it. It is that nothing is so unerring as instinct within its proper field, while reason goes wrong about as often as right -- perhaps oftener. Now those vague beliefs that appear to be indubitable have the same sort of basis as scientific results have. That is to say, they rest on experience -- on the total everyday experience of many generations of multitudinous populations. Such experience is worthless for distinctively scientific purposes, because it does not make the minute distinctions with which science is chiefly concerned; nor does it relate to the recondite subjects of science, although all science, without being aware of it, virtually supposes the truth of the vague results of uncontrolled thought upon such experiences, cannot help doing so, and would have to shut up shop if she should manage to escape accepting them. No "wisdom" could ever have discovered argon; yet within its proper sphere, which embraces objects of universal concern, the instinctive result of human experience ought to have so vastly more weight than any scientific result, that to make laboratory experiments to ascertain, for example, whether there be any uniformity in nature or no, would vie with adding a teaspoonful of saccharine to the ocean in order to sweeten it.

Peirce: CP 5.523 Cross-Ref:††

523. *Doctor Y.* Is there any further peculiarity which distinguishes Critical Common-sensism from that of Reid and Dugald Stewart?

Peirce: CP 5.523 Cross-Ref:††

Pragmaticist. Yes; for it criticizes the critical method, follows its footsteps, tracks it to its lair. To the accusation that Common-Sense accepts a proposition as indubitable because it has not been criticized, the answer is that this confounds two uses of the word "because." Neither the philosophy of Common-Sense nor the man who holds it accepts any belief *on the ground* that it has not been criticized. For, as already said, such beliefs are not "accepted." What happens is that one comes to recognize that one has had the belief-habit as long as one can remember; and to say that no doubt of it has ever arisen is only another way of saying the same thing. But it is quite true that the Common-sensist like everybody else, the Criticist included, believes propositions *because* they have not been criticized in the sense that he does not doubt certain propositions that he would have doubted if he had criticized them. For in the first place, to criticize is *ipso facto* to doubt, and in the second place criticism can only attack a proposition after it has given it some precise sense in which it is impossible entirely to remove the doubt. It is probably true, too, that the Common-sensist believes unquestioningly some propositions that might have been criticized and that are not true. We are all liable to do that; but perhaps he is more in danger of it than other men. Still, as a fact, it is difficult to find a Criticist who does not hold to more fundamental beliefs than any Critical Common-sensist does.

Peirce: CP 5.524 Cross-Ref:††

524. The Critical Philosopher seems to opine that the fact that he has not hitherto doubted a proposition is no reason why he should not henceforth doubt it. (At which Common-Sense whispers that, whether it be "reason" or no, it will be a well-nigh insuperable *obstacle* to doubt.) Accordingly, he will not stop to ask whether he actually does doubt it or not, but at once proceeds to examine it. Now if it happens that he *does* actually doubt the proposition, he does quite right in starting a critical inquiry. But in case he *does not* doubt, he virtually falls into the Cartesian error of supposing that one can doubt at will. A proposition that could be doubted at will is certainly not *believed*. For belief, while it lasts, is a strong habit, and as such, forces the man to believe until some surprise breaks up the habit. The breaking of a belief can only be due to some novel experience, whether external or internal. Now experience which could be summoned up at pleasure would not be experience.

Peirce: CP 5.525 Cross-Ref:††

525. Kant (whom I *more* than admire) is nothing but a somewhat confused pragmatist. A real is anything that is not affected by men's cognitions *about it*; which is a verbal definition, not a doctrine. An external object is anything that is not affected by any cognitions, whether about it or not, of the man to whom it is external. Exaggerate this, in the usual philosopher fashion, and you have the conception of what is not affected by any cognitions at all. Take the converse of this definition and you have the notion of what does not affect cognition, and in this indirect manner you get a hypostatically abstract notion of what the *Ding an sich* would be. In this sense, we also have a notion of a sky-blue demonstration; but in half a dozen ways the *Ding an sich* has been proved to be nonsensical; and here is another way. It has been shown [3.417ff] that in the formal analysis of a proposition, after all that words can convey has been thrown into the predicate, there remains a subject that is

indescribable and that can only be pointed at or otherwise indicated, unless a way, of finding what is referred to, be prescribed. The *Ding an sich*, however, can neither be indicated nor found. Consequently, no proposition can refer to it, and nothing true or false can be predicated of it. Therefore, all references to it must be thrown out as meaningless surplusage. But when that is done, we see clearly that Kant regards Space, Time, and his Categories just as everybody else does, and never doubts or has doubted their objectivity. His limitation of them to possible experience is pragmatism in the general sense; and the pragmatist, as fully as Kant, recognizes the mental ingredient in these concepts. Only (trained by Kant to define), he defines more definitely, and somewhat otherwise, than Kant did, just how much of this ingredient comes from the mind of the individual in whose experience the cognition occurs. The kind of Common-sensism which thus criticizes the Critical Philosophy and recognizes its own affiliation to Kant has surely a certain claim to call itself Critical Common-sensism.

Peirce: CP 5.526 Cross-Ref:††
§3. THE GENERALITY OF THE POSSIBLE

526. *Doctor Z.* You say that no collection of individuals could ever be adequate to the extension of a concept in general, which is, of course, the old peripatetic doctrine. But really I do not quite see how you propose to reconcile that to the proposition that the meaning extends no further than to future embodiments of it.

Peirce: CP 5.526 Cross-Ref:††

Pragmatist. The original paper on pragmatism was completed in September 1877†P1 and appeared in *Popular Science Monthly* for January 1878. At that time, modern investigation of the doctrines of multitude had not begun.†P2 Indeed, there are indications in that paper of an endless series not being regarded as a collection.†1 Yet the philosophical importance of the new studies was fully recognized by the pragmatist from the first.†P3

Peirce: CP 5.527 Cross-Ref:††

527. In 3.527ff the objectivity of possibility was asserted; and the hypothesis defended in vol. 6, Bk. I, chs. 1 and 2 supposes possibility to be real.†2 It was, indeed, *implied* in the scholastic realism maintained in the *N.A. Rev.*, Vol. CXIII (pp. 454 *et seq.*) [vol. 9]. But the paper of January 1878 evidently endeavors to avoid asking the reader to admit a real possibility. The theory of modality is far too great a question to be treated incidentally to any other.†1 But the distinct recognition of real possibility is certainly indispensable to pragmatism.

Peirce: CP 5.528 Cross-Ref:††

528. The pragmatist has always explicitly stated that the intellectual purport of a concept consists in the truth of certain conditional propositions asserting that if the concept be applicable, and the utterer of the proposition or his fellow have a certain purpose in view, he would act in a certain way. A purpose is essentially general, and so is a way of acting; and a conditional proposition is a proposition about a universe of possibility. At the same time, the conditional proposition refers only to possible individual actions. If there be any paradox here, it is partially resolved in the important paper in *The Monist*, Vol. VII [3.526ff], where it is shown that an endless series of experiences, each entirely consistent with those that precede it, cannot itself

be experienced (*as* such endless series), but involves a first dose of ideality, or generality. It is not a perfect general, it is true; but the whole endless series of steps from this to true continuity (which is perfect generality elevated to the mode of conception of the Logic of Relations) are there described. Whoever wishes to complete the theory of modality should set out from the results there demonstrated.

Peirce: CP 5.529 Cross-Ref:††

529. The conditional proposition which the pragmaticist holds to constitute the purport of a concept is in the article of January 1878 spoken of as expressing a "fact"; and it does indeed express the actual state of mind of a person who has made, or is ready to make, a resolve -- not a categorical resolve, but a resolve conditional upon having a certain purpose. But nobody would make any difficulty in admitting that every resolve is limited to future acts, and acts are the most perfectly individual objects there are.

Peirce: CP 5.530 Cross-Ref:††

530. I apprehend that I have thus substantially answered your question; but in order to make my answer a little clearer, I will illustrate it by the consideration of the continuity of Space. In this illustration I shall adopt the Leibnizian conception of Space in place of the Newtonian, which I believe to be the true one. In that Leibnizian view, Space is merely a possibility limited by an impossibility; a possibility of no matter what affections of bodies (determining their relative positions), together with the impossibility of those affections being actualized otherwise than under certain limitations, expressed in the postulates of topical, graphical, and metrical geometry. No collection of points, though it be abnumerable to the billionth degree, could fill a line so that there would be room for no more points; and in that respect the line is truly general; no possible multitude of singulars is adequate to it. Space is thus truly general; and yet it is, so to say, nothing but the way in which actual bodies conduct themselves.

Peirce: CP 5.531 Cross-Ref:††

531. *Doctor Z.* But the idea of Leibnizian Space, if there were such a thing, would not be a concept. It would be a *Vorstellung*, or composite of images. Kant might perhaps have called it a *Schema*, since he defines a *schema* as a determination of intuition by a concept through the reproductive imagination.†1 Of course, it would not be one of those *transcendental* schemata, which he talks of in the *Critik*; but it possesses much the same sort of bastard generality.

Peirce: CP 5.531 Cross-Ref:††

Pragmaticist. The breakneck hurry in which the *C.d.r.V.* was written is its only defence against a charge of slovenly workmanship. Every detail is left in the rough; and there is no more unfinished apartment in the whole glorious edifice than that devoted to the Schematization of the Categories. Kant says that no image, and consequently we may add, no collection of images, is adequate to representing what a schema represents.†2 If that be the case, I should like to know how a schema is not as general as a concept. If I ask him, all he seems to answer is that it is the product of a different "faculty."

Peirce: CP 5.532 Cross-Ref:††

532. But what you would seem to mean by a concept is the meaning of some general symbol, this meaning being conceived as referring to the symbol. Now it is precisely the pragmatist's contention that symbols, owing their origin (on one side) to human conventions, cannot transcend conceivable human occasions. At any rate, it is

plain that no possible collection of single occasions of conduct can be, or adequately represent *all* conceivable occasions. For there is no collection of individuals of any general description which we could not conceive to receive the addition of other individuals of the same description aggregated to it. The generality of the possible, the only true generality, is distributive, not collective. You perhaps do not see how this remark bears upon your question.

Peirce: CP 5.533 Cross-Ref:††
§4. VALUATION

533. *Doctor W.* I should like to know what the attitude of your pragmatism is to the question of whether or no valuation is a factor of all intellectual meaning.

Peirce: CP 5.533 Cross-Ref:††

Pragmatist. Well, collective and distributive universality can bide their time. Considering how it stood in the mid-channel of pragmatistic thought to join ethics to logic, it seems to me strange that we had to wait until 1903 for any pragmatist to assert that logic ought to be based upon ethics. Perhaps some one of us had said it before; but I only know that it was then said in a course of lectures before the Lowell Institute in Boston,†1 and was maintained on the ground that reasoning is thought subjected to self-control, and that the whole operation of logical self-control takes precisely the same quite complicated course which everybody ought to acknowledge is that of effective ethical self-control. Mr. Schiller in the same year published an essay entitled "The Ethical Basis of Metaphysics."†2 The title is promising, but the essay is, for me, reduced to gibberish by the author's talking about the *real*, without the slightest hint of what he means by this word except that it is something the character of which is affected (and it would seem very greatly) by anybody's thinking that it possesses or does not possess that character. In short he treats a verbal definition as a doctrine, and stoutly denying it, leaves the word a mystery. To meet some such fatal blank has more than once been my ill-luck in trying to read Schiller. It is my stupidity no doubt.

Peirce: CP 5.533 Cross-Ref:††

To return to self-control, which I can but slightly sketch, at this time, of course there are inhibitions and coördinations that entirely escape consciousness. There are, in the next place, modes of self-control which seem quite instinctive. Next, there is a kind of self-control which results from training. Next, a man can be his own training-master and thus control his self-control. When this point is reached much or all the training may be conducted in imagination. When a man trains himself, thus controlling control, he must have some moral rule in view, however special and irrational it may be. But next he may undertake to improve this rule; that is, to exercise a control over his control of control. To do this he must have in view something higher than an irrational rule. He must have some sort of moral principle. This, in turn, may be controlled by reference to an esthetic ideal of what is fine. There are certainly more grades than I have enumerated. Perhaps their number is indefinite. The brutes are certainly capable of more than one grade of control; but it seems to me that our superiority to them is more due to our greater number of grades of self-control than it is to our versatility.

Peirce: CP 5.534 Cross-Ref:††

534. *Doctor Y.* Is it not due to our faculty of language?

Peirce: CP 5.534 Cross-Ref:††

Pragmaticist. To my thinking that faculty is itself a phenomenon of self-control. For thinking is a kind of conduct, and is itself controllable, as everybody knows. Now the intellectual control of thinking takes place by thinking about thought. All thinking is by signs; and the brutes use signs. But they perhaps rarely think of them as signs. To do so is manifestly a second step in the use of language. Brutes use language, and seem to exercise some little control over it. But they certainly do not carry this control to anything like the same grade that we do. They do not criticize their thought logically. One extremely important grade of thinking about thought, which my logical analyses have shown to be one of chief, if not the chief, explanation of the power of mathematical reasoning, is a stock topic of ridicule among the wits. This operation is performed when something, that one has thought about any subject, is itself made a subject of thought. You remember how in the last *Intermède* to the *Malade Imaginaire*, the doctor puts a question to the candidate for the medical degree?

Si mihi licentiam dat Dominus Praeses,

Et tanti docti Doctores,

Et assistantes illustres,

Très sçavanti Bacheliero,

Quem estimo et honoro,

Domandabo causam et rationem quare

Opium facit dormire.

To which the candidate replies,

Mihi a docto Doctore

Domandatur causam et rationem quare

Opium facit dormire:

A quoi respondeo,

Quia est in eo

Virtus dormitiva,

Cujus est natura

Sensus assoupire.

Whereupon the chorus bursts out,

Bene, bene, bene, bene respondere,
Dignus, dignus est entrare
In nostro docto corpore.
(Bene, bene respondere.)

Even in this burlesque instance, this operation of hypostatic abstraction is not quite utterly futile. For it does say that there is *some* peculiarity in the opium to which the sleep must be due; and this is not suggested in merely saying that opium puts people to sleep. By the way, John Locke's account †1 of a real function of this sort at Montpellier three years after the play was first performed, with such tragic effect upon Molière, shows that there was more truth than caricature in the *Intermède*. In order to get an inkling -- though a very slight one -- of the importance of this operation in mathematics, it will suffice to remember that a *collection* is an hypostatic abstraction, or *ens rationis*, †2 that *multitude* is the hypostatic abstraction derived from a predicate of a collection, †3 and that a *cardinal number* is an abstraction attached to a multitude. †4 So an *ordinal number* is an abstraction attached to a *place*, †5 which in its turn is a hypostatic abstraction from a relative character of a unit of a *series*, itself an abstraction again. Now, Doctor Z, as well as I can make out, what you mean by a *concept* is a predicate considered by itself, except for its connection with the word or other symbol expressing it, and now regarded as denotative of the concept. Such a concept is not merely prescissively abstracted, but, as being made a subject of thought, is hypostatically abstract. So understood, it is true that it is more removed from the perceptual objects than is the *Vorstellung*, or composite of images. But for all that, its intellectual purport is just the same. It is only the grammatico-logical form that is transmuted.

Peirce: CP 5.535 Cross-Ref: ††

535. And you, Doctor W., will see that since pragmatism makes the purport to consist in a conditional proposition concerning conduct, a sufficiently deliberate consideration of that purport will reflect that the conditional conduct ought to be regulated by an ethical principle, which by further self-criticism may be made to accord with an esthetical ideal. For I cannot admit that any ideal can be too high for a duly transfigured esthetics. So, although I do not think that an esthetic valuation is essentially involved, *actualiter* (so to speak) in every intellectual purport, I do think that it is a *virtual* factor of a duly rationalized purport. That is to say, it really does belong to the purport, since conduct may depend upon its being appealed to. Yet in ordinary cases, it will not be needful that this should be done. Such seem to me to be the facts, phrase them how you may.

Peirce: CP 5.536 Cross-Ref: ††

536. *Doctor W.* I am glad to hear you say so. And what do you think of Humanism?

Peirce: CP 5.536 Cross-Ref: ††

Pragmaticist. Why if you had said *Anthropomorphism*, I should have replied that I heartily embrace most of the clauses of that doctrine, if some right of private

interpretation be allowed me. I hold, for instance, that man is so completely hemmed in by the bounds of his possible practical experience, his mind is so restricted to being the instrument of his needs, that he cannot, in the least, *mean* anything that transcends those limits. The strict consequence of this is, that it is all nonsense to tell him that he must not think in this or that way because to do so would be to transcend the limits of a possible experience. For let him try ever so hard to think anything about what is beyond that limit, it simply cannot be done. You might as well pass a law that no man shall jump over the moon; it wouldn't forbid him to jump just as high as he possibly could.

Peirce: CP 5.536 Cross-Ref:††

For much the same reason, I do not believe that man can have the idea of any cause or agency so stupendous that there is any more adequate way of conceiving it than as vaguely like a man. Therefore, whoever cannot look at the starry heaven without thinking that all this universe must have had an adequate cause, can in my opinion not otherwise think of that cause half so justly than by thinking it is God.

Peirce: CP 5.537 Cross-Ref:††

537. But when you talk of Humanism, I am utterly perplexed to know what it means. One of its clauses seems borrowed from Hegel, on whom how greatly its author [Schiller] dotes is well known. Namely, he apparently does not wish to have phenomena torn to pieces; or at any rate not if that introduces any falsity; and he does not wish us to devote any attention to the effects of conditions that do not occur, or at any rate not to substitute the solution of such a problem for the true problems of nature. For my part, I think such talk shows great ignorance of the conditions of science. Then again, as I understand it, this Humanism is to be a philosophy not purely intellectual because every department of man's nature must be voiced in it. For my part, I beg to be excused from having any dealings with such a philosophy. I wish philosophy to be a strict science, passionless and severely fair. I know very well that science is not the whole of life, but I believe in the division of labor among intellectual agencies. The apostle of Humanism says that professional philosophers "have rendered philosophy like unto themselves, abstruse, arid, abstract, and abhorrent."†1 But I conceive that some branches of science are not in a healthy state if they are *not* abstruse, arid, and abstract, in which case, like the Aristotelianism which is this gentleman's particular *bête noire*, it will be as Shakespeare said (*of it*, remember)

"Not harsh and crabbed, as dull fools suppose,

But musical as is Apollo's lute," etc.†2

Peirce: CP 5.538 Cross-Ref:††

CHAPTER 4

BELIEF AND JUDGMENT

§1. PRACTICAL AND THEORETICAL BELIEFS †1

538. Let us begin by considering practical belief, such as that anthracite is a convenient fuel, leaving purely theoretical belief, such as that the pole of the earth describes an oval of a few rods' diameter, or that there is an imaginary circle which is twice cut by every real circle, for a supplementary study. Let us use the word "habit," throughout this book, not in its narrower, and more proper sense, in which it is opposed to a natural disposition (for the term *acquired habit* will perfectly express that narrower sense), but in its wider and perhaps still more usual sense, in which it denotes such a specialization, original or acquired, of the nature of a man, or an animal, or a vine, or a crystallizable chemical substance, or anything else, that he or it will behave, or always tend to behave, in a way describable in general terms upon every occasion (or upon a considerable proportion of the occasions) that may present itself of a generally describable character. Now to say that a man believes anthracite to be a convenient fuel is to say no more nor less than that if he needs fuel, and no other seems particularly preferable, then, if he acts deliberately, bearing in mind his experiences, considering what he is doing, and exercising self-control, he will often use anthracite. A practical belief may, therefore, be described as a habit of deliberate behavior. The word "deliberate" is hardly completely defined by saying that it implies attention to memories of past experience and to one's present purpose, together with self-control. The acquisition of habits of the nervous system and of the mind is governed by the principle that any special character of a reaction to a given kind of stimulus is (unless fatigue intervenes) more likely to belong to a subsequent reaction to a second stimulus of that kind, than it would be if it had not happened to belong to the former reaction. But habits are sometimes acquired without any previous reactions that are externally manifest. A mere imagination of reacting in a particular way seems to be capable after numerous repetitions of causing the imagined kind of reaction really to take place upon subsequent occurrences of the stimulus. In the formation of habits of deliberate action, we may imagine the occurrence of the stimulus, and think out what the results of different actions will be. One of these will appear particularly satisfactory; and then an action of the soul takes place which is well described by saying that that mode of reaction "receives a deliberate stamp of approval." The result will be that when a similar occasion actually arises for the first time it will be found that the habit of really reacting in that way is already established. I remember that one day at my father's table, my mother spilled some burning spirits on her skirt. Instantly, before the rest of us had had time to think what to do, my brother, Herbert, who was a small boy, had snatched up the rug and smothered the fire. We were astonished at his promptitude, which, as he grew up, proved to be characteristic. I asked him how he came to think of it so quickly. He said, "I had considered on a previous day what I would do in case such an accident should occur." †1 This act of stamping with approval, "endorsing" as one's own, an imaginary line of conduct so that it shall give a general shape to our actual future conduct is what we call a *resolve*. It is not at all essential to the practical belief, but only a somewhat frequent attachment.

Peirce: CP 5.539 Cross-Ref: ††

539. Let us now pass to the consideration of purely theoretical belief. If an opinion can eventually go to the determination of a practical belief, it, in so far, becomes itself a practical belief; and every proposition that is not pure metaphysical jargon and chatter must have some possible bearing upon practice. The diagonal of a square is incommensurable with its side. It is difficult to see what experiential

difference there can be between commensurable and incommensurable magnitudes; but there is this, that it is useless to try to find the exact expression of the diagonal as a rational fraction of the side. Still, it does not follow that because every theoretical belief is, at least indirectly, a practical belief, this is the *whole* meaning of the theoretical belief. Of theoretical beliefs, in so far as they are not practical, we may distinguish between those which are expectations, and those which are not even that. One of the simplest, and for that reason one of the most difficult, of the ideas which it is incumbent upon the author of this book to endeavor to cause the reader to conceive, is that a sense of effort and the experience of any sensation are phenomena of the same kind, equally involving direct experience of the duality of the Without and the Within.†1 The psychology of the sense of effort is not yet satisfactorily made out. It seems to be a sensation which somehow arises when striped muscles are under tension. But though this is the only way of stimulating it, yet an imagination of it is by association called up, upon the occasion of other slight sensations, even when muscles are uncontracted; and this imagination may sometimes be interpreted as a sign of effort. But though the sense of effort is thus merely a sensation, like any other, it is one in which the duality which appears in every sensation is specially prominent. A sense of exertion is at the same time a sense of being resisted. Exertion cannot be experienced without resistance, nor resistance without exertion. It is all one sense, but a sense of duality. Every sensation involves the same sense of duality, though less prominently. This is the direct perception of the external world of Reid and Hamilton.†2 This is the *probatio ambulandi*, which Diogenes Laertius perhaps gets mislocated. An idealist need not deny the reality of the external world, any more than Berkeley did. For the reality of the external world *means* nothing except that real experience of duality. Still, many of them do deny it -- or think they do. Very well; an idealist of that stamp is lounging down Regent Street, thinking of the utter nonsense of the opinion of Reid, and especially of the foolish *probatio ambulandi*, when some drunken fellow who is staggering up the street unexpectedly lets fly his fist and knocks him in the eye. What has become of his philosophical reflections now? Will he be so unable to free himself from prepossessions that no experience can show him the force of that argument? There may be some underlying unity beneath the sudden transition from meditation to astonishment. Grant that: does it follow that that transition did not take place? Is not the transition a direct experience of the duality of the inward past and outward present? A poor analyst is he who cannot see that the Unexpected is a direct experience of duality, that just as there can be no effort without resistance, so there can be no subjectivity of the unexpected without the objectivity of the unexpected, that they are merely two aspects of one experience given together and beyond all criticism. If the idealist should pick himself up and proceed to argue to the striker, saying "you could not have struck me, because you have no independent existence, you know," the striker might answer, "I dare say I have not separate existence enough for that; but I have separate existence enough to make you feel differently from what you were expecting to feel." Whatever strikes the eye or the touch, whatever strikes upon the ear, whatever affects nose or palate, contains something unexpected. Experience of the unexpected forces upon us the idea of duality. Will you say, "Yes, the idea is forced upon us, but it is not directly experienced, because only what is within is directly experienced"? The reply is that *experience* means nothing but just that of a cognitive nature which the history of our lives has forced upon us. It is *indirect*, if the medium of some other experience or thought is required to bring it out. Duality, thought abstractly, no doubt requires the intervention of reflection; but that upon which this reflection is based, the concrete duality, is there in the very experience itself.

Peirce: CP 5.540 Cross-Ref:††

540. In the light of these remarks, we perceive that there is just this difference between a practical belief and an expectation so far as it involves no purpose [or] effort; namely that the former is expectant of muscular sensation, the latter of sensation not muscular. The expectancy consists in the stamp of approval, the act of recognition as one's own, being placed by a deed of the soul upon an imaginary anticipation of experience; so that, if it be fulfilled, though the actual experience will, at all events, contain enough of the unexpected to be recognized as external, yet the person who stands in expectancy will almost claim the event as his due, his triumphant "I told you so" implying a right to expect as much from a justly-regulated world. A man who goes among a barbarous tribe and announces a total eclipse of the sun next day, will expect, not only "his" eclipse from Nature, but due credit for it from that People. In all this, I am endeavoring so to shape what I have to say as to exhibit, besides, the close alliance, the family identity, of the ideas of externality and unexpectedness.

Peirce: CP 5.541 Cross-Ref:††

541. As to purely theoretical beliefs not expectacious, if they are to mean anything, they must be somehow expectative. The word "expect" is now and then applied by careless and ignorant speakers, especially the English, to what is surmised in regard to the past. It is not illogical language: it is only elliptical. "I expect that Adam must have felt a little sore over the extraction of his rib," may be interpreted as meaning that the expectation is, that so it will be found when the secrets of all hearts are laid bare. History would not have the character of a true science if it were not permissible to hope that further evidences may be forthcoming in the future by which the hypotheses of the critics may be tested. A theory which should be capable of being absolutely demonstrated in its entirety by future events, would be no scientific theory but a mere piece of fortune telling. On the other hand, a theory, which goes beyond what may be verified to any degree of approximation by future discoveries is, in so far, metaphysical gabble. To say that a quadratic equation which has no real root has two different imaginary roots does not sound as if it could have any relation to experience. Yet it is strictly expectative. It states what would be expectable if we had to deal with quantities expressing the relations between objects, related to one another like the points of the plane of imaginary quantity. So a belief about the incommensurability of the diagonal relates to what is expectable for a person dealing with fractions; although it means nothing at all in regard to what could be expected in physical measurements, which are, of their very nature, approximate only. Let us examine a highly abstract belief; and see whether there is any expectancy in it. Riemann †1 declared that infinity has nothing to do with the absence of a limit but relates solely to measure. This means that if a bounded surface be measured in a suitable way it will be found infinite, and that if an unbounded surface be measured in a suitable way, it will be found finite. It relates to what is expectable for a person dealing with different systems of measurement. The Roman church requires the faithful to *believe* that the elements of the eucharist are really transformed into flesh and blood, although all their "sensible accidents," that is, all that could be expected from physical experience, remain those of bread and wine. The Protestant episcopal church requires its ministers to teach that the elements remain really bread and wine, although they have miraculous spiritual effects different from those of ordinary bread and wine. "No indeed," say the Romanists, "they not only have those spiritual effects but they really are transmuted." But the layman declares that he cannot understand the difference. "That is not necessary," says the priest, "you can believe it implicitly."

What does that mean? It means that the layman is to trust that if he could understand the matter and know the truth, he would find that the priest was right.†1 But trust -- and the word belief means trust primarily -- essentially refers to the future, or to a contingent future. The implication is that the layman may sometime know, presumably will, in another world; and that he may *expect* that if he ever does come to know, he will find the priest to be right. Thus, analysis shows that even in regard to so excessively metaphysical a matter, the belief, if there can be any belief, has to involve expectation as its very essence.

Peirce: CP 5.542 Cross-Ref:††

542. It now begins to look strongly as if perhaps all belief might involve expectation as its essence. That is as much as can justly be said. We have as yet no assurance that this is true of every kind of belief. One class of accepted truths which we have neglected is that of direct perceptual facts. I lay down a wafer, before me. I look at it, and say to myself, "That wafer looks red." What element of expectation is there in the belief that the wafer *looks* red at this moment?

Peirce: CP 5.542 Cross-Ref:††

In order to handle this question, it is necessary to draw a distinction. Every belief is belief in a proposition. Now every proposition has its predicate which expresses *what* is believed, and its subjects which express *of what* it is believed. The grammarians of today prefer to say that a sentence has but one subject, which is put in the nominative. But from a logical point of view the terminology of the older grammarians was better, who spoke of the subject nominative and the subject accusative. I do not know that they spoke of the subject dative; but in the proposition, "Anthony gave a ring to Cleopatra," Cleopatra is as much a subject of what is meant and expressed as is the ring or Anthony. A proposition, then, has one predicate and any number of subjects. The subjects are either names of objects well known to the utterer and to the interpreter of the proposition (otherwise he could not interpret it) or they are virtually almost directions how to proceed to gain acquaintance with what is referred to. Thus, in the sentence "Every man dies," "Every man" implies that the interpreter is at liberty to pick out a man and consider the proposition as applying to him. In the proposition "Anthony gave a ring to Cleopatra," if the interpreter asks, "What ring?" the answer is that the indefinite article shows that it is a ring which might have been pointed out to the interpreter if he had been on the spot; and that the proposition is only asserted of the suitably chosen ring. The predicate on the other hand is a word or phrase which will call up in the memory or imagination of the interpreter images of things such as he has seen or imagined and may see again. Thus, "gave" is the predicate of the last proposition; and it conveys its meaning because the interpreter has had many experiences in which gifts were made; and a sort of composite photograph of them appears in his imagination. I am told that "Saccharin is 500 times as sweet as cane-sugar." But I never heard of saccharin. On inquiry, I find it is the sulphimide of orthosulphobenzoic acid; that is, it is phthalimide in which one CO group is replaced by SO[2]. I can see on paper that there might be such a body. That it is "500 times sweeter than sugar" produces a rather confused idea of a very familiar general kind. *What* I am to expect is expressed by the predicate, while the subjects inform me *on what occasion* I am to expect it. Diogenes Laertius, Suidas, Plutarch, and an anonymous biographer tell us that Aristotle was unable to pronounce the letter R.†1 I place Aristotle perfectly, of course. He is the author of works I often read and profoundly admire and whose fame far surpasses that of any other logician -- The Prince of Philosophers. I have also met people who could not pronounce R; but in other respects they did not seem to be much like Aristotle -- not even Dundreary.

Should I meet him in the Elysian Fields, I shall know what to expect. That is an impossible supposition; but should I ever meet a great logician, spindle-shanked and pig-eyed, who cannot pronounce R, I shall be interested to see whether he has other characteristics of Aristotle. This example has been selected as one which should seem to a superficial eye to involve no gleam of expectation; and if this testimony of four respectable witnesses, as independent as under the circumstances they could be, is destined never to receive confirmation nor contradiction, nor in any other way to have its probable consequences confronted by future experience, then in truth no expectation does it carry. In that case, it is an idle tale that might, for any practical purpose, have been as well the creation of some ironical poet. In that case, it is, properly speaking, no contribution to knowledge, for at least it is only probability, and probability cannot be reckoned as knowledge, unless it is destined to be indefinitely heightened in the future. Knowledge which should have no possible bearing upon any future experience -- bring no expectation whatever -- would be information concerning a dream. But in truth no such thing can be presumed of any knowledge. We expect that in time it will produce, or reinforce, or weaken some definite expectation. Give science only a hundred more centuries of increase in geometrical progression, and she may be expected to find that the sound waves of Aristotle's voice have somehow recorded themselves. If not, it were better to hand the reports over to the poets to make something pretty of, and thus turn them to some human use. But the right thing to do is to expect the verification. It is the degenerate pronunciation that is to be expected; the occasion is when Aristotle's voice shall become virtually heard again or when we shall have some other information which shall confirm or refute those reports.

Peirce: CP 5.543 Cross-Ref:††

543. Now if the reader should say, "Talk as you please, the assertion that Aristotle was {praulos} simply brings to the mental ear the voice of a man unable to pronounce the letter R, and labels that image with an indication of Aristotle, a man who lived three hundred years before Christ," the author may surprise him and grieve any whom he may have convinced, by declaring "I agree with you entirely"; only this assertion, which is identical with the previous one, though translated into other language, *means* nothing unless it be that Aristotle having been brought, directly or indirectly, to our experience, *will be* found, if found at all, to be incapable of pronouncing the R. Let us distinguish between the *proposition* and the *assertion* of that proposition. We will grant, if you please, that the proposition itself merely represents an image with a label or pointer attached to it. But to *assert* that proposition is to make oneself responsible for it, without any definite forfeit, it is true, but with a forfeit no smaller for being unnamed.†1 Now an *ex post facto* law is forbidden by the Constitution of the United States of America, but an *ex post facto* contract is forbidden by the constitution of things. A man cannot promise what the past shall have been, if he tries. It is evident that to guarantee that, if a piece of work has not already been done right, one will pay for it, and to guarantee that, *if it shall be found* not to have already been done right, one will pay for it, have one and the same meaning. One or other of them therefore must be an elliptical or otherwise unliteral expression, or else both are so. But nobody will maintain that to promise to pay for the work, if it shall be ascertained not to have been already done right, really means to promise to so pay, if it shall in fact not have been already done right, whether it be ascertained or not. It would be equally absurd to say that there was any third meaning which should have reference to an unascertained past. It follows, then, that to contract to pay money if something in the past has been done or not done *can only* mean that

the money shall be paid if it is ascertained that the event has happened or has not happened. But there would be no reason why the literal sense should not be understood if it made any sense. Hence there can be no meaning in making oneself responsible for a past event independent of its future ascertainment. But to assert a proposition is to make oneself responsible for its truth. Consequently, the only meaning which an assertion of a past fact can have is that, if in the future the truth be ascertained, so it shall be ascertained to be. There seems to be no rational escape from this.

Peirce: CP 5.544 Cross-Ref:††

544. Now let us take up the perceptual judgment "This wafer looks red." It takes some time to write this sentence, to utter it, or even to think it. It must refer to the state of the percept at the time that it, the judgment, began to be made. But the judgment does not exist until it is completely made. It thus only refers to a memory of the past; and all memory is possibly fallible and subject to criticism and control. The judgment, then, can only mean that so far as the character of the percept can ever be ascertained, it will be ascertained that the wafer looked red.

Peirce: CP 5.545 Cross-Ref:††

545. Perhaps the matter may be stated less paradoxically. Everybody will agree that it would be perfectly meaningless to say that sulphur had the singular property of turning pink when nobody was looking at it, instantly returning to yellowness before the most rapid glance could catch its pink color, or to say that copper was subject to the law that as long as there was no pressure upon it, it was perfectly yielding, becoming hard in proportion as it was pressed; and generally, a law which never should operate would be an empty formula. Indeed, something not very far from the assertion about copper is contained in all treatises on dynamics, although not limited to any particular substance. Namely, it is set down that no tangential force can be exerted upon a perfect fluid. But no writer puts it forth as a statement of fact; it is given as a definition merely. A law, then, which never will operate has no positive existence. Consequently, a law which has operated for the last time has ceased to exist as a law, except as a mere empty formula which it may be convenient to allow to remain. Hence to assert that a law positively exists is to assert that it will operate, and therefore to refer to the future, even though only conditionally. But to say that a body is hard, or red, or heavy, or of a given weight, or has any other property, is to say that it is subject to law and therefore is a statement referring to the future.

Peirce: CP 5.546 Cross-Ref:††

§2. JUDGMENT AND ASSERTION †1

546. Every new concept first comes to the mind in a judgment. This argument evades the consideration of the difficult question of the logical nature of the judgment, but draws attention to a fact that ordinary speech recognizes; namely, that a judgment is something that *ripens* in the mind, and further that there is a vernacular phrase which betrays a feature of the ripe judgment, the phrase "I says to myself, says I." The phrase indicates the easily verified fact that the ripe judgment, at least, involves an element closely analogous to *assertion*. But what is that? What is the nature of assertion? We have no magnifying-glass that can enlarge its features, and

render them more discernible; but in default of such an instrument we can select for examination a very formal assertion, the features of which have purposely been rendered very prominent, in order to emphasize its solemnity. If a man desires to assert anything very solemnly, he takes such steps as will enable him to go before a magistrate or notary and take a binding oath to it. Taking an oath is not mainly an event of the nature of a setting forth, *Vorstellung*, or representing. It is not mere saying, but is *doing*. The law, I believe, calls it an "act." At any rate, it would be followed by very real effects, in case the substance of what is asserted should be proved untrue. This ingredient, the assuming of responsibility, which is so prominent in solemn assertion, must be present in every genuine assertion. For clearly, every assertion involves an effort to make the intended interpreter believe what is asserted, to which end a reason for believing it must be furnished. But if a lie would not endanger the esteem in which the utterer was held, nor otherwise be apt to entail such real effects as he would avoid, the interpreter would have no reason to believe the assertion. Nobody takes any positive stock in those conventional utterances, such as "I am perfectly delighted to see you," upon whose falsehood no punishment at all is visited. At this point, the reader should call to mind, or, if he does not know it, should make the observations requisite to convince himself, that even in solitary meditation every judgment is an effort to press home, upon the self of the immediate future and of the general future, some truth. It is a genuine assertion, just as the vernacular phrase represents it; and solitary dialectic is still of the nature of dialogue. Consequently it must be equally true that here too there is contained an element of assuming responsibility, of "taking the consequences."

Peirce: CP 5.547 Cross-Ref:††

547. That is the first point of this argument; namely, that the *judgment*, which is the sole vehicle in which a concept can be conveyed to a person's cognizance or acquaintance, is not a purely representitious event, but involves an act, an exertion of energy, and is liable to real consequences, or effects. To this an eager adversary of pragmatism might make answer to the effect that if there be an assumption of responsibility in a judgment, it can only be in a ripe judgment; whereas the concept makes its appearance before the judgment is ripe, when it is still in the problematic or interrogatory mood; and that this shows that the *volitional element* is quite extraneous to the substance, or "meaning," of the concept. But the reply will be that this answer quite mistakes the aim of the argument. For it is no pragmaticistic doctrine that responsibility attaches to a concept; but the argument is that the predication of a concept is capable of becoming the subject of responsibility, since it actually does become so in the act of asserting that predication.

Peirce: CP 5.548 Cross-Ref:††

548. Thereupon it follows that the concept has a capability of having a bearing upon conduct; and this fact will lend it intellectual purport. For it cannot be denied that one, at least, of the functions of intelligence is to adapt conduct to circumstances, so as to subserve desire. If the argument is correct, this applies to any concept whatsoever, unless there be a concept that cannot be predicated.

TRUTH

§1. TRUTH AS CORRESPONDENCE †1

549. A *state of things* is an abstract constituent part of reality, of such a nature that a proposition is needed to represent it. There is but one *individual*, or completely determinate, state of things, namely, the all of reality. A *fact* is so highly a prescissively abstract state of things, that it can be wholly represented in a simple proposition, and the term "simple," here, has no absolute meaning, but is merely a comparative expression.

Peirce: CP 5.550 Cross-Ref:††

550. A *mathematical form* of a state of things is such a representation of that state of things as represents only the samenesses and diversities involved in that state of things, without definitely qualifying the subjects of the samenesses and diversities. It represents not necessarily all of these; but if it does represent all, it is the *complete* mathematical form. Every mathematical form of a state of things is the complete mathematical form of *some* state of things. The complete mathematical form of any state of things, real or fictitious, represents every ingredient of that state of things except the qualities of feeling connected with it. It represents whatever importance or significance those qualities may have; but the qualities themselves it does not represent.

Peirce: CP 5.551 Cross-Ref:††

551. Before any conclusion shall be made to rest upon this almost self-evident proposition, a way of setting it quite beyond doubt shall be explained. As at present enunciated, it is merely put forward as a private opinion of the writer's which will serve to explain the great interest he attaches to the emphatic dualism of the three normative sciences, which may be regarded as being the sciences of the conditions of truth and falsity, of wise and foolish conduct, of attractive and repulsive ideas. Should the reader become convinced that the importance of everything resides entirely in its mathematical form, he, too, will come to regard this dualism as worthy of close attention. Meantime that it exists, and is more marked in these sciences than in any others, is an indisputable fact. To what is this circumstance to be attributed? Skipping the easy reasoning by which it can be shown that this dualism cannot be due to any peculiar quality of feeling that may be connected with these sciences, nor to any intellectual peculiarity of them, which negative propositions will become obtrusively plain at a later stage of our reasoning, we may turn at once to the affirmative reason for attributing the dualism to the reference of the normative sciences to action. It is curious how this reason seems to seek to escape detection, by putting forward an apparent indication that it is not there. For it is evident that it is in esthetics that we ought to seek for the deepest characteristics of normative science, since esthetics, in dealing with the very ideal itself whose mere materialization engrosses the attention of practics and of logic, must contain the heart, soul, and spirit of normative science. But that dualism which is so much marked in the True and False, logic's object of study, and in the Useful and Pernicious of the confessional of Practics, is softened almost to obliteration in esthetics. Nevertheless, it would be the height of stupidity to

say that esthetics knows no good and bad. It must never be forgotten that evil of any kind is none the less bad though *the occurrence of it* be a good. Because in every case the ultimate in some measure abrogates, and ought to abrogate, the penultimate, it does not follow that the penultimate ought not to have abrogated the antepenultimate in due measure. On the contrary, just the opposite follows.

Peirce: CP 5.552 Cross-Ref:††

552. Esthetic good and evil are closely akin to pleasure and pain. They are what would be pleasure or pain to the fully developed superman. What, then, are pleasure and pain? The question has been sufficiently discussed, and the answer ought by this time to be ready. They are secondary feelings or generalizations of such feelings; that is, of feelings attaching themselves to, and excited by, other feelings.†1 A toothache is painful. It is not pain, but pain *accompanies* it; and if you choose to say that pain is an ingredient of it, that is not far wrong. However, the quality of the feeling of toothache is a simple, positive feeling, distinct from pain; though pain accompanies it. To use the old consecrated terms, pleasure is the feeling that a feeling is "sympathetical," pain that it is "antipathetical." The feeling of pain is a symptom of a feeling which repels us; the feeling of pleasure is the symptom of an attractive feeling. Attraction and repulsion are kinds of action. Feelings are pleasurable or painful according to the kind of action which they stimulate. In general, the good is the attractive -- not to everybody, but to the sufficiently matured agent; and the evil is the repulsive to the same. Mr. Ferdinand C.S. Schiller †1 informs us that he and James have made up their minds that the true is simply the satisfactory. No doubt; but to say "satisfactory" is not to complete any predicate whatever. Satisfactory to what end?

Peirce: CP 5.553 Cross-Ref:††

553. That truth is the correspondence of a representation with its object is, as Kant †2 says, merely the nominal definition of it. Truth belongs exclusively to propositions. A proposition has a subject (or set of subjects) and a predicate. The subject is a sign; the predicate is a sign; and the proposition is a sign that the predicate is a sign of that of which the subject is a sign.†3 If it be so, it is true. But what does this correspondence or reference of the sign, to its object, consist in? The pragmaticist answers this question as follows. Suppose, he says, that the angel Gabriel were to descend and communicate to me the answer to this riddle from the breast of omniscience. Is this supposable; or does it involve an essential absurdity to suppose the answer to be brought to human intelligence? In the latter case, "truth," in this sense, is a useless word, which never can express a human thought. It is real, if you will; it belongs to that universe entirely disconnected from human intelligence which we know as the world of utter nonsense. Having no use for this meaning of the word "truth," we had better use the word in another sense presently to be described. But if, on the other hand, it be conceivable that the secret should be disclosed to human intelligence, it will be something that thought can compass. Now thought is of the nature of a sign. In that case, then, if we can find out the right method of thinking and can follow it out -- the right method of transforming signs -- then truth can be nothing more nor less than the last result to which the following out of this method would ultimately carry us. In that case, that to which the representation should conform, is itself something in the nature of a representation, or sign -- something noumenal, intelligible, conceivable, and utterly unlike a thing-in-itself.

Peirce: CP 5.554 Cross-Ref:††

554. Truth is the conformity of a representamen to its object, *its* object, ITS

object, mind you. The International Dictionary at the writer's elbow, the Century Dictionary which he daily studies, the Standard which he would be glad sometimes to consult, all contain the word *yes*; but that word is not true simply because he is going to ask on this eighth of January 1906, in Pike County, Pennsylvania, whether it is snowing. There must be an action of the object upon the sign to render the latter true. Without that, the object is not the representamen's object. If a colonel hands a paper to an orderly and says, "You will go immediately and deliver this to Captain Hanno," and if the orderly does so, we do not say the colonel told the truth; we say the orderly was obedient, since it was not the orderly's conduct which determined the colonel to say what he did, but the colonel's speech which determined the orderly's action. Here is a view of the writer's house: what makes that house to be the object of the view? Surely not the similarity of appearance. There are ten thousand others in the country just like it. No, but the photographer set up the film in such a way that according to the laws of optics, the film was forced to receive an image of this house. What the sign virtually has to do in order to indicate its object -- and make it its -- all it has to do is just to seize its interpreter's eyes and forcibly turn them upon the object meant: it is what a knock at the door does, or an alarm or other bell, or a whistle, a cannon-shot, etc. It is pure physiological compulsion; nothing else.

Peirce: CP 5.554 Cross-Ref:††

So, then, a sign, in order to fulfill its office, to actualize its potency, must be compelled by its object. This is evidently the reason of the dichotomy of the true and the false. For it takes two to make a quarrel, and a compulsion involves as large a dose of quarrel as is requisite to make it quite impossible that there should be compulsion without resistance.

Peirce: CP 5.555 Cross-Ref:††
§2. TRUTH AND SATISFACTION †1

555. It appears that there are certain mummified pedants who have never waked to the truth that the act of knowing a real object alters it. They are curious specimens of humanity, and as I am one of them, it may be amusing to see how I think. It seems that our oblivion to this truth is due to our not having made the acquaintance of a new analysis that the True is simply that in cognition which is Satisfactory. As to this doctrine, if it is meant that True and Satisfactory are synonyms, it strikes me that it is not so much a doctrine of philosophy as it is a new contribution to English lexicography.

Peirce: CP 5.556 Cross-Ref:††

556. But it seems plain that the formula does express a doctrine of philosophy, although quite vaguely; so that the assertion does not concern two words of our language but, attaching some other *meaning* to the True, makes it to be *coextensive with* the Satisfactory in cognition.

Peirce: CP 5.557 Cross-Ref:††

557. In that case, it is indispensable to say what is meant by the True: until this is done the statement has no meaning. I suppose that by the True is meant that at which inquiry aims.

Peirce: CP 5.558 Cross-Ref:††

558. It is equally indispensable to ascertain what is meant by Satisfactory; but this is by no means so easy. Whatever be meant, however, if the doctrine is true at all, it must be necessarily true. For it is the very object, conceived in entertaining the purpose of the inquiry, that is asserted to have the character of satisfactoriness.

Peirce: CP 5.559 Cross-Ref:††

559. Is the Satisfactory meant to be whatever excites a certain peculiar feeling of satisfaction? In that case, the doctrine is simply hedonism in so far as it affects the field of cognition. For when hedonists talk of "pleasure," they do not mean what is so-called in ordinary speech, but what excites a feeling of satisfaction.

Peirce: CP 5.560 Cross-Ref:††

560. But to say that an action or the result of an action is Satisfactory is simply to say that it is congruous to the aim of that action. Consequently, the aim must be determined before it can be determined, either in thought or in fact, to be satisfactory. An action that had no other aim than to be congruous to its aim would have no aim at all, and would not be a deliberate action.

Peirce: CP 5.561 Cross-Ref:††

561. The hedonists do not offer their doctrine as an induction from experience but insist that, *in the nature of things*, that is, from the very essence of the conceptions, an action can have no other aim than "pleasure." Now it is *conceivable* that an action should be disconnected from every other in its aim. Such an action, then, according to hedonistic doctrine, can have no other aim than that of satisfying its own aim, which is absurd.

Peirce: CP 5.562 Cross-Ref:††

562. But if the hedonist replies that his position does not relate to satisfaction, but to a feeling that only arises upon satisfaction, the rejoinder will be that feeling is incomprehensible; so that no necessary truth can be discovered about it. But as a matter of observation we do, now and then, meet with persons who very largely behave with a view of experiencing this or that feeling. These people, however, are exceptional, and are wretched beings sharply marked off from the mass of busy and happy mankind.

Peirce: CP 5.563 Cross-Ref:††

563. It is, however, no doubt true that men act, especially in the action of inquiry, *as if* their sole purpose were to produce a certain state of feeling, in the sense that when that state of feeling is attained, there is no further effort. It was upon that proposition that I originally based pragmatism, laying it down in the article that in November 1877†1 prepared the ground for my argument for the pragmatic doctrine (*Pop. Sci. Monthly* for January, 1878†2). In the case of inquiry, I called that state of feeling "firm belief," and said, "As soon as a firm belief is reached we are entirely satisfied, whether the belief be true or false,"†3 and went on to show how the action of experience consequently was to create the conception of real truth. Early in 1880, in the opening paragraphs of my memoir in Vol. III of the *American Journal of Mathematics*,†4 I referred the matter to the fundamental properties of protoplasm, showing that purposive action must be action *virtually* directed toward the removal of stimulation.

Peirce: CP 5.564 Cross-Ref:††

564. My paper of November 1877, setting out from the proposition that the agitation of a question ceases when satisfaction is attained with the settlement of

belief, and then only, goes on to consider how the conception of truth gradually develops from that principle under the action of experience; beginning with willful belief, or self-mendacity, the most degraded of all intellectual conditions; thence rising to the imposition of beliefs by the authority of organized society; then to the idea of a settlement of opinion as the result of a fermentation of ideas; and finally reaching the idea of truth as overwhelmingly forced upon the mind in experience as the effect of an independent reality.

Peirce: CP 5.565 Cross-Ref:††
§3. DEFINITIONS OF TRUTH †1

565. *Logical.* (1) Truth is a character which attaches to an abstract proposition, such as a person might utter. It essentially depends upon that proposition's not professing to be exactly true. But we hope that in the progress of science its error will indefinitely diminish, just as the error of 3.14159, the value given for π , will indefinitely diminish as the calculation is carried to more and more places of decimals. What we call π is an ideal limit to which no numerical expression can be perfectly true. If our hope is vain; if in respect to some question -- say that of the freedom of the will -- no matter how long the discussion goes on, no matter how scientific our methods may become, there never will be a time when we can fully satisfy ourselves either that the question has no meaning, or that one answer or the other explains the facts, then in regard to that question there certainly is no *truth*. But whether or not there would be perhaps any *reality* is a question for the metaphysician, not the logician. Even if the metaphysician decides that where there is no truth there is no reality, still the distinction between the character of truth and the character of reality is plain and definable. Truth is that concordance of an abstract statement with the ideal limit towards which endless investigation would tend to bring scientific belief, which concordance the abstract statement may possess by virtue of the confession of its inaccuracy and one-sidedness, and this confession is an essential ingredient of truth. A further explanation of what this concordance consists in will be given below. Reality is that mode of being by virtue of which the real thing is as it is, irrespectively of what any mind or any definite collection of minds may represent it to be. The truth of the proposition that Caesar crossed the Rubicon consists in the fact that the further we push our archaeological and other studies, the more strongly will that conclusion force itself on our minds forever -- or would do so, if study were to go on forever. An idealist metaphysician may hold that therein also lies the whole *reality* behind the proposition; for though men may for a time persuade themselves that Caesar did *not* cross the Rubicon, and may contrive to render this belief universal for any number of generations, yet ultimately research -- if it be persisted in -- must bring back the contrary belief. But in holding that doctrine, the idealist necessarily draws the distinction between truth and reality.

Peirce: CP 5.566 Cross-Ref:††

566. In the above we have considered positive scientific truth. But the same definitions equally hold in the normative sciences. If a moralist describes an ideal as the *summum bonum*, in the first place, the perfect truth of his statement requires that it should involve the confession that the perfect doctrine can neither be stated nor conceived. If, with that allowance, the future development of man's moral nature will only lead to a firmer satisfaction with the described ideal, the doctrine is true. A

metaphysician may hold that the fact that the ideal thus forces itself upon the mind, so that minds in their development cannot fail to come to accept it, argues that the ideal is *real*: he may even hold that that fact (if it be one) constitutes a *reality*. But the two ideas, *truth* and *reality*, are distinguished here by the same characters given in the above definitions.

Peirce: CP 5.567 Cross-Ref:††

567. These characters equally apply to pure mathematics. Projective geometry is not pure mathematics, unless it be recognized that whatever is said of rays holds good of every family of curves of which there is one and one only through any two points, and any two of which have a point in common. But even then it is not pure mathematics until for points we put any complete determinations of any two-dimensional continuum. Nor will that be enough. A proposition is not a statement of perfectly pure mathematics until it is devoid of all definite meaning, and comes to this -- that a property of a certain icon is pointed out and is declared to belong to anything like it, of which instances are given. The perfect truth cannot be stated, except in the sense that it confesses its imperfection. The pure mathematician deals exclusively with hypotheses. Whether or not there is any corresponding real thing, he does not care. His hypotheses are creatures of his own imagination; but he discovers in them relations which surprise him sometimes. A metaphysician may hold that this very forcing upon the mathematician's acceptance of propositions for which he was not prepared, proves, or even constitutes, a mode of being independent of the mathematician's thought, and so a *reality*. But whether there is any reality or not, the truth of the pure mathematical proposition is constituted by the impossibility of ever finding a case in which it fails. This, however, is only possible if we confess the impossibility of precisely defining it.

Peirce: CP 5.568 Cross-Ref:††

568. The same definitions hold for the propositions of practical life. A man buys a bay horse, under a warranty that he is sound and free from vice. He brings him home and finds he is dyed, his real colour being undesirable. He complains of false representations; but the seller replies, "I never pretended to state every fact about the horse; what I said was true, so far as it professed to be true." In ordinary life all our statements, it is well understood, are, in the main, rough approximations to what we mean to convey. A tone or gesture is often the most definite part of what is said. Even with regard to perceptual facts, or the immediate judgments we make concerning our single percepts, the same distinction is plain. The percept is the reality. It is not in propositional form. But the most immediate judgment concerning it is abstract. It is therefore essentially unlike the reality, although it must be accepted as true to that reality. Its truth consists in the fact that it is impossible to correct it, and in the fact that it only professes to consider one aspect of the percept.†1

Peirce: CP 5.569 Cross-Ref:††

569. But even if it were impossible to distinguish between truth and reality, that would not in the least prevent our defining what it is that truth consists in. Truth and falsity are characters confined to propositions. A proposition is a sign which separately indicates its object. Thus, a portrait with the name of the original below it is a proposition. It asserts that if anybody looks at it, he can form a reasonably correct idea of how the original looked. A sign is only a sign *in actu* by virtue of its receiving an interpretation, that is, by virtue of its determining another sign of the same object. This is as true of mental judgments as it is of external signs. To say that a proposition is true is to say that every interpretation of it is true. Two propositions are equivalent

when either might have been an interpretant of the other. This equivalence, like others, is by an act of abstraction (in the sense in which forming an abstract noun is abstraction) conceived as identity. And we speak of believing in a proposition, having in mind an entire collection of equivalent propositions with their partial interpretants. Thus, two persons are said to have the same proposition in mind. The interpretant of a proposition is itself a proposition. Any necessary inference from a proposition is an interpretant of it. When we speak of truth and falsity, we refer to the possibility of the proposition being refuted; and this refutation (roughly speaking) takes place in but one way. Namely, an interpretant of the proposition would, if believed, produce the expectation of a certain description of percept on a certain occasion. The occasion arrives: the percept forced upon us is different. This constitutes the falsity of every proposition of which the disappointing prediction was the interpretant.

Peirce: CP 5.569 Cross-Ref:††

Thus, a false proposition is a proposition of which some interpretant represents that, on an occasion which it indicates, a percept will have a certain character, while the immediate perceptual judgment on that occasion is that the percept has not that character. A true proposition is a proposition belief in which would never lead to such disappointment so long as the proposition is not understood otherwise than it was intended.†1

Peirce: CP 5.570 Cross-Ref:††

570. All the above relates to *complex truth*, or the truth of propositions. This is divided into many varieties, among which may be mentioned *ethical truth*, or the conformity of an assertion to the speaker's or writer's belief, otherwise called *veracity*, and *logical truth*, that is, the concordance of a proposition with reality, in such way as is above defined.

Peirce: CP 5.571 Cross-Ref:††

571. (2) The word *truth* has also had great importance in philosophy in widely different senses, in which it is distinguished as *simple truth*, which is that truth which inheres in other subjects than propositions.

Peirce: CP 5.571 Cross-Ref:††

Plato in the *Cratylus* (385B) maintains that words have truth; and some of the scholastics admitted that an incomplex sign, such as a picture, may have truth.

Peirce: CP 5.572 Cross-Ref:††

572. But *truth* is also used in senses in which it is not an affection of a sign, but of things as things. Such truth is called *transcendental truth*. The scholastic maxim was *Ens est unum, verum, bonum*. Among the senses in which transcendental truth was spoken of was that in which it was said that all science has for its object the investigation of *truth*, that is to say, of the real characters of things. It was, in other senses, regarded as a subject of metaphysics exclusively. It is sometimes defined so as to be indistinguishable from reality, or real existence. Another common definition is that truth is the conformity, or conformability, of things to reason. Another definition is that truth is the conformity of things to their essential principles.

Peirce: CP 5.573 Cross-Ref:††

573. (3) *Truth* is also used in logic in a sense in which it inheres only in subjects more complex than propositions. Such is *formal truth*, which belongs to an argumentation which conforms to logical laws.

METHODS FOR ATTAINING TRUTH

§1. THE FIRST RULE OF LOGIC^{††}1

574. Certain methods of mathematical computation correct themselves; so that if an error be committed, it is only necessary to keep right on, and it will be corrected in the end. For instance, I want to extract the cube root of 2. The true answer is 1.25992105. . . . The rule is as follows:

Peirce: CP 5.574 Cross-Ref:††

Form a column of numbers, which for the sake of brevity we may call the A's. The first 3 A's are any 3 numbers taken at will. To form a new A, add the last two A's, triple the sum, add to this sum the last A but two, and set down the result as the next A. Now any A, the lower in the column the better, divided by the following A gives a

$$\sqrt[3]{2.}$$

fraction which increased by 1 is approximately [cube root of 2]
[Click here to view]

<i>Correct</i>	Sum of		<i>Erroneous</i>	Sum of	
<i>Computation</i>	Two	Triple	<i>Computation</i>	Two	Triple
1		1			
0		0			
1	1	3	1	1	3
4	5	15	4	5	15
15	19	57	Error! 16	20	60
58	73	219	61	77	231
223	281	843	235	296	888
858	1081	3243	904	1139	3417

3301 4159 12477 3478 4382 13146
12700 13381

1 3301/12700 1.2599213 1 3478/13381 1.2599208

Error +.0000002

Error -.0000002

Peirce: CP 5.574 Cross-Ref:††

You see the error committed in the second computation, though it seemed to multiply itself greatly, became substantially corrected in the end.

Peirce: CP 5.574 Cross-Ref:††

If you sit down to solve *ten* ordinary linear equations between ten unknown quantities, you will receive materials for a commentary upon the infallibility of mathematical processes. For you will almost infallibly get a wrong solution. I take it as a matter of course that you are not an expert professional computer. He will proceed according to a method which will correct his errors if he makes any.

Peirce: CP 5.575 Cross-Ref:††

575. This calls to mind one of the most wonderful features of reasoning and one of the most important philosophemes in the doctrine of science, of which, however, you will search in vain for any mention in any book I can think of; namely, that reasoning tends to correct itself, and the more so, the more wisely its plan is laid. Nay, it not only corrects its conclusions, it even corrects its premisses. The theory of Aristotle is that a necessary conclusion is just equally as certain as its premisses, while a probable conclusion is somewhat less so. Hence, he was driven to his strange distinction between what is better known to Nature and what is better known to us. But were every probable inference less certain than its premisses, science, which piles inference upon inference, often quite deeply, would soon be in a bad way. Every astronomer, however, is familiar with the fact that the catalogue place of a fundamental star, which is the result of elaborate reasoning, is far more accurate than any of the observations from which it was deduced.

Peirce: CP 5.576 Cross-Ref:††

576. That Induction tends to correct itself, is obvious enough. When a man undertakes to construct a table of mortality upon the basis of the Census, he is engaged in an inductive inquiry. And lo, the very first thing that he will discover from the figures, if he did not know it before, is that those figures are very seriously vitiated by their falsity. The young find it to their advantage to be thought older than they are, and the old to be thought younger than they are. The number of young men who are just 21 is altogether in excess of those who are 20, although in all other cases the ages expressed in round numbers are in great excess. Now the operation of inferring a law in a succession of observed numbers is, broadly speaking, inductive; and therefore we see that a properly conducted Inductive research corrects its own premisses.

Peirce: CP 5.577 Cross-Ref:††

577. That the same thing may be true of a Deductive inquiry our arithmetical example has shown. *Theoretically*, I grant you, there is no possibility of error in necessary reasoning. But to speak thus "theoretically," is to use language in a Pickwickian sense. In practice, and in fact, mathematics is not exempt from that liability to error that affects everything that man does. Strictly speaking, it is not certain that twice two is four. If on an average in every thousand figures obtained by addition by the average man there be one error, and if a thousand million men have each added 2 to 2 ten thousand times, there is still a possibility that they have all committed the same error of addition every time. If everything were fairly taken into account, I do not suppose that twice two is four is more certain than Edmund Gurney †1 held the existence of veridical phantasms of the dying or dead to be. Deductive inquiry, then, has its errors; and it corrects them, too. But it is by no means so sure, or at least so swift to do this as is Inductive science. A celebrated error in the *Mécanique Céleste* concerning the amount of theoretical acceleration of the moon's mean motion deceived the whole world of astronomy for more than half a century. †2 Errors of reasoning in the first book of Euclid's Elements, the logic of which book was for two thousand years subjected to more careful criticism than any other piece of reasoning without exception ever was or probably ever will be, only became known after the non-Euclidean geometry had been developed. The certainty of mathematical reasoning, however, lies in this, that once an error is suspected, the whole world is speedily in accord about it.

Peirce: CP 5.578 Cross-Ref: ††

578. As for Retroductive Inquiries, or the Explanatory Sciences, such as Geology, Evolution, and the like, they always have been and always must be theatres of controversy. These controversies do get settled, after a time, in the minds of candid inquirers; though it does not always happen that the protagonists themselves are able to assent to the justice of the decision. Nor is the general verdict always logical or just.

Peirce: CP 5.579 Cross-Ref: ††

579. So it appears that this marvellous self-correcting property of Reason, which Hegel made so much of, belongs to every sort of science, although it appears as essential, intrinsic, and inevitable only in the highest type of reasoning, which is induction. But the logic of relatives shows that the other types of reasoning, Deduction and Retroduction, are not so thoroughly unlike Induction as they might be thought, and as Deduction, at least, always has been thought to be. Stuart Mill alone among the older logicians in his analysis of the *Pons Asinorum* came very near to the view which the logic of relatives forces us to take. †1 Namely, in the logic of relatives, treated let us say, in order to fix our ideas, by means of those existential graphs of which I gave a slight sketch in the last lecture, †2 [we] begin a Deduction by writing down all the premisses. Those different premisses are then brought into one field of assertion, that is, are *colligated*, as Whewell †3 would say, or joined into one copulative proposition. Thereupon, we proceed attentively to observe the graph. It is just as much an operation of *Observation* as is the observation of bees. This observation leads us to make an *experiment* upon the Graph. Namely, we first duplicate portions of it; and then we erase portions of it, that is, we put out of sight part of the assertion in order to see what the rest of it is. We observe the result of this experiment, and that is our deductive conclusion. Precisely those three things are all that enter into the experiment of any Deduction -- Colligation, Iteration, Erasure. †4 The rest of the process consists of observing the result. It is not, however, in every Deduction that all the three possible elements of the Experiment take place. In

particular, in ordinary syllogism the iteration may be said to be absent. And that is the reason that ordinary syllogism can be worked by a machine.†5 There is but one conclusion of any consequence to be drawn by ordinary syllogism from given premisses. Hence, it is that we fall into the habit of talking of *the* conclusion. But in the logic of relatives there are conclusions of different orders, depending upon how much iteration takes place.†6 What is *the* conclusion deducible from the very simple first principles of number? It is ridiculous to speak of *the* conclusion. *The* conclusion is no less than the aggregate of all the theorems of higher arithmetic that have been discovered or that ever will be discovered. Now let us turn to Induction. This mode of reasoning also begins by a colligation. In fact, it is precisely the colligation that gave induction its name, {epagein} with Socrates,†1 {synagōgē} with Plato,†2 {epagōgē} with Aristotle.†3 It must, by the rule of predesignation,†4 be a deliberate experiment. In ordinary induction we proceed to observe something about each instance. Relative induction is illustrated by the process of making out the law of the arrangement of the scales of a pine-cone. It is necessary to mark a scale taken as an instance, and counting in certain directions to come back to that marked scale. This double observation of the same instance corresponds to Iteration in deduction. Finally, we erase the particular instances and leave the class or system sampled directly connected with the characters, relative or otherwise, which have been found in the sample of it.

Peirce: CP 5.580 Cross-Ref:††

580. We see, then, that Induction and Deduction are after all not so very unlike. It is true that in Induction we commonly make many experiments and in Deduction only one. Yet this is not always the case. The chemist contents himself with a single experiment to establish any qualitative fact. True, he does this because he knows that there is such a uniformity in the behaviour of chemical bodies that another experiment would be a mere repetition of the first in every respect. But it is precisely such a knowledge of a uniformity that leads the mathematician to content himself with one experiment. The inexperienced student in mathematics will mentally perform a number of geometrical experiments, which the veteran would regard as superfluous, before he will permit himself to come to a general conclusion. For example, if the question is, how many rays can cut four rays fixed in space, the experienced mathematician will content himself with imagining that two of the fixed rays intersect and that the other two likewise intersect. He will see, then, that there is one ray through the two intersections and another along the intersection of the two planes of pairs of intersecting fixed rays, and will unhesitatingly declare thereupon that but two rays can cut four fixed rays, unless the fixed rays are so situated that an infinite multitude of rays will cut them all. But I dare say many of you would want to experiment with other arrangements of the four fixed rays, before making any confident pronouncement. A friend of mine who seemed to have difficulties in adding up her accounts was once counselled to add each column five times and adopt the mean of the different results. It is evident that when we run a column of figures down as well as up, as a check, or when we review a demonstration in order to look out for any possible flaw in the reasoning, we are acting precisely as when in an induction we enlarge our sample for the sake of the self-correcting effect of induction.

Peirce: CP 5.581 Cross-Ref:††

581. As for retroduction, it is itself an experiment. A retroductive research is an experimental research; and when we look upon Induction and Deduction from the point of view of Experiment and Observation, we are merely tracing in those types of reasoning their affinity to Retroduction. It begins always with colligation, of course,

of a variety of separately observed facts about the subject of the hypothesis. How remarkable it is, by the way, that the entire army of logicians from Zeno to Whately should have left it to this mineralogist [Whewell] to point out colligation as a generally essential step in reasoning. To return to Retroduction, then, it begins with colligation. Something corresponding to iteration may or may not take place. And then comes an Observation. Not, however, an External observation of the objects as in Induction, nor yet an observation made upon the parts of a diagram, as in Deduction; but for all that just as truly an observation. For what is observation? What is experience? It is the enforced element in the history of our lives. It is that which we are constrained to be conscious of by an occult force residing in an object which we contemplate. The act of observation is the deliberate yielding of ourselves to that *force majeure* -- an early surrender at discretion, due to our foreseeing that we must, whatever we do, be borne down by that power, at last. Now the surrender which we make in Retroduction, is a surrender to the Insistence of an Idea. The hypothesis, as the Frenchman says, *c'est plus fort que moi*. It is irresistible; it is imperative. We must throw open our gates and admit it at any rate for the time being.

Peirce: CP 5.582 Cross-Ref:††

582. Thus it is that inquiry of every type, fully carried out, has the vital power of self-correction and of growth. This is a property so deeply saturating its inmost nature that it may truly be said that there is but one thing needful for learning the truth, and that is a hearty and active desire to learn what is true. If you really want to learn the truth, you will, by however devious a path, be surely led into the way of truth, at last. No matter how erroneous your ideas of the method may be at first, you will be forced at length to correct them so long as your activity is moved by that sincere desire. Nay, no matter if you only half desire it, at first, that desire would at length conquer all others, could experience continue long enough. But the more veraciously truth is described at the outset, the shorter by centuries will the road to it be.

Peirce: CP 5.583 Cross-Ref:††

583. In order to demonstrate that this is so, it is necessary to note what is essentially involved in the Will to Learn. The first thing that the Will to Learn supposes is a dissatisfaction with one's present state of opinion. There lies the secret of why it is that our American universities are so miserably insignificant. What have they done for the advance of civilization? What is the great idea or where is [the] single great man who can truly be said to be the product of an American university? The English universities, rotting with sloth as they always have, have nevertheless in the past given birth to Locke and to Newton, and in our time to Cayley, Sylvester, and Clifford. The German universities have been the light of the whole world. The medieval University of Bologna gave Europe its system of law. The University of Paris and that despised scholasticism took Abelard and made him into Descartes. The reason was that they were institutions of learning while ours are institutions for teaching. In order that a man's whole heart may be in teaching he must be thoroughly imbued with the vital importance and absolute truth of what he has to teach; while in order that he may have any measure of success in learning he must be penetrated with a sense of the unsatisfactoriness of his present condition of knowledge. The two attitudes are almost irreconcilable. But just as it is not the self-righteous man who brings multitudes to a sense of sin, but the man who is most deeply conscious that he is himself a sinner, and it is only by a sense of sin that men can escape its thralldom; so it is not the man, who thinks he knows it all, that can bring other men to feel their need of learning, and it is only a deep sense that one is miserably ignorant that can

spur one on in the toilsome path of learning. That is why, to my very humble apprehension, it cannot but seem that those admirable pedagogical methods, for which the American teacher is distinguished, are of little more consequence than the cut of his coat, that they surely are as nothing compared with that fever for learning that must consume the soul of the man who is to infect others with the same apparent malady. Let me say that of the present condition of Harvard I really know nothing at all except that I know the leaders of the department of philosophy to be all true scholars, particularly marked by eagerness to learn and freedom from dogmatism. And in every age, it can only be the philosophy of that age, such as it may be, which can animate the special sciences to any work that shall really carry forward the human mind to some new and valuable truth. Because the valuable truth is not the detached one, but the one that goes toward enlarging the system of what is already known.

Peirce: CP 5.584 Cross-Ref:††

584. The Inductive Method springs directly out of dissatisfaction with existing knowledge. The great rule of predesignation, which must guide it, is as much as to say that an induction to be valid must be prompted by a definite doubt or at least an interrogation; and what is such an interrogation but first, a sense that we do not know something; second, a desire to know it; and third, an effort -- implying a willingness to labor -- for the sake of seeing how the truth may really be. If that interrogation inspires you, you will be sure to examine the instances; while if it does not, you will pass them by without attention.

Peirce: CP 5.585 Cross-Ref:††

585. I repeat that I know nothing about the Harvard of today, but one of the things which I hope to learn during my stay in Cambridge is the answer to this question, whether the Commonwealth of Massachusetts has set up this university to the end that such young men as can come here may receive a fine education and may thus be able to earn handsome incomes, and have a canvas-back and a bottle of Clos de Vougeot for dinner -- whether this is what she is driving at -- or whether it is that, knowing that all America looks largely to sons of Massachusetts for the solutions of the most urgent problems of each generation, she hopes that in this place something may be studied out which shall be of service in the solutions of those problems. In short, I hope to find out whether Harvard is an educational establishment or whether it is an institution for learning what is not yet thoroughly known, whether it is for the benefit of the individual students or whether it is for the good of the country and for the speedier elevation of man into that rational animal of [which] he is the embryonic form.

Peirce: CP 5.585 Cross-Ref:††

There is one thing that I am sure a Harvard education cannot fail to do, because it did that much even in my time, and for a very insouciant student; I mean that it cannot fail to disabuse the student of the popular notion that modern science is so very great a thing as to be commensurate with Nature and indeed to constitute of itself some account of the universe, and to show him that it is yet, what it appeared to Isaac Newton to be, a child's collection of pebbles gathered upon the beach -- the vast ocean of Being lying there unsounded.

Peirce: CP 5.586 Cross-Ref:††

586. It is not merely that in all our gropings we bump up against problems which we cannot imagine how to attack, why space should have but three dimensions, if it really has but three, why the Listing numbers which define its shape should all equal one, if they really do, or why some of them should be zero, as Listing himself

and many geometers think they are, if that be the truth, of why forces should determine the second derivative of the space rather than the third or fourth, of why matter should consist of about seventy distinct kinds, and all those of each kind apparently exactly alike, and these different kinds having masses nearly in arithmetical progression and yet not exactly so, of why atoms should attract one another at a distance in peculiar ways, if they really do, or if not what produced such vortices, and what gave the vortices such peculiar laws of attraction, of how or by what kind of influence matter came to be sifted out, so that the different kinds occur in considerable aggregations, of why certain motions of the atoms of certain kinds of protoplasm are accompanied by sensation, and so on through the whole list.†1 These things do indeed show us how superficial our science still is; but its littleness is made even more manifest when we consider within how narrow a range all our inquiries have hitherto lain. The instincts connected with the need of nutrition have furnished all animals with some virtual knowledge of space and of force, and made them applied physicists. The instincts connected with sexual reproduction have furnished all animals at all like ourselves with some virtual comprehension of the minds of other animals of their kind, so that they are applied psychists. Now not only our accomplished science, but even our scientific questions have been pretty exclusively limited to the development of those two branches of natural knowledge.†1 There may for aught we know be a thousand other kinds of relationship which have as much to do with connecting phenomena and leading from one to another, as dynamical and social relationships have. Astrology, magic, ghosts, prophecies, serve as suggestions of what such relationships might be.

Peirce: CP 5.587 Cross-Ref:††

587. Not only is our knowledge thus limited in scope, but it is even more important that we should thoroughly realize that the very best of what we, humanly speaking, know [we know] only in an uncertain and inexact way.

Peirce: CP 5.587 Cross-Ref:††

Nobody would dream of contending that because the sun has risen and set every day so far, that afforded any reason at all for supposing that it would go on doing so to all eternity. But when I say that there is not the very slightest reason for thinking that no material atoms ever go out of existence or come into existence, there I fail to carry the average man with me; and I suppose the reason is, that he dimly conceives that there is some reason, other than the pure and simple induction, for holding matter to be ingenerable and indestructible. For it is plain that if it be a mere question of our weighings or other experiences, all that appears is that not more than one atom in a million or ten million becomes annihilated before the deficiency of mass is pretty certain to be balanced by another atom's being created. Now when we are speaking of atoms, a million or ten million is an excessively minute quantity. So that as far as purely inductive evidence is concerned we are very very far from being entitled to think that matter is absolutely permanent. If you put the question to a physicist his reply will probably be, as it certainly ought to be, that physicists only deal with such phenomena as they can either directly or indirectly observe, or are likely to become able to observe until there is some great revolution in science, and to that he will very likely add that any limitation upon the permanence of matter would be a purely gratuitous hypothesis without anything whatever to support it. Now this last part of the physicist's reply is, in regard to the order of considerations which he has in mind, excellent good sense. But from an absolute point of view, I think it leaves something out of account. Do you believe that the fortune of the Rothschilds will endure forever? Certainly not; because although they may be safe enough as far

as the ordinary causes go which engulf fortunes, yet there is always a chance of some revolution or catastrophe which may destroy all property. And no matter how little that chance may be, as far as this decade or this generation goes, yet in limitless decades and generations, it is pretty sure that the pitcher will get broken, at last. There is no danger, however slight, which in an indefinite multitude of occasions does not come as near to absolute certainty as probability can come. The existence of the human race, we may be as good as sure, will come to an end at last. For not to speak of the gradual operation of causes of which we know, the action of the tides, the resisting medium, the dissipation of energy, there is all the time a certain danger that the earth may be struck by a meteor or wandering star so large as to ruin it, or by some poisonous gas. That a purely gratuitous hypothesis should turn out to be true is, indeed, something so exceedingly improbable that we cannot be appreciably wrong in calling it zero. Still, the chance that out of an infinite multitude of gratuitous hypotheses an infinitesimal proportion, which may itself be an infinite multitude, should turn out to be true, is zero multiplied by infinity, which is absolutely indeterminate. That is to say we simply know nothing whatever about it. Now that any single atom should be annihilated is a gratuitous hypothesis. But there are, we may suppose, an infinite multitude of atoms, and a similar hypothesis may be made for each. And thus we return to my original statement that as to whether any finite number or even an infinite number of atoms are annihilated per year, that is something of which we are simply in a state of blank ignorance, unless we have found out some method of reasoning altogether superior to induction. If, therefore, we should detect any general phenomenon of nature which could very well be explained, not by supposing any definite breach of the laws of nature, for that would be no explanation at all, but by supposing that a continual breach of all the laws of nature, every day and every second, was itself one of the laws or habitudes of nature, there would be no power in induction to offer the slightest logical objection to that theory. But as long as we are aware of no such general phenomena tending to show such continual inexactitude in law, then we must remain absolutely without any rational opinion upon the matter *pro* or *con*.

Peirce: CP 5.588 Cross-Ref:††

588. There are various ways in which the natural cocksuredness and conceit of man struggles to escape such confession of total ignorance. But they seem to be all quite futile. One of the commonest, and at the same time the silliest, is the argument that God would for this or that excellent reason never act in such an irregular manner. I think all the men who talk like that must be near-sighted. For to suppose that any man who could see the moving clouds and survey a wide expanse of landscape and note its wonderful complexity, and consider how unimaginably small it all was in comparison to the whole face of the globe, not to speak of the millions of orbs in space, and who would not presume to predict what move Morphy or Steinitz might make in so simple a thing as a game of chess, should undertake to say what God would do, would seem to impeach his sanity. But if instead of its being a God, after whose image we are made, and whom we can, therefore, begin to understand, it were some metaphysical principle of Being, even more incomprehensible, whose action the man pretended to compute, that would seem to be a pitch of absurdity one degree higher yet.

Peirce: CP 5.589 Cross-Ref:††

589. People talk of a hypothesis where there is a *vera causa*. But in such cases the inference is not hypothetical but inductive. A *vera causa* is a state of things known to be present and known partially at least to explain the phenomena, but not known to

explain them with quantitative precision. Thus, when seeing ordinary bodies round us accelerated toward the earth's centre and seeing also the moon, which both in its *albedo* and its volcanic appearance altogether resembles stone, to be likewise accelerated toward the earth, and when finding these two accelerations are in the inverse duplicate ratios of their distances from that centre, we conclude that their nature, whatever it may be, is the same, we are inferring an analogy, which is a type of inference having all the strength of induction and more, besides.†1 For the sake of simplicity, I have said nothing about it in these lectures; but I am here forced to make that remark. Moreover, when we consider that all that we infer about the gravitation of the moon is a continuity between the terrestrial and lunar phenomena, a continuity which is found throughout physics, and when we add to that, the analogies of electrical and magnetical attractions, both of which vary inversely as the square of the distance, we plainly recognize here one of the strongest arguments of which science affords any example. Newton was entirely in the right when he said, *Hypotheses non fingo*.†2 It is they who have criticized the dictum whose logic is at fault. They are attributing an obscure psychological signification to *force*, or *vis insita*, which in physics only connotes a regularity among accelerations. Thus inferences concerning *veræ causæ* are inductions not retroductions, and of course have only such uncertainty and inexactitude as belong to induction. When I say that a reductive inference is not a matter for belief at all, I encounter the difficulty that there are certain inferences which, scientifically considered, are undoubtedly hypotheses and yet which practically are perfectly certain. Such for instance is the inference that Napoleon Bonaparte really lived at about the beginning of this century, a hypothesis which we adopt for the purpose of explaining the concordant testimony of a hundred memoirs, the public records of history, tradition, and numberless monuments and relics. It would surely be downright insanity to entertain a doubt about Napoleon's existence. A still better example is that of the translations of the cuneiform inscriptions which began in mere guesses, in which their authors could have had no real confidence. Yet by piling new conjectures upon former conjectures apparently verified, this science has gone on to produce under our very eyes a result so bound together by the agreement of the readings with one another, with other history, and with known facts of linguistics, that we are unwilling any longer to apply the word *theory* to it. You will ask me how I can reconcile such facts as these with my dictum that hypothesis is not a matter for belief. In order to answer this question I must first examine such inferences in their scientific aspect and afterwards in their practical aspect. The only end of science, as such, is to learn the lesson that the universe has to teach it. In Induction it simply surrenders itself to the force of facts. But it finds, at once -- I am partially inverting the historical order, in order to state the process in its logical order -- it finds I say that this is not enough. It is driven in desperation to call upon its inward sympathy with nature, its instinct for aid, just as we find Galileo at the dawn of modern science making his appeal to *il lume naturale*. But in so far as it does this, the solid ground of fact fails it. It feels from that moment that its position is only provisional. It must then find confirmations or else shift its footing. Even if it does find confirmations, they are only partial. It still is not standing upon the bedrock of fact. It is walking upon a bog, and can only say, this ground seems to hold for the present. Here I will stay till it begins to give way. Moreover, in all its progress, science vaguely feels that it is only learning a lesson. The value of *Facts* to *it*, lies only in this, that they belong to Nature; and Nature is something great, and beautiful, and sacred, and eternal, and real -- the object of its worship and its aspiration. It therein takes an entirely different attitude toward facts from that which Practice takes. For Practice, facts are the arbitrary forces with which it has to reckon and to wrestle.

Science, when it comes to understand itself, regards facts as merely the vehicle of eternal truth, while for Practice they remain the obstacles which it has to turn, the enemy of which it is determined to get the better. Science feeling that there is an arbitrary element in its theories, still continues its studies, confident that so it will gradually become more and more purified from the dross of subjectivity; but practice requires something to go upon, and it will be no consolation to it to know that it is on the path to objective truth -- the actual truth it must have, or when it cannot attain certainty must at least have high probability, that is, must know that, though a few of its ventures may fail, the bulk of them will succeed. Hence the hypothesis which answers the purpose of theory may be perfectly worthless for art. After a while, as Science progresses, it comes upon more solid ground. It is now entitled to reflect: this ground has held a long time without showing signs of yielding. I may hope that it will continue to hold for a great while longer. This reflection, however, is quite aside from the purpose of science. It does not modify its procedure in the least degree. It is extra-scientific. For Practice, however, it is vitally important, quite altering the situation. As Practice apprehends it, the conclusion no longer rests upon mere retrodution, it is inductively supported. For a large sample has now been drawn from the entire collection of occasions in which the theory comes into comparison with fact, and an overwhelming proportion, in fact all the cases that have presented themselves, have been found to bear out the theory. And so, says Practice, I can safely presume that so it will be with the great bulk of the cases in which I shall go upon the theory; especially as they will closely resemble those which have been well tried. In other words there is now reason to believe in the theory, for belief is the willingness to risk a great deal upon a proposition. But this belief is no concern of science, which has nothing at stake on any temporal venture but is in pursuit of eternal verities (not semblances to truth) and looks upon this pursuit, not as the work of one man's life, but as that of generation after generation, indefinitely. Thus those retroductive inferences which at length acquire such high degrees of certainty, so far as they are so probable, are not pure retroductions and do not belong to science, as such; while, so far as they are scientific and are pure retroductions, have no true probability and are not matters for belief. We call them in science established truths, that is, they are propositions into which the economy of endeavor prescribes that, for the time being, further inquiry shall cease.

Peirce: CP 5.590 Cross-Ref:††
 §2. ON SELECTING HYPOTHESES †1

590. If we are to give the names of Deduction, Induction, and Abduction to the three grand classes of inference, then Deduction must include every attempt at mathematical demonstration, whether it relate to single occurrences or to "probabilities," that is, to statistical ratios; Induction must mean the operation that induces an assent, with or without quantitative modification, to a proposition already put forward, this assent or modified assent being regarded as the provisional result of a method that must ultimately bring the truth to light; while Abduction must cover all the operations by which theories and conceptions are engendered.

Peirce: CP 5.591 Cross-Ref:††

591. How is it that man ever came by any correct theories about nature? We know by Induction that man has correct theories; for they produce predictions that are

fulfilled. But by what process of thought were they ever brought to his mind? A chemist notices a surprising phenomenon. Now if he has a high admiration of Mill's *Logic*, as many chemists have, he will remember that Mill tells him that he must work on the principle that, under precisely the same circumstances, like phenomena are produced. Why does he then not note that this phenomenon was produced on such a day of the week, the planets presenting a certain configuration, his daughter having on a blue dress, he having dreamed of a white horse the night before, the milkman having been late that morning, and so on? The answer will be that in early days chemists did use to attend to some such circumstances, but that they have learned better. How have they learned this? By an induction. Very well, that induction must have been based upon a theory which the induction verified. How was it that man was ever led to entertain that true theory? You cannot say that it happened by chance, because the possible theories, if not strictly innumerable, at any rate exceed a trillion -- or the third power of a million; and therefore the chances are too overwhelmingly against the single true theory in the twenty or thirty thousand years during which man has been a thinking animal, ever having come into any man's head. Besides, you cannot seriously think that every little chicken, that is hatched, has to rummage through all possible theories until it lights upon the good idea of picking up something and eating it. On the contrary, you think the chicken has an innate idea of doing this; that is to say, that it can think of this, but has no faculty of thinking anything else. The chicken you say pecks by instinct. But if you are going to think every poor chicken endowed with an innate tendency toward a positive truth, why should you think that to man alone this gift is denied? If you carefully consider with an unbiassed mind all the circumstances of the early history of science and all the other facts bearing on the question, which are far too various to be specifically alluded to in this lecture, I am quite sure that you must be brought to acknowledge that man's mind has a natural adaptation to imagining correct theories of some kinds, and in particular to correct theories about forces, without some glimmer of which he could not form social ties and consequently could not reproduce his kind. In short, the instincts conducive to assimilation of food, and the instincts conducive to reproduction, must have involved from the beginning certain tendencies to think truly about physics, on the one hand, and about psychics, on the other. It is somehow more than a mere figure of speech to say that nature fecundates the mind of man with ideas which, when those ideas grow up, will resemble their father, Nature.

Peirce: CP 5.592 Cross-Ref:††

592. But if that be so, it must be good reasoning to say that a given hypothesis is good, as a hypothesis, because it is a natural one, or one readily embraced by the human mind. It must concern logic in the highest degree to ascertain precisely how far and under what limitations this maxim may be held. For of all beliefs, none is more natural than the belief that it is natural for man to err. The logician ought to find out what the relation is between these two tendencies.

Peirce: CP 5.593 Cross-Ref:††

593. It behooves a man first of all to free his mind of those four idols of which Francis Bacon speaks in the first book of the *Novum Organum*. So much is the dictate of Ethics, itself. But after that, what? Descartes, as you know, maintained that if a man could only get a perfectly clear and distinct idea †1 -- to which Leibniz added the third requirement that it should be adequate †2 -- then that idea must be true. But this is far too severe. For never yet has any man attained to an apprehension perfectly clear and distinct, let alone its being adequate; and yet I suppose that true ideas have been entertained. Ordinary ideas of perception, which Descartes thought

were most horribly confused, have nevertheless something in them that very nearly warrants their truth, if it does not quite so. "Seeing is believing," says the instinct of man.

Peirce: CP 5.594 Cross-Ref:††

594. The question is what theories and conceptions we *ought* to entertain. Now the word "ought" has no meaning except relatively to an *end*. That ought to be done which is conducive to a certain end. The inquiry therefore should begin with searching for the *end* of thinking. What do we think *for*? What is the physiological function of thought? If we say it is action, we must mean the government of action to some end. To what end? It must be something, good or admirable, regardless of any ulterior reason. This can only be the esthetically good. But what is esthetically good? Perhaps we may say the full expression of an idea?†1 Thought, however, is in itself essentially of the nature of a sign. But a sign is not a sign unless it translates itself into another sign in which it is more fully developed. Thought requires achievement for its own development, and without this development it is nothing. Thought must live and grow in incessant new and higher translations, or it proves itself not to be genuine thought.

Peirce: CP 5.595 Cross-Ref:††

595. But the mind loses itself in such general questions and seems to be floating in a limitless vacuity. It is of the very essence of thought and purpose that it should be special, just as truly as it is of the essence of either that it should be general. Yet it illustrates the point that the valuable idea must be eminently fruitful in special applications, while at the same time it is always growing to wider and wider alliances.

Peirce: CP 5.596 Cross-Ref:††

596. Classical antiquity was far too favorable to the sort of concept that was

fortis, et in se ipso totus, teres atque rotundus. †2

I often meet with such theories in philosophical books, especially in the works of theological students and of others who draw their ideas from antiquity. Such is the circular theory, which assumes itself and returns into itself -- the aristocratical theory which holds itself aloof from vulgar facts. Logic has not the least objection to such a view, so long as it maintains its self-sufficiency, keeps itself strictly to itself, as its nobility obliges it to do, makes no pretension of meddling with the world of experience, and does not ask anybody to assent to it.

Peirce: CP 5.597 Cross-Ref:††

597.†1 Auguste Comte, at the other extreme, would condemn every theory that was not "verifiable." Like the majority of Comte's ideas, this is a bad interpretation of a *truth*. An explanatory hypothesis, that is to say, a conception which does not limit its purpose to enabling the mind to grasp into one a variety of facts, but which seeks to connect those facts with our general conceptions of the universe, ought, in one sense, to be *verifiable*; that is to say, it ought to be little more than a ligament of numberless possible predictions concerning future experience, so that if they fail, it fails. Thus, when Schliemann entertained the hypothesis that there really had been a city of Troy and a Trojan War, this meant to his mind among other things that when he should come to make excavations at Hissarlik he would probably find

remains of a city with evidences of a civilization more or less answering to the descriptions of the Iliad, and which would correspond with other probable finds at Mycenae, Ithaca, and elsewhere. So understood, Comte's maxim is sound. Nothing but that *is* an explanatory hypothesis. But Comte's own notion of a *verifiable* hypothesis was that it must not suppose anything that you are not able directly to observe.†2 From such a rule it would be fair to infer that he would permit Mr. Schliemann to suppose he was going to find arms and utensils at Hissarlik, but would forbid him to suppose that they were either made or used by any human being, since no such beings could ever be detected by direct percept. He ought on the same principle to forbid us to suppose that a fossil skeleton had ever belonged to a living ichthyosaurus. This seems to be substantially the opinion of M. Poincaré at this day. The same doctrine would forbid us to believe in our memory of what happened at dinnertime today. I have for many years been an adherent of what is technically called Common Sense in philosophy, myself; and do not think that my Tychistic opinions conflict with that position; but I nevertheless think that such theories as that of Comte and Poincaré about verifiable hypotheses frequently deserve the most serious consideration; and the examination of them is never lost time; for it brings lessons not otherwise so easily learned. Of course with memory would have to go all opinions about everything not at this moment before our senses. You must not believe that you hear me speaking to you, but only that you hear certain sounds while you see before you a spot of black, white, and flesh color; and those sounds somehow seem to suggest certain ideas which you must not connect at all with the black and white spot. A man would have to devote years to training his mind to such habits of thought, and even then it is doubtful whether it would be possible. And what would be gained? If it would alter our beliefs as to what our sensuous experience is going to be, it would certainly be a change for the worse, since we do not find ourselves disappointed in any expectations due to common sense beliefs. If on the other hand it would not make any such difference, as I suppose it would not, why not allow us the harmless convenience of believing in these fictions, if they be fictions? Decidedly we must be allowed these ideas, if only as cement for the matter of our sensations. At the same time, I protest that such permission would not be at all enough. Comte, Poincaré, and Karl Pearson take what they consider to be the first impressions of sense, but which are really nothing of the sort, but are percepts that are products of psychical operations, and they separate these from all the intellectual part of our knowledge, and arbitrarily call the first *real* and the second *fictions*. These two words *real* and *fictive* bear no significations whatever except as marks of *good* and *bad*. But the truth is that what they call *bad* or *fictitious*, or *subjective*, the intellectual part of our knowledge, comprises all that is valuable on its own account, while what they mark *good*, or *real*, or *objective*, is nothing but the pretty vessel that carries the precious thought.

Peirce: CP 5.598 Cross-Ref:††

598. I can excuse a person who has lost a dear companion and whose reason is in danger of giving way under the grief, for trying, on that account, to believe in a future life. I can more than excuse him because his usefulness is at stake, although I myself would not adopt a hypothesis, and would not even take it on probation, simply because the idea was pleasing to me. Without judging others, I should feel, for my own part, that that would be a crime against the integrity of the reason that God has lent to me. But if I had the choice between two hypotheses, the one more ideal and the other more materialistic, I should prefer to take the ideal one upon probation, simply because ideas are fruitful of consequences, while mere sensations are not so; so that the idealistic hypothesis would be the *more verifiable*, that is to say, would *predict*

more, and could be put the more thoroughly to the test.

Peirce: CP 5.598 Cross-Ref:††

Upon this same principle, if two hypotheses present themselves, one of which can be satisfactorily tested in two or three days, while the testing of the other might occupy a month, the former should be tried first, even if its apparent likelihood is a good deal less.

Peirce: CP 5.599 Cross-Ref:††

599. It is a very grave mistake to attach much importance to the antecedent likelihood †1 of hypotheses, except in extreme cases; because likelihoods are mostly merely subjective, and have so little real value, that considering the remarkable opportunities which they will cause us to miss, in the long run attention to them does not pay. Every hypothesis should be put to the test by forcing it to make verifiable predictions. A hypothesis on which no verifiable predictions can be based should never be accepted, except with some mark attached to it to show that it is regarded as a mere convenient vehicle of thought -- a mere matter of form.

Peirce: CP 5.600 Cross-Ref:††

600. In an extreme case, where the likelihood is of an unmistakably objective character, and is strongly supported by good inductions, I would allow it to cause the postponement of the testing of a hypothesis. For example, if a man came to me and pretended to be able to turn lead into gold, I should say to him, "My dear sir, I haven't time to make gold." But even then the likelihood would not weigh with me directly, as such, but because it would become a factor in what really is in all cases the leading consideration in Abduction, which is the question of Economy -- Economy of money, time, thought, and energy.†2

Peirce: CP 5.601 Cross-Ref:††

601. It is Prof. Ernst Mach †3 who has done the most to show the importance in logic of the consideration of Economy although I had written a paper on the subject as early as 1878.†4 But Mach goes altogether too far. For he allows thought no other value than that of economizing experiences. This cannot for an instant be admitted. Sensation, to my thinking, has no value whatever except as a vehicle of thought.

Peirce: CP 5.602 Cross-Ref:††

602. Proposals for hypotheses inundate us in an overwhelming flood, while the process of verification to which each one must be subjected before it can count as at all an item, even of likely knowledge, is so very costly in time, energy, and money -- and consequently in ideas which might have been had for that time, energy, and money, that Economy would override every other consideration even if there were any other serious considerations. In fact there are no others. For abduction commits us to nothing. It merely causes a hypothesis to be set down upon our docket of cases to be tried.

Peirce: CP 5.603 Cross-Ref:††

603. I shall be asked, Do you really mean to say that we ought not to adopt any opinion whatever as an opinion until it has sustained the ordeal of furnishing a prediction that has been verified?

Peirce: CP 5.603 Cross-Ref:††

In order to answer that question, it will be requisite to inquire how an abduction can be justified, here understanding by abduction any mode or degree of acceptance of a proposition as a truth, because a fact or facts have been ascertained

whose occurrence would necessarily or probably result in case that proposition were true. The abduction so defined amounts, you will remark, to observing a fact and then professing to say what idea it was that gave rise to that fact. One would think a man must be privy to the counsels of the Most High so to presume. The only justification possible, other than some such positive fact which would put quite another color upon the matter, is the justification of desperation. That is to say, that if he is not to say such things, he will be quite unable to know anything of positive fact.

Peirce: CP 5.603 Cross-Ref:††

In a general way, this justification certainly holds. If man had not had the gift, which every other animal has, of a mind adapted to his requirements, he not only could not have acquired any knowledge, but he could not have maintained his existence for a single generation. But he is provided with certain instincts, that is, with certain natural beliefs that are true. They relate in part to forces, in part to the action of minds. The manner in which he comes to have this knowledge seems to me tolerably clear. Certain uniformities, that is to say certain general ideas of action, prevail throughout the universe, and the reasoning mind is [it]self a product of this universe. These same laws are thus, by logical necessity, incorporated in his own being. For example, what we call straight lines are nothing but one out of an innumerable multitude of families of nonsingular lines such that through any two points there is one and one only. The particular family of lines called straight has no geometrical properties that distinguish it from any other of the innumerable families of lines of which there is one and one only through any two points. It is a law of *dynamics* that every dynamical relation between two points, no third point being concerned, except by combinations of such pairs, is altogether similar, except in quantity, to every such dynamical relation between any other two points on the same ray, or straight line. It is a consequence of this that a ray or straight line is the shortest distance between two points; whence, light appears to move along such lines; and that being the case, we recognize them by the eye, and call them straight. Thus, the faculty of sight naturally causes us to assign great prominence to such lines; and thus when we come to form a hypothesis about the motion of a particle left uninfluenced by any other, it becomes *natural* for us to suppose that it moves in a straight line. The reason this turns out true is, therefore, that this first law of motion is a corollary from a more general law which, governing all dynamics, governs light, and causes the idea of straightness to be a predominant one in our minds.

Peirce: CP 5.604 Cross-Ref:††

604. In this way, general considerations concerning the universe, strictly philosophical considerations, all but demonstrate that if the universe conforms, with any approach to accuracy, to certain highly pervasive laws, and if man's mind has been developed under the influence of those laws, it is to be expected that he should have a *natural light*, or *light of nature*, or *instinctive insight*, or genius, tending to make him guess those laws aright, or nearly aright. This conclusion is confirmed when we find that every species of animal is endowed with a similar genius. For they not only one and all have some correct notions of force, that is to say, some correct notions, though excessively narrow, of phenomena which we, with our broader conceptions, should call phenomena of force, and some similarly correct notions about the minds of their own kind and of other kinds, which are the two sufficient cotyledons of all our science, but they all have, furthermore, wonderful endowments of genius in other directions. Look at the little birds, of which all species are so nearly identical in their physique, and yet what various forms of genius do they not display in modelling their nests? This would be impossible unless the ideas that are naturally

predominant in their minds were true. It would be too contrary to analogy to suppose that similar gifts were wanting to man. Nor does the proof stop here. The history of science, especially the early history of modern science, on which I had the honor of giving some lectures in this hall some years ago,†1 completes the proof by showing how few were the guesses that men of surpassing genius had to make before they rightly guessed the laws of nature. . . .

Peirce: CP 5.605 Cross-Ref:††
APPENDIX

§ 1. KNOWLEDGE †1

605. This word is used in logic in two senses: (1) as a synonym for Cognition, and (2), and more usefully, to signify a perfect cognition, that is, a cognition fulfilling three conditions: first, that it holds for true a proposition that really is true; second, that it is perfectly self-satisfied and free from the uneasiness of doubt; third, that some character of this satisfaction is such that it would be logically impossible that this character should ever belong to satisfaction in a proposition not true.

Peirce: CP 5.606 Cross-Ref:††

606. Knowledge is divided, firstly, according to whatever classification of the sciences is adopted. Thus, Kantians distinguish formal and material knowledge. Secondly, knowledge is divided according to the different ways in which it is attained, as into immediate and mediate knowledge. Immediate knowledge is a cognition, or objective modification of consciousness, which is borne in upon a man with such resistless force as to constitute a guarantee that it (or a representation of it) will remain permanent in the development of human cognition. Such knowledge is, if its existence be granted, either borne in through an avenue of sense, external or internal, as a percept of an individual, or springs up within the mind as a first principle of reason or as a mystical revelation. Mediate knowledge is that for which there is some guarantee behind itself, although, no matter how far criticism be carried, simple evidency, or direct insistency, of something has to be relied upon. The external guarantee rests ultimately either upon authority, i.e., testimony, or upon observation. In either case mediate knowledge is attained by Reasoning, which see for further divisions.†2 It is only necessary to mention here that the Aristotelians distinguished knowledge {hoti}, or of the facts themselves, and knowledge {dioti}, or of the rational connection of facts, the knowledge of the how and why (cf. the preceding topic). They did not distinguish between the how and the why, because they held that knowledge {dioti} is solely produced by Syllogism †1 in its greatest perfection, as demonstration. The term empirical knowledge is applied to knowledge, mediate or immediate, which rests upon percepts; while the terms philosophical and rational knowledge are applied to knowledge, mediate or immediate, which rests chiefly or wholly upon conclusions or revelations of reason. Thirdly, knowledge is divided, according to the character of the immediate object, into apprehensive and judicative knowledge, the former being of a percept, image, or *Vorstellung*, the latter of the

existence or non-existence of a fact. Fourthly, knowledge is divided, according to the manner in which it is in the mind, into actual, virtual, and habitual knowledge. See Scotus, *Opus Oxoniense*, lib. I, dist. iii. quest. 2, paragraph beginning "Loquendo igitur." Fifthly, knowledge is divided according to its end, into speculative and practical.

Peirce: CP 5.607 Cross-Ref:††
§2. REPRESENTATIONISM †2

607. The doctrine that percepts stand for something behind them.

Peirce: CP 5.607 Cross-Ref:††

In a certain sense it must be admitted, even by presentationists, that percepts only perform the function of conveying knowledge of something else. That is to say, they have to be combined and generalized to become useful knowledge; so that they may be said to represent their own generalizations. In this, representationists and presentationists may agree. But the dispute between them consists in this, that the representationist regards the percept in the light of testimony or a picture, from which by inference, or a mental act analogous to inference, the hidden cause of the percept may become known; while the presentationist holds that perception is a two-sided consciousness in which the percept appears as forcibly acting upon us, so that in perception the consciousness of an active object and of a subject acted on are as indivisible as, in making a muscular effort, the sense of exertion is one with and inseparable from the sense of resistance. The representationist would not allow that there is any bilateral consciousness even in the latter sense, regarding the bilaterality as a quasi-inference, or product of the mind's action; while the presentationist insists that there is nothing intellectual or intelligible in this duality. It is, he says, a hard fact experienced but never understood. A representationist will naturally regard the theory that everything in the outward world is atoms, their masses, motions, and energy, as a statement of the real fact which percepts represent. The presentationist, on the other hand, will more naturally regard it as a formula which is fitted to sum up and reconcile the percepts as the only ultimate facts. These are, however, merely different points of view in which neither ought to find anything absolutely contrary to his own doctrine.

Peirce: CP 5.608 Cross-Ref:††
§3. ULTIMATE †1

608. (1) Last in a series; especially in a series of purposes each, except the last, subsidiary to an *ulterior* one following it in the arrangement considered, or of actions each of which, except the last, leads to the performance of another.

Peirce: CP 5.608 Cross-Ref:††

Thus, the phrase *ultimate signification* implies that a sign determines another sign of the same object, and this another; and so on until something is reached which is a sign only for itself. *Ultimate fact* implies that there is a series of facts each explicable by the one following it, until a fact is reached utterly inexplicable. (Cf.

Hamilton's *Reid*, Note A, V, ii 6, *et seq.*)

Peirce: CP 5.609 Cross-Ref:††

609. (2) Applied also to the limiting state of an endless series of states which approach indefinitely near to the limiting state, and on the whole nearer and nearer, without necessarily ever reaching it; although the word ultimate does not imply a denial of actual attainment.

Peirce: CP 5.609 Cross-Ref:††

Thus, it has been held that a real object is that which will be represented in the ultimate opinion about it. This implies that a series of opinions succeed one another, and that it is hoped that they may ultimately tend more and more towards some limiting opinion, even if they do not reach and rest in a last opinion. Cf. Truth and Error, Logical [Bk. III, ch. 5, §3].

Peirce: CP 5.610 Cross-Ref:††

§4. MR. PETERSON'S PROPOSED DISCUSSION †1

610. Very valuable ideas oftentimes appear so obvious, when once set forth, that high laudation of their inventors would invite ridicule. Such, we are told, was the notion that obsessed C. Colombo, and such is Mr. Peterson's proposal †2 to start in *The Monist* a discussion of philosophical terminology. It may be a very simple proposal, but nobody, as far as one careful reader of *The Monist* remembers, had made it before; and its utility to students of phenomenology, normative science, and metaphysics will have a high coefficient in its proportionality to the advantage they take of it. Duty calls upon us to contribute, each one what he can that will be useful, whether in the way of question or in that of answer. It seems likely that in my lifetime of study I may have learned something of the way to investigate questions such as Mr. Peterson puts; and if so, here is an opportunity to be of aid to other students.

Peirce: CP 5.611 Cross-Ref:††

611. *Experience*, the first term concerning which Mr. Peterson asks for light, is somewhat remarkable for having been employed as nearly as possible in the same sense from Polus the Acragentine (i.e. native of Girgenti) sophist down to Avenarius and Haeckel. As my first step in investigating its meaning, I should look out its equivalent *empeiria*, †P1 in Bonitz's *Index Aristotelicus*. For every serious student of philosophy ought to be able to read the common dialect of Greek at sight, and needs on his shelves the Berlin Aristotle, in the fifth volume of which is that index. On looking out *empeiria* there, what first strikes one is that it is not a very common word with Aristotle, nor yet an unusual one, since Bonitz cites something over a dozen passages in which it occurs. The first (*Post. Anal.* II, xix) runs: "From sense are engendered memories, and from multiplied memory of the same thing is engendered experience; for many memories make up a single experience." Waitz (*Organon*, II, 429) has a minute note on this passage. Another passage to which the Index refers (*Nic. Ethics*, VI, viii) is thus translated by Stewart in his valuable "Notes" on the work: "If we . . . ask why a boy may be a mathematician, but cannot understand philosophy or natural science, we find that it is because the truths of mathematics are abstract" [a bad explanation but that does not affect the evidence as to the meaning of *empeiria*], "whereas the principles of philosophy and natural science are reached

through long *experience*. A boy does not realize the meaning of the principles of philosophy and natural science, but merely repeats by rote the formulæ used to express them." In the *Politics* (A, xi) Aristotle remarks that theorizing is free, while experience is necessitated, and goes on to speak of experience with live stock, etc. In another place in the *Politics* (E, ix) he says that the military commander of greatest experience in strategy is to be preferred, even though his habit of speculation be known; while for the chief of police, or for a treasurer, experience is of no account in comparison with integrity. But the cynosural passage is the first chapter of Book A of the *Metaphysics*; and here he remarks (as he likewise does in the *Ethics*), that experience is a knowledge (*gnosis*)†P1 of singulars. Therein Aristotle's language differs from that of the Socrates of Plato, with whom *empeiria* is the skill that results from long dealings with any matter. Aristotle never intended to say that there is no other cognition of singulars than in experience; for that would directly contradict his doctrine that experience is a mass of memories relating to the same subject. His remark was, however, understood in the Middle Ages to be a *definition* of experience, and was repeated as such, a blunder that was not so unnatural as it would have been if the scholastic doctors had dealt with direct experience. The teachings of the *Aristotelic Index* having been exhausted, I turn to Harper's *Latin Lexicon*, which informs me that no writer of the Golden Age used *experientia* in the general sense, though that acception became common in the Silver Age, especially with Tacitus. The next work that I personally should consult would be my own notes collected during more than forty years. I always carry a pad of the size of a Post Card, of thick papers (50 in a pad, enough to last for two days, at least); and on these I note whatever elements of experience may reach me.†1 I keep these in drawers and boxes like the card catalogue of a library. I arrange and rearrange them from time to time. It is a treasure more valuable than a policy of insurance. I probably have near two hundred thousand such notes. But in order to bring what I have to say to a close, I will quote from the definition of experience given by the father of modern experiential philosophy, Dr. John Locke. In the *Essay concerning Humane †P1 Understanding*, (II, i, 2) we read (and the italics are in the original): "Whence has [the mind] all the materials of reason and knowledge? To this I answer, in one word, from *experience*: in that all our knowledge is founded, and from that it ultimately derives itself. Our observation employed either about *external sensible objects, or about the internal operations of our minds, perceived and reflected on by ourselves, is that which supplies our understanding with all the materials of thinking.*" This definition so formally stated, by such an authority, quite peerless for our present purpose, should be accepted as definitive and as a landmark that it would be a crime to displace or disturb. For in order that philosophy should become a successful science, it must, like biology, have its own vocabulary; and as in biology, it must be the rule that whoever wishes to introduce a new concept is to invent a new word to express it. This is no suggestion of the moment.†2 I am, for my humble part, maturely convinced that philosophy will never be upon the road to sound results until we dismiss our affection for old words and our dislike of newfangled words, and make its vocabulary over after the fashion of taxonomic zoölogy and botany. I limit my recommendation to technical terms; for I can pretend to no competence to give advice about *belles-lettres*. Yet even there I perceive that people read old authors, and admire them for saying what they never meant to say; because the modern readers forget that two or three centuries ago words still familiar suggested quite different ideas from those the same words now suggest.

612. But somebody may object that Locke's definition is vague, being founded on a misconception of the nature of perception. Suppose, the objector will say, that a newborn male infant were to be brought up among a colony of men on a desert island, without ever having seen a woman and barely having heard of such a creature. Suppose that, arrived at the age of twenty, he were to meet on the beach a Pacific Island woman who had swum over from another island. Would not the irresistible, the only possible cognition he could have of this creature be strongly colored by his own instincts? It would be the ineluctable result of "*observation employed concerning an external sensible object.*" The word "experience," however, is employed by Locke chiefly to enable him to say that human cognitions are inscribed by the individual's life-history upon a *tabula rasa*, and are not, like those of the lower animals, gifts of inborn instinct. His definition is vague for the reason that he never realized how important the innate element of our directest perceptions really is.

Peirce: CP 5.613 Cross-Ref:††

613. To such an objector I might say, My dear fellow, you must be joking; for under the guise of an objection you reinforce what I was saying with a new argument for restricting the use of the word "experience" to the expression of that vague idea which Locke so well defines. You make it plain that a distinct word is wanted, or rather two distinct words, to express the two more precise concepts which you suggest. The idea of the word "experience" was to refer to that which is forced upon a man's recognition, will-he nill-he, and shapes his thoughts to something quite different from what they naturally would have been. But the philosophers of experience, like many of other schools, forget to how great a degree it is true that the universe is all of a piece, and that we are all of us natural products, naturally partaking of the characteristics that are found everywhere through nature. It is in some measure nonsensical to talk of a man's nature as opposed to what perceptions force him to think. True, man continually finds himself resisted, both in his active desires and in that passive inertia of thought which causes any new phenomenon to give him a shock of surprise. You may think of an element of knowledge which thus resists his superficial tendencies; but to express precisely that idea you must have a new word: it will not answer the purpose to call it *experience*. You may also reflect that every man's environment is in some measure unfavorable to his development; and so far as this affects his cognitive development, you have there an element that is opposed to the man's nature. But surely the word *experience* would be ill-chosen to express that.

Peirce: CP 5.614 Cross-Ref:††

614. But I am encroaching far too much upon the space of this number, and am taking too much advantage of our good editor's indulgence. I did wish to consider what element of his philosophy Comte had specially in mind in christening it *Positive*. He plainly meant that it should be unlike the metaphysical thought which kneads over and over what we know already, and would be like the sort of material which is furnished by a microscope or by an archæologist's spade. I hope Mr. Peterson's suggestion may bring a whole crop of fruit.

CHARLES SANTIAGO †1 SANDERS PEIRCE.

- Peirce: CP 5.1 Fn 1 p 1
†1 From "Pragmatic and Pragmatism," *Dictionary of Philosophy and Psychology*, ed. by J.M. Baldwin, The Macmillan Co., New York; vol. 2, pp. 321-322 (1902).
- Peirce: CP 5.1 Fn 2 p 1
†2 *Anthropologie in pragmatischer Hinsicht*, Vorrede.
- Peirce: CP 5.2 Fn 3 p 1
†3 This paragraph was contributed by William James.
- Peirce: CP 5.3 Fn 4 p 1
†4 See 402; 526n.
- Peirce: CP 5.3 Fn 1 p 2
†1 *New World*, pp. 327-47; reprinted in 1897 in *The Will to Believe, and Other Essays in Popular Philosophy*.
- Peirce: CP 5.3 Fn 2 p 2
†2 In 1898; *University of California Chronicle*; reprinted in 1920 in *Collected Essays and Reviews*, ed. by R.B. Perry.
- Peirce: CP 5.3 Fn 3 p 2
†3 See 433 and vol. 1, bk. IV, ch. 4.
- Peirce: CP 5.3 Fn 4 p 2
†4 See 402n³, 429.
- Peirce: CP 5.4 Fn 1 p 3
†1 See vol. 6, bk. I, B.
- Peirce: CP 5.5 Fn 2 p 3
†2 From "Pragmatism" [1], c. 1905.
- Peirce: CP 5.5 Fn 3 p 3
†3 See *Kritik der reinen Vernunft*, A832, B860.
- Peirce: CP 5.5 Fn 4 p 3
†4 See 1.294ff.
- Peirce: CP 5.5 Fn 5 p 3
†5 Cf. 27, 469ff.
- Peirce: CP 5.7 Fn 1 p 4
†1 Cf. 1.303ff; 41ff.
- Peirce: CP 5.7 Fn 2 p 4
†2 Cf. 1.322ff; 45ff.
- Peirce: CP 5.7 Fn 3 p 4
†3 Cf. 1.337ff; 59ff.
- Peirce: CP 5.7 Fn 4 p 4
†4 Cf. 3.491.
- Peirce: CP 5.8 Fn 1 p 5

†1 "Linear Associative Algebra," §1, *American Journal of Mathematics*, vol. 4, pp. 97-229 (1881).

Peirce: CP 5.8 Fn 2 p 5

†2 See also 4.233ff.

Peirce: CP 5.11 Fn 1 p 6

†1 From "Pragmatism (Editor [3])," c. 1906. For the remainder, see bk. III, ch. 1.

Peirce: CP 5.12 Fn 1 p 7

†1 To judge from the *Letters of William James*, vol. 2, p. 233, there was a meeting of this club in the autumn of 1874.

Peirce: CP 5.12 Fn 2 p 7

†2 A friend of William James, who worked with Mr. Justice Holmes on Kent's *Commentaries*.

Peirce: CP 5.12 Fn 3 p 7

†3 Cf. *The Emotions and the Will*, ch. 11, p. 505, 3d ed. (1875).

Peirce: CP 5.12 Fn 1 p 8

†1 See 64.

Peirce: CP 5.13 Fn 2 p 8

†2 Author of a textbook on rhetoric in use at Harvard College in Peirce's day.

Peirce: CP 5.13 Fn P1 Para 1/6 p 9 Cross-Ref:††

†P1 **Pragmatism**. It is a singular instance of that over-modesty and unyielding self-underestimate on my part of which I am so justly proud as my principal claim to distinction that I should have omitted *pragmatism*, my own offspring, with which the world resounds. See Baldwin's *Dictionary* where is my original definition of 1878 and an exegesis, not very deep, of William James. Pragmatism is a method in philosophy. *Philosophy* is that branch of positive science (*i.e.*, an investigating theoretical science which inquires what is the fact, in contradistinction to pure mathematics which merely seeks to know what follows from certain hypotheses) which makes no observations but contents itself with so much of experience as pours in upon every man during every hour of his waking life. The study of philosophy consists, therefore, in reflexion, and *pragmatism* is that method of reflexion which is guided by constantly holding in view its purpose and the purpose of the ideas it analyzes, whether these ends be of the nature and uses of action or of thought.

Peirce: CP 5.13 Fn P1 Para 2/6 p 9 Cross-Ref:††

". . . the whole subsequent argument has already had its main lines mapped out by our introductory discussion of that *Weltanschauung* which Professor James has called *pragmatism*." -- F.C.S. Schiller (in *Personal Idealism*, edited by Henry Cecil Sturt, 1902, p. 63).

Peirce: CP 5.13 Fn P1 Para 3/6 p 9 Cross-Ref:††

The passage of Professor James here alluded to is as follows: ". . . Mr. Charles Sanders Peirce has rendered thought a service by disentangling from the particulars of its application the principle by which these men were instinctively guided, and by singling it out as fundamental and giving to it a Greek name. He calls it the principle of *pragmatism*." -- William James, *The Varieties of Religious Experience*, 1902, p.

444.

Peirce: CP 5.13 Fn P1 Para 4/6 p 9 Cross-Ref:††

It will be seen [from the original statement] that *pragmatism* is not a *Weltanschauung* but is a method of reflexion having for its purpose to render ideas clear.

Peirce: CP 5.13 Fn P1 Para 5/6 p 9 Cross-Ref:††

Pragmatic, a., Having the character of pragmatism, as a method in philosophy.

Peirce: CP 5.13 Fn P1 Para 6/6 p 9 Cross-Ref:††

Pragmatist, n., in philosophy, one who professes to practice pragmatism. Thus Schiller of Oxford, author of *Riddles of the Sphinx*, is a pragmatist, although he does not very thoroughly understand the nature of pragmatism. -- From Peirce's personal interleaved copy of the *Century Dictionary*, c. 1902.

Peirce: CP 5.14 Fn 1 p 11

†1 Delivered at Cambridge, Massachusetts, March 26 to May 17, 1903; James described them in his *Pragmatism*, p. 5, as "flashes of brilliant light relieved against Cimmerian darkness." He states that they were delivered at the Lowell Institute; the available records, however, show that they were given in Sever Hall, Harvard, under the auspices of the Harvard department of philosophy.

Peirce: CP 5.21 Fn 1 p 18

†1 Cf. 2.661f.

Peirce: CP 5.22 Fn P1 p 19 Cross-Ref:††

†P1 *I.e.*, he receives $(-2)^{n^2}$ cents if n tails intervene between two successive heads.

Peirce: CP 5.23 Fn 1 p 20

†1 See 19.

Peirce: CP 5.24 Fn 2 p 20

†2 Cf. 1.88, 2.647.

Peirce: CP 5.27 Fn 1 p 21

†1 See 394ff.

Peirce: CP 5.29 Fn 1 p 22

†1 Cf. 2.334, 2.435ff.

Peirce: CP 5.30 Fn 2 p 22

†2 Cf. 2.332ff, 3.432f.

Peirce: CP 5.32 Fn 1 p 23

†1 Cf. 3.203f, 539, 541.

Peirce: CP 5.34 Fn 1 p 24

†1 Cf. 108f., 1.191, 1.574, 1.611f.

Peirce: CP 5.36 Fn 1 p 25

†1 See his *Grammar of Science*, Introduction, pp. 26-27, where he seems to say that society must not allow a bad stock to perpetuate itself.

Peirce: CP 5.36 Fn 2 p 25

†2 Henry Rutgers Marshall's *Instinct and Reason*, p. 569, Macmillan Co. (1898).

Peirce: CP 5.37 Fn 1 p 27

†1 See vol. 1, bk. III for a detailed study of phenomenology.

Peirce: CP 5.38 Fn 2 p 27

†2 Cf. 43, 1.525.

Peirce: CP 5.40 Fn 1 p 28

†1 See 4.232f.

Peirce: CP 5.41 Fn 1 p 29

†1 Second draught. On the first page Peirce wrote "This won't do; it will have to be rewritten"; but no later draught of this part has been found. The third draught is given in 59-65.

Peirce: CP 5.41 Fn 2 p 29

†2 Cf. 1.300ff.

Peirce: CP 5.42 Fn 3 p 29

†3 Cf. 1.43.

Peirce: CP 5.45 Fn 1 p 32

†1 Cf. 1.322ff.

Peirce: CP 5.45 Fn 2 p 32

†2 See his *Philosophiæ Naturalis Principia Mathematica*, liber I, def. IV.

Peirce: CP 5.45 Fn 1 p 33

†1 Cf. 1.321, 3.363, 3.527, 4.157.

Peirce: CP 5.47 Fn 1 p 34

†1 Cf. 1.121, 1.316, 2.750.

Peirce: CP 5.47 Fn P1 p 34 Cross-Ref:††

†P1 I would not have anybody accept any doctrine of logic simply because minute and thorough criticism has resulted in making me perfectly confident of its truth. But I will not allow this scruple to prevent my saying that for my part -- who am characterized in some of the books as a sceptic in philosophy and have even been called a modern Hume ["David Hume Redivivus," Pt. I of "Mr. Charles S. Peirce's Onslaught on the Doctrine of Necessity" by Paul Carus in *The Monist*, vol. 2, pp. 560ff.] -- I have after long years of the severest examination become fully satisfied that, other things being equal, an anthropomorphic conception, whether it makes the best nucleus for a scientific working hypothesis or not, is far more likely to be approximately true than one that is not anthropomorphic. Suppose, for example, it is a question between accepting Telepathy or Spiritualism. The former I dare say is the preferable working hypothesis because it can be more readily subjected to experimental investigation. But as long as there is no reason for believing it except phenomena that Spiritualism is equally competent to explain, I think Spiritualism is much the more likely to be approximately true, as being the more anthropomorphic and natural idea; and in like manner, as between an old-fashioned God and a modern patent Absolute, recommend me to the anthropomorphic conception if it is a question

of which is the more likely to be about the truth. [See vol. 6, bk. II, chs. 4 and 7.]

Peirce: CP 5.52 Fn 1 p 38

†1 Cf. 1.334.

Peirce: CP 5.54 Fn 2 p 38

†2 Cf. 115ff, 151ff, 4.539, 4.541.

Peirce: CP 5.57 Fn 1 p 39

†1 57 and 58 occur as part of a digression at the end of "Lecture IV." That part of the notebook repeats much of the foregoing, and, with the exception of what follows, is not being published.

Peirce: CP 5.59 Fn 1 p 40

†1 Third draught. Cf. 1.337ff.

Peirce: CP 5.60 Fn 1 p 41

†1 Cf. 1.635.

Peirce: CP 5.64 Fn 1 p 43

†1 But see Wright's *Philosophical Discussions*, edited by C.E. Norton (1877).

Peirce: CP 5.64 Fn 1 p 44

†1 Asa Gray, the famous Harvard botanist.

Peirce: CP 5.66 Fn 1 p 47

†1 There were two draughts of this lecture. It is difficult to determine which is the final one. The following is from version "b."

Peirce: CP 5.66 Fn 2 p 47

†2 Cf. 1.527ff.

Peirce: CP 5.67 Fn 3 p 47

†3 Cf. 4.218ff.

Peirce: CP 5.71 Fn P1 p 49 Cross-Ref:††

†P1 This gives an idea of the second degree of degenerate Thirdness. Those of you who have read Professor Royce's Supplementary Essay [in *The World and the Individual*, vol. 1, p. 505, n. 1] will have remarked that he avoids this result, which does not suit his philosophy, by not allowing his map to be continuous. But to exclude continuity is to exclude what is best and most living in Hegel -- from the alternative "a" version.

Peirce: CP 5.71 Fn 1 p 50

†1 See vol. 7.

Peirce: CP 5.73 Fn 2 p 50

†2 Cf. vol 2, bk. II, ch. 2 and ch. 3.

Peirce: CP 5.75 Fn 1 p 52

†1 See "Aesthetics in Washington," in *A Memorial of Horatio Greenough*, by Henry T. Tuckerman, p. 82 (1853).

Peirce: CP 5.77 Fn P1 Para 1/3 p 52 Cross-Ref:††

†P1 Grant me that the three categories of Firstness, Secondness, and Thirdness, or Quality, Reaction, and Representation, have in truth the enormous

importance for thought that I attribute to them, and it would seem that no division of theories of metaphysics could surpass in importance a division based upon the consideration of what ones of the three categories each of different metaphysical systems have fully admitted as real constituents of nature.

Peirce: CP 5.77 Fn P1 Para 2/3 p 52 Cross-Ref: ††

It is, at any rate, a hypothesis easy to try; and the exact logic of hypothesis allots great weight to that consideration. There will be then these seven possible classes:

i. Nihilism, so-called, and idealistic sensualism.

ii. The doctrine of [Wincenty] Lutoslawski and his unpronounceable master [Mickiewicz].

iii. Hegelianism of all shades.

ii iii. Cartesianism of all kinds, Leibnizianism, Spinozism, and the metaphysics of the physicists of today.

i iii. Berkeleyanism.

i ii. Ordinary Nominalism.

ii iii. The metaphysics that recognizes all the categories. It ought to be subdivided, but I shall not stop to consider its subdivisions. It embraces Kantism, Reid's Philosophy, and the Platonic philosophy of which Aristotelianism is a special development.

Peirce: CP 5.77 Fn P1 Para 3/3 p 53 Cross-Ref: ††

A great variety of thinkers call themselves Aristotelians, even the Hegelians, on the strength of special agreements. No modern philosophy, or very little, has any real right to the title. I should call myself an Aristotelian of the scholastic wing, approaching Scotism, but going much further in the direction of scholastic realism. -- From the beginning of "Lecture IV."

Peirce: CP 5.82 Fn 1 p 54

†1 82-87 are from the "a" version; 88-93 follow 81 after an unpublished section which is a duplication of most of 82-87, 2.283f. and 3.423f.

Peirce: CP 5.84 Fn 1 p 58

†1 See 3.641, 4.51f, 4.85.

Peirce: CP 5.85 Fn 2 p 58

†2 See 3.63, 3.421f, 3.468ff.

Peirce: CP 5.85 Fn 3 p 58

†3 *Logik*, §3, 1; see also 2.19-20, 2.151ff.

Peirce: CP 5.93 Fn 1 p 64

†1 In the manuscript, what is here published follows shortly after the note to 77 in Lecture III.

Peirce: CP 5.102 Fn 1 p 67

†1 Cf. 151ff.

Peirce: CP 5.102 Fn 2 p 67

- †2 Cf. 2.367.
- Peirce: CP 5.103 Fn 3 p 67
†3 See 3.547f.
- Peirce: CP 5.106 Fn 1 p 69
†1 See 119.
- Peirce: CP 5.108 Fn 1 p 70
†1 See 2.186f.
- Peirce: CP 5.108 Fn 2 p 70
†2 Cf. 34ff, 440.
- Peirce: CP 5.110 Fn 3 p 70
†3 See 2.39ff.
- Peirce: CP 5.111 Fn 1 p 71
†1 *Introduction to Ethics* by T.S. Jouffroy, trans. by William H. Channing.
- Peirce: CP 5.111 Fn 2 p 71
†2 Dr. James Walker, president of Harvard University, and professor of moral and intellectual philosophy.
- Peirce: CP 5.111 Fn 3 p 71
†3 Probably *The Elements of Morality, including Polity*.
- Peirce: CP 5.112 Fn 1 p 72
†1 Cf. 1.333.
- Peirce: CP 5.115 Fn 1 p 73
†1 Cf. 151ff, 568, 4.539f.
- Peirce: CP 5.118 Fn 1 p 75
†1 §9 of ch. 2, bk. III, vol. 1 follows here in the ms., but apparently was not read.
- Peirce: CP 5.119 Fn 1 p 76
†1 Cf. vol. 6, bk. I, A.
- Peirce: CP 5.120 Fn 1 p 77
†1 The third and final draught; cf. vol. 1, bk. IV.
- Peirce: CP 5.120 Fn 2 p 77
†2 Cf. vol. 1, bk. II, ch. 2, §5.
- Peirce: CP 5.120 Fn 3 p 77
†3 See 61, and 1.126ff.
- Peirce: CP 5.125 Fn 1 p 79
†1 *Théorie mathématique des effets du jeu de billard*, G.G. Coriolis, Paris (1835).
- Peirce: CP 5.126 Fn 1 p 80
†1 See 1.247f, 4.239ff.
- Peirce: CP 5.128 Fn 1 p 81

†1 See *Oeuvres de Descartes*, t. III, lettre 183, A. et P. Tannery, Paris (1897-1910).

Peirce: CP 5.129 Fn 1 p 82
†1 Cf. 1.573ff, 2.196f.

Peirce: CP 5.130 Fn 2 p 82
†2 See 2.186f.

Peirce: CP 5.138 Fn 1 p 87
†1 See 2.317n, 2.393.

Peirce: CP 5.144 Fn 1 p 89
†1 Vol. 2, bk. III, ch. 2, Part III.

Peirce: CP 5.144 Fn 1 p 90
†1 Chapter 25, bk. II.

Peirce: CP 5.147 Fn 1 p 92
†1 See 4.571.

Peirce: CP 5.149 Fn 2 p 92
†2 See his *Logic*, bk. II, ch. 4, §4.

Peirce: CP 5.151 Fn 1 p 94
†1 Cf. 2.367f.

Peirce: CP 5.151 Fn 2 p 94
†2 Cf. 2.440.

Peirce: CP 5.153 Fn 1 p 95
†1 See 448n, 4.539.

Peirce: CP 5.153 Fn 1 p 96
†1 Cf. 2.287n.

Peirce: CP 5.154 Fn 2 p 96
†2 Cf. 2.324, 2.357.

Peirce: CP 5.155 Fn 3 p 96
†3 Cf. 3.532, where a bar is to be inserted over the second 1.

Peirce: CP 5.156 Fn 1 p 97
†1 See 3.532, where the above is interpreted as an instance of subalternation.

Peirce: CP 5.157 Fn 1 p 98
†1 Cf. 3.562B.

Peirce: CP 5.160 Fn 1 p 99
†1 See 2.152ff.

Peirce: CP 5.160 Fn 2 p 99
†2 Cf. 2.654ff.

Peirce: CP 5.161 Fn 3 p 99
†3 Cf. 2.100ff; 2.266ff; 2.619ff.

Peirce: CP 5.162 Fn 1 p 100

- †1 See vol. 4, bk. II, for a detailed study of diagrams.
- Peirce: CP 5.162 Fn 2 p 100
†2 See 3.363f; 3.559, 4.233.
- Peirce: CP 5.163 Fn 3 p 100
†3 See 579, 2.442ff, 4.505f, 4.565f.
- Peirce: CP 5.164 Fn 1 p 101
†1 See vol. 4, bk. II, ch. 2.
- Peirce: CP 5.167 Fn 1 p 102
†1 Cf. vol. 2, bk. III, B.
- Peirce: CP 5.168 Fn 1 p 103
†1 See *Die Schule der Chemie*, Julius A. Stöckhardt, Part I, §6.
- Peirce: CP 5.169 Fn 1 p 104
†1 See *Lettres sur la théorie des probabilités*, 3^{me} lettre.
- Peirce: CP 5.171 Fn 1 p 105
†1 Cf. 1.118, 2.623ff, 2.753f.
- Peirce: CP 5.172 Fn 1 p 107
†1 See 6.307ff.
- Peirce: CP 5.175 Fn 1 p 108
†1 See 166.
- Peirce: CP 5.176 Fn 2 p 108
†2 See e.g., *Kritik der Reinen Vernunft*, A7, B10, 11.
- Peirce: CP 5.177 Fn 1 p 109
†1 *Ibid*, A 656, B 684.
- Peirce: CP 5.178 Fn 2 p 109
†2 Cf. 4.427.
- Peirce: CP 5.178 Fn 1 p 110
†1 See 4.353.
- Peirce: CP 5.178 Fn 2 p 110
†2 See his *Neues Organon*, Bd. I., S. 111ff.
- Peirce: CP 5.178 Fn 3 p 110
†3 *Vorlesungen über die Algebra der Logik (Exakte Logik)*, Bd. III, 12.
- Peirce: CP 5.178 Fn 4 p 110
†4 See vol. 3, No. VII.
- Peirce: CP 5.180 Fn 1 p 112
†1 Peirce was scheduled to deliver six lectures; he seems, however, to have given all seven.
- Peirce: CP 5.181 Fn 2 p 112
†2 See *de Anima*, bk. III, ch. 8.

Peirce: CP 5.181 Fn 3 p 112

†3 See *The Principles of Human Knowledge*, §13.

Peirce: CP 5.187 Fn 1 p 117

†1 Ch. XXI.

Peirce: CP 5.192 Fn 1 p 119

†1 Cf. 280ff.

Peirce: CP 5.194 Fn 1 p 121

†1 See 3.63.

Peirce: CP 5.198 Fn 1 p 123

†1 See *Cours de philosophie positive*, 28^{me} leçon.

Peirce: CP 5.199 Fn 2 p 123

†2 See 170.

Peirce: CP 5.201 Fn 1 p 124

†1 There is a record of the fifth of the Lowell Lectures, "The Doctrine of Multitude, Infinity and Continuity," being delivered on December 7, 1903. It does not seem possible, due to the discrepancy of dates, that this is the lecture meant, but no other has been uncovered. See, however, vol. 4, bk. I, No. VI.

Peirce: CP 5.207 Fn 1 p 127

†1 See 400ff.

Peirce: CP 5.207 Fn 1 p 128

†1 See e.g., 3.217ff.

Peirce: CP 5.207 Fn 2 p 128

†2 See Kirchhoff's *Vorlesungen ü. math. Physik*, Bd. I, Vorrede. Leipzig (1874-6).

Peirce: CP 5 Book 2 Question 1 Fn 1 p 135

†1 *Journal of Speculative Philosophy*, vol. 2, pp. 103-114 (1868); intended as Essay IV of the "Search for a Method," 1893.

Peirce: CP 5.213 Fn P1 p 135 Cross-Ref:††

†P1 The word *intuitus* first occurs as a technical term in St. Anselm's *Monologium*. [*Monologium*, LXVI; Cf. Prantl, III, S. 332, 746n.] He wished to distinguish between our knowledge of God and our knowledge of finite things (and in the next world, of God, also); and thinking of the saying of St. Paul, *Videmus nunc per speculum in ænigmate: tunc autem facie ad faciem*, [LXX], he called the former *speculation* and the latter *intuition*. This use of "speculation" did not take root, because that word already had another exact and widely different meaning. In the middle ages, the term "intuitive cognition" had two principal senses; 1st, as opposed to abstractive cognition, it meant the knowledge of the present as present, and this is its meaning in Anselm; but 2d, as no intuitive cognition was allowed to be determined by a previous cognition, it came to be used as the opposite of discursive cognition (see Scotus, *In sentent.*, lib. 2, dist. 3, qu. 9), and this is nearly the sense in which I employ it. This is also nearly the sense in which Kant uses it, the former distinction being expressed by his *sensuous* and *non-sensuous*. (See *Werke*, herausg. Rosenkranz, Thl. 2, S. 713, 31, 41, 100, u.s.w.) An enumeration of six meanings of

intuition may be found in Hamilton's *Reid*, p. 759.

Peirce: CP 5.215 Fn 1 p 137

†1 See Prantl, II, 73ff.

Peirce: CP 5.215 Fn P1 p 137 Cross-Ref:††

†P1 The proposition of Berengarius is contained in the following quotation from his *De Sacra Cæna*: "*Maximi plane cordis est, per omnia ad dialecticam confugere, quia confugere ad eam ad rationem est confugere, quo qui non confugit, cum secundum rationem sit factus ad imaginem dei, suum honorem reliquit, nec potest renovari de die in diem ad imaginem dei.*" The most striking characteristic of medieval reasoning, in general, is the perpetual resort to authority. When Fredigisus and others wish to prove that darkness is a thing, although they have evidently derived the opinion from nominalistic-Platonistic meditations, they argue the matter thus: "God called the darkness, night;" then, certainly, it is a thing, for otherwise before it had a name, there would have been nothing, not even a fiction to name. [See Prantl, II, 19f.] Abelard [*Ouvrages*, p. 179] thinks it worth while to cite Boëthius, when he says that space has three dimensions, and when he says that an individual cannot be in two places at once. The author of *De Generibus et Speciebus* [*ibid.*, p. 517], a work of a superior order, in arguing against a Platonic doctrine, says that if whatever is universal is eternal, the *form* and matter of Socrates, being severally universal, are both eternal, and that, therefore, Socrates was not created by God, but only put together, "*quod quantum a vero deviet, palam est.*" The authority is the final court of appeal. The same author, where in one place he doubts a statement of Boëthius [*ibid.*, p. 535f], finds it necessary to assign a special reason why in this case it is not absurd to do so. *Exceptio probat regulam in casibus non exceptis*. Recognized authorities were certainly sometimes disputed in the twelfth century; their mutual contradictions insured that; and the authority of philosophers was regarded as inferior to that of theologians. Still, it would be impossible to find a passage where the authority of Aristotle is directly denied upon any logical question. "*Sunt et multi errores eius,*" says John of Salisbury [Metalogicon, Lib. IV, cap. XXVIII], "*qui in scripturis tam ethnicis, quam fidelibus poterunt inveniri; verum in logica parem habuisse non legitur.*" "*Sed nihil adversus Aristotelem,*" says Abelard, and in another place, "*Sed si Aristotelem Peripateticorum principem culpare possumus, quam amplius in hac arte recepimus?*" The idea of going without an authority, or of subordinating authority to reason, does not occur to him.

Peirce: CP 5.219 Fn 1 p 139

†1 *An Essay Towards a New Theory of Vision*, 1709.

Peirce: CP 5.223 Fn P1 p 142 Cross-Ref:††

†P1 *Proceedings of the American Academy*, May 14, 1867. [1.549]

Peirce: CP 5.223 Fn P2 Para 1/3 p 142 Cross-Ref:††

†P2 The above theory of space and time does not conflict with that of Kant so much as it appears to do. They are in fact the solutions of different questions. Kant, it is true, makes space and time intuitions, or rather forms of intuition, but it is not essential to his theory that intuition should mean more than "individual representation." The apprehension of space and time results, according to him, from a mental *process* -- the "Synthesis der Apprehension in der Anschauung." (See *Critik d. reinen Vernunft*. Ed. 1781, pp. 98 *et seq.*) My theory is merely an account of this synthesis.

Peirce: CP 5.223 Fn P2 Para 2/3 p 142 Cross-Ref:††

The gist of Kant's Transcendental Æsthetic is contained in two principles. First, that universal and necessary propositions are not given in experience. Second, that universal and necessary facts are determined by the conditions of experience in general. By a universal proposition is meant merely, one which asserts something of *all* of a sphere -- not necessarily one which all men believe. By a necessary proposition, is meant one which asserts what it does, not merely of the actual condition of things, but of every possible state of things; it is not meant that the proposition is one which we cannot help believing. Experience, in Kant's first principle, cannot be used for a product of the objective understanding, but must be taken for the first impressions of sense with consciousness conjoined and worked up by the imagination into images, together with all which is logically deducible therefrom. In this sense, it may be admitted that universal and necessary propositions are not given in experience. But, in that case, neither are any inductive conclusions which might be drawn from experience, given in it. In fact, it is the peculiar function of induction to produce universal and necessary propositions. Kant points out, indeed, that the universality and necessity of scientific inductions are but the analogues of philosophic universality and necessity; and this is true, in so far as it is never allowable to accept a scientific conclusion without a certain indefinite drawback. But this is owing to the insufficiency in the number of the instances; and whenever instances may be had in as large numbers as we please, *ad infinitum*, a truly universal and necessary proposition is inferable. As for Kant's second principle, that the truth of universal and necessary propositions is dependent upon the conditions of the general experience, it is no more nor less than the principle of Induction. I go to a fair and draw from the "grab-bag" twelve packages. Upon opening them, I find that every one contains a red ball. Here is a universal fact. It depends, then, on the condition of the experience. What is the condition of the experience? It is solely that the balls are the contents of packages drawn from that bag, that is, the only thing which determined the experience, was the drawing from the bag. I infer, then, according to the principle of Kant, that what is drawn from the bag will contain a red ball. This is induction. Apply induction not to any limited experience but to all human experience and you have the Kantian philosophy, so far as it is correctly developed.

Peirce: CP 5.223 Fn P2 Para 3/3 p 143 Cross-Ref:††

Kant's successors, however, have not been content with his doctrine. Nor ought they to have been. For, there is this third principle: "Absolutely universal propositions must be analytic." For whatever is absolutely universal is devoid of all content or determination, for all determination is by negation. The problem, therefore, is not how universal propositions can be synthetical, but how universal propositions appearing to be synthetical can be evolved by thought alone from the purely indeterminate.

Peirce: CP 5.227 Fn P1 p 144 Cross-Ref:††

†P1 *Werke*, vii. (2), 11.

Peirce: CP 5.257 Fn 1 p 152

†1 Cf. *A Treatise Concerning Human Knowledge*, §§1-6.

Peirce: CP 5.262 Fn P1 p 153 Cross-Ref:††

†P1 This argument, however, only covers a part of the question. It does not go to show that there is no cognition undetermined except by another like it.

Peirce: CP 5.264 Fn 1 p 156

†1 *Journal of Speculative Philosophy*, vol. 2, pp. 140-157 (1868); intended as Essay V of the "Search for a Method," 1893.

Peirce: CP 5.269 Fn 1 p 159

†1 Cf. 2.466.

Peirce: CP 5.269 Fn 2 p 159

†2 Cf. 2.470.

Peirce: CP 5.270 Fn 1 p 160

†1 Cf. 2.508ff.

Peirce: CP 5.275 Fn 1 p 163

†1 Cf. 2.623f.

Peirce: CP 5.275 Fn 2 p 163

†2 Cf. his *Analytica Priora*, Bk. III, ch. 23.

Peirce: CP 5.276 Fn P1 Para 1/11 p 164 Cross-Ref:††

†P1 Several persons versed in logic have objected that I have here quite misapplied the term *hypothesis*, and that what I so designate is an argument from *analogy*. It is a sufficient reply to say that the example of the cipher has been given as an apt illustration of hypothesis by Descartes (Rule 10 *Oeuvres choisies*: Paris, 1865, page 334), by Leibniz (*Nouv. Ess.*, lib. 4, ch. 12, §13, Ed. Erdmann, p. 383 *b*), and (as I learn from D. Stewart: *Works*, vol. 3, pp. 305 *et seq.*) by Gravesande, Boscovich, Hartley, and G.L. Le Sage. The term Hypothesis has been used in the following senses: 1. For the theme or proposition forming the subject of discourse. 2. For an assumption. Aristotle divides *theses* or propositions adopted without any reason into definitions and hypotheses. The latter are propositions stating the existence of something. Thus the geometer says, "Let there be a triangle." 3. For a condition in a general sense. We are said to seek other things than happiness {ex hypotheseös}, conditionally. The best republic is the ideally perfect, the second the best on earth, the third the best {ex hypotheseös}, under the circumstances. Freedom is the {hypothesis} or condition of democracy. 4. For the antecedent of a hypothetical proposition. 5. For an oratorical question which assumes facts. 6. In the *Synopsis* of Psellus, for the reference of a subject to the things it denotes. 7. Most commonly in modern times, for the conclusion of an argument from consequence and consequent to antecedent. This is my use of the term. 8. For such a conclusion when too weak to be a theory accepted into the body of a science. [Cf. 2.511n, 2.707.]

Peirce: CP 5.276 Fn P1 Para 2/11 p 164 Cross-Ref:††

I give a few authorities to support the seventh use: *Chauvin. -- Lexicon Rationale*, 1st Ed. -- "Hypothesis est propositio, quæ assumitur ad probandum aliam veritatem incognitam. Requirunt multi, ut hæc hypothesis vera esse cognoscatur, etiam antequam appareat, an alia ex ea deduci possint. Verum aiunt alii, hoc unum desiderari, ut hypothesis pro vera admittatur, quod nempe ex hac talia deducitur, quæ respondent phænomenis, et satisfaciunt omnibus difficultatibus, quæ hac parte in re, et in iis quæ de ea apparent, occurrebant."

Peirce: CP 5.276 Fn P1 Para 3/11 p 164 Cross-Ref:††

Newton. -- "Hactenus phænomena coelorum et maris nostri per vim gravitatis exposui, sed causam gravitatis nondum assignavi . . . Rationem vero harum gravitatis

proprietas ex phænomenis nondum potui deducere, et hypotheses non fingo. Quicquid enim ex phænomenis non deducitur, *hypothesis* vocanda est . . . In hac Philosophiâ Propositiones deducuntur ex phænomenis, et redduntur generales per inductionem." *Principia. Ad fin.*

Peirce: CP 5.276 Fn P1 Para 4/11 p 165 Cross-Ref:††

Sir Wm. Hamilton. -- "*Hypotheses*, that is, propositions which are assumed with probability, in order to explain or prove something else which cannot otherwise be explained or proved." -- *Lectures on Logic* (Am. Ed.), p. 188.

Peirce: CP 5.276 Fn P1 Para 5/11 p 165 Cross-Ref:††

"The name of *hypothesis* is more emphatically given to provisory suppositions, which serve to explain the phenomena in so far as observed, but which are only asserted to be true, if ultimately confirmed by a complete induction." -- *Ibid.*, p. 364.

Peirce: CP 5.276 Fn P1 Para 6/11 p 165 Cross-Ref:††

"When a phenomenon is presented which can be explained by no principle afforded through experience, we feel discontented and uneasy; and there arises an effort to discover some cause which may, at least provisionally, account for the outstanding phenomenon; and this cause is finally recognized as valid and true, if, through it, the given phenomenon is found to obtain a full and perfect explanation. The judgment in which a phenomenon is referred to such a problematic cause, is called a *Hypothesis*." -- *Ibid.*, pp. 449, 450. See also *Lectures on Metaphysics*, p. 117.

Peirce: CP 5.276 Fn P1 Para 7/11 p 165 Cross-Ref:††

J.S. Mill. -- "An hypothesis is any supposition which we make (either without actual evidence, or on evidence avowedly insufficient), in order to endeavor to deduce from it conclusions in accordance with facts which are known to be real; under the idea that if the conclusions to which the hypothesis leads are known truths, the hypothesis itself either must be, or at least is likely to be true." -- *Logic* (6th Ed.), vol. 2, p. 8. [Book III, ch. XIV, §4.]

Peirce: CP 5.276 Fn P1 Para 8/11 p 165 Cross-Ref:††

Kant. -- "*If all the consequents of a cognition are true, the cognition itself is true.* . . . It is allowable, therefore, to conclude from consequent to a reason, but without being able to determine this reason. From the complexus of all consequents alone can we conclude the truth of a determinate reason . . . The difficulty with this *positive* and *direct* mode of inference (*modus ponens*) is that the totality of the consequents cannot be apodeictically recognized, and that we are therefore led by this mode of inference only to a probable and *hypothetically* true cognition (*Hypotheses*)." -- *Logik* by Jäsche; *Werke*, Ed. Rosenk. and Sch., vol. 3, p. 221.

Peirce: CP 5.276 Fn P1 Para 9/11 p 165 Cross-Ref:††

"A hypothesis is the judgment of the truth of a reason on account of the sufficiency of the consequents." -- *Ibid.*, p. 262.

Peirce: CP 5.276 Fn P1 Para 10/11 p 165 Cross-Ref:††

Herbart. -- "We can make hypotheses, thence deduce consequents, and afterwards see whether the latter accord with experience. Such suppositions are termed hypotheses." -- Einleitung; *Werke*, vol. 1, p. 53.

Peirce: CP 5.276 Fn P1 Para 11/11 p 165 Cross-Ref:††

Beneke. -- "Affirmative inferences from consequent to antecedent, or hypotheses." -- *System der Logik*, vol. 2, p. 103.

There would be no difficulty in greatly multiplying these citations.

Peirce: CP 5.277 Fn 1 p 165

†1 See J.S. Mill, *Logic*, bk. II, ch. 3, § 3.

Peirce: CP 5.277 Fn 1 p 166

†1 See 2.513.

Peirce: CP 5.283 Fn 1 p 169

†1 See 233f.

Peirce: CP 5.288 Fn P1 p 172 Cross-Ref:††

†P1 A judgment concerning a minimum of information, for the theory of which see my paper on Comprehension and Extension [2.409ff].

Peirce: CP 5.289 Fn P2 p 172 Cross-Ref:††

†P2 Observe that I say *in itself*. I am not so wild as to deny that my sensation of red today is like my sensation of red yesterday. I only say that the similarity can **consist** only in the physiological force behind consciousness -- which leads me to say, I recognize this feeling the same as the former one, and so does not consist in a community of sensation. [Cf. 1.313, 1.383, 1.388; 3.419, 4.157.]

Peirce: CP 5.289 Fn 1 p 173

†1 See 504n.

Peirce: CP 5.289 Fn P1 p 173 Cross-Ref:††

†P1 Accordingly, just as we say that a body is in motion, and not that motion is in a body we ought to say that we are in thought and not that thoughts are in us.

Peirce: CP 5.290 Fn P1 p 174 Cross-Ref:††

†P1 On quality, relation, and representation, see 1.553f.

Peirce: CP 5.292 Fn 1 p 176

†1 Cf. 2.643.

Peirce: CP 5.294 Fn 1 p 178

†1 Cf. 1.550ff; 3.7, 3.44.

Peirce: CP 5.297 Fn 1 p 179

†1 Cf. 372ff, 394ff, 1.351, 1.390ff, 2.711, 3.155ff.

Peirce: CP 5.299 Fn 1 p 180

†1 Cf. 3.93.

Peirce: CP 5.299 Fn 2 p 180

†2 Berkeley, *Principles of Human Knowledge*, §10 of the Introduction.

Peirce: CP 5.300 Fn 1 p 181

†1 Cf. his *Treatise of Human Nature*, Pt. I, § 3 and Pt. III, § 5.

Peirce: CP 5.300 Fn P1 p 181 Cross-Ref:††

†P1 No person whose native tongue is English will need to be informed that contemplation is essentially (1) protracted, (2) voluntary, and (3) an action, and that it is never used for that which is set forth to the mind in this act. A foreigner can

convince himself of this by the proper study of English writers. Thus, Locke (*Essay concerning Human Understanding*, Book II, chap. 19, § 1) says, "*If* it [an idea] be held there [in view] long under attentive consideration, 'tis *Contemplation*; and again (*ibid.*, Book II, chap. 10, § 1) "keeping the *Idea* which is brought into it [the mind] for some time actually in view, which is called *Contemplation*." This term is therefore unfitted to translate *Anschauung*; for this latter does not imply an act which is necessarily protracted or voluntary, and denotes most usually a mental presentation, sometimes a faculty, less often the reception of an impression in the mind, and seldom, if ever, an action. To the translation of *Anschauung* by intuition, there is, at least, no such insufferable objection. Etymologically, the two words precisely correspond. The original philosophical meaning of intuition was a cognition of the present manifold in that character; and it is now commonly used, as a modern writer says, "to include all the products of the perceptive (external or internal) and imaginative faculties; every act of consciousness, in short, of which the immediate object is an *individual*, thing, act, or state of mind, presented under the condition of distinct existence in space and time." Finally, we have the authority of Kant's own example for translating his *Anschauung* by *Intuitus*; and indeed this is the common usage of Germans writing Latin. Moreover, *intuitiv* frequently replaces *anschauend* or *anschaulich*. If this constitutes a misunderstanding of Kant, it is one which is shared by himself and nearly all his countrymen. [See an anonymous comment on this note in the *Journal of Speculative Philosophy*, vol. II,

p. 191.]

Peirce: CP 5.302 Fn 1 p 182

†1 See 238ff.

Peirce: CP 5.311 Fn P1 p 186 Cross-Ref:††

†P1 By an ideal, I mean the limit which the possible cannot attain.

Peirce: CP 5.311 Fn 1 p 187

†1 Cf. 354f, 2.654f.

Peirce: CP 5.312 Fn P1 p 187 Cross-Ref:††

†P1 Eadem natura est, quæ in existentia per gradum singularitatis est determinata, et in intellectu, hoc est ut habet relationem ad intellectum ut cognitum ad cognoscens, est indeterminata. -- Quaest. Subtillissimæ, lib. 7, qu. 18.

Peirce: CP 5.312 Fn P1 p 188 Cross-Ref:††

†P1 See his argument *Summa logices*, part. 1, cap. 16.

Peirce: CP 5.313 Fn 1 p 188

†1 Cf. 6.270.

Peirce: CP 5.318 Fn 1 p 190

†1 *Journal of Speculative Philosophy*, vol. 2, pp. 193-208 (1868); with corrections of 1893; intended as Essay VI of the "Search for a Method," 1893.

Peirce: CP 5.318 Fn 2 p 190

†2 See 295ff.

Peirce: CP 5.318 Fn 3 p 190

†3 Originally "are."

Peirce: CP 5.318 Fn 1 p 191

†1 See 254, 265.

Peirce: CP 5.319 Fn 2 p 191

†2 Cf. 2.186ff.

Peirce: CP 5.320 Fn P1 p 191 Cross-Ref:††

†P1 The word *suppositio* is one of the useful technical terms of the middle ages which was condemned by the purists of the *renaissance* as incorrect. The early logicians made a distinction between *significatio* and *suppositio*. [Cf. Prantl, II, 286ff; III, 51f.] *Significatio* is defined as "rei per vocem secundum placitum representatio." [*Ibid.*, footnote 199.] It is a mere affair of lexicography, and depends on a special convention (*secundum placitum*), and not on a general principle. *Suppositio* belongs, not directly to the *vox*, but to the *vox* as having this or that *significatio*. "Unde significatio prior est suppositione et differunt in hoc, quia significatio est vocis, suppositio vrto est termini jam compositi ex voce et significatione." [*Ibid.*, footnote 201.] The various *suppositiones* which may belong to one word with one *significatio* are the different senses in which the word may be taken, according to the general principles of the language or of logic. Thus, the word *table* has different *significationes* in the expressions "table of logarithms" and "writing-table"; but the word *man* has one and the same *significatio*, and only different *suppositiones*, in the following sentences: "A man is an animal," "a butcher is a man," "man cooks his food," "man appeared upon the earth at such a date," &c. Some later writers have endeavored to make "*acceptio*" do service for "*suppositio*"; but it seems to me better, now that scientific terminology is no longer forbidden, to revive *supposition*. I should add that as the principles of logic and language for the different uses of the different parts of speech are different, *supposition* must be restricted to the acceptance of a *substantive*. The term *copulatio* was used for the acceptance of an adjective or verb.

Peirce: CP 5.320 Fn 1 p 192

†1 See 311.

Peirce: CP 5.322 Fn P1 p 193 Cross-Ref:††

†P1 "If any one will by ordinary syllogism prove that because every man is an animal, therefore every head of a man is a head of an animal, I shall be ready to -- set him another question." -- De Morgan: "On the Syllogism No. IV. and on the Logic of Relations." [*Transactions, Cambridge Philosophical Society*, X, pt. II, p. 337. Cf. *Principia Mathematica*, *37.62.]

Peirce: CP 5.323 Fn 1 p 194

†1 See 311.

Peirce: CP 5.324 Fn 2 p 194

†2 Originally "and."

Peirce: CP 5.326 Fn 1 p 195

†1 See 2.480f, 2.500ff.

Peirce: CP 5.327 Fn 2 p 195

†2 Not italicized in the original.

Peirce: CP 5.327 Fn P1 p 195 Cross-Ref:††

†P1 That is, in the Kantian sense. -- 1893.

Peirce: CP 5.327 Fn 3 p 195

†3 See 263.

Peirce: CP 5.328 Fn 1 p 196

†1 *An Essay Concerning Human Understanding*, Bk. IV, ch. xvii, §4.

Peirce: CP 5.328 Fn 2 p 196

†2 *E.g.* J.S. Mill; see his *System of Logic*, Bk. II, ch. iii.

Peirce: CP 5.328 Fn 3 p 196

†3 Cf. 2.614.

Peirce: CP 5.328 Fn P1 p 197 Cross-Ref:††

†P1 Mr. Mill thinks the syllogism is merely a formula for recalling forgotten facts. Whether he means to deny, what all logicians since Kant have held, that the syllogism serves to render confused thoughts distinct, or whether he does not know that this is the usual doctrine, does not appear.

Peirce: CP 5.330 Fn 1 p 199

†1 Added, 1893.

Peirce: CP 5.330 Fn 2 p 199

†2 Added, 1893.

Peirce: CP 5.330 Fn 3 p 199

†3 The phrase "that . . . known" was originally, "that everything will be known at some time some number of years into the future."

Peirce: CP 5.330 Fn P1 p 199 Cross-Ref:††

†P1 The difference between the two statements is like that between "Every man is the son of some woman," and "Some woman is the mother of every man." -- 1893.

Peirce: CP 5.331 Fn P1 p 200 Cross-Ref:††

†P1 So far as there is any validity in this conception. -- 1893.

Peirce: CP 5.332 Fn P1 p 201 Cross-Ref:††

†P1 "So zeigt sich jener Schlusssatz dadurch als falsch, obgleich für sich dessen Prämissen und ebenso dessen Consequenz ganz richtig sind." -- Hegel's *Werke*, vol. v., p. 124. [Berlin, 1841.]

Peirce: CP 5.333 Fn 1 p 204

†1 "is . . . undoubted" originally "is dependent on the."

Peirce: CP 5.333 Fn P1 p 204 Cross-Ref:††

†P1 Again the distinction is analogous to that between "Every man is the son of some woman or other," and "Some one woman is the mother of all men." -- 1893.

Peirce: CP 5.335 Fn 1 p 205

†1 Cf. 2.646, 4.121ff, 4.219ff, 6.174ff.

Peirce: CP 5.337 Fn P1 p 206 Cross-Ref:††

†P1 The usage of ordinary language has no relevancy in the matter.

Peirce: CP 5.339 Fn 1 p 209

†1 Cf. 403.

Peirce: CP 5.339 Fn P1 Para 1/2 p 210 Cross-Ref:††

†P1 This seems to me to be the main difficulty of freedom and fate. But the question is overlaid with many others. The Necessitarians seem now to maintain less that every physical event is completely determined by physical causes (which seems to me {in 1869 seemed -- 1893} irrefragable) than that every act of will is determined by the strongest motive. This has never been proved. Its advocates seem to think that it follows from universal causation, but why need the cause of an act lie within the consciousness at all? If I act from a reason at all, I act voluntarily; but which of two reasons shall appear strongest to me on a particular occasion may be owing to what I have eaten for dinner. Unless there is a perfect regularity as to what is the strongest motive with me, to say that I act from the strongest motive is mere tautology. If there is no calculating how a man will act except by taking into account external facts, the character of his motives does not determine how he acts. Mill and others have, therefore, not shown that a man always acts from the strongest motive. Hobbes [*Leviathan*, ch. VI] maintained that a man always acts from a reflection upon what will please him most. This is a very crude opinion. Men are not always thinking of themselves.

Peirce: CP 5.339 Fn P1 Para 2/2 p 210 Cross-Ref:††

Self-control seems to be the capacity for rising to an extended view of a practical subject instead of seeing only temporary urgency. This is the only freedom of which man has any reason to be proud; and it is because love of what is good for all on the whole, which is the widest possible consideration, is the essence of Christianity, that it is said that the service of Christ is perfect freedom.

Peirce: CP 5.340 Fn 1 p 210

†1 Cf. 2.618.

Peirce: CP 5.340 Fn P1 p 212 Cross-Ref:††

†P1 This is the principle which was most usually made the basis of the resolution of the *Insolubilia*. See, for example, Pauli Veneti, *Sophismata Aurea*. Soph. 50. The authority of Aristotle is claimed for this mode of solution. *Sophist. Elench.*, cap. 25. The principal objection which was made to this mode of solution, viz., that the principle that every proposition implies its own truth, cannot be proved, I believe that I have removed. The only arguments against the truth of this principle were based on the imperfect doctrines of *modales* and *obligationes*. Other methods of solution suppose that a part of a proposition cannot denote the whole proposition, or that no intellection is a formal cognition of itself. A solution of this sort will be found in Occam's *Summa Totius Logices*, 3d part of 3d part, cap. 38. Such modern authors as think the solution "very easy" do not understand its difficulties. See Mansel's *Aldrich*, p. 145.

Peirce: CP 5.343 Fn 1 p 214

†1 "With" and "it" were originally transposed.

Peirce: CP 5.345 Fn P1 p 215 Cross-Ref:††

†P1 *Logic*, Book 3, chap. 3, sec. 1. [Cf. 2.761ff.]

Peirce: CP 5.345 Fn P2 Para 1/2 p 215 Cross-Ref:††

†P2 *Ibid.* Book 3, chap. 21, sec. 1. "I am convinced that any one accustomed to abstraction and analysis, who will fairly exert his faculties for the purpose, will, when his imagination has once learnt to entertain the notion, find no difficulty in

conceiving that in some one, for instance, of the many firmaments into which sidereal astronomy divides the universe, events may succeed one another at random, without any fixed law; nor can anything in our experience or mental nature constitute a sufficient, or indeed any, reason for believing that this is nowhere the case.

Peirce: CP 5.345 Fn P2 Para 2/2 p 215 Cross-Ref:††

Were we to suppose (what it is perfectly possible to imagine) that the present order of the universe were brought to an end, and that a chaos succeeded, in which there was no fixed succession of events, and the past gave no assurance of the future," etc.

Peirce: CP 5.345 Fn 1 p 216

†1 Cf. 2.683f, 2.745f, 6.400ff.

Peirce: CP 5.345 Fn P1 p 216 Cross-Ref:††

†P1 Boole (*Laws of Thought*, p. 370) has shown, in a very simple and elegant manner, that an *infinite* number of balls may have characters distributed in such a way, that from the characters of the balls already drawn, we could infer nothing in regard to that of the characters of the next one. The same is true of some arrangements of a finite number of balls, provided the inference takes place after a fixed number of drawings. But this does not invalidate the reasoning above, although it is an important fact without doubt.

Peirce: CP 5.348 Fn 1 p 217

†1 Cf. 2.690ff.

Peirce: CP 5.352 Fn 1 p 219

†1 Cf. 405ff.

Peirce: CP 5.354 Fn 1 p 221

†1 Cf. 2.654f.

Peirce: CP 5.358 Fn 1 p 223 Cross-Ref:††

†1 *Popular Science Monthly*, pp. 1-15, vol. 12 (1877), the first of six papers of a series entitled "Illustrations of the Logic of Science"; with corrections and notes from several revised versions, one of which was intended as chapter 5 of the "Grand Logic" of 1893 and another of which was intended as Essay VII of the "Search for a Method" of 1893. The second paper is no. V of this book, the third, fourth and sixth occur as chapters 6, 7 and 5 respectively of vol. 2, bk. III, while the fifth paper serves as chapter 1 of bk. II, vol. 6. About 1903 the following introduction was attached to this and the following paper: "The two chapters composing this Essay ['My Plea for Pragmatism'] were first published, without any title for the whole [they appeared with a title] in the *Popular Science Monthly* for November 1877 and January 1878. A French version by the author (the second having in fact been first written in French on board a steamer in September 1877) appeared in the *Revue Philosophique*, vols. 6 and 7. They received as little attention as they laid claim to; but some years later the potent pen of Professor James brought their chief thesis to the attention of the philosophic world (pressing it, indeed, further than the tether of their author would reach, who continues to acknowledge, not indeed the Existence, but yet the Reality, of the Absolute, nearly as it has been set forth, for example, by Royce in his *The World and the Individual*, a work not free from faults of logic, yet valid in the main). The doctrine of this pair of chapters had already for some years been known among friends of the writer by the name he had proposed for it, which was 'Pragmatism.'"

- Peirce: CP 5.359 Fn 1 p 224
†1 "On" and "either" originally transposed.
- Peirce: CP 5.360 Fn 2 p 224
†2 See his *Opus Majus*, pars VI.
- Peirce: CP 5.360 Fn 3 p 224
†3 Originally "also."
- Peirce: CP 5.361 Fn P1 p 224 Cross-Ref:††
†P1 Cf. J. Aubrey's *Brief Lives* (Oxford ed. 1898), I, 299.
- Peirce: CP 5.361 Fn 4 p 224
†4 "as . . . said" added c. 1910.
- Peirce: CP 5.362 Fn 5 p 224
†5 Not in the original.
- Peirce: CP 5.362 Fn P2 p 224 Cross-Ref:††
†P2 Not quite so, but as nearly so as can be told in a few words.
- Peirce: CP 5.362 Fn 1 p 225
†1 "and . . . curve" added c. 1910.
- Peirce: CP 5.362 Fn 2 p 225
†2 "but perhaps," originally "and."
- Peirce: CP 5.362 Fn P1 p 225 Cross-Ref:††
†P1 I am ashamed at being obliged to confess that this volume contains a very false and foolish remark about Kepler. When I wrote it, I had never studied the original book as I have since. It is now my deliberate opinion that it is the most marvellous piece of inductive reasoning I have been able to find. -- 1893. [Peirce partially rectifies this error c. 1910 by deleting the expression "in . . . us." See also 1.72ff, 2.96f.]
- Peirce: CP 5.363 Fn 3 p 225
†3 Not in original.
- Peirce: CP 5.363 Fn 4 p 225
†4 Not in original.
- Peirce: CP 5.363 Fn 5 p 225
†5 Originally 'by.'
- Peirce: CP 5.364 Fn P1 p 226 Cross-Ref:††
†P1 What he did, a most instructive illustration of the logic of science, will be described in another chapter [where?]; and we now know what was authoritatively denied when I first suggested it, that he took a hint from Malthus' book on Population. -- 1903.
- Peirce: CP 5.364 Fn 1 p 226
†1 "eight . . . work" inserted c. 1910.
- Peirce: CP 5.364 Fn 2 p 226
†2 Not in original.

Peirce: CP 5.364 Fn 3 p 226

†3 Not in original.

Peirce: CP 5.364 Fn 4 p 226

†4 Not in original.

Peirce: CP 5.365 Fn P2 p 226 Cross-Ref:††

†P2 I.e., be dominated by such a habit as generally to give. -- 1903.

Peirce: CP 5.365 Fn 5 p 226

†5 "facts . . . the" not in original.

Peirce: CP 5.365 Fn 1 p 227

†1 Originally "the conclusion."

Peirce: CP 5.365 Fn 2 p 227

†2 "A . . . be," originally "A is B is."

Peirce: CP 5.366 Fn 3 p 227

†3 The portion within the parentheses was inserted c. 1910.

Peirce: CP 5.366 Fn P1 p 227 Cross-Ref:††

†P1 Let us not, however, be cocksure that natural selection is the only factor of evolution; and until this momentous proposition has been much better proved than as yet it has been, let it not blind us to the force [of] very sound reasoning. -- 1903.

Peirce: CP 5.368 Fn 1 p 228

†1 Originally "which."

Peirce: CP 5.369 Fn 2 p 228

†2 Originally "whether."

Peirce: CP 5.369 Fn 1 p 229

†1 "is . . . follow," originally "follows."

Peirce: CP 5.369 Fn 2 p 229

†2 "such a" originally "that."

Peirce: CP 5.369 Fn 3 p 229

†3 "by" originally at the end of the sentence.

Peirce: CP 5.371 Fn P1 Para 1/2 p 230 Cross-Ref:††

†P1 Let us recall the nature of a sign and ask ourselves how we can know that a feeling of any sort is a sign that we have a habit implanted within us.

Peirce: CP 5.371 Fn P1 Para 2/2 p 230 Cross-Ref:††

We can understand one habit by likening it to another habit. But to understand what any habit is, there must be some habit of which we are directly conscious in its generality. That is to say, we must have a certain generality in our direct consciousness. Bishop Berkeley and a great many clear thinkers laugh at the idea of our being able to imagine a triangle that is neither equilateral, isosceles, nor scalene. They seem to think the object of imagination must be precisely determinate in every respect. But it seems certain that something general we must imagine. I do not intend, in this book, to go into questions of psychology. It is not necessary for us to know in detail how our thinking is done, but only how it can be done. Still, I may as well say,

at once, that I think our direct consciousness covers a duration of time, although only an infinitely brief duration. At any rate, I can see no way of escaping the proposition that to attach any general significance to a sign and to know that we do attach a general significance to it, we must have a direct imagination of something not in all respects determinate. -- 1893. [Cf. 299f.]

Peirce: CP 5.372 Fn P2 p 230 Cross-Ref:††

†P2 In this, it is like any other stimulus. It is true that just as men may, for the sake of the pleasures of the table, like to be hungry and take means to make themselves so, although hunger always involves a desire to fill the stomach, so for the sake of the pleasures of inquiry, men may like to seek out doubts.

Yet, for all that, doubt essentially involves a struggle to escape it. -- 1893.

Peirce: CP 5.372 Fn P3 p 230 Cross-Ref:††

†P3 I am not speaking of secondary effects occasionally produced by the interference of other impulses. ["secondary . . . produced by" changed in 1910 to "accidental . . . superinduced by reflexion or . . ."]

Peirce: CP 5.373 Fn 1 p 231

†1 Originally "a."

Peirce: CP 5.373 Fn 2 p 231

†2 Not in the original.

Peirce: CP 5.373 Fn 3 p 231

†3 Not in the original.

Peirce: CP 5.373 Fn 4 p 231

†4 "Of this sort" originally followed "effect."

Peirce: CP 5.373 Fn 5 p 231

†5 Originally "action."

Peirce: CP 5.373 Fn 6 p 231

†6 Cf. 297, 394ff, 1.351, 1.390ff, 2.711, 3.155ff.

Peirce: CP 5.373 Fn P1 Para 1/2 p 231 Cross-Ref:††

†P1 Doubt, however, is not usually hesitancy about what is to be done then and there. It is anticipated hesitancy about what I shall do hereafter, or a feigned hesitancy about a fictitious state of things. It is the power of making believe we hesitate, together with the pregnant fact that the decision upon the merely make-believe dilemma goes toward forming a bona fide habit that will be operative in a real emergency. It is these two things in conjunction that constitute us intellectual beings.

Peirce: CP 5.373 Fn P1 Para 2/2 p 231 Cross-Ref:††

Every answer to a question that has any meaning is a decision as to how we would act under imagined circumstances, or how the world would be expected to react upon our senses. Thus, suppose I am told that if two straight lines in one plane are cut by a third making the sum of the internal angles on one side less than two right angles, then those lines if sufficiently produced will meet on the side on which the said sum is less than two right angles. This means to me that if I had two lines drawn on a plane and wished to find where they would meet, I could draw a third line cutting them and ascertaining on which side the sum of the two interval angles was less than

two right angles, and should lengthen the lines on that side. In like manner, all doubt is a state of hesitancy about an imagined state of things. -- 1893.

Peirce: CP 5.375 Fn 1 p 232

†1 Originally "any."

Peirce: CP 5.375 Fn P1 p 232 Cross-Ref:††

†P1 Unless, indeed, it leads us to modify our desires. -- 1903.

Peirce: CP 5.375 Fn P2 p 232 Cross-Ref:††

†P2 For truth is neither more nor less than that character of a proposition which consists in this, that belief in the proposition would, with sufficient experience and reflection, lead us to such conduct as would tend to satisfy the desires we should then have. To say that truth means more than this is to say that it has no meaning at all. -- 1903.

Peirce: CP 5.376 Fn 2 p 232

†2 "whether . . . setting" originally "or set."

Peirce: CP 5.376 Fn P3 p 232 Cross-Ref:††

†P3 So long as we cannot put our fingers on our erroneous opinions, they remain our opinions, still. It will be wholesome enough for us to make a general review of the causes of our beliefs; and the result will be that most of them have been taken upon trust and have been held since we were too young to discriminate the credible from the incredible. Such reflections may awaken real doubts about some of our positions. But in cases where no real doubt exists in our minds inquiry will be an idle farce, a mere whitewashing commission which were better let alone. This fault in philosophy was very widespread in those ages in which Disputations were the principal exercises in the universities; that is, from their rise in the thirteenth century down to the middle of the eighteenth, and even to this day in some Catholic institutions. But since those disputations went out of vogue, this philosophic disease is less virulent. -- 1893.

Peirce: CP 5.376 Fn P1 p 233 Cross-Ref:††

†P1 We have to acknowledge that doubts about them may spring up later; but we can find no propositions which are not subject to this contingency. We ought to construct our theories so as to provide for such discoveries; first, by making them rest on as great a variety of different considerations as possible, and second, by leaving room for the modifications which cannot be foreseen but which are pretty sure to prove needful. Some systems are much more open to this criticism than others. All those which repose heavily upon an "inconceivability of the opposite" have proved particularly fragile and short-lived. Those, however, which rest upon positive evidences and which avoid insisting upon the absolute precision of their dogmas are hard to destroy. -- 1893.

Peirce: CP 5.376 Fn P2 p 233 Cross-Ref:††

†P2 Except that of self-criticism. Insert here a section upon self-control and the analogy between Moral and Rational self-control. -- 1903.

Peirce: CP 5.377 Fn 1 p 234

†1 Originally "any."

Peirce: CP 5.377 Fn 2 p 234

†2 Originally "which."

Peirce: CP 5.377 Fn 3 p 234
†3 Originally "which."

Peirce: CP 5.377 Fn P1 p 234 Cross-Ref:††
†P1 Although it certainly may be that it will cause a line of conduct leading to pains that deeper reflection would have avoided. -- 1903.

Peirce: CP 5.380 Fn P1 Para 1/2 p 237 Cross-Ref:††
†P1 Unify them in the sense of Alexander Pope's *Universal Prayer*, and who is the individual whose conceit shall stand up and place his dictum against theirs? These faiths lay claim to divine authorship; and it is true that men have no more *invented* them, than the birds have invented their songs. It is a relapse toward the method of tenacity that segregates them and blinds the ecclesiastic to the value of anything but hatred. Every distinctive creed was as a historical fact invented to harm somebody. Still, the upshot has, on the whole, been success unparalleled. If slavery of opinion is natural and wholesome for men, then slaves they ought to remain.

Peirce: CP 5.380 Fn P1 Para 2/2 p 237 Cross-Ref:††
Every such system was first established by some individual legislator or prophet; and once established it grew of itself. But within this principle of growth lurk germs of decay. The power of individualism becomes extinct; the organization alone has life. Now, in the course of ages old questions pass out of mind: new questions become urgent. The sea advances or recedes; some horde which has always lived by conquest happens to make a conquest of consequence to the world at large. In one way or another, commerce is diverted from its ancient roads. Such change brings novel experiences and new ideas. Men begin to rebel at doings of the authorities to which in former times they would have submitted. Questions never before raised come up for decision; yet an individual legislator would no longer be listened to. Never has the instinct of rulers failed to see that the summoning of a council of the people was a measure fraught with peril to authority. Yet however they strive to avoid it, they in effect invoke public opinion, which is a momentous appeal to a new method of settling opinion. Disturbances occur; knots of men discuss the state of affairs; and a suspicion is kindled, which runs about like a train of gun powder, that the Dicta men have been reverencing, originated in caprice, in the pertinacity of some busybody, in the schemes of an ambitious man, or in other influences which are seen to edify a deliberative assembly. Men now begin to demand that, as the power which maintains the belief has become no longer capricious but public and methodical, so the propositions to be believed shall be determined in a public and methodical manner. -- 1893.

Peirce: CP 5.381 Fn 1 p 238
†1 "Nor . . . candour" originally "And their candor cannot."

Peirce: CP 5.381 Fn 2 p 238
†2 "thus giving" originally "and this gives."

Peirce: CP 5.382 Fn 3 p 238
†3 "A different" originally "And a."

Peirce: CP 5.382 Fn 4 p 238
†4 Originally "Which."

Peirce: CP 5.382 Fn P1 Para 1/6 p 239 Cross-Ref:††

†P1 Let us see in what manner a few of the greatest philosophers have undertaken to settle opinion, and what their success has been. Descartes, who would have a man begin by doubting everything, remarks that there is one thing he will find himself unable to doubt, and that is, that he does doubt; and when he reflects that he doubts, he can no longer doubt that he exists. Then, because he is all the while doubting whether there are any such things as shape and motion, Descartes thinks he must be persuaded that shape and motion do not belong to his nature, or anything else but consciousness. This is taking it for granted that nothing in his nature lies hidden beneath the surface. Next, Descartes asks the doubter to remark that he has the idea of a Being, in the highest degree intelligent, powerful, and perfect. Now a Being would not have these qualities unless he existed necessarily and eternally. By existing necessarily he means existing by virtue of the existence of the idea. Consequently, all doubt as to the existence of this Being must cease. This plainly supposes that belief is to be fixed by what men find in their minds. He is reasoning like this: I find it written in the volume of my mind that there is something X, which is such a sort of thing that the moment it is written down it exists. Plainly, he is aiming at a kind of truth which saying so can make to be so. He gives two further proofs of God's existence. Descartes makes God easier to know than anything else; for whatever we think He is, He is. He fails to remark that this is precisely the definition of a *figment*. In particular, God cannot be a deceiver; whence it follows, that whatever we quite clearly and distinctly think to be true about any subject, must *be* true. Accordingly, if people will thoroughly discuss a subject, and quite clearly and distinctly make up their minds what they think about it, the desired settlement of the question will be reached. I may remark that the world has pretty thoroughly deliberated upon that theory and has quite distinctly come to the conclusion that it is utter nonsense; whence that judgment is indisputably right.

Peirce: CP 5.382 Fn P1 Para 2/6 p 239 Cross-Ref: ††

Many critics have told me that I misrepresent the *a priori* philosophers, when I represent them as adopting whatever opinion there seems to be a natural inclination to adopt. But nobody can say the above does not accurately define the position of Descartes, and upon what does he repose except natural ways of thinking? Perhaps I shall be told however, that since Kant, that vice has been cured. Kant's great boast is that he critically examines into our natural inclinations toward certain opinions. An opinion that something is *universally* true clearly goes further than experience can warrant. An opinion that something is *necessarily* true (that is, not merely is true in the existing state of things, but would be true in every state of things) equally goes further than experience will warrant. Those remarks had been made by Leibniz and admitted by Hume; and Kant reiterates them. Though they are propositions of a nominalistic cast, they can hardly be denied. I may add that whatever is held to be precisely true goes further than experience can possibly warrant. Accepting those criteria of the origin of ideas, Kant proceeds to reason as follows: Geometrical propositions are held to be universally true. Hence, they are not given by experience. Consequently, it must be owing to an inward necessity of man's nature that he sees everything in space. Ergo, the sum of the angles of a triangle will be equal to two right angles for all the objects of our vision. Just that, and nothing more, is Kant's line of thought. But the dry-rot of reason in the seminaries has gone to the point where such stuff is held to be admirable argumentation. I might go through the *Critic of the Pure Reason*, section by section, and show that the thought throughout is precisely of this character. He everywhere shows that ordinary objects, such as trees and gold-pieces, involve elements not contained in the first presentations of sense. But we cannot persuade ourselves to give up the reality of trees and gold-pieces. There is a

general inward insistence upon them, and that is the warrant for swallowing the entire bolus of general belief about them. This is merely accepting without question a belief as soon as it is shown to please a great many people very much. When he comes to the ideas of God, Freedom, and Immortality, he hesitates; because people who think only of bread and butter, pleasure and power, are indifferent to those ideas. He subjects these ideas to a different kind of examination, and finally admits them upon grounds which appear to the seminarists more or less suspicious, but which in the eyes of laboratorists are infinitely stronger than the grounds upon which he has accepted space, time, and causality. Those last grounds amount to nothing but this, that what there is a very decided and general inclination to believe must be true. Had Kant merely said, I shall adopt for the present the belief that the three angles of a triangle are equal to two right angles because nobody but brother Lambert and some Italian has ever called it in question, his attitude would be well enough. But on the contrary, he and those who today represent his school distinctly maintain the proposition is *proved*, and the Lambertists *refuted*, by what comes merely to general disinclination to think with them.

Peirce: CP 5.382 Fn P1 Para 3/6 p 240 Cross-Ref: ††

As for Hegel, who led Germany for a generation, he recognizes clearly what he is about. He simply launches his boat into the current of thought and allows himself to be carried wherever the current leads. He himself calls his method *dialectic*, meaning that a frank discussion of the difficulties to which any opinion spontaneously gives rise will lead to modification after modification until a tenable position is attained. This is a distinct profession of faith in the method of inclinations.

Peirce: CP 5.382 Fn P1 Para 4/6 p 240 Cross-Ref: ††

Other philosophers appeal to "the test of inconceivability of the opposite," to "presuppositions" (by which they mean *Voraussetzungen*, properly translated, *postulates*), and other devices; but all these are but so many systems of rummaging the garret of the skull to find an enduring opinion about the Universe.

Peirce: CP 5.382 Fn P1 Para 5/6 p 241 Cross-Ref: ††

When we pass from the perusal of works upholding the method of authority to those of the philosophers, we not only find ourselves in a vastly higher intellectual atmosphere, but also in a clearer, freer, brighter, and more refreshing moral atmosphere. All this, however, is beside the one significant question of whether the method succeeds in fixing men's opinions. The projects of these authors are most persuasive. One dare swear they should succeed. But in point of fact, up to date they decidedly do not; and the outlook in this direction is most discouraging. The difficulty is that the opinions which today seem most unshakable are found tomorrow to be out of fashion. They are really far more changeable than they appear to a hasty reader to be; since the phrases made to dress out defunct opinions are worn at second hand by their successors.

Peirce: CP 5.382 Fn P1 Para 6/6 p 241 Cross-Ref: ††

We still talk of "cause and effect" although, in the mechanical world, the opinion that phrase was meant to express has been shelved long ago. We now know that the acceleration of a particle at any instant depends upon its position relative to other particles at that same instant; while the old idea was that the past affects the future, while the future does not affect the past. So the "law of demand and supply" has utterly different meanings with different economists. -- 1893.

Peirce: CP 5.382 Fn P1 p 241 Cross-Ref: ††

†P1 An acceptance whose real support has been the opinion that pleasure is the only ultimate good. But this opinion, or even the opinion that pleasure *per se* is any good at all, is only tenable so long as he who holds it remains without any distinct idea of what he means by "good." -- 1903.

Peirce: CP 5.383 Fn 1 p 241

†1 This sentence inserted c. 1910.

Peirce: CP 5.383 Fn 1 p 242

†1 "in . . . least" inserted c. 1910.

Peirce: CP 5.384 Fn 2 p 242

†2 Originally "caused."

Peirce: CP 5.384 Fn P1 p 242 Cross-Ref:††

†P1 But which, on the other hand, unceasingly tends to influence thought; or in other words, by something Real. -- 1903.

Peirce: CP 5.384 Fn P1 p 243 Cross-Ref:††

†P1 Or would be the same if inquiry were sufficiently persisted in. -- 1903.

Peirce: CP 5.384 Fn 1 p 243

†1 Originally "realities."

Peirce: CP 5.384 Fn 2 p 243

†2 Not in the original.

Peirce: CP 5.384 Fn 3 p 243

†3 Not in the original.

Peirce: CP 5.384 Fn 4 p 243

†4 Not in the original.

Peirce: CP 5.384 Fn 5 p 243

†5 Not in the original.

Peirce: CP 5.384 Fn 6 p 243

†6 Originally "realities."

Peirce: CP 5.384 Fn 7 p 243

†7 Cf. 351ff, 405ff.

Peirce: CP 5.384 Fn 8 p 243

†8 Originally "conform."

Peirce: CP 5.384 Fn 9 p 243

†9 Originally "realities."

Peirce: CP 5.384 Fn 10 p 243

†10 Originally "or."

Peirce: CP 5.384 Fn 11 p 243

†11 Originally "me."

Peirce: CP 5.384 Fn 12 p 243

†12 Originally "me."

Peirce: CP 5.384 Fn P1 p 244 Cross-Ref:††

†P1 Changes of opinion are brought about by events beyond human control. All mankind were so firmly of opinion that heavy bodies must fall faster than light ones, that any other view was scouted as absurd, eccentric, and probably insincere. Yet as soon as some of the absurd and eccentric men could succeed in inducing some of the adherents of common sense to look at their experiments -- no easy task -- it became apparent that nature would not follow human opinion, however unanimous. So there was nothing for it but human opinion must move to nature's position. That was a lesson in humility. A few men, the small band of laboratory men, began to see that they had to abandon the pride of an opinion assumed absolutely final in any respect, and to use all their endeavors to yield as unresistingly as possible to the overwhelming tide of experience, which must master them at last, and to listen to what nature seems to be telling us. The trial of this method of experience in natural science for these three centuries -- though bitterly detested by the majority of men -- encourages us to hope that we are approaching nearer and nearer to an opinion which is not destined to be broken down -- though we cannot expect ever quite to reach that ideal goal. -- 1893.

Peirce: CP 5.385 Fn 1 p 245

†1 Not in the original.

Peirce: CP 5.385 Fn 2 p 245

†2 Originally "that."

Peirce: CP 5.385 Fn 3 p 245

†3 Originally "has."

Peirce: CP 5.386 Fn 4 p 245

†4 "Some" deleted in 1893.

Peirce: CP 5.387 Fn 1 p 246

†1 Not in the original.

Peirce: CP 5.387 Fn 1 p 247

†1 "should . . . carry" originally "will carry."

Peirce: CP 5.387 Fn P1 p 247 Cross-Ref:††

†P1 Delete the remainder. -- marginal note, 1893, 1903.

Peirce: CP 5.388 Fn 1 p 248

†1 *Popular Science Monthly*, vol. 12, pp. 286-302 (1878); the second of the papers on the "Illustrations of the Logic of Science"; with corrections and notes from revised versions, one of which was intended as ch. 16 of the "Grand Logic" of 1893 and as Essay IX of the "Search for a Method" of 1893.

Peirce: CP 5.391 Fn P1 p 248 Cross-Ref:††

†P1 One of the treatises upon logic dating from *L'Art de Penser* of the Port Royalists down to very recent times. -- 1893.

Peirce: CP 5.391 Fn 1 p 249

†1 "thought he found" originally "professed to find."

Peirce: CP 5.392 Fn 2 p 249

†2 See 383.

Peirce: CP 5.392 Fn 1 p 250

†1 "two . . . anon" originally "formulas which cannot be denied without self-contradiction."

Peirce: CP 5.392 Fn P1 p 250 Cross-Ref:††

†P1 He was, however, above all, one of the minds that grow; while at first he was an extreme nominalist, like Hobbes, and dabbled in the nonsensical and impotent *Ars magna* of Raymond Lully, he subsequently embraced the law of continuity and other doctrines opposed to nominalism. I speak here of his earlier views. -- 1903.

Peirce: CP 5.392 Fn 2 p 250

†2 Originally "formalities."

Peirce: CP 5.392 Fn 1 p 251

†1 Cf. his "Meditationes de Cognitione," *Die Philosophische Schriften von Leibniz*, her. von C.I. Gerhardt, Bd. IV, S. 422-427; *Nouveaux Essais*, II, 29.

Peirce: CP 5.393 Fn P1 p 251 Cross-Ref:††

†P1 Delete this paragraph. -- 1903.

Peirce: CP 5.393 Fn 2 p 251

†2 Originally "which."

Peirce: CP 5.393 Fn 1 p 252

†1 "is apt to" not in the original.

Peirce: CP 5.393 Fn 2 p 252

†2 "This seems" not in the original, replacing a semicolon.

Peirce: CP 5.393 Fn 3 p 252

†3 Originally "lies."

Peirce: CP 5.394 Fn 4 p 252

†4 "part . . . essay" originally "of these papers."

Peirce: CP 5.394 Fn 5 p 252

†5 "a far," followed "of" in the original.

Peirce: CP 5.394 Fn 6 p 252

†6 Originally "We have there found"

Peirce: CP 5.394 Fn 1 p 253

†1 See 371ff.

Peirce: CP 5.398 Fn 1 p 255

†1 "among themselves" not in the original.

Peirce: CP 5.400 Fn 1 p 257

†1 No matter if contrary to all previous experience. -- marginal note, 1893.

Peirce: CP 5.400 Fn 2 p 257

†2 Not in the original.

Peirce: CP 5.400 Fn 3 p 257

†3 Not in the original.

Peirce: CP 5.401 Fn 4 p 257
†4 "meat and blood" originally "that."

Peirce: CP 5.401 Fn 1 p 258
†1 Originally "or."

Peirce: CP 5.401 Fn 2 p 258
†2 Cf. 541

Peirce: CP 5.402 Fn 3 p 258
†3 Originally "which."

Peirce: CP 5.402 Fn P1 p 258 Cross-Ref:††
†P1 Long addition refuting what comes next. -- 1903. [This seems to refer to the following, which was written ten years earlier on a different sheet.]

Peirce: CP 5.402 Fn P2 Para 1/3 p 258 Cross-Ref:††
†P2 Before we undertake to apply this rule, let us reflect a little upon what it implies. It has been said to be a sceptical and materialistic principle. But it is only an application of the sole principle of logic which was recommended by Jesus; "Ye may know them by their fruits," and it is very intimately allied with the ideas of the gospel. We must certainly guard ourselves against understanding this rule in too individualistic a sense. To say that man accomplishes nothing but that to which his endeavors are directed would be a cruel condemnation of the great bulk of mankind, who never have leisure to labor for anything but the necessities of life for themselves and their families. But, without directly striving for it, far less comprehending it, they perform all that civilization requires, and bring forth another generation to advance history another step. Their fruit is, therefore, collective; it is the achievement of the whole people. What is it, then, that the whole people is about, what is this civilization that is the outcome of history, but is never completed? We cannot expect to attain a complete conception of it; but we can see that it is a gradual process, that it involves a realization of ideas in man's consciousness and in his works, and that it takes place by virtue of man's capacity for learning, and by experience continually pouring upon him ideas he has not yet acquired. We may say that it is the process whereby man, with all his miserable littlenesses, becomes gradually more and more imbued with the Spirit of God, in which Nature and History are rife. We are also told to believe in a world to come; but the idea is itself too vague to contribute much to the perspicuity of ordinary ideas. It is a common observation that those who dwell continually upon their expectations are apt to become oblivious to the requirements of their actual station. The great principle of logic is self-surrender, which does not mean that self is to lay low for the sake of an ultimate triumph. It may turn out so; but that must not be the governing purpose.

Peirce: CP 5.402 Fn P2 Para 2/3 p 259 Cross-Ref:††
When we come to study the great principle of continuity [see vol. 6, Bk. I, B.] and see how all is fluid and every point directly partakes the being of every other, it will appear that individualism and falsity are one and the same. Meantime, we know that man is not whole as long as he is single, that he is essentially a possible member of society. Especially, one man's experience is nothing, if it stands alone. If he sees what others cannot, we call it hallucination. It is not "my" experience, but "our" experience that has to be thought of; and this "us" has indefinite possibilities.

Peirce: CP 5.402 Fn P2 Para 3/3 p 259 Cross-Ref:††

Neither must we understand the practical in any low and sordid sense. Individual action is a means and not our end. Individual pleasure is not our end; we are all putting our shoulders to the wheel for an end that none of us can catch more than a glimpse at -- that which the generations are working out. But we can see that the development of embodied ideas is what it will consist in. -- 1893.

Peirce: CP 5.402 Fn P3 Para 1/3 p 259 Cross-Ref: ††

†P3 Note that in these three lines one finds, "conceivably," "conceive," "conception," "conception," "conception." Now I find there are many people who detect the authorship of my unsigned screeds; and I doubt not that one of the marks of my style by which they do so is my inordinate reluctance to repeat a word. This employment five times over of derivatives of *concipere* must then have had a purpose. In point of fact it had two. One was to show that I was speaking of meaning in no other sense than that of *intellectual purport*. The other was to avoid all danger of being understood as attempting to explain a concept by percepts, images, schemata, or by anything but concepts. I did not, therefore, mean to say that acts, which are more strictly singular than anything, could constitute the purport, or adequate proper interpretation, of any symbol. I compared action to the finale of the symphony of thought, belief being a demi-cadence. Nobody conceives that the few bars at the end of a musical movement are the *purpose* of the movement. They may be called its upshot. But the figure obviously would not bear detailed application. I only mention it to show that the suspicion I myself expressed (Baldwin's *Dictionary* Article, *Pragmatism*) [see 3] after a too hasty rereading of the forgotten magazine paper, that it expressed a stoic, that is, a nominalistic, materialistic, and utterly philistine state of thought, was quite mistaken.

Peirce: CP 5.402 Fn P3 Para 2/3 p 260 Cross-Ref: ††

No doubt, Pragmaticism [see 414] makes thought ultimately *apply* to action exclusively -- to *conceived* action. But between admitting that and either saying that it makes thought, in the sense of the purport of symbols, to consist in acts, or saying that the true ultimate purpose of thinking is action, there is much the same difference as there is between saying that the artist-painter's living art is applied to dabbing paint upon canvas, and saying that that art-life consists in dabbing paint, or that its ultimate aim is dabbing paint. Pragmaticism makes thinking to consist in the living inferential metaboly of symbols whose purport lies in conditional general resolutions to act. As for the ultimate purpose of thought, which must be the purpose of everything, it is beyond human comprehension; but according to the stage of approach which my thought has made to it -- with aid from many persons, among whom I may mention Royce (in his *World and Individual*), Schiller (in his *Riddles of the Sphinx*) as well, by the way, as the famous poet [Friedrich Schiller] (in his *Aesthetische Briefe*), Henry James the elder (in his *Substance and Shadow* and in his conversations), together with Swedenborg himself -- it is by the indefinite replication of self-control upon self-control that the *vir* is begotten, and by action, through thought, he grows an esthetic ideal, not for the behoof of his own poor noddle merely, but as the share which God permits him to have in the work of creation.

Peirce: CP 5.402 Fn P3 Para 3/3 p 260 Cross-Ref: ††

This ideal, by modifying the rules of self-control modifies action, and so experience too -- both the man's own and that of others, and this centrifugal movement thus rebounds in a new centripetal movement, and so on; and the whole is a bit of what has been going on, we may presume, for a time in comparison with which the sum of the geological ages is as the surface of an electron in comparison

with that of a planet. -- From "Consequences of Pragmaticism," 1906.

Peirce: CP 5.403 Fn 1 p 261
†1 But see 453, 457.

Peirce: CP 5.403 Fn 2 p 261
†2 Cf. 339.

Peirce: CP 5.404 Fn P1 p 264 Cross-Ref:††
†P1 Possibly the velocities also have to be taken into account.

Peirce: CP 5.404 Fn 1 p 265
†1 Kirchhoff's *Vorlesungen über math. Physik*, Bd. I, Vorrede.

Peirce: CP 5.406 Fn 1 p 266
†1 In 385.

Peirce: CP 5.406 Fn 2 p 266
†2 Originally "scientific."

Peirce: CP 5.406 Fn 3 p 266
†3 See 377f.

Peirce: CP 5.406 Fn 1 p 267
†1 The parenthetized phrase was not in the original.

Peirce: CP 5.406 Fn 2 p 267
†2 *Patrologica Latina*, vol. 178, p. 114 et seq., (1885).

Peirce: CP 5.406 Fn 3 p 267
†3 See 379f.

Peirce: CP 5.407 Fn 1 p 268
†1 "are . . . hope" originally "are fully persuaded."

Peirce: CP 5.407 Fn 2 p 268
†2 Originally "every."

Peirce: CP 5.407 Fn 3 p 268
†3 "apply it" originally "can be applied."

Peirce: CP 5.407 Fn 4 p 268
†4 "are . . . move" originally "will move."

Peirce: CP 5.407 Fn 5 p 268
†5 Originally "law."

Peirce: CP 5.407 Fn P1 p 268 Cross-Ref:††
†P1 Fate means merely that which is sure to come true, and can nohow be avoided. It is a superstition to suppose that a certain sort of events are ever fated, and it is another to suppose that the word fate can never be freed from its superstitious taint. We are all fated to die.

Peirce: CP 5.410 Fn 1 p 271
†1 See vol. 2, bk. III, ch. 6.

Peirce: CP 5.411 Fn 1 p 272

†1 *The Monist*, vol. 15, pp. 161-181 (1905). The first of three articles. The second article follows this while the third appears as ch. 6 of bk. II, vol. 4.

Peirce: CP 5.411 Fn 1 p 273

†1 *Reflections Suggested by the New Theory of Matter*; Presidential Address, British Association for the Advancement of Science, August 17, 1904.

Peirce: CP 5.413 Fn 1 p 274

†1 Cf. Appendix, §4; vol. 2, bk. II, ch. 1.

Peirce: CP 5.414 Fn 1 p 276

†1 See his *Pragmatism*, p. 47.

Peirce: CP 5.414 Fn 2 p 276

†2 In *Personal Idealism*, ed. by H. Sturt, p. 63 (1902).

Peirce: CP 5.414 Fn P1 p 277 Cross-Ref:††

†P1 To show how recent the general use of the word "pragmatism" is, the writer may mention that, to the best of his belief, he never used it in copy for the press before today, except by particular request, in *Baldwin's Dictionary*. [See 1-4.] Toward the end of 1890, when this part of the *Century Dictionary* appeared, he did not deem that the word had sufficient status to appear in that work. [But see 13n.] But he has used it continually in philosophical conversation since, perhaps, the mid-seventies.

Peirce: CP 5.415 Fn 1 p 278

†1 See vol. 6, bk. I, B.

Peirce: CP 5.416 Fn P1 p 279 Cross-Ref:††

†P1 It is necessary to say that "belief" is throughout used merely as the name of the contrary to doubt, without regard to grades of certainty nor to the nature of the proposition held for true, i.e., "believed."

Peirce: CP 5.419 Fn 1 p 280

†1 Cf. 440; 1.606f.

Peirce: CP 5.422 Fn 1 p 282

†1 See 402.

Peirce: CP 5.424 Fn P1 p 283 Cross-Ref:††

†P1 The writer, like most English logicians, invariably uses the word *proposition* not as the Germans define their equivalent, *Satz*, as the language-expression of a judgment (*Urtheil*), but as that which is related to any assertion, whether mental and self-addressed or outwardly expressed, just as any possibility is related to its actualisation. The difficulty of the, at best, difficult problem of the essential nature of a Proposition has been increased, for the Germans, by their *Urtheil*, confounding, under one designation, the mental *assertion* with the *assertible* [cf. 2.315].

Peirce: CP 5.428 Fn 1 p 285

†1 Apparently a misprint for "expatiates."

Peirce: CP 5.429 Fn 1 p 286

†1 See 3, 402³.

Peirce: CP 5.429 Fn 1 p 287

†1 See 4.537.

Peirce: CP 5.430 Fn 2 p 287

†2 See Prantl, *Geschichte der Logik*, III, 91, Anm. 362.

Peirce: CP 5.431 Fn 1 p 288

†1 Cf. 1.213, 2.149.

Peirce: CP 5.431 Fn 2 p 288

†2 "The Foundations of Geometry," by Paul Carus, *The Monist*, XIII, p. 370.

Peirce: CP 5.434 Fn 1 p 289

†1 See 458ff.

Peirce: CP 5.434 Fn 2 p 289

†2 See 429, 436.

Peirce: CP 5.434 Fn 3 p 289

†3 See 475, 4.536ff.

Peirce: CP 5.435 Fn 1 p 290

†1 *Prior Analytics*, 24b, 28-30.

Peirce: CP 5.435 Fn 2 p 290

†2 See 3.203, 3.493.

Peirce: CP 5.436 Fn 1 p 291

†1 See vol. 6, bk. I.

Peirce: CP 5.436 Fn 2 p 291

†2 See letters to Judge Russell in vol. 9.

Peirce: CP 5.436 Fn 1 p 292

†1 Cf. 77ff.

Peirce: CP 5.438 Fn 1 p 293

†1 *The Monist*, vol. 15, pp. 481-499 (1905). The second article of the series of which the preceding paper is the first.

Peirce: CP 5.438 Fn P1 p 293 Cross-Ref:††

†P1 *Popular Science Monthly*, XII, p. 293; for January 1878. [See 402.] An introductory article opens the volume, in the number for November 1877. [See paper No. IV.]

Peirce: CP 5.440 Fn 1 p 294

†1 See 289.

Peirce: CP 5.440 Fn 2 p 294

†2 The third *published* paper was the "Prolegomena to an Apology for Pragmaticism," vol. 4, bk. II, ch. 6; but see 448n and §1, ch. 5 of bk. III.

Peirce: CP 5.440 Fn 3 p 294

†3 See 394.

Peirce: CP 5.441 Fn P1 p 295 Cross-Ref:††

†P1 But see the experiments of J. Jastrow and me "On Slight Differences of Sensation" in the *Memoirs of the National Academy of Sciences*, vol. III [1884, pp. 1-11; see vol. 8].

Peirce: CP 5.441 Fn 1 p 295
†1 See 2.182ff.

Peirce: CP 5.442 Fn 1 p 296
†1 Cf. 194, 263.

Peirce: CP 5.444 Fn P1 p 297 Cross-Ref:††
†P1 I wish I might hope, after finishing some more difficult work, to be able to resume this study and to go to the bottom of the subject, which needs the qualities of age and does not call upon the powers of youth. A great range of reading is necessary; for it is the belief men *betray* and not that which they *parade* which has to be studied. [No such study has been found.]

Peirce: CP 5.446 Fn 1 p 299
†1 On these terms see 1.191, 2.93, 2.229, 3.430, 4.9.

Peirce: CP 5.447 Fn P1 p 299 Cross-Ref:††
†P1 Hamilton and a few other logicians understood the subject of a universal proposition in the collective sense; but every person who is well-read in logic is familiar with many passages in which the leading logicians explain with an iteration that would be superfluous if all readers were intelligent, that such a subject is distributively not collectively general. A term denoting a collection is singular, and such a term is an "abstraction" or product of the operation of hypostatic abstraction as truly as is the name of the essence. "*Mankind*" is quite as much an abstraction and *ens rationis* as is "humanity." Indeed, every object of a conception is either a signate individual or some kind of indeterminate individual. Nouns in the plural are usually distributive and general; common nouns in the singular are usually indefinite.

Peirce: CP 5.447 Fn 2 p 299
†2 See 2.357.

Peirce: CP 5.447 Fn 3 p 299
†3 Cf. 2.357.

Peirce: CP 5.448 Fn 1 p 300
†1 Cf. 505f.

Peirce: CP 5.448 Fn P1 Para 1/6 p 300 Cross-Ref:††
†P1 These remarks require supplementation. Determination, in general, is not defined at all; and the attempt at defining the determination of a subject with respect to a character only covers (or seems only to cover) explicit propositional determination. The incidental remark [447] to the effect that words whose meaning should be determinate would leave "no latitude of interpretation" is more satisfactory, since the context makes it plain that there must be no such latitude either for the interpreter or for the utterer. The explicitness of the words would leave the utterer no room for explanations of his meaning. This definition has the advantage of being applicable to a command, to a purpose, to a medieval substantial form; in short to anything capable of indeterminacy. (That everything indeterminate is of the nature of a sign can be proved inductively by imagining and analyzing instances of the surdest description. Thus, the indetermination of an event which should happen by pure

chance without cause, *sua sponte*, as the Romans mythologically said, *spontanément* in French (as if what was done of one's own motion were sure to be irrational), does not belong to the event -- say, an explosion -- *per se*, or as explosion. Neither is it by virtue of any real relation: it is by virtue of a relation of reason. Now what is true by virtue of a relation of reason is representative, that is, is of the nature of a sign. A similar consideration applies to the indiscriminate shots and blows of a Kentucky free fight.) Even a future event can only be determinate in so far as it is a consequent. Now the concept of a consequent is a logical concept. It is derived from the concept of the conclusion of an argument. But an argument is a sign of the truth of its conclusion; its conclusion is the rational *interpretation* of the sign. This is in the spirit of the Kantian doctrine that metaphysical concepts are logical concepts applied somewhat differently from their logical application. The difference, however, is not really as great as Kant represents it to be, and as he was obliged to represent it to be, owing to his mistaking the logical and metaphysical correspondents in almost every case.

Peirce: CP 5.448 Fn P1 Para 2/6 p 301 Cross-Ref:††

Another advantage of this definition is that it saves us from the blunder of thinking that a sign is indeterminate simply because there is much to which it makes no reference; that, for example, to say, "C.S. Peirce wrote this article," is indeterminate because it does not say what the color of the ink used was, who made the ink, how old the father of the ink-maker [was] when his son was born, nor what the aspect of the planets was when that father was born. By making the definition turn upon the interpretation, all that is cut off. [Cf. 3.93.]

Peirce: CP 5.448 Fn P1 Para 3/6 p 301 Cross-Ref:††

At the same time, it is tolerably evident that the definition, as it stands, is not sufficiently explicit, and further, that at the present stage of our inquiry cannot be made altogether satisfactory. For what is the interpretation alluded to? To answer that convincingly would be either to establish or to refute the doctrine of pragmatism. Still some explanations may be made. Every sign has a single object, though this single object may be a single set or a single continuum of objects. No general description can identify an object. But the common sense of the interpreter of the sign will assure him that the object must be one of a limited collection of objects. Suppose, for example, two Englishmen to meet in a continental railway carriage. The total number of subjects of which there is any appreciable probability that one will speak to the other perhaps does not exceed a million; and each will have perhaps half that million not far below the surface of consciousness, so that each unit of it is ready to suggest itself. If one mentions Charles the Second, the other need not consider what possible Charles the Second is meant. It is no doubt the English Charles Second. Charles the Second of England was quite a different man on different days; and it might be said that without further specification the subject is not identified. But the two Englishmen have no purpose of splitting hairs in their talk; and the latitude of interpretation which constitutes the indeterminacy of a sign must be understood as a latitude which might affect the achievement of a purpose. For two signs whose meanings are for all possible purposes equivalent are absolutely equivalent. This, to be sure, is rank pragmatism; for a purpose is an affection of action.

Peirce: CP 5.448 Fn P1 Para 4/6 p 302 Cross-Ref:††

What has been said of subjects is as true of predicates. Suppose the chat of our pair of Englishmen had fallen upon the color of Charles II's hair. Now that colors are seen quite differently by different retinas is known. That the chromatic sense is much

more varied than it is positively known to be is quite likely. It is very unlikely that either of the travelers is trained to observe colors or is a master of their nomenclature. But if one says that Charles II had dark auburn hair, the other will understand him quite precisely enough for all their possible purposes; and it will be a determinate predication.

Peirce: CP 5.448 Fn P1 Para 5/6 p 302 Cross-Ref:††

The October remarks [i.e. those in the above paper] made the proper distinction between the two kinds of indeterminacy, viz.: indefiniteness and generality, of which the former consists in the sign's not sufficiently expressing itself to allow of an indubitable determinate interpretation, while the [latter] turns over to the interpreter the right to complete the determination as he please. It seems a strange thing, when one comes to ponder over it, that a sign should leave its interpreter to supply a part of its meaning; but the explanation of the phenomenon lies in the fact that the entire universe -- not merely the universe of existents, but all that wider universe, embracing the universe of existents as a part, the universe which we are all accustomed to refer to as "the truth" -- that all this universe is perfused with signs, if it is not composed exclusively of signs. Let us note this in passing as having a bearing upon the question of pragmatism. [Cf. 4.539.]

Peirce: CP 5.448 Fn P1 Para 6/6 p 302 Cross-Ref:††

The October remarks, with a view to brevity, omitted to mention that both indefiniteness and generality might primarily affect either the logical breadth or the logical depth of the sign to which it belongs. It now becomes pertinent to notice this. When we speak of the depth, or signification, of a sign we are resorting to hypostatic abstraction, that process whereby we regard a thought as a thing, make an interpretant sign the object of a sign. It has been a butt of ridicule since Molière's dying week, and the depth of a writer on philosophy can conveniently be sounded by his disposition to make fun of the basis of voluntary inhibition, which is the chief characteristic of mankind. For cautious thinkers will not be in haste to deride a kind of thinking that is evidently founded upon observation -- namely, upon observation of a sign. At any rate, whenever we speak of a predicate we are representing a thought as a thing, as a *substantia*, since the concepts of *substance* and *subject* are one, its concomitants only being different in the two cases. It is needful to remark this in the present connexion, because, were it not for hypostatic abstraction, there could be no generality of a predicate, since a sign which should make its interpreter its deputy to determine its signification at his pleasure would not signify anything, unless *nothing* be its significate. -- From "Basis of Pragmatism," 1906, following somewhat after 554.

Peirce: CP 5.449 Fn 1 p 302

†1 Cf. 1.549, 2.428.

Peirce: CP 5.449 Fn P1 p 303 Cross-Ref:††

†P1 But unfortunately it has not been in the writer's power to consult the *Oxford Dictionary* concerning these words; so that probably some of the statements in the text might be corrected with the aid of that work.

Peirce: CP 5.450 Fn P1 p 304 Cross-Ref:††

†P1 Thus returning to the writer's original nomenclature, in despite of *Monist* VII, [3.532] where an obviously defective argument was regarded as sufficient to determine a mere matter of terminology. But the Quality of propositions is there regarded from a point of view which seems extrinsic. I have not had time, however, to re-explore all the ramifications of this difficult question by the aid of existential

graphs, and the statement in the text about the last quantifier may need modification.
[See 4.552n.]

Peirce: CP 5.450 Fn 1 p 304
†1 See 2.324.

Peirce: CP 5.450 Fn 1 p 305
†1 3.527ff.

Peirce: CP 5.453 Fn 1 p 306
†1 See 403.

Peirce: CP 5.453 Fn 2 p 306
†2 See 408.

Peirce: CP 5.453 Fn 3 p 306
†3 See 517n.

Peirce: CP 5.454 Fn 1 p 307
†1 Cf. 526ff, 2.382ff, 3.527ff, 4.65ff, 4.573ff.

Peirce: CP 5.458 Fn 1 p 310
†1 Cf. 1.488ff.

Peirce: CP 5.459 Fn 1 p 311
†1 In 450.

Peirce: CP 5.459 Fn 1 p 312
†1 See 450.

Peirce: CP 5.462 Fn 1 p 313
†1 See 244ff.

Peirce: CP 5.464 Fn 1 p 317
†1 From "Pragmatism (Editor [3])" continuing 13.

Peirce: CP 5.466 Fn 1 p 318
†1 See 2.

Peirce: CP 5.467 Fn P1 Para 1/2 p 319 Cross-Ref: ††

†P1 It is in this phrase that I find my sole opportunity of calling attention to the fact that the ratio of frequency with which members of one denumeral collection are found among members of another denumeral, and therefore equal, collection depends upon the order of succession of the members of the latter collection. In order to afford matter for your rumination, my reader, I set down the first members of two endless series, each embracing all integers, and nothing else. The order of succession of the numbers of the first series is that of their occurrence in counting, or (what is the same thing) that of the multitudes they represent. The second series is derived by dividing each member of the first series, first, by 2, and then the final quotient by 3, as often as possible without remainder, and then remultiplying the ultimate quotient by 2 as many times as were the divisions by 3, and the product by 3 as many times as the divisions by 2, and setting down the ultimate product in that place in the second series which in the first series had been occupied by the number originally taken. The consequence will be that the number which, in either series, occupies the $(2^M \cdot 3^N \cdot Q)$ th place will in the other series occupy, and occupy none other than the $(2^N \cdot 3^M \cdot Q)$ th place.

3.Q)th place. Here are the first members:

1.2.3.4.5.6.7.8.9.10.11.12.13.14.15.16.17.18.19.20.21.22.23.24.25.26.27.28.29.30.31.
32.33.34.35.36.37.38.39.40. etc.

1.3.2.9.5.6.7.27.4.15.11.18.13.21.10.81.17.12.19.45.14.33.23.54.25.39.8.63.29.30.31.
243.22.51.35.36.37.57.26.135. etc.

Peirce: CP 5.467 Fn P1 Para 2/2 p 319 Cross-Ref:††

In the upper series every other number is even; in the lower, only every third. In the upper every third number is divisible by 3; in the lower every second is so. It is obvious that the principle involved must overthrow certain arguments about probabilities. Thus, a quarter of a century ago, a London professor put forth the argument (accompanied by the arrogant references then usual, but now disused from obvious motives, toward an American opponent who did not then think it worthwhile to reply, nor would now, but for the illustration of how the general principle is applicable to Probabilities), that because for every form of question to which the true answer is "Yes," another form of the same question is possible to which the true answer is "No," and conversely; therefore, there is an *a priori* probability equal to 1/2 that any given proposition is true. He forgot that, in order to make this out, he would have to show that one form of the question occurred just as often as the other, *in the course of experience*. Whether the fact be so or not is open to doubt. One circumstance that seems to point toward its falsity is that there is, in the English language, no word equivalent to the French "oui," -- as opposed to "si" -- as an answer to the negative form of the question. But, however the fact may be, certain it is that no practical underwriter or actuary would, or ought to, allow an argument based on a form of words to make a dollar's difference in the premium to be paid for a policy.

Peirce: CP 5.468 Fn 1 p 321

†1 *The Nation?*

Peirce: CP 5.469 Fn 2 p 321

†2 Cf. 1.288ff, 3.469ff.

Peirce: CP 5.469 Fn 1 p 322

†1 See 4.309.

Peirce: CP 5.471 Fn 1 p 323

†1 *Metalogicus* II, XX.

Peirce: CP 5.471 Fn 2 p 323

†2 See 2.394ff.

Peirce: CP 5.475 Fn 1 p 326

†1 Cf. 4.536, 4.572.

Peirce: CP 5.479 Fn P1 p 329 Cross-Ref:††

†P1 I must correct a false impression that the above might produce by declaring plainly that I am no conjurer, and am the reverse of adroit. But I have studied a few tricks, and have received minute instructions about others; and know

that many things that are supposed to depend upon difficult manipulations performed with a speed that in reality no human being could approach, really require no extraordinary deftness and are done without hurry. Good conjuring is based on keen practical psychology. The legerdemain, the mechanical devices, etc., merely furnish occasion for the real art, which tells just when the current of thought that, pursued, would lead to discovery, can be turned, and how.

Peirce: CP 5.486 Fn 1 p 333

†1 Cf. 1.341.

Peirce: CP 5.487 Fn P1 p 335 Cross-Ref:††

†P1 I well remember when I was a boy, and my brother Herbert, now our minister at Christiania, was scarce more than a child, one day, as the whole family were at table, some spirit from a "blazer," or "chafing-dish," dropped on the muslin dress of one of the ladies and was kindled; and how instantaneously he jumped up, and did the right thing, and how skillfully each motion was adapted to the purpose. I asked him afterward about it; and he told me that since Mrs. Longfellow's death, it was that he had often run over in imagination all the details of what ought to be done in such an emergency. It was a striking example of a real habit produced by exercises in the imagination. [Cf. 538.]

Peirce: CP 5.489 Fn 1 p 336

†1 *See Proceedings of the Aristotelian Society, 1905-6; Studies in Humanism*, ch. III, 1907.

Peirce: CP 5.490 Fn 1 p 338

†1 "On the Geographical Problem of the Four Colors," *American Journal of Mathematics*, vol. 2, p. 193 (1879).

Peirce: CP 5.490 Fn 1 p 339

†1 "Sixth Memoir on Quantics," *Philosophical Transactions of the Royal Society*, par. 230, v. CXLIX (1859).

Peirce: CP 5.490 Fn P1 p 339 Cross-Ref:††

†P1 This supposes the unbounded 3-dimensional space to have a peculiar shape. For if it had the simplest shape possible or what seems to me such, and what Listing assumed to be the real shape of space, every unbounded surface in it would separate it into two parts, and the unbounded line common to two surfaces would cut any third unbounded surface in an even number of points, since this line would pass alternately from one to the other of the two parts into which the first surface had cut the unbounded solid space. The peculiar shape of the solid space is that of "projective space."

Peirce: CP 5.490 Fn 1 p 340

†1 *Abhandlungen der Königlichen Gesellschaft der W. z. Göttingen*, Band X, §44 (1861-2).

Peirce: CP 5.492 Fn 1 p 342

†1 *Course of Lectures on the Physiology and Pathology of the Central Nervous System* (1860).

Peirce: CP 5.494 Fn 1 p 343

†1 *Pragmatism*, p. 47f.

Peirce: CP 5.494 Fn 2 p 343

†2 *Studies in Humanism*, pp. 7-12 (1906).

Peirce: CP 5.494 Fn 3 p 343

†3 See 405f.

Peirce: CP 5.494 Fn 1 p 344

†1 See *Electricité et Optique*, Introduction, Paris (1890).

Peirce: CP 5.494 Fn 1 p 345

†1 Cf. "What Pragmatism is like," *Popular Science Monthly*, p. 351, vol. 71 (1907).

Peirce: CP 5.497 Fn 1 p 346

†1 "The Basis of Pragmaticism" (short ms.), c. 1905.

Peirce: CP 5.497 Fn P1 p 346 Cross-Ref:††

†P1 I write in the form of a dialogue because it is in that form that my thoughts come to me; and I call my critic Jules in the endeavour to give his objections the solidity equal to those of a subtle Italian opponent of Pragmatism who writes under the *nom de guerre* of Giuliano il Sofista [Giuseppe Prezzolini].

Peirce: CP 5.498 Fn P1 p 347 Cross-Ref:††

†P1 This is a striking instance of the vague's emancipation from the principle of contradiction.

Peirce: CP 5.500 Fn P1 p 349 Cross-Ref:††

†P1 The generic substantive has been invented, as is the usual custom. The thing is called simply *vectis* by George Agricola and by J.A. Comenius.

Peirce: CP 5.501 Fn 1 p 349

†1 Cf. 1.312f.

Peirce: CP 5.501 Fn P1 p 350 Cross-Ref:††

†P1 Such is the circumlocution to which we are driven in consequence of the perversion of the meaning of "objective" characters.

Peirce: CP 5.502 Fn 1 p 351

†1 "Pragmaticism, Prag. [4]" c. 1905. The first page of the ms. is missing.

Peirce: CP 5.502 Fn 2 p 351

†2 See 413.

Peirce: CP 5.504 Fn P1 p 353 Cross-Ref:††

†P1 This was said in 1868, before declaring for pragmaticism, thus: "No present actual thought (which is mere feeling) has any meaning, any intellectual value; for this lies, not in what is actually thought, but in what the thought may be connected with in representation by subsequent thoughts; so that the meaning of a thought is altogether something virtual." [289]. This paper in fact expresses a kind of pragmatism not unlike that of Professor James.

Peirce: CP 5.504 Fn P1 p 354 Cross-Ref:††

†P1 How surprisingly, however, Locke grew in philosophical power during the composition of the *Essay concerning Humane Understanding!*

Peirce: CP 5.505 Fn 1 p 355

†1 *System der Philosophie*, S. IX (1897).

Peirce: CP 5.505 Fn P1 p 356 Cross-Ref:††

†P1 This is illustrated in our use of the phrase "a *certain* man," which means that the determination which is left *uncertain* to the reader or auditor is, nevertheless, or once was, *certain* either to the utterer or to some other person. [Cf. 3.94.]

Peirce: CP 5.505 Fn 1 p 356

†1 "A Memoir on the Theory of Mathematical Forms," *Philosophical Transactions of the Royal Society*, vol. 177, pp. 1-70 (1886). See 3.423f.

Peirce: CP 5.505 Fn 2 p 356

†2 See *Proceedings of the Royal Society*, vol. XLII, pp. 193-196 (1887).

Peirce: CP 5.505 Fn P2 p 356 Cross-Ref:††

†P2 Kempe's word is "single"; but I prefer "singular," from Latin *singularis*, not from *singularis*. His Section 127 throws further light on his meaning of "undistinguished."

Peirce: CP 5.506 Fn 1 p 357

†1 Though the writing is unmistakable this should be "determinate."

Peirce: CP 5.506 Fn 2 p 357

†2 See 4.552n, 4.553n².

Peirce: CP 5.506 Fn 1 p 358

†1 Where?

Peirce: CP 5.508 Fn 2 p 358

†2 See 449.

Peirce: CP 5.508 Fn 3 p 358

†3 *Philosophie analytique de l'histoire* (1896); *Histoire et solution des problèmes métaphysiques* (1901).

Peirce: CP 5.508 Fn 4 p 358

†4 *La matière brute et la matière vivante* (1887).

Peirce: CP 5.508 Fn 5 p 358

†5 *La liberté et le déterminisme* (1872).

Peirce: CP 5.508 Fn 6 p 358

†6 *Pluriverse*, (1920).

Peirce: CP 5.508 Fn 7 p 358

†7 "The Dilemma of Determinism" (1884), reprinted in *The Will to Believe and Other Essays* (1897).

Peirce: CP 5.508 Fn 8 p 358

†8 "The Superstition of Necessity," *The Monist*, vol. 3, pp. 362-379 (1893).

Peirce: CP 5.517 Fn P1 p 363 Cross-Ref:††

†P1 By a *categorical resolution* I mean a representation to oneself that one will behave in a certain general way in a certain expected contingency, this

representation being received with satisfaction, being rehearsed with pleasure, and perhaps exciting a special effort to learn it as a lesson. The purpose toward the accomplishment of which the action tends is taken for granted. By a *conditional resolution*, I mean a similar representation except that the purpose or end in view is only arbitrarily supposed to be aimed at and the circumstances under which the action would take place are not generally expected to arise. By a *conditional habit*, I mean a determination of a man's occult nature tending to cause him to act in a certain general way in case certain general circumstances should arise and in case he should be animated by a certain purpose.

Peirce: CP 5.517 Fn P2 p 363 Cross-Ref:††

†P2 Were it a question of brute force, there were no need of the perhaps. But being a logos-influence, the manifest would promise more efficiency than the occult.

Peirce: CP 5.518 Fn 1 p 364

†1 Peirce was so called by Paul Carus in *The Monist*, vol. 2, p. 561.

Peirce: CP 5.521 Fn 1 p 365

†1 On this term see 1.242n.

Peirce: CP 5.526 Fn P1 p 368 Cross-Ref:††

†P1 This is fixed in the writer's memory by the fact that it was begun after leaving Hoboken on a steamer and was finished a day or two before reaching Plymouth, nothing remaining to be done except to translate it into English. The *Comptes Rendus* of the Europäischer Gradmessung show that on September 29 he was absorbed in quite another discussion in Stuttgart.

Peirce: CP 5.526 Fn P2 p 368 Cross-Ref:††

†P2 It was hardly available for philosophists before June 1883; and was not put into shape even then, Cantor being long occupied, apparently, with his great theory of ordinals, which hardly concerns multitude.

Peirce: CP 5.526 Fn 1 p 368

†1 See, e.g., 395.

Peirce: CP 5.526 Fn P3 p 368 Cross-Ref:††

†P3 C.S. Peirce's paper *On the Logic of Number*, published in the spring of 1881 [vol. 3, no. VII] contained, though not in a perspicuous form, the leading results of Dedekind's classic of 1887.

Peirce: CP 5.527 Fn 2 p 368

†2 See also 454f.

Peirce: CP 5.527 Fn 1 p 369

†1 See 4.65ff, 4.552f, 4.579ff.

Peirce: CP 5.531 Fn 1 p 370

†1 *Kritik der reinen Vernunft*, A140ff, B179ff.

Peirce: CP 5.531 Fn 2 p 370

†2 *Ibid.*, A141; B180.

Peirce: CP 5.533 Fn 1 p 371

†1 See e.g. 1.191, 1.616ff.

- Peirce: CP 5.533 Fn 2 p 371
†2 *The International Journal of Ethics*, July, 1903. This essay is reprinted in his *Humanism, Philosophical Essays* (1903).
- Peirce: CP 5.534 Fn 1 p 373
†1 *Life of John Locke with Extracts from His Correspondence, etc.*, by Lord King, vol. 1, pp. 118-119 (1830).
- Peirce: CP 5.534 Fn 2 p 373
†2 See 4.171.
- Peirce: CP 5.534 Fn 3 p 373
†3 See 4.175.
- Peirce: CP 5.534 Fn 4 p 373
†4 See 3.43, 3.627f.
- Peirce: CP 5.534 Fn 5 p 373
†5 See 3.629f.
- Peirce: CP 5.537 Fn 1 p 375
†1 *Humanism, Philosophical Essays*, XVI (1903).
- Peirce: CP 5.537 Fn 2 p 375
†2 This is from Milton's *Comus*.
- Peirce: CP 5.538 Fn 1 p 376
†1 From "Reason's Rules," c. 1902.
- Peirce: CP 5.538 Fn 1 p 377
†1 Cf. 487n.
- Peirce: CP 5.539 Fn 1 p 378
†1 Cf. 1.332ff.
- Peirce: CP 5.539 Fn 2 p 378
†2 *The Works of Thomas Reid*, ed. by Sir W. Hamilton, Note A, §1; Note C.
- Peirce: CP 5.541 Fn 1 p 380
†1 Art. II, §2, "Ueber die Hypothesen, welche der Geometrie zu Grunde liegen," *Abh. d. König. G.d.W. zu Göttingen*, 13 Bd. (1866-7).
- Peirce: CP 5.541 Fn 1 p 381
†1 Cf. 401.
- Peirce: CP 5.542 Fn 1 p 382
†1 See Zeller's *Die Philosophie der Griechen*, II Th., 2 Abt., Anmerkung.
- Peirce: CP 5.543 Fn 1 p 384
†1 Cf. 2.315, and 546f.
- Peirce: CP 5.546 Fn 1 p 385
†1 From a fragment c. 1908.
- Peirce: CP 5.549 Fn 1 p 388
†1 From the "Basis of Pragmaticism," 1906, continuing 1.574.

- Peirce: CP 5.552 Fn 1 p 389
†1 See 1.376ff, 1.594ff.
- Peirce: CP 5.552 Fn 1 p 390
†1 Cf. *Studies in Humanism*, p. 83.
- Peirce: CP 5.553 Fn 2 p 390
†2 *Kritik der Reinen Vernunft*, A58, B82.
- Peirce: CP 5.553 Fn 3 p 390
†3 2.310ff.
- Peirce: CP 5.555 Fn 1 p 392
†1 Manuscript entitled: "Reflexions upon Pluralistic Pragmatism and upon Cenopythagorean Pragmaticism." c. 1906.
- Peirce: CP 5.563 Fn 1 p 393
†1 Bk. II, No. IV.
- Peirce: CP 5.563 Fn 2 p 393
†2 Bk. II, No. V.
- Peirce: CP 5.563 Fn 3 p 393
†3 See 375.
- Peirce: CP 5.563 Fn 4 p 393
†4 3.154ff.
- Peirce: CP 5.565 Fn 1 p 394
†1 From the article "Truth and Falsity and Error," *Dictionary of Philosophy and Psychology*, ed. J.M. Baldwin, pp. 718-20, vol. 2 (1901). Only the part of the article given here was written by Peirce.
- Peirce: CP 5.568 Fn 1 p 396
†1 Cf. 115ff.
- Peirce: CP 5.569 Fn 1 p 397
†1 Cf. 2.327.
- Peirce: CP 5.574 Fn 1 p 399
†1 Lecture 3 on "Detached Ideas on Vitally Important Topics" of 1898. Lecture 1 is in vol. 1, bk. IV, ch. 5; Lecture 2 is used in the preface to vol. 4.
- Peirce: CP 5.577 Fn 1 p 401
†1 Cf. e.g. his *Phantasms of the Living* (1886).
- Peirce: CP 5.577 Fn 2 p 401
†2 See "On the Secular Variation of the Moon's Mean Motion," J.C. Adams, *Philosophical Transactions*, Royal Society, Pt. I, pp. 397-406, vol. 143 (1853).
- Peirce: CP 5.579 Fn 1 p 402
†1 *Logic*, bk. II, ch. 4, §4.
- Peirce: CP 5.579 Fn 2 p 402
†2 See the Preface and book II of vol. 4.

- Peirce: CP 5.579 Fn 3 p 402
†3 *Novum Organon Renovatum* II, iv.
- Peirce: CP 5.579 Fn 4 p 402
†4 Cf. 163, 2.442ff, 4.505f, 4.565f.
- Peirce: CP 5.579 Fn 5 p 402
†5 See 2.56ff.
- Peirce: CP 5.579 Fn 6 p 402
†6 See e.g., 3.403L.
- Peirce: CP 5.579 Fn 1 p 403
†1 *Timaeus*, 33A.
- Peirce: CP 5.579 Fn 2 p 403
†2 Republic 526D.
- Peirce: CP 5.579 Fn 3 p 403
†3 *An. Post.* I, 1, 4.
- Peirce: CP 5.579 Fn 4 p 403
†4 See 2.735f.
- Peirce: CP 5.586 Fn 1 p 407
†1 Cf. 6.6.
- Peirce: CP 5.586 Fn 1 p 408
†1 See 591, 603; 1.118.
- Peirce: CP 5.589 Fn 1 p 411
†1 Cf. 2.513, 2.733.
- Peirce: CP 5.589 Fn 2 p 411
†2 See 2.707.
- Peirce: CP 5.590 Fn 1 p 413
†1 From the Eighth Lowell Lecture of 1903, entitled "How to Theorize."
- Peirce: CP 5.593 Fn 1 p 415
†1 *Meditations* III; *Method*, Pt. II; *Principles*, Pt. I, 30, 43, etc.
- Peirce: CP 5.593 Fn 2 p 415
†2 Leibniz's *Nouveaux Essais*, Bk. II, ch. 31; *Discours*, XXIV, XXV.
- Peirce: CP 5.594 Fn 1 p 416
†1 Cf. vol. 1, bk. IV, ch. 2, 4.
- Peirce: CP 5.596 Fn 2 p 416
†2 Horace, *Satires*, II, 7, 86.
- Peirce: CP 5.597 Fn 1 p 417
†1 Cf. 2.511n.
- Peirce: CP 5.597 Fn 2 p 417
†2 *Cours de philosophie positive*, 28^{me} leçon.

- Peirce: CP 5.599 Fn 1 p 419
†1 Cf. 2.662f.
- Peirce: CP 5.600 Fn 2 p 419
†2 See 1.122.
- Peirce: CP 5.601 Fn 3 p 419
†3 See, e.g., "The Economical Nature of Physical Inquiry," in the *Popular Scientific Lectures* (1895).
- Peirce: CP 5.601 Fn 4 p 419
†4 Chapter 1 of bk. II, vol. 6?
- Peirce: CP 5.604 Fn 1 p 422
†1 In 1869.
- Peirce: CP 5.605 Fn 1 p 423
†1 Baldwin's *Dictionary of Philosophy and Psychology*, vol. 1, p. 603, Macmillan & Co., N.Y. (1901), signed by C.S.P. and C.L.F. (Christine Ladd-Franklin).
- Peirce: CP 5.606 Fn 2 p 423
†2 2.773ff.
- Peirce: CP 5.606 Fn 1 p 424
†1 See 2.552ff.
- Peirce: CP 5.607 Fn 2 p 424
†2 *Ibid.*, vol. 2, pp. 464-5.
- Peirce: CP 5.608 Fn 1 p 425
†1 *Ibid.*, vol. 2, pp. 723-4.
- Peirce: CP 5.610 Fn 1 p 426
†1 *The Monist*, vol. 16, pp. 147-151 (1906).
- Peirce: CP 5.610 Fn 2 p 426
†2 "Some Philosophical Terms," *The Monist*, vol. 15, pp. 629-633, (1905).
- Peirce: CP 5.611 Fn P1 p 426 Cross-Ref:††
†P1 {empeiria}.
- Peirce: CP 5.611 Fn P1 p 427 Cross-Ref:††
†P1 {gnösis}.
- Peirce: CP 5.611 Fn 1 p 428
†1 See 4.597.
- Peirce: CP 5.611 Fn P1 p 428 Cross-Ref:††
†P1 *Humane* and *human* were one and the same word in Locke's day.
- Peirce: CP 5.611 Fn 2 p 428
†2 See 413ff, and vol. 2, bk. II, ch. 1.
- Peirce: CP 5.614 Fn 1 p 430
†1 The additional name Santiago, St. James in Spanish, was adopted by Peirce

about this time, apparently in honor of his life-long friend, William James.

Peirce: CP 6 Title-Page
COLLECTED PAPERS OF CHARLES SANDERS PEIRCE

EDITED BY
CHARLES HARTSHORNE
AND
PAUL WEISS
VOLUME VI
SCIENTIFIC METAPHYSICS

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Peirce: CP 6 Introduction p iii
INTRODUCTION

Peirce's punctuation and spelling have, wherever possible, been retained. Titles supplied by editors for papers previously published are marked with an *E*, while Peirce's titles for unpublished papers are marked with a *P*. Peirce's titles for previously published papers and the editors' titles for unpublished papers are not

marked. Remarks and additions by the editors are enclosed in light-face square brackets. The editors' footnotes are indicated by various typographical signs, while Peirce's are indicated by numbers. Paragraphs are numbered consecutively throughout the volume. At the top of each page the numbers signify the volume and the first paragraph of that page. All references in the indices are to the numbers of the paragraphs.

Peirce: CP 6 Introduction p iii

The department and the editors desire to express their gratitude to Miss M. Hurst and to Mrs. Charles Hartshorne for their assistance with the proofs, references, and editorial footnotes.

HARVARD UNIVERSITY

APRIL, 1935.

Peirce: CP 6 Ed. Note p v
EDITORIAL NOTE

With the present volume Peirce's philosophical system reaches its culmination in a "scientific" metaphysics, the study of "thirdness as thirdness" or "efficient reasonableness" (5.121).

Peirce: CP 6 Ed. Note p v

Metaphysics, as the third of the philosophic disciplines, has, according to Peirce, three branches--ontology, religion, and cosmology (see 1.192). As he viewed it, it presupposes logic, the topic of volumes II-IV, which in turn rests on ethics, esthetics and phenomenology, discussed in volume I. Though his architectonic prescribes a separate treatment for ontology and cosmology, he never actually separated them. The first book of the present volume, built around five articles first published in 1892-93, embraces both subjects. Of primary interest are its discussions of absolute chance, or tychism, and objective continuity, or synechism (the latter being viewed as a synthesis of the former with pragmatism, the topic of volume V). It is this portion of Peirce's philosophy that most interested his philosophical contemporaries, and which is most pertinent to current cosmological speculations. It relates directly to that type of naturalism which takes scientific laws to be real and immanent--and thus subject to change, and incapable of precise determination. The second book of the volume, devoted to religion or "psychical metaphysics," has rather tenuous connections with the rest of the system, offering, apart from scattered flashes of insight, views which have a sociological or biographical, rather than a fundamental systematic interest.

Peirce: CP 6 Ed. Note p v

For the general student the best approach to the present volume is through those passages in the previous volumes which are referred to in the preface, the most important being 1.487ff; after which the volume should be read in the order

of presentation.

BRYN MAWR COLLEGE
UNIVERSITY OF CHICAGO
APRIL, 1935.

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§1. THE BACKWARD STATE OF METAPHYSICS †1

1. Logic requires that the more abstract sciences should be developed earlier than the more concrete ones.†2 For the more concrete sciences require as fundamental principles the results of the more abstract sciences, while the latter only make use of the results of the former as data; and if one fact is wanting, some other will generally serve to support the same generalization.

Peirce: CP 6.2 Cross-Ref:††

2. But notwithstanding this, there is one highly abstract science which is in a deplorably backward condition. I mean Metaphysics. There is and can be no doubt that this immature condition of Metaphysics has very greatly hampered the progress of one of the two great branches of special science, I mean the Moral or Psychological Sciences. Most immediately has it checked the development of psychology; while the backward state of psychology has been a great disadvantage to all the other psychological sciences, such as linguistics, anthropology, social science, etc. To my mind it is equally clear that defective and bad metaphysics has been almost as injurious to the physical sciences, and is the real reason why all that depends upon the science of the constitution of matter, even physiology, is more or less rolling in the trough of the sea in rudderless fashion. The common opinion has been that Metaphysics is backward because it is intrinsically beyond the reach of human cognition. But that, I think I can clearly discern, is a complete mistake. Why should metaphysics be so difficult? Because it is abstract? But the abstracter a science is, the easier it is, both as a general rule of experience and as a corollary from logical principles. Mathematics, which is far more abstract than metaphysics, is certainly far more developed than any special science; and the same is true, though less tremendously so, of logic. But it will be said that metaphysics is inscrutable because its objects are not open to observation. This is doubtless true of some systems of metaphysics, though not to the extent that it is supposed to be true. The things that any science discovers are beyond the reach of direct observation. We cannot see energy, nor the attraction of gravitation, nor the flying molecules of gases, nor the luminiferous ether, nor the forests of the carbonaceous era, nor the explosions in nerve-cells. It is only the premisses of science, not its conclusions, which are directly observed. But metaphysics, even bad metaphysics, really rests on observations, whether consciously or not; and the only reason that this is not universally recognized is that it rests upon kinds of phenomena with which every man's experience is so saturated that he usually pays no particular attention to them. The data of metaphysics are not less open to observation, but immeasurably more so, than the data, say, of the very highly developed science of astronomy, to make any important addition to whose observations requires an expenditure of many tens of thousands of dollars. No, I think we must abandon the idea that metaphysics is backward owing to any intrinsic difficulty of it.

Peirce: CP 6.3 Cross-Ref:††

3.†1 In my opinion the chief cause of its backward condition is that its leading professors have been theologians. Were they simply Christian ministers the effect of intrusting very important scientific business to their hands would be quite as bad as if the same number of Wall Street promoters and Broad Street brokers were appointed to perform the task. The unfitness in the one case, as in the other, would consist in those persons having no idea of any broader interests than the personal interests of some person or collection of persons. Both classes are practical men. Now it is quite

impossible for a practical man to comprehend what science is about unless he becomes as a little child and is born again. Scientific men are made out of youths who during the plastic period of life are set to study science for a number of years. Most of these develop into mere teachers; only a minority imbibe the spirit of science. The practical man has a definite job which he sets himself to accomplish. For that purpose he has to adopt some consistent plan which must be based upon a theory, and to that theory he must be wedded before the work begins. Even if his practical problem is no more serious than playing a game of whist, when there are only three rounds of a hand to be played, he must go upon the supposition that the cards lie so that he can win the odd trick. If he is a judge presiding over the hearing of a cause, that cause must be decided somehow, no matter how defective the evidence may be; and consequently he is constrained to lay down a rule for the burden of proof. But the idea of science is to pile the ground before the foot of the outworks of truth with the carcasses of this generation, and perhaps of others to come after it, until some future generation, by treading on them, can storm the citadel. The difference comes to this, that the practical man stakes everything he cares for upon the hazard of a die, and must believe with all the force of his manhood that the object for which he strives is good and that the theory of his plan is correct; while the scientific man is above all things desirous of learning the truth and, in order to do so, ardently desires to have his present provisional beliefs (and all his beliefs are merely provisional) swept away, and will work hard to accomplish that object. This is the reason that a good practical man cannot do the best scientific work. The temperaments requisite for the two kinds of business are altogether contrary to one another. This is above all true of the practical teacher [who] has no calling for his work unless he thoroughly believes that he is already in possession of all-important truth, with which he seeks by every physiological means to imbue other minds, so that they shall be unable to give it up. But a scientific man, who has any such immovable beliefs to which he regards himself as religiously bound to be loyal, cannot at the same time desire to have his beliefs altered. In other words he cannot wish to learn the truth. Hence, I say that had the business of metaphysics been intrusted to ordinary parish priests it would have been performed unscientifically enough. But what has in fact been its fate has been far more tragic, in that it has been given over not to parish priests but to the caste of theologians. How much theologians may have contributed to the cause of Christianity, how far their writings and performances may have [been] the instruments of bringing home to men's hearts the truth of the Gospel of Love, or how far, on the other hand, they may have subserved the agencies that work to make Christians forget that truth, it is not in my province to inquire. I once bought and read through Dr. Schaff's three volumes upon the *Creeds of Christendom*†1 for the purpose of ascertaining whether the theologians, who composed them, had ever once, from the first to the last, inserted a single clause in one of them by way of recognition of the principle of love; and I found that such a thing had never been done. But then we must remember that, that principle being fully admitted by all Christians, its insertion would not have served to damn anybody. Now the principal business of theologians is to make men feel the enormity of the slightest departure from the metaphysics they assume to be connected with the standard faith. Upon their religious side, however, I will not pretend to any opinion about the influence of theologians. But since theology pretends to be a science, they must also be judged as scientific men. And in that regard I must say that another so deplorably corrupt an influence as theirs upon the morals of science I do not believe has ever been operative. Theology, I am persuaded, derives its initial impulse from a religious wavering; for there is quite as much, or more, that is mysterious and calculated to awaken scientific curiosity in

the intercourse of men with one another as in their intercourse with God, and it [is] a problem quite analogous to that of theology. Yet we do not find that theologians have cared much for those problems. They have taken human conversation as a matter of course, with rather a remarkable absence of all curiosity about it. But, as far as I can penetrate into the motive of theology, it begins in an effort of men who have joined the Christian army and sworn fidelity to it to silence the suggestions of their hearts that they renounce their allegiance. How far it is successful in that purpose I will not inquire. But nothing can be more unscientific than the attitude of minds who are trying to confirm themselves in early beliefs. The struggle of the scientific man is to try to see the errors of his beliefs -- if he can be said to have any beliefs.†2 The logic which observational science uses is not, like the logic that the books teach, quite independent of the motive and spirit of the reasoner. There is an ethics indissolubly bound up with it -- an ethics of fairness and impartiality -- and a writer, who teaches, by his example, to find arguments for a conclusion which he wishes to believe, saps the very foundations of science by trifling with its morals. To sum up, the case is this:

Peirce: CP 6.4 Cross-Ref:††

4. We should expect to find metaphysics, judging from its position in the scheme of the sciences,†1 to be somewhat more difficult than logic, but still on the whole one of the simplest of sciences, as it is one whose main principles must be settled before very much progress can be gained either in psychics or in physics.

Peirce: CP 6.5 Cross-Ref:††

5. Historically we are astonished to find that it has been a mere arena of ceaseless and trivial disputation. But we also find that it has been pursued in a spirit the very contrary of that of wishing to learn the truth, which is the most essential requirement of the logic of science; and it is worth trying whether by proceeding modestly, recognizing in metaphysics an observational science, and applying to it the universal methods of such science, without caring one straw what kind of conclusions we reach or what their tendencies may be, but just honestly applying induction and hypothesis,†2 we cannot gain some ground for hoping that the disputes and obscurities of the subject may at last disappear.†3

Peirce: CP 6.6 Cross-Ref:††

§2. THE PROBLEMS OF METAPHYSICS †4

6. *Metaphysics* is the proper designation for the third, and completing department of coenoscopia,†5 which in places welds itself into idioscopia,†5 or special science. Its business is to study the most general features of reality and real objects. But in its present condition it is, even more than the other branches of coenoscopia, a puny, rickety, and scrofulous science. It is only too plain that those who pretend to cultivate it carry not the hearts of true men of science within their breast. Instead of striving with might and main to find out what errors they have fallen into, and exulting joyously at every such discovery, they are scared to look Truth in the face. They turn tail and flee her. Only a small number out of the great catalogue of problems which it is their business to solve have they ever taken up at all, and those few most feebly. Here let us set down almost at random a small specimen of the questions of metaphysics which press, not for hasty answers, but for industrious and

solid investigation: Whether or no there be any real indefiniteness, or real possibility and impossibility?†1 Whether or not there is any definite indeterminacy?†2 Whether there be any strictly individual existence?†3 Whether there is any distinction, other than one of more and less, between fact and fancy? Or between the external and the internal worlds?†4 What general explanation or account can be given of the different qualities of feeling and their apparent connection with determinations of mass, space, and time?†5 Do all possible qualities of sensation, including, of course, a much vaster variety of which we have no experience than of those which we know, form one continuous system, as colors seem to do?†6 What external reality do the qualities of sense represent, in general?†7 Is Time a real thing, and if not, what is the nature of the reality that it represents?†8 How about Space, in these regards?†9 How far, and in what respects, is Time external or has immediate contents that are external?†10 Are Time and Space continuous?†11 What numerically are the Chorisys, Cyclosis, Periphaxis, and Apeirys of Space?†12 Has Time, or has Space, any limit or node?†13 Is hylozoism an opinion, actual or conceivable, rather than a senseless vocable; and if so, what is, or would be, that opinion?†14 What is consciousness or mind like; meaning, is it a single continuum like Time and Space, which is for different purposes variously broken up by that which it contains; or is it composed of solid atoms, or is it more like a fluid?†1 Has truth, in Kantian phrase, any "material" characteristics in general, by which it can, with any degree of probability, be recognized?†2 Is there, for example, any general tendency in the course of events, any progress in one direction on the whole?†3

Peirce: CP 6.7 Cross-Ref:††
BOOK I

ONTOLOGY AND COSMOLOGY

SCIENTIFIC METAPHYSICS

A. Tychism

CHAPTER 1

THE ARCHITECTURE OF THEORIES†1

§1. PHILOSOPHIC ARCHITECTONICE

7. Of the fifty or hundred systems of philosophy that have been advanced at different times of the world's history, perhaps the larger number have been, not so much results of historical evolution, as happy thoughts which have accidentally occurred to their authors. An idea which has been found interesting and fruitful has been adopted, developed, and forced to yield explanations of all sorts of phenomena. The English have been particularly given to this way of philosophizing; witness, Hobbes, Hartley, Berkeley, James Mill. Nor has it been by any means useless labor; it shows us what the true nature and value of the ideas developed are, and in that way affords serviceable materials for philosophy. Just as if a man, being seized with the conviction that paper was a good material to make things of, were to go to work to build a *papier mâché* house, with roof of roofing paper, foundations of pasteboard, windows of paraffined paper, chimneys, bath tubs, locks, etc., all of different forms of paper, his experiment would probably afford valuable lessons to builders, while it would certainly make a detestable house, so those one-idea'd philosophies are exceedingly interesting and instructive, and yet are quite unsound.

Peirce: CP 6.8 Cross-Ref:††

8. The remaining systems of philosophy have been of the nature of reforms, sometimes amounting to radical revolutions, suggested by certain difficulties which have been found to beset systems previously in vogue; and such ought certainly to be in large part the motive of any new theory. This is like partially rebuilding a house. The faults that have been committed are, first, that the repairs of the dilapidations have generally not been sufficiently thorough-going, and, second, that not sufficient pains have been taken to bring the additions into deep harmony with the really sound parts of the old structure.

Peirce: CP 6.8 Cross-Ref:††

When a man is about to build a house, what a power of thinking he has to do before he can safely break ground! With what pains he has to excogitate the precise wants that are to be supplied! What a study to ascertain the most available and suitable materials, to determine the mode of construction to which those materials are best adapted, and to answer a hundred such questions! Now without riding the metaphor too far, I think we may safely say that the studies preliminary to the construction of a great theory should be at least as deliberate and thorough as those that are preliminary to the building of a dwelling house.

Peirce: CP 6.9 Cross-Ref:††

9. That systems ought to be constructed architectonically has been preached since Kant, but I do not think the full import of the maxim has by any means been apprehended. What I would recommend is that every person who wishes to form an opinion concerning fundamental problems should first of all make a complete survey of human knowledge, should take note of all the valuable ideas in each branch of science, should observe in just what respect each has been successful and where it has failed, in order that, in the light of the thorough acquaintance so attained of the available materials for a philosophical theory and of the nature and strength of each, he may proceed to the study of what the problem of philosophy consists in, and of the proper way of solving it. I must not be understood as endeavoring to state fully all that these preparatory studies should embrace; on the contrary, I purposely slur over many points, in order to give emphasis to one special recommendation, namely, to make a systematic study of the conceptions out of which a philosophical theory may

be built, in order to ascertain what place each conception may fitly occupy in such a theory, and to what uses it is adapted.

Peirce: CP 6.9 Cross-Ref:††

The adequate treatment of this single point would fill a volume, but I shall endeavor to illustrate my meaning by glancing at several sciences and indicating conceptions in them serviceable for philosophy. As to the results to which long studies thus commenced have led me, I shall just give a hint at their nature.

Peirce: CP 6.10 Cross-Ref:††

10. We may begin with dynamics -- field in our day of perhaps the grandest conquest human science has ever made -- I mean the law of the conservation of energy. But let us revert to the first step taken by modern scientific thought -- and a great stride it was -- the inauguration of dynamics by Galileo. A modern physicist on examining Galileo's works is surprised to find how little experiment had to do with the establishment of the foundations of mechanics. His principal appeal is to common sense and *il lume naturale*. He always assumes that the true theory will be found to be a simple and natural one. And we can see why it should indeed be so in dynamics. For instance, a body left to its own inertia moves in a straight line, and a straight line appears to us the simplest of curves. In *itself*, no curve is simpler than another. A system of straight lines has intersections precisely corresponding to those of a system of like parabolas similarly placed, or to those of any one of an infinity of systems of curves. But the straight line appears to us simple, because, as Euclid says, it lies evenly between its extremities; that is, because viewed endwise it appears as a point. That is, again, because light moves in straight lines. Now, light moves in straight lines because of the part which the straight line plays in the laws of dynamics. Thus it is that, our minds having been formed under the influence of phenomena governed by the laws of mechanics, certain conceptions entering into those laws become implanted in our minds, so that we readily guess at what the laws are.†1 Without such a natural prompting, having to search blindfold for a law which would suit the phenomena, our chance of finding it would be as one to infinity. The further physical studies depart from phenomena which have directly influenced the growth of the mind, the less we can expect to find the laws which govern them "simple," that is, composed of a few conceptions natural to our minds.

Peirce: CP 6.11 Cross-Ref:††

11. The researches of Galileo, followed up by Huygens and others, led to those modern conceptions of *Force* and *Law*, which have revolutionized the intellectual world. The great attention given to mechanics in the seventeenth century soon so emphasized these conceptions as to give rise to the Mechanical Philosophy, or doctrine that all the phenomena of the physical universe are to be explained upon mechanical principles. Newton's great discovery imparted a new impetus to this tendency. The old notion that heat consists in an agitation of corpuscles was now applied to the explanation of the chief properties of gases. The first suggestion in this direction was that the pressure of gases is explained by the battering of the particles against the walls of the containing vessel, which explained Boyle's law of the compressibility of air. Later, the expansion of gases, Avogadro's chemical law, the diffusion and viscosity of gases, and the action of Crookes's radiometer were shown to be consequences of the same kinetical theory; but other phenomena, such as the ratio of the specific heat at constant volume to that at constant pressure, require additional hypotheses, which we have little reason to suppose are simple, so that we find ourselves quite afloat. In like manner with regard to light. That it consists of

vibrations was almost proved by the phenomena of diffraction, while those of polarization showed the excursions of the particles to be perpendicular to the line of propagation; but the phenomena of dispersion, etc., require additional hypotheses which may be very complicated. Thus, the further progress of molecular speculation appears quite uncertain. If hypotheses are to be tried haphazard, or simply because they will suit certain phenomena, it will occupy the mathematical physicists of the world say half a century on the average to bring each theory to the test, and since the number of possible theories may go up into the trillions, only one of which can be true, we have little prospect of making further solid additions to the subject in our time. When we come to atoms, the presumption in favor of a simple law seems very slender. There is room for serious doubt whether the fundamental laws of mechanics hold good for single atoms, and it seems quite likely that they are capable of motion in more than three dimensions.†1

Peirce: CP 6.12 Cross-Ref:††

12. To find out much more about molecules and atoms we must search out a natural history of laws of nature which may fulfill that function which the presumption in favor of simple laws fulfilled in the early days of dynamics, by showing us what kind of laws we have to expect and by answering such questions as this: Can we, with reasonable prospect of not wasting time, try the supposition that atoms attract one another inversely as the seventh power of their distances, or can we not? To suppose universal laws of nature capable of being apprehended by the mind and yet having no reason for their special forms, but standing inexplicable and irrational, is hardly a justifiable position. Uniformities are precisely the sort of facts that need to be accounted for. That a pitched coin should sometimes turn up heads and sometimes tails calls for no particular explanation; but if it shows heads every time, we wish to know how this result has been brought about. Law is *par excellence* the thing that wants a reason.

Peirce: CP 6.13 Cross-Ref:††

§2. THREE THEORIES OF EVOLUTION †1E

13. Now the only possible way of accounting for the laws of nature and for uniformity in general is to suppose them results of evolution. This supposes them not to be absolute, not to be obeyed precisely. It makes an element of indeterminacy, spontaneity, or absolute chance in nature.†2 Just as, when we attempt to verify any physical law, we find our observations cannot be precisely satisfied by it, and rightly attribute the discrepancy to errors of observation, so we must suppose far more minute discrepancies to exist owing to the imperfect cogency of the law itself, to a certain swerving of the facts from any definite formula.

Peirce: CP 6.14 Cross-Ref:††

14. Mr. Herbert Spencer †3 wishes to explain evolution upon mechanical principles. This is illogical, for four reasons. First, because the principle of evolution requires no extraneous cause, since the tendency to growth can be supposed itself to have grown from an infinitesimal germ accidentally started. Second, because law ought more than anything else to be supposed a result of evolution. Third, because exact law obviously never can produce heterogeneity out of homogeneity; and arbitrary heterogeneity is the feature of the universe the most manifest and

characteristic. Fourth, because the law of the conservation of energy is equivalent to the proposition that all operations governed by mechanical laws are reversible; so that an immediate corollary from it is that growth is not explicable by those laws, even if they be not violated in the process of growth. In short, Spencer is not a philosophical evolutionist, but only a half-evolutionist -- or, if you will, only a semi-Spencerian. Now philosophy requires thorough-going evolutionism or none.

Peirce: CP 6.15 Cross-Ref:††

15. The theory of Darwin was that evolution had been brought about by the action of two factors: first, heredity, as a principle making offspring nearly resemble their parents, while yet giving room for "sporting" or accidental variations -- for very slight variations often, for wider ones rarely; and, second, the destruction of breeds or races that are unable to keep the birth rate up to the death rate. This Darwinian principle is plainly capable of great generalization. Wherever there are large numbers of objects having a tendency to retain certain characters unaltered, this tendency, however, not being absolute but giving room for chance variations, then, if the amount of variation is absolutely limited in certain directions by the destruction of everything which reaches those limits, there will be a gradual tendency to change in directions of departure from them. Thus, if a million players sit down to bet at an even game, since one after another will get ruined, the average wealth of those who remain will perpetually increase. Here is indubitably a genuine formula of possible evolution, whether its operation accounts for much or little in the development of animal and vegetable species.

Peirce: CP 6.16 Cross-Ref:††

16. The Lamarckian theory †1 also supposes that the development of species has taken place by a long series of insensible changes, but it supposes that those changes have taken place during the lives of the individuals, in consequence of effort and exercise, and that reproduction plays no part in the process except in preserving these modifications. Thus, the Lamarckian theory only explains the development of characters for which individuals strive, while the Darwinian theory only explains the production of characters really beneficial to the race, though these may be fatal to individuals.†P1 But more broadly and philosophically conceived, Darwinian evolution is evolution by the operation of chance, and the destruction of bad results, while Lamarckian evolution is evolution by the effect of habit and effort.

Peirce: CP 6.17 Cross-Ref:††

17. A third theory of evolution is that of Mr. Clarence King.†1 The testimony of monuments and of rocks is that species are unmodified or scarcely modified, under ordinary circumstances, but are rapidly altered after cataclysms or rapid geological changes. Under novel circumstances, we often see animals and plants sporting excessively in reproduction, and sometimes even undergoing transformations during individual life, phenomena no doubt due partly to the enfeeblement of vitality from the breaking up of habitual modes of life, partly to changed food, partly to direct specific influence of the element in which the organism is immersed. If evolution has been brought about in this way, not only have its single steps not been insensible, as both Darwinians and Lamarckians suppose, but they are furthermore neither haphazard on the one hand, nor yet determined by an inward striving on the other, but on the contrary are effects of the changed environment, and have a positive general tendency to adapt the organism to that environment, since variation will particularly affect organs at once enfeebled and stimulated. This mode of evolution, by external forces and the breaking up of habits, seems to be called for by some of the broadest

and most important facts of biology and paleontology; while it certainly has been the chief factor in the historical evolution of institutions as in that of ideas; and cannot possibly be refused a very prominent place in the process of evolution of the universe in general.

Peirce: CP 6.18 Cross-Ref:††
§3. THE LAW OF HABIT

18. Passing to psychology, we find the elementary phenomena of mind fall into three categories.†2 First, we have Feelings, comprising all that is immediately present, such as pain, blue, cheerfulness, the feeling that arises when we contemplate a consistent theory, etc. A feeling is a state of mind having its own living quality, independent of any other state of mind. Or, a feeling is an element of consciousness which might conceivably override every other state until it monopolized the mind, although such a rudimentary state cannot actually be realized, and would not properly be consciousness. Still it is conceivable, or supposable, that the quality of blue should usurp the whole mind, to the exclusion of the ideas of shape, extension, contrast, commencement and cessation, and all other ideas whatsoever. A feeling is necessarily perfectly simple, *in itself*, for if it had parts these would also be in the mind, whenever the whole was present, and thus the whole could not monopolize the mind.†P1

Peirce: CP 6.19 Cross-Ref:††

19. Besides Feelings, we have Sensations of reaction; as when a person blindfold suddenly runs against a post, when we make a muscular effort, or when any feeling gives way to a new feeling. Suppose I had nothing in my mind but a feeling of blue, which were suddenly to give place to a feeling of red; then, at the instant of transition, there would be a shock, a sense of reaction, my blue life being transmuted into red life. If I were further endowed with a memory, that sense would continue for some time, and there would also be a peculiar feeling or sentiment connected with it. This last feeling might endure (conceivably I mean) after the memory of the occurrence and the feelings of blue and red had passed away. But the *sensation* of reaction cannot exist except in the actual presence of the two feelings blue and red to which it relates. Wherever we have two feelings and pay attention to a relation between them of whatever kind, there is the sensation of which I am speaking. But the sense of action and reaction has two types: it may either be a perception of relation between two ideas, or it may be a sense of action and reaction between feeling and something out of feeling. And this sense of external reaction again has two forms; for it is either a sense of something happening to us, by no act of ours, we being passive in the matter, or it is a sense of resistance, that is, of our expending feeling upon something without. The sense of reaction is thus a sense of connection or comparison between feelings, either, *A*, between one feeling and another, or *B* between feeling and its absence or lower degree; and under *B* we have, first, the sense of the access of feeling, and second, the sense of remission of feeling.

Peirce: CP 6.20 Cross-Ref:††

20. Very different both from feelings and from reaction-sensations or disturbances of feeling are general conceptions. When we think, we are conscious that a connection between feelings is determined by a general rule, we are aware of being

governed by a habit. Intellectual power is nothing but facility in taking habits and in following them in cases essentially analogous to, but in non-essentials widely remote from, the normal cases of connections of feelings under which those habits were formed.

Peirce: CP 6.21 Cross-Ref:††

21. The one primary and fundamental law of mental action consists in a tendency to generalization. Feeling tends to spread; connections between feelings awaken feelings; neighboring feelings become assimilated; ideas are apt to reproduce themselves. These are so many formulations of the one law of the growth of mind. When a disturbance of feeling takes place, we have a consciousness of gain, the gain of experience; and a new disturbance will be apt to assimilate itself to the one that preceded it. Feelings, by being excited, become more easily excited, especially in the ways in which they have previously been excited. The consciousness of such a habit constitutes a general conception.

Peirce: CP 6.22 Cross-Ref:††

22. The cloudiness of psychological notions may be corrected by connecting them with physiological conceptions.†1 Feeling may be supposed to exist wherever a nerve-cell is in an excited condition. The disturbance of feeling, or sense of reaction, accompanies the transmission of disturbance between nerve-cells, or from a nerve-cell to a muscle-cell, or the external stimulation of a nerve-cell. General conceptions arise upon the formation of habits in the nerve-matter, which are molecular changes consequent upon its activity and probably connected with its nutrition.

Peirce: CP 6.23 Cross-Ref:††

23. The law of habit exhibits a striking contrast to all physical laws in the character of its commands. A physical law is absolute. What it requires is an exact relation. Thus, a physical force introduces into a motion a component motion to be combined with the rest by the parallelogram of forces; but the component motion must actually take place exactly as required by the law of force. On the other hand, no exact conformity is required by the mental law. Nay, exact conformity would be in downright conflict with the law; since it would instantly crystallize thought and prevent all further formation of habit. The law of mind only makes a given feeling *more likely* to arise. It thus resembles the "non-conservative" forces of physics, such as viscosity and the like, which are due to statistical uniformities in the chance encounters of trillions of molecules.

Peirce: CP 6.24 Cross-Ref:††

§4. OBJECTIVE IDEALISM †1E

24. The old dualistic notion of mind and matter, so prominent in Cartesianism, as two radically different kinds of substance, will hardly find defenders today. Rejecting this, we are driven to some form of hylopathy, otherwise called monism. Then the question arises whether physical laws on the one hand and the psychical law on the other are to be taken --

Peirce: CP 6.24 Cross-Ref:††

(a) as independent, a doctrine often called *monism*, but which I would name

neutralism; or,

Peirce: CP 6.24 Cross-Ref:††

(b) the psychical law as derived and special, the physical law alone as primordial, which is *materialism*; or,

Peirce: CP 6.24 Cross-Ref:††

(c) the physical law as derived and special, the psychical law alone as primordial, which is *idealism*.

Peirce: CP 6.24 Cross-Ref:††

The materialistic doctrine seems to me quite as repugnant to scientific logic as to common sense; since it requires us to suppose that a certain kind of mechanism will feel, which would be a hypothesis absolutely irreducible to reason -- an ultimate, inexplicable regularity; while the only possible justification of any theory is that it should make things clear and reasonable.

Peirce: CP 6.24 Cross-Ref:††

Neutralism is sufficiently condemned by the logical maxim known as Ockham's razor, *i.e.*, that not more independent elements are to be supposed than necessary. By placing the inward and outward aspects of substance on a par, it seems to render both primordial.

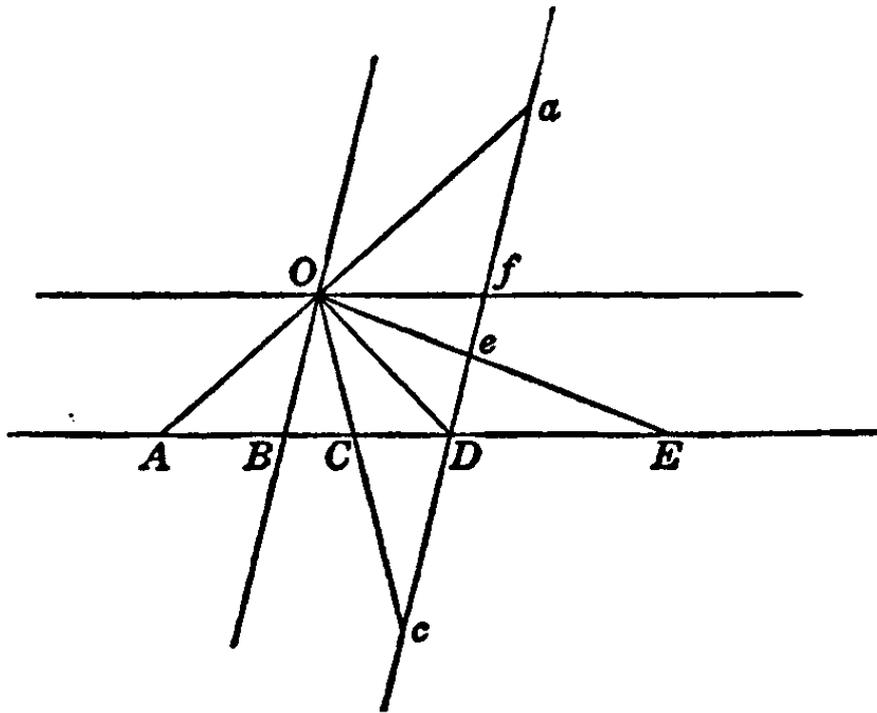
Peirce: CP 6.25 Cross-Ref:††

25. The one intelligible theory of the universe is that of objective idealism, that matter is effete mind, inveterate habits becoming physical laws. But before this can be accepted it must show itself capable of explaining the tri-dimensionality of space, the laws of motion, and the general characteristics of the universe, with mathematical clearness and precision; for no less should be demanded of every philosophy.

Peirce: CP 6.26 Cross-Ref:††

§5. THE NATURE OF SPACE †1E

26. Modern mathematics is replete with ideas which may be applied to philosophy. I can only notice one or two. The manner in which mathematicians generalize is very instructive. Thus, painters are accustomed to think of a picture as consisting geometrically of the intersections of its plane by rays of light from the natural objects to the eye. But geometers use a generalized perspective. For instance, in the figure



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to view]

let O be the eye, let $A B C D E$ be the edgewise view of any plane, and let $a f e D c$ be the edgewise view of another plane. The geometers draw rays through O cutting both these planes, and treat the points of intersection of each ray with one plane as representing the point of intersection of the same ray with the other plane. Thus, e represents E , in the painter's way. D represents itself. C is represented by c , which is farther from the eye; and A is represented by a which is on the other side of the eye. Such generalization is not bound down to sensuous images. Further, according to this mode of representation every point on one plane represents a point on the other, and every point on the latter is represented by a point on the former. But how about the point f which is in a direction from O parallel to the represented plane, and how about the point B which is in a direction parallel to the representing plane? Some will say that these are exceptions; but modern mathematics does not allow exceptions which can be annulled by generalization. As a point moves from C to D and thence to E and off toward infinity, the corresponding point on the other plane moves from c to D and thence to e and toward f . But this second point can pass through f to a ; and when it is there the first point has arrived at A . We therefore say that the first point has passed *through infinity*, and that every line joins in to itself somewhat like an oval. Geometers talk of the parts of lines at an infinite distance as points. This is a kind of generalization very efficient in mathematics.

Peirce: CP 6.27 Cross-Ref:††

27. Modern views of measurement have a philosophical aspect.†1 There is an indefinite number of systems of measuring along a line; thus, a perspective representation of a scale on one line may be taken to measure another, although, of course, such measurements will not agree with what we call the distances of points on

the latter line. To establish a system of measurement on a line we must assign a distinct number to each point of it, and for this purpose we shall plainly have to suppose the numbers carried out into an infinite number of places of decimals. These numbers must be ranged along the line in unbroken sequence. Further, in order that such a scale of numbers should be of any use, it must be capable of being shifted into new positions, each number continuing to be attached to a single distinct point. Now it is found that if this is true for "imaginary" as well as for real points (an expression which I cannot stop to elucidate),^{†2} any such shifting will necessarily leave two numbers attached to the same points as before. So that when the scale is moved over the line by any continuous series of shiftings of one kind, there are two points which no numbers on the scale can ever reach, except the numbers fixed there. This pair of points, thus unattainable in measurement, is called the Absolute. These two points may be distinct and real, or they may coincide, or they may be both imaginary. As an example of a linear quantity with a double absolute we may take probability, which ranges from an unattainable absolute certainty *against* a proposition to an equally unattainable absolute certainty *for* it. A line, according to ordinary notions, we have seen is a linear quantity where the two points at infinity coincide. A velocity is another example. A train going with infinite velocity from Chicago to New York would be at all the points on the line at the very same instant, and if the time of transit were reduced to less than nothing it would be moving in the other direction. An angle is a familiar example of a mode of magnitude with no real immeasurable values. One of the questions philosophy has to consider is whether the development of the universe is like the increase of an angle, so that it proceeds forever without tending toward anything unattained, which I take to be the Epicurean view, or whether the universe sprang from a chaos in the infinitely distant past to tend toward something different in the infinitely distant future, or whether the universe sprang from nothing in the past to go on indefinitely toward a point in the infinitely distant future, which, were it attained, would be the mere nothing from which it set out.^{†1}

Peirce: CP 6.28 Cross-Ref:††

28.^{†2} The doctrine of the absolute applied to space comes to this, that either --

Peirce: CP 6.28 Cross-Ref:††

First, space is, as Euclid teaches, both *unlimited* and *immeasurable*, so that the infinitely distant parts of any plane seen in perspective appear as a straight line, in which case the sum of the three angles of a triangle amounts to 180°; or,

Peirce: CP 6.28 Cross-Ref:††

Second, space is *immeasurable* but *limited*, so that the infinitely distant parts of any plane seen in perspective appear as a circle, beyond which all is blackness, and in this case the sum of the three angles of a triangle is less than 180° by an amount proportional to the area of the triangle; or,

Peirce: CP 6.28 Cross-Ref:††

Third, space is *unlimited* but *finite* (like the surface of a sphere), so that it has no infinitely distant parts; but a finite journey along any straight line would bring one back to his original position, and looking off with an unobstructed view one would see the back of his own head enormously magnified, in which case the sum of the three angles of a triangle exceeds 180° by an amount proportional to the area.

Peirce: CP 6.29 Cross-Ref:††

29. Which of these three hypotheses is true we know not.^{†1} The largest triangles we can measure are such as have the earth's orbit for base, and the distance

of a fixed star for altitude. The angular magnitude resulting from subtracting the sum of the two angles at the base of such a triangle from 180° is called the star's *parallax*. The parallaxes of only about forty stars have been measured as yet. Two of them come out negative, that of Arided (α Cygni), a star of magnitude $1 \frac{1}{2}$, which is $-0''.082$, according to C. H. F. Peters, and that of a star of magnitude $7 \frac{3}{4}$, known as Piazzini III 422, $\dagger 2$ which is $-0''.045$, according to R. S. Ball. But these negative parallaxes are undoubtedly to be attributed to errors of observation; $\dagger 3$ for the probable error of such a determination is about $\pm 0''.075$, and it would be strange indeed if we were to be able to see, as it were, more than half way round space, without being able to see stars with larger negative parallaxes. Indeed, the very fact that of all the parallaxes measured only two come out negative would be a strong argument that the smallest parallaxes really amount to $+0''.1$, were it not for the reflection that the publication of other negative parallaxes may have been suppressed. I think we may feel confident that the parallax of the farthest star lies somewhere between $-0''.05$ and $+0''.15$, and within another century our grandchildren will surely know whether the three angles of a triangle are greater or less than 180° -- that they are *exactly* that amount is what nobody ever can be justified in concluding. It is true that according to the axioms of geometry the sum of the three sides of a triangle are precisely 180° ; but these axioms are now exploded, and geometers confess that they, as geometers, know not the slightest reason for supposing them to be precisely true. $\dagger 4$ They are expressions of our inborn conception of space, and as such are entitled to credit, so far as their truth could have influenced the formation of the mind. But that affords not the slightest reason for supposing them exact.

Peirce: CP 6.30 Cross-Ref: $\dagger\dagger$

30. Now, metaphysics has always been the ape of mathematics. Geometry suggested the idea of a demonstrative system of absolutely certain philosophical principles; and the ideas of the metaphysicians have at all times been in large part drawn from mathematics. The metaphysical axioms are imitations of the geometrical axioms; and now that the latter have been thrown overboard, without doubt the former will be sent after them. It is evident, for instance, that we can have no reason to think that every phenomenon in all its minutest details is precisely determined by law. That there is an arbitrary element in the universe we see -- namely, its variety. This variety must be attributed to spontaneity in some form.

Peirce: CP 6.31 Cross-Ref: $\dagger\dagger$

31. Had I more space, I now ought to show how important for philosophy is the mathematical conception of continuity. Most of what is true in Hegel is a darkling glimmer of a conception which the mathematicians had long before made pretty clear, and which recent researches have still further illustrated.

Peirce: CP 6.32 Cross-Ref: $\dagger\dagger$
 §6. FIRST, SECOND, AND THIRD $\dagger 1E$

32. Among the many principles of Logic which find their application in Philosophy, I can here only mention one. Three conceptions are perpetually turning up at every point in every theory of logic, and in the most rounded systems they occur

in connection with one another. They are conceptions so very broad and consequently indefinite that they are hard to seize and may be easily overlooked. I call them the conceptions of First, Second, Third. First is the conception of being or existing independent of anything else. Second is the conception of being relative to, the conception of reaction with, something else. Third is the conception of mediation, whereby a first and second are brought into relation. To illustrate these ideas, I will show how they enter into those we have been considering. The origin of things, considered not as leading to anything, but in itself, contains the idea of First, the end of things that of Second, the process mediating between them that of Third.†1 A philosophy which emphasizes the idea of the One is generally a dualistic philosophy in which the conception of Second receives exaggerated attention; for this One (though of course involving the idea of First) is always the other of a manifold which is not one. The idea of the Many, because variety is arbitrariness and arbitrariness is repudiation of any Secondness, has for its principal component the conception of First. In psychology Feeling is First, Sense of reaction Second, General conception Third, or mediation.†2 In biology, the idea of arbitrary sporting is First, heredity is Second, the process whereby the accidental characters become fixed is Third.†3 Chance is First, Law is Second, the tendency to take habits is Third.†4 Mind is First, Matter is Second, Evolution is Third.

Peirce: CP 6.33 Cross-Ref:††

33. Such are the materials out of which chiefly a philosophical theory ought to be built, in order to represent the state of knowledge to which the nineteenth century has brought us. Without going into other important questions of philosophical architectonic, we can readily foresee what sort of a metaphysics would appropriately be constructed from those conceptions. Like some of the most ancient and some of the most recent speculations it would be a Cosmogonic Philosophy. It would suppose that in the beginning -- infinitely remote -- there was a chaos of unpersonalized feeling, which being without connection or regularity would properly be without existence. This feeling, sporting here and there in pure arbitrariness, would have started the germ of a generalizing tendency. Its other sportings would be evanescent, but this would have a growing virtue. Thus, the tendency to habit would be started; and from this, with the other principles of evolution, all the regularities of the universe would be evolved. At any time, however, an element of pure chance survives and will remain until the world becomes an absolutely perfect, rational, and symmetrical system, in which mind is at last crystallized in the infinitely distant future.

Peirce: CP 6.34 Cross-Ref:††

34. That idea has been worked out by me with elaboration.†1 It accounts for the main features of the universe as we know it -- the characters of time, space, matter, force, gravitation, electricity, etc. It predicts many more things which new observations can alone bring to the test. May some future student go over this ground again, and have the leisure to give his results to the world.

Peirce: CP 6.35 Cross-Ref:††
CHAPTER 2

THE DOCTRINE OF NECESSITY EXAMINED†1

§1. THE MECHANICAL PHILOSOPHYE

35. In *The Monist* for January, 1891,†2 I endeavored to show what elementary ideas ought to enter into our view of the universe. I may mention that on those considerations I had already grounded a cosmical theory, and from it had deduced a considerable number of consequences capable of being compared with experience. This comparison is now in progress, but under existing circumstances must occupy many years.

Peirce: CP 6.36 Cross-Ref:††

36. I propose here to examine the common belief that every single fact in the universe is precisely determined by law. It must not be supposed that this is a doctrine accepted everywhere and at all times by all rational men. Its first advocate appears to have been Democritus, the atomist, who was led to it, as we are informed, by reflecting upon the "impenetrability, translation, and impact of matter ({antitypia kai phora kai plégé té s hylés})."†3 That is to say, having restricted his attention to a field where no influence other than mechanical constraint could possibly come before his notice, he straightway jumped to the conclusion that throughout the universe that was the sole principle of action -- a style of reasoning so usual in our day with men not unreflecting as to be more than excusable in the infancy of thought. But Epicurus, in revising the atomic doctrine and repairing its defenses, found himself obliged to suppose that atoms swerve from their courses by spontaneous chance; and thereby he conferred upon the theory life and entelechy.†4 For we now see clearly that the peculiar function of the molecular hypothesis in physics is to open an entry for the calculus of probabilities. Already, the prince of philosophers had repeatedly and emphatically condemned the dictum of Democritus (especially in the *Physics*, Book II, chapters 4, 5, 6), holding that events come to pass in three ways, namely, (1) by external compulsion, or the action of efficient causes, (2) by virtue of an inward nature, or the influence of final causes, and (3) irregularly without definite cause, but just by absolute chance; and this doctrine is of the inmost essence of Aristotelianism. It affords, at any rate, a valuable enumeration of the possible ways in which anything can be supposed to have come about. The freedom of the will, too, was admitted both by Aristotle †1 and by Epicurus.†2 But the Stoa,†3 which in every department seized upon the most tangible, hard, and lifeless element, and blindly denied the existence of every other, which, for example, impugned the validity of the inductive method and wished to fill its place with the *reductio ad absurdum*, very naturally became the one school of ancient philosophy to stand by a strict necessitarianism, thus returning to a single principle of Democritus that Epicurus had been unable to swallow. Necessitarianism and materialism with the Stoics went hand in hand, as by affinity they should. At the revival of learning, Stoicism met with considerable favor, partly because it departed just enough from Aristotle to give it the spice of novelty, and partly because its superficialities well adapted it for acceptance by students of literature and art who wanted their philosophy drawn mild. Afterwards, the great discoveries in mechanics inspired the hope that mechanical principles might suffice to explain the universe; and, though without logical justification, this hope has since been continually stimulated by subsequent advances in physics. Nevertheless, the

doctrine was in too evident conflict with the freedom of the will and with miracles to be generally acceptable, at first. But meantime there arose that most widely spread of philosophical blunders, the notion that associationalism belongs intrinsically to the materialistic family of doctrines; and thus was evolved the theory of motives; and libertarianism became weakened. At present, historical criticism has almost exploded the miracles, great and small; so that the doctrine of necessity has never been in so great vogue as now.

Peirce: CP 6.37 Cross-Ref:††

37. The proposition in question is that the state of things existing at any time, together with certain immutable laws, completely determine the state of things at every other time (for a limitation to *future* time is indefensible). Thus, given the state of the universe in the original nebula, and given the laws of mechanics, a sufficiently powerful mind could deduce from these data the precise form of every curlicue of every letter I am now writing.

Peirce: CP 6.38 Cross-Ref:††

38. Whoever holds that every act of the will as well as every idea of the mind is under the rigid governance of a necessity †1 coördinated with that of the physical world will logically be carried to the proposition that minds are part of the physical world in such a sense that the laws of mechanics determine anything that happens according to immutable attractions and repulsions. In that case, that instantaneous state of things, from which every other state of things is calculable, consists in the positions and velocities of all the particles at any instant. This, the usual and most logical form of necessitarianism, is called the mechanical philosophy.

Peirce: CP 6.39 Cross-Ref:††

§2. NECESSITY CONSIDERED AS A POSTULATE

39. When I have asked thinking men what reason they had to believe that every fact in the universe is precisely determined by law, the first answer has usually been that the proposition is a "presupposition" or postulate of scientific reasoning. Well, if that is the best that can be said for it, the belief is doomed. Suppose it be "postulated": that does not make it true, nor so much as afford the slightest rational motive for yielding it any credence. It is as if a man should come to borrow money and, when asked for his security, should reply he "postulated" the loan. To "postulate" a proposition is no more than to hope it is true. There are, indeed, practical emergencies in which we act upon assumptions of certain propositions as true, because if they are not so, it can make no difference how we act. But all such propositions I take to be hypotheses of individual facts. For it is manifest that no universal principle can in its universality be comprised †1 in a special case or can be requisite for the validity of any ordinary inference. To say, for instance, that the demonstration by Archimedes of the property of the lever would fall to the ground if men were endowed with free will is extravagant; yet this is implied by those who make a proposition incompatible with the freedom of the will the postulate of all inference. Considering, too, that the conclusions of science make no pretense to being more than probable, and considering that a probable inference can at most only suppose something to be most frequently, or otherwise approximately, true, but never that anything is precisely true without exception throughout the universe, we see how

far this proposition in truth is from being so postulated.

Peirce: CP 6.40 Cross-Ref:††

40. But the whole notion of a postulate being involved in reasoning appertains to a by-gone and false conception of logic. Non-deductive or ampliative inference is of three kinds: induction, hypothesis, and analogy.†2 If there be any other modes, they must be extremely unusual and highly complicated, and may be assumed with little doubt to be of the same nature as those enumerated. For induction, hypothesis, and analogy, as far as their ampliative character goes, that is, so far as they conclude something not implied in the premisses, depend upon one principle and involve the same procedure. All are essentially inferences from sampling. Suppose a ship arrives at Liverpool laden with wheat in bulk. Suppose that by some machinery the whole cargo be stirred up with great thoroughness. Suppose that twenty-seven thimblefuls be taken equally from the forward, midships, and aft parts, from the starboard, center, and larboard parts, and from the top, half depth, and lower parts of her hold, and that these being mixed and the grains counted, four-fifths of the latter are found to be of quality A. Then we infer, experientially and provisionally, that approximately four-fifths of all the grain in the cargo is of the same quality. I say we infer this *experientially* and *provisionally*. By saying that we infer it *experientially*, I mean that our conclusion makes no pretension to knowledge of wheat-in-itself, our {alétheia}, as the derivation of that word implies, has nothing to do with *latent* wheat. We are dealing only with the matter of possible experience -- experience in the full acceptance of the term as something not merely affecting the senses but also as the subject of thought. If there be any wheat hidden on the ship, so that it can neither turn up in the sample nor be heard of subsequently from purchasers -- or if it be half-hidden, so that it may, indeed, turn up, but is less likely to do so than the rest -- or if it can affect our senses and our pockets, but from some strange cause or causelessness cannot be reasoned about -- all such wheat is to be excluded (or have only its proportional weight) in calculating that true proportion of quality A, to which our inference seeks to approximate. By saying that we draw the inference *provisionally*, I mean that we do not hold that we have reached any assigned degree of approximation as yet, but only hold that if our experience be indefinitely extended, and if every fact of whatever nature, as fast as it presents itself, be duly applied, according to the inductive method, in correcting the inferred ratio, then our approximation will become indefinitely close in the long run; that is to say, close to the experience *to come* (not merely close by the exhaustion of a finite collection) so that if experience in general is to fluctuate irregularly to and fro, in a manner to deprive the ratio sought of all definite value, we shall be able to find out approximately within what limits it fluctuates, and if, after having one definite value, it changes and assumes another, we shall be able to find that out, and in short, whatever may be the variations of this ratio in experience, experience indefinitely extended will enable us to detect them, so as to predict rightly, at last, what its ultimate value may be, if it have any ultimate value, or what the ultimate law of succession of values may be, if there be any such ultimate law, or that it ultimately fluctuates irregularly within certain limits, if it do so ultimately fluctuate. Now our inference, claiming to be no more than thus experiential and provisional, manifestly involves no postulate whatever.

Peirce: CP 6.41 Cross-Ref:††

41. For what is a postulate? It is the formulation of a material fact which we are not entitled to assume as a premiss, but the truth of which is requisite to the validity of an inference. Any fact, then, which might be supposed postulated, must

either be such that it would ultimately present itself in experience, or not. If it will present itself, we need not postulate it now in our provisional inference, since we shall ultimately be entitled to use it as a premiss. But if it never would present itself in experience, our conclusion is valid but for the possibility of this fact being otherwise than assumed, that is, it is valid as far as possible experience goes, and that is all that we claim. Thus, every postulate is cut off, either by the provisionality or by the experientiality of our inference. For instance, it has been said that induction postulates that, if an indefinite succession of samples be drawn, examined, and thrown back each before the next is drawn, then in the long run every grain will be drawn as often as any other, that is to say, postulates that the ratio of the numbers of times in which any two are drawn will indefinitely approximate to unity. But no such postulate is made; for if, on the one hand, we are to have no other experience of the wheat than from such drawings, it is the ratio that presents itself in those drawings and not the ratio which belongs to the wheat in its latent existence that we are endeavoring to determine; while if, on the other hand, there is some other mode by which the wheat is to come under our knowledge, equivalent to another kind of sampling, so that after all our care in stirring up the wheat some experiential grains will present themselves in the first sampling operation more often than others in the long run, this very singular fact will be sure to get discovered by the inductive method, which must avail itself of every sort of experience; and our inference, which was only provisional, corrects itself at last. Again, it has been said, that induction postulates that under like circumstances like events will happen, and that this postulate is at bottom the same as the principle of universal causation.†1 But this is a blunder, or *bévue*, due to thinking exclusively of inductions where the concluded ratio is either 1 or 0. If any such proposition were postulated, it would be that under like circumstances (the circumstances of drawing the different samples) different events occur in the same proportions in all the different sets -- a proposition which is false and even absurd.†1 But in truth no such thing is postulated, the experiential character of the inference reducing the condition of validity to this, that if a certain result does not occur, the opposite result will be manifested, a condition assured by the provisionality of the inference. But it may be asked whether it is not conceivable that every instance of a certain class destined to be ever employed as a datum of induction should have one character, while every instance destined not to be so employed should have the opposite character. The answer is that, in that case, the instances excluded from being subjects of reasoning would not be experienced in the full sense of the word, but would be among these *latent* individuals of which our conclusion does not pretend to speak.

Peirce: CP 6.42 Cross-Ref: ††

42. To this account of the rationale of induction I know of but one objection worth mention: it is that I thus fail to deduce the full degree of force which this mode of inference in fact possesses; that according to my view, no matter how thorough and elaborate the stirring and mixing process had been, the examination of a single handful of grain would not give me any assurance, sufficient to risk money upon, that the next handful would not greatly modify the concluded value of the ratio under inquiry, while, in fact, the assurance would be very high that this ratio was not greatly in error. If the true ratio of grains of quality *A* were 0.80 and the handful contained a thousand grains, nine such handfuls out of every ten would contain from 780 to 820 grains of quality *A*. The answer to this is that the calculation given is correct when we know that the units of this handful and the quality inquired into have the normal independence of one another, if for instance the stirring has been complete and the character sampled for has been settled upon in advance of the examination of the

sample.†2 But in so far as these conditions are not known to be complied with, the above figures cease to be applicable. Random sampling and predesignation of the character sampled for should always be striven after in inductive reasoning, but when they cannot be attained, so long as it is conducted honestly, the inference retains some value. When we cannot ascertain how the sampling has been done or the sample-character selected, induction still has the essential validity which my present account of it shows it to have.

Peirce: CP 6.43 Cross-Ref:††

§3. THE OBSERVATIONAL EVIDENCE FOR NECESSITARIANISME

43. I do not think a man who combines a willingness to be convinced with a power of appreciating an argument upon a difficult subject can resist the reasons which have been given to show that the principle of universal necessity cannot be defended as being a postulate of reasoning. But then the question immediately arises whether it is not proved to be true, or at least rendered highly probable, by observation of nature.

Peirce: CP 6.44 Cross-Ref:††

44. Still, this question ought not long to arrest a person accustomed to reflect upon the force of scientific reasoning. For the essence of the necessitarian position is that certain continuous quantities have certain exact values. Now, how can observation determine the value of such a quantity with a probable error absolutely *nil*? To one who is behind the scenes, and knows that the most refined comparisons of masses, lengths, and angles, far surpassing in precision all other measurements, yet fall behind the accuracy of bank accounts, and that the ordinary determinations of physical constants, such as appear from month to month in the journals, are about on a par with an upholsterer's measurements of carpets and curtains, the idea of mathematical exactitude being demonstrated in the laboratory will appear simply ridiculous. There is a recognized method of estimating the probable magnitudes of errors in physics -- the method of least squares. It is universally admitted that this method makes the errors smaller than they really are; yet even according to that theory an error indefinitely small is indefinitely improbable; so that any statement to the effect that a certain continuous quantity has a certain exact value, if well founded at all, must be founded on something other than observation.

Peirce: CP 6.45 Cross-Ref:††

45. Still, I am obliged to admit that this rule is subject to a certain qualification. Namely, it only applies to continuous †P1 quantity. Now, certain kinds of continuous quantity are discontinuous at one or at two limits, and for such limits the rule must be modified. Thus, the length of a line cannot be less than zero. Suppose, then, the question arises how long a line a certain person had drawn from a marked point on a piece of paper. If no line at all can be seen, the observed length is zero; and the only conclusion this observation warrants is that the length of the line is less than the smallest length visible with the optical power employed. But indirect observations -- for example, that the person supposed to have drawn the line was never within fifty feet of the paper -- may make it probable that no line at all was made, so that the concluded length will be strictly zero. In like manner, experience no doubt would warrant the conclusion that there is absolutely no indigo in a given ear of

wheat, and absolutely no attar in a given lichen. But such inferences can only be rendered valid by positive experiential evidence, direct or remote, and cannot rest upon a mere inability to detect the quantity in question. We have reason to think there is no indigo in the wheat, because we have remarked that wherever indigo is produced it is produced in considerable quantities, to mention only one argument. We have reason to think there is no attar in the lichen, because essential oils seem to be in general peculiar to single species. If the question had been whether there was iron in the wheat or the lichen, though chemical analysis should fail to detect its presence, we should think some of it probably was there, since iron is almost everywhere. Without any such information, one way or the other, we could only abstain from any opinion as to the presence of the substance in question. It cannot, I conceive, be maintained that we are in any *better* position than this in regard to the presence of the element of chance or spontaneous departures from law in nature.

Peirce: CP 6.46 Cross-Ref:††

46. Those observations which are generally adduced in favor of mechanical causation simply prove that there is an element of regularity in nature, and have no bearing whatever upon the question of whether such regularity is exact and universal or not. Nay, in regard to this *exactitude*, all observation is directly *opposed* to it; and the most that can be said is that a good deal of this observation can be explained away. Try to verify any law of nature, and you will find that the more precise your observations, the more certain they will be to show irregular departures from the law. We are accustomed to ascribe these, and I do not say wrongly, to errors of observation; yet we cannot usually account for such errors in any antecedently probable way. Trace their causes back far enough and you will be forced to admit they are always due to arbitrary determination, or chance.

Peirce: CP 6.47 Cross-Ref:††
§4. ABSOLUTE CHANCE

47. But it may be asked whether if there were an element of real chance in the universe it must not occasionally be productive of signal effects such as could not pass unobserved. In answer to this question, without stopping to point out that there is an abundance of great events which one might be tempted to suppose were of that nature, it will be simplest to remark that physicists hold that the particles of gases are moving about irregularly, substantially as if by real chance, and that by the principles of probabilities there must occasionally happen to be concentrations of heat in the gases contrary to the second law of thermodynamics, and these concentrations, occurring in explosive mixtures, must sometimes have tremendous effects. Here, then, is in substance the very situation supposed; yet no phenomena ever have resulted which we are forced to attribute to such chance concentration of heat, or which anybody, wise or foolish, has ever dreamed of accounting for in that manner.

Peirce: CP 6.48 Cross-Ref:††

48. In view of all these considerations, I do not believe that anybody, not in a state of case-hardened ignorance respecting the logic of science, can maintain that the precise and universal conformity of facts to law is clearly proved, or even rendered particularly probable, by any observations hitherto made. In this way, the determined advocate of exact regularity will soon find himself driven to *a priori* reasons to

support his thesis. These received such a socdolager from Stuart Mill †1 in his examination of Hamilton, that holding to them now seems to me to denote a high degree of imperviousness to reason, so that I shall pass them by with little notice.

Peirce: CP 6.49 Cross-Ref:††

49. To say that we cannot help believing a given proposition is no argument, but it is a conclusive fact if it be true; and with the substitution of "I" for "we," it is true in the mouths of several classes of minds: the blindly passionate, the unreflecting and ignorant, and the person who has overwhelming evidence before his eyes. But that which has been inconceivable today has often turned out indisputable on the morrow. Inability to conceive is only a stage through which every man must pass in regard to a number of beliefs -- unless endowed with extraordinary obstinacy and obtuseness. His understanding is enslaved to some blind compulsion which a vigorous mind is pretty sure soon to cast off.

Peirce: CP 6.50 Cross-Ref:††

50.†1 Some seek to back up the *a priori* position with empirical arguments. They say that the exact regularity of the world is a natural belief, and that natural beliefs have generally been confirmed by experience. There is some reason in this. Natural beliefs, however, if they generally have a foundation of truth, also require correction and purification from natural illusions. The principles of mechanics are undoubtedly natural beliefs; but, for all that, the early formulations of them were exceedingly erroneous. The general approximation to truth in natural beliefs is, in fact, a case of the general adaptation of genetic products to recognizable utilities or ends. Now, the adaptations of nature, beautiful and often marvelous as they verily are, are never found to be quite perfect; so that the argument is quite *against* the absolute exactitude of any natural belief, including that of the principle of causation.

Peirce: CP 6.51 Cross-Ref:††

51. Another argument, or convenient commonplace, is that absolute chance is *inconceivable*. This word has eight current significations. The *Century Dictionary*†2 enumerates six. Those who talk like this will hardly be persuaded to say in what sense they mean that chance is inconceivable. Should they do so, it would easily be shown either that they have no sufficient reason for the statement or that the inconceivability is of a kind which does not prove that chance is non-existent.

Peirce: CP 6.52 Cross-Ref:††

52.†3 Another *a priori* argument is that chance is unintelligible; that is to say, while it may perhaps be conceivable, it does not disclose to the eye of reason the how or why of things; and since a hypothesis can only be justified so far as it renders some phenomenon intelligible, we never can have any right to suppose absolute chance to enter into the production of anything in nature. This argument may be considered in connection with two others. Namely, instead of going so far as to say that the supposition of chance can *never* properly be used to explain any observed fact, it may be alleged merely that no facts are known which such a supposition could in any way help in explaining. Or again, the allegation being still further weakened, it may be said that since departures from law are not unmistakably observed, chance is not a *vera causa*,†1 and ought not unnecessarily to be introduced into a hypothesis.

Peirce: CP 6.53 Cross-Ref:††

53. These are no mean arguments, and require us to examine the matter a little more closely. Come, my superior opponent, let me learn from your wisdom. It seems to me that every throw of sixes with a pair of dice is a manifest instance of chance.

Peirce: CP 6.53 Cross-Ref:††

"While you would hold a throw of deuce-ace to be brought about by necessity?" (The opponent's supposed remarks are placed in quotation marks.)

Peirce: CP 6.53 Cross-Ref:††

Clearly one throw is as much chance as another.

Peirce: CP 6.53 Cross-Ref:††

"Do you think throws of dice are of a different nature from other events?"

Peirce: CP 6.53 Cross-Ref:††

I see that I must say that *all* the diversity and specificness of events is attributable to chance.

Peirce: CP 6.54 Cross-Ref:††

54. "Would you, then, deny that there is any regularity in the world?"

Peirce: CP 6.54 Cross-Ref:††

That is clearly undeniable. I must acknowledge there is an approximate regularity, and that every event is influenced by it. But the diversification, specificness, and irregularity of things I suppose is chance. A throw of sixes appears to me a case in which this element is particularly obtrusive.

Peirce: CP 6.54 Cross-Ref:††

"If you reflect more deeply, you will come to see that *chance* is only a name for a cause that is unknown to us."

Peirce: CP 6.54 Cross-Ref:††

Do you mean that we have no idea whatever what kind of causes could bring about a throw of sixes?

Peirce: CP 6.54 Cross-Ref:††

"On the contrary, each die moves under the influence of precise mechanical laws."

Peirce: CP 6.55 Cross-Ref:††

55. But it appears to me that it is not these *laws* which made the die turn up sixes; for these laws act just the same when other throws come up. The chance lies in the diversity of throws; and this diversity cannot be due to laws which are immutable.

Peirce: CP 6.55 Cross-Ref:††

"The diversity is due to the diverse circumstances under which the laws act. The dice lie differently in the box, and the motion given to the box is different. These are the unknown causes which produce the throws, and to which we give the name of chance; not the mechanical law which regulates the operation of these causes. You see you are already beginning to think more clearly about this subject."

Peirce: CP 6.56 Cross-Ref:††

56. Does the operation of mechanical law not increase the diversity?

Peirce: CP 6.56 Cross-Ref:††

"Properly not. You must know that the instantaneous state of a system of particles is defined by six times as many numbers as there are particles, three for the coördinates of each particle's position, and three more for the components of its velocity. This number of numbers, which expresses the amount of diversity in the

system, remains the same at all times. There may be, to be sure, some kind of relation between the coordinates and component velocities of the different particles, by means of which the state of the system might be expressed by a smaller number of numbers. But, if this is the case, a precisely corresponding relationship must exist between the coordinates and component velocities at any other time, though it may doubtless be a relation less obvious to us. Thus, the intrinsic complexity of the system is the same at all times."

Peirce: CP 6.57 Cross-Ref:††

57. Very well, my obliging opponent, we have now reached an issue. You think all the arbitrary specifications of the universe were introduced in one dose, in the beginning, if there was a beginning, and that the variety and complication of nature has always been just as much as it is now. But I, for my part, think that the diversification, the specification, has been continually taking place. Should you condescend to ask me why I so think, I should give my reasons as follows:

Peirce: CP 6.58 Cross-Ref:††

58. (1) Question any science which deals with the course of time. Consider the life of an individual animal or plant, or of a mind. Glance at the history of states, of institutions, of language, of ideas. Examine the successions of forms shown by paleontology, the history of the globe as set forth in geology, of what the astronomer is able to make out concerning the changes of stellar systems. Everywhere the main fact is growth and increasing complexity. Death and corruption are mere accidents or secondary phenomena. Among some of the lower organisms, it is a moot point with biologists whether there be anything which ought to be called death. Races, at any rate, do not die out except under unfavorable circumstances. From these broad and ubiquitous facts we may fairly infer, by the most unexceptionable logic, that there is probably in nature some agency by which the complexity and diversity of things can be increased; and that consequently the rule of mechanical necessity meets in some way with interference.

Peirce: CP 6.59 Cross-Ref:††

59. (2) By thus admitting pure spontaneity or life as a character of the universe, acting always and everywhere though restrained within narrow bounds by law, producing infinitesimal departures from law continually, and great ones with infinite infrequency, I account for all the variety and diversity of the universe, in the only sense in which the really sui generis and new can be said to be accounted for. The ordinary view has to admit the inexhaustible multitudinous variety of the world, has to admit that its mechanical law cannot account for this in the least, that variety can spring only from spontaneity, and yet denies without any evidence or reason the existence of this spontaneity, or else shoves it back to the beginning of time and supposes it dead ever since. The superior logic of my view appears to me not easily controverted.

Peirce: CP 6.60 Cross-Ref:††

60. (3) When I ask the necessitarian how he would explain the diversity and irregularity of the universe, he replies to me out of the treasury of his wisdom that irregularity is something which from the nature of things we must not seek to explain. Abashed at this, I seek to cover my confusion by asking how he would explain the uniformity and regularity of the universe, whereupon he tells me that the laws of nature are immutable and ultimate facts, and no account is to be given of them. But my hypothesis of spontaneity does explain irregularity, in a certain sense; that is, it explains the general fact of irregularity, though not, of course, what each lawless

event is to be. At the same time, by thus loosening the bond of necessity, it gives room for the influence of another kind of causation, such as seems to be operative in the mind in the formation of associations, and enables us to understand how the uniformity of nature could have been brought about. That single events should be hard and unintelligible, logic will permit without difficulty: we do not expect to make the shock of a personally experienced earthquake appear natural and reasonable by any amount of cogitation. But logic does expect things general to be understandable. To say that there is a universal law, and that it is a hard, ultimate, unintelligible fact, the why and wherefore of which can never be inquired into, at this a sound logic will revolt, and will pass over at once to a method of philosophizing which does not thus barricade the road of discovery.

Peirce: CP 6.61 Cross-Ref:††

61. (4) Necessitarianism cannot logically stop short of making the whole action of the mind a part of the physical universe. Our notion that we decide what we are going to do, if, as the necessitarian says, it has been calculable since the earliest times, is reduced to illusion. Indeed, consciousness in general thus becomes a mere illusory aspect of a material system. What we call red, green, and violet are in reality only different rates of vibration. The sole reality is the distribution of qualities of matter in space and time. Brain-matter is protoplasm in a certain degree and kind of complication -- a certain arrangement of mechanical particles. Its feeling is but an inward aspect, a phantom. For, from the positions and velocities of the particles at any one instant, and the knowledge of the immutable forces, the positions at all other times are calculable; so that the universe of space, time, and matter is a rounded system uninterfered with from elsewhere. But, from the state of feeling at any instant, there is no reason to suppose the states of feeling at all other instants are thus exactly calculable; so that feeling is, as I said, a mere fragmentary and illusive aspect of the universe. This is the way, then, that necessitarianism has to make up its accounts. It enters consciousness under the head of sundries, as a forgotten trifle; its scheme of the universe would be more satisfactory if this little fact could be dropped out of sight. On the other hand, by supposing the rigid exactitude of causation to yield, I care not how little -- be it but by a strictly infinitesimal amount -- we gain room to insert mind into our scheme, and to put it into the place where it is needed, into the position which, as the sole self-intelligible thing, it is entitled to occupy, that of the fountain of existence; and in so doing we resolve the problem of the connection of soul and body.†1

Peirce: CP 6.62 Cross-Ref:††

62. (5) But I must leave undeveloped the chief of my reasons, and can only adumbrate it. The hypothesis of chance-spontaneity is one whose inevitable consequences are capable of being traced out with mathematical precision into considerable detail. Much of this I have done and find the consequences to agree with observed facts to an extent which seems to me remarkable.†2 But the matter and methods of reasoning are novel, and I have no right to promise that other mathematicians shall find my deductions as satisfactory as I myself do, so that the strongest reason for my belief must for the present remain a private reason of my own, and cannot influence others. I mention it to explain my own position; and partly to indicate to future mathematical speculators a veritable gold mine, should time and circumstances and the abridger of all joys prevent my opening it to the world.

Peirce: CP 6.63 Cross-Ref:††

63. If now I, in my turn, inquire of the necessitarian why he prefers to suppose

that all specification goes back to the beginning of things, he will answer me with one of those last three arguments which I left unanswered.

Peirce: CP 6.63 Cross-Ref:††

First, he may say that chance is a thing absolutely unintelligible, and therefore that we never can be entitled to make such a supposition. But does not this objection smack of naive impudence? It is not mine, it is his own conception of the universe which leads abruptly up to hard, ultimate, inexplicable, immutable law, on the one hand, and to inexplicable specification and diversification of circumstances on the other. My view, on the contrary, hypothesizes nothing at all, unless it be hypothesis to say that all specification came about in some sense, and is not to be accepted as unaccountable. To undertake to account for anything by saying baldly †3 that it is due to chance would, indeed, be futile. But this I do not do. I make use of chance chiefly to make room for a principle of generalization, or tendency to form habits, which I hold has produced all regularities. The mechanical philosopher leaves the whole specification of the world utterly unaccounted for, which is pretty nearly as bad as to baldly attribute it to chance. I attribute it altogether to chance, it is true, but to chance in the form of a spontaneity which is to some degree regular. It seems to me clear at any rate that one of these two positions must be taken, or else specification must be supposed due to a spontaneity which develops itself in a certain and not in a chance way, by an objective logic like that of Hegel. This last way I leave as an open possibility, for the present; †1 for it is as much opposed to the necessitarian scheme of existence as my own theory is.

Peirce: CP 6.64 Cross-Ref:††

64. Secondly, the necessitarian may say there are, at any rate, no observed phenomena which the hypothesis of chance could aid in explaining. In reply, I point first to the phenomenon of growth and developing complexity, which appears to be universal, and which, though it may possibly be an affair of mechanism perhaps, certainly presents all the appearance of increasing diversification. Then, there is variety itself, beyond comparison the most obtrusive character of the universe: no mechanism can account for this. Then, there is the very fact the necessitarian most insists upon, the regularity of the universe which for him serves only to block the road of inquiry. Then, there are the regular relationships between the laws of nature -- similarities and comparative characters, which appeal to our intelligence as its cousins, and call upon us for a reason. Finally, there is consciousness, feeling, a patent fact enough, but a very inconvenient one to the mechanical philosopher.

Peirce: CP 6.65 Cross-Ref:††

65. Thirdly, the necessitarian may say that chance is not a *vera causa*, that we cannot know positively there is any such element in the universe. But the doctrine of the *vera causa* has nothing to do with elementary conceptions. Pushed to that extreme, it at once cuts off belief in the existence of a material universe; and without that necessitarianism could hardly maintain its ground. Besides, variety is a fact which must be admitted; and the theory of chance merely consists in supposing this diversification does not antedate all time. Moreover, the avoidance of hypotheses involving causes nowhere positively known to act is only a recommendation of logic, not a positive command. It cannot be formulated in any precise terms without at once betraying its untenable character -- I mean as rigid rule, for as a recommendation it is wholesome enough.

Peirce: CP 6.65 Cross-Ref:††

I believe I have thus subjected to fair examination all the important reasons for

adhering to the theory of universal necessity, and have shown their nullity. I earnestly beg that whoever may detect any flaw in my reasoning will point it out to me, either privately or publicly; for, if I am wrong, it much concerns me to be set right speedily. If my argument remains unrefuted, it will be time, I think, to doubt the absolute truth of the principle of universal law; and when once such a doubt has obtained a living root in any man's mind, my cause with him, I am persuaded, is gained.†1

Peirce: CP 6.66 Cross-Ref:††
CHAPTER 3

CAUSATION AND FORCE†1P

§1. PHYSICAL CAUSATION

66. Those who make causality one of the original *uralt* elements in the universe or one of the fundamental categories of thought -- of whom you will find that I am not one -- have one very awkward fact to explain away. It is that men's conceptions of a Cause are in different stages of scientific culture entirely different and inconsistent. The great principle of causation which, we are told, it is absolutely impossible not to believe, has been one proposition at one period of history and an entirely disparate one [at] another and is still a third one for the modern physicist. The only thing about it which has stood, to use my friend Carus's word, a {ktéma ex aei}, *semper eadem*, is the name of it. As Aristotle †2 remarks, what the Ionian philosophers were trying to find out as the principles of things was what they were made of. Aristotle himself, as I need not remind you, recognizes four distinct kinds of cause,†3 which go to determining a fact: the *matter* to which it owes its existence, the *form* to which it owes its nature, the *efficient cause* which acts upon it from past time, and the *final cause* which acts upon it from future time. Oh, but it is commonly said, these are merely verbal distinctions. This to my apprehension is one of those superficial explanations, which pass current till men examine them, and serve, like the elegant bankers' memorandum, *pour donner le change* to the unwary. They seem to me to mark different types of retroductively †4 inferred facts -- facts, which it was supposed, furnished the universal process of Nature the occasions from which different features of the fact were brought about. The conception is that Nature syllogizes from one grand major premiss, and the causes are the different minor premisses of nature's syllogistic development. It is generally held that the word "cause" has simply been narrowed to that one of the four Aristotelian causes which was named from the circumstance that it alone produces an *effect*. But this notion that our conception of cause is that of the Aristotelian efficient cause will hardly bear examination. The efficient cause was, in the first place, generally a thing, not an event; then, something which need not do anything; its mere existence might be sufficient. Neither did the effect always necessarily follow. True when it did follow it was said to be compelled. But it was not necessary in our modern sense. That is, it was not invariable. Even in ancient literature we occasionally meet with the idea that

a cause is an event of such a kind as to be necessarily followed by another event which is the effect. This is the current idea, now. But it is only in the last two centuries that it has become the dominant conception. It is not so with the most accurate thinkers of the time of Descartes.

Peirce: CP 6.67 Cross-Ref:††

67. Those whose admiration for John Stuart Mill knows no bounds consider it one of his most admirable *aperçues* that he regards the cause as the aggregate of all the circumstances under which an event occurs.†1 Whether it be admirable or not, it was certainly a commonplace remark before John Mill ever set pen to paper. But the truth is that the remark is founded upon a misconception. So far as the conception of cause has any validity -- that is to say, as I shall show you -- in a limited domain, the cause and its effect are two *facts*. Now, Mill seems to have thoughtlessly or nominalistically assumed that a fact is the very objective history of the universe for a short time, in its objective state of existence in itself. But that is not what a fact is.†2 A fact is an abstracted element of that. A fact is so much of the reality as is represented in a single *proposition*. If a proposition is true, that which it represents is a *fact*. If, according to a true law of nature as major premiss, it syllogistically follows from the truth of one proposition that another is true, then that abstracted part of the reality which the former proposition represents is the *cause* of the corresponding element of reality represented by the latter proposition. Thus, the fact that a body is moving over a rough surface is the cause of its coming to rest. It is absurd to say that its color is any part of the cause or of the effect. The color is a part of the reality; but it does not belong to those parts of the reality which constitute the two *facts* in question.

Peirce: CP 6.68 Cross-Ref:††

68. But the grand principle of causation which is generally held to be the most certain of all truths and literally beyond the possibility of doubt (so much so that if a scientific man seeks to limit its truth it is thought pertinent to attack his sincerity and moral character generally) involves three propositions to which I beg your particular attention. The first is, that the state of things at any one instant is completely and exactly determined by the state of things at one other instant. The second is that the cause, or determining state of things, precedes the effect or determined state of things in time. The third is that no fact determines a fact *preceding* it in time in the same sense in which it determines a fact *following* it in time. These propositions are generally held to be self-evident truths; but it is further urged that whether they be so or not, they are indubitably proved by modern science. In truth, however, all three of them are in flat contradiction to the principles of mechanics. According to the dominant mechanical philosophy, nothing is real in the physical universe except particles of matter with their *masses*, their relative *positions* in space at different instants of *time*, and the immutable laws of the relations of those three elements of space, time, and matter. Accordingly, at any one *instant* all that is real is the masses and their positions, together with the laws of their motion. But according to Newton's second law of motion the positions of the masses at any one instant are not determined by their positions at any other single instant, even with the aid of the laws. On the contrary, that which is determined is an acceleration. Now an acceleration is the relation of the position at one instant *not* to the position at another instant, but to the positions at a second and a third instant. Let *a*, *b*, *c* be the positions of a particle at three instants very near to one another, and at equal intervals of time, say, for convenience, one second.

Peirce: CP 6.68 Cross-Ref:††

Then we may make a table thus:

<i>Dates</i>	<i>Positions</i>	<i>Velocities</i>	<i>Acceleration</i>
0^s	a	$(b-a)/(1^s-0^s)$ $(c-b)/(2^s-1^s)$	$[(c-b)/(2^s-1^s) - (b-a)/(1^s-0^s)] / (1^{1/2^s} - 0^{1/2^s}) = (c-2b+a)/(1^{1/2^s} - 0^{1/2^s})_2$
1^s	b		
2^s	c		

[Click here to view]

Or if the intervals are not equal:

<i>Dates</i>	<i>Positions</i>	<i>Velocities</i>	<i>Acceleration</i>
t_0	a	$(b-a)/(t_1-t_0)$ $(c-b)/(t_2-t_1)$	$[(c-b)/(t_2-t_1) - (b-a)/(t_1-t_0)] / (\frac{1}{2}(t_2+t_1) - \frac{1}{2}(t_1+t_0)) = [c(t_1-t_0) - b(t_2-t_0) + a(t_2-t_1)] / (t_2-t_1)(t_1-t_0) \cdot \frac{1}{2}(t_2-2t_1+t_0)^*$
t_1	b		
t_2	c		

[Click here to view]†1

Peirce: CP 6.69 Cross-Ref:††

69. It will be perceived that there is an essential *thirdness*, which the principle of causality fails to recognize, so that its first proposition is false. The second proposition, that the cause precedes the effect in time, is equally false. The effect is the acceleration. The cause which produces this effect under the law of force is, according to the doctrine of the conservation of energy, the relative positions of the particles. Now the acceleration which the position requires does not come *later* than the assumption of that position. It is, on the contrary, absolutely simultaneous with it. Thus, the second proposition of the principle of causation is false. The third is equally so. This proposition is that no event determines a previous event in the same sense in which it determines a subsequent one. But, according to the law of the conservation of energy, the position of the particle relative to the center of force, expressed by b , determines what the acceleration shall be at the moment the particle is in that position. That is to say, taking the number b , whose value expresses the position of the particle, we can calculate from this number alone, by the application of a rule supplied by the law of the force, a number which I may denote by Fb , which is the numerical value of the acceleration $(c-2b+a)/((1/2^s-0/2^s)^2)$. So that we have the equation $(c-2b+a)/((1/2^s-0/2^s)^2) = Fb$. Now, if we know the positions, a and b , of the particle at the two earlier dates, this equation does enable us to calculate the position, c , of the particle at the last date. But since a and c enter into this equation in the same way, and since the difference of dates in the denominator is squared, so that if they are interchanged it makes no difference (because the square of the negative of

a number is the [square of the] number itself), it follows that the very same rule, by which we could calculate the value of *c*, that is, the position at the latest of the three dates, from *a* and *b*, those at the two earlier dates, may usually be applied, and in precisely the same form, to calculating the position, *a*, of the particle at the earliest date, from *c* and *b*, its positions at the two later dates. Thus, we see that, according to the law of energy, the positions at the two later instants determine the position at the earliest instant, in precisely the same way, and no other, in which the positions at the two earlier instants determine the position at the latest instant. In short, so far as phenomena governed by the law of the conservation of energy are concerned, the future determines the past in precisely the same way in which the past determines the future; and for those cases, at least, it is a mere human and subjective fashion of looking at things which makes us prefer one of those modes of statement to the other. Thus, all three of the propositions involved in the principle of causation are in flat contradiction to the science of mechanics.

