Fluid Mechanics

Chapter 7:

Boundary Layer Concept

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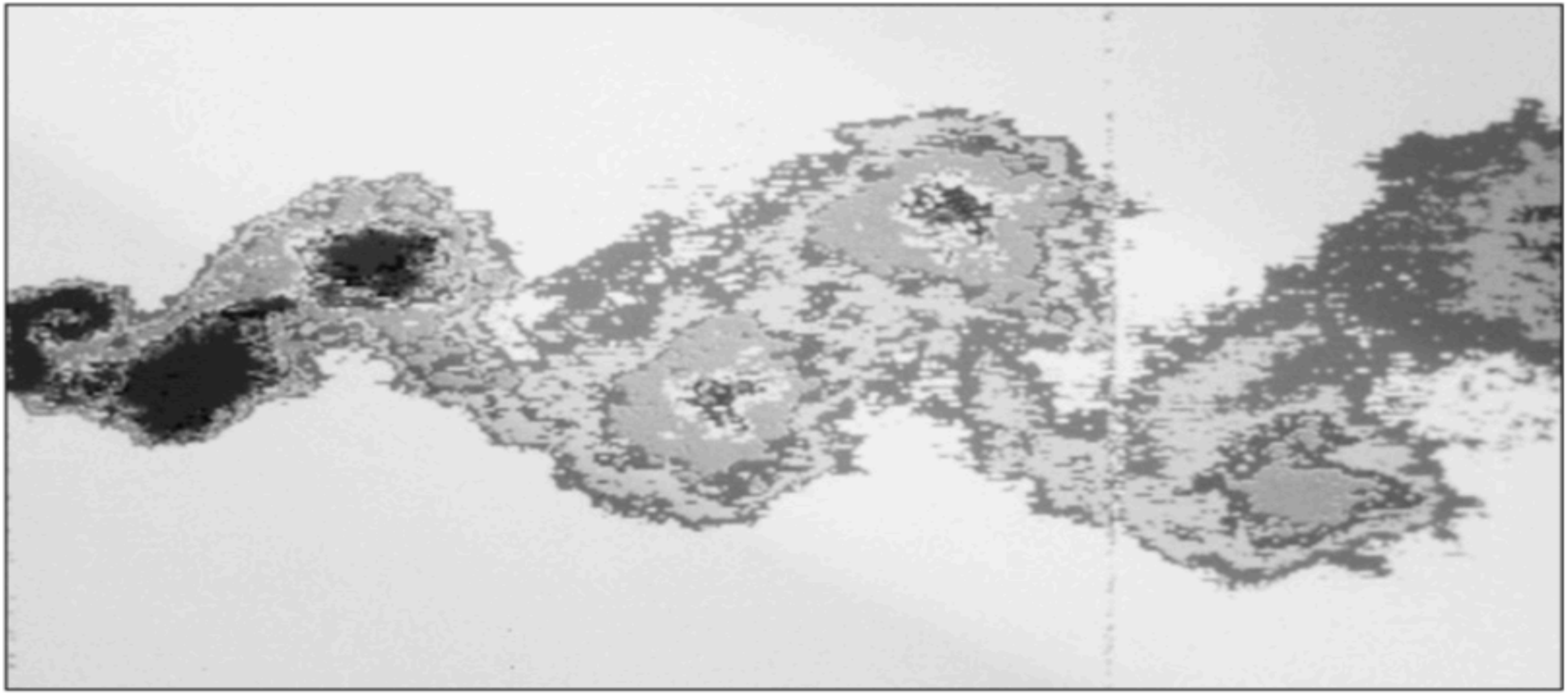


The turbulent wake behind a bluff body immersed in a stream flow is a subject of the present chapter. This is a digitized video image showing the distribution of tracer-dye concentration in the wake of the body. Compare with Fig. 5.2a of the text, which is a laminar wake. *(Courtesy of R. Balachandar, by permission of the American Society of Mechanical Engineers.)*

