Analysis of Rectifier Operation*

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Summary—An analysis of rectifier operation in principal circuits is made. The introduction of linear equivalent diode resistance values permits a simplified and accurate treatment of circuits containing high-vacuum diodes and series resistance. The evaluation of these equivalent resistance values and a discussion of emission characteristics of oxide-coated cathodes precede the circuit analysis.

Generalized curve families for three principal condenser-input circuits are given to permit the rapid solution of rectifier problems in practical circuits without inaccuracies due to idealizing assumptions.

The data presented in this paper have been derived on the basis of a sinusoidal voltage source. It is apparent that the graphic analysis may be applied to circuits with nonsinusoidal voltage sources or intermittent pulse waves.

It is also permissible to consider only the wave section during conduction time and alter the remaining wave form at will. Complicated wave shapes may thus be replaced in many cases by a substantially sinusoidal voltage of higher frequency and intermittent occurrence as indicated by shape and duration of the highest voltage peak.

The applications of these principles have often explained large discrepancies from expected results as being caused by series or diode resistance and excessive peak-current demands.

Practical experience over many years has proved the correctness and accuracy of the generalized characteristics of condenser-input circuits.

Introduction

RECTIFIER circuits, especially of the condenser-input type, are extensively used in radio and television circuits to produce unidirectional currents and voltages. The design of power supplies, grid-current bias circuits, peak voltmeters, detectors and many other circuits in practical equipment is often based on the assumption that rectifier- and power-source resistance are zero, this assumption resulting in serious errors. The rectifier element or diode, furthermore has certain peak-current and power ratings which should not be exceeded. These values vary considerably with the series resistance of the circuit.

General operating characteristics of practical rectifier circuits have been evaluated and used by the writer for design purposes and information since early 1934, but circumstances have delayed publication. Several papers[1–4] have appeared in the meantime treating one or another part of the subject on the assumption of zero series resistance. Practical circuits have resistance and may even require insertion of additional resistance to protect the diode and input condenser against destructive currents. The equivalent diode resistance and the emission from oxide-coated cathodes are, therefore, discussed preceding the general circuit analysis. This analysis is illustrated on graphic constructions establishing a direct link with oscillograph observations on practical circuits. A detailed mathematical discussion requires much space and is dispensed with in favor of graphic solutions, supplemented by generalized operating characteristics.

I. Principles of Rectification

General

Rectification is a process of synchronized switching. The basic rectifier circuit consists of one synchronized switch in series with a single-phase source of single frequency and a resistance load. The switch connection between load terminals and source is closed when source and load terminals have the same polarity, and is open during the time of opposite polarity. The load current consists of half-wave pulses. This simple circuit is unsuitable for most practical purposes, because it does not furnish a smooth load current.

The current may be smoothed by two methods: (a) by increasing the number of phases, and (b) by inserting reactive elements into the circuit. The phase number is limited to two for radio receivers. The circuit analysis which follows later on will treat single- and double-phase rectifier circuits with reactive circuit elements.

Switching in reactive circuits gives rise to "transients." Current and voltage cannot, therefore, be computed according to steady-state methods.

The diode functions as a self-timing electronic switch. It closes the circuit when the plate becomes positive with respect to the cathode and opens the circuit at the instant when the plate current becomes zero.

The diode has an internal resistance which is a function of current. When analyzing rectifier circuits, it is convenient to treat the internal resistance of the diode rectifier as an element, separated from the "switch action" of the diode. Fig. 1 illustrates the three circuit elements so obtained and their respective voltage-current characteristics (see Section II). The diode characteristic is the sum of these characteristics. The resistance r_d is effective only when the switch is closed, i.e., during the conduction period of the diode. The effective diode resistance must, therefore, be measured or evaluated within conduction-time limits. Consider a

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[1] M. B. Stout, "Analysis of rectifier filter circuits," *Elec. Eng. Trans. A.I.E.E.* (*Elec. Eng.*, September, 1935), vol. 54, pp. 977–984; September, 1935.

[2] N. H. Roberts, "The diode as half-wave, full-wave and voltage-doubling rectifier," *Wireless Eng.*, vol. 13, pp. 351–362; July, 1936; and pp. 423–470; August, 1936.

[3] J. C. Frommer, "The determination of operating data and allowable ratings of vacuum-tube rectifiers," Proc. I.R.E., vol. 29, pp. 481–485; September, 1941.

[4] D. L. Waidelich, "The full-wave voltage-doubling rectifier circuit," Proc. I.R.E., vol. 29, pp. 554–558; October, 1941.

switch in series with a fixed resistance and any number of other circuit elements connected to a battery of fixed voltage. The direct current and root-mean-square current which flow in this circuit will depend on the time intervals during which the switch is closed and open; the resistance value is not obtainable from these current values and the battery voltage. The correct value is obtained only when the current and voltage

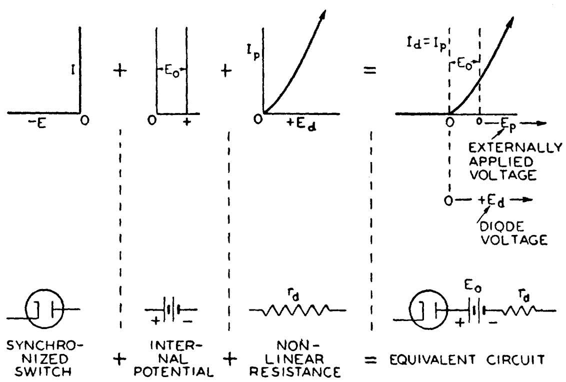


Fig. 1—Characteristics and equivalent circuit for high-vacuum diodes.

drop in the resistance are measured during the time angle ϕ (Fig. 2) when the switch is closed.

The method of analysis of rectifier circuits to be discussed in this paper is based on the principle that the nonlinear effective resistance of the diode may be replaced analytically by an equivalent fixed resistance which will give a diode current equal to that obtained with the actual nonlinear diode resistance. The correct value to be used for the equivalent fixed resistance depends upon whether we are analyzing for peak diode current, average diode current, or root-mean-square diode current.

At the outset of an analysis amplitude and wave shape of the diode current are not known and the diode resistance must, therefore, be determined by successive approximations.

The complexity of repeated calculations, especially on condenser-input circuits, requires that the operating characteristics of the circuit be plotted generally as functions of the circuit constants including series resistance in the diode circuit as a parameter.

Data for these plots (such as Figs. 3 to 7) are to be obtained by general analysis of circuits with linear resistances.

The solution of a practical condenser-input-circuit problem requires the use of three different equivalent linear circuits and diode resistance values.

The resistance values are obtainable from the peak current alone because wave shape can be eliminated as a factor by means of a general relation given by (6). The practical analysis of condenser input circuits thus simplified, is carried out as follows:

The average diode current is estimated roughly and the diode peak current is assumed to be four times the average value. The diode characteristic (Fig. 8) furnishes an initial peak-resistance value and (6) furnishes the other diode resistance values (see $\hat{R}_s$ values in Fig. 9). Direct output voltage and average current are now obtained with the equivalent average value $\overline{R}_s$ from the respective plot (Figs. 3 to 5) as a first approximation. Another chart (Fig. 6) furnishes the peak-to-average-diode-current ratio with the peak value $\hat{R}_s$ and thus the peak current and diode peak resistance in close approximation.

A second approximation gives usually good agreement between initial and obtained resistance values, which are then used to obtain other operating data.

A theoretical treatment of the method just described will be omitted in favor of an analysis of operating characteristics of the rectifier tube itself. The user of tubes may welcome information on the subject of peak emission which is of vital importance in the rating and trouble-free operation of any tube with an oxide-coated cathode.

II. Anode and Cathode Characteristics of Rectifier Tubes

Anode Characteristics

1. Definitions of Resistance Values

The instantaneous resistance (r_d) of a diode is the ratio of the instantaneous plate voltage e_d to the instantaneous plate current i_p at any point on the characteristic measured from the operating point (see Fig. 1). It is expressed by

$$r_d = \frac{e_d}{i_p}. \tag{1}$$

The operating point (0) of a diode is a fixed point on the characteristic, marked by beginning and end of the

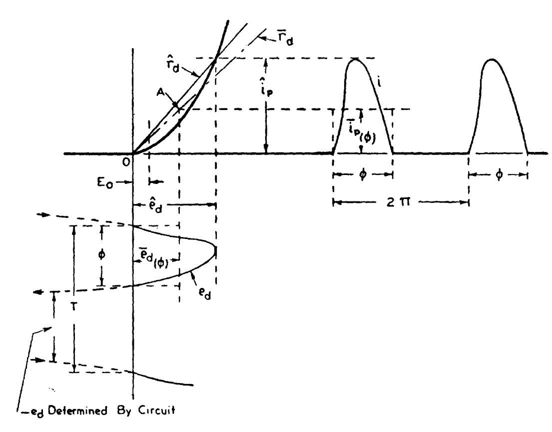


Fig. 2—Graphic evaluation of equivalent diode resistance values.

conduction time. It is, therefore, the cutoff point $I_d = 0$ and $E_d = 0$, as shown in Fig. 1. The operating point is independent of the wave form and of the conduction time ϕ (see Fig. 2).

The peak resistance[5] ($\hat{r}_d$) is a specific value of the instantaneous resistance and is defined as

[5] For system of symbols, see Appendix.

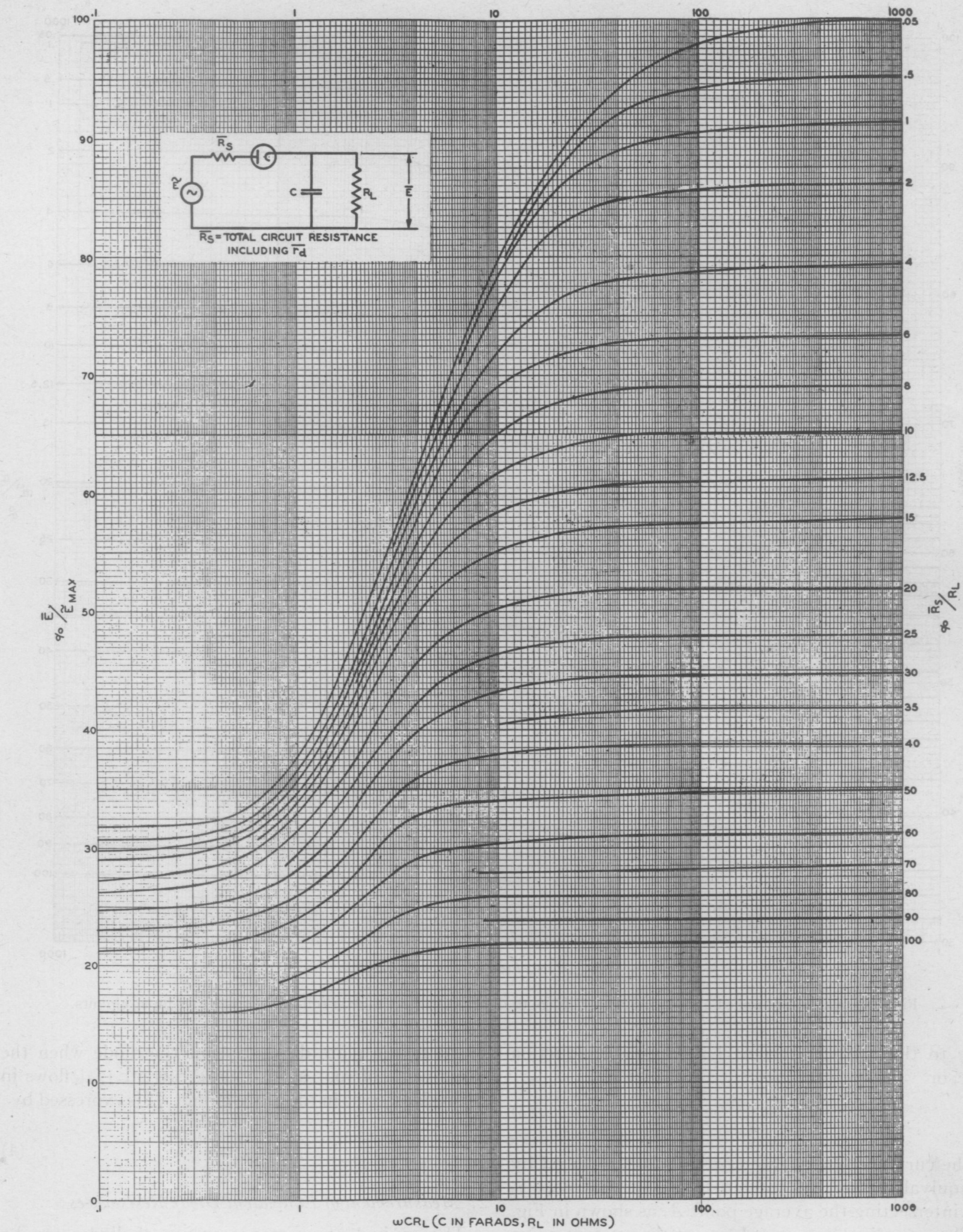


Fig. 3—Relation of applied alternating peak voltage to direct output voltage in half-wave, condenser-input circuits.

$$\hat{r}_d = \frac{\hat{e}_d}{\hat{\imath}_p} \text{ (see Fig. 2).} \tag{2}$$

Peak voltage $\hat{e}_d$ and peak current $\hat{\imath}_p$ are measured from the operating point 0.

