Particle-in-Cell (PIC) kinetic simulations
11. One-dimensional electromagnetic PIC code

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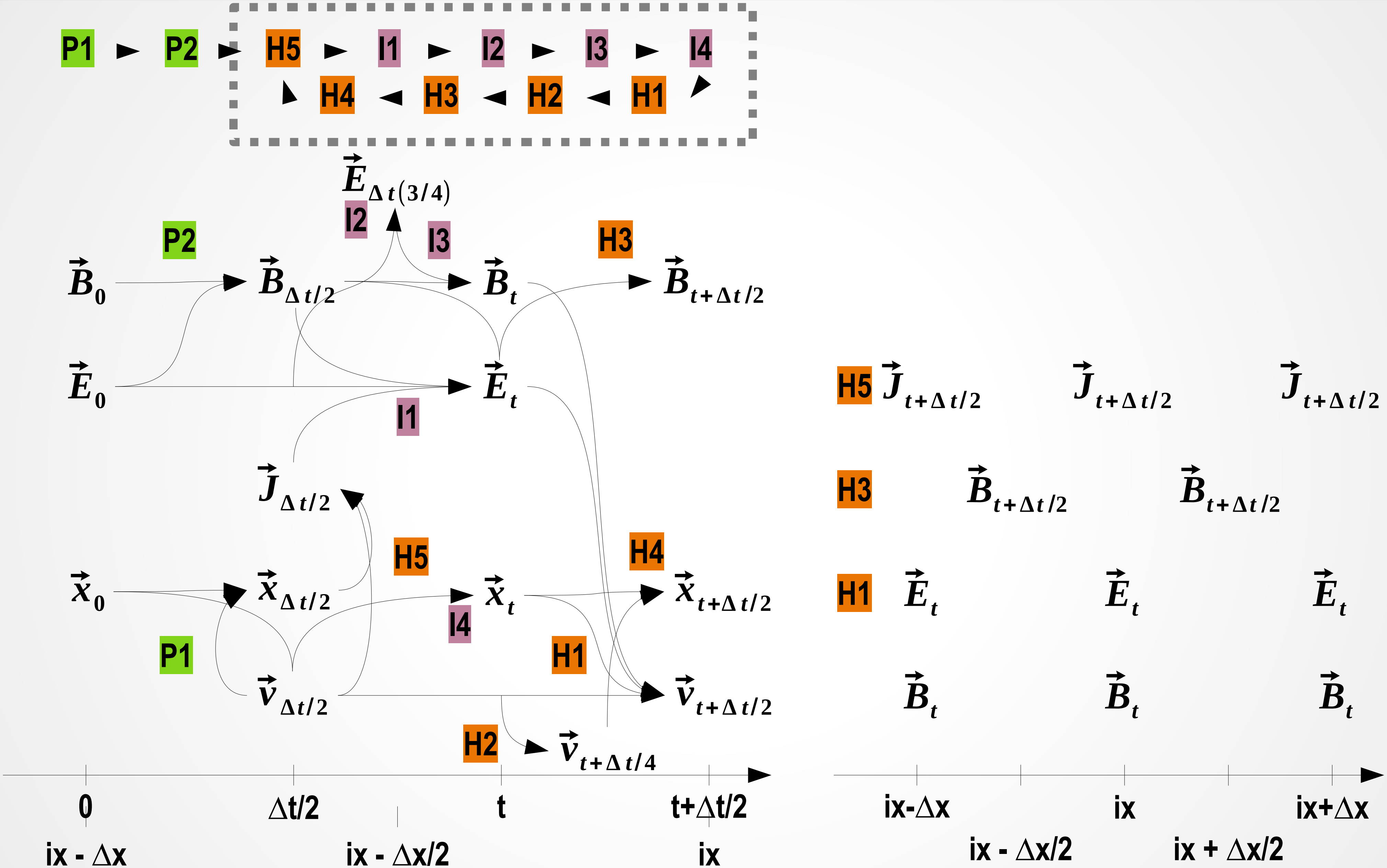
University of São Paulo, 2019.11.25-12.06

www.slido.com code: #B194

Time sequence of electromagnetic code



My first ES code

$$\nabla\cdot\vec{E}=\frac{\rho_c}{\varepsilon_0}$$

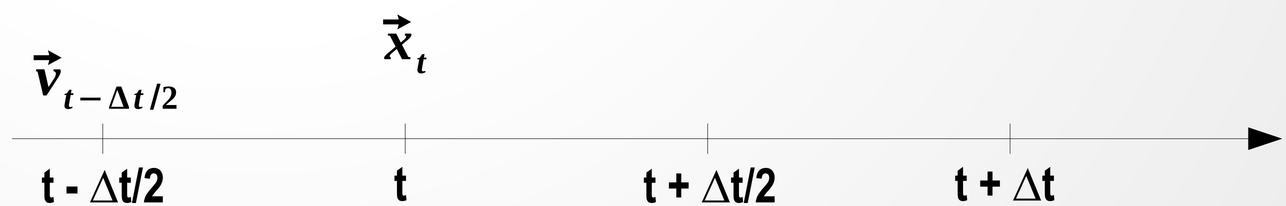
$$\frac{\partial E_x}{\partial x}=\frac{\rho_c}{\varepsilon_0}$$

