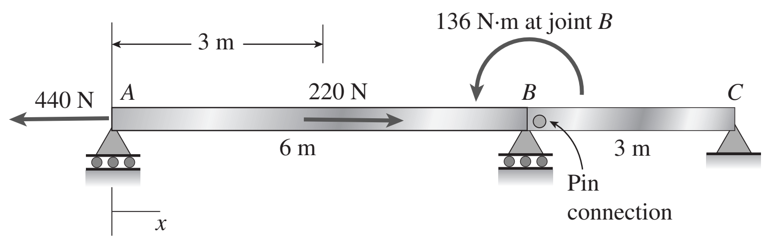
# 1

# Tension, Compression, and Shear

## Statics Review

**Problem 1.2-1** Segments $AB$ and $BC$ of beam $ABC$ are pin connected a small distance to the right of joint $B$ (see figure). Axial loads act at $A$ and at mid-span of $AB$. A concentrated moment is applied at joint $B$.

(a) Find reactions at supports $A$, $B$, and $C$.
(b) Find internal stress resultants $N$, $V$, and $M$ at $x = 4.5$ m.




### Solution 1.2-1

(a) Apply laws of statics

$\Sigma F_x = 0 \qquad C_x = N_A - N_2 = 220\text{ N}$

FBD of $BC$: $\quad \Sigma M_B = 0 \qquad C_y = \dfrac{1}{L_1}(0) = 0$

Entire FBD: $\quad \Sigma M_A = 0 \qquad B_y = \dfrac{1}{L_2}(-M_B) = -22.667\text{ N}$

$\Sigma F_y = 0 \qquad A_y = -B_y = 22.667\text{ N}$

Reactions are $\boxed{A_y = 22.7\text{ N}}$ $\boxed{B_y = -22.7\text{ N}}$ $\boxed{C_x = 220\text{ N}}$ $\boxed{C_y = 0}$

(b) Internal stress resultants $N$, $V$, and $M$ at $x = 4.5$ m

Use FBD of segment from $A$ to $x = 4.5$ m.

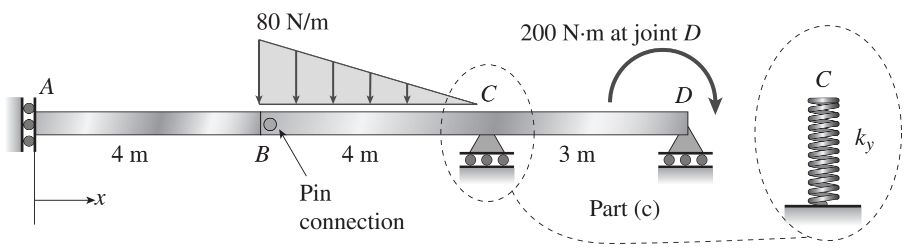
$\Sigma F_x = 0 \qquad \boxed{N_x = N_A - N_2 = 220\text{ N}}$

$\Sigma F_y = 0 \qquad \boxed{V_x = A_y = 22.7\text{ N}}$

$\Sigma M = 0 \qquad \boxed{M_x = A_y(4.5\text{ m}) = 102\text{ N}\cdot\text{m}}$

**Problem 1.2-2** Segments $AB$ and $BCD$ of beam $ABCD$ are pin connected at $x = 4$ m. The beam is supported by a sliding support at $A$ and roller supports at $C$ and $D$ (see figure). A triangularly distributed load with peak intensity of 80 N/m acts on $BC$. A concentrated moment is applied at joint $B$.

(a) Find reactions at supports $A$, $C$, and $D$.
(b) Find internal stress resultants $N$, $V$, and $M$ at $x = 5$ m.
(c) Repeat parts (a) and (b) for the case of the roller support at $C$ replaced by a linear spring of stiffness $k_y = 200$ kN/m.




## Solution 1.2-2

(a) Apply laws of statics

$\Sigma F_x = 0 \qquad A_x = 0$

FBD of $AB$: $\quad \Sigma M_B = 0 \qquad M_A = 0$

Entire FBD: $\quad \Sigma M_C = 0 \qquad D_y = \dfrac{1}{3\text{ m}}\left[200\text{ N}\cdot\text{m} - \dfrac{1}{2}(80\text{ N/m})\,4\text{ m}\left(\dfrac{2}{3}\right)4\text{ m}\right] = -75.556\text{ N}$

$\Sigma F_y = 0 \qquad C_y = \dfrac{1}{2}(80\text{ N/m})\,4\text{ m} - D_y = 235.556\text{ N}$

Reactions are $\boxed{M_A = 0}$ $\quad A_x = 0 \quad$ $\boxed{C_y = 236\text{ N}}$ $\quad$ $\boxed{D_y = -75.6\text{ N}}$

(b) Internal stress resultants $N$, $V$, and $M$ at $x = 5$ m

Use FBD of segment from $A$ to $x = 5$ m; ordinate on triangular load at $x = 5$ m is $\dfrac{3}{4}(80\text{ N/m}) = 60\text{ N/m}$.

$\Sigma F_x = 0 \qquad N_x = -A_x = 0$

$\Sigma F_y = 0 \qquad V = \dfrac{-1}{2}[(80\text{ N/m} + 60\text{ N/m})\,1\text{ m}] = -70\text{ N} \qquad \boxed{V = -70\text{ N}} \qquad$ Upward

$\Sigma M = 0 \qquad M = -M_A - \dfrac{1}{2}(80\text{ N/m})\,1\text{ m}\left(\dfrac{2}{3}1\text{ m}\right) - \dfrac{1}{2}(60\text{ N/m})\,1\text{ m}\left(\dfrac{1}{3}1\text{ m}\right) = -36.667\text{ N}\cdot\text{m}$

(break trapezoidal load into two triangular loads in moment expression)

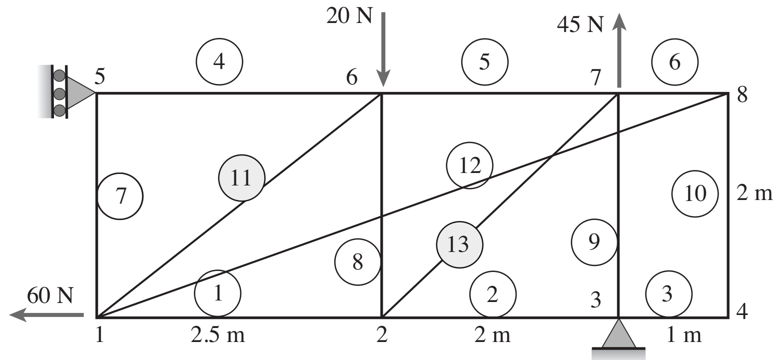
$\boxed{M = -36.7\text{ N}\cdot\text{m}}$ CW

(c) Replace roller support at $C$ with spring support

Structure remains statically determinate so all results above in (a) and (b) are unchanged.

**Problem 1.2-3** Consider the plane truss with a pin support at joint 3 and a vertical roller support at joint 5 (see figure).

(a) Find reactions at support joints 3 and 5.
(b) Find axial forces in truss members 11 and 13.




## Solution 1.2-3

(a) Statics

$\Sigma F_y = 0 \qquad R_{3y} = 20\text{ N} - 45\text{ N} = -25\text{ N}$

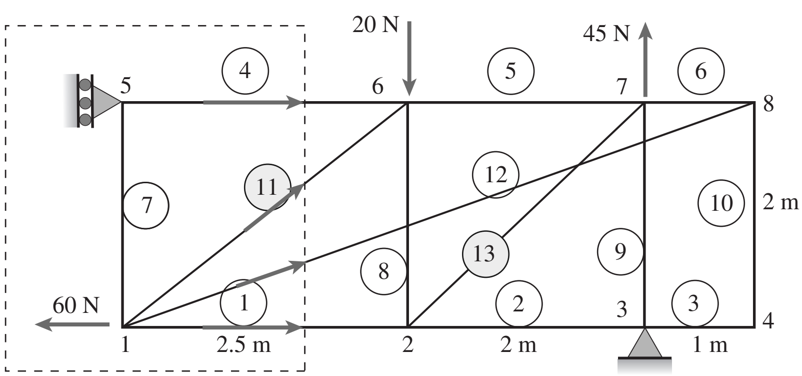
$\Sigma M_3 = 0 \qquad R_{5x} = \dfrac{1}{2\text{ m}}(20\text{ N} \times 2\text{ m}) = 20\text{ N}$

$\Sigma F_x = 0 \qquad R_{3x} = -R_{5x} + 60\text{ N} = 40\text{ N}$

(b) Member forces in members 11 and 13

Number of unknowns: $m = 13 \qquad r = 3 \qquad m + r = 16$

Number of equations: $j = 8 \qquad 2j = 16$ so statically determinate




Section cut for left FBD

Truss analysis

(1) $\Sigma F_V = 0$ at joint 4 so $F_{10} = 0$

(2) $\Sigma F_V = 0$ at joint 8 so $F_{12} = 0$

(3) $\Sigma F_H = 0$ at joint 5 so $F_4 = -R_{5x} = -20\text{ N}$

(4) Cut vertically through 4, 11, 12, and 1; use left FBD; sum moments about joint 2:

$F_{11V} = \dfrac{1}{2.5\text{ m}}(R_{5x} - F_4)$ so $\boxed{F_{11} = 0}$

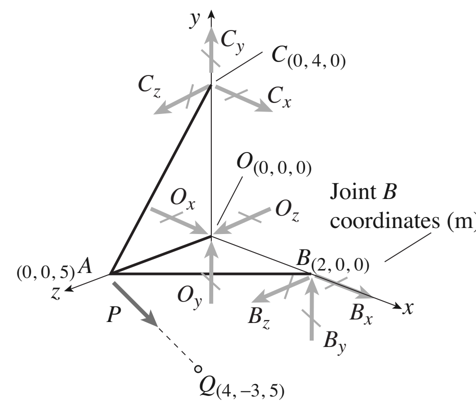
(5) Sum vertical forces at joint 3; $F_9 = R_{3y}$

$F_9 = 25\text{ N}$

(6) Sum vertical forces at joint 7: $\quad F_{13V} = 45\text{ N} - F_9 = 20\text{ N} \quad \boxed{F_{13} = \sqrt{2}F_{13V} = 28.3\text{ N}}$

**Problem 1.2-4** A space truss has three-dimensional pin supports at joints $O$, $B$, and $C$. Load $P$ is applied at joint $A$ and acts toward point $Q$. Coordinates of all joints are given in meters (see figure).

(a) Find reaction force components $B_x$, $B_z$, and $O_z$.
(b) Find the axial force in truss member $AC$.




