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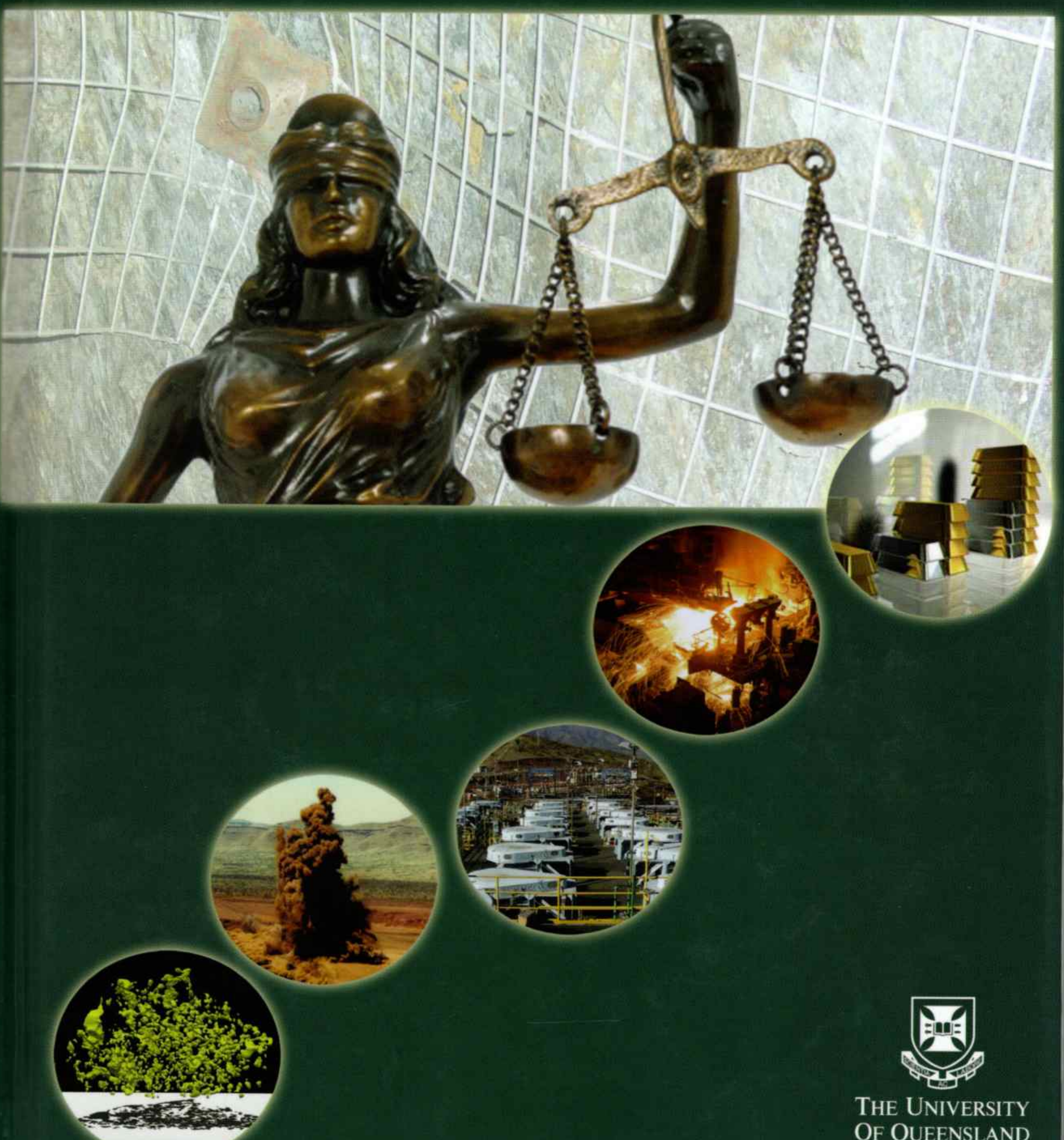
An Introduction to Metal Balancing and Reconciliation



THE UNIVERSITY
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AUSTRALIA

CHAPTER 7

DATA ANALYSIS FOR METAL BALANCING

7.1 INTRODUCTION

It is worth briefly revisiting some of the issues raised in Chapter 1 before addressing the technical issues of data analysis. This may allow us to keep the goals of the process in mind and not become overly focussed on the technical challenges.

Objectives

- We wish to achieve a *"custody transfer"* or a *"metal balance"* which is measured and calculated in a well defined and transparent manner. (See the Glossary for formal definitions.)
- Further, we wish to know (quantitatively) the quality of our balance in terms of both its *"accuracy"* and *"precision"*.
- The process must be credible to both commercial (buyers and sellers) and technical (measurement and analysis) personnel.
- The process should also contain at least some redundant data to facilitate detection of problems with data or with bias as soon as is reasonably possible and to achieve more plausible balance.

Challenges

In some cases data are few and far between (and may be risky - or even dangerous - to collect) while in other cases there will be an apparent excess of data and the main challenge is how best to use it.

Clearly a process which entails many measurements and many different people will break down from time to time. How then can we diagnose the problem quickly so that it can be fixed?

Lastly - and most importantly - what criteria should we set for acceptance of a metal balance by all parties? This of course should be the "normal course of events" in a well designed and well managed system. A system which requires expert input for every transaction is going to be very slow and contentious (and costly).

One other point is well worth noting. A "traditional" purchase contract will often contain commercial bias - usually against the seller. This is a form of risk management which almost guarantees conflict between buyer and seller. However, as a secondary processor (or a toll ore treater) accepts a much higher risk on metal content than the primary producer ("miner"), a discount on payable value based on the accuracy of measurement might provide a more transparent approach. This would also provide an incentive to improve measurement accuracy.

7.2 CHECK IN–CHECK OUT

Quoting from the Code of Practice Section 7.1 (p59).

“A satisfactory metal balance provides a convincing demonstration that:

"What comes into the operation, less any internal accumulation, plus any draw down from internal stock, is sufficiently similar, ie within acceptable limits in mass and analysed components, to what goes out as product(s), losses and rejects."

This is the definition of the Check In-Check Out system. Using the Check In-Check Out system, all streams into and out of the Process or Plant for which the balance is being performed, are measured, sampled and analysed. In terms of this Code, this is the standard procedure to be adopted for Metal Accounting in all operations, and all deviations from this approach must be justified in an Exception Report approved by company management.

The accuracy for each component measured is quantified and the resulting accuracy of the metal balance calculated and reported in order to determine whether it is within the acceptable limits, as defined for the operation by the company’s management. Applying the Check In-Check Out system will normally show a difference between net input and output, due to measurement, sampling and analytical uncertainties, and this imbalance is treated as an unaccounted gain or loss.”

We can consider a complete process (or any sub-section within it) using the Check In-Check Out (CI-CO) method.

Hence the standard equation is:

$$\text{Sum of Outputs} = \text{Sum of Inputs} \pm \text{Change in Inventory} \pm \text{Change in Stock} \qquad (7.1)$$

As all of these measurements are subject to variation, they are not expected to provide an exact numerical match. Hence we need to add a (hopefully) small term to the right hand side which is usually called an unaccounted component or a *“closure error”*. Note that this equation applies equally well to total mass and to each component of interest. If we have estimates of the probability distributions related to measurements of inputs, outputs, stocks and inventory, then we can use the methods outlined in Chapter 2 (The Propagation of

Variance and the Monte Carlo methods) to estimate the probability distributions which we can expect for the unaccounted component or closure errors.

The mean of a series of "unaccounted" components (losses or gains) should always tend to zero - else we have a bias in the system and the *"cusum"* method also outlined in Chapter 2 is appropriate. This also provides a quantitative way to define a discrepancy.

If the *"unaccounted"* component is greater than some specified number of standard deviations from the mean (which should always be zero) then we have a discrepancy for which a reason needs to be identified and the problem rectified. If the unaccounted is normally distributed then differences of more than two standard deviations are worth investigating (this should only happen by chance once out of 20 measurement periods). A difference of more than three standard deviations is highly likely to be a real discrepancy as this should only happen by chance, about once in 100 measurement periods.

Unlike mass balancing, the Check In-Check Out system is not supported by a large body of technical literature. However, an introduction to the method is provided by Wills (2007, Chapter 3).

7.3 MODELLING THE CHECK IN-CHECK OUT PROCESS

As Check In-Check Out is based on an equality, it lends itself to mathematical modelling. In practice, only a few processes can use the simplest form - basically only those where we can measure all inputs and outputs. The following table is drawn from the Guidelines (Table 7.2 p65 of release 3). This table lists seven types of Check In-Check Out strategies. These are considered in more detail in the application sections in Chapter 4 of this book.

A very simple case of Check In-Check Out is covered in Chapter 1, Section 1.7. It may be worth re-reading Sections 1.7 to 1.9 before reading the rest of this chapter.

Table 7.1: Recommended Strategies for Metal Balancing
(Code of Practice Guidelines)

Type of Operation	**Examples**
Type 1: High grade / high value feed. Medium Flow rates. High Value Product. Significant plant inventory. Weigh and sample all inputs: **Check In**. Weigh and sample all outputs: **Check Out**.	**Smelter, Metal Refinery** Recommended strategy is Check In-Check Out. (See Chapters 13 & 16)
Type 2: Low value feed - medium to high flow rates, often of coarse feed. High value product. Low mass, high value plant inventory. Weigh and sample product.	**Gold Operation** Recommended strategy is to reconstitute tailings at measured feed rate and production. (See Chapter 11)
Type 3: Low value feed - medium to high flow rates - sampled after primary grind. Low value product - sampled as concentrate shipments. Low plant inventory. Conveyor weigh-scale or weigh feeder for feed mass into plant. Accurate sampling of fine tailings, feed analyses and tailings analyses at measured feed rate and production rates.	**Base Metal Concentrator** Recommended strategy is to mass balance feed analyses, concentrate and tailings. (See Chapter 10)
Type 4: High tonnage of feed and products. Minor rejects. Low plant inventory. Sometimes direct shipment to customer. Detailed product specifications. International standards for sampling and characterisation. Custody transfer mass will often be a draught survey or a train load out weight.	**Coal Operation, Iron Ore** A commodity sales contract will detail prices and penalties as well as acceptable measurement techniques for both producer and buyer. Recommended strategy is to weigh and sample feed and product streams. (See Chapter 17)
Type 5: High value, low tonnage of feed. High value products. Minor reject streams. Significant plant inventory. Weigh and sample all inputs: **Check In** Weigh and sample all outputs: **Check Out**	**Precious Metals Refineries** Recommended strategy is Check In-Check Out. (See Chapter 16)
Type 6: Low value, high tonnage feed. Minor rejects, usually based on particle size or density. Minimal plant inventory.	**Aggregates** A commodity sales contract will detail prices, usually based on tonnage and size specification. **Recommended strategy is to weigh feed and product streams.**
Type 7: Low grade, high tonnage feed. Residues remain in situ in some cases. Very high, difficult to measure, process inventory. Non steady state, two phase slow reaction kinetics.	**Heap Leach** Weigh, sample and analyse feed and product. Perform accurate solution balance. **Periodic checks of heap solution inventories and heap residue metal contents.** (See Chapter 15)
Type 8: Medium to low value, high tonnage feed. Low process inventory. Require use of Mineralogical analysis. Products may be in bulk, bags, or other containers.	**Industrial Minerals** May be treated as a commodity. May be produced as different grades. **Weigh feed and product streams.** (See Chapter 17)

7.4 A SIMPLE EXAMPLE OF CHECK IN-CHECK OUT

We first consider a (very) simple case and then progress to a more typical case. The simple cases tend to be where all feeds and products can be accurately measured for mass and elemental content. If the custody transfer is of high value material, it is easy to justify expending effort to achieve high physical security and accurate measurement.