$$m\frac{d\vec{v}}{dt}=q\vec{E}$$

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}\vec{E}_t$$

$$\frac{d\vec{x}}{dt}=\vec{v}$$

$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2}$$



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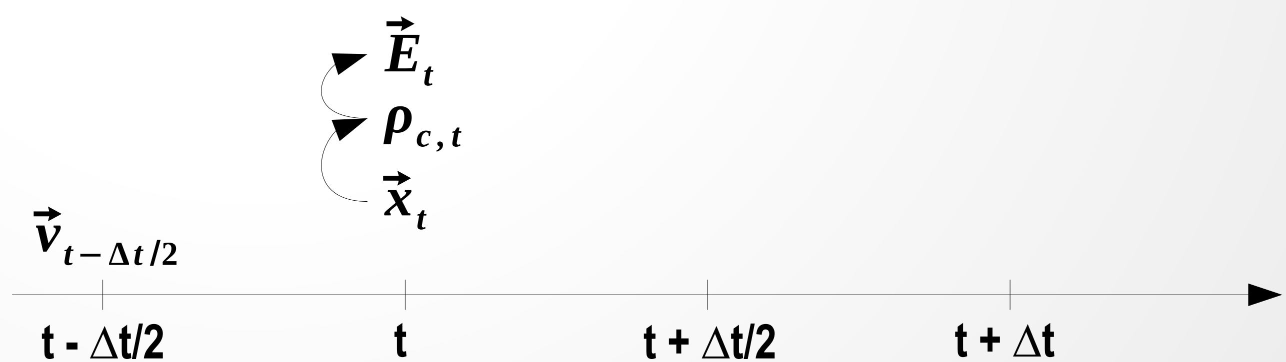
$$\frac{d\vec{x}}{dt}=\vec{v}$$

$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2}$$



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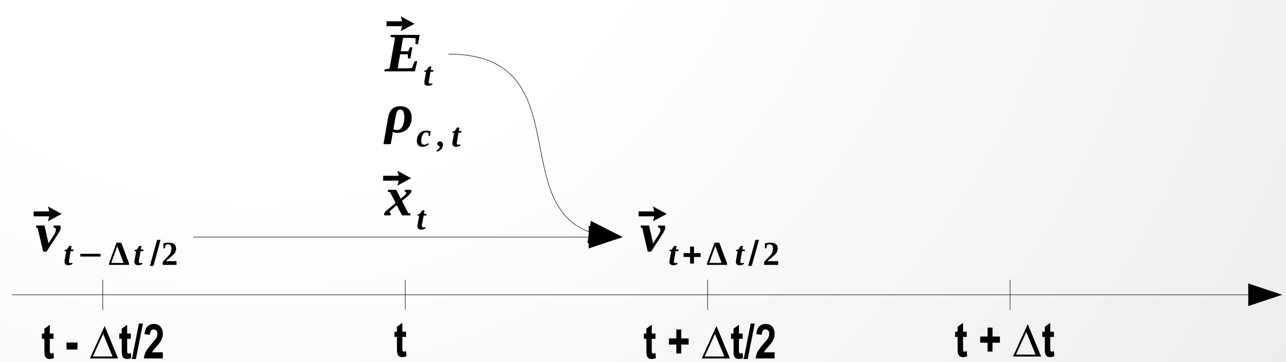
$$\frac{d\vec{x}}{dt}=\vec{v}$$

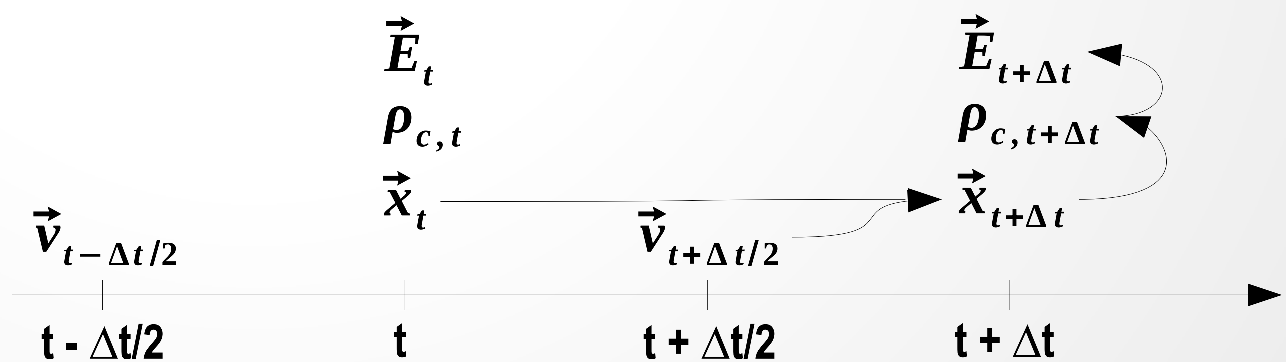
$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2}$$



My first ES code

$$\nabla \cdot \vec{E} = \frac{\rho_c}{\varepsilon_0} \qquad m\frac{d\vec{v}}{dt} = q\vec{E} \qquad \frac{d\vec{x}}{dt} = \vec{v}$$

$$\frac{\partial E_x}{\partial x} = \frac{\rho_c}{\varepsilon_0} \qquad \frac{\vec{v}_{t+\Delta t/2} - \vec{v}_{t-\Delta t/2}}{\Delta t} = \frac{q}{m}\vec{E}_t \qquad \frac{\vec{x}_{t+\Delta t} - \vec{x}_t}{\Delta t} = \vec{v}_{t+\Delta t/2}$$



My first EM code

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}\left(\vec{E}_t+\vec{v}_{t-\Delta t/2}\times\vec{B}_{t-\Delta t/2}\right)$$

$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2}$$

$$-\frac{\partial B_z}{\partial x}=\mu_0 J_y+\frac{1}{c^2}\frac{\partial E_y}{\partial t}$$

$$\frac{\partial B_y}{\partial x}=\mu_0 J_z+\frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

$$\frac{\partial E_y}{\partial x}=-\frac{\partial B_z}{\partial t}$$

$$-\frac{\partial E_z}{\partial x}=-\frac{\partial B_y}{\partial t}$$

$\boldsymbol{\vec{E}_t}$

$\boldsymbol{\vec{B}_{t-\Delta t/2}}$

$\boldsymbol{\vec{x}_t}$

$\boldsymbol{\vec{v}_{t-\Delta t/2}}$

t - Δt/2 | **t** | **t + Δt/2** | **t + Δt**

My first EM code

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

$$-\frac{\partial B_z}{\partial x}=\mu_0 J_y+\frac{1}{c^2}\frac{\partial E_y}{\partial t}$$