**Solution 1.2-4**

(a) Find reactions using statics $\quad m = 3 \quad r = 9 \quad m + r = 12 \quad j = 4 \quad 3j = 12$

$m + r = 3j$ so truss is statically determinate

$$r_{AQ} = \begin{pmatrix} 4 \\ -3 \\ 0 \end{pmatrix} \quad r_{OA} = \begin{pmatrix} 0 \\ 0 \\ 5 \end{pmatrix} \quad e_{AQ} = \frac{r_{AQ}}{|r_{AQ}|} = \begin{pmatrix} 0.8 \\ -0.6 \\ 0 \end{pmatrix} \quad P_A = P e_{AQ} = \begin{pmatrix} 0.8P \\ -0.6P \\ 0 \end{pmatrix} \quad r_{OC} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix} \quad r_{OB} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$$

$\Sigma M = 0$

$$M_O = r_{OA} \times P_A + r_{OC} \times \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} + r_{OB} \times \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} 4C_z + 3.0P \\ 4.0P - 2B_z \\ 2B_y - 4C_x \end{pmatrix} \quad \text{so} \quad \Sigma M_x = 0 \text{ gives } C_z = \frac{-3}{4}P$$

$$\Sigma M_y = 0 \text{ gives } \boxed{B_z = 2\,P}$$

$\Sigma F = 0$

$$R_O = P_A + \begin{pmatrix} O_x \\ O_y \\ O_z \end{pmatrix} + \begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} + \begin{pmatrix} C_x \\ C_y \\ C_z \end{pmatrix} = \begin{pmatrix} B_x + C_x + O_x + 0.8P \\ B_y + C_y + O_y + -0.6P \\ O_z + \dfrac{5P}{4} \end{pmatrix} \quad \text{so} \quad \Sigma M_z = 0 \text{ gives } \boxed{O_z = \frac{-5}{4}P}$$

Method of joints $\quad$ Joint $O \quad \Sigma F_x = 0 \quad O_x = 0 \quad \Sigma F_y = 0 \quad O_y = 0$

Joint $B \quad \Sigma F_y = 0 \quad B_y = 0$

Joint $C \quad \Sigma F_x = 0 \quad C_x = 0$

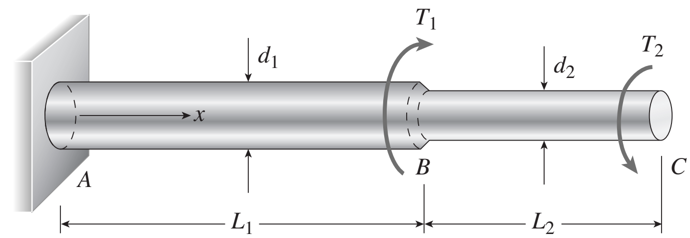
For entire structure: $\Sigma F_x = 0$ gives $\boxed{B_x = -0.8P}$ $\quad \Sigma F_y = 0 \quad C_y = 0.6P - B_y = O_y \quad C_y = 0.6P$

(b) Force in member $AC$

$$\Sigma F_z = 0 \text{ at joint } C: \quad F_{AC} = \frac{\sqrt{4^2 + 5^2}}{5}|C_z| = \frac{3\sqrt{41}\,|P|}{20} \quad \boxed{F_{AC} = \frac{3\sqrt{41}}{20}P} \text{ tension} \quad \frac{3\sqrt{41}}{20} = 0.96$$

**Problem 1.2-5** A stepped shaft $ABC$ consisting of two solid, circular segments is subjected to torques $T_1$ and $T_2$ acting in opposite directions, as shown in the figure. The larger segment of the shaft has a diameter of $d_1 = 58$ mm and a length $L_1 = 0.75$ m; the smaller segment has a diameter $d_2 = 44$ mm and a length $L_2 = 0.5$ m. The torques are $T_1 = 2400$ N·m and $T_2 = 1130$ N·m.

(a) Find reaction torque $T_A$ at support $A$.
(b) Find the internal torque $T(x)$ at two locations: $x = L_1/2$ and $x = L_1 + L_2/2$. Show these internal torques on properly drawn free-body diagrams (FBDs).




## Solution 1.2-5

(a) Apply laws of statics

$\Sigma M_x = 0$ $\boxed{T_A = T_1 - T_2 = 1270 \text{ N·m}}$

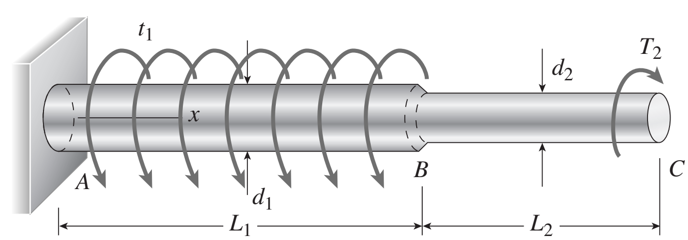
(b) Internal stress resultant $T$ at two locations

Cut shaft at midpoint between $A$ and $B$ at $x = L_1/2$ (use left FBD). $\Sigma M_x = 0$ $\boxed{T_{AB} = -T_A = -1270 \text{ N·m}}$

Cut shaft at midpoint between $B$ and $C$ at $x = L_1 + L_2/2$ (use right FBD). $\Sigma M_x = 0$ $\boxed{T_{BC} = T_2 = 1130 \text{ N·m}}$

**Problem 1.2-6** A stepped shaft $ABC$ consisting of two solid, circular segments is subjected to uniformly distributed torque $t_1$ acting over segment 1 and concentrated torque $T_2$ applied at $C$, as shown in the figure. Segment 1 of the shaft has a diameter of $d_1 = 57$ mm and length of $L_1 = 0.75$ m; segment 2 has a diameter $d_2 = 44$ mm and length $L_2 = 0.5$ m. Torque intensity $t_1 = 3100$ N·m/m and $T_2 = 1100$ N·m.

(a) Find reaction torque $T_A$ at support $A$.
(b) Find the internal torque $T(x)$ at two locations: $x = L_1/2$ and $x = L_1 + L_2/2$. Show these internal torques on properly drawn free-body diagrams (FBDs).




## Solution 1.2-6

(a) Reaction torque at $A$ $L_1 = 0.75$ m $L_2 = 0.75$ m $t_1 = 3100$ N·m/m $T_2 = 1100$ N·m

Statics: $\Sigma M_x = 0$ $T_A = -t_1 L_1 + T_2 = -1225$ N·m $\boxed{T_A = -1225 \text{ N·m}}$

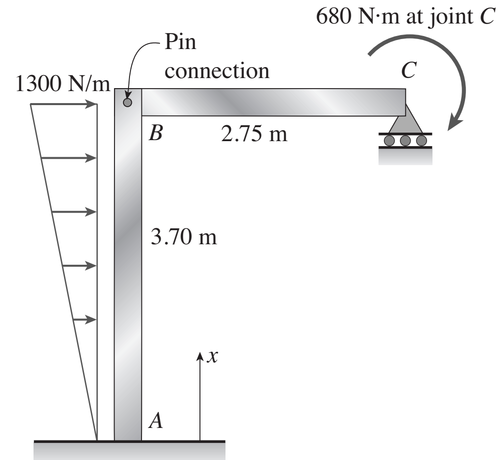
(b) Internal torsional moments at two locations

Cut shaft between $A$ and $B$ (use left FBD). $T_1(x) = -T_A - t_1 x$ $\boxed{T_1\left(\frac{L_1}{2}\right) = 62.5 \text{ N·m}}$

Cut shaft between $B$ and $C$ (use left FBD). $T_2(x) = -T_A - t_1 L_1$ $\boxed{T_2\left(L_1 + \frac{L_2}{2}\right) = -1100 \text{ N·m}}$

**Problem 1.2-7** A plane frame is restrained at joints $A$ and $C$, as shown in the figure. Members $AB$ and $BC$ are pin connected at $B$. A triangularly distributed lateral load with a peak intensity of 1300 N/m acts on $AB$. A concentrated moment is applied at joint $C$.

(a) Find reactions at supports $A$ and $C$.
(b) Find internal stress resultants $N$, $V$, and $M$ at $x = 1.0$ m on column $AB$.




## Solution 1.2-7

(a) Statics

$$\Sigma F_H = 0 \qquad A_x = \frac{-1}{2}\left(1300\ \frac{\text{N}}{\text{m}}\right)3.70\ \text{m} = -2405\ \text{N}$$

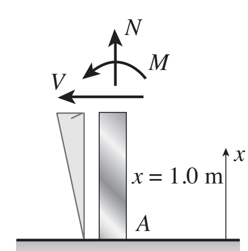
$$\Sigma F_V = 0 \qquad A_y + C_y = 0$$

$$\Sigma M_{FBDBC} = 0 \qquad C_y = \frac{680\ \text{N}\cdot\text{m}}{2.75\ \text{m}} = 247\ \text{N} \qquad A_y = -C_y = -247\ \text{N}$$

$$\Sigma M_A = 0 \qquad M_A = 680\ \text{N}\cdot\text{m} + \frac{1}{2}\left(1300\ \frac{\text{N}}{\text{m}}\right)3.70\ \text{m}\left(\frac{2}{3}\,3.70\ \text{m}\right) - C_y 2.75\ \text{m} = 5932\ \text{N}\cdot\text{m}$$

$$A_x = -2405\ \text{N} \qquad A_y = -247\ \text{N} \qquad M_A = 5932\ \text{N}\cdot\text{m} \qquad C_y = 247\ \text{N} \qquad \leftarrow$$

(b) Internal stress resultants




$$N_x = -A_y = 247\ \text{N}$$

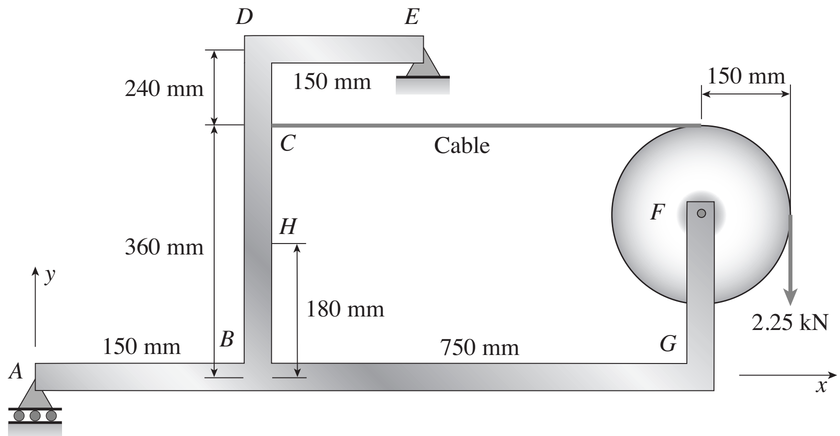
$$V_x = -A_x - \frac{1}{2}\left(\frac{1.0}{3.70}\,1300\ \frac{\text{N}}{\text{m}}\right)1.0\ \text{m} = 2229\ \text{N}$$

$$M_x = -M_A - A_x 1.0\ \text{m} - \frac{1}{2}\left(\frac{1.0}{3.70}\,1300\ \frac{\text{N}}{\text{m}}\right)1.0\ \text{m}\left(\frac{1}{3}\,1.0\ \text{m}\right) = -3586\ \text{N}\cdot\text{m}$$

$$N_x = 247\ \text{N} \qquad V_x = 2229\ \text{N} \qquad M_x = -3586\ \text{N}\cdot\text{m} \qquad \leftarrow$$

**Problem 1.2-8** A plane frame with pin supports at $A$ and $E$ has a cable attached at $C$, which runs over a frictionless pulley at $F$ (see figure). The cable force is known to be 2.25 kN.

(a) Find reactions at supports $A$ and $E$.
(b) Find internal stress resultants, $N$, $V$, and $M$ at point $H$.




## Solution 1.2-8

(a) Statics

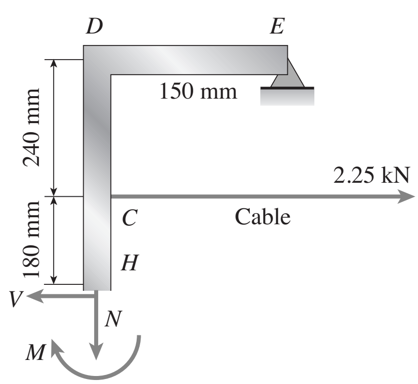
$\Sigma F_x = 0 \qquad E_x = 0$

$\Sigma M_E = 0 \qquad A_y = \dfrac{1}{300\ \text{mm}}[-2250\ \text{N}(750\ \text{mm})] = -5625\ \text{N}$

$\Sigma F_y = 0 \qquad E_y = 2250\ \text{N} - A_y = 7875\ \text{N}$

$A_y = -5625\ \text{N} \qquad E_x = 0 \qquad E_y = 7875\ \text{N} \quad \leftarrow$

(b) Use upper (see figure below) or lower FBD to find stress resultants $N$, $V$, and $M$ at $H$




$\Sigma F_x = 0 \qquad V_x = E_x + 2250\ \text{N} = 2250\ \text{N}$

$\Sigma F_y = 0 \qquad N_x = E_y = 7875\ \text{N}$

$\Sigma M_H = 0$

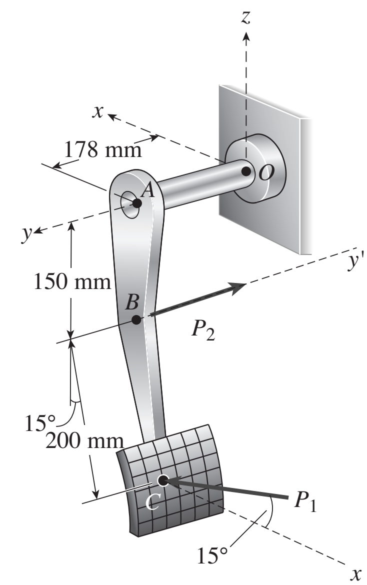
$M_H = -(180\ \text{mm})(2250\ \text{N}) - E_x(180\ \text{mm} + 240\ \text{mm}) + E_y(150\ \text{mm})$

$= 776\ \text{N}\cdot\text{m}$

$N_x = 7875\ \text{N} \qquad V_x = 2250\ \text{N} \qquad M_x = 776\ \text{N}\cdot\text{m} \quad \leftarrow$

**Problem 1.2-9** A special vehicle brake is clamped at $O$, (when the brake force $P_1$ is applied—see figure). Force $P_1 = 220$ N and lies in a plane which is parallel to the $xz$ plane and is applied at $C$ normal to line $BC$. Force $P_2 = 180$ N and is applied at $B$ in the $-y$ direction.