Peirce: CP 6.70 Cross-Ref:††
§2. PSYCHICAL CAUSATION

70. But when from the world of physical force we turn to the psychical world all is entirely different. Here we find no evident trace of any state of mind depending in opposite ways upon two previous states of mind. Every state of mind, acting under an overruling association, produces another state of mind. Or if different states of mind contribute to producing another, they simply act concurrently, and not in opposite ways, as the two earlier positions of a particle of matter do, in determining a third position. I come down in the morning; and the sight of the newspaper makes me think of the *Maine*, the breakfast is brought in, and the sight of something I like puts me into a state of cheerful appetite; and so it goes all day long. Moreover, the effect is not simultaneous with the cause. I do not think of the explosion of the *Maine* simultaneously with seeing the newspaper, but after seeing it, though the interval be but a thirtieth of a second. Furthermore, the relations of the present to the past and to the future, instead of being the same, as in the domain of the Law of Energy, are utterly unlike. I remember the past, but I have absolutely no slightest approach to such knowledge of the future. On the other hand I have considerable *power* over the future, but nobody except the Parisian mob imagines that he can change the past by much or by little. Thus all three propositions of the law of causation are here fully borne out.

Peirce: CP 6.71 Cross-Ref:††
§3. NON-CONSERVATIVE FORCES

71. Even supposing the physical and the psychical laws not to be precisely as they seem to be, yet, though the gulph between the two worlds would not be of so absolute a nature, still in regard to the general features we cannot be mistaken.

Peirce: CP 6.71 Cross-Ref:††

But further than that, we can assert that not only is the psychical world within us governed by the law of causation, but even phenomena of psychical interest

without us, even those of inanimate matter so far as they attract everyday notice, either are, or have the *semblance of being*, under the same governance. In order to bring this highly significant fact into evidence, it will be necessary for me to explain two characteristics of phenomena that are determined by forces obeying the law of the conservation of energy. I am sorry that I shall once more be obliged to employ some very simple algebra. The first of the two characteristics I speak of is this, that if any force obeying the law of the conservation of energy or, as we usually say, any *conservative force* (that is, any force whose value depends exclusively on the situation of the body acted on relatively to the bodies that act on it), if any such force, I say, can produce any *given motion*, then the very same force can equally produce the reverse motion. That is to say, if at any one instant all the particles were to strike fixed plastic plane surfaces, and were to strike them square, so as to rebound in the directions from which they came and with unchanged velocities, each would move backward through precisely the same path that it had moved forward, and with the same velocities, no matter for how long a time the motion might have been going on. This really follows from what I have shown, that conservative force determines the past in the same way that it determines the future. An extremely elementary demonstration would be easy; but I omit it to save time. The other characteristic of conservative force is, as its name implies, that the Energy is conserved, that is, that the *living force* or square of the velocity of a particle is simply a function of the position relative to the interacting particles, the exact function depending on the nature of the force *plus* a quantity constant throughout the motion, which has a value depending on the accidents of the particular case. You are so familiar with this that I will not waste time in proving it. But I will mention that it readily follows from the fact, that a second difference multiplied by the sum of the two adjacent first differences is equal to the difference of the squares of those differences, which is obvious.

For since $\{D\}^2 = \{D\}[2] - \{D\}[1]$ obviously $(\{D\}[1] + \{D\}[2])\{D\}[2]^2 = (\{D\}[2])^2 - (\{D\}[1])^2$.

Peirce: CP 6.72 Cross-Ref:††

72. Now employing these two characteristics, and especially the former, as criteria, we at once recognize that almost all the phenomena of bodies here on earth which attract our familiar notice are non-conservative, that is, are inexplicable by means of the Law of the Conservation of Energy. For they are actions which cannot be reversed. In the language of physics they are irreversible. Such, for instance, is birth, growth, life. Such is all motion resisted by friction or by the viscosity of fluids, as all terrestrial motion is. Such is the conduction of heat, combustion, capillarity, diffusion of fluids. Such is the thunder bolt, the production of high colors by a prism, the flow of rivers, the formations of bars at their mouths, the wearing of their channels; in short, substantially everything that ordinary experience reveals, except the motions of the stars. And even those we do not see to be reversed, though we may well believe them reversible. About the only familiar actions which appear to sense reversible are the motion of a projectile, the bending of a bow or other spring, a freely swinging pendulum, a telephone, a microphone, a galvanic battery, and a dynamo. And all but two of these are unfamiliar to man in his early development. No wonder the doctrine of the conservation of energy was a late discovery.

Peirce: CP 6.73 Cross-Ref:††

73. It is certainly a desideratum in philosophy to unify the phenomena of mind and matter. The logic of retrodution directs us to adopt Monism as a provisional hypothesis of philosophy, whether we think it likely or not; and not to abandon it till the position is stormed and we are forced out of it. In view of this, it becomes exceedingly interesting to inquire how the physicist explains those actions which seem to violate the law of energy. Now such of them as physicists have deeply studied are all explained by the action of *chance*.

Peirce: CP 6.73 Cross-Ref:††

For example, if one horizontal layer of air moves northerly, passing over another layer at rest, the reason why the northerly current will be retarded is that the molecules are flying about in all directions and hence chance will carry a good many of them from one layer to the other. And these chance molecules, so carried from either layer to the other, will be so numerous that it is practically certain that, on the average, they will have as much northerly motion as the average of all the molecules in the layer from which they have emerged. Thus, after a while, the average northerly motion of the molecules in each layer approximates toward that of the other layer. And to say that the average northerly motion of the molecules of the upper layer becomes less is the same as to say that the northerly motion of that layer as a whole becomes less. For the motion of the layer as a whole is nothing but the average motion of its molecules.

Peirce: CP 6.74 Cross-Ref:††

§4. FORTUITOUS DISTRIBUTIONS

74. Now in order that we may make any application of this method, of explaining non-conservative quasi-forces, to psychical phenomena it is necessary to make an exact analysis and description of its essential elements, omitting all circumstances that do not contribute to the effect. To this end, the first requisite is a definition of *Chance*, not as to the causes that produce it, but as to the phenomenon itself. Surely, I need not waste breath in refuting that feeblest of attempts at analysis which makes chance to consist in our ignorance. For that has already been sufficiently done in the *Logic of Chance* of John Venn, a logician, some of whose opinions may be untenable, but whose thought is apt to penetrate beneath the form to the matter he discusses -- and after examining a hundred or two logical treatises one begins to think that a high distinction. It is the operation of chance which produces the retardation of the upper layer of air we were just considering; but surely it is no ignorance of ours that has that effect. Chance, then, as an objective phenomenon, is a property of a *distribution*. That is to say, there is a large collection consisting, say, of colored things and of white things. Chance is a particular manner of distribution of color among all the things. But in order that this phrase should have any meaning, it must refer to some definite arrangement of all the things.

Peirce: CP 6.75 Cross-Ref:††

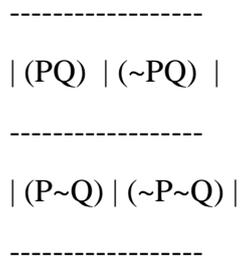
75. Let us begin by supposing that the multitude of colored things is *denumeral*, †1 and that of the white things is likewise *denumeral*. The denumeral multitude as I explained in a former lecture is that of all the whole numbers. Every denumeral collection may be numbered. That is, the number 1 may be affixed to one of its objects, 2 to another, and so on in such a way that every object of the collection

receives a number. When that is done I call the relation of an object, receiving any number but 1, to the object receiving the next lower number, a *generating relation* of the collection. It is by no means indispensable to introduce any mention of the numbers in defining a generating relation. I only do so for the sake of using ideas with which you are familiar and thus save time and trouble. Now I must define the important conception of *independence*, which incessantly recurs in the doctrine of chances.†2 A character, say blueness, is said to be independent of a character, say smoothness, in a given collection if and only if the ratio of the multitude of those objects (PQ) of the collection that are both blue and smooth *to* the multitude of those objects (P~Q) of the collection that are blue but not smooth equals the ratio of the multitude of objects (~PQ) that are not blue but are smooth *to* the multitude of objects (~P~Q) that are not blue and not smooth. Mr. Jevons †1 makes a fuss about proving that if P is independent of Q, so is Q of P. It is because in the proportion

$$(PQ):(P\sim Q) = (\sim PQ):(\sim P\sim Q),$$

we can transpose the means, giving

$$(PQ):(\sim PQ) = (P\sim Q):(\sim P\sim Q)$$



Peirce: CP 6.75 Cross-Ref:††

Now in our collection of denumeral colored things and denumeral white things, let *F* signify a particular generating relation, so that when the objects are numbered, according to that relation, the object numbered *n* + 1 is *F* of the object numbered *n*. Then, I say that a fortuitous distribution of color and whiteness in the collection consists in this, that any object of the collection being colored or not is independent of its being an *F* of a colored thing, and is also independent of its being an *F* of an *F* of a colored thing, and is also independent of its being at once an *F* of a colored thing and an *F* of an *F* of a white thing; and in short that an object's being colored or not is independent of its having or not having any character definable in terms of *F*, color, and whiteness. That satisfactorily defines a *fortuitous distribution* when the colored things and white things are both denumeral.†2

Peirce: CP 6.76 Cross-Ref:††

76. When either or both the two subcollections of colored things and white things are *enumerable*, that is, finite in number, such independence, as the definition requires, becomes impossible. Nevertheless, if both are large enumerable collections, there may be an approximation to the fulfillment of the definition, and then we loosely call the distribution fortuitous. If, for example, there are 500,000 colored things and 500,000 white things, then of all possible modes of sequences of 20 successive objects as to their being colored or white there will be only about one of each example. Therefore we cannot say that an object's being colored or not is independent of the sequence of color and whiteness among the twenty objects that precede it, for one of the four terms of the proportion that defines independence will probably be zero. On the other hand there will be about a thousand occurrences of each possible mode of sequence of 10 objects as to being colored or white and if, from 1 object up to 10 objects, the required proportionality is nearly fulfilled, there will be no harm in calling the distribution fortuitous.

Peirce: CP 6.77 Cross-Ref:††

77. In comparing two infinite collections we have to distinguish between one being *inclusive* of more or less than the other and one being more or less multitudinous than the other. I call a collection *inclusive of* more than another if it includes all the objects of the latter and others besides; but to say that one collection, say the simpletons, is more multitudinous than another, say the sages, means that to every sage a distinct simpleton might be assigned, and assigned to no other sage, while it would be impossible to assign to every simpleton a distinct sage for him alone. Two collections may neither of them be *inclusive* of all the other *includes*, as for example the Buddhists and the Japanese; but they cannot each be *inclusive of ALL* the other includes unless they are identically the same. On the other hand, of any two collections whatsoever, one must be at least as multitudinous as the other, and each *may be* so. That is, they may be equal. Of equal collections one may be inclusive of more than the other; but the less multitudinous of two collections cannot be the more inclusive. All these propositions except one are easily proved; and that one is proved in the *Monist*. †1

Peirce: CP 6.78 Cross-Ref:††

78. If of the two subcollections, the colored things and the white things, one is denumeral while the other is more than denumeral, we may still speak, and sometimes do speak, of a fortuitous distribution. It is true that for a collection more than denumeral there can be no generating relation. But still, unless the total collection is a continuum of more than one dimension, with or without topical singularities, all the objects of it may be placed in a sequence, at any rate by means of a relatively insignificant multitude of ruptures and junctions. It must be understood that the fortuitousness refers to the particular way in which the objects are placed in sequence. It must furthermore be understood that by a definite mode the whole sequence is broken up into a denumeral collection of subcollections, and the fortuitousness is relative to that mode of breaking up, and moreover this mode of dissection must be capable of a particular mode of variation such that the subcollections may be made all at once inclusive of less and less without limit, and the fortuitousness is still further relative to that mode of shrinking. If, then, no matter how small these subcollections are taken, the character of a subcollection containing a blue thing or not containing a blue thing is independent of that subcollection having any character definable in terms of the generating relation of the denumeral collection, of containing a blue thing and of not containing a colored thing, then the distribution is fortuitous. For example, we may say that certain marked points are fortuitously distributed upon an infinitely long

line, meaning that if that line is cut up into a denumeral series of lengths, no matter how small, the lengths containing marked points will be fortuitously distributed along the whole series of lengths.

Peirce: CP 6.78 Cross-Ref:††

We might speak of a finite number of points being fortuitously distributed upon the circumference of a circle, meaning an approximate fortuitous distribution. When we say that a finite number [of] points are distributed *at random* on the circumference, that is quite another matter. We then have in mind a fortuitous distribution, it is true, but it is a fortuitous distribution of the denumeral cases in which a man might, in the course of all time, throw points down upon the circumference.

Peirce: CP 6.78 Cross-Ref:††

I do not say that no sense could be attached to the term *fortuitous distribution* in case both the blue things and the white things were more than denumeral. On the contrary, the difficulty is that several senses might be attached to the phrase, and, having no experience in that line of thought, I am not prepared to say which one would be more appropriate. I therefore pass that case by.

Peirce: CP 6.79 Cross-Ref:††

79. We have now determined precisely what Chance, as an objective phenomenon, consists in. In works on probabilities (of which I particularly recommend that of Laurent †1 as being brief and clear and yet at the same time scientific) very beautiful and valuable properties of the fortuitous distribution will be found traced out, especially that which relates to the probability curve.

Peirce: CP 6.79 Cross-Ref:††

In the fortuitous distribution the colored things and the white things are mixed up together with an irregularity which is perfect. It is the very highest pitch of irregularity. Departures from this, or regularities, may tend in either of two directions. On the one hand they may mix the colored things and white things more perfectly and uniformly, as when colored things and white things alternate, or they may sift them out, as when all the colored things come in one series and all the white things in another. Even the alternation might be called a sifting, for it puts all the colored things into the odd places and all the white things into the even places, and these constitute two distinct series. Still, having the word *regularity* for that, we may as well restrict the word *sifting* so as to enable us to express the less fundamental, but still not altogether unimportant, distinction between leaving the two series mingled and separating them.

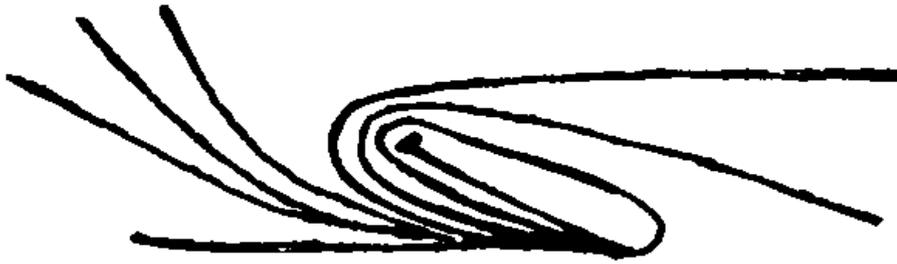
Peirce: CP 6.80 Cross-Ref:††

80. Let us glance for a moment at the ways in which the three states of things -- siftedness, uniform combination, and fortuitous irregularity of mixture -- are in fact brought about in nature; and then in the light of these examples we shall be able to see how they could conceivably be brought about.

Peirce: CP 6.80 Cross-Ref:††

Sifting is performed by any conservative force quite inevitably. For example, a ray of white light strikes upon a prism. The different colors have different refrangibilities and the light is decomposed. The action is conservative because it is reversible. For if the dispersed light were reflected back upon its course it would be recompounded. But this does not happen except in the laboratory and that only

imperfectly, when it is due to the elaborate contrivance of the experimenter. Conservative force, left to itself, can produce no such result, because it depends on the *purposeful* exact adjustment of each pencil of light. Now one of the first things that the mechanical philosophy discovered was that there are no final causes in pure mechanical action. In the same way, were a great number of meteors to start from the same almost infinitely distant point, all moving in the same direction so as to bring them within the sun's strong attraction, but were they to move with various velocities, the sun's attraction would separate their motions so that when they departed again they would all be arranged in the order of their velocities; the one with no velocity returning just as it came, the one with infinite velocity proceeding in a right line unchecked,



[Click here

to view] and all the rest in more or less bent paths.

Peirce: CP 6.81 Cross-Ref:††

81. So much for the sifting. Let us next consider how a state of *fortuitous distribution* is brought about. How, for example, is it that the throws of a dice occur in the utterly irregular way in which they do? It is because when we turn over the dice box, there are slight differences in the motion, and also when we put the dice into the box there are small differences in the motion; and no regularity connects the differences of one kind with those of the other. Still, these circumstances would not in themselves give the character of fortuitous distribution to the throws were there not a fortuitous distribution either in the differences of our motion in putting the dice into the box or else fortuitous distribution in the variations of motion in throwing them out.†1 We see, then, that in this case the fortuitous distribution arises from another fortuitous distribution in one or more of the conditions of the production of the phenomenon. All this has been carefully studied by various writers on the theory of errors. Suppose we put into a jar some hot nitrogen and then some cold oxygen. At first, the molecules of nitrogen will be moving with various *vires vivae* distributed according to a modification of the probability curve and therefore fortuitously, while the molecules of oxygen will likewise have *vires vivae* distributed according to the same general law, but on the average *their* motion will be much slower. In the first state of things, therefore, the distribution of *vires vivae* among all the molecules considered as one collection will *not* be fortuitous. But there will be continual encounters of molecules, which, in these encounters, will be governed by conservative forces, generally attractions. In consequence of the different modes of these encounters being distributed fortuitously, which is itself due to the fortuitous distribution of the molecules in space, and the fortuitous distribution of the directions and velocities of their motions, continual interchanges of *vis viva* will take place, so that as time goes on there will be a closer and closer approximation to one fortuitous distribution of *vis viva* among all the molecules. There we see a fortuitous distribution in process of being brought about. That which happens, happens entirely under the governance of conservative forces; but the character of fortuitous distribution toward which there is a tendency is entirely due to the various fortuitous distributions

existing in the different initial conditions of the motion, with which conservative forces never have anything to do. This is the more remarkable because the peculiar distribution which characterized the initial distribution of *vires vivae* gradually dies out. True, traces of it always remain; but they become fainter and fainter and approach without limit toward complete disappearance. The fortuitous distributions, however, which equally have nothing but initial conditions to sustain them, not only hold their ground, but, wherever the conservative forces act, at once mark their character in the effects. Hence, it is that we find ourselves forced to speak of the "action of chance." . . . †1

Peirce: CP 6.82 Cross-Ref:††

§5. SPACE

82.†2 Would not the human race, supposing that it could survive the shock at all, be pretty sure to develop a new form of intuition in which the things that now appear near would appear far? For what is the real truth of nearness? Who is my neighbor? Is it not he with whom I intimately react? In short, the suggested explanation is that space is that form of intuition in which is presented the law of the mutual reaction of those objects whose mode of existence consists in mutually reacting. Let us see how much this hypothesis will explain. What are its necessary consequences? I must abridge the reasoning to a mere sketch. In the first place space, as a presentation of law, must be *continuous* and without singularities. In the second place, since reaction is essentially *hic et nunc*, or anti-general, it follows that the reacting objects must be entirely independent of one another in their purely spatial determinations. That is, one object being in one particular place in no way requires another object to be in any particular place. From this again it necessarily follows that each object occupies a single point of space, so that matter must consist of Boscovichian atomicules,†1 whatever their multitude may be. On the same principle it furthermore follows that any law among the reactions must involve some other continuum than merely Space alone. Why Time should be that other continuum I shall hope to make clear when we come to consider Time.†2 In the third place, since Space has the mode of being of a law, not that of a reacting existent, it follows that it cannot be the law that, in the absence of reaction, a particle shall adhere to its place; for that would be attributing to it an attraction for that place. Whence it follows that in so far as a particle is not acted upon by another, that which it retains is a relation between space and time. Now it is not logically accurate to say that the law of motion prescribes that a particle, so far as it is not acted upon by forces, continues to move in a straight line, describing equal intervals in equal times. On the contrary the true statement is that straight lines are that family of lines which particles, so far as they are unacted upon, describe, and that equal spaces are such spaces as such a particle describes in equal times. There are some further consequences of this principle the statement of which it will be convenient to postpone for a few minutes.

Peirce: CP 6.82 Cross-Ref:††

In the fourth place, since Space presents a law whose prescriptions are nothing but conditions of reactions, and since reaction is Duality, it follows that the conditions of the prescriptions of space are necessarily Dual. Hence immediately follow five corollaries. The first is that all forces are between pairs of particles. The second is that, when two places of the path of an isolated particle are determined, the law

determines all the other places; so that two different straight lines cannot have two different points in common. The third corollary is that when the places of an isolated body at two instants are given, the law prescribes its places at all other instants. That is, the first differential coefficient, or mere difference between the places at two instants, determines its places at all other instants. That is, the velocity remains constant. From these corollaries again, together with the general principle from which they are derived, it follows that when a body is acted upon by another body that which is directly affected is the uniform velocity in a straight line, and that in such a way that, in so far as the action of the active body remains the same, two velocities or, what comes to the same thing, three positions with their dates, determine all the velocities the particle will take. This explains, therefore, why the force should produce acceleration rather than any other differential coefficient of the space relatively to the time. Hence, it further follows that a force at each moment of time acts to impart to the body a new rectilinear motion; whence it follows that forces will necessarily be compounded according to a parallelogram of forces. In the non-Euclidean geometry this is only so far modified that the parallelograms must be drawn infinitely small. And it further follows that the line of the force is the straight line through the two particles. There is still another apparent consequence; but I am not satisfied with the reasoning, since it rests upon a principle I am unable abstractly to define. I will, however, state it for what it is worth. Namely, since the force acts to impart an acceleration and since the law presented in space is perfectly general and comprehensive, it follows that the acceleration imparted may be different in kind and not merely in amount from the acceleration the particle already possesses. That is the point I consider doubtful. If it be admitted, it certainly follows that space must have at least three dimensions. Moreover, it again follows as a fourth corollary from space being a law of reactive conditions that, except for the quality of the particles themselves, it is the pure spatial determination which prescribes what the reaction of one particular particle upon another shall be; that is, the force between two particles depends only upon their qualities and their places at the instant. Moreover, as a fifth corollary, it follows that the mechanical law only prescribes how a *pair* of particles will act. It does not generally prescribe any relation between the actions of different pairs of particles, nor even of the action of a particle upon particles of the same kind placed differently. Hence, not only may different kinds of pairs of particles act differently, but the law of the variation of the action with the relative positions is left to depend upon the qualities of the particles, and this so completely that there is nothing to prevent a particle exercising different forces on different sides of it.

Peirce: CP 6.82 Cross-Ref:††

In the fifth place, from the fact that space presents a law of reciprocal reactions, several corollaries follow, particularly these two. First, when one particle, *A*, acts on another, *B*, this latter, *B*, will likewise act on *A*; and moreover this action cannot impart to both the same acceleration, because the law is such as to affect their relative places. This follows by the aid of the third principle already enunciated, as we shall see. Hence, it can only impart opposite accelerations to *A* and *B*. Secondly, those two accelerations must be equal, so that the masses of all atomicules are equal. From this, again, it follows that the masses are unchangeable; and further that if two bodies, or aggregates of atomicules, react upon one another in a certain ratio to one another, in that same ratio they will also react upon any third body.

Peirce: CP 6.82 Cross-Ref:††

A sixth principle, concerning the necessity of which some doubt may be entertained, is that, the law presented by space being perfectly general, every motion

must admit of receiving the same kind of changes as every other. From this, if it be admitted, it certainly follows, though the demonstration is far too long to give, that space has either 1, 2 or 4 dimensions. Hence, since 1 and 2 dimensionality have been already excluded, the number of dimensions ought to be just 4.

Peirce: CP 6.82 Cross-Ref:††

I will now mention the postponed corollaries from the third principle. Since space has only the being of a law, its places cannot have distinct identities in themselves, for distinct identity belongs only to existent things. Hence place is only relative. But since, at the same time, different motions must be comparable in quantity, and this comparison cannot be effected by the moving and reacting particles themselves, it follows that another object must be placed in space to which all motion is referred. And since this object compares generally and thus partakes of the nature of law, it must unlike the moving and reacting bodies be continuous. It is a corrected equivalent of that which has been called the body *alpha*. It is the firmament, or Cayley's absolute.†1 Since this is to determine every motion, it follows that it is a locus which every straight line cuts, and because space is a law of twoness only, and for other reasons, every straight line must cut it in two points. It is therefore a real quadratic locus, severing space into two parts, and the space of existence must be infinite and limited in every direction.

Peirce: CP 6.83 Cross-Ref:††

83. I have thus briefly stated one side of my theory of space. That is, without touching upon the question of the derivation of space and its properties, or how accurately it may be supposed to *fulfill its ideal conditions*, I have given a hypothesis from which those ideal properties may be deduced. Many of the properties so deduced are known to be true, at least approximately. Others, I am happy to say, are extremely doubtful. I say I am happy because this gives them the character of predictions and renders the hypothesis capable of experiential confirmation or refutation. One of the doubtful properties, the last mentioned, I have succeeded I think in proving to be true by calculations from the proper motions of the stars. Another, that about atoms attracting differently in different directions, I have succeeded in making highly probable, from chemical facts. Still others have some evidence in their favor. The consequence most opposed to observation is the doubtful one of 4 dimensions.

Peirce: CP 6.84 Cross-Ref:††

84. Endeavoring to generalize the results that have been obtained, we may say that the continuity of space so acts as to cause an object to be affected by modes of existence not its own, not as participating in them but as being opposite to them. For instance, an isolated particle is at any instant at one point; that is its actual state. But it is so affected by the state which is not actual, but belongs to it by a date differing from the actual [in] one way, that, at a date differing from the actual the other way, it takes a state differing in the opposite way from its actual state. So again, when a force acts upon a body the effect of it is that the mean of the states of the body not actual, but indefinitely approximating to the actual, differs from its actual state. So in the action and reaction of bodies, each body is affected by the other body's motion, not as participating in it but as being opposite to it. But if you carefully note the nature of this generalized formula you will see that it is but an imperfect, somewhat particularized restatement of the principle that space presents the law of the reciprocal reactions of existents. Various other such imperfect formulæ might be mentioned.

Peirce: CP 6.85 Cross-Ref:††

85. Let us now consider non-conservative actions. These are all distinguished

by asymptotic approach to a definite state of relative rest. Conservative force can never bring about any state of rest except for an instant. It can only produce, I believe, three permanent changes. Namely, it can permanently change the direction [of] motion of a body, and this it does because the body moves away out of the range of the force, or it can cause one body to rotate round another in an inward spiral, more and more rapidly. And third, a planet like Jupiter may turn the motion of a small body and then move away and leave the small body performing permanently, or quasi-permanently, an orbit round the sun. In course of time, however, Jupiter will come round again in such a way as to throw it out. This is a very curious case. Chance is an important factor of it. But all the non-conservative quasiforces produce states of relative rest. Such, for example, is the effect of viscosity. These states of relative rest are states of uniform distribution which upon minuter inspection turn out to be really states of fortuitous distribution. They betray their real nature by the probability curve, or some modification of it, playing a part in the phenomenon. Such, for example, is the case in the conduction of heat.

Peirce: CP 6.86 Cross-Ref:††
 §6. TIME

86. When we ask why chance produces permanent effects, the natural answer which escapes from our lips is that it is because of the independence of different instants of time. A change having been made, there is no particular reason why it should ever be unmade. If a man has won a napoleon at a gaming table he is no more likely to lose it than he was to lose a napoleon at the outset. But we have no sooner let slip the remark about the independence of the instants of time than we are shocked by it. What can be less independent than the parts of the continuum *par excellence*, through the spectacles of which we envisage every other continuum? And although it may be said that continuity consists in a binding together of things that are different and remain different, so that they are in a measure dependent on one another and yet in a measure independent, yet this is only true of finite parts of the continuum, not of the ultimate elements nor even of the infinitesimal parts. Yet it undoubtedly is true that the permanence of chance effects is due to the independence of the instants of time. How are we to resolve this puzzle? The solution of it lies in this, that time has a point of discontinuity at the present. This discontinuity appears in one form in conservative actions where the actual instant differs from all other instants absolutely, while those others only differ in degree; and the same discontinuity appears in another form in all non-conservative action, where the past is broken off from the future as it is in our consciousness. Thus, although the other instants of time are not independent of one another, independence does appear at the actual instant. It is not an utter, complete independence, but it is absolute independence in certain respects. Perhaps all fortuitous distribution originates from a fortuitous distribution of events in time; and this alone has no other explanation than the Law of Sufficient Reason, that is, is an absolute First. It is a truth well worthy of rumination that all the intellectual development of man rests upon the circumstance that all our action is subject to error. *Errare est humanum* is of all commonplaces the most familiar. Inanimate things do not err at all; and the lower animals very little. Instinct is all but unerring; but reason in all vitally important matters is a treacherous guide.†1 This tendency to error, when you put it under the microscope of reflection, is seen to consist of fortuitous variations of our actions in time. But it is apt to escape our attention that on such fortuitous

variation our intellect is nourished and grows. For without such fortuitous variation, habit-taking would be impossible; and intellect consists in a plasticity of habit.

Peirce: CP 6.87 Cross-Ref:††

87. What is time?†2 Shall we say that it is the form under which the law of logical dependence presents itself to intuition? But what is logical dependence objectively considered? It is nothing but a necessitation which, instead of being brute, is governed by law. Our hypothesis therefore amounts to this, that time is the form under which logic presents itself to objective intuition; and the signification of the discontinuity at the actual instant is that here new premisses, not logically derived by Firsts, are introduced.

Peirce: CP 6.88 Cross-Ref:††
CHAPTER 4

VARIETY AND UNIFORMITY

§1. VARIETY †1

88. . . . The question, what are the shares of uniformity and variety in the phenomena of the universe, is a question which has never been agitated in any public dispute that has attracted general attention. The consequence is that everybody in a semi-unconscious way forms his own opinion about it, usually a pretty vague opinion; and he has the impression that all his neighbors think as he does about it. But, on questioning people closely, it will be found that there are no less than *five* different opinions that are widely spread on this subject.†2 I will briefly state them and point out just how much arbitrariness each supposes.

Peirce: CP 6.89 Cross-Ref:††

89. Beginning with the middling one, in the degree of arbitrariness that it allows, which is, no doubt, the opinion of the largest party among those who know enough of dynamics to entertain such a conception, this view is that the material universe is composed of particles of some kind, each having at any one instant its position in space, and also its velocity, determinate in direction and in amount; and it is held that physical laws are of such a nature that according to the positions of the particles at any instant their velocities are, at that instant, changing at perfectly determinate rates and in perfectly determinate directions. This party holds that, if this statement were rendered definite by indicating just what these accelerations are in each position of the particles, it would be the perfect *résumé* of all the laws of nature. As for consciousness, these persons hold that its states are rigidly dependent upon the instantaneous states of the physical universe, and that it need not be taken into account in saying what will happen in that universe.

Peirce: CP 6.89 Cross-Ref:††

This theory makes the uniformity to be perfectly exact and inflexible. It is

such that, given the positions and velocities of all the particles at any one instant, the positions and velocities at all other instants are precisely determined and with these the exact phenomena of all consciousness and feeling. But it supposes the positions and velocities of all the particles at one instant to be entirely arbitrary. It further regards the law itself, although as a law it is general, [as] yet arbitrary in respect to what its requirements are.

Peirce: CP 6.90 Cross-Ref:††

90. If we designate the five classes of minds who entertain the five opinions by the first five letters of the alphabet, the *A*'s being the persons who admit the least arbitrariness and the *E*'s being those who admit the most, then the opinion just formulated is that of the *C*'s. If you should be out walking on a fine starlit night in company with a *B* and a *C*, and were to point up to the heavens and ask your companions whether they supposed there was any law determining the arrangement of the stars, *C* would smile and would remark that not a single law of that description had ever been discovered yet. Thereupon *B* would exclaim, "What! Do you mean to say there is no regularity in the arrangement of the stars when there is the Milky Way before your eyes!" "Oh," *C* would reply, "there is a rough and irregular compression of the cluster in which we happen to find ourselves. But, assuming the Nebular Hypothesis to be true, that could hardly escape coming about in consequence of the cluster being very much condensed from a former state in which its velocity happened to be a little greater on one side than on the other, which constituted a rotation. But however it came about, the fact that the arrangement is so excessively rough shows at once that it is due to some accident and not to law, since the effects of law are rigidly exact." To this *B* might perhaps reply, "I do not think your explanation very satisfactory. You suppose that a manifest regularity of arrangement among millions of stars is the effect of an accident. It seems much easier to suppose that the regularity of arrangement was once perfect, but that the motions of the separate stars have deranged it." While this dialogue has been going on an adherent of the sect of *A*'s has joined the group. He now says, "But you surely will admit that if the original perfect symmetry of arrangement has been broken up, probably in its passage into some different form of symmetry, the present apparently irregular arrangement must have been fully intended by the Creator." *B* replies, "I do not quite know that I am prepared to admit that the world ever was created. But even if it was, while the positive intentions of the Creator must have been fulfilled, we need not suppose that he expressly intended every relation between facts. If the Dowager Empress of China happens to have a fit of coughing and just at that moment I, on the other side of the globe, happen to take a piece of hoarhound candy, we need not suppose that this coincidence was any part of the Creator's plan." *A* replies, "I believe that Providence overrules every fact and relation however trivial; and even if I were in your state of scepticism, I should still hold it to be inconceivable that any state of facts should fail to conform to some law. You cannot shuffle a pack of cards so that there is no mathematically exact relation between the arrangement before shuffling and the arrangement after shuffling."

Peirce: CP 6.90 Cross-Ref:††

So there you have the three commonest forms of necessitarianism. *A* holds that every feature of all facts conforms to some law. *B* holds that the law fully determines every fact, but thinks that some relations of facts are accidental. *C* holds that uniformity within its jurisdiction is perfect, but confines its application to certain elements of phenomena.

Peirce: CP 6.91 Cross-Ref:††

91. The party of the *D*'s, of which I am myself a member, holds that uniformities are never absolutely exact, so that the variety of the universe is forever increasing. At the same time we hold that even these departures from law are subject to a certain law of probability, and that in the present state of the universe they are far too small to be detected by our observations. We adopt this hypothesis as the only possible escape from making the laws of nature monstrous arbitrary elements. We wish to make the laws themselves subject to law. For that purpose that law of laws must be a law capable of developing itself. Now the only conceivable law of which that is true is an evolutionary law. We therefore suppose that all law is the result of evolution, and to suppose this is to suppose it to be imperfect.

Peirce: CP 6.92 Cross-Ref:††

92. Finally, there are those who suppose nature to be subject to freaks, who believe in miracles not simply as manifestations of superhuman power but as downright violations of the laws of nature, absolutely abnormal. Professor Newcomb, for example, in a series of articles which he contributed to the *Independent*, suggests that the human will has a power of deflecting the motions of particles, in plain violation of the third law of motion. I do not think, by the way, that it is generally known that some of the early Fathers of the Church refused to believe in physical miracles; and apparently attributed them to a superhuman hypnotic power, reminding one of what the Hindoo jugglers have made British officers think they saw. St. Augustine,†1 on the contrary, while holding it impious to think them to be violations of Nature's Laws, regards them apparently as occurrences that are to us what the reading of a letter by a man might seem to a dog to be, namely, a manifestation of some higher mastery of things than would be compatible with his nature.

Peirce: CP 6.93 Cross-Ref:††

93. One fallacy into which the necessitarians of class C generally fall is that they imagine that they can disprove that anything happens by chance by showing that the event has a cause. Thus Boëthius, at the beginning of the fifth book of his *Consolations*, after citing Aristotle as a necessitarian,†2 which is enough to take one's breath away, so monstrous is the blunder or the impudence of it, has a little ode of twelve lines which Mr. Henry Roshier James translates in 24,†3 that imitate the swing of the original very well, but miss the point. By a geographical fiction Boëthius represents that the Tigris and the Euphrates flow from a common lake. Now suppose a boat to be wrecked in that lake and one part of it is carried down the Tigris, the other part down the Euphrates, and where these rivers, after being separate for hundreds of miles, flow together again those two parts of the boats are dashed against one another. There is a fortuitous event if there ever was one; and yet, says Boëthius, the currents forced them to move just as they did so that there was no chance about it. True, the existential events were governed by law. But when we speak of *chance*, it is a question of *cause*. Now it is the ineluctable blunder of a nominalist, as Boëthius was, to talk of the cause of an event. But it is not an existential event that has a cause. It is the *fact*, which is the reference of the event to a general relation, that has a *cause*.†1 The *event*, it is true, was governed by the law of the current. But the *fact* which we are considering is that the two pieces that were dashed together had long before belonged together. That is a fact that would not happen once in ten thousand times, although when you join to this fact various circumstances of the actual event, and so contemplate quite *another* fact, it would happen every time, no doubt. That is to say, nobody can doubt it but an adherent of the *E*'s sect. The example is a very good one

as showing that the causal necessitation of more concrete *fact* does not prevent a more prescindend, †2 or general, *fact* of the same event from being quite fortuitous. The position of Aristotle in this matter is altogether right, and not "veri propinqua ratione" as Boëthius says; but it is a position that nobody can understand who is completely immersed in the state of mind of modern philosophy. Zeller, †3 for example, does not seize it at all.

Peirce: CP 6.94 Cross-Ref: ††

94. But let us drop metaphysics and return to logic. It was Hobbes †4 who first said, referring to and combating Aristotle's doctrine, "Men commonly call that *casual* whereof they do not perceive the necessary cause," for Hobbes was a typical stoic in his philosophy. Leibnitz †5 emphatically agrees with Hobbes. "Fort bien," he says. "J'y consens, si l'on entend parler d'un hasard réel. Car la fortune et le hasard ne sont que des apparences, qui viennent de l'ignorance des causes, ou de l'abstraction qu'on en fait." This has been said a thousand times since with an air as if it explained the whole thing. I do not doubt that that is the impression of almost everybody in this hall. But I am quite sure that most of you will be glad to reexamine the question with me. I will just give you the headings of some thoughts about it, which, if it is not too great a liberty, I would suggest that you take note of and carefully pursue by yourselves when you find leisure. I wish in the latter part of this lecture to make some remarks of great importance in many reasonings; and in order to get any time for those remarks I shall be obliged to make my statement of this part so brief that only the most thorough student of philosophy could fully grasp the meaning of it at the single hearing.

Peirce: CP 6.95 Cross-Ref: ††

95. The first thing to be taken into consideration is the general upshot of Kant's *Critic of the Pure Reason*. The first step of Kant's thought -- the first *moment* of it, if you like that phraseology -- is to recognize that all our knowledge is, and forever must be, relative to human experience and to the nature of the human mind. That conception being well digested, the second moment of the reasoning becomes evident, namely, that as soon as it has been shown concerning any conception that it is essentially involved in the very forms of logic or other forms of knowing, from that moment there can no longer be any rational hesitation about fully accepting that conception as valid for the universe of our possible experience. To repeat an example I have given before, you look at an object and say "That is red." I ask you how you prove that. You tell me you see it. Yes, you see *something*; but you do not see *that it is red*; because *that it is red* is a proposition; and you do not see a proposition. What you see is an image and has no resemblance to a proposition, and there is no logic in saying that your proposition is proved by the image. For a proposition can only be logically based on a premiss and a premiss is a proposition. To this you very properly reply, with Kant's aid, that my objections allege what is perfectly true, but that instead of showing that you have no right to say the thing is red they conclusively prove that you are logically justified in doing so. At this point, the idealist appears before the tribunal of your reason with the suggestion that since these metaphysical conceptions, that repose upon their being involved in the forms of logic, are only valid for experience and since all our knowledge is relative to the human mind, they are not valid for things as they objectively are; and since the conception of *existence* is preeminently a conception of that description, it is a mere fairy tale to say that outward objects *exist*, the only objects of possible experience being our own ideas. Hereupon comes the third moment of Kant's thought, which was only made prominent in the second edition, not, as Kant truly says, that it was not already in the book, but

that it was an idea in which Kant's mind was so completely immersed that he failed to see the necessity of making an explicit statement of it, until Fichte misinterpreted him. It is really a most luminous and central element of Kant's thought. I may say that it is the very sun round which all the rest revolves. This third moment consists in the flat denial that the metaphysical conceptions do not apply to things in themselves. Kant *never* said that. What he said is that these conceptions do not apply beyond the limits of possible experience. But we have *direct experience of things in themselves*. †1 Nothing can be more completely false than that we can experience only our own ideas. That is indeed without exaggeration the very epitome of all falsity. Our knowledge of things in themselves is entirely relative, it is true; but all experience and all knowledge is knowledge of that which is, independently of being represented. Even lies invariably contain this much truth, that they represent themselves to be referring to something whose mode of being is independent of its being represented. †2 This is true even if the proposition relates to an object of representation as such. At the same time, no proposition can relate, or even thoroughly pretend to relate, to any object otherwise than as that object is represented. These things are utterly unintelligible as long as your thoughts are mere dreams. But as soon as you take into account that Secondness that jabs you perpetually in the ribs, you become awake to their truth. Duns Scotus and Kant are the great assertors of this doctrine, for which Thomas Reid deserves some credit too. But Kant failed to work out all the consequences of this third moment of thought and considerable retractions are called for, accordingly, from some of the positions of his Transcendental Dialectic. Nor in other respects must it be supposed that I assent to everything either in Scotus or in Kant. We all commit our blunders.

Peirce: CP 6.96 Cross-Ref: ††

96. To this first consideration, it is necessary to add, in the second place, that of the great difference in the logical status of the future and the past, which Aristotle †1 stated with great emphasis without finding anybody in modern times to comprehend what he said, not even Trendelenburg, †2 who comes the nearest to it. Aristotle is understood by modern critics to be in a childishly naive state of mind on this subject. Now it is quite true that Aristotle was almost the first pioneer in logic and just stood at its threshold. It is also true that there are some monumental follies in his physical books; but the worst of these may fairly be presumed to be insertions made by different students during the thirty years when his manuscripts lay on the shelves of his school for general use. But Aristotle was by many lengths the greatest intellect that human history has to show; and it was precisely in such fields of thought, as this distinction of past and future time, that his mind was the most thoroughly trained. So gigantic is his power of thought that those critics may almost be excused who hold it to be impossible that all of the books that have come down to us as his should all have been produced by one man. I am ashamed to have to confess that I shared the general opinion of Aristotle's childish naïveté in those passages, until the further progress of my own studies forced me to the very substance of what Aristotle says. The past is ended and done; the future is endless and can never have been done. To be sure, if we regard past time as having had no beginning, then, when we make general assertions concerning it, we can only be talking of it as an object of possible experience, that is, of what future researches may bring to light. Hence it might be inferred that the contrast Aristotle speaks of between the past and the future might be merely subjective, having to do with our different attitude toward them. But even a moderate appreciation of the Kantian argument will show that, besides being true in regard to our knowledge of time, it must also be regarded as true of *real* time; and time *is* real, whether we accept Kant's dubious view of it, which he is certainly far from making

evident, as the form of the internal sense, or not. I do not question Time's being a *form*, that is, being of the nature of a Law, and not an Existence; nor its being an Intuition, that is, being at the same time a single object; nor its having a special connection with the internal world. But I doubt very much whether Kant has succeeded in rightly stating the connection between those three features of Time.

Peirce: CP 6.97 Cross-Ref:††

97. Now there are three characters which mark the universe of our experience in a way of their own. They are Variety, Uniformity, and the passage of Variety into Uniformity. By the Passage of Variety into Uniformity, I mean that variety upon being multiplied almost in every department of experience shows a tendency to form *habits*. These habits produce statistical uniformities. When the number of instances entering into the statistics are small compared with the degree of their variation, the law will be extremely rough, but when the number runs up into the trillions, that is to say cubes of millions, or much higher, as in the case of molecules, there are no departures from the law that our senses can take cognizance of.

Peirce: CP 6.98 Cross-Ref:††

§2. UNIFORMITY †1

98. (1) A fact consisting in this: that, of a certain genus of facts, a proportion approaching unity (the whole) belongs, in the course of experience, to a certain species; so that, though of itself the knowledge of this uniformity gives no information concerning a certain thing or character, yet it will strengthen any inductive conclusion of a certain kind.

Peirce: CP 6.98 Cross-Ref:††

It is, therefore, a high objective probability concerning an objective probability. There are, in particular, four classes of uniformities, the knowledge of any of which, or of its falsity, may deductively strengthen or weaken an inductive conclusion. These four kinds of uniformity are as follows:

Peirce: CP 6.98 Cross-Ref:††

i. The members of a class may present an extraordinary resemblance to one another in regard to a certain line of characters. Thus, the Icelanders are said to resemble one another most strikingly in their opinions about general subjects. Knowing this, we should not need to question many Icelanders, if we found that the first few whom we met all shared a common superstition, in order to conclude with considerable confidence that nearly all Icelanders were of the same way of thinking. Philodemus †1 insists strongly upon this kind of uniformity as a support of induction.

Peirce: CP 6.98 Cross-Ref:††

ii. A character may be such that, in whatever genus it occurs at all, it almost always belongs to all the species of that genus; or this uniformity may be lacking. Thus, when only white swans were known, it would have been hazardous to assert that all swans were white, because whiteness is not usually a generic character. It is considerably more safe to assert that all crows are black, because blackness is oftener a generic character. This kind of uniformity is especially emphasized by J. S. Mill as important in inductive inquiries. †2

Peirce: CP 6.98 Cross-Ref:††

iii. A certain set of characters may be intimately connected so as to be usually all present or all absent from certain kinds of objects. Thus, the different chemical reactions of gold are so inseparable that a chemist need only to succeed in getting, say, the purple of Cassius to be confident that the body under examination will show every reaction of gold.

Peirce: CP 6.98 Cross-Ref:††

iv. Of a certain object it may be known that its characteristic is that when it possesses one of a set of characters within a certain group of such sets, it possesses the rest. Thus, it may be known of a certain man that to whatever party he belongs, he is apt to embrace without reserve the entire creed of that party. We shall not, then, need to know many of his opinions, say in regard to politics, in order to infer with great confidence his position upon other political questions.

Peirce: CP 6.99 Cross-Ref:††

99. (2) The word "uniformity" plays such a singular and prominent rôle in the logic of J. S. Mill that it is proper to note it.†3 He was apt to be greatly influenced by Ockham's razor in forming theories which he defended with great logical acumen; but he differed from other men of that way of thinking in that his natural candour led to his making many admissions without perceiving how fatal they were to his negative theories. In addition to that, perhaps more than other philosophers, in endeavouring to embrace several ideas under a common term, he often leaves us at a loss to find any other character common and peculiar to those notions except that of their having received from him that common designation. In one passage †1 of his *System of Logic* (1842), he declares, in reference to the difference in strength between two inductive conclusions, that whoever shall discover the cause of that difference will have discovered the secret of inductive reasoning. When, therefore, he shortly afterwards †2 points out that the distinction between those two inductions is that one of them is supported by a uniformity of the second of the above four classes, while the other is met by a distinct diversity of the same kind, and when he himself gives to that uniformity this designation when he afterwards declares that the validity of induction depends upon uniformity, his reader naturally supposes he means uniformity in that sense. But we find that he employs the word for quite another purpose. Namely, he does not like the word *law*, as applied to an inductive generalization of natural facts -- such as the "law" of gravitation -- because it implies an element in nature, the reality of a general, which no nominalist can admit. He, therefore, desires to call the reality to which a true universal proposition about natural phenomena corresponds a "uniformity."†3

Peirce: CP 6.99 Cross-Ref:††

The implication of the word, thus used, is that the facts are, in themselves, entirely disconnected, and that it is the mind alone which unites them. One stone dropping to the earth has no real connection with another stone dropping to the earth. It is, surely, not difficult to see that this theory of uniformities, far from helping to establish the validity of induction, would be, if consistently admitted, an insuperable objection to such validity. For if two facts, *A* and *B*, are entirely independent in their real nature, then the truth of *B* cannot follow, either necessarily or probably, from the truth of *A*. If I have tried the experiment with a million stones and have found that every one of them fell when allowed to drop, it may be very natural for me to believe that almost any stone will act in the same way. But if it can be proved that there is no real connection between the behaviour of different stones, then there is nothing for it

but to say that it was a chance coincidence that those million stones all behaved in the same way; for if there was any reason for it, and they *really* dropped, there was a *real reason*, that is, a real general. Now, if it is mere chance that they all dropped, that affords no more reason for supposing that the next will drop than my throwing three double sixes successively with a pair of dice is a reason for thinking that the next throw will be double sixes.

Peirce: CP 6.100 Cross-Ref:††

100. (3) But now we find that Mill's good sense and candour will not allow him to take the course which a Hobbes would have taken, and utterly deny the validity of induction; and this leads to a new use of the word *uniformity*, in which he speaks of the "uniformity of nature." Before asking exactly what this phrase means, it may be noted that, whatever it means, the assertion of it is an assent to scholastic realism, except for a difference of emphasis. For to say that throughout the whole course of experience, events always, or even only usually, happen alike under the same conditions (what is usually called the "invariability" of nature) is to assert an agreement (complete or partial) which could not be ascribed to chance without self-contradiction. For chance is merely the possible discrepancy between the character of the limited experience to which it belongs and the whole course of experience. Hence, to say that of the *real*, objective facts some *general* character can be predicated, is to assert the reality of a general. It only differs from scholastic realism in that Mill and his followers treat this aspect of the matter lightly -- that is to say, the objective reality of the general -- while the Scholastics regarded it as a great and vital feature of the universe. Instead of "uniformity" now importing that what others call "laws" are fabrications of the human mind, this "uniformity of nature" is erected by Mill into the greatest of laws and absolutely objective and real.

Peirce: CP 6.100 Cross-Ref:††

Let us now inquire what the "uniformity of nature," with its synonymous expressions that "the future resembles the past," and so forth, can mean. Mill †1 says that it means that if all the circumstances attending two phenomena are the same, they will be alike. But taken strictly this means absolutely nothing, since no two phenomena ever can happen in circumstances precisely alike, nor are two phenomena precisely alike. It is, therefore, necessary to modify the statement in order to give it any meaning at all; and it will be found that, however it may be so modified, the moment it begins to carry a definite meaning, one of three things results: it becomes either, first, grossly false, or, second, an assertion which there is really no good reason to believe even approximately true, or, thirdly, it becomes a quasi-subjective truth, not lending any colour of validity to induction proper. If, for example, we were to say that, under any given species of circumstances presenting any similarity, phenomena of any given genus would be found to have a specific general resemblance in contrast with the specific character of phenomena of the same genus occurring under a different species of circumstances of the same genus, this would be monstrously false, whether intended as an absolutely universal proposition or merely as one approximately true. Let, for example, the genus of phenomena be the values of the throws of a pair of dice in a given series of successive throws indefinitely continued. Let the first species of circumstances be that the ordinal number of a throw in the series is *prime*. It is pretty certain that there would be no general character in the corresponding values of throws to distinguish them from those which would result when the ordinal number is divisible by 2, or by 3, or by any other prime. It thus appears that when we take *any* genus of circumstances, the law turns out false. Suppose, then, that we modify it by saying that, taking any genus of phenomena and

separating this into two species, there will be found in the discoverable circumstances *some* general resemblance for all those attending phenomena of the same species in contrast to those attending phenomena of the other species. This is a proposition which there is not the slightest reason to believe. Take, for example, as the genus of phenomena, the many thousands of Latin descriptions of American species of plants by Asa Gray and his scholars. Now consider the species of this genus of phenomena which agree in this respect, that the two first words of the description have their first vowels the same. There is no reason to suppose that there was any general respect in which the circumstances of that species of the genus of phenomena agree with one another and differ from others, either universally or usually. It is a mere chance result. It is true that some persons will not be inclined to assent to this judgment; but they cannot prove it otherwise. It can afford no adequate basis for induction. We see, then, that when we consider *all* phenomena, there is no way of making the statement sufficiently definite and certain. Suppose, then, that we attempt still another modification of the law, that, of *interesting* resemblances and differences between phenomena, some considerable proportion are accompanied by corresponding resemblances and differences between those of the circumstances which appear to us to be *pertinent*. The proposition is now rather psychological than metaphysical. It would be impossible, with any evidentiary basis, to strengthen the expression "some considerable proportion"; and in other respects the statement is vague enough. Still, there is sufficient truth in it, perhaps, to warrant the presumptive adoption of hypotheses, provided this adoption merely means that they are taken as sufficiently reasonable to justify some expense in experimentation to test their truth by induction; but it gives no warrant at all to induction itself. For, in the first place, induction needs no such dubious support, since it is mathematically certain that the general character of a limited experience will, as that experience is prolonged, approximate to the character of what will be true in the long run, if anything is true in the long run. Now all that induction infers is what would be found true in the usual course of experience, if it were indefinitely prolonged. Since the method of induction must generally approximate to that truth, that is a sufficient justification for the use of that method, although no definite probability attaches to the inductive conclusion. In the second place, the law, as now formulated, neither helps nor hinders the validity of induction proper; for induction proper consists in judging of the relative frequency of a character among all the individuals of a class by the relative frequency of that character among the individuals of a random sample of that class. Now the law, as thus formulated, may tend to make our hypothesis approximately true; but that advantage has been gained before the operation of induction, which merely tests the hypothesis, begins. This inductive operation is just as valid when the hypothesis is bad as when it is good, when the character dealt with is trivial as when it is interesting. The ratio which induction ascertains may be nearer $1/2$, and more remote from 1 or 0, when the characters are uninteresting; and in that case a larger number of instances will usually be requisite for obtaining the ratio with any given degree of precision (for if the ratio is really 1 or 0, it will be almost a miracle if in the sample it is far from that ratio, although this will not be impossible, if the whole class is infinite), but the essential validity of the process of induction remains unaffected by that circumstance.

Peirce: CP 6.100 Cross-Ref:††

What is usually meant by the uniformity of nature probably is that in proportion as the circumstances are alike or unlike, so are any phenomena connected with them alike or unlike. It may be asked to what degree nature is uniform in that sense. The only tenable answer is that it is as little uniform as it possibly could be

imagined to be; for were any considerable proportion of existing uniformities, or laws, of nature destroyed, others would necessarily thereby result.

Peirce: CP 6.100 Cross-Ref:††

In fact, the great characteristic of nature is its diversity. For every uniformity known, there would be no difficulty in pointing out thousands of non-uniformities; but the diversities are usually of small use to us, and attract the attention of poets mainly, while the uniformities are the very staff of life. Hence, the higher and wider are our desires the greater will be the general impression of uniformity produced upon us by the contemplation of nature as it interests us.

Peirce: CP 6.101 Cross-Ref:††

101. (4) There are senses in which nature may not irrationally be held to be uniform; but opinions differ very widely as to the extent and nature of this uniformity. The chief of these are as follows:

Peirce: CP 6.101 Cross-Ref:††

(a) The majority of physicists, at least of the older generation, hold, with regard to the physical universe, that its elements are masses, their positions, and the variations of these positions with time. It is believed that every motion exactly obeys certain laws of attraction and repulsion; and there is no other kind of law, except that each atom or corpuscle is a centre of energy arranged in equipotential surfaces about it, which follow a regular law; and that this is a permanency. But the equations of motion are differential equations of the second order, involving, therefore, two arbitrary constants for each moving atom or corpuscle, and there is no uniformity connected with these constants. At least, no such uniformity is, with the least probability, discoverable. As for the distribution of potential about an atom or corpuscle, it is regular; but there is no ulterior reason for that regularity, or, at least, none is probably discoverable. What is absolutely beyond discovery, whether direct and specific or indirect and general, may be considered to be non-existent.

Peirce: CP 6.101 Cross-Ref:††

From this usual and in some sense standard opinion there are many divergences in both directions. First, in the direction of greater uniformity.

Peirce: CP 6.101 Cross-Ref:††

(b) Some hold that there is some exact uniformity in the arbitrary constants of the motion of the atoms, so that, for example, perhaps at some initial instant they all had some symmetrical or regular arrangement, like a pack of cards unshuffled; and that the velocities at that instant were regular also. But this regularity being of a purely aesthetic or formal kind, and the laws of motion equally formal and unrelated to any purpose, it follows that all kinds of arrangements will be produced, ungoverned by any uniformity, but mere effects of chance. Three stars may, for example, at some instant form an equilateral triangle; but there would be no particular reason for this: it would be merely a casual coincidence.

Peirce: CP 6.101 Cross-Ref:††

(c) Others go farther and maintain that the constants of position and velocity are subject to a law not merely formal, but are governed by final causes in such a way that there is no arrangement or coincidence whatever which was not specially intended by the Creator. To this theory, such words as providence and *fore*-knowledge are ill adapted; because the two constants which each atom or corpuscle has remain constant throughout all time, and ought not to be considered as

having been fixed at any particular epoch. The very idea is that the arrangement is determined by what would be the result of different arrangements at each period of time. If, for example, a given prayer effects rain, it must be supposed that, in view of that prayer, and as its consequence, the different atoms had the appropriate constants; but that these were not given to the atoms at any particular epoch, being permanent values. Any intentional action on the part of a free agent is to be explained in the same way. If an agent is to be supposed really free, it is difficult to see what other physical explanation is compatible with the exactitude of law. This seems to be substantially the notion of most of those who have supported free will.

Peirce: CP 6.101 Cross-Ref:††

On the other hand, many philosophers suppose a less degree of uniformity in nature than is supposed in opinion (*a*). Of these the following have come to the present writer's notice as being actually defended.

Peirce: CP 6.101 Cross-Ref:††

(*d*) Some suppose that while law is absolute, yet there are constantly arising cases analogous to unstable equilibrium in which, owing to a passage of a velocity through infinity or otherwise, the law does not determine what the motion shall be. Thus if one Boscovichian point attracts another inversely as the square of the distance, and they move in one straight line, then when they come together they may move through one another, or move backwards on the same line, or may separate along any other line, without violating the differential equation. Such "singularities," as the mathematicians say, are theoretically possible; and may be supposed to occur very often. But to suppose that free action becomes possible in such a way is very illogical. In the first place, it supposes a direct interaction between "mind" and matter; infinitesimal, no doubt, but none the less real. Why not better suppose a slight but finite action of this kind, and so avoid the following objections? Namely, in the second place, this is to put faith, not scientific credence, in the inductive laws of matter infinitely beyond what induction can ever warrant. We know very well that mind, in some sense, acts on matter, and matter on mind: the question is *how*. It is not in speculations of this fanciful kind that the true answer is likely to be found. In the third place, although this speculation wanders so far beyond all present knowledge, it nevertheless comes into conflict with a legitimate induction, namely, the supposition of any real "singularity" or breach of continuity in nature is in as distinct conflict with all our knowledge as is a miracle.

Peirce: CP 6.101 Cross-Ref:††

(*e*) Sundry far less tenable hypotheses of lacunae between inviolable laws have often been proposed. One opinion frequently met with is that the law of energy does not prescribe the direction of velocity, but only its amount; so that the mind may cause atoms to "swerve," in regular Lucretian fashion.†1 This singular notion has even been embraced by mathematicians, who are thinking of a projectile shot into a curved tube, or other case of an equation of condition. Of course, if mind can construct absolute constraints, it can much easier exert force that is finite. Other writers suppose lacunae, without telling us of what particular description they are; they seem to think law is absolute as far as it goes, but that its jurisdiction is limited.

Peirce: CP 6.101 Cross-Ref:††

(*f*) Much more philosophical and less logically objectionable is the notion of St. Augustine and others (it is near to the opinion of Aristotle) that the only fundamental kind of causation is the action of final causes, and that efficient causation is, in all cases, secondary. Accordingly, when a miracle occurs there is no violation of

the real *cursus naturae*, but only of the apparent course of things.

Peirce: CP 6.101 Cross-Ref:††

(g) The hypothesis suggested by the present writer is that all laws are results of evolution; that underlying all other laws is the only tendency which can grow by its own virtue, the tendency of all things to take habits. Now since this same tendency is the one sole fundamental law of mind, it follows that the physical evolution works towards ends in the same way that mental action works towards ends, and thus in one aspect of the matter it would be perfectly true to say that final causation is alone primary. Yet, on the other hand, the law of habit is a simple formal law, a law of efficient causation; so that either way of regarding the matter is equally true, although the former is more fully intelligent. Meantime, if law is a result of evolution, which is a process lasting through all time, it follows that no law is absolute. That is, we must suppose that the phenomena themselves involve departures from law analogous to errors of observation. But the writer has not supposed that this phenomenon had any connection with free will. In so far as evolution follows a law, the law of habit, instead of being a movement from homogeneity to heterogeneity, is growth from difformity to uniformity. But the chance divergences from law are perpetually acting to increase the variety of the world, and are checked by a sort of natural selection and otherwise (for the writer does not think the selective principle sufficient), so that the general result may be described as "organized heterogeneity," or, better, rationalized variety. In view of the principle of continuity, the supreme guide in framing philosophical hypotheses, we must, under this theory, regard matter as mind whose habits have become fixed so as to lose the powers of forming them and losing them, while mind is to be regarded as a chemical genus of extreme complexity and instability. It has acquired in a remarkable degree a habit of taking and laying aside habits. The fundamental divergences from law must here be most extraordinarily high, although probably very far indeed from attaining any directly observable magnitude. But their effect is to cause the laws of mind to be themselves of so fluid a character as to simulate divergences from law. All this, according to the writer, constitutes a hypothesis capable of being tested by experiment.

Peirce: CP 6.101 Cross-Ref:††

Literature: Besides most treatises on LOGIC (q.v., especially inductive) see Renouvier and Prat, *La nouvelle Monadologie* (1899).

Peirce: CP 6.102 Cross-Ref:††
B. Synechism and Agapism

CHAPTER 5

THE LAW OF MIND†1

§1. INTRODUCTIONE

102. In an article published in *The Monist* for January, 1891,†2 I endeavored to show what ideas ought to form the warp of a system of philosophy, and particularly emphasized that of absolute chance. In the number of April, 1892,†3 I argued further in favor of that way of thinking, which it will be convenient to christen *tychism* (from {tyché}, chance). A serious student of philosophy will be in no haste to accept or reject this doctrine; but he will see in it one of the chief attitudes which speculative thought may take, feeling that it is not for an individual, nor for an age, to pronounce upon a fundamental question of philosophy. That is a task for a whole era to work out. I have begun by showing that *tychism* must give birth to an evolutionary cosmology, in which all the regularities of nature and of mind are regarded as products of growth, and to a Schelling-fashioned idealism which holds matter to be mere specialized and partially deadened mind. I may mention, for the benefit of those who are curious in studying mental biographies, that I was born and reared in the neighborhood of Concord -- I mean in Cambridge -- at the time when Emerson, Hedge, and their friends were disseminating the ideas that they had caught from Schelling, and Schelling from Plotinus, from Boehm, or from God knows what minds stricken with the monstrous mysticism of the East. But the atmosphere of Cambridge held many an antiseptic against Concord transcendentalism; and I am not conscious of having contracted any of that virus. Nevertheless, it is probable that some cultured bacilli, some benignant form of the disease was implanted in my soul, unawares, and that now, after long incubation, it comes to the surface, modified by mathematical conceptions and by training in physical investigations.

Peirce: CP 6.103 Cross-Ref:††

103. The next step in the study of cosmology must be to examine the general law of mental action. In doing this, I shall for the time drop my tychism out of view, in order to allow a free and independent expansion to another conception signaled in my first *Monist* paper as one of the most indispensable to philosophy, though it was not there dwelt upon; I mean the idea of continuity †1 The tendency to regard continuity, in the sense in which I shall define it, as an idea of prime importance in philosophy may conveniently be termed synechism. The present paper is intended chiefly to show what synechism is, and what it leads to. I attempted, a good many years ago, to develop this doctrine in the *Journal of Speculative Philosophy* (Vol. II)†2; but I am able now to improve upon that exposition, in which I was a little blinded by nominalistic prepossessions. I refer to it, because students may possibly find that some points not sufficiently explained in the present paper are cleared up in those earlier ones.

Peirce: CP 6.104 Cross-Ref:††
§2. WHAT THE LAW IS

104. Logical analysis applied to mental phenomena shows that there is but one law of mind, namely, that ideas tend to spread continuously and to affect certain others which stand to them in a peculiar relation of affectibility. In this spreading they lose intensity, and especially the power of affecting others, but gain generality and become welded with other ideas.

Peirce: CP 6.104 Cross-Ref:††

I set down this formula at the beginning, for convenience, and now proceed to comment upon it.

Peirce: CP 6.105 Cross-Ref:††
§3. INDIVIDUALITY OF IDEAS

105. We are accustomed to speak of ideas as reproduced, as passed from mind to mind, as similar or dissimilar to one another, and, in short, as if they were substantial things; nor can any reasonable objection be raised to such expressions. But taking the word "idea" in the sense of an event in an individual consciousness, it is clear that an idea once past is gone forever, and any supposed recurrence of it is another idea. These two ideas are not present in the same state of consciousness, and therefore cannot possibly be compared. To say, therefore, that they are similar can only mean that an occult power from the depths of the soul forces us to connect them in our thoughts after they are both no more. We may note, here, in passing, that of the two generally recognized principles of association, contiguity and similarity, the former is a connection due to a power without, the latter a connection due to a power within.†1

Peirce: CP 6.106 Cross-Ref:††

106. But what can it mean to say that ideas wholly past are thought of at all, any longer? They are utterly unknowable. What distinct meaning can attach to saying that an idea in the past in any way affects an idea in the future, from which it is completely detached? A phrase between the assertion and the denial of which there can in no case be any sensible difference is mere gibberish.

Peirce: CP 6.106 Cross-Ref:††

I will not dwell further upon this point, because it is a commonplace of philosophy.

Peirce: CP 6.107 Cross-Ref:††
§4. CONTINUITY OF IDEAS †2

107. We have here before us a question of difficulty, analogous to the question of nominalism and realism. But when once it has been clearly formulated, logic leaves room for one answer only. How can a past idea be present? Can it be present vicariously? To a certain extent, perhaps, but not merely so; for then the question would arise how the past idea can be related to its vicarious representation. The relation, being between ideas, can only exist in some consciousness: now that past idea was in no consciousness but that past consciousness that alone contained it; and that did not embrace the vicarious idea.

Peirce: CP 6.108 Cross-Ref:††

108. Some minds will here jump to the conclusion that a past idea cannot in any sense be present. But that is hasty and illogical. How extravagant, too, to

pronounce our whole knowledge of the past to be mere delusion! Yet it would seem that the past is as completely beyond the bounds of possible experience as a Kantian thing-in-itself.†1

Peirce: CP 6.109 Cross-Ref:††

109. How can a past idea be present? Not vicariously. Then, only by direct perception. In other words, to be present, it must be *ipso facto* present. That is, it cannot be wholly past; it can only be going, infinitesimally past, less past than any assignable past date. We are thus brought to the conclusion that the present is connected with the past by a series of real infinitesimal steps.

Peirce: CP 6.110 Cross-Ref:††

110. It has already been suggested by psychologists that consciousness necessarily embraces an interval of time. But if a finite time be meant, the opinion is not tenable. If the sensation that precedes the present by half a second were still immediately before me, then, on the same principle, the sensation preceding that would be immediately present, and so *ad infinitum*. Now, since there is a time, say a year, at the end of which an idea is no longer *ipso facto* present, it follows that this is true of any finite interval, however short.

Peirce: CP 6.110 Cross-Ref:††

But yet consciousness must essentially cover an interval of time; for if it did not, we could gain no knowledge of time, and not merely no veracious cognition of it, but no conception whatever. We are, therefore, forced to say that we are immediately conscious through an infinitesimal interval of time.

Peirce: CP 6.111 Cross-Ref:††

111. This is all that is requisite. For, in this infinitesimal interval, not only is consciousness continuous in a subjective sense, that is, considered as a subject or substance having the attribute of duration, but also, because it is immediate consciousness, its object is *ipso facto* continuous. In fact, this infinitesimally spread-out consciousness is a direct feeling of its contents as spread out. This will be further elucidated below. In an infinitesimal interval we directly perceive the temporal sequence of its beginning, middle, and end -- not, of course, in the way of recognition, for recognition is only of the past, but in the way of immediate feeling. Now upon this interval follows another, whose beginning is the middle of the former, and whose middle is the end of the former. Here, we have an immediate perception of the temporal sequence of its beginning, middle, and end, or say of the second, third, and fourth instants. From these two immediate perceptions, we gain a mediate, or inferential, perception of the relation of all four instants. This mediate perception is objectively, or as to the object represented, spread over the four instants; but subjectively, or as itself the subject of duration, it is completely embraced in the second moment. (The reader will observe that I use the word *instant* to mean a point of time, and *moment* to mean an infinitesimal duration.) If it is objected that, upon the theory proposed, we must have more than a mediate perception of the succession of the four instants, I grant it; for the sum of the two infinitesimal intervals is itself infinitesimal, so that it is immediately perceived. It is immediately perceived in the whole interval, but only mediate perceived in the last two-thirds of the interval. Now, let there be an indefinite succession of these inferential acts of comparative perception, and it is plain that the last moment will contain objectively the whole series. Let there be, not merely an indefinite succession, but a continuous flow of inference through a finite time, and the result will be a mediate objective consciousness of the whole time in the last moment. In this last moment, the whole

series will be recognized, or known as known before, except only the last moment, which of course will be absolutely unrecognizable to itself. Indeed, even this last moment will be recognized like the rest, or, at least, be just beginning to be so. There is a little *elenchus*, or appearance of contradiction, here, which the ordinary logic of reflection quite suffices to resolve.

Peirce: CP 6.112 Cross-Ref:††
§5. INFINITY AND CONTINUITY, IN GENERAL

112. Most of the mathematicians who during the last two generations have treated the differential calculus have been of the opinion that an infinitesimal quantity is an absurdity; although, with their habitual caution, they have often added "or, at any rate, the conception of an infinitesimal is so difficult, that we practically cannot reason about it with confidence and security." Accordingly, the doctrine of limits has been invented to evade the difficulty, or, as some say, to explain the signification of the word "infinitesimal."†1 This doctrine, in one form or another, is taught in all the textbooks, though in some of them only as an alternative view of the matter; it answers well enough the purposes of calculation, though even in that application it has its difficulties.

Peirce: CP 6.113 Cross-Ref:††

113. The illumination of the subject by a strict notation for the logic of relatives had shown me clearly and evidently that the idea of an infinitesimal involves no contradiction,†2 before I became acquainted with the writings of Dr. Georg Cantor (though many of these had already appeared in the *Mathematische Annalen* and in *Borchardt's Journal*, if not yet in the *Acta Mathematica*, all mathematical journals of the first distinction), in which the same view is defended with extraordinary genius and penetrating logic.†3

Peirce: CP 6.114 Cross-Ref:††

114. The prevalent opinion is that finite numbers are the only ones that we can reason about, at least, in any ordinary mode of reasoning, or, as some authors express it, they are the only numbers that can be reasoned about mathematically. But this is an irrational prejudice. I long ago †4 showed that finite collections are distinguished from infinite ones only by one circumstance and its consequences, namely that to them is applicable a peculiar and unusual mode of reasoning called by its discoverer, De Morgan, the "syllogism of transposed quantity."†5

Peirce: CP 6.114 Cross-Ref:††

Balzac, in the introduction of his *Physiologie du mariage*, remarks that every young Frenchman boasts of having seduced some French woman. Now, as a woman can only be seduced once, and there are no more French women than Frenchmen, it follows, if these boasts are true, that no French women escape seduction. If their number be finite, the reasoning holds. But since the population is continually increasing, and the seduced are on the average younger than the seducers, the conclusion need not be true. In like manner, De Morgan, as an actuary, might have argued that if an insurance company pays to its insured on an average more than they have ever paid it, including interest, it must lose money. But every modern actuary would see a fallacy in that, since the business is continually on the increase. But should war, or other cataclysm, cause the class of insured to be a finite one, the

conclusion would turn out painfully correct, after all. The above two reasonings are examples of the syllogism of transposed quantity.

Peirce: CP 6.114 Cross-Ref:††

The proposition that finite and infinite collections are distinguished by the applicability to the former of the syllogism of transposed quantity ought to be regarded as the basal one of scientific arithmetic.

Peirce: CP 6.115 Cross-Ref:††

115. If a person does not know how to reason logically, and I must say that a great many fairly good mathematicians -- yea, distinguished ones -- fall under this category, but simply uses a rule of thumb in blindly drawing inferences like other inferences that have turned out well, he will, of course, be continually falling into error about infinite numbers. The truth is such people do not reason at all. But for the few who do reason, reasoning about infinite numbers is easier than about finite numbers, because the complicated syllogism of transposed quantity is not called for. For example, that the whole is greater than its part is not an axiom, as that eminently bad reasoner, Euclid, made it to be. It is a theorem readily proved by means of a syllogism of transposed quantity, but not otherwise. Of finite collections it is true, of infinite collections false. Thus, a part of the whole numbers are even numbers. Yet the even numbers are no fewer than all the numbers; an evident proposition, since if every number in the whole series of whole numbers be doubled, the result will be the series of even numbers.

1, 2, 3, 4, 5, 6, etc. 2, 4, 6, 8, 10, 12, etc.

So for every number there is a distinct even number. In fact, there are as many distinct doubles of numbers as there are of distinct numbers. But the doubles of numbers are all even numbers.

Peirce: CP 6.116 Cross-Ref:††

116. In truth, of infinite collections there are but two grades of magnitude, the *endless* and the *innumerable*.†1 Just as a finite collection is distinguished from an infinite one by the applicability to it of a special mode of reasoning, the syllogism of transposed quantity, so, as I showed in the paper last referred to,†1 a numerable collection is distinguished from an innumerable one by the applicability to it of a certain mode of reasoning, the Fermatian inference, or, as it is sometimes improperly termed, "mathematical induction."†2

Peirce: CP 6.116 Cross-Ref:††

As an example of this reasoning, Euler's demonstration of the binomial theorem for integral powers may be given. The theorem is that $(x+y)^n$, where n is a whole number, may be expanded into the sum of a series of terms of which the first is $x^n y^0$ and each of the others is derived from the next preceding by diminishing the exponent of x by 1 and multiplying by that exponent and at the same time increasing the exponent of y by 1 and dividing by that increased exponent. Now, suppose this

proposition to be true for a certain exponent, $n = M$, then it must also be true for $n = M+1$. For let one of the terms in the expansion of $(x+y)^M$ be written $Ax^p y^q$. Then, this term with the two following will be

$$A x^p y^q + A (p/q+1)x^{(p-1)}y^{(q+1)} + A(p/q+1)(p-1/q+2)x^{(p-2)} y^{(q+2)}$$

Now, when $(x+y)^M$ is multiplied by $x+y$ to give $(x+y)^{M+1}$, we multiply first by x and then by y instead of by x and add the two results. When we multiply by x , the second of the above three terms will be the only one giving a term involving $x^p y^{q+1}$ and the third will be the only one giving a term in $x^{(p-1)} y^{(q+2)}$; and when we multiply by y the first will be the only term giving a term in $x^p y^{(q+1)}$, and the second will be the only term giving a term in $x^{(p-1)} y^{(q+2)}$. Hence, adding like terms, we find that the coefficient of $x^p y^{(q+1)}$ in the expansion of $(x+y)^{(M+1)}$ will be the sum of the coefficients of the first two of the above three terms, and that the coefficient of $x^{(p-1)} y^{(q+2)}$ will be the sum of the coefficients of the last two terms. Hence, two successive terms in the expansion of $(x+y)^{(M+1)}$ will be

$$A(1+(p/q+1))x^p y^{q+1} + A(p/q+1)(1+(p-1/q+2))x^{p-1} y^{q+2} =$$

$$A((p+q+1)/(q+1))x^p y^{(q+1)} + A((p+q+1)/(q+1))(p/(q+2))x^{(p-1)} y^{q+2}.$$

Peirce: CP 6.116 Cross-Ref:††

It is thus seen that the succession of terms follows the rule. Thus if any integral power follows the rule, so also does the next higher power. But the first power obviously follows the rule. Hence, all powers do so.

Peirce: CP 6.116 Cross-Ref:††

Such reasoning holds good of any collection of objects capable of being ranged in a series which, though it may be endless, can be numbered so that each member of it receives a definite integral number. For instance, all the whole numbers constitute such a numerable collection. Again, all numbers resulting from operating according to any definite rule with any finite number of whole numbers form such a collection. For they may be arranged in a series thus. Let F be the symbol of operation. First operate on 1, giving $F(1)$. Then, operate on a second 1, giving $F(1,1)$. Next, introduce 2, giving 3d, $F(2)$; 4th, $F(2,1)$; 5th, $F(1,2)$; 6th, $F(2,2)$. Next use a third variable giving 7th, $F(1,1,1)$; 8th, $F(2,1,1)$; 9th, $F(1,2,1)$; 10th, $F(2,2,1)$; 11th, $F(1,1,2)$; 12th, $F(2,1,2)$; 13th, $F(1,2,2)$; 14th, $F(2,2,2)$. Next introduce 3, and so on, alternately introducing new variables and new figures; and in this way it is plain that every arrangement of integral values of the variables will receive a numbered place in the series. †P1

Peirce: CP 6.117 Cross-Ref:††

117. The class of endless but numerable collections (so called because they can be so ranged that to each one corresponds a distinct whole number) is very large. But there are collections which are certainly innumerable. Such is the collection of all numbers to which endless series of decimals are capable of approximating. It has been recognized since the time of Euclid that certain numbers are surd or incommensurable, and are not exactly expressible by any finite series of decimals, nor

by a circulating decimal. Such is the ratio of the circumference of a circle to its diameter, which we know is nearly 3.1415926. The calculation of this number has been carried to over 700 figures without the slightest appearance of regularity in their sequence. The demonstrations that this and many other numbers are incommensurable are perfect. That the entire collection of incommensurable numbers is innumerable has been clearly proved by Cantor.†1 I omit the demonstration;†1 but it is easy to see that to discriminate one from some other would, in general, require the use of an endless series of numbers. Now if they cannot be exactly expressed and discriminated, clearly they cannot be ranged in a linear series.

Peirce: CP 6.118 Cross-Ref:††

118. It is evident that there are as many points on a line or in an interval of time as there are of real numbers, in all. These are, therefore, innumerable collections. Many mathematicians have incautiously assumed that the points on a surface or in a solid are more than those on a line. But this has been refuted by Cantor.†2 Indeed, it is obvious that for every set of values of coordinates there is a single distinct number. Suppose, for instance, the values of the coordinates all lie between 0 and +1. Then if we compose a number by putting in the first decimal place the first figure of the first coordinate, in the second the first figure of the second coordinate, and so on, and when the first figures are all dealt out go on to the second figures in like manner, it is plain that the values of the coordinates can be read off from the single resulting number, so that a triad or tetrad of numbers, each having innumerable values, has no more values than a single incommensurable number.

Peirce: CP 6.118 Cross-Ref:††

Were the number of dimensions infinite, this would fail; and the collection of infinite sets of numbers, having each innumerable variations, might, therefore, be greater than the simple innumerable collection, and might be called *endlessly infinite*. The single individuals of such a collection could not, however, be designated, even approximately, so that this is indeed a magnitude concerning which it would be possible to reason only in the most general way, if at all.

Peirce: CP 6.119 Cross-Ref:††

119. Although there are but two grades of magnitudes of infinite collections, yet when certain conditions are imposed upon the order in which individuals are taken, distinctions of magnitude arise from that cause. Thus, if a simply endless series be doubled by separating each unit into two parts, the successive first parts and also the second parts being taken in the same order as the units from which they are derived, this double endless series will, so long as it is taken in that order, appear as twice as large as the original series. In like manner the product of two innumerable collections, that is, the collection of possible pairs composed of one individual of each, if the order of continuity is to be maintained, is, by virtue of that order, infinitely greater than either of the component collections.

Peirce: CP 6.120 Cross-Ref:††

120. We now come to the difficult question, What is continuity?†1 Kant †2 confounds it with infinite divisibility, saying that the essential character of a continuous series is that between any two members of it a third can always be found. This is an analysis beautifully clear and definite; but, unfortunately, it breaks down under the first test. For according to this, the entire series of rational fractions arranged in the order of their magnitude would be an infinite series, although the rational fractions are numerable, while the points of a line are innumerable. Nay, worse yet, if from that series of fractions any two with all that lie between them be

excised, and any number of such finite gaps be made, Kant's definition is still true of the series, though it has lost all appearance of continuity.

Peirce: CP 6.121 Cross-Ref:††

121. Cantor defines a continuous series as one which is *concatenated* and *perfect*.†3 By a concatenated series, he means such a one that if any two points are given in it, and any finite distance, however small, it is possible to proceed from the first point to the second through a succession of points of the series each at a distance, from the preceding one, less than the given distance. This is true of the series of rational fractions ranged in the order of their magnitude. By a perfect series, he means one which contains every point such that there is no distance so small that this point has not an infinity of points of the series within that distance of it. This is true of the series of numbers between 0 and 1 capable of being expressed by decimals in which only the digits 0 and 1 occur.

Peirce: CP 6.121 Cross-Ref:††

It must be granted that Cantor's definition includes every series that is continuous; nor can it be objected that it includes any important or indubitable case of a series not continuous. Nevertheless, it has some serious defects. In the first place, it turns upon metrical considerations; while the distinction between a continuous and a discontinuous series is manifestly non-metrical. In the next place, a perfect series is defined as one containing "every point" of a certain description. But no positive idea is conveyed of what all the points are: that is definition by negation, and cannot be admitted. If that sort of thing were allowed, it would be very easy to say, at once, that the continuous linear series of points is one which contains every point of the line between its extremities. Finally, Cantor's definition does not convey a distinct notion of what the components of the conception of continuity are. It ingeniously wraps up its properties in two separate parcels, but does not display them to our intelligence.

Peirce: CP 6.122 Cross-Ref:††

122. Kant's definition expresses one simple property of a continuum; but it allows of gaps in the series. To mend the definition, it is only necessary to notice how these gaps can occur. Let us suppose, then, a linear series of points extending from a point, *A*, to a point, *B*, having a gap from *B* to a third point, *C*, and thence extending to a final limit, *D*; and let us suppose this series conforms to Kant's definition. Then, of the two points, *B* and *C*, one or both must be excluded from the series; for otherwise, by the definition, there would be points between them. That is, if the series contains *C*, though it contains all the points up to *B*, it cannot contain *B*. What is required, therefore, is to state in non-metrical terms that if a series of points up to a limit is included in a continuum the limit is included. It may be remarked that this is the property of a continuum to which Aristotle's attention seems to have been directed when he defines a continuum as something whose parts have a common limit.†1 The property may be exactly stated as follows: If a linear series of points is continuous between two points, *A* and *D*, and if an endless series of points be taken, the first of them between *A* and *D* and each of the others between the last preceding one and *D*, then there is a point of the continuous series between all that endless series of points and *D*, and such that every other point of which this is true lies between this point and *D*. For example, take any number between 0 and 1, as 0.1; then, any number between 0.1 and 1, as 0.11; then any number between 0.11 and 1, as 0.111; and so on, without end. Then, because the series of real numbers between 0 and 1 is continuous, there must be a *least* real number, greater than every number of that endless series. This property, which may be called the Aristotelicity of the series, together with Kant's

property, or its Kanticity, completes the definition of a continuous series.†1

Peirce: CP 6.123 Cross-Ref:††

123. The property of Aristotelicity may be roughly stated thus: a continuum contains the end point belonging to every endless series of points which it contains. An obvious corollary is that every continuum contains its limits. But in using this principle it is necessary to observe that a series may be continuous except in this, that it omits one or both of the limits.

Peirce: CP 6.124 Cross-Ref:††

124. Our ideas will find expression more conveniently if, instead of points upon a line, we speak of real numbers. Every real number is, in one sense, the limit of a series, for it can be indefinitely approximated to. Whether every real number is a limit of a *regular* series may perhaps be open to doubt. But the series referred to in the definition of Aristotelicity must be understood as including all series whether regular or not. Consequently, it is implied that between any two points an innumerable series of points can be taken.

Peirce: CP 6.125 Cross-Ref:††

125. Every number whose expression in decimals requires but a finite number of places of decimals is commensurable. Therefore, incommensurable numbers suppose an infinitieth place of decimals. The word infinitesimal is simply the Latin form of infinitieth; that is, it is an ordinal formed from *infinitum*, as centesimal from *centum*. Thus, continuity supposes infinitesimal quantities.†2 There is nothing contradictory about the idea of such quantities. In adding and multiplying them the continuity must not be broken up, and consequently they are precisely like any other quantities, except that neither the syllogism of transposed quantity, nor the Fermatian inference applies to them.

Peirce: CP 6.125 Cross-Ref:††

If A is a finite quantity and i an infinitesimal, then in a certain sense we may write $A+i = A$. That is to say, this is so for all purposes of measurement. But this principle must not be applied except to get rid of *all* the terms in the highest order of infinitesimals present. As a mathematician, I prefer the method of infinitesimals to that of limits, as far easier and less infested with snares.†1 Indeed, the latter, as stated in some books, involves propositions that are false; but this is not the case with the forms of the method used by Cauchy,†2 Duhamel,†3 and others. As they understand the doctrine of limits, it involves the notion of continuity, and therefore contains in another shape the very same ideas as the doctrine of infinitesimals.

Peirce: CP 6.126 Cross-Ref:††

126. Let us now consider an aspect of the Aristotelical principle which is particularly important in philosophy. Suppose a surface to be part red and part blue; so that every point on it is either red or blue, and, of course, no part can be both red and blue. What, then, is the color of the boundary line between the red and the blue? The answer is that red or blue, to exist at all, must be spread over a surface; and the color of the surface is the color of the surface in the immediate neighborhood †4 of the point. I purposely use a vague form of expression. Now, as the parts of the surface in the immediate neighborhood of any ordinary point upon a curved boundary are half of them red and half blue, it follows that the boundary is half red and half blue. In like manner, we find it necessary to hold that consciousness essentially occupies time; and what is present to the mind at any ordinary instant is what is present during a moment in which that instant occurs. Thus, the present is half past and half to come. Again, the

color of the parts of a surface at any finite distance from a point has nothing to do with its color just at that point; and, in the parallel, the feeling at any finite interval from the present has nothing to do with the present feeling, except vicariously. Take another case: the velocity of a particle at any instant of time is its mean velocity during an infinitesimal instant in which that time is contained. Just so my immediate feeling is my feeling through an infinitesimal duration containing the present instant.

Peirce: CP 6.127 Cross-Ref:††
§6. ANALYSIS OF TIME

127. One of the most marked features about the law of mind is that it makes time to have a definite direction of flow from past to future. The relation of past to future is, in reference to the law of mind, different from the relation of future to past. This makes one of the great contrasts between the law of mind and the law of physical force, where there is no more distinction between the two opposite directions in time than between moving northward and moving southward.

Peirce: CP 6.128 Cross-Ref:††

128. In order, therefore, to analyze the law of mind, we must begin by asking what the flow of time consists in. Now, we find that in reference to any individual state of feeling, all others are of two classes, those which affect this one (or have a tendency to affect it, and what this means we shall inquire shortly),†1 and those which do not. The present is affectible by the past but not by the future.

Peirce: CP 6.129 Cross-Ref:††

129. Moreover, if state **A** is affected by state **B**, and state **B** by state **C**, then **A** is affected by state **C**, though not so much so. It follows, that if **A** is affectible by **B**, **B** is not affectible by **A**.

Peirce: CP 6.130 Cross-Ref:††

130. If, of two states, each is absolutely unaffected by the other, they are to be regarded as parts of the same state. They are contemporaneous.

Peirce: CP 6.131 Cross-Ref:††

131. To say that a state is *between* two states means that it affects one and is affected by the other. Between any two states in this sense lies an innumerable series of states affecting one another; and if a state lies between a given state and any other state which can be reached by inserting states between this state and any third state, these inserted states not immediately affecting or being affected by either, then the second state mentioned immediately affects or is affected by the first, in the sense that in the one the other is *ipso facto* present in a reduced degree.

Peirce: CP 6.131 Cross-Ref:††

These propositions involve a definition of time and of its flow. Over and above this definition they involve a doctrine, namely, that every state of feeling is affectible by every earlier state.

Peirce: CP 6.132 Cross-Ref:††
§7. THAT FEELINGS HAVE INTENSIVE CONTINUITY †1

132. Time with its continuity logically involves some other kind of continuity than its own. †2 Time, as the universal form of change, cannot exist unless there is something to undergo change and to undergo a change continuous in time there must be a continuity of changeable qualities. Of the continuity of intrinsic qualities of feeling we can now form but a feeble conception. The development of the human mind has practically extinguished all feelings, except a few sporadic kinds, sound, colors, smells, warmth, etc., which now appear to be disconnected and disparate. †3 In the case of colors, there is a tridimensional spread of feelings. Originally, all feelings may have been connected in the same way, and the presumption is that the number of dimensions was endless. For development essentially involves a limitation of possibilities. But given a number of dimensions of feeling, all possible varieties are obtainable by varying the intensities of the different elements. Accordingly, time logically supposes a continuous range of intensity in feeling. It follows, then, from the definition of continuity, that when any particular kind of feeling is present, an infinitesimal continuum of all feelings differing infinitesimally from that is present.

Peirce: CP 6.133 Cross-Ref: ††

§8. THAT FEELINGS HAVE SPATIAL EXTENSION †4

133. Consider a gob of protoplasm, say an amoeba or a slime-mould. It does not differ in any radical way from the contents of a nerve-cell, though its functions may be less specialized. There is no doubt that this slime-mould, or this amoeba, or at any rate some similar mass of protoplasm, feels. That is to say, it feels when it is in its excited condition. But note how it behaves. When the whole is quiescent and rigid, a place upon it is irritated. Just at this point, an active motion is set up, and this gradually spreads to other parts. In this action, no unity nor relation to a nucleus, or other unitary organ can be discerned. It is a mere amorphous continuum of protoplasm, with feeling passing from one part to another. Nor is there anything like a wave-motion. The activity does not advance to new parts just as fast as it leaves old parts. Rather, in the beginning, it dies out at a slower rate than that at which it spreads. And while the process is going on, by exciting the mass at another point, a second quite independent state of excitation will be set up. In some places, neither excitation will exist, in others each separately, in still other places, both effects will be added together. Whatever there is in the whole phenomenon to make us think there is feeling in such a mass of protoplasm -- *feeling*, but plainly no *personality* -- goes logically to show that that feeling has a subjective, or substantial, spatial extension, as the excited state has. This is, no doubt, a difficult idea to seize, for the reason that it is a subjective, not an objective, extension. It is not that we have a feeling of bigness; though Professor James, †1 perhaps rightly, teaches that we have. It is that the feeling, as a subject of inhesion, is big. Moreover, our own feelings are focused in attention to such a degree that we are not aware that ideas are not brought to an absolute unity; just as nobody not instructed by special experiment has any idea how very, very little of the field of vision is distinct. Still, we all know how the attention wanders about among our feelings; and this fact shows that those feelings that are not coordinated in attention have a reciprocal externality, although they are present at the same time. But we must not tax introspection to make a phenomenon manifest which essentially

involves externality.

Peirce: CP 6.134 Cross-Ref:††

134. Since space is continuous, it follows that there must be an immediate community of feeling between parts of mind infinitesimally near together. Without this, I believe it would have been impossible for minds external to one another ever to become coordinated, and equally impossible for any coordination to be established in the action of the nerve-matter of one brain.

Peirce: CP 6.135 Cross-Ref:††
§9. AFFECTIONS OF IDEAS

135. But we are met by the question, what is meant by saying that one idea affects another. The unravelment of this problem requires us to trace out phenomena a little further.

Peirce: CP 6.135 Cross-Ref:††

Three elements go to make up an idea. The first is its intrinsic quality as a feeling. The second is the energy with which it affects other ideas, an energy which is infinite in the here-and-nowness of immediate sensation, finite and relative in the recency of the past. The third element is the tendency of an idea to bring along other ideas with it.

Peirce: CP 6.136 Cross-Ref:††

136. As an idea spreads, its power of affecting other ideas gets rapidly reduced; but its intrinsic quality remains nearly unchanged. It is long years now since I last saw a cardinal in his robes; and my memory of their color has become much dimmed. The color itself, however, is not remembered as dim. I have no inclination to call it a dull red. Thus, the intrinsic quality remains little changed; yet more accurate observation will show a slight reduction of it. The third element, on the other hand, has increased. As well as I can recollect, it seems to me the cardinals I used to see wore robes more scarlet than vermilion is, and highly luminous. Still, I know the color commonly called cardinal is on the crimson side of vermilion and of quite moderate luminosity, and the original idea calls up so many other hues with it, and asserts itself so feebly, that I am unable any longer to isolate it.

Peirce: CP 6.137 Cross-Ref:††

137. A finite interval of time generally contains an innumerable series of feelings; and when these become welded together in association, the result is a general idea. For we have just seen how by continuous spreading an idea becomes generalized.

Peirce: CP 6.138 Cross-Ref:††

138. The first character of a general idea so resulting is that it is living feeling. A continuum of this feeling, infinitesimal in duration, but still embracing innumerable parts, and also, though infinitesimal, entirely unlimited, is immediately present. And in its absence of boundedness a vague possibility of more than is present is directly felt.

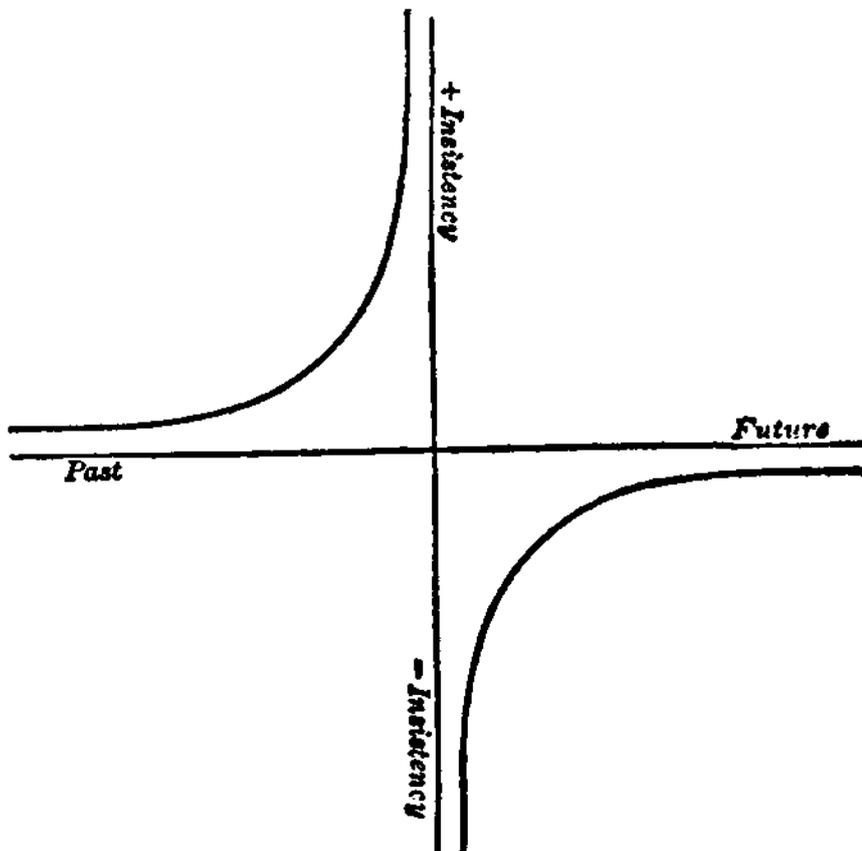
Peirce: CP 6.139 Cross-Ref:††

139. Second, in the presence of this continuity of feeling, nominalistic maxims

appear futile. There is no doubt about one idea affecting another, when we can directly perceive the one gradually modified and shaping itself into the other. Nor can there any longer be any difficulty about one idea resembling another, when we can pass along the continuous field of quality from one to the other and back again to the point which we had marked.

Peirce: CP 6.140 Cross-Ref:††

140. Third, consider the insistency of an idea. The insistency of a past idea with reference to the present is a quantity which is less the further back that past idea is, and rises to infinity as the past idea is brought up into coincidence with the present. Here we must make one of those inductive applications of the law of continuity which have produced such great results in all the positive sciences. We must extend the law of insistency into the future. Plainly, the insistency of a future idea with reference to the present is a quantity affected by the minus sign; for it is the present that affects the future, if there be any effect, not the future that affects the present. Accordingly, the curve of insistency is a sort of equilateral hyperbola.



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Such a conception is none the less mathematical, that its quantification cannot now be exactly specified.

Peirce: CP 6.141 Cross-Ref:††

141. Now consider the induction which we have here been led into. This curve

says that feeling which has not yet emerged into immediate consciousness is already affectible and already affected. In fact, this is habit, by virtue of which an idea is brought up into present consciousness by a bond that had already been established between it and another idea while it was still *in futuro*.

Peirce: CP 6.142 Cross-Ref:††

142. We can now see what the affection of one idea by another consists in. It is that the affected idea is attached as a logical predicate to the affecting idea as subject. So when a feeling emerges into immediate consciousness, it always appears as a modification of a more or less general object already in the mind. The word suggestion is well adapted to expressing this relation. The future is suggested by, or rather is influenced by the suggestions of, the past.

Peirce: CP 6.143 Cross-Ref:††

§10. IDEAS CANNOT BE CONNECTED EXCEPT BY CONTINUITY

143. That ideas can nowise be connected without continuity is sufficiently evident to one who reflects upon the matter. But still the opinion may be entertained that after continuity has once made the connection of ideas possible, then they may get to be connected in other modes than through continuity. Certainly, I cannot see how anyone can deny that the infinite diversity of the universe, which we call chance, may bring ideas into proximity which are not associated in one general idea. It may do this many times. But then the law of continuous spreading will produce a mental association; and this I suppose is an abridged statement of the way the universe has been evolved. But if I am asked whether a blind {ananké} cannot bring ideas together, first I point out that it would not remain blind. There being a continuous connection between the ideas, they would infallibly become associated in a living, feeling, and perceiving general idea. Next, I cannot see what the mustness or necessity of this {ananké} would consist in. In the absolute uniformity of the phenomenon, says the nominalist. Absolute is well put in; for if it merely happened so three times in succession, or three million times in succession, in the absence of any reason, the coincidence could only be attributed to chance. But absolute uniformity must extend over the whole infinite future; and it is idle to talk of that except as an idea. No, I think we can only hold that wherever ideas come together they tend to weld into general ideas; and wherever they are generally connected, general ideas govern the connection; and these general ideas are living feelings spread out.

Peirce: CP 6.144 Cross-Ref:††

§11. MENTAL LAW FOLLOWS THE FORMS OF LOGIC †1

144. The three main classes of logical inference are Deduction, Induction, and Hypothesis. These correspond to three chief modes of action of the human soul. In deduction the mind is under the dominion of a habit or association by virtue of which a general idea suggests in each case a corresponding reaction. But a certain sensation is seen to involve that idea. Consequently, that sensation is followed by that reaction. That is the way the hind legs of a frog, separated from the rest of the body, reason,

when you pinch them. It is the lowest form of psychical manifestation.

Peirce: CP 6.145 Cross-Ref:††

145. By induction, a habit becomes established. Certain sensations, all involving one general idea, are followed each by the same reaction; and an association becomes established, whereby that general idea gets to be followed uniformly by that reaction.

Peirce: CP 6.145 Cross-Ref:††

Habit is that specialization of the law of mind whereby a general idea gains the power of exciting reactions. But in order that the general idea should attain all its functionality, it is necessary, also, that it should become suggestible by sensations. That is accomplished by a psychical process having the form of hypothetic inference. By hypothetic inference, I mean, as I have explained in other writings,†2 an induction from qualities. For example, I know that the kind of man known and classed as a "mugwump" has certain characteristics. He has a high self-respect and places great value upon social distinction. He laments the great part that rowdyism and unrefined good fellowship play in the dealings of American politicians with their constituency. He thinks that the reform which would follow from the abandonment of the system by which the distribution of offices is made to strengthen party organizations and a return to the original and essential conception of office-filling would be found an unmixed good. He holds that monetary considerations should usually be the decisive ones in questions of public policy. He respects the principle of individualism and of *laissez-faire* as the greatest agency of civilization. These views, among others, I know to be obtrusive marks of a "mugwump." Now, suppose I casually meet a man in a railway train, and falling into conversation find that he holds opinions of this sort; I am naturally led to suppose that he is a "mugwump." That is hypothetic inference. That is to say, a number of readily verifiable marks of a mugwump being selected, I find this man has these, and infer that he has all the other characters which go to make a thinker of that stripe. Or let us suppose that I meet a man of a semi-clerical appearance and a sub-pharisaical sniff, who appears to look at things from the point of view of a rather wooden dualism. He cites several texts of Scripture and always with particular attention to their logical implications; and he exhibits a sternness, almost amounting to vindictiveness, towards evil doers in general. I readily conclude that he is a minister of a certain denomination. Now the mind acts in a way similar to this, every time we acquire a power of coördinating reactions in a peculiar way, as in performing any act requiring skill. Thus, most persons have a difficulty in moving the two hands simultaneously and in opposite directions through two parallel circles nearly in the medial plane of the body. To learn to do this, it is necessary to attend, first, to the different actions in different parts of the motion, when suddenly a general conception of the action springs up and it becomes perfectly easy. We think the motion we are trying to do involves this action, and this, and this. Then the general idea comes which unites all those actions, and thereupon the desire to perform the motion calls up the general idea. The same mental process is many times employed whenever we are learning to speak a language or are acquiring any sort of skill.

Peirce: CP 6.146 Cross-Ref:††

146. Thus, by induction, a number of sensations followed by one reaction become united under one general idea followed by the same reaction; while, by the hypothetic process, a number of reactions called for by one occasion get united in a general idea which is called out by the same occasion. By deduction, the habit fulfills its function of calling out certain reactions on certain occasions.

Peirce: CP 6.147 Cross-Ref:††
§12. UNCERTAINTY OF MENTAL ACTION

147. The inductive and hypothetic forms of inference are essentially probable inferences, not necessary; while deduction may be either necessary or probable.

Peirce: CP 6.148 Cross-Ref:††

148. But no mental action seems to be necessary or invariable in its character. In whatever manner the mind has reacted under a given sensation, in that manner it is the more likely to react again; were this, however, an absolute necessity, habits would become wooden and ineradicable and, no room being left for the formation of new habits, intellectual life would come to a speedy close. Thus, the uncertainty of the mental law is no mere defect of it, but is on the contrary of its essence. The truth is, the mind is not subject to "law" in the same rigid sense that matter is. It only experiences gentle forces which merely render it more likely to act in a given way than it otherwise would be. There always remains a certain amount of arbitrary spontaneity in its action, without which it would be dead.

Peirce: CP 6.149 Cross-Ref:††

149. Some psychologists think to reconcile the uncertainty of reactions with the principle of necessary causation by means of the law of fatigue. Truly for a *law*, this law of fatigue is a little lawless. I think it is merely a case of the general principle that an idea in spreading loses its insistency. Put me tarragon into my salad, when I have not tasted it for years, and I exclaim, "What nectar is this!" But add it to every dish I taste for week after week, and a habit of expectation has been created; and in thus spreading into habit, the sensation makes hardly any more impression upon me; or, if it be noticed, it is on a new side, from which it appears as rather a bore. The doctrine that fatigue is one of the primordial phenomena of mind I am much disposed to doubt.†1 It seems a somewhat little thing to be allowed as an exception to the great principle of mental uniformization. For this reason, I prefer to explain it in the manner here indicated, as a special case of that great principle. To consider it as something distinct in its nature, certainly somewhat strengthens the necessitarian position; but even if it be distinct, the hypothesis that all the variety and apparent arbitrariness of mental action ought to be explained away in favor of absolute determinism does not seem to me to recommend itself to a sober and sound judgment, which seeks the guidance of observed facts and not that of prepossessions.

Peirce: CP 6.150 Cross-Ref:††
§13. RESTATEMENT OF THE LAW

150. Let me now try to gather up all these odds and ends of commentary and restate the law of mind, in a unitary way.

Peirce: CP 6.150 Cross-Ref:††

First, then, we find that when we regard ideas from a nominalistic, individualistic, sensualistic way, the simplest facts of mind become utterly

meaningless. That one idea should resemble another or influence another, or that one state of mind should so much as be thought of in another, is, from that standpoint, sheer nonsense.

Peirce: CP 6.151 Cross-Ref:††

151. Second, by this and other means we are driven to perceive, what is quite evident of itself, that instantaneous feelings flow together into a continuum of feeling, which has in a modified degree the peculiar vivacity of feeling and has gained generality. And in reference to such general ideas, or continua of feeling, the difficulties about resemblance and suggestion and reference to the external cease to have any force.

Peirce: CP 6.152 Cross-Ref:††

152. Third, these general ideas are not mere words, nor do they consist in this, that certain concrete facts will every time happen under certain descriptions of conditions; but they are just as much, or rather far more, living realities than the feelings themselves out of which they are concentered. And to say that mental phenomena are governed by law does not mean merely that they are describable by a general formula; but that there is a living idea, a conscious continuum of feeling, which pervades them, and to which they are docile.

Peirce: CP 6.153 Cross-Ref:††

153. Fourth, this supreme law, which is the celestial and living harmony, does not so much as demand that the special ideas shall surrender their peculiar arbitrariness and caprice entirely; for that would be self-destructive. It only requires that they shall influence and be influenced by one another.

Peirce: CP 6.154 Cross-Ref:††

154. Fifth, in what measure this unification acts, seems to be regulated only by special rules; or, at least, we cannot in our present knowledge say how far it goes. But it may be said that, judging by appearances, the amount of arbitrariness in the phenomena of human minds is neither altogether trifling nor very prominent.

Peirce: CP 6.155 Cross-Ref:††
§14. PERSONALITY

155. Having thus endeavored to state the law of mind, in general, I descend to the consideration of a particular phenomenon which is remarkably prominent in our own consciousnesses, that of personality. A strong light is thrown upon this subject by recent observations of double and multiple personality. The theory, which at one time seemed plausible, that two persons in one body corresponded to the two halves of the brain will, I take it, now be universally acknowledged to be insufficient. But that which these cases make quite manifest is that personality is some kind of coordination or connection of ideas. Not much to say, this, perhaps. Yet when we consider that, according to the principle which we are tracing out, a connection between ideas is itself a general idea, and that a general idea is a living feeling, it is plain that we have at least taken an appreciable step toward the understanding of personality. This personality, like any general idea, is not a thing to be apprehended in an instant. It has to be lived in time; nor can any finite time embrace it in all its fullness. Yet in each infinitesimal interval it is present and living, though specially

colored by the immediate feelings of that moment. Personality, so far as it is apprehended in a moment, is immediate self-consciousness.

Peirce: CP 6.156 Cross-Ref:††

156. But the word coordination implies somewhat more than this; it implies a teleological harmony in ideas, and in the case of personality this teleology is more than a mere purposive pursuit of a predeterminate end; it is a developmental teleology. This is personal character. A general idea, living and conscious now, it is already determinative of acts in the future to an extent to which it is not now conscious.

Peirce: CP 6.157 Cross-Ref:††

157. This reference to the future is an essential element of personality. Were the ends of a person already explicit, there would be no room for development, for growth, for life; and consequently there would be no personality. The mere carrying out of predetermined purposes is mechanical. This remark has an application to the philosophy of religion. It is that a genuine evolutionary philosophy, that is, one that makes the principle of growth a primordial element of the universe, is so far from being antagonistic to the idea of a personal creator that it is really inseparable from that idea;†1 while a necessitarian religion is in an altogether false position and is destined to become disintegrated. But a pseudo-evolutionism which enthrones mechanical law above the principle of growth is at once scientifically unsatisfactory, as giving no possible hint of how the universe has come about, and hostile to all hopes of personal relations to God.

Peirce: CP 6.158 Cross-Ref:††

§15. COMMUNICATION

158. Consistently with the doctrine laid down in the beginning of this paper, I am bound to maintain that an idea can only be affected by an idea in continuous connection with it. By anything but an idea, it cannot be affected at all. This obliges me to say, as I do say, on other grounds, that what we call matter is not completely dead, but is merely mind hidebound with habits. It still retains the element of diversification; and in that diversification there is life. When an idea is conveyed from one mind to another, it is by forms of combination of the diverse elements of nature, say by some curious symmetry, or by some union of a tender color with a refined odor. To such forms the law of mechanical energy has no application. If they are eternal, it is in the spirit they embody; and their origin cannot be accounted for by any mechanical necessity. They are embodied ideas; and so only can they convey ideas. Precisely how primary sensations, as colors and tones, are excited, we cannot tell, in the present state of psychology. But in our ignorance, I think that we are at liberty to suppose that they arise in essentially the same manner as the other feelings, called secondary. As far as sight and hearing are in question, we know that they are only excited by vibrations of inconceivable complexity; and the chemical senses are probably not more simple. Even the least psychical of peripheral sensations, that of pressure, has in its excitation conditions which, though apparently simple, are seen to be complicated enough when we consider the molecules and their attractions. The principle with which I set out requires me to maintain that these feelings are communicated to the nerves by continuity, so that there must be something like them

in the excitants themselves. If this seems extravagant, it is to be remembered that it is the sole possible way of reaching any explanation of sensation, which otherwise must be pronounced a general fact, absolutely inexplicable and ultimate. Now absolute inexplicability is a hypothesis which sound logic refuses under any circumstances to justify.

Peirce: CP 6.159 Cross-Ref:††

159. I may be asked whether my theory would be favorable or otherwise to telepathy.†1 I have no decided answer to give to this. At first sight, it seems unfavorable. Yet there may be other modes of continuous connection between minds other than those of time and space.

Peirce: CP 6.160 Cross-Ref:††

160. The recognition by one person of another's personality takes place by means to some extent identical with the means by which he is conscious of his own personality. The idea of the second personality, which is as much as to say that second personality itself, enters within the field of direct consciousness of the first person, and is as immediately perceived as his ego, though less strongly. At the same time, the opposition between the two persons is perceived, so that the externality of the second is recognized.

Peirce: CP 6.161 Cross-Ref:††

161. The psychological phenomena of intercommunication between two minds have been unfortunately little studied. So that it is impossible to say, for certain, whether they are favorable to this theory or not. But the very extraordinary insight which some persons are able to gain of others from indications so slight that it is difficult to ascertain what they are is certainly rendered more comprehensible by the view here taken.

Peirce: CP 6.162 Cross-Ref:††

162. A difficulty which confronts the synechistic philosophy is this. In considering personality, that philosophy is forced to accept the doctrine of a personal God; but in considering communication, it cannot but admit that if there is a personal God, we must have a direct perception of that person and indeed be in personal communication with him. Now, if that be the case, the question arises how it is possible that the existence of this being should ever have been doubted by anybody. The only answer that I can at present make is that facts that stand before our face and eyes and stare us in the face are far from being, in all cases, the ones most easily discerned. That has been remarked from time immemorial.

Peirce: CP 6.163 Cross-Ref:††

§16. CONCLUSION

163. I have thus developed as well as I could in a little space the *synechistic* philosophy, as applied to mind. I think that I have succeeded in making it clear that this doctrine gives room for explanations of many facts which without it are absolutely and hopelessly inexplicable; and further that it carries along with it the following doctrines: first, a logical realism of the most pronounced type; second, objective idealism; third, tychism, with its consequent thorough-going evolutionism. We also notice that the doctrine presents no hindrances to spiritual influences, such as

some philosophies are felt to do.

Peirce: CP 6.164 Cross-Ref:††

CHAPTER 6

THE CONTINUUM

§1. KANT'S DEFINITION †1

164. [*Continuous* means] in mathematics and philosophy a connection of points (or other elements) as intimate as that of the instants or points of an interval of time: thus, the *continuity* of space consists in this, that a point can move from any one position to any other so that at each instant it shall have a definite and distinct position in space. This statement is not, however, a proper definition of *continuity*, but only an exemplification drawn from time. The old definitions -- the fact that adjacent parts have their limits in common (Aristotle), infinite divisibility (Kant), the fact that between any two points there is a third (which is true of the system of rational numbers) -- are inadequate.†2 The less unsatisfactory definition is that of G. Cantor, that continuity is the *perfect concatenation* of a system of points -- words which must be understood in special senses.†3 Cantor calls a system of points *concatenated* when any two of them being given, and also any finite distance, however small, it is always possible to find a finite number of other points of the system through which by successive steps, each less than the given distance, it would be possible to proceed from one of the given points to the other. He terms a system of points *perfect* when, whatever point belonging to the system be given, it is not possible to find a finite distance so small that there are not an infinite number of points of the system within that distance of the given point. As examples of a concatenated system not perfect, Cantor gives the rational and also the irrational numbers in any interval. As an example of a perfect system not concatenated, he gives all the numbers whose expression in decimals, however far carried out, would contain no figures except 0 and 9.

Peirce: CP 6.165 Cross-Ref:††

165. Cantor's definition of continuity is unsatisfactory as involving a vague reference to *all* the points, and one knows not what that may mean. It seems to me to point to this: that it is impossible to get the idea of continuity without two dimensions. An oval line is continuous, because it is impossible to pass from the inside to the outside without passing a point of the curve.

Peirce: CP 6.166 Cross-Ref:††

166. Subsequent to writing the above [164] I made a new definition, according to which continuity consists in *Kanticity* and *Aristotelicity*.†1 The Kanticity is having a point between any two points. The Aristotelicity is having every point that is a limit to an infinite series of points that belong to the system.

Peirce: CP 6.167 Cross-Ref:††

167. I here slightly modify Cantor's definition of a perfect system. Namely, he defines it as such that it contains every point in the neighborhood of an infinity of points and no other. But the latter is a character of a concatenated system; hence I omit it as a character of a perfect system.

Peirce: CP 6.168 Cross-Ref:††

168. But further study of the subject has proved that this definition is wrong. It involves a misunderstanding of Kant's definition which *he himself* likewise fell into. Namely he defines a continuum as that all of whose parts have parts of the same kind.†2 He himself, and I after him, understood that to mean infinite divisibility, which plainly is not what constitutes continuity since the series of rational fractional values is infinitely divisible but is not by anybody regarded as continuous. Kant's real definition implies that a continuous line contains no points. Now if we are to accept the common sense idea of continuity (after correcting its vagueness and fixing it to mean something) we must either say that a continuous line contains no points or we must say that the principle of excluded middle does not hold of these points. The principle of excluded middle only applies to an individual (for it is not true that "Any man is wise" nor that "Any man is not wise"†3). But places, being mere possibles without actual existence, are not individuals. Hence a point or indivisible place really does not exist unless there actually be something there to mark it, which, if there is, interrupts the continuity. I, therefore, think that Kant's definition correctly defines the common sense idea, although there are great difficulties with it. I certainly think that on any line whatever, on the common sense idea, there is room for any multitude of points however great. If so, the *analytical continuity* of the theory of functions, which implies there is but a single point for each distance from the origin, defined by a quantity expressible to indefinitely close approximation by a decimal carried out to an indefinitely great number of places, is certainly not the continuity of common sense, since the whole multitude of such quantities is only the first abnumeral multitude, and there is an infinite series of higher grades.†1 On the whole, therefore, I think we must say that continuity is the relation of the parts of an unbroken space or time. The precise definition is still in doubt; but Kant's definition, that a continuum is that of which every part has itself parts of the same kind, seems to be correct. This must not be confounded (as Kant himself confounded it) with infinite divisibility, but implies that a line, for example, contains no points until the continuity is broken by marking the points. In accordance with this it seems necessary to say that a continuum, where it is continuous and unbroken, contains no definite parts; that its parts are created in the act of defining them and the precise definition of them breaks the continuity. In the calculus and theory of functions it is assumed that between any two rational points (or points at distances along the line expressed by rational fractions) there are rational points and that further for every convergent series of such fractions (such as 3.1, 3.14, 3.141, 3.1415, 3.14159, etc.) there is just one limiting point; and such a collection of points is called *continuous*. But this does not seem to be the common sense idea of continuity. It is only a collection of independent points. Breaking grains of sand more and more will only make the sand more broken. It will not weld the grains into unbroken continuity.

Peirce: CP 6.169 Cross-Ref:††

§2. SYNECHISM †1

169. [Synechism is] that tendency of philosophical thought which insists upon the idea of continuity as of prime importance in philosophy and, in particular, upon the necessity of hypotheses involving true continuity.

Peirce: CP 6.170 Cross-Ref:††

170. A true continuum is something whose possibilities of determination no multitude of individuals can exhaust. Thus, no collection of points placed upon a truly continuous line can fill the line so as to leave no room for others, although that collection had a point for every value towards which numbers, endlessly continued into the decimal places, could approximate; nor if it contained a point for every possible permutation of all such values. It would be in the general spirit of synechism to hold that time ought to be supposed truly continuous in that sense. The term was suggested and used by C. S. Peirce in 1892.†2 (Cf. Pragmatism, passim.†3)

Peirce: CP 6.171 Cross-Ref:††

171. The general motive is to avoid the hypothesis that this or that is inexplicable. For the synechist maintains that the only possible justification for so much as entertaining a hypothesis is that it affords an explanation of the phenomena. Now, to suppose a thing inexplicable is not only to fail to explain it, and so to make an unjustifiable hypothesis, but, much worse, it is to set up a barrier across the road of science, and to forbid all attempt to understand the phenomenon.

Peirce: CP 6.172 Cross-Ref:††

172. To be sure, the synechist cannot deny that there is an element of the inexplicable and ultimate, because it is directly forced upon him; nor does he abstain from generalizing from this experience. True generality is, in fact, nothing but a rudimentary form of true continuity. Continuity is nothing but perfect generality of a law of relationship.

Peirce: CP 6.173 Cross-Ref:††

173. It would, therefore, be most contrary to his own principle for the synechist not to generalize from that which experience forces upon him, especially since it is only so far as facts can be generalized that they can be understood; and the very reality, in his way of looking at the matter, is nothing else than the way in which facts must ultimately come to be understood. There would be a contradiction here, if this ultimacy were looked upon as something to be absolutely realized; but the synechist cannot consistently so regard it. Synechism is not an ultimate and absolute metaphysical doctrine; it is a regulative principle of logic, prescribing what sort of hypothesis is fit to be entertained and examined. The synechist, for example, would never be satisfied with the hypothesis that matter is composed of atoms, all spherical and exactly alike. If this is the only hypothesis that the mathematicians are as yet in condition to handle, it may be supposed that it may have features of resemblance with the truth. But neither the eternity of the atoms nor their precise resemblance is, in the synechist's view, an element of the hypothesis that is even admissible hypothetically. For that would be to attempt to explain the phenomena by means of an absolute inexplicability. In like manner, it is not a hypothesis fit to be entertained that any given law is absolutely accurate. It is not, upon synechist principles, a question to be asked, whether the three angles of a triangle amount precisely to two right angles, but only whether the sum is greater or less. So the synechist will not believe that some things are conscious and some unconscious, unless by consciousness be meant a

certain grade of feeling. He will rather ask what are the circumstances which raise this grade; nor will he consider that a chemical formula for protoplasm would be a sufficient answer. In short, synechism amounts to the principle that inexplicabilities are not to be considered as possible explanations; that whatever is supposed to be ultimate is supposed to be inexplicable; that continuity is the absence of ultimate parts in that which is divisible; and that the form under which alone anything can be understood is the form of generality, which is the same thing as continuity.

Peirce: CP 6.174 Cross-Ref:††
§3. CONTINUITY REDEFINED †1E

174. I feel that I ought to make amends for my blundering treatment of Continuity in a paper entitled "The Law of Mind," in Vol. II of *The Monist*, †2 by here redefining it after close and long study of the question. Whatever is continuous has *material parts*.

Peirce: CP 6.174 Cross-Ref:††

I begin by defining these thus: The *material parts* of a thing or other object, *W*, that is composed of such parts, are whatever things are, firstly, each and every one of them, other than *W*; secondly, are all of some one internal nature (for example, are all places, or all times, or all spatial realities, or are all spiritual realities, or are all ideas, or are all characters, or are all relations, or are all external representations, etc.); thirdly, form together a collection of objects in which no one occurs twice over and, fourthly, are such that the Being of each of them together with the modes of connexion between all subcollections of them, constitute the being of *W*. Almost everything which has material parts has different sets of such parts, often various *ad libitum*. Nothing which has an Essence (such as an essential purpose or use, like the jackknife of the celebrated poser) has any material parts in the strict sense just defined. But the term "material parts" may, without confusion (if a little care be exerted), be used in a somewhat looser sense. Namely, if the Being (generally, a Concept) of an object, *T*, essentially involves something *C* which prevents it from having any material parts in the strict sense, and if there be something, *W*, which differs from *T* only in the absence of *C* and of any other such hindrances, so that *W* has material parts, then the material parts of *W* may loosely be termed material parts of *T*; but in such case the concept of *W* so derived from *T* is nearly or quite always somewhat vague, so that either the material parts will be so too, or else they must be conceived as merely the parts of some *state* of it, and very likely of an instantaneous state that is an *ens rationis* closely approximating to the nature of a fiction. It will be seen that the definition of Material Parts involves the concept of *Connexion*, even if there be no other connexion between them than co-being; and in case no other connexion be essential to the concept of *W*, this latter is called a *Collection*, concerning which I have merely to say that my reflexions on Mr. Alfred Bray Kempe's invaluable, very profound, and marvellously strong contribution to the science of Logic in the *Philosophical Transactions* [of the Royal Society, v. 177] for 1886 (which, by the way, seems to have proved too strong food for the mewling, etc., creatures who write the treatises on the science) have led me to believe it to be indecomposable. But I dare not be positive thereanent.

Peirce: CP 6.175 Cross-Ref:††

175. I must here give the substance of a far-famed definition of *equality in multitude* which was originally due to Bernardo Bolzano.†1 This writer was a Catholic priest in Buda-Pesth who published a treatise on Logic in four volumes, and a work entitled *Paradoxes of the Infinite*. In one or other of these he certainly laid the foundations of the great modern *exact logic of quantity*, which has so far been developed under the lead of the immortal Dr. Georg Cantor. Though I have never seen either work I do not hesitate to say that Bolzano put Human Reason under an eternal debt by laying the foundation of this science, since his definition of equality sufficed of itself to do that; and I need hardly say that the Catholic Church, which carries consistency as far as is consistent with any life at all, visited condign punishment upon the priest for such outrageous violation of loyalty to Her as the giving of aid and comfort to Human Reason -- and most traitorous of all to Reasoning about Infinity! -- was felt to be by Her and by all the world except the poor simple soul who committed that foul offense. I gave the substance of the definition in a former paper,†2 going on to other matters of importance which I need not here touch upon. But owing to my having then a very imperfect understanding of graphs, I expressed the definition in the insufficiently analytic language of my Algebra of Dyadic Relations (the same that is mainly employed in Schröder's third volume). I am continually obliged to make elementary explanations owing to the disgracefully unscientific state of Logic, which is quite as much behind its condition six centuries ago in some particulars as it is in advance of that state in others. As for contemporary text-books in our language, they are the merest rubbish on the whole. The very best that can be said of them is that a few have merits in particular directions. They are all amateurish and encourage amateurish views of the universe and of life. In comparison with the state of all the non-philosophical sciences, they are downright puerile; and a green scum grows over them year by year. If our people were at all aware of this blot upon our civilization, it would be possible for a scientific student of the subject of some real strength to put forth at least a primer of the science. But it is a condition of the success of any such student in penetrating to the true science that he should make himself a recluse. He is thus out of the swim, and is crowded out of all opportunities to be of much service; whereby Spencerism, Agnosticism, and other amateurisms, whose professors lose precious little time in arduous research, are able to gain the exclusive ear of the ignorant persons whom they court. In the fourteenth century Nominalism was rendered a respectable opinion by the halting realism of Scotus and by the extravagant unpragmatism of his followers. But after physical science has discovered so many general principles in Nature, nominalism becomes a disgraceful habitude of thought.

Peirce: CP 6.176 Cross-Ref:††

176. But now I define a *pseudo-continuum* as that which modern writers on the theory of functions call a *continuum*. But this is fully represented by, and according to G. Cantor stands in one-to-one correspondence with, the totality of real values, rational and irrational; and these are iconized, in their turn, according to these writers [by the] entire body of decimal expressions carried out to the right to all finite powers of 1/10 without going on to Cantor's {ö}th place of decimals.

Peirce: CP 6.176 Cross-Ref:††

For it is a principle continually employed in the reasoning of the universally accepted "doctrine of limits" that two values, that differ at all, differ by a *finite value*, which would not be true if the {ö}th place of decimals were supposed to be included in their exact expressions; and indeed the whole purpose of the doctrine of limits is to avoid acknowledging that that place is concerned. Consequently the denumeral rows

of figures which, by virtue of a simple general principle, are in one-to-one correspondence with the values, have relations among themselves, quite regardless of their denoting those values that perfectly agree in form with the relations between the values; and consequently these unlimited decimal fractions themselves, apart from their significations, constitute a pseudo-continuum. This consideration renders it easy to define a pseudo-continuum. It is in the first place a collection of objects absolutely distinct from one another. Now from the fact that Cantor and others call it a "continuum," as well as from other things they say about it, I am led to suspect that they do not regard the pseudo-continuum of unlimited decimal expressions as [having members] all absolutely distinct from any other, for the reason that, taking any one of them, it does not possess any one elementary and definite non-relative character which is not possessed by any other of them. But this is not what I mean, nor what is generally meant, by a collection of absolutely independent members. What I mean by that expression is that every member is distinguished from every other by possessing some one or another elementary and definite non-relative character which that other does not possess; and that this is the usual acceptance of the expression is evidenced by the fact that the majority of logicians are in the habit of conceiving of a universe of absolutely distinct individual objects, by which they only mean that every individual is in every respect, of a certain universe of respects, determined in one or other of two ways and that every individual is differently determined from every other in some of those respects; and they do not generally conceive that every individual object has a determination in any one elementary and definite respect, while all the other individuals are determined in the opposite way.

Peirce: CP 6.177 Cross-Ref:††
 §4. ACHILLES AND THE TORTOISE †1

177. . . . Three times in my life it has happened to me to receive visits from writers, each of them illustrious as compared with myself (and one of them was certainly one of the most famous men of his time), who came to talk with me about the paradox of Achilles and the Tortoise. Now since this ridiculous little catch presents no difficulty at all to a mind adequately trained in mathematics and in logic, but is one of those which is very apt to excite minds of a certain class to an obstinate determination to believe a given proposition -- if a high degree of courtesy, not to say veneration (hard for an unsocial student to observe with veracity) has ever been found wanting in those who have endeavoured to set them right -- I will make use of my experience gained in those three interviews to illustrate the danger of mistaking an accidental and temporary inability to doubt a proposition for such an absolute inability as can safely be considered an excuse for allowing oneself to become wedded to a belief.

Peirce: CP 6.178 Cross-Ref:††

178. All three of my interviewers entered upon the subject of the paradox by asking me to state my view of it; and I answered each of them more or less as follows: "A uniform speed is a quantity which multiplied into a time will give, as the product, the distance which an object moving uniformly with that speed will pass over in that time. So, if we use a capital L to denote the adopted unit of length, a capital T for that

of time, and the lower case italics n, m, l, k , etc., for any abstract numbers whole or fractional, of which it may be convenient to speak without specifying their values, we can denote a uniform speed that would cover any number, n , of units of length, in any number, m , of units of time, by (n/m)

(L/T) , or $(nL)/(mT)$, or $L/((m/n)T)$, etc.

This speed kept up for the time kT will result in covering a space of $(n/m)(L/T) \cdot kT = ((n \cdot k)/m)L$; and the same speed will cover l units of length in the time $lL/(n/m)(L/T) = (lm/n)T$, or $l|(n/m)$ units of time.

Peirce: CP 6.178 Cross-Ref:††

Now the start, allowed to the tortoise before the race begins, may be measured, say, by sL or a number, s , of units of length. If we express the uniform speeds of the two runners during the race by the numbers of units of length they respectively run in each unit of time, writing their initials, a for Achilles and t for tortoise to express this number, their speeds will be aL/T and tL/T . It cannot be doubted that this famous race was run in southern Thessaly, say in latitude 39° where, by the rotation of the earth, everything is moving due east at a rate of nearly $6\ 3/4$ miles a minute. Nevertheless, the point of the compass toward which they ran is not stated; so that it is to be assumed that it would make no difference, as, indeed, common sense would have testified that it could not, if that guide had not been dismissed in the very act of taking up the consideration of this problem. If, then, a velocity of 6.71 miles a minute makes no difference in the result, we may safely assume that it would make none if the race were run on a deck which was floating from the front to the back of the runners at the same rate as the tortoise was running forward. The effect of this would be to diminish the resultant velocity of each runner by tL/T , so that the resultant speed of Achilles would be $(a-t)L/T$, while that of the tortoise would disappear entirely: he would really be at rest. Since then at the instant when the race began Achilles was sL behind the tortoise, the time required for him to reach that stationary racer must have been $sL/[(a-t)L/T] = (s/(a-t))T$, or $s/(a-t)$ units of time. Zeno's method of approximating to the time required for Achilles to overtake the tortoise manifests a want of arithmetical skill that was inevitable at his time. Yet, after all, it coincides in principle, and in many cases in the arithmetical result, with the method of long division. Namely, he calculates the quantity that is really $s/(a-t)$, where t is much smaller than a , by taking as his first approximation (s/a) , or the time which would be correct if $t=0$, that is, if the tortoise did not move. But since during this time the tortoise moves over the distance $(s/a)t$, he adds to his first approximation the time required for Achilles to run this distance, which is $(s/a^2)t$, and so gets his second approximation $s/a + (s/a^2)t$. Proceeding in this way he obtains, as his final result, the endless series $s/a + (s/a^2)t + (s/a^3)t^2 + (s/a^4)t^3 + \text{etc.}$. . . Now let us calculate $s/(a-t)$ by long division.

Peirce: CP 6.178 Cross-Ref:††

Here is the work:

$$a-t)s \quad (s/a + (s/a^2)t + (s/a^3)t^2 + (s/a^4)t^3 + \text{etc.})$$

$$s - (s/a)t$$

$$(s/a)t$$

$$(s/a)t - (s/a^2)t^2$$

$$(s/a^2)t^2$$

$$(s/a^2)t^2 - (s/a^3)t^3$$

$$(s/a^3)t^3$$

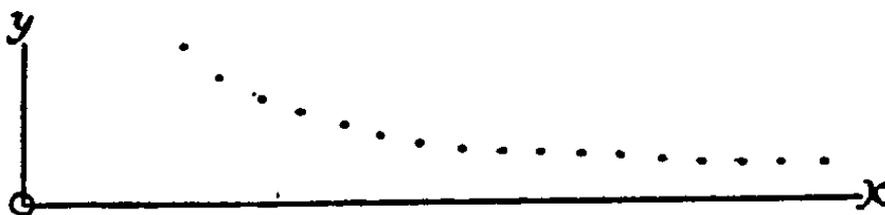
Peirce: CP 6.178 Cross-Ref:††

Arithmetically, of course, long division will come to an end, in case the divisor has no other factor, that the dividend has not, than such as are products of powers of factors of the base of numeration; and indeed, it may *always* be brought to an end in several ways. One of these is by adopting a smaller and appropriate unit of time. Another is by adopting a different and appropriate base of numeration. These devices show how entirely the *nodus* of the paradox of Achilles and the tortoise is a difficulty of the arithmetician who is awkward in finding an appropriate *expression* of that which Achilles does without the least embarrassment.

Peirce: CP 6.179 Cross-Ref:††

179. I now at length come to the ways in which different writers on this paradox have mistaken an impotence of thought, really due to their own insufficient study, for an impossibility of human thought universal. Zeno himself concluded that motion was altogether inconceivable, in spite of the fact that all men, himself included -- the very babe in the cradle included -- conceive it well enough, though they may become occasionally a little confused about it. Others have thought it inconceivable that Achilles should pass through a series of points essentially endless. One of my interviewers raised this point; and I am inclined to admit, though I do not feel absolutely sure of it, that a man could not, in a finite time, make an endless series of distinct volitions. But this does not embarrass Achilles the least in the world, for his final effort carries him through a whole infinity of the points into which Zeno's unskillful method would (could it be carried out) divide the space which that last bound of the hero covered. It seemed to me that when I showed my interviewer, by a comparison, that a series might be endless in respect to its succession of members and yet very short in another respect, that this simple reflexion took him by surprise and was quite a new idea to him, though he quickly exercised a will to continue believing in the inconceivability of passing over an endless succession in a finite time, and determined not to be shaken from it. My illustration was simply a succession of points on squared paper, so that the *x*-coördinates should be 4, 5, 6, 7, 8, 9, 10, etc., and supposed to extend endlessly while the *y*-coördinates were $y = 16/x$; and so the series would be endless, if it were completed, yet the whole measure of its *ys* would be only

4.



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to view]

Peirce: CP 6.180 Cross-Ref:††

180. The difficulty of another of my interviewers seemed to me less silly. He said, "Is not every one of the parts into which Zeno cuts up the time finite?" "Certainly," said I. "And are they not infinite in number?" "In multitude, yes." "Well, to me, it is perfectly inconceivable that an infinite multitude of objects, each finite, should not be infinite!" I told him it was *next to* inconceivable to me that the thing should remain inconceivable to him much longer. "Do you not fully agree that the sum of a lot of lengths is equal to their number multiplied into their average length?" I asked. "Yes," he said, "but what is the average length of these parts?" It was, however, easy to prove to him that fixing upon any definite value, however small, this value must be much greater than the average of all the successive terms of Zeno's series. For with each term of that series that is larger than the assumed value (the total number of which will be finite) one might associate a million less one of terms, each less than the quotient assumed value, divided by a billion times the number of all the terms greater than the assumed value; and this could evidently be done without using any term twice. Thus, we should find that, of each of the lots of terms each including one of terms larger than the assumed value, the average value would be less than the assumed value; and consequently the average value of all would be less than that value. So I proved that the average value of all the terms would be less than any assumed value; and consequently the fact that their multitude is infinite is no proof that their sum is infinite. It will be observed that I felt obliged to reason in the inelegant and awkward way I did, instead of directly proving the rule for the summation of a geometrical progression because I was dealing with one of those minds who, not detecting the fallacy (which always consists in their not being able to conceive of something which is, however, true) of a certain reasoning, directly opposed to a sound mathematical demonstration, passionately take the former to their bosoms, and not only regard all who trust to the latter as criminals, but think no act that tends to suppress such criminals is wrong. Those who have not had to do with this class of persons will think my statement exaggerated. I can only say to them, "Consult some other person who has had experience, before you rashly argue with these persons as you would with others." For that reason I felt I must avoid bringing in the rule for the summation of a geometrical series, and so condemned myself to the awkward argument I used. But all reasoning is quite thrown away upon a person who has once set his teeth and has resolved to believe in a definite proposition.

Peirce: CP 6.181 Cross-Ref:††

181. There is one intellectual habit which I have laboured very seriously to cultivate, and of which I have a number of times experienced advantages enough, each one of them, to repay all the work I have done toward acquiring it: I mean the

habit, when I have been upon the point of assenting, in my own mind, to some conclusion, [and when] I knew that some other mind (whose ways of thinking were very unlike my own, but whom I had known to have reached, in his way, truths not easy to reach) had considered the matter and had reached a conclusion inconsistent with the one that was recommending itself to me, of pausing, endeavoring to put myself in that other's point of view, reconsidering more minutely my whole reasoning, seeking to weld it to other reasonings and reflexions which all sound thinkers would approve, and doing my best to find weak points in the reasoning I came so near to embracing. I should not venture to recommend the cultivation of this habit to any of those who set up their own accidental impossibility of conceiving, as a permanent and essential one, before which all other men ought to crook the knee; since the very essence of their mental malady consists in an exaggerated loyalty to their own principles, *i.e.* a heartfelt and rather intolerant religion whose divinity is their past mental selves. Those who are really acquainted with this folk will recognize the portrait.

Peirce: CP 6.182 Cross-Ref:††

182. No man could be closer to the antipode of their model than America's and the world's highest respected and closest beloved philosophic soul, William James. Nobody has a better right to testify to the morality of his attitude toward his own thoughts than I, who knew and loved him for forty-nine or fifty years. But owing to his almost unexampled incapacity for mathematical thought, combined with intense hatred for logic -- probably for its pedantry, its insistence on minute exactitude -- the *gêne* of its barbarous formulations, etc. rendered him an easy victim to Zeno and the Achilles; and he had, I fear, a right to be offended at the contemptuous language that I thought it my duty to use when talking of this paradox to the young men; though if he did feel offended, he never showed it to me. In what I have said here on the subject, I have endeavoured to substitute serious and courteous remonstrance for the tone I used at Harvard.†1

Peirce: CP 6.182 Cross-Ref:††

Although he is now gone from us, I thoroughly believe he is looking over my shoulder this minute as I write; and I hope he will be able to guide my pen to greater delicacy than I am capable of. He thought that the Achilles disproved Dedekind's theory of continuity, which I take to be generally believed by mathematicians, though it is beyond the jurisdiction of Pure Mathematics, which deals exclusively with the consequences deducible from hypotheses arbitrarily posited.†2 Personally, I agree entirely with James, against Dedekind's view; and hold that there would be no actually existent points in an existent continuum, and that if a point were placed in a continuum it would constitute a breach of the continuity. Of course, there is a possible, or potential, point-place wherever a point might be placed; but that which only *may be* is necessarily thereby *indefinite*, and as such, and in so far, and in those respects, as it is such, it is not subject to the principle of contradiction, just as the negation of a may-be, which is of course a *must-be*, (I mean that if "S *may be* P" is untrue, then "S *must be* non-P" is true), in those respects in which it is such, is not subject to the principle of excluded middle.†3 This renders may-be's and must-be's very delicate objects for thought to handle, and propositions concerning them that sound absurd sometimes express plain facts. This, however, is a matter that I cannot pretend to have got to the bottom of; and logic here seems to touch metaphysics. But while I am in full accord with James's conclusion, and indeed am inclined to think that it was I who first drew his attention to Kant's sometimes speaking, in the *C. d. r. V.*,†1 as if this was his opinion, though other of his expressions are against it, I cannot

in the least agree with him that the Achilles argument proves that it is so. What seems to me to prove that it cannot do so is that the whole state of things supposed in that paradox might perfectly well be true if, in place of time and space, there were a series of instants and a series of points corresponding to all the values of (mathematically speaking) real and rational quantities. That is to say, if all those places in space were abolished whose distances from a fixed point are not expressible as some fraction, proper or improper, of an inch -- that is to say, whose distances from a selected point, the same for all, did not stand to the inch in the same ratio as that of one whole number to another -- and the same work of destruction had been done to time (and we have absolutely no evidence, as far as I see, that such is not the actual truth as to space and time both), then the race between Achilles and Tortoise could have taken place exactly as it did, and no one the wiser. Yet it could not be said *then* that there were no points in space; for it would be nothing but points without the least continuity. In fact, in order to prove that such is not in fact the constitution of actual time, at least (for rotation might introduce strictly infinitesimal difficulties about space), it seems to me that we should be driven to considerations of the same nature as some I introduced in perhaps the crudest of my struggles with such subjects, a paper (regretted as soon as published) entitled "The Law of Mind." †2 William James, in one of his last talks with me, expressed the opinion that that paper was, perhaps, the best I had ever written. I mention this in the hope that it may lead to somebody's using what truth there may be in it, in new and far better treatment of the continuity of time and of consciousness. My notion is that we directly perceive the continuity of consciousness; and if anybody objects, that which is not really continuous may *seem* so, I reply, "Aye, but it could not *seem* so, if there were not *some* consciousness that is so." †1 I should like to see a good criticism of that reply.

Peirce: CP 6.182 Cross-Ref: ††

I am not, at present, prepared to believe, as William James did, that he was, permanently and as a finality, incapable of conceiving that Achilles could traverse an infinite succession of points, [although Achilles] certainly would have no notion that there were any such points there. (There were doubtless a lot of pebbles and grains of sand along his path, judging from the little I saw, in passing in a braganza over a road in Phthiotis in the night. Now I do not think that if each pebble were broken into a million pieces the difficulty of getting over the road would necessarily have been increased; and I don't see why it should if one of those millions -- or all of them -- had been multiplied into an infinity.)

Peirce: CP 6.183 Cross-Ref: ††

183. After studying William James on the intellectual side for half a century -- for I was not acquainted with him as a boy -- I must testify that I believe him to be, and always to have been during my acquaintance with him, about as perfect a lover of truth as it is possible for a man to be; and I do not believe there is any definite limit to man's capacity for loving the truth. If you ask me what that ugly word "about" signifies in my statement of James's love of truth, as I believe that love to have been, I reply that I conceive the imperfection of man's devotion to anything -- at any rate to any such perfect ideal as Truth -- to be very different from his incapacity to attain exactitude in reproducing a metre or a kilo, inasmuch as in these latter cases, what he is liable to do is to make his copy either too large or too small, with an equal liability -- after making a constant allowance, one way or the other, for his tendency to make it a bit different from "nature" (as the artists call the real thing they aim to imitate, at least to a certain difference, *près*) -- I repeat, after [we have made] such allowances [for] an equal liability, to err in excess and in defect, in such a case we are just as

likely, and indeed a little likelier, to hit the truth as near as our last place of decimals goes, as we are to make a small error one way or the other. Indeed we are infinitesimally likelier to do so. But when it comes to efforts to attain an ideal that it would be an absurdity to talk of surpassing and an impossibility actually to reach, we ought to measure our shortcomings by the logarithms of [our] mechanical, blockhead, measures of those shortcomings. Exactly of what nature these "blockhead measures" would be, it would be a study to ascertain; but all measurement of the errors that are only known through those erroneous measurements are pretty rough; and a slight error in the mode of measuring is very unlikely to be of serious consequence.

Peirce: CP 6.184 Cross-Ref:††

184. In speaking, then, of William James as I do, I am saying the most that I could of any man's intellectual morality; and with him this was but one of a whole diadem of virtues. Though it is entirely out of place in this connexion, and I must beg the reader's pardon for so wandering from the point under consideration, I really lack the self-command to repress my reflexions when I have once set down his name. Though his lectures were delightful, they not at all exhibited the man at his best. It was his unstudied common behaviour that did so by the perfection of his manners, in their perfect freedom from expressing flattery or anything else false or inappropriate to the occasion. He did not express himself very easily, because rhetoric was his antipathy and logic an inconvenience to him. One always felt that the pencil, not the pen, was the lever with which he ought to have moved the world; and yet no! it was not the externals of things but their souls he could have pictured.

Peirce: CP 6.184 Cross-Ref:††

His comprehension of men to the very core was most wonderful. Who, for example, could be of a nature so different from his as I? He so concrete, so living; I a mere table of contents, so abstract, a very snarl of twine. Yet in all my life I found scarce any soul that seemed to comprehend, naturally, [not] my concepts, but the mainspring of my life better than he did. He was even greater [in the] practice than in the theory of psychology.

Peirce: CP 6.185 Cross-Ref:††

CHAPTER 7

THE LOGIC OF CONTINUITY†1

§1. POTENTIAL AGGREGATES

185. By the *limit* of an endless series of successive objects we mean an object which comes after all the objects of that series, but so that every *other* object which comes after all those objects comes after the limit also. When I say that the series of abnumeral multitudes has no limit, I mean that it has no limit among multitudes of distinct individuals. It will have a limit if there is properly speaking, any meaning in saying that something that is *not* a multitude of distinct individuals is *more* than every

multitude of distinct individuals. But, you will ask, can there be any sense in that? I answer, yes, there can, in this way. That which is possible is in so far *general* and, as general, it ceases to be individual. Hence, remembering that the word "potential" means *indeterminate yet capable of determination in any special case*, there may be a potential aggregate of all the possibilities that are consistent with certain general conditions; and this may be such that given any collection of distinct individuals whatsoever, out of that potential aggregate there may be actualized a more multitudinous collection than the given collection. Thus the potential aggregate is, with the strictest exactitude, greater in multitude than any possible multitude of individuals. But being a potential aggregate only, it does not contain any individuals at all. It only contains general conditions which *permit* the determination of individuals.

Peirce: CP 6.186 Cross-Ref:††

186. The logic of this may be illustrated by considering an analogous case. You know very well that $2/3$ is not a whole number. In the whole collection of whole numbers you will not find $2/3$. That you know. Therefore, you know something about the entire collection of whole numbers. But what is the nature of your conception of this collection? It is general. It is potential. It is vague, but yet with such a vagueness as permits of its accurate determination in regard to any particular object proposed for examination. Very well, that being granted, I proceed to the analogy with what we have been saying. Every whole number considered as a multitude is capable of being completely counted. Nor does its being aggregated with or added to any other whole number in the least degree interfere with the completion of the count. Yet the aggregate of *all* whole numbers cannot be completely counted. For the completion would suppose the *last* whole number was included, whereas there is no last whole number. But though the aggregate of all whole numbers cannot be completely counted, that does not prevent our having a distinct idea of the multitude of all whole numbers. We have a conception of the entire collection of whole numbers. It is a *potential* collection, indeterminate yet determinable. And we see that the entire collection of whole numbers is more multitudinous than any whole number.

Peirce: CP 6.187 Cross-Ref:††

187. In like manner the potential aggregate of all the abnumeral multitudes is more multitudinous than any multitude. This potential aggregate cannot be a multitude of distinct individuals any more than the aggregate of all the whole numbers can be completely counted. But it is a distinct general conception for all that -- a conception of a potentiality.

Peirce: CP 6.188 Cross-Ref:††

188. A potential collection, more multitudinous than any collection of distinct individuals can be, cannot be entirely vague. For the potentiality supposes that the individuals are determinable in every multitude. That is, they are determinable as distinct. But there cannot be a distinctive quality for each individual; for these qualities would form a collection too multitudinous for them to remain distinct. It must therefore be by means of relations that the individuals are distinguishable from one another. . . . No perfect continuum can be defined by a [asymmetrical] dyadic relation [since the origin and terminus would be points of discontinuity].†1 But if we take instead a triadic relation, and say *A* is *r* to *B* for *C*, say, to fix our ideas, that proceeding from *A* in a particular way, say to the right, you reach *B* before *C*, it is quite evident that a continuum will result like a self-returning line with no discontinuity whatever. . . .

Peirce: CP 6.189 Cross-Ref:††
§2. THE LOGIC OF THE UNIVERSE

189. Every attempt to understand anything -- every research -- supposes, or at least *hopes*, that the very objects of study themselves are subject to a logic more or less identical with that which we employ.

Peirce: CP 6.189 Cross-Ref:††

That the logic of the universe is more rudimentary than our subjective logic is a hypothesis which may be worth examination in some stage of culture, but it is too violently at war with all the lessons which this age has learned for any man nowadays to embrace it with that ardor with which a man must embrace the theory which he is to devote his best powers to developing and bringing to the test of experience. Whatever else may be said for or against that hypothesis, that which we of these times ought to try is rather the hypothesis that the logic of the universe is one to which our own aspires, rather than attains.

Peirce: CP 6.190 Cross-Ref:††

190. Now continuity is shown by the logic of relations to be nothing but a higher type of that which we know as generality. It is relational generality.

Peirce: CP 6.191 Cross-Ref:††

191. How then can a continuum have been derived? Has it for example been put together? Have the separated points become welded, or what?

Peirce: CP 6.191 Cross-Ref:††

Looking upon the course of logic as a whole we see that it proceeds from the question to the answer -- from the vague to the definite. And so likewise all the evolution we know of proceeds from the vague to the definite. The indeterminate future becomes the irrevocable past. In Spencer's phrase the undifferentiated differentiates itself. The homogeneous puts on heterogeneity. However it may be in special cases, then, we must suppose that as a rule the continuum has been derived from a more general continuum, a continuum of higher generality.

Peirce: CP 6.192 Cross-Ref:††

192. From this point of view we must suppose that the existing universe, with all its arbitrary secondness, is an offshoot from, or an arbitrary determination of, a world of ideas, a Platonic world; not that our superior logic has enabled us to reach up to a world of forms to which the real universe, with its feebler logic, was inadequate.

Peirce: CP 6.193 Cross-Ref:††

193. If this be correct, we cannot suppose the process of derivation, a process which extends from before time and from before logic, we cannot suppose that it began elsewhere than in the utter vagueness of completely undetermined and dimensionless potentiality.

Peirce: CP 6.194 Cross-Ref:††

194. The evolutionary process is, therefore, not a mere evolution of the

existing universe, but rather a process by which the very Platonic forms themselves have become or are becoming developed.

Peirce: CP 6.195 Cross-Ref:††

195. We shall naturally suppose, of course, that existence is a stage of evolution. *This existence* is presumably but a *special* existence. We need not suppose that every form needs for its evolution to emerge into this world, but only that it needs to enter into *some* theatre of reactions, of which this is one.

Peirce: CP 6.196 Cross-Ref:††

196. The evolution of forms begins or, at any rate, has for an early stage of it, a vague potentiality; and that either is or is followed by a continuum of forms having a multitude of dimensions too great for the individual dimensions to be distinct. It must be by a contraction of the vagueness of that potentiality of everything in general, but of nothing in particular, that the world of forms comes about.

Peirce: CP 6.197 Cross-Ref:††

197. We can hardly but suppose that those sense-qualities that we now experience, colors, odors, sounds, feelings of every description, loves, griefs, surprise, are but the relics of an ancient ruined continuum of qualities, like a few columns standing here and there in testimony that here some old-world forum with its basilica and temples had once made a magnificent *ensemble*. And just as that forum, before it was actually built, had had a vague underexistence in the mind of him who planned its construction, so too the cosmos of sense-qualities, which I would have you to suppose in some early stage of being was as real as your personal life is this minute, had in an antecedent stage of development a vaguer being, before the relations of its dimensions became definite and contracted.

Peirce: CP 6.198 Cross-Ref:††

198. The sense-quality is a feeling. Even if you say it is a *slumbering* feeling, that does not make it less intense; perhaps the reverse. For it is the absence of *reaction* -- of feeling *another* -- that constitutes slumber, not the absence of the immediate feeling that is all that it is in its immediacy. Imagine a magenta color. Now imagine that all the rest of your consciousness -- memory, thought, everything except this feeling of magenta -- is utterly wiped out, and with that is erased all possibility of comparing the magenta with anything else or of estimating it as more or less bright. That is what you must think the pure sense-quality to be. Such a definite potentiality can emerge from the indefinite potentiality only by its own vital Firstness and spontaneity. Here is this magenta color. What originally made such a quality of feeling possible? Evidently nothing but itself. It is a First.

Peirce: CP 6.199 Cross-Ref:††

199. Yet we must not assume that the qualities arose separate and came into relation afterward. It was just the reverse. The general indefinite potentiality became limited and heterogeneous. Those who express the idea to themselves by saying that the Divine Creator determined so and so may be incautiously clothing the idea in a *garb* that is open to criticism, but it is, after all, substantially the only philosophical answer to the problem. Namely, they represent the ideas as springing into a preliminary stage of being by their own inherent firstness. But so springing up, they do not spring up isolated; for if they did, nothing could unite them. They spring up in reaction upon one another, and thus into a kind of existence. This reaction and this existence these persons call the mind of God. I really think there is no objection to this except that it is wrapped up in figures of speech, instead of having the

explicitness that we desire in science. For all you know of "minds" is from the actions of animals with brains or ganglia like yourselves, or at furthest like a cockroach. To apply such a word to *God* is precisely like the old pictures which show him like an aged man leaning over to look out from above a cloud. Considering the *vague intention* of it, as conceived by the *non-theological* artist, it cannot be called false, but rather ludicrously figurative.

Peirce: CP 6.200 Cross-Ref:††

200. In short, if we are going to regard the universe as a result of evolution at all, we must think that not merely the existing universe, that locus in the cosmos to which our reactions are limited, but the whole Platonic world, which in itself is equally real, is evolutionary in its origin, too. And among the things so resulting are time and logic. The very first and most fundamental element that we have to assume is a Freedom, or Chance, or Spontaneity, by virtue of which the general vague nothing-in-particular-ness that preceded the chaos took a thousand definite qualities. The *second* element we have to assume is that there could be accidental reactions between those qualities. The qualities themselves are mere eternal possibilities. But these reactions we must think of as *events*. Not that *Time* was. But still, they had all the here-and-nowness of events. I really do not see how the metaphysician can explain either of these elements as results, further than this, that it may be said that the accidental reaction was, at first, one of the special determinations that came about by pure spontaneity or chance.

Peirce: CP 6.201 Cross-Ref:††

201. Let me here say one word about Tychism, or the doctrine that absolute chance is a factor of the universe. There is one class of objectors to it who are so impressed with what they have read in popular books about the triumphs of science that they really imagine that science has *proved* that the universe is regulated by law down to every detail. Such men are theologians, perhaps, or perhaps they have been brought up in surroundings where everything was so minutely regulated that they have come to believe that every tendency that exists at all in Nature must be carried to its furthest limit. Or, there is I know not what other explanation of their state of mind; but I do know one thing: they cannot be real students of physical science -- they cannot be *chemists*, for example. They are wrong in their logic. But there is *another* class of objectors for whom I have more respect. They are shocked at the atheism of Lucretius and his great master. They do not perceive that that which offends them is not the Firstness in the swerving atoms, because they themselves are just as much advocates of Firstness as the ancient Atomists were. But what they cannot accept is the attribution of this firstness to things perfectly dead and material. Now I am quite with them there. I think too that whatever is First is *ipso facto* sentient. If I make atoms swerve -- as I do -- I make them swerve but very very little, because I conceive they are not absolutely dead. And by that I do not mean exactly that I hold them to be physically such as the materialists hold them to be, only with a small dose of sentiency superadded. For that, I grant, would be feeble enough. But what I mean is, that all that there is, is First, Feelings; Second, Efforts; Third, Habits -- all of which are more familiar to us on their psychical side than on their physical side; and that dead matter would be merely the final result of the complete induration of habit reducing the free play of feeling and the brute irrationality of effort to complete death. Now I would suppose that that result of evolution is not quite complete even in our beakers and crucibles. Thus, when I speak of chance, I only employ a mathematical term to express with accuracy the characteristics of freedom or spontaneity.

Peirce: CP 6.202 Cross-Ref:††

202. Permit me further to say that I object to having my metaphysical system as a whole called Tychism. For although tychism does enter into it, it only enters as subsidiary to that which is really, as I regard it, the characteristic of my doctrine, namely, that I chiefly insist upon continuity, or Thirdness, and, in order to secure to thirdness its really commanding function, I find it indispensable fully [to] recognize that it is a third, and that Firstness, or chance, and Secondness, or Brute reaction, are other elements, without the independence of which Thirdness would not have anything upon which to operate. Accordingly, I like to call my theory Synechism, because it rests on the study of continuity. I would not object to Tritism. And if anybody can prove that it is *trite*, that would delight me [in] the chiefest degree.

Peirce: CP 6.203 Cross-Ref:††

203. All that I have been saying about the beginnings of creation seems wildly confused enough. Now let me give you such slight indication, as brevity permits, of the clue to which I trust to guide us through the maze.

Peirce: CP 6.203 Cross-Ref:††

Let the clean blackboard be a sort of diagram of the original vague potentiality, or at any rate of some early stage of its determination. This is something more than a figure of speech; for after all continuity is generality. This blackboard is a continuum of two dimensions, while that which it stands for is a continuum of some indefinite multitude of dimensions. This blackboard is a continuum of possible points; while that is a continuum of possible dimensions of quality, or is a continuum of possible dimensions of a continuum of possible dimensions of quality, or something of that sort. There are no points on this blackboard. There are no dimensions in that continuum. I draw a chalk line on the board. This discontinuity is one of those brute acts by which alone the original vagueness could have made a step towards definiteness. There is a certain element of continuity in this line. Where did this continuity come from? It is nothing but the original continuity of the blackboard which makes everything upon it continuous. What I have really drawn there is an oval line. For this white chalk-mark is not a *line*, it is a plane figure in Euclid's sense -- a *surface*, and the only line there, is the line which forms the *limit* between the black surface and the white surface. Thus the discontinuity can only be produced upon that blackboard by the reaction between two continuous surfaces into which it is separated, the white surface and the black surface. The whiteness is a Firstness -- a springing up of something new. But the boundary between the black and white is neither black, nor white, nor neither, nor both. It is the pairedness of the two. It is for the white the active Secondness of the black; for the black the active Secondness of the white.

Peirce: CP 6.204 Cross-Ref:††

204. Now the clue, that I mentioned, consists in making our thought diagrammatic and mathematical, by treating generality from the point of view of geometrical continuity, and by experimenting upon the diagram.

Peirce: CP 6.204 Cross-Ref:††

We see the original generality like the ovum of the universe segmentated by this mark. However, the mark is a mere accident, and as such may be erased. It will not interfere with another mark drawn in quite another way. There need be no consistency between the two. But no further progress beyond this can be made, until a mark will *stay* for a little while; that is, until some beginning of a *habit* has been

established by virtue of which the accident acquires some incipient staying quality, some tendency toward consistency.

Peirce: CP 6.204 Cross-Ref:††

This habit is a generalizing tendency, and as such a generalization, and as such a general, and as such a continuum or continuity. It must have its origin in the original continuity which is inherent in potentiality. Continuity, as generality, is inherent in potentiality, which is essentially general.

Peirce: CP 6.205 Cross-Ref:††

205. The whiteness or blackness, the Firstness, is essentially indifferent as to continuity. It lends itself readily to generalization but is not itself general. The limit between the whiteness and blackness is essentially discontinuous, or antigeneral. It is insistently *this here*. The original potentiality is essentially continuous, or general.

Peirce: CP 6.206 Cross-Ref:††

206. Once the line will stay a little after it is marked, another line may be drawn beside it. Very soon our eye persuades us there is a *new* line, the envelope of those others. This rather prettily illustrates the logical process which we may suppose takes place in things, in which the generalizing tendency builds up new habits from chance occurrences. The new curve, although it is new in its distinctive character, yet derives its continuity from the continuity of the blackboard itself. The original potentiality is the Aristotelian matter or indeterminacy from which the universe is formed. The straight lines as they multiply themselves under the habit of being tangent to the envelope gradually tend to lose their individuality. They become in a measure more and more obliterated and sink into mere adjuncts to the new cosmos in which they are individuals.

Peirce: CP 6.207 Cross-Ref:††

207. Many such reacting systems may spring up in the original continuum; and each of these may itself act as a first line from which a larger system may be built, in which it in turn will merge its individuality.

Peirce: CP 6.208 Cross-Ref:††

208. At the same time all this, be it remembered, is not of the order of the existing universe, but is merely a Platonic world, of which we are, therefore, to conceive that there are many, both coordinated and subordinated to one another; until finally out of one of these Platonic worlds is differentiated the particular actual universe of existence in which we happen to be.

Peirce: CP 6.209 Cross-Ref:††

209. There is, therefore, every reason in logic why this here universe should be replete with accidental characters, for each of which, in its particularity, there is no other reason than that it is one of the ways in which the original vague potentiality has happened to get differentiated.

Peirce: CP 6.209 Cross-Ref:††

But, for all that, it will be found that if we suppose the laws of nature to have been formed under the influence of a universal tendency of things to take habits, there are certain characters that those laws will necessarily possess.

Peirce: CP 6.209 Cross-Ref:††

As for attempting to set forth the series of deductions I have made upon this subject, that would be out of the question. All that I have any thought of doing is to

illustrate, by a specimen or two, chosen among those which need the least explanation, some of the methods by which such reasoning may be conducted.

Peirce: CP 6.210 Cross-Ref:††
§3. CIRCULAR CONTINUA; TIME AND SPACE

210. Various continua, to which the inquirer's attention will be directed in the course of this investigation, must be assumed to be devoid of all topical singularities. For any such singularity is a locus of discontinuity; and from the nature of the continuum there may be no room to suppose any such secondness. But now, a continuum which is without singularities must, in the first place, return into itself. Here is a remarkable consequence.

Peirce: CP 6.210 Cross-Ref:††

Take, for example, Time. It makes no difference what singularities you may see reason to impose upon this continuum. You may, for example, say that all evolution began at this instant, which you may call the infinite past, and comes to a close at that other instant, which you may call the infinite future. But all this is quite extrinsic to time itself. Let it be, if you please, that evolutionary time, our section of time, *is* contained between those limits. Nevertheless, it cannot be denied that time itself, unless it be discontinuous, as we have every reason to suppose it is not, stretches on beyond those limits, infinite though they be, returns into itself, and begins again.†1 Your metaphysics must be shaped to accord with that.

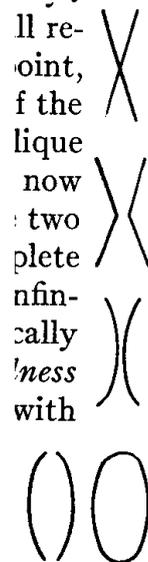
Peirce: CP 6.211 Cross-Ref:††

211. Again, the lowest Listing number,†2 the number of separate pieces, cannot be *zero*; for such a hypothesis would annul the whole continuum. Nor can the highest Listing number †3 be *zero*, unless the continuum has singularities. But the intermediate Listing numbers †1 may be *zero* or almost any numbers. If metaphysics is really to be made a definite science, and not child's play, the first inquiry concerning any general must be, first, what its dimensionality is, and secondly, what these intermediate Listing numbers are; and whatever your answer is, it will generally be found to lead you into those difficult but definite questions out of which we are accustomed in inductive science to think that the true theory is pretty sure to grow. It is one of the great merits of the method of thought, that the logic of relatives inculcates, that it leads to such definite questions.

Peirce: CP 6.211 Cross-Ref:††

For example, take the continuum of all possible sense qualities after this has been so far restricted that the dimensions are distinct. This is a continuum in which firstness is the prevailing character. It is also highly primitive; and therefore we ought to suppose, till the contrary is proved, that the intermediate Listing numbers are all *unity*. For *zero* is distinctly a dualistic idea. It is mathematically $A - A$, *i.e.* the result of the inverse process of subtraction. Now an inverse process is a *Second* process. It is true that there is another sort of zero which is a *limit*. Such is the vague zero of indeterminacy. But a limit involves Secondness prominently, and besides that, Thirdness. In fact, the *generality* of indeterminacy *marks* its Thirdness. Accordingly, *zero* being an idea of Secondness, we find, as we should expect, that any continuum

whose intermediate Listing numbers are zero is equivalent to a pair of continua whose Listing numbers are 1. For instance, a perspective plane has a cyclosis equal to 1, while a ball has a cyclosis equal to 0. Now a ball is, topically speaking, of the same shape as two planes after the singularity of the pair has been removed. I will show you that this is true. Let the one plane be that of the blackboard, and let the other be oblique to it. Let this mark represent their ray of intersection. This ray is a singular line upon the two planes considered as one surface. In order to remove this singularity, we must split it down, so as to leave the right-hand side of the blackboard plane joined along the right-hand parts of the split line to that part of the oblique plane that is in front, while the left-hand part of the blackboard plane is joined along the left-hand parts of the split line to the part of the oblique plane behind the blackboard. Thus the ray becomes two rays. But two rays *intersect*. So that a singular



point still remains. [\[Click here to view\]](#) We must, then, cut through that singular point, making two points of it; and leaving the right side of the blackboard plane joined to the forward part of the oblique plane and the left side joined to the other part. We now move apart those two hyperbolic branches, that the two rays have made, until they have made nearly a complete circuit of the plane. They no longer cut the ray at infinity, and we have an egg-shaped solid which is topically just like a ball. Thus I have shown how *secondness* enters into the *zero* cyclosis. It is the same with the other intermediate Listing numbers; and we must assume that all the Listing numbers of the continuum of sense-qualities are equal to 1. This is confirmed by carrying the evolution of the continuum and its definiteness a step further. Namely, we will now suppose that each quality has acquired a settled identity in all its different degrees, so that the continuum is ready for the application of measurement. This measurement is a network figure imposed upon the blank continuum. It is true that it is in large measure arbitrary. It is *our* creation. Nevertheless, we shall adapt our creation as far as possible to the real properties of the continuum itself. Besides, there are certain modes of measurement which are impossible without breach of continuity in certain shapes of continua. For example, anybody can see that the same system of coordinates which could be applied to defining positions of points on a sphere, say latitude and longitude, would have to be modified in order to apply it to the definition of positions on an anchor-ring. On the sphere, longitude returns into itself after every 360°, and there are two points, the poles, whose longitudes are indeterminate; while latitude extends through 180° and then stops. But on the ring there will be one series of lines which will go round the bar of the ring without ever cutting one another, and another series going round the hole of the ring without cutting one another. This is a

much simpler system of measurement than any that is possible on the sphere. Now in the network figure of coordinates which conforms best to the properties of the continuum of pure quality there is a line for each quality, along which line that quality only varies in intensity. All these lines come together at the absolute zero of quality. For in the *zero* of intensity, quality is indistinguishable in its inmost nature. But those lines meet nowhere else. In the infinite degree qualities may dazzle our senses; but in themselves they are different. Hence, the continuum of quality is such that unlimited lines may cut one another an odd number of times; namely, *once* only. Now this would be impossible were the intermediate Listing numbers *even*, say zero. Our hypothesis that they are odd is therefore confirmed. (I must add that the measurement of quality is evidently hyperbolic, which weakens considerably the force of the last argument.)

Peirce: CP 6.212 Cross-Ref:††

212. As another example, consider the continuum of Space. In my lecture †1 on the subject I pointed out to you how though it is a continuum, and therefore a Thirdness, the whole nature and function of space refers to Secondness. It is the theatre of the reactions of particles, and reaction is Secondness in its purity. For this and other reasons, which I omit for the sake of brevity, we must, as our first retrodution, assume that the intermediate Listing numbers for space are all *zero*. When we come to consider the principles of hydrodynamics we find that view confirmed. I cannot enter into details; but the motions of a frictionless incompressible fluid is as though it were composed of interpenetrating parts, shot out in straight lines from sources and disappearing into sinks. But that implies that all the straight lines radiating from a single point will meet again in another single point which supposes the Cyclosis and Periphraxis of Space to be *zero*. There will be some difficulties connected with this view, but I do not think them serious; and at any rate this will serve as another illustration of the manner in which reasoning about continuity can be applied to give real vitality to metaphysical reasoning, and to cure it of its deathly impotency.

Peirce: CP 6.213 Cross-Ref:††

213. I should have been glad if I could have set forth all this in greater detail; but that would have required more mathematics. I should have liked to interest you in a number of my scientifically important and philosophically significant results which I have been obliged to leave altogether unmentioned. I wish I could also have expounded some theories of other thinkers which, although I cannot accept them, seem to me to be well worthy of the most careful consideration. But to treat a theory like this, the whole life of which lies in minute diagrammatic reasoning, in eight lectures was inevitably to make it seem excessively abstruse and, at the same time, to do no more than exhibit a fragment here and there selected as being comparatively easy of presentation. The subject of mathematical metaphysics, or Cosmology, is not so very difficult, provided it be properly expanded and displayed. It deeply concerns both physicist and psychist. The physicist ought to direct his attention to it, in order that he may be led to contemplate the intellectual side of his own science. Especially the chemist, whose attention is forced to theory, needs above all to study the theory of theorizing. Psychologists have not yet dropped their excellent habit of studying philosophy; but I venture to think that they are not fully alive to all the value for their science of certain higher mathematics and to the virtues of mathematical thinking. The failure of Herbart, whose attempt was made before either Mathematics or Psychology was ripe for it, does not argue that no success can be attained in that line. I have presented -- or no, I have not presented anything in these lectures, but I have

talked about the most abstract parts of Cosmology; but the subject embraces many topics which have not that character, such as the question of the present state of the evidences of the Conservation of Energy and the question of the nature of the influences which hold together the constituent elements of chemical compounds. In short, there is a great variety of different ways in which Cosmology is both curious and useful for widely different classes of minds. We all know the kind of man who is warranted never to be interested in it, the man who lays out a system of ideas in his youth and stands on his platform with stalwart constancy like Casabianca on the burning deck. But if a mind is not absolutely argon and helium, but is capable of being drawn by any means within an alien sphere of attraction, no study is more calculated to bring about that event than this. It is decidedly a difficult subject on which to break ground for oneself. Economy of time, avoidance of a terrible waste, requires the student to take counsel of the experience here of a mathematician, there of a logician, again of a physicist or chemist, and continually of a psychologist. It is, by the way, precisely in psychology where you are the strongest that I have to confess myself the weakest. For that reason, in these lectures I have touched as little as possible upon psychology, preferring to deal with topics of Cosmology where I should be more at home, although you were less so. Crabbed and confused as all these circumstances have caused these conferences to become, you have been kind enough to listen to them, and really I dare not acknowledge, as it is in my heart to do, the whole warmth of my thanks, for fear you might think it out of measure. But should it happen to any of you to select for his life's explorations a region very little trodden, he will, as a matter of course, have the pleasure of making a good many discoveries of more fundamental importance than at all remain to be made in any ground that has long been highly cultivated. But on the other hand, he will find that he has condemned himself to an isolation like that of Alexander Selkirk. He must be prepared for almost a lifetime of work with scarce one greeting, and I can assure him that if, as his day is sinking, a rare good fortune should bring a dozen men of real intellect, some men of great promise, others of great achievement, together to listen to so much of what he has learned as his long habit of silence shall have left him the power of expressing in the compass of eight lectures, he will know then an almost untasted joy and will comprehend then what gratitude I feel at this moment.

Peirce: CP 6.214 Cross-Ref: ††
CHAPTER 8

OBJECTIVE LOGIC

§1. THE ORIGIN OF THE UNIVERSE †1

214. Metaphysics has to account for the whole universe of being. It has, therefore, to do something like supposing a state of things in which that universe did not exist, and consider how it could have arisen. However, this statement needs amendment. For time is itself an organized something, having its law or regularity; so

that time itself is a part of that universe whose origin is to be considered. We have therefore to suppose a state of things before time was organized. Accordingly, when we speak of the universe as "arising" we do not mean that literally. We mean to speak of some kind of sequence, say an objective logical sequence; but we do not mean in speaking of the first stages of creation before time was organized, to use "before," "after," "arising," and such words in the temporal sense. But for the sake of the commodity of speech we may avail ourselves of these words.

Peirce: CP 6.215 Cross-Ref:††

215. The initial condition, before the universe existed, was not a state of pure abstract being. On the contrary it was a state of just nothing at all, not even a state of emptiness, for even emptiness is something.

Peirce: CP 6.215 Cross-Ref:††

If we are to proceed in a logical and scientific manner, we must, in order to account for the whole universe, suppose an initial condition in which the whole universe was non-existent, and therefore a state of absolute nothing.

Peirce: CP 6.216 Cross-Ref:††

216. You must not let this interfere with or be interfered with by any religious belief. Religion is a practical matter. Its beliefs are formulae you will go upon. But a scientific proposition is merely something you take up provisionally as being the proper hypothesis to try first and endeavor to refute. The only belief you -- as a purely scientific man -- have about it is that it is adopted in accordance with a method which must lead to the truth in the long run. It is a damnable absurdity indeed to say that one thing is true in theology and another in science. But it is perfectly true that the belief which I shall do well to embrace in my practical affairs, such as my religion, may not accord with the proposition which a sound scientific method requires me provisionally to adopt at this stage of my investigation. Later, both the one proposition and the other may very likely be modified; but how, or which comes nearer to the ultimate conclusion, not being a prophet or a magician, I cannot yet say.

Peirce: CP 6.217 Cross-Ref:††

217. We start, then, with nothing, pure zero. But this is not the nothing of negation. For *not* means *other than*, and *other* is merely a synonym of the ordinal numeral *second*. As such it implies a first; while the present pure zero is prior to every first. The nothing of negation is the nothing of death, which comes *second* to, or after, everything. But this pure zero is the nothing of not having been born. There is no individual thing, no compulsion, outward nor inward, no law. It is the germinal nothing, in which the whole universe is involved or foreshadowed. As such, it is absolutely undefined and unlimited possibility -- boundless possibility. There is no compulsion and no law. It is boundless freedom.

Peirce: CP 6.217 Cross-Ref:††

So of *potential* being there was in that initial state no lack.

Peirce: CP 6.218 Cross-Ref:††

218. Now the question arises, what necessarily resulted from that state of things? But the only sane answer is that where freedom was boundless nothing in particular necessarily resulted.

Peirce: CP 6.218 Cross-Ref:††

In this proposition lies the prime difference between my objective logic and that of Hegel.†1 He says, if there is any sense in philosophy at all, the whole universe

and every feature of it, however minute, is rational, and was constrained to be as it is by the logic of events, so that there is no principle of action in the universe but reason. But I reply, this line of thought, though it begins rightly, is not exact. A logical slip is committed; and the conclusion reached is manifestly at variance with observation. It is true that the whole universe and every feature of it must be regarded as rational, that is as brought about by the logic of events. But it does not follow that it is *constrained* to be as it is by the logic of events; for the logic of evolution and of life need not be supposed to be of that wooden kind that absolutely constrains a given conclusion. The logic may be that of the inductive or hypothetic inference.

Peirce: CP 6.218 Cross-Ref:††

This may-be is at once converted into must-be when we reflect that among the facts to be accounted for are such as that, for example, red things look red and not blue and *vice versa*. It is obvious that that cannot be a necessary consequence of abstract being.

Peirce: CP 6.218 Cross-Ref:††

The effect of this error of Hegel is that he is forced to deny [the] fundamental character of two elements of experience which cannot result from deductive logic. What these elements are will appear in the sequel.

Peirce: CP 6.219 Cross-Ref:††

219. I say that nothing *necessarily* resulted from the Nothing of boundless freedom. That is, nothing according to deductive logic. But such is not the logic of freedom or possibility. The logic of freedom, or potentiality, is that it shall annul itself. For if it does not annul itself, it remains a completely idle and do-nothing potentiality; and a completely idle potentiality is annulled by its complete idleness.

Peirce: CP 6.220 Cross-Ref:††

220. I do not mean that potentiality immediately results in actuality. Mediate perhaps it does; but what immediately resulted was that unbounded potentiality became potentiality of this or that sort -- that is, of some *quality*.

Peirce: CP 6.220 Cross-Ref:††

Thus the zero of bare possibility, by evolutionary logic, leapt into the *unit* of some quality. This was hypothetic inference. Its form was:

Something is possible,
Red is something;
∴ Red is possible.

Peirce: CP 6.221 Cross-Ref:††

221. Now a *quality* is a consciousness. I do not say a *waking* consciousness -- but still, something of the nature of consciousness. A *sleeping* consciousness, perhaps.

Peirce: CP 6.221 Cross-Ref:††

A possibility, then, or potentiality, is a particular *tinge* of consciousness. I do not say the possibility is exactly a consciousness; but it is a *tinge* of consciousness, a potential consciousness. However, the distinction is little more than verbal.

Peirce: CP 6.221 Cross-Ref:††

But let us consider more closely what the *quale*-consciousness is.

Peirce: CP 6.222 Cross-Ref:††
§2. QUALE-CONSCIOUSNESS †1

222. If a man is blind to the *red* and *violet* elements of light and only sees the *green* element, then all things appear of one color to him, and that color is a green of colorific intensity beyond anything that we with normal eyes can see or imagine. Such is the color that all things look to him. Yet since all things look alike in this respect, it never attracts his attention in the least. He may be said to be dead to it. If the man is at the same time deaf, without smell and taste, and devoid of skin sensations, then it is probable the green will be still more chromatic; for I suppose colors are for us somewhat diluted by skin sensations. But for the very reason that it is his own kind of sensation, he will only be the more completely oblivious of its *quale*. Yet for all that, that *is* the way things look to him, more intensely green than any scarlet or magenta is red to us.

Peirce: CP 6.222 Cross-Ref:††

This illustration puts into a high light the distinction between two kinds of consciousness, the *quale*-consciousness and that kind of consciousness which is intensified by attention, which objectively considered, I call *vividness*, †2 and as a faculty we may call *liveliness*.

Peirce: CP 6.223 Cross-Ref:††

223. The *quale*-consciousness is not confined to simple sensations. There is a peculiar *quale* to *purple*, though it be only a mixture of red and blue. There is a distinctive *quale* to every combination of sensations so far as it is really synthetized -- a distinctive *quale* to every work of art -- a distinctive *quale* to this moment as it is to me -- a distinctive *quale* to every day and every week -- a peculiar *quale* to my whole personal consciousness. I appeal to your introspection to bear me out in this.

Peirce: CP 6.224 Cross-Ref:††

224. Each *quale* is in itself what it is for itself, without reference to any other. It is absurd to say that one *quale* in itself considered is like or unlike another. Nevertheless, comparing consciousness does pronounce them to be alike. They are alike to the comparing consciousness, though neither alike nor unlike in themselves.

Peirce: CP 6.225 Cross-Ref:††

225. And now I enunciate a truth. It is this. In so far as *qualia* can be said to have anything in common, that which belongs to one and all is *unity*; and the various synthetical unities which Kant attributes to the different operations of the mind, as well as the unity of logical consistency, or *specific* unity, and also the unity of the individual object, all these unities †1 originate, not in the operations of the intellect, but in the *quale*-consciousness upon which the intellect operates.

Peirce: CP 6.226 Cross-Ref:††

226. But here the critic will interpose an objection based upon a dilemma. He will ask me whether I intend that "truth" which I have enunciated so pretentiously for a *logical* truth or a *psychological* truth. Because he will say, if you mean it for a logical truth then the only unity it can account for is the unity of non-contradiction, while if you intend it for a psychological truth it must depend upon the peculiar construction of the nervous system and consequently cannot be transferred to metaphysics, in general, which is the use to which you probably intend to put it.

Peirce: CP 6.226 Cross-Ref:††

This is a cunning objection. But it assumes that logic is purely subjective in a sense that is not only pre-Hegelian but pre-Kantian.

Peirce: CP 6.227 Cross-Ref:††

227. All those synthetic unities considered cerebrally are but the unity which comes out when association channels are closed and the inflowing excitation of a part of the brain is pent up and intensified.

Peirce: CP 6.227 Cross-Ref:††

But it ought to be evident that no *unity* can *originate* in concentration. If there is no unity in a mass of gas, it cannot acquire unity merely by being condensed to half its volume. But any unity there was there already may, in that way, be many times intensified.

Peirce: CP 6.227 Cross-Ref:††

If you say those unities are not all of this one species, then how unaccountable it is that so many different operations should give rise to one and the same character of *unity*.

Peirce: CP 6.228 Cross-Ref:††

228. Perhaps it may be thought that hypnotic phenomena show that subconscious feelings are not unified. But I maintain on the contrary that those phenomena exhibit the very opposite peculiarity. They *are* unified so far as they are brought into one *quale*-consciousness at all; and that is why different personalities are formed. Of course, each personality is based upon a "bundle of habits," as the saying is that a man is a bundle of habits. But a bundle of habits would not have the unity of self-consciousness. That unity must be given as a centre for the habits.

Peirce: CP 6.229 Cross-Ref:††

229. The brain shows no central cell. The unity of consciousness is therefore not of physiological origin. It can only be metaphysical. So far as feelings have any continuity, it is the metaphysical nature of feeling to have a *unity*.

Peirce: CP 6.230 Cross-Ref:††

230. I say then that this unity is logical in this sense, that to feel, to be immediately conscious, so far as possible, without any action and reaction nor any reflection, logically supposes one consciousness and not two nor more.

Peirce: CP 6.230 Cross-Ref:††

To be conscious of scarlet and magenta at the same time is either to compare them or set them over against one another and so to introduce another kind of consciousness than the mere *quale*-consciousness, or it is to merge them into one general feeling in which the scarlet and magenta are not separately recognized, and

thus preserve the unity of *quale*-consciousness.

Peirce: CP 6.230 Cross-Ref:††

This is so true that it is next to impossible to express it in language without appearing to utter a tautologous sentence. If I were to say any quality considered in itself is considered apart from anything else and is therefore without parts, since the parts of a sensation are really other sensations, my auditors would object that this was a mere identical proposition, since *in itself* means nothing but "apart from anything else." But the illustration of color-blindness shows that there is such a thing as a *quale*-consciousness, and when I describe this consciousness as the idea of a quality in itself, I merely, by the laws of speech, am forced to seize upon the character of separateness in order to let another person know what sort of consciousness I have in view.

Peirce: CP 6.231 Cross-Ref:††

231. My speech may be tautologous, but I have a positive meaning; and the very fact that it is so difficult to express this meaning without tautology is a mark of its veraciousness. I may express myself thus:

Peirce: CP 6.231 Cross-Ref:††

In *quale*-consciousness there is but one quality, but one element. It is entirely simple.

Peirce: CP 6.231 Cross-Ref:††

There are evidences of this on every hand.

Peirce: CP 6.231 Cross-Ref:††

Thus consciousness, so far as it can be contained in an instant of time, is an example of *quale*-consciousness. Now everybody who has begun to think about consciousness at all has remarked that the present so conceived is absolutely severed from past and future. That is, the past and the future are utterly absent in the sense in which I am conscious of the *now*.

Peirce: CP 6.231 Cross-Ref:††

So I might express my truth by saying --

Peirce: CP 6.231 Cross-Ref:††

The Now is one, and but one.

Peirce: CP 6.231 Cross-Ref:††

The principle of contradiction may be regarded as a formalistic result of the same thing. Any object, *A*, cannot be blue and not blue at once.

Peirce: CP 6.231 Cross-Ref:††

It can be blue and hard, because blueness and hardness are not thought of as joined in *quale*-consciousness, one appealing to one experiment and the other to another. But *A* cannot be blue and yellow, because these would blend and so the color would cease to be blue or yellow either. Thus, the positive truth in the principle of contradiction is that *quale*-consciousness has but one element.

Peirce: CP 6.232 Cross-Ref:††

232. If *quale*-consciousness were double, it would be like a case of double consciousness. One might pronounce the object to be blue that the other said was not blue and the principle of contradiction would only assert that one judge must be set

above those two. But where would be the strife requiring a judge if the *quale*-consciousness were double?

Peirce: CP 6.232 Cross-Ref:††

Our truth might therefore be expressed by saying:

Peirce: CP 6.232 Cross-Ref:††

The *quale*-consciousness is not a consciousness of strife, or duality.

Peirce: CP 6.232 Cross-Ref:††

The quality itself is nothing in the world but a *quale*-consciousness of a composite photograph or general average of experience.

Peirce: CP 6.232 Cross-Ref:††

And if the quality can be double, the principle of contradiction falls to the ground.

Peirce: CP 6.233 Cross-Ref:††

233. All the operations of the intellect consist in taking composite photographs of *quale*-consciousnesses. Instead of introducing any unity, they only introduce conflict that was not in the *quale*-consciousness itself.

Peirce: CP 6.233 Cross-Ref:††

Such unity as remains is nothing but the unity and simplicity of the *quale*-consciousness persisting in spite of all this multiplication and diversity.

Peirce: CP 6.234 Cross-Ref:††

234. Another expression of our truth is this:

Peirce: CP 6.234 Cross-Ref:††

Quality or *quale*-consciousness is all that it is in and for itself.

Peirce: CP 6.234 Cross-Ref:††

It is essentially solitary and celibate, a dweller in the desert.

Peirce: CP 6.235 Cross-Ref:††

235. Another expression of it is:

Peirce: CP 6.235 Cross-Ref:††

Quale-consciousness cannot blend with *quale*-consciousness without loss of its identity.

Peirce: CP 6.236 Cross-Ref:††

236. So much for the meaning of the proposition. I now call attention to a remarkable consequence of it. Namely it follows that there is no check upon the utmost variety and diversity of *quale*-consciousness as it appears to the comparing intellect. For if consciousness is to blend with consciousness, there must be common elements. But if it has nothing in itself but just itself, it is *sui generis* and is cut loose from all need of agreeing with anything. Whatever is absolutely simple must be absolutely free; for a law over it must apply to some common feature of it. And if it has no features, no law can seize upon it. It is *totus, teres, atque rotundus*.

Peirce: CP 6.236 Cross-Ref:††

And thus it is that that very same logical element of experience, the *quale*-element, which appears upon the inside as unity, when viewed from the outside

is seen as variety.

Peirce: CP 6.237 Cross-Ref:††

237. This ought not to appear as a paradox. Suppose for example that one of you proposes to live by yourself and to suit yourself alone, away from all constraints of law and custom, according to your own whimses, your simple plan of life being to do whatever came into your head. If you are disposed to do this, why should not thousands of others have similar views of life? They certainly will. Not by force of any logical *necessity*, but yet by a logic which is just as certain as if it were necessary, an analogical inference. If nothing prevents such a man as you from existing, nothing will prevent a million such men from existing.

Peirce: CP 6.237 Cross-Ref:††

Now if everyone acts really irrespective of all the rest, there will be no regularity nor common character in their behaviour. There will be a Babel of strange tongues and a Bedlam of strange deeds. The units, each *totus, teres, atque rotundus* will make a chaos of fortuitously wandering atoms.

Peirce: CP 6.237 Cross-Ref:††

This is the logic by which the unity of *quale*-consciousness, implying simplicity, and through simplicity, freedom, necessarily results in endless multiplicity and variety. All that is a perfectly ostensive result of logic and involves no paradox whatsoever. . . .

Peirce: CP 6.238 Cross-Ref:††
CHAPTER 9

MAN'S GLASSY ESSENCE†1

§1. THE CONSTITUTION OF MATTER†E

238. In *The Monist* for January, 1891†2 I tried to show what conceptions ought to form the brick and mortar of a philosophical system. Chief among these was that of absolute chance †P1 for which I argued again in last April's number.†3 In July,†4 I applied another fundamental idea, that of continuity, to the law of mind. Next in order, I have to elucidate, from the point of view chosen, the relation between the psychical and physical aspects of a substance.

Peirce: CP 6.239 Cross-Ref:††

239. The first step towards this ought, I think, to be the framing of a molecular theory of protoplasm. But before doing that, it seems indispensable to glance at the constitution of matter in general. We shall, thus, unavoidably make a long detour; but, after all, our pains will not be wasted, for the problems of the papers that are to follow in the series will call for the consideration of the same question.†5

Peirce: CP 6.240 Cross-Ref:††

240. All physicists are rightly agreed the evidence is overwhelming which shows all sensible matter is composed of molecules in swift motion and exerting enormous mutual attractions, and perhaps repulsions, too. Even Sir William Thomson,†6 Lord Kelvin, who wishes to explode action at a distance and return to the doctrine of a plenum, not only speaks of molecules, but undertakes to assign definite magnitudes to them. The brilliant Judge Stallo, a man who did not always rightly estimate his own qualities in accepting tasks for himself, declared war upon the atomic theory in a book †1 well worth careful perusal. To the old arguments in favor of atoms which he found in Fechner's †2 monograph, he was able to make replies of considerable force, though they were not sufficient to destroy those arguments. But against modern proofs he made no headway at all. These set out from the mechanical theory of heat. Rumford's experiments †3 showed that heat is not a substance. Joule †4 demonstrated that it was a form of energy. The heating of gases under constant volume, and other facts instanced by Rankine,†5 proved that it could not be an energy of strain. This drove physicists to the conclusion that it was a mode of motion. Then it was remembered that John Bernoulli †6 had shown that the pressure of gases could be accounted for by assuming their molecules to be moving uniformly in rectilinear paths. The same hypothesis was now seen to account for Avogadro's law, that in equal volumes of different kinds of gases exposed to the same pressure and temperature are contained equal numbers of molecules. Shortly after, it was found to account for the laws of diffusion and viscosity of gases, and for the numerical relation between these properties. Finally, Crookes's radiometer †7 furnished the last link in the strongest chain of evidence which supports any physical hypothesis.

Peirce: CP 6.241 Cross-Ref:††

241. Such being the constitution of gases, liquids must clearly be bodies in which the molecules wander in curvilinear paths, while in solids they move in orbits or quasi-orbits. (See my definition *solid II*, 1, in the *Century Dictionary*.)†P1

Peirce: CP 6.242 Cross-Ref:††

242. We see that the resistance to compression and to interpenetration between sensible bodies is, by one of the prime propositions of the molecular theory, due in large measure to the kinetical energy of the particles, which must be supposed to be quite remote from one another, on the average, even in solids. This resistance is no doubt influenced by finite attractions and repulsions between the molecules. All the impenetrability of bodies which we can observe is, therefore, a limited impenetrability due to kinetic and positional energy. This being the case, we have no logical right to suppose that absolute impenetrability, or the exclusive occupancy of space, belongs to molecules or to atoms. It is an unwarranted hypothesis, not a *vera causa*.†P1 Unless we are to give up the theory of energy, finite positional attractions and repulsions between molecules must be admitted. Absolute impenetrability would amount to an infinite repulsion at a certain distance. No analogy of known phenomena exists to excuse such a wanton violation of the principle of continuity as such a hypothesis is. In short, we are logically bound to adopt the Boscovichian †1 idea that an atom is simply a distribution of component potential energy throughout space (this distribution being absolutely rigid) combined with inertia. The potential energy belongs to two molecules, and is to be conceived as different between molecules *A* and *B* from what it is between molecules *A* and *C*. The distribution of energy is not necessarily spherical. Nay, a molecule may conceivably have more than one center; it may even have a central curve, returning into itself. But I do not think there are any observed facts pointing to such multiple or linear centers. On the other hand, many facts relating to crystals, especially those observed by Voigt,†P1 go to show that the

distribution of energy is harmonical but not concentric. We can easily calculate the forces which such atoms must exert upon one another by considering †P2 that they are equivalent to aggregations of pairs of electrically positive and negative points infinitely near to one another. About such an atom there would be regions of positive and of negative potential, and the number and distribution of such regions would determine the valency of the atom, a number which it is easy to see would in many cases be somewhat indeterminate. I must not dwell further upon this hypothesis, at present. In another paper, its consequences will be further considered. †2

Peirce: CP 6.243 Cross-Ref: ††

243. I cannot assume that the students of philosophy who read this magazine are thoroughly versed in modern molecular physics, and therefore it is proper to mention that the governing principle in this branch of science is Clausius's law of the virial. I will first state the law, and then explain the peculiar terms of the statement. This statement is that the total kinetic energy of the particles of a system in stationary motion is equal to the total virial. By a *system* is here meant a number of particles acting upon one another. †P1 Stationary motion is a quasi-orbital motion among a system of particles so that none of them are removed to indefinitely great distances nor acquire indefinitely great velocities. The kinetic energy of a particle is the work which would be required to bring it to rest, independently of any forces which may be acting upon it. The virial of a pair of particles is half the work which the force which actually operates between them would do if, being independent of the distance, it were to bring them together. The equation of the virial is

$$1/2 \sum m v^2 = 1/2 \sum \sum R r.$$

Here *m* is the mass of a particle, *v* its velocity, *R* is the attraction between two particles, and *r* is the distance between them. The sign \sum on the left-hand side signifies that the values of *m v*² are to be summed for all the particles, and $\sum \sum$ on the right-hand side signifies that the values of *R r* are to be summed for all the pairs of particles. If there is an external pressure *P* (as from the atmosphere) upon the system, and the volume of vacant space within the boundary of that pressure is *V*, then the virial must be understood as including 3/2 *P V*, so that the equation is

$$1/2 \sum m v^2 = 3/2 P V + 1/2 \sum \sum R r.$$

There is strong (if not demonstrative) reason for thinking that the temperature of any body above the absolute zero (-273• C.) is proportional to the average kinetic energy of its molecules, or say $\alpha \Theta$, where α is a constant and Θ is the absolute temperature. Hence, we may write the equation

$$\alpha \Theta = 1/2 m v^2 = 3/2 P V + 1/2 \sum R r$$

where the heavy lines above the different expressions signify that the average values for single molecules are to be taken. In 1872, a student in the University of Leyden, Van der Waals,^{†1} propounded in his thesis for the doctorate a specialization of the equation of the virial which has since attracted great attention. Namely, he writes it

$$\alpha\Theta = (\overline{P} + c/V^2)(V - b)$$

The quantity b is the volume of a molecule, which he supposes to be an impenetrable body, and all the virtue of the equation lies in this term which makes the equation a cubic in V , which is required to account for the shape of certain isothermal curves.^{†P1} But if the idea of an impenetrable atom is illogical, that of an impenetrable molecule is almost absurd. For the kinetical theory of matter teaches us that a molecule is like a solar system or star-cluster in miniature. Unless we suppose that in all heating of gases and vapors internal work is performed upon the molecules, implying that their atoms are at considerable distances, the whole kinetical theory of gases falls to the ground. As for the term added to \overline{P} , there is no more than a partial and roughly approximative justification for it. Namely, let us imagine two spheres described round a particle as their center, the radius of the larger being so great as to include all the particles whose action upon the center is sensible, while the radius of the smaller is so large that a good many molecules are included within it. The possibility of describing such a sphere as the outer one implies that the attraction of the particles varies at some distances inversely as some higher power of the distance than the cube, or, to speak more clearly, that the attraction multiplied by the cube of the distance diminishes as the distance increases; for the number of particles at a given distance from any one particle is proportionate to the square of that distance and each of these gives a term of the virial which is the product of the attraction into the distance. Consequently, unless the attraction multiplied by the cube of the distance diminished so rapidly with the distance as soon to become insensible, no such outer sphere as is supposed could be described. However, ordinary experience shows that such a sphere is possible; and consequently there must be distances at which the attraction does thus rapidly diminish as the distance increases. The two spheres, then, being so drawn, consider the virial of the central particle due to the particles between them. Let the density of the substance be increased, say, N times. Then, for every term, Rr , of the virial before the condensation, there will be N terms of the same magnitude after the condensation. Hence, the virial of each particle will be proportional to the density, and the equation of the virial becomes

$$\alpha\Theta = \overline{P}V + c/V.$$

This omits the virial within the inner sphere, the radius of which is so taken that within that distance the number of particles is not proportional to the number in a large sphere. For Van der Waals this radius is the diameter of his hard molecules, which assumption gives his equation. But it is plain that the attraction between the

molecules must to a certain extent modify their distribution, unless some peculiar conditions are fulfilled. The equation of Van der Waals can be approximately true, therefore, only for a gas. In a solid or liquid condition, in which the removal of a small amount of pressure has little effect on the volume, and where consequently the virial must be much greater than

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PV ,

the virial must increase with the volume. For suppose we had a substance in a critical condition in which an increase of the volume would diminish the virial more than it would increase

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$3/2 PV$.

If we were forcibly to diminish the volume of such a substance, when the temperature became equalized, the pressure which it could withstand would be less than before, and it would be still further condensed, and this would go on indefinitely until a condition were reached in which an increase of volume would increase

—

$3/2 PV$ more than it would decrease the virial. In the case of solids, at least, P may be zero; so that the state reached would be one in which the virial increases with the volume or the attraction between the particles does not increase so fast with a diminution of their distance as it would if the attraction were inversely as the distance.

Peirce: CP 6.244 Cross-Ref:††

244. Almost contemporaneously with Van der Waals's paper, another remarkable thesis for the doctorate was presented at Paris by Amagat. It related to the elasticity and expansion of gases, and to this subject the superb experimenter, its author, has devoted his whole subsequent life. Especially interesting are his observations of the volumes of ethylene and of carbonic acid at temperatures from 20• to 100• and at pressures ranging from an ounce to five thousand pounds to the square inch.†1 As soon as Amagat had obtained these results, he remarked that the "coefficient of expansion at constant volume" as it is absurdly called, that is, the rate of variation of the pressure with the temperature, was very nearly constant for each volume. This accords with the equation of the virial, which gives

— —

$$d p/d\Theta = \alpha/V - d\Sigma Rr/d\Theta$$

Now, the virial must be nearly independent of the temperature, and therefore the last term almost disappears. The virial would not be quite independent of the temperature because, if the temperature (*i.e.* the square of the velocity of the molecules) is lowered, and the pressure correspondingly lowered, so as to make the volume the

same, the attractions of the molecules will have more time to produce their effects, and consequently, the pairs of molecules the closest together will be held together longer and closer; so that the virial will generally be increased by a decrease of temperature. Now, Amagat's experiments do show an excessively minute effect of this sort, at least, when the volumes are not too small. However, the observations are well enough satisfied by assuming the "coefficient of expansion at constant volume" to consist wholly of the first term,

$$-\alpha/V.$$

Thus, Amagat's experiments enable us to determine the values of α and thence to calculate the virial; and this we find varies for carbonic acid gas nearly inversely to

$$V^{0.9}.$$

There is, thus, a rough approximation to satisfying Van der Waals's equation. But the most interesting result of Amagat's experiments, for our purpose at any rate, is that the quantity α , though nearly constant for any one volume, differs considerably with the volume, nearly doubling when the volume is reduced fivefold. This can only indicate that the mean kinetic energy of a given mass of the gas for a given temperature is greater the more the gas is compressed. But the laws of mechanics appear to enjoin that the mean kinetic energy of a moving particle shall be constant at any given temperature. The only escape from contradiction, then, is to suppose that the mean mass of a moving particle diminishes upon the condensation of the gas. In other words, many of the molecules are dissociated, or broken up into atoms or sub-molecules. The idea that dissociation should be favored by diminishing the volume will be pronounced by physicists, at first blush, as contrary to all our experience. But it must be remembered that the circumstances we are speaking of, that of a gas under fifty or more atmospheres pressure, are also unusual. That the "coefficient of expansion under constant volume" when multiplied by the volumes should increase with a decrement of the volume is also quite contrary to ordinary experience; yet it undoubtedly takes place in all gases under great pressure. Again, the doctrine of Arrhenius $\dagger\dagger$ is now generally accepted, that the molecular conductivity of an electrolyte is proportional to the dissociation of ions. Now the molecular conductivity of a fused electrolyte is usually superior to that of a solution. Here is a case, then, in which diminution of volume is accompanied by increased dissociation.

Peirce: CP 6.245 Cross-Ref: $\dagger\dagger$

245. The truth is that several different kinds of dissociation have to be distinguished. In the first place, there is the dissociation of a chemical molecule to form chemical molecules under the regular action of chemical laws. This may be a double decomposition, as when iodhydric acid is dissociated, according to the formula



or it may be a simple decomposition, as when pentachloride of phosphorus is dissociated according to the formula



All these dissociations require, according to the laws of thermo-chemistry, an elevated temperature. In the second place, there is the dissociation of a physically polymeric molecule, that is, of several chemical molecules joined by physical attractions. This I am inclined to suppose is a common concomitant of the heating of solids and liquids; for in these bodies there is no increase of compressibility with the temperature at all comparable with the increase of the expansibility. But, in the third place, there is the dissociation with which we are now concerned, which must be supposed to be a throwing off of unsaturated sub-molecules or atoms from the molecule. The molecule may, as I have said, be roughly likened to a solar system. As such, molecules are able to produce perturbations of one another's internal motions; and in this way a planet, *i.e.*, a sub-molecule, will occasionally get thrown off and wander about by itself, till it finds another unsaturated sub-molecule with which it can unite. Such dissociation by perturbation will naturally be favored by the proximity of the molecules to one another.

Peirce: CP 6.246 Cross-Ref: ††
§2. PROTOPLASM †1E

246. Let us now pass to the consideration of that special substance, or rather class of substances, whose properties form the chief subject of botany and of zoölogy, as truly as those of the silicates form the chief subject of mineralogy: I mean the life-slimes, or protoplasm. Let us begin by cataloguing the general characters of these slimes. They one and all exist in two states of aggregation, a solid or nearly solid state and a liquid or nearly liquid state; but they do not pass from the former to the latter by ordinary fusion. They are readily decomposed by heat, especially in the liquid state; nor will they bear any considerable degree of cold. All their vital actions take place at temperatures very little below the point of decomposition. This extreme instability is one of numerous facts which demonstrate the chemical complexity of protoplasm. Every chemist will agree that they are far more complicated than the albumens. Now, albumen is estimated to contain in each molecule about a thousand atoms; so that it is natural to suppose that the protoplasms contain several thousands. We know that while they are chiefly composed of oxygen, hydrogen, carbon, and nitrogen, a large number of other elements enter into living bodies in small proportions; and it is likely that most of these enter into the composition of protoplasms. Now, since the numbers of chemical varieties increase at an enormous rate with the number of atoms per molecule, so that there are certainly hundreds of thousands of substances whose molecules contain twenty atoms or fewer, we may well suppose that the number of

protoplasmic substances runs into the billions or trillions. Professor Cayley †1 has given a mathematical theory of "trees," with a view of throwing a light upon such questions; and in that light the estimate of trillions (in the English sense) seems immoderately moderate. It is true that an opinion has been emitted, and defended among biologists, that there is but one kind of protoplasm; †2 but the observations of biologists themselves have almost exploded that hypothesis, which from a chemical standpoint appears utterly incredible. The anticipation of the chemist would decidedly be that enough different chemical substances having protoplasmic characters might be formed to account, not only for the differences between nerve-slime and muscle-slime, between whale-slime and lion-slime, but also for those minuter pervasive variations which characterize different breeds and single individuals.

Peirce: CP 6.247 Cross-Ref: ††

247. Protoplasm, when quiescent, is, broadly speaking, solid; but when it is disturbed in an appropriate way, or sometimes even spontaneously without external disturbance, it becomes, broadly speaking, liquid. A moner in this state is seen under the microscope to have streams within its matter; a slime-mould slowly flows by force of gravity. The liquefaction starts from the point of disturbance and spreads through the mass. This spreading, however, is not uniform in all directions; on the contrary, it takes at one time one course, at another another, through the homogeneous mass, in a manner that seems a little mysterious. The cause of disturbance being removed, these motions gradually (with higher kinds of protoplasm, quickly) cease, and the slime returns to its solid condition.

Peirce: CP 6.248 Cross-Ref: ††

248. The liquefaction of protoplasm is accompanied by a mechanical phenomenon. Namely, some kinds exhibit a tendency to draw themselves up into a globular form. This happens particularly with the contents of muscle-cells. The prevalent opinion, founded on some of the most exquisite experimental investigations that the history of science can show, is undoubtedly that the contraction of muscle-cells is due to osmotic pressure; and it must be allowed that that is a factor in producing the effect. But it does not seem to me that it satisfactorily accounts even for the phenomena of muscular contraction; and besides, even naked slimes often draw up in the same way. In this case, we seem to recognize an increase of the surface-tension. In some cases, too, the reverse action takes place, extraordinary pseudopodia being put forth, as if the surface-tension were diminished in spots. Indeed, such a slime always has a sort of skin, due no doubt to surface-tension, and this seems to give way at the point where a pseudopodium is put forth.

Peirce: CP 6.249 Cross-Ref: ††

249. Long-continued or frequently repeated liquefaction of the protoplasm results in an obstinate retention of the solid state, which we call fatigue. †1 On the other hand, repose in this state, if not too much prolonged, restores the liquefiability. These are both important functions.

Peirce: CP 6.250 Cross-Ref: ††

250. The life-slimes have, further, the peculiar property of growing. Crystals also grow; their growth, however, consists merely in attracting matter like their own from the circumambient fluid. To suppose the growth of protoplasm of the same nature would be to suppose this substance to be spontaneously generated in copious supplies wherever food is in solution. Certainly, it must be granted that protoplasm is but a chemical substance, and that there is no reason why it should not be formed synthetically like any other chemical substance. Indeed, Clifford has clearly shown †2

that we have overwhelming evidence that it is so formed. But to say that such formation is as regular and frequent as the assimilation of food is quite another matter. It is more consonant with the facts of observation to suppose that assimilated protoplasm is formed at the instant of assimilation, under the influence of the protoplasm already present. For each slime in its growth preserves its distinctive characters with wonderful truth, nerve-slime growing nerve-slime and muscle-slime, muscle-slime, lion-slime growing lion-slime, and all the varieties of breeds and even individual characters being preserved in the growth. Now it is too much to suppose there are billions of different kinds of protoplasm floating about wherever there is food.

Peirce: CP 6.251 Cross-Ref:††

251. The frequent liquefaction of protoplasm increases its power of assimilating food; so much so, indeed, that it is questionable whether in the solid form it possesses this power.

Peirce: CP 6.252 Cross-Ref:††

252. The life-slime wastes as well as grows; and this too takes place chiefly if not exclusively in its liquid phases.

Peirce: CP 6.253 Cross-Ref:††

253. Closely connected with growth is reproduction; and though in higher forms this is a specialized function, it is universally true that wherever there is protoplasm, there is, will be, or has been a power of reproducing that same kind of protoplasm in a separated organism. Reproduction seems to involve the union of two sexes; though it is not demonstrable that this is always requisite.

Peirce: CP 6.254 Cross-Ref:††

254. Another physical property of protoplasm is that of taking habits. The course which the spread of liquefaction has taken in the past is rendered thereby more likely to be taken in the future; although there is no absolute certainty that the same path will be followed again.

Peirce: CP 6.255 Cross-Ref:††

255. Very extraordinary, certainly, are all these properties of protoplasm; as extraordinary as indubitable. But the one which has next to be mentioned, while equally undeniable, is infinitely more wonderful. It is that protoplasm feels. We have no direct evidence that this is true of protoplasm universally, and certainly some kinds feel far more than others. But there is a fair analogical inference that all protoplasm feels. It not only feels but exercises all the functions of mind.

Peirce: CP 6.256 Cross-Ref:††

256. Such are the properties of protoplasm. The problem is to find a hypothesis of the molecular constitution of this compound which will account for these properties, one and all.

Peirce: CP 6.256 Cross-Ref:††

Some of them are obvious results of the excessively complicated constitution of the protoplasm molecule. All very complicated substances are unstable; and plainly a molecule of several thousand atoms may be separated in many ways into two parts, in each of which the polar chemical forces are very nearly saturated. In the solid protoplasm, as in other solids, the molecules must be supposed to be moving as it were in orbits, or, at least, so as not to wander indefinitely. But this solid cannot be melted, for the same reason that starch cannot be melted; because an amount of heat

insufficient to make the entire molecules wander is sufficient to break them up completely and cause them to form new and simpler molecules. But when one of the molecules is disturbed, even if it be not quite thrown out of its orbit at first, sub-molecules of perhaps several hundred atoms each are thrown off from it. These will soon acquire the same mean kinetic energy as the others, and therefore velocities several times as great. They will naturally begin to wander, and in wandering will perturb a great many other molecules and cause them in their turn to behave like the one originally deranged. So many molecules will thus be broken up that even those that are intact will no longer be restrained within orbits, but will wander about freely. This is the usual condition of a liquid, as modern chemists understand it; for in all electrolytic liquids there is considerable dissociation.

Peirce: CP 6.256 Cross-Ref:††

But this process necessarily chills the substance, not merely on account of the heat of chemical combination, but still more because the number of separate particles being greatly increased, the mean kinetic energy must be less. The substance being a bad conductor, this heat is not at once restored. Now the particles moving more slowly, the attractions between them have time to take effect, and they approach the condition of equilibrium. But their dynamic equilibrium is found in the restoration of the solid condition, which therefore takes place, if the disturbance is not kept up.

Peirce: CP 6.257 Cross-Ref:††

257. When a body is in the solid condition, most of its molecules must be moving at the same rate, or, at least, at certain regular sets of rates; otherwise the orbital motion would not be preserved. The distances of neighboring molecules must always be kept between a certain maximum and a certain minimum value. But if, without absorption of heat, the body be thrown into a liquid condition, the distances of neighboring molecules will be far more unequally distributed, and an effect upon the virial will result. The chilling of protoplasm upon its liquefaction must also be taken into account. The ordinary effect will no doubt be to increase the cohesion and with that the surface-tension, so that the mass will tend to draw itself up. But in special cases, the virial will be increased so much that the surface-tension will be diminished at points where the temperature is first restored. In that case, the outer film will give way and the tension at other places will aid in causing the general fluid to be poured out at those points, forming pseudopodia.

Peirce: CP 6.258 Cross-Ref:††

258. When the protoplasm is in a liquid state, and then only, a solution of food is able to penetrate its mass by diffusion. The protoplasm is then considerably dissociated; and so is the food, like all dissolved matter. If then the separated and unsaturated sub-molecules of the food happen to be of the same chemical species as sub-molecules of the protoplasm, they may unite with other sub-molecules of the protoplasm to form new molecules, in such a fashion that when the solid state is resumed there may be more molecules of protoplasm than there were at the beginning. It is like the jackknife whose blade and handle, after having been severally lost and replaced, were found and put together to make a new knife.

259. We have seen that protoplasm is chilled by liquefaction, and that this brings it back to the solid state, when the heat is recovered. This series of operations must be very rapid in the case of nerve-slime and even of muscle-slime, and may account for the unsteady or vibratory character of their action. Of course, if assimilation takes place, the heat of combination, which is probably trifling, is gained. On the other hand, if work is done, whether by nerve or by muscle, loss of energy must take place. In the case of the muscle, the mode by which the instantaneous part of the fatigue is brought about is easily traced out. If when the muscle contracts it be under stress, it will contract less than it otherwise would do, and there will be a loss of heat. It is like an engine which should work by dissolving salt in water and using the contraction during the solution to lift a weight, the salt being recovered afterwards by distillation. But the major part of fatigue has nothing to do with the correlation of forces. A man must labor hard to do in a quarter of an hour the work which draws from him enough heat to cool his body by a single degree. Meantime, he will be getting heated, he will be pouring out extra products of combustion, perspiration, etc. and he will be driving the blood at an accelerated rate through minute tubes at great expense. Yet all this will have little to do with his fatigue. He may sit quietly at his table writing, doing practically no physical work at all, and yet in a few hours be terribly fagged. This seems to be owing to the deranged sub-molecules of the nerve-slime not having had time to settle back into their proper combinations. When such sub-molecules are thrown out, as they must be from time to time, there is so much waste of material.

Peirce: CP 6.260 Cross-Ref:††

260. In order that a sub-molecule of food may be thoroughly and firmly assimilated into a broken molecule of protoplasm, it is necessary not only that it should have precisely the right chemical composition, but also that it should be at precisely the right spot at the right time and should be moving in precisely the right direction with precisely the right velocity. If all these conditions are not fulfilled, it will be more loosely retained than the other parts of the molecule; and every time it comes round into the situation in which it was drawn in, relatively to the other parts of that molecule and to such others as were near enough to be factors in the action, it will be in special danger of being thrown out again. Thus, when a partial liquefaction of the protoplasm takes place many times to about the same extent, it will, each time, be pretty nearly the same molecules that were last drawn in that are now thrown out. They will be thrown out, too, in about the same way, as to position, direction of motion, and velocity, in which they were drawn in; and this will be in about the same course that the ones last before them were thrown out. Not exactly, however; for the very cause of their being thrown off so easily is their not having fulfilled precisely the conditions of stable retention. Thus, the law of habit is accounted for, and with it its peculiar characteristic of not acting with exactitude.

Peirce: CP 6.261 Cross-Ref:††

261. It seems to me that this explanation of habit, aside from the question of its truth or falsity, has a certain value as an addition to our little store of mechanical examples of actions analogous to habit. All the others, so far as I know, are either statical or else involve forces which, taking only the sensible motions into account, violate the law of energy. It is so with the stream that wears its own bed. Here, the sand is carried to its most stable situation and left there. The law of energy forbids this; for when anything reaches a position of stable equilibrium, its momentum will be at a maximum, so that it can, according to this law, only be left at rest in an unstable

situation. In all the statical illustrations, too, things are brought into certain states and left there. A garment receives folds and keeps them; that is, its limit of elasticity is exceeded. This failure to spring back is again an apparent violation of the law of energy; for the substance will not only not spring back of itself (which might be due to an unstable equilibrium being reached) but will not even do so when an impulse that way is applied to it. Accordingly, Professor James says, "the phenomena of habit . . . are due to the plasticity of the . . . materials."†1 Now, plasticity of materials means the having of a low limit of elasticity. (See the *Century Dictionary*, under *solid*.†2) But the hypothetical constitution of protoplasm here proposed involves no forces but attractions and repulsions strictly following the law of energy. The action here, that is, the throwing of an atom out of its orbit in a molecule, and the entering of a new atom into nearly, but not quite the same orbit, is somewhat similar to the molecular actions which may be supposed to take place in a solid strained beyond its limit of elasticity. Namely, in that case certain molecules must be thrown out of their orbits, to settle down again shortly after into new orbits. In short, the plastic solid resembles protoplasm in being partially and temporarily liquefied by a slight mechanical force. But the taking of a set by a solid body has but a moderate resemblance to the taking of a habit, inasmuch as the characteristic feature of the latter, its inexactitude and want of complete determinacy, is not so marked in the former, if it can be said to be present there at all.

Peirce: CP 6.262 Cross-Ref:††

262. The truth is that, though the molecular explanation of habit is pretty vague on the mathematical side, there can be no doubt that systems of atoms having polar forces would act substantially in that manner, and the explanation is even too satisfactory to suit the convenience of an advocate of tychism. For it may fairly be urged that since the phenomena of habit may thus result from a purely mechanical arrangement, it is unnecessary to suppose that habit-taking is a primordial principle of the universe. But one fact remains unexplained mechanically, which concerns not only the facts of habit, but all cases of actions apparently violating the law of energy; it is that all these phenomena depend upon aggregations of trillions of molecules in one and the same condition and neighborhood; and it is by no means clear how they could have all been brought and left in the same place and state by any conservative forces. But let the mechanical explanation be as perfect as it may, the state of things which it supposes presents evidence of a primordial habit-taking tendency. For it shows us like things acting in like ways because they are alike. Now, those who insist on the doctrine of necessity will for the most part insist that the physical world is entirely individual. Yet law involves an element of generality. Now to say that generality is primordial, but generalization not, is like saying that diversity is primordial but diversification not. It turns logic upside down. At any rate, it is clear that nothing but a principle of habit, itself due to the growth by habit of an infinitesimal chance tendency toward habit-taking, is the only bridge that can span the chasm between the chance-medley of chaos and the cosmos of order and law.

Peirce: CP 6.263 Cross-Ref:††

263. I shall not attempt a molecular explanation of the phenomena of reproduction, because that would require a subsidiary hypothesis, and carry me away from my main object. Such phenomena, universally diffused though they be, appear to depend upon special conditions; and we do not find that all protoplasm has reproductive powers.

264. But what is to be said of the property of feeling? If consciousness belongs to all protoplasm, by what mechanical constitution is this to be accounted for? The slime is nothing but a chemical compound. There is no inherent impossibility in its being formed synthetically in the laboratory, out of its chemical elements; and if it were so made, it would present all the characters of natural protoplasm. No doubt, then, it would feel. To hesitate to admit this would be puerile and ultra-puerile. By what element of the molecular arrangement, then, would that feeling be caused? This question cannot be evaded or pooh-poohed. Protoplasm certainly does feel; and unless we are to accept a weak dualism, the property must be shown to arise from some peculiarity of the mechanical system. Yet the attempt to deduce it from the three laws of mechanics, applied to never so ingenious a mechanical contrivance, would obviously be futile. It can never be explained, unless we admit that physical events are but degraded or undeveloped forms of psychical events. But once grant that the phenomena of matter are but the result of the sensibly complete sway of habits upon mind, and it only remains to explain why in the protoplasm these habits are to some slight extent broken up, so that, according to the law of mind, in that special clause of it sometimes called the principle of accommodation,†P1 feeling becomes intensified. Now the manner in which habits generally get broken up is this. Reactions usually terminate in the removal of a stimulus; for the excitation continues as long as the stimulus is present. Accordingly, habits are general ways of behaviour which are associated with the removal of stimuli. But when the expected removal of the stimulus fails to occur, the excitation continues and increases, and non-habitual reactions take place; and these tend to weaken the habit. If, then, we suppose that matter never does obey its ideal laws with absolute precision, but that there are almost insensible fortuitous departures from regularity, these will produce, in general, equally minute effects. But protoplasm is in an excessively unstable condition; and it is the characteristic of unstable equilibrium that near that point excessively minute causes may produce startlingly large effects. Here then, the usual departures from regularity will be followed by others that are very great; and the large fortuitous departures from law so produced will tend still further to break up the laws, supposing that these are of the nature of habits. Now, this breaking up of habit and renewed fortuitous spontaneity will, according to the law of mind, be accompanied by an intensification of feeling. The nerve-protoplasm is, without doubt, in the most unstable condition of any kind of matter; and consequently there the resulting feeling is the most manifest.

Peirce: CP 6.265 Cross-Ref:††

265. Thus we see that the idealist has no need to dread a mechanical theory of life. On the contrary, such a theory, fully developed, is bound to call in a tychistic idealism as its indispensable adjunct. Wherever chance-spontaneity is found, there in the same proportion feeling exists. In fact, chance is but the outward aspect of that which within itself is feeling. I long ago showed that real existence, or thing-ness, consists in regularities.†1 So, that primeval chaos in which there was no regularity was mere nothing, from a physical aspect. Yet it was not a blank zero; for there was an intensity of consciousness there, in comparison with which all that we ever feel is but as the struggling of a molecule or two to throw off a little of the force of law to an endless and innumerable diversity of chance utterly unlimited.†2

Peirce: CP 6.266 Cross-Ref:††

266. But after some atoms of the protoplasm have thus become partially emancipated from law, what happens next to them? To understand this we have to remember that no mental tendency is so easily strengthened by the action of habit as is the tendency to take habits. Now, in the higher kinds of protoplasm, especially, the atoms in question have not only long belonged to one molecule or another of the particular mass of slime of which they are parts; but before that, they were constituents of food of a protoplasmic constitution. During all this time they have been liable to lose habits and to recover them again; so that now, when the stimulus is removed, and the foregone habits tend to reassert themselves, they do so in the case of such atoms with great promptness. Indeed, the return is so prompt that there is nothing but the feeling to show conclusively that the bonds of law have ever been relaxed.

Peirce: CP 6.267 Cross-Ref:††

267. In short, diversification is the vestige of chance spontaneity; and wherever diversity is increasing, there chance must be operative. On the other hand, wherever uniformity is increasing, habit must be operative. But wherever actions take place under an established uniformity, there, so much feeling as there may be, takes the mode of a sense of reaction. That is the manner in which I am led to define the relation between the fundamental elements of consciousness and their physical equivalents.

Peirce: CP 6.268 Cross-Ref:††
§5. THE NATURE OF PERSONALITY †1E

268. It remains to consider the physical relations of general ideas. It may be well here to reflect that if matter has no existence except as a specialization of mind, it follows that whatever affects matter according to regular laws is itself matter. But all mind is directly or indirectly connected with all matter, and acts in a more or less regular way; so that all mind more or less partakes of the nature of matter. Hence, it would be a mistake to conceive of the psychical and the physical aspects of matter as two aspects absolutely distinct. Viewing a thing from the outside, considering its relations of action and reaction with other things, it appears as matter. Viewing it from the inside, looking at its immediate character as feeling, it appears as consciousness. These two views are combined when we remember that mechanical laws are nothing but acquired habits, like all the regularities of mind, including the tendency to take habits, itself; and that this action of habit is nothing but generalization, and generalization is nothing but the spreading of feelings. But the question is, how do general ideas appear in the molecular theory of protoplasm?

Peirce: CP 6.269 Cross-Ref:††

269. The consciousness of a habit involves a general idea. In each action of that habit certain atoms get thrown out of their orbit, and replaced by others. Upon all the different occasions it is different atoms that are thrown off, but they are analogous from a physical point of view, and there is an inward sense of their being analogous. Every time one of the associated feelings recurs, there is a more or less vague sense that there are others, that it has a general character, and of about what this general character is. We ought not, I think, to hold that in protoplasm habit never acts in any

other than the particular way suggested above. On the contrary, if habit be a primary property of mind, it must be equally so of matter, as a kind of mind. We can hardly refuse to admit that wherever chance motions have general characters, there is a tendency for this generality to spread and to perfect itself. In that case, a general idea is a certain modification of consciousness which accompanies any regularity or general relation between chance actions.

Peirce: CP 6.270 Cross-Ref:††

270. The consciousness of a general idea has a certain "unity of the ego," in it, which is identical when it passes from one mind to another. It is, therefore, quite analogous to a person; and, indeed, a person is only a particular kind of general idea. Long ago, in the *Journal of Speculative Philosophy* (Vol. II, p. 156),†1 I pointed out that a person is nothing but a symbol involving a general idea; but my views were, then, too nominalistic to enable me to see that every general idea has the unified living feeling of a person.

Peirce: CP 6.271 Cross-Ref:††

271. All that is necessary, upon this theory, to the existence of a person is that the feelings out of which he is constructed should be in close enough connection to influence one another. Here we can draw a consequence which it may be possible to submit to experimental test. Namely, if this be the case, there should be something like personal consciousness in bodies of men who are in intimate and intensely sympathetic communion. It is true that when the generalization of feeling has been carried so far as to include all within a person, a stopping-place, in a certain sense, has been attained; and further generalization will have a less lively character. But we must not think it will cease. *Esprit de corps*, national sentiment, sympathy, are no mere metaphors. None of us can fully realize what the minds of corporations are, any more than one of my brain cells can know what the whole brain is thinking. But the law of mind clearly points to the existence of such personalities, and there are many ordinary observations which, if they were critically examined and supplemented by special experiments, might, as first appearances promise, give evidence of the influence of such greater persons upon individuals. It is often remarked that on one day half a dozen people, strangers to one another, will take it into their heads to do one and the same strange deed, whether it be a physical experiment, a crime, or an act of virtue. When the thirty thousand young people of the society for Christian Endeavor were in New York, there seemed to me to be some mysterious diffusion of sweetness and light. If such a fact is capable of being made out anywhere, it should be in the church. The Christians have always been ready to risk their lives for the sake of having prayers in common, of getting together and praying simultaneously with great energy, and especially for their common body, for "the whole state of Christ's church militant here in earth," as one of the missals has it. This practice they have been keeping up everywhere, weekly, for many centuries. Surely, a personality ought to have developed in that church, in that "bride of Christ," as they call it, or else there is a strange break in the action of mind, and I shall have to acknowledge my views are much mistaken. Would not the societies for psychical research be more likely to break through the clouds, in seeking evidences of such corporate personality, than in seeking evidences of telepathy, which, upon the same theory, should be a far weaker phenomenon?†1

MIND AND MATTER

§1. THE CONNECTION BETWEEN MIND AND MATTER^{P†1}

272. Several different theories have been urged during the past months to account for the mutual action between mind and matter in volition and in sensation. Little positive evidence has been gathered as yet for any of them; yet three or four hint at some possible eventual solution of this famous, deeply interesting, and dark problem.

Peirce: CP 6.273 Cross-Ref:††

273. It will do no harm to glance, first, at Leibnitz's theory of Preëstablished Harmony, if only to serve as foil to the modern hypotheses. This was possibly evoked by the phenomena of the mutual influence of two pendulums. These phenomena are like the following. A clock with a heavy pendulum had been solidly fastened to a brick wall. About six feet away was a shelf attached to the same wall. A Hardy's noddy, or little inverted pendulum swinging by the bending of a spring, having been adjusted to the period of oscillation of the clock-pendulum, was placed upon that shelf, when it immediately began to swing in unmistakable synchronism with the clock; and this motion it kept up for months. Again, two pendulums exactly alike and weighing some twenty pounds each, were suspended from a two-inch plank which was supported all along its eighteen-inch ends by quarter-inch plates of vertically corrugated iron attached to the floor. The direction of swinging of both pendulums was that of the length of the plank. Both being at rest, one of them was started into oscillation. Almost immediately, the other began to swing, and at the end of about three minutes the one first started had come to rest, while the one that had not been touched was swinging with nearly the same amplitude that the first had in the beginning. But no sooner had the latter come to rest than it began to start up again, while the arcs of the other became less; and in three minutes more, substantially the original condition of things had been restored. Of course, the real explanation of these phenomena is that the support of the heavy pendulum -- the brick wall in the one case, the corrugated iron in the other -- yielded under the swaying of the pendulums, and so communicated an impulse from each to the other, increasing the motion of the one that was behind and diminishing that of the one that was ahead; and though this effect was in itself imperceptible, yet when it had been multiplied one or two hundredfold, or as many times as the repetitions of its cause, the total effect became great. Meantime (the pendulum-support in such experiments, not being seen to sway, and being recognized to be almost quite rigid) each pendulum always looks as if it were going through its curious succession of motions of its own accord, although in a sort of inverse sympathy with the other. Nor is this a mere ocular illusion; for when the differential equations which define the forces are integrated, it is found that the motion of each pendulum is simply the sum of two perfectly simple and regular funipendulous motions of slightly different periods, and is, in fact, exactly like that of a cork floating upon still water traversed by two series of waves of slightly different

lengths. The remarkable thing is that the motion of each pendulum is expressed as perfectly regular, without reference to the other. And what is so manifest in regard to pendulums is equally true of any other dynamical phenomenon. Namely, the forces are expressed by differential equations; but those equations have their integrals, whether mathematicians have yet discovered them or not; and these integrals express the motion of each particle as perfectly regular and determinate, and express it independently of all other particles. Now, Leibnitz was a professed nominalist. For him, a law was nothing more than a regularity, a regularity nothing less than a law. Since, therefore, each atom was experiencing a series of motions (and, as he presumed, also a series of feelings) which were perfectly regular, independently of every other, it followed, for him, that such was the law of its being; and the action between the different atoms consisted merely in certain relations -- a certain harmony -- between the laws having been preestablished by the great Author of those laws; and from this point of view there was no more difficulty in actions between mind and matter than between matter and matter.

Peirce: CP 6.273 Cross-Ref:††

The fault of this explanation is the capital fault which attaches to all nominalistic explanations, namely that they merely restate the fact to be explained under another aspect; or, if they add anything to it, add only something from which no definite consequences can be deduced. A scientific explanation ought to consist in the assertion of some positive matter of fact, other than the fact to be explained, but from which this fact necessarily follows; and if the explanation be hypothetical, the proof of it lies in the experiential verification of predictions deduced from it as necessary consequences. Leibnitz's explanation merely comes to this, that the motions and changes of state of atoms are relative to one another, because God made them so in the beginning. But nothing can be deduced from this theory, since it is impossible for man to predict what God might see fit to do. This stamps the theory as one of those to which Auguste Comte applied the epithet *metaphysical*, that is, unverifiable.†1 To accept it as sufficient would be to block the road of inquiry.

Peirce: CP 6.274 Cross-Ref:††

274. Among the modern theories not open to this objection, the simplest is the materialistic hypothesis. According to this, nothing really exists but matter: feelings are nothing but the way matter appears to itself. The gist of this theory, be it remarked, is that the Whole is governed by mechanical forces that are determined by the state of things at the instant they act, without any reference to a purpose of bringing about any determinate state of things in the future. The distinguishing merits of this theory are its simplicity, together with its loyal adhesion to that wholesome maxim *entia non sunt multiplicanda praeter necessitatem*, which, though urged by an illustrious nominalist, nobody surely can deny. Nor can the theory be absolutely refuted. Still, there are so many facts which have all the appearance of being opposed to it, that, notwithstanding the strong bias in favor of it which Ockham's razor justly produces in the minds of scientific men, few of these who have duly considered the facts have been able to bring themselves to hold it for true. No doubt, all nervous physiology shows the dependence of mind upon body; but that is not in question. The question is whether mental phenomena are exclusively controlled by blind mechanical law, as they certainly must be if mind be but an aspect of matter and matter is governed by such a law. The very circumstance that we can foretell how we shall act seems to militate against the hypothesis; nor is it easy to divine how the hypothesis and this fact are to be reconciled. Then the fact that our knowledge of the future is of so different a kind [from] our knowledge of the past seems to be hopelessly in conflict

with materialism; since the laws of mechanics, as they are now understood, make the dynamical relation of the past to the future exactly the same as that of the future to the past. For, given a system of forces, and the positions and velocities of all the particles at any one instant, and all the previous positions and velocities are determined in precisely the same manner as are the subsequent positions and velocities. These are among the more obvious objections to the theory; others even more fatal have been urged.†1

Peirce: CP 6.275 Cross-Ref:††

275. Another theory, which, though not new, has newly been revived, is that by a mental effort certain material particles can be made temporarily to attract and repel one another. One naturally asks why this would not result in perpetual motion contrary to the principle of the conservation of energy, in case, for example, of the repelling particles being made to do work till they were brought to rest, and then, after attraction between them had been induced, being made to do additional work until restored to their original positions and velocities. The only answer possible would be that fatigue would, in that case, set in; so that a certain amount of work having been done the power of making the particles attract and repel would be exhausted until restored by nutrition. But such exhaustion could not be identified with that fatigue of which we are sensible; for the latter has little relation to the amount of mechanical work performed. It must, then, be an exhaustion which is not otherwise manifested; so that, comparing this theory with that of the materialist, we see that, in regard to muscular exertion, the two almost coincide, the logical advantage, however, being altogether with the materialist. For, brought up against the fact of exhaustion, the materialist fetches out his microscopes and chemical apparatus and endeavors to verify his theory by detecting some histological or chemical facts on which exhaustion may depend; while the other theorist attributes it to a metaphysical agency out of the realm of verification. Note, by the way, this singular feature of the theory under examination, that though, according to it, the mind only sets up now an attraction and now a repulsion between a pair of atoms, yet the exhaustion is not all occasioned by this conversion of attraction into repulsion and of repulsion into attraction, but only arises when these atoms happen to meet with a physical force against which they can do work, or happen to be so moving that their velocities are increased. If to answer the objection that the exhaustion which the theory supposes is a metaphysical butterfly it be maintained that the energy can be traced throughout in its transformations, then not only are mechanical and mental energy correlated, and the latter made measurable in foot-pounds, but it is held that this can be exhibited as so measurable. Now, it is undeniable that mental energy may be expended in producing a purely intellectual result; and mental energy that might be expended in muscular effort may, if we choose, be devoted to thought instead. If, therefore, the mental energy expended in muscular work is really transformed into that work, as this theory has to suppose, in order to preserve any semblance of a conservation of energy, it is necessary to admit that the purely intellectual product upon which that mental energy may be expended has also its equivalent in foot-pounds. Supposing, for example, this energy to be expended in writing a newspaper article, that paper would have to be supposed to contain that energy. So, if the editor does not like it and throws it into the fire, the theorist must either hold that all those foot-pounds have been destroyed, or else that the fire burns hotter on account of the mental energy expended on the composition. In short, this hypothesis falls either into the Scylla of absurdity or into the Charybdis of unverifiable metaphysics. Besides, its application to the action of matter upon mind in sensation is unsatisfactory. For if the energy of the mind is exhausted in volition, sensation ought to share with nutrition the function

of restoring it; but this we do not find to be the case.

Peirce: CP 6.276 Cross-Ref:††

276. For these and other obvious reasons, all the physicists who have examined into the question of an interchange of energy between mind and matter have come to the conclusion that no such phenomenon takes place. Hence the suggestion is often made that while mind creates a mechanical force between particles this force only acts at right angles to the directions of their motion, so as to leave the mechanical energy unaffected. If the motion of a mass were to be deflected, irrespective of any other, the law of action and reaction would be violated. But this is not what the advocates of this theory mean. Their true meaning will to a mathematician be most clearly expressed by saying that the effect of mind is supposed to be, not to alter the equation of motion, but only to add, at will, an equation of condition. The general reader will have to accept the statement that the theory comes to this, that the mind attaches at will, what it can at will remove, to certain pairs of portions of matter, pairs of absolutely rigid surfaces impenetrable to one another -- a surface to each of the two portions of matter -- so that these surfaces, being in contact, hamper the motions. But the theory may very advantageously be specialized a little, as follows. Atoms are usually assumed to be absolutely rigid bodies perfectly elastic. Let it be supposed that the mind has the power of altering the shape and size of certain of the atoms, with the proviso that their centers of gravity and inertia-properties be not disturbed in the act. No immediate dynamical result will take place, but only an indirect effect as soon as the atoms next impinge upon one another. This theory seems highly artificial, and is at present sadly in need of evidence to sustain it. Nevertheless, it fulfills the conditions of the problem, and is eminently deserving of being kept in view as one of the possibilities. The manner in which it applies to the action of matter upon mind in sensation is interesting. It is philosophical to presume that this action is analogous to the action of mind upon matter in volition. Upon this theory, then, it must be supposed that in sensation the action of the mental law of association goes on undisturbed, but that an absolute restraint is placed upon the ideas which may present themselves, so that the sensation is really a violent suggestion. But a further development of this idea would probably lead to another theory of the connection between soul and body, which has recently been advocated and which is the last that we shall mention.

Peirce: CP 6.277 Cross-Ref:††

277.†1 Observations upon living naked protoplasm seem to show that mind, or feeling, has a continuous extension in space. Nobody doubts that it has a continuity in time, nor that the consciousness in one instant directly influences, or spreads over into, the succeeding instant. In like manner, the feeling at any point of space appears to spread and to assimilate to its own quality, though with reduced intensity, the feelings in the closely surrounding places. In this way, feeling seems directly to act upon feeling continuous with it. Now, in obedience to the principle, or maxim, of continuity, that we ought to assume things to be continuous as far as we can, it has been urged that we ought to suppose a continuity between the characters of mind and matter, so that matter would be nothing but mind that had such indurated habits as to cause it to act with a peculiarly high degree of mechanical regularity, or routine. Supposing this to be the case, the reaction between mind and matter would be of no essentially different kind from the action between parts of mind that are in continuous union, and would thus come directly under the great law of mental association, just as the theory last mentioned makes sensation to do. This hypothesis might be called materialistic, since it attributes to mind one of the recognized properties of matter,

extension, and attributes to all matter a certain excessively low degree of feeling, together with a certain power of taking habits. But it differs essentially from materialism, in that, instead of supposing mind to be governed by blind mechanical law, it supposes the one original law to be the recognized law of mind, the law of association, of which the laws of matter are regarded as mere special results. This theory has been ridiculed by theologians as the merest whimsey while philosophers have pronounced it to be absurd upon metaphysical grounds; but students of physical and natural science are somewhat more favorable to it. Its advocates maintain that it is a perfectly consistent and legitimate working hypothesis, that it unmistakably commits itself to certain predictions and predesignations, that its truth or falsity ought to be judged exclusively from the comparison of these consequences of it with observation, and that, as far as it has been carried, this comparison has been quite favorable to the theory.

Peirce: CP 6.278 Cross-Ref:††

§2. THE MATERIALISTIC ASPECT OF REASONING^{†1}

278. The class of chemical substances having the most complicated molecules is, without doubt, that of the protoplasm.†P1 This chemical complexity is, in my opinion, sufficient to account for the extraordinary properties of those substances by virtue of which they grow into animals and plants. In particular, the laws of nervous action are, as I think, traceable to the chemical characters of the protoplasm of which the contents of nerve-cells are composed.†2

Peirce: CP 6.279 Cross-Ref:††

279. When a group of nerves are stimulated, it is certain that the ganglions with which the group is most intimately connected on the whole are thrown into an active state. This in its turn usually occasions movements of the body. Those movements are often intelligent; that is to say, what is to be accomplished determines what is done. Now, as all mechanical action is determined by the conditions at the instant, the question arises how is the tendency of nervous reactions towards ends to be accounted for. Suppose, then, that in the beginning, the reflex movements were not intelligent. In that case, the stimulation continuing, the irritation would spread from ganglion to ganglion, while increasing in intensity. Meantime, the ganglions first excited would begin to be fatigued, and their action would flag; and thus for a double reason the bodily activity would be of a changing kind. This would happen again and again, until at last some motion would remove the stimulus; and as soon as this was withdrawn, the excitement would quickly subside.

Peirce: CP 6.280 Cross-Ref:††

280.†3 Now it seems to be a universal property of protoplasm, intimately connected with the property of growth, that it takes habits. That is to say, first, when a lump of protoplasm is disturbed, say by a prick, at a given point, a so-called excited state, in which the matter is more fluid, is brought on; and this condition spreads. But, second, it does not spread uniformly, but very differently in different directions, and precisely what direction the spreading will take seems to be as uncertain as a throw of dice. Nevertheless, third, there is a preponderance of cases in which the path of spreading is the same as it had been the last time a similar stimulation of the same point occurred, or as it had been in the majority of cases.

Peirce: CP 6.281 Cross-Ref:††

281. The nerves are particularly ready to take and to change their habits. Consequently, in the case we have been considering, if, after the withdrawal of the stimulus and the consequent cessation of the excitement, the stimulation should be repeated, the last mode of reflex action, which removed the stimulus, is more likely to occur at once than any other; and in case it does not occur at once, the action will as before go on until a reaction takes place which shall remove the stimulus. In this way, a habit is pretty certain to be speedily acquired of so reacting from any stimulation as to remove the stimulus.

Peirce: CP 6.282 Cross-Ref:††

282. In fact, the greater part of intelligent actions are directed toward causing the cessation of some irritation. We eat to get rid of hunger, etc. Even when the eye of an infant rolls to the light, the action is perhaps of this kind; for the field of distinct vision on the retina is less sensitive to light than other parts. When we stop and listen to a sound, there may be a different principle; but then, any sensation, when it is interpreted, is diminished in intensity in immediate consciousness.

Peirce: CP 6.283 Cross-Ref:††

283. But other principles of intelligent action may probably be deduced from the primitive characters of protoplasm. There are many circumstances which lead us to believe that habit-taking is intimately connected with nutrition. Protoplasm grows; and that not as a crystal in a supersaturated or highly concentrated solution grows, by simply attracting matter like itself. It grows by chemically transforming other substances into its own chemical kind. This I believe to be due to the excessive complexity of its molecule. Chemists have estimated that the number of atoms in ordinary egg-albumen is nearly a thousand; and there are several circumstances which show that it must be nearly that. The most conclusive of these is the fact that a solution of albumen may be enclosed in the merest film of coagulum, and will float in water without bursting its sac.†P1 But albumen is not protoplasm. Albumen is dead; protoplasm is essentially alive. Hence, it is not too much to suppose that protoplasm, even of a low order, has several thousands of atoms in each molecule; and any high order of protoplasm probably has ten thousand. Such a molecule must be excessively unstable; and I believe that in the excited condition a considerable percentage of the molecules of protoplasm are partially decomposed. The peripheral stimulus deranges one or more molecules (which must be imagined as something like little solar systems, only vastly more complex) and an errant fragment from one of these enters another such system and perturbs that. But after the stimulus is removed they gradually settle down again, some molecules being destroyed, but others being recomposed with groups of atoms coming from food, while still others take up fragments which had been thrown off from neighboring molecules. I think it is pretty clear that the new portions thus taken in would be a very long time in acquiring the ideally stable places in the molecule; and until they did so they would be more likely to be thrown out than other portions of the same molecules; and so a new excitation would be likely to repeat approximately the phenomena of the previous one; and the spreading of the disturbance would be likely to take the same course as before.

Peirce: CP 6.284 Cross-Ref:††

284. If this theory be true, different modes of spreading might differ greatly in regard to the amount of nutrition that would accompany them; and since the recomposed molecules would be the ones most likely to be deranged, those habits would be most likely to be formed which would result in the greatest nutritive gain.

Thus, the animal would appear to exhibit a preference for modes of action involving the formation of new molecules of protoplasm. Were there a feeling of pain at every breaking of a molecule, and a pleasure at every recomposition of such a system, the animal would have a preference for pleasurable actions, and it would seem to him as if this pleasure, or the anticipation of it, were the cause of his acting in one way rather than in another.

Peirce: CP 6.285 Cross-Ref:††

285. This is a mode in which it would seem perhaps possible that a tendency to act intelligently, that is, so as to bring about a certain result, might arise in a mere mechanical system. Although it has not been shown that observed phenomena of intelligence could be thus accounted for, but only that they perhaps might be accounted for, and although the theory presents at one point a monstrous absurdity, that of supposing a piece of dead mechanism to feel pain and pleasure, yet, after all, this does not touch the main point, and I feel quite sure that the hypothesis affords an instructive point of view from which to contemplate the general question.

Peirce: CP 6.286 Cross-Ref:††

286. It is plain that intelligence does not consist in feeling in a certain way, but in acting in a certain way. Only, we must acknowledge that there are inward actions -- what might be called *potential* actions, that is, actions which do not take place, but which somehow influence the formation of habits.†1 Certain stimuli, commonly visceral in their origin, throw the brain into an activity which simulates the effect of peripheral excitations of the senses. The reactions from such stimuli have the same internal character; an inward action removes the inward stimulus. A fancied conjuncture leads us to fancy an appropriate line of behaviour. Day-dreams are often spoken of as mere idleness; and so they would be, but for the remarkable fact that they go to form habits, by virtue of which when a similar real conjuncture arises we really behave in the manner we had dreamed of doing.

"Some say the soft Ideal that we wooed
Confronts us fiercely, foe-beset, pursued,
And cries reproachful, "Was it, then, my praise
And not myself was loved? Prove now thy truth;
I claim of thee the promise of thy youth;
Give me thy life, or cower in empty phrase,
The victim of thy genius, not its mate!"

People who build castles in the air do not, for the most part, accomplish much, it is true; but every man who does accomplish great things is given to building elaborate castles in the air and then painfully copying them on solid ground. Indeed, the whole business of ratiocination, and all that makes us intellectual beings, is performed in imagination. Vigorous men are wont to hold mere imagination in contempt; and in that they would be quite right if there were such a thing. How we feel is no matter; the question is what we shall do. But that feeling which is subservient to action and to the

intelligence of action is correspondingly important; and all inward life is more or less so subservient. Mere imagination would indeed be mere trifling; only no imagination is *mere*. "More than all that is in thy custody, watch over thy phantasy," said Solomon, "For out of it are the issues of life." *Omni custodia serva cor tuum, quia ex ipso vita procedit.* †1

Peirce: CP 6.286 Cross-Ref: ††

A decapitated frog almost reasons. †2 The habit that is in his cerebellum serves as a major premiss. The excitation of a drop of acid is his minor premiss. And his conclusion is the act of wiping it away. All that is of any value in the operation of ratiocination is there, except only one thing. What he lacks is the power of preparatory meditation.

Peirce: CP 6.287 Cross-Ref: ††

CHAPTER 11

EVOLUTIONARY LOVE †1

§1. AT FIRST BLUSH. COUNTER-GOSPELS

287. Philosophy, when just escaping from its golden pupa-skin, mythology, proclaimed the great evolutionary agency of the universe to be Love. Or, since this pirate-lingo, English, is poor in such-like words, let us say Eros, the exuberance-love. Afterwards, Empedocles †2 set up passionate love and hate as the two coördinate powers of the universe. In some passages, kindness is the word. But certainly, in any sense in which it has an opposite, to be senior partner of that opposite, is the highest position that love can attain. Nevertheless, the ontological gospeller, in whose days those views were familiar topics, made the One Supreme Being, by whom all things have been made out of nothing, to be cherishing-love. What, then, can he say to hate? Never mind, at this time, what the scribe of the Apocalypse, if he were John, stung at length by persecution into a rage, unable to distinguish suggestions of evil from visions of heaven, and so become the Slanderer of God to men, may have dreamed. The question is rather what the sane John thought, or ought to have thought, in order to carry out his idea consistently. His statement that God is love seems aimed at that saying of Ecclesiastes that we cannot tell whether God bears us love or hatred. "Nay," says John, "we can tell, and very simply! We know and have trusted the love which God hath in us. God is love." There is no logic in this, unless it means that God loves all men. In the preceding paragraph, he had said, "God is light and in him is no darkness at all." We are to understand, then, that as darkness is merely the defect of light, so hatred and evil are mere imperfect stages of {agapé} and {agathon}, love and loveliness. This concords with that utterance reported in John's Gospel: "God sent not the Son into the world to judge the world; but that the world should through him be saved. He that believeth on him is not judged: he that believeth not hath been judged already. . . . And this is the judgment, that the light is come into the world, and that men loved darkness rather than the light." That is to say, God visits no punishment on them; they punish themselves, by their natural affinity for the defective. Thus, the

love that God is, is not a love of which hatred is the contrary; otherwise Satan would be a coordinate power; but it is a love which embraces hatred as an imperfect stage of it, an Anteros -- yea, even needs hatred and hatefulness as its object. For self-love is no love; so if God's self is love, that which he loves must be defect of love; just as a luminary can light up only that which otherwise would be dark. Henry James, the Swedenborgian, says: "It is no doubt very tolerable finite or creaturely love to love one's own in another, to love another for his conformity to one's self: but nothing can be in more flagrant contrast with the creative Love, all whose tenderness *ex vi termini* must be reserved only for what intrinsically is most bitterly hostile and negative to itself." This is from *Substance and Shadow: An Essay on the Physics of Creation*. †1 It is a pity he had not filled his pages with things like this, as he was able easily to do, instead of scolding at his reader and at people generally, until the physics of creation was well-nigh forgot. I must deduct, however, from what I just wrote: obviously no genius could make his every sentence as sublime as one which discloses for the problem of evil its everlasting solution.

Peirce: CP 6.288 Cross-Ref: ††

288. The movement of love is circular, at one and the same impulse projecting creations into independency and drawing them into harmony. This seems complicated when stated so; but it is fully summed up in the simple formula we call the Golden Rule. This does not, of course, say, Do everything possible to gratify the egoistic impulses of others, but it says, Sacrifice your own perfection to the perfectionment of your neighbor. Nor must it for a moment be confounded with the Benthamite, or Helvetian, or Beccarian motto, Act for the greatest good of the greatest number. Love is not directed to abstractions but to persons; not to persons we do not know, nor to numbers of people, but to our own dear ones, our family and neighbors. "Our neighbor," we remember, is one whom we live near, not locally perhaps but in life and feeling.

Peirce: CP 6.289 Cross-Ref: ††

289. Everybody can see that the statement of St. John is the formula of an evolutionary philosophy, which teaches that growth comes only from love, from I will not say self-*sacrifice*, but from the ardent impulse to fulfill another's highest impulse. Suppose, for example, that I have an idea that interests me. It is my creation. It is my creature; for as shown in last July's *Monist*, †1 it is a little person. I love it; and I will sink myself in perfecting it. It is not by dealing out cold justice to the circle of my ideas that I can make them grow, but by cherishing and tending them as I would the flowers in my garden. The philosophy we draw from John's gospel is that this is the way mind develops; and as for the cosmos, only so far as it yet is mind, and so has life, is it capable of further evolution. Love, recognizing germs of loveliness in the hateful, gradually warms it into life, and makes it lovely. That is the sort of evolution which every careful student of my essay "The Law of Mind" †2 must see that *synechism* calls for.

Peirce: CP 6.290 Cross-Ref: ††

290. The nineteenth century is now fast sinking into the grave, and we all begin to review its doings and to think what character it is destined to bear as compared with other centuries in the minds of future historians. It will be called, I guess, the Economical Century; for political economy has more direct relations with all the branches of its activity than has any other science. Well, political economy has its formula of redemption, too. It is this: Intelligence in the service of greed ensures the justest prices, the fairest contracts, the most enlightened conduct of all the

dealings between men, and leads to the *summum bonum*, food in plenty and perfect comfort. Food for whom? Why, for the greedy master of intelligence. I do not mean to say that this is one of the legitimate conclusions of political economy, the scientific character of which I fully acknowledge. But the study of doctrines, themselves true, will often temporarily encourage generalizations extremely false, as the study of physics has encouraged necessitarianism. What I say, then, is that the great attention paid to economical questions during our century has induced an exaggeration of the beneficial effects of greed and of the unfortunate results of sentiment, until there has resulted a philosophy which comes unwittingly to this, that greed is the great agent in the elevation of the human race and in the evolution of the universe.

Peirce: CP 6.291 Cross-Ref:††

291. I open a handbook of political economy †1 -- the most typical and middling one I have at hand -- and there find some remarks of which I will here make a brief analysis. I omit qualifications, sops thrown to Cerberus, phrases to placate Christian prejudice, trappings which serve to hide from author and reader alike the ugly nakedness of the greed-god. But I have surveyed my position. The author enumerates "three motives to human action:†2

Peirce: CP 6.291 Cross-Ref:††

The love of self;

Peirce: CP 6.291 Cross-Ref:††

The love of a limited class having common interests and feelings with one's self;

Peirce: CP 6.291 Cross-Ref:††

The love of mankind at large."

Peirce: CP 6.291 Cross-Ref:††

Remark, at the outset, what obsequious title is bestowed on greed -- "the love of self." Love! The second motive is love. In place of "a limited class" put "certain persons," and you have a fair description. Taking "class" in the old-fashioned sense, a weak kind of love is described. In the sequel, there seems to be some haziness as to the delimitation of this motive. By the love of mankind at large, the author does not mean that deep, subconscious passion that is properly so called; but merely public-spirit, perhaps little more than a fidget about pushing ideas. The author proceeds to a comparative estimate of the worth of these motives. Greed, says he, but using, of course, another word, "is not so great an evil as is commonly supposed. . . . Every man can promote his own interests a great deal more effectively than he can promote any one else's, or than any one else can promote his." Besides, as he remarks on another page, the more miserly a man is, the more good he does. The second motive "is the most dangerous one to which society is exposed." Love is all very pretty: "no higher or purer source of human happiness exists." (Ahem!) But it is a "source of enduring injury," and, in short, should be overruled by something wiser. What is this wiser motive? We shall see.

Peirce: CP 6.291 Cross-Ref:††

As for public spirit, it is rendered nugatory by the "difficulties in the way of its effective operation." For example, it might suggest putting checks upon the fecundity of the poor and the vicious; and "no measure of repression would be too severe," in the case of criminals. The hint is broad. But unfortunately, you cannot induce legislatures to take such measures, owing to the pestiferous "tender sentiments of man

towards man." It thus appears that public-spirit, or Benthamism, is not strong enough to be the effective tutor of love (I am skipping to another page) which must, therefore, be handed over to "the motives which animate men in the pursuit of wealth," in which alone we can confide, and which "are in the highest degree beneficent."†P1 Yes, in the "highest degree" without exception are they beneficent to the being upon whom all their blessings are poured out, namely, the Self, whose "sole object," says the writer, in accumulating wealth is his individual "sustenance and enjoyment." Plainly, the author holds the notion that some other motive might be in a higher degree beneficent, even for the man's self, to be a paradox wanting in good sense. He seeks to gloze and modify his doctrine; but he lets the perspicacious reader see what his animating principle is; and when, holding the opinions I have repeated, he at the same time acknowledges that society could not exist upon a basis of intelligent greed alone, he simply pigeon-holes himself as one of the eclectics of inharmonious opinions. He wants his mammon flavored with a *souçon* of god.

Peirce: CP 6.292 Cross-Ref:††

292. The economists accuse those, to whom the enunciation of their atrocious villainies communicates a thrill of horror, of being *sentimentalists*. It may be so: I willingly confess to having some tincture of sentimentalism in me, God be thanked! Ever since the French Revolution brought this leaning of thought into ill repute -- and not altogether undeservedly, I must admit, true, beautiful, and good as that great movement was -- it has been the tradition to picture sentimentalists as persons incapable of logical thought and unwilling to look facts in the eyes. This tradition may be classed with the French tradition that an Englishman says godam at every second sentence, the English tradition that an American talks about "Britishers," and the American tradition that a Frenchman carries forms of etiquette to an inconvenient extreme; in short, with all those traditions which survive simply because the men who use their eyes and ears are few and far between. Doubtless some excuse there was for all those opinions in days gone by; and sentimentalism, when it was the fashionable amusement to spend one's evenings in a flood of tears over a woeful performance on a candle-litten stage, sometimes made itself a little ridiculous. But what after all is sentimentalism? It is an *ism*, a doctrine, namely, the doctrine that great respect should be paid to the natural judgments of the sensible heart. This is what sentimentalism precisely is; and I entreat the reader to consider whether to condemn it is not of all blasphemies the most degrading. Yet the nineteenth century has steadily contemned it, because it brought about the Reign of Terror. That it did so is true. Still, the whole question is one of *how much*. The Reign of Terror was very bad; but now the Gradgrind banner has been this century long flaunting in the face of heaven, with an insolence to provoke the very skies to scowl and rumble. Soon a flash and quick peal will shake economists quite out of their complacency, too late. The twentieth century, in its latter half, shall surely see the deluge-tempest burst upon the social order -- to clear upon a world as deep in ruin as that greed-philosophy has long plunged it into guilt. No post-thermidorian high jinks then!

Peirce: CP 6.292 Cross-Ref:††

So a miser is a beneficent power in a community, is he? With the same reason precisely, only in a much higher degree, you might pronounce the Wall Street sharp to be a good angel, who takes money from heedless persons not likely to guard it properly, who wrecks feeble enterprises better stopped, and who administers wholesome lessons to unwary scientific men, by passing worthless checks upon them -- as you did, the other day, to me, my millionaire Master in glomery, when you thought you saw your way to using my process without paying for it, and of so

bequeathing to your children something to boast about of their father -- and who by a thousand wiles puts money at the service of intelligent greed, in his own person. Bernard Mandeville, in his *Fable of the Bees*,†1 maintains that private vices of all descriptions are public benefits, and proves it, too, quite as cogently as the economist proves his point concerning the miser. He even argues, with no slight force, that but for vice civilization would never have existed. In the same spirit, it has been strongly maintained and is today widely believed that all acts of charity and benevolence, private and public, go seriously to degrade the human race.

Peirce: CP 6.293 Cross-Ref:††

293. The *Origin of Species* of Darwin merely extends politico-economical views of progress to the entire realm of animal and vegetable life. The vast majority of our contemporary naturalists hold the opinion that the true cause of those exquisite and marvelous adaptations of nature for which, when I was a boy, men used to extol the divine wisdom, is that creatures are so crowded together that those of them that happen to have the slightest advantage force those less pushing into situations unfavorable to multiplication or even kill them before they reach the age of reproduction. Among animals, the mere mechanical individualism is vastly reënfined as a power making for good by the animal's ruthless greed. As Darwin puts it on his title-page, it is the struggle for existence; and he should have added for his motto: Every individual for himself, and the Devil take the hindmost! Jesus, in his sermon on the Mount, expressed a different opinion.

Peirce: CP 6.294 Cross-Ref:††

294. Here, then, is the issue. The gospel of Christ says that progress comes from every individual merging his individuality in sympathy with his neighbors. On the other side, the conviction of the nineteenth century is that progress takes place by virtue of every individual's striving for himself with all his might and trampling his neighbor under foot whenever he gets a chance to do so. This may accurately be called the Gospel of Greed.

Peirce: CP 6.295 Cross-Ref:††

295. Much is to be said on both sides. I have not concealed, I could not conceal, my own passionate predilection. Such a confession will probably shock my scientific brethren. Yet the strong feeling is in itself, I think, an argument of some weight in favor of the agapastic theory of evolution -- so far as it may be presumed to bespeak the normal judgment of the Sensible Heart. Certainly, if it were possible to believe in agapasm without believing it warmly, that fact would be an argument against the truth of the doctrine. At any rate, since the warmth of feeling exists, it should on every account be candidly confessed; especially since it creates a liability to one-sidedness on my part against which it behooves my readers and me to be severally on our guard.

Peirce: CP 6.296 Cross-Ref:††
§2. SECOND THOUGHTS. IRENICA

296. Let us try to define the logical affinities of the different theories of evolution.†1 Natural selection, as conceived by Darwin, is a mode of evolution in which the only positive agent of change in the whole passage from moner to man is fortuitous variation. To secure advance in a definite direction chance has to be

seconded by some action that shall hinder the propagation of some varieties or stimulate that of others. In natural selection, strictly so called, it is the crowding out of the weak. In sexual selection, it is the attraction of beauty, mainly.

Peirce: CP 6.297 Cross-Ref:††

297. The *Origin of Species* was published toward the end of the year 1859. The preceding years since 1846 had been one of the most productive seasons -- or if extended so as to cover the great book we are considering, the most productive period of equal length in the entire history of science from its beginnings until now. The idea that chance begets order, which is one of the corner-stones of modern physics (although Dr. Carus †2 considers it "the weakest point in Mr. Peirce's system ") was at that time put into its clearest light. Quetelet had opened the discussion by his *Letters on the Application of Probabilities to the Moral and Political Sciences*,†3 a work which deeply impressed the best minds of that day, and to which Sir John Herschel †1 had drawn general attention in Great Britain. In 1857, the first volume of Buckle's *History of Civilisation* had created a tremendous sensation, owing to the use he made of this same idea. Meantime, the "statistical method" had, under that very name, been applied with brilliant success to molecular physics. Dr. John Herapath, an English chemist, had in 1847 outlined the kinetical theory of gases in his *Mathematical Physics*; and the interest the theory excited had been refreshed in 1856 by notable memoirs by Clausius †2 and Krönig.†3 In the very summer preceding Darwin's publication, Maxwell had read before the British Association the first and most important of his researches on this subject.†4 The consequence was that the idea that fortuitous events may result in a physical law, and further that this is the way in which those laws which appear to conflict with the principle of the conservation of energy are to be explained, had taken a strong hold upon the minds of all who were abreast of the leaders of thought. By such minds, it was inevitable that the *Origin of Species*, whose teaching was simply the application of the same principle to the explanation of another "non-conservative"†5 action, that of organic development, should be hailed and welcomed. The sublime discovery of the conservation of energy by Helmholtz in 1847,†6 and that of the mechanical theory of heat by Clausius †7 and by Rankine,†8 independently, in 1850, had decidedly overawed all those who might have been inclined to sneer at physical science. Thereafter a belated poet still harping upon "science peddling with the names of things" would fail of his effect. Mechanism was now known to be all, or very nearly so. All this time, utilitarianism -- that improved substitute for the Gospel -- was in its fullest feather; and was a natural ally of an individualistic theory. Dean Mansell's injudicious advocacy had led to mutiny among the bondsmen of Sir William Hamilton, and the nominalism of Mill had profited accordingly; and although the real science that Darwin was leading men to was sure some day to give a death-blow to the sham-science of Mill, yet there were several elements of the Darwinian theory which were sure to charm the followers of Mill.†1 Another thing: anæsthetics had been in use for thirteen years. Already, people's acquaintance with suffering had dropped off very much; and as a consequence, that unlovely hardness, by which our times are so contrasted with those that immediately preceded them, had already set in, and inclined people to relish a ruthless theory. The reader would quite mistake the drift of what I am saying if he were to understand me as wishing to suggest that any of those things (except perhaps Malthus) influenced Darwin himself. What I mean is that his hypothesis, while without dispute one of the most ingenious and pretty ever devised, and while argued with a wealth of knowledge, a strength of logic, a charm of rhetoric, and above all with a certain magnetic genuineness that was almost irresistible, did not appear, at first, at all near to being proved; and to a sober mind its case looks less hopeful now

than it did twenty years ago; but the extraordinarily favorable reception it met with was plainly owing, in large measure, to its ideas being those toward which the age was favorably disposed, especially, because of the encouragement it gave to the greed-philosophy.

Peirce: CP 6.298 Cross-Ref:††

298. Diametrically opposed to evolution by chance are those theories which attribute all progress to an inward necessary principle, or other form of necessity. Many naturalists have thought that if an egg is destined to go through a certain series of embryological transformations, from which it is perfectly certain not to deviate, and if in geological time almost exactly the same forms appear successively, one replacing another in the same order, the strong presumption is that this latter succession was as predeterminate and certain to take place as the former. So, Nägeli,†1 for instance, conceives that it somehow follows from the first law of motion and the peculiar, but unknown, molecular constitution of protoplasm, that forms must complicate themselves more and more. Kölliker †2 makes one form generate another after a certain maturation has been accomplished. Weismann,†3 too, though he calls himself a Darwinian, holds that nothing is due to chance, but that all forms are simple mechanical resultants of the heredity from two parents.†P1 It is very noticeable that all these different sectaries seek to import into their science a mechanical necessity to which the facts that come under their observation do not point. Those geologists who think that the variation of species is due to cataclysmic alterations of climate or of the chemical constitution of the air and water are also making mechanical necessity chief factor of evolution.

Peirce: CP 6.299 Cross-Ref:††

299. Evolution by sporting and evolution by mechanical necessity are conceptions warring against one another. A third method, which supersedes their strife, lies enwrapped in the theory of Lamarck.†4 According to his view, all that distinguishes the highest organic forms from the most rudimentary has been brought about by little hypertrophies or atrophies which have affected individuals early in their lives, and have been transmitted to their offspring. Such a transmission of acquired characters is of the general nature of habit-taking, and this is the representative and derivative within the physiological domain of the law of mind. Its action is essentially dissimilar to that of a physical force; and that is the secret of the repugnance of such necessitarians as Weismann to admitting its existence. The Lamarckians further suppose that, although some of the modifications of form so transmitted were originally due to mechanical causes, yet the chief factors of their first production were the straining of endeavor and the overgrowth superinduced by exercise, together with the opposite actions. Now, endeavor, since it is directed toward an end, is essentially psychical, even though it be sometimes unconscious; and the growth due to exercise, as I argued in my last paper,†1 follows a law of a character quite contrary to that of mechanics.

Peirce: CP 6.300 Cross-Ref:††

300. Lamarckian evolution is thus evolution by the force of habit. -- That sentence slipped off my pen while one of those neighbors whose function in the social cosmos seems to be that of an Interrupter was asking me a question. Of course, it is nonsense. Habit is mere inertia, a resting on one's oars, not a propulsion. Now it is energetic projaculation (lucky there is such a word, or this untried hand might have been put to inventing one) by which in the typical instances of Lamarckian evolution the new elements of form are first created. Habit, however, forces them to take

practical shapes, compatible with the structures they affect, and, in the form of heredity and otherwise, gradually replaces the spontaneous energy that sustains them. Thus, habit plays a double part; it serves to establish the new features, and also to bring them into harmony with the general morphology and function of the animals and plants to which they belong. But if the reader will now kindly give himself the trouble of turning back a page or two, he will see that this account of Lamarckian evolution coincides with the general description of the action of love, to which, I suppose, he yielded his assent.

Peirce: CP 6.301 Cross-Ref:††

301. Remembering that all matter is really mind, remembering, too, the continuity of mind, let us ask what aspect Lamarckian evolution takes on within the domain of consciousness. Direct endeavor can achieve almost nothing. It is as easy by taking thought to add a cubit to one's stature as it is to produce an idea acceptable to any of the Muses by merely straining for it before it is ready to come. We haunt in vain the sacred well and throne of Mnemosyne; the deeper workings of the spirit take place in their own slow way, without our connivance. Let but their bugle sound, and we may then make our effort, sure of an oblation for the altar of whatsoever divinity its savour gratifies. Besides this inward process, there is the operation of the environment, which goes to break up habits destined to be broken up and so to render the mind lively. Everybody knows that the long continuance of a routine of habit makes us lethargic, while a succession of surprises wonderfully brightens the ideas. Where there is a motion, where history is a-making, there is the focus of mental activity, and it has been said that the arts and sciences reside within the temple of Janus, waking when that is open, but slumbering when it is closed. Few psychologists have perceived how fundamental a fact this is. A portion of mind, abundantly commissured to other portions, works almost mechanically. It sinks to a condition of a railway junction. But a portion of mind almost isolated, a spiritual peninsula, or *cul-de-sac*, is like a railway terminus. Now mental commissures are habits. Where they abound, originality is not needed and is not found; but where they are in defect spontaneity is set free. Thus, the first step in the Lamarckian evolution of mind is the putting of sundry thoughts into situations in which they are free to play. As to growth by exercise, I have already shown, in discussing "Man's Glassy Essence,"†1 in last October's *Monist*, what its *modus operandi* must be conceived to be, at least, until a second equally definite hypothesis shall have been offered. Namely, it consists of the flying asunder of molecules, and the reparation of the parts by new matter. It is, thus, a sort of reproduction. It takes place only during exercise, because the activity of protoplasm consists in the molecular disturbance which is its necessary condition. Growth by exercise takes place also in the mind. Indeed, that is what it is to *learn*. But the most perfect illustration is the development of a philosophical idea by being put into practice. The conception which appeared, at first, as unitary splits up into special cases; and into each of these new thought must enter to make a practicable idea. This new thought, however, follows pretty closely the model of the parent conception; and thus a homogeneous development takes place. The parallel between this and the course of molecular occurrences is apparent. Patient attention will be able to trace all these elements in the transaction called learning.

Peirce: CP 6.302 Cross-Ref:††

302. Three modes of evolution have thus been brought before us: evolution by fortuitous variation, evolution by mechanical necessity, and evolution by creative love. We may term them *tychastic* evolution, or *tychasm*, *anancastic* evolution, or *anancasm*, and *agapastic* evolution, or *agapasm*. The doctrines which represent these

as severally of principal importance we may term *tychasticism*, *anancasticism*, and *agapasticism*. On the other hand the mere propositions that absolute chance, mechanical necessity, and the law of love are severally operative in the cosmos may receive the names of *tychism*, *anancism*, and *agapism*.

Peirce: CP 6.303 Cross-Ref:††

303. All three modes of evolution are composed of the same general elements. Agapasm exhibits them the most clearly. The good result is here brought to pass, first, by the bestowal of spontaneous energy by the parent upon the offspring, and, second, by the disposition of the latter to catch the general idea of those about it and thus to subserve the general purpose. In order to express the relation that tychasm and anancasm bear to agapasm let me borrow a word from geometry. An ellipse crossed by a straight line is a sort of cubic curve; for a cubic is a curve which is cut thrice by a straight line; now a straight line might cut the ellipse twice and its associated straight line a third time. Still the ellipse with the straight line across it would not have the characteristics of a cubic. It would have, for instance, no contrary flexure, which no true cubic wants; and it would have two nodes, which no true cubic has. The geometers say that it is a *degenerate* cubic. Just so, tychasm and anancasm are degenerate forms of agapasm.

Peirce: CP 6.304 Cross-Ref:††

304. Men who seek to reconcile the Darwinian idea with Christianity will remark that tychastic evolution, like the agapastic, depends upon a reproductive creation, the forms preserved being those that use the spontaneity conferred upon them in such wise as to be drawn into harmony with their original, quite after the Christian scheme. Very good! This only shows that just as love cannot have a contrary, but must embrace what is most opposed to it, as a degenerate case of it, so tychasm is a kind of agapasm. Only, in the tychastic evolution, progress is solely owing to the distribution of the napkin-hidden talent of the rejected servant among those not rejected, just as ruined gamblers leave their money on the table to make those not yet ruined so much the richer. It makes the felicity of the lambs just the damnation of the goats, transposed to the other side of the equation. In genuine agapasm, on the other hand, advance takes place by virtue of a positive sympathy among the created springing from continuity of mind. This is the idea which tychasticism knows not how to manage.

Peirce: CP 6.305 Cross-Ref:††

305. The anancasticist might here interpose, claiming that the mode of evolution for which he contends agrees with agapasm at the point at which tychasm departs from it. For it makes development go through certain phases, having its inevitable ebbs and flows, yet tending on the whole to a fore-ordained perfection. Bare existence by this its destiny betrays an intrinsic affinity for the good. Herein, it must be admitted, anancasm shows itself to be in a broad acceptance a species of agapasm. Some forms of it might easily be mistaken for the genuine agapasm. The Hegelian philosophy is such an anancasticism. With its revelatory religion, with its synechism (however imperfectly set forth), with its "reflection," the whole idea of the theory is superb, almost sublime. Yet, after all, living freedom is practically omitted from its method. The whole movement is that of a vast engine, impelled by a *vis a tergo*, with a blind and mysterious fate of arriving at a lofty goal. I mean that such an engine it *would* be, if it really worked; but in point of fact, it is a Keely motor.†1 Grant that it really acts as it professes to act, and there is nothing to do but accept the philosophy. But never was there seen such an example of a long chain of reasoning --

shall I say with a flaw in every link? -- no, with every link a handful of sand, squeezed into shape in a dream. Or say, it is a pasteboard model of a philosophy that in reality does not exist. If we use the one precious thing it contains, the idea of it, introducing the tychism which the arbitrariness of its every step suggests, and make that the support of a vital freedom which is the breath of the spirit of love, we may be able to produce that genuine agapasticism at which Hegel was aiming.

Peirce: CP 6.306 Cross-Ref:††
§3. A THIRD ASPECT. DISCRIMINATION

306. In the very nature of things, the line of demarcation between the three modes of evolution is not perfectly sharp. That does not prevent its being quite real; perhaps it is rather a mark of its reality. There is in the nature of things no sharp line of demarcation between the three fundamental colors, red, green, and violet. But for all that they are really different. The main question is whether three radically different evolutionary elements have been operative; and the second question is what are the most striking characteristics of whatever elements have been operative.

Peirce: CP 6.307 Cross-Ref:††

307. I propose to devote a few pages to a very slight examination of these questions in their relation to the historical development of human thought. I first formulate for the reader's convenience the briefest possible definitions of the three conceivable modes of development of thought, distinguishing also two varieties of anancasm and three of agapasm. The tychastic development of thought, then, will consist in slight departures from habitual ideas in different directions indifferently, quite purposeless and quite unconstrained whether by outward circumstances or by force of logic, these new departures being followed by unforeseen results which tend to fix some of them as habits more than others. The anancastic development of thought will consist of new ideas adopted without foreseeing whither they tend, but having a character determined by causes either external to the mind, such as changed circumstances of life, or internal to the mind as logical developments of ideas already accepted, such as generalizations. The agapastic development of thought is the adoption of certain mental tendencies, not altogether heedlessly, as in tychasm, nor quite blindly by the mere force of circumstances or of logic, as in anancasm, but by an immediate attraction for the idea itself, whose nature is divined before the mind possesses it, by the power of sympathy, that is, by virtue of the continuity of mind; and this mental tendency may be of three varieties, as follows. First, it may affect a whole people or community in its collective personality, and be thence communicated to such individuals as are in powerfully sympathetic connection with the collective people, although they may be intellectually incapable of attaining the idea by their private understandings or even perhaps of consciously apprehending it. Second, it may affect a private person directly, yet so that he is only enabled to apprehend the idea, or to appreciate its attractiveness, by virtue of his sympathy with his neighbors, under the influence of a striking experience or development of thought. The conversion of St. Paul may be taken as an example of what is meant. Third, it may affect an individual, independently of his human affections, by virtue of an attraction it exercises upon his mind, even before he has comprehended it. This is the

phenomenon which has been well called the *divination* of genius; for it is due to the continuity between the man's mind and the Most High.

Peirce: CP 6.308 Cross-Ref:††

308. Let us next consider by means of what tests we can discriminate between these different categories of evolution. No absolute criterion is possible in the nature of things, since in the nature of things there is no sharp line of demarcation between the different classes. Nevertheless, quantitative symptoms may be found by which a sagacious and sympathetic judge of human nature may be able to estimate the approximate proportions in which the different kinds of influence are commingled.

Peirce: CP 6.309 Cross-Ref:††

309. So far as the historical evolution of human thought has been tyochastic, it should have proceeded by insensible or minute steps; for such is the nature of chances when so multiplied as to show phenomena of regularity. For example, assume that of the native-born white adult males of the United States in 1880, one-fourth part were below 5 feet 4 inches in stature and one-fourth part above 5 feet 8 inches. Then by the principles of probability, among the whole population, we should expect

216 under 4 feet 6 inches 216 above 6 feet 6 inches

48 under 4 feet 5 inches 48 above 6 feet 7 inches

9 under 4 feet 5 inches 9 above 6 feet 8 inches

less than 2 under 4 feet 3 inches less than 2 above 6 feet 9 inches

Peirce: CP 6.309 Cross-Ref:††

I set down these figures to show how insignificantly few are the cases in which anything very far out of the common run presents itself by chance. Though the stature of only every second man is included within the four inches between 5 feet 4 inches and 5 feet 8 inches, yet if this interval be extended by thrice four inches above and below, it will embrace all our 8 millions odd of native-born adult white males (of 1880), except only 9 taller and 9 shorter.

Peirce: CP 6.310 Cross-Ref:††

310. The test of minute variation, if *not* satisfied, absolutely negatives tychasm. If it *is* satisfied, we shall find that it negatives anacasm but not agapasm. We want a positive test, satisfied by tychasm, only. Now wherever we find men's thought taking by imperceptible degrees a turn contrary to the purposes which animate them, in spite of their highest impulses, there, we may safely conclude, there has been a tyochastic action.

Peirce: CP 6.311 Cross-Ref:††

311. Students of the history of mind there be of an erudition to fill an imperfect scholar like me with envy adulcorated by joyous admiration, who maintain that ideas when just started are and can be little more than freaks, since they cannot yet have been critically examined, and further that everywhere and at all times progress has been so gradual that it is difficult to make out distinctly what original step any given man has taken. It would follow that tychasm has been the sole method of intellectual development. I have to confess I cannot read history so; I cannot help

thinking that while tychasm has sometimes been operative, at others great steps covering nearly the same ground and made by different men independently have been mistaken for a succession of small steps, and further that students have been reluctant to admit a real entitative "spirit" of an age or of a people, under the mistaken and unscrutinized impression that they should thus be opening the door to wild and unnatural hypotheses. I find, on the contrary, that, however it may be with the education of individual minds, the historical development of thought has seldom been of a tychastic nature, and exclusively in backward and barbarizing movements. I desire to speak with the extreme modesty which befits a student of logic who is required to survey so very wide a field of human thought that he can cover it only by a reconnaissance, to which only the greatest skill and most adroit methods can impart any value at all; but, after all, I can only express my own opinions and not those of anybody else; and in my humble judgment, the largest example of tychasm is afforded by the history of Christianity, from about its establishment by Constantine to, say, the time of the Irish monasteries, an era or eon of about 500 years. Undoubtedly the external circumstance, which more than all others at first inclined men to accept Christianity in its loveliness and tenderness, was the fearful extent to which society was broken up into units by the unmitigated greed and hard-heartedness into which the Romans had seduced the world. And yet it was that very same fact, more than any other external circumstance, that fostered that bitterness against the wicked world of which the primitive gospel of Mark contains not a single trace. At least, I do not detect it in the remark about the blasphemy against the Holy Ghost,†1 where nothing is said about vengeance, nor even in that speech †2 where the closing lines of Isaiah †3 are quoted, about the worm and the fire that feed upon the "carcasses of the men that have transgressed against me." But little by little the bitterness increases until, in the last book of the New Testament, its poor distracted author represents that all the time Christ was talking about having come to save the world, the secret design was to catch the entire human race, with the exception of a paltry 144,000, and souse them all in a brimstone lake, and as the smoke of their torment went up forever and ever, to turn and remark, "There is no curse any more." Would it be an insensible smirk or a fiendish grin that should accompany such an utterance? I wish I could believe St. John did not write it; but it is his gospel which tells about the "resurrection unto condemnation" -- that is of men's being resuscitated just for the sake of torturing them -- and at any rate, the Revelation is a very ancient composition. One can understand that the early Christians were like men trying with all their might to climb a steep declivity of smooth wet clay; the deepest and truest element of their life, animating both heart and head, was universal love; but they were continually, and against their wills, slipping into a party spirit, every slip serving as a precedent, in a fashion but too familiar to every man. This party feeling insensibly grew until by about A.D. 330 the luster of the pristine integrity that in St. Mark reflects the white spirit of light was so far tarnished that Eusebius (the Jared Sparks of that day), in the preface to his History, could announce his intention of exaggerating everything that tended to the glory of the church and of suppressing whatever might disgrace it.†1 His Latin contemporary Lactantius †2 is worse still; and so the darkling went on increasing until before the end of the century the great library of Alexandria was destroyed by Theophilus,†P1 until Gregory the Great, two centuries later, burnt the great library of Rome,†3 proclaiming that "Ignorance is the mother of devotion" (which is true, just as oppression and injustice is the mother of spirituality), until a sober description of the state of the church would be a thing our not too nice newspapers would treat as "unfit for publication." All this movement is shown by the application of the test given above to have been tychastic. Another very much like it on a small scale, only a

hundred times swifter, for the study of which there are documents by the library-full, is to be found in the history of the French Revolution.

Peirce: CP 6.312 Cross-Ref:††

312. Anancastic evolution advances by successive strides with pauses between. The reason is that, in this process, a habit of thought, having been overthrown, is supplanted by the next strongest. Now this next strongest is sure to be widely disparate from the first, and as often as not is its direct contrary. It reminds one of our old rule of making the second candidate vice-president. This character, therefore, clearly distinguishes anancasm from tychasm. The character which distinguishes it from agapasm is its purposelessness. But external and internal anancasm have to be examined separately. Development under the pressure of external circumstances, or cataclasmic evolution, is in most cases unmistakable enough. It has numberless degrees of intensity, from the brute force, the plain war, which has more than once turned the current of the world's thought, down to the hard fact of evidence, or what has been taken for it, which has been known to convince men by hordes. The only hesitation that can subsist in the presence of such a history is a quantitative one. Never are external influences the only ones which affect the mind, and therefore it must be a matter of judgment for which it would scarcely be worth while to attempt to set rules, whether a given movement is to be regarded as principally governed from without or not. In the rise of medieval thought, I mean scholasticism and the synchronistic art developments, undoubtedly the crusades and the discovery of the writings of Aristotle were powerful influences. The development of scholasticism from Roscellin to Albertus Magnus closely follows the successive steps in the knowledge of Aristotle. Prantl thinks that that is the whole story,†1 and few men have thumbed more books than Carl Prantl. He has done good solid work, notwithstanding his slap-dash judgments. But we shall never make so much as a good beginning of comprehending scholasticism until the whole has been systematically explored and digested by a company of students regularly organized and held under rule for that purpose. But as for the period we are now specially considering, that which synchronized the Romanesque architecture, the literature is easily mastered. It does not quite justify Prantl's dicta as to the slavish dependence of these authors upon their authorities. Moreover, they kept a definite purpose steadily before their minds, throughout all their studies. I am, therefore, unable to offer this period of scholasticism as an example of pure external anancasm, which seems to be the fluorine of the intellectual elements. Perhaps the recent Japanese reception of western ideas is the purest instance of it in history. Yet in combination with other elements, nothing is commoner. If the development of ideas under the influence of the study of external facts be considered as external anancasm -- it is on the border between the external and the internal forms -- it is, of course, the principal thing in modern learning. But Whewell, whose masterly comprehension of the history of science critics have been too ignorant properly to appreciate, clearly shows that it is far from being the overwhelmingly preponderant influence, even there.

Peirce: CP 6.313 Cross-Ref:††

313. Internal anancasm, or logical groping, which advances upon a predestined line without being able to foresee whither it is to be carried nor to steer its course, this is the rule of development of philosophy. Hegel first made the world understand this; and he seeks to make logic not merely the subjective guide and monitor of thought, which was all it had been ambitioning before, but to be the very mainspring of thinking, and not merely of individual thinking but of discussion, of the history of the development of thought, of all history, of all development. This

involves a positive, clearly demonstrable error. Let the logic in question be of whatever kind it may, a logic of necessary inference or a logic of probable inference (the theory might perhaps be shaped to fit either), in any case it supposes that logic is sufficient of itself to determine what conclusion follows from given premisses; for unless it will do so much, it will not suffice to explain why an individual train of reasoning should take just the course it does take, to say nothing of other kinds of development. It thus supposes that from given premisses, only one conclusion can logically be drawn, and that there is no scope at all for free choice. That from given premisses only one conclusion can logically be drawn is one of the false notions which have come from logicians' confining their attention to that Nantucket of thought, the logic of non-relative terms. In the logic of relatives, it does not hold good.†1

Peirce: CP 6.314 Cross-Ref:††

314. One remark occurs to me. If the evolution of history is in considerable part of the nature of internal anacasm, it resembles the development of individual men; and just as thirty-three years is a rough but natural unit of time for individuals, being the average age at which man has issue, so there should be an approximate period at the end of which one great historical movement ought to be likely to be supplanted by another. Let us see if we can make out anything of the kind. Take the governmental development of Rome as being sufficiently long and set down the principal dates.

B.C. 753, Foundation of Rome.

B.C. 510, Expulsion of the Tarquins.

B.C. 27, Octavius assumes title Augustus.

A.D. 476, End of Western Empire.

A.D. 962, Holy Roman Empire.

A.D. 1453, Fall of Constantinople.

The last event was one of the most significant in history, especially for Italy. The intervals are 243, 483, 502, 486 491, years. All are rather curiously near equal, except the first which is half the others. Successive reigns of kings would not commonly be so near equal. Let us set down a few dates in the history of thought.

B.C. 585, Eclipse of Thales. Beginning of Greek philosophy.

A.D. 30, The crucifixion.

A.D. 529, Closing of Athenian schools. End of Greek philosophy. A.D. 1125, (Approximate) Rise of the Universities of Bologna and Paris.

A.D. 1543, Publication of the *De Revolutionibus* of Copernicus. Beginning of Modern Science.

The intervals are 615, 499, 596, 418 years. In the history of metaphysics, we may take the following:

B.C. 322, Death of Aristotle.

A.D. 1274, Death of Aquinas.

A.D. 1804, Death of Kant.

The intervals are 1595 and 530 years. The former is about thrice the latter.

Peirce: CP 6.314 Cross-Ref:††

From these figures, no conclusion can fairly be drawn. At the same time, they suggest that perhaps there may be a rough natural era of about 500 years. Should there be any independent evidence of this, the intervals noticed may gain some significance.

Peirce: CP 6.315 Cross-Ref:††

315. The agapastic development of thought should, if it exists, be distinguished by its purposive character, this purpose being the development of an idea. We should have a direct agapic or sympathetic comprehension and recognition of it by virtue of the continuity of thought. I here take it for granted that such continuity of thought has been sufficiently proved by the arguments used in my paper on the "Law of Mind" in *The Monist* of last July [Chapter 5]. Even if those arguments are not quite convincing in themselves, yet if they are reënforced by an apparent agapasm in the history of thought, the two propositions will lend one another mutual aid. The reader will, I trust, be too well grounded in logic to mistake such mutual support for a vicious circle in reasoning. If it could be shown directly that there is such an entity as the "spirit of an age" or of a people, and that mere individual intelligence will not account for all the phenomena, this would be proof enough at once of agapasticism and of synechism. I must acknowledge that I am unable to produce a cogent demonstration of this; but I am, I believe, able to adduce such arguments as will serve to confirm those which have been drawn from other facts. I believe that all the greatest achievements of mind have been beyond the powers of unaided individuals; and I find, apart from the support this opinion receives from synechistic considerations, and from the purposive character of many great movements, direct reason for so thinking in the sublimity of the ideas and in their occurring simultaneously and independently to a number of individuals of no extraordinary general powers. The pointed Gothic architecture in several of its developments appears to me to be of such a character. All attempts to imitate it by modern architects of the greatest learning and genius appear flat and tame, and are felt by their authors to be so. Yet at the time the style was living, there was quite an abundance of men capable of producing works of this kind of gigantic sublimity and power. In more than one case, extant documents show that the cathedral chapters, in the selection of architects, treated high artistic genius as a secondary consideration, as if there were no lack of persons able to supply that; and the results justify their confidence. Were individuals in general, then, in those ages possessed of such lofty natures and high intellect? Such an opinion would break down under the first examination.

Peirce: CP 6.316 Cross-Ref:††

316. How many times have men now in middle life seen great discoveries made independently and almost simultaneously! The first instance I remember was the prediction of a planet exterior to Uranus by Leverrier †1 and Adams.†2 One hardly knows to whom the principle of the conservation of energy ought to be attributed, although it may reasonably be considered as the greatest discovery science has ever made. The mechanical theory of heat was set forth by Rankine †3 and by Clausius †4 during the same month of February, 1850; and there are eminent men who attribute this great step to Thomson.†P1 The kinetical theory of gases, after being started by John Bernoulli †1 and long buried in oblivion, was reinvented and applied to the explanation not merely of the laws of Boyle, Charles, and Avogadro, but also of diffusion and viscosity, by at least three modern physicists separately. It is well known that the doctrine of natural selection was presented by Wallace and by Darwin at the same meeting of the British Association; and Darwin in his "Historical Sketch" prefixed to the later editions of his book shows that both were anticipated by obscure forerunners. The method of spectrum analysis was claimed for Swan as well as for Kirchhoff, and there were others who perhaps had still better claims. The authorship of the Periodical Law of the Chemical Elements is disputed between a Russian, a German, and an Englishman;†2 although there is no room for doubt that the principal merit belongs to the first. These are nearly all the greatest discoveries of our times. It is the same with the inventions. It may not be surprising that the telegraph should have been independently made by several inventors, because it was an easy corollary from scientific facts well made out before. But it was not so with the telephone and other inventions. Ether, the first anæsthetic, was introduced independently by three different New England physicians †3. Now ether had been a common article for a century. It had been in one of the pharmacopoeias three centuries before. It is quite incredible that its anaesthetic property should not have been known; it was known. It had probably passed from mouth to ear as a secret from the days of Basil Valentine; but for long it had been a secret of the Punchinello kind. In New England, for many years, boys had used it for amusement. Why then had it not been put to its serious use? No reason can be given, except that the motive to do so was not strong enough. The motives to doing so could only have been desire for gain and philanthropy. About 1846, the date of the introduction, philanthropy was undoubtedly in an unusually active condition. That sensibility, or sentimentalism, which had been introduced in the previous century, had undergone a ripening process, in consequence of which, though now less intense than it had previously been, it was more likely to influence unreflecting people than it had ever been. All three of the ether-claimants had probably been influenced by the desire for gain; but nevertheless they were certainly not insensible to the agapic influences.

Peirce: CP 6.317 Cross-Ref:††

317. I doubt if any of the great discoveries ought, properly, to be considered as altogether individual achievements; and I think many will share this doubt. Yet, if not, what an argument for the continuity of mind, and for agapasticism is here! I do not wish to be very strenuous. If thinkers will only be persuaded to lay aside their prejudices and apply themselves to studying the evidences of this doctrine, I shall be fully content to await the final decision.

NOTES ON METAPHYSICS

§1. RELATIONS AND RELATIONSHIPS †1

318. I have, since 1870, written much about the logic of relations. In those writings, I have *usually* restricted the terms "relations" and "relationships" to *existential* relations and relationships. By a relationship I understand the conception of a fact about a set of things abstracted from the representation of the things themselves or, in other words, a predicate which requires more than one subject to complete a proposition, or conception of a fact. A "relation" only differs from a "relationship" in that one of the subjects is regarded as being taken account of first, and is usually called the *subject nominative*, while the others are called the *direct* and *indirect objects*. †2 In other words a relation is a predicate requiring one subject nominative and one or more objects in a definite sequence. In my earlier papers [in Volume 3] I use the conception of relation chiefly; in my later ones that of relationship. The difference is little more than trifling. An *existential* relation or relationship is distinguished from others by two marks. †3 In the first place, its different subjects all belong to one universe; which distinguishes it very strikingly from such relations as that which subsists between a thing and its qualities, and that which subsists between portions of matter and the form into which they are built; as for example between the cells of a living body and the whole body, and often times between the different singulars of a plural and the plural itself. In the second place, an existential relation or relationship differs from some other relations and relationships in a respect which may be described in two ways, according as we employ collective or distributive forms of expression and thought. Speaking collectively, the one logical universe, to which all the correlates of an existential relationship belong, is ultimately composed of *units*, or subjects, none of which is in any sense separable into parts that are members of the same universe. For example, no relation between different lapses of time -- say, between the age of Agamemnon and that of Homer -- can be an existential relation, if we conceive every lapse of time to be made up of lapses of time, so that there are no indivisible units of time. To state the same thing distributively, every correlate of an existential relation is a single object which may be indefinite, or may be distributed; †1 that is, may be chosen from a class by the interpreter of the assertion of which the relation or relationship is the predicate, or may be designated by a proper name, but in itself, though in some guise or under some mask, it can always be perceived, yet never can it be unmistakably identified by any sign whatever, without collateral observation. Far less can it be defined. It is *existent*, in that its being does not consist in any *qualities*, but in its effects -- in its actually acting and being acted on, so long as this action and suffering endures. Those who experience its effects perceive and know it in that action; and just that constitutes its very being. It is not in perceiving its qualities that they know it, but in hefting its insistency then and there, which Duns called its *haecceitas* -- or, if he didn't, it was this that he was groping after. However, let me not lapse further into metaphysics just

now.

Peirce: CP 6.319 Cross-Ref:††

319. My reasons for mostly limiting the scope of my logical studies of relations to the existential class were, firstly, that these are very tangible and are logically tractable; secondly that the great body of other sorts of relations differ from these merely in being indeterminate in some respects in which existential relations or some species of these are determinate, so that the logical theory of these virtually puts the student into possession of the logical theory of all but a very few recondite relations; thirdly, that when one takes up a virgin subject with a view to clearing the ground and erecting upon it a scientific structure, it is necessary to begin with some part of the work; and fourthly and finally, that to my perhaps dull apprehension it seemed that no sane mind could, after mature deliberation, make choice of another part of the whole task to begin with than that which I chose. I may add that it does not convince me that that seeming was illusory, that those who think it was so, instead of pointing out some better way of treating the problem, have been led to despair of the possibility of erecting any logic of relations at all. But while they have been occupying themselves with these doleful prognostications, I have taken hold of the work of erection itself and have brought it to a promising stage of advance. But just as there are many fogies nowadays -- old and young -- who with idle conservatism dispute the value of my work, so, unless the whole congregation of logicians experiences a regeneration, I expect the day will come when another generation of old and young fogies will be equally indisposed to admit that there is any corner of the whole field that I have not turned up, and put into the right condition. Yet I have faithfully tried to do my share in putting an end to all such unscientific attitudes among logicians, and am confident that the new blood that has been brought into our house is going to insure its modicum of scientific health to the logical stock of the next generation.

Peirce: CP 6.320 Cross-Ref:††

320. The intellectual life of thought resides in its forms -- its patterns. Now there is one pattern which must always be supreme in thought, because it is essentially the pattern of reasoning itself. It ought to be called the Aristotelian pattern, because it was first formally emphasized in Aristotle's definition of universal predication -- the *dictum de omni*, or "universal predication."†1 That definition runs thus: "We say that a term, *P*, is predicated universally, when there is nothing of which its subject is predicated of which the term *P* is not likewise predicated."†2 . . . The negative of an Aristotelian relationship is of a different pattern; and since Reasoning is essentially critical, it is almost as important to consider what does not follow as what does follow, consequently, the pattern of the negative of an Aristotelian -- or of a *neg-Aristotelian*, as we may call it -- has an importance second only to that of the Aristotelian pattern. This neg-Aristotelian pattern will be sufficiently illustrated in a single example: "A, even though he does not love *B*, may love something unloved by *B*."†1 Of special importance is such a combination of the Aristotelian and neg-Aristotelian patterns as not to produce a self-contradiction. Such is: "A loves whatever *B* may love and something besides."†2 This is the pattern of any *transitive difference*; that is, of any transitive relation in which nothing can stand to itself. It may be called the *alio-transitive* pattern. . . . The *alio-transitive* pattern may also be called the *linear-pattern*; since among points lying upon a line, if one, *A*, is further advanced -- say, for example, further along the line from one end of it -- than another, *B*, it will be further advanced than any point than which *B* may be further advanced, and is also further advanced than some (perhaps unactualized, *i.e.* unmarked) point

than which **B** is not further advanced. This pattern is the logically simplest of any that is at once needful and sufficient as a basis for exact reasoning, and consequently is involved in all exact reasoning. This statement, faulty as it is in being vague, and inevident, is nevertheless a correct statement of the reason of the supreme importance of number.

Peirce: CP 6.321 Cross-Ref:††

321. That reasoning is of a triadic constitution has generally been perceived since Aristotle, though not generally quite definitely and accurately. In some external particulars this character has even been exaggerated by Kant and his school, who not only defines reasoning as "judgment by means of a mediate attribute," thus introducing the triadic character correctly enough, but also goes so far as to insist that a [Vernunft] *Schluss* must have two premisses;†3 giving a different title to what usually goes by the name of "immediate inference"; which is a purely verbal distinction. But even an immediate inference -- if it really be an inference, and not a mere rewording, like, "No man is mortal," therefore "No mortal is man"†4 -- involves a triadic relation. Take, for example, the inference from, "A certain woman is adored by all catholic men," and therefore, "Every catholic man adores some woman or other."†5 In order to show the structure of this reasoning, it should be stated in some such fashion as this: "I could name such a woman that you could not find a catholic man that did not adore her," and therefore, "Specify what catholic man you will and I can mention some woman whom he adores." This turns on a relation between, first, what you can designate, secondly, what I can designate, and thirdly, the result. If you choose to say that there are *more* than three correlates, that is a matter of indifference; for every relationship of more than three correlates is equivalent to a logical composite of two or more triadic relationships; but a triadic relationship cannot be built up from dyadic relationships.†1 Whoever thinks it can be so composed has overlooked the fact that *composition* is itself a triadic relationship, between the two (or more) components and the composite whole.

Peirce: CP 6.322 Cross-Ref:††

322. For forty years, that is, since the beginning of the year 1867, I have been constantly on the alert to find a *genuine* triadic relation -- that is, one that does not consist in a mere collocation of dyadic relations, or the negative of such, etc. (I prefer not to attempt a perfectly definite definition) -- which is not either an intellectual relation or a relation concerned with the less comprehensible phenomena of life. I have not met with one which could not reasonably be supposed to belong to one or other of these two classes. As a case as nearly brute and inorganic as any, I may mention the form of relationship involved in any screw-form which is definitely of the right-hand, or occidental, mode, or is definitely of the Japanese, or left-handed, mode. Such a relation exists in every carbon-atom whose four valencies are saturated by combination with four atoms of as many different kinds. But where the action of chance determines whether the screw be a right-handed or a left-handed one, the two forms will, in the long run, be produced in equal proportions, and the general result will not be definitely, or decisively, of either kind. We know no case of a definitely right-handed or left-handed screw-phenomenon, where the decision is not certainly due to the intervention of a definitely one-sided screw in the conditions of that decision, except in cases where the choice of a living being determines it; as when Pasteur picked out under the microscope the two kinds of crystals of a tartrate, and shoved those of one kind to the right and those of the other kind to the left.†1 We do not know the mechanism of such choice, and cannot say whether it be determined by an antecedent separation of left-handed screws from right-handed screws or not. No

doubt, all that chance is competent to destroy, it may, once in a long, long time, produce; but it is a question whether absolute chance -- pure tychism -- ought not to be regarded as a product of freedom, and therefore of life, not necessarily physiological. It could not be caused, apparently, by the inorganic action of dynamical law. For the only way in which the laws of dynamics involve triadic relations is by their reference to second differentials of positions.†2 But though a second differential generally involves a triadic relation, yet owing to the law of the conservation of energy, which has been sufficiently proved for purely inorganic phenomena, the dynamic laws for such phenomena are expressible in terms of first differentials. It is, therefore, a non-genuine, or, as I phrase it, a "degenerate" form of triadic relationship †3 which is involved in such case. In short, the problem of how genuine triadic relationships first arose in the world is a better, because more definite, formulation of the problem of how life first came about; and no explanation has ever been offered except that of pure chance, which we must suspect to be no explanation, owing to the suspicion that pure chance may itself be a vital phenomenon. In that case, life in the physiological sense would be due to life in the metaphysical sense. Of course, the fact that a given individual has been persuaded of the truth of a proposition is the very slenderest possible argument for its truth; nevertheless, the fact that I, a person of the strongest possible physiocistic prejudices, should, as the result of forty years of questionings, have been brought to the deep conviction that there is some essentially and irreducibly other element in the universe than pure dynamism may have sufficient interest to excuse my devoting a single sentence to its expression. For you may be sure that I had reasons that withstood severe, not to say hostile criticism; and if I live to do it, I shall embody them in a volume.†4

Peirce: CP 6.323 Cross-Ref:††

323. A tetradic, pentadic, etc. relationship is of no higher nature than a triadic relationship; in the sense that it consists of triadic relationships and is constituted of them. But a triadic relationship is of an essentially higher nature than a dyadic relationship, in the sense that while it involves three dyadic relationships, it is not constituted by them. If **A** gives **B** to **C**, he, **A**, acts upon **B**, and acts upon **C**; and **B** acts upon **C**. Perhaps, for example, he lays down **B**, whereupon **C** takes **B** up, and is benefited by **A**. But these three acts might take place without that essentially intellectual operation of transferring the legal right of possession, which axiomatically cannot be brought about by any pure dyadic relationships whatsoever. Just as much, but no more, is a dyadic relation -- or the sort of fact expressed by a two-subject predicate -- of a higher nature than any fact expressed by a one-subject predicate, such as "is blue." For the two-subject fact involves two one-subject facts, but is not constituted by them. If **A** acts upon **B** in any way, something analogous to a strain, or stress, takes place within **A**, and something of the same sort in **B**; but these two happenings might take place irrespectively of one another, without any action of **A** on **B**. In that sense the action is a higher sort of fact than the one-subject stress. A man cannot conceive of a one-subject fact otherwise than as more or less vaguely analogous to a feeling of his own. He cannot conceive of a two-subject fact otherwise than as analogous to an action of his own. A three-subject fact is comprehensible and is analogous to an utterance, a speech, a thought.

Peirce: CP 6.324 Cross-Ref:††

324. It is astonishing how human minds seem naturally to pervert the interrelations of these three categories of facts. The triadic fact takes place in thought. I do not say in anybody's thinking, but in pure abstract thought; while the dyadic fact is existential. With that comparison plainly before them, our minds perversely regard

the dyadic fact as superior in reality to the "mere" relation of thought which is the triadic fact. We forget that thinking implies existential action, though it does not consist in that; or if we remember that thought implies the action of forces upon a brain or something like it, we still more perversely regard that as lowering the dignity of thought, and as making it a "mere" existential event; whereas the truth is just the opposite. In that thought requires existential acting, and further requires something else beside that, it ought to be plain enough that it exceeds the existential acting. The ruler of a nation depends upon his cook and his secretaries. That does not place him lower than they, but higher. Thought is higher than brute fact in much the same way that a statesman is higher than his secretaries: namely, it needs the existential facts, but regulates them. It is only imitation-thought to which the adjective "mere" is appropriate. . . .

Peirce: CP 6.325 Cross-Ref:††
§2. MATHEMATICAL AND REAL TIME

325. If the pure mathematician speaks of "time," he by no means refers to the time of which we have experience, but to an arbitrarily imagined object whose characters are analogous to those of experiential time, so far as the characters of the latter are known. The mathematician's time is an arbitrarily supposed object in some respects analogous (this I insert to give a temporary support, or scaffolding, to the reader's conception) to the instantaneous condition of the water of some river whose water should be perfectly homogeneous and not composed of molecules, supposing however that we quite disregard the dimensions of depth and breadth of the river. But understand me: I mean this comparison with the river merely to afford a temporary support to your mind, Reader; a scaffolding that shall be convenient until the mathematical concept of Time has been erected in your field of thought; but being no part of that concept, it is afterward to be broken up and thrown away, unless its fragments should be serviceable in erecting some other concept. Analogies are never perfect, for an analogy that should be made perfect would be more than an analogy. The most important of the respects in which the ideal river differs from Time is that the former is the idea of a form that is imagined to exist, while the hypothesis of Time (for however closely it may agree with something in experience, which experience has indeed occasioned the hypothesis, nevertheless the mathematician's "time" is a purely arbitrary hypothesis, and makes no appeal to any evidences), the supposed "Time," far from being imagined to be anything existent, that is, anything that can react with the other existents, is imagined to be a mere *possibleness* -- in forming which word I assume that "possible" is taken, not as relative to this or that condition, but as absolutely supposable in consistent thought. This point of contrast between the river and Time entrains consequences that it would be obviously fatal to sound thought to overlook. The chiefest of these, to my thinking, are the two I am going to mention. I will remark that although it is a Heraclitan river -- is that river I am supposing to be supposed (*i.e.*, one of those rivers that one can only cross once, because it is the water in its instantaneous place) -- yet this does not prevent the recognition of its relation to other Heraclitan states of the same water; so that, notwithstanding that a quite instantaneous state composes this river, every drop has its temporal relation to the state of the water at another indefinitely near instant; so that while instantaneous, every part of it has a definite velocity. Now then, I might imagine that this flowing water comes into existence at a certain section of the stream,

and is annihilated at another section, or I might, if I chose, imagine it to be sourceless and mouthless, an unlimited Heraclitan river. I might imagine that these creations and annihilations were many along the river. I might even imagine that the water never comes into complete existence, but [is] instantly annihilated at the very instant of its instantaneous creation, [so] that it consists of a series of lengthless cross sections; and so [that] the whole would have but an inchoate existence. All that is easily supposable in the case of the river, because the river is imagined to be existent, that is, to have a mode of being quite independent of any rationality, but consisting only in certain brute action. This gives room for supposing that a predicate is neither wholly true of it nor wholly false, nor has the limit between the true and the false parts at any definitely described cross sections. We can suppose those limits to be at some cross sections without saying what ones, nor even saying that it would be possible exactly to define them. But with mathematical Time all this is quite different, owing to its being a possibility consisting in freedom from self-contradiction, without any supposed experience forcing ideas upon the mind from the external world.

Peirce: CP 6.326 Cross-Ref:††

326. If on a Monday an idea be possible, in the sense of involving no contradiction within itself regardless of all mere *circumstances*, then it will be possible on Tuesdays, on Wednesdays, and on Fridays; in short it will be possible forever and ever, unless the idea of the circumstance should come into *definite* rational contradiction to the idea in question.†1 Consequently, mathematical Time cannot have an arbitrary beginning nor end. For it is but a possibility; and what is possible at all is possible without limit, unless there be some kind of a limit which comes into *definite* rational contradiction with the idea of Time. For the *indefinite* inhabits only the realm of experience, being, or having been, forced upon us brutally and without reason. That a certain idea may be existentially impossible may be a brute fact; but that it should be rationally impossible requires a definite reason. In like manner, if there be a "lapse," or determination of time, that is at once "after" one lapse **B**, and "before" another **Y**, then (but for a restriction which I shall insert before I reach the end of this statement) there must be any multitude you please of lapses that are at once "after" **B** and "before" **Y**; and since I long ago proved (*Monist*, Vol. 7, [3.548f]) that there is no maximum multitude of objects *distinct from one another* it will follow that in Time *itself* there is no such thing as an absolute *instant*, or absolutely definite before-and-afterness relatively to all other instants, and that an instant can only come about as the consequence of some brute existential fact related to time; but all this is subject to the following restriction: if it be true that some multitude -- say, for example, the second abnumerable multitude, which is the multitude of all possible different collections of irrational quantities -- is of such a nature that the idea of arranging the singulars of any plural of that multitude in a linear series would involve a self-contradiction, then the consequence falls to the ground, and it only remains that between **B** and **Y** there must be a plural of lapses whose multitude is the largest that is capable of linear arrangement.†P1 . . .

Peirce: CP 6.327 Cross-Ref:††

§3. EXTERNALITY AND REALITY †1

327. What is meant by calling anything *real*? I can tell you in what sense I always use the word. According to my use of it, there is a certain resemblance

between the *Real* and the *External* which renders the discrimination of each from the other important for right reason. Any object whose attributes, *i.e.* all that may truly be predicated, or asserted, of it, will, and always would, remain exactly what they are, unchanged, though you or I or any man or men should think or should have thought as variously as you please, I term *external*, in contradistinction to *mental*. For example, a dream is mental, because it depends upon what passed in the thoughts of the dreamer whether it be true that the dream was of a dog or was of the Round Table of King Arthur or of anything else. On the other hand, the colors of objects of human experience and in particular the contrast between the color of the petals of a Jaqueminot rose and that of the leaves of the bush, although it is relative to the sense of sight, is not *mental*, in my sense of that word. It is true that all colors are relative to the sense of sight. Yet there is a difference between a color and a sensation of color. For a color is a quality of a thing which remains the same whether it be exposed to one kind of illumination or to another, and whether it be seen by a normal or by a color-blind eye. Such is the established signification of the word "color." For we do not say that the petals and leaves of a rose-bush have the same color when they are viewed by color-blind people: on the contrary, we say that such people cannot distinguish those colors. Color is also something essentially vague; for the color of an object is the admixture of lights necessary to producing the same chromatic effect (1) upon a normal eye, in the absence of any perturbing cause, and (2) under a moderate white illumination. Now even normal eyes receive quite different chromatic sensations from the same objects; and two white illuminations may be quite indistinguishable and yet may be produced by two lights of such different compositions that it is possible to find two colored objects such that the one will to all normal eyes plainly appear to be of the warmer color under one of the two white illuminations, while the other object will as obviously appear to be of the warmer color under the other illumination; though the two illuminations are absolutely indistinguishable when viewed directly without being reflected. I once got up an apparatus to exhibit this, and showed the striking phenomenon to all visitors for months. Few persons are aware how very striking such phenomena can be made. Color, therefore, is a quite remarkably vague quality, as well as being relative to the normal sense of sight. If by "normal" were meant merely the average (or any other kind of mean) of actually occurring instances, say the average sensation of all the inhabitants of the globe on a certain date, then this might have been modified by some disease affecting a large part of the people who happened to be living at the time; and since "color" refers to normal chromatic sense, it would depend upon what passed in the minds of a certain body of men. But, in fact, the "normal" is not the average (or any other kind of mean) of what actually occurs, but of what *would*, in the long run, occur under certain circumstances. Now what *would* be, can, it is true, only be learned through observation of what happens to be; but nevertheless no collection of happenings can constitute one trillionth of one *per cent* of what might be, and would be under supposable conditions; and therefore, though it might conceivably prevent many generations from rightly determining what is normal, it could not affect the true -- and ultimately ascertainable (provided there were anybody to ascertain it) -- mean and normal; and thus, the result is that no such accident could affect the normal or the true color. So, in general, what I mean by the *external* might vary with how persons of a given general description *would* think under supposable circumstances; but it will not vary with how any finite body of individuals have thought, do now think, or will actually think.

328. So much for what I mean by the *external*. The main difference between the *external*, as I use the term, and the *real*, as I employ *that* term, seems to be that the question whether anything is *external* or not is the question of what a word or other symbol or concept (for thinking proper is always conducted in general signs of some sort) is, I say, a question of what a symbol *signifies*; while the question of whether anything is *real* or is a *figment* is the question what a word or other symbol or concept *denotes*. If the attributes of or possible true assertions about an object could vary according to the way in which you or I or any man or actual body of single men, living at any time or times, might think *about that object*, then that object is what I call a *figment*. But if even although its attributes, or what is true of it, should possibly vary according to what some man or men might think, yet if no attribute could vary between being true and being false, according to what any plural of single men could think *about that thing*, then, and though it were accordingly not *external* but *mental*, it would nevertheless be *real*, since precisely that is what I mean by calling an object real. . . . If two contradictory assertions about the same definite subject be true that subject cannot be real, is the principle of the *reductio ad absurdum*. If a subject of two assertions conflicting in form be indefinite (as, for example, "One might have one's pocket picked of real property" and "One cannot have one's pocket picked of real property") those assertions are not, properly speaking, *contradictory*, nor are they usually called so. They are, in logical parlance, *subcontrary*. (In the example, one assertion means "One can have one's pocket picked of real property in *some* sense," and "One cannot have one's pocket picked of real property in *some* sense.") That is, they are related like "Some *S* is *P*" and "Some *S* is not *P*." Such, then, is the sense to which I restrict the word "real"; and I believe that my definition comes as near to expressing the principal sense of our vernacular word in the mouths of the clearest-headed talkers as any definition could. Except in the legal sense, the vernacular word was derived from *realis*, which (except in the legal sense) was a vocable invented by medieval metaphysicians for their own purposes. Especially, it is a prominent word in the works of Duns Scotus, of which I have been an attentive and meditative student. Now Duns, generally believed to be the birthplace of this great thinker (and I have scarce a doubt of it), is less than ten statute miles north of the Tweed in Berwick, so that in the last quarter of the thirteenth century the logician was doubtless as almost genuinely an English boy as he would be if born there today. A northern dialect of Middle English was his mother tongue. No medieval logician influenced the present English of the market-place so much as he did; and my definition is thoroughly imbued with the spirit of Scotism. These considerations confirm my confidence that my definition not only, as that of a term of philosophy, best conforms to sound canons of terminology,†1 but that it expresses, as nearly as any definition could, the principal sense of the vernacular word, which, if this be true, is a supererogatory virtue. . . .

Peirce: CP 6.329 Cross-Ref:††
 §4. DYADIC AND TRIADIC ACTION

329. There has been during the nineteenth century a decided leaning of scientific opinion to discredit any other sort of action in the external world than that of dynamical force; to understand a dynamical force to be a purely brute force with no element of inherent reasonableness in it, but merely to be the only force that scientific

research could discover. Especially, at a date about a generation after the enunciation of the doctrine of the conservation of energy (or "of forces," as the phrase was, until the conception of energy became widespread), and half a generation after the setting forth of the theory of natural selection, this current of opinion was mighty strong, wide, and deep in the scientific world. It has since sensibly abated. It certainly cannot be said that this conviction is, or was, quite without any sound reason to back it; for there were two such. The principal argument runs as follows: Scientific research has hitherto made plainly apparent no other cause of changes in the physical universe than brute force; and therefore it is presumable that there is no other. This is a sort of induction resting upon the principle that whatever error it may lead us into the very same argument will in time correct, if it be persisted in long enough. We still occasionally are obliged to take refuge in this argument; and doubtless always shall be so obliged; but it is the very weakest of all those forms of argumentation that have any validity at all.†1 It is good for nothing against the least sound objection; and in this case there is the conclusive objection that exact logical analysis shows dynamic causation (if every element of it be considered) is more than the mere brute force, the dyadic action, that it appears to superficial thinkers to be. For it is governed by law; and to him who bends his ear to that law it articulately testifies, though in a whisper, to the existential might of reasonableness. The secondary argument is therefore in truth somewhat the stronger of the two. It is that in all recent debate the contentions of the champions of spiritual force in Nature are pretty constantly marked with sophistries, exaggerations, unfairness, fogginess, velleities of persecution; phenomena each of which is an infallible mark of a faith which is rotting in the breasts of its most insistent defenders. Except that "pretty constantly" is perhaps an exaggeration, this is entirely true. But it is no argument whatever against the reality of reasonableness: it only illustrates how the materialistic view recommends itself the more to those minds who are the further from sharing the spirit of science.