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$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}\left(\vec{E}_t+\vec{v}_{t-\Delta t/2}\times\vec{B}_{t-\Delta t/2}\right)$$

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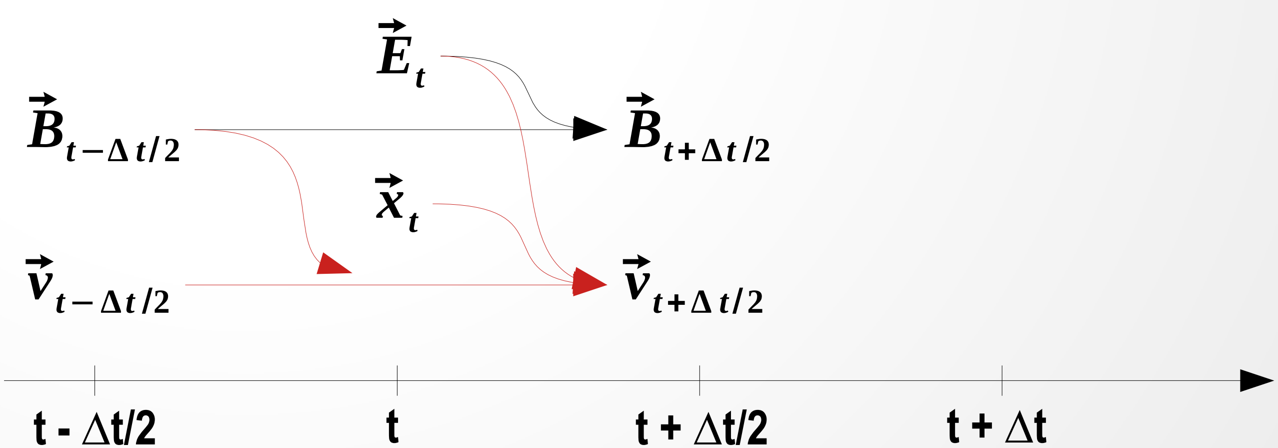

My first EM code

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

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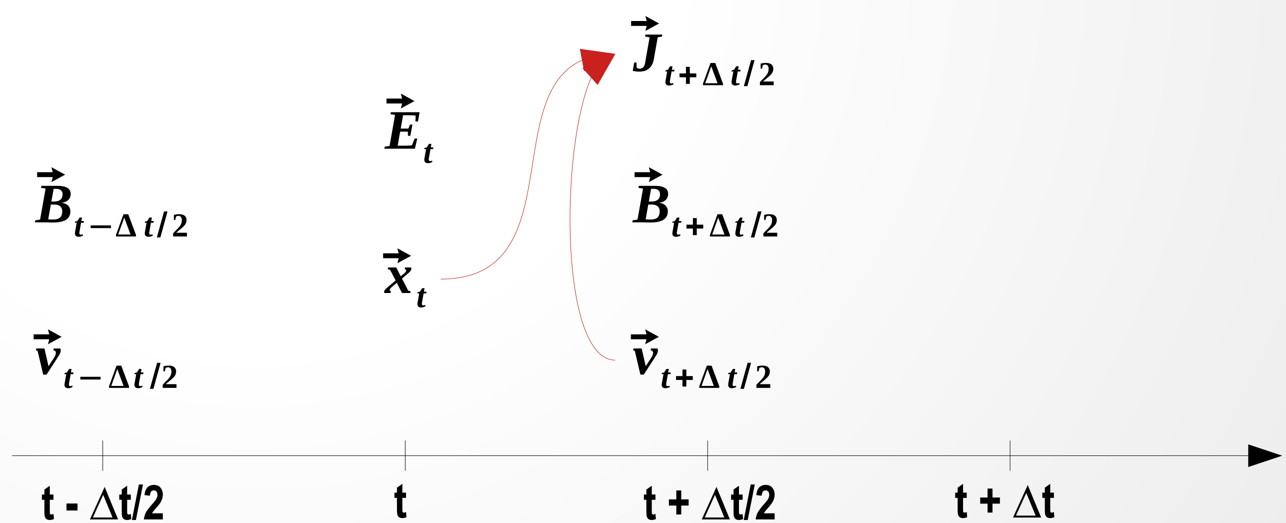
$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2}$$



My first EM code

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

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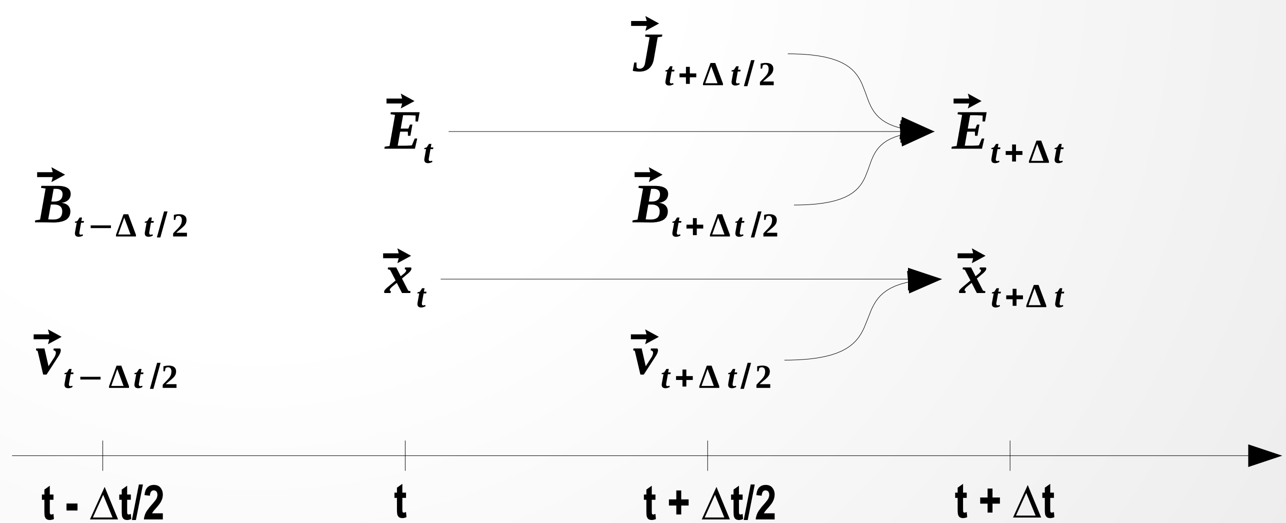
$$\frac{\partial B_y}{\partial x}=\mu_0 J_z+\frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

