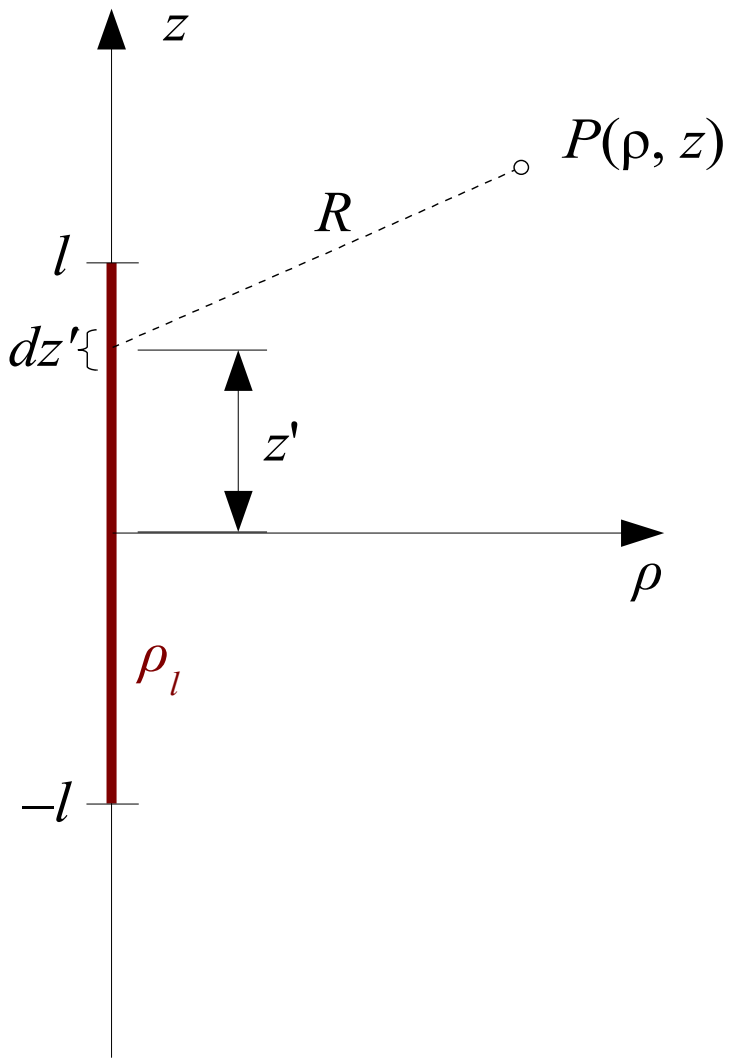
Campo de fonte linear



Campo de fonte linear

$$\varphi(\rho, z) = \int_{-l}^{l} \frac{\rho_l \, dz'}{4\pi\varepsilon R} = \int_{-l}^{l} \frac{\rho_l \, dz'}{4\pi\varepsilon\sqrt{(z-z')^2+\rho^2}} =$$

$$= \frac{-\rho_l}{4\pi\varepsilon} \ln\left[(z-z') + \sqrt{(z-z')^2+\rho^2}\right]\Bigg|_{z'=-l}^{z'=l}$$

$$\varphi(\rho, z) = \frac{\rho_l}{4\pi\varepsilon} \ln\left[\frac{(z+l)+\sqrt{(z+l)^2+\rho^2}}{(z-l)+\sqrt{(z-l)^2+\rho^2}}\right]$$