Peirce: CP 6.330 Cross-Ref:††

330. Any dynamic action -- say, the attraction by one particle of another -- is in itself *dyadic*. It is governed by a law; but that law no more furnishes a correlate to the relation than the vote of a legislator which insures a bill's becoming a statute makes him a participator in the blow of the swordsman who, in obedience to the warrant issued after conviction according to that statute, strikes off the head of a condemned man. In the law, *per se*, there is no physical force nor other compulsion. It is nothing but a formula, a maxim. The particles follow the law simply because, being sprung from the stock of reason, they naturally incline to obey reason. It is true that the attraction of one particle for another acts through continuous Time and Space, both of which are of triadic constitution. Yes; but this continuous Time and Space merely serve to weld together (while imparting form to the welded whole) instantaneous impulses in which there is neither continuous Time, Space, nor any third correlate; and it is such instantaneous impulse that I say is dyadic. However, the dyadic action is not the whole action; and the whole action is, in a way, triadic.

Peirce: CP 6.331 Cross-Ref:††

331.†1 Every triadic relationship involves three dyadic relationships and three monadic characters; just as every dyadic action involves two monadic characters. A monadic character involves nothing dyadic or triadic; nor does a dyadic action involve anything triadic. But a triad always involves three dyads and three monads; and a dyad involves two monads.

Peirce: CP 6.332 Cross-Ref:††

332. That whatever action is brute, unintelligent, and unconcerned with the result of it is purely dyadic is either demonstrable or is too evident to be demonstrable. But in case that dyadic action is merely a member of a triadic action, then so far from its furnishing the least shade of presumption that all the action in the physical universe is dyadic, on the contrary, the entire and triadic action justifies a guess that there may be other and more marked examples in the universe of the triadic pattern. No sooner is the guess made than instances swarm upon us amply verifying it, and refuting the agnostic position; while others present new problems for our study. With the refutation of agnosticism, the agnostic is shown to be a superficial neophyte in philosophy, entitled at most to an occasional audience on special points, yet infinitely more respectable than those who seek to bolster up what is really true by sophistical arguments -- the traitors to truth that they are. . . .

Peirce: CP 6.333 Cross-Ref:††
§5. ESSENCE AND EXISTENCE

333. There are two grades or constituents of Being: the *Essence*, and the *Existence*. Each of these terms has an epistemological and metaphysical force. I consider Existence first, and to begin with, in its epistemological aspect. When a new image, optical, acoustical, or other, appears in the mind, one subjects it to various tests in order to ascertain whether it be of internal or of external provenance. These tests may be distributed into three classes, according to their strength when they testify to externality of origin (which I call being "affirmative") and according to their strength when they testify to internality of origin (which I call being "negative").

Peirce: CP 6.334 Cross-Ref:††

334. The following scheme shows the classes:

Peirce: CP 6.334 Cross-Ref:††
TESTS OF EXTERNALITY

Class I. *Affirmatively, the strongest; negatively, the weakest.*

Peirce: CP 6.334 Cross-Ref:††

Tests by physical concomitants; as by photography, phonography, seismography, chemical test-papers; and a great variety of other physical apparatus and observations.

Class II. *Affirmatively and negatively of middling value.*

Peirce: CP 6.334 Cross-Ref:††

Tests by the testified experience of other observers, or even of oneself at another time, placed in nearly the same circumstances.

Class III. *Affirmatively, the weakest; negatively, the strongest.*

Peirce: CP 6.334 Cross-Ref:††

Criticism of all the circumstances of the apparition, ending with the readiest and, in case its evidence should be negative, the most conclusive of all single tests, namely, that of making a direct inward effort to suppress the apparition.

Peirce: CP 6.335 Cross-Ref:††

335. The word "insistency," which I have used, wrongly conveys the suggestion of a more or less. But the above tests, critically applied, avail to separate apparitions into two discrete classes, with none of an intermediate character: those which are due to the normal action of the senses, and those which are due to some derangements of the health of the person to whom the appearance comes. The former are really *experienced*, the latter are deceptive hallucinations or illusions. All to which the senses normally testify without room for critical reasoning is usually and properly said to be "experienced"; and all that is truly experienced is, in the epistemological sense, *existent*. But experience only informs us that single objects exist, and that each of these at each single date exists only in a single place. These, no doubt, are what Aristotle meant by {to kath' hekaston}†1 and by {ai prōtai ousiai}†2 in his earlier works, particularly the *Predicaments*. For {ousia} there plainly means existent, and {to ti einai} is existence. (I cannot satisfy myself that this was his meaning in his later writings; nor do I think it possible that Aristotle was such a dolt as never to modify his metaphysical opinions.) But {to atomon}†1 was, I think, the strict logical individual, determinate in every respect.†P1

Peirce: CP 6.336 Cross-Ref:††

336. In the metaphysical sense, *existence* is that mode of being which consists in the resultant genuine dyadic relation of a strict individual with all the other such individuals of the same universe. . . .

Peirce: CP 6.337 Cross-Ref:††

337. Coming to Essence, this in its epistemological force is that intelligible character which truly defines what a general or indefinite, that is, what an indeterminate monadic predicate primarily asserts, so that all else that it asserts is the necessary consequence of this epistemological essence. It is easy to state what the essences of artificial objects are: The essence of a stove is that it is intended to diffuse warmth. But as to the essence of natural objects, if they have any, we are unable as yet to give them. We are only able to state the essence of our common names for such things. The metaphysical essence is the intelligible element of the possibility of its Being, or so much of that as is not a mere consequence of the rest. . . .

Peirce: CP 6.338 Cross-Ref:††

§6. MODES OF BEING

338. All thinking is dialogic in form. Your self of one instant appeals to your deeper self for his assent. Consequently, all thinking is conducted in signs that are mainly of the same general structure as words; those which are not so, being of the nature of those signs of which we have need now and then in our converse with one another to eke out the defects of words, or *symbols*.†2 These non-symbolic

thought-signs are of two classes: first, pictures or diagrams or other images (I call them *Icons*†2) such as have to be used to explain the significations of words; and secondly, signs more or less analogous to symptoms (I call them *Indices*†2) of which the collateral observations, by which we know what a man is talking about, are examples. The Icons chiefly illustrate the significations of predicate-thoughts, the Indices the denotations of subject-thoughts. The substance of thoughts consists of these three species of ingredients.

Peirce: CP 6.339 Cross-Ref:††

339. The next step consists in considering why it is that thoughts should take those three different forms. You will observe that each kind of sign serves to bring before the mind objects of a different kind from those revealed by the other species of signs. The key to the solution of this question is that what we think of cannot possibly be of a different nature from thought itself. For the thought thinking and the immediate thought-object are the very same thing regarded from different points of view. Therefore, Berkeley †1 was, so far, entirely in the right; although he blundered when from that manifest truth he inferred his idealism -- and it was a blunder for just the reason pointed out by Kant in the second edition of the *Kritik der Reinen Vernunft*. †2 We must conclude, then, that the reason why different things have to be differently thought of is that their modes of metaphysical being are different.

Peirce: CP 6.340 Cross-Ref:††

340. Aristotle, however, failed to strike the nail squarely on the head when he said that generals are known by reason and singulars by sense.†3 Generals are predicates. Now while the *structure*, not only of predicates, but of all kinds of thought, is known by reason, that is, by symbols, like words, the *matter* of predicates, *simple* predicates, is *not* known by reason, but by the senses and by other feelings. A subject of every judgment -- and it is *the* subject *par excellence* -- is a singular;†4 and every singular, as Aristotle †5 himself says, is a subject. But to say that a singular is known by sense is a confusion of thought. It is not known by the feeling-element of sense, but by the compulsion, the insistency, that characterizes experience. For the singular subject is real; and reality is insistency. That is what we mean by "reality." It is the brute irrational insistency that forces us to acknowledge the reality of what we experience, that gives us our conviction of any singular.

Peirce: CP 6.341 Cross-Ref:††

341. The mode of being of the composition of thought, which is always of the nature of the attribution of a predicate to a subject, is the living intelligence which is the creator of all intelligible reality, as well as of the knowledge of such reality. It is the *entelechy*, or perfection of being.

Peirce: CP 6.342 Cross-Ref:††

342. So, then, there are these three modes of being: first, the being of a feeling, in itself, unattached to any subject, which is merely an atmospheric possibility, a possibility floating *in vacuo*, not rational yet capable of rationalization; secondly, there is the being that consists in arbitrary brute action upon other things, not only irrational but anti-rational, since to rationalize it would be to destroy its being; and thirdly, there is living intelligence from which all reality and all power are derived; which is rational necessity and necessitation.

Peirce: CP 6.343 Cross-Ref:††

343. A feeling is what it is, positively, regardless of anything else. Its being is in it alone, and it is a mere potentiality. A brute force, as, for example, an existent

particle, on the other hand, is nothing for itself; whatever it is, it is for what it is attracting and what it is repelling: its being is actual, consists in action, is dyadic. That is what I call *existence*. A reason has its being in bringing other things into connexion with each other; its essence is to compose: it is triadic, and it alone has a real power.

Peirce: CP 6.344 Cross-Ref:††

344. Signs, the only things with which a human being can, without derogation, consent to have any transaction, being a sign himself,†1 are triadic; since a sign denotes a subject, and signifies a form of fact, which latter it brings into connexion with the former.†2 "But what," some listener, not you, dear Reader, may say, "are we not to occupy ourselves at all with earthquakes, droughts, and pestilence?" To which I reply, if those earthquakes, droughts, and pestilences are subject to *laws*, those laws being of the nature of signs, then, no doubt being signs of those laws they are thereby made worthy of human attention; but if they be mere arbitrary brute interruptions of our course of life, let us wrap our cloaks about us, and endure them as we may; for they cannot injure us, though they may strike us down. "But," some other peerer over the fence may continue -- yet no! I will pursue the argument no further; for such cobwebs will not hold them prisoners, once they really desire the liberty of truth.

Peirce: CP 6.345 Cross-Ref:††

345. The division of modes of Being needs, for our purposes, to be carried a little further. A feeling so long as it remains a mere feeling is absolutely simple. For if it had parts, those parts would be something different from the whole, in the presence of which the being of the whole would consist. Consequently, the being of the feeling would consist of something beside itself, and in a relation. Thus it would violate the definition of feeling as that mode of consciousness whose being lies wholly in itself and not in any relation to anything else. In short, a pure feeling can be nothing but the total unanalyzed impression of the *tout ensemble* of consciousness. Such a mode of being may be called *simple monadic Being*.

Peirce: CP 6.346 Cross-Ref:††

346. *Blind existential being* may possibly not occur at all; since we know nothing with absolute certainty of existent things, and are especially in the dark as to their modes of being, and above all know extremely little about the ultimate parts of matter, beyond the fact that electricity, itself a most mysterious sort of existent, is an ingredient of them. In the book about God and religion upon which I have been at work for several years, and hope to write, one of the questions which will come up for fair consideration is whether either the monotheistic, absolute God or the polytheistic, finite God of the pseudo-pragmatists could know the nature of blind existence, as He must, if he had created it. It is an unexplored passage in the mammoth cave of metaphysics; and various questions concerning it suggest themselves. This much, however, seems clear about such existence; namely, that there ought to be two grades of it; a lower kind, approximating to the inner being of a simple quality, yet existential, instead of being merely potential, consisting in the action of the thing upon itself, a sort of embryonic self-consciousness; and a higher grade consisting in the action of a thing upon all the other things of the same universe, and measuring by its intensity its remoteness from each of them. A whole universe of such existents can only have the lower, or internal grade of existence.

Peirce: CP 6.347 Cross-Ref:††

347. Of triadic Being the multitude of forms is so terrific that I have usually shrunk from the task of enumerating them; and for the present purpose such an enumeration would be worse than superfluous: it would be a great inconvenience. In

another paper, I intend to give the formal definition of a sign, which I have worked out by arduous and long labour.†1 I will omit the explanation of it here. Suffice it to say that a sign endeavours to represent, in part at least, an Object, which is therefore in a sense the cause, or determinant, of the sign even if the sign represents its object falsely. But to say that it represents its Object implies that it affects a mind, and so affects it as, in some respect, to determine in that mind something that is mediately due to the Object. That determination of which the immediate cause, or determinant, is the Sign, and of which the mediate cause is the Object may be termed the *Interpretant*. . . .

Peirce: CP 6.348 Cross-Ref:††

348. Modern thought has been extravagantly Ockhamistic, owing to the accidental circumstance that, at the revival of learning, the obscurantists, the fogeys, were adherents of Duns, of whom the politician Ockham was the typical opponent. But this had come about because, in those days of precise, if shallow, thinking, the Scotistic doctrine had emerged triumphant from all the scholastic disputations, of which the reformers of learning had but the dimmest idea. Get rid, thoughtful Reader, of the Ockhamistic prejudice of political partizanship that in thought, in being, and in development the indefinite is due to a degeneration from a primary state of perfect definiteness. The truth is rather on the side of the scholastic realists that the unsettled is the primal state, and that definiteness and determinateness, the two poles of settledness, are, in the large, approximations, developmentally, epistemologically, and metaphysically.

Peirce: CP 6.349 Cross-Ref:††
§7. REALITY AND EXISTENCE †2

349. C. S. P. In the first place, I call your attention to the fact that *reality* and *existence* are two different things.

Peirce: CP 6.349 Cross-Ref:††

VELIAN. How, then, would you define existence?

Peirce: CP 6.349 Cross-Ref:††

C. S. P. It will not be necessary to go into that question, which is one of great delicacy. It will be sufficient to point out certain respects in which reality and existence differ. Let us suppose two seeds to be exactly alike. I do not say that two seeds ever are so; but we are now merely considering the meanings of two words, and, therefore, we are free to imagine any state of things we can. We will suppose, then, that not merely to our senses, but to any conceivable senses, those seeds are precisely alike, except that they are in different places. But now we will suppose that I am really resolved to plant those two seeds in such different soil, and to treat them so differently, that they will grow into plants whose flowers will have different colors. They really will be different, whatever anybody may say or think. I have made certain dispositions, so that I myself could not now have it otherwise. Their future difference is then a *reality*, already. For the time has already passed at which anybody's *dictum* could make the fact otherwise. Yet I have not decided what the colors of the flowers of each are to be; for one of the two seeds will be taken *at random*, and placed in one soil and the other in another. Now, when it comes to the *existence* of those flowers, the colors will be absolutely what they will be. There can be no uncertainty or

ambiguity about existence. The reality, however, of my determination of the colors is not altogether certain. Existence, then, is a special mode of reality, which, whatever other characteristics it possesses, has that of being absolutely determinate. Reality, in its turn, is a special mode of being, the characteristic of which is that things that are real are whatever they really are, independently of any assertion about them. If Man is the measure of things, as Protagoras said, then there is no complete reality; but being there certainly is, even then.

Peirce: CP 6.349 Cross-Ref:††

My dear Velian, some god suggests to me that, thousands of years after you and I have left this earth, a physicist by the name of Thomson †1 will prove that all matter consists of corpuscles. Now each of those corpuscles must, I suppose, have a center of mass, which is an absolute point. But no matter how infinite the multitude of those centers, they cannot in all time pass through every place. There must, then, remain places where no center has been or ever will be. Yet, for my part, I believe that the laws of motion, the law of gravitation, etc. are as real in those places as anywhere. I may be wrong; but we are considering only the meanings of words; and if my belief has any sense or meaning, although no matter ever exists in those places, those laws are real there. There are many other kinds of reality distinct from existence. But have I not said enough to show that the meanings of the three terms, being, reality, and existence, are distinguishable? VELIAN. More than enough.

Peirce: CP 6.350 Cross-Ref:††
§8. TRUTH, BEING, AND NOTHING

350. C. S. P. Now let us consider the nature of truth. Before anything can be true or false, it is necessary, is it not, that something should be said, whether by writing, by speech, or in thought?

Peirce: CP 6.350 Cross-Ref:††
VELIAN. Undoubtedly.

Peirce: CP 6.350 Cross-Ref:††
C. S. P. And this must be said concerning something, some *subject*, must it not?

Peirce: CP 6.350 Cross-Ref:††
VELIAN. Yes.

Peirce: CP 6.350 Cross-Ref:††
C. S. P. And something definite must be said of that subject, some *predicate*, must it not?

Peirce: CP 6.350 Cross-Ref:††
VELIAN. Yes.

Peirce: CP 6.350 Cross-Ref:††
C. S. P. The subject must be designated by a word or other sign, must it not?

Peirce: CP 6.350 Cross-Ref:††
VELIAN. Yes.

Peirce: CP 6.350 Cross-Ref:††

C. S. P. And the predicate must be signified by some word or other sign, must it not?

Peirce: CP 6.350 Cross-Ref:††

VELIAN. Yes.

Peirce: CP 6.350 Cross-Ref:††

C. S. P. If it is said that the predicate-sign is [in-]applicable to something to which the subject-sign is applicable, that must be true or false, must it not?

Peirce: CP 6.350 Cross-Ref:††

VELIAN. Yes.

Peirce: CP 6.350 Cross-Ref:††

C. S. P. If this is false, then whatever there may be to which the subject-sign is applicable the predicate-sign is also applicable, is this not so?

Peirce: CP 6.350 Cross-Ref:††

VELIAN. Yes.

Peirce: CP 6.350 Cross-Ref:††

C. S. P. While if the former is true, the latter is false?

Peirce: CP 6.350 Cross-Ref:††

VELIAN. Yes.

Peirce: CP 6.350 Cross-Ref:††

C. S. P. Thus, if it is said that to whatever there may be to which the subject-sign is applicable the predicate-sign is also applicable, this must be either true or false.

Peirce: CP 6.350 Cross-Ref:††

VELIAN. Yes.

Peirce: CP 6.350 Cross-Ref:††

C. S. P. Now is there anything which is true or false which is not of one or other of those forms of assertion, or else of a form a mixture of those two?

Peirce: CP 6.350 Cross-Ref:††

VELIAN. What of the assertion "It rains"?

Peirce: CP 6.350 Cross-Ref:††

C. S. P. In order to be true or false, this assertion must refer to some time and place, and the circumstances under which the assertion was made must have indicated a time and place. That indicating circumstance, of which speaker and auditor had experience, was the subject-sign; and we may presume that the assertion was in meaning equivalent to these two: first, there is some time and place indicated by these circumstances to which the description "it rains" is applicable; and secondly, whatever time and place these circumstances indicate is an occasion to which the description "it rains" is applicable.

Peirce: CP 6.350 Cross-Ref:††

VELIAN. That will do; but what of the assertion "If I had upset my inkstand I should have spoiled my manuscript"?

Peirce: CP 6.350 Cross-Ref:††

C. S. P. The first clause suggests that a certain past occasion or series of past occasions have been otherwise indicated to which the meaning is limited, and it further calls to mind the proposition "I upset my inkstand"; and now it asserts positively one thing and virtually asserts another. The first is that whatever connected series of occurrences there may be among the series of occurrences alluded to, to which the description "I upset my inkstand" would be applicable, is a series of occurrences to which "I spoiled my Ms." is applicable. But the second virtual assertion modifies this by adding that whatever series of past occurrences there may be is a series to which the description "I did not upset my inkstand" is applicable.

Peirce: CP 6.351 Cross-Ref:††

351. VELIAN. Let your analysis of that which is true or false be granted, provisionally, what then?

Peirce: CP 6.351 Cross-Ref:††

C. S. P. Then it follows that that which can be true or false must be one or the other.

Peirce: CP 6.351 Cross-Ref:††

VELIAN. How so?

Peirce: CP 6.351 Cross-Ref:††

C. S. P. In order that my explanation may be fully understood, a little preface is desirable. Suppose that this is true: "Under some circumstances, every possible course of action will prove fatal." If that is true, then it is true that every possible course of action will prove fatal, is it not?

Peirce: CP 6.351 Cross-Ref:††

VELIAN. Only under some very peculiar circumstances.

Peirce: CP 6.351 Cross-Ref:††

C. S. P. Yes; but I wish my description of what is true or false, to apply to what is not only true or false generally, but also to what is true or false under conditions already assumed. Whatever may be the limitations previously imposed, that to which the truth or falsity is limited may be called the *universe of discourse*. †1 For example, at the mention of a certain name, every person initiated into the Eleusinian mysteries invariably experiences a feeling of awe. This is true. It is therefore true that every person initiated into the Eleusinian mysteries always experiences a sentiment of awe; not universally, but only under the limitations already understood before this is said.

Peirce: CP 6.351 Cross-Ref:††

VELIAN. That is clear.

Peirce: CP 6.351 Cross-Ref:††

C. S. P. Another point to be noticed is that given any sign whatever, which we may call *P*, we can always frame a sign which shall be applicable to every object of the universe of discourse to which *P* is inapplicable and which shall be inapplicable to every object to which *P* is applicable. If *P* is a word, this sign may be formed by simply prefixing not, {mé}; as *man, not-man; righteous, not-righteous; I, not-I*. The relation between *P* and not-*P*, therefore, is that that to which *P* and not-*P* both apply is not in the universe of discourse, and that to everything in the universe of discourse

either *P* or not-*P* is applicable.

Peirce: CP 6.351 Cross-Ref:††

VELIAN. I do not know that that is true.

Peirce: CP 6.351 Cross-Ref:††

C. S. P. But you must allow me to define my own terms as I choose. I propose to define not-*P* as such a sign that it is applicable to everything in the universe of discourse unless *P* is applicable to it; while it shall be inapplicable to everything in the universe of discourse to which *P* is applicable. That is not a statement of fact, but simply of my use of the word "not." If you dispute it, you must show that I do not so use the word "not." . . .†2

Peirce: CP 6.351 Cross-Ref:††

Now, *S* and *P* being any two signs, I propose, in case there really is something in the universe of discourse to which both *S* and *P* are applicable, that is, something to which *S* is applicable but not-*P* inapplicable, to use the word *true* in such a sense that it is "true" that to something to which *S* is applicable *P* is applicable; and to use the phrase *particularly true* in such a sense that in this case *P* shall be said to be "particularly true" of *S*; while I propose to use the word *false* in such a sense that under the same circumstances it shall be "false" that to whatever there may be to which *S* is applicable [not-]*P* is applicable. But if there is not really any object of the universe of discourse to which *S* and *P* are both applicable, that is, if there is nothing to which *S* is applicable unless not-*P* be also applicable to it, then I propose to use the word *false* in such a sense that it shall be "false" that to something to which *S* is applicable *P* is applicable, and I propose to use the word *true* in such a sense that in that case it shall be "true" that to whatever there may be to which *S* is applicable, not-*P* is applicable; and I propose to use the phrase *universally true* in such a sense that not-*P* shall be said to be "universally true" of *S*. Furthermore, if *A* and *B* are two assertions, and if there is a third assertion *C* which is equivalent to asserting both *A* and *B*, then I say that the copulative assertion, *C*, is true in case both *A* is true and *B* is true, but is false in every other case, whether *A* is false or *B* is false; but if *D* is equivalent to asserting that *C* is false, then I say that the *disjunctive* assertion, *D*, is false in case both not-*A* and not-*B* are false, but is true in every other case whether not-*A* or not-*B* be true.

Peirce: CP 6.351 Cross-Ref:††

VELIAN. What next?

Peirce: CP 6.352 Cross-Ref:††

352. C. S. P. Now let us suppose that in the whole universe of discourse there is not really a single black tulip. In that case, according to the rules just laid down, green, blue, white, reality, non-existence, and anything else you please are universally true of black tulip, while not even being black or being a tulip is particularly true of black tulip. Everything is universally true of it, but universally nothing is false of it. Nothing is particularly true of it, but particularly everything is false of it. You assent?

Peirce: CP 6.352 Cross-Ref:††

VELIAN. Yes.

Peirce: CP 6.352 Cross-Ref:††

C. S. P. All this is so, because in the case supposed a black tulip is *nothing*. Therefore, instead of nothing being unutterable, and all that, as you say, universally it

is real and non-real, utterable and unutterable, etc. But particularly it is none of these.

Peirce: CP 6.352 Cross-Ref:††

VELIAN. In that case, of Being everything should be universally false, and particularly true.

Peirce: CP 6.352 Cross-Ref:††

C. S. P. My dear Velian, leave rattles to babes, and jingles of words to Germans; for to an Italian and a Greek, reason is more becoming. Since of nothing, everything is true, it follows that of everything, being is true universally, but not at all that everything is universally true of being.†1 Moreover being is particularly true of everything of which anything is particularly true; but not of everything. But if you choose to extend the name *being* to everything which can be invented or suggested *or cannot*, you will have the notion of a non-notion, which is not even universally itself, and of which what you say may be feigned to be true. For the whole thing does not even rise to the level of a figment. It is a dream within a dream.

Peirce: CP 6.352 Cross-Ref:††

VELIAN. Why, Pure Being is the very foundation of all wisdom.

Peirce: CP 6.352 Cross-Ref:††

C. S. P. Say rather of that love which winds itself up in needless and senseless paradoxes. But to return to nothing. Good father Parmenides was quite right when he said "You will never get over this, that nothing is." Why should we wish to? Whatever is nothing *is*, of course. But when he adds the advice to keep your mind from the path of a certain inquiry, that is bad advice for a philosopher, no matter what the inquiry is.

Peirce: CP 6.352 Cross-Ref:††

VELIAN. But to say that nothing is, is a contradiction in terms!

Peirce: CP 6.352 Cross-Ref:††

C. S. P. Of course, why should it not be?

Peirce: CP 6.352 Cross-Ref:††

VELIAN. But [it] is absurd!

Peirce: CP 6.352 Cross-Ref:††

C. S. P. It is certainly absurd that nothing should be. Being nothing, it is not adequately described until it is shown to be absurd. But there is nothing absurd in saying so. In geometry we often prove the non-existence of something by showing that it would have contrary attributes. That proved, it follows that it is nothing, because nothing, and nothing alone, possesses contrary attributes. It is, therefore, an important truth, and not absurd, that nothing is absurd.

Peirce: CP 6.352 Cross-Ref:††

VELIAN. You astonish me.

Peirce: CP 6.352 Cross-Ref:††

C. S. P. You say that the name "nothing," {mé on}, is not applicable to anything that is. I grant you that, with pleasure. Particularly, everything is false of it. Moreover, you say, the word "something" refers exclusively to existing things. You are quite right there. It is the same thing, otherwise expressed. But when you conclude that the word "nothing" expresses nothing, you are entirely wrong. What a thing "expresses," {legei}, is whatever is universally true of it. Thus, to say that something

is a man is to say that it is an animal and that it reasons. And it is still more absurd to say that he who uses the word "nothing" does not speak. It would be far truer to say that he who uses the word "being" does not speak; for to say that whatever may be a man *is*, may be said, exactly or approximately, to convey no information, true or false; so that it is all the same as if nothing were said.

Peirce: CP 6.352 Cross-Ref:††

VELIAN. As if *nothing* were said! So you confess that saying nothing is saying nothing.

Peirce: CP 6.352 Cross-Ref:††

C. S. P. My dear Velian, does your whole stock in trade consist of such bagatelles as that? Or are you really upon such an intellectual level as to think there is anything in such a confusion?

Peirce: CP 6.352 Cross-Ref:††

VELIAN. No; I admit that having nothing as what was uttered and having nothing as one's meaning may be different.

Peirce: CP 6.352 Cross-Ref:††

C. S. P. You do not draw the distinction accurately; but let it go. You proceeded to say that nothing which exists is in a relation to a thing that does not exist. Understood in a "particular" sense, that is true. For example, loving is a relation, and lovers of females are fewer than lovers of animals; because there are other animals besides females who are loved. Lovers of women are still fewer; and of lovers of pea-green women there are none, because whatever there may be which is a pea-green woman is nothing. But taking what you say in a "universal" sense, it is not true. Thus, there are many men each of whom loves all women; but those who love all females, including female mosquitoes, are much fewer, and when it comes to loving whatever pea-green woman there may be, this may be said of every object in the universe. For since there is no falsity in it, and every assertion not false is true, this is true. Again, you say number is the totality of things that exist. You can use number in that sense, if you like. Only in that sense, *number* is not a sign but a collection of things. In the sense in which *number* is an attribute, it is a sign, and, like other signs, it does not prove that a thing exists because this sign would be applicable to it if it did exist. But even if it did, that would not prevent number from being true of "nothing," since existence and everything else is true of nothing. Thus, your proof, that whatever be said of nothing is not true, utterly fails. Whatever is said universally of nothing is true: whatever is said particularly of nothing is false.

Peirce: CP 6.352 Cross-Ref:††

VELIAN. But let us take an object, say an inkstand. Take away from it its visibility, its impenetrability, and every character by which it could manifest itself to any sentient being with our senses or with any conceivable senses; so that the very gods could not perceive it or any effects of it. It would be reduced to nothing; and it would be false and not true that it was visible and impenetrable, and equally false that it was invisible or penetrable.

Peirce: CP 6.352 Cross-Ref:††

C. S. P. Yes; I grant that. But this is because you are making those assertions in the particular sense. In the very act of denying that these things are true in that sense, you are asserting them universally. To say that it is not true that there is an invisible nothing, and not true that there is a visible nothing, is just the same as to say

that it is true that whatever there may be that is nothing is both visible and invisible.

Peirce: CP 6.352 Cross-Ref:††

VELIAN. But it is nonsense to say that it is false that the inkstand deprived of existence is invisible and at the same time that it is false to say that it is visible. For everything is either visible or invisible.

Peirce: CP 6.352 Cross-Ref:††

C. S. P. Everything is either visible or invisible, and every nothing there may be is both visible and invisible. The rule only applies to things of which it is true that they exist and false that they do not exist. There are even realities, if we admit the reality of generals (which is, at least, not to be refuted by mere definition), which are indeterminate in respect to the applicability of many signs.†1

Peirce: CP 6.353 Cross-Ref:††

§9. MATTER AND FORM †1

353. The word matter (Lat. *materia*, which was used to translate the Gr. {hylé}) is often employed where the more appropriate Greek word would be {söma} *corpus*, body; or {to hypokeimenon}, *subjectum*, or even {hé hypostasis}, translated *person* in theology. Form (Lat. *forma*, used to translate the Gr. {morphé} and {eidós}), though the latter is more exactly represented by *species*) is often employed where {schéma} figure, or {typos}, shape, would be near equivalents. The Greek expressions {morphé, paradeigma, eidós, idea, to ti esti, to ti en einai} are pretty nearly synonymous.

Peirce: CP 6.354 Cross-Ref:††

354. The distinction of matter and form was first made, apparently, by Aristotle. It almost involves his metaphysical doctrine; and as long as his reign lasted it was dominant. Afterwards it was in disfavour; but Kant applied the terms, as he did many others drawn from the same source, to an analogous but widely different distinction. In many special phrases the Aristotelian and Kantian senses almost coalesce, in others they are quite disconnected. It will, therefore, be convenient to consider: (1) the Aristotelian distinction; (2) the Kantian distinction; and (3) special applications.

Peirce: CP 6.355 Cross-Ref:††

355. *The Aristotelian distinction.* Not only was the distinction originated by Aristotle, but one of the two conceptions, that of *matter*, is largely due to him. Indeed, it is perhaps true that the Greek word for matter in the sense of material, {hylé}, was never understood in that general sense before Aristotle came to Athens. For the first unquestionable cases of that meaning occur in certain dialogues of Plato, concerning which -- though there are no dates that are not open to dispute -- it seems to the present writer that it is as certain as any such fact in the history of Greek philosophy that the earliest of them was written about the time of Aristotle's arrival. It is true that, as Aristotle himself says, *matter* was the earliest philosophical conception.†2 For the first Ionian philosophers directed their thoughts to the question what the world was made of. But the extreme vagueness of the notion with them is shown by their calling it {he arché} the beginning, by the nonsense of the question, and by many more special symptoms. If the philosophical conception of matter distinguished the

metaphysics of Aristotle that of Plato had been no less marked by its extraordinary development of the notion of form, to which the mixed morality and questioning spirit of Socrates had naturally led up; the morality, because the form is the complex of characters that a thing ought to have; the questioning, because it drew attention to the difference between those elements of truth which experience brutally forces upon us, and those of which reason persuades us, which latter make up the form. But Aristotle's distinction set form, as well as matter, in a new light.

Peirce: CP 6.356 Cross-Ref:††

356. It must not be forgotten that Aristotle was an Asclepiad, that is, that he belonged to a family which for generation after generation, from prehistoric times, had had their attention turned to vital phenomena; and he is almost as remarkable for his capacity as a naturalist as he is for his incapacity in physics and mathematics. He must have had prominently before his mind the fact that all eggs are very much alike, and all seeds are very much alike, while the animals that grow out of the one, the plants that grow out of the other, are as different as possible. Accordingly, his *dunamis* is germinal being, not amounting to existence; while his *entelechy* is the perfect thing that ought to grow out of that germ. Matter, which he associates with stuff, timber, metal, is that undifferentiated element of a thing which it must possess to have even germinal being. Since matter is, in itself, indeterminate, it is also in itself unknowable; but it is both determinable by form and knowable, even sensible, through form. The notion that the form can antecede matter is, to Aristotle, perfectly ridiculous.†1 It is the result of the development of matter. He looks upon the problem from the point of view of a naturalist. In particular, the soul is an outgrowth of the body.†2

Peirce: CP 6.357 Cross-Ref:††

357. The scholastics, who regarded Aristotle as all but infallible, yet to whom the ideas of a naturalist were utterly foreign, who were thoroughly theological in their notions, admitted that the soul was a form. But then they had great difficulty with those opinions of their master which depended upon his conceiving of matter as more primitive than form. Their notions of form were rather allied to those of Plato. The mode of being that, in some sense, anteceded individual existence, they would have held to be one in which there was form without matter, if awe of Aristotle had not caused them to modify the proposition in one way or another. A question, for example, which exercised them greatly was, how the form was restricted to individual existence? For Aristotle there could not be any such question, because he did not conceive of a form taking on individuality, but of an undifferentiated matter taking on, or rather developing, form, and individuality, perhaps, with it (412a, 7).

Peirce: CP 6.358 Cross-Ref:††

358. *The Kantian distinction.* Aristotle refuses to consider any proposition as *science* which is not universal. He does not go so far as to say that all knowledge involves synthesis, but he often approaches doing so. In particular, he holds that matter is something in itself beyond our knowledge, but the existence of which has to be assumed in order to synthesize the opposites that are involved in all change. He expressly defines that as the function of the conception of matter.†1 With Kant, the view that all knowledge involves synthesis -- various acts of synthesis one over another -- is vastly more developed; and he, too, employs the terms *matter* and *form* as called for by such synthesis. But it is curious that while with Aristotle it is matter that is the quasi-hypothesis imported into the facts that the mind may synthesize, with Kant, on the other hand, it is form which performs this function. The matter of

cognition consists of those elements which are brutally and severally forced upon us by experience. By the form he means the rational or intelligible elements of cognition, which he wishes, as far as possible, to regard as independent contributions of the mind itself, which we have no right to suppose are duplicated by anything corresponding to them in the thing. For the Aristotelian, all pure matter is exactly alike, equally devoid of all predicates, while the forms make all the variety of the universe. For the Kantian, on the other hand, matter is the manifold, while the pure forms are the few different modes of unity. Nevertheless, the Kantians -- indeed, Kant himself (see the *Critic of the Pure Reason*, 1st ed., 266) -- argued that they were using the terms in their old and accepted sense. What enabled them to give some speciousness to their contention was the circumstance that during the full century and more of neglect of the Aristotelian doctrine that had intervened, certain secondary senses of the term *matter*, especially that of corporeal matter, and that of a species of corporeal matter, had become relatively prominent.

Peirce: CP 6.359 Cross-Ref:††

359. *Special senses.* Although there is only one *first* or *primary* matter, absolutely indeterminate, yet Aristotle often uses the term in a modified sense as that which is relatively indeterminate; so that the *last* or *second* matter is the same as the form. But these phrases are also used in quite other senses, which need not here be specially noticed. Matter being taken relatively, the same thing can have this or that as its matter in different respects; and so matter is distinguished into *materia ex qua*, *in qua*, and *circa quam*. *Materia ex qua* is the material; silver is the *materia ex qua* of a dime. *Materia in qua* is the subject in which the form inheres; *materia circa quam* is the object. Aquinas †1 illustrates the distinction by virtue, which is a form, and, as such, has no *materia ex qua*; but it has a subject in which it inheres and an object upon which it is exercised. Aquinas introduced the term *signate matter*. †2 Matter of composition, or *proximate matter*, is that of which a thing consists; matter of generation, or *remote matter*, that from which it is developed, as a seed or egg.

Peirce: CP 6.360 Cross-Ref:††

360. The varieties of form are so numerous that they may best be taken in alphabetical order.

Peirce: CP 6.360 Cross-Ref:††

Absolute form: form abstracted from matter.

Peirce: CP 6.360 Cross-Ref:††

Accidental form: an accident, or that the presence of which constitutes an accident; as music is the accidental form of the musician.

Peirce: CP 6.360 Cross-Ref:††

Advenient form: a form subsequent to the final form.

Peirce: CP 6.360 Cross-Ref:††

Apprehended form = apprehended species.

Peirce: CP 6.360 Cross-Ref:††

Artificial form: a form superinduced by art.

Peirce: CP 6.360 Cross-Ref:††

Assistant form: an agent aiding in the realization of a form, especially of that whose essential character is to move; as the angel who turns the heavens round once every twenty-four hours, or the captain of a ship.

Peirce: CP 6.360 Cross-Ref:††

Astral form. According to Gilbert (*De Magnete*), phenomena of electricity are produced by a material effluvium, while the action of a magnet takes place directly at a distance. Whatever it may be then which constitutes the magnetic field, not being matter, must be called form. Gilbert names it *forma prima radicalis et astralis*.

Peirce: CP 6.360 Cross-Ref:††

Common form: a form belonging to a species.

Peirce: CP 6.360 Cross-Ref:††

Completive form: used by Aquinas †1 in the sense of the last of the series of forms which gradually bring a thing to fully developed existence. By Aristotle called *last form*. †2

Peirce: CP 6.360 Cross-Ref:††

Composite form: the form of a collective whole, so far as it is different from its parts.

Peirce: CP 6.360 Cross-Ref:††

Corporeal form: a form of a corporeal nature. This is used by Aquinas, *Summa Theologica*, pars I. qu. lxxv. art. 4. See *Material form*.

Peirce: CP 6.360 Cross-Ref:††

Disponent form: a form rendering matter apt to receive another, *principal*, form. Thus, dryness in wood disposes it to receive combustibility.

Peirce: CP 6.360 Cross-Ref:††

Elementary form: one of the four combinations of hot and cold with moist and dry which were supposed to characterize the four elements.

Peirce: CP 6.360 Cross-Ref:††

Exemplar form: an idea.

Peirce: CP 6.360 Cross-Ref:††

Final form: see *Completive form*.

Peirce: CP 6.360 Cross-Ref:††

General form: the form of a genus; as we should now say a *generic* form.

Peirce: CP 6.360 Cross-Ref:††

Immaterial form: a form which neither depends upon matter while it is being made nor after it is made; a term employed in the theological doctrine of creation.

Peirce: CP 6.360 Cross-Ref:††

Incorruptible form: a form not subject to corruption.

Peirce: CP 6.360 Cross-Ref:††

Individual form: in one of the theories of individuation, was a form which by existing in matter acquired the power of individuating another form.

Peirce: CP 6.360 Cross-Ref:††

Informant form: a form which is a part of the thing of which it is the form.

Peirce: CP 6.360 Cross-Ref:††

Inherent form: a form which can only exist in a state of inherence in matter.

Peirce: CP 6.360 Cross-Ref:††

Intellective form: the mind as form.

Peirce: CP 6.360 Cross-Ref:††

Intelligible form: see *Sensible form*.

Peirce: CP 6.360 Cross-Ref:††

Intermediate form: a form having a middle position between an elementary and a complete form.

Peirce: CP 6.360 Cross-Ref:††

Material form: a term of Scotus, who defines it as follows: "Formam materialem dico esse omnem illam, quae ex natura sua necessario inclinatur naturaliter, ut sit actus materiae, sive sit substantialis, sive accidentalis." (*Op. Oxon.*, IV, i, 1); "Ideo dici potest tertio modo." But elsewhere (*ibid.*, I Post. qu. ii.) he distinguishes two senses of the term: "Forma materialis potest intelligi dupliciter. Uno modo dicitur, quae educitur de potentia materiae, vel quia utitur organo corporeo in operando: et isto modo forma intellectiva non est forma materialis. Alio modo dicitur forma materialis, quia perfectio materiae, et isto modo anima intellectiva est forma materialis, ideo aliquam variationem potest accipere a materia, quam perficit, quia ex materia et forma fit vere unum." Perhaps the most accessible book from which to gain a hint of the nature of the difficulty which gives rise to this distinction is Bridges' edition of what is called *The Opus Majus of Roger Bacon*, vol. II, pp. 507-511, cap. ii.

Peirce: CP 6.360 Cross-Ref:††

Mathematical form: an object of mathematical contemplation, and the result of mathematical abstraction.

Peirce: CP 6.360 Cross-Ref:††

Metaphysical form: form in the philosophical sense.

Peirce: CP 6.360 Cross-Ref:††

Native or natural form, forma in natura existens, forma naturae, form of a nature, is a term going back to John of Salisbury (*Opera*, ed. Giles, v. 92), and closely connected, if not synonymous, with **material form**. Certain questions started by Aristotle †1 in Book V of the *Metaphysics* (of which there is an admirable periphrastic translation by *Grote*, Aristotle, 2d ed., pp. 619 ff.) gave rise to discussions in which the doctrine was compared with Christian beliefs; and the **natural form** plays a considerable part in such discussions.

Peirce: CP 6.360 Cross-Ref:††

Bacon †2 adopted the term **forma naturae**. He did not grossly depart from the received meaning of the term but, owing to his occupying himself with inquiries quite antipodal to those of the scholastics, the two parties did not understand one another. Bacon means the physical explanation of a phenomenon, its occult **modus operandi**. Among the followers of Bacon we, at first, hear a great deal about forms. Boyle wrote whole books about them. But the distinction of matter and form was not calculated to further such inquiries as theirs. It is adapted to expressing phenomena of life. It might be twisted to such a purpose as Gilbert put it to (see Astral form), but it was not suited to the mechanical philosophy of Boyle, and only led to wordy and fruitless discussions.

Peirce: CP 6.360 Cross-Ref:††

Participate form: a form considered as it is united with matter.

Peirce: CP 6.360 Cross-Ref:††

Preparatory form: a term used by Boyle where *disponent* form would be more technical. He says, "The preparatory form is but (if I may so speak) a harbinger that disposes the matter to receive a more perfect form, which, if it be not to be succeeded by any other more noble, is entitled the specific form of that body; as in the embryo, the vegetative and the sensitive soul is but preparatory to the rational, which alone is said to be the specific form of man" (*Free Considerations about Subordinate Forms*).

Peirce: CP 6.360 Cross-Ref:††

Physical form: such forms as may form the object of physical inquiries. Of course, the term was very differently understood during scholastic times and in the 17th century. But the above definition covers both uses.

Peirce: CP 6.360 Cross-Ref:††

Primary form. There is no such well-recognized term of metaphysics; but a remark of William Gilbert leads us to suppose that medical men attached some meaning to it.

Peirce: CP 6.360 Cross-Ref:††

Principal form is that which *per se* constitutes a species. Called also *specific form*.

Peirce: CP 6.360 Cross-Ref:††

Radical form: see *Astral form*.

Peirce: CP 6.360 Cross-Ref:††

Sensible form. Though it chanced that Aristotle nowhere distinguishes {morphé} into {aisphété} and {noété}, yet his followers did. Sensible forms are those which the outward senses distinguish; intelligible are those which the intellect alone can distinguish.

Peirce: CP 6.360 Cross-Ref:††

Significate form: a Thomistic term, a form distinguished by a name.

Peirce: CP 6.360 Cross-Ref:††

Simple form: form without matter. "Forma simplex, quae est purus actus, est solus deus," says St. Thomas.†1

Peirce: CP 6.360 Cross-Ref:††

Specific form: see *Principal form*.

Peirce: CP 6.360 Cross-Ref:††

Subsistent form: a form capable of existing separate from matter, as Aquinas †1 holds that the angels and departed spirits are.

Peirce: CP 6.361 Cross-Ref:††

361. **Substantial form:** a form which constitutes a *nature*, i.e. a species or genus. Thus, the accidental form of a musician is music; but his substantial form is the rational soul which makes him a man. When men's thoughts became turned from theology to the investigation of physics, those who were animated by the new spirit

found themselves confronted with objections based upon allegations of substantial forms. That these substantial forms, so used, were merely a hindrance to the progress of science, was quite plain to them. But the objections were urged with a logical accuracy, born of centuries of study, with which the new men were utterly incapable of coping. Their proper course would have been quietly to pursue their own inquiries, and leave the theologians to square their results with philosophy as best they could. But circumstances did not permit this. The theologians had the popular intelligence and the arm of power on their side; and, when an apparent opposition arose, they naturally exerted themselves to put it down. Thus, the innovators were led to protest against these senseless and harmful substantial forms; and they had to formulate their objections to them -- a business for which they were entirely unfitted. But since the discoveries of the physicists were plainly adding to man's knowledge and power, while their antagonists were simply obstructive, the former soon carried the day in the general opinion of mankind. The history proves that there was something vicious about the theological application of *substantial forms*; but it in no degree goes to show that the physicists accurately defined the objection to that application. In reviewing the arguments at the present day, when the position of the mechanical philosophers is becoming almost as obsolete as that of the scholastic doctors, we first note that when the new men denied that the substantial forms were "entities," what they really had in mind was that those forms had not such a mode of being as would confer upon them the power dynamical to react upon things. The Scotists, for it was they upon whom, as being in possession of the universities, the brunt of the battle fell, had in fact never called the substantial forms "entities," a word sounding like a Scotistic term, but in fact the mere caricature of such a term. But had they used the word, nothing more innocent than the only meaning it could bear for them could be imagined. To call a form an "entity" could hardly mean more than to call it an abstraction. If the distinction of matter and form could have any value at all, it was the substantial forms that were, properly speaking, forms. If the Scotists could really specify any natural class, say man -- and physics was at that time in no condition to raise any just doubt upon that score -- then they were perfectly justified in giving a name to the intelligible characteristic of that class, and that was all the substantial form made any pretension to being. But the Scotists were guilty of two faults. The first -- great enough, certainly, but relatively inconsiderable -- was often referred to, though not distinctly analyzed and brought home to them. It was that they were utterly uncritical in accepting classes as natural, and seemed to think that ordinary language was a sufficient guarantee in the matter. Their other and principal fault, which may with justice be called a sin, since it involved a certain moral delinquency, was that they set up their idle logical distinctions as precluding all physical inquiry. The physicists and Scotists, being intent upon widely discrepant purposes, could not understand one another. There was a tolerably good excuse for the physicist, since the intention of the Scotist was of an abstract and technical kind, not easily understood. But there was no other excuse for the Scotist than that he was so drugged with his metaphysics that ordinary human needs had lost all appeal to him. All through the eighteenth century and a large part of the nineteenth, exclamations against the monstrousness of the scholastic dogma that substantial forms were entities continued to be part of the stock-in-trade of metaphysicians, and it accorded with the prevalent nominalism. But nowadays, when it is clearly seen that physical science gives its assent much more to scholastic realism (limited closely to its formal statement) than it does to nominalism, a view of the history more like that here put forward is beginning to prevail.

362. In the following terms, mostly Kantian, prepositional phrases express the qualifications.

Peirce: CP 6.362 Cross-Ref:††

Form of corporeity: a very common term of scholasticism, originating with Avicenna, and used by Aquinas (*Summa Theol.*, pars i. cap. lxvi. art. 2), but more particularly by Scotus (in his great discussion *Opus Oxon.*, IV. dist. xi. 9.3, beginning "De secundo articulo dico") and by all his followers. The point is, that the rational soul, being purely spiritual, cannot confer corporeity upon the human body, but a special form, the form of corporeity, is requisite. Suarez and others, generally Thomists, as well as Henry of Ghent, denied this on the ground that a species has but one form. Thus a great metaphysical dispute arose. It sprung from the study of the doctrine of transubstantiation. See Cavellus, *Suppl. ad quaest. Scoti in De Anima*, disp. i, which is in the Lyons ed. of Scotus, tom. ii.

Peirce: CP 6.362 Cross-Ref:††

Form of cognition, in Kant's doctrine, is that element of knowledge which the matter of experience must assume in order to be apprehended by the mind. Kant seems to have been thinking of legal forms which must be complied with in order to give standing before a court. So an English sovereign, in order to be crowned, must, as a "matter of form," swear to an intensity of loathing for Romish dogmas which he probably regards with great coolness. Kant's definitions are chiefly the following:

Peirce: CP 6.362 Cross-Ref:††

"In the phenomenon, that which corresponds to the impression of sense, I call the matter of it; while that which constitutes the fact that manifoldness of the phenomenon is intuited as ordered in certain relations, I call the form of the phenomenon" (*Krit. d. Reinen Vernunft*, 1st ed., p. 20).

Peirce: CP 6.362 Cross-Ref:††

"All cognition requires a concept, be it as imperfect and dark as you will; and this, in respect to its form, is always a universal which serves as a rule" (*ibid.*, p. 106).

Peirce: CP 6.362 Cross-Ref:††

"The transcendental unity of the synthesis of the imagination is the pure form of all possible cognition, through which, consequently, all objects of possible experience must *a priori* be represented" (*ibid.*, p. 118).

Peirce: CP 6.362 Cross-Ref:††

"There are two factors in cognition: first, the concept by which any object is thought -- that is, the category; and secondly, the intuition by which that object is given. For if the concept had had no corresponding intuition, it would be a thought, no doubt, as far as its form goes; but having no object, no cognition whatsoever [he means, whether true or false] of anything would be possible by it; since, so far as I should know, there would be nothing, and perhaps could be nothing, to which such a concept would be applicable" (2d ed. of the Deduction of the Categories, §22).

Peirce: CP 6.362 Cross-Ref:††

"It is not more surprising that the laws of phenomena in nature must agree with the understanding and its *a priori* form, *i.e.* with its power of combining any manifold, than that the phenomena themselves must agree with the *a priori* form of sensuous intuition. For just as phenomena have no existence in themselves, but are merely relative to the mind, as having senses, so laws do not exist in the phenomena,

but are merely relative to the mind in which the phenomena inhere, that mind exercising understanding" (and see the rest of this passage, *ibid.*, §26).

Peirce: CP 6.362 Cross-Ref:††

Form of forms. Francis Bacon says "the soul may be called the form of forms," which would be a pretty conceit, were it not plagiarized from the serious doctrine of Aristotle: {ho nous eidos eidōn} (432 a, 2).

Peirce: CP 6.363 Cross-Ref:††

363. The terms *matter* and *form* are used in certain peculiar ways in logic. Speaking *materialiter*, the matter of a proposition is said to be its subject and predicate, while the copula is its form. But speaking *formaliter*, the matter of a proposition is, as we familiarly say, the "matter of fact" to which the proposition relates; or as defined by the scholastics, "habitus extremorum adinvicem." The second tractate of the *Summulae* of Petrus Hispanus begins with the words: "Propositionum triplex est materia; scilicet, naturalis, contingens, et remota. Naturalis est illa in qua praedicatum essentia subiecti vel proprium eius; ut, homo est animal; vel, homo est risibilis. Contingens est illa in qua praedicatum potest adesse et abesse subiecto praeter subiecti corruptionem; ut, homo est albus, homo non est albus. Remota est illa in qua praedicatum non potest convenire cum subiecto; ut, homo est asinus."

Peirce: CP 6.363 Cross-Ref:††

Of a syllogism, the proximate matter is the three propositions; the remote, the three terms. The form, which ought to be the *ergo*, by the same right by which the copula is recognized as the form of the proposition, is said to be "apta trium propositionum dispositio ad conclusionem ex praemissis necessario colligendam." But Kant, in the *Logik* by Jäsche, §59, makes the premisses the matter, and the conclusion the form.

Peirce: CP 6.364 Cross-Ref:††

§10. POSSIBILITY, IMPOSSIBILITY, AND POSSIBLE †1

364. The term is used to express a variety of meanings which, although distinct in themselves, yet flow readily into one another. These meanings may best be grouped according as they have (1) an ontological objective value, or a logical subjective value; and (2) according as they are used antithetically to actuality or necessity. The antithetical point of view is the most convenient from which to begin.

Peirce: CP 6.365 Cross-Ref:††

365. Possibility may mean that something is (1) not actual, or (2) that, while it possesses actual existence, that existence lacks causal or rational necessity.

Peirce: CP 6.365 Cross-Ref:††

(1) As opposed to the actual, the phrase has again a double meaning. (a) Taken objectively, it may mean something as yet undeveloped, since not presenting itself in actually objectified form, but capable of doing so at some future time, when all the conditions of its realization occur: latent, potential being. This implies capacity for realization; and, if this capacity be taken in an active sense, connotes some inherent tendency to actuality, which, if not thwarted, leads to final completeness of being.

This involves the active sense of potentiality, of force, etc. It is close to the literal sense of the term (*posse, can* be). This is the dominating sense in Greek philosophy, being connected with Aristotle's teleological theory of development. (b) Taken logically, it denotes that there is some ground for asserting actuality, but not sufficient to justify a positive statement: *may*, as distinct from *can*, be. Thus, possibly it will rain tomorrow. It has to do with degrees of certainty in judging.

Peirce: CP 6.366 Cross-Ref:††

366. (2) As opposed to the necessary, the term has also a double sense. (a) It may mean chance, contingency, as an objective fact. Chance again, has a double meaning: (i) something not derivable or explainable causally by reference to antecedent facts. There are those who assert the reality of such chance. On this view there are many *possibilities* in store in the future which no amount of knowledge would enable us to foresee or forestall. Indeterministic theories of the will assert possibilities of this sort also. (ii) Chance may mean that which, while necessary causally, is not necessary teleologically; the unplanned, the fatalistic. From this point of view the "possible" is that which unexpectedly prevents the carrying-out of a purpose or intention. It leads up to the logical sense (b), according to which the possible, as opposed to the necessary, is anything whose existence cannot be derived from reason; that, the existence of which, rationally speaking, might be otherwise. It is opposed to mathematical or metaphysical necessity, where existence cannot be otherwise than as it is. In this sense the objective actual may be only (logically) possible; the present rain-storm is actual, but since it does not follow from a necessity of thought, but only from empirical antecedents, it is not necessary, and hence just a contingent possibility. This distinction goes back also to Aristotle, being found in his logical writings,†1 as the possible, as potential meaning, is found in his metaphysical.†2 It has played a large part in modern rationalism, especially in the philosophy of Leibnitz,†3 being identical with his distinction of "truths of reason" and "truths of fact." In the sphere of mathematics, logic, and metaphysics there is no possibility in the strict sense; all that exists exists of necessity. In the physical and practical spheres which deal with the space and time world the notion of possibility has full sway. Everything is possible which does not contradict the laws of reason; that which is inconceivable, which violates the law of reason, is impossible. The impossible is the self-contradictory. Kant's †4 criticism of rational conceivability as a criterion of truth, to the effect that it is only formal, resting upon the principle of identity and contradiction, and when applied to existence must be supplemented by appeal to sense, made Leibnitz's distinctions of hardly more than historic interest.

Peirce: CP 6.367 Cross-Ref:††

367. The nominalistic definition (nominalistic in its real character, though generally admitted by realists, as Scotus, i. dist. 7, qu. unica) that that is possible which is not known not to be true in a real or assumed state of information is, like many nominalistic definitions, extremely helpful up to a certain point, while in the end proving itself quite superficial. It is not that certain things are possible because they are not known not to be true, but that they are not known not to be true because they are, more or less clearly, seen to be possible.

Peirce: CP 6.367 Cross-Ref:††

For example, one collection may be said to be greater than another if, and only if, there is no *possible* relation in which every member of the former collection stands to a member of the latter, to which no other member of the former stands in the same relation. Now, the question arises -- whether or not it is possible for two collections to

be, under this definition, each greater than the other. In advance of an investigation, the *proposition* is possibly true, in the sense that we do not know that it is impossible. But is the *fact* possible? That is, can we in any way suppose such a state of things without involving ourselves in contradiction? It is that positive supposition which will constitute the possibility, not the mere ignorance of whether such a supposition can be made or not. In order to make two such collections possible, we must make some positive assumption in regard to the possibility of collections; while in order to make such a relation between two collections impossible, we have to make a positive assumption of the possibility of a certain description of relation. It is not a question of ignorance, since nothing but pure hypothesis is concerned. The question is whether it is possible in every case to suppose distinct pairs, each composed of a member of either collection and such as completely to exhaust one of the collections. If this is always possible, then two collections each greater than the other are impossible. It is evidently desirable to state the logical principles of this general kind of possibility, which does not consist in ignorance, but, as it would seem, in hypothetical indetermination or disjunctive determination.

Peirce: CP 6.368 Cross-Ref:††

368. Nominalists uniformly speak of Aristotle's view of future contingents as really absurd. It may be so; but it is certainly the only doctrine which their principles leave room for. A certain event either will happen or it will not. There is nothing now in existence to constitute the truth of its being about to happen, or of its being about not to happen, unless it be certain circumstances to which only a law or uniformity can lend efficacy. But that law or uniformity, the nominalists say, has no real being; it is only a mental representation. If so, neither the being about to happen nor the being about not to happen has any reality at present; and the most that we can say is that the disjunction is true, but neither of the alternatives. If, however, we admit that the law has a real being, not of the mode of being of an individual, but even more real, then the future necessary consequent of a present state of things is as real and true as that present state of things itself.

Peirce: CP 6.369 Cross-Ref:††

369. By the old logicians, possibility is usually defined as non-repugnancy to existence. Kant defines it as that which satisfies the formal conditions of experience (*Krit. d. Reinen Vernunft*, 1st ed., pp. 218, 234).

Peirce: CP 6.370 Cross-Ref:††

370. The possible proposition, or problematic judgment, as it is called by German logicians, is said by many logicians, especially Sigwart, not to be any proposition at all, because it does not draw a sharp line between truth and falsity. It seems to be necessary to distinguish between a proposition which asserts that under such and such general conditions a certain thing is possible, of which an example is the proposition that of any two collections one is not greater than the other, and a proposition which pretends to be no more than a conjecture. If a conjecture can be absolutely baseless, which may be doubted, a proposition which pretended to be no more than that may be said to be no proposition at all. But it can hardly be maintained that, when Poincaré says that there is no physical law whatever which will not be rendered more certain by every new confirmatory experiment, he is depriving those laws of all meaning as propositions.

Peirce: CP 6.371 Cross-Ref:††

371. **Logical possibility:** that of a hypothesis not involving any self-contradiction.

Peirce: CP 6.371 Cross-Ref:††

Mere possibility: that of a state of things which might come to pass, but, in point of fact, never will. In common language, exaggerated to the "merest possibility."

Peirce: CP 6.371 Cross-Ref:††

Metaphysical possibility ought to mean a possibility of existence, nearly a potentiality; but the phrase does not seem to be used in that sense, but rather in the sense of possibility by supernatural power.

Peirce: CP 6.371 Cross-Ref:††

Moral possibility one might expect should be the opposite of moral impossibility, meaning, therefore, something reasonably free from extreme improbability. But, in fact, it seems to be used to mean what is morally permissible.

Peirce: CP 6.371 Cross-Ref:††

Physical possibility: (1) that which a knowledge of the laws of nature would not enable a person to be sure was not true; (2) that which might be brought about if psychological and spiritual conditions did not prevent, such as the Pope's pronouncing *ex cathedra* as an article of faith the fallibility of all his own utterances.

Peirce: CP 6.371 Cross-Ref:††

Practical possibility: that which lies within the power of a person or combination of persons under external conditions likely to be fulfilled, and questionable chiefly because internal conditions may not be fulfilled.

Peirce: CP 6.371 Cross-Ref:††

Proximate possibility. It is very difficult to make out what is meant by this; but the phrase is evidently modelled on *potentia proxima*, which is a state of high preparedness for existence; so that proximate possibility would be a high grade of possibility in a proposition amounting almost to positive assertion.

Peirce: CP 6.371 Cross-Ref:††

Real possibility is possibility in the thing, as contradistinguished from mere logical possibility (Scotus, *Opus Oxon.*, I. ii. 7, *Ad secundam probationem maioris*).

Peirce: CP 6.371 Cross-Ref:††

Remote possibility: the possibility of a proposition which is far from being positively asserted. Also used in common speech.

Peirce: CP 6.371 Cross-Ref:††

Substantive possibility: the admissibility of a pure hypothesis (as illustrated above).

Peirce: CP 6.372 Cross-Ref:††

§11. VIRTUAL †1

372. (1) A virtual *X* (where *X* is a common noun) is something, not an *X*, which has the efficiency (*virtus*) of an *X*.

Peirce: CP 6.372 Cross-Ref:††

This is the proper meaning of the word; but (2) it has been seriously confounded with "potential," which is almost its contrary. For the potential *X* is of the nature of *X*, but is without actual efficiency. A virtual velocity is something *not* a velocity, but a displacement; but equivalent to a velocity in the formula, "what is gained in velocity is lost in power."

Peirce: CP 6.372 Cross-Ref:††

So *virtual representation* was the non-representation of the American colonies in the British Parliament, which was supposed to be replaced by something. So Milton asks whether the angels have *virtual* or immediate touch. So, too, the sun was said to be *virtualiter* on earth, that is, in its efficiency.

Peirce: CP 6.372 Cross-Ref:††

(3) *Virtual* is sometimes used to mean pertaining to virtue in the sense of an ethical habit.

Peirce: CP 6.372 Cross-Ref:††

Virtual knowledge: a term of Scotus defined by him (*Opus Oxon.*, Pt. I. iii. 3). . . .

Peirce: CP 6.373 Cross-Ref:††
§12. UNITY AND PLURALITY †1

373. (1) A universally accepted understanding as to the formation of Latin words would require *unity* to mean that which is essential to the number one.

Peirce: CP 6.373 Cross-Ref:††

If we consider the number one, irrespective of the possibility of two, three, etc., it involves no idea of number (and therefore not of totality or collection), nor even any idea of relation. The idea would, therefore, be found in a pure state only in an immediate consciousness which should make no distinction of any kind, whether between subject and object, or of the parts of the object. But the term is never used in this sense, unless with an accompanying explanation.

Peirce: CP 6.373 Cross-Ref:††

(2) The oneness element of experience which involves a positive assignment of the number one, and which must be originally one, and not a total.

Peirce: CP 6.374 Cross-Ref:††

374. Much may be said for the opinion that we are thus brought to the event of acting and being acted upon; for that must be *one*, and is the only element of experience whose essential peculiarity is entirely lost in any generalization of it. The negative oneness of immediate consciousness -- as, for example, it appears in the idea of any particular colour -- does not resist generalization, the idea of redness in general having the same sort of unity as that of the scarlet of mercuric iodide or the colour of aniline red (magenta). But the moment I pass from the idea of a particular item of my experience, such as seeing a boat with a couple of men going over Niagara, to the slightest generalization of it, such as that of the *memory* of seeing the event, or the general conception of going over Niagara, the positive oneness entirely disappears.

Peirce: CP 6.375 Cross-Ref:††

375. Nevertheless, the word unity is seldom applied to this sort of oneness, which goes by the name of *individuality*. There is no individuality in a quality of immediate consciousness such as magenta or solferino, because there is no setting of object over against subject, which is requisite before oneness can be positively assigned to an idea (positive oneness thus involving duality); but neither is there any generality in the immediate consciousness, as long as it remains nothing more than what it first was. The purest oneness does not involve the least reference to plurality, and is therefore not positively opposed to generalization, and is not destroyed when generalization takes place. But positive and insistent oneness necessarily involves, or rather springs out of, the idea of duality, and distinctly denies it; so that generalization destroys it; it is the otherness of otherness, the negation of negation.

Peirce: CP 6.376 Cross-Ref:††

376. (3) The idea which the word unity is usually employed in philosophy to express is that of a general (in the most general sense) in its relation to particulars, which would be much more accurately called "totality."

Peirce: CP 6.376 Cross-Ref:††

Unity is thus used, not to express pure oneness, nor yet positive oneness, but to express the negation of multitude in the object to which it is attributed. Thus it involves a distinct reference to the possibility, not of duality merely, as positive unity does, but of *plurality* (in the sense of more than two). The first unity might be named *simplicity* or *firstness*; the second is very appropriately termed *individuality*; the third, which is nearly what Kant †1 terms *synthetical unity*, ought to have some better designation than *totality* or *universality*.

Peirce: CP 6.377 Cross-Ref:††

377. Unity in certain old books (as in the *Institutiones Metaphysicae* of Burgersdicius, 1647) is divided into *singular* and *universal unity*, the former belonging to singulars, the latter to universals. Singular unity is either *material* (or *numerical*) or *formal*. *Material unity* is defined as that which belongs to the singulars in so far as they are considered as units with individualizing differences; which is an awkward attempt to define individuality. *Formal unity* is "the unity which belongs to singulars abstractedly from their individualizing differences." These definitions depend upon a particular theory of individuation. *Universal unity* is distinguished from *communicability*, which is the reference to the plurality of singulars; and these two elements together make up *universality*. *Numerical unity* implies repugnance to multiplication; *formal unity*, indifference to multiplication; universal unity, non-repugnance to multiplication. The nature of the *fundamentum universalitatis* distinguishes the mediaeval realist from the nominalist. The nominalists generally do not admit that there is any similarity in things apart from the mind; but they may admit that this exists, provided they deny that it constitutes any unity among the things apart from the mind. They cannot admit the latter and remain consistent nominalists. Thus, a nominalist may admit that there is in the events themselves an agreement consisting in the uniformity with which all stones dropped from the hand fall to the ground; but if he admits that there is anything at all, except the mere fact that they happen to do so, that should in any sense *determine* the different stones to fall every time they are dropped, he ceases to be a good nominalist and becomes a mediaeval realist. *Unity* is further divided (by Burgersdicius, whom we continue to follow) into *unity of simplicity* and *unity of composition*. *Simplicity* is said to be unity devoid of all multitude; *composition* is the union of a plurality of things. This definition of simplicity is confessedly defective in representing this mode of unity as

having a reference to multitude. *Composition* is divided into *real*, *rational*, and *modal*. *Real composition* is the union of distinct entities in the real thing itself. It is either *actual* or *potential*. *Actual composition* is either *per compositionem*, as when water and alcohol are mixed, or *per aggregationem* (as in an army). *Potential composition* is when one thing is united in potentia to another. It is either *per informationem* or *per inhaerentiam*; a distinction peculiar to a certain kind of Aristotelianism. *Rational composition* is either of things which differ by reason alone, or of things brought together in one concept; it includes, firstly, genera, species, etc.; secondly, equality, similitude, etc.; thirdly, agreement in effects, external causes, etc. *Modal composition* is composition from a thing and a mode. Most of the above distinctions go back to Scotus, and a few are still earlier.

Peirce: CP 6.378 Cross-Ref:††

378. *Unity* is divided by Kant into *analytical* and *synthetical*. He never defines or explains these terms; but if we remember that, in his use of words, multiplicity of elements is essential to unity, it is easy to see that what he means by analytical unity is the unity of that which is given in its combined state and is analyzed by ordinary reflection. Thus we perceive a fact; and in order to express or think it we analyze it, and the relation of the percept to the elements resulting from this analysis is very inappropriately called *analytical unity*. But when in the formation of the percept different sense impressions are put together, so that ordinary thought cannot proceed from whole to parts, but an operation more or less subconscious is supposed to manufacture the whole out of the parts, the relation of the whole to the parts is called *synthetical unity*. Three kinds of transcendental synthesis are recognized in the first edition of the *Krit. d. Reinen Vernunft*, each resulting in a *synthetical unity*: they are the synthesis of apprehension in the intuition, which produces one representation; the synthesis of reproduction in the imagination; and the synthesis of recognition in the concept, which gives the unity of the rule. The *transcendental unity of apperception* is the unity which belongs to all my cognitions arising from a correlative *unity* of consciousness. It is transcendental, objective, and original. Besides these modes of transcendental unity, Kant recognizes other kinds of synthetical unity, some of which are empirical and contingent. There are also different modes of rational unity, some speculative, others moral.

Peirce: CP 6.378 Cross-Ref:††

That which the Scholastics meant by *transcendental unity* was unity in the sense in which it is said *Quodlibet ens est unum*, that is, is self-consistent.

Peirce: CP 6.379 Cross-Ref:††

379. We must not forget the three *dramatic unities of time, place, and action*, requiring the events represented to be included in one day, in one scene, and all to relate to one plot. Unity of action is set forth by Aristotle (*Poetics*, chaps. viii, ix, xviii). Unity of time is mentioned by him. That is, the action of a tragedy should, when convenient, be compressed into one day (*Poetics*, chap. v). The unity of place was remarked by French critics to have been observed by the Greek tragedians.

Peirce: CP 6.380 Cross-Ref:††

380. A *unity* in mathematics is a quantity which multiplied by any other gives that other. There can thus be but one unity in an algebra, although there may be many units.

381. The old definition is: "Totum est quod constat plurium rerum unione." Psychologically, whatever is treated as a single object, though capable of treatment as two or more objects (parts of the whole): by "treated" meaning "thought of," "attended to," or otherwise "acted upon."†2

Peirce: CP 6.382 Cross-Ref:††

382. We may say that a whole is an *ens rationis* whose being consists in the copulate being of certain other things, either not *entia rationis* or not so much so as the whole; so that a whole is analogous to a *collection*, which is, in fact, a special kind of whole. There can be no doubt that the word whole always brings before the mind the image of a collection, and that we interpret the word whole by analogy with collection. The idea of a collection is itself, however, by no means an easy one to analyze. It is an *ens rationis*, abstraction, or fictitious subject (but the adjective must be understood in a broad sense, to be considered below), which is individual, and by means of which we are enabled to transform universal propositions into singular propositions. Thus, the proposition "all men are mortal," with a new subject and new predicate, appears as "The collection of men is a collection of mortals"; just as, for other purposes, and by means of other abstractions, we transform the same proposition into "The character of mortality is possessed by every man"; and the members of the collection are regarded as less fictitious than the collection. It very often happens that an object given indirect perception as an individual is, on closer scrutiny, seen to be identifiable with a collection of parts. But it does not seem to be strictly accurate to say that the larger object of perception is identical with that abstraction, the collection of the smaller objects. It is rather something perceived which agrees in its relations with the abstraction so well that, for convenience, it is regarded as the same thing. No doubt the parts of a perceived object are virtually objects of consciousness in the first percept; but it is useless to try to extend logical relations to the sort of thought which antecedes the completion of the percept. By the time we conceive an object as a collection, we conceive that the first reality belongs to the members of the collection and that the collection itself is a mere intellectual aspect, or way of regarding these members, justified, in ordinary cases, by certain facts. We may, therefore, define a collection as a fictitious (thought) individual, whose being consists in the being of certain less fictitious individuals.†1

Peirce: CP 6.383 Cross-Ref:††

383. Many adjectives are used to distinguish different kinds of wholes. Certain of the phrases may be defined.

Peirce: CP 6.383 Cross-Ref:††

Actual whole: any whole which cannot exist without the existence of its parts. Usually identified with the *Constitute whole*. Monboddo's definition (*Ancient Met.*, i. 479) is not quite accurate.

Peirce: CP 6.383 Cross-Ref:††

Collective whole, or *aggregate whole*: defined by Chauvin as "that which has material parts separate and accidentally thrown together into one, as an army," etc. But the example shows that organization does not disqualify a whole from being

called *collective*, although the term *totum per aggregationem* will no longer be applied to it, in that case. In so far as a whole is collective, any other relation between its parts is put out of view.

Peirce: CP 6.383 Cross-Ref:††

Composite whole: a term of Burgersdicius, who (*Inst. Met.*, I. xxii. §7) defines it as "quod ex duabus partibus constat quarum una est in potentia ad alterum et altera vice versa actus est alterius." It includes the whole by information and the whole by inherence.

Peirce: CP 6.383 Cross-Ref:††

Comprehensive whole: a whole of logical comprehension.

Peirce: CP 6.383 Cross-Ref:††

Constituent whole: a whole which is essential to its parts.

Peirce: CP 6.383 Cross-Ref:††

Constitute whole: a whole whose parts are essential to it. See Actual whole (above).

Peirce: CP 6.383 Cross-Ref:††

Continuous whole: a continuum regarded as a whole. In order to define it, it would first be necessary to define continuity. Now we have, perhaps, not yet succeeded in analyzing the conception of continuity; for what the mathematicians call by that name, such as the relations of all real quantities capable of being designated to an indefinite degree of approximation by means of a whole number and a decimal, does not answer the requisites of the problem.

Peirce: CP 6.383 Cross-Ref:††

Copulative whole: a whole consisting of a sign which is essentially applicable to whatever certain signs, called its parts, are all applicable, but is essentially inapplicable to anything to which any one of these signs is inapplicable.

Peirce: CP 6.383 Cross-Ref:††

Definite whole: a whole constituted by genus and difference.

Peirce: CP 6.383 Cross-Ref:††

Definitive whole: see *Definite whole* (above).

Peirce: CP 6.383 Cross-Ref:††

Discrete whole: the same as a *Collective whole* (above).

Peirce: CP 6.383 Cross-Ref:††

Disjunctive whole: a whole consisting of a sign which is essentially applicable to whatever any one of certain signs, called its parts, is applicable, but is essentially inapplicable to anything to which none of these parts is applicable.

Peirce: CP 6.383 Cross-Ref:††

Dissimilar whole: same as *Heterogeneous whole* (below).

Peirce: CP 6.383 Cross-Ref:††

Essential whole: great confusion exists in the use of this very common expression. Aquinas (*Summa Theol.*, Pt. I. lxxvi. 8) uses it in a broad sense which would make it about equivalent to Burgersdicius' composite whole, or perhaps broader. On the other hand, it is sometimes restricted to the whole *per*

informationem, and this is perhaps the best settled use. But others make it include the physical and the metaphysical whole as its two species.

Peirce: CP 6.383 Cross-Ref:††

Extensive whole: a whole of logical extension, usually called a subjective or logical whole.

Peirce: CP 6.383 Cross-Ref:††

Formal whole: a comprehensive whole, especially of essential comprehension. See **Actual whole** (above).

Peirce: CP 6.383 Cross-Ref:††

Heterogeneous whole: a term of Aquinas; a whole whose parts are dissimilar from the whole.

Peirce: CP 6.383 Cross-Ref:††

Homogeneous whole: a term of Aquinas; a whole whose parts are similar to the whole, as the parts of a whole of water are.

Peirce: CP 6.383 Cross-Ref:††

Integral whole (a term in common use since Abélard's time): Blundevile (1599) says, "Whole integral is that which consisteth of integral parts, which though they cleave together, yet they are distinct and severall in number, as man's body, consisting of head, brest, belly, legs, etc." The usual definition is "quod habet partem extra partem," which restricts it to space. Burgersdicius, however, says that parts which differ in their ordinal places are *partes extra partes*.

Peirce: CP 6.383 Cross-Ref:††

Integrate whole: a pedantic variant of **Integral whole** (above).

Peirce: CP 6.383 Cross-Ref:††

Logical whole: same as Universal whole (below).

Peirce: CP 6.383 Cross-Ref:††

Mathematical whole: same as **Integral whole** (above).

Peirce: CP 6.383 Cross-Ref:††

Metaphysical whole: a whole in that respect in which a species is the whole of its genus and difference. See **Formal whole** (above).

Peirce: CP 6.383 Cross-Ref:††

Natural whole: a term proposed by Hamilton †1 to replace Comprehensive or Metaphysical whole; as if that were not sufficiently provided with aliases under which to hide itself.

Peirce: CP 6.383 Cross-Ref:††

Negative whole: a unit regarded as a whole, as in the phrases "deus totus est ubique," and "anima est tota in toto corpore."

Peirce: CP 6.383 Cross-Ref:††

Physical whole: a whole compounded of substance and accident; but some say of matter and form; and some that both come to the same thing. In the peripatetic view, however, substance is form, not matter.

Peirce: CP 6.383 Cross-Ref:††

Positive whole: a whole consisting of parts. See **Negative whole** (above).

Peirce: CP 6.383 Cross-Ref:††

Potential whole: same as **Universal whole** (below); so called because the genus does not actually, but only potentially, contain the species, etc.

Peirce: CP 6.383 Cross-Ref:††

Potestative whole: a term of Aquinas; equivalent to **Potential whole** (above).

Peirce: CP 6.383 Cross-Ref:††

Predicative whole: a whole of logical depth.

Peirce: CP 6.383 Cross-Ref:††

Quantitative whole: same as **Integral whole** (above).

Peirce: CP 6.383 Cross-Ref:††

Similar whole: see **Homogeneous whole** (above).

Peirce: CP 6.383 Cross-Ref:††

Subject whole: same as **Subjective whole** (below).

Peirce: CP 6.383 Cross-Ref:††

Subjective whole: a very venerable name for **Universal whole** (below).

Peirce: CP 6.383 Cross-Ref:††

Substantial whole: a whole of logical breadth.

Peirce: CP 6.383 Cross-Ref:††

Universal whole: see Universal.

Peirce: CP 6.383 Cross-Ref:††

Whole by accident: such a whole as neither essentially belongs to the parts nor the parts to it.

Peirce: CP 6.383 Cross-Ref:††

Whole by aggregation or aggregative whole: same as **Collective whole** (above) in an exclusive sense.

Peirce: CP 6.383 Cross-Ref:††

Whole by information: a compound of act and power in the same kind, such as man, according to the Aristotelian theory of the soul.

Peirce: CP 6.383 Cross-Ref:††

Whole by inherence: same as **Physical whole** (above).

Peirce: CP 6.383 Cross-Ref:††

Whole by itself or per se: a whole which essentially belongs to its parts or its parts to it.

Peirce: CP 6.384 Cross-Ref:††

§14. KIND †1

384. Before "class" acquired its logical signification in Queen Anne's reign,

kind was sometimes used for any collection of objects having a common and peculiar general character, simple or complex.

Peirce: CP 6.384 Cross-Ref:††

Thus, in Blundevile's *Arte of Logicke*, we read: "*Genus* is a generall kind which may be spoken of many things differing in speciall kind." At other times, and more accurately, it was restricted to the species, or narrowest recognized class, or that which was supposed to be derived from one stock. Thus Wilson's *Rule of Reason* (1551) has: "*Genus* is a generall woorde vnder the whiche diuerse kindes or sortes of thinges are comprehended."

Peirce: CP 6.384 Cross-Ref:††

But before persons who picked their words had become ready to use "class" as a mere logical extension, they had begun to avoid "kind," except when the emphasis of attention was placed upon the logical depth rather than the breadth.†1 Watts's *Logick* (1724) illustrates this. This last is the ordinary popular sense of the word today; so that "of this kind," "of this nature," "of this character" are interchangeable phrases. J. S. Mill, however, in his *System of Logic*, Bk. I, chap. vii, §4, erected the word into a technical term of logic, at the same time introducing the term "real kind." His meaning, so far as it was determinate, was that classes are of two orders, the first comprising those which, over and above the characters which are involved in their definitions and which serve to delimit their extension, have, at most, but a limited number of others, and those following as "consequences, under laws of nature," of the defining characters; and the second, the real kinds, comprising those each of which has innumerable common properties independent of one another. As instances of real kinds, he mentions the class of animals and the class of sulphur; as an instance of a kind not real, the class of white things. It is important for the understanding of Mill's thought here, as throughout his work, to note that when he talks of "properties," he has in mind, mainly, characters interesting to us. Otherwise, it would not be true that all white things have few properties in common. By a "law of nature" he means any absolute uniformity; so that it is hardly enough to assert that if all white things had any property *P*, this would be a "consequence, under a law of nature," of their whiteness; for it would be itself an absolute and ultimate uniformity. Mill says that if the common properties of a class thus follow from a small number of primary characters "which, as the phrase is, *account* for all the rest," it is not a real kind. He does not remark that the man of science is bent upon ultimately thus accounting for each and every property that he studies. The following definition might be proposed: Any class which, in addition to its defining character, has another that is of permanent interest and is common and peculiar to its members, is destined to be conserved in that ultimate conception of the universe at which we aim, and is accordingly to be called "real."

Peirce: CP 6.385 Cross-Ref:††
§15. PERSEITY AND PER SE †1

385. Scotus says there are two kinds of "perseity," that of a demonstration and that of a predicate which belongs immediately to its subject.

Peirce: CP 6.385 Cross-Ref:††

Per se translates {kath' auto, kath' auton}, etc. Similar phrases occur in ordinary Greek. Plato, for example, in the *Theaetetus*, †2 speaks of {epistémé en ekeinö tö onomati, ho ti pot' echei hé psyché hotan auté kath' autén pragmateuétai peri ta onta}. But in Aristotle it first becomes a term of art (see Bonitz under {heautou}). He enumerates four or five different meanings of it, from which we are led to infer that he did not himself invent it. Two such passages are *Met.*, {D}. xviii. 2, and *Anal. Post.*, iv. There are others, but they are less clear. *Per se* cannot very well be understood without some understanding of the phrase *secundum quid* ({kath' ho}). Aristotle †3 says:

Peirce: CP 6.385 Cross-Ref:††

"*Secundum quid* is said in several senses. In one sense it is the species ({eidos}) and essence of anything; thus, that *secundum quid* a man is good is itself good. Another sense is in what anything first comes into existence, as colour in a surface. In the first sense the *secundum quid* is the form ({eidos}); in the second it is the matter and first subject of anything. And, generally speaking, *secundum quid* refers to a cause. '*Secundum quid* comes a man' is 'on what account comes he?' and '*secundum quid* does he paralogize,' or 'does he syllogize,' is 'what is the cause of the paralogism' or 'the syllogism?' Furthermore, *secundum quid* is said in reference to position in space; as '*secundum quid* stands he,' or '*secundum quid* is he walking.' In such phrases it denotes position and place.

Peirce: CP 6.385 Cross-Ref:††

"Consequently, *per se* is necessarily said in different senses. In one sense, *per se* refers to the essence ({to ti én einai}) of anything; as 'Callias is *per se* Callias,' that is, the very essence of Callias. It also refers to whatever is involved in the definition of anything ({hosa en töi ti estin hyparchei}), as 'Callias is *per se* an animal'; that is, that he is an animal is implied in the word, or animal is what Callias is. The phrase is further applied in case anything in its origin assumes any character in itself or in what belongs to it ({ei en autöi dedektai prötöi é tön auton tini}); thus white is *per se* a surface, and man is *per se* alive, since the soul, which is part of man, receives life in its very origin. Further, that is *per se* which has nothing else as its cause. Thus there are many causes of man, such as being an animal, being biped, etc.; yet still man is *per se* man. Further, whatever belongs to one thing alone, and in so far as it is alone, is *per se*; so that what is abstract ({kechörismenon}) is *per se*."

Peirce: CP 6.385 Cross-Ref:††

These five senses are, then: (1) that a substance exists *per se* and not *per accidens*; (2) that an analytical proposition is true *per se*, or formally, and not as matter of fact; (3) that any character which a thing necessarily assumes by virtue of existing, belongs to it *per se*, and not *secundum quid*; (4) that which a thing causes of itself it does *per se*, and not *per aliud*; and (5) that which any abstraction, *qua* that which it is, is, does, or suffers, is *per se* and not *secundum quid*.

Peirce: CP 6.385 Cross-Ref:††

The second of the above senses is called *per se primo modo*; the third is called *per se secundo modo*; but a different explanation from the above is often given. In reliance particularly on a passage in Aristotle's *Met.*, {Z}. v, it is said that a predication is *per se secundo modo* where the definition of the predicate contains the subject.

Peirce: CP 6.385 Cross-Ref:††

Another important expression is "known *per se*." A proposition is known *per se* if, and only if, it is cognoscible from its own terms but not cognoscible in any other way. For instance, that the letters on this page are black is not known *per se*, because it may be proved by testimony. Nor, on the other hand, is the doctrine of the Trinity *per se*, though it cannot be proved; for it is not self-evident. It has to be received on faith. But there was a great controversy between the Thomists on the one hand and the Scotists with the Nominalists on the other, as to whether, in the above definition, the word "terms" was to be taken *objective* or *formaliter*. See the Conimbricenses in I. *Anal. Post.*, iii.

Peirce: CP 6.386 Cross-Ref:††
§16. PRIORITY, PRIOR, AND PRIUS †1

386. These words are used in about a dozen different senses in philosophy, although only five are specially recognized. They are enumerated in the mnemonic verses,

"Tempore, Natura, prius Ordine, dic et Honore,
Effecto Causam dicimus esse prius."

Peirce: CP 6.387 Cross-Ref:††

387. (1) *Priority in time* is considered by Kant to be dependent upon the peculiar constitution of the internal sense (though he does not attempt any inquiry into the constitution further than that it places objects in time). Now, so far as effects in the outer world are due to forces, it seems to be proved that they follow the law of energy. In that case, though connection and continuity in time are important, yet the flow of time one way rather than in the reverse way is unmeaning. There is no effect that follows after its cause. The law of energy amounts to this, that the instantaneous accelerations of the motions of particles depend solely upon the relative positions of those particles at that same instant; and what follows after depends upon what now is, in the same way precisely, and is calculated by the same laws, as what went before depends upon what now is. Thus, in respect to the direction of its flow, time seems to be, if not purely a psychological affair, at any rate not purely a dynamical affair. Those physical phenomena which proceed in one direction and not in the reverse direction, and which seem to be well explained, such as the viscosity, diffusion, and conduction of gases, may all be explained by principles of probability.

Peirce: CP 6.387 Cross-Ref:††

From the point of view of causality exercised by our ideas and upon our ideas, the relations of *prius* and *posterius* present a different problem. Our wishes and endeavors cannot change the past in the least degree; and the future cannot affect our senses. The past affects the senses, and more and more strongly the nearer it is; our will can affect the future, and more and more strongly the nearer it is. The consequence is that the whole procedure of investigating the past and the future is different from the problem as regards real time.

Peirce: CP 6.387 Cross-Ref:††

This kind of priority is divided by the schoolmen into priority *quoad existentiam* and *quoad generationem* (that is, the older of two is the prior).

Peirce: CP 6.388 Cross-Ref:††

388. (2) In a meaning allied to temporal priority, Aristotle sometimes speaks of sense as *prior* to reason.†1

Peirce: CP 6.388 Cross-Ref:††

(3) That which is at an earlier stage of development is also called prior to that which is more matured; boyhood is said by Aristotle †2 to be prior to manhood.

Peirce: CP 6.388 Cross-Ref:††

(4) So matter is prior to form; and potency to energy.

Peirce: CP 6.388 Cross-Ref:††

(5) The simple is prior to the complex; as a point to a line, a line to a surface, a surface to a solid.

Peirce: CP 6.388 Cross-Ref:††

(6) The rudimentary is prior to the recondite in order of exposition.

Peirce: CP 6.388 Cross-Ref:††

(7) In order of arrangement, the thing reached sooner is prior to that reached later.

Peirce: CP 6.388 Cross-Ref:††

(8) The relatively independent is prior to the relatively dependent, as substance to accident, and parts to whole.

Peirce: CP 6.388 Cross-Ref:††

(9) That caused thing which is nearer the cause, in any of the four senses of cause, is prior to that which is further from the cause.

Peirce: CP 6.388 Cross-Ref:††

(10) That is "prior in illation" from which the posterior follows as a rational consequence.

Peirce: CP 6.388 Cross-Ref:††

(11) The more general is prior to the more special.

Peirce: CP 6.388 Cross-Ref:††

(12) That which is more honorable or higher in rank or dignity is prior to that which is less so.

Peirce: CP 6.389 Cross-Ref:††

389. *Prius natura*, as practically used by Aristotle,†3 seems often to convey no clear notion. But he certainly calls the *prius dignitate* and *prius causalitate* both *prius natura*. The usage of the Aristotelians is to call that *prius natura* which is prior in consecution or in causality. That is prior in consecution which is such that if something else is supposed it is supposed, but which being supposed something else is not thereby supposed. Thus, if two are supposed, one is supposed; but one being supposed, two is not thereby supposed. Hence, one is prior to two. *Prius causalitate* is either *prius natura generantis* or *prius natura intendente*. *Prius natura*

generantis is the priority of the simple to the complex, as of the parts to the whole; *prius natura intendente* is the priority of the perfect to the imperfect, as of the whole to the parts. But this hardly seems to agree with Aristotle (646 a 25). *Prius nobis* ({pros hémas proteron}) is what is prior in the order of learning, or more easily known.

Peirce: CP 6.390 Cross-Ref:††
§17. PROXIMATE †1

390. Lat. past participle of *proximare*, to approach, but it is used to translate *proximus*, next. The word occurs in Glanvil's *Vanity of Dogmatizing*, but in no English treatise on logic before Watts. In philosophy, synonymous with IMMEDIATE, though not so strong.

Peirce: CP 6.391 Cross-Ref:††

391. *Proximate cause and effect*: an obscure term, like most of the terms of Aristotelianism, which acquired some practical importance owing to the courts holding that a man was responsible for the proximate effects of his actions, not for their remote effects. This ought to determine what should be meant by *proximate cause and effect*; namely, that that which a man ought to have foreseen might result from his action is its *proximate effect*. The idea of making the payment of considerable damages dependent upon a term of Aristotelian logic or metaphysics is most shocking to any student of those subjects, and well illustrates the value of PRAGMATISM. Burgersdicius, who is one of the clearest of the Aristotelians, says: "*Proximate cause* is taken in two senses, to wit *in suo genere* and absolutely. An absolutely proximate cause is one which constitutes its effect, not merely immediately, but by its mere existence; so that, if it exists, its effect (*causatum*, for Burgersdicius is not limiting his remarks to efficient causes) necessarily exists. The proximate cause *in suo genere* is that which immediately constitutes its effect, that is to say, without the intervention of anything else of the same order concurring to produce the effect." Interpreting this in the light of pragmatism, the man should be held responsible for what might naturally be expected, or feared, as the result of his action; but not for effects depending upon subsequent occurrences which he could not anticipate. Burgersdicius continues: "One thing may have many causes, proximate *in suo genere*, but only one absolutely proximate. . . . So the proximate material cause of man is his body; the efficient, his father; the formal, his rational soul; the final, *bene esse*."

Peirce: CP 6.392 Cross-Ref:††

392. *Proximate knowledge* is direct knowledge of a thing, not knowledge through something else. Better called *direct knowledge*.

Peirce: CP 6.392 Cross-Ref:††

Proximate matter is matter in a state in which it is prepared for the reception of a form. The proximate matter of a syllogism consists in its propositions, as distinct from the remote matter, which consists in the terms.

Peirce: CP 6.392 Cross-Ref:††

Proximate object of a directive (as we now say, normative) science is a certain one of the objects of practice, as distinguished from the object of doctrine. In

speculative sciences there is only one object, the object of doctrine. In practical sciences there is besides an object of practice, which is that upon which it is designed to produce an effect. In a normative science, such as logic, there are two objects of practice -- *the proximate*, which is the operation or action which is regulated, such as reasoning, and the remote, which is that in which that action takes place, such as a mind or a science conducted by many minds.

Peirce: CP 6.392 Cross-Ref:††

Proximate witness, testimony. There is hardly any such thing in English law. It is the witness who testifies, not to his own experience, but to facts which he knows by the immediate testimony of others.

Peirce: CP 6.393 Cross-Ref:††
§18. SUFFICIENT REASON †1

393. (1) This phrase was made a term of philosophy, if not invented, by Leibnitz. In the *Principes de la Nature et de la Grâce*, he says (but this is far from being the first time in which he signalizes the principle): "It is necessary to resort to metaphysics and to make use of a *great principle*, not much employed, to the effect that nothing takes place without reason (*rien ne se fait sans raison suffisante*); that is to say, that nothing occurs for which one having sufficient knowledge might not be able to give a reason sufficient to determine why it is as it is and not otherwise."

Peirce: CP 6.393 Cross-Ref:††

It is impossible to understand what Leibnitz means by this, without careful study of his works. There are two difficulties. In the first place, Leibnitz confounded under this phrase two entirely different ideas which he failed to discriminate. In the second place, in order to understand Leibnitz's position here, it is necessary to take into account, on the one hand, the thorough individualistic nominalism, with which he began his philosophical life and never consciously surrendered, and on the other hand his recognition of intellectual relations in the universe of which that nominalistic metaphysics involves the denial. His singular and complicated metaphysics is the outcome of his struggle to reconcile those two incompatible positions.

Peirce: CP 6.393 Cross-Ref:††

His sufficient reason is not an efficient cause, but a utility, or, in a broad sense, a final cause. But a nominalist cannot admit that an immediate final cause *exists*. Leibnitz, however, makes it *true*. For a realist, the real is nothing but the immediate object of that which is true. But Leibnitz has another notion of truth. Thus, in a letter to Arnauld (quoted in Latta's accurate and convenient exposition, p. 61, *note* beginning p. 60), he says: "Always in every true affirmative proposition, whether necessary or contingent, universal or singular, the notion of the predicate is in some way comprehended in that of the subject, *praedicatum inest subiecto*; otherwise I know not what truth is"; and in other passages he shows that for him truth is a relation between notions. Yet, as a nominalist, he could not hold that those notions immediately correspond to anything real. Consequently, he does not say that there really is a sufficient reason, but that anybody favorably situated would be able to render a sufficient reason. There is nothing real that corresponds to it immediately. Remotely, the purpose of God may correspond to it. Thus, the world of reality and the

world of truth are completely sundered; for the former, Leibnitz is a pure individualistic nominalist; for the latter, on the contrary, he is an intellectualist. When he says, for example, that that which has no sufficient reason is "necessarily" non-existent, he uses the adverb of logical not of metaphysical modality. He does not hold that real things are either emanations or entelechies of anything corresponding to a sufficient reason, but that is how the mind is affected. But when he comes to the ultimate sufficient reason of contingent truths, which is God, he ceases to draw the distinction between the world of thought and the world of being; and this exception introduces difficulties into his system. But Leibnitz confounds two things under his word "reason." The idea which principally governs his doctrine is that a reason is an explanation of the utility of that of which it is a reason; but he includes under the same word any explanation of the logical necessity of the object, the why it follows from a general law. Hence, in many cases, his sufficient reason fulfills the function of an efficient cause. It would be quite possible to quote passages from Leibnitz which conflict with this account of his conception. In order that the reader should apprehend it as he did, it would be requisite that his mind should be in the same unclear condition, which is not possible after one has once attained a superior grade of clearness. We can account for his implicit contradictions, but cannot reproduce his apprehension of them when we once see them to be contradictions.

Peirce: CP 6.393 Cross-Ref:††

It is to be remarked that Renouvier and Prat, in their rehabilitation of Leibnitzianism, reject the principle of sufficient reason (*La Nouvelle Monadologie*, p. 41, note 29).

Peirce: CP 6.394 Cross-Ref:††

394. The principle of sufficient reason may very well be understood to express our natural expectation or hope to find each unexpected phenomenon to be subject to reason and so to be intelligible. But to entertain this hope for *each* is not necessarily to entertain it for *all*. At any rate, it is easy to see that, however strong the tendency may be, it does not amount to any such absolute and ineluctable necessity as attaches to the law of contradiction, by the side of which Leibnitz and many Germans have placed it. Moreover, however important this tendency of thought and this truth about the universe may be in reference to the development of science, nevertheless, like the principle of the uniformity of nature, its strictly logical application to add force to arguments is very limited indeed. The *modus ponens* and *modus tollens* stand in no need of any such general principle to be perfectly apodictic. It is essential to no broad division of reasoning. As a general rule, when we infer that a particular phenomenon, or set of phenomena, which seemed surprising at first, is to be explained as a consequence of a fact or law not directly observable, the argument is not appreciably strengthened by a separate assumption that the phenomenon has *some* explanation; although there are special cases in which it can be fortified by a similar, but more definite, premiss.

Peirce: CP 6.395 Cross-Ref:††

BOOK II

RELIGION

CHAPTER 1

THE ORDER OF NATURE†1

§1. THE SIGNIFICANCE OF ORDERE

395. Any proposition whatever concerning the order of Nature must touch more or less upon religion. In our day, belief, even in these matters, depends more and more upon the observation of facts. If a remarkable and universal orderliness be found in the universe, there must be some cause for this regularity, and science has to consider what hypotheses might account for the phenomenon. One way of accounting for it, certainly, would be to suppose that the world is ordered by a superior power. But if there is nothing in the universal subjection of phenomena to laws, nor in the character of those laws themselves (as being benevolent, beautiful, economical, etc.), which goes to prove the existence of a governor of the universe, it is hardly to be anticipated that any other sort of evidence will be found to weigh very much with minds emancipated from the tyranny of tradition.

Peirce: CP 6.396 Cross-Ref:††

396. Nevertheless, it cannot truly be said that even an absolutely negative decision of that question could altogether destroy religion, inasmuch as there are faiths in which, however much they differ from our own, we recognize those essential characters which make them worthy to be called religions, and which, nevertheless, do not postulate an actually existing Deity. That one, for instance, which has had the most numerous and by no means the least intelligent following of any on earth, teaches that the Divinity in his highest perfection is wrapped away from the world in a state of profound and eternal sleep, which really does not differ from non-existence, whether it be called by that name or not. No candid mind who has followed the writings of M. Vacherot can well deny that his religion is as earnest as can be. He worships the Perfect, the Supreme Ideal; but he conceives that the very notion of the Ideal is repugnant to its real existence.†1 In fact, M. Vacherot finds it agreeable to his reason to assert that nonexistence is an essential character of the perfect, just as St. Anselm and Descartes found it agreeable to theirs to assert the extreme opposite. I confess that there is one respect in which either of these positions seems to me more congruous with the religious attitude than that of a theology which stands upon evidences; for as soon as the Deity presents himself to either Anselm or Vacherot, and manifests his glorious attributes, whether it be in a vision of the night or day, either of them recognizes his adorable God, and sinks upon his knees at once; whereas the theologian of evidences will first demand that the divine apparition shall identify himself, and only after having scrutinized his credentials and weighed the probabilities of his being found among the totality of existences will he finally render his circumspect homage, thinking that no characters can be adorable but those which

belong to a real thing.

Peirce: CP 6.397 Cross-Ref:††

397. If we could find out any general characteristic of the universe, any mannerism in the ways of Nature, any law everywhere applicable and universally valid, such a discovery would be of such singular assistance to us in all our future reasoning that it would deserve a place almost at the head of the principles of logic. On the other hand, if it can be shown that there is nothing of the sort to find out, but that every discoverable regularity is of limited range, this again will be of logical importance. What sort of a conception we ought to have of the universe, how to think of the *ensemble* of things, is a fundamental problem in the theory of reasoning.

Peirce: CP 6.398 Cross-Ref:††
§2. UNIFORMITIESE

398. It is the legitimate endeavor of scientific men now, as it was twenty-three hundred years ago, to account for the formation of the solar system and of the cluster of stars which forms the galaxy, by the fortuitous concourse of atoms. The greatest expounder of this theory, when asked how he could write an immense book on the system of the world without one mention of its author, replied, very logically, "Je n'avais pas besoin de cette hypothèse-là." But, in truth, there is nothing atheistical in the theory, any more than there was in this answer. Matter is supposed to be composed of molecules which obey the laws of mechanics and exert certain attractions upon one another; and it is to these regularities (which there is no attempt to account for) that general arrangement of the solar system would be due, and not to hazard.

Peirce: CP 6.399 Cross-Ref:††

399. If anyone has ever maintained that the universe is a pure throw of the dice, the theologians have abundantly refuted him. "How often," says Archbishop Tillotson, "might a man, after he had jumbled a set of letters in a bag, fling them out upon the ground before they would fall into an exact poem, yea, or so much as make a good discourse in prose! And may not a little book be as easily made by chance as this great volume of the world?"†1 The chance-world, here shown to be so different from that in which we live, would be one in which there were no laws, the characters of different things being entirely independent; so that, should a sample of any kind of objects ever show a prevalent character, it could only be by accident, and no general proposition could ever be established. Whatever further conclusions we may come to in regard to the order of the universe, this much may be regarded as solidly established, that the world is not a mere chance-medley.

Peirce: CP 6.399 Cross-Ref:††

But whether the world makes an exact poem or not, is another question. When we look up at the heavens at night, we readily perceive that the stars are not simply splashed onto the celestial vault; but there does not seem to be any precise system in their arrangement either. It will be worth our while, then, to inquire into the degree of orderliness in the universe; and, to begin, let us ask whether the world we live in is any more orderly than a purely chance-world would be.

Peirce: CP 6.400 Cross-Ref:††

400. Any uniformity, or law of Nature, may be stated in the form, "Every *A* is *B*"; as, every ray of light is a non-curved line, every body is accelerated toward the earth's center, etc. This is the same as to say, "There does not exist any *A* which is not *B*"; there is no curved ray; there is no body not accelerated toward the earth; so that the uniformity consists in the non-occurrence in Nature of a certain combination of characters (in this case, the combination of being *A* with being non-*B*). †P1 And, conversely, every case of the non-occurrence of a combination of characters would constitute a uniformity in Nature. Thus, suppose the quality *A* is never found in combination with the quality *C*: for example, suppose the quality of idiocy is never found in combination with that of having a well-developed brain. Then nothing of the sort *A* is of the sort *C*, or everything of the sort *A* is of the sort non-*C* (or say, every idiot has an ill-developed brain), which, being something universally true of the *A*'s, is a uniformity in the world. Thus we see that, in a world where there were no uniformities, no logically possible combination of characters would be excluded, but every combination would exist in some object. But two objects not identical must differ in some of their characters, though it be only in the character of being in such and such a place. Hence, precisely the same combination of characters could not be found in two different objects; and, consequently, in a chance-world every combination involving either the positive or negative of every character would belong to just one thing. Thus, if there were but five simple characters in such a world, †P2 we might denote them by *A*, *B*, *C*, *D*, *E*, and their negatives by *a*, *b*, *c*, *d*, *e*; and then, as there would be 2⁵ or 32 different combinations of these characters, completely determinate in reference to each of them, that world would have just 32 objects in it, their characters being as in the following table:

TABLE I

ABCDE	AbCDE	aBCDE	abCDE
ABCDe	AbCDe	aBCDe	abCDe
ABCdE	AbCdEa	BCdEab	CdEabCdE
ABCde	AbCde	aBCde	abCde
ABcDE	AbcDEa	BcDEabc	DEabcDE
ABcDe	AbcDe	aBcDe	abcDe
ABcdE	ABcdE	aBcdE	abcdE
ABcde	Abcde	aBcde	abcde

Peirce: CP 6.400 Cross-Ref: ††

For example, if the five primary characters were *hard*, *sweet*, *fragrant*, *green*, *bright*, there would be one object which reunited all these qualities, one which was hard, sweet, fragrant, and green, but not bright; one which was hard, sweet, fragrant, and bright, but not green; one which was hard, sweet, and fragrant, but neither green nor bright; and so on through all the combinations.

Peirce: CP 6.401 Cross-Ref:††

401. This is what a thoroughly chance-world would be like, and certainly nothing could be imagined more systematic. When a quantity of letters are poured out of a bag, the appearance of disorder is due to the circumstance that the phenomena are only partly fortuitous. The laws of space are supposed, in that case, to be rigidly preserved, and there is also a certain amount of regularity in the formation of the letters. The result is that some elements are orderly and some are disorderly, which is precisely what we observe in the actual world. Tillotson, in the passage of which a part has been quoted, goes on to ask, "How long might 20,000 blind men which should be sent out from the several remote parts of England, wander up and down before they would all meet upon Salisbury Plains, and fall into rank and file in the exact order of an army? And yet this is much more easy to be imagined than how the innumerable blind parts of matter should rendezvous themselves into a world."†1 This is very true, but in the actual world the *blind men* are, as far as we can see, *not* drawn up in any particular order at all. And, in short, while a certain amount of order exists in the world, it would seem that the world is not so orderly as it might be, and, for instance, not so much so as a world of pure chance would be.

Peirce: CP 6.402 Cross-Ref:††

402. But we can never get to the bottom of this question until we take account of a highly-important logical principle †P1 which I now proceed to enounce. This principle is that any plurality or lot of objects whatever have some character in common (no matter how insignificant) which is peculiar to them and not shared by anything else. The word "character" here is taken in such a sense as to include negative characters, such as incivility, inequality, etc., as well as their positives, civility, equality, etc. To prove the theorem, I will show what character any two things, *A* and *B*, have in common, not shared by anything else. The things, *A* and *B*, are each distinguished from all other things by the possession of certain characters which may be named *A*-ness and *B*-ness. Corresponding to these positive characters are the negative characters un-*A*-ness, which is possessed by everything except *A*, and un-*B*-ness, which is possessed by everything except *B*. These two characters are united in everything except *A* and *B*; and this union of the characters un-*A*-ness and un-*B*-ness makes a compound character which may be termed *A-B*-lessness. This is not possessed by either *A* or *B*, but it is possessed by everything else. This character, like every other, has its corresponding negative un-*A-B*-lessness, and this last is the character possessed by both *A* and *B*, and by nothing else. It is obvious that what has thus been shown true of two things is *mutatis mutandis*, true of any number of things. Q. E. D.

Peirce: CP 6.403 Cross-Ref:††

403. In any world whatever, then, there must be a character peculiar to each possible group of objects. If, as a matter of nomenclature, characters peculiar to the same group be regarded as only different aspects of the same character, then we may say that there will be precisely one character for each possible group of objects. Thus, suppose a world to contain five things, $\alpha, \beta, \gamma, \delta, \epsilon$. Then it will have a separate character for each of the 31 groups (with *non-existence* making up 32 or 2⁵) shown in the following table:

Peirce: CP 6.403 Cross-Ref:††

TABLE II

{ab} {abg} {abgd} {abgde}
 {a} {ag} {abd} {abge}
 {b} {ad} {abe} {abde}
 {g} {ae} {agd} {agde}
 {d} {bg} {age} {bgde}
 {e} {bd} {ade}
 {be} {bgd}
 {gd} {bge}
 {ge} {bde}
 {de} {gde}

Peirce: CP 6.404 Cross-Ref:††

404. This shows that a contradiction is involved in the very idea of a chance-world, for in a world of 32 things, instead of there being only 3^5 or 243 characters, as we have seen that the notion of a chance-world requires, there would, in fact, be no less than 2^{32} , or 4,294,967,296 characters, which would not be all independent, but would have all possible relations with one another.

Peirce: CP 6.405 Cross-Ref:††

405. We further see that so long as we regard characters abstractly, without regard to their relative importance, etc., there is no possibility of a more or less degree of orderliness in the world, the whole system of relationship between the different characters being given by mere logic; that is, being implied in those facts which are tacitly admitted as soon as we admit that there is any such thing as reasoning.

Peirce: CP 6.406 Cross-Ref:††

406. In order to descend from this abstract point of view, it is requisite to consider the characters of things as relative to the perceptions and active powers of living beings. Instead, then, of attempting to imagine a world in which there should be no uniformities, let us suppose one in which none of the uniformities should have reference to characters interesting or important to us. In the first place, there would be nothing to puzzle us in such a world. The small number of qualities which would directly meet the senses would be the ones which would afford the key to everything which could possibly interest us. The whole universe would have such an air of system and perfect regularity that there would be nothing to ask. In the next place, no action of ours, and no event of Nature, would have important consequences in such a world. We should be perfectly free from all responsibility, and there would be nothing to do but to enjoy or suffer whatever happened to come along. Thus there would be nothing to stimulate or develop either the mind or the will, and we consequently should neither act nor think. We should have no memory, because that depends on a law of our organization. Even if we had any senses, we should be situated toward such a world precisely as inanimate objects are toward the present one, provided we

suppose that these objects have an absolutely transitory and instantaneous consciousness without memory -- a supposition which is a mere mode of speech, for that would be no consciousness at all. We may, therefore, say that a world of chance is simply our actual world viewed from the standpoint of an animal at the very vanishing-point of intelligence. The actual world is almost a chance-medley to the mind of a polyp. The interest which the uniformities of Nature have for an animal measures his place in the scale of intelligence.

Peirce: CP 6.407 Cross-Ref:††

407. Thus, nothing can be made out from the orderliness of Nature in regard to the existence of a God, unless it be maintained that the existence of a finite mind proves the existence of an infinite one.

Peirce: CP 6.408 Cross-Ref:††
§3. INDUCTIONE

408. In the last of these papers †1 we examined the nature of inductive or synthetic reasoning. We found it to be a process of sampling. A number of specimens of a class are taken, not by selection within that class, but at random. These specimens will agree in a great number of respects. If, now, it were likely that a second lot would agree with the first in the majority of these respects, we might base on this consideration an inference in regard to any one of these characters. But such an inference would neither be of the nature of induction, nor would it (except in special cases) be valid, because the vast majority of points of agreement in the first sample drawn would generally be entirely accidental, as well as insignificant. To illustrate this, I take the ages at death of the first five poets given in Wheeler's *Biographical Dictionary*. They are:

Aagard, 48.

Abeille, 70.

Abulola, 84.

Abunowas, 48.

Accords, 45.

These five ages have the following characters in common:

Peirce: CP 6.408 Cross-Ref:††

1. The difference of the two digits composing the number, divided by three, leaves a remainder of *one*.

Peirce: CP 6.408 Cross-Ref:††

2. The first digit raised to the power indicated by the second, and divided by three, leaves a remainder of *one*.

Peirce: CP 6.408 Cross-Ref:††

3. The sum of the prime factors of each age, including one, is divisible by three.

Peirce: CP 6.409 Cross-Ref:††

409. It is easy to see that the number of accidental agreements of this sort would be quite endless. But suppose that, instead of considering a character because of its prevalence in the sample, we designate a character before taking the sample, selecting it for its importance, obviousness, or other point of interest. Then two considerable samples drawn at random are extremely likely to agree approximately in regard to the proportion of occurrences of a character so chosen. ***The inference that a previously designated character has nearly the same frequency of occurrence in the whole of a class that it has in a sample drawn at random out of that class is induction.*** If the character be not previously designated, then a sample in which it is found to be prevalent can only serve to suggest that it may be prevalent in the whole class. We may consider this surmise as an inference if we please -- an inference of possibility; but a second sample must be drawn to test the question of whether the character actually is prevalent. Instead of designating beforehand a single character in reference to which we will examine a sample, we may designate two, and use the same sample to determine the relative frequencies of both. This will be making two inductive inferences at once; and, of course, we are less certain that both will yield correct conclusions than we should be that either separately would do so. What is true of two characters is true of any limited number. Now, the number of characters which have any considerable interest for us in reference to any class of objects is more moderate than might be supposed. As we shall be sure to examine any sample with reference to these characters, they may be regarded not exactly as predesignated, but as predetermined (which amounts to the same thing); and we may infer that the sample represents the class in all these respects if we please, remembering only that this is not so secure an inference as if the particular quality to be looked for had been fixed upon beforehand.

Peirce: CP 6.410 Cross-Ref:††

410. The demonstration of this theory of induction rests upon principles and follows methods which are accepted by all those who display in other matters the particular knowledge and force of mind which qualify them to judge of this. The theory itself, however, quite unaccountably seems never to have occurred to any of the writers who have undertaken to explain synthetic reasoning. The most widely-spread opinion in the matter is one which was much promoted by Mr. John Stuart Mill †1 -- namely, that induction depends for its validity upon the uniformity of Nature -- that is, on the principle that what happens once will, under a sufficient degree of similarity of circumstances, happen again as often as the same circumstances recur. The application is this: The fact that different things belong to the same class constitutes the similarity of circumstances, and the induction is good, provided this similarity is "sufficient." What happens once is, that a number of these things are found to have a certain character; what may be expected, then, to happen again as often as the circumstances recur consists in this, that all things belonging to the same class should have the same character.

Peirce: CP 6.411 Cross-Ref:††

411. This analysis of induction has, I venture to think, various imperfections, to some of which it may be useful to call attention.†2 In the first place, when I put my hand in a bag and draw out a handful of beans, and, finding three-quarters of them black, infer that about three-quarters of all in the bag are black, my inference is

obviously of the same kind as if I had found any larger proportion, or the whole, of the sample black, and had assumed that it represented in that respect the rest of the contents of the bag. But the analysis in question hardly seems adapted to the explanation of this *proportionate* induction, where the conclusion, instead of being that a certain event uniformly happens under certain circumstances, is precisely that it does not uniformly occur, but only happens in a certain proportion of cases. It is true that the whole sample may be regarded as a single object, and the inference may be brought under the formula proposed by considering the conclusion to be that any similar sample will show a similar proportion among its constituents. But this is to treat the induction as if it rested on a single instance, which gives a very false idea of its probability.

Peirce: CP 6.412 Cross-Ref:††

412. In the second place, if the uniformity of Nature were the sole warrant of induction, we should have no right to draw one in regard to a character whose constancy we knew nothing about. Accordingly, Mr. Mill says †1 that, though none but white swans were known to Europeans for thousands of years, yet the inference that all swans were white was "not a good induction," because it was not known that color was a usual generic character (it, in fact, not being so by any means). But it is mathematically demonstrable that an inductive inference may have as high a degree of probability as you please independent of any antecedent knowledge of the constancy of the character inferred. Before it was known that color is not usually a character of genera, there was certainly a considerable probability that all swans were white. But the further study of the genera of animals led to the induction of their non-uniformity in regard to color. A deductive application of this general proposition would have gone far to overcome the probability of the universal whiteness of swans before the black species was discovered. When we do know anything in regard to the general constancy or inconstancy of a character, the application of that general knowledge to the particular class to which any induction relates, though it serves to increase or diminish the force of the induction, is, like every application of general knowledge to particular cases, deductive in its nature and not inductive.

Peirce: CP 6.413 Cross-Ref:††

413. In the third place, to say that inductions are true because similar events happen in similar circumstances -- or, what is the same thing, because objects similar in some respects are likely to be similar in others -- is to overlook those conditions which really are essential to the validity of inductions. When we take all the characters into account, any pair of objects resemble one another in just as many particulars as any other pair. If we limit ourselves to such characters as have for us any importance, interest, or obviousness, then a synthetic conclusion may be drawn, but only on condition that the specimens by which we judge have been taken at random from the class in regard to which we are to form a judgment, and not selected as belonging to any sub-class. The induction only has its full force when the character concerned has been designated before examining the sample. These are the essentials of induction, and they are not recognized in attributing the validity of induction to the uniformity of Nature. The explanation of induction by the doctrine of probabilities, given in the last of these papers, is not a mere metaphysical formula, but is one from which all the rules of synthetic reasoning can be deduced systematically and with mathematical cogency. But the account of the matter by a principle of Nature, even if it were in other respects satisfactory, presents the fatal disadvantage of leaving us quite as much afloat as before in regard to the proper method of induction. It does not surprise me, therefore, that those who adopt this theory have given erroneous rules for

the conduct of reasoning, nor that the greater number of examples put forward by Mr. Mill in his first edition, as models of what inductions should be, proved in the light of further scientific progress so particularly unfortunate that they had to be replaced by others in later editions. One would have supposed that Mr. Mill might have based an induction on this circumstance, especially as it is his avowed principle that, if the conclusion of an induction turns out false, it cannot have been a good induction. Nevertheless, neither he nor any of his scholars seem to have been led to suspect, in the least, the perfect solidity of the framework which he devised for securely supporting the mind in its passage from the known to the unknown, although at its first trial it did not answer quite so well as had been expected.

Peirce: CP 6.414 Cross-Ref:††
§4. MIND AND NATURE E

414. When we have drawn any statistical induction -- such, for instance, as that one-half of all births are of male children -- it is always possible to discover, by investigation sufficiently prolonged, a class of which the same predicate may be affirmed universally; to find out, for instance, *what sort of* births are of male children. The truth of this principle follows immediately from the theorem that there is a character peculiar to every possible group of objects. The form in which the principle is usually stated is, that *every event must have a cause*.

Peirce: CP 6.415 Cross-Ref:††

415. But, though there exists a cause for every event, and that of a kind which is capable of being discovered, yet if there be nothing to guide us to the discovery; if we have to hunt among all the events in the world without any scent; if, for instance, the sex of a child might equally be supposed to depend on the configuration of the planets, on what was going on at the antipodes, or on anything else -- then the discovery would have no chance of ever getting made.

Peirce: CP 6.416 Cross-Ref:††

416. That we ever do discover the precise causes of things, that any induction whatever is absolutely without exception, is what we have no right to assume. On the contrary, it is an easy corollary, from the theorem just referred to, that every empirical rule has an exception. But there are certain of our inductions which present an approach to universality so extraordinary that, even if we are to suppose that they are not strictly universal truths, we cannot possibly think that they have been reached merely by accident. The most remarkable laws of this kind are those of *time* and *space*. With reference to space, Bishop Berkeley first showed, in a very conclusive manner, that it was not a thing *seen*, but a thing *inferred*.†1 Berkeley chiefly insists on the impossibility of directly seeing the third dimension of space, since the retina of the eye is a surface. But, in point of fact, the retina is not even a surface; it is a conglomeration of nerve-needles directed toward the light and having only their extreme points sensitive, these points lying at considerable distances from one another compared with their areas. Now, of these points, certainly the excitation of no one singly can produce the perception of a surface, and consequently not the aggregate of all the sensations can amount to this. But certain relations subsist between the excitations of different nerve-points, and these constitute the premisses upon which the hypothesis of space is founded, and from which it is inferred.†2 That space is not

immediately perceived is now universally admitted; and a mediate cognition is what is called an inference, and is subject to the criticism of logic. But what are we to say to the fact of every chicken as soon as it is hatched solving a problem whose data are of a complexity sufficient to try the greatest mathematical powers? It would be insane to deny that the tendency to light upon the conception of space is inborn in the mind of the chicken and of every animal. The same thing is equally true of time. That time is not directly perceived is evident, since no lapse of time is present, and we only perceive what is present. That, not having the idea of time, we should never be able to perceive the flow in our sensations without some particular aptitude for it, will probably also be admitted. The idea of force -- at least, in its rudiments -- is another conception so early arrived at, and found in animals so low in the scale of intelligence, that it must be supposed innate. But the innateness of an idea admits of degree, for it consists in the tendency of that idea to present itself to the mind. Some ideas, like that of space, do so present themselves irresistibly at the very dawn of intelligence, and take possession of the mind on small provocation, while of other conceptions we are prepossessed, indeed, but not so strongly, down a scale which is greatly extended. The tendency to personify every thing, and to attribute human characters to it, may be said to be innate; but it is a tendency which is very soon overcome by civilized man in regard to the greater part of the objects about him. Take such a conception as that of gravitation varying inversely as the square of the distance. It is a very simple law. But to say that it is simple is merely to say that it is one which the mind is particularly adapted to apprehend with facility. Suppose the idea of a quantity multiplied into another had been no more easy to the mind than that of a quantity raised to the power indicated by itself -- should we ever have discovered the law of the solar system?

Peirce: CP 6.417 Cross-Ref:††

417. It seems incontestable, therefore, that the mind of man is strongly adapted to the comprehension of the world; at least, so far as this goes, that certain conceptions, highly important for such a comprehension, naturally arise in his mind; and, without such a tendency, the mind could never have had any development at all.

Peirce: CP 6.418 Cross-Ref:††

418.†1 How are we to explain this adaptation? The great utility and indispensableness of the conceptions of time, space, and force, even to the lowest intelligence, are such as to suggest that they are the results of natural selection. Without something like geometrical, kinetical, and mechanical conceptions, no animal could seize his food or do anything which might be necessary for the preservation of the species. He might, it is true, be provided with an instinct which would generally have the same effect; that is to say, he might have conceptions different from those of time, space, and force, but which coincided with them in regard to the ordinary cases of the animal's experience. But, as that animal would have an immense advantage in the struggle for life whose mechanical conceptions did not break down in a novel situation (such as development must bring about), there would be a constant selection in favor of more and more correct ideas of these matters. Thus would be attained the knowledge of that fundamental law upon which all science rolls; namely, that forces depend upon relations of time, space, and mass. When this idea was once sufficiently clear, it would require no more than a comprehensible degree of genius to discover the exact nature of these relations. Such an hypothesis naturally suggests itself, but it must be admitted that it does not seem sufficient to account for the extraordinary accuracy with which these conceptions apply to the phenomena of Nature, and it is probable that there is some secret here

which remains to be discovered.

Peirce: CP 6.419 Cross-Ref:††
§5. DESIGNÆ

419. Some important questions of logic depend upon whether we are to consider the material universe as of limited extent and finite age, or quite boundless in space and in time. In the former case, it is conceivable that a general plan or design embracing the whole universe should be discovered, and it would be proper to be on the alert for some traces of such a unity. In the latter case, since the proportion of the world of which we can have any experience is less than the smallest assignable fraction, it follows that we never could discover any *pattern* in the universe except a repeating one; any design embracing the whole would be beyond our powers to discern, and beyond the united powers of all intellects during all time. Now, what is absolutely incapable of being known is, as we have seen in a former paper,†1 not real at all. An absolutely incognizable existence is a nonsensical phrase. If, therefore, the universe is infinite, the attempt to find in it any design embracing it as a whole is futile, and involves a false way of looking at the subject. If the universe never had any beginning, and if in space world stretches beyond world without limit, there is no *whole* of material things, and consequently no general character to the universe, and no need or possibility of any governor for it. But if there was a time before which absolutely no matter existed, if there are certain absolute bounds to the region of things outside of which there is a mere void, then we naturally seek for an explanation of it, and, since we cannot look for it among material things, the hypothesis of a great disembodied animal, the creator and governor of the world, is natural enough.

Peirce: CP 6.420 Cross-Ref:††

420. The actual state of the evidence as to the limitation of the universe is as follows: As to time, we find on our earth a constant progress of development since the planet was a red-hot ball; the solar system seems to have resulted from the condensation of a nebula, and the process appears to be still going on. We sometimes see stars (presumably with systems of worlds) destroyed and apparently resolved back into the nebulous condition, but we have no evidence of any existence of the world previous to the nebulous stage from which it seems to have been evolved. All this rather favors the idea of a beginning than otherwise. As for limits in space, we cannot be sure that we see anything outside of the system of the Milky Way. Minds of theological predilections have therefore no need of distorting the facts to reconcile them with their views.

Peirce: CP 6.421 Cross-Ref:††

421. But the only scientific presumption is, that the unknown parts of space and time are like the known parts, occupied; that, as we see cycles of life and death in all development which we can trace out to the end, the same holds good in regard to solar systems; that as enormous distances lie between the different planets of our solar system, relatively to their diameters, and as still more enormous distances lie between our system relatively to its diameter and other systems, so it may be supposed that other galactic clusters exist so remote from ours as not to be recognized as such with certainty. I do not say that these are strong inductions; I only say that they are the presumptions which, in our ignorance of the facts, should be preferred to hypotheses

which involve conceptions of things and occurrences totally different in their character from any of which we have had any experience, such as disembodied spirits, the creation of matter, infringements of the laws of mechanics, etc.

Peirce: CP 6.422 Cross-Ref:††

422. The universe ought to be presumed too vast to have any character. When it is claimed that the arrangements of Nature are benevolent, or just, or wise, or of any other peculiar kind, we ought to be prejudiced against such opinions, as being the offspring of an ill-founded notion of the finitude of the world. And examination has hitherto shown that such beneficences, justice, etc., are of a most limited kind -- limited in degree and limited in range.

Peirce: CP 6.423 Cross-Ref:††

423. In like manner, if anyone claims to have discovered a plan in the structure of organized beings, or a scheme in their classification, or a regular arrangement among natural objects, or a system of proportionality in the human form, or an order of development, or a correspondence between conjunctions of the planets and human events, or a significance in numbers, or a key to dreams, the first thing we have to ask is whether such relations are susceptible of explanation on mechanical principles, and if not they should be looked upon with disfavor as having already a strong presumption against them; and examination has generally exploded all such theories.

Peirce: CP 6.424 Cross-Ref:††

424. There are minds to whom every prejudice, every presumption, seems unfair. It is easy to say what minds these are. They are those who never have known what it is to draw a well-grounded induction, and who imagine that other people's knowledge is as nebulous as their own. That all science rolls upon presumption (not of a formal but of a real kind) is no argument with them, because they cannot imagine that there is anything solid in human knowledge. These are the people who waste their time and money upon perpetual motions and other such rubbish.

Peirce: CP 6.425 Cross-Ref:††

425. But there are better minds who take up mystical theories (by which I mean all those which have no possibility of being mechanically explained). These are persons who are strongly prejudiced in favor of such theories. We all have natural tendencies to believe in such things; our education often strengthens this tendency; and the result is, that to many minds nothing seems so antecedently probable as a theory of this kind. Such persons find evidence enough in favor of their views, and in the absence of any recognized logic of induction they cannot be driven from their belief.

Peirce: CP 6.425 Cross-Ref:††

But to the mind of a physicist there ought to be a strong presumption against every mystical theory; and therefore it seems to me that those scientific men who have sought to make out that science was not hostile to theology have not been so clear-sighted as their opponents.

Peirce: CP 6.426 Cross-Ref:††

426. It would be extravagant to say that science can at present disprove religion; but it does seem to me that the spirit of science is hostile to any religion except such a one as that of M. Vacherot. Our appointed teachers inform us that Buddhism is a miserable and atheistical faith, shorn of the most glorious and needful

attributes of a religion; that its priests can be of no use to agriculture by praying for rain, nor to war by commanding the sun to stand still. We also hear the remonstrances of those who warn us that to shake the general belief in the living God would be to shake the general morals, public and private. This, too, must be admitted; such a revolution of thought could no more be accomplished without waste and desolation than a plantation of trees could be transferred to new ground, however wholesome in itself, without all of them languishing for a time, and many of them dying. Nor is it, by the way, a thing to be presumed that a man would have taken part in a movement having a possible atheistical issue without having taken serious and adequate counsel in regard to that responsibility. But, let the consequences of such a belief be as dire as they may, one thing is certain: that the state of the facts, whatever it may be, will surely get found out, and no human prudence can long arrest the triumphal car of truth -- no, not if the discovery were such as to drive every individual of our race to suicide!

Peirce: CP 6.427 Cross-Ref: ††

427. But it would be folly to suppose that any metaphysical theory in regard to the mode of being of the perfect is to destroy that aspiration toward the perfect which constitutes the essence of religion. It is true that, if the priests of any particular form of religion succeed in making it generally believed that religion cannot exist without the acceptance of certain formulas, or if they succeed in so interweaving certain dogmas with the popular religion that the people can see no essential analogy between a religion which accepts these points of faith and one which rejects them, the result may very well be to render those who cannot believe these things irreligious. Nor can we ever hope that any body of priests should consider themselves more teachers of religion in general than of the particular system of theology advocated by their own party. But no man need be excluded from participation in the common feelings, nor from so much of the public expression of them as is open to all the laity, by the unphilosophical narrowness of those who guard the mysteries of worship. Am I to be prevented from joining in that common joy at the revelation of enlightened principles of religion which we celebrate at Easter and Christmas because I think that certain scientific, logical, and metaphysical ideas which have been mixed up with these principles are untenable? No; to do so would be to estimate those errors as of more consequence than the truth -- an opinion which few would admit. People who do not believe what are really the fundamental principles of Christianity are rare to find, and all but these few ought to feel at home in the churches.

Peirce: CP 6.428 Cross-Ref: ††
CHAPTER 2

A RELIGION OF SCIENCE

§1. THE MARRIAGE OF RELIGION AND SCIENCE †1

428. What is science? The dictionary will say that it is systematized knowledge. Dictionary definitions, however, are too apt to repose upon derivations; which is as much as to say that they neglect too much the later steps in the evolution of meanings. Mere knowledge, though it be systematized, may be a dead memory; while by science we all habitually mean a living and growing body of truth. We might even say that knowledge is not necessary to science. The astronomical researches of Ptolemy, though they are in great measure false, must be acknowledged by every modern mathematician who reads them to be truly and genuinely scientific. That which constitutes science, then, is not so much correct conclusions, as it is a correct method. But the method of science is itself a scientific result. It did not spring out of the brain of a beginner: it was a historic attainment and a scientific achievement. So that not even this method ought to be regarded as essential to the beginnings of science. That which is essential, however, is the scientific spirit, which is determined not to rest satisfied with existing opinions, but to press on to the real truth of nature. To science once enthroned in this sense, among any people, science in every other sense is heir apparent.

Peirce: CP 6.429 Cross-Ref:††

429. And what is religion? In each individual it is a sort of sentiment, or obscure perception, a deep recognition of a something in the circumambient All, which, if he strives to express it, will clothe itself in forms more or less extravagant, more or less accidental, but ever acknowledging the first and last, the {A} and {Ö}, as well as a relation to that Absolute of the individual's self, as a relative being. But religion cannot reside in its totality in a single individual. Like every species of reality, it is essentially a social, a public affair. It is the idea of a whole church, welding all its members together in one organic, systemic perception of the Glory of the Highest -- an idea having a growth from generation to generation and claiming a supremacy in the determination of all conduct, private and public.

Peirce: CP 6.430 Cross-Ref:††

430. Now, as science grows, it becomes more and more perfect, considered as science; and no religionist can easily so narrow himself as to deny this. But as religion goes through the different stages of its history, it has, I fear we must confess, seldom been seen so vitalized as to become more and more perfect, even as judged from its own standpoint. Like a plucked flower, its destiny is to wilt and fade. The vital sentiment that gave it birth loses gradually its pristine purity and strength, till some new creed treads it down. Thus it happens quite naturally that those who are animated with the spirit of science are for hurrying forward, while those who have the interests of religion at heart are apt to press back.

Peirce: CP 6.431 Cross-Ref:††

431. While this double change has been taking place, religion has found herself compelled to define her position; and, in doing so, has inevitably committed herself to sundry propositions, which, one by one, have been, first questioned, then assailed, and finally overthrown by advancing science. Seeing such a chasm open before her feet, religion has at first violently recoiled, and at last has leapt it; satisfying herself as best she might with an altered creed. In most cases the leap has not seemed to hurt her; yet internal injuries may have been sustained. Who can doubt that the church really did suffer from the discovery of the Copernican system, although infallibility, by a narrow loophole, managed to escape? In this way, science and religion become forced into hostile attitudes. Science, to specialists, may seem to have little or nothing to say that directly concerns religion; but it certainly encourages

a philosophy which, if in no other respect, is at any rate opposed to the prevalent tendency of religion, in being animated by a progressive spirit. There arises, too, a tendency to pooh-pooh at things unseen.

Peirce: CP 6.432 Cross-Ref:††

432. It would be ridiculous to ask to whose fault this situation is chargeable. You cannot lay blame upon elemental forces. Religion, from the nature of things, refuses to go through her successive transformations with sufficient celerity to keep always in accord with the convictions of scientific philosophy. The day has come, however, when the man whom religious experience most devoutly moves can recognize the state of the case. While adhering to the essence of religion, and so far as possible to the church, which is all but essential, say, penessential, to it, he will cast aside that religious timidity that is forever prompting the church to recoil from the paths into which the Governor of history is leading the minds of men, a cowardice that has stood through the ages as the landmark and limit of her little faith, and will gladly go forward, sure that truth is not split into two warring doctrines, and that any change that knowledge can work in his faith can only affect its expression, but not the deep mystery expressed.

Peirce: CP 6.433 Cross-Ref:††

433. Such a state of mind may properly be called a religion of science. Not that it is a religion to which science or the scientific spirit has itself given birth; for religion, in the proper sense of the term, can arise from nothing but the religious sensibility. But it is a religion, so true to itself, that it becomes animated by the scientific spirit, confident that all the conquests of science will be triumphs of its own, and accepting all the results of science, as scientific men themselves accept them, as steps toward the truth, which may appear for a time to be in conflict with other truths, but which in such cases merely await adjustments which time is sure to effect. This attitude, be it observed, is one which religion will assume not at the dictate of science, still less by way of a compromise, but simply and solely out of a bolder confidence in herself and in her own destiny.

Peirce: CP 6.434 Cross-Ref:††

434. Meantime, science goes unswervingly its own gait. What is to be its goal is precisely what it must not seek to determine for itself, but let itself be guided by nature's strong hand. Teleological considerations, that is to say ideals, must be left to religion; science can allow itself to be swayed only by efficient causes; and philosophy, in her character of queen of the sciences, must not care, or must not seem to care, whether her conclusions be wholesome or dangerous.

Peirce: CP 6.435 Cross-Ref:††
§2. WHAT IS CHRISTIAN FAITH?†1

435. It is easy to chop logic about matters of which you have no experience whatever. Men colour blind have more than once learnedly discussed the laws of colour-sensation, and have made interesting deductions from those laws. But when it comes to positive knowledge, such knowledge as a lawyer has of the practice of the courts, that can only rest on long experience, direct or indirect. So, a man may be an accomplished theologian without ever having felt the stirring of the spirit; but he cannot answer the simple question at the head of this article except out of his own

religious experience.

Peirce: CP 6.436 Cross-Ref:††

436. There is in the dictionary a word, *solipsism*, meaning the belief that the believer is the only existing person. Were anybody to adopt such a belief, it might be difficult to argue him out of it. But when a person finds himself in the society of others, he is just as sure of their existence as of his own, though he may entertain a metaphysical theory that they are all hypostatically the same *ego*. In like manner, when a man has that experience with which religion sets out, he has as good reason -- putting aside metaphysical subtleties -- to believe in the living personality of God as he has to believe in his own. Indeed, *belief* is a word inappropriate to such direct perception.

Peirce: CP 6.437 Cross-Ref:††

437. Seldom do we pass a single hour of our waking lives away from the companionship of men (including books); and even the thoughts of that solitary hour are filled with ideas which have grown in society. Prayer, on the other hand, occupies but little of our time; and, of course, if solemnity and ceremony are to be made indispensable to it (though why observe manners toward the Heavenly Father that an earthly father would resent as priggish?) nothing more is practicable. Consequently, religious ideas never come to form the warp and woof of our mental constitution, as do social ideas. They are easily doubted, and are open to various reasons for doubt, which reasons may all be comprehended under one, namely, that the religious phenomenon is sporadic, not incessant.

Peirce: CP 6.438 Cross-Ref:††

438. This causes a degeneration in religion from a perception to a trust, from a trust to a belief, and a belief continually becoming more and more abstract. Then, after a religion has become a public affair, quarrels arise, to settle which watchwords are drawn up. This business gets into the hands of theologians: and the ideas of theologians always appreciably differ from those of the universal church. They swamp religion in fallacious logical disputations. Thus, the natural tendency is to the continual drawing tighter and tighter of the narrowing bounds of doctrine, with less and less attention to the living essence of religion, until, after some *symbolum quodcumque* has declared that the salvation of each individual absolutely and almost exclusively depends upon his entertaining a correct metaphysics of the godhead, the vital spark of inspiration becomes finally quite extinct.

Peirce: CP 6.439 Cross-Ref:††

439. Yet it is absurd to say that religion is a mere belief. You might as well call society a belief, or politics a belief, or civilization a belief. Religion is a life, and can be identified with a belief only provided that belief be a living belief -- a thing to be lived rather than said or thought.

Peirce: CP 6.440 Cross-Ref:††

440. The Christian religion, if it has anything distinctive -- and must not aspire to be the necessary ultimate outcome of *every* path of religious progress -- is distinguished from other religions by its precept about the Way of Life. I appeal to the typical Christian to answer out of the abundance of his spirit, without dictation from priests, whether this be not so. In the recently discovered book, *The Teaching of the Twelve Apostles*, †P1 which dates from about A.D. 100, we see that, long before the Apostles' or any other creed was insisted upon, or at all used, the teaching of the Lord was considered to consist in the doctrine of the Two Ways -- the Way of Life and the

Way of Death. This it was that at that date was regarded as the saving faith -- not a lot of metaphysical propositions. This is what Jesus Christ taught; and to believe in Christ is to believe what he taught.

Peirce: CP 6.441 Cross-Ref:††

441. Now what is this way of life? Again I appeal to the universal Christian conscience to testify that it is simply love. As far as it is contracted to a rule of ethics, it is: Love God, and love your neighbour; "on these two commandments hang all the law and the prophets." It may be regarded in a higher point of view with St. John as the universal evolutionary formula. But in whatever light it be regarded or in whatever direction developed, the belief in the law of love is the Christian faith.

Peirce: CP 6.442 Cross-Ref:††

442. "Oh," but it may be said, "that is not distinctive of Christianity! That very idea was anticipated by the early Egyptians, by the Stoics, by the Buddhists, and by Confucius." So it was; nor can the not insignificant difference between the negative and the positive precept be properly estimated as sufficient for a discrimination between religions. Christians may, indeed, claim that Christianity possesses that earmark of divine truth -- namely, that it was anticipated from primitive ages. The higher a religion the more catholic.

Peirce: CP 6.443 Cross-Ref:††

443. Man's highest developments are social; and religion, though it begins in a seminal individual inspiration, only comes to full flower in a great church coextensive with a civilization. This is true of every religion, but supereminently so of the religion of love. Its ideal is that the whole world shall be united in the bond of a common love of God accomplished by each man's loving his neighbour. Without a church, the religion of love can have but a rudimentary existence; and a narrow, little exclusive church is almost worse than none. A great catholic church is wanted.

Peirce: CP 6.444 Cross-Ref:††

444. The invisible church does now embrace all Christendom. Every man who has been brought up in the bosom of Christian civilization does really believe in some form of the principle of love, whether he is aware of doing so, or not.

Peirce: CP 6.445 Cross-Ref:††

445. Let us, at any rate, get all the good from the vital element in which we are all at one that it can yield: and the good that it can yield is simply all that is anyway possible, and richer than is easily conceivable. Let us endeavour, then, with all our might to draw together the whole body of believers in the law of love into sympathetic unity of consciousness. Discountenance as immoral all movements that exaggerate differences, or that go to make fellowship depend on formulas invented to exclude some Christians from communion with others.

Peirce: CP 6.446 Cross-Ref:††

446. A sapient critic has recently blamed me for defective cocksureness in my metaphysical views. That is no less than an indictment for practicing just as I have always preached. *Absurd* was the epithet ever coming to my tongue for persons very confident in opinions which other minds, as good as they, denied. Can you induce the philosophic world to agree upon any assignable creed, or in condemning any specified item in the current creeds of Christendom? I believe not; though doubtless you can gather a sequacious little flock, quite disposed to follow their bell-bearer into every vagary -- if you will be satisfied so. For my part, I should think it more lovely

to patch up such peace as might be with the great religious world. This happens to be easy to an individual whose unbiased study of scientific logic has led him to conclusions not discordant with traditional dogmas. Unfortunately, such a case is exceptional; and guilt rests on you who insist on so tautening the lines of churches as to close them against the great body of educated and thinking men, pure and undefiled though the religion of many of them (you are obliged to acknowledge it) be. Surely another generation will witness a sweeping reform in this respect. You will not be permitted to make of those churches a permanent laughing-stock for coming ages. Many things are essential to religion which yet ought not to be insisted on: the law of love is not the rule of angry and bullying insistence. Thus, it seems plain to me, I confess, that miracles are intrinsic elements of a genuine religion.†1 But it is not half so important to emphasize this as it is to draw into our loving communion almost the entire collection of men who unite clear thought with intellectual integrity. And who are you, anyway, who are so zealous to keep the churches small and exclusive? Do you number among your party the great scholars and the great saints? Are you not, on the other hand, egged on by all the notorious humbugs -- votaries of Mammon or of Ward McAllister -- who deem the attitude of a church-caryatid to be a respectable or a genteel thing? Your voting-power, too, is repleted with many who, as soon as they are a little better informed and educated, will drop away from you; and in these days that education will come speedily.

Peirce: CP 6.447 Cross-Ref:††

447. To those who for the present are excluded from the churches, and who, in the passionate intensity of their religious desire, are talking of setting up a church for the scientifically educated, a man of my stripe must say, Wait, if you can; it will be but a few years longer; but if you cannot wait, why then Godspeed! Only, do not, in your turn, go and draw lines so as to exclude such as believe a little less -- or, still worse, to exclude such as believe a little more -- than yourselves. Doubtless, a lot of superstition clings to the historical churches; but superstition is the grime upon the venerable pavement of the sacred edifice, and he who would wash that pavement clean should be willing to get down on his knees to his work inside the church.

Peirce: CP 6.448 Cross-Ref:††

448. A religious organization is a somewhat idle affair unless it be sworn in as a regiment of that great army that takes life in hand, with all its delights, in grimmest fight to put down the principle of self-seeking, and to make the principle of love triumphant. It has something more serious to think about than the phraseology of the articles of war. Fall into the ranks then; follow your colonel. Keep your one purpose steadily and alone in view, and you may promise yourself the attainment of your sole desire, which is to hasten the chariot wheels of redeeming love!

Peirce: CP 6.449 Cross-Ref:††

§3. THE CHURCH †1

449. Many a scientific man and student of philosophy recognizes that it is the Christian church which has made him a man among men. To it he owes consolations, enjoyments, escapes from great perils, and whatever rectitude of heart and purpose may be his. To the monks of the medieval church he owes the preservation of ancient literature; and without the revival of learning he can hardly see how the revival of

science would have been possible. To them he owes the framework of his intellectual system, and if he speaks English, a most important part of his daily speech. The law of love which, however little it be obeyed, he holds to be the soul of civilization, came to Europe through Christianity. Besides, religion is a great, perhaps the greatest, factor of that social life which extends beyond one's own circle of personal friends. That life is everything for elevated, and humane, and democratic civilization; and if one renounces the Church, in what other way can one as satisfactorily exercise the faculty of fraternizing with all one's neighbours?

Peirce: CP 6.450 Cross-Ref:††

450. On the other hand, owe what one may to the Church, the truth claims paramount allegiance; and above the importance of any particular truth, or body of truths, is that of the right methods of reaching the truth. Now the Church requires subscription to a platform -- a Creed. And how has that platform been made? With strict party regularity, no doubt. Yet whether it be that of Trent, Lambeth, Geneva, or what, there is not one plank in it that has not, as a matter of historical fact, been inserted with a view of proclaiming the damnation and of procuring the persecution of some body of convinced Christians. Hence it is that the central doctrine of love is not to be found in any one of them. Granting for a moment that exact theology is a vital matter, as all creeds agree to make it (though had it been so deemed by the founder of Christianity he would have laid down his own formula in set terms), can anybody who understands the procedure of science, or has so much as read the first book of Bacon's *Novum Organum*, assent for a moment to the idea that any science, be it theology or any other, can be rightly developed under the impulses of ecclesiastical ambition and the *odium* of priests? Truth is the fruit of free inquiry and of such docility toward facts as shall make us always willing to acknowledge that we are wrong, and anxious to discover that we have been so.

Peirce: CP 6.451 Cross-Ref:††

451. The *raison d'être* of a church is to confer upon men a life broader than their narrow personalities, a life rooted in the very truth of being. To do that it must be based upon and refer to a definite and public experience. Fears of hell and hopes of paradise have no such reference; they are matters all sane men confess they know nothing about. Even for the greatest saints, the active motives were not such hopes and fears, but the prospect of leaving behind them fertile seeds of desirable fruits here on earth. It is not the question whether miracles and answers to prayer are abstractly possible. The question is whether they are appreciable constituents of human experiences, worth taking into account in comparison with those great facts of life that no man either doubts or ever will doubt.

Peirce: CP 6.452 Cross-Ref:††

CHAPTER 3

A NEGLECTED ARGUMENT FOR THE REALITY OF GOD†1

§1. MUSEMENTE

452. The word "God," so "capitalized" (as we Americans say), is *the* definable proper name, signifying *Ens necessarium*; in my belief Really creator of all three Universes of Experience.

Peirce: CP 6.452 Cross-Ref:††

Some words shall herein be capitalized when used, not as vernacular, but as terms defined. Thus an "idea" is the substance of an actual unitary thought or fancy; but "Idea," nearer Plato's idea of {idea}, denotes anything whose Being consists in its mere capacity for getting fully represented, regardless of any person's faculty or impotence to represent it.

Peirce: CP 6.453 Cross-Ref:††

453. "Real" is a word invented in the thirteenth century to signify having Properties, *i.e.* characters sufficing to identify their subject, and possessing these whether they be anywise attributed to it by any single man or group of men, or not. Thus, the substance of a dream is not Real, since it was such as it was, merely in that a dreamer so dreamed it; but the fact of the dream is Real, if it was dreamed; since if so, its date, the name of the dreamer, etc. make up a set of circumstances sufficient to distinguish it from all other events; and these belong to it, *i.e.* would be true if predicated of it, whether *A*, *B*, or *C* Actually ascertain them or not. The "Actual" is that which is met with in the past, present, or future.

Peirce: CP 6.454 Cross-Ref:††

454. An "Experience" is a brutally produced conscious effect that contributes to a habit, self-controlled, yet so satisfying, on deliberation, as to be destructible by no positive exercise of internal vigour. I use the word "self-controlled" for "controlled by the thinker's self," and not for "uncontrolled" except in its own spontaneous, *i.e.* automatic, self-development, as Professor J. M. Baldwin †2 uses the word. Take for illustration the sensation undergone by a child that puts its forefinger into a flame with the acquisition of a habit of keeping all its members out of all flames. A compulsion is "Brute," whose immediate efficacy nowise consists in conformity to rule or reason.

Peirce: CP 6.455 Cross-Ref:††

455. Of the three Universes of Experience familiar to us all †1, the first comprises all mere Ideas, those airy nothings to which the mind of poet, pure mathematician, or another *might* give local habitation and a name within that mind. Their very airy-nothingness, the fact that their Being consists in mere capability of getting thought, not in anybody's Actually thinking them, saves their Reality. The second Universe is that of the Brute Actuality of things and facts. I am confident that their Being consists in reactions against Brute forces, notwithstanding objections redoubtable until they are closely and fairly examined. The third Universe comprises everything whose being consists in active power to establish connections between different objects, especially between objects in different Universes. Such is everything which is essentially a Sign -- not the mere body of the Sign, which is not essentially such, but, so to speak, the Sign's Soul, which has its Being in its power of serving as intermediary between its Object and a Mind. Such, too, is a living consciousness, and such the life, the power of growth, of a plant. Such is a living constitution -- a daily newspaper, a great fortune, a social "movement."

Peirce: CP 6.456 Cross-Ref:††

456. An "Argument" is any process of thought reasonably tending to produce a definite belief.†2 An "Argumentation" is an Argument proceeding upon definitely formulated premisses.

Peirce: CP 6.457 Cross-Ref:††

457. If God Really be, and be benign, then, in view of the generally conceded truth that religion, were it but proved, would be a good outweighing all others, we should naturally expect that there would be some Argument for His Reality that should be obvious to all minds, high and low alike, that should earnestly strive to find the truth of the matter; and further, that this Argument should present its conclusion, not as a proposition of metaphysical theology, but in a form directly applicable to the conduct of life, and full of nutrition for man's highest growth. What I shall refer to as the N.A. -- the Neglected Argument -- seems to me best to fulfill this condition, and I should not wonder if the majority of those whose own reflections have harvested belief in God must bless the radiance of the N.A. for that wealth. Its persuasiveness is no less than extraordinary; while it is not unknown to anybody. Nevertheless, of all those theologians (within my little range of reading) who, with commendable assiduity, scrape together all the sound reasons they can find or concoct to prove the first proposition of theology, few mention this one, and they most briefly. They probably share those current notions of logic which recognize no other Arguments than Argumentations.

Peirce: CP 6.458 Cross-Ref:††

458. There is a certain agreeable occupation of mind which, from its having no distinctive name, I infer is not as commonly practiced as it deserves to be; for indulged in moderately -- say through some five to six per cent of one's waking time, perhaps during a stroll -- it is refreshing enough more than to repay the expenditure. Because it involves no purpose save that of casting aside all serious purpose, I have sometimes been half-inclined to call it reverie with some qualification; but for a frame of mind so antipodal to vacancy and dreaminess such a designation would be too excruciating a misfit. In fact, it is Pure Play. Now, Play, we all know, is a lively exercise of one's powers. Pure Play has no rules, except this very law of liberty. It bloweth where it listeth. It has no purpose, unless recreation. The particular occupation I mean -- a *petite bouchée* with the Universes -- may take either the form of aesthetic contemplation, or that of distant castle-building (whether in Spain or within one's own moral training), or that of considering some wonder in one of the Universes, or some connection between two of the three, with speculation concerning its cause. It is this last kind -- I will call it "Musement" on the whole -- that I particularly recommend, because it will in time flower into the N.A. One who sits down with the purpose of becoming convinced of the truth of religion is plainly not inquiring in scientific singleness of heart, and must always suspect himself of reasoning unfairly. So he can never attain the entirety even of a physicist's belief in electrons, although this is avowedly but provisional. But let religious meditation be allowed to grow up spontaneously out of Pure Play without any breach of continuity, and the Muser will retain the perfect candour proper to Musement.

Peirce: CP 6.459 Cross-Ref:††

459. If one who had determined to make trial of Musement as a favorite recreation were to ask me for advice, I should reply as follows: The dawn and the gloaming most invite one to Musement; but I have found no watch of the nychthemeron that has not its own advantages for the pursuit. It begins passively enough with drinking in the impression of some nook in one of the three Universes.

But impression soon passes into attentive observation, observation into musing, musing into a lively give and take of communion between self and self. If one's observations and reflections are allowed to specialize themselves too much, the Play will be converted into scientific study; and that cannot be pursued in odd half hours.

Peirce: CP 6.460 Cross-Ref:††

460. I should add: Adhere to the one ordinance of Play, the law of liberty. I can testify that the last half century, at least, has never lacked tribes of Sir Oracles, colporting brocards to bar off one or another roadway of inquiry; and a Rabelais would be needed to bring out all the fun that has been packed in their airs of infallibility. Auguste Comte, notwithstanding his having apparently produced some unquestionably genuine thinking, was long the chief of such a band. The vogue of each particular maxim of theirs was necessarily brief. For what distinction can be gained by repeating saws heard from all mouths? No bygone fashion seems more grotesque than a *panache* of obsolete wisdom. I remember the days when a pronouncement all the rage was that no science must borrow the methods of another; the geologist must not use a microscope, nor the astronomer a spectroscope. Optics must not meddle with electricity, nor logic with algebra. But twenty years later, if you aspired to pass for a commanding intellect, you would have to pull a long face and declare that "It is not the business of science to search for origins." This maxim was a masterpiece, since no timid soul, in dread of being thought naive, would dare inquire what "origins" were, albeit the secret confessor within his breast compelled the awful self-acknowledgment of his having no idea into what else than "origins" of phenomena (in some sense of that indefinite word) man can inquire. That human reason can comprehend some causes is past denial, and once we are forced to recognize a given element in experience, it is reasonable to await positive evidence before we complicate our acknowledgment with qualifications. Otherwise, why venture beyond direct observation? Illustrations of this principle abound in physical science. Since, then, it is certain that man is able to understand the laws and the causes of some phenomena, it is reasonable to assume, in regard to any given problem, that it would get rightly solved by man, if a sufficiency of time and attention were devoted to it. Moreover, those problems that at first blush appear utterly insoluble receive, in that very circumstance, as Edgar Poe remarked †1 in his "The Murders in the Rue Morgue," their smoothly-fitting keys. This particularly adapts them to the Play of Musement.

Peirce: CP 6.461 Cross-Ref:††

461. Forty or fifty minutes of vigorous and unslackened analytic thought bestowed upon one of them usually suffices to educe from it all there is to educe, its general solution. There is no kind of reasoning that I should wish to discourage in Musement; and I should lament to find anybody confining it to a method of such moderate fertility as logical analysis. Only, the Player should bear in mind that the higher weapons in the arsenal of thought are not playthings but edge-tools. In any mere Play they can be used by way of exercise alone; while logical analysis can be put to its full efficiency in Musement. So, continuing the counsels that had been asked of me, I should say, "Enter your skiff of Musement, push off into the lake of thought, and leave the breath of heaven to swell your sail. With your eyes open, awake to what is about or within you, and open conversation with yourself; for such is all meditation." It is, however, not a conversation in words alone, but is illustrated, like a lecture, with diagrams and with experiments.

Peirce: CP 6.462 Cross-Ref:††

462. Different people have such wonderfully different ways of thinking that it would be far beyond my competence to say what courses Musement might not take; but a brain endowed with automatic control, as man's indirectly is, is so naturally and rightly interested in its own faculties that some psychological and semi-psychological questions would doubtless get touched; such, in the latter class, as this: Darwinians, with truly surprising ingenuity, have concocted, and with still more astonishing confidence have accepted as proved, one explanation for the diverse and delicate beauties of flowers, another for those of butterflies, and so on; but why is all nature -- the forms of trees, the compositions of sunsets -- suffused with such beauties throughout, and not nature only, but the other two Universes as well? Among more purely psychological questions, the nature of pleasure and pain will be likely to attract attention. Are they mere qualities of feeling, or are they rather motor instincts attracting us to some feelings and repelling others? Have pleasure and pain the same sort of constitution, or are they contrasted in this respect, pleasure arising upon the forming or strengthening of an association by resemblance, and pain upon the weakening or disruption of such a habit or conception?† 1

Peirce: CP 6.463 Cross-Ref:††

463. Psychological speculations will naturally lead on to musings upon metaphysical problems proper, good exercise for a mind with a turn for exact thought. It is here that one finds those questions that at first seem to offer no handle for reason's clutch, but which readily yield to logical analysis. But problems of metaphysics will inevitably present themselves that logical analysis will not suffice to solve. Some of the best will be motivated by a desire to comprehend universe-wide aggregates of unformulated but partly experienced phenomena. I would suggest that the Muser be not too impatient to analyze these, lest some significant ingredient be lost in the process; but that he begin by pondering them from every point of view, until he seems to read some truth beneath the phenomena.

Peirce: CP 6.464 Cross-Ref:††

464. At this point a trained mind will demand that an examination be made of the truth of the interpretation; and the first step in such examination must be a logical analysis of the theory. But strict examination would be a task a little too serious for the Musement of hour fractions, and if it is postponed there will be ample remuneration even in the suggestions that there is not time to examine; especially since a few of them will appeal to reason as all but certain.

Peirce: CP 6.464 Cross-Ref:††

Let the Muser, for example, after well appreciating, in its breadth and depth, the unspeakable variety of each Universe, turn to those phenomena that are of the nature of homogeneities of connectedness in each; and what a spectacle will unroll itself! As a mere hint of them I may point out that every small part of space, however remote, is bounded by just such neighbouring parts as every other, without a single exception throughout immensity. The matter of Nature is in every star of the same elementary kinds, and (except for variations of circumstance), what is more wonderful still, throughout the whole visible universe, about the same proportions of the different chemical elements prevail. Though the mere catalogue of known carbon-compounds alone would fill an unwieldy volume, and perhaps, if the truth were known, the number of amino-acids alone is greater, yet it is unlikely that there are in all more than about 600 elements, of which 500 dart through space too swiftly to be held down by the earth's gravitation, coronium being the slowest-moving of these. This small number bespeaks comparative simplicity of structure. Yet no

mathematician but will confess the present hopelessness of attempting to comprehend the constitution of the hydrogen-atom, the simplest of the elements that can be held to earth.

Peirce: CP 6.465 Cross-Ref:††

465. From speculations on the homogeneities of each Universe, the Muser will naturally pass to the consideration of homogeneities and connections between two different Universes, or all three. Especially in them all we find one type of occurrence, that of growth, itself consisting in the homogeneities of small parts. This is evident in the growth of motion into displacement, and the growth of force into motion. In growth, too, we find that the three Universes conspire; and a universal feature of it is provision for later stages in earlier ones. This is a specimen of certain lines of reflection which will inevitably suggest the hypothesis of God's Reality. It is not that such phenomena might not be capable of being accounted for, in one sense, by the action of chance with the smallest conceivable dose of a higher element; for if by God be meant the *Ens necessarium*, that very hypothesis requires that such should be the case. But the point is that that sort of explanation leaves a mental explanation just as needful as before. Tell me, upon sufficient authority, that all cerebration depends upon movements of neurites that strictly obey certain physical laws, and that thus all expressions of thought, both external and internal, receive a physical explanation, and I shall be ready to believe you. But if you go on to say that this explodes the theory that my neighbour and myself are governed by reason, and are thinking beings, I must frankly say that it will not give me a high opinion of your intelligence. But however that may be, in the Pure Play of Musement the idea of God's Reality will be sure sooner or later to be found an attractive fancy, which the Muser will develop in various ways. The more he ponders it, the more it will find response in every part of his mind, for its beauty, for its supplying an ideal of life, and for its thoroughly satisfactory explanation of his whole threefold environment.

Peirce: CP 6.466 Cross-Ref:††

§2. THE HYPOTHESIS OF GODE

466. The hypothesis of God is a peculiar one, in that it supposes an infinitely incomprehensible object, although every hypothesis, as such, supposes its object to be truly conceived in the hypothesis. This leaves the hypothesis but one way of understanding itself; namely, as vague yet as true so far as it is definite, and as continually tending to define itself more and more, and without limit. The hypothesis, being thus itself inevitably subject to the law of growth, appears in its vagueness to represent God as so, albeit this is directly contradicted in the hypothesis from its very first phase. But this apparent attribution of growth to God, since it is ineradicable from the hypothesis, cannot, according to the hypothesis, be flatly false. Its implications concerning the Universes will be maintained in the hypothesis, while its implications concerning God will be partly disavowed, and yet held to be less false than their denial would be. Thus the hypothesis will lead to our thinking of features of each Universe as purposed; and this will stand or fall with the hypothesis. Yet a purpose essentially involves growth, and so cannot be attributed to God. Still it will, according to the hypothesis, be less false to speak so than to represent God as purposeless.

Peirce: CP 6.467 Cross-Ref:††

467. Assured as I am from my own personal experience that every man capable of so controlling his attention as to perform a little exact thinking will, if he examines Zeno's argument about Achilles and the tortoise, come to think, as I do, that it is nothing but a contemptible catch,†1 I do not think that I either am or ought to be less assured, from what I know of the effects of Musement on myself and others, that any normal man who considers the three Universes in the light of the hypothesis of God's Reality, and pursues that line of reflection in scientific singleness of heart, will come to be stirred to the depths of his nature by the beauty of the idea and by its august practicality, even to the point of earnestly loving and adoring his strictly hypothetical God, and to that of desiring above all things to shape the whole conduct of life and all the springs of action into conformity with that hypothesis. Now to be deliberately and thoroughly prepared to shape one's conduct into conformity with a proposition is neither more nor less than the state of mind called Believing that proposition, however long the conscious classification of it under that head be postponed.†2

Peirce: CP 6.468 Cross-Ref:††
§3. THE THREE STAGES OF INQUIRY \mathcal{E}

468. There is my poor sketch of the Neglected Argument, greatly cut down to bring it within the limits assigned to this article. Next should come the discussion of its logicity; but nothing readable at a sitting could possibly bring home to readers my full proof of the principal points of such an examination. I can only hope to make the residue of this paper a sort of table of contents, from which some may possibly guess what I have to say; or to lay down a series of plausible points through which the reader will have to construct the continuous line of reasoning for himself. In my own mind the proof is elaborated, and I am exerting my energies to getting it submitted to public censure. My present abstract will divide itself into three unequal parts. The first shall give the headings of the different steps of every well-conducted and complete inquiry, without noticing possible divergencies from the norm. I shall have to mention some steps which have nothing to do with the Neglected Argument in order to show that they add no jot nor tittle to the truth which is invariably brought just as the Neglected Argument brings it. The second part shall very briefly state, without argument (for which there is no room), just wherein lies the logical validity of the reasoning characteristic of each of the main stages of inquiry. The third part shall indicate the place of the Neglected Argument in a complete inquiry into the Reality of God, and shall show how well it would fill that place, and what its logical value is supposing the inquiry to be limited to this; and I shall add a few words to show how it might be supplemented.

Peirce: CP 6.469 Cross-Ref:††

469. Every inquiry whatsoever takes its rise in the observation, in one or another of the three Universes, of some surprising phenomenon, some experience which either disappoints an expectation, or breaks in upon some habit of expectation of the *inquisiturus*; and each apparent exception to this rule only confirms it. There are obvious distinctions between the objects of surprise in different cases; but throughout this slight sketch of inquiry such details will be unnoticed, especially since

it is upon such that the logic-books descant. The inquiry begins with pondering these phenomena in all their aspects, in the search of some point of view whence the wonder shall be resolved. At length a conjecture arises that furnishes a possible Explanation, by which I mean a syllogism exhibiting the surprising fact as necessarily consequent upon the circumstances of its occurrence together with the truth of the credible conjecture, as premisses.†1 On account of this Explanation, the inquirer is led to regard his conjecture, or hypothesis, with favor. As I phrase it, he provisionally holds it to be "Plausible"; this acceptance ranges in different cases -- and reasonably so -- from a mere expression of it in the interrogative mood, as a question meriting attention and reply, up through all appraisals of Plausibility, to uncontrollable inclination to believe. The whole series of mental performances between the notice of the wonderful phenomenon and the acceptance of the hypothesis, during which the usually docile understanding seems to hold the bit between its teeth and to have us at its mercy, the search for pertinent circumstances and the laying hold of them, sometimes without our cognizance, the scrutiny of them, the dark laboring, the bursting out of the startling conjecture, the remarking of its smooth fitting to the anomaly, as it is turned back and forth like a key in a lock, and the final estimation of its Plausibility, I reckon as composing the First Stage of Inquiry. Its characteristic formula of reasoning I term Retroduction,†1 *i.e.* reasoning from consequent to antecedent. In one respect the designation seems inappropriate; for in most instances where conjecture mounts the high peaks of Plausibility -- and is *really* most worthy of confidence -- the inquirer is unable definitely to formulate just what the explained wonder is; or can only do so in the light of the hypothesis. In short, it is a form of Argument rather than of Argumentation.

Peirce: CP 6.470 Cross-Ref:††

470. Retroduction does not afford security. The hypothesis must be tested.

Peirce: CP 6.470 Cross-Ref:††

This testing, to be logically valid, must honestly start, not as Retroduction starts, with scrutiny of the phenomena, but with examination of the hypothesis, and a muster of all sorts of conditional experiential consequences which would follow from its truth. This constitutes the Second Stage of Inquiry. For its characteristic form of reasoning our language has, for two centuries, been happily provided with the name Deduction.

Peirce: CP 6.471 Cross-Ref:††

471. Deduction has two parts. For its first step must be by logical analysis to Explicate the hypothesis, *i.e.* to render it as perfectly distinct as possible. This process, like Retroduction, is Argument that is not Argumentation. But unlike Retroduction, it cannot go wrong from lack of experience, but so long as it proceeds rightly must reach a true conclusion. Explication is followed by Demonstration, or Deductive Argumentation. Its procedure is best learned from Book I of Euclid's *Elements*, a masterpiece which in real insight is far superior to Aristotle's *Analytics*; and its numerous fallacies render it all the more instructive to a close student. It invariably requires something of the nature of a diagram; that is, an "Icon," or Sign that represents its Object in resembling it. It usually, too, needs "Indices," or Signs that represent their Objects by being actually connected with them. But it is mainly composed of "Symbols," or Signs that represent their Objects essentially because they will be so interpreted. Demonstration should be *Corollarial* when it can. An accurate definition of Corollarial Demonstration would require a long explanation; but it will suffice to say that it limits itself to considerations already introduced or else involved

in the Explication of its conclusion; while *Theorematic* Demonstration resorts to a more complicated process of thought.†1

Peirce: CP 6.472 Cross-Ref:††

472. The purpose of Deduction, that of collecting consequents of the hypothesis, having been sufficiently carried out, the inquiry enters upon its Third Stage, that of ascertaining how far those consequents accord with Experience, and of judging accordingly whether the hypothesis is sensibly correct, or requires some inessential modification, or must be entirely rejected. Its characteristic way of reasoning is Induction. This stage has three parts. For it must begin with Classification, which is an Inductive Non-argumentational kind of Argument, by which general Ideas are attached to objects of Experience; or rather by which the latter are subordinated to the former. Following this will come the testing-argumentations, the Probations; and the whole inquiry will be wound up with the Sentential part of the Third Stage, which, by Inductive reasonings, appraises the different Probations singly, then their combinations, then makes self-appraisal of these very appraisals themselves, and passes final judgment on the whole result.

Peirce: CP 6.473 Cross-Ref:††

473. The Probations, or direct Inductive Argumentations, are of two kinds. The first is that which Bacon ill described as "*inductio illa quæ procedit per enumerationem simplicem*." So at least he has been understood. For an enumeration of instances is not essential to the argument that, for example, there are no such beings as fairies, or no such events as miracles. The point is that there is no well-established instance of such a thing. I call this Crude Induction.†2 It is the only Induction which concludes a logically Universal Proposition. It is the weakest of arguments, being liable to be demolished in a moment, as happened toward the end of the eighteenth century to the opinion of the scientific world that no stones fall from the sky. The other kind is Gradual Induction,†1 which makes a new estimate of the proportion of truth in the hypothesis with every new instance; and given any degree of error there will *sometime* be an estimate (or would be, if the probation were persisted in) which will be absolutely the last to be infected with so much falsity. Gradual Induction is either Qualitative or Quantitative and the latter either depends on measurements, or on statistics, or on countings.

Peirce: CP 6.474 Cross-Ref:††

§4. THE VALIDITY OF THE THREE STAGES^E

474. Concerning the question of the nature of the logical validity possessed by Deduction, Induction, and Retroduction, which is still an arena of controversy, I shall confine myself to stating the opinions which I am prepared to defend by positive proofs. The validity of Deduction was correctly, if not very clearly, analyzed by Kant.†2 This kind of reasoning deals exclusively with Pure Ideas attaching primarily to Symbols and derivatively to other Signs of our own creation; and the fact that man has a power of Explicating his own meaning renders Deduction valid. Induction is a kind of reasoning that may lead us into error; but that it follows a method which, sufficiently persisted in, will be Inductively Certain (the sort of certainty we have that a perfect coin, pitched up often enough, will *sometime* turn up heads) to diminish the error below any predesignate degree, is assured by man's power of perceiving

Inductive Certainty. In all this I am inviting the reader to peep through the big end of the telescope; there is a wealth of pertinent detail that must here be passed over.

Peirce: CP 6.475 Cross-Ref:††

475. Finally comes the bottom question of logical Critic,†3 What sort of validity can be attributed to the First Stage of inquiry? Observe that neither Deduction nor Induction contributes the smallest positive item to the final conclusion of the inquiry. They render the indefinite definite; Deduction Explicates; Induction evaluates: that is all. Over the chasm that yawns between the ultimate goal of science and such ideas of Man's environment as, coming over him during his primeval wanderings in the forest, while yet his very notion of error was of the vaguest, he managed to communicate to some fellow, we are building a cantilever bridge of induction, held together by scientific struts and ties. Yet every plank of its advance is first laid by Retroduction alone, that is to say, by the spontaneous conjectures of instinctive reason; and neither Deduction nor Induction contributes a single new concept to the structure. Nor is this less true or less important for those inquiries that self-interest prompts.

Peirce: CP 6.476 Cross-Ref:††

476. The first answer we naturally give to this question is that we *cannot help* accepting the conjecture at such a valuation as that at which we do accept it; whether as a simple interrogation, or as more or less Plausible, or, occasionally, as an irresistible belief. But far from constituting, by itself, a logical justification such as it becomes a rational being to put forth, this pleading, that we cannot help yielding to the suggestion, amounts to nothing more than a confession of having failed to train ourselves to control our thoughts. It is more to the purpose, however, to urge that the strength of the impulse is a symptom of its being instinctive. Animals of all races rise far above the general level of their intelligence in those performances that are their proper function, such as flying and nest-building for ordinary birds; and what is man's proper function if it be not to embody general ideas in art-creations, in utilities, and above all in theoretical cognition? To give the lie to his own consciousness of divining the reasons of phenomena would be as silly in a man as it would be in a fledgling bird to refuse to trust to its wings and leave the nest, because the poor little thing had read Babinet,†1 and judged aerostation to be impossible on hydrodynamical grounds. Yes; it must be confessed that *if we knew* that the impulse to prefer one hypothesis to another really were analogous to the instincts of birds and wasps, it would be foolish not to give it play, within the bounds of reason; especially since we must entertain some hypothesis, or else forego all further knowledge than that which we have already gained by that very means. But is it a fact that man possesses this magical faculty? Not, I reply, to the extent of guessing right the first time, nor perhaps the second; but that the well-prepared mind has wonderfully soon guessed each secret of nature is historical truth. All the theories of science have been so obtained. But may they not have come fortuitously, or by some such modification of chance as the Darwinian supposes? I answer that three or four independent methods of computation show that it would be ridiculous to suppose our science to have so come to pass. Nevertheless, suppose that it can be so "explained," just as that any purposed act of mine is supposed by materialistic necessitarians to have come about. Still, what of it? Does that materialistic explanation, supposing it granted, show that reason has nothing to do with my actions? Even the parallelists will admit that the one explanation leaves the same need of the other that there was before it was given; and this is certainly sound logic. There is a reason, an interpretation, a logic, in the course of scientific advance, and this indisputably proves to him who has perceptions of

rational or significant relations, that man's mind must have been attuned to the truth of things in order to discover what he has discovered. It is the very bedrock of logical truth.

Peirce: CP 6.477 Cross-Ref:††

477. Modern science has been builded after the model of Galileo, who founded it, on *il lume naturale*. That truly inspired prophet had said that, of two hypotheses, the *simpler* is to be preferred;†1 but I was formerly one of those who, in our dull self-conceit fancying ourselves more sly than he, twisted the maxim to mean the *logically* simpler, the one that adds the least to what has been observed, in spite of three obvious objections: first, that so there was no support for any hypothesis; secondly, that by the same token we ought to content ourselves with simply formulating the special observations actually made; and thirdly, that every advance of science that further opens the truth to our view discloses a world of unexpected complications. It was not until long experience forced me to realize that subsequent discoveries were every time showing I had been wrong, while those who understood the maxim as Galileo had done, early unlocked the secret, that the scales fell from my eyes and my mind awoke to the broad and flaming daylight that it is the simpler Hypothesis in the sense of the more facile and natural, the one that instinct suggests, that must be preferred; for the reason that, unless man have a natural bent in accordance with nature's, he has no chance of understanding nature at all. Many tests of this principal and positive fact, relating as well to my own studies as to the researches of others, have confirmed me in this opinion; and when I shall come to set them forth in a book, their array will convince everybody. Oh, no! I am forgetting that armour, impenetrable by accurate thought, in which the rank and file of minds are clad! They may, for example, get the notion that my proposition involves a denial of the rigidity of the laws of association: it would be quite on a par with much that is current. I do not mean that logical simplicity is a consideration of no value at all, but only that its value is badly secondary to that of simplicity in the other sense.

Peirce: CP 6.477 Cross-Ref:††

If, however, the maxim is correct in Galileo's sense, whence it follows that man has, in some degree, a divinatory power, primary or derived, like that of a wasp or a bird, then instances swarm to show that a certain altogether peculiar confidence in a hypothesis, not to be confounded with rash cocksureness, has a very appreciable value as a sign of the truth of the hypothesis. I regret I cannot give an account of certain interesting and almost convincing cases. The N.A. excites this peculiar confidence in the very highest degree.

Peirce: CP 6.478 Cross-Ref:††
§5. PRAGMATICISM †1E

478. We have now to apply these principles to the evaluation of the N.A. Had I space I would put this into the shape of imagining how it is likely to be esteemed by three types of men: the first of small instruction with corresponding natural breadth, intimately acquainted with the N.A., but to whom logic is all Greek; the second, inflated with current notions of logic, but prodigiously informed about the N.A.; the third, a trained man of science who, in the modern spirit, has added to his specialty an exact theoretical and practical study of reasoning and the elements of thought, so that

psychologists account him a sort of psychologist, and mathematicians a sort of mathematician.

Peirce: CP 6.479 Cross-Ref:††

479. I should, then, show how the first would have learned that nothing has any kind of value in itself -- whether æsthetic, moral, or scientific -- but only in its place in the whole production to which it appertains; and that an individual soul with its petty agitations and calamities is a zero except as filling its infinitesimal place, and accepting his little futility as his entire treasure. He will see that though his God would not really (in a certain sense) adapt means to ends, it is nevertheless quite true that there are relations among phenomena which finite intelligence must interpret, and truly interpret, as such adaptations; and he will macarize himself for his own bitterest griefs, and bless God for the law of growth with all the fighting it imposes upon him -- Evil, *i.e.* what it is man's duty to fight, being one of the major perfections of the Universe. In that fight he will endeavour to perform just the duty laid upon him and no more. Though his desperate struggles should issue in the horrors of his rout, and he should see the innocents who are dearest to his heart exposed to torments, frenzy and despair, destined to be smirched with filth, and stunted in their intelligence, still he may hope that it be best *for them*, and will tell himself that in any case the secret design of God will be perfected through their agency; and even while still hot from the battle, will submit with adoration to His Holy will. He will not worry because the Universes were not constructed to suit the scheme of some silly scold.

Peirce: CP 6.480 Cross-Ref:††

480. The context of this I must leave the reader to imagine. I will only add that the third man, considering the complex process of self-control, will see that the hypothesis, irresistible though it be to first intention, yet needs Probation; and that though an infinite being is not tied down to any consistency, yet man, like any other animal, is gifted with power of understanding sufficient for the conduct of life. This brings him, for testing the hypothesis, to taking his stand upon Pragmaticism, which implies faith in common sense and in instinct, though only as they issue from the cupel-furnace of measured criticism. In short, he will say that the N.A. is the First Stage of a scientific inquiry, resulting in a hypothesis of the very highest Plausibility, whose ultimate test must lie in its value in the self-controlled growth of man's conduct of life.

Peirce: CP 6.481 Cross-Ref:††

481.†1 Since I have employed the word *Pragmaticism*, and shall have occasion to use it once more, it may perhaps be well to explain it. About forty years ago, my studies of Berkeley, Kant, and others led me, after convincing myself that all thinking is performed in Signs, and that meditation takes the form of a dialogue, so that it is proper to speak of the "meaning" of a concept, to conclude that to acquire full mastery of that meaning it is requisite, in the first place, to learn to recognize the concept under every disguise, through extensive familiarity with instances of it. But this, after all, does not imply any true understanding of it; so that it is further requisite that we should make an abstract logical analysis of it into its ultimate elements, or as complete an analysis as we can compass. But, even so, we may still be without any living comprehension of it; and the only way to complete our knowledge of its nature is to discover and recognize just what general habits of conduct a belief in the truth of the concept (of any conceivable subject, and under any conceivable circumstances) would reasonably develop; that is to say, what habits would ultimately result from a sufficient consideration of such truth. It is necessary to understand the word

"conduct," here, in the broadest sense. If, for example, the predication of a given concept were to lead to our admitting that a given form of reasoning concerning the subject of which it was affirmed was valid, when it would not otherwise be valid, the recognition of that effect in our reasoning would decidedly be a habit of conduct.

Peirce: CP 6.482 Cross-Ref:††

482. In 1871, in a Metaphysical Club in Cambridge, Massachusetts, I used to preach this principle as a sort of logical gospel, representing the unformulated method followed by Berkeley, and in conversation about it I called it "Pragmatism."†2 In December [November] 1877 and January 1878 I set forth the doctrine in the *Popular Science Monthly*; and the two parts of my essay were printed in French in the *Revue Philosophique*, volumes vi and vii.†1 Of course, the doctrine attracted no particular attention, for, as I had remarked in my opening sentence, very few people care for logic. But in 1897 Professor James remodelled the matter, and transmogrified it into a doctrine of philosophy,†2 some parts of which I highly approved, while other and more prominent parts I regarded, and still regard, as opposed to sound logic. About the time Professor Papini †3 discovered, to the delight of the Pragmatist school, that this doctrine was incapable of definition, which would certainly seem to distinguish it from every other doctrine in whatever branch of science, I was coming to the conclusion that my poor little maxim should be called by another name; and accordingly, in April, 1905 I renamed it *Pragmaticism*.†4 I had never before dignified it by any name in print, except that, at Professor Baldwin's request, I wrote a definition of it for his *Dictionary of Psychology and Philosophy*.†5 I did not insert the word in the *Century Dictionary*, though I had charge of the philosophical definitions of that work;†6 for I have a perhaps exaggerated dislike of *réclame*.

Peirce: CP 6.483 Cross-Ref:††

483. It is that course of meditation upon the three Universes which gives birth to the hypothesis and ultimately to the belief that they, or at any rate two of the three, have a Creator independent of them, that I have throughout this article called the N.A., because I think the theologians ought to have recognized it as a line of thought reasonably productive of belief. This is the "humble" argument, the innermost of the nest.†7 In the mind of a metaphysician it will have a metaphysical tinge; but that seems to me rather to detract from its force than to add anything to it. It is just as good an argument, if not better, in the form it takes in the mind of the clodhopper.

Peirce: CP 6.484 Cross-Ref:††

484. The theologians could not have *presented* the N.A.; because that is a living course of thought of very various forms. But they might and ought to have *described* it, and should have defended it, too, as far as they could, without going into original logical researches, which could not be justly expected of them. They are accustomed to make use of the principle that that which convinces a normal man must be presumed to be sound reasoning; and therefore they ought to say whatever can truly be advanced to show that the N.A., if sufficiently developed, will convince any normal man. Unfortunately, it happens that there is very little established fact to show that this is the case. I have not pretended to have any other ground for my belief that it is so than my assumption, which each one of us makes, that my own intellectual disposition is normal. I am forced to confess that no pessimist will agree with me. I do not admit that pessimists are, at the same time, thoroughly sane, and in addition are endowed in normal measure with intellectual vigour; and my reasons for thinking so are two. The first is, that the difference between a pessimistic and an optimistic mind is of such controlling importance in regard to every intellectual function, and

especially for the conduct of life, that it is out of the question to admit that both are normal, and the great majority of mankind are naturally optimistic. Now, the majority of every race depart but little from the norm of that race. In order to present my other reason, I am obliged to recognize three types of pessimists. The first type is often found in exquisite and noble natures of great force of original intellect whose own lives are dreadful histories of torment due to some physical malady. Leopardi is a famous example. We cannot but believe, against their earnest protests, that if such men had had ordinary health, life would have worn for them the same colour as for the rest of us. Meantime, one meets too few pessimists of this type to affect the present question. The second is the misanthropical type, the type that makes itself heard. It suffices to call to mind the conduct of the famous pessimists of this kind, Diogenes the Cynic, Schopenhauer, Carlyle, and their kin with Shakespeare's Timon of Athens, to recognize them as diseased minds. The third is the philanthropical type, people whose lively sympathies, easily excited, become roused to anger at what they consider the stupid injustices of life. Being easily interested in everything, without being overloaded with exact thought of any kind, they are excellent raw material for *littérateurs*: witness Voltaire. No individual remotely approaching the calibre of a Leibnitz is to be found among them.

Peirce: CP 6.485 Cross-Ref:††

485. The third argument, enclosing and defending the other two, consists in the development of those principles of logic according to which the humble argument is the first stage of a scientific inquiry into the origin of the three Universes, but of an inquiry which produces, not merely scientific belief, which is always provisional, but also a living, practical belief, logically justified in crossing the Rubicon with all the freightage of eternity. The presentation of this argument would require the establishment of several principles of logic that the logicians have hardly dreamed of, and particularly a strict proof of the correctness of the maxim of Pragmaticism. My original essay, having been written for a popular monthly, assumes, for no better reason than that real inquiry cannot begin until a state of real doubt arises and ends as soon as Belief is attained, that "a settlement of Belief," or, in other words, a state of *satisfaction*, is all that Truth, or the aim of inquiry, consists in.†1 The reason I gave for this was so flimsy, while the inference was so nearly the gist of Pragmaticism, that I must confess the argument of that essay might with some justice be said to beg the question. The first part of the essay,†2 however, is occupied with showing that, if Truth consists in satisfaction, it cannot be any *actual* satisfaction, but must be the satisfaction which *would* ultimately be found if the inquiry were pushed to its ultimate and indefeasible issue. This, I beg to point out, is a very different position from that of Mr. Schiller and the pragmatists of today.†3 I trust I shall be believed when I say that it is only a desire to avoid being misunderstood in consequence of my relations with pragmatism, and by no means as arrogating any superior immunity from error which I have too good reason to know that I do not enjoy, that leads me to express my personal sentiments about their tenets. Their avowedly undefinable position, if it be not capable of logical characterization, seems to me to be characterized by an angry hatred of strict logic, and even some disposition to rate any exact thought which interferes with their doctrines as all humbug. At the same time, it seems to me clear that their approximate acceptance of the Pragmaticist principle, and even that very casting aside of difficult distinctions (although I cannot approve of it), has helped them to a mightily clear discernment of some fundamental truths that other philosophers have seen but through a mist, and most of them not at all. Among such truths -- all of them old, of course, yet acknowledged by few -- I reckon their denial of necessitarianism; their rejection of any "consciousness" different from a visceral or

other external sensation; their acknowledgment that there are, in a Pragmatistical sense, Real habits (which Really *would* produce effects, under circumstances that may not happen to get actualized, and are thus Real generals); and their insistence upon interpreting all hypostatic abstractions in terms of what they *would* or *might* (not actually *will*) come to in the concrete. It seems to me a pity they should allow a philosophy so instinct with life to become infected with seeds of death in such notions as that of the unreality of all ideas of infinity †1 and that of the mutability of truth, †2 and in such confusions of thought as that of active willing (willing to control thought, to doubt, and to weigh reasons) with willing not to exert the will (willing to believe). †3

Peirce: CP 6.486 Cross-Ref: ††
§6. ADDITAMENT †4P

486. A nest of three arguments for the Reality of God has now been sketched, though none of them could, in the limits of a single article, be fairly presented. The first is that entirely honest, sincere and unaffected, because unpremeditated, meditation upon the Idea of God, into which the Play of Musement will inevitably sooner or later lead, and which, by developing a deep sense of the adorability of that Idea, will produce a truly religious Belief in His Reality and His nearness. It is a reasonable argument, because it naturally results in the most intense and living determination (*Bestimmung*) of the soul toward shaping the Muser's whole conduct into conformity with the Hypothesis that God is Real and very near; and such a determination of the soul in regard to any proposition is the very essence of a living Belief in such proposition. This is that "humble argument," open to every honest man, which I surmise to have made more worshippers of God than any other.

Peirce: CP 6.487 Cross-Ref: ††

487. The second of the nest is the argument which seems to me to have been "neglected" by writers upon natural theology, consisting in showing that the humble argument is the natural fruit of free meditation, since every heart will be ravished by the beauty and adorability of the Idea, when it is so pursued. Were the theologians able to perceive the force of this argument, they would make it such a presentation of universal human nature as to show that a latent tendency toward belief in God is a fundamental ingredient of the soul, and that, far from being a vicious or superstitious ingredient, it is simply the natural precipitate of meditation upon the origin of the Three Universes. Of course, it could not, any more than any other theological argumentation, have the value or the religious vitality of the "Humble Argument"; for it would only be an apology -- a vindicatory description -- of the mental operations which the Humble Argument actually and actively lives out. Though this is properly the neglected argument, yet I have sometimes used the abbreviation "the N.A." for the whole nest of three.

Peirce: CP 6.488 Cross-Ref: ††

488. The third argument of the nest consists in a study of logical methodic, illuminated by the light of a first-hand acquaintance with genuine scientific thought -- the sort of thought whose tools literally comprise not merely Ideas of mathematical exactitude, but also the apparatus of the skilled manipulator, actually in use. The student, applying to his own trained habits of research the art of logical analysis -- an

art as elaborate and methodical as that of the chemical analyst, compares the process of thought of the Muser upon the Three Universes with certain parts of the work of scientific discovery, and finds that the "Humble Argument" is nothing but an instance of the first stage of all such work, the stage of observing the facts, or variously rearranging them, and of pondering them until, by their reactions with the results of previous scientific experience, there is "evolved" (as the chemists word it) an explanatory hypothesis. He will note, however, that this instance of Retroduction, undeniable as this character is, departs widely from the ordinary run of instances, especially in three respects. In the first place, the Plausibility of the hypothesis reaches an almost unparalleled height among deliberately formed hypotheses. So hard is it to doubt God's Reality, when the Idea has sprung from Musements, that there is great danger that the investigation will stop at this first stage, owing to the indifference of the Muser to any further proof of it. At the same time, this very Plausibility is undoubtedly an argument of no small weight in favor of the truth of the hypothesis.

Peirce: CP 6.489 Cross-Ref:††

489. In the second place, although it is a chief function of an explanatory hypothesis (and some philosophers say the only one) to excite a clear image in the mind by means of which experiential consequences of ascertainable conditions may be predicted, yet in this instance the hypothesis can only be apprehended so very obscurely that in exceptional cases alone can any definite and direct deduction from its ordinary abstract interpretation be made. How, for example, can we ever expect to be able to predict what the conduct would be, even of [an] omniscient being, governing no more than one poor solar system for only a million years or so? How much less if, being also omnipotent, he be thereby freed from all experience, all desire, all intention! Since God, in His essential character of *Ens necessarium*, is a disembodied spirit, and since there is strong reason to hold that what we call consciousness is either merely the general sensation of the brain or some part of it, or at all events some visceral or bodily sensation, God probably has no consciousness. Most of us are in the habit of thinking that consciousness and psychic life are the same thing and otherwise greatly to overrate the functions of consciousness. (See James's paper "Does 'Consciousness' Exist?" in *Jour. Phil., Psy., and Sci. Meth.* I, 477; 1904, Sep. 1. But the negative reply is, in itself, no novelty.)

Peirce: CP 6.490 Cross-Ref:††

490. The effects of the second peculiarity of the hypothesis are counteracted by a third, which consists in its commanding influence over the whole conduct of life of its believers. According to that logical doctrine which the present writer first formulated in 1873†1 and named Pragmatism, the true meaning of any product of the intellect lies in whatever unitary determination it would impart to practical conduct under any and every conceivable circumstance, supposing such conduct to be guided by reflexion carried to an ultimate limit. It appears to have been virtually the philosophy of Socrates. But although it is "an old way of thinking," in the sense that it was practiced by Spinoza, Berkeley, and Kant, I am not aware of its having been definitely formulated, whether as a maxim of logical analysis or otherwise, by anybody before my publication of it in 1878. Naturally, nobody ever heard of pragmatism. People don't care for methods! they want results. Give them all the diamonds you make, and you may have the method of making them for your own. So it was not until in 1898†2 -- Professor James took hold of the old thing, dignified it by calling it by its name in print (which I had never done even when I was in charge of the philosophical part of the *Century Dictionary*), furbished it up, and turned it into a

philosophical doctrine -- that it had any vogue at all. It did not, however, shine with its present effulgence until Professor Papini †3 made the discovery that it cannot be defined -- a circumstance which, I believe, distinguishes it from all other doctrines, of whatsoever natures they may be, that were ever promulgated. Thereupon I thought it high time to give my method a less distinguished designation; and I rechristened it *pragmaticism*. †4 Pragmaticism, then, is a theory of logical analysis, or true definition; and its merits are greatest in its application to the highest metaphysical conceptions. At the same time, these merits can only be appreciated as the result of long training. A full exposition of the pragmaticistic definition of *Ens necessarium* would require many pages; but some hints toward it may be given. A disembodied spirit, or pure mind, has its being out of time, since all that it is destined to think is fully in its being at any and every previous time. But in endless time it is destined to think all that it is capable of thinking. Order is simply thought embodied in arrangement; and thought embodied in any other way appears objectively as a character that is a generalization of order, and that, in the lack of any word for it, we may call for the nonce, "Super-order." It is something like uniformity. The idea may be caught if it is described as that of which order and uniformity are particular varieties. Pure mind, as creative of thought, must, so far as it is manifested in time, appear as having a character related to the habit-taking capacity, just as super-order is related to uniformity. Now imagine, in such vague way as such a thing can be imagined, a perfect cosmology of the three universes. It would prove all in relation to that subject that reason could desiderate; and of course all that it would prove must, in actual fact, now be true. But reason would desiderate that that should be proved from which would follow all that is in fact true of the three universes; and the postulate from which all this would follow must not state any matter of fact, since such fact would thereby be left unexplained. That perfect cosmology must therefore show that the whole history of the three universes, as it has been and is to be, would follow from a premiss which would not suppose them to exist at all. Moreover, such premiss must in actual fact be true. But that premiss must represent a state of things in which the three universes were completely nil. Consequently, whether in time or not, the three universes must actually be absolutely necessary results of a state of utter nothingness. We cannot ourselves conceive of such a state of nility; but we can easily conceive that there should be a mind that could conceive it, since, after all, no contradiction can be involved in mere non-existence. A state in which there should be absolutely no super-order whatsoever would be such a state of nility. For all Being involves some kind of super-order. For example, to suppose a thing to have any particular character is to suppose a conditional proposition to be true of it, which proposition would express some kind of super-order, as any formulation of a general fact does. To suppose it to have elasticity of volume is to suppose that if it were subjected to pressure its volume would diminish until at a certain point the full pressure was attained within and without its periphery. This is a super-order, a law expressible by a differential equation. Any such super-order would be a super-habit. Any general state of things whatsoever would be a super-order and a super-habit. In that state of absolute nility, in or out of time, that is, before or after the evolution of time, there must then have been a *tohu bohu* of which nothing whatever affirmative or negative was true universally. There must have been, therefore, a little of everything conceivable. There must have been here and there a little undifferentiated tendency to take super-habits. But such a state must tend to *increase itself*. For a tendency to act in any way, combined with a tendency to take habits, must increase the tendency to act in that way. Now substitute in this general statement for "tendency to act in any way" a tendency to take habits, and we see that that tendency would grow. It would

also become differentiated in various ways. But there are some habits that carried beyond a certain point eliminate their subjects from the universe. There are many ways in which this may happen. Thus a tendency to lose mass will end in a total loss of mass. A tendency to lose energy will end in removing its subject from perceptible existence. A tendency to gain energy will end in the body's shooting through the universe too rapidly to produce any effect, etc.

Peirce: CP 6.491 Cross-Ref:††

491. Among the many pertinent considerations which have been crowded out of this article, I may just mention that it could have been shown that the hypothesis of God's Reality is logically not so isolated a conclusion as it may seem. On the contrary, it is connected so with a theory of the nature of thinking that if this be proved so is that. Now there is no such difficulty in tracing experiential consequences of this theory of thinking as there are in attempting directly to trace out other consequences of God's reality. In so short an article, it could not be expected that I should take notice of objections. Yet objections, such as they are, are obvious enough, and a few of them wear at first sight a redoubtable aspect. For example, it may be said that since I compare man's power of guessing at the truth with the instincts of animals, I ought to have noticed that these are entirely explained by the action of natural selection in endowing animals with such powers as contribute to the preservation of their different stocks; and that there is evidence that man's power of penetrating the secrets of nature depends upon this, in the fact that all the successful sciences have been either mechanical in respect to their theories or psychological. Now, some notions of mechanics are needed by all animals to enable them to get food, and are needed most by man; while correct ideas of what passes in his neighbours' minds are needed for the existence of society, and therefore for the propagation of his kind.†1 Metaphysics, however, cannot adapt the human race to maintaining itself, and therefore the presumption [is] that man has no such genius for discoveries about God, Freedom, and Immortality, as he has for physical and psychical science.

Peirce: CP 6.492 Cross-Ref:††
§7. KNOWLEDGE OF GOD †2

492. [We] can know nothing except what we *directly* experience. So all that we can anyway know relates to experience. All the creations of our mind are but patchworks from experience. So that all our ideas are but ideas of real or transposed experiences. A word can mean nothing except the idea it calls up. So that we cannot even *talk* about anything but a knowable object. The unknowable about which Hamilton and the agnostics talk can be nothing but an Unknowable Knowable. The absolutely unknowable is a non-existent existence.†3 The Unknowable is a nominalistic heresy. The nominalists in giving their adherence to that doctrine which is really held by all philosophers of all stripes, namely, that experience is all we know, understand experience in their nominalistic sense as the mere first impressions of sense. These "first impressions of sense" are hypothetical creations of nominalistic metaphysics: I for one deny their existence. But anyway even if they exist, it is not in them that experience consists. By experience must be understood the entire mental product. Some psychologists whom I hold in respect will stop me here to say that, while they admit that experience is more than mere sensation, they cannot extend it to the whole mental product, since that would include hallucinations, delusions,

superstitious imaginations and fallacies of all kinds; and that they would limit experience to sense-perceptions. But I reply that my statement is the logical one. Hallucinations, delusions, superstitious imaginations, and fallacies of all kinds are experiences, but experiences misunderstood; while to say that all our knowledge relates merely to sense perception is to say that we can know nothing -- not even mistakenly -- about higher matters, as honor, aspirations, and love.

Peirce: CP 6.493 Cross-Ref:††

493. Where would such an idea, say as that of God, come from, if not from direct experience? Would you make it a result of some kind of reasoning, good or bad? Why, reasoning can supply the mind with nothing in the world except an estimate of the value of a statistical ratio, that is, how often certain kinds of things are found in certain combinations in the ordinary course of experience. And scepticism, in the sense of doubt of the validity of elementary ideas -- which is really a proposal to turn an idea out of court and permit no inquiry into its applicability -- is doubly condemned by the fundamental principle of scientific method -- condemned first as obstructing inquiry, and condemned second because it is treating some other than a statistical ratio as a thing to be argued about. No: as to God, open your eyes -- and your heart, which is also a perceptive organ -- and you see him. But you may ask, Don't you admit there are any delusions? Yes: I may think a thing is black, and on close examination it may turn out to be bottle-green. But I cannot think a thing is black if there is no such thing to be seen as black. Neither can I think that a certain action is self-sacrificing, if no such thing as self-sacrifice exists, although it may be very rare. It is the nominalists, and the nominalists alone, who indulge in such scepticism, which the scientific method utterly condemns.

Peirce: CP 6.494 Cross-Ref:††
CHAPTER 4

ANSWERS TO QUESTIONS CONCERNING MY BELIEF IN GOD†1P

§1. THE REALITY OF GOD

494. The questions can be answered without very long explanations. "Do you believe in the existence of a Supreme Being?" Hume, in his *Dialogues Concerning Natural Religion*, †2 justly points out that the phrase "Supreme Being" is not an equivalent of "God," since it neither implies infinity nor any of the other attributes of God, excepting only Being and Supremacy. This is important; and another distinction between the two designations is still more so. Namely, "God" is a vernacular word and, like all such words, but more than almost any, is *vague*. No words are so well understood as vernacular words, in one way; yet they are invariably vague; and of many of them it is true that, let the logician do his best to substitute precise equivalents in their places, still the vernacular words alone, for all their vagueness, answer the principal purposes. This is emphatically the case with the very vague word

"God," which is not made less vague by saying that it imports "infinity," etc., since those attributes are at least as vague. I shall, therefore, if you please, substitute "God," for "Supreme Being" in the question.

Peirce: CP 6.495 Cross-Ref:††

495. I will also take the liberty of substituting "reality" for "existence." This is perhaps overscrupulosity; but I myself always use *exist* in its strict philosophical sense of "react with the other like things in the environment."†3 Of course, in that sense, it would be fetichism to say that God "exists." The word "reality," on the contrary, is used in ordinary parlance in its correct philosophical sense. It is curious that its legal meaning, in which we speak of "real estate," is the earliest, occurring early in the twelfth century. Albertus Magnus, who, as a high ecclesiastic, must have had to do with such matters, imported it into philosophy.†1 But it did not become at all common until Duns Scotus, in the latter part of the thirteenth century began to use it freely.†2 I define the *real* as that which holds its characters on such a tenure that it makes not the slightest difference what any man or men may have *thought* them to be, or ever will have *thought* them to be, here using thought to include, imagining, opining, and willing (as long as forcible *means* are not used); but the real thing's characters will remain absolutely untouched.

Peirce: CP 6.496 Cross-Ref:††

496. Of any kind of figment, this is not true. So, then, the question being whether I believe in the reality of God, I answer, Yes. I further opine that pretty nearly everybody more or less believes this, including many of the scientific men of my generation who are accustomed to think the belief is entirely unfounded. The reason they fall into this extraordinary error about their own belief is that they precise †3 (or render precise) the conception, and, in doing so, inevitably change it; and such precise conception is easily shown not to be warranted, even if it cannot be quite refuted. Every concept that is vague is liable to be self-contradictory in those respects in which it is vague.†4 *No* concept, not even those of mathematics, is absolutely precise; and some of the most important for everyday use are extremely vague. Nevertheless, our instinctive beliefs involving such concepts are far more trustworthy than the best established results of science, if these be precisely understood. For instance, we all think that there is an element of order in the universe. Could any laboratory experiments render that proposition more certain than instinct or common sense leaves it? It is ridiculous to broach such a question. But when anybody undertakes to say *precisely* what that order consists in, he will quickly find he outruns all logical warrant. Men who are given to defining too much inevitably run themselves into confusion in dealing with the vague concepts of common sense.

Peirce: CP 6.497 Cross-Ref:††

497. They generally make the matter worse by erroneous, not to say absurd, notions of the function of reasoning. Every race of animals is provided with instincts well adapted to its needs, and especially to strengthening the stock. It is wonderful how unerring these instincts are. Man is no exception in this respect; but man is so continually getting himself into novel situations that he needs, and is supplied with, a subsidiary faculty of *reasoning* for bringing instinct to bear upon situations to which it does not directly apply. This faculty is a very imperfect one in respect to fallibility; but then it is only needed to bridge short gaps. Every step has to be reviewed and criticized; and indeed this is so essential that it is best to call an uncriticized step of inference by another name. If one does not at all know how one's belief comes about, it cannot be called even by the name of inference. If, with St. Augustine,†1 we draw

the inference "I think; therefore, I am," but, when asked how we justify this inference, can only say that we are *compelled to think* that, since we think, we are, this uncriticized inference ought not to be called reasoning, which at the very least conceives its inference to be one of a general class of possible inferences on the same model, and all equally valid. But one must go back and criticize the premisses and the *principles* that guide the drawing of the conclusions. If it could be made out that all the ultimate (or first) premisses were percepts; and that all the ultimate logical principles were as clear as the principle of contradiction, then one might say that one's conclusion was *perfectly* rational. Strictly speaking, it would not be quite so, because it is quite possible for perception itself to deceive us, and it is much more possible for us to be mistaken about the indubitableness of logical principles. But as a matter of fact, as far as logicians have hitherto been able to push their analyses, we have *in no single case*, concerning a matter of *fact*, as distinguished from a matter of mathematical conditional possibility, been able to reach this point. We are in every case either forced by the inexorable critic, sooner or later, to declare, "such and such a proposition or mode of inference I *cannot doubt*; it seems perfectly clear that it is so, but I can't say *why*," or else the critic himself tires before the criticism has been pushed to its very end.

Peirce: CP 6.498 Cross-Ref:††

498. If you absolutely cannot doubt a proposition -- cannot bring yourself, upon deliberation, to entertain the least suspicion of the truth of it, it is plain that there is no room to desire anything more.†1 Many and many a philosopher seems to think that taking a piece of paper and writing down "I doubt that" is doubting it, or that it is a thing he can do in a minute as soon as he decides what he wants to doubt. Descartes convinced himself that the safest way was to "begin" by doubting everything, and accordingly he tells us he straightway did so, except only his *je pense*, which he borrowed from St. Augustine. Well I guess not; for genuine doubt does not talk of *beginning* with doubting. The pragmatist knows that doubt is an art which has to be acquired with difficulty; and his genuine doubts will go much further than those of any Cartesian. What he does not doubt, about ordinary matters of everybody's life, he is apt to find that no well matured man doubts. They are part of our instincts. Instincts are now known not to be nearly so unchangeable as used to be supposed; and the present "mutation"-theory, which I have *always* insisted must be the way in which species have arisen,†2 is, I am confident, the first beginning of the correct theory, and shows that it is no disproof of the instinctive character of a belief that it relates to concepts which the primitive man cannot be supposed to have had. Now, this is no confirmation of what one does not doubt. For what one does not doubt cannot be rendered more satisfactory than it already is. Yet while I may entertain, as far as I can search my mind, no perceptible doubt whatever of any one of a hundred propositions, I may suspect that, among so many, some one that is not true may have slipped in; and, if so, the marvellous inerrancy of instinct may perhaps add a little to my *general* confidence in the whole lot. However, I am far from insisting upon the point. I think the consideration is better adapted to helping us to detect the counterfeit paper doubts, of which so many are in circulation.

Peirce: CP 6.499 Cross-Ref:††

499. All the instinctive beliefs, I notice, are vague. The moment they are precided, the pragmatist will begin to doubt them.

Peirce: CP 6.500 Cross-Ref:††

500. The fourth part of the first book of Hume's *Treatise of Human Nature*

affords a strong argument for the correctness of my view that reason is a mere succedaneum to be used where instinct is wanting, by exhibiting the intensely ridiculous way in which a man winds himself up in silly paper doubts if he undertakes to throw common sense, *i.e.* instinct, overboard and be perfectly rational. Bradley's *Appearance and Reality* is another example of the same thing, although Bradley is at the opposite pole from Hume in what he does admit. But Bradley is in no way as good a case as Hume. Hume endeavours to modify his conclusion by not stating it in the extreme length to which it ought to carry him. But a careful reader will see that if he proves anything at all by all his reasoning, it is that reasoning, as such, is *ipso facto* and essentially illogical, "illegitimate," and unreasonable. And the reason it is so is that either it is bad reasoning, or rests on doubtful premisses, or else that those premisses have not been thoroughly criticized. Of course not. The moment you come to a proposition which is perfectly satisfactory, so that you can entertain not the smallest suspicion of it, this fact debars you from making any genuine criticism of it. So that what Hume's argument would lead him to is that reasoning is "illegitimate" because its premisses are perfectly satisfactory. He candidly confesses that they are satisfactory to himself. But he seems to be dissatisfied with himself for being satisfied. It is easy to see, however, that he pats himself on the back, and is very well satisfied with himself for being so dissatisfied with being satisfied. Bradley's position is equally ridiculous. Another circumstance which goes toward confirming my view that instinct is the great internal source of all wisdom and of all knowledge is that all the "triumphs of science," of which that poor old nineteenth century used to be so vain, have been confined to two directions. They either consist in physical -- that is, ultimately, dynamical -- explanations of phenomena, or else in explaining things on the basis of our common sense knowledge of human nature. Now dynamics is nothing but an elaboration of common sense; its experiments are mere imaginary experiments. So it all comes down to common sense in these two branches, of which the one is founded on those instincts about physical forces that are required for the feeding impulsion and the other upon those instincts about our fellows that are required for the satisfaction of the reproductive impulse. Thus, then all science is nothing but an outgrowth from these two instincts. †1

Peirce: CP 6.500 Cross-Ref: ††

You will see that all I have been saying is not preparatory to any argument for the reality of God. It is intended as an apology for resting the belief upon instinct as the very bedrock on which all reasoning must be built.

Peirce: CP 6.501 Cross-Ref: ††

501. I have often occasion to walk at night, for about a mile, over an entirely untravelled road, much of it between open fields without a house in sight. The circumstances are not favorable to severe study, but are so to calm meditation. If the sky is clear, I look at the stars in the silence, thinking how each successive increase in the aperture of a telescope makes many more of them visible than all that had been visible before. The fact that the heavens do not show a sheet of light proves that there are vastly more dark bodies, say planets, than there are suns. They must be inhabited, and most likely millions of them with beings much more intelligent than we are. For on the whole, the solar system seems one of the simplest; and presumably under more complicated phenomena greater intellectual power will be developed. What must be the social phenomena of such a world! How extraordinary are the minds even of the lower animals. We cannot appreciate our own powers any more than a writer can appreciate his own style, or a thinker the peculiar quality of his own thought. I don't mean that a Dante did not know that he expressed himself with fewer words than

other men do, but he could not admire himself as we admire him; nor can we wonder at human intelligence as we do at that of wasps. Let a man drink in such thoughts as come to him in contemplating the physico-psychical universe without any special purpose of his own; especially the universe of mind which coincides with the universe of matter. The idea of there being a God over it all of course will be often suggested; and the more he considers it, the more he will be enwrapt with Love of this idea. He will ask himself whether or not there really is a God. If he allows instinct to speak, and searches his own heart, he will at length find that he cannot help believing it. I cannot tell how every man will think. I know the majority of men, especially educated men, are so full of pedantries -- especially the male sex -- that they cannot think straight about these things. But I can tell how a man must think if he is a pragmatist. Now the shower of communications that I have been getting during the last two months causes me to share the expectation that I find so many good judges are entertaining, that pragmatism is going to be the dominant philosophical opinion of the twentieth century. . . .

Peirce: CP 6.502 Cross-Ref:††

502. If a pragmatist is asked what he means by the word "God," he can only say that just as long acquaintance with a man of great character may deeply influence one's whole manner of conduct, so that a glance at his portrait may make a difference, just as almost living with Dr. Johnson enabled poor Boswell to write an immortal book and a really sublime book, just as long study of the works of Aristotle may make him an acquaintance, so if contemplation and study of the physico-psychical universe can imbue a man with principles of conduct analogous to the influence of a great man's works or conversation, then that analogue of a mind -- for it is impossible to say that *any* human attribute is *literally* applicable -- is what he means by "God." Of course, various great theologians explain that one cannot attribute *reason* to God, nor perception (which always involves an element of surprise and of learning what one did not know), and, in short, that his "mind" is necessarily so unlike ours, that some -- though wrongly -- high in the church say that it is only negatively, as being entirely different from everything else, that we can attach any meaning to the Name. This is not so; because the discoveries of science, their enabling us to *predict* what will be the course of nature, is proof conclusive that, though we cannot think any thought of God's, we can catch a fragment of His Thought, as it were.

Peirce: CP 6.503 Cross-Ref:††

503. Now such being the pragmatist's answer to the question what he means by the word "God," the question whether there really *is* such a being is the question whether all physical science is merely the figment -- the arbitrary figment -- of the students of nature, and further whether the *one* lesson the Gautama Boodha, Confucius, Socrates, and all who from any point of view have had their ways of conduct determined by meditation upon the physico-psychical universe, be only their arbitrary notion or be the Truth behind the appearances which the frivolous man does not think of; and whether the superhuman courage which such contemplation has conferred upon priests who go to pass their lives with lepers and refuse all offers of rescue is mere silly fanaticism, the passion of a baby, or whether it is strength derived from the power of the truth. Now the only guide to the answer to this question lies in the power of the passion of love which more or less overmasters every agnostic scientist and everybody who seriously and deeply considers the universe. But whatever there may be of *argument* in all this is as nothing, the merest nothing, in comparison to its force as an appeal to one's own instinct, which is to argument what substance is to shadow, what bed-rock is to the built foundations of a cathedral.

Peirce: CP 6.504 Cross-Ref:††

504. Caldecott's *Philosophy of Religion* explains thirteen different *types* of reasons for believing in God, with different varieties of several of them.†1 I have examined them all with care, and think each one proves *something*. But I do not think their conclusions always have much to do with *religion*.

Peirce: CP 6.505 Cross-Ref:††
§2. CREATION

505. "Do you believe this Supreme Being to have been the creator of the universe?" Not so much *to have been* as to be now creating the universe, concerning which see my articles in the first three volumes of *The Monist*;†2 and much the same opinion has been entertained by others, especially by Renouvier,†3 the French protagonist of [the monadistic] philosophy. But I object to Renouvier's philosophy as nominalistic and otherwise not thorough. Still, his *Essais de Critique* and particularly his *Nouvelle Monadologie* are very strong books in many respects, which no thoughtful reader can forget. I think that, vain as it is to attempt to bring to light any definite meaning from the idea, it is nevertheless true that *all reality* is due to the creative power of God.

Peirce: CP 6.506 Cross-Ref:††

506. I am inclined to think (though I admit that there is no necessity of taking that view) that the process of creation has been going on for an infinite time in the past, and further, during *all* past time, and, further, that past time had no definite beginning, yet came about by a process which in a generalized sense, of which we cannot *easily* get much idea, was a development.†1 I believe Time to be a reality, and not the figment which Kant's nominalism proposes to explain it as being. As reality, it is due to creative power. People who have had no practice in higher logical analysis are apt to be sceptical as to anybody's being able to attach any idea to such propositions. They are even dumbfounded to hear one say that a part is not necessarily less than its whole; while after one has learned how to think of such things, the marvel is that anybody should ever have deliberately said that the part is necessarily less than the whole or ever should have said "so fast eternity comes on," meaning by "eternity" the infinitely distant future, as if the part of the future that will remain future tomorrow were not just as long as today's or yesterday's future.

Peirce: CP 6.506 Cross-Ref:††

I think we must regard Creative Activity as an inseparable attribute of God.

Peirce: CP 6.507 Cross-Ref:††
§3. GOD'S PURPOSE

507. "What do you imagine the present functions of this Supreme Being toward the universe to be?" Creation, as just said; and much may also be learned from the book *Substance and Shadow* (1863) by Henry James, the father. The book was presented to me, by the way, by Miss Maria Fay, a very interesting and spiritual lady.

In particular, the obvious solution of the problem of evil is there pointed out.†2
Columbus's egg was not simpler. In general, God is perpetually creating us, that is
developing our real manhood, our spiritual reality. Like a good teacher, He is engaged
in detaching us from a False dependence upon Him.

Peirce: CP 6.508 Cross-Ref:††
§4. OMNISCIENCE

508. "Do you believe Him to be omniscient?" Yes, in a vague sense. Of
course, God's knowledge is something so utterly unlike our own that it is more like
willing than knowing.†1 I do not see why we may not assume that He refrains from
knowing much. For this thought is creative. But perhaps the wisest way is to say that
we do not know how God's thought is performed and that [it] is simply vain to
attempt it. We cannot so much as frame any notion of what the phrase "the
performance of God's mind" *means*. Not the faintest! The question is gabble.

Peirce: CP 6.509 Cross-Ref:††
§5. OMNIPOTENCE

509. "Do you believe Him to be Omnipotent?" Undoubtedly He is so, vaguely
speaking; but there are many questions that might be put of no profit except to the
student of logic. Some of the scholastic commentaries consider them. Leibnitz
thought that this was the best of "all possible" worlds. That seems to imply some
limitation upon Omnipotence. Unless the others were created too, it would seem that,
all things considered, this universe was the only possible one. Perhaps others do exist.
But we only wildly gabble about such things.

Peirce: CP 6.510 Cross-Ref:††
§6. INFALLIBILITY

510. "Do you believe Him to be infallible?" If omniscient, how not? But
perhaps this is a slip of the typewriter for *impeccable*. Theologians insist upon sundry
questions which are in the highest degree displeasing to me, not to say offensive. I do
not presume to know anything about it, but it seems to me that the very meaning of
the word "God" implies, not *surely* morality, for He seems to me to be above all
self-restraint or law, but to imply aesthetic spiritual perfection.

Peirce: CP 6.511 Cross-Ref:††
§7. MIRACLES

511. "Do you believe that He ever modifies or changes the laws of nature or interferes with the course of events in individual cases?" I call your attention to the circumstance that some of the most respected theologians, such as St. Augustine,†2 and others before him, St. Thomas Aquinas,†3 Bishop Joseph Butler,†1 are decidedly of the opinion that God never interfered with what they call the *cursus naturæ*, which is what we call the operations of the laws of nature, "laws of nature" meaning with them the items of the *jus naturæ*, or something which my unlegal ignorance is unable to distinguish from that. Miracles are for them simply what no man can do without special aid from on high, or which at least are signs of some special authority, without being in reality deviations from the regular uniformities of the world. However, my own doctrine of Tychism,†2 like Renouvier's somewhat similar theory,†3 and those of Fouillée,†4 Delboeuf,†5 and others, must, in so far as it is accepted, somewhat weaken that view.

Peirce: CP 6.512 Cross-Ref:††

512. I also call your attention to the fact that Hume's †6 argument against miracles †7 has nothing at all to do with whether they are or are not violations of the laws of nature. The argument is based upon a misunderstanding of the doctrine of probabilities, of which some of the early treatises had appeared in his day. It might be corrected, but it would still rest on a complete misunderstanding of the true logic of the criticism of ancient history.†8

Peirce: CP 6.513 Cross-Ref:††

513. But the German critics (I speak only of those who treat of the history of philosophy, for I have never looked into the Biblical criticisms) are as illogical as Hume and in much the same way.†9 Hence, whenever their conclusions have been tested by the spade of the archeologist it has been to their complete discomfiture. Hume's argument is in no particularly intimate relation to the rest of his book, and was evidently inserted as a bid for popularity. For while he was a young fellow of fifteen to seventeen, miracles had been vehemently attacked by a clergyman of the name of Woolston,†10 who took the ground of Origen †1 and other early fathers of the church that the stories in the gospel were simply allegorical. His books had the most stupendous sale in England, completely demonstrating the general disbelief in miracles at that day. In point of fact, there never was a period of history in which the general tone of thought was so absolutely contrary to the supernatural. The state of opinion about the [time of the] French Revolution, and that about 1875, when "agnosticism" was at its [crudest], were pious in comparison with 1730. Therefore, Hume who sacrificed the best parts of his system to make his *Inquiry* popular, undoubtedly stuck in his argument against miracles for that purpose.

Peirce: CP 6.514 Cross-Ref:††

514. For my part, I do not see how we can ascertain *a priori* whether *miracles* (be they violations of the laws of nature or not) and special providences take place or not. In so far Hume is entirely in the right. It is simply a question of evidence. His argument has a certain weight. If there are no miracles nowadays, there is a strong presumption against those which took place amidst a rabble of Galileans. But are

there no miracles nowadays? I do not feel so sure of it. There is Mrs. Piper and Perry. I do not think it rational not [to] think, for us who know Perry, that that case is of *tremendous*, almost conclusive, weight. There is the blood of St. Januarius which Sir Humphrey Davy -- *of his own motion*, and not forced into it at all -- undertook to investigate and was given every facility he could think of, and who declared he could not find the least symptom of fraud about the thing. Take such men as Sir William Crookes †2 and Lord Rayleigh †3 -- well even Hodgson †4 -- one must confess the case is very strong; so strong that but for one circumstance I should unhesitatingly accept it. That circumstance is that every surprising discovery of science -- as for example when Becquerel found those photographic plates which he had put away in a drawer to be affected by the uranium salt that was wrapped up in black paper and accidentally laid upon them -- every such event, is soon followed by others closely connected with it, so that all possible doubt is swept away together with all surprise at the occurrence. Miracles, on the contrary, are always *sui generis*. The only ones that were not so, the falling of stones out of the heavens, lost all their prestige when it was found how common the occurrence was. The isolatedness of the miracle is really no argument against its reality. It is nearly the same with works of great genius. You have Rafael and Michelangelo together, and then for a long time nothing surprising. Dante stands all alone. Byron was unparalleled before or since; for A. de Musset is surely not to be compared with him. Indeed every branch of art and science can furnish such examples. The isolation, then, is no argument *against* miracles, but it effectively prevents our ever having sufficient evidence of them. I must confess that the gospel miracles appear at this date very far from impressive. It is curious that Origen, no further from Jesus in history than we from the expulsion of James II from England, should have found them so difficult to believe.

Peirce: CP 6.515 Cross-Ref: ††
 §8. PRAYER

515. "Do you believe in the efficacy of prayer?" The only thing connected with that, that I am quite satisfied about, is that the clergy do not believe in it. I mean the influential clergy. The conclusive proof of that is that when Tyndall †1 proposed to put the matter to the test of experiment, although they had the record of the somewhat similar proposal of the King of Samaria and Elijah's †2 perfectly frank response, they backed down and pretended that it would be blasphemous. So it is blasphemy to inquire into the truth of religion, is it? No living man thinks it disrespectful to inquire into the authenticity of his signature; and the higher clergy are far more sensitive to their own dignity than God's, and very justly so, since it is quite possible to be disrespectful to an ecclesiastic, while it is absolutely impossible *really* to think of God without awe mingled with love.

Peirce: CP 6.516 Cross-Ref: ††

516. But what business is it of mine whether my prayers are to be efficacious or not? We, one and all of us, have an instinct to pray; and this fact constitutes an invitation from God to pray. And in fact there is found to be not only *soulagement* in prayer, but great spiritual good and moral strength. I do not see why prayer may not be efficacious, or if not the prayer exactly, the state of mind of which the prayer is nothing more than the expression, namely the soul's consciousness of its relation to God, which is nothing more than precisely the pragmatistic meaning of the name of

God; so that, in that sense, prayer is simply calling upon the name of the Lord. To pray for specific things, not merely for the {epioution}, bread, but that it may be better baked than yesterday's, is childish, of course; yet innocent.

Peirce: CP 6.517 Cross-Ref:††

517. "Why does not this Omniscient Being see the need and interpose the Omnipotent and Supreme Authority to meet the needs prayed for? Is it because of a vanity which is one of the attributes of fallible man?" I remember two passages in my writings in which I made as much fun as politeness would allow of writers who undertook to tell us what was "conducive to our welfare." Once it was Simon Newcomb who was talking like that in his book on Political Economy;†1 and I remarked that an economist, far from having any qualifications for exploring this most occult of all matters, was particularly unfit for the task owing to his habit of taking it for granted that wealth was desirable.†2 The other time it was Karl Pearson, who wanted to found the rules of logic upon that, and I remarked †3 that, for my part, if ever I undertook the supremely difficult inquiry of what was conducive to our welfare I should feel that I needed to arm myself beforehand with whatever resources logic could afford, to speak of no others. What are our "needs"? We know what we have an impulse to seek, and if we have considered the matter deliberately we are convinced that those things are far from being the same as our true needs. Yet if we are going to pray for anything specific, which is once in a long time, on some supreme occasion, a permissible frailty, surely we shall add something like, "Fulfill now, O Lord, the desires and petitions of Thy servants, *as may be most expedient for them.*" Not to do so, would, as you seem to suggest, be vanity indeed.

Peirce: CP 6.518 Cross-Ref:††

518. "Do you believe that the prayers of several persons for one end are more potent than those of one?" I know of no experiments to ascertain how this may be; but I certainly think that common prayers have some peculiar virtues of their own. As I say, the inquiry into efficacy is distasteful to me because that is not the motive of my prayers. Still, I should like to have an inquiry instituted into the matter.

Peirce: CP 6.519 Cross-Ref:††

§9. IMMORTALITY †1

519. "Do you believe in a future life?" *Some* kind of a future life there can be no doubt of. A man of character leaves an influence living after him. It is living: it is personal. In my opinion, it is quite proper to call that a future life. Jesus so spoke of it when he said he would always be with us. It is in some respects more fit to be made the subject of a promise than any other kind of future life. For it is something we all desire; while other kinds present nothing alluring that is not excessively vague or else unwholesome and antipractical. In the next place its vivacity and endurance are proportional to the spirituality of the man. How many instances have we seen of that! Beyond that, I simply am content to be in God's hands. If I am in another life it is sure to be most interesting; but I cannot imagine how it is going to be *me*. At the same time, I really don't know anything about it.

Peirce: CP 6.520 Cross-Ref:††

520. "Is not every act of memory in the human being the result of the action of that being's material brain. . . . ? If this is true, on the death of the material body . . . does not the memory cease?"†2

Peirce: CP 6.520 Cross-Ref:††

This is commonly assumed to be the case; and owing to my slight interest in the matter it may well be that there are some facts bearing upon the question that I am not aware of. But my impression is that there is no positive reason for believing it except the general facts of the dependence of mental action on the brain. For instance, when Broca's convolution is much diseased we always find the use of language is greatly affected. But I am sure this is not a strong positive reason for an affirmative answer to the first of the two questions. It undoubtedly warrants the assumption in science, until facts to the contrary appear. But your questions are not scientific, but practical questions. From that standpoint I think I must say that the matter is open to some doubt. When a part of the brain is extirpated we find the result is that certain faculties are lost. But after a time they are recovered. How can this be? The answer given is that other parts of the brain learn to perform these functions. But after all, we do not know more than that if anything happens to the hemispheres, memory is deranged. It is a most wonderful thing if all we remember is really preserved in the cells of the cerebrum. However, there can be no doubt, I think, that upon death we soon lose consciousness, at least for the time being.

Peirce: CP 6.520 Cross-Ref:††

You will observe that the essential immortality of the soul is not exactly the Christian doctrine, which is that the body is reproduced, and with it presumably the memory. There is nothing at all to prove it except that it was a belief clung to by St. Paul and founded by him upon the resurrection of Jesus.

Peirce: CP 6.521 Cross-Ref:††

521. "If the power to remember dies with the material body, has the question of any single person's future life after death any particular interest for him?" As you put the question, it is not whether the matter ought rationally to have an interest, but whether as a fact it has; and perhaps this is the proper question, trusting as it seems to do, rather to instinct, than to reason. Now if we had a drug which would abolish memory for a while, and you were going to be cut for the stone, suppose the surgeon were to say, "You will suffer damnably, but I will administer this drug so that you will during that suffering lose all memory of your previous life. Now you have of course no particular interest in your sufferings as long as you will not remember your present and past life, you know, have you?"†1

Peirce: CP 6.522 Cross-Ref:††
CHAPTER 5

HUME ON MIRACLES†1P

§1. THE NATURE OF HYPOTHESIS

522. The science of legitimate inference can only be of practical value provided its propositions are proved with absolute completeness. For they have to carry weight enough to override our instinctive judgments of what is good reasoning. If they cannot do that, they are of no use. But even apparently convincing proofs may be mistaken; and the maxims of this science may be suspected of being so, if they conflict with our instinctive logic.^{†2} They must, therefore, not only come to us supported by full proofs, but also by the recommendation of those who have had long experience in the use of them.

Peirce: CP 6.522 Cross-Ref:††

In a brief article it is impossible to make the proofs clearly evident; and it would be unjust to them to attempt such a thing, which could only convey the idea that they were of an unintelligible and unconvincing nature. Yet to discuss the legitimacy of inferences on any other than the true scientific grounds is mere trifling. The difficulty is insuperable. All that can be done is to lay down the correct principles, and postpone the proofs to another occasion. It may perhaps be permissible to give some hints as to what the general nature of the proofs is.

Peirce: CP 6.522 Cross-Ref:††

All our knowledge may be said to rest upon *observed facts*. It is true that there are psychological states which antecede our observing facts as such. Thus, it is a fact that I see an inkstand before me; but before I can say that I am obliged to have impressions of sense into which no idea of an inkstand, or of any separate object, or of an "I," or of seeing, enter at all; and it is true that my judging that I see an inkstand before me is the product of mental operations upon these impressions of sense. But it is only when the cognition has become worked up into a proposition, or judgment of a fact, that I can exercise any direct control over the process; and it is idle to discuss the "legitimacy" of that which cannot be controlled. Observations of fact have, therefore, to be accepted as they occur.

Peirce: CP 6.523 Cross-Ref:††

523. But observed facts relate exclusively to the particular circumstances that happened to exist when they were observed. They do not relate to any future occasions upon which we may be in doubt how we ought to act. They, therefore, do not, in themselves, contain any practical knowledge.

Peirce: CP 6.523 Cross-Ref:††

Such knowledge must involve additions to the facts observed. The making of those additions is an operation which we can control; and it is evidently a process during which error is liable to creep in.

Peirce: CP 6.524 Cross-Ref:††

524. Any proposition added to observed facts, tending to make them applicable in any way to other circumstances than those under which they were observed, may be called a hypothesis. A hypothesis ought, at first, to be entertained interrogatively. Thereupon, it ought to be tested by experiment so far as practicable. There are two distinct processes, both of which may be performed rightly or wrongly. We may go wrong and be wasting time in so much as entertaining a hypothesis, even as a question. That is a subject for criticism in every case. There are some hypotheses which are of such a nature that they never can be tested at all. Whether such hypotheses ought to be entertained at all, and if so in what sense, is a serious

question; but it hardly concerns our present inquiry. The hypotheses with which we shall have in this paper to deal are capable of being put to the test. How this is to be done is a question of extreme importance; but my intention is to consider it only in a very cursory manner, at present. There are, moreover, many hypotheses in regard to which knowledge already in our possession may, at once, quite justifiably either raise them to the rank of opinions, or even positive beliefs, or cause their immediate rejection. This also is a matter to be considered. But it is the first process, that of entertaining the question, which will here be of foremost importance.

Peirce: CP 6.525 Cross-Ref:††

525. Before we go further, let us get the points stated above quite clear. By a *hypothesis*, I mean, not merely a supposition about an observed object, as when I suppose that a man is a Catholic priest because that would explain his dress, expression of countenance, and bearing, but also any other supposed truth from which would result such facts as have been observed, as when van't Hoff, having remarked that the osmotic pressure of one per cent solutions of a number of chemical substances was inversely proportional to their atomic weights, thought that perhaps the same relation would be found to exist between the same properties of any other chemical substance. The first starting of a hypothesis and the entertaining of it, whether as a simple interrogation or with any degree of confidence, is an inferential step which I propose to call *abduction*. This will include a preference for any one hypothesis over others which would equally explain the facts, so long as this preference is not based upon any previous knowledge bearing upon the truth of the hypotheses, nor on any testing of any of the hypotheses, after having admitted them on probation. I call all such inference by the peculiar name, *abduction*, because its legitimacy depends upon altogether different principles from those of other kinds of inference.

Peirce: CP 6.526 Cross-Ref:††

§2. THE TESTING OF HYPOTHESES †1

526. The operation of testing a hypothesis by experiment, which consists in remarking that, if it is true, observations made under certain conditions ought to have certain results, and then causing those conditions to be fulfilled, and noting the results, and, if they are favorable, extending a certain confidence to the hypothesis, I call *induction*. For example, suppose that I have been led to surmise that among our colored population there is a greater tendency toward female births than among our whites. I say, if that be so, the last census must show it. I examine the last census report and find that, sure enough, there was a somewhat greater proportion of female births among colored births than among white births in that census year. To accord a certain faith to my hypothesis on that account is legitimate. It is a strong induction. I have taken all the births of that year as a sample of all the births of years in general, so long as general conditions remain as they were then. It is a very large sample, quite unnecessarily so, were it not that the excess of the one ratio over the other is quite small. All induction whatever may be regarded as the inference that throughout a whole class a ratio will have about the same value that it has in a random sample of that class, provided the nature of the ratio for which the sample is to be examined is specified (or virtually specified) in advance of the examination.†1 So long as the class sampled consists of units, and the ratio in question is a ratio between counts of

occurrences, induction is a comparatively simple affair. But suppose we wish to test the hypothesis that a man is a Catholic priest, that is, has all the characters that are common to Catholic priests and peculiar to them. Now characters are not units, nor do they consist of units, nor can they be counted, in such a sense that one count is right and every other wrong. Characters have to be estimated according to their significance. The consequence is that there will be a certain element of guess-work in such an induction; so that I call it an *abductory induction*. †2 I might say to myself, let me think of some other character that belongs to Catholic priests, beside those that I have remarked in this man, a character which I can ascertain whether he possesses or not. All Catholic priests are more or less familiar with Latin pronounced in the Italian manner. If, then, this man is a Catholic priest, and I make some remark in Latin which a person not accustomed to the Italian pronunciation would not at once understand, and I pronounce it in that way, then if that man is a Catholic priest he will be so surprised that he cannot but betray his understanding of it. I make such a remark; and I notice that he does understand it. But how much weight am I to attach to that test? After all, it does not touch an essential characteristic of a priest or even of a Catholic. It must be acknowledged that it is but a weak confirmation, and all the more so, because it is quite uncertain how much weight should be attached to it. Nevertheless, it does and ought to incline me to believe that the man is a Catholic priest. It is an induction, because it is a test of the hypothesis by means of a prediction, which has been verified. But it is only an abductory induction, because it was a sampling of the characters of priests to see what proportion of them this man possessed, when characters cannot be counted, nor even weighed, except by guess-work. It also partakes of the nature of abduction in involving an original suggestion; while typical induction has no originality in it, but only tests a suggestion already made.

Peirce: CP 6.527 Cross-Ref: ††

527. In induction, it is not the fact predicted that in any degree necessitates the truth of the hypothesis or even renders it probable. It is the fact that it has been predicted successfully and that it is a haphazard specimen of all the predictions which might be based on the hypothesis and which constitute its practical truth. But it frequently happens that there are facts which, merely as facts, apart from the manner in which they have presented themselves, necessitate the truth, or the falsity, or the probability in some definite degree, of the hypothesis. For example, suppose the hypothesis to be that a man believes in the infallibility of the Pope. Then, if we ascertain in any way that he believes in the immaculate conception, in the confessional, and in prayers for the dead, or on the other hand that he disbelieves all or some of these things, either fact will be almost decisive of the truth or falsity of the proposition. Such inference is *deduction*. So if we ascertain that the man in question is a violent partisan in politics and in many other subjects. If, then, we find that he has given money toward a Catholic institution, we may fairly reason that such a man would not do that unless he believed in the Pope's infallibility. Or again, we might learn that he is one of five brothers whose opinions are identical on almost all subjects. If, then, we find that the other four all believe in the Pope's infallibility or all disbelieve it, this will affect our confidence in the hypothesis. This consideration will be strengthened by our general experience that while different members of a large family usually differ about most subjects, yet it mostly happens that they are either all Catholics or all Protestants. Those are four different varieties of deductive considerations which may legitimately influence our belief in a hypothesis.

Peirce: CP 6.528 Cross-Ref: ††

528. These distinctions are perfectly clear in principle, which is all that is

necessary, although it might sometimes be a nice question to say to which class a given inference belongs. It is to be remarked that, in pure abduction, it can never be justifiable to accept the hypothesis otherwise than as an interrogation. But as long as that condition is observed, no positive falsity is to be feared; and therefore the whole question of what one out of a number of possible hypotheses ought to be entertained becomes purely a question of economy.

Peirce: CP 6.529 Cross-Ref:††

529. Let us suppose that there are thirty-two different possible ways of explaining a set of phenomena. Then, thirty-one hypotheses must be rejected. The most economical procedure, when it is practicable, will be to find some observable fact which, under conditions easily brought about, would result from sixteen of the hypotheses and not from any of the other sixteen. Such an experiment, if it can be devised, at once halves the number of hypotheses. Or if the experiment might give any one of four results each of which would be the necessary consequence of the truth of any one of eight of the hypotheses, the single experiment would divide the number of admissible hypotheses by four. When such an experiment, or anything approaching such an experiment, is possible, it is clear that it is unwise to adopt any other course. But unfortunately, it commonly happens that this method becomes exhausted before the hypotheses are reduced to a single one, so that nothing remains but to test the remainder each by itself.

Peirce: CP 6.530 Cross-Ref:††

530. Now the testing of a hypothesis is usually more or less costly. Not infrequently the whole life's labor of a number of able men is required to disprove a single hypothesis and get rid of it. Meantime the number of possible hypotheses concerning the truth or falsity of which we really know nothing, or next to nothing, may be very great. In questions of physics there is sometimes an infinite multitude of such possible hypotheses. The question of economy is clearly a very grave one.†1

Peirce: CP 6.530 Cross-Ref:††

In very many questions, the situation before us is this: We shall do better to abandon the whole attempt to learn the truth, however urgent may be our need of ascertaining it, unless we can trust to the human mind's having such a power of guessing right that before very many hypotheses shall have been tried, intelligent guessing may be expected to lead us to the one which will support all tests, leaving the vast majority of possible hypotheses unexamined. Of course, it will be understood that in the testing process itself there need be no such assumption of mysterious guessing-powers. It is only in selecting the hypothesis to be tested that we are to be guided by that assumption.

Peirce: CP 6.531 Cross-Ref:††

531. If we subject the hypothesis, that the human mind has such a power in some degree, to inductive tests, we find that there are two classes of subjects in regard to which such an instinctive scent for the truth seems to be proved. One of these is in regard to the general modes of action [of] mechanical forces, including the doctrine of geometry; the other is in regard to the ways in which human beings and some quadrupeds think and feel. In fact, the two great branches of human science, physics and psychics, are but developments of that guessing-instinct under the corrective action of induction.†1

Peirce: CP 6.532 Cross-Ref:††

532. In those subjects, we may, with great confidence, follow the rule that that

one of all admissible hypotheses which seems the simplest to the human mind ought to be taken up for examination first. Perhaps we cannot do better than to extend this rule to all subjects where a very simple hypothesis is at all admissible.

Peirce: CP 6.532 Cross-Ref:††

This rule has another advantage, which is that the simplest hypotheses are those of which the consequences are most readily deduced and compared with observation; so that, if they are wrong, they can be eliminated at less expense than any others.†2

Peirce: CP 6.533 Cross-Ref:††

533. This remark at once suggests another rule, namely, that if there be any hypothesis which we happen to be well provided with means for testing, or which, for any reason, promises not to detain us long, unless it be true, that hypothesis ought to be taken up early for examination. Sometimes, the very fact that a hypothesis is improbable recommends it for provisional acceptance on probation.

Peirce: CP 6.534 Cross-Ref:††

534. On the other hand, if one of the admissible hypotheses presents a marked probability of the nature of an objective fact, it may in the long run promote economy to give it an early trial. By an objective probability I mean one which could be used to guarantee an insurance company or gamester against loss, because it expresses the real fact that among occurrences of a certain genus a certain proportion are of a certain species. Such is the probability of one/six that a die will turn up any particular face. Such a probability must be distinguished from a mere likelihood †1 which is nothing better than the expression of our preconceived ideas. The confusion between those two kinds of probability is one of the main sources of human errors, especially in abduction, in which yielding to judgments of likelihood is a fertile source of waste of time and energy.

Peirce: CP 6.535 Cross-Ref:††

535. In some departments of science, where experimentation is easy, the testing of hypotheses may be performed with some promptitude. In other departments, especially in ancient history, it will extend beyond a human life, so that for the individual the result of the abduction is all that he can hope to live to see. So long as the scientific hypothesis does not offer any particular dangers to the individual, he will do well to content himself with that hypothesis which the wise application of principles of economy recommends to undying scientific research. On the other hand, if there are such dangers, the individual may, as a scientific man, entertain one hypothesis for probation, while he allows probabilities greater weight in deciding upon what hypothesis he shall base his individual behaviour. Thus, in metaphysics, the maxim called Ockham's razor, to the effect that more elements must not be introduced into a hypothesis until it is absolutely proved that fewer are not sufficient, is a sound economic principle which ought to guide the scientific metaphysician. But centuries before it is absolutely proved that the simpler hypothesis is inadequate, it may have been made extremely probable that it is so, and the individual's behaviour may reasonably be based upon what the ultimate conclusion of science is likely to be.

Peirce: CP 6.536 Cross-Ref:††

536. In the department of ancient history, what is called "higher criticism" -- that is to say, that particular color of non-textual criticism which has been dominant during the nineteenth century, especially in Germany -- has placed, and though it has of late years retreated from many of its positions, still continues to place, great

reliance upon likelihoods. To such a pitch is this carried that, although we can have no knowledge of ancient history independent of Greek (and Latin) authors, yet the critics do not hesitate utterly to reject narratives attested sometimes by as many as a dozen ancient authorities -- all the testimony there is, at any rate -- because the events narrated do not seem to persons living in modern Germany to be likely. I could write a whole book,†1 and not an unentertaining one, in illustration of this point. But scientific archaeology has, in our day, subjected those hypotheses to objective tests; and the uniform result has been to show that what seemed likelihoods to German professors were all but quite uniformly wrong and the ancient testimonies right.†2 Thus the maxim of exact logical analysis, that no regard at all, or very little indeed, ought to be paid to subjective likelihoods in abduction, has been fully confirmed by inductive tests.

Peirce: CP 6.537 Cross-Ref:††
§3. THE MEANING OF MIRACLES

537. Hume's argument against miracles is substantially based upon the assumption that we ought to judge of testimony by balancing the likelihood that the witnesses tell the truth against the likelihood that no such event as that to which they testify ever took place.†3 It is true that Hume gives a metaphysical definition of a miracle based upon the definition of Aquinas.†4 But his argument in no way turns upon that. The definition he virtually *uses* is that a miracle is something the like of which has never been known to happen. He has completely mistaken the nature of the true logic of abduction.

Peirce: CP 6.538 Cross-Ref:††

538. I beg to say that I go no farther than that. I do not assent to the contention of many theologians that the miracles of Jesus can properly convince a modern man of the divinity of Jesus. On the contrary, all the evidence which can now be presented for them is quite insufficient, unless the general divinity of the Christian religion be assumed. The evidence which may have been overwhelming for eye witnesses and persons near them is of a very different and inferior character to that which may weigh with a modern Christian.

Peirce: CP 6.539 Cross-Ref:††

539. Now, laying aside the question of how one ought to think, let us ask what effect was, in fact, produced upon Hume's contemporaries by his argument against miracles; and also, what effect was produced by Hume's introducing a definition of a miracle which introduced the idea of a *Law of Nature*.†1 For it is a fact that Hume defined a miracle as a violation of a law of nature, and that the great bulk of his contemporaries were not sufficiently at home in philosophy to see that he was giving the definition a metaphysical turn that was quite uncalled for.

Peirce: CP 6.540 Cross-Ref:††

540. The fathers of the church had introduced no more metaphysics into their definitions of a miracle than had the simple folks who had witnessed miracles, or thought they had witnessed them. For both classes a miracle was nothing more than a great wonder.

Peirce: CP 6.541 Cross-Ref:††

541. The miracle remained nothing more than a great wonder, until the scholastic doctors, in their desire to give exactitude to theology, began to define it metaphysically. Aquinas said that a miracle was an interruption of the order of nature; and that remained the regular definition for the scholastics. When Hume took up the subject of miracles, he endeavoured to conform to the definition of the theologians, although for the purposes of his argument it was a matter of indifference how a miracle should be defined. All he needed was that it should be something the like of which was perfectly unexampled in experience. He was perfectly willing to adopt whatever definition the theologians preferred. But Hume was a literary man, and one of the characteristics of his philosophical style was that he was continually endeavouring to clothe philosophical ideas in fresh and modern phraseology. He probably referred either to Aquinas or to some theologian influenced by Aquinas; but he thought the definition would come home to his readers more if, instead of defining a miracle as an interruption of the order of nature, he defined it as a violation of a law of nature; for "law of nature" had become a familiar phrase. There was nothing in this modification of language which was particularly favorable to Hume's argument.

Peirce: CP 6.542 Cross-Ref:††

542. Aquinas had not spoken of a violation of a law of nature, because the phrase "law of nature" bore, in his day, no such meaning as Hume attached to it. The phrase itself is very old. It occurs in the early Greek poet Pindar,†1 and in Plato.†2 In Latin it is met with in the early poet Lucretius.†3 But until modern times, it had meant a rule of natural morality. For a scholastic, therefore, it would have been simple nonsense to say that a violation of a law of nature would be a miracle. That is why he spoke of the "order of nature" which meant for him substantially what we mean by a "law of nature." If there was any difference, it lay in this, that to the majority of scholastics, the order of nature was something absolutely real, having a being in or behind the very essence of nature; while for the majority of modern thinkers a law of nature is relative to the human understanding. The medieval "order of nature" would, therefore, have been more inviolable than the modern "law of nature," were it not for the fact that the marvellous appealed to the childish mind of the Middle Ages, while the scientific regularity appeals to the modern mind. But after men had become generally pretty thoroughly disgusted with scholasticism, Descartes came forward with a new philosophy which answered their new wants. Now it was a part of this philosophy, not, it is true, much insisted on by Descartes †4 himself, but so much in the line of his ideas that his followers would be sure to insist upon it, as in fact they did, that nothing takes place in the world without the direct assistance of the Deity. There was no other force in causation, according to them, than that it is the will of God that events shall follow one another in certain definite ways. It was under the influence of this conception that Boyle,†5 a natural philosopher of strong theological tendencies, and at the same time of a decided physical turn of thought, began to speak of "laws of nature," very much in the modern sense, except that with him, no doubt, there was an implication that they were divine decrees. The phrase had met with favor and, by Hume's time, had become familiar to all English ears. He adopted it, therefore, in his definition of a miracle, although it rather suggested the possibility of miracles than otherwise. Still, as close a reasoner as Hume was could easily see that it would not materially affect the force of his argument to admit that miracles do, from time to time, occur; for in regard to any special miracle it would still remain more likely that the miracle had not occurred and that the witnesses had not given exact testimony than that the witnesses had been exact and that the particular miracle in question had occurred. The truth is that, as soon as it is granted that it is proper to judge of testimony by the balance of likelihoods and unlikelihoods, Hume's reasoning

about miracles has been substantially admitted.

Peirce: CP 6.543 Cross-Ref:††

543. In estimating the effect of Hume's argument upon his contemporaries it should be remembered that attacks upon the miracles of Jesus presented no novelty whatever at the time when Hume's celebrated essay was written. For all through the period of Hume's boyhood the whole island of Great Britain had resounded with the violent attacks continually renewed upon them by Thomas Woolston, whose books had had a popular sale that was quite unprecedented. Woolston was a thoroughly sincere Christian; but he maintained that the fathers of the church had unanimously held the gospel miracles to be parables and types of the greater spiritual miracles that the religion of Christ was destined to accomplish.†1 He offered no general argument against miracles, but simply took up the narrative of each one and undertook to show, first, that the circumstances were such as to render its literal happening incredible, and secondly, that even if it did happen it would not tend to prove the divinity of Jesus. As an example of his style of reasoning we may take the miracle of casting out the devils into the herd of swine. Woolston said that it could not have happened, because it had for centuries been against the law in Judæa to keep swine; so that no herd of swine could have been on the public road. He added that if it had happened, so far from evidencing the divinity of Jesus, it would simply have evinced a total disregard for the rights of property, for which any jury in England would have awarded damages. Woolston kept up a hot fire of such arguments all his life, and after his death his follower Annet †1 had taken up the cudgels.

Peirce: CP 6.544 Cross-Ref:††

544. This discussion had been made familiar to every man in England and Scotland; and it is therefore safe to say that by the time Hume wrote, whoever was to [be] persuaded that the miracles of Jesus were unliteral had already been persuaded. But I must say that I cannot but believe that Woolston had carried substantially the whole population with him on his main contention, which was that whatever views are taken of the miracles they cannot suffice by themselves, in modern days, to prove the Christian religion. I believe that those who adhered to the literal miracles did so because they were inclined to believe in the Christian religion, and that they had, but for rare exceptions, failed to be brought to believe in the Christian religion because of an antecedent conviction of the reality of the gospel miracles.

Peirce: CP 6.545 Cross-Ref:††

545. What Hume mainly did was to supply those who were disposed to reject the miracles with an expeditious way of disposing of the evidence at one blow. At the same time, he probably shook the belief of some who had been accustomed to regard legal evidence as a matter to be discussed by the method of balancing likelihoods and unlikelihoods -- a method which had met considerable favour among lawyers.

Peirce: CP 6.546 Cross-Ref:††

546. Of course, formalism was rife among the men of that generation. There was in many minds such a worship of obstinacy that they would naturally look upon the immutability of a divine decree as an attribute of Deity, regard[less] of the circumstances of any particular case. The extreme irrationality of many rules of law fostered such sentiments. It may have been, therefore, that as soon as some men of that description were credibly informed that a miracle was a violation of a law of nature, or divine decree, they would be unable to conceive that such a Deity as they could worship, the personification of obstinacy, would ever consent to such a thing; and they may perhaps have read that argument into Hume, although it is one which

Hume himself could only look upon with contempt. I can, at any rate, imagine no other way in which Hume's definition of a miracle could have specially weakened faith in miracles.

Peirce: CP 6.547 Cross-Ref:††
§4. BUTLER'S ANALOGY

547. It was while Hume was engaged in writing his first treatise, and long before he touched upon miracles, that Bishop Butler's *Analogy of Reason with Nature* was published.†1 This work contains an interesting application of the then current notion that the order of nature is a law to the doctrine of miracles. Butler †2 remarks that if we could know what the laws of nature really are it would perhaps be seen that they positively require the occurrence of miracles. For if there are any "laws" of nature, they must be supposed to be supremely reasonable. Now the supreme reasonableness of a "law" will consist in its advancing a rational purpose in every particular case. Hence, if there is really a need of an apparently exceptional phenomenon, it will not be contrary to real analogies, but on the contrary required by them, that that apparently exceptional phenomenon should occur. On the surface of it, at any rate, this view creates no objection to Hume's real argument; but it clearly does show that to look upon the order of nature as being of the nature of a "law" is to adopt a view which is really favorable to miracles, rather than the reverse.

Peirce: CP 6.547 Cross-Ref:††

But when we come to penetrate the spirit of Butler's remark, we recognize that it has, hidden in the depths of it, an idea which has only to be developed to refute all such reasonings as that of Hume about miracles, and the similar but far more extravagant conclusions of the "higher critics" of ancient history, and which is in remarkable consonance with the higher teachings of modern science.

Peirce: CP 6.548 Cross-Ref:††
CHAPTER 6

SCIENCE AND IMMORTALITY†1

§1. PSYCHIC RESEARCH

548. What is the bearing of positively ascertained facts upon the doctrine of a future life? By the doctrine of a future life, I understand the proposition that after death we shall retain or recover our individual consciousness, feeling, volition, memory, and, in short (barring an unhappy contingency), all our mental powers unimpaired. The question is, laying aside all higher aspects of this doctrine, its sacredness and sentiment -- concerning which a scientific man is not, as such, entitled

to an opinion -- and judging it in the same cold way in which a proposition in physics would have to be judged, what facts are there leading us to believe or to disbelieve it?

Peirce: CP 6.549 Cross-Ref:††

549. Under the head of direct positive evidence to the affirmative would be placed that of religious miracles, of spiritualistic marvels, and of ghosts, etc. I have little to say to all this. I take the modern Catholic miracles to be the best attested. Three members of the English Psychical Research Society have lately published a vast book of fourteen hundred pages, large octavo, under the title of *Phantasms of the Living*. This work gives some seven hundred cases of apparitions, etc. of a dying person to another person at a distance. The phenomenon of telepathy, or perception under conditions which forbid ordinary perception, though not fully established, is supported by some remarkable observations. But the authors of the book I am speaking of -- Messrs. Gurney, Myers, and Podmore -- think they have proved a kind of telepathy by which dying persons appear to others at great distances. Their most imposing arguments are based upon the doctrine of probabilities, and these I have examined with care. I am fully satisfied that these arguments are worthless, partly because of the uncertainty and error of the numerical data, and partly because the authors have been astonishingly careless in the admission of cases ruled out by the conditions of the argumentation.

Peirce: CP 6.550 Cross-Ref:††

550. But, granting all the ghost stories that ever were told, and the reality of all spiritual manifestation, what would they prove? These ghosts and spirits exhibit but a remnant of mind. Their stupidity is remarkable. They seem like the lower animals. If I believed in them, I should conclude that, while the soul was not always at once extinguished on the death of the body, yet it was reduced to a pitiable shade, a mere ghost, as we say, of its former self. Then these spirits and apparitions are so painfully solemn. I fancy that, were I suddenly to find myself liberated from all the trials and responsibilities of this life, my probation over, and my destiny put beyond marring or making, I should feel as I do when I find myself on an ocean steamer, and know that for ten days no business can turn up, and nothing can happen. I should regard the situation as a stupendous frolic, should be at the summit of gayety, and should only be too glad to leave the vale of tears behind. Instead of that, these starveling souls come mooning back to their former haunts, to cry over spilled milk.

Peirce: CP 6.551 Cross-Ref:††

551. Under the head of positive evidence apparently unfavorable to the doctrine, we may reckon ordinary observations of the dependence of healthy mind-action upon the state of the body. There are, also, those rare cases of double consciousness where personal identity is utterly destroyed or changed, even in this life. If a man or woman, who is one day one person, another day another, is to live hereafter, pray tell me which of the two persons that inhabit the one body is destined to survive?

Peirce: CP 6.552 Cross-Ref:††

552. There is certainly a large and formidable mass of facts, which, though not bearing directly upon the question of a future life, yet inclines us to a general conception of the universe which does not harmonize with that belief. We judge of the possibility of the unseen by its analogy with the seen. We smile at Aladdin's lamp or the elixir of life, because they are extremely unlike all that has come under our observation. Those of us who have never met with spirits, or any fact at all analogous to immortality among the things that we indubitably know, must be excused if we

smile at that doctrine. As far as we see, forms of beauty, of sentiment, and of intelligence are the most evanescent of phenomena.

"The flower that once has bloomed forever dies."

Besides, scientific studies have taught us that human testimony, when not hedged about with elaborate checks, is a weak kind of evidence. In short, the utter unlikeness of an immortal soul to anything we cannot doubt, and the slightness of all the old arguments of its existence, appear to me to have tremendous weight.

Peirce: CP 6.553 Cross-Ref:††

§2. THE BREAKDOWN OF THE MECHANICAL PHILOSOPHY

553. On the other hand, the theory of another life is very likely to be strengthened, along with spiritualistic views generally, when the palpable falsity of that mechanical philosophy of the universe which dominates the modern world shall be recognized. It is sufficient to go out into the air and open one's eyes to see that the world is not governed altogether by mechanism, as Spencer, in accord with greater minds, would have us believe. The endless variety in the world has not been created by law. It is not of the nature of uniformity to originate variation, nor of law to beget circumstance. When we gaze upon the multifariousness of nature we are looking straight into the face of a living spontaneity.†1 A day's ramble in the country ought to bring that home to us.

Peirce: CP 6.554 Cross-Ref:††

554. Then there is the great fact of growth, of evolution. I know that Herbert Spencer endeavours to show that evolution is a consequence of the mechanical principle of the conservation of energy. But his chapter †2 on the subject is mathematically absurd, and convicts him of being a man who will talk pretentiously of what he knows nothing about. The principle of the conservation of energy may, as is well known, be stated in this form: whatever changes can be brought about by forces can equally happen in the reverse order (all the movements taking place with the same velocities, but in the reverse directions), under the government of the same forces. Now, the essential of growth is that it takes place in one determinate direction, which is *not* reversed. Boys grow into men, but not men into boys. It is thus an immediate corollary from the doctrine of the conservation of energy that growth is not the effect of force alone.

Peirce: CP 6.555 Cross-Ref:††

555. The world, then, is evidently not governed by blind law. Its leading characteristics are absolutely irreconcilable with that view. When scientific men first began to understand dynamics, and had applied it with great success to the explanation of some phenomena, they jumped to the anticipation that the universe could be explained in that way; and thus what was called the Mechanical Philosophy was set up. But a further study of the nature of force has shown that it has this conservative character, which absolutely refutes that mechanical notion of the universe. As well as I can read the signs of the times, the doom of necessitarian

metaphysics is sealed. The world has done with it. It must now give place to more spiritualistic views, and it is very natural now to anticipate that a further study of nature may establish the reality of a future life.

Peirce: CP 6.556 Cross-Ref:††

556. For my part, I cannot admit the proposition of Kant -- that there are certain impassable bounds to human knowledge; and, even if there are such bounds in regard to the infinite and absolute, the question of a future life, as distinct from the question of immortality, does not transcend them. The history of science affords illustrations enough of the folly of saying that this, that, or the other can never be found out. Auguste Comte said that it was clearly impossible for man ever to learn anything of the chemical constitution of the fixed stars,†1 but before his book had reached its readers the discovery which he announced as impossible had been made. Legendre said of a certain proposition in the theory of numbers that, while it appeared to be true, it was most likely beyond the powers of the human mind to prove it; yet the next writer on the subject gave six independent demonstrations of the theorem. I really cannot see why the dwellers upon earth should not, in some future day, find out for certain whether there is a future life or not. But at present I apprehend that there are not facts enough in our possession to warrant our building any practical conclusion upon them. If any one likes to believe in a future life, either out of affection for the venerable creed of Christendom or for his private consolation, he does well. But I do not think it would be wise to draw from that religious or sentimental proposition any practical deduction whatever -- as, for instance, that human happiness and human rights are of little account, that all our thoughts ought to be turned away from the things of this world, etc. -- unless such deduction has the independent sanction of good sense.

Peirce: CP 6.557 Cross-Ref:††
CHAPTER 7

LOGIC AND SPIRITUALISM†1P

557. Facts, new or newly published, rappings, table-turnings, with different predispositions opining differently, started controversy concerning Spiritualism. In course of time, other facts, planchette, public exhibitions, mind-reading, trances, apparitions, physical manifestations in great variety, many hundred well-attested strange experiences, attempts at scientific experimentation -- contrariwise, important mediums and mind-readers detected rogues, new psychological laws explanatory of various illusions -- all these facts doubtless had influence, one or other way, upon men's opinions. Meantime, a mighty flood, literature and talk, deluged the subject -- observations highly judicious, delicious satire, ingenious speculations, a large part sadly rash, a very little too timorous. But doubted whether all this comment has changed one individual's mind.

Peirce: CP 6.558 Cross-Ref:††

558. In this impotence of argumentation, sole hope of contributing anything

useful to the discussion lies in breathing into it spirit so candid, unsophisticated, direct, yielding, that the impartial mind, he who alone can get good from such reading, he who looks upon speculative opinions as so many objects of natural history, calculated to excite lively interest by curious relationships and affinities, more so perhaps being false than being true, but who lays them upon his dissection-table as things not calling for sympathy, as vivisection-subjects whose vehement logic-squirmings need excite no concern whatever -- that this reader may be aided in picking to pieces, disentangling, studying, the intellectual component impulses urgent to the opinion in hand, in appreciating them, in considering their just limits of action, not so much himself to form definitive judgment *pro* or *con* (which mostly is not safe while controversy rages) as to assign it schematic place in the natural history of opinion.

Peirce: CP 6.559 Cross-Ref:††

559. First, I state plainly what I dispute. Hypnotism I question not, nor double and triple personality. That these things yet remain imperfectly classified is admitted, too; and not alone these phenomena, but much in everyday life, in communication of ideas in ordinary conversation. Only vague, doubtful explanations are deliverable for phenomena resembling clairvoyance. Not altogether improbably, unrecognized avenues of sense may exist. Possibly *so* the blind avoid trees and walls. Phenomena in abundance await explanation from future science about every stock and stone; how much more about mind? But here, with parting salutation, I diverge from spiritualistic paths, for I think no mind with which man can communicate can act or feel otherwise than through its residential nerve matter, which in turn can act and react upon external bodies only according to recognized laws of mechanics. Not that telepathy is absurd or in its nature impossible, but, in the coarse form it has been imagined impracticable as voyage to planet Mars. Belief in telepathy ought to be ranked as variation of spiritualism.

Peirce: CP 6.560 Cross-Ref:††

560. I run up my colors and confess myself scientific specialist. Spiritualists do not take kindly to scientific men, and never forego opportunities of instancing scientific follies. Though eminent scientists be their allies, they would not have spiritualism judged by the scientific kind of intelligence, surely anticipating disfavor from such judgment. For scientific men, we may as well acknowledge it, are, as such, mere specialists. That stigma! We are blind to our own blindness; but the world seems to declare us simply incapable of rising from narrowness and specialism to take broad view of any facts whatsoever.

Peirce: CP 6.561 Cross-Ref:††

561. "Myopy" and "presbyopy": inability to focus objects too far away; corresponding inability for objects too near. I suppose we scientific specialists, technical sharps, connoisseurs, travelers, scholars, are myopic minds, seeing microscopically, but only things under our several noses. Presbyopic minds, with defective accommodation, would be able to see only what stands open to all men's apprehension: these are fogies, average board-members, men whom one believes to be very wise, but whom one perceives to be very ignorant.

Peirce: CP 6.562 Cross-Ref:††

562. Is there a corresponding contrast between objects themselves? Some paintings are not easily made out because done in miniature, requiring narrow examination. Others, large and executed broadly, when looked at closely show only brush marks, the design quite invisible. So, some experiences are inapprehensible

because minute and recondite. They are (*a*) scientific observations, only feasible with special instrumental aids, under special precautions, by virtue of special skill; they are (*b*) strange adventures, happenings dependent on rare chances fallen to few people, unrepeatably at pleasure. In contrast with these phenomena, remote from everyday life, others (let us hasten to acknowledge) are as hard or harder to see, simply because they surround us on every hand; we are immersed in them and have no background against which to view them. A person's heart stops beating; he perceives it; but let it keep on its regular course, and he knows not he has a heart. People do not hear how their own voices sound, nor feel their own manners. Writers are unaware of peculiar impressions produced by their own styles. What is the most obvious characteristic of the universe we live in puzzles one to answer otherwise than by rote.

Peirce: CP 6.563 Cross-Ref:††

563. Curious how little impression experience too familiar makes upon men's minds, how little attention is paid to it. With an oversecure, not to say ridiculous, contempt, I bethink me, are we despising everyday experience, we specialists and half the world besides -- except where its lessons are followed irreflectively. Recondite experiences, whether scientific or autobiographic, are cherished as very precious. They are rare; the means for acquiring them have been costly; they distinguish their possessor over other men; they are all that many a man has to show for life's labor. Have we professional men often been found *underrating* importance of special orders of facts we have spent our lives in acquiring and in learning how to acquire? Has it happened to any of us, I wonder, to detect smiles on circumspect faces at our contrary tendency? Or has it been generally remarked that persons who have gazed upon the midnight sun, or attended Nijni-Novgorod fair, or seen the effigies in Westminster Abbey, have set *less* store by these experiences than their untraveled interlocutors would seem inclined to do?

Peirce: CP 6.564 Cross-Ref:††

564. Let us ask ourselves whether not only scientific specialists, not only professional men, not only all educated men, but whether the bulk of mankind do not place too much stress on particulars, and regard too little the universal. Two persons casually meeting, with wish to develop mild sympathy, call one another's attention to the fair weather or the foul weather -- insignificant details. The deeper ground of common feeling, that it is day-time, nobody is ever asked to remark; still less the good cheer that earth contains fellow creatures, heaven a Father. Commonplaces these? Granted. But what are commonplaces but universal experiences?

Peirce: CP 6.564 Cross-Ref:††

Ask a thoughtful company the general question where lies the intellectual superiority of one set of experiences over another, and reply will be made with some concord; facts are important only, first, as they are massed and concentrated upon one or more positions, and, secondly, as these positions are themselves important.

Peirce: CP 6.564 Cross-Ref:††

Go on, however, to inquire of relative values of experiences familiar and recondite, and differences will emerge. Respectability will remark that worth of anything is equivalent of trouble requisite to supply it, familiar experiences, like air and water, commanding no price -- answer veritably redolent of the frankincense and the myrrh of the temple of Solomon. Science will hold scientific experiences more capable of systematic marshaling to great ends than civilian facts. Young America will call familiar phenomena squeezed lemons, whatever they had to teach already learned, things to be left behind in pressing on to things new; and it will recall

dazzling inventions sprung from recondite experiences, gunpowder, mariner's compass, steam engine, electric telegraph, India rubber, anæsthetics, sewing machine, telephone, electric light.

Peirce: CP 6.565 Cross-Ref:††

565. But all these voices will not drown those that decry and revile specialism, extolling and magnifying perfections of the all-pervading. These will be heard to say that those things are not most intellectually helpful which most dazzle imagination; that great facts of nature which familiar experiences embody are not of the number of those things which can have their juices sucked out of them and be cast aside; that (returning to the principle upon which alone the question can be properly answered) the very circumstance which renders facts familiar is their being grouped into uniformed hordes, in consequence of which no collection of scientific observations can well be vast enough to withstand their concurrent testimony.

Peirce: CP 6.566 Cross-Ref:††

566. These protestants against worship of scientific specialties, deeper-thinking of the spiritualists included, will be averse from admitting that the discovery of phenomena of electricity, establishment of its laws, determination of its constants, and application to the uses of life, rank as the greatest triumph of modern knowledge. That distinction they will rather reserve for the evolution of the principle of conservation of energy, *résumé* of all that man has ever learned about force, great governing principle of all physics, whose history reaches from Archimedes to our day. That history shows that this great investigation has, from first to last, rested almost exclusively upon familiar experiences.

Peirce: CP 6.566 Cross-Ref:††

Its completion was involved in the discovery that heat, instead of being something ingenerable, indestructible, is but mechanical work transformed. Humanity wanted time to master that subtilty, energy being yet only letter in algebraic formula and even *vis viva* no commanding feature of ordinary conceptions of mechanics. Accordingly, the doctrine having been accepted, a vestige of mental confusion remained in disagreements as to nature of the evidence that had demonstrated it. Tyndall †1 and Clausius, †2 sound logicians, uphold Mayer's †3 proof drawn from facts to be reckoned as familiar; but Tait, †4 patriotic Scot, finds adherents for attribution to more special considerations adduced by countrymen. Others, however, had found it out long before, Rumford, Sadi Carnot, †5 and, if I am rightly informed, Uriah Boyden. †6

Peirce: CP 6.567 Cross-Ref:††

567. Be those doubts what they may, there can be none that previous steps used familiar experiences as almost their sole premisses. Archimedes proves property of lever, Stevinus that of inclined plane, Huygens that of moments of inertia, by mathematical reasoning from propositions assumed as self-evident -- dictates of common sense. Galileo, demonstrating mechanical parallelogram, merely asks imagination how a body would move upon a vessel itself moving, no outward experiment demanded. Newton, establishing the law of action and reaction, treats his facts as matters of course. Even Galileo arguing substantially first two laws of motion in teeth of supposed established facts, though keen observer, though experimenter, rests almost exclusively on familiar experience, and "il lume naturale," adducing but few simple experiments, after all not needed.

Peirce: CP 6.568 Cross-Ref:††

568. Dr. Thomas Young,†1 name to conjure with among physicists, thought, in the first quarter of this century, scientific experimentation had gone far enough and should stop till facts already collected were digested. Every scientist will jeer. My individual notion, doubtless warped by specialism, is this. Reasoning is strictly experimentation. Euclid, having constructed a diagram according to prescription, draws an extra line, whereupon his mind's eye observes new relations not among those prescribed quite as surprising as new metals or new stars. Experimentation is strictly appeal to reason. Chemist sets up retort, introduces ingredients, lights fire, awaits result. Why so confident? Because he trusts that what happens once happens always; nature follows general laws, in other words, has a reason. Successful research -- say Faraday's -- is conversation with nature; the macrocosmic reason, the equally occult microcosmic law, must act together or alternately, till the mind is in tune with nature. This, the distinctively scientific procedure, linked experimentation and reasoning (suppose we say *indagation*), essentially involves special, new experience. A scientific man is simply one who has been trained to conduct observations of some special kind, with which his distinctive business begins and ends. Nevertheless, reasoning from familiar experience plays a great rôle in science: it lays the indispensable foundation, is needful in frequent later conjunctures. The part so built is the strongest of the structure, upholding the rest.

Peirce: CP 6.569 Cross-Ref:††

569. Such reasoning is sometimes elaborate, self-critical; but at its best it is simple, sleepy. The doctrine of Descartes,†1 that the mind consists solely of that which directly asserts itself in unitary consciousness, modern scientific psychologists altogether reject. Swarming facts positively leave no doubt that vivid consciousness, subject to attention and control, embraces at any one moment a mere scrap of our psychical activity. Without attempting accuracy of statement demanding long explanations, and irrelevant to present purposes, three propositions may be laid down. (1) The obscure part of the mind is the principal part. (2) It acts with far more unerring accuracy than the rest. (3) It is almost infinitely more delicate in its sensibilities. Man's fully-conscious inferences have no quantitative delicacy, except where they repose on arithmetic and measurement, which are mechanical processes; and they are almost as likely as not to be downright blunders. But unconscious or semi-conscious irreflective judgments of mother-wit, like instinctive inferences of brutes, answer questions of "how much" with curious accuracy; and are seldom totally mistaken.

Peirce: CP 6.570 Cross-Ref:††

570. Conclusions men reach they know not how are better than those fortified by unscientific logic. By logic *Aquinas*, if not *Calvin*, persuaded himself that one of the chief joys of the blest will be to peer over heaven's parapet and watch the damned writhing in torments and rage below:†P1 by instinct, or half-conscious inference, a poor peasant girl will inwardly reject the doctrine, for all revered pastor may say. No moral sentiment more universally violent than reprobation of intermarriage of near relatives. Assassin will shudder at thought of incest. But had a man to depend upon conscious reasoning to instruct conscience in this matter, while he might be led to condemn the act, he would be unlikely to regard it with the extreme horror in which actually all share. Generation after generation has, in almost unconscious mode, taken measure of ordinary experiences about family relationships, has transmitted its impression to the next, partly by tradition, partly, one guesses, by congenital bequest, this next has made its observations and discussions, has modified in some insensible degree the sentiment it derived from its fathers, and so at last our strong feeling has

been developed. That races tolerating occasional incest have died out and that *so* horror of it has been bred, there is scant room to believe.

Peirce: CP 6.571 Cross-Ref:††

571. This transmission from father to son of dictates of good judgment makes the growth of common sense. Based on large, ordinary experience, far more valuable reservoir of truth than the aggregate of man's special experiences (scientific and extraordinary), worked up in that part of the mind that functions the most delicately and unerringly, reconsidered and revised by countless generations, such conclusion, if unequivocal and pertaining to matter plainly within the competency of good sense, who shall dare to dispute?

Peirce: CP 6.572 Cross-Ref:††

572. Let not conscious reason look down upon it as inadequate to problems high, intellectual, intricate. From data of sensations proper to hundreds of nerve-terminals in the optic retina, combined with certain muscular sensations -- premisses more tangled and confused than tongue can tell or brain can think -- common sense has extricated the marvelously clear and beautiful conception, Space. †1 What simple theory, reducing to order what infinitely complicated facts! Can whole history of science show any discovery whatever half so practically important, half so intrinsically difficult, half so intellectually interesting? It is conceivable that future science should find some principles of geometry to be measurably erroneous. Such discovery would be the most remarkable ever made by science. Yet what insignificant detail compared with that which common sense has taught us of space!

Peirce: CP 6.573 Cross-Ref:††

573. Common sense corrects itself, improves its conclusions. The history of the science of dynamics is that of gradual correction by inference from familiar experience (essentially an operation of good sense) of primitive conceptions of "force" and "matter." There, however, the reasoning was of the self-conscious kind. But we see social, political, religious common sense modifying itself insensibly in course of generations, ideas of rights of man acquiring new meaning, thaumaturgic elements of Christianity sinking, spiritual rising in religious consciousness.

Peirce: CP 6.574 Cross-Ref:††

574. Common sense improves; it does not, then, attain infallibility. Then, its decisions are subject to review. But in case there be evidence that such a conclusion is definitive, not a mere stage in a changing estimate; if it appears to have been formed under guidance of general experience and to be of the kind such experience can warrant; finally, if its substance is in harmony with individual good judgment from general experience, then the authority of common sense as to the practical truth of the conclusion (subject to minute modification) is so weighty that special experience can hardly attain sufficient strength to overthrow it.

Peirce: CP 6.575 Cross-Ref:††

575. How will this rule work in practice? Dr. Zöllner, eminent astronomer and mathematical physicist, man of true genius, keen and subtle, has Mr. Slade, celebrated medium, as visitor in his house. †1 One night he ties the ends of a string together, putting seal upon the knot. Next day, he hands this string to Slade, who thereupon before his eyes makes (or seems to make) a knot in the single string (in contradistinction, I mean, to the double string), and hands it back for examination. The ends of the string not being free, this was impossible according to common sense.

But had space a fourth dimension, additional to its three of length, breadth, and thickness, there would be no such impossibility. Hence Dr. Zöllner concludes that space really has four dimensions. Now, it must be admitted that no experiences, familiar or otherwise, are absolutely inconsistent with space having four dimensions. For example, this refutation might be proposed: steam can be subjected to great pressure in a boiler; now if space were open in a fourth dimension, there would be ways round from inside to outside of the boiler, and why should not the steam escape? It might, however, be replied that the molecules having no component velocities in that direction, and there being no component pressure in that direction, there would be no tendency to motion in that direction; indeed various other loop-holes in the argument are discoverable. Only suppose, then, that space really has fourth dimension, and suppose that one single muscle-cell of Slade's had somehow got displaced so as to project in that direction, and force is thus supplied in that direction which, in total absence of resistance in that direction, would suffice to carry Slade's fingers and with them the string round by that path so as to tie the knot; and here we have explanation, simple and beautiful, of the phenomenon; a gentlemanly explanation, too, not unnecessarily offensive to Mr. Slade's honour. Should it be urged that all experience is against space having fourth dimension, because on that hypothesis phenomena similar to that tying of knot ought to be more common, ready reply comes: If space has fourth dimension there is no determining *a priori* how often it would happen that something would project into it; experience seems to show it happens so rarely that Mr. Slade furnishes the first conclusive instance of it. Now, it is certainly true that no experience whatever can furnish the slightest reason for thinking that an event of any conceivable kind will *absolutely never* happen. Take a thousand people at random among the inhabitants of the United States, and upon inquiry it may probably be found that not one of them will ever have read a line of Martin Farquhar Tupper. It will be fair to conclude that not one in a thousand of all the people in the country ever read line of this poet; but not to conclude he has no readers in the country, since it might be not one in a thousand were readers of his works, while still those poems were devoured by sixty thousand people. Upon same principle, all accumulable experience will never furnish any smallest reason for thinking that no one, or no million for that matter, of all the bodies in the universe juts out into fourth dimension. Nay, presumption rather holds that this does somewhere occur, since every rule has exceptions; for how could an absolutely universal law ever come about? But the whole of our personal experience, itself an amazing flood, together with the experience of all history as embodied in common sense, compels us to hold such jutting to be so excessively infrequent that the probability of its occurrence in any particular case, as in the person of Mr. Slade, is beyond all compare smaller than the probability of trick, even were we at a loss to conceive how trick could be.

Peirce: CP 6.576 Cross-Ref:††

576. Of course, popular belief has often fallen into gross errors. Primitive man peopled woods, streams, earth, air, clouds, stars, with spirits. If intercourse with these beings could be shown to have been believed customary, happening every day, it would be an inference of a kind legitimately to be drawn from familiar experience, and we should have to inquire seriously into its truth. But I fancy intercourse with spirits was never considered matter of course. Belief in it was not formed under guidance of experience, but was hot, extravagant fancy, classable with those superstitions that have inspired or terrified mankind -- fountain of perpetual youth, philosopher's stone, fairies, ogres, ghosts, magic, personal devil, jinns sealed up by Solomon, archei, oracles of Apollo, Eleusinian mysteries, metempsychosis, and all

other romances about substantial spirits.

Peirce: CP 6.577 Cross-Ref:††

577. Faith in these things is fading out; where people are enlightened mere traces of it remain. The essence of these rapidly-decadent beliefs is the doctrine that soul (such as we can know) is able to feel and act independently of its animal body. State the proposition in the abstract, and most men will subscribe to it. Find a practical case, and willingness to risk great interests upon the truth of the principle will commonly be deemed symptomatic of aberration of mind.

Peirce: CP 6.578 Cross-Ref:††

578. Common sense is coming to reject the doctrine, good sense does reject it. All ordinary phenomena of life, which crowd upon us every minute, together with such familiar matters as sleep, faintings, bodily illness, insanity, death, show as plainly, as conclusively, dependence of mind upon body, as familiar facts of lifeless things show first law of motion. That law which Galileo †1 substantially first told the world is that a moving body left to itself will move on with no diminution, no increase of speed, in one straight line forever. They say the first thing that made him think so was seeing a lamp hung by chain from roof of the beautiful cathedral at Pisa, just before the choir, swinging backward and forward, through a small arc, once in about four seconds, and continuing so to move all through high Mass, without any perceptible decrement of the amplitude of its oscillations. I can well believe this true. Performance of good judgment is a sluggish movement, a mental peristalsis, slow, obscure, that is favored by beautiful and peaceful surroundings, even by luxurious tedium, which, in most satisfying of cathedrals, ceremonies performed with an elegance in manner, a refinement in spirit, caught it might seem from the architecture, would well produce in mind of boy attending them too often. I remember myself giving that same lamp, as no doubt it was, a small impulse shortly before a function more than ordinarily prolonged -- obsequies of prince of the church -- and to have watched its grave, impressive, though soft, assertion of the first law of motion, all through service, wondering whether the obvious lesson of mortality which that corpse, whole scene, was bearing in upon me could be less true, less infallible than that.

Peirce: CP 6.579 Cross-Ref:††

579. Completely satisfactory discussion of the question of Spiritualism would involve satisfactory theory of connection of soul and body, which is not perhaps forthcoming.

Peirce: CP 6.580 Cross-Ref:††

580. The obsolete Cartesian dualism, that soul and body are two substances, distinct, independent, untenable as positing double absolute, rendering connection of soul and body absolutely inexplicable either on mechanical or on psychological principles, had a single element of philosophical strength, its recognition of real reaction between *ego* and *non-ego*. Development of this naturally leads to thinking that minds can communicate only through bodies -- doctrine unfavorable to Spiritualism.

Peirce: CP 6.581 Cross-Ref:††

581. Philosophy tries to understand. In so doing, it is committed to the assumption that things are intelligible, that the process of nature and the process of reason are one. Its explanation must be derivation. Explanation, derivation, involve suggestion of a starting-point -- starting-point in its own nature not requiring

explanation nor admitting of derivation. Also, there is suggestion of goal or stopping-point, where the process of reason and nature is perfected. A principle of movement must be assumed to be universal. It cannot be supposed that things ever actually reached the stopping-point, for there movement would stop and the principle of movement would not be universal; and similarly with the starting-point. Starting-point and stopping-point can only be ideal, like the two points where the hyperbola leaves one asymptote and where it joins the other.

Peirce: CP 6.582 Cross-Ref:††

582.†1 In regard to the principle of movement, three philosophies are possible.

Peirce: CP 6.582 Cross-Ref:††

1. Elliptic philosophy. Starting-point and stopping-point are not even ideal. Movement of nature recedes from no point, advances towards no point, has no definite tendency, but only flits from position to position.

Peirce: CP 6.582 Cross-Ref:††

2. Parabolic philosophy. Reason or nature develops itself according to one universal formula; but the point toward which that development tends is the very same nothingness from which it advances.

Peirce: CP 6.582 Cross-Ref:††

3. Hyperbolic philosophy. Reason marches from premisses to conclusion; nature has ideal end different from its origin.

Peirce: CP 6.583 Cross-Ref:††

583. The choice of elliptic philosophy, which refuses to acknowledge the ideal, supposes more interest in nature than in reason. The philosophy which sees nothing in nature but the washing of waves on a beach cannot consistently regard mind as primordial, must rather take mind to be a specialization of matter. Bent on outward studies, it will find the statement that nerve-matter feels, just as carmine is red, a convenient disposition of a troublesome question. Elliptic philosophy is irreconcilable with Spiritualism.

Peirce: CP 6.584 Cross-Ref:††

584. He who feels himself and his neighbors under the constraints of overwhelming power, from which they long to take refuge in annihilation -- situation less common in this country and age than in other places and times -- viewing this little life as rounded with a sleep, readily accepts the idea that the world, too, sprang out of the womb of nothingness to evolve its destiny, and into nothingness back to return. Such life as this philosophy recognizes -- a fatal struggle, a mere death-throe -- it should extend throughout nature. Soul should be a mere aspect of the body, not tied to it, therefore, but identical with it. Nothing can be more hostile to Spiritualism than this Parabolic philosophy.

Peirce: CP 6.585 Cross-Ref:††

585.†2 Hyperbolic philosophy has to assume for starting-point something *free*, as neither requiring explanation nor admitting derivation. The free is living; the immediately living is *feeling*. Feeling, then, is assumed as starting-point; but feeling uncoordinated, having its manifoldness implicit. For principle of progress or growth, something must be taken not in the starting-point, but which from infinitesimal beginning will strengthen itself continually. This can only be a principle of growth of principles, a tendency to generalization. Assume, then, that feeling tends to be associated with and assimilated to feeling, action under general formula or habit

tending to replace the living freedom and inward intensity of feeling. This tendency to take habits will itself increase by habit. Habit tends to coordinate feelings, which are thus brought into the order of Time, into the order of Space. Feelings coordinated in a certain way, to a certain degree, constitute a person; on their being dissociated (as habits do sometimes get broken up), the personality disappears. Feelings over whose relations to their neighbors habit has acquired such an empire that we detect no trace of spontaneity in their actions, are known as dead matter. The hypothesis here sketched, whose consequences, traceable with precision to considerable detail in various directions, appear to accord with observation, to an extent of which I can here give no idea, affords a rational account of the connection of body and soul. This theory, so far as I have been able as yet to trace its consequences, gives little or no countenance to Spiritualism. Still, it is evidently less unfavorable than any other reasonable philosophy.

Peirce: CP 6.586 Cross-Ref:††

586. The myriad strange stories prove nothing. Tell me a marvel; I cannot explain it. Does that teach you or me anything? True, you offer explanation, the spiritualistic one; but that is in conflict with good sense, while we know so little of the mind, at present, that it is not surprising that many things are yet inexplicable. Taking these stories in the gross, the only profitable way, we can roughly compare the phenomena with the general facts at our command for their explanation. These facts are four. First, the fact that all men are liars. Secondly, the fact of deranged imagination, hypnotism, hysteria. Thirdly, the fact that we may receive and act upon indications of which we are quite unconscious, and which, owing to the low sensibility of the conscious part of the mind, seem impossible. Fourthly, the fact that a certain number of coincidences will occur by chance. The result of such rough comparison is that, notwithstanding these four considerations, there are some stories truly surprising. If you have already admitted the general proposition of Spiritualism, you will naturally be inclined to use it to explain some of these stories. If, on the other hand, your judgment is that general experience is emphatically opposed to that proposition, these stories will assuredly not shake that judgment.

Peirce: CP 6.587 Cross-Ref:††

587. Meantime, those who are engaged in psychical research should receive every encouragement. They may have reached little or no result, so far; perhaps will not till they dismiss the phantom of telepathy from their minds. But scientific men, working in something like scientific ways, must ultimately reach scientific results. Psychology is destined to be the most important experimental research of the twentieth century; fifty years hence its wonders may be expected to occupy popular imagination as wonders of electricity do now.

Peirce: CP 6.588 Cross-Ref:††
APPENDIX A

REJOINDER TO DR. CARUS

588. In *The Monist* for January, 1891, and in the number for April, 1892, I attacked the doctrine that every event is precisely determined by law.†2 Like everybody else, I admit that there is regularity: I go further; I maintain the existence of law as something *real and general*. But I hold there is no reason to think that there are general formulæ to which the phenomena of nature *always* conform, or to which they *precisely* conform. At the end of my second paper, the partisans of the doctrine of necessity were courteously challenged and besought to attempt to answer my arguments. This, so far as I can learn, Dr. Carus alone, in *The Monist* of July and October, 1892, has publicly vouchsafed to do. For this I owe him my particular thanks and a careful rejoinder.

Peirce: CP 6.589 Cross-Ref:††

589. I number the paragraphs of his papers consecutively. The following index shows the pages [of *The Monist*] on which those paragraphs commence, and the numbered sections of this rejoinder in which they are noticed.

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Peirce: CP 6.590 Cross-Ref:††

590. Dr. Carus' philosophy is hard to understand. Some phrases which he frequently uses lead the reader to imagine that he is listening to an old-fashioned Königsberg Kantian. What, then, is our surprise when we find (PARASYMBOLQX 14) that he sneers at the Kantian, Sir William Hamilton (whom he calls Mr Hamilton) as having "no adequate conception of the *a priori*." In his *Ursache, Grund und Zweck* (1883), an admirably clear and systematic exposition of much of his thought, he takes a Schleiermacherian view of the *a priori*. He admits it to be founded in the universal conditions of cognition; but he thinks it is among the objective rather than the subjective conditions. This is an opinion to which Hamilton is also at times inclined. It is a weak conception, unless the whole distinction between the inward and the outward world be reformed in the light of agapastic and synechistic ontology. For to deny that the *a priori* is subjective is to remove its essential character; and to make it both subjective and objective (otherwise than in the sense in which Kant himself makes it objective) is uncalled for, and is cut off by Ockham's razor. But when synechism has united the two worlds, this view gains new life.

Peirce: CP 6.590 Cross-Ref:††

Another thing which has astonished me is Dr. Carus's extravagant laudation (PARASYMBOLQX 17) of Venn's highly enlightened and remarkably bright-thinking, yet blundering little book, *The Logic of Chance*. †P1 This is the way he speaks of it: "This admirable work, we will make bold to say, marks a new epoch in the study of logic." He adds that it "paves the way which Mr. Peirce has actually followed." But the question of the nature of probability had long before that publication engaged the attention of some of the most powerful intellects in England; and my opinion concerning it was fully made up before I saw the book. †1 I do not

think I learned anything from that except a classification of the philosophies of probability. However, after all his eulogy, Dr. Carus only uses the book to quote from it Mill's rewording of Kant's definition of causation,†2 which he would better have quoted direct.

Peirce: CP 6.590 Cross-Ref:††

Let me say, not to Dr. Carus, but to the younger generation of readers, that if they imagine that Hamilton, because he is antiquated, is not worth reading, they are much mistaken. The Scotistic elements of his philosophy, and his method in the notes on Reid, are especially worthy of attention. As for Mill, though his philosophy was not profound, it is, at least in his *Examination of Hamilton*, admirably set forth. Whoever wishes to appeal to the American philosophical mind needs to be quite familiar with the writings of these two men.

Peirce: CP 6.590 Cross-Ref:††

Dr. Carus himself accepts all that I hold for erroneous in Kant's definition of causation as universal and necessary sequence.†1 Mill merely substitutes the exacter words *invariable* for "universal," and *unconditional* for "necessary."†P1 In giving his form of the definition, Mill shows †2 why it is not applicable to the sequence of day and night, namely, that that is not necessary. Yet Dr. Carus writes (PARASYMBOLQX 18) of this very same sequence as if it came under Mill's definition.†P2

Peirce: CP 6.590 Cross-Ref:††

Again, why should he make it "the immortal merit of the great Scotchman" (PARASYMBOLQX 22), that is, of Hume, that he admitted the truth of Leibnitz's principle?

Peirce: CP 6.590 Cross-Ref:††

The famed puzzle of causation is peculiarly understood by Dr. Carus. The difficulties which the perusal of Hume suggested to the mind of Kant,†P3 were such as belonged to all categories, or general conceptions of the understanding. The pre-critical Kant inherited a very decided nominalism from Leibnitz and Wolf; and the puzzle for him was simply the usual difficulty that plagues nominalism when it finds itself confronted with a reality which has an element of generality. Necessity is, I need hardly say, but a particular variety of universality. But Dr. Carus (PARASYMBOLQX 24) passes over this, to dwell upon an entirely different objection to causation, namely, that it seems to be a creation out of nothing, and a miracle.

Peirce: CP 6.590 Cross-Ref:††

I find myself equally at cross-purposes with him, when in PARASYMBOLQX PARASYMBOLQX71-77, he speaks of the prevalent views of logicians concerning *comprehension*. This word, in logic, measures the amount of predicates or marks attached to a conception;†3 but Dr. Carus's criticisms seem to be based upon the idea that by comprehension is meant logical breadth, or the amount of subjects to which the conception is applicable.

Peirce: CP 6.590 Cross-Ref:††

I am simply gravelled by his remarks (PARASYMBOLQX 95) concerning sundry English words.

Peirce: CP 6.590 Cross-Ref:††

No more do I know what to make of his praise (PARASYMBOLQX 123) of

the German translation of a French phrase used in the theory of functions, meaning *univocally determined*.

Peirce: CP 6.591 Cross-Ref:††

591. One habit which goes far to obscure Dr. Carus's meaning is that whenever he finds his opinion at variance with a familiar saying, instead of rejecting that formula, he retains it and changes the meaning. This is calculated to throw the whole discussion into confusion. Thus, nothing is more certain than that the so-called "law of identity," or A is A , was intended to express the fact that every term is predicable of itself.†1 But Dr. Carus, simply because he finds that "meaningless and useless" (PARASYMBOLQX 96), thinks himself authorized to confuse the terminology of logic by making this formula, $A = A$, under the same old name, mean that things to which the same name is applicable are for some purpose equivalent.

Peirce: CP 6.591 Cross-Ref:††

In like manner, he changes the meaning of the word *freedom* (PARASYMBOLQX 165), so that the distinction between those who maintain and those who oppose the freedom of the will may, in words, disappear. It seems scarcely defensible for a thoroughgoing necessitarian, such as he is, to fly the flag of Free Will.

Peirce: CP 6.591 Cross-Ref:††

He also changes the meaning of *spontaneity* so far that, according to him, "masses gravitate spontaneously" (PARASYMBOLQX 191), and so pretends that his doctrine does not suppress the spontaneity of nature!

Peirce: CP 6.592 Cross-Ref:††

592. There are other questions of terminology in which I am unable to agree with Dr. Carus. Thus, when I define †2 necessitarianism as "the theory that the will is subject to the general mechanical law of cause and effect," Dr. Carus (PARASYMBOLQX 139) wishes to delete "mechanical." But the result would be to define a doctrine to which the advocates of free will would generally subscribe, as readily as their opponents. In order properly to limit the definition, it is quite requisite to exclude "free causation." By "mechanical" causation, I mean a causation entirely determinative, like that of dynamics, but not necessarily operating upon matter.

Peirce: CP 6.592 Cross-Ref:††

Dr. Carus mentions (PARASYMBOLQX 84) that there are several different ideas to which the term necessity is applied. It seems to me that what lies at the bottom of all of them is the experience of reaction against one's will. In the simplest form, this gives the sense of reality.†1 Dr. Carus himself admits (PARASYMBOLQX 46) that reality involves the idea of inevitable fate. Yet philosophical necessity is a special case of universality. But the universality, or better, the generality, of a pure form involves no necessity. It is only when the form is materialized that the distinction between necessity and freedom makes itself plain. These ideas are, therefore, as it seems to me, of a mixed nature. Dr. Carus (PARASYMBOLQX PARASYMBOLQX 91-94) insists that by the necessary, he wishes to be understood to mean in all cases the *inevitable*. This is the idea of *fate*, †2 and is not the conception which determinists usually attach to the term necessity. Yet he does not appear to be quite consistent. At one time (PARASYMBOLQX 88), he carefully distinguishes necessity from fate. At another time (PARASYMBOLQX 163), every element of compulsion is to be excluded from the conception of necessity.

Peirce: CP 6.593 Cross-Ref:††

593. One important key to Dr. Carus's opinions is the recognition of the fact that, like many other philosophers, he is a nominalist tinged with realistic opinions.

Peirce: CP 6.593 Cross-Ref:††

He says (PARASYMBOLQX 103) that "there is no need of discussing the truism that, properly speaking, there is no absolute sameness." Now, upon the nominalistic theory, there is not only no absolute or numerical identity, but there are not even any real agreements or likenesses between individuals; for likeness consists merely in the calling of several individuals by one name, or (in some systems) in their exciting one idea. On the other hand, upon the realistic theory, the fact that identity is a relation of reason does not in the least prevent it from being real. On that theory, it is real unless it is *false* that anything is itself. Thus, upon either theory, identity is just as real as similarity. But Dr. Carus, being a nominalist leaning toward realism, is inclined to make dynamical relations real, and second-intentional ones unreal. This opinion, I think, is a transitional one.

Peirce: CP 6.593 Cross-Ref:††

The declaration (PARASYMBOLQX 198) that "natural laws are simply a description of nature as nature is," and that "the facts of nature express the character of nature," are nominalistic. But in another place (PARASYMBOLQXPARASYMBOLQX 107-116) he says distinctly that uniformities are real.

Peirce: CP 6.593 Cross-Ref:††

He says (PARASYMBOLQX 70), "Mr. Peirce attempts to explain natural laws as if they were concrete and *single facts*." This is eminently nominalistic. The nominalist alone makes this sharp distinction between the abstract and the concrete,†P1 which must not be confounded with Hegel's distinction for which the same words are used. The nominalist alone falls into the absurdity of talking of "single facts," or *individual generals*. Yet Dr. Carus says (PARASYMBOLQX 68) that natural laws describe the facts of nature *sub specie aeternitatis*. Now I understand Spinoza to be a realist. In PARASYMBOLQX 117 he considers it "settled" "that there are samenesses." This is realistic. But in PARASYMBOLQX 120, he holds "the whole business of science to be to systematize the samenesses of experience," which is nominalistic.

Peirce: CP 6.594 Cross-Ref:††

594. Dr. Carus seems to be in some doubt as to how far evolutionism ought to be carried. In PARASYMBOLQXPARASYMBOLQX 48-51, he seems to side with my contention that it should be thoroughgoing. In PARASYMBOLQX 116, he makes intellect an evolution from feeling. Yet he is sometimes (PARASYMBOLQX125) "inclined" to say the world never was a chaos; he sometimes (PARASYMBOLQX 61) thinks it weak to suppose that real chance begets order; and he sometimes (PARASYMBOLQX 68) goes so far as to pronounce eternity to be the *conditio sine qua non* of natural law.

Peirce: CP 6.595 Cross-Ref:††

595. Every reader of *The Monist* knows that our good editor's great word is "formal law." The clearest statement he has ever made of this doctrine I find in the following two sentences (PARASYMBOLQX 127):

Peirce: CP 6.595 Cross-Ref:††

"The *a priori systems* of thought are . . . constructions raised out of the recognition of the formal, *i.e.* relational samenesses that appear in experience. All possibilities of a certain class of relations can be exhausted and formulated in theorems."†P2

Peirce: CP 6.595 Cross-Ref:††

This is perspicuous. For example, of pairs, we can easily show that there are but two forms $A:A$ and $A:B$. This proposition--theorem if you will -- exhausts the possibilities. If we make believe there is no danger of falling into error in mathematical reasoning -- and one danger, though not, perhaps, a very serious one, *is* eliminated -- then this proposition is absolutely certain. But I will say, at once, that such a proposition is not, in a proper sense, synthetic. It is a mere corollary from the definition of a pair. Moreover, its application to experience, or to possible experience, opens the door to probability, and shuts out absolute necessity and certainty, *in toto*.

Peirce: CP 6.595 Cross-Ref:††

Concerning points like this, Dr. Carus, in company with the general body of thinkers, is labouring under a great disadvantage from not understanding the logic of relatives. It is a subject I have been studying for a great many years, and I feel and know that I have an important report that I ought to make upon it.†1 This branch of logic is, however, so abstruse, that I have never been able to find the leisure to translate my conclusions into a form in which their significance would be manifest even to a powerful thinker whose thoughts had not long been turned in that direction. I shall succeed in doing so, whenever I can find myself in a situation where I need think of nothing else for months, and not before. That may not be for thirty years; but I believe it is the intention of providence that it should be. Meantime, I will testify, and the reader can take my testimony for what he thinks it is worth, that all deductive reasoning, except that kind which is so childishly simple that acute minds have doubted whether there was any reasoning there -- I mean non-relative syllogism -- requires an act of choice; because from a given premiss, several conclusions -- in some cases an infinite number -- can be drawn. Hence, Dr. Carus is altogether too hasty in his confidence (PARASYMBOLQXPARASYMBOLQX 195, 196) that general thinking machines "are not impossibilities." An act of original and arbitrary determination would be required; and it seems almost evident that no machine could perform such an act except within narrow limits, thought out beforehand and embodied in its construction.†2 Moreover, positive observation is called for in all inference, even the simplest -- though in deduction it is only observation of an object of imagination. Moreover, a peculiar act which may properly be called *abstraction*†P1 is usually required, consisting in seizing evanescent elements of thought and holding them before the mind as "substantive" objects, to borrow a phrase from William James. At the same time, the process I am describing, that is, relative deduction, is perfectly general and demonstrative, and depends upon the truth of the assumed premisses, and not, like inductive reasoning, upon the manner in which those premisses present themselves.

Peirce: CP 6.595 Cross-Ref:††

But the application of the logic of relatives shows that the propositions of arithmetic, which Dr. Carus usually adduces as examples of formal law (PARASYMBOLQX 15), are, in fact, only corollaries from definitions. They are certain only as applied to ideal constructions and, in such application, they are merely analytical.

Peirce: CP 6.595 Cross-Ref:††

The truth is our ideas about the distinction between analytical and synthetical judgments is much modified by the logic of relatives, and by the logic of probable inference. An analytical proposition is a definition or a proposition *deducible* from definitions; a synthetical proposition is a proposition not analytical.†1 Deduction, or analytical reasoning, is, as I have shown in my "Theory of Probable Reasoning,"†2 a reasoning in which the conclusion follows (necessarily, or probably) from the state of things expressed in the premisses, in contradistinction to scientific or synthetical, reasoning, which is a reasoning in which the conclusion follows probably and approximately from the premisses, owing to the conditions under which the latter have been observed, or otherwise ascertained. The two classes of reasoning present, besides, some other contrasts that need not be insisted upon in this place. They also present some significant resemblances. Deduction is really a matter of perception and of experimentation, just as induction and hypothetic inference are; only, the perception and experimentation are concerned with imaginary objects instead of with real ones. The operations of perception and of experimentation are subject to error, and therefore it is only in a Pickwickian sense that mathematical reasoning can be said to be perfectly certain. It is so only under the condition that no error creeps into it; yet, after all, it is susceptible of attaining a practical certainty. So, for that matter, is scientific reasoning; but not so readily. Again, mathematics brings to light results as truly occult †P1 and unexpected as those of chemistry; only they are results dependent upon the action of reason in the depths of our own consciousness, instead of being dependent, like those of chemistry, upon the action of Cosmical Reason, or Law. Or, stating the matter under another aspect, analytical reasoning depends upon associations of similarity, synthetical reasoning upon associations of contiguity.†1 The logic of relatives, which justifies these assertions, shows accordingly that deductive reasoning is really quite different from what it was supposed by Kant to be; and this explains how it is that he and others have taken various mathematical propositions to be synthetical which in their ideal sense, as propositions of pure mathematics, are in truth only analytical.

Peirce: CP 6.595 Cross-Ref:††

Descending from things I can demonstrate to things of which various facts, in the light of those demonstrations, fully persuade me, I will say that in my opinion there are many synthetical propositions which, if not *a priori* in Dr. Carus's sense, are, at least, innate (notwithstanding his frequent denials of this, as in PARASYMBOLQX 15), though he is quite right in saying that their abstract and distinct formulation comes very late. (PARASYMBOLQX 126.)†2 But turn the facts as I will, I cannot see that they afford the slightest reason for thinking that such propositions are ever absolutely universal, exact, or necessary in their truth. On the contrary, the principles of probable inference show this to be impossible.

Peirce: CP 6.595 Cross-Ref:††

Dr. Carus adduces the instance of a geometrical proposition, namely, "that two congruent regular tetrahedrons, when put together, will form a hexahedron." (PARASYMBOLQX 25.) This, he says, seems to be "a very wonderful thing"; for why should not a larger tetrahedron be formed, just as two heaps of flour make a large heap of flour? Yet, he continues, the probability that the two tetrahedrons do always make a hexahedron is 1, "which means certainty." (PARASYMBOLQX 27.) But as it happens, the proposition, in the form stated, is quite erroneous. What is true is this. If two tetrahedra are so placed that one face of each is coincident with one face of the other, while all the other faces are inclined to one another, and if of the 8 faces, the 2 that are coincident are not counted, there remain to be counted $8 - 2 = 6$ faces. But

there is nothing more wonderful about this than that $8 - 2 = 6$, which is an easy corollary from definitions. Very few propositions in mathematics that appear "marvellous" will hold water; and those few excite our astonishment only because the real complexity of the conditions are masked in an intuitional presentation of them.

Peirce: CP 6.595 Cross-Ref:††

Dr. Carus holds (PARASYMBOLQX 15) that formal knowledge is absolutely universal, exact, and necessary. In some cases, as where he says that, given the number of dimensions of space, the entire geometry could be deduced (PARASYMBOLQX 35), the boasted infallibility will prove on examination to be downright error. In all other cases, the propositions only relate to ideal constructions, and their applicability to the real world is at the best doubtful and, as I think, false; while in their ideal purity they are not synthetical.

Peirce: CP 6.595 Cross-Ref:††

Thus, my good friend and antagonist holds that the combination of oxygen and hydrogen to produce water is not "different in principle" from that of the tetrahedra to produce a hexahedron. (PARASYMBOLQX 26.) There is all the difference between the ideal and the real; which to my Scotistic mind is very important. But this is not the only passage in which he speaks as if form were the principle of individuation.

Peirce: CP 6.596 Cross-Ref:††

596. Dr. Carus's position is even weaker than that of Kant, who makes space, for example, a necessary form of thought (in a broad sense of that term). But Dr. Carus appears to consider space as an absolute reality. For he says (PARASYMBOLQX 119) that "every single point of space has its special and individual qualities." Here again form is made the principle of individuation; whence the queer phrase "individual qualities."

Peirce: CP 6.597 Cross-Ref:††

597. Dr. Carus argues that whatever is unequivocally determinate is necessary. (PARASYMBOLQX 124.) Were the determination spoken of real dynamic determination, this would be a mere truism. But the expression used, *eindeutig bestimmt*, merely expresses a mathematical determination, and therefore no real necessity ensues. The equation

$$x^2 - 23x + 132 \frac{1}{4} = 0$$

determines x to be either 11.477 or 11.523. In this sense, x has necessarily one value or the other. The equation

$$x^2 - 12x + 6 = 0$$

determines x to be either 11.477 or 0.523. Together, the two equations uniquely determine x to be 11.477. This shows how much that argument amounts to.

Peirce: CP 6.598 Cross-Ref:††

598. By "sameness," Dr. Carus means equivalence for a given purpose. (PARASYMBOLQXPARASYMBOLQX 102, 106.) By the "idea of sameness," he means (PARASYMBOLQXPARASYMBOLQX 77, 96) the principle that things having a common character are for some purpose equivalent. This, he says, "has a solid basis in the facts of experience." By a "world of sameness" (PARASYMBOLQX 113), he seems to mean one in which any two given concrete things are in some respect equivalent. He argues (PARASYMBOLQX 122) that a "world of sameness is a world in which necessity rules." I do not see this. It seems to me so bald a *non sequitur* that I cannot but suppose the thought escapes my apprehension. If there were anything in the argument, it would seem to be a marvellously expeditious way of settling the whole dispute; and therefore it would have been worth the trouble of stating, so as to bring it within the purview of minds like mine.

Peirce: CP 6.599 Cross-Ref:††

599. My candid opponent sometimes endorses emphatically the Leibnizian principle. "Necessitarianism must be founded on something other than observation. Observation is *a posteriori*; it has reference to single facts, to particulars; yet the doctrine of necessity . . . is of universal application. The doctrine of necessity . . . is of an *a priori* nature." (PARASYMBOLQX 11.) "Millions of single experiences . . . cannot establish a solid belief in necessity." (PARASYMBOLQX 14.) "No amount of experience is sufficient to constitute causation by a mere synthesis of sequences." (PARASYMBOLQX 22.) "Millions of millions of cases" constitute "no proof" that a proposition "is always so." (PARASYMBOLQX 29.)

Peirce: CP 6.599 Cross-Ref:††

Nevertheless, he holds that the law of "the conservation of matter and energy" so conclusively proves necessary causation that the obstinacy of Hume, himself, could not have withstood the argument. (PARASYMBOLQX 23.) One wonders, then, what is supposed to prove this "law of the conservation of matter and energy," if no amount of experience can prove it.

Peirce: CP 6.599 Cross-Ref:††

But the *a priori* itself can "be based on the firm ground of experience." (PARASYMBOLQX 14.) In that case, it is not prior to experience, after all! "The idea of necessity is based upon the conception of sameness, and . . . the existence of sameness is a fact of experience." (PARASYMBOLQX 87.) If absolute necessity can be irrefragably demonstrated from the fact that two things are alike, it is a pity Dr. Carus should not state this demonstration in a form that I, and men like me, can understand. That would be more to the purpose than merely saying it can be proved. Absolute chance is rejected as "involving a violation of laws well established by *positive evidence*." (PARASYMBOLQX 149.)

Peirce: CP 6.599 Cross-Ref:††

All these denials that absolute necessity can be established and absolute chance refuted by experiential evidence, mixed with as clear *assertions* of the same things, when taken together, have the appearance of an attempt, as the politicians say, to "straddle" the question.

Peirce: CP 6.600 Cross-Ref:††

600.†1 But the ingenious Doctor seeks to bolster up necessity by introducing the confused notion of "causation."

Peirce: CP 6.600 Cross-Ref:††

I do not know where the idea originated that a cause is an instantaneous state of things, perfectly determinative of every subsequent state. It seems to be at the bottom of Kant's discourse on the subject; yet it accords neither with the original conception of a cause, nor with the principles of mechanics. The original idea of an efficient cause is that of an agent, more or less like a man. It is prior to the effect, in the sense of having come into being before the latter; but it is not transformed into the effect. In this sense, it may happen that an event is a cause of a subsequent event; seldom, however, is it the principal cause. Far less are events the only causes. The modern mechanical conception, on the other hand, is that the relative positions of particles determine their accelerations at the instants when they occupy those positions. In other words, if the positions of all the particles are given at *two* instants (together with the law of force), then the positions at all other instants may be deduced.†P1 This doctrine conflicts with Kant's second analogy of experience, as interpreted by him, in no less than four essential particulars. In the first place, far from involving any principle that could properly be termed generation, or *Erzeugung*, which is Kant's word for the sequence of effect from cause, the modern mechanical doctrine is a doctrine of persistence, and, as I have repeatedly explained, positively prohibits any real growth. In the second place, one state of things (*i.e.* one configuration of the system) is not sufficient to determine a second; it is two that determine a third. To whomsoever may think that this is an inconsiderable divergence of opinions, let me say, study the logic of relatives, and you will think so no longer. In the third place, the two determining configurations, according to mechanics, may be taken at almost any two instants, and the determined configuration be taken at any third instant we like. ***There is no mechanical truth in saying that the past determines the future, rather than the future the past.*** We habitually follow tradition in continuing to use that form of expression, but every mathematician knows that it is nothing but a form of expression. We continue, for convenience, to talk of mechanical phenomena as if they were regulated, in the same manner in which our intentions regulate our actions (which is essentially a determination of the future by the past), although we are quite aware that it is not really so. Remark how Kant reasons:†1

Peirce: CP 6.600 Cross-Ref:††

"If it is a necessary law of our sensibility, and consequently a *formal condition* of all perceptions, that the preceding time determines the following (since I can only come to the following through the preceding), then is it also an indispensable *law of the empirical representation* of the time-series that the appearances of the preceding time determine every occurrence in the following."

Peirce: CP 6.600 Cross-Ref:††

What this leads to is a causality like that of mental phenomena, where it *is* the past which determines the future, and *not* (in the same sense) the reverse; but the doctrine of the conservation of energy consists precisely in the denial that anything like this occurs in the domain of physics. Had Kant studied the psychological phenomena more attentively and generalized them more broadly, he would have seen that in the mind causation is not absolute, but follows such a curve as is traced in my essay towards "The Law of Mind."†1 Does our judicious editor deem it ungracious to find fault with Kant for not doing so much more than he did, considering what that hero-like achievement was? We must seem to carp, as long as thinkers can hold that achievement for sufficient. In the fourth place, Kant's "Analogy" ignores that continuity which is the life blood of mathematical thought. He deals with those

awkward chunks of phenomenon called "events." He represents one such "event" as determined by certain others, definitely, while the rest have nothing to do with it. It is impossible to cement such thought as this into hermetic continuity with the refined conceptions of modern dynamics. The statement that every instantaneous state of things determines precisely all subsequent states, and not at all any previous states, could, I rather think, be shown to involve a contradiction.

Peirce: CP 6.600 Cross-Ref:††

The notion which Dr. Carus holds of a cause seems to be that it is a state, embracing all the positions and velocities of all the masses at one instant, the effect being a similar state for any subsequent instant. (PARASYMBOLQXPARASYMBOLQX 21, 24.) This breaks at once with common parlance, with dynamics, and with philosophical logic. In common parlance, we do not say that the position and upward velocity of a missile is the cause of its being at a subsequent instant lower down and moving with a greater downward velocity.†P1 In dynamics, it is the fixed force, gravitation, or whatever else, together with those relative positions of the bodies that determine the intensity and direction of the forces, that is regarded as the cause. But these causes are not previous to, but simultaneous with, their effects, which are the instantaneous accelerations. Finally, logic opposes our calling one of two states which equally determine one another (as any two states of a system do, if the velocities are taken to be included in these states) *the* determinator, or cause, simply because of the circumstance that it precedes the other in time --a circumstance that is upon the principles of dynamics plainly insignificant and irrelevant.

Peirce: CP 6.600 Cross-Ref:††

Everybody will make slips in the use of words that have been on his lips from before the time when he learned to think; but the practice which I endeavor to follow in regard to the word *cause* is to use it in the Aristotelian sense of an *efficient cause*, in all its crudeness. In short, I refuse to use it at all as a philosophical word. When my conception is of a dynamical character, I endeavour to employ the accepted terminology of dynamics;†P1 and when my idea is a more general and logical one, I prefer to speak of the *explanation*.

Peirce: CP 6.601 Cross-Ref:††

601. Dr. Carus thinks the element of necessity in causation can be demonstrated by considering the process as a transformation. "It is a sequence of two states which belong together as an initial and final aspect of one and the same event." (PARASYMBOLQX 21. Compare PARASYMBOLQXPARASYMBOLQX 20, 24.) He neglects to explain how he brings under this formula the inward causation of the will and character, as set forth by him in PARASYMBOLQXPARASYMBOLQX 163-167.

Peirce: CP 6.601 Cross-Ref:††

It is unnecessary for me to reply, at length, to an argument so manifestly inconclusive. On the one hand, it conflicts with the principle that absolute necessity cannot be proved from experience; and, on the other hand, it leaves room for an imperfect necessity.

Peirce: CP 6.601 Cross-Ref:††

Professor Tait †1 has done an ill office to thought in countenancing the idea that the conservation of energy is of the same nature as the "conservation," or rather perduration, of matter. Dr. Carus says (PARASYMBOLQX 121) that

Peirce: CP 6.601 Cross-Ref:††

"The law of the conservation of matter and energy rests upon the experience (corroborated by experiments) that causation is transformation. It states that the total amount of matter and the total amount of energy remain constant. There is no creation out of nothing and no conversion of something into nothing."

Peirce: CP 6.601 Cross-Ref:††

The historical part of this statement contains only a small grain of truth; but that I will not stop to criticize. The point I wish to make is that the law of the conservation of energy is here represented under a false aspect. The true substance of the law is that the accelerations, or rates of change, of the motions of the particles at any instant depend solely on their relative positions at those instants. The equation which expresses the law under this form is a differential equation of the second order; that is, it involves the rates of change of the rates of change of positions, together with the positions themselves. Now, because of the purely analytic proposition of the differential calculus that

$$D[t]^2s = 1/2 D[s](D[t]s)^2,$$

the first integral of the differential equation of the second order, that is, the differential equation of the first order which expresses the same state of things, equates half the sum of the masses, each multiplied by the square of its velocity, to a function of the relative positions of the particles *plus* an arbitrary constant.†P1 In order to fix our ideas, let us take a very simple example, that of a single particle accelerated towards an infinite plane, at a rate proportional to the *n*th power of its distance from the plane. In this case, if *s* be the distance, the second differential equation will be

$$D[t]^2s = -a s^n$$

and the first integral of it will be

$$(D[t]s)^2 = -(2a/n+1)s^{(n+1)} + C.$$

By the first law of motion, and the Pythagorean proposition, the part of the velocity-square depending on the horizontal component is also constant.

Peirce: CP 6.601 Cross-Ref:††

The arbitrary constant, *C*, plainly has its genesis in the fact that forces do not determine velocities, but only accelerations. Its value will be fixed as soon as the velocity at any instant is known. This quantity would exist, just the same, and be independent of the time, and would therefore be "conserved" whether the forces were "conservative," that is, simply positional, or not. Now, this constant is the energy; or

rather, the energy is composed of this constant increased by another which is absolutely indeterminable, being merely supposed large enough to make the sum positive.

Peirce: CP 6.601 Cross-Ref:††

Thus, the law of energy does not prescribe that the total amount of energy shall remain constant; for this would be so in any case by virtue of the second law of motion; but what it prescribes is that the total energy diminished by the living force shall give a remainder which depends upon the relative positions of the particles and not upon the time or the velocities. It is also to be noticed that the energy has no particular magnitude, or quantity. Furthermore, in transformations of kinetical energy into positional energy, and the reverse, the different portions of energy do not retain their identity, any more than, in bookkeeping, the identity of the amounts of different items is preserved. In short, the conservation of energy (I do not mean the *law* of conservation) is a mere result of algebra. Very different is it with the "conservation" of matter. For, in the first place, the total mass is a perfectly definite quantity; and, in the second place, in all its transformations not only is the *total* amount constant, but all the different parts preserve their identity. To speak, therefore, of "the conservation of matter and energy" is to assimilate facts of essentially contrary natures; and to say that the law of the conservation of energy makes the total amount of energy constant is to attribute to this law a phenomenon really due to another law, and to overlook what this law really does determine, namely, *that the total energy less the kinetic energy gives a remainder which is exclusively positional.*

Peirce: CP 6.602 Cross-Ref:††

602. Dr. Carus does not make it clear what he means by *chance*. He does, indeed, say (PARASYMBOLQXPARASYMBOLQX 145, 146):

Peirce: CP 6.602 Cross-Ref:††

"What is chance?"

Peirce: CP 6.602 Cross-Ref:††

Chance is any event not especially intended, either not calculated, or, with a given and limited stock of knowledge, incalculable."

Peirce: CP 6.602 Cross-Ref:††

This defines what he means by a chance event, in the concrete; what he understands by probability we are left to conjecture. But from what he says in PARASYMBOLQX 147, I infer that he regards it as dependent upon the state of our ignorance, and therefore nothing real.

Peirce: CP 6.602 Cross-Ref:††

I am, therefore, much puzzled when I find him expressing a conviction (PARASYMBOLQXPARASYMBOLQX 88, 156) that chance plays an important part in the real world. He explains very distinctly that "when we call a throw of the dice pure chance, we mean that the incidents which condition the turning up of these or those special faces of the dice have not been, or cannot be, calculated." (PARASYMBOLQX 147.) This is the commonest, because the shallowest, philosophy of chance. Even Venn might teach him better than that. However, according to that view, when he writes of "the important part that chance plays in the world -- not absolute chance . . . but that same chance of which the throw of the die is a typical instance" (PARASYMBOLQX 88), he can be understood to mean no more than that many things happen which we are not in condition to calculate or predict.

This is not playing a part *in the world*, one would say -- at least, not in the natural world; it is only playing a part in our ignorance.

Peirce: CP 6.602 Cross-Ref:††

Dr. Carus frequently uses phrases which make us suspect he penetrates deeper. Thus, he says, "We do not believe in absolute chance, but we believe in chance" (PARASYMBOLQX 144); and again, "Every man is the architect of his own fortune -- but not entirely. There are sometimes coincidences determining the fates of men." (PARASYMBOLQX 161.) But when we remark the consecution of PARASYMBOLQXPARASYMBOLQX 137-162, we feel pretty sure he really sees no further. To do so would have been to perceive that indefinitely varied specificness is chance.