$$\frac{\partial E_y}{\partial x}=-\frac{\partial B_z}{\partial t}$$

$$-\frac{\partial E_z}{\partial x}=-\frac{\partial B_y}{\partial t}$$

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}\left(\vec{E}_t+\vec{v}_{t-\Delta t/2}\times\vec{B}_{t-\Delta t/2}\right)$$

$$\frac{\vec{x}_{t+\Delta t}-\vec{x}_t}{\Delta t}=\vec{v}_{t+\Delta t/2}$$



My first EM code

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

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$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}\left(\vec{E}_t+\vec{v}_{t-\Delta t/2}\times\vec{B}_{t-\Delta t/2}\right)$$

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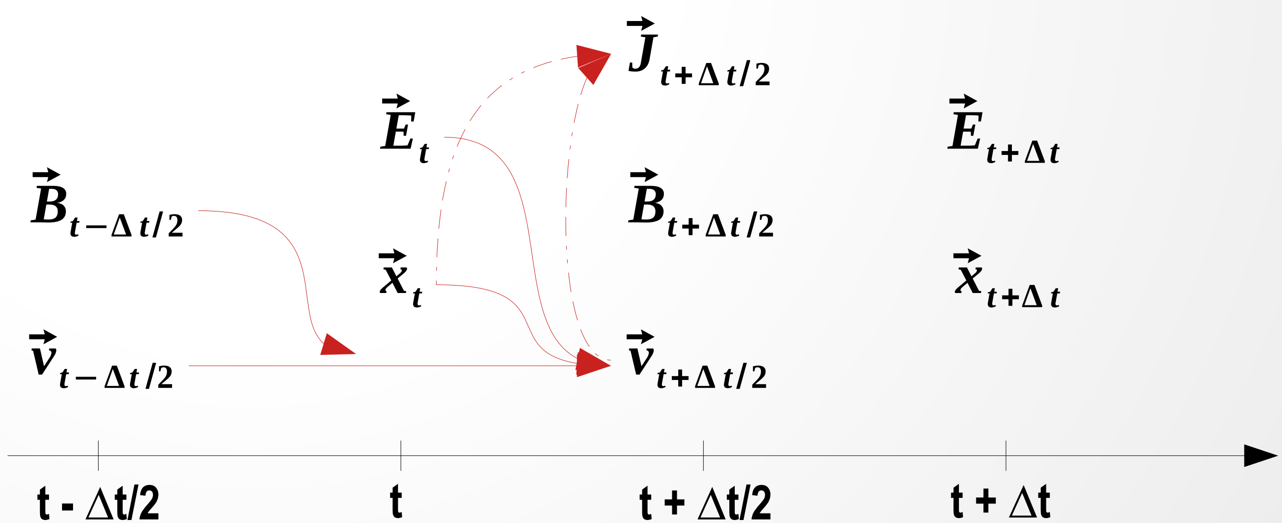

My second EM code (refereed to KEMPO1)

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

$$-\frac{\partial B_z}{\partial x}=\mu_0 J_y+\frac{1}{c^2}\frac{\partial E_y}{\partial t}$$

$$\frac{\partial B_y}{\partial x}=\mu_0 J_z+\frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

$$\frac{\partial E_y}{\partial x}=-\frac{\partial B_z}{\partial t}$$

$$-\frac{\partial E_z}{\partial x}=-\frac{\partial B_y}{\partial t}$$

$$\frac{d}{dt}(\gamma m_o \vec{v})=q(\vec{E}+\vec{v}\times\vec{B})$$

$$\frac{d\vec{x}}{dt}=\vec{v}$$

$\vec{E}_t$

$\vec{B}_t$

$\vec{v}_{t-\Delta t/2}$

$\vec{x}_t$

t - Δt/2 | t | t + Δt/2 | t + Δt

My second EM code (refereed to KEMPO1)

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

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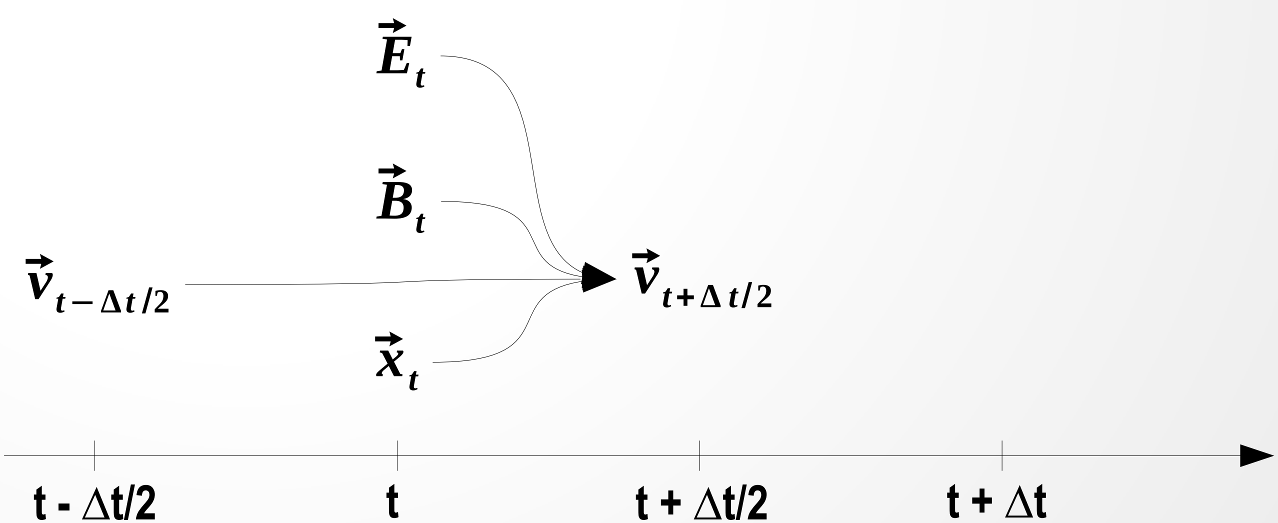
$$\frac{\partial B_y}{\partial x}=\mu_0 J_z+\frac{1}{c^2}\frac{\partial E_z}{\partial t}$$