Superfícies equipotenciais

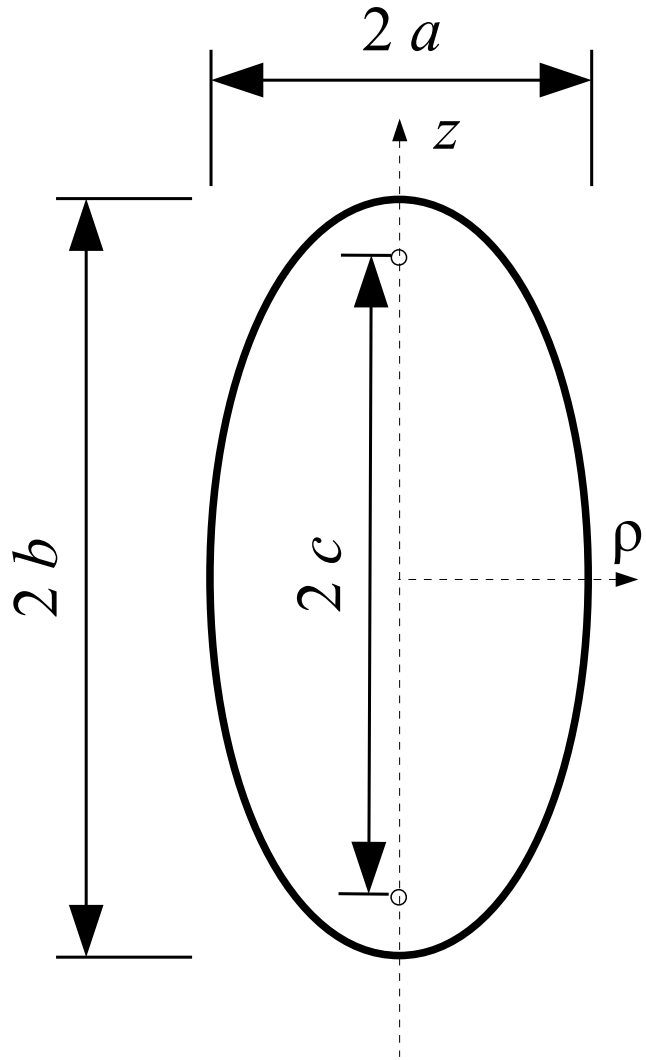
$$\varphi = V \Rightarrow \left[\frac{(z+l)+\sqrt{(z+l)^2+\rho^2}}{(z-l)+\sqrt{(z-l)^2+\rho^2}} \right] = e^{\frac{4\pi\varepsilon V}{\rho_l}} = \alpha \; ; \quad 1<\alpha<\infty$$

Que pode ser reescrita como

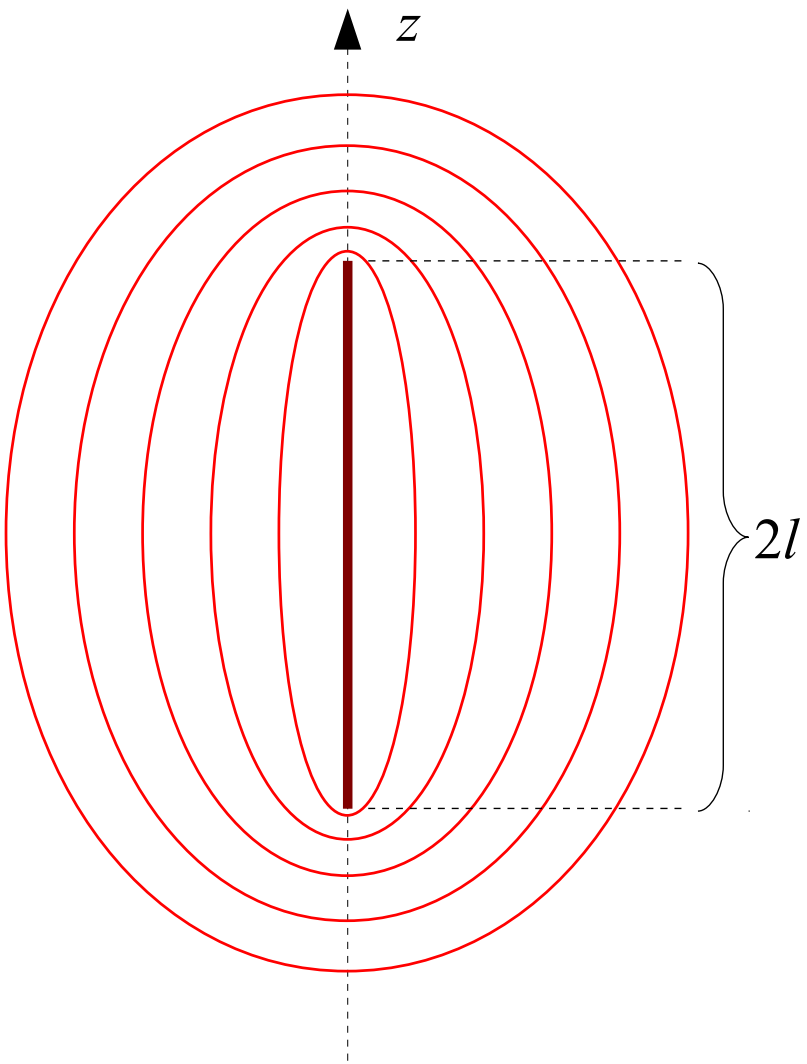
$$\frac{z^2}{b^2}+\frac{\rho^2}{a^2}=1 \qquad b^2 = l^2 \frac{(\alpha+1)^2}{(\alpha-1)^2}, \quad a^2 = l^2 \frac{4\alpha}{(\alpha-1)^2}$$