The equivalent average resistance ($\bar{r}_d$) is defined on the basis of circuit performance as a resistance value determining the magnitude of the average current in the circuit. The value $\bar{r}_d$ is, therefore, the ratio of the average voltage drop $\bar{e}_{d(\phi)}$ in the diode during conduction

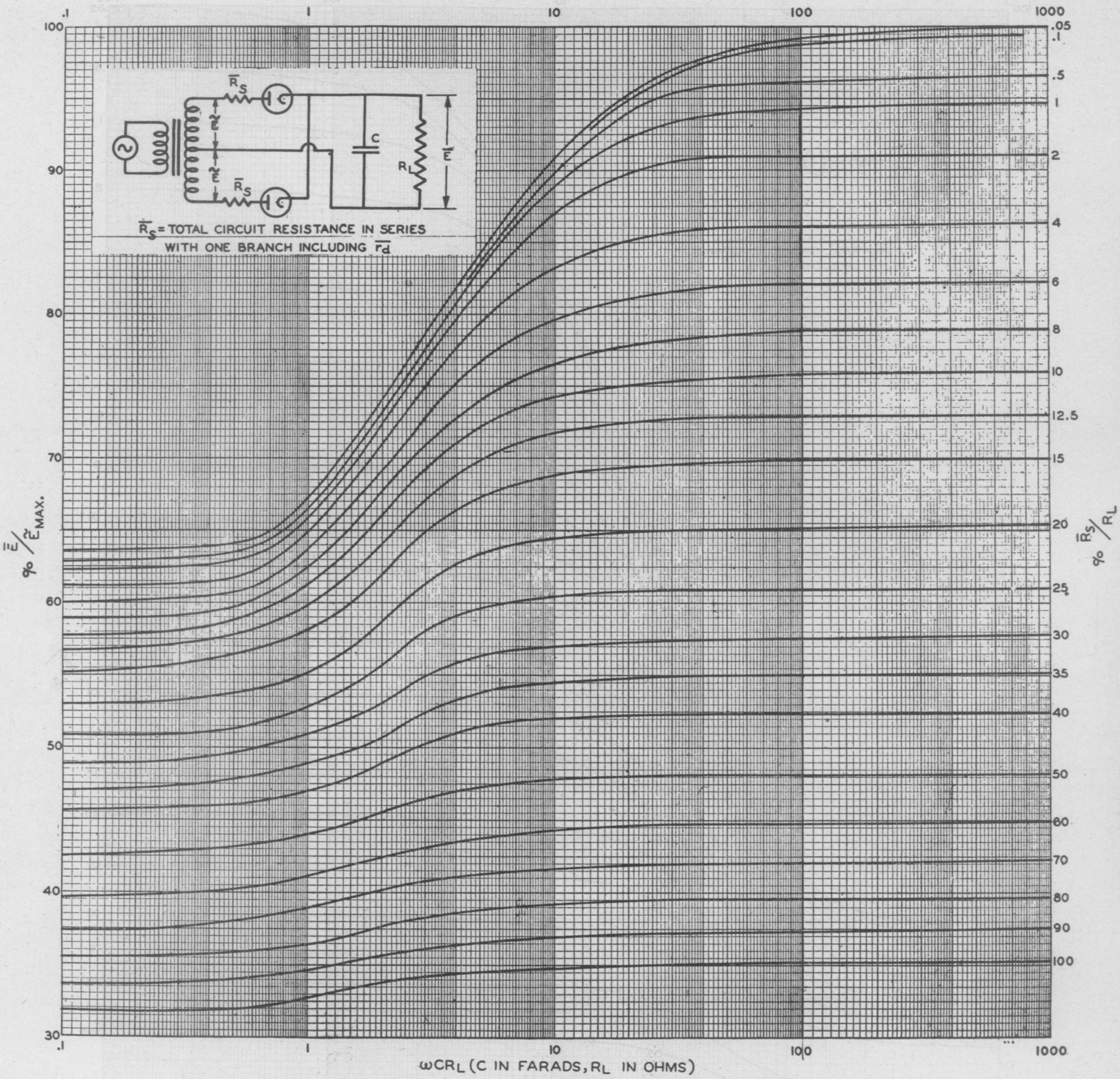


Fig. 4—Relation of applied alternating peak voltage to direct output voltage in full-wave, condenser-input circuits.

time to the average current $\bar{\imath}_{p(\phi)}$ during conduction time, or

$$\bar{r}_d = \frac{\bar{e}_{d(\phi)}}{\bar{\imath}_{p(\phi)}}. \tag{3}$$

The curved diode characteristic is thus replaced by an equivalent linear characteristic having the slope $\bar{r}_d$ and intersecting the average point A, as shown in Fig. 2. The co-ordinates $\bar{e}_{d(\phi)}$ and $\bar{\imath}_{p(\phi)}$ of the average point depend on the shape of voltage and current within the time angle ϕ. The analysis of rectifier circuits shows that the shape of the current pulse in actual circuits varies considerably between different circuit types.

The equivalent root-mean-square resistance ($|r_d|$) is defined as the resistance in which the power loss P_d is equal to the plate dissipation of the diode when the same value of root-mean-square current $|I_d|$ flows in the resistance as in the diode circuit. It is expressed by

$$|r_d| = \frac{P_d}{|I_d|^2}. \tag{4}$$

2. *Measurement of Equivalent Diode Resistances*

The equivalent resistance values of diodes can be measured by direct substitution under actual operating conditions. The circuit arrangement is shown in Fig. 10. Because the diode under test must be replaced as a whole by an adjustable resistance of known value, a second switch (a mercury-vapor diode identified in the figure as the ideal diode) with negligible resistance

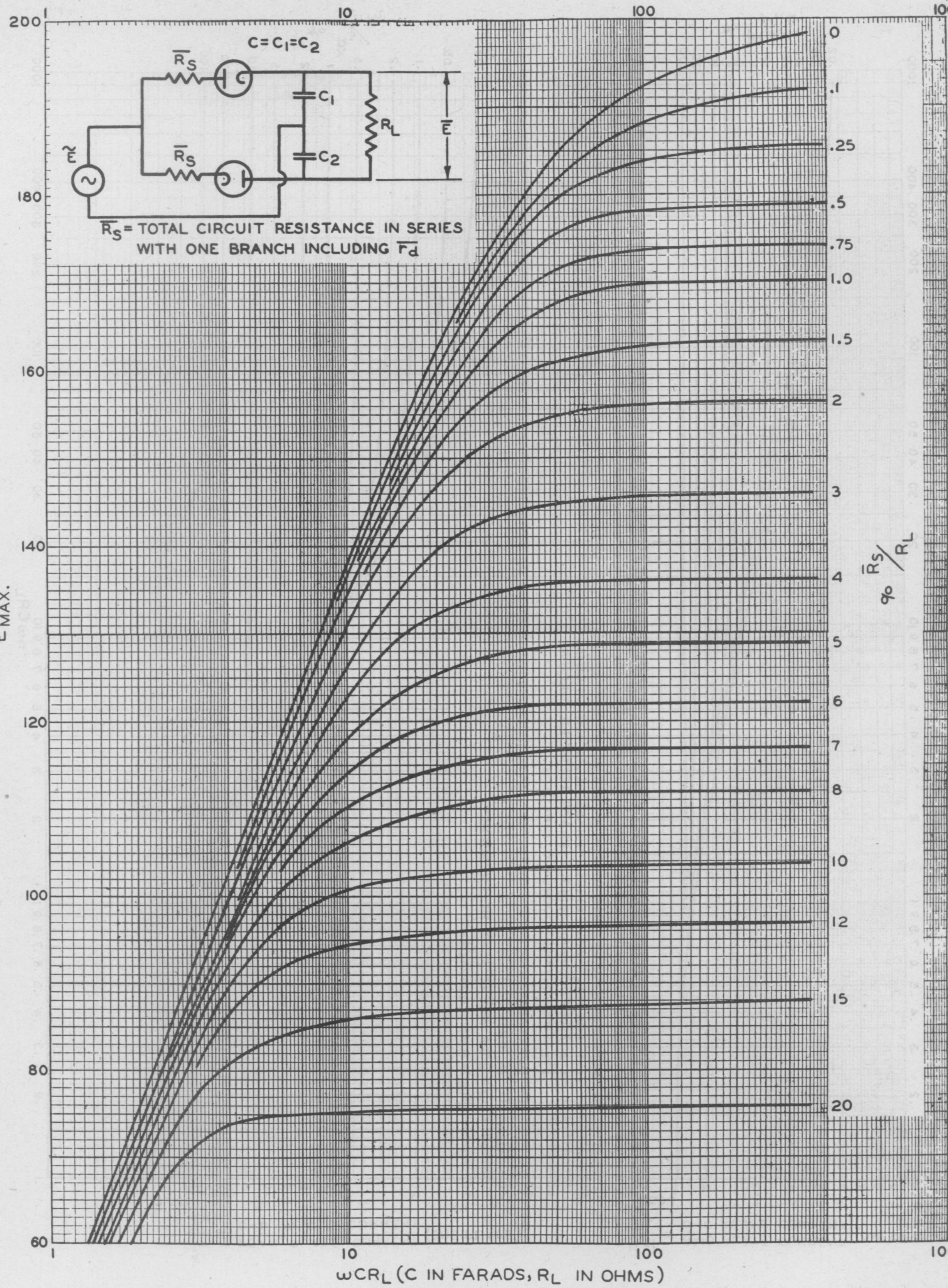


Fig. 5—Relation of applied alternating peak voltage to direct output voltage in condenser-input, voltage-doubling circuits.

must be inserted in order to preserve the switch-action in the circuit.

When a measurement is being made, the resistor R_d is varied until the particular voltage or current under observation remains unchanged for both positions of the switch S. We observe (1) that it is impossible to find one single value of R_d which will duplicate conditions of the actual tube circuit, i.e., give the same

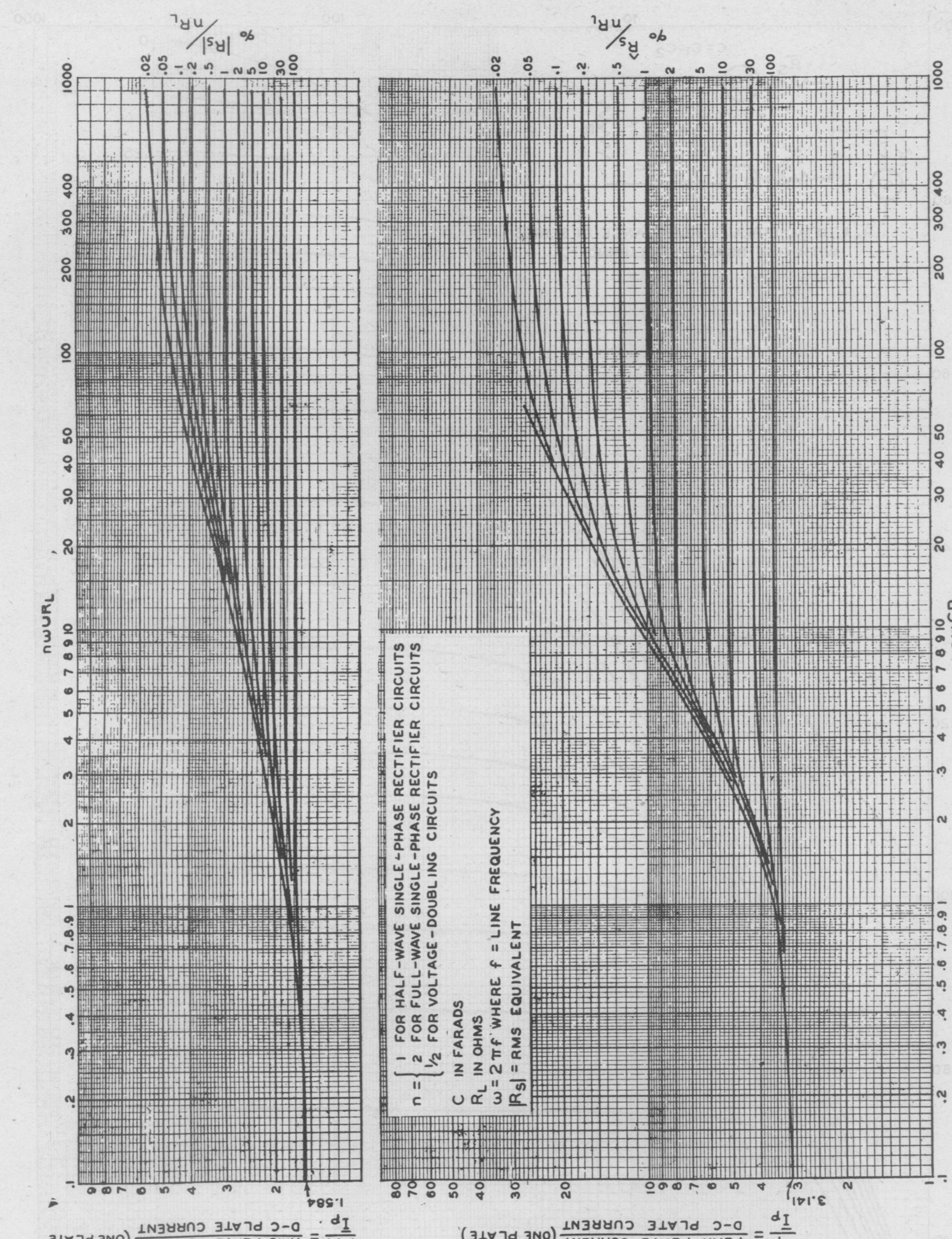


Fig. 6—Relation of peak, average, and root-mean-square diode current in condenser-input circuits.

values of peak, average, and root-mean-square current in the circuit; (2) that the ratio of these three "equivalent" resistance values of the diode varies for different combinations of circuit elements; and (3) that the resistance values are functions of the current amplitude and wave shape.