Let us consider a group of ten dore (gold alloyed with other metals such as silver) bars produced by a concentrator and received by a refinery. The bars are of two types - gravity dore and CIL dore. Based on experience, the latter are highly consistent in composition while the former are much less so.

In the gold room of the process plant each bar is stamped with a serial number - that is, a number in sequence. However, gravity dore and CIL dore use different sequences. Each bar is weighed and drilled to generate an assay sample. The drillings are also weighed. We can easily establish the *"standard deviation"* of the scale by repeated weighing and the accuracy by checking against a certified weight. The standard deviation of the sampling and gold assaying process can also be established by occasional repeated drilling. The weight *sd* can be a combined value while gravity and CIL dore assays can be expected to have different standard deviations.

In some cases, we may wish to carry out other assays such as silver.

For any particular shipment, we can total the mass for each kind of bar and estimate the variance (sd^2) for the total mass based on the sum of the variances for each of the bars. If the bars are about equal in mass, then the relative *sd* (or standard error) of the total mass will be the standard error for one bar divided by the square root of the number of bars.

$$StdError(Total\ Mass) = StdError(one\ dore\ bar) \,/\, \sqrt{n} \tag{7.2}$$

If each set of bars came from a single smelt (of that type), then the *sd* of the assays for each bar should be a good estimate of the *sd* of the mean value. Note that two assays will usually be carried out on each sample.

If the dore bars were produced by different process routes, then the bars from each type of process should be treated as separate sets.

Now we can calculate the total mass of gold and an associated *sd.* This is the Check Out value.

When the refinery receives the bars, it will weigh each one again and possibly sample a few bars of each type. The sample will be composited and assayed several times for a broad range of elements - to assist with the refining requirements. Based on these measurements and an agreed gold price, the refiner will make the supplier a monetary offer. If these values are in reasonable agreement with the gold room values, the supplier will accept the offer.

This becomes the Check In value at the refinery and the final Check Out value at the gold plant.

There will rarely be a disagreement based on weight but if there is an assay discrepancy, the bars will be sampled again and the samples sent to an independent (umpire) laboratory. Sometimes this is part of standard practice. Usually whichever assay (buyer or seller) is closer to the umpire assay will be accepted as the true value for the commercial transfer. This value can be used to reconcile back to the inputs and outputs of the gold plant. However, the inherent delays in this process of sale and assaying mean that short term metal balances will usually be reported on a provisional basis as detailed in Chapter 11.

This very simple example captures the essence of the Check In-Check Out method. Each party – buyer and seller – generates an estimate of the transfer of value associated with the transfer of custody and there is an agreed procedure to establish a single commercial value for this transaction. Both buyer and seller can then reconcile through their flowsheets based on that transfer of mass, components and value.

7.5 A MORE GENERAL CASE

Figure 7.1 is a generic metal accounting flowsheet for a "typical" processing plant. For this example, upper case letters are used to denote total dry flows within an accounting period and lower case letter are used for assays of the stream with the same letter. Note that the indices for feed, hold-ups and products do NOT indicate any sort of "special linkage". They are simply counters which make the equations easier to read.

Our process plant receives feed from several sources (F_1, F_2, F_3 and F_4 dry tonnes) which are stored in separate stockpiles (S_1 to S_4). They are reclaimed into the process in a ratio which tries to keep the different sections inside the plant in mass flow balance. That is, tries not to exceed maximum throughput for any products P_1, P_2 and P_3 in any section of the plant.

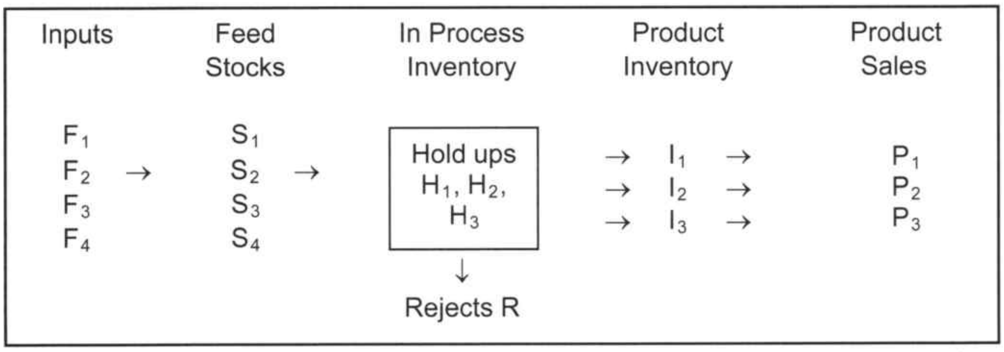

Figure 7.1: A "generic" metal accounting flowsheet for a "typical" mineral processing plant

Products are stored as product inventories (I_1, I_2, and I_3) for reclaim as product sales P_1, P_2 and P_3. In some cases, the product stockpiles may be blended to generate other saleable products.

The plant also produces a net reject stream R.

There are three components of economic interest with assays of c_1, c_2 and c_3 and two more contaminants which detract from the saleable value (c_4 and c_5) of the concentrates.

The various feeds come into the plant over a certified weighbridge and are sampled during the transfer to each stockpile.

The process plant contains a number of intermediate stockpiles which act as process hold ups H_1, H_2 and H_3 or process inventory. One further complication is that the increments of feed contain variable levels of moisture - which will dry out to some degree on the stockpiles.

This flow sheet may seem overly complex but it is not at all unusual. It might represent a gold operation selling gravity concentrate, a sulphide concentrate and a CIL or CIP dore. It could also be a mineral sands dry plant or a lead zinc concentrator producing separate lead, bulk and zinc concentrates.

Instead of considering the fraction of moisture in each stream let us consider the fraction of solids s_i in each feed stream. The values which can reasonably be expected to add up (to be self consistent) for reporting purposes are dry tonnages. If need be they can easily be converted to wet tonnages.

Then the measured (wet) mass inputs are:

$$\text{Total Mass In} = F_1/s_1 + F_2/s_2 + F_3/s_3 + F_4/s_4 \tag{7.3}$$

Each of these mass estimates is made up of a series of $_j$ net truck weights (gross - empty) hauled to each stockpile:

$$\text{That is,}\quad F_1/s_1 = \sum_j t_j \tag{7.4}$$

where $t_j = (\text{Gross}_j - \text{Tare}_j)/s_j$

Gross and Tare are the loaded and empty truck weights.

Truck Factors

A much simpler approach - and one that is in common use – is to define a "Truck Factor" which is simply multiplied by the number of loaded trucks to provide an estimate of the total (wet or dry) tonnage hauled. If this factor is based on a reasonable number of actual measurements and operating practice is consistent, it can provide a reasonable estimate – which can be checked against reclaim tonnages.

7.6 LONGER TERM BALANCES

For a typical accounting balance of four weeks or a calendar month, the in-process inventory (internal hold ups H_1, H_2 and H_3) should become very small relative to the plant throughput. This is probably not true for feed or product stocks which could be substantial.

Hence our Check In equations are:

$$\begin{aligned}\text{Check In Mass} = &\sum F_1 + \sum F_2 + \sum F_3 + \sum F_4\\ &+ S_1\ \text{Opening Stock} - S_1\ \text{Closing Stock}\\ &+ S_2\ \text{Opening Stock} - S_2\ \text{Closing Stock}\\ &+ S_3\ \text{Opening Stock} - S_3\ \text{Closing Stock}\\ &+ S_4\ \text{Opening Stock} - S_4\ \text{Closing Stock}\end{aligned} \tag{7.5}$$

where $\sum$ designates the sum of all of the measurement periods for this accounting period.

Opening Stock is the beginning of the first period and Closing Stock is at the end of the last period. This Closing Stock obviously becomes the Opening Stock for the next accounting period.

For each component of interest, we have a similar set of equations:

$$\begin{aligned}&\text{Component Check In}\\ &= \sum c_1F_1 + \sum c_1F_2 + \sum c_1F_3 + \sum c_1F_4\\ &\quad + c_{1o}S_1\ \text{Open} - c_{1c}S_1\ \text{Close}\\ &\quad + c_{1o}S_2\ \text{Open} - c_{1c}S_2\ \text{Close}\\ &\quad + c_{1o}S_3\ \text{Open} - c_{1c}S_3\ \text{Close}\\ &\quad + c_{1o}S_4\ \text{Open} - c_{1c}S_4\ \text{Close}\end{aligned} \tag{7.6}$$

and so on for each component c_2, c_3, c_4 and c_5 where the additional subscripts indicate our best estimate of the grade of the stockpile for that component. However, based on those estimates, there is no real constraint to force (or even encourage) the long term CI-CO component sums to be consistent.

As for the total mass, closing stock for each component is the opening stock for the following period. In many cases, the blended feed will be weighed and sampled on its way into the process. This does provide us with a total which can be compared with the feed material which has been delivered over a long period. If we can manage our stockpiles to be empty at the start/end of each (or at least some) measurement period, then we do have a practical constraint on stockpile contents of mass and components.

In actual practice, a running total is kept of what went in less what was withdrawn during each period for mass and components. This is less contentious for input than output. If we have some measure of metal content of the feed, that measure provides the assays to be multiplied by mass to estimate component inputs. However, unless the outputs are sampled and assayed, our estimates of the assays of the outputs are dependant on the mixing behaviour of the stockpiles.

If the inputs and outputs are managed by truck dump and front end loader reclaim, most stockpiles can be managed for minimal mixing. By way of contrast, the mixing and hang up behaviour of large conical stockpiles (which are typically reclaimed from below) is very difficult to measure and may be controlled by the profile of the stockpile. A practical strategy is to assume some proportional split between mixing and plug flow behaviour. Because of the potential for damage to the ore feeders under these stockpiles, there is usually very little enthusiasm for regular empty stockpile conditions. Hence there is plenty of scope for illusory metal holdup in such stockpiles.

Enforcing the constraint noted earlier re the combined feed and assuming the closing stock has the same grade as the last period of withdrawal will keep such problems in check.

7.7 ESTIMATING ACCURACY

We can use the rule for the *"Propagation of Variance"* - provided the relative error of each measurement is less than a few percent. This requirement may be a problem for the stockpiles - particularly the estimates of the components present in each.

Some of the following concepts may not be immediately obvious. Chapter 13 provides several examples, as do Cutler and Eksteen (2006).

For the mass Check In, we assume the measurements are independent and only the derivative of that measurement with respect to tonnage will be non-zero and its value will be unity (Equation 7.5).

Note that the Opening Stock is the Closing Stock from the previous period and is fixed. Hence, all Opening Stock derivatives are zero.

$$\frac{\partial(\text{Check In Mass})}{\partial F_1\,\text{(for a particular measurement period)}} = 1 + 0 + 0 + 0 \tag{7.7}$$

$$\frac{\partial(\text{Check In Mass})}{\partial S_1\,\text{(Opening Stock)}} = 0 + 0 + 0 + 0 \tag{7.8}$$

$$\frac{\partial(\text{Check In Mass})}{\partial S_1(\text{Closing Stock})} \begin{aligned} &= 0 + 0 + 0 + 0 \\ &\quad -1 + 0 + 0 + 0 \end{aligned} \tag{7.9}$$

Hence the total Variance of the Check In Mass Total is the accumulation of the variances of all of the input masses except the Opening Stock. For the Feeds, this process will cause a decrease in relative standard deviation. For the stockpiles changes, we will have a difference - possibly between two quite large measurements. The absolute variances will accumulate while the dividing mass gets smaller.