Elipsoide de raios *a* e *b* e distância focal *c*:

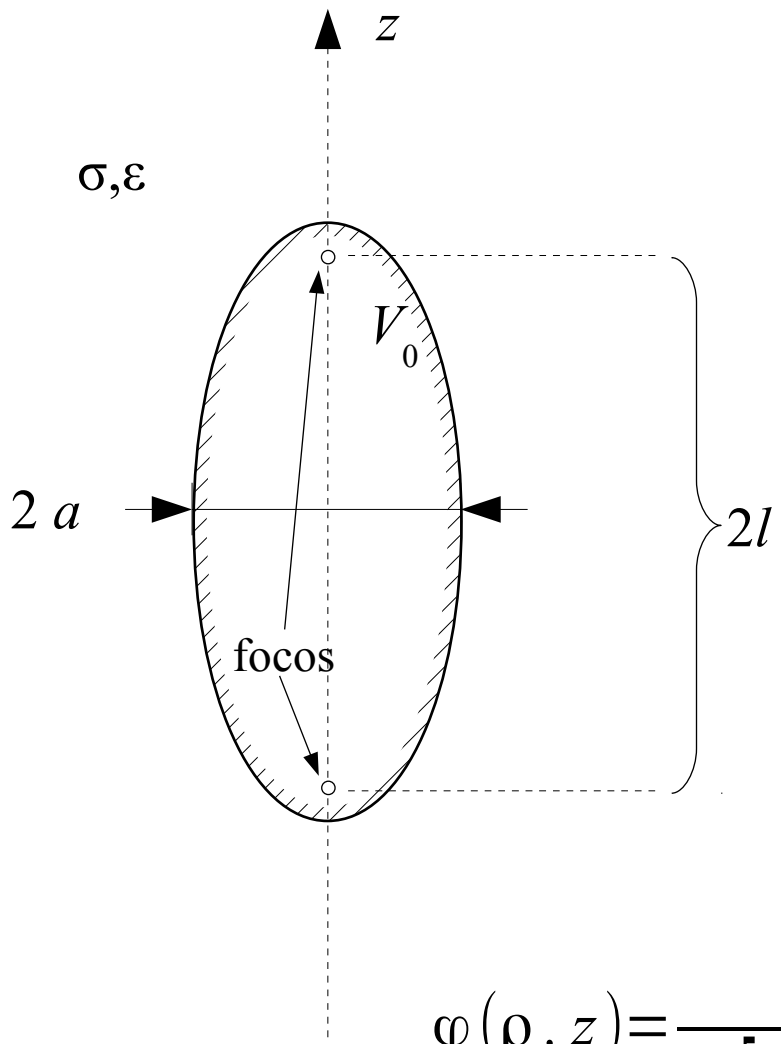
$$c^2 = b^2 - a^2 = l^2$$



Superfícies equipotenciais



Elipsoide condutor



$$\varphi(a,0)=\frac{\rho_l}{4\pi\varepsilon}\ln\left[\frac{\sqrt{l^2+a^2}+l}{\sqrt{l^2+a^2}-l}\right]=V_0\Rightarrow$$

$$\Rightarrow\rho_l=\frac{4\pi\varepsilon V_0}{\ln\left[\frac{\sqrt{l^2+a^2}+l}{\sqrt{l^2+a^2}-l}\right]}$$

$$\varphi(\rho,z)=\frac{V_0}{\ln\left[\frac{\sqrt{l^2+a^2}+l}{\sqrt{l^2+a^2}-l}\right]}\ln\left[\frac{(z+l)+\sqrt{(z+l)^2+\rho^2}}{(z-l)+\sqrt{(z-l)^2+\rho^2}}\right]$$

Elipsoide condutor

Carga do elipsoide:

$$Q = 2l \frac{4\pi\varepsilon V_0}{\ln\left[\frac{\sqrt{l^2+a^2}+l}{\sqrt{l^2+a^2}-l}\right]} = \frac{8\pi\varepsilon l V_0}{\ln\left[\frac{\sqrt{l^2+a^2}+l}{\sqrt{l^2+a^2}-l}\right]}$$