Peirce: CP 6.602 Cross-Ref:††

For a long time, I myself strove to make chance that diversity in the universe which laws leave room for, instead of a violation of law, or lawlessness. That was truly believing in chance that was not absolute chance. It was recognizing that chance does play a part in the real world, apart from what we may know or be ignorant of. But it was a transitional belief which I have passed through, while Dr. Carus seems not to have reached it.

Peirce: CP 6.602 Cross-Ref:††

As for absolute chance, Dr. Carus makes the momentous admission that it is "not unimaginable." (PARASYMBOLQX 150.) If so, its negation, or absolute necessity, cannot be a formal principle.

Peirce: CP 6.603 Cross-Ref:††

603. But it is time for me to leave the consideration of Dr. Carus's system and to take up his strictures upon mine. His philosophy is one eminently enlightened by modern ideas, which it synthetizes to an unusual extent. It is distinguished for its freedom from the vice of one-sidedness, and displays every facet of the gem of philosophical inquiry, except the one on which it rests, the question of absolute law. Its prominent faults, which I feel sure must have struck every competent reader, are that it shows little trace of meditation upon the thoughts of the great idealists, and that there is a certain want of congruity between different elements of it. How strangely it sounds, for instance, to find an apriorian, and one who is dinging "formal laws" so perpetually into our ears, one who holds that "in order to weave the woof of the *a posteriori* elements into coherent cloth we want the warp of the *a priori*" (PARASYMBOLQX 15), to find this man declaring for a positivism "which accepts no doctrine, theory, or law, unless it be a formulation of facts," and proclaiming that "the whole business of science is to systematize the samenesses of experience, and to present them in convenient formulas." (PARASYMBOLQX 120.) Now there is just one way of bringing such warring elements into harmony, and curing the greatest defect of the system -- and it is a way which would also bring the whole into far better concordance with natural science. It is to lop off the heads of all absolute propositions whose subject is not the Absolute, and reduce them to the level of probable and approximate statements.†1 Were that defalcation performed, Dr. Carus's philosophy would, in its general features, offer no violent opposition to my opinions. Moreover, the Doctor has at heart the conciliation of religion and science. I confess such serious concern makes me smile; for I think the atonement he desires is a thing which will come to pass of itself when time is ripe, and that our efforts to hasten it have just that slight effect that our efforts to hasten the ripening of apples on a tree may have. Besides, natural ripening is the best. Let science and religion each have stout faith in

itself, and refuse to compromise with alien and secondary purposes, but push the development of its own thought on its own line; and then, when reconciliation comes -- as come it surely will -- it will have a positive value, and be an unmixed good. But since our accomplished editor thinks himself called upon to assist in this birth of time, let me ask him whether of all the conditions of such peace the first is not that religious thought should abandon that extravagant absoluteness of assertion which is proper to the state of intellectual infancy, but which it has so long been too timid to let go? This pragmatism and unneeded absoluteness it is which is most deeply contrary to the method, the results, and the whole spirit of science; and no error can be greater than to fancy that science, or scientific men, rest upon it or readily tolerate it.

Peirce: CP 6.604 Cross-Ref:††

604. Dr. Carus (PARASYMBOLQXPARASYMBOLQX 56-64) condemns my method of investigation as contrary to that by which science has been advanced; and holds that a radically different, and thoroughly positivistic method is requisite -- a method so intensely positivistic as to exclude all originality. I suppose he will not object to my forming an opinion concerning the methods of science. I was brought up in an atmosphere of scientific inquiry, and have all my life chiefly lived among scientific men. For the last thirty years, the study which has constantly been before my mind has been upon the nature, strength, and history of methods of scientific thought. I have no space here to argue the question. In its logical aspect, I have partly considered it in various publications;†1 and in its historical aspect I have long been engaged upon a treatise about it.†2 My critic says (PARASYMBOLQX 57) that I am "very positivistic in my logic of science." This is a singular misapprehension. Few of the great scientific minds with whom I have come into personal contact, and from whom I endeavoured to learn, were disposed to condemn originality or the ideal part of the mind's work in investigation; and those few, it was easy to see, really breathed an atmosphere of ideas which were so incessantly present that they were unconscious of them. Were I to name those of my teachers who were most positivistic in theory a smile would be excited. My own historical studies, which have been somewhat minutely critical, have, on the whole, confirmed the views of Whewell, the only man of philosophical power conjoined with scientific training who had made a comprehensive survey of the whole course of science, that progress in science depends upon the observation of the right facts by minds *furnished with appropriate ideas*.†3 Finally, my long investigation of the logical process of scientific reasoning led me many years ago to the conclusion that science is nothing but a development of our natural instincts. So much for my *theory* of scientific logic. It is as totally opposed as anything can be to Dr. Carus's theory (PARASYMBOLQX 69, note; and *Ursache, Grund und Zweck*, p. 2) that originality is out of place in science.

Peirce: CP 6.604 Cross-Ref:††

But in my *practice* of scientific reasoning, Dr. Carus accuses me of being what he calls a "constructionist"; that is, a theorizer unguided by indications from observation or accepted facts. To a mind upon whom that celebrated and splendid chapter of Kant upon the architectonical method failed to make a deep impression I may appear so; but *travesty* is in truth hardly too strong a word to describe the account of my method by Dr. Carus.

Peirce: CP 6.604 Cross-Ref:††

Perhaps exaggeration is not without its value. If so, let me sum up the method Dr. Carus recommends. Eschew originality is its pious formula; do not think for yourself, nor countenance results obtained by original minds. Distrust them; they are

not safe men. Leave originality to mathematicians and their breed, to poets, and to all those who seek the sad notoriety of having unsettled belief.†P1 Flee all philosophies which smack of this aberrant nineteenth century.†P2 This theory of Dr. Carus condemns itself; for it is highly original, and soars into the free ether untrammelled by historic facts.

Peirce: CP 6.604 Cross-Ref:††

Kepler comes very close to realizing my ideal of the scientific method; and he is one of the few thinkers who have taken their readers fully into their confidence as to what their method really has been.†P3 I should not feel justified in inflicting upon mine an autobiographical account of my own course of thought; but some things Dr. Carus's accusation forces me to mention. My method of attacking all problems has ever been to begin with an historical and rational inquiry into the special method adapted to the special problem. This is the essence of my architectonical proceeding upon which Dr. Carus has commented very severely. To look an inch before one's nose involves originality: therefore, it is wrong to have a conscious method. But further, in regard to philosophy, not only the methods, but the elementary ideas which are to enter into those methods, should be subjected to careful preliminary examination. This, especially, Dr. Carus finds very unscientific. (PARASYMBOLQX 64, and elsewhere.) It is, undoubtedly, the most characteristic feature of my procedure. Certainly it was not a notion hastily or irreflectively caught up; but is the maturest fruit of a lifetime of reflection upon the methods of science, including those of philosophy; and if it shall be found that one contribution to thought on my part has proved of permanent service, that, I expect, will be the one. This method in nowise teaches that the method and materials for thought are not to be modified in the course of the study of the subject-matter. But instead of taking ideas at haphazard, or being satisfied with those that have been handed down from the good old times, as a mind keenly alive to the dangers of originality would have done, I have undertaken to make a systematic survey of human knowledge (a very slight sketch of which composed the substance of my paper on the "Architecture of Theories")†1 in order to find what ideas have, as a fact, proved most fruitful, and to observe the special utilities they have severally fulfilled. A subsidiary object of this survey was to note what the great obstacles are today in the way of the further advance of the different branches of science. In my "Architecture of Theories," I never professed to do more than make a slight sketch of a small portion of my preliminary studies, devoting thirteen lines to some hints as to the nature of the results.†2 In the four following papers I have given a selection of a few of these results. Among those which remain to be reported are some of much more immediate importance than any of those hitherto set forth. If anybody has been surprised to find my subsequent papers developing thoughts which they were unable to foresee from my first, it is only what I warned people from the outset that they would find to happen. Nor have the greatest of these surprises yet been reached.†P1

Peirce: CP 6.604 Cross-Ref:††

The next series of facts reviewed was that of the history of philosophy. I waded right into this fearful slough of "originality," in order to gather what seemed to throw a light upon the subject. Finally, I reviewed the general facts of the universe.

Peirce: CP 6.604 Cross-Ref:††

I now found myself forced by a great many different indications to the conclusion that an evolutionary philosophy of some kind must be accepted -- including among such philosophies systems like those of Aristotle and of Hegel. From

this point the reasoning was more rapid. Evolution had been a prominent study for half a generation; and much light had been thrown upon the conditions for a fruitful evolutionary philosophy. The first question was, how far shall this evolution go back? What shall we suppose *not* to be a product of growth? I fancy it is this cautious reflectiveness of my procedure which especially displeases Dr. Carus. It is not positivistic: it is architectonic. But the answer to the question was not far to seek. If an evolutionary explanation is to be adopted, philosophy, logic, and the economy of research all dictate that in the first essay, at least, that style of explanation be carried as far back as explanation is called for. What elements of the universe require no explanation? This was a simple question, capable of being decided by logic with as much facility and certainty as a suitable problem is solved by differential calculus. Being, and the uniformity in which being consists, require to be explained. The only thing that does not require it is non-existent spontaneity. This was soon seen to mean absolute chance. The conclusion so reached was clinched by a careful reëxamination of the office of chance in science generally, and especially in the doctrines of evolution. Arrived at this point, the next question was, what is the principle by which the development is to proceed? It was a difficult inquiry, and involved researches from different points of view.

Peirce: CP 6.604 Cross-Ref:††

But I will not trouble the reader with further autobiographical details. I have given enough to show that my method has neither been in theory purely empirical, nor in practice mere brain-spinning; and that, in short, my friend Dr. Carus's account of it has been as incorrect as can be.

Peirce: CP 6.605 Cross-Ref:††

605. The learned doctor (PARASYMBOLQXPARASYMBOLQX 6, 7, 8) pronounces me to be an imitator of David Hume, or, at least, classes my opinions as closely allied to his. Yet be it known that never, during the thirty years in which I have been writing on philosophical questions, have I failed in my allegiance to realistic opinions and to certain Scotistic ideas; while all that Hume has to say is said at the instance and in the interest of the extremest nominalism. Moreover, instead of being a purely negative critic, like Hume, seeking to annul a fundamental conception generally admitted, I am a positive critic, pleading for the admission to a place in our scheme of the universe for an idea generally rejected. In the first paper of this series, in which I gave a preliminary sketch of such of my ideas as could be so presented, I carefully recorded my opposition to all philosophies which deny the reality of the Absolute, and asserted that "the one intelligible theory of the universe is that of objective idealism, that matter is effete mind."†1 This is as much as to say that I am a Schellingian, of some stripe; so that, on the whole, I do not think Dr. Carus has made a very happy hit in likening me to Hume, to whose whole method and style of philosophizing I have always been perhaps too intensely averse. Yet, notwithstanding my present disclaimer, I have little doubt apriorians will continue to describe me as belonging to the sceptical school. They have their wonderful ways of arriving at truth, without stooping to confront their conclusions with facts; and it is amusing to see how sincerely they are convinced that nobody can have science at heart, without denying all they uphold.†P1

Peirce: CP 6.605 Cross-Ref:††

My opponent has a habit of throwing out surprising opinions without the least attempt to illuminate them with the effulgence of reason. Thus he says (PARASYMBOLQX 8): "If Kant's answer to Hume had been satisfactory, Mr. Peirce

would probably not have renewed the attack." What attack? All that Hume attacked I defend, namely, law as a reality. How could a defense of that which I defend as essential to my position, cause me to surrender that position, namely, that real regularity is imperfect? In any sense in which Hume could have admitted the possibility of law, it must be precisely followed; since its existence could consist only in the conformity of facts unto it. But perhaps Dr. Carus means that if one question had been completely settled, I should probably have confined myself to talking about that, instead of broaching a new one.

Peirce: CP 6.606 Cross-Ref:††

606. Another misunderstanding of my position on the part of Dr. Carus (PARASYMBOLQX PARASYMBOLQX 12, 13) is simply due to "boldly" having been twice printed where the reading should have been "baldly," in my paper on "The Doctrine of Necessity." (*The Monist*, vol. II, p. 336, lines 20 and 25 [63].) I wish printers would learn that I never use the word *bold*. I have so little of the quality, that I don't know what it means. As I read the "revise," as usual, it was presumably my fault that the *erratum* occurred. At any rate, had my meaning been clearly expressed, the proofreader would not have been misled by my defective chirography. What I was trying to say was, in substance, this: Absolute chance is a hypothesis; and, like every hypothesis, can only be defended as explaining certain phenomena.†P1 Yet to suppose that an event is brought about by absolute chance is utterly illogical, since as a hypothesis it could only be admitted on the ground of its explaining observed facts; now from mere non-law nothing necessarily follows, and therefore nothing can be explained; for to explain a fact is to show that it is a necessary or, at least, a probable result from another fact, known or supposed. Why is not this a complete refutation of the theory of absolute chance? *Answer*: because the *existence* of absolute chance, as well as many of its characters, are not themselves absolute chances, or sporadic events, unsubject to general law. On the contrary, these things *are* general laws. Everybody is familiar with the fact that chance has laws, and that statistical results follow therefrom. Very well: I do not propose to explain anything as due to the action of chance, that is, as being lawless. I do not countenance the idea that Bible stories, for instance, show that nature's laws were violated; though they may help to show that nature's laws are not so mechanical as we are accustomed to think. But I only propose to explain the regularities of nature as consequences of the only uniformity, or general fact, there was in the chaos, namely, the general absence of any determinate law.†P1 In fact, after the first step is taken, I only use *chance* to give room for the development of law by means of the law of habits.

Peirce: CP 6.607 Cross-Ref:††

607. In PARASYMBOLQX 28, I read: "Mr. Peirce does not object to necessity in certain cases; he objects to necessity being a universal feature of the world." This is correctly stated, and so it is in PARASYMBOLQX 203. I object to necessity being universal, as well as to its ever being exact. In short, I object to absolute universality, absolute exactitude, absolute necessity, being attributed to any proposition that does not deal with the {A} and the {Ö}, in the which I do not include any object of ordinary knowledge. But it is careless to write (PARASYMBOLQX 193) that I "describe the domain of mind as the absence of law." Is not one of my papers entitled "The Law of Mind"? It is true that I make the law of mind essentially different in its mode of action from the law of mechanics, inasmuch as it requires its own violation; but it is law, not chance uncontrolled. That it is not "an undetermined and indeterminable sporting" should have been obvious from my expressly stating that its ultimate result must be the entire elimination of chance from the universe.

That directly negatives the adjective "indeterminable," and hence also the adjective "undetermined." Still more unwarranted is the statement (PARASYMBOLQX 205) that I deny "that there are samenesses in this world." If the slightest excuse for such an accusation can be found in all my writings I shall be mightily surprised.

Peirce: CP 6.608 Cross-Ref:††

608. Dr. Carus fully admits (PARASYMBOLQX 9) the justice of my first reply to the argument that necessity is postulated in all scientific reasoning, which reply is that to postulate necessity does not make it true. As this reply, if correct, is complete, Dr. Carus was bound after that admission to drop the postulate-argument in favor of necessity.†P1 But he takes no notice, at all, of my four-page argument to show that scientific reasoning does *not* postulate absolute universality, exactitude, or necessity (*The Monist*, vol. II, pp. 324-327 [39-43]); but calmly asserts, four or five times over (PARASYMBOLQX PARASYMBOLQX 5, 11, 16, 62, 79), without one scintilla of argumentation, that that postulate *is* made, and uses this as an argument in favor of necessity.

Peirce: CP 6.609 Cross-Ref:††

609. He also fully admits (PARASYMBOLQX PARASYMBOLQX 11, 14, 22) the justice of my argument that the absoluteness of universality, exactitude, and necessity, cannot be proved, nor rendered probable, by arguments from observation. That argument consisted in assuming that all arguments from observation are probable arguments, and in showing that probable inferences are always affected with probable errors.

Peirce: CP 6.609 Cross-Ref:††

Had I deemed it requisite, I might easily have fortified that argument by a more profound analysis of scientific reasoning. Such an analysis I had formerly given in my "Theory of Probable Inference" [vol. 2, bk. III, ch. 8] (in *Studies in Logic*, Boston, Little and Brown).

Peirce: CP 6.609 Cross-Ref:††

But, notwithstanding his admissions, Dr. Carus sets up his *ipse dixit* against my argumentation. "We deny most positively," says the editorial Elohim, "that the calculus of probabilities is applicable to the order of the world, as to whether it may or may not be universal." (PARASYMBOLQX PARASYMBOLQX 27, 31.)

Peirce: CP 6.609 Cross-Ref:††

To support this, he cites (PARASYMBOLQX PARASYMBOLQX 31-34) four passages from articles written by me sixteen years ago. I hope my mind has not been stationary during all these years; yet there is little in those old articles which I now think positively erroneous, and nothing in the passages cited. My present views had, at that time, already begun to urge themselves on my mind; but they were not ripe for public avowal. In the first of the passages cited, I express the opinion, which I first uttered in my earlier lectures before the Lowell Institute, in 1866, afterwards in the *Popular Science Monthly* in 1877†1 [1878], in still fuller elaboration in my "Theory of Probable Inference" in 1882,†2 and maintain now as strongly as ever, that no definite probability can be assigned to any general arrangement of nature. To speak of an *antecedent* probability would imply that there was a statistical science of different universes; and a *deduced* probability requires an antecedent probability for one of its data.†P1 This consideration only goes to fortify my present position, that we cannot conclude from observed facts with any degree of probability, and therefore *a fortiori* not with certainty, that any proposition is absolutely universal, exact, or necessary. In

the absence of any weight of probability in favor of any particular exact statement, the formal presumption is altogether against any one out of innumerable possible statements of that kind.

Peirce: CP 6.609 Cross-Ref:††

The second passage cited is one in which I argue that the universe is not a chaos, or chance-medley. Now Dr. Carus admits (PARASYMBOLQX 28) that I do not today maintain that it is a chance-medley.

Peirce: CP 6.609 Cross-Ref:††

The third passage cited is this: "A contradiction is involved in the very idea of a chance-world." This is in entire harmony with my present position that "a chaos . . . being without connection or regularity would properly be without existence." ("Architecture of Theories," *The Monist*, vol. I, p. 176 [33].)

Peirce: CP 6.609 Cross-Ref:††

The fourth passage is to the effect that "the interest which the uniformities of nature have for an animal measures his place in the scale of intelligence." This I still believe.

Peirce: CP 6.609 Cross-Ref:††

So much for my supposed contradictions. If I am not mistaken, our amiable editor, whose admirable editorship springs so largely out of his amiability, in copying out these passages was really not half so much intent on showing me to be wrong at present, as on showing me to have been right formerly. However hard he hits, he contrives to honey his sockdolagers, and sincerely cares more to make the reader admire his antagonist when he is right than to condemn him when he is wrong. There is a touch of art in this that proclaims the born editor, and which I can hardly hope to imitate.

Peirce: CP 6.609 Cross-Ref:††

Though Dr. Carus admits over and over again that necessity cannot be based on observation, he often slips back to the idea that it can be so based. He says (PARASYMBOLQX 30) that "form is a quality of this world, not of some samples of it, but throughout, so far as we know of existence in even the most superficial way." But does he not see that all we *do* know, and all we *shall* tomorrow, or at any date know, is nothing but a sample of our possible experience -- nay, is but a sample of what we are in the future to have already experienced? I have characterized inductive inference as reasoning from samples; but the most usual way of sampling a class is by examining *all* the instances of it that have come under our observation, or which we can at once collect.

Peirce: CP 6.610 Cross-Ref:††

610. Dr. Carus (PARASYMBOLQX PARASYMBOLQX 44, 46) holds that from my social theory of reality, namely, that the real is the idea in which the community ultimately settles down, the existence of something inevitable is to be inferred. I confess I never anticipated that anybody would urge that. I thought just the reverse might be objected, namely, that all absoluteness was removed from reality by that theory; and it was many years ago that, in my "Theory of Probable Inference,"†1 I admitted the obvious justice, as it seemed to me, of that objection. We cannot be quite sure that the community ever will settle down to an unalterable conclusion upon any given question. Even if they do so for the most part, we have no reason to think the unanimity will be quite complete, nor can we rationally presume any

overwhelming *consensus* of opinion will be reached upon every question. All that we are entitled to assume is in the form of a *hope* that such conclusion may be substantially reached concerning the particular questions with which our inquiries are busied.

Peirce: CP 6.610 Cross-Ref:††

Such, at least, are the results to which the consideration of the doctrine of probability brings my mind irresistibly. So that the social theory of reality, far from being incompatible with tychism, inevitably leads up to that form of philosophy. Socialistic, or as I prefer to term it, agapastic ontology seems to me likely to find favour with many minds at an early day, because it is a natural path by which the nominalist may be led into the realistic ways of thought, ways toward which many facts and inward forces impel him. It is well, therefore, to call attention to the circumstances that the realism to which it leads is a doctrine which declares general truths to be real -- independent of the opinions of any particular collection of minds -- but not to be destined, in a strictly universal, exact, and sure acceptation, to be so settled, and established. Now to assert that general truths are objectively real, but to deny that they are strictly universal, exact, and certain, is to embrace the doctrine of absolute chance. Thus it is that the agapastic ontologist who endeavours to escape tychism will find himself "led into" that "inextricable confusion" which Dr. Carus (PARASYMBOLQX 4) has taken a contract to show that I am led into.

Peirce: CP 6.611 Cross-Ref:††

611. Conservatism is wholesome and necessary; the most convinced radical must admit the wisdom of it, in the abstract; and a conservative will be in no haste to espouse the doctrine of absolute chance. I, myself, pondered over it for long years before doing so. But I am persuaded, at length, that mankind will before very long take up with it; and I do not believe philosophers will be found tagging on to the tail of the general procession.

Peirce: CP 6.611 Cross-Ref:††

My little dialogue between the tychist and the necessitarian (*The Monist*, vol. II, pp. 331-333 [53-57]) seems to have represented pretty fairly the views of the latter; for Dr. Carus, in PARASYMBOLQX PARASYMBOLQX 151-155, does little more than reiterate them, without much, if at all, reinforcing them. His PARASYMBOLQX PARASYMBOLQX 158-160 merely work out, in a form perhaps not quite clear, what is manifest from the elementary principles of dynamics, and was considered in my dialogue.

Peirce: CP 6.611 Cross-Ref:††

His arguments in this connection, apart from those already noticed, are that absolute chance is something which if it existed would require explanation, that the manifold specificness of nature is explained by law without any aid from chance, and that absolute chance if it existed, in the sense in which it is supposed to exist in my chaos, could not possibly breed law as supposed by me. To the consideration of these arguments I proceed to apply myself.

Peirce: CP 6.612 Cross-Ref:††

612. One of the architectonic--and, therefore, I suppose, by Dr. Carus considered as highly reprehensible--features of my theory, is that, instead of saying off-hand what elements strike me as requiring explanation and what as not doing so, which seems to be his way, I have devoted a long time to the study of the whole logical doctrine of explanation, and of the history of explanations, and have based

upon the general principles so ascertained my conclusions as to what things do and what do not require to be explained.

Peirce: CP 6.612 Cross-Ref:††

Dr. Carus (PARASYMBOLQX 67) defines *explanation* as a description of a special process of nature in such a way that the process is recognized as a transformation. This I cannot quite grant. First, I cannot admit that "special processes of nature" are the only things to be explained. For instance, if I were to meet a gentleman who seemed to conform scrupulously to all the usages of good society, except that he wore to an evening party an emerald satin vest, that would be a fact calling for explanation, although it would not be a "special process of nature." Second, I cannot admit that an explanation is a description of the fact explained. It is true that in the setting forth of some explanations, it is convenient to restate the fact explained, so as to set it under another aspect; but, even in these cases, the statement of other facts is essential. In all cases, it is *other facts*, usually hypothetical, which constitute the explanation; and the process of explaining is a process by which from those other facts the fact to be explained is shown to follow as a consequence, by virtue of a general principle, or otherwise. Thus, a "special process of nature" calling for explanation is the circumstance that the planet Mars, while moving in a general way from west to east among the fixed stars, yet retrogrades a part of the time, so as to describe loops in the heavens. The explanation is that Mars revolves in one approximate circle and we in another. Again, it has been stated that a warm spring in Europe is usually followed by a cool autumn, and the explanation has been offered that so many more icebergs than usual are liberated during a warm spring, that they subsequently lower sensibly the temperature of Europe. I care little whether the fact and the explanation are correct or no. The case illustrates, at any rate, my point that an explanation is a special fact, supposed or known, from which the fact to be explained follows as a consequence. Third, I cannot admit that every description which recognizes the fact described as a transformation is an explanation; far less that "it is complete and exhaustive." (PARASYMBOLQX 67.) A magician transforms a watch into a dove. Recognize it as a transformation and the trick is explained, is it? This is delightfully facile. Describe the change from a caterpillar to a butterfly as a transformation, and does that explain it? Fourth, I cannot admit that every explanation recognizes the fact explained as a transformation. The explanation of the loops in the motion of Mars is not of that nature. But I willingly recognize in Dr. Carus's definition an attempt -- more or less successful -- to formulate one of the great offices of scientific inquiry, that of bridging over the gap between the familiar and the unfamiliar.

Peirce: CP 6.612 Cross-Ref:††

Explanation, however, properly speaking, is the replacement of a complex predicate, or one which seems improbable or extraordinary, by a simple predicate from which the complex predicate follows on known principles.†1 In like manner, a *reason*, in one sense, is the replacement of a multiple subject of an observational proposition by a general subject, which by the very conditions of the special experience is predicable of the multiple subject.†P1 Such a reason may be called an explanation in a loose sense.

Peirce: CP 6.612 Cross-Ref:††

Accordingly, that which alone requires an explanation is a coincidence.

Peirce: CP 6.612 Cross-Ref:††

Hence, I say that a uniformity, or law, is *par excellence*, the thing that requires

explanation. And Dr. Carus (PARASYMBOLQX 51) admits that this "is perfectly true."

Peirce: CP 6.612 Cross-Ref:††

But I cannot imagine anything further from the truth than his statement (PARASYMBOLQX 66) that "the only thing in the world of which we cannot and need not give account is the existence of facts itself." I should say, on the contrary, that the existence of facts is the only thing of which we need give account. Forms may indulge in whatever eccentricities they please in the world of dreams, without responsibility; but when they attempt that kind of thing in the world of real existence, they must expect to have their conduct inquired into. But should Dr. Carus reply that I mistake his meaning, that it is only "being in general" (PARASYMBOLQX 66) that he holds unaccountable, I reply that this is simply expressing scepticism as to the possibility and need of philosophy. In a certain sense, my theory of reality, namely that reality is the dynamical reaction of certain forms upon the mind of the community, is a proposed explanation of being in general; and be it remarked that the mind of the community, itself, is the thing the nature of whose being this explanation first of all puts upon an idealistic footing.

Peirce: CP 6.612 Cross-Ref:††

Chance, according to me, or irregularity--that is, the absence of any coincidence--calls for no explanation. If *law* calls for a particular explanation, as Dr. Carus admits it does, surely the mere absence of law calls for no further explanation than is afforded by the mere absence of any particular circumstance necessitating the result. An explanation is the conception of a fact as a necessary result, thereby accounting for the coincidence it presents. It would be highly absurd to say that the absence of any definite character must be accounted for, as if it were a peculiar phenomenon, simply because the imperfection of language leads us so to talk of it. Quite unfounded, therefore, is Dr. Carus's opinion that "chance needs exactly as much explanation as anything else" (PARASYMBOLQX 53) -- an opinion which, so far as I can see, rests on no defensible principle.

Peirce: CP 6.612 Cross-Ref:††

Equally hasty is his oft-repeated objection (PARASYMBOLQX PARASYMBOLQX 55, 58, 61) that my absolute chance is something ultimate and inexplicable. I go back to a chaos so irregular that in strictness the word existence is not applicable to its merely germinal state of being; and here I reach a region in which the objection to ultimate causes loses its force. But I do not stop there. Even this nothingness, though it antecedes the infinitely distant absolute beginning of time, is traced back to a nothingness more rudimentary still, in which there is no variety, but only an indefinite specificability, which is nothing but a tendency to the diversification of the nothing, while leaving it as nothing as it was before.†1 What objectionable ultimacy is here? The objection to an ultimate consists in its raising a barrier across the path of inquiry, in its specifying a phenomenon at which questions must stop, contrary to the postulate, or hope, of logic. But what question to which any meaning can be attached am I forbidding by my absolute chance? If what is demanded is a theological backing, or rational antecedent, to the chaos, that my theory fully supplies. The chaos is a state of intensest feeling, although, memory and habit being totally absent, it is sheer nothing still. Feeling has existence only so far as it is welded into feeling. Now the welding of this feeling to the great whole of feeling is accomplished only by the reflection of a later date. In itself, therefore, it is nothing; but in its relation to the end it is everything.

Peirce: CP 6.612 Cross-Ref:††

More unreasonable yet is Dr. Carus's pretension that the manifold specificness, which is what I mean by chance, is capable of explanation (PARASYMBOLQX PARASYMBOLQX 142, 143) by his own philosophic method. He may explain one particularity by another, of course; but to explain specificness itself would be to show that a specific predicate is a necessary consequence of a generic one, or that a whole is without ambiguity a part of its part. Remark, reader, at this point, that chance, whether it be absolute or not, is not the mere creature of our ignorance. It is that diversity and variety of things and events which law does not prevent. Such is that real chance upon which the kinetical theory of gases, and the doctrines of political economy, depend. To say that it is not absolute is to say that it -- this diversity, this specificness -- can be explained as a consequence of law. But this, as we have seen, is logically absurd.

Peirce: CP 6.612 Cross-Ref:††

Dr. Carus admits that absolute chance is "not unimaginable." (PARASYMBOLQX 150.) Chance itself pours in at every avenue of sense: it is of all things the most obtrusive. That it is absolute is the most manifest of all intellectual perceptions. That it is a being, living and conscious, is what all the dullness that belongs to ratiocination's self can scarce muster hardihood to deny.

Peirce: CP 6.612 Cross-Ref:††

Almost as unthinking is the objection (PARASYMBOLQX 61) that absolute chance could never beget order. I have noticed elsewhere the historic obliviscence of this objection.†1 Must I once again repeat that the tendency to take habits, being itself a habit, has *eo ipso* a tendency to grow; so that only a slightest germ is needed? A realist, such as I am, can find no difficulty in the production of that first infinitesimal germ of habit-taking by chance, provided he thinks chance could act at all. This seems, at first blush, to be explaining something as a chance-result. But exact analysis will show it is not so.

Peirce: CP 6.612 Cross-Ref:††

In like manner, when the eminent thinker who does me the honor to notice my speculation objects that I do not, after all, escape making law absolute, since the tendency to take habits which I propose to make universal is itself a law, I confess I can find only words without ideas in the objection. Law is a word found convenient, I grant, in describing that tendency; but is there no difference between a law the essence of which is to be inviolable (which is the nominalistic conception of mechanical law, whose being, they say, lies in its action) and that mental law the violation of which is so included in its essence that unless it were violated it would cease to exist? In my essay "The Law of Mind" I have so described that law. [See 148.] In so describing it, I make it a law, but not an absolute law; and thus I clearly escape the contradiction attributed to me.†1

Peirce: CP 6.613 Cross-Ref:††

613. In my attack on "The Doctrine of Necessity" I offered four positive arguments for believing in real chance. They were as follows:

Peirce: CP 6.613 Cross-Ref:††

1. The general prevalence of growth, which seems to be opposed to the conservation of energy.

Peirce: CP 6.613 Cross-Ref:††

2. The variety of the universe, which is chance, and is manifestly inexplicable.

Peirce: CP 6.613 Cross-Ref:††

3. Law, which requires to be explained, and like everything which is to be explained must be explained by something else, that is, by non-law or real chance.

Peirce: CP 6.613 Cross-Ref:††

4. Feeling, for which room cannot be found if the conservation of energy is maintained.

Peirce: CP 6.613 Cross-Ref:††

In a brief conversation I had with him, my friend remarked (and if it was an inconsiderate concession, I certainly do not wish to hold him to it) that, while the theory of tychism had some attractive features, its weakness consisted in the absence of any positive reasons in its favor. I infer from this that I did not properly state the above four arguments. I therefore desire once more to call attention to them, especially in their relations to one another.

Peirce: CP 6.613 Cross-Ref:††

Mathematicians are familiar with the theorem that if a system of particles is subject only to positional forces, it is such that if at any instant the velocities were all suddenly reversed, without being altered in quantity, the whole previous history of the system would be repeated in inverse succession. Hence, when physicists find themselves confronted with a phenomenon which takes place only in one order of succession and never in the reverse order -- of which no better illustration could be found than the phenomena of growth, for nobody ever heard of an animal growing back into an egg -- they always take refuge in the laws of probability as preventive of the velocities ever getting so reversed. To understand my argument number 1, it is necessary to make this method of escape from apparent violations of the law of energy quite familiar to oneself. For example, according to the law of energy, it seems to follow (and by the aid of the accepted theory of light it does follow) that if a prism, or a grating, disperses white light into a spectrum, then the colors of the spectrum falling upon the prism or grating at the same angles, and in the same proportions, will be recombined into white light; and everybody knows that this does in fact happen. Nevertheless, the usual and prevalent effect of prisms and gratings is to produce colored spectra. Why? Evidently, because, by the principles of probability, it will rarely happen that colored lights converging from different directions will fall at just the right angles and in just the right proportions to be recombined into white light. So, when physicists meet with the phenomena of frictional and viscous resistance to a body in motion, although, according to their doctrine, if the molecules were to move with the same velocities in opposite directions the moving body would be accelerated, yet they say that the laws of probability, applied to the trillions of molecules concerned, render this practically certain not to occur. I do no more, then, than follow the usual method of the physicists, in calling in chance to explain the apparent violation of the law of energy which is presented by the phenomena of growth: only instead of chance, as they understand it, I call in absolute chance. For many months I endeavoured to satisfy the data of the case with ordinary *quasi* chance; but it would not do. I believe that in a broad view of the universe a simulation of a given elementary mode of action can hardly be explained except by supposing the genuine mode of action somewhere has place. If it is improbable that colored lights should fall together in just such a way as to give a white ray, is it not an equally extraordinary thing that they should all be generated in such a way as to produce a white ray? If it is incredible that trillions of molecules in a fluid should strike a solid body moving

through it so as to accelerate it, is it not marvellous that trillions of trillions of molecules all alike should ever have got so segregated as to create a state of things in which they should be practically certain to retard the body? It is far from easy to understand how mere positional forces could ever have brought about those vast congregations of similar atoms which we suppose to exist in every mass of gas, and by which we account for the apparent violations of the law of energy in the phenomena of the viscosity of the gas. There is no difficulty in seeing how sulphuric acid acting on marble may produce an aggregation of molecules of carbonic anhydride, because there are similar aggregations in the acid and in the marble; but how were such aggregations brought about in the first place? I will not go so far as to say that such a result is manifestly impossible with positional forces alone; but I do say that we cannot help suspecting that the simulated violation of the law of energy has a real violation of the same law as its ultimate explanation. Now, growth *appears* to violate the law of energy. To explain it, we must, at least, suppose a simulated, or *quasi*, chance, such as Darwin calls in to produce his fortuitous variations from strict heredity. It may be there is no real violation of the law, and no real chance; but even if there be nothing of the sort in the immediate phenomenon can the conditions upon which the phenomenon depends have been brought about except by real chance? It is conceivable, again, that the law of the conservation of forces is not strictly accurate, and that, nevertheless, there is no absolute chance. But I think so much has been done to put the law of the conservation of forces upon the level of the other mechanical laws that when one is led to entertain a serious doubt of the exactitude of that, one will be inclined to question the others.

Peirce: CP 6.613 Cross-Ref:††

Besides, few psychologists will deny the very intimate connection which seems to subsist between the law, or quasi-law, of growth and the law of habit, which is the principal, if not (as I hold it to be) the sole, law of mental action. Now, this law of habit seems to be quite radically different in its general form from mechanical law, inasmuch as it would at once cease to operate if it were rigidly obeyed: since in that case all habits would at once become so fixed as to give room for no further formation of habits. In this point of view, then, growth seems to indicate a positive violation of law.

Peirce: CP 6.613 Cross-Ref:††

Let us now consider argument number 3: and remark how it fortifies number 1. Physical laws that appear to be radically different yet present some striking analogies. Electrical force appears to be polar. Its polarity is explained away by Franklin's one-fluid theory, but in that view the force is a repulsion. Now, gravitation is an attraction, and is, therefore, essentially different from electricity. Yet both vary inversely as the square of their distance. Radiation likewise follows the same formula. In this last case, the formula, in one aspect of it, follows from the conservation of energy. In another aspect of it, it results from the principle of probability, and does not hold good, in a certain sense, when the light is concentrated by a lens free from spherical aberration. But neither the conservation of energy nor the principle of probability seems to afford any possible explanation of the application of this theory to gravitation nor to electricity. How, then, are such analogies to be explained? The law of the conservation of energy and that of the perduration of matter present so striking an analogy that it has blinded some powerful intellects to their radically different nature. The law of action and reaction, again, has often been stated as the law of the conservation of momentum. Yet it is not only an independent law, but is even of a contrary nature, inasmuch as it is only the algebraical sum of opposite

momenta that is "conserved."†P1 How is this striking analogy between three fundamental laws to be explained? Consider the still more obvious analogy between space and time. Newton †1 argues that the laws of mechanics prove space and time to be absolute entities. Leibnitz,†2 on the other hand, takes them as laws of nature. Either view calls for an explanation of the analogy between them, which no such reflection as the impossibility of motion without that analogy can supply. Kant's †3 theory seems to hint at the possibility of an explanation from both being derived from the nature of the same mind. Any three orthogonal directions †P1 in space are exactly alike, yet are dynamically independent.

Peirce: CP 6.613 Cross-Ref:††

These things call for explanation; yet no explanation of them can be given, if the laws are fundamentally original and absolute.

Peirce: CP 6.613 Cross-Ref:††

Moreover, law itself calls for explanation. But how is it to be explained if it is as fundamentally original and absolute as it is commonly supposed to be? Yet if it is not so absolute, there is such a phenomenon as absolute chance.

Peirce: CP 6.613 Cross-Ref:††

Thus, the chance which growth calls for is now seen to be absolute, not *quasi* chance.

Peirce: CP 6.613 Cross-Ref:††

Now consider argument number 2. The variety of the universe so far as it consists of unlikenesses between things calls for no explanation. But so far as it is a general character it ought to be explained. The manifold diversity or specificness, in general, which we see whenever and wherever we open our eyes, constitutes its liveliness, or vivacity. The perception of it is a direct, though darkling, perception of God.†1 Further explanation in that direction is uncalled for. But the question is whether this manifold specificness was put into the universe at the outset, whether God created the universe in the infinitely distant past and has left it to its own machinery ever since, or whether there is an incessant influx of specificness. Some of us are evolutionists; that is, we are so impressed with the pervasiveness of growth, whose course seems only here and there to be interrupted, that it seems to us that the universe as a whole, so far as anything can possibly be conceived or logically opined of the whole, should be conceived as growing. But others say, though parts of the universe simulate growth at intervals, yet there really is no growth on the whole -- no passage from a simpler to a more complex state of things, no increasing diversity.

Peirce: CP 6.613 Cross-Ref:††

Now, my argument is that, according to the principles of logic, we never have a right to conclude that anything is absolutely inexplicable or unaccountable. For such a conclusion goes beyond what can be directly observed, and we have no right to conclude what goes beyond what we observe, except so far as it explains or accounts for what we observe. But it is no explanation or account of a fact to pronounce it inexplicable or unaccountable, or to pronounce any other fact so. Now, to say no process of diversification takes place in nature leaves the infinite diversity of nature unaccounted for; while to say the diversity is the result of a general tendency to diversification is a perfectly logical probable inference. Suppose there be a general tendency to diversification; what would be the consequence? Evidently, a high degree of diversity. But this is just what we find in nature. It does not answer the purpose to say there is diversity because God made it so, for we cannot tell what God would do,

nor penetrate his counsels. We see what He *does* do, and nothing more. For the same reason one cannot logically infer the existence of God; one can only know Him by direct perception.

Peirce: CP 6.613 Cross-Ref:††

It is to be noted that a general tendency to diversification does not explain diversity in its specific characters; nor is this called for. Neither can such a tendency explain any specific fact. Any attempt to make use of the principle in that manner would be utterly illogical. But it can be used to explain universal facts, just as *quasi* chance is used to explain statistical facts. Now, the diversity of nature is a universal fact.

Peirce: CP 6.613 Cross-Ref:††

To explain diversity is to go behind the chaos, to the original undiversified nothing. Diversificacity was the first germ.

Peirce: CP 6.613 Cross-Ref:††

Argument number 4 was, upon its negative side, sufficiently well presented in my "Doctrine of Necessity Examined." Mechanical causation, if absolute, leaves nothing for consciousness to do in the world of matter; and if the world of mind is merely a transcript of that of matter, there is nothing for consciousness to do even in the mental realm. The account of matters would be better if it could be left out of account. But the positive part of the argument, showing what can be done to reinstate consciousness as a factor of the universe when once tychism is admitted is reinforced in the later papers. This ought to commend itself to Dr. Carus, who shows himself fully alive to the importance of that part of the task of science which consists in bridging gaps. But consciousness, for the reason just stated, is not to be so reinstated without tychism; nor can the work be accomplished by assigning to the mind an occult power, as in two theories to be considered in the section following this. As might be anticipated (and a presumption of this kind is rarely falsified in metaphysics), to bridge the gap synechism is required. Supposing matter to be but mind under the slavery of inveterate habit, the law of mind still applies to it. According to that law, consciousness subsides as habit becomes established, and is excited again at the breaking up of habit. But the highest quality of mind involves a great readiness to take habits, and a great readiness to lose them; and this implies a degree of feeling neither very intense nor very feeble.

Peirce: CP 6.613 Cross-Ref:††

I have noticed above (594) Dr. Carus's dubious attitude toward the first argument. I considered in the last section his attempted reply to the second. To the third argument, he replies (PARASYMBOLQX 165) that law ought to be accounted for by the principle of sufficient reason. But, of course, that principle cannot recommend itself to me, a realist; for it is nothing but the lame attempt of a nominalist to wriggle out of his difficulties. Reasons explain nothing, except upon some theistic hypothesis which may be pardoned to the yearning heart of man, but which must appear doubtful in the eyes of philosophy, since it comes to this, that Tom, Dick, and Harry are competent to pry into the counsels of the Most High, and can invite in their cousins and sweethearts and sweethearts' cousins to look over the original designs of the Ancient of Days.

Peirce: CP 6.614 Cross-Ref:††

614. My fourth argument it is which seems to have made most impression upon Dr. Carus's mind (PARASYMBOLQX 85), and his reply is rather elaborate.

Peirce: CP 6.614 Cross-Ref:††

While embracing unequivocally the necessitarian dogma, equally for mind and for matter (PARASYMBOLQX 193), Dr. Carus wishes utterly to repudiate materialism and the mechanical philosophy (PARASYMBOLQX 133). To facilitate his thus walking the slack-rope, he makes (PARASYMBOLQX 168) a division of events into "(1) mechanical, (2) physical, (3) chemical, (4) physiological, and (5) psychical events." The first three (PARASYMBOLQX PARASYMBOLQX 169-171) are merely distinguished by the magnitude of the moving masses, so that, for philosophical purposes, they do not differ at all. As for physiological events, though he devotes a paragraph (PARASYMBOLQX 172) to their definition, he utterly fails to distinguish them from the mechanical (including the physical and chemical) on the one hand, or from the psychical on the other. Dr. Carus seems to think (PARASYMBOLQX 176) that by this division he has separated himself entirely from the materialists; but this is an illusion, for nobody denies the existence of feelings.

Peirce: CP 6.614 Cross-Ref:††

The truth is, he distinctly enrolls himself in the mechanical army when he asserts that mental laws are of the same necessitarian character as mechanical laws (PARASYMBOLQX 193). The only question that remains as to his position is whether he is a materialist or not. He instances (PARASYMBOLQX 185) the case of a general receiving a written dispatch and being stimulated into great activity by its perusal, and causing great motions to be made and missiles to be sped in consequence. Now, the dilemma is this. Will Dr. Carus, on the one hand, say that the motion of those missiles was determined by mechanical laws alone, in which case, it would only be necessary to state all the positions and velocities of particles concerned, a hundred years before, to determine just how those bullets would move and, consequently, whether the guns were to be fired or not, and this would constitute him a materialist, or will he say that the laws of motion do not suffice to determine motions of matter, in which case, since they formally certainly do so suffice, they must be *violated*, and he will be giving to mind a direct dynamical power which is open to every objection that can be urged against tychism?

Peirce: CP 6.614 Cross-Ref:††

Now admire the decision with which he cuts the Gordian knot!

Peirce: CP 6.614 Cross-Ref:††

"THERE ARE NO PURELY MECHANICAL PHENOMENA."
(PARASYMBOLQX 175.)

Peirce: CP 6.614 Cross-Ref:††

That is,

Peirce: CP 6.614 Cross-Ref:††

"*The laws of motion ARE applicable to and will explain all motions.*"
(PARASYMBOLQX 177.)

Peirce: CP 6.614 Cross-Ref:††

But hold!

Peirce: CP 6.614 Cross-Ref:††

"The mechanical philosopher . . . feels warranted in the hope that . . . the actions of man . . . can be explained by the laws of motion. . . . We may anticipate that this conclusion will prove ERRONEOUS. *And so it is.*" (PARASYMBOLQX 176.)

Peirce: CP 6.614 Cross-Ref:††

At the same time,

Peirce: CP 6.614 Cross-Ref:††

"No OBJECTION CAN BE MADE *to the possibility of explaining the delicate motions in the nervous substance of the brain by the laws of molar or molecular mechanics.*" (PARASYMBOLQX 178.)

Peirce: CP 6.614 Cross-Ref:††

Yet,

"*The simplest* psychical reflexes, including those *physiological reflexes* which we must suppose to have originated by conscious adaptation, . . . CANNOT *be explained from mechanical* or physical *laws* alone." (PARASYMBOLQX 186.)
However,

Peirce: CP 6.614 Cross-Ref:††

"*We do NOT say that there are motions . . . in the brain . . . which form exceptions to the laws of mechanics.*" (PARASYMBOLQX 187.)

Peirce: CP 6.614 Cross-Ref:††

Nevertheless,

"The brain-atoms are possessed of the same spontaneity as the atoms of a gravitating stone. Yet there is present an additional feature; there are present states of awareness. . . . Neither states of awareness nor their meanings can be weighed on any scales, be they ever so delicate, nor are they determinable in footpounds." (PARASYMBOLQX 192.)

Peirce: CP 6.614 Cross-Ref:††

Clearness is the first merit of a philosopher; and what PARASYMBOLQX 192 comes to is crystal-clear. Dr. Carus wants to have the three laws of motion always obeyed; but he wishes the forces between the molecules to be varied according to the momentary states of awareness. All right: he is entitled to suppose whatever he likes, so long as the supposition is self-consistent, as this supposition is. It conflicts with the law of energy, it is true; for that law is that the forces depend on the situations of the particles alone, and not on the time. It is liable to give rise to perpetual motion. It was intended, no doubt, to be an improvement on my molecular theory of protoplasm,†1 earlier in the same number. It escapes materialism. It supposes a direct dynamical action between mind and matter, such as has not been supposed by any eminent philosopher that I know of for centuries. I am sorry to say that it shows a dangerous leaning toward originality. The argument for thus rejecting the law of the conservation of energy I leave to others to be weighed. It seems to suppose a much larger falsification of that law than my doctrine; but it is a pretty clever attempt to escape my conclusions. It rejects what has to be rejected, the law of the conservation of energy; and is far more intelligent than the theory of those (like Oliver and Lodge)†2 who wish to give to mind a power of deflecting atoms, which would satisfy the conservation of energy while violating the law of action and reaction. If it can have due consideration, I doubt not it will accelerate the acceptance of my views. Meantime, I do not see where that "inextricable confusion" into which I was to be led is to come in. (PARASYMBOLQX 4.)

Peirce: CP 6.615 Cross-Ref:††

615. Little more requires to be noticed in Dr. Carus's articles. He admits (PARASYMBOLQX 2) that indeterminism is the more natural belief, which is no slight argument in its favor.

Peirce: CP 6.616 Cross-Ref:††

616. The remarks upon the theological bearings of the theories, if they are found somewhat wide of the mark, are explained by the haste of the editor to show just what all the affiliations of my views were, before I had had time to explain what those views are. The remarks to which I refer will be found in PARASYMBOLQXPARASYMBOLQX 3, 36, 81, 82, 83, 128, 203, 204. They are worth putting together.

Peirce: CP 6.617 Cross-Ref:††

617. The doctrine of symbolism, to which Dr. Carus has recourse, seems to be similar to that of my essay "Some Consequences of Four Incapacities" (*Journal of Speculative Philosophy*, vol. II).†1 (PARASYMBOLQXPARASYMBOLQX 180, 183, 199.) On this head, I can only approve of his ideas.

Peirce: CP 6.618 Cross-Ref:††

618. It is true that I wrote many definitions for one of the "encyclopedic lexicons." But they were necessarily rather vaguely expressed, in order to include the popular use of terms, and in some cases were modified by proofreaders or editors; and, for reasons not needful here to explain, they are hardly such as I should give in a Philosophical Dictionary proper.

Peirce: CP 6 Appendix B - Ed. Note
APPENDIX B

NOMINALISM *versus* REALISM †1

[We print below some strictures upon the position assumed in our last number with reference to M. Janet's version of Hegel's doctrine of the "Becoming." We hope that these acute statements which have been written, for the most part, in the form of queries, will receive a careful reading, especially by those who have differed from our own views hitherto expressed. They seem to us the most profound and compendious statement of the anti-speculative standpoint as related to the Science of Pure Thought (*Prima Philosophia*), that we have seen. But for this very reason we are fain to believe that the defects of the formalism relied upon are all the more visible. We have endeavoured to answer these queries with the same spirit of candour that animates their author. -- *Editor*.]

Peirce: CP 6.619 Cross-Ref:††

Mr. Editor of the Journal of Speculative Philosophy:

619. I should like to make some inquiries in regard to your meaning in the paragraph beginning "Being is the pure Simple," vol. I, p. 255.

Peirce: CP 6.619 Cross-Ref:††

I will begin by stating how much of it I already understand, as I believe. I understand that "Being" and "Nothing," as used by you, are two abstract, and not two general terms. That Being is the abstraction belonging in common and exclusively to the objects of the concrete term, whose extension is unlimited or all-embracing, and whose comprehension is null. I understand that you use Nothing, also, as an abstract term = nothingness; for otherwise to say that Being is Nothing is like saying that humanity is non-man, and does not imply at all that Being is in any opposition with itself, since it would only say "Das Sein ist nicht Seiendes," not "Sein ist nicht Sein." By Nothing, then, I understand the abstract term corresponding to a (possible) concrete term, which is the logical contradictory of the concrete term corresponding to "Being." And since the logical contradictory of any term has no extension in common with that term, the *concrete* nothing is the term which has no extension. I understand, that, when you say "Being has no content," and "Being is wholly undetermined," you mean, simply, that its corresponding concrete has no logical comprehension, or, at least, that what you mean follows from this, and this, conversely, from what you mean.

Peirce: CP 6.619 Cross-Ref:††

I come now to what I do not understand, and I have some questions to ask, which I have endeavoured so to state that all can see that the Hegelian is bound to answer them, for they simply ask what you mean, whether this or that; they simply ask you to be explicit upon points upon which you have used ambiguous expressions. They are not put forward as arguments, however, but only as inquiries.

Peirce: CP 6.620 Cross-Ref:††

620. Abstract terms, according to the doctrine of modern times, are only a device for expressing in another way the meaning of concrete terms. To say that whiteness inheres in an object is the same as to say that an object is white. To say that whiteness is a color is the same as to say that the white is colored, and that this is implied in the very meanings of the words.

Peirce: CP 6.620 Cross-Ref:††

But, you will undoubtedly admit that there is a difference between a hundred dollars in my pocket *Being or not Being*, and so in any other particular case. You, therefore, admit that there is nothing which is, which is also not. Therefore, it follows that *what is*, and *what is not*, are mutually exclusive and not coëxtensive.

Peirce: CP 6.620 Cross-Ref:††

Since, then, you nevertheless say that the corresponding abstractions, Being and Nothingness, are absolutely the same (although you at the same time hold that it is not so, at all), it is plain that you find some other meaning in abstract terms than that which other logicians find. I would, therefore, ask what you mean by an abstraction, and how you propose to find out what is true of abstractions.†1

Peirce: CP 6.621 Cross-Ref:††

621. You say, in effect,

Being has no determination;

Ergo, It is nothing.

Peirce: CP 6.621 Cross-Ref:††

Now, it certainly appears that the contrary conclusion follows from this premiss, namely: that it is not nothingness. I suppose that you have suppressed one of your premisses, and that you mean to argue thus:

Indetermination in respect to any character is the negation of that character;

Being is indeterminate in respect to every character;

Ergo, Being is negative of every character.

In short, you seem to imply that to abstract from a character is to deny it. Is this the manner in which your argument is to be completed, or how else?

Peirce: CP 6.622 Cross-Ref:††

622. This suggests another question. You say that nothing has no determination. It is plain that it would not follow from this that Being is nothing, but only that Nothing is being, or rather that Any non-being is a being, thus reducing non-being (*nicht-seiende*) to an absurdity. This would be nothing new (for Albertus Magnus quotes Avicenna to this effect) and in my opinion would be perfectly true. *Non-ens*, or "the not being," is a self-contradictory expression. Still, though I thus see no monstrous consequences of saying that nothing has no determination, I see no proof at all that it is so. It might be said, indeed, that the things which are not have no characters in common, and that therefore *what is not* has no logical comprehension and Being -- not no determination. I would ask, then, have you proved that nothing has no determination? Do not suppose that I am endeavouring to drive you into contradiction; for I understand Hegelians profess to be self-contradictory. I only wish to ascertain whether they have an equal disregard for those logical maxims which relate to ambiguities.

Peirce: CP 6.623 Cross-Ref:††

623. You say, in effect,

Difference is determination,

Being has no determination;

Ergo, Being has no difference from nothing;

Ergo, Being is nothing.

Peirce: CP 6.623 Cross-Ref:††

It is incontestable that difference from anything is determination in respect to being or not being that thing. A monkey, in differing from a man, is determined (negatively) in respect to humanity. Difference, then, in any respect, is determination in that respect. This, I take it, is what you mean. Now let us parallel the above argument:

Difference in any respect is determination in that respect;

Animality, in general, is not determined in respect to humanity;

Ergo, Animality, in general, has no difference from humanity;

Ergo, Animality, in general, is humanity. This is plainly sophistical. For to say that an abstraction, in general, is undetermined, has two different senses; one resulting from a strict analysis of the language, and the other reposing upon the ordinary use of language. Strictly, to say that an abstraction is undetermined would mean that it may be this or may be that abstraction; that is, that the abstract word by which it is expressed may have any one of a variety of meanings. What is ordinarily meant by the phrase, however, is that the object of the corresponding concrete term is undetermined, so that neither of a certain pair of mutually contradictory predicates are *universally* true of that concrete. Now, it is true to say that animality is undetermined in respect to humanity, or that being is not determined at all, only in the latter of these senses, to wit: that not every animal is a man, and not every animal is not a man, and (in the other case) that there is no predicate which can be truly affirmed or denied of all beings. For in the other sense we should imply that the abstractions themselves were vague, and that being, for example, has no precise meaning. In the only true sense, therefore, the premiss is, in the one case, that "Animal, simply, is undetermined," and, in the other, that "*Ens (seiende)* is undetermined"; and what follows is, in the one case, that "not every animal differs from a man," and, in the other, that "not every being differs from any nothing." This latter amounts merely to saying that there is nothing from which every being differs, or that a nothing is an absurdity. These correct conclusions do not in the least imply that animality is humanity, or that being is nothingness. To reach the latter conclusions, it would be necessary (in the first place) to use the premisses in the other and false sense; but even then all that would be legitimately inferable would be that "humanity, *in some sense*, is animality," and that "being, *in some sense*, is nothing." Only by a second fallacy could it be concluded that animality, in the sense intended, is humanity, or that being, in the sense intended, is humanity, or that being, in the sense intended, is nothing. Now, I would inquire whether you inadvertently fell into these ambiguities, or, if not, wherein the force of your argumentation lies?

Peirce: CP 6.624 Cross-Ref:††

624. Finally, I would inquire whether, in your opinion, the maxims of (ordinary) logic relating to contradictions lack even a *prima facie* presumption in their favor? Whether the burden of proof is or is not upon the Hegelians to show that the assumption of their falsity is a more tenable position than the assumption of their truth? For in the present state of the question, it seems to me more probable that subtle fallacies lurk in the Hegelian reasoning than that such fallacies lurk in all other reasoning whatsoever.

Peirce: CP 6.625 Cross-Ref:††

APPENDIX C

WHAT IS MEANT BY "DETERMINED"?†1

To the Editor of the Journal of Speculative Philosophy:

625. Your remarks upon my inquiries concerning Being and Nothing are very kind and courteous. Considered as replies, they are less satisfactory than they might have been had I succeeded better in making my difficulties understood.

Peirce: CP 6.625 Cross-Ref:††

I suspect that there must be some misunderstanding between us of the meaning of the various terms cognate with "determined." Perhaps, therefore, I shall do well to state more fully than I did before, the manner in which I understand Hegel (in common with all other logicians) to use them. Possibly, the original signification of *bestimmt* was "settled by vote"; or it may have been "pitched to a key." Thus its origin was quite different from that of "determined"; yet I believe that as philosophical terms their equivalence is exact. In general, they mean "fixed to be *this* (or *thus*), in contradistinction to being this, that, or the other (or in some way or other)." When it is a concept or term, such as is expressed by a concrete noun or adjective which is said to be more determinate than another, the sense sometimes is that the logical extension of the former concept or term is a part and only a part of that of the latter; but more usually the sense is that the logical comprehension of the latter is a part and only a part of that of the former.

Peirce: CP 6.626 Cross-Ref:††

626. In my former letter (623) I sufficiently expressed my own understanding of "determined" as applied to a concept or term such as is expressed by an abstract noun. Determinate is also used either in express application or with implicit reference to a second intention or term of second denomination. In such an acceptation, we may speak either of a singular as indeterminate, or of a conception of Being, in general, as determinate. Every singular is in one sense perfectly determinate, since there is no pair of contradictory characters of which it does not possess one. Yet if the extension of the term be limited, not by additions to its comprehension, but *by a reflection upon the term itself* -- namely, that it shall denote but one -- it is called an indeterminate singular. In this sense, "some one horse" is an indeterminate individual, while "Dexter" is a determinate individual. In a somewhat similar way, every universal conception of Being is quite indeterminate in the sense of not signifying any particular character. Yet, if the reflection is explicitly made (*gesetzt*) that every thing to which it applies has its particular characters, it is called by Hegel *determinate being*. Hegel teaches that the whole series of categories or universal conceptions can be *evolved* from one -- that is, from *Seyn* -- by a certain process, the effect of which is to make actually thought that which was virtually latent in the thought. So that this reflection which constitutes *Daseyn* lies implicitly even in *Seyn*, and it is by explicitly evolving it from *Seyn* that *Daseyn* is evolved from *Seyn*. (Hegel's *Werke*, Bd. 3, S. 107.) The term "What is" has reference to pure *Seyn* only; the term "What is somehow" has reference to *Daseyn*. This is my understanding of the term "determinate." It must differ from yours, or you would not say that animality, *in general*, is determined in respect to humanity: so when you say that were animality and humanity, in general, undetermined with respect to each other they would be identical, I take the example of "highness of pitch in general" and "loudness of sound in general," and I conclude again that we are taking the word "determine" in different senses. May I ask you to reperuse my fourth question? (623.)

Peirce: CP 6.627 Cross-Ref:††

627. You have apparently understood me as applying the term "abstract" to

any concept the result of abstraction. But, as I intimated (620), I adopt that acceptance in which "whiteness" is said to be abstract and "white" concrete. For this use of the terms, I refer to the following authorities: Andrews and Stoddard's *Latin Grammar*, §26, 5; Scotus, *Super Prædicamenta*, qu. 8; Durandus à Sancto Porciano, *In Sentent.*, lib. 1, dist. 34, qu. 1; Ockham, *Summa Logices*, pars 1, cap. 5; Chauvin, *Lexicon Rationale*, sub. V. *Abstractum*; Mill, *Logic*, bk. 1, cap. 2, §4; Trendelenburg, *Elementa Logices Arist.*, 6th ed., p. 117, note; Uberweg, *Logik*, §51 (where Wolff, also, is cited); Hoppe, *Logik*, §§256, 257. This misapprehension affects the relevancy of most of your remarks.

Peirce: CP 6.628 Cross-Ref:††

628. I think that I have not, as you suppose, greatly mistaken the sense in which Hegelians use the term Pure Being.

Peirce: CP 6.628 Cross-Ref:††

At least, my definition seems to be in accord with the explanations of almost all, if not all, the commentators and expositors of Hegel. I would submit respectfully, that your own remarks upon p. 117 of vol. 1, of this Journal contradict, almost in terms, what you say (vol. II, p. 57) in reply to me.

Peirce: CP 6.629 Cross-Ref:††

629. Once or twice you use such expressions as "We do not profess to speak for Hegelians," "Hegelians may understand this as they please," etc. Have I been wrong, then, in supposing that the passage to which my queries related was a professed defense of Hegelian doctrine?

Peirce: CP 6.630 Cross-Ref:††

630. I am sorry to learn that I have done you injustice in saying that you profess to be self-contradictory. Yet I do not see in what sense you object to the remark. To say that a man is self-contradictory is, of course, but a way of saying that what he believes is self-contradictory. You believe that "finite things contradict themselves"; that is, as I understand it, that contradictions exist. Therefore, what you believe in appears to be self-contradictory. Nor can I see how a person "escapes self-contradiction by not attempting to set up non-contradiction as the first principle of things"; that is, by not professing to be otherwise than self-contradictory.

Peirce: CP 6.630 Cross-Ref:††

I do not see that you notice query 3.

Peirce: CP 6.1 Fn 1 p 1

†1 From "The Logic of Events", 1898, the last of a proposed set of eight lectures. See 212n. Cf. also ch. 7, which appears to be an alternative draft.

Peirce: CP 6.1 Fn 2 p 1

†2 See 1.244ff.

Peirce: CP 6.3 Fn 1 p 2

†1 Cf. 216ff., 1.619ff.

Peirce: CP 6.3 Fn 1 p 4

†1 *The Creeds of Christendom*, by Philip Schaff, 2d edition, 1878.

Peirce: CP 6.3 Fn 2 p 4

†2 See 5.589.

Peirce: CP 6.4 Fn 1 p 5

†1 See 1.186.

Peirce: CP 6.5 Fn 2 p 5

†2 See vol. 2, bk. III, b for a detailed discussion of the nature of induction and hypothesis (or abduction).

Peirce: CP 6.5 Fn 3 p 5

†3 For the rest of this manuscript, see 214ff.

Peirce: CP 6.6 Fn 4 p 5

†4 From an unidentified fragment, c. 1903.

Peirce: CP 6.6 Fn 5 p 5

†5 See 1.241n, 1.242n for definitions of these terms.

Peirce: CP 6.6 Fn 1 p 5

†5 See 1.241n, 1.242n for definitions of these terms.

Peirce: CP 6.6 Fn 2 p 6

†1 See 185f, 364ff, 3.527, 4.65ff, 4.172, 4.514, 5.447f, 5.453ff.

Peirce: CP 6.6 Fn 3 p 6

†2 See 185ff, 4.61, 5.447f.

Peirce: CP 6.6 Fn 4 p 6

†3 See 1.433f, 3.93, 3.216, 4.172, 5.299ff, 5.429, 5.448n.

Peirce: CP 6.6 Fn 5 p 6

†4 See 1.321, 3.527f, 5.45, 5.474, 5.487, 5.493, 5.539.

Peirce: CP 6.6 Fn 6 p 6

†5 See 132f, 1.305f, 1.412f, 1.549.

Peirce: CP 6.6 Fn 7 p 6

†6 See 132f, 197, 211, 1.311.

Peirce: CP 6.6 Fn 8 p 6

†7 See 1.422.

Peirce: CP 6.6 Fn 9 p 6

†8 See 87, 96, 127f, 506, 1.489ff.

Peirce: CP 6.6 Fn 10 p 6

†9 See 82f, 1.501ff, 3.134n.

Peirce: CP 6.6 Fn 11 p 6

†10 See 1.492f, 1.495f.

Peirce: CP 6.6 Fn 12 p 6

†11 See 82, 86, 127ff, 133ff, 1.170, 1.499, 5.530.

Peirce: CP 6.6 Fn 13 p 6

†12 See 211, 4.222ff.

Peirce: CP 6.6 Fn 14 p 6

†13 See 28, 82, 210, 419f, 1.274ff, 4.223.

Peirce: CP 6.6 Fn 15 p 6

†14 See 24f, 264ff.

Peirce: CP 6.6 Fn 1 p 7

†1 See 268f.

Peirce: CP 6.6 Fn 2 p 7

†2 See 350f, 5.142, 5.549ff.

Peirce: CP 6.6 Fn 3 p 7

†3 See 27, 91, 189f, 311ff, 582.

Peirce: CP 6.7 Fn 1 p 11

†1 *The Monist*, vol. 1, pp. 161-176 (1891); the first of a series of five articles. The second appears as ch. 2, the third as ch. 5, the fourth as ch. 9, and the fifth as ch. 11 of this book.

Peirce: CP 6.10 Fn 1 p 13

†1 Cf. 1.118, 5.47, 5.586, 5.591, 5.603.

Peirce: CP 6.11 Fn 1 p 14

†1 Cf. 575.

Peirce: CP 6.13 Fn 1 p 15

†1 Cf. 296ff, 1.103f, 1.173f.

Peirce: CP 6.13 Fn 2 p 15

†2 See 553f, for Peirce's earliest statement of this view.

Peirce: CP 6.14 Fn 3 p 15

†3 See his *First Principles*, part 2, ch. 18.

Peirce: CP 6.16 Fn 1 p 16

†1 See Lamarck, *Philosophie Zoologique*, vol. 1, ch. VII, Paris (1873).

Peirce: CP 6.16 Fn P1 p 16 Cross-Ref:††

†P1 The neo-Darwinian, Weismann, has shown that mortality would almost necessarily result from the action of the Darwinian principle. [See his *Essays upon Heredity*, I, "The Duration of Life" (1889).]

Peirce: CP 6.17 Fn 1 p 17

†1 See his *Catastrophism and the Evolution of Environment*, 1877.

Peirce: CP 6.18 Fn 2 p 17

†2 These categories are discussed in detail in vol. 1, bk. III.

Peirce: CP 6.18 Fn P1 p 18 Cross-Ref:††

†P1 A feeling may certainly be compound, but only in virtue of a perception which is not that feeling nor any feeling at all.

Peirce: CP 6.22 Fn 1 p 19

†1 Cf. 246ff, 280, 1.385ff.

- Peirce: CP 6.24 Fn 1 p 20
†1 Cf. 264ff, 4.551.
- Peirce: CP 6.26 Fn 1 p 21
†1 Cf. 82f.
- Peirce: CP 6.27 Fn 1 p 22
†1 Cf. 1.275f, 4.142ff.
- Peirce: CP 6.27 Fn 2 p 22
†2 See 4.145ff.
- Peirce: CP 6.27 Fn 1 p 23
†1 See 189f, 582, 1.362.
- Peirce: CP 6.28 Fn 2 p 23
†2 Cf. 4.143.
- Peirce: CP 6.29 Fn 1 p 24
†1 But see 82f.
- Peirce: CP 6.29 Fn 2 p 24
†2 242?
- Peirce: CP 6.29 Fn 3 p 24
†3 According to the Yale Observatory, the above figures "are affected by errors nearly one hundred times as great as good modern determinations."
- Peirce: CP 6.29 Fn 4 p 24
†4 Cf. 1.130ff, 1.401f.
- Peirce: CP 6.32 Fn 1 p 25
†1 Cf. vol. I, bk. 3.
- Peirce: CP 6.32 Fn 1 p 26
†1 Cf. 214ff.
- Peirce: CP 6.32 Fn 2 p 26
†2 Cf. 1.374ff.
- Peirce: CP 6.32 Fn 3 p 26
†3 Cf. 1.395ff.
- Peirce: CP 6.32 Fn 4 p 26
†4 Cf. 1.409.
- Peirce: CP 6.34 Fn 1 p 27
†1 See chs. 7 and 8.
- Peirce: CP 6.35 Fn 1 p 28
†1 *The Monist*, vol. 2, pp. 321-337 (1892); the second paper of a series of five. See note to ch. 1.
- Peirce: CP 6.35 Fn 2 p 28
†2 Ch. 1.
- Peirce: CP 6.36 Fn 3 p 28

†3 See H. Diels, *Die Fragmente der Vorsokratiker*, c. 55, A66.

Peirce: CP 6.36 Fn 4 p 28

†4 See Aetius, *Placita*, I, 12, 15.

Peirce: CP 6.36 Fn 1 p 29

†1 See Aristotle, *De Interpretatione*, 18b, 31: 19a, 7. *Ethica Nicomachea*, 1112a, 7-10.

Peirce: CP 6.36 Fn 2 p 29

†2 Epicurus, *Epistle*, III, 133-134.

Peirce: CP 6.36 Fn 3 p 29

†3 See Cleanthes, in Epictetus, *Manual*, ch. 53; Seneca, *De Providentia*, V, 8.

Peirce: CP 6.38 Fn 1 p 30

†1 Peirce gives a list of various uses of the term "necessity" in the *Century Dictionary* (1889) and in Baldwin's *Dictionary of Philosophy and Psychology*, vol. 2 (1901).

Peirce: CP 6.39 Fn 1 p 31

†1 The published paper has "compromised".

Peirce: CP 6.40 Fn 2 p 31

†2 See 2.508-513.

Peirce: CP 6.41 Fn 1 p 33

†1 Cf. 2.761ff.

Peirce: CP 6.41 Fn 1 p 34

†1 Cf. 2.683f.

Peirce: CP 6.42 Fn 2 p 34

†2 See 2.725ff.

Peirce: CP 6.45 Fn P1 p 35 Cross-Ref:††

†P1 *Continuous* is not exactly the right word, but I let it go to avoid a long and irrelevant discussion.

Peirce: CP 6.48 Fn 1 p 37

†1 See his *Examination of Sir William Hamilton's Philosophy*, ch. 16.

Peirce: CP 6.50 Fn 1 p 38

†1 Cf. 5.445; 5.508f.

Peirce: CP 6.51 Fn 2 p 38

†2 See page 3042, edition of 1889, for Peirce's definitions.

Peirce: CP 6.52 Fn 3 p 38

†3 Cf. 322.

Peirce: CP 6.52 Fn 1 p 39

†1 See 242n for a definition of this term.

Peirce: CP 6.61 Fn 1 p 43

†1 Cf. ch. 10; 4.611.

Peirce: CP 6.62 Fn 2 p 43

†2 The editors have been unable to discover any manuscript whose contents clearly answer to the foregoing description.

Peirce: CP 6.63 Fn 3 p 43

†3 See 606.

Peirce: CP 6.63 Fn 1 p 44

†1 See chs. 7 and 8.

Peirce: CP 6.65 Fn 1 p 45

†1 See Appendix A for Peirce's published reply to Dr. Paul Carus's criticisms of this paper.

Peirce: CP 6.66 Fn 1 p 46

†1 Lecture No. 4 on "Detached Ideas on Vitally Important Topics," 1898. See vol. 1, bk. IV, ch. 5; vol. 4, Preface; vol. 5, bk. III, ch. 6, §1 for the first three lectures.

Peirce: CP 6.66 Fn 2 p 46

†2 *Metaphysica*, 983b, 6-8.

Peirce: CP 6.66 Fn 3 p 46

†3 *Ibid.*, 983a, 24-34.

Peirce: CP 6.66 Fn 4 p 46

†4 "Retructive" is an alternative expression for "abductive," for which see vol. 2, *passim*.

Peirce: CP 6.67 Fn 1 p 47

†1 See his *System of Logic*, bk. III, ch. 5, §3.

Peirce: CP 6.67 Fn 2 p 47

†2 Cf. 1.427ff.

Peirce: CP 6.68 Fn 1 p 49

†1 ($t[2]-2t[1]+t[0]$) should be ($t[2]-t[0]$).

Peirce: CP 6.75 Fn 1 p 54

†1 See 4.188ff.

Peirce: CP 6.75 Fn 2 p 54

†2 Cf. 3.21, 3.33.

Peirce: CP 6.75 Fn 1 p 55

†1 See his *Pure Logic, and Other Minor Works*, London (1890), p. 189.

Peirce: CP 6.75 Fn 2 p 55

†2 Peirce has here a marginal note -- "Repeat with 'follows next after' [substituted] for *F*."

Peirce: CP 6.77 Fn 1 p 56

†1 See 3.546ff.

Peirce: CP 6.79 Fn 1 p 57

†1 *Traité du calcul des probabilités*, Paris, 1873.

- Peirce: CP 6.81 Fn 1 p 59
†1 Cf. 53f.
- Peirce: CP 6.81 Fn 1 p 60
†1 Four manuscript pages are missing here.
- Peirce: CP 6.82 Fn 2 p 60
†2 Cf. 1.501f.
- Peirce: CP 6.82 Fn 1 p 61
†1 See P. R. J. Boscovich, *Theoria philosophiae naturalis*, §7, 81, Vienna (1758).
- Peirce: CP 6.82 Fn 2 p 61
†2 See 86f.
- Peirce: CP 6.82 Fn 1 p 63
†1 Cf. 4.145.
- Peirce: CP 6.86 Fn 1 p 66
†1 See 1.652ff.
- Peirce: CP 6.87 Fn 2 p 66
†2 Cf. 1.489ff.
- Peirce: CP 6.88 Fn 1 p 67
†1 From the sixth Lowell Lecture of 1903.
- Peirce: CP 6.88 Fn 2 p 67
†2 Cf. 101.
- Peirce: CP 6.92 Fn 1 p 70
†1 *De Trinitate*, bk. III, ch. 10.
- Peirce: CP 6.93 Fn 2 p 70
†2 Boëthius, *De Consolatione Philosophiae*, Liber v, 1, cites Aristotle's *Physica*, II, 5, 196b.
- Peirce: CP 6.93 Fn 3 p 70
†3 *Consolations of Boëthius*, translated by H. R. James, pp. 229-230, London (1897).
- Peirce: CP 6.93 Fn 1 p 71
†1 Cf. 67.
- Peirce: CP 6.93 Fn 2 p 71
†2 See 1.549n and 2.428 for a definition of this term.
- Peirce: CP 6.93 Fn 3 p 71
†3 See his *Die Philosophie der Griechen*, 2^{te} Auf., pp. 252-55.
- Peirce: CP 6.94 Fn 4 p 71
†4 *Elements of Philosophy*, Pt. II, ch. 10.
- Peirce: CP 6.94 Fn 5 p 71
†5 See the 2^{me} Appendice to his *Théodicée*, §5.

- Peirce: CP 6.95 Fn 1 p 73
†1 Cf. 108.
- Peirce: CP 6.95 Fn 2 p 73
†2 Cf. 5.340.
- Peirce: CP 6.96 Fn 1 p 74
†1 *De Interpretatione*, 18a, 28-19b, 4.
- Peirce: CP 6.96 Fn 2 p 74
†2 See *Historische Beiträge zur Philosophie*, 2^{te} Bd., S. 168, Berlin (1855).
- Peirce: CP 6.98 Fn 1 p 75
†1 *Baldwin's Dictionary*, vol. 2, pp. 727-731, The Macmillan Co., New York (1902).
- Peirce: CP 6.98 Fn 1 p 76
†1 See Theodor Gomperz, *Herculanische Studien*, Pt. I (1865). Cf. 2.741, 2.761.
- Peirce: CP 6.98 Fn 2 p 76
†2 *System of Logic*, bk. III, ch. 4, §2.
- Peirce: CP 6.99 Fn 3 p 76
†3 Cf. 2.761f.
- Peirce: CP 6.99 Fn 1 p 77
†1 *Op. cit.*, bk. III, ch. 3, §3.
- Peirce: CP 6.99 Fn 2 p 77
†2 *Ibid.*, bk. III, ch. 4, §2.
- Peirce: CP 6.99 Fn 3 p 77
†3 *Ibid.*, bk. III, ch. 4, §1.
- Peirce: CP 6.100 Fn 1 p 78
†1 *Op. cit.*, bk. III, ch. 3, §1.
- Peirce: CP 6.101 Fn 1 p 83
†1 See *De rerum natura*, bk. II, 11. 284-293.
- Peirce: CP 6.102 Fn 1 p 86
†1 *The Monist*, vol. II, pp. 533-559 (1892); the third paper of a series.
- Peirce: CP 6.102 Fn 2 p 86
†2 See ch. 1.
- Peirce: CP 6.102 Fn 3 p 86
†3 See ch. 2.
- Peirce: CP 6.103 Fn 1 p 87
†1 See 31.
- Peirce: CP 6.103 Fn 2 p 87
†2 See 5.263, 5.311ff.

- Peirce: CP 6.105 Fn 1 p 88
†1 See 1.383, 5.7, 5.288f, 5.307.
- Peirce: CP 6.107 Fn 2 p 88
†2 Cf. 182.
- Peirce: CP 6.108 Fn 1 p 89
†1 Cf. 95.
- Peirce: CP 6.112 Fn 1 p 91
†1 See 4.118, 4.125.
- Peirce: CP 6.113 Fn 2 p 91
†2 See 3.565ff.
- Peirce: CP 6.113 Fn 3 p 91
†3 *Gesammelte Abhandlungen*, S. 139-140.
- Peirce: CP 6.114 Fn 4 p 91
†4 See 3.286f.
- Peirce: CP 6.114 Fn 5 p 91
†5 See his *Formal Logic* (1847), pp. 165ff.
- Peirce: CP 6.116 Fn 1 p 92
†1 Cf. 4.113ff.
- Peirce: CP 6.116 Fn 1 p 93
†1 See 3.258f.
- Peirce: CP 6.116 Fn 2 p 93
†2 See Fermat, *Opera Omnia* (Leipzig, 1911), vol. 1, §§340-351.
- Peirce: CP 6.116 Fn P1 p 94 Cross-Ref:††
†P1 This proposition is substantially the same as a theorem of Cantor [*Gesammelte Abhandlungen*, S. 115ff], though it is enunciated in a much more general form.
- Peirce: CP 6.117 Fn 1 p 94
†1 *Gesammelte Abhandlungen*, S. 278.
- Peirce: CP 6.117 Fn 1 p 95
†1 See 4.639.
- Peirce: CP 6.118 Fn 2 p 95
†2 *Op. cit.*, S. 289 (13) and (14).
- Peirce: CP 6.120 Fn 1 p 96
†1 See 174ff., 4.121, 4.642.
- Peirce: CP 6.120 Fn 2 p 96
†2 *Kritik der Reinen Vernunft*, A 169, B 211; cf. 168.
- Peirce: CP 6.121 Fn 3 p 96
†3 *Gesammelte Abhandlungen*, S. 194.

- Peirce: CP 6.122 Fn 1 p 97
†1 See his *Physica*, 227a, 10; *Metaphysica*, 1069a, 5f.
- Peirce: CP 6.122 Fn 1 p 98
†1 Cf. 4.121f.
- Peirce: CP 6.125 Fn 2 p 98
†2 See 3.568, 4.674.
- Peirce: CP 6.125 Fn 1 p 99
†1 See 3.563f, 4.113ff, 4.118n.
- Peirce: CP 6.125 Fn 2 p 99
†2 See his *Leçons sur les applications du calcul infinitésimal a la géométrie*, Paris (1826).
- Peirce: CP 6.125 Fn 3 p 99
†3 See his *Eléments de calcul infinitésimal*, Paris (1856).
- Peirce: CP 6.126 Fn 4 p 99
†4 See 4.125ff for a discussion of this concept.
- Peirce: CP 6.128 Fn 1 p 100
†1 See 135ff.
- Peirce: CP 6.132 Fn 1 p 101
†1 Cf. 197.
- Peirce: CP 6.132 Fn 2 p 101
†2 See 1.170, 4.172.
- Peirce: CP 6.132 Fn 3 p 101
†3 Cf. 197, 1.312.
- Peirce: CP 6.133 Fn 4 p 101
†4 Cf. 264.
- Peirce: CP 6.133 Fn 1 p 102
†1 See his *Principles of Psychology*, vol. 2, ch. 20 (1890).
- Peirce: CP 6.144 Fn 1 p 106
†1 Cf. 2.643, 2.711f, 3.154ff, 5.223.
- Peirce: CP 6.145 Fn 2 p 106
†2 See 2.514f, 2.632.
- Peirce: CP 6.149 Fn 1 p 108
†1 See 275, 1.390.
- Peirce: CP 6.157 Fn 1 p 111
†1 See 553, 613f, 5.536.
- Peirce: CP 6.159 Fn 1 p 112
†1 See 559.
- Peirce: CP 6.164 Fn 1 p 114
†1 164 is from the *Century Dictionary* (1889). 165-167 is a marginal note

dated September 18, 1903, in Peirce's personal copy of the *Dictionary*, now in the Treasure Room at the Widener Library, Cambridge, Mass.

Peirce: CP 6.164 Fn 2 p 114

†2 See 120f and 166.

Peirce: CP 6.164 Fn 3 p 114

†3 See 121.

Peirce: CP 6.166 Fn 1 p 115

†1 See 120-124.

Peirce: CP 6.168 Fn 2 p 115

†2 *Kritik der Reinen Vernunft*, A 169; B 211.

Peirce: CP 6.168 Fn 3 p 115

†3 See 1.434, 3.612, 5.448, 5.505.

Peirce: CP 6.168 Fn 1 p 116

†1 See 4.213ff, 4.639, 4.654.

Peirce: CP 6.169 Fn 1 p 117

†1 Baldwin's *Dictionary of Philosophy and Psychology*, vol. 2, p. 657, The Macmillan Co., New York (1902).

Peirce: CP 6.170 Fn 2 p 117

†2 See 103.

Peirce: CP 6.170 Fn 3 p 117

†3 See 5.3.

Peirce: CP 6.174 Fn 1 p 118

†1 From "The Bedrock beneath Pragmatism," 1906; continuing 4.561n.

Peirce: CP 6.174 Fn 2 p 118

†2 See 120f.

Peirce: CP 6.175 Fn 1 p 120

†1 *Paradoxien des Unendlichen*, §22, Leipzig (1851). Cf. 651.

Peirce: CP 6.175 Fn 2 p 120

†2 See 3.537.

Peirce: CP 6.177 Fn 1 p 122

†1 From "A Sketch of Logical Critic," c. 1911.

Peirce: CP 6.182 Fn 1 p 128

†1 In the Pragmatism Lectures of 1903? See 5.181, 5.202.

Peirce: CP 6.182 Fn 2 p 128

†2 See 4.229ff.

Peirce: CP 6.182 Fn 3 p 128

†3 See 168.

Peirce: CP 6.182 Fn 1 p 129

†1 A 169, B 211.

Peirce: CP 6.182 Fn 2 p 129

†2 See ch. 5.

Peirce: CP 6.182 Fn 1 p 130

†1 Cf. 1.36ff.

Peirce: CP 6.185 Fn 1 p 132

†1 The last of a proposed set of eight lectures, 1898. See 212n. Cf. also Preface and ch. 8.

Peirce: CP 6.188 Fn 1 p 133

†1 A symmetrical dyadic relation, for Peirce, does not enable one to distinguish the individuals related.

Peirce: CP 6.210 Fn 1 p 141

†1 See 1.274ff.

Peirce: CP 6.211 Fn 2 p 141

†2 Usually called *chorisis* by Peirce. See 4.222.

Peirce: CP 6.211 Fn 3 p 141

†3 Called *periphraxis* in Listing's *Census Räumlicher Complexe* (Göttingen, 1862), where it is described as "*Eigenschaft einer Fläche oder eines Raumes, wenn sie allseitig zusammenhängen u. einen Complex oder Complextheil rings umhüllen*" (p. 182). See also Peirce's definition of *Periphractic* in the Century Dictionary (1889). Peirce's terminology, which is largely that of Listing, has not been preserved in the development of topology. See Veblen, *Analysis Situs*, New York (1931); or Seifert and Threlfall, *Lehrbuch der Topologie*, Teubner (1934).

Peirce: CP 6.211 Fn 1 p 142

†1 Called *cyclosis* by Listing (*op. cit.*, Art. 9, p. 181), and paraphrased by him as "ringmässiger Zusammenhang, Anastomose." In modern terms, the connectivity minus one. See 4.225.

Peirce: CP 6.212 Fn 1 p 144

†1 Apparently not delivered; the series of lectures on "Detached Ideas on Vitally Important Topics" (see ch. 3) apparently having been given by request instead of the proposed eight lectures of which the present chapter and that on the continuity of space are parts.

Peirce: CP 6.214 Fn 1 p 147

†1 From "The Logic of Events," 1898, continuing 5.

Peirce: CP 6.218 Fn 1 p 148

†1 Cf. 5.79.

Peirce: CP 6.222 Fn 1 p 150

†1 "Notes for Eight Lectures," which Peirce specifically says are to follow the preceding.

Peirce: CP 6.222 Fn 2 p 150

†2 Cf. 1.308f.

Peirce: CP 6.225 Fn 1 p 151

†1 Cf. 376f.

Peirce: CP 6.238 Fn 1 p 155

†1 *The Monist*, vol. 3, pp. 1-22 (1892). The fourth paper of a series of five.

Peirce: CP 6.238 Fn 2 p 155

†2 Chapter 1.

Peirce: CP 6.238 Fn P1 p 155 Cross-Ref:††

†P1 I am rejoiced to find, since my last paper was printed, that a philosopher as subtle and profound as Dr. Edmund Montgomery ["The Dependence of Quality on Specific Energies," *Mind*, O. S., vol. 5, pp. 1-29] has long been arguing for the same element in the universe. Other world-renowned thinkers, as M. Renouvier [see *Essais de Critique Générale*, appendice ix] and M. Delboeuf [see "Déterminisme et Liberté," *Revue Philosophique*, vol. 13, pp. 453- 480, 608-638; vol. 14, pp. 158-189 (1882)], appear to share this opinion.

Peirce: CP 6.238 Fn 3 p 155

†3 Chapter 2.

Peirce: CP 6.238 Fn 4 p 155

†4 Chapter 5.

Peirce: CP 6.239 Fn 5 p 155

†5 See Chapter 11.

Peirce: CP 6.240 Fn 6 p 155

†6 See "The Size of Atoms," *Popular Lectures and Addresses*, p. 218, London (1889).

Peirce: CP 6.240 Fn 1 p 156

†1 *The Concepts and Theories of Modern Physics*, Chapter 7, New York (1882).

Peirce: CP 6.240 Fn 2 p 156

†2 *Ueber die Physikalische und Philosophische Atomenlehre*, Leipzig (1864).

Peirce: CP 6.240 Fn 3 p 156

†3 See *Complete Works*, vol. 1, pp. 471-493ff; vol. 2, pp. 1-22, 166-187, 208-221.

Peirce: CP 6.240 Fn 4 p 156

†4 See *Scientific Papers*, pp. 149ff, London (1884).

Peirce: CP 6.240 Fn 5 p 156

†5 See *Transactions of the Royal Society of Edinburgh*, vol. 20, p. 192.

Peirce: CP 6.240 Fn 6 p 156

†6 See Daniel Bernoulli, *Hydrodynamica*, Section X.

Peirce: CP 6.240 Fn 7 p 156

†7 See *Philosophical Transactions of the Royal Society*, vol. 165, p. 519 (1875).

Peirce: CP 6.241 Fn P1 p 156 Cross-Ref:††

†P1 SOLID -- A body which throughout its mass (and not merely at its

surface) resists for an indefinite time a sufficiently small force that tends to alter its equilibrium figure, always springing back into shape after the force is removed; a body possessing elasticity of figure. Every such body has limits of elasticity and if subjected to a strain exceeding these limits, it takes a set and does not return to its original shape on being let go. This property is called *plasticity*. The minimum energy required to give a set to a body of definite form and size measures its resilience. When the resilience of a body is small and masks its springiness the body is called *soft*. Even fluids transmit shearing forces if time be allowed and many substances will yield indefinitely to very small (but not indefinitely small) forces applied for great lengths of time. So solids that have received a small set will sometimes partially recover their figures after a long time. This property in fluids is called *viscosity*, in solids *after-effect* (German *nachwirkung*). The phenomenon is connected with a regrouping of the molecules, and indicates the essential difference between a solid and a liquid. In fluids diffusion is continually active, and in gases it produces phenomena of viscosity. In liquids it is not rapid enough to give rise to sensible viscosity, but the free motion of the molecules makes the body fluid, while the tendency of sets of molecules to continue for a while associated makes the fluidity imperfect. In solids, on the other hand (at least when not under strain), there is no diffusion, and the molecules are consequently in stationary motion or describing quasi-orbits. They thus become grouped in the mode in which they have least positional energy consistent with their kinetic energy. When this grouping is slightly disturbed it tends to restore itself; but when the disturbance is greater, some of the molecules will tend to return to their old places and others to move on to new situations, and this may give rise to a new permanent grouping, and exhibit the phenomenon of plasticity. But if not quite sufficient for this, disturbances of the molecular motions somewhat similar to the secular perturbations of the planets will result, from which there will be no restoration for a very long time. Solid bodies are very strongly cohesive, showing that the molecules attract one another on the whole; and they are generally capable of crystallization, showing that the attractions of the molecules are different in different directions. -- *Century Dictionary*, p. 5759, edition of 1889.

Peirce: CP 6.242 Fn P1 p 157 Cross-Ref:††

†P1 By a *vera causa*, in the logic of science, is meant a state of things known to exist in some cases and supposed to exist in other cases, because it would account for observed phenomena.

Peirce: CP 6.242 Fn 1 p 158

†1 See *Theoria Philosophicae Naturalis*, Sections 8ff, 81ff, 132ff.

Peirce: CP 6.242 Fn P1 p 158 Cross-Ref:††

†P1 Wiedemann, *Annalen*, 1887-89. [Vol. 30, p. 190; 31, pp. 141, 544; 32, p. 526; 34, p. 981; 35, pp. 76, 370; 36, p. 743; 38, p. 573.]

Peirce: CP 6.242 Fn P2 p 158 Cross-Ref:††

†P2 See Maxwell on Spherical Harmonics, in his *Electricity and Magnetism*. [Vol. 2, pt. I, ch. 9, p. 179.]

Peirce: CP 6.242 Fn 2 p 158

†2 This does not seem to have been done.

Peirce: CP 6.243 Fn P1 p 159 Cross-Ref:††

†P1 The word *system* has three peculiar meanings in mathematics. (A) It

means an orderly exposition of the truths of astronomy, and hence a theory of the motions of the stars, as the Ptolemaic *system*, the Copernican *system*. This is much like the sense in which we speak of the Calvinistic *system* of theology, the Kantian *system* of philosophy, etc. (B) It means the aggregate of the planets considered as all moving in somewhat the same way, as the solar *system*, and hence any aggregate of particles moving under mutual forces. (C) It means a number of forces acting simultaneously upon a number of particles.

Peirce: CP 6.243 Fn 1 p 160

†1 "Over de continuïtet van den gas en vloeistof-toestand," *Academisch Proefschrift*, Leiden (1873).

Peirce: CP 6.243 Fn P1 p 160 Cross-Ref:††

†P1 But, in fact, an inspection of these curves is sufficient to show that they are of a higher degree than the third. For they have the line $V=0$, or some line V a constant for an asymptote, while for small values of P , the values of $d^2P/(dV)^2$ are positive.

Peirce: CP 6.244 Fn 1 p 162

†1 See "Mémoire sur la compressibilité des gaz à des pressions élevées," *Annales de Chimie et de Physique*, vol. 19, pp. 345-385 (1880).

Peirce: CP 6.244 Fn 1 p 163

†1 "Ueber die Dissociation der in Wasser gelösten Stoffe," *Zeitschrift für Physikalische Chemie*, Bd. 1, pp. 631-648 (1887).

Peirce: CP 6.244 Fn P1 p 163 Cross-Ref:††

†P1 Anticipated by Clausius ["Ueber die Elektrizitätsleitung in Elektrolyten," *Poggendorff's Annalen*, Bd. 101, pp. 338-360], as long ago as 1857; and by Williamson ["Ueber die Theorie der Aetherbildung," *Annalen der Chemie und Pharmacie*, Bd. 77, pp. 37-49] in 1851.

Peirce: CP 6.246 Fn 1 p 164

†1 Cf. 1.351, 1.386ff.

Peirce: CP 6.246 Fn 1 p 165

†1 "On the Theory of the Analytical Forms called Trees," *American Journal of Mathematics*, vol. 4, pp. 266-268 (1881).

Peirce: CP 6.246 Fn 2 p 165

†2 See 278n.

Peirce: CP 6.249 Fn 1 p 166

†1 See 149, 275.

Peirce: CP 6.250 Fn 2 p 166

†2 *Lectures and Essays*, vol. 2, pp. 311-316 (1879).

Peirce: CP 6.259 Fn 1 p 169

†1 Cf. 23, 280f, 1.390.

Peirce: CP 6.261 Fn 1 p 171

†1 *Principles of Psychology*, vol. 1, p. 105 (1890).

Peirce: CP 6.261 Fn 2 p 171

†2 See 241n.

Peirce: CP 6.264 Fn 1 p 172

†1 Cf. 24f.

Peirce: CP 6.264 Fn P1 p 173 Cross-Ref:††

†P1 "Physiologically, . . . accommodation means the breaking up of a habit. . . . Psychologically, it means reviving consciousness." Baldwin, *Psychology*, Part III, ch. 1, §5.

Peirce: CP 6.265 Fn 1 p 174

†1 Cf. 5.311.

Peirce: CP 6.265 Fn 2 p 174

†2 Cf. 215ff, 613.

Peirce: CP 6.268 Fn 1 p 175

†1 Cf. 135ff, 155ff.

Peirce: CP 6.270 Fn 1 p 176

†1 See 5.313-14.

Peirce: CP 6.271 Fn 1 p 177

†1 Cf. 559, 587.

Peirce: CP 6.272 Fn 1 p 178

†1 c. 1893.

Peirce: CP 6.273 Fn 1 p 180

†1 See *System of Positive Polity*, vol. 1, pp. 421-422, Paris (1851).

Peirce: CP 6.274 Fn 1 p 181

†1 Cf. 68ff.

Peirce: CP 6.277 Fn 1 p 184

†1 Cf. 133ff.

Peirce: CP 6.278 Fn 1 p 185

†1 From ch. 3 (Div. II), of the *Grand Logic* (1893). Cf. 3.154ff.

Peirce: CP 6.278 Fn P1 p 185 Cross-Ref:††

†P1 The theory that there is but one protoplasm shall be considered in our chapter on fallacies. [That chapter is not being published.]

Peirce: CP 6.278 Fn 2 p 185

†2 See 246f, 1.386ff.

Peirce: CP 6.280 Fn 3 p 185

†3 Cf. 259f, 1.386f.

Peirce: CP 6.283 Fn P1 p 187 Cross-Ref:††

†P1 I assume for the percentage composition of albumen the following:

Oxygen 21.5 = 1.34 times at. wt.

Hydrogen 7 = 7. times at. wt.

Nitrogen 16.5 = 1.18 times at. wt.

Carbon 54 = 4.5 times at. wt.

Sulphur 1 = 0.03 times at. wt.

— —
100 14.05

I assume the solution having an osmotic pressure of 1/2 at. to be of 12.5 p. c. I assume the Sp. Gr. Albumen to be 1.25. Then by the laws of osmotic pressure, there would be 980 atoms to a molecule.

Peirce: CP 6.286 Fn 1 p 188
†1 Cf. 5.479.

Peirce: CP 6.286 Fn 1 p 189
†1 *Proverbs*, 4.23.

Peirce: CP 6.286 Fn 2 p 189
†2 Cf. 2.711.

Peirce: CP 6.287 Fn 1 p 190
†1 *The Monist*, vol. 3, pp. 176-200 (1893); the last paper of a series of five.
See note to ch. 1.

Peirce: CP 6.287 Fn 2 p 190
†2 See H. Diels, *Fragmente der Vorsokratiker*, vol. 1, 21B.

Peirce: CP 6.287 Fn 1 p 191
†1 p. 442.

Peirce: CP 6.289 Fn 1 p 192
†1 See 270.

Peirce: CP 6.289 Fn 2 p 192
†2 See ch. 5.

Peirce: CP 6.291 Fn 1 p 193
†1 Simon Newcomb, *Principles of Political Economy*, New York (1886).

Peirce: CP 6.291 Fn 2 p 193
†2 *Ibid.*, p. 534.

Peirce: CP 6.291 Fn P1 p 194 Cross-Ref:††
†P1 How can a writer have any respect for science, as such, who is capable of confounding with the scientific propositions of political economy, which have nothing to say concerning what is "beneficent," such brummagem generalizations as this?

Peirce: CP 6.292 Fn 1 p 196
†1 See especially Remark G.

Peirce: CP 6.296 Fn 1 p 197

†1 Cf. 13ff.

Peirce: CP 6.297 Fn 2 p 197

†2 See "Mr. Charles S. Peirce's Onslaught on the Doctrine of Necessity," *The Monist*, vol. 2, p. 576.

Peirce: CP 6.297 Fn 3 p 197

†3 Bruxelles, 1846. Translation by O. G. Downes. London, 1849.

Peirce: CP 6.297 Fn 1 p 198

†1 "Quetelet on Probabilities," *Edinburgh Review*, vol. 42, pp. 1-57 (1850).

Peirce: CP 6.297 Fn 2 p 198

†2 "Ueber die Art der Bewegung welche wir Wärme nennen," *Poggendorff's Annalen*, Bd. 100, S. 365 (1857).

Peirce: CP 6.297 Fn 3 p 198

†3 "Grundzüge einer Theorie der Gase," *Poggendorff's Annalen*, Bd. 99, S. 315 (1856).

Peirce: CP 6.297 Fn 4 p 198

†4 "Illustrations of the Dynamical Theory of Gases," *Philos. Magazine* IV, p. 22 (1860). Reprinted in *Collected Papers*, vol. 1, p. 377.

Peirce: CP 6.297 Fn 5 p 198

†5 See 71.

Peirce: CP 6.297 Fn 6 p 198

†6 *Ueber die Erhaltung der Kraft*. Introduction to a series of lectures delivered at Karlsruhe 1862-63. Translated in *Popular Scientific Lectures*, vol. 1, pp. 316-362, New York, (1885).

Peirce: CP 6.297 Fn 7 p 198

†7 "Ueber die bewegende Kraft der Wärme," *Poggendorff's Annalen*, Bd. 79, S. 368.

Peirce: CP 6.297 Fn 8 p 198

†8 *Transactions of the Royal Society of Edinburgh*, vol. 20, p. 192.

Peirce: CP 6.297 Fn 1 p 199

†1 Cf. 5.64.

Peirce: CP 6.298 Fn 1 p 200

†1 In his *Mechanisch-physiologische Theorie der Abstammungslehre*. Einleitung, S. 14ff. München and Leipzig (1884).

Peirce: CP 6.298 Fn 2 p 200

†2 *Entwicklungsgeschichte des Menschen und der Höheren Thiere*, Einleitung §1, Leipzig (1879).

Peirce: CP 6.298 Fn 3 p 200

†3 See *Essays on Heredity*, vol. 1, essay 2.

Peirce: CP 6.298 Fn P1 p 200 Cross-Ref:††

†P1 I am happy to find that Dr. Carus ["The Soul of Man," *Open Court*, 1891, p. 215], too, ranks Weismann among the opponents of Darwin, notwithstanding his

flying that flag.

Peirce: CP 6.299 Fn 4 p 200

†4 *Philosophie Zoologique*, Pt. I, ch. 7, Paris (1873).

Peirce: CP 6.299 Fn 1 p 201

†1 See 261ff.

Peirce: CP 6.301 Fn 1 p 202

†1 See 250ff.

Peirce: CP 6.305 Fn 1 p 204

†1 An engine "invented" in 1874 by J. E. W. Keely, supposed to produce power by responding to the intermolecular vibrations of the ether.

Peirce: CP 6.311 Fn 1 p 208

†1 See Mark 3, 29.

Peirce: CP 6.311 Fn 2 p 208

†2 See Mark 9, 48.

Peirce: CP 6.311 Fn 3 p 208

†3 See Isaiah 66, 24.

Peirce: CP 6.311 Fn 1 p 209

†1 *Ecclesiastical History*, vol. 8, p. 2.

Peirce: CP 6.311 Fn 2 p 209

†2 See "Of the False Wisdom of Philosophers." *The Divine Institutes*, Bk. III.

Peirce: CP 6.311 Fn P1 p 209 Cross-Ref:††

†P1 See Draper's *History of Intellectual Development*, ch. 10.

Peirce: CP 6.311 Fn 3 p 209

†3 See John of Salisbury, *Policraticus*, ii, 26; viii, 19.

Peirce: CP 6.312 Fn 1 p 210

†1 *Geschichte der Logik in Abendlande*, Leipzig (1867), Dritter Band, 17 Abschn., S. 2.

Peirce: CP 6.313 Fn 1 p 211

†1 See 3.396ff, 3.506ff, 3.641.

Peirce: CP 6.316 Fn 1 p 213

†1 "Recherches sur les mouvements de la planète Herschel, dite Uranus." *Connaissances des temps*, 1849.

Peirce: CP 6.316 Fn 2 p 213

†2 See *Nautical Almanac*, 1851, p. 3.

Peirce: CP 6.316 Fn 3 p 213

†3 *Transactions of the Royal Society of Edinburgh*, vol. 20, p. 192.

Peirce: CP 6.316 Fn 4 p 213

†4 "Ueber die bewegende Kraft der Wärme," *Poggendorff's Annalen*, Bd. 79, S. 368.

- Peirce: CP 6.316 Fn P1 p 214 Cross-Ref:††
 †P1 Thomson, himself, in his article *Heat* in the *Encyclopedia Britannica* [edition of 1875-89] never once mentions the name of Clausius.
- Peirce: CP 6.316 Fn 1 p 214
 †1 *Daniel Bernoulli, Hydrodynamica*, Section X (1738).
- Peirce: CP 6.316 Fn 2 p 214
 †2 Mendeléeff, Lothar Meyer, and J. A. R. Newlands.
- Peirce: CP 6.316 Fn 3 p 214
 †3 W. T. G. Morton, C. T. Jackson, J. C. Warren.
- Peirce: CP 6.318 Fn 1 p 216
 †1 §§1-6 are from "Some Amazing Mazes, Fourth Curiosity" (c. 1909). See 4.647.
- Peirce: CP 6.318 Fn 2 p 216
 †2 See 3.466f, 3.571.
- Peirce: CP 6.318 Fn 3 p 216
 †3 See 3.573f.
- Peirce: CP 6.318 Fn 1 p 217
 †1 See 2.357.
- Peirce: CP 6.320 Fn 1 p 218
 †1 Cf. 2.590f, 4.355.
- Peirce: CP 6.320 Fn 2 p 218
 †2 *Anal. priora* I, 24b, 28.
- Peirce: CP 6.320 Fn 1 p 219
 †1 Cf. 3.575 (6), 3.595.
- Peirce: CP 6.320 Fn 2 p 219
 †2 Cf. 3.575 (8), 3.597.
- Peirce: CP 6.321 Fn 3 p 219
 †3 *Kritik der Reinen Vernunft*, A 303, B 360.
- Peirce: CP 6.321 Fn 4 p 219
 †4 Cf. 2.496.
- Peirce: CP 6.321 Fn 5 p 219
 †5 *I.e.*, $\Sigma[w]\pi[c] cAw \rightarrow \pi[c] \Sigma[w] cAw$.
- Peirce: CP 6.321 Fn 1 p 220
 †1 See 3.63, 3.144, 3.421, 3.483f.
- Peirce: CP 6.322 Fn 1 p 221
 †1 See *OEuvres de Pasteur*, vol. 1, pp. 83ff, Paris (1922).
- Peirce: CP 6.322 Fn 2 p 221
 †2 See 68.

Peirce: CP 6.322 Fn 3 p 221
†3 Cf. 1.473, 2.274.

Peirce: CP 6.322 Fn 4 p 221
†4 Peirce died in 1914, about five years after he wrote this paper, the last detailed study he seems to have made.

Peirce: CP 6.326 Fn 1 p 225
†1 Cf. 3.64f, 3.527.

Peirce: CP 6.326 Fn P1 p 225 Cross-Ref:††
†P1 I know no question of metaphysics so pressing as this of whether or not there be a maximum multitude capable of linear arrangement. There appears to be a proof that there is none; but owing to the extreme liability to fallacy of reasoning concerning this question, I have some doubt of its validity. I found what appeared to be a proof to the same effect. After careful and repeated scrutiny, I discovered no fallacy in it; and there was no great intricacy in the proof. I submitted it to two persons, each of whom had devoted so much study to the general subject that I had a right to presume that not a dozen persons living were as competent in the premisses as he. Neither could discover any fallacy. Yet, I subsequently found, myself, that my proof was fallacious. Such an experience naturally renders me very cautious. [Cf. 3.549n. 4.218, 4.652.]

Peirce: CP 6.327 Fn 1 p 226
†1 Cf. 5.405f, 5.430f.

Peirce: CP 6.328 Fn 1 p 229
†1 See vol. 2, bk. II, ch. 1.

Peirce: CP 6.329 Fn 1 p 230
†1 Cf. 2.269, 2.756f.

Peirce: CP 6.331 Fn 1 p 231
†1 Cf. 1.471ff, 1.536ff.

Peirce: CP 6.335 Fn 1 p 232
†1 *Analytica Posteriora*, II, 100a, 18.

Peirce: CP 6.335 Fn 2 p 232
†2 *Categoriae*, 2b, 5.

Peirce: CP 6.335 Fn 1 p 233
†1 *Analytica Posteriora*, II, 91b, 34; *Categoriae*, 1b, 6.

Peirce: CP 6.335 Fn P1 p 233 Cross-Ref:††
†P1 But I am here expressing myself upon points which I have not reexamined for a great many years. I might hold different opinions, if I were to revise my judgments.

Peirce: CP 6.338 Fn 2 p 233
†2 See 2.247, 2.274ff.

Peirce: CP 6.338 Fn 2 p 233
†2 See 2.247, 2.274ff.

Peirce: CP 6.338 Fn 2 p 233
†2 See 2.247, 2.274ff.

Peirce: CP 6.339 Fn 1 p 234
†1 *Principles of Human Knowledge*, §3.

Peirce: CP 6.339 Fn 2 p 234
†2 B 274-279.

Peirce: CP 6.340 Fn 3 p 234
†3 *Anal. Post.*, I, 31, 87b, 27-32; II, 19, 100a, 14-64.

Peirce: CP 6.340 Fn 4 p 234
†4 Cf. 4.539.

Peirce: CP 6.340 Fn 5 p 234
†5 *Categoriæ*, 1b, 3.

Peirce: CP 6.344 Fn 1 p 235
†1 Cf. 5.310ff.

Peirce: CP 6.344 Fn 2 p 235
†2 See 2.251, 2.310.

Peirce: CP 6.347 Fn 1 p 237
†1 See 2.230, 2.274.

Peirce: CP 6.349 Fn 2 p 237
†2 §§7 and 8 form a digression in ch. 4 of the *Minute Logic* (1902-3). The Velian is the stranger of Plato's *Sophist*, a dialogue which Peirce characterizes in the preceding, unpublished portion of the manuscript (see 1.584n) as being "purely a logical dialogue" with "all Hegel's faults and more than a glimmer of Hegel's merit." The present section is part of an attempt to give the Velian stranger "a little dose of his own cathartic."

Peirce: CP 6.349 Fn 1 p 238
†1 "Cathode Rays," *Philosophical Magazine*, October 1897, pp. 293-310; "On Bodies Smaller than Atoms," *Popular Science Monthly*, August 1901, pp. 323-337.

Peirce: CP 6.351 Fn 1 p 241
†1 See 2.536.

Peirce: CP 6.351 Fn 2 p 241
†2 Cf. 2.487n1, 2.550, 2.596f.

Peirce: CP 6.352 Fn 1 p 243
†1 See 2.412.

Peirce: CP 6.352 Fn 1 p 245
†1 Cf. 5.505.

Peirce: CP 6.353 Fn 1 p 246
†1 From Baldwin's *Dictionary of Philosophy and Psychology*, The Macmillan Co., New York, vol. 2, pp. 50-55 (1902).

- Peirce: CP 6.355 Fn 2 p 246
†2 See *Metaphysica*, {A}3, 983b, 7.
- Peirce: CP 6.356 Fn 1 p 247
†1 *Metaphysica*, {Theta}, 8, 1050a, 15.
- Peirce: CP 6.356 Fn 2 p 247
†2 *De Anima*, 415a, 25.
- Peirce: CP 6.358 Fn 1 p 248
†1 *Metaphysica*, {É}, 1, 1042a31.
- Peirce: CP 6.359 Fn 1 p 249
†1 *Summa Theologica*, I, II, 55, 4, c.
- Peirce: CP 6.359 Fn 2 p 249
†2 *Summa Theologica*, I, 75, 4, c.
- Peirce: CP 6.360 Fn 1 p 250
†1 *Ibid.*, I, II, 50, 2, c.
- Peirce: CP 6.360 Fn 2 p 250
†2 *Metaphysica*, {D}, 10, 1018b 5. K., 3, 1061a 23.
- Peirce: CP 6.360 Fn 1 p 251
†1 *Metaphysica*, {E}, 1, 1025b 30, 1026a 27.
- Peirce: CP 6.360 Fn 2 p 251
†2 *Novum Organum*, II, 5.
- Peirce: CP 6.360 Fn 1 p 252
†1 *Summa Theologica*, I, 54, 3, ad. 2.
- Peirce: CP 6.360 Fn 1 p 253
†1 *Ibid.*, I, 61, 1.
- Peirce: CP 6.364 Fn 1 p 257
†1 Baldwin's *Dictionary of Philosophy and Psychology*, vol. 2, pp. 313-315.
Cf. 3.527, 4.65ff.
- Peirce: CP 6.366 Fn 1 p 258
†1 *Analytica Priora*, I, 13, 32a, 18.
- Peirce: CP 6.366 Fn 2 p 258
†2 *Metaphysica*, {D} 12, 1019a, 32.
- Peirce: CP 6.366 Fn 3 p 258
†3 *Monadology*, 33.
- Peirce: CP 6.366 Fn 4 p 258
†4 *Kritik der Reinen Vernunft*, A151, B191.
- Peirce: CP 6.372 Fn 1 p 261
†1 Baldwin's *Dictionary of Philosophy and Psychology*, vol. 2, pp. 763-764.
- Peirce: CP 6.373 Fn 1 p 262

- †1 *Ibid.*, vol. 2, pp. 734-736.
- Peirce: CP 6.376 Fn 1 p 263
 †1 *Kritik der Reinen Vernunft*, A118, B131.
- Peirce: CP 6.381 Fn 1 p 266
 †1 Baldwin's *Dictionary of Philosophy and Psychology*, vol. 2, pp. 814-815.
- Peirce: CP 6.381 Fn 2 p 266
 †2 This paragraph is signed by Peirce, J. M. Baldwin, and G. F. Stout.
- Peirce: CP 6.382 Fn 1 p 267
 †1 Cf. 3.537n.
- Peirce: CP 6.383 Fn 1 p 268
 †1 *Lectures on Metaphysics*, XXXVII, vol. 2, p. 340, Edinburgh (1858).
- Peirce: CP 6.384 Fn 1 p 269
 †1 Baldwin's *Dictionary of Philosophy and Psychology*, vol. 1, p. 600 (1901).
- Peirce: CP 6.384 Fn 1 p 270
 †1 See 2.407ff.
- Peirce: CP 6.385 Fn 1 p 271
 †1 *Ibid.*, vol. 2, pp. 281-282.
- Peirce: CP 6.385 Fn 2 p 271
 †2 187A.
- Peirce: CP 6.385 Fn 3 p 271
 †3 *Metaphysica*, {D}, c. XVIII; 1022a 14-36.
- Peirce: CP 6.386 Fn 1 p 273
 †1 Baldwin's *Dictionary of Philosophy and Psychology*, vol. 2, pp. 342-343.
- Peirce: CP 6.388 Fn 1 p 274
 †1 *Anal. Post.*, I, 2, 72a 1.
- Peirce: CP 6.388 Fn 2 p 274
 †2 *Metaphysica*, {D}, 11, 1018b 21.
- Peirce: CP 6.389 Fn 3 p 274
 †3 *Metaphysica* {A}, 8, 989a 16; *Categoriae*, 12, 14b 3.
- Peirce: CP 6.390 Fn 1 p 275
 †1 Baldwin's *Dictionary of Philosophy and Psychology*, vol 2, pp. 281-282.
- Peirce: CP 6.393 Fn 1 p 276
 †1 *Ibid.*, vol. 2, pp. 616-617.
- Peirce: CP 6.395 Fn 1 p 283
 †1 *Popular Science Monthly*, vol. 13, pp. 203-217 (1878); the fifth of a series of six articles called "Illustrations of the Logic of Science." For the first and second articles see vol. 5, bk. II, chs. 4 and 5; for the third, fourth, and sixth see vol. 2, bk. III, chs. 6, 7, and 5.
- Peirce: CP 6.396 Fn 1 p 284

†1 Cf. *La Religion* (1869), bk. II, ch. 5.

Peirce: CP 6.399 Fn 1 p 285

†1 *Works*, vol. 1, p. 346, London (1820).

Peirce: CP 6.400 Fn P1 p 286 Cross-Ref:††

†P1 For the present purpose, the negative of a character is to be considered as much a character as the positive, for a uniformity may either be affirmative or negative. I do not say that no distinction can be drawn between positive and negative uniformities.

Peirce: CP 6.400 Fn P2 p 286 Cross-Ref:††

†P2 There being 5 simple characters, with their negatives, they could be compounded in various ways so as to make 241 characters in all, without counting the characters *existence* and *non-existence*, which make up 243 or 3⁵.

Peirce: CP 6.401 Fn 1 p 287

†1 *Op. cit.*, p. 347.

Peirce: CP 6.402 Fn P1 p 287 Cross-Ref:††

†P1 This principle was, I believe, first stated by Mr. De Morgan. [See his "On the Syllogism, no. V, etc." *Transactions of the Cambridge Philosophical Society*, vol. 10, pp. 456, 467 (1864); *Formal Logic*, p. 39, London (1847).]

Peirce: CP 6.408 Fn 1 p 290

†1 See vol. 2, bk. III, ch. 7.

Peirce: CP 6.410 Fn 1 p 291

†1 *A System of Logic*, bk. III, ch. 3, §1.

Peirce: CP 6.411 Fn 2 p 291

†2 Cf. 2.766f.

Peirce: CP 6.412 Fn 1 p 292

†1 *Ibid.*, bk. III, ch. 3, §3.

Peirce: CP 6.416 Fn 1 p 294

†1 *A New Theory of Vision*, Sections 2 and 3.

Peirce: CP 6.416 Fn 2 p 294

†2 Cf. 5.223.

Peirce: CP 6.418 Fn 1 p 295

†1 Cf. 491, 1.118, 5.47, 5.586, 5.591.

Peirce: CP 6.419 Fn 1 p 296

†1 5.405ff.

Peirce: CP 6.428 Fn 1 p 302

†1 *The Open Court*, vol. 7, pp. 3559-60 (1893).

Peirce: CP 6.435 Fn 1 p 305

†1 *The Open Court*, vol. 7, pp. 3743-45 (1893).

Peirce: CP 6.440 Fn P1 p 306 Cross-Ref:††

†P1 Edited with translation and notes by Roswell D. Hitchcock and Francis

Brown, New York, Scribners (1884). Also, by Philip Schaff, 3d edition, New York, Funk and Wagnalls (1890).

Peirce: CP 6.446 Fn 1 p 308

†1 Cf. 511ff, 522ff.

Peirce: CP 6.449 Fn 1 p 309

†1 From "Religion and Politics," c. 1895; apparently a proposed letter to a newspaper.

Peirce: CP 6.452 Fn 1 p 311

†1 *Hibbert Journal*, vol. 7, pp. 90-112 (1908).

Peirce: CP 6.454 Fn 2 p 311

†2 See his *Thought and Things*, p. 261, London (1906).

Peirce: CP 6.455 Fn 1 p 312

†1 Cf. 4.545ff.

Peirce: CP 6.456 Fn 2 p 312

†2 Cf. 2.266ff, 3.160.

Peirce: CP 6.460 Fn 1 p 315

†1 "It appears to me that this mystery is considered insoluble for the very reason which should cause it to be regarded as easy of solution. I mean the outré character of its features."

Peirce: CP 6.462 Fn 1 p 316

†1 Cf. 1.333.

Peirce: CP 6.467 Fn 1 p 319

†1 See 177ff.

Peirce: CP 6.467 Fn 2 p 319

†2 See 5.397ff.

Peirce: CP 6.469 Fn 1 p 320

†1 Cf. 2.622ff.

Peirce: CP 6.469 Fn 1 p 321

†1 Or Abduction. See 2.708ff, 2.755 and vol. 5, bk. I, ch. 7.

Peirce: CP 6.471 Fn 1 p 322

†1 Cf. 2.267.

Peirce: CP 6.473 Fn 2 p 322

†2 Cf. 2.756ff.

Peirce: CP 6.473 Fn 1 p 323

†1 Cf. 2.758f.

Peirce: CP 6.474 Fn 2 p 323

†2 *Kritik der Reinen Vernunft*, A154-158; B193-197.

Peirce: CP 6.475 Fn 3 p 323

†3 See 2.93.

Peirce: CP 6.476 Fn 1 p 324

†1 Jacques Babinet (1794-1872), a popular writer on hydrodynamics and many other scientific subjects.

Peirce: CP 6.477 Fn 1 p 325

†1 See "Dialogues Concerning the Two Great Systems of the World," in *Mathematical Collections and Translations of Thomas Salisbury*, vol. 1, p. 301, London (1661).

Peirce: CP 6.478 Fn 1 p 326 Cross-Ref: ††

†1 In a letter to William James, November 17, 1908, Peirce says, "I had never contemplated the possibility of the last section's being published."

Peirce: CP 6.481 Fn 1 p 328

†1 Cf. 3.457, 5.388ff.

Peirce: CP 6.482 Fn 2 p 328

†2 See 5.12f.

Peirce: CP 6.482 Fn 1 p 329

†1 See note to vol. 5, bk. II, Paper No. IV.

Peirce: CP 6.482 Fn 2 p 329

†2 See *The Will to Believe and Other Essays in Popular Philosophy* (1897).

Peirce: CP 6.482 Fn 3 p 329

†3 See "What Pragmatism is Like," *Popular Science Monthly*, vol. 71, p. 351 (1907).

Peirce: CP 6.482 Fn 4 p 329

†4 See 5.414.

Peirce: CP 6.482 Fn 5 p 329

†5 See 5.1f.

Peirce: CP 6.482 Fn 6 p 329

†6 See 1.106n, 5.13n.

Peirce: CP 6.483 Fn 7 p 329

†7 See 486.

Peirce: CP 6.485 Fn 1 p 331

†1 See 5.375.

Peirce: CP 6.485 Fn 2 p 331

†2 See 5.365ff.

Peirce: CP 6.485 Fn 3 p 331

†3 See 5.552, 5.555f.

Peirce: CP 6.485 Fn 1 p 332

†1 F. C. S. Schiller, *Humanism*, p. 314, note, London (1903); *Studies in Humanism*, p. 295, London (1907).

Peirce: CP 6.485 Fn 2 p 332

†2 William James, *Pragmatism*, p. 59ff, New York (1908).

- Peirce: CP 6.485 Fn 3 p 332
†3 William James, *The Will to Believe*, p. 11, New York (1899).
- Peirce: CP 6.486 Fn 4 p 332
†4 c. 1910; 491 is from an alternative draft.
- Peirce: CP 6.490 Fn 1 p 335
†1 See 482 and p. v of the Preface to vol. 5.
- Peirce: CP 6.490 Fn 2 p 335
†2 "Philosophical Conceptions and Practical Results," *The University of California Chronicle*, pp. 24ff (1898); reprinted in *Collected Essays and Reviews*, pp. 406-437 (1920).
- Peirce: CP 6.490 Fn 3 p 335
†3 "What Pragmatism is Like," *Popular Science Monthly*, p. 351, vol. 71 (1907).
- Peirce: CP 6.490 Fn 4 p 335
†4 See 5.414.
- Peirce: CP 6.491 Fn 1 p 338
†1 See 418.
- Peirce: CP 6.492 Fn 2 p 338
†2 From an unpaginated fragment, c. 1896.
- Peirce: CP 6.492 Fn 3 p 338
†3 See 5.255f.
- Peirce: CP 6.494 Fn 1 p 340
†1 c. 1906.
- Peirce: CP 6.494 Fn 2 p 340
†2 See Part IV *ad init.*
- Peirce: CP 6.495 Fn 3 p 340
†3 See *e.g.* 336, 5.503.
- Peirce: CP 6.495 Fn 1 p 341
†1 *Physicorum Liber*, I, 1, I.
- Peirce: CP 6.495 Fn 2 p 341
†2 See *Sententiarum Libri*, III, Distinctio 34.
- Peirce: CP 6.496 Fn 3 p 341
†3 See 1.549n.
- Peirce: CP 6.496 Fn 4 p 341
†4 See 5.448.
- Peirce: CP 6.497 Fn 1 p 342
†1 *De civitate Dei*, XI, 26.
- Peirce: CP 6.498 Fn 1 p 343
†1 See *e.g.* 5.265, 5.416ff, 5.438ff.

- Peirce: CP 6.498 Fn 2 p 343
†2 See 17, 33, 296ff, 1.104.
- Peirce: CP 6.500 Fn 1 p 345
†1 Cf. 418, 491.
- Peirce: CP 6.504 Fn 1 p 347
†1 A. Caldecott, *The Philosophy of Religion in England and America*, p. 9, Macmillan, New York (1901).
- Peirce: CP 6.505 Fn 2 p 347
†2 See chs. 1, 2, 5, 9, 11, in bk. I of the present volume.
- Peirce: CP 6.505 Fn 3 p 347
†3 See *La Nouvelle Monadologie*, Art. CXXX, p. 163, Paris (1899).
- Peirce: CP 6.506 Fn 1 p 348
†1 Cf. 189ff.
- Peirce: CP 6.507 Fn 2 p 348
†2 See 287.
- Peirce: CP 6.508 Fn 1 p 349
†1 Cf. 4.67, 4.583.
- Peirce: CP 6.511 Fn 2 p 349
†2 *Contra Faustum*, bk. XXVI, ch. 3.
- Peirce: CP 6.511 Fn 3 p 349
†3 *Summa Theologica*, I, 105, 6.
- Peirce: CP 6.511 Fn 1 p 350
†1 *The Analogy*, pt. I, ch. 7.
- Peirce: CP 6.511 Fn 2 p 350
†2 See 47ff, 264, 302.
- Peirce: CP 6.511 Fn 3 p 350
†3 *Essais de critique générale*, appendice IX (1854-64).
- Peirce: CP 6.511 Fn 4 p 350
†4 *La Liberté et le déterminisme*, Paris (1872).
- Peirce: CP 6.511 Fn 5 p 350
†5 "Déterminisme et liberté," *Revue Philosophique*, vol. 13, pp. 433-480, 608-638; vol. 14, pp. 158-189 (1882).
- Peirce: CP 6.512 Fn 6 p 350
†6 *Inquiry*, Section X.
- Peirce: CP 6.512 Fn 7 p 350
†7 See the next chapter for a detailed consideration of this question.
- Peirce: CP 6.512 Fn 8 p 350
†8 See the long discussion of historical evidence in vol. 8.

Peirce: CP 6.513 Fn 9 p 350

†9 See e.g. 1.617.

Peirce: CP 6.513 Fn 10 p 350

†10 *Discourses on the Miracles of Our Savior*, etc., London (1727-29).

Peirce: CP 6.513 Fn 1 p 351

†1 *Commentaries* on the Gospels of Matthew and John. See e.g. on John IV, 46; also *in Exod. Hom.* v. 7.

Peirce: CP 6.514 Fn 2 p 351

†2 See *Researches in the Phenomena of Spiritualism*, London (1874).

Peirce: CP 6.514 Fn 3 p 351

†3 For Rayleigh's attitude see A. C. Doyle, *History of Spiritualism*, vol. 1, p. 181, London (1926).

Peirce: CP 6.514 Fn 4 p 351

†4 "A Further Record of Observations of Certain Phenomena of Trance," *Proceedings of the S. P. R.*, vol. 13, pp. 285-582 (1897-98).

Peirce: CP 6.515 Fn 1 p 352

†1 "The Prayer for the Sick, Hints Towards a Serious Attempt to Estimate Its Value," *Contemporary Review*, vol. 20 (1872).

Peirce: CP 6.515 Fn 2 p 352

†2 I Kings XVIII.

Peirce: CP 6.517 Fn 1 p 353

†1 *Principles of Political Economy*, New York (1886).

Peirce: CP 6.517 Fn 2 p 353

†2 See 290ff.

Peirce: CP 6.517 Fn 3 p 353

†3 See Peirce's review of Pearson's *Grammar of Science* in *Popular Science Monthly*, vol. 58, pp. 296-306 (1901); to be reprinted in vol. 9.

Peirce: CP 6.519 Fn 1 p 354

†1 Cf. chapter 6.

Peirce: CP 6.520 Fn 2 p 354

†2 The dots, indicating omissions, occur in the manuscript.

Peirce: CP 6.521 Fn 1 p 355

†1 The manuscript ends at this point.

Peirce: CP 6.522 Fn 1 p 356

†1 c. 1901

Peirce: CP 6.522 Fn 2 p 356

†2 Cf. 2.186ff.

Peirce: CP 6.526 Fn 1 p 358

†1 Cf. 2.708ff, 2.786, 5.171.

- Peirce: CP 6.526 Fn 1 p 359
†1 Cf. 2.735ff.
- Peirce: CP 6.526 Fn 2 p 359
†2 Cf. 2.759, 2.772.
- Peirce: CP 6.530 Fn 1 p 361
†1 Cf. 1.122ff, 5.600ff.
- Peirce: CP 6.531 Fn 1 p 362
†1 See 418, 491.
- Peirce: CP 6.532 Fn 2 p 362
†2 Cf. 2.740.
- Peirce: CP 6.534 Fn 1 p 363
†1 See 2.101, 2.662f, 2.777.
- Peirce: CP 6.536 Fn 1 p 364
†1 See vol. 8.
- Peirce: CP 6.536 Fn 2 p 364
†2 See 513.
- Peirce: CP 6.537 Fn 3 p 364
†3 *Inquiry*, Section X.
- Peirce: CP 6.537 Fn 4 p 364
†4 *Summa Theol.*, I, *qu.* 110, *art.* 4, *ad.* 2.
- Peirce: CP 6.539 Fn 1 p 365
†1 *Inquiry*, X, I, 90.
- Peirce: CP 6.542 Fn 1 p 366
†1 *Fragment*, 169 (151); see Plato's *Gorgias*, 484 B, 488 B; *Laws*, 690 B, 715 A.
- Peirce: CP 6.542 Fn 2 p 366
†2 *Gorgias*, 483 E.
- Peirce: CP 6.542 Fn 3 p 366
†3 See *De rerum natura*, I, 148.
- Peirce: CP 6.542 Fn 4 p 366
†4 See *Meditation* III.
- Peirce: CP 6.542 Fn 5 p 366
†5 See *Works*, vol. 5, p. 52, London (1672).
- Peirce: CP 6.543 Fn 1 p 367
†1 See his *Discourses on the Miracles of Our Savior*, etc., London (1727-29).
- Peirce: CP 6.543 Fn 1 p 368
†1 See *A Collection of the Tracts of a Certain Free Inquirer*, etc., London (1739).

Peirce: CP 6.547 Fn 1 p 369

†1 1733.

Peirce: CP 6.547 Fn 2 p 369

†2 *Analogy*, Part II, bk. IV (1736). Hume's *Treatise* was published in 1739-40.

Peirce: CP 6.548 Fn 1 p 370

†1 First published in a Symposium in the *Christian Register*, Boston, April 7, 1887. Reprinted from *Science and Immortality*, The Christian Register Symposium, Revised and Enlarged, edited by S. J. Barrows; Geo. H. Ellis, Boston (1887).

Peirce: CP 6.553 Fn 1 p 372

†1 Cf. 58ff.

Peirce: CP 6.554 Fn 2 p 372

†2 *First Principles*, bk. II, ch. 18.

Peirce: CP 6.556 Fn 1 p 373

†1 *Cours de philosophie positive*, 19^e leçon. vol. 2, p. 8, Paris (1835).

Peirce: CP 6.557 Fn 1 p 375

†1 Exactly as written and corrected by Peirce. Intended for *The Forum*, c. 1905.

Peirce: CP 6.566 Fn 1 p 379

†1 See his *Heat Considered as a Mode of Motion*, Lecture V (1862).

Peirce: CP 6.566 Fn 2 p 379

†2 "Ueber einen Grundsatz der Mechanischen Wärmetheorie," *Annalen der Physik u. Chemie*, Bd. CXX, pp. 426-452 (1863).

Peirce: CP 6.566 Fn 3 p 379

†3 See "Remarks on the Mechanical Equivalent of Heat," *Philosophical Magazine*, Series IV, vol. 25, p. 493 (1863).

Peirce: CP 6.566 Fn 4 p 379

†4 *Sketch of Thermodynamics*, §§30-39 (1868 and 1877).

Peirce: CP 6.566 Fn 5 p 379

†5 *Réflexions sur la puissance motrice du feu*, 1824.

Peirce: CP 6.566 Fn 6 p 379

†6 An American inventor and scientist (1804-79) who does not seem to have published his theoretic results.

Peirce: CP 6.568 Fn 1 p 380

†1 "On the Theory of Light and Colours," *Philosophical Transactions*, 1802, p. 12.

Peirce: CP 6.569 Fn 1 p 381

†1 *Meditation II. Principles*, Part I, 9.

Peirce: CP 6.570 Fn P1 p 381 Cross-Ref:††

†P1 Thus in the *Scriptum in quartum librum sententiarum Magistri Petri*

Lombardi (Distinctio 50, questio 3, articulus 4), he says: Dicendum . . . quod . . . ut beatitudo sanctorum eis magis complaceat et de ea uberiores gratias Deo agant, dantur eis ut poenam impiorum perfecte intueantur. In his 8th Quodlibet, question 7, article 16, he says: Dicendum quod videre miseriam damnatorum omnino erit sanctis ad gloriam, gaudebunt enim de justicia Dei et de sua evasione. And in another place: laetantur de justa punitione. "A horrible doctrine, I confess," says Calvin [*Institutes of the Christian Religion*, bk. III, chapter 23, section 7] of election to damnation; but logic forces him to it.

Peirce: CP 6.572 Fn 1 p 382

†1 See e.g. 5.223.

Peirce: CP 6.575 Fn 1 p 383

†1 See Zöllner, *Transcendental Physics*, trans. by W. H. Harrison, London (1880).

Peirce: CP 6.578 Fn 1 p 385

†1 *Dialogues Concerning Two New Sciences*, "Fourth Day" *ad init.* (1638).

Peirce: CP 6.582 Fn 1 p 387

†1 Cf. 27, 1.362.

Peirce: CP 6.585 Fn 2 p 387

†2 Cf. 260ff.

Peirce: CP 6.588 Fn 1 p 390

†1 *The Monist*, vol. 3, pp. 526-570 (1893).

Peirce: CP 6.588 Fn 2 p 390

†2 See bk I, chaps. 1 and 2.

Peirce: CP 6.590 Fn P1 p 393 Cross-Ref:††

†P1 J. S. Mill had in the first edition of his *Logic* decisively taken an objective conception of chance and probability; but in his second edition he had become puzzled and had retracted [see bk. III, ch. 18], leaving that chapter, and with it his whole logic, a melancholy wreck, over which the qualified reader sighs, "And this once seemed intelligible!" Venn in the first edition of his book set forth the same objective conception with great clearness, and for that he was entitled to high praise, notwithstanding his manifest inadequacy to the problems treated. But in his second edition [see ch. 6] he too has fallen away from his first and correct view, and has adopted a theory which I shall some day show to be untenable. Venn's whole method in logic, as well as his system, is in my opinion of the weakest.

Peirce: CP 6.590 Fn 1 p 393

†1 See 3.14ff.

Peirce: CP 6.590 Fn 2 p 393

†2 *System of Logic*, bk. III, ch. 5, §5.

Peirce: CP 6.590 Fn 1 p 394

†1 See e.g. *Kritik der Reinen Vernunft*, A91, B124.

Peirce: CP 6.590 Fn P1 p 394 Cross-Ref:††

†P1 Mill often did good service in substituting precise terms for ambiguous ones; as when in speaking of mathematical conclusions he prefers to say they are

legitimate deductions rather than that they are necessary. [*System of Logic*, bk. II, ch. 5 §1.]

Peirce: CP 6.590 Fn 2 p 394

†2 In *A System of Logic*, bk. III, ch. 5, §5.

Peirce: CP 6.590 Fn P2 p 394 Cross-Ref:††

†P2 In his *Ursache, Grund und Zweck*, Dr. Carus alludes to this passage [*Ursache, Grund und Zweck*, p. 37, Dresden (1883)]. But he prefers the treatment of the question by Reid, whom he calls Mill's opponent (*Gegner*).

Peirce: CP 6.590 Fn P3 p 394 Cross-Ref:††

†P3 It is of comparatively little consequence what Hume really meant. The main interest is in what Kant thought he meant.

Peirce: CP 6.590 Fn 3 p 394

†3 See 2.407ff.

Peirce: CP 6.591 Fn 1 p 395

†1 Cf. 2.315, 2.594, 3.182.

Peirce: CP 6.592 Fn 2 p 395

†2 In the *Century Dictionary* of 1889; see also 38.

Peirce: CP 6.592 Fn 1 p 396

†1 See 1.322ff.

Peirce: CP 6.592 Fn 2 p 396

†2 See 4.547n, 5.339, 5.403.

Peirce: CP 6.593 Fn P1 p 397 Cross-Ref:††

†P1 Along with the distinction, I would of course do away with this use of the words *abstract* and *concrete* to which no clear idea can be attached, as far as I can see.

Peirce: CP 6.595 Fn P2 p 397 Cross-Ref:††

†P2 I cannot but disapprove of this use of the word "construction" to mean a studied theory, because the word is imperatively required in the theory of cognition to denote a mathematical diagram framed according to a general precept.

Peirce: CP 6.595 Fn 1 p 398

†1 See Paper No. XVI of vol. 3.

Peirce: CP 6.595 Fn 2 p 398

†2 Cf. 2.56ff.

Peirce: CP 6.595 Fn P1 p 399 Cross-Ref:††

†P1 I apply this term because it is essentially like the passage from the concrete "virtuous" to the abstract "virtue," or from the concrete "white" (adjective) to the abstract "whiteness," or "white" (substantive). [Cf. 2.428.]

Peirce: CP 6.595 Fn 1 p 399

†1 Cf. 3.85ff.

Peirce: CP 6.595 Fn 2 p 399

†2 See 2.694ff.

Peirce: CP 6.595 Fn P1 p 400 Cross-Ref:††

†P1 I can never use this word without thinking of the explanation of it given by Petrus Peregrinus in his *Epistole de Magnete*. [See vol. 7.] He says that physical properties are occult in the sense that they are only brought out by experimentation, and are not to be deduced from admixtures of *hot* and *cold*, *moist* and *dry*.

Peirce: CP 6.595 Fn 1 p 400

†1 Cf. 1.383.

Peirce: CP 6.595 Fn 2 p 400

†2 See 5.504, 5.591.

Peirce: CP 6.600 Fn 1 p 403

†1 Cf. 67ff.

Peirce: CP 6.600 Fn P1 p 404 Cross-Ref:††

†P1 It follows as a corollary from this that if the positions of the particles at any one instant, together with the velocities at that instant, and the law of force, are given, the positions at all instants can be calculated. Of course, to give the positions and velocities at one instant is a special case of the giving of the positions at two instants. The two instants may be such that there will be more than one solution of the problem; but this is an insignificant detail.

Peirce: CP 6.600 Fn 1 p 404

†1 *Kritik der Reinen Vernunft*, A199, B244.

Peirce: CP 6.600 Fn 1 p 405

†1 See 139.

Peirce: CP 6.600 Fn P1 p 405 Cross-Ref:††

†P1 It would seem to follow from his notion that in uniform motion each minute's motion is the cause of that of the next. Yet he says (PARASYMBOLQX 19) "there is no cause that is equal to its effect."

Peirce: CP 6.600 Fn P1 p 406 Cross-Ref:††

†P1 But, as I have elsewhere said, I should like to persuade mathematicians to speak of "positional energy" as *Kinetic potency*, the *vis viva* as *Kinetic energy*, and the total "energy" as the *Kinetic entelechy*.

Peirce: CP 6.601 Fn 1 p 406

†1 *Lectures on Some Recent Advances in Physical Science*, p. 17 (1876).

Peirce: CP 6.601 Fn P1 p 407 Cross-Ref:††

†P1 The differential equation being an ordinary, not a partial one, this is an absolute constant, determined by initial (or final, or any instantaneous) conditions.

Peirce: CP 6.603 Fn 1 p 410

†1 Cf. 1.135ff.

Peirce: CP 6.604 Fn 1 p 411

†1 See vol. 2, bk. III, B.

Peirce: CP 6.604 Fn 2 p 411

†2 See vol. 1, bk. I, ch. 2.

Peirce: CP 6.604 Fn 3 p 411

†3 *History of the Inductive Sciences*, p. 6, third edition, London (1837).

Peirce: CP 6.604 Fn P1 p 412 Cross-Ref:††

†P1 Dr. Carus calls attention to the connection between my doctrine of the fixation of opinion and his anti-originalism.

Peirce: CP 6.604 Fn P2 p 412 Cross-Ref:††

†P2 Dr. Carus passes a sweeping judgment on post-Kantian philosophy as being original.

Peirce: CP 6.604 Fn P3 p 412 Cross-Ref:††

†P3 This was a remark of my father's.

Peirce: CP 6.604 Fn 1 p 413

†1 See 7ff.

Peirce: CP 6.604 Fn 2 p 413

†2 See 33.

Peirce: CP 6.604 Fn P1 p 413 Cross-Ref:††

†P1 A person in the last *Monist* [G. M. McCrie, "The Issues of Synechism," vol. 3, pp. 380-401] breaks in upon my series of articles to foretell what the "issues of synechism" will be. Were he able to do so, it would certainly be the height of ill-manners thus to take the words out of my mouth.

Peirce: CP 6.605 Fn 1 p 415

†1 See 25.

Peirce: CP 6.605 Fn P1 p 415 Cross-Ref:††

†P1 As I am writing, I am shown a letter, in which the writer says: "Peirce with all his materialistic ideas, yet," etc. I never promulgated a materialistic idea in my life. The writer simply assumes that science is materialistic. As I am correcting the proofs, I notice that Mr. B. C. Burt, in his new *History of Modern Philosophy*, sets me down as sceptical, though doubtfully. There are a good many inaccuracies in the work. This was inevitable in a first edition. But the ingenious plan of the book admirably adapts it to the wants of just that class of students who cannot understand that no repertory of facts ever can be trusted implicitly.

Peirce: CP 6.606 Fn P1 p 416 Cross-Ref:††

†P1 Its being hypothetical will not prevent its being established with a very high degree of certainty. Thus, all history is of the nature of hypothesis; since its facts cannot be directly observed, but are only supposed to be true to account for the characters of the monuments and other documents.

Peirce: CP 6.606 Fn P1 p 417 Cross-Ref:††

†P1 Somebody may notice that I here admit a proposition as absolutely true. Undoubtedly; because it relates to the Absolute.

Peirce: CP 6.608 Fn P1 p 418 Cross-Ref:††

†P1 Indeed, to admit that reply is all but to admit the non-absolute grade of necessity.

Peirce: CP 6.609 Fn 1 p 418

†1 See 2.683f.

Peirce: CP 6.609 Fn 2 p 418

†2 See 2.745f.

Peirce: CP 6.609 Fn P1 p 419 Cross-Ref:††

†P1 I rightly go somewhat further in my *Theory of Probable Inference*; but that has no bearing on the present discussion.

Peirce: CP 6.610 Fn 1 p 420

†1 See 2.749.

Peirce: CP 6.612 Fn 1 p 423

†1 Cf. 2.690, 2.716.

Peirce: CP 6.612 Fn P1 p 423 Cross-Ref:††

†P1 Dr. Carus, in his *Ursache, Grund und Zweck*, well says that *reasons* are discovered by induction, in the strict sense. It is often admitted that *causes* can only be inferred by hypothetic reasoning.

Peirce: CP 6.612 Fn 1 p 424

†1 See 214ff.

Peirce: CP 6.612 Fn 1 p 425

†1 See 297.

Peirce: CP 6.612 Fn 1 p 426

†1 Dr. Carus says of this: "Mr. Peirce's escape is like the disappearance of a medium from a room without doors. He must have got out through the fourth dimension. The argument is so subtle that I cannot see it." (*The Monist*, vol. 3, p. 599.)

Peirce: CP 6.613 Fn P1 p 429 Cross-Ref:††

†P1 The conservation of a vortex, which consists of the preservation of a certain character of motion by the same particles, though derived from the coöperation of other laws, is, in form, quite different.

Peirce: CP 6.613 Fn 1 p 429

†1 *Principia*, I, 9ff.

Peirce: CP 6.613 Fn 2 p 429

†2 3^{me} Lettre à Clarke 4. Gerhardt, vol. 7, p. 363, Berlin (1890).

Peirce: CP 6.613 Fn 3 p 429

†3 See *Kritik der Reinen Vernunft*, A33, B50.

Peirce: CP 6.613 Fn P1 p 430 Cross-Ref:††

†P1 In speaking of directions, we assume the Euclidean hypothesis that the angles of a triangle are equal to two right angles.

Peirce: CP 6.613 Fn 1 p 430

†1 Cf. 553.

Peirce: CP 6.614 Fn 1 p 434

†1 See 239-263.

Peirce: CP 6.614 Fn 2 p 434

†2 Sir Oliver Lodge. See his "Force and determinism," *Nature*, vol. 43, p. 491; Vol. 44, pp. 198, 272; (1890-91).

Peirce: CP 6.617 Fn 1 p 435

†1 See vol. 5, bk. II, no. II.

Peirce: CP 6 Appendix B - Ed. Note Fn 1 p 436

†1 A letter in *The Journal of Speculative Philosophy*, vol. 2, pp. 57-61 (1868) which, to judge from internal evidence, was written by Peirce.

Peirce: CP 6.620 Fn 1 p 437

†1 Comments of the editor of the *Journal* follow here.

Peirce: CP 6.625 Fn 1 p 441

†1 A letter continuing the discussion begun in Appendix B. From *The Journal of Speculative Philosophy*, vol. 2, pp. 190-191 (1868).

Peirce: CP 7 Title-Page
COLLECTED PAPERS OF CHARLES SANDERS PEIRCE

VOLUME VII
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Peirce: CP 7 Preface p v

PREFACE

When the first volume of the *Collected Papers* of Charles Peirce was published in 1931, its Introduction predicted some ten volumes in the series, those beyond the sixth being expected to contain Peirce's "writings on physics and psychology, as well as his reviews, letters, and biography." Footnotes to the subsequent text even cited some of those last volumes. Nevertheless, it was only of the six volumes that the Introduction spoke in the present tense, and these, under the editorship of Drs. Charles Hartshorne and Paul Weiss, appeared as promised, ending with Volume VI in 1935. Selection, preparation, and publication of further material was at that time impractical, and for the next twenty years the remaining papers in Harvard's custody were accessible only to such scholars as could consult them in Cambridge. In 1954, however, the Harvard Department of Philosophy was able to renew the enterprise. The Rockefeller Foundation granted to the University a subvention for the costs of further editing, and the Department was fortunate to enlist Professor Arthur W. Burks, of the University of Michigan, to perform that peculiarly exacting task. The Department is glad of this occasion to acknowledge its debt, and that of the scholarly community in general, both to the Foundation and to Professor Burks, and to remind the reader that since the expense of the further actual book-making is defrayed from royalties from the earlier volumes, which were subsidized by gifts of the late Professor James H. Woods and anonymous friends, we are still their beneficiaries too.

Peirce: CP 7 Preface p v

The present publication comprises two volumes instead of the four of the old estimate. Mr. Burks's Introduction, which follows on page xi, indicates how nearly he is completing the original plan. A substantial addition is his extraordinarily searching bibliography, with its introduction comparing the sum of published material with what still remains only in manuscript.

Peirce: CP 7 Preface p v

Besides joining in the editor's acknowledgements of those who assisted him, the Department and Publishers thank the Houghton Library for permission to print previously unpublished correspondence between Peirce and James, Edith Davidson Harris and the Hoose Library of Philosophy at the University of Southern California for permission to print a letter from Peirce to William T. Harris, and the following for the use of copyrighted material as indicated: Helen G. Baldwin, definitions from James Mark Baldwin's *Dictionary of Philosophy and Psychology*, Vol. II; *The Nation*, several reviews; *Popular Science Monthly*, Peirce's review of Pearson's *Grammar of Science*; Whitlock's, Inc., parts of two letters reprinted from Irwin C. Lieb's *Charles S. Peirce's Letters to Lady Welby*; William James, correspondence from Peirce to his father, William James, and quotations from a Peirce manuscript, "Questions on William James's Principles of Psychology," previously printed in Ralph Barton Perry's *Thought and Character of William James*.

Harvard University

August, 1957

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INTRODUCTION

The first six volumes of the series, *Collected Papers of Charles Sanders Peirce*, included Peirce's main writings in general philosophy, logic (deductive, inductive, and symbolic), pragmatism, and metaphysics. The present two volumes are a continuation of this series. Volume VII is organized in three books containing papers on experimental science, scientific method, and philosophy of mind, respectively. Volume VIII contains selections from Peirce's reviews and correspondence and a bibliography of his published works.

Peirce: CP 7 Introduction p xi

Since Book I of Volume VII, "Experimental Science," is the only book in the series not on philosophy, its inclusion may require special comment. Much of Peirce's life was devoted to experimental science. In fact, the only permanent position Peirce held was with the United States Coast Survey, where he was employed to do research in astronomy and geodetics. Though this position allowed him considerable time for philosophy, it is nevertheless true that for more than half of his mature life his main responsibility was to conduct scientific investigation.†1 His efforts in the physical sciences resulted in a large number of published papers, several of considerable length, as well as his only published book, *Photometric Researches*, 1878.†2 Moreover, Peirce's experimental work had an important influence on his philosophy. His pragmatic theory of meaning is a generalization from scientific practice, his laboratory experience having influenced his discovery of that theory (see 5.411-412). His indeterminism was connected with his work for the Coast Survey on precise measurement.†3 And his empirical investigations in psychology (see 7.21-35) influenced his theory of mind. Thus Peirce's scientific work played an important role in his life and in the formation of his philosophy, and it is for these reasons that we have reprinted his only published article in experimental psychology and two short pieces on gravity. We have limited ourselves to this small sample of his work in physical science because, though his astronomical researches showed originality and his gravimetric work was well respected by eminent men in geodetics, Peirce remained a minor figure in these fields.

Peirce: CP 7 Introduction p xii

The editors of the earlier volumes made some introductory remarks about the Peirce manuscripts and their policies in editing them (Vol. I, p. iv ff.). The present editor has continued their practice of publishing only parts of some of the works, omitting large portions altogether. He has also continued the policy of selecting or compiling a draft for publication whenever there were several drafts available. The justification for these procedures is to be found in certain aspects of the Peirce manuscripts. Many of the manuscripts are internally variable in quality: one frequently finds publishable sections in the midst of material which either is of little contemporary interest or presents ideas better treated in other manuscripts. Moreover, there are often alternative and sometimes incomplete versions of the same work to choose from; and in some cases no final version is discernible. The editor must then make selections and, if necessary, piece together drafts from the hodgepodge of

partial drafts. These characteristics are so typical that any satisfactory edition of Peirce's papers must contain a great deal of fragmentary material.

Peirce: CP 7 Introduction p xii

The present editor has also continued the plan of organization pursued in the previous volumes, breaking up manuscripts, books, and series of articles, and arranging the resultant materials primarily by subject matter rather than chronologically. Though this plan tends to bring together under one heading passages on a given topic, it makes Peirce's writings appear to be more disorganized than they actually are. To mitigate this shortcoming, there is included in Volume VIII as complete a bibliography of Peirce's published works as the editor could compile. This bibliography is arranged for the most part chronologically, with a connected series of articles, parts of a single manuscript, or a series of lectures grouped together under one listing. The bibliography will also assist the reader in locating any published works not reprinted in this series.

Peirce: CP 7 Introduction p xiii

The chapter and section headings have been chosen by the editor; when these are the same as Peirce's, the footnotes so indicate. Editorial alterations are enclosed in brackets, and the editor's footnotes are indicated by "(Ed.)." Peirce's punctuation, spelling, and underlining for emphasis have, in general, been retained. Obvious errors, however, have been corrected; and where clarity of presentation has required them, minor changes have been made in format, capitalization, abbreviations, italicizing of titles, etc., without any indication. Likewise, questions of manuscript interpretation which made no significant difference in the meaning have been settled without any indication.

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The editor wishes to thank the Rockefeller Foundation and the Department of Philosophy of Harvard University for their aid in the preparation and publication of these two volumes. While giving him the kindest and fullest assistance, both left him completely free in his work and have no responsibility for any shortcomings in it. The editor is also grateful for a grant for editorial assistance from the Horace H. Rackham School of Graduate Studies of the University of Michigan.

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For assistance in locating and evaluating various Peirce materials, the editor is personally indebted to William Alston, Jackson Cope, Carolyn Eisele, Mrs. E. L. C. Hales, Murray Murphey, W. E. Schlaretzki, James F. Sheridan, Manley Thompson, and Philip Wiener; to the previous editors, Charles Hartshorne and Paul Weiss; and especially to Max Fisch, who also read the manuscript and made many valuable suggestions. In all cases the final decision as to what materials were to be included was made by the editor, and the responsibility for the choices is his alone.

Peirce: CP 7 Introduction p xiv

The editor expresses his deep thanks to Grace L. Wood, who assisted greatly in the preparation of the bibliography and did much of the editorial work for the volumes, and to his wife, Alice, for her helpful advice.

Arthur W. Burks

Ann Arbor, Michigan

Peirce: CP 7.1 Cross-Ref:††
BOOK I

EXPERIMENTAL SCIENCE

CHAPTER 1

MEASUREMENT OF THE FORCE OF GRAVITY

§1. A SOURCE OF ERROR IN PENDULUM MEASUREMENTS †1

1. The fact that the rate of a pendulum might be largely influenced by the elastic yielding of its support was first pointed out by Dr. Thomas Young in his article on Tides in the Encyclopædia Britannica, where he gave a correct mathematical analysis of the problem. Kater made use of the *noddy*, or inverted pendulum of Hardy, to assure himself that its support was sufficiently steady.

Peirce: CP 7.2 Cross-Ref:††

2. Hardy's noddy is a pendulum turning with a reed spring and provided with an adjustable bob. It differs from an ordinary pendulum, first, in being upside down, that is, having its center of mass above its point of support; and second, in having a spring so strong as to act a little more strongly than gravity. The force tending to bring the pendulum to the vertical is then the excess of the force of the spring over the moment of gravity. In this way the noddy is easily adjusted so as to have the same period of oscillation as the pendulum used to determine gravity, while its moment of inertia is very small. In a note at the end of this paper I give the mathematical analysis of this state of things, from which it will be seen that Kater might have constructed his noddy in such a manner as to detect any amount of flexure sufficient to have a serious effect upon the period of his pendulum.

Peirce: CP 7.3 Cross-Ref:††

3. Bessel, at the end of §3 of his great memoir on the length of the second's pendulum at Königsberg, states that he also used Hardy's noddy, and that he swung his pendulum again after stiffening the support. He adds that the effect on the period would probably be the same for his long pendulum as for his short one -- a very just remark -- which made it less necessary for him to attend to the rigidity of the stand.

Peirce: CP 7.4 Cross-Ref:††

4. The construction of English pendulum supports, that of Bassevi, for

example, shows that in that country this source of error was never overlooked. It is noticed even in brief accounts in English of the process of measuring gravity. Thus, a writer in the *Encyclopædia Britannica* proposed to make use of two different reversible pendulums of the same form but of different weights, in order to take account of the flexure, an idea lately borrowed by M. Cellierier.

Peirce: CP 7.5 Cross-Ref:††

5. When the reversible pendulum came into use the study of the writings of the older observers seems to have been neglected,†2 and the grave errors due to flexure were never suspected until Albrecht found a value of gravity at Berlin differing by nearly 2 millimeters from that of Bessel. So little was the true cause of this discrepancy at first suspected that it was paradoxically attributed to the neglect of a buoyancy correction.

Peirce: CP 7.6 Cross-Ref:††

6. In 1875, however, General Baeyer gravely suspected that the period of a pendulum swinging upon a Repsold tripod was affected by the oscillation of the latter, and in a circular addressed to the members of the committee on the pendulum of the International Geodetic Congress, he wrote: "The necessity of suspending the pendulum from a stand is a source of error, since a pendulum swinging on a stand sets the latter into oscillation and so influences the rate of the former. The effect could be diminished by the use of a shorter pendulum and smaller stand; but whether it would be rendered entirely insensible is open to question."

Peirce: CP 7.7 Cross-Ref:††

7. It was at this time that I first received the Repsold apparatus from the makers, of whom it had been ordered two years before, on the occasion of my first being charged with the pendulum operations of the Coast Survey. Becoming acquainted with General Baeyer's doubts, I determined to settle the question by measuring the flexibility of the Repsold tripod at the earliest opportunity. This I did at Geneva, where, though I only made a rough measurement, I found that the flexure was fully sufficient to account for the discrepancy between the determinations of Bessel and of Albrecht.

Peirce: CP 7.8 Cross-Ref:††

8. On September 25 of the same year I communicated my result to the standing committee of the Geodetical Congress.†3 At the same sitting the reports of the different members of the pendulum committee were read. Dr. Bruhns said: "The question whether the stand is set into oscillation, and whether the rate of the pendulum is influenced thereby is, in my opinion, well worth investigation. But I should suppose that the stand could be made so stiff as to eliminate this source of error for a pendulum used only as a relative instrument." The views of M. Hirsch, who is so much occupied with the going of timekeepers, are interesting. He said: "The fear that the tripod of suspension may also enter into oscillation, unless it be a fact established by direct observations, seems to me unfounded. Indeed, it cannot be supposed that there are any true oscillations of a body of such a form resting on three points. Besides, the movement of the pendulum whose mechanical moment (*moment mécanique*) is slight on account of its small velocity, could only be communicated to the tripod by the friction of the knife on the supporting plane. Now, this friction is insignificant, as the slowness of the decrement of the amplitude shows, this being almost entirely due to the resistance of the air." It may be observed that the rolling friction of the knife edge is, in truth, very slight, but the amount of the sliding friction is sufficient to hold the knife in place on the supporting plane. Dr. von Oppolzer, the

designer of the Repsold tripod in its definitive form, said that the construction of the stand rendered any serious flexure *a priori* improbable; but he did not support this opinion by any calculations.

Peirce: CP 7.9 Cross-Ref:††

9. During the spring of 1876, having already measured the flexibility of the tripod in Paris, I remeasured it in Berlin, where my experiments were witnessed by General Baeyer and a party of gentlemen attached to the Prussian Survey.

Peirce: CP 7.10 Cross-Ref:††

10. In October, 1876, at the meeting of the standing committee of the International Geodetical Union at Brussels, the result of my experiments was announced by General Baeyer.†4 M. Hirsch described certain experimental researches undertaken by him to ascertain whether there was any such flexure in the case of the Swiss tripod. He had, in the first place, employed an extremely sensitive level, which had not entered into oscillation while the pendulum was swinging upon it. It is not clear why M. Hirsch employed a very sensitive level, the natural time of oscillation of which would differ much more from the period of the pendulum than that of a less sensitive level would do. He also used an artificial horizon in the same way. M. Hirsch's conclusion is that "there remains no doubt that the Swiss stand is free from every trace of such oscillations." Dr. von Oppolzer entirely agreed with the views of M. Hirsch.

Peirce: CP 7.11 Cross-Ref:††

11. In the following summer I addressed to M. Plantamour a paper upon the subject, to be submitted to the next meeting of the Geodetical Congress.†5 In this note, which is reprinted at the end of the present report, I first give a mathematical analysis of the problem. I next show experimentally that the motion of the knife-edge support is not a translation, but is a rotation, so that different parts of the head of the tripod, only a few centimeters distant from one another, move through very different distances. Consequently, measures of the flexure made anywhere except at the center of the knife-edge plane require an important correction before they can be used to correct the periods. This is confirmed by experiments with a mirror while the pendulum is in motion. I next give a brief *résumé* of my statical measures of the flexure. I then give measures of the actual flexure under the oscillation of the pendulum, and show that the statical and dynamical flexibilities are approximately equal. Finally, I swing the same pendulum upon the Repsold support and upon another having seven times the rigidity of that one, and I show that the difference of the periods of oscillation agrees with the theory.

Peirce: CP 7.12 Cross-Ref:††

12. Immediately upon the reception of my manuscript, MM. Hirsch and Plantamour commenced new researches, designed to form an "étude approfondie de ce phénomène." These were embodied in a paper by M. Plantamour, which was read to the Geodetical Congress, and which has since been expanded into a memoir entitled "Recherches expérimentales sur le mouvement simultané d'un pendule et de ses supports."†6 M. Plantamour finds fault with me, first, for having measured the flexure with a force five or ten times that of the deflecting force of the pendulum; and second, for measuring the elasticity statically instead of dynamically. The reply to the first objection is that the properties of metals are known to a great extent, that elasticity is not "une force capricieuse," and that no fact is better established than that an elastic strain is proportional to the stress up to near the limit of elasticity, which limit was not approached in the author's experiments. As to the second objection, I

had shown by experiment that the statical and dynamical flexures are nearly equal; and I am willing to leave it to time to show whether this will not be assumed in future measures of the flexure of future pendulum supports. M. Plantamour caused a fine point fixed into the head of the tripod to press against a little mirror, mounted on an axis; and then observed the reflection of a scale in a telescope. The length of the path of light from the scale to the telescope divided by the distance of the bearing point from the axis of the mirror he calls the *grossissement*; so that had he used a fixed star in place of his scale, the *grossissement* would have been virtually infinite. From the given length of the lever it would appear that a movement of $0\{u\}.03$ in the point would turn the mirror 4". The aperture of the mirror is not stated, but it cannot be supposed that the error of observation would be less than this. It does not seem to me that the use of this mode of measurement, which magnifies the motion but little more than my method, is conducive to accuracy, especially in investigating the difference between statical and dynamical flexure. A certain finite force presses together the point and the lever. Dividing this force by the minute area of pressure, we find the pressure upon the metal is very great, approaching the crushing pressure. Now, the behavior of metals under great pressure is greatly influenced by the time. But my objection is not merely theoretical; I have myself made experiments upon this method, and, making them as skillfully as I could, I still found great uncertainty in the results.

Peirce: CP 7.13 Cross-Ref:††

§2. SIX REASONS FOR THE PROSECUTION OF PENDULUM EXPERIMENTS

†7

13. [Reason 1.] The first scientific object of a geodetical survey is unquestionably the determination of the earth's figure. Now, it appears probable that pendulum experiments afford the best method of determining the amount of oblateness of the spheroid of the earth; for the calculated probable error in the determination of the quantity in question from the pendulum work already executed does not exceed that of the best determination from triangulation and latitude observations, and the former determination will shortly be considerably improved. Besides, the measurements of astronomical arcs upon the surface of the earth cover only limited districts, and the oblateness deduced from them is necessarily largely affected, as Mr. Schott has remarked, by the old arc of Peru, the real error of which no doubt greatly exceeds that which the calculation attributes to it, so that we cannot really hold it probable that the error of this method is so small as it is calculated by least squares to be. On the other hand, the pendulum determinations are subject to no great errors of a kind which least squares cannot ascertain; they are widely scattered over the surface of the earth; they are very numerous; they are combined to obtain the ellipticity by a simple arithmetical process; and, all things considered, the calculated probable error of the oblateness deduced from them is worthy of unusual confidence. In this connection it is very significant, as pointed out by Colonel Clarke (*Geodesy*, p. vi), that while the value derived from pendulum work has for a long time remained nearly constant, that derived from measurements of arcs has altered as more data have been accumulated, and the change has continually been in the direction of accord with the other method. It is needless to say that the comparison of the expense of the two methods of obtaining this important quantity is immensely in favor of pendulum

work.

Peirce: CP 7.14 Cross-Ref:††

14. [Reason 2.] Recent investigations also lead us to attach increased importance to experiments with the pendulum in their connection with metrology. The plan of preserving and transmitting to posterity an exact knowledge of the length of the yard after the metallic bar itself should have undergone such changes as the vicissitudes of time bring to all material objects, was at one time adopted by the British Government. It was afterwards abandoned because pendulum operations had fallen into desuetude, and because doubts had been thrown upon the accuracy of Kater's original measure of the length of the second's pendulum. Yet I do not hesitate to say that this plan should now be revived, for the following reasons.

Peirce: CP 7.15 Cross-Ref:††

15. First, because measurements of the length of the second's pendulum, although formerly subject to grave uncertainties, are now secure against all but very small errors. Indeed, we now know that the determinations by Kater and his contemporaries, after receiving certain necessary corrections, are by no means so inaccurate as they were formerly suspected to be. Secondly, metallic bars have now been proved, by the investigations of Professor Hilgard †8 and others, to undergo unexpected spontaneous alterations of their length, so that some check upon these must be resorted to. To this end the late Henri Ste. Claire Deville and Mascart constructed for the International Geodetical Association a metre ruled upon a sort of bottle of platin-iridium, with the idea that the cubic contents of this bottle should be determined from time to time, so as to ascertain whether its dimensions had undergone any change. I am myself charged with, and have nearly completed, a very exact comparison of the length of a metre bar with that of a wave of light, for the same purpose.†9 Neither of these two methods is infallible, however, for the platin-iridium bottle may change its three dimensions unequally, and the solar system may move into a region of space in which the luminiferous ether may have a slightly different density (or elasticity), so that the wave length of the ray of light used would be different. These two methods should therefore be supplemented by the comparatively simple and easy one of accurately comparing the length of the second's pendulum with the metre or yard bar. Thirdly, I do not think it can be gainsaid by any one who examines the facts that the measurements of the length of the second's pendulum by Borda and by Biot in Paris and by Bessel in Berlin do, as a matter of fact, afford us a better and more secure knowledge of the length of their standard bars than we can attain in any other way. So also I have more confidence in the value of the ratio of the yard to the metre obtained by the comparison of the measurements of the length of the second's pendulum at the Kew observatory by Heaviside in terms of the yard and by myself in terms of the metre than I have in all the elaborate and laborious comparisons of bars which have been directed to the same end. I will even go so far as to say that a physicist in any remote station could ascertain the length of the metre accurately to a one hundred thousandth part more safely and easily by experiments with an invariable reversible pendulum than by the transportation of an ordinary metallic bar.

Peirce: CP 7.16 Cross-Ref:††

16. A new application of the pendulum to metrology is now being put into practice by me. Namely, I am to oscillate simultaneously a yard reversible pendulum and a metre reversible pendulum. I shall thus ascertain with great precision the ratio of their lengths without any of those multiform comparisons which would be

necessary if this were done by the usual method. These two pendulums will be swung, the yard one in the office of the Survey, at a temperature above 62°F., which is the standard temperature of the yard, the other nearly at 0°C., which is the standard temperature of the metre; and thus we shall have two bars compared at widely different temperatures, which, according to ordinary processes, is a matter of great difficulty. The knife-edges of the pendulums will be interchanged and the experiments repeated. Finally, the yard pendulum will be compared with a yard bar and the metre pendulum with a metre bar, and last of all the yard pendulum with its yard bar will be sent to England, the metre pendulum with its metre bar to France, for comparison with the primary standards; and thus it is believed the ratio of yard to metre will be ascertained with the highest present attainable exactitude.

Peirce: CP 7.17 Cross-Ref:††

17. [Reason 3.] Geologists affirm that from the values of gravity at different points useful inferences can be drawn in regard to the geological constitution of the underlying strata. For instance, it has been found that when the gravity upon high lands and mountains is corrected for difference of centrifugal force and distance from the earth's centre, it is very little greater than at the sea-level. Consequently it cannot be that there is an amount of extra matter under these elevated stations equal to the amount of rock which projects above the sea-level; and the inference is that the elevations have been mainly produced by vertical and not by horizontal displacements of material. On the other hand, Mendenhall has found that gravity on Fujisan, the well-known volcanic cone of Japan, which is about 12,000 feet high, and which is said to have been upheaved in a single night, about 300 B.C., is as much less than that in Tokio as if the mountain had been wholly produced by horizontal transfer. This conclusion, if correct, must plainly have a decisive bearing upon certain theories of volcanic action. Again, it has long been known that gravity is in excess upon islands, and I have shown that this excess is fully equal to the attraction of the sea-water. This shows that the interior of the earth is not so liquid and incompressible that the weight of the sea has pressed away to the sides the underlying matter. But in certain seas gravity is even more in excess than can be due to the attraction of the ocean, as if they had been receptacles of additional matter washed down from the land. It is evident that only the paucity of existing data prevents inferences like these from being carried much further. On the two sides of the great fault in the Rocky Mountains gravity must be very different, and if we knew how great this difference was we should learn something more about the geology of this region; and many such examples might be cited.

Peirce: CP 7.18 Cross-Ref:††

18. [Reason 4.] Gravity is extensively employed as a unit in the measurement of forces. Thus, the pressure of the atmosphere is, in the barometer, balanced against the weight of a measured column of mercury; the mechanical equivalent of heat is measured in foot-pounds, etc. All such measurements refer to a standard which is different in different localities, and it becomes more and more important to determine the amounts of these differences as the exactitude of measurement is improved.

Peirce: CP 7.19 Cross-Ref:††

19. [Reason 5.] It may be hoped that as our knowledge of the constitution of the earth's crust becomes, by the aid of the pendulum investigations, more perfected, we shall be able to establish methods by which we can securely infer from the vertical attractions of mountains, etc., what their horizontal attractions and the resulting deflections of the plumb-line must be.

Peirce: CP 7.20 Cross-Ref:††

20. [Reason 6.] Although in laying out the plan of a geodetical survey the relative utility of the knowledge of different quantities ought to be taken into account, and such account must be favorable to pendulum work, yet it is also true that nothing appertaining to such a survey ought to be neglected, and that too great stress ought not to be put upon the demands of the practically useful. The knowledge of the force of gravity is not a mere matter of utility alone, it is also one of the fundamental kinds of quantity which it is the business of a geodetical survey to measure. Astronomical latitudes and longitudes are determinations of the direction of gravity; pendulum experiments determine its amount. The force of gravity is related in the same way to the latitude and longitude as the intensity of magnetic force is related to magnetical declination and inclination; and as a magnetical survey would be held to be imperfect in which measurements of intensity were omitted, to the same extent must a geodetical survey be held to be imperfect in which the determinations of gravity had been omitted; and such would be the universal judgment of the scientific world.

Peirce: CP 7.21 Cross-Ref:††
CHAPTER 2

SMALL DIFFERENCES OF SENSATION

§1. ORIGINAL PAPER †1

21. The physiological psychologists assume that two nerve excitations alike in quality will only produce distinguishable sensations provided they differ in intensity by an amount greater than a fixed ratio. The least perceptible difference of the excitations divided by half their sum is what they call the *Unterschiedsschwelle*. Fechner †2 gives an experiment to prove the fact assumed, namely: He finds that two very dim lights placed nearly in line with the edge of an opaque body show but one shadow of the edge. It will be found, however, that this phenomenon is not a clearly marked one, unless the lights are nearly in range. If the experiment is performed with lateral shifting of one of the lights, and with a knowledge of the effects of a telescope upon the appearance of terrestrial objects at night, it will be found very far from conclusive.

Peirce: CP 7.22 Cross-Ref:††

22. The conception of the psychologists is certainly a difficult one to seize. According to their own doctrine, in which the observed facts seem fully to bear them out, the intensity of the sensation increases continuously with the excitation, so that the least increase of the latter must produce a corresponding increase of the former. And, indeed, the hypothesis that a continuous increase of the excitation would be accompanied by successive discrete increments of the sensation, gratuitous as it would be, would not be sufficient to account for a constant *Unterschiedsschwelle*. We are therefore forced to conclude that if there be such a phenomenon, it has its origin,

not in the faculty of sensation, but in that of comparing sensations. In short, if the phenomenon were established, we should be forced to say that there was a least perceptible difference of sensation -- a difference which, though existing in sensation, could not be brought into consciousness by any effort of attention. But the errors of our judgments in comparing our sensations seem sufficiently accounted for by the slow and doubtless complicated process by which the impression is conveyed from the periphery to the brain; for this must be liable to more or less accidental derangement at every step of its progress. Accordingly we find that the frequencies of errors of different magnitudes follow the probability curve, which is the law of an effect brought about by the sum of an infinite number of infinitesimal causes. This theory, however, does not admit of an *Unterschiedsschwelle*. On the contrary, it leads to the method of least squares, according to which the multiplication of observations will indefinitely reduce the error of their mean, so that if of two excitations one were ever so little the more intense, in the long run it would be judged to be the more intense the majority of times. It is true that the astronomers themselves have not usually supposed that this would be the case, because (apart from constant errors, which have no relevancy to the present question) they have supposed this extreme result to be contrary to common sense. But it has seemed to us that the most satisfactory course would be to subject the question to the test of direct experiment. If there be a least perceptible difference, then when two excitations differing by less than this are presented to us, and we are asked to judge which is the greater, we ought to answer wrong as often as right in the long run. Whereas, if the theory of least squares is correct, we not only ought to answer right oftener than wrong, but we ought to do so in a predictable ratio of cases.†3

Peirce: CP 7.23 Cross-Ref:††

23. We have experimented with the pressure sense, observing the proportion of errors among judgments as to which is the greater of two pressures, when it is known that the two are two stated pressures, and the question presented for the decision of the observer is, which is which? From the probability, thus ascertained, of committing an error of a given magnitude, the probable error of a judgment can be calculated according to the mathematical theory of errors. If, now, we find that when the ratio of the two pressures is smaller than a certain ratio, the erroneous judgments number one-half of the whole, while the mathematical theory requires them to be sensibly fewer, then this theory is plainly disproved, and the maximum ratio at which this phenomenon is observed the so-called *Unterschiedsschwelle*. If, on the other hand, the values obtained for the probable error are the same for errors varying from three times to one-fourth of the probable error (the smallest for which it is easy to collect sufficient observations), then the theory of the method of least squares is shown to hold good within those limits, the presumption will be that it extends still further, and it is possible that it holds for the smallest differences of excitation. But, further, if this law is shown to hold good for difference so slight that the observer is not conscious of being able to discriminate between the sensations at all, all reason for believing in an *Unterschiedsschwelle* is destroyed. The mathematical theory has the advantage of yielding conceptions of greater definiteness than that of the physiologists, and will thus tend to improve methods of observation. Moreover, it affords a ready method for determining the sensibility or fineness of perception and allows of a comparison of one observer's results with the results of others; for, knowing the number of errors in a certain number of experiments, and accepting the conclusions of this paper, the calculated ratio to the total excitation of that variation of excitation, in judging which we should err one time out of four, measures the sensibility. Incidentally our experiments will afford additional information upon the

value of the normal average sensibility for the pressure sense, which they seem to make a finer sense than it has hitherto been believed to be. But in this regard two things have to be noted: (1) Our value relates to the probable error or the value for the point at which an error is committed half the time; (2) in our experiments there were two opportunities for judging, for the initial weight was either first increased and then diminished, or *vice versa*, the subject having to say which of these two double changes was made. It would seem at first blush that the value thus obtained ought to be multiplied by $\sqrt{2}(1.414)$ to get the error of a single judgment. Yet this would hardly be correct, because the judgment, in point of fact, depended almost exclusively on the sensation of increase of pressure, the decrease being felt very much less. The ratio $\sqrt{2}(1.414)$ would therefore be too great, and 1.2 would perhaps be about correct. The advantage of having two changes in one experiment consists in this: If only one change were employed, then some of the experiments would have an increase of excitation only and the others a decrease only; and since the former would yield a far greater amount of sensation than the latter, the nature of the results would be greatly complicated; but when each experiment embraces a double change this difference in the amount of sensation caused by an increase and decrease of pressure affects every experiment alike, and the liability to error is constant.†4

Peirce: CP 7.24 Cross-Ref:††

24. Throughout our observations we noted the degree of confidence with which the observer gave his judgment upon a scale of four degrees, as follows:

0 denoted absence of any preference for one answer over its opposite, so that it seemed nonsensical to answer at all.

1 denoted a distinct leaning to one alternative.

2 denoted some little confidence of being right.

3 denoted as strong a confidence as one would have about such sensations.

Peirce: CP 7.24 Cross-Ref:††

We do not mean to say that when zero was the recorded confidence, there was absolutely no sensation of preference for the answer given. We only mean that there was no sensation that the observer noticed when attending to his feelings of this sort as closely as he conveniently could, namely, closely enough to mark them on this scale. The scale of confidence fluctuated considerably. Thus, when Mr. Jastrow passed from experiments upon differences of weight of 60, 30, and 15 on the thousand to differences of 20, 10, and 5 on the thousand, although the accuracy of his judgments was decidedly improved, his confidence fell off very greatly, owing to his no longer having the sensation produced by a difference of 60 present to his memory. The estimations of confidence were also rough, and might be improved in future work. The average marks seem to conform to the formula--

$$m = c \log (p/1-p)$$

where *m* denotes the degree of confidence on the scale, *p* denotes the probability of the answer being right, and *c* is a constant which may be called the index of

confidence.

Peirce: CP 7.25 Cross-Ref:††

25. To show that this formula approximates to the truth, we compare it with the average marks assigned to estimates of differences for which more than a hundred experiments were made. Mr. Jastrow's experiments are separated into groups, which will be explained below.

Peirce, observer. Jastrow, observer.

Ratio of pressures. $c = 1.25.$ First group. Second group.
 $c = 1.5.$ $c = 0.0.$

Mean confidence. Mean confidence. Mean confidence.

Observed. Calculated. Observed. Calculated. Observed. Calculated.

1.015 .. 0.14	0.10	0.30	0.2	0.34	0.2
1.030 .. 0.30	0.35	0.40	0.42	0.55	0.56
1.060 .. 0.70	0.70	0.85	0.87	1.02	1.22

Jastrow, observer.

Ratio of pressures. $c = 0.25.$ Third group. Fourth group.
 $c = 0.4.$

Mean confidence. Mean confidence.

Observed. Calculated. Observed. Calculated.

1.005	0.00	0.03	0.00	0.06
1.010	0.07	0.06	0.05	0.12
1.020	0.12	0.12	0.50	0.39

Peirce: CP 7.26 Cross-Ref:††

26. The judgments enunciated with any given degree of confidence were more likely to be right with greater differences than with smaller differences. To show this, we give the frequency of the different marks in Mr. Jastrow's second, third, and fourth groups.†5

Peirce: CP 7.27 Cross-Ref:††

27. The apparatus used was an adaptation of a "Fairbanks" post-office scale; upon the end of the beam of which was fixed a square enlargement (about one-half inch square), with a flat top, which served to convey the pressure to the finger in a manner to be presently described. This was tightly covered with an India-rubber cap, to prevent sensations of cold, etc., from contact with the metal. A kilogram placed in the pan of the balance brought a pressure of one-fourth of its weight upon the finger. The differential pressure was produced by lowering upon the pan of the balance a smaller pan into which the proper weights could be firmly fixed; this little pan had its bottom of cork, and was placed upon a piece of flannel which constantly remained in the pan of the balance. It was lifted off and on by means of a fine India-rubber thread, which was so much stretched by the weight as certainly to avoid any noise or jar from the momentum of the descending pan. A sufficient weight could also be hung on the beam of the balance, so as to take off the entire pressure from the finger at the end of each experiment. This weight could be applied or removed by means of a cam acting upon a lever; and its bearings upon the beam were guarded by India-rubber. It was found that the use of this arrangement, which removed all annoying irregularities of sensation connected with the removal and replacement of the greater (initial) pressure, rendered the results more uniform and diminished the probable error. It also shortened the time necessary for performing the experiments, so that a series of 25 experiments was concluded before the effects of fatigue were noticeable. It may be mentioned that certain causes tended to the constant decrease of the probable error as the experiments went on, these mainly being an increased skill on the part of the *operator* and an education of the sensibility of the *subject*. The finger was supported in such a way as to be lightly but firmly held in position, all the muscles of the arm being relaxed; and the India-rubber top of the brass enlargement at the end of the beam of the balance was never actually separated from the finger. The projecting arm of a filter-stand (the height of which could be adjusted) with some attachments not necessary to detail, gently prevented the finger from moving upwards under the pressure exerted by the weight in the pan. In the case of Mr. Peirce as subject (it may be noted that Mr. Peirce is left-handed, while Mr. Jastrow is strongly right-handed) the tip of forefinger, and in the case of Mr. Jastrow of the middle finger, of the left hand were used. In addition, a screen served to prevent the subject from having any indications whatever of the movements of the operator. It is hardly necessary to say that we were fully on guard against unconsciously received indications.

Peirce: CP 7.28 Cross-Ref:††

28. The observations were conducted in the following manner: At each sitting three differential weights were employed. At first we always began and ended with the heaviest, but at a later period the plan was to begin on alternate days with the lightest and heaviest. When we began with the heaviest 25 observations †6 were made with that; then 25 with the middle one, and then 25 with the lightest; this constituted

one-half of the sitting. It was completed by three more sets of 25, the order of the weights being reversed. When we began with the lightest the heaviest was used for the third and fourth sets. In this way 150 experiments on each of us were taken at one sitting of two hours.

Peirce: CP 7.29 Cross-Ref:††

29. A pack of 25 cards were taken, 12 red and 13 black, or *vice versa*, so that in the 50 experiments made at one sitting with a given differential weight, 25 red and 25 black cards should be used. These cards were cut exactly square and their corners were distinguished by holes punched in them so as to indicate the scale of numbers (0, 1, 2, 3) used to designate the degree of confidence of the judgment. The backs of these cards were distinguished from their faces. They were, in fact, made of ordinary playing-cards. At the beginning of a set of 25, the pack was well shuffled, and, the operator and subject having taken their places, the operator was governed by the color of the successive cards in choosing whether he should first diminish the weight and then increase it, or *vice versa*. If the weight was to be first increased and then diminished the operator brought the pressure exerted by the kilogram alone upon the finger of the subject by means of the lever and cam mentioned above, and when the subject said "change" he gently lowered the differential weight, resting in the small pan, upon the pan of the balance. The subject, having appreciated the sensation, again said "change," whereupon the operator removed the differential weight. If, on the other hand, the color of the card directed the weight to be first diminished and then increased, the operator had the differential weight already on the pan of the balance before the pressure was brought to bear on the finger, and made the reverse changes at the command of the subject. The subject then stated his judgment and also his degree of confidence, whereupon the total pressure was at once removed by the cam, and the card that had been used to direct the change was placed face down or face up according as the answer was right or wrong, and with corner indicating the degree of confidence in a determinate position. By means of these trifling devices the important object of rapidity was secured, and any possible psychological guessing of what change the operator was likely to select was avoided. A slight disadvantage in this mode of proceeding arises from the long runs of one particular kind of change, which would occasionally be produced by chance and would tend to confuse the mind of the subject. But it seems clear that this disadvantage was less than that which would have been occasioned by his knowing that there would be no such long runs if any means had been taken to prevent them. At the end of each set the results were of course entered into a book.†7

Peirce: CP 7.30 Cross-Ref:††

30. The following tables show the results of the observations for each day:

Ratios of pressures. [Subject: Mr. Peirce.]

Date. 1.100 1.080 1.060 1.050 1.040 1.030 1.015

Dec. 10 2 errors 13 errors

Dec. 13 4 errors 8 errors 15 errors

Dec. 17	11	20 errors
Dec. 20	7	16 21 errors
Jan. 3	14	20 28
Jan. 15	15	29 17
Jan. 22	12	16 20
Jan. 24	6	15 22

Means 2 4 10.4±1.0 13 15 19.3±1.4 21.6±1.1

Calculated

from

probable

error

=0.051 4.6±1.0 7.2±1.6 10.7±0.8 12.7±2.1 14.9±2.2 17.2±0.9 21.0±1.1.

Average

confidence.

Observed 1.9 0.9 0.7 0.8 0.3 0.3 0.2

Calculated 1.3 1.0 0.7 0.6 0.5 0.3 0.2

Peirce: CP 7.30 Cross-Ref:††

The numbers in the columns show the number of errors in fifty experiments. With the average number of errors in a set of fifty we compare the theoretical value of this average as calculated by the method of least squares. The number .051 thus obtained in this case best satisfies the mean number of errors. The numbers affixed with a sign denote, in the upper row the observed (*a posteriori*) probable error of the mean value as given, in the lower row the calculated (*a priori*) probable error. The last two lines give the average confidence observed and calculated with each variation of the ratios of pressure. It will be seen that the correspondence between the real and theoretical numbers is close, and closest when the number of sets is large. The probable errors also closely correspond, the observed being, as is natural, slightly larger than the calculated probable errors.

Peirce: CP 7.31 Cross-Ref:††

31. The following is a similar table for Mr. Jastrow as subject:

Ratios of pressures.

Date. 1.100 1.080 1.060 1.050 1.040 1.030 1.020 1.015 1.010 1.005

Dec. 10 5 19
 Dec. 13 9 15 15
 Dec. 17 14 23
 Dec. 20 10 17 25
 Jan. 3 8 14 24
 Jan. 10 7 13 17
 Jan. 15 12 6 22
 Jan. 22 11 10 16
 Jan. 24 4 11 18
 Feb. 11 1 7 18
 Feb. 17 2 10 17
 Feb. 18 2 11 17
 Feb. 24 2 8 15
 March 4 13 16
 March 5 13 17
 March 18..... 14 19 29
 March 19..... 11 21 18
 March 23..... 14 17 18
 March 25..... 12 16 18
 March 30..... 11 16 21
 March 31..... 10 15 21
 April 2 11 17 21
 April 3 9 18 20
 April 6 12 15 21
 April 7 0 5 7 14 15 17

Means 5 9 6.6 19 15.0 11.6 11.4 18.9 16.8 20.5

Peirce: CP 7.32 Cross-Ref:††

32. It would obviously be unfair to compare these numbers with any set of theoretical numbers, since the probable error is on the decrease throughout, owing to effects of practice, etc. For various reasons we can conveniently group these experiments into four groups. The first will include the experiments from December 10 to January 22, inclusive; the second from January 24 to February 24, inclusive; the third from March 4 to March 25, inclusive; the fourth from March 30 to the end of the work.

Peirce: CP 7.33 Cross-Ref:††

33. The mean results for the different groups are exhibited in the following tables:

First group.

[Probable error=0.05.]

Average number of		Average confidence.			
errors.					
-----		-----			
Calculated					
from					
Ratios of	Number of	probable			
pressures.	sets of 50.	Observed.	error.	Observed.	Calculated.

1.100	1	5	4.4±1.4	0.9	1.5
1.080	1	9	7.0±1.7	0.9	1.2
1.060	7	11.0±0.7	10.4±0.7	0.85	0.9
1.050	1	19	12.5±2.1	0.35	0.7
1.040	1	15	14.7±2.2	0.3	0.6
1.030	6	13.8±1.5	17.0±0.9	0.5	0.4
1.015	5	20.8±1.1	21.0±1.1	0.3	0.2

Second group.

[Probable error=0.0235.]

1.060	5	2.2±0.3	2.1±0.4	1.0	1.2
1.030	5	9.4±0.6	9.6±0.8	0.55	0.6
1.015	5	17.0±0.3	16.6±1.0	0.3	0.3

Third group.

[Probable error=0.02.]

Average number of

errors.

Average confidence.

Calculated

from

Ratios of Number of probable
pressures. sets of 50. Observed. error. Observed. Calculated.

1.020	6	12.8±0.3	12.5±0.8	0.12	0.12
1.010	6	17.7±0.6	18.3±0.9	0.07	0.06
1.005	4	20.7±1.7	21.6±1.2	0.00	0.03

Fourth group.

[Probable error = 0.0155.]

1.060	1	0	0.8±0.6	1.6
1.030	1	5	4.8±1.4	0.5	0.4
1.020	6	10.0±0.5	9.6±0.8	0.1	0.2

1.015	1	14	12.8±2.1	0.1	0.13
1.010	6	16.0±0.3	16.5±0.9	0.05	0.12
1.005	6	20.8±0.4	20.6±1.0	0.00	0.06

Peirce: CP 7.34 Cross-Ref:††

34. The tables show that the numbers of errors follow, as far as we can conveniently trace them, the numbers assigned by the probability curve,†8 and therefore destroy all presumption in favor of an *Unterschiedsschwelle*. The introduction and retention of this false notion can only confuse thought, while the conception of the mathematician must exercise a favorable influence on psychological experimentation.†9

Peirce: CP 7.35 Cross-Ref:††

35. The quantity which we have called the degree of confidence was probably the secondary sensation of a difference between the primary sensations compared. The evidence of our experiments seems clearly to be that this sensation has no *Schwelle*, and vanishes only when the difference to which it refers vanishes. At the same time we found the subject often overlooked this element of his field of sensation, although his attention was directed with a certain strength toward it, so that he marked his confidence as *zero*. This happened in cases where the judgments were so much affected by the difference of pressures as to be correct three times out of five. The general fact has highly important practical bearings, since it gives new reason for believing that we gather what is passing in one another's minds in large measure from sensations so faint that we are not fairly aware of having them, and can give no account of how we reach our conclusions about such matters.†10 The insight of females as well as certain "telepathic" phenomena may be explained in this way.†11 Such faint sensations ought to be fully studied by the psychologist and assiduously cultivated by every man.

Peirce: CP 7.36 Cross-Ref:††
 §2. LATER REFLECTIONS †12

36. Our knowledge of any subject never goes beyond collecting observations and forming some half-conscious expectations, until we find ourselves confronted with some experience contrary to those expectations.†13 That at once rouses us to consciousness: we turn over our recollections of observed facts; we endeavour so to rearrange them, to view them in such new perspective that the unexpected experience shall no longer appear surprising. This is what we call explaining it, which always consists in supposing that the surprising facts that we have observed are only one part of a larger system of facts, of which the other part has not come within the field of our experience, which larger system, taken in its entirety, would present a certain character of reasonableness, that inclines us to accept the surmise as true, or likely. For example, let a person entering a large room for the first time, see upon a wall projecting from behind a large map that has been pinned up there, three-quarters of an admirably executed copy in fresco of one of Rafael's most familiar cartoons. In this

instance the explanation flashes so naturally upon the mind and is so fully accepted, that the spectator quite forgets how surprising those facts are which alone are presented to his view; namely, that so exquisite a reproduction of one of Rafael's grandest compositions should omit one-quarter of it. He guesses that that quarter is there, though hidden by the map; and six months later he will, maybe, be ready to swear that he saw the whole. This will be a case under a logico-psychical law of great importance, to which we may find occasion to revert soon, that a fully accepted, simple, and interesting inference tends to obliterate all recognition of the uninteresting and complex premisses from which it was derived. The brighter the observer's intelligence (unless some circumstance has raised a doubt), the more confident he will soon be that he saw the entire composition. Yet, in fact, the idea of the whole's being on that wall will be merely evolved from his *Ichheit*: it will be a surmise, conjecture, or guess.

Peirce: CP 7.37 Cross-Ref:††

37. We may be aided by previous knowledge in forming our hypotheses. In that case they will not be pure guesses but will be compounds of deductions from general rules we already know, applied to the facts under observation, for one ingredient, and pure guess for the other ingredient. Thus, suppose the surprising facts which puzzle us are the actions of a certain man on a certain occasion; and our conjecture relates to the state of belief that caused such conduct. If we have no previous knowledge of the man, any one state of belief that would account for his conduct might be as good a guess as any other; but if we know that he is particularly inclined, or particularly disinclined, to extravagant beliefs or to any other special kind of belief, we still have to guess; only we shall select our guess from a smaller number of possible hypotheses.

Peirce: CP 7.38 Cross-Ref:††

38. In the evolution of science, guessing plays the same part that variations in reproduction take in the evolution of biological forms, according to the Darwinian theory.†14 For just as, according to that theory, the whole tremendous gulf, or ocean rather, between the moner and man has been spanned by a succession of infinitesimal fortuitous variations at birth, so the whole noble organism of science has been built up out of propositions which were originally simple guesses. For my part I refuse to believe that either the one or the other were *fortuitous*; and indeed I gravely doubt whether there be any tenable meaning in calling them so. As to the biological variations, I will spare the reader my reasons for not believing them fortuitous. For it would only lead us away from our subject. But as to the first guesses out of which science has been developed, I will say a word or two. It is well within bounds to reckon that there are a billion (i.e., a million million) hypotheses that a fantastic being might guess would account for any given phenomenon. For this phenomenon would certainly be more or less connected in the mind of such a being with a million other phenomena (for he would not be restricted to contemporaneous events) and it might be supposed that the special determination of each was connected with the special determinations of each of the others in order to produce the observed phenomenon. I will not carry out this idea further: it suffices to show that according to the doctrine of chances it would be practically impossible for any being, by pure chance, to guess the cause of any phenomenon.

Peirce: CP 7.39 Cross-Ref:††

39. There are, indeed, puzzles, and one might well say mysteries, connected with the mental operation of guessing; -- yes; more than one. There can, I think, be no

reasonable doubt that man's mind, having been developed under the influence of the laws of nature, for that reason naturally thinks somewhat after nature's pattern. This vague explanation is but a surmise; but there is no room to believe that it was merely by luck that Galileo and other masters of science reached the true theories after so few wrong guesses as they did. This power of divining the truths of physics, -- for such it is, although it is somewhat imperfect, -- is certainly an aid to the instinct for obtaining food, an instinct whose wonders throughout the animal kingdom are exceeded only by that of producing and rearing offspring.

Peirce: CP 7.40 Cross-Ref:††

40. This latter function requires all the higher animals to have some insight into what is passing in the minds of their fellows. Man shows a remarkable faculty for guessing at that. Its full powers are only brought out under critical circumstances. . . .†15

Peirce: CP 7.41 Cross-Ref:††

41. All the above, be it understood is sober truth, sedulously freed from all exaggeration and colour. If any reader should incline to deem the narrative apocryphal, it will certainly not be the psychologist, equally versed in the theory of his science and skilled in the application of it; for to him the incidents will present no extraordinary features. I suppose almost everybody has had similar experiences. But however frequently such facts may be encountered, there is certainly something a little mysterious in them; they demand explanation. That explanation must itself be conjectural, and must remain so until exact investigation has tested its sufficiency; and unless some new school of psychology should make its appearance, I do not believe that scientific testing of the theory is likely to be performed in our time.

Peirce: CP 7.42 Cross-Ref:††

42. I am going to point out a *vera causa* -- a known agency which tends to produce effects like the facts to be explained. But whether it would, under the circumstances described, be sufficient to produce the somewhat surprising facts, or whether it was aided by some other agency that has not suggested itself to my mind, I will not presume to opine.

Peirce: CP 7.43 Cross-Ref:††

43. My surmise is that at the bottom of the little mystery is buried a principle often enough asserted but never, I believe, supported by scientific observation, until Professor Joseph Jastrow and I carried through, at the Johns Hopkins University, a certain series of experiments.†16 These experiments were mainly designed for quite another purpose, namely, in order to test Fechner's hypothesis of the "Differenzschwelle," which in no wise concerns us now. I proceed to describe, in outline, the essentials of the experiments. Of the two persons engaged in them, the one acted as experimenter and recorder, while the other, who could neither see nor hear the former, was the "subject" or victim of the experimentation. The latter said, "Ready." Thereupon an automatic arrangement, namely, by exposing a card from a well shuffled pack, indicated to the experimenter what pressure he was to bring to bear upon the finger of the subject, who carefully observed the degree of his feeling of pressure. When he was satisfied, perhaps after from five to twenty seconds, he said "Change." Thereupon by an exceedingly delicate contrivance (to avoid any sudden change or shock), the experimenter, according to an automatic operation of chance, either increased or diminished the pressure by less than one *per cent* of itself. The subject observed the new feeling of pressure, and again said "Change," whereupon the first pressure was brought back. These experiments were interspersed (by the

automatic chance arrangement which was intended, of course, to exclude, as far as possible, mental action on the part of the experimenter), by others in which the changes of pressure were somewhat more considerable. The subject having observed the three states of feeling of pressure (of which the first and last were equal), first pronounced one or another of the four numerals, Naught, One, Two, Three. "Three" would mean that he was sure, or almost sure, of being able to say whether the middle pressure was greater or less than the other two. "Two" would mean that he was by no means sure, yet inclined to think he could tell. "One" would mean that he did not think he really perceived any difference; yet suspected that he perhaps might. "Naught" would mean that he was sure he could not perceive the slightest variation of pressure. Having thus indicated the degree of his confidence, he was *obliged to say* whether the middle pressure was greater or less than the others. In case his confidence was zero, this declaration would be (to his own consciousness) a purely random one, though he would avoid any particular regularity in his declarations, or any great preponderance of either "greater" or "lesser." Of course he never received the slightest intimation of whether he was right or wrong.

Peirce: CP 7.44 Cross-Ref:††

44. When our course of experiments had been carried on two hours daily (with such precautions against fatigue as the imperfect psychology of twenty-five years ago prescribed), and for about a month it was found that of the answers supposed to be given at random, which were a good half of the whole number and must, I think (I have not before me the record, which is given in Vol. III of the Memoirs of the U. S. National Academy of Sciences),†17 have approached a thousand in number, about three out of every five were correct. That is to say, among all those cases in which the subject, after carefully searching his consciousness, felt quite sure he had experienced no variation of the sense of pressure, though a change and reverse change had really been made; and had accordingly said, quite at random, as he thought, that the middle pressure was greater or less than the first and last, what he so said agreed with the real fact half as often again as it disagreed. A reader inexpert in dealing with probabilities may think that so small a preponderance of true answers might have come about by chance. But in truth it is among the most certain things that we know that this was not so. So much is demonstrated truth, quite unquestionable. But if you go on to ask me upon what principle I would explain the fact that a person who, after the closest scrutiny of his consciousness, had pronounced that there was no trace of perceptible difference between two sensations of pressure, should in the very next breath have correctly said which of them was the greater, in three cases out of every five, my confidence largely evaporates. I can, indeed, mention a cause which undoubtedly exists and which must have acted toward producing that indubitable fact; but I cannot say whether that cause would or would not have been sufficient, by itself, for that result.

Peirce: CP 7.45 Cross-Ref:††

45. Everybody knows how self-consciousness makes one awkward and may even quite paralyze the mind. Nobody can have failed to remark that mental performances that are gone through with lightly are apt to be more adroit than those in which every little detail is studied while the action is proceeding, nor how a great effort -- say to write a particularly witty letter -- or even to recall a word or name that has slipped one's memory may spoil one's success. Perhaps it is because in trying very hard we are thinking about our effort instead of about the problem in hand. At any rate my own experience is that self-consciousness, and especially conscious effort, are apt to carry me to the verge of idiocy and that those things that I have done

spontaneously were the best done. Now, in the experiments I have described, the so-called "subject," the victim of the experimentation, would not seldom sit in the darkened and silent room, straining with all his might for two or three minutes, to detect the slightest difference between two pressures. Finding himself unable to do so he would utter his "zero" that this inability might be recorded. Thereupon all straining ceased; for all it then remained for him to do was mention at random which one of the pressures he would mark as the heavier -- and here his perfect unconsciousness greatly increased his power of discrimination -- a discrimination below the surface of consciousness, and not recognized as a real judgment, yet in very truth a genuine discrimination, as the statistical results showed. The circumstances of my talking with the waiters on the boat were almost identical. While I was going through the row, chatting a little with each, I held myself in as passive and receptive a state as I could. When I had gone through the row I made a great effort to detect in my consciousness some symptoms of the thief, and this effort, I suppose, prevented my success. But then finding I could detect nothing I said to myself, "Well, anyway, I *must* fasten on someone, though it be but a random choice," and instantly I *knew* which of the men it was. . . .

Peirce: CP 7.46 Cross-Ref:††

46. I could tell many other true tales of successful guessings; but I have mentioned here two principles which I have been led to conjecture furnish at least a partial explanation of the mystery that overhangs this singular guessing instinct. I infer in the first place that man divines something of the secret principles of the universe because his mind has developed as a part of the universe and under the influence of these same secret principles; and secondly, that we often derive from observation strong intimations of truth, without being able to specify what were the circumstances we had observed which conveyed those intimations.

Peirce: CP 7.47 Cross-Ref:††

47. It is a chapter of the art of inquiry.

Peirce: CP 7.48 Cross-Ref:††

48. Our faculty of guessing corresponds to a bird's musical and aeronautic powers; that is, it is to us, as those are to them, the loftiest of our merely instinctive powers. I suppose that if one were sure of being able to discriminate between the intimations of this instinct and the self-flatteries of personal desire, one would always trust to the former. For I should not rate high either the wisdom or the courage of a fledgling bird, if, when the proper time had come, the little agnostic should hesitate long to take his leap from the nest on account of doubts about the theory of aerodynamics.

Peirce: CP 7.49 Cross-Ref:††

BOOK II

SCIENTIFIC METHOD

CHAPTER 1

SCIENTIFIC METHOD

§1. SCIENCE †1

49. What is Science? We cannot define the word with the precision and concision with which we define *Circle*, or *Equation*, any more than we can so define *Money*, *Government*, *Stone*, *Life*. The idea, like these, and more than some of them, is too vastly complex and diversified. It embodies the epitome of man's intellectual development. We can only single out some leading properties of it, and different people will select these differently. To most men, including all who are outside of the world of science, the term means a particular kind of knowledge. Wherein lies the essential peculiarity of this knowledge? Some thinkers agree with the ancient Greeks in making it consist in the *Method* of knowing, the manner in which the truth is laid hold on. But the majority of modern writers regard the Systematic character of the doctrine itself as more characteristic. Both marks of scientific knowledge are exceedingly important; but the former is deeper cut, and because it is at present less noticed, more needs to be emphasized. Plato is quite right in saying that a true belief is not necessarily knowledge. A man may be willing to stake his life upon the truth of a doctrine which was instilled into his mind before his earliest memories without knowing at all why it is worthy of credence; and while such a faith might just as easily be attached to a gross superstition as to a noble truth, it may, by good luck, happen to be perfectly true. But can he be said to *know* it? By no means: to render the word knowledge applicable to his belief, he must not only believe it, but must know, -- I will not say, with the ancients, the rationale of the real fact, as a reality, -- but must know what justifies the belief, and just WHY and HOW the justification is sufficient. I beg that the reader will turn this over in his mind and satisfy himself as to how far what I am saying is true. For it is not a very simple point but is one that I intend to insist upon.

Peirce: CP 7.50 Cross-Ref: ††

50. Before knowledge of any subject can be put to any extensive use, it is almost indispensable that it should be made as thorough and complete as possible, until every detail and feature of the matter is spread out as in a German handbook. But if I am asked to what the wonderful success of modern science is due, I shall suggest that to gain the secret of that, it is necessary to consider science as living, and therefore not as knowledge already acquired but as the concrete life of the men who are working to find out the truth. Given a body of men devoting the sum of their energies to refuting their present errors, doing away with their present ignorance, and that not so much for themselves as for future generations, and all other requisites for the ascertainment of truth are insured by that one. Strictly speaking, one need not ask for so much as that. Given the oxygen, hydrogen, carbon, nitrogen, sulphur, phosphorus, etc., in sufficient quantities and under proper radiations, and living protoplasm will be produced, will develop, will gain power of self-control, and the

scientific passion is sure to be generated. Such is my guess. Science was preordained, perhaps, on the Sunday of the *Fiat lux*.

Peirce: CP 7.51 Cross-Ref:††

51. Coming down to the more immediate and more pertinent causes of the triumph of modern science, the considerable numbers of the workers, and the singleness of heart with which, -- (we may forget that there are a few self-seekers who succeed in gaining the power to make themselves more despised than they naturally would be; they are so few,) -- they cast their whole being into the service of science lead, of course, to their unreserved discussions with one another, to each being fully informed about the work of his neighbour, and availing himself of that neighbour's results; and thus in storming the stronghold of truth one mounts upon the shoulders of another who has to ordinary apprehension failed, but has in truth succeeded by virtue of the lessons of his failure. This is the veritable essence of science. It is in the memory of these concrete living gests that we gain the speaking portraiture of true science in all her life and beauty.

Peirce: CP 7.52 Cross-Ref:††

52. The point of view just explained enables us to perceive that a particular branch of science, such as Physical Chemistry or Mediterranean Archeology, is no mere word, manufactured by the arbitrary definition of some academic pedant, but is a real object, being the very concrete life of a social group constituted by real facts of inter-relation, -- as real an object as a human carcass, which is made one by the inter-relations of its millions of cells. Any two of these groups (and with them the sciences, which are their lives,) may be related, as to the matter of the groups in either of the three modes of relationship of material wholes; that is, either by *Inclusion*, one being a part of another; or by *Intersection*, when each has one part in common with the other, and another part foreign to the other; or by *Exclusion*, when the two have no part in common. But of greater importance are the dynamical relations between the different sciences, by which I mean that one often acts upon another, not by bringing forward any reason or principle, but as it were with a compulsive quality of action. Thus one group may stimulate another by demanding the solution of some problem. In this way, the practical sciences incessantly egg on researches into theory. For considerable parts of chemical discovery we have to thank the desire to find a substitute for quinine or to make quinine itself synthetically, to obtain novel and brilliant dye-stuffs, and the like. The mechanical theory of heat grew out of the difficulties of steam navigation. For it was first broached by Rankine while he was studying how best to design marine engines. Then again, one group of scientists sometimes urges some overlooked phenomenon upon the attention of another group. It was a botanist who called van't Hoff's attention to the dependence of the pressure of sap in plants upon the strength of the solution, and thus almost instantaneously gave a tremendous impulse to physical chemistry. In 1820, Kästner, a manufacturer of cream of tartar in Mulhouse, called the attention of chemists to the occasional, though rare, occurrence in the wine casks of a modification of tartaric acid, since named racemic acid; and from the impulse so given has resulted a most important doctrine of chemistry, that of the unsymmetric carbon atom, as well as the chief discoveries of Pasteur, with their far-reaching blessings to the human species.

Peirce: CP 7.53 Cross-Ref:††

53. It is now time to explain the classification of this chapter, what it aims to be, by what means that aim has been pursued, and how nearly it seems to have been

attained. Two questions have to be answered at the outset: What is here meant by *science*? And what is meant by *a* science, one of the unit species out of which the system is built up? The spirit of this book is always to look upon those aspects of things which exhibit whatever of living and active there is in them.

Peirce: CP 7.54 Cross-Ref:††

54. The prevalent definition of a science, the definition of Coleridge, which influenced all Europe through the Encyclopaedia Metropolitana, that science is systematized knowledge, is an improvement upon a statement of Kant (Metaphysische Anfangsgründe der Naturwissenschaft: 1786): "Eine jede Lehre, wenn sie ein System, dass ist, ein nach Principien geordnetes Ganzes der Erkenntniss sein soll, heisst Wissenschaft."†2 Yet it is to be noted that knowledge may be systematic or "organized," without being organized by means of general principles. Kant's definition, however, is only a modification of the ancient view that science is the knowledge of a thing through its causes, -- the comprehension of it, as we might say, -- as being the only perfect knowledge of it. In short, the Coleridgian definition is nothing but the last development of that sort of philosophy that strives to draw knowledge out of the depths of the *Ich-heit*. If, on the other hand, one opens the works of Francis Bacon, one remarks that, with all the astounding *greenness* and inexperience of his views of science, in some respects he is really a scientific man himself. He met his death as the consequence of an experiment. True, it was rather a foolish one; but what a monument to the genuineness of his intelligence, that he, a great legal light, should, at the age of sixty-six, have perished from his zeal in performing disagreeable and dangerous laboratory work that he thought might go toward teaching him something of the nature of true science! For him man is nature's interpreter; and in spite of the crudity of some anticipations, the idea of science is, in his mind, inseparably bound up with that of a life devoted to singleminded inquiry. That is also the way in which every scientific man thinks of science. That is the sense in which the word is to be understood in this chapter. Science is to mean for us a mode of life whose single animating purpose is to find out the real truth, which pursues this purpose by a well-considered method, founded on thorough acquaintance with such scientific results already ascertained by others as may be available, and which seeks coöperation in the hope that the truth may be found, if not by any of the actual inquirers, yet ultimately by those who come after them and who shall make use of their results. It makes no difference how imperfect a man's knowledge may be, how mixed with error and prejudice; from the moment that he engages in an inquiry in the spirit described, that which occupies him is *science*, as the word will here be used.

Peirce: CP 7.55 Cross-Ref:††

55. By a specific science will be meant a group of connected inquiries of sufficient scope and affinity fitly to occupy a number of independent inquirers for life, but not capable of being broken up into smaller coexclusive groups of this description. For since we are to consider science in general as a mode of life, it is proper to take as the unit science the scientific mode of life fit for an individual person. But science being essentially a mode of life that seeks coöperation, the unit science must, apparently, be fit to be pursued by a number of inquirers.

Peirce: CP 7.56 Cross-Ref:††

56. It seems plain that, with these definitions, the classification cannot be concerned with all possible sciences, but must be confined to actually realized sciences. If, however, this limitation is to be maintained, the question will arise, To what date or stage of scientific development is the classification to relate? According

to the general spirit of this book, which values everything in its relation to Life, knowledge which is altogether inapplicable to the future is nugatory. Consequently, our classification ought to have reference to the science of the future, so far as we are now able to foresee what the future of science is to be. It will therefore be upon the soil of the near future of science that we shall endeavor to plant our flag. If it be objected that we cannot know enough of the science of the future to classify it accurately, the reply would be that even if all faults of classification could be eliminated by remaining on the threshold of the future, it would still be necessary to advance further. For all the applicability of any writing, though it be not (like this,) the fruit of near half a century of study, must evidently be subsequent to its composition, and all its significance for that time has reference to a time still later. But when the objector comes to see the various imperfections that will have to be confessed in that part of the classification which concerns the present state of science, he will probably be disposed himself to acknowledge that its standard will not be much lowered by the danger of mistake about what is likely soon to be discovered.

Peirce: CP 7.57 Cross-Ref:††

57. Meantime, let it not be understood that the classification is to ignore the scientific discoveries of the past. For the memoirs of that work are not so poor as not to merit being read critically, precisely as we shall read the memoirs of tomorrow. Such reading is, therefore, of the nature of scientific inquiry. True, it is not original research; but there is original research still to be done in the same specific science. For none of the sciences of the past is finished. If it be one of the positive sciences that is in question, there is not a single conclusion belonging to it which has in the past been made sufficiently precise or sufficiently indubitable. If it be a branch of mathematics, its propositions require to be further generalized, as well as to be more accurately limited. For these reasons all the old science that still stands is to be retained in the classification, but in its most modern forms.

Peirce: CP 7.58 Cross-Ref:††

58. The only remaining instinct on our list is the Gnostic Instinct, or curiosity. In one sense, the sciences that are practically ministrant to this are the Theoretical Sciences; but this remark leads us to signalize a distinction the neglect of which is the source of several of the most fatal errors into which philosophers have fallen. It is quite true that the Gnostic Instinct is the cause of all purely theoretical inquiry, and that every discovery of science is a gratification of curiosity. But it is not true that pure science is or can be successfully pursued *for the sake* of gratifying this instinct. Indeed, if it were so pursued, it would not be true that this instinct was the cause of it. Its motive would then be the Gust-Instinct, or love of pleasure. One wish may be that another wish should be gratified; but no wish can be that that very wish should be gratified. For in that case, the wish would not have any object at all, and having no object it would not be a wish. The case is precisely like that of an assertion which should have no other subject than itself. For a wish is a sort of proposition. To long for anything is to judge it to be good and urgently good. No doubt every assertion implies that it is itself true;†3 but it cannot consist of that alone; and so every wish that is reflective wishes itself gratified; but it must wish something else, besides. Hence, the hedonist, who opines that man can wish for nothing but pleasure, has fallen into a damnable error from a mere confusion of thought. We should commit the same error if we supposed the gratification of curiosity were the sole, or the principal, object of theoretical science. Curiosity is their motive; but the gratification of

curiosity is not their aim.

Peirce: CP 7.59 Cross-Ref:††
§2. LOGIC AND SCIENTIFIC METHOD †4

59. It might be supposed that logic taught that much was to be accomplished by mere rumination, though every one knows that experiment, observation, comparison, active scrutiny of facts, are what is wanted, and that mere *thinking* will accomplish nothing even in mathematics. Logic had certainly been defined as the "art of thinking," and as the "science of the normative laws of thought." But those are not true definitions. "*Dyalectica*," says the logical text-book of the middle ages, "*est ars artium et scientia scientiarum, ad omnium aliarum scientiarum methodorum principia viam habens*,"†5 and although the logic of our day must naturally be utterly different from that of the Plantagenet epoch, yet this general conception that it is the *art of devising methods of research*, -- the *method of methods*, -- is the true and worthy idea of the science. Logic will not undertake to inform you what kind of experiments you ought to make in order best to determine the acceleration of gravity, or the value of the Ohm; but it will tell you how to proceed to form a plan of experimentation.

Peirce: CP 7.60 Cross-Ref:††

60. It is impossible to maintain that the superiority of the science of the moderns over that of the ancients is due to anything but a better *logic*. No one can think that the Greeks were inferior to any modern people whatever in natural aptitude for science. We may grant that their opportunities for research were less; and it may be said that ancient astronomy could make no progress beyond the Ptolemaic system until sufficient time had elapsed to prove the insufficiency of Ptolemy's tables. The ancients could have no dynamics so long as no important dynamical problem had presented itself; they could have no theory of heat without the steam-engine, etc. Of course, these causes had their influence, and of course they were not the main reason of the defects of the ancient civilization. Ten years' astronomical observations with instruments such as the ancients could have constructed would have sufficed to overthrow the old astronomy. The great mechanical discoveries of Galileo were made with no apparatus to speak of. If, in any direction whatever, the ancients had once commenced research by right methods, opportunities for new advances would have been brought along in the train of those that went before. But read the logical treatise of Philodemus; see how he strenuously argues that inductive reasoning is not utterly without value, and you see where the fault lay. When such an elementary point as that needed serious argumentation it is clear that the conception of scientific method was almost entirely wanting.

Peirce: CP 7.61 Cross-Ref:††

61. Modern methods have created modern science; and this century, and especially the last twenty-five years, have done more to create new methods than any former equal period. We live in the very age of methods. Even mathematics and astronomy have put on new faces. Chemistry and physics are on completely new tracks. Linguistics, history, mythology, sociology, biology, are all getting studied in new ways. Jurisprudence and law have begun to feel the impulse, and must in the future be more and more rapidly influenced by it.

Peirce: CP 7.62 Cross-Ref:††

62. This is the age of methods; and the university which is to be the exponent of the living condition of the human mind, must be the university of methods.

Peirce: CP 7.63 Cross-Ref:††

63. Now I grant you that to say that this is the age of the development of new methods of research is so far from saying that it is the age of the theory of methods, that it is almost to say the reverse. Unfortunately practice generally precedes theory, and it is the usual fate of mankind to get things done in some boggling way first, and find out afterward how they could have been done much more easily and perfectly. And it must be confessed that we students of the science of modern methods are as yet but a voice crying in the wilderness, and saying prepare ye the way for this lord of the sciences which is to come.

Peirce: CP 7.64 Cross-Ref:††

64. Yet even now we can do a little more than that. The theory of any act in no wise aids the doing of it, so long as what is to be done is of a narrow description, so that it can be governed by the unconscious part of our organism. For such purposes, rules of thumb or no rules at all are the best. You cannot play billiards by analytical mechanics nor keep shop by political economy. But when new paths have to be struck out, a spinal cord is not enough; a brain is needed, and that brain an organ of mind, and that mind perfected by a liberal education. And a liberal education -- so far as its relation to the understanding goes -- means *logic*. That is indispensable to it, and no other one thing is.

Peirce: CP 7.65 Cross-Ref:††

65. I do not need to be told that science consists of specialties. I know all that, for I belong to the guild of science, have learned one of its trades and am saturated with its current notions.†6 But in my judgment there are scientific men, all whose training has only served to belittle them, and I do not see that a mere scientific specialist stands intellectually much higher than an artisan. I am quite sure that a young man who spends his time exclusively in the laboratory of physics or chemistry or biology, is in danger of profiting but little more from his work than if he were an apprentice in a machine shop.

Peirce: CP 7.66 Cross-Ref:††

66. The scientific specialists -- pendulum swingers †7 and the like -- are doing a great and useful work; each one very little, but altogether something vast. But the higher places in science in the coming years are for those who succeed in adapting the methods of one science to the investigation of another. That is what the greatest progress of the passing generation has consisted in. Darwin adapted to biology the methods of Malthus and the economists; Maxwell adapted to the theory of gases the methods of the doctrine of chances, and to electricity the methods of hydrodynamics. Wundt adapts to psychology the methods of physiology;†8 Galton adapts to the same study the methods of the theory of errors; Morgan adapted to history a method from biology; Cournot adapted to political economy the calculus of variations. The philologists have adapted to their science the methods of the decipherers of dispatches. The astronomers have learned the methods of chemistry; radiant heat is investigated with an ear trumpet; the mental temperament is read off on a vernier.

Peirce: CP 7.67 Cross-Ref:††

67. Now although a man needs not the theory of a method in order to apply it

as it has been applied already, yet in order to adapt to his own science the method of another with which he is less familiar, and to properly modify it so as to suit it to its new use, an acquaintance with the principles upon which it depends will be of the greatest benefit. For that sort of work a man needs to be more than a mere specialist; he needs such a general training of his mind, and such knowledge as shall show him how to make his powers most effective in a new direction. That knowledge is logic.

Peirce: CP 7.68 Cross-Ref:††

68. In short, if my view is the true one, a young man wants a physical education and an aesthetic education, an education in the ways of the world and a moral education, and with all these logic has nothing in particular to do; but so far as he wants an intellectual education, it is precisely logic that he wants; and whether he be in one lecture-room or another, his ultimate purpose is to improve his logical power and his knowledge of methods. To this great end a young man's attention ought to be directed when he first comes to the university; he ought to keep it steadily in view during the whole period of his studies; and finally, he will do well to review his whole work in the light which an education in logic throws upon it.

Peirce: CP 7.69 Cross-Ref:††

69. I should be the very first to insist that logic can never be learned from logic-books or logic lectures. The material of positive science must form its basis and its vehicle. Only relatively little could be done by the lecturer on method even were he master of the whole circle of the sciences. Nevertheless, I do think that I can impart to you something of real utility, and that the theory of method will shed much light on all your other studies.

Peirce: CP 7.70 Cross-Ref:††

70. The impression is rife that success in logic requires a mathematical head. But this is not true. The habit of looking at questions in a mathematical way is, I must say, of great advantage, and thus a turn for mathematics is of more or less service in any science, physical or moral. But no brilliant talent for mathematics is at all necessary for the study of logic.

Peirce: CP 7.71 Cross-Ref:††

71. The course which I am to give this year begins with some necessary preliminaries upon the theory of cognition.†9 For it is requisite to form a clear idea at the outset of what knowledge consists of, and to consider a little what are the operations of the mind by which it is produced. But I abridge this part of the course as much as possible, partly because it will be treated by other instructors, and partly because I desire to push on to my main subject, the method of science.

Peirce: CP 7.72 Cross-Ref:††

72. I next take up syllogism, the lowest and most rudimentary of all forms of reasoning, but very fundamental because it is rudimentary.†10 I treat this after the general style of De Morgan, with references to the old traditional doctrine. Next comes the logical algebra of Boole, a subject in itself extremely easy, but very useful both from a theoretical point of view and also as giving a method of solving certain rather frequently occurring and puzzling problems. From this subject, I am naturally led to the consideration of relative terms. The logic of relatives, so far as it has been investigated, is clear and easy, and at the same time it furnishes the key to many of the difficulties of logic, and has already served as the instrument of some discoveries in mathematics. An easy application of this branch of logic is to the doctrine of breadth and depth or the relations between objects and characters. I next introduce the

conception of number, and after showing how to treat certain statistical problems, I take up the doctrine of chances. A very simple and elegant mathematical method of treating equations of finite differences puts the student into possession of a powerful instrument for the solution of all problems of probability that do not import difficulties extraneous to the theory of probability itself.

Peirce: CP 7.73 Cross-Ref:††

73. We thus arrive at the study of that kind of probable inference that is really distinctive; that is to say, Induction in its broadest sense -- Scientific Reasoning. The general theory of the subject is carefully worked out with the aid of real examples in great variety, and rules for the performance of the operation are given. These rules have not been picked up by hazard, nor are they merely such as experience recommends; they are deduced methodically from the general theory.

Peirce: CP 7.74 Cross-Ref:††

74. Finally, it is desirable to illustrate a long concatenation of scientific inferences. For this purpose we take up Kepler's great work, *De Motu Stellae Martis*, the greatest piece of inductive reasoning ever produced. Owing to the admirable and exceptional manner in which the work is written, it is possible to follow Kepler's whole course of investigation from beginning to end, and to show the application of all the maxims of induction already laid down.

Peirce: CP 7.75 Cross-Ref:††

75. In order to illustrate the method of reasoning about a subject of a more metaphysical kind, I shall then take up the scientific theories of the constitution of matter.

Peirce: CP 7.76 Cross-Ref:††

76. Last of all, I shall give a few lectures to show what are the lessons that a study of scientific procedure teaches with reference to philosophical questions, such as the conception of causation and the like.

Peirce: CP 7.77 Cross-Ref:††

77. I will assume, then, that scientific doubt never gets completely set to rest in regard to any question until, at last, the very truth about that question becomes established.†11 Taking the phenomenon as a whole, then, without considering how it is brought about, science is foredestined to reach the truth of every problem with as unerring an infallibility as the instincts of animals do their work, this latter result like the former being brought about by some process of which we are as yet unable to give any account. It is, we will say, the working of the human instinct. It is not (always considering it in its entirety,) of a rational nature, since, being infallible, it is not open to criticism, while "rational" means essentially self-criticizing, self-controlling and self-controlled, and therefore open to incessant question. But this instinctive infallibility is brought about by the exercise of reason, which is all along subject to blunder and to go wrong. The manner in which this comes about may be, I will not quite say illustrated, but may be rendered intelligible, by the following skeletal example. I call it skeletal because it involves the one character of research which is here to be considered, while attempting no representation of it in other respects. Let us suppose, then, that you have a die which may, for all you know, be loaded; and that you proceed to experiment upon it by throwing it repeatedly, counting as you go the total number of throws and also the number of them which turn up the ace side. For

the sake of simplicity, I will suppose that the die is really perfect, although you do not know that it is so. After you have thrown it six times, it will be more likely to give either no ace or more than one ace than to give just one. Namely, there is one chance in three that there will be no ace in the first six throws, there are two chances in five that there will be just one, one chance in five that there will be just two; and there will remain one chance in fifteen that there will be more than two aces. Suppose you go on throwing the die a great many times, and after each throw you divide the number of aces that have turned up by the whole number of throws so far. The quotient will be [the] result for the probability of throwing an ace with this die. You will get a new and amended, though not always a really improved, result after every throw. Now although the throws are purely fortuitous, so that to most questions about them only probable answers can be given, yet one thing will certainly happen. Namely, sooner or later, probably very soon, but it may be only very late, yet certainly at length, a time will come after which all your values for the probability of throwing an ace with this die will be correct in the first figure after the decimal point. A later time there will be after which all the successive determinations will be correct in the first two figures, and so on. You will never be certain that that time has come, but it certainly some time will have come. Thus to the question, What is the first figure of the probability?; to the question, What are the first two figures, etc.; all the answer you will obtain will after a time be free from error. This will be the necessary result. Now that which is necessarily inerrant may in a somewhat indefinite sense be fairly called *infallible*. Thus, a skillful use of fortuitous events will bring infallibly correct replies to an endless series of questions. This kind of infallibility, which may [be], for aught we know, not to say quite probably is, the infallibility of the instinct of animals, is certainly the only kind of infallibility that can be attributed to the results of science, inasmuch as we can so little know when the very truth is reached that even the second law of motion is at this moment under indictment. Moreover, when we come to subject the processes of science to criticism, we shall find it impossible to deny that a conditional form of this kind of infallibility must be attributed to science.

Peirce: CP 7.78 Cross-Ref: ††

78. In the light of what has been said, what are we to say to that logical fatalism whose stock in trade is the argument that I have already indicated? I mean the argument that science is predestined to reach the truth, and that it can therefore make no difference whether she observes carefully or carelessly nor what sort of formulae she treats as reasons. The answer to it is that the only kind of predestination of the attainment of truth by science is an eventual predestination, -- a predestination *aliquando denique*. Sooner or later it will attain the truth, nothing more. It means that if you take the most pigheaded and passionate of men who has sworn by all the gods that he never will allow himself to believe the earth is round, and give him time enough, and cram that time with experience in the pertinent sphere, and he will surely come to and rest in the truth about the form of the earth. Such is the infallibility of science. But the secret of the matter is that the man's wilfulness and prejudice will break down before such experience. Such, at least, must be our assumption, if we are to adhere to our faith in the infallibility of science. So far as this assumption goes beyond ordinary everyday experience, it rests on the deeper assumption that that which experience has done for generations of men, who a thousand years ago were substantially in that man's plight, it would do for an individual who were to go through the experiences that those generations have gone through. If one does not believe in this, then the present question does not arise. Our belief in the infallibility of science, which alone prompts the fatalistic suggestion, rests upon our experience of the overwhelming rationalizing power of experience. As long as the man keeps to his

determination to exclude from his thoughts whatever might tend to make him assent to the proposition that the earth is round, he certainly will not come to that truth. Granting, therefore, that it is of the nature of experience to develop albuminous matter into rational brain, and to make the mind unceasingly agitate doubt until it finally comes to repose in the true belief, -- which is only a more developed way of formulating our belief in the infallibility of science, it is entirely uncertain *when* the truth will be reached. It will be reached; but only after the investigator has come, first, to a conception of the nature of truth, and to a worship of it as the purest emanation of That which is creating the universe, and *then*, to an understanding of the right method to absorb it from the universe of experience. It will infallibly be reached sooner or later, if favorable conditions continue; but man having a short life, and even mankind not a very long one, the question is urgent, How soon? And the answer is, as soon as a sane logic has had time to control conclusions. Everything thus depends upon rational methods of inquiry. They will make that result as speedy as possible, which otherwise would have kicked its heels in the anteroom of chance. Let us remember, then, that the precise practical service of sound theory of logic is to abbreviate the time of waiting to know the truth, to expedite the predestined result. But I here use the words 'abbreviate' and 'expedite' in a peculiar sense. Imagine a derelict wreck to be floating about on the ocean; and suppose that it will be driven hither and thither until it chances to be cast upon a shore. Then, a vessel which should go and take that derelict in tow and deliberately strand it upon the nearest shore, would be "abbreviating" or "expediting" the fulfillment of the destiny of that derelict in the same sense in which I hold that logic "abbreviates" inquiry, and "expedites" its result. It changes a fortuitous event which may take weeks or may take many decennia into an operation governed by intelligence, which will be finished within a month. This is the sense in which logic "abbreviates" and "expedites" the attainment of truth.

Peirce: CP 7.79 Cross-Ref:††
 §3. SCIENTIFIC METHOD †12

79. Scientific Method: The general method of successful scientific research. The following are some of its characteristics. Cf. *Science*. †13

Peirce: CP 7.80 Cross-Ref:††

80. (1) The student's first step is to form a perfectly definite and consistent idea of what the problem really is; then he ought to develop the mathematics of the subject in hand as far as possible; and to establish a mathematical method appropriate to the particular problem, if it be one which allows exact treatment. As examples and models of what is meant, may be mentioned Maxwell's researches on colour sensation in the *Philos. Trans.* for 1860, Flinders Petrie's book *Inductive Metrology*, the last chapters of Pearson's *Grammar of Science*. Of course, as the student's understanding of the matter advances, he will return to this first task, and continually improve upon his first essays.

Peirce: CP 7.81 Cross-Ref:††

81. The second step will be to consider the logic and methodetic of the research in hand, unless it is itself a question of pure mathematics, where the logic is inseparable from the mathematics. He will do well to study the manner in which questions somewhat analogous to his own have been successfully resolved in widely

different fields; for the greatest advantage has accrued from the extension of methods from one subject to a widely different one, especially from simple to intricate matters.

Peirce: CP 7.82 Cross-Ref:††

82. The third step should be to reform his metaphysics, if the question is a broad one. Perhaps he thinks he has no metaphysics, and does not wish to have any. That will be a sure sign that he is badly handicapped with metaphysics of the crudest quality. The only way to disburden himself of it is to direct his attention to it. But he cannot reduce himself to anything like absolute scepticism in metaphysics without arresting his work.

Peirce: CP 7.83 Cross-Ref:††

83. The fourth step will be to study the laws of the phenomena dealt with, so far as they can be made out at this stage. The general order of discovery in the nomological sciences is first to pick up the phenomena by excursions in those fields in which they are to be found, with alertness of observation, with those clear ideas that make the new fact instantly recognizable as new, and with the energy that seizes upon the faint trace and follows it up. Witness the manner in which all the new phenomena of radiation have been brought to light during the last generation: cathode rays, X rays, Becquerel rays, etc. After making some acquaintance with the phenomena, the next discovery is of their laws (nomological). In the light of one's metaphysics and general conception of the department of truth dealt with, one considers what different hypotheses have any claims to investigation. The leading considerations here will be those of the 'economics' of research.†14 If, for example, a hypothesis would necessitate an experimental result that can be cheaply refuted if it is not true, or would be greatly at variance with preconceived ideas, that hypothesis has a strong claim to early examination. But one must not give up a hypothesis too readily. Many a discovery has been missed by that fault. Gravitation would have been known a decade earlier if Newton had not hastily thought it refuted, and so set back all the subsequent history of physics by something like that amount of time lost. It is likely that thousands of persons more will die of consumption -- as remote as that may seem -- than would have died if he had not made that error. The testing of the hypothesis proceeds by deducing from it experimental consequences almost incredible, and finding that they really happen, or that some modification of the theory is required, or else that it must be entirely abandoned. The law of the phenomena once made out, it only remains to measure with precision the values of the coefficients in the equation which expresses it.

Peirce: CP 7.84 Cross-Ref:††

84. The problem under investigation may not be of a nomological kind. Not that the phenomena are not conceivably subject to law, so that the subject may ultimately be received into the nomological sciences, -- as chemistry, for example, promises some day to mature into a nomological science; but in the present state of knowledge the question, we will suppose, cannot be so studied. Still, a certain amount of nomological study is a necessary preliminary to engaging with the problem itself. Biology calls for aid from physiology. The student who is studying the growth of languages must avail himself of all the knowledge that there is about the physics of speech sounds. In case, then, the question has not yet reached the nomological stage, the sixth step in the work will be of a classificatory nature. Such order, of a more or less imperfect kind, as can be traced in the phenomena must be made out. Students of the classificatory sciences like to call such regularities laws. The tendency is a symptom of health; because it shows that law is their ideal, and that they are striving

to bring their sciences to the nomological stage. But such orderlinesses as 'Grimm's Law' (see *Gender*) and 'Mendeléeef's Law' are not laws in the sense in which the association of ideas and the three laws of motion are laws. They are not satisfactory for a minute. They are nothing that can blend with our metaphysics; they are not of a universal kind; and they are not precise. You may imagine that there might be a chain of more and more universal, precise, and reasonable regularities leading from these to those. But there is, in fact, a great gap, which has to be acknowledged. A hypothesis may be made about the cause of the three laws of motion; but we can have no present hopes of satisfactorily proving the truth of such a thing; while we at once set to work with great hopes of making considerable steps towards explaining Mendeléeef's Law and Grimm's Law. But the most important distinction between true laws and such regularities lies in the very different way in which we proceed to the discovery of the one and of the other. The whole attitude of mind is so different that it is difficult to believe that the same man would have great success in the two tasks. We have seen in our day the establishment of a grand example of each kind, the Law of the *Conservation of Energy* (q.v.) and the Periodic Law. The one dealt with a small number of observations. Exactitude was the main thing. The hypothesis itself sprang almost immediately from the natural light of reason. In the other case, it was necessary with a positive effort to put ideas of exactitude aside and to find order in a great tangle of facts.

Peirce: CP 7.85 Cross-Ref:††

85. Perhaps the problem in hand relates to one of those sciences basely called descriptive, that is, sciences which study, not classes of facts, but individual facts, such as history, descriptive astronomy, geography. No science is merely descriptive. These sciences are investigations of causes. The historian's facts of observation are not those contained in his text, but those mentioned in the foot-notes -- the documents and monuments. †15 It is the supposed causes of these which make the text. Nor is he contented with a mere chronicle of striking public events; he endeavours to show what the hidden causes of them were. So the astronomer's real business is to prove the *Nebular Hypothesis* (q.v.) or whatever ought to replace it. The geologist does not merely make a geological map, but shows how the existing state of things must have come to pass. To do this the historian has to be a profound psychologist, the geologist a master of physics and dynamics. Just as the classificatory sciences tend to become nomological, so the descriptive, or explanatory, sciences tend to become classificatory. The astronomer finds so many examples of systems in formation, that he can formulate the cycle of events through which they generally pass; as the historian formulates cycles through which communities usually pass, and the geologist formulates cycles through which continents commonly pass. These are analogous to the cyclical laws of the classificatory sciences.

Peirce: CP 7.86 Cross-Ref:††

86. But perhaps the problem before the student is not one of theoretical physics or of theoretical psychics, but a practical problem. He wishes to invent. In that case he ought to have a great knowledge both of facts about men's minds and of facts about matter; for he has to adapt the one to the other. He ought to know more than any pure scientist can be expected to know. Of course, as the world goes, he does not.

Peirce: CP 7.87 Cross-Ref:††

87. (2) The most vital factors in the method of modern science have not been the following of this or that logical prescription -- although these have had their value

too -- but they have been the moral factors. First of these has been the genuine love of truth and conviction that nothing else could long endure. Given that men strive after the truth, and, in the nature of things, they will get it in a measure. The greatest difference between the scientific state of the modern scientific era from Copernicus and the middle ages, is that now the whole concern of students is to find out the truth; while then it was to put into a rational light the faith of which they were already possessed. The chief obstacle to the advance of science among students of science in the modern era has been that they were teachers, and feared the effect of this or that theory. But the salvation from this danger has been the fact that there was no vast institution which anybody for a moment hoped could withstand the mighty tide of fact. The next most vital factor of the method of modern science is that it has been made social. On the one hand, what a scientific man recognizes as a fact of science must be something open to anybody to observe, provided he fulfils the necessary conditions, external and internal. As long as only one man has been able to see a marking upon the planet Venus, it is not an established fact. Ghost stories and all that cannot become the subject of genuine science until they can in some way be welded to ordinary experience.†16 On the other hand, the method of modern science is social in respect to the solidarity of its efforts. The scientific world is like a colony of insects, in that the individual strives to produce that which he himself cannot hope to enjoy. One generation collects premises in order that a distant generation may discover what they mean. When a problem comes before the scientific world, a hundred men immediately set all their energies to work upon it. One contributes this, another that. Another company, standing upon the shoulders of the first, strike a little higher, until at last the parapet is attained. Still another moral factor of the method of science, perhaps even more vital than the last, is the self-confidence of it. In order to appreciate this, it is to be remembered that the entire fabric of science has to be built up out of surmises at truth. All that experiment can do is to tell us when we have surmised wrong. The right surmise is left for us to produce. The ancient world under these circumstances, with the exception of a few men born out of their time, looked upon physics as something about which only vague surmises could be made, and upon which close study would be thrown away. So, venturing nothing, they naturally could gain nothing. But modern science has never faltered in its confidence that it would ultimately find out the truth concerning any question in which it could apply the check of experiment.

Peirce: CP 7.88 Cross-Ref:††

88. These are some of the more vital factors of the method of modern science. For the purely logical elements the reader should consult special topics, e.g. *Reasoning*, †17 *Probable Inference*, †18 *Psychophysical Methods*, *Errors of Observation*, *Empirical Logic*, *Variation*, etc.

Peirce: CP 7.89 Cross-Ref:††

89. Verification: It is desirable to understand by a verifiable hypothesis one which presents an abundance of necessary consequences open to experimental tests, and which involves no more than is necessary to furnish a source of those consequences. The verification will not consist in searching the facts in order to find features that accord or disagree with the hypothesis. That is to no purpose whatsoever. The verification, on the contrary, must consist in basing upon the hypothesis predictions as to the results of experiments, especially those of such predictions as appear to be otherwise least likely to be true, and in instituting experiments in order to

ascertain whether they will be true or not.

Peirce: CP 7.90 Cross-Ref:††

90. These experiments need not be experiments in the narrow and technical sense, involving considerable preparation. That preparation may be as simple as it may. The essential thing is that it shall not be known beforehand, otherwise than through conviction of the truth of the hypothesis, how these experiments will turn out. It does not need any long series of experiments, so long as every feature of the hypothesis is covered, to render it worthy of positive scientific credence. What is of much greater importance is that the experiments should be independent, that is, such that from the results of some, the result of no other should be capable of reasonable surmise, except through the hypothesis. But throughout the process of verification the exigencies of the economy of research should be carefully studied from the point of view of its abstract theory.

Peirce: CP 7.91 Cross-Ref:††

91. When, in 1839, Auguste Comte laid down the rule that no hypothesis ought to be entertained which was not capable of verification, it was far from receiving general acceptance. But this was chiefly because Comte did not make it clear, nor did he apparently understand, what verification consisted in. He seemed to think, and it was generally understood, that what was meant was that the hypothesis should contain no facts of a kind not open to direct observation. That position would leave the memory of the past as something not so much as to be entertained as plausible.

Peirce: CP 7.92 Cross-Ref:††

§4. SIMPLICITY †19

92. Parsimony (law of): Ockham's razor, i.e. the maxim 'Entia non sunt multiplicanda praeter necessitatem.' The meaning is, that it is bad scientific method to introduce, at once, independent hypotheses to explain the same facts of observation.

Peirce: CP 7.93 Cross-Ref:††

93. Though the maxim was first put forward by nominalists, its validity must be admitted on all hands, with one limitation; namely, it may happen that there are two theories which, so far as can be seen, without further investigation, seem to account for a certain order of facts. One of these theories has the merit of superior simplicity. The other, though less simple, is on the whole more likely. But this second one cannot be thoroughly tested by a deeper penetration into the facts without doing almost all the work that would be required to test the former. In that case, although it is good scientific method to adopt the simpler hypothesis to guide systematic observations, yet it may be better judgment, in advance of more thorough knowledge, to suppose the more complex hypothesis to be true. For example, I know that men's motives are generally mixed. If, then, I see a man pursuing a line of conduct which apparently might be explained as thoroughly selfish, and yet might be explained as partly selfish and partly benevolent, then, since absolutely selfish characters are somewhat rare, it will be safer for me in my dealings with the man to assume the more complex hypothesis to be true; although were I to undertake an elaborate examination of the question, I ought to begin by ascertaining whether the hypothesis of pure selfishness would quite account for all he does.

Peirce: CP 7.94 Cross-Ref:††

94. The whole aim of science is to find out facts, and to work out a satisfactory theory of them. Still, a theory does not necessarily lose its utility by not being altogether true. . . .

Peirce: CP 7.95 Cross-Ref:††

95. No theory in the positive sciences can be supposed to satisfy every feature of the facts. Although we know that the law of gravitation is one of the most perfect of theories, yet still, if bodies were to attract one another inversely as a power of the distance whose exponent were not 2, but 2.000001, the only observable effect would be a very slow rotation of the line of apsides of each planet. Now the lines of apsides all do rotate in consequence of perturbations, which virtually do alter slightly the sun's attraction, and thus such an effect would probably only produce slight discrepancies in the values obtained for the masses of the planets. In very many cases, especially in practical problems, we deliberately go upon theories which we know are not exactly true, but which have the advantage of a simplicity which enables us to deduce their consequences. This is true of almost every theory used by engineers of all kinds. The most extraordinary departure from the known facts occurs when hydrodynamics is applied, where the theory is in striking opposition to facts which obtrude themselves upon every spectator of moving water. Nevertheless, even in this case, the theory is not useless.

Peirce: CP 7.96 Cross-Ref:††

96. In all the explanatory sciences theories far more simple than the real facts are of the utmost service in enabling us to analyse the phenomena, and it may truly be said that physics could not possibly deal even with its relatively simple facts without such analytic procedure. Thus, the kinetical theory of gases, when first propounded, was obliged to assume that all the molecules were elastic spheres, which nobody could believe to be true. If this is necessary even in physics, it is far more indispensable in every other science, and most of all in the moral sciences, such as political economy. Here the sane method is to begin by considering persons placed in situations of extreme simplicity, in the utmost contrast to those of all human society, and animated by motives and by reasoning powers equally unlike those of real men. Nevertheless, in this way alone can a base be obtained from which to proceed to the consideration of the effects of different complications. Owing to the necessity of making theories far more simple than the real facts, we are obliged to be cautious in accepting any extreme consequences of them, and to be also upon our guard against apparent refutations of them based upon such extreme consequences.

Peirce: CP 7.97 Cross-Ref:††

§5. KINDS OF REASONING †20

97. *First* of all I must establish, as well as I can, the proposition that all Reasoning is either Deduction, Induction, or Retroduction.†21

Peirce: CP 7.98 Cross-Ref:††

98. Unfortunately, I am unable to make this as *evident* as would be desirable,

although I think there is very little room for doubting it, since in the course of a long life of active study of reasonings, during which I have never met with any argument not of a familiar type without carefully analyzing and studying it, I have constantly since 1860, or 50 years, had this question prominently in mind, and if I had ever met with an argument not of one of these three kinds, I must certainly have perceived it. But I never have found any such kind of argument except Analogy, which, as I have shown, is of a nature, -- a mixture of the three recognized kinds. Therefore, it may be taken as substantially certain that I have never in 50 years met with a reasoning of any fourth type.

Peirce: CP 7.99 Cross-Ref:††

99. Now I have not been the only man whose attention would have been roused by the appearance of any such reasoning; and if anybody in the civilized world had found such an argument, I should have heard of it.

Peirce: CP 7.100 Cross-Ref:††

100. Now it is of the nature of a genus of reasoning that it applies to any kind of matter in inexhaustible variety. It is therefore very difficult to believe that there is any kind of reasoning that has not been familiarly employed and known by all the world from time immemorial. On the whole, then, I think my negative experience ought to be pretty convincing, inductively.

Peirce: CP 7.101 Cross-Ref:††

101. Though I do not profess to render it strictly speaking, *evident* that there are but the three types of reasoning, yet it will be interesting to see how nearly I can approach that desideratum.

Peirce: CP 7.102 Cross-Ref:††

102. A sound reasoning justifies us in some kind of belief in the truth of a proposition that in the absence of the reasoning we should not have been so much justified in believing.

Peirce: CP 7.103 Cross-Ref:††

103. In reasoning, one is obliged to *think to oneself*. In order to recognize what is needful for doing this it is necessary to recognize, first of all, what "oneself" is. One is not twice in precisely the same mental state. One is *virtually* (i.e. for pertinent purposes, the same as if one were) a somewhat different person, to whom one's present thought has to be communicated. Consequently, one has to express one's thought so that that virtually other person may understand it. One may, with great advantage, however, employ a language, in thinking to oneself, that is free from much explanation that would be needed in explaining oneself to quite a different person. One can establish conventions with oneself, which enable one to express the essence of what [one] has to communicate free from signs that are not essential. For that reason for example a mathematician has, in thinking of mathematical subjects, an immense advantage. Thus if he has to express to himself a force he will think of $D(2/t)S$, which, he will remember, or can readily see if he should not remember it, is the same as $Ds[1/2(DtS)^2]$.†22 Or he may express the same thing by means of a geometrical diagram, and that in any one of various forms. In like mathematical fashion Existential Graphs †23 enable me here and there greatly to abridge the labor and increase the exactitude of my thought by putting intricate logical relations in the forms that display to me precisely what they involve.

Peirce: CP 7.104 Cross-Ref:††

104. In particular, [the system of] Existential Graphs shows clearly that all logical relations are compounds of the relation of *consequence, provided we look upon identity as so composed*. But Existential Graphs does *not* so regard Identity. That is, it does not assert that to say that the Battle of Waterloo was the final downfall of Napoleon is precisely the same as to say, that if the Battle of Waterloo was the final downfall of Napoleon then for Napoleon to lose that battle as completely as he did, necessarily involved his final overthrow, while if he had not so lost that battle, he would not then and there have been finally overthrown.

Peirce: CP 7.105 Cross-Ref:††

105. My reason in constructing the system of Existential Graphs for not allowing such an identity was that no single *actual event* can follow as logically consequent upon any other, since if it [were] otherwise in the smallest particular, it would be a different event. If in the Battle of Waterloo one man's wound were shifted a hundredth of an inch, or if it had occurred a tenth of a second earlier or later, the Battle would not have been that actual event that did take place; and we never can be in a situation to affirm that under specified circumstances that which did take place must have taken place with such absolute precision; and it is the merest moonshine to claim to know that only as any describable circumstances had taken place the Battle of Waterloo or any other actual historical event *must* have taken place *precisely* as it did. It is a pretty theory although there are grave objections to its precise truth, but to claim to know it is a pretension that I do not think any sober minded man who sufficiently considers the subject will allow himself to make. It has all the ear-marks of the *doctrinaire*, the man who is willing to accept theories as absolutely true. All the difficulties into which metaphysicians contrive to snarl themselves up are traceable to just that doctrinaire disposition. Certainly, I will take care that my system of logic is not inoculated with that easily avoidable but fatal infection.

Peirce: CP 7.106 Cross-Ref:††

106. Therefore, the System of Graphs is so constructed that nothing can be recognized as an apodictic proof that in any circumstances defined in general terms, an event *must* have happened precisely as it did.

Peirce: CP 7.107 Cross-Ref:††

107. But as long as we have to do with general states of things, Existential Graphs analyzes all logical relations into cases of the one relation of *consequence*, that is the relation between one general description of event, A, an antecedent, and another general description of event, C, a consequent, the relation consisting in the fact that whenever A is realized, C will be realized. All known laws of dynamics as well as all other truths consist of such relations.

Peirce: CP 7.108 Cross-Ref:††

108. I will not, therefore, admit that we know anything whatever with *absolute certainty*. †24 It is possible that twice two is not four. For a computer might commit an error in the multiplication of 2 by 2; and whatever might happen once might happen again. Now 2 has never been multiplied by 2 but a finite number of times; and consequently all such multiplications may have been wrong in the same way. It is true that it would be difficult to imagine a greater folly than to attach any serious importance to such a doubt. Still foolish as that would be, its folly would not be so great as to assert that there is some number of repetitions of a multiplication that renders their result, if all agree, absolutely certain. For if this be the case there is some number which is the *least* that is sufficient to produce certainty. Let this number be

denoted by N. Then N-1 repetitions of the multiplication do not yield an absolutely certain result, but one more, if it agree with all the others, will have that result. Consequently a single multiplication will be sufficient to give us *absolute certainty*, that the result is the same, unless some other one of N-1 repetitions should give a different result. Thus, disregarding the particular proposition in question one is driven to maintaining that a single experiment is capable of giving us certain knowledge as to the result of any number of experiments. This is sufficient to show that such an assumption is dangerous in the extreme. It is also absurd from various points of view. The only safety is to say that man is incapable of absolute certainty.

Peirce: CP 7.109 Cross-Ref:††

109. But some one will ask me, "Do you, then, really entertain any doubt that twice two is four?" To this I must answer, "No, as well as I can perceive, there is not the slightest real doubt of it in my mind."†25 "But," he will say, "how can that be? You say it is not certain. Ought you not then, to entertain a doubt of it; and if you feel that it ought to be doubted, do you not, *ipso facto*, actually doubt it?" I reply: "Doubt is a certain kind of feeling. It has not only grades of intensity, but also varieties of quality. Now if I were able to modify my state of mind by a sufficiently slight tincture of the right kind of doubt, I ought to do so. But if I were to attempt really to feel any doubt at all, I should certainly either feel none at all or else millions upon millions of times too much. For I could not in the least recognize a tincture so small nor even one that should be millions of times too great. If I were to devote my whole life to the useless task of trying to make such slight distinctions in my feelings, I could not come near to the requisite delicacy. My feeling of doubt is one of the coarser of my sensations; and there would be no practical use in making it more delicate than it is, for it is already so far more delicate than that of almost all the persons with whom I converse, that I often find an insuperable difficulty in making them comprehend the slighter grades of my feeling, and there is no practical difference in my conduct whether, say, 3/8 or 5/13 be the proper degree of doubt about a matter not measurable. It would be a waste of time to adjust my feeling of doubt more accurately, since it neither would have, nor ought to have, any effect upon my scientific conduct. Instead of wasting effort on my feeling, I devote my energies to learning more about the subjects concerning which I have any considerable doubts, while very small doubts I neglect until I can reduce the amount of my doubt concerning subjects of greater importance."

Peirce: CP 7.110 Cross-Ref:††
§6. KINDS OF INDUCTION †26

110. Suppose we define Inductive reasoning as that reasoning whose conclusion is justified not by there being any necessity of its being true or approximately true but by its being the result of a method which if steadily persisted in must bring the reasoner to the truth of the matter or must cause his conclusion in its changes to converge to the truth as its limit. Adopting this definition, I find that there are three orders of induction of very different degrees of cogency although they are all three indispensable.

Peirce: CP 7.111 Cross-Ref:††

111. The first order of induction, which I will call *Rudimentary Induction*, or

the Pooh-pooh argument, proceeds from the premiss that the reasoner has no evidence of the existence of any fact of a given description and concludes that there never was, is not, and never will be any such thing. The justification of this is that it goes by such light as we have, and that truth is bound eventually to come to light; and therefore if this mode of reasoning temporarily leads us away from the truth, yet steadily pursued, it will lead to the truth at last. This is certainly very weak justification; and were it possible to dispense with this method of reasoning, I would certainly not recommend it. But the strong point of it is that it *is* indispensable. It goes upon the roughest kind of information, upon merely negative information; but that is the only information we can have concerning the great majority of subjects.

Peirce: CP 7.112 Cross-Ref:††

112. I find myself introduced to a man without any previous warning. Now if I knew that he had married his grandmother and had subsequently buried her alive, I might decline his acquaintance; but since I have never heard the slightest suspicion of his doing such a thing, and I have no time to investigate idle surmises, I presume he never did anything of the sort. I know a great many men, however, whose whole stock of reasoning seems to consist in this argument, which they continue to use where there *is* positive evidence and where this argument consequently loses all force. If you ask such a man whether he believes in the liquefaction of the blood of St. Januarius, he will say *no*. Why not? Well, nothing of that kind ever came within the range of my experience. But it did come within the range of Sir Humphrey Davy's experience, who was granted every facility for the thorough investigation of it. His careful report simply confirms the usual allegations with more circumstantial details. You are not justified in pooh-poohing such observations; and that the fact is contrary to the apparent ordinary course of nature is no argument whatever. You are bound to believe it, until you can bring some positive reason for disbelieving it.

Peirce: CP 7.113 Cross-Ref:††

113. In short this rudimentary kind of induction is justified where there is no other way of reasoning; but it is of all sound arguments the very weakest and must disappear as soon as any positive evidence is forthcoming.

Peirce: CP 7.114 Cross-Ref:††

114. The second order of induction consists in the argument from the fulfillment of predictions. After a hypothesis has been suggested to us by the agreement between its consequences and observed fact, there are two different lines that our further studies of it may pursue. In the first place, we may look through the known facts and scrutinize them carefully to see how far they agree with the hypothesis and how far they call for modifications of it. That is a very proper and needful inquiry. But it is Abduction, not Induction, and proves nothing but the ingenuity with which the hypothesis has been adapted to the facts of the case. To take this for Induction, as a great proportion of students do, is one of the greatest errors of reasoning that can be made. It is the *post hoc ergo propter hoc* fallacy, if so understood. But if understood to be a process antecedent to the application of induction, not intended to test the hypothesis, but intended to aid in perfecting that hypothesis and making it more definite, this proceeding is an essential part of a well-conducted inquiry.

Peirce: CP 7.115 Cross-Ref:††

115. The other line which our studies of the relation of the hypothesis to experience may pursue, consists in directing our attention, not primarily to the facts, but primarily to the hypothesis, and in studying out what effect that hypothesis, if

embraced, must have in modifying our expectations in regard to future experience. Thereupon we make experiments, or quasi-experiments,†27 in order to find out how far these new conditional expectations are going to be fulfilled. In so far as they greatly modify our former expectations of experience and in so far as we find them, nevertheless, to be fulfilled, we accord to the hypothesis a due weight in determining all our future conduct and thought. It is true that the observed conformity of the facts to the requirements of the hypothesis may have been fortuitous. But if so, we have only to persist in this same method of research and we shall gradually be brought around to the truth. This gradual process of rectification is in great contrast to what takes place with rudimentary induction where the correction comes with a bang. The strength of any argument of the Second Order depends upon how much the confirmation of the prediction runs counter to what our expectation would have been without the hypothesis. It is entirely a question of how much; and yet there is no measurable quantity. For when such measure is possible the argument assumes quite another complexion, and becomes an induction of the Third Order. Inductions of the second order are of two varieties, that are logically quite distinct.

Peirce: CP 7.116 Cross-Ref:††

116. The weaker of these is where the predictions that are fulfilled are merely of the continuance in future experience of the same phenomena which originally suggested and recommended the hypothesis, expectations directly involved in holding the hypothesis. Even such confirmation may have considerable weight. This, for example, is the way in which the undulatory theory of light stood before Maxwell. The phenomena of interference suggested undulations, which measures of the velocity of light in different media confirmed; and the phenomena of polarization suggested transverse vibrations. All the direct expectations involved in the hypothesis were confirmed, except that there no phenomena due to longitudinal vibrations were found. But all physicists felt that it was a weakness of the theory that no unexpected predictions occurred. The rotation of the plane of polarization was an outstanding fact not accounted for.

Peirce: CP 7.117 Cross-Ref:††

117. The other variety of the argument from the fulfillment of predictions is where truths ascertained subsequently to the provisional adoption of the hypothesis or, at least, not at all seen to have any bearing upon it, lead to new predictions being based upon the hypothesis of an entirely different kind from those originally contemplated and these new predictions are equally found to be verified.

Peirce: CP 7.118 Cross-Ref:††

118. Thus Maxwell, noticing that the velocity of light had the same value as a certain fundamental constant relating to electricity, was led to the hypothesis that light was an electromagnetic oscillation. This explained the magnetic rotation of the plane of polarization, and predicted the Hertzian waves. Not only that, but it further led to the prediction of the mechanical pressure of light, which had not at first been contemplated.

Peirce: CP 7.119 Cross-Ref:††

119. The second order of induction only infers that a theory is very much like the truth, because we are so far from ever being authorized to conclude that a theory is the *very truth* itself, that we can never so much as understand what that means. Light is electro-magnetic vibrations; that is to say, it [is] something very like that. In order to say that it is precisely that, we should have to know precisely what we mean by electro-magnetic vibrations. Now we never can know precisely what we mean by

any description whatever.

Peirce: CP 7.120 Cross-Ref:††

120. The third order of induction, which may be called Statistical Induction, differs entirely from the other two in that it assigns a definite value to a quantity. It draws a sample of a class, finds a numerical expression for a predesignate character of that sample and extends this evaluation, under proper qualification, to the entire class, by the aid of the doctrine of chances. The doctrine of chances is, in itself, purely deductive. It draws necessary conclusions only. The third order of induction takes advantage of the information thus deduced to render induction exact.

Peirce: CP 7.121 Cross-Ref:††

121. This family of inductions has three different kinds quite distinct logically. Beginning with the lowest and least certain, we have cases in which a class of individuals recur in endless succession and we do not know in advance whether the occurrences are entirely independent of one another or not. But we have some reason to suppose that they would be independent and perhaps that they have some given ratio of frequency. Then what has to be done is to apply all sorts of consequences of independence and see whether the statistics support the assumption. For instance, the value of the ratio of the circumference of a circle to its diameter, a number usually called π has been calculated in the decimal notation, to over seven hundred figures. Now as there is not the slightest reason to suppose that any law expressible in a finite time connects the value of π with the decimal notation or with any whole number, we may presume that the recurrences of any figure say 5 in that succession are independent of one another and that there is simply a probability of 1/10 that any figure will be a 5.

Peirce: CP 7.122 Cross-Ref:††

122. In order to illustrate this mode of induction, I have made a few observations on the calculated number. There ought to be, in 350 successive figures, about 35 fives. The odds are about 2 to 1 that there will be 30-39 [and] 3 to 1 that there will be 29-41. Now I find in the first 350 figures 33 fives, and in the second 350, 28 fives, which is not particularly unlikely under the supposition of a chance distribution. During the process of counting these 5's, it occurred to me that as the expression of a rational fraction in decimals takes the form of a circulating decimal in which the figures recur with perfect regularity, so in the expression of a quantity like π , it was naturally to be expected that the 5's, or any other figure, should recur with some approach to regularity. In order to find out whether anything of this kind was discernible I counted the fives in 70 successive sets of 10 successive figures each. Now were there no regularity at all in the recurrence of the 5's, there ought among these 70 sets of ten numbers each to be 27 that contained just one five each; and the odds against there being more than 32 of the seventy sets that contain just one five each is about 5 to 1. Now it turns out upon examination that there are 33 of the sets of ten figures which contain just one 5. It thus seems as if my surmise were right that the figures will be a little more regularly distributed than they would be if they were entirely independent of one another. But there is not much certainty about it. This will serve to illustrate what this kind of induction is like, in which the question to be decided is how far a given succession of occurrences are independent of one another and if they are not independent what the nature of the law of their succession is.

Peirce: CP 7.123 Cross-Ref:††

123. In the second variety of statistical induction, we are supposed to know

whether the occurrences are independent or not, and if not, exactly how they are connected, and the inquiry is limited to ascertaining what the ratio of frequency is, after the effects of the law of succession have been eliminated. As a very simple example, I will take the following. The dice that are sold in the toy shops as apparatus for games . . . are usually excessively irregular. It is no great fault, but rather enhances the Christmas gaiety. Suppose, however, that some old frump with an insatiable appetite for statistics [were to] get hold of a die of that sort, and he will spend his Christmas in throwing it and recording the throws in order to find out the relative frequency with which the different faces turn up. He assumes that the different throws are independent of one another and that the ten thousand or so which he makes will give the same relative frequencies of the different faces as would be found among any similar large number of throws until the die gets worn down. At least he can safely assume that this will be the case as long as the die is thrown out of the same box by the same person in the same fashion.

Peirce: CP 7.124 Cross-Ref:††

124. This second variety is the usual and typical case of statistical induction. But it occasionally happens that we can sample a finite collection of objects by such a method that in the long run any one object of the collection would be taken as often as every other and any one succession as often as any other. This may [be] termed a *random* selection. It is obviously possible only in the case of an enumerable collection. When this sort of induction is possible it far surpasses every other in certainty and may closely approach that of demonstration itself.

Peirce: CP 7.125 Cross-Ref:††

125. I have now passed in review all the modes of pure induction with which I am acquainted. Induction may, of course, be strengthened or weakened by the addition of other modes of argument leading to the same conclusion or to a contrary conclusion. It may also be strengthened or weakened by arguments which do not directly affect the conclusion of the induction but which increase or diminish the strength of its procedure. There are in particular four kinds of uniformities which may greatly affect an induction.

Peirce: CP 7.126 Cross-Ref:††

126. In the first place the members of a class may present a greater or less general resemblance as regards certain kinds of characters. Birds for example are, generally speaking, much more alike than are fishes or mammals; and that will strengthen any induction about birds. Orchids, on the other hand, are extraordinarily various.

Peirce: CP 7.127 Cross-Ref:††

127. In the second place a character may have a greater or less tendency to be present or absent throughout the whole of certain kinds of groups. Thus, coloration often differs within one species, while the number of the principal bones of the skeleton, and almost all characters which are developed early in individual life and which persist to maturity are common to all the members of large classes.

Peirce: CP 7.128 Cross-Ref:††

128. In the third place, a certain set of characters may be more or less intimately connected, so as probably to be present or absent together in certain kinds of objects. Thus, we generally associate insistency upon minute forms with narrowness of mind, cleanliness with godliness, and so on.

Peirce: CP 7.129 Cross-Ref:††

129. In the fourth place, an object may have more or less tendency to possess the whole of certain sets of characters when it possesses any of them. Thus, one meets one man whose views whatever they may be are extreme, while the opinions of another form a strange mosaic.

Peirce: CP 7.130 Cross-Ref:††

130. From the knowledge of a uniformity of any one of these four classes or from the knowledge of the lack of such uniformity it may be deductively inferred that a given induction is either stronger or weaker than it otherwise would be.

Peirce: CP 7.131 Cross-Ref:††
§7. UNIFORMITY OF NATURE †28

131. There is still another sense in which we might speak of the uniformity of nature. If we select a good many objects on the principle that they shall belong to a certain class and then find that they all have some common character, pretty much the whole class will generally be found to have that character. Or if we take a good many of the characters of a thing at random, and afterwards find a thing which has all these characters, we shall generally find that the second thing is pretty near the same as the first.

Peirce: CP 7.132 Cross-Ref:††

132. It seems to me that it is this pair of facts rather than any others which are properly expressed by saying that nature is uniform. We shall see that it is they which are the leading principles of scientific inference.

Peirce: CP 7.132 Cross-Ref:††

Let us ask, then, whether these facts are statements of a particular constitution of the world so as to be properly speaking matters of fact or whether they are purely formal propositions, laws of logic, having no more application to one state of things than they would have to any other.

Peirce: CP 7.133 Cross-Ref:††

133. In the first place, I would call your attention to the quantitative indeterminateness of both propositions. The first speaks of a *good many* samples being selected, and of *pretty much* all the things in the class from which they are taken being like them, and of this occurring *almost* always. The second speaks of a *good many* characters of a thing being taken, and of any thing found to have them being *pretty near* the same thing, and of this happening *almost always*. We have no means whatsoever of defining the propositions in either of the three respects in which they are thus seen to be so utterly vague.

Peirce: CP 7.134 Cross-Ref:††

134. Now you know how a malicious person [who] wishes to say something ill of another, prefers *insinuation*; that is, he speaks so vaguely that he suggests a great deal while he expressly says nothing at all. In this way he avoids being confronted by fact. It is the same way with these principles of scientific inference. They are so vague that you cannot bring them to any touch-stone of experience. They rather insinuate a uniformity in nature than state it. And as insinuation always

expresses the state of feeling of the person who uses it rather than anything concerning its object, so we may suppose these principles express rather the scientific attitude than a scientific result.

Peirce: CP 7.135 Cross-Ref:††

135. But what if we were in a world of chance? How would it be with these principles then, or, to simplify the matter, with the first principle? In that case, it would be extremely seldom that, having selected a number of objects as having certain characters, we should find that they had any other common character; and thus there would be very little applicability for this principle. But, we have seen that the proportion of cases where this principle applies is indefinitely small in our present world. Cases might occur, doubtless would in a world of chance and when they did occur the principle doubtless would hold true.

Peirce: CP 7.136 Cross-Ref:††

136. It is a mistake to suppose that there would be no laws in a world of chance. At least, so I should think. Suppose we were to throw a die any number of times and set down the numbers thrown in a column. I could show you that there would be some very curious laws in reference to those numbers. They would appear quite surprising. So that *chance* is not the abrogation of all laws.

Peirce: CP 7.137 Cross-Ref:††

137. But there is a peculiarity about those laws that chance does not abrogate; suppose that in throwing the die other numbers had turned up from those which actually turned up, so that the row of numbers would have been somewhat different; still the laws would have held; they would hold with one set of numbers as well as with another. Whereas if we were to give a whale legs or a woman wings, the laws of the animal kingdom would be interfered with. So that there are two kinds of laws, those which in a different state of things would continue to hold good and those which in a different state of things would not hold good. The former we call *formal* laws, the latter *material* laws. The formal laws do not depend on any particular state of things, and hence we say we have not derived them from experience; that is to say, any other experience would have furnished the premisses for them as well as that which we have experienced; while to discover the material laws we require to have known just such facts as we did. But as the laws which we have mentioned, that as is sample so is the whole and that the sameness of a number of characters manifests identity, are laws which would hold so long as there were *any* laws, though only formal ones, it is plain that no alteration in the constitution of the world would abrogate them, so that they are themselves formal laws, and therefore not laws of *nature* but of the conditions of knowledge in general.

Peirce: CP 7.138 Cross-Ref:††

138. Two classes of thinkers wish to make the difference between formal and material laws merely relative; namely, those who would reduce all formal laws to material laws, and those who would reduce all material laws to formal laws. But neither can deny that there is a great difference between what we must consider formal and what we must consider material laws. Those who would reduce all material laws to formal laws, have indeed shown that what we call material laws are only those which *we cannot discover* to be formal; and thus that all material laws may be formal; and in so doing they have cut anyone off from saying that there is a peculiar uniformity of nature consisting in its material laws. On the other hand, those who would reduce formal laws to material laws, among whom is Mr. Mill, have shown that laws may be thought to be formal, that is to be such that a violation of

them is unimaginable, owing to a want of imaginative power in us arising from a defective experience, and they infer from that that *all* formal laws may be material. But so long as there are any laws whatsoever, *these* laws that the whole is as the sample and that identity goes with similarity in respects [not] chosen to make out the similarity, *these* laws I say must exist. For these are but as much as to say that there *is* law. That we shall see in future lectures. Now all law may, in one sense, be contingent. But that there should be knowledge without the existence of law, that there should be intelligence without anything intelligible, all admit to be impossible. These laws therefore cannot be abrogated without abrogating knowledge; and thus are the formal conditions of all knowledge.

Peirce: CP 7.139 Cross-Ref:††
CHAPTER 2

ECONOMY OF RESEARCH

§1. ORIGINAL PAPER †1

139. When a research is of a quantitative nature, the progress of it is marked by the diminution of the probable error. The results of non-quantitative researches also have an inexactitude or indeterminacy which is analogous to the probable error of quantitative determinations. To this inexactitude, although it be not numerically expressed, the term "probable error" may be conveniently extended.

Peirce: CP 7.140 Cross-Ref:††

140. The doctrine of economy, in general, treats of the relations between utility and cost. That branch of it which relates to research considers the relations between the utility and the cost of diminishing the probable error of our knowledge. Its main problem is, how, with a given expenditure of money, time, and energy, to obtain the most valuable addition to our knowledge.

Peirce: CP 7.141 Cross-Ref:††

141. Let r denote the probable error of any result, and write $s = 1/r$. Let $U \cdot dr$ denote the infinitesimal utility of any infinitesimal diminution, dr , of r . Let $V \cdot ds$ denote the infinitesimal cost of any infinitesimal increase, ds , of s . The letters U and V are here used as functional symbols. Let subscript letters be attached to r , s , U , and V , to distinguish the different problems into which investigations are made. Then, the total cost of any series of researches will be

and their total utility will be $\sum [i] U[i] \cdot dr[i];$

$$\sum [i] V[i] \cdot ds[i];$$

$$\Sigma[i] \partial U[i] r[i] \cdot dr[i]$$

The problem will be to make the second expression a maximum by varying the inferior limits of its integrations, on the condition that the first expression remains of constant value.

Peirce: CP 7.142 Cross-Ref:††

142. The functions U and V will be different for different researches. Let us consider their general and usual properties. And, first, as to the relation between the exactitude of knowledge and its utility. The utility of knowledge consists in its capability of being combined with other knowledge so as to enable us to calculate how we should act. If the knowledge is uncertain, we are obliged to do more than is really necessary, in order to cover this uncertainty. And, thus, the utility of any increase of knowledge is measured by the amount of wasted effort it saves us, multiplied by the specific cost of that species of effort. Now, we know, from the theory of errors, that the uncertainty in the calculated amount of effort necessary to be put forth may be represented by an expression of the form

$$c\sqrt{a+r^2}$$

where a and c are constants. And, therefore, the differential coefficient of this, multiplied by the specific cost of the effort in question, say h/c , gives

$$Ur = h \cdot (r/\sqrt{a+r^2})$$

When a is very small compared with r this becomes nearly constant, and in the reverse case it is nearly proportional to r . An analogous proposition must hold for non-quantitative research.

Peirce: CP 7.143 Cross-Ref:††

143. Let us next consider the relation between the exactitude of a result and the cost of attaining it. When we increase our exactitude by multiplying observations, the different observations being independent of one another as to their cost, we know from the theory of errors

that ∂

$\partial V_s \cdot ds$ is proportional to s^2 , and that consequently V_s is proportional to s . If the costs of the different observations are not independent (which usually happens), the cost will not increase so fast relatively to the accuracy; but if the errors of the observations are not independent (which also usually happens), the cost will increase faster relatively to the accuracy; and these two perturbing influences may be supposed, in the long run, to balance one another. We may, therefore, take $V_s = ks$,

where k represents the specific cost of the investigation.

Peirce: CP 7.144 Cross-Ref:††

144. We thus see that when an investigation is commenced, after the initial expenses are once paid, at little cost we improve our knowledge, and improvement then is especially valuable; but as the investigation goes on, additions to our knowledge cost more and more, and, at the same time, are of less and less worth. Thus, when chemistry sprang into being, Dr. Wollaston, with a few test tubes and phials on a tea-tray, was able to make new discoveries of the greatest moment. In our day, a thousand chemists, with the most elaborate appliances, are not able to reach results which are comparable in interest with those early ones. All the sciences exhibit the same phenomenon, and so does the course of life. At first we learn very easily, and the interest of experience is very great; but it becomes harder and harder, and less and less worth while, until we are glad to sleep in death.

Peirce: CP 7.145 Cross-Ref:††

145. Let us now apply the expressions obtained for Ur and Vs to the economic problem of research. The question is, having certain means at our disposal, to which of two studies they should be applied. The general answer is that we should study that problem for which the economic urgency, or the ratio of the utility to the cost

$$(Ur \cdot dr)/(Vs \cdot ds) = r^2(Ur/Vs) = (h/k)(r^4/\sqrt{a+r^2})$$

is a maximum. When the investigation has been carried to a certain point this fraction will be reduced to the same value which it has for another research, and the two must then be carried on together, until finally, we shall be carrying on, at once, researches into a great number of questions, with such relative energies as to keep the urgency-fraction of equal values for all of them. When new and promising problems arise they should receive our attention to the exclusion of the old ones, until their urgency becomes no greater than that of others. It will be remarked that our ignorance of a question is a consideration which has between three and four times the economic importance of either the specific value of the solution or the specific cost of the investigation in deciding upon its urgency.

Peirce: CP 7.146 Cross-Ref:††

146. In order to solve an economical problem, we may use as variables

$$x = \int Vs \cdot ds,$$

or the total cost of an inquiry, and

$$y = (Ur \cdot dr)/(Vs \cdot ds),$$

or the economic urgency. Then, C being the total amount we have to spend in certain researches, our equations will be

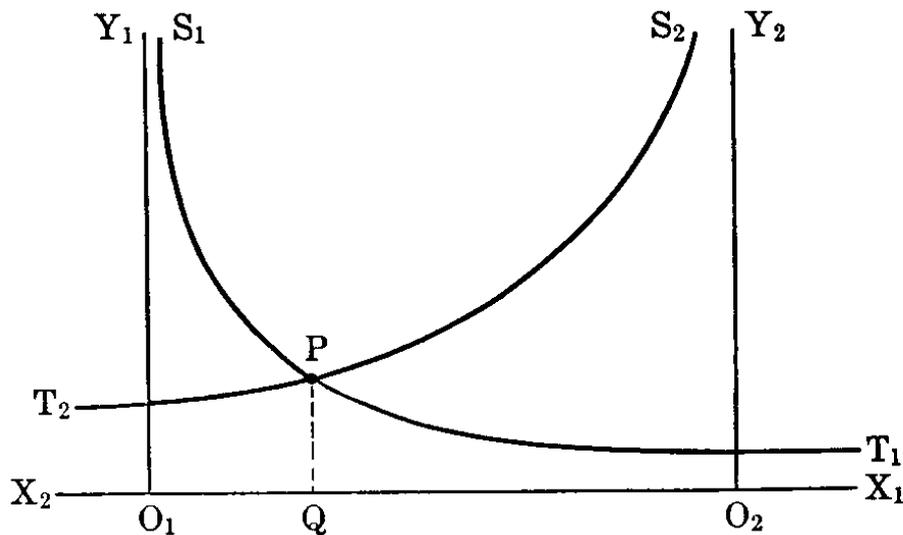
$$C = x[1] + x[2] + x[3] + \text{etc.}$$

$$y[1] = y[2] = y[3] = \text{etc.}$$

Then, expressing each y in terms of x , we shall have as many equations as unknown quantities.

Peirce: CP 7.147 Cross-Ref:††

147. When we have to choose between two researches only, the solution may be represented graphically, as follows:



[Click here

to view]

Peirce: CP 7.148 Cross-Ref:††

148. From any point $O[1]$ taken as an origin, draw the axis of abscissas $O[1] X[1]$, along which $x[1]$, the total cost of the first investigation, is to be measured. Draw also the axis of ordinates $O[1] Y[1]$, along which $Y[1]$, the economic urgency of the first investigation, is to be measured. Draw the curve $S[1] T[1]$ to represent the relations of $x[1]$ and $y[1]$. Take, on the axis $O[1] X[1]$, a point $O[2]$ such that $O[1] O[2]$ shall measure the total cost of the two investigations. Let $x[2]$, the total cost of the second investigation, be measured on the same axis as $x[1]$, but in the opposite direction. From $O[2]$ draw the axis of ordinates $O[2] Y[2]$ parallel to $O[1] Y[1]$, and measure $y[2]$, the economic urgency of the second investigation, along this axis. Draw the curve $S[2] T[2]$ to represent the relations of $x[2]$ and $Y[2]$. Then, the two curves $S[1] T[1]$ and $S[2] T[2]$ will generally cut one another at one point, and only one, between the axes $O[1] Y1$ and $O2 Y2$. From this point, say P , draw the ordinate PQ , and the abscissas $O[1]Q$ and $O[2]Q$ will measure the amounts which ought to be

expended on the two inquiries.

Peirce: CP 7.149 Cross-Ref:††

149. According to the usual values of U and V, we shall have

$$y = (1/4)(hk/x\sqrt{ax^2 + 1/2 kx})$$

Peirce: CP 7.150 Cross-Ref:††

150. In this case, when there are two inquiries, the equation to determine x_1 will be a biquadratic. Two of its roots will be imaginary, one will give a negative value of either x_1 or x_2 , and the fourth, which is the significant one, will give positive values of both.

Peirce: CP 7.151 Cross-Ref:††

151. Let us now consider the economic relations of different researches to one another. 1st, as alternative methods of reaching the same result, and 2d, as contributing different premises to the same argument.

Peirce: CP 7.152 Cross-Ref:††

152. Suppose we have two different methods of determining the same quantity. Each of these methods is supposed to have an accidental probable error and a constant probable error, so that the probable errors, as derived from n observations in the two ways, are:

$$r[1] = \sqrt{R[1]^2 + (\{r\}[1]^2/n)} \text{ and } r[2] = \sqrt{R[2]^2 + (\{r\}[2]^2/n)}$$

The probable error of their weighted mean is

$$1/\sqrt{1/(r[1]^2 + 1/r[2]^2)}$$

if their constant probable errors are known. The sole utility of any observation of either is to reduce the error of the weighted mean; hence,

$$Ur[1] = Dr[1](r[1]^{-2} + r[2]^{-2})^{-1/2} =$$
$$r[1]^{-2} + r[2]^{-2})^{-3/2} r[1]^{-3}.$$

And as the cost is proportional to the number of observations

$$V_{S_1} = k_1 \frac{1}{D_{n_1} s_1} = \frac{k_1}{D_{n_1} (R_1^2 + \rho_1^2 n_1^{-1})^{-\frac{1}{2}}} = \frac{2 k_1 r_1^3 n_1^2}{\rho_1^2} .$$

[Click here to view]

Hence, the urgency is (omitting a factor common to the values for the two methods)

$$r_1^2 \frac{U r_1}{V_{S_1}} = \frac{1}{k_1 \rho_1^2 \left(1 + n_1 \frac{R_1^2}{\rho_1^2} \right)^2} .$$

[Click here to view]

And, as the urgency of the two methods ought to be the same at the conclusion of the work, we should have

$$\sqrt{k_1 \cdot \rho_1 \left(1 + n_1 \frac{R_1^2}{\rho_1^2} \right)} = \sqrt{k_2 \cdot \rho_2 \left(1 + n_2 \frac{R_2^2}{\rho_2^2} \right)} ,$$

[Click here to view]

which equation serves to determine the relative values of $n[1]$ and $n[2]$. We again perceive that the cost is the smallest consideration. The method which has the smallest accidental probable error is the one which is to be oftenest used in case only a small number of observations are made; but if a large number are taken the method with the larger accidental probable error is to be oftenest used, unless it has so much greater a probable constant error as to countervail this consideration. If one of the two methods has only $(1/p)$ th the accidental probable error of the other, but costs p^2 times as much, the rule should be to make the total cost of the two methods inversely proportional to the squares of their constant errors.

Peirce: CP 7.153 Cross-Ref:††

153. Let us now consider the case in which two quantities $x[1]$ and $x[2]$ are observed, the knowledge of which serves only to determine a certain function of them, y .†2 In this case the probable error $[dy]$ of y is

$$\sqrt{D_{x_1}y \cdot r_1^2 + D_{x_2}y \cdot r_2^2}$$

[Click here to view]

and [since $Ur[1] = Dr[1](dy)$] we shall have

$$Ur_1 = \frac{D_{x_1}y \cdot r_1}{\sqrt{D_{x_1}y \cdot r_1^2 + D_{x_2}y \cdot r_2^2}}.$$

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$Vs[1]$ will have the same value as before; but neglecting now the constant error, we may write

$$Vs[1] = 2k[1]\{r\}[1]n[1]^{1/2}$$

Then the urgency (with omission of the common factor) is

$$\frac{\rho_1^2}{k_1 n_1^2} \cdot D_{x_1}y,$$

[Click here to view]

and, as the two urgencies must be equal, we have

$$\frac{n_1}{n_2} = \frac{\rho_1}{\rho_2} \sqrt{\frac{k_2}{k_1} \cdot \frac{D_{x_1} y}{D_{x_2} y}}.$$

[Click here to view]

Peirce: CP 7.154 Cross-Ref:††

154. The following is an example of the practical application of the theory of economy in research: Given a certain amount of time, which is to be expended in swinging a reversible pendulum, how much should be devoted to experiments with the heavy end up, and how much to those with the heavy end down?†3

Peirce: CP 7.155 Cross-Ref:††

155. Let $T[d]$ be the period of oscillation with heavy end down, $T[u]$ the same with heavy end up. Let $h[d]$ and $h[u]$ be the distances of the center of mass from the points of support of the pendulum in the two positions. Then the object of the experiments is to ascertain a quantity proportional to †4

$$h[d] T[d] - h[u] T[u].$$

Accordingly, if $dT[d]$ and $dT[u]$ are the probable errors of $T[d]$ and $T[u]$, that of the quantity sought will be

$$\sqrt{h[d]^2 (dT[d])^2 + h[u]^2 (dT[u])^2}.$$

Peirce: CP 7.156 Cross-Ref:††

156. We will suppose that it has been ascertained, by experiment, that the whole duration of the swinging being C , and the excess of the duration of the swinging with heavy end down over that with heavy end up being x , the probable errors of the results are

$$dT_d = \sqrt{a + \left(b + \frac{c}{h_d^2}\right) \frac{1}{C+x}}$$

$$dT_u = \sqrt{a + \left(b + \frac{c}{h_u^2}\right) \frac{1}{C-x}}$$

to view]

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where a , b , and c are constants. Then, the square of the probable error of the quantity sought will be

$$a(h_d^2 + h_u^2) + (bh_d^2 + c) \frac{1}{C+x} + (bh_u^2 + c) \frac{1}{C-x} .$$

to view]

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The differential coefficient of this relatively to x is

$$- (bh_d^2 + c) \frac{1}{(C+x)^2} + (bh_u^2 + c) \frac{1}{(C-x)^2} .$$

to view]

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Putting this equal to zero and solving, we find for the only significant root,

$$\frac{x}{C} = \frac{b(h_d^2 + h_u^2) + 2c}{b(h_d^2 - h_u^2)} - \sqrt{\left(\frac{b(h_d^2 + h_u^2) + 2c}{b(h_d^2 - h_u^2)}\right)^2 - 1}$$

to view]

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when b vanishes, x reduces to zero, and the pendulum should be swung equally long in the two positions. When c vanishes, as it would if the pendulum experiment were made absolutely free from certain disturbing influences, we have

$$x/C = (h[d]-h[u])/(h[d]+h[u]),$$

so that the duration of an experiment ought to be proportional to the distance of the center of mass from the point of support. This would be effected by beginning and ending the experiments in the two positions with the same amplitudes of oscillation.

Peirce: CP 7.157 Cross-Ref:††

157. It is to be remarked that the theory here given rests on the supposition that the object of the investigation is the ascertainment of truth. When an investigation is made for the purpose of attaining personal distinction, the economics of the problem are entirely different. But that seems to be well enough understood by those engaged in that sort of investigation.

Peirce: CP 7.158 Cross-Ref:††
 §2. LATER REFLECTIONS †5

158. In all economics the laws are ideal formulae from which there are large deviations, even statistically. In the economics of research the "laws" are mere general tendencies to which exceptions are frequent. The laws being so indefinite, at best, there is little advantage in very accurate definitions of such terms as "amount of knowledge." It is, however, possible to attach a definite conception to one increment of knowledge being greater than another. To work this out will be the first business of the memoir. I also establish a definite meaning for the amount of an increment in diffusion of knowledge. I then consider the relation of each of these to the expenditure of energy and value required to produce them in varying conditions of the advancement or diffusion of knowledge already attained. Comparing knowledge with a material commodity, we know that in the latter case a given small increment in the supply is very expensive, in most cases, when the supply is very small, that as the supply increases, it sinks to a minimum, from which it increases to a very large but finite value of the supply where no further increment would be possible at any finite cost. Putting instead of supply, the amount of knowledge attained, we find that there is a "law," or general tendency, subject to similar large irregularities as in the case of the supply of a material commodity, but here even greater. The final increase of cost of an increment with the increase of attainment already achieved is marked, on the whole, in almost all cases, while in many cases, at least, there is a point of attainment where the cost of an increment is at a minimum. The same general tendency appears in reference to the diffusion of knowledge; but there is this striking difference, that attainments in advance of sciences are very commonly actually on the upward slope where increments are costing more and more, while there are few branches of knowledge whose diffusion is already so great that a given increment of the diffusion will cost more and more, as the diffusion is increased.

Peirce: CP 7.159 Cross-Ref:††

159. I shall next pass to a study of the variation of the *utility* (meaning, generally, the *scientific* utility) of given small increments of scientific knowledge and of the diffusion of knowledge in varying states of attainment. This is to be compared with the variation of the total amount that will be paid for a commodity for a fixed

small increment of the *demand*, or amount thrown upon the market to fetch what it will, with varying amounts of that demand. Here, the additional total amount that will be paid for the small increment of amount sold will correspond to the utility of the small fixed increase of scientific knowledge or of the diffusion of knowledge; while, the demand being equal to the supply, this demand, or total amount that is sold, will correspond as before to the amount of attainment in scientific knowledge or in the diffusion of knowledge. For a material commodity we know that if it is given away people will only carry home a finite amount. One would have to pay them to carry away more. On the other hand there is probably some maximum price for most things, above which none at all would be sold. It necessarily follows that beyond a certain amount thrown upon the market, a small increment in that amount would actually diminish the total receipts from the sale of it, while for any smaller amount the increment of receipts for a given small increment of amount sent to market would be less and less. With regard to the scientific utility of a small fixed advance of knowledge, the "law" is certainly very different from that. In the first place, there is no degree of knowledge of which a small increase would be worse than useless, and while the general tendency is that the utility of such fixed increase becomes less and less, yet this curve is rather saw shaped, since like Rayleigh's small addition to our knowledge of the density of nitrogen, now and then a small increment will be of great utility and will then immediately sink to its former level. The scientific advantage of the diffusion of knowledge is difficult to determine. It cannot be believed that any increment of diffusion is positively unfavorable to science. It is favorable in two ways; first, by preparing more men to be eminent researchers; and secondly, by increasing general wealth, and therefore the money bestowed on science. I am inclined to think that the general tendency is that a given increment of diffusion is less and less advantageous to science the greater the attained diffusion. But I am not confident that this is so, at any rate without very important deflexions. The general effect, however, is nearly the same for the advancement as for the diffusion of knowledge. Namely, beginning with dense ignorance, the first increments cost more than they come to. That is, knowledge is increased but scientific energy is spent and not at once recovered. But we very soon reach a state of knowledge which is profitable to science, that is, not only is knowledge increased, but the facility of increasing knowledge gives us a return of more available means for research than we had before the necessary scientific energy was spent. This increases to a maximum, diminishes, and finally, there is no further gain. Yet still, in the case of energy expended upon research, if it is persisted in, a fortunate discovery may result in a new means of research. I shall analyze as far as I can the relative advantages, *for pure science* exclusively, of expending energy (which is of such a kind as to be equally capable of being directed either way) to the direct advancement of knowledge and to the diffusion of knowledge. I find the latter so overwhelmingly more important (although all my personal sympathies are the other way) that it appears to me that, for the present, to give to research, in money, one or two per cent of what is spent upon education is enough. Research must contrive to do business at a profit; by which I mean that it must produce more effective scientific energy than it expends. No doubt, it already does so. But it would do well to become conscious of its economical position and contrive ways of living upon it.

Peirce: CP 7.160 Cross-Ref:††

160. Many years ago I published a little paper on the Economy of Research, in which I considered this problem.†6 Somebody furnishes a fund to be expended upon research without restrictions. What sort of researches should it be expended upon? My answer, to which I still adhere, was this. Researches for which men have been

trained, instruments procured, and a plant established, should be continued while those conditions subsist. But the new money should mainly go to opening up new fields; because new fields will probably be more profitable, and, at any rate, will be profitable longer.

Peirce: CP 7.161 Cross-Ref:††

161. I shall remark in the course of the memoir that economical science is particularly profitable to science; and that of all the branches of economy, the economy of research is perhaps the most profitable; that logical methodic and logic in general are specially valuable for science, costing little beyond the energies of the researcher, and helping the economy of every other science. It was in the middle of the 13th century that a man distinguished enough to become pope opened his work on logic with the words, "Dialectica est ars artium et scientia scientiarum, ad omnium methodorum principia viam habens."†7 This memorable sentence, whose gothic ornamentation proves upon scrutiny to involve no meaningless expression nor redundant clause, began a work wherein the idea of this sentence was executed satisfactorily enough for the dominant science of the middle ages. Jevons adopted the sentence as the motto of his most scientific contribution to logic; and it would express the purpose of my memoirs, which is, upon the ground well prepared by Jevons and his teacher De Morgan, and by the other great English researchers, especially Boole, Whewell, Berkeley, Glanvill, Ockham, and Duns Scotus, to lay a solid foundation upon which may be erected a new logic fit for the life of twentieth century science.

Peirce: CP 7.162 Cross-Ref:††
CHAPTER 3

THE LOGIC OF DRAWING HISTORY FROM ANCIENT DOCUMENTS

§1. ABSTRACT †1

162. Mr. C. S. Peirce gave an abstract of a long paper on the logic of the process of drawing history from ancient testimonies. He endeavored to show that the method of balancing the veracity of a witness against the improbability of his narrative, although it may be defended upon the principles of the calculus of probabilities under certain conditions, is nevertheless in the great majority of cases illogical, because there is not the roughest approximation to fulfillment of those conditions. For the testimonies are seldom even approximately independent, and still more seldom independent of the antecedent probability of the narrative; and moreover there is no determinate objective probability that a witness will tell the truth, and often no objective probability that the fact is as stated. Furthermore, we ought not to seek probabilities in such problems, but ought to pursue a method which must ultimately lead to the truth. Nor can such a method pursue the path of mathematical demonstration, which all reasoning that deduces a probability is, at its very best. But the probabilities upon which the critics of history rely are not objective, but are mere

expressions of their preconceived notions, than which no guide can be less trustworthy.

Peirce: CP 7.163 Cross-Ref:††

163. Mr. Peirce then undertook to develop the principles upon which ancient historical research ought to proceed in order to be scientific. He pointed out that the logic of scientific investigation, in which a conclusion is not established for many years, perhaps not for generations, cannot be assumed to be the same as the proper logical procedure for an individual who seeks a practical basis for present action. Although the latter has to be hastily performed, the accurate theory of it is far more difficult than is the logic of scientific procedure, to which the present paper limited itself. Here the question is, what is the process by which the truth will be attained most speedily. It does not content itself with probabilities, although absolute certainty can never be fully attained; nor has it anything to do with belief, which is a practical concern. This scientific procedure consists, according to Mr. Peirce, in carefully framing a hypothesis, in tracing out the experiential consequences of that hypothesis, and in testing those consequences by comparison with facts not taken into account in the formation of the hypothesis. If the facts refute the hypothesis, it must be abandoned and another substituted; but if the predictions based upon it are verified, it will be entitled to be received as a scientific result until we find deductions from it which are contrary to the facts.

Peirce: CP 7.164 Cross-Ref:††

§2. THE THEORY OF BALANCING LIKELIHOODS †2

164. Ancient history is drawn partly from documents and partly from monuments. The last generation has afforded so many examples of the refutation by archeology of the conclusions of the critics of documents as to suggest the question whether the whole logical procedure of the latter class of students has not been radically wrong. The purpose of the present paper is to show that this is the case; that the logical theory upon which the critics proceed is as bad as logic can be; to set forth and defend the true logical method of treating ancient historical documents; and to set this new theory in a clear light by applying it to two or three examples, including a case where the testimonies are comparatively strong and another where the testimony is at best very feeble.

Peirce: CP 7.165 Cross-Ref:††

165. The theory of the logic of testimony which forms the basis of the procedure of historical critics today is, I suppose, old. But it can only have taken a distinct form when the doctrine of probabilities was developed, in the early years of the eighteenth century. A popular statement of it was, I believe, first given by Hume, in his essay on Miracles, in 1748. Hume's statement is, mathematically considered, excessively crude. It seems evident that he had been reading either De Moivre's *Doctrine of Chances* (first edition, 1716; second enlarged edition, 1735) or De Montmort's *Essai d'Analyse sur les Jeux de Hazard* (1708; second edition, 1713). For Jacob Bernoulli's posthumous *Ars Conjectandi* (1713) would have been beyond him. Whatever work he read he did not understand; yet in a confused and untenable form, he put forth ideas of his own of considerable value. I may restate Hume's doctrine, correcting such errors as are not inseparable from it, as follows. When a

reputable witness makes, or witnesses make, an assertion which experience renders highly improbable, or when there are other independent arguments in its favor, each independent argument *pro* or *con* produces a certain impression upon the mind of the wise man, dependent for its quantity upon the frequency with which arguments of those kinds lead to the truth, and the algebraical sum of these impressions is the resultant impression that measures the wise man's state of opinion on the whole. For example, if there are a number of independent arguments, *pro*, such that, in general, such arguments lead to the truth, p1 times, p2 times, p3 times, etc., respectively, for every q1 times, q2 times, q3 times, etc., that they lead to error; and if there are arguments *con*, which lead to the truth q5 times, q6 times, q7 times, etc., for every p5 times, p6 times, p7 times, etc., that they lead to error, then the probability that the arguments *pro* all lead to the truth, and the arguments *con* all lead to error will be

$$p1/p1+q1 \cdot p2/p2+q2 \cdot p3/p3+q3 \cdot \text{etc.}$$

$$\times p5/p5+q5 \cdot p6/p6+q6 \cdot p7/p7+q7 \cdot \text{etc.};$$

and the probability that all the arguments *pro* lead to error while all the arguments *con* lead to truth, will be

$$q1/p1+q1 \cdot q2/p2+q2 \cdot q3/p3+q3 \cdot \text{etc.}$$

$$\times q5/p5+q5 \cdot q6/p6+q6 \cdot q7/p7+q7 \cdot \text{etc.}$$

But one or [the] other of these two alternatives must be the case; so that the *odds* or ratio of favorable to unfavorable probability on the whole is simply

$$(p1 \cdot p2 \cdot p3 \cdot \text{etc.} \cdot p5 \cdot p6 \cdot p7 \cdot \text{etc.}) / (q1 \cdot q2 \cdot q3 \cdot \text{etc.} \cdot q5 \cdot q6 \cdot q7 \cdot \text{etc.}).$$

Now if we suppose that the impression made on the mind of the wise man is proportional to the logarithm of the *odds* as its exciting cause, then the total impression will be

$$\log.(p1/q1 \cdot p2/q2 \cdot p3/q3 \cdot \text{etc.} \cdot p5/q5 \cdot p6/q6 \cdot p7/q7 \cdot \text{etc.})=$$

$$\log. p1/q1 + \log. p2/q2 + \log. p3/q3 + \text{etc.}$$

$$+ \log. p5/q5 + \log. p6/q6 + \log. p7/q7 + \text{etc.}$$

Peirce: CP 7.166 Cross-Ref:††

166. This is *Hume's Theory Improved*, by merely being disembarassed of blunders. If we strip the mathematics from it, we have the simple *theory of balancing likelihoods*, which is the theory that Hume undertook to elaborate and to render scientific. It really hardly differs from Hume's Theory Improved except in its vagueness. At any rate, it involves the notion that the different arguments have likelihoods, that they are quantities upon an algebraical scale, and that they are to be combined as independent.

Peirce: CP 7.167 Cross-Ref:††

167. Now the practice of those modern German critics of ancient history whose works I have read, particularly those who treat of the history of philosophy, and whose methods are generally extolled, is based upon the theory of balancing likelihoods. In so far as their general logical method departs from that of Hume, it is only less refined. The principal difference between Hume and them is that the word 'Proof' is continually in their mouths, a word which Hume scrupulously avoided in speaking of the minor facts of ancient history. He recognized the question as purely one of probabilities. They seem to be discontented with mere probability; and are always in search of an argument that something "must" be. The necessity which enters into the conclusion of such an argument as part of its subject matter is confounded by them with the necessity of a mathematical demonstration, in the conclusion of which the word "must" does not frequently occur. Now since it happens ten times that we can argue that testimony *must* be false to every once that we can argue that it *must* be true, it naturally follows and is a fact, that these critics show far greater favor to views which reject all the historical evidence in our possession than they do to views which are based on some part of the evidence. "That, however, is not *proved*," is their usual comment upon any such hypothesis. Another particular in which they depart from Hume is in applying to history generally the canon of Bentley concerning the criticism of texts, that, in general, the more difficult reading is to be preferred. In like manner, they hold that that narrative which was least likely to be invented, owing to its improbability, is to be preferred. They are thus provided with two defences against historical testimony. If the story told appears to them in any degree unlikely, they reject it without scruple; while if there is no taint of improbability in it, it will fall under the heavier accusation of being too probable; and in this way, they preserve a noble freedom in manufacturing history to suit their subjective impressions.

Peirce: CP 7.168 Cross-Ref:††

§3. CRITICISM OF THE THEORY OF BALANCING LIKELIHOODS

168. I now propose to show some weighty reasons for holding that the theory of balancing likelihoods, however it may be worked out, and though there are, undoubtedly, special cases where it ought to be followed, is nevertheless, as a general method of treating ancient documents, a bad one. In cases where objective and somewhat definite probabilities can be attributed to all the different arguments on both sides, and where they are, as arguments, independent of one another, it seems to be incontestable that Hume's method improved is sound. In the ordinary text books on

the Doctrine of Chances, so much of this theory as is given at all is only given in their chapters on the probability of testimony; and I will mention that Professor F. Y. Edgeworth says that in extending it to all independent arguments that have definite general probabilities I am "confusing" testimonies with arguments. But however obliging his attribution to me of this extension may be, it is mistaken; for the same extension has been made by several writers, among them one whom Professor Edgeworth holds in peculiar respect, Augustus De Morgan, who gave the necessary demonstration as far back as 1846 (*Cambridge Phil. Trans.* VIII. 393.). . . . But I will now set down the reasons which compel me to acknowledge the justice of Hume's method when improved as above, and applied to all independent arguments to which definite general probabilities can be attributed. But I will first call attention to a confusion of thought which might easily lead a man to infer that the theory in question was not applicable to arguments in general, unless his mind had been brought to an edge worth assiduous care before one presumes to discuss questions of probability. Taking the time-honored urn from which balls are drawn at random and thrown back after each drawing, I will suppose, that every ball is, in fact, a box, and that out of every 7 of them 3 contain gold and 4 lead. I will also suppose that I have two expert witnesses, one of whom judges by the color, and is right 3 times to every time he fails, while the other judges by the weight, and is right 9 times for every 5 failures. Let us suppose the testimony is independent, the color-expert being just as proportionally often right when the material-expert is right as when he is wrong. In order to fix our ideas, let us suppose the numbers are as follows:

Auriferous. Plumbiferous.

Heavy Light Heavy Light

Yellow, aAa 15 pAa 35 aPa 14 pPa 6

Grey, aAp 21 pAp 1 aPp 10 pPp 66

The total number of auriferous balls is $A=72$: of plumbiferous is $P=96$.

The total number of heavy auriferous and light plumbiferous, $aA+pP= 108$: that of the light auriferous and heavy plumbiferous is $pA+aP=60$.

The total number of yellow auriferous and grey plumbiferous, $Aa+Pp= 126$: that of the grey auriferous and yellow plumbiferous is $Ap+Pa=42$.

Now both witnesses report a ball to be auriferous; and according to the rule, I infer that the odds are $9/5 \cdot 3/1=27/5$ that it is gold; and that is correct since $(aAa+pPp)/(pAp+aPa)= 81/15=27/5$. But suppose that the witnesses, instead of testifying to the ball being auriferous, or otherwise, testify, the one that it is heavy, and the other that it is yellow, and leave the inference to me. Then the argument from its being heavy will be true 3 times to every 2 times that it is false, whether the color test succeed or fail; for $aAa:aPp=15:10=3:2$ and $aAp:aPa=21:14=3:2$, and in like manner the argument from its being yellow will hold 5 times for every 2 failures; for $aAa:pPa= 15:6=5:2$ and $pAa:aPa=35:14=5:2$. But if, following the rule, I were to infer that the odds that the ball was auriferous were $3/2 \cdot 5/2= 15/4$ I should be wrong; for

the true odds are $aAa:aPa=15:14$. If I should take into account the argument that $3/7$ of all the balls are auriferous, and say the odds were $3/2.5/2.3/4=45/16$, I should still be wrong. From this an unskilled person might suppose that the rule did not hold in case of arguments. But two errors would be involved. In the first place the odds in favor of a sign's signifying a fact are equal to the ratio of the probability of the occurrence of the sign when the fact takes place to the probability of the occurrence of the sign when the fact does not take place; and in the second place the independence of two signs, considered as signifying the same fact, consists in the one occurring with the same proportionate frequency whether the other occurs or not, and when the fact takes place, and further, with the same proportionate frequency whether the other occurs or not, when the fact does not take place. But it is not necessary that the one should occur with the same proportionate frequency whether the other occurs or not, in general, without reference to whether the fact occurs or not. The required independence is not found in the above numbers. It is, however, found in the following:

<i>Auriferous.</i>	<i>Plumbiferous.</i>
Heavy Light	Heavy Light

Yellow,	$aAa = 21$	$pAa = 3$	$aPa = 10$	$pPa = 2$
Grey,	$aAp = 14$	$pAp = 2$	$aPp = 15$	$pPp = 3$

The odds in favor of a ball being auriferous:

Antecedently are	$40 : 30 = 4:3$
As yellow are	$24/40 : 12/30 = 3:2$
As heavy are	$35/40 : 25/30 = 21:20$

Hence, on the whole, the odds in favor of a heavy yellow ball being auriferous are $4/3 \cdot 3/2 \cdot 21/20 = 21/10$, which is, of course, correct. The demonstration that this is always so, is now extremely simple. Write x for aAp/pAp and y for pAa/pAp . Also $\{x\}$ for aPp/pPp and $\{e\}$ for pPa/pPp . Then the condition of independence is that

$$aAa/pAp = xy \text{ and } aPa/pPp = \{x\}\{e\}$$

Then the odds

Antecedently will be

$$pAp(1+x)(1+y):pPp(1+\{x\})(1+\{e\});$$

As yellow will be $y(1+\{e\})/(1+y)\{e\}$;

As heavy will be $x(1+\{x\})/(1+x)\{x\}$;

and the product of the three will be $(pAp\ xy)/(pPp\ \{x\}\{e\}) = aAa/aPa$, as it should be.

Peirce: CP 7.169 Cross-Ref:††

169. Thus, when the essential conditions are fulfilled, this method is perfectly correct. Nor is it requisite that they should be fulfilled with any exactitude. A rough approximation is sufficient to give the conclusion some value. But the further from fulfilment the conditions are, the further from any scientific value is the conclusion; and with sufficient time and space I would undertake to show that, in reference to ancient history, they are, in a large majority of those cases in which there is any room for two opinions, so far from fulfilment, that it not only becomes utter nonsense to talk of "proof" and "perfect demonstration," -- phrases perpetually in the mouths of the critics, -- but, were there no better way of investigation, this method, taken as the general and regular method of treating questions of ancient history, must sink it in all its details to the rank of idle surmise. In this paper, however, I shall only give an outline of what this argument would be, because it is here not my principal object to refute the method now prevalent, but to expound a different logical theory, and to show what method of study results from it.

Peirce: CP 7.170 Cross-Ref:††

170. Let it be clearly understood, then, that what I attack is the method of deciding questions of fact by weighing, that is by algebraically adding, the feelings of approval produced in the mind by the different testimonies and other arguments pertinent to the case. I acknowledge that this method is supported, under abstract conditions, by the doctrine of chances, and that there are cases in which it is useful. But I maintain that these conditions are not often even roughly fulfilled in questions of ancient history; so that in those investigations it commonly has no value worth consideration.

Peirce: CP 7.171 Cross-Ref:††

171. Let us first make sure that we take proper account of everything that can be urged in favor of the method. Now, as far as I am aware, beyond its foundation in the doctrine of chances, the argument which was stated with such consummate skill by Hume, there are only two things to be said in favor of this method.

Peirce: CP 7.172 Cross-Ref:††

172. The first is that every science must develop its own method out of the natural reason of man; and that is the very way in which this method has been developed. Balancing reasons *pro* and *con* is the natural procedure of every man. No man can avoid doing so continually; and if he could, he would only have trained himself to the observance of rules having no foundation in reason. For reason is

nothing but man's natural way of thinking, carefully and consistently observed.

Peirce: CP 7.173 Cross-Ref:††

173. The remaining argument in favor of this method is that the only alternatives are that of using this method and that of swallowing uncriticized all the incredible tales with which ancient history abounds.

Peirce: CP 7.174 Cross-Ref:††

174. This last argument need not detain us, because I shall in this paper develop a different method, which, instead of being less critical than that of balancing likelihoods, is much more so. But I repeat that I do not maintain that the ordinary method is never to be employed, but that its use should be restricted to exceptional cases, instead of being made the regular and standard procedure.

Peirce: CP 7.175 Cross-Ref:††

175. Now as to this method's being natural, I admit that there is some foundation for that. There is no kind of fallacious reasoning to which mankind is liable for which as much as that might not be said. But I appeal to modern psychologists to support me in the assertion, that it is not at all natural for men to employ this method as a usual procedure. On the contrary, the natural thing is to believe anything that one may hear said, until it is found that that assumption leads to difficulties; and when it is found to lead to difficulties, the most natural impulse is to make further inquiries, to cross-examine, etc. The occasions when we naturally balance reasons *pro* and *con* mostly relate to what we prefer to do, not to questions of fact. But, in the next place, I demur to the principle that what is natural is necessarily reasonable. It is one of the consequences of German preeminence in science and philosophy, which I hope will not last much longer, that subjective ways of deciding questions are, at this time, far too highly esteemed. Logic itself is made a pure question of feeling by Sigwart, whose treatise is now more in vogue than any other.†3 The Anglo-Saxon mind will never assent to that. I am sorry to say that it has been only too true that, under the German lead, the methods of reasoning in the different branches of science and philosophy have been left to grow up pretty much as they naturally would; and sooner or later, no doubt, natural tendencies do bring them right; but that result would be brought about much more quickly if methods were subjected to a more continual and strict criticism from exact logic; and what I mean by this, I must hope that this very paper may illustrate.

Peirce: CP 7.176 Cross-Ref:††

176. Passing now to the objections to the method of balancing likelihoods in the study of ancient history, the most obvious, perhaps, although not the most important, is, that the different testimonies and other arguments, are not commonly even in a rough sense independent, as the only rational basis for the method requires that they should be. Circumstantial evidences are, no doubt, often sufficiently independent; but direct testimony seldom is so. The same circumstances which lead one witness into error are likely to operate to deceive another. Nor does this want of independence always lead them into agreement. It may, frequently, be the cause of disagreement. Conflict of testimony in the vast majority of cases is *not* principally a mere chance result, as the theory supposes it to be. That concordance of testimony commonly has some other cause than its mere tendency to truth, is too obvious to need saying. The method of balancing likelihoods not only supposes that the testimonies are independent but also that each of them is independent of the antecedent probability of the story; and since it is far more difficult to make allowance for a violation of this requirement than of that of the independence of the

testimonies, it becomes a much more serious matter. But how very remote from the real state of things it is to suppose that the narration of an ancient event is independent of the likelihood of the story told! Roughly speaking, it may be said that all detached stories of Greece and Rome, were told chiefly because the writer had something marvellous to recount; so that we may almost say that ancient history is simply the narrative of all the unlikely events that happened during the centuries it covers. It is evident that this circumstance in itself almost destroys the legitimate weight of any argument from the antecedent improbability, unless that improbability is so great as to render the story absolutely incredible. Examples are useful at this point. It is well-known that three ancient authorities give the story that Pythagoras had a golden thigh; and the custom of modern critics is simply to pass it by, hardly mentioned. Now had any historian asserted that the thigh of Pythagoras was a metallic gold to the centre while his lower leg and foot were solid flesh, that would unquestionably have been a case in which the method under consideration might very properly have been employed to reject the testimony. I may mention, however, that one of the authorities affords an illustration of the opposite kind of influence of antecedent probability upon the matter of testimony. For when Diogenes Laertius softens the story as he does, it is in my opinion, in order to avoid extreme improbability. As another example, let us take a story the extreme improbability of which has caused almost, if not quite, every modern critic to over-rule the testimony of a baker's dozen of the greatest authorities that antiquity can boast. This story is that the mathematician Thales once stumbled and fell into a ditch while he was showing an old woman the constellations. Zeller, one of the few modern writers who so much as condescends to show reasons for almost giving Aristotle, Plato, Cicero, and all the rest the lie, says that it is utterly incredible that Thales should have been such an impracticable theorist. Considering that pretty much all we know about the personality of Thales is that the Greeks considered him as the first of the wise men, and that eccentricity was, according to the Greek conception, essential to the character of a philosopher, that reason of Zeller's shows a wonderful depth of psychological insight. Of all the modern mathematicians whom I have known, there have been perhaps not over one in five, of whom I should not hesitate to believe such a thing. But I should like to know how the story ever came to be so generally stated, both by ancient writers, and by all modern writers until the days of modern criticism, if it is not that the whole thing, both Thales stumbling and old woman's and Zeller's contempt for him for doing so, is too richly true to human nature. If it is not historical, it must surely have been its extreme antecedent likelihood which caused so many authorities to assert that it was true. Many more examples are needed in order to show how very far ancient testimony is from being independent of the antecedent probability of its matter. But I leave this point, in order to hasten to another which is more important.

Peirce: CP 7.177 Cross-Ref:††

177. The theory of probabilities has been called the logic of the modern exact sciences;†4 and it is known to be the basis of the vast business of insurance; and therefore when a literary man learns that the method which he has been pursuing has the sanction of such a great mathematical doctrine, he begins to feel that he is a very scientific person. I notice that this sense of personal scientificity is far more developed in men who write second-hand commentaries on ancient authors than it is in the Faradays, the Helmholtzes, and the Mendeléefs. It is, therefore, well to point out to such persons that the word *probability*, taken in the sense in which the insurance business uses it, means a well-founded statistical generalization. Nor are probabilities assumed in the exact sciences without either a statistical basis or else a

thoroughly criticized assurance that no serious error can result. But if by "probability" be meant the degree to which a hypothesis in regard to what happened in ancient Greece recommends itself to a professor in a German university town, then there is no mathematical theory of probabilities which will withstand the artillery of modern mathematical criticism. A probability, in that sense, is nothing but the degree to which a hypothesis accords with one's preconceived notions; and its value depends entirely upon how those notions have been formed, and upon how much objectivity they can lay a solid claim to. If a man bring me a collection of sphygmograph-tracings accompanied with notes of the circumstances under which they were taken, and tells me that he thinks they prove that the pulse of a man is affected by the mental state of another man on the other side of a brick wall, I confess that his hypothesis is so contrary to my preconceived notions that I shall not easily be persuaded to interrupt my work to make a study of the case. But those preconceived notions I hold to have a far more solid basis than those which ordinarily influence historical critics to pronounce an ancient narrative improbable. Yet even so, it is only my practical conduct which I allow to be influenced by that improbability. My action has to be decided one way or the other, and without loss of time; and "rough and ready" is unavoidably the character of the majority of practical decisions. But were I once to undertake the study of the sphygmograph-tracings, I would endeavor to get to the bottom of the question, without reference to my preconceived notions. For preconceived notions are only a fit basis for applications of science, not for science itself.

Peirce: CP 7.178 Cross-Ref:††

178. Thus everything that is put into one pan of the balance in weighing historical probabilities is utterly uncertain. Yet, if possible, what goes into the other pan is worse still. This consists of the "credibilities" of the testimonies. The inappropriateness of the application of the conception of probability here is striking. In playing a game, say with dice, there is this good reason for the calculation of chances, that any one face turns up as often as any other, quite independently of the result of any other throw, and the cause of the die turning up any particular face at any particular throw is quite beyond our powers of analysis. It is probably due to the combination of many little influences. In like manner, in insurance, though the cause of any one man's death might be ascertained, yet that would have no relation to the purposes of insurance, and why it is that out of a thousand men insured at the age of thirty, just so many will die each year afterward, is a question not to be answered, except that it is due to the coöperation of many causes. It is this which makes the calculation of chances appropriate. So, in making astronomical observations, why it is that out of a thousand observations, just so many will have just such an amount of error, can only be answered by saying that it is due to the summation of many small effects. But now on the other hand, take a question of history. We do not care to know how many times a witness would report a given fact correctly, because he reports that fact but once. If he misstates the matter, there is no coöperation of myriad causes. It is on the contrary, due to some one cause which, if it cannot often be ascertained with certainty, can at any rate be very plausibly guessed in most cases, if the circumstances are closely inquired into; and it is most pertinent to the business of the historical critic to consider how a mistake, if mistake there be, might credibly have arisen. A mere general ratio of true statements to false, would be utterly insufficient for his purpose, even if it really existed. But it does not exist. In the case of the die, we know that one throw in every six will bring up an ace in the future as it has done in the past; and so it is with insurance, and with the errors of observations. However complex the causes, that simple law *will be* followed, we are sure. But nothing of the kind is true in the

case of a witness. His new statements, if he makes any, will necessarily relate to different topics from his old ones which he has exhausted; and his personal relation to them will be different. There is, therefore, no arguing from what his credibility was in one case, to what it will be in another, as there would be if the error were a sum due to the recurrence of myriad small effects. There thus neither is any such quantity as a real, general, and predictive truthfulness of a witness; nor if there were, would it answer the purposes of the historian to deal with it. For he does not want to know merely how many of a witness's statements out of a hundred are wrong; but just which ones of them are wrong.

Peirce: CP 7.179 Cross-Ref:††

179. This objection goes vastly deeper and is vastly weightier than that based on the want of independence of the arguments. Yet even this objection is downright insignificant as compared with the principal one, which I now proceed to state, although I have already hinted at it.

Peirce: CP 7.180 Cross-Ref:††

180. All mathematical reasoning, even although it relates to probability, is of the nature of necessary reasoning. All necessary reasoning consists of tracing out what is virtually asserted in the assumed premisses. While some of these may be new observations, yet the principal ones relate to states of things not capable of being directly observed. As has often been said, especially since Kant, such reasoning really does not amplify our positive knowledge; although it may render our understanding of our own assumptions more perfect. It is the kind of reasoning proper for any application of science. For example, it is by such reasoning that, assuming the law of gravitation to have been scientifically established, we go on to predict the time and place of an eclipse of the sun. Or, if our desire is to rectify our theory of the moon, we may do so by comparing such predictions, regarded as conditional, with observations. If, in making the correction, we assume that there can be no error discoverable by these observations except in the values of one or two constants employed, the correction is itself made by a mere application of principles assumed to be already scientifically established; and although it will be called a contribution to science, it leaves the frame-work of the theory untouched, and merely consists in incorporating the new observations into the places provided for them in our existing assumptions, so that there really is, in the logician's sense, no enlargement of our knowledge, but merely an arrangement or preservation of the systematization of knowledge already established. In applying observations to the fundamental correction of a theory, as Kepler applied Tycho's observations to the correction of the crude Copernican system, a kind of reasoning comes in which is not purely mathematical demonstration. If I remember rightly, there were only three points in the orbit of Mars, -- I am sure there were only about three or four, -- where Kepler ascertained the position of Mars in space by positive triangulation. Even those triangulations involved hypothetical elements, such as the assumption that the orbit was the same at every revolution, which might very well not have been true; and even had they been absolutely positive, they were altogether inadequate to determining the form of the orbit. These, therefore, as well as all the other items of his argument, were merely of this nature, that all simpler theories having been proved inadequate, all the predictions which could be based upon the theory of the elliptical orbit were verified by the observations well within the limits of possible, and even of not apparently unlikely, error. This was not mathematical demonstration; and all the subsequent work upon the solar system has merely multiplied and made more precise the same kind of proof, but has not changed its character. It is not now mathematical demonstration any more than it was

then. Empirical science can never be enlarged by mathematical demonstration or any other kind of necessary reasoning; although when nomological science has advanced to a certain point, a mathematical theory can be based upon it which will be useful, not only for nomological science itself, but also for the classificatory and descriptive sciences which depend upon it.