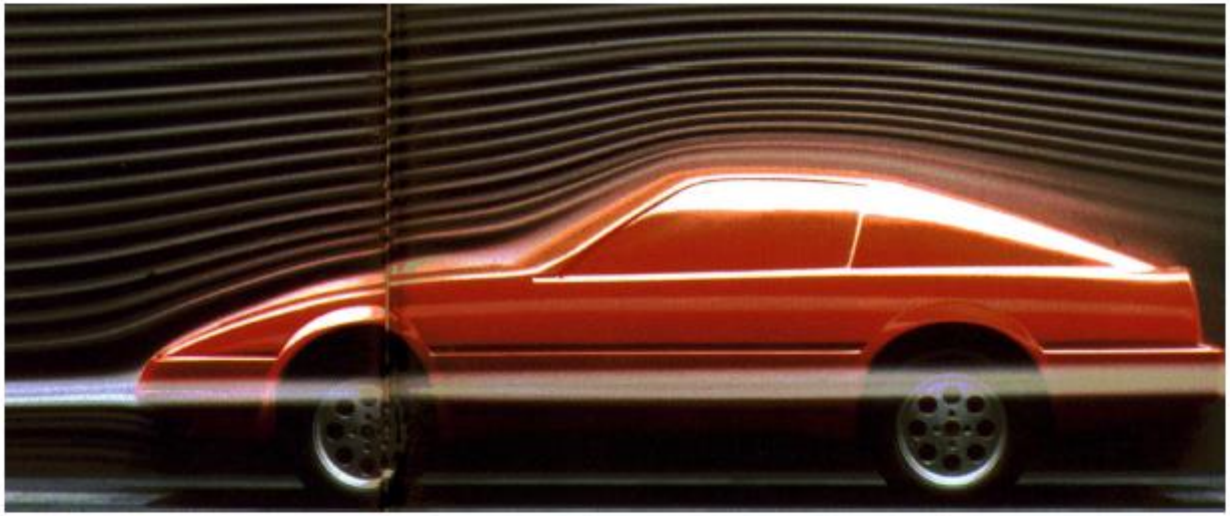
Introduction

- External flows past objects encompass an extremely wide variety of fluid mechanics phenomena. Clearly the character of the flow field is a function of the shape of the body.
- For a given shaped object, the characteristics of the flow depend very strongly on various parameters such as size, orientation, speed, and fluid properties.
- According to dimensional analysis arguments, the character of the flow should depend on the various dimensionless parameters involved.
- For typical external flows the most important of these parameters are the Reynolds number, $Re = UL/v$, where L– is characteristic dimension of the body.

Introduction

- For many high-Reynolds-number flows the flow field may be divided into two region
  i. A viscous boundary layer adjacent to the surface
  ii. The essentially inviscid flow outside the boundary layer
- W know that fluids adhere the solid walls and they take the solid wall velocity. When the wall does not move also the velocity of fluid on the wall is zero.
- In region near the wall the velocity of fluid particles increases from a value of zero at the wall to the value that corresponds to the external ”frictionless” flow outside the boundary layer

Introduction



- Figure 6.1: Visualization of the flow around the car. It is visible the thin layer along the body cause by viscosity of the fluid. The flow outside the narrow region near the solid boundary can be considered as ideal (inviscid).

Introduction

- The concept of boundary layer was first introduced by a German engineer, Prandtl in 1904.
- According to Prandtl theory, when a real fluid flows past a stationary solid boundary at large values of the Reynolds number, the flow will be divided into two regions.

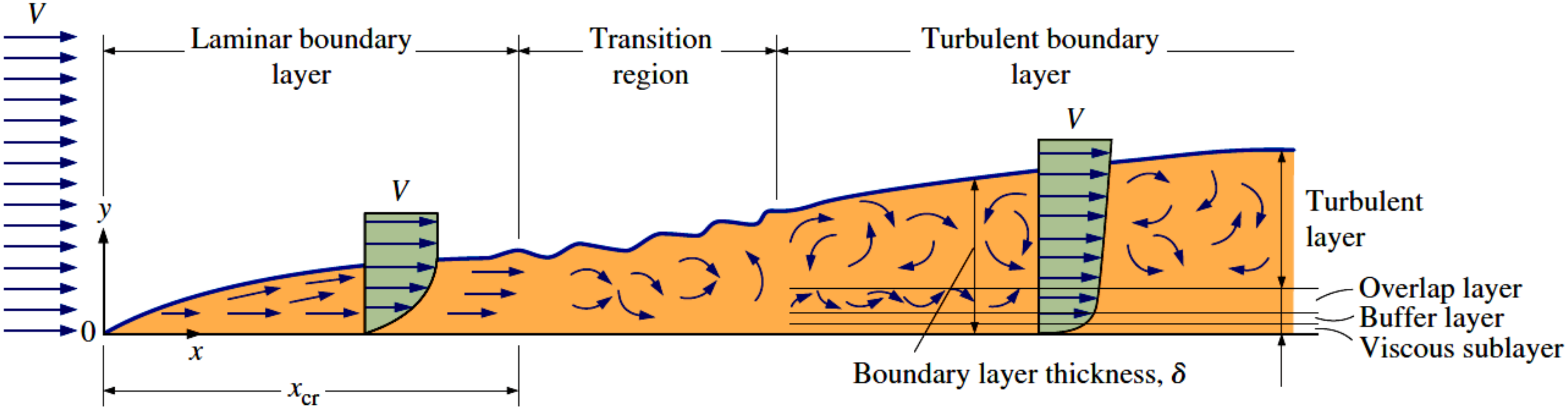
i. A thin layer adjoining the solid boundary, called the boundary layer, where the viscous effects and rotation cannot be neglected.

ii. An outer region away from the surface of the object where the viscous effects are very small and can be neglected. The flow behavior is similar to the upstream flow. In this case a potential flow can be assumed.