Hence the relative measurement error can easily become very large. Somewhat paradoxically, a useful strategy to overcome this problem is to have an empty stockpile at the beginning/end of each accounting period. The terms for that stockpile are then not present in the Check In Mass (or component) equation.

We can easily carry out a similar calculation for each component (based on equation 7.6) except that the derivative is now that of a product

$$\frac{\partial(\text{Total Check In Component 1})}{\partial c_1} \begin{aligned} &= F_1 + F_2 + F_3 + F_4 \\ &\quad S_{1c} - S_{2c} + S_{3c} - S_{4c} \end{aligned} \tag{7.10}$$

for each measurement period. Note that the summation terms survive.

As before, we need an *sd* estimate for each complete sampling and assaying process to allow us to complete our variance estimates for the variance of each of the component totals. This would be a somewhat daunting task to calculate by hand but it is not one which provides any challenge at all for a modern PC.

As for the masses, the relative errors associated with component changes in stockpile contents can be very large indeed - an "empty pile to empty pile" strategy can be valuable for detection of bias.

The Check Out equations for mass and components are very similar to the Check In equation with the addition of the tailings or residue stream.

$$\begin{aligned} \text{Mass Check Out} = {} & \sum P_1 + \sum P_2 + \sum P_3 \\ & + I_1 os - I_1 cs \\ & + I_2 os - I_2 cs \\ & + I_3 os - I_3 cs \\ & + \sum R \end{aligned} \tag{7.11}$$

It is unusual for the tailings mass flow to be measured directly but it is almost always sampled to measure the concentration of each component. Hence we will often be faced with calculating the tailings mass based on mass difference. As this stream by definition is of low commercial value, this inaccuracy is often not perceived as a problem. This is a good common sense argument but this lack of data can easily limit the accuracy of performance estimates and conceal any shortcomings in the mass measurement process.

7.8 THE TWO PRODUCT FORMULA

An alternative approach which is in common use is the two product formula. This approach is feasible when a separation process imposes some information on the composition of each stream.

Note: a process which simply achieves a mass split does NOT impose any information on the stream composition and these techniques do not work for splitters - or for separation processes which make little difference to the stream composition.

We will return to this topic in much more detail in Chapter 8. For the moment consider a three stream process with mass increments over a measurement period of F(eed), P(roduct) and R(eject) and mean compositions of f, p and r of a selected component. This Separation Process might represent a single flotation cell or a sizing screen or a complete flotation plant or even a smelter. As noted earlier, its key feature is that the product and reject have significantly different compositions from the feed.

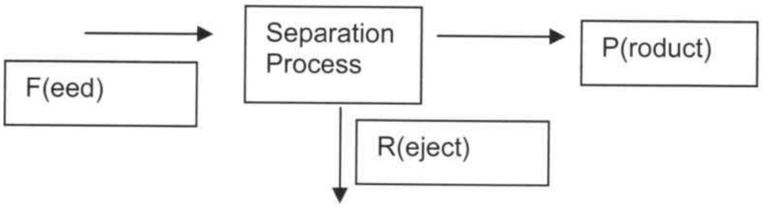

Figure 7.2: Two product separation process

If we assume the data are "perfect" we can say:

$$
\begin{aligned}
F &= P + R \text{ or } R = F - P \quad \text{Total Mass Balance} \\
Ff &= Pp + Rr \quad \text{Component Mass Balance} \\
&= Pp + (F - P)\, r \\
&= rF + P\,(p - r) \\
F\,(f - r) &= P\,(p - r) \\
P/F &= (f - r) / (p - r) \qquad (7.12)
\end{aligned}
$$

That is, we can estimate the mass split to the reject stream (or the product) based on the assays of any component which the process concentrates into a particular stream.

As usual, we can use the Propagation of Variance to estimate the variance of R if we have estimates of the variance of each assay and the Feed during our measurement interval.

However, it is worth working out as an exercise just what effect a significant change in hold up will have on this calculation and its associated variance and potential for bias. Changes in process hold ups for an accounting period need to be as small as possible (preferably < 1%) compared with the mass flow in the smallest stream to justify using this approach. In many cases, they will be sufficiently small for longer accounting periods.

There are some complications. We can easily use mass weighted assays of our feed and product assay errors if we are measuring the mass flows. Mass weighting the reject assay with the flowrate is more difficult as that flowrate is what we would like to estimate. Internal hold ups can make it very difficult to get any useful results by applying the formula to short measurement periods.

However, as what most process plants actually do is attempt to produce a constant reject we can easily check the tailings assays for variation by measurement period. If this variation is small, we can use the mean of the Reject assays, as the mass weighting will have very little effect.

Using this approach we can generate a mass split to reject which will have a much smaller calculated *sd* than our initial strategy of many differences.

The two product formula has one other particular use.

If we know the mass split to a particular product we only need to multiply it by the product grade and divide by the feed grade (in the appropriate component) to estimate the recovery of that component.

$$\begin{matrix}\text{Recovery} \\ \text{(to Reject)}\end{matrix} = \frac{(f-p)}{(r-p)} * \left(\frac{r}{f}\right) \tag{7.13}$$

or

$$\begin{matrix}\text{Recovery} \\ \text{(to Product)}\end{matrix} = \frac{(f-r)}{(p-r)} * \left(\frac{p}{f}\right) \tag{7.14}$$

(Historical Note: This formula was very useful indeed when the only measured mass in the overall system was the product.)

As before, we can use the rule of the Propagation of Variance to devise formulas to estimate the *sd*s of our calculated mass split and recovery.

The derivatives for recovery to product are:

$$\frac{\partial \mathrm{Rec}}{\partial \mathrm{f}} = \frac{\mathrm{Rec}^* \mathrm{r}}{\mathrm{f}(\mathrm{f}-\mathrm{p})} \tag{7.15}$$

$$\frac{\partial \mathrm{Rec}}{\partial \mathrm{p}} = \mathrm{Rec}\left[\frac{1}{\mathrm{p}} - \frac{1}{\mathrm{p}-\mathrm{r}}\right] \tag{7.16}$$

$$\frac{\partial \mathrm{Rec}}{\partial \mathrm{r}} = \mathrm{Rec}\left[\frac{1}{\mathrm{p}} + \frac{\mathrm{r}}{\mathrm{p}^2} - \frac{1}{\mathrm{f}} - \frac{\mathrm{f}}{\mathrm{r}^2}\right] \tag{7.17}$$

The derivatives can be used for propagation of variance. See Chapters 2 and 13 or Cutler and Eksteen (2006) which is also reproduced with the permission of The Southern African Institute of Mining and Metallurgy as Appendix G of the Code of Practice and Guidelines.

7.9 THE MULTI-COMPONENT TWO PRODUCT FORMULA

The multi-component (or assay) case is common enough to be useful and offers an introduction to the more complex methods of mass balancing which are considered in Chapter 8. The single process node which is considered here might represent a single flotation cell or a complete flotation plant – or a coal processing jig or any concentrator treating a single feed to produce a concentrate and a tailing.

Therefore, we will use somewhat more general nomenclature which can be extended to more complex cases. A, B and C are the total flows over any specified period for feed, concentrate and tailings respectively. Each stream is samples over that period and assayed for a series of "n" metals which are designated by lower case and sub-script as a_1, b_1, c_1 to a_n, b_n and c_n or a_i, b_i and c_i to represent any member by its index.

We can use the two product formula to estimate the mass split beta β to concentrate based on any of these assays although the mineral which is the target of the separation will usually have the largest differences in assay between streams and produce the best defined estimate of the mass split.

A spreadsheet is ideal for this type of analysis. However, it is a good idea to be as systematic as possible. We can set up the preliminary balance around our separator in MS Excel following the general structure shown below in Figure 7.3.

	Feed Assay	**Product Assay**	**Reject Assay**	**(ai - ci)**	**(bi-ci)**	**Beta β (ai - ci)/(bi-ci)**
	A	B	C			
Flows						
Assay i						
Assay 1	'	'	'			
Assay 2	'	'	'			
Assay 3	'	'	'			
Assay n						

Figure 7.3: A standardised structure for analysis of a two product node using a spreadsheet

However, the focussing question is: "What is the best value of Beta to use for our mass split?" If the flowrate of the concentrate stream has been measured, that will often be the best mass to use. However, that may not happen at all or only happen for the combined product of many shifts of operation.

Table 7.2: A typical spreadsheet result for a single node with two products

		Float Feed	**Lead Conc**	**Lead Tail**			
Meas	t/h	1502.00					
Rel Flow		1	Beta	1 – Beta			β for each assay
		ai	bi	ci	(ai-ci)	(bi-ci)	(ai-ci)/(bi-ci)
Meas	Au g/t	0.90	20.25	0.36	0.54	19.89	0.027
Meas	Ag g/t	8.16	323.8	1.33	6.83	322.47	0.021
Meas	Pb %	0.78	35.11	0.10	0.68	35.01	0.019
Meas	Zn %	1.27	14.96	0.83	0.44	14.13	0.031
Meas	Cu %	0.029	0.69	0.014	0.02	0.68	0.022
Meas	Fe %	5.090	7.18	4.830	0.26	2.35	0.111

The statistical ideas outlined in Chapter 2 suggest that we should seek a solution which in some way minimises the required adjustments to the measured data which would generate a set of completely self consistent "adjusted data" as defined by *"adjustments"* in the glossary.

For mass balancing, the problem can usually be split into two stages:

- finding the best fit flow rates or flow splits; and
- finding the minimum adjustments for the measured assays.

The first stage is often called the "Minimisation of Mass Flow Errors".

Each set of measured values can be written as a component balance equation, with a "mass flow error" for each measured assay or component.

$$\Delta_i = a_i - \beta b_i - (1-\beta)c_i \tag{7.18}$$

Note that only one β value is required for all of the n assays and that Δ_i should be zero for perfect data. Hence, a process which makes a large difference to the stream assays will have a better defined flow split than one which only makes a small difference such as a final cleaner or scavenger bank,

Similarly if a splitter is working well, it is essential to measure or estimate its flow split as mass balancing will not be helpful.

Transposing and simplifying we get for each component:

$$\Delta_i = (a_i - c_i) - \beta\,(b_i - c_i) \tag{7.19}$$

If we square both sides and sum up all of the components:

$$SSQ = \sum_i \Delta_i^2 = \left[(a_i - c_i) - \beta(b_i - c_i)\right]^2 \tag{7.20}$$

We might reasonably expect the value of β which minimises SSQ to be a reasonable estimate of the true value of beta (β* or beta star). As this best value is an estimator of a central tendency we can call it $\overline{\beta}$ (or beta bar). As before, we can use a quite generic spread sheet format.

Estimated Beta	Value to Adjust						
	Feed Assay	Product Assay	Reject Assay	(ai - ci)	β*(bi-ci)	Δi	Δi2
Measured	A	B	C				
Flows							
Assay i							
Assay 1	'	'	'				
Assay 2	'	'	'				
Assay 3	'	'	'				
Assay 4							
Assay n					Total	SSQ =	Target

Figure 7.4: A standardised structure for analysis of a two product node using the "Method of Mass Flow Errors"

Set up this spreadsheet as shown in Figure 7.4. Then work through it with estimates of β of 0, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2 as we know from the "average" flow splits that the mass split is about 2%.