Peirce: CP 7.181 Cross-Ref:††

181. Now ancient history occupies a place among the psychical sciences somewhat analogous to that of astronomy among the physical sciences. The one is a description of what is distant in the world of mind, as the other is a description of what is distant in the world of matter; and curiously enough, or significantly enough, an ancient alliance exists between the two sciences through chronology. Yet the amount of aid which physical astronomy can derive from mathematics is quite moderate, notwithstanding the mathematical perfection of nomological physics.†5 Anybody can convince himself that the reasoning of physical astronomy is not of a demonstrative kind by simply running over any text-book on the subject. But the science of nomological psychics,†6 -- psychology, as we call it, -- is still far too backward to afford any distinguished aid to history; and consequently, the demonstrative part of rightly reasoned history, exclusive of mere chronology, must, for a long time, remain very small. History, however, is as much more worthy than astronomy of being studied scientifically as mind is more worthy of our attention than matter. The use we should desire to make of ancient history is to *learn* from the study of it, and not to carry our preconceived notions into it, until they can be put upon a much more scientific basis than at present they can. Consequently, the staple of our reasoning in ancient history should not be of the demonstrative kind, as it is, as long as it remains, at best, an application of the mathematical doctrine of chances. If somebody replies that in weighing arguments *pro* and *con* critics make no use of the mathematical calculus of probabilities, the rejoinder will be that their proceeding only differs from that by its greater vagueness, and that a vague and inexact use of probabilities has no logical advantage over a more critical employment of them. If it is said that, as far as possible, the critics avoid likelihoods, and aim at positive certainty, the answer will be that they endeavor to do this by the employment of apodictic arguments, which only mark a still less exact grade of the same kind of demonstrative reasoning. Fully to appreciate the force of this argument one must have a well-matured comprehension of the logic of science; but when it is fairly apprehended, it cannot but be deemed quite conclusive.

Peirce: CP 7.182 Cross-Ref:††

182. Nevertheless, there still remains a further objection to the method of balancing likelihoods in the study of ancient history which is worthy of attention even after what has been said. We all know that as soon as a hypothesis has been settled upon as preferable to others, the next business in order is to commence deducing from it whatever experiential predictions are extremest and most unlikely among those deducible from it, in order to subject them to the test of experiment, and thus either quite to refute the hypothesis or make such corrections of it as may be called for by the experiments; and the hypothesis must ultimately stand or fall by the result of such experiments. Now what is true of any single hypothesis should equally hold good for any method of constructing many hypotheses. It, too, should have its consequences experimentally tested, and must stand or fall by the ultimate result. Now within the last half of the nineteenth century the merits of the procedure of the historical critics have been many times subjected to the test of archeological exploration; and what has been the result? I have not the necessary knowledge myself to sum it up in a

magisterial manner; but from what I have casually heard about the relation of Egyptian exploration to the critics' previous opinion of Manetho and even of Herodotus, about the explorations in the Troiad and in Mycenae, and much else, I gather that, on the whole, it has been shown that the critics were found to be more or less fundamentally wrong in nearly every case, and in particular that their fashion of throwing all the positive evidence overboard in favor of their notions of what was likely, stands condemned by those tests. If this be so, it is no slight modification, but a complete revolution, of their logic which is called for; because, considering their great learning and competence, and the absolute confidence which they attached to their conclusions, as perfectly ineluctable, there is no middle course between pronouncing those men to have been a pack of charlatans and concluding that their method was wrong in principle. If it were not so, their pretensions to scientific determinacy of those conclusions would have been simply disgraceful.†7

Peirce: CP 7.183 Cross-Ref:††
§4. THE LOGIC OF SCIENCE

183. Having thus outlined the argument for the timeliness of a new logical theory of the proper method of dealing with ancient testimonies, I proceed to show how the question appears from the point of view of the "Minute Logic"†8 of which I am a defender.

Peirce: CP 7.184 Cross-Ref:††

184. I could not present the reason which has the greatest weight to my own mind, so that it should be convincing, unless I were to write a paper more than fifteen times the length of this one. That has to be foregone. Meantime, the secondary considerations that remain will be strong enough to maintain the position successfully.

Peirce: CP 7.185 Cross-Ref:††

185. To begin with, let me say that I propose to confine myself exclusively to the consideration of the proper *scientific* procedure concerning the documents in question. I do not propose to touch upon the question of miracles in so far as it is a practical religious question for an individual man.†9 This is not from timidity or any indisposition to express myself, could I have my whole say; but it is because it would expand this paper beyond all bounds of convenience in all respects. A practical belief is what a man proposes to go upon. A decision is more or less pressing. What ought it to be? That must depend upon what the purpose of his action is. What then, is the purpose of a man? That is the question of pure ethics, a very great question which must be disposed of before the logic of practical belief can be entered upon to any good effect. With science it is entirely different. A problem started today may not reach any scientific solution for generations. The man who begins the inquiry does not expect to learn, in this life, what conclusion it is to which his labors are tending. Strictly speaking, the inquiry never will be completely closed. Even without any logical method at all, the gradual accumulation of knowledge might probably ultimately bring a sufficient solution. Consequently, the object of a logical method is to bring about more speedily and at less expense the result which is destined, in any case, ultimately to be reached, but which, even with the best logic, will not probably come in our day. Really the word belief is out of place in the vocabulary of science. If

an engineer or other practical man takes a scientific result, and makes it the basis for action, it is he who converts it into a belief. In pure science, it is merely the formula reached in the existing state of scientific progress. The question of what rules scientific inference ought to follow in order to accelerate the progress of science to the utmost is a comparatively simple one, and may be treated by itself. The question of how a given man, with not much time to give to the subject, had best proceed to form his hasty decision, involves other very serious difficulties, which make it a distinct inquiry. The former question, taken by itself, will be enough for the present communication.

Peirce: CP 7.186 Cross-Ref:††

186. I have said that in order to determine what the logic of the individual man should be, it would be necessary to consider what his purpose was. The same remark applies to the logic of science. It is easier to determine the purpose of science. It does not involve opening the question of ethics. Yet it is not a perfectly simple matter, either. Several definitions of the purpose of science that I have met with made it the business of science to ascertain that certain things were so, to reach foregone conclusions. Nothing could be more contrary to the spirit of science. Science seeks to discover whatever there may be that is true. I am inclined to think that even single perceptual facts are of intrinsic value in its eyes, although their value in themselves is so small that one cannot be quite sure that there is any. But every truth which will prevent a future fact of perception from surprising us, which will give the means of predicting it, or the means of conditionally predicting what would be perceived were anybody to be in a situation to perceive it, this it is, beyond doubt, that which science values. Although some will contradict me, I am bound to say that, as I conceive the matter, science will value these truths for themselves, and not merely as useful. Mathematics appears to me to be a science as much as any science, although it may not contain all the ingredients of the complete idea of a science. But it is a science, as far as it goes; the spirit and purpose of the mathematician are acknowledged by other scientific men to be substantially the same as their own. Yet the greater part of the propositions of mathematics do not correspond to any perceptual facts that are regarded as even being possible. The diagonal of the square is incommensurable with its side; but how could perception ever distinguish between the commensurable and the incommensurable? The mathematical interest of the imaginary inflections of plane curves is quite as great as that of the real inflections. Yet we cannot say that the scientific man's interest is in mere ideas, like a poet's or a musician's. Indeed, we may go so far as to say that he cares for nothing which could not conceivably come to have a bearing on some practical question. Whether a magnitude is commensurable or not has a practical bearing on the mathematician's action. On the other hand, it cannot be said that there is any kind of proportion between the scientific interest of a fact and its probability of becoming practically interesting. So far is that from being the case, that, although we are taught in many ways the lesson [of] the Petersburg problem, -- so stupidly obscured by the extraneous consideration of moral expectation, -- the lesson that we utterly neglect minute probabilities, yet for all that, facts whose probabilities of ever becoming practical are next to nothing are still regarded with keen scientific interest, not only by scientific men, but even by a large public. Here, then, are the facts to be reconciled in order to determine what the purpose of science, what scientific interest, consists in. First, every truth which affords the means of predicting what would be perceived under any conceivable conditions is scientifically interesting; and nothing which has not conceivable bearing upon practice is so, unless it be the perceptual facts themselves. But, second, the scientific interest does not lie in the application of those truths for the sake of such predictions. Nor, thirdly, is it true

that the scientific interest is a mere poetical interest in the ideas as images; but solid truth, or reality, is demanded, though not necessarily existential reality. Carefully comparing these three conditions, we find ourselves forced to conclude that scientific interest lies in finding what we roughly call generality or rationality or law to be true, independently of whether you and I and any generations of men think it to be so or not. I might enunciate and prove this with more accuracy and evidence; but since I am not now undertaking to present the subject with the strictest method, I think what I have said will answer my purpose. But however this question be argued, it is one of those concerning which

A man convinced against his will

Is of his own opinion still,

(a current corruption worth dozens of such as its original). The dry light of intelligence is manifestly not sufficient to determine a great purpose: the whole man goes into it. So the fact that logic depends upon such a question is sufficient to account for the endless disputes of which logic is still the theatre.

Peirce: CP 7.187 Cross-Ref:††

187. Confining ourselves to science, inference, in the broadest sense, is coextensive with the deliberate adoption, in any measure, of an assertion as true. For deliberation implies that the adoption is voluntary; and consequently, the observation of perceptual facts that are forced upon us in experience is excluded. General principles, on the other hand, if deliberately adopted, must have been subjected to criticism; and any criticism of them that can be called scientific and that results in their acceptance must involve an argument in favor of their truth. My statement was that an inference, in the broadest sense, is a deliberate adoption, *in any measure*, of an assertion as true. The phrase "in any measure" is not as clear as might be wished. "Measure," here translates *modus*. The modes of acceptance of an assertion that are traditionally recognized are the necessary, the possible, and the contingent. But we shall learn more accurately, as our inquiry proceeds, how the different measures of acceptance are to be enumerated and defined. Then, as to the word "true," I may be asked what this means. Now the different sciences deal with different kinds of truth; mathematical truth is one thing, ethical truth is another, the actually existing state of the universe is a third; but all those different conceptions have in common something very marked and clear. We all hope that the different scientific inquiries in which we are severally engaged are going ultimately to lead to some definitely established conclusion, which conclusion we endeavor to anticipate in some measure. Agreement with that ultimate proposition that we look forward to, -- agreement with that, whatever it may turn out to be, is the scientific truth.

Peirce: CP 7.188 Cross-Ref:††

188. Perhaps there will here be no harm in indulging in a little diagrammatic psychology after the manner of the old writers' discussions concerning the *primum cognitum*; for however worthless it may be as psychology, it is not a bad way to get orientated in our logic. No man can recall the time when he had not yet begun a theory of the universe, when any particular course of things was so little expected that nothing could surprise him, even though it startled him. The first surprise would naturally be the first thing that would offer sufficient handle for memory to draw it

forth from the general background. It was something new. Of course, nothing can appear as definitely new without being contrasted with a background of the old. At this, the infantile scientific impulse, -- what becomes developed later into various kinds of intelligence, but we will call it the scientific impulse because it is science that we are now endeavoring to get a general notion of, -- this infantile scientific impulse must strive to reconcile the new to the old. The first new feature of this first surprise is, for example, that it is a surprise; and the only way of accounting for that is that there had been before an expectation. Thus it is that all knowledge begins by the discovery that there has been an erroneous expectation of which we had before hardly been conscious. Each branch of science begins with a new phenomenon which violates a sort of negative subconscious expectation, like the frog's legs of Signore Galvani.

Peirce: CP 7.189 Cross-Ref:††
§5. REGULARITY AND EXPLANATION

189. What, then, is that element of a phenomenon that renders it surprising, in the sense that an explanation for it is demanded? *Par excellence*, it is irregularity, says Dr. Paul Carus, in substance.†10 I cannot but think that there is a faulty analysis here. Nobody is surprised that the trees in a forest do not form a regular pattern, or asks for any explanation of such a fact. So, irregularity does not prompt us to ask for an explanation. Nor can it be said that it is because the explanation is obvious; for there is, on the contrary, no explanation to be given, except that there is no particular reason why there should be a regular pattern, -- or rather that there is no sufficient reason, because there must be a tendency for large trees to grow where there is most room, which tendency, if it were strong enough and undisturbed enough, would produce a regular pattern. I mention this to show that, so far is mere irregularity [from being] a motive for demanding an explanation, that, even when there is a slight reason for expecting a regularity and we find irregularity, we do not ask for an explanation; whereas if it were an equally unexpected *regularity* that we had met with, we certainly should have asked for an explanation. I am, for reasons similar to this, as well as for others, confident that mere irregularity, where no definite regularity is expected, creates no surprise nor excites any curiosity. Why should it, when irregularity is the overwhelmingly preponderant rule of experience, and regularity only the strange exception? In what a state of amazement should I pass my life, if I were to wonder why there was no regularity connecting days upon which I receive an even number of letters by mail and nights on which I notice an even number of shooting stars! But who would seek explanations for irregularities like that?

Peirce: CP 7.190 Cross-Ref:††

190. Let me not, however, be understood to make the strength of an emotion of surprise the measure of a logical need for explanation. The emotion is merely the instinctive indication of the logical situation. It is evolution ({physis}) that has provided us with the emotion. The situation is what we have to study.

Peirce: CP 7.191 Cross-Ref:††

191. Before dismissing irregularity, I may note, as aiding to clear the matter up, that a breach of an existing regularity always stimulates a demand for an explanation; but where, having expected regularity, we only find irregularity without

any breach of regularity, we are only induced to revise our reasons for expecting anything. Irregularity, be it noted, cannot be expected, as such. For an expectation is, in every case, founded upon some regularity. For the same reason, merely not finding regularity where no particular regularity was expected, occasions no surprise.

Peirce: CP 7.192 Cross-Ref:††

192. In order to define the circumstances under which a scientific explanation is really needed, the best way is to ask in what way explanation subserves the purpose of science. We shall then see what the evil situation is which it remedies, or what the need is which it may be expected to supply. Now what an explanation of a phenomenon does is to supply a proposition which, if it had been known to be true before the phenomenon presented itself, would have rendered that phenomenon predictable, if not with certainty, at least as something very likely to occur. It thus renders that phenomenon rational, -- that is, makes it a logical consequence, necessary or probable. Consequently, if without any particular explanation, a phenomenon is such as must have occurred, there is no room at all for explanation. If the phenomenon is such as need not have occurred on the special occasion, but must occur on occasions differing in no discoverable and exactly assignable pertinent respect from the special occasion on which the phenomenon in question actually occurs, still there is nothing for explanation to do, until it is ascertained in what respects, if any, the individual occasion differs from those other occasions. For example, I throw a die, and it turns up ace. Now I know already that this die will turn up ace once in six times; and I am persuaded that it would be hopeless to attempt, at present, to find any pertinent conditions fulfilled on this occasion which are not fulfilled every time the die is thrown. Hence, no proposed explanation of the die's turning up an ace can be in order, unless we can discover some peculiar and pertinent feature about the present occasion. Why should my lottery-ticket have drawn a blank, and somebody else's a prize? No explanation is called for. The question is silly.

Peirce: CP 7.193 Cross-Ref:††

193. Let us now pass to the case of a phenomenon in which, apart from a particular explanation, there was antecedently no reason for expecting it, and as little for expecting it not to happen. Suppose, for example, that on the day of the Lisbon earthquake the brightest new star had appeared in the heavens. There might possibly have been some explanation for this; but there would have been no motive for searching for one. To have done so would, indeed, have been a foolish proceeding, for reasons we need not now consider.

Peirce: CP 7.194 Cross-Ref:††

194. Thus, the only case in which this method of investigation, namely, by the study of how an explanation can further the purpose of science, leads to the conclusion that an explanation is positively called for, is the case in which a phenomenon presents itself which, without some special explanation, there would be reason to expect would *not* present itself; and the logical demand for an explanation is the greater, the stronger the reason for expecting it not to occur was.

Peirce: CP 7.195 Cross-Ref:††

195. Since it is never prudent to rely upon reasoning that is largely deductive, without a check upon its accuracy, especially where the conclusion is disputed, as this is, I will select a few examples calculated to refute it, if it is to be refuted; and examine its application to them. First, suppose the phenomenon observed consists simply in irregularity; then, if there were no ground for anticipating any particular regularity, there is simply nothing to explain (irregularity being the prevailing

character of experience generally). This agrees with our natural judgment. But if we anticipate a regularity, and find simple irregularity, but no *breach* of regularity, -- as for example if we were to expect that an attentive observation of a forest would show something like a pattern, then there is nothing to explain except the singular fact that we should have anticipated something that has not been realized. Here, by our theory, there is need of an explanation, not of an objective, but of a subjective phenomenon (pardon the jargon, -- slang jargon, at that). This again agrees with our natural judgment; for in such a case we straightway commence reviewing our logic to find how our error is to be explained.

Peirce: CP 7.196 Cross-Ref:††

196. Street cars are famous *ateliers* for speculative modelling. Detained there, with no business to occupy him, one sets to scrutinizing the people opposite, and to working up biographies to fit them. I see a woman of forty. Her countenance is so sinister as scarcely to be matched among a thousand, almost to the border of insanity, yet with a grimace of amiability that few even of her sex are sufficiently trained to command: -- along with it, those two ugly lines, right and left of the compressed lips, chronicling years of severe discipline. An expression of servility and hypocrisy there is, too abject for a domestic; while a certain low, yet not quite vulgar, kind of education that is evinced, together with a taste in dress neither gross nor meretricious, but still by no means elevated, bespeak companionship with something superior, beyond any mere contact as of a maid with her mistress. The whole combination, although not striking at first glance, is seen upon close inspection to be a very unusual one. Here our theory declares an explanation is called for; and I should not be long in guessing that the woman was an ex-nun.

Peirce: CP 7.197 Cross-Ref:††

197. In this last case, the emotion of surprise is not felt, because the cognitive part of the mind must be uppermost in order to recognize the rarity of the phenomenon. There are cases in which the most familiar of facts seem to call for explanation. I am myself, for example, fond of urging that no theory of space can be satisfactory which does not explain why it should have three dimensions. Perhaps all will not agree with me on this point. They will say, it must have some number of dimensions; why not three as well as any number? Or I may be asked what number of dimensions I should expect space to have. My reply is, that if I did not know what number of dimensions space really had, and was obliged to investigate the question as we usually investigate scientific questions, by trying successive hypotheses until I found one that experiment would not refute, I should soon see that one dimension would not be sufficient; and I should try two as giving, not only the simplest, but by far the most comprehensible, of continua. I should guess that it was similar to the field of imaginary quantity. When that was refuted, I should pass to the next most comprehensible continuum, that of the field of real quaternions, quadridimensional space. Although the reasons for those numbers are not at all apodictic, yet I should, I am sure, be much surprised to learn that its dimensionality was three, which is so much more difficult to conceive than four. No doubt, it may be said that rationality has nothing to do with the question; and I have to confess that the fact that space has three dimensions has the air, at least, of proving that rationality has, in fact, nothing to do with it. But if it has not, still it seems to me that three is a number one would decidedly not expect. For triads mostly have some connection with Rationality; while things that are not governed by Rational considerations very seldom have three elements. I say all this, because it seems to me that this is almost a crucial instance for my theory of what it is that demands explanation. For, to the majority of minds, who

would not definitely expect one number of dimensions rather than another, the fact that space has three dimensions does not seem to call for any particular explanation. That this is the fact seems to be proved by the circumstance that, of all the philosophers who have elaborated theories of space, hardly one has paid the smallest attention to the number of its dimensions, or regarded it as at all significant. But in me we have an instance of a mind to which it does seem that this feature of space calls for some definite explanation; and this same mind we find differs from the others in that it would decidedly have expected antecedently some other number. Certainly, my theory of what it is that demands explanation appears to be remarkably verified in this instance.

Peirce: CP 7.198 Cross-Ref:††

198. It is singular that there are not many logicians who attempt to define the circumstances which render an explanation of a phenomenon desirable or urgent. The majority of them seem tacitly to assume that any one fact calls for explanation as much as any other. Mr. Venn, however, in his *Empirical Logic*, †11 states, without much discussion, that it is the *isolation* of a fact that creates the need of an explanation. This approaches pretty close to my opinion, since the work of reason consists in finding connections between facts. Still, the distinctions between the two doctrines are manifold, too. All facts are more or less connected and more or less separated; so that Mr. Venn ought to say, and probably would say, that all facts call for explanation more or less. According to me, however, the demand for explanation is a more definite demand. All conceivable facts are divisible into those which, upon examination, would be found to call for explanation and those which would not. For if any fact would call for explanation, then if that which was ascertained in the consequent investigation was no more nor less than the falsity of that supposed fact, this latter would not call for explanation. Although I have not bestowed upon Mr. Venn's whole volume the minute study which it merits, so that I may be mistaken, I think I can account for this discrepancy. Mr. Venn belongs to a school which considers the logical process as starting at the percepts, if not at impressions of sense. Mr. Venn is himself so candid and so acute that he may perhaps have seen the error of this. But supposing that he has followed his school, the discrepancy between him and me would easily be accounted for, because there can be no doubt that every *percept* does involve elements that call for explanation. But I maintain that logical criticism cannot go behind *perceptual facts*, which are the first judgments which we make concerning percepts. A perceptual fact is therefore an abstract affair. Each such fact covers only certain features of the percept. I look at an object and think that it seems white. That is my judgment of the object perceived, or my judgment concerning the percept, but not the percept itself; and it is idle to attempt to criticize by any logic that part of the performance of the intellect which draws that judgment from the percept, for the excellent reason that it is involuntary and cannot be prevented or corrected. Such a fact which represents the percept in a very meagre way, although it is, in itself, a relatively isolated fact, -- as isolated as any fact can be, -- nevertheless does not, in itself, call for any explanation. On the contrary, it can only do that when it has been connected with other facts which taken by themselves would justify an expectation of the contrary of this fact. For example, if we should find that this object which seemed white, in the first place *was* white, and then that it was a crow, and finally that all the crows known were black, then the fact of this seeming and really being white would require explanation. It might be an albino, or it might be some new species or variety of crow. But perhaps it will be insisted that this thing's appearing white *does* call for an explanation; -- that we want to know the cause of its being white. To this I reply that it has always been agreed that the tendency of the understanding was merely

toward synthesis, or unification. Now no fact could possibly be more unified and simple than the fact that this is white, taken in itself. It would seem, therefore, that, if we consider this fact isolated from all others, it completely accomplishes the tendency of reason. To find a cause for the whiteness would only be to complicate our conception of the matter; and I never heard it suggested that intelligence *per se* demands complexity and multiplicity. But I suspect that when Mr. Venn speaks of *isolation*, he is thinking of there being other facts from which the given fact is separated; and that it is not *isolation* that he means, but *separation*. Now separation is itself a kind of connection; so that if that be his meaning, the state of things which calls for explanation is a connection which is not satisfactory to the mind. In that case, it is incumbent on Mr. Venn to explain himself more precisely, and to say in what respect it is unsatisfactory. If he were to say, "unsatisfactory in being contrary to what ought to be expected," he would come to my position, precisely.

Peirce: CP 7.199 Cross-Ref: ††

199. Further light on the question may be obtained by considering the different types of explanation, of which Mr. Venn admits three after Mill, although he says of the third that it is to be received with reserve. I so far agree with him in this, that I think if the second type is accurately defined, it will be seen to include the third as a special and not important variety. The others I fully accept, though with my own definitions of them. I will take the examples of each which Mr. Venn has himself proposed. In illustration of the first type, he says: "We notice a plant that is flagging on a hot summer day: next morning it stands up again fresh and green. 'Why has it revived in the morning?' -- 'Oh they always do.'" One may smile at the naïveté of this; and certainly, it is not an explanation in the proper sense of the word. Still, its general function is the same as that of explanation; namely, it renders the fact a conclusion, necessary or probable, from what is already well known. It might be called a *regularization*, explanation and regularization being the two types of *rationalization*. The regularization, stated in full, would be,

Plants of a certain class usually revive in the morning;

This plant belongs to that class;

∴ This plant might be expected to revive in the morning.

Now it is true that the effect of the regularization is that the fact observed is less isolated than before; but the purpose of the regularization is, I think, much more accurately said to be to show that it might have been expected, had the facts been fully known. That the demand for regularization is due to the contrary being expected is shown by the fact that when that contrary expectation is very strong indeed, a regularization which even leaves the event quite improbable will in great measure satisfy the mind. When my father, Benjamin Peirce, stated (as Leverrier himself also did, at first) that the planet discovered by Galle was not that predicted by Leverrier, †12 people generally, who imagined that, in the absence of any prediction, the entire sphere of the heavens might have had to be swept to find the planet, asked, "How, then, was it that Galle found it in the very telescopic field in which Leverrier located his planet?" This was a challenge for a regularization; to which the response was that Galle's planet was about 50 minutes of longitude distant from Leverrier's place, and that this would occur by pure chance once in two hundred times. It was,

therefore, about as extraordinary as that a given man of 75 should live to be a hundred. But the popular notion was that its probability was as one square degree is to the surface of the sphere, or as one to 41254. It is plain that the partial satisfaction which such a regularization affords is due to the great diminution of the unexpectedness.

Peirce: CP 7.200 Cross-Ref:††

200. The other type of demand for explanation is exemplified by Mr. Venn by the question, "Why is it so difficult to walk on ice?" He gives several supposed attempts at explanation; but the one he pronounces satisfactory is, 'Because, owing to the absence of friction, there is no horizontal reaction to the impulse of the feet,' which except for the misuse of the word impulse is correct, but I fear not very perspicuous to anybody who really needed the explanation! If we endeavor to place ourselves into the shoes of such a person, we must imagine ourselves noticing how easy it looks to skate upon ice, and to have remarked some such fact as that if a wagon receives a push from the land to the ice, it moves with the utmost ease on the ice. All these hazy ideas about the ice-surface, jostle one another in the mind in a perplexing way. It is, therefore, not the simple fact that ice is hard to walk on which creates the demand for an explanation: it is, on the contrary, a puzzling complexus of facts. Tell a man who never saw ice that frozen water is very hard to walk on, and he may ask whether the feet stick to it, or put other questions in order to figure to himself what you mean; but as long as the fact is apprehended by him as a simple one, he will no more ask why it should be so than a common man asks why lead should be heavy. The fact is entirely sufficient as long as it is simple and isolated. It is when the difficulty of walking on ice is compared with the extraordinary distance that a ball can be bowled upon it, or with such other facts as would naturally lead one to expect that ice would be particularly easy to walk on, that a scientific explanation is sought. This is shown by the rarity of the inquiry why it is tiresome to walk on sand. Everybody knows that it is hard to make a vehicle go over sand; and so it seems, to minds in the state of ignorance supposed, to be quite natural that walking on sand should be tiresome; and thus no explanation is asked for, although, in other respects, the question is so similar to that about ice. An isolated fact is precisely what a demand for an explanation proper never refers to; it always applies to some fact connected with other facts which seem to render it improbable.

Peirce: CP 7.201 Cross-Ref:††

201. I think I have now said enough to show that my theory -- that that which makes the need, in science, of an explanation, or in general of any rationalization of any fact, is that without such rationalization the contrary of the fact would be anticipated, so that reason and experience would be at variance, contrary to the purpose of science -- [that this theory] is correct, or as nearly so as we can make any theory of the matter at present. I will add, however, one more argument. Mr. Venn has felt the need of accounting for that desire of getting rid of isolated facts, to which he attributes the demand for an explanation; and he does so by remarking that isolated facts are dangerous. Now how, I should be glad to know, are isolated facts dangerous? The only way in which they would appear to be so, and it is the only way which Mr. Venn points out, is that in their presence we do not know what to expect. But if this is so, getting rid of the isolation of facts is not, after all, the ultimate motive of seeking an explanation; but on the contrary, an ulterior purpose has reference to expectation. And what is this condition described as being full of risk, of not knowing what to expect? It is not a mere negation of all expectation, -- the state of mind in which a man takes his Sunday afternoon's stroll. It is a state in which a man seems to

have ground for expecting certain things, and yet has evidence that those expectations may be falsified. Now this precisely describes the conditions under which according to my theory rationalization is called for. It may, however, be objected that if we are to go back to the ultimate motive for explanation, I should have asked what the danger is to which error would expose us. I reply that were I investigating the practical logic of the individual man, then, as I have already remarked, the question of pure ethics would have to be taken up, namely, the question 'What can a man deliberately accept as his ultimate purpose?' But restricting myself, as I do, to scientific reasoning, I need not go behind the recognized purpose of science, which stops at knowledge.

Peirce: CP 7.202 Cross-Ref:††

§6. ABDUCTION, INDUCTION, AND DEDUCTION †13

202. Accepting the conclusion that an explanation is needed when facts contrary to what we should expect emerge, it follows that the explanation must be such a proposition as would lead to the prediction of the observed facts, either as necessary consequences or at least as very probable under the circumstances. A hypothesis then, has to be adopted, which is likely in itself, and renders the facts likely. This step of adopting a hypothesis as being suggested by the facts, is what I call *abduction*. I reckon it as a form of inference, however problematical the hypothesis may be held. What are to be the logical rules to which we are to conform in taking this step? There would be no logic in imposing rules, and saying that they *ought* to be followed, until it is made out that the purpose of hypothesis requires them. Accordingly, it appears that the early scientists, Thales, Anaximander, and their brethren, seemed to think the work of science was done when a likely hypothesis was suggested. I applaud their sound logical instinct for that. Even Plato, in the *Timaeus* and elsewhere, does not hesitate roundly to assert the truth of anything, if it seems to render the world reasonable; and this same procedure, in a more refined modification, is the essence of modern historical criticism. It is all right as long as it is not found to interfere with the usefulness of the hypothesis. Aristotle departs a little from that method. His physical hypotheses are equally unfounded; but he always adds a 'perhaps.' That, I take it, was because Aristotle had been a great reader of other philosophers, and it had struck him that there are various inconsistent ways of explaining the same facts. Ultimately, the circumstance that a hypothesis, although it may lead us to expect some facts to be as they are, may in the future lead us to erroneous expectations about other facts, -- this circumstance, which anybody must have admitted as soon as it was brought home to him, was brought home to scientific men so forcibly, first in astronomy, and then in other sciences, that it became axiomatical that a hypothesis adopted by abduction could only be adopted on probation, and must be tested.

Peirce: CP 7.203 Cross-Ref:††

203. When this is duly recognized, the first thing that will be done, as soon as a hypothesis has been adopted, will be to trace out its necessary and probable experiential consequences. This step is *deduction*. Here I may notice a rule of abduction much insisted upon by Auguste Comte, to the effect that metaphysical hypotheses should be excluded; and by a metaphysical hypothesis he means, as he tells us, a hypothesis which has no experiential consequences. I suppose a partially

metaphysical hypothesis would be one that had, among its consequences, some not relating to possible experience; and that from those Comte would wish us to tear away the metaphysical part. I have no particular objection to Comte's rule. Indeed, I think it would obviously be fully justified by a consideration of the purposes of hypothesis. Only I beg to remark that its positive utility is limited by the circumstance that such [a] thing as a hypothesis which is either wholly or partially metaphysical cannot be constructed. I may be asked what I should say to the proposition that

The warranted genuine Snark has a taste

Which is meagre and hollow, but crisp;

Like a coat that is rather too tight in the waist,

With a flavor of Will-o-the-wisp.

I reply that it is not a metaphysical proposition, because it is no proposition at all, but only an imitation proposition. For a proposition is a sign separately indicating what it is a sign of; and analysis shows that this amounts to saying that it represents that an image is similar to something to which actual experience forces the attention. Consequently a proposition cannot predicate a character not capable of sensuous presentation; nor can it refer to anything with which experience does not connect us. A metaphysical proposition in Comte's sense would, therefore, be a grammatical arrangement of words simulating a proposition, but in fact, not a proposition, because destitute of meaning. Comte's use of the word metaphysical, in a sense which makes it synonymous with nonsense, simply marks the nominalistic tendency of Comte's time, from which he was unable to free himself, although the general tendency of his philosophy is rather opposed to it. However, be that as it may, the entire meaning of a hypothesis lies in its conditional experiential predictions: if all its predictions are true, the hypothesis is wholly true.

Peirce: CP 7.204 Cross-Ref:††

204. This appears to be in harmony with Kant's view of deduction, namely, that it merely explicates what is implicitly asserted in the premisses. This is what is called a half-truth. Deductions are of two kinds, which I call *corollarial* and *theorematic*. The corollarial are those reasonings by which all corollaries and the majority of what are called theorems are deduced; the theorematic are those by which the major theorems are deduced. If you take the thesis of a corollary, -- i.e. the proposition to be proved, and carefully analyze its meaning, by substituting for each term its definition, you will find that its truth follows, in a straightforward manner, from previous propositions similarly analyzed. But when it comes to proving a major theorem, you will very often find you have need of a *lemma*, which is a demonstrable proposition about something outside the subject of inquiry; and even if a lemma does not have to be demonstrated, it is necessary to introduce the definition of something which the *thesis* of the theorem does not contemplate. In the most remarkable cases, this is some abstraction; that is to say, a subject whose existence *consists* in some fact about other things. Such, for example, are operations considered as in themselves subject to operation; *lines*, which are nothing but descriptions of the motion of a particle, considered as being themselves movable; collections; numbers; and the like. When the reform of mathematical reasoning now going on is complete, it will be seen

that every such supposition ought to be supported by a proper postulate. At any rate Kant himself ought to admit, and would admit if he were alive today, that the conclusion of reasoning of this kind, although it is strictly deductive, does not flow from definitions alone, but that postulates are requisite for it.

Peirce: CP 7.205 Cross-Ref:††

205. Deduction, of course, relates exclusively to an ideal state of things. A hypothesis presents such an ideal state of things, and asserts that it is the icon, or analogue of an experience.

Peirce: CP 7.206 Cross-Ref:††

206. Having, then, by means of deduction, drawn from a hypothesis predictions as to what the results of experiment will be, we proceed to test the hypothesis by making the experiments and comparing those predictions with the actual results of the experiment. Experiment is very expensive business, in money, in time, and in thought; so that it will be a saving of expense, to begin with that positive prediction from the hypothesis which seems least likely to be verified. For a single experiment may absolutely refute the most valuable of hypotheses, while a hypothesis must be a trifling one indeed if a single experiment could establish it.†14 When, however, we find that prediction after prediction, notwithstanding a preference for putting the most unlikely ones to the test, is verified by experiment, whether without modification or with a merely quantitative modification, we begin to accord to the hypothesis a standing among scientific results. This sort of inference it is, from experiments testing predictions based on a hypothesis, that is alone properly entitled to be called *induction*.

Peirce: CP 7.207 Cross-Ref:††

207. I may as well say that arguments which I cannot now stop to set forth ought to remove all doubt that, accepting the term induction in this sense, the critical distinction, that is, the distinction in respect to the nature of their validity between deduction and induction consists in this, -- namely, deduction professes to show that certain admitted facts could not exist, even in an ideal world constructed for the purpose, either without the existence of the very fact concluded, or without the occurrence of this fact in the long run in that proportion of cases of the fulfilment of certain objective conditions in which it is concluded that it will occur, or in other words, without its having the concluded objective probability. In either case, deductive reasoning is necessary reasoning, although, in the latter case, its subject matter is probability. Induction, on the other hand, is not justified by any relation between the facts stated in the premisses and the fact stated in the conclusion; and it does not infer that the latter fact is either necessary or objectively probable. But the justification of its conclusion is that that conclusion is reached by a method which, steadily persisted in, must lead to true knowledge in the long run of cases of its application, whether to the existing world or to any imaginable world whatsoever. Deduction cannot make any such claim as this; since it does not lead to any positive knowledge at all, but only traces out the ideal consequences of hypotheses.

Peirce: CP 7.208 Cross-Ref:††
§7. THREE KINDS OF INDUCTION †15

208. It is desirable to consider a large range of inductions, with a view to

distinguishing accurately between induction and abduction, which have generally been much confounded. I will, therefore, mention that, in the present state of my studies, I think I recognize three distinct genera of induction. I somewhat hesitate to publish this division; but it might take more years than I have to live to render it as satisfactory as I could wish. It is not that there seems to be any very serious want of clearness in it, or that the reasons for maintaining it are wanting in conclusiveness, or that I have any particular reason to doubt, either the conclusion or the correctness of the reasoning; but it is simply that the factor of safety is too small. I have not so thoroughly considered the subject as to be quite secure against possible oversights of one kind or another; nor have I collected a sufficient surplus of proofs so that they will hold even although oversights there be. Consequently, I am not yet willing to incorporate this division with the body of results of this investigation. But with this warning, I now state the division.

Peirce: CP 7.209 Cross-Ref:††

209. The first genus of induction is where we judge what approximate proportion of the members of a collection have a predesignate character by a sample drawn under one or other of the following three conditions, forming three species of this genus. First, the sample be a *random* one, an expression to which I attach a peculiar meaning. Namely, I mean by a random sample, a sample drawn from the whole class by a method which if it were applied over and over again would in the long run draw any one possible collection of the members of the whole collection sampled as often as any other collection of the same size, and would produce them in any one order as often as in any other. In this peculiar sense of the term *random sample*, it is only from a finite collection that a random sample can be drawn. And here it will be well to call to mind the exact meaning of a few terms relating to multitudes.†16 . . . The multitude of every collection is either *enumerable* or finite, *denumeral* or indefinite, *abnumerable* or transfinite. The scientific definitions of these terms were, I believe, first given by me in 1881;†17 but Dedekind gave, perhaps independently, in 1888, substantially the same definition of a finite multitude. An *enumerable* multitude is the multitude of a collection, -- say the A's, -- if and only if, no matter what relation {r} may be, either it is not true that every A is in the relation, {r}, to some A to which no other A stands in the relation, {r}, or else to every A some one A, and no other, stands in the relation {r}. This is as much as to say that the A's form a non-enumerable or infinite collection, if and only if there be some relation, {r}, such that every A stands in the relation {r} to an A to which no other A stands in the relation {r}, although there is an A of which it is not true that any A stands in the relation {r}, while no other A does so. Now according to our definition of the relation of being as small as, this is the same as to say, that a collection, say the A's, is non-enumerable or infinite, if and only if, the entire collection of A's is as small as a collection of A's from which some A is excluded; and this again is obviously identical with Dedekind's definition of an infinite collection which is that a collection is infinite if and only if the whole of it is as small as some part of it not the whole. But a more readily intelligible definition, coming to the same thing, is to say that an enumerable multitude is a multitude less than that of all the finite whole numbers. The *denumeral* multitude, for there is but one, is the multitude of a collection, say the A's, if and only if, there is a relation, say {s}, and an A, say A0, such that every A stands in the relation {s} to some A to which no other A stands in this relation, and no A stands in the relation {s} to A0, and taking any predicate, P, whatever, either every A has this character, P, or A0 does not possess the character P, or there is an A having the character P which is not in the relation {s} to any A that has the character, P. This comes to the same thing as saying that the denumeral multitude is the multitude of the

finite whole numbers. Every denumeral collection is numerable; that is to every member of it a separate ordinal number may be assigned; and this may be done in such a way as to exhaust the entire collection of finite whole numbers. Every denumeral collection, therefore, either has an order or may receive an order. . . . To show that the whole is no more numerous than a part, or all the numbers than the even numbers, it suffices to write down

1 2 3 4 5 6 7 8 9, etc.

2 4 6 8 10 12 14 16 18, etc.

Of course, there is no number without a double, and each double is an even number. An *abnumerable* multitude is one of a denumeral succession of multitudes greater than the denumeral multitude; each of these being the multitude of the different possible collections of members of a collection of the next lower abnumerable multitude. I have proved that there is no multitude greater than every abnumerable multitude; and it seems to follow from a theorem of Cantor's about ordinal numbers that there is no multitude intermediate between two abnumerable multitudes. It will, therefore, suffice to define an abnumerable multitude as a multitude greater than that of all the finite whole numbers. If there is room on a line for any multitude of points, however great, a genuine continuity implies, then, that the aggregate of points on a line is too great to form a collection: the points lose their identity; or rather, they never had any numerical identity, for the reason that they are only possibilities, and therefore are essentially general. They only become individual when they are separately marked on the line; and however many be separately marked, there is room to mark more in any multitude.

Peirce: CP 7.210 Cross-Ref:††

210. Returning to the first genus of induction, it now becomes plain that a random sample, in the exact sense defined, can only be drawn from a finite collection. For the definition contains the phrase "the long run." Now what is meant by "the long run?" The phrase is only used in saying that the ratio of frequency of an event has such and such a value in the long run. The meaning is that if the occasion referred to upon which the event might happen were to recur indefinitely, and if tallies were to be kept of the occurrences and the non-occurrences, then [the] ratio of the one number to the other, as the occasions went on, would indefinitely converge toward a definite limit. The word 'converge' is here used in a different sense from that which is usual in mathematics. The common definition is that a series of values, $x[1]$, $x[2]$, $x[3]$, etc., converges toward a limiting value x , provided, after any discrepancy $\{e\}$ has been named it is possible to find one of the members of the series $x\{n\}$ such that, for every value of n greater than $\{n\}$, $(x[n]-x)^2 < \{e\}^2$. This ought to be called *definite* convergence. No such member $x[n]$ can, in the indefinite convergence with which we have to do, be fixed in advance of the experiment. Nevertheless, there will be some such value.

Peirce: CP 7.211 Cross-Ref:††

211. Such being the nature of a long run, we see that the idea of a random sample supposes that in a denumeral series of trials all possible samples of the class sampled are to be capable of being drawn, and that in every possible order *inter se*.

But all possible orders in which all possible samples, however small, could be drawn from a denumeral collection, would be abnumerable, and therefore not to be completed in a long run. It follows that it is only a finite or enumerable class from which a random sample in the sense of the definition can be drawn. It is, indeed, evident that one cannot take even a single whole number at random; for a whole number taken at random would be infinitely more likely to be larger than any predesignate number than not.

Peirce: CP 7.212 Cross-Ref:††

212. Let us now consider another species of the first genus of induction. It had better be mentioned, by the way, that no multitude not enumerable is increased by being multiplied by itself; so that a denumeral collection of denumeral collections makes up a denumeral collection of the members of the latter collections. Let us now suppose that we are about to sample a denumeral collection in order to ascertain the proportionate frequency with which its members have a certain character designated in advance of the examination. Usually, there is no sense in speaking of a definite finite proportion of a denumeral collection; but I am going to suppose that this collection has an order which gives it a sense. The sample is to be drawn under the guidance of a precept under which we can enlarge any sample drawn indefinitely and can also draw an indefinite number of samples. Now I shall suppose that in some way, no matter how, we become assured that a relation exists between four correlates, to wit, the predesignate character, the precept of sampling, the collection sampled, and the future course of experience, this relation being such that, in the long run, the distribution of the predesignate character in samples drawn under the precept will be the same as if they had been drawn strictly at random from an indefinitely large finite collection composing all our future experience of members of the same collection. Then, as before, we can infer inductively the proportional frequency of that character in future experiences of members of the same collection; and the induction must approximate indefinitely, though irregularly, to the true proportion. As an example, take a certain die. All the falls of it shall form the denumeral collection. In future experience this die will probably be thrown a very large but finite number of times. Let me sample the throws in order to find out (since it may be loaded or badly made) with what relative frequency it will turn up an ace. My precept shall be to throw it from the dicebox after shaking, replace it, and go on in the same way. I will not stop to inquire how I know that my sample throws will, as far as the distribution of aces is concerned, be determined as if they had been drawn strictly at random among all future throws, because this question has, at this stage, no relevancy, and would only divert our minds from our point. And besides, the elements of the difficulty will find their solution in questions we have presently to take up.

Peirce: CP 7.213 Cross-Ref:††

213. Perhaps we may reckon, as a third species under the first genus of induction, those cases in which we find a denumeral series in an objective order of succession, and wish to know what the law of occurrence of a certain character among its members is, without at the outset so much as knowing whether it has any definite frequency in the long run or not. As an example, I will make a very slight examination of the occurrence of the figure 5 in the endless decimal that would express the value of π . Since the enormous labor has been performed of calculating this number to over seven hundred places, it seems a pity that no use whatever should be made of it. An instructor having a class in probabilities might very well give out as an exercise the examination of the calculated figures with a view to drawing such inferences as might be drawn by the doctrine of chances. I shall confine myself to

illustrating this sort of induction by beginning an inquiry whether the figure 5 occurs in a purely chance way. I do not know why I chose this particular figure: I did so before I looked at the value of π . Taking the first 700 places, I separate them into the first 350 and the second 350. If these are quasirandom samples of the whole and all the figures occur equally often, there ought not to be far from 35 fives in each set. The odds that the number will fall in the thirties are about 2 to 1. The odds that it will fall between 28 and 42 *exclusive* is just 3 to 1. We find, in fact, that there are 33 fives in the first 350, and 28 in the second 350. Since the odds against this are only about 2 to 1, we conclude that the *fives*, and presumably the other figures, either occur by chance or very nearly so. It would, however, perhaps not be surprising if they were to occur with a little more approach to regularity than if they were purely fortuitous. Therefore, as a further illustration of this kind of induction, I have counted the number of 5's in each of the seventy sets of ten successive figures. In these seventy sets, the normal number of those having one 5 should be $27 \frac{1}{8}$. But we suspect there will be more owing to the fives coming a little more regularly than merely by chance. We will ask then what is the probability that there will be no more than 32 tens containing just one 5. It is about $\frac{5}{6}$; but the actual number of such tens is 33. There is, therefore, a doubtful indication of such regularity.

Peirce: CP 7.214 Cross-Ref:††

214. These are all the species I can mention of the first type of induction, in which we ascertain the value of a ratio and are morally certain to approximate to it indefinitely in the long run for each problem. By "morally certain," I mean that the probability of that event is 1. Of course, there is a difference between probability 1 and absolute certainty. In like manner, "bare possibility" should mean the possibility of that whose probability is zero. It is barely possible that a well made pair of dice should turn up doublets every time they were thrown: it is a conceivable chance, though morally certain not to happen. But that a pair of dice will not turn up *sevens* is absolutely certain; it is not possible.

Peirce: CP 7.215 Cross-Ref:††

215. The second genus of induction comprises those cases in which the inductive method if persisted in will certainly in time correct any error that it may have led us into; but it will not do so gradually, inasmuch as it is not quantitative; -- not but that it may relate to quantity, but it is not a quantitative induction. It does not discover a ratio of frequency. The first species under this genus is where the collection to be sampled is an objective series of which some members have been experienced, while the rest remain to be experienced, and we simply conclude that future experience will be like the past. We may take Quetelet's well-known example of the ancient Greek who, never having heard of the flux and reflux of the ocean, should have wandered to the shores of the Bay of Biscay and should have there seen the tide rise for *m* successive half-days. I need hardly say that I utterly reject the doctrine that there is any consequent definite probability that the event will happen during the next half-day. That doctrine has been absolutely disproved. If the Greek's conclusion is that the tide rises about once every half-day, it is an induction of the first genus, second species. He may say that the indication is that the frequency is somewhere between $m - \frac{1}{2}/m$ and $m + \frac{1}{2}/m$ although this is only a rough approximation. He may, thence, deduce the conclusion that a tide will not be wanting in the following *m* half-days, although he ought not to risk much upon it. Beyond that it cannot be said that the quantitative induction warrants such a prediction. But if the Greek had seen the tide rise just often enough to suggest to him that it would rise every half-day forever, and had proposed then to make observations to test this

hypothesis, had done so, and finding the predictions successful, had provisionally accepted the theory that the tide would never cease to rise every half-day, there would be just this justification for this conclusion, that it was the result of a method which, if it be persisted in, must correct its result if it were wrong. For if the tide was going to skip a half-day, he must discover it, if he continued his observations long enough. This degree of justification and no more he would have whether he made a dozen trials, or half a dozen, or three, or two, or one only, or even none at all. The argument would have precisely the same justification in either case. The method would infallibly correct itself, provided he continued this series of experiments; but not if he dropped it and subsequently commenced another series, as would be the case with quantitative induction. For this induction not being quantitative, does not conclude that the probability of the tides rising is 1; but that it raises every half-day without exception. It has nothing to do with probabilities or improbabilities; and if the series of observations skips a single day, that day may be the very day of the exceptional fact. This kind of induction further differs from quantitative induction, inasmuch as there is no probable indication in advance, if its conclusion is to break down; so that, as long as it does not break down, there is nothing to be said but that no reason appears as yet for giving up the hypothesis. It gives, therefore, but a very slight and merely negative support to the hypothesis. It is a proper answer enough to gratuitous hypotheses. It is impossible to avoid making some use of it for that purpose. But it must be set down as the weakest possible of inductive arguments. I have confined myself to cases in which the series of occasions considered is objective. But I am unable to perceive that there is any intrinsic logical distinction between these cases and those in which the series results from our own subdivision of a continuum. It might, for example, be suggested that the action of gravitation may be intermittent, either with a very short period, or without any definite period. In that case, a body moving for a considerable time would show merely the average acceleration; but two molecules might, during the interval of their encounter, either undergo no acceleration, so that it would be as if there were no encounter, or they might undergo acceleration many times that of average gravitation; and this might account for there seeming to be greater attraction at small distances than the law of average gravitation would account for. Moreover, greater masses moving slower than smaller ones, a periodic relation between atomic weights and attractions of atoms might be expected. Now, as a test of that hypothesis, it might be proposed to shorten the period of a pendulum more and more, and try to observe some irregularities of its amplitudes. If we found that, as far as we could go, say with pendulums oscillating 50 times a second, there were no observable irregularities of amplitude, and were to infer that there was no intermittency of gravitation, I am unable to see that the argument would differ from the argument that the tide will rise on every half-day forever, because it has been seen to do so on several successive half-days. It is true that this latter argument is weakened by the consideration that states of things not universal usually come to an end; while the other is strengthened by the consideration that, time being continuous, it is reasonable to suppose that, in sufficiently short intervals, there will be no further variation of any given phenomenon. But both of these are extraneous considerations. As far as the mere argument that what has not been found need not be expected is concerned, there seems to be no logical distinction between them. Let us consider one more example. By means of a well-constructed color-box, two adjacent rectangles are illuminated, each with nearly homogeneous violet light, of the same apparent luminosity; the one of a wave-length of about 404 micromicrons, the other 402 micromicrons, and the observer who knows only what he sees, is asked which is the redder. He says he sees no difference. But the operator insists upon his deciding

for one or the other; and with reluctance he names one, as it seems to him quite at random. However, the experiment having been repeated several hundred times, it is found that in each set of a hundred answers, a decided majority makes the more refrangible the redder. Now then, what do we proceed to infer from this, -- that there is no such thing as a *Differenz-Schwelle*, but that no matter how small the difference of excitations a sufficient number of answers would betray a difference of sensation? The only justification for this would be that it is the result of a method that persisted in must eventually correct any error that it leads us into. I may mention, that the argument that there is no *Differenz-Schwelle* is, in reality, stronger than this. But a negative induction of this sort, a refusal to expect what is contrary to experience, will rightly be resorted to when gratuitous objections are raised to any induction.

Peirce: CP 7.216 Cross-Ref:††

216. I seem to recognize a third genus of inductions where we draw a sample of an aggregate which can not be considered as a collection, since it does not consist of units capable of being either counted or measured, however roughly; and where probability therefore cannot enter; but where we can draw the distinction of much and little, so that we can conceive of measurement being established; and where we may expect that any error into which the sampling will lead us, though it may not be corrected by a mere enlargement of the sample, or even by drawing other similar samples, yet must be brought to light, and that gradually, by persistence in the same general method. This kind of reasoning may be described in slightly different terms by saying that it tests a hypothesis by sampling the possible predictions that may be based upon it. Predictions are not units; for they may be more or less detailed. One can say roughly that one is more significant than another; but no approach to actual weighing of their significance can, in most cases, be made. Consequently, we cannot say that a collection of predictions drawn from a hypothesis constitutes a strictly random sample of all that can be drawn. Sometimes we can say that it appears to constitute a very fair, or even a severe, sample of the possible predictions; while in other cases, we cannot even say that, but only that it comprises all the predictions which we can as yet draw and put to the test. Those two classes of cases may be taken as constituting two species under this genus. We cannot ordinarily hope that our hypothesis will pass through the fire of induction, absolutely unmodified. Consequently, we ought not to conclude that it is absolutely correct, but only that it very much resembles the truth. In so far as further induction will modify it, as it must be expected that it will do, if it is not to meet with downright refutation, it can hardly fail that the modification should come about gradually. We shall first find facts, reconcilable yet unexpected. These will be discovered in greater volume, until they show that a modification of the theory is necessary. The familiar history of the kinetical theory of gases well illustrates this. It began with a number of spheres almost infinitesimally small occasionally colliding. It was afterward so far modified that the forces between the spheres, instead of merely separating them, were mainly attractive, that the molecules were not spheres, but systems, and that the part of space within which their motions are free is appreciably less than the entire volume of the gas. There was no new hypothetical element in these modifications. They were partly merely quantitative, and partly such as to make the formal hypothesis represent better what was really supposed to be the case, but which had been simplified for mathematical simplicity. There was, besides, an important modification which was imposed by mathematical necessity. So far as these modifications were introduced in order to bring the hypothesis into better accord with the facts, they were indicated and suspected long before the need of them became quite apparent; so that this genus of induction shares with the first the advantage that where the inductive conclusion errs,

it will be but slightly, and the discovery, instead of being shot like a bolt out of the blue, creeps upon us as a dawning day.

Peirce: CP 7.217 Cross-Ref:††

217. The reasonings of science are for the most part complex. Their parts are so put together as to increase their strength. Our attention has been confined to the elements out of which scientific argumentations are built up. We have now passed in review all the logically distinct forms of pure induction. It has been seen that one and all are mere processes for testing hypotheses already in hand. The induction adds nothing. At the very most it corrects the value of a ratio or slightly modifies a hypothesis in a way which had already been contemplated as possible.

Peirce: CP 7.218 Cross-Ref:††
§8. ABDUCTION

218. Abduction, on the other hand, is merely preparatory. It is the first step of scientific reasoning, as induction is the concluding step. Nothing has so much contributed to present chaotic or erroneous ideas of the logic of science as failure to distinguish the essentially different characters of different elements of scientific reasoning; and one of the worst of these confusions, as well as one of the commonest, consists in regarding abduction and induction taken together (often mixed also with deduction) as a simple argument. Abduction and induction have, to be sure, this common feature, that both lead to the acceptance of a hypothesis because observed facts are such as would necessarily or probably result as consequences of that hypothesis. But for all that, they are the opposite poles of reason, the one the most ineffective, the other the most effective of arguments. The method of either is the very reverse of the other's. Abduction makes its start from the facts, without, at the outset, having any particular theory in view, though it is motivated by the feeling that a theory is needed to explain the surprising facts. Induction makes its start from a hypothesis which seems to recommend itself, without at the outset having any particular facts in view, though it feels the need of facts to support the theory. Abduction seeks a theory. Induction seeks for facts. In abduction the consideration of the facts suggests the hypothesis. In induction the study of the hypothesis suggests the experiments which bring to light the very facts to which the hypothesis had pointed. The mode of suggestion by which, in abduction, the facts suggest the hypothesis is by *resemblance*, -- the resemblance of the facts to the consequences of the hypothesis. The mode of suggestion by which in induction the hypothesis suggests the facts is by *contiguity*, -- familiar knowledge that the conditions of the hypothesis can be realized in certain experimental ways.

Peirce: CP 7.219 Cross-Ref:††

219. I now proceed to consider what principles should guide us in abduction, or the process of choosing a hypothesis. Underlying all such principles there is a fundamental and primary abduction, a hypothesis which we must embrace at the outset, however destitute of evidentiary support it may be. That hypothesis is that the facts in hand admit of rationalization, and of rationalization by us. That we must hope they do, for the same reason that a general who has to capture a position or see his country ruined, must go on the hypothesis that there is some way in which he can and shall capture it. We must be animated by that hope concerning the problem we have

in hand, whether we extend it to a general postulate covering all facts, or not. Now, that the matter of no new truth can come from induction or from deduction, we have seen. It can only come from abduction; and abduction is, after all, nothing but guessing. We are therefore bound to hope that, although the possible explanations of our facts may be strictly innumerable, yet our mind will be able, in some finite number of guesses, to guess the sole true explanation of them. *That* we are bound to assume, independently of any evidence that it is true. Animated by that hope, we are to proceed to the construction of a hypothesis.

Peirce: CP 7.220 Cross-Ref:††

220. Now the only way to discover the principles upon which anything ought to be constructed is to consider what is to be done with the constructed thing after it is constructed. That which is to be done with the hypothesis is to trace out its consequences by deduction, to compare them with results of experiment by induction, and to discard the hypothesis, and try another, as soon as the first has been refuted; as it presumably will be. How long it will be before we light upon the hypothesis which shall resist all tests we cannot tell; but we hope we shall do so, at last. In view of this prospect, it is plain that three considerations should determine our choice of a hypothesis. In the first place, it must be capable of being subjected to experimental testing. It must consist of experiential consequences with only so much logical cement as is needed to render them rational. In the second place, the hypothesis must be such that it will explain the surprising facts we have before us which it is the whole motive of our inquiry to rationalize. This explanation may consist in making the observed facts natural chance results, as the kinetical theory of gases explains facts; or it may render the facts necessary, and in the latter case as implicitly asserting them or as the ground for a mathematical demonstration of their truth. In the third place, quite as necessary a consideration as either of those I have mentioned, in view of the fact that the true hypothesis is only one out of innumerable possible false ones, in view, too, of the enormous expensiveness of experimentation in money, time, energy, and thought, is the consideration of economy.†18 Now economy, in general, depends upon three kinds of factors: cost; the value of the thing proposed, in itself; and its effect upon other projects. Under the head of cost, if a hypothesis can be put to the test of experiment with very little expense of any kind, that should be regarded as a recommendation for giving it precedence in the inductive procedure. For even if it be barely admissible for other reasons, still it may clear the ground to have disposed of it. In the beginning of the wonderful reasonings by which the cuneiform inscriptions were made legible, one or two hypotheses which were never considered likely were taken up and soon refuted with great advantage. Under the head of value, we must place those considerations which tend toward an expectation that a given hypothesis may be true. These are of two kinds, the purely instinctive and the reasoned. In regard to instinctive considerations, I have already pointed out that it is a primary hypothesis underlying all abduction that the human mind is akin to the truth in the sense that in a finite number of guesses it will light upon the correct hypothesis. Now inductive experience supports that hypothesis in a remarkable measure. For if there were no tendency of that kind, if when a surprising phenomenon presented itself in our laboratory, we had to make random shots at the determining conditions, trying such hypotheses as that the aspect of the planets had something to do with it, or what the dowager empress had been doing just five hours previously, if such hypotheses had as good a chance of being true as those which seem marked by good sense, then we never could have made any progress in science at all. But that we have made solid gains in knowledge is indisputable; and moreover, the history of science proves that when the phenomena were properly analyzed, upon fundamental points, at least, it has

seldom been necessary to try more than two or three hypotheses made by clear genius before the right one was found. I have heard it said that Kepler tried nineteen orbits for Mars before he hit upon the right one; but in the first place, I cannot admit that that is a fair description of his elaborate series of inductions, and in the second place the subject of the hypothesis was not of the fundamental class. We cannot go so far as to say that high human intelligence is more often right than wrong in its guesses; but we can say that, after due analysis, and unswerved by prepossessions, it has been, and no doubt will be, not very many times more likely to be wrong than right. As we advance further and further into science, the aid that we can derive from the natural light of reason becomes, no doubt, less and less; but still science will cease to progress if ever we shall reach the point where there is no longer an infinite saving of expense in experimentation to be effected by care that our hypotheses are such as naturally recommend themselves to the mind, and make upon us the impression of simplicity, -- which here means facility of comprehension by the human mind, -- of aptness, of reasonableness, of good sense. For the existence of a natural instinct for truth is, after all, the sheet-anchor of science. From the instinctive, we pass to reasoned, marks of truth in the hypothesis. Of course, if we know any positive facts which render a given hypothesis objectively probable, they recommend it for inductive testing. When this is not the case, but the hypothesis seems to us likely, or unlikely, this likelihood is an indication that the hypothesis accords or discords with our preconceived ideas; and since those ideas are presumably based upon some experience, it follows that, other things being equal, there will be, in the long run, some economy in giving the hypothesis a place in the order of precedence in accordance with this indication. But experience must be our chart in economical navigation; and experience shows that likelihoods are treacherous guides. Nothing has caused so much waste of time and means, in all sorts of researches, as inquirers' becoming so wedded to certain likelihoods as to forget all the other factors of the economy of research; so that, unless it be very solidly grounded, likelihood is far better disregarded, or nearly so; and even when it seems solidly grounded, it should be proceeded upon with a cautious tread, with an eye to other considerations, and a recollection of the disasters it has caused. The third category of factors of economy, those arising from the relation of what is proposed to other projects, is especially important in abduction, because very rarely can we positively expect a given hypothesis to prove entirely satisfactory; and we must always consider what will happen when the hypothesis proposed breaks down. The qualities which these considerations induce us to value in a hypothesis are three, which I may entitle Caution, Breadth, and Incomplexity. In respect to caution, the game of twenty questions is instructive. In this game, one party thinks of some individual object, real or fictitious, which is well-known to all educated people. The other party is entitled to answers to any twenty interrogatories they propound which can be answered by *Yes* or *No*, and are then to guess what was thought of, if they can. If the questioning is skillful, the object will invariably be guessed; but if the questioners allow themselves to be led astray by the will-o-the-wisp of any prepossession, they will almost as infallibly come to grief. The uniform success of good questioners is based upon the circumstance that the entire collection of individual objects well-known to all the world does not amount to a million. If, therefore, each question could exactly bisect the possibilities, so that *yes* and *no* were equally probable, the right object would be identified among a collection numbering 2^{20} . Now the logarithm of 2 being 0.30103, that of its twentieth power is 6.0206, which is the logarithm of about 1,000,000 ($1+.02 \times 2.3$) ($1+.0006 \times 2.3$) or over one million and forty-seven thousand, or more than the entire number of objects from which the selection has been made. Thus,

twenty skillful hypotheses will ascertain what two hundred thousand stupid ones might fail to do. The secret of the business lies in the caution which breaks a hypothesis up into its smallest logical components, and only risks one of them at a time. What a world of futile controversy and of confused experimentation might have been saved if this principle had guided investigations into the theory of light! The ancient and medieval notion was that sight starts from the eye, is shot to the object from which it is reflected, and returned to the eye. This idea had, no doubt, been entirely given up before Römer showed that it took light a quarter of an hour to traverse the earth's orbit, a discovery which would have refuted it by the experiment of opening the closed eyes and looking at the stars. The next point in order was to ascertain of what the ray of light consisted. But this not being answerable by *yes* or *no*, the first question should have been 'Is the ray homogeneous along its length?' Diffraction showed that it was not so. That being established, the next question should have been 'Is the ray homogeneous on all sides?' Had that question been put to experiment, polarization must have been speedily discovered; and the same sort of procedure would have developed the whole theory with a gain of half a century.

Peirce: CP 7.221 Cross-Ref:††

221. Correlative to the quality of caution is that of breadth. For when we break the hypothesis into elementary parts, we may, and should, inquire how far the same explanation accounts for the same phenomenon when it appears in other subjects. For example, the kinetical theory of gases, although it was originally proposed with a view merely to explaining the law of Boyle, never attracted much attention, nor was there any good reason why it should, until the conservation of energy was brought to light, and it was found that the kinetical theory would account, in a remarkably satisfactory way, for non-conservative phenomena. It accounts for those phenomena, so far as it does account for them, by representing that they are results of chance; or, if you please, of the law of high numbers; for it is remarkable that chance operates in one way and not in the opposite way. Under those circumstances, the economical consideration which we now have in view, would recommend that we at once inquire into non-conservative phenomena, generally, in order to see whether the same sort of explanation is equally admissible in all cases, or whether we are thus led to some broad category of conditions under which non-conservative phenomena appear, or whether there are several distinct ways in which they are brought about. For great economy must result in whichever way this question is answered, provided it can be answered at not too great an expense. Thus, if we find that there are several explanations of non-conservative phenomena, we have only to trace out their several consequences, and we shall have criteria for distinguishing them; while if we find there is but one cause, we at once reach a wide generalization which will save repetitious work. It is, therefore, good economy, other things being equal, to make our hypotheses as broad as possible. But, of course, one consideration has to be balanced against another. There still remains one more economic consideration in reference to a hypothesis; namely, that it may give a good "leave," as the billiard-players say. If it does not suit the facts, still the comparison with the facts may be instructive with reference to the next hypothesis. For example, I might be inclined to surmise that an observable quantity y was such a function of a quantity x , determined by the conditions of experiments, as to be expressible in the form $y = a + bx^2$. Still, as I am not sure of this, perhaps it would be wise first to try how well the experiments could be satisfied by $y = cx$; because the residuals will be more readily interpretable in the latter case. As a provisional hypothesis, it will, for this reason, other considerations apart, be better to assume something very simple, even although we imagine that by

complicating the hypothesis it could be brought nearer the truth. Let us suppose, for example, that I wished to find some mathematical relation between the Atomic weights and the succession of chemical elements according to Mendeléeff's system. In point of fact, I hardly think that the time has yet come when it is worth while to take up that question for its own sake. The discoveries of Gallium, Germanium, and Scandium have proved that there is some truth in one part of Mendeléeff's theory; but the non-discovery of hecamagnesium inclines me, I must say, to think that Chromium and Magnesium are exceptional elements; and it seems to me that the groups, unless it be the extreme ones, are founded on pretty superficial characters. And if we are to separate all elements into what Mendeléeff calls groups, it seems to me that the rare earths seem to afford symptoms that an additional group must be admitted, say perhaps between the group of Zinc and the group of Gallium. However, if we are to retain Mendeléeff's system, let us suppose, as a first rough approximation to the truth, that, in the absence of disturbing conditions of which we know nothing, the atomic weights of the elements would increase from $K = 39$ by $2 \frac{1}{2}$ units at each step. Even if it is of little service as chemistry, it will, at least, serve as a tolerable illustration of the point of logic we have under consideration to compare the numbers required by this hypothesis with the numbers found. The latter depend upon the purity of the materials, of which the sole and insufficient guarantee is that atomic weights of material from different sources and subjected to different chemical operations agree. The table [next page] shows the comparison.

Peirce: CP 7.222 Cross-Ref:††

222. There are 16 consecutive elements undiscovered, according to Mendeléeff's theory. Those that we know of the same groups, that is those in the table just above and just below the vacant spaces, may very well be contaminated with the unknown elements. The 24 first, not open to this suspicion, mostly differ from our calculation by not more than 1. Those just above the missing elements have on the average atomic weights 2 units too great, those just below are on the average about $\frac{1}{2}$ a unit too small. Thus, this very incomplex and even rough hypothesis has done for us what a more elaborate one would almost surely have failed to do, namely, it has brought to light an indication that all the elements from Ruthenium up are probably largely contaminated with undiscovered elements, which contaminations have mostly atomic weights between 146 and 182, but are partly of large atomic weights, say from 211 up. Since this perturbation is largest in tellurium and barium, we should naturally look in these elements, especially, for admixtures of substances of higher atomic weight. In tellurium they have been sought, in vain; yet one cannot say that the negative has been rendered altogether improbable. In barium, on the other hand, there are indications of something of the sort, though whether it is sufficient to account for the large atomic weight, it is impossible to decide at present. I will not say that the hypothesis merits much attention, for the reason that it rests upon the acceptance of Mendeléeff's arrangement, and that arrangement is itself in considerable doubt.

Peirce: CP 7.223 Cross-Ref:††
§9. THE LOGIC OF HISTORY

223. Having now passed in review all the elements of merit of an hypothesis, I ought, in regular procedure, to consider the general principles of synthesis of these elements. But I think that that would delay us to no advantage; for once it is granted

that the elements I have enumerated are the points to consider, the mode in which they are to be combined in the case of ancient history is too obvious for dispute. The elements are as follows:

Experiential character of the hypothesis.

{ as natural concomitants.

Its explaining all the facts {

{ as deductions { Corrollarial.

{ Theorematic.

{ Cheapness.

{

{ Intrinsic Value { Naturalness.

{ Likelihood.

Economical considerations {

{

{ Relation of { Caution

{ Hypotheses { Breadth

{ Incomplexity.

Element	K	Ca	Sc	Ti	V	Cr	Mn
Calc	39	41 1/2	44	46 1/2	49	51 1/2	54
Obs	39.1	40.0	44	48.2	51.4	52.1	55.0
O-C	+0	-1 1/2	0	+1 1/2	+2 1/2	+1/2	+1

Element	Fe	Co	Ni	Cu	Zn	Ga	Ge
Calc	56 1/2	59	61 1/2	64	66 1/2	69	71 1/2
Obs	55.9	59.0	58.7	63.6	65.4	70.0	72.5
O-C	-1/2	0	-3	-1/2	-1	+1	+1

Element	As	Se	Br	Kr	Rb	Sr	Y
Calc	74	76 1/2	79	81 1/2	84	86 1/2	89
Obs	75.0	79.2	80.0	81.6	85.4	87.7	89.0
O-C	+1	+2 1/2	+1	+0	+1 1/2	+1	0

Element	Zr	Cb	Mo		Ru	Rh	Pd
Calc	91 1/2	94	96 1/2	99	101 1/2	104	106 1/2
Obs	90.6	94	96.0	..	101.7	103.0	106.5
O-C	-1	0	-1/2	..	+0	-1	0

Element	Ag	Cd	In	Sn	Sb	Te	I
Calc	109	111 1/2	114	116 1/2	119	121 1/2	124
Obs	107.9	112.3	114	119.0	120.0	127.5	126.8
O-C	-1	+1	0	+2 1/2	+1	+6	+3

Element	X	Cs	Ba	La	Ce	Pr	Nd
Calc		126 1/2	129	131 1/2	134	136 1/2	139
Obs		128.0	132.9	137.4	138.5	140	140.5
O-C		+4	+6	+4 1/2	+3 1/2	+1 1/2	+2

Element							
Calc	144	146 1/2	149	151 1/2	154	156 1/2	159
Obs
O-C

Element							
Calc	161 1/2	164	166 1/2	169	171 1/2	174	176 1/2
Obs
O-C

Element		Ta	W		Os	Ir
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Calc	179	181 1/2	184	186 1/2	189	191 1/2	194
Obs	183	184	..	190.8	193.0
O-C	-1	-2 1/2	..	- 1/2	-1

Element	Pt	Au	Hg	Tl	Pb	Bi
Calc	196 1/2	199	201 1/2	204	206 1/2	209 211 1/2
Obs	195.2	197.3	200.0	204.2	206.9	208 ..
O-C	-1	-1 1/2	-1 1/2	0	+ 1/2	-1 ..

Element

Calc	214	216 1/2
Obs
O-C

Peirce: CP 7.224 Cross-Ref:††

224. In the case of ancient history, the facts to be explained are, in part, of the nature of monuments, among which are to be reckoned the manuscripts; but the facts in greater part are documentary; that is, they are assertions and virtual assertions which we read either in the manuscripts or upon inscriptions. This latter class of facts is so much in excess, that ancient history may be said to consist in the interpretation of testimonies, occasionally supported or refuted by the indirect evidence of the monuments.

Peirce: CP 7.225 Cross-Ref:††

225. Now the first rule which we should set up is that our hypothesis ought to explain *all* the related facts. It is not sufficient to say that testimony is not true, it is our business to explain how it came to be such as it is.

Peirce: CP 7.226 Cross-Ref:††

226. The second rule is that our first hypothesis should be that the principal testimonies are true; and this hypothesis should not be abandoned until it is conclusively refuted. No practice is more wasteful than that of abandoning a hypothesis once taken up, until it becomes evident that it is quite untenable. An excellent method in the great majority of those cases in which it is applicable and in which it leads to any unequivocal results is to give precedence to that hypothesis which reposes upon a deep and primary instinct, such as is the instinct to believe testimony, without which human society could not exist. There is no surer mark of inexperience in dealing with witnesses than a tendency to believe that they are falsifying, without any definite, objective, and strong reason for the suspicion. But especially in ancient history, where the only facts we have are, in most cases, testimonies, the extremely bad economy of supposing those testimonies false, before we have first thoroughly tried the hypothesis that they are true, and have found it

quite inadmissible, is so obvious, that it is difficult to repress a certain contempt for the reasoning powers of those critics who are given to this procedure.

Peirce: CP 7.227 Cross-Ref:††

227. The third rule will be that probabilities that are strictly objective and at the same time very great, although they can never be absolutely conclusive, ought nevertheless to influence our preference for one hypothesis over another; but slight probabilities, even if objective, are not worth consideration; and merely subjective likelihoods should be disregarded altogether. For they are merely expressions of our preconceived notions. Now one of the main purposes of studying history ought to be to free us from the tyranny of our preconceived notions.

Peirce: CP 7.228 Cross-Ref:††

228. The fourth rule will be that we should split up a hypothesis into its items as much as possible, so as to test each one singly.

Peirce: CP 7.229 Cross-Ref:††

229. The fifth rule will be that when we are in doubt which of two hypotheses ought to have precedence, we should try whether, by enlarging the field of facts which they are to explain, a good reason will not appear for giving one of them a decided preference over the other.

Peirce: CP 7.230 Cross-Ref:††

230. The sixth rule will be that if the work of testing a particular hypothesis will have substantially or largely to be done in any case, in the process of testing another hypothesis, that circumstance should, other things being equal, give this hypothesis which thus involves little or no extra expense, a preference over another which would require special work of no value except for testing it.

Peirce: CP 7.231 Cross-Ref:††

231. A hypothesis having been adopted on probation, the process of testing it will consist, not in examining the facts, in order to see how well they accord with the hypothesis, but on the contrary in examining such of the probable consequences of the hypothesis as would be capable of direct verification, especially those consequences which would be very unlikely or surprising in case the hypothesis were not true. It is not easy to enumerate the different kinds of consequences; but among them may be, that the hypothesis would render the present existence of a monument probable, or would result in giving a known monument a certain character; that if it were true, certain ancient documents ought to contain some allusion to it; that if it is misstated by some authority not considered in the selection of the hypothesis, that misstatement would be likely to be of a certain kind; that if the hypothesis is true, and an assertion or allusion found in an ancient work is to be explained by the author's knowing it to be true, he must have had certain other knowledge, etc. When the hypothesis has sustained a testing as severe as the present state of our knowledge of the particular branch of history to which it belongs renders imperative, it will be admitted provisionally into the list of our present historical results, subject of course to reconsideration along with all those other results, when we are in a condition to insist upon a higher grade of security. In order to make the difference between this method and that usually pursued quite clear, I propose to give three illustrative examples. I shall draw them from the history of philosophy, with which I am better acquainted than I am with political history. I shall endeavor to make the examples illustrative of different kinds of questions, and in departments of history where various grades of probability can be insisted upon. I shall, in each case, first show how the question

would be treated in accordance with the method of this paper; and then I shall show how some one or more of the best critics actually have treated it. I shall not notice the theories of those who carry higher criticism to its last extravagances, but shall confine myself to those who are most esteemed for their sobriety and thoroughness. It will be necessary to confine our illustrations to some minor points, because these are the only ones which can be discussed within moderate limits.

Peirce: CP 7.232 Cross-Ref:††

232. If the necessity of this rule is not at once apparent, it will soon become so; for I am going to begin by breaking it in taking up the question of the authenticity of the writings of Aristotle, -- a question which has formed the subject of several laborious books, and which I can here consider only in the most insufficient manner.

Peirce: CP 7.233 Cross-Ref:††

§10. APPLICATION OF THE METHOD

233. The facts to be explained may be outlined as follows. The works in our edition of Aristotle are forty-six in number, containing a million words, equal, if translated, to perhaps ten volumes of Herbert Spencer. In addition, a hundred ancient authors give some six hundred fragments from some sixty other works. Diogenes Laertius gives a catalogue of the writings of Aristotle, probably made in Alexandria, (but the date of it is a matter for conjecture,) which contains 146 titles, which Diogenes says had 445,270 lines, which would be more than three times the size of our edition. Yet this catalogue does not seem to refer to any large part of the substance of the works that we possess. Moreover, down to the time of Cicero, Aristotle, whose reputation was more Roman than Greek, was chiefly known by works which we do not possess. The style of those works, as it is described, and as we see it in the fragments, was highly artistic; while that of the general mass of the works we possess is harsh and excessively condensed. They seem like notes for books rather than finished books. We find, however, in them a few long passages in the same polished style as the fragments, as if Jeremy Bentham had allowed Ruskin to write a few pages for him. The works we possess are also extremely repetitious. More than once, a whole book, or even a whole work, is substantially written again. And of the lost works, some, we are told by those who had seen them, agreed in substance with some of those in our possession. Shorter passages, and especially definitions, are often repeated almost verbatim. Our Aristotle abounds in references to his own works, sometimes to lost works, but mostly apparently to the works we know under other titles, but the nomenclature of the titles in the references presents no fixity; the same work will receive from Aristotle himself various names, -- unless he means to refer to some works that we do not know. Two different works will refer reciprocally to one another, and a work will contain a reference to a part further on in the same work. In one case, at least, it seems pretty clear that the reference involves a misunderstanding of the passage referred to; but I do not know why a voluminous author may not fall into a misunderstanding of a passage of his own writing. The surprising circumstances will in part be explained if we trust to a narrative given by Strabo in the XIIIth book of his Geography, of which we know that the IVth book, or a passage of it, was composed A.D. 19, -- and Strabo is supposed, on tolerably good grounds, to have been born 66 B.C. This narrative is confirmed by Strabo's younger contemporary Plutarch in his life of Sulla, and in great part by Athenaeus in the IIIrd century of our

era, and in part by Porphyry, also of the IIIrd century. The story is as follows.

Peirce: CP 7.234 Cross-Ref:††

234. Alexander died in June 323 B.C., Aristotle's school having been running in Athens for twelve years, and in the later years Aristotle was supposed to be almost the secret boss of Athens, was enormously rich, and was decidedly unpopular. Athens and Macedonia were instantly at war, and Aristotle retired to Chalcis in Euboea where there was a Macedonian garrison, leaving Theophrastus in charge of his school. Athens was subdued in September 322 B.C.; but just about that time Aristotle died. His will does not mention his precious library nor his personal papers; but it seems they passed into the possession of Theophrastus. Theophrastus lived until 287 B.C., and at his death bequeathed his books to a friend, Neleus, who lived at Scepsis in Aeolis, Asia Minor. Strabo is of opinion that this loss of Aristotle's writings was a severe blow to the peripatetic school; for many of the writings, it would seem, had not been published. At any rate, it seems to be a fact that the school became rather colorless. Meantime, about 250 B.C., the kings of Pergamus, to whom Scepsis belonged, began to collect a vast library, by the simple method of seizing books wherever they could lay hands upon them. The consequence was that the heirs of Neleus hid the books in a cellar. The cellar was damp; but there the books remained until, in 133 B.C., the last of that dynasty of kings bequeathed his kingdom to the Roman people. Soon after this, it would seem that the peripatetic school had been somewhat drawn into politics, although this is an obscure point. However, it is certain that a peripatetic, Aristion, or Athenion, became tyrant of Athens; and a peripatetic friend of his, Apellicon, who was so fond of books that he would steal them from the archives of Greek cities, and who had become rich by plunder, as an agent of Aristion, -- this Apellicon, purchased the library of Neleus and brought it back to Athens. The papers of Aristotle had suffered grievously, during their century and a half of incarceration, from damp and insects, and were in places illegible; but Apellicon, in the intervals of his raids, occupied himself with copying and editing them. But in 87 B.C., Apellicon having died shortly before, Sulla took Athens, executed Aristion, and carried away the library to Rome. Without this accident, Aristotle would probably be to us today a name like Democritus; for the Greeks never regarded him as a supremely great philosopher. In mind and in breeding he was not exactly a Greek, but, like Democritus, was a Thracian. At any rate, Sulla took the library to Rome, and handed it over to the learned peripatetic Tyrannion, whose library finally amounted, according to Suidas, to 30,000 volumes. Tyrannion found that the editing of Apellicon was excessively bad. Ultimately, the peripatetic scholiarch Andronicus of Rhodes undertook the arrangement of the papers, the correction of the text, and the publication of a new edition.

Peirce: CP 7.235 Cross-Ref:††

235. The hypothesis that this story is true is so natural, and it ought to be so easily refuted if it is not true, that I am strongly inclined, in spite of the jeers of all the great German critics, to admit it on probation.†19 When I ask what, then, would be the source of information of Strabo, I find that he was the personal scholar of Tyrannion, and that the labors of Andronicus, which would have made considerable noise in the world of letters, were probably performed in his life-time. Still, before admitting the hypothesis to probation, we must ask whether it would explain all the principal facts. It explains the two styles. The one was for the outer public, the other for use in the school, whether as notes for lectures only, or for general reference, we have not sufficient positive evidence concerning the methods of instruction there to say, or not. The question has been put, as an unanswerable one, how it could be,

according to this hypothesis, that some of the works of our collection were known previously to Andronicus. But this seems to mark a great eagerness to raise difficulties. Even if Strabo had said that none of these works had been published before, and on the contrary, he implies that some of them had been published, we should still be inclined to think that this could not be true in every sense. For how could the notes of students upon Aristotle's lectures have been prevented from circulating? We shall undoubtedly naturally conclude that the publication of Andronicus would be of *Opera Inedita*, including all works of which a decidedly new recension was found; but naturally of Aristotle's polished and finished productions no such text would be found. Strabo, it is true, surmises that the decadence of the peripatetic school was due to their want of Aristotle's writings; but that, from the nature of things, could only be Strabo's surmise. We are at liberty, on our side, if we think it best to do so, to suppose that it was because Aristotelianism was a little outside the general current of Greek thought. Nevertheless, if in place of imperfect notes by students, the school had had the works of Aristotle which we have, we too may surmise, if we find reason for it, that the school would have been somewhat better able to stem the current of thought, and not have been so swept into it as it seems to have been.

Peirce: CP 7.236 Cross-Ref:††

236. We are also asked how, upon this hypothesis can the cross-references have been inserted? But to make a difficulty of this does seem a little childish. The manuscripts had been upon the average six years in the school at Aristotle's death, and remained there for 35 years longer. Was not this time enough to insert any cross-references which had not been inserted in the first writing?

Peirce: CP 7.237 Cross-Ref:††

237. A more difficult question is how, upon the hypothesis, we are to explain the insertion of some spurious works; and how we are to explain the fact that Andronicus is said to have himself believed that the book of the categories was spurious. It seems, certainly, to be probable that there had originally been autographs of all the writings, except perhaps some of the most polished ones, which do not concern us, and perhaps some of the memoranda of facts. But those which touched upon subjects often gone through by classes would naturally be revised and copied; especially since papyrus, judging by what was made in Sicily in 1870, is not nearly so durable as modern paper. Thus, a considerable part of the works, and those of evident authenticity, would not be autographs. Consequently, if a book were marked as being by Aristotle, which might accidentally have happened by error, it could not be excluded. Other manuscripts were perhaps neither autographs nor distinctly attributed to Aristotle; and in such case, Andronicus would have been guided by such information as he could obtain. [It is certain that chirography could not have been, at that time, so characteristic of the individual as now.]†20

Peirce: CP 7.238 Cross-Ref:††

238. Another apparent difficulty is that Athenaeus, in one passage, says that Ptolemy Philadelphus purchased the books of Aristotle from Theophrastus; while, in another passage, he says the same king purchased the same books from Neleus. But Athenaeus must not be supposed to be acquainted with all the details of a transaction that happened five centuries before his time, especially since he seems not even to know that Theophrastus died before Ptolemy Philadelphus acceded to the throne. It is also to be noted that it was for the interest of the heirs of Neleus to say that Ptolemy Philadelphus had bought all the books, since it was about the end of his reign that

they secreted the books lest Eumenes, King of Pergamus, should grab them. It is further to be noted that the reputation of Aristotle as a philosopher was not, at that time, very high; and such reputation as he had was chiefly that of a rhetorician. If there was a purchase it would probably be of copies. At any rate, copies would be returned to the seller, according to practice in other cases. What the agents of the libraries would want of the writings of the rhetorician Aristotle would be finished works. They would not care for lecture notes nor for memoranda.

Peirce: CP 7.239 Cross-Ref:††

239. Having thus reviewed all the supposed difficulties of this hypothesis and having found that they are not serious, we may admit it upon probation, and proceed to trace out its consequences. In the first place, then, supposing it to be true, the works of Aristotle which we possess should be expected to belong to the following classes: first, unfinished works intended for publication; second, lecture notes, or memoirs of investigations; third, brief outlines of theories to be studied further; fourth, memoranda of facts concerning subjects upon which Aristotle had not completed any theory nor written any book; fifth, jottings of ideas. In the third of these classes, that of briefs, almost every writer has a tendency to write nearly to the end of his last sheet, often crowding a little at the end; or, if his sheets are large, he may cut the last into halves and use but one half. In order to ascertain whether or not Aristotle had this habit, I have counted the number of lines in the Berlin edition in nineteen of the shortest books. I have added ten to each number to allow for the heading; and comparing the numbers so increased I find that in the majority of cases, namely in 11 out of the 19, they differ from the nearest multiples of 68 by less than 1/8 of 68, which, of course, ought by chance to happen only once in 4 times. The probability of its occurring 11 times out of 19, is less than 1/500. But in addition to that these eleven cases embrace all but one of the books whose length is short enough to afford decided indications, say less than 6 times 68 lines. Moreover, of the remaining 8 cases all but two are equally near to being multiples of half 68. Finally, the only two real exceptions are both spurious works. The following table gives the numbers. I think we may conclude with reasonable certainty that Aristotle generally wrote about 70 Berlin lines on a sheet. I put it at 70, because I think the method would naturally tend to give too small a number. The amount would be determined by the average size of a sheet of papyrus and the average size of Aristotle's writing, both of which would vary considerably.

Lines + 10

De Ventorum situs et appell. . . .	60 + 8	= 1 X 68
De Divinatione per somnum	134 + 2	= 2 X 68
De longi. et brevitate vitae . . .	198 + 6	= 3 X 68
De juventute et senectute	206 - 2	= 3 X 68
De memoria et reminiscentia	266 + 6	= 4 X 68
De insomniis	277 - 5	= 4 X 68
De somno et vigiliis	341 - 1	= 5 X 68

De insecalibus lineis	324 + 16 = 5 X 68
De audibilibus	408 = 6 X 68
De Xenoph. Lenone et Gorgia . . .	489 - 13 = 7 X 68
De respiratione	676 + 4 = 10 X 68
De animalium incessu	686 - 6 = 10 X 68
Physiognomica	679 + 1 = 10 X 68
De spiritu	444 - 2 = 6 1/2 X 68
De animalium motione	434 + 8 = 6 1/2 X 68
De coloribus	576 + 2 = 8 1/2 X 68
De plantio I	577 + 1 = 8 1/2 X 68
De plantio II	656 - 8 = 9 1/2 X 68
De Mundo	643 + 3 = 9 1/2 X 68

Peirce: CP 7.240 Cross-Ref:††

240. I may add that a cursory examination leads me to think that Aristotle liked to fill a sheet with the whole of a chapter; for a large part of the chapters are of about that length.

Peirce: CP 7.241 Cross-Ref:††

241. Now let us consider what would be done with the sheets. When the work was finished, they would be pasted together, rolled up, and sent to the copyist. They would not be pasted together before the work was quite written, since that might prove inconvenient; for insertions might be desirable, or even rearrangement. Now our collection probably does not contain any finished works. For although it does contain a few which were published during Aristotle's life-time, yet the quoted fragments of them indicate that what we now possess come from MSS. in more or less unfinished states. At any rate the great bulk of what we have are either short essays, lecture notes or notes of researches, memoranda of facts, or memoranda of ideas. It is unlikely that the sheets were pasted together. They must have been rolled up; because the method of keeping papers flat is so superior, that had it once come into use, rolls would have been almost given up, as they are today. Before being rolled up, they would have been arranged in order, with the end at the back; and they would be rolled up face inwards, and most likely put into leather cases. The manuscripts were in very bad condition, and it would be the outside of the roll which would be most exposed to injuries, which would often penetrate several sheets, so that bad places would occur at intervals of about seventy Berlin lines. Cocoons would also be deposited on the insides of the rolls, unless they were wound tight about their sticks; so that, in some cases, the beginnings of books would be injured.

Peirce: CP 7.242 Cross-Ref:††

242. For the sake of brevity, I pass over my reasons for thinking that the heirs

of Neleus disposed of all the works that appeared to be finished, and that neither Neleus nor anybody else meddled much with the MSS. of those that have come down to us, until they passed into the hands of that Apellicon who corrected them so stupidly. We need to take account of the character of Apellicon. He was a peripatetic and a great book-collector. He stole a number of books from the archives of different cities; and having been detected, was obliged to abscond. He joined himself to another peripatetic scoundrel, Aristion, or Athenion, by whom he was sent to loot the sacred treasury of Delos. This he succeeded in doing, and both conspirators were made enormously rich, although by the extreme recklessness and carelessness of Apellicon his army was destroyed. Apellicon then bought the library of Neleus, while Aristion at once made himself tyrant of Athens, where he distinguished himself by his frightful cruelty, in which Apellicon was his right hand man. It was during the brief tyranny of Aristion at Athens that Apellicon's work upon the MSS. of Aristotle was done. It must have been marked by extreme carelessness and utter want of conscience, though we are told that its stupidity was its most striking characteristic.

Peirce: CP 7.243 Cross-Ref:††

243. Owing to the subsequent editing by Andronicus, the traces of Apellicon's work would naturally be obliterated in great measure. But we cannot doubt that such a character as we see him to have been would not hesitate to write over the bad places, so as to make what he judged to be sense; and in some cases, Andronicus must have been forced to accept what Apellicon had written, although, by close attention, we may be led to very strong suspicion that the text is not what Aristotle wrote. If Apellicon had any pet doctrine of philosophy, nothing but want of ingenuity would stand in the way of his altering the text of Aristotle, so as to get that philosopher's apparent support for his own views.

Peirce: CP 7.244 Cross-Ref:††

244. At any rate, some of the phenomena to which our hypothesis points as probable are as follows:

Peirce: CP 7.244 Cross-Ref:††

1st, strange stupidities, or commonplaces, where what went before led us to think that a remarkable idea was to be developed.

Peirce: CP 7.244 Cross-Ref:††

2nd, stupidities, commonplaces, and puzzling places occurring toward the ends of books at intervals of about 70 Berlin lines or multiples thereof; at least when Aristotle's autographs and not copies were there.

Peirce: CP 7.244 Cross-Ref:††

3rd, the appearance of transpositions of passages of about 70 Berlin lines, under the same limiting condition.

Peirce: CP 7.244 Cross-Ref:††

4th, omissions of matter which it was in the line of Aristotle's highly systematic thought to insert, and which he would probably have treated in about 70 Berlin lines or a multiple.

Peirce: CP 7.245 Cross-Ref:††

245. Now I have noted the first of these four phenomena, as I suppose every reader has done. But I have not had time to make any search for the others. It is,

perhaps, just as well that I have not; because their predictive character is thus made manifest. Only since drawing up this list of predictions, I have had the curiosity to make a hasty examination of one book in order to see whether any of the phenomena would present themselves. I have chosen the second book of the Prior Analytics, in which my attention had already been directed to a circumstance which has influenced me to give the name of abduction to the process of selecting a hypothesis to be tested.

Peirce: CP 7.246 Cross-Ref:††

246. In the beginning of this book, I find nothing suspicious unless it be the word method (*{methodon}*) in the fifth line, for a syllogistic figure. As the text stands, Aristotle is made to say that he has already done precisely what it is his intention in the present book to do. The first four chapters are devoted to considering how true conclusions may happen to be drawn from false premisses. Then follow three chapters, of about twice 70, or exactly 146 lines, concerning demonstration in a circle. These appear to me to be out of place, for the reason that Aristotle has not yet concluded his purely formal investigation which these interrupt; and for the further reason that as soon as these inquiries are brought to a close, with the 15th chapter, Aristotle has a chapter on the *Petitio Principii*, which is almost exactly the same thing as circular demonstration; and I do not think that so systematic a thinker would have separated them. The next three chapters, VIII, IX, X, relate to the apagogical contraposition of syllogisms, which come in here very well, but a subsequent chapter, the XXIInd, of 63 lines, on almost exactly the same subject should have followed them immediately. Next follow four chapters on the *Reductio ad absurdum*; and after these is placed a chapter, which, being of purely formal interest, ought to have preceded them. This chapter relates to syllogisms from two premisses which contradict one another, like this:

No A is B

Some A is B

∴ Some A is not A.

I am all but certain that Aristotle, with his mind directed, as it was, to contrapositions of syllogisms, which is the main subject of this book, would not have treated syllogisms which conclude [to] logical absurdities, without also considering syllogisms which take truisms as premisses, such as:

No A is B All A is A

All B is B Some A is B

∴ No B is A ∴ Some B is A.

I therefore suspect that this matter was originally there. If we suppose that Aristotle would have treated these in 50 lines, which I estimate to be correct from a minute study, the chapter would have been brought up from its present length of 87 lines to 137, or two sheets of papyrus, which I am inclined to suspect originally preceded

those on the *Reductio ad absurdum*. After a chapter on reasonings that conclude that such and such circumstance explains the non-occurrence of an expected event, there are a series of chapters on false reasonings and disputations. Then comes the XXIInd chapter which, as just observed, seems to be misplaced; and the rest of the book is devoted to Induction in a broad sense, except one chapter on Objections, which is perhaps not out of place. This chapter on Objections ends with the remark "We have to consider objections adduced from what is contrary, what is like, and what is according to opinion; and further whether a particular objection can be urged in the first or second figure." Evidently the intention was to take up these subjects, at once. But nothing further is said about objection, although some things in the following chapter, which is the last of the book, may be understood as virtually fulfilling the promise. But we cannot believe that Aristotle intended this XXVIth chapter to end as it does. We thus find abundant reason for suspecting omissions, and also transpositions of sheets.

Peirce: CP 7.247 Cross-Ref:††

247. Let us now look for corrupt passages. The last chapter but one being apparently incomplete, we must suppose that the last chapter began a sheet. It is 72 lines long, and therefore would just fill the sheet. The unfinished penultimate chapter XXVI was probably written on a sheet, being only 43 lines long. Chapters XXIII, XXIV, XXV, which are intimately connected, fill just 70 Berlin lines, and, no doubt, just filled a sheet, which would be the third from the end. There is some doubt what it was which originally preceded them.

Peirce: CP 7.248 Cross-Ref:††

248. In chapters XXIII, XXIV, XXV, I strongly suspect two blunders. As an example of an induction, Aristotle supposes that by a simple induction we prove that all animals without gall are long-lived. Using the letters A, B, {G}, he says that he is to prove that the predicate A belongs [to] B, by an induction from {G}. Then the text now reads: "Let A be long-lived, B without gall, {G} the single long-lived animals, as man, horse, mule. Then to the whole of {G} belongs A, because everything without gall is long-lived." It is perfectly evident to anybody but another Apellicon, that {G} is the single animals without gall. That is, he has put {makrobion} where he ought to read {acholon}, unless Aristotle or a copyist made the blunder.

Peirce: CP 7.249 Cross-Ref:††

249. The other blunder requires much more careful study to assure oneself of it. It is to be observed that Aristotle's theory of induction, in the narrow sense, is that it is the inference of the major premiss of a syllogism in Barbara or Celarent, from the minor premiss and conclusion, as data, or premisses. I may add that this is, as far as it goes, the correct theory. Only it is not from any syllogism in Barbara or Celarent, but from a statistical deduction in which the distinctions between Barbara, Celarent, Darii, and Ferio, disappear. Having treated of induction proper, Aristotle goes on to {Paradeigma}, or Analogy, which he regards as a modification of induction proper. Now, Aristotle is throughout the Prior Analytics, especially, so unfailingly thorough in examining every case which is formally analogous to other cases treated by him, that we cannot doubt for an instant that, having remarked that induction, {Epagögé} is the inference of the major premiss of a syllogism in Barbara or Celarent from its other two propositions as data, [he] would have asked himself whether the minor premiss of such a syllogism is not sometimes inferred from its other two propositions as data. Certainly, he would not be Aristotle, to have overlooked that question; and it would no sooner be asked than he would perceive that such inferences are very common.

Accordingly, when he opens the next chapter with the word {Apagögé} a word evidently chosen to form a pendant to {Epagögé} we feel sure that this is what he is coming to. In the excessively abridged and obscure style of the Analytics, he begins as follows: "Abduction, {apagögé} is when it is well-known that the major term is true of the middle, and that the middle is true of the last is not known, but yet is antecedently more credible than is the conclusion." He should have added, "which conclusion we find to be a fact," but he overlooks that, in his wish to add the clause, "and if moreover the middles between the middle and the minor term be few; for thus it will be decidedly nearer to a thorough comprehension of the matter, {epistémé}."

Peirce: CP 7.250 Cross-Ref:††

250. To translate this into our ordinary conceptions, as nearly as possible, it means that it will be better if the minor premiss which is not known to be true but is so easy to believe, is such that it seems as if little were needed to render it evident; and the object of the proceeding is to approach the thorough comprehension of things. In order to make sure of his meaning, we need examples; and it is to be observed that Aristotle's examples are almost always arguments well-known at his time to have been actually employed. He immediately proceeds to give the needed examples. The first is this: "Let {A} be capable of being taught, {didakton}; {B}, science or comprehension, {epistémé}; {G}, righteousness, {dikaiosyné}. Now that comprehension is capable of being taught is plain; but that virtue is comprehension is not known. If, however, this is as antecedently likely or more so, than that virtue should be capable of being taught [which, it seems needless to say, everybody knows to be the fact],†21 then there is ground for the abduction; since we are brought by the hypothesis, '{to proeiléphenai}' nearer to a comprehension of virtue being capable of being taught, than we were before." This seems very clear. He is giving as his illustration the incessant argument of Socrates that virtue must be comprehension, since how otherwise could one explain the patent fact that it can be taught. I have translated {epistémé}, comprehension, because this is what Aristotle meant by it; and the ordinary translation *science* conveys an utterly wrong idea to the modern mind. It must be admitted, however, that before Aristotle wrote this men had paradoxically doubted righteousness being taught. Consequently, this example is neutral, favoring neither the old interpretation nor mine more than the other. He now gives another example to illustrate the case in which the hypothesis brings us nearer to comprehension because, to use his phrase, "the middles are few"; that is, it seems near to first principles. It is here that the text seems to me corrupt. It reads as follows:

Peirce: CP 7.250 Cross-Ref:††

"Let {D} be capable of being squared; {E}, rectilinear; {Z}, the circle. If there is only one middle to {EZ}, that the circle is equal to a rectilinear figure, then the circles being equal by lunes to a rectilinear figure, is near to being known."

Peirce: CP 7.251 Cross-Ref:††

251. Let us endeavor to make sense out of that. The reference plainly is to the discovery of Hippocrates of Chios that certain lunes, or figures bounded by two arcs of circles, were equal to rectilinear figures and capable of being squared; and Aristotle plainly meant that this fact justified the hope, which we know was entertained on this ground, that the circle could be squared. There was "only one middle," or remove from knowledge, concerning the circle's being equal to a constructible rectilinear figure, since it is evidently equal to *some* square. Mathematics was not Aristotle's strong point, and possibly he did not clearly understand that it was only two or three special lunes that Hippocrates had squared. It

is likely, however, that he understood the argument to be the inference of the minor premiss of the following syllogism from its other two propositions:

Whatever is equal to a constructible rectilinear figure is equal to a sum of lunes;

The circle is equal to a constructible rectilinear figure;

∴ The circle is equal to a sum of lunes.

To make this out, we have to change just one word of the text. In place of saying that the major term is {tetralōnizesthai} we have to put {ison méniskois}. This change of a single word of the text, not only renders the whole chapter intelligible; but gives it the very meaning which it ought to have in the development of Aristotle's doctrine. Such a singular corruption of the text as I suppose could hardly have taken place without an Apellicon; but with him, it was easy enough.†22 If we do not suppose this corruption to have taken place, we are reduced to accepting the text as it stands; and if we accept the text as it stands we must accept the usual interpretation of it. This interpretation is that abduction is nothing but an ordinary syllogism of the first figure, when we are not sure of the minor premiss, but still are more inclined to admit it than we should be to admit the conclusion if the latter were not a necessary consequence of the former. The abstract description at the beginning of the chapter will bear this construction perfectly well; only it makes the chapter an impertinent obtruder at this point, and not in the style of Aristotle's thought. But when we come to the examples, the ordinary interpretation reduces the latter, at least, to nonsense. The first becomes,

Comprehension can be taught,

Virtue is comprehension;

∴ Virtue can be taught.

In the first place, this is a *petitio principii*, or very near to one since there is no way of proving that virtue is comprehension, except by its being taught. In the next place, few in Aristotle's time had used this absurd argument; it had scarcely been seriously doubted, what all experience shows, that virtue can be taught. A very few ethical writers of modern times have denied it; but it had hardly been denied then, except as a temporary shift in debate. A philosopher who, like Socrates, maintained that it was better to do wrong, knowing it was wrong, than not knowing this, could not doubt that righteousness could be taught.

Peirce: CP 7.252 Cross-Ref:††

252. The other example is still worse. It becomes,

Whatever is equal to a rectilinear figure can be squared,

Every circle is equal to a rectilinear figure;

∴ Every circle can be squared.

We here naturally understand by "equal to a rectilinear figure," equal to a rectilinear figure, constructible or inconstructible. But in that case, the minor premiss, instead of being not known, is the most evident thing in the world; while the major premiss which ought to be manifest, is far from being so; for if a figure cannot be constructed it cannot be squared. Supposing however that by a rectilinear figure is meant one that can be constructed, which must have been the meaning, since Aristotle says that it is almost known through lunes, who ever used such a ridiculous argument? And how can Aristotle say, as he does, that lunes in any way help the matter, or are at all relevant? Whatever bearing lunes were supposed to have upon the quadrature of the circle disappears entirely from this representation. Nothing can be more utterly unlike Aristotle's usual examples, which bring up in vivid aptness actual reasonings well known to his scholars.

Peirce: CP 7.253 Cross-Ref:††

253. I think, then, that my interpretation of the passage, considering its being what the current of thought demands, considering how the word {Apagögé} balances {Epagögé} and considering that it renders both the examples their real historic forms, comes within a tolerably close approach to certainty. If we accept it, it affords a remarkable confirmation of the Scepsis story; because of the bold insertion it supposes to have been made in the text.

Peirce: CP 7.254 Cross-Ref:††

254. I have looked forward seventy lines from each of the two corruptions I have mentioned; but the measure falls in each case upon a passage so plain, that had it been totally obliterated, not even an Apellicon could fail to restore it correctly. I have said enough to show how I think this hypothesis should be treated.

Peirce: CP 7.255 Cross-Ref:††

255. All the best critics of Germany, on the other hand, utterly reject the Scepsis story. Their reasons I have already indicated. They are so weak that I think I am justified in surmising that the real motive of their rejection is a reluctance to accepting any ancient testimony without dressing it up and putting a new face upon it. At any rate, their general principle is that they think the story told by Strabo is less likely than that Strabo should tell a false story; and that principle seems to me to amount to believing whatever they are inclined to believe.†23

Peirce: CP 7.256 Cross-Ref:††

CHAPTER 4

NOTES ON SCIENCE

§1. THE STUDY OF GREAT MEN †1

256. Having been asked to write something about the productiveness of the nineteenth century in great men, I ought to begin by frankly confessing just what the facts are that, being known, will lead the reader to attach a certain value, and no more, to my opinions on this subject. In the year 1883, having charge of the instruction in logic in the Johns Hopkins University, I cast about for a subject that might afford valuable training in such inductive investigation [as] the members of my class might need in future life and which they would not be likely to acquire in their other studies. I wished it to be a subject susceptible of mathematical treatment, since an inductive investigation so treated may throw abundant light on the proper logical procedure where mathematics is not available, while the converse can hardly be true. Yet there were several reasons for selecting a subject concerning which no exact observations could be made. Much more logical caution is requisite in such a field; and it was desirable to explode the ordinary notions that mathematical treatment is of no advantage when observations are devoid of precision and that no scientific use can be made of very inexact observations. Besides, very little training is required in the purely observational part of the business of making observations that can be rendered extremely precise; while great training is requisite for the making of the very observations themselves when the observations are of the kind which can never be made precise. In order to make my meaning clear, I will take an example of each kind. The matching of two colors is an observation that can be rendered precise. In order to learn to perform this observation what is chiefly requisite is to appreciate the fact that the two surfaces to be compared have to be put in precisely the same light, and that, unless various precautions are taken, they will not usually receive light of the same intensity and color though they be held quite close together. The mere observation itself of whether they match or not will very soon be made so accurately that all the effort has to be expended, not upon that pure observational part of the work, but on the experimentation. On the other hand, an example of an observation that can never be rendered precise is that of saying which of two different colors, say a red and a blue, is the more luminous. Here, no extraordinary experimental precautions are called for. The thing is to look at the colors and disregarding entirely their extreme difference of hue, as well as the circumstance that red is a very high color while blue is comparatively greyish, to just note the relative impressions of luminousness that they make upon us. A great deal of training is required before a person can do this well enough to give any uniformity to his judgments. It is not an education of the eye or of part of the brain particularly connected with the eye, so much as it is of the mind. A certain person upon hearing a note struck upon the piano was utterly unable to pick out any harmonic by ear. He then went through a course of training with a color-box, involving observations of the general kind I have mentioned, until he acquired a good deal of skill. Upon now returning casually to the observation upon the piano-tone he found to his surprise that he could pick out three or four harmonics without difficulty. Now this pure observational power is of great value in life. It is true that the only thing that is disclosed by such pure observation is one's own feelings. But then different persons' feelings are so closely similar, that that is of the utmost use in a world of men and women. It is this that the word *tact* ought to denote independent of adroitness in playing upon people. A Sherlock Holmes's habit of making deductions from minor circumstances is all very well, in itself; but if attention to these circumstances is to draw it away from the Lavaterian, artist-like, direct, and pure observation of the suspected person, it will do more harm than good. I should be afraid to tell what I have known this power to accomplish, because it is

unpleasant to have one's veracity doubted. Lavater's 'Essays on Physiognomy' is a book very much discredited; and I cannot say that I am a strong believer in such notions, as that a large and prominent nose is associated with push and energy. But matter of that sort makes but a small part of the work, the diligent study of which, in a good edition, will, I can testify, stimulate a young person to train the faculty of which I am speaking. At any rate, ever since I read it, I have been convinced that psychology would assume its legitimate dignity among the sciences from the day on which it should be recognized as based mainly upon the pure observation of which I speak, and not before. The sciences of objective nature do not train this power, because they can make no use of the sort of data it affords. The exact sciences, physics and chemistry, do not teach any observation of any kind, to speak of; but only manipulation and experimentation. The purely observational business is confined to such trifles as bisecting stars, putting crosswires upon spectral lines, reading verniers and the like. Natural history undoubtedly trains a certain kind of observational power, the taking note of circumstances that would escape an untrained attention. But that is not the power I had in view.†2 These are among the reasons that led to my pitching upon the study of great men as affording useful training for my class. I have dwelt upon these reasons at some length, simply because they throw some light upon the matter with which I seek to acquaint the reader, the degree of my qualifications to give an opinion upon a subject upon which any opinion must be largely subjective and rough, even though it should attain a certain degree of validity. . . .

Peirce: CP 7.257 Cross-Ref:††

257. The first thing we did was to make what I called an impressionist list of great men, -- *impressionist*, because the admission to it of any name was to depend upon our pure observation of the impression of greatness which we received from the contemplation of his life and labors, while carefully abstaining from any analysis of greatness or of the reason for the impression we felt, since one of the very purposes of the list was to serve as a test of any theory of the nature of greatness and the cause of the impression it makes upon us. A preliminary list of nearly a thousand names was first formed; and these were considered and reconsidered, three or four times with increasing care, until we finally settled upon 288 great men. I was desirous of having this include substantially all the great men of history. Yet I was less concerned that it should omit none whom it ought to contain than that those that it did contain should form a fair sample of what great men were like. Since we were all students, no doubt we had a bias in favor of men of intellect; but against this we were on our guard. There was naturally some moral leaning, and the social atmosphere of Baltimore must have affected our judgments. Moreover, it is humanly impossible in such a selection to do justice to contemporaries, compatriots, and acquaintances, whose greatness we are too close by to discern. I could now improve the list in details, besides doing something toward bringing it down to date. But it was formed with so much care that I would not venture to touch it short of a good six months solid preparatory study.

Peirce: CP 7.258 Cross-Ref:††

258. The list having been formed was found to be too large for so small a class to study to advantage, and we therefore restricted ourselves to the consideration of every sixth name. I do not intend to weary the reader with any account of the elaborate inductive inquiries, about seventy in number, which we undertook in regard to those men, and which I have followed out with some diligence all through the seventeen years which have since elapsed. But there is one point which it is important for my present purpose that I should explain. Everybody knows, I suppose, that the ancient astronomers divided the stars visible to the naked eye into six orders of

brightness, called "magnitudes," the first magnitude containing the brightest stars, and the sixth those that are barely visible in an ordinary atmosphere on a clear and moonless night. Ptolemy already subdivided each magnitude into thirds, and in our century they were first divided into tenths, and after the introduction of photometrics into hundredths. Now when photometry came into use, it was found that there was a nearly constant ratio of light between the light of average stars of successive magnitudes. Thus, in my book entitled 'Photometric Researches' p. 47,†3 I show that the differences between the light of the faintest stars normally referred by Ptolemy to the first four magnitudes, expressed upon a scale of magnitudes which makes the ratio of light strictly equable, are 0.94, 1.07, 0.99, which are practically equal. This remarkable fact is connected with Fechner's psychophysical law (at which the utter ignorance of German philosophical professors of the mathematical theory of metrics, their fondness for expressing opinions about matters of which they are ignorant, and the awe with which they are copied by Americans, has made it the fashion to sneer) according to which equal ratios of excitation produce equal differences of sensation. The old astronomers assigned successive numbers to stars which gave equal differences of sensation, and these, when the stars are not so faint that another influence, the nature of which is made clear in my book, interferes, correspond to a geometrical progression of intensities of light measured physically, so as to make the intensity of light proportional to the square of the amplitude of vibration. The scale of star-magnitudes, having been found to lend itself perfectly to mathematical treatment, was imitated by us in expressing our impressions of the greatness of the different men. That is to say, we marked the greatest man on our list 1 and the least 6. Some member of the class to whom the duty had been assigned would read an account drawn up by him of one of the men, and each member would then secretly jot down in units and tenths, his evaluation of the impression produced upon him. The ballots giving these numbers would then be handed up to me and the mean would be adopted as the "magnitude" of that man. As this method was no novelty to me, I having often applied it to all sorts of feelings, so that I knew about how well the numbers would agree, I never took the trouble to preserve the individual estimates; so that I can now only put before the reader the results of the ballotings in three cases which occurred at an informal meeting when the whole class was not present, and when I calculated the means upon the margin of a paper which has been preserved for another purpose. These three ballotings thus casually preserved are as follows:

<i>Bolivar</i>	<i>Julian</i>	<i>Swedenborg</i>
4.2	3.6	4.0
4.2	3.8	4.5
4.0	3.8	4.3
3.8	4.8†4	3.8
3.9		
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<i>Means</i> 4.0	4.0	4.1

Peirce: CP 7.259 Cross-Ref:††

259. I do not think that there was any man for whom the extreme estimates exceeded two magnitudes. Such a discrepancy could only occur in the case of some vaguely known and semi-mythical hero, such as King David. It is obvious, therefore, that if what we mean by a judgment being "objectively valid" is that all the world will agree in it, and after all Kant's discussion that is about what it comes to, then there was a satisfactory degree of "objectivity" in the mean magnitudes we assigned, although they referred, not to the man as he really was, but to the man as he was presented in the account read to the class, and although the marking could not escape a large "subjective" percentage due to our common, but not thoroughly catholic, culture and environment.

Peirce: CP 7.260 Cross-Ref:††

260. At the meeting I have just spoken of, we tried estimating, in the same way, the "magnitude" of the average man. This was a matter of difficulty and uncertainty, for two reasons. The value was so far removed from the part of the scale to which we were habituated that slight differences in the value of the unit of "magnitude," as employed by different members of the class would necessarily be exaggerated. This, however, was probably of little account compared with the uncertainty as to what kind of a person the truly "average man," with whom we were none of us accustomed to associate, really was. It is not surprising, therefore, that the ballots, which here follow were pretty wild.

Average Man

10.5

13

13

11

Mean 11.9

Peirce: CP 7.261 Cross-Ref:††

261. At the same meeting we balloted, with very little preparatory discussion, for the "magnitude" of the leader of the Baltimore bar, with the following results:

Leader of the bar

8

8.8

9

8.5

Mean 8.4

Peirce: CP 7.262 Cross-Ref:††

262. Just as a young gentleman nine years of age looks upon a lad of eight as a person of little knowledge of the world or experience of life, and as altogether inferior in intellectual grasp, so the nineteenth century has had the habit of looking down upon the eighteenth; and no doubt the judgment is a sound one in both cases. But when we ask whether the nineteenth century had shown any vastly superior productiveness of great personalities, we have a difficult question before us. A pretty careful sampling enables one to say that if we were to enumerate the men whose achievements in art, in practical life, or in science, can never be forgotten by history, we should find upwards of a thousand in the nineteenth century against not more than five hundred in the eighteenth. But there is a vast difference between a man who accomplishes something great, -- say, the introduction of the first anaesthetic, ether, -- and the man who, upon the whole, impresses us as a great man; and if we confine ourselves to great men, we shall find only about seventy in the nineteenth century against some sixty in the eighteenth, -- a very marked falling off, when the increase in the number who can read and write, and in the opportunities for distinction, generally, is considered.

Peirce: CP 7.263 Cross-Ref:††

263. It is an observation as true as it is trite that the nineteenth century has been an era of machinery, -- not of machinery of steel merely, but of machinery in politics and in business, such as trusts and trade-unions, machinery in all the methods of research, physical, philological, historical, philosophical, mathematical, even of machinery in art and in poetry. Consider Mathematics, as a field where, if anywhere, it might be supposed that machinery would be of little avail. In the early eighteenth century the greatest geometers in Europe were still "stumping" one another with problems, and the discovery of a theorem might raise a man to greatness, -- witness Taylor's theorem, published in 1717. Subsequently, it must be a *method*, no longer a mere *theorem*, to impress the world so powerfully. Nowadays, methods of the greatest profundity and power are turned out at such an astonishing rate that nobody but professional mathematicians ever hear of them singly, at all. Hermann Schubert's Calculus of Geometry, which enables us, for example, by a brief computation, to determine that the number of cubic curves each of which shall touch any twelve given spheroids, is just 5 billion 819,539 million, 783,680, hardly makes a ripple in the ocean of modern mathematics. In Addison's time, the man who could write graceful English was a prodigy. Now, everybody writes decently well: the leader-writer of the yellowest journal has a better command of his pen than the accomplished Shaftesbury or the learned Toland. An age which in every department has offered a thousand new appliances to enable a puny man to do a giant's work has been an age of machinery, indeed.

Peirce: CP 7.264 Cross-Ref:††

264. Whether machinery, organization, and a great development of methods and of the *methodus methodorum*, †5 with their fruits of incessant new discoveries, inventions, improvements of all kinds, ought *a priori* to be unfavorable to the production of great personalities is a problem for a more sagacious man than I can pretend to be, to compute with any confidence. One might incline to think not; for, on

the one hand, just as we know that plants transferred to new soil are the more apt to sport, so it might be expected that under novel social conditions the proportion of births of extraordinary minds would be increased, while on the other hand, once born, one would suppose they would find in novel situations just the opportunities that were needed to bring their superiority into exercise. History, too, seems to confirm this, in always showing us a wealth of great men at every great social transformation. The list of heroes of our own country shows this. Rigidly exclude every man not unquestionably great, and how few will be left outside the groups that cluster about our two cataclysmic upheavals, the revolution and the rebellion!

Peirce: CP 7.265 Cross-Ref:††

265. Yet somehow, the nineteenth century has certainly lacked its due quota of great personalities. When about a fifth of the century was still in the future, I drew up, with the coöperation of a class at the Johns Hopkins University, and with much outside assistance, what I called an Impressionist List of about 300 Great Men. It was one of those matchless classes, -- the very salt of the earth, -- which it was my privilege to enjoy in Baltimore. Almost every member of it has since signally distinguished himself, and those who have less obtrusively done so are among those whose observations I now, after twenty years, most frequently recall as forcible and just. I called it an impressionist list, not that it was at all hastily drawn up, but because, in the process of sifting out the names from a much larger list, we were to set aside all preconceived ideas of what greatness consists in, and were simply to estimate the impression made upon us by a fair review of each man's life and labors. Being all of us students, no doubt, we had some bias in favor of the philosophers; but against this we endeavored, as much as possible, to be upon our guard. The social atmosphere of Baltimore, too, probably affected us somewhat; and I think we had a human prejudice against monsters of iniquity and against men of greed. Everybody would, I know, find fault with our list. Not a few of its omissions were violently counter to my own personal impressions. Still, it would be very difficult, if not impossible, to draw up a much better list. I may mention that this list formed the basis for much subsequent study; and one of my main purposes was to train the men to the nice observation of their own sensations, to show them that feelings are capable of direct evaluation with sufficient precision to serve a scientific purpose, and to admit of mathematical treatment, and to demonstrate that they do not, for the most part, differ extravagantly among different persons in the same environment. For instance, after an account of a man had been read to the class, we would each estimate the impression of greatness produced upon him, somewhat as astronomers estimate the magnitude of a star, calling Pythagoras, as he is represented in the life of Iamblichus, a first magnitude man, and Cola Rienzi a tenth magnitude man. Each of us would, after hearing the account, secretly mark upon paper his estimate of the man's magnitude. The papers would then be collected and the values copied in a column upon the blackboard. The extreme variation would not ordinarily exceed two magnitudes, so that we all came to feel pretty confident that the averages of our estimates would for the most part be pretty close to those that would be produced in the average man of the same general breeding and culture, by the same biographies. I have, since the list was first made, never allowed it entirely to drop out of mind, but have spent a good deal of time, all told, in the further study of the 288 great men it includes, and recently, with all the aid I could get, in adding such names as seemed to [be] required by the history of [the] last twenty years.

Peirce: CP 7.266 Cross-Ref:††

266. This long explanation was necessary in order to show what degree of

objectivity might fairly be attributed to my impressions of the productiveness of the century in historic personalities. This objectivity is, like Mercutio's wound, not as wide as a church door, yet 'twill serve. It will not be valid for every normal mind as my matching of a color might be valid for every normal eye; but it tolerably represents how an average student of science sixty years old, not unobservant of human life, and not very narrow in his interests, would be likely to be impressed.†6

Peirce: CP 7.267 Cross-Ref:††
§2. THE HISTORY OF SCIENCE †7

267. I have now expounded to you as much of the history of science as I found myself able to do in 12 hours. Of course, a great deal remains to be considered; but even the few facts we have collected will do something to answer the questions with which I set out.†8

Peirce: CP 7.268 Cross-Ref:††

268. We have found as I suggested at the outset that there are three ways by which Human Thought grows, by the formation of habits, by the violent breaking up of habits, and by the action of innumerable fortuitous variations of ideas combined with differences in the fecundity of different variations.

Peirce: CP 7.269 Cross-Ref:††

269. As for the last mode of Development which I have called Darwinian, however important it may be in reference to some of the growths of mind, -- and I will say that in my opinion we should find it a considerable factor in individual thinking, -- yet in the history of science it has made as far as we have been able to see, no figure at all, except in retrograde movements. In all these cases it betrays itself infallibly by its two symptoms of proceeding by insensible steps and of proceeding in a direction different from that of any strivings. Whether or not it may not be more or less influential in other cases, in which its action is masked, the means of investigation which I have so far been able to bring to bear fail to disclose.

Peirce: CP 7.270 Cross-Ref:††

270. The manner in which the great and startling advances in scientific thought have been made appears very clearly. It is by the violent breaking up of certain habits, combined with the action of other habits not broken up. Thus, the highest level of Egyptian thought seems to have been reached at a very early age. So it appears to us, and so it always appeared to the Egyptians, for they always reverence the ideas of antiquity, as superior to those of their own time. Now the great factor in the development of the Egyptian mind was undoubtedly the physical geography of the country which probably produced its effects in a reasonably small number of generations after it was first felt. So with the Greeks. Their thought remained in its primeval condition until the extension of commerce brought them within the sphere of influence of other peoples, the Phoenicians, the Egyptians, and the Babylonians, and then within a few generations they made great strides in thought, to be succeeded by a slower movement of another kind. At first, we have a rather servile copying of the ideas of those countries, a syncretism such as we see in Pythagoras. But soon the foreign ideas begin to react with the ideas and faculties peculiar to the Greeks, and a great original life commences. So it was again, when in the 13th century, the ideas of the Dark Ages were rudely shaken up by contact with the more civilized Saracens;

although as far as science was concerned that movement was quickly stifled by the rapid development of theological ideas.

Peirce: CP 7.271 Cross-Ref:††

271. The renaissance in Italy was of slower growth, because foreign ideas had been slowly filtering in since the thirteenth century uninterrupted. However, after the fall of Constantinople in 1454, there was a much more rapid movement. That movement was first strongest in the direction of art, which I take to be a mark of rapidly growing minds, of minds receiving nutrition too rapidly to be packed down into the forms of science. But the scientific development came later. Galileo was born the very day of Michelangelo's death.

Peirce: CP 7.272 Cross-Ref:††

272. In this early development of science there were two great factors. In the first place, the direct strivings of the astronomers, the European successors of the Arabians, who brought to astronomy more masculine intellects than the Arabians had, had brought out at length a world-shaking idea, the Copernican conception. In the existing state of the church, this was more easily accomplished in Northern Europe, and there it was brought to its perfection by Kepler, and I have traced out the birth of this conception with some minuteness because it is remarkable as being a birth from within, not an influence from without. Although the authors of this, Copernicus, Tycho, and Kepler were all Teutons, the value of their work was better understood and more accurately appreciated in Italy than north of the Alps.

Peirce: CP 7.273 Cross-Ref:††

273. The other great factor, which chiefly influenced the development of dynamics, was the study of the works of Archimedes; and a strongly Hellenic color is apparent everywhere in that branch of science down to the time of Newton. It is shown in the great fondness for demonstrations from axioms, in the desire to put all special experimentation out of sight, and to rely on the Light of Nature. It is also shown in the geometrical methods which are preferentially employed.

Peirce: CP 7.274 Cross-Ref:††

274. As to the third mode of intellectual development, we should see more of it if we were to trace out the history of science into its later era. Though it is not so startlingly manifest, it is certainly the method of the ordinary successful prosecution of scientific inquiry. We see its action clearly in the history of astronomy at all periods, and especially in Kepler's gigantic work. It is growth by exercise, or by direct efforts in the direction of the growth. If we have seen little of it, it is because I felt it necessary to the understanding of the subject to begin at the beginning and I could not in twelve hours carry you on to the point in which science, except in astronomy and to some extent in the last developments of dynamics, was really settled down to its work. I will mention, however, that in the January number of the *Monist*, †9 I have endeavored to give an analysis of this kind of evolution, and especially have connected it with the Christian theory of the way in which the world is to be made better and wiser.

Peirce: CP 7.275 Cross-Ref:††

275. I have to thank the company very gratefully for the patience and kind indulgence with which my lectures have been listened to. I have done what lay in my power to present as much of the History of Science as I have been able to treat in a lucid manner, and to show that it is governed by Law like other departments of nature. But these laws are not of the nature of mechanical forces, such that the

individual and the spirit of man is swallowed up in cosmical movements, but on the contrary it is a law by virtue of which lofty results require for their attainment lofty thinkers of original power and individual value. You cannot silence or stifle or starve a single one of them without a loss of civilization from which it never can wholly recover. It is not more certain that the inches of a man's stature will be affected all his life by an attack of fever as a baby, than that we are now less happy because of the many great geniuses whom untoward circumstances have put down. The country that can first find the means not to provide the million with miscellaneous reading matter, and elementary education, but to utilize its superior intellects for the general good, will experience a wonderful acceleration of civilization from which the benefit of the million, in much more valuable ways, will come about of itself.

Peirce: CP 7.276 Cross-Ref:††

276. Although by far the greater part of almost every treatise on psychology is devoted to the department of Cognition, yet it is curious that less is known about thought than about either of the other two modes of consciousness. What I have to say about it is even more inadequate than what I have said of Primisense and Altersense.†10 In order to make out what the main processes of Thought, or varieties of Medisense, are, I naturally turn to Logic, which has been my chief study for the last forty years. In the process of inference, or the self-controlled formation of new belief on the basis of Knowledge already possessed, I remark three chief steps. They are, first, the putting together of facts which it had not occurred to us to consider in their bearings upon one another, second, experimentation, observation, and experimental analysis, which is substantially the same process whether it be performed with physical apparatus such as the chemist uses or with an apparatus of diagrams of our own creation, such as the mathematician employs, and third, the generalization of experimental results, that is, the recognition of the general conditions governing the experiment, and the formation of a habit of thought under the influence of it. If we turn to the history of the physical sciences, as the most perfect example of the successful application of thought to the external world, we find that they have all gone through five stages. First, an interesting phenomenon has attracted attention. Here, inquiry has often come to a stop for a long time. It does not proceed until, Secondly, somebody invents an instrument or a method by means of which the elements of the phenomenon can be subjected to experiment. Third, a process of experimental analysis has been carried out, resulting in the ascertainment of a law, or exact relation between the different elements of the phenomenon. Thus, the lodestone must have been known for a long time, before somebody invented the idea of experimenting upon it with iron rings. This rude instrument led to the discovery that a second ring would be supported by the first, that a third could be hung to that, and so on. Nothing more was discovered until Petrus Peregrinus invented a new method of experimentation.†11 Namely, he shaped the lodestone into a terella or ball, and then applied to it short pieces of iron needles. With this apparatus he at once discovered the poles of the magnet and their properties. Then, discovery stuck fast again; because nothing more could be made out with those instruments; until Gilbert invented a new instrument consisting of an iron needle balanced on a point, and free to turn round.†12 Such has been the history of every science. After a while a Fourth class of studies has commenced, consisting in the exact measurement of the constants concerned. In optics there was, for example, the velocity of light. Finally, Fifthly, has come the construction of mechanical theories by which the causes of the phenomena were probably assigned.

Peirce: CP 7.277 Cross-Ref:††

277. Comparing the teachings of the history of science, on the one hand, with those of logic, on the other, we notice a certain agreement between the two. The interesting phenomenon which gives the first impulse to scientific thought corresponds with that interesting colligation of facts, which is the first step of the inferential procedure. Experimental analysis plays a great part in both. Finally, the generalization of the experimental result which completes the inference is represented in physical inquiry by speculation into the mechanical causes of the phenomenon. For as the mechanical explanation of the physicist consists in a reference of the experimental result to the higher principle of dynamics, so the generalization of the reasoner is the reference of the experimental result to the higher principle of mathematical necessity or probability.

Peirce: CP 7.278 Cross-Ref:††

278. Perhaps, just as the study of a physical phenomenon must depend upon applying such instruments as we can find, so in our attempt to enumerate the processes of thought, we cannot do better than to begin by laying hold of the suggestions that are offered by these analogies between the process of inference and the history of physical science.

Peirce: CP 7.279 Cross-Ref:††

279. Some thirty years ago I made a remark upon the course of physical studies which has been repeated by very competent men, to the effect that there were three stages; first, the observation and miscellaneous research into the phenomena; secondly, the analysis of the phenomena and formulation of their laws, including hypothetical explanations of them; thirdly, the determination of the constants. Nothing intellectual, -- no casts of mind, -- more diverse than those needed for these three steps can well be imagined within the sphere of physics. These three steps occur in every branch. True, ordinary observation supplies many of the phenomena in optics and acoustics; but not all, by any means. Not circular polarization; not the zones of silence about a fog-horn. In electricity, in galvanism, in X-rays, and other radiations, in fluorescence, the first breaking of ground required a peculiar genius, not to be undervalued, yet not for an instant to be likened to that which analyzes the phenomena, -- the work of a Galileo, a Kepler, a Faraday, a Maxwell. Then the painful and technical business of accurately determining the constants is not one of genius or insight but of perfect thoroughness and flawless technique. It is the work of the Regnaults, the Michelsons, the Rowlands, -- the accomplished experts.

Peirce: CP 7.280 Cross-Ref:††
§3. MEASUREMENT †13

280. *Definition.* A *character* is a possible fact regarded as concerning a particular thing or things.

Peirce: CP 7.280 Cross-Ref:††

Illustration. The moon may come between the sun and the earth so as to cast a shadow upon the latter. That is a possible fact. Now, if we think of this fact as

something that concerns, or modifies what we can say of, the sun, we call it an eclipse of the sun; and so considered, it is what we mean by a *character* of the sun at such an instant.

Peirce: CP 7.281 Cross-Ref:††

281. *Explanation.* I say a *possible* fact, because if a character is not actually true of a given thing, that is not sufficient to prevent its being a character. Thus, when the sun is not eclipsed, it does not possess that character, but we do not say that there is no such character. If it could be shown that the supposition of the moon coming between the sun and the earth so as to cast a shadow on the latter, could only be carried out in ways which, being examined, would prove all of them to involve contradictions, then we should say this is no real character, but only a phrase, which cannot be realized in any imaginable manner. In short, the character in itself does not pretend to belong to the world of experience; to be a character it only needs to have a place in the realm of ideas. When the character is attributed to any particular thing, that thing is something having its place in experience. It is not needful, for the purposes of mathematics, to inquire particularly what experience is. It may be said, however, that it is something which is forced upon us; so that one element of it is its insistence, whether we like it or not. And it is also something which forces itself not merely momentarily upon me; but upon me and you alike, at various times, so that it has a certain consistency and extension in its forcefulness. Finally, it is something of which our knowledge can never be complete; so that there is always a difference between the experienced thing and our idea of it. But since the character, to be a character, need not really belong to the particular thing, but it is sufficient if we can really ask whether or not it belongs to that thing, it will be seen that we do not need to go very far into a study of its nature.

Peirce: CP 7.282 Cross-Ref:††

282. *Analysis.* Characters may be more or less precisely defined. A general character may be conceived as a multitude of precise characters. Mathematical thought consists in the study of precise relationships between ideal objects. But a possible fact may vary in an indefinite multitude of different ways. The moon coming between the earth and the sun, may not only have a multitude of different positions, but it may have a multitude of different shapes and colors and chemical compositions; and so may the sun and earth. For the purposes of mathematics, it is necessary, in the first place, to abstract from most of those differences, and consider them as insignificant and, for our purposes, nil. Then, those varieties which still, for our purposes, are different, have to be arranged, as far as possible, dimensionally. To do this, we imagine any one precise variety of the character, -- precise, I mean, after the proper abstractions have been made, -- and conceive that as undergoing a multitude of variations in time, from the infinitely distant past to the infinitely distant future, such that in the course of all time it will be continually changing; and we prefer to take this series so that the character will ultimately return to its initial state, and just barely return to that state. If this series of changes does not include all the variations, we think of the whole multitude of states through which the character passes in all time, as belonging at one instant to an equal multitude of objects; and then we conceive the characters of those objects to undergo in the whole course of time a continuous series of changes of a different kind, so as to be entirely distinct. We call this a second dimension, or series of variations. We can make any number of these dimensions; and as far as possible we thus seek to give some precise arrangement to all the variations of characters. When this method fails us, we can resort to other systems of arrangement, of some of which we shall have examples in geometry.

Peirce: CP 7.283 Cross-Ref:††

283. Any one of those dimensions is such that the characters can pass through the whole series of states in the course of time. It is easy to imagine multitudes of variations so related to one another that one precise character could not even in all time pass through them all by insensible gradations. For example, colors differ from one another, not merely in hue, but also in luminosity, and in chroma, or intensity of departure from grey. Now, starting with a color of some precise hue, precise luminosity, and precise chroma, if it is to change its hue gradually, for each precise hue that it takes, it will have just one sole luminosity and one sole chroma; so that, when it has gone through the whole cycle of hues, it will have had for each of them but one single luminosity and but one single chroma. Though it should pass through the cycle of hues times without end, it would still not have begun to exhaust the possible luminosities and chromas for each hue. The student may admit that it might be possible (and, in fact, it might be shown to be possible) that if the color were so to jump from hue to hue, from luminosity to luminosity, and from chroma to chroma, that taking any two instants, no matter how near to one another, if during the interval between them the variations should embrace the whole cycle of hues, the whole range of luminosity, and the whole range of chroma, then the color might in the course of time precisely assume for an instant every special variety of color. But if the variation is to take place *gradually*, then it is not possible that the color should in the course of time assume every possible variation.

Peirce: CP 7.284 Cross-Ref:††

284. The meaning of this seems to be clear. That is, it possesses the first grade of clearness of ideas, that of containing no element which perfect familiarity does not enable us to use with entire confidence.†14 But that grade of clearness is not sufficient for precision of statement, and logical security. For that purpose, we must say what we mean by "gradually." In attempting to state this, it first occurs to us to say that we mean by a gradual change of hue, such a change that in passing from one exact hue to another we pass through all intermediate hues. There are two reflections to be made upon this statement. First, it supposes that the different hues are so related in our minds that we are able to say what ones are, and what ones are not, intermediate between any given pair of hues. That is to say, we must have a precise idea of what it means to say that the hues are mentally arranged in a line. But if that be so, we need not introduce the conception of a change in time; for that was only a device to enable us to describe what we mean by a line of variations of character. In truth, though the introduction of the idea of time gives sensuous clearness to our idea, it contributes not in the least to logical clearness. The second reflection which has to be made upon our attempt to define gradual change of hue is that the hues form a circle, the so-called color-circle; so that it is possible to pass from any one to any other by going either way round the circle; and thus there is no particular hue that we need pass through. To define a linear arrangement, the line being permitted to return into itself, it is necessary to speak of four points on the line.

Peirce: CP 7.285 Cross-Ref:††

285. **Definition.** A *state* is an exact character, that is, one which, certain understood abstractions being made, admits of no varieties.

Peirce: CP 7.286 Cross-Ref:††

286. **Definition.** A *line of variations of states* is a continuous multitude of states such that, taking any four of them all different, there are in the nature of the characters two that are adjacent to any one, or else it would be so were two that are

extremes made adjacent to one another; and any fifth character of the same line occupies in its nature a definite position between two adjacent characters of the first four.

Peirce: CP 7.287 Cross-Ref:††

287. **Definition.** A *circuit of states* is a line of variation of states which returns into itself and has no extreme states.

Peirce: CP 7.287 Cross-Ref:††

Illustrations. Suppose a light to be increasing in intensity from that of a fire-fly toward that of the planet Venus at such a rate that it would at a certain instant, say Midnight, attain that brightness. Suppose next that it increases at a more and more rapid rate, so that after a while its rate of brightening is such that at that rate it would attain the brightness of Venus at 11 o'clock. Suppose it continues to increase more rapidly, so that after a while, its rate of brightening is such that it would attain the brightness of Venus at 10 o'clock. Finally, suppose that its increase becomes so rapid that at that rate it would become as bright as Venus at 9 o'clock. Thus, there is this order among these characters: such a change of brightness as would make it equal Venus at 12, at 11, at 10, at 9. But when it was increasing at a rate to make it equal Venus at 12, its increase might become slower and slower so that the time at which it would equal Venus would become later and later. If it ceased to increase, the time would be thrown into the indefinite future. If it began slowly to decrease, its change would be as if it had been as bright as Venus a long time before. If it decreased faster, this time might become nearer and nearer, until at some time after 9, it was decreasing as if it had been as bright as Venus at 9 o'clock. At a little after 10, it might be at ten o'clock, at a little after 11, at 11 o'clock, and at a little after 12, at 12 o'clock. Thus the order would be reversed. But there is no imaginable way in which the time could gradually change from 12 o'clock to 10 o'clock without passing through either 11 o'clock or 9 o'clock. This shows that the instants of time form a circuit.

Peirce: CP 7.288 Cross-Ref:††

288. **Definition.** A *Quality* is that character of a character which consists in its belonging to a particular line of variation of states.

Peirce: CP 7.288 Cross-Ref:††

Illustrations. Thus, temperature, probability, wealth, the happening earlier or later than something else are qualities.

Peirce: CP 7.289 Cross-Ref:††

289. **Scholium.** The order of states in a line of variation may be shown by attaching to sensibly different states different numbers. For if the line of variation forms a circuit, its states are related to one another like the real numbers, rational and irrational, positive and negative, including ∞ , except that the states may perhaps be so multitudinous that it is impossible to assign distinct numbers to them all. Whether any example can be given of a quality in which there are sensible variations too multitudinous for numbers to discriminate or not, there can be no doubt that such a quality might exist.†15

Peirce: CP 7.290 Cross-Ref:††

290. In mathematics, we have to deal with an ideal condition of things. We imagine ourselves to be in possession of a general method of working by which definite states can be assigned, in the first place, to all rational numbers, in their order. That is, the states in their own nature shall have the same order of succession as the

values of the rational numbers. We then suppose states for the irrational numbers to be interposed in their orders. States which the particular method of assignment of numbers may leave unnumbered, between the numbered states, are for this method not sensibly different from the irrational numbers that are near them. A new distribution of numbers by a different method might possibly distinguish some of these, and in doing so it might, or might not, leave others undistinguished.

Peirce: CP 7.291 Cross-Ref:††

291. The numbers may occur in every assignable part of the circuit, or may be contained between two limits, or a part of the series of numbers may cover the whole circuit. In the last case, we suppose the remaining numbers to be assigned to the circuit taken over and over again in regular arithmetical progression. In the second case, we are at liberty to fill up the vacant part of the circuit with a second series of numbers which will be distinguished by having a quantity not a number added to it. But in doing this, we shall assume that the numbers are so assigned that taking any three states A, B, C, a state, D, can be found whose number diminished by that of C equals the number of B diminished by that of A.

Peirce: CP 7.292 Cross-Ref:††

292. **Definition.** A *method of measurement* upon a circuit of variation is a general rule according to which, it is possible to assign each rational number to an exact state and to but one; and conversely, given the numbers assigned to any three states, it is possible to ascertain whether any fourth rational number is exactly assignable to a given state or not, and if not in what one of the four intervals between the four numbers, the number that ought to be assigned to the given state falls. [*Corollary.* This affords the means of vaguely assigning states to the irrational numbers.]†16 Numbers so assigned to states may be called *state-numbers*. States to which definitely different places in the scale of numbers cannot be assigned are said not to be *measurably* distinct, according to the particular system of measurement employed. If the whole circuit of numbers does not precisely correspond to the whole circuit of states, it is assumed that either, on the one hand, the same state receives different numbers or, on the other hand, that different states receive the same numbers distinguished by the addition of a non-numerical quantity. But in all such cases the numbers are to be so distributed that if any three states have *a, b, c* for state-numbers not infinite (in the same system of measurement) then there shall be one state and but one which has for a state-number, $a + b - c$.

Peirce: CP 7.293 Cross-Ref:††

293. **Scholium.** Were we to say, at once, that there is a state for every syzygy of two state-numbers, we should no longer have a line of variation, in case a non-numerical increment is required for part of the circuit.

Peirce: CP 7.294 Cross-Ref:††

294. **Scholium.** Suppose we have two objects both capable of taking states of the same quality and of changing those states, but only in the inherent order of the circuit of variation. Suppose further that the whole pair has but a single degree of freedom of changes, so that for each state of the one there is one and but one state of the other. Finally, suppose that one member of the pair can always be changed to the state which before the change was that of the other. If, then, we affix numbers to the states in such a way that every change in the state-number of one of the pair shall be equal to that of the other, these numbers will fulfil the necessary conditions. But if there is a region over which the pair cannot be moved, then it will be necessary to

have as many such changeable pairs as there are regions. Each pair may conveniently consist of one object in one region and one in another.

Peirce: CP 7.295 Cross-Ref:††

295. Another method of measurement would be obtained if we had a multitude of objects so that there was just one for each state of the quality in the whole circuit, and if the whole multitude had but a single degree of freedom of change and two positions of the whole multitude were given.

Peirce: CP 7.296 Cross-Ref:††

296. Of course, it would be still more convenient if we had given all states of such a multitude of objects. But in that case, they must conform [to] the condition that, let A, B, C, D, be what different states they may, if a change of one member of the multitude from A to B is synchronous with the change of another member from C to D, then every change of any member from A to B is simultaneous with the change of the member in state C to D.

Peirce: CP 7.297 Cross-Ref:††

297. **Definition.** A *standard of measurement* upon a circuit of variation is a pair of objects subject to the quality of the circuit, such that either can be changed to the state of the other in respect to that quality, but all its changes must follow the inherent order of the states, and the whole pair has but one degree of freedom of position, so that if one of the pair is in any given state there is but one state in which the other can be; and the method of measurement is such that station-numbers of the states of the two objects change by equal differences simultaneously.

Peirce: CP 7.298 Cross-Ref:††

298. **Definition.** A *metron* is an object composed of parts in one to one correspondence with all the states of a circuit of variation, such that any two form a standard of measurement, and the whole has *steady displacements*, that is, if the change of one part from state A to state B is synchronous with the change of the part at first in state C to state D, then the change of any other part from state A to state B is synchronous with the change of the part at first in state C to state D.

Peirce: CP 7.299 Cross-Ref:††

299. **Definition.** The *quantity*, or *modular quantity*, of an interval between two states in a line of variation is that which there is in the inherent difference between two states which justifies their receiving state-numbers whose difference has this or that amount. Otherwise, it is that whose *measure* is the difference of their state-numbers multiplied by a unit expressive of the unit of difference.

Peirce: CP 7.300 Cross-Ref:††

300. **Division.** Measurement is either

Peirce: CP 7.300 Cross-Ref:††

A. **Parabolic**, when the circuit of real numbers corresponds to the circuit of variation of states, or

Peirce: CP 7.300 Cross-Ref:††

B. **Non-parabolic**, when the two circuits are not coincident. **Non-parabolic** measurement is of two kinds, viz.: --

a. **Elliptic**, when the entire circuit of states coincides with a finite part of the circuit of numbers.

b. *Hyperbolic*, when the entire line of finite numbers occupies but a portion of the circuit of variation, and leaves a portion vacant.

Peirce: CP 7.301 Cross-Ref:††

301. *Definition.* An *absolute*, or *firmamental*, state is a state to which or from which no member of the metron can change.

Peirce: CP 7.302 Cross-Ref:††

302. *Theorem.* *In parabolic measurement upon one circuit there is just one firmamental state.*

Peirce: CP 7.302 Cross-Ref:††

Demonstration. In every measurable change of the metron, the station-numbers of all members are changed by the same finite increment. Now, a finite number added to any number gives a different number with one sole exception. Namely, if the number increased is infinite, the sum has the same value. But only one state has a given state-number. Therefore, the state of that member of the metron which has ∞ for its state-number is never changed. Hence, since every change can be reversed, no member is by any measurable change carried to the state whose state-number is infinite.

Peirce: CP 7.303 Cross-Ref:††

303. *Theorem.* *In elliptical motion there is no firmamental state, and the modular quantity of the whole circuit is finite, and the measurement proceeds in one annular order.*

Peirce: CP 7.303 Cross-Ref:††

Demonstration. If the entire circuit of variation is covered by a finite part of the scale of numbers, let the metron receive any measurable change. Then, the state-numbers of all its members will receive the same finite increment. Consequently, those members whose state-numbers were sufficiently great will be made greater than the finite difference of numbers of the whole circuit, and will be made greater than any of the numbers were at first. Since no state-number is infinite no member will have its state unchanged. And every member will have an arithmetical progression of state-numbers, the difference of all these progressions being the same. That difference will be the modular quantity of the circuit.

Peirce: CP 7.304 Cross-Ref:††

304. *Theorem.* *In hyperbolic motion there are just two firmamental states, and in both the regions into which they sever the circuit the state-numbers increase toward one of these and away from the other; and the quantity of the whole circuit is not infinite. . . .†17*

Peirce: CP 7.305 Cross-Ref:††

305. *Analysis.* Our measures of space are of various descriptions. Some of them, as square measure, cubic measure, etc. are composite. Thus, square measure depends upon long measures of length and breadth. Other kinds of measure are derivative. Thus, the distance of a point from a surface is the distance of the point from the nearest point of the surface. The measures which are neither composite nor derivative are two: long measure between pairs of points and angular measure between pairs of planes.

Peirce: CP 7.306 Cross-Ref:††

306. There are many different sorts of standards which might be assumed for long measure. From the point of view of theoretical physics the average distance that a molecule of hydrogen at a standard temperature would move in a day would have something to recommend it. But the primitive and most usual sort of standard, as well as the most convenient for the purposes of geometry, is a rigid body which is carried about from place to place, such as a yard-stick.

Peirce: CP 7.307 Cross-Ref:††

307. Let us ask what we mean by a rigid body. If we say that it is a body whose measures remain fixed, somebody may reply that if it is taken as the standard of measure it means nothing to say that its measure relatively to itself remains the measure of itself. Some care is, therefore, required in saying what we mean by a rigid body. If a rigid body were a single body there would be less meaning in the word rigid than there is when it is a kind of body which the working of Nature makes to be a usual kind, or a kind toward the ideal properties of which many objects closely approximate. Our general experience leads us to think that solid bodies come very near to being bodies which at fixed temperature and free from external influences have certain properties which we proceed to consider. First, every particle of such a body, or part of it occupying an indivisible place, always occupies a single indivisible place. It never goes out of existence or out of space; nor does it separate so as to occupy a number of points, nor does it enlarge so as to occupy a line or a surface or a solid space. Second, every flat film of such a body, or part of it occupying a plane, always continues (there being no stress upon the body) to occupy a plane. It neither breaks nor bends. Third, in any straight fibre of such a body, or part of it occupying a ray, (and which must always continue to occupy a ray, since every flat film in which it lies continues to lie in a plane) the order and continuity of the particles remain always the same, and round such straight fibre the order and continuity of the flat films through it remain always the same. These three properties, taken together, may be termed the optical property of the rigid body.

Peirce: CP 7.308 Cross-Ref:††

308. In the next place, our general experience leads us to believe that a perfectly rigid body, which ordinary solid bodies sensibly resemble, is such that if a given particle of it be brought to a point [M] and the straight filament through that particle and a second particle be brought into a given ray through [M], then there are just two points to which that second particle can be brought. In like manner, if of three particles two are brought to two points [M] and [N], there are, in each plane through {MN} just two points to which the third particle might be brought. But there is a difference between the two cases. Namely, as long as the straight filament remains in the ray, if at one time one particle is at the point [M] and another at the point [N], then, no matter how the filament is moved in the ray, every time the first particle is brought back to [M] the second particle can be brought to no other point than [N]. But in the plane, though the flat film remain in all its movements in that plane, yet if the first particle is at first at [M] the second at [N] and the third at [P], if the first two particles are carried along {MN} until they have performed an entire circuit, while the third always remains in the plane, it will not return to the same point [P] (supposing it has ever left that point) but will return to a fourth point [Q] and it will not be until a second circuit has been performed that it returns to [P]. We shall demonstrate that this must be so. It is mentioned now for a special purpose. We shall find the difference is owing to the ray having an odd number, and the plane an even number of dimensions. Hence, in three dimensions if of four particles of a rigid body three are restored to their initial positions, there is only one point to which the fourth

can be brought, although there is another point which it could perfectly well occupy (as shown by its looking glass image) if there were only any fourth dimension through which it could be carried to that point. On this account, it is convenient in geometry to imagine ourselves to be in possession of a rigid body to be used as a standard of measurement which shall have the purely imaginary property that, while any three of its particles remain fixed, it shall be capable of being *perverted*, that is, of being suddenly converted into its looking glass image. On account of this property, we will denominate the thing, not a rigid body, but a rigid image.

Peirce: CP 7.309 Cross-Ref:††

309. Finally, we are led to believe that if any number of parts of a rigid body occupy at one time certain positions and at another time certain other positions, then any other parts which may at any time occupy the first set of positions may be carried into the second set of positions.

Peirce: CP 7.310 Cross-Ref:††

310. There is another property of rigid bodies which makes all long measure to be parabolical and all angular measure to be elliptical; but it is best, at first, to consider the consequences of the other properties, and to reserve the consideration of this until later.

Peirce: CP 7.311 Cross-Ref:††

311. **Definition.** A *metrical image* is a continuous multitude of particles, straight filaments, and flat films having the following properties:

Peirce: CP 7.311 Cross-Ref:††

1st, it has the optical property that whatever part occupies at one instant a point or a plane occupies the same kind of place (whichever it may be) at every instant, and with the same connective relations to straight filaments.

Peirce: CP 7.311 Cross-Ref:††

2nd, the fixation of a particle of such image at a point or of a flat film in a plane diminishes by unity the freedom of motion of all other such parts of the same metrical image.

Peirce: CP 7.311 Cross-Ref:††

3rd, when the particles and flat films of [a] metrical image are subject to such conditions as just suffice to reduce the freedom of motion to zero, there are just two positions, said to be *perverse* of one another, in which it can fulfill those conditions.

Peirce: CP 7.311 Cross-Ref:††

4th, if one metrical image or part of such image can occupy one place at one instant and another at another instant, then every such image or part of an image which can occupy the former place at one instant can occupy the latter place at another instant.

Peirce: CP 7.312 Cross-Ref:††

312. **Definition.** Two places (whether the same or different) are said to be equal which can be occupied by the same metrical image or by the same part of one metrical image. And the order of the occupied parts makes no difference.†18

THE LOGIC OF 1873†1

§1. INVESTIGATION †2

313. The very first of distinctions which logic supposes is between doubt and belief, a question and a proposition. Doubt and belief are two states of mind which feel different, so that we can distinguish them by immediate sensation. We almost always know without any experiment when we are in doubt and when we are convinced. This is such a difference as there is between red and blue, or pleasure and pain. Were this the whole distinction, it would be almost without significance. But in point of fact the mere sensible distinguishability is attended with an important practical difference.†3 When we believe, there is a proposition which according to some rule determines our actions, so that our belief being known, the way in which we shall behave may be surely deduced, but in the case of doubt we have such a proposition more or less distinctly in our minds but do not act from it. There is something further removed from belief than doubt, that is to say not to conceive the proposition at all. Nor is doubt wholly without effect upon our conduct. It makes us waver. Conviction determines us to act in a particular way while pure unconscious ignorance alone which is the true contrary of belief has no effect at all.

Peirce: CP 7.314 Cross-Ref:††

314. Belief and doubt may be conceived to be distinguished only in degree.†4

Peirce: CP 7.315 Cross-Ref:††

315. Living doubt is the life of investigation. When doubt is set at rest inquiry must stop. . . .†5

Peirce: CP 7.316 Cross-Ref:††

316. From this conception springs the desire to get a settlement of opinion [that] is some conclusion which shall be independent of all individual limitations, independent of caprice, of tyranny, of accidents of situation . . . , -- a conclusion to which every man would come who should pursue the same method and push it far enough. The effort to produce such a settlement of opinion is called *investigation*. Logic is the science which teaches whether such efforts are rightly directed or not.

Peirce: CP 7.317 Cross-Ref:††

317. There is an important difference between the settlement of opinion which results from investigation and every other such settlement. It is that investigation will not fix one answer to a question as well as another, but on the contrary it tends to unsettle opinions at first, to change them and to confirm a certain opinion which depends only on the nature of investigation itself. The method of producing fixity of belief by adhering obstinately to one's belief, tends only to fix such opinions as each man already holds. The method of persecution tends only to spread the opinions which happen to be approved by rulers; and except so far as rulers are likely to adopt views of a certain cast does not determine at all what opinions shall become settled.

The method of public opinion tends to develop a particular body of doctrine in every community. Some more widely spread and deeply rooted conviction will gradually drive out the opposing opinions, becoming itself in the strife somewhat modified by these. But different communities, removed from mutual influence, will develop very different bodies of doctrine, and in the same community there will be a constant tendency to sporting which may at any time carry the whole public. What we know of growth, in general, shows that this will take place; and history confirms us. The early history of sciences before they begin to be really investigated, especially of psychology, metaphysics, etc., illustrates as well as anything the pure effect of this method of fixing opinions. The numerous well-defined species of doctrines which have existed on such subjects and their progressive historical succession give the science of the history of philosophy considerable resemblance to that of paleontology.

Peirce: CP 7.318 Cross-Ref:††

318. Thus no one of these methods can as a matter of fact attain its end of settling opinions. Men's opinions will act upon one another and the method of obstinacy will infallibly be succeeded by the method of persecution and this will yield in time to the method of public opinion and this produces no stable result.

Peirce: CP 7.319 Cross-Ref:††

319. Investigation differs entirely from these methods in that the nature of the final conclusion to which it leads is in every case destined from the beginning, without reference to the initial state of belief. Let any two minds investigate any question independently and if they carry the process far enough they will come to an agreement which no further investigation will disturb.

Peirce: CP 7.320 Cross-Ref:††

320. But this will not be true for any process which anybody may choose to call investigation, but only for investigation which is made in accordance with appropriate rules. Here, therefore, we find there is a distinction between good and bad investigation. This distinction is the subject of study in logic. Some persons will doubt whether any sort of investigation will settle all questions. I refrain, however, from arguing the matter, because I should thus be led to anticipate what comes later, and because after any demonstration I might give I should still rest on *some* assumption and it is as easy to see that investigation assumes its own success as that it assumes anything else.

Peirce: CP 7.321 Cross-Ref:††

321. Logic is the doctrine of truth, its nature and the manner in which it is to be discovered.

Peirce: CP 7.322 Cross-Ref:††

322. The first condition of learning is to know that we are ignorant. A man begins to inquire and to reason with himself as soon as he really questions anything and when he is convinced he reasons no more. Elementary geometry produces formal proofs of propositions which nobody doubts, but that cannot properly be called reasoning which does not carry us from the known to the unknown, and the only value in the first demonstrations of geometry is that they exhibit the dependence of certain theorems on certain axioms, a thing which is not clear without the demonstrations. When two men discuss a question, each first endeavors to raise a doubt in the mind of the other, and that is often half the battle. When the doubt ceases there is no use in further discussion. Thus real inquiry begins when genuine doubt begins and ends when this doubt ends. And the premises of the reasoning are facts not

doubted. It is therefore idle to tell a man to begin by doubting familiar beliefs, unless you say something which shall cause him really to doubt them. Again, it is false to say that reasoning must rest either on first principles or on ultimate facts. For we cannot go behind what we are unable to doubt, but it would be unphilosophical to suppose that any particular fact will never be brought into doubt.

Peirce: CP 7.323 Cross-Ref:††

323. It is easy to see what truth would be for a mind which could not doubt. That mind could not regard anything as possible except what it believed in. By all existing things it would mean only what it thought existed, and everything else would be what it would mean by *nonexistent*. It would, therefore, be omniscient in its universe. To say that an omniscient being is necessarily destitute of the faculty of reason, sounds paradoxical; yet if the act of reasoning must be directed to an end, when that end is attained the act naturally becomes impossible.

Peirce: CP 7.324 Cross-Ref:††

324. The only justification for reasoning is that it settles doubts, and when doubt finally ceases, no matter how, the end of reasoning is attained. Let a man resolve never to change his existing opinions, let him obstinately shut his eyes to all evidence against them, and if his will is strong enough so that he actually does not waver in his faith, he has no motive for reasoning at all, and it would be absurd for him to do it. That is method number one for attaining the end of reasoning, and it is a method which has been much practised and highly approved, especially by people whose experience has been that reasoning only leads from doubt to doubt. There is no valid objection to this procedure if it only succeeds. It is true, it is utterly irrational; that is to say it is foolish from the point of view of those who do reason. But to assume that point of view is to beg the question. In fact, however, it does not succeed; and the first cause of failure is that different people have different opinions and the man who sees this begins to feel uncertain. It is therefore desirable to produce unanimity of opinion and this gives rise to method number two, which is to force people by fire and sword to adopt one belief, to massacre all who dissent from it and burn their books. This way of bringing about a catholic consent has proved highly successful for centuries in some cases, but it is not practicable in our days. A modification of this is method number three, to cultivate a public opinion by oratory and preaching and by fostering certain sentiments and passions in the minds of the young. This method is the most generally successful in our day. The fourth and last method is that of reasoning. It will never be adopted when any of the others will succeed and it has itself been successful only in certain spheres of thought. Nevertheless those who reason think that it must be successful in the end, and so it would if all men could reason. There is this to be said in favor of it. He who reasons will regard the opinions of the majority of mankind with contemptuous indifference; they will not in the least disturb his opinions. He will also neglect the beliefs of those who are not informed, and among the small residue he may fairly expect some unanimity on many questions.

Peirce: CP 7.325 Cross-Ref:††

325. I hope it will now be plain to the reader, that the only rational ground for preferring the method of reasoning to the other methods is that it fixes belief more surely. A man who proposes to adopt the first method may consistently do so simply because he chooses to do so. But if we are to decide in favor of reasoning, we ought to do so on rational grounds. Now if belief is fixed, no matter how, doubt has as a matter of fact ceased, and there is no motive, rational or other, for reasoning any

more. Any settlement of opinion, therefore, if it is full and perfect, is entirely satisfactory and nothing could be better. It is the peculiarity of the method of reasoning, that if a man thinks that it will not burn him to put his hand in the fire, reasoning will not confirm that belief but will change it. This is a vast advantage to the mind of a rationalist. But the advocate of any one of the first three methods, will be able to say (if either of these methods will yield a fixed belief) either that he *knows* by his method that fire will burn, so that reasoning is inferior to his method in that it may permit a man for a moment to doubt this, or else that he *knows* that fire will not burn, so that reasoning leads all astray. In either case therefore he will conceive that which to the rationalist seems the great advantage of reasoning, to be a great fault. Thus the only ground of a fair decision between the methods must be that one actually succeeds while the others break up and dissolve. Peirce expresses the philosophy of the matter perfectly:

Truth struck to earth shall rise again
The eternal years of God are hers
While error . . . writhes in pain
And dies amidst her worshippers.

Peirce: CP 7.326 Cross-Ref:††
§2. LOGIC †6

326. It is the business of the logician to study the nature of the fourth method of inquiry and to discover the rules for conducting it with success. The whole subject will in the exposition of it here offered to the reader be divided into three parts. The first shall treat of the essence of investigation in general, by whatever mind it is conducted and to whatever subject it is applied. The second shall treat of those maxims of investigation which become necessary owing to the peculiar constitution of man in his senses, and his mental nature. The third shall give some slight outline of the special methods of research which are applicable in the different branches of science, and which arise from the peculiarities of the matter investigated. In this first part then we have, broadly speaking, nothing to do with the nature of the human mind. Only as there are some faculties which must belong to any mind which can investigate at all, these must come under our consideration. All inquiry, for example, presupposes a passage from a state of doubt to a state of belief; and therefore there must be a succession of time in the thoughts of any mind which is able to inquire. In the fourth method of inquiry a certain predetermined though not pre-known belief is sure to result from the process; no matter what may have been the opinion of the inquirer at the outset. It follows that during the investigation elements of thought must have sprung up in the mind which were not caused by any thought which was present at the time the investigation was commenced. Such new ideas springing up in the mind and not produced by anything in the mind, are called sensations. Every mind capable of investigation must therefore have a capacity for sensations. But were all thoughts of this kind investigation would be almost an involuntary process. We might will to investigate but we could not change the course which investigation should

take. There would therefore be no distinction between a right and a wrong method of investigation. Now we have seen in the last chapter, that such a distinction is essential to the fourth method of inquiry and is, in fact, the only thing which distinguishes it from the third. There must be thoughts therefore which are determined by previous thoughts. And such a faculty of producing thoughts from others must belong to every mind which can investigate. Without a succession of ideas in time it is clear that no reasoning is possible. I shall proceed to show that without it and without the determination of one idea by another no *thought* in any proper sense of the word is possible.

Peirce: CP 7.327 Cross-Ref:††
§3. OBSERVATION AND REASONING †7

327. Because the only purpose of inquiry is the settlement of opinion, we have seen that everyone who investigates, that is, pursues an inquiry by the fourth method assumes that that process will, if carried far enough, lead him to a certain conclusion, he knows not what beforehand, but which no further investigation will change. No matter what his opinion at the outset may be, it is assumed that he will end in one predestinated belief. Hence it appears that in the process of investigation wholly new ideas and elements of belief must spring up in the mind which were not there before.

Peirce: CP 7.328 Cross-Ref:††

328. Some thoughts are produced by previous thoughts according to regular laws of association,†8 so that if the previous thoughts be known, and the rule of association be given, the thought which is so produced may be predicted. This is the elaborative operation of thought, or thinking *par excellence*. But when an idea comes up in the mind which has no such relation to former ideas, but is something new to us, we say that it is caused by something out of the mind, and we call the process by which such thoughts spring up, sensation. And those parts of investigation which consist chiefly in supplying such materials for thought to work over, combine and analyze, are termed observations. The first thing to be noted then is that since investigation leads us from whatever state of opinion we may happen to have to an opinion which is predetermined, it must be that investigation involves observation as one part of it, and, in fact, the conclusion to which we finally come ultimately depends entirely upon the observations.

Peirce: CP 7.329 Cross-Ref:††

329. We may pause here to make a practical application of this principle. No argument can possibly be a correct one which pretends to disclose to us a fact wholly new without being based on evidence which is new. The metaphysicians are given to this kind of reasoning; even those of them who are the most energetic in maintaining that all our knowledge comes from sense. Writers upon the nature of the human mind, especially, have built up a great body of doctrine without the aid of any observations or facts, except such as are familiar to all the world. Such things justly excite our suspicion. When Hobbes, for example, would persuade us that no man can act otherwise than for the sake of pleasure, it is clear that this belief would deeply modify our conceptions of men, and our plans of life; but when on asking what supports this momentous conclusion we learn that it is but the simple fact -- if it can be dignified by that name -- that every man desires to do what he does do, we are led at once to

suspect that there is some sophistry in the process by which so novel a conclusion can be drawn from so familiar a premise. So, when modern necessitarians maintain that every act of the will proceeds from the strongest motive, they lay down a principle which should be expected to give rise to a psychological science as exact as mechanics, and capable of reducing human actions to precise calculation. But when we find that the advocates of this principle have made no experiments to test their law, we are strongly inclined to think that there has been some juggle of reasoning which has enabled them thus to create something out of nothing.

Peirce: CP 7.330 Cross-Ref:††

330. An observation, as we have defined it, is merely an idea arising in the mind, and not produced by previous ideas. This is not the complete description of observation as understood by scientific men, and we must be careful that the word does not lead us to conclusions which we are not yet warranted in drawing. For example a dream, a presentiment or some fancied inspiration from on high, might, as far as we have yet seen, involve entirely new elements of thought, and, therefore, be an observation in the sense of our definition, so that we are not yet warranted in saying that such things cannot be the ground of legitimate reasoning. This is a question which we shall have again to examine when we come to consider those maxims of inference which depend upon the peculiar constitution of man.

Peirce: CP 7.331 Cross-Ref:††

331. But Observation alone cannot constitute investigation; for if it did the only active part which we should have to play in this method of inquiry would be simply the willing to observe, and there would be no distinction of a wrong method and a right method of investigation. But we have seen that such a distinction is essential to the idea of investigation, and that it is, in fact the only thing which separates this from the third method of inquiry. Accordingly, besides observation it must be that there is also an elaborative process of thought by which the ideas given by observation produce others in the mind.†9 Besides, the observations are most varied and are never exactly repeated or reproduced so that they cannot constitute that settled opinion to which investigation leads. Two men, for example, agree in an opinion, and if you ask upon what their opinions rest they will perhaps allege the same fact. But trace the matter back further; ask them upon what grounds they believe that fact again and you will eventually come to premises that are different. Two minds, for example, may have formed the same judgment of a certain person's character and yet may have based their opinions on observing his behavior on different occasions. The rotation of the earth was at first inferred from the movement of the heavenly bodies; but afterwards the manner in which a long pendulum when allowed to swing would gradually turn around and change its direction of oscillation, afforded an entirely new proof; and there are certain very small movements of the stars, which, if they were capable of sufficiently exact observation, would show another ground for the same conclusion. Indeed, the fact which one man observes, is in no case precisely the same as the fact which another man observes. One astronomer observes that the moon passes over a star so as to hide it at a certain instant at his observatory, another astronomer observes that the same star is occulted at a certain instant at his observatory. These two facts are not the same, because they relate to different stations of observation. What is so plain in regard to astronomical observation, because we are accustomed to precision of thought about this, is equally true in regard to the most familiar facts. You and I both see an ink-stand on the table; but what you observe, is that there is a certain appearance from where you sit, and what I observe, is that there is a certain appearance from where I sit. The fact in

which we agree, that there is an ink-stand there, is what we conclude from the different appearances which we each severally observe. We may change places and still we shall fail to get each other's observations; for the difference of time then comes in. I may observe that there is such an appearance now as you describe as having existed a few moments before; but I cannot observe that there was such an appearance before I took your place. It is needless to multiply these examples, because the slightest reflection will supply them in any number; but what have been adduced are sufficient to show that observations are for every man wholly private and peculiar. And not only can no man make another man's observations, or reproduce them; but he cannot even make at one time those observations which he himself made at another time. They belong to the particular situation of the observer, and the particular instant of time.

Peirce: CP 7.332 Cross-Ref:††

332. Indeed, if we carefully distinguish that which is first given by sensation, from the conclusion which we immediately draw from it, it is not difficult to see that different observations are not in themselves even so much as alike; for what does the resemblance between the two observations consist in? What does it mean to say that two thoughts are alike? It can only mean that any mind that should compare them together, would pronounce them to be alike. But that comparison would be an act of thought not included in the two observations severally; for the two observations existing at different times, perhaps in different minds, cannot be brought together to be compared directly in themselves, but only by the aid of the memory, or some other process which makes a thought out of previous thoughts, and which is, therefore, not observation. Since, therefore, the likeness of these thoughts consists entirely in the result of comparison, and comparison is not observation, it follows that observations are not alike except so far as there is a possibility of some mental process besides observation.

Peirce: CP 7.333 Cross-Ref:††

333. Without however insisting upon this point which may be found too subtle, the fact remains that the observations are not the same in the sense in which the conclusions to which they give rise are the same. All astronomers, for example, will agree that the earth is ninety-two or ninety-three millions of miles from the sun. And yet, one would base his conclusion on observations of the passage of Venus across the sun's disk; another upon observations of the planet Mars; another upon experiments upon light combined with observations of the satellites of Jupiter. And the same thing is equally true in regard to most of the ordinary affairs of life.

Peirce: CP 7.334 Cross-Ref:††

334. Now how is it that the springing up into the mind of thoughts so dissimilar should lead us inevitably though sometimes not until after a long time to one fixed conclusion? Disputes undoubtedly occur among those who pursue a proper method of investigation. But these disputes come to an end. At least that is the assumption upon which we go in entering into the discussion at all, for unless investigation is to lead to settled opinion it is of no service to us whatever. We do believe then in regard to every question which we try to investigate that the observations though they may be as varied and as unlike in themselves as possible, yet have some power of bringing about in our minds a predetermined state of belief. This reminds us of the species of necessity which is known as fate. The fairy stories are full of such examples as this: A king shuts his daughter up in a tower because he has been warned that she is destined to suffer some misfortune from falling in love

before a certain age and it turns out that the very means which he has employed to prevent it is just what brings the prophecy to fulfillment. Had he pursued a different course, the idea seems to be that that would equally have brought about the destined result. Fate then is that necessity by which a certain result will surely be brought to pass according to the natural course of events however we may vary the particular circumstances which precede the event. In the same manner we seem fated to come to the final conclusion. For whatever be the circumstances under which the observations are made and by which they are modified they will inevitably carry us at last to this belief.

Peirce: CP 7.335 Cross-Ref:††

335. The strangeness of this fact disappears entirely when we adopt the conception of external realities. We say that the observations are the result of the action upon the mind of outward things, and that their diversity is due to the diversity of our relations to these things; while the identity of the conclusion to which the mind is led by them is owing to the identity of the things observed, the reasoning process serving to separate from the many different observations that we make of the same thing the constant element which depends upon the thing itself from the differing and variable elements which depend on our varying relations to the thing. This hypothesis I say removes the strangeness of the fact that observations however different yield one identical result. It removes the strangeness of this fact by putting it in a form and under an aspect in which it resembles other facts with which we are familiar. We are accustomed very rightly to think that causes always precede their effects and to disbelieve in fate, which is a fancied necessity by which some future event as it were forces the conditions which precede to be such as would bring it about. That there is no such intrinsic and unconditional necessity to bring about events Western nations are fully and rightly convinced. This is why it seems strange to assert that the final conclusion of the investigation is predestined and why it is satisfactory to the mind to find a hypothesis which shall assign a cause preceding the final belief which would account for the production of it, and of the truth of this conception of external realities there can be no doubt. Even the idealists, if their doctrines are rightly understood have not usually denied the existence of real external things. But though the conception involves no error and is convenient for certain purposes, it does not follow that it affords the point of view from which it is proper to look at the matter in order to understand its true philosophy. It removes the strangeness of a certain fact by assimilating it to other familiar facts; but is not that fact that investigation leads to a definite conclusion really of so different a character from the ordinary events in the world to which we apply the conception of causation that such an assimilation and classification of it really puts it in a light which, though not absolutely false, fails nevertheless to bring into due prominence the real peculiarity of its nature? That observation and reasoning produce a settled belief which we call the truth seems a principle to be placed at the head of all special truths which are only the particular beliefs to which observation and reasoning in such cases lead. And it is hardly desirable to merge it among the rest by an analogy which serves no other purpose.

Peirce: CP 7.336 Cross-Ref:††

§4. REALITY †10

336. The question is, "Whether corresponding to our thoughts and sensations,

and represented in some sense by them, there are realities, which are not only independent of the thought of you, and me, and any number of men, but which are absolutely independent of thought altogether." The objective final opinion is independent of the thoughts of any particular men, but is not independent of thought *in general*.†11 That is to say, if there were no thought, there would be no opinion, and therefore, no final opinion.

Peirce: CP 7.337 Cross-Ref:††

337. All that we directly experience is our thought -- what passes through our minds; and that only, at the moment at which it is passing through. We here see thoughts determining and causing other thoughts, and a chain of reasoning or of association is produced. But the beginning and the end of this chain, are not distinctly perceived. A current is another image under which thought is often spoken of, and perhaps more suitably. We have particularly drawn attention to the point to which thought flows, and that it finally reaches: a certain level, as it were -- a certain basin, where reality becomes unchanging. It has reached its destination, and that permanency, that fixed reality, which every thought strives to represent and image, we have placed in this objective point, towards which the current of thought flows.

Peirce: CP 7.338 Cross-Ref:††

338. But the matter has often been regarded from an opposite point of view; attention being particularly drawn to the spring, and origin of thought. It is said that all other thoughts are ultimately derived from sensations; that all conclusions of reasoning are valid only so far as they are true to the sensations; that the real cause of sensation therefore, is the reality which thought presents. Now such a reality, which causes all thought, would seem to be wholly external to the mind -- at least to the thinking part of the mind, as distinguished from the feeling part; for it might be conceived to be, in some way, dependent upon sensation.

Peirce: CP 7.339 Cross-Ref:††

339. Here then are two opposite modes of conceiving reality. The one which has before been developed at some length, and which naturally results from the principles which have been set forth in the previous chapters of this book is an idea which was obscurely in the minds of the medieval realists; while the other was the motive principle of nominalism. I do not think that the two views are absolutely irreconcilable, although they are taken from very widely separated stand-points. The realistic view emphasizes particularly the permanence and fixity of reality; the nominalistic view emphasizes its externality. But the realists need not, and should not, deny that the reality exists externally to the mind; nor have they historically done so, as a general thing. That is external to the mind, which is what it is, whatever our thoughts may be on any subject; just as that is real which is what it is, whatever our thoughts may be concerning that particular thing. Thus an emotion of the mind is real, in the sense that it exists in the mind whether we are distinctly conscious of it or not. But it is not external because although it does not depend upon what we think about it, it does depend upon the state of our thoughts about something. Now the object of the final opinion which we have seen to be independent of what any particular person thinks, may very well be external to the mind. And there is no objection to saying that this external reality causes the sensation, and through the sensation has caused all that line of thought which has finally led to the belief.

Peirce: CP 7.340 Cross-Ref:††

340. At first sight it seems no doubt a paradoxical statement that, "The object of final belief which exists only in consequence of the belief, should itself produce the

belief"; but there have been a great many instances in which we have adopted a conception of existence similar to this. The object of the belief exists it is true, only because the belief exists; but this is not the same as to say that it begins to exist first when the belief begins to exist. We say that a diamond is hard. And in what does the hardness consist? It consists merely in the fact that nothing will scratch it; therefore its hardness is entirely constituted by the fact of something rubbing against it with force without scratching it. And were it impossible that anything should rub against it in this way, it would be quite without meaning, to say that it was hard, just as it is entirely without meaning to say that virtue or any other abstraction is hard.†12 But though the hardness is entirely constituted by the fact of another stone rubbing against the diamond yet we do not conceive of it as beginning to be hard when the other stone is rubbed against it; on the contrary, we say that it is really hard the whole time, and has been hard since it began to be a diamond. And yet there was no fact, no event, nothing whatever, which made it different from any other thing which is not so hard, until the other stone was rubbed against it.

Peirce: CP 7.341 Cross-Ref:††

341. So we say that the inkstand upon the table is heavy. And what do we mean by that? We only mean, that if its support be removed it will fall to the ground. This may perhaps never happen to it at all -- and yet we say that it is really heavy all the time; though there is no respect whatever, in which it is different from what it would be if it were not heavy, until that support is taken away from it. The same is true in regard to the existence of any other force. It exists only by virtue of a condition, that something will happen under certain circumstances; but we do not conceive it as first beginning to exist when these circumstances arise; on the contrary, it will exist though the circumstances should never happen to arise. And now, what is matter itself? The physicist is perfectly accustomed to conceive of it as merely the centre of the forces. It exists, therefore, only so far as these forces exist. Since, therefore, these forces exist only by virtue of the fact, that something will happen under certain circumstances, it follows that matter itself only exists in this way.

Peirce: CP 7.342 Cross-Ref:††

342. Nor is this conception one which is peculiar to the physicists and to our views of the external world. A man is said to know a foreign language. And what does that mean? Only that if the occasion arises, the words of that language will come into his mind; it does not mean that they are actually in his mind all the time. And yet we do not say that he only knows the language at the moment that the particular words occur to him that he is to say; for in that way he never could be certain of knowing the whole language if he only knew the particular word necessary at the time. So that his knowledge of the thing which exists all the time, exists only by virtue of the fact that when a certain occasion arises a certain idea will come into his mind.

Peirce: CP 7.343 Cross-Ref:††

343. A man is said to possess certain mental powers and susceptibilities, and we conceive of him as constantly endowed with these faculties; but they only consist in the fact that he will have certain ideas in his mind under certain circumstances; and not in the fact of his having certain ideas in his mind all the time. It is perfectly conceivable that the man should have faculties which are never called forth: in which case the existence of the faculties depends upon a condition which never occurs. But what is the mind itself but the focus of all the faculties? and what does the existence of the mind consist in but in these faculties? Does the mind cease to exist when it

sleeps? and is it a new man who wakes every morning?

Peirce: CP 7.344 Cross-Ref:††

344. It appears then that the existence of mind equally with that of matter according to these arguments which have led to this view which is held by all psychologists, as well as physicists, depends only upon certain hypothetical conditions which may first occur in the future, or which may not occur at all. There is nothing extraordinary therefore in saying that the existence of external realities depends upon the fact, that opinion will finally settle in the belief in them. And yet that these realities existed before the belief took rise, and were even the cause of that belief, just as the force of gravity is the cause of the falling of the inkstand -- although the force of gravity consists merely in the fact that the inkstand and other objects will fall.

Peirce: CP 7.345 Cross-Ref:††

345. But if it be asked us, whether some realities do not exist, which are entirely independent of thought; I would in turn ask, what is meant by such an expression and what can be meant by it. What idea can be attached to that of which there is no idea? For if there be an idea of such a reality, it is the object of that idea of which we are speaking, and which is not independent of thought. It is clear that it is quite beyond the power of the mind to have an idea of something entirely independent of thought -- it would have to extract itself from itself for that purpose; and since there is no such idea there is no meaning in the expression.†13 The experience of ignorance, or of error, which we have, and which we gain by means of correcting our errors, or enlarging our knowledge, does enable us to experience and conceive something which is independent of our own limited views; but as there can be no correction of the sum total of opinions, and no enlargement of the sum total of knowledge, we have no such means, and can have no such means of acquiring a conception of something independent of all opinion and thought.

Peirce: CP 7.346 Cross-Ref:††
§5. TIME AND THOUGHT †14

346. Any mind which has the power of investigation, and which therefore passes from doubt to belief, must have its ideas follow after one another in time. And if there is to be any distinction of a right and a wrong method of investigation, it must have some control over the process. So that there must be such a thing as the production of one idea from another which was previously in the mind. This is what takes place in reasoning, where the conclusion is brought into the mind by the premises.

Peirce: CP 7.347 Cross-Ref:††

347. We may imagine a mind which should reason and never know that it reasoned; never being aware that its conclusion was a conclusion, or was derived from anything which went before. For such a mind there might be a right and a wrong method of thinking; but it could not be aware that there was such a distinction, nor criticise in any degree its own operations. To be capable of logical criticism, the mind must be aware that one idea is determined by another.

Peirce: CP 7.348 Cross-Ref:††

348. Now when this happens, after the first idea comes the second. There is a process which can only take place in a space of time; but an idea is not present to the mind during a space of time -- at least not during a space of time in which this idea is replaced by another; for when the moment of its being present is passed, it is no longer in the mind at all. Therefore, the fact that one idea succeeds another is not a thing which in itself can be present to the mind, any more than the experiences of a whole day or of a year can be said to be present to the mind. It is something which can be lived through; but not be present in any one instant; and therefore, which can not be present to the mind at all; for nothing is present but the passing moment, and what it contains. The only way therefore in which we can be aware of a process of inference, or of any other process, is by its producing some idea in us. Not only therefore is it necessary that one idea should produce another; but it is also requisite that a mental process should produce an idea. These three things must be found in every logical mind: First, ideas; second, determinations of ideas by previous ideas; third, determinations of ideas by previous processes. And nothing will be found which does not come under one of these three heads.

Peirce: CP 7.349 Cross-Ref:††

349. The determination of one thing by another, implies that the former not only follows after the latter, but follows after it according to a general rule, in consequence of which, every such idea would be followed by such a second one. There can therefore be no determination of one idea by another except so far as ideas can be distributed into classes, or have some resemblances. But how can one idea resemble another? An idea can contain nothing but what is present to the mind in that idea. Two ideas exist at different times; consequently what is present to the mind in one is present only at that time, and is absent at the time when the other idea is present. Literally, therefore, one idea contains nothing of another idea; and in themselves they can have no resemblance. They certainly do not resemble one another except so far as the mind can detect a resemblance; for they exist only in the mind, and are nothing but what they are thought to be. Now when each is present to the mind the other is not in the mind at all. No reference to it is in the mind, and no idea of it is in the mind. Neither idea therefore when it is in the mind, is thought to resemble the other which is not present in the mind. And an idea can not be thought, except when it is present in the mind. And, therefore, one idea can not be thought to resemble another, strictly speaking.

Peirce: CP 7.350 Cross-Ref:††

350. In order to escape from this paradox, let us see how we have been led into it. Causation supposes a general rule, and therefore similarity. Now so long as we suppose that what is present to the mind at one time is absolutely distinct from what is present to the mind at another time, our ideas are absolutely individual, and without any similarity. It is necessary, therefore, that we should conceive a process as present to the mind. And this process consists of parts existing at different times and absolutely distinct. And during the time that one part is in the mind, the other is not in the mind. To unite them, we have to suppose that there is a consciousness running through the time. So that of the succession of ideas which occur in a second of time, there is but one consciousness, and of the succession of ideas which occurs in a minute of time there is another consciousness, and so on, perhaps indefinitely. So that there may be a consciousness of the events that happened in a whole day or a whole life time.

Peirce: CP 7.351 Cross-Ref:††

351. According to this, two parts of a process separated in time -- though they are absolutely separate, in so far as there is a consciousness of the one, from which the other is entirely excluded -- are yet so far not separate, that there is a more general consciousness of the two together. This conception of consciousness is something which takes up time. It seems forced upon us to escape the contradictions which we have just encountered. And if consciousness has a duration, then there is no such thing as an instantaneous consciousness; but all consciousness relates to a process. And no thought, however simple, is at any instant present to the mind in its entirety, but it is something which we live through or experience as we do the events of a day. And as the experiences of a day are made up of the experiences of shorter spaces of time so any thought whatever is made up of more special thoughts which in their turn are themselves made up by others and so on indefinitely.

Peirce: CP 7.352 Cross-Ref:††

352. It may indeed very likely be that there is some minimum space of time within which in some sense only an indivisible thought can exist and as we know nothing of such a fact at present we may content ourselves with the simpler conception of an indefinite continuity in consciousness. It will easily be seen that when this conception is once grasped the process of the determination of one idea by another becomes explicable. What is present to the mind during the whole of an interval of time is something generally consisting of what there was in common in what was present to the mind during the parts of that interval. And this may be the same with what is present to the mind during any interval of time; or if not the same, at least similar -- that is, the two may be such that they have much in common. These two thoughts which are similar may be followed by others that are similar and according to a general law by which every thought similar to either of these is followed by another similar to those by which they are followed. If a succession of thoughts have anything in common this may belong to every part of these thoughts however minute, and therefore it may be said to be present at every instant. This element of consciousness which belongs to a whole only so far as it belongs to its parts is termed the matter of thought.

Peirce: CP 7.353 Cross-Ref:††

353. There is besides this a causation running through our consciousness by which the thought of any one moment determines the thought of the next moment no matter how minute these moments may be. And this causation is necessarily of the nature of a reproduction; because if a thought of a certain kind continues for a certain length of time as it must do to come into consciousness the immediate effect produced by this causality must also be present during the whole time, so that it is a part of that thought. Therefore when this thought ceases, that which continues after it by virtue of this action is a part of the thought itself. In addition to this there must be an effect produced by the following of one idea after a different idea; otherwise there would be no process of inference except that of the reproduction of the premises.

Peirce: CP 7.354 Cross-Ref:††
§6. BELIEF †16

354. We have seen that an inference is the process by which one belief determines another. But a belief is itself a habit of the mind by virtue of which one

idea gives rise to another. When I say that I know the French language, I do not mean that as long as I know it I have all the words which compose it in my mind, or a single one of them. But only that when I think of an object, the French word for it will occur to me, and that when a French word is brought to my attention I shall think of the object it signifies. What is true of knowledge is equally true of belief, since the truth or falsehood of the cognition does not alter its character in this respect. I believe that prussic acid is poison, and always have believed it. This does not mean that I have always had the idea of prussic acid in my mind, but only that on the proper occasion, on thinking of drinking it, for example, the idea of poison and all the other ideas that that idea would bring up, would arise in my mind.

Peirce: CP 7.355 Cross-Ref:††

355. Thus there are three elements of cognition: thoughts, the habitual connection between thoughts, and processes establishing a habitual connection between thoughts. We have seen already that an idea cannot be instantaneously present, that consciousness occupies time, and that we have no consciousness in an instant. So that at no time have we a thought. But now it further appears that in reference to a belief not only can we not have it in an instant, but it can not be present to the mind in any period of time. It does not consist in anything which is present to the mind, but in an habitual connection among the things which are successively present. That is to say, it consists in ideas succeeding one another according to a general rule; but not in the mere thinking of this general rule, nor in the mere succession of ideas one upon another, nor in both together. A thought must therefore be a sign of a belief; but is never the belief itself. The same thing is obviously true in regard to an inference; and even a simple idea is of intellectual value to us not for what it is in itself but as standing for some object to which it relates. Now a thing which stands for another thing is a representation or sign. So that it appears that every species of actual cognition is of the nature of a sign. It will be found highly advantageous to consider the subject from this point of view, because many general properties of signs can be discovered by a set of words and the like which are free from the intricacies which perplex us in the direct study of thought.

Peirce: CP 7.356 Cross-Ref:††

356. Let us examine some of the characters of signs in general. A sign must in the first place have some qualities in itself which serve to distinguish it, a word must have a peculiar sound different from the sound of another word; but it makes no difference what the sound is, so long as it is something distinguishable. In the next place, a sign must have a real physical connection with the thing it signifies so as to be affected by that thing. A weather-cock, which is a sign of the direction of the wind, must really turn with the wind. This word in this connection is an indirect one; but unless there be some way or other which shall connect words with the things they signify, and shall ensure their correspondence with them, they have no value as signs of those things. Whatever has these two characters is fit to become a sign. It is at least a symptom, but it is not actually a sign unless it is used as such; that is unless it is interpreted to thought and addresses itself to some mind. As thought is itself a sign we may express this by saying that the sign must be interpreted as another sign.† 16 Let us see however, whether this is true of thought itself that it must address itself to some other thought. There are some cases in which it is not difficult to see that this must be the case. I have no belief that prussic acid is poisonous unless when the particular occasion comes up I am led to the further belief that that particular acid is poisonous; and unless I am further led to the belief that it is a thing to avoid drinking. For all these things are necessary to my acting on my belief. A belief which will not be acted

on ceases to be a belief.

Peirce: CP 7.357 Cross-Ref:††

357. It may be that I shall finally come to a belief which is a motive for action directly without the intervention of a more special belief. In this case how does the belief address itself to a sign? When a person is said to act upon a certain belief the meaning is that his actions have a certain consistency; that is to say, that they possess a certain intellectual unity. But this implies that they are interpreted in the light of thought. So that even if a belief is a direct motive to action it still is a belief only because that action is interpretable again. And thus the intellectual character of beliefs at least are dependent upon the capability of the endless translation of sign into sign. An inference translates itself directly into a belief. A thought which is not capable of affecting belief in any way, obviously has no signification or intellectual value at all. If it does affect belief it is then translated from one sign to another as the belief itself is interpreted. And therefore this character of signs that they must be capable of interpretation in every sense belongs to every kind of cognition. And consequently no cognition is such or has an intellectual significance for what it is in itself, but only for what it is in its effects upon other thoughts. And the existence of a cognition is not something actual, but consists in the fact that under certain circumstances some other cognition will arise.

Peirce: CP 7.358 Cross-Ref:††

§7. PRAGMATISM †17

358. In every logical mind there must be 1st, ideas; 2nd, general rules according to which one idea determines another, or habits of mind which connect ideas; and, 3rd, processes whereby such habitual connections are established.

Peirce: CP 7.359 Cross-Ref:††

359. A belief is an habitual connection of ideas. For example, to say that I believe prussic acid is a poison is to say that when the idea of drinking it occurs to me, the idea of it as a poison with all the other ideas which follow in the train of this will arise in my mind. Among these ideas, or objects present to me, is the sense of refusing to drink it. This, if I am in a normal condition, will be followed by an action of the nerves when needed which will remove the cup from my lips. It seems probable that every habitual connection of ideas may produce such an effect upon the will. If this is actually so, a belief and an habitual connection of ideas are one and the same.

Peirce: CP 7.360 Cross-Ref:††

360. In a mind which is capable of logical criticism of its beliefs, there must be a sensation of believing, which shall serve to show what ideas are connected. The recognition that two objects present belong together as one is a judgment. All ideas arise in judgments. This is clearly the case if they are caused by previous ideas. If they are sensations then they at once cause other ideas and are connected with these in judgments. The intellectual value of ideas lies evidently in their relations to one another in judgments and not to their qualities in themselves.†18 All that seems blue to me might seem red and *vice versa* and yet all that I now find true of those objects I should equally find true then, if nothing else were changed. I should still perceive the same distinctions of things that I do now. The intellectual significance of beliefs lies wholly in the conclusions which may be drawn from them, and ultimately in their

effects upon our conduct. For there does not seem to be any important distinction between two propositions which never can yield different practical results.†19 Only the difference in the facility with which a conclusion can be reached from two propositions must be regarded as a difference in their effects upon our actions.

Peirce: CP 7.361 Cross-Ref:††

361. It appears then that the intellectual significance of all thought ultimately lies in its effect upon our actions. Now in what does the intellectual character of conduct consist? Clearly in its harmony to the eye of reason; that is in the fact that the mind in contemplating it shall find a harmony of purposes in it. In other words it must be capable of rational interpretation to a future thought. Thus thought is rational only so far as it recommends itself to a possible future thought. Or in other words the rationality of thought lies in its reference to a possible future.†20

Peirce: CP 7.362 Cross-Ref:††
BOOK III

PHILOSOPHY OF MIND

CHAPTER 1

PSYCHOGNOSY†1

§1. INTRODUCTION

362. The next business before us is to run through the Subclass of Psychognosy once more, and note its Families, etc.†2 The first order is Nomological Psychognosy, which coincides with what is called Psychology, except that under this latter head, sundry classificatory studies, such as of Criminals, Insects, Great Men, Devil-fishes, Insanity, Sexual, Professional, and Racial characters, are usually included. Such studies cannot be pursued to advantage without a good knowledge of psychology; and here and there, they do contribute a few useful facts to the psychologist. Still, they are not generally pursued by the regular psychologists; nor is the elucidation of the laws of mind their aim, -- unless it be a distant and dubious one; so that to a fair and discriminating mind they hardly appear to belong naturally to psychology. It is merely the influence of the abstract definition which has caused them to be called psychological; and we must be on our guard against the deceptions of abstract definitions.

Peirce: CP 7.363 Cross-Ref:††

363. We have already recognized two Suborders of Psychology, -- General Psychology and Special Psychology.†3 But here we meet with the serious difficulty that we find ourselves in disaccord with the psychologist's conception of his own science. Not that the psychologist will have any objection to a division into General and Special Psychology. But he will draw the line in quite another place from that in which we have drawn it. We have conceived of General Psychology as the study of the law of final causation. The psychologists will disapprove of that. They will say that purpose is characteristic only of a special department of mind, and that what they are studying is the phenomenon of consciousness generally. Now it is my intention in another chapter to examine seriously the question which of us is right.†4 I do not think that such a discussion would be quite relevant here. But when an author (it is my case), having for many years pushed a line of investigation over a path, very old indeed, -- *uralt*, as the Germans say, -- yet for centuries unused and grown over with brush, finds that in order to set forth his results, he has to persuade the reader to look at matters from points of view from which everything must seem to him unfamiliar and paradoxical; it will be an advantage, when the two come to the serious argumentation, if the author has already made the reader somewhat acquainted with the propositions he desires to defend. He will do well (so it has seemed to me) to expose what he has to offer to the sunlight, that its garish colors may fade a little, before the time comes for deciding whether it is to be accepted or rejected. To be sure, this will involve considerable repetition; but the repetition will not be without a purpose, and I hope not without its convenience for both parties. Let it be understood, then, that what I say here on the subject of my difference with the modern psychologists, for the scientific character of whose work I have a high degree of respect, is not my argument. That will be presented later; it is not needed here. All I wish to bring before the reader here is the fact that there is a very different opinion from that now current among the psychologists, and an opinion which a man who has anxiously examined into the question, fully alive to the perils of pet-theories and knowing well that no other force than that of the truth that comes from observation can possibly cause an opinion to endure, can entertain with modest confidence as the more conformed to the facts.†5

Peirce: CP 7.364 Cross-Ref:††
§2. CONSCIOUSNESS AND PURPOSE

364. To begin with the psychologists have not yet made it clear what Mind is. I do not mean its substratum; but they have not even made it clear what a psychical phenomenon is. Far less has any notion of mind been established and generally acknowledged which can compare for an instant in distinctness to the dynamical conception of matter. Almost all the psychologists still tell us that mind is consciousness. But to my apprehension Hartmann has proved conclusively that unconscious mind exists. What is meant by consciousness is really in itself nothing but feeling. Gay and Hartley were quite right about that; and though there may be, and probably is, something of the general nature of feeling almost everywhere, yet feeling in any ascertainable degree is a mere property of protoplasm, perhaps only of nerve matter. Now it so happens that biological organisms, and especially a nervous system are favorably conditioned for exhibiting the phenomena of mind also; and therefore it is not surprising that mind and feeling should be confounded. But I do not believe that psychology can be set to rights until the importance of Hartmann's

argument is acknowledged, and it is seen that feeling is nothing but the inward aspect of things, while mind on the contrary is essentially an external phenomenon. The error is very much like that which was so long prevalent that an electrical current moved through the metallic wire; while it is now known that that is just the only place from which it is cut off, being wholly external to the wire. Again, the psychologists undertake to locate various mental powers in the brain; and above all consider it as quite certain that the faculty of language resides in a certain lobe; but I believe it comes decidedly nearer the truth (though not really true) that language resides in the tongue. In my opinion it is much more true that the thoughts of a living writer are in any printed copy of his book than that they are in his brain.

Peirce: CP 7.365 Cross-Ref:††

365. What the psychologists study is mind, not consciousness exclusively. Their mistake upon this point has had a singularly disastrous result, because consciousness is a very simple thing. Only take care not to make the blunder of supposing that Self-consciousness is meant, and it will be seen that consciousness is nothing but Feeling, in general, -- not feeling in the German sense, but more generally, the immediate element of experience generalized to its utmost. Mind, on the contrary, when you once grasp the truth that it is not consciousness nor proportionate in any way to consciousness, is a very difficult thing to analyze. I am not speaking of Soul, the metaphysical substratum of Mind (if it has any), but of Mind phenomenally understood. To get such a conception of Mind, or mental phenomena, as the science of Dynamics affords of Matter, or material events, is a business which can only be accomplished by resolute scientific investigation. But the psychologists have been prevented from making that investigation by their delusion that Mind is just Consciousness, a simple affair, as far as the mere phenomenon goes, about which there is no room for error or doubt.

Peirce: CP 7.366 Cross-Ref:††

366. The psychologists say that consciousness is the essential attribute of mind; and that purpose is only a special modification. I hold that purpose, or rather, final causation, of which purpose is the conscious modification, is the essential subject of psychologists' own studies; and that consciousness is a special, and not a universal, accompaniment of mind. Von Hartmann, as long ago as 1869, proved conclusively that unconscious mind exists.†6 True, we may suppose that, in the cases instanced by him, there is a rudiment of consciousness; but such an objection would not meet his argument, which goes to show that the mental phenomena may be strong where the consciousness, if there be any, is almost nil, and where there is reason to believe that more consciousness would be rather unfavorable than otherwise to the action of mind. A psychologist cuts out a lobe of my brain (*nihil animale me alienum puto*) and then, when I find I cannot express myself, he says, "You see your faculty of language was localized in that lobe." No doubt it was; and so, if he had filched my inkstand, I should not have been able to continue my discussion until I had got another. Yea, the very thoughts would not come to me. So my faculty of discussion is equally localized in my inkstand. It is localization in a sense in which a thing may be in two places at once. On the theory that the distinction between psychical and physical phenomena is the distinction between final and efficient causation, it is plain enough that the inkstand and the brain-lobe have the same general relation to the functions of the mind. I suppose that if I were to ask a modern psychologist whether he holds that the mind "resides" in the brain, he would pronounce that to be a crude expression; and yet he holds that the protoplasmal content of a brain-cell feels, I suppose: there is every evidence that it does so. This feeling, however, is

consciousness. Consciousness, *per se*, is nothing else: and consciousness, he maintains, is Mind. So that he really does hold that Mind resides in, or is a property of, the brain-matter. The early students of electricity, who assumed that an electrical current resides in the metallic circuit, had infinitely more reason for their mistaken opinion. Yes, without exaggeration, infinitely more; for the ratio of something to nothing is infinite.

Peirce: CP 7.367 Cross-Ref:††

367. No doubt, it seems an extraordinary piece of presumption for a man to tell a large body of scientific men for whom he professes high respect that they do not know what are the problems which they are endeavoring to solve; that while they think they are trying to make clear the phenomena of consciousness, it is really something quite different that they are trying to do.†7 I admit that the notion that phenomena of consciousness are the objects of psychology has caused a disproportionate development of certain departments, and has caused other departments to be much neglected. Nevertheless, I hold my ground. For if psychology were restricted to phenomena of consciousness, the establishment of mental associations, the taking of habits, which is the very market-place of psychology, would be outside its boulevards. To say of such departments of psychology, -- from every point of view, the most essential parts of it, -- that they are studies of phenomena of consciousness, is as if an ichthyologist were to define his science as a study of water.

Peirce: CP 7.368 Cross-Ref:††

§3. MIND AND BODY

368. There can be no better touch-stone of a psychology than the question of the relation between soul and body. Here the current psychology, instead of producing a scientific theory, finds itself driven into metaphysics. If you want to know how the doctrine of psychophysical parallelism will appear when it comes to be viewed [from] a distance of a couple of centuries, there is a closely similar theory from which the lesson may be learned, -- the theory of Pre-established Harmony. What does good sense say to this today? The doctrine of Parallelism, which is that that which when viewed from the outside appears as material, when viewed from the inside appears as mental, encounters apparently insuperable difficulties when it is examined in its application to details. It has been adopted as the only means of reconciling the contradiction that the law of dynamics is never violated and that the law of mind is never violated. Neither of the members of this contradiction are rigidly proved to be true. Here, then, are two pretty serious defects in this theory of parallelism. Still, it is conceivable that satisfactory answers should be found to both objections. There is, however, a third objection which seems to knock the ground from under it, once for all; and this is that the two propositions which are supposed to be in contradiction with one another, unless metaphysics is brought in to reconcile them in a superphenomenal manner, can, in fact, be seen to be not in the least in conflict with one another, if we scrutinize them with a logical microscope. The facts which are supposed to conflict with one another are these. First, ideas produce material effects. A whisper in the ear may cause motions on the earth's surface sufficient to attract the attention of the inhabitants of the planet Venus. And reciprocally motions of matter affect ideas. That is one side of the truth; and no sane man can question it. But,

secondly, according to the law of dynamics, no change of motion can take place except through accelerations which are dependent exclusively upon the mutual positions of particles or parts of matter; and according to the law of mind, no idea can arise except by virtue of an association. These propositions cannot properly be said to be proved; but they are postulated; and for the sake of argument we may admit that they are strictly true. Indeed, we should not be justified in believing that they are so very false as it would be necessary to suppose them to be, if this were the only way of preventing their clashing with the obvious fact of the reaction between mind and matter. But in point of fact, there is no contradiction in the propositions stated.

Peirce: CP 7.369 Cross-Ref:††

369. Let us imagine a case where matter, the external world, acts upon mind and where mind responsively acts upon matter. Let it be an extremely simple case, yet not so rudimentary as not to display the principal features of such cases. Suppose that while I am sitting writing in my study, which has an outside door, my dog comes and touches me with his nose; which I know is his sign that he wishes to go out. I do not mind the slight interruption of getting up and opening the door for him; but there is a newly made garden-patch out there, where I do not wish him to go; and I object to standing at the door to watch him. However, I know that the first thing he will do, if he is allowed to go out, will be to run to another door to see if my wife is there, so that he can greet her; and I remember that on this day of the week, at this hour, it being a fine day, she will be seated on a certain balcony where she will hear the dog; and I further know that, under those circumstances, she will look out and call him away from the garden-patch, if he starts to go there. Consequently, I get up, and go to the door, let the dog out, shut the door, and return to my table and to my writing. Now I ask the reader whether he will believe that those motions of my body were purely automatic, my mental reflexions being mere surplusage, provided I can show him that there is no necessary violation of the laws of dynamics in supposing that the mental operations were an essential factor of the phenomenon? Do not ask me in return whether I hold it to be inconceivable that an automaton should be made that would act like that; because that is not the question, for two reasons. The first is, that if such an automaton were to be constructed, a vast expenditure of thought would be necessary to design it. While here, the question being whether or not mind is an essential factor in the motions of matter, the theory of parallelism requires us to suppose, not that such an automaton gets constructed, but that it comes into being without any design at all, or automatically and independently of any design. The second reason why the supposed question would not be relevant is that the true question is not what is conceivable but what is credible. Here am I counting upon what the dog will do, and upon what my wife will do. That is, future events determine my present action. True, it is possible that something may interfere to falsify my expectation; but still experience assures us that such expectations are reasonably sure. Such being the fact, those modes of inference upon which we always rely in science lead us, lead the man whose good sense is not sophisticated by a metaphysical theory, to believe that it was my reflections which caused me first to hesitate and then to go and open the door. Nobody would doubt that that was the true account of the matter, were it not that it is contrary to the law of dynamics that mind should act on matter and contrary to the law of purpose that matter should act on mind. Grant that it is so: but the man who regards that as an objection to the common sense theory is simply allowing himself to be taken in by the Achilles sophism.

Peirce: CP 7.370 Cross-Ref:††

370. Which do you prefer; one of those ghost-like hypotheses about

things-in-themselves which anybody can set up but nobody can refute; or a flesh-and-blood hypothesis that nothing prevents you from wrestling with and flinging it to the ground by any one of a hundred experimental tricks, except that, when you come to try them, they one and all unexpectedly turn out just the other way? I will bring you a hypothesis that is ready to try a bout with you, if you like. This hypothesis may prove false in all its details; I rather guess it will. But in one respect, it certainly is not false; namely, insofar as it shows that matter's acting immediately solely on matter and mind's acting immediately solely on mind in no wise conflicts with matter's truly and literally acting on mind and mind's acting on matter. I will suppose, then, -- and this is only one of innumerable hypotheses of the same general type, and leading to the same conclusion, -- that when the dog touches his nose to my person, he sets up sound waves in the matter of my body, which, of course, will be propagated with the velocity of sound. I will suppose, however, that, owing to the viscosity of the matter in which these waves reside, a part of the motion is quickly converted into heat, which is a motion of the molecules, atoms, and corpuscles of this matter. But these atoms are really vortices in an ether, and along with the heat there will be vibrations of this ether, which will constitute light and be propagated with the velocity of light. I will further suppose that this ether is not devoid of viscosity, so that part of this vibratory and vortical motion will be converted into what we may call a heating of the ether, a motion of *its* atoms. But these atoms, I will suppose, are really vortices in an ether's ether; and along with this heat of the ether, vibrations of this ether's ether will be excited, and be propagated at a velocity as much greater than that of light as the velocity of light is greater than that of sound. I will further suppose that this ether's ether is not devoid of viscosity; so that a part of those vibrations will be converted just as before. I will suppose that there is an endless series of these ethers below ethers, and that in consequence of the increase of velocity as the motion passes from each to the next, the entire infinite series of transformations of motion will be accomplished in a fraction of a second of time. All this motion will be purely dynamical; the mind will have nothing to do with it.

Peirce: CP 7.371 Cross-Ref:††

371. The third of our diagrams of spirals will be useful here, in helping the reader to grasp my meaning.†8 Let the radius vector measure the time, beginning at the outermost point, as the instant when the dog's nose touches me and proceeding inwards. Let each coil of the spiral represent the transformation of the motion from one ether to the next. At the end of the period of time represented by one inch of the radius, all that infinite series of transformations will be complete. Now let us suppose that the inner series of coils of the spiral, which, instead of being endless, is beginningless in terms of the coils, though not in time, represents operations governed exclusively by final causation, and therefore purely mental. Let us suppose that, although mental, they are not noticeably conscious until the innermost end of the coil is approached. Here begin those reflections of which I am able to give any account, although from ever so early in the second series of coils the mind was acting rationally, in the sense in which unconscious, and therefore uncritical, action can be called rational. Finally, at the innermost end of the spiral will occur my volition to let the dog out. Another similar diagram would be required to show what happens next. If the dog is to be let out, the door must be opened; if the door is to be opened, I must open it. But if I am to open it, I must go to it; if I am to go to it, I must walk; if I am to walk, I must stand; if I am to stand, I must rise; if I am to rise, I had better put down my pen; and there consciousness becomes dim. But there must be an infinite series of such ratiocinations if the mind only acts rationally. Take any instant after the work of the mind has been done, and at that instant, an infinite series of dynamical

transformations will have taken place which are to terminate in the door being opened.

Peirce: CP 7.372 Cross-Ref:††

372. I repeat that I will not here argue the question. All I have said is not intended as argument. But neither is it vain talk. It is the necessary preparation for an inquiry. How the inquiry itself is to be conducted shall be shown in another chapter.†9 All I wish now to convince the reader of is that, notwithstanding the admirable, conscientious, and most useful studies for which we have to be grateful to Wilhelm Wundt and his hundreds of disciples, -- for all modern psychologists are his loyal disciples, -- the science is still too unsettled even to understand its own motives; so that a classification which should not look at all beyond the present state of the study could hardly be expected to be really helpful. In this place, therefore, by exception, I shall allow myself to anticipate, as far as such a thing is possible, a state of the science in the near future.

Peirce: CP 7.373 Cross-Ref:††
§4. NOMOLOGICAL PSYCHOGNOSY

373. I shall, then, recognize that Nomological Psychognosy must separate into two Suborders; the first of General Psychonomy, the second of Special Psychonomy, or Nomological Psychology. The former will study the law of final causation and seek to formulate it with exactitude, while the latter will study subordinate laws of mind, of which that of Association is the first.

Peirce: CP 7.374 Cross-Ref:††

374. General Psychonomy will have four Families.†10 Family 1 will define the essence of Mind and the law of final Causation, together with its application to non-biological phenomena. We may term it phenomenalistic Pneumatology. Family 2 will show how final causation works in the development of biological stocks. Family 3 will study the law in its application to biological individuals and to consciousness. Family 4 may be called Demonomy, although it should study the application of the law, not merely to Societies, but also to mere Associations, such as that of a profession. This Family has a truly remarkable analogy to that Family of Dynamics which treats of stationary motion. One Subfamily (or possibly only a Genus) of it will work out the general laws of concert and strife. A second will apply these principles to special phenomena. One Genus will apply them to Law and the Constitution of Society, in Subgenera; another to Wealth.

Peirce: CP 7.375 Cross-Ref:††

375. Nomological Psychology will treat of laws of mind subordinate to the general principles of final causation. I recognize in it but two Families, of which the one is devoted to the study of the great law of Association (including, of course, Fusion) or that of the mutual attraction of all ideas, the analogue of Gravitation in the Physical world, while the other analyzes the laws of the connection of body and mind. The former Family seems to involve three Subfamilies of which one treats of Association *per se*, a second association as modified by dissociation, etc., and a third, the laws of the growth of mind in individual and in society or stock. The second of these Subfamilies will have Genera relating, the first to Habit, the second to Imagination and Vividness, the third to Generalization and Reason, the fourth to

Recognition and Belief. The Subfamily of the laws of Growth of Mind must have four Genera relating, the first to General Laws; the second to growth, the morphogeny, etc.; the third to growth in the conscious mind, with three Subgenera relating to natural immaturity (with Infant Psychology and Child Psychology as species), to growth under education, and to growth by experience; and the fourth Genus to the laws of growth of the Social Consciousness. Family 2 will have two Subfamilies treating respectively of elementary laws and of the laws of peculiar states of mind. The first Subfamily will have three Genera. Genus 1 will study the general law of the reaction between Body and Mind. Genus 2, Psychophysics, will treat of Sensation and whatever may be analogous to it. Genus 3 will treat of Volition, with three species relating to Impulse, Control, and Controlled Volition, each of these having one Subspecies for direct, or outward volition, another for attention. Perhaps the first of these species must have a third Subspecies for Emotion. The Subfamily relating to laws of special states of mind would have three Genera. Genus 1 will treat of the laws of dissolution and double consciousness, with five Subgenera studying the general law *per se*, studying fatigue, studying sleepiness and sleep, normal and abnormal, studying dreams and hallucinations, normal and abnormal, and studying double consciousness, normal and abnormal. For it is commonly recognized that there is a multiple personality that is perfectly normal. Genus 2 will treat of the laws of "credenciveness," including what is carelessly called "suggestion," a term which was already preoccupied for one of the general phenomena of association. Genus 3 will treat of the laws of the Passions.

Peirce: CP 7.376 Cross-Ref:††
 §5. PSYCHOLOGY

376. All this is but a preliminary sketch of what I should imagine might be the division of a psychology of the future, subject to modifications to adapt it to existing psychology. The usual division now is into Introspective, Experimental, and Physiological Psychology. These divisions exist; but they are unimportant, and can only be regarded as constituting species under several of the genera. Let us consider them a moment, beginning with Physiological Psychology. This term may be taken in a strict sense; but it is almost invariably understood in a loose sense. In a strict sense, Physiological Psychology means making experiments on the brain, and from the results of these experiments drawing inferences concerning the Mind. The one fair inference to be drawn from those experiments is that the connection between the mind and the brain is an accidental one. When a lobe of the brain is extirpated, we find that the connection between certain organs of the body is deranged or broken. Of course, for the time being, that prevents mental action, just as if you abstracted my inkstand you would find me transformed for a time from a student working sixteen hours a day into an ardent devotee of amusement. The injury may be such that there is no recovery. But usually recovery does take place. Other parts of the brain are made to do the work, after a fashion, with perhaps other parts of the body. The remarkable thing is, that those very actions, now performed with other organs, show the same mental idiosyncrasies, down to minute details that they did before. The man who has had to learn to write with his other hand directed by another part of the brain, may write pretty badly, but his very handwriting will show the same inimitable characteristics it had before. A few such general truths as that may be established by Physiological Psychology proper. They may be summed up in the proposition: The

brain has no radically peculiar relation to mind. But this is only a negative proposition, and, as such, of no positive importance. But what is ordinarily meant by physiological psychology is not these supremely difficult and uncertain experiments, but work done very commodiously with a diagrammatic figure of the brain and its connections. What psychology thus derives from anatomy rather than from physiology is suggestions of theories. When it is conscious mind that is to be studied, no doubt those suggestions are often of great value. In other departments of psychology, they are largely fallacious. But in any case, it is not a method of investigation, but only a help in forming those hypotheses from which investigation takes its start. It cannot, then, constitute one of the higher groups of psychology. As for Experimental Psychology, it is commonly said that this can only be practised in conjunction with introspection. I cannot admit the truth of this. In many experiments there is nothing that any form of statement can torture into introspection.†11 In others, there is no introspection unless nice observation, -- such as saying for example which of two colors is the brighter, -- is introspection. But this is in reality nothing but the attentive observation of an outward object. It is nonsense to call attention to an outward object by the name of introspection. Introspection is direct observation of the operations of the mind as mental operations; because, as for feelings, they are always referred to some object, and there is no observation of feelings except as characters of objects. I will say a word more about this presently. The type of true psychological experiment consists of putting a man in peculiar and carefully defined conditions, requesting him to do something, typically, of a natural and ordinary kind, and then carefully measuring a certain feature of his action. That will be a scientific observation which may serve to explode a psychological hypothesis. It is the only scientific basis of psychology. There is no introspection in it. Introspective Psychology is the old false psychology which ought not to be countenanced. It is true that a good deal is known about the mind from ordinary observation. There is a considerable store of wisdom concerning man before scientific psychology begins its work. But so far as this is anything more than instinctive knowledge, it really rests on experiments. Only, they are such experiments as any man can perform, by simply resolving to watch all cases in his experience where certain conditions are fulfilled, and noting how far certain results accompany them. As for the mind's watching its own operations, no such thing is possible. It is pure delusion. Take, for example, a train of ideas. A man may recall some of the ideas of the train. But what are they? They are objects, imaginary objects, -- *products* of the mind's operation, but not the movement of mind itself. Whether even these objects can afterward be described or known as they actually presented themselves during the process, -- whether they are not really subsequent creations, -- is usually very doubtful. It is certain that unless the train of thought was arrested at any one of them long enough to make such an object as can be described, in which case it is not at all the operation of the mind but only an elaborate product that is known, then we cannot afterward know them as they were in the fleeting thought. And even our falsified knowledge of them is not knowledge of thought's movement, but only of a product which it stops in order to throw off. Moreover, even of these, it is experimentally certain that only a small part can be recovered. Thus, the real operation of thought is something purely inferential, at best; and I shall give reason for thinking that it is commonly mistaken. But the Introspective Psychologists, while they acknowledge that there is some force in these objections, though not so much as I think, insist that we have an introspective knowledge of the characters of feelings. They say, for example, that we can perceive that red and green are simple sensations, that yellow is a third simple sensation, not a mixture of the sensations of red and green, but that orange is not a simple sensation,

but is a mixture of red and yellow. In weighing the truth of this assertion, the first thing to be clearly understood is that red, orange, yellow, and green, are not pure feelings, but are generalizations of feelings. Red and green lights falling together upon the *fovea* do produce the sensation of yellow. It is commonly supposed that this is only very roughly true. But this is because proper precautions are not taken in performing the experiment. If it is accurately done, although there is a slight whitishness in the yellow produced, it is only slight. It is very likely, however, that this varies with different eyes, because it probably depends upon the number of rods in that part of the retina. If there were no rods, it might likely be exactly true. For some reason we put the yellow into a different class from red or green. The introspectionists, however, admit that yellow is more like red and more like green than red and green are like one another. If that be so, then we class red and yellow together, and see in them some common element, and we also class yellow and green together, and see in them some common element. Therefore, in yellow we do see a mixture of an element of red and an element of green. They therefore fall into self-contradiction when they say that we perceive that yellow is a simple sensation. It is true that we do not see red in yellow, nor do we see green in it, but only an element of red-likeness and an element of green-likeness; and furthermore we see something peculiar in yellow which predominates over its red-likeness and its green-likeness. But then this is more or less the case whenever from any motive we are led to regard a certain class of phenomena as a class by itself. Any collection of objects in the world, no matter how artificial, has some character common and peculiar to its members. It must be so, or we could not have thought of them as forming a collection. Now in cases where it seems important to regard the objects as a class, this character in which they are utterly dissimilar from all other objects, assumes a mental predominance. But yellow is an important kind of sensation for the reason that any color when highly illuminated looks more yellow. Now since the illumination of surfaces is constantly changing, this yellowishness has to be allowed for specially in classing the color of the surface; and thus it is very natural that the peculiar character yellow should acquire a special importance. But this does not constitute any simplicity of it as a feeling which does not belong to any class of sensations in proportion as it acquires importance. It must be so, in accordance with the general laws of mental phenomena. Orange, on the contrary, is not particularly significant, and therefore its reddishness and its yellowishness seem to us sufficiently to describe it. But if we look at it accurately, we shall see that it is not red and not yellow, but only red-like and yellow-like, and with a peculiar character of its own. The truth is that the phrase *simple sensation* is devoid of all meaning, unless by simple be meant important. For if the phrase simple sensation means anything, it must mean that no other two sensations are like this in different respects; that is, all others can only differ from it in one way. For if sensations vary along a continuous line, tell me if you can, what it can mean to take a single point of that line and say it is more simple than any of the other points. If you mean that some circumstance gives it a special importance, that I can understand. But that circumstance is probably some fact of astronomy or geognosy, not essentially related to consciousness. If, then, the psychologists wish me to admit merely that yellow seems to be a peculiar color, I grant it is a fact about our minds that they are educated to regard it so. But in that same sense, the sensation of nakedness is a simple sensation; and the sensation of a letter of the alphabet is a simple sensation. These considerations ought to convince anybody who has carefully gone through the introspectional psychology of the present day that it never can again be an important department of the science. It is merely a preliminary study in several Genera.

Peirce: CP 7.377 Cross-Ref:††

377. In the rough sketch given above of my impression of what psychology ought to be, I have omitted, by negligence partly, several topics upon which books enough have been written to bend our shelves. The remarkable thing is how valuable they are on the average. It shows what virgin soil psychology is that with so little exertion of strength good fruit can be obtained in this field. Reviewing my preliminary scheme and comparing it with current ideas, the first thing that strikes me is the absence of the terms relating to association which the Germans and their lackeys have introduced; such as Fusion, Assimilation, Synthesis, Complication, Apperception. These are connected with valuable contributions to psychology, no doubt. But I think that the conceptions of the English Associationalists, which are now met with a smile, are of vastly greater importance; such as the distinction between the radically different processes, now generally overlooked, between association and associative suggestion, and the capital distinction between Association by Resemblance and Association by Contiguity, which goes down to the root of the matter as none of the distinctions based on speculations about the brain begin to do.†12 Another modern term that I miss is synesthesia, a trifling variety of imagination. But there is Memory, concerning which there is a whole library of books of exceptional average foolishness. No doubt, under the Genus of Imagination and Vividness, a Species must be devoted to Memory. Attention is a faculty which many excellent psychologists regard as of fundamental importance, not at all to be relegated to subspecies of Volition, as I have relegated it. Of course, I may be wrong; but I adhere to my opinion on this point. I have not said a word about Telepathy; and in general I have not admitted that abnormal phenomena differ deeply from normal phenomena. I cannot admit that there is any special nomology of the abnormal. It is the condition of a science of laws that it is restricted to what it can find to be regular.†13

Peirce: CP 7.378 Cross-Ref:††

§6. CLASSIFICATORY PSYCHOGNOSY †14

378. Of Classificatory Psychology, which for brevity I term Psychotaxy, two Suborders have already been designated, studying respectively Kinds of Performances and Kinds of Individuals. The word performance must here be taken in a broad sense to include actions that are simple and involuntary. I have been tempted to substitute the word "faculties" and have not done so because this word would certainly be positively misunderstood, while "performance" has no greater fault than unintelligibility at first sight. The Classification of Performances seems, on the whole, to consist of two Families, the one of Elements of Performance, the other of Systems of Performance. But this is one of those cases in which my conclusion is reached by that process of balancing one consideration with another which the history of science, and, as we shall find, the theory of logic, alike stamp as extremely apt to lead to error.†15 The first Family consists (such is my hesitating conclusion) of two Genera relating respectively to Kinds of Sensation and their relations, and to Kinds of Emotion and their relations. It [is] questionable, I admit, whether I ought thus cut the studies of Sensation and of Emotion into two parts belonging to different Orders. If I am wrong about this, I would, at any rate, retain the two Genera of the Classificatory Order. Whether, however, they are separate Genera, rather than Subgenera of one

Genus, is another question which I may have answered wrongly. One naturally baulks, too, at admitting that this Family has no Genera studying any other mental faculties than those of Feeling; yet this is a point upon which I permit myself a certain degree of confidence. The second Family, that of Systems of Performance, has quite clearly two Subfamilies. Both the one and the other are, substantially, and in the gross, confined to animal and human performances, including organized associations. One Subfamily studies those systems of performance which are mainly confined to the brutes, to the lower animals generally, being in the instinctive stage of development; so that there is an inborn faculty by virtue of which the performance is determined in almost all its details. The Instincts are said to be all adaptive, although it would seem to be in very small measure that play is so. However, the instincts seem to be of two distinct kinds, those which are adapted to the preservation of the stock, if at all, †16 through preserving the individual in whom the instinct acts, and those which are social, and therefore, so far as they are adaptive, are adaptive primarily to the advantage of some other individual or individuals than the agent. Association may happen to be of advantage to the associating individuals; but each individual's instinct brings no more advantage to him than the sum of all the advantages that it brings to so many others. It is double-entry book-keeping; and the sides of the ledger must balance. But then, over and above this, association is generally connected with reproduction, and is therefore advantageous to the stock quite independently of its advantage to the individual. In many cases, the social instincts are expensive to the individual, even dangerous, sometimes fatal. It appears to me that this Subfamily has but one Genus and that this Genus has two Subgenera relating to the two kinds of instincts. There is a certain difficulty in the fact that instincts for war are on the one hand social, since war is a sort of social reaction, and is moreover often dangerous, sometimes certainly fatal, yet on the other hand, it seems improper to distinguish war from preying, and preying is generally an affair of bread-winning, adapted to the preservation of the agent. It seems to me, however, that in war the enemy is not looked upon as fellow-creatures, but is treated as a thing, and I believe that though the instinct brings danger, it is nevertheless a selfish one, and ought to be classed with those which go to preserve the agent. This is inconvenient, it is true, as regards duels motivated by jealousy, which is the chief kind of fighting among herbivora. The leading instincts studied in the first Subgenus are those of Feeding and Food Getting. The study of these will make the first Species. The study of instincts of War will be the second, with two Subspecies, the study of instincts of active war making one, the study of self-preservation including the instincts of self-concealment being the other. Other species will study the minor instincts, such as those of personal Cleanliness, of Medicine (dogs eating grass, etc.), of hibernation, and the singular instinct for collecting and hoarding all sorts of useless things, as seen in rats, in mag-pies, and some other animals, a genuine collecting mania, apparently quite useless. Then I am inclined to think that we must admit a Species for the study of instincts of working materials, such as the tree-felling instinct of beavers, the instinct of the wood-pecker. With this I would reckon all instinctive mechanical skill.

Peirce: CP 7.379 Cross-Ref: ††

379. The Subgenus of Social Instinct has as its nucleus, so to speak, the Reproductive Instincts proper, to which the first Species will be devoted. Next in importance will be the Instincts of Communication; for some kind of language there is among nearly all animals. Not only do animals of the same species convey their assertions, but different classes of animals do so, as when a snake hypnotizes a bird. Two particularly important varieties of this Species of study will relate to Cries and Songs (among mammals and birds chiefly) and to facial expression among

mammals.†17 As belonging to this Species I would include all studies of Instincts for understanding mind. But the subdivision of this Species requires much further study. A third Species will relate to the Architectural Instincts which I place here, although they are sometimes a bachelor establishment or even only a machine, like a cobweb. Passing by as unimportant the Instincts for Clothing, a fourth species will study Instincts of Locomotion and Migration. A fifth Species will relate to Instincts for Games; a sixth, to [the] Instinct for Adornment and Decoration, an Instinct which though associated with the Reproductive Instinct is quite distinct from it. The horse is delighted with a handsome harness. The poodle just trimmed goes to display himself to all his human friends without any teaching.

Peirce: CP 7.380 Cross-Ref:††

380. Passing to the second Subfamily of the Psychotaxy of Systems of Performance which relates to such systems in minds that are too highly developed for much wealth of Instinct, let me say, at once, that I doubt very much whether the Instinctive mind could ever develop into a Rational mind. I should expect the reverse process sooner. The Rational mind is the Progressive mind, and as such, by its very capacity for growth, seems more infantile than the Instinctive mind. Still, it would seem that Progressive minds must have, in some mysterious way, probably by arrested development, grown from Instinctive minds; and they are certainly enormously higher. The Deity of the Théodicée of Leibniz is as high an Instinctive mind as can well be imagined; but it impresses a scientific reader as distinctly inferior to the human mind. It reminds one of the view of the Greeks that Infinitude is a defect; for although Leibniz imagines that he is making the Divine Mind infinite, by making its knowledge Perfect and Complete, he fails to see that in thus refusing it the powers of thought and the possibility of improvement he is in fact taking away something far higher than knowledge. It is the human mind that is infinite. One of the most remarkable distinctions between the Instinctive mind of animals and the Rational mind of man is that animals rarely make mistakes, while the human mind almost invariably blunders at first, and repeatedly, where it is really exercised in the manner that is distinctive of it. If you look upon this as a defect, you ought to find an Instinctive mind higher than a Rational one, and probably, if you cross-examine yourself, you will find you do. The greatness of the human mind lies in its ability to discover truth notwithstanding its not having Instincts strong enough to exempt it from error.†18 This comes out strongly in almost any concrete instance. Frederic the Great is a notable example. Kant's power of making use of confused conceptions and working out so much truth as he did in spite of them illustrates this virtue. Hardly any really great inventor thoroughly comprehends his own invention until long after it is achieved.

Peirce: CP 7.381 Cross-Ref:††

381. The conception of the Rational Mind as an Unmatured Instinctive Mind which takes another development precisely because of its childlike character is confirmed, not only by the prolonged childhood of men, but also by the fact that all systems of rational performances have had instinct for their first germ. Not only has instinct been the first germ, but every step in the development of those systems of performance comes from instinct.†19 It is precisely because this Instinct is a weak, uncertain Instinct that it becomes infinitely plastic, and never reaches an ultimate state beyond which it cannot progress. Uncertain tendencies, unstable states of equilibrium are conditions *sine qua non* for the manifestation of Mind.

Peirce: CP 7.382 Cross-Ref:††

382. It does not appear that all the animal instincts have produced great systems of human performance.†20 In a general way we can see that, in much the same sense in which it would be true that, according to the theory of Natural Selection, every step of the development from the moner to man has been due to fortuitous variations in reproduction, we can equally say that every step of the progress of physiognosy has been due to a guess prompted by one of those instincts studied in the First Subgenus of the science of Instinct, or as we may call them, the mechanical instincts, and that every step in the progress of psychical science has been due to a guess prompted by one of the social instincts. But science is, after all, but a small part of the rational developments that have had instinct for what we may call their efficient cause; that is, their cause so long as we consider them only as aggregates of their smallest parts, neglecting all that we do neglect when we say that Natural Selection makes man the result of fortuitous variations at birth; in one word, neglecting the rational, the integral, element altogether.

Peirce: CP 7.383 Cross-Ref:††

383. But the importance of the different Intellectual Systems of Performance which have developed from the different instincts have had not the slightest relation to the strength or to the adaptiveness of those instincts. In a general way, the useful arts have mostly developed from the Selfish Instincts. But of all these instincts perhaps those of Food getting and Feeding are the strongest. What has developed from these? Agriculture is by far the most respectable of their offspring. The Instincts of War have given rise to the Arts of War. Mechanical Instincts connected with the human hand, and therefore peculiar to man, have probably affected tool-making. But it is a general mechanical instinct, common to almost all animals, which has produced physics and the physical arts generally. Self-preservation has been a motive, or final cause, of the Art of Medicine; but it does not seem to have contributed anything to Medicine as an efficient effect of this instinct.

Peirce: CP 7.384 Cross-Ref:††

384. The Social Instincts were more sympathetic to Reason; and it is they that have been the efficient cause of most human performances, and of the higher ones. For although it happens that perspective at this time makes Physiognosy seem very great, it is not in the nature of things that it should be so. True, [it] is pure; it is unblotted by all the ignoble passions which disfigure other elements of civilization. That is its most admirable feature; and it is not a merely negative one. But at the end of the twenty-first century that will no longer be the achievement upon which man can most plume himself. Of the Social Instincts, by far the most violent are of course the Reproductive Instincts; and in man they strongly color all but one of the Social Instincts, to such a degree as almost to weld them all together. The earliest Art of man, after that of trapping animals, was doubtless the Art of making a home. A cluster of arts there are there; as architecture, heating, lighting, etc. Families became communities; and there gradually arose a notion of what customs and behaviour were tolerable. It must have been early in this state of things when men were leading an easy and joyous life, -- perhaps in the flush of some great victory which brought them slaves, -- or perhaps having recently domesticated cows, -- that language began to take a grammatical form, so that sentences could be constructed, and to acquire a relatively copious vocabulary. I have never seen a satisfactory account of how it came about; but the time is ripe for such an account. Next month's *Popular Science Monthly* may tell us that some Italian has accomplished the feat. Language gave man distinct conceptions, and awakened in him the idea of understanding things. The first fruit of the scientific spirit must have been a Theology, and some confused

Cosmogony;†21 for it is man's way to attack the most difficult questions first, and attempt detailed answers to them. What the first religion was like one would give something to know. To tell us would be a suitable task for a Shakespeare and a Browning, in collaboration with a Darwin, a Spencer, and a Hegel. Records we have none of any faith that has not undergone corruption and reform; senility and rejuvenation by fards, hair-dyes, and false teeth; revolution and restoration; death and galvanization, over and over again.

Peirce: CP 7.385 Cross-Ref:††

385. Let us see how far we can utilize such reflexions in classifying the Classificatory Sciences of Intellectual Systems of Performance. In the light of what has been said, there seem to be about seven Genera. The first embraces studies of the kinds of purely Material Arts; the second, studies of Ways of Individual Life; the third, studies of Customs and Laws and of Constitutions of Society; the fourth, of Religions; the fifth, of Classifications of Sciences, the very subject upon which we are now engaged, so that we are in the somewhat embarrassing position of having to criticize the very thing we are doing before it can get done; the sixth, of Fine Arts; the seventh, of Language†22

Peirce: CP 7.386 Cross-Ref:††

386. The second Suborder of Classificatory Psychognosy deals with kinds of minds. This, very evidently, if we exclude the minds of plants, and of communities, divides into two Families treating respectively of Animal Instinct and of Types of Men. A fair beginning has been made of the study of animal minds. We really seem to be penetrating the psychical nature of the Hymenoptera, -- souls so utterly unlike our own. It is most desirable that we should add to this an understanding of some third kind of mind different from both the others, -- say, for example, that of the octopus, whose passions are betrayed by the waves of color that pass over his person. For one can hardly make a beginning of generalization until three instances are at hand. The worms, echinoderms, coelenterates and sponges do not seem to be very interesting psychically. But the squids have a sort of horrible fascination

Peirce: CP 7.387 Cross-Ref:††

387. Of types of men, serious investigations have been made of genius, of criminals, of men of science, and of great men, generally.†23 Theophrastus began the study of types of character; and different authors have, in various ages and countries, displayed the most splendid power for this sort of research. But unfortunately, they have all been seduced by the glamour of literary glory, -- even St. Simon, (the ancestor, I mean), whose memoirs were not published until three quarters of a century after his death; so that a field of science of whose cultivation there is the most urgent need, both theoretically and practically has been for millennia lying fallow ground awaiting a conscientious ploughman.†24

Peirce: CP 7.388 Cross-Ref:††

CHAPTER 2

ASSOCIATION†1

§1. GENERAL CHARACTERISTICS OF MENTAL ACTION

388. In the absence of external impressions of interest, thoughts begin to dance through the mind, each leading in another by the hand, like a train of Bacchants on a Grecian vase, as Hegel says. After a while the clear train of thought breaks, and for a time ideas are scattered, soon, however, to take places again in another train.

Peirce: CP 7.388 Cross-Ref:††

There is a *law* in this succession of ideas. We may roughly say it is the law of habit. It is the great "Law of the Association of Ideas," -- the one law of all psychical action.

Peirce: CP 7.389 Cross-Ref:††

389. Many psychologists hold that this law as strictly necessitates what idea shall rise on a given occasion as the law of mechanics necessitates how a body in a given relative position to other bodies endowed with given forces shall have its motion altered. This is a theory hard to disprove; but it is a mere forejudgment, or prejudice: no observed facts afford the slightest warrant for it. I do not mean to condemn the trial of it, in psychology, as a working hypothesis: on the contrary, logic fully approves of it in that sense. But many things are worth trial, which do not seem at any time probably true. What is, far more than unyielding uniformity, characteristic of the phenomenon of *suggestion*, as the calling up of an idea through association is called,†2 is its *gentleness*. There is another natural operation that, from every point of view, seems much nearer allied to the psychological sequence than is mechanical causation; I mean heredity. Now heredity, powerful as it is, leaves room for sporting, or variations. Remark that the development of species, whether by Natural Selection or by Artificial Breeding, would be impossible were variations not to occur in the teeth of heredity; and in like manner without the element of spontaneous originality, or something acting like it, the development of thought would be instantly arrested.†3 Hume, whose cogitations led up to the recognition of Association as the one law of mind, most judiciously remarks, "This uniting principle among ideas is not to be considered as an inseparable connexion; for that has been already excluded from the imagination: Nor yet are we to conclude, that without it the mind cannot join two ideas; for nothing is more free than that faculty: but we are only to regard it as a gentle force, which commonly prevails."†4 That phrase "a *gentle* force which *commonly* prevails" describes the phenomenon to perfection.

Peirce: CP 7.390 Cross-Ref:††

390. But it is hopeless to expect men to agree about the question of spontaneity. The observed facts all tend to support it; but some men *will* deny it, because they are disinclined to that view; thus exercising that very freedom that they refuse to recognize.

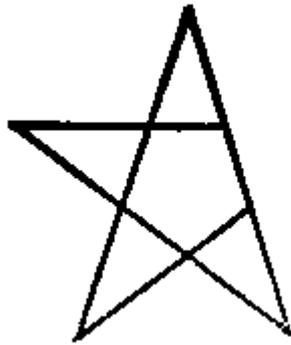
Peirce: CP 7.391 Cross-Ref:††

§2. CONTIGUITY AND RESEMBLANCE

391. Psychologists differ in regard to what they call the primary principles of association; but the usual doctrine is that suggestion takes place either by *Contiguity* or by *Resemblance*. These are Hume's terms, which psychologists have agreed to retain as convenient names, although as descriptions they are acknowledged to be faulty enough. Suggestion by contiguity means that when an idea is familiar to us as part of a system of ideas, that idea may call the system to our minds, and from the system, one of the other ideas may, for some reason, detach itself and come to be thought of by itself. Thus, mention "the judicial branch of government," and you suggest the idea of a government composed of an executive, a legislature, and a judiciary; and shortly after, the person to whom you are speaking, if you are boring him, will have his ideas running, it may be, upon executive power, it may be upon the conduct of a legislative assembly. So, mention a *wife*, and your hearer will think of husband and wife, and thereupon, most likely, of a married man. Mention a knife-blade and the whole knife will be thought of, and thence a knife-handle. Thus, suggestion by contiguity may be defined as the suggestion by an idea of another, which has been associated with it, not by the nature of thought, but by *experience*, or the course of life.†5

Peirce: CP 7.392 Cross-Ref:††

392. Suggestion by *resemblance* is easily enough understood, as soon as the conception is once grasped that the similarity of two ideas *consists* in the fact that the mind naturally joins them in thought in a certain way. For instance, yesterday I saw a blue color; and here *is* a blue color. I recall that sensation of yesterday, and I observe that of today. I find myself disposed to say the two are closely allied; *in that disposition their similarity consists*. For they are two different ideas. One was in my mind yesterday, and consequently that identical idea is not present now. However, I accept the impression it has left on my memory as probably about right. I look again at the color before me. The idea of yesterday and that of today are two ideas; they have nothing in common, unless it be that the mind naturally throws them together. Some beginner may object that they have both a *blueness* in them; but I reply that blueness is nothing but the idea of these sensations and of others I have had, thrown together and indistinctly thought at once. Blueness is the idea of the *class*. It is absurd to say that different things which cannot be compared are alike, except in the sense that they act alike. Now, two ideas are compared only in the idea of the class, lot, or set to which they belong; and they act alike only in so far as they have one and the same relation to that connecting idea. Resemblance, then, is a mode of association by the inward nature of ideas and of mind.†6 There are other modes of such association. Thus, contrary ideas are thrown into pairs by [the] inward nature of ideas and of mind. Numbers run into a succession, or sequence, by the same force.



[Click here to view] What does this figure show? The answer will be a broken star. That answer shows how the mind naturally looks at those lines from the point of view of a *set*, or regular figure, to which they do not even conform. As experience clusters certain ideas into *sets*, so does the mind too, by its occult nature, †7 cluster certain ideas into *sets*. These sets have various forms of connection. †8 The simplest are sets of things all on one footing and agreeing in each belonging to the set. Such a set is a *class*. The clustering of ideas into *classes* is the simplest form which the association of ideas by the occult nature of ideas, or of the mind, can take. Now, just as in association by contiguity an idea calls up the idea of the *set* in which experience has placed it, and thence one of the other ideas of that set, so in association by resemblance an idea calls up the idea of the *set* in which the mind's occult virtue places it, and that conception perhaps gives, owing to some other circumstance, another of the particular ideas of the same set. Everybody has heard in conversation a person remark, "What you say puts me in mind of a similar occurrence." That is suggestion by resemblance. Association by contrast is a case of association by resemblance, which is so called after its most prominent variety. Suggestion by resemblance means, let it be repeated, the indirect suggestion by one idea of another which has, by virtue of the occult nature of ideas or of the mind, been associated with it into one set. All the suggestions of pure mathematics, of which there is a vast body, are associations by resemblance. Some psychologists refuse to acknowledge association by resemblance. The reason is that they conceive that two different principles of association break the law of association into two; now the idea that there must be *one* law for a given group of phenomena is an idea so natural to the mind, that for the sake of it, those psychologists allow themselves to wrench all the facts, thus illustrating by their own action the strength of the kind of association which they refuse to acknowledge †9

Peirce: CP 7.393 Cross-Ref: ††

§3. DEFENCE OF AUTHOR'S OPINIONS STATED IN ART. 2 [SEC. 2]

393. The reader, having thus been railroaded through the history of the doctrine of association, will perceive that the present writer differs from an important minority of the authorities in holding to two principles of association, and that he differs from all the English psychologists while agreeing with the Herbartians, in splitting the suggestion of B by A into two operations, one leading from A to AB and the other from AB to B.

Peirce: CP 7.394 Cross-Ref:††

394. As a sufficient defence of association by resemblance, as a fundamental mode of association, he offers the remark that resemblance *consists* in an association due to the occult substratum of thought.

Peirce: CP 7.395 Cross-Ref:††

395. The defence of the division of suggestion into two steps cannot be quite so summary. In the first place, there are many cases in association in which parts of the train of thought do not appear in memory. Different psychologists have different ways of treating this difficulty. Some maintain that they are performed with lightning rapidity and also with a low degree of consciousness. This is very improbable. Others, as Leibniz, Hamilton, Hartmann,†10 Samuel Butler,†11 and others maintain that they are totally unconscious, or practically so. The present writer will not countenance the rejection of a theory because of metaphysical difficulties. The great object of the metaphysics of Duns Scotus is so to state the results of ordinary experience, that it shall not close any positive experimental inquiry, or pronounce anything possibly observable to be *a priori* impossible. In Scotus, this naturally led to loyalty to Authority, then the recognized fountain of truth; in our day it will produce unfaltering faith in Observation. Still others are of opinion that parts of a train of association may be altogether suppressed. We cannot deny the possibility of such a thing.

Peirce: CP 7.396 Cross-Ref:††

396. The present writer has made a good many quantitative experiments to ascertain what he could of the nature of consciousness and of attention. It would not be convenient to set these forth in this place.†12 But the conclusions to which they lead will be mentioned. An excitement of the nerves will spread, and affect more and more nerve matter. The total excitement may considerably increase, in this. No equation is preserved. It may also wear itself out. Considerable excitements expend themselves *out of* the spinal nerves, either in contractions of voluntary muscles, attended mostly with a sober activity of mind, or in glandular secretions, actions on involuntary muscles, probably through the sympathetic nerves, and these cases are apt to be accompanied by emotional excitement of the soul. Ideas tending to either of these modes of suggestion present the phenomenon of interest. The intensity of ideas is of two kinds, an *objective* and a *subjective intensity*. A high color, a loud sound, a burn have, *per se*, high objective intensity. I have not been to Niagara for three or four years. But I well recall the tremendous roar; objectively the intensity of the idea is great, as great as it was when it was present (possibly more so), but subjectively my memory of it is getting now a little dim. I remember some time about 37 years ago sitting in the dark room of my class-mate, Albert Stickney, he having just carried away the lamp, I believe. I suddenly saw everything lit up and quickly looking out the window I caught sight of a great meteor. Objectively, my memory of it is of an intense light; subjectively, it is very dim.†13 Ideas of great objective intensity have their subjective intensity, other things being equal, greatly reinforced; but the distinction is unmistakable. The word *interest* probably *means* suggestiveness of action or emotion; but it is accompanied with considerable subjective intensity of direct consciousness. The phenomenon of the heightening of the subjective intensity of an idea as it draws into the vortex of an interesting suggestion, is called *attention*. Ribot and others wish to make attention a fundamental element of consciousness. I grant the fundamental element that they are groping for really exists, and that to some extent it affects attention; but attention itself does not consist in that, nor does that element show out with remarkable clearness in attention. When we try to say what is

immediately in consciousness, we have a difficult task. We have to tear down a whole structure which the mind has built round it; and throw that away, while leaving the original elements. Nothing requires greater energy, and energetic determination to be passive; nothing is more fatiguing; and no mental process is so slow. Suppose, you are asked of two colors which is the more agreeable. If you have no training in introspective observation, you may answer quickly; so you may, if there is a marked difference in agreeability. But otherwise, it may take a minute or more of fatiguing strain to reply. Of two pressures brought upon the skin, which seems the stronger, is a direct question of introspective observation. But it is difficult. Sensations differing so slightly, that an exhausting effort of attention, under the most favorable surroundings, leaves us a feeling of utter inability to make out any difference, can, nevertheless, be proved to affect the mind. For instance, it is found that in such cases, if the observer forces himself to say which is the stronger sensation, though his answer seems to him to be given at random, yet taking a dozen sets of a hundred answers, in each set there will be a decided preponderance of true answers.† 14 Pass now to sensations differing quite markedly, -- or which according to the difference of excitations ought to do so, -- and try thousands of times asking which is the stronger. It will be found that erroneous answers are given, oftener than anybody would believe possible in advance of the experiment. That this is not due entirely to the sensations not being what the excitations ought to produce is shown by other mental effects they produce. Take a series of efforts of given degree; for instance, in frames of mind which are marked by the observer on a scale as seeming to have about the same vigor, let a given time say 30 seconds be given to an introspective effort; let this effort be applied to the attempt to discriminate two sensations, and let the resulting feeling of confidence that they are correctly distinguished be marked on a scale. Now out of a thousand observations, where the difference of excitations was the same, the mean difference of sensations will be less than in another thousand observations where the difference of excitations was twice as great. We can then count the proportion of cases in which the degree of confidence in the reply reached a certain point on the scale. Thence, we can, by the usual mathematical methods, find a formula connecting the proportion of cases in which the given degree of confidence is reached with the difference of excitation. Then, we uniformly find that if that degree of confidence is not so high as to invalidate the formula, owing to the small number of cases in which it is observed, [then] it makes no difference what its degree may be in respect to this: that the proportion of cases in which it occurs only sinks to zero when the difference of excitations sinks to zero. It follows, I think, that there is no sensation which can affect the mind at all, which cannot be detected in consciousness by a sufficient exertion of attention.

Peirce: CP 7.397 Cross-Ref:††

397. The result of the study of the above formulae and of many others (which I have never published, because no psychologist has paid the slightest attention to those I have published) is that the contents of immediate consciousness range all the way from feelings which an indefinitely great effort is required to reduce to [a] given grade of subjective intensity to feelings which an indefinitely great effort is required to magnify to any given grade of subjective intensity. If we assume, as a convenient scale of measurement, that the measure of subjective intensity of an idea before such effort is applied to it is, other things being equal, proportional to a base raised to a power expressing the degree of effort required to lower its subjective intensity to an assumed standard, then I find that no feelings affecting the mind have the measures of their subjective intensities 0 or ∞ ; but that they approach indefinitely to those limits.

Without any effort of attention at all, certain feelings have sufficient subjective intensity to affect us in certain ways, for instance, to cause us, in an off-hand answer, to reply that we are affected by them. The subjective intensity of many a feeling is sufficient for *that* without being sufficient to rouse us to decided exertion. On the other hand, the subjective intensity of many a feeling, though insufficient for that, is sufficient to affect our actions and color our emotions strongly.

Peirce: CP 7.398 Cross-Ref:††

398. Ideas become subjectively intense, if they are objectively intense. They become for a moment subjectively intense and directly after dim, if their suggestions are interesting. They become subjectively intense in small sets, especially sets of two; and become subjectively faint in large sets. They rapidly lose subjective intensity with time, while at the same time they spread into sets.

Peirce: CP 7.399 Cross-Ref:††

399. It follows from the above that the compound idea AB may be operative as an intermediate stage between A and B, notwithstanding our seldom being able to detect it in contiguous suggestion.

Peirce: CP 7.399 Cross-Ref:††

Very often this intermediate stage *can* be directly detected. In others, [it] is demonstrably there, as in ideas of motion where the preservation of the identity in idea of the mobile [is such] that one stage must have been imagined before the imagination of the previous stage was relinquished.

Peirce: CP 7.399 Cross-Ref:††

In other cases, whether the compound is present as intermediacy or not can only be a matter of hypothesis. That theory must be adopted which (1) best and (2) most simply explains the observed facts, is (3) in the closest analogy with the rest of our knowledge, and (4) is attributable to causes known to operate or most likely to be operative.

Peirce: CP 7.400 Cross-Ref:††

400. The theory here advocated is called, for convenience, the "new" theory. It is not really new; but it has been furbished up so as to be as good as new; and the designation is handy.†15

Peirce: CP 7.401 Cross-Ref:††

401. (1) It is true that the new theory does not so well explain the observed facts in the majority of suggestions by contiguity, because it supposes an idea to pass unnoticed. See (11), below.

Peirce: CP 7.401 Cross-Ref:††

But it has the advantage in those cases of contiguity where it is observed that A suggests AB. For this kind of suggestion does not come under the old formula. And it is difficult to explain how AB should be sometimes suggested by A and not always suggested.

Peirce: CP 7.401 Cross-Ref:††

In the case of association by resemblance, on the other hand, all the testimony of consciousness is that A almost always suggests AB, and perhaps never directly B. See (12), below.

Peirce: CP 7.402 Cross-Ref:††

402. (2) In regard to the simplicity of the theory a trifold distinction has to be made. In the sense of presenting many more features requiring study, and suggesting many more questions, the new theory is infinitely more complicated than the old one. See (21), below.

Peirce: CP 7.402 Cross-Ref:††

In the sense of being a more natural conception, it is doubtful which theory is superior. But this point is not an important one. See (22), below.

Peirce: CP 7.402 Cross-Ref:††

In the sense [of] giving a more unitary account of all the facts, the new theory is superior. See (23), below.

Peirce: CP 7.403 Cross-Ref:††

403. (3) In the general analogy with attraction the superiority is with the new theory. See (31), below.

Peirce: CP 7.403 Cross-Ref:††

In the general analogy with other changes of state, the superiority is with the new theory. See (32), below.

Peirce: CP 7.403 Cross-Ref:††

The new theory is more in conformity with current views of philologists about changes in the signification of words. See (33), below.

Peirce: CP 7.403 Cross-Ref:††

The new theory is favored by those modern psychologists who have made the most thorough study of association. See (34), below.

Peirce: CP 7.404 Cross-Ref:††

404. (4) There is a psychological cause which would probably produce the process supposed in the new theory. See (41), below.

Peirce: CP 7.405 Cross-Ref:††

405. Let us now consider the ten points just mentioned a little more closely.†16

Peirce: CP 7.406 Cross-Ref:††

406. (11) The new theory supposes a fact of consciousness usually to pass unnoticed in suggestions by contiguity, namely that when A suggests B, the compound idea AB intervenes.

Peirce: CP 7.406 Cross-Ref:††

But, then, it is to be observed that this is a sort of fact of consciousness very likely to pass unnoticed. It is well-known that ideas containing peculiar elements often do pass through the mind unnoticed. But in this case, there is no peculiar element. What is supposed is that instead of A jumping out of consciousness at the very instant B opens the door, quicker than a mouse could run into its hole, it stays until it finds itself *de trop*. Since it is necessary to suppose that the star of A is on the wane, very little interest would be had in it; and its remaining would not be noticed at the time, and still less be remembered. When you are entertaining a bore, if a visitor of consequence enters the room; and the bore behaves modestly, nobody can say afterwards whether he took his departure at that moment or not. It is, therefore, certain that the number of cases in which we remember positively that A did suggest AB and only later B, which cases are after all numerous, *cannot*, according to known

principles of attention, be but a small percentage of all the cases in which this happened. But dividing the observed frequency of the intermediate suggestion by this small percentage we certainly get a large percentage, and it may be unity. Not knowing the percentages numerically, we cannot positively say that the quotient *is* unity; but it is not at all unlikely that it is so.

Peirce: CP 7.407 Cross-Ref:††

407. (12) We have seen †17 that Bain, as high an authority as any desultory observer can be, and there are as yet too few scientific observations of association to be of much importance, holds that generalization is the direct effect of "an effort of similarity." Why not say, at once, it is the first half of a suggestion by similarity? I am trying to recall the precise hue of a certain emerald that my mother used to wear. A sequence of shades runs through my mind. Perhaps they run into a continuum; but that makes no difference. They are a multitude of colors suggested by that one color. Conceived under what Kant imperfectly describes as a rule or schema, they constitute a *general* conception of a green something like that emerald. The old-fashioned nominalists would say nothing was in my mind but a *word*, or other *symbol*. For my part, I am not quite prepared to say what precisely is in my consciousness; but of this I am sure, that every memory of a sensation is more or less vague, that is, general. Every *memory*! Why, the sensation itself, when present for a few moments, is so; as every person who has made careful photometric measures is aware. In working with a photometer, I have confined myself to observations of a square centimetre some 10 inches from the eye; and have found the most elaborate arrangements hardly adequate to make the illumination of that small surface sensibly uniform. What, then, must be the vagueness of our observation when we look at a whole sheet of paper and pronounce it to be of one shade throughout? How is it possible to reconcile our notions of the origin of errors of observation with the doctrine that the sensation is absolutely free from all vagueness, all generality? But if it be granted that every memory of a sensation is more or less general, the distinction of intuitive and symbolical knowledge breaks down, as an absolute distinction. The vague memory of a sensation is just an aggregate, whether continuous or not makes no difference, of ideas which are called up together by a suggesting idea. The apparent direct suggestion of a single, or anything that can be taken for a single, idea by resemblance, is so rare, if not altogether dubious, a phenomenon, that William James declares that "there is no tendency on the part of simple ideas or qualities to remind us of their like."†18

Peirce: CP 7.408 Cross-Ref:††

408. (21) If A remains in consciousness after it has called in B, we have to inquire in what form A and B coexist in consciousness at such times and what their relation is, a question which we escape, of course, if A vanishes in the act of bringing in B. Nor is this question so innocent and easy as it may appear at first blush; on the contrary, it leads to considerable discussion and creates an extensive theory. Moreover, when A and B are both present, a new problem presents itself. Namely, how A drops out; and that phenomenon, examined closely, turns out to be far from the simple thing one might fancy it would be. Besides, the cases are few in which the *set* of associated ideas †19 consists of a pair. Most often it is what the country-people in my neighborhood call "quite a few," that is, almost a good many. At other times, though more rarely, it is three or more. Now if A calls up ABCD etc., it is a serious question how all these coexist in consciousness, and another why and how ACD etc. all disappear. Do they all depart in company, or do they sneak out one by one? And in the latter case, do those that remain have to rearrange themselves at each departure?

And *why* should this desertion take place? And what determines who is to be left as IT, to use the terminology of children's games? All these questions are avoided on the old theory.

Peirce: CP 7.409 Cross-Ref:††

409. (22) In a question like this, it really makes little difference whether the hypothesis adopted is a natural one or not. We ought to give high authority to natural, instinctive conceptions of the mind, so far as they are of any practical utility. For natural selection, or whatever the principle of evolution may be, is there to adapt them to the welfare of the species. But in questions like that now on the *tapis*, natural psychology is quite beyond its depth. We will not stop, therefore, to argue this question.

Peirce: CP 7.410 Cross-Ref:††

410. (23) The purpose of a theory may be said to be to embrace the manifold of observed facts in one statement, and other things being equal that theory best fulfils its function which brings the most facts under a single formula. Besides, it will be shown in the chapter †20 devoted to the reckoning of the probabilities about such reasoning that the credibility of the more unitary hypothesis is far superior.

Peirce: CP 7.410 Cross-Ref:††

Now the new theory makes the whole action of the soul, so far as it is subject to law, consist of nothing but taking up and letting drop in ceaseless alternation. The old theory is obliged to recognize the same processes, and besides supposes, quite unnecessarily, an additional elementary process of passing over from A to B by a saltus. *Entia non sunt multiplicanda praeter necessitatem* is a maxim of science, which, we shall find, applies to forbid all unnecessary elements of hypothesis.

Peirce: CP 7.411 Cross-Ref:††

411. (31) Several philosophers of high rank consider the analogy between association and attraction to be a true and intrinsic one. Whether it really is so or not is doubtful; besides attraction is a little out of fashion among the physicists of our day. Studies of elasticity, in particular, have undermined its philosophical ascendancy. Still, taking the analogy for what it is worth, it is certainly much closer with the new theory. For according to the old theory, while A attracts B, B not only repels A, but repels it with such violence that A goes as soon as B begins to appear. The law of action and reaction would forbid such a phenomenon in physics. We there see one thing strike against another and knock it out; but we do not see it do this by virtue of the attraction of that second thing. On the new theory on the other hand, a wave of condensation brings B up to A, when the completion of the undulation carries A away.

Peirce: CP 7.412 Cross-Ref:††

412. (32) Of a good deal more significance are analogies with physical changes in general; and these are two. Under (321), we consider the saltus involved in the old theory; under (322) the rhythm involved in the new theory.

Peirce: CP 7.413 Cross-Ref:††

413. (321) Nowhere in nature is there the slightest reason to believe that any *saltus* takes place during changes. The more we learn of physics, the more we are led to exclude such hypotheses. Nor is there the slightest appearance of the phenomena of the mind being more sudden than those of matter. On the contrary the general evidence of experimental psychology is that mental actions are particularly gradual

and gentle.

Peirce: CP 7.413 Cross-Ref:††

The old theory, understood as opposed to the new, supposes that at every instant up to a certain instant A is present and B absent, and at every instant after the same instant A is absent and B present.

Peirce: CP 7.413 Cross-Ref:††

According to the doctrine of limits, used in the calculus, that is, according to the geometer's conception of time and space, it would follow that *at* the limiting instant A and B were both present and absent. Now, it may be that time is not really continuous in the way in which the mathematician takes it to be; but logic will not justify the hypothesis that it is not continuous.

Peirce: CP 7.413 Cross-Ref:††

Everywhere in nature things that appear and disappear do so little by little. For instance, we can suppose the idea A fades out of attention and out of consciousness, while B quickly grows prominent. But, there, both must be present at once, or else A must be entirely extinct before B begins to appear. It is far more rational to suppose both are present, at once; first, because it is more natural and analogical to suppose A acts as a cause while it is in existence, second, because quantitative experiment shows that A would not naturally fade out quickly until B is already present to accelerate the fading.

Peirce: CP 7.414 Cross-Ref:††

414. (322) Everywhere in the world of life there is an alternating motion, an inspiration and expiration. Now according to the new theory, the whole activity of the mind consists of a drawing in and dropping out. One thing's moving in just as another moves out might be supposed in the case of an incompressible fluid not subject to diffusion; but there is no reason to think it really takes place in any case.

Peirce: CP 7.415 Cross-Ref:††

415. (33) The changes in the significations of words obey, of course, the law of association; and having been studied until of comparatively recent years as facts of observation, in themselves, without prejudice from psychological theory, they are independent evidence. Now, philologists, from their observations of such changes, have reduced them to successive broadenings and narrowings. The broadenings consist in the taking in of new ideas, the narrowings in the dropping out of little-used ideas.

Peirce: CP 7.416 Cross-Ref:††

416. (34) The Herbartians have made the most elaborate and minute studies of association of any psychologists, except the somewhat desultory but very extended studies of the English associationalists. Now, the Herbartian theory of apperception is precisely the theory that A suggests AB. It perfectly agrees with the present theory, only that the Herbartians do not admit that A is ever dropped, again. AB becomes what may be figured as AB, the A becoming faint; but even this is, according to them, unusual. They hold that the ordinary phenomenon is rather AB.

Peirce: CP 7.417 Cross-Ref:††

417. (41) Finally, there is a good reason why A should not disappear, until B has been present for some time. For ideas persist in consciousness for a long time after they are gone from the field of easy attention. See Hartley's Proposition 3, a doctrine elaborately studied by the modern school.†21

Peirce: CP 7.418 Cross-Ref:††
§4. PSYCHOLOGICAL TRUTHS NEEDED IN LOGIC

418. Reasoning is performed by the mind. Hence, the logician must not be entirely neglectful of the science of mind. This science, more than any other, is embarrassed by metaphysical puzzles. The attempt to avoid these only leads to careless and noxious solutions of them. What are the observed facts of psychology? They are the behaviour of men, especially what they *say*, together with our observation of what takes place within us, termed *introspection*.

Peirce: CP 7.419 Cross-Ref:††

419. What is the nature of introspection? The pre-scientific psychologists, such as Hamilton, believed that introspection was infallible. "To know, and to know that we know, are one and the same thing," said that philosopher.†22 They also had the naïve opinion that the things which are immediately present to consciousness are the things which introspection reveals. If those psychologists had been at all acquainted with the experimental sciences, they would have seen (what they would, doubtless, readily have acknowledged) that in these respects psychology, as they conceived it, was radically unlike any other observational science. The things we observe in a physical science, say in astronomy, are not the elementary facts, at all. Kepler, for example, was not, as even J.S. Mill seems to fancy, provided, in the observations of Tycho, with the real places of the planet Mars, by the study of which he made out two of his three laws. No astronomer can directly *observe* the situation of a planet relatively to the sun. He only observes, the secondary and derivative fact, that the planet as viewed from the earth, and subject to aberration and the equation of time, is in such and such a *direction* at such a time. According to the method of observation, this direction will be more or less affected by refraction, etc. Moreover, the astronomer is forced to recognize that every single observation he makes is more or less affected by *error*. Those errors have to be corrected by reasoning whose only premises are the erroneous observations themselves.

Peirce: CP 7.420 Cross-Ref:††

420. Now the truth is that the data of introspection are in these respects altogether analogous to those of external observation. Introspection does not directly reveal what is immediately present to consciousness, at all; but only what seems to have been present from the standpoint of subsequent reflection. It does not even tell what the normal appearance from this subsequent standpoint is, without its testimony being falsified at all times with serious accidental errors.†23

Peirce: CP 7.420 Cross-Ref:††

We cannot directly observe even so much as that there is such a thing as present consciousness.

Peirce: CP 7.421 Cross-Ref:††

421. We set out with a mass of opinions about the mind, which are not testified to by introspection. They are simply man's natural psychology. It is instinctive, no doubt, in part; for we can see that the lower animals have virtually such a psychology. In part also, it grows up nearly the same in all men as the natural result of similar experiences acting upon similar understandings. There is not the smallest

reason for supposing that that natural psychology is at all accurately true; on the contrary, it probably involves great errors. At the same time, its authority must be allowed to be very high indeed in regard to all features which are of importance in the conduct of life; for, on the whole, man has prospered under such beliefs.

Peirce: CP 7.422 Cross-Ref:††

422. A part of this instinctive science, as we may call it, is that events succeed one another in time, that the past, when not too remote, is remembered, that the future, when not too remote, can be with some probability conjectured or anticipated, and that a single moment between the past and the future, (that is, *some* facts belonging to that *moment*), is directly before the mind.

Peirce: CP 7.423 Cross-Ref:††

423. This is the first item of our instinctive psychology. It makes a distribution of knowledge which has the closest bearing upon practical life; and to doubt its practical truth would be idle. At the same time, we have no reason at all for presuming it to be minutely true. Indeed, criticism shows that minutely examined it will not hold water; it is at issue with itself. The justification for this statement will be adduced in a subsequent chapter.

Peirce: CP 7.424 Cross-Ref:††

424. Meantime, we shall have to assume that, practically speaking, there is a flow of ideas through the mind, that is, of *objects*, of which we have the barest glimpse while they are with us, but which are reported by memory after they have been associated together and considerably transformed; and this report, though not very accurate, is substantially acceptable as correct.

Peirce: CP 7.425 Cross-Ref:††

425. Still, we must never forget that when we say, this idea is *the same* [that] I had yesterday, this idea *resembles* that, this idea *involves* or *contains* that, these are not things that are true of the ideas in their presentation. They suppose a mind, -- *our* mind, -- in which it seems that the workings of ideas involve those judgments. Perhaps, some of them would be true for all minds, and in that sense of the ideas themselves; but that cannot be averred at the outset. The point to remember is, that whatever we say of ideas as they are in consciousness is said of something unknowable in its immediacy. The only thought that is really present to us is a thought we can neither think about nor talk about. "Of thine eye I am eyebeam," says the Sphinx. We have no reason to deny the dicta of introspection; but we have to remember that they are all results of association, are all theoretical, bits of instinctive psychology. We accept them, but not as literally true; only as expressive of the impression which has naturally been made upon our understandings.

Peirce: CP 7.426 Cross-Ref:††

426. By the time we can examine our ideas at all, we find the process of combining them into sets has begun. But we seem to be able to discriminate roughly between a *matter* of cognition, as Kant calls it, forced upon us by the mysterious power without and within, and a skeleton of a *set*, in providing which we feel as if we have had a *comparative* freedom, which skeleton is nearly what Kant means by the *form* of cognition. For example, I hear at this moment a bird calling. I think he is on a lilac-tree close by the verandah. Every time he calls, I seem to see the bird. It is not much like *seeing*, but still it is a visual idea. Now that visual idea I think of as the bird *itself*, and the *call* I think of as something appertaining to that idea. Though the association is quite involuntary, I could banish the visual idea if I chose. Yet *that*, I

recognize as forced upon my belief by experience. I cannot help believing there *is* a bird there, that would look something as I imagine it to look. But I have besides the visual idea of the bird and the sound of the call, a skeleton idea of *connection between two things*. It is a dim idea in itself; but if I want to think about it, I have a

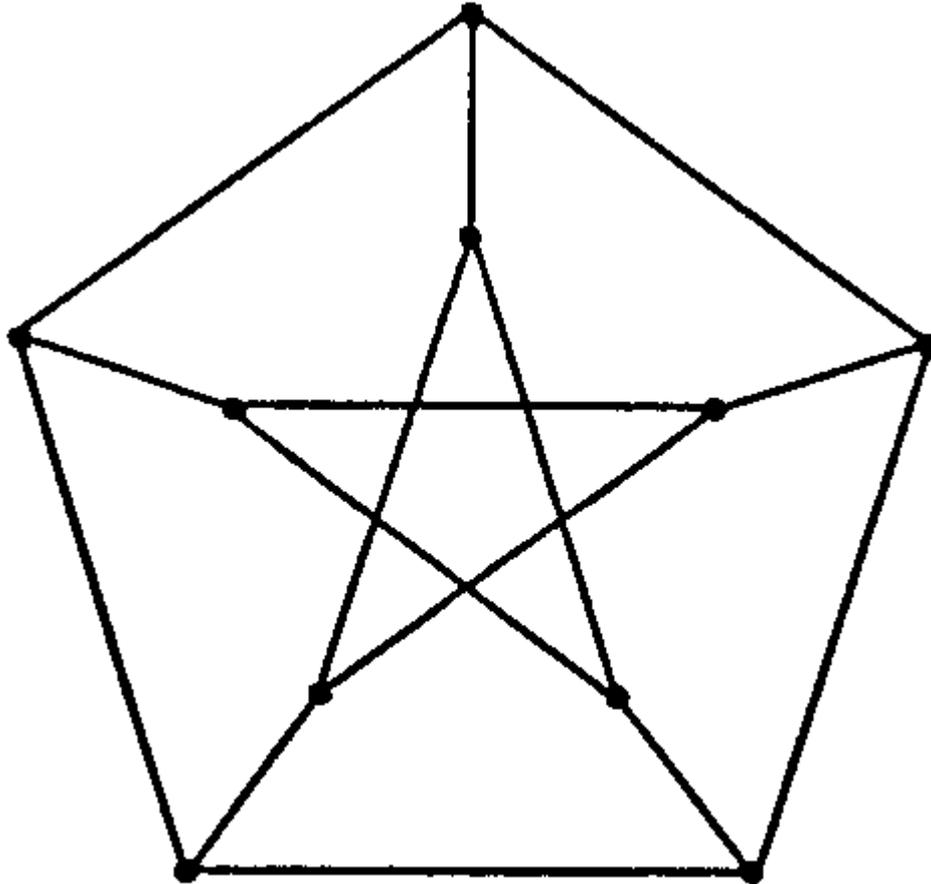


visual idea [\[Click here to view\]](#) of two dots connected by a line, or of a knot in a string. However, when I just think of the bird calling, I do not think the idea of connection so distinctly. Nevertheless, I *do* think it, and think of the call and the visual bird as belonging to it. Under ordinary circumstances, I might not remark the idea of connection; but *potentially* it would be there, that is, it would be all ready to be called into existence, as soon as there should be need of it. Before me on the table is a nearly cubical box, containing a photometer. As I look at it, I see three faces. I not only see them, at once, which associates them in contiguity; but I regard them as coming up to form a square corner, and thus associate them by means of a skeleton

idea  [\[Click here to view\]](#) of a triplet.

Peirce: CP 7.427 Cross-Ref:††

427. In all association, even by contiguity, the potential idea of the *form* of the set is *operative*. It is the instrument without which the association would take no hold upon the mind. It is not necessary that the formal idea should be clearly apprehended. As to the metaphysics of the causation, I do not care a straw for that. If any nominalist fancies it is more philosophical to say that the force of mind which can produce the idea of the set, has to operate to produce the association, I am not concerned at present to enter into that discussion. Only this must be insisted upon, that the skeleton of the set is something of which a mathematical diagram can be made. It is something in itself intelligible; though it is not necessary that it should emerge into the field of easy attention. For example, if a mind is under the influence of the skeleton idea of connection shown in the accompanying figure of the ten dots,



[Click here to view] then upon seeing two triangles in perspective,†24 without thinking at all of this figure, or much of any form of connection, that mind will be pretty sure to regard the six rays making the two triangles, the three rays to the eye, or centre of perspective, and the ray of intersection of the two planes as forming a set of ten rays, an idea which certainly would not occur to a mind which was not so dominated by the form of connection.

Peirce: CP 7.427 Cross-Ref:††

By subsequent examination of the set of associated ideas, namely, the ten lines, the form of their connection might be perceived.

Peirce: CP 7.428 Cross-Ref:††

428. Looking out of the window, I see the cow whose milk we generally drink. There are certain difficulties which have occasioned a good deal of thought, so that I imagine I see a boy sitting by the cow milking her. The boy, and the stool, and the pail are added to my idea. Thence, I imagine that boy carrying the pail to the house. The cow and stool have dropped out. The straining of the milk presents itself to my imagination. A bowl is there and the pail. The boy is standing by; but I lose sight of him. I am following along the train dramatically, that is, following the interesting history.

Peirce: CP 7.429 Cross-Ref:††

429. As I followed that series of events in my mind (as I did; for I do not take make-believe observations), there was always something identical carried along. The boy going up to the house with the pail, was thought as that same boy, the pail that

same pail, and the occasion that same occasion that I had just before been thinking of. The new ideas must, therefore, have always been taken in before the old ones were allowed to drop. By the time the milk was straining in imagination I had already begun to think that it would be good for my wife, who is threatened with nervous prostration.

Peirce: CP 7.430 Cross-Ref:††

430. To one skeleton-set another is added to form a compound set. Then, the first, perhaps, is dropped and the ideas which remain are viewed in a new light.

Peirce: CP 7.431 Cross-Ref:††

431. I saw a lady yesterday. I had not seen her since one evening when she and her husband drove up as my wife and I were standing at the well. A handsome man! He and she are both very fond of his nephew who seems to us too a charming fellow; and only yesterday my wife showed me a newspaper-paragraph that he had been arrested for debt. So his centi-millionaire cousin paid his tailor's bill of \$5000. The last time I saw him he was hardly presentable. So my thoughts ran on in spite of me. First, the lady. Then, she in a set with her husband, my wife, myself, the buggy, and the well. Then mingles with this set another, the lady, her husband, my wife, myself, the nephew, and his agreeability; now that nephew brings in something concentrating him, my wife, myself, the newspaper. I forget the others. Then, him, and that Newport house, and the Croesus cousin. The skeleton-sets themselves I do not think particularly about; but they are operative. The marriage relation, the familiar intercourse of people in the country, the relation of handsome nephew to a message; newspaper-publications about people; the relation of a man of stupendous fortune to his cousins. All of these skeleton-sets, though not attended to, influenced my thought; and they followed one another by the same alternating process of taking up and dropping.

Peirce: CP 7.432 Cross-Ref:††

432. When these skeleton-sets were joined intermediately to the passage from one to another, these connections of them had *their* skeleton-sets. But these latter were all of that simple form expressible by the sign +, and had no specific character. There are, however, cases in which the connections of the skeleton-sets have skeleton-sets of more interest.

Peirce: CP 7.433 Cross-Ref:††
§5. THEORETICAL INTEREST

433. Interest has been spoken of as either connected with voluntary action or as emotional. We must not, however, lose sight of the fact that there is such a thing as *theoretical* interest, which has reference not to outward action but to the voluntary agency we put forth in directing our own ideas. Whether or not this is muscular is, as far as I can see, of no great importance for our purpose. It would be a pretty large hypothesis to suppose that we could contract and expand the arteries of our brains by a direct exertion. Let us rather content ourselves with acknowledging that this sort of voluntary action is of a nature not understood. It exists; there is a corresponding kind of interest referring to it; and there is a kind of attention, or heightening of consciousness in the initial stage of suggestions interesting in that way.

Peirce: CP 7.434 Cross-Ref:††

434. An immense number of associations are formed, and remain as long as they endure, in the background of consciousness, that is, in subjective obscurity. But as soon as a cerebro-motor suggestion is made, -- that is a suggestion of the idea of voluntarily exercising thought, -- the whole set brightens up. At the same time the action of forming and annulling sets, -- say the metabolism of thought, -- becomes more active. So that our instinctive psychological explanation is that the heightened consciousness is an *agent* that performs the action. If there is nothing in this but a word, we need not quarrel about it; but it would seem that as a matter of dynamics heightened consciousness, or attention, is nothing but a concomitant of the idea of voluntary action, and that it is the approach to a discharge of the excitement out of the cerebro-spinal nerves (although *upon* the same nerves again,) which accelerates the movement of thought, somewhat as the current of a stream is accelerated in approaching a cataract.

Peirce: CP 7.435 Cross-Ref:††

435. But when we undertake to give an account of any train of thought, as I have done in a few examples, when we say such and such ideas *occurred* to us, it is not at all true that they then came into consciousness. All that is true is, that at those moments they became connected with the idea of a voluntary act of thought, and consequently, became subjectively vivid enough to find a place in our narrative. In my train of thought about the cow, I have no doubt the idea of *doing something* to help my wife was what made me notice the creature at all, and caused my thought to be active in that direction. The set *wife-milk* was in the deeper shaded part of consciousness. The set *cow-milk* joined itself to this and gave *wife-milk-cow*, and thence *wife-cow*. This did not emerge into the glare of attention but was working all the time.

Peirce: CP 7.436 Cross-Ref:††

436. The subjective vividness which an idea gets from exciting interest is very transient. If it connects itself permanently with an enduring interest, it is very soon found that it has *less* vividness than it would naturally have had if it had not been connected with that interest. For instance, we have the most erroneous notions of how words sound in our own mouths and those of others. If we did not know the language the mere total impression of the peculiarity of sound, I do not say the analysis of the articulations, but the general character of the noise, would be much more vividly apprehended. This may be explained by remembering that the impression we speak of means our *recollection* of the impression; now this is the sum of the effect it makes while it is present. But if the idea is interesting, it suggests another so quickly, that it is present itself but a very short time; and its total effect does not amount to much. It is like a thermometer dipped for a moment into very hot water and immediately withdrawn. It is hardly affected at all. Still, however the fact be explained, it remains true that old ideas, while they have generally gained in interest and suggestiveness, have lost subjective vividness. On the other hand, an idea that is new and fresh seems quickly to gain vividness from that circumstance. Hence we say that as the excitation spreads it loses intensity.

437. The examples which have hitherto been dwelt upon are instances of the free play of imagination. But in order to become aware of the whole range of association, or of that side of it which is of most interest to the logician, we have to consider *experience*.

Peirce: CP 7.437 Cross-Ref:††

Experience may be defined as the sum of ideas which have been irresistibly borne in upon us, overwhelming all free-play of thought, by the tenor of our lives. The authority of experience consists in the fact that its power cannot be resisted; it is a flood against which nothing can stand. The maxim that we ought to be "guided" by experience amounts to this, that what we have got to yield to at last we shall economically do well to be submissive to from the first. "Guided" is too egotistical a word.

Peirce: CP 7.438 Cross-Ref:††

438. We naturally make all our distinctions too absolute. We are accustomed to speak of an external universe and an inner world of thought. But they are merely vicinities with no real boundary line between them. It comes to this: there are some ideas, --*objects*, be it remembered, -- which will have their own way, and we cannot swerve them much, and the little effect we can produce upon them we only produce indirectly. They make up or indicate the outward world. There are other ideas which seem very docile, they are just as we think they ought to be. They form the inner world. Yet it will be found that the inner world has its surprises for us, sometimes. It isn't so exactly as we would have it as we fancy. It is rather our wishes which conform to it, Mahomet that repairs to the mountain. Neither is the moderate amount of control which we exercise upon the world of ideas nearly so direct as we fancy it to be. We go about instinctively, and without being aware how circuitously we proceed to change the current of thought. There is an intermediate world, our own neighborhood, household, and persons, which belongs to us, which we sometimes feel inclined to class with the outer world and sometimes with the inner world.

Peirce: CP 7.439 Cross-Ref:††

439. Experience being something forced upon us, belongs to the external type. Yet in so far as it is I or you who experiences the constraint, the experience is *mine* or *yours*, and thus belongs to the inner world.

Peirce: CP 7.440 Cross-Ref:††

440. Experience is double, as much as reality is. That is, there is an *outward* and an *inward experience*. Under the latter head ought particularly to be reckoned a mathematical experience, not usually so called, which has compelled the development of pure thought to take a determinate course.

Peirce: CP 7.441 Cross-Ref:††

441. There is also an emotional experience, which has all the authority of any experience, provided it is equally irresistible. But experience and its irresistibility has a *public* character, which we shall study in another chapter.

Peirce: CP 7.442 Cross-Ref:††

442. Under the influence of association, the lash of experience needs only to be shown to us to cause us to submit. Now, there are indications by which we recognize the experiential character of certain ideas. One of these is the glowing blaze

of their subjective vividness; but there are others, besides.

Peirce: CP 7.443 Cross-Ref:††

443. When an idea bearing the stamp of experience suggests another, that other in many cases itself carries that same stamp, which is carried forward in suggestion and thus a derivative authority from experience is conferred upon an idea which may have neither the vividness nor the other marks of directer experience. This sort of suggestion is *inference*. The law of association will divide inference into inferences by contiguity and inferences by resemblance, meaning by these latter inference from the occult inward nature of ideas or of the soul.

Peirce: CP 7.444 Cross-Ref:††

§7. UNCONTROLLED INFERENCE

444. All inferences are really performed under the influence of the law of association. But all psychical actions divide into two great classes, those which are performed under the *uncontrolled* governance of association and those in which by the "agency" of consciousness, -- whatever that may mean, -- the actions come under self-criticism and self-control. The latter class of actions may be pronounced *good* or *bad*; the former could not be otherwise than they were.

Peirce: CP 7.445 Cross-Ref:††

445. Uncontrolled inference from contiguity, or experiential connection, is the most rudimentary of all reasoning. The lower animals so reason. A dog, when he hears his master's voice, runs expecting to see him; and if he does not find him, will manifest surprise, or, at any rate, perplexity.

Peirce: CP 7.446 Cross-Ref:††

446. Inference from resemblance perhaps implies a higher degree of self-consciousness than any of the brutes possess. It involves somewhat steady attention to qualities, as such; and this must rest on a capacity, at least, for language, if not on language, itself. Primitive man is very industrious in this sort of reasoning. Mythology, says Major Powell, is the philosophy of the Savage. It is certainly composed of inferences from resemblance. Our ancestors saw something man-like in the Sun, and could even tell what sort of a character the Sun-God's was.

Peirce: CP 7.447 Cross-Ref:††

447. Our daily life is full of involuntary determinations of belief. It is the egotism of the ego, or field of attention, which imposes upon [us] with its High German modest conviction that whatever is known is known through it. It is not so. I converse with a man and learn how he is thinking: I fancy he has *told me*, that is, has "stated" the fact in accurate forms of speech. But he has not, and how I have found out his thought is too subtle a process for this psychologist writing to find out. You hear a new slang word: you never ask for a definition of it; and you never get one. You do not get even any simple example of its use; you only hear it in ironical, twisted, humorous sentences whose meaning is turned inside out and tied in a hard knot; yet you know what that word means much better than any abstract definition could have informed you. In riding a horse; rider and ridden understand one another in [a] way of which the former can no more give an account than the latter.

Peirce: CP 7.448 Cross-Ref:††

448. Such inferences are beyond the jurisdiction of criticism. It is the part of psychology to explain their processes as it can; but, as long as they are out of the focal plane of consciousness, they are out of our control; and to call them good or bad were idle. The ordinary business of life is, however, best conducted without too much self-criticism. Respiration, circulation, and digestion are, depend upon it, better carried on as they are, without any meddling by Reason; and the countless little inferences we are continually making, -- be they ever so defective, -- are, at any rate, less ill performed unconsciously than they would be under the regimen of a captious and hypochondriac logic.

Peirce: CP 7.449 Cross-Ref:††

449. Quite otherwise is it with the actions which carry out our grander purposes. Here all must be voluntary, thoroughly conscious, based on critical reflection. Logic is wanted here, to pull inferences to pieces, to show whether they be sound or not, to advise how they may be strengthened, to consider by what methods they ought to proceed.

Peirce: CP 7.450 Cross-Ref:††

450. Intermediate between the lesser and the greater inferences lies a class which are best governed by habits, yet by habits formed or corrected under conscious criticism. Within a man's own special profession, his habits of thinking will in the natural course of things have been subjected to a good deal of criticism, -- perhaps not remarkably intelligent, yet at all events based on experience. Outside of that narrow beaten path, if he has not studied logic, his habits of thinking will have been carelessly formed. If he has been educated under the traditional logic, they will have been formed under an influence positively baleful.

Peirce: CP 7.451 Cross-Ref:††
§8. ASSOCIATION AND INFERENCE †25

451. In the absence of external impressions, thoughts chase one another through the mind in a sort of Bacchic train. Each suggests another. After a while, the clear train of thought is broken, the ideas remain scattered for a time, and then reconcentrate in another train. Psychologists recognize that the suggestion of one idea by another may take place according to either one of two different principles; for an idea may suggest another like it, or it may suggest another which has been connected with it in experience. Thus, the thought of Niagara may suggest a hero or anything else that is grand, and so similar to the cataract, or it may suggest a crowd of importunate hackdrivers, which is connected with the place in every visitor's experience.

Peirce: CP 7.452 Cross-Ref:††

452. Association of the latter kind, association by *contiguity* as it is called, is the more typical. In it the characteristics of mental association are more strongly marked. Association by similarity is related to association by contiguity somewhat as our inward consciousness is related to outward experience; the one association is due to a connection in outward experience, the other to a connection in our feelings. Many psychologists have proposed to reduce association by similarity to a special kind of

association by contiguity; few have been inclined to reduce the latter principle to the former.

Peirce: CP 7.453 Cross-Ref:††

453. Suggestions of these two kinds characterize not merely dreams and dreamy meditations, but also thoughts referred to the real world, or in technical language *categorical judgments*. Association is the only force which exists within the intellect, and whatever power of controlling the thoughts there may be can be exercised only by utilizing these forces; indeed, the power, and even the wish, to control ourselves can come about only by the action of the same principles. Still, the force of association in its native strength and wildness is seen best in persons whose understandings are so little developed that they can hardly be said to reason at all. Believing one thing puts it into their heads to believe in another thing; but they know not how they come by their beliefs, and can exercise no control over the inferential process. These unconscious and uncontrolled reasonings hardly merit that name; although they are very often truer than if they were regulated by an imperfect logic, showing in this the usual superiority of instinct over reason, and of practice over theory.†26 They take place like other mental suggestions according to the two principles of similarity and connection in experience.

Peirce: CP 7.454 Cross-Ref:††

454. Inference from connection in experience is most rudimentary of all reasoning. The lower animals plainly reason in this way. The dog, when he hears the voice of his master runs expecting to see him, and if he does not find him will manifest surprize, or at any rate perplexity. This is as good an example of inference from connection in experience as could easily be given.

Peirce: CP 7.455 Cross-Ref:††

455. Inference from resemblance probably implies a higher degree of self-consciousness than any of the brutes possess. It involves a somewhat steady attention to qualities as such; and this must rest on the capacity for language, if not on language itself. Primitive man, however, reasons in this way; for mythology is built of such inferences. Our ancestors saw something manlike in the sun, and could even tell what kind of a man the sun-god was.

Peirce: CP 7.456 Cross-Ref:††

456. But we need not go to the lower animals nor to savages for examples of associational determinations of belief. Our daily life is full of such phenomena. We have the naïve idea that our beliefs are principally determined by the exercise of our conscious intellect; but it is not so. I converse with a man and learn how he is thinking: I fancy that he has told me, but he has not. I hear a new slang word, but I need not ask for a definition of it; I understand its meaning much better without a definition, I know not how, than I should with one. In riding a horse, I understand him and he understands me; but how we understand one another I know hardly better than he.

Peirce: CP 7.457 Cross-Ref:††

457. All such inferences are, of course, beyond the jurisdiction of criticism. It is the part of psychology to explain their processes as it can; but as long as they are out of consciousness, they are out of our control and it is idle to call them good or bad. We may, however, say that the ordinary business of life is best conducted without too much self-criticism. Just as our respiration, circulation, and digestion are far better carried on by involuntary than they could possibly be by voluntary actions,

so the countless little reasonings which we are continually making, -- although they may often be defective, -- are nevertheless much better performed unconsciously than they would be if we were to try to interfere with them by a captious and hypochondriac logic. It is very different with the actions which we undertake in order to carry out our grander purposes. Here, all must be voluntary, thoroughly conscious, and based upon the most critical reflection. Here logic is wanted, to pull to pieces our inferences, to show whether they are good or bad, how they can be strengthened, and by what methods they ought to proceed. Intermediate between the lesser and the greater inferences, lies an intermediate class which are best governed by habits, yet by habits formed or corrected under conscious criticism. Within a man's own special profession, his habits of thinking will have been subjected to a good deal of criticism (not necessarily the most intelligent, but based on experience); outside of that, if he has not studied logic, his habits of thinking will have been carelessly formed, or if he has studied the traditional logic, they will have been formed under an influence truly baleful.

Peirce: CP 7.458 Cross-Ref:††

458. Reasoning unconsciously can hardly be called reasoning at all. As long as I simply find myself seized with a belief, without being able to give any account of how I came by it, logic has nothing to say except to warn me of the extreme danger that I shall err.

Peirce: CP 7.459 Cross-Ref:††

459. Reasoning proper begins when I am conscious that the judgment I reach is the effect in my mind of a certain judgment which I had formed before. The judgment which is the cause is called the *premise*, that which is the effect is called the *conclusion*. When I am aware that a certain conclusion which I draw is determined by a certain premise, there are three things which I have more or less clearly in my mind. First, I have a peculiar sense of constraint to believe the conclusion, connected with a sense that that constraint comes from the premise; second, I have a conception that there is a whole class of possible analogous inferences (though I may not be able to define the class) in which a similar constraint would be felt by me; and third I have a present belief that all of these inferences, or at least the great body of them would be true.

Peirce: CP 7.460 Cross-Ref:††

460. The lowest kind of conscious reasoning is where I know what the premise is from which my belief in the conclusion follows, and I feel that it follows upon some principle, -- technically called the *leading principle* of the inference, -- but I do not distinctly know what that principle is. Such an inference is called a *simple consequence*.

Peirce: CP 7.461 Cross-Ref:††

461. Such inferences are common enough. Uneducated people seldom reason in a higher way; and educated people reason so very often. Since the object of reasoning is merely to arrive at the truth, if the leading principle of our inference be really true, it is not necessary for us to know it, for in that case the mode of inference based upon it can in no case carry us away from a true premise to a false conclusion. Such a mode of inference is *valid*, that is, its leading principle is true; but it is only *materially valid*, that is, valid because as a matter of fact the leading principle happens to be true; it is not *logically valid*, that is, the leading principle might be

false. Although the simple consequence may be valid, logic condemns it. A conclusion may be true, though the inference by which it was reached was invalid; that is, it may chance to be true. But the inference is condemned because other conclusions similarly drawn may be false. In the same way, a simple consequence may happen to be valid; but since the leading principle is not recognized, there is no security for its validity, and the next simple consequence drawn, though indistinguishable from the first (the leading principle being unrecognized), may very likely fail.

Peirce: CP 7.462 Cross-Ref:††

462. Simple consequences have occasionally been introduced into philosophy. The most remarkable instance is the *Cogito, ergo sum* of Descartes, who wished every philosopher to begin by doubting everything without exception. But even in doubting everything he must, says Descartes, be aware that he doubts, that he thinks. Now from this belief that he thinks he is led by a blind but irresistible constraint to believe in his own existence. This inference, "I think, therefore I am," is a simple consequence, for as long as the philosopher doubts everything he can have no fuller reason to give. He cannot, for example, say that thought supposes a thinker, for he is bound as a Cartesian to doubt that among other things at the outset. Such is the doctrine of Descartes. Of course, were there nothing to check the absolutely resistless force of a belief, logic must be silent, *Leges silent inter arma*. But in point of fact, no belief is found thus absolutely irresistible. There is always room for the reflection that an error may have been committed.†27†28

Peirce: CP 7.463 Cross-Ref:††

§9. ASSOCIATION AND THE LAW OF MIND †29

463. In reflecting upon the work which has hitherto been done upon association, we are led to remark how many writers have been led astray by futile attempts to seek guidance for their psychological studies in physiological hypotheses. Their idea has, no doubt, been that physical facts are more tangible, certain, and easily intelligible than psychical facts; and that knowledge of the less easily ascertained truths ought to be based upon an acquaintance with more easily ascertained truths. How far is this just? The present writer holds that in advance of positive knowledge, the presumption ought to be that there is such a unity in the universe that the difference between mental and natural phenomena is only a difference of degree. Presumably, the same elements are in both; and if so, *so far* there is no essential difference in their intelligibility. But upon the opposite supposition, namely that the phenomena of mind are essentially more complicated than those of matter, the study of mind through physiology must necessarily be misleading. It may be said that, at all events, we do understand physical phenomena the best. This, however, is open to doubt. Those physical phenomena which we really understand, the motions of the planets and the like, are quite exceptionally simple; and the assumption that all physical phenomena are regulated by the laws of mechanics, which since Helmholtz has been the ruling idea of science, appears in the light of the logical rules to be developed in this treatise, as a good "working hypothesis," but as a dogmatic proposition, -- in which way it is used, when a method of psychological investigation is founded upon it, -- as exceedingly improbable. Nothing is less understood than the action of the nerves, in the sense in which they would require to be understood for

such a purpose. The psychological phenomena, on the other hand, are revealed to us in the complicated process of "introspection," in language, in behaviour, and in our instinctive, and (we may presume) approximately correct, notions of the way of the mind, with an eminent degree of clearness. This method is far more accurate and scientific than the other, at least, in the present stage of the inquiry.

Peirce: CP 7.464 Cross-Ref:††

464. Our object is to formulate the law of mind. We have to consider all mental action whatsoever and, generalizing it, to say not what all the elements of it are, but what that element of it is which is *legislative*. "Generalization," is the old answer. Pretty well that for a first essay; but open to the following objections. First, it is logical, it contemplates only the product, while what we aim at is the psychological law, relating to the action itself; second, it confines itself to the logic of non-relative terms, and is therein very imperfect, indeed; third, it is a meagre word, while we want the whole process with all its characteristics at large. Still, we remark that all attempts to say what mind does, that it reduces the manifold of sense to unity, that it assimilates, that it shapes action to purposes, have one thing in common.

Peirce: CP 7.465 Cross-Ref:††

465. A feeling is an element of consciousness just as it immediately is in the moment when it is there for itself and not as delegate of some other feeling not present. Such a feeling is not a psychological datum. The data are highly complex. That there is a cream colored surface with black characters on it is as near as I can readily describe the datum of my consciousness at this minute, -- but in truth the moment I pick it to pieces, as I must do to describe it, it ceases to be a datum. As for the pure feeling, *that* is a hypothetical entity, and is as completely veiled from me by its own immediacy as a material particle, as it exists in itself, is veiled by the somewhat absurd requirement that it shall be considered in itself. The truth is there are no data. We have a lot of inferences from data, liable to error, and these we have to correct as best we can by putting them together. The state of the case is quite similar to that of a physical science, say astronomy. All we have to go upon in astronomy is observations, and all those observations are erroneous. But we collect them and take their means and find a general description of the path of the observed object; and from this we can calculate an ephemeris, and finally, if there is any interest in doing so, ascertain what those observations ought to have been. We can no more start with immediate feelings in psychology than we can start with accurate places of the planets, as affected by parallax, aberration, refraction, etc. in astronomy. We start with mediate data, subject to error, and requiring correction.

Peirce: CP 7.466 Cross-Ref:††

466. The mind pronounces that what I see now resembles something I saw yesterday. The whole aspect of things as flowing in time is, it is plain, virtually a *theory* of the mind's creation. But, for the present, we take that theory as *true*, that is, as a stable one. Taking it as true, it seems to provide no possible means by which the mind could compare what is present to it now with what is past and gone and done with. This compels us to say that the time idea, -- at least, in its first crude shape, -- needs correction, like an erroneous observation in astronomy. Examining it more carefully, we observe that the idea is that the series of instants of time is *continuous*. Analyzing this idea of continuity, as we shall do in a future chapter with the most minute accuracy which an improved art of logic puts at our disposition,†30 we find that it implies that there are instants *infinitesimally* close together; that is that there are durations of time so short, that every one such starting with a given date has a

character exactly like the one before it in some respect, without any limitation to this rule, while yet a time a little later does *not* possess that character. This enables us to suppose that the consciousness is not limited to a single instant but that it *immediately* and objectively extends over a lapse of time, without thereby extending over any sensible lapse of time. We are thus able to suppose that consciousness is carried along from one time to another, and is able to compare what is present to it at different times. Such we may suppose to be the process of memory; and this is the account of it which best squares with those natural beliefs which are all the data the psychologist can possibly have upon which to found his science, corresponding, as they do, to the observations of the astronomer.

Peirce: CP 7.467 Cross-Ref:††

467. But granting that memory is thus justified, -- while errors may, of course, creep in during the process, -- it still remains that when the mind declares that what it sees now, or remembers to have seen yesterday, is *like* what it remembers to have seen last week, the *likeness*, which though accompanied like all mental processes with a peculiar and characteristic sensation, is mainly a *fact*, a mental fact, and the sensation of it is of no consequence except as an advertisement of that fact. That fact is that by virtue of the occult working of the depths within us, those two feelings coalesce into one notion. For the sake of calling this by a familiar name, I call this association by similarity. But the ideas united by virtue of an occult inward power, are not always regarded as *similar*. Contraries are also so joined. Ideas and feelings are so joined which are neither merely declared by the mind to be similar nor to be contrary. Such, for instance, are length, breadth, and thickness. The mind delights in triads. In general, what the mind pronounces is that the feeling or idea of yesterday and that of today belong to one system, of which it forms a conception. A concept is not a mere jumble of particulars, -- that is only its crudest species. A concept is the living influence upon us of a *diagram*, or *icon*, with whose several parts are connected in thought an equal number of feelings or ideas. The law of mind is that feelings and ideas attach themselves in thought so as to form systems. But the icon is not always clearly apprehended. We may not know at all what it is; or we may have learned it by the observation of nature.

Peirce: CP 7.468 Cross-Ref:††

CHAPTER 3

HABIT†1

§1. LAWS OF PHYSICS

468. The books on physics are replete with examples of what they call "empirical laws," that is to say, formulas which are satisfied as nearly as men have succeeded in observing the facts and under certain limited circumstances, but which nobody supposes go down to the roots of existence, or to exhibit the general forms of

all phenomena. They are, on the contrary, supposed to be merely special modifications which the universal formulæ assume under special conditions. Of such a pseudo-law centrifugal force affords a good example. When a railway-train moves round a curve, there is always a pressure away from the centre of curvature. It must be so; for since a body not subjected to any force naturally moves in a right line, while this railway-train does not so move, it follows that the guiding rail exerts a force upon it in a direction toward the centre of curvature; and consequently by virtue of the law of action and reaction, the train must exert an equal and opposite force upon the rail. This is a perfectly real force. Namely, it is the elastic force of the iron rail which is strained by the tendency of the train to preserve a rectilinear motion. If you examine the rail you will detect manifestations of the reality of the centrifugal pressure; or if you whirl a sling, you will actually feel the centrifugal force. But now certain natural philosophers extend the formula of centrifugal force, which is a genuine force where the motion is constrained by a rigid guide to cases where there is no such constraint. They say that a planet is held to its circular orbit by the balance between centrifugal and centripetal forces. In this case, centrifugal force is a mere formula, -- a formula which is undoubtedly quite correct as far as the effect goes, while yet the centrifugal force is a merely formal affair with nothing at all corresponding to it in nature. It is very much as if between two men, A and B, there has been a single transaction consisting in A lending B \$5. Now if B were to keep his books in such a manner that the state of the account as entered on those books made A owe him \$100 with \$105 on the opposite side of the account, the entries would in effect be correct; but yet that hundred dollars would be a fiction of bookkeeping. In like manner the centrifugal force of a planet is a fiction due to using polar coördinates in place of rectangular coördinates. It is true that were the gravitation of the sun suddenly to be annihilated there would be at the first instant an acceleration of the planet away from the circular orbit equal to the centrifugal force; and it is certainly true that what we call *force* in Dynamics is nothing more than the product of an acceleration multiplied by a mass. Only, this acceleration away from the circular motion of the natural motion of the planet were it suddenly emancipated from gravitation is nothing in the world but the entry we have to make on one side of our accounts to balance that fictitious entry which we have virtually made on the other side when we took the circular motion as the standard or origin from which to reckon accelerations.

Peirce: CP 7.469 Cross-Ref:††

469. Now the question is, whether or not there is any ratiocinative method by which we can assure ourselves that any law which we may discover by the observation of nature is not like centrifugal force a mere fiction of bookkeeping but represents a real and a living action in nature. Many nominalistic logicians will deny at once that any such distinction can be made; but in doing so, they will be merely adhering to preconceived metaphysical opinions. They have no real evidence to offer upon the subject. Of absolute knowledge there can be no question. But if we see that as soon as circumstances are somewhat varied, the form of the law is lost, the inference would seem to be that it is not a universal or living mode of action. If on the other hand, we find that as soon as the form is prevented from manifestation in one shape it immediately reappears in another shape, and especially if it shows a power of spreading and of reproducing itself, these phenomena may be considered as evidence of genuine vitality and fundamental reality in the form of the law.

Peirce: CP 7.469 Cross-Ref:††

But I confess I think it will, and ought to be, harder to convince you of the truth of this general principle than it will be to assure you of the consequence which

leads me to formulate it. Namely, what I wish to show is that causation, as distinct from the action of conservative force, is a real, fundamental, and vital element both in the outer and in the inner world.

Peirce: CP 7.470 Cross-Ref:††

470. As to those explanations which the physicists propose for irreversible phenomena by means of the doctrine of chances as applied to trillions of molecules, I accept them fully as one of the finest achievements of science. Judge Stallo †2 performed an acceptable service in his earnest assault upon them, which was conducted with as much ability as so poor a cause could possibly be expected to command. Other writers have recently attempted to reinforce the attack, one of them with some understanding of the subject. But the judgment of a really scientific logic must be altogether in favor of the accepted theory. Its explanation of the facts is altogether admirable and is fortified by a variety of new phenomena which were not known at the time the theory was first proposed, but which fit into their places like the pieces of a boy's dissected map, after he has once begun to put a few of them rightly together. This explanation demonstrates that the agency of energy is disseminated through every department of physical phenomena. But in one thing it fails; namely, it fails to show the absence of a very different kind of agency; and it not only fails to show its absence, but even supplies the means of proving its presence.

Peirce: CP 7.471 Cross-Ref:††

§2. NON-CONSERVATIVE ACTIONS

471. Those non-conservative actions which seem to violate the law of energy, and which physics explains away as due to chance-action among trillions of molecules, are one and all marked by two characters. The first is that they act in one determinate direction and tend asymptotically toward bringing about an ultimate state of things. If teleological is too strong a word to apply to them, we might invent the word *finious*, to express their tendency toward a final state. The other character of non-conservative actions is that they are *irreversible*. †3 If a falling stone, which moves under the conservative force of gravity, were suddenly to strike a perfectly elastic horizontal fixed surface, its motion would be reversed and it would move upwards to the point from which it fell with precisely the velocities it had in falling, only in reverse order. So it would be if every planet in the solar system suddenly had its motions reversed. Whatever motion conservative forces can effect, the very reverse of that motion they are equally capable of effecting.

Peirce: CP 7.472 Cross-Ref:††

472. There is some objection to taking either of the two characters of finiosity and irreversibility as *criteria* of the conservative or non-conservative character of an action. That which strictly constitutes an action as conservative is that the forces depend solely on the relative positions of the particles, and do not depend on the velocities. But *theoretically* that which makes an action irreversible is that the forces do not depend upon odd powers of the velocities. *Practically*, however, the irreversibility is an infallible criterion. For example, the friction of sliding motion is altogether independent of the velocity; so that according to the definition it is a conservative action. The velocity of a sliding motion is retarded by friction according to precisely the same formula as the velocity of a body shot vertically upwards. The

only difference is that when the instantaneous state of rest is reached, a new kind of friction, rest-friction, suddenly begins to act and breaks the continuity of the motion. Sliding friction is a unique example of a non-conservative action that simulates conservative action. The reason that it does so undoubtedly is that conservative action enters into it in a singularly uniform manner. When one solid body is set down upon another, there will be many points at which they come into contact, and where this occurs the paths of the atoms, -- for I do not half believe in the molecules of solids, will begin to be interlaced. The result is that when one begins to slide over the other, many ruptures have to be made, and before the ruptured parts have attained their positions of equilibrium they will on the average come into new contacts with the other body and thus there is a perpetual average state of elastic strain. The elastic stress of this strain is the friction, and it really is a conservative force. The parts of the action which are non-conservative are two, first and most important the ruptures, by which the elastic potential is at once converted into heat, and second and less important the contacts. You will observe that by friction the energy of molar motion is not immediately converted into heat but into elastic potential and it is only after the action is over that this becomes converted into heat, and that fact explains why friction acts like a conservative force.

Peirce: CP 7.473 Cross-Ref:††

473. The resistance of a fluid according to the analysis of Newton and his contemporaries is proportional to an even power of the velocity, namely the square. It ought therefore to be reversible; and probably it would be so in part for a moment. But the truth is [that] the whole analysis is an example of the unskillful application of mathematics, the hypotheses being too unlike the real facts to be useful. Of course, no resistance proper can be reversible.

Peirce: CP 7.474 Cross-Ref:††

474. The other character of non-conservative action, namely, its finiosity, is, as a criterion, open on the theoretical side to still more serious objection. Namely, it is not true that only non-conservative forces can bring about enduring states of things.

Peirce: CP 7.475 Cross-Ref:††

475. In the first place, let me remark that it is not generally true that a particle moving about an attracting centre describes any fixed orbit. In order that that should be the case, it is requisite that the law of the force should be subject to peculiar numerical conditions. We know that if the attraction is inversely as the square of the distance, and the velocity is not too great, the moving particle will describe an ellipse having the attracting centre at the focus. If, however, at the smaller distances the attraction is a little greater than the law of the inverse square of the distance requires, the result will mainly be that the ellipse itself will revolve slowly about the centre in the same direction in which the moving particle revolves. If there is any commensurable ratio between the periods of the two revolutions, the motion will finally return into itself; otherwise not.

Peirce: CP 7.476 Cross-Ref:††

476. If the attraction is inversely as the cube of the distance, the revolving orbit will make infinitely many revolutions while the moving body is making one-half revolution in that orbit; so that it will describe a spiral line having in general an outer and an inner boundary. The outer boundary may however be at an infinite distance or even further away. Here then we have a case in which conservative action asymptotically moves toward a final and ultimate state of things. Suppose the inner limit be distant from the centre by an insensibly minute interval. Then, it will appear

to remain fixed in one spot, although it will really be in tremendously rapid motion. The fact that tremendously rapid or even infinitely rapid motion may simulate rest is what makes the conservative action simulate the finiosity of non-conservative action.

Peirce: CP 7.477 Cross-Ref:††

477. The attraction may vary according to such a law that the moving body winds in indefinitely near to the centre without ever passing out or passing through the centre. It certainly seems as if the atoms of the chemical elements may have been formed by some such aggregation. For in that way Prout's law could be accounted for.

Peirce: CP 7.478 Cross-Ref:††

478. It is important to remark that even if the attraction varies inversely as the cube of the distance, and still more easily if it varies more rapidly, the moving particle may pass through or, at any rate, *to* the centre. And this it will generally do by performing infinitely many revolutions in an infinitesimal moment of time. What the motion will be when it does arrive at the centre it is hard to say. My father †4 in his *Analytic Mechanics* says that after that the body will proceed in a straight line. This, of course, would violate the principle of areas. He does not mention the circumstance that the direction of that straight line would in many cases be indeterminate. It appears to me that a general law being essentially continuous, to suppose an infinite velocity, or any other discontinuity in the action is to suppose that general law to be violated. If therefore a general law is such that it essentially involves such a phenomenon, the law is, in so far, self-contradictory. Still, the contradiction only amounts to this, that there is a point of discontinuity in the continuum. It is only a slight departure from generality in one particular instance. It is not that the state of facts supposed is self-contradictory; but only that it is self-contradictory to suppose such a phenomenon to be a result of a perfectly general law.

Peirce: CP 7.479 Cross-Ref:††

479. If such an event can happen then it follows as a necessary consequence that there is such a thing as an absolutely chance event. For even an infinitesimal variation in the conditions will make a finite difference in the result.

Peirce: CP 7.480 Cross-Ref:††

480. But as to whether or not there is any such law, inquiry in that direction is absolutely barricaded and brought to an eternal standstill, unless there has been some logical process in nature whereby the laws of nature have been brought about. Since, therefore, it is a corollary from the First Rule of Reasoning †5 that we must not make hypotheses that will absolutely stop inquiry, it follows that we are bound to hope that such a logical process of the evolution of law in nature may be discovered and that it is our duty as scientific men to search for it.

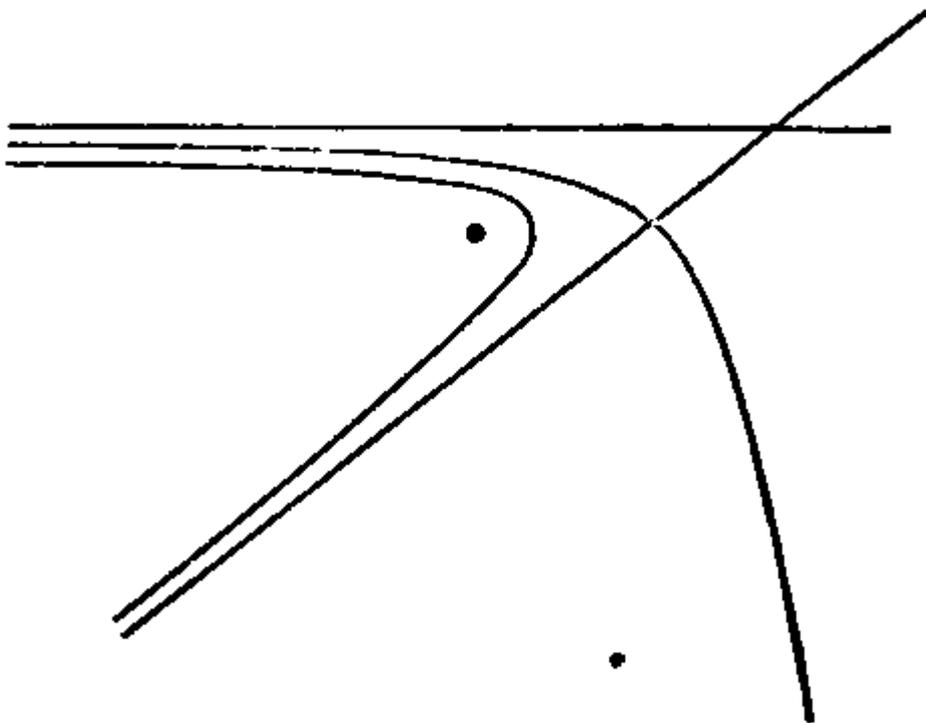
Peirce: CP 7.481 Cross-Ref:††

481. But let us return to those spiral motions which reach the centre only at the end of an infinite time. It must be confessed that here the simulation of non-conservative action by a conservative action is not a false or extrinsic simulation, but is true and intrinsic. It is just one of those extreme cases which throw the most light on philosophical problems and to which a powerfully solvent method of reasoning must pay particular attention. We note in the first place that the simulation depends in part on the bringing together into one infinitesimal moment motions which undo one another, and in declining to analyze this moment because it is absolutely infinitesimal. Thus the velocities in that moment, though instantaneously infinite are in their resultant zero; and with the attractive forces the same thing is true. From this

point of view, it becomes absurd to say that an attraction varies inversely as the cube or any higher power of the distance down to the very centre of attraction. Indeed, a somewhat similar difficulty arises whenever there is any attraction at all at the centre.

Peirce: CP 7.482 Cross-Ref:††

482. This leads me to remark that the finiosity of non-conservative action is also manifested in hyperbolic orbits under the attraction inversely as the square of the distance.



[Click

here to view] That is to say a moving body which starts from an infinite distance in one direction reaches at the end of infinite time an infinite distance in another direction. This finiosity might be regarded as due to the circumstance that time has an absolute limit. For could the motion continue beyond the infinitely distant instant of time it would continue through the infinitely distant line in the plane and complete the closed hyperbolic orbit. But I do not think that this simple way of solving the difficulty ought to be regarded as satisfactory. At any rate, if a similar solution be sought for the spiral, one is led to imaginaries, which seems to show that the mathematical hypothesis does not correspond to the facts.

Peirce: CP 7.483 Cross-Ref:††

483. It will be remarked that both these cases, that of the spiral orbit and that of the hyperbolic orbit, are connected with angular displacements. Any kind of rectilinear motion is continued by virtue of momentum, and from this circumstance arises the result that conservative forces affect not directly the velocities, but only the accelerations; and in the fact that such forces depend upon the relative positions of the particles lies their conservative character. But it is different with rotations. There is *no* momentum continuing an angular displacement as such, but only so far as that angular displacement involves rectilinear displacements. The mere rotation of an absolute particle, strictly occupying a single point, has no momentum at all. So if a

rectilinear displacement is effected as in the hyperbolic orbit by means of motions which in their limits become radial, the momentum has no tendency to continue the angular motion. Thus angular motion *per se* is not a conservative action. If, for example, atomicules are Boscovichian points, the attractions of those atomicules may be different on different sides of them. Many facts in elaterics, crystallography, and chemistry render it almost certain, for reasons which it would be too long here to discuss, that, as far as *atoms* are concerned, this is actually true. Suppose for the moment that it be also true of atomicules. What would be the result? As far as the mutual action of two atomicules was concerned, they would instantly turn those sides to one another which gave the minimum potential energy; and in the absence of all momentum, there would be no tendency to swing beyond that point. Those two sides would always be turned toward one another. But when there were *three* such points, the face which one atomicule turned toward another, and consequently its attraction for that other, would depend in part upon the position of the third atomicule. In this case, although the motions of translation would be conservative, the rotations of the atomicules would be regulated by the old formula of causation.

Peirce: CP 7.484 Cross-Ref:††
§3. RELATIVE AND ABSOLUTE MOTION

484. Now from our modern point of view of the non-Euclidean geometry, it appears that, strictly speaking, there is no kind of motion having the properties which we associate with *translation*. That is to say there is no motion which is merely relative. It would not be convenient to attempt to explain this here, before we have examined further into the nature of Continuity.

Peirce: CP 7.485 Cross-Ref:††

485. But let me here say a word about the attempt of Ernst Mach to show that all motion, even rotation, is merely relative. Mach belongs to that school of *soi disant* experiential philosophers whose aim it is to emancipate themselves from all metaphysics and go straight to the facts. This attempt would be highly laudable, -- were it possible to carry it out. But experience shows that the experientialists are just as metaphysical as any other philosophers, with this difference, however, that their pre-conceived ideas not being recognized by them as such, are much more insidious and much more apt to fly in the face of all the facts of observation.

Peirce: CP 7.486 Cross-Ref:††

486. Newton in his *Principia* maintains that Time and Space are substances, or in the jargon of French philosophers that they are *Entities*. The doctrine was a new one, well-recognized as such by Newton. Mach seems to think it was a blunder which Newton fell into inadvertently. It was nothing of the sort. We have historical testimony to show that Newton himself and his contemporaries regarded it as a peculiar, definite, and deliberate theory. Newton does not overtly argue the question in the *Principia* for the reason that he was a stickler for the traditions of mathematical exposition; and that tradition compelled him to confine himself to demonstrations and comments upon demonstrations. But he contrives to make his reason plain enough. That reason is that the laws of motion make velocity of rotation to be something absolute and not merely relative. Now velocity is the ratio of the amount of a space-displacement to the amount of time in which this displacement takes place. If

therefore, argued Newton, velocity is not merely *relative*, neither is a displacement in space nor a lapse of time relative; and therefore Space and Time are not mere relations but are absolute subjects or substances. Now this reasoning is founded on positive facts of observation; and it appears to me to be sound reasoning. I will not say that it draws a *necessary* conclusion; but I do say it is an excellent hypothesis to account for the facts.

Peirce: CP 7.487 Cross-Ref:††

487. Mach on the other hand lays it down as an Axiom that Space and Time are merely relative. No facts lend any support whatever to such an assertion. The most that could be said, -- more than is really true, -- is that facts concerning the composition of motions of Translation go to show that space-position has an element that is merely relative. Mach's struggles to define angular motion as motion relative to the mean position of all the bodies in the universe are not only struggling against all observation, and not only involve the absurdity that the centrifugal force of sling would be influenced by the angular motion of stars very far away, and more influenced by the more remote than by the nearer stars, contrary to his own conception of space as an image of dynamical relations; but, what is still worse, this gratuitous theory is in mathematical contradiction to the point he most insists upon, namely that rectilinear motion is purely relative.