(a) Find reactions at support $O$.
(b) Find internal stress resultants $N$, $V$, $T$, and $M$ at the midpoint of segment $OA$.




## Solution 1.2-9

(a) Statics $\quad P_1 = 220$ N $\quad P_2 = 180$ N

$\Sigma F_x = 0 \qquad O_x = -P_1\cos(15°) = -212.5\text{ N} \qquad \Sigma F_y = 0 \qquad O_y = P_2 = 180\text{ N}$

$\Sigma F_z = 0 \qquad O_z = P_1\sin(15°) = 56.94\text{ N}$

$\Sigma M_x = 0 \qquad M_{Ox} = P_2\,150\text{ mm} + P_1\sin(15°)(178\text{ mm}) = 37.1\text{ N·m}$

$\Sigma M_y = 0 \qquad M_{Oy} = P_1\sin(15°)(200\text{ mm}\sin(15°)) + P_1\cos(15°)(150\text{ mm} + 200\text{ mm}\cos(15°))$

$M_{Oy} = 75.9\text{ N·m}$

$\Sigma M_z = 0 \qquad M_{Oz} = -P_1\cos(15°)(178\text{ mm}) = -37.8\text{ N·m}$

$O_x = -213\text{ N} \qquad O_y = 180\text{ N} \qquad O_z = 56.9\text{ N}$

$M_{Ox} = 37.1\text{ N·m} \qquad M_{Oy} = 75.9\text{ N·m} \qquad M_{Oz} = -37.8\text{ N·m} \qquad \leftarrow$

(b) Internal stress resultants at midpoint of $OA$

$N_x = -O_y = -180\text{ N}$

$V_x = -O_x = 212.5\text{ N} \qquad V_z = -O_z = -56.9\text{ N} \qquad V_{res} = \sqrt{V_x^2 + V_z^2} = 220\text{ N}$

$T_x = -M_{Oy} = -75.9\text{ N·m}$

$M_x = -M_{Ox} = -37.1\text{ N·m} \qquad M_z = -M_{Oz} = 37.8\text{ N·m} \qquad M_{res} = \sqrt{M_x^2 + M_z^2} = 53\text{ N·m}$

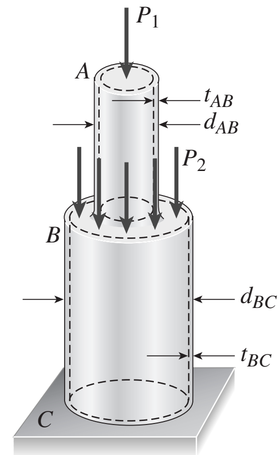
$N_x = -180\text{ N} \qquad V_{res} = 220\text{ N}$

$T_x = -75.9\text{ N·m} \qquad M_{res} = 53\text{ N·m} \qquad \leftarrow$

## Normal Stress and Strain

**Problem 1.3-1** A hollow circular post $ABC$ (see figure) supports a load $p_1 = 7.5$ KN acting at the top. A second load $P_2$ is uniformly distributed around the cap plate at $B$. The diameters and thicknesses of the upper and lower parts of the post are $d_{AB} = 32$ mm, $t_{AB} = 12$ mm, $d_{BC} = 57$ mm, and $t_{BC} = 9$ mm, respectively.

(a) Calculate the normal stress $\sigma_{AB}$ in the upper part of the post.
(b) If it is desired that the lower part of the post have the same compressive stress as the upper part, what should be the magnitude of the load $P_2$?
(c) If $P_1$ remains at 7.5 kN and $P_2$ is now set at 10 kN, what new thickness of $BC$ will result in the same compressive stress in both parts?




### Solution 1.3-1

PART (a)

$P_1 = 75$ kN $\quad d_{AB} = 32$ mm $\quad t_{AB} = 12$ mm

$d_{BC} = 57$ mm $\quad t_{BC} = 9$ mm

$$A_{AB} = \frac{\pi[\,d_{AB}{}^2 - (d_{AB} - 2\,t_{AB})^2]}{4}$$

$$A_{AB} = 7.54 \times 10^{-4}\ \text{m}^2 \qquad \sigma_{AB} = \frac{P_1}{A_{AB}}$$

$$\sigma_{AB} = 9.95\ \text{MPa} \quad \longleftarrow$$

PART (b)

$$A_{BC} = \frac{\pi[d_{BC}{}^2 - (d_{BC} - 2t_{BC})^2]}{4}$$

$$A_{BC} = 1.357 \times 10^{-3}\ \text{m}^2 \qquad P_2 = \sigma_{AB}A_{BC} - P_1$$

$$P_2 = 6\ \text{kN} \quad \longleftarrow$$

CHECK: $\dfrac{P_1 + P_2}{A_{BC}} = 9.947 \times 10^6$ Pa

PART (c)

$$P_2 = 10\ \text{kN} \qquad \frac{P_1 + P_2}{\sigma_{AB}} = A_{BC}$$

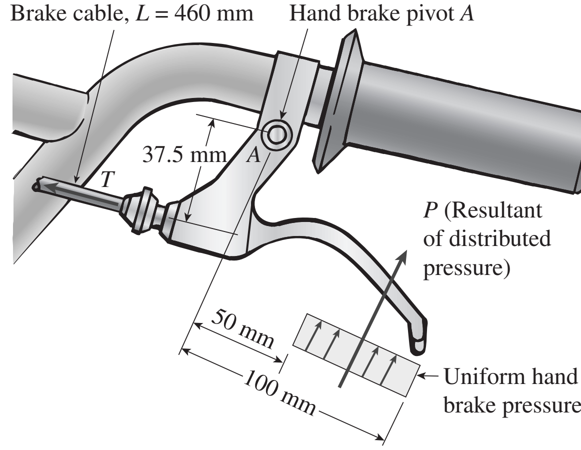
$$(d_{BC} - 2t_{BC})^2 = d_{BC}{}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)$$

$$d_{BC} = 2t_{BC} = \sqrt{d_{BC}^2 - \frac{4}{\pi}\left(\frac{P_1 + P_2}{\sigma_{AB}}\right)}$$

$$t_{BC} = \frac{d_{BC} - \sqrt{d_{BC}^2 - \dfrac{4}{\pi}\left(\dfrac{P_1 + P_2}{\sigma_{AB}}\right)}}{2}$$

$$t_{BC} = 12.62\ \text{mm} \quad \longleftarrow$$

**Problem 1.3-2** A force $P$ of 70 N is applied by a rider to the front hand brake of a bicycle ($P$ is the resultant of an evenly distributed pressure). As the hand brake pivots at $A$, a tension $T$ develops in the 460-mm long brake cable ($A_e = 1.075\ \text{mm}^2$) which elongates by $\delta = 0.214$ mm. Find normal stress $\sigma$ and strain $\varepsilon$ in the brake cable.




## Solution 1.3-2

$P = 70$ N $\qquad A_e = 1.075\ \text{mm}^2$

$L = 460$ mm $\qquad \delta = 0.214$ mm

Statics: sum moments about $A$ to get $T = 2P$

$$\sigma = \frac{T}{A_e} \qquad \sigma = 103.2\ \text{MPa} \qquad \leftarrow$$

$$\varepsilon = \frac{\delta}{L} \qquad \varepsilon = 4.65 \times 10^{-4} \qquad \leftarrow$$

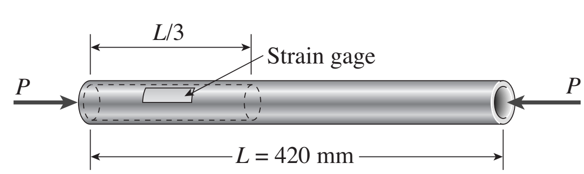
$$E = \frac{\sigma}{\varepsilon} = 1.4 \times 10^5\ \text{MPa}$$

**NOTE**: ($E$ for cables is approximately 140 GPa.)

---

**Problem 1.3-3** A circular aluminum tube of length $L = 420$ mm is loaded in compression by forces $P$ (see figure). The hollow segment of length $L/3$ has outside and inside diameters of 60 mm and 35 mm, respectively. The solid segment of length $2L/3$ has diameter of 60 mm. A strain gage is placed on the outside of the hollow segment of the bar to measure normal strains in the longitudinal direction.

(a) If the measured strain in the hollow segment is $\varepsilon_h = 470 \times 10^{-6}$, what is the strain $\varepsilon_s$ in the solid part? (*Hint*: The strain in the solid segment is equal to that in the hollow segment multiplied by the ratio of the area of the hollow to that of the solid segment).
(b) What is the overall shortening $\delta$ of the bar?
(c) If the compressive stress in the bar cannot exceed 48 MPa, what is the maximum permissible value of load $P$?



**Solution 1.3-3**

$L = 420\text{ mm} \qquad d_2 = 60\text{ mm} \qquad d_1 = 35\text{ mm} \qquad \varepsilon_h = 470\,(10^{-6}) \qquad \sigma_a = 48\text{ MPa}$

PART (a)

$$A_s = \frac{\pi}{4}d_2^{\,2} = 2.827 \times 10^{-3}\text{m}^2 \qquad A_h = \frac{\pi}{4}\left(d_2^{\,2} - d_1^{\,2}\right) = 1.865 \times 10^{-3}\text{ m}^2$$

$$\varepsilon_h = \frac{A_h}{A_s}\varepsilon_h = 3.101 \times 10^{-4}$$

PART (b)

$$\delta = \varepsilon_h\,\frac{L}{3} + \varepsilon_s\left(\frac{2L}{3}\right) = 0.1526\text{ mm} \qquad \varepsilon_h\,\frac{L}{3} = 0.066\text{ mm} \qquad \varepsilon_s\left(\frac{2L}{3}\right) = 0.087\text{ mm}$$

PART (c)

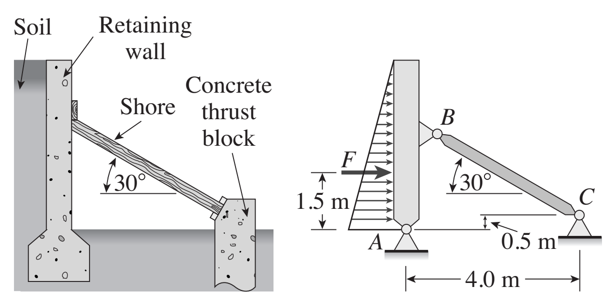
$$P_{\text{maxh}} = \sigma_a A_h = 89.535\text{ kN} \qquad P_{\text{maxs}} = \sigma_a A_s = 135.717\text{ kN} \qquad < \text{lesser value controls}$$

$$\boxed{P_{\text{max}} = P_{\text{maxh}} = 89.5\text{ kN}}$$

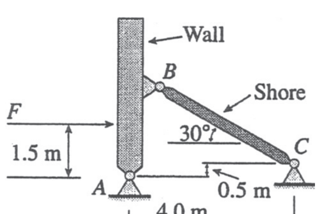
**Problem 1.3-4** A long retaining wall is braced by wood shores set at an angle of 30° and supported by concrete thrust blocks, as shown in the first part of the figure. The shores are evenly spaced, 3 m apart.