Capacitância:

$$C = \frac{8\pi\varepsilon l}{\ln\left[\frac{\sqrt{l^2+a^2}+l}{\sqrt{l^2+a^2}-l}\right]}$$

Resistência:

$$R = \frac{1}{8\pi\sigma l} \ln\left[\frac{\sqrt{l^2+a^2}+l}{\sqrt{l^2+a^2}-l}\right]$$

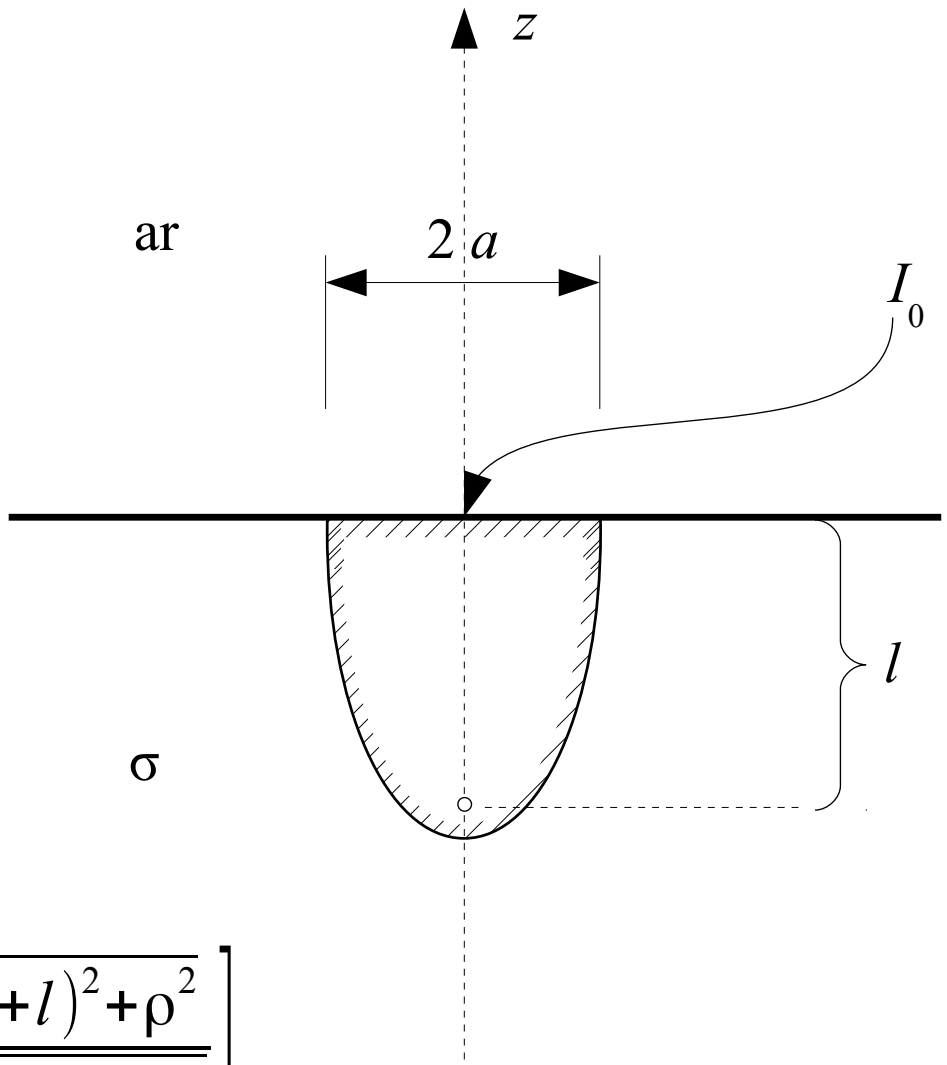
Elipsoide condutor semienterrado

- Condições de contorno:

$$\varphi\left(S_{elipsoide}\right)=V_0$$

$$\varphi\left(z,\rho\rightarrow\infty\right)=0$$

$$\left.\frac{\partial\varphi}{\partial z}\right|_{z=0}=0$$



- Solução:

$$\varphi\left(\rho,z\right)=\frac{V_0}{\ln\left[\frac{\sqrt{l^2+a^2}+l}{\sqrt{l^2+a^2}-l}\right]}\ln\left[\frac{\left(z+l\right)+\sqrt{\left(z+l\right)^2+\rho^2}}{\left(z-l\right)+\sqrt{\left(z-l\right)^2+\rho^2}}\right]$$