Introduction

- Since the fluid at the boundaries has zero velocity, there is a steep velocity gradient from the boundary into the flow. This velocity gradient in a real fluid sets up shear forces near the boundary that reduce the flow speed to that of the boundary.
- That fluid layer which has had its velocity affected by the boundary shear is called ***the boundary layer.***
- For smooth upstream boundaries the boundary layer starts out as a ***laminar boundary layer*** in which the fluid particles move in smooth layers.
- As the laminar boundary layer increases in thickness, it becomes unstable and finally transforms into a ***turbulent boundary layer*** in which the fluid particles move in haphazard paths.
- When the boundary layer has become turbulent, there is still a very thon layer next to the boundary layer that has laminar motion. It is called the ***laminar sublayer.***

Introduction



- Fig. 6.2 The development of the boundary layer for flow over a flat plate, and the different flow regimes. The vertical scale has been greatly exaggerated and horizontal scale has been shortened.

Introduction

- The turbulent boundary layer can be considered to consist of four regions, characterized by the distance from the wall.
- The very thin layer next to the wall where viscous effects are dominant is the **viscous sublayer.** The velocity profile in this layer is very nearly *linear*, and the flow is nearly parallel.
- Next to the viscous sublayer is the **buffer layer,** in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.
- Above the buffer layer is the **overlap layer,** in which the turbulent effects are much more significant, but still not dominant.
- Above that is the **turbulent** (or **outer**) **layer** in which turbulent effects dominate over viscous effects.

Boundary layer thickness, δ

- The boundary layer thickness is defined as the vertical distance from a flat plate to a point where the flow velocity reaches 99 per cent of the velocity of the free stream.
- Another definition of boundary layer are the
  - *Boundary layer displacement thickness,* δ^*
  - *Boundary layer momentum thickness,* θ

Boundary layer displacement thickness, δ*

- Consider two types of fluid flow past a stationary horizontal plate with velocity U as shown in Fig. 6.3. Since there is no viscosity for the case of ideal fluid (Fig. 6.3a), a uniform velocity profile is developed above the solid wall.
- However, the velocity gradient is developed in the boundary layer region for the case of real fluid with the presence of viscosity and no-slip at the wall (Fig. 6.3b).

Boundary layer displacement thickness, δ*

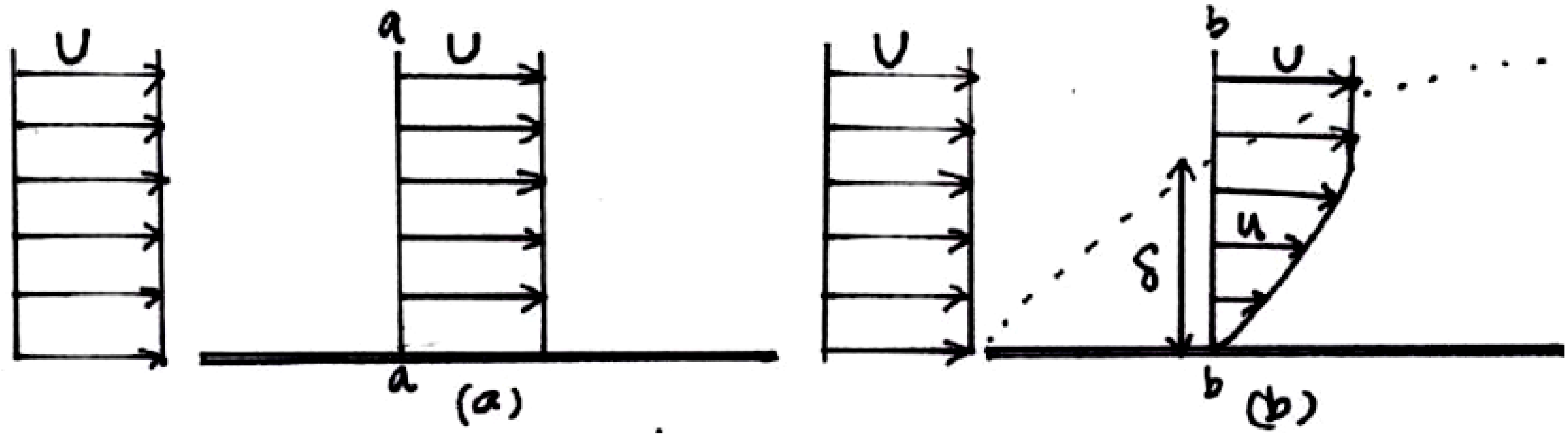


Figure 6.3 Flow over a horizontal solid surface for the case of (a) Ideal fluid (b) Real fluid

- The velocity deficits through the element strip of cross section b-b is U - u . Then the reduction of mass flow rate is obtained as $\rho(U-u)bdy$ where b is the plate width.
- The total mass reduction due to the presence of viscosity compared to the case of ideal fluid

$$\int_0^{\delta} \rho(U-u)bdy \qquad (6.1)$$

Boundary layer displacement thickness, δ*

- However, if we displace the plate upward by a distance δ^* at section a-a to give mass reduction of $\rho U b \delta^*$, then the deficit of flow rates for the both cases will be identical if