For analysis purposes, the wall and shores are idealized as shown in the second part of the figure. Note that the base of the wall and both ends of the shores are assumed to be pinned. The pressure of the soil against the wall is assumed to be triangularly distributed, and the resultant force acting on a 3-meter length of the wall is $F = 190$ kN.

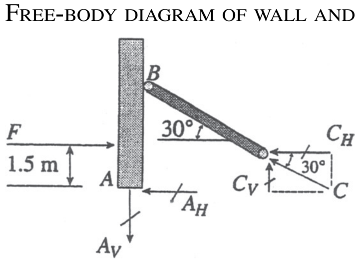
If each shore has a 150 mm × 150 mm square cross section, what is the compressive stress $\sigma_c$ in the shores?




**Solution 1.3-4 Retaining wall braced by wood shores**




FREE-BODY DIAGRAM OF WALL AND SHORE




$C$ = compressive force in wood shore

$C_H$ = horizontal component of $C$

$C_V$ = vertical component of $C$

$C_H = C\cos 30°$

$C_V = C\sin 30°$

$F = 190$ kN

$A$ = area of one shore

$A = (150\text{ mm})(150\text{ mm})$

$\quad = 22{,}500\text{ mm}^2$

$\quad = 0.0225\text{ m}^2$

SUMMATION OF MOMENTS ABOUT POINT $A$

$\Sigma M_A = 0$

$-F(1.5\text{ m}) + C_V(4.0\text{ m}) + C_H(0.5\text{ m}) = 0$

or

$-(190\text{ kN})(1.5\text{ m}) + C(\sin 30°)(4.0\text{ m})$
$\quad + C(\cos 30°)(0.5\text{ m}) = 0$

$\therefore\quad C = 117.14\text{ kN}$

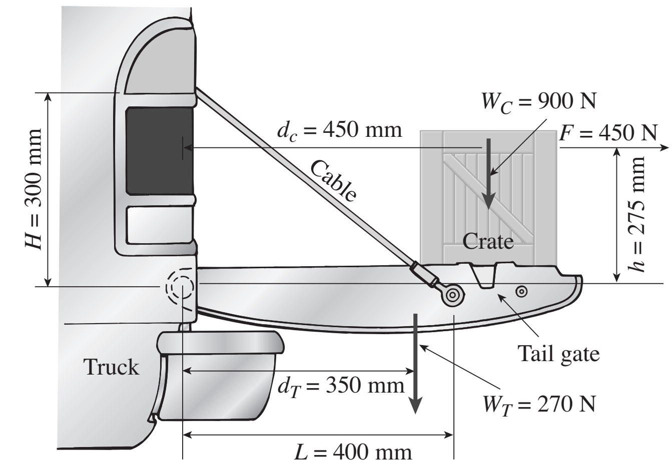
COMPRESSIVE STRESS IN THE SHORES

$$\sigma_c = \frac{C}{A} = \frac{117.14\text{ kN}}{0.0225\text{ m}^2}$$

$$= 5.21\text{ MPa} \qquad \leftarrow$$

**Problem 1.3-5** A pickup truck tailgate supports a crate ($W_C = 900$ N), as shown in the figure. The tailgate weighs $W_T = 270$ N and is supported by two cables (only one is shown in the figure). Each cable has an effective cross-sectional area $A_e = 11\text{ mm}^2$.

(a) Find the tensile force $T$ and normal stress $\sigma$ in each cable.
(b) If each cable elongates $\delta = 0.42$ mm due to the weight of both the crate and the tailgate, what is the average strain in the cable?




**Solution 1.3-5**

$F_h = 450\text{ N} \qquad h = 275\text{ mm}$

$W_c = 900\text{ N}$

$A_c = 11\text{ mm}^2$

$W_T = 270\text{ N}$

$\delta = 0.42\text{ mm}$

$d_c = 450\text{ mm}$

$d_T = 350\text{ mm}$

$H = 300\text{ mm}$

$L = 400\text{ mm}$

$L_e = \sqrt{L^2 + H^2} \qquad L_c = 0.5\text{ m}$

$\Sigma M_{\text{hinge}} = 0 \qquad 2T_v L = W_c d_c + W_T d_T + F_h h$

$$T_v = \frac{W_c d_c + W_T d_T + F_h h}{2L} \qquad T_v = 779.063\text{ N}$$

$$T_h = \frac{L}{H} T_v \qquad T_h = 1.039 \times 10^3\text{ N}$$

$$T = \sqrt{T_v^2 + T_h^2} \qquad T = 1.298\text{ kN} \qquad \longleftarrow$$

(a) $$\sigma_{\text{cable}} = \frac{T}{A_c} \qquad \sigma_{\text{cable}} = 118.0\text{ MPa} \qquad \longleftarrow$$

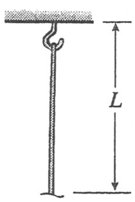
(b) $$\varepsilon_{\text{cable}} = \frac{\delta}{L_c} \qquad \varepsilon_{\text{cable}} = 8.4 \times 10^{-4} \qquad \longleftarrow$$

## Mechanical Properties and Stress-Strain Diagrams

**Problem 1.4-1** Imagine that a long steel wire hangs vertically from a high-altitude balloon.

(a) What is the greatest length (meters) it can have without yielding if the steel yields at 260 MPa?
(b) If the same wire hangs from a ship at sea, what is the greatest length? (Obtain the weight densities of steel and sea water from Table H-1, Appendix H.)

### Solution 1.4-1




$W$ = total weight of steel wire

$\gamma_S$ = weight density of steel

$\gamma_S = 77 \text{ kN/m}^3$

$\gamma_w$ = weight density of sea water

$\gamma_w = 10 \text{ kN/m}^3$

$A$ = cross-sectional area of wire

$\sigma_{\max} = 260 \text{ MPa}$

(a) Wire hanging in air

$$W = \gamma_S AL$$

$$\sigma_y = \frac{W}{A} = \gamma_S L$$

$$L_{\max} = \frac{\sigma_y}{\gamma_S} = 3377 \text{ m}$$ <hanging from balloon

(b) Wire hanging in sea water

$F$ = tensile force at top of wire

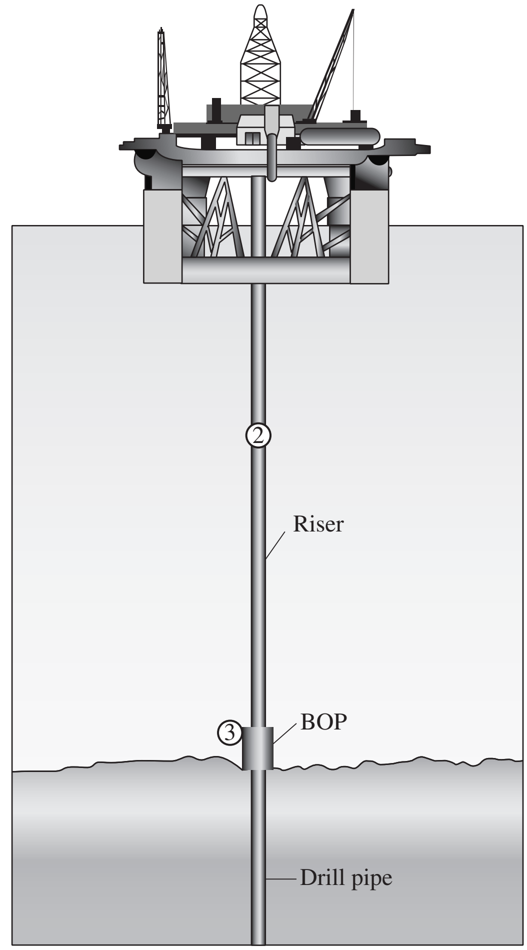
$$F = (\gamma_S - \gamma_W)AL \quad \sigma_y = \frac{F}{A} = (\gamma_S - \gamma_W)L$$

$$L_{\max} = \frac{\sigma_y}{\gamma_S - \gamma_W}$$

$$= 3881 \text{ m}$$ <hanging from ship at sea

**Problem 1.4-2** Steel riser pipe hangs from a drill rig located offshore in deep water (see figure).

(a) What is the greatest length (meters) it can have without breaking if the pipe is suspended in air and the ultimate strength (or breaking strength) is 550 MPa?
(b) If the same riser pipe hangs from a drill rig at sea, what is the greatest length? (Obtain the weight densities of steel and sea water from Appendix H, Table H-1. Neglect the effect of buoyant foam casings on the pipe.)




**Solution 1.4-2**

(a) Pipe suspended in air

$$\sigma_U = 550\ \text{MPa}$$

$$\gamma_s = 77\ \text{kN/m}^3$$

$$W = \gamma_s AL$$

$$\boxed{L_{max} = \frac{\sigma_U}{\gamma_s} = 7143\,\text{m}}$$

(b) Pipe suspended in sea water

$$\gamma_w = 10\ \text{kN/m}^3$$

Force at top of pipe: $F = (\gamma_s - \gamma_w)AL$

Stress at top of pipe:

$$\sigma_{max} = \frac{F}{A} \qquad \sigma_{max} = (\gamma_s - \gamma_w)L$$

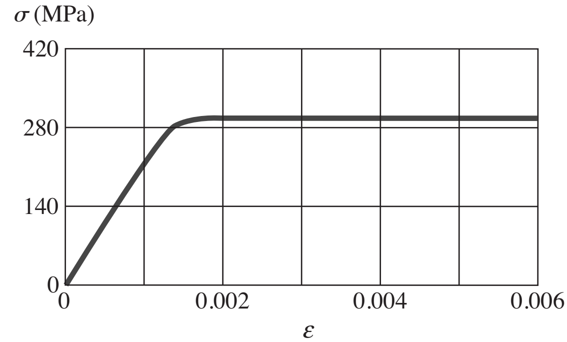
Set max stress equal to ultimate and then solve for $L_{max}$:

$$\boxed{L_{max} = \frac{\sigma_U}{(\gamma_s - \gamma_w)} = 8209\,\text{m}}$$

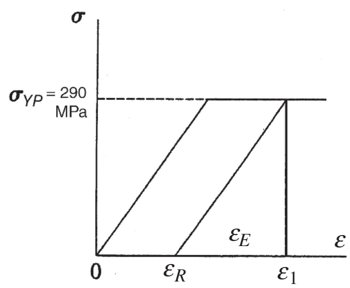
## Elasticity, Plasticity, and Creep

**Problem 1.5-1** A bar made of structural steel having the stress-strain diagram shown in the figure has a length of 1.5 m. The yield stress of the steel is 290 MPa and the slope of the initial linear part of the stress-strain curve (modulus of elasticity) is 207 GPa. The bar is loaded axially until it elongates 7.6 mm, and then the load is removed.

How does the final length of the bar compare with its original length? (*Hint*: Use the concepts illustrated in Fig. 1-18b.)




**Solution 1.5-1 Steel bar in tension**




$L = 1.5\text{ m} = 1500\text{ mm}$

Yield stress $\sigma_Y = 290\text{ MPa}$

Slope $= 207\text{ GPa}$

$\delta = 7.6\text{ mm}$

$$\varepsilon_1 = \frac{\delta}{L} = \frac{7.6\text{ mm}}{1500\text{ mm}} = 0.00507$$

Elastic recovery $\varepsilon_E = \varepsilon_I - \varepsilon_R$

From $\sigma-\varepsilon$ diagram,

$$\varepsilon_E = \frac{\sigma_{YP}}{\text{Slope}} = \frac{290\text{ MPa}}{207\text{ GPa}} = 0.00140$$

Residual strain

$$\varepsilon_R = \varepsilon_1 - \varepsilon_E = 0.0057 - 0.00140 = 0.00367$$

Permanent set

$$\varepsilon_R L = (0.00367)(1500\text{ mm}) = 5.5\text{ mm}$$

Final length of bar is 5.5 mm greater than the original length. ←

**Problem 1.5-2** A circular bar of magnesium alloy is 750 mm long. The stress-strain diagram for the material is shown in the figure. The bar is loaded in tension to an elongation of 6.0 mm, and then the load is removed.