If your set up is correct, the SSQ should go from 47.6 at 0.0 to 3382.6 at 0.2 with a minimum value of 0.19 at a beta value of about 0.02.

It is also a useful exercise to tabulate β and plot it against SSQ as shown in Figure 7.5. This graph shows a well defined minimum at β at approximately 0.01. It is well defined because making a small change (± 0.005) in β makes a ± 10% change in the SSQ. However, it is necessary to plot the log of the SSQ to be able to see the minimum.

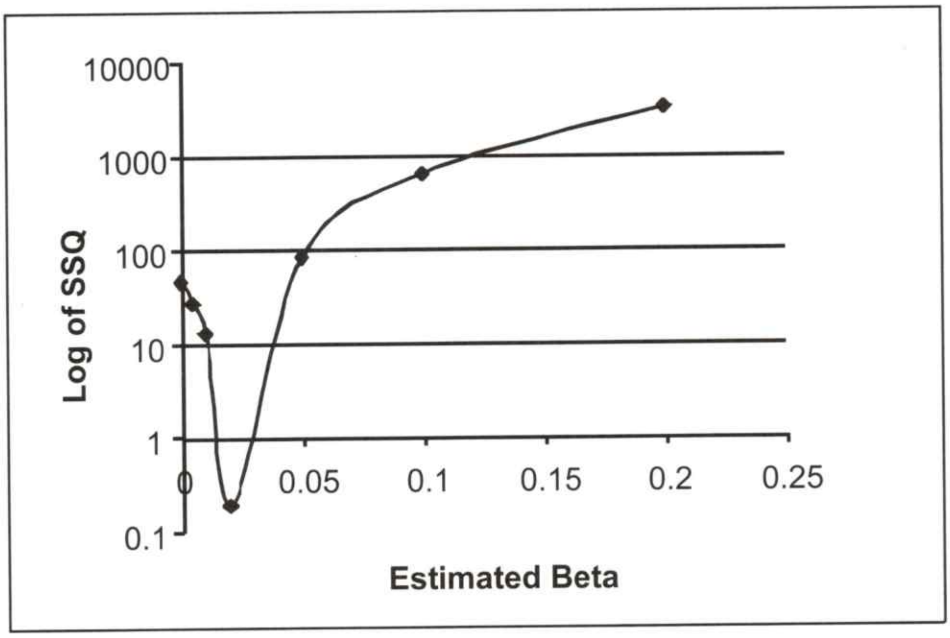

Figure 7.5: Variation in the sum of squares of mass flow errors for a range of estimates of mass split - Beta

Go through the β sequence again and watch how the SSQ or $\sum(\Delta_i)^2$ for each assay varies. They do NOT all have a minimum at the same value of β. How should we decide which value of β to use? Obviously, the value which has the most accurately measured data should dominate. Let us first consider the case where each of the minima is quite similar. Clearly the weighting will have very little effect on the value of the flow split. In this case, there is little room for debate about the value of the flow split.

Table 7.3: Typical "best fit" mass split results based on mass flow errors

Est Beta =	0.0212	A	B	C			
Stream		Float Feed	Lead Conc	Lead Tail			
Meas	t/h	1502.00	Beta	1 - Beta	Beta for		
Rel Flow		1	0.021	0.979	each assay	Δ	Δ^2
		ai	bi	ci	(ai-ci)/(bi-ci)	ai-B*bi-(1-B)*ci	
Meas	Au g/t	0.90	20.25	0.36	0.027	0.118	
Meas	Ag g/t	8.16	323.8	1.33	0.021	-0.006	0.014
Meas	Pb %	0.78	35.11	0.10	0.019	-0.062	0.000
Meas	Zn %	1.27	14.96	0.83	0.031	0.140	0.004
Meas	Cu %	0.029	0.69	0.014	0.022	0.001	0.020
Meas	Fe %	5.090	7.18	4.830	0.111	0.210	0.000
						SSQ	0.044

An Analytical Solution for Multi-component Balancing

In the days before PCs and SOLVER, a simple analytical solution was very useful. Recalling some high school calculus, if we wish to find the minimum (or maximum) of a function, we take the first derivative with respect to the variable of interest and set the result to zero.

$$SSQ = \sum_i \left[(a_i - c_i) - \beta (b_i - c_i) \right]^2$$

$$\frac{dSSQ}{d\beta} = 2\sum_i \left[(a_i - c_i) - \beta (b_i - c_i) \right] (b_i - c_i)$$

$$0 = \sum_i \left[(a_i - c_i)(b_i - c_i) - \beta (b_i - c_i)^2 \right]$$

$$\beta = \frac{\sum (a_i - c_i)(b_i - c_i)}{\sum (b_i - c_i)^2} \tag{7.21}$$

This is an interesting result. When $a_i = b_i = c_i$ it is also undefined for example, in the case of a perfect splitter. The assays with the largest difference will dominate each sum. Hence the target of the separation process (for example, lead in a lead circuit and zinc in a zinc circuit) provides the best defined balance. As for the earlier cases, this approach is easy to set up in a spread sheet. This is left to the reader as an exercise.

7.10 GENERATING NUMERICALLY "EXACT" DATA FOR FURTHER ANALYSIS

Given that we now have estimates of the flow split and its accuracy, we can consider various ways to generate an "exact" data set.

Reconstitution

If the flow split is well defined, adding the reject and concentrate together in the flow split ratio has much to recommend it. If the weighted sum estimate of the feed assays is within ± one standard deviation of the feed measurement, this realisation will be quite adequate for further analysis.

In practice, we can insert a single column to the right of the feed column and add β times the concentrate plus $(1 - \beta)$ times the tailing assay.

Note that the summation constraints are satisfied.

This strategy has the advantage that two thirds of our numbers are exactly the ones that we measured.

Table 7.4: A set of self consistent data generated by reconstitution of the feed in the ratio of the best fit mass split ratio "beta"

Est Beta =	0.0212	A	Recon	B	C
Stream		Float Feed	Float Feed	Lead Conc	Lead Tail
Meas	t/h	1502.00	1502.00	Beta	1 - Beta
Rel Flow		1	1.000	0.021	0.9788
		ai (meas)	ai	bi	ci
Recon	Au g/t	0.90	0.782	20.25	0.36
Recon	Ag g/t	8.16	8.166	323.80	1.33
Recon	Pb %	0.78	0.842	35.11	0.1
Recon	Zn %	1.27	1.130	14.96	0.83
Recon	Cu %	0.029	0.028	0.690	0.014
Recon	Fe %	5.090	4.880	7.180	4.83

Data Adjustment

Persons of a statistical bent will favour least squares adjustment of all of the assays. This does lead us back towards the general case, so it is useful as an exercise. We can start with the unit weighted case.

Recalling equation (7.18):

$$\Delta_i = a_i - \beta b_i - (1-\beta) c_i$$

If we want an exact solution, we need some strategy to apportion Δ_i across the measured assays. Strictly speaking, we should use the best fit value of beta - $\overline{\beta}$. However, as we are going to do a combined minimisation a little later, using $\overline{\beta}$ is probably confusing.

Some possible simple solutions are to divide delta by three or, to proportion delta according to the flow in each stream.

The mathematically preferred approach is to minimise the required adjustment in some way - and the sum of squares is a good general approach. Let Δa, Δb and Δc be the minimum adjustments and omit the i - as we can consider each type of assay as a separate case. Then our adjusted data must balance,

$$0 = (a-\Delta b)-\beta(b-\Delta b)-(1-\beta)(c-\Delta c) \qquad (7.22)$$

subject to the constraint that

$$\Delta = \Delta a-\beta\Delta b-(1-\beta)\Delta c \qquad (7.23)$$

and minimise

$$\text{SSQ}=(\Delta a)^2+(\Delta b)^2+(\Delta c)^2 \qquad (7.24)$$

We can use a Lagrange multiplier λ to impose the constraint by setting the constraint equation to zero adding it to the SSQ. This is called a "modified" sum of squares. We then minimise each adjustment and λ as well.

These multipliers are named after their inventor. Possibly a more instructive name would be "unknown" multipliers. We are seeking a solution where each of the first derivatives is equal to zero and the required constraints are also satisfied. Hence the multiplier is always one result from a set of simultaneous equations. The technique is very unusual in that it is easier to use than to understand intuitively. Most applied maths texts consider Lagrange multipliers – a useful reference is Boas (1966):

$$SSQ = (\Delta a)^2 + (\Delta b)^2 + (\Delta c)^2 + 2\lambda(-\Delta + \Delta a - \beta\Delta b - (1-\beta)\Delta c) \qquad (7.25)$$

Then

$$\frac{\partial SSQ}{\partial \Delta a} = 2\Delta a - 2\lambda = 0$$

$$\frac{\partial SSQ}{\partial \Delta b} = 2\overset{\checkmark}{\beta}\Delta b + 2\lambda\beta = 0$$

$$\frac{\partial SSQ}{\partial \Delta c} = 2\Delta c + 2\lambda(1-\beta) = 0$$

$$\frac{\partial SSQ}{\partial \lambda} = 2\left(-\Delta + \Delta a - \beta\Delta b - (1-\beta)\Delta c\right) = 0$$

We can drop all of the "2"s

$$\Delta a = \lambda$$
$$\Delta b = -\beta\lambda$$
$$\Delta c = -(1-\beta)\lambda$$

and substitute into the last equation:

$$\beta = \lambda + \beta^2\lambda + (1-\beta)^2\lambda$$

or $$\lambda = \Delta / \left(1+\beta^2+(1-\beta)^2\right) \tag{7.26}$$

As we already know the value of β and Δ for each assay, we can easily calculate a set of adjustments.

For this case, the adjustment turns out to be proportional to the flow of that component (assay) in each stream.

The table below shows the adjusted data and provides a check that it actually does add up. Examining the adjusted values for lead and zinc shows that in this case it would be very difficult to distinguish the likely change in the tailings assays as a result of the concentrate assays – that is – the difference imposed by the process. By way of contrast, copper, iron and silver are all changed significantly.

Table 7.5: A set of fully self consistent data generated by least squares of data adjustments in each component

Beta	0.0212	A	B	C		1.958499		
Stream		Float Feed	Lead Conc	Lead Tail				
Meas	tonnes	1502.00	0.00					
Rel Flow		1	0.021	0.979			Check	Beta
		ai	bi	ci	Delta	Lambda	Delta	Check
Adj	Au g/t	0.84	20.25	0.42	0.1183	0.06	0	0.0212
Adj	Ag g/t	8.16	323.80	1.33	-0.0064	0.00	0	0.0212
Adj	Pb %	0.81	35.11	0.07	-0.0622	-0.03	0	0.0212
Adj	Zn %	1.20	14.96	0.90	0.1404	0.07	0	0.0212
Adj	Cu %	0.03	0.69	0.01	0.0007	0.00	0	0.0212
Adj	Fe %	4.98	7.18	4.94	0.2102	0.11	0	0.0212

Checking the Balance

We can see from Table 7.5 that the required adjustments are actually quite small. However, a plot of measured against balanced assays (and flowrates where those are measured) provides a very convenient way to look at the data for self consistency.

Figure 7.6 plots measured against balanced for each type of assay. As we are using both grams per tonne and percent, logarithmic scales are useful.