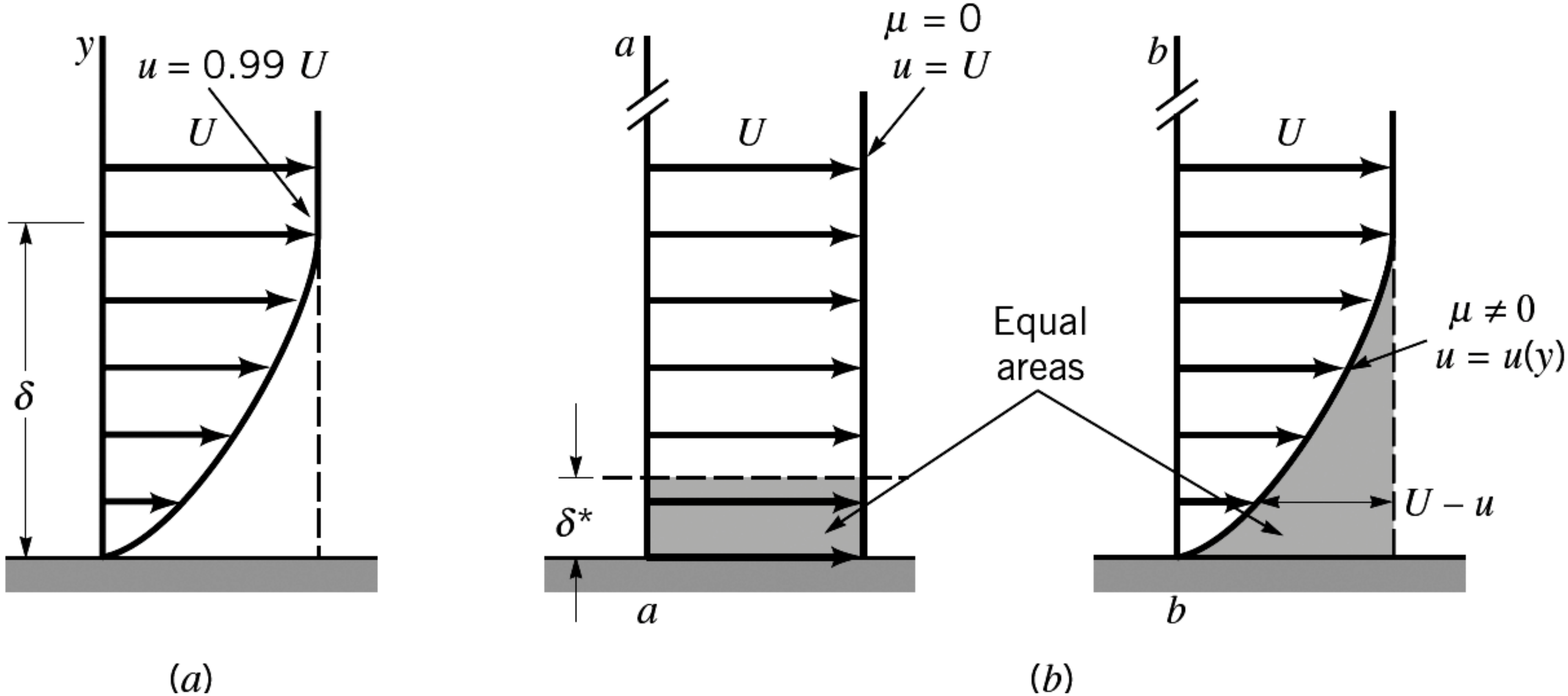
$$\int_0^\delta \rho (U - u) b\,dy = \rho U b \delta^*$$

and

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy \qquad (6.2)$$

- Here, δ^* is known as the boundary layer displacement thickness.

Boundary layer displacement thickness, δ*



- Figure 6.4: Definition of boundary layer thickness:(a) standard boundary layer(*u = 99%U),(b)* boundary layer displacement thickness .

Boundary layer displacement thickness, δ*

- The displacement thickness represents the vertical distance that the solid boundary must be displaced upward so that the ideal fluid has the same mass flow rate as the real fluid.