$$\frac{\partial E_y}{\partial x}=-\frac{\partial B_z}{\partial t}$$

$$-\frac{\partial E_z}{\partial x}=-\frac{\partial B_y}{\partial t}$$

$$\frac{d}{dt}(\gamma m_o \vec{v})=q(\vec{E}+\vec{v}\times\vec{B})$$

$$\frac{d\vec{x}}{dt}=\vec{v}$$



My second EM code (refereed to KEMPO1)

$$0 = \mu_0 J_x + \frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

$$-\frac{\partial B_z}{\partial x} = \mu_0 J_y + \frac{1}{c^2}\frac{\partial E_y}{\partial t}$$

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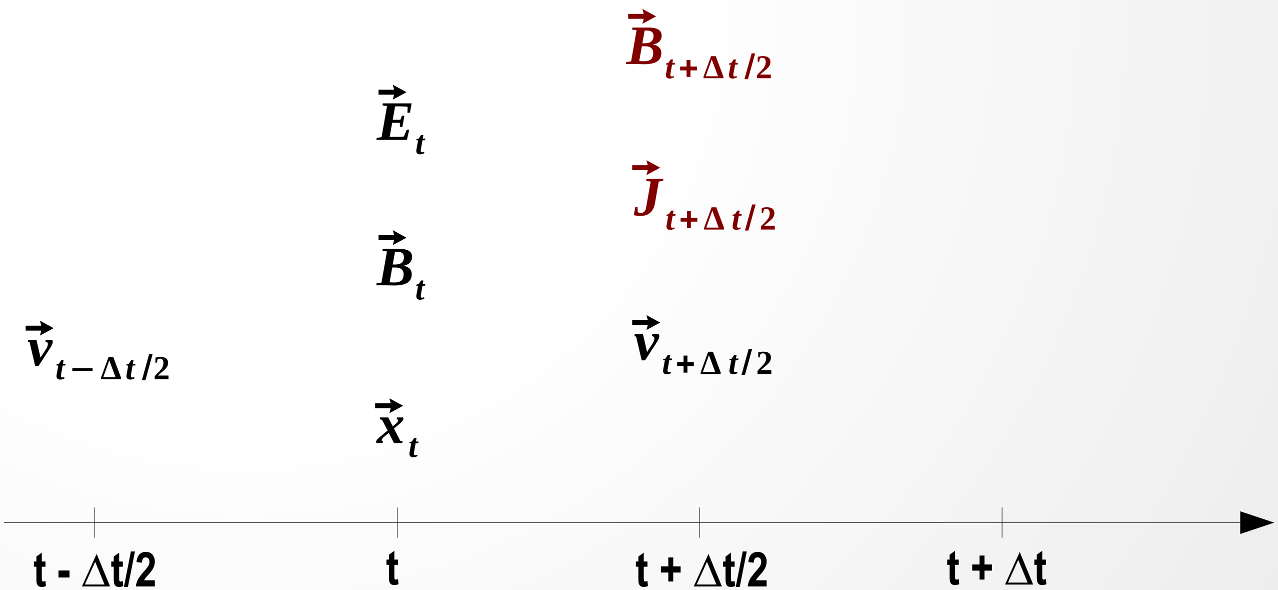
$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

$$-\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$$

$$\frac{d}{dt}(\gamma m_o \vec{v}) = q(\vec{E} + \vec{v}\times\vec{B})$$

$$\frac{d\vec{x}}{dt} = \vec{v}$$



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$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

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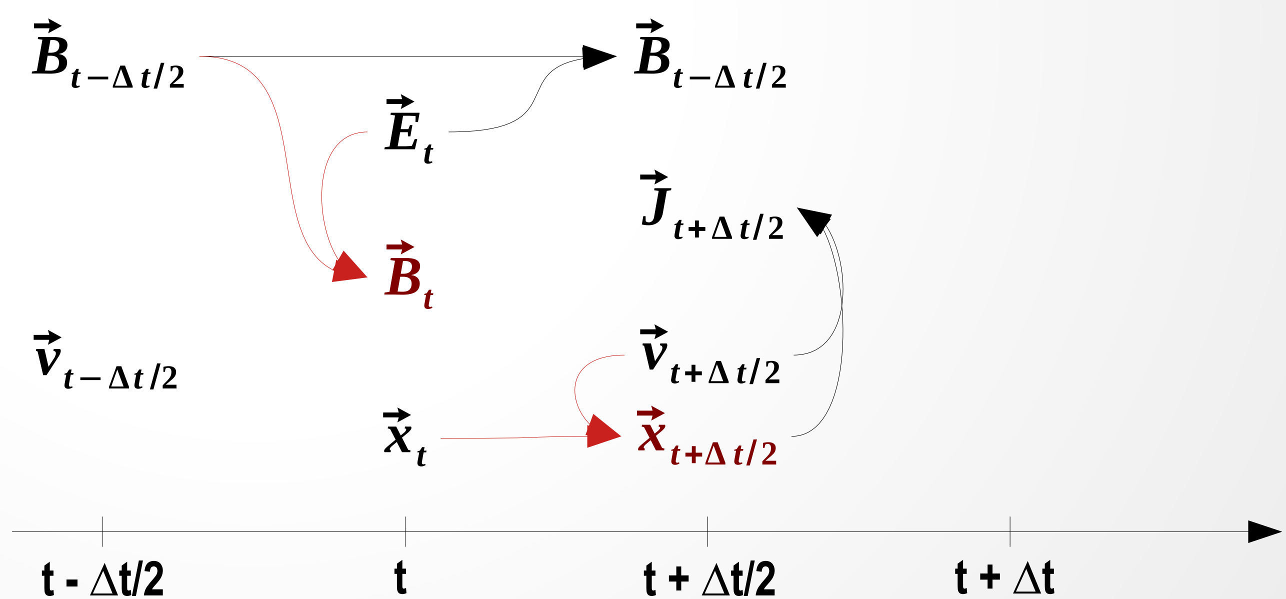
$$\frac{d}{dt}(\gamma m_o \vec{v})=q(\vec{E}+\vec{v}\times\vec{B})$$