This parity graph (a graph where all points are expected to follow the x = y line) suggests that there may be a problem with the iron assays but the rest are highly self consistent. The reconstituted data of Table 7.4 also suggest that there may be a problem with the iron assay in the feed – but this is speculation. We will return to the diagnostic process presently.

However, for the moment, we have a set of self consistent data which we can use to generate a shift report for the lead circuit. If a zinc circuit follows the lead circuit, we could carry out the flow split (or beta) analysis for that node as well and reconstitute back to the total feed.

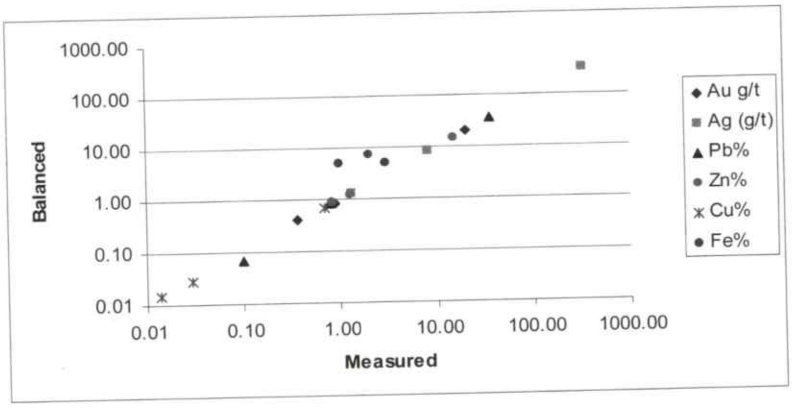

Figure 7.6: A parity chart of measured assays compared with balanced and adjusted assays

7.11 MINERAL BALANCING

When an element is present in more than one mineral, there is a reasonable theoretical basis for carrying out a mineral balance and then converting it back to elements. If the minerals have well defined elemental compositions, this strategy is worth testing. It makes essentially no difference for elements which are present in only one mineral.

Minerals

Many separation processes depend on mineral composition not elemental composition. If an element appears in only one mineral, it will not matter if elements or minerals are used. For example, lead is often only present in galena. Many other cases are not so clear cut. For example, iron can be present in chalcopyrite, pyrite, phyrrhotite and iron silicates - or in varying proportions in sphalerite.

If we have consistent assay ratios, we can convert our assays into equivalent minerals and use the mineral assays for balancing. This will often give results which are better defined and easier to understand – and are more suitable for modelling. In this case, we might consider converting the copper to chalcopyrite and the remaining iron to pyrite or phyrrhotite. However, this task is left as an exercise.

In some cases, such as several species of copper minerals or a variable Zn:Fe ratio in sphalerite, quantitative mineralogy will become essential – each elemental assay will generate a flow split estimate but they will not define mineral behaviour.

For smelting and hydrometallurgy, there is often no choice other than to use elemental values. These processes can use another technique for balancing which is rarely applicable to physical processing. If an element essentially takes no part in the key chemical reactions of a process, its flowrate does not change within the sections where it is unaffected. Hence, if we can measure its ratio to other elements (which are transformed into other phases or compounds in the process) we can estimate their flowrates by ratio to the *"tie element"*.

This strategy is also amenable to least squares or simulation balancing. Chapter 12 includes an example of hydromet balancing.

7.12 DISCUSSION

We now have two approaches to the calculation of the complete Check In and Check Out Totals across the process. We should be able to compare our Check In estimate for the period with that of our supplier(s) and our Check Out estimate with the Check In estimate(s) of our customers for this accounting period.

We also have an estimate of the expected variance for Check In-Check Out total masses and masses of components of interest.

Hence we are in a position to decide whether a difference between Check In and Check Out values is within the expected unaccounted range (and is to be expected) or large enough to be likely to be an actual discrepancy - or perhaps worse - unaccounted plus a bias.

However, to make the relationships and calculations as simple as possible, we have assumed that all of the measurements have similar absolute errors – or that the *sds* for every measurement are all equal to unity – or are at least similar to each other. For good data, this is often an adequate assumption. However, in many cases, it is essential to allow each measurement adjustment to be scaled by an estimate of its accuracy – usually its *sd*. These issues are addressed in Chapter 8.

7.13 BALANCING A MORE REALISTIC CIRCUIT

As suggested above, we can consider two of these balances in succession without a great deal more effort. The Tables below extend this style of calculation to a zinc circuit treating the tailing of the lead circuit.

The errors for each element around each node have been omitted to keep the tables as brief as possible but should generally be included to aid in identifying assay errors or transposition of assays or samples.

Table 7.6: A flow split shift balance for a lead - zinc flotation circuit

0.01090	Gamma								
0.02120	A	B	C			D	E		
Stream	Float Feed	Lead Conc	Lead Tail			Zinc Conc	Final Tail		
Tones	1502.0	Beta	1 - Beta			Gamma	1-Beta-Gamma		
Rel Flow	1	0.021	0.979	Delta	Delta Sqd	0.0109	0.9679	Delta	Delta Sqd
Measured	ai	bi	ci	Lead		di	ei	Zinc	
Au g/t	0.90	20.25	0.36	0.118	0.014	4.5	0.26	0.051631	0.002666
Ag g/t	8.16	323.8	1.33	-0.008	0.000	43.2	0.85	0.007881	6.21E-05
Pb %	0.78	35.11	0.10	-0.062	0.004	1.31	0.04	0.044875	0.002014
Zn %	1.27	14.96	0.83	0.140	0.020	52.4	0.27	-0.02049	0.00042
Cu %	0.029	0.69	0.014	0.001	0.000	0.51	0.008	0.000397	1.58E-07
Fe %	5.090	7.18	4.830	0.210	0.044	7.91	4.65	0.140624	0.019775
				SSQ	0.082			SSQ	0.024937
							Total	SSQ	0.107

A second flow split factor called "gamma" is added. Gamma is the proportion of the mass flow of new feed which becomes the mass flow of zinc concentrate. The Solver Tool of MS Excel has been used to find the minimum sum of mass flow (squared) errors. Based on the flow errors in each component around each node (Table 7.6), the measured data are quite self consistent.

As before, we can minimise the sum of squares for each assay adjustment separately with two constraints or use MS Solver to seek a full set of adjustments subject to all of the *constraints as shown in Tables 7.7 and 7.8. Table 7.8 shows the required data adjustments* for each component.

Table 7.7: Data adjustments required for each component

Est Beta = 0.021	A	B	C	D	E	
Stream	Float Feed	Lead Conc	Lead Tail	Zinc Conc	Final Tail	
Measured Tones	1502.00	Beta	1 - Beta	Gamma	1-Beta-Gamma	
Rel Flows	1	0.021	0.979	0.011	0.968	SSQs
Adj Delta	ai	bi	ci	di	ei	
Au g/t	0.098	-0.002	-0.021	-0.001	-0.074	0.01553536
Ag g/t	-0.003	0.000	0.005	0.000	-0.003	4.52748E-05
Pb %	-0.027	0.001	0.036	0.000	-0.010	0.002133592
Zn %	0.088	-0.002	-0.053	0.000	-0.033	0.011685926
Cu %	0.001	0.000	0.000	0.000	0.000	5.91441E-07
Fe %	0.191	-0.004	-0.020	-0.002	-0.165	0.064099774
			Sum of adjustments		0.00	0.093500519

Table 7.8: Shows the adjusted data and provides a zero check on each node for each element. This value is used to provide a constraint for the adjustments in MS Solver

Est Beta = 0.0211949	A	B	C	D	E		
Stream	Float Feed	Lead Conc	Lead Tail	Zinc Conc	Final Tail		
Meas tonnes	1502.00	Beta	1 - Beta	Gamma	1-Beta-Gamma		
Rel Flow	1	0.021	0.979	0.011	0.968	Check on	Check on
	ai	bi	ci	di	ei	Node Pb	Node Zn
Au g/t	0.80	20.25	0.38	4.50	0.33	0.000	0.000
Ag g/t	8.16	323.80	1.32	43.20	0.85	0.000	0.000
Pb %	0.81	35.11	0.06	1.31	0.05	0.000	0.000
Zn %	1.18	14.96	0.88	52.40	0.30	0.000	0.000
Cu %	0.03	0.69	0.01	0.51	0.01	0.000	0.000
Fe %	4.90	7.18	4.85	7.91	4.82	0.000	0.000

If we wish to report the results for this period of operation, we also need to consider the distribution of metal into tailings and products. The most important criteron will be what proportion of a particular metal in the feed has reported to the product in which it will attract the best financial return. In some cases, misplaced metal (that is, contained in the wrong concentrate) has almost no value.

For successive separation processes like a lead zinc circuit, it is usual to consider elemental recovery based on the actual feed to that circuit. Table 7.9 shows the results of our analysis of the shift data converted into two distribution "matrices". The one on the left shows metal distribution across the complete circuit and the one in the right hand two columns shows the distribution of metal in the zinc circuit based on the tailings from the lead circuit.

Table 7.9: Shows the distribution of metal over the complete circuit based on new feed and zinc circuit feed

Est Beta = 0.0211949	A	B	C	D	E	D(ZnFeed)	E(ZnFeed)
Stream	Float Feed	Lead Conc	Lead Tail	Zinc Conc	Final Tail	Zinc Conc	Final Tail
Measured tonnes	1502.00	Beta	1 - Beta	Gamma	1-Beta-Gamma		
Rel Flows	1	0.021	0.979	0.011	0.968	0.011	0.989
Bal tonnes	1502.00	31.85	1470.15	16.38	1453.77	16.38	1453.77
Recovery	ai	bi	ci	di	ei	di (Zn Feed)	ei (Zn Feed)
Au g/t	100.0%	53.5%	46.5%	6.1%	40.3%	13.2%	86.8%
Ag g/t	100.0%	84.1%	15.9%	5.8%	10.1%	36.3%	63.7%
Pb %	100.0%	92.3%	7.7%	1.8%	6.0%	22.9%	77.1%
Zn %	100.0%	26.8%	73.2%	48.4%	24.8%	66.1%	33.9%
Cu %	100.0%	51.5%	48.5%	19.6%	28.9%	40.4%	59.6%
Fe %	100.0%	3.1%	96.9%	1.8%	95.1%	1.8%	98.2%

Tables 7.8 and 7.9 provide fully self consistent data for essentially any performance measure. If we use the measured data, we can estimate different results for recovery based on measured feed. These tables also show the essential features for Shift and Daily performance data. That is, key flowrates, assays and metal recoveries across the complete circuit. In practice, Shift and Daily reports will usually include planned production, feed and concentrate grades and target recoveries for each section of the concentrator. These plan items and targets will usually be restricted to elements of economic interest – saleable and penalty elements.

Table 7.10 shows a typical shift report based on the data contained in Tables 7.8 and 7.9. Chapter 9 provides some further examples. One of the objectives of report design is that key comparative data are as close to adjacent as possible. This can be difficult when more than one or two elements are of interest.