3. *Wave Forms and Equivalent Resistance Ratios for Practical Circuit Calculations*

The form of the current pulse in practical rectifier circuits is determined by the power factor of the load circuit and the phase number. Practical circuits may be

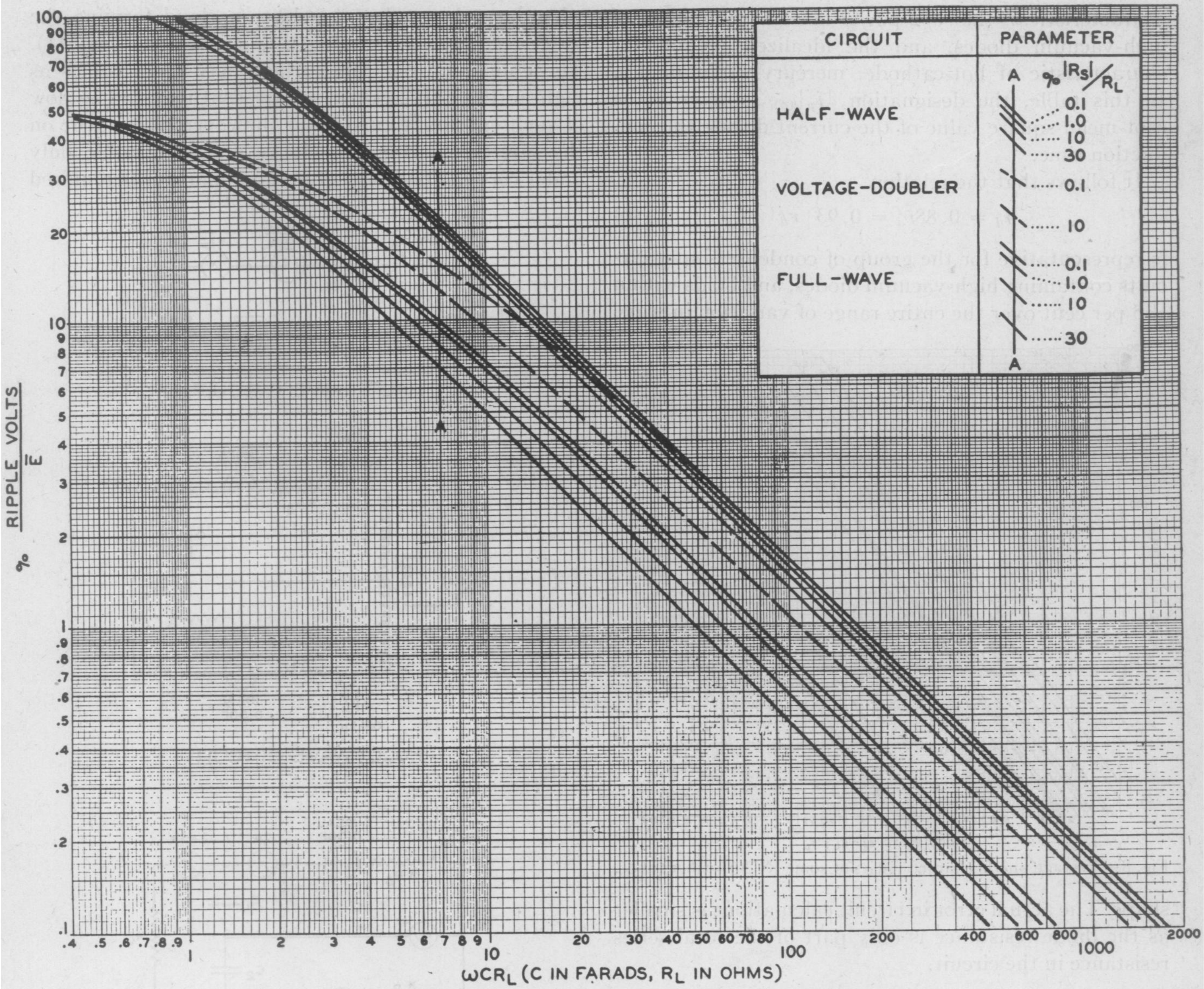


Fig. 7—Root-mean-square ripple voltage of condenser-input circuits.

divided into two main groups: (a) circuits with choke-input filter; and (b) circuits with condenser-input filter.

The diode current pulse in choke-input circuits has a rectangular form on which is superimposed one cycle of the lowest ripple frequency. In most practical circuits, this fluctuation is small as compared with the average amplitude of the wave and may be neglected when determining the equivalent diode resistances. It is apparent then that the equivalent diode resistance values are all equal and independent of the type of diode characteristics for square-wave forms. Hence, for choke-input circuits, we have

$$\hat{r}_d = \bar{r}_d = |r_d|. \tag{5}$$

The diode current pulse in condenser-input circuits is the summation of a sine-wave section and a current having an exponential decay. It varies from a triangular form for $\phi < 20$ degrees to a full half cycle ($\phi = 180$ degrees) as the other extreme. In Table I are given the ratios of voltages, currents, and resistance values during conduction time for two principal types of rectifier

TABLE I

Conduction Time Angle ϕ	Wave Shape	$\frac{\bar{i}_p(\phi)}{\hat{i}_p}$	$\frac{\lvert i_p\rvert(\phi)}{\hat{i}_p}$	3/2-Power Rectifier Characteristic $\frac{\bar{e}_d\phi}{\hat{e}_d}$	$\frac{\bar{r}_d}{\hat{r}_d}$	$\frac{\lvert r_d\rvert}{\hat{r}_d}$	Rectangular Characteristic $\frac{\bar{e}_d(\phi)}{\hat{e}_d}$	$\frac{\bar{r}_d}{\hat{r}_d}$	$\frac{\lvert r_d\rvert}{\hat{r}_d}$
Condenser-Input Circuits									
Degrees ≦20		0.500	0.577	0.593	1.185	1.120	1.0	2.00	1.500
90 and 180		0.637	0.707	0.715	1.120	1.057	1.0	1.57	1.272
130		0.725	0.780	0.787	1.085	1.030	1.0	1.38	1.190
Choke-Input Circuits									
180		1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

characteristics: the 3/2-power-law characteristic of high-vacuum diodes, and the idealized rectangular characteristic of hot-cathode, mercury-vapor diodes. In this table, the designation $|i_p|_{(\phi)}$ represents the root-mean-square value of the current during the conduction time.

It follows that the relation

$$\hat{r}_d = 0.88\bar{r}_d = 0.93 \mid r_d \mid \qquad (6)$$

is representative for the group of condenser-input circuits containing high-vacuum diodes, and holds within ± 5 per cent over the entire range of variation in wave

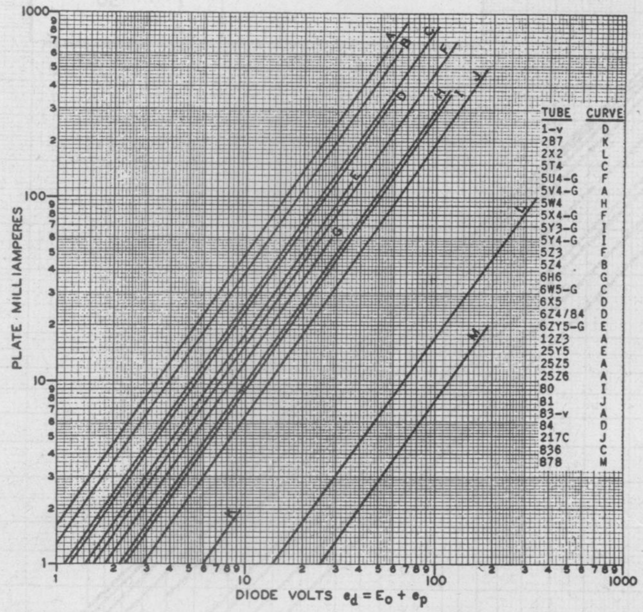


Fig. 8—Average anode characteristics of some RCA rectifier tubes.

shape. The actual error in circuit calculations is smaller as the diode resistance is only part of the total series resistance in the circuit.

Cathode Characteristics

Peak-Emission and Saturation of Oxide-Coated Cathodes

The normal operating range of diodes (including instantaneous peak values) is below the saturation potential because the plate dissipation rises rapidly to dangerous values if this potential is exceeded. Saturation is definitely recognized in diodes with tungsten or thoriated-tungsten cathodes as it does not depend on the time of measurement, provided the plate dissipation is not excessive. The characteristics of such diodes are single-valued even in the saturated range, i.e., the range in which the same value of current is obtained at a given voltage whether the voltage has been increased or decreased to the particular value.

Diodes with oxide-coated cathodes may have double-valued characteristics because of the coating characteristic. The cathode coating has resistance and capacitance, both of which are a function of temperature, current, and the degree of "activation."

A highly emitting monatomic layer of barium on oxygen is formed on the surface of the coating, which, when heated, supplies the electron cloud forming the space charge above the coating surface (see Fig. 11). The emission from this surface may have values as high as 100 amperes per square centimeter. The flow of such enormous currents is, however, dependent on the internal-coating impedance and is possible only under certain conditions. Special apparatus is required

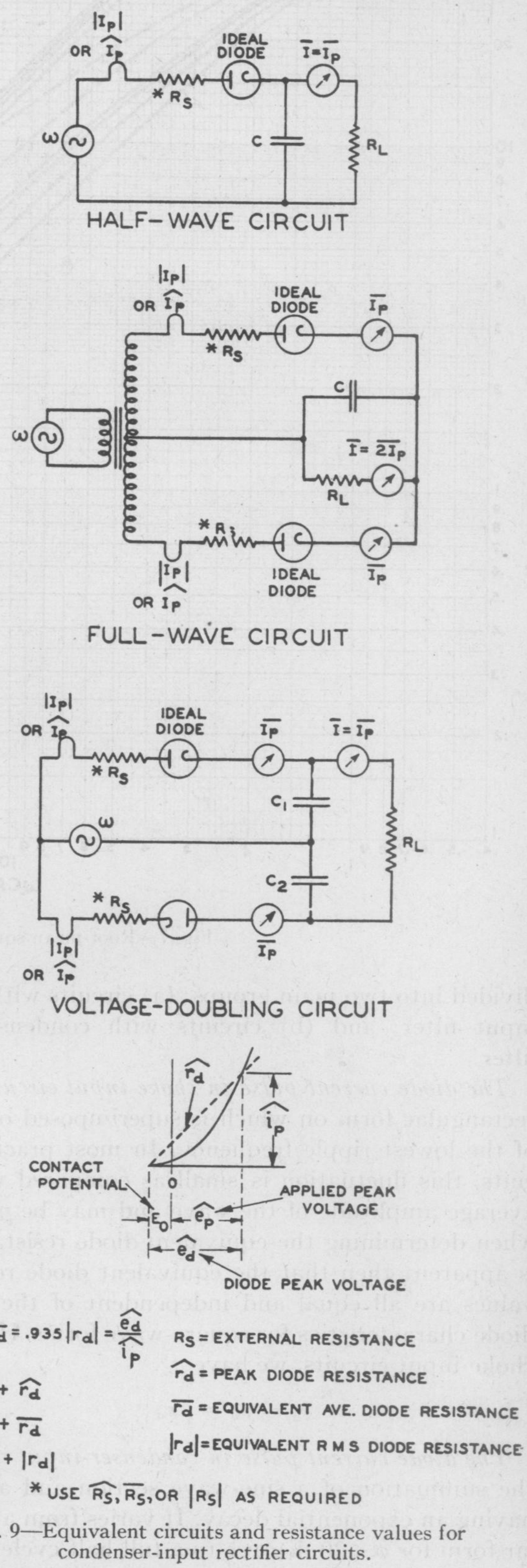


Fig. 9—Equivalent circuits and resistance values for condenser-input rectifier circuits.

to permit observation of high current values which, to prevent harm to the tube, can be maintained only over very short time intervals determined by the thermal capacity of the plate and coating. For example, an instantaneous power of 15 kilowatts must be dissipated in the close-spaced diode type 83-v at a current of 25 amperes from its cathode surface of only 1 square centimeter.

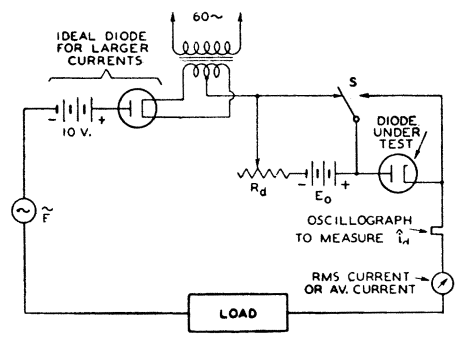


Fig. 10—Circuit for measuring equivalent diode resistance values.

Equipment for such observations was built in June, 1937, by the author after data obtained in 1935 on a low-powered curve tracer[6] indicated the need for equipment having a power source of very low internal impedance for measurements on even relatively small diodes.

1. Measurement of Diode Characteristics and Peak Emission

The circuit principle is shown in Fig. 12. The secondary voltage of a 2-kilovolt-ampere transformer T_m is adjustable from zero to 2 kilovolts by means of an autotransformer T_A. Transformer and line reactances are eliminated for short-time surge currents by a large condenser load ($C=20$ to 80 microfarads). The large reactive current is "tuned out" by a choke L of considerable size. The voltage is applied through a large mercury diode and a synchronous contact arrangement m to the tube under test in series with a resistance box R_s and a condenser input load C_L and R_L. This load

[6] Demonstrated, Rochester Fall Meeting, Rochester, N. Y., November 18, 1935.