$$\frac{d\vec{x}}{dt}=\vec{v}$$



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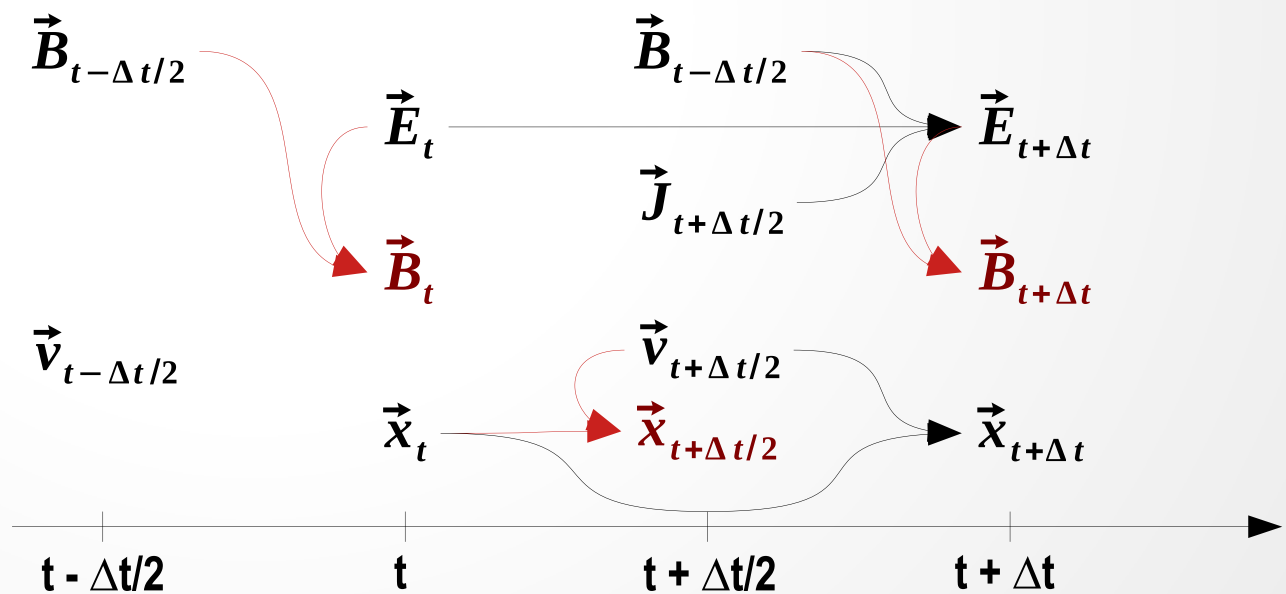

My third EM code??

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

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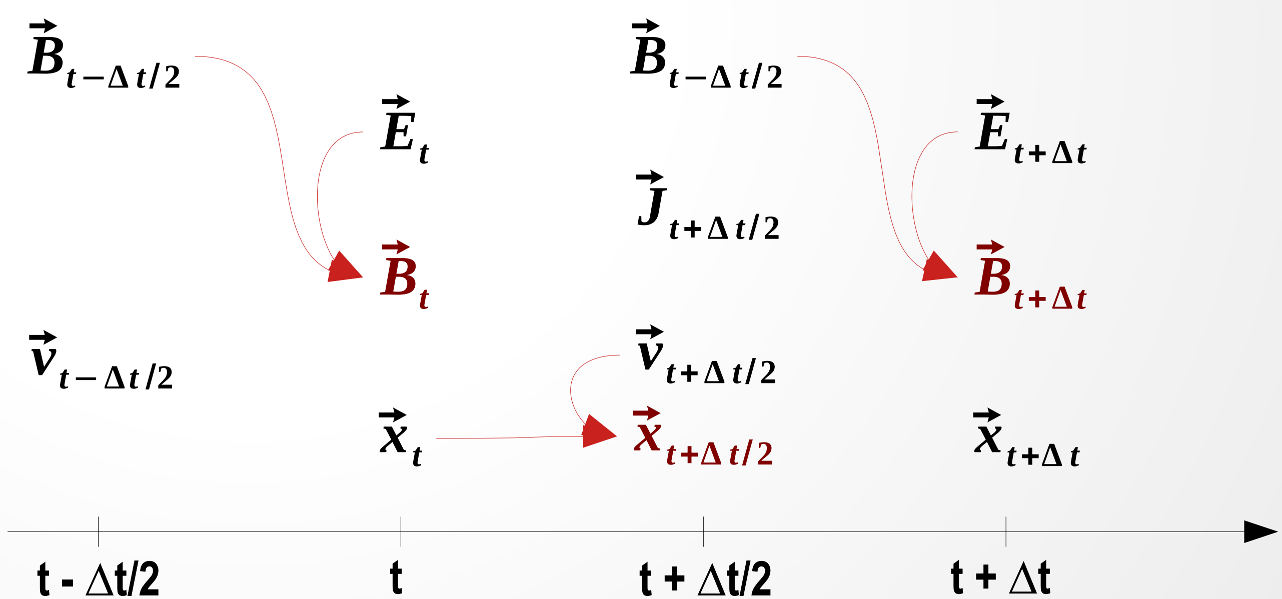

My third EM code??