Table 7.10: A typical shift report based on the data in Tables 7.8 and 7.9

Date	1/01/08	Shift	DAY				
Stream	Float Feed	Lead Conc	Lead Tail	Zinc Conc	Final Tail		
Dry tonnes	1502.0	31.9	1470.1	16.4	1453.8		
%Solids	97	85	35	90	68		
Plan tonnes	1500	35		15			
Au g/t	0.80	20.25	0.38	4.50	0.33		
Ag g/t	8.16	323.80	1.32	43.20	0.85		
Pb %	0.81	35.11	0.06	1.31	0.05		
Pb % Plan	0.75	35.00		1.00			
Zn %	1.18	14.96	0.88	52.40	0.30		
Zn % Plan	1.00	12.00		55.00			
Cu %	0.03	0.69	0.01	0.51	0.01		
Fe %	4.90	7.18	4.85	7.91	4.82		
Distribution	Pb Feed	Basis				Zn Feed	Basis
Stream	Float Feed	Lead Conc	Lead Tail	Zinc Conc	Final Tail	Zinc Conc	Final Tail
Dry tonnes	1502.0	31.9	1470.1	16.4	1453.8	16.4	1453.8
Mass recovery	100%	2.1%	98%	1.1%	97%	1.1%	99%
Au	100.0%	53.5%	46.5%	6.1%	40.3%	13.2%	86.8%
Ag	100.0%	84.1%	15.9%	5.8%	10.1%	36.3%	63.7%
Ag Plan		85%					
Pb	100.0%	92.3%	7.7%	1.8%	6.0%	22.9%	77.1%
Pb Plan		93%				20.0%	
Zn	100.0%	26.8%	73.2%	48.4%	24.8%	66.1%	33.9%
Zn Plan		25%				67%	
Cu	100.0%	51.5%	48.5%	19.6%	28.9%	40.4%	59.6%
Fe	100.0%	3.1%	96.9%	1.8%	95.1%	1.8%	98.2%

7.14 ACCURACY CONSIDERATIONS

In order to make sound decisions based on metal accounting results, it should be essential to have at least some indication of the credibility of the results of our analysis. Chapter 8 looks at this issue in much more detail. For a simple balance where the process makes a large difference to the assays of each stream, a simpler analysis may suffice – although a full error analysis will always provide further insight.

Tables 7.6 through 7.9 provide an "automated" balance, data adjustment and recovery calculations for a set of shift flow and assay measurements. As detailed in Chapter 2, if we have some knowledge of (or can make plausible estimates) the accuracy of our measured data, we can use the Method of Propagation of Variance or a Monte Carlo method to estimate the standard deviation of any calculated numbers.

For this data set, we can reasonably assume that the feed belt weigher has an *sd* of 0.5%, the feed assays (to each circuit) have *sd*s of 5% relative, the concentrate assays 3% relative and the tailings assays of 10% relative. The Propagation of Error calculations can become quite complicated – even for this simple case. Chapter 13 provides a detailed example. However, it is straight forward to impose some variation on the measured circuit feed rate and rerun the balance for a reasonable number of times (10 to 20). There is one trap to be careful to avoid. The Monte Carlo results will provide both means and *sd*s of each calculated number in our balances and recovery distributions. However, only the *sd* estimates can be used to estimate the *sd*s of our calculated results. As the actual error distribution of our measured data is unlikely to be symmetrical, the means from our Monte Carlo analysis can easily be biased. To be able to use these means, we would need to take multiple measurements to establish the shape of each probability distribution and then to simulate that shape to generate our synthetic variation.

Tables 7.11 and 7.12 show the results (means and *sd*s) for seven periods of synthetically varied data. Undertaking this exercise in MS Excel is quite tedious because the pseudo random numbers update every time the spreadsheet updates. There are several add-ins for MS Excel (Palisade @RISK; RiskAmp) which can greatly simplify this procedure or you can construct a VBA routine which saves the original values and replaces them with successive sets of perturbed values while accumulating the variation in selected output values. As a check on the process, the mean and *sd* of each perturbed value should also be accumulated and stored.

Note that the relative standard deviation of the flow splits is very small compared with that of the estimated flow rates of the products. The latter contain the belt weigher variation but the former do not! Hence, the important KPI of recovery of saleable components may be better defined by the Shift balance than the critically important KPI of production.

Hence there is substantial benefit in measuring the mass of concentrate produced when ever possible.

Tables 7.11: Means of perturbed data – based on *seven sets of simulated data*

		A	B	C	D	E
Stream		Float Feed	Lead Conc	Lead Tail	Zinc Conc	Final Tail
Means	tonnes	1502.40	Beta	1 - Beta	Gamma	1-Bta-Gma
Rel Flow		1.000	0.021	0.979	0.012	0.967
		1502.40	31.93	1470.47	17.70	1452.77
Means	Au g/t	0.80	20.34	0.38	4.43	0.33
Means	Ag g/t	8.19	321.76	1.38	42.57	0.88
Means	Pb %	0.80	35.07	0.06	1.31	0.04
Means	Zn %	1.19	14.93	0.90	51.70	0.28
Means	Cu %	0.03	0.70	0.01	0.50	0.01
Means	Fe %	4.92	7.24	4.87	7.85	4.84

Table 7.12 (a): Absolute standard deviations – based on seven sets of simulated data

		A	B	C	D	E
Stream		Float Feed	Lead Conc	Lead Tail	Zinc Conc	Final Tail
sd	tonnes		Beta	1 - Beta	Gamma	1-Bta-Gma
Rel Flow *sd*		0.000%	0.112%	0.112%	0.148%	0.104%
Bal Flow *sd*	tonnes	4.44	1.73	3.79	2.20	4.78
sd	Au g/t	0.024	0.427	0.024	0.152	0.022
sd	Ag g/t	0.400	9.987	0.103	0.950	0.105
sd	Pb %	0.031	0.752	0.012	0.061	0.011
sd	Zn %	0.039	0.537	0.059	0.884	0.038
sd	Cu %	0.001	0.025	0.001	0.010	0.000
sd	Fe %	0.248	0.202	0.251	0.161	0.252

Table 7.12 (b): Relative standard deviations – based on seven sets of simulated data

		A	B	C	D	E
Stream		Float Feed	Lead Conc	Lead Tail	Zinc Conc	Final Tail
Bal Flow rel *sd*	tonnes	0.30%	5.42%	0.26%	12.41%	0.33%
rel *sd*	Au g/t	2.987%	2.098%	6.276%	3.419%	6.838%
rel *sd*	Ag g/t	4.884%	3.104%	7.439%	2.232%	11.945%
rel *sd*	Pb %	3.908%	2.144%	19.646%	4.651%	26.338%
rel *sd*	Zn %	3.274%	3.595%	6.584%	1.711%	13.848%
rel *sd*	Cu %	3.470%	3.565%	5.807%	2.052%	2.543%
rel *sd*	Fe %	5.034%	2.796%	5.146%	2.052%	5.220%

If we accumulate successive sets of shift balances – assuming there are no significant biases – then the standard error (relative standard deviation) estimated in Table 7.11 should decrease by about ${}^{1}\!/\!_{\sqrt{n}}$ where n is the number of sets of shift data which are being accumulated. This gives us an estimate of the *sd*s of the sums of mass and metal entering and leaving the concentrator for comparison with CI-CO estimates.

7.15 A WORKED EXAMPLE

We are now able to use a hypothetical lead/zinc processing plant to illustrate some of the practical issues. Figure 7.7 provides the flowsheet of the imaginary operation.

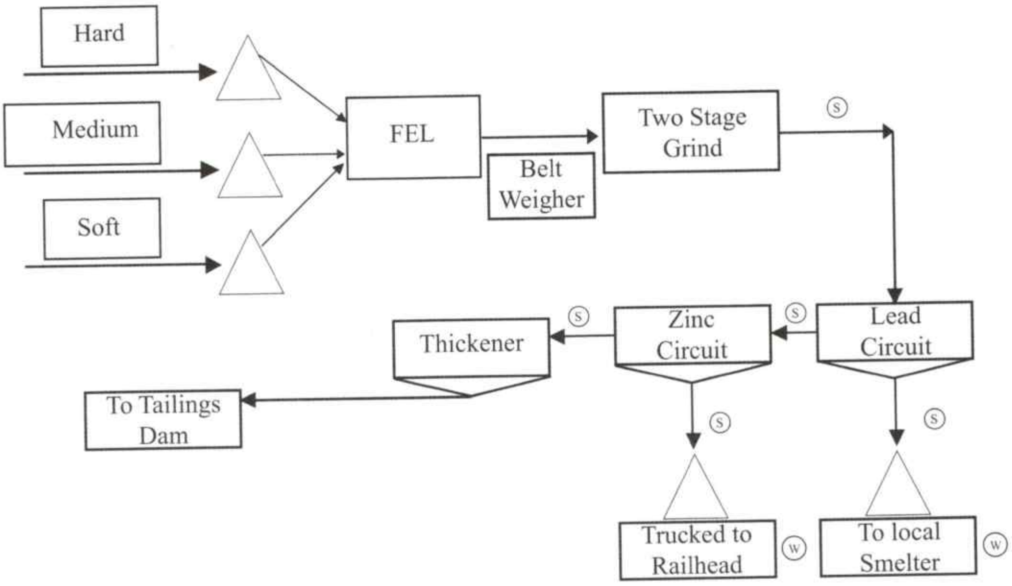

Figure 7.7: Example flow sheet

The plant receives ore from three pits (hard, medium and soft). The ore is stored in separate stockpiles. The trucks which carry the ore are check weighed several times a shift and are tared (weighed empty) near the beginning of each 12 hour shift.

A large front end loader is used to reclaim from the stockpiles to a small mill feed stockpile using a "recipe" which has the objective of blending grade and ore feed hardness to stabilise plant operation.

Mill feed is reclaimed via feeders under the stockpile to a conveyor which passes over a belt weigher on its way to the primary mill. Occasional belt cuts *("shovel tests")* are taken to provide a rough check on the belt weigher and a more precise check on the moisture and grade of each ore source. After two stages of grinding, flotation circuit feed is correctly sampled, composited across a shift and assayed for lead, zinc, iron, silver and gold. A similar sampling and assaying strategy is applied to lead concentrate, lead circuit tailings, zinc concentrate and final tailings.

The product concentrates are filtered and then stored in separate sheds where they dry to some degree. Each concentrate shed can hold up to a week of typical production.

Lead concentrate is trucked across a certified weigh bridge to a local smelter where it is reweighed and unloaded into a hopper. The concentrate tucks are weighed before unloading and tared immediately afterwards. The concentrate is reclaimed from the hopper by a conveyor, weighed by a belt weigher and correctly sampled for metal values and moisture.

Zinc concentrate is trucked across the same weigh bridge and unloaded to two large bins (silos) at a nearby railhead. Each silo can hold about 600 dry tonnes or about 20 days of production. As each train is loaded, the reclaimed concentrate is weighed by a certified belt weigher, correctly sampled and assayed for metal content and moisture. Each train carries about 1000 wet tonnes of concentrate. That is, about one month of production. At the zinc smelter, the contents of each train are reweighed and sampled for moisture and metal content.

Residence time within the concentrator is less than an hour so shift and daily balances can be carried out using feed tonnage (measured by the belt weigher integrator) and an average moisture estimate, a grade sample for monitoring and composite samples from correctly designed and installed automatic samples for flotation feed, concentrate and tailings.

The shift and daily balances are very simple to calculate. We Check In the integrated feed mass measurement less the measured or estimated moisture and multiply by the composite (measured or adjusted) metal grade for mass and metal Check In. The various mass balancing approaches detailed in Section 7.10 provide estimates of mass split and recovery to each concentrate. The strategies outlined in Section 7.11 provide estimates of the *sd*s of concentrate mass and metal contents.

Tables 7.8 and 7.9 show the essential features for Shift and Daily performance data. That is, key flowrates, assays and metal recoveries across the complete circuit. In practice, Shift and Daily reports will usually include planned production, feed and concentrate grades and target recoveries for each section of the concentrator. These plan items and targets will usually be restricted to elements of economic interest - saleable and penalty elements.