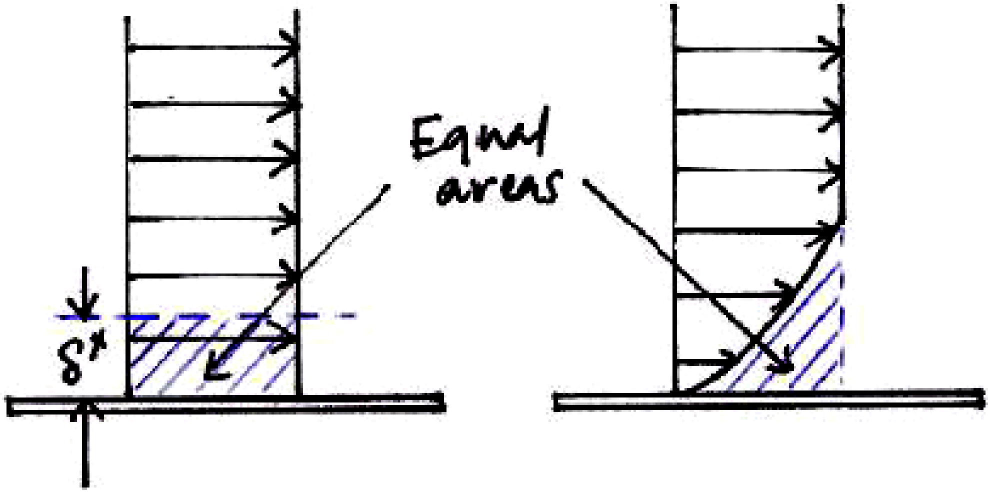


Figure 6.5 boundary layer displacement thickness

Boundary layer momentum thickness, θ

- Another definition of boundary layer thickness, the boundary layer momentum thickness θ, is often used to predict the drag force on the object surface.
- By referring to Fig. 6.3, again the velocity deficit through the element strip of cross section b-b contributes to deficit in momentum flux as

$$\rho u(U-u)bdy \qquad (6.3)$$

- Thus, the total momentum reductions

$$\int_0^\delta \rho u(U-u)bdy$$

- However, if we displace the plate upward by a distance θ at section a-a to give momentum reduction of $\rho U^2 b\theta$, then the momentum deficit for the both cases will be identical if

Boundary layer momentum thickness, θ

$$\int_0^{\delta} \rho u (U - u) b\,dy = \rho U^2 b \theta$$

and

$$\theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad (6.4)$$

- Here, θ is known as the boundary layer momentum thickness.
- The momentum thickness represents the vertical distance that the solid boundary must be displaced upward so that the ideal fluid has the same mass momentum as the real fluid

Reynolds Number and Geometry Effects

- The technique of boundary layer (BL) analysis can be used to compute viscous effects near solid walls and to “patch” these onto the outer inviscid motion.
- This patching is more successful as the body Reynolds number becomes larger, as shown in Fig. 6.6.
- In Fig. 6.6 a uniform stream U moves parallel to a sharp flat plate of length L. If the Reynolds number UL/ν is low *(Fig. 6.6a)*, the viscous region is very broad and extends far ahead and to the sides of the plate. The plate retards the oncoming stream greatly, and small changes in flow parameters cause large changes in the pressure distribution along the plate.
- There is no existing simple theory for external flow analysis at Reynolds numbers from 1 to about 1000. Such thick-shear-layer flows are typically studied by experiment or by numerical modeling of the flow field on a computer