Peirce: CP 7.488 Cross-Ref:††

488. It is true that Space, in so far as it is a continuum, is a mere law, -- a mere Thirdness.†6 But that does not stand in the way of its being a *thing* too. If besides its continuity it presents arbitrary *thisness*,†7 we must admit that it is something more than a mere law. The question of the relativity of motion is a question of the measurement of space, not of the nature of space itself; and therefore, although motion be not relative, it would not necessarily follow that space *itself* is non-relative, however good the inference may be, considered as a retroduction. But there are characters belonging to space *per se* which seem to involve *thisness*, such as its having three dimensions, -- which is an arbitrary limitation. Its *cyclosis* and *periphaxis*,†8 whether these be supposed equal to 0 or to 1 are apparently arbitrary Facts. You cannot reduce them to mere formalities without supposing that space has some kind of topical singularity, -- which is still more manifestly an arbitrary fact of existence. As to the Fourth Listing number, all must admit that its value is 1. That is to say, a body filling all space could not by gradual degrees shrink to a point without being ruptured, while the slightest explosion which should separate the body entirely from a single place however small, -- the smallest vacuous cist in it, -- would suffice to enable the collapse to take place. This I believe nobody who has carefully considered the matter has doubted or is likely to doubt, -- at least unless it be supposed that space has modes of connection of which observation affords not the slightest trace. Here again, then, is an arbitrary existential fact about Space, which is simply the way it insists upon being, without any logical necessity. Now insistence upon being in some quite arbitrary way is Secondness, which is the characteristic of the actually existing thing. It is its self-willedness.

Peirce: CP 7.489 Cross-Ref:††

489. Now if you examine the matter more closely than I have time to do in this lecture, you will find that it is precisely in those respects in which Space shows such indications of Secondness that motions act as though governed by the law of causality, while in those respects in which Space preserves all its Thirdness the motions preserve their dynamical character.

Peirce: CP 7.490 Cross-Ref:††

490. Let us next consider actions of which the space-element is not an intrinsic part. For example, I slip a nickel into the hand of a mendicant. One might say that this was a space motion. But the non-conservative friction is so great that neither the beggar nor the giver remarks any effects of *momentum*. The coin is not thrown but pushed along, and the dynamical part of the action is altogether insignificant. The fact that there is any space motion at all is accidental as far as the determination of the events goes. The money is caused to become the beggar's and remains his. Take the purest kind of temporal action. The very flow of time itself. The event passes out of the problematical state of futurity into the state of a *fait accompli*. All psychical action has this character. A question is answered, and answered it remains. A mere duality, a passage from a first state into a second state, here takes the place of that determination of a relation between three states which characterizes physical dynamics.

Peirce: CP 7.491 Cross-Ref:††

491. My father, Benjamin Peirce, drew my attention to the psychological peculiarity of an experiment which I am going to show you. I do not now remember how he formulated the matter. It is that a mathematical analysis of the conditions of motions often gives an expression of what happens conceived under one aspect; while anybody looking at the experiment would instinctively express what he saw under quite another aspect. The dictum of the eye is that one of the pendulums is ahead of the other in its oscillations by half an oscillation, i.e. by a quarter of a vibration, and that the oscillations [of] this pendulum are continually losing their amplitude and transferring it to the other. This is quite true, too. But analytic mechanics looks upon the fact from quite a different point of view. According to it each pendulum oscillates in two different ways at once. One of the components of its oscillation has the period of the two pendulums when they are swinging together and both pendulums partake equally of this component, while the other has the period of the two pendulums swinging opposite ways and the two pendulums are opposite one another in the phases of this component. I remember distinctly that my father remarked that while the view of analytic mechanics corresponds to the formula

$$\{\theta\}_1 = \{\theta\} \cos [(a-b)t] + \{\Theta\} \cos [(a+b)t]$$

$$\{\theta\}_2 = \{\theta\} \cos [(a-b)t] - \{\Theta\} \cos [(a+b)t]$$

the instinctive, or intuitional, view corresponds to the formula

$$\{\theta\}_1 = 2 \{\Theta\} \cos at \cdot \sin (bt+90^\circ)$$

$$\{\theta\}_2 = 2 \{\Theta\} \sin at \cdot \sin bt.$$

And I further remember his remarking that the decided choice of this last view showed a peculiarity of our mental constitution. But I cannot remember that he attempted to formulate this peculiarity. It is, however, clear to me that it is nothing but our natural tendency to prefer the formula of causation. We regard the pendulum

which is ahead as the agent and the one which lags behind as the patient.

Peirce: CP 7.492 Cross-Ref:††

492. To the reason of the mathematician the intuitional mode of conception is singularly crooked and unphilosophical. It happens to be pretty simple because the two pendulums are of equal weight and equal length. Were they not so, the phenomenon would appear very complicated from that point of view, though almost as simple as before from the point of view of analytical mechanics.

Peirce: CP 7.493 Cross-Ref:††

§4. PSYCHICAL ACTION

493. It is now time to inquire whether psychical action be of the conservative or the causal type. You know I make no pretension to competing with the profound psychologists under whom you sit here in Harvard; and I do not promise to bring the question to a satisfactory conclusion. But I shall hope in the few minutes that I can devote to it to make you all understand what the question is, and I hope the provisional reply I make to it may recommend itself as provisional good sense.

Peirce: CP 7.494 Cross-Ref:††

494. I read out to you the rules of philosophical terminology that seem to me to recommend themselves at once to the logic of science and to the ethics of science.†9 Those reasons for adopting them were so weighty, that I would not range any other consideration alongside of them. But now that I am no longer arguing the question, let me add that I for one entertain a deep feeling of reverence for the traditions of the English language. It has not the amazing psychical and especially emotional wealth of German. It has not half as many words for tools and manipulations as French; nor has it the delightful social *finesse* of French. But in all that concerns logic and reasoning, it has a spirit of accuracy which is due to the fact that the language spoken in State Street and other market places preserves to an extraordinary degree the sharp distinctions of the scholastic lore of the middle ages; and where those distinctions are not available, our vernacular language still preserves the spirit of them. I regret very much that those who of late years have written in English upon philosophy and psychology seem most of them to have a contempt for all English thought and English speech so great that it produces an utter insensibility to the distinctions of the language. The French language has long been cut off from medieval traditions; and moreover it is the genius of the French to rely upon skillful phraseology to express their precise thoughts rather than upon accurate terminology. But notwithstanding this, large numbers of French words which happen to be spelled like English words but which bear quite different meanings have by recent writers been used in their French meanings threatening an utter break-down of the spirit of English speech and of English thought. For example, the word *Entartung*, having been translated into French by *dégénération*, becomes "degeneration" in English, although what is meant is degeneracy, which is an entirely different thing. So *spontanée* becomes in this new lingo *spontaneous*, which is almost the reverse of the correct *English* meaning of *spontaneous*. *Suggestion* becomes "suggestion," regardless of the fact that suggestion was already an exact term of philosophy in English in a different sense. The German *Association* is rendered by "association," although if ever there was a school of writers who by the clearness of their definitions

and the accuracy of their thought deserved to have their usage of terms respected, it was the English Associationalists. I might expend the rest of the hour on this theme. When these neologists have succeeded in thus dishonoring their mother-tongue, till no vestige of her pristine virtue remains, they will by the same act have hopelessly corrupted all the old virility and health of English thought.

Peirce: CP 7.495 Cross-Ref:††

495. However, putting aside all such regrets, which are probably futile, -- and saying no more about vernacular speech, I am still obliged in the interest of the logic of science to employ a scientific terminology; and this must follow the only rules by which confusion can possibly be avoided. According to those rules I am bound to use scientific terms in the senses in which they first became terms of science. Accordingly, the English associationalists having first made *association* a term of science, and they having been careful never to extend it to the operation or event whereby one idea calls up another into the mind, but to restrict it primarily to a *habit* or *disposition* of mind in consequence of which an idea of one description is likely to bring into comparative vividness of consciousness an idea of another description, or, when they applied the term association to any operation or event, to designate by it only that process of habituation by which such a habit or disposition of mind acquires strength, they having been punctilious in this matter, my code of rules obliges me logically and morally, to follow them. As for that mental event which corresponds, as we suppose, to the nervous discharge of one part of the cortex upon another, -- or the action of one idea to render another idea which is associated with it vivid, -- for that they employed the term *suggestion*. This word is now applied mostly to motor phenomena or to such manifestations of mind as can be observed from without; and therefore, although the two meanings doubtless are in real facts connected together, the meanings themselves are different. But here a compromise is possible; for I shall violate no rule of terminology by speaking of the "suggestion" of the associationalists as *associational suggestion* and that of the hypnotists as *nervous suggestion*. The adjectives may be dropped, -- especially the former, -- in cases where there is no possibility of the meaning being mistaken.

Peirce: CP 7.496 Cross-Ref:††

496. I next remark that different sense-qualities have different degrees of intensity. The sound of thunder is more intense than the sound of a dozen people clapping their hands; and the light of an electric arc is more intense than that of a star. It is also true that the sound of thunder is more intense than the light of a star, and that the electric arc light is more intense than the sound of a dozen people clapping their hands. It is not at random that I say this. Besides this intensity of the sense-qualities, ideas have another mode of intensity, -- their *vividness*. The contrary of *vividness* we call *dimness*. Although my personal imagination and memory of colors is very dim compared with that of most persons, yet it is decidedly above the average in accuracy; and in matching a color by memory I am no more likely to select a paler or darker color than I am to select a higher or more luminous color. This *vividness*, which is so much more intense in my memory of the red pencil which I saw this afternoon than it is in my memory of a certain red fan which I possessed when I was nine years old, appears, as far as I have been able to experiment, to be entirely distinct from the intensity of the qualities remembered; although, no doubt, other things being equal my memory of an intense sensation is likely to be more vivid than my memory of a faint sensation. It does not belong to the *Firstness* of the quality, but to the *Secondness* or insistency of the particular apparition of that quality.

Peirce: CP 7.497 Cross-Ref:††

497. At any one time I have a great multitude of ideas in my consciousness of different degrees of vividness. How vivid the most vivid of them are depends upon how wide awake I am. In any given state of mental wakefulness or alertness, there is a certain maximum limit of vividness which none of my ideas surpass, but which a few of them always attain. There is only room in my consciousness for a few at this highest level of vividness. If others force themselves up, some of those that were at the surface must subside. Below these there are others less vivid, and still deeper others that are so dim that only by intense effort, perhaps by no effort that I can possibly exert, can I assure myself of their presence. And yet it may be proved indirectly that they are really there. For example, I have occupied myself for weeks in answering questions about the relative intensity of excitations of sense when with the most vigorous effort I could not seem to detect the slightest difference between them, so that my answers seemed quite random guesses; and yet the decided majority of the answers would be right every day, thus proving that sensations were capable of affecting my answers although I could not seem to be aware of them at all.†10 Moreover, ideas of which we do not seem to be aware will sometimes suggest or call up others by association, these others being vivid enough. I have endeavored to ascertain whether there is in any ordinary state of consciousness a definite minimum degree of vividness, as there certainly is a maximum degree. But all my experiments upon careful mathematical discussion point to the presence of ideas so very *dim*, or wanting in *vividness*, that I am strongly inclined to say, as a first approximation at any rate, that the vividness ranges all the way down to *zero*, and that every cell that ever can be sentient is in some degree sentient as long as it is alive at all.

Peirce: CP 7.498 Cross-Ref:††

§5. ASSOCIATION

498. Association is of two kinds.†11 For, on the one hand, it may be a natural disposition, which was from birth destined to develop itself whatever the child's outward experiences might be, so long as he was not maimed nor virtually maimed, say by being imprisoned. This sort of association by virtue of which certain kinds of ideas become naturally allied, as *crimson* and *scarlet*, is called *association by resemblance*. The name is not a good one, since it implies that the resemblance causes the association, while in point of fact it is the association which constitutes the resemblance. In themselves considered any two sense-qualities are what they are to themselves alone and have no relation to one another. But could they be compared by a mind that brought no tinge of its own nature into the comparison, any two ideas would appear somewhat alike and somewhat different. But the human mind attaches a peculiar value and emphasis to some resemblances, and that consists in this, that when one quality is brought vividly to consciousness, others will at once have their vividness increased, some more, some less. Thus, an idea which may be roughly compared to a composite photograph surges up into vividness, and this composite idea may be called a *general idea*. It is not properly a *conception*; because a conception is not an idea at all, but a *habit*. But the repeated occurrence of a general idea and the experience of its *utility*, results in the formation or strengthening of that habit which is the conception; or if the conception is already a habit thoroughly compacted, the general idea is the *mark* of the habit. Some psychologists deny the

existence of association by resemblance, or say that it is at bottom merely a special case of association by contiguity. To the arguments in defence of its fundamental character which are to be found in common books, I will add three. The first is that it is incredible that man is so constituted that no paths of nervous discharge between parts of the cortex are naturally more or less resistant than others. But those that are less resistant must correspond to natural associations, and ideas naturally associated will resemble one another. The second argument is that without association by resemblance there could be no general ideas and no resemblances. The third argument is this. Suppose I have long been puzzling over some problem, -- say how to construct a really good typewriter. Now there are several ideas dimly in my mind from time, none of which taken by itself has any particular analogy with my grand problem. But someday these ideas, all present in consciousness together but yet all very dim deep in the depths of subconscious thought, chance to get joined together in a particular way such that the combination does present a close analogy to my difficulty. That combination almost instantly flashes out into vividness. Now it cannot be contiguity; for the combination is altogether a new idea. It never occurred to me before; and consequently cannot be subject to any *acquired habit*. It must be, as it appears to be, its *analogy*, or resemblance in form, to the nodus of my problem which brings it into vividness. Now what can that be but pure fundamental association by resemblance?

Peirce: CP 7.499 Cross-Ref:††

499. On the other hand, the association, instead of being a natural disposition of mind, may be an acquired habit of mind. That supposes that similar ideas have been conjoined in experience until they have become associated. That is termed *association by contiguity*. Of course, psychologists have not been wanting who sought to show that there is no such thing as association by contiguity, or that it is merely a special case of association by resemblance. It is a long time since I read the work of Gay who first gave the idea of associationalism to Hartley. But I seem dimly to remember that he had a notion of that kind. A number of other principles of association have been proposed, such as contrast and causation. Association by contrast ought to be regarded as a case of association by resemblance, not in the narrow sense in which the reduction is often made, but by generalizing the conception of resemblance in accordance with the logic of relatives until it embraces all high degrees of logical relations between ideas. Contrast is a particular form, an especially prominent and familiar form, of what may be called *relational resemblance* by which I do not mean a resemblance of relations, but a connection of the kind which in the logic of relatives is shown to belong to the same class of relations to which the relation of resemblance belongs. Association by causation is an ill-defined conception embracing associations of different natures. But besides that reiterated coöccurrence which helps to consolidate an association by contiguity, another factor which plays a great part in accomplishing the association, is the experience that the combination of the ideas has important consequences. When we learn that white cats with blue eyes are deaf and have peculiar habits, such as that of following their masters like dogs, we no sooner see a white cat than we want to know what colored eyes she has. This may be called association by *relational contiguity*. That is to say not only have the two ideas frequently been experienced together, but their union has often been accompanied in experience with a third idea of an interesting kind. Another kind of association which is very important is that which makes an idea *interesting*. I propose to term it *association by interest*. An idea occurs to us in such a way that it would, other things being equal, be very dim. For example, it may result from a fortuitous putting together of two other ideas both of which are sunk deep in the subconscious

mind. But if the new idea happens to be *interesting*, it will promptly become vivid. Why is this? Clearly it is because the objective self-consciousness, or the idea which a man has of himself, consists in large measure of what may be roughly described as a composite of ideas of his aims and purposes, including all problems which exercise him. Now the separate components of this composite may for the most part be dim; but the total idea is perhaps the most vivid in consciousness at all times. Now an interesting idea is one which has an analogy, or resemblance in form, to this composite of the man's aims. It is, therefore, drawn into vividness by the vividness of that composite.

Peirce: CP 7.500 Cross-Ref:††
§6. LAW OF ACTION OF IDEAS

500. Let us now make an attempt to formulate the law of action of ideas. In the first place, an idea left to itself does not retain its vividness but sinks more and more into dimness. In the second place associated ideas in consciousness together soon undergo alterations of vividness, the dimmer ones becoming more vivid, and the more vivid ones dimmer, according to the strength of the associations. The dimmer idea never becomes more vivid than the more vivid idea had been before the change; but it may become more vivid than the idea originally the more vivid is after the change; for otherwise it would hardly be possible to explain idea chasing after idea. But in the third place the action of associative suggestion does not take place instantly as soon as the two ideas are in consciousness together. There are continual changes going on in the *connections* of ideas in consciousness; and the action of associative suggestion does not take place until chance has brought the two ideas into suitable connection for acting upon one another. Thus, I stand before an emblem wondering what it means. It is vividly in my mind. Perhaps the meaning is dimly in my consciousness; but it is not until by the movements in consciousness, chance has thrown the idea of the emblem and the idea of its meaning into the right sort of connection, that they suddenly change in vividness, the idea of the emblem becoming much dimmer and that of its meaning much more vivid.

Peirce: CP 7.501 Cross-Ref:††

501. In the fourth place, this interchange of vividness is accompanied by another event which takes place altogether outside my consciousness, though there is a sign of it in consciousness. Namely, the association between the two ideas becomes strengthened, in such a way that the more vivid idea becomes more likely to call up the less vivid one on another occasion. At the same time, in the fifth place, certain other associations become weakened.

Peirce: CP 7.502 Cross-Ref:††

502. Now that the mental action, as so described, is upon the surface, at least, causal and not conservative is quite obvious. There is no reversibility in it, and the traces of anything like momentum are slight and doubtful. At the same time, it would be possible to suppose that it was a conservative action affected to such a degree by resistances, that the momentum had no sensible effect.

Peirce: CP 7.503 Cross-Ref:††

503. The established cerebral theory will easily account for all the five features of mental action which I have mentioned; and that theory is favorable to the

view that while the action is of a mixed nature, the non-conservative elements are the predominant ones. For there can hardly be a doubt that the peculiar properties of protoplasm depend upon the enormous complexity of its molecules, upon those molecules being frequently broken up and reunited in new connections, and upon the circumstance that in the quiescent state the molecules are in stationary motion, while in the active state they are partly broken up and the fragments are wandering. Now all this may be summarized by saying that its properties depend upon Bernoulli's law of high numbers, and every action depending upon that law is, so far as it is so dependent, purely causal and not conservative.

Peirce: CP 7.504 Cross-Ref:††

504. Although the cerebral theory is established and although it is of priceless value to psychology in its present stage of development, it by no means follows that it will never be superseded. That method may perhaps lead to a purely psychical way of investigating the mind. We must wait and see whether it will or not; but meantime for various reasons which I cannot now enter upon that is what I am inclined to expect.

Peirce: CP 7.505 Cross-Ref:††
§7. PHYSICS AND PSYCHICS

505. We have, then, these two modes of action, the conservative and the causal, the former rather the dominant one in pure physics, the latter dominant in psychics. Our logical impulse, which prompts us to try to understand the universe, and as an essential condition of doing so to bring all its action under a single principle, this impulse, I say, compels us to *hope* that it may in some way be shown either that all causal action is conservative at bottom or that all conservative action is causal at bottom.

Peirce: CP 7.506 Cross-Ref:††

506. But I am quite sure that, as far as I personally am concerned, if I had not been moved by any consideration which touched me more nearly than such a vast and shadowy hope can do, I never should have been moved to do all the hard work I have done for the last fifteen years in trying to reason this matter out. I must confess that for me a living motive must have smaller dimensions than that very general hope. But I am a physicist and a chemist, and as such eager to push investigation in the direction of a better acquaintance with the minute anatomy and physiology of matter. What led me into these metaphysical speculations, to which I had not before been inclined, I being up to that time mainly a student of the methods of science, was my asking myself, how are we ever going to find out anything more than we now [know] about molecules and atoms? How shall we lay out a broad plan for any further grand advance?

Peirce: CP 7.507 Cross-Ref:††

507. As a first step toward the solution of that question, I began by asking myself what were the means by which we had attained so much knowledge of molecules and ether as we already had attained. I cannot here go through the analysis, although it is very interesting. But that knowledge has been based on the assumption that the molecules and ether are like large masses of ordinary matter. Evidently, however, that similarity has its limits. We already have positive proof that there are also wide dissimilarities; and furthermore it seems clear that nearly all that method

could teach has been already learned.

Peirce: CP 7.508 Cross-Ref:††

508. We now seem launched upon a boundless ocean of possibilities. We have speculations put forth by the greatest masters of physical theorizing of which we can only say that the mere testing of any one of them would occupy a large company of able mathematicians for their whole lives; and that no one such theory seems to have an antecedent probability of being true that exceeds say one chance in a million. When we theorized about molar dynamics we were guided by our instincts. Those instincts had some tendency to be true; because they had been formed under the influence of the very laws that we were investigating. But as we penetrate further and further from the surface of nature, instinct ceases to give any decided answers; and if it did there would no longer be any reason to suppose its answers approximated to the truth. We thus seem to be reduced to this alternative. Either we must make some very broad generalization as to the character of Nature's ways, which may at least tell us that one theory about molecules and ether is better worth trying than another theory, or else we had better abandon altogether a line of inquiry, -- I mean into the inmost constitution of matter, -- which is likely to prove a mere waste of time.

Peirce: CP 7.509 Cross-Ref:††

509. But meantime our scientific curiosity is stimulated to the highest degree by the very remarkable relations which we discover between the different laws of nature, -- relations which cry out for rational explanation. That the intensity of light should vary inversely as the square of the distance, is easily understood, although not in that superficial way in which the elementary books explain it, as if it were a mere question of the same thing being spread over a larger and larger surface. I cannot stop to give the true explanation, but I will just give you two hints. The first is that the basis of the measurement of light is the convention that we will call the light of two candles double the light of one. The other hint is that according to the superficial explanation of the school-books, you would expect the brightness of the image of a star made by a perfect lens to be proportional to the area of the lens, while in point of fact it is proportional to the square of that area. But grant that the law of variation of light with the distance is known, what an extraordinary fact it is that the force of gravitation should vary according to the same law! When both have a law which appeals to our reasons as so extraordinarily simple, it would seem that there must be some *reason* for it. Gravitation is certainly not spread out on thinner and thinner surfaces. If anything is so spread it is the potential energy of gravitation. Now *that* varies not as the inverse square but simply [as] the distance. Then electricity repels itself according to the very same formula. Here is a fluid; for electricity is really something like a fluid. It is not a mode of motion. Here is a fluid repelling itself but not at all as a gas seems to repel itself, but following that same law of the inverse square. I have not time to instance other extraordinary relations between laws of nature. But I cannot refrain from alluding to that most extraordinary law of Mendeléef.

Peirce: CP 7.510 Cross-Ref:††

510. According to the strictest principles of logic these relations call for explanation. In order to find such explanation, you must deduce the fundamental laws of the physical universe as necessary consequences of something. That is you must explain those laws altogether.

Peirce: CP 7.511 Cross-Ref:††

511. Now were it merely a question of the *form* of the law, you might hope for

a purely *rational* explanation, -- something in Hegel's line, for example. But it is *not* merely that. Those laws involve *constants*. Light for example moves over 300,000,000 centimetres *per* second. A mass at a distance of one centimetre from a gramme of matter receives in consequence of gravitation an increment of velocity toward that mass every second of . . . †12 centimetre per second. The explanation of the laws of nature must be of such a nature that it shall explain why these quantities should have the particular values they have. But these particular values have nothing rational about them. They are mere arbitrary *Secondnesses*. The explanation cannot then be a purely rational one. And there are numberless other facts about nature which, if my logic is not quite at fault, absolutely and decisively refute the notion that there can be any purely rational explanation.

Peirce: CP 7.512 Cross-Ref: ††
§8. EVOLUTION OF THE LAWS OF NATURE

512. What kind of an explanation can there be then? I answer, we may still hope for an evolutionary explanation. We may suppose that the laws of nature are results of an evolutionary process. In the course of this process of evolution, light, let us suppose, age by age moves faster and faster, and we have now arrived at the stage of the process in which it moves just so fast. Now logic does not demand any further explanation than that. The same applies to gravitation. You might ask me whether the relation between the velocity of light and the modulus of gravitation does not require explanation. I answer that it does not because the dimensions of the quantities are different. One involves the unit of mass and the other does not. But two universal constants are as many as can be allowed without explanation of their relations, except that there may be besides a constant of space.

Peirce: CP 7.513 Cross-Ref: ††

513. By a process of reasoning, then, of the nature of which I thus give you some hint, though given in full it would be seen to be drawn from a great variety of different evidences, I reached the conclusion that a theory of the evolution of the laws of nature must be sought. †13

Peirce: CP 7.514 Cross-Ref: ††

514. But if the laws of nature are the result of evolution, this evolutionary process must be supposed to be still in progress. For it cannot be complete as long as the constants of the laws have reached no ultimate possible limit. Besides, there are other reasons for this conclusion. But if the laws of nature are still in process of evolution from a state of things in the infinitely distant past in which there were no laws, it must be that events are not even now absolutely regulated by law. It must be that just as when we attempt to verify any law of nature our observations show irregular departures from law owing to our errors, so there are in the very facts themselves absolutely fortuitous departures from law trillions of trillions of times smaller no doubt, but which nevertheless must manifest themselves in some indirect way on account of their continual occurrence. I do not mean to say that it is a strictly necessary consequence that there should be this element of absolute chance in nature, and my first theory attempted to avoid it. But *as* I went on, I found other reasons to support this view of which I will endeavor to give you some idea in the next lecture. †14

Peirce: CP 7.515 Cross-Ref:††

515. But if the laws of nature are results of evolution, this evolution must proceed according to some principle; and this principle will itself be of the nature of a law. But it must be such a law that it can evolve or develop itself. Not that if absolutely absent it would create itself perhaps, but such that it would strengthen itself, and looking back into the past we should be looking back through times in which its strength was less than any given strength, and so that at the limit of the infinitely distant past it should vanish altogether. Then the problem was to imagine any kind of a law or tendency which would thus have a tendency to strengthen itself. Evidently it must be a tendency toward generalization, -- a generalizing tendency. But any fundamental universal tendency ought to manifest itself in nature. Where shall we look for it? We could not expect to find it in such phenomena as gravitation where the evolution has so nearly approached its ultimate limit, that nothing even simulating irregularity can be found in it. But we must search for this generalizing tendency rather in such departments of nature where we find plasticity and evolution still at work. The most plastic of all things is the human mind, and next after that comes the organic world, the world of protoplasm. Now the generalizing tendency is the great law of mind, the law of association, the law of habit taking. We also find in all active protoplasm a tendency to take habits. Hence I was led to the hypothesis that the laws of the universe have been formed under a universal tendency of all things toward generalization and habit-taking.

Peirce: CP 7.516 Cross-Ref:††

516. The next problem was to find a method of reasoning by which I could deduce with mathematical certainty the exact nature and formulae of the laws which would be formed under the influence of such a tendency and having deduced them to compare them with nature and thus see whether the theory was tenable or not.

Peirce: CP 7.517 Cross-Ref:††

517. Now I have had some remarkable successes in this line; and have also been led to make some remarkable predictions which remain yet to be compared with observation. Of the method of reasoning I have used I shall give you some slight idea in the next lecture.†15

Peirce: CP 7.518 Cross-Ref:††
§9. CHANCE AND LAW †16

518. *Uniform* distribution presents to a superficial view diverse characters. There are just so many suicides every year; of children born every year just so many develop into giants and just as many into dwarfs. An insurance company stakes almost its existence upon the expectation that just so many losses will occur each year. The relation between temperature, pressure, and volume upon which the whole cosmos of business reposes, insofar as it depends on the regular working of steam-engines, is another case of a uniformity which is simply a necessary corollary of a fortuitous distribution. But in many cases of *uniform* distribution, so far as we can see, *fortuitous* distribution plays no part. Thus, the two kinds of electricity tend to unite in a certain fixed proportion. This is simply because one kind attracts what the other repels and these two forces vary with the distance in precisely the same way. Both are conservative forces; and the uniform distribution of the two electricities is

due to the very peculiarly adjusted relation between the two conservative forces. In chemical combinations we have a very marked example of uniform distribution. We do not know by what sort of forces chemical compounds are held together. Even apart from the circumstance that some of the most readily formed bodies, such as acetylene, are endothermic, there are other considerations which show that those forces are not altogether conservative. But the bonds of atoms and their atomicities are sufficient warrant for the assertion that the forces must be exceedingly complicated and specially related to one another. I might say much more both about chemical forces and about the conditions of uniform distribution in general; but in the limits of one lecture I think it best to confine myself to the two clearer cases.

Peirce: CP 7.519 Cross-Ref:††

519. I have said that a uniformity, or regular law, may be a mere consequence of a fortuitous distribution. But if you examine any such case critically, you will find that after all, this only results because of some regularity in the conditions. Take, for example, Boyle's law that if the density of a gas is doubled its pressure will be exactly doubled. This is because if there are twice as many molecules in the space, twice as many in a given time will pound upon the wall of the receptacle. But this results not from fortuitous distribution alone, but from fortuitous distribution conjoined with the circumstance that the paths of the molecules are all very nearly rectilinear. I will not stop to prove this, which you will find set forth both in Watson's little treatise and in the more generally interesting volume of Oscar Emil Meyer. Suffice it to say that it *is* an essential condition. Now this is something which, being true of *all* the molecules, is a regularity. The simplicity of the law is due to the simplicity of this regularity. You will find, if you analyze the problem, that it must always be the case when a regularity results from a fortuitous distribution that some uniformity of the objects of the collection must come into play, and further that any simplicity the resulting law may exhibit must be due to the simplicity of that uniformity.

Peirce: CP 7.520 Cross-Ref:††

520. On the other hand, in regard to fortuitous distribution, while you may undoubtedly suppose that it arises simply from the absence of any sufficient reason to the contrary, -- not that I accept the principle of sufficient reason as a general one by any means, but in this case, it amounts merely to supposing the fortuitous distribution is a pure First, without any cause or reason whatsoever, -- while this you may of course suppose, yet if you suppose it to have been in any case a necessary result, this necessity certainly implies that some law of uniformity is at work, but for all that it will be quite evident that the uniformity has not *per se* of its own nature produced the irregularity, but that this irregularity is due to some other irregularity, some other fortuitous distribution, in the initial conditions.

Peirce: CP 7.521 Cross-Ref:††

521. Thus it is that uniformity, or necessary law, can only spring from another law; while fortuitous distribution can only spring from another fortuitous distribution. Law begets law; and chance begets chance; and these elements in the phenomena of nature must of their very nature be primordial and radically distinct stocks. Or if we are to escape this duality at all, urged to do so by the principle of retroduction, according to which we ought to begin by pressing the hypothesis of unity as far as we can, the only possible way of doing so is to suppose that the first germ of law was an *entity*, which itself arose by chance, that is as a First. For it is of the nature of Chance to be First and that which is First is Chance; and fortuitous distribution, that is, utter irregularity, is the only thing which it is legitimate to explain by the absence of any

reason to the contrary.

Peirce: CP 7.522 Cross-Ref:††

522. These things having become clear to us, let us now, remembering that the whole aim of this discussion is to find some clue by which physical and psychical action may be unified, examine, a little, certain other features of the two classes of phenomena governed respectively by conservative forces and by the principle of causality, and see how bright or how darkling a light is shed upon them by what we have thus far made out.

Peirce: CP 7.523 Cross-Ref:††

523. Looking first at conservative forces, we remark that they govern nothing but the space relations of particles. They are the law of the mutual reactions of particles in space. And the first fact that demands our attention is that, other things being equal, particles react upon one another more strongly the nearer they are to one another. How shall we explain this fact? We shall get the right hint if we ask ourselves what would happen in case all this were suddenly reversed and particles were to act most and most directly on those particles which were most distant from them.

Peirce: CP 7.524 Cross-Ref:††
CHAPTER 4

CONSCIOUSNESS

§1. CATEGORIES OF EXPERIENCE †1

524. If the whole business of mathematics consists in deducing the properties of hypothetical constructions, mathematics is the one science to which a science of logic is not pertinent. For nothing can be more evident than its own unaided reasonings. On the contrary logic is an experiential, or positive, science. Not that it needs to make any special observations, but it does rest upon a part of our experience that is common to all men. Pure deductive logic, insofar as it is restricted to mathematical hypotheses, is, indeed, mere mathematics. But when logic tells us that we can reason about the real world in the same way with security, it tells us a positive fact about the universe. As for induction, it is generally admitted that it rests upon some such fact. But all facts of this sort are irrelevant to the deduction of the properties of purely hypothetical constructions.

Peirce: CP 7.525 Cross-Ref:††

525. But there is a part of the business of the mathematician where a science of logic is required. Namely, the mathematician is called in to consider a state of facts which are presented in a confused mass. Out of this state of things he has at the outset to build his hypothesis. Thus, the question of topical geometry is suggested by ordinary observations. In order definitely to state its hypothesis, the mathematician,

before he comes to his proper business, must define what continuity, for the purpose of topics, consists in; and this requires logical analysis of the utmost subtlety. Mathematicians still survive who are so little versed in reasoning as to deny that we can reason mathematically about infinity, although the hypothesis of an endless series of whole numbers involves infinity and the hypothesis of transcendental irrational quantities involves an infinity of another kind. If we cannot reason mathematically about infinity, *a fortiori* we cannot reason mathematically about continuity, and any exact mathematics of topical geometry becomes impossible. To clear up these difficulties, some consideration of logical matters is indispensable.

Peirce: CP 7.526 Cross-Ref:††

526. Logic is a branch of philosophy. That is to say it is an experiential, or positive science, but a science which rests on no special observations, made by special observational means, but on phenomena which lie open to the observation of every man, every day and hour. There are two main branches of philosophy, Logic, or the philosophy of thought, and Metaphysics, or the philosophy of being. Still more general than these is High Philosophy which brings to light certain truths applicable alike to logic and to metaphysics. It is with this high philosophy that we have at first to deal.

Peirce: CP 7.527 Cross-Ref:††

527. What is the *experience* upon which high philosophy is based? For any one of the special sciences, experience is that which the observational art of that science directly reveals. This is connected with and assimilated to knowledge already in our possession and otherwise derived, and thereby receives an interpretation, or theory. But in philosophy there is no special observational art, and there is no knowledge antecedently acquired in the light of which experience is to be interpreted. The interpretation itself is experience. Even logic, however, the higher of the two main branches of philosophy, draws a distinction between truth and falsehood. But in high philosophy, experience is the entire cognitive result of living, and illusion is, for its purposes, just as much experience as is real perception. With this understanding, I proceed to make evident the following proposition.

Peirce: CP 7.528 Cross-Ref:††

528. All the elements of experience belong to three classes, which, since they are best defined in terms of numbers, may be termed *Kainopythagorean categories*. †2 Namely, experience is composed of

Peirce: CP 7.528 Cross-Ref:††

1st, *monadic experiences*, or *simples*, being elements each of such a nature that it might without inconsistency be what it is though there were nothing else in all experience;

Peirce: CP 7.528 Cross-Ref:††

2nd, *dyadic experiences*, or *recurrences*, each a direct experience of an opposing pair of objects;

Peirce: CP 7.528 Cross-Ref:††

3rd, *triadic experiences*, or *comprehensions*, each a direct experience which connects other possible experiences.

Peirce: CP 7.529 Cross-Ref:††

529. In order to prove this proposition I have, first, to invite every reader to note certain phenomena in experience and make certain simple generalizations from

those observations; second, to point out in those generalized phenomena the essential characters in the above definitions of the categories; third, to point out certain other characteristics of those phenomena and show how they are related to the essential characters of the categories; fourth, to exemplify the wide range of each category in experience; fifth, to show by comparison of the characters already ascertained that none of the categories can be resolved into the others, but that all are distinct from one another; sixth, and most difficult, to prove that there can be no element in experience not included in the three categories.

Peirce: CP 7.530 Cross-Ref:††

530. A *quality* of feeling, say for example of [a] certain purple color, might be imagined to constitute the whole of some being's experience without any sense of beginning, ending, or continuance, without any self-consciousness distinct from the feeling of the color, without comparison with other feelings; and still it might be the very color we see. This is a conclusion which anybody can reach by comparing his different states of feeling; but we cannot actually observe a quality of feeling in its purity; it is always mixed with other elements which modify it greatly. Were a feeling thus to usurp the whole consciousness, it would necessarily be perfectly simple; for the perception of different elements in it is a comparison of feelings. Moreover, with us every feeling has its degree of vividness, which does not affect its quality, but is apparently the degree of disturbance it produces. It is necessary to speak vaguely, because it is not settled precisely what vividness consists in. But [were] the feeling uncomplicated by anything else, no particular degree of vividness would attach to it. The quality of feeling would then be the whole feeling. Qualities, then, constitute the first category. A quality of feeling is perfectly simple, in itself; though a quality thought over and thus mixed with other elements, may be compared with others and analyzed. A quality of feeling, in itself, is no object and is attached to no object. It is a mere tone of consciousness. But qualities of feeling may be attached to objects. A quality of feeling, in itself, has no generality; but it is susceptible of generalization without losing its character; and indeed all the qualities of feeling we are able to recognize are more or less generalized. In a mathematical hypothesis the qualities of feeling are so subordinate as to be scarcely noticeable.

Peirce: CP 7.531 Cross-Ref:††

531. That we cannot have an experience of exertion without a direct experience therein of resistance to our exertion is plain. By an experience of exertion, I do not mean a consciousness of resolving to do something, nor the collection of our force preparatory to an effort, but merely what we experience in the very act of doing. This being understood, I contend that it is equally true that we cannot have an experience of being affected by anything without having therein a direct experience of our resisting the effect. Take hold of one end of a lexicon and lift it, while one edge remains on the table or floor. You experience its resistance. But when the centre of gravity has passed beyond the vertical plane of stationary edge, what you feel is that the dictionary is acting upon you. Yet the only difference is that different muscles are now called into play, which are elongating instead of contracting. Lay your forearm on the table and place the book on your palm. Though the sensation is somewhat different, you still have an experience of being overborne, and thus of holding out against the compression. A series of such experiments, with variations needless to describe, will convince the reader that there is a common character in the experience of acting upon anything by a muscular contraction and an experience of being acted on whether by a relaxation of a muscle or by a sensation received upon the organs of sense. That experience gives us at once a direct consciousness of something inward

and an equally direct consciousness of something outward. In fact, these two are one and the same consciousness. They are inseparable. The same two-sided consciousness appears when by direct effort I bring to the surface of recollection a name that I dimly remember, and when I make distinct to myself a confused conception.†3 If the purple color which we just supposed made up the whole consciousness of a being were suddenly to change, then, still supposing the idea of continuance is either absent or not prominent, that being will have a two-sided consciousness. The sense of what has been will be a rudimentary *ego*, the sense of what comes about will be a rudimentary *non-ego*. For past experience is for each of us *ours*, and that which the future brings is not ours, which becomes present only in the instant of assimilation. That being could have no sense of change except by experiencing the two colors together. The instant change would involve a sort of shock consisting in the two-sided consciousness. This experience of *reaction* is the second Kainopythagorean category. It is impossible to find any element of experience directly involving two objects, and no more, -- those two embraced in any third, such as a pair, but standing in their naked otherness, -- other than an experience of reaction.

Peirce: CP 7.532 Cross-Ref:††

532. A reaction is something which occurs *hic et nunc*. It happens but once. If it is repeated, that makes two reactions. If it is continued for some time, that, as will be shown below, involves the third category. It is an individual event, and I shall show that it is the root of all logical individuality. A reaction cannot be generalized without entirely losing its character as a reaction. A generalized reaction is a law. But a law, by itself without the addition of a living reaction to carry it out on each separate occasion, is as impotent as a judge without a sheriff. It is an idle formula entirely different from a reaction. A reaction may be ever so conformable to law or reason, that is, it may occur when law or reason calls for it. But, in itself, as reaction it is arbitrary, blind, and brute exertion of force. To express the fact that a reaction thus resists all generalization, I say that it is *anti-general*. In this respect it contrasts with a quality of feeling, which though not in itself general is susceptible of generalization without losing its character as quality of feeling. It is remarkable that Reaction, which is the Dyad category, should have an aggressive unity that Quality, the Monad category, does not exhibit. But the explanation of it is that the quality involves no reference to anything else and so is one without any special emphasis, since it could not be otherwise; while reaction consists in the congress of two things, that might not come together, and every concurrence of them makes a distinct reaction. It will be found that the third category also has a mode of unity which does not belong to either of the others.

Peirce: CP 7.533 Cross-Ref:††

533. A quality of feeling does not in itself involve any reaction. But an experience of reaction does involve two qualities of feeling. It consists in the conjunction of two qualities of feeling; and in this conjunction those two qualities of feeling become more than mere qualities. In being thus set over against one another they acquire the concreteness and actuality of feelings. The one purple color absorbing the entire consciousness of our supposed being was a mere tone of life. But when a sudden change occurs setting two against one another, they become objects.

Peirce: CP 7.534 Cross-Ref:††

534. Although in all direct experience of reaction, an *ego*, a something within, is one member of the pair, yet we attribute reactions to objects outside of us. When we say that a thing *exists*, what we mean is that it reacts upon other things.†4 That we

are transferring to it our direct experience of reaction is shown by our saying that one thing *acts* upon another. It is our hypothesis to explain the phenomena, -- a hypothesis, which like the working hypothesis of a scientific inquiry, we may not believe to be altogether true, but which is useful in enabling us to conceive of what takes place.

Peirce: CP 7.535 Cross-Ref:††

535. Now if we ask ourselves what else we observe in every experience (taking experience in its broadest sense to include experience of ideal worlds and of the real world as we interpret its phenomena) besides qualities and reactions, the answer will readily come that there remain the regularities, the continuities, the significances. These are essentially of one kind. That continuity is only a variation of regularity, or, if we please so to regard it, that regularity is only a special case of continuity, will appear below, when we come to analyze the conception of continuity. It is already quite plain that any continuum we can think of is perfectly regular in its way as far as its continuity extends. No doubt, a line may be say an arc of a circle up to a certain point and beyond that point it may be straight. Then it is in one sense continuous and without a break, while in another sense, it does not all follow one law. But in so far as it is continuous, it everywhere follows a law; that is, the same thing is true of every portion of it; while in the sense in which it is irregular its continuity is broken. In short, the idea of continuity is the idea of a homogeneity, or sameness, which is a regularity. On the other hand, just as a continuous line is one which affords room for any multitude of points, no matter how great,†5 so all regularity affords scope for any multitude of variant particulars; so that the idea [of] continuity is an extension of the idea of regularity. Regularity implies generality;†6 and generality is an intellectual relation essentially the same as significance, as is shown by the contention of the nominalists that all generals are names. Even if generals have a being independent of actual thought, their being consists in their being possible objects of thought whereby particulars can be thought. Now that which brings another thing before the mind is a representation; so that generality and regularity are essentially the same as significance. Thus, continuity, regularity, and significance are essentially the same idea with merely subsidiary differences. That this element is found in experience is shown by the fact that all experience involves time. Now the flow of time is conceived as continuous. No matter whether this continuity is a datum of sense, or a quasi-hypothesis imported by the mind into experience, or even an illusion; in any case it remains a direct experience. For experience is not what analysis discovers but the raw material upon which analysis works. This element then is an element of direct experience.

Peirce: CP 7.536 Cross-Ref:††

536. It remains to be shown that this element is the third Kainopythagorean category. All flow of time involves learning; and all learning involves the flow of time. Now no continuum can be apprehended except by a mental generation of it, by thinking of something as moving through it, or in some way equivalent to this, and founded upon it. For a mere dull staring at a superficies does not involve the positive apprehension of continuity. All that is given in such staring is a feeling which serves as a sign that the object might be apprehended as a continuum. Thus, all apprehension of continuity involves a consciousness of learning. In the next place, all learning is virtually reasoning; that is to say, if not reasoning, it only differs therefrom in being too low in consciousness to be controllable and in consequently not being subject to criticism as good or bad, -- no doubt, a most important distinction for logical purposes, but not affecting the nature of the elements of experience that it contains. In

order to convince ourselves that all learning is virtually reasoning, we have only to reflect that the mere experience of a sense-reaction is not learning. That is only something from which something can be learned, by interpreting it. The interpretation is the learning. If it is objected that there must be a first thing learned, I reply that this is like saying that there must be a first rational fraction, in the order of magnitudes, greater than zero. There is no minimum time that an experience of learning must occupy. At least, we do not conceive it so, in conceiving time as continuous; for every flow of time, however short, is an experience of learning. It may be replied that this only shows that not all learning is reasoning, inasmuch as every train of reasoning whatever consists of a finite number of discrete steps. But my rejoinder is that if by an argument we mean an attempt to state a step in reasoning, then the simplest step in reasoning is incapable of being completely stated by any finite series of arguments. For every step in reasoning has a premiss, P, and a conclusion, C; and the reasoning consists in the perception that if P is found true as it has been found true, then *must* C be always or mostly true; and this "must" means that not only [is] C true (or probable) unless P is false (or not found true in the way supposed) but that every analogous premiss and conclusion are in the same relation. That is to say, in the reasoning we observe that P has a certain general character and C is related to it in a certain general way, and further that given any proposition whatever of that general character, the proposition related to it in that general way is true unless the former proposition is false; whence it necessarily follows of C and P, that either the former is true or the latter is false. But this is a second argument involved in the reasoning. For the first argument was

P is true,

Hence, C must be true;

while the second argument is

P has a general character P' and C has a relation r to P;

But given any proposition having the character P', the proposition having the relation r to it is true unless the former is false;

Hence, C is true unless P is false.

Peirce: CP 7.536 Cross-Ref:††

Thus, every reasoning involves another reasoning, which in its turn involves another, and so on *ad infinitum*. Every reasoning connects something that has just been learned with knowledge already acquired so that we thereby learn what has been unknown. It is thus that the present is so welded to what is just past as to render what is just coming about inevitable. The consciousness of the present, as the boundary between past and future, involves them both. Reasoning is a new experience which

involves something old and something hitherto unknown. The past as above remarked is the *ego*. My recent past is my uppermost *ego*; my distant past is my more generalized *ego*. The past of the community is *our ego*. In attributing a flow of time to unknown events we impute a quasi-*ego* to the universe. The present is the immediate representation we are just learning that brings the future, or non-ego, to be assimilated into the *ego*. It is thus seen that learning, or representation, is the third Kainopythagorean category.

Peirce: CP 7.537 Cross-Ref:††

537. There are no more Kainopythagorean categories than these three. For the first category is nonrelative experience, the second is experience of a dyadic relation, and the third is experience of a triadic relation. It is impossible to analyze a triadic relation, or fact about three objects, into dyadic relations; for the very idea of a compound supposes two parts, at least, and a whole, or three objects, at least, in all. On the other hand, every tetradic relation, or fact about four objects can be analyzed into a compound of triadic relations. This can be shown by an example. Suppose a seller, S, sells a thing, T, to a buyer, B, for a sum of money, M. This sale is a tetradic relation. But if we define precisely what it consists in, we shall find it to be a compound of six triadic relations, as follows:

Peirce: CP 7.537 Cross-Ref:††

1st, S is the subject of a certain receipt of money, R, in return for the performance of a certain act As;

Peirce: CP 7.537 Cross-Ref:††

2nd, This performance of the act As effects a certain delivery, D, according to a certain contract, or agreement, C;

Peirce: CP 7.537 Cross-Ref:††

3rd, B is the subject of a certain acquisition of good, G, in return for the performance of a certain act, Ab;

Peirce: CP 7.537 Cross-Ref:††

4th, This performance of the act Ab effects a certain payment, P, according to the aforesaid contract C;

Peirce: CP 7.537 Cross-Ref:††

5th, The delivery, D, renders T the object of the acquisition of good G;

Peirce: CP 7.537 Cross-Ref:††

6th, The payment, P, renders M the object of the receipt of money, R.

Or we may define a sale as the execution of contract of sale. The contract of sale has two clauses. The first clause provides for a giving and a receiving. The giving is by the seller of the commodity; the receiving is by the buyer of the same commodity. The second clause provides for a giving and a receiving. The giving is by the buyer of the price; the receiving is by the seller of the same price. The execution is of the first clause and of the second, etc. But I do not think this latter definition as good as the other, since it introduces several unnecessary elements and also covertly brings in four pentadic relations, such as the relation of the buyer to the first and second clauses of the contract and to the separate executions of them.

Peirce: CP 7.538 Cross-Ref:††

538. Let me now resume the argument. To begin with, it is to be remarked that I use the word "experience" in a much broader sense than it carries in the special sciences. For those sciences, experience is that which their special means of observation directly bring to light, and it is contrasted with the interpretations of those observations which are effected by connecting these experiences with what we otherwise know. But for philosophy, which is the science which sets in order those observations which lie open to every man every day and hour, experience can only mean the total cognitive result of living, and includes interpretations quite as truly as it does the matter of sense. Even more truly, since this matter of sense is a hypothetical something which we never can seize as such, free from all interpretative working over. Such being what is here meant by experience, my argument is of the utmost simplicity. It consists merely in begging the reader to notice certain phenomena which he will find, I believe, in every corner of experience and to draw the simplest generalizations from them. The first phenomenon that I ask him to observe is, that he can detect elements in experience which are whatever they are each in its own simplicity. Namely, he will perceive that this is true of colors, smells, emotions, tones of mood, the characteristic flavors, if I may use this expression, attaching to certain ideas. Look, for instance, on anything yellow. That yellow quality is not in itself, as that mere quality, to be explained by anything else, or defined in terms of anything else; nor does it involve or imply anything else. This is surely evident. True, we know by experiment that a yellow color can be produced by mixing green and red light. But the yellow, as a quality of feeling, involves no reference to any other color. Every quality of feeling, as such, is perfectly simple and irrespective of anything else. The second phenomenon that I ask the reader to observe is that there are in experience occurrences; and in every experience of an occurrence two things are directly given as opposed, namely, what there was before the occurrence, which now appears as an *ego*, and what the occurrence forces upon the *ego*, a *non-ego*. This is particularly obvious in voluntary acts; but it is equally true of reactions of sense. If the latter are intense, or violent, the sense of reaction is particularly strong. There is a certain quality of feeling here, a brute arbitrariness, as I may call it, though it cannot be described any more than yellow can be described. But it is not this quality of feeling to which I wish to direct attention as peculiar, but the actual taking place. This actual taking place essentially involves two things, what there was before and what the occurrence introduces. I ask the reader to remark that such an occurrence cannot possibly be resolved into qualities of feeling. For in the first place, a quality of feeling is, in itself, simple and irrespective of anything else; so that anything compound necessarily involves something besides a quality of feeling. Secondly, a quality is merely something that *might* be realized, while an occurrence is something that *actually* takes place. The character of brute exertion that attaches to every occurrence is, no doubt, a quality of feeling; but experience of the occurrence itself, is something else. Such an element of experience I term a *reaction* in order to emphasize its essentially dual character. Thirdly, a reaction has an individuality. It happens only once. If it is repeated, the repetition is another occurrence, no matter how like the first it may be. It is anti-general. A quality, on the other hand, has no individuality. Two qualities are different only so far as they are unlike. Individuality is an aggressive unity, arising from an absolute refusal to be in any degree responsible for anything else. This a quality cannot have since it is too utterly irrespective of anything else even to deny it. A reaction, on the other hand, is an opposition, or pairedness of objects that are existentially correlative, neither existing except by virtue of this opposition.

539. I propose to review the ideas of the Nineteenth Century; and as an introduction to that review it will be well to glance at the general tendencies of the times in their influence upon human nature in general. In order to do that it will be convenient first to enumerate the departments of mental action.

Peirce: CP 7.540 Cross-Ref:††

540. Almost all the philosophers of this century have agreed to name Feeling, Knowledge, and Will, as the parts of the mind, or to speak more accurately as the three classes of states of mind. Few of those who make use of this enumeration pretend that it is exactly scientific; but it has served a good purpose. It is usually attributed to the Father of German philosophy, Immanuel Kant, who died in the last year of the last century. Kant borrowed it from his master Tetens; but in doing so he quite changed the boundaries of the department of *Feeling*. Take whatever is directly and immediately in consciousness at any instant,†8 just as it is, without regard to what it signifies, to what its parts are, to what causes it, or any of its relations to anything else, and that is what Tetens means by Feeling; and I shall invariably use the word in that same sense. For example, here we are in this pleasant room, sitting before the fire, listening to my reading. Now take what is in your consciousness at any one single moment. There is in the first place a general consciousness of life. Then, there is the collection of little skin-sensations of your clothes. Then, there is the sense of cheerfulness of the room. Then, there is a social consciousness, or feeling of sympathy with one another. Then, there is the light. Then, there is the warmth of the fire. Then, there is the sound of my voice, which in any one instant will merely be a note. In addition, there [are] a hundred things in the background of consciousness. This is the best way in which I can describe what is in your consciousness in a single moment. But it has taken me a considerable time to describe them. I cannot, therefore, have described them as they are in your mind; for precisely what I am trying to describe is the consciousness of a moment. By the very nature of language, I am obliged to pick them to pieces to describe them. This requires reflection; and reflection occupies time. But the consciousness of a moment as it is in that very moment is not reflected upon, and not pulled to pieces. As it is in that very moment, all these elements of feeling are together and they are one undivided feeling without parts. What I have described as elements of the feeling are not really parts of the feeling as it is in the very moment when it is present; they are what appears to have been in it, when we reflect upon it, after it is past. As it is felt at the moment itself these parts are not yet recognized, and therefore they do not exist in the feeling itself. I have assured myself that this is so, by frequent repetitions of the following experiment. Namely, sitting in a perfectly dark room with my eyes directed to a piece of paper upon which some pretty simple figure had been drawn, I knew not what, I have caused the paper to be instantaneously illuminated by a single electric spark. The spark was practically instantaneous; but the impression upon the retina would last nearly a quarter of a second. But I always found I had to sit reflecting for several seconds before I could tell at all what I had seen. Undoubtedly, the feeling was actually present for a good quarter of a second; and it was followed by a vivid memory that for some seconds was, I will not say nearly as intense as the feeling itself, but yet very perfect for a memory. But until I had had time to pick the memory to pieces, I found I could not say what I had seen. After experimenting in this way for

some time, I became struck with the fact that there was, after all, conclusive proof of the thing, without the experiment. In the very moment of receiving an impression, it is impossible that we should say what the parts of it are, because in order that we should do that we must attend to one part and another separately, and to carry the attention from one part to another requires time. The third of a second is sufficient time for me to say that I have seen a light, not felt a jab; for some reflection can be accomplished in a third of a second, or much less. For in watching a pendulum swing over twenty millimetres with a scale of millimetres behind it, I can accurately observe the extreme point of the swing to a millimetre and estimate to a tenth part the fraction of the millimetre, although the pendulum is not there for a twentieth of a second. Now this requires reflection. So that it is proved that a not very simple reflection can be performed in the twentieth part of a second. But reflection cannot be performed instantaneously; and the evidence is quite satisfactory that the feeling of a moment cannot be at all analyzed in that moment. I trust then that I have made clear what I mean, and what Tetens meant by Feeling. It is the consciousness of a moment as it is in its singleness, without regard to its relations whether to its own elements or to anything else. Of course, this feeling although it can exist in a moment, can also be protracted for some time. For example, if we are close to an engine with a powerful whistle, and this suddenly shrieks, the intensity of it seems to paralyze me; and for several seconds my mind seems to have hardly anything in it but that shriek. Even when our thoughts are active, at each instant we have a feeling; and in the midst of changes of thought not a whole feeling but an element of feeling, which I shall also call a Feeling, may endure. Kant, in order to make the enumeration of Tetens fit into his own philosophical system, limited the word Feeling to feelings of pleasure and pain; and the majority of philosophical writers of this century have followed him in this. I think this has been unfortunate, and has hindered the perception of the real relations of [the] triad.

Peirce: CP 7.541 Cross-Ref:††

541. It may be asked where Tetens got his idea that Feelings, Cognitions or Knowledges, and Volitions or acts of willing made up the mind. I have never seen this question answered. Yet the answer is not far to seek. He took it from the ancient writers upon rhetoric. For they instruct the orator to begin his discourse by creating a proper state of feeling in the minds of his auditors, to follow this with whatever he has to address to their understandings, that is, to produce cognitions, and finally to inflame them to action of the will. For the rhetoricians, therefore, the triad names three states of mind; and most of the psychologists of our century have considered Feeling, Cognition, and Volition to be three general states of mind.†9

Peirce: CP 7.542 Cross-Ref:††

542. But in my opinion, by a slight modification the triad may be made to stand for three radically different kinds of elements of all consciousness, the only elements of consciousness, which are respectively predominant in the three whole states of mind which are usually called Feeling, Knowing, and Willing. It is thus raised from a mere loose grouping into a scientific and fundamental analysis of the constituents of consciousness.

Peirce: CP 7.543 Cross-Ref:††

543. The modification which I propose relates to the department to which *Sensation* is to be assigned. It will be best to explain what I mean by *sensation*. If you look at a flame, you observe that it is orange, and that orange color as it was seen on that particular occasion and was attributed to a reality then and there before you,

and not called up in memory, was a sensation. So if you hear a cry, whether it is real or a hallucination, if you take it for a reality then and there present, every vowel and consonant of it is a separate sensation. But if it is only called up by your own act of imagination it is not a sensation. A sensation is not a feeling; but an element of feeling is one part of it. Here is a little bottle with some green spicular crystals in it. When I look at it, I experience a sensation of greenness. Were that greenness to fill my whole field of vision, while I became momentarily deaf, lost my skin-sensations, and my memory, it would be a total feeling. For it would be my life for the moment, and would not be attributed to anything in particular without me or within me. As it is, it is an element of my feeling while I am looking at the bottle. But to make up the sensation, along with this feeling there is a consciousness of being irresistibly compelled to see it when I look at it. I cast my eyes upon it, without any intention of imagining such a thing, and there it is more vivid than any imagination could be. The sight forces itself upon me. The sensation has two parts: first, the feeling, and second, the sense of its assertiveness, of my being compelled to have it. The consequence is that remembering a sensation is not at all the same thing as having it. For though there is some vestige of compulsiveness, even in the memory, it is not at all comparable to the compulsiveness of the actual sensation. But if I remember, or imagine a feeling, whatever I remember or imagine is a feeling, and I cannot remember or imagine or anywise represent to myself a feeling without having that very feeling then and there. All the existence a feeling can have is had the moment it is thought. But a sensation is not had until I am really acted upon by something out of my control. I have, thus, made clear, I hope, what I mean by a sensation. It is an event which has to happen at a particular moment. For that reason there is great need of a verb to correspond with the noun; and I shall use the expression to *sense* the greenness. It translates the German verb *empfinden*. The verb to *feel* is quite superfluous; for to think a feeling at all is to feel it. But the verb to *sense* is indispensable, because to actually *sense* a sensation is very different from remembering or imagining it. Now, it is usual to put sensation under the head of Knowledge. This may or may not be correct language; I do not propose to be led into any verbal dispute. But I refuse to classify sensation in that way, because I aim to enumerate the different kinds of elements of consciousness. Now, sensation contains two radically different kinds of consciousness. One part is feeling and the other part is the consciousness of being compelled to feel upon that particular occasion. This consciousness of compulsion has a general resemblance to the consciousness of willing. Willing is the consciousness of exerting a force upon something without the consciousness, or at least outside of that part of consciousness. But the consciousness of exerting force and the consciousness of suffering the effect of a force are one and inseparable. Suppose I try to exercise my strength in lifting a huge dumb-bell. If I strive to lift it, I feel that it is drawing my arm down. If I suffer no consciousness of having my arm pulled down, I can have no consciousness of exerting force in lifting the dumb-bell. To be conscious of exerting force and to be conscious [of] having force used upon me are the same consciousness. Hence, the compulsive element of sensation must be classed along with the consciousness of willing. Both are particular events which must happen at definite times; and to dream of exercising the will is as different from actually willing as dreaming of a sensation is from actually sensing. Feeling is neither over against me nor in me. If I have any momentary consciousness of self, that is a part of the feeling. So that I am, or at any rate my immediate self-consciousness is, a part of my total feeling. Any element of feeling has the same relation to self-consciousness that it has to any other element of the same total feeling; that is to say, it is quite independent of it. An element of Feeling is neither a part of self-consciousness nor is set up over against

self-consciousness. But the consciousness of compulsion in sensation as well as the consciousness of willing necessarily involves self-consciousness and also the consciousness of some exterior force. The self and the not-self are separated in this sort of consciousness. The sense of reaction or struggle between self and another is just what this consciousness consists in. Hence, to give it a name, I propose to call it *altersense*. To avoid circumlocution, I will speak of the *altersense* element of sensation, as *Sensation* simply. Thus, *Altersense* has two varieties, Sensation and Will. The difference between them is that Sensation is an event in which a feeling is forced upon the mind; while Volition or Willing, is an event in which a desire is satisfied, that is, an intense state of feeling is reduced. In Sensation, a feeling is forced upon us; in Willing, feeling forces its way out from us.

Peirce: CP 7.544 Cross-Ref:††

544. The removal of sensation from the department of *cognition*, or Knowledge, leaves nothing remaining in that department except what are called Mediate Cognitions, that is, Knowledges through some third idea or process different from either the Knowing self or the Known object. For the sake of giving this Mediate Cognition, or rather the peculiar kind of element of consciousness it involves a single name, I will call it *medisense*, that is, the consciousness of a middle term, or process, by which something not-self is set up over against the consciousness. All consciousness of a process belongs to this *medisense*. It has several varieties. In the first place there is a separative process, the centrifugal tendency of thought, by which any idea by following out its own development becomes separated from those with which it is connected. We see this in attention. When we see the little bottle with green crystals, the green idea detaches itself from the remaining ideas, the spicular form, the being bunched together in a little tube, etc. and leads to a thought which is accurately expressed by the sentence "these crystals are green," where the green stands off from the remaining ideas which remain confused together. It is the liveliness of the green idea which brings this about. And in all cases it is the idea which has vigor which spontaneously detaches itself from the rest. We may call this variety of *medisense* by the name of *Abstraction*.

Peirce: CP 7.545 Cross-Ref:††

545. Before I go further, there are one or two points which require explanation. Everybody knows that recollections gradually become "*dim*"; and also that attention makes Feelings "*vivid*," which before attention was applied to them were relatively "dim." This does not mean that we recollect a bright color as a dull color, or a loud sound as a faint sound; nor that attention makes an olive color seem an apple green or a whisper seem to be a bellow. It is several years since I was at Niagara Falls; but I have no doubt my recollection of the roar is of its being about as loud as it really is. I remember when I returned from a long absence, I found my old room was much smaller than I had recollected it. The first cup of coffee I tasted more than fifty years ago seems to me of a higher flavor than any coffee in the world really is. Though my imagination is very much the reverse of a vivid one, I can carry colors in my mind with more than usual accuracy. Dim as my imaginations and memories are, they do not represent high colors by low colors. We must say then that feelings have two kinds of intensity. One is the intensity of the feeling itself,†10 by which loud sounds are distinguished from faint ones, luminous colors from dark ones, highly chromatic colors from almost neutral tints, etc. The other is the intensity of consciousness that lays hold of the feeling, which makes the ticking of a watch actually heard infinitely more vivid than a cannon shot remembered to have been heard a few minutes ago. I shall not stop to discuss the difficult question of what the

distinction between those two kinds of intensity consists in, about which three or four opinions are held. I shall simply say that in my opinion the first kind of intensity, distinguishing bright colors from dim ones, is the intensity of feeling-consciousness; while the second kind, distinguishing sensation from imagination, is the intensity of *altersense* or of the assertiveness of the feeling. I shall call this second *vividness*.

Peirce: CP 7.546 Cross-Ref:††

546. Now, in order that you may understand what I am about to say, I must tell you about some experiments which I have conducted.†11 They have been several times repeated. But each series of experiments lasted for from three to five weeks, at a time when the weather was steady and my health or that of the subject of experiment was good; and I or he, as the case might be, took pains to lead a regular, cheerful life without agitation. The experiments would last for an hour, in some series for an hour and a half, at the same time every day. Extraordinary precautions were taken to have all the mental influences, light, warmth, etc. precisely the same every day, and the same throughout the time of experimentation. The attention would be exercised for seven minutes, in some series for ten; and then five minutes, sometimes three minutes, rest would be taken of just the same kind, generally a little conversation about the experiments. A habit was acquired of making two different efforts of attention of just the same degree of effort of attention at every experiment. One of these would be a very light effort, the other a vigorous one. I did not assume that these two degrees of attention would remain exactly the same throughout the month, but I did assume, first, that they would not change extravagantly, and second, that if, say, six or eight kinds of experiments were made, each ten times every day, then on the average of the whole month, the degree of effort made would be the same in one kind of experiment as in another, especially when the subject could not tell which were which. In that way, I was able to produce a fixed average intensity of attention. That attention was directed to certain differences of sensation. The sensations were of two general kinds in different series, some being colors and others sensations of pressure. In each case, there was a very carefully devised apparatus for producing the precise sensations desired, and for measuring them. I may mention that my color-box cost me \$2000. The pressure apparatus was cheap; but there no conceivable refinement was left out of account. I have mentioned that the sensations were measured. Now many psychologists deny that sensation can be measured. That is because they are not mathematicians, and are unacquainted with the mathematical theory of measurement; of which they have the crudest notions. I repeat that the sensations *were* scientifically measured, and on such a scale that zero meant no sensation. To show how accurate they were I may say that a piece of specially selected fuzzy black paper much blacker than black velvet was found to be 30 measurable degrees from absolute blackness. To get absolute blackness a closet about ten feet by ten was constructed and painted with lamp black in the inside. This was placed in a large hall with little light in it. Then, a small hole about the size of a cent in the thin wall of the closet when looked at from the outside appeared to be absolutely black. Now the experiments would be like this. The subject would put his eye to the eye-piece of the color-box and would see a small rectangle which at first



sight would appear to be all one color. [\[Click here to view\]](#) But after an effort of attention continuing from 5 seconds to a minute, he would see that it was not all one color, that it was sharply divided into two parts whose colors were



different. [\[Click here to view\]](#) One part might be a very little yellower than the other. After he had exerted his attention for a certain number of seconds, the whole would disappear and he would tell a second person, the operator, what he thought he had seen, and would also give a number marking his degree of confidence that he had seen it. The operator would record the result, in absolute silence without the slightest sign and without being seen. A tap mechanically made of precisely the same intensity would inform the subject that another rectangle was to be seen, and he would say "tp" when he put his eye to the eyepiece. I need not describe exactly how the pressure-experiments were made.†12 As a result there would be several thousand experiments showing the effect of two degrees of attention exerted for several different lengths of time upon discriminating between sensations differing by several known amounts. The results of experiments would all be expressed in two sets [of] numbers, one showing the percentage of errors and the other the feeling of confidence, in the attempts at discrimination under different circumstances. Those numbers were then subjected to mathematical discussion, according to the established principles of such work; and from them a law was deduced. It was found that the feeling of confidence did not begin to show itself at all until the real power of discrimination had reached a considerable strength. There were a very large number of cases in which the confidence was *zero*, so that the answers given appeared to the subject to be mere random guesses. Yet a decided majority of them were, in all cases, correct. It was also found that during the period of attention, the difference of sensation was continually increasing in vividness. Sensations that differed less, no matter how little, could still be discriminated just as well as sensations that differed more, if the effort of attention were greater, or if it were longer continued. The law was so accurately fulfilled, that it was safe to infer that the point at which infinite attention would be required for the slightest preponderance of right answers over wrong ones was just the point where the two sensations differed not at all. The difference of sensation for the discrimination of which no attention whatever would be required appeared to be infinite; though this result was less certain than the other.

Peirce: CP 7.547 Cross-Ref:††

547. The general upshot of all these experiments, together with others which I have not time to describe, was that when you ask yourself what is in your mind at any moment, and give yourself an answer, even after a searching scrutiny of the field of consciousness, you have not begun to tell yourself the whole truth. For it is one thing to feel a thing and it is another thing to have a reflex feeling, that *there is* a feeling; and my experiments conclusively show that the consciousness must reach a considerable vividness before the least reflex feeling of it is produced. That it is really felt is shown by the fact that a greater effort of attention would detect it. There is as it were, an upper layer of consciousness to which reflex consciousness, or self-consciousness, is attached. A moderate effort of attention for a second or two only brings a few items into that upper layer. But all the time the attention lasts, thousands of other ideas, at different depths of consciousness, so to speak, that is, literally, of different degrees of vividness, are moving upwards. These may influence our other thoughts long before they reach the upper layer of reflex consciousness. There are such vast numbers of ideas in consciousness of low degrees of vividness, that I think it may be true, -- and at any rate is roughly true, as a necessary consequence of my experiments, -- that our whole past experience is continually in our consciousness, though most of it sunk to a great depth of dimness. I think of consciousness as a bottomless lake, whose waters seem transparent, yet into which we can clearly see but a little way. But in this water there are countless objects at different depths; and certain influences will give certain kinds of those objects an upward impulse which may be intense enough and continue long enough to bring them into the upper visible layer. After the impulse ceases they commence to sink downwards.†13

Peirce: CP 7.548 Cross-Ref:††

548. I have spoken †14 of the first kind of *medisense*, abstraction, which breaks one idea away from another. There is an opposite influence by which when one idea has its vividness increased it gives an upward impulse to a number of other ideas with which it is connected so that it forms one set with them. The law of this is often called the law of the association of ideas. That is well enough. But it is inaccurate to call this phenomenon *association*, as Germans especially often do. Association is a different thing.†15 More accurate German writers call the action of which we are now speaking *reproduction*. But even that is not free from objection. For the *idea* which receives an upward impulse, making it grow more vivid, was not necessarily ever so near the surface of consciousness before; or if it was, that circumstance has nothing to do with it. It is a great mistake to suppose that ideas only become associated into sets in the upper layer of consciousness, although such action is more lively there. Most English and American psychologists of today use the term *Reproduction*; but I prefer the older English word *Suggestion*, to which some of the very best writers still adhere. The only objection to it is that the word is used in another sense in reference to the phenomena of hypnotism.

Peirce: CP 7.549 Cross-Ref:††

549. What takes place in suggestion is that an idea when it rises gives an upward motion to all other ideas belonging to the same set. For example, if the idea of *husband* is mentioned, since husband and wife form a set, the idea of *wife* will receive an upward motion and a few minutes later when fatigue has caused the husband idea to commence sinking, that is to become dim, the *wife* idea will be uppermost. I will some other day talk to you about this action of suggestion.

Peirce: CP 7.550 Cross-Ref:††

550. At present I hurry on to the third form of *medisense* which is that of the formation of sets of ideas, or association proper. A great many associations of ideas are inherited. Others grow up spontaneously. The rest depend upon the principle that ideas once brought together into a set remain in that set. Many associations are merely accidental. A child acquires a distaste for a particular kind of food merely because it ate it when it was sick. The idea of that food and the feeling of sickness are brought into a set; and the consequence is that every time the idea of that food reaches a high degree of vividness, the feeling of sickness gets a swift upward motion. Other associations cannot be called accidental because it was in the nature of things that they should appear in sets. Thus, light and warm get associated in our minds because they are associated in Nature.

Peirce: CP 7.551 Cross-Ref:††

551. There are no other forms of consciousness except the three that have been mentioned, Feeling, Altersense, and Medisense. They form a sort of system. Feeling is the momentarily present contents of consciousness taken in its pristine simplicity, apart from anything else. It is consciousness in its first state, and might be called *primisense*. *Altersense* is the consciousness of a directly present other or second, withstanding us. *Medisense* is the consciousness of a thirdness, or medium between primisense and altersense, leading from the former to the latter. It is the consciousness of a process of bringing to mind. Feeling, or *primisense*, is the consciousness of firstness; altersense is consciousness of otherness or secondness; medisense is the consciousness of means or thirdness. Of *primisense* there is but one fundamental mode. *Altersense* has two modes, Sensation and Will. *Medisense* has three modes, *Abstraction, Suggestion, Association*. †16

Peirce: CP 7.552 Cross-Ref:††

552. The only element of the mind that this enumeration omits is the phenomenon of *Fatigue*. It is a highly important matter. Our mental life and health depend to a far greater extent than would at first be guessed upon the action of fatigue. But it finds no place in this system. The only defence that I can offer for this is that there is no direct consciousness of fatigue. Whether that is an adequate excuse or not, I am not yet quite decided.

Peirce: CP 7.553 Cross-Ref:††

§3. CONSCIOUSNESS AND REASONING †17

553. We are going to shock the physiological psychologists, for once, by attempting, not an account of a hypothesis about the brain, but a description of an image which shall correspond, point by point, to the different features of the phenomena of consciousness. Consciousness is like a bottomless lake in which ideas are suspended at different depths. †18 Indeed, these ideas themselves constitute the very medium of consciousness itself. †19 Percepts alone are uncovered by the medium. We must imagine that there is a continual fall of rain upon the lake; which images the constant inflow of percepts in experience. All ideas other than percepts are more or less deep, and we may conceive that there is a force of gravitation, so that the deeper ideas are, the more work will be required to bring them to the surface. This virtual work, which the mathematicians call the 'potentials' of the particles, is the negative of the 'potential energy'; and the potential energy is that feature of the image

which corresponds to the degree of vividness of the idea. Or we may see that the potential, or depth, represents the degree of energy of attention that is requisite to discern the idea at that depth. But it must not be thought that an idea actually has to be brought to the surface of consciousness before it can be discerned. To bring it to the surface of consciousness would be to produce a hallucination. Not only do all ideas tend to gravitate toward oblivion, but we are to imagine that various ideas react upon one another by selective attractions. This images the associations between ideas which tend to agglomerate them into single ideas. Just as our idea of spatial distance consists in the sense of time that it would take with a given effort to pass from one object to another, so the distance between ideas is measured by the time it will take to unite them. One tries to think of the French for *shark* or for *linchpin*. The time that it will take to recover the forgotten word depends upon the force of association between the ideas of the English and French words and upon circumstances which we image by their *distance*. This, it must be confessed, is exceedingly vague; as vague as would be our notion of spatial distance if we lived in the body of an ocean, and were destitute of anything rigid to measure with, being ourselves mere portions of fluid.

Peirce: CP 7.554 Cross-Ref:††

554. Consciousness is rather like a bottomless lake in which ideas are suspended, at different depths. Percepts alone are uncovered by the medium. The meaning of this metaphor is that those which [are] deeper are discernible only by a greater effort, and controlled only by much greater effort. These ideas suspended in the medium of consciousness, or rather themselves parts of the fluid, are attracted to one another by associational habits and dispositions, -- the former in association by contiguity, the latter in association by resemblance. An idea near the surface will attract an idea that is very deep only so slightly that the action must continue for some time before the latter is brought to a level of easy discernment. Meantime the former is sinking to dimmer consciousness. There seems to be a factor like momentum, so that the idea originally dimmer becomes more vivid than the one which brought it up. In addition, the mind has but a finite area at each level; so that the bringing of a mass of ideas up inevitably involves the carrying of other ideas down. Still another factor seems to be a certain degree of buoyancy or association with whatever idea may be vivid, which belongs to those ideas that we call purposes, by virtue of which they are particularly apt to be brought up and held up near the surface by the inflowing percepts and thus to hold up any ideas with which they may be associated. The control which we exercise over our thoughts in reasoning consists in our purpose holding certain thoughts up where they may be scrutinized. The levels of easily controlled ideas are those that are so near the surface as to be strongly affected by present purposes. The aptness of this metaphor is very great.

Peirce: CP 7.555 Cross-Ref:††

555. If the question is asked in what the processes of contemplation and of fixation of the attention consist, this question being psychological, it is necessary, before answering it, to describe some phenomena of the mind. Be it known, then, that consciousness, or feeling, has been ascertained by careful observations mathematically discussed to have the properties now to be stated.†20 Feeling, by which is here meant that of which we are supposed to be immediately conscious, is subject to degrees. That is to say, besides the *objective intensity* which distinguishes a loud sound from a faint sound, there is a *subjective intensity* which distinguishes a lively consciousness of a sound, from a dull consciousness of it. Though the two

kinds of intensity are apt to go together, yet it is possible for a person at the same time to recall the tick of a watch and the sound of a neighboring cannon, and to have a livelier consciousness of the former than of the latter, without however remembering the latter [as] a fainter sound than the former. Feelings of slight subjective intensity act upon one another, undergo transformations, and affect the emotions and the voluntary actions; although they do all this less decisively than they would do if they were more intense. They are also, *other things being equal*, much less under control than more subjectively intense feelings. This remark needs explanation. A feeling may be forced upon the mind through the senses, or by experience, directly or indirectly, and bear down the power of the will; and those feelings are the most subjectively intense we have. Why they should be so, will soon be explained.†21 But when a feeling is *not* thus forced upon us our being conscious of it sufficiently to attract attention makes it act more upon us, and at the same time enables us to affect its transformations, more than if it were scarcely perceptible. Now there are certain combinations of feelings which are specially interesting. These are those which tend toward a reaction between mind and body, whether in sense, in the action of the glands, in contractions of involuntary muscles, in coördinated voluntary deeds, or, finally, in discharges of an extraordinary kind of one part of the nerves upon another. Interesting combinations of ideas are more active than others, both in the way of suggestion, and in the way of subjective intensity. The action of thought is all the time going on, not merely in that part of consciousness which thrusts itself on the attention, and which is the most under discipline, but also in its deeply shaded parts, of which we are in some measure conscious but not sufficiently so to be strongly affected by what is there. But when in the uncontrolled play of that part of thought, an interesting combination occurs, its subjective intensity increases for a short time with great rapidity.†22 This is what constitutes the fixation of the attention. Contemplation consists in using our self-control to remove us from the forcible intrusion of other thoughts, and in considering the interesting bearings of what may lie hidden in the icon, so as to cause the subjective intensity of it to increase.

Peirce: CP 7.556 Cross-Ref:††

556. The observation of the icon may be ordinary direct observation, or it may be scientific observation aided by the apparatus of logical algebra and other technical means.

Peirce: CP 7.557 Cross-Ref:††

557. A third step in inference is performed upon the indices. Thus, an index may be dropped from consideration. If there are two selective indices, one universal, the other particular, and the latter selection is made last, the order of the selections can be reversed. But all these changes in the indices are justified only by considering icons. We may, therefore, say that excepting the colligation of different beliefs the whole of inference consists in *observation*, namely in the observation of icons. Even the colligations well up from the depths of consciousness in precisely the same manner as that in which the special features of icons are remarked.

Peirce: CP 7.558 Cross-Ref:††

558. Thus, all knowledge comes to us by observation, part of it forced upon us from without from Nature's mind and part coming from the depths of that inward aspect of mind, which we egotistically call *ours*; though in truth it is we who float upon its surface and belong to it more than it belongs to us. Nor can we affirm that the inwardly seen mind is altogether independent of the outward mind which is its Creator.

Peirce: CP 7.559 Cross-Ref:††
§4. WHAT IS THE USE OF CONSCIOUSNESS?†23

559. In the last chapter,†24 I assumed the reader would occupy the position of Common Sense, which makes the real things in this world blind unconscious objects working by mechanical laws together with a consciousness as idle spectator. I pointed out that this spectator cannot have part or lot even in the intelligence and purpose of the business; for intelligence does not consist in feeling intelligently but in acting so that one's deeds are concentrated upon a result.

Peirce: CP 7.560 Cross-Ref:††

560. This makes the universe a muddle. According to it consciousness is perfectly impotent and is not the original of the material world; nor on the other hand can material forces ever have given birth to feeling, for all they do is to accelerate the motions of particles. Nay, that they should so much as give rise to sensations in that consciousness is more than incomprehensible, it is manifestly impossible. There is no room for reaction between mind and matter. The only consistent position for such a philosophy is flatly to deny that there is any such thing as consciousness. Even were that denial made, the question would be insoluble -- not for us merely, but in its own nature, -- how all the laws of mechanics came about, or why they should have the curious relationships they exhibit. Then, were that impossibility disregarded, just look out of your window, reader, and see this world in all its infinite manifoldness, and say whether you are content to take it wholly unaccounted for, as something that always has been, and always has been as complex as it is now. For mechanical forces never produce any new diversity, but only transform one diversity into another diversity.

Peirce: CP 7.561 Cross-Ref:††

561. The whole of this suicide of Common Sense results from its incautious assumption that it is one thing to look red or green and another thing to see red or green. Now metaphysicians never have agreed, or at least never have perceived that they agreed, about anything; but I believe that every man who has ever reflected deeply about knowledge has come to the conclusion that there is something wrong about that assumption.†25

Peirce: CP 7.562 Cross-Ref:††

562. Grant that that assumption is somehow wrong, though we may not, at first, see how exactly, and the muddle begins to clarify itself. The spectator is no longer on one side of the footlights, and the world on the other. He is, in so far as he sees, at one with the poet of the piece. To act intelligently and to see intelligently become at bottom one. And in the matter of auditing the account of the universe, its wealth and its government, we gain the liberty of drawing on the bank of thought.

Peirce: CP 7.563 Cross-Ref:††

563. This method promises to render the totality of things thinkable; and it is plain there is no other way of explaining anything than to show how it traces its lineage to the womb of thought.

Peirce: CP 7.564 Cross-Ref:††

564. This is what is called Idealism. As soon, however, as we seek preciser

statement, difficulties arise, -- by no means insuperable ones, yet calling for patient study based upon a thorough understanding of logic. All this must be postponed. Yet one very obvious and easily answered objection may be noticed. It will be said that the identification of knowledge and being threatens to deprive us of our Ignorance and Error. Let me hasten to swear that no act of mine shall lay hands on those sacramenta.

Peirce: CP 7.565 Cross-Ref:††
§5. SYNECHISM AND IMMORTALITY †26

565. The word *synechism* is the English form of the Greek {synechismos}, from {synechés}, continuous.†27 For two centuries we have been affixing -ist and -ism to words, in order to note sects which exalt the importance of those elements which the stemwords signify. Thus, *materialism* is the doctrine that matter is everything, *idealism* the doctrine that ideas are everything, *dualism* the philosophy which splits everything in two. In like manner, I have proposed to make *synechism* mean the tendency to regard everything as continuous.†28

Peirce: CP 7.566 Cross-Ref:††

566. For many years I have been endeavoring to develop this idea, and have, of late, given some of my results in the *Monist*.†29 I carry the doctrine so far as to maintain that continuity governs the whole domain of experience in every element of it. Accordingly, every proposition, except so far as it relates to an unattainable limit of experience (which I call the Absolute,) is to be taken with an indefinite qualification; for a proposition which has no relation whatever to experience is devoid of all meaning.

Peirce: CP 7.567 Cross-Ref:††

567. I propose here, without going into the extremely difficult question of the evidences of this doctrine, to give a specimen of the manner in which it can be applied to religious questions. I cannot here treat in full of the method of its application. It readily yields corollaries which appear at first highly enigmatic; but their meaning is cleared up by a more thoroughgoing application of the principle. This principle is, of course, itself to be understood in a synechistic sense; and, so understood, it in no wise contradicts itself. Consequently, it must lead to definite results, if the deductions are accurately performed.

Peirce: CP 7.568 Cross-Ref:††

568. Thoroughgoing synechism will not permit us to say that the sum of the angles of a triangle exactly equals two right angles, but only that it equals that quantity plus or minus some quantity which is excessively small for all the triangles we can measure.†30 We must not accept the proposition that space has three dimensions as certainly strictly accurate; but can only say that any movements of bodies out of the three dimensions are at most exceedingly minute. We must not say that phenomena are perfectly regular, but only that the degree of their regularity is very high indeed.

Peirce: CP 7.569 Cross-Ref:††

569. There is a famous saying of Parmenides {esti gar einai, méden d' ouk einai}, "being is, and not-being is nothing." This sounds plausible; yet synechism

flatly denies it, declaring that being is a matter of more or less, so as to merge insensibly into nothing. How this can appear when we consider that to say that a thing *is* is to say that in the upshot of intellectual progress it will attain a permanent status in the realm of ideas. Now, as no experiential question can be answered with absolute certainty, so we never can have reason to think that any given idea will either become unshakably established or be forever exploded.†31 But to say that neither of these two events will come to pass definitively is to say that the object has an imperfect and qualified existence. Surely, no reader will suppose that this principle is intended to apply only to some phenomena and not to others, -- only, for instance, to the little province of matter and not to the rest of the great empire of ideas. Nor must it be understood only of phenomena to the exclusion of their underlying substrates. Synechism certainly has no concern with any incognizable; but it will not admit a sharp sundering of phenomena from substrates. That which underlies a phenomenon and determines it, thereby is, itself, in a measure, a phenomenon.

Peirce: CP 7.570 Cross-Ref:††

570. Synechism, even in its less stalwart forms, can never abide dualism, properly so called. It does not wish to exterminate the conception of twoness, nor can any of these philosophic cranks who preach crusades against this or that fundamental conception find the slightest comfort in this doctrine. But dualism in its broadest legitimate meaning as the philosophy which performs its analyses with an axe, leaving as the ultimate elements, unrelated chunks of being, this is most hostile to synechism. In particular, the synechist will not admit that physical and psychical phenomena are entirely distinct, -- whether as belonging to different categories of substance, or as entirely separate sides of one shield, -- but will insist that all phenomena are of one character, though some are more mental and spontaneous, others more material and regular. Still, all alike present that mixture of freedom and constraint, which allows them to be, nay, makes them to be teleological, or purposive.

Peirce: CP 7.571 Cross-Ref:††

571. Nor must any synechist say, "I am altogether myself, and not at all you." If you embrace synechism, you must abjure this metaphysics of wickedness. In the first place, your neighbors are, in a measure, yourself, and in far greater measure than, without deep studies in psychology, you would believe. Really, the selfhood you like to attribute to yourself is, for the most part, the vulgarest delusion of vanity. In the second place, all men who resemble you and are in analogous circumstances are, in a measure, yourself, though not quite in the same way in which your neighbors are you.

Peirce: CP 7.572 Cross-Ref:††

572. There is still another direction in which the barbaric conception of personal identity must be broadened. A Brahmanical hymn begins as follows: "I am that pure and infinite Self, who am bliss, eternal, manifest, all-pervading, and who am the substrate of all that owns name and form." This expresses more than humiliation, -- the utter swallowing up of the poor individual self in the Spirit of prayer. All communication from mind to mind is through continuity of being. A man is capable of having assigned to him a *rôle* in the drama of creation, and so far as he loses himself in that *rôle*, -- no matter how humble it may be, -- so far he identifies himself with its Author.

Peirce: CP 7.573 Cross-Ref:††

573. Synechism denies that there are any immeasurable differences between phenomena; and by the same token, there can be no immeasurable difference between waking and sleeping. When you sleep, you are not so largely asleep as you fancy that

you be.

Peirce: CP 7.574 Cross-Ref:††

574. Synechism refuses to believe that when death comes, even the carnal consciousness ceases quickly. How it is to be, it is hard to say, in the all but entire lack of observational data. Here, as elsewhere, the synechistic oracle is enigmatic. Possibly, the suggestion of that powerful fiction "Dreams of the Dead," recently published, †32 may be the truth.

Peirce: CP 7.575 Cross-Ref:††

575. But, further, synechism recognizes that the carnal consciousness is but a small part of the man. There is, in the second place, the social consciousness, by which a man's spirit is embodied in others, and which continues to live and breathe and have its being very much longer than superficial observers think. Our readers need not be told how superbly this is set forth in Freytag's *Lost Manuscript*. †33

Peirce: CP 7.576 Cross-Ref:††

576. Nor is this, by any means, all. A man is capable of a spiritual consciousness, which constitutes him one of the eternal verities, which is embodied in the universe as a whole. This as an archetypal idea can never fail; and in the world to come is destined to a special spiritual embodiment.

Peirce: CP 7.577 Cross-Ref:††

577. A friend of mine, in consequence of a fever, totally lost his sense of hearing. He had been very fond of music before his calamity; and, strange to say, even afterwards would love to stand by the piano when a good performer played. So then, I said to him, after all you can hear a little. Absolutely not at all, he replied; but I can *feel* the music all over my body. Why, I exclaimed, how is it possible for a new sense to be developed in a few months! It is not a new sense, he answered. Now that my hearing is gone I can recognize that I always possessed this mode of consciousness, which I formerly, with other people, mistook for hearing. In the same manner, when the carnal consciousness passes away in death, we shall at once perceive that we have had all along a lively spiritual consciousness which we have been confusing with something different.

Peirce: CP 7.578 Cross-Ref:††

578. I have said enough, I think, to show that, though synechism is not religion, but, on the contrary, is a purely scientific philosophy, yet should it become generally accepted, as I confidently anticipate, it may play a part in the onement of religion and Science.

Peirce: CP 7.579 Cross-Ref:††

§6. CONSCIOUSNESS AND LANGUAGE †34

579. Philosophy is the attempt, -- for as the word itself implies it is and must be imperfect -- is the attempt to form a general informed conception of the *All*. All men philosophize; and as Aristotle says we must do so if only to prove the futility of philosophy. Those who neglect philosophy have metaphysical theories as much as others -- only they [have] rude, false, and wordy theories. Some think to avoid the influence of metaphysical errors, by paying no attention to metaphysics; but

experience shows that these men beyond all others are held in an iron vice of metaphysical theory, because by theories that they have never called in question. No man is so enthralled by metaphysics as the totally uneducated; no man is so free from its dominion as the metaphysician himself. Since, then, everyone must have conceptions of things in general, it is most important that they should be carefully constructed.

Peirce: CP 7.580 Cross-Ref:††

580. I shall enter into no criticism of the different methods of metaphysical research, but shall merely say that in the opinion of several great thinkers, the only successful mode yet lighted upon is that of adopting our logic as our metaphysics. In the last lecture,†35 I endeavored to show how logic furnishes us with a classification of the elements of consciousness. We found that all modifications of consciousness are inferences and that all inferences are valid inferences. At the same time we found that there were three kinds of inference: 1st, Intellectual inference with its three varieties Hypothesis, Induction and Deduction; 2nd, Judgments of sensation, emotions, and instinctive motions which are hypotheses whose predicates are unanalyzed in comprehension; and 3rd, Habits, which are Inductions whose subjects are unanalyzed in extension. This division leads us to three elements of consciousness: 1st, *Feelings* or Elements of comprehension; 2nd, *Efforts* or Elements of extension; and 3rd, *Notions* or Elements of information,†36 which is the union of extension and comprehension. I regret that the time does not permit me to dwell further upon this theory; but I wish to pass to a loftier and more practical question of metaphysics in order to put in a still stronger light the advantages of the study of logic. The question which I shall select is "what is man?" I think I may state the prevalent conception thus: Man is essentially a soul, that is, a thing occupying a mathematical point of space, not thought itself but the subject of inhesion of thought, without parts, and exerting a certain material force called volition. I presume that most people consider this belief as intuitive, or, at least, as planted in man's nature and more or less distinctly held by all men, always and everywhere.†37 On the contrary, the doctrine is a very modern one. All the ancients and many of the scholastics, held that man is compact of several souls; three was the usual number assigned, sometimes two, four, or five. Every attentive reader of St. Paul is aware that according to him, man has a threefold being. We derive the notion of the soul's being single from Descartes. But with him, thought itself makes the man; whereas with us consciousness *is* not the man but is in man. Descartes, also, does not admit that the will of man exerts any force upon matter; as we mostly believe. In fact, the prevalent view of the present day is a heterogeneous hodgepodge of the most contradictory theories; its doctrines are borrowed from different philosophers while the premisses by which alone those philosophers were able to support their doctrines are denied; the theory thus finds itself totally unsupported by facts and in several particulars at war with itself. And this is admitted by most of those who have subjected it to rigid criticism.†38

Peirce: CP 7.581 Cross-Ref:††

581. One source of all this diversity of opinion, has been the want of an accurate discrimination between an inductive and a hypothetic explanation of the facts of human life. We have seen that every fact requires two kinds of explanation; the one proceeds by induction to replace its subject by a wider one, the other proceeds by hypothesis to replace its predicate by a deeper one.†39 We have seen that these two explanations never coincide, that both are indispensable, and that quarrels have sprung up even in physical science where there are so few disagreements in

consequence of trying to make one theory perform both functions. Let us take care that we do not confound these two separate inquiries in reference to the soul. The hypothetic explanation will inform us of the causes or necessary antecedents of the phenomena of human life. These phenomena may be regarded internally or externally. Regarded internally they require an internal explanation by internal necessary antecedents, that is by *premises*; and this explanation was given in the last lecture. If they are regarded externally or physically they require a physical explanation by physical antecedents, and this inquiry must be turned over unreservedly to the physiologists. They will find the truth of the matter, and we may rest satisfied that no explanation which is based squarely upon legitimate hypothesis from the facts of nature, can possibly conflict with a purely inductive explanation of man. It is true that the question for the physiologist is what are the physical antecedents of man's actions, that is what sort of an automaton is man; so that it is assumed as the condition of the problem that man is an automaton. For, automatism in this connection, consciousness being of course admitted, means nothing but regular physical antecedence; -- implies only that nature is uniform; and this as we have seen is not a mere law of nature or fact of observation, but is a postulate of all thought, which no man consistently or persistently denies.†40 Yet this automatism seems, no doubt, to many to conflict with the notion of man as a RESPONSIBLE and IMMORTAL soul. But, then, we should remember that in our minds, the essential conceptions of responsibility and immortality are covered over with a mass of parasitical reflection derived from every philosophy and every religion of past time; so that if *we* cannot reconcile the doctrines of responsibility and immortality with the postulates of thought or with themselves, this is sufficiently accounted for by the obscurity and confusion of our notions on this subject, and we are by no means forced to adopt that which is the only other alternative and say that these doctrines are essentially false. These doctrines are a part of our religion; and one [of] them -- if not both -- are among its most precious consolations, which it would be difficult indeed to wring from the breast of a people which has entertained them for a thousand years. Talk as they please, of the weariness of unceasing day, of the balm of an eternal sleep, of the nobility lent to humanity by regarding it as capable of struggling and suffering for that which transcends its own responsibility and life-time; we still cling by nature original or acquired, to the dear hopes of our ancient religion. But while I have thought it proper to dwell for a moment upon the possibility of our being unable to reconcile responsibility and immortality with physical necessity, I must add that in fact we are not driven to that point, at all. On the contrary the philosophers of the Brownist school have shown uncontrovertibly that they are capable of being reconciled, and their arguments are very forcibly stated in an American work called *Liberty and Necessity*, written by Judge [Henry] Carleton of Louisiana and published in Philadelphia in 1857. There may be other modes of reconciling these conceptions †41 besides that which they have pointed out but still they have shown that a rational reconciliation is possible.

Peirce: CP 7.582 Cross-Ref:††

582. Thus, the hypothetic explanation of human nature stands by itself and will present no contradiction to the inductive explanation, which is what we desire when we ask what is man? To what real kind does the thinking, feeling, and willing being belong? We know that externally considered man belongs to the animal kingdom, to the branch of vertebrates, and the class of mammals; but what we seek is his place when considered internally, disregarding his muscles, glands, and nerves and considering only his feelings, efforts, and conceptions.

Peirce: CP 7.583 Cross-Ref:††

583. We have already seen that every state of consciousness [is] an inference; so that life is but a sequence of inferences or a train of thought. At any instant then man is a thought, and as thought is a species of symbol, the general answer to the question what is man? is that he is a symbol. To find a more specific answer we should compare man with some other symbol.

Peirce: CP 7.584 Cross-Ref:††

584. I write here the word *Six*. Now let [us] ask ourselves in what respects a *man* differs from that word. In the first place, the body of man is a wonderful mechanism, that of the word nothing but a line of chalk. In the second place, the meaning of the word is very simple, the meaning of a man a very Sphinx's question. These two differences are very obvious, -- they lie upon the surface. But what other difference is there?

Peirce: CP 7.585 Cross-Ref:††

585. A man has consciousness; a word has not. †42 What do we mean by consciousness, for it is rather an ambiguous term. There is that emotion which accompanies the reflection that we have animal life. A consciousness which is dimmed when animal life is at its ebb, in age or sleep, but which is not dimmed when the spiritual life is at its ebb; which is more lively the better animal a man is, but is not so the better man he is. You can all distinguish this sensation I am sure; we attribute it to all animals but not to words, because we have reason to believe that it depends upon the possession of an animal body. And, therefore, this difference is included under the first that we mentioned and is not an additional one. In the second place, consciousness is used to mean the knowledge which we have of what is in our minds; the fact that our thought is an index for itself of itself on the ground of a complete identity with itself. But so is any word or indeed any thing, so that this constitutes no difference between the word and the man. In the third place, consciousness is used to denote the *I think*, the unity of thought; but the unity of thought is nothing but the unity of symbolization; consistency, in a word -- the implication of being -- and belongs to every word whatever. It is very easy to think we have a *clear* notion of what we mean by consciousness, and yet it may be that the word excites no thought but only a sensation, a mental word within us; and then because we are not accustomed to allow the word written on the board to excite that sensation, we may think we distinguish between the man and the word when we do not.

"Most ignorant of what we're most assured

Our glassy essence!"

Peirce: CP 7.586 Cross-Ref:††

586. Consciousness is, also, used to denote what I call *feeling*; as by Mr. Bain whom I mention in order to say that he recognizes the unity of sensation and emotion under this term although he has not carried out the conception consistently. Has that word feeling? Man, say the sensationalists, is a series of feelings; at any one moment, then, *is a feeling*. How is it with the word? Feelings, we all know, depend upon the bodily organism. The blind man from birth has no such feelings as red, blue, or any other colour; and without any body at all, it is probable we should have no feelings at all; and the word which has no animal body probably therefore has no animal

feelings, and of course if we restrict the word feeling to meaning animal feeling the word has no feelings. But has it not something corresponding to feeling? Every feeling is cognitive -- is a sensation, and a sensation is a mental sign or word. Now the word has a word; it has itself; and so if man is an animal feeling, the word is just as much a written feeling.

Peirce: CP 7.587 Cross-Ref:††

587. But is there not this difference. Man's feelings are perceptions, he is affected by objects. He sees, hears, etc. A word does not. Yes; that is true, but perception, plainly, depends upon having an animal organism and therefore there is here no further difference beyond the obvious two mentioned at first. Yet even here, there is a correspondence between the word and the man. Perception is the possibility of acquiring information, of meaning more; now a word may learn. How much more the word *electricity* means now than it did in the days of Franklin; how much more the term planet means now than it did in the time [of] Hipparchus. These words have acquired information; just as a man's thought does by further perception. But is there not a difference, since a man makes the word and the word means nothing which some man has not made it mean and that only to that man? This is true; but since man can think only by means of words or other external symbols, words might turn round and say, You mean nothing which we have not taught you and then only so far as you address some word as the interpretant of your thought. In fact, therefore, men and words reciprocally educate each other; each increase of a man's information is at the same time the increase of a word's information and *vice versa*. So that there is no difference even here.

Peirce: CP 7.588 Cross-Ref:††

588. You see that remote and dissimilar as the word and the man appear, it is exceedingly difficult to state any essential difference between them except a physiological one. A man has a moral nature, a word apparently has none. Yet morals relate primarily to what we ought to do; and therefore as words are physiologically incapacitated to act we should not consider this as a separate point of distinction. But if we consider morality as the conformity to a law of fitness of things, -- a principle of what is suitable in thought, not in order to make it true but as a prerequisite to make it spiritual, to make it rational, to make it more truly thought at all; we have something extremely analogous in the good grammar of a word or sentence. Good grammar is that excellence of a word by which it comes to have a good conscience, to be satisfactory not merely [with] reference being had to the actual state of things which it denotes, not merely to the consequences of the act, but to it in its own internal determination. Beauty and truth belong to the mind and word alike. The third excellence is morality on the one hand, Grammar on the other.

Peirce: CP 7.589 Cross-Ref:††

589. Man has the power of effort or attention; but as we have seen that this is nothing but the power of denotation, it is possessed by the word also.

Peirce: CP 7.590 Cross-Ref:††

590. Perhaps the most marvellous faculty of humanity is one which it possesses in common with all animals and in one sense with all plants, I mean that of procreation. I do not allude to the physiological wonders, which are great enough, but to the fact of the production of a new human soul. Has the word any such relation as that of father and son? If I write "Let *Kax* denote a gas furnace," this sentence is a symbol which is creating another within itself. Here we have a certain analogy with paternity; just as much and no more as when an author speaks of his writings as his

offspring, -- an expression which should be regarded not as metaphorical but merely as general. Cuvier said that Metaphysics is nothing but Metaphor; an identity which is prettily typified in those acted charades, [in] the first of which two doctors come in at opposite sides of the stage, shake hands and go out for the first scene, then repeat the same thing for the second scene and again for the whole word; and then do the same thing three times for the three scenes of the second word; the two words are of course metaphysician and metaphor; and their identity suggests that the characters must have been the invention of some one who thought with Cuvier that Metaphysics is another term for Metaphor. If metaphor be taken literally to mean an expression of a similitude when the sign of predication is employed instead of the sign of likeness -- as when we say this man *is* a fox instead of this man is like a fox, -- I deny entirely that metaphysicians are given to metaphor; on the contrary, no other writers can compare with them for precision of language; but if Cuvier was only using a metaphor himself, and meant by metaphor broad comparison on the ground of characters of a formal and highly abstract kind, -- then, indeed, metaphysics professes to be metaphor -- that is just its merit -- as it was Cuvier's own merit in Zoölogy

Peirce: CP 7.591 Cross-Ref:††

591. Enough has now been said, I think, to show a true analogy between a man and a word. I dare say this seems very paradoxical to you; I remember it did to me, at first. But having thought it over repeatedly, it has come to seem merest truism. A man denotes whatever is the object of his attention at the moment; he connotes whatever he knows or feels of this object, and is the incarnation of this form or intelligible species; his interpretant is the future memory of this cognition, his future self, or another person he addresses, or a sentence he writes, or a child he gets. In what does the identity of man consist and where is the seat of the soul? It seems to me that these questions usually receive a very narrow answer. Why we used to read that the soul resides in a little organ of the brain no bigger than a pin's head. Most anthropologists now more rationally say that the soul is either spread over the whole body or is all in all and all in every part. But are we shut up in a box of flesh and blood? When I communicate my thought and my sentiments to a friend with whom I am in full sympathy, so that my feelings pass into him and I am conscious of what he feels, do I not live in his brain as well as in my own -- most literally? True, my animal life is not there but my soul, my feeling thought attention are. If this be not so, a man is not a word, it is true, but is something much poorer. There is a miserable material and barbarian notion according to which a man cannot be in two places at once; as though he were a *thing*! A word may be in several places at once, Six Six, because its essence is spiritual; and I believe that a man is no whit inferior to the word in this respect. Each man has an identity which far transcends the mere animal; -- an essence, a *meaning* subtle as it may be. He cannot know his own essential significance; of his eye it is eyebeam. But that he truly has this outreaching identity -- such as a word has -- is the true and exact expression of the fact of sympathy, fellow feeling -- together with all unselfish interests -- and all that makes us feel that he has an absolute worth. Some one will ask me for *proof* of this. It seems to me that I have already given both the proof and the confirmation. The whole proof is very long but its principal *lemmas* were these: 1st, "What is man?" is an inductive question in its present sense; 2nd, The inductive explanation is only the general expression of the phenomena, and makes no hypothesis; 3rd, Whatever man is he is at each instant; 4th, At each instant the only internal phenomena he presents are feeling, thought, attention; 5th, Feelings, thought, attention are all cognitive; 6th, All cognition is general, there is no intuition;†43 7th, A general representation is a symbol; 8th, Every symbol has an essential comprehension which determines its identity. The confirmation I offered was the fact

that man is conscious of his interpretant, -- his own thought in another mind -- I do not say immediately conscious -- is happy in it, feels himself in some degree to be there. So that I believe that nothing but an undue ascendancy of the animal life can prevent the reception of this truth.

Peirce: CP 7.592 Cross-Ref:††

592. This essence of which I speak is not the whole soul of man; it is only his core which carries with it all the information which constitutes the development of the man, his total feelings, intentions, thoughts. When I, that is my thoughts, enter into another man, I do not necessarily carry my whole self, but what I do carry is the seed of [the] part that I do not carry -- and if I carry the seed of my whole essence, then of my whole self actual and potential. I may write upon paper and thus impress a part of my being there; that part of my being may involve only what I have in common with all men and then I should have carried the soul of the race, but not my individual soul into the word there written. Thus every man's soul is a special determination of the generic soul of the family, the class, the nation, the race to which he belongs

Peirce: CP 7.593 Cross-Ref:††

593. The principle that the essence of a symbol is formal, not material, has one or two important consequences. Suppose I rub out this word (Six) and write Six. Here is not a second word but the first over again; they are identical. Now can identity be interrupted or ought we to say that the word existed although it was unwritten? This word *six* implies that *twice three* is *five and one*. This is eternal truth; a truth which always is and must be; which would be though there were not six things in the universe to number, since it would still remain true that *five and one* would have been *twice three*. Now this *truth* is the word, six; if by six we mean not this chalk line, but that wherein *six, sex, {hex}, sechs, zes, seis, sei* agree. Truth, it is said, is never without a witness; and, indeed, the fact itself -- the state of things -- is a symbol of the general fact through the principles of induction; so that the true symbol has an interpretant so long as it is true. And as it is identical with its interpretant, it always exists. Thus, the necessary and true symbol is immortal. And man must also be so, provided he is vivified by the truth. This is an immortality very different indeed from what most people hope for, although it does not conflict with the latter. I do not *know* that the Mohammedan paradise is not true, only I have no evidence that it is. Animal existence is certainly a pleasure, though some speak of being weary of it; but I think it is confessed by the most cultivated peoples that it is not immortal; otherwise, they would consider the brutes as immortal. Spiritual existence, such as a man has in him, whom he carries along with him in his opinions and sentiments; sympathy, love; this is what serves as evidence of man's absolute worth -- and this is the existence which logic finds to be certainly immortal. It is not an impersonal existence; for personality lies in the unity of the *I think* -- which is the unity of symbolization -- the unity of consistency -- and belongs to every symbol. It is not an existence, cut off from the external world, for feeling and attention are essential elements of the symbol itself. It is, however, a changed existence; one in which there are no longer any of the glories of hearing and vision, for sounds and colors require an animal eye; and in the same way all the feelings will be different.

Peirce: CP 7.594 Cross-Ref:††

594. This immortality is one which depends upon the man's being a true symbol. If instead of *six*, we had written *Jove* we should have had a symbol which has but a contingent existence; it has no everlasting witness in the nature of things and will pass away or remain only in men's memories without exciting any response in

their hearts. It is, indeed, true so far as it means a *supreme being*; its generic soul is true and eternal, but its specific and individual soul is but a shadow.

Peirce: CP 7.595 Cross-Ref:††

595. Each man has his own peculiar character. It enters into all he does. It is in his consciousness and not a mere mechanical trick, and therefore it is by the principles of the last lecture a cognition; but as it enters into all his cognition, it is a cognition of *things in general*. It is therefore the man's philosophy, his way of regarding things; not a philosophy of the head alone -- but one which pervades the whole man. This idiosyncrasy is the idea of the man; and if this idea is true he lives forever; if false, his individual soul has but a contingent existence.

Peirce: CP 7.596 Cross-Ref:††

596. Gentlemen and ladies, I announce to you this theory of immortality for the first time. It is poorly said, poorly thought; but its foundation is the rock of truth. And at least it will serve to illustrate what use might be made by mightier hands of this reviled science, logic, *nec ad melius vivendum, nec ad commodius disserendum*.

Peirce: CP 7.597 Cross-Ref:††
CHAPTER 5

TELEPATHY AND PERCEPTION†1

§1. TELEPATHY

597. A little passage at arms between a physicist of standing and a celebrated psychologist has lately focussed the public glance once more for a moment upon this subject.†2 The remarks of Prof. Trowbridge express, more or less successfully, the typical physicist attitude toward the hypothesis of telepathy and toward psychical research. I shall consider rather that typical attitude than the special points which this excellent physicist has made. For I was trained from boyhood in physics, have mostly associated with physicists, and fully share their prejudices, whether legitimate or illegitimate. Upon the formation of the Society for Psychical Research, I disapproved of physicists lending it their countenance, on the ground that in doing so they would be encouraging young men to devote the best years of their lives to an inquiry of little promise in which those men would be in great danger of compromising themselves, since it would appear that they must either have some positive result to defend after so much time had been spent or else be reduced to a distressing acknowledgment of failure. When the large book "Phantasms of the Living" appeared, I wrote a long and minute criticism of it,†3 arguing that it only showed that no scientific conclusion could be reached by the examination of stories of sporadic and unanalyzed phenomena. Thus I have little need of consulting with others to do full justice to the typical physicist's disapproval of the hypothesis and the methods of the telepathists. On the other hand, ever since Wundt inaugurated the modern science of psychology about 1862 (the date of the collected publication of his *Beiträge zur Theorie der*

Sinneswahrnehmung,) I have pursued that study both experimentally and speculatively, and am thus better able than some physicists to appreciate the opinions of the psychologists.

Peirce: CP 7.598 Cross-Ref:††

598. Let me endeavor to explain the attitude of the physicist. Many people imagine that there is a certain class of facts which it is repugnant to the physicist to acknowledge. This is not so. If there were such a class of facts, the phenomena connected with radium would fall within it. Yet there has been no disposition to ignore these phenomena. But the physicist recognizes that a phenomenon is of no use to him unless both it and its conditions can be subjected to exact analysis. Moreover, the only theories that can have any value for him are those from which can be deduced exact predictions capable of exact verification. As long as a fact stands isolated and strange, it is next to impossible to make sure that it is a fact, and quite impossible to render it useful to science. Physicists are doubtless inclined to discredit facts of which they, as physicists, could make no use. But that is a mere matter of personal *belief* with them. It cannot affect their conduct as physicists. Thus, in regard to aerolites, as long as there were merely scattered tales of stones having fallen from heaven at long intervals, although some of them were very strongly attested, as when a shower of them were dashed into a public square of Siena, it must be admitted that physicists of the very highest genius were too much inclined to pooh-pooh the matter. But this they did not do in their character as physicists, since there seemed to be no way by which a physicist could inquire into the matter. It was merely a "belief." One physicist, however, Chladni, bethought him to draw up a catalogue of all these tales, -- using, not "post-card evidence," since post-cards were not yet in use, but testimonies that, taken singly, were mostly far from affording satisfactory scientific security. The result was that it appeared that stones fall from heaven every day (we now know that thousands fall daily). Thus, the subject became opened to scientific investigation, and the personal adverse "beliefs" of the physicists, gave way to the general acknowledgment that the fact had been established.

Peirce: CP 7.599 Cross-Ref:††

599. Suppose we knew nothing at all about electricity except for the stories of fire-balls. Then it could make no difference to physicists, as physical inquirers, whether those stories were true or not, since there would be no way by which further physical inquiry into their nature could be pursued. It would not be their concern, professionally; but personally their "belief" would mostly be that they were fancies.

Peirce: CP 7.600 Cross-Ref:††

600. The theory of telepathy is that in some cases one mind acts upon another, whether directly or not, at any rate by means fundamentally different from those that every-day experience renders familiar. As a scientific theory, this almost condemns itself. For to say that a phenomenon is fundamentally different from anything in ordinary experience is almost to say that it is of such a nature as to preclude the deduction from it of manifold exact predictions verifiable by ordinary perception. Pretty nearly the sole support claimed for this theory consists of certain tremendous experiences that are said to have happened to a minute fraction of mankind. If such be the only facts in the case, they are facts with which science can have nothing to do, since science is the business of finding out Law, i.e., what always happens.

Peirce: CP 7.601 Cross-Ref:††

601. Suppose it were true that those marvellous stories proved the doctrine of telepathy; then what would it be that had been proved? Why, that very rarely mind

acts upon mind in a way utterly unlike the normal way. This would be no contribution to science. It would, in the case supposed, have been ascertained that sometimes a marvel, an impenetrable mystery, occurs. The concern of science, however, is with intelligible facts. Science no more denies that there are miracles and mysteries than it asserts them. But it is a Postulate, -- a hope, -- of science and of all sound reasoning that *any given fact* to which our attention may be directed shall turn out to be intelligible. †4 It is justly held criminal for a commander-in-chief to "despair of the republic"; and it would be as monstrous for an inquirer to despair of the comprehensibility of his problem. The psychicists are fond of reminding us that the most familiar facts are full of mystery. So they are, in a metaphysical sense; and that is the very reason why the physicist eschews metaphysical inquiry and seeks to understand phenomena only in the sense of establishing their exact relations to one another. Hence, what is absolutely severed and sundered from the body of ordinary experience is absolutely beyond scientific comprehension.

Peirce: CP 7.602 Cross-Ref: ††

602. Sharing these sentiments common to all physicists, when 'Phantasms of the Living' appeared, -- that vast collection of well-sifted stories by which Messrs. Gurney, Myers, and Podmore endeavored to prove the theory of telepathy, -- I felt that I would far sooner believe in spiritualism. For according to this latter theory, we *all* pass into another life; nor would this experience common to us all be much more wonderful than the development that we all undergo when the child becomes a grown person. If telepathy be a fact, why should it have the strange character of being excessively infrequent, so contrary to the ubiquity of all the other agencies of nature? One cannot suppose it otherwise than extremely infrequent without having long ago become well-known and beyond doubt. Nor have the psychicists succeeded in making out the slightest trace of it in ordinary people. If, on the other hand, we suppose there are communications from the other world, there is nothing surprising in their being infrequent. For with what disgust and indifference must the departed spirit recall the pettiness and blindness of the ends, the villainy, meanness, and filth of the methods of the inhabitants of this [world], who, as he well sees, are at the bottom of their hearts glad to get rid of him, and who have to work out their own salvation without interference, -- the nasty little imps. †5 Surely, the canyon that spiritualism has to suppose between the two worlds has a gentle slope compared with the abyss that telepathy opens in the midst of experience between the ordinary and extraordinary intercommunion between minds.

Peirce: CP 7.603 Cross-Ref: ††

603. Messrs. Gurney, Myers, and Podmore in asking human reason to admit that a class of phenomena was sundered from ordinary experience vastly more than any phenomena had been believed to be, seemed to me to be all but asking reason to admit that its hope of understanding things, -- its only aim, -- was futile; and I was accordingly moved to write an elaborate criticism of their book to show that hallucinations were so very common, while hallucinations coincident with truth beyond the ken of sense were so very rare, after the suspicious cases had been weeded out, that these coincidences might very well be supposed to be fortuitous. At the same time, I essayed to determine by logical analysis what were the conditions to which a census of hallucinations must conform in order to afford sound support to the contention of the telepathists that the veridical †6 hallucinations were too numerous to be accounted chance coincidences. The Society at once set about making a new census. I wish I could pay them the compliment of averring that, in doing so, they were influenced by my reasons or by any better considered reasons. But I am obliged

to say that they so far failed to conform to the conditions which I had shown to be requisite, -- and in the direction of favoring their doctrine of telepathy, -- that now the question stands as it did before, a question which each man will answer according to his predilections, whether these owe their birth to his general experience of the ways of nature or to some episode of his private life.

Peirce: CP 7.604 Cross-Ref:††
§2. THE SCIENTIFIC ATTITUDE

604. I am sorry. For the question at issue between the physicist and the psychologist in the recent passage of arms was how far the psychical researchers have the character of men of science. Now however much ratiocinative analysis may help toward a decision upon this point, the answer must ultimately rest, like every other judgment concerning men's character, upon the testimony of instinctive impressions; and one of my purposes in this article is to render the testimony of my own impressions upon the matter. Thus finding myself in a situation that calls for truth-telling, truth will I tell, without exaggeration or extenuation, as accurately as I can, whatever it costs me.

Peirce: CP 7.605 Cross-Ref:††

605. There are two qualifications which every true man of science possesses, and which, if a man possesses them, he is sure to develop into a scientific man in the course of time, if he ought not fairly to be called such already. First, the dominant passion of his whole soul must be to find out the truth in some department, regardless of what the color of that truth may be. Secondly, he must have a natural gift for reasoning, for severely critical thought. Perhaps a man who had drunk of the fountain of perpetual youth need not, at the outset of his career, possess either of those qualifications: he would infallibly become a man of science at last, because the incessant action of experience would ultimately produce those two qualities in him. For we see that, in a measure, that effect has been produced in the course of history upon civilized man at large.

Peirce: CP 7.606 Cross-Ref:††

606. In a measure, I say, for the love of truth is still far from mighty; and a gift for reasoning is still as rare a talent as a gift for music. Most men are incapable of strong control over their minds. Their thoughts are such as instinct, habit, association suggest, mainly. Their criticism of their thoughts is confined to reconsideration and to asking themselves whether their ideas seem reasonable. I do not call this reasoning: I call it instinctive reflexion. For most purposes it is the best way to think; for instinct blunders far less than reason. Reasoners are in danger of falling into sophistry and pedantry. Our instinctive ways of thinking have become adapted to ordinary practical life, just as the rest of our physiology has become adapted to our environment. Wisdom lies in nicely discriminating the occasions for reasoning and the occasions for going by instinct. Some of my most valued friends have been almost incapable of reasoning; and yet they have been men of singularly sound judgment, penetrating and sagacious. So much more important is it, on the whole, to feel right than to reason deeply. But in science instinct can play but a secondary *rôle*. The reason of this is that our instincts are adapted to the continuance of the race and thus to individual life. But science has an indefinite future before it; and what it aims at is to gain the greatest

possible advance in knowledge in five centuries or ten. Instinct not being adapted to this purpose, the methods of science must be artificial. As Professor Trowbridge hints, pure science has nothing to do with *belief*. What I *believe* is what I am prepared to go on today. Imagine a general besieging a city. He sits in his tent at night preparing the details of his plan of action for the morrow. He finds that what his orders ought to be and perhaps the whole fate of his army depend upon a certain question of topography concerning which he is in need of information. He sends for his best engineer officer, -- a highly scientific man, -- and asks how he is to ascertain the fact in question. The officer replies, "There is only one possible way of ascertaining that. So and so must be done." "How long will that take?" "Two or three months." The general dismisses the man of science, -- as Napoleon dismissed Laplace, -- and sends for another officer, not half so scientific, but good at guessing. What this officer shall say, the general will go by. He will adopt it as his belief.

Peirce: CP 7.607 Cross-Ref:††

607. Beyond the two general qualifications mentioned, every scientific man needs a special training in his particular branch of research; and moreover this branch will probably call for the possession of some special mental, physiological, and instrumental advantages for the acquisition of a special line of facts. I do not set down "power of observation" among the general qualifications, because this phrase covers a variety of capacities having nothing in common, of which a naturalist will want one, an astronomer another, a linguist a third, a psychologist a fourth, and so on. Besides, there has been some exaggeration about the observing powers of scientific men; and in that quality which the phrase seems best to fit, the power of noting one's precise sensations unaffected by any interpretation of them, the scientific men of most branches are decidedly inferior to the artists. I should not, therefore, attribute any very high importance to great general powers of observation as distinguishing men of science. But I must say that the best psychical researchers rank high in this respect as compared with scientific men generally.

Peirce: CP 7.608 Cross-Ref:††

608. In considering how far they possess the essential characters of scientists, I first note that amateurs, idlers, and people not extra wise naturally flock to the Society for Psychical Research and make up an even greater proportion of it than they do of other large scientific associations. But these are not the persons under consideration. It is only the men who have devoted all the strenuous energies of their lives to the work, some of whom have succumbed to the strain of it, whom I am to endeavor to class.

Peirce: CP 7.609 Cross-Ref:††

609. As to their devotion to truth, I have to remember that, as I have known them, they have been serious and foreseeing men who would not embark upon any enterprise without carefully reckoning up its contingencies. Therefore, at the moment when any one of those men deliberately devoted his life and his whole being to this inquiry, as several have done, he certainly had distinctly before his mind the following considerations:

that it would be hard and incessant work, mostly drudgery, requiring him to be occupied mostly with knaves and fools;

that it would cost him a great deal of money, considering all that it would prevent him from earning;

that it would never bring him much honor, but would put a certain stamp of obloquy upon him;

that even among the company of those who professed to love the truth, and who ought to press him to their hearts as a brother, there would be found in the more richly endowed sciences, individuals who would treat him in the narrowest and most despicable spirit of the east wind;

that after his whole life had been poured out into the inquiry, it was not unlikely that he might find that he had not found out anything.

Peirce: CP 7.610 Cross-Ref:††

610. These considerations go to show that, whatever those men have been aiming at, they have aimed at in a single-hearted manner. Were they desirous of believing in the particular doctrine of telepathy, or were they bent on finding out the truth, whatever it might be?

Peirce: CP 7.611 Cross-Ref:††

611. I cannot see that there is anything particularly comfortable about a belief in telepathy. To be sure, it cuts off, or greatly weakens, the force of such evidence as there might seem to be for spiritualism; and, no doubt, for many reflecting persons, the prospect of another life is rather unpleasant. But who will believe that the main motive of the telepathists has been to escape thoughts of another world? Such thoughts are not troublesome enough for that.

Peirce: CP 7.612 Cross-Ref:††

612. I had a somewhat prolonged controversy with Edmund Gurney which was only interrupted by his death; and this brought me into fine touch with the spirit of the man. I was most strongly impressed with the purity of his devotion to truth. This impression has been thoroughly convincing in the cases of others whom I have met personally, but whom I do not name because they are still living.

Peirce: CP 7.613 Cross-Ref:††

613. The power of reasoning of the leaders is certainly much above that of the average of men. It does not seem to me to have been altogether sufficient for their problem. The hypothesis of telepathy does not seem to me to have much merit, as a specimen of scientific method. I believe that if the researchers had been better reasoners they would not have spent so much time in sifting tales of extraordinary happenings but would rather have thrown their energies into an endeavor to connect the dubious extraordinary phenomena with ordinary experience. In this direction, the little that they have done seems to me feeble and to show defect of analysis.

Peirce: CP 7.614 Cross-Ref:††

614. As for scientific training and special qualifications, the psychicists seem to me to have been amply provided with all that their method gave room for. Their exposure of several frauds has been neat and workmanlike, and they have made use of numbers as far as numbers have been applicable. Prof. Trowbridge complains that they make no measurements; and that coincides with my complaint that they have not sufficiently endeavored to bring their marvels into relation to ordinary experience. For, in order to do that, the phenomena would have had to be analyzed; and then, and not before, would measurements have been applicable. Prof. Trowbridge will not forget that in the early stages of physics, there were no measures. Galileo, Gilbert and others made considerable progress before they arrived at a stage at which they were

able to make any measurements to speak of. But they did exhibit great skill in analyzing the phenomena; and I agree that the psychicists have not exhibited signal ability in doing that. It must be remembered, however, that they are only breaking ground in a perfectly virgin soil intractable and thicket-tangled. They are doing good honest work, not shrinking from labor, and proceeding intelligently, if not with high genius. The farm of physics has long been under cultivation, the costliest implements and the most refined methods are here in use. Here have come these new settlers, occupying land not easy to till and poor in resources. Come, let us turn up our noses at them, gossip about their suspicious ways, and let them feel the difference between them and us. Who knows whether they are honest?

Peirce: CP 7.615 Cross-Ref:††
§3. PERCEPTION

615. If there were only some way of reconciling the usual order of nature, as it is familiar to us, with the possibility of rare cases of quasi-vision beyond the ken of sense, then I grant that the testimonies adduced in "Phantasms of the Living" would suffice to render it extremely likely that such rare quasi-vision actually takes place. For, after all, the theory of chance coincidence puts a certain strain upon our instinct of receiving testimony; and natural instinct is not a thing to be lightly disregarded.

Peirce: CP 7.616 Cross-Ref:††

616. Is it quite certain that such an occasional, but very rare, determinedly veridical vision of things beyond the senses' ken would be altogether unlike every-day experience? As to that I have a suggestion to make.

Peirce: CP 7.617 Cross-Ref:††

617. It is not uncommon to hear a man or woman say, "I believe what I see, and nothing more." But little cross-questioning would commonly be needed to show that it is an exaggeration. There are not many persons, even though they boast themselves to be materialists, who do not really believe much that they do not themselves think that they directly perceive by any sense, peripheral or visceral; -- polonium, the sacred city of Thibet, George Washington, the glacial period, that motions will tomorrow conform to Newton's three laws. But I myself happen, in common with a small but select circle, to be a pragmatist, or "radical empiricist,"†7 and as such, do not believe in anything that I do not (as I think) perceive: and I am far from believing in the whole of that.

Peirce: CP 7.618 Cross-Ref:††

618. Only, the question arises, What do we perceive? It would not serve our turn to answer scholastically with an arbitrary definition which might be ill-considered. Let us rather set out from familiar instances, and having noticed what their relation is to the formation of scientific opinions, found upon that a definition which shall cover all that is so related to knowledge and shall cover nothing else.

Peirce: CP 7.619 Cross-Ref:††

619. Let us say that, as I sit here writing, I see on the other side of my table, a yellow chair with a green cushion. That will be what psychologists term a "percept" (*res percepta*). They also frequently call it an "image." With this term I shall pick no quarrel. Only one must be on one's guard against a false impression that it might

insinuate. Namely, an "image" usually means something intended to represent, -- virtually professing to represent, -- something else, real or ideal. So understood, the word "image" would be a misnomer for a percept. The chair I appear to see makes no professions of any kind, essentially embodies no intentions of any kind, does not stand for anything. It obtrudes itself upon my gaze; but not as a deputy for anything else, not "as" anything. It simply knocks at the portal of my soul and stands there in the doorway.

Peirce: CP 7.620 Cross-Ref:††

620. It is very insistent, for all its silence. It would be useless for me to attempt to pooh-pooh it, and say, "Oh come, I don't believe in the chair." I am forced to confess that it appears. Not only does it appear, but it disturbs me, more or less. I cannot think the appearance is not there, nor dismiss it as I would a fancy. I can only get rid of it by an exertion of physical force.

Peirce: CP 7.621 Cross-Ref:††

621. It is a forceful thing. Yet it offers no reason, defence, nor excuse for its presence. It does not pretend to any right to be there. It silently forces itself upon me.

Peirce: CP 7.622 Cross-Ref:††

622. Such is the percept. Now what is its logical bearing upon knowledge and belief. This may be summed up in three items, as follows:

Peirce: CP 7.622 Cross-Ref:††

1st, it contributes something positive. (Thus, the chair has its four legs, seat, and back, its yellow color, its green cushion, etc. To learn this is a contribution to knowledge.)

Peirce: CP 7.622 Cross-Ref:††

2nd, it *compels* the perceiver to acknowledge it.

Peirce: CP 7.622 Cross-Ref:††

3rd, it neither offers any reason for such acknowledgment nor makes any pretension to reasonableness. This last point distinguishes the percept from an axiom. I am a total disbeliever in axioms; but so far as the proposition, say, that a straight line is the shortest distance between two points even so much as *seems* to be self-evident, it seems to be *reasonable*. It is as founded in reason or in the nature of things, or as *founded* in something, that it recommends itself. The percept, on the contrary, is absolutely dumb. It acts upon us, it forces itself upon us; but it does not address the reason, nor *appeal* to anything for support.

Peirce: CP 7.623 Cross-Ref:††

623. Let us say, then, that anything is, for the purposes of logic, to be classed under the species of perception wherein a positive qualitative content is forced upon one's acknowledgment without any reason or pretension to reason. There will be a wider genus of things *partaking* of the character of perception, if there be any matter of cognition which exerts a force upon us *tending* to make us acknowledge it without any *adequate* reason.

Peirce: CP 7.623 Cross-Ref:††

But in order fully to satisfy ourselves of the justice of classing in this species and in this genus all that I propose to refer to them, it is desirable to examine a little more closely the characters of the percept.

Peirce: CP 7.624 Cross-Ref:††

624. A visual percept obtrudes itself upon me in its entirety. I am not therein conscious of any mental process by which the image has been constructed. The psychologists, however, are able to give some account of the matter. Since 1709, they have been in possession of sufficient proof (as most of them agree,) that, notwithstanding its apparent primitiveness, every percept is the product of mental processes, or at all events of processes for all intents and purposes mental, except that we are not directly aware of them; and these are processes of no little complexity. The psychologists very reasonably argue that the first impressions made upon sense must have been feelings of sense qualities, -- say colors, sounds, etc. -- disconnected from one another, and not appearing to stand over against a self as objects; and it would seem that this must have been true of the *very* first impressions ever made upon sense in the history of mental development, however far the sense of the individual man of today may have been rendered capable of immediately apprehending the complex. But this is quite inferential. We are, of course, directly aware of positive sense-qualities in the percept (although in the percept itself they are in no wise separate from the whole object); but as for their being at first disconnected and not objectified, that is psychological theory.

Peirce: CP 7.625 Cross-Ref:††

625. Thus, two utterly different kinds of elements go to compose any percept. In the first place, there are the qualities of feeling or sensation, each of which is something positive and *sui generis*, being such as it is quite regardless of how or what anything else is. On account of this self-sufficiency, it is convenient to call these the elements of "Firstness."†8 In the percept, these elements of Firstness are perceived to be connected in definite ways. A visual percept of a chair has a definite shape. If it is yellow with a green cushion, that is quite different from being green with a yellow cushion. These connectives are directly perceived, and the perception of each of them is a perception at once of two opposed objects, -- a double awareness. In respect to each of these connections, one part of the percept appears as it does *relatively to a second part*. Hence, it is convenient to call them elements of "Secondness." The vividness with which a percept stands out is an element of secondness; because the percept is vivid in proportion to the intensity of its effect upon the perceiver. These elements of secondness bring with them the peculiar singleness of the percept. This singleness consists in a double definiteness. For on the one hand, the percept contains no blank gaps which, in representing it, we are free to fill as we like. What I mean will be seen if we consider any knowledge we can have of the future. I heard somebody say that the Brooklyn bridge would fall some day. The only way in which he could even think he knew that would be by knowing that any bridge I might select that should be constructed in a certain way would fall. There is no such universality about the percept. It is quite individual. On the other hand, the definiteness of the percept is of a perfectly explicit kind. In any knowledge of the past something is, as it were, held in reserve. There is an indicated gap which we are not free to fill but which further information may fill. We know that the Sphinx was made by *some* king of Egypt. But what one? The percept, however, exhibits itself in full. These two kinds of definiteness, first, that the percept offers no range of freedom to anybody who may undertake to represent it, and secondly, that it reserves no freedom to itself to be one way or another way, taken together, constitute that utter absence of "range" which is called the *singularity*, or singleness, of the percept, the one making it individual and the other positive. The percept is, besides, whole and undivided. It has parts, in the sense that in thought it can be separated; but it does not represent itself to have parts.

In its mode of being as a percept it is one single and undivided whole.

Peirce: CP 7.626 Cross-Ref:††

626. The percept is not the only thing that we ordinarily say we "perceive"; and when I professed to believe only what I perceived, of course I did not mean *percepts*, since percepts are not subjects of belief or disbelief. I meant *perceptual judgments*. Given a percept, this percept does not describe itself; for description involves analysis, while the percept is whole and undivided. But once having a percept, I may contemplate it, and say to myself, "That appears to be a yellow chair"; and our usual language is that we "perceive" it to be a yellow chair, although this is not a percept, but a judgment about a present percept.

Peirce: CP 7.627 Cross-Ref:††

627. The perceptual judgment is all but in the same relation to knowledge and belief as is the percept. It is true that I may, by an effort of will, abstain from thinking about the color of the chair, so that the judgment 'the chair appears yellow' is not unconditionally forced upon me, and thus might seem not quite fully to partake of the character of perception. One can, however, escape the percept itself by shutting one's eyes. If one *sees*, one cannot avoid the percept; and if one *looks*, one cannot avoid the perceptual judgment. Once apprehended, it absolutely compels assent. Its defect in forcefulness is thus excessively slight and of no logical importance.

Peirce: CP 7.628 Cross-Ref:††

628. To about the same degree its forcefulness falls short of the utter irrationality of that of the percept. The perceptual judgment professes to represent the percept. A logical defence of it would therefore have to be founded either on the percept as a premiss of that logical defence, or else on the percept as a fact represented by such premiss. But the percept cannot be a premiss, since it is not a proposition; and a statement of the character of the percept would have to rest on the perceptual judgment, instead of this on that. Thus, the perceptual judgment does not represent the percept *logically*. In what intelligible manner, then, does it represent the percept? It cannot be a *copy* of it; for, as will presently appear, it does not resemble the percept at all. There remains but one way in which it can represent the percept; namely, as an index, or true symptom, just as a weather-cock indicates the direction of the wind or a thermometer the temperature. There is no warrant for saying that the perceptual judgment actually *is* such an index of the percept, other than the *ipse dixit* of the perceptual judgment itself. And even if it be so, what is an index, or true symptom? It is something which, without any rational necessitation, is forced by blind fact to correspond to its object. To say, then, that the perceptual judgment is an infallible symptom of the character of the percept means only that in some unaccountable manner we find ourselves impotent to refuse our assent to it in the presence of the percept, and that there is no appeal from it. Thus, the forcefulness of the perceptual judgment falls short of the pure unreasonableness of the percept only to this extent, that it does profess to represent the percept, while the perfection of the percept's surdity consists in its not so much as professing anything.

Peirce: CP 7.629 Cross-Ref:††

629. The perceptual judgment, then, does not quite accurately fulfill the condition of forcefulness nor that of irrationality, as it should do to be strictly entitled to be considered a product of perception. But the differences are so minute and so unimportant logically that it will be convenient to neglect them. Perhaps I might be permitted to invent the term *percipuum*†9 to include both percept and perceptual judgment.

Peirce: CP 7.630 Cross-Ref:††

630. I promised to show that a perceptual judgment is entirely unlike a percept. If it be true, as my analysis makes it to be, that a percept contains only two kinds of elements, those of firstness and those of secondness, then the great overshadowing point of difference is that the perceptual judgment professes to represent something, and thereby does represent something, whether truly or falsely. This is a very important difference, since the idea of representation is essentially what may be termed an element of "Thirdness," that is, involves the idea of determining one thing to refer to another. The element of secondness in the percept consists in one part being relative to another. But the percept presents itself ready made, and contains no idea of any state of things being brought about. There is a rigid mathematical demonstration (which I cannot give here) that the idea of Firstness, or that of a positive suchness, and the idea of Secondness, or that of one thing's referring to another, can in no way be combined so as to produce the idea of one thing A, referring to a second, B, in the very act of referring to a third, C.†10 This is the element of Thirdness, or mediation, which the conception of the representation *of* something *to* somebody obviously involves. In a perceptual judgment the mind professes to tell the mind's future self what the character of the present percept is.†11 The percept, on the contrary, stands on its own legs and makes no professions of any kind.

Peirce: CP 7.631 Cross-Ref:††

631. There are several other points of contrast between the perceptual judgment and the percept that are calculated to exhibit their disparateness. The judgment, "This chair appears yellow," separates the color from the chair, making the one predicate and the other subject. The percept, on the other hand, presents the chair in its entirety and makes no analysis whatever.

Peirce: CP 7.632 Cross-Ref:††

632. We have seen that the "singularity" of the percept is a composite of two modes of definiteness. The first consists in this, that its faithful and full interpreter has no freedom allowed to him, but all is prescribed. But the perceptual judgment 'This chair appears yellow' has vaguely in mind a whole lot of yellow things, of which some have been seen, and no end of others may be or might be seen; and what it means to say is, 'Take any yellow thing you like, and you will find, on comparing it with this chair, that they agree pretty well in color.' It thus directly invites the exercise of a freedom of choice on the part of the interpreter (any one yellow thing answering as well as any other) which freedom the percept sternly and stupidly precludes.

Peirce: CP 7.633 Cross-Ref:††

633. The other mode of definiteness of the percept consists in its being perfectly explicit. The perceptual judgment carelessly pronounces the chair yellow. What the particular shade, hue, and purity of the yellow may be it does not consider. The percept, on the other hand, is so scrupulously specific that it makes this chair different from every other in the world; or rather, it would do so if it indulged in any comparisons.

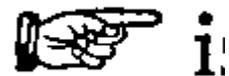
Peirce: CP 7.634 Cross-Ref:††

634. It may be objected that the terms of the judgment resemble the percept. Let us consider, first, the predicate, 'yellow' in the judgment that 'this chair appears yellow.' This predicate is not the sensation involved in the percept, because it is general. It does not even refer particularly to this percept but to a sort of composite

photograph of all the yellows that have been seen. If it *resembles* the sensational element of the percept, this resemblance consists only in the fact that a new judgment will predicate it of the percept, just as this judgment does. It also awakens in the mind an imagination involving a sensational element. But taking all these facts together, we find that there is no relation between the predicate of the perceptual judgment and the sensational element of the percept, except forceful connections.

Peirce: CP 7.635 Cross-Ref:††

635. As for the subject of the perceptual judgment, as subject it is a sign. But it belongs to a considerable class of mental signs of which introspection can give hardly any account. It ought not to be expected that it should do so, since the qualities of these signs as objects have no relevancy to their significative character; for these signs merely play the part of demonstrative and relative pronouns, like "that," or like the A, B, C, of which a lawyer or a mathematician avails himself in making complicated statements. In fact, the perceptual judgment which I have translated into



"that chair is yellow" would be more accurately represented thus: "[\[Click here to view\]](#) is yellow," a pointing index-finger taking the place of the subject. On the whole, it is plain enough that the perceptual judgment is not a copy, icon, or diagram of the percept, however rough. It may be reckoned as a higher grade of the operation of perception.

Peirce: CP 7.636 Cross-Ref:††

636. In order not to prolong the discussion, I leave some possible objections to what I have said unanswered. The most serious of these is that a perceptual judgment may be revised; so that there is a certain check upon it. The examination of this point would be lengthy, and it does not seem to me likely to appear to anybody to be of fundamental importance. Full justice to it would involve complications which few would have the patience to follow.†12

Peirce: CP 7.637 Cross-Ref:††
§4. A PROGRAMME

637. Having thus formed some acquaintance with the characteristics of perception, we go on to a line of thought of which the programme may be set down in advance.

Peirce: CP 7.638 Cross-Ref:††

638. No scientifically valuable classification can draw a line of demarcation between forms which differ indefinitely little. I shall begin by showing that this principle, which I shall refer to as the "serial principle,"†13 obliges us to class as perceptions many ideas not popularly regarded as belonging to this class.

Peirce: CP 7.639 Cross-Ref:††

639. By the continued application of the same principle, I shall widen more and more our notion of what perception includes. In particular, I shall endeavor to bring into a clear light the truth that although what I have already said implies the truth of that doctrine of the direct, or "immediate," perception of the external world

which is taught by the Aristotelians, by Kant, and by the philosophers of the Scotch school, yet we cannot refuse the name of perception to much which we rightly reject as unreal; as indeed, dreams and hallucinations are quite commonly classed as perceptions.

Peirce: CP 7.640 Cross-Ref:††

640. In this way, I shall lead up to the consideration of a certain group of ideas, -- I beg permission to withhold the specification of what group it is, for the moment, -- which ordinary instinctive language speaks of as perception, but which, as soon as one undertakes to be extra precise in one's speech, one usually never thinks of so classing; and I hope to give good reason for holding that, in this particular, instinct is right and pedantry wrong. Of this group of ideas, however, the great majority are illusory, the small minority determinedly veridical and not merely true by chance coincidence. The physicist would stultify himself if he were not the last of all men to call this into question.

Peirce: CP 7.641 Cross-Ref:††

641. With this group of ideas I proceed to compare the entire host of hallucinations, which there is no good reason to separate into the veridical and the non-veridical, and which there *is* good reason to account far more frequent than the census of the Society for Psychological Research admits. They shade off into *bévués* of which ordinary people commit from one to dozens daily. In view of this comparison and of all the characteristics that have been developed of ordinary perceptions in their now enlarged domain, I shall inquire, to begin [at] the end, whether it will be more in accordance with general experience to expect that some fraction of hallucinations should have a pronounced tendency to being determinedly veridical or that none should be so. Thence I shall go on to the two questions that are really pertinent to the present positions of opinion,

first, whether, in case we were to admit, what is very far from commanding any reasonable assent, that all the stories which the telepathists consider marvellous and that are supported by testimony of real weight are real histories, there would be any ground for admitting a mode of communication between minds, in those cases, differing radically from modes that are ordinary;

secondly, whether it be consonant with ordinary experience to be so extremely sceptical about the determinedly veridical character of occasional visions of what is beyond the ken of the senses as the rank and file of the physicists are apt to be.

Peirce: CP 7.642 Cross-Ref:††

§5. THE PERCIPUUM

642. If there is any novelty in the suggestion I am about to make, -- and I must confess I fear there is, -- it lies only in the juxtaposition of ideas. The facts which I shall adduce are to the psychologist the merest commonplaces. In the classification of them I endeavor to eliminate the pedantries and formalisms which are retained in the books for convenience, but to which psychologists attach no particular importance; and this I do by the application of the above principle of forming almost continuous series, which principle is the principal clew of which the biological taxonomists, -- the great masters of classification and of terminology, -- are accustomed to avail

themselves. Addressing, as I do, a reader who I dare say has read a book or two about psychology but who has not dwelt enough upon the subject to prevent his being impressed by the more tangible, wooden, and dead ideas, -- or corpses of ideas, -- rather than by the more elusive, fluid, and living ones, my principal care will be to correct such notions. If the reader comes with a fresh mind unsophisticated by any such reading, he will follow my meaning all the easier.

Peirce: CP 7.643 Cross-Ref:††

643. We know nothing about the percept otherwise than by testimony of the perceptual judgment, excepting that we feel the blow of it, the reaction of it against us, and we see the contents of it arranged into an object, in its totality, -- excepting also, of course, what the psychologists are able to make out inferentially. But the moment we fix our minds upon it and *think* the least thing about the percept, it is the perceptual judgment that tells us what we so "perceive." For this and other reasons, I propose to consider the percept as it is immediately interpreted in the perceptual judgment, under the name of the "percipuum." The percipuum, then, is what forces itself upon your acknowledgment, without any why or wherefore, so that if anybody asks you why you should regard it as appearing so and so, all you can say is, 'I can't help it. That is how I see it.' For example, one of the foolish questions with which treatises on physics used to abound was why things look right side up, when the images on the two retinas are upside down; and sundry sapient reasons more or less abstruse were given for their looking as they do. Now such arguments might have proved that things really are right side up, or perhaps they might have shown what physiological and psychical agencies cause us to regard them as right side up; but if anybody were to ask you why you *should* regard the visual images as right side up like the things themselves, rather than upside down like the optical images on the retinae, how you were justified in doing so, your only possible answer would be "They *do* look so, and I cannot make them look otherwise," whether it is reasonable for them to look so or not. Sometimes when I have been seated in a railway-car that was stationary and another train has been slowly passing by, I have been vexed at the unreasonableness of its appearing to me that our train was moving and the other train was at rest. I have reasoned with my perception. I have asked, "Is there jarring such as there is when one is in a moving car?" No. "Is there any noise of the wheels?" No. "Is there anything at all in the looks of either train that is more as if we were moving rather than they?" Quite the reverse. "Then why do I have the idea that that train is at rest and that we are moving?" There is no answer except that such is the percipuum, and I cannot help it.

Peirce: CP 7.644 Cross-Ref:††

644. There is no difference between a real perception and a hallucination, taken in themselves; or if there be, it is altogether inconsiderable. The difference is that rational predictions based upon the hallucination will be apt to be falsified, -- as for example, if the person having the hallucination expects another person to see the same thing; while truly sound predictions based on real perceptions are supposed *never* to be falsified, although we have no positive reason for assuming so much as that. But this difference between hallucinations and real perceptions is a difference in respect to the relations of the two cases to other perceptions: it is not a difference in the presentations themselves. For the purposes of physiological psychology it may be proper and needful to put them into different classes. That is a branch of science with which I am not concerned. But for logical purposes, that is, in regard to their relations to knowledge and belief, which is the concern of this whole paper, they should be regarded as one and the same phenomenon, in themselves.

Peirce: CP 7.645 Cross-Ref:††

645. So likewise all hallucinations, the non-veridical, the fortuitously veridical, and the determinedly veridical, are, for our purposes, all one and the same phenomenon, in themselves.

Peirce: CP 7.646 Cross-Ref:††

646. But this is not all. The serial principle will not permit us to draw a hard and fast line of demarcation between perception and imagination. Physiological psychology may be justified or compelled to separate them, for aught I know or care. But, in regard to their relation to knowledge and belief, the percipuum is nothing but an extreme case of the fancy.

Peirce: CP 7.647 Cross-Ref:††

647. We all know, only too well, how terribly insistent perception may be; and yet, for all that, in its most insistent degrees, it may be utterly false, -- that is, may not fit into the general mass of experience, but be a wretched hallucination. In other cases, the insistence of the percipuum may be something of a sham. That is, it may resist all unskillful attempts to overthrow it, be they ever so energetic; and yet when one knows the right trick it will be curious to see how easily it is downed. For example, that moving train that appears stationary will not move however one may try to force it to do so. Yet if one only looks down and watches the wheels turn, in a very few seconds it will seem to start up. The percipuum, insistent as it is, is not nearly so persistent and thing-like as one is apt to think. There is a well-known outline figure of a straight flight of steps against a wall seen in perspective from the side. It is called 'Schroeder's Stair.'^{†14} When you look at it you seem to be looking at the stairs from above. You cannot conceive it otherwise. Continue to gaze at it, and after two or three minutes the back wall of the stairs will jump forward and you will now be looking at the under side of them from below, and again cannot see the figure otherwise. After a shorter interval, the upper wall, which is now nearer to you, will spring back, and you will again be looking from above. These changes will take place more and more rapidly, the aspect from above always lasting longer, until at length, you will find you can at will make it look either way. But put it aside for a quarter of an hour or less and you will find you have lost this power. Doubtless frequent repetition of the experiment would give one complete control over it. You will thus have converted an uncontrollable percipuum into a controllable imagination by a brief process of education. It is one of the recognized difficulties of all psycho-physical measurement that the faculties rapidly become educated to an extraordinary degree. Thus, contrast-colors, when properly exhibited, are incredibly vivid. One is not easily persuaded that they are not real. Yet the experimenter becomes, in time, almost incapable of perceiving them. This is a case in which the same educational course which gives control over appearances which sometimes do and sometimes do not accord with the mass of experiences, only serves to strengthen the forcefulness of those appearances which always do so accord.

Peirce: CP 7.648 Cross-Ref:††

648. It is a difficult question whether the serial principle permits us to draw sharp lines of demarcation between the percept and the near anticipation, or say the *antecept*, and between the percept and the recent memory (may I be permitted to call this the *ponecept*, a distant and dubious memory being perhaps quite another thing?), or whether the percept is at once but an extreme case of an antecept and an extreme case of a ponecept. Or rather, -- I beg the reader's pardon for my awkwardness of statement, -- the precise question is not about percept, antecept, and ponecept, but

about percipuum, *antecipuum*, and *ponecipuum*, the direct and uncontrollable interpretations of percept, antecept, and ponecept. There can surely be no objection to our beginning the discussion by asking what the deliverance of a flow of time, say for example of the motion of a shooting star, is concerning the matter; for this inquiry cannot result in our knowing less about the main question than we did before. The psychologists think it will not cause us to know any more, either; because the question is not what the percipuum represents the nature of the transformations of antecipuum into percipuum and of percipuum into ponecipuum to be, but what in fact they really are. It will be observed that, in so putting the question, the psychologists are assuming that time really is as our common-sense metaphysics, or rather as their own interpretation of common-sense metaphysics, represents it to be. What they doubt is whether the percipuum accords with the facts; that is, with their interpretation of the general mass of experience; or, since experience is nothing but the interpretation of the percipua, what they doubt is whether the percipuum accords with an interpretation of an interpretation of itself, just as one asks whether the Greek testament faithfully accords with the Authorized Version, whether Jesus was a sound christian and could sign the thirty-nine articles, whether Plato, Isocrates, Thucydides and the rest of the prosaists of that period wrote good grammar, and the like.†15 I fear I shall not be able to follow them far into this deep and dark exploration.

Peirce: CP 7.649 Cross-Ref:††

649. If we wish to know what the percipuum of the course of time is, all we have to do is to abstain from sophisticating it, and it will be plain enough. No more than the present moment directly confronts us. The future, however little future it may be, is known only by generalization. The past, however little past it may be, lacks the explicitness of the present. Nevertheless, in the present moment we are directly aware of the flow of time, or in other words that things can change.†16 Several other points are clearly delivered in the percipuum, but they are implied in what has been stated. Sundry schools of philosophy hold this testimony of the percipuum to be contrary to sound reason, and accordingly either set it down as false, or conclude that the facts really are absurd, or both. But it strikes me that while it is not the business of facts to conform to what M or N may approve as sound reasoning, it is the business of M's and N's logic to conform to facts.

Peirce: CP 7.650 Cross-Ref:††

650. Fully to unfold all the implications of the deliverance of the percipuum so as to demonstrate the accuracy of the interpretation would require a small treatise, if written in the English style, or three stout octavos in German. But I will endeavor to summarize what the upshot of the discussion would be so as to render it tolerably perspicuous.

Peirce: CP 7.651 Cross-Ref:††

651. Kant squarely hit the nail on the head when he said that every part of a lapse of time was a lapse of time. But here as in many parts of his philosophy, Kant did not quite understand himself, and imagined that in saying that every part of a time is a time he had only said that time was infinitely divisible.†17 . . . †18

Peirce: CP 7.652 Cross-Ref:††

652. But although Kant unwarily confused the idea that time, as "intuited" (to use his language, for he himself translates his *Anschauung* by the Latin *intuitus*), has no parts that are not themselves times with the very different idea that there *is* a way of dividing time so as never to reach an ultimate part (though such parts there may be), yet his reasoning, taken as a whole, is consistent only with the idea that time as

intuited has no ultimate parts, or instants. That opinion I share, substituting the percipuum for his intuition. Thus, that which the mathematicians call "continuity" becomes, for me, "pseudo-continuity."

Peirce: CP 7.653 Cross-Ref:††

653. Of course, if there is no such thing as an absolute instant, there is nothing *absolutely present* either temporarily or in the sense of confrontation. In fact, we are thus brought close to the doctrine of Synechism, which is that elements of Thirdness cannot entirely be escaped. The present moment will be a lapse of time, highly confrontital, when looked at as a whole, seeming absolutely so, but when regarded closely, seen not to be absolutely so, its earlier parts being somewhat of the nature of memory, a little vague, and its later parts somewhat of the nature of anticipation, a little generalized. It contains a central part which is still more present, still more confrontital, but which presents the same features. There is nothing at all that is absolutely confrontital; although it is quite true that the confrontital is continually flowing in upon us.

Peirce: CP 7.654 Cross-Ref:††

654. I aver, but do not show, that if my statement of what I call the essential part of the delivery of the percipuum be true to it, then the interpretation given is demonstrably the only one possible. But though I do not actually demonstrate it, I will mention some secondary features of the deliverance of the percipuum which strongly tend to confirm the interpretation.

Peirce: CP 7.655 Cross-Ref:††

655. One such deliverance is that any multitude of changes not too great to be successive in any sense might take place in any lapse of time however short. Now two things are demonstrable (although again I withhold the demonstrations). One is that *no* multitude is so great as to prevent a collection of objects of that multitude from being linearly arranged.†19 The other is that there is no maximum multitude. It follows, then, from the deliverance just stated, that the possible mutually exclusive divisions of any time, however short, exceed all multitude. In that case, time can not only not have merely the pseudo-continuity of quantity, -- since the multitude of quantities †20 is well-known to be only the second of an endless series of grades of infinite multitude, -- but it cannot be composed of instants at all (as it might very well be and still enormously exceed the differentiation of quantity) since the entire collection of such instants would have a multitude.

Peirce: CP 7.656 Cross-Ref:††

656. Another plain deliverance of the percipuum is that moment melts into moment. That is to say, moments may be so related as not to be entirely separate and yet not be the same. Obviously, this would be so according to our interpretation. But if time consists of instants, each instant is exactly what it is and is absolutely not any other. In particular, any two real quantities differ by a finite amount.

Peirce: CP 7.657 Cross-Ref:††

657. Still another deliverance of the percipuum is that every interval of time has a beginning and an end, an initial moment and a terminal moment; that is, a moment before which there is no moment of the interval and a moment after which there is no moment of the interval. This again is obviously true of our interpretation; and were the terminal moment thrown off, that is, not reckoned as belonging to the interval, still what was left would have a terminal moment. If only one moment were thrown off at the end, the new terminal moment would not be altogether different

from what had been the terminal moment. Of course, the expression "only one moment" can only be understood in a relative sense. This precisely agrees with the percipuum, according to which if from a terminated line, the terminal point be thrown off, what remains still has a terminal which is substantially identical with the old terminal. I appeal to the reader's consciousness to say if that be not so. But if time is composed of instants, and the last instant of an interval be thrown off, the interval will be left *without any last*. For if it had any, then before the last was thrown off, there would be no instant between this original last and the instant which becomes the last after the throwing off. But to say that there is not a moment between any two distinct moments is too monstrously in conflict with the percipuum for any reasoning man to maintain. Such a notion would be exploded in a dozen ways.

Peirce: CP 7.658 Cross-Ref:††

658. By this time, while I have not exhausted my stock of reasons for thinking that the deliverance of the percipuum concerning the flow of time must be interpreted as I say, yet I have said enough to convince any superior reasoner, -- or as nearly to convince him as his caution will allow him to be convinced until he has had time for reconsideration. Further confirmation will soon appear; but we may turn now to the question of whether the testimony of the percipuum is truthful or not. It is obvious that a *percept* cannot be false, since it makes no assertion and is not a proposition, whether indicative, interrogative, optative, imperative, or in any mood whatever; and few philosophers will today think otherwise. It is indeed one of the stock remarks that even a hallucination is not false. It would save me trouble in my present argumentation if I could take that short cut to the truth; but I fear I cannot quite do so.

Peirce: CP 7.659 Cross-Ref:††

659. We must enter for a few moments into the field of metaphysics. For we are inquiring how things really are whatever we may think. What is reality? There would not be any such thing as truth unless there were something which is as it is independently of how we may think it to be.†21 That is the reality, and we have to inquire what its nature is. We speak of *hard facts*. We wish our knowledge to conform to hard facts. Now, the "hardness" of fact lies in the insistency of the percept, its entirely irrational insistency, -- the element of Secondness in it. That is a very important factor of reality. But this factor is not confined to the percept. We can know nothing about the percept, -- but only experience it in its totality, -- except through the perceptual judgment, and this likewise compels acceptance without any assignable reason. This indefensible compulsiveness of the perceptual judgment is precisely what constitutes the cogency of mathematical demonstration. One may be surprised that I should pigeon-hole mathematical demonstration with things unreasonably compulsory. But it is the truth that the nodus of any mathematical proof consists precisely in a judgment in every respect similar to the perceptual judgment except only that instead of referring to a percept forced upon our perception, it refers to an imagination of our creation. There is no more why or wherefore about it than about the perceptual judgment, "This which is before my eyes looks yellow." To show this I must take an example of a mathematical proof; and for the reader's comfort I will take an extremely simple one. At the same time it must be of as abstract a nature as possible, or it might be said that whether intuitional mathematics were perceptual or not, this was not the nature of all mathematics.

Peirce: CP 7.660 Cross-Ref:††

660. I will start with the following premiss, which is true of whole numbers: If any predicate, P, be true of the number 0, *zero*, but not of all numbers, then there must

be two numbers M and N such that N is next greater than M, and P, while true of M, is not true of N.

Peirce: CP 7.661 Cross-Ref:††

661. From this I proceed to prove, first, that there is no number except zero that is not next greater than some number other than itself. For if there were any such number, call it A. Then the predicate "is not A" would be true of 0 (since A is supposed other than 0) but not of all numbers (since it would not be true of A). Yet there would be no number of which it would be true that it was not A while there was a number next greater than this number of which this was not true (since A is, by hypothesis, not next greater than any other number than itself).

Peirce: CP 7.662 Cross-Ref:††

662. I will further prove from the same premiss that there is no number except zero that cannot be reached from zero by a finite multitude of successive steps, each passing from a number to [a] number next greater than it. By a "finite" multitude is meant the multitude of any collection [for which, that collection] being substituted for "Hottentot" in the following syllogism, this syllogism would be valid:

Every Hottentot kills a Hottentot;

No Hottentot is killed by more than one Hottentot;

Therefore every Hottentot must be killed by a Hottentot.

Peirce: CP 7.663 Cross-Ref:††

663. I will first demonstrate that if a single individual is joined to a finite collection, the collection will remain finite. For that purpose, I first remark that the premisses of the above syllogism say nothing about the individual differences of the single Hottentots; and therefore if a newcomer were substituted for one of the Hottentots and the conclusion logically followed from the premisses before the substitution, so they would after the substitution. (There are other ways of making this evident.) Now suppose the tribe were increased by a new man. If the collection thereby ceased to be finite, the premisses might continue to be true of the new Hottentots (that is, of the tribe as so increased) and yet some man might escape being killed. We may suppose for convenience that this is the newcomer (since we have seen that a mere substitution as to who kills who makes no difference). Then no Hottentot kills the newcomer; but every Hottentot still kills some Hottentot. Hence he must kill one of the old Hottentots. So as before every old Hottentot kills an old Hottentot and the old Hottentots being a finite collection, every old Hottentot gets killed by an old Hottentot. But no man is killed by two different men; so that there is nobody for the newcomer to kill. Thus the premisses cannot be true of the increased collection unless the conclusion be true; which is as much as to say that the collection remains finite.

Peirce: CP 7.664 Cross-Ref:††

664. Now, to return to the numbers, suppose there be such a number as I undertake to prove there is not. Call it A (or if there be more than one, call any one of them A). Let the predicate, P, be "is either zero or can be reached from zero by a finite collection of steps from [a] number to [a] next greater number." This predicate

is true of zero, but not of A. Yet there is no number of which it is true and yet not of a next greater, since to suppose this would be to suppose a finite collection of steps would cease to be finite on the adjunction of one more. Thus the supposition that there is such a number as A is absurd, and the proposition is proved.

Peirce: CP 7.665 Cross-Ref:††

665. These proofs are founded on nothing but an abstract proposition. But an abstract proposition is a subject for observation as much as anything else. The proofs consist in remarking implications which were not thought of in the proposition as first adopted. Our premiss said that no matter what predicate and what number be chosen, either that predicate is true of that number or not true of *zero* or else two numbers could be found of one of which, which was next greater than the other, the predicate would not be true while it would be true of the other. In thinking this, we did not think whether or not it was possible that there should be a number not greater than any other. But when this is suggested, we are led to remark that being not next greater than any number and yet not zero is itself a predicate and the negative of a predicate. This is the kind of observation peculiar to mathematics, the looking at things substantively that have only been regarded transitively, or transitorily, the operation of abstraction.†22 How do we know that is a predicate? Precisely as we know that what is before our eyes is yellow. We can, after the fact, invent a formula to cover the case; but it will still leave something undefended. It is really a compulsion similar to that of the perceptual judgment with which we have to do.

Peirce: CP 7.665 Cross-Ref:††

Both the arguments happen to be stated in the form of the *reductio ad absurdum*, which is very appropriate to mathematics as bringing out its ultimately irrational character. But it is a mere form of statement. Every *reductio ad absurdum* can be stated as a direct argument. The reduction of the figures of [the] syllogism illustrates, if it does not virtually demonstrate, this truth.

Peirce: CP 7.666 Cross-Ref:††

666. But it is not in respect to this immediate insistence alone that the real is as it is independently of how we think it to be. The future is real, as well as the present, in so far as it is predetermined; and who but a mad man will deny that it largely is predetermined, at least to some degree, if not irrevocably. Indeed, the tendency of modern philosophy has been to follow the stoical doctrine that the future is what it is to be, independently -- of something; I do not think it is clearly said of what. The explanation of this state of mind, as far as the stoics were concerned, is not difficult. The early stoics in their efforts to make the universe comprehensible endeavored to discard elements of Thirdness, with a more or less unconscious feeling of Ockham's razor. The resulting hard dualism suited the hard lot of slaves and of the oppressed; and as time went on and this class and their descendants became more influential, a stern morality, which is essentially dualistic, naturally flourished amid the hideous riot of that age, while thirdness was utterly shoved into the background. The chief rival of stoicism, Epicureanism, sought to get along in its philosophy even without elements of secondness. About 1600, it was the scholastic realists who were the defenders of thirdness, and their dunsical opposition to the new learning and their dreadful corruption of the university disgusted the new men. Hence the fight was between Gassendi, the Epicurean, and Descartes, the real though unconscious stoic. I refuse to believe the future is entirely predetermined; but it certainly is so in great measure; and in so far as it is so, it is independent of all that we can think, wish, or do. It has the kind of compulsiveness that belongs to inductive reasoning, or experimental

inquiry, really the most mighty cogency there is. For experimental inquiry sets out with a hypothesis; upon which it bases predictions as to the issue of experiments, and it is left to the future experiment to bring forth the conclusion from the womb of the future. This factor of reality is specially prominent in the reality of personality. It is what the man is destined to do, what of the future is wrapped up in him, that makes him what he is.

Peirce: CP 7.667 Cross-Ref:††

667. The difference between the insistency of what is before us and the power of predestination is manifest enough. But there is a third factor of reality, different from either of those. The past also is real, -- *something* in it, at least. The future weeds it out; but the positive element is peculiar. Memory would be nothing but a dream if it were not that predictions are based on it that get verified. When we think how slight and entangled must be the ultimate bits of feeling out of which memory constructs her mosaic, we are compelled to liken it to conjecture. It is a wonderful power of constructing quasi-conjectures or dreams that will get borne out by future experience. The power of performing this feat, which is the power of the past, is a gentle compulsiveness.

Peirce: CP 7.668 Cross-Ref:††

668. There are the three elements of reality: that by which ideas spring up that have concealed within them an accord with the mass of ideas; that by which one idea acts directly on another; that force from without that weeds out a part of the ideas and strengthens the rest.

Peirce: CP 7.669 Cross-Ref:††

669. To state the matter otherwise: An idea, a surmise springs up in my mind. It recommends itself to me more or less forcibly as reasonable. The fact that it recommends itself to me more or less surely warrants its pretty near accord with what will recommend itself to reasonable minds as well as to the quasi-mind behind the issues of the future. That idea acts upon other ideas and absolutely forces me to say that it requires certain things to happen in the future. The future events come to pass and in part negative my surmise, in part confirm it. I do not know what idea we can form of reality except that it is that threefold force; or what the real can be except that which the whole process tends, as we hope, to induce our thoughts to rest upon.

Peirce: CP 7.670 Cross-Ref:††

670. Such being the nature of the real, since the percipuum confesses itself to contain a *soupeçon* of memory, -- that is, of conjecture, -- as well as a *soupeçon* of oversight, or watching to see if the future comes as expected, it must be confessed that according to the percipuum's own account of itself, not only may this or that percipuum be false, -- and should be so regarded in the case of blunders, if not of hallucinations, -- but it is even conceivable that all percipua should contain a false element, perpetually refuted by oncoming fresh percipua, although these fresh percipua perpetually introduce the like falsity anew.

Peirce: CP 7.671 Cross-Ref:††

671. But it is remarkable that in case we do not accept the percipuum's own account of itself, but maintain that time is composed of absolute instants, then it would seem that there is nothing that empirical truth can mean except accordance with what is given in those instants, which in this case, in no way testify concerning one another or in any way refer to one another. If that be so, a percept must be absolutely true. Here, then, we have a witness who testifies, "There is a certain dose

of falsity in my testimony, I know," and a large and influential party of philosophers who protest, "Oh, no such thing! Whatever you testify to is absolute truth."

Peirce: CP 7.672 Cross-Ref:††

672. Suppose, however, that some fallacy lurks here, and that the doctrine of instantaneous consciousness does not logically require absolute assent to all the representations of perception. Still, it must be admitted that the only method of ascertaining the truth is to repeat this trio of operations: conjecture; deductions of predictions from the conjecture; testing the predictions by experimentation (not necessarily what is technically so called, but essentially the same thing, -- trial).†23 We, thus, necessarily repose upon man's power of guessing the truth. Let him analyze the question as far as practicable; and give him guesses enough (not very many) upon each simple item and eventually he will guess right. Since we thus unavoidably assume that the mind has a certain power of evolving the truth from its own entrails, natural ideas, such as the deliverances of the percipua, have a right of precedence, and should be adhered to until fact flatly contradicts them. But thus far all the facts of observation are in remarkable accord with the deliverance of the percipuum as it is above stated.

Peirce: CP 7.673 Cross-Ref:††

673. If my view is right, time is of the nature of a general. That is, it may be a day; and a day is, by virtue of there being a forenoon and an afternoon; and a forenoon or afternoon is, by virtue of the different hours; the hour, by virtue of its minutes; and so on endlessly. But you never can find an indivisible time.

Peirce: CP 7.674 Cross-Ref:††

674. It is true that we may imagine a pendulous motion. The motion to the right ceases and the motion to the left begins. If the pendulum is an absolutely rigid body all whose parts are constrained to move in precisely the same way, then, were time composed of rational dates only, there need not be any date at which the right hand motion ended and the left hand motion began. In a true continuum there must be a common moment, but not an absolute instant, independent of all that is before and after. Looking at matters through the wrong end of a telescope, as it were, -- that is, aggregating the parts, -- there certainly is something in a moment altogether independent of past and future. But examining the moment under a microscope we find this independent element divided up into portions, less independent of one another. Finally, we come to this, that while there are elements of secondness, -- of irrational compulsion -- they flow in upon us continuously, thus being subjected from the very first to thirdness. Take away considerable time, -- as a day; -- and doubtless much therein happens that could not have been expected. But if we divide the day into hours, we find that much that was unexpected on the whole is no more than might have been anticipated from a part; and so we are led to say that the unexpected comes, not only in driblets, but in inappreciable flow.

Peirce: CP 7.675 Cross-Ref:††

675. On the whole, then, the percipuum is not an absolute event. There is no span of present time so short as not to contain something remembered, that is, taken as a reasonable conjecture, not without containing something expected for the confirmation which we are waiting. The peculiar element of the present, that it confronts us with ideas which it forces upon us without reason, is something which accumulates in wholes of time and dissipates the more minutely the course of time is scrutinized.

Peirce: CP 7.676 Cross-Ref:††

676. There is no percipuum so absolute as not to be subject to possible error.

Peirce: CP 7.677 Cross-Ref:††

677. The percipuum is a recognition of the character of what is past, the percept which we think we remember. The interpretation is forced upon us; but no reason for it can be given.

Peirce: CP 7.678 Cross-Ref:††

678. But just so when we experience a long series of systematically connected phenomena, suddenly the idea of the mode of connection, of the system, springs up in our minds, is forced upon us, and there is no warrant for it and no apparent explanation of how we were led so to view it. You may say that we put this and that together; but what brought those ideas out of the depths of consciousness? On this idea, which springs out upon experience of part of the system we immediately build expectations of what is to come and assume the attitude of watching for them.

Peirce: CP 7.679 Cross-Ref:††

679. It is in this way that science is built up; and science would be impossible if man did not possess a tendency to conjecture rightly.

Peirce: CP 7.680 Cross-Ref:††

680. It is idle to say that the doctrine of chances would account for man's ultimately guessing right. For if there were only a limited number n of hypotheses that man could form, so that $1/n$ would be the chance of the first hypothesis being right, still it would be a remarkable fact that man only could form n hypotheses, including in the number the hypothesis that future experimentation would confirm. Why should man's n hypotheses include the right one? The doctrine of chances could never account for that until it was in possession of statistics of the hypotheses that are inconceivable by man. But even that is not the real state of things. It is hard to say how many hypotheses a physicist could conceive to account for a phenomenon in his laboratory. He might suppose that the conjunctions of the planets had something to do with it, or some relation between the phases of variability of the stars in {ö} *Centauri*, or the fact of the Dowager empress having blown her nose 1 day 2 hours 34 minutes and 56 seconds after an inhabitant of Mars had died. The truth is that very few hypotheses will appear to the physicist to be reasonable; and the one true hypothesis is usually of this small number. Why is that? It may be answered, very truly, that experience has taught us that astrology, correspondences, magic, and many hypotheses formerly considered reasonable are to be put aside. Yes, but if primitive man had not had, at the very outset, some decided tendency toward preferring truthful hypotheses, no length of time, -- absolutely none, -- would have been sufficient to educate him even to the state of mind of Aristotle in his *Book of Physical Auscultations*, ridiculous as all that now seems to us. No, it is absolutely necessary to admit some original connection between human ideas, and the events that the future was destined to unfold.

Peirce: CP 7.681 Cross-Ref:††

681. But that is something very like telepathy. What would telepathy amount to, if it were an established fact. It would then be proved that people not very infrequently have hallucinations, and that one hallucination out of a great number (but more frequently than chance coincidence could account for) coincides with subsequent experience to such a degree as to attract attention; for even if there really

be telepathy we must suppose, from what we know of human nature, that the accordance with truth is apt to be greatly exaggerated. In this case, telepathy would be a phenomenon somewhat more remote from perception than the conjectures by which physicists so often hit upon the truth.

Peirce: CP 7.682 Cross-Ref:††
§6. CONCLUSION

682. Very well, then; what does all this rambling lead to? Nobody, of course, denies the *phenomena*, which the telepathists bring before us; that is to say, that such and such stories are told. The only question is whether they are to be accounted for by the operation of causes usually active in such phenomena.

Peirce: CP 7.683 Cross-Ref:††

683. The doctrine of telepathy is not established, nor far on the way toward establishment, as a scientific truth. For understood as the telepathists make a point of defining it, it denies the hope on which all science is based. If, however, we amend it, in order to avoid that positively anti-scientific character, what does it amount to? That very rarely a person at a distance from another has a hallucination or a positive conviction apparently irrational which represents something as having happened to that other person, and this turns out to be veridical with a frequency and to a degree which we know not how to explain, and which seems mysterious. But science can make no use of a proposition so vague as that. Where is the phenomenon, or the feature of a phenomenon, in which, if we examine closely, we shall not find much that our science cannot yet explain?

Peirce: CP 7.684 Cross-Ref:††

684. There is, however, no part of the work of science that ought to rank higher than that which brings a phenomenon to the cognizance of science. That sort of work, which Chladni did for the falling stones, is what the psychicists are endeavoring to do for veridical hallucinations and the like; and they will no doubt persevere until they succeed. I do not think they have displayed great genius for research; but they ought to be respected for the thorough single-heartedness, conscientiousness, and constancy with which they have pursued this purpose.

Peirce: CP 7.685 Cross-Ref:††

685. The general public has no accurate conception of what the work of a scientific investigator consists in; for the books of "popular science" give no idea of it. But the general public is no fool in judging of human nature; and the general public is decidedly of the opinion that there is such a thing as a scientific pedantry that swells with complaisance when it can sneer at popular observations, not always wisely. I must confess that in the past generations scientists of great eminence have sometimes been betrayed into this fault; but it will not be so hereafter. As for Sir William Crooks his most exquisite experimental researches were still in the future when he first displayed an openness of mind more admirable still. Neither Newcomb nor Langley was a worn-out and super-annuated physicist when he took a prominent position in the Psychical Research Society; neither had at that time achieved his most brilliant experimental triumphs.

Peirce: CP 7.686 Cross-Ref:††

686. What school of philosophers is it that attributes to the human mind the most wonderful powers? One might guess that it would be the idealists and spiritualists; but strange to say it is those who boast themselves to be materialists and who insist that nothing is real except *mass* and *motion*. For in order to maintain that, they are obliged to say that *law* is nothing but a figment of the human mind. Now *law* can certainly not be jammed into the pigeon-hole of mass, nor into that of motion. That indeed is their real reason for making it a fiction. Now is it not of all things the most wonderful, that the mind should be able to create an idea for which there is no prototype in nature, nor anything in the least resembling it, and that by means of this utter fiction it should manage to predict the results of future experiments and by means of that power should during the nineteenth century have transformed the face of the globe? Telepathy, with its infrequency and usual deceptiveness (for there is no reason for separating veridical from non-veridical hallucinations, as phenomena essentially different,) would be an insignificant faculty in comparison.

Peirce: CP 7.687 Cross-Ref:††

687. For my part, I cannot accept such a theory. It attributes to man powers which he knows too well that he does not possess. It seems to me that the only admissible view is that the reasonableness, or idea of law, in a man's mind, being an idea by which objective predictions are effected, -- for all physical theories originate in human conjectures, and experiment only lops off what is erroneous and determines exact values, -- must be in the mind as a consequence of its being in the real world. Then the reasonableness of the mind and that of nature being essentially the same, it is not surprising that the mind, after a limited number of guesses, should be able to conjecture what the law of any natural phenomenon is. How far this power of conjecture may go we certainly do not know. We do know that it goes far enough to have enabled men to make already considerable progress in science. Whether or not it extends so far that very rarely one mind can know what passes in another at a distance, would seem to be a question to be investigated as soon as we can see our way to doing so intelligently. I do not believe that questions can be permanently settled by pooh-poohing one or another alternative.

Peirce: CP 7.688 Cross-Ref:††

688. It is curious to see how the materialists, -- or those who are flattered at being so called, -- plume themselves on being free from "beliefs." Really, one would suppose they imagined it possible for a man to conduct his life on a basis of scientifically established doctrine. It is true that an engineer, civil, mechanical, electrical, or chemical, is able to do this in the present advanced state of science, so far as he has to deal with materials. But even a member of one of those four professions, the only ones that can be called thoroughly reduced to the applications of scientific principles, even he still finds that exact science fails him in dealing with men, -- that is to say in considering the use to which his construction is to be put, in considering financial questions, in considering his relations to those who are to execute his plans. For everything else in life unscientific beliefs have to be relied upon for the present; and in particular, it is precisely what we are to *believe* about telepathy that we all are curious to know.†24

†1 He was employed by the Coast Survey from September 21, 1859, to June 1, 1860, and from July 1, 1861, to December 31, 1891.

Peirce: CP 7 Introduction p xi Fn 2 p xi

†2 See the Bibliography in Vol. VIII, especially the years 1872 through 1886; Peirce produced little in the way of results for the Coast Survey after he moved to Milford, Pa., in the spring of 1887.

Peirce: CP 7 Introduction p xi Fn 3 p xii

†3 Cf. 6.44, 6.46. Peirce's most original contribution to geodetics was the discovery of a new source of error in gravity measurement; this is described at 7.1-12. Peirce also did research for the Coast Survey on standards of measurement.

Peirce: CP 7.1 Fn 1 Para 1/2 p 3

†1 (Ed.) From pp. 359-361 of Appendix 14, "On the Flexure of Pendulum Supports," [CS 1881] 1883, pp. 359-441.

Peirce: CP 7.1 Fn 1 Para 2/2 p 3

This gives a succinct account, in Peirce's own words, of his most original work in the field of pendulum measurements.

Peirce: CP 7.5 Fn 2 p 4

†2 Thus, Bessel's idea of directly measuring the position of the center of mass was supposed by the Swiss *savans* to belong to M. Cellierier.

Peirce: CP 7.8 Fn 3 p 5

†3 (Ed.) [Bibliography] G-1875-5. The quotations from the circular by Baeyer, given in 7.6, and from the reports, given in this paragraph, are in [International Geodetic Comm., Paris, 1875] 1875, pp. 90-91, 93, and 96.

Peirce: CP 7.10 Fn 4 p 6

†4 (Ed.) See [Bibliography] G-1875-5. The quotation in this paragraph from Hirsch is in [International Geodetic Comm., Brussels, 1876] 1877, p. 18.

Peirce: CP 7.11 Fn 5 p 6

†5 (Ed.) [Bibliography] G-1877-3.

Peirce: CP 7.12 Fn 6 p 7

†6 (Ed.) See [Bibliography] G-1877-3.

Peirce: CP 7.13 Fn 7 Para 1/2 p 8

†7 (Ed.) "Six Reasons for the Prosecution of Pendulum Experiments," pp. 506-508 of Appendix 22, "Report of a Conference on Gravity Determinations, Held in Washington, in May, 1882," [CS 1882] 1883, as corrected from a reprint marked "Copy read by C. S. Peirce," Widener IC2c.

Peirce: CP 7.13 Fn 7 Para 2/2 p 8

This is reprinted here because it is a good statement by Peirce of what he thought the value of his pendulum work to be.

Peirce: CP 7.15 Fn 8 p 9

†8 (Ed.) J. E. Hilgard was Superintendent of the United States Coast and Geodetic Survey at this time.

Peirce: CP 7.15 Fn 9 p 10

†9 (Ed.) Cf. [Bibliography] G-1879-1, G-1881-5, G-1882-12.

Peirce: CP 7.21 Fn 1 Para 1/3 p 13

†1 (Ed.) "On Small Differences of Sensation," *Memoirs of the National Academy of Sciences* 3, Part I (1884) 73-83, with corrections from a reprint marked, "Prof. John W. Langley, With the regards of C. S. Peirce," Widener IC1a.

Peirce: CP 7.21 Fn 1 Para 2/3 p 13

The article was written with Joseph Jastrow, a student of Peirce at Johns Hopkins University. In *The Nation* 98 (14 May 1914) 571 Jastrow says of it: "It was Mr. Peirce who introduced me to the possibility of an experimental study of a psychological problem. He provided the problem, the instruments which I set up in my room, the method, and the mode of reaching the results; these were printed over our joint names." See also Joseph Jastrow, "Charles S. Peirce as a Teacher," *The Journal of Philosophy, Psychology, and Scientific Method* 13 (21 Dec. 1916) 724.

Peirce: CP 7.21 Fn 1 Para 3/3 p 13

Edwin G. Boring, in his *A History of Experimental Psychology*, p. 529, refers to "On Small Differences of Sensation" as "an important paper on the method of determining the differential limen . . ."

Peirce: CP 7.21 Fn 2 p 13

†2 *Elemente der Psychophysik*, I, p. 242.

Peirce: CP 7.22 Fn 3 p 15

†3 The rule for finding this ratio is as follows: Divide the logarithm of the ratio of excitations by the probable error and multiply the quotient by 0.477. Call this product t . Enter it in the table of the integral $\int_0^t e^{-x^2} dx$, given in most works on probabilities; $\int_0^t e^{-x^2} dx$ is the proportion of cases in which the error will be less than the difference between the given excitations. In all these cases, of course, we shall answer correctly, and also by chance in one-half of the remaining cases. The proportion of erroneous answers is therefore $(1 - \int_0^t e^{-x^2} dx)/2$. In the following table the first column gives the quotient of the logarithm of the ratio of excitation, divided by the probable error, and the second column shows the proportion of erroneous judgments:

0.0	0.50
0.05	0.49
0.1	0.47
0.25	0.43
0.5	0.37
1.0	0.25

To guess the correct card out of a pack of fifty-two once in eleven times it would be necessary to have a sensation amounting to 0.37 of the probable error. This would be a sensation of which we should probably never become aware, as will appear below.

Peirce: CP 7.23 Fn 4 p 17

†4 The number of errors, when an increase of weight was followed by a

decrease, was slightly less than when the first change was a decrease of pressure.

Peirce: CP 7.26 Fn 5 Para 1/4 p 18

†5 The result of our observations on the confidence connected with the judgments is as follows:

[Subject, Mr Peirce.]

Variations.	Average confidence.	Number of sets of 50.
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Grams.

6067	7
3028	6
1515	5

[Subject, Mr. Jastrow.]

60.....	.90	13
30.....	.51	12
15.....	.30	12
20.....	.11	12
10.....	.06	12
5.....	.00	10

Peirce: CP 7.26 Fn 5 Para 2/4 p 18

In 1,125 experiments (subject, Mr. Peirce) -- variations 15, 30, and 60 grams -- there occurred confidence of 3, 35 times (3 per cent.); of 2, 102 times (9 per cent.); of 1, 282 times (25 per cent.); of 0, 706 times (63 per cent.). In these experiments there were 332 (29 per cent.) errors committed, of which 1 (0.3 per cent.) was made in connection with a confidence 3; 10 (3 per cent.) with a confidence 2; 51 (15 per cent.) with a confidence 1; 270 (81 per cent.) with a confidence 0. From which we find that in connection with a confidence of 3 there occurred 1 error in 35 cases (3 per cent.);

with a confidence of 2, 10 errors in 102 cases (10 per cent.); with a confidence of 1, 51 errors in 282 cases (18 per cent.); with a confidence of 0, 270 errors in 706 cases (38 per cent.).

Peirce: CP 7.26 Fn 5 Para 3/4 p 18

In 1,975 experiments (subject, Mr. Jastrow) -- variations 15, 30, and 60 grams -- there occurred confidence of 3, 62 times (3 per cent.); of 2, 196 times (10 per cent.); of 1, 594 times (30 per cent.); of 0, 1,123 times (57 per cent.). In these experiments there were 451 (23 per cent.) errors committed, of which 2 (0.4 per cent.) were made in connection with a confidence of 3; 12 (3 per cent.) with a confidence of 2; 97 (22 per cent.) with a confidence of 1; 340 (75 per cent.) with a confidence of 0. Again, in connection with a confidence of 3, errors occurred twice in 62 cases (3 per cent.); with a confidence of 2, 12 times in 196 cases (6 per cent.); with a confidence of 1, 97 times in 504 cases (16 per cent.); with a confidence of 0, 340 times in 1,123 cases (30 per cent.).

Peirce: CP 7.26 Fn 5 Para 4/4 p 18

In 1,675 experiments (subject, Mr. Jastrow) -- variations 5, 10, and 20 grams -- there occurred confidences of 3, none; of 2, none; of 1, 115 times (7 per cent.); of 0, 1,560 times (93 per cent.). In these experiments there were 538 (32 per cent.) errors committed, of which 16 (3 per cent.) occurred in connection with a confidence of 1; 522 (97 per cent.) with a confidence of 0. Again, in connection with a confidence of 1, errors occurred 16 times in 115 cases (14 per cent.); with a confidence of 0, 522 times in 1,560 cases (34 per cent.).

Second group.

Ratio of

weights. Mark 0. Mark 1. Mark 2. Mark 3.

1.015.....{ 110 right 51 right 3 right 1 right

 { 66 wrong 17 wrong 2 wrong 0 wrong

1.030.....{ 106 right 72 right 23 right 2 right

 { 35 wrong 11 wrong 1 wrong 0 wrong

1.060.....{ 86 right 75 right 54 right 24 right

 { 8 wrong 1 wrong 2 wrong 0 wrong

Third and fourth groups.

[Marks 2 and 3 do not occur.]

Ratio of weights. Mark 0. Mark 1.

1.005.....	{294 right	2 right
	{203 wrong	1 wrong
1.010.....	{366 right	32 right
	{192 wrong	30 wrong
1.020.....	{395 right	68 right
	{131 wrong	6 wrong

Peirce: CP 7.28 Fn 6 p 21

†6 At first a short pause was made in the set of 25, at the option of the subject; later this was dispensed with.

Peirce: CP 7.29 Fn 7 p 22

†7 In the experiments of December, 1883, and January, 1884, the method as above described was not fully perfected, the most important fault being that the total weight instead of being removed and replaced by a mechanical device, was taken off by the operator pressing with his finger upon the beam of the balance.

Peirce: CP 7.34 Fn 8 p 26

†8 In the tables of the third and fourth groups, there is a marked divergence between the *a priori* and *a posteriori* probable error, for the average number of errors in 50, making the observed probable error too small. This can only be partly accounted for by the fact that the subject formed the unconscious habit of retaining the number of each kind of experiment in a set and answering according to that knowledge. In point of fact the plus errors and minus errors separately do not exhibit the singular uniformity of their sums, for which we are quite unable to account. Thus in the fourth group we have:

Number of + and - errors.

Date.	1.020	1.010	1.005
March 30.....	-4,+ 7	-6,+10	-13,+ 8
March 31.....	-7,+ 3	-5,+10	- 6,+15
April 2.....	-1,+10	-8,+ 9	- 8,+13
April 3.....	-4,+ 5	-4,+14	-10,+10
April 6.....	-6,+ 6	-8,+ 7	-10,+11
April 7.....	-5,+ 9	-8,+ 7	- 8,+ 9

Peirce: CP 7.34 Fn 9 Para 1/2 p 26

†9 The conclusions of this paper are strengthened by the results of a series of experiments on the color sense, made with the use of a photometer by Mr. Jastrow. The object was to determine the number of errors of a given magnitude, and compare the numbers thus ascertained with the theoretical numbers given by the probability curve. A thousand experiments were made. Dividing the magnitude of the errors from 0 to the largest error, made into 5 parts, the number of errors, as observed and calculated, that occur in each part are as follows:

Observed 199 181 217 213 190

Calculated 213 197 209 181 200

Peirce: CP 7.34 Fn 9 Para 2/2 p 26

These numbers would be in closer accordance if the probable error were the same throughout, as it is not owing to the effects of practice, etc. Moreover, the experiments were made on different colors -- 300 on white and 100 each on yellow, blue, dove, pink, green, orange, and brown. These experiments were not continuous.

Peirce: CP 7.35 Fn 10 p 27

†10 (Ed.) See the following section of the present chapter.

Peirce: CP 7.35 Fn 11 p 27

†11 (Ed.) Cf. [Bibliography]G-1887-3, p. 194.

Peirce: CP 7.36 Fn 12 Para 1/3 p 27

†12 (Ed.) From a manuscript, "Guessing," with a quotation in 36n13 from a fragmentary alternative draft. These manuscripts are in Widener IB2-12, and are dated c.1907 on the basis of internal references and statements in the letter referred to in 44n17.

Peirce: CP 7.36 Fn 12 Para 2/3 p 27

The manuscript, "Guessing," was published in *The Hound and Horn* 2 (April-June 1929) 267-282.

Peirce: CP 7.36 Fn 12 Para 3/3 p 27

In the passages reprinted here Peirce gives a non-technical summary of the previous paper and discusses some philosophical implications of it.

Peirce: CP 7.36 Fn 13 p 27

†13 (Ed.) The fragmentary manuscript (see 36n12) begins: "All our knowledge starts from perception, and consequently perception ought never to be doubted, and indeed never *can* be doubted. That is to say, that we cannot doubt that that really *seems* which *seems to seem*" and then the manuscript proceeds very much like the version printed above. Cf. 5.157.

Peirce: CP 7.38 Fn 14 p 29

†14 (Ed.) Cf. 2.638, 2.753, 2.755.

Peirce: CP 7.40 Fn 15 p 30

†15 (Ed.) Peirce here recounts a personal anecdote, concerning the theft of his coat and a valuable watch from his stateroom on a Boston to New York boat. He says that he made all the waiters stand in a row and after talking briefly with each, but without consciously getting any clue, he made a guess as to which one was guilty. The upshot of the story is that after many difficulties, and by making more successful guesses, he proved that his original guess had been correct.

Peirce: CP 7.43 Fn 16 p 31

†16 (Ed.) See the first section of this chapter.

Peirce: CP 7.44 Fn 17 p 32

†17 (Ed.) Reprinted as the first section of this chapter.

Peirce says in a letter to William James of 16 July 1907 (Houghton Library) that there were 3389 guesses given with a confidence of "Naught" and about two thirds of these were correct (see 26n5).

Peirce: CP 7.49 Fn 1 p 37

†1 (Ed.) Paragraphs 49-52 are from manuscript L, undated (but cf. 59n4), Widener IB2-9.

Paragraphs 53-58 are from "Of the Classification of the Sciences. Second Paper. Of the Practical Sciences," Widener II. Paragraphs 53-57, 381n19 and 58 come from the manuscript in that order. This manuscript is dated c.1902 on the basis of references in it.

Peirce: CP 7.54 Fn 2 p 40

†2 (Ed.) This statement appears on page 3 in the edition edited by Alois Höfler, published by C. E. M. Pfeffer, Leipzig, 1900.

Peirce: CP 7.58 Fn 3 p 43

†3 (Ed.) Cf. 5.340.

Peirce: CP 7.59 Fn 4 p 43

†4 (Ed.) Paragraphs 59-76 are "Introductory Lecture on the Study of Logic," [JHUC] 2(Nov 1882)11-12, with two preliminary paragraphs omitted.

Paragraphs 77-78 are from manuscript N, Widener IB2-9, undated, but the manuscript contains results from the census of 1900. This manuscript and manuscript L (cf. 49n1) are probably parts of the same work.

Peirce: CP 7.59 Fn 5 p 44

†5 (Ed.) Orbellis (Nicholaus de), *Expositio super textu Petri Hispani, Super libro Peryhermenias*, Venice, 1500, fol. a3v.

Peirce: CP 7.65 Fn 6 p 46

†6 (Ed.) See Book I of the present volume.

Peirce: CP 7.66 Fn 7 p 46

†7 (Ed.) Peirce's main task in the United States Coast Survey was to measure the force of gravity by swinging a pendulum.

Peirce: CP 7.66 Fn 8 p 46

†8 (Ed.) See the review of Wundt's book in [CP] VIII, Book I, Review 14.

Peirce: CP 7.71 Fn 9 p 47
†9 (Ed.) See [CP] V.

Peirce: CP 7.72 Fn 10 p 48
†10 (Ed.) See the rest of the present book and [CP] II for discussions of most of the topics mentioned in this and the following paragraphs.

Peirce: CP 7.77 Fn 11 p 49
†11 (Ed.) Cf. 5.383ff.

Peirce: CP 7.79 Fn 12 p 52
†12 (Ed.) Paragraphs 79-88 are "Scientific Method," *Dictionary of Philosophy and Psychology* (edited by James Mark Baldwin), Vol. II, 1902, pp. 500-503. Paragraphs 89-91 are "Verification," *ibid.*, pp. 761-762.

Peirce: CP 7.79 Fn 13 p 52
†13 (Ed.) Peirce did not define this term for Baldwin's *Dictionary*, but see Section 1 of the present chapter (49ff.).

Peirce: CP 7.83 Fn 14 p 54
†14 (Ed.) See Chapter 2, "Economy of Research," in the present book.

Peirce: CP 7.85 Fn 15 p 55
†15 (Ed.) See Chapter 3, "The Logic of Drawing History from Ancient Documents," in the present book.

Peirce: CP 7.87 Fn 16 p 57
†16 (Ed.) Cf. Chapter 5, "Telepathy and Perception," in Book III of the present volume.

Peirce: CP 7.88 Fn 17 p 57
†17 (Ed.) 2.773-778.

Peirce: CP 7.88 Fn 18 p 58
†18 (Ed.) 2.783-787.

Peirce: CP 7.92 Fn 19 p 59
†19 (Ed.) Paragraphs 92-93 are "Parsimony (law of)," *Dictionary of Philosophy and Psychology* (edited by James Mark Baldwin), Vol. II, 1902, p. 264. Paragraphs 94-96 are from "Theory," *ibid.*, pp. 693-694.

Peirce: CP 7.97 Fn 20 p 61
†20 (Ed.) "Notes for my Logical Criticism of Articles of the Christian Creed," Widener IB3. Judging by the reference to 1860 in the second paragraph, this is to be dated c.1910.

Peirce: CP 7.97 Fn 21 p 61
†21 (Ed.) Peirce also uses "Abduction" and "Hypothesis" for what he here calls "Retroduction."

Peirce: CP 7.103 Fn 22 p 62
†22 (Ed.) That is, in the case of a unit mass, the force is equal to the acceleration, and it is also equal to the derivative of the energy with respect to distance.

Peirce: CP 7.103 Fn 23 p 62
†23 (Ed.) See [CP] IV, Book II.

Peirce: CP 7.108 Fn 24 p 64
†24 (Ed.) Peirce's fallibilism, the doctrine that there is no absolute certainty in knowledge, is discussed at 1.8ff., and elsewhere in [CP] I.

Peirce: CP 7.109 Fn 25 p 64
†25 (Ed.) Cf. the discussion of unreal doubt as contrasted to genuine doubt at 5.265 and elsewhere in [CP] V.

Peirce: CP 7.110 Fn 26 p 65
†26 (Ed.) From Vol. I of Lecture 7 of the Lowell Lectures of 1903, Widener IB2-4.

Cf. 2.755-760 and 7.208-217 for related treatments of the same topic.

Peirce: CP 7.115 Fn 27 p 67
†27 (Ed.) "The Deductions which we base upon the hypothesis which has resulted from Abduction produce conditional predictions concerning our future experience. That is to say, we infer by Deduction that if the hypothesis be true, any future phenomena of certain descriptions must present such and such characters. We now institute a course of quasi-experimentation in order to bring these predictions to the test, and thus to form our final estimate of the value of the hypothesis, and this whole proceeding I term Induction. I speak of quasi-experimentation because the term *experiment* is, according to the usage of scientific men, restricted to the operation of bringing about certain conditions. The noting of the results of experiments or of anything

else to which our attention is directed in advance of our noting it, is called *Observation*. But by quasi-experimentation I mean the entire operation either of producing or of searching out a state of things to which the conditional predictions deduced from hypothesis shall be applicable and of noting how far the prediction is fulfilled." From an earlier passage of the same lecture (110n26).

Peirce: CP 7.131 Fn 28 p 72
†28 (Ed.) From Lecture IV (c.1866) of the same series from which 7.579-596 are taken, Widener IB2-10; cf. 7.579n34.

Peirce: CP 7.139 Fn 1 p 76
†1 (Ed.) "Note on the Theory of the Economy of Research," [CS 1876] 1879, pp. 197-201, with some corrections from a manuscript version in Widener IC2a.

Cf. 5.600ff.; the paper referred to at 5.601n§ is probably this one.

Peirce: CP 7.153 Fn 2 p 81
†2 (Ed.) In this paragraph the editor has corrected the second formula according to the manuscript, and rewritten the notation somewhat to make it uniform.

Peirce: CP 7.154 Fn 3 p 82
†3 (Ed.) In the manuscript version (see 139n1) Peirce says that this problem suggested his speculations on the economy of research.

Peirce: CP 7.155 Fn 4 p 82
†4 (Ed.) See [Bibliography] G-1879-5d, p. 260.

Peirce: CP 7.158 Fn 5 Para 1/4 p 84

†5 (Ed.) "[Memoir] No. 28, *On the Economics of Research*," from an application by Peirce for a grant from the Carnegie Institution (stamped by the Carnegie Institution 30 July 1902) with an added quotation in the present footnote from an alternative draft, both in Widener VB5. Peirce proposed to write thirty-six memoirs on logic, most of them of approximately 20,000 words, a few much shorter, and some of 50,000 words.

Peirce: CP 7.158 Fn 5 Para 2/4 p 84

The following are quoted from other parts of the application:

"Therefore, what I hereby solicit the aid of the Carnegie Institution to enable me to do is *to draw up some three dozen memoirs, each complete in itself, yet the whole forming a unitary system of logic in all its parts*, which memoirs shall present in a form quite convincing to a candid mind the results to which I have found that the scientific method unequivocally leads, adding, in each case, rational explanations of how opposing opinions have come about; the whole putting logic, as far as my studies of it have gone, upon the undeniable footing of a science," pp. 4-5.

Peirce: CP 7.158 Fn 5 Para 3/4 p 84

"It is my belief that science is approaching a critical point in which the influence of a truly scientific logic will be exceptionally desirable. Science, as the outlook seems to me, is coming to something not unlike the age of puberty. Its old and purely materialistic conceptions will no longer suffice; while yet the great danger involved in the admission of any others, ineluctable as such admission is, is manifest enough. The influence of the conceptions of methodetic will at that moment be decisive," p. 57.

Peirce: CP 7.158 Fn 5 Para 4/4 p 84

In an alternative draft of this application he says:

"What are the researches of which I speak?

"They are the work of my life, that which I seem to have been put into the world to do."

The application was rejected.

Peirce: CP 7.160 Fn 6 p 87

†6 (Ed.) Section 1 of the present chapter.

Peirce: CP 7.161 Fn 7 p 87

†7 (Ed.) See the same quotation in 7.59.

Peirce: CP 7.162 Fn 1 p 89

†1 (Ed.) From a draft of a report on the meeting of the National Academy of Sciences in November, 1901, Widener IV; see [Bibliography] N-1901-16.

Cf. [Bibliography] G-1901-2b and 2c, both of which touch on the topic of the present chapter and which were written about the same time.

Peirce: CP 7.164 Fn 2 p 90

†2 (Ed.) The remainder of this chapter is from a typed manuscript, "On the

Logic of drawing History from Ancient Documents especially from Testimonies," with two quotations added at 182n7 and 220n18. The material from the manuscript includes additions in Peirce's hand and a few minor editorial corrections taken from a corresponding handwritten manuscript; the quotation in 220n18 is from some alternate pages of the handwritten manuscript. These manuscripts are at Widener IB2-12 and are dated c.1901 on the basis of Peirce's National Academy of Sciences paper, [Bibliography] G-1901-4.

Peirce: CP 7.175 Fn 3 p 99

†3 (Ed.) Cf. 2.19ff.

Peirce: CP 7.177 Fn 4 p 101

†4 (Ed.) Peirce's theory of probability is presented in [CP] II, Part B of Book III.

Peirce: CP 7.181 Fn 5 p 105

†5 (Ed.) See 1.188, 1.260, 1.502.

Peirce: CP 7.181 Fn 6 p 105

†6 (Ed.) See 1.189.

Peirce: CP 7.182 Fn 7 p 107 Cross-Ref:††

†7 (Ed.) "On five occasions in my life, and on five occasions *only*, I have had an opportunity of testing my Abductions about historical facts, by the fulfillment of my predictions in subsequent archeological or other discoveries; and on each one of those five occasions my conclusions, which in every case ran counter to that of the highest authorities, turned out to be correct. The last two cases were these. Prof. Petrie published a history of Egypt in which he treated the first three dynasties as mythical. I was just about writing a history of science and in the first chapter I showed why those Dynasties including the name of Menes and other facts ought to be considered historical. Before my book was near completion Petrie himself found the tomb of Menes. Again a few years ago I wrote in the *Nation*, where there was no room for details, that the Babylonians had high scientific genius and that there was reason to conjecture that Alexander sent home a Babylonian celestial globe dating from 2300 years B.C. Now the newest finds show that at that very date they were accomplished astronomers." From the Lowell Lectures of 1903, Lecture VIII, following shortly after 5.604; Widener IB2-4. See [Bibliography] G-1903-2a.

Peirce: CP 7.183 Fn 8 p 107

†8 (Ed.) [Bibliography] G-c.1902-2.

Peirce: CP 7.185 Fn 9 p 107

†9 (Ed.) Cf. [Bibliography] G-1901-2.

Peirce: CP 7.189 Fn 10 p 112

†10 (Ed.) Cf. [Bibliography] G-1891-1.

Peirce: CP 7.198 Fn 11 p 116

†11 (Ed.) *The Principles of Empirical or Inductive Logic*, London, 1889. The quotations given by Peirce in paragraphs 199 and 200 are taken from pages 495 and 498.

Peirce: CP 7.199 Fn 12 p 119

†12 (Ed.) See R. C. Archibald, *Benjamin Peirce*, p. 14, for an account of this.

Peirce: CP 7.202 Fn 13 p 121

†13 (Ed.) These subjects are also treated in [CP] II.

Peirce: CP 7.206 Fn 14 p 125

†14 (Ed.) See Chapter 2, "Economy of Research," in the present book.

Peirce: CP 7.208 Fn 15 p 126

†15 (Ed.) Cf. 7.110-130.

Peirce: CP 7.209 Fn 16 p 126

†16 (Ed.) There follows a long discussion of collections (sets), of which we have printed here only that part necessary to understand the discussion below.

Peirce: CP 7.209 Fn 17 p 127

†17 (Ed.) [Bibliography] G-1881-7 ([AJM]). See also [Bibliography] G-1885-3 ([AJM]).

Peirce: CP 7.220 Fn 18 Para 1/4 p 138

†18 (Ed.) "[Abduction] alone can propose every proposition. . . . induction is the sole court of last resort in every case.

Peirce: CP 7.220 Fn 18 Para 2/4 p 138

"This being the case, what does it matter how the work of abduction is performed? It matters much, for the reason that it originates every proposition. It is true that, however carelessly the abduction is performed, the true hypothesis will get suggested at last. But the aid which a correct logic can afford to science consists in enabling that to be done at small expenditure of every kind which, at any rate, is bound to get done somehow. The whole service of logic to science, whatever the nature of its services to individuals may be, is of the nature of an economy. So much truth, -- and more than this, -- I concede to the doctrine of Ernst Mach, although I cannot approve of the extreme length to which he carries the theory of economy. . . .

Peirce: CP 7.220 Fn 18 Para 3/4 p 138

"The principles upon which abduction ought to be conducted ought to be determined exclusively by considerations of what purpose it subserves and how it may best subserve that purpose. Since, therefore, in scientific investigation abduction can subserve no other purpose than economy, it follows that the rules of scientific abduction ought to be based exclusively upon the economy of research." From a fragmentary alternative draft (see 164n2).

Peirce: CP 7.220 Fn 18 Para 4/4 p 138

Cf. Chapter 2, "Economy of Research," in the present book (7.139ff.).

Peirce: CP 7.235 Fn 19 p 151

†19 (Ed.) The essentials of this story are credited in John Edwin Sandys, *A History of Classical Scholarship*, Vol. 1, 3rd edition, University Press, Cambridge, 1921, pp. 85-86.

Peirce: CP 7.237 Fn 20 p 153

†20 (Ed.) Peirce's brackets.

Peirce: CP 7.250 Fn 21 p 161

†21 (Ed.) Peirce's brackets.

Peirce: CP 7.251 Fn 22 p 162

†22 (Ed.) Peirce advanced the same theory at 2.776, but at 8.209 (c.1905) he confesses that it is doubtful.

Peirce: CP 7.255 Fn 23 p 164

†23 (Ed.) Peirce then proceeds to give two more illustrative examples. The first is a problem of the chronology of Plato's dialogues, and the second concerns the life of Pythagoras. This part of the manuscript is more than half as long as the part reprinted here.

Peirce: CP 7.256 Fn 1 Para 1/2 p 165

†1 (Ed.) Paragraphs 256-261 are from one draft of a manuscript on the productiveness of the nineteenth century in great men; paragraphs 262-266 are from an alternative draft. Both drafts are in Widener IC1b, and are dated c.1900 on the basis of their resemblance to [Bibliography] G-1901-5a and internal references to dates.

Peirce: CP 7.256 Fn 1 Para 2/2 p 165

For additional information on Peirce's study of great men see [Fisch-Cope] 290-291 and Joseph Jastrow, "Charles S. Peirce as a Teacher," *The Journal of Philosophy, Psychology, and Scientific Method* 13 (21 Dec 1916) 724-725.

Peirce: CP 7.256 Fn 2 Para 1/2 p 167

†2 The fact that some of the finest observations of the naturalist were made by a blind man sufficiently shows that that science does not call for any extraordinary amount of the power I have in view.

Peirce: CP 7.256 Fn 2 Para 2/2 p 167

(Ed.) Peirce is probably referring here to François Huber, 1750-1831, a Swiss naturalist who became blind at age fifteen. He was noted for his investigations of bees.

Peirce: CP 7.258 Fn 3 p 169

†3 (Ed.) [Bibliography] G-1878-6.

Peirce: CP 7.258 Fn 4 p 170

†4 This vote was probably influenced by religious bias.

Peirce: CP 7.264 Fn 5 p 172

†5 (Ed.) Cf. Peirce's "Introductory Lecture on the Study of Logic," 7.59-76.

Peirce: CP 7.266 Fn 6 p 174

†6 (Ed.) Peirce then proceeds to discuss the great men of the nineteenth century. He discusses the century's great men in science in [Bibliography] G-1901-5a.

Peirce: CP 7.267 Fn 7 Para 1/3 p 175

†7 (Ed.) Paragraphs 267-275 are the concluding remarks to a series of lectures on the history of science, Widener IC1b, with quotations added at 267n8. The first paragraph of 267n8 is from Lecture V of this series, Widener IC1b. These are most probably the lectures delivered on "The History of Science" to the Lowell Institute, 1892-1893. On the basis of this probability and internal references, Lecture V is dated c.1892, and the concluding remarks 1893.

Peirce: CP 7.267 Fn 7 Para 2/3 p 175

Paragraphs 276-278 are Section 13, "Varieties of Medisense," of an incomplete manuscript, undated, Widener IB2-10. This manuscript has some

resemblance to 7.539-552, [Bibliography] G-undated-9, and may have been written about the same time.

Peirce: CP 7.267 Fn 7 Para 3/3 p 175

Paragraph 279 is from Section 1, "Classification of the Sciences," of *Minute Logic*, Chapter 2, "Prelogical Notions," Widener IB2-2. The marginal sidehead is "Three Stages of Physical Research." One of the manuscript pages is stamped "Mar 12 1902." This selection lies between 7.374n10 and 7.362; cf. 7.362n1.

Peirce: CP 7.267 Fn 8 Para 1/2 p 175 Cross-Ref:††

†8 (Ed.) In Lecture V of the series (see 267n7), Peirce says: "For my part, I am quite sure that, however it may be with the rank and file of the great army of general readers, those who come here will be interested in the history of science not as a mere Wonder Book, but as an instance, a specimen, of how the laws of growth apply to the human mind. As this Century is drawing to a close, it is interesting to pause and look about us and to ask ourselves in what great questions science is now most interested. The answer must be that *the* question that everybody is now asking, in metaphysics, in the theory of reasoning, in psychology, in general history, in philology, in sociology, in astronomy, perhaps even in molecular physics, is the question *How things grow*; and by far the most interesting aspect of the history of science, is that it shows how an important department of human thought has been developed from generation to generation, with a view of comparing this growth with the historical development of art, of religion, of politics, and of institutions generally, and not only with historical development but also with the growth of the individual mind, and not only of mind, but of organisms both in their geological succession and in their individual development, and with the formation of worlds, and even with the gradual coming into being and crystallization of the fundamental laws of matter and of mind, -- from all of which facts taken together we are to expect in the future a grand cosmogony or philosophy of creation."

Peirce: CP 7.267 Fn 8 Para 2/2 p 175 Cross-Ref:††

In the "Introduction" to a planned history of science, Widener IC1b (undated), Peirce says: "For that which the author had at heart throughout his studies of the history of science was to gain an understanding of the whole logic of every pathway to the truth."

Peirce: CP 7.274 Fn 9 p 178

†9 (Ed.) [Bibliography] G-1891-1e (1893).

Peirce: CP 7.276 Fn 10 p 179

†10 (Ed.) Cf. 7.539-552.

Peirce: CP 7.276 Fn 11 p 179

†11 (Ed.) Cf. [Bibliography] G-c.1893-4.

Peirce: CP 7.276 Fn 12 p 180

†12 (Ed.) Cf. [Bibliography] N-1894-4.

Peirce: CP 7.280 Fn 13 p 181

†13 (Ed.) From "Of the Nature of Measurement," an undated, incomplete manuscript in Widener IA-4. Cf. 4.142-152 which are on the same topic.

Peirce: CP 7.284 Fn 14 p 184

†14 (Ed.) Cf. 5.389.

Peirce: CP 7.289 Fn 15 p 186
†15 (Ed.) Cf. 3.567ff., 4.639ff., 6.174ff.

Peirce: CP 7.292 Fn 16 p 187
†16 (Ed.) Brackets in the original.

Peirce: CP 7.304 Fn 17 p 190
†17 (Ed.) The proof, which is quite long, and a scholium are omitted.

The title, "Of Space Measurement," precedes the following paragraphs.

Peirce: CP 7.312 Fn 18 p 193
†18 (Ed.) The manuscript continues for several pages and then breaks off in the middle of a sentence.

Peirce: CP 7.313 Fn 1 Para 1/2 p 194 Cross-Ref:††
†1 (Ed.) This chapter is from a set of associated manuscripts in Widener IB2-8, with the addition of a quotation in the present footnote. These manuscripts seem to be a number of partial drafts of a book to be titled "Logic." One set of chapters includes several bearing dates in March, 1873, and one chapter of another set is dated "1873 July 1." Others may have been written in 1872; in a letter to his brother Henry dated November 24, 1872, William James says, "Charles Peirce . . . read us an admirable introductory chapter to his book on logic the other day" ([Perry] I, 332). Some of the manuscripts have no heading, but probably belong to this undertaking. The portions bearing no date are dated here c.1873.

Peirce: CP 7.313 Fn 1 Para 2/2 p 194 Cross-Ref:††
The contents of the present chapter should be compared with "The Fixation of Belief" (1877), 5.358-387, and "How to Make Our Ideas Clear" (1878), 5.388-410. Several pages in the set of manuscripts from which the present chapter is taken contain passages to be found in the first of these two articles. In 1909 and 1910 Peirce worked on a revision of these two articles, to be published under a single title. A draft for this work is titled, "Essays Toward the Interpretation of Our Thoughts (Provisional Title of volume); My Pragmatism (Provisional Title of the Essay), set forth in Two Chapters," Widener IB2-11. On page 1 of the draft, in a preface for the work, Peirce says: "The main part of this Essay, -- the characterizations of Belief and of Doubt, the argument as to the effective aim of inquiry, the description of four methods directed toward that aim, with the criticisms of them, the discussion of the proper function of thinking, and the consequent maxim for attaining clear concepts, -- reproduces almost verbatim a paper I read, -- it must have been in 1872, -- to a group of young men who used, at that time, to meet once a fortnight in Cambridge, Mass., under the name of 'The Metaphysical Club,'-a name chosen to alienate such as it would alienate." This page is headed "1909 Apr 6 2AM, MEANING, Pragmatism." Cf. 5.13 and [Bibliography] G-1909-1.

Peirce: CP 7.313 Fn 2 p 194
†2 (Ed.) Paragraphs 313-314 are "Chapter 1 (Enlarged abstract)," with added quotations at 313n3 and 314n4. Paragraphs 315-316 are from an abstract of the first few chapters. Paragraphs 317-325 are the contents of an untitled manuscript. See 313n1.

Peirce: CP 7.313 Fn 3 p 195
†3 (Ed.) ". . . the characters of belief are three. First, there is a certain feeling

with regard to a proposition. Second, there is a disposition to be satisfied with the proposition. And third, there is a clear impulse to act in certain ways, in consequence." From "*Of Reality*" (see 313n1).

Peirce: CP 7.314 Fn 4 p 195

†4 (Ed.) "Doubt has degrees and may approximate indefinitely to belief, but when I doubt, the effect of the mental judgment will not be seen in my conduct as invariably or to the full extent that it will when I believe. Thus, if I am perfectly confident that an insurance company will fulfill their engagements I will pay them a certain sum for a policy, but if I think there is a risk of their breaking, I shall not pay them so much." From a fragment (see 313n1).

Peirce: CP 7.315 Fn 5 Para 1/3 p 195

†5 (Ed.) In the omitted portion of the manuscript Peirce briefly outlines three of his four "methods of effecting a settlement of opinion." The first is "obstinate adhering to whatever happens to be one's existing opinions." The second is by persecution. The third is "by the natural development of opinion," which fails when "one community comes in contact with another. Then it is seen that the result is quite accidental and dependent on surrounding circumstances and initial conditions and belief gets all unsettled.

Peirce: CP 7.315 Fn 5 Para 2/3 p 195

"In this way once more the conviction is forced on man that another's opinion, if derived by the same process as his own, is as good as his own, and that other's opinion is taken by him for his own. Then he says *we* in the sense of the learned world."

Peirce: CP 7.315 Fn 5 Para 3/3 p 195

Peirce's fourth method is discussed in our following paragraph.

Peirce: CP 7.326 Fn 6 p 200

†6 (Ed.) From "LOGIC, Chap. 4 (--- draft)" (see 313n1).

Peirce: CP 7.327 Fn 7 p 201

†7 (Ed.) From "LOGIC, Chap. 4. -- Of Reality (1st draft)," with an added quotation in 331n9. See 313n1.

Peirce: CP 7.328 Fn 8 p 201

†8 (Ed.) See Book III, Chapter 2, "Association," in the present volume.

Peirce: CP 7.331 Fn 9 p 203

†9 (Ed.) Investigation involves, besides sensation, "the production of new beliefs out of old ones according to logical laws. This process is the *logical process*, but by an extension of the meaning of a familiar word I call it also *inference*." From "*Of Reality*," the same manuscript quoted at 313n3.

Peirce: CP 7.336 Fn 10 p 207

†10 (Ed.) An untitled manuscript originally in one paragraph, with an added quotation in 336n11. See 313n1.

Peirce: CP 7.336 Fn 11 p 207 Cross-Ref:††

†11 (Ed.) "The final settled opinion is not any particular cognition, in such and such a mind, at such and such a time, although an individual opinion may chance to coincide with it. If an opinion coincides with the final settled opinion, it is because the general current of investigation will not affect it. The object of that individual opinion

is whatever is thought at that time. But if anything else than that one thing is thought, the object of that opinion changes and it thereby ceases to coincide with the object of the final opinion which does not change. The perversity or ignorance of mankind may make this thing or that to be held for true, for any number of generations, but it can not affect what would be the result of sufficient experience and reasoning. And this it is which is meant by the final settled opinion. This therefore is no particular opinion but is entirely independent of what you, I, or any number of men may think about it; and therefore it directly satisfies the definition of reality." From "**Logic**, Chap. 6th," March 10, 1873 (see 313n1).

Peirce: CP 7.340 Fn 12 p 209
†12 (Ed.) Cf. 5.403.

Peirce: CP 7.345 Fn 13 p 211
†13 (Ed.) Cf. 5.255.

Peirce: CP 7.346 Fn 14 p 212
†14 (Ed.) An untitled manuscript originally in one paragraph. The date is hardly legible, but is probably March 6, 1873. A manuscript dated March 8, 1873, seems to be an alternative draft, but in the opinion of the editor the manuscript printed here is superior to the later draft. See 313n1.

Peirce: CP 7.354 Fn 15 p 215
†15 (Ed.) "**Logic**, Chap. 5th," March 10, 1873, originally in one paragraph. See 313n1.

Peirce: CP 7.356 Fn 16 p 217
†16 Cf. 5.253.

Peirce: CP 7.358 Fn 17 p 218
†17 (Ed.) Chapter V, "***That the significance of thought lies in its reference to the future.***" A draft of Chapter IV, "***The Conception of Time essential in Logic,***" dated "1873 July 1," seems to belong with this Chapter V as parts of one series. See 313n1.

Peirce: CP 7.360 Fn 18 p 218
†18 (Ed.) Cf. 5.287ff.

Peirce: CP 7.360 Fn 19 p 219
†19 (Ed.) Cf. 8.33 (1871) and 5.400 (1878).

Peirce: CP 7.361 Fn 20 p 219
†20 (Ed.) The manuscript ends without a period.

Peirce: CP 7.362 Fn 1 Para 1/3 p 223
†1 (Ed.) Peirce uses the terms "psychognosy" and "physiognosy" for the psychical and physical sciences, respectively (cf. 1.242).

Peirce: CP 7.362 Fn 1 Para 2/3 p 223
This chapter is from Section 1, "Classification of the Sciences," of ***Minute Logic***, Chapter 2, "Prelogical Notions," including some alternative pages, Widener IB2-2, with a quotation added in 381n19. The manuscript has dates of early 1902 stamped in the margin of several sheets. All but one of Peirce's marginal sideheads are omitted, and some of his marginal notes have been treated as footnotes rather than as insertions in the text.

Peirce: CP 7.362 Fn 1 Para 3/3 p 223

1.203-283 are from the first part of this manuscript. Paragraph 374n10 follows shortly after 1.283. The rest of the present chapter follows considerably after 374n10; in the manuscript between 374n10 and 362, Peirce deals with the subdivisions of Physiognosy. Cf. 7.267n7.

Peirce: CP 7.362 Fn 2 p 223

†2 (Ed.) Cf. 1.269-272.

Peirce: CP 7.363 Fn 3 p 224

†3 (Ed.) 1.269-270.

Peirce: CP 7.363 Fn 4 p 224

†4 (Ed.) No such chapter is among the manuscripts. A list of all the chapters for which there are manuscripts is given at [Bibliography] G-c.1902-2.

Peirce: CP 7.363 Fn 5 p 225

†5 (Ed.) The text continues at paragraph 366. Paragraph 364 is from some alternative sheets, and 365 is from some other alternative sheets (see 362n1).

Peirce: CP 7.366 Fn 6 p 226

†6 Not being a German university professor, his arguments were not fairly considered by German university professors, and were treated with contempt by those who pin their faith upon the philosophical fashions that happen for the moment to prevail in those universities. [Peirce's marginal insert.]

Peirce: CP 7.367 Fn 7 p 227

†7 Professor Baldwin in the preface of his Dictionary distinctly places himself upon the platform that philosophical and scientific questions ought to be settled by majorities. 'We are many; you are one,' he says. But in the history of science majorities short of unanimity have more often been wrong than right. Majorities do not form their opinions rationally. [Peirce's marginal insert.]

Peirce: CP 7.371 Fn 8 p 231

†8 (Ed.) Cf. 1276, 8.122n19, and 8.274.

Peirce: CP 7.372 Fn 9 p 232

†9 (Ed.) No such chapter is among the manuscripts. Cf. 363n4.

Peirce: CP 7.374 Fn 10 Para 1/3 p 233

†10 (Ed.) "As the different Orders are distinguished by the different conceptions which govern them, so it has seemed to me that the different Families are distinguished by the differences of their methods of investigation, when these different methods are applied to different problems. For if there are two different methods, both of them sound and scientific, which are applicable to the same problem, they ought to be employed jointly; or if not, they at any rate are too closely associated to make different families of science. Nothing, for example, can be in stronger contrast than the method of investigating ancient history from monuments and from documents. But the only proper course is to use both methods conjointly. It is true that one man may not be strong enough to work in both ways to advantage; but still he will thoroughly know that his own work is only a result of the division of labour and that it has to be joined to another man's work by a third workman, before anything can be settled. On the other hand, a mere difference of problems, where the methods of investigation are identical does not constitute a division that can rank as a

division between Families of science.

Peirce: CP 7.374 Fn 10 Para 2/3 p 233

"Different Genera of the same Family of science are studies which have precisely the same general character, but of which each is strongly distinguished by some marked feature. This is a sufficiently vague statement, not to have the effect of imposing an artificial classification, while it is clear enough to be of material service in enabling us to determine whether a given subdivision is a division into Families or into Genera.

Peirce: CP 7.374 Fn 10 Para 3/3 p 233

"Finally, the different Species of one Genus of science, are studies which, though morphologically identical in all their features, even similar in their proportions, and undistinguished by any leading peculiarities, lie nevertheless not so exactly in one path that the man who is everywhere prepared for the pursuit of the one will be quite *ipso facto* expert in the other, without having anything to learn of new kinds of precaution; in sharpening his attention to unfamiliar observations, in manipulating an instrument that has been strange to him, or in putting a familiar one to unfamiliar uses." From the manuscript, following shortly after 1.283 (see 362n1).

Peirce: CP 7.376 Fn 11 p 236

†11 Take, as an example, Cattell's observations of the motions of a reader's eyeball. [Peirce's marginal insert.]

Peirce: CP 7.377 Fn 12 p 240

†12 (Ed.) See the following chapter, "Association."

Peirce: CP 7.377 Fn 13 p 240

†13 (Ed.) See Chapter 5, "Telepathy and Perception," in the present book.

Peirce: CP 7.378 Fn 14 p 240

†14 (Ed.) Cf. 1.271.

Peirce: CP 7.378 Fn 15 p 241

†15 (Ed.) Cf. Chapter 3, "The Logic of Drawing History from Ancient Documents," Book II, present volume.

Peirce: CP 7.378 Fn 16 p 242

†16 Of course, if they are not so, they are not what the Darwinian means by "adaptive." For example, Weismann says that death is an adaptive character.

Peirce: CP 7.379 Fn 17 p 243

†17 I can tell by the expression of face the state of mind of my horse just as unmistakably as I can that of my dog or my wife.

Peirce: CP 7.380 Fn 18 p 244

†18 This is the marvel and admirable in it; and this essentially supposes a generous portion of capacity for blundering. [Peirce's marginal insert.]

Peirce: CP 7.381 Fn 19 p 244 Cross-Ref:††

†19 (Ed.) "Descriptive Definition of a Human Instinct, as the term will here be used. An animal instinct is a natural disposition, or inborn determination of the individual's Nature (his 'nature' being that within him which causes his behaviour to be such as it is), manifested by a certain unity of quasi-purpose in his behaviour. In man, at least, this behaviour is always conscious, and not purely spasmodic. More

than that, unless he is under some extraordinary stress, the behaviour is always partially controlled by the deliberate exercise of imagination and reflexion; so much so that to the man himself his action appears to be entirely rational, so far is it from being merely sensori-motor. General analogy and many special phenomena warrant the presumption that the same thing is true of the lower animals, though they are undoubtedly far less reflective than men. Yet the adaptation of the behaviour to its quasi-purpose in some definite part overleaps all control So then the three essential characters of instinctive conduct are that it is conscious, is determined to a quasi-purpose, and that in definite respects it escapes all control." From "Of the Classification of the Sciences. Second Paper. Of the Practical Sciences," Widener II, c.1902 (see 7.49n1).

Peirce: CP 7.382 Fn 20 p 245

†20 (Ed.) The marginal sidehead "Science is but a Development of Instinct" appears opposite this paragraph.

Peirce: CP 7.384 Fn 21 Para 1/2 p 247

†21 As I write, the *Monist* XII.321 brings me the account of such a thing dating from the XVIth century before Christ, or earlier. Several sciences are known to have been earlier than that; but they were probably looked upon as Useful Arts. This seems purely speculative.

Peirce: CP 7.384 Fn 21 Para 2/2 p 247

(Ed.) The reference is to James H. Breasted, "The First Philosopher," *The Monist* 12(April, 1902)321-336.

Peirce: CP 7.385 Fn 22 p 247

†22 (Ed.) We omit a long section on linguistics and anthropology. The following is from an alternate page of this part of the manuscript (see 362n1): "The Indo-European languages are singular in having the common noun distinctly and fully developed as a separate part of speech, and by more or less development even of abstract nouns. I do not mean to say that the common noun is not fully developed in any other language; but only that such a phenomenon is exceptional in every other great family of speech. This requires and evidences considerable power of thought on the part of those who use these languages. With the exercise of a little ingenuity it is possible to express anything in these languages, provided no higher relations than dyadic ones enter. Only very simple propositions can be expressed involving higher relations; and those whose mental education is limited by the powers of these languages are unable to grasp the meaning of a complex triple relation; . . ."

Peirce: CP 7.387 Fn 23 p 248

†23 (Ed.) Cf. 7.256-266.

Peirce: CP 7.387 Fn 24 p 248

†24 (Ed.) The manuscript continues, "In Descriptive Psychognosy we have recognized two Suborders, the one relating to Situations, the other to Events." Cf. 1.272. Peirce then discusses bibliography, books of travels, numismatics, metrology, history, etc.

Peirce: CP 7.388 Fn 1 p 249

†1 (Ed.) Sections 1-7 are "Introduction, Association of Ideas," c.1893, from *Grand Logic*, Widener IB2-1, with the omission of Art. 3, a long section devoted to the history of the doctrine of association, of which only a few sentences are printed (417n21), and with an added quotation in 392n7. Sections 1 and 2 are Peirce's Art. 1

and 2, respectively, and sections 3 through 7 are Peirce's Art. 4 through Art. 8, respectively. The titles of all these sections, except that of Section 5, are taken from a partial table of contents in Widener IB2-1, with some changes in capitalization.

Peirce: CP 7.389 Fn 2 p 250

†2 It is now more often called *reproduction*, which is the German term. I prefer to write English; and certainly no reader will suppose that I am speaking of suggestion in its hypnotic connection.

Peirce: CP 7.389 Fn 3 p 250

†3 (Ed.) Cf. 6.60.

Peirce: CP 7.389 Fn 4 p 250

†4 (Ed.) *A Treatise of Human Nature*. In the Everyman's Library edition, No. 548, 1939, edited by Ernest Rhys, this passage appears in Vol. I, page 19.

Peirce: CP 7.391 Fn 5 p 251

†5 (Ed.) Cf. 1.383.

Peirce: CP 7.392 Fn 6 p 252

†6 (Ed.) Cf. 4.157.

Peirce: CP 7.392 Fn 7 p 252 Cross-Ref:††

†7 (Ed.) "An 'occult property' is a property which is only brought to light by experiment. 'Occult Science' means, therefore, precisely experimental science. The reason these properties were called occult was that they could not be deduced after the manner of Aristotle from the prime qualities hot and cold, moist and dry." A footnote, p. 16, from Peirce's Prospectus of *The Treatise of Petrus Peregrinus on the Lodestone*, [Bibliography] G-c.1893-4.

Peirce: CP 7.392 Fn 8 p 252

†8 See Mr. A.B. Kempe's Memoir upon Mathematical Form.

Peirce: CP 7.392 Fn 9 p 253

†9 (Ed.) Art. 3, on the history of the doctrine of association, is deleted here (see 388n1).

Peirce: CP 7.395 Fn 10 p 253

†10 Hartmann's account of association is well worth reading.

Peirce: CP 7.395 Fn 11 p 253

†11 *Unconscious Memory*.

Peirce: CP 7.396 Fn 12 p 254

†12 They are written out, but as for seeing the light, even if they were printed, that does in no wise depend on their merit. Men do not want to know the truth and they ought to have their way. (Ed.) But cf. 7.546, 7.215.

Peirce: CP 7.396 Fn 13 p 254

†13 The distinction is virtually made by James Mill. J.S. Mill, and I think Bain too, mistakenly calls subjective intensity *emotional* intensity.

Peirce: CP 7.396 Fn 14 p 255

†14 (Ed.) See Chapter 2, "Small Differences of Sensation," of Book I of the present volume.

Peirce: CP 7.400 Fn 15 p 257

†15 (Ed.) In the following paragraphs, Peirce shows the inter-relationship between the points he is making by numbering them in a special way. We have placed these numbers in parentheses to separate them from the paragraph numbers. Note that (22) is the second point under (2), (32) is the second point subordinate to (3), (321) the first sub-item under (32), etc.

Peirce: CP 7.405 Fn 16 p 258

†16 (Ed.) In the manuscript paragraph 404 was inserted between the lines after paragraph 405. We have adopted the present order so that there will be references to ten numbered points, (11), (12), (21), (22), (23), (31), (32), (33), (34), and (41), preceding paragraph 405.

Peirce: CP 7.407 Fn 17 p 259

†17 (Ed.) In the deleted Art. 3; see 392n9.

Peirce: CP 7.407 Fn 18 Para 1/2 p 260

†18 Professor James seems to think it to the purpose to remark that "the similarity of two things does not exist till both are there." That may be; it is so, in a sense. But how he infers that "it is meaningless to talk of it as an agent of production" I cannot see. Nobody means that the *appearance* of resemblance is an agent but that what there is in the depths of the soul that makes the appearance also makes the association.

Peirce: CP 7.407 Fn 18 Para 2/2 p 260

(Ed.) The quotation from James in the text is from *The Principles of Psychology*, Vol. 1, Henry Holt and Company, New York, 1890, p. 579; the quotation in this footnote is from the same volume, p. 591.

Peirce: CP 7.408 Fn 19 p 261

†19 James wants us to say *things*. I reply that *ideas* were always meant as *objects*, direct objects, not matters of psychology, by those who talked of them. When he says *things* he cannot mean the real external things; for they are beyond the power of thought. He can only mean the perceived *objects*, which are precisely what is meant by "ideas." What is *perceived* is an *idea*, in contradistinction to a raw sensation.

Peirce: CP 7.410 Fn 20 p 261

†20 (Ed.) Chapters XVIII (2.645-660) and XIX of the *Grand Logic* were to be on probability and induction, respectively; no manuscript of Chapter XIX has been found.

Peirce: CP 7.417 Fn 21 p 264

†21 (Ed.) In the discussion of Hartley in the omitted Art. 3, Peirce says: "Proposition 3: sensations remain in the soul for a brief interval of time after the object is removed. This seems like a genuine psychological observation; and the acuteness of the physician is certainly shown in giving it this foremost place. But the proof offered is perfectly futile."

Peirce: CP 7.419 Fn 22 p 264

†22 (Ed.) In *Discussions on Philosophy and Literature, Education and University Reform*, Harper and Brothers, New York, 1861, p. 53, Sir William Hamilton says: "*We know*; and *We know that we know*: -- these propositions,

logically distinct, are *really* identical; each implies the other The attempt to analyze the cognition *I know*, and the cognition *I know that I know*, into the separate energies of distinct faculties, is therefore vain."

Peirce: CP 7.420 Fn 23 p 265
†23 (Ed.) Cf. 5.213ff.

Peirce: CP 7.427 Fn 24 p 268
†24 (Ed.) The editor suggests that the two triangles in perspective constitute a figure distinct from the figure of the ten dots; and that the association is between ten dots in the original figure, and ten rays in the second figure.

Peirce: CP 7.451 Fn 25 Para 1/2 p 275
†25 (Ed.) From *Qualitative Logic*, a partially completed book intended for young people, Widener IB2-9, undated. Paragraphs 451-457 are a draft of Chapter I, "The Association of Ideas." Paragraphs 458-462 are a draft of Chapter II, "The Simple Consequence."

Peirce: CP 7.451 Fn 25 Para 2/2 p 275
The reader's attention is called to the similarity between 7.454-457 of this section and 7.445-450 which come from *Grand Logic*. Paragraphs 7.457 and 7.448-450 are nearly identical. This similarity provides some information as to when *Qualitative Logic* was written.

Peirce: CP 7.453 Fn 26 p 276
†26 (Ed.) Cf. 1.626ff.

Peirce: CP 7.462 Fn 27 p 279
†27 Absolute doubt is also impossible.

Peirce: CP 7.462 Fn 28 p 279
†28 (Ed.) The manuscript then continues with Chapter III, "The Modus Ponens"; Chapter IV, "The Traditional Syllogistic"; etc.

Peirce: CP 7.463 Fn 29 p 280
†29 (Ed.) From Chapter VI of *Grand Logic*, Widener IB2-1, coming after 4.52. This is dated 1893 on the basis of 4.21n*.

Peirce: CP 7.466 Fn 30 p 282
†30 (Ed.) Cf. 4.121ff.

Peirce: CP 7.468 Fn 1 p 284
†1 (Ed.) Sections 1-8 are "Habit," Widener IB3, c.1898, with a quotation added in 494n9. The manuscript, "Habit," is probably a version of Lecture 7 of the Cambridge series, [Bibliography] G-1898-1. The manuscript from which 494n9 is taken is probably a version of Lecture 2 of this series. Section 9 is from Lecture 4 of the Cambridge series; cf. 518n16.

Peirce: CP 7.470 Fn 2 p 286
†2 (Ed.) Cf. 6.240.

Peirce: CP 7.471 Fn 3 p 287
†3 (Ed.) See also 6.23, 6.72, and [Bibliography] N-1890-3.

Peirce: CP 7.478 Fn 4 p 289
†4 (Ed.) Benjamin Peirce.

Peirce: CP 7.480 Fn 5 p 290
†5 (Ed.) Cf. 1.135.

Peirce: CP 7.488 Fn 6 p 294
†6 (Ed.) Cf. 6.82ff. Peirce's categories of First, Second, and Third are discussed especially in [CP] I.

Peirce: CP 7.488 Fn 7 p 294
†7 (Ed.) See the discussion of indices in [CP] II and elsewhere.

Peirce: CP 7.488 Fn 8 p 294
†8 (Ed.) Cf. 6.6, 6.211-212.

Peirce: CP 7.494 Fn 9 Para 1/5 p 296
†9 (Ed.) This is probably a reference to the following rules (cf. 2.226): "Rule I. Assign to every scientific conception a scientific name of its own, preferably a new word rather than one already appropriated to an unscientific and dubious conception.

Peirce: CP 7.494 Fn 9 Para 2/5 p 296
"That was the practice adopted by the scholastic doctors, how advantageously every student will testify. The *renaissance*, on the other hand, condemned the scholastic terms as not being Ciceronian, with the result of making *renaissance* philosophy as soft and savorless as a sage pudding. There is a rule of good writing higher than Ciceronian purity, that of expressing your thought, both accurately and *concisely*. More than a rule of good writing, this is a fundamental condition of scientific thinking; for man cannot think *at all* without formulas, nor think *powerfully* without concise formulas

Peirce: CP 7.494 Fn 9 Para 3/5 p 296
"Rule II. The author of a scientific conception has the first right to name it; and his name ought to be accepted, unless there are grave substantial objections to it. But if *he* fails to give it a scientific name, somebody else must do so; and in that case the earliest good scientific name shall be employed

Peirce: CP 7.494 Fn 9 Para 4/5 p 296
"Rule III. After a scientific conception has once received a suitable name, let it not be called by any other scientific name, old or new

Peirce: CP 7.494 Fn 9 Para 5/5 p 296
"Rule IV. As far as practicable, let the terms of philosophy be modelled after those of scholasticism" From "Detached Ideas; Induction, Deduction, and Hypothesis," Widener IB2-10, c.1898, probably a draft of one of the lectures (possibly the second) on "Detached Ideas on Vitally Important Topics" of the Cambridge series (see 468n1).

Peirce: CP 7.497 Fn 10 p 300
†10 (Ed.) See Chapter 2, "On Small Differences of Sensation," in Book I of the present volume.

Peirce: CP 7.498 Fn 11 p 300
†11 (Ed.) See also the preceding chapter, "Association."

Peirce: CP 7.511 Fn 12 p 307
†12 (Ed.) The dots indicate a blank in the manuscript, where presumably

Peirce meant to write in the value of the gravitational constant. This value is approximately 6.7×10^{-8} in the centimeter-gram-second system of units.

Peirce: CP 7.513 Fn 13 p 308

†13 (Ed.) Cf. [Bibliography] G-1891-1.

Peirce: CP 7.514 Fn 14 p 309

†14 (Ed.) Presumably Lecture 8, "The Logic of Continuity." There is a manuscript, "The Logic of Continuity," in Widener IB3 which is very probably the lecture referred to; 6.185-213 are from this manuscript.

Peirce: CP 7.517 Fn 15 p 310

†15 (Ed.) See the preceding footnote.

Peirce: CP 7.518 Fn 16 p 310

†16 (Ed.) This final section is the four missing manuscript pages referred to at 6.81n* (p. 60); these pages are now with the rest of the manuscript at Widener IB2-10. They are published here to fill the gap in "Causation and Force," 6.66-87, Lecture 4 of the Cambridge series, [Bibliography] G-1898-1. This lecture deals with some of the topics discussed in "Habit," Sections 1-8 above.

Peirce: CP 7.524 Fn 1 p 313

†1 (Ed.) From Section 2, "Some Logical Prolegomena," the final section of a manuscript, "On Topical Geometry, in General," Widener IA-2, undated, with added quotations in 534n4 and 535n6. A partial draft and a complete draft of Section 2 have a common first page. Paragraphs 524-529 are from the partial draft; paragraphs 530-538 are the last part of the complete draft.

Peirce: CP 7.528 Fn 2 p 315

†2 (Ed.) These are the psychological versions of Peirce's categories, First, Second, and Third, which are treated in general in [CP] I, Book III.

Peirce: CP 7.531 Fn 3 p 317

†3 But I do not mean to say that bare striving usually does any more good in these cases than in the case of such a physical difficulty as turning a key that does not fit very well in its lock. If a moderate effort does not suffice, some contrivance has to be employed.

Peirce: CP 7.534 Fn 4 p 318 Cross-Ref:††

†4 (Ed.) "When the idea of space forms itself in our minds, or being inborn, connects itself with sensations, those things come to be regarded as near together which act strongly on one another, and are intimately related in regard to forces; while those things come to be regarded as remote which, as far as those forces of which we have any primitive experience are concerned, have little to do with one another. Hence, when gravitation was found to act at a distance, and even at vast distances, men were astounded; they thought it could not be. It seemed somehow to involve absurdity. The absurdity which was obscurely felt was that the idea [of] nearness was the idea of close connection; and therefore men felt that immediate connection supposed confinity. But that instinctive idea has been corrected. Any particle may be regarded as extending throughout space. It is where it acts. But it acts extremely little except in a very little space; and its place is peculiarly where it acts the most." From a fragment in Widener IA-8, undated.

Peirce: CP 7.535 Fn 5 p 319

†5 (Ed.) Cf. 3.567, 4.639-640.

Peirce: CP 7.535 Fn 6 Para 1/2 p 319 Cross-Ref:††

†6 (Ed.) "I begin by defining a *part* of any *whole*, in a sense of the [term] much wider [than] any in current use, though it is not obsolete in the vocabulary of philosophy. In this broadest sense, [it] is anything that is (1) other than its whole, and (2) . . . such that if the whole were really to be, no matter what else might be true, then the part must under all conceivable circumstances itself really be, in the same 'universe of discourse,' though by no means necessarily in the same one of those three Universes with which experience makes us all more or less acquainted. Thus, light is a *part* of vision

Peirce: CP 7.535 Fn 6 Para 2/2 p 319 Cross-Ref:††

"A perfect continuum belongs to the genus, of a whole all whose parts without any exception whatsoever conform to one general law to which same law conform likewise all the parts of each single part. *Continuity* is thus a special kind of *generality*, or conformity to one Idea. More specifically, it is a *homogeneity*, or generality among all of a certain kind of parts of one whole. Still more specifically, the characters which are the same in all the parts are a certain kind of relationship of each part to all the coördinate parts; that is, it is a *regularity*. The step of specification which seems called for next, as appropriate to our purpose of defining, or logically analyzing the Idea of continuity, is that of asking ourselves what kind [of] relationship between parts it is that constitutes the regularity a continuity; and the first, and therefore doubtless the best answer for our purpose, not as the ultimate answer, but as the proximate one, is that it is the relation or relations of *contiguity*; for continuity is unbrokenness (whatever that may be,) and this seems to imply a *passage* from one part to a contiguous part. What is this 'passage'? This passage seems to be an act of turning the attention from one part to another part; in short an actual event in the mind. This seems decidedly unfortunate, since an event can only take place in Time, and Time is a continuum; so that the prospect is that we shall rise from our analysis with a definition of continuity in general in terms of a special continuity. However, it is possible that this objection will disappear as we proceed." From "Supplement. 1908 May 24," Widener IA-3, an addendum alternative to 4.642 ([Bibliography] G-1908-1b).

Peirce: CP 7.539 Fn 7 p 325

†7 (Ed.) An untitled manuscript, Widener IB1-2, undated. The quotations in 540n8 and 541n9 are from what appears to be an alternative partial draft, Widener IC1-a,b, undated. In this alternative draft it is stated that the paper was originally written to be read to a group that met Sunday afternoons. It is likely that these manuscripts were written about 1900; see paragraphs 539, 540 and 545 and [Bibliography] G-1901-5, but note that Kant died in 1804 and not "in the last year of the last century," as Peirce says in paragraph 540.

Peirce: CP 7.540 Fn 8 p 325

†8 (Ed.) "*Direct* in philosophical language without anything intervening. Thus this house directly abuts upon the street. I am talking to you directly. *Immediate* involves the same idea carried further. It denies every kind of separation by a boundary, by a difference of place or time. What is immediately in consciousness is what consciousness is made of." A footnote from the alternative draft (see 539n7).

Peirce: CP 7.541 Fn 9 p 328

†9 (Ed.) "No sharp line of demarcation can be drawn between different

integral states of mind; certainly not between such states as feeling, knowing, and willing. It is plain that we are actively knowing in all our waking minutes, and actually feeling, too. If we are not always willing, we are, at least, at all times consciously reacting against the outer world. Strumpell's celebrated experiment, fully confirmed by others, shows that as soon as outward stimuli are entirely removed, the person falls asleep." From the alternative draft (see 539n7).

Peirce: CP 7.545 Fn 10 p 332

†10 (Ed.) Peirce first had "One is an *objective* intensity" and then substituted "One is the intensity of the feeling itself." Cf. 555.

Peirce: CP 7.546 Fn 11 p 332

†11 (Ed.) Cf. 7.215.

Peirce: CP 7.546 Fn 12 p 334

†12 (Ed.) See Chapter 2, "Small Differences of Sensation," Book I, present volume.

Peirce: CP 7.547 Fn 13 p 335

†13 (Ed.) Cf. 553-554.

Peirce: CP 7.548 Fn 14 p 335

†14 (Ed.) In 544.

Peirce: CP 7.548 Fn 15 p 336

†15 (Ed.) See Chapter 2, "Association," in the present book.

Peirce: CP 7.551 Fn 16 p 337

†16 (Ed.) See further 7.276-278.

Peirce: CP 7.553 Fn 17 Para 1/2 p 337

†17 (Ed.) Paragraph 553 is an undated fragment; paragraph 554 is from an alternative draft. Both are in Widener IC1-a,b.

Peirce: CP 7.553 Fn 17 Para 2/2 p 337

Paragraphs 555-558 are from Chapter I, "Of Reasoning in General," of the "Short Logic," Widener IB2-10. This manuscript is dated c.1893 in [CP] II. Paragraph 555 follows immediately after the last sentence of 2.443. The part of the manuscript from which 555-558 and 2.444 are taken is not continuous; paragraph 2.444 is from manuscript pages beyond 555-558, but the last paragraph of 2.444 is similar to 558.

Peirce: CP 7.553 Fn 18 p 338

†18 (Ed.) Cf. 547.

Peirce: CP 7.553 Fn 19 p 338

†19 (Ed.) "An idea is nothing but a portion of consciousness having in itself no definite boundaries, except so far as it may be of a different quality from contiguous ideas." Peirce's marginal comment.

Peirce: CP 7.555 Fn 20 p 340

†20 (Ed.) Cf. 546.

Peirce: CP 7.555 Fn 21 p 340

†21 (Ed.) No such explanation is given in the extant manuscript.

Peirce: CP 7.555 Fn 22 p 341

†22 It may be remarked that this is a very temporary condition, and shortly after the interesting idea will be found to have a lower subjective intensity than it would have had if it had not been interesting.

Peirce: CP 7.559 Fn 23 p 341

†23 (Ed.) "What is the Use of Consciousness?" Chapter IV, c.1893, of *Grand Logic*, Widener IB2-1.

Peirce: CP 7.559 Fn 24 p 341

†24 (Ed.) "The Materialistic Aspect of Reasoning," 6.278-286.

Peirce: CP 7.561 Fn 25 p 342

†25 Take, for instance, that superlatively cunning defense of common sense, the doctrine of immediate perception, -- a doctrine so subtle that it has eluded the grasp of many a fine logician, -- and what is it, after all, but a confession that to see and to be seen are one and the same fact.

Peirce: CP 7.565 Fn 26 p 343

†26 (Ed.) "Immortality in the Light of Synechism," Widener IB3. "For the Open Court" is written on the manuscript. It is dated c.1892 on the basis of the bibliographic references in the article.

Peirce: CP 7.565 Fn 27 p 343

†27 (Ed.) Synechism is treated in detail in [CP] VI, Part B of Book I.

Peirce: CP 7.565 Fn 28 p 343

†28 The Greek word means continuity of parts brought about by surgery.

Peirce: CP 7.566 Fn 29 p 343

†29 (Ed.) [Bibliography] G-1891-1c (1892). This paper gives a definition of "synechism," 6.103.

Peirce: CP 7.568 Fn 30 p 344

†30 (Ed.) Cf. Chapter 5, "On Non-Euclidean Geometry," [CP] VIII, Book I.

Peirce: CP 7.569 Fn 31 p 344

†31 (Ed.) Cf. 1.171.

Peirce: CP 7.574 Fn 32 p 346

†32 (Ed.) Edward Stanton Huntington, *Dreams of the Dead*, by Edward Stanton (pseud.), Boston, 1892. Cf. [Bibliography] N-1892-14.

Peirce: CP 7.575 Fn 33 p 346

†33 (Ed.) Gustav Freytag, *Die Verlorene Handschrift*, Leipzig, 1869.

Peirce: CP 7.579 Fn 34 p 347

†34 (Ed.) From Lecture XI of an incomplete series of lectures in Widener IB2-10. The editor has been unable to make a positive identification of these lectures, but the internal references and the general tone of the writing indicate that they are probably the manuscripts for the Lowell Institute lectures of 1866-67 ([Bibliography] G-1866-2a). On this basis Lecture XI is dated c.1867. In Widener IB2-10 these lectures are mixed with the manuscripts of a different series, which are probably those of the Harvard series of 1865-66, [Bibliography] G-1864-3.

Peirce: CP 7.580 Fn 35 p 348

†35 (Ed.) Lecture X. Only fragments of it have been found.

Peirce: CP 7.580 Fn 36 p 348
†36 (Ed.) Cf. 2.407ff.

Peirce: CP 7.580 Fn 37 p 348
†37 (Ed.) The following was pencilled in the margin with a line to indicate that it was to be inserted here:

"Most ignorant of what he's most assured
His glassy essence."

Peirce: CP 7.580 Fn 38 p 349
†38 (Ed.) Peirce here inserted "Upon the diversities of theories of the soul there are some pretty lines by Sir John Davies in his poem on Psychology" and referred to Sir William Hamilton's edition of Thomas Reid's *Works*, p. 203.

Peirce: CP 7.581 Fn 39 p 349
†39 (Ed.) Induction and hypothesis are discussed in Book II of the present volume and at 2.508ff. (1867).

Peirce: CP 7.581 Fn 40 p 350
†40 (Ed.) Cf. 7.131-138, which are from Lecture IV of the same series (see 579n34).

Peirce: CP 7.581 Fn 41 p 351
†41 (Ed.) Cf. 5.339.

Peirce: CP 7.585 Fn 42 p 351
†42 (Ed.) Compare the following paragraphs with 5.313ff. (1868).

Peirce: CP 7.591 Fn 43 p 356
†43 (Ed.) Cf. [Bibliography] G-1868-2a (5.213-263).

Peirce: CP 7.597 Fn 1 Para 1/3 p 359 Cross-Ref:††
†1 (Ed.) "Telepathy," including some alternative pages, Widener IC1-a,b, with deletions, and with an added quotation in 597n3. This manuscript is dated 1903 on the basis of the dates given in the next footnote, references to a manuscript on telepathy in the Peirce-James correspondence (James collection, Houghton Library), what appear to be James's handwritten comments in the margin of the first part of the manuscript, and a crossed-out date in the last section of the manuscript. This was a first draft from which Peirce intended to extract a magazine article.

Peirce: CP 7.597 Fn 1 Para 2/3 p 359 Cross-Ref:††
Peirce divided the manuscript into six sections (with no titles); the sections of the present chapter are the same as Peirce's, but the titles have been supplied by the editor.

Peirce: CP 7.597 Fn 1 Para 3/3 p 359 Cross-Ref:††
Cf. 6.548-587 and [Bibliography] G-1887-3 for other writings by Peirce on the subject of psychical research.

Peirce: CP 7.597 Fn 2 p 359
†2 (Ed.) John Trowbridge, "Telepathy," *The Nation* 76 (16 April

1903)308309; and William James, "Telepathy Once More," *ibid.*, (23 April 1903) 330.

Peirce: CP 7.597 Fn 3 p 360

†3 (Ed.) [Bibliography] G-1887-3, "Criticism on 'Phantasms of the Living'. . ."
In a fragment in Houghton Library, Peirce says: "Phenomena for which psychical researchers invoke telepathy can be explained by unconscious mental action, after severe scrutiny of testimony, and with aid from the doctrine of chances." Cf. [Bibliography] G-1887-3, p. 194. The fragment concludes, "Eight years ago on the appearance of the laborious work, Phantasms of the Living, I examined the question with care, pronounced it very doubtful, but provisionally rejected the whole theory. Reviewing the state of the question, this year, I find no decided change in either direction." This internal reference dates the fragment as c.1895.

Peirce: CP 7.601 Fn 4 p 362

†4 (Ed.) Cf. 5.354ff., 2.654ff.

Peirce: CP 7.602 Fn 5 p 363

†5 It would be different if the theory of Hobbes, LaRoche foucauld, and other thinkers in the boyhood of modern philosophy, were true, and man could not act otherwise than selfishly. But this theory which rests on a logical fallacy is supported by no fact and refuted by many.

Peirce: CP 7.603 Fn 6 p 363

†6 The word "veridical" was first made a term of art by Mr. Myers in the sense of coinciding with the truth, whether fortuitously or causedly. It is contrary to the ethics of terminology to change the meaning of a term of art without necessity. Its author has no more right to do so than anybody else. Yet in Baldwin's Dictionary, where accuracy of definition ought to have been the first consideration, Mrs. Sidgwick limits the term to that which is determined by a general cause to be true, with no hint that it had ever been used in any other sense. Looseness of speech and looseness of thought are wife and husband. I shall speak of the 'simply veridical' and the 'determinedly veridical,' in the wider and narrower senses.

Peirce: CP 7.617 Fn 7 p 369

†7 (Ed.) See [CP] V, *Pragmatism and Pragmaticism*.

Peirce: CP 7.625 Fn 8 p 371

†8 (Ed.) The psychological versions of Peirce's categories of First, Second, and Third are discussed in 7.524-538, and the categories in general are treated in [CP] I.

Peirce: CP 7.629 Fn 9 p 374

†9 Formed from *percipio* on the analogy of *praecipuum* from *praecipio*. I am quite sure that it would be well if philosophers were bolder in forming new words instead of giving old ones so many meanings. What if we were to use words ending in *-cept* for different kinds of acquisition of cognition? There would be of good Latin words accept, antecept, decept, except, incept, intercept, ocept, precept, suscept, besides many others quite supportable.

Peirce: CP 7.630 Fn 10 p 374

†10 (Ed.) Cf. 7.537.

Peirce: CP 7.630 Fn 11 p 375

†11 There is no objection to saying that 'The chair appears yellow' means 'The chair appears *to me* yellow'; but the reference to the future self is more pertinent.

Peirce: CP 7.636 Fn 12 p 376

†12 (Ed.) "It may be objected that a perceptual judgment is not so utterly beyond all control or check as I say; since it may be revised. I may negligently think 'this chair appears scarlet,' when if I had looked more carefully, I should have said 'it appears vermilion.' I reply that no doubt mistakes can arise from inadvertency; and possibly, in order to take due account of that phenomenon, some complication of my statement would be required. But it cannot be that, on this account, it is fundamentally wrong. A perceptual judgment can only refer to a single percept which can never re-exist; and if I judge that it appears red when it did not appear red, it must, at least, be acknowledged that it *appeared to appear* red. I do not think it worth while to follow out the objection further." From alternate pages of the manuscript (see 597n1).

Peirce: CP 7.638 Fn 13 p 377

†13 (Ed.) Cf. Peirce's doctrine of synechism, 7.565ff.

Peirce: CP 7.647 Fn 14 p 381

†14 It was discovered by H.G.F. Schroeder in 1858. Poggendorff Annalen CV, 298.

Peirce: CP 7.648 Fn 15 p 382

†15 I myself was severely taken to task, sat upon, squelched, marked down to my real insignificance, by an eminent French logician, for presuming to speak disparagingly of so sublime a work of genius as the algebra of dual relatives, this algebra happening to be a thing of my own invention.

Peirce: CP 7.649 Fn 16 p 383

†16 (Ed.) Cf. 5.205, 6.109-110.

Peirce: CP 7.651 Fn 17 p 383

†17 In the *Monist* for July, 1892, I followed him in this misunderstanding, my notion of continuity at that date being far from clear. (Ed.) Cf. 6.120ff.

Peirce: CP 7.651 Fn 18 p 383

†18 (Ed.) The editor has deleted a long passage on continuity, which presents material already covered in 6.120ff. and 6.174ff.

Peirce: CP 7.655 Fn 19 p 384

†19 (Ed.) Cf. 3.567, 4.639ff.

Peirce: CP 7.655 Fn 20 p 384

†20 (Ed.) That is, real numbers.

Peirce: CP 7.659 Fn 21 p 386

†21 (Ed.) Cf. 8.12ff.

Peirce: CP 7.665 Fn 22 p 389

†22 (Ed.) Peirce's observational theory of mathematics is discussed also in various places in [CP] IV.

Peirce: CP 7.672 Fn 23 p 391

†23 (Ed.) See Book II, "Scientific Method," in the present volume.

Peirce: CP 7.688 Fn 24 p 397

†24 (Ed.) The manuscript breaks off shortly after this point.

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COLLECTED PAPERS OF CHARLES SANDERS PEIRCE

VOLUME VIII
REVIEWS, CORRESPONDENCE, AND BIBLIOGRAPHY
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Peirce: CP 8 Preface p v
PREFACE

When the first volume of the *Collected Papers* of Charles Peirce was published in 1931, its Introduction predicted some ten volumes in the series, those beyond the sixth being expected to contain Peirce's "writings on physics and psychology, as well as his reviews, letters, and biography." Footnotes to the subsequent text even cited some of those last volumes. Nevertheless, it was only of the six volumes that the Introduction spoke in the present tense, and these, under the editorship of Drs. Charles Hartshorne and Paul Weiss, appeared as promised, ending with Volume VI in 1935. Selection, preparation, and publication of further material was at that time impractical, and for the next twenty years the remaining papers in Harvard's custody were accessible only to such scholars as could consult them in Cambridge. In 1954, however, the Harvard Department of Philosophy was able to

renew the enterprise. The Rockefeller Foundation granted to the University a subvention for the costs of further editing, and the Department was fortunate to enlist Professor Arthur W. Burks, of the University of Michigan, to perform that peculiarly exacting task. The Department is glad of this occasion to acknowledge its debt, and that of the scholarly community in general, both to the Foundation and to Professor Burks, and to remind the reader that since the expense of the further actual book-making is defrayed from royalties from the earlier volumes, which were subsidized by gifts of the late Professor James H. Woods and anonymous friends, we are still their beneficiaries too.

Peirce: CP 8 Preface p v

The present publication comprises two volumes instead of the four of the old estimate. Mr. Burks's Introduction, which follows on page ix, indicates how nearly he is completing the original plan. A substantial addition is his extraordinarily searching bibliography, with its introduction comparing the sum of published material with what still remains only in manuscript.

Peirce: CP 8 Preface p v

Besides joining in the editor's acknowledgements of those who assisted him, the Department and Publishers thank the Houghton Library for permission to print previously unpublished correspondence between Peirce and James, Edith Davidson Harris and the Hoose Library of Philosophy at the University of Southern California for permission to print a letter from Peirce to William T. Harris, and the following for the use of copyrighted material as indicated: Helen G. Baldwin, definitions from James Mark Baldwin's *Dictionary of Philosophy and Psychology*, Vol. II; *The Nation*, several reviews; *Popular Science Monthly*, Peirce's review of Pearson's *Grammar of Science*; Whitlock's, Inc., parts of two letters reprinted from Irwin C. Lieb's *Charles S. Peirce's Letters to Lady Welby*; William James, correspondence from Peirce to his father, William James, and quotations from a Peirce manuscript, "Questions on William James's Principles of Psychology," previously printed in Ralph Barton Perry's *Thought and Character of William James*.

Harvard University

August, 1957

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INTRODUCTION

The first six volumes of the series, *Collected Papers of Charles Sanders Peirce*, included Peirce's main writings in general philosophy, logic (deductive, inductive, and symbolic), pragmatism, and metaphysics. The present two volumes are a continuation of this series. Volume VII is organized in three books containing papers on experimental science, scientific method, and philosophy of mind, respectively. Volume VIII contains selections from Peirce's reviews and correspondence and a bibliography of his published works.

Peirce: CP 8 Introduction p ix

Since Book I of Volume VII, "Experimental Science," is the only book in the series not on philosophy, its inclusion may require special comment. Much of Peirce's life was devoted to experimental science. In fact, the only permanent position Peirce held was with the United States Coast Survey, where he was employed to do research in astronomy and geodetics. Though this position allowed him considerable time for philosophy, it is nevertheless true that for more than half of his mature life his main responsibility was to conduct scientific investigations.†1 His efforts in the physical sciences resulted in a large number of published papers, several of considerable length, as well as his only published book, *Photometric Researches*, 1878.†2 Moreover, Peirce's experimental work had an important influence on his philosophy. His pragmatic theory of meaning is a generalization from scientific practice, his laboratory experience having influenced his discovery of that theory (see 5.411-412). His indeterminism was connected with his work for the Coast Survey on precise measurement.†3 And his empirical investigations in psychology (see 7.21-35) influenced his theory of mind. Thus Peirce's scientific work played an important role in his life and in the formation of his philosophy, and it is for these reasons that we have reprinted his only published article in experimental psychology and two short pieces on gravity. We have limited ourselves to this small sample of his work in physical science because, though his astronomical researches showed originality and his gravimetric work was well respected by eminent men in geodetics, Peirce remained a minor figure in these fields.

Peirce: CP 8 Introduction p x

The editors of the earlier volumes made some introductory remarks about the Peirce manuscripts and their policies in editing them (Vol. I, p. iv ff.). The present

editor has continued their practice of publishing only parts of some of the works, omitting large portions altogether. He has also continued the policy of selecting or compiling a draft for publication whenever there were several drafts available. The justification for these procedures is to be found in certain aspects of the Peirce manuscripts. Many of the manuscripts are internally variable in quality: one frequently finds publishable sections in the midst of material which either is of little contemporary interest or presents ideas better treated in other manuscripts. Moreover, there are often alternative and sometimes incomplete versions of the same work to choose from; and in some cases no final version is discernible. The editor must then make selections and, if necessary, piece together drafts from the hodgepodge of partial drafts. These characteristics are so typical that any satisfactory edition of Peirce's papers must contain a great deal of fragmentary material.

Peirce: CP 8 Introduction p x

The present editor has also continued the plan of organization pursued in the previous volumes, breaking up manuscripts, books, and series of articles, and arranging the resultant materials primarily by subject matter rather than chronologically. Though this plan tends to bring together under one heading passages on a given topic, it makes Peirce's writings appear to be more disorganized than they actually are. To mitigate this shortcoming, there is included in Volume VIII as complete a bibliography of Peirce's published works as the editor could compile. This bibliography is arranged for the most part chronologically, with a connected series of articles, parts of a single manuscript, or a series of lectures grouped together under one listing. The bibliography will also assist the reader in locating any published works not reprinted in this series.

Peirce: CP 8 Introduction p xi

The chapter and section headings have been chosen by the editor; when these are the same as Peirce's, the footnotes so indicate. Editorial alterations are enclosed in brackets, and the editor's footnotes are indicated by "(Ed.)." Peirce's punctuation, spelling, and underlining for emphasis have, in general, been retained. Obvious errors, however, have been corrected; and where clarity of presentation has required them, minor changes have been made in format, capitalization, abbreviations, italicizing of titles, etc., without any indication. Likewise, questions of manuscript interpretation which made no significant difference in the meaning have been settled without any indication.

Peirce: CP 8 Introduction p xi

The editor wishes to thank the Rockefeller Foundation and the Department of Philosophy of Harvard University for their aid in the preparation and publication of these two volumes. While giving him the kindest and fullest assistance, both left him completely free in his work and have no responsibility for any shortcomings in it. The editor is also grateful for a grant for editorial assistance from the Horace H. Rackham School of Graduate Studies of the University of Michigan.

Peirce: CP 8 Introduction p ix

For assistance in locating and evaluating various Peirce materials, the editor is personally indebted to William Alston, Jackson Cope, Carolyn Eisele, Mrs. E.L.C. Hales, Murray Murphey, W.E. Schlaretzki, James F. Sheridan, Manley Thompson, and Philip Wiener; to the previous editors, Charles Hartshorne and Paul Weiss; and especially to Max Fisch, who also read the manuscript and made many valuable suggestions. In all cases the final decision as to what materials were to be included was made by the editor, and the responsibility for the choices is his alone.

Peirce: CP 8 Introduction p x

The editor expresses his deep thanks to Grace L. Wood, who assisted greatly in the preparation of the bibliography and did much of the editorial work for the volumes, and to his wife, Alice, for her helpful advice.

Arthur W. Burks

Ann Arbor, Michigan

Peirce: CP 8.1 Cross-Ref:††
BOOK I

REVIEWS

CHAPTER 1

JOHN VENN, *THE LOGIC OF CHANCE*†1

1. Here is a book which should be read by every thinking man. Great changes have taken place of late years in the philosophy of chances. Mr. Venn remarks, with great ingenuity and penetration, that this doctrine has had its realistic, conceptualistic, and nominalistic stages. The logic of the Middle Ages is almost coextensive with demonstrative logic; but our age of science opened with a discussion of probable argument (in the *Novum Organum*), and this part of the subject has given the chief interest to modern studies of logic. What is called the doctrine of chances is, to be sure, but a small part of this field of inquiry; but it is a part where the varieties in the conceptions of probability have been most evident. When this doctrine was first studied, probability seems to have been regarded as something inhering in the singular events, so that it was possible for Bernouilli to enounce it as a *theorem* (and not merely as an identical proposition), that events happen with frequencies proportional to their probabilities. That was a realistic view. Afterwards it was said that probability does not exist in the singular events, but consists in the degree of credence which ought to be reposed in the occurrence of an event. This is conceptualistic. Finally, probability is regarded as the ratio of the number of events in a certain part of an aggregate of them to the number in the whole aggregate. This is the nominalistic view.

Peirce: CP 8.2 Cross-Ref:††

2. This last is the position of Mr. Venn and of the most advanced writers on the subject. The theory was perhaps first put forth by Mr. Stuart Mill; but his head became involved in clouds, and he relapsed into the conceptualistic opinion. Yet the

arguments upon the modern side are overwhelming. The question is by no means one of words; but if we were to inquire into the manner in which the terms *probable*, *likely*, and so forth, have been used, we should find that they always refer to a determination of a genus of argument. See, for example, Locke on the Understanding, Book IV. ch. 14, §1. There we find it stated that a thing is probable when it is supported by reasons *such as* lead to a true conclusion. These words *such as* plainly refer to a genus of argument. Now, what constitutes the validity of a genus of argument? The necessity of thinking the conclusion, say the conceptualists. But a madman may be under a necessity of thinking fallaciously, and (as Bacon suggests) all mankind may be mad after one uniform fashion. Hence the nominalist answers the question thus: A genus of argument is valid when from true premises it will yield a true conclusion -- invariably if demonstrative, generally if probable. The conceptualist says, that probability is the degree of credence which *ought* to be placed in the occurrence of an event. Here is an allusion to an entry on the debtor side of man's ledger. What is this entry? What is the meaning of this *ought*? Since probability is not an affair of morals, the *ought* must refer to an alternative to be avoided. Now the reasoner has nothing to fear but error. Probability will accordingly be the degree of credence which it is necessary to repose in a proposition in order to escape error. Conceptualists have not undertaken to say what is meant by "degree of credence." They would probably pronounce it indefinable and indescribable. Their philosophy deals much with the indefinable and indescribable. But propositions are either absolutely true or absolutely false. There is nothing *in the facts* which corresponds at all to a degree of credence, except that a genus of argument may yield a certain proportion of true conclusions from true premises. Thus, the following form of argument would, in the long run, yield (from true premises) a true conclusion two thirds of the time:--

A is taken at random from among the B's;

2/3 of the B's are C;

∴ A is C.

Peirce: CP 8.3 Cross-Ref:††

3. Truth being, then, the agreement of a representation with its object, and there being nothing *in re* answering to a degree of credence, a modification of a judgment in that respect cannot make it more true, although it may indicate the proportion of *such* judgments which are true *in the long run*. That is, indeed, the precise and only use or significance of these fractions termed probabilities: they give security in the long run. Now, in order that the degree of credence should correspond to any truth in the long run, it must be the representation of a general statistical fact, -- a real, objective fact. And then, as it is the fact which is said to be probable, and not the belief, the introduction of "degree of credence" at all into the definition of probability is as superfluous as the introduction of a reflection upon a mental process into any other definition would be, -- as though we were to define man as "that which (if the essence of the name is to be apprehended) ought to be conceived as a rational animal."

Peirce: CP 8.4 Cross-Ref:††

4. To say that the conceptualistic and nominalistic theories are both true at once, is mere ignorance, because their numerical results conflict. A conceptualist might hesitate, perhaps, to say that the probability of a proposition of which he knows absolutely nothing is $1/2$, although this would be, in one sense, justifiable for the nominalist, inasmuch as one half of all possible propositions (being contradictions of the other half) are true; but he does not hesitate to assume events to be equally probable when he does not know anything about their probabilities, and this is for the nominalist an utterly unwarrantable procedure. A probability is a statistical fact, and cannot be assumed arbitrarily. Boole first did away with this absurdity, and thereby brought the mathematical doctrine of probabilities into harmony with the modern logical doctrine of probable inference. But Boole (owing to the *needs* of his calculus) admitted the assumption that simple events whose probabilities are given are independent, -- an assumption of the same vicious character. Mr. Venn strikes down this last remnant of conceptualism with a very vigorous hand.

Peirce: CP 8.5 Cross-Ref:††

5. He has, however, fallen into some conceptualistic errors of his own; and these are specially manifest in his "applications to moral and social science." The most important of these is contained in the chapter "on the credibility of extraordinary stories"; but it is defended with so much ingenuity as almost to give it the value of a real contribution to science. It is maintained that the credibility of an extraordinary story depends either entirely upon the veracity of the witness, or, in more extraordinary cases, entirely upon the *a priori* credibility of the story; but that these considerations cannot, under any circumstances, be combined, unless arbitrarily. In order to support this opinion, the author invents an illustration. He supposes that statistics were to have shown that nine out of ten consumptives who go to the island of Madeira live through the first year, and that nine out of ten Englishmen who go to the same island die the first year; what, then, would be the just rate of insurance for the first year of a consumptive Englishman who is about to go to that island? There are no certain data for the least approximation to the proportion of consumptive Englishmen who die in Madeira during the first year. But it is certain that an insurance company which insured only Englishmen in Madeira during the first year, or only consumptives under the same circumstances, would be warranted (a certain moral fact being neglected) in taking the consumptive Englishman at its ordinary rate. Hence, Mr. Venn thinks that an insurance company which insured all sorts of men could with safety and fairness insure the consumptive Englishman either as Englishman or as consumptive.†2 Now, the case of an extraordinary story is parallel to this: for such a story is, 1st, told by a certain person, who tells a known proportion of true stories, -- say nine out of ten; and, 2d, is of a certain sort (as a fish story), of which a known proportion are true, -- say one in ten. Then, as much as before, we come out right, in the long run, by considering such a story under either of the two classes to which it belongs. Hence, says Mr. Venn, we must repose such belief in the story as the veracity of the witness alone, or the antecedent probability alone, requires, or else arbitrarily modify one or other of these degrees of credence. In examining this theory, let us first remark, that there are two principal phrases in which the word probability occurs: for, first, we may speak of the probability of an event or proposition, and then we express ourselves incompletely, inasmuch as we refer to the frequency of true conclusions in the genus of arguments by which the event or proposition in question may have been inferred, without indicating what genus of argument that is; and, secondly, we may speak of the probability that any individual of a certain class has a certain character, when we mean the ratio of the number of those of that class that have that character to the total number in the class. Now it is

this latter phrase which we use when we speak of the probability that a story of a certain sort, told by a certain man, is true. And since there is nothing in the data to show what this ratio is, the probability in question is unknown. But a "degree of credence" or "credibility," to be logically determined, must, as we have seen, be an expression of probability in the nominalistic sense; and therefore this "degree of credence" (supposing it to exist) is unknown. "We know not what to believe," is the ordinary and logically correct expression in such cases of perplexity.

Peirce: CP 8.6 Cross-Ref:††

6. Credence and expectation cannot be represented by single numbers. Probability is not always known; and then the probability of each degree of probability must enter into the credence. Perhaps this again is not known; then there will be a probability of each degree of probability of each degree of probability; and so on. In the same way, when a risk is run, the expectation is composed of the probabilities of each possible issue, but is not a single number, as the Petersburg problem shows. Suppose the capitalists of the world were to owe me a hundred dollars, and were to offer to pay in either of the following ways: 1st, a coin should be pitched up until it turned up heads (or else a hundred times, if it did not come up heads sooner), and I should be paid two dollars if the head came up the first time, four if the second time, eight if the third time, etc.; or, 2d, a coin should be turned up a hundred times, and I should receive two dollars for every head. Each of these offers would be worth a hundred dollars, *in the long run*; that is to say, if repeated often enough, I should receive on the average a hundred dollars at each trial. But if the trial were to be made but once, I should infinitely prefer the second alternative, on account of its greater security. Mere certainty is worth a great deal. We wish to know our fate. How much it is worth is a question of political economy. It must go into the market, where its worth is what it will fetch. And since security may be of many kinds (according to the distribution of the probabilities of each sum of money and of each loss, in prospect), the value of the various kinds will fluctuate among one another with the ratio of demand and supply, -- the demand varying with the moral and intellectual state of the community, -- and thus no single and constant number can represent the value of any kind.

Peirce: CP 8.7 Cross-Ref:††

CHAPTER 2

FRASER'S EDITION OF *THE WORKS OF GEORGE BERKELEY* †1

§1. INTRODUCTION

7. This new edition of Berkeley's works is much superior to any of the former ones. It contains some writings not in any of the other editions, and the rest are given with a more carefully edited text. The editor has done his work well. The introductions to the several pieces contain analyses of their contents which will be

found of the greatest service to the reader. On the other hand, the explanatory notes which disfigure every page seem to us altogether unnecessary and useless.

Peirce: CP 8.8 Cross-Ref:††

8. Berkeley's metaphysical theories have at first sight an air of paradox and levity very unbecoming to a bishop. He denies the existence of matter, our ability to see distance, and the possibility of forming the simplest general conception; while he admits the existence of Platonic ideas; and argues the whole with a cleverness which every reader admits, but which few are convinced by. His disciples seem to think the present moment a favorable one for obtaining for their philosophy a more patient hearing than it has yet got. It is true that we of this day are sceptical and not given to metaphysics, but so, say they, was the generation which Berkeley addressed, and for which his style was chosen; while it is hoped that the spirit of calm and thorough inquiry which is now, for once, almost the fashion, will save the theory from the perverse misrepresentations which formerly assailed it, and lead to a fair examination of the arguments which, in the minds of his sectators, put the truth of it beyond all doubt. But above all, it is anticipated that the Berkeleyan treatment of that question of the validity of human knowledge and of the inductive process of science, which is now so much studied, is such as to command the attention of scientific men to the idealistic system. To us these hopes seem vain. The truth is that the minds from whom the spirit of the age emanates have now no interest in the only problems that metaphysics ever pretended to solve. The abstract acknowledgment of God, Freedom, and Immortality, apart from those other religious beliefs (which cannot possibly rest on metaphysical grounds) which alone may animate this, is now seen to have no practical consequence whatever. The world is getting to think of these creatures of metaphysics, as Aristotle of the Platonic ideas: {teretismata gar esti, kai ei estin, ouden pros ton logon estin}. The question of the grounds of the validity of induction has, it is true, excited an interest, and may continue to do so (though the argument is now become too difficult for popular apprehension); but whatever interest it has had has been due to a hope that the solution of it would afford the basis for sure and useful maxims concerning the logic of induction, -- a hope which would be destroyed so soon as it were shown that the question was a purely metaphysical one. This is the prevalent feeling, among advanced minds. It may not be just; but it exists. And its existence is an effectual bar (if there were no other) to the general acceptance of Berkeley's system. The few who do now care for metaphysics are not of that bold order of minds who delight to hold a position so unsheltered by the prejudices of common sense as that of the good bishop.

Peirce: CP 8.9 Cross-Ref:††

9. As a matter of history, however, philosophy must always be interesting. It is the best representative of the mental development of each age. It is so even of ours, if we think what really is our philosophy. Metaphysical history is one of the chief branches of history, and ought to be expounded side by side with the history of society, of government, and of war; for in its relations with these we trace the significance of events for the human mind. The history of philosophy in the British Isles is a subject possessing more unity and entirety within itself than has usually been recognized in it. The influence of Descartes was never so great in England as that of traditional conceptions, and we can trace a continuity between modern and mediaeval thought there, which is wanting in the history of France, and still more, if possible, in that of Germany.

Peirce: CP 8.10 Cross-Ref:††

10. From very early times, it has been the chief intellectual characteristic of the English to wish to effect everything by the plainest and directest means, without unnecessary contrivance. In war, for example, they rely more than any other people in Europe upon sheer hardihood, and rather despise military science. The main peculiarities of their system of law arise from the fact that every evil has been rectified as it became intolerable, without any thoroughgoing measure. The bill for legalizing marriage with a deceased wife's sister is yearly pressed because it supplies a remedy for an inconvenience actually felt; but nobody has proposed a bill to legalize marriage with a deceased husband's brother. In philosophy, this national tendency appears as a strong preference for the simplest theories, and a resistance to any complication of the theory as long as there is the least possibility that the facts can be explained in the simpler way. And, accordingly, British philosophers have always desired to weed out of philosophy all conceptions which could not be made perfectly definite and easily intelligible, and have shown strong nominalistic tendencies since the time of Edward I., or even earlier. Berkeley is an admirable illustration of this national character, as well as of that strange union of nominalism with Platonism, which has repeatedly appeared in history, and has been such a stumbling-block to the historians of philosophy.

Peirce: CP 8.11 Cross-Ref:††

11. The mediaeval metaphysic is so entirely forgotten, and has so close a historic connection with modern English philosophy, and so much bearing upon the truth of Berkeley's doctrine, that we may perhaps be pardoned a few pages on the nature of the celebrated controversy concerning universals. And first let us set down a few dates. It was at the very end of the eleventh century that the dispute concerning nominalism and realism, which had existed in a vague way before, began to attain extraordinary proportions. During the twelfth century it was the matter of most interest to logicians, when William of Champeaux, Abélard, John of Salisbury, Gilbert de la Porrée, and many others, defended as many different opinions. But there was no historic connection between this controversy and those of scholasticism proper, the scholasticism of Aquinas, Scotus, and Ockam. For about the end of the twelfth century a great revolution of thought took place in Europe. What the influences were which produced it requires new historical researches to say. No doubt, it was partly due to the Crusades. But a great awakening of intelligence did take place at that time. It requires, it is true, some examination to distinguish this particular movement from a general awakening which had begun a century earlier, and had been growing ever since. But now there was an accelerated impulse. Commerce was attaining new importance, and was inventing some of her chief conveniences and safeguards. Law, which had hitherto been utterly barbaric, began to be a profession. The civil law was adopted in Europe, the canon law was digested; the common law took some form. The Church, under Innocent III., was assuming the sublime functions of a moderator over kings. And those orders of mendicant friars were established, two of which did so much for the development of the scholastic philosophy. Art felt the spirit of a new age, and there could hardly be a greater change than from the highly ornate round-arched architecture of the twelfth century to the comparatively simple Gothic of the thirteenth. Indeed, if any one wishes to know what a scholastic commentary is like, and what the tone of thought in it is, he has only to contemplate a Gothic cathedral. The first quality of either is a religious devotion, truly heroic. One feels that the men who did these works did really believe in religion as we believe in nothing. We cannot easily understand how Thomas Aquinas can speculate so much on the nature of angels, and whether ten thousand of them could dance on a needle's point. But it was simply because he held them for real. If they are

real, why are they not more interesting than the bewildering varieties of insects which naturalists study; or why should the orbits of double stars attract more attention than spiritual intelligences? It will be said that we have no means of knowing anything about them. But that is on a par with censuring the schoolmen for referring questions to the authority of the Bible and of the Church. If they really believed in their religion, as they did, what better could they do? And if they found in these authorities testimony concerning angels, how could they avoid admitting it. Indeed, objections of this sort only make it appear still more clearly how much those were the ages of faith. And if the spirit was not altogether admirable, it is only because faith itself has its faults as a foundation for the intellectual character. The men of that time did fully believe and did think that, for the sake of giving themselves up absolutely to their great task of building or of writing, it was well worth while to resign all the joys of life. Think of the spirit in which Duns Scotus must have worked, who wrote his thirteen volumes in folio, in a style as condensed as the most condensed parts of Aristotle, before the age of thirty-four. Nothing is more striking in either of the great intellectual products of that age, than the complete absence of self-conceit on the part of the artist or philosopher. That anything of value can be added to his sacred and catholic work by its having the smack of individuality about it, is what he has never conceived. His work is not designed to embody *his* ideas, but the universal truth; there will not be one thing in it however minute, for which you will not find that he has his authority; and whatever originality emerges is of that inborn kind which so saturates a man that he cannot himself perceive it. The individual feels his own worthlessness in comparison with his task, and does not dare to introduce his vanity into the doing of it. Then there is no machine-work, no unthinking repetition about the thing. Every part is worked out for itself as a separate problem, no matter how analogous it may be in general to another part. And no matter how small and hidden a detail may be, it has been conscientiously studied, as though it were intended for the eye of God. Allied to this character is a detestation of antithesis or the studied balancing of one thing against another, and of a too geometrical grouping, -- a hatred of posing which is as much a moral trait as the others. Finally, there is nothing in which the scholastic philosophy and the Gothic architecture resemble one another more than in the gradually increasing sense of immensity which impresses the mind of the student as he learns to appreciate the real dimensions and cost of each. It is very unfortunate that the thirteenth, fourteenth, and fifteenth centuries should, under the name of Middle Ages, be confounded with others, which they are in every respect as unlike as the Renaissance is from modern times. In the history of logic, the break between the twelfth and thirteenth centuries is so great that only one author of the former age is ever quoted in the latter. If this is to be attributed to the fuller acquaintance with the works of Aristotle, to what, we would ask, is this profounder study itself to be attributed, since it is now known that the knowledge of those works was not imported from the Arabs? The thirteenth century was realistic, but the question concerning universals was not as much agitated as several others. Until about the end of the century, scholasticism was somewhat vague, immature, and unconscious of its own power. Its greatest glory was in the first half of the fourteenth century. Then Duns Scotus, †2 a Briton (for whether Scotch, Irish, or English is disputed), first stated the realistic position consistently, and developed it with great fulness and applied it to all the different questions which depend upon it. His theory of "formalities" was the subtlest, except perhaps Hegel's logic, ever broached, and he was separated from nominalism only by the division of a hair. It is not therefore surprising that the nominalistic position was soon adopted by several writers, especially by the celebrated William of Ockam, who took the lead of this party by the

thoroughgoing and masterly way in which he treated the theory and combined it with a then rather recent but now forgotten addition to the doctrine of logical terms. With Ockam, who died in 1347, scholasticism may be said to have culminated. After him the scholastic philosophy showed a tendency to separate itself from the religious element which alone could dignify it, and sunk first into extreme formalism and fancifulness, and then into the merited contempt of all men; just as the Gothic architecture had a very similar fate, at about the same time, and for much the same reasons.

Peirce: CP 8.12 Cross-Ref:††
§2. FORMULATION OF REALISM

12. The current explanations of the realist-nominalist controversy are equally false and unintelligible. They are said to be derived ultimately from Bayle's Dictionary; at any rate, they are not based on a study of the authors. "Few, very few, for a hundred years past," says Hallam, with truth, "have broken the repose of the immense works of the schoolmen." Yet it is perfectly possible so to state the matter that no one shall fail to comprehend what the question was, and how there might be two opinions about it. Are universals real? We have only to stop and consider a moment what was meant by the word *real*, when the whole issue soon becomes apparent. Objects are divided into figments, dreams, etc., on the one hand, and realities on the other. The former are those which exist only inasmuch as you or I or some man imagines them; the latter are those which have an existence independent of your mind or mine or that of any number of persons. The real is that which is not whatever we happen to think it, but is unaffected by what we may think of it.†3 The question, therefore, is whether *man*, *horse*, and other names of natural classes, correspond with anything which all men, or all horses, really have in common, independent of our thought, or whether these classes are constituted simply by a likeness in the way in which our minds are affected by individual objects which have in themselves no resemblance or relationship whatsoever. Now that this is a real question which different minds will naturally answer in opposite ways, becomes clear when we think that there are two widely separated points of view, from which *reality*, as just defined, may be regarded. Where is the real, the thing independent of how we think it, to be found? There must be such a thing, for we find our opinions constrained; there is something, therefore, which influences our thoughts, and is not created by them. We have, it is true, nothing immediately present to us but thoughts. These thoughts, however, have been caused by sensations, and those sensations are constrained by something out of the mind. This thing out of the mind, which directly influences sensation, and through sensation thought, because it *is* out of the mind, is independent of how we think it, and is, in short, the real. Here is one view of reality, a very familiar one. And from this point of view it is clear that the nominalistic answer must be given to the question concerning universals. For, while from this standpoint it may be admitted to be true as a rough statement that one man is like another, the exact sense being that the realities external to the mind produce sensations which may be embraced under one conception, yet it can by no means be admitted that the two real men have really anything in common, for to say that they are both men is only to say that the one mental term or thought-sign "man" stands indifferently for either of the sensible objects caused by the two external realities; so that not even the two sensations have in themselves anything in common, and far less is it to be inferred

that the external realities have. This conception of reality is so familiar, that it is unnecessary to dwell upon it; but the other, or realist conception, if less familiar, is even more natural and obvious. All human thought and opinion contains an arbitrary, accidental element, dependent on the limitations in circumstances, power, and bent of the individual; an element of error, in short. But human opinion universally tends in the long run to a definite form, which is the truth. Let any human being have enough information and exert enough thought upon any question, and the result will be that he will arrive at a certain definite conclusion, which is the same that any other mind will reach under sufficiently favorable circumstances. Suppose two men, one deaf, the other blind. One hears a man declare he means to kill another, hears the report of the pistol, and hears the victim cry; the other sees the murder done. Their sensations are affected in the highest degree with their individual peculiarities. The first information that their sensations will give them, their first inferences, will be more nearly alike, but still different; the one having, for example, the idea of a man shouting, the other of a man with a threatening aspect; but their final conclusions, the thought the remotest from sense, will be identical and free from the one-sidedness of their idiosyncrasies. There is, then, to every question a true answer, a final conclusion, to which the opinion of every man is constantly gravitating. He may for a time recede from it, but give him more experience and time for consideration, and he will finally approach it. The individual may not live to reach the truth; there is a residuum of error in every individual's opinions. No matter; it remains that there is a definite opinion to which the mind of man is, on the whole and in the long run, tending. On many questions the final agreement is already reached, on all it will be reached if time enough is given. The arbitrary will or other individual peculiarities of a sufficiently large number of minds may postpone the general agreement in that opinion indefinitely; but it cannot affect what the character of that opinion shall be when it is reached. This final opinion, then, is independent, not indeed of thought in general, but of all that is arbitrary and individual in thought; is quite independent of how you, or I, or any number of men think.†4 Everything, therefore, which will be thought to exist in the final opinion is real, and nothing else. What is the POWER of external things, to affect the senses? To say that people sleep after taking opium because it has a soporific *power*, is that to say anything in the world but that people sleep after taking opium because they sleep after taking opium? To assert the existence of a power or potency, is it to assert the existence of anything actual? Or to say that a thing has a potential existence, is it to say that it has an actual existence? In other words, is the present existence of a power anything in the world but a regularity in future events relating to a certain thing regarded as an element which is to be taken account of beforehand, in the conception of that thing? If not, to assert that there are external things which can be known only as exerting a power on our sense, is nothing different from asserting that there is a general *drift* in the history of human thought which will lead it to one general agreement, one catholic consent. And any truth more perfect than this destined conclusion, any reality more absolute than what is thought in it, is a fiction of metaphysics. It is obvious how this way of thinking harmonizes with a belief in an infallible Church, and how much more natural it would be in the Middle Ages than in Protestant or positivist times.

Peirce: CP 8.13 Cross-Ref: ††

13. This theory of reality is instantly fatal to the idea of a thing in itself, -- a thing existing independent of all relation to the mind's conception of it. Yet it would by no means forbid, but rather encourage us, to regard the appearances of sense as only signs of the realities. Only, the realities which they represent would not be the unknowable cause of sensation, but *noumena*, or intelligible conceptions which are

the last products of the mental action which is set in motion by sensation. The matter of sensation is altogether accidental; precisely the same information, practically, being capable of communication through different senses. And the catholic consent which constitutes the truth is by no means to be limited to men in this earthly life or to the human race, but extends to the whole communion of minds to which we belong, including some probably whose senses are very different from ours, so that in that consent no predication of a sensible quality can enter, except as an admission that so certain sorts of senses are affected. This theory is also highly favorable to a belief in external realities. It will, to be sure, deny that there is any reality which is absolutely incognizable in itself, so that it cannot be taken into the mind. But observing that "the external" means simply that which is independent of what phenomenon is immediately present, that is of how we may think or feel; just as "the real" means that which is independent of how we may think or feel *about it*; it must be granted that there are many objects of true science which are external, because there are many objects of thought which, if they are independent of that thinking whereby they are thought (that is, if they are real), are indisputably independent of all *other* thoughts and feelings.

Peirce: CP 8.14 Cross-Ref:††

14. It is plain that this view of reality is inevitably realistic; because general conceptions enter into all judgments, and therefore into true opinions. Consequently a thing in the general is as real as in the concrete. It is perfectly true that all white things have whiteness in them, for that is only saying, in another form of words, that all white things are white; but since it is true that real things possess whiteness, whiteness is real. It is a real which only exists by virtue of an act of thought knowing it, but that thought is not an arbitrary or accidental one dependent on any idiosyncrasies, but one which will hold in the final opinion.

Peirce: CP 8.15 Cross-Ref:††

15. This theory involves a phenomenalism. But it is the phenomenalism of Kant, and not that of Hume. Indeed, what Kant called his Copernican step was precisely the passage from the nominalistic to the realistic view of reality. It was the essence of his philosophy to regard the real object as determined by the mind. That was nothing else than to consider every conception and intuition which enters necessarily into the experience of an object, and which is not transitory and accidental, as having objective validity. In short, it was to regard the reality as the normal product of mental action, and not as the incognizable cause of it.

Peirce: CP 8.16 Cross-Ref:††

16. This realistic theory is thus a highly practical and common-sense position. Wherever universal agreement prevails, the realist will not be the one to disturb the general belief by idle and fictitious doubts. For according to him it is a consensus or common confession which constitutes reality. What he wants, therefore, is to see questions put to rest. And if a general belief, which is perfectly stable and immovable, can in any way be produced, though it be by the fagot and the rack, to talk of any error in such belief is utterly absurd. The realist will hold that the very same objects which are immediately present in our minds in experience really exist just as they are experienced out of the mind; that is, he will maintain a doctrine of immediate perception. He will not, therefore, sunder existence out of the mind and being in the mind as two wholly improportionable modes. When a thing is in such relation to the individual mind that that mind cognizes it, it is in the mind; and its being so in the mind will not in the least diminish its external existence. For he does not think of the

mind as a receptacle, which if a thing is in, it ceases to be out of. To make a distinction between the true conception of a thing and the thing itself is, he will say, only to regard one and the same thing from two different points of view; for the immediate object of thought in a true judgment *is* the reality. The realist will, therefore, believe in the objectivity of all necessary conceptions, space, time, relation, cause, and the like.

Peirce: CP 8.17 Cross-Ref:††

17. No realist or nominalist ever expressed so definitely, perhaps, as is here done, his conception of reality. It is difficult to give a clear notion of an opinion of a past age, without exaggerating its distinctness. But careful examination of the works of the schoolmen will show that the distinction between these two views of the real -- one as the fountain of the current of human thought, the other as the unmoving form to which it is flowing -- is what really occasions their disagreement on the question concerning universals. The gist of all the nominalist's arguments will be found to relate to a *res extra animam*, while the realist defends his position only by assuming that the immediate object of thought in a true judgment is real. The notion that the controversy between realism and nominalism had anything to do with Platonic ideas is a mere product of the imagination, which the slightest examination of the books would suffice to disprove. But to prove that the statement here given of the essence of these positions is historically true and not a fancy sketch, it will be well to add a brief analysis of the opinions of Scotus and Ockam.

Peirce: CP 8.18 Cross-Ref:††

§3. SCOTUS, OCKAM, AND HOBBS

18. Scotus sees several questions confounded together under the usual *utrum universale est aliquid in rebus*. In the first place, there is the question concerning the Platonic forms. But putting Platonism aside as at least incapable of proof, and as a self-contradictory opinion if the archetypes are supposed to be strictly universal, there is the celebrated dispute among Aristotelians as to whether the universal is really in things or only derives its existence from the mind. Universality is a relation of a predicate to the subjects of which it is predicated. That can exist only in the mind, wherein alone the coupling of subject and predicate takes place. But the word *universal* is also used to denote what are named by such terms as *a man* or *a horse*; these are called universals, because a man is not necessarily this man, nor a horse this horse. In such a sense it is plain universals are real; there really is a man and there really is a horse. The whole difficulty is with the actually indeterminate universal, that which not only is not necessarily *this*, but which, being one single object of thought, is predicable of many things. In regard to this it may be asked, first, is it necessary to its existence that it should be in the mind; and, second, does it exist *in re*? There are two ways in which a thing may be in the mind, -- *habitualiter* and *actualiter*. A notion is in the mind *actualiter* when it is actually conceived; it is in the mind *habitualiter* when it can directly produce a conception. It is by virtue of mental association (we moderns should say), that things are in the mind *habitualiter*. In the Aristotelian philosophy, the intellect is regarded as being to the soul what the eye is to the body. The mind *perceives* likenesses and other relations in the objects of sense, and thus just as sense affords sensible images of things, so the intellect affords intelligible images of them. It is as such a *species intelligibilis* that Scotus supposes

that a conception exists which is in the mind *habitualiter*, not *actualiter*. This *species* is in the mind, in the sense of being the immediate object of knowledge, but its existence in the mind is independent of *consciousness*. Now that the *actual* cognition of the universal is necessary to its existence, Scotus denies. The subject of science is universal; and if the existence of [the] universal were dependent upon what we happened to be thinking, science would not relate to anything real. On the other hand, he admits that the universal must be in the mind *habitualiter*, so that if a thing be considered as it is independent of its being cognized, there is no universality in it. For there is *in re extra* no one intelligible object attributed to different things. He holds, therefore, that such natures (i.e. sorts of things) as a *man* and a *horse*, which are real, and are not of themselves necessarily *this* man or *this* horse, though they cannot exist *in re* without being some particular man or horse, are in the *species intelligibilis* always represented positively indeterminate, it being the nature of the mind so to represent things. Accordingly any such nature is to be regarded as something which is of itself neither universal nor singular, but is universal in the mind, singular in things out of the mind. If there were nothing in the different men or horses which was not of itself singular, there would be no real unity except the numerical unity of the singulars; which would involve such absurd consequences as that the only real difference would be a numerical difference, and that there would be no real likenesses among things. If, therefore, it is asked whether the universal is in things, the answer is, that the nature which in the mind is universal, and is not in itself singular, exists in things. It is the very same nature which in the mind is universal and *in re* is singular; for if it were not, in knowing anything of a universal we should be knowing nothing of things, but only of our own thoughts, and our opinion would not be converted from true to false by a change in things. This nature is actually indeterminate only so far as it is in the mind. But to say that an object is in the mind is only a metaphorical way of saying that it stands to the intellect in the relation of known to knower. The truth is, therefore, that that real nature which exists *in re*, apart from all action of the intellect, though in itself, apart from its relations, it be singular, yet is actually universal as it exists in relation to the mind. But this universal only differs from the singular in the manner of its being conceived (*formaliter*), but not in the manner of its existence (*realiter*).

Peirce: CP 8.19 Cross-Ref:††

19. Though this is the slightest possible sketch of the realism of Scotus, and leaves a number of important points unnoticed, yet it is sufficient to show the general manner of his thought and how subtle and difficult his doctrine is. That about one and the same nature being in the grade of singularity in existence, and in the grade of universality in the mind, gave rise to an extensive doctrine concerning the various kinds of identity and difference, called the doctrine of the *formalitates*; and this is the point against which Ockam directed his attack.

Peirce: CP 8.20 Cross-Ref:††

20. Ockam's nominalism may be said to be the next stage in English opinion. As Scotus's mind is always running on forms, so Ockam's is on logical terms; and all the subtle distinctions which Scotus effects by his *formalitates*, Ockam explains by implied syncategorematics (or adverbial expressions, such as *per se*, etc.) in terms. Ockam always thinks of a mental conception as a logical term, which, instead of existing on paper, or in the voice, is in the mind, but is of the same general nature, namely, a *sign*. The conception and the word differ in two respects: first, a word is arbitrarily imposed, while a conception is a natural sign; second, a word signifies whatever it signifies only indirectly, through the conception which signifies the same

thing directly. Ockam enunciates his nominalism as follows: "It should be known that *singular* may be taken in two senses. In one sense, it signifies that which is one and not many; and in this sense those who hold that the universal is a quality of mind predicable of many, standing however in this predication, not for itself, but for those many (i.e. the nominalists), have to say that every universal is truly and really singular; because as every word, however general we may agree to consider it, is truly and really singular and one in number, because it is one and not many, so every universal is singular. In another sense, the name *singular* is used to denote whatever is one and not many, is a sign of something which is singular in the first sense, and is not fit to be the sign of many. Whence, using the word *universal* for that which is not one in number, -- an acceptance many attribute to it, -- I say that there is no universal; unless perchance you abuse the word and say that *people* is not one in number and is universal. But that would be puerile. It is to be maintained, therefore, that every universal is one singular thing, and therefore there is no universal except by signification, that is, by its being the sign of many."†5 The arguments by which he supports this position present nothing of interest.†6 Against Scotus's doctrine that universals are without the mind in individuals, but are not really distinct from the individuals, but only formally so, he objects that it is impossible there should be any distinction existing out of the mind except between things really distinct. Yet he does not think of denying that an individual consists of matter and form, for these, though inseparable, are really distinct things; though a modern nominalist might ask in what sense things could be said to be distinct independently of any action of the mind, which are so inseparable as matter and form. But as to *relation*, he most emphatically and clearly denies that it exists as anything different from the things related; and this denial he expressly extends to relations of agreement and likeness as well as to those of opposition. While, therefore, he admits the real existence of qualities, he denies that these real qualities are respects in which things agree or differ; but things which agree or differ agree or differ in themselves and in no respect *extra animam*. He allows that things without the mind are similar, but this similarity consists merely in the fact that the mind can abstract one notion from the contemplation of them. A resemblance, therefore, consists solely in the property of the mind by which it naturally imposes one mental sign upon the resembling things. Yet he allows there is something in the things to which this mental sign corresponds.

Peirce: CP 8.21 Cross-Ref:††

21. This is the nominalism of Ockam so far as it can be sketched in a single paragraph, and without entering into the complexities of the Aristotelian psychology nor of the *parva logicalia*. He is not so thoroughgoing as he might be, yet compared with Durandus and other contemporary nominalists he seems very radical and profound. He is truly the *venerabilis inceptor* of a new way of philosophizing which has now broadened, perhaps deepened also, into English empiricism.

Peirce: CP 8.22 Cross-Ref:††

22. England never forgot these teachings. During that Renaissance period when men could think that human knowledge was to be advanced by the use of Cicero's Commonplaces, we naturally see little effect from them; but one of the earliest prominent figures in modern philosophy is a man who carried the nominalistic spirit into everything -- religion, ethics, psychology, and physics, the *plusquam nominalis*, Thomas Hobbes of Malmesbury. His razor cuts off, not merely substantial forms, but every incorporeal substance. As for universals, he not only denies their real existence, but even that there are any universal conceptions except so far as we conceive names. In every part of his logic, names and speech play an extraordinarily

important part. Truth and falsity, he says, have no place but among such creatures as use speech, for a true proposition is simply one whose predicate is the name of everything of which the subject is the name. "From hence, also, this may be deduced, that the first truths were arbitrarily made by those that first of all imposed names upon things, or received them from the imposition of others. For it is true (for example), that *man is a living creature*, but it is for this *reason* that it pleased men to impose both those names on the same thing."†7 The difference between true religion and superstition is simply that the state recognizes the former and not the latter.

Peirce: CP 8.23 Cross-Ref:††

23. The nominalistic love of simple theories is seen also in his opinion, that every event is a movement, and that the sensible qualities exist only in sensible beings, and in his doctrine that man is at bottom purely selfish in his actions.

Peirce: CP 8.24 Cross-Ref:††

24. His views concerning matter are worthy of notice, because Berkeley is known to have been a student of Hobbes, as Hobbes confesses himself to have been of Ockam. The following paragraph gives his opinion:--

Peirce: CP 8.24 Cross-Ref:††

"And as for that matter which is common to all things, and which philosophers, following Aristotle, usually call *materia prima*, that is, *first matter*, it is not a body distinct from all other bodies, nor is it one of them. What then is it? A mere name; yet a name which is not of vain use; for it signifies a conception of body without the consideration of any form or other accident except only magnitude or extension, and aptness to receive form and other accident. So that whensoever we have use of the name *body in general*, if we use that of *materia prima*, we do well. For when a man, not knowing which was first, water or ice, would find out which of the two were the matter of both, he would be fain to suppose some third matter which were neither of these two; so he that would find out what is the matter of all things ought to suppose such as is not the matter of anything that exists. Wherefore *materia prima* is nothing; and therefore they do not attribute to it form or any other accident, besides quantity; whereas all singular things have their forms and accidents certain.

Peirce: CP 8.24 Cross-Ref:††

"*Materia prima* therefore is body in general, that is, body considered universally, not as having neither form nor any accident, but in which no form nor any other accident but quantity are at all considered, that is, they are not drawn into argumentation." -- p. 118.†8

Peirce: CP 8.25 Cross-Ref:††

25. The next great name in English philosophy is Locke's. His philosophy is nominalistic, but does not regard things from a logical point of view at all. Nominalism, however, appears in psychology as sensationalism; for nominalism arises from taking that view of reality which regards whatever is in thought as caused by something in sense, and whatever is in sense as caused by something without the mind. But everybody knows that this is the character of Locke's philosophy. He believed that every idea springs from sensation and from his (vaguely explained) reflection.

Peirce: CP 8.26 Cross-Ref:††
§4. BERKELEY'S PHILOSOPHY

26. Berkeley is undoubtedly more the offspring of Locke than of any other philosopher. Yet the influence of Hobbes with him is very evident and great; and Malebranche doubtless contributed to his thought. But he was by nature a radical and a nominalist. His whole philosophy rests upon an extreme nominalism of a sensationalistic type. He sets out with the proposition (supposed to have been already proved by Locke), that all the ideas in our minds are simply reproductions of sensations, external and internal. He maintains, moreover, that sensations can only be thus reproduced in such combinations as might have been given in immediate perception. We can conceive a man without a head, because there is nothing in the nature of sense to prevent our seeing such a thing; but we cannot conceive a sound without any pitch, because the two things are necessarily united in perception. On this principle he denies that we can have any abstract general ideas, that is, that universals can exist in the mind; if I think of a man it must be either of a short or a long or a middle-sized man, because if I see a man he must be one or the other of these. In the first draft of the Introduction of the Principles of Human Knowledge, which is now for the first time printed, he even goes so far as to censure Ockam for admitting that we can have general terms in our mind; Ockam's opinion being that we have in our minds conceptions, which are singular themselves, but are *signs* of many things.†9 But Berkeley probably knew only of Ockam from hearsay, and perhaps thought he occupied a position like that of Locke. Locke had a very singular opinion on the subject of general conceptions. He says:--

"If we nicely reflect upon them, we shall find that general ideas are fictions, and contrivances of the mind, that carry difficulty with them, and do not so easily offer themselves as we are apt to imagine. For example, does it not require some pains and skill to form the general idea of a triangle (which is none of the most abstract, comprehensive, and difficult); for it must be neither oblique nor rectangle, neither equilateral, equicrural, nor scalenon, but all and none of these at once? In effect, is something imperfect that cannot exist, an idea wherein some parts of several different and inconsistent ideas are put together."†10

Peirce: CP 8.27 Cross-Ref:††

27. To this Berkeley replies:--

"Much is here said of the difficulty that abstract ideas carry with them, and the pains and skill requisite in forming them. And it is on all hands agreed that there is need of great toil and labor of the mind to emancipate our thoughts from particular objects, and raise them to those sublime speculations that are conversant about abstract ideas. From all which the natural consequence should seem to be, that so difficult a thing as the forming of abstract ideas was not necessary to communication, which is so easy and familiar to all sort of men. But we are told, if they seem obvious and easy to grown men, it is only because by constant and familiar use they are made

so. Now, I would fain know at what time it is men are employed in surmounting that difficulty [and furnishing themselves with those necessary helps for discourse]. It cannot be when they are grown up, for then it seems they are not conscious of such painstaking; it remains, therefore, to be the business of their childhood. And surely the great and multiplied labor of framing abstract notions will be found a hard task at that tender age. Is it not a hard thing to imagine that a couple of children cannot prate together of their sugar-plums and rattles, and the rest of their little trinkets, till they have first tacked together numberless inconsistencies, and so formed in their minds abstract general ideas, and annexed them to every common name they make use of?"†11

Peirce: CP 8.28 Cross-Ref:††

28. In his private note-book Berkeley has the following:--

"*Mem.* To bring the killing blow at the last, e.g. in the matter of abstraction to bring Locke's general triangle in the last."†12

Peirce: CP 8.28 Cross-Ref:††

There was certainly an opportunity for a splendid blow here, and he gave it.

Peirce: CP 8.29 Cross-Ref:††

29. From this nominalism he deduces his idealistic doctrine. And he puts it beyond any doubt that, if this principle be admitted, the existence of matter must be denied. Nothing that we can know or even think can exist without the mind, for we can only think reproductions of sensations, and the *esse* of these is *percipi*. To put it another way, we cannot think of a thing as existing unperceived, for we cannot separate in thought what cannot be separated in perception. It is true, I can think of a tree in a park without anybody by to see it; but I cannot think of it without anybody to imagine it; for I am aware that I am imagining it all the time. Syllogistically: trees, mountains, rivers, and all sensible things are perceived; and anything which is perceived is a sensation; now for a sensation to exist without being perceived is impossible; therefore, for any sensible thing to exist out of perception is impossible. Nor can there be anything out of the mind which *resembles* a sensible object, for the conception of likeness cannot be separated from likeness between ideas, because that is the only likeness which can be given in perception. An idea can be nothing but an idea, and it is absurd to say that anything inaudible can resemble a sound, or that anything invisible can resemble a color. But what exists without the mind can neither be heard nor seen; for we perceive only sensations within the mind. It is said that *Matter* exists without the mind. But what is meant by matter? It is acknowledged to be known only as *supporting* the accidents of bodies; and this word 'supporting' in this connection is a word without meaning. Nor is there any necessity for the hypothesis of external bodies. What we observe is that we have ideas. Were there any use in supposing external things it would be to account for this fact. But grant that bodies exist, and no one can say how they can possibly affect the mind; so that instead of removing a difficulty, the hypothesis only makes a new one.

Peirce: CP 8.30 Cross-Ref:††

30. But though Berkeley thinks we know nothing out of the mind, he by no means holds that all our experience is of a merely phantasmagoric character. It is not all a dream; for there are two things which distinguish experience from imagination: one is the superior vividness of experience; the other and most important is its connected character. Its parts hang together in the most intimate and intricate conjunction, in consequence of which we can infer the future from the past. "These

two things it is," says Berkeley, in effect, "which constitute reality. I do not, therefore, deny the reality of common experience, although I deny its externality." Here we seem to have a third new conception of reality, different from either of those which we have insisted are characteristic of the nominalist and realist respectively, or if this is to be identified with either of those, it is with the realist view. Is not this something quite unexpected from so extreme a nominalist? To us, at least, it seems that this conception is indeed required to give an air of common sense to Berkeley's theory, but that it is of a totally different complexion from the rest. It seems to be something imported into his philosophy from without. We shall glance at this point again presently. He goes on to say that ideas are perfectly inert and passive. One idea does not make another and there is no power or agency in it. Hence, as there must be some cause of the succession of ideas, it must be *Spirit*. There is no *idea* of a spirit. But I have a consciousness of the operations of my spirit, what he calls a *notion* of my activity in calling up ideas at pleasure, and so have a relative knowledge of myself as an active being. But there is a succession of ideas not dependent on my will, the ideas of perception. Real things do not depend on my thought, but have an existence distinct from being perceived by me; but the *esse* of everything is *percipi*; therefore, *there must be some other mind wherein they exist*. "As sure, therefore, as the sensible world really exists, so sure do there an infinite omnipotent Spirit who contains and supports it."†13 This puts the keystone into the arch of Berkeleian idealism, and gives a theory of the relation of the mind to external nature which, compared with the Cartesian Divine Assistance, is very satisfactory. It has been well remarked that, if the Cartesian dualism be admitted, no divine *assistance* can enable things to affect the mind or the mind things, but divine power must do the whole work. Berkeley's philosophy, like so many others, has partly originated in an attempt to escape the inconveniences of the Cartesian dualism. God, who has created our spirits, has the power immediately to raise ideas in them; and out of his wisdom and benevolence, he does this with such regularity that these ideas may serve as signs of one another. Hence, the laws of nature. Berkeley does not explain how our wills act on our bodies, but perhaps he would say that to a certain limited extent we can produce ideas in the mind of God as he does in ours. But a material thing being only an idea, exists only so long as it is in some mind. Should every mind cease to think it for a while, for so long it ceases to exist. Its permanent existence is kept up by its being an idea in the mind of God. Here we see how superficially the just-mentioned theory of reality is laid over the body of his thought. If the reality of a thing consists in its harmony with the body of realities, it is a quite needless extravagance to say that it ceases to exist as soon as it is no longer thought of. For the coherence of an idea with experience in general does not depend at all upon its being actually present to the mind all the time. But it is clear that when Berkeley says that reality consists in the connection of experience, he is simply using the word *reality* in a sense of his own. That *an object's independence of our thought about it* is constituted by its connection with experience in general, he has never conceived. On the contrary, that, according to him, is effected by its being in the mind of God. In the usual sense of the word *reality*, therefore, Berkeley's doctrine is that the reality of sensible things resides only in their archetypes in the divine mind. This is Platonistic, but it is not realistic. On the contrary, since it places reality wholly out of the mind in the cause of sensations, and since it denies reality (in the true sense of the word) to sensible things in so far as they are sensible, it is distinctly nominalistic. Historically there have been prominent examples of an alliance between nominalism and Platonism. Abélard and John of Salisbury, the only two defenders of nominalism of the time of the great controversy whose works remain to us, are both Platonists; and Roscellin, the famous

author of the *sententia de flatu vocis*, the first man in the Middle Ages who carried attention to nominalism, is said and believed (all his writings are lost) to have been a follower of Scotus Erigena, the great Platonist of the ninth century. The reason of this odd conjunction of doctrines may perhaps be guessed at. The nominalist, by isolating his reality so entirely from mental influence as he has done, has made it something which the mind cannot conceive; he has created the so often talked of "improportion between the mind and the thing in itself." And it is to overcome the various difficulties to which this gives rise, that he supposes this *noumenon*, which, being totally unknown, the imagination can play about as it pleases, to be the emanation of archetypal ideas. The reality thus receives an intelligible nature again, and the peculiar inconveniences of nominalism are to some degree avoided.

Peirce: CP 8.31 Cross-Ref:††

31. It does not seem to us strange that Berkeley's idealistic writings have not been received with much favor. They contain a great deal of argumentation of doubtful soundness, the dazzling character of which puts us more on our guard against it. They appear to be the productions of a most brilliant, original, powerful, but not thoroughly disciplined mind. He is apt to set out with wildly radical propositions, which he qualifies when they lead him to consequences he is not prepared to accept, without seeing how great the importance of his admissions is. He plainly begins his principles of human knowledge with the assumption that we have nothing in our minds but sensations, external and internal, and reproductions of them in the imagination. This goes far beyond Locke; it can be maintained only by the help of that "mental chemistry" started by Hartley. But soon we find him admitting various *notions* which are not *ideas*, or reproductions of sensations, the most striking of which is the notion of a cause, which he leaves himself no way of accounting for experientially. Again, he lays down the principle that we can have no ideas in which the sensations are reproduced in an order or combination different from what could have occurred in experience; and that therefore we have no abstract conceptions. But he very soon grants that we can consider a triangle, without attending to whether it is equilateral, isosceles, or scalene; and does not reflect that such exclusive attention constitutes a species of abstraction. His want of profound study is also shown in his wholly mistaking, as he does, the function of the hypothesis of matter. He thinks its only purpose is to account for the production of ideas in our minds, so occupied is he with the Cartesian problem. But the real part that material substance has to play is to account for (or formulate) the constant connection between the accidents. In his theory, this office is performed by the wisdom and benevolence of God in exciting ideas with such regularity that we can know what to expect. This makes the unity of accidents a rational unity, the material theory makes it a unity not of a *directly* intellectual origin. The question is, then, which does experience, which does science decide for? Does it appear that in nature all regularities are directly rational, all causes final causes; or does it appear that regularities extend beyond the requirement of a rational purpose, and are brought about by mechanical causes. Now science, as we all know, is generally hostile to the final causes, the operation of which it would restrict within certain spheres, and it finds decidedly an other than directly intellectual regularity in the universe. Accordingly the claim which Mr. Collyns Simon, Professor Fraser, and Mr. Archer Butler make for Berkeleyanism, that it is especially fit to harmonize with scientific thought, is as far as possible from the truth. The sort of science that his idealism would foster would be one which should consist in saying what each natural production was made for. Berkeley's own remarks about natural philosophy show how little he sympathized with physicists. They should all be read; we have only room to quote a detached sentence or two:--

Peirce: CP 8.31 Cross-Ref:††

"To endeavor to explain the production of colors or sound by figure, motion, magnitude, and the like, must needs be labor in vain In the business of gravitation or mutual attraction, because it appears in many instances, some are straightway for pronouncing it *universal*; and that to attract and be attracted by every body is an essential quality inherent in all bodies whatever There is nothing necessary or essential in the case, but it depends entirely on the will of the Governing Spirit, who causes certain bodies to cleave together or tend towards each other according to various laws, whilst he keeps others at a fixed distance; and to some he gives a quite contrary tendency, to fly asunder just as he sees convenient First, it is plain philosophers amuse themselves in vain, when they inquire for any natural efficient cause, distinct from *mind* or *spirit*. Secondly, considering the whole creation is the workmanship of a *wise and good Agent*, it should seem to become philosophers to employ their thoughts (contrary to what some hold) about the final causes of things; and I must confess I see no reason why pointing out the various ends to which natural things are adapted, and for which they were originally with unspeakable wisdom contrived, should not be thought one good way of accounting for them, and altogether worthy of a philosopher." -- Vol. I. p. 466.†14

Peirce: CP 8.32 Cross-Ref:††

32. After this how can his disciples say "*that the true logic of physics is the first conclusion from his system*"!

Peirce: CP 8.33 Cross-Ref:††

33. As for that argument which is so much used by Berkeley and others, that such and such a thing cannot exist because we cannot so much as frame the idea of such a thing, -- that matter, for example, is impossible because it is an abstract idea, and we have no abstract ideas, -- it appears to us to be a mode of reasoning which is to be used with extreme caution. Are the facts such, that if we could have an idea of the thing in question, we should infer its existence, or are they not? If not, no argument is necessary against its existence, until something is found out to make us suspect it exists. But if we ought to infer that it exists, if we only could frame the idea of it, why should we allow our mental incapacity to prevent us from adopting the proposition which logic requires? If such arguments had prevailed in mathematics (and Berkeley was equally strenuous in advocating them there), and if everything about negative quantities, the square root of *minus*, and infinitesimals, had been excluded from the subject on the ground that we can form no idea of such things, the science would have been simplified no doubt, simplified by never advancing to the more difficult matters. A better rule for avoiding the deceits of language is this: Do things fulfil the same function practically? Then let them be signified by the same word. Do they not? Then let them be distinguished. If I have learned a formula in gibberish which in any way jogs my memory so as to enable me in each single case to act as though I had a general idea, what possible utility is there in distinguishing between such a gibberish and formula and an idea? Why use the term *a general idea* in such a sense as to separate things which, for all experiential purposes, are the same?†15

Peirce: CP 8.34 Cross-Ref:††

34. The great inconsistency of the Berkeleyan theory, which prevents his nominalistic principles from appearing in their true colors, is that he has not treated mind and matter in the same way. All that he has said against the existence of matter might be said against the existence of mind; and the only thing which prevented his seeing that, was the vagueness of the Lockian *reflection*, or faculty of internal perception. It was not until after he had published his systematic exposition of his doctrine, that this objection ever occurred to him. He alludes to it in one of his dialogues, but his answer to it is very lame. Hume seized upon this point, and, developing it, equally denied the existence of mind and matter, maintaining that only appearances exist. Hume's philosophy is nothing but Berkeley's, with this change made in it, and written by a mind of a more sceptical tendency. The innocent bishop generated Hume; and as no one disputes that Hume gave rise to all modern philosophy of every kind, Berkeley ought to have a far more important place in the history of philosophy than has usually been assigned to him. His doctrine was the half-way station, or necessary resting-place between Locke's and Hume's.