Elipsoide condutor semienterrado

- Para mesmo V_0:
  - Mesmo campo elétrico E
  - Mesma densidade de corrente J
  - Metade da corrente
  - Dobro da resistência:

$$R = \frac{1}{4\pi\sigma l} \ln\left[\frac{\sqrt{l^2+a^2}+l}{\sqrt{l^2+a^2}-l}\right]$$

Elipsoide condutor semienterrado

Aproximação para $a << l$:

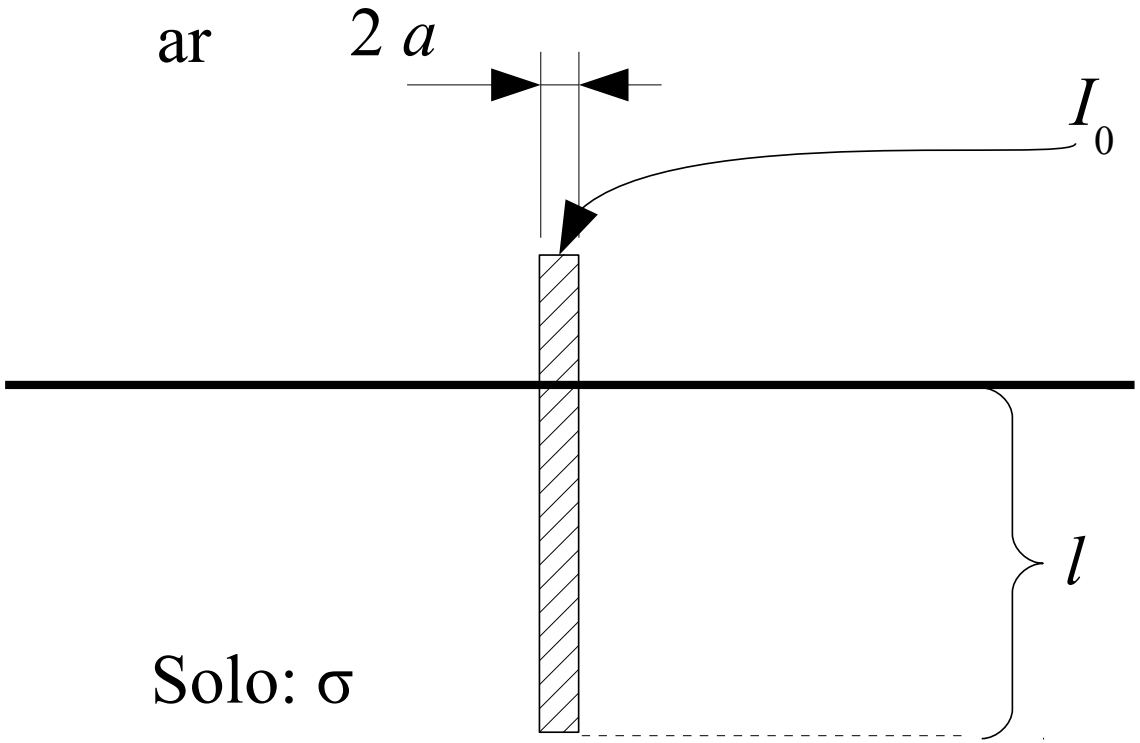
$$R = \frac{1}{4\pi\sigma l} \ln\left[\frac{\sqrt{1+(a/l)^2}+1}{\sqrt{1+(a/l)^2}-1}\right] \approx$$

$$\approx \frac{1}{4\pi\sigma l} \ln\left[\frac{1+(a/l)^2/2+1}{1+(a/l)^2/2-1}\right] \approx$$

$$\approx \frac{1}{4\pi\sigma l} \ln\left[\frac{2}{(a/l)^2/2}\right] =$$

$$= \frac{1}{4\pi\sigma l} \ln\left[\frac{4l^2}{a^2}\right] = \frac{1}{2\pi\sigma l} \ln\left[\frac{2l}{a}\right]$$

Estaca cilíndrica condutora



$$R \approx \frac{1}{2\pi\sigma l} \ln\left[\frac{2l}{a}\right]$$

$2\,a = 5$ cm; $l = 1$ m; $\sigma = 0{,}01$ S/m:
$R \approx 70\ \Omega$