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t}$$

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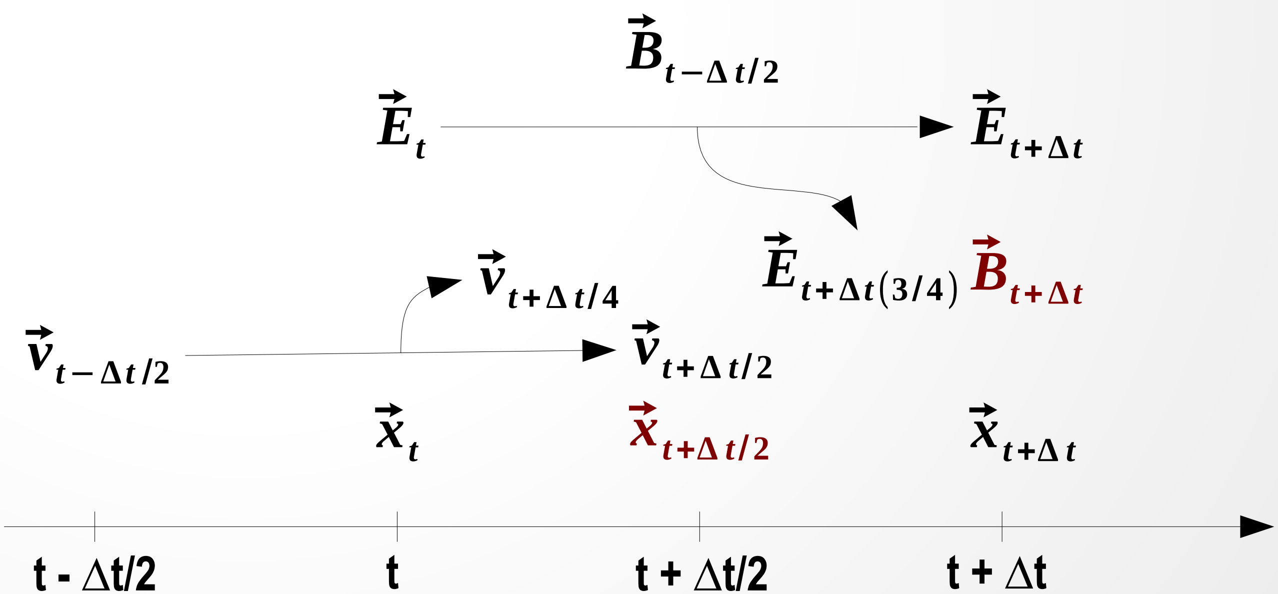

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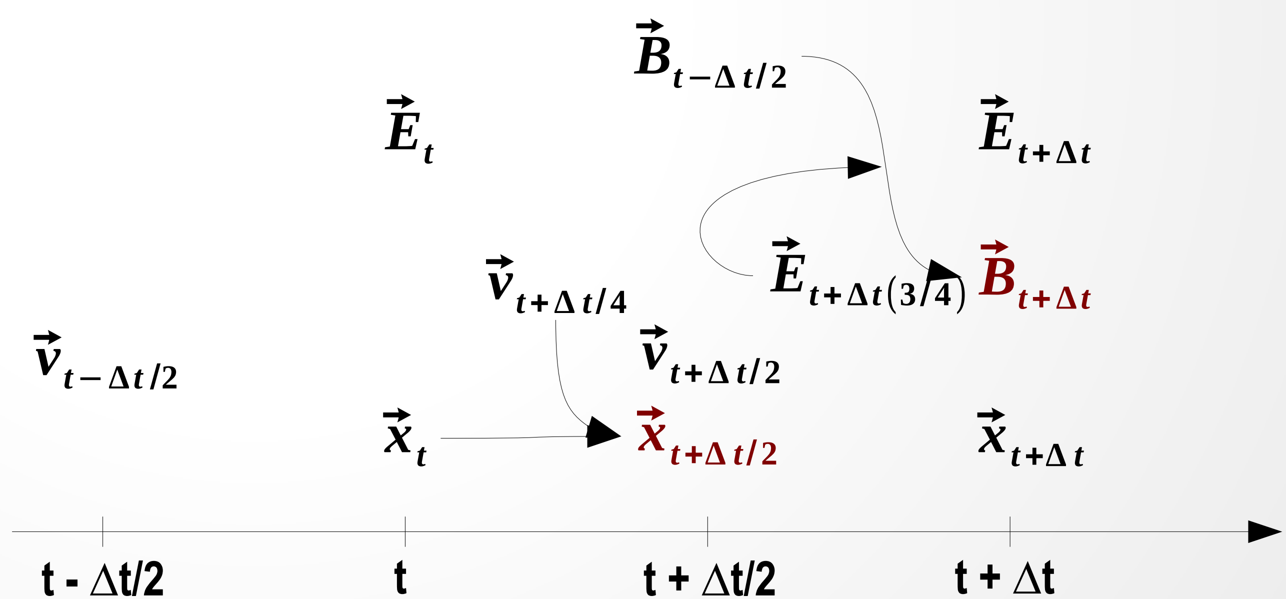
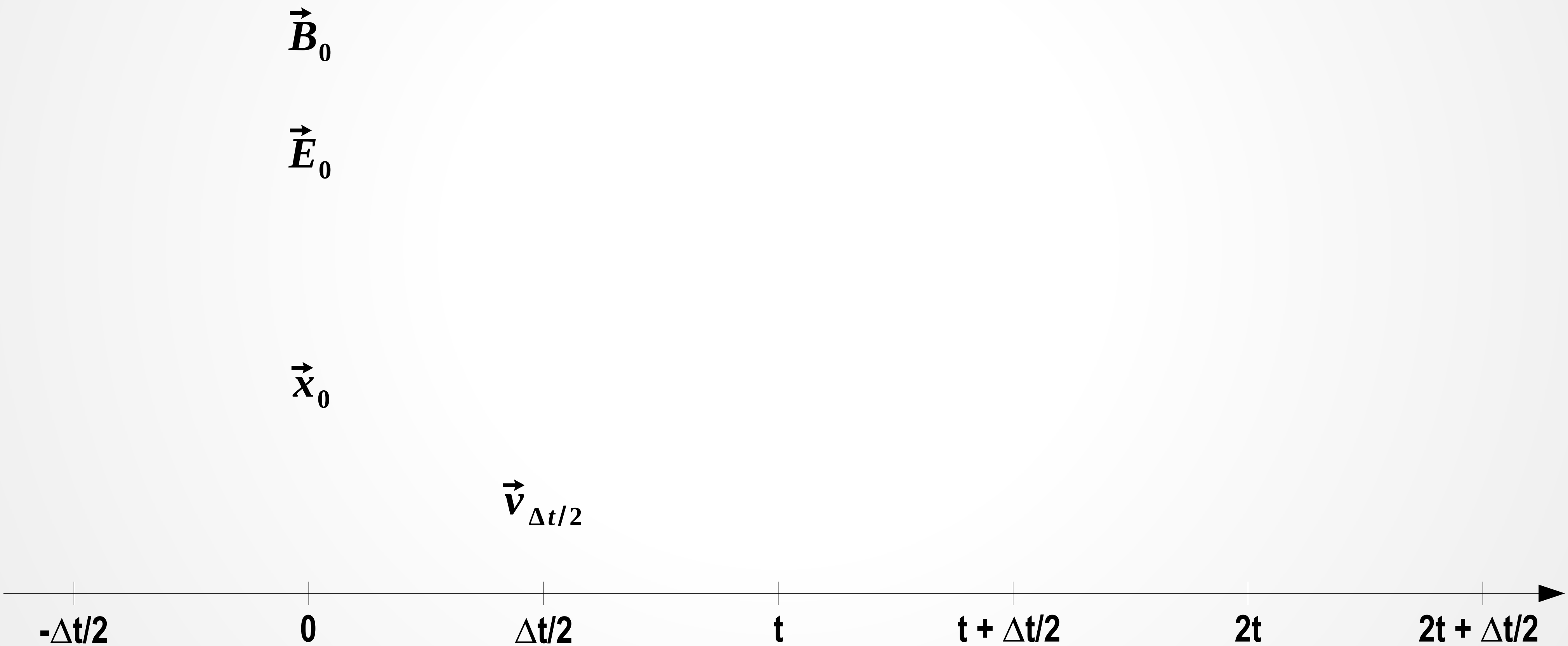