Reynolds Number and Geometry Effects

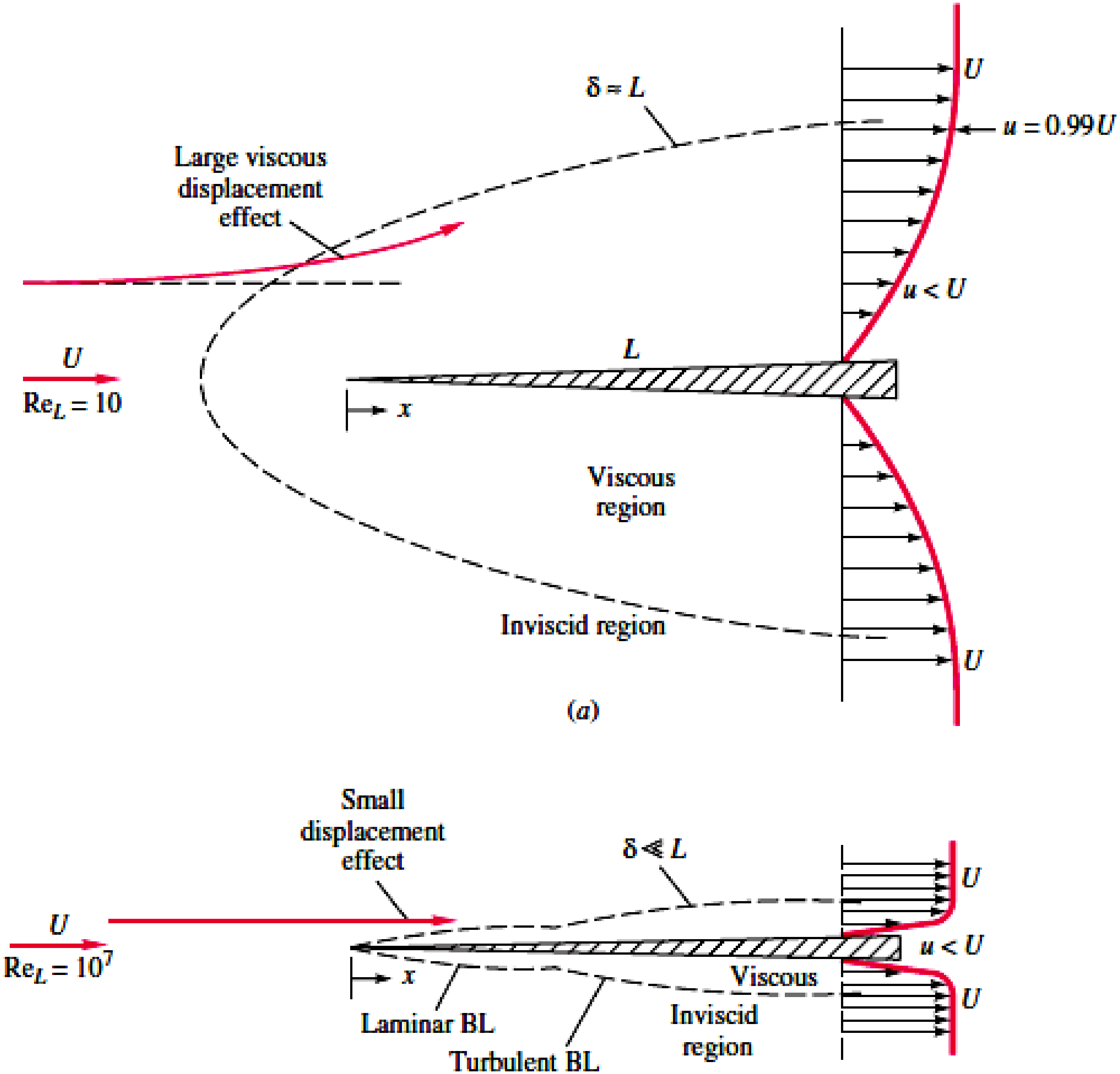


Fig. 6.6. Comparison of flow past a sharp flat plate at low and high Reynolds numbers: (*a*) *laminar*, low-Re flow; (*b*) *high-Re flow.*

Reynolds Number and Geometry Effects

- A high-Reynolds-number flow (Fig. 6.6*b)* is much more amenable to boundary layer patching, as first pointed out by Prandtl in 1904.
- The viscous layers, either laminar or turbulent, are very thin, thinner even than the drawing shows.
- We define the boundary layer thickness δ as the locus of points where the velocity *u* parallel to the plate reaches 99 percent of the external velocity *U*.
- The accepted formulas for flat-plate flow, and their approximate ranges, are

$$\frac{\delta}{x} \approx \begin{cases} \dfrac{5.0}{\mathrm{Re}_x^{1/2}} & \text{laminar} \qquad 10^3 < \mathrm{Re}_x < 10^6 \\ \dfrac{0.16}{\mathrm{Re}_x^{1/7}} & \text{turbulent} \qquad 10^6 < \mathrm{Re}_x \end{cases} \qquad (6.5)$$

- .

Reynolds Number and Geometry Effects

- where $Re_x = Ux/\nu$ is called the *local Reynolds number* of the flow along the plate surface. The turbulent flow formula applies for Re_x greater than approximately 10^6 .
- Some computed values are shown below

Re_x	10^4	10^5	10^6	10^7	10^8
$(\delta/x)_{lam}$	0.050	0.016	0.005		
$(\delta/x)_{turb}$			0.022	0.016	0.011

- The blanks indicate that the formula is not applicable. In all cases these boundary layers are so thin that their displacement effect on the outer inviscid layer is negligible.
- Thus the pressure distribution along the plate can be computed from inviscid theory as if the boundary layer were not even there.

Example 1

- A long, thin flat plate is placed parallel to a 20-ft/s stream of water at 68F. At what distance x from the leading edge will the boundary layer thickness be 1 in?

Solution

- Approach: Guess laminar flow first. If contradictory, try turbulent flow.
- Property values: From Table for water at 68F, ν =*1.082E-5 ft²/s.*
- Solution step 1: With δ = 1 in = 1/12 ft, try laminar flow

$$\frac{\delta}{x}\bigg|_{\text{lam}} = \frac{5}{(Ux/\nu)^{1/2}} \quad \text{or} \quad \frac{1/12\ \text{ft}}{x} = \frac{5}{\left[(20\ \text{ft/s})x/(1.082\text{E-5 ft}^2/\text{s})\right]^{1/2}}$$

Solve for $x \approx 513$ ft

Pretty long plate! This does not sound right. Check the local Reynolds number:

$$\text{Re}_x = \frac{Ux}{\nu} = \frac{(20\ \text{ft/s})(513\ \text{ft})}{1.082\text{E-5 ft}^2/\text{s}} = 9.5\text{E8} \qquad (!)$$