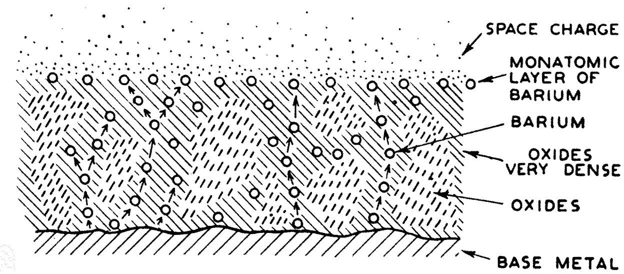


Fig. 11—Representation of cathode coating.

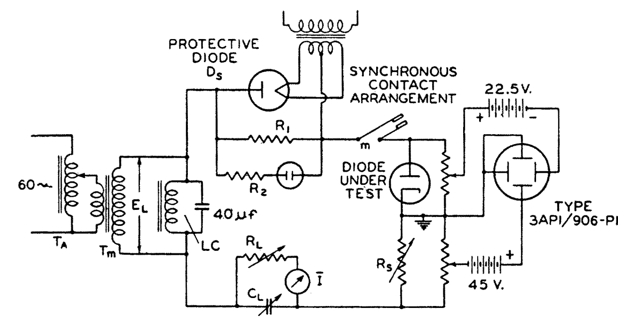


Fig. 12—Peak emission test circuit.

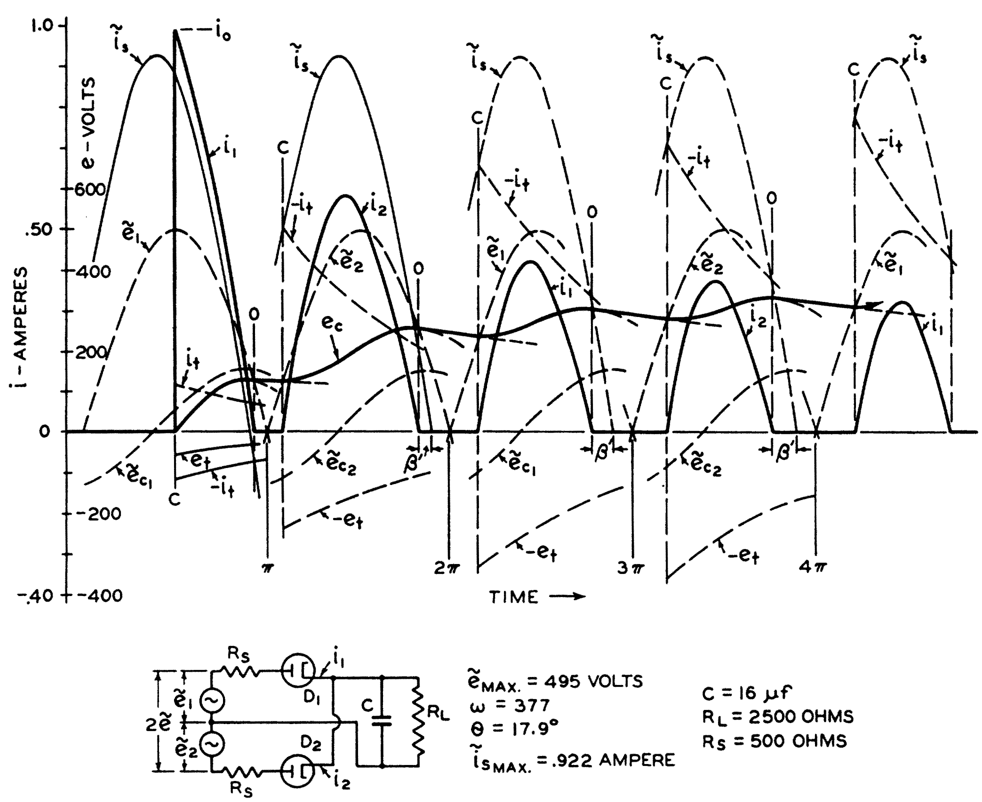


Fig. 13—Starting conditions in a full-wave, condenser-input circuit with large series resistance.

permits adjustment of the peak-to-average current ratio. Variation of R_L changes the average current. Variation of C_L and phasing of the synchronous contact m with respect to the 60-cycle line voltage permit regulation, within wide limits, of the rate of change and duration of the current pulses.

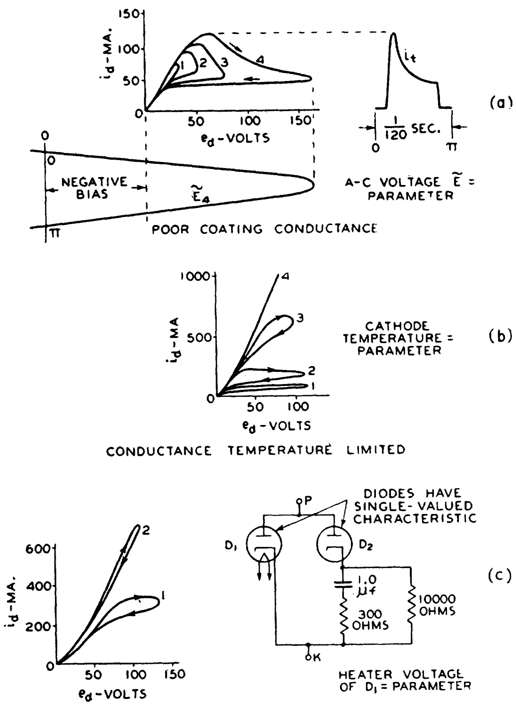


Fig. 14—Double-valued characteristics of actual and artificial diodes showing coating saturation.

The dynamic voltage-current characteristic of the tube under test is observed on a cathode-ray oscillograph connected in the conventional manner. Calibration deflections are inserted (not shown) by other synchronous contacts to provide accurate and simultaneously visible substitution co-ordinates which may be moved to any point in the characteristic.

The motor-driven synchronous contactor closes the circuit at a desired instant of the line-voltage cycle. The circuit may then be maintained closed for approximately 30 cycles to allow decay of the starting transient (see Fig. 13). It is then opened for approximately 70 cycles to allow time for the discharge of condenser C_L. This cycle repeats continuously. The diode D_S in series with the tube under test protects it against damage in case it breaks down or arcs, because the diode takes up the inverse voltage if a given small reversed current determined by R_1 is exceeded. This condition is indicated by a small glow tube in shunt with D_S.

2. Coating Characteristics

A theory of electron movement and conditions in oxide coatings has been formulated after careful analysis of saturation characteristics observed on the curve tracer. As saturated coatings produce closed reactive loops in the characteristic, it is found necessary to assume the existence of a capacitance in the diode itself. Because of its large value (see Fig. 14 (c)), this capacitance requires a dielectric thickness approaching crystal spacing and, hence, must be located inside the coating. It is beyond the scope of this paper to report the many investigations which led to this particular conception.

The oxide coating is an insulator at room temperature. At increased temperatures, it becomes conductive (normal operating temperatures are between 1000 and 1100 degrees Kelvin). Electronic conduction may be thought of as occurring by relay movement of electrons under the influence of electrostatic potentials in the coating, which is a layer containing insulating oxide crystals (shaded areas in Fig. 11) interposed with metal atoms and ions (circles). These have been produced during the activation and aging processes by high cathode temperature and electrolysis. The required potential gradients can be produced by rather small potentials because of the minute distances in the structure; the potential drop throughout the coating, therefore, is low under normal conditions.

The conduction is high, when a sufficient number of relay paths not broken by oxides have been formed and when electron movement is facilitated by the loosening of the atomic structure which takes place at increased temperatures.

The coating is not necessarily a homogeneous conductor as it may consist of many sections operating in parallel but having different conductance values with

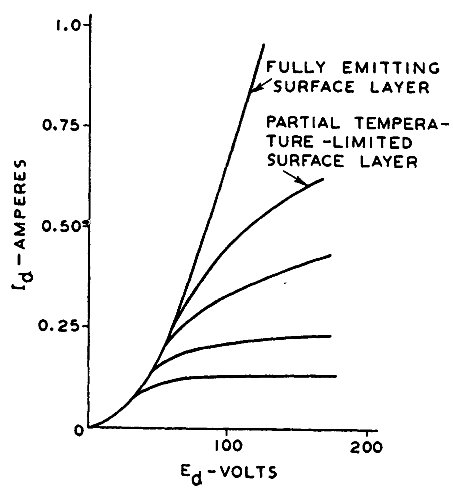


Fig. 15—Single-valued diode characteristics.

individual temperature parameters. At increased plate potentials, poorly conducting sections tend to saturate, the section potential becoming more positive towards the surface. Negative-grid action of neighboring sections with higher conductivity may tend to limit emission from the surface over the poor section but the increased positive gradient towards the saturating section causes it to draw electrons from the surrounding coating towards its surface. Further increase in current demand may then saturate the better conducting paths and may even fuse them, thus forcing current through poorer sections. Forced electron flow results in local power dissipation and temperature increases and may cause ionization and electrolysis accompanied by liberation of gas (oxygen) and formation of barium metal; i.e., it causes an accelerated activation process.

These conditions in the diode coating, therefore, should furnish a voltage-current characteristic of purely ohmic character as long as activation-gas liberation is substantially absent. Characteristics of this type are single valued. Single-valued characteristics indicate, however, unsaturated ohmic coating conductance and limiting surface emission when moderate-current densities are involved as will be apparent from the following discussion. As cathode and coating temperatures are relatively slowly varying parameters, characteristics such as shown in Fig. 15 are observed on the cathode-ray curve tracer. The characteristic

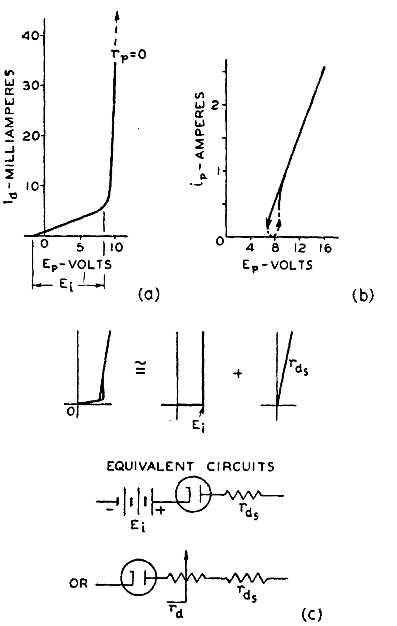


Fig. 16—Characteristics and equivalent circuit for hot-cathode, mercury-vapor diodes.

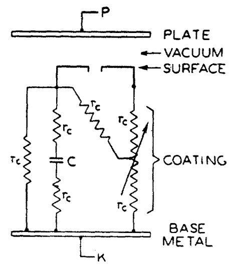
of diodes containing larger amounts of gas exhibits a discontinuity or "gas loop" (compare Fig. 16 (b)) which is recognized by the fact that corresponding current values after ionization require less diode potential than before "breakdown." The characteristic, hence, is steeper than normal.

3. Transient Emission

Let us now consider the action of insulating oxides in the coating. They block many possible electron paths to sections of the surface layer which, therefore, cannot emit steady electron currents. However, electrons can be moved to the oxide surfaces and a displacement current can flow in these coating sections allowing transient-emission currents to be drawn from the corresponding surface sections.

The displacement current in the coating and the corresponding transient surface emission represent a certain fraction of the total diode current, which may permit a total emission current of short duration much in excess of the possible steady-state conduction current. The "transient-emission" current depends on the effective capacitance value of the blocking oxides, their series and shunt conductance in the coating, the emission and area of corresponding surface elements to the plate as well as on the external plate-circuit impedance, and the wave form of the applied plate voltage.

For the purpose of analysis, therefore, we may draw representative networks such as shown in Figs. 17 or



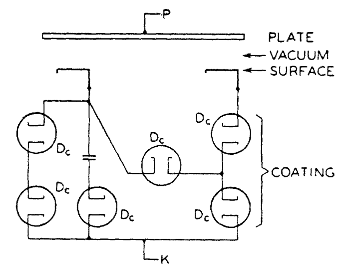


Fig. 17 (*left*)—Circuit network representing the coating impedance in high-vacuum diodes.
Fig. 18 (*right*)—Same as Fig. 17 with resistances replaced by special diodes.

18 and show the temperature-controlled coating conductances r_c as a network of "close-spaced diodes" which may conduct in two directions, each one having a single-valued characteristic which may be unsaturated or saturated depending on the assumed conditions in the coating; the conductance values of these "diodes" depend on the number of parallel or series paths they represent.

The diode contains, therefore, in its coating, a type of condenser-input load circuit, which is analyzed later on in this paper; its action explains double-valued voltage-current characteristics obtainable from the diode alone.

Consider a high plate voltage suddenly applied by means of a switch to a diode as in the circuit of Fig. 19.

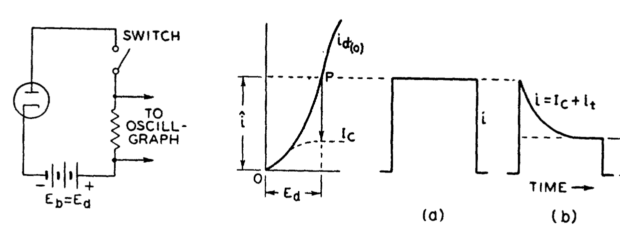


Fig. 19—Circuit for observation of peak emission transients.

If the coating is not limiting, the current obtained is that at a point P on the corresponding diode characteristic. Hence, the current wave form in the circuit is as shown in Fig. 19(a). If the surface emission is assumed to be unchanged, but the coating conductance is limited, due to an insufficient number of "coating diodes" and too many nonconducting oxide groups, the wave form of Fig. 19 (b) is obtained. At the instant when the switch is closed the current value $\hat{\imath}$ is demanded by E_d from the surface layer; the conduction current in the coating is limited to the value I_c by saturation of the "coating diodes." Because of the oxide capacitance, a displacement current can flow and charge up the oxides, but their charge may be limited by hypothetical series diodes.