For a well instrumented, measured and sampled base metal concentrator, adding the mass and metal contents of each shift balance together is often quite adequate for daily and even weekly balances. However, as the length of the accounting period increases, the effect of variations in hold up are reduced and full Check In-Check Out balances can be undertaken. These balances should be supported by cusums based on economic elements wherever these are available from more or less independent sources.

We can still use the flotation balance as the core of our calculations. The required extensions are to sum the inputs of ore, stockpile changes, product stock changes and shipped production – exactly as described in Section 7.5 and Figure 7.1. Clearly, it is essential to estimate the stocks of feed and products at the beginning/end of each accounting period. For a concentrator like this one, those estimates (surveys) would certainly be made monthly and may even be made on a weekly basis. Some semi-automated techniques for volumetric measurement are emerging which may allow volumetric surveys to be carried out daily with minimal effort.

As has been mentioned many times in this text, an empty stockpile at the end/start of an accounting period makes any systematic bias in mass measurement much easier to detect.

For Check Out, we can add the estimated concentrate mass and metal to concentrate stocks and subtract mass and metal which have been shipped, thereby updating our estimate of stored mass and metal in concentrate.

It is generally a poor idea to try to estimate a water balance across the concentrate pile as it will vary unevenly due to drying and be very difficult to sample as a result. What we will actually get paid for is the mass of contained metal.

At the end of each week, the mill feed and concentrate stockpiles are surveyed (ie the volumes are estimated) for a weekly balance from mine to concentrate sales or concentrate shipped. We will usually also have access to the Check In figures for mass and grade measured by the customer as that will be what they would prefer to pay for.

Now we are in a position to set up a balance and analysis of longer periods of operation. These balances will become better defined as more tonnage is processed relative to internal hold ups but can be expected to show some (hopefully small) unaccounted mass and metal content. In some cases, there may also be bias in mass or metal. Detecting bias is covered in section 7.17 of this chapter.

Tables 7.13 and 7.14 show a simple and reasonably consistent structure which can be used for CI-CO accounting on a weekly or monthly basis. The entries in the tables are for 30 days of 12 hour shifts.

Table 7.13: Shows a monthly mine production report

MONTH -		MINE	REPORT								
Ore	Number of	Truck	Dry Tonnes	Pit grade	estimates			Contained	Metal		
Source	Trucks	Factor		Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
Hard	293	97	28421	0.60	7.50	0.60	1.00	17052.6	213157.5	170.526	284.21
OS-CS*			450	0.60	7.50	0.60	1.00	270	3375	2.7	4.5
Medium	415	100	41500	0.80	8.10	0.80	1.20	33287.16	336150	332	498
OS-CS			-200	0.80	8.10	0.80	1.20	-160.42	-1620	-1.6	-2.4
Soft	204	103	21012	0.90	11.00	1.00	1.30	18910.8	231132	210.12	273.156
OS-CS			-330	0.90	11.00	1.00	1.30	-297	-3630	-3.3	-4.29
Monthly				Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
Production	912		90853	0.76	8.57	0.78	1.16	69063.1	778564.5	710.4	1053.2

Table 7.14: Shows a monthly mill production report

MONTH -		MILL	REPORT								
		Process						Contained	Metal		
		Stream	Feed (dry t)	Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
		Feed	90120	0.80	8.16	0.81	1.18	72285.28	735641.6	727.23	1065.05
		Lead Conc	1911	20.25	323.80	35.11	14.96	38703.3	618807.1	670.97	285.93
		OS-CS*	240	20.25	323.80	35.11	14.96	4860.5	77712.0	84.26	35.91
		Zinc Conc	983	4.50	43.20	1.31	52.40	4424.3	42465.6	12.88	515.10
		OS-CS*	-150	4.50	43.20	1.31	52.40	-675.1	-6480.0	-1.97	-78.60
		Tailings	87226	0.33	0.85	0.05	0.30	29157.7	74368.9	43.38	264.02
Monthly				Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
Production			90210	0.848	8.944	0.897	1.133	76470.7	806873.7	809.5	1022.4

Table 7.15 shows the comparison between these two reports and considers the difference in mass and metal between mine and mill in terms of the estimated *sd*s of the accumulated concentrator shift results. At first glance, there appear to be serious problems but, as we do not know what the actual *sd*s for the mine results might be, there is no provable inconsistency between the two sets of results. It is interesting to note that the discrepancy is of the same order as the difference in the stockpiles but that is probably a matter of chance.

Table 7.15: Reconciles the monthly mine and mill production reports

		Unaccounted		Mine	less	Mill					
			Feed (dry t)	Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
			-642.9	-11.52	-2457.03	-15.41	267.48	7407.5	28309.2	99.1	-30.8
Simple	Simple	Error	Analysis								
			Feed (dry t)	Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
rel *sds*	based on	One shift	0.50%					2.987%	4.884%	3.908%	3.274%
	based on	30 shifts	0.09%					0.55%	0.89%	0.71%	0.60%
one *sd*	Mill	Mill	82.4					417.1	7194.1	5.8	6.1
		Production	Feed (dry t)					Au (g)	Ag (g)	Pb (t)	Zn (t)
Unaccounted/*sd*			-7.8					17.8	3.9	17.2	-5.0

Various ways of reconciling the "unaccounted" are considered in Chapter 8. However, if the ore resource model is considered to be "absolute", then the mined values will be reconciled across the concentrator results. More rationally, the concentrator balance would be reconciled back to the mine results. But before doing either, it is more useful to consider the product balances to customers as those results will determine the revenue of the operation. We are interested in whether the discrepancies are within the expected range of statistical variation or really are discrepancies at some level of confidence.

Table 7.16 considers a Lead Concentrate CI-CO balance. There are two measurements of mass in transit and a more credible sampling process for moisture. Therefore the smelter results for dry mass are likely to be more precise and more accurate. As before, we can compare our expected mass and metal *sd*s for concentrate production with the estimated discrepancy.

Table 7.16: Check In-Check Out balance for Lead Concentrate production

MONTH -		LEAD	CONCENTRATE			Mill	less	Smelter			
YEAR -			REPORT								
	Source							Contained	Metal		
			Feed (dry t)	Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
Shift	Lead	Conc	1911	20.25	323.80	35.11	14.96	38703	618807	671	286
Data		OS-CS*	240	20.25	323.80	35.11	14.96	4860	77712	84	36
		Total	2151	20.25	323.80	35.11	14.96	43564	696519	755	322
	Wet	% Solids									
Mill	2120	87	1844								
W/scale											
Smelter	2213	86.4	1912	20.10	321.10	35.30	15.02	38432	613953	675	287
Receipts											
Monthly	Wet			Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
Unaccounted	93		-239	21.47	345.40	33.59	14.50	-5132	-82566	-80	-35

Table 7.17 provides a simple error analysis of the lead concentrate discrepancy. In this case we do not need a t-test. The concentrator production estimates are significantly lower than the quantities for which the smelter will be prepared to pay. However, the reasonable agreement between wet shipped and wet received suggests that concentrate production is indeed lower than our estimate. If we accept that the shipped mass is lower in reality, that shortfall will account for about half of the deficit in the metal values – all of which are also low.

To deduce possible reasons for this problem, we can consider mass measurement (the feed weigher) and assays (new feed, lead concentrate and lead circuit tailings). Only a tailings grade which is less than it should be, will result in a higher estimate of both concentrate mass and contained metal.

Table 7.17: Simple Error Analysis for Lead Concentrate

Simple	Simple	Error	Analysis								
Lead Concentrate			Feed (dry t)	Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
rel *sds*	based on	one shift	5.42%					2.098%	3.104%	2.144%	3.595%
	based on	30 shifts	0.99%					0.38%	0.57%	0.39%	0.66%
one *sd*	Lead	Conc	21.3					166.8	3946.9	3.0	2.1
		(dry t)						Au (g)	Ag (g)	Pb (t)	Zn (t)
Unaccounted/*sd*			-11.2					-30.8	-20.9	-27.2	-16.4

Comparing the assays requires a bit more thought. If the flowrate estimates for each shift are a little high, that small increase will affect the metal estimate in about the same proportion and the built up assay estimates (accumulated metal divided by accumulated mass) should be less affected. In this case, the differences between the assays are not as large. Hence, while reasonable agreement between metal composition for buyer and seller will eliminate disagreements re quality and levels of bonus and penalty elements, it does not necessarily offer any useful assurance re contained metal!

Table 7.18 shows the same approach applied to the zinc concentrate. It looks superficially more self consistent. However, for the case of the rail load out silos, simply including the difference between opening and closing stock is not adequate because these silos can hold more than a month of production. Hence, we need to manage these transfers to have much chance of closing a CI-CO system in a useful manner.

Table 7.18: Check In-Check Out balance for Zinc Concentrate production
(Note: not recommended – see text)

MONTH -		ZINC	CONCENTRATE			Mill	less	Railed			
			REPORT								
	Source							Contained	Metal		
			Feed (dry t)	Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
Shift		Zinc Conc	983	4.50	43.20	1.31	52.40	4424	42466	13	515
Data		OS-CS*	-150	4.50	43.20	1.31	52.40	-675	-6480	-2	-79
		Total	833	4.50	43.20	1.31	52.40	3749	35986	11	436
	Wet	% Solids									
Mill	1125	90	1013								
W/scale											
Rail Loaded	1006	91.5	920	4.30	39.20	1.35	53.50	3958	36083	12	492
OS-CS*	225	90	203	4.30	39.20	1.35	53.50	871	7938	3	108
Wet		Dry									
Total	1231	Total	1123	4.30	39.20	1.35	53.50	4829	44021	15	601
Monthly	Wet			Au g/t	Ag g/t	Pb%	Zn%	Au (g)	Ag (g)	Pb (t)	Zn (t)
Unaccounted	-119		87	2.39	1.12	1.73	63.97	209	98	2	56

One possible strategy is to treat the rail head silos as a single "train load". This will ensure the silos are empty (or are very close to empty) after each train is loaded. This means accumulating all of the zinc concentrate production in the shed for a few days until the train is loaded and then catching up over the next few weeks. At the end of the catch up period, the silos should contain one train load and the shed should be as close as possible to empty.

The usual strategy of alternating between silos is flawed in this case as trains are usually loaded as quickly as possible with all available silos feeding the train load out system. Issues like this one are relatively easy to solve at design time and often very difficult and expensive to overcome once a system is in operation. The design strategy should include both the equipment and a plan for how it is to be operated. Hence, a "metal accounting review" can be a very beneficial part of a design review.

If we use the strategy suggested above, the CI-CO report becomes much simpler. We know exactly what period of production corresponds to each train load and the unaccounted becomes the difference. From daily results, we can assign the production and discrepancy across the monthly reporting period.

Each of the concentrate CI-CO strategies considered above has the same problem. We do not have a reliable measure of concentrate production within the concentrator. Hence, the operation is obliged to wait for up to a month until a problem can be recognised and perhaps for several more to have any confidence that the problem has been fixed. One straight forward way to reduce such problems is to include a reasonably accurate measure of concentrate production in the system. This might be a belt weigher for filter concentrate with a sampler for moisture and assays. Pressure filters which can dump several tonnes of concentrate within a few seconds are not well suited to this strategy.