(a) What is the permanent set of the bar?
(b) If the bar is reloaded, what is the proportional limit? (*Hint:* Use the concepts illustrated in Figs. 1-36b and 1-37.)

**Solution 1.5-2**

NUMERICAL DATA $L = 750\text{ mm}$ $\delta = 6\text{ mm}$

$$E_t(\varepsilon) = \frac{d}{d\varepsilon}\frac{\alpha\,\varepsilon}{1 + \beta\varepsilon} \qquad E_t(0) = 41{,}000\text{ MPa}$$

(or 41 GPa > magnesium alloy)

$\varepsilon_B = \dfrac{\delta}{L} = 8 \times 10^{-3}$ $\sigma_B 65.6\text{ MPa}$ < from curve (see figure)

$\varepsilon_E = 0.0023$ < elastic recovery (see figure)

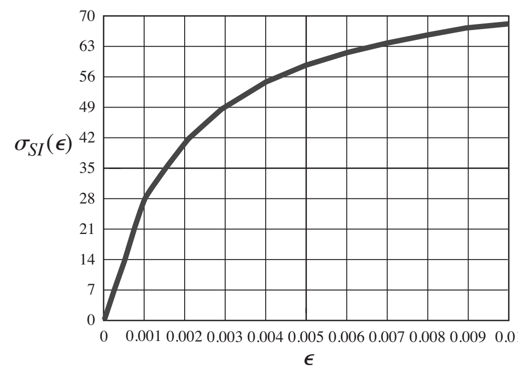
$\varepsilon_R = \varepsilon_B - \varepsilon_E = 5.7 \times 10^{-3}$ < residual strain

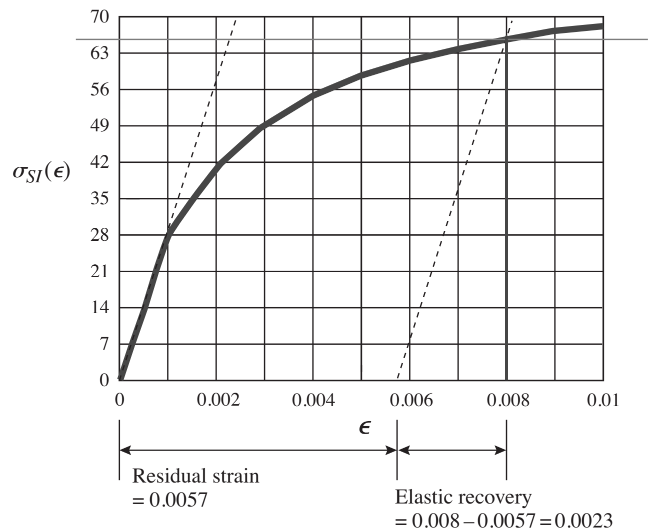
(a) PERMANENT SET

$\delta_{pset} = \varepsilon_R L = 4.275$ $\boxed{\delta_{pset} = 4.28\text{ mm}}$

(b) PROPORTIONAL LIMIT WHEN RELOADED

$\boxed{\sigma_B = 65.6\text{ MPa}}$



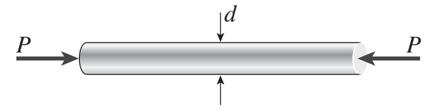



## Linear Elasticity, Hooke's Law, and Poisson's Ratio

*When solving the problems for Section 1.6, assume that the material behaves linearly elastically.*

**Problem 1.6-1** A high-strength steel bar used in a large crane has diameter $d = 50$ mm (see figure). The steel has modulus of elasticity $E = 200$ GPa and Poisson's ratio $\nu = 0.3$. Because of clearance requirements, the diameter of the bar is limited to 50.025 mm when it is compressed by axial forces.

What is the largest compressive load $P_{\max}$ that is permitted?




### Solution 1.6-1 Steel bar in compression

STEEL BAR

$E = 200$ GPa  max. $\Delta d = 0.025$ mm

$d = 50$ mm  $\nu = 0.3$

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d}{d} \quad \text{(increase in diameter)}$$

AXIAL STRAIN

$$\varepsilon = -\frac{\varepsilon'}{\nu} = -\frac{\Delta d}{\nu d} \quad \text{(decrease in length)}$$

Assume Hooke's law is valid for the material.

AXIAL STRESS

$$\sigma = E\varepsilon = \frac{E\Delta d}{\nu d} \quad \text{(compressive stress)}$$

MAXIMUM PERMISSIBLE COMPRESSIVE LOAD

$$P = \sigma A = \frac{EA\Delta d}{\text{n}d}$$

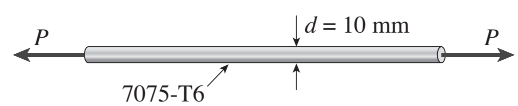
SUBSTITUTE NUMERICAL VALUES:

$$P = \frac{(200\text{ GPa})(\pi/4)(50\text{ mm})(0.025\text{ mm})}{(0.3)(50\text{ mm})}$$

$$= 654\text{ kN} \quad \leftarrow$$

**Problem 1.6-2** A round bar of 10 mm diameter is made of aluminum alloy 7075-T6 (see figure). When the bar is stretched by axial forces $P$, its diameter decreases by 0.016 mm.

Find the magnitude of the load $P$. (Obtain the material properties from Appendix H.)




### Solution 1.6-2 Aluminum bar in tension

$d = 10$ mm  $\Delta d = 0.016$ mm

(Decrease in diameter)

7075-T6

From Table H-2: $E = 72$ GPa  $\nu = 0.33$

From Table H-3: Yield stress $\sigma_Y = 480$ MPa

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d}{d} = \frac{-0.016\text{ mm}}{10\text{ mm}} = -0.0016$$

AXIAL STRAIN

$$\varepsilon = -\frac{\varepsilon'}{\nu} = \frac{0.0016}{0.33}$$

$$= 0.004848 \quad \text{(Elongation)}$$

AXIAL STRESS

$$\sigma = E\varepsilon = (72\text{ GPa})(0.004848)$$

$$= 349.1\text{ MPa (Tension)}$$

Because $\sigma < \sigma_Y$, Hooke's law is valid.

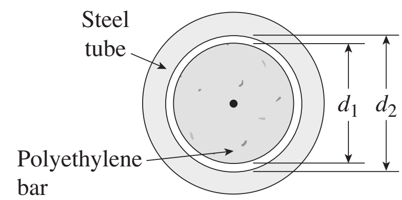
LOAD $P$ (TENSILE FORCE)

$$P = \sigma A = (349.1\text{ MPa})\left(\frac{\pi}{4}\right)(10\text{ mm})^2$$

$$= 27.4\text{ kN} \quad \leftarrow$$

**Problem 1.6-3** A polyethylene bar having diameter $d_1 = 70$ mm is placed inside a steel tube having inner diameter $d_2 = 70.2$ mm (see figure). The polyethylene bar is then compressed by an axial force $P$.

At what value of the force $P$ will the space between the polyethylene bar and the steel tube be closed? (For polyethylene, assume $E = 1.4$ GPa and $\nu = 0.4$.)




**Solution 1.6-3**

NUMERICAL DATA

$d_1 = 70$ mm $\quad d_2 = 70.2$ mm $\quad E = 1.4$ GPa

$\nu = 0.4 \quad \Delta d_1 = d_2 - d_1$

$$A_1 = \frac{\pi}{4} d_1^2 \qquad A_2 = \frac{\pi}{4} d_2^2$$

$A_1 = 3.848 \times 10^{-3}\ \text{m}^2$

$A_2 = 3.87 \times 10^{-3}\ \text{m}^2$

LATERAL STRAIN

$$\varepsilon' = \frac{\Delta d_1}{d_1} \qquad \varepsilon' = \frac{0.01}{4} \qquad \varepsilon' = 2.5 \times 10^{-3}$$

NORMAL STRAIN

$$\varepsilon_1 = \frac{-\varepsilon'}{\nu} \qquad \varepsilon_1 = -6.25 \times 10^{-3}$$

AXIAL STRESS

$\sigma_1 = E\varepsilon_1 \qquad \sigma_1 = -8.75 \times 10^6$ Pa

COMPRESSION FORCE

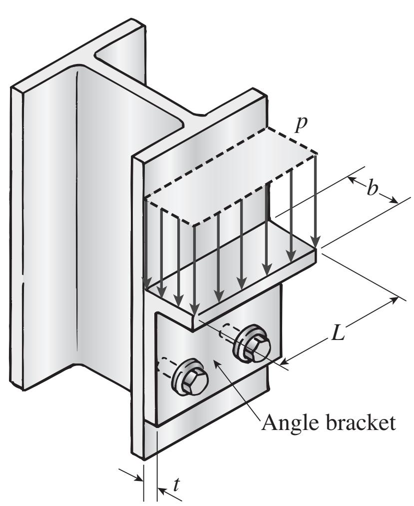
$P = EA_1\varepsilon_1$

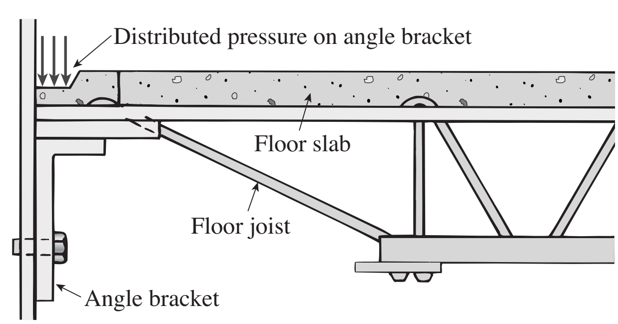
$P = -33.7$ kN $\quad \leftarrow$

## Shear Stress and Strain

**Problem 1.7-1** An angle bracket having thickness $t = 19$ mm is attached to the flange of a column by two 16 mm diameter bolts (see figure). A uniformly distributed load from a floor joist acts on the top face of the bracket with a pressure $p = 1.9$ MPa. The top face of the bracket has length $L = 200$ mm and width $b = 75$ mm.

Determine the average bearing pressure $\sigma_b$ between the angle bracket and the bolts and the average shear stress $\tau_{aver}$ in the bolts. (Disregard friction between the bracket and the column.)



**Solution 1.7-1**

NUMERICAL DATA

$t = 19$ mm $\quad L = 200$ mm

$b = 75$ mm $\quad p = 1.9$ MPa $\quad d = 16$ mm

BEARING FORCE

$F = pbL \quad F = 2.85 \times 10^4$ N

SHEAR AND BEARING AREAS

$$A_S = \frac{\pi}{4} d^2 \qquad A_S = 2.011 \times 10^{-4} \text{ m}^2$$

$$A_b = dt \qquad A_b = 3.04 \times 10^{-4} \text{ m}^2$$

BEARING STRESS

$$\sigma_b = \frac{F}{2A_b} \qquad \sigma_b = 46.9 \text{ MPa} \quad \leftarrow$$

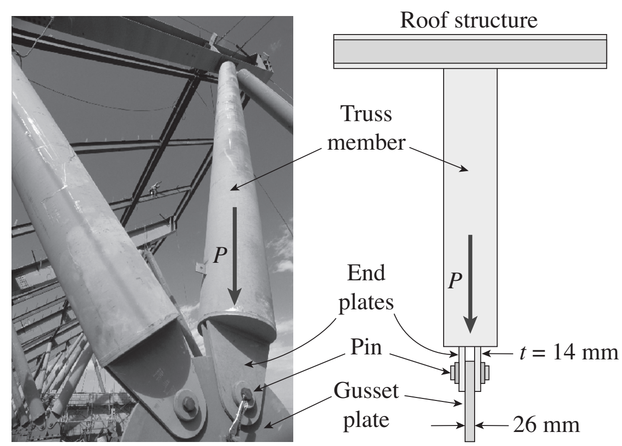
SHEAR STRESS

$$\tau_{ave} = \frac{F}{2A_S} \qquad \tau_{ave} = 70.9 \text{ MPa} \quad \leftarrow$$

**Problem 1.7-2** Truss members supporting a roof are connected to a 26-mm-thick gusset plate by a 22-mm diameter pin as shown in the figure and photo. The two end plates on the truss members are each 14 mm thick.