The coating resistance is extremely low[7] below saturation, but becomes infinite when the conduction current is saturated; the charging current must then flow in the plate circuit (external) of the diode. The total plate current is, therefore, the sum of the conduction current I_c and a "transient-emission" current. The "coating transient" decays to zero the same as normal transients at a rate depending on the actual shunt-conductance value and the total series resistance in the circuit (Fig. 19(b)). The decay can be changed by adding external resistance in the plate circuit. When the surface emission is good, i.e., as long as the total vacuum-space plate current is space-charge-limited, the current will rise initially to the value (point P) determined by the applied potential, but will then decay to the saturation value determined by the coating conductance.

The condition of oxide-coated cathodes can, therefore, not be judged alone by their capability of furnishing high peak currents, but the time of current flow and the current wave form must also be carefully considered, because the diode characteristic may not be single-valued. Fig. 14 shows characteristics which are not single-valued. It should be noted that the characteristic loops are formed in the opposite sense as gas loops. Their extent depends on the time interval involved and the current value exceeding the unsaturated conductance current. An artificially produced characteristic of this type is also shown in Fig. 14(c). The loop size can be varied by adjusting the cathode temperature of the shunting diode. Both diodes had single-valued characteristics.

4. *Current Overload and Sputter*

The degree of activation is not stable during the life of the cathode. Coating conductance and surface emission change. Factors affecting the change are the coating substances, the evaporation rate of barium which depends on the base material, and the operating conditions to which the cathode is subjected. This life history of the cathode is the basis on which current ratings are established. Rectifier tubes especially are subject to severe operating conditions. If a diode is operated with too high a current in a rectifier circuit and its surface emission is decreased to the saturation value, then the tube-voltage drop will increase rapidly and cause excessive plate dissipation and destruction of the tube. Should the coating conductance in this diode decrease to a value which limits the demanded current, power is dissipated in the now-saturated coating with the result that the coating-voltage drop and coating temperature are raised. The voltage and temperature rise in the coating may cause reactivation but also may become cumulative and melt the coating material. We may consider that good conducting paths are fused or that a dielectric breakdown of oxide capacitance occurs; in any event vapor or gas discharges result from saturated coatings. In most cases breakdown occurs during one of the following inverse voltage cycles as observed on the curve tracer. A saturation loop is first formed as shown in Fig. 14 and a certain time must be allowed for diffusion of the gas into the vacuum space. Fusion of coating material may also occur during the conduction period. These breakdowns are known as "sputter," and in usual circuits destroy the cathode.

A second type of sputter is caused by the intense electrostatic field to which projecting "high spots" on the plate or cathode are subjected. The resulting current concentration causes these spots to vaporize with the result that an arc may be started. Hundreds of scintillating small spots can be observed at first at very high applied surge potentials, but may be cleared after a relatively short time.

Transient peak currents of 25 amperes per square centimeter have been observed from well-activated oxide-coated cathodes. The stable peak emission over an extended period is usually less than one third of this value.

5. *Hot-Cathode Mercury-Vapor Diodes*

The breakdown voltage E_i of mercury vapor for cumulative ionization is a function of the gas pressure and temperature. It is approximately 10 volts in the RCA-83 and similar tubes. A small electron current begins to flow at $E_p=0$ (see Fig. 16), and causes ionization of the mercury vapor. This action decreases the variational diode resistance r_p to a very low value. The ionization becomes cumulative at a certain current value ($r_p=0$ at 40 milliamperes in Fig. 16(a)), and causes a discontinuity in the characteristic. Hence, it is not single-valued within a certain voltage range. Beyond this range (see Fig. 16(b)), the slope (r_p) of the characteristic becomes again positive until saturation of the emitter is reached.

For circuit analysis, the mercury-vapor diode may be replaced by a bucking battery having the voltage E_i and a fixed resistance as shown in Fig. 16(c); or the diode characteristic may be replaced by an ideal rectangular characteristic and its equivalent resistance values and the series resistance r_{ds} as shown.

The first representation is adequate for most practical calculations. The value r_{ds} is in the order of 4 ohms for small rectifier tubes. The low series resistance and the small constant-voltage drop E_i are distinct advantages for choke-input filters, as they cause very good regulation; the low resistance, however, will give rise to enormously high starting transients in condenser-input circuits, in case all other series resistances are also small. The destruction of the coating in

[7] Its magnitude depends on the number of series diodes and, hence on the barium content and thickness of the coating.

mercury-vapor diodes is caused by concentration of current to small sections of the coating surface and not by heat dissipation in the coating. Mercury-vapor diodes as well as high-perveance (close-spaced), high-vacuum diodes having oxide cathodes should, therefore, be protected against transient-current overloads when they are started in low-resistance circuits to prevent destruction of the cathode coating.

6. Protective Resistance Values.

Very high instantaneous peak currents may occur in noninductive condenser-input circuits when the circuit is opened long enough to discharge the condenser, but reclosed before the cathode temperature of the diode has decreased substantially. The maximum peak current $\hat{I}_{\max}$ occurs when closing the circuit at peak line voltage. At the instant of switching, C is a short circuit and the current $\hat{I}_{\max}$ is limited only by the series resistance (including diode) of the circuit,

$$\hat{I}_{\max} = \frac{\tilde{e}_{\max}}{\hat{R}_s}.$$

For a given maximum diode current $\hat{I}_{d\max}$ and the corresponding diode peak voltage $\hat{E}_{d\max}$, the minimum effective series resistance R_s in the circuit must hence be

$$R_s = \frac{\tilde{e}_{\max} - \hat{E}_{d\max}}{\hat{I}_{d\max}}.$$

This limiting resistance must be inserted in series with low-impedance sources (power line in transformerless sets). Commercial power transformers for radio receivers have often sufficient resistance besides some leakage reactance to limit starting currents to safe values.

III. Circuit Analysis

General

The rectifier diode is a switch operated in synchronism with the applied alternating-current frequency. Switching in reactive circuits causes transients. The total current in the circuit may be regarded as the sum of all steady-state currents and transient currents within the time between two switching operations. Steady-state voltages (e_s) and currents (i_s) in the particular circuit before and after switching are determined without difficulty. It is very helpful to draw them approximately to scale and with proper phase relation.

The switching time of the diode is then located on the graph. Currents change at switching time t_0 from i_1 to $i_2 = i_{s(2)} + i_t$ and voltages from e_1 to $e_2 = e_{s(2)} + e_t$. The transients i_t or e_t are zero, when the current change does not occur in an inductive circuit or when a voltage change is not required on a capacitance at the time of switching. A sudden change Δi_L or Δe_c demanded at t_0 causes transients. They initially cancel the change Δi_L or Δe_c because an inductance offers infinite impedance to an instantaneous change in total current and a capacitance offers zero impedance to an instantaneous voltage change.

The initial transient values are, therefore,

$$i_{t(0)} \text{ in } L = -\Delta i_L$$

and

$$e_{t(0)} \text{ on } C = -\Delta e_c.$$

The transients decay exponentially from their initial value.

According to the decay time of the transients, fundamental rectifier circuits may be classified into two principal groups: (1) circuits with repeating transients in which the energy stored in reactive elements decreases to zero between conduction periods of the diode; and (2) circuits with chain transients in which (a) the magnetic energy stored in the inductance of the circuit remains above zero value, and (b) the electric energy stored in the capacitance of the circuit remains above zero value. The much used "choke-input" and "condenser-input" circuits fall under the second group.

We shall analyze the operation in important circuits, i.e., the full-wave choke-input circuit and condenser-input circuits.

1. The Full-Wave Choke-Input Circuit

a) Operation of circuits with L and R_s in the common branch circuit

Circuit and operation are shown in Fig. 20. The analysis is made by considering first one of the diodes short-circuited to obtain the phase relation of the alternating voltage $\tilde{e}$, and the steady-state current $\tilde{\imath}_s$, as shown. If we assume that the diode D_1 closes the circuit I at the time $\tilde{e}=0$, a transient i_t with the initial value $i_{t(0)} = -\tilde{\imath}_{s(0)}$ will flow in the circuit. The total current i is the sum of the currents $\tilde{\imath}_{s1} + i_t$. It starts, therefore, at zero and rises as shown until the second switching operation occurs at the commutation time $t=\pi$ when the second diode D_2 receives a positive plate voltage. The total current i in circuit II after $t=\pi$ is again the sum of currents $\tilde{\imath}_{s2} + i_t$ ($\tilde{\imath}_{s2}$ has reversed polarity with respect to $\tilde{\imath}_{s1}$ and is not shown in Fig. 20) but the initial value $i_{t(0)}$ of the second transient is increased by the value $i_{(\pi)}$ now flowing in the common circuit inductance L.

The current $i_{t(0)}$ increases, therefore, at every new switching time until the decay of the transient $i_{t(n)}$, during the time $t=\pi$, is numerically equal to the steady-state current rise $2\tilde{\imath}_{s(0)}$. For the final operating current at the nth commutation time (see right side of Fig. 20)

$$(i_{(n\pi)} - \tilde{\imath}_{s(0)})(1 - \epsilon^{-R_s/2FL}) = -2\tilde{\imath}_{s(0)}$$

$$i_{(n\pi)} = \tilde{\imath}_{s(0)} - (2\tilde{\imath}_{s(0)}/1 - \epsilon^{-R_s/2FL}). \tag{7}$$

A broken line is shown connecting all commutation-current values. This line represents closely the average current $\bar{I}$ in the common circuit branch. The final average current $\bar{I}$ in the load resistance R_s is given by

(7), when the transient decay $i_{t(n)}$ during the time π (Fig. 20) can be regarded as linear (low steady-state power factor of circuit). The average plate current per diode is $\bar{I}_p = 0.5\bar{I}$, since each diode conducts

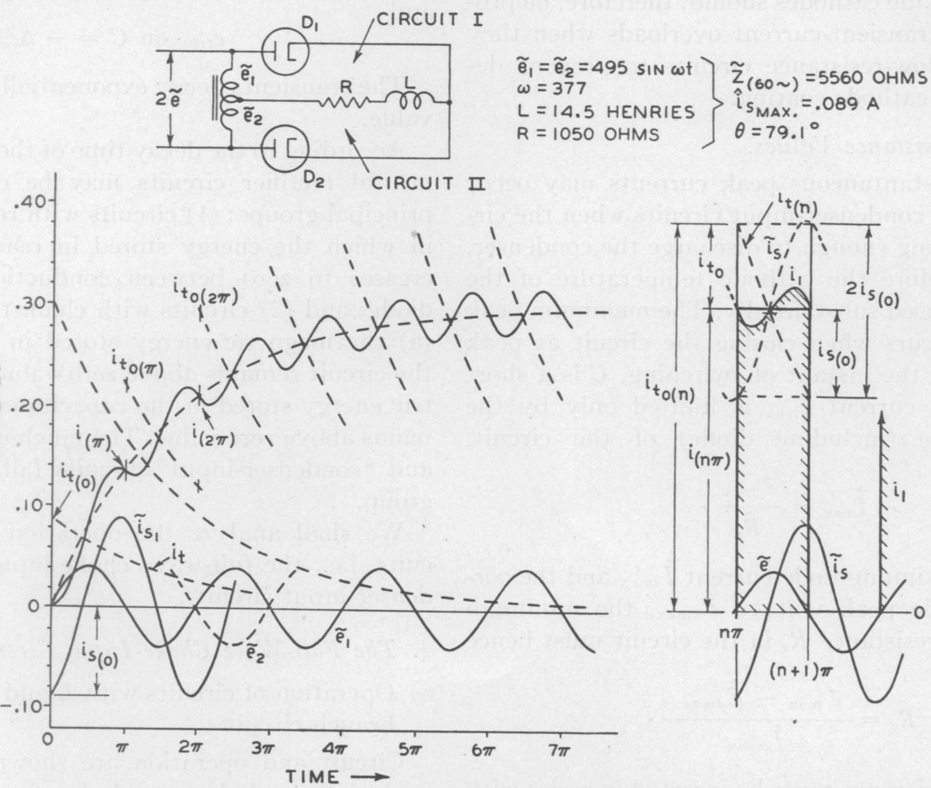


Fig. 20—Starting and operating conditions of an aperiodic full-wave, choke-input rectifier circuit.

alternately, and passes a current pulse shown by the shaded area in Fig. 20. With the numerical values of the circuit Fig. 20 substituted in (7) we obtain

$$i_{(n\pi)} \cong \bar{I} = 0.298 \text{ ampere.}$$

The oscillogram in Fig. 21 was taken on circuit Fig. 20.