Example 1

- This is impossible, since laminar boundary layer flow only persists up to about 10^6 (or, with special care to avoid disturbances, up to 3×10^6).
- *Solution step 2: Try turbulent flow*

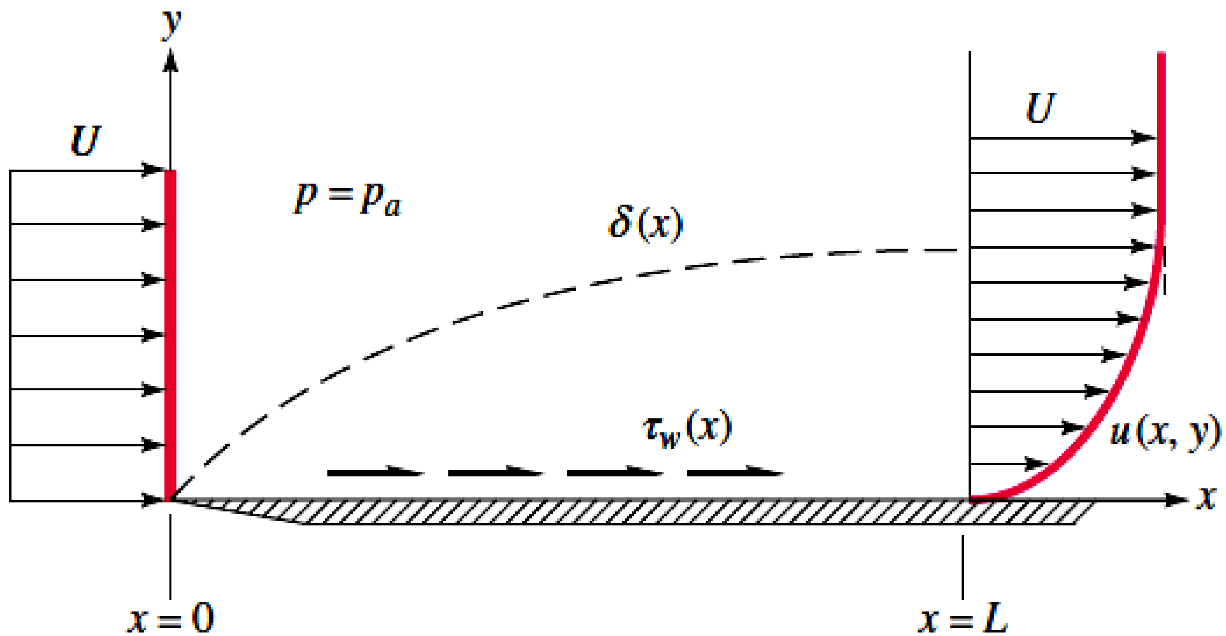
$$\frac{\delta}{x} = \frac{0.16}{(Ux/\nu)^{1/7}} \quad \text{or} \quad \frac{1/12 \text{ ft}}{x} = \frac{0.16}{[(20 \text{ ft/s})x/(1.082\text{E-5 ft}^2/\text{s})]^{1/7}}$$

$$\text{Solve for} \quad x \approx 5.17 \text{ ft}$$

Check $Re_x = (20 \text{ ft/s})(5.17 \text{ ft})/(1.082\text{E-5 ft}^2/\text{s}) = 9.6\text{E6} > 10^6$. OK, turbulent flow.

Boundary Layer: Momentum Integral Estimates

- A shear layer of unknown thickness grows along the sharp flat plate in Fig. 6.7. The no-slip wall condition retards the flow, making it into a rounded profile *u(x,y)*, which merges into the external velocity *U* = constant at a "thickness" y = δ(x).



- Fig. 6.7 Growth of a boundary layer on a flat plate.

Boundary Layer: Momentum Integral Estimates

- The drag force on the plate is given by the following momentum integral across the exit plane:

$$D(x) = \rho b \int_0^{\delta(x)} u(U - u)\, dy \qquad (6.6)$$

- where b is the plate width into the paper and the integration is carried out along a vertical plane x = constant.
- Equation (6.6) was derived in 1921 by Kármán, who wrote it in the convenient form of the *momentum thickness as*:

$$D(x) = \rho b U^2 \theta \qquad \theta = \int_0^{\delta} \frac{u}{U}\left(1 - \frac{u}{U}\right) dy \qquad (6.7)$$

- Momentum thickness is a measure of total plate drag which also equals the integrated wall shear stress along the plate:

Boundary Layer: Momentum Integral Estimates

$$D(x) = b\int_0^x \tau_w(x)\,dx$$

or

$$\frac{dD}{dx} = b\tau_w \qquad (6.8)$$

- Meanwhile, the derivative of Eq. (6.7), with U = *constant, is*

$$\frac{dD}{dx} = \rho b U^2 \frac{d\theta}{dx}$$

- By comparing this with eq. (6.8), the momentum integral relation for flat-plate boundary layer flow is given by

$$\tau_w = \rho U^2 \frac{d\theta}{dx} \qquad (6.9)$$

- It is valid for either laminar or turbulent flat-plate flow.