More economically, a good quality flowmeter and a digital density gauge can be installed on the pumped line feeding the filters from the concentrate storage tank of thickened concentrate. Arranging for this line to be switched to recycle allows for accurate calibration of flow and density. The mass flow integrating system will need to sense whether concentrate is recycling or going to the filters. A small correction for carry over and clean up going back to the thicker will also be required. If the return stream flowrate can be manually checked and the percent solids sampled, the assumption can be checked on an occasional basis.

Either of the above strategies will provide a mass Check Out measure which is available on a shift basis. Given typical hold ups in thickening and storage, it may require a week of operation and a measure of storage tank level before the mass value provides a sufficiently precise check. These results will be invaluable for detection of bias which is covered in the next section.

For our worked example, the lead smelter results may be available in time to include in the concentrator monthly report but it is unlikely that the zinc rail out figures will be available. Hence, even monthly reports will often be provisional to some degree and will need to be updated when actual sales figures become available.

For the final version of the monthly report, we might subtract paid mass and metal from the measured feed stream to calculate the tailing stream. The magnitude of the expected losses or gains in tailings mass and metal can be considered as before but will be subject to a higher degree of relative variation because they will be the difference between two large accumulations of mass and metal.

More sophisticated approaches to reconciliation are considered in Chapter 8.

The balancing process described in this worked example meets most of the requirements for a CI-CO strategy as defined in the Code of Practice with the exception of bias detection which is considered in Section 7.17.

7.16 ACCOUNTING OVER MULTIPLE MEASUREMENT PERIODS

In practice, any operation will carry out balances over a range of time periods. These balances will often be at very different levels of detail. In some cases, the different levels of detail are necessary because a shift or daily balance is a short period compared with the residence time of product in the process plant. A good example of this is gold plant accounting which is described in Chapter 11.

Where the hold ups are reasonably short compared with the measurement period, we can either accumulate the new data and redo the balance on that basis or accumulate the balanced data. If the plant throughput and feed grade are reasonably consistent, these strategies will produce very similar answers.

If there is significant fluctuation, the results can be affected by choice of assumptions and balancing strategy. As outlined in Chapter 2, cumulative totals become steadily better defined (relatively – see equation 7.2) with regard to measurement error but they may still accumulate bias.

Hence the challenge for accounting over longer periods is not only to detect errors in measurement and analysis but also to identify bias as soon as possible. The sale of product to a third party can often assist in this process as they will rarely agree to pay for more metal than they receive – although the reverse may not be true.

7.17 AN EXAMPLE OF BIAS

Consider a case where the concentrator feed weigher in our worked example (Figure 7.6) develops a small negative bias. This is not at all uncommon as a build up of ore on the weigh frame will provide just such an error.

The equation for the CI-CO system can be modified to model bias:

$$\text{Sum of Outputs} = \text{Sum of Inputs} \pm \text{Change in Inventory} \pm \text{Change in Stock} \pm \text{Bias in measurements} \pm \text{random error} \qquad (7.17)$$

This equation will work equally well for mass or for metals. In most systems, cusums can only be applied monthly. Therefore problem detection tends to be "well after the fact" and require an even longer wait before we can be sure that a problem has been rectified or has simply gone away. If the weigh frame mentioned above gets an occasional coating of slurry and an equally occasional clean up, a monthly cusum has little chance of detecting it. A daily inspection is a much more effective strategy. The same comments apply for bias in assays where occasional composites of certified reference material may be able to flush out a bias quickly and definitively. However, cusums do provide a very useful long term checking process – particularly in precious metal accounting where the discrepancy might be due to small and regular theft.

Now that we have some idea of the error inherent in the Check In-Check Out system, we can estimate our chances for detecting a bias. (For a more detailed explanation of these issues for a smelter, see Cutler and Eksteen, 2006 which is reproduced as Appendix G of the Guidelines to the Code of Practice.)

Let us suppose an unusually "wet" batch of ore overflows the belt and sticky slurry flows over the weigh frame. This slurry will dry on initially and may cake and crack off later. Further, let us suppose that this extra load on the weigh frame adds 2% (or a little more than 2 t/hour) to our reading. At a set point of 125 dry t/h the concentrator will now only receive 123 dry t/h. This will reduce production by 2% or about two thirds of a tonne of lead concentrate per shift and 0.3 t of zinc concentrate per shift.

As our mass split estimate is based on assays only, there is absolutely no chance of detecting this problem. We must rely on the concentrate load out measurements. However, at other than empty pile conditions the estimate of surveyed concentrate volume is unlikely to be less than 5% relative. Hence, a few hundred tonnes of concentrate in either shed will provide a survey error (*sd*) which will be very effective in helping to "mask" the accumulating short fall.

If we manage the concentrate stockpile to achieve frequent "zero points" or "empty shed" points by filling one concentrate stockpile while loading trucks from the other, we have a much improved chance of detection. This means operating the concentrate shed as two smaller sheds – or from silo to silo as described earlier. After one month, ore processed will be short by some 1800 t, lead production by about 40 t and zinc concentrate by about 20 t. Based on our Monte Carlo estimates of one *sd* (Table 7.12), we should be just about able to detect those errors. However, the comparison with the smelter shows that would be a very optimistic view point, even more so, if estimated hold up contents are inaccurate. After a few months, a comparison of mass and metal between concentrator, smelter and load out should reveal a problem. Hopefully the weigh frame has long since been cleaned.

The cusum model is quite amenable to statistical analysis but the standard way to use it is to plot the cumulative sum of the difference between input and output for each metal in arbitrary units (Bartlett, 2005).

Figure 7.8 shows a weekly cusum for concentrator lead production vs smelter lead receipts with a 2% high reading on the feed to the concentrator – assuming that an empty shed has been achieved at the end of each week. The zinc cusum has only random variation – no bias – but it has much more variation.

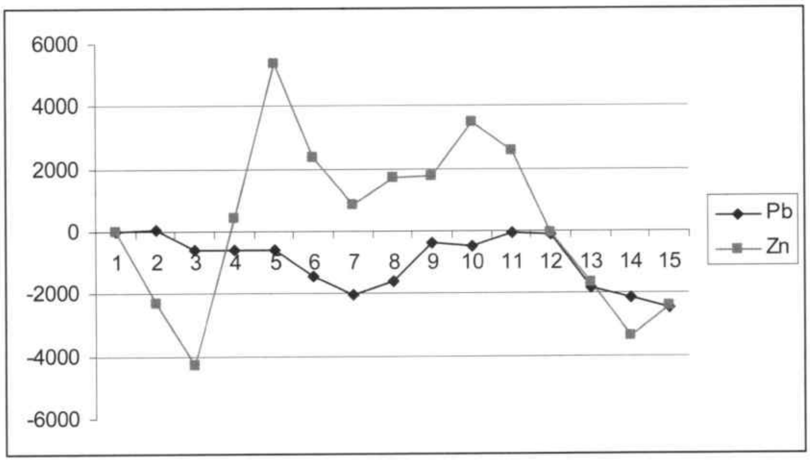

Figure 7.8: Cusums for weekly lead and zinc concentrate production. Differences between lead smelter receipts and concentrator production in arbitrary units of metal plotted against weeks of production.

In short, any capital or operating cost saved by not having appropriate mass and moisture measurement (as well as correct sampling for assay) of the product will be insignificant compared with the potential for lost production.

Given the difficulty of detection of bias in the short term, an almost standard operating strategy is to operate at a small margin of production above "plan". This provides some protection against a shortfall in metal production and cash flow.

7.18 THE "HOME AND AWAY" STRATEGY

A reasonably common approach for product transfer is called the *"Home and Away System"*. In this case, the Glossary definition provides sufficient explanation.

Home and Away System (Double Sampling and Analysis) – A system where batches/lots are weighed and sampled by both the sender and receiver. Data from the analysis of the samples can then be statistically analysed to establish the presence of biases in the sampling and/or analytical processes. In the "Half Home and Away System" samples from even numbered batches of material are analysed by the sampling sites laboratory (ie sender at sender's laboratory and receiver at receiver's laboratory) and odd numbered batches are analysed at the other laboratory (ie sender at receiver's laboratory and receiver at sender's laboratory). Statistical analysis of the data (by paired t-test) can then establish the level of bias in either the sampling or analytical procedures.

7.19 ACCOUNTABILITY

Where a process has more than one input or more than one output and the masses and grades of each are measured, the Check In and Check Out totals can be separately summed and compared.

The CODE definition of *"Accountability"* is:

Accountability – The comparison of the total output of a plant, or section of a plant, to its total input, normally expressed as a percentage.

If we have some way of estimating the standard deviation of each mass and assay for input and outputs, we can use the rule for Propagation of Variance or a Monte Carlo simulation to test accountability at various levels of confidence to establish whether a numerical discrepancy indicates a possible measurement problem or a reasonably expected statistical deviation.

A somewhat unfortunate consequence of this approach is that it leads to multiple versions of KPIs such as recovery. For example, "recovery based on inputs" may appear to be better than "recovery based on outputs". This kind of flexibility is to be avoided if possible. Where it cannot be avoided, the calculation method should be reported as well as the result.

7.20 USING ADDITIONAL DATA – PRIMARY AND SECONDARY ACCOUNTING DATA

Thus far, this chapter has focussed on using a minimum of measurement to calculate a single result. The likely *sd* of the calculated result can also be estimated. The set of data required for this process are often referred to as *"Primary Accounting"* data.

Primary Accounting – The metal balance across an entire plant as determined by Check In-Check Out.

Other measured information which might contribute to the balance is called *"Secondary Accounting"* data.

Secondary Accounting – Conducting mass balances over smaller sections of the total plant circuit, to identify where lock-ups are occurring, time lags are involved or where any measurement problems exist.

However, this distinction between data types is more likely to be associated with production of precious metals (Chapters 11 and 16) than base metals and is almost never used in commodity accounting (Chapter 17). In some cases, different divisions within an organisation take responsibility for each type of data, including its collection, analysis and reporting.

For some precious metal operations and most of those producing base metals and commodities, it is common for each sample to be assayed for multiple elements or minerals. Similarly, there may be many mass measurement points within a processing circuit.

Mathematical techniques which can utilise redundant data are also called mass balancing technologies. They are more complex than Check In-Check Out methods but the redundant data allows estimates to be made of the self consistency of each data point.

These techniques are also useful for reconciliation which is considered in detail in Chapter 8. The fundamental idea behind the use of redundant data is that combining as much of the available data as possible can substantially reduce the variance of the calculated estimates of mass flow rates with opportunities to more rapidly identify self consistent data and to pinpoint areas which have problems. This capacity for problem identification makes these methods useful for almost all types of metal accounting.

7.21 CONCLUSIONS

For some applications (as noted in Table 7.8) analysis using redundant data is the preferred method. In the author's opinion, a truly robust system will use Check In-Check Out for reportable calculations and statistical mass balancing techniques for quality assurance of Check In-Check Out values and to generate a fully self consistent data set for use as the basis of KPIs which will only result a single value based on that set of data.

The strategies described in this chapter can achieve the objectives stated in Section 7.1. Measurement and calculation are straight forward and therefore could be exposed to the buyer. We have estimates of the expected variation – although they are subject to quite a few assumptions. For the redundant data approach, the criteria for acceptance of Check In–Check Out values is that they fall within a specified confidence interval of the mass balanced results.

Bias, however, is very difficult to detect in the short term. Therefore, reasonably accurate mass measurement of concentrate production in the short term will substantially improve the accountability of the balance and reduce the risk of an undetected bias. Chapter 8 will consider how to include these measurements in the balancing process.

7.22 REFERENCES

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