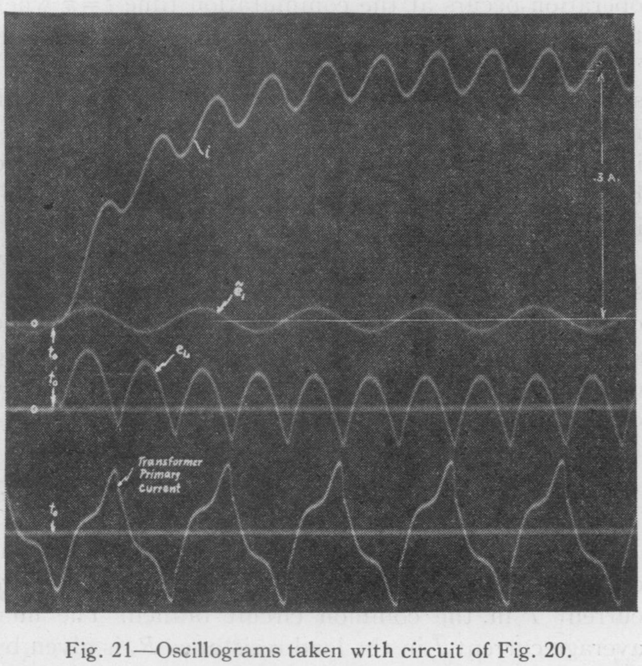


Fig. 21—Oscillograms taken with circuit of Fig. 20.

b) The full-wave choke input circuit with capacitance-shunted resistance load

For large capacitance values the by-passed load resistance R_L of practical circuits is equivalent to a battery having a voltage $\bar{E}_B = \bar{I}R_L$, where $\bar{I}$ is the average load current or battery-charging current. The circuit operation (see Fig. 22) is described by obtaining $\bar{I}$ as a function of $\bar{E}_B$. The final commutation current $i_{(n\pi)}$ which is closely the average current $\bar{I}$ is given by

$$\bar{I} \cong i_{(n\pi)} = (\bar{I}_B + \tilde{\imath}_{s(0)}) - 2\tilde{\imath}_{s(0)}/(1 - \epsilon^{-R_s/2FL}) \quad (7b)$$

and similar to (7) except for an increase of the transient term due to the battery current $\bar{I}_B = \bar{E}_B/R_s$.

Equation (7b) is valid only over a range of load or battery voltage ($\bar{E}_B$) in which switching time and conduction period of the diodes are constant ($\phi = \pi$). This range is shown by the solid part of curve F in Fig. 22 and ends at a particular current and voltage of the circuit characteristic marked the "critical point."

The critical point is the operating condition at which the instantaneous current i in the common branch circuit has zero value at one instant. An analysis shows that in the range $\bar{E}_B = \tilde{e}_{\max}$ to $\bar{E}_B = E_B'$ each diode circuit operates independently as a half-wave rectifier circuit (battery-charger operation, curve H in Fig. 22). Current commutation begins at E_B'; the diode circuits begin to interact, but the conduction angle is still $\phi < \pi$.

The conduction angle increases from $\phi = 0$ at $\bar{E}_B = \tilde{e}_{\max}$ to $\phi = \pi$ at the critical point E_B'' which marks the beginning of chain current operation.

The critical operating condition is obtained by solving for $i=0$ with $\phi=\pi$ or by equating the direct current to the negative peak value of the total alternating current in L. The critical point is hence specified by a certain current or by a certain ratio K of direct-current resistance to alternating-current impedance in the circuit. With reference to the equivalent circuit treated in the following section, a relation to the fundamental alternating-current component of the rectified current (see (10)), i.e., to the impedance $Z_{(2F)}$, at double line frequency is more useful. We set, therefore,

$$\frac{(R_s + R_L)}{Z_{(2F)}} = K \tag{8}$$

and determine significant values of K for particular circuit impedance conditions.

If we neglect harmonics higher than $2F$, which contribute little to the peak value because of phase shift and increasing attenuation in L, the peak ripple current (equation (10)) becomes

$$i_{\min} = 4/3\pi(\tilde{e}_{\max}/Z_{(2F)})$$

and setting it equal to the average current

$$\bar{I} = 2/\pi(\tilde{e}_{\max}/(R_s+R_L))$$

we obtain $K=1.5$ for the ratio as shown in (8).

The exact solution for the critical current can be obtained from a graphic analysis by simple reasoning for the case $R_s=0$. The general solution will only be indicated. It is obtained by drawing the complementary curve $(1-i_t)$ of the total transient beginning at the time $\tilde{e}=0$ (see Figs. 23 and 24) and shifting it upward until it touches the current $\tilde{\imath}_s$, thus solving

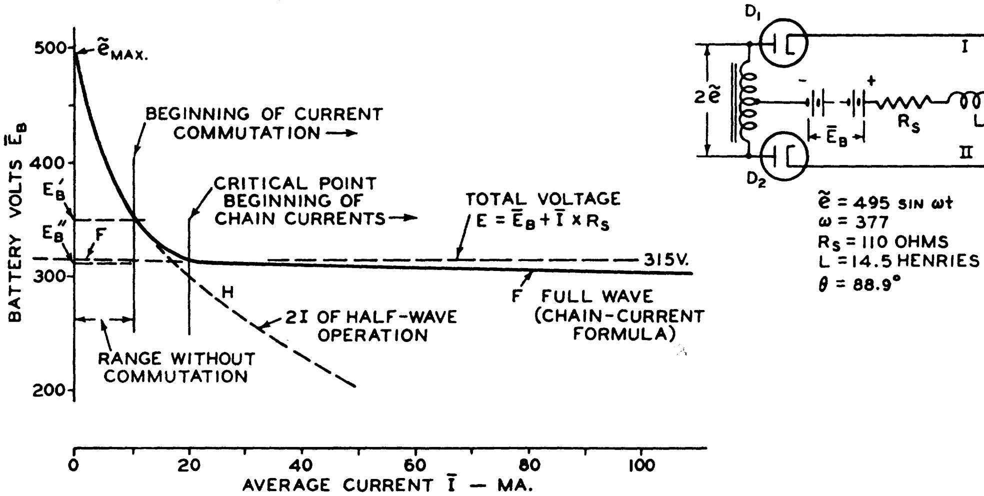


Fig. 22—Operating characteristic of a full-wave, choke-input rectifier circuit with battery load $\bar{E}_B$ or resistance load $R_L = \bar{E}_B/\bar{I}$ shunted by a large capacitance.

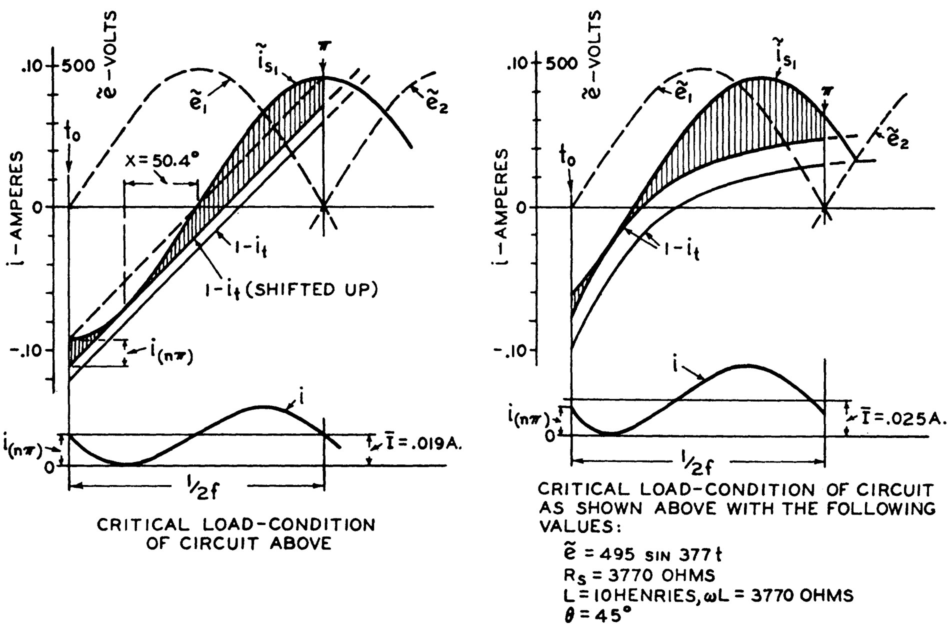


Fig. 23 (*left*)—Graphic solution for the critical load condition with negligible series resistance.
Fig. 24 (*right*)—Same as Fig. 23 but with large series resistance.

for $i=0$ at the point of contact. Note that $i_{(n\pi)}$ is the same at t_0 and π in both cases shown.

For $R_s=0$, the transient section becomes a straight line having the slope $2/\pi$ and running parallel to the peak-to-peak connecting line of $\tilde{\imath}_s$. The sine-wave slope $2/\pi=-\cos x$ gives the point of contact at $X=50.4$ degrees (Fig. 23), and the peak ripple current is obtained from

$$i_{\min} = \tilde{\imath}_{s\max}\left(\sin 50.4° - \frac{50.4}{90}\right) = 0.211\tilde{\imath}_{s\max}$$

$$= 0.211\ \frac{\tilde{e}_{\max}}{\omega L}\cdot$$

Equating this value to the average current given by (10), we obtain the value $K=1/0.211=1.51$ for cir-

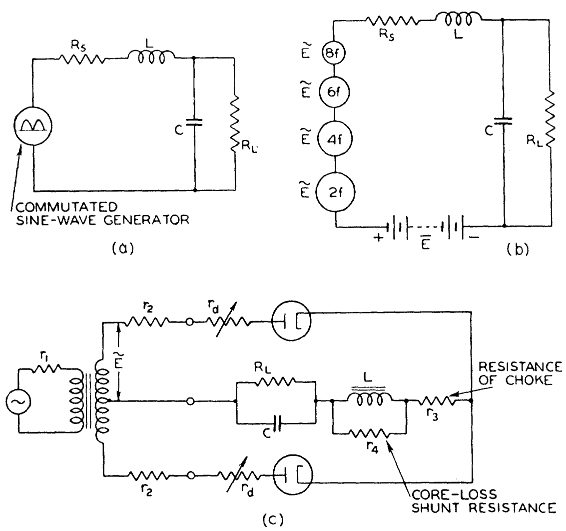


Fig. 25—Components of equivalent and practical full-wave, choke-input circuits.

cuits with $R_s=0$. The graphic analysis of circuits with larger resistance (see Fig. 24) furnishes K values sufficiently close to 1.5 to justify the use of this constant for all practical purposes. For practical circuits with $2\omega L \gg 1/2\omega C$ we may further write $Z_{(2F)} \cong 2\omega L$ and obtain the *critical inductance*[8]

$$L_0 \cong (R_s = R_L)/2\omega K = (R_s + R_L)/6\pi F. \qquad (9)$$

c) Equivalent circuit for the chain current operating range ($\phi=\pi$ or $(R_s+R_L)<1.5Z_{(2F)}$)

Inspection of (7b) shows that average and commutation current are directly proportional to the sum of battery current $\bar{I}_B$ and a term having a constant current value "I_K" for a given circuit and constant line voltage. Equation (7b) can be changed into the form

$$\bar{I} = (I_K R_s)/(R_L + R_s),$$

indicating that the secondary circuit may be replaced by an equivalent circuit without switches and energized by a voltage which contains a constant direct-current component $\bar{E}=I_K R_s$. The equivalent voltage in the circuit is the commutated sine wave resulting from the sequence of positive half cycles $+\tilde{e}_1$ and $+\tilde{e}_2$ in the range $\phi=\pi$. The equivalent circuit is shown in Fig. 25(a). The single generator may be replaced by a battery and a series of sine-wave generators (Fig. 25(b)) having amplitudes and frequencies as given by the following equation of the commutated sine wave:

$$e=\frac{2\tilde{e}_{\max}}{\pi}\left(1-\frac{2\cos 2F}{1\cdot 3}-\frac{2\cos 4F}{3\cdot 5}-\frac{2\cos 6F}{5\cdot 7}-\cdots\right). \qquad (10)$$

All current components in the circuit may now be computed separately by steady-state methods; the direct-current component is the total average voltage $\bar{E}$ in the circuit.

Some useful relations of voltage components are: Line voltage induced in one half of the secondary winding (root-mean-square)

$$|\tilde{E}| = 1.1\bar{E}$$

Total average voltage

$$\bar{E} = \begin{cases} 0.90|\tilde{E}| \\ 0.637\tilde{e}_{\max} \end{cases}$$

Voltage of frequency $2F$ (root-mean-square)

$$|\tilde{E}|_{2F} = \begin{cases} 0.424|\tilde{E}| \\ 0.471\bar{E} \end{cases} \qquad (11)$$

Voltage of frequency $4F$ (root-mean-square)

$$|\tilde{E}|_{4F} = \begin{cases} 0.085|\tilde{E}| \\ 0.0945\bar{E} \end{cases}$$

Total choke voltage (root-mean-square)

$$|E|_L = \begin{cases} \sqrt{|\tilde{E}|^2 - \bar{E}^2} \\ 0.482\bar{E} \end{cases}$$

The current components in the common circuit branch are calculated from the above voltages divided by the impedance of one branch circuit at the particular frequency. Because the current is commutated every half cycle of the line frequency from one to the other branch circuit, the average current in each diode circuit is one half of the total average current; and root-mean-square values of currents or current components in each branch circuit are obtained by multiplying the root-mean-square current values in the common circuit branch by $1/\sqrt{2}$. The peak current in each diode circuit has the same value as in the common circuit branch.

Average load current

$$\bar{I} = \frac{\bar{E}}{R_s + R_L}$$

Average plate current (per diode)

$$\bar{I}_p = 0.5\bar{I} \qquad (12a)$$

[8] The relation $L_0=R_L/1000$ was given on an empirical basis for $\omega=377$ by F. S. Dellenbaugh, Jr., and R. S. Quinby, "The important first choke in high-voltage rectifier circuits," *QST*, vol. 16; pp. 14–19; February, 1932.

Double-frequency current (root-mean-square) in common circuit branch

$$|\tilde{I}|_{2F} = \frac{|\tilde{E}|_{2F}}{Z_{(2F)}}$$

Total current (root-mean-square) in common circuit branch

$$|I|_L = \sqrt{\bar{I}^2 + |I|^2_{2F}}$$

Root-mean-square diode current or root-mean-square current per transformer winding

$$|I|_p = \frac{|I|_L}{\sqrt{2}} \qquad (12b)$$

Peak diode current

$$\hat{\imath}_d = \bar{I} + (|\tilde{I}|_{2F} \times \sqrt{2})$$

The regulation curve for a circuit with high-vacuum diodes is the sum of the 3/2-power-law diode characteristic and the ohmic series resistance r_2 of one branch circuit as shown in Fig. 26. The curve is correct for constant voltage $\tilde{e}$ and beyond the critical current value. In practical circuits, the voltage source $\tilde{e}$ has a certain equivalent resistance, which must be added to r_2. The regulation curve Fig. 26 is invalid below the critical current value and must be replaced by a curve following the laws discussed for Fig. 22.