Peirce: CP 8.35 Cross-Ref:††

35. Hume's greatness consists in the fact that he was the man who had the courage to carry out his principles to their utmost consequences, without regard to the character of the conclusions he reached. But neither he nor any other one has set forth nominalism in an absolutely thoroughgoing manner; and it is safe to say that no one ever will, unless it be to reduce it to absurdity.

Peirce: CP 8.36 Cross-Ref:††

36. We ought to say one word about Berkeley's theory of vision. It was undoubtedly an extraordinary piece of reasoning, and might have served for the basis of the modern science. Historically it has not had that fortune, because the modern science has been chiefly created in Germany, where Berkeley is little known and greatly misunderstood. We may fairly say that Berkeley taught the English some of the most essential principles of that hypothesis of sight which is now getting to prevail, more than a century before they were known to the rest of the world. This is much; but what is claimed by some of his advocates is astounding. One writer says that Berkeley's theory has been accepted by the leaders of all schools of thought! Professor Fraser admits that it has attracted no attention in Germany, but thinks the German mind too *a priori* to like Berkeley's reasoning. But Helmholtz, who has done more than any other man to bring the empiricist theory into favor, says: "Our knowledge of the phenomena of vision is not so complete as to allow only one theory and exclude every other. It seems to me that the choice which different *savans* make between different theories of vision has thus far been governed more by their metaphysical inclinations than by any constraining power which the facts have had."†16 The best authorities, however, prefer the empiricist hypothesis; the fundamental proposition of which, as it is of Berkeley's, is that the sensations which we have in seeing are signs of the relations of things whose interpretation has to be discovered inductively. In the enumeration of the signs and of their uses, Berkeley shows considerable power in that sort of investigation, though there is naturally no very close resemblance between his and the modern accounts of the matter. There is no modern physiologist who would not think that Berkeley had greatly exaggerated the part that the muscular sense plays in vision.

Peirce: CP 8.37 Cross-Ref:††

37. Berkeley's theory of vision was an important step in the development of the associationalist psychology. He thought all our conceptions of body and of space

were simply reproductions in the imagination of sensations of touch (including the muscular sense). This, if it were true, would be a most surprising case of mental chemistry, that is of a sensation being felt and yet so mixed with others that we cannot by an act of simple attention recognize it. Doubtless this theory had its influence in the production of Hartley's system.

Peirce: CP 8.37 Cross-Ref:††

Hume's phenomenalism and Hartley's associationalism were put forth almost contemporaneously about 1750. They contain the fundamental positions of the current English "positivism." From 1750 down to 1830 -- eighty years -- nothing of particular importance was added to the nominalistic doctrine. At the beginning of this period Hume was toning down his earlier radicalism, and Smith's theory of Moral Sentiments appeared. Later came Priestley's materialism, but there was nothing new in that; and just at the end of the period, Brown's Lectures on the Human Mind. The great body of the philosophy of those eighty years is of the Scotch common-sense school. It is a weak sort of realistic reaction, for which there is no adequate explanation within the sphere of the history of philosophy. It would be curious to inquire whether anything in the history of society could account for it. In 1829 appeared James Mill's Analysis of the Human Mind, a really great nominalistic book again. This was followed by Stuart Mill's Logic in 1843. Since then, the school has produced nothing of the first importance; and it will very likely lose its distinctive character now for a time, by being merged in an empiricism of a less metaphysical and more working kind. Already in Stuart Mill the nominalism is less salient than in the classical writers; though it is quite unmistakable.

Peirce: CP 8.38 Cross-Ref:††
§5. SCIENCE AND REALISM

38. Thus we see how large a part of the metaphysical ideas of today have come to us by inheritance from very early times, Berkeley being one of the intellectual ancestors whose labors did as much as any one's to enhance the value of the bequest. The realistic philosophy of the last century has now lost all its popularity, except with the most conservative minds. And science as well as philosophy is nominalistic. The doctrine of the correlation of forces, the discoveries of Helmholtz, and the hypotheses of Liebig and of Darwin, have all that character of explaining familiar phenomena apparently of a peculiar kind by extending the operation of simple mechanical principles, which belongs to nominalism. Or if the nominalistic character of these doctrines themselves cannot be detected, it will at least be admitted that they are observed to carry along with them those daughters of nominalism, -- sensationalism, phenomenalism, individualism, and materialism. That physical science is necessarily connected with doctrines of a debasing moral tendency will be believed by few. But if we hold that such an effect will not be produced by these doctrines on a mind which really understands them, we are accepting this belief, not on experience, which is rather against it, but on the strength of our general faith that what is really true it is good to believe and evil to reject. On the other hand, it is allowable to suppose that science has no essential affinity with the philosophical views with which it seems to be every year more associated. History cannot be held to exclude this supposition; and science as it exists is certainly much less nominalistic than the nominalists think it should be. Whewell represents it quite as well as Mill.

Yet a man who enters into the scientific thought of the day and has not materialistic tendencies, is getting to be an impossibility. So long as there is a dispute between nominalism and realism, so long as the position we hold on the question is not determined by any proof *indisputable*, but is more or less a matter of inclination, a man as he gradually comes to feel the profound hostility of the two tendencies will, if he is not less than man, become engaged with one or other and can no more obey both than he can serve God and Mammon. If the two impulses are neutralized within him, the result simply is that he is left without any great intellectual motive. There is, indeed, no reason to suppose the logical question is in its own nature unsusceptible of solution. But that path out of the difficulty lies through the thorniest mazes of a science as dry as mathematics. Now there is a demand for mathematics; it helps to build bridges and drive engines, and therefore it becomes somebody's business to study it severely. But to have a philosophy is a matter of luxury; the only use of that is to make us feel comfortable and easy. It is a study for leisure hours; and we want it supplied in an elegant, an agreeable, an interesting form. The law of natural selection, which is the precise analogue in another realm of the law of supply and demand, has the most immediate effect in fostering the other faculties of the understanding, for the men of mental power succeed in the struggle for life; but the faculty of philosophizing, except in the literary way, is not called for; and therefore a difficult question cannot be expected to reach solution until it takes some practical form. If anybody should have the good luck to find out the solution, nobody else would take the trouble to understand it. But though the question of realism and nominalism has its roots in the technicalities of logic, its branches reach about our life. The question whether the *genus homo* has any existence except as individuals, is the question whether there is anything of any more dignity, worth, and importance than individual happiness, individual aspirations, and individual life. Whether men really have anything in common, so that the *community* is to be considered as an end in itself, and if so, what the relative value of the two factors is, is the most fundamental practical question in regard to every public institution the constitution of which we have it in our power to influence.

Peirce: CP 8.39 Cross-Ref: ††
CHAPTER 3

JOSIAH ROYCE, *THE RELIGIOUS ASPECT OF PHILOSOPHY* †1

§1. THE CONCEPT OF REALITY

39. Dr. Royce has produced a work which will form a good introduction to Hegel. His language and his thought are equally lucid and within the capacity of ordinary minds, -- his style is animated and readable, and in passages rises without effort to true philosophical eloquence. His method is a dialectic one; that is to say, it proceeds by the criticism of opinions, first, destructively to absolute scepticism, and then finds hidden in that scepticism itself the highest truth. It differs, however, very

decidedly from the dialectic of Hegel, -- and in its simplicity and general tone reminds us rather of the reasoning of Plato.

Peirce: CP 8.40 Cross-Ref:††

40. But before we examine the method, let us glance at the philosophical upshot of the book. This is, that the reality of whatever really exists consists in the real thing being thought by God.†2 Ordinary people think that things exist by the *will* of God; and if thought be taken in so wide a sense as to include volition, they have no difficulty in admitting the proposition which Dr. Royce has borrowed from Hegel and Schelling. But ordinary people say that not merely the real but all that can possibly enter into the mind of man must be within the thought of God in some sense; so that it must be some particular kind of divine thought which constitutes reality; and that particular kind of thought must be distinguished by a volitional element. In short, ordinary people make at once the very same criticism that the profoundest students of philosophy have made, namely, that the Hegelian school overlooks the importance of the will as an element of thought.

Peirce: CP 8.41 Cross-Ref:††

41. A certain writer has suggested that reality, the fact that there is such a thing as a true answer to a question, consists in this: that human inquiries, -- human reasoning and observation, -- tend toward the settlement of disputes and ultimate agreement in definite conclusions which are independent of the particular stand-points from which the different inquirers may have set out; so that the real is that which any man would believe in, and be ready to act upon, if his investigations were to be pushed sufficiently far.†3 Upon the luckless putter-forth of this opinion Dr. Royce is extremely severe. He will not even name him (perhaps to spare the family), but refers to him by various satirical nick-names, especially as "*Thrasymachus*,"†4 -- a foolish character introduced into the Republic and another dialogue of Plato for the purpose of showing how vastly such an ignorant pretender to philosophy is inferior to Socrates (that is, to Plato himself) in every quality of mind and heart, and especially in good manners. But I must with shame confess that if I understand what the opinion of this poor, Royce-forsaken Thrasymachus is, I coincide with it exactly. I ask any man, Suppose you could be miraculously assured that a certain answer to any question that interests you would be the one in which, were your life and mental vigor to be indefinitely prolonged, you must eventually rest, would you not cease all inquiries at once, and be content with that answer now, as being the very thing you had been striving after? This question Dr. Royce answers explicitly in the negative. "No barely possible judge," he says, "who *would* see the error, *if* he were there, will do for us."†5 Yet if I were to represent Dr. Royce as preferring to believe for a little while that which a certain Being -- no matter who -- imagines, rather than to come at once to the belief to which investigation is destined at last to carry him, I should probably be doing him injustice; because I suppose he would say that the thing which God imagines, and the opinion to which investigation would ultimately lead him, in point of fact, coincide. If, however, these two things coincide, I fail to understand why he should be so cruel to the childish Thrasymachus; since after all there is no real difference between them, but only a formal one, -- each maintaining as a theorem that which the other adopts as a definition. As was just remarked, the Hegelian school does not sufficiently take into account the volitional element of cognition. Dr. Royce admits in words that belief is what a man will act from; but he does not seem to have taken the truth of this proposition home to him, or else he would see that the whole end of inquiry is the settlement of belief; so that a man shall not war against himself, nor undo tomorrow that which he begins to do today. Dr. Royce's main argument in

support of his own opinion, to the confusion of Thrasymachus, is drawn from the existence of error. Namely, the subject of an erroneous proposition could not be identified with the subject of the corresponding true proposition, except by being completely known, and in that knowledge no error would be possible. The truth must, therefore, be present to the actual consciousness of a living being. This is an argument drawn from Formal logic, for Formal logic it is which inquires how different propositions are made to refer to the same subject, and the like. German metaphysics has, since Kant, drawn its best arguments from Formal logic; and it is quite right in doing so, for the conceptions which are proved to be indispensable in Formal logic, must have been already rooted in the nature of the mind when reasoning first began, and are, so far, *a priori*. But one would surely have supposed that when the German philosophers were thus drawing their arguments from formal logic, they would have postponed their venturesome flights into the thin air of theology and the vacuum of pure reason, until they had carefully tried the strength of every part of that logical machine on which they were to depend. Instead of that, they have left the great work of creating a true system of formal logic to English authors, who, while they have done most excellent work, have (with the insignificant exception of the present writer) been quite indifferent to the transcendental bearings of their results. Kant gives a half dozen only of his brief pages to the development of the system of logic upon which his whole philosophy rests; and though many valuable treatises on the science have appeared in Germany, there is hardly one of them which is not more or less marred by some arrant absurdity, acknowledged to be so by all the others; Grassmann and Schroeder alone pursuing the one method which will yield positive results properly secured against error. We must not, therefore, wonder that Dr. Royce's argument from formal logic overlooks one of the most important discoveries that have lately resulted from the study of that exact branch of philosophy. He seems to think that the real subject of a proposition can be denoted by a general term of the proposition; that is, that precisely what it is that you are talking about can be distinguished from other things by giving a general description of it. Kant already showed, in a celebrated passage of his cataclysmic work, that this is not so; and recent studies in formal logic †6 have put it in a clearer light. We now find that, besides general terms, two other kinds of signs are perfectly indispensable in all reasoning. One of these kinds is the *index*, which like a pointing finger exercises a real physiological *force* over the attention, like the power of a mesmerizer, and directs it to a particular object of sense. †7 One such index at least must enter into every proposition, its function being to designate the subject of discourse. Now observe that Dr. Royce does not merely say that there are no means by which an erroneous proposition can be produced; what he says is that the conception of an erroneous proposition (without an actual including consciousness) is absurd. If the subject of discourse had to be distinguished from other things, if at all, by a general term, that is, by its peculiar characters, it would be quite true that its complete segregation would require a full knowledge of its characters and would preclude ignorance. But the index, which in point of fact alone can designate the subject of a proposition, designates it without implying any characters at all. A blinding flash of lightning forces my attention and directs it to a certain moment of time with an emphatic "Now!" Directly following it, I may judge that there will be a terrific peal of thunder, and if it does not come I acknowledge an error. One instant of time is, in itself, exactly like any other instant, one point of space like any other point; nevertheless dates and positions can be approximately distinguished. And how are they so distinguished? By *intuition* says Kant; perhaps not in so many words; but it is because of this property that he distinguishes Space and Time from the general

conceptions of the understanding and sets them off by themselves under the head of intuition. But I should prefer to say that it is by volitional acts that dates and positions are distinguished. The element of feeling is so prominent in sensations, that we do not observe that something like Will enters into them, too. You may quarrel with the word *volition* if you like; I wish I had a more general one at my hand. But what I mean is that that strong, clear, and voluntary consciousness in which we act upon our muscles is nothing more than the most marked variety of a kind of consciousness which enters into many other phenomena of our life, a consciousness of duality or dual consciousness. Feeling is simple consciousness, the consciousness that can be contained within an instant of time, the consciousness of the excitation of nerve-cells; it has no parts and no unity.†8 What I call volition is the consciousness of the discharge of nerve-cells, either into the muscles, etc., or into other nerve-cells; it does not involve the sense of time (i.e. not of a continuum) but it does involve the sense of action and reaction, resistance, externality, otherness, pair-edness. It is the sense that something has hit me or that I am hitting something; it might be called the sense of collision or clash. It has an outward and an inward variety, corresponding to Kant's outer and inner sense, to will and self-control, to nerve-action and inhibition, to the two logical types A:B and A:A. The capital error of Hegel which permeates his whole system in every part of it is that he almost altogether ignores the Outward Clash.†9 Besides the lower consciousness of feeling and the higher consciousness of nutrition, this direct consciousness of hitting and of getting hit enters into all cognition and serves to make it mean something real. It is formal logic which teaches us this; not that of a Whateley or a Jevons, but formal logic in its new development, drawing nutriment from physiology and from history without leaving the solid ground of logical forms.

Peirce: CP 8.42 Cross-Ref:††

42. An objection different from that of Dr. Royce might be raised. Namely it might be asked *how* two different men can know they are speaking of the same thing. Suppose, for instance, one man should say a flash of lightning was followed by thunder and another should deny it. How would they know they meant the same flash? The answer is that they would compare notes somewhat as follows. One would say, "I mean that very brilliant flash which was preceded by three slight flashes, you know." The second man would recognize the mark, and thus by a probable and approximate inference they would conclude they meant the same flash.

Peirce: CP 8.43 Cross-Ref:††

43. Dr. Royce in describing the opinion of Thrasymachus has selected the expression "a *barely* possible judge."†10 Is there not an ambiguity in this mode of speech which is unfair to Thrasymachus? The final opinion which would be sure to result from sufficient investigation may possibly, in reference to a given question, never be actually attained, owing to a final extinction of intellectual life or for some other reason. In that sense, this final judgment is not *certain* but only possible. But when Dr. Royce says "bare possibility is blank nothingness,"†11 he would seem to be speaking of mere logical possibility, and not a possibility which differs but a hair's breadth from entire certainty. Let us consider what probability there is that a given question, say one capable of being answered by *yes* or *no*, will never get answered. Let us reason upon this matter by inductive logic. Dr. Royce and his school, I am well aware, consider inductive reasoning to be radically vicious; so that we unhappily cannot carry them along with us. (They often deny this, by the way, and say they rest entirely on experience. This is because they so overlook the Outward Clash, that they do not know what experience is. They are like Roger Bacon, who after stating in

eloquent terms that all knowledge comes from experience, goes on to mention spiritual illumination from on high as one of the most valuable kinds of experiences.) But they will not succeed in exploding the method of modern science; and there is no reason why those who believe in induction at all, should not be willing to apply it to the subject now in hand. In the first place, then, upon innumerable questions, we have already reached the final opinion. How do we know that? Do we fancy ourselves infallible? Not at all; but throwing off as probably erroneous a thousandth or even a hundredth of all the beliefs established beyond present doubt, there must remain a vast multitude in which the final opinion has been reached. Every directory, guide-book, dictionary, history, and work of science is crammed with such facts. In the history of science, it has sometimes occurred that a really wise man has said concerning one question or another that there was reason to believe it never would be answered. The proportion of these which have in point of fact been conclusively settled very soon after the prediction has been surprisingly large. Our experience in this direction warrants us in saying with the highest degree of empirical confidence that questions that are either practical or could conceivably become so are susceptible of receiving final solutions provided the existence of the human race be indefinitely prolonged and the particular question excite sufficient interest. As for questions which have no conceivable practical bearings, as the question whether force is an entity, they mean nothing and may be answered as we like, without error.†12 We may take it as certain that the human race will ultimately be extirpated; because there is a certain chance of it every year, and in an indefinitely long time the chance of survival compounds itself nearer and nearer zero. But, on the other hand, we may take it as certain that other intellectual races exist on other planets, -- if not of our solar system, then of others; and also that innumerable new intellectual races have yet to be developed; so that on the whole, it may be regarded as most certain that intellectual life in the universe will never finally cease. The problem whether a given question will ever get answered or not is not so simple; the number of questions asked is constantly increasing, and the capacity for answering them is also on the increase. If the rate of the latter increase is greater than that of the [former] the probability is unity that any given question will be answered; otherwise the probability is *zero*. Considerations too long to be explained here lead me to think that the former state of things is the actual one. In that case, there is but an infinitesimal proportion of questions which do not get answered, although the multitude of unanswered questions is forever on the increase. It plainly is not fair to call a judgment which is certain to be made a "barely possible" one. But I will admit (if the reader thinks the admission has any meaning, and is not an empty proposition) that some finite number of questions, we can never know which ones, will escape getting answered forever. Nor must I forget that I have not given the reader my proof that of the questions asked at any time the proportion that will never be answered is infinitesimal; so that he may be in doubt upon this point. That is not a thing to be regretted; for scepticism about the reality of things, -- provided it be genuine and sincere, and not a sham, -- is a healthful and growing stage of mental development. Let us suppose, then, for the sake of argument, that some questions eventually get settled, and that some others, indistinguishable from the former by any marks, never do. In that case, I should say that that conception of reality was rather a faulty one, for while there is a real so far as a question that will get settled goes, there is none for a question that will never be settled; for an unknowable reality is nonsense. The non-idealistic reader will start at this last assertion; but consider the matter from a practical point of view. You say that real things are manifested by their effects. True; for example, if the timbers of my house are inwardly rotting, it will some day fall down, and thus there will be a

practical effect for me, whether I know the beams be rotten or not. Well, but if all the effects consistently point to the theory that the beams are rotting, it will come to be admitted at last that they are so; and if nothing is ever settled about the matter, it will be because the phenomena do not consistently point to any theory; and in that case there is a want of that "uniformity of nature" (to use a popular but very loose expression) which constitutes reality, and makes it differ from a dream. In that way, if we think that some questions are never going to get settled, we ought to admit that our conception of nature as absolutely real is only partially correct. Still, we shall have to be governed by it practically; because there is nothing to distinguish the unanswerable questions from the answerable ones, so that investigation will have to proceed as if all were answerable. In ordinary life, no matter how much we believe in questions ultimately getting answered, we shall always put aside an innumerable throng of them as beyond our powers. We shall not in our day seek to know whether the centre of the sun is distant from that of the earth by an odd or an even number of miles on the average; we shall act as if neither man nor God could ever ascertain it. There is, however, an economy of thought, in assuming that it is an answerable question. From this practical and economical point of view, it really makes no difference whether or not all questions are actually answered, by man or by God, so long as we are satisfied that investigation has a universal tendency toward the settlement of opinion; and this I conceive to be the position of Thrasymachus.

Peirce: CP 8.44 Cross-Ref:††

44. If there be any advantage to religion in supposing God to be omniscient, this sort of scepticism about reality can do no practical harm. We can still suppose that He knows all that there is of real to be known. On the theory of Dr. Royce, the real existence of God would consist in his imagining or positing Himself; it would thus be, according to him, of the same nature as the reality of anything else. For my part, I hold another theory, which I intend to take an early opportunity of putting into print.†13 I think that the existence of God, as well as we can conceive of it consists in this, that a tendency toward ends is so necessary a constituent of the universe that the mere action of chance upon innumerable atoms has an inevitable teleological result. One of the ends so brought about is the development of intelligence and of knowledge; and therefore I should say that God's omniscience, humanly conceived, consists in the fact that knowledge in its development leaves no question unanswered. The scepticism just spoken of would admit this omniscience as a regulative but not a speculative conception. I believe that even that view is more religiously fruitful than the opinion of Dr. Royce.

Peirce: CP 8.45 Cross-Ref:††

§2. COMMENTS ON ROYCE'S PHILOSOPHY

45. Let us now turn to the examination of Dr. Royce's peculiar method of reasoning; for that is always the most important element in every system of philosophy. His work is divided into a brief introduction and two books, the first entitled "The Search for a Moral Ideal"; the second "The Search for a Religious Truth." These titles seem to me to point, at the outset, to a fault of method. The pursuit of a conscience, if one hasn't one already, or of a religion, which is the subjective basis of conscience, seems to me an aimless and hypochondriac pursuit. If a man finds himself under no sense of obligation, let him congratulate himself. For

such a man to hanker after a bondage to conscience, is as if a man with a good digestion should cast about for a regimen of food. A conscience, too, is not a theorem or a piece of information which may be acquired by reading a book; it must be bred in a man from infancy or it will be a poor imitation of the genuine article. If a man has a conscience, it may be an article of faith with him, that he should reflect upon that conscience, and thus it may receive a further development. But it never will do him the least good to get up a make-believe scepticism and pretend to himself not to believe what he really does believe. In point of fact, every man born and reared in a christian community, however little he may believe the dogmas of the Church, does find himself believing with the strongest conviction in the moral code of christendom. He has a horror of murder and incest, a disapproval of lying, etc., which he cannot escape from. The modern dialectician (if he will pardon a touch of exaggeration) would have such a man say to himself, Now I am going to be sceptical, but only provisionally so, in order to return to my faith with renewed conviction! But the whole history of thought shows that men cannot doubt at pleasure or merely because they find they have no positive reason for the belief they already hold. Reasons concern the man who is coming to believe, not the man who believes already. It has often been remarked that metaphysics is an imitation of mathematics; and it may be added that the philosophic doubt is an imitation of the absurd procedure of elementary geometry, which begins by giving worthless demonstrations of propositions nobody ever questions. When Hegel tells me that thought has three stages, that of naïve acceptance, that of reaction and criticism, and that of rational conviction; in a general sense, I agree to it. And a down-right living scepticism without *arrière-pensée*, may be beneficial. It is not perhaps easy to see why an imaginary scepticism might not sometimes serve the same purpose; but experience shows that in questions of magnitude men haven't imagination enough to put themselves in the true doubter's shoes. But be that as it may, the idea that the mere reaction of assent and doubt, the mere play of thought, the heat-lightning of the brain, is going to settle anything in this real world to which we appertain, -- such an idea only shows again how the Hegelians overlook the facts of volitional action and reaction in the development of thought. I find myself in a world of forces which act upon me, and it is they and not the logical transformations of my thought which determine what I shall ultimately believe.

Peirce: CP 8.46 Cross-Ref:††

46. Dr. Royce seems to hold that at least in the philosophy of morals and religion a mere contemplation of our own crude beliefs will lead us to absolute scepticism and that then a mere contemplation of our own absolute scepticism will lead us back to rational conviction. Neither I nor the readers of the *Popular Science Monthly* can possibly believe that, in advance. But let us see how the method will work when applied to the discussion of ethics.

Peirce: CP 8.47 Cross-Ref:††

47. The moral stand-point from which every man with a christian training sets out, even if he be a dogmatic atheist, is pretty nearly the same. He has a horror of certain crimes and a disapproval of certain lesser sins. He is also more or less touched with the spirit of christian love, which he believes should be his beacon, and which in point of fact, by its power in his heart, shall and will govern him in all questions of disputed morals. More or less, in all of us, this sentiment replaces and abolishes conscience; like Huckleberry Finn, we act from christian charity without caring very much whether conscience approves of the act or not.

Peirce: CP 8.47 Cross-Ref:††

This is the state of mind of the ordinary man or woman who will open Dr. Royce's book. And now Dr. Royce proposes that this person shall ask himself the question, what validity or truth is there in the distinction of right and wrong. To me, it plainly appears that such a person, if he have a clear head, will at once reply, right and wrong are nothing to me except so far as they are connected with certain rules of living by which I am enabled to satisfy a real impulse which works in my heart; and this impulse is the love of my neighbor elevated into a love of an ideal and divine humanity which I identify with the providence that governs the world. But Dr. Royce says that different people will answer the question in different ways; some will take the position of the 'moral realist' and say that moral distinctions are founded on some matter of fact (say a decree from Sinai), while others will take the position of the 'moral idealist' and say that these distinctions are founded on an inward sentiment, -- an ideal.†14 Two such persons come into collision; they find by mutual criticism that both positions are unsatisfactory; external fact can only determine what *is*, not what *ought* to be; while inward sentiment cannot be a resting-place, because it is only individual caprice and has no authority for another man. From this criticism the only outcome is ethical scepticism.

Peirce: CP 8.48 Cross-Ref:††

48. This is a fair specimen of Dr. Royce's logical method, which is a mere apotheosis of the dilemma, as the great instrument of thought. As compared with syllogistic method of the middle ages (which survives in certain quarters, yet) it is certainly wonderfully superior; but as compared with the mathematical reasoning upon which modern science is built, it is inefficacious and restricted.

Peirce: CP 8.49 Cross-Ref:††

49. In the particular case in hand, it appears to me that the ordinary christian does not find himself caught in Dr. Royce's dilemma at all. He is a moral idealist; yet far from being shaken by the spectacle of different men having different passions, he feels that every man may come to the same passion which animates him by a mere enlargement of his horizon, and that his is the only sentiment in which all others may be reconciled. For altruism is but a developed egoism; that same sensitiveness which in its lowest state is selfishness, first transforms itself into *esprit de corps* or collective selfishness; then, passing from feeling for others collectively to feeling for them individually, it becomes philanthropy, pity, sympathy tossed hither and thither rudderless on the ocean of human misery; finally, steadying itself by the conception of an ideal humanity and a divine providence, it passes into christian charity, which gathers up all selfishnesses and all pities and is ready to give each its due measure.

Peirce: CP 8.50 Cross-Ref:††

50. The author having stated the above argument with admirable clearness, fills a hundred pages with a perhaps not altogether necessary, though a charmingly written and highly interesting elaboration and illustration of it. He here passes in review a goodly number of the ethical theories which have been proposed at different times. After the Sophists, Plato, Aristotle, and the Stoics, he criticizes what he conceives to be the ethics of Jesus.†15 Every christian will tell him that he makes the mistake of viewing that as a *theory* or speculation which is really a spiritual *experience*; -- another example of his neglect of the volitional element. For instance, he asks, "If I feel not the love of God, how prove to me that I ought to feel it?"†16 The answer to that need not be pointed out.

Peirce: CP 8.51 Cross-Ref:††

51. In what he says about Herbert Spencer, he seems to forget that Mr.

Spencer is not addressing a body of moral sceptics but readers animated by the sentiments which, in our day, animate every man who reads at all.†17

Peirce: CP 8.52 Cross-Ref:††

52. At last he takes up the thread of his argument as follows. The conflict between moral realism and idealism can only lead to moral scepticism. Now what is this scepticism? It is the contemplation of two opposing aims. Here he adduces the testimony of modern psychologists to show that we cannot think of willing without actually willing. (But for all that, I fancy I notice a difference sometimes in cold weather between thinking of willing to take my morning dash of cold water and actually willing to take it.) Scepticism, then, shares at once these opposing aims, or strives to share them. It has thus itself an aim, namely, to reconcile opposing aims. So absolute moral scepticism is self-destructive. "Possibly this result may be somewhat unexpected,"†18 says our author. Not at all unexpected to one who does not believe in the dialectical method. You started with a hypochondriac hankering after an aim; and now you have acquired it. *Eureka!* Well, what is it, this aim which you have at last got? Why, to have an aim! But that is nothing but the old nonsensical longing with which you set out. Like Kant's dove, you have been winging [in] a vacuum, without remarking that you never advanced an inch. I do not misrepresent the author. "For behold," he says, "made practical, brought down from its lonesome height, my Ideal very simply means the Will to direct my acts *towards* the attainment of universal Harmony."†19 But this, I must insist, was obviously implied in the original fantastic desire to have an aim. When I say that this is a fantastic desire, I do not of course mean to deny that there may be such an operation as the *choice of an aim*, if by that aim he meant a secondary or derived one; but I do say that it is absurd to speak of choosing an original and ultimate aim. That is something which, if you haven't it, you have nothing to do but wait till the grace of God confers it on you. I should think, however, that were it once admitted to be a rational performance to go a hunting for an ultimate aim or end, the first preliminary would be to recognize the axiom that such an end must have unity, after which the hunt might begin. But Royce, calling this axiom the 'ideal of ideals,'†20 as it certainly is, in a sense, exclaims 'Here I have the aim I wanted, and the hunt is over.' If one might be permitted to enliven a dry subject with a little folly, I should say that it reminded me of the surveyor Phoenix, who after purchasing 365 solar compasses and a vast amount of other paraphernalia, in order to ascertain the distance between San Francisco and the Mission of Dolores, stepped into a grocery and inquired how far it was, and returned "much pleased at so easily acquiring so much valuable information." If Dr. Royce merely means that it can be shown that a man who fancies he has no moral ideal really has one, I heartily grant it; and I will further admit that dialectic is the proper instrument to show this. But then a very lowly kind of dialectic will do; and a rather more definite ideal may be pointed out.

Peirce: CP 8.53 Cross-Ref:††

53. The rest of Book I is occupied, as it seems to me, with illicitly slipping some content into an empty formula. Much of this part of the book is splendidly said. But other passages seem to me to preach, in a way quite uncalled for by the premises, an ethics of the evil eye. "It is well that we should feel . . . joy whenever pride has a fall In all such ways . . . we must show *no mercy*."†21 "When the hedonist gives us his picture of a peaceful society, where, in the midst of [universal] good humor, his ideal, the happiness of everybody concerned, is steadfastly pursued, we find ourselves disappointed and contemptuous Who cares whether that [really] wretched set . . . think themselves happy or not?"†22 "The appearance of anybody who pretends to be

content with himself must be the signal not for admiration at the sight of his success, but for a good deal of contempt"†23 etc. Some of the students to whom this ethics is taught at Harvard may upon reflection think that christian charity is not so much lower a frame of mind after all.

Peirce: CP 8.54 Cross-Ref:††

54. In Book II, Dr. Royce undertakes, by the same dialectic procedure to establish the existence of a God. Space does not permit me to enter into a criticism of the second book; nor is it necessary, for it consists only of an application of the same method to a subject to which dialectics is far less suited. Besides, to the reader who has had the kindness and the resolution to follow me to this point I can say, 'You are the man to enjoy Dr. Royce's own book, which I can promise you you shall find, in comparison with [the] harsh and crabbed matter you have been reading here, to be "as musical as is Apollo's lute."

Peirce: CP 8.55 Cross-Ref:††
CHAPTER 4

WILLIAM JAMES, *THE PRINCIPLES OF PSYCHOLOGY*

§1. REVIEW IN THE NATION †1

55. Upon this vast work no definitive judgment can be passed for a long time; yet it is probably safe to say that it is the most important contribution that has been made to the subject for many years. Certainly it is one of the most weighty productions of American thought. The directness and sharpness with which we shall state some objections to it must be understood as a tribute of respect.

Peirce: CP 8.56 Cross-Ref:††

56. Beginning with the most external and insignificant characters, we cannot much admire it as a piece of bookmaking; for it misses the unity of an essay, and almost that of a connected series of essays, while not attaining the completeness of a thorough treatise. It is a large assortment of somewhat heterogeneous articles loosely tied up in one bag, with tendencies towards sprawling.

Peirce: CP 8.57 Cross-Ref:††

57. With an extraordinarily racy and forcible style, Prof. James is continually wresting words and phrases of exact import to unauthorized and unsuitable uses. He indulges himself with idiosyncrasies of diction and tricks of language such as usually spring up in households of great talent. To illustrate what we mean, we will open one of the volumes at random, and we come upon this: "A statement *ad hominem* meant as part of a reduction to the absurd."†2 Now a *reductio ad absurdum* is a species of demonstration, and as such can contain no *argumentum ad hominem*, which is merely something a man is obliged by his personal interests to admit. On the next page, we read: "This dynamic (we had almost written dynamitic) way of representing

knowledge." On the next page: "They talk as if, with this miraculous tying or 'relating,' the Ego's duties were done." It is the same with the technical terms of psychology. Speaking of certain theories, our author says they "carry us back to times when the soul as vehicle of consciousness was not discriminated, as it now is, from the vital principle presiding over the formation of the body."†3 How can anybody write so who knows the technical meaning of *vehicle*? On the same page occurs this phrase, "If unextended, it is absurd to speak of its having space relations at all," which sounds like a general attack on the geometry of points.

Peirce: CP 8.58 Cross-Ref:††

58. Prof. James's thought is highly original, or at least novel; but it is originality of the destructive kind. To prove that we do not know what it has been generally supposed that we did know, that given premises do not justify the conclusions which all other thinkers hold they do justify, is his peculiar function. For this reason the book should have been preceded by an introduction discussing the strange positions in logic upon which all its arguments turn. Even when new theories are proposed, they are based on similar negative or sceptical considerations, and the one thing upon which Prof. James seems to pin his faith is the general incomprehensibility of things. He clings as passionately to that as the old lady of the anecdote did to her total depravity. Of course, he is materialistic to the core -- that is to say, in a methodical sense, but not religiously, since he does not deny a separable soul nor a future life; for materialism is that form of philosophy which may safely be relied upon to leave the universe as incomprehensible as it finds it. It is possible that Prof. James would protest against this characterization of his cast of mind. Brought up under the guidance of an eloquent apostle of a form of Swedenborgianism,†4 which is materialism driven deep and clinched on the inside, and educated to the materialistic profession, it can only be by great natural breadth of mind that he can know what materialism is, by having experienced some thoughts that are not materialistic. He inclines towards Cartesian dualism, which is of the true strain of the incomprehensibles and modern materialism's own mother. There is no form of idealism with which he will condescend to argue. Even evolutionism, which has idealistic affinities, seems to be held for suspect. It is his *métier* to subject to severe investigation any doctrine whatever which smells of intelligibility.

Peirce: CP 8.59 Cross-Ref:††

59. The keynote of this is struck in the preface, in these words:

"I have kept close to the point of view of natural science throughout the book. Every natural science assumes certain data uncritically, and declines to challenge the elements between which its own 'laws' obtain, and from which its deductions are carried on. Psychology, the science of finite individual minds, assumes as its data (1) *thoughts and feelings*, and (2) *a physical world* in time and space with which they coexist and which (3) *they know*. Of course these data themselves are discussable; but the discussion of them (as of other elements) is called metaphysics, and falls outside the province of this book. This book, assuming that thoughts and feelings exist, and are the vehicles of knowledge, thereupon contends that Psychology, when she has ascertained the empirical correlation of the various sorts of thought and feeling with definite conditions of the brain, can go no farther -- can go no farther, that is, as a natural science. If she goes farther, she becomes metaphysical. All attempts to *explain* our phenomenally given thoughts as products of deeper-lying entities (whether the latter be named 'Soul,' 'Transcendental Ego,' 'Ideas,' or 'Elementary Units of Consciousness') are metaphysical. This book consequently rejects both the

associationist and the spiritualist theories; and in this strictly positivistic point of view consists the only feature of it for which I feel tempted to claim originality."†5

Peirce: CP 8.60 Cross-Ref:††

60. This is certainly well put -- considered as prestigation. But when we remember that a natural science is not a person, and consequently does not "decline" to do anything, the argument evaporates. It is only the students of the science who can "decline," and they are not banded together to repress any species of inquiry. Each investigator does what in him lies; and declines to do a thousand things most pertinent to the subject. To call a branch of an inquiry "metaphysical" is merely a mode of objurgation, which signifies nothing but the author's personal distaste for that part of his subject. It does not in the least prove that considerations of that sort can throw no light on the questions he has to consider. Indeed, we suspect it might be difficult to show in any way that any two branches of knowledge should be allowed to throw no light on one another. Far less can calling one question scientific and another metaphysical warrant Prof. James in "consequently *rejecting*" certain conclusions, against which he has nothing better to object. Nor is it in the least true that physicists confine themselves to such a "strictly positivistic point of view." Students of heat are not deterred by the impossibility of directly observing molecules from considering and accepting the kinetical theory; students of light do not brand speculations on the luminiferous ether as metaphysical; and the substantiality of matter itself is called in question in the vortex theory, which is nevertheless considered as perfectly germane to physics. All these are "attempts to explain phenomenally given elements as products of deeper-lying entities." In fact, this phrase describes, as well as loose language can, the general character of scientific hypotheses.

Peirce: CP 8.61 Cross-Ref:††

61. Remark, too, that it is not merely nor chiefly the "soul" and the "transcendental ego" for which incomprehensibles he has some tenderness, that Prof. James proposes to banish from psychology, but especially *ideas* which their adherents maintain are direct data of consciousness. In short, not only does he propose, by the simple expedient of declaring certain inquiries extra-psychological, to reverse the conclusions of the science upon many important points, but also by the same negative means to decide upon the character of its data. Indeed, when we come to examine the book, we find it is precisely this which is the main use the author makes of his new principle. The notion that the natural sciences accept their data *uncritically* we hold to be a serious mistake. It is true, scientific men do not subject their observations to the kind of criticism practised by the high-flying philosophers, because they do not believe that method of criticism sound. If they really believed in idealism, they would bring it to bear upon physics as much as possible. But in fact they find it a wordy doctrine, not susceptible of any scientific applications. When, however, a physicist has to investigate, say, such a subject as the scintillation of the stars, the first thing he does is to subject the phenomena to rigid criticism to find whether these phenomena are objective or subjective, whether they are in the light itself, or arise in the eye, or in original principles of mental action, or in idiosyncrasies of the imagination, etc. The principle of the uncritical acceptance of data, to which Prof. James clings, practically amounts to a claim to a new kind of liberty of thought, which would make a complete rupture with accepted methods of psychology and of science in general. The truth of this is seen in the chief application that has been made of the new method, in the author's theory of space-perception. And into the enterprise of thus revolutionizing scientific method he enters with a light heart, without any exhaustive scrutiny of his new logic in its generality, relying only on the resources of the moment. He distinctly

discourages a separate study of the method. "No rules can be laid down in advance. Comparative observations, to be definite, must usually be made to test some preëxisting hypothesis; and the only thing then is to use as much sagacity as you possess, and to be as candid as you can."†6

Peirce: CP 8.62 Cross-Ref:††

62. We have no space for any analysis of the contents of this work, nor is that necessary, for everybody interested in the subject must and will read the book. It discusses most of the topics of psychology in an extremely unequal way, but always interesting and always entertaining. We will endeavor to give a fair specimen of the author's critical method (for the work is essentially a criticism and exposition of critical principles), with a running commentary, to aid a judgment. For this purpose we will select a short section entitled "Is Perception Unconscious Inference?"†7 Perception in its most characteristic features is, of course, a matter of association in a wide sense of that term. If two spots of light are thrown upon the wall of a dark room so as to be adjacent, and one of these is made red while the other remains white, the white one will appear greenish by contrast. If they are viewed through a narrow tube, and this is moved so that the red spot goes out of view, still the white one will continue to look green. But if the red light, now unseen, be extinguished and we then remove the tube from the eye, so as to take a new look, as it were, the apparent greenness will suddenly vanish. This is an example of a thousand phenomena which have led several German psychologists to declare that the process of perception is one of reasoning in a generalized sense of that term.

Peirce: CP 8.63 Cross-Ref:††

63. It is possible some of the earlier writers held it to be reasoning, strictly speaking. But most have called it "unconscious inference," and unconscious inference differs essentially from inference in the narrow sense, all our control over which depends upon this, that it involves a conscious, though it may be an indistinct, reference to a genus of arguments. These German writers must also not be understood as meaning that the perceptive process is any more inferential than are the rest of the processes which the English have so long explained by association -- a theory which until quite recently played little part in German psychology. The German writers alluded to explain an ordinary suggestion productive of belief, or any cognition tantamount to belief, as inference conscious or unconscious, as a matter of course. As German writers are generally weak in their formal logic, they would be apt to formulate the inference wrongly; but the correct formulation is as follows:

Peirce: CP 8.64 Cross-Ref:††

64. A well-recognized kind of object, M, has for its ordinary predicates P[1], P[2], P[3], etc., indistinctly recognized.

The suggesting object, S, has these same predicates, P[1], P[2], P[3], etc.

Hence, S is of the kind M.

Peirce: CP 8.65 Cross-Ref:††

65. This is hypothetic inference in form. The first premise is not actually thought, though it is in the mind habitually. This, of itself, would not make the inference unconscious. But it is so because it is not recognized as an inference; the conclusion is accepted without our knowing how. In perception, the conclusion has the peculiarity of not being abstractly thought, but actually seen, so that it is not

exactly a judgment, though it is tantamount to one. The advantage of this method of explaining the process is conceived to be this: To explain any process not understood is simply to show that it is a special case of a wider description of process which is more intelligible. Now nothing is so intelligible as the reasoning process. This is shown by the fact that all explanation assimilates the process to be explained to reasoning. Hence, the logical method of explaining the process of association is looked upon as the most perfect explanation possible. It certainly does not exclude the materialistic English explanation by a property of the nerves. The monist school, to which the modern psychologists mostly belong, conceives the intellectual process of inference and the process of mechanical causation to be only the inside and outside views of the same process. But the idealistic tendency, which tinctures almost all German thought not very recent, would be to regard the logical explanation as the more perfect, under the assumption that the materialistic explanation requires itself ultimately to be explained in terms of the reasoning process. But Prof. James is naturally averse to the logical explanation. Let us see, then, how he argues the point. His first remark is as follows:

Peirce: CP 8.65 Cross-Ref:††

"If every time a present sign suggests an absent reality to our mind, we make an inference; and if every time we make an inference, we reason, then perception is indubitably reasoning."

Peirce: CP 8.66 Cross-Ref:††

66. Of course, every psychological suggestion is regarded as of the general nature of inference, but only in a far more general sense than that in which perception is so called. This should be well known to Prof. James, and he would have dealt more satisfactorily with his readers if he had not kept it back. Namely, perception attains a virtual judgment, it subsumes something under a class, and not only so, but virtually attaches to the proposition the seal of assent -- two strong resemblances to inference which are wanting in ordinary suggestions. However, Prof. James admits that the process *is* inference in a broad sense. What, then, has he to object to [in] the theory under consideration?

Peirce: CP 8.67 Cross-Ref:††

67.

"Only one sees no room in it for any unconscious part. Both associates, the present sign and the contiguous things which it suggests, are above board, and no intermediary ideas are required."

Peirce: CP 8.67 Cross-Ref:††

Here are two errors. In the first place, "unconscious inference" does not, either with other logicians or with the advocates of the theory in question, mean an inference in which any proposition or term of the argument is unconscious, any more than "conscious inference" implies that both premises are conscious. But unconscious inference means inference in which the reasoner is not conscious of making an inference. He may be conscious of the premise, but he is not conscious that his acceptance of the conclusion is inferential. He does not make that side-thought which enters into all inference strictly so called: "and so it would be in every analogous case (or in most cases.)" There is no doubt, therefore, that ordinary suggestion, regarded as inference, is of the unconscious variety. But Prof. James further forgets his logic in hinting, what he soon expresses more clearly, that such an inference is to be regarded as a mere "immediate inference," because it has no middle term. We might suppose he

had never heard of the *modus ponens*, the form of which, A and B being any proposition, is

If A, then B;

But A:

Hence, B.

Those who think a light is thrown upon the ordinary process of suggestion by assimilating it to reasoning, assimilate it to the *modus ponens*. The proposition "If A, then B," is represented by the association itself, which is not present to consciousness, but exists in the mind in the form of a habit, as all beliefs and general propositions do. The second premise A is the suggesting idea, the conclusion B is the suggested idea.

Peirce: CP 8.68 Cross-Ref:††

68. Already quite off the track, our author now plunges into the jungle in this fashion:

"Most of those who have upheld the thesis in question have, however, made a more complex supposition. What they have meant is that perception is a *mediate* inference, and that the middle term is unconscious. When the sensation which I have called 'this' is felt, they think that some process like the following runs through the mind:

'This' is M;

but M is A;

therefore 'this' is A."

Peirce: CP 8.68 Cross-Ref:††

Those who have upheld the thesis are not in dispute among themselves, as represented. They make no supposition throughout not admitted by all the world. To represent any process of inference now as a *modus ponens*, now as a syllogism with a middle term, is not necessarily taking antagonistic views. As for the syllogism given, it is the weakest mode of supporting the thesis, far more open to attack than the form first given above. But Prof. James makes no headway, even against this. He says:

Peirce: CP 8.68 Cross-Ref:††

"Now there seems no good grounds for supposing this additional wheelwork in the mind. The classification of '*this*' as M is itself an act of perception, and should, if all perception were inference, require a still earlier syllogism for its performance, and so backwards *ad infinitum*."

Peirce: CP 8.69 Cross-Ref:††

69. Not one of the authors whom we have consulted makes the M entirely unconscious; but Prof. James says they do. If so, when he insists that "this is M" is an act of perception, he must mean some ultra-Leibnitzian *unconscious* perception! Has

he ever found the German authors maintaining that that kind of perception is inferential? If not, where is his *regressus ad infinitum*? What those authors do say is that M, and with it the two premises, are thrown into the background and shade of consciousness; that "this is M" is a perception, sometimes in the strict sense, sometimes only in that sense in which perception embraces every sensation. They do not hold sensation to be inferential, and consequently do not suppose a *regressus ad infinitum*. But even if they did, there would be no *reductio ad absurdum*, since it is well known to mathematicians that any finite interval contains an infinite number of finite intervals; so that supposing there is no finite limit to the shortness of time required for an intellectual process, an infinite number of them, each occupying a finite time, may be crowded into any time, however short.

Peirce: CP 8.70 Cross-Ref:††

70. The Professor concludes:

"So far, then, from perception being a species of reasoning, properly so called, both it and reasoning are coördinate varieties of that deeper sort of process known psychologically as the association of ideas, and -- "

Peirce: CP 8.70 Cross-Ref:††

We break the sentence, which goes on to something else, in order to remark that "a species of reasoning properly so called" must be a slip of the pen. For otherwise there would be an *ignoratio elenchi*; nobody ever having claimed that perception is inference in the strict sense of conscious inference. Instead of "a species of reasoning properly so called," we must read "reasoning in a generalized sense." Remembering also that Prof. James began by insisting on extending the controversy to association in general, we may put association in place of perception, and thus the conclusion will be, "so far from association being reasoning in a generalized sense, reasoning is a special kind of association." Who does not see that to say that perception and reasoning are coördinate varieties of association, is to say something in entire harmony with the thesis which Prof. James is endeavoring to combat? To resume:

Peirce: CP 8.71 Cross-Ref:††

71.

"-- physiologically as the law of habit in the brain. To call perception unconscious reasoning is thus either a useless metaphor or a positively misleading confusion between two different things."

Peirce: CP 8.71 Cross-Ref:††

Here the section ends, and in these last words, for the first time in the whole discussion, the real question at issue is at length touched, and it is dismissed with an *ipse dixit*. There is no room for doubt that perception and, more generally, associative suggestion, may truthfully be considered as inference in a generalized sense; the only question is whether there is any use in so considering them. Had Prof. James succeeded in establishing his *regressus ad infinitum*, he would have refuted himself effectually, since it would then have been shown that an important consequence, not otherwise known, had been drawn from the theory. As it is, he says nothing pertinent either pro or con. But a little before, when an unconscious predication was called perception, was this perception "properly so called"? And if not, was calling it by that name a "useless metaphor," or was it a "positively misleading confusion between two

different things"?

Peirce: CP 8.72 Cross-Ref:††

§2. QUESTIONS ON WILLIAM JAMES'S *THE PRINCIPLES OF PSYCHOLOGY*†8

72. Qu: 3 *p. 66*. "The cortex is the sole organ of consciousness in man." The reasoning seems pretty loose for settling all the important positions implied in this statement. What is consciousness anyway?

Peirce: CP 8.73 Cross-Ref:††

73. Qu: 5 *p. 80*. Is not the conscious element of any conception, -- as Kant would say, its *matter*, -- pretty accidental and unimportant? It must, no doubt, be there, but will not anything there do? Shall we not take *tongue* sensations as the skeleton or *corpus* of our conception of language, etc.?

Peirce: CP 8.74 Cross-Ref:††

74. Qu: 12 *p. 137*. ["Psychology is a mere natural science, accepting certain terms uncritically as her data, and stopping short of metaphysical reconstruction. Like physics, she must be *naïve*; and if she finds that in her very peculiar field of study ideas *seem* to be causes, she had better continue to talk of them as such. She gains absolutely nothing by a breach with common-sense in this matter, and she loses, to say the least, all naturalness of speech."] Had physics taken the course you wish psychology to take would she not have stuck to the idea of explaining everything by hot and cold, moist and dry? Is not the lesson of physics rather not to attack the most difficult problems first?

Peirce: CP 8.75 Cross-Ref:††

75. Qu: 14 *p. 144*. ["But if pleasures and pains have no efficacy, one does not see (without some such *à priori* rational harmony as would be scouted by the 'scientific' champions of the automaton-theory) why the most noxious acts, such as burning, might not give thrills of delight, and the most necessary ones, such as breathing, cause agony."] Why would it not be equally logical to say, "if pleasures and pains have no efficacy, one does not see why men should not shun the pleasurable as much as the painful." But the obvious answer would be, *because, as this fact shows, pleasure and pain are more than pure monadic feelings*. Is not this the answer to the question that *is* put?

Peirce: CP 8.76 Cross-Ref:††

76. Qu: 21 *p. 215*. ["The truth is that if the thinking principle is extended we neither know its form nor its seat; whilst if unextended, it is absurd to speak of its having any space-relations at all. Space-relations we shall see hereafter to be *sensible* things. The only objects that can have mutual relations of position are objects that are perceived coexisting in the same felt space. A thing not perceived at all, such as the inextended soul must be, cannot coexist with any perceived objects in this way. No lines can be felt stretching from it to the other objects. It can form no terminus to any space-interval. It can therefore in no intelligible sense enjoy position. Its relations cannot be spatial, but must be exclusively cognitive or dynamic, as we have seen. So far as they are dynamic, to talk of the soul being 'present' is only a figure of speech. Hamilton's doctrine that the soul is present to the whole body is at any rate false: for

cognitively its presence extends far beyond the body, and dynamically it does not extend beyond the brain."] The two centres of gyration of a reversible pendulum are *unextended points*. No lines can be felt stretching from them to other objects. They form no termini to any space-interval. Will you then say they "can have no mutual relations of position," or that "in no intelligible sense can they 'enjoy' position"?

Peirce: CP 8.77 Cross-Ref:††

77. Qu: 22 *p. 215*. Is anything "present" in space except in the sense of being in dynamic reaction with other objects in space? If so, in what does the figure of speech consist?

Peirce: CP 8.78 Cross-Ref:††

78. Qu: 23 *p. 215*. There is an attempt in the last sentence of the text of this page (and the idea has been vaguely running along) to establish a great contrast between the mode of the mind's *cognitive* reactions with things and its *dynamic* reactions. The former is direct, or there is, at least, no sense in calling it indirect. The latter is direct only with the brain, and mainly indirect. Is this tenable? The soul reacts dynamically with the future, cognitively with the past. Both are mediate. In the immediate present, volition and experience are indistinguishable, are they not? What is the distinction that can exist in that instant? If I am right here, is there not a pretty accurate correspondence between our dealings with the Future and the Past, as far as mediacy is concerned, at any rate?

Peirce: CP 8.79 Cross-Ref:††

79. Qu: 29 *p. 222*. ["Through feelings we become acquainted with things, but only by our thoughts do we know about them. Feelings are the germ and starting point of cognition, thoughts the developed tree."] "Through feelings we become acquainted with things." This seems to me to be at the root of a good deal of bad metaphysics. On the contrary, the feelings are matters of indifference (in their qualities). It is by the *reactions* of ourselves upon things and of their parts on one another that we become acquainted with things, as it seems to me.

Peirce: CP 8.80 Cross-Ref:††

80. Qu: 30 *p. 222*. Is this classification of "mental states" as feelings and thoughts sufficiently scientific? Is it not better to adopt the *logical* division not of "mental states" but of mental elements, into feeling-qualities, reactions (volition and experience), and habit-taking?

Peirce: CP 8.81 Cross-Ref:††

81. Qu: 31 *p. 226*. "No thought even comes into direct sight of a thought in another personal consciousness than its own. Absolute insulation, irreducible pluralism, is the law." Is not the direct contrary nearer observed facts? Is not this pure metaphysical speculation? You think there *must* be such isolation, because you confound thoughts with feeling-qualities; but all observation is against you. There are some small particulars that a man can keep to himself. He exaggerates them and his personality sadly.

Peirce: CP 8.82 Cross-Ref:††

82. Qu: 32 *p. 226*. ["It seems as if the elementary psychic fact were not *thought* or *this thought* or *that thought*, but *my thought*, every thought being *owned*. Neither contemporaneity, nor proximity in space, nor similarity of quality and content are able to fuse thoughts together which are sundered by this barrier of belonging to different personal minds. The breaches between such thoughts are the most absolute

breaches in nature. Everyone will recognize this to be true, so long as the existence of *something* corresponding to the term 'personal mind' is all that is insisted on, without any particular view of its nature being implied. On these terms the personal self rather than the thought might be treated as the immediate datum in psychology."] Everybody will admit a personal self exists in the same sense in which a snark exists; that is, there is a phenomenon to which that name is given. It is an illusory phenomenon; but still it is a phenomenon. It is not quite *purely* illusory, but only *mainly* so. It is true, for instance, that men are *selfish*, that is, that they are really deluded into supposing themselves to have some isolated existence; and in so far, they *have* it. To deny the reality of personality is not anti-spiritualistic; it is only anti-nominalistic. It is true that there are certain phenomena, really quite slight and insignificant, but exaggerated, because they are connected with the tongue, which may be described as personality. The agility of the tongue is shown in its insisting that the world depends upon it. The phenomena of personality consist mainly in ability to hold the tongue. This is what the tongue brags so about. †9

Peirce: CP 8.83 Cross-Ref: ††

83. But all this business will appear dark and mysterious until the three categories are mastered and applied. †10

Peirce: CP 8.84 Cross-Ref: ††

84. Meantime, physicians are highly privileged that they can ask to see people's tongues; for this is inspecting the very organ of personality. It is largely because this organ is so sensitive that personality is so vivid. But it is more because it is so agile and complex a muscle. Its muscular habits are the basis of personality, which need not be lodged in the brain. The inhibition however which makes the strong personality comes from some exterior ganglion, no doubt.

Peirce: CP 8.85 Cross-Ref: ††

85. This is a specimen of how other "thoughts" ought to be conceived. They are readily adoptable habits, taken, lost, replaced continually, and felt, no matter how. Mostly no doubt lodged in nerve matter, but not necessarily so.

Peirce: CP 8.86 Cross-Ref: ††

86. The cases of double personality show that the cunning right hand can in a measure replace the tongue. But till a personality can control the tongue, it is very obscure. The principal personality resides there. Its superiority is shown by this that if cut out the person soon gets along and talks very well, with the remaining fragments. Farmers sometimes slit the tongues of self milking cows. But they soon learn to make use of the slit tongue just the same. So if a man's right hand is cut off, it is marvellous how much he can do with the stump. But the hand altogether lacks the extreme subtilty of the tongue. The school-boy writes with his tongue. That is the tongue teaching the fingers language. Some people roll up their tongues, or bite them, or shove them down when they do something sly or tricky. Some people stick them into their cheeks. These are the gestures of pure egotism. The tobacco chewer shifts his quid when he betrays his vanity.

Peirce: CP 8.86 Cross-Ref: ††

All animals capable of domestication have good tongues.

Peirce: CP 8.87 Cross-Ref: ††

87. Qu: 33 *p. 231*. ["Are not the sensations we get from the same object, for example, always the same? Does not the same piano-key, struck with the same force,

make us hear in the same way? Does not the same grass give us the same feeling of green, the same sky the same feeling of blue, and do we not get the same olfactory sensation no matter how many times we put our nose to the same flask of cologne? It seems a piece of metaphysical sophistry to suggest that we do not; and yet a close attention to the matter shows that *there is no proof that the same bodily sensation is ever got by us twice.*

Peirce: CP 8.87 Cross-Ref:††

"*What is got twice is the same* OBJECT. We hear the same *note* over and over again; we see the same *quality* of green, or smell the same objective perfume, or experience the same *species* of pain." Is it not plain that two feelings cannot be compared as they are as pure feelings? If so can a "likeness" between two feelings possibly consist in anything but their being naturally associated? That granted, is it not certain that feelings ever so much alike *do*, in that only possible sense, recur? As for *sameness*, this is a relation which by its nature is restricted to individuals. Feelings are in so far the same as they are alike.

Peirce: CP 8.88 Cross-Ref:††

88. Qu: 36 *p.* 235. I should be glad to know what possible relevancy all that has been so skillfully said about the total states of mind, that its commonplace is forgotten, has with the proposition that no two ideas can ever be exactly the same. This seems perfectly absurd. The essence of thought lies in the law of relationship that it implies. Do you mean to say that I never can have again my present view of the essence of the system of whole numbers? *That* is what it means to say I have the same idea I had yesterday.

Peirce: CP 8.89 Cross-Ref:††

89. Qu: 41 *p.* 243. [*Let us call the resting-places the 'substantive parts,' and the places of flight the 'transitive parts,' of the stream of thought.* It then appears that the main end of our thinking is at all times the attainment of some other substantive part than the one from which we have just been dislodged. And we may say that the main use of the transitive parts is to lead us from one substantive conclusion to another."] This is one of the finest, if not the finest, passage in the whole book. It is a direful pity the author could not have sufficient acquaintance with the history of words, and of knowledge of their importance, to avoid two of the most objectionable terms he could possibly have selected, for the trade marks of his invention. Why could he not have said "transitory"†11 instead of taking a word already overburdened with ambiguities. Not that still better terms might not have been discovered. As for "substantive," it wouldn't have been much worse if he had called it "absolute." . . .

Peirce: CP 8.90 Cross-Ref:††

90. Qu: 42 *p.* 244. [*Let anyone try to cut a thought across in the middle and get a look at its section, and he will see how difficult the introspective observation of the transitive tracts is. The rush of the thought is so headlong that it almost always brings us up at the conclusion before we can arrest it.*] To cut a thought across and look at the section requires no introspection. It is one of the principal methods in mathematics, which is in no degree introspective. Treating operations as quantities is one of a hundred familiar examples.

ON NON-EUCLIDEAN GEOMETRY†1

91. Lobachevski's little book, 'Geometrische Untersuchungen,' marks an epoch in the history of thought, that of the overthrow of the axioms of geometry. The philosophical consequences of this are undoubtedly momentous, and there are thinkers who hold that it must lead to a new conception of nature, less mechanical than that which has guided the steps of science since Newton's discovery. The book has been published many years -- in fact, the essence of it was set forth before 1830; so long does it take a pure idea to make its way, unbacked by any interest more aggressive than the love of truth. In this case, the idea is lucid, easy, and convincing. Nobody with enough mathematical capacity to be able to understand the first book of geometry need fear the least difficulty in mastering Lobachevski's tract; and really it is high time that every thinking man and woman should know what is in it.

Peirce: CP 8.92 Cross-Ref:††

92. In the pre-Lobachevskian days, elementary geometry was universally regarded as the very exemplar of conclusive reasoning carried to great lengths. It had been the ideal of speculative thinkers in all ages. Metaphysics, indeed, as an historical fact, has been nothing but an attempt to copy, in thinking about substances, the geometer's reasoning about shapes. This is shown by the declarations of Plato and others, by the spatial origin of many metaphysical conceptions and of the terms appropriated to them, such as *abstract, form, analogy*, etc., and by the love of donning the outer clothing of geometry, even when no fit for philosophy. For instance, one of the remarkable features of geometry is the small number of premises from which galaxies of theorems result; and accordingly it has been an effort of almost all metaphysicians to reduce their first principles to the fewest possible, even if they had to crowd disparate thoughts into one formula. It did not seem to occur to them that since a list of first principles is a work of analysis, it would not be a small number of elementary propositions so much as a large number that would bespeak its thoroughness. Admiration for the elements of geometry was not, however, confined to metaphysicians. Euclid's treatise was acknowledged by all kinds of minds to be all but absolutely perfect in its reasoning, and the very type of what science should aim at as to form and matter.

Peirce: CP 8.93 Cross-Ref:††

93. In the empyrean of geometry there was but one little speck -- the theory of parallels. Euclid had had a difficulty in proving the sum of the angles of a triangle to be not less than two right angles. His treatment of the subject betrays a very profound study of it; for instead of slipping over the difficulty unaware, as forty-nine out of fifty mathematicians would have done, instead of even bringing the necessary assumption to a persuasive shape, he takes as his fifth postulate (or 11th axiom, in incorrect editions) a proposition that begs the question in the frankest manner -- namely, if two straight lines in a plane are met by a third making the sum of the inner angles on one side of this third less than two right angles, then these two lines will meet on that side if sufficiently produced. Innumerable attempts were made to demonstrate this; but, at length, the efforts of Legendre and others made it pretty clear

that this proposition could be deduced only from some other nearly equivalent. The least unsatisfactory assumption ever proposed was that of Playfair, that if of three unlimited straight lines lying in one plane two intersect, the third must cross one or both. It was at this point that Lobachevski cut the knot by supposing Euclid's postulate untrue, and showing that the result was a perfectly consistent system of geometry which may, for all we can yet observe, be the system of nature.†2 All this time, Euclid's proof (Elements, Bk. I., props. 16 and 17) of what substantially amounts to the proposition, that the sum of the three angles of a triangle is *not greater* than two right angles, was regarded as perfect. It was not till 1854 that Riemann first discovered that, though accepted for two thousand years as conclusive (and it stands to-day unchanged in almost all the text-books), this pretended proof is really quite fallacious. It is plain that it is so, because it uses no premises not as true in the case of spherical as in that of plane triangles; and yet the conclusion drawn from those premises is known to be false of spherical triangles.†3

Peirce: CP 8.94 Cross-Ref:††

94. The truth is, that elementary geometry, instead of being the perfection of human reasoning, is riddled with fallacies, and is thoroughly unmathematical in its method of development. It has in some measure confused all mathematics, by leaving unnoticed most of the really fundamental propositions, while raising to an undue rank certain others almost arbitrarily selected. It leads young men into bad logical ways; and it causes pupil and teacher to think that whoever has difficulty with this sophisticated logic is wanting in aptitude for the apprehension of mathematics. The study of geometry ought to begin with the theory of perspective. Let a man be supposed to stand on an unbroken sandy plain. Let him fix a needle upon a post, and set up a plate of glass in a steady position, and draw a perspective picture upon the glass by placing his eye so as to bring the needle-point over each point in the sand to be represented and marking it on the glass in the same line of sight. The horizon is where the lines of sight just skim the surface of the rounded earth. These lines of sight form a cone, and their perspective representation will be the section of this cone by the plane of the glass. But for simplicity let it be supposed that the earth is flat and indefinitely extended, so that the *plain* is also a *plane*, and an unbounded one. Then every straight line in the sand will have a straight line for its picture, for all the lines of sight from the needle-point to points in that straight line will lie in one plane; and this plane will cut the plane of the glass in a straight line.

Peirce: CP 8.95 Cross-Ref:††

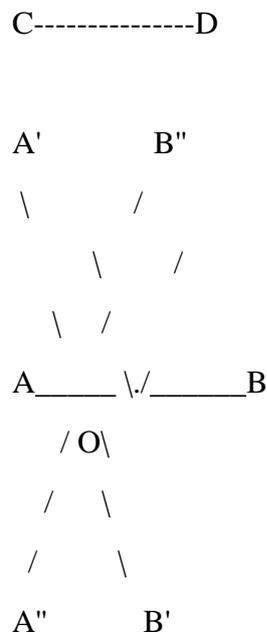
95. Lobachevski and Riemann cast no manner of doubt upon the geometry of perspective, so far as this is confined to questions of incidence and coincidence. But when it comes to the measurement of distances and angles, their objections begin. According to the Euclidean notions, the infinitely distant parts of an unbounded plane would be represented in perspective by a straight horizon or vanishing line. But Lobachevski says we cannot be sure that this line would be straight, that maybe it would be a hyperbola like the perspective of the terrestrial horizon; and, in fact, the straight line being only a special case of the hyperbola, it is proper to say that such is its form. Riemann, however, points out that we cannot even be sure there would be any such line at all, for we cannot be sure that space has any infinitely distant parts, since it may be that if we were to move off in any direction in a straight line, we might find that, after traversing a sufficient distance, we had got around to our starting-point again.

Peirce: CP 8.96 Cross-Ref:††

96. Prof. Halsted's translation (which, while our notice has been waiting, has reached, we are glad to see, a fourth edition) is excellent; his useful bibliography of non-Euclidean geometry was already well known. We could only wish there were a more copious appendix. The work of Lobachevski, though simple and convincing, is not what would now be considered a scientific presentation of the subject, and is open to a good deal of criticism. A new synthetic exposition is much needed, and might well accompany a collection of the contributions of Lobachevski, Bolyai, Riemann, Cayley, Klein, and Clifford.

Peirce: CP 8.97 Cross-Ref:††

97. We have an *opinion* or natural idea of space, which by some kind of evolution has come to be very closely in accord with observations. But we find in regard to our natural ideas, in general, that while they do accord in some measure with fact, they by no means do so to such a point that we can dispense with correcting them by comparison with observations.



Peirce: CP 8.98 Cross-Ref:††

98. Given a line CD and a point O. Our natural (Euclidean) notion is that

1st there is a line AB through O in the plane OCD which will not meet CD at any finite distance from O.

2nd that if any line A'B' or A''B'' through O in the plane OCD be inclined by any finite angle, however small, to AB, it will meet CD at some finite distance from O.

Peirce: CP 8.99 Cross-Ref:††

99. Is this natural notion *exactly* true?

Peirce: CP 8.99 Cross-Ref:††

A. This is not certain.

Peirce: CP 8.99 Cross-Ref:††

B. We have no probable reason to believe it so.

Peirce: CP 8.99 Cross-Ref:††

C. We *never* can have positive evidence lending it any degree of likelihood. It *may* be disproved in the future.

Peirce: CP 8.99 Cross-Ref:††

D. It may be true, perhaps. But since the chance of this is as $1:\infty$ or $0/1$, the logical presumption is, and must ever remain, that it is not true.

Peirce: CP 8.99 Cross-Ref:††

E. If there is some influence in evolution tending to adapt the mind to nature, it would probably not be completed yet. And we find other natural ideas require correction. Why not this, too? Thus, there is some reason to think this natural idea is *not* exact.

Peirce: CP 8.99 Cross-Ref:††

F. I have a theory which fits all the facts as far as I can compare them, which would explain how the natural notion came to be so closely approximate as it is, and how space came to have the properties we find it has. According to this theory, this natural notion would not be exact.

Peirce: CP 8.99 Cross-Ref:††

To give room for the non-Euclidean geometry, it is sufficient to admit the first of these propositions.

Peirce: CP 8.100 Cross-Ref:††

CHAPTER 6

JOSIAH ROYCE, *THE WORLD AND THE INDIVIDUAL*

§1. FIRST SERIES: THE FOUR HISTORICAL CONCEPTIONS OF BEING †1

100. We can do no more than explain in untechnical language what this important book is about. Its purpose is to say what it is that we aim at when we make any inquiry or investigation -- not what our ulterior purpose may be, nor yet what our special effort is in any particular case, but what the direct and common aim of all search for knowledge is. This is a question of fact. Prof. Royce has clothed the matter in such academical guise that a reader untrained in philosophy might suppose it was a mere dispute about a definition, and therefore a profitless discussion; but, stripping off technicalities, we find this question of fact beneath them.

Peirce: CP 8.101 Cross-Ref:††

101. The only opinion on this subject generally held at this day that Prof. Royce considers to be essentially different from his own, is one which may be attributed to Bishop Berkeley more justly than to any other individual. It is the opinion of Possible Experience. Though this has taken slightly different shapes with different thinkers, it will suffice, in order to explain the purport of Prof. Royce's book, to state it in one of its forms. The answer, then, generally given, or virtually given, to the question what any inquiry is instituted for, is approximately that it is intended to settle doubt on the subject. Did Sir Philip Francis write the Junius letters? I can imagine, as the handwriting experts say, that he did. I can imagine, as most of the recent inquirers say, that he did not. I feel no compulsion to attach either idea to my mental representation of the historic world. There are some images which I am forced, whether I would or no, to attach to mental objects -- such as a dark skin and jealousy to Othello. The course of life has developed certain compulsions of thought which we speak of collectively as Experience. Moreover, the inquirer more or less vaguely identifies himself in sentiment with a Community of which he is a member, and which includes, for example, besides his momentary self, his self of ten years hence; and he speaks of the resultant cognitive compulsions of the course of life of that community as Our Experience. He says "we" find that terrestrial bodies have a component acceleration towards the earth of 980 centimetres per second, though neither he nor many of his acquaintances have ever made the experiment.

Peirce: CP 8.102 Cross-Ref:††

102. Now, such being his state of mind, two hopes motive his inquiry: the first is, that the course of "our" experience may ultimately compel the attachment of a settled idea to the mental subject of the inquiry; and the second is, that the inquiry itself may compel him to think that he anticipates what that destined ultimate idea is to be.†2

Peirce: CP 8.103 Cross-Ref:††

103. Such, approximately, is the ordinary opinion of Possible Experience, in one of its modes of statement. According to it every inquiry is directed toward the resultant of certain compulsions; and, therefore, so far as a sense of compulsion is an immediate knowledge of something outside of self, exerting a brute force on self, this opinion is that every inquiry relates to a brute something without the mind. It was substantially on this ground that Kant opposed the anti-materialism of Berkeley. But, regarded from another side, this opinion is that the only object to which inquiry seeks to make our opinion conform is itself something of the nature of thought; namely, it is the predestined ultimate idea, which is independent of what you, I, or any number of men may persist, for however long, in thinking, yet which remains thought, after all. The whole course of life within which the experiential compulsions appear is a purely psychical development. For the gist of the opinion is that the flow of time consists in a continual assimilation into "our" inwardness, the Past, of a non-ego that is nothing but the ego that is to be -- the Future. The Past acts upon the Future intelligibly, logically. But those blind compulsions are glimpses of an unknown object. Now, the unknown, according to this theory, is nothing but what is bound, as our hope is, to emerge in the future. Those blind compulsions, then, can be regarded as actions of the future on the past. From that point of view, it is seen that they can but be brute and blind, and, further, that in the course of time they must be seen to rationalize themselves and fall into place as the cognition develops.

Peirce: CP 8.104 Cross-Ref:††

104. To Prof. Royce's thinking, this opinion is unsatisfactory. He finds four

faults with it, and sets them before us with his own argumentative lucidity and admirable mastery of the subject. Of the nature of three of them -- that the opinion under examination makes the object of knowledge to be no more than a "would-be"; that its "experience" is no experience for an inquirer; that it seats an abstraction on a throne of reality -- we can here find room for no clearer hint than those phrases may convey. Whatever solid skeleton the three objections may clothe is pretty much the same as that of the fourth and strongest, that if the non-ego to which the inquirer seeks to make his ideas conform is merely an idea in the future, that future idea must have for its object an idea future to it, and so on *ad infinitum*. There is no escaping the admission that the ultimate end of inquiry -- the essential, not ulterior end -- the mould to which we endeavor to shape our opinions, cannot itself be of the nature of an opinion. Could it be realized, it would rather be like an insistent image, not referring to anything else, and in that sense concrete. Passing from the consideration of a single inquiry to that of the aggregate of all possible inquiries, the phantom ultimate issue of them all would be the real universe. To be that, however, it must include the mental world as well as the physical, and must set forth to itself all laws and modes of conception. It must, above all, exhibit to itself the whole course of time, with that process of complete rationalization of ideas upon the assumption of which the very hypothesis of a fated ultimate destination of opinion is based. It must, therefore, be conceived as a perfect rational consciousness. In short, it is such a conception of Deity (necessarily a one-sided one) as considerations limited to the Theory of Cognition could reasonably be expected to yield.

Peirce: CP 8.105 Cross-Ref:††

105. This inevitable outcome of the doctrine of Possible Experience is the very same goal, roughly speaking, to which Prof. Royce's explorations have brought him, too, by a path nearly parallel to that for which we have set up a sign-post for whoever may care to follow it out, though the hedgerows of thought may prevent the traveller over the one from being aware how close he is to the other. Prof. Royce reaches his conclusion by analyzing the nature of the purpose of an idea. Now this same conception of the purpose of an idea ought equally to be seized as the guiding thread to the doctrine of Possible Experience, although Prof. Royce believes his position to be quite foreign, even hostile, to that. One divergence is, that where another thinker might speak of a hope, as we have done above, Prof. Royce would substitute a *reductio ad absurdum* of the contrary opinion -- a diminution of man's natural sublime attitude to a sorry "A is A." Fortunately the logic of those arguments is never impeccable, so that the hopes retain their matter and are not reduced to mere formulae.

Peirce: CP 8.106 Cross-Ref:††

106. Two other views are examined. One is that of cognitive Dualism, which Professor Royce calls by the objectionable name Realism (as if the Dualists alone admitted outward realities). The other is that of Mysticism, which is less an opinion than an attitude of mind, of which Professor Royce gives an exceedingly penetrating analysis. There is a long and technical supplementary essay on the One, the Many, and the Infinite, which is very important.

Peirce: CP 8.107 Cross-Ref:††

107. The dress of the book is as charming as that of one so sure of being long and often perused ought to be.

Peirce: CP 8.108 Cross-Ref:††

108. Does the reader dabble in metaphysics? If he does, we make no sort of doubt that his opinions on such matters are nearer correct than those of any other human being; for we have talked with a hundred metaphysicians without ever yet meeting one who was not vastly superior to all the rest. So, *a fortiori*, the same superiority must be enjoyed by the gracious reader. But *present company excepted*, we do not know that there lives a second metaphysician as strong as Prof. Royce. It need scarcely be said that no other theoretical science is at all comparable with philosophy in respect to the deep and large reading that its study absolutely makes exigent, lacking which the most splendid natural powers will leave their possessor a mere child in this science. For it is humanly impossible to know whither a given proposition in philosophy will inevitably lead without tracing out its historic development. Nor for this purpose will mere information suffice. It is requisite to enter into the range of ideas and spirit of each doctrine and thoroughly to assimilate it. In other sciences, only the true theories require close attention; but in philosophy the false ones are even more important, since it is precisely the thin and light soil of one-sided and extreme opinions that is easiest turned over to bring up the absurdities that lurk beneath the surface of their assumptions. Perhaps the greatest difficulty of the study arises from the circumstance that in youth one lacks the patience to sit down and soak one's mind in views from which one entirely dissents; so that by the time the preparation for original work is accomplished, the *élan* and agility of intellect that are nowhere more needed have been lost in the wear of advancing age. Now Royce has not only read all the great systems but he shows a truly admirable power of throwing himself into the mind of each philosopher and of appreciating with the greatest nicety just how each thinker has thought. The present volume contains a striking illustration of this in an exposition of the central position of Brahminic mysticism. It is a revelation almost too complete for the author's purpose, in that it comes perilously near to persuading the reader that that which the mystic affirms is undeniably true, and may almost indispose him to listen to its refutation.

Peirce: CP 8.109 Cross-Ref:††

109. The scientific world has now expended more than two centuries of concerted endeavor in the attempt to explain the phenomena of nature by means of the attractions and repulsions of discrete particles, and it is beginning to look strongly as if that hypothesis were insufficient. Some of the best authorities, for instance, now profess to demonstrate that phenomena as relatively simple as those of the elasticity of solid bodies cannot be so explained. Accordingly, new hypotheses infinitely more difficult to deal with are getting proposed, such as that the universe is filled with a homogeneous fluid whose vortices constitute ordinary matter. It would be a pity, would it not, to turn the speculative energy of the world into such a channel only to find, at the expiration of two centuries, or more likely five, that it had a foolish hypothesis from which little or nothing ought ever to have been expected. If we only had at our command at this moment a really scientific logic and metaphysics, which might serve as guides in the choice of a hypothesis, such a doctrine might at this time be of the utmost service to science. But unfortunately the profounder sects of philosophy have sprung out of theology. Their adherents are tainted with the vicious intellectual diathesis of the seminaries. No class of persons above day-laborers has less comprehension of what science really is. As for mathematics, which ought to be the log-line and binnacle of the metaphysician, they are afraid to touch it, or when they do so venture, only make themselves ridiculous. This is particularly unfortunate because the main Hegelian idea is virtually an attempt to introduce the conception of

continuity into philosophical doctrine, a conception which the mathematicians, on their side, have been engaged, since the birth of the differential calculus, in endeavoring to render distinct, hitherto without complete success; so that could a concerted assault be made upon it from opposite sides, it might be greatly to the advantage of both assailants. Now Prof. Royce is doing good service toward bringing this about, since, while he enjoys the esteem and sympathy of the theological metaphysicians, he is thoroughly alive to the ideas of science, and is thoroughly versed in all the more philosophical part of modern mathematical speculation. A "Supplementary Essay," appended to these lectures shows this very clearly. Meantime, in respect to that subtlety which seizes with accuracy upon the precise essence of a philosophical problem and disembarasses it from all irrelevant considerations, which is the prime quality of a metaphysical mind, it may well be doubted whether any of the great figures of the history of philosophy have exhibited greater power than our American Plato.

Peirce: CP 8.110 Cross-Ref:††

110. It is now time to indicate what we venture to conceive to be Prof. Royce's greatest fault as a philosophical thinker. Metaphysicians have always taken mathematics as their exemplar in reasoning, without remarking the essential difference between that science and their own. Mathematical reasoning has for its object to ascertain what would be true in a hypothetical world which the mathematician has created for himself, -- not altogether arbitrarily, it is true, but nevertheless, so that it can contain no element which he has not himself deliberately introduced into it.†3 All that his sort of reasoning, therefore, has to do is to develop a preconceived idea; and it never reaches any conclusion at all as to what is or is not true of the world of existences. The metaphysician, on the other hand, is engaged in the investigation of matters of fact, and the only way to matters of fact is the way of experience. The only essential difference between metaphysics and meteorology, linguistics, or chemistry, is that it does not avail itself of microscopes, telescopes, voyages, or other means of acquiring recondite experiences, but contents itself with ascertaining all that can be ascertained from such experience as every man undergoes every day and hour of his life.†4 All other differences between philosophy and the special sciences are mere consequences of this one. It follows, that deductive, or mathematical, reasoning, although in metaphysics it may oftener "take the stage" than in the drama of special research, yet after all, has precisely the same *rôle* to enact, and nothing more. All genuine advance must come from real observation and inductive reasoning. Yet Dr. Royce cannot free himself from the Hegelian notion that the one satisfactory method in philosophy is to examine an opinion and to detect in it some hidden denial of itself, -- which is nothing but the *reductio ad absurdum*. Strange that that method of reasoning to which mathematicians are often forced to resort, but which they always dislike because it does not exhibit the *rationalité* of the proposition it proves, should by philosophers be made the standard of excellence. Such refutations in metaphysics are most frequently downright fallacies due to the loose habits of thinking prevalent in the theological seminaries. When their plight is not quite as bad as that, the very fact that the contradiction has to be sought in some obscure corner of the opinion refuted, shows that this opinion only needs to be modified in an inessential detail in order to escape the refutation. In the rare instances in which such refutations are really decisive, what happens is, that the refuter, without himself remarking it, slips into his reasoning some experiential fact. If, before publishing his proof, he were to search out that fact and bring it forward explicitly, he would not only make his reasoning more truly logical, though no longer purely deductive, but he would also render it infinitely more persuasive. For very seldom is anybody really

convinced by the Socratic style of dialectic. Rather point out to a man a new fact, or one that he had overlooked; and then he himself, seeing it to be pertinent, will straightway begin to revise his opinion. The volume before us contains a remarkable illustration of this *penchant* of Prof. Royce in a proposed refutation of the opinion that, not merely is the element of existence a brute and non-intellectual element, but further that there exist things and facts about them which are as they are irrespective of any reason or idea. That this opinion can very readily be disposed of nobody knows better than Dr. Royce. But he is not satisfied with any mode of disproving it other than a *reductio ad absurdum*; and in order to effect the refutation in that way, he forces upon the notion of independence (or being irrespective of reason) elements that nobody who holds the opinion to be disproved will, for a moment, admit that it involves. With that hint, we refer the reader to Lecture III in the book itself.

Peirce: CP 8.111 Cross-Ref:††

111. The main purpose of this volume is to show the inadequacy of the doctrine that whatever we know is either a direct experience or a possibility of experience, -- a doctrine that Berkeley, more than any other person, introduced into philosophy and upon which Kant built his system of critic, -- and further to propose in place of this doctrine a substitute [which], while Dr. Royce would certainly not claim it to be altogether novel, is undoubtedly a distinct improvement upon previous conceptions. Everybody who has reflected deeply upon the Berkeleyan principle must have seen that it leaves something to be desired. It will aid the reader to understand precisely what Dr. Royce's work is designed to effect, and how far it does actually effect it, if we state that doctrine of Possible Experience in one of its more modern forms.

Peirce: CP 8.112 Cross-Ref:††

112. Remembering, then, that philosophy is a science based upon everyday experience, we must not fall into the absurdity of setting down as a datum and starting-point of philosophy any abstract and simple idea, as Hegel did when he began his logic with pure Being; but we must set out from ideas familiar and complex, as Hegel began his greater masterpiece by considering a man sitting under a tree in a garden in the afternoon. We must not begin by talking of pure ideas, -- vagabond thoughts that tramp the public roads without any human habitation, -- but must begin with men and their conversation. We are familiar with the phenomenon of a man's expressing an opinion, sometimes decidedly, often otherwise. Perhaps it will be a mere suggestion, a mere question. Any such suggestion that may be expressed and understood relates to some common experience of the interlocutors, or, if there is a misunderstanding, they may think they refer to some common experience when, in fact, they refer to quite different experiences. A man reasoning with himself is liable to just such a misunderstanding. About this common experience the speaker has something to suggest which is supposed to be new to his auditor. Now this suggestion will be found inductively, by the examination of instances, to consist invariably in this, that if the auditor or any other man will act in a certain way, more or less vaguely described, he will find that common experience to connect itself with a new experience after a fashion analogous to other connections of experiences, which have made this mode of connection familiar to both parties. For example, if example be needed, suppose a man to go out of his house at night and see the light of a distant fire in the sky. He meets a neighbor and remarks, "There is a fire."†5 If he had only said "a fire exists," he would have conveyed next to no meaning at all. Not quite no meaning, since the remark would even so refer to that universe that is familiarly known to both men. But in saying "There is a fire," he refers to the common

experience of that very place and time, and virtually says that if the second person will raise his eyes and look about him, he will find the common experience of that place and time to connect itself with the experience of a light as of a fire, the mode of connection being the familiar one that the speaker indicated. Let us take another example. Let the second man, having seen the fire, ask "Would you say, now, that that fire was about three miles away?" This virtually suggests that if the first man or any other man will fill his purse, and take ship, and go to Westminster, and break into the houses of parliament, and bring away the standard yard, and lay it down repeatedly on the ground from where the two stand to where the fire is, and utter the cardinal numbers in their order as the successive layings down proceed, or if he will perform any other experiment virtually amounting to that, then the last number uttered might be 5280, and if it should prove to be a number near to that, he might not be surprised. Extensive experience leads us to expect that if an experiment virtually amounting to that were tried a hundred times, different numbers would be obtained which would cluster about one of them, and that among a million trials the clustering would be still more marked, according to a law well-known to mathematicians. It is possible, no doubt, that if our experience were still more extensive, we should find that if the experiment were tried, say, more than a billion times, then a new phenomenon would emerge and the oftener it was tried the *less* marked might grow the clustering. Our *hope*, however, in endeavoring to make a measurement extremely precise, is that there is a certain value toward which the resultant of all the experiments would approximate more and more, without limitation. Having that hope, the Berkeleyan theory is, that whenever we endeavor to state the distance, all that we aim at is to state as nearly as possible what that ultimate result of experience would be. We do not aim at anything quite beyond experience, but only at the limiting result toward which all experience will approximate, -- or, at any rate, *would* approximate, were the inquiry to be prosecuted without cessation. And the theory is that so it is with all attempts at knowing anything more than what we immediately experience. This might be called the doctrine of the Non-relativity of Knowledge, since it eliminates any non-notional correlate of knowledge.

Peirce: CP 8.113 Cross-Ref:††

113. Prof. Royce seems to think that this doctrine is unsatisfactory because it talks about what would be,†6 although the event may never come to pass. It may be he is right in this criticism; yet to our apprehension this "would be" is readily resolved into a hope for *will be*. For what we mean by saying that any event, B, *would* happen under conditions, A, that are never fulfilled, is that the ultimate opinion which will, as we hope, actually be attained concerning any given question (though not in any finite time concerning *all* questions), will accept certain general laws from which a formal logical consequence will be that conditions, A, in any other world in which they may be fulfilled will, those laws still obtaining, involve the happening of the event, B. In short, we have only to conceive that the ultimate opinion about those general laws is attained before the attainment of the ultimate opinion that the condition, A, is never anywhere fulfilled. Let us not, then, too hastily accede to that criticism. On the other hand, it would be difficult to resist the criticism that the theory is unsatisfactory because it talks too exclusively of what *will be*. It is obvious that this *will be* is the very heart of the theory. The familiar notion of the flow of time is one of the most important data of metaphysical experience. In the special sciences facts are set over against theories, because it is the business of those sciences to connect the special phenomena which they discover with the general experience they derive from other sources. But philosophy embraces all experience. Its direct data are the familiar phenomena found everywhere which, from the point of view of psychology are

quasi-theories, that is, are supposed to be worked up by the mind from simple elements that are not at all familiar to us, or even for the most part observable by themselves. It is of no consequence to the metaphysician whether psychology may teach that our sensations actually have a flow in time or whether we are only conscious in a series of detached instants, like the separate pictures of a zoëtrope. Whether Prof. Royce be right or wrong in asserting that we have an immediate consciousness of a finite span of time is equally unimportant.†7 The only important thing here is our metaphysical phenomenon, or familiar notion, that the past is a matter for knowledge but not for endeavor, that the future is an object that we may hope to influence, but which cannot affect us except through our anticipations, and that the present is a moment immeasurably small through which, as their limit, past and future can alone act upon one another. Whether this be an illusion or not, it is the phenomenon of which the metaphysician has to give an account. Now the Berkeleyan idea, when we come to reflect upon it, amounts to this, that past experience is in some sense, my *ego*, that future experience is my sole *non-ego*, continually being assimilated by the *ego* through the present, and that that in this future *non-ego* which is destined at last to remain unrevoked in the *ego* is the only exemplar to which we desire that our ideas should conform.

Peirce: CP 8.114 Cross-Ref:††

114. Prof. Royce admits, as we think justly, that this doctrine of Possible Experience is true as far as it goes. But still he holds it to be unsatisfactory; and so it is, inasmuch as it regards time as a mere order of succession and fails to do justice to the *continuity* of the flow, which makes of all time an individual object. That time is not a mere order of succession among a multitude of instants is shown by the following considerations. Any multitude of instants, however great, will find room for their succession in any lapse of time; so that if time were the mere order among a multitude of instants, that multitude would have to be a maximum than which no multitude could be greater. But it can be proved that there is no such maximum multitude. Consequently, there is more in time than any or all multitudes.†8 Time is such, says Kant, that every part of it has similar parts, -- a proposition very different from merely saying that time is infinitely divisible, though Kant himself did not perceive the distinction. This continuity, or similarity of parts in respect to having parts, necessarily makes time an individual whole (though precisely how we lack space to explain); and thus it is that we shall never have a satisfactory account of what we virtually aim at in seeking to know, until we recognize the individual character of the object of search.

Peirce: CP 8.115 Cross-Ref:††

115. This individual character is what Prof. Royce desires to bring out by his new definition of the object.†9 He reaches this aspect of the matter through the conception of Purpose. Every idea he says has its purpose, which he calls its "internal meaning." We wish that in place of the vague word "idea," he had substituted *judgment* or *virtual judgment*; for since he is considering cognition in its truth or falsity, and only judgments have truth or falsity, he would thus have made himself more explicit and clear. Reality belongs primarily to *facts*, and attaches to *things* only as elements of facts. A judgment asserts that certain consequences would result from more or less vaguely indicated lines of action, which would be so many experiments. Now by the internal meaning or purpose of an idea Prof. Royce, if we rightly gather his intention, understands all the experiments which would verify it. We can hardly believe that he is so entirely won over to the extreme pragmatism of his colleague, James, as to hold that Doing is the ultimate purpose of life. Nor is this necessary; for

the purpose of an experiment is to learn, and the performance of it is only a means to that end. This internal meaning calls, then, for more and more definiteness without cessation; and the limit toward which it thus tends but never fully attains is the knowledge of an individual, in short, of God. All this part of the discussion is susceptible of much improvement, which will come when the idea of continuity has been more fully analyzed. But, as it stands, it is a very notable contribution to the *prima philosophia*; and we need not say that in the book itself the thought shines out far more brightly and vivifyingly than it can shine through the cranny of our poor notice of it.

Peirce: CP 8.116 Cross-Ref:††

116. One word about the appearance of the book. The press-work is not quite uniform. Some pages are splendid. Others have greyish areas. The shape of the page is singularly pleasing, and we have been led to compare it with other serious octavos, in search of some reason for the sense of pleasure it imparts. We find that in English octavos, not mathematical, the diagonal of the rectangle of solid text divides the right angle between the vertical and horizontal edges into $1/3$ and $2/3$, or 30° and 60° . In American octavos, which are perhaps a little handsomer in shape, the same slope drawn from a bottom corner will intersect the opposite vertical edge half way between the text and the running title. Mathematical octavos are made broader, in order to accommodate long formulae; and they are far more pleasing in appearance. Now it is well-known that owing to the lesser strength of the muscles that cause the vertical motions of the eyeball, a rectangle that looks square really has a height less than its breadth by about one tenth part. In order, therefore, to find the ratio of height to breadth in a rectangle whose diagonal shall seem to the eye to divide the right angle into a third and a two-thirds, we must take $9/10$ of the tangent of 60° , which gives 1.559. In this pleasing page of Royce's volume, we find the height of the solid text is 1.556 times the breadth, which agrees as closely as the measurement can be made. Two diagonals drawn across the page in pencil will seem to make two equilateral triangles.

Peirce: CP 8.117 Cross-Ref:††

§2. SECOND SERIES: NATURE, MAN, AND THE MORAL ORDER †10

117. Professor Royce's second and concluding volume discusses questions of intimate interest to everybody. It is more persuasive than the first, of which it enhances the significance. The design of the whole now comes out -- to introduce into the Hegelian philosophy of religion such rectifications as must result from recognition of scientific conceptions worked out during the century now completing itself since that philosophy first appeared. Of these new conceptions, some are psychological, some logical; but the chief of them are the new mathematical ideas which cluster about that of an infinite multitude. Mathematicians, perhaps, still linger on the stage, who, in their best days, used to be quite positive that one cannot reason mathematically about infinity, and used to feel, like the old lady about her total depravity, that, this cherished inability being taken away, the bottom would fall out of the calculus. Such notions are obsolete. Various degrees of infinity are to-day conceived with perfect definiteness; and the utter misapprehension of the metaphysicians about it, above all of Hegel, glares. As a first serious attempt to apply to philosophical subjects the exactitude of thought that reigns in the mathematical

sciences, and this, not on the part of some obscure recluse whose results do not become known to the public, but on that of an eminent professor in a great university, to whom the world is disposed to listen with attention, Royce's "The World and the Individual" will stand a prominent milestone upon the highway of philosophy.†11 Agitating problems to which no man can be indifferent, offering us, at any rate, a sublime conception of the relation of man to God, a fit trellis for [a] vine of religion that might appease the longings of the heart in life and in the hour of death, it is a book about which little fuller information is proper here than concerning any ordinary essay in ontology. . . .†12

Peirce: CP 8.118 Cross-Ref:††

118. Metaphysics is not yet a subject concerning which magisterial judgments can be wise; but surveying its present situation from the standpoint of a greater respect for physical science than for theology, and for the objective style of thought of the English with its faint traditional odor of scholasticism rather than for German subjectivity, the reviewer has been brought to believe that metaphysics has at length reached a point in its disorderly march at which it can now discern, through the haze upon the distant hill, the place at which it is destined to join company with the orderly army of science. Surely, that reunion must take place sometime. All human research must come to be conducted upon some unitary plan. The pendulum of dispute may swing long; but we must hope it will at last come to rest. To workers for that end this book is an encouraging signal. For only let exact diagrammatic conceptions, like those of mathematics, once take the place of the vague discourse that has prevailed in modern philosophy since it threw off those wholesome obligations of debate (which kept the scholastics to precise points and insured their precise arguments' meeting precise criticism), and what more will be needed to make metaphysics a science? One vital condition must be satisfied. The scientific man hangs upon the lips of nature, in order to learn wherein he is ignorant and mistaken: the whole character of the scientific procedure springs from that disposition. The metaphysician begins with a resolve to make out the truth of a foregone conclusion that he has really never doubted for an instant. Hegel was frank enough to avow that so it was in his case. His "voyage of discovery" was undertaken in order to recover the very fleece that it professed to bring home. The development of the metaphysician's thought is a continual breeding in and in; its destined outcome, sterility. The experiment was fairly tried with Hegelianism through an entire generation of Germans. The metaphysician is a worshipper of his own prepossessions. As Royce expresses it, he is intent upon developing his own purpose. The scientific man is eager to submit himself, his ideas, and his purpose, to the Great Power which, no doubt, penetrates his own being, but is yet all but wholly external to him and beyond anything that his poor present notion could ever, of itself, develop unfructified. The Absolute Knowledge of Hegel is nothing but G.W.F. Hegel's idea of himself; and it has not taught him the very first true lesson in philosophy, that "whoever shall choose to seek his own purpose and idea shall miss it, and whoever shall abandon his own purpose and idea to adopt the purpose and idea of the Author of nature shall accomplish that, and his own long-abandoned purpose and idea along with it." If the idealist school will add to their superior earnestness the diligence of the mathematician about details, one will be glad to hope that it may be they who shall make metaphysics one of the true sciences. Prof. Royce himself has yet his best years of philosophical reflexion before him. The time seems all but ripe for the achievement of this great benefit to mankind; and he beyond any other now living seems to be the man fit for the undertaking. But it cannot be brought to accomplishment until Hegel is *aufgehoben*, with his mere rotation on his axis. Inquiry must react against experience in order that the ship may be propelled

through the ocean of thought

Peirce: CP 8.119 Cross-Ref: ††

119. Prof. Royce's theory, roughly sketched, is this. "An Idea is any state of mind that has a conscious meaning." As for the 'meaning,' logicians have recognized since Abélard's day and earlier that there is one thing which any sign, external or internal, stands for, and another thing which it signifies; its denoted breadth, its "connoted" depth. They have further generally held, in regard to the most important signs, that the depth, or signification, is intrinsic, the breadth extrinsic. Prof. Royce applying this doctrine to Ideas, notices their Internal Meaning and their External Meaning. He conceives of the internal meaning in a peculiar way. Another writer, a quarter of a century ago, laid down this maxim: "Consider what effects that might conceivably have practical bearings, we conceive the object of our conception to have. Then our conception of those effects is the *whole* of our conception of the object." †13 In the same pragmatistic spirit, Prof. Royce holds that the Internal Meaning of an idea is a Purpose, obscurely recognized in consciousness, partially fulfilled in being recognized but mainly unfulfilled and ill-understood in itself. The external meaning lies in the fulfillment of the purpose. In the opinion [of] some students who have succeeded in rendering the doctrines of logic more precise than they used to be, it is better to divide the difficulty of defining the meaning of an "idea," by first analyzing the nature of a sign in general. For an "idea," as having a meaning, is of the nature of a sign. After the general nature of a sign is once mastered, the problem of determining in what the meaning of an idea consists will evidently be stripped of a portion of its difficulty, and, as it turns out, of the greater portion. But in analyzing the general nature of a sign, it will be needful, to distinguish radically different kinds of signs. A sign may serve as a sign simply because it happens to resemble its object. †14 This resemblance will, then, constitute its internal meaning. But it cannot be said to have any external meaning, since it does not profess to represent anything; for if it did, that would be a manner of signifying its object, not consisting in merely resembling it. There are other signs which become such by virtue of being really connected with their objects. †15 Such is a symptom of disease, or the letters attached to parts of a diagram. The external meaning of such a sign is its most prominent feature. Its internal meaning may be vanishingly small, as in the case of the letters on the diagram, without ever utterly disappearing. There is, however, a third totally different order of signs, which become such, not by virtue of any character of their own as things, nor by virtue of any real connection with their objects, but simply by virtue of being represented to be signs. †16 Thus, the word 'cuckoo' does present a resemblance to the bird; but its onomatopoeia is a mere accident of its origin. It is further most used when cuckoos, or some effects of cuckoos, are really present; but that slight real connection with the birds is insignificant. It is constituted a name for the genus of birds exclusively by the circumstance that an English-speaking hearer of the word will so understand it. Such signs may have little or much internal meaning and external meaning but they have a third kind of meaning which consists in the character of the interpretant signs which they determine. This is their principal meaning. What Prof. Royce calls an "idea" is a sign of this class. For when he defines an idea as a state of mind which consciously "means" something, the whole context shows, as he would admit, that it "means" something in the sense of intending or purposing something. Now a purposive state of mind is one that signifies something by virtue of intending to be interpreted in a deed. Therefore, although an idea certainly has its internal and its external meaning, yet its principal meaning is of a different kind from either of those.

Peirce: CP 8.120 Cross-Ref:††

120. That the object of an idea, then, its external meaning, is of the nature of a sign could hardly be gainsaid. But Professor Royce finds it not only a sign but an idea; not only one idea but a "concrete" idea in the Hegelian sense, and that, not relatively, but perfectly, and so of the nature of life; and not only life, but an entire life. "The Being of the real object of which you now think, means a life that expresses the fulfilment of just your present plan."†17 We omit an inadmissible qualification †18 and remark that Prof. Royce's theory even if it were proved would not afford the slightest rational assurance that there is any such thing as a reality. In that respect it is, however, no worse than other theories, except in professing to be better.

Peirce: CP 8.121 Cross-Ref:††

121. An arbitrary feature of the theory, and a most regrettable blemish, where the author, too, parts company with Hegel, is that he insists that the object of an idea must be something other, -- or as he writes it, Other, -- than the idea itself. Not having space to criticize this, we shall simply ignore it in the further account

Peirce: CP 8.122 Cross-Ref:††

122. But how, it will be asked, can the meaning of a single idea be an entire life? An idea being a state of mind involving a purpose not fully realized, its internal meaning being that purpose so far as it is defined, we can understand that that purpose becomes more and more definite, until, being a sincere purpose, free from *arrière-pensée*, at the moment when it becomes in all respects determinate, it is transformed into an act But how can it become a complete life? The answer to this is very simple. Royce evidently thinks that a purpose cannot be fully definite, until all the circumstances of the entire life are taken into account; so that, however meagre the internal meaning of an idea may be, as long as it remains general and "abstract," yet when that internal meaning is fully accomplished by its becoming in every respect determined, the external meaning will cover the whole life of the individual. Certainly, it is conceivable that such might be the result; but to prove that such would be the result, a far more exact examination of the question would be requisite than the author attempts. There is another difficulty which he removes very happily. How, he supposes his reader to ask, can an idea, which is so microscopic a part of a life, contain within its implication a distinct feature corresponding to every feature of the entire life of which it is only a part? Here, he resorts to Gauss's conception of an *Abbild*, which has played so important a part in mathematics. That is to say, he likens the idea representing the entire life to a map of a country lying upon the ground in that country. Imagine that upon the soil of England, there lies somewhere a perfect map of England, showing every detail, however small. Upon this map, then, will be shown that very ground where the map lies, with the map itself in all its minutest details. There will be a part fully representing its whole, just as the idea is supposed to represent the entire life. On that map will be shown the map itself, and the map of the map will again show a map of itself, and so on endlessly. But each of these successive maps lies well inside the one which it immediately represents. Unless, therefore, there is a hole in the map within which no point represents a point otherwise unrepresented, this series of maps must all converge to a single point which represents itself throughout all the maps of the series. In the case of the idea, that point would be the self-consciousness of the idea. Since an idea is a state of mind with a conscious purpose, it obviously must be self-conscious.†19 Here, therefore, is a beautiful and needed, though not complete, confirmation of the idea's really being so related to the entire life. Singularly enough, however, for some reason, Prof. Royce

here draws back and supposes the analogy with the map to break down in this respect
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Peirce: CP 8.123 Cross-Ref:††

123. It will be perceived that, according to Prof. Royce's theory, if an idea fails of being a Self, it is only because it is general and not fully determined. Its implicit or germinal inward meaning is a little Self representing the entire man as its external meaning. In a similar way, the Self of the man is perhaps included within a larger Self of the community. On the other hand, the man's Self embraces intermediate selves, the domestic Self, the Self of business, the better Self, the evil spirit that sometimes possesses him. Here the author draws support from the psychological doctrine of what he calls the "time-span," a doctrine which, so far as it has really been placed beyond doubt, amounts to little more than that our image of the events of the few seconds last past is, or is very like, a direct perception, while our representation of what happened a minute ago partakes far less of the perceptive character.†20 The phenomenon had already been seized upon by several idealistic writers as affording a refutation of dualism; but the large calibre of Royce's thought cannot be better appreciated than by comparing their style of putting the phenomenon to the service of metaphysics with his.

Peirce: CP 8.124 Cross-Ref:††

124. He imagines that greater selves will naturally have vastly longer time-spans than lesser selves. Now a consciousness whose time-span was a thousandth of a second or a thousand years would not ordinarily be recognized by us, as observers of its external manifestations, as being a consciousness, at all. The time-span of the All-seeing must cover all time; and thus foreknowledge and freewill become more clearly reconcilable after the fashion of Boethius, St. Augustine, and others.

Peirce: CP 8.125 Cross-Ref:††

125. Every reality, then, is a Self; and the Selves are intimately connected, as if they formed a continuum. Each one is, so to say, a delineation, -- with mathematical truth, incongruous as the metaphor is, we may say that each is a quasimap of the organic aggregate of all the Selves, which is itself a Self, the Absolute Idea of Hegel, or God. It is a flagrant offence to use this name in philosophy. It is like inviting a man to see the body of his wife dissected. There is also a pretension in it that the philosophy of religion can be religion. But things shocking to right feeling are sometimes necessary in philosophy, as they are in science. It will be observed that if the Selves did form a continuum, each would be distinguished by its own point of Self-consciousness. This would not generally be the same as the point of self-consciousness of an idea within self, since each idea is distinguished by its own exclusive self-consciousness. The systems of delineation must be different. Here we see an inadequacy in the metaphor of the map; for what, more than anything else, makes my ideas mine is that they appeal to me, and are, or tend to become, represented in my general consciousness as representations. But, of course, the map-metaphor must be inadequate, since a map wants several of the essential characters of the class of signs to which ideas belong. Again, in the map the boundaries of the selves are somewhat indeterminate; each must embrace no more nor less than a complete map of the whole surface; but the boundary of any one can be considered to be drawn in any way which fulfills this condition, the boundaries of the others being drawn accordingly, just as on the Mercator's chart, which gives an endless series of representations of the whole globe, any one line from pole to pole

may be taken as the boundary of the globe as represented in each chart. But the boundaries between Selves are not so indeterminate, because all that is in one Self appeals by a continuum of representations to that Self's self-consciousness. It will be necessary, therefore, to replace the idea of a map by that of a continuum of maps overlying one another. A map is a section of a projection of which the surface mapped is another section. The projection itself is a sheaf of lines which diverge from one point. Instead of saying that a Self is a map, a more adequate metaphor would call it a projection of the reality, of which projection any one idea of the Self is a section. At any rate, it is plain that the map-metaphor requires deep emendation in order to answer the purposes of philosophy. At the same time, it is a considerable aid even as it is; and the initiating of the introduction of such exact ideas into philosophy is one of the momentous events in its history.†21

Peirce: CP 8.126 Cross-Ref:††

126. All reasoning goes upon the assumption that there is a true answer to whatever question may be under discussion, which answer cannot be rendered false by anything that the disputants may say or think about it; and further, that the denial of that true answer is false.†22 This makes an apparent difficulty for idealism. For if all reality is of the nature of an actual idea, there seems to be no room for possibility or any lower mode than actuality, among the categories of being. (Hegel includes modality only in his Subjective Logic.) But what, then, can be the mode of being of a representation or meaning unequivocally false? For Hegel, the false is the bad, that which is out of harmony with its own essence; and since, in his view, contradiction is the great form of activity of the world, he has no difficulty in admitting that an idea may be out of harmony with itself. Prof. Royce, however, seems almost to resent the idea that anybody could suppose that he denied the validity of the distinction of truth and falsehood. He is fairly outspoken in pronouncing sundry doctrines false (a word Hegel hardly uses), even if we do not quite hear his foot come down; and nothing does he hold more false than the usual form of stating the distinction now in question, namely, that a true proposition corresponds to a *real matter of fact*, by which is meant a state of things, definite and individual, which *does not consist merely in being represented (in any particular representation) to be as it is*. For example, if I dream that I find I can float in the air, this matter of dream is not matter of fact, for the reason that the only sense in which I can float in the air is that so my dream represented the matter. Now Prof. Royce offers to demonstrate by necessary reasoning that the statement -- or, as he expresses it, that "to be real means to be independent of ideas which relate to that being"†23 -- is false. His argument to this effect will serve as a sufficiently characteristic, but rather favorable sample of his general style of argumentation.

Peirce: CP 8.127 Cross-Ref:††

127. Having given us to understand that he is going to disprove the proposition, he opens his argumentation by declaring that he does not know what the proposition means. Thereupon, he proceeds to propound a general maxim of procedure for all cases in which one has to refute a proposition without knowing what it means. It is to begin by assigning to it its "most extreme form." This certainly does not signify the most extremely defensible meaning, but rather the most extremely indefensible meaning that the language will bear. The proposition having been refuted in this extreme sense, it will only be necessary afterwards to argue that other interpretations make no essential difference. This maxim, one would suppose, would prove very serviceable to anybody who should have any large amount of that sort of refutation to perform. In accordance with this maxim, Prof. Royce begins by

assuming that realists hold that no idea in the slightest degree determines the real object of it, whether causally or in any other manner. Whether this does not overstep the limits of admissible interpretation, seeing that a realist who meant this would deny that any promise can really be kept, or that any purpose can influence the real result, the reader must say. At any rate, it would not seem to be a difficult position to refute.

Peirce: CP 8.128 Cross-Ref:††

128. Now in order that he may get the realist where he wants him, there are two acknowledgments which Professor Royce endeavors to extort from him. To bring him to the first, the author assumes the principle that all causal action is reciprocal, or of the nature of reaction. This is evidently contrary to popular opinion, which holds that while the past has exerted some efficient causality upon the future, the future cannot have any *effect*, in the strict sense of that word, upon the past; and that while the future may have influenced the past by final, or ideal, causation, the past cannot possibly influence the future as the aim of the future. The reader may judge whether a realist of so extreme a type as that which Professor Royce has set up would or would not admit that the real object of an idea cannot have influenced the idea, in the absence of any attempt on the part of Professor Royce to prove his general principle of reciprocity. If he would not, old-fashioned logic (which Hegelians, it is true, hold in high contempt) would pronounce the attempted demonstration to be a bald *petitio principii*.

Peirce: CP 8.129 Cross-Ref:††

129. In order to extract the second acknowledgment from the realist, Professor Royce produces an argument which would seem to have as much force for one kind of realist as for another. He supposes two objects, B and R, to be related to one another as the realist supposes the Being, or the real object of an idea, and the Representation, in the form of an idea of that object, to be related; and he undertakes to define the relation between them. "The definition in question," he says, "is, as a mere abstract statement, easy."†24 One would think so. The realist simply says that B is not constituted by its being represented in R; that is, he says that the fact that B is as it is, would be logically consistent with R's representing it to be otherwise. But in place of this easily comprehensible relation, what fantastic attempt do we find at the definition that was pronounced to be so easy! Professor Royce will have it that the realist holds that the relation is such that no matter how R may be metamorphosed, it is logically possible for B to remain unchanged. In such a sense, what two things in the world are independent? Change the problematic madness of Hamlet into the pacification of the Philippines, and it will become logically inconsistent with the continuance of great disturbances there. But change the doubtful *representation* by Shakspeare that the fictitious Hamlet was unhinged into the *representation* that the Philippines were pacificated in 1901, and it will not have the slightest logical consequence for the real state of things. The truth is, that Professor Royce is blind to a fact which all ordinary people will see plainly enough; that the essence of the realist's opinion is that it is one thing to *be* and another thing to *be represented*; and the cause of this cecity is that the Professor is completely immersed in his absolute idealism, which precisely consists in denying that distinction. It is his element, and there is total reflection at its surface. That, however, is what Professor Royce asks the realist to admit as a premise. The conclusion which he deduces from it is that if B is linked as cause to any determination of R, there must be a *tertium quid* by the mediation of which the causation takes place. Now the premise is absurd; and the formal rule is that from an absurd premise every conclusion must be allowed to be logical; that is to say, it is needless to dispute its logicity, the premise being false. The argument,

therefore, cannot be called formally bad; nor can we object that a few lines below, in a restatement of the conclusion, B's being linked as cause gets changed into B's having any causal *or other* linkage.

Peirce: CP 8.130 Cross-Ref:††

130. Professor Royce, armed with his wrong definition of realism, goes on to a dilemma to show that, whether the realist says that real things are one or are many, he equally involves himself in contradiction. But, although the characteristics of his style of argumentation become even more prominent in that dilemma, the exigencies of space forbid our following him further. But we should like to say one word to this powerful and accurate thinker who has been so completely led astray in his argumentation by his Hegelian logic: Absolute idealism depends, as Hegel saw that it did, upon assuming that position at the outset. If your refutation of realism is addressed to students who are already absolute idealists at heart, we will not undertake to say whether it will be serviceable for the development of that doctrine, or not. But if it is addressed to realists themselves, it must conform to the logical principles recognized by realists, or be nugatory. Now you know very well that realists do not admit that matter of fact can be apodeictically demonstrated. You ought to know, and surely you do know, that if you drive them into a corner, they will simply modify their admissions so far as may be necessary to avoid self-contradiction, and that from the very nature of apodeictic proof it is absolutely impossible to close off such escape in arguing about matter of fact. The history of the doctrine of parallels illustrates what logic shows to be necessarily the state of the case.†25 But the question of realism is a question of hard fact, if ever there was a hard fact; and therefore your method must be revolutionized if you are ever to convince any master of logic.†26

Peirce: CP 8.131 Cross-Ref:††

131. Now let us address a few words to the author. A healthy religious spirit will not allow its religion to be disturbed by all the philosophy in the world. Nevertheless, a philosophy of religion deeply concerns us all. It is not a religious, but an intellectual need to bring our ideas into some harmony. Prof. Royce has inaugurated a vast reform, affecting not only the philosophy of religion but every department of metaphysics, and consisting in sweeping away all the vaguenesses and vagaries that now prevail in that science and replacing them by such exact ideas as Weierstrass and Cantor have begun to introduce into mathematics. No other man in the world, prominently before the public, is half so capable of working this matter out as he. What he has done is merely a preliminary essay. It is a pity that it fills a thousand pages. We want another book of about the same size; only instead of being written in the loose form of lectures, we want it to be a condensed and severe treatise, in which the innumerable vague and unsatisfactory points of the present exposition shall be minutely examined, in which *all* the new conceptions of multitude and continuity, and not merely that of the endless series, shall be applied not merely in the single narrow way in which that one is here applied, but in every way, not merely to the one matter to which it is here applied but to every subject of metaphysics from top to bottom, together with whatsoever other exact diagrammatic conceptions can be produced, and the whole reasoning, so far as it is demonstrative, be rendered diagrammatic,†27 and so far as it relates to questions of fact be made scientific. To illustrate what various applications may be made of the idea of the endless series, it may be noted that admitting the actuality of this, it does not follow that because A (or mind) cannot act directly upon B (or matter), . . . A cannot act upon B without the intervention of a *tertium quid*.†28 The bearing of this simple remark upon the theory

of Parallelism and upon the philosophy of conduct is dynamic.†29 This is the work which it is Prof. Royce's duty to give to the world, and the world's bounden duty to aid Royce to produce, no matter how many dimes, cents, and dollars it may cost.

Peirce: CP 8.132 Cross-Ref:††
CHAPTER 7

KARL PEARSON, *THE GRAMMAR OF SCIENCE*†1

§1. THE JUSTIFICATION OF SCIENTIFIC RESEARCH

132. If any follower of Dr. Pearson thinks that in the observations I am about to make I am not sufficiently respectful to his master, I can assure him that without a high opinion of his powers I should not have taken the trouble to make these annotations, and without a higher opinion still, I should not have used the bluntness which becomes the impersonal discussions of mathematicians.

Peirce: CP 8.133 Cross-Ref:††

133. An introductory chapter of ethical content sounds the dominant note of the book. The author opens with the declaration that our conduct ought to be regulated by the Darwinian theory. Since that theory is an attempt to show how natural causes tend to impart to stocks of animals and plants characters which, in the long run, promote reproduction and thus insure the continuance of those stocks, it would seem that making Darwinism the guide of conduct ought to mean that the continuance of the race is to be taken as the *summum bonum*, and '*Multiplicamini*' as the epitome of the moral law. Professor Pearson, however, understands the matter a little differently, expressing himself thus: "The sole reason [for encouraging] any form of human activity . . . lies in this: [its] existence tends to promote the welfare of human society, to increase social happiness, or to strengthen social stability. In the spirit of the age we are bound to question the value of science; to ask in what way it increases the happiness of mankind or promotes social efficiency."†2

Peirce: CP 8.134 Cross-Ref:††

134. The second of these two statements omits the phrase, 'the welfare of human society,' which conveys no definite meaning; and we may, therefore, regard it as a mere diluent, adding nothing to the essence of what is laid down. Strict adherence to Darwinian principles would preclude the admission of the 'happiness of mankind' as an ultimate aim. For on those principles everything is directed to the continuance of the stock, and the individual is utterly of no account, except in so far as he is an agent of reproduction. Now there is no other happiness of mankind than the happiness of individual men. We must, therefore, regard this clause as logically deleterious to the purity of the doctrine. As to 'social stability,' we all know very well what ideas this phrase is intended to convey to English apprehensions; and it must be admitted that Darwinism, generalized in due measure, may apply to English society the same principles that Darwin applied to breeds. A family in which the standards of that

society are not traditional will go under and die out, and thus 'social stability' tends to be maintained.

Peirce: CP 8.135 Cross-Ref:††

135. But against the doctrine that social stability is the sole justification of scientific research, whether this doctrine be adulterated or not with the utilitarian clause, I have to object, first, that it is historically false, in that it does not accord with the predominant sentiment of scientific men; second, that it is bad ethics; and, third, that its propagation would retard the progress of science.

Peirce: CP 8.136 Cross-Ref:††

136. Professor Pearson does not, indeed, pretend that that which effectually animates the labors of scientific men is any desire 'to strengthen social stability.' Such a proposition would be too grotesque. Yet if it was his business, in treating of the grammar of science, to set forth the legitimate motive to research -- as he has deemed it to be -- it was certainly also his business, especially in view of the splendid successes of science, to show what has, in fact, moved such men. They have, at all events, not been inspired by a wish either to 'support social stability' or, in the main, to increase the sum of men's pleasures. The man of science has received a deep impression of the majesty of truth, as that to which, sooner or later, every knee must bow.†3 He has further found that his own mind is sufficiently akin to that truth, to enable him, on condition of submissive observation, to interpret it in some measure. As he gradually becomes better and better acquainted with the character of cosmical truth, and learns that human reason is its issue and can be brought step by step into accord with it, he conceives a passion for its fuller revelation. He is keenly aware of his own ignorance, and knows that personally he can make but small steps in discovery. Yet, small as they are, he deems them precious; and he hopes that by conscientiously pursuing the methods of science he may erect a foundation upon which his successors may climb higher. This, for him, is what makes life worth living and what makes the human race worth perpetuation. The very being of law, general truth, reason -- call it what you will -- consists in its expressing itself in a cosmos and in intellects which reflect it, and in doing this progressively; and that which makes progressive creation worth doing -- so the researcher comes to feel -- is precisely the reason, the law, the general truth for the sake of which it takes place.

Peirce: CP 8.137 Cross-Ref:††

137. Such, I believe, as a matter of fact, is the motive which effectually works in the man of science. That granted, we have next to inquire which motive is the more rational, the one just described or that which Professor Pearson recommends. The ethical text-books offer us classifications of human motives. But for our present purpose it will suffice to pass in rapid review some of the more prominent ethical classes of motives.

Peirce: CP 8.138 Cross-Ref:††

138. A man may act with reference only to the momentary occasion, either from unrestrained desire, or from preference for one desideratum over another, or from provision against future desires, or from persuasion, or from imitative instinct, or from dread of blame, or in awed obedience to an instant command; or he may act according to some general rule restricted to his own wishes, such as the pursuit of pleasure, or self-preservation, or good-will toward an acquaintance, or attachment to home and surroundings, or conformity to the customs of his tribe, or reverence for a law; or, becoming a moralist, he may aim at bringing about an ideal state of things definitely conceived, such as one in which everybody attends exclusively to his own

business and interest (individualism), or in which the maximum total pleasure of all beings capable of pleasure is attained (utilitarianism), or in which altruistic sentiments universally prevail (altruism), or in which his community is placed out of all danger (patriotism), or in which the ways of nature are as little modified as possible (naturalism); or he may aim at hastening some result not otherwise known in advance than as that, whatever it may turn out to be, to which some process seeming to him good must inevitably lead, such as whatever the dictates of the human heart may approve (sentimentalism), or whatever would result from every man's duly weighing, before action, the advantages of his every purpose (to which I will attach the nonce-name *entelism*, distinguishing it and others below by italics), or whatever the historical evolution of public sentiment may decree (*historicism*), or whatever the operation of cosmical causes may be destined to bring about (evolutionism); or he may be devoted to truth, and may be determined to do nothing not pronounced reasonable, either by his own cogitations (rationalism), or by public discussion (dialecticism), or by crucial experiment; or he may feel that the only thing really worth striving for is the generalizing or assimilating elements in truth, and that either as the sole object in which the mind can ultimately recognize its veritable aim (educationalism), or that which alone is destined to gain universal sway (pancratism), or, finally, he may be filled with the idea that the only reason that can reasonably be admitted as ultimate is that living reason for the sake of which the psychical and physical universe is in process of creation (*religionism*).†4

Peirce: CP 8.139 Cross-Ref:††

139. This list of ethical classes of motives may, it is hoped, serve as a tolerable sample upon which to base reflections upon the acceptability as ultimate of different kinds of human motives; and it makes no pretension to any higher value. The enumeration has been so ordered as to bring into view the various degrees of generality of motives. It would conduce to our purpose, however, to compare them in other respects. Thus, we might arrange them in reference to the degree to which an impulse of dependence enters into them, from express obedience, generalized obedience, conformity to an external exemplar, action for the sake of an object regarded as external, the adoption of a motive centering on something which is partially opposed to what is present, the balancing of one consideration against another, until we reach such motives as unrestrained desire, the pursuit of pleasure, individualism, sentimentalism, rationalism, educationalism, religionism, in which the element of otherness is reduced to a minimum. Again, we might arrange the classes of motives according to the degree in which immediate qualities of feeling appear in them, from unrestrained desire, through desire present but restrained, action for self, action for pleasure generalized beyond self, motives involving a retro-consciousness of self in outward things, the personification of the community, to such motives as direct obedience, reverence, naturalism, evolutionism, experimentalism, pancratism, religionism, in which the element of self-feeling is reduced to a minimum. But the important thing is to make ourselves thoroughly acquainted, as far as possible from the inside, with a variety of human motives ranging over the whole field of ethics.

Peirce: CP 8.140 Cross-Ref:††

140. I will not go further into ethics than simply to remark that all motives that are directed toward pleasure or self-satisfaction, of however high a type, will be pronounced by every experienced person to be inevitably destined to miss the satisfaction at which they aim. This is true even of the highest of such motives, that which Josiah Royce develops in his 'World and Individual.'†5 On the other hand, every motive involving dependence on some other leads us to ask for some ulterior

reason. The only desirable object which is quite satisfactory in itself without any ulterior reason for desiring it, is the reasonable itself. I do not mean to put this forward as a demonstration; because, like all demonstrations about such matters, it would be a mere quibble, a sheaf of fallacies. I maintain simply that it is an experiential truth.

Peirce: CP 8.141 Cross-Ref:††

141. The only ethically sound motive is the most general one; and the motive that actually inspires the man of science, if not quite that, is very near to it -- nearer, I venture to believe, than that of any other equally common type of humanity. On the other hand, Professor Pearson's aim, 'the stability of society,' which is nothing but a narrow British patriotism, prompts the *cui bono* at once. I am willing to grant that England has been for two or three centuries a most precious factor of human development. But there were and are *reasons* for this. To demand that man should aim at the stability of British society, or of society at large, or the perpetuation of the race, as an *ultimate* end, is too much. The human species will be extirpated sometime; and when the time comes the universe will, no doubt, be well rid of it. Professor Pearson's ethics are not at all improved by being adulterated with utilitarianism, which is a lower motive still. Utilitarianism is one of the few theoretical motives which has unquestionably had an extremely beneficial influence. But the greatest happiness of the greatest number, as expounded by Bentham, resolves itself into merely superinducing the quality of pleasure upon men's immediate feelings. Now, if the pursuit of pleasure is not a satisfactory ultimate motive for me, why should I enslave myself to procuring it for others? Leslie Stephen's book was far from uttering the last word upon ethics; but it is difficult to comprehend how anybody who has read it reflectively can continue to hold the mixed doctrine that no action is to be encouraged for any other reason than that it either tends to the stability of society or to general happiness.

Peirce: CP 8.142 Cross-Ref:††

142. Ethics, as such, is extraneous to a Grammar of Science; but it is a serious fault in such a book to inculcate reasons for scientific research the acceptance of which must tend to lower the character of such research. Science is, upon the whole, at present in a very healthy condition. It would not remain so if the motives of scientific men were lowered. The worst feature of the present state of things is that the great majority of the members of many scientific societies, and a large part of others, are men whose chief interest in science is as a means of gaining money, and who have a contempt, or half-contempt, for pure science. Now, to declare that the sole reason for scientific research is the good of society is to encourage those pseudo-scientists to claim, and the general public to admit, that they, who deal with the applications of knowledge, are the true men of science, and that the theoreticians are little better than idlers.

Peirce: CP 8.143 Cross-Ref:††

143. In Chapter II., entitled 'The Facts of Science,' we find that the 'stability of society' is not only to regulate our conduct, but, also, that our opinions have to be squared to it. In section 10 we are told that we must not believe a certain purely theoretical proposition because it is 'anti-social' to do so, and because to do so 'is opposed to the interests of society.' As to the 'canons of legitimate inference' themselves, that are laid down by Professor Pearson, I have no great objection to them. They certainly involve important truths. They are excessively vague and capable of being twisted to support illogical opinions, as they are twisted by their

author, and they leave much ground uncovered. But I will not pursue these objections. I do say, however, that truth is truth, whether it is opposed to the interests of society to admit it or not -- and that the notion that we must deny what it is not conducive to the stability of British society to affirm is the mainspring of the mendacity and hypocrisy which Englishmen so commonly regard as virtues. I must confess that I belong to that class of scallawags who purpose, with God's help, to look the truth in the face, whether doing so be conducive to the interests of society or not. Moreover, if I should ever attack that excessively difficult problem, 'What is for the true interest of society?' I should feel that I stood in need of a great deal of help from the science of legitimate inference; and, therefore, to avoid running round a circle, I will endeavor to base my theory of legitimate inference upon something less questionable -- as well as more germane to the subject -- than the true interest of society.

Peirce: CP 8.144 Cross-Ref:††
§2. NATURAL LAW

144. The remainder of this chapter on the 'Facts of Science' is taken up with a theory of cognition, in which the author falls into the too common error of confounding psychology with logic. He will have it that knowledge is built up out of sense-impressions -- a correct enough statement of a conclusion of psychology. Understood, however, as Professor Pearson understands and applies it, as a statement of the nature of our logical data, of 'the facts of science,' it is altogether incorrect. He tells us that each of us is like the operator at a central telephone office, shut out from the external world, of which he is informed only by sense-impressions. Not at all! Few things are more completely hidden from my observation than those hypothetical elements of thought which the psychologist finds reason to pronounce 'immediate,' in his sense.†6 But the starting point of all our reasoning is not in those sense-impressions, but in our percepts. When we first wake up to the fact that we are thinking beings and can exercise some control over our reasonings, we have to set out upon our intellectual travels from the home where we already find ourselves. Now, this home is the parish of percepts. It is not inside our skulls, either, but out in the open. It is the external world that we directly observe. What passes within we only know as it is mirrored in external objects. In a certain sense, there is such a thing as introspection; but it consists in an interpretation of phenomena presenting themselves as external percepts. We first see blue and red things. It is quite a discovery when we find the eye has anything to do with them, and a discovery still more recondite when we learn that there is an *ego* behind the eye, to which these qualities properly belong. Our logically initial data are percepts. Those percepts are undoubtedly purely psychical, altogether of the nature of thought. They involve three kinds of psychical elements, their qualities of feelings, their reaction against my will, and their generalizing or associating element. But all that we find out afterward. I see an inkstand on the table: that is a percept. Moving my head, I get a different percept of the inkstand. It coalesces with the other. What I call the inkstand is a generalized percept, a quasi-inference from percepts, perhaps I might say a composite-photograph of percepts. In this psychical product is involved an element of resistance to me, which I am obscurely conscious of from the first. Subsequently, when I accept the hypothesis of an inward subject for my thoughts, I yield to that consciousness of resistance and admit the inkstand to the standing of an external object. Still later, I may call this in question. But as soon as I do that, I find that the inkstand appears

there in spite of me. If I turn away my eyes, other witnesses will tell me that it still remains. If we all leave the room and dismiss the matter from our thoughts, still a photographic camera would show the inkstand still there, with the same roundness, polish and transparency, and with the same opaque liquid within. Thus, or otherwise, I confirm myself in the opinion that its characters are what they are, and persist at every opportunity in revealing themselves, regardless of what you, or I, or any man, or generation of men, may think that they are. That conclusion to which I find myself driven, struggle against it as I may, I briefly express by saying that the inkstand is a *real* thing. Of course, in being real and external, it does not in the least cease to be a purely psychical product, a generalized percept, like everything of which I can take any sort of cognizance.

Peirce: CP 8.145 Cross-Ref:††

145. It might not be a very serious error to say that the facts of science are sense-impressions, did it not lead to dire confusion upon other points. We see this in Chapter III.,†7 in whose long meanderings through irrelevant subjects, in the endeavor to make out that there is no rational element in nature, and that the rational element of natural laws is imported into them by the minds of their discoverers, it would be impossible for the author to lose sight entirely of the bearing of the question which he himself has distinctly formulated, if he were not laboring with the confusing effects of his notion that the data of science are the sense-impressions. It does not occur to him that he is laboring to prove that the mind has a marvelous power of creating an element absolutely supernatural -- a power that would go far toward establishing a dualism quite antagonistic to the spirit of his philosophy. He evidently imagines that those who believe in the reality of law, or the rational element in nature, fail to apprehend that the data of science are of a psychical nature. He even devotes a section to proving that natural law does not belong to things-in-themselves, as if it were possible to find any philosopher who ever thought it did. Certainly, Kant, who first decked out philosophy with these chaste ornaments of things-in-themselves, was not of that opinion; nor could anybody well hold it after what he wrote. In point of fact, it is not Professor Pearson's opponents but he himself who has not thoroughly assimilated the truth that everything we can in any way take cognizance of is purely mental. This is betrayed in many little ways, as, for instance, when he makes his answer to the question, whether the law of gravitation ruled the motion of the planets before Newton was born, to turn upon the circumstance that the law of gravitation is a formula expressive of the motion of the planets 'in terms of a purely mental conception,' as if there could be a conception of anything not purely mental. Repeatedly, when he has proved the content of an idea to be mental, he seems to think he has proved its object to be of human origin. He goes to no end of trouble to prove in various ways, what his opponent would have granted with the utmost cheerfulness at the outset, that laws of nature are rational; and, having got so far, he seems to think nothing more is requisite than to seize a logical maxim as a leaping pole and lightly skip to the conclusion that the laws of nature are of human provenance. If he had thoroughly accepted the truth that all realities, as well as all figments, are alike of purely mental composition, he would have seen that the question was, not whether natural law is of an intellectual nature or not, but whether it is of the number of those intellectual objects that are destined ultimately to be exploded from the spectacle of our universe, or whether, as far as we can judge, it has the stuff to stand its ground in spite of all attacks. In other words, is there anything that is really and truly a law of nature, or are all pretended laws of nature figments, in which latter case, all natural science is a delusion, and the writing of a grammar of science a very idle pastime?

Peirce: CP 8.146 Cross-Ref:††

146. Professor Pearson's theory of natural law is characterized by a singular vagueness and by a defect so glaring as to remind one of the second book of the *Novum Organum* or of some strong chess-player whose attention has been so riveted upon a part of the board that a fatal danger has, as it were, been held upon the blind-spot of his mental retina. The manner in which the current of thought passes from the woods into the open plain and back again into the woods, over and over again, betrays the amount of labor that has been expended upon the chapter. The author calls attention to the sifting action both of our perceptive and of our reflective faculties. I think that I myself extracted from that vein of thought pretty much all that is valuable in reference to the regularity of nature in the *POPULAR SCIENCE MONTHLY* for June, 1878, (p. 208).†8 I there remarked that the degree to which nature seems to present a general regularity depends upon the fact that the regularities in it are of interest and importance to us, while the irregularities are without practical use or significance; and in the same article I endeavored to show that it is impossible to conceive of nature's being markedly less regular, taking it, 'by and large,' than it actually is. But I am confident, from having repeatedly returned to that line of thought that it is impossible legitimately to deduce from any such considerations the unreality of natural law. 'As a pure suggestion and nothing more,' toward the end of the chapter, after his whole plea has been put in, Dr. Pearson brings forward the idea that a transcendental operation of the perceptive faculty may reject a mass of sensation altogether and arrange the rest in place and time, and that to this the laws in nature may be attributable -- a notion to which Kant undoubtedly leaned at one time. The mere emission of such a theory, after his argument has been fully set forth, almost amounts to a confession of failure to prove his proposition. Granting, by way of waiver, that such a theory is intelligible and is more than a nonsensical juxtaposition of terms, so far from helping Professor Pearson's contention at all, the acceptance of it would at once decide the case against him, as every student of the *Critic of the Pure Reason* will at once perceive. For the theory sets the rationality in nature upon a rock perfectly impregnable by you, me or any company of men.

Peirce: CP 8.147 Cross-Ref:††

147. Although that theory is only problematically put forth by Professor Pearson, yet at the very outset of his argumentation he insists upon the relativity of regularity to our faculties, as if that were in some way pertinent to the question. "Our law of tides," he says, "could have no meaning for a blind worm on the shore, for whom the moon had no existence."†9 Quite so; but would that truism in any manner help to prove that the moon was a figment and no reality? On the contrary, it could only help to show that there may be more things in heaven and earth than your philosophy has dreamed of. Now the *moon*, on the one hand, and the *law of the tides*, on the other, stand in entirely analogous positions relatively to the remark, which can no more help to prove the unreality of the one than of the other. So, too, the final decisive stroke of the whole argumentation consists in urging substantially the same idea in the terrible shape of a syllogism, which the reader may examine in section 11. I will make no comment upon it.

Peirce: CP 8.148 Cross-Ref:††

148. Professor Pearson's argumentation rests upon three legs. The first is the fact that both our perceptive and our reflective faculties reject part of what is presented to them, and 'sort out' the rest. Upon that, I remark that our minds are not, and cannot be, positively mendacious. To suppose them so is to misunderstand what

we all mean by truth and reality. Our eyes tell us that some things in nature are red and others blue; and so they really are. For the real world is the world of insistent generalized percepts. It is true that the best physical idea which we can at present fit to the real world, has nothing but longer and shorter waves to correspond to red and blue. But this is evidently owing to the acknowledged circumstance that the physical theory is to the last degree incomplete, if not to its being, no doubt, in some measure, erroneous. For surely the completed theory will have to account for the extraordinary contrast between red and blue. In a word, it is the business of a physical theory to account for the percepts; and it would be absurd to accuse the percepts -- that is to say, the facts -- of mendacity because they do not square with the theory.

Peirce: CP 8.149 Cross-Ref:††

149. The second leg of the argumentation is that the mind projects its worked-over impressions into an object, and then projects into that object the comparisons, etc., that are the results of its own work. I admit, of course, that errors and delusions are everyday phenomena, and hallucinations not rare. We have just three means at our command for detecting any unreality, that is, lack of insistency, in a notion. First, many ideas yield at once to a direct effort of the will. We call them *fancies*. Secondly, we can call in other witnesses, including ourselves under new conditions. Sometimes dialectic disputation will dispel an error. At any rate, it may be voted down so overwhelmingly as to convince even the person whom it affects. Thirdly, the last resort is prediction and experimentation. Note that these two are equally essential parts of this method, which Professor Pearson keeps -- I had almost said sedulously -- out of sight in his discussion of the rationality of nature. He only alludes to it when he comes to his transcendental 'pure suggestion.' Nothing is more notorious than that this method of prediction and experimentation has proved the master-key to science; and yet, in Chapter IV., †10 Professor Pearson tries to persuade us that prediction is no part of science, which must only describe sense-impressions. (A sense-impression cannot be described.) He does not say that he would permit generalization of the facts. He ought not to do so, since generalization inevitably involves prediction.

Peirce: CP 8.150 Cross-Ref:††

150. The third leg of the argumentation is that human beings are so much alike that what one man perceives and infers another man will be likely to perceive and infer. This is a recognized weakness of the second of the above methods. It is by no means sufficient to destroy that method, but along with other defects it does render resort to the third method imperative. When I see Dr. Pearson passing over without notice the first and third of the only three possible ways of distinguishing whether the rationality of nature is real or not, and giving a lame excuse for reversing the verdict of the second, so that his decision seems to spring from antecedent predilection, I cannot recommend his procedure as affording such an exemplar of the logic of science as one might expect to find in a grammar of science.

Peirce: CP 8.151 Cross-Ref:††

151. An ignorant sailor on a desert island lights in some way upon the idea of the parallelogram of forces, and sets to work making experiments to see whether the actions of bodies conform to that formula. He finds that they do so, as nearly as he can observe, in many trials invariably. He wonders why inanimate things should thus conform to a widely general intellectual formula. Just then, a disciple of Professor Pearson lands on the island and the sailor asks him what he thinks about it. "It is very simple," says the disciple, "you see you made the formula and then you projected it

into the phenomena." *Sailor*: What are the phenomena? *Pearsonist*: The motions of the stones you experimented with. *Sailor*: But I could not tell until afterward whether the stones had acted according to the rule or not. *Pearsonist*: That makes no difference. You made the rule by looking at some stones, and all stones are alike. *Sailor*: But those I used were very unlike, and I want to know what made them all move exactly according to one rule. *Pearsonist*: Well, maybe your mind is not in time, and so you made all the things behave the same way at all times. Mind, I don't say it is so; but it may be. *Sailor*: Is that all you know about it? Why not say the stones are made to move as they do by something *like* my mind?

Peirce: CP 8.152 Cross-Ref:††

152. When the disciple gets home, he consults Dr. Pearson. "Why," says Dr. Pearson, "you must not deny that the facts are really concatenated; only there is no rationality about that." "Dear me," says the disciple, "then there really is a concatenation that makes all the component accelerations of all the bodies scattered through space conform to the formula that Newton, or Lami, or Varignon invented?" "Well, the formula is the device of one of those men, and it conforms to the facts." "To the facts its inventor knew, and also to those he only predicted?" "As for prediction, it is unscientific business." "Still the prediction and the facts predicted agree." "Yes." "Then," says the disciple, "it appears to me that there really is in nature something extremely like action in conformity with a highly general intellectual principle." "Perhaps so," I suppose Dr. Pearson would say, "but nothing in the least like rationality." "Oh," says the disciple, "I thought rationality was conformity to a widely general principle."

Peirce: CP 8.153 Cross-Ref:††

153. To sum up my objections. Prof. Pearson mistakes sense-impressions, which are psychological inferences, for the logical data, and is thus led to confuse his thought in this chapter with matters totally irrelevant to the question which he clearly puts. He fancies that his antagonists fail to apprehend the psychical side of the subject; but in fact it is he himself who has not thoroughly assimilated the truth that everything that we can in any way take cognizance of is purely mental. This is a truth, because every object of thought is either a percept or a generalization, that is, an inference from percepts. I am conscious that my meaning here is but vaguely expressed, because I use the word "generalization" in a generalized sense. Unfortunately, I cannot explain myself without tedious developments of exact logic into which I cannot here enter. Meantime, though my meaning can, perhaps, be but dimly apprehended, still it can be sufficiently understood for the purpose in hand. Prof. Pearson, not having fully assimilated the truth that every object is purely mental or psychical, thinks that when he has shown that the content of natural law is intellectual, he is entitled to conclude that it is of human origin. But every scientific research goes upon the assumption, the hope, that, in reference to its particular question, there is some true answer. That which that truth represents is a reality. This reality being cognizable and comprehensible, is of the nature of thought. Wherein, then, does its reality consist? In the fact that, though it has no being out of thought, yet it is as it is, whether you or I or any group of men think it to be so or not. The question of whether Hamlet was insane is the question whether Shakespeare conceived him to be insane. Consequently, Hamlet is a figment and not a reality. But as to the inkstand being on my table, though I should succeed in persuading myself

and all who have seen it that it is a mere optical illusion, yet there will be a limit to this, and by the photographic camera, the balance, new witnesses, etc., it will, at last, I guess, force its recognition upon the world. If so, it has the characteristic which we call reality. There is a blind force about the inkstand by which it crowds its way into our universe in spite of all we can do. Prof. Pearson has no difficulty in showing, what his opponents are eager to have him commit himself to, that law is of an intellectual or rational nature; but he slides swiftly and lightly over the passage from that position to its having been introduced into the object by the scientist's own mind. But here is the whole question. Is law real or is it figment? Psychological of course it is; for every thing we can cognize is purely psychological. Intellectual or rational it plainly is. But the question is whether it is among those intellectual objects that are destined ultimately to be exploded from the spectacle of the universe, or whether, as far as we can judge, it has the force to stand its ground indefinitely. It seems clear, to begin with, that to prove law a figment would be to prove all science to be a delusion and a Grammar of Science an idle pastime. Prof. Pearson is very likely quite right when, in a later chapter, he suggests that the law of the parallelogram of forces is not perfectly true. His reasons have great weight. I, for my part, do not believe that any law is perfectly satisfied. If I am right in this, the reality of law is diminished; but it is not thereby abolished. But my argument to show that law is reality and not figment, -- is in nature independently of any connivance of ours, -- is that predictions are verified. Nobody will maintain that these verifications are chance coincidences. Nor can Prof. Pearson explain how Newton and Laplace have been influential in producing eclipses at the moments they were called for by theory. He does not attempt it. He tells us he admits that phenomena are "concatenated," but that he can see in that nothing that, in any intelligible sense, can be called rational. Here again, I take issue. "Concatenation" is not a fair word in this connection. For "concatenation" implies contiguity in some sense, through which some unintelligible action and reaction can take place. But the different cases in which a law is verified are not connected by contiguity but by resemblance, and that of a very abstract kind. Their connection consists merely in this, that wherever it may be that a certain very broad but definite resemblance occurs, there also resemblance in another definite respect occurs. Now how would you define a reason if not as a very broad definite character which makes us expect the occurrence of another definite character? If Prof. Pearson does not accept this statement, then, since he maintains that there is nothing at all mysterious about law, it was his business to say how he proposed to account for what he very ill describes as the concatenation of sense-impressions, that is to say, the conformity of widely scattered phenomena to the predictions of the scientist. Not to account for it at all, is simply to leave it as a conformity to a rational formula, and therefore as a real reasonableness in nature.

Peirce: CP 8.154 Cross-Ref:††

154. From most of the other chapters in the book I should altogether dissent, and most especially from the account of probability in Chapter IV. But I shall, at this time, notice only two small points connected with the above remarks.

Peirce: CP 8.155 Cross-Ref:††

155. In Chapter IV., Prof. Pearson declares that the sole business of science is to describe past experience and not at all to predict the future. This is entirely contrary to the universal opinion of men of science, in whose eyes prediction is the seal of success. Neither can it please those who have been led to expect from the introductory chapter that science would be of some practical service. It would be a maxim utterly blighting to all further progress of science, were it accepted, since it is only by

predictions that men are led to devise new experiments. According to this doctrine, scientists should print their notebooks, and do no more. But evidently, science has, not so much to describe experience, as to generalize it. To generalize it is to comprehend it. Moreover, generalization refuses to limit itself to the past, but involves virtual prediction.

Peirce: CP 8.156 Cross-Ref:††

156. In the same chapter, the author says that the *why* of things remains a mystery. He quotes with approval a sentence from Kirchhoff's *Mechanik* to the effect that dynamics is the description, in the simplest terms, of motions. This, except for its indeterminacy, is well enough. But he omits to mention that Kirchhoff goes on to say that what Force is must remain a mystery. But according to my notions there can be no mystery in the universe, in the sense of a real fact to which no approach to knowledge can ever be gained. For a reality is an idea that insists upon proclaiming itself, whether we like it or not. There may be a question that no amount of research can ever answer. If so, there is a *lacuna* in the completeness of reality. But these things usually called mysteries are simply cases in which questions cannot be answered for the reason that no definite meaning can be attached to them. If, for example, we know exactly under what circumstances bodies are accelerated, and what the resulting changes of velocity and position are, you must say definitely what further experience you wish to predict before you talk about a mystery.

Peirce: CP 8.157 Cross-Ref:††
CHAPTER 8

***REVIEW OF A BOOK ON ETHICS*†1**

157. Professor Mezes of the University of Texas has been known to the general public as a scholar of Howison, and as one of the four authors of the sympotic book, 'The Conception of God.' He there produced upon us a mixed impression, for his intellect seemed not to have quite so keen an edge as is called for in philosophy; and yet here and there conceptions appeared so simple and obvious, and yet so novel, that one ransacked one's memory in the endeavor to recall any anticipation of the remark. Much the same impression is renewed by the present book. Hard work and solid has been put into it; and, of course, the harvest must have proportionate value. Parts of the treatise are admirably worked out, and are, at any rate, instructive, even if their conclusions are rejected. But hard work is not all that is required in dealing with such a subject.

Peirce: CP 8.158 Cross-Ref:††

158. In aim and method the present work is fully as original as it ought to be. The author belongs to that school of ethics which is probably nearest right -- that is to say, to the school which makes tribal tradition a main factor of morality, and which is thus enabled to frame an evolutionary theory of it. But although the author is thus in the van of ethical exploration, a certain old-fashioned and conservative color -- attributable, perhaps, to temperament and Texan environment -- strongly tinges his

theory. Now, conservatism in morals is most needful in practice, and, of course, is theoretically defensible.†2 But that defence itself is not conservative: on the contrary, it is rationalistic; and in pure theory, especially in a theory of aims, conservatism is irrational and out of place. The writer effects a reconciliation of his conservatism (which is very likely unconscious) with his advanced views by exaggerating more than usual a prevalent tendency which we venture to think that the majority of philosophers of our day carry too far -- we mean the tendency to base everything in philosophy upon the psychical sciences. The immense success of scientific psychology during the last forty years has very naturally given it a weight in men's minds that ought not in philosophy to be accorded to any merely special science, which is precisely what psychology has all along been striving and struggling to be.†3 On the contrary, it is now generally admitted that psychology, like general physics, necessarily takes for granted a *Weltanschauung* or outline system of metaphysics. Now, metaphysics can have no satisfactory grounding except upon a scientific logic; and logic rests on ethics to a degree that few are aware of. So if there be no other basis for ethics than psychology, which is a third story above it, the whole erection floats on air. Ethics as a positive science must rest on observed facts. But it is quite a different thing to make it rest on special scientific observation, and still more so to base it upon scientific conclusions. The only solid foundation for ethics lies in those facts of everyday life which no skeptical philosopher ever yet really called in question.

Peirce: CP 8.159 Cross-Ref:††

159. Now, Mr. Mezes is so far from taking this view that he maintains that the whole business of the moralist consists in saying what men mean by morality, in describing what they hold to be moral, and in explaining how they come to do so. This is a most interesting and valuable study, but it is ethical anthropology, not pure ethics; and to limit ethics in this way is to be faithless to the first duty of a moralist, as such. "Ethical writers do not in any proper sense," he says, meaning that they overstep the bounds of their province when they do, "judge conduct or issue pronouncements as to what is right or wrong. Their more modest task is to discover and record men's genuine judgments as to what is right or wrong."†4 Let us see how this view of ethics works. A judge, let us suppose, has brought before him a case in which a man has suffered injury for which he claims damages of another. Whether damages ought to be paid in such a case is often, we know, a delicate and puzzling question. We will follow Professor Mezes in using a much too simple illustration, which ought to puzzle nobody. "Take," he says, "the case where A's cattle break out of their enclosure, in spite of A's having used all the care he reasonably could have used, or could learn to use, and destroy B's valuable crop in an adjoining field."†5 This case (or rather another far more difficult) puzzles the judge, and he takes it under advisement. He naturally looks into works on ethics, and, finding nothing pertinent in modern books, is driven to the scholastic treatises. Now, there is nothing in the whole scholastic logic more justly an object of derision for any modern thinker than its weak confusion of thought in its doctrine of causes; nor in that whole doctrine is there any more manifest absurdity than the distinction between a *proximate* and a *remote* cause. When we meet with an application of it in the scholastic commentary on the Sentences, it stands out as so much more nonsensical than the rest as to be comical; but that anybody should be made to suffer because of any consequence of such metaphysical jargon is outrageous flippancy. Yet it is just this outrage that the judge is driven to commit, or to pretend to commit, because the ethical writers have not expounded right and wrong in a sufficiently luminous and reasonable form.

Peirce: CP 8.160 Cross-Ref:††

160. Professor Mezes follows them. He maintains that A, the owner of the cattle, ought to reimburse B for the injury done by them to his crop, because A is the *proximate cause* of B's suffering. If he would not follow the decisions of Texas courts as the ultimate evidence concerning right and wrong, he could not fail to see that the real reason why the judge awards damages to B is that to allow a private person to undertake a business humanly sure in the long run to injure his neighbors (and all the more so if he "cannot learn to use" suitable preventive measures), and then to allow him to pocket all the profits, and make his neighbors pay for incidental losses, would be to bring himself and his court into public contempt and into no little danger. That was the judge's real reason. But in days gone by (perhaps not yet in Texas) if a judge could decide a case justly, and yet by a process of metaphysical reasoning the less intelligible the better, he was regarded with awe by the vulgar; and that was one motive for his seizing upon that argument when he could get no modern light.

Peirce: CP 8.161 Cross-Ref:††

161. One of the distinctive features of Professor Mezes's book is a seventy-page chapter on Justice, in which legal decisions are followed, often in a way which will be repugnant to right-minded readers, and yet not so exclusively that the chapter can be said to constitute an exposition of the traditional legal conception of justice. Professor Mezes defines ultimate good as "the welfare of all sentient beings," but he is doubtful whether it is worth while to have any regard for the welfare either of *bacilli* (are these sentient beings in Texas?) or of criminals of all classes. The last exclusion is characteristic, we are sorry to say. But when we ask what he means by "welfare," in place of a *definition*, nothing is vouchsafed but a *division* of "welfare," in which there are two or three dozen items, such as "easy activity," "sense of personal attractiveness," "sense of solvency," "satisfaction from social standing," "sense of divine favor," "national pride," "self-control," "a body of well-poised spontaneous activities," "systematic ideas of rights and duties," "sagacity."†6 There are those who will think that all this is on a pretty low plane, and we do not see much in the list about the welfare of earthworms, etc., notwithstanding the insistence upon "all sentient beings."

Peirce: CP 8.162 Cross-Ref:††

162. The best thing in the book is the psychological analysis of conscience, which is decidedly noticeable. We could hardly have expected the terminology to be reformed. The scholastic writers mark two things which they distinguish by the terms *synderesis* and *conscience* (the latter nearly in the sense in which it is a household word). The interest of progress in ethical discussion calls upon us to come to agreement about the use of technical terms. But each of us is attached to his own habit, and will not surrender it unless it can be shown clearly to violate a law to which he has given in his allegiance. A code of rules is needed, in framing which we cannot do better than to be guided by the taxonomists, who have had, of all men, most experience in dealing with similar difficulties. If we do that, our first rule, subject, perhaps, to a few general but well-defined classes of exceptions (the fewer the better), will certainly be that every technical term of philosophy ought to be used in that sense in which it first became a technical term of philosophy. This will, generally speaking, result in the greatest accord between the language of philosophy and the vernacular, of which the word *conscience* will be an example. As for that other thing which a good many moralists call conscience, some other name ought to be given to it, preferably a new word. At any rate, not *synderesis*, of which the original meaning, we

are convinced, is not that which Siebeck assigns to it. Professor Mezes, whose definitions are mostly of doubtful accuracy, distinguishes between conscience about others' acts and conscience about one's own. But a stay-at-home conscience does the most to render earth habitable.

Peirce: CP 8.163 Cross-Ref:††

163. As we rise from the reading of the whole book, we find ourselves saying, If *this* is what morality is, we are disposed to sympathize with Henry James, the elder, in his very limited respect for morality.

Peirce: CP 8.164 Cross-Ref:††
CHAPTER 9

J. M. BALDWIN, *DICTIONARY OF PHILOSOPHY AND PSYCHOLOGY*, VOL.
II †1

164. Many evidences of different kinds reach us of the good service that this work is already rendering, notwithstanding the imperfections inevitable in any such composite book, and notwithstanding its lack of those formal perfections and uniformities upon which our American dictionaries and cyclopaedias are apt to insist to the neglect of the weightier matters of the law, to the point of leaving them dry, innutritious, and unvitalizing. Professor Baldwin, in the preface of this concluding volume (of the Dictionary proper), puts forth more an excuse than a defence for one of the few features of it that have been disapproved in almost every quarter; urging that the diminutive biographical notices which he has scattered through the vocabulary are that half-loaf that is said to be better than no bread. This hardly meets the stricture commonly made, which was to the effect that the entire omission of these supererogatory crumbs would have left room that might either have been filled to better purpose, or to better purpose have lightened the avoirdupois of the volumes.

Peirce: CP 8.165 Cross-Ref:††

165. But a more interesting question suggests itself. Upwards of seventy of the most reputable philosophers †2 whose services a distinguished editor could secure, have here set down their opinions upon the special points of philosophy of which they are reputed best qualified to treat. They have not argued their doctrines, since this is a dictionary, not a cyclopaedia; but they have defined them. All the principal groups of schools are more or less represented in the assemblage of contributors; even the idealists, whose showing is probably the least adequate. One naturally peruses their utterances to see what impression one can derive from them as to the prevalent tendencies of philosophy at the opening of the twentieth century; for surely this is an aspect under which it may be hoped that this dictionary will never lose its interest.

Peirce: CP 8.166 Cross-Ref:††

166. The most prominent of the philosophical signs of the times, as here displayed -- so it strikes us, at least -- is the manifest strenuous endeavor of the students of every department of philosophy to impart a "scientific" character each to

his own particular branch, *i.e.*, to make it conform to the conditions which have caused the success of the modern acknowledged sciences. The progress is satisfactory. At least one branch of psychology has already taken its place among the special sciences, whose array others are well upon the way toward joining. The movement is not confined to psychology. There is much of a scientific character in ethics; and the critical part of logic has, in some hands at least, come to submit itself to the same criteria as those that have long been acknowledged in science. There seems every reason for hope concerning metaphysics and other branches.

Peirce: CP 8.167 Cross-Ref:††

167. Another mark of our philosophy is the disposition to make psychology the key to philosophy -- categories, aesthetics, ethics, logic, and metaphysics. Something of this has existed since Descartes; but since about 1863 every student of philosophy, even though he be one of those who consider the present psychological tendency excessive, has placed a new and higher estimate than before upon the scientific value of psychology. Here was seen one science, than which no branch of philosophy, in the days when men disputed about the *primum cognitum*, was more enveloped in metaphysical fog, which yet almost suddenly, that mist lifting, had come out bright and clear as a June forenoon. How could it but happen, as it certainly did, that men should think that the best way to resolve any problem of philosophy would be to reduce it to a question of psychology? The future must determine precisely what the value of this method may be. It has its opponents. For some years after the movement once became general, no strong voice was raised against it; and ten or fifteen years ago psychologists of the first rank could dream of establishing the truths of their science without any metaphysical assumptions whatsoever. Some writers use such language even yet; but careful examination has convinced the better part that even physics has its metaphysical postulates, and that psychology is peculiarly dependent upon them.†3 That being the case, some writers urge that if psychology needs to rest upon metaphysics, and metaphysics upon logic, especially if, as some contend, logic rests upon ethics, then to found ethics, logic, and metaphysics in their turn upon a basis of psychology, this self-supporting cycle would rest on nothing. The reply is that the philosophical sciences will support each other, like two drunken sailors. Suffice it to say that the mutual support theory and with it the theory that psychology is the proper foundation for philosophy are not now without vigorous opponents.

Peirce: CP 8.168 Cross-Ref:††

168. A third symptom of the philosophy of the day is a reaction against the general agnostic tendency of a generation ago. Many are beginning to feel that the only possible justification for a hypothesis is that it renders the facts comprehensible, and that to suppose them absolutely incomprehensible (which is what the doctrine of the Unknowable comes to) is not rendering them comprehensible. This seems to point toward some new incarnation of the idea of the old philosophy of common sense. In this connection it may be noticed that the theory of psychophysical parallelism is distinctly losing followers. Minds cannot reconcile themselves to the notion that consciousness stands and idle spectator of human conduct. Besides, the new logic of quantity, which Cantor, Whitehead and others have made irrefragable, shows that even though matter acts directly only upon matter and thought acts directly only upon thought, and though there be no *tertium quid*, it by no means follows that thought does not act upon matter nor that matter does not act upon thought. What if it should turn out that the atoms of matter were vortices of an ether, which ether is itself comprised of atoms each a vortex of an ether's ether; and so on *ad infinitum*; and

what if mind had a similar constitution? It then might happen that upon an endless series of physical operations occurring in a fraction of a second should ensue a beginningless series of mental operations. Now it is to no purpose to say that this is improbable. If it is possible, as it certainly is, that suffices to show that mind and matter might, without contradiction, interact, although each could directly act only upon its own kind of substance. To the same general tendency belongs an opinion, now very common, that it is unscientific to inquire whether there be a God; the only rational question being what sort of God there is. With this is naturally associated the further opinion that instead of its being shallow philosophy to suppose an "anthropomorphic" God, if by "anthropomorphic" be meant *mental*, it is far more consonant with the method of science to formulate the problem by asking what sort of a mind God is; and if we cannot in some measure understand God's mind, all science, it is said with some color of justice, must be a delusion and a snare.

Peirce: CP 8.169 Cross-Ref:††

169. There is one more lineament of contemporary philosophy which, trivial as it may seem, is worth mention when a dictionary is our theme. It is that the days of literary style in philosophy seem to be numbered. The philosophy of the future must, like the other sciences, be put forth chiefly in the form of memoirs; and it is a truism to say that a memoir written in an ornate style would be as ridiculous as if it were in rhyming pentameters. From this follows another truism, that there is a good style and a bad style for a scientific memoir. Philosophy cannot become scientifically healthy without an immense technical vocabulary. We can hardly imagine our great-grandsons turning over the leaves of this dictionary without amusement over the paucity of words with which their grandsires attempted to handle metaphysics and logic. Long before that day, it will have become indispensably requisite, too, that each of these terms should be confined to a single meaning which, however broad, must be free from all vagueness. This will involve a revolution in terminology; for in its present condition a philosophical thought of any precision can seldom be expressed without lengthy explanations. Already, when philosophy is only just beginning to resemble science, the influx of new terms is getting to be considerable. One of the chief purposes of this dictionary seems to have been to fix the use of them. Before long philosophers will find themselves confronted with a Babel such as zoölogists and botanists have had to contend with; and scientific progress will be hampered until something like uniformity of usage has been attained. What is to be done? Shall we go on, *laisant faire*, until we find our terminology in an inextricable snarl, and then call in an Alexander to cut the knot with some Volapük system? Such would perhaps be the dictate of our glorious Anglo-Saxon genius, which has endowed us, for example, with the word *bushel*, whose meaning in any given State of the Union, for any given commodity, can be ascertained by simply consulting a table of double-entry, and has given us that admirable word *inch*, which cannot now be changed at a cost of less than a hundred million dollars. Or shall [we] take time by the forelock, imitate the French, with their metric system, their Academy, their Code Napoleon, their Guyton de Morveau (with his chemical nomenclature, now universal, after modification), their minute regulation of everything, and agree upon how the battle shall be fought before we find ourselves actually engaged in it? In the weightier matters of the law the French are none too moral a people; but for the ethics of the mint and cummin, the ought and ought not of manner and fashions, and other forms of expression, they lead, and the rest of the world, after duly prolonged demurs, generally ends by following them.

Peirce: CP 8.170 Cross-Ref:††

170. We must expect arduous labours yet to be performed before philosophy can work its way out of the jungle and emerge upon the high road of science. But the prospect is no longer so desperately gloomy, if philosophers will only resign themselves to the toilsome procedure of science, and recognize that a single generation can make little headway, but yet may faithfully clear away a few obstacles, and lying down to die, resign the axe to their successors.

Peirce: CP 8.171 Cross-Ref:††
CHAPTER 10

LADY WELBY, *WHAT IS MEANING?*†1

171. Lady Victoria Welby's little volume is not what one would understand by a scientific book. It is not a treatise, and is free from the slightest shade of pedantry or pretension. Different people will estimate its value very differently. It is a feminine book, and a too masculine mind might think parts of it painfully weak. We should recommend the male reader to peruse chapters xxii. to xxv. before he reads the whole consecutively, for they will bear a second reading. The question discussed in these chapters is how primitive men ever came to believe in their absurd superstitions. This has generally been supposed to be the simplest of questions. Lady Victoria does not deign to mention La Fontaine's pretty fable (the sixth of the ninth book; the whole of it is worth rereading if you have forgotten it) of the sculptor and his statue of Jove:

"L'artisan exprima si bien
Le caractère de l'Idole,
Qu'on trouva qu'il ne manquait rien
A Jupiter que la parole.

"Même l'on dit que l'ouvrier
Eut à peine achevé l'image,
Qu'on le vit frémir le premier,
Et redouter son propre ouvrage.

.

"Il était enfant en ceci:

Les enfants n'ont l'âme occupée
Que du continuel souci
Qu'on ne fâche point leur poupée.

"Le coeur suit aisément l'esprit.
De cette source est descendue
L'erreur payenne qui se vit
Chez tant de peuples répandue.

"Chacun tourne en réalités,
Autant qu'il peut, ses propres songes.
L'homme est de glace aux vérités;
Il est de feu pour les mensonges."

Peirce: CP 8.172 Cross-Ref:††

172. La Fontaine's theory is somewhat complex, and allows more to the artistic impulse than modern ethnologists have done. They make mythology rather an attempt at a philosophical explanation of phenomena. But the authoress shows by a painstaking analysis that all such theories -- La Fontaine's and the new current ones alike -- are fatally irreconcilable with those traits of the primitive mind that have struck Tylor, Spencer, and ethnologists generally, as the deepest graven. In place of them she offers a hypothesis of her own, and the reader is tempted to lose patience with her for regarding it only as provisional, so strongly does it recommend itself, until she presents quite another view which one must admit has its plausibility.

Peirce: CP 8.173 Cross-Ref:††

173. The greatest service the book can render is that of bringing home the question which forms its title, a very fundamental question of logic, which has commonly received superficial, formalistic replies. Its vital and far-reaching significance has been even more ignored than usually happens with matters of universal and ubiquitous concern. To direct attention to the subject as one requiring study, both on its theoretical and on its practical side, is the essential purpose of the work. But in doing this the authoress has incidentally made a contribution towards the answer to the question, in pointing out three orders of signification. She has wisely abstained from any attempt at formal definitions of these three modes of signification. She tells us what she means only in the lowest of those three senses. To have gone further would have shunted her off upon a long and needless discussion.

Peirce: CP 8.174 Cross-Ref:††

174. One can see, though she does not remark it, that her three kinds of meaning correspond roughly to Hegel's three stages of thought. Her distinction, too, partly coincides with what was long ago said, that to understand a word or formula

may, in the first place, consist in such familiarity with it as will enable one to apply it correctly; or secondly, may consist in an abstract analysis of the conception or understanding of its intellectual relations to other concepts; or, thirdly, may consist in a knowledge of the possible phenomenal and practical upshot of the assertion of the concept.†2 We might point out other interesting affiliations of her thought, sufficient to show that she must be on the right track.

Peirce: CP 8.175 Cross-Ref:††

175. Lady Victoria, however, does not wish the matter to be agitated in the logician's study alone. She urges that people do not sufficiently take to heart the ethics of language. She thinks that modern conceptions call for a modern imagery of speech. But we fear that she does not realize how deep the knife would have to go into the body of speech to make it really scientific. We should have to form words like those the chemists use -- if they can be called words. In particular, she preaches making logic -- "significs," she calls it, but it would be logic -- the basis or core of education. All those ideals deserve to be pondered. The book is very rich in illustrations drawn from contemporary writing.

Peirce: CP 8.176 Cross-Ref:††

176. A little book by Lady Victoria Welby has lately appeared, entitled "What is Meaning." The book has sundry merits, among them that of showing that there are three modes of meaning. But the best feature of it is that it presses home the question "What is Meaning." A word has meaning for us in so far as we are able to make use of it in communicating our knowledge to others and in getting at the knowledge that these others seek to communicate to us. That is the lowest grade of meaning. The *meaning* of a word is more fully the sum total of all the conditional predictions which the person who uses it *intends* to make himself responsible for or intends to deny. That conscious or quasi-conscious *intention* in using the word is the second grade of meaning. But besides the consequences to which the person who accepts a word knowingly commits himself to, there is a vast ocean of unforeseen consequences which the acceptance of the word is destined to bring about, not merely consequences of knowing but perhaps revolutions of society. One cannot tell what power there may be in a word or a phrase to change the face of the world; and the sum of these consequences makes up the third grade of meaning.†3

Peirce: CP 8.177 Cross-Ref:††

177. [My definition of a sign is:] A Sign is a Cognizable that, on the one hand, is so determined (i.e., specialized, *bestimmt*.) by something *other than itself*, called its Object,†4 while, on the other hand, it so determines some actual or potential Mind, the determination whereof I term the Interpretant created by the Sign, that that Interpreting Mind is therein determined mediately by the Object.†5

Peirce: CP 8.178 Cross-Ref:††

178. This involves regarding the matter in an unfamiliar way. It may be asked, for example, how a lying or erroneous Sign is determined by its Object, or how if, as not infrequently happens, the Object is brought into existence by the Sign. To be puzzled by this is an indication of the word determine being taken in too narrow a sense. A person who says Napoleon was a lethargic creature has evidently his mind determined by Napoleon. For otherwise he could not attend to him at all. But here is a

paradoxical circumstance. The person who interprets that sentence (or any other Sign whatsoever) must be determined by the Object of it through collateral observation quite independently of the action of the Sign. Otherwise he will not be determined to thought of that object. If he never heard of Napoleon before, the sentence will mean no more to him than that some person or thing to which the name "Napoleon" has been attached was a lethargic creature. For Napoleon cannot determine his mind unless the word in the sentence calls his attention to the right man and that can only be if, independently, [a] habit has been established in him by which that word calls up a variety of attributes of Napoleon the man. Much the same thing is true in regard to any sign. In the sentence instanced Napoleon is not the only Object. Another Partial Object is Lethargy; and the sentence cannot convey its meaning unless collateral experience has taught its Interpreter what Lethargy is, or what that is that 'lethargy' means in this sentence. The Object of a Sign may be something to be created by the sign. For the Object of "Napoleon" is the Universe of Existence so far as it is determined by the fact of Napoleon being a Member of it. The Object of the sentence "Hamlet was insane" is the Universe of Shakespeare's Creation so far as it is determined by Hamlet being a part of it. The Object of the Command "Ground arms!" is the immediately subsequent action of the soldiers so far as it is affected by the molition †6 expressed in the command. It cannot be understood unless collateral observation shows the speaker's relation to the rank of soldiers. You may say, if you like, that the Object is in the Universe of things desired by the Commanding Captain at that moment. Or since the obedience is fully expected, it is in the Universe of his expectation. At any rate, it determines the Sign although it is to be created by the Sign by the circumstance that its Universe is relative to the momentary state of mind of the officer.

Peirce: CP 8.179 Cross-Ref: ††

179. Now let us pass to the Interpretant. I am far from having fully explained what the Object of a Sign is; but I have reached the point where further explanation must suppose some understanding of what the Interpretant is. The Sign creates something in the Mind of the Interpreter, which something, in that it has been so created by the sign, has been, in a mediate and *relative* way, also created by the Object of the Sign, although the Object is essentially other than the Sign. And this creature of the sign is called the Interpretant. It is created by the Sign; but not by the Sign quâ member of whichever of the Universes it belongs to; but it has been created by the Sign in its capacity of bearing the determination by the Object. It is created in a Mind (how far this mind must be real we shall see). All that part of the understanding of the Sign which the Interpreting Mind has needed collateral observation for is outside the Interpretant. I do not mean by "collateral observation" acquaintance with the system of signs. What is so gathered is *not* COLLATERAL. It is on the contrary the prerequisite for getting any idea signified by the sign. But by collateral observation, I mean previous acquaintance with what the sign denotes. Thus if the Sign be the sentence "Hamlet was mad," to understand what this means one must know that men are sometimes in that strange state; one must have seen madmen or read about them; and it will be all the better if one specifically knows (and need not be driven to *presume*) what Shakespeare's notion of insanity was. All that is collateral observation and is no part of the Interpretant. But to put together the different subjects as the sign represents them as related -- that is the main [i.e., force] of the Interpretant-forming. Take as an example of a Sign a *genre* painting. There is usually a lot in such a picture which can only be understood by virtue of acquaintance with customs. The style of the dresses for example, is no part of the *significance*, i.e. the deliverance, of the painting. It only tells what the *subject* of it is. *Subject* and *Object*

are the same thing except for trifling distinctions But that which the writer aimed to point out to you, presuming you to have all the requisite collateral information, that is to say just the quality of the sympathetic element of the situation, generally a very familiar one -- a something you probably never did so clearly realize before -- *that* is the Interpretant of the Sign, -- its "significance."

Peirce: CP 8.180 Cross-Ref:††

180. Now all this is, so far, very muddled for the lack of certain distinctions which I proceed to point out, though it will be hard to make them fully comprehended.

Peirce: CP 8.181 Cross-Ref:††

181. In the first place, it should be observed that so far as the Sign denotes the Object, it calls for no particular *intelligence* or *Reason* on the part of its Interpreter. To read the Sign at all, and distinguish one Sign from another, what is requisite is delicate perceptions and acquaintance with what the usual concomitants of such appearances are, and what the conventions of the system of signs are. To know the Object, what is requisite is previous experience of that Individual Object. The Object of every sign is an Individual, usually an Individual Collection of Individuals. Its *Subjects*, i.e., the Parts of the Sign that denote the *Partial Objects*, are either *directions* for *finding the Objects* or are Cyrioids, i.e., signs of single Objects Such for example are all *abstract* nouns, which are names of single characters, the personal pronouns, and the demonstrative and relative pronouns, etc. By directions for finding the Objects, for which I have as yet invented no other word than "Selectives," I mean such as "Any" (i.e., any you please), "Some" (i.e., one properly selected), etc. To know the Interpretant, which is what the sign itself expresses, may require the highest power of reasoning.

Peirce: CP 8.182 Cross-Ref:††

182. In the second place, to get more distinct notions of what the Object of a Sign in general is, and what the Interpretant in general is, it is needful to distinguish two senses of "Object" and three of "Interpretant." It would be better to carry the division further; but these two divisions are enough to occupy my remaining years . . .

Peirce: CP 8.183 Cross-Ref:††

183. As to the Object, that may mean the Object as cognized in the Sign and therefore an Idea, or it may be the Object as it is regardless of any particular aspect of it, the Object in such relations as unlimited and final study would show it to be. The former I call the *Immediate* Object, the latter the *Dynamical* Object. For the latter is the Object that Dynamical Science (or what at this day would be called "Objective" science,) can investigate. Take for example, the sentence "the Sun is blue." Its Objects are "the Sun" and "blueness." If by "blueness" be meant the Immediate Object, which is the quality of the sensation, it can only be known by Feeling. But if it means that "Real," existential condition, which causes the emitted light to have short mean wave-length, Langley has already proved that the proposition is true. So the "Sun" may mean the occasion of sundry sensations, and so is Immediate Object, or it may mean our usual interpretation of such sensations in terms of place, of mass, etc., when it is the Dynamical Object. It is true of both Immediate and Dynamical Object that acquaintance cannot be given by a Picture or a Description, nor by any other sign which has the Sun for its Object. If a person points to it and says, See there! *That* is what we call the "Sun," the Sun is *not* the Object of that sign. It is the *Sign* of the sun,

the *word* "sun" that his declaration is about; and that *word* we must become acquainted with by collateral experience. Suppose a teacher of French says to an English-speaking pupil, who asks "comment appelle-t-on ça?" pointing to the Sun, . . . "C'est le soleil," he begins to furnish that collateral experience by speaking in French of the Sun itself. Suppose, on the other hand, he says "Notre mot est 'soleil'" then instead of expressing himself in language and *describing* the word he offers a pure *Icon* of it. Now the Object of an Icon is entirely indefinite, equivalent to "something." He virtually says "our word is like this:" and makes the sound. He informs the pupil that the word, (meaning, of course, a certain *habit*) has an effect which he *pictures* acoustically. But a pure picture without a legend only says "something is like this:". True he attaches what amounts to a legend. But that only makes his sentence analogous to a portrait we will say of Leopardi with Leopardi written below it. It conveys its information to a person who knows who Leopardi was, and to anybody else it only says "something called Leopardi looked like this." The pupil is in the state of a person who was pretty sure there was a man Leopardi; for he is pretty sure there must be a word in French for the sun and thus is already acquainted with it, only he does not know how it sounds when spoken nor how it looks when written. I think by this time you must understand what I mean when I say that no sign can be understood -- or at least that no *proposition* can be understood -- unless the interpreter has "collateral acquaintance" with every Object of it. As for a mere *substantive*, it must be borne in mind that it is not an indispensable part of speech. The Semitic languages seem to be descendants of a language that had no "common nouns." Such a word is really nothing but a *blank form* of proposition and the Subject is the blank, and a blank can only mean "something" or something even more indefinite. So now I believe I can leave you to consider carefully whether my doctrine is correct or not.

Peirce: CP 8.184 Cross-Ref:††

184. As to the Interpretant, i.e., the "signification," or "interpretation" rather, of a sign, we must distinguish an Immediate and a Dynamical, as we must the Immediate and Dynamical Objects. But we must also note that there is certainly a third kind of Interpretant, which I call the Final Interpretant, because it is that which *would finally* be decided to be the true interpretation if consideration of the matter were carried so far that an ultimate opinion were reached. My friend Lady Welby has, she tells me, devoted her whole life to the study of *significs*, which is what I should describe as the study of the relation of signs to their interpretants; but it seems to me that she chiefly occupies herself with the study of words. She also reaches the conclusion that there are three senses in which words may be interpreted. She calls them *Sense*, *Meaning*, and *Significance*. Significance is the deepest and most lofty of these, and thus agrees with my *Final Interpretant*; and Significance seems to be an excellent name for it. *Sense* seems to be the logical analysis or definition, for which I should prefer to stick to the old term *Acception* or *Acceptation*. By *Meaning* she means the *intention* of the utterer.

Peirce: CP 8.185 Cross-Ref:††

185. But it appears to me that all symptoms of disease, signs of weather, etc., have no utterer. For I do not think we can properly say that God *utters* any sign when He is the Creator of all things. But when [Lady Welby] says, as she does, that this is connected with Volition, I at once note that the volitional element of Interpretation is the *Dynamical Interpretant*. In the Second Part of my Essay on Pragmatism, in *The Popular Science Monthly* of 1877 Nov. and 1878 Jan., I made three grades of clearness of Interpretation.†7 The first was such familiarity as gave a person familiarity with a sign and readiness in using it or interpreting it. In his consciousness

he seemed to himself to be quite *at home* with the Sign. In short, it is Interpretation *in Feeling*. The second was Logical Analysis = Lady Welby's *Sense*. The third, . . . Pragmatic Analysis, would seem to be a Dynamical Analysis, but [is] identified with the Final Interpretant. †8

Peirce: CP 8.186 Cross-Ref: ††
CHAPTER 11

C. A. STRONG, *WHY THE MIND HAS A BODY* †1

186. Today, the animating endeavour of the younger philosophers is to bring their queen within the circle of the genuine sciences, -- those careful and prudent sciences whose occasional leaps and strides are rendered achievable by their habitual training in picking their steps, slowly and laboriously, so as to make sure of each foothold. It is this commendable spirit, and this alone, that justifies high hopes for the future of philosophy; and the past twelvemonth has brought no worthier pattern of it than Prof. Strong's 'Why the Mind has a Body' -- a remark that, to our knowledge, has fallen from more than one or two pairs of weighty lips. A cool and painstaking attempt is here made to set forth the contemporary issues of the question of the nature of the connection between mind and matter, to subject the different contentions to brief but penetrating criticism, and to develop by original studies the panpsychism of Clifford and Paulsen. A plainly marked stadium is thus set up whence for some years all discussions of this subject must set out. Not that any real solution of the problem has been reached. Let us hope that such may some day appear; but at present, no peering into the future descries its features nor yields us any confidence out of what quarter of the horizon it shall first loom. Meantime, the information that a thorough student of philosophy, highly intelligent, and exceptionally impartial, understands such and such to be the present state of the discussion will be a great help to sober inquirers. Doubtless Prof. Strong himself anticipates that not a few additions and corrections will be found needed in this first draught of his analysis. He will want, for example, to supply some proof that interactionism, automatism, and parallelism are really conflicting doctrines and not mere ways of apprehending the same facts. He will also desire to take some notice of the pragmatist position to which so many minds are flocking. †2 He has, to be sure, a section entitled 'Thoroughgoing Phenomenalism,' -- a phrase that ought, we should think, to denote pragmatism; but the author does not seem to see that thorough-going phenomenalism must be phenomenalism *aufgehoben*; that it must involve the opinion that the reader of this page directly perceives the very page itself some ten inches from his eye, and that another person, looking over his shoulder, will see the very same object, although under a different angle, and although each sees the real object, not in its entirety, but only as it is related to his own view-point, literal and tropical. Not only must the pragmatist entertain this opinion, but he must hold that no other can be held by anybody, except in a sense in which a self-contradictory opinion is possible. For all men acknowledge that the statement that the reader sees the real page answers all human purposes, prompts the suitable conduct on each occasion on which it gives any

prompting. Now the pragmatist maintains that there is no other conception of reality to be by any means had than the conception of what must ultimately appear to answer human purposes, where '*human*' means belonging to the communion of mankind. He holds that one who thinks he believes that anything is real for something more than human purposes, in reality merely believes that it is true for human purposes that something is real for more than human purpose, -- which may perhaps be quite true in the only sense it can have, namely, that for human purposes so it is. Pragmatism makes or ought to make no pretension to throwing positive light on any problem. It is merely a logical maxim for laying the dust of pseudoproblems, and thus enabling us to discern what pertinent facts the phenomena may present. But this is a good half of the task of philosophy.

Peirce: CP 8.187 Cross-Ref:††

187. We must here content ourselves with a further remark or two which we trust may prove useful to a critical reader of the volume. Prof. Strong has much to say about the conservation of energy. The ordinary abstract statement of this doctrine is ill-adapted for philosophical discussion and is apt to be more or less misunderstood by metaphysicians. Better suited to their purposes is a recognized mathematical equivalent. Namely, the doctrine precisely amounts to this, that those motions of particles of which all physical events, considered as purely physical, are composed, undergo no "accelerations" (under which term are included all states of undergoing changes of motion) except such as are determined, according to fixed laws, *exclusively by the relative positions* of those particles at the very instants of those accelerations. The accelerations, which are the immediate effects, are absolutely simultaneous with the positions which are their causes. Yet Prof. Strong never fails to insist, in connection with the doctrine of energy, that causes precede their effects; -- which is true of much causation, but is emphatically false of "conservative" forces.†3 It is, indeed, a mathematical consequence of the doctrine of conservation that if the velocities of all the particles were at any instant precisely reversed, all those particles would move back over their former paths with precisely the same, though reversed, velocities as before. Thus, the laws of motion do not favor any one determinate direction in an entire course of change, rather than the reverse direction. The physical universe is full of changes regularly taking place in determinate directions; -- so full that this might almost be said to be the predominant character of nature. But with such features of phenomena the doctrine of the conservation of energy has as much and as little to do as has the Monroe doctrine. When light strikes upon a glass prism, *some* of the rays *usually* emerge more highly colored than they entered the prism, which is an effect of chance. So when a man is exposed to natural agencies, *some* of these *usually* emerge from his organism highly marked with a purposive character. We know not exactly how this comes about; but one thing we may be sure of: the conservation of energy has nothing to do with it. Another point to be noted about physical causation is that *acceleration*, the effect, and *relative position*, the cause, are of disparate natures, not to be measured in terms of the same units. Yet Prof. Strong holds it to be an argument against the interaction of body and soul that their natures are disparate. Had he said that every effect is disparate to its cause he would have been nearer the mark. There is still another important suggestion to be derived from dynamics. In treating any ordinary problem in analytic mechanics, -- say, for example, that of two pendulums swinging from one yielding support, -- we begin by expressing the state of facts by means of that form of the law of energy enunciated above.†4 This furnishes a differential equation which represents the interactions of the different parts of the system, (say the two pendulums). We now subject this equation to a mathematical transformation. It continues to express the same facts, or a portion of them; but from

the new form of statement the conception of interaction has disappeared and each part of the system is represented as moving under a regularity of its own, independently of every other part. Each pendulum, to recur to our example, is now asserted to perform a regular harmonic motion consisting of two simple oscillations like one train of waves passing over another such train, while the other pendulum is ignored in the statement. This is a "parallelistic" form of stating the same facts that were at first stated "interactionistically." Hence we see that "interactionism" and "parallelism" may be merely two forms of expressing the same truth. Only the dynamist would hold that it is the original interactionist form that expresses the facts in their real relations to the general course of nature, while the parallelistic form is merely a partial expression [of the] facts which happens to be convenient for a particular purpose. However enigmatic the assertion may appear (and it is impossible here to defend it) it is certain that interactionism is in no logical conflict with the assumption that every transformation of motion is determined by physical conditions exclusively.†5

Peirce: CP 8.188 Cross-Ref:††
CHAPTER 12

JOHN DEWEY, *STUDIES IN LOGICAL THEORY*†1

188. The volume of which Professor Dewey is the father forms a part of the University of Chicago's exhibit of an impressive decade's work, and is a worthy part of it, being the monument of what he has done in his own department. Here are eleven essays, four by himself, defining his conception of the business of the logician, seven by the students whom he has helped to form and set upon their own intellectual legs. It affords conclusive proof of the service he has rendered to these accomplished thinkers and, no doubt, to others; and they in their turn will render to another generation services of the same nature. Whatever there was to be gained by contact with a sincere student of philosophy, as such, they have manifestly gained. Are there any further services that logic could be expected to perform? Are any logical questions now being agitated in the different sciences? Is there any such question as to the constitution of matter, the value of mechanical hypotheses, now open in physics? Are there any methods as to more or less statistical methods of philological and historical criticism? If there are such questions, has past experience gone to show that there was any help to be had from broader sweeps of study than specialists can make? Is it worth while to examine at all into the questions here asked; and if it be, is it best to carry to them vague impressions, or the exactest conceptions that studies specially directed to them have been able to evoke?

Peirce: CP 8.189 Cross-Ref:††

189. There are specialists who are disposed to think any inquiries from the outside into their methods are impertinent. They say, with perfect justice, that they understand fully their own business, and wish to be let alone. Unquestionably, they must be right. There is, however, another class of specialists whose aims are of such a nature that they can sometimes make good use of ideas which have grown up in other studies. Such specialists, when they have created, say, physical chemistry, the new

astronomy, physiological psychology, stylometry, etc., have sometimes gained a certain measure of esteem even from those of straiter sects. It has often happened that general studies of logic have resulted in such applications of one science to another. Analytical geometry was first conferred upon the human race as an illustrative example of the 'Discours de la Méthode.' The group of writers whom, abandoning all attempt at finding a descriptive designation, we may roughly call the English school of logicians, meaning, for example, Boole, De Morgan, Whewell, J.S. Mill, Jevons, Venn, Pearson, MacColl, etc., while pursuing studies often purely theoretical, are nevertheless taking a road which may be expected to lead to results of high value for the positive sciences. Those whom we may as roughly call the German school of logicians, meaning such writers as Christoph Sigwart, Wundt, Schuppe, Benno Erdmann, Julius Bergmann, Glogau, Husserl, etc., are engaged upon problems which must be acknowledged to underlie the others, but attack them in a manner which the exact logicians regard as entirely irrelevant, because they make *truth*, which is a matter of fact, to be a matter of a way of thinking or even of linguistic expression. The Chicago school or group are manifestly in radical opposition to the exact logicians, and are not making any studies which anybody in his senses can expect, directly or indirectly, in any considerable degree, to influence twentieth-century science.

Peirce: CP 8.190 Cross-Ref:††

190. Prof. Dewey regards himself as radically opposed to the German school, and explains how he is so. We must confess that had he not put so much emphasis upon it, we should hardly have deemed the point of difference so important; but we suppose he must know what his own affiliations are and are not. He seems to regard what he calls "logic" as a natural history of thought.†2 If such a natural history can be worked out, it will undoubtedly form valuable knowledge; and with all our heart we wish the Chicago school godspeed in their enterprise of discovery. But their task will call for such extreme subtlety, precision, and definiteness of thought that we hope their new science will not disdain to take a lesson, if not from any of the older logicians of the country, nor from that American thinker who first essayed to use his great powers of observation to establish a natural history of mental products -- we mean Dr. James Rush -- at least from the well-established natural history of Nature, chemistry, botany, and zoology; the lesson, to wit, that a natural history can hope to begin a successful course of discovery only from the day when it abandons altogether the trivial language of practical life, and sets up a thoroughly new glossary of words exclusively its own, thereby not confusing our meagre philosophical vocabulary with the burden of added meanings to old words. If calling the new natural history by the name of "logic" (a suspicious beginning) is to be a way of prejudging the question of whether or not there be a logic which is more than a mere natural history, inasmuch as it would pronounce one proceeding of thought to be sound and valid and another to be otherwise, then we should regard this appropriation of that name to be itself fresh confirmation of our opinion of the urgent need of such a normative science at this day.†3

ON PRAGMATISM, FROM A REVIEW OF A BOOK ON COSMOLOGY†1

191. No criticism of such a book, no characterization of it, not even as slight a one as that here to be attempted, can have any meaning until the standpoint of the critic's observations be recognized. Our standpoint will be pragmatism;†2 but this word has been so loosely used, that a partial explanation of its nature is needful, with some indications of the intricate process by which those who hold it become assured of its truth. If philosophy is ever to become a sound science, its students must submit themselves to that same ethics of terminology that students of chemistry and taxonomic biology observe; and when a word has been invented for the declared purpose of conveying a precisely defined meaning, they must give up their habit of using it for every other purpose that may happen to hit their fancy at the moment. The word *pragmatism* was invented to express a certain maxim of logic, which, as was shown at its first enunciation, involves a whole system of philosophy. The maxim is intended to furnish a method for the analysis of concepts. A concept is something having the mode of being of a general type which is, or may be made, the rational part of the purport of a word. A more precise or fuller definition cannot here be attempted. The method prescribed in the maxim is to trace out in the imagination the conceivable practical consequences, -- that is, the consequences for deliberate, self-controlled conduct, -- of the affirmation or denial of the concept; and the assertion of the maxim is that herein lies the *whole* of the purport of the word, the *entire* concept. The sedulous exclusion from this statement of all reference to sensation is specially to be remarked. Such a distinction as that between red and blue is held to form no part of the concept. This maxim is put forth neither as a handy tool to serve so far as it may be found serviceable, nor as a self-evident truth, but as a far-reaching theorem solidly grounded upon an elaborate study of the nature of signs.†3 Every thought, or cognitive representation, is of the nature of a sign. "Representation" and "sign" are synonyms. The whole purpose of a sign is that it shall be interpreted in another sign; and its whole purport lies in the special character which it imparts to that interpretation. When a sign determines an interpretation of itself in another sign, it produces an effect external to itself, a physical effect, though the sign producing the effect may itself be not an existent object but merely a type. It produces this effect, not in this or that metaphysical sense, but in an indisputable sense. As to this, it is to be remarked that actions beyond the reach of self-control are not subjects of blame. Thinking is a kind of action, and reasoning is a kind of deliberate action; and to call an argument illogical, or a proposition false, is a special kind of moral judgment, and as such is inapplicable to what we cannot help. This does not deny that what cannot be conceived today may be conceivable tomorrow. But just as long as we cannot help adopting a mode of thought, so long it must be thoroughly accepted as true. Any doubt of it is idle make-believe and irredeemable paper. Now we all do regard, and cannot help regarding, signs as *affecting* their interpretant signs. It is by a patient examination of the various modes (some of them quite disparate) of interpretations of signs, and of the connections between these (an exploration in which one ought, if possible, to provide himself with a guide, or, if that cannot be, to prepare his courage to see one conception that will have to be mastered peering over the head of another, and soon another peering over that, and so on, until he shall begin to think there is to be no end of it, or that life will not be long enough to complete the study) that the pragmatist has at length, to his great astonishment, emerged from the disheartening labyrinth with this simple maxim in his hand. In distrust of so surprising a result he

has searched for some flaw in its method, and for some case in which it should break down, but after every deep-laid plot for disproving it that long-working ingenuity could devise has recoiled upon his own head, and all doubts he could start have been exhausted, he has been forced at last to acknowledge its truth. This maxim once accepted, -- intelligently accepted, in the light of the evidence of its truth, -- speedily sweeps all metaphysical rubbish out of one's house. Each abstraction is either pronounced to be gibberish or is provided with a plain, practical definition. The general leaning of the results is toward what the idealists call the naïve, toward common sense, toward anthropomorphism. Thus, for example, the *real* becomes that which is such as it is regardless of what you or I or any of our folks may think it to be.†4 The *external* becomes that element which is such as it is regardless of what somebody thinks, feels, or does, whether about that external object or about anything else. Accordingly, the external is necessarily real, while the real may or may not be external; nor is anything absolutely external nor absolutely devoid of externality. Every assertory proposition refers to something external, and even a dream withstands us sufficiently for one description to be true of it and another not. The *existent* is that which reacts against other things. Consequently, the external world, (that is, the world that is comparatively external) does not consist of existent objects merely, nor merely of these and their reactions; but on the contrary, its most important reals have the mode of being of what the nominalist calls "mere" words, that is, general types and would-bes. The nominalist is right in saying that they are substantially of the nature of words; but his "mere" reveals a complete misunderstanding of what our everyday world consists of.

Peirce: CP 8.192 Cross-Ref:††

192. With this preface, let us examine a fair specimen of Dr. Nichols's power of analytic thought, which is the first requirement of a philosopher. This specimen shall consist of his definition of scientific law, or, as he prefers to term it, of "lawfulness." We all know that John Mill banished the word 'law' and substituted 'uniformity' for it, as more precisely expressing what is meant. But pragmatism discovers a serious error here. For while uniformity is a character which might be realized, in all its fulness, in a short series of past events, law, on the other hand, is essentially a character of an indefinite future; and while uniformity involves a regularity exact and exceptionless, law only requires an approach to uniformity in a decided majority of cases. This appears as follows: when the pragmatist puts to himself his stereotyped question, How could law ever reasonably affect human conduct, the answer that reflection brings him is that law could affect such conduct only through the knowledge of it creating and warranting anticipations of future experience. Now this sort of influence upon reasonable conduct requires no more than that those predictions should *ordinarily* be fulfilled; nor does it in the least preclude their being vague to almost any degree. But what the answer to the pragmatist's self-question does require is that the law should be a truth expressible as a conditional proposition whose antecedent and consequent express experiences *in a future tense*, and further, that, as long as the law retains the character of a law, there should be possible occasions in an indefinite future when events of the kind described in the antecedent may come to pass. Such, then, *ought* to be our conception of law, whether it has been so or not. But upon examining the usage of physicists, we find there are not a few truths called laws by the most careful terminologists which are not of an exact nature and which present downright exceptions. Such is Dulong and Petit's law, such is the periodic law of the chemical elements; and everybody can name others, galore. But in all the range of science there is no single proposition that goes by the name of a law, from which conditional predictions as to future experiences may not

be deduced.

Peirce: CP 8.193 Cross-Ref:††

193. Dr. Nichols devotes a chapter of thirteen of his large pages to making out what "lawfulness" consists in, -- full double what any pragmatist would require for the most convincing elucidation. But Dr. Nichols wastes part of his space on trifles, and occupies a good deal with old-fashioned declamatory rhetoric against "hypostatized entities." What was the use of this? Nobody will now-a-days dispute that the writers who are the target of all these philippics, those who used to talk so much about "causation" and "immediate causes" and the like, were at fault. Only, their real fault was of no other nature than precisely that of the eloquent declaimers against "entities" and "hypostatizing." Namely, in each case, it is not so much any positively erroneous doctrine, as it is that having before them ill-apprehended phenomena, instead of patiently sitting down to analyze the same, as they should have done, they content themselves with applying names of redoubtable sound but of little or no pragmatic purport.

Peirce: CP 8.194 Cross-Ref:††

194. A practical attitude of mind concerns itself primarily with the living future, and pays no regard to the dead past or even the present except so far as it may indicate what the future will be. Thus, the pragmatist is obliged to hold that whatever means anything means that something will happen (provided certain conditions are fulfilled), and to hold that the future alone has primary reality. The fact that Napoleon did run his marvellous career *consists* in the fact that anybody who looks for them will find a thousand and one vestiges of that career. A questioner to whom pragmatism comes as a novelty will naturally ask, "Do you mean to say that you do not believe there has been any past?" To which the pragmatist will reply, -- and note well his answer, because it is analogous to the answer he will give to a host of questions to which no further allusion will be made, -- "Why, I believe in the reality of the past just as completely as you do, and just in the way that you do, except that either you or I perhaps do not describe correctly the intellectual side of [its] real meaning. To any memory [of] the past, there attaches a certain color, -- a certain quality of feeling, -- just as there does to the sight of a Jacqueminot rose.†5 Ontological metaphysicians usually say that 'secondary sensations,' such as colors, are delusive and false; but not so the Pragmatist. He insists that the rose really *is* red; for *red* is, by the meaning of the word, an appearance; and to say that a Jacqueminot rose really is red means, and can mean, nothing but that if such a rose is put before a normal eye, in the daylight, it will look red. Just so, the feeling qualities attaching to memories are entirely true and real, though obviously relative, as pastness itself obviously is relative."

Peirce: CP 8.195 Cross-Ref:††

195. "But what say you to the myriad details of Napoleon's life of which no vestige remains, -- his having winked, let us suppose, one night when he was in absolute darkness. Did those events not occur?" So the questioner: to which the Pragmatist will reply, "You speak of a wink as if it were a small event. How many trillions of corpuscles are involved in the action, through how many million times their diameters they move, and during how many billions of their revolutions in their orbits the action endures, I will not undertake to calculate. But certainly you cannot yourself think that so vast an operation will have had no physical effects, or that they will cease for ages yet to come. Certainly, when you talk of an actual event leaving at

a subsequent time absolutely no consequences whatever, I confess that I can attach no meaning at all to your words, and I believe that for you yourself it is simply a formula into which by some form of logic you have transformed a proposition that had a real meaning while overlooking the circumstance that the transformation has left no real meaning in it, unless one calls it a meaning that you continue vaguely to associate the memory-feeling with this empty form of words. At any rate, you will remark that in all important respects you and I think alike about time, and that it is only in regard to metaphysical statements of no earthly consequence to anybody that we differ, -- you allowing them to confuse and litter up your mind, while I sweep them out of doors." "But why look upon past and future so differently?" "The intellectual meaning of a statement is precisely the same whether it refers to past or future time. To say that a piece of porcelain is soft before it is baked is equivalent to saying that if anybody during that period tries to scratch it with a knife he will succeed, and to say this is again equivalent to saying that every experiment which is logically necessitated, if this be true, to turn out in a certain way, will turn out in that way; and this last statement has a corresponding equivalent, and so on endlessly. But of this endless series of equivalent propositions there is one which my situation in time makes to be the practical one for me, and that one becomes for me the primary meaning. As long as the porcelain is not yet baked, I mean by calling it soft that if anyone tries to scratch it with a knife he will readily succeed. But after it has been baked, and nobody has taken occasion to try that experiment, it is a different experiment among the endless series of equivalents that now expresses my primary meaning. The nature of the fact does not change; but my relation to it and consequent mode of conceiving do change, although I all the time recognize the equivalence of the different meanings." "Then you maintain, do you, that when you directly act upon a thing in making an experiment, this direct action consists entirely in the fact that subsequent experimental investigators will ultimately be led to the conclusion that you did act upon it?" "Ah, that I have not said, but have carefully guarded against such an interpretation by saying that it is only of *conceptions*, that is, of the intellectual part of meaning that I was speaking. The pragmaticist need not deny that such ideas as those of action, of actual happening, of individuality, of existence, etc., involve something like a reminiscence of an exertion of brute force which is decidedly anti-intellectual, which is an all-important ingredient of the practical, although the pragmat[ic]istic interpretation leaves it out of account. Yet while he may admit that this idea of brute thereness, -- or whatever best names it, -- is quite distinct from any concept, yet he is bound to maintain that this does not suffice to make an idea of practical reality."

Peirce: CP 8.196 Cross-Ref:††
CHAPTER 14

WILHELM WUNDT, *PRINCIPLES OF PHYSIOLOGICAL PSYCHOLOGY*†1

196. When, in 1862, two years after Fechner's 'Psychophysik,' Wundt emerged from the physiological laboratory with his 'Beiträge zur Theorie der Sinneswahrnehmung,' students in this country there were who saw in the little volume

the harbinger of a new science of experimental psychology; and the next year their hopes seemed to be crowned in the same author's 'Vorlesungen über die Menschen- und Thierseele,' concerning which, by the way, it had better be noted that, like other of Wundt's books, it has lost most of its original flavor in a second, reconsidered edition, and that the English translation represents this later edition. Without this explanation, the sensation it first caused would be incomprehensible. Its readers heard in it the promise that the new science should keep pace with the other strictly experimental sciences, and should quickly outstrip all those sciences (more numerous than now) in which experimentation had not become practicable. Alas, today we are forty years wiser, and a chilling shade settles on hearts of enthusiasts of the sixties who now compare the advance that psychology has achieved -- indisputable, but how modest! -- with the unheard-of leaps that every other science has performed, be it an experimental one or not. Since 1860 the foundations of pure mathematics have been reconstructed; exact logic has been developed; physics has gained an optico-electrical theory, and radically new conceptions of molecular forces have been established; organic chemistry has followed out the doctrine of the aromatic compounds, and has been enriched by the doctrine of the unsymmetrical carbon atom; in its inorganic division the classification of the elements has been laid bare, the group of helium-argon elements has been added, and Mme. Curie has pronounced her magical "Open, sesame!" Besides all that, a new and more scientific kind of chemistry has been opened up. Biology has been equally revolutionized; astronomy has its new astro-physics, and geognosy has kept pace with the other sciences. Even on the psychical wing, linguistics, ethnology, archaeology, the history of high antiquity, have all found and matured new methods. In short, there is not a science that has not left psychology lingering in the rear; and the burning question of to-day is, why this should be so? Who will diagnose the malady of psychology?

Peirce: CP 8.197 Cross-Ref:††

197. It has been remarked that, at present, there is nothing which for the psychical wing of science fulfils that function which the science of dynamics fulfils on the physical side. Everybody knows what that function is. Every attempt to explain any phenomenon physically consists in first proposing some hypothesis as to the existence of designated dynamical conditions from which, according to the principles of dynamics, phenomena such as have been observed would take place, and then going on to put the hypothesis to the test of making it the basis of predictions concerning untried experiments.

Peirce: CP 8.198 Cross-Ref:††

198. Now it is a circumstance most significant for the logic of science, that this science of dynamics, upon which all the physical sciences repose, when defined in the strict way in which its founders understood it, and not as embracing the law of the conservation of energy, neither is nor ever was one of the special sciences that aim at the discovery of novel phenomena, but merely consists in the analysis of truths which universal experience has compelled every man of us to acknowledge. Thus, the proof by Archimedes of the principle of the lever, upon which Lagrange substantially bases the whole statical branch of the science, consists in showing that that principle is virtually assumed in our ordinary conception of two bodies of equal weight. Such universal experiences may not be true to microscopical exactitude, but that they are true in the main is assumed by everybody who devises an experiment, and is therefore more certain than any result of a laboratory experiment.

Peirce: CP 8.199 Cross-Ref:††

199. The sort of science that is founded upon the common experience of all men was recognized by Jeremy Bentham under the name of *cenoscopy*, in opposition to *idioscopy*, which discovers new phenomena. But long before Bentham's day the situation was sufficiently understood to set up a movement in the more enlightened countries to supply the psychological sciences with an analogous analytical foundation. The innumerable grades in the distinctness of thought prevent us from assigning dates, but one may say that the idea is struggling to the light in Locke's 'Essay' of 1689, and that its development was the best fruit of the eighteenth century. It moved in Italy, in France, and especially in Scotland. The analytical economics of Adam Smith and of Ricardo were examples of it. The whole doctrine in its totality is properly termed the Philosophy of Common Sense, of which analytical mechanics and analytical economics are branches. That Pragmatism of which so much has been said of late years is only an endeavor to give the philosophy of common sense a more exact development, especially by emphasizing the point that there is no intellectual value in mere feeling *per se*, but that the whole function of thinking consists in the regulation of conduct.†2 All this it is most needful to comprehend in order to assign to Wundt his proper rating in the history of philosophy.

Peirce: CP 8.200 Cross-Ref:††

200. The 'Physiological Psychology' is Wundt's most imposing and monumental work, but no man of science will call it his *chef-d'oeuvre*. That rank can be accorded to one production alone, his 'Untersuchungen zur Mechanik der Nerven und Nervencentren,' of which the first part appeared in 1871; the second, which is less fundamental but perhaps not less important, having been delayed by accidental causes until 1876, after the first edition of the 'Physiological Psychology' had appeared. Four traits of the 'Mechanik der Nerven' command admiration. One of them is a natural gift; two are results of scientific training; and one is a moral virtue. The gift is an astonishing sagacity about nerve-physiology -- a subterconscious susceptibility to the noeto-meteorological premonitions of a hailstorm of evidence that, when it bursts, will be cold, hard, and cutting enough.

Peirce: CP 8.201 Cross-Ref:††

201. Of the two scientific perfections the more striking is the mature prestudy of the methods that were or might have been pursued in the investigation. The other is the vigilant scrutiny of all details of the phenomena, especially of such as, being unlooked for, might easily have been overlooked. But the most admirable trait of all -- that self-respecting quality of Wundt's which no foibles can obscure -- is his genuine anxiety to correct the opinions which he at the time entertains, and to cast away his most brilliant theories the instant the dicta of experience seem to be against them -- a quality in which he so contrasts with all the metaphysical charlatans and self-admirers and with every other quintessential extract of littleness.†3 Wundt's great service to man, aside from that special research described in the 'Mechanik der Nerven,' has consisted in teaching the students of cenoscopy the beauty of those virtues upon which the students of idioscopy, especially those on the physical wing, have always insisted -- virtues that will necessarily result from any well-considered desire to know the truth. That such service has been Wundt's undoubtedly remains true, notwithstanding some lapses.

Peirce: CP 8.202 Cross-Ref:††

202. But the work of which Professor Titchener is publishing his translation is not to be classed as a performance of idioscopy, and little given is idioscopy to expressing itself in big books. It is not work of heurctic science of any kind. It is a

product of that useful industry of collecting, arranging, and digesting the deductions of mathematics, the analyses of cenoscopy, and the discoveries of idioscopy -- a service of which the Germans have assumed the burden, and which, as being the "systematization of knowledge," they as well as the general public are too apt to mistake for the business of science. From the date of the publication of this work, Wundt has turned a corner in his career, and has pursued a course not determined by the intrinsic affinities of his previous work. His principal publications (aside from revisions and from papers in his periodical *Philosophische Studien*) have consisted in an extensive treatise on logic, another on ethics, and a 'System der Philosophie.' These are subjects to which the majority of their devotees have been led by a desire to settle their beliefs about God, freedom, and immortality. But students of science are a good deal given to thinking that high theory is more apt to lead men wrong than right about religion, while religion has never done theory more good than harm. The doubts which impelled the few men of science who have been led to any thorough study of philosophy have almost always been concerned with the limits of trustworthiness of scientific results. But Wundt has never entertained any such general doubts. He explicitly says that whatever is not based upon the results of the special sciences has no real basis at all. He makes no exception in favor of dynamics, on the truth of which all his own work reposes. But, for him, common sense is nothing but an imperfect kind of science; and it is remarkable that his physiology recognizes no very fundamental difference between the functions of the cerebral cortex and those of the organs at the base of the brain. To the question what could have been Wundt's motive in putting himself forward as a leader in philosophy, for which he had never displayed any genius, but rather the reverse, the answer to which the study of his writings must lead is that the results of experimental psychology, meagre though they be as compared with those of other sciences, so dazzled the imagination of Wundt as to make him think that that study alone must be set up as the queen of the sciences, and prompted him to try to prove that logic, ethics, and philosophy could be securely based on that special science.

Peirce: CP 8.203 Cross-Ref:††

203. Wundt's philosophical publications have not met the acclamations that he undoubtedly at first expected; nor can it be said that the two scientific merits above mentioned are here one whit better exemplified than in the general run of second-rate philosophical treatises of the time. They rather fall below that average. In the matter of the deliberate preselection of methods, for example, one will not often meet with anything weaker than Wundt's admission that it seems self-evident that metaphysics should not be made to depend on the results of special science, while defending himself by saying that, having come to philosophy from physical science by the route of experimental psychology, it is natural that he should be unable to pursue philosophical investigations by any other method than that which his own sequence of study suggested to him. ("Ship ahoy! -- Where are you bound?" "For the port of Philosophy." "Then why, in Heaven's name, are you sailing on that course, Captain Wundt?" "Well, the truth is, this is the way the vessel was heading at the time it occurred to me to make that port.") Other equally gross departures from the two scientific ideals could easily be pointed out. Whether or not, if Wundt had possessed any analytical strength, it would have been possible for him to imagine that he could base such matters as dynamics, geometry, and arithmetic upon his physiological experiments, or whether in that case he could have failed to perceive the value of the pragmatist analysis in binding together nerve-physiology and psychology, must remain matters of opinion. But, unfortunately for his good fame, there exist departments of logic upon which he has touched that no more fall within the marches

of opinion than does the principle of the lever or the doctrine of limits; and here he simply places himself where Hobbes placed himself by his attempts at reasoning on exact subjects; and those who, nevertheless, talk of Hobbes as a "great logician" will be free to entertain the same opinion of Wundt -- and of Lord Timothy Dexter.

Peirce: CP 8.204 Cross-Ref:††

204. As for the 'Ethics' and the 'System of Philosophy,' we shall simply say that no person of discrimination would prove that quality by ranking them among works of the first order. We say no more, because such deviations from a great career are too unpleasant to contemplate. Of course, even in the 'Logic' there are brilliant chapters; it could not be otherwise, their author having achieved such things as he had, though in a distant field. As to the 'Physiological Psychology,' there will probably be no break in the unanimity that it is the most important monument of the new experimental psychology. Professor Titchener's translation has been eagerly awaited for long years. He explains the delay in his preface. It appears that he has made three complete translations of the work which have twice been superseded by revisions of the original. He is himself of opinion that his third is the least good of the three, but one does not see how that could possibly be. His unusual skill in making agreeable English of a faithful rendering from disagreeable German had already been proved -- a psychological accomplishment which Oxford training, the experience of the psychological laboratory, and practice in this very thing have perfected. It is not comprised in the verbal expression. Unerring judgment has been exercised in the editing both of the present volume and of others. The author's slips, if not too numerous, have to be corrected, with or without mention, according to circumstances. Whether the lettering of diagrams shall continue to represent German words or not, whether or not bad figures shall be replaced by better ones, etc., are questions about which the least talent for judging wrong would have betrayed itself if it had lurked in the translator. The present volume, the first of three, includes only the first and perhaps the most interesting of the six divisions of the original work. It relates to the subject in which Wundt's opinions have the greatest weight; and it is a subject whose practical corollaries will be obvious to every reader -- "the bodily substrate of the mental life."

Peirce: CP 8.205 Cross-Ref:††
BOOK II

CORRESPONDENCE

CHAPTER 1

TO SIGNOR CALDERONI, ON PRAGMATISM†1

205. I have delayed thanking you, as I now very warmly do, for sending me the three numbers of *Leonardo*,†2 and for your too flattering references to my formulation. In the April number of the *Monist*†3 I proposed that the word "pragmatism" should hereafter be used somewhat loosely to signify affiliation with Schiller, James, Dewey, Royce, and the rest of us, while the particular doctrine which I invented the word to denote, which is your first kind of pragmatism, should be called "*pragmaticism*." The extra syllable will indicate the narrower meaning.

Peirce: CP 8.206 Cross-Ref:††

206. Pragmaticism is not a system of philosophy. It is only a method of thinking; and your correspondent, Juliano il Sofista,†4 is quite right in saying that it is not a new way of thinking. If it were so, that, to my mind, would be almost sufficient to condemn it. It is only the formulation of it which was new thirty years ago, unless your correspondent is prepared to cite the volume and page on which an equivalent formulation had already been given. From his tone, I infer that he is quite prepared to do this; and I shall thus congratulate myself on an unknown fellow-thinker. Of those who have used this way of thinking Berkeley is the clearest example, though Locke (especially in the fourth book of his Essay), Spinoza, and Kant may be claimed as adherents of it.

Peirce: CP 8.207 Cross-Ref:††

207. Although pragmaticism is not a philosophy, yet, as you rightly say, it best comports with the English philosophy, and more particularly with the Scotch doctrine of common sense.

Peirce: CP 8.208 Cross-Ref:††

208. In an article which should have appeared in the July *Monist*.†5 but which seems to have been crowded out by matters of superior importance, magic squares and the like, I specify six errors which I find in the Scotch doctrine of common sense, of which the most important is that those philosophers failed to remark the extreme vagueness of our indubitable beliefs. For example, everybody's actions show that it is impossible to doubt that there is an element of order in the world; but the moment we attempt to define that orderliness we find room for doubt. There is, besides, another respect in which pragmaticism is at issue not only with English philosophy more particularly, but with all modern philosophy more or less, even with Hegel; and that is that it involves a complete rupture with nominalism. Even Duns Scotus is too nominalistic when he says that universals are contracted to the mode of individuality in singulars, meaning, as he does, by singulars, ordinary existing things. The pragmaticist cannot admit that. I myself went too far in the direction of nominalism when I said that it was a mere question of the convenience of speech whether we say that a diamond is hard when it is not pressed upon, or whether we say that it is soft until it is pressed upon.†6 I *now* say that experiment will prove that the diamond is hard, as a positive fact. That is, it is a real fact that it *would* resist pressure, which amounts to extreme scholastic realism. I deny that pragmaticism as originally defined by me made the intellectual purport of symbols to consist in our conduct. On the contrary, I was most careful to say that it consists in our *concept* of what our conduct *would* be upon *conceivable* occasions. For I had long before declared that absolute individuals were *entia rationis*, and not realities. A concept determinate in all respects is as fictitious as a concept definite in all respects. I do not think we can ever have a logical right to infer, even as probable, the existence of anything entirely

contrary in its nature to all that we can experience or imagine. But a nominalist must do this. For he must say that all future events are the total of all that will have happened and therefore that the future is not endless; and therefore, that there will be an event not followed by any event. This *may* be, inconceivable as it is; but the nominalist must say that it *will* be, else he will make the future to be endless, that is, to have a mode of being consisting in the truth of a general law. For every future event will have been completed, but the endless future will not have been completed. There are many other turns that may be given to this argument; and the conclusion of it is that it is only the general which we can understand. What we commonly designate by pointing at it or otherwise indicating it we assume to be singular. But so far as we can comprehend it, it will be found not to be so. We can only *indicate* the real universe; if we are asked to describe it, we can only say that it includes whatever there may be that really is. This is a universal, not a singular.

Peirce: CP 8.209 Cross-Ref:††

209. The truth of pragmatism may be proved in various ways. I would conduct the argument somewhat as follows. In the first place, there are but three elementary kinds of reasoning. The first, which I call *abduction* (on the theory, the doubtful theory, I confess, that the meaning of the XXVth chapter of the second book of the Prior Analytics has been completely diverted from Aristotle's meaning by a single wrong word having been inserted by Apellicon where the original word was illegible) consists in examining a mass of facts and in allowing these facts to suggest a theory. In this way we gain new ideas; but there is no force in the reasoning. The second kind of reasoning is *deduction*, or necessary reasoning. It is applicable only to an ideal state of things, or to a state of things in so far as it may conform to an ideal. It merely gives a new aspect to the premisses. It consists in constructing an image or diagram in accordance with a general precept, in observing in that image certain relations of parts not explicitly laid down in the precept, and in convincing oneself that the same relations will always occur when that precept is followed out. For example, having convinced ourselves of the truth of the *pons asinorum* with the aid of a diagram drawn with a common lead pencil, we are quite sure it would be the same with a diagram drawn in red; and a form of syllogism which is certain in black is equally so in red. A phenomenon having been observed in a laboratory, though we may not know on what conditions it depends, yet we are quite sure that it would make no difference whether the number of degrees of the longitude of the planet Eros just one week previous were a prime or composite number. The third way of reasoning is *induction*, or experimental research. Its procedure is this. Abduction having suggested a theory, we employ *deduction* to deduce from that ideal theory a promiscuous variety of consequences to the effect that if we perform certain acts, we shall find ourselves confronted with certain experiences. We then proceed to try these experiments, and if the predictions of the theory are verified, we have a proportionate confidence that the experiments that remain to be tried will confirm the theory. I say that these three are the only elementary modes of reasoning there are. I am convinced of it both *a priori* and *a posteriori*. The *a priori* reasoning is contained in my paper in the Proceedings of the American Academy of Arts and Sciences for April 9, 1867.†7 I will not repeat it. But I will mention that it turns in part upon the fact that induction is, as Aristotle says, the inference of the truth of the major premiss of a syllogism of which the minor premiss is made to be true and the conclusion is found to be true, while abduction is the inference of the truth of the minor premiss of a syllogism of which the major premiss is selected as known already to be true while the conclusion is found to be true. Abduction furnishes all our ideas concerning real things, beyond

what are given in perception, but is mere conjecture, without probative force. Deduction is certain but relates only to ideal objects. Induction gives us the only approach to certainty concerning the real that we can have. In forty years diligent study of arguments, I have never found one which did not consist of those elements. The successes of modern science ought to convince us that induction is the only capable *imperator* of truth-seeking. Now pragmatism is simply the doctrine that the inductive method is the only essential to the ascertainment of the intellectual purport of any symbol.

Peirce: CP 8.210 Cross-Ref:††

210. This argument must be supplemented by examples of the wholesome effect of pragmatistic interpretations. Among the most signal of these is the explanation of probability. We begin by asking, what is the use of calculations of probabilities; and the answer is that the great business of insurance rests upon such calculations. The probability upon which this business proceeds consists in the practical certainty that for every ten thousand dollars paid in about a certain number of dollars will have to be paid out. In the rare, the very rare, case in which decidedly more must be paid out, there are not only reserves more than ample, but there is the knowledge that such large payments will cause a great increase in the amounts paid in. A probability, therefore, is the known ratio of frequency of a specific future event to a generic future event which includes it. That is what probability must mean in order to have any importance for business. What, then, does it mean to say that if a man sees a phenomenon occur on m successive days, the probability is $m + 1/m + 2$ that the same phenomenon will appear on the next following day? Does it mean that if we put a large number of universes in a bag, shake them up well, and draw out one at random this will be the average result? It plainly means nothing at all of any consequence.

Peirce: CP 8.211 Cross-Ref:††

211. But all this neither proves, nor tends to prove, the whole proposition. It goes to show that the practical consequences are *much*, but not that they are *all* the meaning of a concept. A new argument must supplement the above. All the more active functions of animals are adaptive characters calculated to insure the continuance of the stock. Can there be the slightest hesitation in saying, then, that the human intellect is implanted in man, either by a creator or by a quasi-intentional effect of the struggle for existence, virtually in order, and solely in order, to insure the continuance of mankind? But how can it have such effect except by regulating human conduct? Shall we not conclude then that the conduct of men is the sole purpose and sense of thinking, and that if it be asked *why* should the human stock be continued, the only answer is that that is among the inscrutable purposes of God or the virtual purposes of nature which for the present remain secrets to us?

Peirce: CP 8.212 Cross-Ref:††

212. So it would seem. But this conclusion is too vastly far-reaching to be admitted without further examination. Man seems to himself to have some glimmer of co-understanding with God, or with Nature. The fact that he has been able in some degree to predict how Nature will act, to formulate general "laws" to which future events conform, seems to furnish inductive proof that man really penetrates in some measure the ideas that govern creation. Now man cannot believe that creation has not some ideal purpose. If so, it is not mere action, but the development of an idea which is the purpose of thought; and so a doubt is cast upon the ultra pragmatic notion that action is the *sole* end and purpose of thought.

Peirce: CP 8.213 Cross-Ref:††

213. It was in the desperate endeavor to make a beginning of penetrating into that riddle that on May 14, 1867, after three years of almost insanely concentrated thought, hardly interrupted even by sleep, I produced my one contribution to philosophy in the "New List of Categories" in the Proceedings of the American Academy of Arts and Sciences, Vol. VII, pp. 287-298.†8 Tell your friend Julian that this is, if possible, even less original than my maxim of pragmatism; and that I take pride in the entire absence of originality in all that I have ever sought to bring to the attention of logicians and metaphysicians. My three categories are nothing but Hegel's three grades of thinking. I know very well that there are other categories, those which Hegel calls by that name. But I never succeeded in satisfying myself with any list of *them*. We may classify objects according to their matter; as wooden things, iron things, silver things, ivory things, etc. But classification according to structure is generally more important. And it is the same with ideas. Much as I would like to see Hegel's list of categories reformed, I hold that a classification of the elements of thought and consciousness according to their formal structure is more important. I believe in inventing new philosophical words in order to avoid the ambiguities of the familiar words. I use the word *phaneron* to mean all that is present to the mind in any sense or in any way whatsoever, regardless of whether it be fact or figment. I examine the phaneron and I endeavor to sort out its elements according to the complexity of their structure. I thus reach my three categories.†9

Peirce: CP 8.214 Cross-Ref:††

CHAPTER 2

TO PAUL CARUS, ON "ILLUSTRATIONS OF THE LOGIC OF SCIENCE"†1

214. Ever since I was paid that money by you and Mrs. Carus, I have been engaged with all my energy, allowing only for such as I had to expend upon my wife's health and upon getting this house habitable and in salable condition, in trying to write an article or articles for you upon the second grade of clearness, i.e., that which results from analytic definition, and upon corrections to the errors and other faults of the articles of mine that appeared in *The Popular Science Monthly* in 1877 and 1878,†2 to which I should be glad if you would add a reprint of the article of January, 1901,†3 which requires no correction.

Peirce: CP 8.215 Cross-Ref:††

215. I have written a great deal but am satisfied with but the smaller part of it . . . Since I got your letter I have . . . gradually been forced to the conclusion that since you are very reasonably impatient, my best course is simply to write a preface in which I state in general terms how what I then say ought to be altered, and I will here . . . try to indicate the points I should make.

Peirce: CP 8.216 Cross-Ref:††

216. In regard to the first Essay consisting of the first two articles,†4 the

principal positive error is its nominalism, especially illustrated by what I said about Gray's stanza, "Full many a gem" etc., . . . †5 I must show that the *will be's*, the actually *is's*, and the *have beens* are not the sum of the reals. They only cover actuality. There are besides *would be's* and *can be's* that are real. The distinction is that the *actual* is subject both to the principles of contradiction and of excluded middle; and in *one* way so are the *would be's* and *can be's*. In *that* way a *would be* is but the negation of a *can be* and conversely. But in another way a *would be* is not subject to the principle of excluded middle; both *would be X* and *would be not X* may be false. And in this latter way a *can be* may be defined as that which is not subject to the principle of contradiction. On the contrary, if of anything it is only true that it *can be X* [then] it *can be not X* as well.

Peirce: CP 8.217 Cross-Ref:††

217. It certainly can be proved very clearly that the Universe does contain both *would be's* and *can be's*.

Peirce: CP 8.218 Cross-Ref:††

218. Then in regard to the second article, I ought to say that my three grades of clearness are *not*, as I seemed then to think, such that either the first or the second are superseded by the third, although we may say that they are acquired, mostly, in the order of those numbers. I ought to describe, if only in a paragraph, how to train oneself and one's children in the first grade of clearness, so that, for example, one will recognize a millimetre length when one meets with it, and so with colors. I have done a great deal of work in training myself to this kind of clearness. It would if put together amount to two or three years of industry; and I should recommend systematic exercises of the sort to everybody. †6 Useful as that is, however, I don't hesitate to say that the second grade of clearness is far more important, and all my writings of late years illustrate that. Still, I continue to admit that the third grade is the most important of all and a good example of it is William James who is so phenomenally weak in the second grade, yet ever so high above most men in the third. But there is no reason why all three should not be symmetrically developed.

Peirce: CP 8.219 Cross-Ref:††

219. The bulk of these *Popular Science* articles, after the first two, are occupied with a criticism of the underlying principles of Laplace's *Théorie Analytique des Probabilités* and Mill's *System of Logic* -- two writers of a high order which have had and still have a great and deplorable influence.

Peirce: CP 8.220 Cross-Ref:††

220. Before the third article, on probability, I should like to insert a short and easy account of my existential graphs †7 because when that system is well in hand, it becomes so much easier to show great faults of Laplace and Mill; and that shorter account I could now easily write.

Peirce: CP 8.221 Cross-Ref:††

221. It would also be well to show how all numbers involve *essentially* nothing but ideas of *succession*. Then I should like to point out how utterly Laplace fails to define what he means by *probabilité*, his account of it resting upon what he calls the *également possible*, which I maintain has none but the vaguest meaning. I ought on my side to define *probability*. †8 For that purpose, I should have to begin by distinguishing three ways -- three quite different *directions* so to speak, as different as the X, Y, Z of a system of orthogonal co-ordinates -- in which cognitions can fall short of absolute certainty, or rather of *mathematical* certainty, which is not absolute,

because blunders may have been committed in reaching it.†9

Peirce: CP 8.222 Cross-Ref:††

222. The names which I would propose for general adoption for the three different kinds of acceptability of propositions are plausibility, verisimilitude, probability

Peirce: CP 8.223 Cross-Ref:††

223. The last alone seems to be capable of a certain degree of exactitude or measurement. By plausibility, I mean the degree to which a theory ought to recommend itself to our belief independently of any kind of evidence other than our instinct urging us to regard it favorably. All the other races of animals certainly have such instincts; why refuse them to mankind? Have not all men some notions of right and wrong as well as purely theoretical instincts? For example, if any man finds that an object of no great size in his chamber behaves in any surprising manner, he wonders what makes it do so; and his instinct suggests that the cause, most plausibly, is also in his chamber or in the neighbourhood. It is true that the alchemists used to think it might be some configuration of the planets, but in my opinion this was due to a special derangement of natural instinct. Physicists certainly today continue largely to be influenced by such plausibilities in selecting which of several hypotheses they will first put to the test.

Peirce: CP 8.224 Cross-Ref:††

224. By verisimilitude I mean that kind of recommendation of a proposition which consists in evidence which is insufficient because there is not enough of it, but which will amount to proof if that evidence which is not yet examined continues to be of the same virtue as that already examined, or if the evidence not at hand and that never will be complete, should be like that which is at hand. All determinations of probability ultimately rest on such verisimilitudes. I mean that if we throw a die 216 times in order to ascertain whether the probability of its turning up a six at any one throw differs decidedly from 1/6 or not, our conclusion is an affair not of probability as Laplace would have it, by assuming that the antecedent probabilities of the different values of the probability are equal, but is a verisimilitude or as we say a "likelihood." That Laplace is wrong can be demonstrated, since his theory leads to contradictory results. But perhaps the easiest way to show it is wrong is to point out that there is no more reason for assuming that all the values of the probability are equally probable than for assuming that all the values of the *odds* are so; or that all the values of the logarithms of the odds are so, since this is our instinctive way of judging of probabilities, as is shown by our "balancing the probabilities."

Peirce: CP 8.225 Cross-Ref:††

225. Having thus defined plausibility and verisimilitude, I come to define probability. None of the books contain a definition of mathematical probability (which is what I mean by "probability" however measured) which will hold water. For the sake of simplicity, I will define it in a particular example. If, then, I say that the probability that if a certain die be thrown in the usual way it will turn up a number divisible by 3 (i.e., either 3 or 6) is 1/3, what do I mean? I mean, of course, to state that that die has a certain habit or disposition of behaviour in its present state of wear.†10 It is a *would be* and does not consist in actualities or single events in any multitude finite or infinite. Nevertheless a habit does consist in what *would* happen under certain circumstances if it should remain unchanged throughout an endless series of actual occurrences. I must therefore define that habit of the die in question which we express by saying that there is a probability of 1/3 (or odds of 1 to 2) that if

it be thrown it will turn up a number divisible by 3 by saying how it *would* behave if, while remaining with its shape, etc. just as they are now, it *were to be* thrown an endless succession of times. Now it is very true that it is quite impossible that it should be thrown an infinite succession of times. But this is no objection to my supposing it, since that impossibility is merely a physical, or if you please, a metaphysical one, and is not due to any logical impossibility to the occurrence in a finite time of an endless succession of events each occupying a finite time. For when Achilles overtook the tortoise he had to go through such an endless series (endless *in the series*, but not endless *in time*) and supposedly actually did so.

Peirce: CP 8.226 Cross-Ref:††

226. Very well, I will further suppose that tallies are kept during the throwings, one tally of the throws turning up 6 or 3, but the other tally of the throws turning up 1, 2, 4, 5, and further I will suppose that after each throw the number that the latter tally has reached shall be divided by the number that the former tally has reached. I will use the expression that this quotient *changes its value* at every new throw, instead of saying that a new quotient differs from the last. When the quotient changes from being greater than 2 [or] being less than 2 . . . to being either just 2 or on the opposite side of 2 to what it was before, as for example if it passes from being 21:10 to being 21:11 or from 21:11 to 22:11 or from 25:12 to 25:13, etc., I shall say it touches 2 (meaning strictly that it either comes to 2 or passes across 2). Then after the first throw it will be either 0 or ∞ and there it may remain for any number of throws. But after it has once moved away it never will return to either of these values, but after it has finally recovered from the effects of the first throws it will oscillate in a very irregular way, and soon it will "touch" (or pass over) some other values *for the last time* although nobody can *know* that it is to prove to have been for the last time; and then values still nearer to 2 will be touched or traversed for the last time. And in its endless series there will be no value that it would not touch or traverse for the last time excepting only the value 2. And this "would be" is what constitutes the habit which we state in saying that the odds against its turning up a number divisible by 3 are 2:1 or that the probability of its turning up a 6 or a 3 is $1/3$

Peirce: CP 8.227 Cross-Ref:††

227. In order to get this matter straightened out, I think it would be well to change the place of the sixth paper and place it directly after the third.†11 Then I would append a *Correction* in which I would state that the division of the elementary kinds of reasoning into three heads was made by me in my first lectures and was published in 1869 in Harris's *Journal of Speculative Philosophy*.†12 I still consider that it had a sound basis. Only in almost everything I printed before the beginning of this century I more or less mixed up Hypothesis and Induction

Peirce: CP 8.228 Cross-Ref:††

228. The general body of logicians had also at all times come very near recognizing the trichotomy. They only failed to do so by having so narrow and formalistic a conception of inference (as necessarily having formulated judgments for its premises) that they did not recognize Hypothesis (or, as I now term it, *retroduction*)†13 as an *inference*

Peirce: CP 8.229 Cross-Ref:††

229. When one contemplates a surprising or otherwise perplexing state of things (often so perplexing that he cannot definitely state what the perplexing character is) he may formulate it into a judgment or many apparently connected

judgments; he will often finally strike out a hypothesis, or problematical judgment, as a mere possibility, from which he either fully perceives or more or less suspects that the perplexing phenomenon would be a necessary or quite probable consequence.

Peirce: CP 8.230 Cross-Ref:††

230. That is a retroduction. Now three lines of reasoning are open to him. First, he may proceed by mathematical or syllogistic reasoning at once to demonstrate that consequence. That of course will be deduction.

Peirce: CP 8.231 Cross-Ref:††

231. Or, second, he may proceed still further to study the phenomenon in order to find other features that the hypothesis will *explain* (i.e. in the English sense of explain, to deduce the facts from the hypothesis as its necessary or *probable* consequences). That will be to continue reasoning retroductively, i.e., by hypothesis.

Peirce: CP 8.232 Cross-Ref:††

232. Or, what is usually the best way, he may turn to the consideration of the hypothesis, study it thoroughly and deduce miscellaneous observable consequences, and *then* return to the phenomena to find how nearly these consequences agree with the actual facts.

Peirce: CP 8.233 Cross-Ref:††

233. This is not essentially different from induction. Only it is most usually an induction from instances which are not discrete and numerable. I now call it Qualitative Induction. It is this which I used to confound with the second line of procedure, or at least not to distinguish it sharply.

Peirce: CP 8.234 Cross-Ref:††

234. A good account of Quantitative Induction is given in my paper in *Studies in Logic, By Members of the Johns Hopkins University*, †14 and its two rules are there well developed. But what I there call hypothesis is so far from being that, that it is rather Quantitative than Qualitative Induction. At any rate, it is treated *mostly* as Quantitative. Hypothesis proper is in that paper only touched upon in the last section.

Peirce: CP 8.235 Cross-Ref:††

235. There is a third kind of Induction. In order to show this, it is requisite to define Induction.

Peirce: CP 8.236 Cross-Ref:††

236. Now the essential character of induction is that it infers a *would-be* from actual singulars. These singulars must, in general, be finite in multitude and then, as I show in my Johns Hopkins paper, the inductive conclusion can be (usually) but *indefinite*, and can never be *certain*

Peirce: CP 8.237 Cross-Ref:††

237. But in ordinary cases an induction would become both *precise* and *certain*, -- though even then it would not be apodictic certainty, if the instances were of denumeral (or simply endless) multitude. Therefore, defining induction as the sort of inference which produces verisimilitude or likelihood (that is, which regards an endless series of actualities as conclusive evidence of a *would-be* since it is the best evidence possible when we are not behind the scenes), . . . any plausible proposition that is supported by instances in every respect is justifiable so long as one keeps on the alert for the first exception. Of course, such an induction has the very minimum of likelihood, yet it has *some*; and we very often find ourselves driven to accept it. The

world has always turned on its axis so far as we know about once every 24 hours and therefore we presume (vaguely) that it always will continue to do so. In every case that has been sufficiently inquired into, every human being has been born of a woman not a maiden. So almost everybody feels sure it always will be found so. People are far more confident of it than they have any right to be. All former generations of men have died off. Therefore, people say, they always will. In one sense I suppose this is certain. But that they always *would* even if there were no accidents, seems to me as weak an inference as any that I would not positively condemn as utterly worthless. I call this kind of thing crude induction. I must confess that although my explanation of the validity of induction seems to me to be far superior to any other, I am not altogether satisfied with it, or rather with its results. Quantitative Induction depends upon the possibility of making a truly representative sample. That is to say, the examples composing it must be chosen as possessing the conditional character, which is easy enough, but also so that the choice of them shall not be influenced one way or the other, by whether or not they possess the consequent character. They must be such on the whole in that course of experience to which the induction is to be applied. That, to be sure, cannot but be the case, should the entire class sampled be alike in respect to the consequent character. But the further this ideal state of things is from being realized the more extremely difficult it becomes to get a truly representative sample, and the result, after every precaution has been taken, is that we cannot expect any great precision in inductive conclusions when the class is anywhere near being equally divided between individuals that do and that do not possess the consequent character. However, this is not owing to any falsity in my theory, but to the essential imperfection of induction itself when applied to these cases.

Peirce: CP 8.238 Cross-Ref:††

238. As for the validity of the hypothesis, the retrodution, there seems at first to be no room at all for the question of what supports it, since from an actual fact it only infers a *may-be* (*may-be* and *may-be not*). But there is a decided leaning to the affirmative side and the frequency with which that turns out to be an actual fact is to me quite the most surprising of all the wonders of the universe.

Peirce: CP 8.239 Cross-Ref:††
CHAPTER 3

TO JOHN DEWEY, ON THE NATURE OF LOGIC†1

239. I mean, if I can manage it, to get some notice of the book of your logical school into the *Nation*. †2 But the editor fights very shy of the subject as I write about it and it is necessary to dilute and decorate it so that the result has not much value for serious students. I will therefore write to express how your position appears as viewed from mine. I am struck with the literary tone of your men, a sort of maturity which bespeaks the advantage of studying under you and thoroughly applaud your efforts to set them on their own legs. All that is admirable and warms my heart. But I must say to you that your style of reasoning about reasoning has, to my mind, the usual fault that when men touch on this subject, they seem to think that no reasoning can be too

loose, that indeed there is a merit in such slipshod arguments as they themselves would not dream of using in any other branch of science. You propose to substitute for the Normative Science which in my judgment is the greatest need of our age a "Natural History" of thought or of experience. Far be it from me to do anything to hinder a man's finding out whatever kind of truth he is on the way to finding out. But I do not think anything like a natural history can answer the terrible need that I see of checking the awful waste of thought, of time, of energy, going on, in consequence of men's not understanding the theory of inference. Though you use the expression "Natural History," yet of the two branches of Natural History, physiology and anatomy, which are as sharply sundered today as ever they were, you seem to be alluding only to the latter, since you speak of its being revolutionized by conceptions of evolution. Now the doctrine of evolution has not affected physiology either much or little, unless by lending a competing interest to anatomy †3 and thus *weakening* physiology. It has certainly neither directly, nor indirectly, strengthened it. So, using the word anatomy without reference to its etymological suggestions, but simply as a designation of the sort of business that Comparative Anatomists are engaged in, you seem to conceive your occupation to be the studying out of the Anatomy of Thought. Thereupon, I remark that the "thought" of which you speak cannot be the "thought" of normative logic. For it is one of the characteristics of all normative science that it does not concern itself in the least with what actually takes place in the universe, barring always its assumption that what is before the mind always has those characteristics that are found there and which Phänomenologie is assumed to have made out. But as to particular and variable facts, no normative science has any concern with them, further than to remark that they form a constant constituent of the phenomenon. Now nothing like the study the Comparative Anatomists are occupied with can be made of *mere possibilities*. It absolutely requires a rich experimental field. If it were not so, one could have an anatomy of Higher Plane Curves; and upon a superficial examination it might seem as if that were possible. But more thorough study will show that such a thing would be entirely artificial. There is no anatomy of possibilities because one can say in advance how pure possibilities vary and diverge from one another. Namely, they do so in every possible way. What renders a Comparative Anatomy possible is that certain conceivable forms do not occur. Only a minute proportion of them occur. Thus we have a comparative anatomy of the chemical elements, because though Mendeléef's Table *roughly* describes what elements there are *in part*, yet each element has peculiarities which that table does not account for and besides there are no elements except Manganese in one column and the rare earths do not differ from one another in any such way as the table predicts and besides the table sets no limits to the atomic weights; but we find that elements of very high atomic weights are radioactive in every column of the table and beyond these we find no elements at all. Thus there is in the list of chemical elements just that experiential diversity and absence of most possible forms that renders the kind of study called anatomical possible. If then you have a "Natural History" (i.e. a comparative anatomy) of thought, -- it is not the merely *possible* thought that Normative Science studies, but thought as it presents itself in an *apparently* inexplicable and irrational experience.

Peirce: CP 8.240 Cross-Ref: ††

240. The effect of teaching that such a Natural History can take the place of a normative science of thought must be to render the rules of reasoning lax; and in fact I find you and your students greatly given over to what to me seems like a debauch of loose reasoning. Chicago hasn't the reputation of being a moral place; but I should think that the effect of living there upon a man like you would be to make you feel all

the more the necessity for Dyadic distinctions, -- Right and Wrong, Truth and Falsity. These are only to be kept up by self control. Now just as Moral Conduct is Self-controlled conduct so Logical Thought is Moral, or Self-controlled, thought. The Germans have always been in favor of giving thought the rein. What is taught in German Universities bespeaks only the fashion of the day. No doubt a slow evolutionary process will gradually bring them round to the truth. But that is the Wild Oats doctrine applied to thought. It involves unspeakable waste.

Peirce: CP 8.241 Cross-Ref:††

241. Although I am strongly in favor of your Pragmatic views, I find the whole volume penetrated with this spirit of intellectual licentiousness, that does not see that anything is so very false. Of course you will understand that I should not write in such underscored terms to any man with whom I did not feel a very deep respect and sympathy. I am simply *projecting upon the horizon*, where distance gets magnified indefinitely, the *direction* of your standpoint as viewed from mine.

Peirce: CP 8.242 Cross-Ref:††

242. There are three sciences according to me to which Logic ought to appeal for principles, because they do not depend upon Logic. They are Mathematics, Phenomenology, and Ethics. There are several sciences to which logicians often make appeal by arguments which would be circular if they rose to the degree of correctness necessary to that kind of fallacy. They are Metaphysical Philosophy, Psychology, Linguistics (of which they barely know that of the Aryan Languages, -- and not Gaelic which does not ordinarily give a sentence a subject nominative), History, etc.

Peirce: CP 8.243 Cross-Ref:††

243. Your letter about my April Monist article †4 gave me keen pleasure, -- all the more so because I was somewhat surprised to learn you found so much good in what I said. For your Studies in Logical Theory certainly forbids all such researches as those which I have been absorbed in for the last eighteen years. That is what I liked least in those four papers. First, because it is contrary to a maxim I never infringe "Never permanently bar the road of any true inquiry," and my studies are so real that they compel me to say that certain highly esteemed and "genetical" methods are leading to false conclusions, certain others that are despised are most precious. Secondly, because your mode of arguing that every inquiry ought to be conducted genetically is a wretched method, considering the extreme importance of the conclusion. Thirdly, because some of your premisses are entirely contrary to the convictions which half a dozen years of careful historical studies and my personal experience force upon me. For according to my studies there are some sciences which can be and ought to be studied genetically, while others cannot be so studied without rendering them perfectly futile. Such, for example, is pure mathematics; such are dynamics and general physics; such is chemistry; such is physiology proper. Again, you take as premiss of a confirmatory argument that any non-genetic logic will reach no conclusions that have any meaning in their real applications. But all my studies are conducted in full view of actual scientific memoirs and other records of scientific inquiry, in which they lead to denials of conclusions to which bad logic has led their authors; and some of my non-genetical studies have led directly to discoveries in mathematics and others to instituting experimental researches about the reality, if not the solidity, of which there can be no question; and in short I should like to know what genetic logician ever came to have such close quarters with actual science as I have done.

Peirce: CP 8.244 Cross-Ref:††

244. If it were not for this uncalled for intolerance of your logical theory, I should have no serious objection to it; and there are parts of it that seem to me admirable and of great value. I regretted your making everything turn on Lotze, as if he were a Hume. He was in his day a very careful, serious inquirer. But he was never a thinker of great subtilty, and he is now so entirely left behind, that I thought you might have left his doctrine to be disposed of by Jones and men of that calibre; and that he was rather small game for you. Whenever I come across a dilemma, I look out for the fallacy, my experience having shown me there almost always was one. Your reasoning generally is that either Lotze or you must be right, now Lotze isn't, etc. But you in no case, or in *one* at most, convince me at all that these are the only alternatives. In short, I think you could have made a stronger argument if you had let Lotze alone. That would not have been *genetic*, perhaps! But if instead of this argumentation which I can but believe to be artificial, you had just narrated how as historic fact you arrived at your opinion, that would have been genetic, and I venture to think that it would have seemed to me a conclusive proof, not of precisely all your propositions but of your main contention; and the errors would easily have been separated as merely the exaggerations of over-precision. The fallacy of over-precision which consists not in taking an ell when one has a right to an inch, but in stretching a warrant for a percentage of a micro-micron to more than the sum of all macro-kilometres, may be called the Philosopher's Fallacy. The first maxim of my "Synechism" runs: "Let us not preclude our conclusions beyond what our premisses definitely warrant." What you had a right to say was that for certain logical problems the entire development of cognition and along with it that of its object become pertinent, and therefore should be taken into account. What you do say is that no inquiry for which this development is not pertinent should be permitted.

Peirce: CP 8.245 Cross-Ref:††

CHAPTER 4

TO WILLIAM T. HARRIS, ON MIND†1

245. I send to you today two proof-sheets.†2 I should have sent the first one on some days ago but was ill when it came.

Peirce: CP 8.246 Cross-Ref:††

246. I suppose you saw that I struck out the paragraph referring to Hegelians. I intended no *slur* on them, or any appeal to the ignorant against them. What I meant was to protest respectfully but energetically *to* them against a certain tendency in their philosophy. In fact with all the disposition of this school to find every philosophical doctrine true for its time and stage of development, yet if their categories should happen not to be true it is plain that to classify men according to them may be one of the most unfair things in the world.

Peirce: CP 8.247 Cross-Ref:††

247. I have considered your remark that you do not see the drift of my making man entirely ignorant of his own states of mind. I suppose I have not written very clearly for one thing, -- and that I have tried to correct in the proof. But the real difficulty is that the article is truncated. I had intended to wind up with a long discussion about the metaphysics -- the ontology of the soul. I left this off on account of the length of the article. But now I find by your criticism that it is wanted, and I have endeavored to put it into the briefest and most meagre form and send it to you, in hopes you will be able to tack it on to the end of the article.

Peirce: CP 8.248 Cross-Ref:††

248. I do not say that we are ignorant of our states of mind. What I say is that the mind is virtual, not in a series of moments, not capable of existing except in a space of time -- nothing so far as it is at any one moment.†3

Peirce: CP 8.249 Cross-Ref:††
CHAPTER 5

TO WILLIAM JAMES†1

§1. PRAGMATISM

249. [March 13, 1897] Your letter and the dedication and the book gave me more delight than you would be apt to believe.†2 The note came day before yesterday. I got the book last night. I have read the first essay which is of great value, and I don't see that it is so very "elementary" as you say, unless you mean that it is very easy to read and comprehend, and it is a masterpiece in that respect.

Peirce: CP 8.250 Cross-Ref:††

250. That everything is to be tested by its practical results was the great text of my early papers;†3 so, as far as I get your general aim in so much of the book as I have looked at, I am quite with you in the main. In my later papers,†4 I have seen more thoroughly than I used to do that it is not mere action as brute exercise of strength that is the purpose of all, but say generalization, such action as tends toward regularization, and the actualization of the thought which without action remains unthought

Peirce: CP 8.251 Cross-Ref:††

251. As to "belief" and "making up one's mind," if they mean anything more than this, that we have a plan of procedure, and that according to that plan we will try a given description of behaviour, I am inclined to think they do more harm than good. "Faith," in the sense that one will adhere consistently to a given line of conduct, is highly necessary in affairs.†5 But if it means you are not going to be alert for indications that the moment has come to change your tactics, I think it ruinous in practice. If an opportunity occurs to do business with a man, and the success of it depends on his integrity, then if I decide to go into the transaction, I must go on the

hypothesis he is an honest man, and there is no sense at all in halting between two lines of conduct. But that won't prevent my collecting further evidence with haste and energy, because it may show me it is time to change my plan. That is the sort of "faith" that seems useful. The hypothesis to be taken up is not necessarily a probable one. The cuneiform inscriptions could never have been deciphered if very unlikely hypotheses had not been tried. You must have a consistent plan of procedure, and the hypothesis you try is the one which comes next in turn to be tried according to that plan.†6 This justifies giving nominalism a fair trial before you go on to realism; because it is a simple theory which if it doesn't work will have afforded indications of what kind of realism ought to be tried first. I do not say probability ought not to be considered. It will be a prominent factor in a well considered plan of research. Probability is simply absurd and nonsensical in reference to a matter of "supreme interest," and any decision of such a question on probable grounds is illogical. But wherein does the illogicality lie? Simply in considering any interest as supreme. No man can be logical who reckons his personal well-being as a matter of overwhelming moment

Peirce: CP 8.252 Cross-Ref:††

252. I am much encouraged at your thinking well of "tychism."†7 But tychism is only a part and corollary of the general principle of Synechism. That is what I have been studying these last fifteen years, and I become more and more encouraged and delighted with the way it seems to fit all the wards of your lock. It was a truly sweet thing, my dear William, to dedicate your book to me.

Peirce: CP 8.253 Cross-Ref:††

253. [November 10, 1900] Now, however, I have a particular occasion to write. Baldwin, arrived at J in his dictionary, suddenly calls on me to do the rest of the logic, in the utmost haste, and various questions of terminology come up.

Peirce: CP 8.253 Cross-Ref:††

Who originated the term *pragmatism*, I or you? Where did it first appear in print? What do you understand by it?†8

Peirce: CP 8.254 Cross-Ref:††

254. [November 25, 1902] You feel, as I do, that the importance of pragmatism is not confined to philosophy. The country is at this moment in imminent danger on which I need not expatiate. In philosophy those who think themselves pragmatists, like Mr. Schiller, miss the very point of it, that one simply can't form any conception that is other than pragmatistic.

Peirce: CP 8.255 Cross-Ref:††

255. But I seem to myself to be the sole depositary at present of the completely developed system, which all hangs together and cannot receive any proper presentation in fragments. My own view in 1877 was crude. Even when I gave my Cambridge lectures †9 I had not really got to the bottom of it or seen the unity of the whole thing. It was not until after that that I obtained the proof that logic must be founded on ethics,†10 of which it is a higher development. Even then, I was for some time so stupid as not to see that ethics rests in the same manner on a foundation of esthetics, -- by which, it is needless to say, I don't mean milk and water and sugar.

Peirce: CP 8.256 Cross-Ref:††

256. These three normative sciences correspond to my three categories, which in their psychological aspect, appear as Feeling, Reaction, Thought.†11 I have advanced my understanding of these categories much since Cambridge days; and can now put them in a much clearer light and more convincingly. The true nature of pragmatism cannot be understood without them. It does not, as I seem to have thought at first, take Reaction as the be-all, but it takes the end-all as the be-all, and the End is something that gives its sanction to action. It is of the third category. Only one must not take a nominalistic view of Thought as if it were something that a man had in his consciousness. Consciousness may mean any one of the three categories. But if it is to mean Thought it is more without us than within. It is we that are in it, rather than it in any of us. Of course I can't explain myself in a few words; but I think it would do the psychologists a great service to explain to them my conception of the nature of thought.

Peirce: CP 8.257 Cross-Ref:††

257. This then leads to synechism,†12 which is the keystone of the arch.

Peirce: CP 8.258 Cross-Ref:††

258. [March 7, 1904]†13 I want to thank you for your kind reference to me in your piece about Schiller's *Humanism*.†14 . . . The humanistic element of pragmatism is very true and important and impressive; but I do not think that the doctrine can be *proved* in that way. The present generation likes to skip proofs. I am tempted to write a little book of 150 pages about pragmatism, just outlining my views of the matter, and appending to it some of my old pieces with critical notes.†15 You and Schiller carry pragmatism too far for me. I don't want to exaggerate it but keep it within the bounds to which the evidences of it are limited. The most important consequence of it, by far, on which I have always insisted, as for example in my notice of Fraser's Berkeley in the *North American Review* of October, 1871,†16 is that under that conception of reality we must abandon nominalism. That in my opinion is the great need of philosophy. Notwithstanding what Royce says, Hegel appears to me to be on the whole a nominalist with patches of realism rather than a real realist.

Peirce: CP 8.259 Cross-Ref:††

259. I also want to say that after all pragmatism solves no real problem. It only shows that supposed problems are not real problems. But when one comes to such questions as immortality, the nature of the connection of mind and matter (further than that mind acts on matter not like a *cause* but like a *law*) we are left completely in the dark. The effect of pragmatism here is simply to open our minds to receiving any evidence, not to furnish evidence.

Peirce: CP 8.260 Cross-Ref:††

260. [July 23, 1905]†17 To begin with I want to emphasize my particular gratitude for your papers, as well as for the copies of Leonardo and a paper from Prof. Vailati no doubt sent at your prompting.†18 . . . I read the French paper †19 first of this batch you have last sent. I found it entirely clear as well as beautifully written. When you write English (it is better to say the disagreeable thing) I can seldom at all satisfy myself that I know what you are driving at. Your writing would, I can see, be

immensely forcible if one knew what you meant; but one (No. 1) doesn't. Now, for example, when you talk about doubting whether "consciousness" exists, you drive me at once to consulting a lot of books (in that particular case just 23 without counting the dictionaries of Baldwin, Eisler, etc.) to see what you could mean; and they left me as much in the dark as ever.†20 But now that you are tied down to the rules of French rhetoric, you are perfectly perspicuous; and I wish, and I am sure lots of others do, that you would consider yourself so tied down habitually. Because one sees that it only aids your force of style. Of course, you can smile at my undertaking to advise you about anything whatever. The fact that you can do so, if you like, emboldens me to say what I say.

Peirce: CP 8.261 Cross-Ref:††

261. I also agree to every word you say in this French article to the full, with one exception. That is that I am quite sure the doctrine is not at all so novel as you say. Of course it is all the better for not being novel. My recent delvings in the psychologies showed me that. Besides, it is nothing in the world but the well-known doctrine of immediate perception (followed out, of course, into other fields). This same thing, therefore, was held by our old friend Sir Wm. Hamilton, who had the same unfounded idea of its novelty. Not only Reid, but Kant, in his refutation of Berkeley, explicitly accepts it. As for the scholastics, no doubt so far as they were influenced by St. Augustine they were medium-ists, if I may coin the word for the purpose. But insofar as they followed Aristotle, I do not believe they were. I could easily mention several moderns who agree with you, and I have myself preached immediate perception as you know; -- and you can't find a place where I distinguish the objective and subjective sides of things. I think I will mail you a paper of mine that was printed Jan., 1901, in the *Popular Science Monthly*†21 where you will see this, -- not developed in the beautiful way you do, but plainly enough stated, I think. I refer you particularly to p. 301 et seq. I will quote a few phrases, though of course it is the continuous text that talks: "He tells us that each of us is like the operator at a central telephone office. . . Not at all! . . . When we first wake up to the fact that we are thinking beings . . . we have to set out upon our intellectual travels from the home where we already find ourselves. Now this home is the parish of Percepts. It is not inside our skulls but out in the open. It is the external world that we directly observe . . . The inkstand is a *real* thing. Of course, in being real and external, it does not in the least cease to be a purely psychical product, a generalized percept."†22 If I had had the least idea that I was uttering anything newer than the doctrine of immediate perception, I should have argued the matter more closely. Of course, this doctrine of immediate perception is a *corollary from the corollary* of pragmatism that the object perceived is the immediate object of the destined ultimate opinion, -- not of course, identical as a psychological phenomenon, for there never will be a necessarily ultimate opinion as a psychological phenomenon, but identical logically and metaphysically. I am quite sure that lots of others have held the same view, some of them pragmatists and some not. (I hope the word "pragmatism" may be accepted, as I suggest, as the term expressive of these things, -- perhaps we cannot be sure just what they are -- in which the group of us are in agreement, as to the interpretation of thought.)

Peirce: CP 8.262 Cross-Ref:††

262. As for humanism, it appears to me to be an allied doctrine, in perfect harmony with pragmatism, but not relating exactly to the same question. Indeed, since Schiller identifies it with the old humanism, I prefer the word "anthropomorphism" as expressive of *the scientific opinion*. For the old humanism was not a scientific

opinion but an aim; and whether in harmony with scientific aims or not, quite exterior to the scientific aim. To Schiller's anthropomorphism I subscribe in the main. And in particular if it implies *theism*, I am an anthropomorphist. But the God of my theism is not finite. That won't do at all. For to begin with, existence is reaction, and therefore no existent can be *clear supreme*. On the contrary, a finite being, without much doubt, and at any rate *by presumption*, is one of a genus; so that it would, to my mind, involve polytheism. In the next place, anthropomorphism for me implies above all that the true Ideal is a living power, which is a variation of the ontological proof due, I believe, to Moncure Conway's predecessor, William Johnson (*not* James) Fox. That is, the esthetic ideal, that which we *all* love and adore, the altogether admirable, has, *as ideal*, necessarily a mode of being to be called living. Because our ideas of the infinite are necessarily extremely vague and become contradictory the moment we attempt to make them precise. But still they are not utterly unmeaning, though they can only be interpreted in our religious adoration and the consequent effects upon conduct. This I think is good sound solid strong pragmatism. Now the Ideal is not a finite existent. Moreover, the human mind and the human heart have a filiation to God. That to me is the most comfortable doctrine. At least I find it most wonderfully so every day in contemplating all my misdeeds and shortcomings. Pluralism, on the other hand, does not satisfy either my head or my heart. I am as sure as I am of anything that the logical doctrines connected with it, -- Achilles and the Tortoise etc., -- are utterly false.

Peirce: CP 8.263 Cross-Ref:††

263. As for the "problem of evil," and the like, I see in them only blasphemous attempts to define the purposes of the Most High, -- or rather that is what I think of such disturbances of religious consciousness generally; but that particular problem has received the most beautiful and satisfactory solution in *Substance and Shadow*. We had a tramp working for us for a few days not long ago. One day he started the problem of evil. In twenty words I put before him the *Substance and Shadow* solution. He saw it, at once, did my tramp; and after a few moments' reflexion he looked up and said to me, "Yes, I guess that is just it." There is, however, nothing more wholesome for us than to find problems that quite transcend our powers, and I must say, too, that it imparts a delicious sense of being cradled in the waters of the deep, -- a feeling I always have at sea. It is, for example, entirely inscrutable to me why my three categories have been made so luminous to me without my being given the power to make them understood by those who alone are in a condition to see their meaning, -- i.e. my fellow-pragmatists. It seems to me that you all must have a strange blind spot on your mental retina not to see what others see and what pragmatism ought to make so much plainer; . . .

Peirce: CP 8.264 Cross-Ref:††

§2. CATEGORIES

264. [June 8, 1903] It rather annoys me to be told that there is anything novel in my three categories;†23 for if they have not, however confusedly, been recognized by men since men began to think, that condemns them at once. To make them as distinct as it is in their nature to be is, however, no small task. I do not suppose they are so in my own mind; and evidently, it is not in their nature to be sharp as ordinary concepts. But I am going to try to make here a brief statement that, I think, will do

something for them.

Peirce: CP 8.265 Cross-Ref:††

265. By the *phenomenon* I mean whatever is before our minds in any sense. The three categories are supposed to be the three kinds of elements that attentive perception can make out in the phenomenon.

Peirce: CP 8.266 Cross-Ref:††

266. The practical exigencies of life render Secondness the most prominent of the three. This is not a conception, nor is it a peculiar quality. It is an experience. It comes out most fully in the shock of reaction between ego and non-ego. It is there the double consciousness of effort and resistance. That is something which cannot properly be conceived. For to conceive it is to generalize it; and to generalize it is to miss altogether the *hereness* and *nowness* which is its essence. According to me, the idea of a reaction is not the idea of two *plus* forcefulness. On the contrary to think of two dots as two is to have a little experience of reaction and then to tell ourselves that that is to be taken only in a Pickwickian sense, as a mere reaction within the world of ideas, the experience of reaction itself at once leading us to think of a world of seconds or existences and a world of mere tame ideas; the one resistant, the other subject to our wills. We also find ourselves thinking of the things without us, as acting on one another, as really connected. Now it is your business as a psychologist to say how that comes about, not mine. I merely look at the phenomenon, and say that all idea of real relation, or connection, has in it that same element of irrational reaction. All the *actual* character of consciousness is merely the sense of the shock of the non-ego upon us. Just as a calm sea sleeps except where its rollers dash upon the land.

Peirce: CP 8.267 Cross-Ref:††

267. If we imagine that feeling retains its positive character but absolutely loses all relation, (and thereby all *vividness*, which is only the sense of shock), it no longer is exactly what we call feeling. It is a mere sense of quality. It is the sort of element that makes *red* to be such as it is, whatever anything else may be. I do not see how that can be described except as being such as it is, positively, of itself, while secondness is such as it is relatively to something else. Anything familiar gains a peculiar positive quality of feeling of its own; and that I think is the connection between Firstness and Hegel's first stage of thought. The second stage agrees better with Secondness.

Peirce: CP 8.268 Cross-Ref:††

268. The third stage is very close indeed to Thirdness, which is substantially Hegel's *Begriff*. Hegel, of course, blunders monstrously, as we shall all be seen to do; but to my mind the one fatal disease of his philosophy is that, seeing that the Begriff in a sense implies Secondness and Firstness, he failed to see that nevertheless they are elements of the phenomenon not to be *aufgehoben*, but as real and able to stand their ground as the Begriff itself. The third element of the phenomenon is that we perceive it to be intelligible, that is, to be subject to law, or capable of being represented by a general sign or Symbol. But I say the same element is in all signs. The essential thing is that it is capable of being represented. Whatever is capable of being represented is itself of a representative nature. The idea of representation involves infinity, since a representation is not really such unless it be interpreted in another representation. But infinity is nothing but a peculiar twist given to generality.†24 There is not anything truly general that does not actually make irrational existences conform to itself. That

is the very heart of the idea.

Peirce: CP 8.269 Cross-Ref:††

269. That is a very bald statement. An immense number of items might be added. But I endeavor so to draw it up that these ideas may appear less of the nature of will-o'-the-wisps to you, -- as steady lights. The more you reflect upon them the steadier they will become; -- at least, such is my experience.

Peirce: CP 8.270 Cross-Ref:††

§3. CONSCIOUSNESS

270. [June 12, 1902] There is a point of psychology which has been interesting me. I should like to know from you whether there is any book which can give me aid about it. My own notion, which I dare say is crude, is this: The question is what passes in consciousness, especially what emotional and irritational states of feeling, in the course of forming a new belief. The man has some belief at the outset. This belief is, as to its principal constituent, a habit of expectation. Some experience which this habit leads him to expect turns out differently; and the emotion of *surprise* suddenly appears. Under the influence of *fatigue* (is this right?) this emotion passes into an irritational feeling, which, for want of a better name, I may call *curiosity*. I should define it as a feeling causing a reaction which is directed toward the invention of some *possible* account, or *possible* information, that might take away the astonishing and fragmentary character of the experience by rounding it out. (Of course, we want later to get a *real* explanation; but at first it seems to me that we merely say, "What *can* it be?") When such possible explanation is suggested, the idea of it instantly sets up a second peculiar emotion of "Gad! I shouldn't wonder!" Fatigue (?) again transforms this into a second irritational feeling which might perhaps be called *suspicion*. I should define it as a feeling causing a reaction directed toward unearthing the fault by which the original belief that encountered the surprise became erroneous in the respect in which it is now suspected to be erroneous. When this weak point in the process is discovered, it at once and suddenly causes an emotion of "Bah!" Fatigue (?) transforms this into the irritational feeling called *doubt*, i.e. a feeling producing a reaction tending to the establishment of a new habit of expectation. This object attained, there is a new sudden emotion of "Eureka" passing on fatigue into a desire to find an occasion to try it.

Peirce: CP 8.271 Cross-Ref:††

271. I had got to that point, when the expressman came in bringing me the copy of your new book.†25 I have spent five minutes turning over the leaves. I can see what the general feature of your position is, sufficiently to say that I am heartily in accord with you.

Peirce: CP 8.272 Cross-Ref:††

272. I say to people, -- imaginary interlocutors, for I have nobody to talk to, -- you think that the proposition that truth and justice are the greatest powers in this world, is metaphorical. Well, I, for my part, hold it to be *true*. No doubt Truth has to have defenders to uphold it. But truth creates its defenders and gives them strength. The mode in which the idea of truth influences the world is essentially the same as that in which my desire to have the fire poked causes me to get up and poke it. There

is efficient causation and there is final, or ideal, causation. If either of them is to be set down as a metaphor, it is rather the former. Pragmatism is correct doctrine only in so far as it is recognized that material action is the mere husk of ideas. The brute element exists and must not be explained away as Hegel seeks to do. But the end of thought is action only in so far as the end of action is another thought. Far better abandon the word thought and talk of representation and then *define* what kind of a representation it is that constitutes consciousness.

Peirce: CP 8.273 Cross-Ref:††

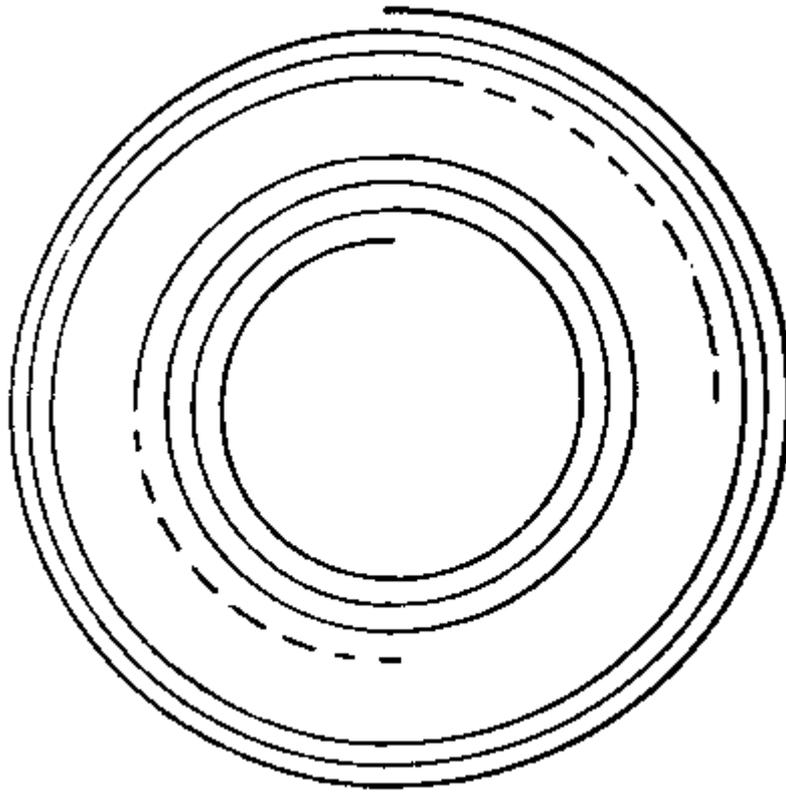
273. But I want to tell you that you should study the new ideas about multitude and continuity (I alone as yet understand continuity, and have published nothing since I mastered it). Ah, my logic will give a tremendous boost to spiritual views! I hope it will get finished, although *personally* it makes mighty little odds to me.

Peirce: CP 8.274 Cross-Ref:††

274. Consider the plane spiral curve whose equation in polar co-ordinates is
†26

$$(r^2-4r+3)/r-2 = C\Theta$$

That curve will start at $r = 1$ and coil outwards toward $r = 2$ making an endless series of revolutions before it reaches $r = 2$. Then it will keep right on and perform an endless series of revolutions before r becomes $2 + e$, no matter how small a distance e may be. Finally, when r becomes 3 the curve will come to an abrupt stop. This shows that although it be true that Being immediately acts only on Being and Representation immediately acts only on Representation, still there may be two endless series, whereby Being and Representation act on one another without any *tertium quid*.



view] [\[Click here to](#)

Peirce: CP 8.275 Cross-Ref:††

275. If atoms are vortices in an ether, which ether is composed of atoms themselves, vortices in another ether, and so on *ad infinitum*, as may possibly be the case (and we care only for possibilities, since we are only refuting a supposed necessity), then it is very likely that the sound waves of my voice should be converted into heat, and this heat into the *ether's* heat, and so on, and that the whole infinite series should be traversed in a fraction of a second, after which they will be in the form of *thoughts* in your mind and so you will come to understand the meaning of those sounds. My logic will open up a world for investigation and show how to set about it.

Peirce: CP 8.276 Cross-Ref:††

276. With your notions of spiritual influence, why don't you join the Church? Surely you won't allow metaphysical formulae, dead as the dust of the catacombs, to deprive you of your RIGHT to the influences of Church.

Peirce: CP 8.277 Cross-Ref:††

277. I have been studying Royce's book.†27 The ideas are very beautiful. The logic is most execrable. I don't think it very good taste to stuff it so full of the name of God. The Absolute is strictly speaking only God, in a Pickwickian sense, that is, in a sense that has no effect.

Peirce: CP 8.278 Cross-Ref:††

278. Forgive the garrulity that comes of my eremitical life and God bless you!

Peirce: CP 8.279 Cross-Ref:††

279. [September 28, 1904] Your article about consciousness †28 comes to me very *a propos* as I am writing about consciousness and have been reading up about it as well as my library (!) permits. But your paper floors me at the very opening and I wish you would do me the favor (I suppose it to be a simple matter) of explaining what you mean by saying that consciousness is often regarded as an "entity." I do not think you capable of setting up a man of straw and have no doubt you can tell me just how any given writer regards consciousness as an "entity." But this word, in modern philosophy, has never conveyed to my mind any idea except that it is a sign the writer is setting up some man of straw whom he imagines to entertain opinions too absurd for definite statement. Now I do not think anybody has any such opinions.

Peirce: CP 8.279 Cross-Ref:††

Therefore much as I am concerned to read your article, it is barred to me until I can find out what it is that you are opposing.

Peirce: CP 8.280 Cross-Ref:††

280. It appears to me that many writers think, or have influential vestiges of having formerly thought, that we have before us at each moment something far more detailed and determinate than any picture, and also think that all we are in any sense aware of is somehow in that image.

Peirce: CP 8.281 Cross-Ref:††

281. For my part, †29 I think first that feelings, -- say red, -- are present when they are present in a peculiar positive self-contained way; so that although contrast makes us attend to them, contrast in no way constitutes their peculiarities; and feelings are thus present to us wholly, -- wholly within consciousness in such a way that one might if one chose very well limit the meaning of consciousness to the feeling of an instant, -- though then it would be something that we only know by analysis.

Peirce: CP 8.282 Cross-Ref:††

282. In the next place there is a two-sided consciousness, in which we separate the element under control from the element we cannot help, -- *although in this mode of consciousness* there is no inseparable reflection that this is done. We separate the past and the present. The past is the inner world, the present the outer world. Now, this joined with feeling (which it involves or requires) might be called consciousness and would be the world, were it not for the phenomena of *error* and *ignorance*, which force us to reflect that there were two worlds in that two-sided consciousness. This consciousness furnishes all our facts. It is this that makes them facts.

Peirce: CP 8.283 Cross-Ref:††

283. Then we have in our minds as the main body of its contents, what never can be in consciousness in either of these senses and never can be in existence or be distinctly supposed to exist. This is the whole world of triadic relations, thought. We are aware of it, and thus it might be included in consciousness.

Peirce: CP 8.284 Cross-Ref:††

284. Second, I think there are writers who limit consciousness to what we know of the *past* which they mistake for the *present* and who thus think it to be a

question whether we are to say the external world alone is *real* and the internal world *fiction* or whether we shall say that the internal world is the real and the external world a fiction. While the true idealism, the pragmatistic idealism, is that reality consists in the *future*. By mellonization (Gr. {mellön} the being about to do, to be, or to suffer) I mean that operation of logic by which what is conceived as having been (which I call conceived as *parelelythose*) is conceived as repeated or extended indefinitely into what always will be (or what will some day be, that is, its absence will *not* always be, which equally involves mellonization, which does not *assert* anything but is merely a mode of *conceiving*). †30 The conception of the *real* is derived by a mellonization of the constraint-side of double-sided consciousness. Therefore to say that it is the world of thought that is real is, when properly understood, to assert emphatically the reality of the public world of the indefinite future as against our past opinions of what it was to be.

Peirce: CP 8.285 Cross-Ref: ††

285. This long and vaporous letter is all intended to ask the simple question what you mean when you say that some people regard consciousness as an *entity*. †31

Peirce: CP 8.286 Cross-Ref: ††

286. [October 3, 1904] Now I will speak first of my meaning of which you "don't understand a word" and then of what you say.

Peirce: CP 8.287 Cross-Ref: ††

287. It is very vexatious to be told at every turn that I am utterly incomprehensible, notwithstanding my careful study of language. When I say it is vexatious, I don't mean that I don't wish to be told so. On the contrary, I am aware that my modes of thought and of expression are peculiar and gauche, and that twenty years of a recluse life have made them more so, and am grateful to people who help me by correcting me. But when, as in the present case, I am able to show that the accusation is a mere auto-suggestion due to your having told yourself that everything that Peirce says is unintelligible, and really having commanded yourself not to understand, it gives me a certain glee to feel authorized to yield to my natural vexation. You will be gratified, with your truly kind nature, to have afforded me so much innocent pleasure. Questions being usually answered at cross-purposes, in asking you what you meant by saying you did not believe that consciousness was an entity, I set down, -- I mean I "sot" down, -- one of the commonplaces of psychology, not that I thought you could have lost sight of it, but that I thought it very likely you might think that I had done so. It was that "consciousness," -- the word, -- is used by different psychologists in three senses, and that connected therewith are three doctrines about the thing or things. Well, you "don't understand a word" of it. Listen, then, and see how the same things precisely, only less explicitly stated, will sound when scratched from other pens.

Peirce: CP 8.288 Cross-Ref: ††

288. 1st. *Consciousness means feeling.*

Peirce: CP 8.288 Cross-Ref: ††

"Whenever there is any kind of feeling, there consciousness exists It is needless to point out that, from the very nature of an infinite series, it cannot be a present modification of consciousness. We may, I think, confidently assert †32 that the object of thought is never a content of our finite consciousness." ([George F.]

Stout, *Analytic Psychology*, I, [1896], pp. 1 and 45.)

Peirce: CP 8.288 Cross-Ref:††

"To feel an idea and to be conscious of that feeling are not two things; the feeling and the consciousness are but two names for the same thing." (James Mill, *Analysis*, New Ed., I, 225.)†33

Peirce: CP 8.288 Cross-Ref:††

"Perhaps as good a definition as can be given of consciousness would be: a knowledge of a feeling." (Lester F. Ward, *The Psychic Factors of Civilization*, [1893], p. 298.)

Peirce: CP 8.289 Cross-Ref:††

289. These people are all sensationalists. They analyze psychic phenomena into their smallest portions, just as a physicist does physical phenomena, -- and just as the latter, if you ask him what composes the physical universe, will say "matter and motion," so these sensationalists find nothing present to the mind but feelings. If you say to the physicist "What! Nothing but matter and motion? Are there no signs?" he will reply, "Undoubtedly, but they are psychical phenomena." Just so if you say to a sensationalist "What! Nothing but feeling? Are there no signs?" he will say, "Undoubtedly, but they are cerebral connections, -- purely physical phenomena."

Peirce: CP 8.290 Cross-Ref:††

290. *2nd. Consciousness is a dual affair* (therefore *not* feeling which has no duality) *and just how makes little odds. Some say because all is relative in consciousness: some say because of the distinction of subject and object that is always present.*

Peirce: CP 8.290 Cross-Ref:††

"Consciousness is the widest word in our vocabulary. By common consent it embraces everything that 'mind' embraces. . . . We speak of the object-consciousness as our attitude in being cognisant of the extended universe; while our attitude under feeling and thought we call subject-consciousness or mind." (Bain, Note to Mill's *Analysis*, I, 226.)†34

Peirce: CP 8.290 Cross-Ref:††

"Consciousness without contrast is impossible." ([William DeWitt] Hyde, *Practical Idealism*, p. 18.)

Peirce: CP 8.290 Cross-Ref:††

Hamilton's hammering on the *ego* and *non-ego*, we'll take for granted.

Peirce: CP 8.290 Cross-Ref:††

"Are the sensationalists right in asserting that sensational and affective elements of consciousness are the only ones? The writer [of this book] is convinced that the introspective analysis of sensationalists is inexact What James calls the feelings of 'and' and of 'but' -- that is, the consciousness of connection and of opposition," etc. (Mary W. Calkins, *An Introduction to Psychology*, [1901], pp. 130, 131.)†35

Peirce: CP 8.291 Cross-Ref:††

291. 3rd. *More or less explicitly, some writers, namely the Thomists, the Hegelians, and other Intellectualists, together with some scientific thinkers not too much sophisticated by reading philosophy, recognize with me (until I shall have*

studied your views, which I don't believe will carry me entirely away from this anchorage) three modes of consciousness, that of feeling, that of EXPERIENCE (experience meaning precisely that which the history of my life has FORCED me to think; so that the idea of a struggle, of not mere twoness but active oppugnancy is in it), and thirdly the consciousness of the future (whether veridical or not is aside from the question) in expectation, which enters into all general ideas according to my variety of pragmatism. †36

Peirce: CP 8.291 Cross-Ref: ††

"In its widest sense, consciousness, as opposed to unconsciousness, denotes all modes of mental life. It comprises all cognitive, emotional, and appetitive states which are capable of being apprehended; it is, in fact, synonymous with the sum-total of our psychical existence. In its second sense, it signifies the mind's direct, intuitive, or immediate knowledge either of its own operations, or of something other than itself acting upon it In its third meaning the word is limited," etc. etc. ([Michael] Maher, *Psychology, Empirical and Rational*, 4th Ed., [1900], p. 26.)

Peirce: CP 8.291 Cross-Ref: ††

"In truth the field of science is much more consciousness than an external world." (Karl Pearson, *The Grammar of Science*, 1st Ed., [1892], p. 63; 2nd Ed., [1900], p. 52.)

Peirce: CP 8.291 Cross-Ref: ††

"If one listens to any simple rhythm, such as the ticking of a watch, one can note how the succession of separate ticks is viewed by our consciousness in such a way that the successive beats do not stand as merely separate facts, but are always elements in the whole experienced rhythm to which they seem to belong, while the successive presentations of the rhythm form a sort of stream of events, each one of which gradually dies out of mind as the new event occurs. †37 In consciousness there is no such thing as an indivisible present moment." ([Josiah] Royce, *Outlines of Psychology*, [1903], p. 83.)

Peirce: CP 8.292 Cross-Ref: ††

292. I submit that this is all as clear as your New Hampshire air of October, and as familiar "comme votre poche," and that I said it plainly enough; only it had to be unintelligible because I said it.

Peirce: CP 8.293 Cross-Ref: ††

293. Now as to what you say, of course it is perfectly clear (or seems to be) NOW. But see how far from clear it is in your article. You note that the sentence "I deny that the word stands for an entity" was put forward *as an explanation*; -- as clearly explaining how your view differs from the usual one, so that it implies that most persons, or most students of philosophy are in the habit of saying "Consciousness is an entity." So you still say, since you say that *you* don't know what "entity" means, and presumably don't believe it means anything clearly; so that it must be the other fellows' statement. I take your word for it, as your reading is immense, that they do say so. Yet I declare I do not remember to have met the expression, and am very much surprised to learn that "entity" can be commonly used to mean "a constituent principle of all experience." My difficulty lay in this terrible word "entity," and I cannot help thinking that it was not a perspicuous way of expressing the opinion you combat

Peirce: CP 8.294 Cross-Ref: ††

294. I shall probably have very different notions after having studied your articles from those I entertain now. Hitherto I have been with the other fellows as to all three kinds of consciousness. I don't believe much that we shouldn't hear the music of spheres if there were any. I have been inclined to think that there is a certain tinge or tone of feeling connected with living and being awake, though we cannot *attend* to it, for want of a background. That would answer your description of a "constituent principle of all experience," would it not? I ask to make sure of my understanding of the opinion. In the second place, *experience*, from the very essence of the word, consists of our belief about a universe, -- "the truth," -- over against our opinions and beliefs, which are thought of as fallible and ignorant. Consequently, I do not see how you can possibly deny that consciousness in the second sense is a "constituent principle of experience." In the third place we carry about a sort of a bass counterpoint melody of beliefs in all our living. Now as I understand pragmatism it is of the very essence of it that belief is expectation of the future in all cases. Consequently it seems to me that the third kind of consciousness is also a "constituent principle of all" our life, and *a fortiori* of all experience.

Peirce: CP 8.295 Cross-Ref:††

295. As I understand you, then, the proposition which you are arguing is a proposition in what I have called *phenomenology*, that is, just the analysis of what kind of constituents there are in our thoughts and lives, (whether these be valid or invalid being quite aside from the question). It is a branch of philosophy I am most deeply interested in and which I have worked upon almost as much as I have upon logic.†38 It has nothing to do with psychology.

Peirce: CP 8.296 Cross-Ref:††

296. Your mind and mine are as little adapted to understanding one another as two minds could be, and therefore I always feel that I have more to learn from you than from anybody. At the same time, it gives great weight in my mind to our numerous agreements of opinion.

Peirce: CP 8.297 Cross-Ref:††

297. Perhaps the most important aspect of the series of papers of which the one you send me is the first, will prove to be that it shows so clearly that phenomenology is one science and psychology a very different one. I know that you are not inclined to see much value in distinguishing between one science and another. But my opinion is that it is absolutely necessary to any progress. The standards of certainty must be different in different sciences, the principles to which one science appeals altogether different from those of the other. From the point of view of logic and methodical development the distinctions are of the greatest concern. Phenomenology has no right to appeal to logic, except to deductive logic. On the contrary, logic must be founded on phenomenology. Psychology, you may say, observes the same facts as phenomenology does. No. It does not *observe* the same facts. It looks upon the same world; -- the same world that the astronomer looks at. But what it *observes* in that world is different. Psychology of all sciences stands most in need of the discoveries of the logician, which he makes by the aid of the phenomenologist.

Peirce: CP 8.298 Cross-Ref:††

298. I am not sure that it will do to call this science *phenomenology* owing to Hegel's *Phänomenologie* being somewhat different. But I am not sure that Hegel ought not to have it named after his attempt.

Peirce: CP 8.299 Cross-Ref:††

299. At the top of p. 483 you speak of various worlds. But the number is not so great. F.E. Abbot, one of the strongest thinkers I ever encountered, first showed me that there were just three; the outer, the inner, and the logical world. The others are not distinct worlds.

Peirce: CP 8.300 Cross-Ref:††

300. On the same page, a little below, you mention as a difficulty people will have in understanding this doctrine, that they are so accustomed to think of percepts as the only realities. "To think of realities as similar to percepts," I should amend this. That is the chief reason why people do not understand me, or I them without a special effort. For I am thoroughly accustomed to think of percepts or rather of perceptual judgments as the data of all knowledge, and as such having a certain imperfect reality. They *exist*, -- the percepts themselves do. But developed reality only belongs to signs of a certain description. Percepts are signs for psychology; but they are not so for phenomenology.

Peirce: CP 8.301 Cross-Ref:††

301. What you call "pure experience" is not experience at all and certainly ought to have a name. It is downright bad morals so to misuse words, for it prevents philosophy from becoming a science. One of the things I urge in my forthcoming *Monist* paper †39 is that it is an indispensable requisite of science that it should have a recognized technical vocabulary composed of words so unattractive that loose thinkers are not tempted to use them, and a recognized and legitimated way of making up new words freely when a new conception is introduced, and that it is vital for science that he who introduces a new conception should be held to have a *duty* imposed upon him to invent a sufficiently disagreeable series of words to express it. I wish you would reflect seriously upon the moral aspect of terminology. My "phenomenon" for which I must invent a new word is very near your "pure experience" but not quite since I do not exclude time and also speak of only *one* "phenomenon."

Peirce: CP 8.302 Cross-Ref:††

302. [December 17, 1909]†40 I was and had been long working as hard as I could upon my "System of Logic, from the point of view of Semiotic," when Juliette was enabled to start up the repairs which may enable me to finish that book, by keeping us alive. Then Carus having written me already offering me \$200 or \$250 (I forget which) for the copyright of my 6 articles in the *Popular Science Monthly*†41 for 1877 and '78, I agreed for \$250 to allow him to print *one edition*, with a revision that I would furnish together with an Article for the *Monist* on my method of performing logical analyses. Owing to a lot of interruptions, . . . I either forgot, or never comprehended that Carus particularly cared to have that Article a *separate* one, and I had been working on it as a chapter of my first essay,†42 consisting of the two articles with which I began in the *Popular Science Monthly* called, -- "The Settlement of Opinion" and "How to make our Ideas clear." Of course Definition, which is the end of Logical Analysis, is the first step, (after general familiarity in use,) toward making Ideas clear. However, he has lately written, remonstrating on my delay, and in consequence I am going immediately to write that article, which I think will be a really helpful one to many people.

Peirce: CP 8.303 Cross-Ref:††

303. I mean to begin by drawing a distinction between what I call "Psychology Proper," meaning an account of how the mind functions, develops, and decays, together with the explanation of all this by motions and changes of the brain, or, in default of this kind of explanation, by generalizations of psychical phenomena, so as to account for all the workings of the soul in the sense of reducing them to combinations of a few typical workings, -- in short a sort of physiology of the mind, on the one hand, -- and what I call "Phaneroscopy" on the other, or a description of what is before the mind or in consciousness, as it *appears*, in the different kinds of consciousness,†43 which I rank under . . . three headings . . . *First*, "Qualisense," which means that element of Feeling which consists in consciousness of the Quality of the Feeling, but omitting the element of Vividness, which does not alter the Quality (thus a faint memory of a highly luminous, and chromatic vermilion does not appear less luminous or less high colored, for all its dimness) and omitting all other concomitants of present feeling that are absent from a correct recollection of the same Quality. *Second Heading*: what I call *Molition*, which is volition minus all desire and purpose, the mere consciousness of *exertion* of any kind. *Third Heading*: the recognition of Habit of any kind in consciousness.

Peirce: CP 8.304 Cross-Ref:††

304. Then as preliminary to an argument tending to show that there are conscious elements that do not come under one of these three heads, I shall go on to show reason for thinking, first, that these three kinds of consciousness are entirely unlike; second, that they are connected with the ideas of *one*, *two*, *three* which are the three elementary forms with which logical analysis has to deal. *One* being the form of a simple idea, *two* that of an ordinary relative idea, and *three* the only *simple* form of combination of a direct union of more than two ideas, but being incapable itself of reduction to a pair of pairs but involving the idea expressed by "and" which always unites a triad or higher collection. Qualisense is the sort of consciousness of any whole regardless of anything else, and therefore regardless of the parts of that whole. Molition is a double consciousness of exertion and resistance. Consciousness of habit is a consciousness at once of the substance of the habit, the special case of application, and the union of the two.

Peirce: CP 8.305 Cross-Ref:††

305. Now third, I argue that there seem to be no other modes of consciousness by taking up some of the most difficult and analyzing them, which will at the same time illustrate my method of analysis. I shall show that a Concept is a Sign and shall define a Sign and show its triadic form. I shall define the Modality of a Sign and show that in this respect every Object is either a Can-be, an Actual, or a Would-be.†44 I shall show (as generally recognized,) that an Actual cannot be defined and that the Can-be's and Would-be's when accurately discriminated are only definable in different senses. There is no use of going through these headings, however, because they are unintelligible until they are defined at length. I don't pretend that my argument that there are only three kinds of consciousness does more than raise a presumption by the precision with which I succeed in defining a great variety of terms without calling in any fourth element. It will remain for those who question the conclusion to find a term I cannot define with this apparatus. After all this I shall undertake to show (still somewhat imperfectly) that concepts are capable of such phaneroscopic analysis, or in common parlance "logical analysis"; but there are only a few cases in which I pretend as yet to carry the analysis so far as to resolve the

concept into its ultimate *elements*. After a few more such questions have been discussed, I show how to go to work to perform the analysis; and then I proceed to show that a definition constructed according to my method at once clears up various puzzles relating to the concept.

Peirce: CP 8.306 Cross-Ref:††
§4. FREE WILL

306. [March 18, 1897] I have been much struck with the *Dilemma of Determinism*.†45 I do not mean that there was any new thought to me in it, because this matter has been the subject of a very serious analysis on my part, a work much more elaborate than anybody would suppose from anything I have printed. But I was surprised to see how far you had penetrated into the logical analysis so long ago as 1884.

Peirce: CP 8.307 Cross-Ref:††

307. Two points particularly struck me. One was your resolving the matter altogether into a question of plurality, which is another name for my "variety" of nature. About that I need say no more, because I have developed the idea in print.

Peirce: CP 8.308 Cross-Ref:††

308. The other was your remark that the question is, is possibility a mode of being. Good. Precisely so. As I remarked in the last *Monist*,†46 my old definition of the possible as that which we do not know not to be true (in some state of information real or feigned) is an anacoluthon. The possible is a positive universe, and the two negations happen to fit it, but that is all. Of course, there *is* a general logical possible that is no more than I defined it. But there is also a possible which [is] something else. I reached this truth by studying the question of possible grades of multitude, where I found myself arrested until I could form a whole logic of possibility, -- a very difficult and laborious task. You would not have reached it that way. You must have some short cut, which I am curious to know more about.

Peirce: CP 8.309 Cross-Ref:††

309. Very well. You have said the whole question of determinism is the question of ultimate plurality; and you have also said the same question is the question of positive possibility. But you have not said anything to show that you perceive how these two statements agree.

Peirce: CP 8.310 Cross-Ref:††

310. I have never read the *Logique* of Charles Renouvier. Do you possess a copy of it; and if so could you spare it long enough for me to read it?

Peirce: CP 8.311 Cross-Ref:††

311. There are some things in your *Dilemma of Determinism* that I cannot assent to. I cannot admit the *will* is free in any appreciable measure, for reasons that may be found in my *Man's Glassy Essence*.†47 Namely, chance can only amount to much in a state of things closely approximating to unstable equilibrium. Now in the act of willing there is no such state of things. The freedom lies in the *choice* which long antecedes the will. *There* a state of nearly unstable equilibrium is found. But this makes a great difference in your doctrine.

Peirce: CP 8.312 Cross-Ref:††

312. As for the note about God being out of time, it seems to me probable that it was hastily penned. For it appears plain that (as has been often said, -- by Kant for instance) if God is out of time the difficulty is removed. And in my opinion the scholastics were right in holding that, putting God into time, there is no contradiction between Foreknowledge and Free Will.

Peirce: CP 8.312 Cross-Ref:††

I forget what your father said about it, but I remember being much struck with it. Your father saw a long way in a certain direction.

Peirce: CP 8.313 Cross-Ref:††

§5. SIGNS

313. [January 22, 1905]†48 Now an assertion belongs to the class of phenomena like going before a notary and making an affidavit, executing a deed, signing a note, of which the essence is that one voluntarily puts oneself into a situation in which penalties will be incurred unless some proposition is true. One may maintain that every proposition involves an assertion. Very likely that may be true as a psychological truth; but if so the element of assertion is frequently altogether or in great degree inhibited and disavowed. I have nothing further to say about assertion. What I want to come to is the question what a proposition consists in, when the element of assertion is, as far as possible, removed from it. It is of course a kind of sign. . . . What kind of a sign is a proposition? A symptom is not a proposition although it justifies a proposition. The reason is that it lacks what is most essential to propositions and to various other kinds of signs, that of *professing* something, making a *pretension*, representing itself to be such and such. It professes to be a certain kind of sign, namely that kind that is a sign by virtue of being *really* connected with its object, which means that it has that kind of relation to its object which subsists in those two correlates regardless of all else. So, then, according to me, reality is a conception that every man has because it is involved in every proposition; and since every man makes assertions he deals with propositions. (Of course, I have not fully defined a proposition, because I have not discriminated the proposition from the individual sign which is the embodiment of the proposition. By a proposition, as something which can be repeated over and over again, translated into another language, embodied in a logical graph or algebraical formula, and still be one and the same proposition, we do not mean any existing individual object but a type, a general, which does not exist but governs existents, to which individuals conform.)

Peirce: CP 8.314 Cross-Ref:††

314. [March 14, 1909] We must distinguish between the Immediate Object, -- i.e. the Object as represented in the sign, -- and the Real (no, because perhaps the Object is altogether fictive, I must choose a different term, therefore), say rather the Dynamical Object, which, from the nature of things, the Sign *cannot* express, which it can only *indicate* and leave the interpreter to find out by *collateral experience*. For instance, I point my finger to what I mean, but I can't make my companion know what I mean, if he can't see it, or if seeing it, it does not, to his mind, separate itself from

the surrounding objects in the field of vision. It is useless to attempt to discuss the genuineness and possession of a personality beneath the histrionic presentation of Theodore Roosevelt with a person who recently has come from Mars and never heard of Theodore before. A similar distinction must be made as to the Interpretant. But in respect to *that* Interpretant, the dichotomy is not enough by any means. For instance, suppose I awake in the morning before my wife, and that afterwards she wakes up and inquires, "What sort of a day is it?" *This* is a sign, whose Object, as expressed, is the weather at that time, but whose Dynamical Object is the *impression which I have presumably derived from peeping between the window-curtains*. Whose Interpretant, as expressed, is the quality of the weather, but whose Dynamical Interpretant, is *my answering her question*. But beyond that, there is a third Interpretant. The *Immediate Interpretant* is what the Question expresses, *all* that it immediately expresses, which I have imperfectly restated above. The *Dynamical Interpretant* is the actual effect that it has upon me, its interpreter. But the Significance of it, the *Ultimate, or Final, Interpretant* is her *purpose* in asking it, what effect its answer will have as to her plans for the ensuing day. I reply, let us suppose: "It is a stormy day." Here is another sign. Its *Immediate Object* is the notion of the present weather so far as this is common to her mind and mine -- not the *character* of it, but the *identity* of it. The *Dynamical Object* is the *identity* of the actual or Real meteorological conditions at the moment. The *Immediate Interpretant* is the *schema* in her imagination, i.e. the vague Image or what there is in common to the different Images of a stormy day. The *Dynamical Interpretant* is the disappointment or whatever actual effect it at once has upon her. The *Final Interpretant* is the sum of the *Lessons* of the reply, Moral, Scientific, etc. Now it is easy to see that my attempt to draw this three-way, "trivialis" distinction, relates to a real and important three-way distinction, and yet that it is quite hazy and needs a vast deal of study before it is rendered perfect. Lady Welby has got hold of the same real distinction in her "Sense, Meaning, Significance," but conceives it as imperfectly as I do, but imperfectly in other ways. Her *Sense* is the *Impression* made or normally to be made. Her *meaning* is what is intended, its purpose. Her Significance is the real upshot. †49

Peirce: CP 8.315 Cross-Ref: ††

315. [April 1, 1909] . . . let me give a *little* fuller explanation of my distinction between the Immediate, the Dynamical, and the Final Interpretants. . . The Dynamical Interpretant is whatever interpretation any mind actually makes of a sign. This Interpretant derives its character from the Dyadic category, the category of Action. This has two aspects, the Active and the Passive, which are not merely opposite aspects but make relative contrasts between different influences of this Category as More Active and More Passive. In psychology this category marks Molition †50 in its active aspect of a force and its passive aspect as a resistance. When an imagination, a day-dream fires a young man's ambition or any other active passion, that is a more Active variety of his Dynamical Interpretation of the dream. When a novelty excites his surprise, -- and the scepticism that goes along with surprise, -- this is a more Passive variety of Dynamical Interpretant. I am not speaking of the *feelings* of passion or of surprise as *qualities*. For those *qualities* are no part of the dynamic Interpretant. But the *agitations* of passion and of surprise are the actual dynamic Interpretants. So surprise again has its Active and its Passive variety; -- the former when what one perceives positively *conflicts* with expectation, the latter when having no positive expectation but only the absence of any suspicion of anything out of the common something quite unexpected occurs, -- such as a total eclipse of the sun

which one had not anticipated. Any surprise involves a resistance to accepting the fact. One rubs one's eyes, as Shaler used to do, determined not to admit the observation until it is plain one will be compelled to do so. Thus every actual interpretation is dyadic [As] pragmatism says . . . (one part of pragmatism, for Pragmatism is not exclusively an opinion about the Dynamic Interpretant), . . . it says, for one thing, that the meaning of any sign for anybody consists in the way he reacts to the sign. When the captain of infantry gives the word "Ground arms!" the dynamic Interpretant is in the thump of the muskets on the ground, or rather it is the Act of their Minds. In its Active/Passive forms, the Dynamical Interpretant indefinitely approaches the character of the Final/Immediate Interpretant; and yet the distinction is absolute. The Final Interpretant does not consist in the way in which any mind does act but in the way in which every mind would act. That is, it consists in a truth which might be expressed in a conditional proposition of this type: "If so and so were to happen to any mind this sign would determine that mind to such and such *conduct*." By "conduct" I mean *action* under an intention of self-control. No event that occurs to any mind, no action of any mind can constitute the truth of that conditional proposition. The Immediate Interpretant consists in the *Quality* of the Impression that a sign is fit to produce, not to any actual reaction. Thus the Immediate and Final Interpretants seem to me absolutely distinct from the Dynamical Interpretant and from each other. And if there be any fourth kind of Interpretant on the same footing as those three, there must be a dreadful rupture of my mental retina, for I can't see it at all.

Peirce: CP 8.316 Cross-Ref:††
CHAPTER 6

TO CHRISTINE LADD-FRANKLIN, ON COSMOLOGY†1

316. My work in philosophy has consisted in an accurate analysis of concepts, showing what is and what is not essential to the subject of analysis. Particularly, in logic, my motive for studying the algebra of the subject, has been the desire to find out with accuracy what are the essential ingredients of reasoning in general and of its principal kinds. To make a powerful calculus has not been my care.

Peirce: CP 8.317 Cross-Ref:††

317. I may mention that my chief avocation in the last ten years has been to develop my cosmology.†2 This theory is that the evolution of the world is *hyperbolic*, that is, proceeds from one state of things in the infinite past, to a different state of things in the infinite future. The state of things in the infinite past is chaos, *tohu bohu*, the nothingness of which consists in the total absence of regularity. The state of things in the infinite future is death, the nothingness of which consists in the complete triumph of law and absence of all spontaneity.†3 Between these, we have on *our* side a state of things in which there is some absolute spontaneity counter to all law, and some degree of conformity to law, which is constantly on the increase owing to the growth of *habit*. The tendency to form habits or tendency to generalize, is something which grows by its own action, by the habit of taking habits itself growing. Its first

germs arose from pure chance. There were slight tendencies to obey rules that had been followed, and these tendencies were rules which were more and more obeyed by their own action. There were also slight tendencies to do otherwise than previously, and these destroyed themselves. To be sure, they would sometimes be strengthened by the opposite tendency, but the stronger they became the more they would tend to destroy themselves. As to the part of time on the further side of eternity which leads back from the infinite future to the infinite past, it evidently proceeds by contraries.

Peirce: CP 8.318 Cross-Ref:††

318. I believe the law of habit to be purely psychical. But then I suppose matter is merely mind deadened by the development of habit. While every physical process can be reversed without violation of the law of mechanics, the law of habit forbids such reversal. Accordingly, time may have been evolved by the action of habit. At first sight, it seems absurd or mysterious to speak of time being evolved, for evolution presupposes time. But after all, this is no serious objection, and nothing can be simpler. Time consists in a regularity in the relations of interacting feelings. The first chaos consisted in an infinite multitude of unrelated feelings. As there was no continuity about them, it was, as it were, a powder of feelings. It was worse than that, for of particles of powder some are nearer together, others farther apart, while these feelings had no relations, for relations are general. Now you must not ask me what happened first. This would be as absurd as to ask what is the smallest finite number. But springing away from the infinitely distant past to a very very distant past, we find already evolution had been going on for an infinitely long time. But this "time" is only our way of saying that something had been going on. There was no real time so far as there was no regularity, but there is no more falsity in using the language of time than in saying that a quantity is zero. In this chaos of feelings, bits of similitude had appeared, been swallowed up again. Had reappeared by chance. A slight tendency to generalization had here and there lighted up and been quenched. Had reappeared, had strengthened itself. Like had begun to produce like. Then even pairs of unlike feelings had begun to have similars, and then these had begun to generalize. And thus relations of contiguity, that is connections other than similarities, had sprung up. All this went on in ways I cannot now detail till the feelings were so bound together that a passable approximation to a real time was established. It is not to be supposed that the ideally perfect time has even yet been realized. There are no doubt occasional lacunae and derailments.†4

Peirce: CP 8.319 Cross-Ref:††

CHAPTER 7

TO F.C.S. SCHILLER, ON PRAGMATISM†1

319. . . . I think the very first application that should be made of pragmatism of any stripe is to define words. Renouvier (in his 'Essais de Critique Philosophique' or elsewhere) has well said that as regards definitions in philosophy there are two classes of words. The one class consists [of] technical terms which ought to be defined before they are used and which ought to be supplanted by *new terms* and not

by new definitions of old terms; the other class consists of the words of the vernacular, representing the vague ideas of common sense, which it is a part of the business of philosophy *to discover* the definitions of. You consider "real" to belong to the latter class. I hold that it ought to be kept in the former class where it was put by Duns Scotus who brought the word into common use, Albertus Magnus having only occasionally employed it, borrowing it from the law phrase 'real property.' As to the plasticity of the real, I am, on one side, entirely with you, having in 1892 and 1893 [argued] . . . that it is presumable that the laws of nature are not absolutely rigid.†2 And whether they be so or not, it is to *my* mind quite certain that there are general signs, -- namely, laws of nature, -- which influence, or determine, actual events, and equally certain there are also other general signs which, having been shaped in human reasoning, further influence, or determine, muscular contractions, and through these, other actual events . . .

Peirce: CP 8.320 Cross-Ref:††

320. I do not know whether or not you will approve of my particular way of denying Necessitarianism. But as it is certain that the proposition that every physical event is directly determined by dynamical non-telic conditions and laws alone while every mental representation is directly determined by logical and, as such, telic conditions and laws alone, does not conflict with the proposition that physical events are determined by mental representations and mental representations by physical events †3 (as every student of G. Cantor will perceive); so on the other hand the propositions that the laws of nature are not absolute and that important physical events are due to human reasoning are far from proving that human action is (in any important degree) free, except in the sense that a man is a machine with automatic controls, one over another, for five or six grades, at least. I, for my part, am very dubious as to man's having more freedom than that, nor do I see what pragmatic meaning there is in saying that he has more. The power of self-control is certainly not a power over what one is doing at the very instant the operation of self-control is commenced. It consists (to mention only the leading constituents) first, in comparing one's past deeds with standards, second, in rational deliberation concerning how one will act in the future, in itself a highly complicated operation, third, in the formation of a resolve, fourth, in the creation, on the basis of the resolve, of a strong determination, or modification of habit. This operation of self-control is a process in which logical sequence is converted into mechanical sequence or something of the sort. How this happens, we are in my opinion as yet entirely ignorant. There is a class of signs in which the logical sequence *is* at the same time a mechanical sequence and very likely this fact enters into the explanation.

Peirce: CP 8.321 Cross-Ref:††

321. Let me thank you very particularly for sending me a copy of your last article †4 which I have read with profit and entertainment, as I do all of your articles that I am so fortunate as to see.

Peirce: CP 8.322 Cross-Ref:††

322. Of course I agree entirely to most of what you say, -- as well as I can understand it without having seen any writing of Taylor. For example, I agree that of the two implications of pragmatism that concepts are purposive, and that their meaning lies in their conceivable practical bearings, the former is the more fundamental. I think, however, that the doctrine would be quite *estropiée* without the latter point. By "practical" I mean apt to affect conduct; and by conduct, voluntary

action that is self-controlled, i.e. controlled by adequate deliberation. But the neater definition you put into a footnote is worth fully all you claim for it.†5

Peirce: CP 8.323 Cross-Ref:††

323. However, it would be idle to write merely to note points of agreement. They are too many. Let me rather note that some of the ends which you mention as going to the meanings of concepts seem to me to form no part of those meanings. What the hundredth decimal figure of π means consists to my mind in just what any other figure means. For it is quite *conceivable* that it should be an important practical quantity. It is one of the beauties of pragmatism that it gives some symbols much more meaning than others, and the hundredth figure of π certainly has precious little. A much better question is, What on pragmatist principles is the difference between a rational and an irrational quantity or what it means to say that the diagonal of a square is incommensurable with its side? It is interpreted in the conduct of the arithmetician as such. . .

Peirce: CP 8.324 Cross-Ref:††

324. As for Cantor's cardinal transfinities, though called numbers by him, they are not properly so called but are *multitudes*, or many-nesses of infinite collections. The first is the multitude of the objects of an endless series of objects. I call it the *denumeral* multitude. The next is the multitude of all collections of objects involved in an endless series (by collection I mean simply a plural). The rest are each the multitude of all collections involved in a collection of the next preceding multitude. I call these the *abnumerable* multitudes. . .

Peirce: CP 8.325 Cross-Ref:††

325. These abnumerable multitudes are describable intelligibly and exactly, but only in general terms. No precise idea can be formed of the simplest of them; and they increase in difficulty at a frightful rate (that is in the characters that *would* make difficulty if it were surmountable at all). If anything violates the principle of pragmatism it is these. But I have no doubt whatever of the validity of the concepts. They are interpretable in the conduct of the logician or logico-mathematician in dealing with them. If they were not exact, so as [to] lay definite logical obligations upon him they would be meaningless, or without definite meaning.

Peirce: CP 8.326 Cross-Ref:††

326. When you say that Logical consequences cannot be separated from psychological effects, etc. in my opinion you are merely adopting a mode of expression highly inconvenient which cannot help, but can only confuse, any sound argumentation. It is a part of nominalism which is utterly antipragmatic, as I think, and mere refusal to make use of valuable forms of thought.

Peirce: CP 8.327 Cross-Ref:††

CHAPTER 8

TO LADY WELBY

§1. ON SIGNS AND THE CATEGORIES †1

327. But I wanted to write to you about signs, which in your opinion and mine are matters of so much concern. More in mine, I think, than in yours. For in mine, the highest grade of reality is only reached by signs; that is by such ideas as those of Truth and Right and the rest. It sounds paradoxical; but when I have devolved to you my whole theory of signs, it will seem less so. I think that I will today explain the outlines of my classification of signs.

Peirce: CP 8.328 Cross-Ref:††

328. You know that I particularly approve of inventing new words for new ideas. I do not know that the study I call *Ideoscopy* can be called a new idea, but the word *phenomenology* is used in a different sense.†2 *Ideoscopy* consists in describing and classifying the ideas that belong to ordinary experience or that naturally arise in connection with ordinary life, without regard to their being valid or invalid or to their psychology. In pursuing this study I was long ago (1867) led, after only three or four years' study, to throw all ideas into the three classes of Firstness, of Secondness, and of Thirdness.†3 This sort of notion is as distasteful to me as to anybody; and for years, I endeavored to pooh-pooh and refute it; but it long ago conquered me completely. Disagreeable as it is to attribute such meaning to numbers, and to a triad above all, it is as true as it is disagreeable. The ideas of Firstness, Secondness, and Thirdness are simple enough. Giving to being the broadest possible sense, to include ideas as well as things, and ideas that we fancy we have just as much as ideas we do have, I should define Firstness, Secondness, and Thirdness thus:

Peirce: CP 8.328 Cross-Ref:††

Firstness is the mode of being of that which is such as it is, positively and without reference to anything else.

Peirce: CP 8.328 Cross-Ref:††

Secondness is the mode of being of that which is such as it is, with respect to a second but regardless of any third.

Peirce: CP 8.328 Cross-Ref:††

Thirdness is the mode of being of that which is such as it is, in bringing a second and third into relation to each other.

Peirce: CP 8.328 Cross-Ref:††

I call these three ideas the cenopythagorean categories.

Peirce: CP 8.329 Cross-Ref:††

329. The typical ideas of firstness are qualities of feeling, or mere appearances. The scarlet of your royal liveries, the quality itself, independently of its being perceived or remembered, is an example, by which I do not mean that you are to imagine that you *do not* perceive or remember it, but that you are to drop out of account that which may be attached to it in perceiving or in remembering, but which does not belong to the quality. For example, when you remember it, your idea is said to be *dim* and when it is before your eyes, it is *vivid*. But dimness or vividness do not belong to your idea of the quality. They *might* no doubt, if considered simply as a feeling; but when you think of vividness you do not consider it from that point of

view. You think of it as a degree of disturbance of your consciousness. The quality of red is not thought of as belonging to you, or as attached to liveries. It is simply a peculiar positive possibility regardless of anything else. If you ask a mineralogist what hardness is, he will say that it is what one predicates of a body that one cannot scratch with a knife. But a simple person will think of hardness as a simple positive possibility the *realization* of which causes a body to be like a flint. That idea of hardness is an idea of Firstness. The unanalyzed total impression made by any manifold not thought of as actual fact, but simply as a quality, as simple positive possibility of appearance, is an idea of Firstness. Notice the *naïveté* of Firstness. The cenopythagorean categories are doubtless another attempt to characterize what Hegel sought to characterize as his three stages of thought. They also correspond to the three categories of each of the four triads of Kant's table. But the fact that these different attempts were independent of one another (the resemblance of these Categories to Hegel's stages was not remarked for many years after the list had been under study, owing to my antipathy to Hegel) only goes to show that there really are three such elements. The idea of the present instant, which, whether it exists or not, is naturally thought as a point of time in which no thought can take place or any detail be separated, is an idea of Firstness.

Peirce: CP 8.330 Cross-Ref:††

330. The type of an idea of Secondness is the experience of effort, prescinded from the idea of a purpose. It may be said that there is no such experience, that a purpose is always in view as long as the effort is cognized. This may be open to doubt; for in sustained effort we soon let the purpose drop out of view. However, I abstain from psychology which has nothing to do with ideoscopy. The existence of the word *effort* is sufficient proof that people think they have such an idea; and that is enough. The experience of effort cannot exist without the experience of resistance. Effort only is effort by virtue of its being opposed; and no third element enters. Note that I speak of the *experience*, not of the *feeling*, of effort. Imagine yourself to be seated alone at night in the basket of a balloon, far above earth, calmly enjoying the absolute calm and stillness. Suddenly the piercing shriek of a steam-whistle breaks upon you, and continues for a good while. The impression of stillness was an idea of Firstness, a quality of feeling. The piercing whistle does not allow you to think or do anything but suffer. So that too is absolutely simple. Another Firstness. But the breaking of the silence by the noise was an experience. The person in his inertness identifies himself with the precedent state of feeling, and the new feeling which comes in spite of him is the non-ego. He has a two-sided consciousness of an ego and a non-ego. That consciousness of the action of a new feeling in destroying the old feeling is what I call an *experience*. Experience generally is what the course of life has *compelled* me to think. Secondness is either *genuine* or *degenerate*. There are many degrees of genuineness. Generally speaking genuine secondness consists in one thing acting upon another, -- brute action. I say brute, because so far as the idea of any *law* or *reason* comes in, Thirdness comes in. When a stone falls to the ground, the law of gravitation does not act to make it fall. The law of gravitation is the judge upon the bench who may pronounce the law till doomsday, but unless the strong arm of the law, the brutal sheriff, gives effect to the law, it amounts to nothing. True, the judge can create a sheriff if need be; but he must have one. The stone's actually falling is purely the affair of the stone and the earth at the time. This is a case of *reaction*. So is *existence* which is the mode of being of that which reacts with other things. But there is also action without reaction. *Such is the action of the previous upon the subsequent.* †4 It is a difficult question whether the idea of this one-sided

determination is a pure idea of secondness or whether it involves thirdness. At present, the former view seems to me correct. I suppose that when Kant made Time a form of the internal sense alone, he was influenced by some such considerations as the following. The relation between the previous and the subsequent consists in the previous being determinate and fixed for the subsequent, and the subsequent being indeterminate for the previous. But indeterminacy belongs only to ideas; the existent is determinate in every respect; and this is just what the law of causation consists in. Accordingly, the relation of time concerns only ideas. It may also be argued that, according to the law of the conservation of energy, there is nothing in the physical universe corresponding to our idea that the previous determines the subsequent in any way in which the subsequent does not determine the previous. For, according to that law, all that happens in the physical universe consists in the exchange of just so much *vis viva* $\frac{1}{2}m(ds/dt)^2$ for so much displacement. Now the square of a negative quantity being positive, it follows that if all the velocities were reversed at any instant, everything would go on just the same, only time going backward as it were. Everything that had happened would happen again in reversed order. These seem to me to be strong arguments to prove that temporal causation (a very different thing from physical dynamic action) is an action upon ideas and not upon existents.†5 But since our idea of the past is precisely the idea of that which is absolutely determinate, fixed, *fait accompli*, and dead, as against the future which is living, plastic, and determinable, it appears to me that the idea of one-sided action, in so far as it concerns the being of the determinate, is a pure idea of Secondness; and I think that great errors of metaphysics are due to looking at the future as something that will have been past. I cannot admit that the idea of the future can be so translated into the Secundal ideas of the past. To say that a given kind of event never will happen is to deny that there is any date at which its happening will be past; but it is not equivalent to any affirmation about a past relative to any assignable date. When we pass from the idea of an event to saying that it never will happen, or will happen in endless repetition, or introduce in any way the idea of endless repetition, I will say the idea is *mellonized* ({mellön}), about to be, do, or suffer). When I conceive a fact as acting but not capable of being acted upon, I will say that it is *parelelythose* ({parelélythös}, past) and the mode of being which consists in such action I will call *parelelythosine* (-ine = {einai}, being); I regard the former as an idea of Thirdness, the latter as an idea of Secondness. I consider the idea of any dyadic relation not involving any third as an idea of Secondness; and I should not call any completely degenerate except the relation of identity. But similarity which is the only possible identity of Firsts is very near to that. Dyadic relations have been classified by me in a great variety of ways; but the most important are, first, with regard to the nature of the Second in itself and, second, with regard to the nature of its First. The Second, or *Relate*,†6 is, in itself, either a *Referate*, if it is intrinsically a possibility, such as a quality, or it is a *Revelate* if it is of its own nature an Existent. In respect to its First, the Second is divisible either in regard to the dynamic first or to the immediate first. In regard to its dynamic first, a Second is determined either by virtue of its own intrinsic nature, or by virtue of a real relation to that second (an action). Its immediate second is either a Quality or an Existent.