(a) If the load $P = 80$ kN, what is the largest bearing stress acting on the pin?
(b) If the ultimate shear stress for the pin is 190 MPa, what force $P_{ult}$ is required to cause the pin to fail in shear?

(Disregard friction between the plates.)




**Solution 1.7-2**

NUMERICAL DATA

$t_{ep} = 14$ mm

$t_{gp} = 26$ mm

$P = 80$ kN

$d_p = 22$ mm

$\tau_{ult} = 190$ MPa

(a) BEARING STRESS ON PIN

$$\sigma_b = \frac{P}{d_p t_{gp}}$$ gusset plate is thinner than ($2t_{ep}$) so gusset plate controls

$$\sigma_b = 139.9 \text{ MPa} \quad \leftarrow$$

(b) ULTIMATE FORCE IN SHEAR

Cross-sectional area of pin:

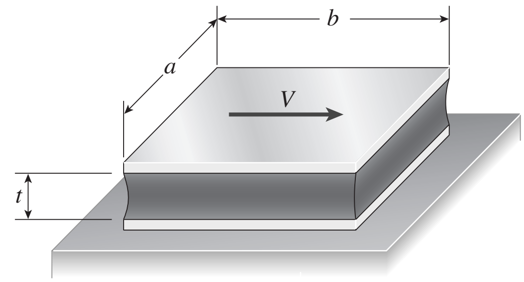
$$A_p = \frac{\pi d_p^2}{4}$$

$$A_p = 380.133 \text{ mm}^2$$

$$P_{ult} = 2\tau_{ult} A_p \qquad P_{ult} = 144.4 \text{ kN} \quad \leftarrow$$

**Problem 1.7-3** An elastomeric bearing pad consisting of two steel plates bonded to a chloroprene elastomer (an artificial rubber) is subjected to a shear force $V$ during a static loading test (see figure). The pad has dimensions $a = 125$ mm and $b = 240$ mm, and the elastomer has thickness $t = 50$ mm. When the force $V$ equals 12 kN, the top plate is found to have displaced laterally 8.0 mm with respect to the bottom plate.

What is the shear modulus of elasticity $G$ of the chloroprene?




**Solution 1.7-3**

NUMERICAL DATA

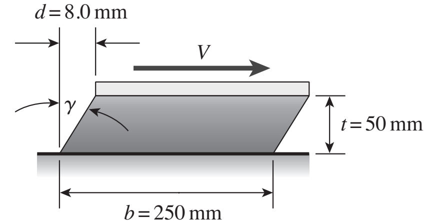
$V = 12$ kN $\quad a = 125$ mm

$b = 240$ mm $\quad t = 50$ mm $\quad d = 8$ mm

AVERAGE SHEAR STRESS

$$\tau_{ave} = \frac{V}{ab} \qquad \tau_{ave} = 0.4 \text{ MPa}$$

AVERAGE SHEAR STRAIN $\quad \gamma_{ave} = \dfrac{d}{t} \qquad \gamma_{ave} = 0.16$




SHEAR MODULUS $G$ $\quad G = \dfrac{\tau_{ave}}{\gamma_{ave}}$

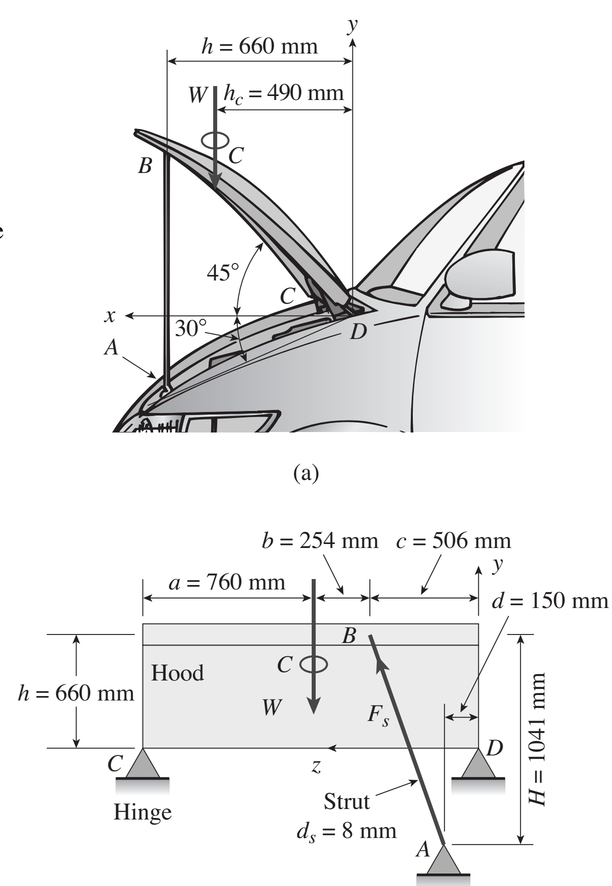
$G = 2.5$ MPa ←

**Problem 1.7-4** A single steel strut $AB$ with diameter $d_s = 8$ mm supports the vehicle engine hood of mass 20 kg, which pivots about hinges at $C$ and $D$ [see figures (a) and (b)]. The strut is bent into a loop at its end and then attached to a bolt at $A$ with diameter $d_b = 10$ mm. Strut $AB$ lies in a vertical plane.

(a) Find the strut force $F_s$ and average normal stress $\sigma$ in the strut.

(b) Find the average shear stress $\tau_{aver}$ in the bolt at $A$.

(c) Find the average bearing stress $\sigma_b$ on the bolt at $A$.



## Solution 1.7-4

NUMERICAL DATA

$d_s = 8 \text{ mm} \qquad d_b = 10 \text{ mm} \qquad m = 20 \text{ kg}$

$a = 760 \text{ mm} \qquad b = 254 \text{ mm}$

$c = 506 \text{ mm} \qquad d = 150 \text{ mm}$

$h = 660 \text{ mm} \qquad h_c = 490 \text{ mm}$

$$H = h\left(\tan\left(30\frac{\pi}{180}\right) + \tan\left(45\frac{\pi}{180}\right)\right)$$

$H = 1041 \text{ mm}$

$W = m\,(9.81 \text{ m/s}^2) \qquad W = 196.2 \text{ N}$

$$\frac{a + b + c}{2} = 760 \text{ mm}$$

VECTOR $r_{AB}$

$$r_{AB} = \begin{pmatrix} 0 \\ \text{H} \\ \text{c} - \text{d} \end{pmatrix} \qquad r_{AB} = \begin{pmatrix} 0 \\ 1.041 \times 10^3 \\ 356 \end{pmatrix}$$

UNIT VECTOR $e_{AB}$

$$e_{AB} = \frac{r_{AB}}{|r_{AB}|} \qquad e_{AB} = \begin{pmatrix} 0 \\ 0.946 \\ 0.324 \end{pmatrix} \qquad |e_{AB}| = 1$$

$$W = \begin{pmatrix} 0 \\ -W \\ 0 \end{pmatrix} \qquad W = \begin{pmatrix} 0 \\ -196.2 \\ 0 \end{pmatrix}$$

$$r_{DC} = \begin{pmatrix} h_c \\ h_c \\ b + c \end{pmatrix} \qquad r_{DC} = \begin{pmatrix} 490 \\ 490 \\ 760 \end{pmatrix}$$

$$\sum M_D \qquad M_D = r_{DB} \times F_s e_{AB} + W \times r_{DC}$$

(ignore force at hinge $C$ since it will vanish with moment about line $DC$)

$$F_{sx} = 0 \qquad F_{sy} = \frac{H}{\sqrt{H^2 + (c - d)^2}} F_s$$

$$F_{sz} = \frac{c - d}{\sqrt{H^2 + (c - d)^2}} F_s$$

where

$$\frac{H}{\sqrt{H^2 + (c - d)^2}} = 0.946$$

$$\frac{c - d}{\sqrt{H^2 + (c - d)^2}} = 0.324$$

(a) FIND THE STRUT FORCE $F_S$ AND AVERAGE NORMAL STRESS $\sigma$ IN THE STRUT

$$\sum M_{\text{line}DC} = 0 \qquad F_{sy} = \frac{|W| h_c}{h}$$

$F_{sy} = 145.664$

$$F_s = \frac{F_{sy}}{\dfrac{H}{\sqrt{H^2 + (c - d)^2}}} \qquad F_s = 153.9 \text{ N} \quad \leftarrow$$

$$A_{\text{strut}} = \frac{\pi}{4} d_s^2 \qquad A_{\text{strut}} = 50.265 \text{ mm}^2$$

$$\sigma = \frac{F_s}{A_{\text{strut}}} \qquad \sigma = 3.06 \text{ MPa} \quad \leftarrow$$

(b) FIND THE AVERAGE SHEAR STRESS $\tau_{\text{ave}}$ IN THE BOLT AT $A$

$d_b = 10 \text{ mm}$

$$A_s = \frac{\pi}{4} d_b^2 \qquad A_s = 78.54 \text{ mm}^2$$

$$\tau_{\text{ave}} = \frac{F_s}{A_s} \qquad \tau_{\text{ave}} = 1.96 \text{ MPa} \quad \leftarrow$$

(c) FIND THE BEARING STRESS $\sigma_b$ ON THE BOLT AT $A$

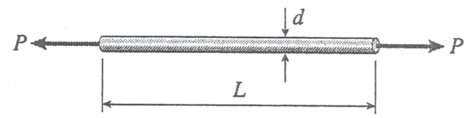
$$A_b = d_s d_b \qquad A_b = 80 \text{ mm}^2$$

$$\sigma_b = \frac{F_s}{A_b} \qquad \sigma_b = 1.924 \text{ MPa} \quad \leftarrow$$

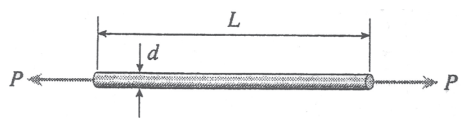
## Allowable Stresses and Allowable Loads

**Problem 1.8-1** A bar of solid circular cross section is loaded in tension by forces $P$ (see figure). The bar has length $L = 380$ mm and diameter $d = 6$ mm. The material is a magnesium alloy having modulus of elasticity $E = 42.7$ GPa. The allowable stress in tension is $\sigma_{allow} = 89.6$ GPa, and the elongation of the bar must not exceed 0.08 mm.

What is the allowable value of the forces $P$?