Boundary Layer: Momentum Integral Estimates

- To get a numerical result for laminar flow, assuming that the velocity profiles have an approximately parabolic shape

$$u(x, y) \approx U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right) \qquad 0 \le y \le \delta(x) \qquad (6.10)$$

- which makes it possible to estimate both momentum thickness and wall shear:

$$\theta = \int_0^{\delta} \left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)\left(1 - \frac{2y}{\delta} + \frac{y^2}{\delta^2}\right) dy \approx \frac{2}{15}\delta$$

$$\tau_w = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} \approx \frac{2\mu U}{\delta} \qquad (6.11)$$

- By substituting these values into the momentum integral relation (eq. (6.9) and rearranging we obtain

$$\delta \, d\delta \approx 15 \frac{\nu}{U} dx \qquad (6.12)$$

Boundary Layer: Momentum Integral Estimates

- where $\nu = \mu / \rho$. We can integrate from 0 to x, assuming that $\delta = 0$ at $x = 0$, the leading edge

$$\frac{1}{2}\delta^2 = \frac{15\nu x}{U}$$

or

$$\frac{\delta}{x} \approx 5.5\left(\frac{\nu}{Ux}\right)^{1/2} = \frac{5.5}{\mathrm{Re}_x^{1/2}} \qquad (6.13)$$

- This is the desired thickness estimate. It is only 10 percent higher than the known accepted solution for laminar flat-plate flow (eq. (6.5)).
- We can also obtain a shear stress estimate along the plate from the above relations

$$c_f = \frac{2\tau_w}{\rho U^2} \approx \left(\frac{\frac{8}{15}}{\mathrm{Re}_x}\right)^{1/2} = \frac{0.73}{\mathrm{Re}_x^{1/2}} \qquad (6.14)$$

Boundary Layer: Momentum Integral Estimates

- This is only 10 percent higher than the known exact laminar-plate-flow solution $c_f = 0.664/Re_x^{1/2}$
- The dimensionless quantity c_f, called *the skin friction coefficient*, *is* analogous to the friction factor *f* in ducts.
- A boundary layer can be judged as "thin" if, say, the ratio δ/x *is* less than about 0.1. This occurs at $\delta/x = 0.1 = 5.0/Re_x^{1/2}$ *or at* $Re_x = 2500$.
- For Re_x less than 2500 we can estimate that boundary layer theory fails because the thick layer has a significant effect on the outer inviscid flow.
- The upper limit on Re_x for *laminar flow* is about 3 x10^6, where measurements on a smooth flat plate show that the flow undergoes transition to a turbulent boundary layer.
- From 3 x10^6 upward the turbulent Reynolds number may be arbitrarily large, and a practical limit at present is 5 x 10^{10} for oil supertankers

Boundary Layer: Momentum Integral Estimates

- For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the *friction drag coefficient*, or simply the *friction coefficient*).
- Once the average friction coefficient C_f is available, the drag (or friction) force over the surface is determined from

$$F_D = F_f = \tfrac{1}{2} C_f A \rho V^2$$

- where A is the surface area of the plate exposed to fluid flow. When both sides of a thin plate are subjected to flow, A becomes the total area of the top and bottom surfaces.

Example 2

- Are low-speed, small-scale air and water boundary layers really thin? Consider flow at *U* =1 ft/s past a flat plate 1 ft long. Compute the boundary layer thickness at the trailing edge for (a) air and (b) water at 68F.

Solution

- From Table ν_{air} = 1.61 E-4 ft^2/s. The trailing-edge Reynolds number thus is

$$\mathrm{Re}_L = \frac{UL}{\nu} = \frac{(1\ \mathrm{ft/s})(1\ \mathrm{ft})}{1.61\ \mathrm{E\text{-}4\ ft^2/s}} = 6200$$

- Since this is less than 10^6, the flow is presumed laminar, and since it is greater than 2500, the boundary layer is reasonably thin. The predicted laminar thickness is

Example 2

$$\frac{\delta}{x} = \frac{5.0}{\sqrt{6200}} = 0.0634$$

or, at $x = 1$ ft, $\delta = 0.0634 \text{ ft} \approx 0.76 \text{ in}$

- From Table $\nu_{water} = 1.08$ E-5 ft^2/s. The trailing-edge Reynolds number is

$$Re_L = \frac{(1 \text{ ft/s})(1 \text{ ft})}{1.08 \text{ E-5 ft}^2/\text{s}} \approx 92{,}600$$

- This again satisfies the laminar and thinness conditions. The boundary layer thickness is

$$\frac{\delta}{x} \approx \frac{5.0}{\sqrt{92{,}600}} = 0.0164$$

or, at $x = 1$ ft, $\delta = 0.0164 \text{ ft} \approx 0.20 \text{ in}$