Peirce: CP 8.331 Cross-Ref:††

331. I now come to Thirdness. To me, who have for forty years considered the matter from every point of view that I could discover, the inadequacy of Secondness to cover all that is in our minds is so evident that I scarce know how to begin to persuade any person of it who is not already convinced of it. Yet I see a great many

thinkers who are trying to construct a system without putting any thirdness into it. Among them are some of my best friends who acknowledge themselves indebted to me for ideas but have never learned the principal lesson. Very well. It is highly proper that Secondness should be searched to its very bottom. Thus only can the indispensableness and irreducibility of thirdness be made out, although for him who has the mind to grasp it, it is sufficient to say that no branching of a line can result from putting one line on the end of another.†7 My friend Schröder fell in love with my algebra of dyadic relations. The few pages I gave to it in my Note B in the 'Studies in Logic by Members of the Johns Hopkins University' were proportionate to its importance.†8 His book is profound,†9 but its profundity only makes it more clear that Secondness cannot compass Thirdness. (He is careful to avoid ever saying that it can, but he does go so far as to say that Secondness is the more important. So it is, considering that Thirdness cannot be understood without Secondness. But as to its application, it is so inferior to Thirdness as to be in that aspect quite in a different world.) Even in the most degenerate form of Thirdness, and thirdness has two grades of degeneracy, something may be detected which is not mere secondness. If you take any ordinary triadic relation, you will always find a *mental* element in it. Brute action is secondness, any mentality involves thirdness. Analyze for instance the relation involved in 'A gives B to C.' Now what is giving? It does not consist [in] A's putting B away from him and C's subsequently taking B up. It is not necessary that any material transfer should take place. It consists in A's making C the possessor according to *Law*. There must be some kind of law before there can be any kind of giving, -- be it but the law of the strongest. But now suppose that giving *did* consist merely in A's laying down the B which C subsequently picks up. That would be a degenerate form of Thirdness in which the thirdness is externally appended. In A's putting away B, there is no thirdness. In C's taking B, there is no thirdness. But if you say that these two acts constitute a single operation by virtue of the identity of the B, you transcend the mere brute fact, you introduce a mental element The criticism which I make on [my] algebra of dyadic relations, with which I am by no means in love, though I think it is a pretty thing, is that the very triadic relations which it does not recognize, it does itself employ. For every combination of relatives to make a new relative is a triadic relation irreducible to dyadic relations. Its *inadequacy* is shown in other ways, but in this way it is in a conflict with itself *if it be regarded*, as I never did regard it, *as sufficient for the expression of all relations*. My universal algebra of relations, with the subjacent indices and Σ and π , is susceptible of being enlarged so as to comprise everything; and so, still better, though not to ideal perfection, is the system of *existential graphs*.†10

Peirce: CP 8.332 Cross-Ref:††

332. I have not sufficiently applied myself to the study of the degenerate forms of Thirdness, though I think I see that it has two distinct grades of degeneracy. In its genuine form, Thirdness is the triadic relation existing between a sign, its object, and the interpreting thought, itself a sign, considered as constituting the mode of being of a sign.†11 A sign mediates between the *interpretant* sign and its object. Taking sign in its broadest sense, its interpretant is not necessarily a sign. Any concept is a sign, of course. Ockham, Hobbes, and Leibniz have sufficiently said that. But we may take a sign in so broad a sense that the interpretant of it is not a thought, but an action or experience, or we may even so enlarge the meaning of sign that its interpretant is a mere quality of feeling. A *Third* is something which brings a First into relation to a Second. A sign is a sort of Third. How shall we characterize it? Shall we say that a Sign brings a Second, its Object, into *cognitive* relation to a Third? That

a Sign brings a Second into the same relation to a first in which it stands itself to that First? If we insist on *consciousness*, we must say what we mean by consciousness of an object. Shall we say we mean Feeling? Shall we say we mean association, or Habit? These are, on the face of them, psychological distinctions, which I am particular to avoid. What is the essential difference between a sign that is communicated to a mind, and one that is not so communicated? If the question were simply what we *do* mean by a sign, it might soon be resolved. But that is not the point. We are in the situation of a zoölogist who wants to know what ought to be the meaning of "fish" in order to make fishes one of the great classes of vertebrates. It appears to me that the essential function of a sign is to render inefficient relations efficient, -- not to set them into action, but to establish a habit or general rule whereby they will act on occasion. According to the physical doctrine, nothing ever happens but the continued rectilinear velocities with the accelerations that accompany different relative positions of the particles. All other relations, of which we know so many, are inefficient. Knowledge in some way renders them efficient; and a sign is something by knowing which we know something more. With the exception of knowledge, in the present instant, of the contents of consciousness in that instant (the existence of which knowledge is open to doubt) all our thought and knowledge is by signs. A sign therefore is an object which is in relation to its object on the one hand and to an interpretant on the other, in such a way as to bring the interpretant into a relation to the object, corresponding to its own relation to the object. I might say 'similar to its own' for a correspondence consists in a similarity; but perhaps correspondence is narrower.

Peirce: CP 8.333 Cross-Ref:††

333. I am now prepared to give my division of signs, as soon as I have pointed out that a sign has two objects, its object as it is represented and its object in itself. It has also three interpretants, its interpretant as represented or meant to be understood, its interpretant as it is produced, and its interpretant in itself. Now signs may be divided as to their own material nature, as to their relations to their objects, and as to their relations to their interpretants.

Peirce: CP 8.334 Cross-Ref:††

334. As it is in itself, a sign is either of the nature of an appearance, when I call it a *qualisign*; or secondly, it is an individual object or event, when I call it a *sinsign* (the syllable *sin* being the first syllable of *semel*, *simul*, *singular*, etc.); or thirdly, it is of the nature of a general type, when I call it a *legisign*. As we use the term 'word' in most cases, saying that 'the' is one 'word' and 'an' is a second 'word,' a 'word' is a legisign. But when we say of a page in a book, that it has 250 'words' upon it, of which twenty are 'the's, the 'word' is a sinsign. A sinsign so embodying a legisign, I term a 'replica' of the legisign. The difference between a legisign and a qualisign, neither of which is an individual thing, is that a legisign has a definite identity, though usually admitting a great variety of appearances. Thus, &, *and*, and the sound are all one word. The qualisign, on the other hand, has no identity. It is the mere quality of an appearance and is not exactly the same throughout a second. Instead of identity, it has *great similarity*, and cannot differ much without being called quite another qualisign.

Peirce: CP 8.335 Cross-Ref:††

335. In respect to their relations to their dynamic objects, I divide signs into Icons, Indices, and Symbols (a division I gave in 1867).†12 I define an Icon as a sign which is determined by its dynamic object by virtue of its own internal nature. Such is

any qualisign, like a vision, -- or the sentiment excited by a piece of music considered as representing what the composer intended. Such may be a sinsign, like an individual diagram; say a curve of the distribution of errors. I define an Index as a sign determined by its dynamic object by virtue of being in a real relation to it. Such is a Proper Name (a legisign); such is the occurrence of a symptom of a disease. (The symptom itself is a legisign, a general type of a definite character. The occurrence in a particular case is a sinsign.) I define a Symbol as a sign which is determined by its dynamic object only in the sense that it will be so interpreted. It thus depends either upon a convention, a habit, or a natural disposition of its interpretant or of the field of its interpretant (that of which the interpretant is a determination). Every symbol is necessarily a legisign; for it is inaccurate to call a replica of a legisign a symbol.

Peirce: CP 8.336 Cross-Ref:††

336. In respect to its immediate object a sign may either be a sign of a quality, of an existent, or of a law.

Peirce: CP 8.337 Cross-Ref:††

337. In regard to its relation to its signified interpretant, a sign is either a Rheme, a Dicent, or an Argument. This corresponds to the old division, Term, Proposition, and Argument, modified so as to be applicable to signs generally. A **Term** is simply a class-name or proper-name. I do not regard the common noun as an essentially necessary part of speech. Indeed, it is only fully developed as a separate part of speech in the Aryan languages and the Basque, -- possibly in some other out of the way tongues. In the Shemitic languages it is generally in form a verbal affair, and usually is so in substance, too. As well as I can make out, such it is in most languages. In my universal algebra of logic there is no common noun. A rheme is any sign that is not true nor false, like almost any single word except 'yes' and 'no,' which are almost peculiar to modern languages. A **proposition** as I use that term, is a dicent symbol. A dicent is not an assertion, but is a sign **capable** of being asserted. But an assertion is a dicent. According to my present view (I may see more light in future) the act of assertion is not a pure act of signification. It is an exhibition of the fact that one subjects oneself to the penalties visited on a liar if the proposition asserted is not true. An act of judgment is the self-recognition of a belief; and a belief consists in the deliberate acceptance of a proposition as a basis of conduct. But I think this position is open to doubt. It is simply a question of which view gives the simplest view of the nature of the proposition. Holding, then, that a Dicent does not assert, I naturally hold that an Argument need not actually be submitted or urged. I therefore define an argument as a sign which is represented in its signified interpretant not as a Sign of that interpretant (the conclusion) [for that would be to urge or submit it]†13 but **as if** it were a Sign of the Interpretant or perhaps as if it were a Sign of the state of the universe to which it refers, in which the premisses are taken for granted. I define a dicent as a sign represented in its signified interpretant **as if it were** in a Real Relation to its Object. (Or as being so, if it is asserted.) A rheme is defined as a sign which is represented in its signified interpretant **as if it were** a character or mark (or as being so).

Peirce: CP 8.338 Cross-Ref:††

338. According to my present view, a sign may appeal to its dynamic interpretant in three ways:

1st, an argument only may be **submitted** to its interpretant, as something the reasonableness

of which will be acknowledged.

2nd, an argument or dicent may be *urged* upon the interpretant by an act of insistence.

3rd, argument or dicent may be, and a rheme can only be, presented to the interpretant for *contemplation*.

Peirce: CP 8.339 Cross-Ref:††

339. Finally, in its relation to its immediate interpretant, I would divide signs into three classes as follows:

1st, those which are interpretable in thoughts or other signs of the same kind in infinite series,

2nd, those which are interpretable in actual experiences,

3rd, those which are interpretable in qualities of feelings or appearances.

Peirce: CP 8.340 Cross-Ref:††

340. Now if you think on the whole (as I do) that there is much valuable truth in all this, I should be gratified if you cared to append it to the next edition of your book, after editing it and of course cutting out personalities of a disagreeable kind, **ESPECIALLY IF [IT WERE] ACCOMPANIED BY ONE OR MORE** (running or other) **CLOSE CRITICISMS**; for I haven't a doubt there is more or less error involved
. . . .

Peirce: CP 8.341 Cross-Ref:††

341. P.S. On the whole, then, I should say there were ten principal classes of signs

1. Qualisigns
2. Iconic Sinsigns
3. Iconic Legisigns
4. *Vestiges*, or Rhematic Indexical Sinsigns
5. *Proper Names*, or Rhematic Indexical Legisigns
6. Rhematic Symbols
7. Dicent Sinsigns (as a portrait with a legend)
8. Dicent Indexical Legisigns
9. *Propositions*, or Dicent Symbols
10. Arguments.†14

Peirce: CP 8.342 Cross-Ref:††

§2. ON THE CLASSIFICATION OF SIGNS †15

342. The publishers of the Britannica have given an unequivocal earnest of their determination to make every edition of their encyclopaedia maintain its supereminence in employing editors who would enlist you for an epitome of your exploration of "significs."†16 It greatly encourages me in my endeavours, since, as well as I can make out, what you call "significs" is equivalent to the study that I entitle logic. In my paper of 1867 May 14 (Proc. Am. Acad. of Arts & Sci., Vol. VII, p. 295) I said, "We come to this, that logic treats of the reference of symbols in general to their objects. In this view it is one of a trivium of conceivable sciences. The first would treat of the formal conditions of symbols having meaning, that is of the reference of symbols in general to their grounds, or imputed characters; and this might be called Formal Grammar [the *grammatica speculativa* of Duns]. The second, logic, would treat of the formal conditions of the truth of symbols. The third would treat of the formal conditions of the force of symbols, or their power of appealing to a mind, that is, of their reference in general to interpretants, and this might be called formal rhetoric."†17 I should still opine that in the future there probably will be three such sciences. But I have learned that the only natural lines of demarcation between nearly related sciences are the divisions between the social groups of devotees of those sciences; and for the present the cenoscopic studies (i.e., those studies which do not depend upon new special observations) of all signs remain one undivided science, -- a conclusion I had come to before I made your acquaintance, but which the warm interest that you and I have in each other's researches in spite of the difference in their lines, decidedly confirms.

Peirce: CP 8.343 Cross-Ref:††

343. It seems to me that one of the first useful steps toward a science of *semeiotic* ({sémeiōtiké}), or the cenoscopic science of signs, must be the accurate definition, or logical analysis, of the concepts of the science.†18 I define a *Sign* as anything which on the one hand is so determined by an Object and on the other hand so determines an idea in a person's mind, that this latter determination, which I term the *Interpretant* of the sign, is thereby mediately determined by that Object. A sign, therefore, has a triadic relation to its Object and to its Interpretant. But it is necessary to distinguish the *Immediate Object*, or the Object as the Sign represents it, from the *Dynamical Object*, or really efficient but not immediately present Object. It is likewise requisite to distinguish the *Immediate Interpretant*, i.e. the Interpretant represented or signified in the Sign, from the *Dynamic Interpretant*, or effect actually produced on the mind by the Sign; and both of these from the *Normal Interpretant*, or effect that would be produced on the mind by the Sign after sufficient development of thought. On these considerations I base a recognition of ten respects in which Signs may be divided. I do not say that these divisions are enough. But since every one of them turns out to be a trichotomy, it follows that in order to decide what classes of signs result from them, I have 3^{10} or 59049, difficult questions to carefully consider; and therefore I will not undertake to carry my systematical division of signs any further, but will leave that for future explorers.

Peirce: CP 8.344 Cross-Ref:††

344. The ten respects according to which the chief divisions of signs are determined are as follows:

1st, According to the Mode of Apprehension of the Sign itself,

- 2nd, According to the Mode of Presentation of the Immediate Object,
- 3rd, According to the Mode of Being of the Dynamical Object,
- 4th, According to the Relation of the Sign to its Dynamical Object,
- 5th, According to the Mode of Presentation of the Immediate Interpretant,
- 6th, According to the Mode of Being of the Dynamical Interpretant,
- 7th, According to the Relation of the Sign to the Dynamical Interpretant,
- 8th, According to the Nature of the Normal Interpretant,
- 9th, According to the Relation of the Sign to the Normal Interpretant,
- 10th, According to the Triadic Relation of the Sign to its Dynamical Object and to its Normal Interpretant.

Peirce: CP 8.345 Cross-Ref:††

345. The ten divisions appear to me to be all Trichotomies; but it is possible that some of them are not properly so. Of these Ten Trichotomies, I have a clear apprehension of some, (which I mark {d} for {délos}), an unsatisfactory and doubtful notion of others (which I mark {a} for {adélos}), and a tolerable but not thoroughly tried conception of others (which I mark {m} for {metrios}, {s} for {schedon}, almost clear, {ch} for {chalepös} hardly better than {a}).

The Ten Main Trichotomies of Signs

(as they are apprehended by me 1908 Dec. 24)

Peirce: CP 8.346 Cross-Ref:††

346. I. A Sign is necessarily in itself present to the Mind of its Interpreter. Now there are three entirely different ways in which Objects are present to minds:†19

Peirce: CP 8.346 Cross-Ref:††

First, in themselves as they are in themselves. Namely, Feelings are so present. At the first instant of waking from profound sleep when thought, or even distinct perception, is not yet awake, if one has gone to bed more asleep than awake in a large, strange room with one dim candle. At the instant of waking the *tout ensemble* is felt as a unit. The feeling of the skylark's song in the morning, of one's first hearing of the English nightingale.

Peirce: CP 8.346 Cross-Ref:††

Secondly, the sense of something opposing one's Effort, something preventing one from opening a door slightly ajar; which is known in its individuality by the actual shock, the Surprising element, in any Experience which makes it *sui generis*.

Peirce: CP 8.346 Cross-Ref:††

Thirdly, that which is stored away in one's Memory; Familiar, and as such, General.

Peirce: CP 8.347 Cross-Ref:††

347. Consequently, Signs, in respect to their Modes of possible Presentation, are divisible (Σ) into

Peirce: CP 8.347 Cross-Ref:††

A. *Potissigns*, or Objects which are signs so far as they are merely possible, but felt to be positively possible; as, for example, the seventh ray that passes through the three intersections of opposite sides of Pascal's hexagram.

Peirce: CP 8.347 Cross-Ref:††

B. *Actisigns*, or Objects which are Signs as Experienced *hic et nunc*; such as any single word in a single place in a single sentence of a single paragraph of a single page of a single copy of a book. There may be repetition of the whole paragraph, this word included, in another place. But that other occurrence is not *this* word. The book may be printed in an edition of ten thousand; but *THIS* word is only in my copy.

Peirce: CP 8.347 Cross-Ref:††

C. *Famisigns*, familiar signs, which must be General, as General signs must be familiar or composed of Familiar signs. (I speak of signs which are "general," not in the sense of *signifying* Generals, but as being *themselves* general; just as Charlemagne is general, in that it occurs many times with one and the same denotation.)

Peirce: CP 8.348 Cross-Ref:††

348. I think I might as well have marked this division {d} instead of {s}, except that perhaps the question may arise whether I ought not to have recognized a division according as the sign is a *natural sign*, which has no party to the dialogue as its author, or whether it be an *uttered sign*, and in the latter case, is the very sign that is getting uttered or another. But it seems to me that this division turns upon the question of whether or not the sign uttered is a sign of a sign as its Object. For must not every sign, in order to become a sign, get uttered?

Peirce: CP 8.349 Cross-Ref:††

349. II Objects may be presented in three ways, thus:

1st, As mere Ideas, or what might be if things were not as they are; such as a geometrical surface, or an absolutely definite or distinct notion.

2nd, As brutally compelling attention.

3rd, As Rationally recommending themselves, or as Habitudes to which one is already reconciled.

Peirce: CP 8.350 Cross-Ref:††

350. Adopting this enumeration as a basis of a division of Signs, I obtain

Peirce: CP 8.350 Cross-Ref:††

A. *Descriptives*, which determine their Objects by stating the characters of the latter.

Peirce: CP 8.350 Cross-Ref:††

B. *Designatives* (or *Denotatives*), or Indicatives, *Denominatives*, which like a Demonstrative pronoun, or a pointing finger, brutally direct the mental eyeballs of the interpreter to the object in question, which in this case cannot be given by

independent reasoning.

Peirce: CP 8.350 Cross-Ref:††

C. *Copulants*, which neither describe nor denote their Objects, but merely express the logical relations of these latter to something otherwise referred to.†20 Such, among linguistic signs, as "If -- then --," " -- is --," " -- causes --," " -- would be --," " -- is relative to -- for --" "Whatever" etc.

Peirce: CP 8.351 Cross-Ref:††

351. Shall I appoint this famous distinction (as I have stated it, or modified [it]) to the governance of my Second way of dividing Signs, or shall I yield this place to a distinction prominent in every language on earth, that between the three 'persons,' *amo, amas, amat*? If I and *thou* are the Objects, we say *We*; if *thou* and *he* are the Objects, we say *Ye*. But if I and *We* are the Objects to the exclusion of *Thee*, I know no other linguistic form than the French expression "*Nous autres.*" *I, Thou, and He* can be expressed by the Tri-*al* and *Quadral* numbers of Polynesian languages. In English we can only say "*We all of us.*" Thus there ought, logically one would say, to be *seven* grammatical persons, if any at all. But none at all are needed, if we have the Designative pronouns *I, Thou, He*. But hold! When I say there are only 7 persons I forget the differences between Thou and I are Anglo-Saxon. Thou and I are correspondents. Thou and I are endurer and endured. Thou and I are admired and admirer. Thou, he, and I are accuser to and of, accuser of and to, accused by and to, accused to and by, informed of by, informed by of. In short this distinction does not require any special form of sign, nor could any form be adequate without numerous variations.

Peirce: CP 8.352 Cross-Ref:††

352. On the other hand [is] the distinction of *Designatives* such as concrete subjects of signs or essentially nominative signs, [and] *Descriptives* such as Predicates and Predicative Signs (such as a portrait with a legend designating the person represented), [with] Abstract nouns to be reckoned among Descriptives. The copulants are likewise indispensable and have the property of being *Continuant*. What I mean is that the sign 'A is red' can be decomposed so as to separate 'is red' into a Copulative and a Descriptive, thus: 'A possesses the character of redness.' But if we attempt to analyze 'possesses the character' in like manner, we get 'A possesses the character of the possession of the character of Redness'; and so on *ad infinitum*. So it is, with 'A implies B,' 'A implies its implication of B,' etc. So with 'It rains and hails,' 'It rains concurrently with hailing,' 'It rains concurrently with the concurrence of hailing,' and so forth. I call all such sign Continuants. They are all Copulants and are the only *pure* copulants. These signs *cannot be explicated*: they must convey Familiar universal elementary relations of logic. We do not derive these notions from observation, nor by any sense of being opposed, but from our own reason. This trichotomy, then, sustains criticism and must be marked ({m}) at least. I would mark it ({d}) if I were satisfied with the distinction between Descriptives and Denominatives.

Peirce: CP 8.353 Cross-Ref:††

353. Before proceeding to the third trichotomy, let [us] inquire what relations, if any, are found between the two that have been brought to light. What I mean precisely by *between these relations* is whether or not the three members of the first trichotomy, which we may for the moment denote as 11, 12, 13, are or are not independent of the three members of the second, which we may denote by 21, 22, 23;

so that they form nine classes, which, if we use a dot to mean "which is," will be denoted by

11·21 11·22 11·23

12·21 12·22 12·23

13·21 13·22 13·23

Peirce: CP 8.354 Cross-Ref:††

354. The inquiry ought, one would expect, to be an easy one, since both trichotomies depend on there being three Modes of Presence to the mind, which we may term

The Immediate, -- The Direct, -- The Familiar

Mode of Presence.

The difference between the two trichotomies is that the one refers to the Presence to the Mind of the Sign and the other to that of the Immediate Object. The Sign may have any Modality of Being, i.e., may belong to any one of the three Universes; its Immediate Object must be in some sense, in which the Sign need not be, Internal.

Peirce: CP 8.355 Cross-Ref:††

355. To begin, then, it is evident that an Actisign, or one that belongs to the Universe of Experience, which Brutely acts on the person, can also be a Denominative, that is, that its Immediate Object is represented as belonging to the same Universe; so that 12·22, the central class of our block of nine, is possible. Indeed, a pointing finger is a familiar example of a Sign of that class. Let us next ask whether all the four corner classes of the block are possible. We fully expect to find that a Potisign can be Descriptive and that a Famisign can be Copulant. But we may well doubt whether a Potisign can be Copulant or a Famisign can be Descriptive. Let us see.

Peirce: CP 8.356 Cross-Ref:††

356. Before taking up the cases, let me notice a source of possible confusion. By a "General Sign," or a "General Term," we do not, in the ordinary language of Logic, mean, as might be supposed, a Famisign. For we do not mean that the Sign *itself* is General: we only mean that its Object is so.

Peirce: CP 8.357 Cross-Ref:††

357. The Northern United States are full of I know not how many thousand "villages," as they are called in the State of New York, "towns" as they are called in New England, which are governed in a simple way by "town meetings" or otherwise; and in Pennsylvania "boroughs," whose head is a "chief burgess"; and there are also countless little places somewhat larger (especially in the West) called "cities." In the middle of any one of these where one might wander he would find a small green of an acre or two and in the middle of this will be a stone statue, often of granite, representing a common soldier standing in his regulation overcoat and resting on his grounded musket. Nothing imaginable could be more devoid of imagination, less idealized, less artistically beautiful. They are eye-sores to all cultivated people; but

not to me. For I know that that means that almost every family in that place . . . had in the war of the southern rebellion sent its flower, who had no military instinct whatever, much less any hatred for southern people, to the war, bitterly contrary to all his instincts but simply from a sense of duty; and only a fraction of them came back. The very fact of their vulgarity, which the statue proclaims above all else, makes this universal self-sacrifice on the altar of the abstraction which we call the "general government" pathetically sublime. To each such family, that very realistic statue represents the mourned one who fell in the war. That statue is one piece of granite, and not a Famisign. Yet it is what we call a "General" sign, meaning that it is *applicable* to many singulars. It is not *itself* General; it is its Object which is taken to be General. And yet this Object is not truly Universal, in the sense of implying a truth of the kind of 'Any S is P'; it only expresses 'Some S is P.' This makes it *not* a *Copulant* (Copulative) but only a *Descriptive*. This needs to be borne in mind. And this warning having been noticed, we can proceed to inquire about the corners of our block of supposed classes, which I will designate according to the usual map that has N above, S below, E to the right, W to the left.

Peirce: CP 8.358 Cross-Ref:††

358. As to the NW corner, a Geometrical diagram is always capable of being *imagined*, seldom or never of *existing*; since the limits of solid bodies are the loci at which forces of cohesion are neither very great nor very small, which being vague, has not the character of a geometrical surface. The diagram is therefore a *Potisign*. It is clearly Descriptive; and therefore 11·21 is possible.

Peirce: CP 8.359 Cross-Ref:††

359. The verbal expression "If -- , then -- " is a Famisign, as all words are (in the sense in which two that are just alike are the very same "word"). It is also a Copulative since it expresses a universal sequence, 'If A, then C,' meaning that in every state of things whatever, either not-A or is-C is true. So that the SE corner 13·23 is possible.

Peirce: CP 8.360 Cross-Ref:††

360. "Given any 4 rays in space; then either there can be only 2 rays, at most, that cut them all, or there can be any number." True or not, this is a Copulant; and any single expression of it is an Actisign. It is also expressible in Existential Graphs in the form of a geometrical diagram, which is a Potisign. Therefore, the NE corner 11·23 is possible.

Peirce: CP 8.361 Cross-Ref:††

361. But can a Famisign be Descriptive? Everybody will make haste to cry, "Of course, it can: of course, a description can be expressed in words, when even a universal can." Yet, while I am more than usually sensible of the danger of my being mistaken, I venture, for the present, the opinion that it is not so. The proper way to pursue the inquiry is to start from the definition already given of the triadic relation of Sign-Object-Interpretant. We thus learn that the Object determines (i.e. renders definitely to be such as it will be,) the Sign in a particular manner. Now it is of the essence of the Sign to determine certain Ideas, i.e. certain Possibles; and it is the essence of any Tendency to determine Occurrences. Therefore, an Actisign or a Potisign may be a Copulative. But no Occurrence or collection of occurrences can logically determine a Habit or other Tendency. Thus, if wishing to test a *die* to see whether it is loaded (whether intentionally or not) I throw it say 900 times. If the different faces come up with as equal frequency as they could be expected to do, what can I infer? Only that *as long as the habit or tendency of the die remains what it is,*

it will probably not bring the different faces up so unequally as to show decisively in 900 throws. That is, I base my inference on the assumption that *there is some habit*. Or take a simpler case. If I positively knew (what I cannot *know*) that a certain shilling had a habit when pitched to turn up *heads* and *tails* with equal frequency, then I should positively know that if it were pitched often enough, it would *sometime* turn up "heads." For if it *always* would turn up tails, that would constitute a habit contrary to the habit supposed to be known.†21 That known habit may be defined thus: Let a tally be kept of the heads as they occur and another of the tails; and after each throw let the exact quotient of the number of heads divided by the number of tails be calculated. Then, given any positive number (not zero therefore) there will *certainly* come a time after which none of the quotients will differ by as much as that number from 1. The value 1 being the only one about which the values of the quotients will never cease to oscillate. Thus the tendency consists entirely in what *will be*; and what *has been* has nothing to do with it. But what *will be* is not an Actual Occurrence. It is true that physiological and some other habits are determined by what has been done; but not by those occurrences *of themselves*, but only because there is a *special Tendency* by virtue of which what has been done *will be* done oftener than what has not been done. In general, it is of the essence of a Real Tendency that no Actual Occurrence can of itself determine it in any way. Whence a Denominative cannot be a Famisign. Whence the middle of the S side of the block 13·22 is impossible. But an Actual Occurrence always determines the Possibility of its character; whence no Descriptive can be a Famisign; or [i.e.] the SW corner of the block 13·21 is impossible. As an example of this, no number of Descriptive propositions of the type "Some S is P" can ever determine the truth of a Copulative Proposition "Any S is P." It is, if possible, still more obvious that Possibility can never determine Actuality and therefore a Descriptive cannot be an Actisign, or [i.e.] the middle of the W side of the block 12·21 is impossible. The remaining six classes are possible, i.e.,

Copulative Potisigns

Denominative Potisigns Copulative Actisigns

Descriptive Potisigns Denominative Actisigns Copulative Famisigns

Peirce: CP 8.362 Cross-Ref:††

362. There are four objections that would probably be raised against my doctrine; but I will not lengthen this letter with the refutations of them. I have carefully considered them, and have found them to be unsound.

Peirce: CP 8.363 Cross-Ref:††

363. From the summer of 1905 to the same time in 1906, I devoted much study to my ten trichotomies of signs. It is time I reverted to the subject, as I know I could now make it much clearer. But I dare say some of my former names are better than those I now use. I formerly called a Potisign a Tinge or Tone, an Actisign a Token, a Famisign a Type; -- a Descriptive an Indefinite (but this was bad), a Denominative a Designation, a Copulative (which is bad) a Distributive (which is much better).

Peirce: CP 8.364 Cross-Ref:††

364. I think Potisign, Actisign, [and] Famisign might be called Mark, Token, [and] Type(?)†22 [respectively], while Descriptive, Denominative, [and] Copulative might be called Descriptive, Denominative, [and] Distributive, [respectively].

Peirce: CP 8.365 Cross-Ref:††

365. I have now given as much time to this letter as I can afford and I cannot now re-examine the remaining trichotomies, although I must do so as soon as possible. So I just give them as they stood two years and more ago. In particular, the relations I assumed between the different classes were the wildest guesses, and cannot be altogether right I think.

Peirce: CP 8.366 Cross-Ref:††

366. III. In respect to the Nature of their Dynamical Objects, Signs I found to be either

1. Signs of Possibles. That is *Abstractives* such as Color, Mass, Whiteness, etc.
2. Signs of Occurrences. That is *Concretives* such as Man, Charlemagne.
3. Signs of Collections. That is *Collectives* such as Mankind, the Human Race, etc.

Peirce: CP 8.366 Cross-Ref:††

By *Abstractives* I meant signs of *immediate* abstractions; but was in some doubt what to do with abstractions resulting from experiment. I thought it would be requisite to study subdivisions of these classes but never went into that research.

Peirce: CP 8.367 Cross-Ref:††

367. I was of the opinion that if the Dynamical Object be a mere Possible the Immediate Object could only be of the same nature, while if the Immediate Object were a Tendency or Habit then the Dynamical Object must be of the same nature. Consequently an Abstractive must be a Mark, while a Type must be a Collective, which shows how I conceived Abstractives and Collectives.

Peirce: CP 8.368 Cross-Ref:††

368. IV. The fourth Trichotomy is the one which I most frequently use: Icon, Index,†23 Symbol.

Peirce: CP 8.368 Cross-Ref:††

All the remaining six trichotomies have to do with the Interpretants, which you have, I imagine, studied much more thoroughly than I have done.

Peirce: CP 8.369 Cross-Ref:††

369. V. As to the nature of the Immediate (or Felt?) Interpretant, a sign may be: Ejaculative, or merely giving utterance to feeling; Imperative, including, of course, Interrogatives; Significative.

Peirce: CP 8.369 Cross-Ref:††

But later I made this the 7th Trichotomy and for the fifth substituted -- with great hesitation--: Hypothetic, Categorical, Relative.

Peirce: CP 8.370 Cross-Ref:††

370. VI. As to the Nature of the Dynamical Interpretant: Sympathetic, or Congruentive; Shocking, or Percussive; Usual.

Peirce: CP 8.371 Cross-Ref:††

371. VII. As to the Manner of Appeal to the Dynamic Interpretant: Suggestive, Imperative, Indicative.

Peirce: CP 8.372 Cross-Ref:††

372. VIII. According to the Purpose of the Eventual Interpretant: Gratific; To produce action; To produce self-control.

Peirce: CP 8.373 Cross-Ref:††

373. IX. As to the Nature of the Influence of the Sign: Seme, like a simple sign; Pheme, with antecedent and consequent; Delome, with antecedent, consequent, and principle of sequence.

Peirce: CP 8.374 Cross-Ref:††

374. X. As to the Nature of the Assurance of the Utterance: assurance of Instinct; assurance of Experience; assurance of Form.

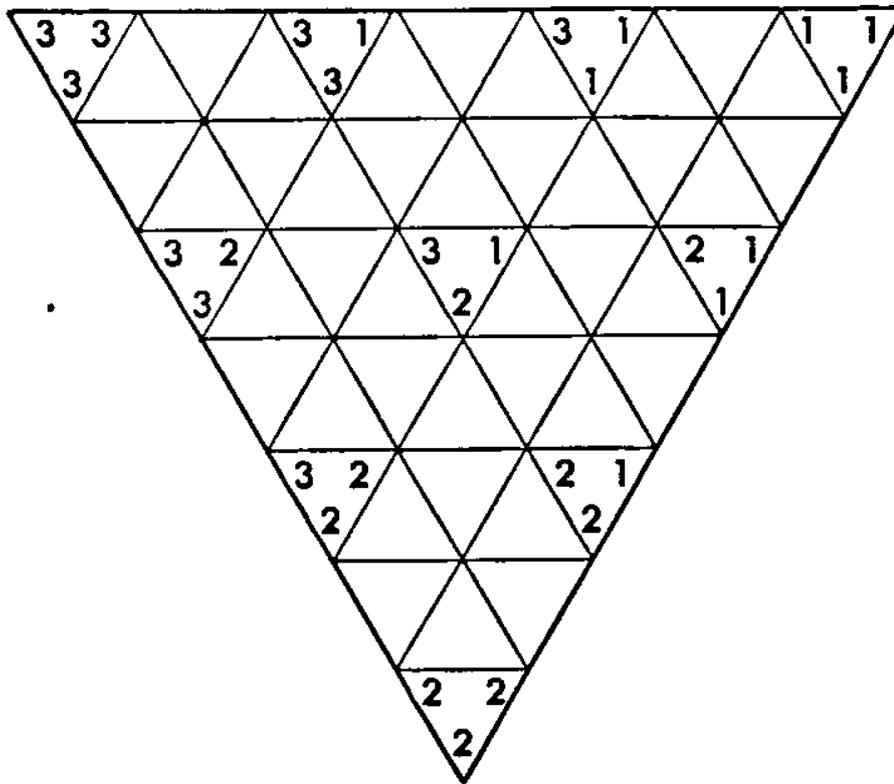
Peirce: CP 8.375 Cross-Ref:††

375. I don't know whether these trichotomies will suggest anything to you or not. No doubt you have studied relations to Interpretants in some directions much further than I

Peirce: CP 8.376 Cross-Ref:††

376. P.S. 1908 Dec 28. Well, dear Lady Welby, you deserve this infliction, for having spoken of my having "always been kindly [! ! !] interested in the work to which my life is devoted,"†24 when I have myself been entirely absorbed in the very same subject since 1863, without meeting, before I made your acquaintance, a single mind to whom it did not seem very like bosh. I add some scraps.

Signs divided into Ten Classes ²⁵



to view]

[Click here

The number above to the left describes the Object of the Sign. That above to the right describes its Interpretant. That below describes the Sign itself. 1 signifies the Possible Modality, that of an Idea. 2 signifies the Actual Modality, that of an Occurrence. 3 signifies the Necessary Modality, that of a Habit

Peirce: CP 8.377 Cross-Ref:††

377. I have often thought that if it were not that it would sound too German (and I have an utter contempt for German logic) I would entitle my logic-book (which is now coming on) "Logic considered as Semeiotic" (or probably *Semeiotic* without the *i*;) but everybody would think I was translating *als Semeiotik betrachtet*, which I couldn't stand.

Peirce: CP 8.378 Cross-Ref:††

378. The fact that I have entertained this idea shows how near together you and I are. "Significs" sounds to me narrower than Semeiotic, since *signification* is only *one* of the *two* chief functions of signs; as the elegant and correct John of Salisbury notices, in referring to "quod fere in omnium ore celebre est, aliud scilicet esse quod appellatiua significant, et aliud esse quod nominant. Nominantur singularia, sed uniuersalia significantur." (*Metalogicus* II. xx. I copy from the ed. of 1620.)†26 So *significs* appears to be limited to the study of the relations of Signs to their Interpretants; and I presume you do so limit it. On the other hand Logic is more interested in the *Truth* of Signs, i.e. in their relation to their Objects. But I am

satisfied that in the present state of the subject, there is but one General science of the nature of Signs. If we were to separate it into two, -- then, according to my idea that a "science," as scientific men use the word, implies a social group of devotees, we should be in imminent danger of erecting two groups of one member each! Whereas, if you and I stick together, we are, at least, two of us. I remember in my college days that the Statutes of Harvard defined a "group" as *three* persons or more convening together. We shall have to try to seduce one of the linguists to our more fundamental study. Max Müller was, in a feeble way, perhaps one of our group. I hope in your Britannica article you will adhere to the stern method of treatment proper to an Encyclopaedia, and show the reader that distinct positive discovery is what we are laboring upon. I remember one day, when I was in the twenties, on my way to the post-office I fell in with the novelist Wm. D. Howells, who began criticizing one of my articles from the point of view of rhetorical elegance. I said to him, "Mr. Howells, it is no part of the purpose of my writings to give readers pleasure." Such an idea was quite out of his horizon; and I heard of his repeating it as very amusing. People do not consult an encyclopaedia to be amused, but to receive definite instruction as condensed as clearness permits. I hope your article will cause readers to appreciate Significs as a study of grave importance not merely from the point of view of Morals, but also from that of Truth. But I am absolutely sure your article will be a valuable one however far it may depart from what I should wish the article to be. It must be so, simply because you have been a devoted student of the subject. It is your own message that you are to deliver and nobody else's.

Peirce: CP 8.379 Cross-Ref:††

379. It is a remarkable thing that no other people but the Greeks ever so felt the desire to avoid errors as to strike out a logic for themselves. All other logic down to the invention of the Doctrine of Chances was made on the Greek model. The Greeks so strongly felt that need that they erected no less than 5 independent systems! Namely, 1st, though Prantl's "Megaric logic" is poppy-cock, there are unmistakable indications of early strivings for a logical system; 2nd, came the logic of Socrates, who had a much clearer idea of what logic should be than such writers as Wundt, Chr. Sigwart, Jerusalem, Bradley, Bosanquet, Dewey, etc. etc., -- not to notice the Joseph's and such quasi-minds, -- have today; 3rd, passing by Plato, there was Aristotle; 4th, the Epicurean logic, very closely allied to Mill's system, yet distinctly different, as shown by the Herculaneum Papyrus of the {peri sémeiön kai sémeiöseön} of Philodemus; 5th, there was the logic illustrated by the strict methods of the mathematicians, -- which was not merely a *practice* of reasoning, but was a definite theory of logic greatly superior to Aristotle's, as far as deductive reasoning is concerned. Here are my 5 without including either Zeno of {Helis} †27 nor Chrisippus, both original logicians.

Peirce: CP 8.380 Cross-Ref:††
CHAPTER 9

TO F.A. WOODS, ON "WOULD BE"†1

380. I have duly received the copy of your new work,†2 and have already read pp. iii-xiii, 1-46, and 196-417, and have found it perfectly convincing; so that my only criticism relates to Chap. XVIII,†3 which, though convincing to me, might, I think, be improved by introducing, in some form, some considerations like the following: A conditional proposition, -- say "If A, then B" is equivalent to saying that "*Any* state of things in which A should be true, *would* (within limits) *be* a state of things in which B is true."†4 It is therefore essentially an assertion of a *general* nature, the statement of a "*would-be*." But when the antecedent supposes an *existential* fact to be different from what it actually is or was, the conditional proposition does not accurately state anything; and if it conveys any meaning, i.e. if it is calculated to produce any state of mind, in a person who trusts in it, it must be that it establishes a *habit* in that mind, using the word "habit" in the original sense, as meaning only that the person or thing that has the habit, *would* behave (or usually behave) in a certain way *whenever* a certain occasion should arise. But if this occasion did in actuality *not* arise, such habit of thought as the conditional proposition might produce would be a nullity pragmatically and practically. A historian simply talks nonsense when he says "If Napoleon had not done as he did before the battle of Leipzig (specifying in what respect his behaviour is supposed different from what it was) he would have won that battle." Such historian may have meant something; but he utterly fails to express any meaning.

Peirce: CP 8.381 Cross-Ref:††

381. To this it might be objected, that when a man deliberates as to what line of conduct he shall pursue, he will very advantageously consider conditional propositions whose antecedents are of the form "If I should do so and so." To this conceivable objection I should reply that the case of the deliberating mind and that of the futile historian's are essentially different in that the former reasons at some stage of his deliberations as follows (that is, he will unless he decides to verify the antecedent in question by adopting the contemplated line of conduct):

If I should do so and so, I should bring about such and such a result,

Now I will not bring about that result,

Ergo, I shall not do "so and so." He thus comes to a profitable conclusion, provided he had not already fully made up his mind not to "do so and so," in which case he would be pursuing an idle dream, and a bad dream at that.

Peirce: CP 8.382 Cross-Ref:††

382. But the supposed historian knows well already that the supposition of the antecedent is false; and therefore he has no occasion to seek a proof that it is so; and there is no intelligent purpose to which he or anybody can put his conditional proposition, unless he can find enough similar instances about other persons belonging to the same class as Napoleon so that he will be able to make an *induction* serving as a *maxim* by which other men may profitably regulate their conduct. For instance, if Napoleon before the battle slept with his head close to a red hot stove, and was "not himself," as we say, during the battle, a historian may be excused for putting the two facts together. But unless there is some *general* antecedent to be gained in a reasonable manner, a conditional proposition with a singular antecedent known to be false, is worse than a puerility. It is downright nonsense, a series of words without meaning

Peirce: CP 8.383 Cross-Ref:††

383. . . . logicians generally almost always confine what they have to say about reasoning to its "correctness," by which they mean its leaving an absolute inability to doubt the truth of a conclusion so long as the premisses are assumed to be true. But that amounts to confining their study to deduction. When one thinks as long about certainty as I have done, one does not very often find oneself in this state of inability to doubt

Peirce: CP 8.384 Cross-Ref:††

384. I think logicians should have two principal aims: 1st, to bring out the amount and kind of *security* (approach to certainty) of each kind of reasoning, and 2nd, to bring out the possible and esperable uberty, or value in productiveness, of each kind.

Peirce: CP 8.385 Cross-Ref:††

385. I have always, since early in the sixties, recognized three different types of reasoning, viz: 1st, *Deduction* which depends on our confidence in our ability to analyze the meanings of the signs in or by which we think; 2nd, *Induction*, which depends upon our confidence that a run of one kind of experience will not be changed or cease without some indication before it ceases; and 3rd, *Retroduction*, or Hypothetic Inference, which depends on our hope, sooner or later, to guess at the conditions under which a given kind of phenomenon will present itself.†5

Peirce: CP 8.386 Cross-Ref:††

386. Each of these three types occurs in different forms requiring special studies.

Peirce: CP 8.387 Cross-Ref:††

387. From the 1st type to the 3rd the security decreases greatly, while the uberty as greatly increases

Peirce: CP 8.388 Cross-Ref:††

388. I don't think the adoption of a hypothesis on probation can properly be called induction; and yet it is *reasoning* and though its *security* is low, its *uberty* is high.

Peirce: CP 8 Bibliography Introduction
BIBLIOGRAPHY OF THE WORKS OF CHARLES SANDERS PEIRCE

INTRODUCTION

This bibliography has three main sections: (I) General, (II) Items from *The Nation*, and (III) Miscellaneous. The first section includes all Peirce's works which have been published from manuscripts in the eight volumes of *Collected Papers of*

Charles Sanders Peirce and all his known works which were presented as addresses or were printed in publications other than *The Nation* during his lifetime. The second section consists of all the works published in *The Nation* that have been identified as Peirce's contributions. The third section consists of correspondence by Peirce, and also works by other authors which quote or describe manuscripts by Peirce that are not published in *Collected Papers*, volumes I-VIII.

Peirce: CP 8 Bibliography Introduction

Sections I and II are arranged primarily in chronological order. The date under which an item appears is the year of its first public appearance, either in print or as an address, except for manuscripts not published during Peirce's life. For these latter cases the item appears under the date when the manuscript was written, where this was known or could be inferred from the available evidence; the dates of subsequent "revisions," including corrections and marginal notes, are given in the same entry. Undated manuscripts are listed at the end of Section I. Various public presentations which there is reason to believe are the same material are listed together if the dates of the public appearances fall within the same calendar year (see, for example, the first item of 1879 in Section I); otherwise they are listed separately with appropriate back references (for example, the sixth item of 1878 in Section I). Functionally related series of articles, lectures, and associated items are grouped together and listed by the *year of the first*. The third section of the bibliography is arranged alphabetically according to the name of the recipient of the correspondence or the name of the author or editor of the work cited; these names are numbered.

Peirce: CP 8 Bibliography Introduction

References and cross-references to items of the bibliography are made as follows: to an item in the General section, [Bibliography] G-year-item number, e.g., [Bibliography] G-c.1901-3b; to an item in *The Nation* section, [Bibliography] N-year-item number, e.g., [Bibliography] N-1892-1; and to an item in the Miscellaneous section, [Bibliography] M-name number, e.g., [Bibliography]M-6. A list of the abbreviations for publication titles precedes the bibliography proper.

Peirce: CP 8 Bibliography Introduction

An item that has been reprinted in *Collected Papers* is followed by the designation "[CP]" and the volume and paragraph numbers. For example, in [Bibliography] G-1908-1a the reference [CP] 4.585-593 indicates that the article described in this item ("Some Amazing Mazes") is reprinted at paragraphs 585-593 of Volume 4 of *Collected Papers*. It should be noted that the [CP] reference may follow explanatory material concerning the item, as in this example.

Peirce: CP 8 Bibliography Introduction

A cross-reference index from the paragraphs in *Collected Papers* to the entries of the bibliography is given at the end of the bibliography. The reader's attention is also invited to the list of errata for *Collected Papers*, volumes I-VI, which concludes this introduction.

Peirce: CP 8 Bibliography Introduction

Sources for the Bibliography. The bibliographies by Cohen and the one by Fisch and Haskell, the information in *Collected Papers*, volumes I-VI, and personal communications from Max Fisch constituted the original basis for the present bibliography. In connection with the process of going through Peirce's published

works and the unpublished manuscripts at Harvard University and elsewhere, considerable effort was devoted to finding new items and acquiring more information about known items. There are undoubtedly Peirce publications not discovered in this search, especially unsigned contributions, but in the editor's opinion it is not likely that any of these are of great importance.

Peirce: CP 8 Bibliography Introduction

Much, though not all, of the information derived from the sources mentioned above was checked before it was incorporated. Every item published while Peirce was alive, with the exception of articles in the *New York Evening Post* (access to which is difficult) and an unlocated item, has been examined and the information given in the present bibliography confirmed. With regard to manuscripts or parts of manuscripts not published during Peirce's lifetime, but included in *Collected Papers*, volumes I-VI, the information in the published volumes was accepted as given; in particular, the present editor has not attempted to re-date the manuscripts printed there. In the course of going through the Harvard manuscripts, however, a considerable amount of this information has been checked and the necessary corrections have been incorporated in the present bibliography.

Peirce: CP 8 Bibliography Introduction

Knight McMahon, in connection with his preparation of a catalogue of the Widener manuscripts, was able in some instances to unite manuscripts that were in separate pieces when *Collected Papers*, volumes I-VI, were edited; and the present editor has also succeeded in recombining some fragments. In cases where different dates have been assigned to the respective parts of a work, the work as a whole is listed under one of the dates, and the respective parts, with the date appropriate to each, are listed under this heading. Concerning the problem of dating Peirce's work, it is worth noting that, especially after he moved to Milford (1887), Peirce was constantly reworking drafts of articles and books, with the result that many of the manuscripts are confused jumbles of various drafts, some of them written at quite different times.

Peirce: CP 8 Bibliography Introduction

Many of Peirce's published works are unsigned -- this is true of most of the items published in *The Nation* -- and in a few cases they bear signatures other than Peirce's, though there is conclusive evidence in the Harvard manuscripts and correspondence that these are by him. In many such instances the present editor has accepted the authority of other bibliographies concerning the identity of the items, after examining each one (with the exception, as before, of articles in the *New York Evening Post* and one unlocated item) for internal evidence that it was written by Peirce. Unless otherwise indicated, the items from *The Nation* were unsigned and the identification has been taken from Haskell's *Index to the Nation*, which is based on the records of *The Nation*.

Peirce: CP 8 Bibliography Introduction

The present editor has sometimes made identifications on the basis of correspondence, manuscript drafts, a list in Widener VA² of some of Peirce's reviews in volumes 50 through 57 of *The Nation*, and proof sheets. Rather strict criteria of identification were used. Except for two or three items where there seemed good independent reason to believe the publication was by Peirce, the mere presence of proof sheets alone among the manuscripts was not accepted as sufficient evidence of authorship; however, in some cases identification was based on scraps of manuscript.

It would be possible to establish Peirce's authorship, with varying degrees of probability, of a number of additional reviews by the use of proof sheets, inconclusive references in correspondence, and internal evidence; but in every such instance that came to this editor's attention, the review was of such little significance that it seemed better to confine the bibliography to items where the evidence was more conclusive. Even so, the reader should be aware of the nature of the evidence on which the identifications were based. He should further understand that many of Peirce's reviews in *The Nation* were modified by the editor of that journal. (See [CP] 8.117n10 and [Fisch-Haskell]377.) Where the identification is the work of the present editor on the basis of Harvard manuscripts, the expression, "Identification based on Harvard mss.," is used.

Peirce: CP 8 Bibliography Introduction

Peirce Manuscript Sources. The most important collection of Peirce manuscripts, the source of almost all of the previously unpublished papers in *Collected Papers*, is in the Archives of Widener Library at Harvard University. It consists of sixty-one boxes and bundles, listed under the call number HUG 1680.402. A catalogue for this collection, prepared by Knight W. McMahon, *Catalogue of C. S. Peirce Manuscripts*, December, 1941, typed, 99 pp., is available at the Archives. While this catalogue is not complete, it gives a list of the contents of most of the boxes, and it is extremely useful. The collection is divided according to Peirce's classification of sciences as follows, with the number of boxes and bundles listed after each division:

I. Science of Discovery	
A. Mathematics	8
B. Philosophy	
1. Pragmatism and the Categories	2
2. Normative Sciences (Logic)	12
3. Metaphysics	2
C. Idioscopy	8
II. Science of Review: Classification of the Sciences	1
III. Practical Science and Miscellaneous	3
IV. Book Reviews	2
V. Life and Letters	
Unclassified: V{a}, V{b}, . . . , V{z}	6
A. Biography	2
B. Correspondence	
1. Personal	6

2. Professional	5
3. Business	2
4. Official: Coast Survey	1
5. Applications	1

These manuscripts are identified in the present volume by designations, such as "Widener IB2-7," "Widener VA2," etc., following the above classification.

Peirce: CP 8 Bibliography Introduction

There is also a large collection of Peirce manuscripts in Houghton Library at Harvard University. This consists of about twenty-one boxes and bundles of material, mainly fragments and items of minor importance, which are unclassified and uncatalogued.

Peirce: CP 8 Bibliography Introduction

Considerable effort was made to locate other Peirce materials, and many sources were uncovered. In each case the source was inspected by the editor or by someone known to him to insure that all manuscripts of general importance were included in the series. A complete list of all known sources of Peirce manuscripts is given below. It is evident from this list and from the description given above of the Harvard materials that the total quantity of unpublished Peirce manuscripts is tremendous. It may be of interest to Peirce scholars to have a general estimate of the value of the manuscripts not included in *Collected Papers*. In the present editor's opinion, *Collected Papers*, volumes I-VIII, include all manuscripts that would be of interest to the general philosophic reader; they also include sufficient material for the Peirce specialist to trace the development of Peirce's thought in some detail. There still remains among the unpublished manuscripts much material of value for a personal biography of Peirce and for very detailed studies of Peirce's intellectual development.

Peirce: CP 8 Bibliography Introduction Sources 1

SOURCES OF PEIRCE MANUSCRIPTS

1. Archives, Widener Library, Harvard University.

(a) The main Peirce collection in the Archives, referred to above.

(b) The class book of 1859 (Call No. HUD 259.714F) and the photographs of the class of 1859 (Call No. HUD 259.704F).

(c) Peirce mss. and related materials: HUG 1680.402.10, HUG 1680.402.12.

(d) Correspondence in the papers of Josiah Royce and F. E. Abbot.

Peirce: CP 8 Bibliography Introduction Sources 2

2. Houghton Library, Harvard University.

(a) The material referred to above.

(b) Letters between Peirce and James in the William James Collection.

(c) Peirce's interleaved and annotated copy of the *Century Dictionary*; cf. [Bibliography] G-1889-3.

(d) Approximately six additional items listed in the catalogues of Widener and Houghton.

(e) A letter from Peirce in the C. E. Norton papers.

Peirce: CP 8 Bibliography Introduction Sources 3

3. Other Harvard University sources.

(a) There are some of Peirce's copies of his own reprints in Widener Library. There are many of Peirce's books in Widener Library and at least one in the Philosophy Library, but these are not identified as Peirce's in the card catalogue and no record of them has been found. A record of Peirce's class, *Harvard College Records of the Class of 1859*, Edward W. Wheeler, Cambridge, 1896 (Call No. HUD 259.775), is in Widener Library.

(b) The records of the Harvard College Observatory during the period Peirce was associated with it are stored in the Harvard University Deposit Library. Volumes N11 through N19, and perhaps others, contain records of Peirce's observations. There are also eight volumes of Peirce's "Photometric Observations" and a volume of his "Experiments with Photometer."

Peirce: CP 8 Bibliography Introduction Sources 4

4. National Archives, Washington, D. C. Letters and papers in the records of the United States Coast and Geodetic Survey.

Peirce: CP 8 Bibliography Introduction Sources 5

5. Library of Congress, Washington, D. C., Manuscript Division. Peirce's side of a correspondence with Simon Newcomb; Newcomb's letters to Peirce are at Harvard. Cf. [Bibliography] M-6c.

Peirce: CP 8 Bibliography Introduction Sources 6

6. Johns Hopkins University Library, Baltimore.

(a) Letters and papers in the Daniel C. Gilman Collection.

(b) Letters and papers in the records of the *American Journal of Mathematics*.

(c) Books and reprints that belonged to Peirce. Peirce sold several hundred books to this library; a list of these is in the Accession Book of the Hopkins Library for February 7-9, 1881. Cf. [Fisch-Cope] 292ff.

Peirce: CP 8 Bibliography Introduction Sources 7

7. Smithsonian Institution, Washington, D. C. Manuscripts and correspondence published by Wiener; cf. [Bibliography] G-1901-2. In addition there are other manuscripts and about twenty-five letters; cf. [Bibliography] M-6b.

Peirce: CP 8 Bibliography Introduction Sources 8

8. Yale University Library, New Haven.

(a) Photostat copies of the correspondence with Lady Welby; cf. [Bibliography] M-20a, p. 34, ftn. 79.

(b) Rare Book Room, Sterling Library. A letter from Peirce to Josiah Gibbs. (Also a letter from Melusina Fay Peirce, Peirce's wife, to George Eliot and letters from Benjamin and James Mills Peirce to people at Yale.)

Peirce: CP 8 Bibliography Introduction Sources 9

9. Hoose Library of Philosophy, University of Southern California, Los Angeles. About eighteen letters from Peirce to William T. Harris.

Peirce: CP 8 Bibliography Introduction Sources 10

10. Columbia University, New York City.

(a) Some Peirce reprints; cf. [Bibliography] M-15b, p. 718.

(b) A few items in the Plimpton Collection and in the David Eugene Smith Library; cf. [Bibliography] M-6a.

Peirce: CP 8 Bibliography Introduction Sources 11

11. Private possession of Mrs. D. P. Abbot. A letter; cf. [Bibliography] M-21b, p. 252.

Peirce: CP 8 Bibliography Introduction Sources 12

12. St. John's College, Cambridge University, Cambridge, England. About eight letters from Peirce to J. J. Sylvester regarding nonions. Cf. [Bibliography] G-1883-1.

Peirce: CP 8 Bibliography Introduction Sources 13

13. Private possession of André Lalande. A letter from Peirce to Lalande. (There is a typed copy in Widener VB2.)

Peirce: CP 8 Bibliography Introduction Sources 14

14. Open Court Publishing Company, La Salle, Illinois. At least ten letters from Peirce to Paul Carus and numerous carbon copies of Carus's letters to Peirce.

Peirce: CP 8 Bibliography Introduction Sources 15

15. Private possession of Sylvia Wright. Some minor biographical items. (There is a description in Widener VA².)

Peirce: CP 8 Abbreviations
LIST OF ABBREVIATIONS AND SYMBOLS

* As a guide to the general reader of philosophical literature, those items which, in the editor's opinion, are Peirce's main contributions to philosophy (exclusive of symbolic logic) are marked with an asterisk.

Peirce: CP 8 Abbreviations

† Of the items which have been published but are not included in *Collected Papers*, those most interesting to specialists in Peirce's philosophy have been marked with a dagger.

Peirce: CP 8 Abbreviations

[AJM] *American Journal of Mathematics*.

Peirce: CP 8 Abbreviations

[BPSW] *Bulletin of the Philosophical Society of Washington*.

Peirce: CP 8 Abbreviations

[Cohen] Morris R. Cohen, "Charles S. Peirce and a Tentative Bibliography of his Published Writings," *The Journal of Philosophy, Psychology, and Scientific Methods* 13 (21 Dec 1916) 726-737; "Bibliography," pp. 733-737. Additions were published in *ibid.*, 15(10 Oct 1918)578-584. Both were reprinted with modifications in "Bibliography of Peirce's Published Writing," *Chance, Love, and Logic* (edited by Morris R. Cohen), Harcourt, Brace, and Company, 1923, and Peter Smith, 1949, pp. 309-318. The abbreviation designates all these publications collectively.

Peirce: CP 8 Abbreviations

[CP] *Collected Papers of Charles Sanders Peirce*, 8 vols. *Collected Papers* I-VI were edited by Charles Hartshorne and Paul Weiss, Harvard University Press, Cambridge, 1931-1935: Vol. I, *Principles of Philosophy*; Vol. II, *Elements of Logic*; Vol. III, *Exact Logic*; Vol. IV, *The Simplest Mathematics*; Vol. V, *Pragmatism and Pragmaticism*; Vol. VI, *Scientific Metaphysics*. Vol. VII, *Science and Philosophy*, and the present volume, Vol. VIII, *Reviews, Correspondence, and Bibliography*, were edited by Arthur W. Burks, Harvard University Press, Cambridge, 1958. References to paragraphs employ the decimal notation used in *Collected Papers*, I-VI, e.g., "[CP] 5.431" refers to [CP], Vol. V, paragraph 431. Page references are given by the designation [CP] and the volume and page number, e.g., [CP] IV, 291.

Peirce: CP 8 Abbreviations

[CS --]. . . . *Report of the Superintendent of the United States Coast Survey Showing the Progress of the Work for the Fiscal Year Ending with June, ---*. Wash.: Gov. Printing Office, (Title pages for some years are unimportant variations of this.) Items from these reports are dated by year of publication and not by the fiscal year of the report.

Peirce: CP 8 Abbreviations

[Fisch] Max H. Fisch, "Alexander Bain and the Genealogy of Pragmatism," *Journal of the History of Ideas* 15 (June 1954) 413-444.

Peirce: CP 8 Abbreviations

[Fisch-Cope] Max H. Fisch and Jackson I. Cope, "Peirce at the Johns Hopkins," *Studies in the Philosophy of Charles Sanders Peirce* (edited by Philip P. Wiener and Frederick H. Young), Harvard University Press, 1952, pp. 277-311, 355-360, 363-374.

Peirce: CP 8 Abbreviations

[Fisch-Haskell] Max H. Fisch and Daniel C. Haskell, "Some Additions to Morris R. Cohen's Bibliography of Peirce's Published Writings," *ibid.*, pp. 375-381. ("During the time when the *Nation* was in effect a weekly edition of the *Post*, it will

probably be found that most of Peirce's reviews in the *Nation* had already appeared in the *Post*." We have included the *Post* references given here, but have made no further search.)

Peirce: CP 8 Abbreviations

[Harvard --] *A Catalogue of the Officers and Students of Harvard University for the Academical Year --* , Cambridge.

Peirce: CP 8 Abbreviations

[Haskell] Daniel C. Haskell, *Index to the Nation* (Volumes 1-105, 1865-1917), The New York Public Library, New York: Volume 1, *Index of Titles*, 1951; Volume 2, *Index of Contributors*, 1953.

Peirce: CP 8 Abbreviations

[International Geodetic Comm. ---, ---]. . . . Proceedings of the meeting of the Permanent Commission of the International Geodetic Association held at -- in the year -- , published in the year For example, '[International Geodetic Comm., Hamburg, 1878]1879' abbreviates: *Comptes-Rendus des Séances de la Commission Permanente de l'Association Géodésique Internationale pour la Mesure des Degrés en Europe Réunie à Hambourg du 4 au 8 Septembre 1878*, rédiges par les secrétaires C. Bruhns and A. Hirsch, George Reimer, Berlin, 1879. These volumes are listed on Library of Congress cards under the heading: Internationale erdmessung, Permanente commission.

Peirce: CP 8 Abbreviations

[International Geodetic Conference ---, ---]. . . . Proceedings of the International Geodetic Conference which met at -- in the year -- , published in the year For example, '[International Geodetic Conference, Stuttgart, 1877]1878' abbreviates: *Comptes-Rendus des Séances de la Cinquième Conférence Géodésique Internationale pour la Mesure des Degrés en Europe Réunie à Stuttgart du 27 Septembre au 2 Octobre 1877*, rédiges par les secrétaires C. Bruhns and A. Hirsch, Georg Reimer, Berlin, 1878. These volumes are listed on Library of Congress cards under the heading: Internationale erdmessung, Allgemaine Conferenz.

Peirce: CP 8 Abbreviations

[JHUC] *Johns Hopkins University Circulars*.

Peirce: CP 8 Abbreviations

[JHUR --]. . . . *Annual Report of Johns Hopkins University for the year --* , published in the year. . . . (Title pages for some years are unimportant variations of this.)

Peirce: CP 8 Abbreviations

[PAAAS] *Proceedings of the American Academy of Arts and Sciences*.

Peirce: CP 8 Abbreviations

[Perry] Ralph Barton Perry, *The Thought and Character of William James*, 2 vols., Little, Brown and Company, Boston, 1935.

Peirce: CP 8 Abbreviations

[RNA --] *Report of the National Academy of Sciences*, for the year -- .

Peirce: CP 8 Bibliography General 1860 [G-1860-1]
I. GENERAL

1860

1. Reference to Peirce's azimuth and magnetic observations. [CS 1859] 1860, p. 36.

Peirce: CP 8 Bibliography General 1861 [G-1861-1]
1861

1. Reference to Peirce's work on triangulation. [CS 1860] 1861, p. 86.

Peirce: CP 8 Bibliography General 1862 [G-1862-1]
1862

1. Reference to Peirce's having checked computations of Benjamin Peirce on occultations of the Pleiades. [CS 1861] 1862, p. 220. Cf. pp. 196-221.

Peirce: CP 8 Bibliography General 1863 [G-1863-1]
1863

- † 1. "The Chemical Theory of Interpenetration," dated "Dec. 1862," *The American Journal of Science and Arts* 2nd series 35(Jan 1863)78-82.

Peirce: CP 8 Bibliography General 1863 [G-1863-2]

2. Extracts from "The Place of Our Age in the History of Civilization," presented at the reunion of the Cambridge High School Association 12 Nov 1863, *Cambridge Chronicle* (a Cambridge, Mass., weekly newspaper) Vol. 18, No. 47, 21 Nov 1863, p. 1, columns 1-5.

Peirce: CP 8 Bibliography General 1864 [G-1864-1]
1864

1. "Shakespearian Pronunciation," (with J. B. Noyes) unsigned, *The North American Review* 98(April 1864)342-369 A note in a bound volume of Peirce's

reprints at Johns Hopkins University (*Papers on Logic*, Call No. BC10.P2) states that this paper "was written in collaboration with John Buttrick Noyes."

Peirce: CP 8 Bibliography General 1864 [G-1864-2]

2. Reference to Peirce's having made computations for Benjamin Peirce on occultations of the Pleiades. [CS 1863] 1864, pp. 146-154.

Peirce: CP 8 Bibliography General 1864 [G-1864-3]

3. Reference to a series of University Lectures, "On the Logic of Science," given by Peirce at Harvard during 1864-65. [Harvard 1865-66] 104; cf. [CP] 7.579n34. In a letter to F. E. Abbot dated March 17, 1865 (F. E. Abbot papers, Widener Archives), Peirce says "My lectures fell through for want of an audience."

Peirce: CP 8 Bibliography General 1866 [G-1866-1]
1866

1. Reference to Peirce's work on computations for longitude from occultations of the Pleiades. [CS 1864] 1866, p. 114.

Peirce: CP 8 Bibliography General 1866 [G-1866-2a]

2. (a) Reference to twelve lectures on "The Logic of Science and Induction," given by Peirce at the Lowell Institute during the winter of 1866-67. Harriette Knight Smith, *The History of the Lowell Institute*, 1898, p. 63. Cf. [CP] 2.641n1. [CP] 7.131-138 (c.1866) and 7.579-596 (c.1867), Widener IB2-10, are probably parts of Lectures IV and XI, respectively; cf. [CP] 7.579n34.

Peirce: CP 8 Bibliography General 1866 [G-1866-2b]

(b) *Memoranda Concerning the Aristotelian Syllogism*, privately printed and distributed at the Lowell Institute, Nov 1866, perhaps in connection with the above lectures. [CP] 2.792-807.

Peirce: CP 8 Bibliography General 1867 [G-1867-1]
1867

1. [PAAAS] series on logic. [CP] 4.2-4 contain comments on this series.

Peirce: CP 8 Bibliography General 1867 [G-1867-1a]

(a) "On an improvement in Boole's Calculus of Logic," 7(presented 12 March 1867)250-261. [CP] 3.1-19 with revisions of 1870.

Peirce: CP 8 Bibliography General 1867 [G-1867-1b]

* (b) "On the Natural Classification of Arguments," 7(presented 9 April 1867)261-287. Part II is in large part a restatement of [Bibliography] G-1866-2b. [CP]

2.461-516 with revisions of 1893.

Peirce: CP 8 Bibliography General 1867 [G-1867-1c]

* (c) "On a New List of Categories," 7(presented 14 May 1867)287298. [CP] 1.545-559 except 549n1.

Peirce: CP 8 Bibliography General 1867 [G-1867-1d]

(d) "Upon the Logic of Mathematics," 7(presented 10 Sept 1867) 402-412. [CP] 3.20-44.

Peirce: CP 8 Bibliography General 1867 [G-1867-1e]

* (e) "Upon Logical Comprehension and Extension," 7(presented 13 Nov 1867)416-432. [Bibliography] G-1893-7 is a supplement to it. [CP] 2.391-426 with revisions of c.1870 and 1893.

Peirce: CP 8 Bibliography General 1867 [G-1867-2]

* 2. Review of John Venn's *The Logic of Chance*, unsigned, *The North American Review* 105(July 1867)317-321. Identified in [Cohen]. Peirce refers to it in [CP] 3.19n1. [CP] 8.1-6.

Peirce: CP 8 Bibliography General 1868 [G-1868-1a]
1868

1. (a) "Nominalism *versus* Realism," an unsigned letter, *The Journal of Speculative Philosophy* 2(1868)57-61. Actually this is a portion of a letter from Peirce to William T. Harris, dated 24 Jan 1868, the original of which is in the Hoose Library of Philosophy, University of Southern California. This portion criticizes Harris' article, *ibid.*, 1(1867)250-256, and is interspersed with Harris' reply. [CP] 6.619-624, preceded by an introductory comment by Harris but without his reply.

Peirce: CP 8 Bibliography General 1868 [G-1868-1b]

(b) "What is Meant by 'Determined'?" an unsigned letter, *The Journal of Speculative Philosophy* 2(1868)190-191. This letter gives Peirce's rebuttal to Harris' reply to "Nominalism *versus* Realism" and contains Harris' rejoinder in footnotes. [CP] 6.625-630 without Harris' rejoinder.

Peirce: CP 8 Bibliography General 1868 [G-1868-2]

* 2. *Journal of Speculative Philosophy* 1868-69 series on intuitive knowledge.

Peirce: CP 8 Bibliography General 1868 [G-1868-2a]

(a) "Questions Concerning Certain Faculties Claimed for Man," 2(1868) 103-114. [CP] 5.213-263.

Peirce: CP 8 Bibliography General 1868 [G-1868-2b]

(b) "Some Consequences of Four Incapacities," 2(1868)140-157. [CP] 5.264-317.

Peirce: CP 8 Bibliography General 1868 [G-1868-2c]

(c) "Grounds of Validity of the Laws of Logic: Further Consequences of Four Incapacities," 2(1869)193-208. [CP] 5.318-357 with revisions of 1893.

Peirce: CP 8 Bibliography General c.1868 [G-c.1868-1]
c.1868

1. "A Philosopher's Political Diagnosis" (on proportional representation), *New York: A Symphonic Study*, by Melusina Fay Peirce, The Neale Publishing Co., New York, 1918, pp. 100-104. This comment is quoted from Peirce (p. 99) and was made between 1865 and 1870.

Peirce: CP 8 Bibliography General 1869 [G-1869-1]
1869

1. Reference to Peirce's latitude observations. [CS 1867] 1869, pp. 19-20.

Peirce: CP 8 Bibliography General 1869 [G-1869-2]

2. References to a series of nine lectures on "British Logicians," given by Peirce at Harvard during 1869-70. [Harvard 1869-70]102; [Perry]I, 321. [CP] 1.28-34 is from Lecture I, "Early Nominalism and Realism."

Peirce: CP 8 Bibliography General 1870 [G-1870-1]
1870

1. "Description of a Notation for the Logic of Relatives, resulting from an Amplification of the Conceptions of Boole's Calculus of Logic," *Memoirs of the American Academy of Arts and Sciences* ns 9(presented 26 Jan 1870) 317-378. Reprinted separately by Welch, Bigelow and Company, Cambridge, Mass., 1870, 62 pp. [CP] 3.45-149, except 45n*, with revisions from Peirce's own copy.

Peirce: CP 8 Bibliography General 1870 [G-1870-2]

† 2. Review of Henry James's *The Secret of Swedenborg: being an Elucidation of his Doctrine of the Divine Natural Humanity*, unsigned, *The North American Review* 110 (April 1870)463-468. Identified by [Perry] I, 533, and in a personal communication from Max Fisch.

Peirce: CP 8 Bibliography General 1871 [G-1871-1]
1871

* 1. Review of *The Works of GEORGE BERKELEY, D.D., formerly Bishop*

of Cloyne: including many of his writings hitherto unpublished (edited by Alexander Campbell Fraser), signed "C.S.P.," *The North American Review* 113 (Oct 1871) 449-472. [CP] 8.7-38.

Peirce: CP 8 Bibliography General 1871 [G-1871-2]

2. Reference to a series of thirty-five University Lectures on "Logic," given by Peirce at Harvard during the second term of 1870-71. [Harvard 1870-71]109.

Peirce: CP 8 Bibliography General 1871 [G-1871-3]

3. Reference to "On the Appearance of Encke's Comet as Seen at Harvard College Observatory," presented 16 Dec 1871. [BPSW]1(1874)35.

Peirce: CP 8 Bibliography General 1872 [G-1872-1]
1872

1. Items in [CS 1869]1872.

Peirce: CP 8 Bibliography General 1872 [G-1872-1a]

(a) References to Peirce's spectroscopic eclipse observations, pp. 39, 126, 187.

Peirce: CP 8 Bibliography General 1872 [G-1872-1b]

(b) Letter concerning the above, dated 20 Aug 1869, pp. 126-127.

Peirce: CP 8 Bibliography General 1872 [G-1872-1c]

(c) "Report on the Results of the Reduction of the Measures of the Photographs of the Partial Phases of the Eclipse of August 7, 1869 . . .," pp. 181-185.

Peirce: CP 8 Bibliography General 1872 [G-1872-2]

2. Reference to Peirce's presenting a paper on the photometric measurements of the stars and exhibiting an instrument for this purpose devised by Zöllner. [PAAAS]8 (12 March 1872)412.

Peirce: CP 8 Bibliography General 1872 [G-1872-2]

Reference to "On Stellar Photometry," presented 19 Oct 1872. [BPSW] 1 (1874)63.

Peirce: CP 8 Bibliography General 1872 [G-1872-3]

3. Abstract of paper, "On the Coincidence of the Geographical Distribution of Rainfall and of Illiteracy, as shown by the Statistical Maps of the Ninth Census Reports," presented 21 Dec 1872, [BPSW]1(1874)68. Cf. [CP] 2.752.

Peirce: CP 8 Bibliography General 1873 [G-1873-1]
1873

1. Reference to "On Logical Algebra," presented 17 May 1873.
[BPSW]1(1874)88; cf. p. 87. Cf. [Bibliography] G-1870-1.

Peirce: CP 8 Bibliography General 1873 [G-1873-2]
2. Items in [CS 1870] 1873.

Peirce: CP 8 Bibliography General 1873 [G-1873-2a]
(a) Letter to Benjamin Peirce (Superintendent, Coast Survey) reporting Peirce's observations of the total solar eclipse of 22 Dec 1870 (made at Catania, Sicily), p. 125.

This is followed by a letter from Mrs. Charles S. Peirce, reporting on her observations of the eclipse, pp. 125-127, with a sketch near the end of the volume.

Peirce: CP 8 Bibliography General 1873 [G-1873-2]
Cf. also references to Peirce on pp. 137, 230, 231.

Peirce: CP 8 Bibliography General 1873 [G-1873-2b]
(b) Appendix No. 21, "On the Theory of Errors of Observations," pp. 200-224 (with errata on a slip inserted after p. 200) and Diagram No. 27.

Peirce: CP 8 Bibliography General c.1873 [G-c.1873-1]
c.1873

* 1. "Logic," Widener IB2-8. [CP] 7.313-361, except 313n1, are from it. Cf. [CP] 7.313n1.

Peirce: CP 8 Bibliography General 1874 [G-1874-1]
1874

1. Reference to "On Quaternions, as Developed from the General Theory of the Logic of Relatives," presented 3 Jan 1874. [BPSW] 1 (1874) 94.

Peirce: CP 8 Bibliography General 1874 [G-1874-2]
2. Reference to "On Various Hypotheses in Reference to Space," presented 14 March 1874. [BPSW]1(1874)97.

Peirce: CP 8 Bibliography General 1874 [G-1874-3]
3. References to the work of Peirce and his wife in connection with the solar eclipse of 22 Dec 1870. [CS 1871] 1874, pp. 10, 11, 182. (Pages 10 and 11 are the same as [CS 1870]1873, pp. 230, 231.)

Peirce: CP 8 Bibliography General 1875 [G-1875-1]
1875

1. Reference to "Photometric Measurements of the Stars." [PAAAS]ns 2, ws 10(9 March 1875)473. Cf. [Bibliography] G-1872-2.

Peirce: CP 8 Bibliography General 1875 [G-1875-2]

2. "On the Application of Logical Analysis to Multiple Algebra," [PAAAS]ns 2, ws 10(presented 11 May 1875)392-394. [CP] 3.150-151. See [CP] 3.648 for a possible reference to this paper.

Peirce: CP 8 Bibliography General 1875 [G-1875-3]

3. "A Plan and an Illustration" (on proportional representation), *The Democratic Party; A Political Study*, by a Political Zero (Melusina Fay Peirce), John Wilson and Son, Cambridge, 1875, pp. 36-37. Both the whole work and Peirce's contribution are anonymous, but these are identified in [Fisch-Haskell].

Peirce: CP 8 Bibliography General 1875 [G-1875-4]

4. Items in [CS 1873] 1875. See p. 60 and also [CS 1872] 1875, p. 50, for biographical remarks.

Peirce: CP 8 Bibliography General 1875 [G-1875-4a]

(a) Reference to Peirce's pendulum experiments, p. 14.

Peirce: CP 8 Bibliography General 1875 [G-1875-4b]

(b) Appendix No. 14, "A List of Stars for Observations of Latitude," prepared under Peirce's direction, pp. 138-174.

Peirce: CP 8 Bibliography General 1875 [G-1875-4c]

(c) Appendix No. 15, "Errata in the Heis Catalogue of Stars," prepared under Peirce's direction, pp. 175-180.

Peirce: CP 8 Bibliography General 1875 [G-1875-5]

5. References to Peirce's participation in the meetings of the Permanent Committee of the International Geodetic Association, 1885. [International Geodetic Comm., Paris, 1875]1875, pp. 13, 16, 19-23, 32, 51, 54, 58-61, 71. Cf. [CS 1876]1879, pp. 8-9 and [CS 1881]1883, p. 360. See also [International Geodetic Comm., Brussels, 1876] 1877, pp. 12, 18, 19, 41-42, 47-49, for references to Peirce's work. Cf. [CS 1881]1883, p. 360.

Peirce: CP 8 Bibliography General c.1875 [G-c.1875-1]
c.1875

1. "Third," a fragment. [CP] 1.337.

Peirce: CP 8 Bibliography General 1876 [G-1876-1]
1876

1. "Logical Contraposition and Conversion," *Mind* 1(July 1876)424-425. This is a comment on a statement by the Editor (George Croom Robertson), *ibid.*, (Jan 1876)148; the Editor's reply is at *ibid.*, 1(July 1876)425. [CP] 2.550.

Peirce: CP 8 Bibliography General 1876 [G-1876-2]

2. Reference to "On a new edition of Ptolemy's Catalogue of Stars." [PAAAS]ns 4, ws 12(11 Oct 1876)283.

Peirce: CP 8 Bibliography General 1877 [G-1877-1]
1877

1. "Note on the Sensation of Color," *The American Journal of Science and Arts* 3rd series 13(April 1877)247-251. Reprinted with slight omissions in *Philosophical Magazine and Journal of Science* 5th series 3(1877) 543-547.

Peirce: CP 8 Bibliography General 1877 [G-1877-2]

2. The unsigned obituary of Nicholas St. John Green in [PAAAS]ns 4, ws 12(9 May 1877)289-291, contains a quotation (pp. 290-291) from a letter from "one who was familiar with his modes of thought" (p. 290) about Green. Max Fisch suggests this letter was written by Peirce.

Peirce: CP 8 Bibliography General 1877 [G-1877-3]

3. "De l'Influence de la Flexibilité du Trépied sur l'Oscillation du Pendule à Réversion," [International Geodetic Conference, Stuttgart, 1877]1878, pp. 171-187, with a letter of introduction by E. Plantamour (pp. 171-172). See pp. 188-190 of the proceedings for comments by Th. von Oppolzer on Peirce's paper. See pp. 4, 20, 23, 100-101, 118, 139, for references to Peirce's attendance and remarks at the conference. E. Plantamour, *Recherches Expérimentales sur le Mouvement Simultané d'un Pendule et de ses Supports* is an appendix. See pp. 3-5 for references to Peirce. A translation by Peirce is printed at [CS 1881] 1883, pp. 427-436, with revisions and notes of 1882; [Bibliography] G-1883-5b.

Peirce: CP 8 Bibliography General 1877 [G-1877-4]

4. "Note on Grassmann's Calculus of Extension," [PAAAS] ns 5, ws 13(presented 10 Oct 1877)115-116. [CP] 3.152-153 with revisions.

Peirce: CP 8 Bibliography General 1877 [G-1877-5]
* 5. "Illustrations of the Logic of Science," *The Popular Science Monthly*, 1877-1878.

Peirce: CP 8 Bibliography General 1877 [G-1877-5a]
(a) "The Fixation of Belief," 12(Nov 1877) 1-15. [CP] 5.358-387, except 358n*, with revisions and notes of 1893, 1903, and c.1910.

Peirce: CP 8 Bibliography General 1877 [G-1877-5b]
(b) "How to Make Our Ideas Clear," 12(Jan 1878) 286-302. [CP] 5.388-410, except 402n3, with revisions and notes of 1893 and 1903.

Peirce: CP 8 Bibliography General 1877 [G-1877-5c]
(c) "The Doctrine of Chances," 12(March 1878) 604-615. [CP] 2.645-660 with revisions of 1893 and a note of 1910.

Peirce: CP 8 Bibliography General 1877 [G-1877-5d]
(d) "The Probability of Induction," 12(April 1878) 705-718. [CP] 2.669-693.

Peirce: CP 8 Bibliography General 1877 [G-1877-5e]
(e) "The Order of Nature," 13(June 1878)203-217. [CP] 6.395-427.

Peirce: CP 8 Bibliography General 1877 [G-1877-5f]
(f) "Deduction, Induction, and Hypothesis," 13(Aug 1878)470-482. [CP] 2.619-644.

Peirce: CP 8 Bibliography General 1877 [G-1877-6]
6. Reference to Peirce's pendulum observations. [CS 1874] 1877, p. 18.

Peirce: CP 8 Bibliography General 1878 [G-1878-1]
1878

1. "On the Influence of Internal Friction upon the Correction of the Length of the Second's Pendulum for the Flexibility of the Support," [PAAAS]ns 5, ws 13 (presented 13 March 1878)396-401; cf. p. 433. Reprinted at [CS 1881]1883, pp. 437-441; [Bibliography] G-1883-5b.

Peirce: CP 8 Bibliography General 1878 [G-1878-2]
2. Review of Annibale Ferrero's *Esposizione del Metodo dei Minimi Quadrati*, [AJM] 1 (1878) 59-63.

Peirce: CP 8 Bibliography General 1878 [G-1878-3]
3. Reference to "On the acceleration of gravity at initial stations," presented 5-8 Nov 1878. [RNA 1883]49.

Peirce: CP 8 Bibliography General 1878 [G-1878-4]

4. A brief statement of Peirce's experimental pendulum work during 1878, [International Geodetic Comm., Hamburg, 1878]1879, pp. 116-120. "The details of all this work will be given in my forthcoming memoir upon the comparison of the European and American pendulum work" (p. 120). Cf. [Bibliography] G-1879-5d. See also [International Geodetic Comm., Geneva, 1879] 1880, pp. 9, 19-20, for references to Peirce's work.

Peirce: CP 8 Bibliography General 1878 [G-1878-5]

5. Items in [CS 1875]1878.

Peirce: CP 8 Bibliography General 1878 [G-1878-5a]

(a) Reference to Peirce's pendulum observations, p. 19.

Peirce: CP 8 Bibliography General 1878 [G-1878-5b]

(b) Appendix No. 15, "Description of an Apparatus for Recording the Mean of the Times of a Set of Observations," pp. 249-253, with a plate facing p. 250.

Peirce: CP 8 Bibliography General 1878 [G-1878-6]

6. *Photometric Researches*, Vol. 9 of *Annals of the Astronomical Observatory of Harvard College*, Leipzig, 1878, vi + 181 pp. + errata sheet + five diagrams. Cf. [Bibliography] G-1872-2 and G-1875-1.

Peirce: CP 8 Bibliography General 1878 [G-1878-6]

"The chief end of observations of the magnitudes of stars is to determine the form of the cluster in which our sun is situated. . . . I shall . . . [endeavor] to show the general forms of the surfaces of equal star density throughout the cluster"
Photometric Researches, p. 174.

Peirce: CP 8 Bibliography General 1878 [G-1878-6]

Solon I. Bailey (in *The History and Work of Harvard Observatory, 1839-1927*, Harvard Observatory Monograph No. 4, McGraw-Hill, New York, 1931) says of the work reported in *Photometric Researches*: "The first attempt at the Harvard Observatory to determine the form of the Milky Way, or the galactic system, was made by Charles S. Peirce. . . . The investigation was of a pioneer nature, founded on scant data" (pp. 198-199). Cf. pp. 52, 53, 86, 95, 110, 124, 125, 196, 240, 260, 274, for further references to Peirce.

Peirce: CP 8 Bibliography General 1878 [G-1878-7]

7. "La Logique de la Science," *Revue Philosophique*, 1878-1879.

Peirce: CP 8 Bibliography General 1878 [G-1878-7a]

(a) "Comment se fixe la croyance," 6(Dec 1878)553-569. This is a republication with some changes of [Bibliography] G-1877-5a. The last paragraph is omitted, and "or to a reformed Catholic . . . Bible" is deleted from the penultimate paragraph.

Peirce: CP 8 Bibliography General 1878 [G-1878-7b]

(b) "Comment rendre nos idées claires," 7(Jan 1879) 39-57. This is a republication with some changes of [Bibliography] G-1877-5b. Everything is deleted after "I will not trouble the reader with any more Ontology at this moment" which occurs in the penultimate paragraph of the English version. Erratum: delete "ne . . . pas" from line 22 of page 56. See [Fisch-Cope] ftn. 35 for Peirce's comment on the two articles of this series.

Peirce: CP 8 Bibliography General 1879 [G-1879-1]
1879

1. "Note on the Progress of Experiments for comparing a Wave-length with a Meter," *The American Journal of Science and Arts* 3rd series 18(July 1879)51.

Peirce: CP 8 Bibliography General 1879 [G-1879-1]

Reference to "Comparisons of the meter with wave lengths," presented 15-18 April 1879. [RNA 1883]50.

Peirce: CP 8 Bibliography General 1879 [G-1879-1]

Reference to "On the Reference of the Unit of Length to the WaveLengths of Light." [PAAAS]ns 7, ws 15 (presented 11 June 1879)370.

Peirce: CP 8 Bibliography General 1879 [G-1879-2]

2. "On a method of Swinging Pendulums for the Determination of Gravity, proposed by M. Faye," *The American Journal of Science and Arts* 3rd series 18, ws 118(Aug 1879)112-119.

Peirce: CP 8 Bibliography General 1879 [G-1879-2]

Reference to "On the errors of pendulum experiments, and on the method of swinging pendulums proposed by M. Faye," presented 17 April 1879. [RNA 1883]50.

Peirce: CP 8 Bibliography General 1879 [G-1879-3]

3. "On the Ghosts in Rutherford's Diffraction-Spectra," [AJM]2 (1879)330-347.

Peirce: CP 8 Bibliography General 1879 [G-1879-3]

Reference to "Ghosts in the diffraction spectra," presented 15-18 April 1879. [RNA 1883] 50.

Peirce: CP 8 Bibliography General 1879 [G-1879-4]

4. "A Quincuncial Projection of the Sphere," [AJM]2(1879) 394-396 with one plate. Erratum, *ibid.*, 3(1880)v.

Peirce: CP 8 Bibliography General 1879 [G-1879-4]

Reference to "On the projections of the sphere which preserve the angles," presented 15-18 April 1879. [RNA 1883]50.

Peirce: CP 8 Bibliography General 1879 [G-1879-5]

5. Items in [CS 1876]1879.

Peirce: CP 8 Bibliography General 1879 [G-1879-5a]

(a) Reference to Peirce's pendulum observations at European stations, pp. 6-9.

Peirce: CP 8 Bibliography General 1879 [G-1879-5b]

(b) The list of stars given in Appendix No. 7, "A Catalogue of Stars for Observations of Latitude," pp. 83-129, was selected under the direction of Peirce. The data from Ptolemy's catalogue "is based on Mr. Peirce's transcript of the Paris manuscript, an account of which he has presented to the American Academy of Arts and Sciences" (p. 84). Cf. [Bibliography] G-1876-2. Reprinted with the additional heading, "Methods and Results," Washington, 1879.

Peirce: CP 8 Bibliography General 1879 [G-1879-5c]

* (c) Appendix No. 14, "Note on the Theory of the Economy of Research," pp. 197-201. [CP] 7.139-157 with some corrections from a ms. version in Widener IC2a.

Peirce: CP 8 Bibliography General 1879 [G-1879-5d]

(d) Appendix No. 15, "Measurements of Gravity at Initial Stations in America and Europe," dated 13 Dec 1878, pp. 202-337, with fifteen plates, and pp. 410-416. Cf. [Bibliography] G-1878-3. See [CS 1883]1884, p. 476 (ftn.) for a correction. See *The American Journal of Science* 3rd series 20(Oct 1880)327 for an erratum. Reprinted with the additional heading, "Methods and Results," Washington, 1879.

Peirce: CP 8 Bibliography General 1879 [G-1879-6]

6. Reference to "Geographical Problem of the Four Colors," presented to the Scientific Association, 5 Nov and 3 Dec 1879. [JHUC] 1 (Jan 1880) 16.

Peirce: CP 8 Bibliography General 1879 [G-1879-7]

7. Abstract of a paper, "Questions concerning certain Faculties claimed for Man," presented to the Metaphysical Club, 11 Nov 1879, [JHUC]1 (Jan 1880)18. Cf. [Bibliography] G-1868-2a.

Peirce: CP 8 Bibliography General 1880 [G-1880-1]
1880

1. References to Peirce's courses at Johns Hopkins University for 1879-80. [JHUC]1(Dec 1879)6, 7; 1(Jan 1880)12; 1(Feb 1880)25; 1(May 1880)62; 1(Aug 1880)71; [JHUR 1880] 1880, p. 50.

Peirce: CP 8 Bibliography General 1880 [G-1880-2]

2. Reference to Peirce's comments on Alan Marquand's talk on a treatise of Philodemus at the Metaphysical Club, 13 Jan 1880. [JHUC]1(Feb. 1880)34.

Peirce: CP 8 Bibliography General 1880 [G-1880-3]

3. Reference to Peirce's discussion of Stringham's "A Generalization, for n -fold Space, of Euler's Equation for Polyhedra," at the Mathematical Seminary, 21 Jan 1880. [JHUC]1(Feb 1880)35.

Peirce: CP 8 Bibliography General 1880 [G-1880-4]

4. Abstract of a comment on Jevons and Schroeder made at the Metaphysical Club, 9 March 1880, [JHUC]1(April 1880)49.

Peirce: CP 8 Bibliography General 1880 [G-1880-5]

5. Abstract of "On Kant's 'Critic of the Pure Reason' in the light of Modern Logic," presented to the Metaphysical Club, 9 March 1880, [JHUC] 1 (April 1880)49.

Peirce: CP 8 Bibliography General 1880 [G-1880-6]

6. "Results of Pendulum Experiments," *The American Journal of Science* 3rd series 20, ws 120(Oct 1880)327. This note gives the main results of [Bibliography] G-1879-5d. Reprinted in *Philosophical Magazine and Journal of Science* 5th series 10(Nov 1880)387.

Peirce: CP 8 Bibliography General 1880 [G-1880-7]

7. References to Peirce's courses at Johns Hopkins University for 1880-81. [JHUC]1(Dec 1880)76, 77; 1(April 1881)124, 125; [JHUR 1881]1882, p. 16; Ellery W. Davis, "Charles Peirce at Johns Hopkins," *The Mid-West Quarterly* 2(Oct 1914)48-56.

Peirce: CP 8 Bibliography General 1880 [G-1880-8]

8. "On the Algebra of Logic," [AJM]3(1880)15-57. The original ends with "to be continued"; cf. same title [Bibliography] G-1885-3. [CP] 3.154-251, except 154n1 and 200n* (p. 128), with revisions of 1880 and c.1882 and undated revisions.

Peirce: CP 8 Bibliography General 1880 [G-1880-9]

9. "Sur la valeur de la pesanteur à Paris," *Comptes Rendus des Séances de l'Académie des Sciences* 90 (14 June 1880)1401-1403. See comments by M. Faye, *ibid.*, (21 June 1880) 1443, 1463-1466. A translation by Peirce is printed at [CS 1881] 1883, pp. 461-462; [Bibliography] G-1883-5e.

Peirce: CP 8 Bibliography General 1880 [G-1880-10]

10. Reference to "On the ellipticity of the earth as deduced from the pendulum experiments," presented 16-19 Nov 1880. [RNA 1883]53.

Peirce: CP 8 Bibliography General 1880 [G-1880-11]

11. Items in [CS 1877]1880.

Peirce: CP 8 Bibliography General 1880 [G-1880-11a]

(a) Reference to Peirce's pendulum experiments, pp. 17-18.

Peirce: CP 8 Bibliography General 1880 [G-1880-11b]

(b) Appendix No. 15, "A Quincuncial Projection of the Sphere," pp. 191-192, and sketch 25. The same as [Bibliography] G-1879-4 ([AJM]). Reprinted in Thomas Craig, *A Treatise on Projections*, Govt. Printing Office, Washington, 1882, pp. 132, 247.

Peirce: CP 8 Bibliography General 1880 [G-1880-12]

12. Letter to M. Faye concerning Peirce's gravity work, [International Geodetic Conference, Munich, 1880]1881, pp. 30-32; repeated on pp. 84-86. See pp. 43, 96, and Appendix IIa, pp. 5-8, for references to Peirce. See also the following references to Peirce's work: [International Geodetic Conference, Rome, 1883]1884, pp. 41, 44-45, 50-52, 59-60, of Appendix VIb; [International Geodetic Comm., Nice, 1887]1888, pp. 1, 2, 15-16, Table IV of Appendix IIa and pp. 1, 3, 15-17, Table IV of Appendix IIc; [International Geodetic Conference, Paris, 1900]1901, Vol. 2, pp. 330-335 of Appendix IX.

Peirce: CP 8 Bibliography General c.1880 [G-c.1880-1]
c.1880

1. On a Boolean algebra with one constant. [CP] 4.12-20.

Peirce: CP 8 Bibliography General c.1880 [G-c.1880-2]

2. "One, Two, Three." [CP] 1.353 is from it.

Peirce: CP 8 Bibliography General 1881 [G-1881-1]
1881

1. Abstract of "A New Computation of the Compression of the Earth, from Pendulum Experiments," presented to the Scientific Association, Feb 1881, [JHUC]1(April 1881)128.

Peirce: CP 8 Bibliography General 1881 [G-1881-2]

2. Reference to Peirce's comments on O. H. Mitchell's "On Binomial Congruences" at the Mathematical Seminary, March 1881. [JHUC] 1 (April 1881)132.

Peirce: CP 8 Bibliography General 1881 [G-1881-3]

3. Reference to "On the progress of pendulum work," presented 19-22 April 1881. [RNA 1883]53. Cf. [Bibliography] G-1880-6.

Peirce: CP 8 Bibliography General 1881 [G-1881-4]

4. Reference to "On Relations between Sensations," presented to the Metaphysical Club, April 1881. [JHUC]1 (July 1881)150.

Peirce: CP 8 Bibliography General 1881 [G-1881-5]

5. "Width of Mr. Rutherford's Rulings," *Nature* 24(21 July 1881)262.

Peirce: CP 8 Bibliography General 1881 [G-1881-5]

"There is a solar spectral line, well suited for precise observation . . . I would propose that this line be adopted as a standard of reference by such observers of wave-lengths as desire to escape the arduous operation of measuring the mean width of their rulings . . ."

Peirce: CP 8 Bibliography General 1881 [G-1881-6]

6. References to Peirce's courses at Johns Hopkins University for 1881-82. [JHUC]1(July 1881)139, 142; 1(Dec 1881) 157, 159, 160, 161; 1(March 1882)195, 196; 1(July 1882)218, 233, 234; [JHUR 1882]1882, pp. 17, 47, 61.

Peirce: CP 8 Bibliography General 1881 [G-1881-7]

7. "On the Logic of Number," [AJM]4(1881)85-95. [CP] 3.252-288 with undated revisions.

Peirce: CP 8 Bibliography General 1881 [G-1881-7]

Reference to "On the logic of number," presented 15-17 Nov 1881. [RNA 1883]54.

Peirce: CP 8 Bibliography General 1881 [G-1881-8]

8. References to "A Fallacy of Induction," presented to the Scientific Association, Nov 1881. [JHUC]1(Feb 1882)172; Ellery W. Davis, "Charles Peirce at Johns Hopkins," *The Mid-West Quarterly* 2(Oct 1914) 52-53.

Peirce: CP 8 Bibliography General 1881 [G-1881-9]

9. Reference to Peirce's comments on B. I. Gilman's "Theories of Induction," at the Metaphysical Club, Nov 1881. [JHUC]1(Feb 1882)177.

Peirce: CP 8 Bibliography General 1881 [G-1881-10]

10. Notes and two addenda to Benjamin Peirce's "Linear Associative Algebra," [AJM]4(1881)97-229. Errata for the notes, *ibid.*, p. iv. The two addenda are: Addendum II, "On the Relative Forms of the Algebras," pp. 221-225; Addendum III, "On the Algebras in which Division is Unambiguous," pp. 225-229. [CP]

3.289-305 contain the two addenda. *Linear Associative Algebra*, including Peirce's notes, was reprinted by van Nostrand, New York, 1882.

Peirce: CP 8 Bibliography General 1881 [G-1881-10]

"Proof that there are only Three Linear Associative Algebras in which Division is an Unambiguous Process," presented to the Mathematical Seminary, Jan 1881. [JHUC]1(April 1881) 131.

Peirce: CP 8 Bibliography General 1881 [G-1881-11]

11. Reference to Peirce's pendulum experiments and work on meter. [CS 1878]1881, p. 18. "The records of the various experiments mentioned in this abstract are contained in twenty-six volumes."

Peirce: CP 8 Bibliography General 1881 [G-1881-12]

12. Report on Peirce's researches on pendula, using light as a standard of length, and projections, with quotations from Peirce, [CS 1879]1881, pp. 27-29.

Peirce: CP 8 Bibliography General 1882 [G-1882-1]
1882

1. *A Brief Description of the Algebra of Relatives*, Baltimore, 7 Jan 1882, 6 pp., with a postscript of 16 Jan 1882. A privately printed brochure; cf. [CP] 3.294, 3.306. [CP] 3.306-322.

Peirce: CP 8 Bibliography General 1882 [G-1882-1]

Reference to "On the logic of relatives," read 14-17 Nov 1882. [RNA 1883]55.

Peirce: CP 8 Bibliography General 1882 [G-1882-2]

2. Reference to "J. S. Mill's Logic," presented to the Metaphysical Club, Jan 1882. [JHUC]1(Feb 1882)178.

Peirce: CP 8 Bibliography General 1882 [G-1882-3]

3. Abstract of "On the Relative Forms of Quaternions," presented to the Mathematical Seminary, Jan 1882, [JHUC]1(Feb 1882)179. [CP] 3.323.

Peirce: CP 8 Bibliography General 1882 [G-1882-4]

4. Reference to "On a fallacy in induction," presented 18-21 April 1882. [RNA 1883]54. Cf. [Bibliography] G-1881-8.

Peirce: CP 8 Bibliography General 1882 [G-1882-5]

5. Abstract of remarks on B. I. Gilman's "On Propositions and the Syllogism," both presented to the Metaphysical Club, April 1882; both published in [JHUC]1(Aug 1882)240.

Peirce: CP 8 Bibliography General 1882 [G-1882-6]

6. References to Peirce's course at Johns Hopkins University for 1882-83. [JHUC]1(July 1882)234; 2(Nov 1882) 18; 2(Dec 1882)39; 2(April 1883)93; [JHUR 1883]1883, p. 35.

Peirce: CP 8 Bibliography General 1882 [G-1882-6]

*An abstract of one lecture of the course, "Introductory Lecture on the Study of Logic," presented Sept 1882, [JHUC]2(Nov 1882)11-12. [CP] 7.59-76 are from it.

Peirce: CP 8 Bibliography General 1882 [G-1882-7]

7. "On Irregularities in the Amplitude of Oscillation of Pendulums," *The American Journal of Science* 3rd series 24(Oct 1882)254-255.

Peirce: CP 8 Bibliography General 1882 [G-1882-8]

8. Abstract of "On a Class of Multiple Algebras," presented to the University Mathematical Society, 18 Oct 1882, with a note of 30 Oct, [JHUC]2(Nov 1882)3-4. [CP] 3.324-327.

Peirce: CP 8 Bibliography General 1882 [G-1882-9]

9. Brief summary of opening remarks on the object of the Metaphysical Club, presented 14 Nov 1882, [JHUC]2(Dec 1882)38.

Peirce: CP 8 Bibliography General 1882 [G-1882-10]

10. Reference to "On the determination of the figure of the earth by the variations of gravity," presented 14-17 Nov 1882. [RNA 1883]55. Cf. [Bibliography] G-1880-10.

Peirce: CP 8 Bibliography General 1882 [G-1882-11]

11. Reference to "On Ptolemy's catalogue of stars," presented 14-17 Nov 1882. [RNA 1883]55. Cf. [Bibliography] G-1879-5b.

Peirce: CP 8 Bibliography General 1882 [G-1882-12]

12. Reference to Peirce's pendulum observations and comparison of the meter with a wave length of light. [CS 1880] 1882, pp. 19-20.

Peirce: CP 8 Bibliography General 1882 [G-1882-13]

13. References to Peirce's telescopic observations made during 1866-1872.

Joseph Winlock and Edward C. Pickering, *Micrometric Measurements*, Part I of Vol. 13 of *Annals of the Astronomical Observatory of Harvard College*, Cambridge, 1882, pp. iv, 19ff., 28ff., 63ff., 86ff., 172ff., 184ff.

Peirce: CP 8 Bibliography General 1882 [G-1882-14]

14. Reference to "Report on the spectrum meter (U. S. Superintendent of Weights and Measures, Report, in press)." [JHUR 1882]1882, p. 112. The publication date is given as 1882 in *Bibliographia Hopkinsiensis*, 1876-1893, Part IV, Physics, p. 18 (volume page 120). The editor has been unable to find such a publication and questions that the report was published.

Peirce: CP 8 Bibliography General 1883 [G-1883-1]
1883

1. "A Communication from Mr. Peirce," [JHUC]2(April 1883) 86-88. This is Peirce's answer to J. J. Sylvester's "Erratum," *ibid.*, (Feb 1883)46. Sylvester's rebuttal is at *ibid.*, (April 1883)86. See in this connection the footnote reference to Sylvester's comment of *ibid.*, 1(May 1882)203 supplied by Peirce, and Sylvester's comment on Peirce, *ibid.*, 1(Feb 1882) 180. [CP] 3.646-648.

Peirce: CP 8 Bibliography General 1883 [G-1883-2]

2. References to Peirce's courses at Johns Hopkins University for 1883-84. [JHUC]2(June 1883)119, 136; 3(Nov 1883)27, 28; 3(March 1884)69; 3(June 1884)101, 119; [JHUR 1884]1884, p. 38; [Fisch-Cope] 290-291.

Peirce: CP 8 Bibliography General 1883 [G-1883-3]

3. Reference to "Reply to Professor Morris on 'Life'," presented to the Metaphysical Club, 13 Nov 1883. [JHUC] 3(Jan 1884)46.

Peirce: CP 8 Bibliography General 1883 [G-1883-4]

4. "A New Rule for Division in Arithmetic," *Science* 2 (21 Dec 1883) 788-789.

Peirce: CP 8 Bibliography General 1883 [G-1883-5]

5. Items in [CS 1881] 1883.

Peirce: CP 8 Bibliography General 1883 [G-1883-5a]

(a) Reference to Peirce's pendulum observations, p. 26.

Peirce: CP 8 Bibliography General 1883 [G-1883-5b]

(b) Appendix No. 14, "On the Flexure of Pendulum Supports," pp. 359-441. Pages 427-436 contain a translation by Peirce of [Bibliography] G-1877-3 with

revisions and notes of 1882. Pages 437-441 contain a reprint of [Bibliography] G-1878-1. Erratum, p. 463. [CP] 7.1-12 are from pp. 359-361.

Peirce: CP 8 Bibliography General 1883 [G-1883-5c]

(c) Appendix No. 15, "On the Deductions of the Ellipticity of the Earth from Pendulum Experiments," pp. 442-456. Cf. [Bibliography] G-1880-10 and G-1882-10.

Peirce: CP 8 Bibliography General 1883 [G-1883-5d]

(d) Appendix No. 16, "On a Method of Observing the Coincidence of Vibration of Two Pendulums," dated 2 Aug 1878 with a note dated 20 Feb 1883, pp. 457-460.

Peirce: CP 8 Bibliography General 1883 [G-1883-5e]

(e) Appendix No. 17, "On the Value of Gravity at Paris," pp. 461-463. Pages 461-462 contain a translation of [Bibliography] G-1880-9.

Peirce: CP 8 Bibliography General 1883 [G-1883-6]

6. Items in [CS 1882]1883.

Peirce: CP 8 Bibliography General 1883 [G-1883-6a]

(a) References to Peirce's pendulum work, pp. 4, 19, 32-33, 557.

Peirce: CP 8 Bibliography General 1883 [G-1883-6b]

(b) Appendix No. 22, "Report of a Conference on Gravity Determinations, Held at Washington, in May, 1882," pp. 503-516. Contributions by Peirce at pp. 506-508 ("Six Reasons for the Prosecution of Pendulum Experiments"), 508, 509, 512-516, with errata in the quarter sheet between pp. 502-503. Reprinted with some changes and with the additional heading, "Methods and Results," Washington, 1883. [CP] 7.13-20, "Six Reasons for the Prosecution of Pendulum Experiments," as corrected from a reprint in Widener IC2c.

Peirce: CP 8 Bibliography General 1883 [G-1883-7]

7. *Studies in Logic, By Members of the Johns Hopkins University* (edited by Peirce), Little, Brown and Company, Boston, 1883, vi+203 pp.; also published by University Press, John Wilson and Son, Cambridge (copyright 1883 by C. S. Peirce). The following are by Peirce.

Peirce: CP 8 Bibliography General 1883 [G-1883-7a]

(a) "Preface," pp. iii-vi. Reprinted in [JHUC]2(Dec 1882)34 with slight differences.

Peirce: CP 8 Bibliography General 1883 [G-1883-7b]

* (b) "A Theory of Probable Inference," pp. 126-181; two maps between pp. 178-179. [CP] 2.694-754 with the omission of maps and the addition of changes from Peirce's own copy. Cf. [CP] 2.752 with [Bibliography] G-1872-3.

Peirce: CP 8 Bibliography General 1883 [G-1883-7c]

(c) Note A: "Extension of the Aristotelian Syllogistic," pp. 182-186. [CP] 2.517-531, as rewritten in 1893 for *Grand Logic*.

Peirce: CP 8 Bibliography General 1883 [G-1883-7d]

(d) Note B: "The Logic of Relatives," pp. 187-203. Cf. [Bibliography]

G-1882-1. [CP] 3.328-358 with a marginal note and indications of the revisions of 1893 for *Grand Logic*.

Peirce: CP 8 Bibliography General 1883 [G-1883-8]

8. References to two lectures, "The Observational Element in Mathematics," and "The *a priori* Element in Physics," presented in a pedagogical series given 1883-84. [JHUC]3(Jan 1884)32 and 3(June 1884)119.

Peirce: CP 8 Bibliography General 1884 [G-1884-1]
1884

1. References to "On the Mode of Representing Negative Quantity in the Logic of Relatives," presented to the Mathematical Society 16 Jan 1884. [JHUC]3(March 1884)70 and 3(June 1884)102.

Peirce: CP 8 Bibliography General 1884 [G-1884-2]

2. Reference to "Design and Chance," presented to the Metaphysical Club, 17 Jan 1884. [JHUC]3(March 1884)70.

Peirce: CP 8 Bibliography General 1884 [G-1884-3]

3. Reference to "The Logic of Religion," presented to the Metaphysical Club, 13 May 1884. [JHUC]3(July 1884)138.

Peirce: CP 8 Bibliography General 1884 [G-1884-4]

4. Reference to "On Gravitation Survey," presented 14-17 Oct 1884. [RNA 1884]12.

Peirce: CP 8 Bibliography General 1884 [G-1884-5]

5. Reference to "On the Algebra of Logic," presented 14-17 Oct 1884. [RNA 1884]13.

Peirce: CP 8 Bibliography General 1884 [G-1884-6]

6. "The numerical measure of the success of predictions," a letter to the Editor, *Science* 4(14 Nov 1884) 453-454.

Peirce: CP 8 Bibliography General 1884 [G-1884-7]

7. Reference to "On 'The Magnet' a fourteenth century manuscript of Petrus Peregrinus," presented to the Metaphysical Club, 18 Nov 1884. [JHUC]4(Dec 1884)28. Cf. [Bibliography] G-c.1893-4.

Peirce: CP 8 Bibliography General 1884 [G-1884-8]

8. "The 'Old Stone Mill' at Newport," *Science* 4(5 Dec 1884) 512-514.

Peirce: CP 8 Bibliography General 1884 [G-1884-9]

9. Items in [CS 1883]1884.

Peirce: CP 8 Bibliography General 1884 [G-1884-9a]

(a) References to Peirce's pendulum work, and determination of longitude, pp. 27, 36-37, 41-42, 42. Cf. p. 97.

Peirce: CP 8 Bibliography General 1884 [G-1884-9b]

(b) Appendix No. 19, "Determinations of Gravity at Allegheny, Ebensburg, and York, Pa., in 1879 and 1880," pp. 473-487. Apparently this is the paper promised under the title, "Experimental Researches on the Force of Gravity," [Bibliography] G-1883-6a, p. 557. Reprinted with the heading, *Methods and Results, Determinations of Gravity at Stations in Pennsylvania*, Govt. Printing Office, Washington, 1884, 58 pp. Pages 1-15 are identical with pp. 473-487 of the appendix (except for a number omitted on p. 3); pp. 16-58 contain tables of data not included in the main report.

Peirce: CP 8 Bibliography General 1884 [G-1884-10]

10. Reference to "On Minimum Differences of Sensibility," (with J. Jastrow), presented 14-17 Oct 1884. [RNA 1884] 12. Published under title, "On Small Differences of Sensation," (with J. Jastrow), *Memoirs of the National Academy of Sciences* 3, Part I(1884)73-83. Cf. [CP] 7.21n1 for references to statements by Jastrow about this article. [CP] 7.21-35 with corrections from a reprint in Widener IC1a.

Peirce: CP 8 Bibliography General 1884 [G-1884-11]

11. Reference to a paper on pendulum experiments, presented to the American Metrological Society, 30 Dec 1884. *Proceedings of the American Metrological Society* 5(Dec 1884) 46. See also pp. 47-48. See further [CS 1885]1886, p. 38.

Peirce: CP 8 Bibliography General 1885 [G-1885-1]
1885

1. Peirce's testimony concerning the office of weights and measures, presented 24 Jan 1885, *Testimony before the Joint Commission to Consider the Present Organizations of the Signal Service, Geological Survey, Coast and Geodetic Survey, . . .* 49th Congress, 1st Session, Mis. Doc. No. 82, Govt. Printing Office, Washington, 1886, pp. 370-378. See pp. 839 and 852 for references to Peirce's expenditures.

Peirce: CP 8 Bibliography General 1885 [G-1885-2a]

2. (a) "The Coast Survey Investigation," *New York Evening Post* (14 Aug 1885)3:3. Cf. next item.

Peirce: CP 8 Bibliography General 1885 [G-1885-2b]

(b) A signed note concerning Peirce's work for the Coast Survey, *Science* 6(21 Aug 1885)158.

Peirce: CP 8 Bibliography General 1885 [G-1885-3]

3. "On the Algebra of Logic: A Contribution to the Philosophy of Notation," [AJM]7(1885)180-202. Cf. [Bibliography] G-1880-8 of which this item may have been intended as a continuation. Cf. [Bibliography] G-1884-5. [CP] 3.359-403, except 396n† (p. 230), with an undated marginal note, 384n1. [CP] 3.403A-403M (c.1885) are a note on the article.

Peirce: CP 8 Bibliography General 1885 [G-1885-4]

4. Items in [CS 1884]1885.

Peirce: CP 8 Bibliography General 1885 [G-1885-4a]

(a) References to Peirce's pendulum work and comparisons of standards in Europe and in the United States, pp. 40, 80, 81, 89, 93; cf. p. 2. Page 81 contains a quotation from Peirce on the ratio of the meter to the yard.

Peirce: CP 8 Bibliography General 1885 [G-1885-4b]

(b) A letter and a statement by Peirce concerning corrections to the thermometers of the Kater pendulum apparatus, Appendix No. 14, pp. 442-443.

Peirce: CP 8 Bibliography General 1885 [G-1885-4c]

(c) Appendix No. 15, "On the Use of the Noddy for Measuring the Amplitude of Swaying in a Pendulum Support," pp. 475-482, with corrections on a half-sheet inserted between pp. 474 and 475.

Peirce: CP 8 Bibliography General 1885 [G-1885-4d]

(d) Appendix No. 16, "Note on the Effect of the Flexure of a Pendulum upon its Period of Oscillation," pp. 483-485, with corrections on a half-sheet inserted between pp. 474 and 475. Printed separately with the title, *Methods and Results, Gravity Research, Effect of the Flexure . . .*, Govt. Printing Office, Washington, 1885.

Peirce: CP 8 Bibliography General c.1885 [G-c.1885-1]
c.1885

1. "One, Two, Three: Fundamental Categories of Thought and of Nature." [CP] 1.369-372, 1.376-378 are from it.

Peirce: CP 8 Bibliography General c.1885 [G-c.1885-2]

2. An unpublished, uncompleted review of T. K. Abbott's translation of Kant's *Introduction to Logic*, 1885. [CP] 1.35.

Peirce: CP 8 Bibliography General c.1885 [G-c.1885-3]

3. An unpublished, incomplete manuscript of a review of Josiah Royce's *The Religious Aspect of Philosophy* (1885), Houghton Library. [CP] 8.39-54.

Peirce: CP 8 Bibliography General 1886 [G-1886-1]
1886

1. Reference to "Review of Paper on Color Contrast," presented 20-23 April 1886. *Proceedings of the National Academy of Sciences*, Vol. 1, Part 3(1895)269.

Peirce: CP 8 Bibliography General 1886 [G-1886-2]

2. Items in [CS 1885]1886.

Peirce: CP 8 Bibliography General 1886 [G-1886-2a]

(a) References to Peirce's gravity work and experimental researches, pp. 37-38, 46, 83, 84; cf. p. 99.

Peirce: CP 8 Bibliography General 1886 [G-1886-2b]

(b) Appendix No. 15, "Note on a Device for Abbreviating Time Reductions," pp. 503-508.

Peirce: CP 8 Bibliography General 1886 [G-1886-2c]

(c) Appendix No. 16, "On the Influence of a Noddy on the Period of a Pendulum," pp. 509-510.

Peirce: CP 8 Bibliography General 1886 [G-1886-2d]

(d) Appendix No. 17, "On the Effect of Unequal Temperature upon a Reversible Pendulum," pp. 511-512.

Peirce: CP 8 Bibliography General 1887 [G-1887-1]
1887

1. "Science and Immortality," *The Christian Register* 66 (7 April 1887) 214. The journal, *ibid.*, p. 210, had solicited "the opinions of some of the most prominent scientific men in this country. . ." The above was Peirce's contribution. Reprinted in *Science and Immortality* (The Christian Register Symposium, Revised and Enlarged, edited by S. J. Barrows), George H. Ellis, Boston, 1887, with revisions. [CP] 6.548-556, as in *Science and Immortality*.

Peirce: CP 8 Bibliography General 1887 [G-1887-2]

† 2. "Logical Machines," *The American Journal of Psychology* 1(Nov 1887) 165-170.

Peirce: CP 8 Bibliography General 1887 [G-1887-3]

† 3. "Criticism on 'Phantasms of the Living,' An Examination of an Argument of Messrs. Gurney, Myers, and Podmore," *Proceedings of the American Society for Psychical Research* 1(Dec 1887)150-157. "Mr. Peirce's Rejoinder," *ibid.*, pp. 180-215. Replies by the authors are (1) Edmund Gurney, "Remarks on Professor Peirce's Paper," *ibid.*, pp. 157-179; (2) Edmund Gurney, "Remarks on Mr. Peirce's Rejoinder," *ibid.*, (March 1889)286-300; (3) Frederic W. H. Myers, "Postscript to Mr. Gurney's Reply to Professor Peirce," *ibid.*, pp. 300-301. Cf. [Bibliography] N-1894-14 and N-1897-2; also see [CP] 6.549, 7.597n3.

Peirce: CP 8 Bibliography General 1887 [G-1887-4]

4. References to Peirce's pendulum experiments. [CS 1886]1887, pp. 41, 49, 85, 86, 99, 100, 103. (On pp. 135 and 137 are references to records and books turned in to the office by Peirce.)

Peirce: CP 8 Bibliography General 1888 [G-1888-1]
1888

1. Items in Adolphus W. Greely, *Report on the Proceedings of the United States Expedition to Lady Franklin Bay, Grinnell Land*, 2 vols., Govt. Printing Office, Washington, 1888. Cf. pp. 14-15, 50, 61, 104, 107 of Vol. 1 and pp. 716-729 and chart opposite p. 724 of Vol. 2. (See further Adolphus W. Greely, *Three Years of Arctic Service*, 2 vols., Charles Scribners Sons, New York, 1886; pp. xii, 130-132, 180 of Vol. 1; pp. 6768, 135-136 of Vol. 2.)

Peirce: CP 8 Bibliography General 1888 [G-1888-1a]

(a) "Pendulum Observations," Vol. 2, pp. 701-714.

Peirce: CP 8 Bibliography General 1888 [G-1888-1b]

(b) An explanatory note, half-sheet opposite p. 714 of Vol. 2, replying to Greely's comments on (a) above, Vol. 2, p. 715.

Peirce: CP 8 Bibliography General c.1888 [G-c.1888-1]
c.1888

1. *Proceedings of the Assay Commission of 1888; also, Laws of the United States Relating to the Annual Assay, and Rules for the Organization and Government of the Board of Assay Commissioners*, Treasury Department, Document No. 1089, Director of the Mint, 25 pp. (There are copies in Houghton Library.) Peirce was designated by Grover Cleveland to be one of several "Commissioners to test and examine the weight and fineness of the coins reserved at the several mints during the

year 1887" (p. 3). "Report of the Committee on Weighing," pp. 8-12, "Report of the Committee on Counting," pp. 12-13, and the general report, signed at p. 16, are signed by Peirce and others.

Peirce: CP 8 Bibliography General 1889 [G-1889-1]
1889

1. Reference to "On Sensations of Color," presented 16-19 April 1889. [RNA 1889]6.

Peirce: CP 8 Bibliography General 1889 [G-1889-2]

2. Reference to "On Determinations of Gravity," presented 16-19 April 1889. [RNA 1889]6.

Peirce: CP 8 Bibliography General 1889 [G-1889-3]

3. Contributions to *The Century Dictionary and Cyclopaedia*; first edition, 6 vols., The Century Co., New York, 1889-1891. Peirce is listed in front as a contributor, but the entries are unsigned. Information concerning his contributions is given in the dictionary, and further information is given at [CP] 1.106, 1.209, 1.559n, 3.416, 3.427 (cf. [Bibliography] N-1902-2, p. 179), 6.38n, 6.51n, 6.164n*, 6.211n, 6.592, 6.618. *The Monist* 3:80, [Fisch] ftm.64, [Fisch-Cope] 278, and Peirce's personal interleaved copy of the dictionary (in 24 parts) in Houghton Library, with indications by him of what definitions he wrote.

Peirce: CP 8 Bibliography General 1889 [G-1889-3]

The following published contributions are reprinted in *Collected Papers*: Continuous, 6.164 (cf. below); Method or doctrine of limits, 4.118n1; Solid (1890-91), 6.241n1; Syntheme (1891), 3.396n†.

Peirce: CP 8 Bibliography General 1889 [G-1889-3]

The following notes from Peirce's personal copy are reprinted in *Collected Papers*: on pragmatism (c.1902), 5.13n1; on the continuum. 6.165-168. (6.164n* assigns a date of 1903 to 6.165-167 and does not identify 6.168. The dating in Peirce's copy is not clear, but it seems to the editor that "Sept. 18, 1903" applies only to 6.168 and not to 6.165-167.)

Peirce: CP 8 Bibliography General 1889 [G-1889-3]

In *The Century Dictionary Supplement*, 2 vols., 1909 (republished with revisions and additions in the 1911 edition of the complete dictionary), the definition of "phenoscopy" is signed by Peirce. Cf. "firstness," "secondness," "thirdness," "phenomenology (Caenopythagorean phenomenology)," "phaneron," and "pragmaticism" in the supplement.

Peirce: CP 8 Bibliography General 1889 [G-1889-4]

4. Reference to Peirce's pendulum experiments. [CS 1887] 1889, pp. 116-117; cf. p. 88.

Peirce: CP 8 Bibliography General 1890 [G-1890-1]
1890

1. References to Peirce's work. [CS 1889]1890, pp. 100 and 179.

Peirce: CP 8 Bibliography General c.1890 [G-c.1890-1]
c.1890

* 1. A Guess at the Riddle. [CP] 1.354-368, 1.373-375, 1.379-416. [CP] 1.1-2 are from an alternative version of Sec. 1 (c.1898); see [CP] 1.355n†. According to [CP] I, this uncompleted work was to have nine sections: Sec. 2, 8, 9 were unwritten; Sec. 3 only partly written; pages of Sec. 4 are missing.

Peirce: CP 8 Bibliography General c.1890 [G-c.1890-2]

2. "Notes on the Question of the Existence of an External World." [CP] 1.37-38. [CP] 1.36 and 1.39 are from fragmentary alternative mss. of the same date. Cf. [CP] 1.35n*.

Peirce: CP 8 Bibliography General 1891 [G-1891-1]
1891

* 1. *The Monist* series of 1891-1893 on metaphysics with a reply to a criticism. See [CP] 5.436 for a reference to an additional article on "the principle of continuity" planned for this series; cf. [CP] 6.239 and 6.242.

Peirce: CP 8 Bibliography General 1891 [G-1891-1a]

(a) "The Architecture of Theories," 1(Jan 1891) 161-176. [CP] 6.7-34.

Peirce: CP 8 Bibliography General 1891 [G-1891-1b]

(b) "The Doctrine of Necessity Examined," 2(April 1892)321-337. [CP] 6.35-65 with two minor corrections ([CP] 6.63) from "Reply to the Necessitarians," (f) below.

Peirce: CP 8 Bibliography General 1891 [G-1891-1c]

(c) "The Law of Mind," 2(July 1892)533-559. [CP] 6.102-163.

Peirce: CP 8 Bibliography General 1891 [G-1891-1d]

(d) "Man's Glassy Essence," 3(Oct 1892) 1-22. [CP] 6.238-271, except 241n1.

Peirce: CP 8 Bibliography General 1891 [G-1891-1e]

(e) "Evolutionary Love," 3(Jan 1893)176-200. [CP] 6.287-317.

Peirce: CP 8 Bibliography General 1891 [G-1891-1f]

(f) "Reply to the Necessitarians. Rejoinder to Dr. Carus," 3(July 1893)526-570. [CP] 6.588-618. This is a reply to the following two articles by Paul Carus, which criticize mainly (b) above: (1) "Mr. Charles S. Peirce's Onslaught on the Doctrine of Necessity," 2(July 1892)560-582; (2) "The Idea of Necessity, Its Basis and its Scope," 3(Oct 1892)68-96. Carus replied to Peirce in "The Founder of Tychism, His Methods, Philosophy, and Criticisms, In Reply to Mr. Charles S.

Peirce," 3(July 1893) 571-622. This contains a quotation from a letter from Peirce, p. 571. Cf. the following two items by Carus: (1) "The Criterion of Truth," 1(Jan 1891)229-244; (2) "Mr. Charles S. Peirce on Necessity," 2(April 1892)442.

Peirce: CP 8 Bibliography General 1891 [G-1891-2]

2. Reference to "Astronomical Methods of Determining the Curvature of Space," presented 10-12 Nov 1891. [RNA 1891] 16.

Peirce: CP 8 Bibliography General 1891 [G-1891-3]

3. Reference to Peirce's gravity research. [CS 1890]1891, p. 104.

Peirce: CP 8 Bibliography General c.1891 [G-c.1891-1]
c.1891

1. "Questions on William James's Principles of Psychology," Widener IC1a. [CP] 8.72-90 are from it. Cf. [Perry] II, 105-108, where this manuscript is quoted in part.

Peirce: CP 8 Bibliography General 1892 [G-1892-1]
1892

1. *The Open Court* series of 1892 on the methods of reasoning. See the announcement and the advertisement at 6(1 Sept 1892)3374.

Peirce: CP 8 Bibliography General 1892 [G-1892-1a]

(a) "Pythagorics," 6(8 Sept 1892)3375-3377.

Peirce: CP 8 Bibliography General 1892 [G-1892-1b]

(b) "The Critic of Arguments." "I. Exact Thinking," 6(22 Sept 1892) 3391-3394. [CP] 3.404-414. "II. The Reader is Introduced to Relatives," 6(13 Oct 1892)3415-3418. [CP] 3.415-424. Only two papers were published, but more were planned; see [CP] 3.422. [CP] 4.187n1 is from "The Critic of Arguments," III (1892). It seems likely that this is from a third paper in this series.

Peirce: CP 8 Bibliography General 1892 [G-1892-2]

2. "Dmesis," *The Open Court* 6(29 Sept 1892)3399-3402.

Peirce: CP 8 Bibliography General 1892 [G-1892-3]

3. Reference to Peirce's gravity research. [CS 1891] 1892, Part I, p. 97.

Peirce: CP 8 Bibliography General 1892 [G-1892-4]

4. Reference to twelve lectures delivered by Peirce on "The History of Science," at the Lowell Institute, 1892-1893. Harriette Knight Smith, *The History of the Lowell Institute*, 1898, p. 88. [CP] 7.267-275, except 267n8, are the concluding remarks (1893) and one quotation in [CP] 7.267n8 is from Lecture V (c.1892), both in Widener IC1b, of a series which very probably is this Lowell Institute series. See [CP] 7.267n7.

Peirce: CP 8 Bibliography General c.1892 [G-c.1892-1]
c.1892

1. Two passages on Hegel from separate fragments. (a) [CP] 1.40. (b) [CP] 1.41-42.

Peirce: CP 8 Bibliography General c.1892 [G-c.1892-2]

2. "Immortality in the Light of Synechism," Widener IB3. [CP] 7.565-578.

Peirce: CP 8 Bibliography General 1893 [G-1893-1]
1893

1. "The Marriage of Religion and Science," *The Open Court* 7(16 Feb 1893)3559-3560. [CP] 6.428-434.

Peirce: CP 8 Bibliography General 1893 [G-1893-2]

2. "Cogito Ergo Sum," a letter to the Editor, *The Open Court* 7(15 June 1893)3702.

Peirce: CP 8 Bibliography General 1893 [G-1893-3]

3. "What is Christian Faith?" *The Open Court* 7(27 July 1893)37433745. [CP] 6.435-448.

Peirce: CP 8 Bibliography General 1893 [G-1893-4]

4. Review of Arthur Lévy's *Napoléon Intime*, unsigned, *The Independent* 45(21 Dec 1893)1725-1726 and (28 Dec 1893)1760. Identification based on Harvard mss.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

* 5. *Grand Logic*, or, an alternative title, *How to Reason: A Critick of*

Arguments, a completed but unpublished book. The quotations below are from two drafts of an advertisement written by Peirce, c.1893, Widener IB2-1. The organization of the book given further below is a reconstruction from data found in a table of contents (c.1893) and ms. at Widener IB2-1 (where the material printed in [CP] VII is located) and from data in *Collected Papers* I-VI.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

"This work is distinguished from other logics, 1st, by the way it makes the nature of inquiry into real facts illuminate that of demonstration from fixed assumptions, and *vice versa*; 2nd, by drawing, not from any 'cannot-help-thinking,' but from an accurate analysis of inference, as its unavoidable consequences, rules that resolve the most obstinate logical doubts; and 3rd, by accepting (here is the upshot of the whole discussion) the principle of continuity for *lucerna pedibus* in all the dark paths of scientific and philosophical exploration." [There follows a description of the contents; cf. the table of contents below.]

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

"But if, however, the field of possibility is not continuous, absolutely exact conclusions may be warranted. For this reason (among others), it is proper to consider the evidences for the reality of continuity. That we have a perfectly consistent *conception* of continuity has been shown. But what evidence is there that it is real? The author maintains that it is given in direct presentation. In this he is sustained by the psychological studies of Professor James; and he adds sundry arguments of his own. Besides, even if continuity is not given intuitively, its reality answers the logical conditions of a good theory.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

"The reality of continuity once admitted, the next question is what are we to regard as continuous and what as discontinuous? It is shown that to say that anything is continuous is to leave possibilities open which are closed by asserting that it is discontinuous. Accordingly a regulative principle of logic requires us to hold anything as continuous until it is proved discontinuous. But absolute discontinuity cannot be proved to be real, nor can any good reason for believing it real be alleged. We thus reach the conclusion that as a regulative principle, at least, ultimate continuity ought to be presumed everywhere.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

"The reality of continuity appears most clearly in reference to mental phenomena; and it is shown that every general concept is, in reference to its individuals, strictly a continuum. This (though asserted by Kant and others) did not appear quite evident as long as the doctrine of generals was restricted to non-relative terms. But in the light of the logic of relatives, the general is seen to be precisely the continuous. Therefore, the doctrine of the reality of continuity is simply that doctrine the scholastics called realism; and though as they held it, it was a crude notion enough, yet as Dr. F. E. Abbot has proved, in another dress it is the doctrine of all modern science.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

"This point reached, a massive foundation has been laid for a philosophy which shall not take for its first axiom a principle utterly irreconcilable with all spiritual truth, and with some lighter matters the volume is brought to a close."

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

The contents included the following chapters of which some had been published earlier, and some were revised in 1893. Cf. [CP] 4.88n* and 6.278n1.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

Book I, Of Reasoning in General. Introduction, The Association of Ideas; [CP] 7.388-450 (c.1893), except 392n7, are from it.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

Division I, Reasoning Formally Studied. Chapter I, The Categories; [CP] 1.545-559, except 549n1; [Bibliography] G-1867-1c. Chapter II, Signs. (A manuscript in Widener IB2-1, with a heading, "The Art of Reasoning," and a title, "Ch. 2. What is a Sign?" is probably chapter two of a draft of the *Grand Logic*. [CP] 2.281, 2.285, 2.297-302 are from this manuscript, c.1895.)

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

Division II, Transcendental Logic. Chapter III, The Materialistic Aspect of Reasoning; [CP] 6.278-286. Chapter IV, What is the Use of Consciousness?; [CP] 7.559-564 (c.1893). Chapter V, The Fixation of Belief; [CP] 5.358-387; [Bibliography] G-1877-5a.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

Division III, Reasoning Substantially Studied. Chapter VI, The Essence of Reasoning; [CP] 4.21-52 and 7.463-467 are from one draft, in that order; [CP] 4.53-79 are from an alternative draft with deletions.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

Book II, Demonstrative Reasoning. Introduction, Chapter VII, Analysis of Propositions.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

Division I, Stecheology. Part 1, Non Relative. Chapter VIII, The Algebra of the Copula. Chapter IX, The Aristotelian Syllogistic; [CP] 2.445-460 with deletions. Chapter X, Extension of the Aristotelian Syllogistic; [CP] 2.532-535 with deletions. Chapter XI, The Boolean Calculus. Part 2, Relative. Chapter XII, The Logic of Relatives. Chapter XIII, Simplification for Dual Relatives. (According to [CP] 2.517n†* and 3.345n this chapter contained rewritten versions of [Bibliography] G-1883-7c and 7d, [CP] 2.517-531 and 3.328-358, respectively.) Chapter XIV, Second Intentional Logic; [CP] 4.80-84.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

Division II, Methodology. Chapter XV, Breadth and Depth; [CP] 2.391-426; [Bibliography] G-1867-1e. ("Terminology," [CP] 2.427-430, [Bibliography] G-1893-7, is a supplement to it.) Chapter XVI, Clearness of Ideas; [CP] 5.388-410; [Bibliography] G-1877-5b.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

Book III, Quantitative Logic. Chapter XVII, Logic of Quantity; [CP] 4.85-152 with deletions and a marginal note. Chapter XVIII, The Doctrine of Chances; [CP] 2.645-660; [Bibliography] G-1877-5c. Chapter XIX, Induction etc.

Peirce: CP 8 Bibliography General 1893 [G-1893-5]

Appendix I, Recreations and Exercises. Glossarial Index.

Peirce: CP 8 Bibliography General 1893 [G-1893-6]

* 6. *Search for a Method*, an unpublished book. The contents included the following "Essays," all of which had been published earlier and some of which have revisions of 1893.

I. On the Natural Classification of Arguments. [CP] 2.461-516; [Bibliography] G-1867-1b.

II. On a New List of Categories. [CP] 1.545-559, except 549n1; [Bibliography] G-1867-1c.

III. Upon Logical Comprehension and Extension. [CP] 2.391-426; [Bibliography] G-1867-1e. ("Terminology," [CP] 2.427-430, [Bibliography] G-1893-7, is a supplement to it.)

Essays IV, V, and VI consisted of the series on intuitive knowledge in the original order: [CP] 5.213-263, 5.264-317, 5.318-357; [Bibliography] G-1868-2a, 2b, 2c.

Essays VII and IX through XIII consisted of the series on the logic of science in the original order: [CP] 5.358-387, 5.388-410, 2.645-660, 2.669-693, 6.395-427, 2.619-644; [Bibliography] G-1877-5a through 5f. (This identification of Essay XII is suggested by Manley Thompson, *The Pragmatic Philosophy of C. S. Peirce*, p. 279; it is not made in [CP].)

XIV. A Theory of Probable Inference. [CP] 2.694-754; [Bibliography] G-1883-7b.

Peirce: CP 8 Bibliography General 1893 [G-1893-7]

7. "Terminology," a supplement to [Bibliography] G-1867-1e. [CP] 2.427-430.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-1]
c.1893

1. A fragment on axioms. [CP] 1.130-132.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-2]

2. "The Connection between Mind and Matter." [CP] 6.272-277.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-3]

3. Chapter I, "Of Reasoning in general," from "Short Logic," Widener IB2-10. No further chapters have been found. [CP] 2.282, 2.286-291, 2.295-296 (with 2.295 continuing 2.291), 2.435-443, 7.555-558 (with 7.555 continuing 2.443), and 2.444 are from it in that order; cf. [CP] 7.553n17.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-4]

4. Prospectus of *The Treatise of Petrus Peregrinus on the Lodestone*, 16 pp. There is a copy in Widener, catalogued under Phys. 5.1. Cf. [Bibliography] N-1894-1 and N-1894-4. The prospectus is dated c.1893 on the basis of the items in *The Nation* and of information in Harvard mss. Peirce's edition of Peregrinus was never published as far as can be discovered. [CP] 7.392n7 contains a footnote from p. 16. A quotation from pp. 1-6 of the prospectus is given below because it well illustrates Peirce's aim in studying the history of science, to gain an understanding of the nature of scientific method.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-4]

"The brief treatise on the lodestone by Petrus Peregrinus, dated 1269, occupies a unique position in the history of the human mind, being without exception the earliest work of experimental science that has come down to us. Nor can we learn that anything of this sort had been written earlier. No doubt experiments had been made earlier. The medical papyrus contains a prescription said to have been given for the mother of King Thoth; and something had been accomplished in optics. But no ancient experiments can be considered as scientific, for several reasons. First, they were not made for the simple purpose of learning the truth, but with a *parti pris*, except where they were merely accidental. In the second place, in no single case was a piece of apparatus devised by any person before Peregrinus -- at least there is no evidence of such a thing -- for the purpose of obtaining an experimental answer to a question. Now, with instruments already existing we may make casual observations; but unless they are much more complicated than those of the ancients were, we can hardly make with them genuine experiments. At any rate, the total absence of experiments made with apparatus devised for the purpose, betrays a complete absence of the spirit of experimental inquiry. The quintessence of science, however, consists precisely in, and its success depends upon, the spirit with which it is prosecuted. In the third place, no law or general proposition of ancient science was consciously based upon experiment. In the fourth place, no ancient experiments were ever conducted in one connected series, each one after the first based on the truth the previous one had established. The work of Petrus is absolutely the first that fulfils a single one of the conditions here indicated; and it fulfils them all. The sole direct purpose of his experiments was evidently the analysis of the properties of the lodestone and the ascertainment of their laws. His experiments are made with several distinct pieces of apparatus constructed and devised for no other end. Upon his experiments he bases a general theory of the lodestone, which, though in part not correct, yet remains in error only because, as he virtually confesses, he had not sufficient mechanical skill to construct an apparatus described by him and proposed for the purpose of making the crucial experiment which, had it been made, would have corrected his error. His experiments all do follow in sequence, each reposing on the result of the one that went before. Not only does Peregrinus in practice follow these four conditions, but he is fully alive to the importance of each one of them. Thus, on the whole, this little book must be considered as one of the most important monuments of human progress. The experimentation of Gilbert, who has often been considered as the founder of magnetical science, is in large part downright plagiarism upon Peregrinus; and though his real merits are far from inconsiderable, yet he is not at all upon the plane of importance of the earlier writer, even without making allowance for the more advanced state of civilization of his century.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-4]

"Time was when educated people could imagine that the idea of inductive science could spring full grown from the brain of Francis Bacon; but in our days we have learned better the natural course of development of ideas. Thoughts of that comprehensive kind do not start up in the mushroom-beds of individual brains. They require the broader fields of societies; and generations must pass by before they can acquire any maturity of strength. There is plenty of testimony both of Peregrinus and of his pupil, Roger Bacon, from which we may securely infer that they were acquainted with older physicists, although those elders may perhaps not have committed their experiments to writing. But whether it will be possible ever to make out with any plausibility the earlier history of experimental science, previous and prelusive to the work of Peregrinus, time alone can show. Certain it is that the early school of physicists, of which Peregrinus and Roger Bacon remain for us the only representatives, was shortly overwhelmed by the rising tide of theology, between which on the one side and popular superstition on the other, no place was left for it. Was the flame of scientific inquiry, then, utterly quenched, or did a spark remain alive from which, in historical fact, the sacred fire was rekindled in the sixteenth century? To the solution of this problem the introduction of this volume will offer some contributions.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-4]

"The text of the treatise here presented is substantially that of a contemporary MS. in the Paris Library.* [Footnote: "*MSS. latins 7378."] All deviations from that authority are noted. Three other important texts have, however, been carefully collated, together with several that are incomplete. The work, though much written about, has, it is believed, never been printed, except at Augsburg in 1558, and that edition is of the extremest rarity. True, Libri, in his valuable work on the history of the mathematical sciences in Italy, did attempt a transcription of the very MS. here used; but, owing to its extreme illegibility, though he invoked the aid of the most expert paleographers, he has hardly been able to make perfect sense out of a single sentence, not to speak of places where his text suggests a wrong meaning, nor of innumerable lesser errors."

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

5. *The Principles of Philosophy: or, Logic, Physics, and Psychics, considered as a unity, in the Light of the Nineteenth Century*, a planned and partly executed work of twelve volumes. [CP] 1.176-179 (c.1896) is apparently a foreword.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

The following is almost all of a printed prospectus in Widener VA². Cf. [Bibliography] N-1894-1 on the basis of which this is dated c.1893.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"This philosophy, the elaboration of which has been the chief labor of the author for thirty years, is of the nature of a Working Hypothesis for use in all branches of experiential inquiry. Unmistakable consequences can be deduced from it, whose truth is not yet known but can be ascertained by observation, so as to put the theory to the test. It is thus at once a philosophy and a scientific explanation of observed facts.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"The actual comparison of its consequences with observation can by Mr.

Peirce himself only be commenced. He will, however, carry the operation far enough to convince the most skeptical of its entire feasibility.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Both logically and dynamically the whole doctrine develops out of the *desire to know*, or philosophia, which carries with it the confession that we do not know already. In those branches of knowledge that are the most perfect no self-respecting man puts forth a statement without affixing to it his estimate of its *probable error*, while in branches where arbitrary opinion is uncurbed authors are unwilling to confess that the smallest doubt hangs over their conclusions. Nothing can be more completely contrary to a philosophy the fruit of a scientific life than infallibilism, whether arrayed in its old ecclesiastical trappings, or under its recent 'scientistic' disguise. Mr. Peirce will, therefore, not be understood himself to make any such pretensions. He hopes some power of truth is in his theory, because it has been conceived in a spirit of utter surrender to the *force majeure* of Experience, or the Course of Life; and it is through such self-abnegation that all Power comes. But how far this hope is fulfilled must be determined by the success or failure of such *predictions* as are deducible from the theory.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"The principles supported by Mr. Peirce bear a close affinity with those of Hegel; perhaps are what Hegel's might have been had he been educated in a physical laboratory instead of in a theological seminary. Thus, Mr. Peirce acknowledges an objective logic (though its movement differs from the Hegelian dialectic), and like Hegel endeavors to assimilate truth got from many a looted system.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"The entelechy and soul of the work, from which every part of its contents manifestly flows, is the *principle of continuity*, which has been the guiding star of exact science from the beginning, but of which novel and unexpected applications are now made. The logical ground of this principle is examined and its precise formula established.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"The principle of continuity leads directly to Evolutionism, and naturally to a hearty acceptance of many of the conclusions of Spencer, Fiske, and others. Only, Matter, Space, and Energy will not be assumed eternal, since their properties are mathematically explicable as products of an evolution from a primeval (and infinitely long past) chaos of unpersonalized feeling. This modified doctrine, so much in harmony with the general spirit of evolutionism, quite knocks the ground from under both materialism and necessitarianism.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"In religion, the new philosophy would teach us to await and expect definite and tangible facts of experience, actually undergone. While details of dogma are beyond its province, it would favor rather old-fashioned Christianity, than any attempt to make a christianoidal metaphysics serve in lieu of religion. Still less could it accept a theology of phrases which should label an abstraction 'God' and influence with posterity 'A future life.' It distinctly upholds a *Christian Sentimentalism*, as contra-distinguished from a gospel of salvation through intelligent greed."

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

The following is all but the title of a printed prospectus in Widener,

catalogued under Phys. 5.1.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. I. (Nearly ready.) **Review of the Leading Ideas of the Nineteenth Century.** Defines the essential ideas involved in and sentiments fostered by political economy, machinery and modern inventions, labor unions, socialism, scientific associations, centennials, nationalism, emigration, various forms of idealism, Hegel's objective logic, the historical method, modern mathematics and its imaginaries, the theory of heat and conservation of energy, statistical methods of research, the kinetical theory of gases, Darwinism, etc. It is believed that these analyses will be found valuable, apart from the conclusions drawn from them. Next, a definite affinity is traced between all these ideas, and is shown to lie in the principle of *continuity*. The idea of continuity traced through the history of the Human Mind, and shown to be the great idea which has been working itself out. (The author's papers in the *North American Review* are here used.) Modern science due to it exclusively. A great part, if not all, of evolution in all departments, and at all times, probably to be ascribed to the action of this principle. The urgent needs of our time may, we have strong reason to hope, be met by the further application of it. Sketch of a thoroughgoing philosophy of continuity. The great opponent of this philosophy has been in history, and is in logic, infallibilism, whether in its milder ecclesiastical form, or in its more dire scientific and materialistic apparitions.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. II. (Substantially ready.) **Theory of Demonstrative Reasoning.** The first part of this volume contains a plain, elementary account of formal logic, ordinary and relative. It has been very carefully adapted to the use of young persons of mediocre capacities, and has been subjected to experimental tests with success. This is followed by more intricate developments for persons having a turn for such matters, and others may skip this part. (The author's papers in the *Memoirs of the American Academy* and in the *Journal of Mathematics* are here made use of.) Deductive reasoning having thus been accurately described and the working of it taught, the third part of the volume makes a careful analysis of it, and shows what the natures of its different ingredients are. The principle of continuity is shown to be the crown of the logic of relatives.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. III. **The Philosophy of Probability.** After an analysis of the nature of probability, the principles of the calculus are set forth. The doctrine of inverse probabilities refuted. The theory of inductive and hypothetic inference set forth nearly as in the Johns Hopkins 'Studies in Logic,' but the position there taken is reinforced with powerful new arguments. Mr. Peirce's rules for inductive reasoning are the strictest that have been advocated. New illustrations are given to show the absurdly bad reasoning into which those fall who follow looser rules. A few inferences admitted by Mr. Peirce as valid are disallowed by some writers. Their inconsistency in this shown, and that those writers simply maintain an unreasonable skepticism concerning some questions which they do not extend to others quite analogous.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. IV. **Plato's World: An Elucidation of the Ideas of Modern Mathematics.** A lucid analysis of the logic and conceptions of the calculus, imaginaries, the theory of functions, and the non-Euclidean geometry. The conceptions of infinity and continuity are now accurately analyzed. The notion that

we cannot reason mathematically about infinity refuted. The doctrine of limits as stated by some authors inadequate to its purpose; as stated by others, really involves reasoning about infinity. It is impossible to assign any reason for the dogma that we cannot reason mathematically about infinity; one might as well say we cannot reason mathematically about imaginaries.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol V. *Scientific Metaphysics*. Begins with the theory of cognition. The nature of reality discussed as in the author's papers in the *Popular Science Monthly*; but the position taken is now set forth more clearly, fully, and in psychological detail. The reality of the external world. Primary and secondary qualities. The evidence of the real existence of continuity. The question of nominalism and realism from the point of view of continuity. Continuity and evolution. Necessitarianism refuted. Further corollaries from the principle of continuity.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. VI. *Soul and Body*. Begins with an analysis of the law of association, which is somewhat generalized. The question of fatigue and its law. Review of psychological phenomena. The apparent discontinuity of sense-qualities considered. Definition of the soul, following out ideas put forth by the author in the *Journal of Speculative Philosophy*. The 'unity of consciousness' admits of degrees, and is probably in many cases very low. Phenomena of anaesthesia considered. The author's theory of universal evolution, which supposes matter and its laws to be the result of evolution, is now set forth more systematically and argumentatively. Still, it is to be regarded for the present as no more than a working hypothesis. Explanation of the method of reasoning by which a multitude of unmistakable consequences can be rigidly deduced from the hypothesis. A considerable number of these are shown to be true, while none are known to be false. One prediction of a fact hitherto unknown is shown to be supported by observation. Others remain to be tested by future experience, and the theory will have to stand or fall by the result.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. VII. *Evolutionary Chemistry*. The working out of the consequences of the theory of universal evolution into chemistry. Mendeléeff's law.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. VIII. *Continuity in the Psychological and Moral Sciences*. Mathematical economics. Precisely similar considerations supposed by utilitarians to determine individual action. But, this being granted, Marshall and Walras's theorem leads to a mathematical demonstration of free will. Refutation of the theory of motives. The true psychology of action expounded.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. IX. *Studies in Comparative Biography*. The application of mathematical principles in a new way to this study.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. X. *The Regeneration of the Church*. The philosophy of continuity is peculiar in leading unequivocally to Christian sentiments. But there it stops. This metaphysics is only an appendix to physics; it has nothing positive to say in regard to religion. It does, however, lead to this, that religion can rest only on positive observed facts, and that such facts may prove a sufficient support for it. As it must rest upon positive facts, so it must itself have a positive content. A series of plays upon words

will not answer for a religion. This philosophy shows that there is no philosophical objection to the positive dogmas of Christianity; but the question as to their truth lies out of its province.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. XI. A *Philosophical Encyclopaedia*. The philosophy of continuity leads to an objective logic, similar to that of Hegel, and to triadic categories. But the movement seems not to accord with Hegel's dialectic, and consequently the form of the scheme of categories is essentially different. Systematic perfection seems to be for the present neither requisite nor attainable; but something like Hegel's Encyclopaedia is proposed.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Vol. XII. *Index raisonné* of ideas and words.

Peirce: CP 8 Bibliography General c.1893 [G-c.1893-5]

"Mr. Peirce does not hold himself pledged to follow precisely the above syllabus, which, on the contrary, he expects to modify as the work progresses. He will only promise that he will not depart from this programme except to improve upon it. The work is to be published by subscription at \$2.50 per volume. Address: Mr. C. S. Peirce, 'Arisbe,' Milford, Pa."

Peirce: CP 8 Bibliography General 1894 [G-1894-1]
1894

1. Reference to Peirce at the 7 April 1894 meeting where he exhibited a 1424 arithmetic by Rollandus, and in the discussion of a certain paper proposed the term "galileo" for the c.g.s. unit of acceleration. *Bulletin of the New York Mathematical Society* 3(May 1894)199-200. See *New York Times* 43(8 April 1894)8, column 1, for a report on the first item.

Peirce: CP 8 Bibliography General 1894 [G-1894-2]

2. Reference to "Rough Notes on geometry, Constitution of real space," presented 24 Nov 1894. *Bulletin of the American Mathematical Society* 1(Dec 1894)77. Cf. [Bibliography]M-1b.

Peirce: CP 8 Bibliography General c.1894 [G-c.1894-1]
c.1894

1. "The List of Categories: A Second Essay." [CP] 1.300-301, 1.293, 1.303, 1.326-329 are from it in this order. "The List of Categories: A Second Essay, X." [CP] 1.302 is from it.

Peirce: CP 8 Bibliography General c.1895 [G-c.1895-1]
c.1895

1. "That Categorical and Hypothetical Propositions are one in essence, with some connected matters." [CP] 2.332-339, 2.278-280, 1.564-567 (c.1899), 2.340-356 are from it in this order.

Peirce: CP 8 Bibliography General c.1895 [G-c.1895-2]

2. "Religion and Politics," apparently a proposed letter to a newspaper. [CP] 6.449-451 are from it.

Peirce: CP 8 Bibliography General c.1895 [G-c.1895-3]

3. "Thirdness," a fragment. [CP] 1.340-342 are from it.

Peirce: CP 8 Bibliography General c.1895 [G-c.1895-4]

4. A fragment on telepathy, Houghton Library. [CP] 7.597n3 is from it.

Peirce: CP 8 Bibliography General 1896 [G-1896-1]
1896

1. Reference to "On the Logic of Quantity," presented 21-24 April 1896. [RNA 1896] 9.

Peirce: CP 8 Bibliography General 1896 [G-1896-2]

† 2. Review of Andrew Dickson White's *A History of the Warfare of Science with Theology in Christendom*, unsigned, *The American Historical Review* 2(Oct 1896)107-113. Identification based on Harvard mss.

Peirce: CP 8 Bibliography General 1896 [G-1896-3]

3. Reference to "A Graphical Method of Logic," presented 17-18 Nov 1896. [RNA 1896]11.

Peirce: CP 8 Bibliography General 1896 [G-1896-4]

4. Reference to "Mathematical Infinity," presented 17-18 Nov 1896. [RNA 1896]11.

Peirce: CP 8 Bibliography General 1896 [G-1896-5]

5. Translation of William Hirsch's *Genius and Degeneration*, D. Appleton and Co., New York, 1896. No translator is given. Identification based on Harvard mss.

Peirce: CP 8 Bibliography General 1896 [G-1896-6]

* 6. Review in *The Monist*, 1896-1897, of Ernst Schroeder's *Algebra und Logik der Relative*, Part I, Vol. 3, *Vorlesungen über die Algebra der Logik (exakte Logik)*. Cf. [Bibliography] N-1896-3.

Peirce: CP 8 Bibliography General 1896 [G-1896-6a]

(a) "The Regenerated Logic," 7(Oct 1896)19-40. [CP] 3.425-455.

Peirce: CP 8 Bibliography General 1896 [G-1896-6b]

(b) "The Logic of Relatives," 7(Jan 1897)161-217. [CP] 3.456-552 with revisions of 1908.

Peirce: CP 8 Bibliography General 1896 [G-1896-7]

7. Reference to a lecture, "Number: A Study of the Methods of Exact Philosophical Thought," given before the Mathematics Department, Bryn Mawr College, during the year 1896-97. *Annual Report of the President of Bryn Mawr College*, 1896-97, p. 35.

Peirce: CP 8 Bibliography General c.1896 [G-c.1896-1]
c.1896

1. A fragment on knowledge of God. [CP] 6.492-493 are from it.

Peirce: CP 8 Bibliography General c.1896 [G-c.1896-2]

* 2. "The Logic of Mathematics; An Attempt to Develop My Categories from Within." First four pages of ms. missing. [CP] 1.417-520.

Peirce: CP 8 Bibliography General c.1896 [G-c.1896-3]

3. "Lessons of the History of Science." [CP] 1.43-125 are from it.

Peirce: CP 8 Bibliography General 1897 [G-1897-1]
1897

1. "Multitude and Number," apparently a lecture ([CP] 4.217). [CP] 4.170-226, except 187n1, with deletions.

Peirce: CP 8 Bibliography General c.1897 [G-c.1897-1]
c.1897

1. A fragment of biographical comments. [CP] 1.3-7.

Peirce: CP 8 Bibliography General c.1897 [G-c.1897-2]

2. A fragment on Peirce's philosophy. [CP] 1.8-14 are from it.

Peirce: CP 8 Bibliography General c.1897 [G-c.1897-3]

3. A fragment on semiotics. [CP] 2.227-229, 2.444n1 are from it.

Peirce: CP 8 Bibliography General c.1897 [G-c.1897-4]

4. "Recreations in Reasoning." [CP] 4.153-169, "A Theory about Quantity," with deletions, are from it.

Peirce: CP 8 Bibliography General c.1897 [G-c.1897-5]

5. An untitled ms. (or mss.) apparently intended as part of a lecture ([CP] 1.141n, 1.155n). [CP] 1.141-175, on fallibilism, continuity, and evolution, are from it.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]
1898

* 1. Cambridge lectures. A number of different sets of lectures were prepared. Cf. [Perry] II, 418-421. There are mss. at Widener IB2-10 and IB3.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]

The following is from a printed announcement at Widener VB3b.
"CAMBRIDGE CONFERENCES, Revised Announcement: Mr. CHARLES SANDERS PEIRCE of Milford, Pennsylvania will give a course of Eight Class Lectures on REASONING AND THE LOGIC OF THINGS, at the rooms of the Cambridge Conferences, Studio House, 168 Brattle Street, on Monday and Thursday Evenings in February and March, 1898, at eight o'clock. The special topics and dates will be as follows:

Peirce: CP 8 Bibliography General 1898 [G-1898-1]

February 10. Philosophy and the Conduct of Life.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]

February 14. Types of Reasoning.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]

February 17. The Logic of Relatives.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]

February 21. The First Rule of Logic.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]
February 24. Training in Reasoning.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]
February 28. Causation and Force.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]
March 3. Habit.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]
March 7. The Logic of Continuity.

The course herein outlined will be of unusual interest and value to students and teachers of Philosophy. It is hoped that many will avail themselves of the privilege of attending. . . ."

Peirce: CP 8 Bibliography General 1898 [G-1898-1]
Selections from a set of eight lectures, originally written for the occasion on which the above lectures were given ([CP] 1.622, 6.212n*), are printed in Collected Papers, Volume VI as follows: Lecture 8, "Logic of Events," 6.1-5, 6.214-221 (with 6.214 continuing 6.5), with minor deletions; "Notes for Eight Lectures," 6.222-237 with deletion, cf. 6.222n*; "The Logic of Continuity," an alternative draft of Lecture 8 (6.1n*), 6.185-213 with deletions (cf. 7.514n14).

Peirce: CP 8 Bibliography General 1898 [G-1898-1]
The following lectures are also associated with this occasion. It is implied at [CP] 6.212n* and elsewhere in [CP] I-VI that they were delivered under the title "Detached Ideas on Vitally Important Topics." Lecture 1. Philosophy and the Conduct of Life. [CP] 1.616-648 are from it. [CP] 1.649-677 are from an alternative version, "On Detached Ideas in general and on Vitally Important Topics as such," with deletions. Lecture 2. Detached Ideas. [CP] 4.1-5 are from it. Cf. [Bibliography] G-c.1898-1. Lecture 3. The First Rule of Logic. [CP] 5.574-589. Cf. [Bibliography] G-c.1899-1. Lecture 4. Causation and Force. [CP] 6.6681, 7.518-523, 6.82-87 in that order are all but the first paragraph. Cf. [CP] 7.518n16.

Peirce: CP 8 Bibliography General 1898 [G-1898-1]
The following very probably belong to this series. Lecture 2(?). "Detached Ideas; Induction, Deduction, and Hypothesis." [CP] 7.494n9 (c.1898) is from it. There is also a ms. in Widener IB2-10 entitled "Types of Reasoning," which may be a version of the second lecture. † Lecture 5. Training in Reasoning. A ms. at Widener IB2-10 bears this title, and there is good evidence that it is a draft of the fifth lecture, c.1898. This ms. was published, with deletions and pages missing, under the title, "Training in Reasoning," *The Hound and Horn* 2 (July-Sept 1929) 398-416. Lecture 7. Habit. A ms. at Widener IB3 bears this title, and there is good evidence that it is a draft of this lecture, c.1898. [CP] 7.468-517 are this manuscript.

Peirce: CP 8 Bibliography General 1898 [G-1898-2]
2. "The Logic of Mathematics in Relation to Education," *Educational Review* 15(March 1898)209-216. (This ends with "to be continued," but no further articles were published in *ibid.*) [CP] 3.553-562. [CP] 3.562A-562I are taken from paginated page proofs of the original article, apparently not published for lack of space. The proof sheet ends with the note "to be continued," but no further ms. has been found

([CP] 3.562In).

Peirce: CP 8 Bibliography General 1898 [G-1898-3]

3. Review of *The "Opus Majus" of Roger Bacon* (edited by John Henry Bridges), unsigned, *The American Historical Review* 3(April 1898)526-528.

Identification based on Harvard mss.

Peirce: CP 8 Bibliography General 1898 [G-1898-4]

† 4. "Note on the Age of Basil Valentine," *Science* ns 8 (12 Aug 1898) 169-176.

Peirce: CP 8 Bibliography General c.1898 [G-c.1898-1]
c.1898

1. Comments on "On a New List of Categories," from a fragment of a proposed "DI" lecture. [CP] 1.563. Cf. "DI" with the title of [Bibliography] G-1898-1. Cf. [Bibliography] G-1867-1c.

Peirce: CP 8 Bibliography General 1899 [G-1899-1]
1899

1. Reference to "The map-coloring problem," presented 15 Nov 1899. [RNA 1899] 13.

Peirce: CP 8 Bibliography General 1899 [G-1899-2]

2. "Professor Bunsen," unsigned, *Progressive Age; Gas-Electricity-Water* 17(1 Sept 1899)393-394. Reprinted from the *New York Evening Post*. Probably by Peirce; identification based on Harvard mss.

Peirce: CP 8 Bibliography General c.1899 [G-c.1899-1]
c.1899

1. "F.R.L." [CP] 1.135-140 are from it. Both the ms. from which [CP] 1.135-140 are taken and the ms. from which 5.574-589 ([Bibliography] G-1898-1, Lecture 3) are taken have "F.R.L." written in the corner of each sheet and there are other similarities.

Peirce: CP 8 Bibliography General 1900 [G-1900-1]

1900

1. "Infinitesimals," a letter to the Editor, *Science* ns 11 (16 March 1900)430-433. [CP] 3.563-570.

Peirce: CP 8 Bibliography General 1900 [G-1900-2]

2. Review of *Clark University, 1889-1899, Decennial Celebration*, *Science* ns 11(20 April 1900)620-622.

Peirce: CP 8 Bibliography General 1900 [G-1900-3]

3. Review of George A. Coe's *The Spiritual Life* and Frank Thilly's *Introduction to Ethics*, signed "Jordan Brown," *The Bookman* 11(July 1900)491-492. Identification based on Harvard mss. Cf. [Bibliography] G-c.1900-1.

Peirce: CP 8 Bibliography General c.1900 [G-c.1900-1]
c.1900

1. Review of Frank Thilly's *Introduction to Ethics*, an alternative draft of the review in *The Nation* ([Bibliography] N-1900-20). [CP] 1.589-590 (c.1903).

Peirce: CP 8 Bibliography General c.1900 [G-c.1900-2]

2. Unpublished review of Josiah Royce's *The World and the Individual*, Vol. 1, Widener IV. [CP] 8.108-116. It is not known whether or not this draft was intended for *The Nation*; cf. [CP] 8.100-107, [Bibliography] N-1900-15, for a published review. See [Bibliography] N-1902-10 for a review of Vol. 2.

Peirce: CP 8 Bibliography General 1901 [G-1901-1]
1901

1. "Pearson's Grammar of Science," *The Popular Science Monthly* 58(Jan 1901)296-306. [CP] 8.132-152, except 136n3 and 138n4. [CP] 8.153-156 and 8.136n3 are from a partial draft of this review, c.1900, in Widener IV.

Peirce: CP 8 Bibliography General 1901 [G-1901-2]

2. Three manuscripts written for Samuel P. Langley on Hume and miracles. See Philip P. Wiener, "The Peirce-Langley Correspondence and Peirce's Manuscript on Hume and the Laws of Nature," *Proceedings of the American Philosophical Society* 91(5 April 1947)201-228.

Peirce: CP 8 Bibliography General 1901 [G-1901-2a]

(a) "The Idea of a Law of Nature among the contemporaries of David Hume and among advanced thinkers of the present day." [CP] 1.133-134 are from it.

Peirce: CP 8 Bibliography General 1901 [G-1901-2b]

(b) "Hume on Miracles." [CP] 6.522-547.

Peirce: CP 8 Bibliography General 1901 [G-1901-2c]

† (c) "Hume on Miracles and Laws of Nature." Langley suggested changes and Peirce finished a revised version under the title, "The Laws of Nature and Hume's Argument Against Miracles." Wiener, *op. cit.*, pp. 212-228, gives the final draft together with deviations from the first draft and Langley's suggestions; see also letters between Peirce and Langley, 1 April 1901 to 6 May 1902, *ibid.*, pp. 205-211, 214.

Peirce: CP 8 Bibliography General 1901 [G-1901-3]

3. "Campanus," *Science* ns 13(24 May 1901)809-811.

Peirce: CP 8 Bibliography General 1901 [G-1901-4]

4. Reference to "On the Logic of Research into Ancient History," presented 12-14 Nov 1901. [RNA 1901]16. Cf. [Bibliography] N-1901-16.

Peirce: CP 8 Bibliography General 1901 [G-1901-4]

* "On the Logic of drawing History from Ancient Documents especially from Testimonies," c.1901, Widener IB2-12. [CP] 7.164-255, except 182n7, are from it. Cf. [CP] 7.164n2.

Peirce: CP 8 Bibliography General 1901 [G-1901-5]

5. Studies of great men. Cf. the references in [CP] 7.256n1.

Peirce: CP 8 Bibliography General 1901 [G-1901-5a]

† (a) "The Century's Great Men in Science," *Annual Report . . . of the Smithsonian Institution . . . for the Year Ending June 30, 1900*, Govt. Printing Office, Washington, 1901, pp. 693-699. Reprinted from *New York Evening Post* (12 Jan 1901) Sec. IV, "Review of the Nineteenth Century."

Peirce: CP 8 Bibliography General 1901 [G-1901-5b]

(b) On the productiveness of the nineteenth century in great men, c.1900, Widener IC1b. [CP] 7.256-261 are from one ms.; 7.262-266 are from an alternative draft.

Peirce: CP 8 Bibliography General 1901 [G-1901-6]

* 6. Contributions to *Dictionary of Philosophy and Psychology* (edited by James Mark Baldwin), 3 vols., The Macmillan Co., New York. Vol. I, 1901; Vol. II, 1902; a new edition, with corrections, of Vols. I and II, 1911. The new edition apparently differs from the original only with regard to minor corrections; the introductions, pagination, etc., are the same.

Peirce: CP 8 Bibliography General 1901 [G-1901-6]

The following information comes from the Editor's Preface, pp. xi-xii. The contributors are not responsible for the recommendations as to foreign equivalents. Joint authorship is indicated by connecting the two sets of initials by a hyphen. A comma between two signatures "as in '(A.B.C., X.Y.Z.)' indicates that the article was written by A.B.C. and accepted without alteration by X.Y.Z., who thus adds the weight of his authority to it" (p. xii). The following contributions were written by Christine Ladd-Franklin, not by Peirce, though they were accepted by him: Signification (and Application, in logic), in part (see below for a contribution to this article that is by Peirce), Vol. II, p. 528, reprinted at [CP] 2.431433; and Transposition, Vol. II, p. 713, reprinted at 3.644-645.

Peirce: CP 8 Bibliography General 1901 [G-1901-6]

Peirce's contributions to the articles constitute in many cases only parts of articles, and sometimes these are not consecutive parts of an article. The references below to his works are given as follows. The term defined is given first, followed by the pagination of Peirce's contribution in the dictionary; if the contribution is joint, this is indicated in parentheses following the page reference. If the contribution is reprinted in *Collected Papers*, the paragraph reference follows a semicolon; if Peirce's contribution was not reprinted in its entirety, this is indicated.

Peirce: CP 8 Bibliography General 1901 [G-1901-6]

Volume I: Economy (logical principle of), 309. Empirical Logic, 318 (with R. Adamson). Equipollence or -cy, 338. Genus (in logic), 411. Given, 414. Imaging (in logic), 518-519; [CP] 3.609-610. Implicit (in logic), 525-526; [CP] 2.603-604. Inconsistency, 529; [CP] 2.609-611. Independence, 530. Index (in exact logic), 531-532; [CP] 2.305-306. Individual (in logic), 537-538; [CP] 3.611-613. Inference, 542-543. Insolubilia, 554; [CP] 2.618. Intention, 561; [CP] 2.548. Involution, 574; [CP] 3.614-615. Kind, 600-601; [CP] 6.384. Knowledge (in logic), 603; [CP] 5.605-606. Laws of Thought, 641-643, 644; [CP] 2.593-600 (641-643 only).

Peirce: CP 8 Bibliography General 1901 [G-1901-6]

Volume II: Leading of Proof, 1. Leading Principle, 1-2; [CP] 2.588-589. Lemma, 3. Light of Nature, 6. Limitative, 6-7; [CP] 2.381 (deletion). Logic, 20-23; [CP] 2.203-218 (deletion). Logic (exact), 23-27; [CP] 3.616-625 (deletions). Logical, 27-28; [CP] 2.537-543 (deletions). Logical Diagram (or Graph), 28; [CP] 4.347-349. Logomachy, 30. Major and Minor (in logic), 37. Mark, 43. Material Fallacy, 44; [CP] 2.617. Material Logic, 44-45; [CP] 2.549. Mathematical Logic, 47. Matter and Form, 50-55; [CP] 6.353-363. Maxim (in logic), 55. Method and Methodology, or Methodeutic, 75. Middle term (and Middle), 77; [CP] 2.581. Mixed, 87. Mnemonic Verses and Words (in logic), 87-89; [CP] 2.584. Modality, 89-93; [CP] 2.382-390. Modulus, 94. Modus ponens and Modus tollens, 94. Multitude (in mathematics), 117-118; [CP] 3.626-631. Name (in logic), 127-128. Necessary (in logic), 143. Necessity, 145-146. Negation, 146-147; [CP] 2.378-380 (deletion). Negative, 148; [CP] 2.580 is from it. Nominal, 179. Nomology, 180. Non-A (in logic), 180. Non-contradiction, 181. Non-sequitur, 181; [CP] 2.613. Norm (and Normality), 182. Nota notae, 183; [CP] 2.590-592. Numerical, 190. Observation, 198; [CP] 2.605-606. Obversion, 199; [CP] 2.551. Opposition, 206; [CP] 2.608. Organon, 219; [CP] 2.547 (deletion). P (in logic), 253. Paradox, 258. Parallogism, 259. Parity, 263. Parsimony (law of), 264; [CP] 7.92-93. Partial, 265. Particular, 265-266; [CP] 2.372-373. Particulate, 266. Parva Logicalia, 266. Per accidens, 276. Perseity and Per se, 281-282; [CP] 6.385. Perspicuity, 287. Pertinent, 287; [CP] 2.602. Petitio Principii,

287-288; [CP] 2.614. Philosopheme, 290; [CP] 2.615. Plurality of Causes, 306-307. Poly-, 309. Port Royal Logic, 310. Positive, 311-312. Possibility, Impossibility, and Possible, 313-315; [CP] 6.364-371. Postpredicament, 315. Postulate, 315-316; [CP] 3.632-634. Pragmatic and Pragmatism, 321-322; [CP] 5.1-4. (The contribution to this article by James Mark Baldwin contains on p. 322: "In the words of Peirce (comment on this article): 'Nominalism, up to that of Hegel, looks at reality retrospectively. What all modern philosophy does is to deny that there is any *esse in futuro*.'") Precise, 323. Precision, 323-324; [CP] 1.549n1 (deletion). Predesignate, 324-325; [CP] 2.788-790. Predicable, 325. Predicament, 325. Predicate, 325-326; [CP] 2.358. Predication, 326-329; [CP] 2.359-361 (deletions). Predicative Proposition, 329. Premise (and Premiss), 330-331; [CP] 2.582-583. Presumption, 337; [CP] 2.791. Presupposition, 338; [CP] 3.635. Prime, 341. Primum cognitum, 341. Principal, 341. Priority (with Prior and Prius), 342-343; [CP] 6.386-389. Privation, 343. Probable Inference, 353-355; [CP] 2.783-787. Problem, 355. Problematic, 355-356. Progressive, 358. Proof, 359; [CP] 2.782. Proposition, 361-362 (with J. M. Baldwin). Protasis, 371. Provisional, 373. Proximate, 373-374; [CP] 6.390-392. Pure (in philosophy), 401-402; [CP] 2.544-546 (deletions). Quality (in grammar and logic), 408-409; [CP] 2.374-377. Quantity (in logic and mathematics), 410-412; [CP] 2.362-366. Ratio, 415. Ratiocination, 415. Rational, 415. Reasoning, 426-428; [CP] 2.773-778. Reductio ad absurdum, 434; [CP] 2.612. Reduction, 435; [CP] 2.585-587. Regular, 439; [CP] 2.601 is from it. Relatives (logic of), 447-450; [CP] 3.636-643 (deletion). Remote, 463. Represent, 464; [CP] 2.273. Representationism, 464-465; [CP] 5.607. Repugnance, 466. Residues (method of), 467-468. Rule, 481 (part with J. M. Baldwin). S (in logic), 483. Saltus, 484; [CP] 2.616 is from it. Scientific Method, 500-503; [CP] 7.79-88. Scope, 503. Secundum quid, 504. Series, 521. Sign, 527-528; [CP] 2.303-304 (deletion). Signification (and Application, in logic), 528-529; [CP] 2.434. Similar (with Similarity, Similitude), 530. Simple, 531-532 (parts with J. M. Baldwin). Singular, 533. Solution, 554. Some (in logic), 555. Sophism, 556. Sorites, 557. Species (and Specific Marks, in logic), 567 (with J. M. Baldwin). Spurious Proposition, 588; [CP] 2.607. State (and condition), 593. Subalternation, 606. Subcontrary, 607. Subject (in logic), 609-610; [CP] 2.357. Sublation, 611. Substitution (in logic), 614-615. Subsumption, 615. Sufficient Reason, 616-617; [CP] 6.393-394 (deletion). Summum Genus, 621. Supposition, 624-625. Syllogism, 628-629, 633-639; [CP] 2.552-554, 2.555-579 (deletions). Symbol, 640; [CP] 2.307 (deletion). Symbolic Logic or Algebra of Logic, 645-650; [CP] 4.372-393. Symbolical, 651. Synechism, 657; [CP] 6.169-173. Synthetic (-al), 658-659. Tautology, 663. Term, 676-677. Testimony, 686 (with J. M. Baldwin). Thema, 691-692; [CP] 2.308. Theorem, 693. Theory, 693-694; [CP] 7.94-96 are from it. Thesis, 695. Tree of Porphyry, 714. Trilemma, 715. Trivium, 716. Truth and Falsity and Error, 718-720; [CP] 5.565-573. Ultimate, 723-724; [CP] 5.608-609. Uniformity, 727-731; [CP] 6.98-101. Unity (and Plurality), 734-736; [CP] 6.373-380. Universal (and Universality), 737-741; [CP] 2.367-371 (deletions). Universe, 742; [CP] 2.536 (deletions). Vague (in logic), 748. Validity, 748-749; [CP] 2.779-781. Verification, 761-762; [CP] 7.89-91. Virtual, 763-764; [CP] 6.372 (deletion). Whole (and Parts), 814 (with J. M. Baldwin), 814-815; [CP] 6.381, 6.382-383.

Peirce: CP 8 Bibliography General 1901 [G-1901-7]

7. A draft of a report of the meeting of the National Academy of Sciences in Nov 1901, Widener IV. [CP] 7.162-163 are from it. See [Bibliography] N-1901-16 for the published report of the meeting.

Peirce: CP 8 Bibliography General 1901 [G-1901-8]

8. A draft of a review of Sidney E. Mezes' *Ethics: Descriptive and Explanatory*, Widener IV. [CP] 8.158n3 is from it. See [CP] 8.157-163, [Bibliography] N-1901-15, for the published review.

Peirce: CP 8 Bibliography General 1901 [G-1901-9]

9. Review of Henry Stevens's *Thomas Hariot, the Mathematician, the Philosopher, and the Scholar*, *The American Historical Review* 6(April 1901)557-561.

Peirce: CP 8 Bibliography General 1902 [G-1902-1]
1902

1. Reference to "The Classification of the Sciences," presented 15-17 April 1902. [RNA 1902]13.

Peirce: CP 8 Bibliography General 1902 [G-1902-2]

2. Reference to "The Postulates of Geometry," presented 15-17 April 1902. [RNA 1902]13.

Peirce: CP 8 Bibliography General 1902 [G-1902-3]

3. Reference to "The Color System," presented 15-17 April 1902. [RNA 1902]13. Cf. [Bibliography] N-1902-5.

Peirce: CP 8 Bibliography General 1902 [G-1902-4]

4. A section revised and rewritten by Peirce (pp. 280-286 in the chapter on units and measures) of Thomas J. McCormack's translation of Ernst Mach's *The Science of Mechanics*, Open Court, Chicago, 1902.

Peirce: CP 8 Bibliography General 1902 [G-1902-5]

5. Translation of J. Marey's "The History of Chronophotography," *Annual Report . . . of the Smithsonian Institution . . . for the Year Ending June 30, 1901*, Govt. Printing Office, Washington, 1902, pp. 317340. Pages 337-340 may not have been translated by Peirce.

Peirce: CP 8 Bibliography General 1902 [G-1902-6]

6. Application for a grant from the Carnegie Institution, Widener VB5. [CP] 7.158-161 and 8.176n3 are from it. [CP] 7.158n5 contains a quotation from an

alternative draft.

Peirce: CP 8 Bibliography General c.1902 [G-c.1902-1]
c.1902

1. A fragment on induction. [CP] 2.757n1 is from it.

Peirce: CP 8 Bibliography General c.1902 [G-c.1902-2]

* 2. *Minute Logic*, an uncompleted book. Ch. 1-3 and part of Ch. 4 were finished, and further chapters were planned ([CP] 1.274, 1.277, 1.283n{tt}, 1.584n*; [CP]II, iii; [CP] 2.197, 4.227, 4.242, 4.244, 4.274, 4.323, 6.349n†).

Peirce: CP 8 Bibliography General c.1902 [G-c.1902-2]

Chapter 1. Intended Characters of this Treatise. [CP] 2.1-118 (1902) with deletions. (See [Perry]II, 422 concerning an earlier version of [CP] 2.9.)

Peirce: CP 8 Bibliography General c.1902 [G-c.1902-2]

Chapter 2. Prelogical Notions. Section 1. Classification of the Sciences. Widener IB2-2, 1902. [CP] 1.203-283, 7.374n10, 7.279, and 7.362-387 except 381n19, are from it in that order; cf. 7.362n1, 7.363n5. Section 2. Why Study Logic. [CP] 2.119-202.

Peirce: CP 8 Bibliography General c.1902 [G-c.1902-2]

Chapter 3. The Simplest Mathematics. [CP] 4.227-323 (1902) with deletions.

Peirce: CP 8 Bibliography General c.1902 [G-c.1902-2]

Chapter 4. Ethics. [CP] 1.575-584 and 6.349-352 (both dated 1902/1903) are from it.

Peirce: CP 8 Bibliography General c.1902 [G-c.1902-3]

3. "Reason's Rules." [CP] 5.538-545 are from it.

Peirce: CP 8 Bibliography General c.1902 [G-c.1902-4]

4. Various partial drafts of a review of Josiah Royce's *The World and the Individual*, Vol. 2, Widener IV. [CP] 8.117 (in part), 8.118-119, 8.120 (in part), 8.121-125 (except 122n19) and 8.131 are from them. See [Bibliography] N-1902-10 for the published review, parts of which are reprinted with the above. See [Bibliography] N-1900-15 for a review of Vol. 1.

Peirce: CP 8 Bibliography General c.1902 [G-c.1902-5]

5. "Of the Classification of the Sciences. Second Paper. Of the Practical Sciences." Widener II. [CP] 7.53-57, 7.381n19, and 7.58 are from it in that order.

Peirce: CP 8 Bibliography General 1903 [G-1903-1]
1903

- * 1. A series of lectures on pragmatism, delivered at Harvard University,

March-May, 1903.

Peirce: CP 8 Bibliography General 1903 [G-1903-1]

Lecture I. On pragmatism and the normative sciences. [CP] 5.14-40.

Peirce: CP 8 Bibliography General 1903 [G-1903-1]

Lecture II. Draft 1, "On Phenomenology"; [CP] 1.322-323 (c.1903) are from it. Draft 2, "On Phenomenology"; [CP] 5.41-56 (cf. 5.41n*) are from it. Draft 3, "On Phenomenology, or the Categories"; [CP] 5.59-65.

Peirce: CP 8 Bibliography General 1903 [G-1903-1]

Lecture III. On the categories. (Cf. [CP] 5.66n*, 5.82n*.) Version "a," "The Categories continued"; [CP] 5.71n1 and 5.82-87 are from it. Version "b," "The Categories Defended"; [CP] 5.66-81 (except 5.71n1, 5.77n1) and 5.88-92 are from it.

Peirce: CP 8 Bibliography General 1903 [G-1903-1]

Lecture IV. "The Seven Systems of Metaphysics." [CP] 5.77n1, from the beginning; 5.93-119, following shortly after 5.77n1; 5.57-58, part of a digression at the end (5.57n*); 1.314-316, an apparently undelivered passage (1.314n*, 5.118n*).

Peirce: CP 8 Bibliography General 1903 [G-1903-1]

Lecture V. On three kinds of goodness. [CP] 5.120-150, from the third and final draft.

Peirce: CP 8 Bibliography General 1903 [G-1903-1]

Lecture VI. On three types of reasoning. [CP] 5.151-179 with a deletion.

Peirce: CP 8 Bibliography General 1903 [G-1903-1]

Lecture VII. On pragmatism and abduction. [CP] 5.180-212. It is not certain that this lecture was given. According to the Peirce-James correspondence ([Perry]II, 426-427) and [CP] 5.180n*, only six lectures were planned. [CP] 5.180-212 are taken from a notebook which is marked "Pragmatism -- Lecture VII" and which seems to belong to the set of notebooks from which the other lectures of this series printed in *Collected Papers* are taken. At the beginning of this notebook Peirce says this is an extra lecture.

Peirce: CP 8 Bibliography General 1903 [G-1903-1]

In Lecture VII ([CP] 5.201) Peirce refers to "tomorrow evening's lecture on multitude and continuity." There is in Houghton Library a notebook labeled "Multitude and Continuity. A lecture to be delivered . . . in Harvard University, 1903 May 15." It should be noted in this connection that while [CP]V (n* preceding 5.14) gives March 26 to May 17, 1903, as the dates of the lectures, [Perry]II, 426, gives March 26 to May 14, 1903. The editor knows of no evidence other than that offered by the notebook that this eighth lecture was given.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

* 2. Lowell lectures of 1903 and supplementary materials.

Peirce: CP 8 Bibliography General 1903 [G-1903-2a]

(a) The eight lectures were given the general title, "Some Topics of Logic Bearing on Questions Now Vexed." This title and the titles and dates for the individual lectures were taken from a ticket in Widener VA²; the material printed in [CP] VII is in Widener IB2-4. Cf. [Perry]II, 426.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

Lecture I. What Makes a Reasoning Sound? Nov. 23. The following are from it: [CP] 1.591-610 from Vol. 1, draft 3; 1.611-615 and 8.176 (except 176n3), following shortly after 1.611-615, from Vol. 2, draft 2, a continuation of Vol. 1, draft 3.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

Lecture II. A System of Diagrams for Studying Logical Relations. Exposition of it begun. Nov. 27.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

Lecture III. The Three Universal Categories and their Utility. Nov. 30. The following are from it: [CP] 1.15-26 from Lecture IIIa; 1.324 from Lecture III, Vol. 1, draft 3; 1.343-349 from Lecture III, Vol. 1, draft 3; 1.521-544 from Lecture III, Vol. 2, draft 3, following 1.349.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

Lecture IV. Exposition of the System of Diagrams Completed. Dec. 3. [CP] 4.510-529 with deletions; 4.529n* contains material found on a separate sheet, apparently for use in a similar lecture.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

Lecture V. The Doctrine of Multitude, Infinity and Continuity. Dec. 7. Cf. reference at [CP] 5.201n*.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

Lecture VI. What is Chance? Dec. 10. [CP] 6.88-97 are from it.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

Lecture VII. Induction as Doing, not mere Cogitation. Dec. 14. [CP] 7.110-130 are from Vol. 1.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

Lecture VIII. How to Theorize. Dec. 17. [CP] 5.590-604 and 7.182n7, following shortly after 5.604, are from it.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

Two quotations from these lectures are given at [CP] 3.45n* and 3.154n1.

Peirce: CP 8 Bibliography General 1903 [G-1903-2b]

(b) *A Syllabus of Certain Topics of Logic*, Alfred Mudge & Son, Boston, 1903. Cf. Peirce's notice of it, [Bibliography] N-1905-9. Reprinted in part as follows: "Preface," [CP] 1.180n*, "This syllabus has for its object to supplement a course of eight lectures to be delivered at the Lowell Institute . . ."; pp. 5-9, "An Outline Classification of the Sciences," [CP] 1.180-202; pp. 10-14, "The Ethics of Terminology," [CP] 2.219-226 continuing 1.202; pp. 15-23, "Existential Graphs," [CP] 4.394-417, continuing 2.226, with a revision of c.1910.

Peirce: CP 8 Bibliography General 1903 [G-1903-2c]

(c) A partly printed work in two parts:

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

"Nomenclature and Divisions of Dyadic Relations," printed separately in eight pages (c.1903). [CP] 3.571-587. [CP] 3.588-608 are from the ms. (c.1903).

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

"Nomenclature and Divisions of Triadic Relations, as far as they are determined." [CP] 2.233-272, from the ms. (c.1903), continuing 3.608.

Peirce: CP 8 Bibliography General 1903 [G-1903-2]

This two-part work was apparently intended as the second part of the previous item. See references: [CP] 3.571n*, 3.587n{tt}, 3.598n* (p. 383), 3.608n{tt};, 2.233n*.

Peirce: CP 8 Bibliography General 1903 [G-1903-2d]

(d) "Syllabus" (c.1902). [CP] 2.274-277, 2.283-284, 2.292-294, 2.309331 (with 2.309 continuing 2.294) are from it. There is a ms. in Widener IB2-4 which contains both this and most of the printed syllabus, (b) above. In the ms., as contrasted to the printed syllabus, this material comes between that of [CP] 2.219-226 and 4.394-417, but it is not adjacent to either.

Peirce: CP 8 Bibliography General 1903 [G-1903-3]

3. Translation of Victor Schumann's *On the Absorption and Emission of Air and its Ingredients for Light of Wave-Lengths from 250 {mm} to 100 {mm}*, Smithsonian Institution, Washington, 1903, iv + 30 pp. + 4 plates.

Peirce: CP 8 Bibliography General 1903 [G-1903-4]

4. Comment by Peirce on his work on logic. [CP] II, xii.

Peirce: CP 8 Bibliography General 1903 [G-1903-5]

5. "Telepathy," Widener IC1-a,b. [CP] 7.597-688, except 597n3, with deletions.

Peirce: CP 8 Bibliography General c.1903 [G-c.1903-1]
c.1903

1. A fragment on the classification of ends. [CP] 1.585-588 are from it.

Peirce: CP 8 Bibliography General c.1903 [G-c.1903-2]

* 2. "Logical Tracts, No. 2." [CP] 4.418-509, "On Existential Graphs, Euler's Diagrams, and Logical Algebra," are from it. For a statement about "Logical Tracts, No. 1," see [CP] 4.418n.

Peirce: CP 8 Bibliography General c.1903 [G-c.1903-2]

Peirce referred to his system of existential graphs as "My Chef d'Oeuvre." [CP]IV, 291 (undated).

Peirce: CP 8 Bibliography General c.1903 [G-c.1903-3]

3. "Graphs." [CP] 4.350-371 are from it.

Peirce: CP 8 Bibliography General c.1903 [G-c.1903-4]
4. A fragment on metaphysics. [CP] 6.6 is from it.

Peirce: CP 8 Bibliography General c.1903 [G-c.1903-5]
5. A draft of a review of James Mark Baldwin's *Dictionary of Philosophy and Psychology*, Widener IV. [CP] 8.167 (in part) and 168-170 are from it. See [Bibliography] N-1903-10 for the published review, reprinted in part at [CP] 8.164-166 and 167 (in part).

Peirce: CP 8 Bibliography General c.1903 [G-c.1903-6]
6. Unpublished review of C. A. Strong's *Why the Mind Has a Body* (Macmillan, 1903, 355 pp.), Widener IV. [CP] 8.186-187.

Peirce: CP 8 Bibliography General 1904 [G-1904-1]
1904

1. "French Academy of Science," *New York Evening Post*, 5 March 1904.

Peirce: CP 8 Bibliography General 1904 [G-1904-2]
2. Reference to "Note on the Simplest Possible Branch of Mathematics," presented 19-21 April 1904. [RNA 1904] 14.

Peirce: CP 8 Bibliography General 1904 [G-1904-3]
3. Reference to "On Topical Geometry," presented 15-16 Nov 1904. [RNA 1904]16. This paper was described in [Bibliography] N-1904-20. Cf. [Cohen]737.

Peirce: CP 8 Bibliography General 1904 [G-1904-4]
4. An untitled manuscript, Widener V{beta}. [CP] 8.225n10 is from it.

Peirce: CP 8 Bibliography General c.1904 [G-c.1904-1]
c.1904

1. "Notes on the List of Postulates of Dr. Huntington's Section 2." [CP] 4.324-330.

Peirce: CP 8 Bibliography General c.1904 [G-c.1904-2]

2. "Logic viewed as Semeiotics, Introduction Number 2, Phaneroscopy." [CP] 1.285-287, 1.304 (continuing 1.287) are from it.

Peirce: CP 8 Bibliography General c.1904 [G-c.1904-3]

3. A draft of a review of Herbert Nichols' *A Treatise on Cosmology*, Vol. 1, *Introduction*, Widener IV. [CP] 8.191-193 are from it. Cf. [Bibliography] G-1905-2.

Peirce: CP 8 Bibliography General c.1904 [G-c.1904-3]

"Nichols' Cosmology and Pragmaticism," Widener IB1-2. [CP] 8.194-195 are from it. Cf. [CP] 8.191n1.

Peirce: CP 8 Bibliography General 1905 [G-1905-1]
1905

* 1. *The Monist* 1905-1906 series on pragmatism, plus drafts (items (d) through (h) below) of an unpublished paper intended as a fourth article to appear Jan 1907. See [CP] 1.305n*, 1.306n*, 4.540, 4.540n*, 4.541, 4.572, 5.440, 8.191n1.

Peirce: CP 8 Bibliography General 1905 [G-1905-1a]

(a) "What Pragmatism Is," 15(April 1905)161-181. [CP] 5.411-437.

Peirce: CP 8 Bibliography General 1905 [G-1905-1b]

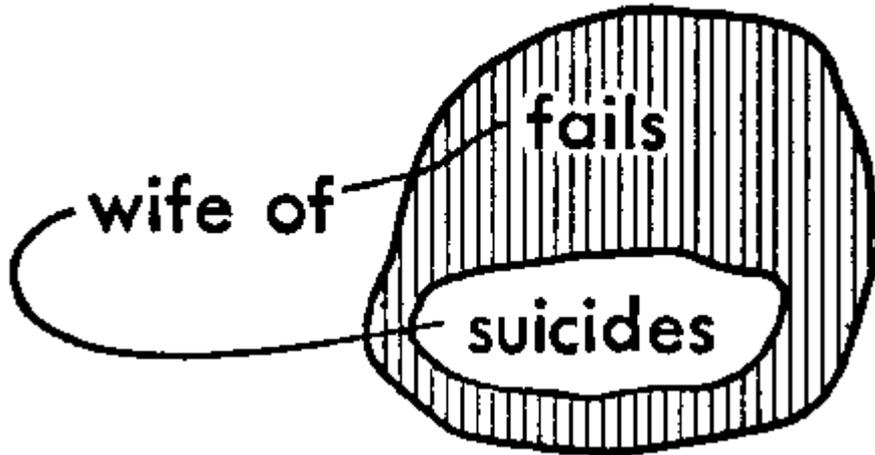
(b) "Issues of Pragmatism," 15(Oct 1905)481-499. [CP] 5.438-463, except 448n1. [CP] 5.402n3 (1906) is from a ms., "Issues of Pragmatism" (not "Consequences of Pragmatism"), Widener IB1-1, which is similar to the printed article.

Peirce: CP 8 Bibliography General 1905 [G-1905-1c]

(c) "Prolegomena to an Apology for Pragmatism," 16(Oct 1906) 492-546. Errata, 17(Jan 1907)160. [CP] 4.530-572 with the published corrections and with material from (e), (f), and (h) below in footnotes. [CP] 1.288-292 are from draft {pl} of the ms. (c.1908).

Peirce: CP 8 Bibliography General 1905 [G-1905-1c]

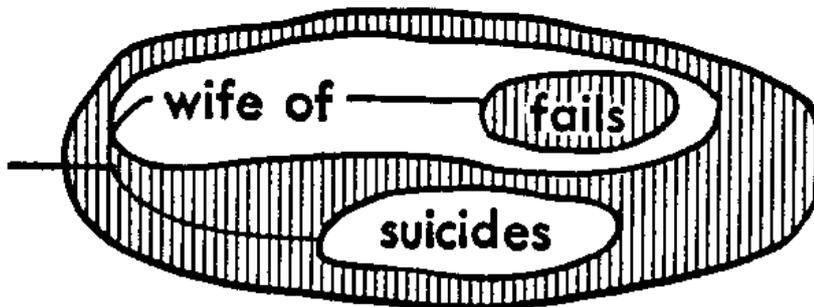
In a letter to F. A. Woods, [Bibliography]M-22, Peirce says that the material in [CP] 4.569, from "For the sake of illustrating this . . ." up to the statement of the Fourth Permission, is wrong. He says: "Instead of scribing



[Click

here to view]

as I did, I should have scribed



[Click here

to view]

Peirce: CP 8 Bibliography General 1905 [G-1905-1c]

". . . [This fallacy] cost me the trouble of my nonsensical 'tinctures' and heraldry.

Peirce: CP 8 Bibliography General 1905 [G-1905-1c]

"I am also sceptical as to the universal validity of my '4th permission.'"

Peirce: CP 8 Bibliography General 1905 [G-1905-1d]

(d) "Basis of Pragmatism." Portions dated 1906 published at: [CP] 1.573-574; 5.549-554, continuing 1.574; 5.448n1, following somewhat after 5.554.

Peirce: CP 8 Bibliography General 1905 [G-1905-1e]

(e) "The Bedrock beneath Pragmatism." Portions dated c.1906 published at: [CP] 4.553n2; 4.561n1; 6.174-176

Peirce: CP 8 Bibliography General 1905 [G-1905-1f]
(f) "Copy T." [CP] 4.564n1 (c.1906) is from it.

Peirce: CP 8 Bibliography General 1905 [G-1905-1g]
(g) "An Apology for Pragmatism." [CP] 1.305 is from it.

Peirce: CP 8 Bibliography General 1905 [G-1905-1h]
(h) "Phanerostopy {phan}." Portions dated c.1906 published at: [CP] 1.306-311, 4.6-11, 4.534n1, 4.553n1 (p. 441).

Peirce: CP 8 Bibliography General 1905 [G-1905-2]
2. Review of Herbert Nichols' *A Treatise on Cosmology*, Vol. 1, signed "Cusp.," *The Monist* 15(Jan 1905)157-158. Identification based on Harvard mss.

Peirce: CP 8 Bibliography General 1905 [G-1905-3]
3. "Substitution in Logic," *The Monist* 15(April 1905)294-295. This article was signed by "Francis C. Russell," but, judging by correspondence in Widener, it was written by Peirce though it was approved and perhaps modified by Russell.

Peirce: CP 8 Bibliography General 1905 [G-1905-4]
4. Reference to "The Relation of Betweenness and Royce's O-Collections," presented 14-15 Nov 1905. [RNA 1905] 15.

Peirce: CP 8 Bibliography General 1905 [G-1905-5]
5. "Adirondack Lectures, 1905." [CP] 1.284 is from it.

Peirce: CP 8 Bibliography General 1905 [G-1905-6]
6. "Analysis of Some Demonstrations Concerning Definite Positive Integers." [CP] 4.341-346.

Peirce: CP 8 Bibliography General c.1905 [G-c.1905-1]
c.1905

1. Ms. G of "A Neglected Argument for the Reality of God." [CP] 2.755-772 (on induction), except 757n1, are from it. Cf. [Bibliography] G-1908-2, especially [CP] 6.473.

Peirce: CP 8 Bibliography General c.1905 [G-c.1905-2]
2. "Logic and Spiritualism," intended for *The Forum*. [CP] 6.557-587.

Peirce: CP 8 Bibliography General c.1905 [G-c.1905-3]

3. On ordinals. [CP] 4.331-340. [CP] 4.331-334 are from a proposed lecture to the American Academy of Arts and Sciences; 4.335-340 are from "Topics," a revised version of the latter part of the lecture.

Peirce: CP 8 Bibliography General c.1905 [G-c.1905-4]

4. "Phaneroscopy or the Natural History of Concepts." [CP] 1.332336 are from it.

Peirce: CP 8 Bibliography General c.1905 [G-c.1905-5]

5. "The Basis of Pragmatism." [CP] 1.294-299, 1.313n1 are from Notebook I. [CP] 1.313, 1.350-352 (undated) are from Notebook II.

Peirce: CP 8 Bibliography General c.1905 [G-c.1905-6]

6. "Sketch of Some Proposed Chapters on the Sect of Philosophy Called Pragmatism," a notebook. [CP] 1.126-129 are from "Introduction showing the point of view from which Philosophy appears to the author to be an interesting subject to a man of common-sense."

Peirce: CP 8 Bibliography General c.1905 [G-c.1905-7]

7. "The Basis of Pragmaticism." [CP] 5.497-501 with deletions.

Peirce: CP 8 Bibliography General c.1905 [G-c.1905-8]

* 8. "Pragmaticism, Prag. [4]." The first page of the ms. is missing. [CP] 5.502-537 with a deletion.

Peirce: CP 8 Bibliography General c.1905 [G-c.1905-9]

9. A draft of a review of Wilhelm Wundt's *Principles of Physiological Psychology*, Widener IV. [CP] 8.201n3 is from it. See [CP] 8.196-204, [Bibliography] N-1905-19, for the published review.

Peirce: CP 8 Bibliography General 1906 [G-1906-1]
1906

1. "Mr. Peterson's Proposed Discussion," *The Monist* 16(Jan 1906) 147-151. Erratum, *ibid.*, 16(April 1906)20. This is a discussion of James B. Peterson's "Some Philosophical Terms," *ibid.*, 15(Oct 1905)629633. [CP] 5.610-614.

Peirce: CP 8 Bibliography General 1906 [G-1906-2]

2. Reference to "Recent Developments of Existential Graphs and their Consequences for Logic," presented 16-18 April 1906. [RNA 1906] 15. This paper is described in [Bibliography] N-1906-4. [CP] 4.573-584 are from it.

Peirce: CP 8 Bibliography General 1906 [G-1906-3]

3. Reference to "Phanerescopy, or Natural History of Signs, Relations, Categories, etc.: A method of investigating this subject expounded and illustrated," presented 20-22 Nov 1906. [RNA 1906]18.

Peirce: CP 8 Bibliography General c.1906 [G-c.1906-1]
c.1906

1. "Reflexions upon Pluralistic Pragmatism and upon Cenopythagorean Pragmaticism." [CP] 5.555-564.

Peirce: CP 8 Bibliography General c.1906 [G-c.1906-2]

2. "Answers to Questions Concerning My Belief in God." [CP] 6.494-521.

Peirce: CP 8 Bibliography General 1907 [G-1907-1]
1907

1. References to Peirce's two lectures on scientific method at the Philosophy Club at Harvard, June 1907. [Perry]II, 436; Paul Weiss, "Charles Sanders Peirce," *Dictionary of American Biography*, Vol. 14, New York, 1937, p. 402.

Peirce: CP 8 Bibliography General c.1907 [G-c.1907-1]
c.1907

* 1. An unpublished letter-article to the Editor of *The Nation* on pragmatism. The following are grouped together since there is considerable evidence that the manuscripts from which they come are different attempts at the same article. This is dated c.1907 since F.C.S. Schiller's *Studies in Humanism*, apparently referred to at [CP] 5.494 (see (c) below), was published in 1907 ([Fisch]413). Cf. [Bibliography] M-21b, pp. 20-21; see also [Fisch] 443-444.

Peirce: CP 8 Bibliography General c.1907 [G-c.1907-1a]

(a) "Pragmatism (Prag. [J])," comments on "On a New List of Categories," [Bibliography] G-1867-1c. [CP] 1.560-562 (c.1905).

Peirce: CP 8 Bibliography General c.1907 [G-c.1907-1b]

(b) "Pragmatism [1]." [CP] 5.5-10 (c.1905) are from it.

Peirce: CP 8 Bibliography General c.1907 [G-c.1907-1c]

(c) "Pragmatism (Editor [3])." [CP] 5.11-13, except 5.13n1, and 5.464 496, with 5.464 continuing 5.13, (both dated c.1906) are from it. [CP] 5.11-13 were previously printed in *The Hound and Horn* 2(April/June 1929)282-285, under the title, "The Founding of Pragmatism."

Peirce: CP 8 Bibliography General c.1907 [G-c.1907-2]

2. "Guessing," Widener IB2-12. [CP] 7.36-48, except 36n13, are from it. The ms. is published in *The Hound and Horn* 2(April-June 1929) 267-282. [CP] 7.36n13 is from an alternative fragmentary ms. in Widener IB2-12.

Peirce: CP 8 Bibliography General 1908 [G-1908-1]
1908

1. *The Monist* 1908-1909 series on mazes, plus unpublished mss.

Peirce: CP 8 Bibliography General 1908 [G-1908-1a]

(a) "Some Amazing Mazes," 18(April 1908)227-241. Cf. Francis C. Russell, "Hints for the Elucidation of Mr. Peirce's Logical Work," *The Monist* 18(July 1908)406-415. In a letter to Russell, dated 6 July 1908 (Widener VB2a), Peirce says of this, "I must thank you for your very ingenious defense of me." [CP] 4.585-593.

Peirce: CP 8 Bibliography General 1908 [G-1908-1b]

(b) "Some Amazing Mazes, [Conclusion], Explanation of Curiosity the First," 18(July 1908)416-464. [CP] 4.594-642. "*Supplement*. 1908 May 24," Widener IA-3. [CP] 7.535n6 is from it.

Peirce: CP 8 Bibliography General 1908 [G-1908-1c]

(c) "Some Amazing Mazes, A Second Curiosity," 19(Jan 1909)36-45. [CP] 4.643-646.

Peirce: CP 8 Bibliography General 1908 [G-1908-1d]

(d) Reference to "The Third Curiosity" at [CP] 4.647n*.

Peirce: CP 8 Bibliography General 1908 [G-1908-1e]

(e) "Some Amazing Mazes, Fourth Curiosity," (c.1909). The following are from it: [CP] 6.318-348, on various topics in metaphysics; 4.647-681, on classes and numbers, with 4.647 following shortly after 6.348.

Peirce: CP 8 Bibliography General 1908 [G-1908-2]

* 2. "A Neglected Argument for the Reality of God," *The Hibbert Journal* 7(Oct 1908)90-112. [CP] 6.452-485.

Peirce: CP 8 Bibliography General c.1908 [G-c.1908-1]
c.1908

1. A fragment on assertion. [CP] 5.546-548 are from it.

Peirce: CP 8 Bibliography General 1909 [G-1909-1]
1909

1. A set of manuscripts with various titles, but all having "Meaning" and the date written in the upper left-hand corner of each page, Widener IB2-11. [CP] 1.27 and 7.313n1 are from these. The quote from Peirce in [CP] 5.358n* is, except for minor editorial changes, identical with a passage in one of these mss., "Studies in Meaning; The Import of Thought: An Essay in Two Chapters." Cf. [CP] 8.214ff., 8.302ff.

Peirce: CP 8 Bibliography General 1910 [G-1910-1]
1910

1. "Definition." [CP] 1.312 is from it.

Peirce: CP 8 Bibliography General 1910 [G-1910-2]

* 2. A comment on "The Doctrine of Chances," [Bibliography] G-1877-5c. [CP] 2.661-668.

Peirce: CP 8 Bibliography General 1910 [G-1910-3]

3. "Meaning." [CP] 2.230-232 are from it.

Peirce: CP 8 Bibliography General 1910 [G-1910-4]

† 4. On trichotomies. [CP] 1.568-572. See [CP] 1.568n for a reference to a book of which this may have been a part. Several more pages of the ms. have been found since [CP] 1.572n* was written.

Peirce: CP 8 Bibliography General c.1910 [G-c.1910-1]
c.1910

1. "Additament," comments on "A Neglected Argument for the Reality of God," [Bibliography] G-1908-2. [CP] 6.486-490. [CP] 6.491 is from an alternative draft.

Peirce: CP 8 Bibliography General c.1910 [G-c.1910-2]
2. "Pragmatism," Fragment 2. [CP] 1.317-321 are from it.

Peirce: CP 8 Bibliography General c.1910 [G-c.1910-3]
3. "Notes for my Logical Criticism of Articles of the Christian Creed,"
Widener IB3. [CP] 7.97-109.

Peirce: CP 8 Bibliography General 1911 [G-1911-1]
1911

1. Reference to "A method of computation," presented 21-22 Nov 1911. [RNA
1911]24.

Peirce: CP 8 Bibliography General 1911 [G-1911-2]
2. Reference to "The reasons of reasoning, or grounds of inferring," presented
21-22 Nov 1911. [RNA 1911]24.

Peirce: CP 8 Bibliography General c.1911 [G-c.1911-1]
c.1911

1. "A Sketch of Logical Critic." [CP] 6.177-184 are from it.

Peirce: CP 8 Bibliography General Undated [G-Undated-1]
Undated

1. On secondness. [CP] 1.325.

Peirce: CP 8 Bibliography General Undated [G-Undated-2]
2. On secondness. [CP] 1.330-331 are from it.

Peirce: CP 8 Bibliography General Undated [G-Undated-3]
3. On thirdness. [CP] 1.338-339 are from it.

Peirce: CP 8 Bibliography General Undated [G-Undated-4]

4. "Of the Nature of Measurement," Widener IA-4. [CP] 7.280-312 are from it.

Peirce: CP 8 Bibliography General Undated [G-Undated-5]

5. Quotation from the "Introduction" to a planned history of science, Widener IC1b. [CP] 7.267n8.

Peirce: CP 8 Bibliography General Undated [G-Undated-6]

6. A fragment on non-Euclidean geometry, Widener IA7. [CP] 8.93n2.

Peirce: CP 8 Bibliography General Undated [G-Undated-7]

7. "The Non Euclidean Geometry made Easy," Widener IA2. [CP] 8.97-99 are from it.

Peirce: CP 8 Bibliography General Undated [G-Undated-8]

8. A manuscript on signs, Widener IB3a. This is part of a letter. [CP] 8.177-185 are from it.

Peirce: CP 8 Bibliography General Undated [G-Undated-9]

9. An untitled ms. on primisense, altersense, and medisense, Widener IB1-2. [CP] 7.539-552, with 540n8 and 541n9 from an alternate draft at Widener IC1-a,b; cf. 7.539n7.

Peirce: CP 8 Bibliography General Undated [G-Undated-10]

10. An incomplete ms., Widener IB2-10. [CP] 7.276-278 are Section 13, "Varieties of Medisense." This ms., has some resemblance to [Bibliography] G-Undated-9, and may have been written at about the same time.

Peirce: CP 8 Bibliography General Undated [G-Undated-11]

11. *Qualitative Logic*, a partially completed book, Widener IB2-9. [CP] 7.451-462 are from it. Cf. [CP] 7.451n25 and 7.462n28.

Peirce: CP 8 Bibliography General Undated [G-Undated-12]

12. "On Topical Geometry, in General," Widener IA-2. [CP] 7.524538, except 534n4 and 535n6, are from it.

Peirce: CP 8 Bibliography General Undated [G-Undated-13]

13. Mss. L and N, Widener IB2-9. [CP] 7.49-52 and 7.77-78, respectively, are from them. Cf. [CP] 7.59n4.

Peirce: CP 8 Bibliography General Undated [G-Undated-14]

14. Two fragmentary drafts on consciousness, Widener ICl-a,b. [CP] 7.553 is one of these; 7.554 is from the other.

Peirce: CP 8 Bibliography General Undated [G-Undated-15]

15. A fragment on matter, Widener IA-8. [CP] 7.534n4 is from it.

Peirce: CP 8 Bibliography General Undated [G-Undated-16]

16. "Notes on Topical Geometry," Widener IA-2. [CP] 8.368n23 is from it.

Peirce: CP 8 Bibliography General Undated [G-Undated-17]

17. On "would be," Widener IA-8. [CP] 8.380n4 is from it.

Peirce: CP 8 Bibliography *The Nation* 1869 [N-1869-1]

II. ITEMS FROM *THE NATION*

1869

† 1. Review of Noah Porter's *The Human Intellect*, 8(18 March 1869) 211-213.

Peirce: CP 8 Bibliography *The Nation* 1869 [N-1869-2]

2. Review of Henry E. Roscoe's *Spectrum Analysis*, 9(22 July 1869) 73-74.

Peirce: CP 8 Bibliography *The Nation* 1869 [N-1869-3]

† 3. "The English Doctrine of Ideas," a review of James Mill's *Analysis of the Phenomena of the Human Mind* (edited by John Stuart Mill), 2 vols., 9(25 Nov 1869)461-462.

Peirce: CP 8 Bibliography *The Nation* 1871 [N-1871-1]

1871

1. "Mr. Peirce and the Realists," a signed letter, 13(14 Dec 1871)386. This is a reply to editorial comments in *The Nation* 13(2 Nov 1871)294 and 13(30 Nov

1871)355-356, the latter by Chauncey Wright, on [Bibliography] G-1871-1.

Peirce: CP 8 Bibliography *The Nation* 1873 [N-1873-1]
1873

1. Review of H. M. Lazelle's *One Law in Nature*, 17(10 July 1873) 28-29.

Peirce: CP 8 Bibliography *The Nation* 1878 [N-1878-1]
1878

1. Review of Simon Newcomb's *Popular Astronomy*, 27(1 Aug 1878)74.

Peirce: CP 8 Bibliography *The Nation* 1879 [N-1879-1]
1879

1. Review of Carveth Read's *The Theory of Logic*, 28(3 April 1879) 234-235.

Peirce: CP 8 Bibliography *The Nation* 1879 [N-1879-2]

2. Review of Ogden N. Rood's *Modern Chromatics*, 29(16 Oct 1879) 260. All but the last two paragraphs are by Peirce.

Peirce: CP 8 Bibliography *The Nation* 1879 [N-1879-3]

3. Notes on the *American Journal of Mathematics*, 29(25 Dec 1879) 440.

Peirce: CP 8 Bibliography *The Nation* 1881 [N-1881-1]
1881

- † 1. Review of W. Stanley Jevons' *Studies in Deductive Logic*, 32(31 March 1881)227.

Peirce: CP 8 Bibliography *The Nation* 1884 [N-1884-1]
1884

1. "The Reciprocity Treaty with Spain," a signed letter, 39(18 Dec 1884) 521.

Peirce: CP 8 Bibliography *The Nation* 1885 [N-1885-1]
1885

1. "The Spanish Treaty Once More," a signed letter, 40(1 Jan 1885) 12.

Peirce: CP 8 Bibliography *The Nation* 1889 [N-1889-1]
1889

1. "The Century Dictionary," a signed letter, 48(20 June 1889)504505. This is a reply to a letter from S. Newcomb, *The Nation* 48(13 June 1889)488. Newcomb's rejoinder is at *ibid.*, (27 June 1889)524.

Peirce: CP 8 Bibliography *The Nation* 1890 [N-1890-1]
1890

1. Review of E. Noel's *The Science of Metrology*, 50(27 Feb 1890)184. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1890 [N-1890-2]

2. Review of F. Howard Collins' *Epitome of the Synthetic Philosophy*, 50(27 March 1890)265. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1890 [N-1890-3]

- † 3. Review of Th. Ribot's *The Psychology of Attention*, 50(19 June 1890)492-493.

Peirce: CP 8 Bibliography *The Nation* 1890 [N-1890-4]

- † 4. Review of W. Stanley Jevons' *Pure Logic, and Other Minor Works*, 51(3 July 1890)16. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1890 [N-1890-5]

- † 5. Review of Paul Carus' *Fundamental Problems*, 51(7 Aug 1890) 118-119. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1890 [N-1890-6]

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8. Review of R. O. Williams' *Our Dictionaries, and Other English-Language Topics*, 51(30 Oct 1890)349. Identification based on Harvard mss.

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† 3. Review of Herbert Spencer's *Essays, Scientific, Political, and Speculative*, 53(8 Oct 1891)283. Identification based on Harvard mss.

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5. Review of William J. McClelland's *A Treatise on the Geometry of the Circle*, 54(24 March 1892)237.

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9. Review of William Ridgeway's *The Origin of Metallic Currency and Weight Standards*, 54(23 June 1892)472-473. Identification based on Harvard mss.

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10. Review of Karl Pearson's *The Grammar of Science*, first ed., 55(7 July 1892)15. Identification based on Harvard mss.

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11. Review of S. S. Curry's *The Province of Expression*, 55(14 July 1892)35. Identification based on Harvard mss.

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† 7. Review of David G. Ritchie's *Darwin and Hegel*, 57(23 Nov 1893)
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[Bibliography] G-c.1893-5 for quotations from the prospectus for the first work, and
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† 11. "Hemholtz," an article, 59(13 Sept 1894)191-193.

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† 13. Review of Benedict de Spinoza's *Ethic*, 59(8 Nov 1894) 344-345.

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14. "Hallucinations," a signed letter, 59(22 Nov 1894)381. This is a comment on a note in *The Nation* 59(8 Nov 1894) 343. Cf. [Bibliography] G-1887-3.

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Peirce: CP 8 Bibliography *The Nation* 1895 [N-1895-2]
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2. Review of George John Romanes' *Mind and Matter, and Monism*, 62(26 March 1896)261-262. Probably by Peirce; identification based on Harvard mss.

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5. Review of H. Durége's *Elements of the Theory of Functions of a Complex Variable*, 63(3 Sept 1896)181-182. Identification based on Harvard mss.

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3. Notice of Reye's *Geometrie der Lage*, Part 1, 67(14 July 1898)31.

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5. Review of Alfred Russel Wallace's *The Wonderful Century*, 67(22 Sept 1898)228-229.

Peirce: CP 8 Bibliography *The Nation* 1898 [N-1898-6]

6. Notice of Richard Kerr's *Wireless Telegraphy*, 67(29 Sept 1898)242.

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10. Review of Noah Brooks' *The Story of Marco Polo*, 67(24 Nov 1898)397.

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8. Review of David P. Todd's *Stars and Telescopes*, 68(22 June 1899)
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11. Notice of W. Watson's *A Text-Book of Physics*, 70(1 March 1900) 163.

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14. Review of David Eugene Smith's *The Teaching of Elementary Mathematics*, 70(22 March 1900)230.

Peirce: CP 8 Bibliography *The Nation* 1900 [N-1900-15]

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23. Review of Andrew Russell Forsyth's *Theory of Differential Equations*, 3 vols., 71(19 July 1900)59.

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27. Review of Édouard Feret's *Bordeaux and its Wines, Classed by Order of Merit*, 71(20 Sept 1900)235-236.

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31. Review of Nathaniel Southgate Shaler's *The Individual*, 71(22 Nov 1900)410-411.

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† 33. Review of Henry Smith Williams' *The Story of Nineteenth Century Science*, 71(27 Dec 1900)515-516.

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2. Review of James Sime's *William Herschel and His Work*, 72(24 Jan 1901)72-73.

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6. Review of John M. Bacon's *By Land and Sea*, 72(28 March 1901) 258-259.

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12. Review of Alfred Caldecott's *The Philosophy of Religion in England and America*, 73(15 Aug 1901)139-140.

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† 14. Review of Michael Maher, S. J., *Psychology: Empirical and Rational*, 73(3 Oct 1901)267-268.

Peirce: CP 8 Bibliography *The Nation* 1901 [N-1901-15]

15. Review of Sidney Edward Mezes' *Ethics: Descriptive and Explanatory*, 73(24 Oct 1901)325-326. [CP] 8.157-163, except 158n3 ([Bibliography] G-1901-8).

Peirce: CP 8 Bibliography *The Nation* 1901 [N-1901-16]

16. "The National Academy at Philadelphia," an article, signed "M.D.," 73(21 Nov 1901)393-395. This is included in both [Cohen] and [Haskell]. See [CP] 7.162-163, [Bibliography] G-1901-7, which are from a draft of this report in Widener IV.

Peirce: CP 8 Bibliography *The Nation* 1901 [N-1901-17]

17. Notice of Arthur H. Wall's *Concise French Grammar*, 73(28 Nov 1901)415.

Peirce: CP 8 Bibliography *The Nation* 1901 [N-1901-18]

18. Review of Frank J. Addyman's *Practical X-Ray Work*, 73(12 Dec 1901)462.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-1]
1902

1. Review of John Beattie Crozier's *History of Intellectual Development on the Lines of Modern Evolution*, Vol. 3, 74(23 Jan 1902)78-79.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-2]

2. Review of Ernest Cushing Richardson's *Classification, Theoretical and Practical*, 74(27 Feb 1902)178-179.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-3]

† 3. Review of René Vallery-Radot's *The Life of Pasteur*, 2 vols., 74(6 March 1902)192-194.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-4]

4. Review of Franklin Henry Giddings' *Inductive Sociology*, 74(3 April 1902)273-274.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-5]

5. "The National Academy of Sciences," an article, 74(24 April 1902)

322-324.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-6]

6. Review of Edward R. Emerson's *The Story of the Vine*, 74(29 May 1902)433-434.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-7]

7. Notice of Delta's *Charades*, 75(10 July 1902)31.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-8]

8. Review of Harold H. Joachim's *A Study of the Ethics of Spinoza*, 75(10 July 1902)36-37.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-9]

9. Review of *Studies in Physiological Chemistry* (edited by R. H. Chittenden); Albert B. Prescott and Otis C. Johnson's *Qualitative Chemical Analysis*; Victor von Richter's *Organic Chemistry*, 2 vols.; and Harry C. Jones's *The Elements of Physical Chemistry*, 75(24 July 1902)79.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-10]

10. Review of Josiah Royce's *The World and the Individual*, Vol. 2, 75(31 July 1902) 94-96. [CP] 8.117 (in part), except 117n10 and 117n12, 8.120 (in part), and 8.126-130 are from the published review. See [Bibliography] G-c.1902-4 for selections from alternative drafts printed with the above. Cf. [Bibliography] N-1900-15 for the review of Vol. 1.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-11]

11. Notice of the *Annales* of the Paris International Congress of 1900, the report of the fifth section on the history of science, 75(7 Aug 1902) 115.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-12]

12. Review of T. E. Thorpe's *Essays in Historical Chemistry*, 75(21 Aug 1902)153-154.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-13]

† 13. Review of Friedrich Paulsen's *Immanuel Kant: His Life and Doctrine*, 75(11 Sept 1902)209-211.

"Accordingly Kant's great engine and distinction is accurate analysis. But absolute completeness of logical analysis is no less unattainable [than] is omniscience. Carry it as far as you please, and something will always remain unanalyzed." From a ms.

review of this book in Widener IV.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-14]

† 14. Review of Herbert Austin Aikins' *The Principles of Logic*, 75 (18 Sept 1902) 229-230.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-15]

15. Review of Paul Drude's *The Theory of Optics*, 75(2 Oct 1902)273.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-16]

16. Review of E. Seton Valentine and F. L. Tomlinson's *Travels in Space* and Frederic Walker's *Aerial Navigation*, 75(23 Oct 1902)329-330.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-17]

17. Review of J. B. Baillie's *The Origin and Significance of Hegel's Logic*, 75(13 Nov 1902)390.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-18]

18. Review of Andrew Russell Forsyth's *Theory of Differential Equations*, Vol. 4, 75(27 Nov 1902)430.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-19]

19. Review of George H. Ellwanger's *The Pleasures of the Table*, 75(18 Dec 1902)485-486.

Peirce: CP 8 Bibliography *The Nation* 1902 [N-1902-20]

20. Review of Alice Morse Earle's *Sundials and Roses of Yesterday*, 75(25 Dec 1902)506-507.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-1]
1903

1. Review of Thomas Smith's *Euclid: His Life and System*, 76(29 Jan 1903)99-100.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-2]

2. Notice of *The Physical Papers of Henry Augustus Rowland*, 76(5 March 1903) 194.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-3]

3. Untitled statement, 76(19 March 1903)226. This is a reply to a letter by J. S. Ames, *The Nation* 76(19 March 1903)226, criticizing Peirce's notice of item 2 above. That this is by Peirce is evident from the following remark inserted between Ames's letter and the untitled reply: "[We have received from our contributor the following statement concerning this matter. -- ED. NATION.] "

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-4]

4. Notice of Fiske's *Cosmic Philosophy*, 4 vols., with an introduction by Royce, 76(2 April 1903)269.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-5]

5. "The National Academy Meeting," an article, 76(30 April 1903) 349-351.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-6]

6. Notice of J. W. Mellor's *Higher Mathematics for Students of Chemistry and Physics*, 76(21 May 1903)418.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-7]

7. Review of John Grier Hibben's *Hegel's Logic*, 76(21 May 1903) 419-420.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-8]

8. Notice of E. T. Whittaker's *A Course of Modern Analysis*, 76(28 May 1903)436.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-9]

9. Review of *Personal Idealism*, 76(4 June 1903)462-463.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-10]

10. Review of James Mark Baldwin's *Dictionary of Philosophy and Psychology*, Vol. 2, 76(11 June 1903)482. [CP] 8.164-166 and 167 (in part) are from this review. See [CP] 8.167 (in part) and 168-170, [Bibliography] G-c.1903-5, from an alternative draft.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-11]

11. Notice of Kant's *Prolegomena to any Future Metaphysics* (edited in English by Paul Carus), 76(18 June 1903)497-498.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-12]

12. Review of Norman Smith's *Studies in the Cartesian Philosophy*, 77(16 July 1903)57-58.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-13]

13. Review of J. I. D. Hinds's *Inorganic Chemistry*, 77(23 July 1903) 81-82.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-14]

† 14. Review of Agnes M. Clerke's *Problems in Astrophysics*, 77(30 July 1903)98-99.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-15]

15. Review of A. A. Michelson's *Light Waves and their Uses* and J. A. Fleming's *Waves and Ripples in Water, Air, and Aether*, 77(13 Aug 1903) 141.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-16]

16. Notice of Alfred I. Cohn's *Tests and Reagents*, 77(3 Sept 1903)189. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-17]

17. Notice of C. Krauch's *Testing of Chemical Reagents for Purity*, 77(10 Sept 1903)208. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-18]

18. Notice of F. A. C. Perrine's *Conductors for Electrical Distribution*, 77(10 Sept 1903)208. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-19]

19. Untitled note on Sir Norman Lockyer, 77(17 Sept 1903)229.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-20]

20. "British and American Science," an article, 77(1 Oct 1903)263-264. Cf. Peirce's untitled note (item above) and a letter signed "H. T.," *The Nation* 77(1 Oct 1903) 265, commenting on this note.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-21]

21. Review of V. Welby's *What is Meaning?* and Bertrand Russell's *The*

Principles of Mathematics, Vol. 1, 77(15 Oct 1903)308-309. [CP] 8.171-175.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-22]

22. Untitled statement, 77(22 Oct 1903)320. This is a reply to a letter by James McMahan entitled "Practical Application of the Theory of Functions," *The Nation* 77(22 Oct 1903)320, commenting on [Bibliography] N-1903-20 above.

Peirce: CP 8 Bibliography *The Nation* 1903 [N-1903-23]

23. "Francis Ellingwood Abbot," a letter signed "C.S.P.," 77(5 Nov 1903) 360.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-1]
1904

† 1. Review of J. J. Fahie's *Galileo*, 78(11 Feb 1904)113-115.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-2]

2. Notice of Frederick J. E. Woodbridge's *The Philosophy of Hobbes in Extracts and Notes collected from his Writings*, 78(17 March 1901)211.
Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-3]

3. Review of Frederick A. Halsey's *The Metric Fallacy* and Samuel S. Dale's *The Metric Failure in the Textile Industry* (bound together), 78(17 March 1904)215-216.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-4]

4. Review of Simon Newcomb's *The Reminiscences of an Astronomer*, 78(24 March 1904)237.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-5]

5. Review of M. E. Boole's *Lectures on the Logic of Arithmetic* and Joseph Bowden's *Elements of the Theory of Integers*, 78(14 April 1904) 298.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-6]

6. "The National Academy Meeting," an article, 78(28 April 1904)328-330.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-7]

† 7. Review of L. Lévy-Bruhl's *The Philosophy of Auguste Comte*, 78(28

April 1904)335-336.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-8]

8. Notice of A. Clement Jones's *Notes on Analytical Geometry: An Appendix*, 78(19 May 1904)393.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-9]

9. Notice of John Hall Ryder's *Electric Traction*, 78(26 May 1904)411.
Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-10]

10. Notice of Hawkins' and Wallis' *The Dynamo*, 78(26 May 1904)411.
Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-11]

† 11. Review of William Turner's *History of Philosophy*, 79(7 July 1904)
15-16.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-12]

† 12. Review of Robert A. Duff's *Spinoza's Political and Ethical Philosophy*,
79(21 July 1904)63.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-13]

13. Review of T. Clifford Allbutt's *Notes on the Composition of Scientific
Papers*, 79(28 July 1904)84-85.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-14]

14. Notice of Roberdeau Buchanan's *The Mathematical Theory of Eclipses*,
79(25 Aug 1904)162.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-15]

† 15. Review of *The Collected Mathematical Papers of James Joseph
Sylvester*, Vol. 1, 79(8 Sept 1904)203-204.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-16]

16. Review of Ch. Renouvier's *Les Derniers Entretiens* and John Dewey's
Studies in Logical Theory, 79(15 Sept 1904)219-220. [CP] 8.188-190 are the review
of Dewey's *Studies in Logical Theory*.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-17]

17. Review of Josiah Royce's *Outlines of Psychology*, 79(29 Sept 1904) 264-265.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-18]

† 18. Review of George Malcolm Stratton's *Experimental Psychology and its Bearing upon Culture*, 79(17 Nov 1904)402-403.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-19]

19. Notices of J. Clark Murray's *Introduction to Psychology* and Florian Cajori's *Introduction to the Modern Theory of Equations*, 79(17 Nov 1904)396. Probably by Peirce; identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-20]

20. "The National Academy in New York," an article, 79(1 Dec 1904) 432-434.

Peirce: CP 8 Bibliography *The Nation* 1904 [N-1904-21]

21. Notice of J. C. Olsen's *Text-book of Quantitative Chemical Analysis*, 79(8 Dec 1904)460.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-1]
1905

1. Review of M. E. Boole's *The Preparation of the Child for Science*, 80(5 Jan 1905)18-19.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-2]

† 2. Review of Josiah Royce's *Herbert Spencer*, 80(26 Jan 1905)71-72.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-3]

† 3. Review of the Hon. R. J. Strutt's *The Becquerel Rays and the Properties of Radium*, 80(2 Feb 1905)100.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-4]

4. Review of Arthur Schuster's *An Introduction to the Theory of Optics*, 80(9 March 1905)198-199.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-5]
5. Review of R. Mullineux Walmsley's *Modern Practical Electricity*, 80(16 March 1905)218-219.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-6]
6. Notice of Gaston Laurent's *Les Grands Écrivains Scientifiques*, 80(23 March 1905)231.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-7]
7. Review of Alex. Findlay's *The Phase Rule and its Application*, 80(30 March 1905)255-256.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-8]
8. "The National Academy of Sciences," an article, 80(27 April 1905) 327-328.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-9]
9. Review of Robert Flint's *Philosophy as Scientia Scientiarum* and C. S. Peirce's *A Syllabus of Certain Topics of Logic*, 80(4 May 1905) 360-361. Identified in [Cohen]. Cf. [Bibliography] G-1903-2b.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-10]
10. Notice of J. Garcin's *N-Rays*, 80(11 May 1905)374.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-11]
11. Notice of Mendeleeff's *Principles of Chemistry*, 80(1 June 1905) 438.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-12]
12. Review of Robert Brandon Arnold's *Scientific Fact and Metaphysical Reality*, 80(1 June 1905)444-445.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-13]
† 13. Notice of Santayana's *The Life of Reason*, Vols. 1 and 2, 80(8 June 1905)461.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-14]
14. Notice of Ida Freund's *The Study of Chemical Composition*, 80(22 June

1905)503.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-15]

15. Review of Andrew Carnegie's *James Watt*, 80(29 June 1905) 527-528.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-16]

16. Notice of Gertrude Bacon's *Balloons, Airships, and Flying Machines*, 81(13 July 1905)33.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-17]

17. Notice of W. Hampson's *Radium Explained*, 81(13 July 1905)33-34.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-18]

† 18. Review of *Sociological Papers* (published by the Sociological Society) and Edward Alsworth Ross's *Foundations of Sociology*, 81(13 July 1905)42-43.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-19]

19. Review of Wilhelm Wundt's *Principles of Physiological Psychology*, Vol. 1, 81(20 July 1905)56-57. Also in *New York Evening Post* (21 July 1905)4:1-3. [CP] 8.196-204, except 201n3 ([Bibliography] G-c.1905-9) which is from a draft in Widener.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-20]

20. Notice of William James's "La Notion de Conscience," 81(3 Aug 1905)97. Also in *New York Evening Post* (31 July 1905) 4:3-4.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-21]

21. Review of Maurice de Fleury's *Nos Enfants au Collège*, 81(7 Sept 1905)205.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-22]

22. Review of Sir H. E. Roscoe and C. Schorlemmer's *A Treatise on Chemistry*, Vol. 1, 81(7 Sept 1905)205-206.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-23]

† 23. Review of Henry de Varigny's *La Nature et la Vie*, 81(5 Oct 1905)286-287.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-24]
24. Note on G. W. Hill's contributions to the theory of the moon, 81(19 Oct 1905)321.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-25]
25. Notice of Shields' *Philosophia Ultima*, 81(26 Oct 1905)340.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-26]
26. Notice of H. Carton de Wiart's *La Cité Ardente* 81(9 Nov 1905) 382.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-27]
27. "The National Academy of Sciences at New Haven," an article, 81(23 Nov 1905)417-419.

Peirce: CP 8 Bibliography *The Nation* 1905 [N-1905-28]
28. Review of Edmund Gosse's *Sir Thomas Browne*, 81(14 Dec 1905) 486-488.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-1]
1906

† 1. Review of E. Rutherford's *Radio-Activity*, 82(18 Jan 1906)61.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-2]
2. Review of Alfred Russel Wallace's *My Life*, 82(22 Feb 1906)160161.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-3]
† 3. Review of Elizabeth S. Haldane's *Descartes*, 82(22 March 1906) 242-243.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-4]
4. "Meeting of the National Academy of Sciences," an article, 82(26 April 1906)341-342.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-5]
5. Review of *Congress of Arts and Sciences, Universal Exposition, St. Louis, 1904*, Vol. 1, 82(7 June 1906) 475-476.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-6]

6. Review of Jacques Loeb's *The Dynamics of Living Matter* and Gustav Mann's *Chemistry of the Proteids*, 83(5 July 1906) 17-18.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-7]

7. Review of H. E. Roscoe's *The Life and Experiences of Sir Henry Enfield Roscoe*, 83(12 July 1906)43.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-8]

8. Notice of Agnes M. Clerke's *System of the Stars*, 83(26 July 1906) 78. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-9]

9. Review of Thomas Marshall's *Aristotle's Theory of Conduct*, 83(13 Sept 1906)226-227.

Peirce: CP 8 Bibliography *The Nation* 1906 [N-1906-10]

10. Review of Horace William Brindley Joseph's *An Introduction to Logic*, 83(25 Oct 1906)353-354.

Peirce: CP 8 Bibliography *The Nation* 1907 [N-1907-1]
1907

1. Review of *The Scientific Papers of J. Willard Gibbs*, 2 vols., 84(24 Jan 1907)92.

Peirce: CP 8 Bibliography *The Nation* 1907 [N-1907-2]

2. Review of Ernest von Meyer's *A History of Chemistry*, 84(21 Feb 1907)181-182. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1907 [N-1907-3]

† 3. Review of James Mark Baldwin's *Functional Logic, or Genetic Theory of Knowledge*, Vol. 1 of *Thought and Things: A Study of the Development and Meaning of Thought; or Genetic Logic*, 84(28 Feb 1907)203-204. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1907 [N-1907-4]

4. Review of *The Collected Mathematical Works of George William Hill*, Vol. IV, 85(17 Oct 1907)355. At 85(31 Oct 1907)396 Peirce replies briefly to a letter from Simon Newcomb criticizing the review. Identification based on Harvard mss.

Peirce: CP 8 Bibliography *The Nation* 1908 [N-1908-1]
1908

† 1. Review of James Mark Baldwin's *Experimental Logic, or Genetic Theory of Thought*, Vol. 2 of *Thought and Things: A Study of the Development and Meaning of Thought, or Genetic Logic*, 87(20 Aug 1908) 164-165. Identification based on Harvard mss.

Peirce: CP 8 Bibliography Misc. [M-1a]
III. MISCELLANEOUS

1. Archibald, Raymond C.

(a) "Benjamin Peirce's Linear Associative Algebra and C. S. Peirce," *The American Mathematical Monthly* 34(Dec 1927)525-527. This contains a note by Peirce on Jordan's *Traité des Substitutions et des Equations Algébriques*, dated 28 June 1910.

Peirce: CP 8 Bibliography Misc. [M-1b]

(b) *Semicentennial History of the American Mathematical Society, 1888-1938*, American Mathematical Society, New York, 1938, pp. 6 and 7. This contains information concerning Peirce's contributions to various newspapers.

Peirce: CP 8 Bibliography Misc. [M-2]

2. Burks, Arthur W., "Icon, Index, and Symbol," *Philosophy and Phenomenological Research* 9(June 1949)674. This contains a brief description of a manuscript in iconic handwriting at Widener.

Peirce: CP 8 Bibliography Misc. [M-3]

3. Calderoni. A draft of a letter to "Signor Calderoni" (probably Mario Calderoni), c.1905, in Widener VB2a. [CP] 8.205-213 are from it.

Peirce: CP 8 Bibliography Misc. [M-4a]

4. Carus, Paul.

(a) "The Founder of Tychism, His Methods, Philosophy, and Criticisms, In Reply to Mr. Charles S. Peirce," *The Monist* 3(July 1893)571-622. On p. 571 there is part of a letter from Peirce concerning his "Reply to the Necessitarians . . .," [Bibliography] G-1891-1f.

Peirce: CP 8 Bibliography Misc. [M-4b]

† (b) "The Nature of Logical and Mathematical Thought," *The Monist* 20(Jan 1910)33-75. On p. 45 there is a fragment on non-Aristotelian logic from a letter "on sundry topics of modern logic" from Peirce to Francis C. Russell. There is a comment on this in a letter from Peirce to Carus, quoted in Paul Carus, "Non-Aristotelian Logic," *ibid.*, p. 158.

Peirce: CP 8 Bibliography Misc. [M-4c]

(c) Two letters in Widener VB2a. One of these letters bears no date but was probably written c.1910. [CP] 8.214-238 except 214n1 and 225n10 are from it. The other letter is dated 19 July 1910. [CP] 8.214n1 contains a quotation from it.

Peirce: CP 8 Bibliography Misc. [M-5]

5. Dewey, John. Two letters in Widener VB2a. One of the letters is dated 9 June 1904. [CP] 8.239-242 are from it. The other letter is c.1905. [CP] 8.243-244 are from it. Part of the c.1905 letter is printed in [Bibliography] M-16a, pp. 215-216.

Peirce: CP 8 Bibliography Misc. [M-6a]

6. Eisele, Carolyn.

(a) "The *Liber Abaci* through the Eyes of Charles S. Peirce," *Scripta Mathematica* 17(Sept-Dec 1951)236-259. This contains a letter dated 1901 May 18 concerning Leonardo Fibonacci's *Liber Abaci* (from the Plimpton Collection at Columbia University), pp. 242-254 and 256-257; and a letter of 1903 (from the David Eugene Smith Library at Columbia University), pp. 254-255 and 258. See p. 255 for a reference to a card by Peirce in the David Eugene Smith Library.

Peirce: CP 8 Bibliography Misc. [M-6b]

(b) "The Scientist-Philosopher C. S. Peirce at the Smithsonian," *Journal of the History of Ideas* 18(Oct 1957)537-547. This contains correspondence between Samuel P. Langley and Peirce and material on related Peirce mss. See [Bibliography] M-21a for another reference to PeirceLangley correspondence.

Peirce: CP 8 Bibliography Misc. [M-6c]

(c) "The Charles S. Peirce-Simon Newcomb Correspondence," *Proceedings of the American Philosophical Society* 101(Oct 1957)409-433. Besides the correspondence this contains material on related Peirce mss.

Peirce: CP 8 Bibliography Misc. [M-7a]

7. Fisch, Max H.

(a) Reviews of W. H. Werkmeister's *A History of Philosophical Ideas in*

America and of Philip P. Wiener's *Evolution and the Founders of Pragmatism*, *American Literature* 22(May 1950)185-189. This contains quotations concerning the Metaphysical Club at Cambridge from Peirce's unpublished papers.

Peirce: CP 8 Bibliography Misc. [M-7b]

(b) "General Introduction," *Classic American Philosophers* (edited by Max H. Fisch, et al.), Appleton-Century-Crofts, Inc., New York, 1951, pp. 1-39. This contains a quote from a letter to D. C. Gilman, p. 10, and references (including quotations) to mss., pp. 16-17, 24, 26, 29, and 32-33.

Peirce: CP 8 Bibliography Misc. [M-7c]

† (c) [Fisch]. This contains quotations from and references to Peirce mss., pp. 414, 415, 417, 440, and 443-444.

Peirce: CP 8 Bibliography Misc. [M-7d]

† (d) [Fisch-Cope]. This contains many references to and quotations from correspondence and mss. The following appendices also contain source materials: (II), Peirce to Gilman concerning the Johns Hopkins physics department, 13 Jan 1878, pp. 365-368; (III), Peirce's courses at Johns Hopkins University, pp. 369-370 (it is not clear from [JHUC] that Peirce taught Mill's Logic in 1880 as is indicated here on p. 369, and it seems to the present editor that the intention in [JHUC] is to state that Marquand taught this course); (IV), titles of papers read at the Metaphysical Club at Johns Hopkins University, pp. 371-374.

Peirce: CP 8 Bibliography Misc. [M-8]

8. Garrison, Wendell Phillips. *Letters and Memorials of Wendell Phillips Garrison*, Houghton Mifflin, Boston and New York, 1909. This contains a letter to Garrison of 29 May 1905, p. 156; cf. p. 140.

Peirce: CP 8 Bibliography Misc. [M-9]

9. Gilman, Daniel C. Jackson I. Cope, "William James' Correspondence with Daniel Coit Gilman, 1877-1881," *Journal of the History of Ideas* 12(Oct 1951)609-627. This contains a letter from Peirce to Gilman, dated 13 Sept 1877, pp. 615-616. See [Bibliography]M-7b and M-7d for other references to correspondence with Gilman.

Peirce: CP 8 Bibliography Misc. [M-10]

10. Goudge, Thomas A., *The Thought of C. S. Peirce*, University of Toronto Press, 1950. Peirce's entry in the Harvard class book of 1859 (at the time of his graduation) is reprinted at pp. 347-349. The information contained here differs from that of Peirce's biography in the class book, *Harvard College, Records of the Class of 1859*, Cambridge, 1896, pp. 49-50 (cf. pp. 9, 74, 76, and 77).

Peirce: CP 8 Bibliography Misc. [M-10]

Further biographical data, probably supplied by Peirce, are to be found in *American Men of Science*, 1906, p. 248, and 1910, p. 364, and in *The National Cyclopaedia of American Biography*, Vol. 8, 1900, p. 409.

Peirce: CP 8 Bibliography Misc. [M-10]

Additional biographical information is given in Paul Weiss, "Charles Sanders Peirce," *Dictionary of American Biography*, Vol. 14, Charles Scribner's Sons, New York, 1937, pp. 398-403.

Peirce: CP 8 Bibliography Misc. [M-11]

11. Harris, William T. A letter of 30 Nov 1868 in the Hoose Library of Philosophy, University of Southern California. [CP] 8.245-248. Cf. [Bibliography] G-1868-1a.

Peirce: CP 8 Bibliography Misc. [M-12]

12. Huntington, Edward V., "Sets of Independent Postulates for the Algebra of Logic," *Transactions of the American Mathematical Society* 5(1904)288-309. On pp. 300-302 Huntington gives a proof of a distributive principle of which he says: "This demonstration is borrowed, almost verbatim, from a letter of Mr. C. S. Peirce's, dated December 24, 1903 . . ." (p. 300n*). There is also a quotation from a letter from Peirce, dated 14 Feb 1904, concerning this proof, p. 300n*. This quotation is reprinted at [CP] 3.200n* (p. 128); cf. 3.384n1 concerning this proof.

Peirce: CP 8 Bibliography Misc. [M-13]

13. James, Henry. A letter to William James's son, Henry, dated 21 Sept 1910, is given in part in [Perry] II, 286-287.

Peirce: CP 8 Bibliography Misc. [M-14a]

14. James, William.

(a) [Perry]. This contains excerpts from 28 letters to William James: Vol. I: 536-538; Vol. II: 222-224, 407n5, 413-440. Cf. [CP] 8.249n1.

Peirce: CP 8 Bibliography Misc. [M-14b]

(b) Correspondence, 1897-1909, in the William James Collection, Houghton Library. [CP] 6.478n*, 8.117n10, and 8.249-315 are from it. Cf. [CP] 7.44n17 and 8.39n1.

Peirce: CP 8 Bibliography Misc. [M-15a]

15. Ladd-Franklin, Christine.

(a) A letter, dated 29 Aug 1891, in Widener VB2a. [CP] 8.316-318 are from it.

Peirce: CP 8 Bibliography Misc. [M-15b]

† (b) "Charles S. Peirce at the Johns Hopkins," *The Journal of Philosophy, Psychology and Scientific Methods* 13(21 Dec 1916)715-722. This contains four letters to Christine Ladd-Franklin, written in 1900 and later.

Peirce: CP 8 Bibliography Misc. [M-16a]

16. Royce, Josiah.

(a) James Harry Cotton, *Royce on the Human Self*, Harvard University Press, Cambridge, 1954. This contains part of the correspondence listed under [Bibliography] M-5 and parts of letters to Josiah Royce, pp. 300-301.

Peirce: CP 8 Bibliography Misc. [M-16b]

(b) Two letters in Widener VB2a. One of these letters is dated 27 May 1902. Quotations from it are at [CP] 8.117n10, 8.117n12, and 8.122n19. The other letter is dated 28 May 1902. A quotation from it is at [CP] 8.122n19.

Peirce: CP 8 Bibliography Misc. [M-16c]

(c) Josiah Royce and Fergus Kernan, "Charles Sanders Peirce," *The Journal of Philosophy, Psychology, and Scientific Methods* 13(21 Dec 1916)701-709. This contains information about the Harvard mss.

Peirce: CP 8 Bibliography Misc. [M-17]

17. Russell, Francis C. A letter dated 14 July 1905 in Widener VB2a. A quotation from it is at [CP] 8.138n4. [Bibliography]M-4b contains a reference to another letter to Russell. See also [CP] 4.85n*.

Peirce: CP 8 Bibliography Misc. [M-18]

18. Schiller, F. C. S. Two letters in Widener VB2. [CP] 8.319-320 are from an undated letter. [CP] 8.321-326 are from a letter of 10 Sept 1906.

Peirce: CP 8 Bibliography Misc. [M-19]

19. Stokes, G. G. *Memoir and Scientific Correspondence of the Late Sir George Gabriel Stokes* (edited by Joseph Larmor), University Press, Cambridge, 1907, 2 vols. In Vol. 2, pp. 309-313, there is a letter to Stokes, dated 14 Jan 1886 and Stokes's replies of 19 May and 17 June 1886.

Peirce: CP 8 Bibliography Misc. [M-20a]

20. Welby, Victoria (Lady).

† (a) *Charles S. Peirce's Letters to Lady Welby* (edited by Irwin C. Lieb), Whitlock's, Inc., New Haven, 1953. This contains 22 letters which are dated from 7 June 1903 to 25 July 1911, pp. 1-48. Photostat copies of these letters, including the deleted existential graphs of the letter of 31 Jan, 24 Feb, 14 March, 1909, are on deposit in the Yale University Library. See also Appendix B, pp. 51-55, on Peirce's classification of signs, and in this connection see Paul Weiss and Arthur W. Burks, "Peirce's Sixty-Six Signs," *The Journal of Philosophy* 42(5 July 1945) 383-388, which contains names of kinds of signs derived from the Harvard mss.

Peirce: CP 8 Bibliography Misc. [M-20a]

See *Other Dimensions: A Selection from the Later Correspondence of Victoria Lady Welby* (edited by Mrs. Henry Cust), Jonathan Cape, London, 1931, which contains some of Lady Welby's side of the Peirce-Welby correspondence as well as some of the correspondence in the book by Lieb cited above, pp. 144-150,

154-158, 161-164, 296-314. Part of the correspondence published in Lieb's book is also to be found in C. K. Ogden and I. A. Richards, *The Meaning of Meaning*, Harcourt, Brace and Company, New York, 1930, pp. 279-290.

Peirce: CP 8 Bibliography Misc. [M-20b]

* (b) Photostat copies of two letters in the Yale University Library. One of these letters, on signs and the categories, is dated 12 Oct 1904. [CP] 8.327-341 are from it. The other letter is dated 31 Jan, 24 Feb, and 14 March 1909. [CP] 8.342n15 contains a brief quotation from it. These letters are published in full in Lieb's book, (a) above.

Peirce: CP 8 Bibliography Misc. [M-20b]

A draft of a letter, on the classification of signs, in Widener IB3a, dated 24, 25, 28 Dec 1908. [CP] 8.342-379, except 368n23, are from it.

Peirce: CP 8 Bibliography Misc. [M-21a]

† 21. Wiener, Philip P.

(a) "The Peirce-Langley Correspondence and Peirce's Manuscript on Hume and the Laws of Nature," *Proceedings of the American Philosophical Society* 91(5 April 1947)201-228. See [Bibliography] G-1901-2.

Peirce: CP 8 Bibliography Misc. [M-21b]

† (b) *Evolution and the Founders of Pragmatism*, Harvard University Press, Cambridge, 1949. This contains quotations from an unpublished letter on pragmatism to the editor of *The Sun*, pp. 20-21; and quotations from and references to correspondence and mss., pp. 21, 72-75, 81-82, 86, 95, 221-222, 252, and 260-261.

Peirce: CP 8 Bibliography Misc. [M-21c]

(c) *Values in a Universe of Chance*, to be published in 1958 by Doubleday & Company, Inc., New York, and by Stanford University Press, Stanford University. This is to contain some previously unpublished selections from the Peirce mss.

Peirce: CP 8 Bibliography Misc. [M-22]

22. Woods, Frederick A. A letter, written during Oct and Nov 1913 in Widener VB2a. [CP] 8.380-388, except 380n4, are from it. See also [Bibliography] G-1905-1c for a quotation from this letter.

Peirce: CP 8 Cross-Reference Index **CROSS-REFERENCE INDEX**

This index enables the reader to pass directly from any paragraph of *Collected Papers* I-VIII to the place or places in the chronologically arranged bibliography where the paragraph is listed. In those cases where several items of different dates have been grouped together in the bibliography under one date, the dates appropriate to the particular items, not including revisions, are given in parentheses. Hence, the

present index serves the auxiliary purpose of enabling a reader to determine the date assigned to a paragraph of *Collected Papers* I-VIII without further effort. It should be noted that sometimes a footnote and the paragraph to which it is attached have different bibliography references, e.g., [CP] 3.200 and 3.200n*.

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 243-244 M-5 (c.1905)
 245-248 M-11 (1868)
 249-315 M-14b (1897-1909)
 316-318 M-15a (1891)
 319-320 M-18 (undated)
 321-326 M-18 (1906)
 327-341 M-20b (1904)
 342-379 M-20b (1908)
 342n15 M-20b (1909)
 368n23 G-undated-16
 380-388 M-22 (1913)
 380n4 G-undated-17

Peirce: CP 8 Introduction Fn 1 p ix

†1 He was employed by the Coast Survey from September 21, 1859, to June 1, 1860, and from July 1, 1861, to December 31, 1891.

Peirce: CP 8 Introduction Fn 2 p ix

†2 See the Bibliography in Vol. VIII, especially the years 1872 through 1886; Peirce produced little in the way of results for the Coast Survey after he moved to Milford, Pa., in the spring of 1887.

Peirce: CP 8 Introduction Fn 3 p x

†3 Cf. 6.44, 6.46. Peirce's most original contribution to geodetics was the discovery of a new source of error in gravity measurement; this is described at 7.1-12.

Peirce also did research for the Coast Survey on standards of measurement.

Peirce: CP 8.1 Fn 1 p 3

†1 (Ed.) Review of John Venn's *The Logic of Chance, An Essay on the Foundations and Province of the Theory of Probability, with especial Reference to its Application to Moral and Social Science* (London and Cambridge, 1866), *The North American Review* 105 (July 1867) 317-321. This is Peirce's earliest full statement of his theory of probability; cf. 3.19 (1867). His later views are discussed in [CP] II, Book III, and [CP] VII, Book II.

Peirce: CP 8.5 Fn 2 p 6

†2 This is an error. For supposing every man to be insured for the same amount, which we may take as our unit of value, and adopting the notation,

(c,e) = number of consumptive Englishmen insured.

(c,{e}) = " " consumptives not English insured.

({c},e) = " " not consumptive English insured.

x = unknown ratio of consumptive English who *do not die* in the first year. The amount paid out yearly by the company would be, in the long run,

$$1/10(c,\{e\}) + 9/10(\{c\},e) + x(c,e),$$

and x is unknown. This objection to Venn's theory may, however, be waived.

Peirce: CP 8.7 Fn 1 p 9

†1 (Ed.) Review of Alexander Campbell Fraser's *The Works of George Berkeley, D.D., formerly Bishop of Cloyne: including many of his Writings hitherto unpublished*, four volumes (Clarendon Press, Oxford, 1871), *The North American Review* 113 (Oct. 1871) 449-472.

Cf. Peirce's review of the 1901 edition, [Bibliography] N-1901-10.

Peirce: CP 8.11 Fn 2 p 14

†2 Died 1308.

Peirce: CP 8.12 Fn 3 p 15

†3 (Ed.) Cf. 5.311.

Peirce: CP 8.12 Fn 4 p 17

†4 (Ed.) Cf. 5.311.

Peirce: CP 8.20 Fn 5 p 23

†5 (Ed.) See William Ockham, *Summa Logicae, Pars Prima*, Philotheus Boehner, ed., St. Bonaventure, New York, 1951, p. 44; cf. Ernest A. Moody, *The Logic of William of Ockham*, Sheed and Ward, Inc., New York, 1935, footnotes pp. 81-82, for a somewhat different version of this passage.

Peirce: CP 8.20 Fn 6 p 23

†6 The *entia non sunt multiplicanda praeter necessitatem* is the argument of Durand de St. Pourcain. But any given piece of popular information about scholasticism may be safely assumed to be wrong.

Peirce: CP 8.22 Fn 7 p 24

†7 (Ed.) *The English Works of Thomas Hobbes of Malmesbury*, Sir William Molesworth, ed., Vol. I, London, 1839, p. 36.

Peirce: CP 8.24 Fn 8 p 25

†8 (Ed.) *The English Works of Thomas Hobbes of Malmesbury*, Sir William Molesworth, ed., Vol. I, London, 1839, pp. 118-119.

Peirce: CP 8.26 Fn 9 p 26

†9 The sole difference between Ockam and Hobbes is that the former admits the universal signs in the mind to be natural, while the latter thinks they only follow instituted language. The consequence of this difference is that, while Ockam regards all truth as depending on the mind's naturally imposing the same sign on two things, Hobbes will have it that the first truths were established by convention. But both would doubtless allow that there is something *in re* to which such truths corresponded. But the sense of Berkeley's implication would be that there are no universal thought-signs at all. Whence it would follow that there is no truth and no judgments but propositions spoken or on paper.

Peirce: CP 8.26 Fn 10 p 27

†10 (Ed.) See *An Essay Concerning Human Understanding* by John Locke, edited by Alexander Campbell Fraser, Vol. II, Clarendon Press, Oxford, 1894, p. 247, §9.

Peirce: CP 8.27 Fn 11 p 27

†11 (Ed.) In the work under review this passage from the introduction to "A Treatise Concerning the Principles of Human Knowledge" is to be found in Vol. I, p. 146, §14. The portion in brackets was omitted by Peirce without notice.

Peirce: CP 8.28 Fn 12 p 28

†12 (Ed.) In the work under review this passage is in "Commonplace Book of Occasional Metaphysical Thoughts," Vol. IV, p. 448.

Peirce: CP 8.30 Fn 13 p 29

†13 (Ed.) In the work reviewed this passage from "The Second Dialogue between Hylas and Philonous" is in Vol. I, p. 304. There the passage reads: "As sure, therefore, as the sensible world really exists, so sure is there an infinite omnipresent Spirit, who contains and supports it."

Peirce: CP 8.31 Fn 14 p 33

†14 (Ed.) In the work reviewed this passage from "A Treatise Concerning the Principles of Human Knowledge," Part I, is in Vol. I, p. 208 (§102), p. 210 (§106), and pp. 210-211 (§107).

Peirce: CP 8.33 Fn 15 p 34

†15 (Ed.) This is an early anticipation of Peirce's pragmatism, which is discussed in detail in [CP] V, *Pragmatism and Pragmaticism*. See especially 5.402, 5.453, 5.504n1 (p. 353). Cf. also 7.360.

Peirce: CP 8.36 Fn 16 p 35

†16 (Ed.) See Helmholtz's *Treatise on Physiological Optics*, §33.

Peirce: CP 8.39 Fn 1 Para 1/2 p 39

†1 (Ed.) Review of Josiah Royce's *The Religious Aspect of Philosophy* (Houghton Mifflin Company, Boston, 1885, 484 pp.), hereafter referred to as [RAP], from a manuscript in Houghton Library. The first three pages (half-sheets) of this manuscript are missing and there is no explicit statement in the remainder that this is a review of [RAP], but all of the quotations from the work under review have been located there.

Peirce: CP 8.39 Fn 1 Para 2/2 p 39

In a letter to James, dated 28 October 1885 (James Collection, Houghton Library), Peirce says that he wrote a review of Royce's book for Youmans, who would not take it. (E. L. Youmans and W.J. Youmans edited *Popular Scientific Monthly* at this time. Peirce refers to this journal in the text below.) The dates of the letter and [RAP] establish the date of the review as c.1885. It is the case, however, that some of the pages and many of the corrections in the manuscript are in a different ink and finer handwriting from the others, and hence they may be of a somewhat different date.

Peirce: CP 8.40 Fn 2 p 39

†2 (Ed.) [RAP] Ch. X, "Idealism."

Peirce: CP 8.41 Fn 3 p 40

†3 (Ed.) Cf. 5.311ff.

Peirce: CP 8.41 Fn 4 p 40

†4 (Ed.) [RAP] 393-394, 426ff.

Peirce: CP 8.41 Fn 5 p 41

†5 (Ed.) [RAP] 427.

Peirce: CP 8.41 Fn 6 p 42

†6 *Mitchell* in Logical Studies by members of the Johns Hopkins University, and *Peirce* in the American Journal of Mathematics, vol. vii.

(Ed.) These items are listed at [Bibliography] G-1883-7 and G-1885-3.

Peirce: CP 8.41 Fn 7 p 42

†7 (Ed.) Cf. 2.248 and elsewhere in [CP] II.

Peirce: CP 8.41 Fn 8 p 43

†8 (Ed.) Feeling and volition are instances of Peirce's categories of First and Second, respectively. See [CP] I, VII.

Peirce: CP 8.41 Fn 9 p 44

†9 "We must be in contact with our subject-matter," says he in one place, whether it be by means of our external senses, *or, what is better*, by our profounder mind and our innermost self-consciousness.

Peirce: CP 8.43 Fn 10 p 44

†10 (Ed.) Italics not in [RAP]; cf. 41.

Peirce: CP 8.43 Fn 11 p 44

†11 (Ed.) [RAP] 430.

Peirce: CP 8.43 Fn 12 p 45

†12 (Ed.) Pragmatism is treated in [CP] V.

Peirce: CP 8.44 Fn 13 p 47

†13 (Ed.) Cf. [CP] V, Book II, "Religion."

Peirce: CP 8.47 Fn 14 p 50

†14 (Ed.) [RAP] 21ff.

Peirce: CP 8.50 Fn 15 p 51

†15 (Ed.) [RAP] Ch. III.

Peirce: CP 8.50 Fn 16 p 51

†16 (Ed.) [RAP] 48. The full statement in [RAP] is: "'If I feel not the love of God,' the objector will say, 'how prove to me that I ought to feel it?'"

Peirce: CP 8.51 Fn 17 p 51

†17 (Ed.) [RAP] 82ff.

Peirce: CP 8.52 Fn 18 p 52

†18 (Ed.) [RAP] 138.

Peirce: CP 8.52 Fn 19 p 52

†19 (Ed.) [RAP] 140-141.

Peirce: CP 8.52 Fn 20 p 52

†20 (Ed.) [RAP] 144.

Peirce: CP 8.53 Fn 21 p 53

†21 (Ed.) "Therefore it is well that we should feel not a selfish but a righteous joy whenever pride has a fall, whenever the man who thinks that he is something discovers of a truth that he is nothing In all such ways we must ask and we must show no mercy, save when these keen pains . . ." [RAP] 181-182.

Peirce: CP 8.53 Fn 22 p 53

†22 (Ed.) [RAP] 187. The words in brackets are in [RAP], but were omitted by Peirce without notice.

Peirce: CP 8.53 Fn 23 p 53

†23 (Ed.) ". . . whereas the rule of life for one's own person is simply to get all the satisfaction that one can, the appearance of anybody else who pretends to be content with *himself* must be the signal not for admiration at the sight of his success, but for a good deal of contempt." [RAP] 196-197.

Peirce: CP 8.55 Fn 1 p 54

†1 (Ed.) The review of William James's *The Principles of Psychology* (American Science Series, Advanced Course, Henry Holt and Company, 1890), 2 vols. Paragraphs 55-61, *The Nation* 53 (2 July 1891) 15; paragraphs 62-71, *The Nation* 53 (9 July 1891) 32-33.

Peirce: CP 8.57 Fn 2 p 54

†2 (Ed.) Vol. I, p. 368.

Peirce: CP 8.57 Fn 3 p 55

†3 (Ed.) Vol. I, p. 215.

Peirce: CP 8.58 Fn 4 p 55

†4 (Ed.) Cf. Peirce's review of a book by James's father on Swedenborg, [Bibliography] G-1870-2.

Peirce: CP 8.59 Fn 5 p 56
†5 (Ed.) Vol. I, pp. v-vi.

Peirce: CP 8.61 Fn 6 p 58
†6 (Ed.) Vol. I, p. 194.

Peirce: CP 8.62 Fn 7 p 58
†7 (Ed.) Vol. II, pp. 111-114. Cf. 5.115, 5.181ff.

Peirce: CP 8.72 Fn 8 p 64
†8 (Ed.) "Questions on William James's Principles of Psychology," Widener IC1a. This manuscript is dated c.1891 since it was probably written when Peirce composed his review (see 55n1). The questions are numbered from 1 to 44, with two questions bearing the number 40. All the questions concern Vol. I of the two volume work; where necessary, the editor has inserted in brackets the relevant passages. The questions published here are on the following chapters: 3 and 5 are on Ch. II, "The Functions of the Brain"; 12 and 14, Ch. V, "The Automaton-Theory"; 21, 22, 23, 29, and 30, Ch. VIII, "The Relations of Minds to other Things"; and 31, 32, 33, 36, 41, and 42, Ch. IX, "The Stream of Thought." [Bibliography] M-14a quotes some of the questions not printed here.

Peirce: CP 8.82 Fn 9 p 67
†9 (Ed.) Cf. 5.313ff.

Peirce: CP 8.83 Fn 10 p 67
†10 (Ed.) The psychological versions of Peirce's categories of First, Second, and Third are discussed in 7.524-538, and the categories in general are treated in [CP] I. See also 5.290.

Peirce: CP 8.89 Fn 11 p 69
†11 (Ed.) This word is not clearly legible in the manuscript. Cf. [Perry] II, 413-416.

Peirce: CP 8.91 Fn 1 p 70
†1 (Ed.) Paragraphs 91-96 are the review of Nicholas Lobatchewsky's *Geometrical Researches on the Theory of Parallels* (translated by George Bruce Halsted, Austin, 1891), *The Nation* 54(11 Feb 1892)116, with an added quotation in 93n2.

Paragraphs 97-99 are from an undated manuscript, "The Non Euclidean Geometry made Easy," Widener IA-2.

Cf. 1.130, 3.134n1, 3.557.

Peirce: CP 8.93 Fn 2 p 71 Cross-Ref:††
†2 (Ed.) Peirce worked on the problem of determining which geometry holds of physical space. ". . . the physical geometry of celestial triangles needs examination, in order to ascertain whether the constant of space may not have a sensible magnitude. I have undertaken such an examination. I began by forming a list of all possible methods of determining this quantity by means of the following observations: 1st, the parallaxes of stars; 2nd, the numbers of stars of each parallax; 3rd, the proper motions of stars; 4th, the numbers of stars of different proper motions;

5th, the spectroscopic determinations of the motions of stars in the line of sight; 6th, the magnitudes of stars; 7th, the numbers of stars of each magnitude. My list of possible methods was long. All of them, it is true, involved some hypothetical element; but that is true of any research, whatever, into the value of a physical quantity; and it is possible so to modify the methods that the hypotheses that appear the most dangerous may probably be eliminated. I applied several methods: they seemed to indicate a hyperbolic space with a constant far from insignificant." From an undated fragment, Widener IA-7.

Peirce: CP 8.93 Fn 3 p 72

†3 (Ed.) Cf. *The Thirteen Books of Euclid's Elements*, edited by Thomas L. Heath, Second Edition (Dover, 1956), Vol. I, p. 280.

Peirce: CP 8.100 Fn 1 p 75

†1 (Ed.) Paragraphs 100-107 are the review of Josiah Royce's *The World and the Individual*: Gifford Lectures delivered before the University of Aberdeen, First Series: The Four Historical Conceptions of Being (Macmillan, 1900, 588 pp.), *The Nation* 70 (5 April 1900) 267.

Paragraphs 108-116 are a review of the same work, Widener IV, dated c.1900 on the basis of the dates of the book and the published review.

Peirce: CP 8.102 Fn 2 p 76

†2 (Ed.) Cf. 5.357.

Peirce: CP 8.110 Fn 3 p 81

†3 (Ed.) Cf. 3.363 and elsewhere in [CP] III.

Peirce: CP 8.110 Fn 4 p 81

†4 (Ed.) Cf. 6.2.

Peirce: CP 8.112 Fn 5 p 83

†5 (Ed.) This expression is indexical; cf. 2.305ff.

Peirce: CP 8.113 Fn 6 p 84

†6 (Ed.) Cf. 1.420, 2.661-668, 4.580, 5.453, 5.467, 5.528, 6.327.

Peirce: CP 8.113 Fn 7 p 85

†7 (Ed.) This question is further discussed in the following section.

Peirce: CP 8.114 Fn 8 p 86

†8 (Ed.) Cf. 4.639.

Peirce: CP 8.115 Fn 9 p 86

†9 But we must say that his attempt at defining an individual is surprisingly feeble; nor is this the only fault of this kind the book contains. But the truth is that the intellectual life of Harvard has, ever since Dr. Walker's death, been languishing more and more for want of a good sharp logician. To one who visits it once in every four or five years this is more noticeable than to a man living there.

Peirce: CP 8.117 Fn 10 Para 1/3 p 88 Cross-Ref:††

†10 (Ed.) Paragraphs 117 (in part), 120 (in part), and 126-130 are from the review of Josiah Royce's *The World and the Individual*: Gifford Lectures [on Natural Religion] delivered before the University of Aberdeen, Second Series: Nature, Man, and the Moral Order (Macmillan, 1901, 480 pp.), *The Nation* 75 (31 July 1902)94-96,

with added quotations in the present footnote. Paragraphs 117 (in part), 118-119, 120 (in part), 121-125, and 131 are from various partial drafts of this review, Widener IV, dated c.1902 on the basis of the dates of the book and of the published review, with added quotations in 117n12 and 122n19.

Peirce: CP 8.117 Fn 10 Para 2/3 p 88 Cross-Ref:††

In a letter to James dated 25 November 1902 (James Collection, Houghton Library) Peirce remarks: "As for the *Nation*, I get \$250 a year from it on which we live; and therefore I cannot speak above a whisper about it. But the way my bits (bad enough, at best) are cut is awful. I was really wounded at the way all the praise was cut out of my notice of Royce."

Peirce: CP 8.117 Fn 10 Para 3/3 p 88 Cross-Ref:††

In a letter to "My dear Prof. Royce," dated 27 May 1902, Widener VB2a, Peirce says: "I am going to try to say what should be said of your second Volume in the *Nation*. I shall send Garrison something which is too long for anybody to read and too short to express what I try to cram into it; and Garrison will cut it down so as to leave what will strike the afternoon businessman on his way uptown hanging on to a strap, as smartly said, and whether or not it will mean anything to you I can't say. So in case it shouldn't get said there, as I hope it will, I will say *hic et nunc* that the volume has cut off a big piece of the road that it remains for Philosophy to travel before she will join company with the rest of the peaceable sciences. That junction must be made or Philosophy is a humbug. Your best years of philosophic reflection are still before you. The time is ripe and you are the very man to accomplish the great achievement of covering that distance. Yet you could not do it with your present views of logic, antagonistic to all that is possible for progressive science. My entreaty is that you will study logic."

Peirce: CP 8.117 Fn 11 p 89

†11 (Ed.) The remainder of this paragraph and the following two paragraphs are from manuscript (see 117n10).

Peirce: CP 8.117 Fn 12 p 89 Cross-Ref:††

†12 (Ed.) The closing paragraph of the letter to Royce, 27 May 1902 (see 117n10), reads: "Underneath your logic which I cannot approve there is a nearly parallel stream of thought perfectly sound and in fact without doubt this was really what has kept you straight so that, -- of course, I am saying what *seems to me*, -- the affirmatory clauses of your conclusions are approximately right. Your statement of the relation of the individual to God is sublime and fit to satisfy the soul in life and in the hour of death. It must stand for age after age. My feeling is that the individual just fills his little place in the revelation of the universal and except for the sake of what fragment of universal meaning he bears is no account. Like the word 'to' which fills out 'Be or not be' and so helps the effect of the drama of Hamlet. If there is so much glee in heaven over one sinner that repenteth, what must be the deep ineffable felicity to Carnegie of picking up a newspaper in the elevated and so saving his copper. Individuals are cells."

Cf. [Bibliography] M-16a, pp. 300-301.

Peirce: CP 8.119 Fn 13 p 91

†13 (Ed.) 5.402; italics not in the text quoted.

Peirce: CP 8.119 Fn 14 p 92

†14 (Ed.) Peirce generally calls such signs "icons"; see [CP] II.

Peirce: CP 8.119 Fn 15 p 92

†15 (Ed.) These are generally called "indices" by Peirce; see [CP] II.

Peirce: CP 8.119 Fn 16 p 92

†16 (Ed.) These are generally called "symbols" by Peirce; see [CP] II.

Peirce: CP 8.120 Fn 17 p 93

†17 (Ed.) The preceding part of this paragraph is from *The Nation*; the following part and subsequent paragraphs are from the manuscripts (see 117n10).

Peirce: CP 8.120 Fn 18 p 93

†18 (Ed.) *The World and the Individual*, First Series, p. 359. The full statement reads: "And the Being of the real object of which you now think means a life that expresses the fulfilment of just your present plan, in the greatest measure in which your plan itself is logically capable of fulfilment."

Peirce: CP 8.122 Fn 19 Para 1/2 p 94 Cross-Ref:††

†19 (Ed.) The first paragraph of the letter to Royce, 27 May 1902 (see 117n10 and 117n12), contains the following statement: "Perhaps the most suggestive phrase in your book is your 'dynamo of ideas.' Imagine each ether to be composed of atoms which are vortices in another ether of similar constitution and if the velocity of transmission increases as from sound to light, the whole endless series might be traversed in a fraction of a second, with purely dynamical causation, passing then into an endless series of soul-ethers under causation purely final or formal. No matter how improbable such [a] hypothesis, its mere possibility refutes the principal argument for 'Parallelism.' That is, it shows that though matter cannot act *immediately* upon mind or t'other way it may act all the same upon it. That self-control, self consciousness, involve endless series is clear. There are other *modes* of application, not merely other applications." Cf. 7.370.

Peirce: CP 8.122 Fn 19 Para 2/2 p 94 Cross-Ref:††

The following is the closing paragraph of a letter to "My dear Professor Royce," dated 28 May 1902, Widener VB2a: "I wish you would tell me precisely why it is that you object to making anything its own purpose, or the sign of itself. It seems to me clear that that is just what consciousness is; and if that were admitted, the obstacle to the introduction of higher ideas, which we have but of which you admit no realization in God, would be removed."

Peirce: CP 8.123 Fn 20 p 95

†20 (Ed.) In one of the partial drafts (see 117n10), Peirce says in this connection: "Abridge our theory as we may and must, there is one detail which it will not do to omit. That is the use which is made of the psychological doctrine of the 'time span.' That singularly accurate observer, Thomas Reid, whose lessons have not yet been thoroughly learned by psychologists, seems to have been the first distinctly to recognize that we have something very like a direct perception of duration or, at least, of motion; and he drew the needful distinction between the lapse of time during the act of perception and the lapse of time represented in the percept. There are sundry unsettled questions, such as whether there is any consciousness in an instant of time and whether the time that is directly perceived seems to be present or not, but the best modern psychologists even from their precursor, Herbart, and more and more yearly since James's great work, recognize that our image of the last six to twelve seconds past is almost or quite of the nature of a percept, while the remoter past and the future are represented in a more mediated way. One opinion which has been put

forward and which seems, at any rate, to be tenable and to harmonize with the modern logico-mathematical conceptions, is that our image of the flow of events receives, in a strictly continuous time, strictly continual accessions on the side of the future, while fading in a gradual manner on the side of the past, and that thus the absolutely immediate present is gradually transformed by an immediately given change into a continuum of the reality of which we are thus assured. The argument is that in this way, and apparently in this way only, our having the idea of a true continuum can be accounted for."

Peirce: CP 8.125 Fn 21 p 97

†21 (Ed.) The following paragraphs are from *The Nation* (see 117n10).

Peirce: CP 8.126 Fn 22 p 97

†22 (Ed.) Cf. 5.384 and Chapter 2 on *The Works of George Berkeley* in the present book.

Peirce: CP 8.126 Fn 23 p 98

†23 (Ed.) ". . . to be real means to be independent of ideas which, while other than a given real being, still relate to that being." *The World and the Individual*, First Series, pp. 92-93.

Peirce: CP 8.129 Fn 24 p 99

†24 (Ed.) *The World and the Individual*, First Series, p. 118.

Peirce: CP 8.130 Fn 25 p 101

†25 (Ed.) Cf. Chapter 5, "On Non-Euclidean Geometry," in the present book.

Peirce: CP 8.130 Fn 26 p 101

†26 (Ed.) The following paragraph is from manuscript (see 117n10).

Peirce: CP 8.131 Fn 27 p 102

†27 (Ed.) Cf. [CP] IV, Book II, "Existential Graphs."

Peirce: CP 8.131 Fn 28 p 102

†28 (Ed.) Cf. 8.168.

Peirce: CP 8.131 Fn 29 p 102

†29 (Ed.) Cf. 122n19.

Peirce: CP 8.132 Fn 1 p 103

†1 (Ed.) Paragraphs 132-152 are the review, "Pearson's Grammar of Science, Annotations on the First Three Chapters" (second edition, Adams and Charles Black, London, 1900, 548 pp.), *The Popular Science Monthly* 58(Jan 1901)296-306, with added quotations in 136n3 and 138n4. Paragraphs 153-156 and 136n3 are from a partial draft of this review, Widener IV, dated c.1900 on the basis of the dates of the book and the published review.

Cf. Peirce's review of the first edition, [Bibliography] N-1892-10 and the brief notice of the second edition, [Bibliography] N-1900-12.

Peirce: CP 8.133 Fn 2 p 104

†2 (Ed.) *The Grammar of Science*, p. 8; the brackets are Peirce's.

Peirce: CP 8.136 Fn 3 Para 1/4 p 105 Cross-Ref:††

†3 (Ed.) "The scientific man is deeply impressed with the majesty of truth, as something reasonable or intelligible which is bound sooner or later to force itself

upon every mind. It is not too much to say that he worships the divine majesty of the power of reasonableness behind the fact. From that sentiment springs his ardent desire to further the discovery of truth. If he cannot discover it himself he wishes to lay a sure foundation from which some successor may come to the truth; -- and the more far-reaching and general the particular question that he aims [at], the more it inspires him. It may be that all that he himself expects to ascertain is a minute fact, -- say the parallax of a star. But he anticipates that this fact along with many others will ultimately lead to a great discovery. Will not every scientific researcher acknowledge the substantial accuracy of this statement of his motive?

Peirce: CP 8.136 Fn 3 Para 2/4 p 105 Cross-Ref:††

"That it is a better motive than that which Prof. Pearson gives as the 'sole reason' for encouraging any form of human activity is easily shown. Every object which ever has been proposed as desirable in itself without any ulterior reason belongs to one or other of three classes. Namely it either consists

A. in superinducing upon feeling a particular quality, say pleasure; or

B. in extending the existence of some well-known thing, whether one's own life, or some known Creed or community, or what not, -- or

C. in furthering the realization of some ideal description of a state of things.

Peirce: CP 8.136 Fn 3 Para 3/4 p 105 Cross-Ref:††

"The desire for the stability of a particular social organization, say that of Great Britain, is a motive belonging to class B. The desire to further the discovery of truth, by whomsoever may be in a condition to discover it, belongs to class C. So also does the utilitarian end with which Prof. Pearson, following Herbert Spencer, adulterates his Darwinism; for it is highly unjust to confuse utilitarianism with simple hedonism of the pursuit of pleasure. That love for an individual thing, such as the British community, which is necessarily full of faults, is a less rational motive than the desire to realize an ideal state of things is almost too plain for argument. But it may be observed that if pleasure ought to be desired for itself, it is because it is desired; and whether that is a good reason or not, it is a reason; so that it cannot be said that pleasure ought to be desired without any reason. And if we ought to desire the extended existence of any particular object, it must be because that object has some good character, so that again there is a reason. In order to judge of the desire for the realization of a universal ideal, it is necessary to distinguish the character of that ideal; and here again there are three classes:

I. The ideal is one which recommends itself to immediate feeling. Such is the utilitarian ideal of the greatest pleasure of the greatest number. But if pleasure itself is good only for a reason, so *à fortiori* is such a generalization of it.

II. The ideal is a generalization of some familiar kind of good; such as the government of altruistic motives, or a state of society in which nothing is wasted. But if such characters are good, it must be because of some feature which renders them so. Besides, they would cease to be good if carried too far.

III. The ideal is one whose character cannot be known in advance, so that it can only be defined as the result, whatever it may be, of a process recognized as productive of good.

Peirce: CP 8.136 Fn 3 Para 4/4 p 105 Cross-Ref:††

"In order to judge of this third kind of ideal, it is requisite again to subdivide. Any such ideal belongs to one or other of three classes, as follows:

i. The natural development of feeling may be recognized as good and its ultimate dictum as the ideal. This is sentimentalism. But if the natural man is so good, it is by virtue of a contingent fact, which constitutes a reason for it.

ii. A developmental process of the world of experience may be recognized as good and its ultimate limit as the ideal.

iii. Reasoning may be recognized as good, and the reasonable as the ideal." From the partial draft (see 132n1).

Peirce: CP 8.138 Fn 4 p 107 Cross-Ref:††

†4 (Ed.) In a letter of 14 July 1905 with the salutation, "My dear Russell," Widener VB2a, Peirce says: "Decidedly I must send you my article of Jan 1901 [the present review]. Your *summum bonum*, 'life,' is probably at bottom about the same as mine, though I view it more concretely. I look upon creation as going on and I believe that such vague idea as we can have of the power of creation is best identified with the idea of theism. So then the ideal would be to be fulfilling our appropriate offices in the work of creation. Or to come down to the practical, every man sees some task cut out for him. Let him do it, and feel that he is doing what God made him in order that he should do."

The letter was written to Francis C. Russell, a Chicago attorney and friend.

Peirce: CP 8.140 Fn 5 p 108

†5 (Ed.) Cf. Peirce's review of this work, Chapter 6 in the present book.

Peirce: CP 8.144 Fn 6 p 111

†6 (Ed.) Cf. 5.213ff., [Bibliography] G-1868-2.

Peirce: CP 8.145 Fn 7 p 112

†7 (Ed.) The title of Chapter III is "The Scientific Law."

Peirce: CP 8.146 Fn 8 p 114

†8 (Ed.) 6.405ff.

Peirce: CP 8.147 Fn 9 p 115

†9 (Ed.) *The Grammar of Science*, p. 85.

Peirce: CP 8.149 Fn 10 p 116

†10 (Ed.) The title of Chapter IV is "Cause and Effect -- Probability."

Peirce: CP 8.157 Fn 1 p 121

†1 (Ed.) Review of Sidney Edward Mezes' *Ethics: Descriptive and Explanatory* (Macmillan, 1901, 435 pp.), *The Nation* 73(24 Oct 1901)325-326, with an added quotation in 158n3 from an alternative draft, Widener IV, dated 1901 on the basis of the dates of the book and the published review.

Peirce: CP 8.158 Fn 2 p 121

†2 (Ed.) Cf. 1.661ff.

Peirce: CP 8.158 Fn 3 p 122 Cross-Ref:††

†3 (Ed.) "The immense success of scientific psychology during the last forty years has naturally given it a weight that a merely special science, which is precisely

what it has all along been aiming to be, ought not to have in philosophy. The special sciences must be built upon philosophy; and consequently if philosophy has no deeper support than a special science, the whole rests on air. The first duty of ethics is to show us what we really do desire and are willing to accept as good, without any ulterior reason. This is a question of fact, and the solution of the problem must be based upon experience. But it is no recondite scientific experience which is wanted; but what is well-known and accepted by all men, philosophers, scientific men, and all others, unanimously. It is not a question of how the mind acts. On the contrary, it is only after the moralist has shown us what is our ultimate aim that the logician can tell how we ought to think in order to conform to that end. It is only on a scientific logic that a trustworthy metaphysics can be erected; and that the psychologist does and must take a metaphysics for granted is now generally admitted. Thus, the fundamental part, at least, of ethics -- what might be called pure ethics, -- ought to precede psychology. Prof. Mezes, however, is not content even to base ethics upon psychology, but must needs go to the still more special science of anthropology to find a support for it." From the alternative draft (see 157n1).

Peirce: CP 8.159 Fn 4 p 123

†4 (Ed.) *Ethics*, p. 7.

Peirce: CP 8.159 Fn 5 p 123

†5 (Ed.) *Ethics*, p. 34.

Peirce: CP 8.161 Fn 6 p 124

†6 (Ed.) *Ethics*, Chapter XV, "Welfare," pp. 383-409; see especially "Table of Components of Individual Welfare," p. 400.

Peirce: CP 8.164 Fn 1 p 126

†1 (Ed.) Paragraphs 164-166 and 167 (in part) are from the review of James Mark Baldwin's *Dictionary of Philosophy and Psychology*, Vol. II (Macmillan, 1902, 892 pp.), *The Nation* 76(11 June 1903)482. Paragraphs 167 (in part) and 168-170 are from an alternative draft, Widener IV, dated c.1903 on the basis of the dates of the book and of the published review.

Peirce: CP 8.165 Fn 2 p 126

†2 (Ed.) Peirce was one of the contributors; cf. [Bibliography] G-1901-6.

Peirce: CP 8.167 Fn 3 p 128

†3 (Ed.) The remainder of the review is from parts of the manuscript, rearranged by the editor.

Peirce: CP 8.171 Fn 1 Para 1/4 p 131

†1 (Ed.) Paragraphs 171-175 are the review of Lady Victoria Welby's *What is Meaning?* (Macmillan, 1903, 321 pp.), *The Nation* 77(15 Oct 1903)308-309. Paragraph 176 is from the Lowell Lectures of 1903 (from Lecture I, Vol. 2, following shortly after 1.611-615), Widener IB2-4, with an added quotation in 176n3.

Peirce: CP 8.171 Fn 1 Para 2/4 p 131

Paragraphs 177-185 are from a long manuscript, undated, in Widener IB3a. The references in it indicate that this manuscript is part of a letter, but the extant part contains neither salutation nor signature. This manuscript required more editorial changes in punctuation, etc., than most of the manuscripts printed in the present volume.

Peirce: CP 8.171 Fn 1 Para 3/4 p 131

Cf. the correspondence with Lady Welby in Book II of the present volume.

Peirce: CP 8.171 Fn 1 Para 4/4 p 131

The review of Lady Welby's book in *The Nation* was combined with a brief mention of Bertrand Russell's *The Principles of Mathematics*, Vol. I (University Press, Cambridge; Macmillan, New York, 1903, 534 pp.). The combined review begins with the following paragraph: "Two really important works on logic are these; or, at any rate, they deserve to become so, if readers will only do their part towards it. Yet it is almost grotesque to name them together, so utterly disparate are their characters. This is not the place to speak of Mr. Russell's book, which can hardly be called literature. That he should continue these most severe and scholastic labors for so long, bespeaks a grit and industry, as well as a high intelligence, for which more than one of his ancestors have been famed. Whoever wishes a convenient introduction to the remarkable researches into the logic of mathematics that have been made during the last sixty years, and that have thrown an entirely new light both upon mathematics and upon logic, will do well to take up this book. But he will not find it easy reading. Indeed, the matter of the second volume will probably consist, at least nine-tenths of it, of rows of symbols." The remainder of the review is printed below.

Peirce: CP 8.174 Fn 2 p 133

†2 (Ed.) Cf. "How to Make Our Ideas Clear," 5.388-410.

Peirce: CP 8.176 Fn 3 p 134 Cross-Ref:††

†3 (Ed.) Cf. "How to Make Our Ideas Clear," 5.388-410.

In his application for a grant from the Carnegie Institution, 1902, Widener VB5, Peirce describes his proposed thirty-second memoir, *On Definition and the Clearness of Ideas*, as follows: "In January, 1878, I published a brief sketch of this subject wherein I enunciated a certain maxim of 'Pragmatism,' which has of late attracted some attention, as indeed, it had when it appeared in the *Journal Philosophique*. I still adhere to that doctrine; but it needs more accurate definition in order to meet certain objections and to avoid certain misapplications. Moreover, my paper of 1878 was imperfect in tacitly leaving it to appear that the maxim of pragmatism led to the last stage of clearness. I wish now to show that this is not the case and to find a series of Categories of clearness."

Peirce: CP 8.177 Fn 4 p 135

†4 (Ed.) The following occurs here in parentheses: "(or, in some cases, as if the Sign be the sentence 'Cain killed Abel,' in which Cain and Abel are equally Partial Objects, it may be more convenient to say that that which determines the Sign is the Complexus, or Totality, of Partial Objects. And in every case the Object is accurately the Universe of which the Special Object is member, or part)."

Peirce: CP 8.177 Fn 5 p 135

†5 (Ed.) Cf. 2.228ff. for another discussion of signs.

Peirce: CP 8.178 Fn 6 p 136

†6 (Ed.) Cf. 8.303.

Peirce: CP 8.185 Fn 7 p 140

†7 (Ed.) See [Bibliography] G-1877-5a and 5b, 5.358-387 and 5.388-410, respectively. The three kinds of clearness are discussed in the second of these two

articles. The two articles did not form a unit in the original series, but in later years Peirce considered republishing them as two parts of a single essay (cf. [Bibliography] G-1909-1).

Peirce: CP 8.185 Fn 8 p 140
†8 (Ed.) Cf. 5.476, 5.491.

Peirce: CP 8.186 Fn 1 p 141
†1 (Ed.) Review of C.A. Strong's *Why The Mind Has A Body* (Macmillan, 1903, 355 pp.), Widener IV, dated c.1903 on the basis of the date of the book.

Peirce: CP 8.186 Fn 2 p 142
†2 (Ed.) See [CP] V, *Pragmatism and Pragmaticism*.

Peirce: CP 8.187 Fn 3 p 143
†3 (Ed.) See [CP] VI, Chapter 3, "Causation and Force."

Peirce: CP 8.187 Fn 4 p 144
†4 (Ed.) See 6.272ff., 7.491.

Peirce: CP 8.187 Fn 5 p 144
†5 (Ed.) Cf. 7.370, 8.274.

Peirce: CP 8.188 Fn 1 p 145
†1 (Ed.) Review of John Dewey's *Studies in Logical Theory* (The Decennial Publications, Second Series, Volume XI, University of Chicago Press, 1903, 378 pp.), *The Nation* 79(15 Sept 1904)219-220.

Peirce: CP 8.190 Fn 2 p 146
†2 (Ed.) See the correspondence to Dewey in the present volume, Book II.

Peirce: CP 8.190 Fn 3 p 147
†3 (Ed.) Logic as a normative science is discussed in [CP] I, Book IV, "The Normative Sciences."

Peirce: CP 8.191 Fn 1 Para 1/2 p 148
†1 (Ed.) Paragraphs 191-193 are from a draft of a review of Herbert Nichols's *A Treatise on Cosmology, Vol. I, Introduction* (The University Press, Cambridge, Mass., 1904, 455 pp.), Widener IV, dated c.1904 from the date of the book.

Peirce: CP 8.191 Fn 1 Para 2/2 p 148
Paragraphs 194-195 are from Section 2, "What Pragmatism is," of a manuscript entitled, "Nichols' Cosmology and Pragmaticism," Widener IB1-2, dated c.1904 from the date of the book. In this manuscript, however, Peirce deals only slightly with Nichols's book. Section 2 is similar in many respects to the published article of the same title, [Bibliography] G-1905-1a (5.411-437). Cf. [Bibliography] G-1905-2.

Peirce: CP 8.191 Fn 2 p 148
†2 (Ed.) See [CP] V, *Pragmatism and Pragmaticism*.

Peirce: CP 8.191 Fn 3 p 149
†3 (Ed.) See 2.227ff.

Peirce: CP 8.191 Fn 4 p 150
†4 (Ed.) The notions of reality, externality, existence, law, etc., are discussed

in many places; see especially [CP] I, V, and VI.

Peirce: CP 8.194 Fn 5 p 152

†5 (Ed.) Cf. 5.458ff.

Peirce: CP 8.196 Fn 1 p 155

†1 (Ed.) Review of Wilhelm Wundt's *Principles of Physiological Psychology*, translated from the fifth German edition (1902) by Edward Bradford Titchener, Vol. I (Macmillan, 1904), *The Nation* 81(20 July 1905) 56-57, with an added quotation in 201n3 from a draft in Widener IV, dated c.1905 on the basis of the dates of the book and the published review.

Peirce: CP 8.199 Fn 2 p 157

†2 (Ed.) See "Pragmatism and Critical Common-Sensism," 5.497-501, and elsewhere in [CP] V.

Peirce: CP 8.201 Fn 3 p 158 Cross-Ref:††

†3 (Ed.) "Endeavoring to sum up the results of this elaborate investigation so far as they concern psychology in such imperfect fashion as they can be reduced to one simple sentence, we may say that Wundt finds that the function of our thinking-organ lies in its regulation of motor reactions. Now this is neither more nor less than the substance of pragmatism in the dress of physiology. The original definition of pragmatism put it into this form of maxim: 'Consider what effects that might conceivably have practical bearings you conceive the object of your conception to have. Then, your conception of those effects is THE WHOLE of your conception of the object.' What is that than to say that the sole function of thought is to regulate motor reactions?" From a fragmentary draft (see 196n1).

Peirce: CP 8.205 Fn 1 p 165

†1 (Ed.) From an unsigned letter with the salutation, "Dear Signor Calderoni," Widener VB2a. The internal bibliographical references indicate that the letter was written c.1905. Cf. 8.260.

The intended recipient was probably Mario Calderoni, an Italian pragmatist. Cf. Giovanni Papini, *Le démon m'a dit . . .*, Payot, Paris, 1923, "Mario Calderoni," pp. 181-189; M. Calderoni and G. Vailati, *Il Pragmatismo*, Lanciano, 1920.

Peirce: CP 8.205 Fn 2 p 165

†2 (Ed.) An Italian philosophical journal to which Mario Calderoni contributed.

Peirce: CP 8.205 Fn 3 p 165

†3 (Ed.) [Bibliography] G-1905-1a.

Peirce: CP 8.206 Fn 4 p 165

†4 (Ed.) Giuliano il Sofista was a frequent contributor to *Leonardo*.

Peirce: CP 8.208 Fn 5 p 166

†5 (Ed.) [Bibliography] G-1905-1b.

Peirce: CP 8.208 Fn 6 p 166

†6 (Ed.) At 5.403.

Peirce: CP 8.209 Fn 7 p 168

†7 (Ed.) [Bibliography] G-1867-1b.

Peirce: CP 8.213 Fn 8 p 170

†8 (Ed.) [Bibliography] G-1867-1c.

Peirce: CP 8.213 Fn 9 p 170

†9 (Ed.) Peirce then begins a long discussion of the categories and signs (cf. [CP] I, II).

Peirce: CP 8.214 Fn 1 Para 1/2 p 171 Cross-Ref:††

†1 (Ed.) From an incomplete draft of a letter, bearing no date, to "My Dear Doctor Carus," Widener VB2a, with added quotations below in the present footnote and in 225n10. The similarity of this letter to 2.661-668 (1910) and to the letter described below in this footnote (19 July 1910) indicates that it was probably written c.1910. This draft was written when Peirce was in ill health, and it is in very rough form. Considerable editing has been done on it. The Carus papers at the Open Court Publishing Company, LaSalle, Illinois, contain what appears to be a typed copy of the letter of which this is a draft.

Peirce: CP 8.214 Fn 1 Para 2/2 p 171 Cross-Ref:††

In a draft of a letter of July 19, 1910, to "My dear Doctor Carus," Widener VB2a, Peirce discusses the republication of [Bibliography] G-1877-5a and 5b, "The Fixation of Belief" and "How to Make Our Ideas Clear," as two parts of a single essay with the title, "Pragmatic Clearness of Thought." His main point is that "The error of the Essay lies in its *Nominalism*." He further states that 1873 "is the date of my formulating the opinion expressed in the two articles that are the two parts of that Essay on Pragmatic Clearness . . ." Cf. [Bibliography] G-1909-1.

Peirce: CP 8.214 Fn 2 p 171

†2 (Ed.) [Bibliography] G-1877-5, "Illustrations of the Logic of Science," *The Popular Science Monthly*, 1877-1878.

Peirce: CP 8.214 Fn 3 p 171

†3 (Ed.) [Bibliography] G-1901-1, a review of Pearson's *The Grammar of Science*.

Peirce: CP 8.216 Fn 4 p 172

†4 (Ed.) "The Fixation of Belief" and "How to Make Our Ideas Clear."

Peirce: CP 8.216 Fn 5 p 172

†5 (Ed.) 5.409. The stanza referred to is the fourteenth in "Elegy Written in a Country Church-yard," by Thomas Gray.

Peirce: CP 8.218 Fn 6 p 172

†6 (Ed.) Cf. [Bibliography] G-1898-1, Lecture 5, "Training in Reasoning."

Peirce: CP 8.220 Fn 7 p 173

†7 (Ed.) Existential graphs are discussed in [CP] IV, Book II.

Peirce: CP 8.221 Fn 8 p 173

†8 (Ed.) Peirce's theory of probability is discussed mainly in [CP] II.

Peirce: CP 8.221 Fn 9 p 173

†9 (Ed.) Cf. 7.108ff.

Peirce: CP 8.225 Fn 10 p 175 Cross-Ref:††

†10 (Ed.) "No sign can function as such except so far as it is interpreted in

another sign (for example, in a "thought," whatever that may be). Consequently it is absolutely essential to a sign that it should *affect* another sign. In using this causal word, 'affect,' I do not refer to invariable accompaniment or sequence, merely, or necessarily. What I mean is that when there is a sign there *will be* an interpretation in another sign. The essence of the relation is in the conditional futurity; but it is not essential that there should be absolutely no exception. If, for example, in the 'long run' (that is, in an endless series of experiences taken in their experiential order) there WOULD BE as many cases of interpreted signs as of signs, I should say that this 'would be' constitutes a causal relation, even though there were, as there might be, an infinite number of exceptions. If the exceptions are, as they occur, as many or nearly as many as the cases of following the rule, the causality would be in my terminology 'very weak.' But if there is any WOULD BE at all, there is more or less causation; for that is all I mean by causation. I do not pretend that this is an accurate analysis of the ordinary conception, or a parlance to be recommended. It is simply what I mean in this connection. It leaves the whole question of what there may be of a *metaphysical* character quite open." From a sheet which is very probably part of an incomplete letter, dated July, 1904, Widener Vβ. Cf. 2.661ff.

Peirce: CP 8.227 Fn 11 p 176

†11 (Ed.) Peirce is proposing to place "Deduction, Induction, and Hypothesis" between "The Doctrine of Chances" and "The Probability of Induction."

Peirce: CP 8.227 Fn 12 p 176

†12 (Ed.) [Bibliography] G-1868-2c.

Peirce: CP 8.228 Fn 13 p 177

†13 (Ed.) Peirce often calls this "abduction."

Peirce: CP 8.234 Fn 14 p 177

†14 (Ed.) [Bibliography] G-1883-7b.

Peirce: CP 8.239 Fn 1 p 180

†1 (Ed.) Paragraphs 239-242 are a letter dated "1904 June 9"; paragraphs 243-244 are from an incomplete letter, bearing no date, but from the evidence given in 243n4 it was written c.1905. Both letters are in Widener VB2a, and both have the salutation, "My dear Prof. Dewey."

Peirce: CP 8.239 Fn 2 p 180

†2 (Ed.) [Bibliography] N-1904-16, reprinted in Book I of the present volume.

Dewey had written from the University of Chicago, saying that he was sending Peirce a copy of the book and expressing his general indebtedness to Peirce. The letter, dated 11 January 1904, is in Widener VB2a.

Peirce: CP 8.239 Fn 3 p 181

†3 (Ed.) A marginal note on the first page of this letter reads: "The idea that two such elements as Evolution and Function can in the same sense depend upon one another seems to me *absurd*."

Peirce: CP 8.243 Fn 4 p 183

†4 (Ed.) [Bibliography] G-1905-1a. Dewey had written from Columbia University, praising this article. The letter, dated 11 April 1905, is in Widener VB2a. The date of the article and the date of Dewey's letter establish the date of Peirce's letter as c.1905 (see 239n1).

Peirce: CP 8.245 Fn 1 p 185

†1 (Ed.) A letter to "Wm. T. Harris Esq.," dated "1868 Nov. 30," in the Hoose Library at the University of Southern California. Harris was the editor of the *Journal of Speculative Philosophy*.

Peirce: CP 8.245 Fn 2 p 185

†2 (Ed.) For one of the articles, probably the second, of the series on intuitive knowledge in the *Journal of Speculative Philosophy*, 1868-1869; [Bibliography] G-1868-2.

Peirce: CP 8.248 Fn 3 p 185

†3 (Ed.) Cf. 5.289, 5.313.

Peirce: CP 8.249 Fn 1 p 186

†1 (Ed.) From a series of letters from Peirce to James in the William James Collection in Houghton Library. The date of each letter is given in brackets at the beginning of the first paragraph. Peirce's salutation is "My dear William," with but one exception (the letter of November 10, 1900) when he uses "My dear Willie." No letter is printed in full.

There is considerable overlap between the contents of this chapter and [Perry] II, but each contains Peirce-James correspondence not to be found in the other.

Peirce: CP 8.249 Fn 2 p 186

†2 (Ed.) James, *The Will to Believe*, 1897, was dedicated "To My Old Friend, CHARLES SANDERS PEIRCE, to whose philosophic comradeship in old times and to whose writings in more recent years I owe more incitement and help than I can express or repay."

Peirce: CP 8.250 Fn 3 p 186

†3 (Ed.) [Bibliography] G-1877-5.

Peirce: CP 8.250 Fn 4 p 186

†4 (Ed.) Cf. [Bibliography] G-1891-1.

Peirce: CP 8.251 Fn 5 p 187

†5 (Ed.) Cf. 1.616ff.

Peirce: CP 8.251 Fn 6 p 187

†6 (Ed.) Cf. [CP] VII, Book II, "Scientific Method."

Peirce: CP 8.252 Fn 7 p 187

†7 (Ed.) Cf. 6.102.

Peirce: CP 8.253 Fn 8 p 188

†8 (Ed.) On a post card dated November 26, 1900, Widener VB2a, James replies: "You invented 'pragmatism' for which I gave you full credit in a lecture entitled 'Philosophical conceptions and practical results' of which I sent you 2 (unacknowledged) copies a couple of years ago."

Peirce: CP 8.255 Fn 9 p 188

†9 (Ed.) [Bibliography] G-1898-1.

Peirce: CP 8.255 Fn 10 p 188

†10 (Ed.) Cf. 1.573ff.

- Peirce: CP 8.256 Fn 11 p 188
†11 (Ed.) See Chapter 4, "Consciousness," [CP] VII, Book III.
- Peirce: CP 8.257 Fn 12 p 189
†12 (Ed.) Cf. 6.102ff.
- Peirce: CP 8.258 Fn 13 p 189
†13 (Ed.) The letter bears dates of March 1 and 7, 1904, but the passage quoted comes under the latter date.
- Peirce: CP 8.258 Fn 14 p 189
†14 (Ed.) "Quite recently the word 'pragmatism,' first used thirty years ago by our American philosopher, C.S. Peirce, has become fashionable as the designation of a novel way of looking at the mind's relations to reality." From p. 175 of James's review of F.C.S. Schiller's *Humanism*, in *The Nation* 78(3 March 1904)175-176. (This is identified as James's by [Haskell].)
- Peirce: CP 8.258 Fn 15 p 189
†15 (Ed.) Cf. [Bibliography] G-1909-1.
- Peirce: CP 8.258 Fn 16 p 189
†16 (Ed.) 8.7-38.
- Peirce: CP 8.260 Fn 17 p 190
†17 (Ed.) Some of the paragraphing in this letter is the editor's.
- Peirce: CP 8.260 Fn 18 p 190
†18 (Ed.) Cf. 8.205.
- Peirce: CP 8.260 Fn 19 p 190
†19 (Ed.) "La Notion de Conscience," *Archives de Psychologie* 5(June 1905) 1-12.
- Peirce: CP 8.260 Fn 20 p 190
†20 (Ed.) See 270-305.
- Peirce: CP 8.261 Fn 21 p 191
†21 (Ed.) 8.132ff.
- Peirce: CP 8.261 Fn 22 p 191
†22 (Ed.) From 8.144; Peirce's deletions.
- Peirce: CP 8.264 Fn 23 p 193
†23 (Ed.) Cf. 7.524ff. and [CP] I, Book III, "Phenomenology."
- Peirce: CP 8.268 Fn 24 p 195
†24 (Ed.) Cf. 7.535.
- Peirce: CP 8.271 Fn 25 p 196
†25 (Ed.) *The Varieties of Religious Experience*, 1902.
- Peirce: CP 8.274 Fn 26 p 197
†26 (Ed.) In the manuscript "{theta}" is an exponent on the constant "C." Cf. 7.370.
- Peirce: CP 8.277 Fn 27 p 198

†27 (Ed.) *The World and the Individual*. Cf. 8.117ff.

Peirce: CP 8.279 Fn 28 p 198

†28 (Ed.) "Does 'Consciousness' Exist?", *The Journal of Philosophy, Psychology, and Scientific Methods* 1(1 Sept 1904)477-491.

Peirce: CP 8.281 Fn 29 p 199

†29 (Ed.) Cf. Chapter 4, "Consciousness," [CP] VII, Book III.

Peirce: CP 8.284 Fn 30 p 200

†30 (Ed.) Cf. 8.330.

Peirce: CP 8.285 Fn 31 p 200

†31 (Ed.) James replied in a letter dated September 30 [1904] (James Collection, Houghton Library) as follows:

"I have to confess that I don't understand a word of your letter, . . .

"As for what entity may mean in general I know not, except it be some imperceptible kind of being. In my article it meant a *constituent principle* of all experience, as contrasted with a certain *function* or *relation* between particular parts of experience. The distinction seems to me plain enough.

"I will shortly send you a couple more articles which build out that one farther."

Peirce: CP 8.288 Fn 32 p 201

†32 (Ed.) In the text quoted this word is "affirm."

Peirce: CP 8.288 Fn 33 p 201

†33 (Ed.) *Analysis of the Phenomena of the Human Mind by James Mill* with notes illustrative and critical by Alexander Bain, Andrew Findlater, and George Grote, 2 vols., second edition, London, 1878.

Peirce: CP 8.290 Fn 34 p 202

†34 (Ed.) Note 74 by Alexander Bain in *Analysis of the Phenomena of the Human Mind by James Mill* with notes illustrative and critical by Alexander Bain, Andrew Findlater, and George Grote, 2 vols., second edition, London, 1878.

Peirce: CP 8.290 Fn 35 p 202

†35 (Ed.) The words in brackets appear in the quoted text, but were omitted without notice by Peirce. In the quoted text the last word of the second sentence is "inadequate" where Peirce has "inexact." The first words of the quotation appear on p. 129 of the work.

Peirce: CP 8.291 Fn 36 p 202

†36 (Ed.) Cf. 7.539ff.

Peirce: CP 8.291 Fn 37 p 203

†37 (Ed.) In the quoted text this word is "enters."

Peirce: CP 8.295 Fn 38 p 204

†38 (Ed.) Cf. [CP] I, Book III, "Phenomenology," and [CP] VII, Book III, "Philosophy of Mind."

Peirce: CP 8.301 Fn 39 p 206

†39 (Ed.) 5.413.

Peirce: CP 8.302 Fn 40 p 206

†40 (Ed.) Some of the paragraphing is the editor's.

Peirce: CP 8.302 Fn 41 p 206

†41 (Ed.) [Bibliography] G-1877-5.

Peirce: CP 8.302 Fn 42 p 206

†42 (Ed.) Cf. [Bibliography] G-1909-1.

Peirce: CP 8.303 Fn 43 p 207

†43 (Ed.) Cf. 7.539ff.

Peirce: CP 8.305 Fn 44 p 208

†44 (Ed.) Cf. 8.216.

Peirce: CP 8.306 Fn 45 p 208

†45 (Ed.) James had recently sent Peirce a copy of *The Will to Believe*, which contains this essay; cf. 249.

Peirce: CP 8.308 Fn 46 p 209

†46 (Ed.) 3.527.

Peirce: CP 8.311 Fn 47 p 209

†47 (Ed.) 6.238-271.

Peirce: CP 8.313 Fn 48 p 210

†48 (Ed.) This letter is unsigned and has some scratch work on the last page.

Peirce: CP 8.314 Fn 49 p 212

†49 (Ed.) Cf. 8.171ff. and 8.342ff.

Peirce: CP 8.315 Fn 50 p 212

†50 (Ed.) Cf. 303.

Peirce: CP 8.316 Fn 1 p 214

†1 (Ed.) From an incomplete letter to "My dear Mrs. Franklin," dated "1891 Aug 29," Widener VB2a.

Peirce: CP 8.317 Fn 2 p 214

†2 (Ed.) See *The Monist* series of 1891-1893, [Bibliography] G-1891-1.

Peirce: CP 8.317 Fn 3 p 214

†3 (Ed.) Cf. 6.33.

Peirce: CP 8.318 Fn 4 p 215

†4 (Ed.) The letter breaks off here without a period.

Peirce: CP 8.319 Fn 1 p 216

†1 (Ed.) Paragraphs 319-320 (originally one paragraph) are from an undated letter; paragraphs 321-326 are from a letter dated Sept. 10, 1906. Both letters are in Widener VB2a and bear the salutation, "My dear Mr. Schiller." Neither is signed.

Peirce: CP 8.319 Fn 2 p 216

†2 (Ed.) In [Bibliography] G-1891-1.

Peirce: CP 8.320 Fn 3 p 217

†3 (Ed.) Cf. 7.370.

Peirce: CP 8.321 Fn 4 p 217

†4 (Ed.) "Pragmatism and Pseudo-Pragmatism," *Mind* n.s. 15(July 1906) 375-390. This is the last paper in a controversy between F.C.S. Schiller and A.E. Taylor in *Mind*, n.s. vols. 14 and 15.

Peirce: CP 8.322 Fn 5 p 218

†5 (Ed.) Schiller defines "the practical as '*whatever tends to the control of events*'" in *Mind* n.s. 15(July 1906)386.

Peirce: CP 8.327 Fn 1 Para 1/2 p 220

†1 (Ed.) From a letter dated "1904 Oct 12" to "My dear Lady Welby." A photostat copy of the original letter is in the Yale University Library. The complete letter is also in [Bibliography] M-20a, pp. 7-14, published by Whitlock's, Inc., New Haven, Conn., with whose permission the parts given here and the quotations in 330n4 and 330n6 are reprinted.

Peirce: CP 8.327 Fn 1 Para 2/2 p 220

Lady Victoria Welby was an English semanticist, at one time Maid of Honour to Queen Victoria. For Peirce's review of her *What is Meaning?* see Book I of the present volume. For additional correspondence see [Bibliography] M-20.

Peirce: CP 8.328 Fn 2 p 220

†2 (Ed.) Peirce's phenomenology and categories are discussed at various places in [CP], especially in [CP] I, Book III. See also 7.524-538.

Peirce: CP 8.328 Fn 3 p 220

†3 (Ed.) In [Bibliography] G-1867-1c, 1.545-559.

Peirce: CP 8.330 Fn 4 p 223

†4 (Ed.) "The italicized sentence is, in manuscript, underlined in pencil. Perhaps it was underlined by Lady Welby, yet it was not her habit to annotate Peirce's letters." From [Bibliography] M-20a, p. 9.

Peirce: CP 8.330 Fn 5 p 224

†5 (Ed.) Reversible and irreversible actions are discussed further in Chapter 3, "Habit," [CP] VII, Book III.

Peirce: CP 8.330 Fn 6 p 224

†6 (Ed.) "'Relate', in manuscript, is underlined in pencil." From [Bibliography] M-20a, p. 10.

Peirce: CP 8.331 Fn 7 p 225

†7 (Ed.) Cf. 1.347, 3.421.

Peirce: CP 8.331 Fn 8 p 225

†8 (Ed.) [Bibliography] G-1883-7d, 3.328-358.

Peirce: CP 8.331 Fn 9 p 225

†9 (Ed.) Cf. [Bibliography] G-1896-6.

Peirce: CP 8.331 Fn 10 p 226

†10 (Ed.) This is treated at length in [CP] IV, Book II.

Peirce: CP 8.332 Fn 11 p 226

†11 (Ed.) Signs are discussed at various places in [CP]. See 8.313ff., 2.227ff., and the letter following the present one. See also [Bibliography] M-20a.

Peirce: CP 8.335 Fn 12 p 228

†12 (Ed.) 1.558 ([Bibliography] G-1867-1c).

Peirce: CP 8.337 Fn 13 p 229

†13 (Ed.) The brackets are Peirce's.

Peirce: CP 8.341 Fn 14 p 231

†14 (Ed.) Cf. 2.254 and [Bibliography] M-20a, Appendix B.

Peirce: CP 8.342 Fn 15 Para 1/3 p 231 Cross-Ref:††

†15 (Ed.) From a partial draft of a letter to Lady Welby, bearing dates of 24, 25, and 28 December 1908, Widener IB3a, with an added quotation in 368n23. The editor has made more than the usual number of alterations in punctuation, capitalization, etc., in both this and the previous letter, and in the present letter he has made several changes in format.

Peirce: CP 8.342 Fn 15 Para 2/3 p 231 Cross-Ref:††

In a letter to Lady Welby, with dates 31 January, 24 February, and 14 March 1909, Peirce says: "I find in my portfolio some part of a letter, if not the whole, dated December 28. I suppose I sent you that" [Bibliography] M-20a, p. 35. It is probable that in this passage Peirce is referring to the letter printed here. This letter never reached Lady Welby; cf. *Other Dimensions: A Selection from the Later Correspondence of Victoria Lady Welby*, p. 309n1 (listed with [Bibliography] M-20a).

Peirce: CP 8.342 Fn 15 Para 3/3 p 231 Cross-Ref:††

The letter printed here resembles, but is distinct from, a letter with dates of 14 and 23 December 1908, printed in [Bibliography] M-20a, pp. 22-32, q.v.

Peirce: CP 8.342 Fn 16 p 231

†16 (Ed.) Lady Welby wrote "Significs" for the 11th edition of *The Encyclopedia Britannica*, 1910-11.

Peirce: CP 8.342 Fn 17 p 232

†17 (Ed.) 1.559 ([Bibliography] G-1867-1c). The brackets are in the manuscript.

Peirce: CP 8.343 Fn 18 p 232

†18 (Ed.) For references to other discussions of signs, see 332n11.

Peirce: CP 8.346 Fn 19 p 233

†19 (Ed.) For references to discussions of Peirce's categories, see 328n2.

Peirce: CP 8.350 Fn 20 p 235

†20 (Ed.) Peirce had revised the manuscript here, but since his revision is unintelligible, we give the text as it was originally.

Peirce: CP 8.361 Fn 21 p 239

†21 (Ed.) See 8.225ff.

Peirce: CP 8.364 Fn 22 p 240

†22 (Ed.) It is clear from the format of the manuscript that the question mark applies only to the term "type."

Peirce: CP 8.368 Fn 23 p 241 Cross-Ref:††

†23 (Ed.) "An *index* represents an object by virtue of its connection with it. It makes no difference whether the connection is natural, or artificial, or merely mental. There is, however, an important distinction between two classes of indices. Namely, some merely stand for things or individual quasi-things with which the interpreting mind is already acquainted, while others may be used to ascertain facts. Of the former class, which may be termed *designations*, personal, demonstrative, and relative pronouns, proper names, the letters attached to a geometrical figure, and the ordinary letters of algebra are examples. They act to force the attention to the thing intended. Designations are absolutely indispensable both to communication and to thought. No assertion has any meaning unless there is some designation to show whether the universe of reality or what universe of fiction is referred to. The other class of indices may be called *reagents*. Thus water placed in a vessel with a shaving of camphor thrown upon it will show whether the vessel is clean or not. If I say that I live two and a half miles from Milford, I mean that a rigid bar that would just reach from one line to another upon a certain bar in Westminster, might be successively laid down on the road from my house to Milford, 13200 times, and so laid down on my reader's road would give him a knowledge of the distance between my house and Milford. Thus, the expression "two miles and a half" is, not exactly a reagent, but a description of a reagent. A scream for help is not only intended to force upon the mind the knowledge that help is wanted, but also to force the will to accord it. It is, therefore, a reagent used rhetorically. Just as a designation can denote nothing unless the interpreting mind is already acquainted with the thing it denotes, so a reagent can indicate nothing unless the mind is already acquainted with its connection with the phenomenon it indicates." From "Notes on Topical Geometry," undated, Widener IA-2.

Peirce: CP 8.376 Fn 24 p 243

†24 (Ed.) The brackets are Peirce's.

Peirce: CP 8.376 Fn 25 p 243

†25 (Ed.) Cf. [Bibliography] M-20a, Appendix B.

Peirce: CP 8.378 Fn 26 p 244

†26 (Ed.) See Clemens C.I. Webb, *Ioannis Saresberiensis Episcopi Carnotensis Metalogicon*, Oxford University Press, 1929, Liber II, Cap. 20, p. 104.

Peirce: CP 8.379 Fn 27 p 245

†27 (Ed.) In early Greek this word was written with an initial symbol called digamma followed by {alis}. In the manuscript Peirce wrote a digamma followed by {élis}. We have substituted the conventional spelling since the digamma is no longer used.

Peirce: CP 8.380 Fn 1 p 246

†1 (Ed.) From a long letter to "My dear Dr. Woods," written over a period between 14 October 1913 and 19 November 1913, with an added quotation in 380n4. The letter was sent to Woods, but it is now in Widener VB2a.

Frederick Adams Woods, M.D., was a lecturer in biology at the Massachusetts Institute of Technology.

Peirce: CP 8.380 Fn 2 p 246

†2 (Ed.) *The Influence of Monarchs, Steps in a New Science of History*, The Macmillan Co., New York, 1913, xiii + 422 pp.

Peirce: CP 8.380 Fn 3 p 246

†3 (Ed.) This chapter is titled, "Causation in History."

Peirce: CP 8.380 Fn 4 p 246 Cross-Ref:††

†4 (Ed.) "An ordinary *hypothetical proposition*, as propositions containing an antecedent condition and a consequent result are called, relates to what *would* occur in states of things not all coincident with the existing state of things. Suppose I say 'If I were to upset the inkstand the table cloth would be injured.' This means that of all the different courses of events that might occur, the disposition of things in the room being what it is, every one is one in which either the inkstand is not upset or the tablecloth is spoiled." From an undated fragment, Widener IA-8. Cf. 5.453, 6.327, and 8.216.

Peirce: CP 8.385 Fn 5 p 248

†5 (Ed.) These types of reasoning are discussed in more detail in [CP] VII, Book II, Chapter 3, "The Logic of Drawing History from Ancient Documents."