The equivalent internal resistance of the rectifier circuit as a direct-current supply source is the slope of the regulation curve at the current value under consideration. This value should be used for steady-output conditions only, since the reactances in the load circuit cause transients at the instant of sudden load changes.

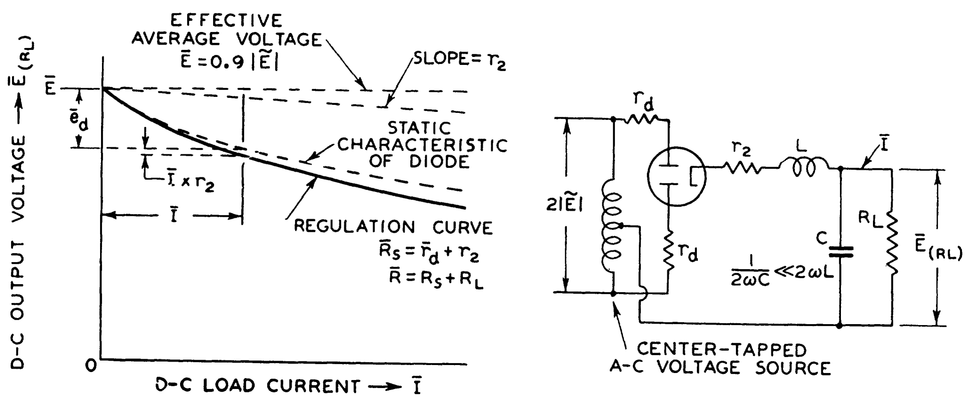


Fig. 26—Regulation characteristic of a full-wave, choke-circuit with high-vacuum diode.

The total power dissipated in diode and load circuits of the practical secondary circuit shown in Fig. 25(c) is the sum of the power losses in the circuit resistances. In equation form, it is

Total power = series-resistance loss
+ choke-core loss
+ direct-current power in load.

The plate dissipation per diode is given by

$$P_d = 0.5|I|^2_L \times |r_d|. \qquad (13)$$

With reference to (5), we have

$$P_d = 0.5|I|_L^2 \times \frac{\bar{e}_d}{\bar{I}} \qquad (14)$$

where $\bar{e}_d$ is the diode voltage taken from the static diode characteristic at the output-current value $\bar{I}$.

d) Regulation

The regulation of choke-input circuits is determined by the total series resistance $\bar{R}_S$, since the voltage $\bar{E}$ in the circuit is constant in the useful chain current range for an energizing alternating voltage of constant value. Thus, the regulation curve has the slope $\bar{R}_S$ (see Fig. 26), which includes the diode resistance.

2. *The Condenser-Input Circuit*

In rectifier circuits with shunt-condenser-input loads, the condenser is alternately charged and discharged. In the final state of operation, charge and discharge are balanced. The graphic analysis of such circuits is comparatively simple and readily followed. Formulas for the calculation of specific circuit conditions are easily derived from the constructions.

a) Circuits without series resistance

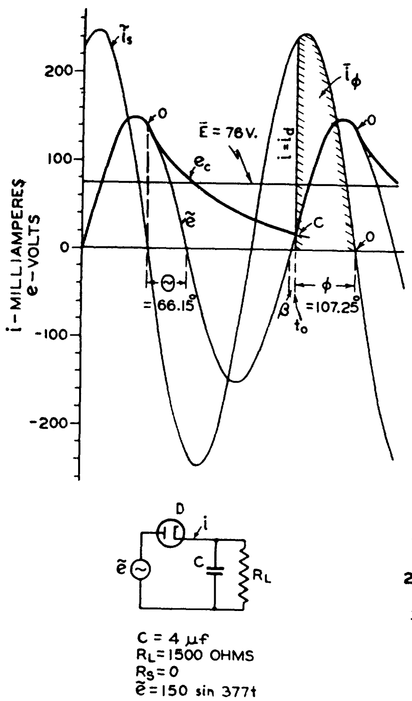
The graphic analysis of a half-wave rectifier circuit without series resistance (R_S) is illustrated in Fig. 27. Steady-state voltage $\tilde{e}$ and current $\tilde{\imath}_s$ are constructed on the assumption that the diode is short-circuited. The steady-state condenser voltage $\tilde{e}_c$ coincides with $\tilde{e}$ because $R_S = 0$.

The diode timing is as follows:

The diode opens the circuit at point 0 when the diode current becomes zero.

Since the condenser-discharge circuit consists of C and R_L, the condenser voltage decays exponentially as shown. At point C it has become equal to the energizing voltage $\tilde{e}$. The diode becomes conducting and closes the circuit. Because there is no potential difference between the steady-stage voltages $\tilde{e}$ and $\tilde{e}_c$, the condenser does not receive a transient charge. The current, therefore, rises instantly to the steady-state value of the $\tilde{\imath}_s$ curve and follows it until zero at point 0.

The timing of the full-wave circuit in Fig. 28 is quite similar. The time for the condenser discharge through R_L is reduced since e_c meets the positive half cycle $\tilde{e}_2$ and thus closes the circuit through D_2. Point C in Fig. 28 is located at a higher value of $\tilde{e}$ than in Fig. 27. The conduction angle ϕ is consequently reduced although C, R_L, and Θ have the same values in both circuits. The average current in the full-wave circuit is, therefore, smaller than twice that of the half-wave circuit.



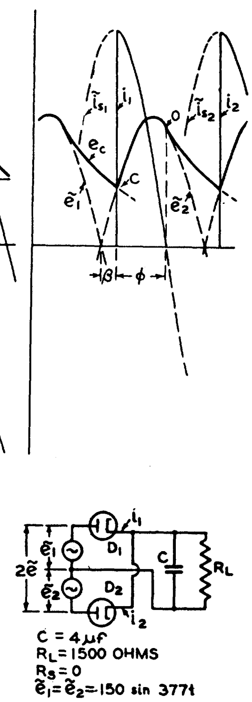


Fig. 27 (*left*)—Graphic solution of operation for a half-wave, condenser-input circuit without series resistance.

Fig. 28 (*right*)—Graphic solution of operation for a full-wave, condenser-input circuit without series resistance.

Some of the relations obtainable directly from Figs. 27 and 28 are

i. the conduction angle $\phi = 180° - \Theta - \beta$. (15)

The intersection of $\tilde{e}$ with the decaying voltage e_t furnishes for half-wave operation ($n=1$) and full-wave operation ($n=2$)

$$\left.\begin{array}{llll} \text{ii.} & \sin\beta = \sin\Theta\epsilon^{-(\pi+\Theta+\beta)/\omega CR_L} & \text{for} & n=1 \\ \text{and} & \sin\beta = \sin\Theta\epsilon^{-(\Theta+\beta)/\omega CR/_L} & \text{for} & n=2 \end{array}\right\} \quad (16)$$

where π, Θ, and β in the exponents are in radius. This equation may be solved graphically or by trial and error, varying β.

iii. The average current during conduction time is

$$\bar{I}_{(\phi)} = I_s(1 - \cos\phi)/\phi.$$

It is the area under a sine-wave section divided by its base. Hence, the average plate current is as shown in (iv).

$$\text{iv.} \qquad \bar{I}_p = \bar{\imath}_{(\phi)}\frac{\phi}{2\pi} = \frac{\hat{\imath}_s}{2\pi}(1 - \cos\phi). \qquad (17)$$

v. Average current $\bar{I}$ and voltage $\bar{E}$ in the load resistor are

$$\left.\begin{array}{ll} \bar{I} = \bar{I}_p & \text{for } n=1 \\ \bar{I} = 2\bar{I}_p & \text{for } n=2 \\ \bar{E} = \bar{I}R_L & \end{array}\right\}. \qquad (18)$$

vi. The diode peak current $\hat{\imath}_p$ is, obviously

$$\left.\begin{array}{lll} & \hat{\imath}_p = \hat{\imath}_s & \text{for } \phi > 90° \\ \text{and} & \hat{\imath}_p = \hat{\imath}_s \sin\phi & \text{for } \phi < 90° \end{array}\right\}. \qquad (19)$$

The performance of these circuits, hence, is determined by their power factor ωCR_L and the phase number n. It will be evident from the following that the series resistance R_s of practical circuits appears as an additional parameter which cannot be neglected.

b) Circuits with series resistance

In circuits with series resistance, the steady-state condenser voltage $\tilde{e}_c$ does not coincide with the supply

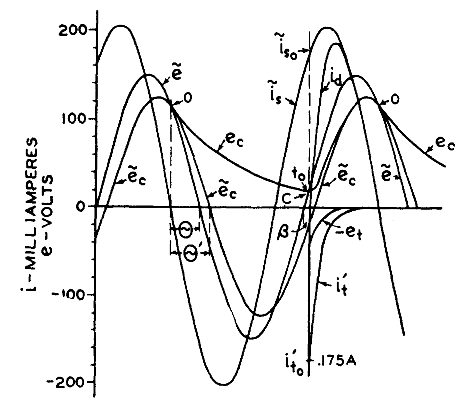


Fig. 29—Graphic solution of operation for a half-wave, condenser-input circuit with series resistance.

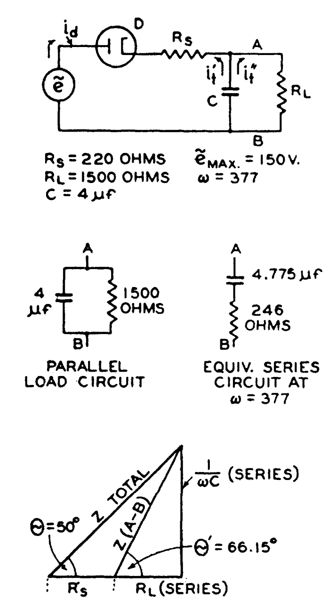


Fig. 30—Equivalent series circuit for the analysis of half-wave, condenser-input circuits with $R_s>0$.

voltage $\tilde{e}$, as illustrated in Figs. 29 and 30. Phase displacement and magnitudes of current and voltage under steady-state conditions are required for analysis of the circuit and are computed in the conventional manner. The parallel circuit $C||R_L$ is converted into an equivalent series circuit to determine the angles Θ and Θ' by which $\tilde{i}_s$ is leading $\tilde{e}_c$ and $\tilde{e}$, respectively. The steady-state condenser voltage $\tilde{e}_c$ in the parallel circuit equals the voltage across the equivalent circuit as shown by the vector diagram in Fig. 30.

The diode opens the circuit at the instant $i_d=0$. For circuit constants as in Fig. 30, the diode current i_d substantially equals $\tilde{i}_s$ at the time of circuit interruption because the transient component i_t' of the current, as shown later, has decayed to a negligible value. Point 0 is thus easily located. In circuits with large series resistance, however, $i_d=0$ does not coincide with $\tilde{i}_s=0$ due to slow decay of the transient i_t'. In both cases the condenser voltage $e_{c(0)}$ equals the voltage $\tilde{e}_{(0)}$ at the time 0, because $i_d=0$ and consequently there is no potential difference on R_S and transients do not occur at 0. The condenser voltage decays exponentially on R_L from its initial value at 0, as discussed for circuits with $R_S=0$, and meets the supply voltage $\tilde{e}$ again at point C. At this instant (t_0), the diode closes the circuit. Current and voltage, however, do not rise to their steady-state values as in circuits with $R_S=0$, because the steady-state voltage $\tilde{e}_{c(0)}$ differs from the line voltage $\tilde{e}_{(0)}$ by the amount $\Delta e_c=\tilde{i}_{s(0)}R_S$. A transient voltage of initial value $e_{t(0)}=-(\tilde{i}_{s(0)}R_S)$ occurs on C. It drives transient currents $i_{t'}$ and $i_{t''}$ determined by Ohms law through the resistances R_S and R_L respectively. (See Fig. 30.)

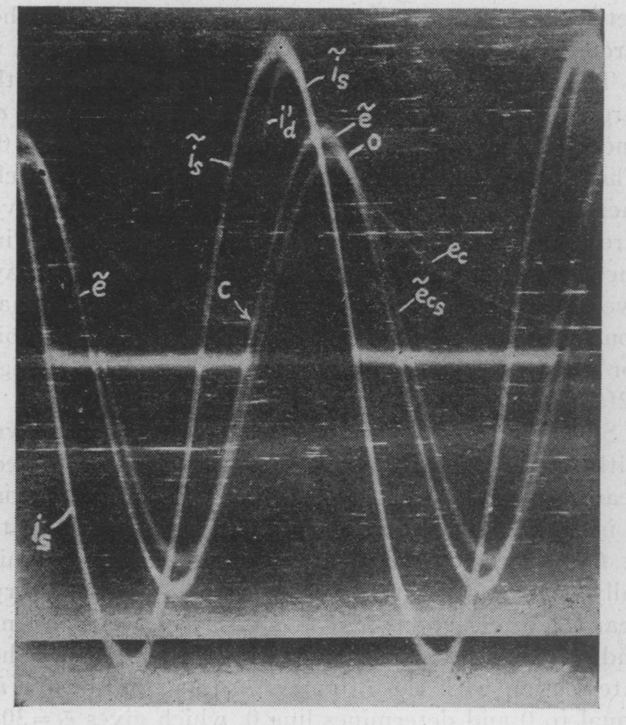


Fig. 31—Oscillograms taken with circuit of Fig. 30.