**Solution 1.8-1 Magnesium bar in tension**




$L = 380$ mm $\qquad d = 6$ mm

$E = 42.7$ GPa

$\sigma_{allow} = 89.6$ MPa $\qquad \delta_{max} = 0.8$ mm

$$\varepsilon_{max} = \frac{\delta_{max}}{L} = \frac{0.8 \text{ mm}}{380 \text{ mm}} = 0.002$$

Maximum stress based upon elongation

$$\sigma_{max} = E\varepsilon_{max} = (44{,}850 \text{ MPa})(0.00200)$$
$$= 89.7 \text{ MPa}$$

Maximum stress based upon elongation

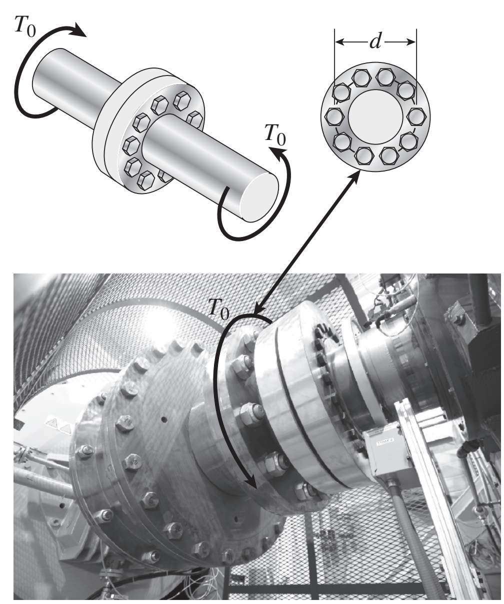
$\sigma_{allow} = 89.6$ MPa

Elongation governs

$$P_{allow} = \sigma_{max}A = (89.7 \text{ MPa})\left(\frac{\pi}{4}\right)(6 \text{ mm})^2$$
$$= 2.53 \text{ kN} \quad \leftarrow$$

**Problem 1.8-2** A torque $T_0$ is transmitted between two flanged shafts by means of ten 20-mm bolts (see figure and photo). The diameter of the bolt circle is $d = 250$ mm.

If the allowable shear stress in the bolts is 90 MPa, what is the maximum permissible torque? (Disregard friction between the flanges.)



**Solution 1.8-2 Shafts with flanges**

NUMERICAL DATA

$r = 10 \quad d = 250$ mm

^ bolts ^ flange

$A_s = \pi r^2$

$A_s = 314.159\ \text{m}^2$

$\tau_a = 85$ MPa

MAXIMUM PERMISSIBLE TORQUE

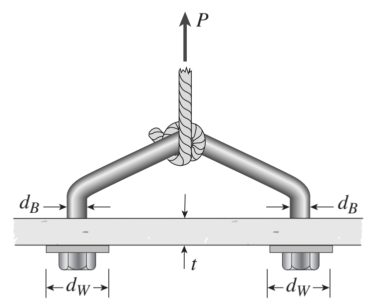
$$T_{\max} = \tau_a A_s \left( r \frac{d}{2} \right)$$

$T_{\max} = 3.338 \times 10^7$ N·mm

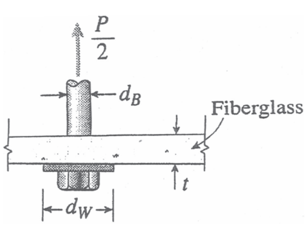
$T_{\max} = 33.4$ kN·m ←

**Problem 1.8-3** A tie-down on the deck of a sailboat consists of a bent bar bolted at both ends, as shown in the figure. The diameter $d_B$ of the bar is 6 mm, the diameter $d_W$ of the washers is 22 mm, and the thickness $t$ of the fiberglass deck is 10 mm.

If the allowable shear stress in the fiberglass is 2.1 MPa, and the allowable bearing pressure between the washer and the fiberglass is 3.8 MPa, what is the allowable load $P_{\text{allow}}$ on the tie-down?




**Solution 1.8-3 Bolts through fiberglass**




$d_B = 6$ mm

$d_W = 22$ mm

$t = 10$ mm

ALLOWABLE LOAD BASED UPON SHEAR STRESS IN FIBERGLASS

$\tau_{\text{allow}} = 2.1$ MPa

Shear area $A_s = \pi d_W t$

$$\frac{P_1}{2} = \tau_{\text{allow}} A_s = \tau_{\text{allow}}(\pi d_W t)$$

$$= (2.1\ \text{MPa})(\pi)(22\ \text{mm})(10\ \text{mm})$$

$$= 1451\ \text{N}$$

$P_1 = 2(1451\ \text{N})$

$P_1 = 2902$ N

ALLOWABLE LOAD BASED UPON BEARING PRESSURE

$\sigma_b = 3.8$ MPa

Bearing area $A_b = \frac{\pi}{4}(d_W^2 - d_B^2)$

$$\frac{P_2}{2} = \sigma_b A_b = (3.8\ \text{MPa})\left(\frac{\pi}{4}\right)[(22\ \text{mm})^2 - (6\ \text{mm})^2]$$

$$= 1337\ \text{N}$$

$P_2 = 2(1337\ \text{N})$

$P_2 = 2674$ N

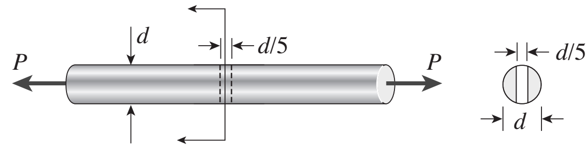
ALLOWABLE LOAD

Bearing pressure governs.

$P_{\text{allow}} = 2.67$ kN ←

**Problem 1.8-4** A solid bar of circular cross section (diameter $d$) has a hole of diameter $d/5$ drilled laterally through the center of the bar (see figure). The allowable average tensile stress on the net cross section of the bar is $\sigma_{\text{allow}}$.

(a) Obtain a formula for the allowable load $P_{\text{allow}}$ that the bar can carry in tension.
(b) Calculate the value of $P_{\text{allow}}$ if the bar is made of brass with diameter $d = 45$ mm and $\sigma_{\text{allow}} = 83$ MPa.
(*Hint*: Use the formulas of Case 15, Appendix D.)




## Solution 1.8-4

NUMERICAL DATA

$d = 45 \text{ mm} \qquad \sigma_a = 83 \text{ MPa}$

(a) FORMULA FOR $P_{\text{allow}}$ IN TENSION

From Case 15, Appendix D:

$$A = 2r^2\left(a - \frac{ab}{r^2}\right) \qquad r = \frac{d}{2} \qquad a = \frac{d}{10}$$

$$\alpha = \arccos\left(\frac{a}{r}\right) \quad r = 0.023 \text{ m} \quad a = 4.5 \times 10^{-3} \text{ m}$$

$$\alpha = 78.463°$$

$$b = \sqrt{r^2 - a^2}$$

$$b = \sqrt{\left[\left(\frac{d}{2}\right)^2 - \left(\frac{d}{10}\right)^2\right]}$$

$$b = \sqrt{\left(\frac{6}{25}d^2\right)} \qquad b = \frac{d}{5}\sqrt{6}$$

$$P_a = \sigma_a A$$

$$P_a = \sigma_a\left[\frac{1}{2}d^2\left(\arccos\left(\frac{1}{5}\right) - \frac{2}{25}\sqrt{6}\right)\right]$$

$$\frac{\arccos\left(\frac{1}{5}\right) - \frac{2}{25}\sqrt{6}}{2} = 0.587$$

$$P_a = \sigma_a(0.587d^2) \quad \leftarrow$$

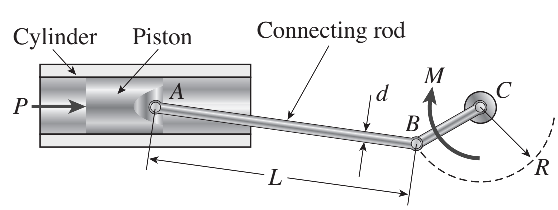
(b) EVALUATE NUMERICAL RESULT

$d = 0.045 \text{ m} \qquad \sigma_a = 83 \text{ MPa}$

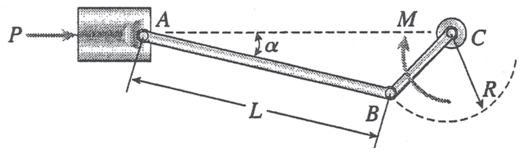
$P_a = 98.7 \text{ kN} \quad \leftarrow$

**Problem 1.8-5** The piston in an engine is attached to a connecting rod $AB$, which in turn is connected to a crank arm $BC$ (see figure). The piston slides without friction in a cylinder and is subjected to a force $P$ (assumed to be constant) while moving to the right in the figure. The connecting rod, which has diameter $d$ and length $L$, is attached at both ends by pins. The crank arm rotates about the axle at $C$ with the pin at $B$ moving in a circle of radius $R$. The axle at $C$, which is supported by bearings, exerts a resisting moment $M$ against the crank arm.

(a) Obtain a formula for the maximum permissible force $P_{\text{allow}}$ based upon an allowable compressive stress $\sigma_c$ in the connecting rod.
(b) Calculate the force $P_{\text{allow}}$ for the following data: $\sigma_c = 160$ MPa, $d = 9.00$ mm, and $R = 0.28L$.

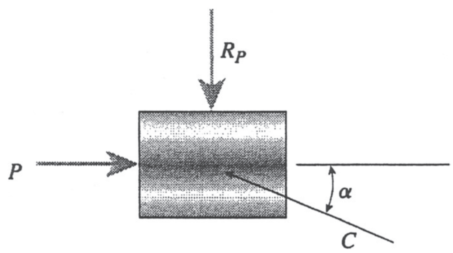


**Solution 1.8-5**




$d$ = diameter of rod $AB$

FREE-BODY DIAGRAM OF PISTON




$P$ = applied force (constant)

$C$ = compressive force in connecting rod

$RP$ = resultant of reaction forces between cylinder and piston (no friction)

$$\sum F_{\text{horiz}} = 0 \underset{+}{\rightarrow} \underset{-}{\leftarrow}$$

$$P - C\cos\alpha = 0$$

$$P = C\cos\alpha$$

MAXIMUM COMPRESSIVE FORCE $C$ IN CONNECTING ROD

$$C_{\max} = \sigma_c A_c$$

in which $A_c$ = area of connecting rod

$$A_c = \frac{\pi d^2}{4}$$

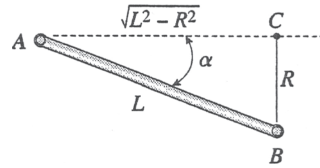
MAXIMUM ALLOWABLE FORCE $P$

$$P = C_{\max}\cos\alpha$$

$$= \sigma_c A_c \cos\alpha$$

The maximun allowable force $P$ occurs when cos α has its smallest value, which means that α has its largest value.

LARGEST VALUE OF α




The largest value of α occurs when point $B$ is the farthest distance from line $AC$. The farthest distance is the radius $R$ of the crank arm.

Therefore,

$$\overline{BC} = R$$

Also, $\overline{AC} = \sqrt{L^2 - R^2}$

$$\cos\alpha = \frac{\sqrt{L^2 - R^2}}{L} = \sqrt{1 - \left(\frac{R}{L}\right)^2}$$

(a) MAXIMUM ALLOWABLE FORCE $P$

$$P_{\text{allow}} = \sigma_c A_c \cos\alpha$$

$$= \sigma_c\left(\frac{\pi d^2}{4}\right)\sqrt{1 - \left(\frac{R}{L}\right)^2} \quad \leftarrow$$

(b) SUBSTITUTE NUMERICAL VALUES

$\sigma_c$ = 160 MPa $\quad d$ = 9.00 mm

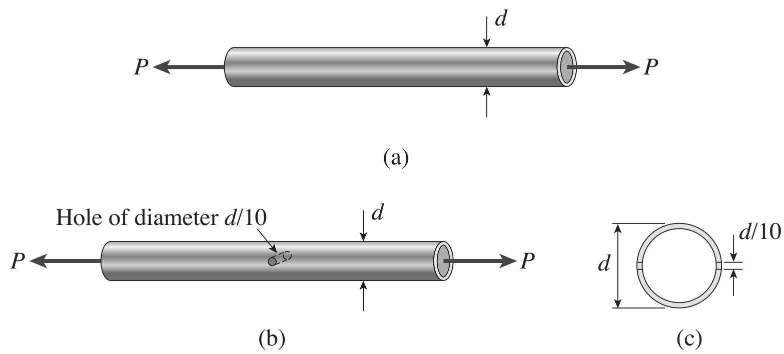
$R = 0.28L \quad R/L = 0.28$

$P_{\text{allow}}$ = 9.77 kN $\leftarrow$

## Design for Axial Loads and Direct Shear

**Problem 1.9-1** An aluminum tube is required to transmit an axial tensile force $P = 148$ kN (see figure part a). The thickness of the wall of the tube is to be 6 mm.

(a) What is the minimum required outer diameter $d_{min}$ if the allowable tensile stress is 84 MPa?
(b) Repeat part (a) if the tube will have a hole of diameter $d/10$ at mid-length (see figure parts b and c).




### Solution 1.9-1

NUMERICAL DATA

$P = 148$ kN $\quad t = 6$ mm $\quad \sigma_a = 84$ MPa

(a) MIN. DIAMETER OF TUBE (NO HOLES)

$$A_1 = \frac{\pi}{4}\left[d^2-(d-2t)^2\right] \qquad A_2 = \frac{P}{\sigma_a}$$

$$A_2 = 1.762 \times 10^{-3}\text{m}^2$$

Equating $A_1$ and $A_2$ and solving for $d$:

$$d = \frac{P}{\pi\sigma_a t} + t \qquad d = 99.5 \text{ mm} \quad \leftarrow$$

(b) MIN. DIAMETER OF TUBE (WITH HOLES)

$$A_1 = \left[\frac{\pi}{4}\left[d^2-(d-2t)^2\right]-2\left(\frac{d}{10}\right)t\right]$$

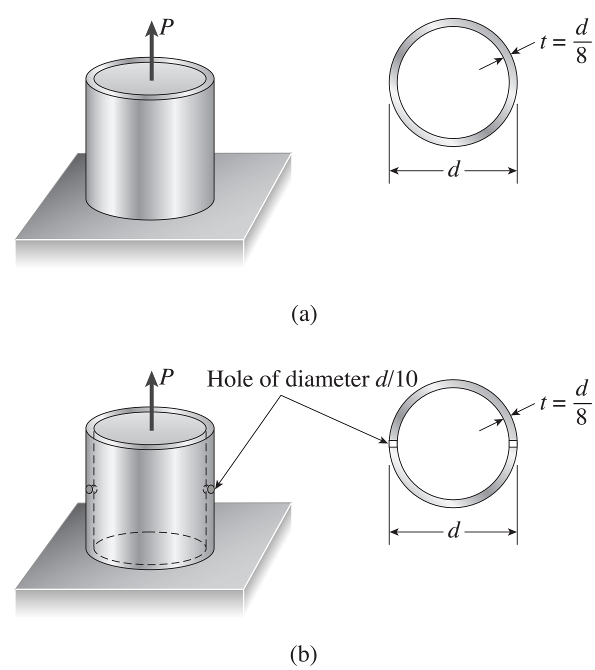
$$A_1 = d\left(\pi t - \frac{t}{5}\right) - \pi t^2$$

Equating $A_1$ and $A_2$ and solving for $d$:

$$d = \frac{\frac{P}{\sigma_a} + \pi t^2}{\pi t - \frac{t}{5}} \qquad d = 106.2 \text{ mm} \quad \leftarrow$$

**Problem 1.9-2** A copper alloy pipe having yield stress $\sigma_Y = 290$ MPa is to carry an axial tensile load $P = 1500$ kN [see figure part (a)]. A factor of safety of 1.8 against yielding is to be used.

(a) If the thickness $t$ of the pipe is to be one-eighth of its outer diameter, what is the minimum required outer diameter $d_{min}$?
(b) Repeat part (a) if the tube has a hole of diameter $d/10$ drilled through the entire tube as shown in the figure [part (b)].



**Solution 1.9-2**

Numerical data

$\sigma_Y = 290$ MPa

$P = 1500$ kN

$FS_y = 1.8$

(a) Minimum diameter (no holes)

$$A_1 = \frac{\pi}{4}\left[d^2 - \left(d - \frac{d}{4}\right)^2\right]$$

$$A_1 = \frac{7}{64}\pi d^2$$

$$A_2 = \frac{P}{\dfrac{\sigma_Y}{FS_y}} \qquad A_2 = 9.31 \times 10^3 \text{ mm}^2$$

Equate $A_1$ and $A_2$ and solve for $d$:

$$d^2 = \frac{7}{64\pi}\left(\frac{P}{\dfrac{\sigma_Y}{FS_y}}\right)$$

$$d_{\min} = \sqrt{\frac{7}{64\pi}\left(\frac{P}{\dfrac{\sigma_Y}{FS_y}}\right)}$$

$$d_{\min} = 164.6 \text{ mm} \quad \leftarrow$$

(b) Minimum diameter (with holes)

Redefine $A_1$—subtract area for two holes—then equate to $A_2$:

$$A_1 = \left[\frac{\pi}{4}\left[d^2 - \left(d - \frac{d}{4}\right)^2\right] - 2\left(\frac{d}{10}\right)\left(\frac{d}{8}\right)\right]$$

$$A_1 = \frac{7}{64}\pi d^2 - \frac{1}{40}d^2$$

$$A_1 = d^2\left(\frac{7}{64}\pi - \frac{1}{40}\right) \qquad \frac{7}{64}\pi - \frac{1}{40} = 0.319$$

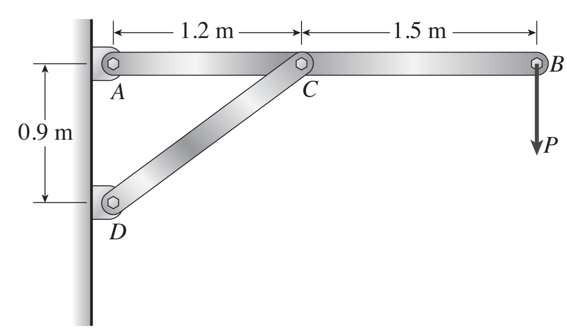
$$d^2 = \frac{\left(\dfrac{P}{\dfrac{\sigma_Y}{FS_y}}\right)}{\left(\dfrac{7}{64}\pi - \dfrac{1}{40}\right)}$$

$$d_{\min} = \sqrt{\frac{\left(\dfrac{P}{\dfrac{\sigma_Y}{FS_y}}\right)}{\left(\dfrac{7}{64}\pi - \dfrac{1}{40}\right)}} \qquad d_{\min} = 170.9 \text{ mm} \quad \leftarrow$$

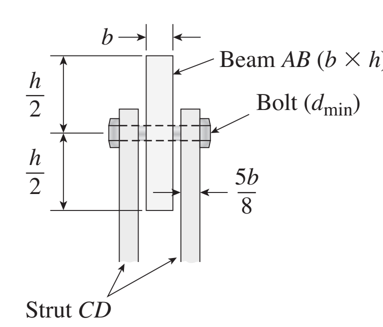
---

**Problem 1.9-3** A horizontal beam $AB$ with cross-sectional dimensions ($b = 19$ mm) $\times$ ($h = 200$ mm) is supported by an inclined strut $CD$ and carries a load $P = 12$ kN at joint $B$ (see figure part a). The strut, which consists of two bars each of thickness $5b/8$, is connected to the beam by a bolt passing through the three bars meeting at joint $C$ (see figure part b).

(a) If the allowable shear stress in the bolt is 90 MPa, what is the minimum required diameter $d_{\min}$ of the bolt at $C$?
(b) If the allowable bearing stress in the bolt is 130 MPa, what is the minimum required diameter $d_{\min}$ of the bolt at $C$?

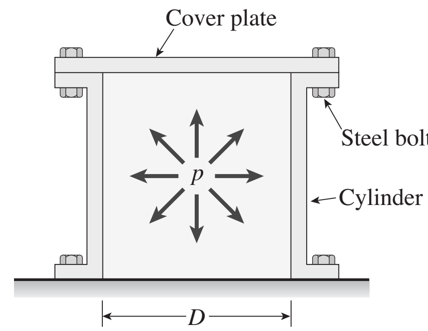



(a)

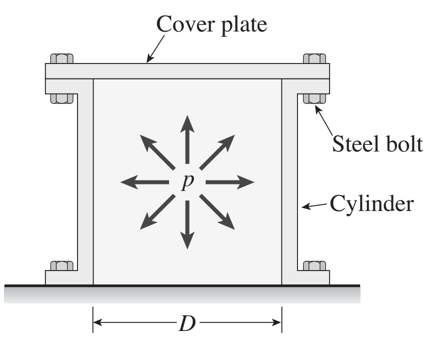


**Problem 1.9-4** A pressurized circular cylinder has a sealed cover plate fastened with steel bolts (see figure). The pressure $p$ of the gas in the cylinder is 1900 kPa, the inside diameter $D$ of the cylinder is 250 mm, and the diameter $d_B$ of the bolts is 12 mm.

If the allowable tensile stress in the bolts is 70 MPa, find the number $n$ of bolts needed to fasten the cover.




**Solution 1.9-4 Pressurized cylinder**




$p = 1900$ kPa $\quad D = 250$ mm $\quad d_b = 12$ mm

$\sigma_{\text{allow}} = 70$ kPa $\quad n =$ number of bolts

$F =$ total force acting on the cover plate from the internal pressure

$$F = p\left(\frac{\pi D^2}{4}\right)$$

NUMBER OF BOLTS

$P =$ tensile force in one bolt

$$P = \frac{F}{n} = \frac{\pi p D^2}{4n}$$

$$A_b = \text{area of one bolt} = \frac{\pi}{4} d_b^2$$

$$P = \sigma_{\text{allow}} A_b$$

$$\sigma_{\text{allow}} = \frac{P}{A_b} = \frac{\pi p D^2}{(4n)(\frac{\pi}{4})d_b^2} = \frac{pD^2}{n d_b^2}$$

$$n = \frac{pD^2}{d_b^2 \sigma_{\text{allow}}}$$

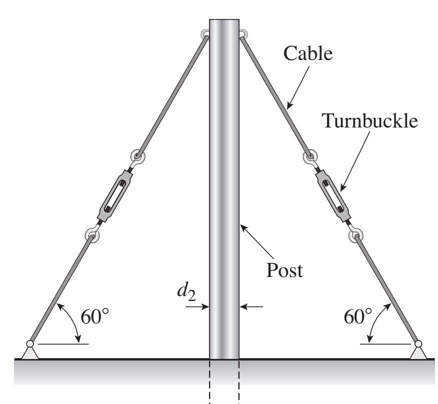
SUBSTITUTE NUMERICAL VALUES:

$$n = \frac{(1900\text{ kPa})(250\text{ mm})^2}{(12\text{ mm})^2(70\text{ MPa})} = 11.8$$

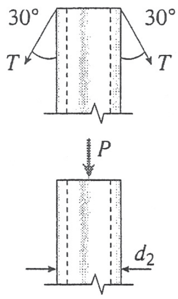
Use 12 bolts. $\leftarrow$

**Problem 1.9-5** A tubular post of outer diameter $d_2$ is guyed by two cables fitted with turnbuckles (see figure). The cables are tightened by rotating the turnbuckles, thus producing tension in the cables and compression in the post. Both cables are tightened to a tensile force of 110 kN. Also, the angle between the cables and the ground is 60°, and the allowable compressive stress in the post is $\sigma_c = 35$ MPa.

If the wall thickness of the post is 15 mm, what is the minimum permissible value of the outer diameter $d_2$?



## Solution 1.9-5 Tubular post with guy cables




$d_2$ = outer diameter

$d_1$ = inner diameter

$t$ = wall thickness
= 15 mm

$T$ = tensile force in a cable
= 110 kN

$\sigma_{allow}$ = 35 MPa

$P$ = compressive force in post
= $2T\cos 30°$

REQUIRED AREA OF POST

$$A = \frac{P}{\sigma_{allow}} = \frac{2T\cos 30°}{\sigma_{allow}}$$

AREA OF POST

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}[d_2^2 - (d_2 - 2t)^2]$$

$$= \pi t(d_2 - t)$$

EQUATE AREAS AND SOLVE FOR $d_2$:

$$\frac{2T\cos 30°}{\sigma_{allow}} = \pi t(d_2 - t)$$

$$d_2 = \frac{2T\cos 30°}{\pi t\sigma_{allow}} + t \quad \leftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$(d_2)_{min} = 131$ mm $\quad \leftarrow$