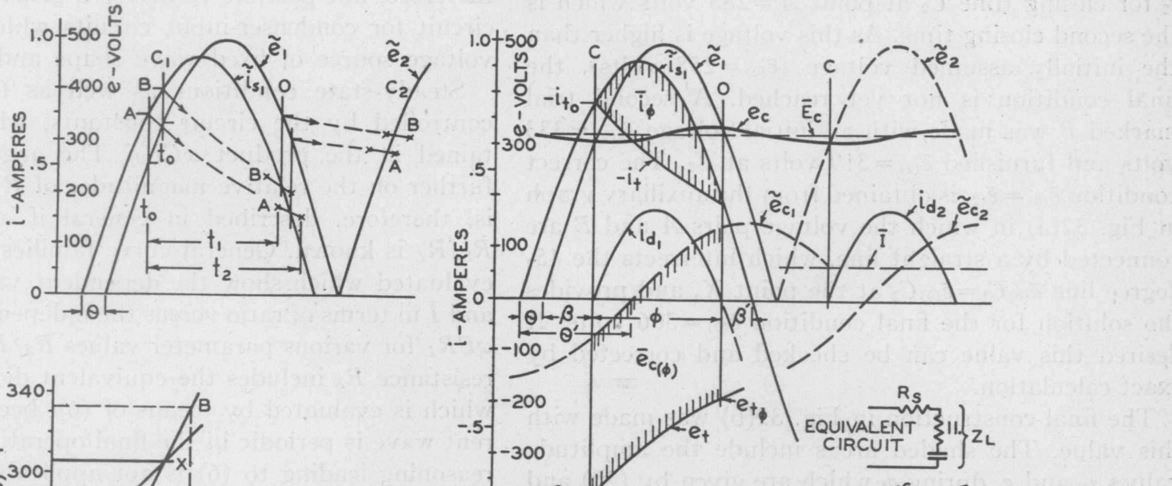


Fig. 32—Graphic solution of final operating conditions for circuit in Fig. 13.

The transients e_t and i_t' prevent voltage and current from following the steady-state wave forms, as

$$i_d = \tilde{i}_s + i_t' = \tilde{i}_s - \tilde{i}_{s(0)}\epsilon^{-t/(R_s||R_L)C} \tag{20}$$

and

$$e_c = \tilde{e}_c + e_t = \tilde{e}_c + R_s\tilde{i}_{s(0)}\epsilon^{-t/(R_s||R_L)C} \tag{21}$$

between the time t_0 and the opening time at 0.

For small values R_S and C, the transient decay is rapid as shown in Fig. 29 and point 0 is readily determined. The oscillogram Fig. 31 was taken on the circuit Fig. 30 and checks the graphic construction.

The solution of operating conditions in circuits with large time constants requires additional steps, as e_c and i_d do not reach steady-state values before $\tilde{\imath}_s=0$. The diode opens the circuit earlier at an angle β', which increases from cycle to cycle as shown for a full-wave circuit in Fig. 13. The condenser voltage e_c rises in successive conduction periods until its numerical decay over R_L equals the numerical rise during ϕ. This final condition is shown in Fig. 32(b). The graphic solution for the final operating condition is illustrated in Fig. 32(a) and is made as follows:

Steady-state current $\tilde{\imath}_s$ and voltage $\tilde{e}_s$ are drawn with proper phase relation. A closing time t_0 is assumed near the estimated average output voltage, condition A in Fig. 32(a) assumes $\tilde{\imath}_{s(0)}=0.7A$ and $\tilde{e}_{(0)}=258$ volts at t_0. The current transient i_t' is subtracted graphically from $\tilde{\imath}_s$. Only two points t_1 and t_2 are necessary near the intersection; t_1 gives a decay of 57.4 per cent and then t_2 gives a decay of 50 per cent from $\tilde{\imath}_{s(0)}$. The intersection with the $\tilde{\imath}_{s1}$ curve gives a solution for i_p equal to 0 and determines line 0, which gives $\tilde{e}_1=308$ volts which is also the voltage e_c. This voltage decays now over R_L until it intersects the following half cycle $\tilde{e}_2$ for closing time C_2 at point $A=283$ volts which is the second closing time. As this voltage is higher than the initially assumed voltage ($\tilde{e}_{(0)}=258$ volts), the final condition is not yet reached. A second trial marked B was made with an initial voltage $\tilde{e}_{s(0)}=333$ volts and furnished $\tilde{e}_{(2)}=319$ volts at C_2. The correct condition $\tilde{e}_{(0)}=\tilde{e}_{(2)}$ is obtained from the auxiliary graph in Fig. 32(a) in which the voltage pairs A and B are connected by a straight line, which intersects the 45-degree line $\tilde{e}_{(0)}C_1=\tilde{e}_{(0)}C_2$ at the point X, and provides the solution for the final condition $\tilde{e}_{(0)}=306$ volts. If desired this value can be checked and corrected by exact calculation.

The final construction in Fig. 32(b) was made with this value. The shaded areas include the amplitude values i_d and e_c during ϕ which are given by (20) and (21).

The average current during ϕ is the area under the sine-wave section minus the area under the exponential curve i_t, both divided by the base. This furnishes

$$\bar{\imath}_{d(\phi)} = \tilde{\imath}_{s\max}\left[(\cos\beta' - \cos(\phi+\beta')) - (\omega CR'(1-\epsilon^{-\phi/\omega CR'})\sin(\Theta+\beta)\right]/\phi \qquad (22)$$

with $R'=R_S||R_L$ and ϕ, β and β' determined graphically from the construction or by trial of values. The average plate current per diode is again

$$\bar{I}_p = i_{d(\phi)'}\phi^\circ/360^\circ$$

and the direct load current in this full-wave circuit is $\bar{I}=2\bar{I}_p$. In case of large time constants, as in the example, the average condenser voltage $\bar{E}_c$ is quite accurately obtained from

$$\bar{E}_c = 0.5(e_{c(0)} + e_{c(\phi)}) \qquad (23)$$

and the load current by Ohms law $\bar{I}=\bar{E}_c/R_L$.

The root-mean-square values of ripple voltage and diode current are needed for many calculations. They may be obtained for all cases from

$$|E|_{(\text{ripple})} = 0.321(e_{c\max} - e_{c(\min)}) \qquad (24)$$

and

$$|I_p| = 1.1\bar{I}_p\sqrt{\frac{360^\circ}{\phi^\circ}}\,. \qquad (25)$$

Equation (24) holds within 10 per cent for wave shapes varying from a sine-wave to a saw-tooth and (25) gives better than 5 per cent accuracy for all wave shapes occurring in condenser-input circuits.

c) Generalized operation characteristics (steady-state operation)

It has been shown that the conduction angle is a function of the circuit constants in condenser-input circuits. The section of the energizing voltage $\tilde{e}$ utilized during conduction time has, therefore, no fixed value as in choke-input circuits where $\phi=180$ degrees and where the voltage $\tilde{e}$ during ϕ is a half sine wave. It is, therefore, not possible to derive a general equivalent circuit for condenser-input circuits which contains a voltage source of fixed wave shape and magnitude.[9]

Steady-state conditions as well as transients are controlled by the circuit constants, which are contained in the product ωCR_L. The angle ϕ depends further on the relative magnitudes of R_L and R_S and is, therefore, described in general if also the ratio R_S/R_L is known. General curve families may thus be evaluated which show the dependent variables $\bar{E}$, $\hat{\imath}$, and $\bar{I}$ in terms of ratio versus the independent variable ωCR_L for various parameter values R_S/R_L. The series resistance R_S includes the equivalent diode resistance which is evaluated by means of (6), because the current wave is periodic in the final operating state. The reasoning leading to (6) is not applicable to a single transient, as obtained for starting conditions of rectifier circuits.

Generalized characteristics have been evaluated for the three types of circuits shown in Fig. 9. The characteristics in Figs. 3, 4, and 5 show the average voltage $\bar{E}$ across the load resistance R_L as a function of ωCR_L and $\bar{R}_S$ for half-wave, full-wave, and voltage-doubling circuits. They permit the solution of the reversed problem to determine the magnitude of the applied voltage necessary to give a certain average voltage output for a given load. The series-resistance value $\bar{R}_S$ includes the equivalent average resistance $\bar{r}_d$ of one diode and the power-transformer resistances as reflected into one secondary winding. As their complete

[9] The equivalent voltage may be expressed by a Fourier series for each individual case as shown for the simplest case $R_s=0$ by M. B. Stout in footnote reference 1; the method, however, is hardly suitable for practical circuit analysis.

calculation required too much time, the characteristics were plotted from accurately measured values. The measurements were made on circuits of negligible inductive reactance. Series-resistance values in these circuits were determined accurately by the method shown in Fig. 10. Table II gives a number of calculated values which show the accuracy of the curves to be approximately 5 per cent or better.

TABLE II

Type of Condenser-Input Circuit	$n\omega CR_L$	$\overline{R}_S / n R_L$	θ degrees	ϕ degrees	$\overline{E} / \tilde{e}_{max}$	$\hat{\imath}_d / \overline{I}_p$	$\lvert I_p \rvert / \overline{I}_p$
Half-Wave $n=1$	0.5	0	26.5	153.5	0.335	3.33	1.69
	1.	0	45.0	134.0	0.384	3.68	1.81
	2.	0	63.4	111.6	0.486	4.61	2.00
	2.26	0	66.15	106.4	0.503	4.91	2.02
	4.	0	75.9	87.1	0.623	6.60	2.24
	8.	0	82.9	65.1	0.742	9.86	2.60
	16.	0	86.4	48.6	0.862	13.92	3.00
	32.	0	88.2	35.3	0.930	19.90	3.51
	64.	0	89.1	25.1	0.996	27.5?	4.16
	2.	0.10	—	121.	0.434	4.48	1.9
	2.26	0.147	50.	123.	0.428	4.42	1.88
	4.	0.05	65.1	99.3	0.632	5.28	2.1
	4.	0.10	56.	108.4	0.537	5.14	2.0
Full-Wave $n=2$	1.	0	26.5	142.5	0.644	3.47	1.75
	2.	0	45.0	121.0	0.678	4.17	1.90
	4.	0	63.4	92.6	0.740	6.06	2.17
	4.52	0	66.15	86.8	0.744	6.55	2.24
	8.	0	75.9	67.0	0.816	9.30	2.55
	16.	0	83.0	49.0	0.885	13.74	3.00
	32.	0	86.4	35.6	0.945	19.70	3.50
	64.	0	88.2	25.4	0.999	27.1?	4.15
	4.	0.05	—	104.	0.671	5.43	2.05
	4.52	0.0735	50.	105.	0.636	5.35	2.04
	8.	0.05	56.	90.	0.710	6.20	2.20
	30.2	0.10	17.9	100.6	0.646	5.39	2.08

In compiling the data for the current-ratio characteristics in Fig. 6, it was found that the three rectifier-circuit types could be shown by a single family after a "charge factor" n was added to the product of the circuit constants ωCR_L and to R_S as shown in Table II. The factor n is unity for the half-wave circuit. For the full-wave circuit, n is 2 because the condenser C is charged twice during one cycle. For the voltage-doubling circuit, n is $\frac{1}{2}$ because the two condensers require together twice the charge to deliver the same average current at double voltage. The values in the table indicate that the factor n is actually not a constant. The mean value of the current ratios does, however, not depart more than 5 per cent from the true value, the error being a maximum in the steep portion of the curves and decreasing to zero at both ends. The upper section of Fig. 6 shows the ratio of root-mean-square current to average current per diode plate. This family is of special interest in the design of power transformers and for computation of diode plate dissipation.

Fig. 7 shows the root-mean-square value of the ripple voltage across R_L in per cent of the average voltage.

The voltage-doubling circuit shown with the other two condenser-input circuits in Fig. 9 may be regarded in principle as a series connection of two half-wave rectifier circuits. Each condenser is charged separately during conduction time of one diode, but is discharged in series with the other condenser during the time of nonconduction of its associated diode. The analysis of operation is made according to the method discussed but will not be treated. The average anode characteristics of RCA rectifiers are shown in Fig. 8. The method of carrying out a practical analysis by use of these curve families has been outlined in the first section of this paper.

APPENDIX

System of Symbols

The number of special symbols and multiple indexing have been greatly reduced by introducing four special signs for use with any symbol.

1) The symbols in general are of standard notation, lower case letters i, r, indicate instantaneous, sectional, or variable values and capital letters I and R indicate steady values.

2) Special values
   a) *Sinusoidal voltages or currents* are indicated by a sine-wave sign above the symbol $\tilde{e}$, $\tilde{\imath}$, $\tilde{E}$. Their maximum values are indicated by index, $\tilde{e}_{max}$, $\tilde{E}_{max}$.
   b) *Peak values* are indicated by a circumflex; $\hat{e}_c$, $\hat{\imath}$, $\hat{r}_d$, maximum peak values are written $\hat{\imath}_{max}$, etc.
   c) *Average values* are indicated by a horizontal bar; $\overline{E}$, $\overline{I}$, $\overline{R}$.
   d) Root-mean-square values are indicated by vertical bars $\lvert E \rvert$, $\lvert I \rvert$, $\lvert R_s \rvert$.

3) An index in parenthesis specifies the time at which the symbol is valid, i.e., its numerical value. Hence, $\tilde{\imath}_{s(\pi)}$ is the steady-state alternating-current value at the time π and $i_{t(0)}$ is the transient current at the time 0. When used with an average or root-mean-square value, the time index specifies the period over which average or root-mean-square-values are taken, such as $\bar{\imath}_{(\phi)}$, $\lvert i_p \rvert_{(\phi)}$. A conduction time index (ϕ) on resistance values such as $\bar{r}_d$, $\overline{R}_s$ is unnecessary. (See definition.)