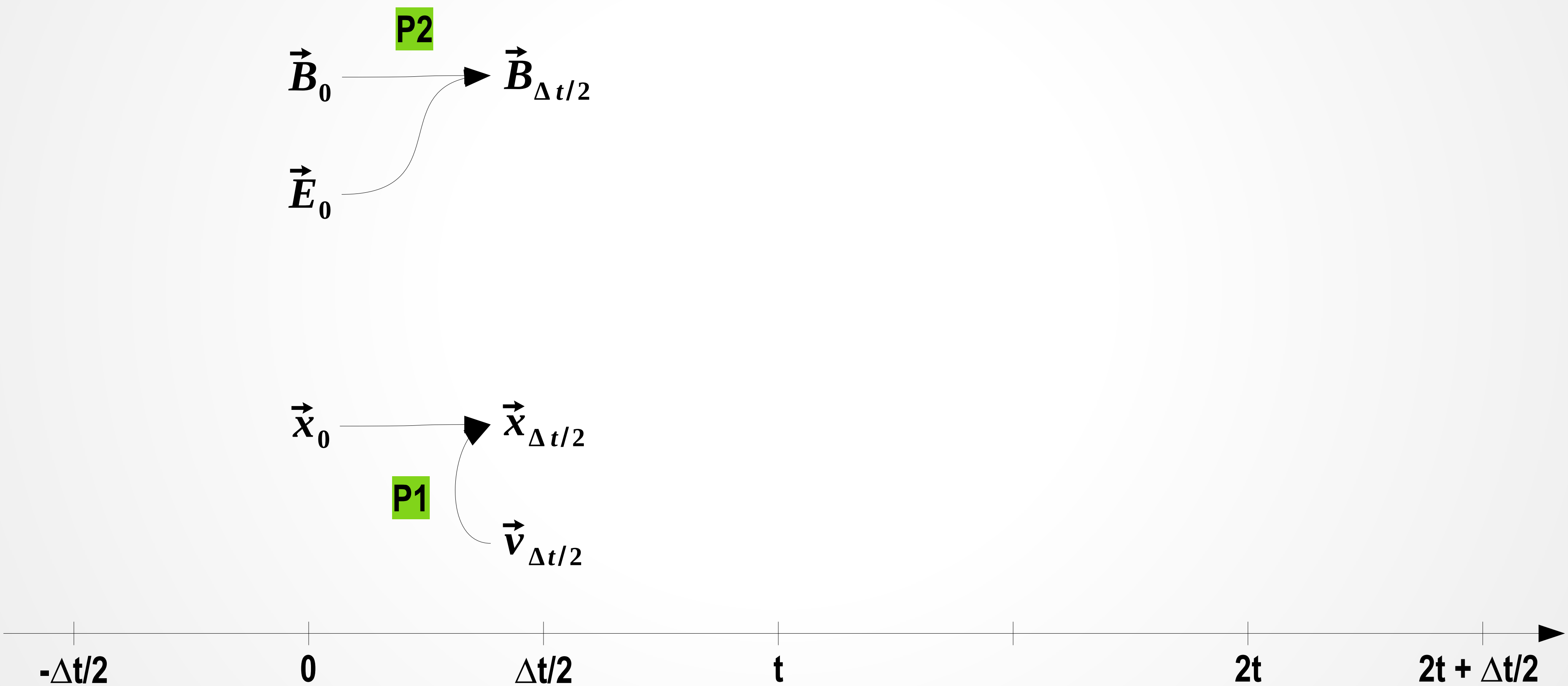
My third EM code



My third EM code

P1 $$\frac{d\vec{x}}{dt} = \vec{v}$$

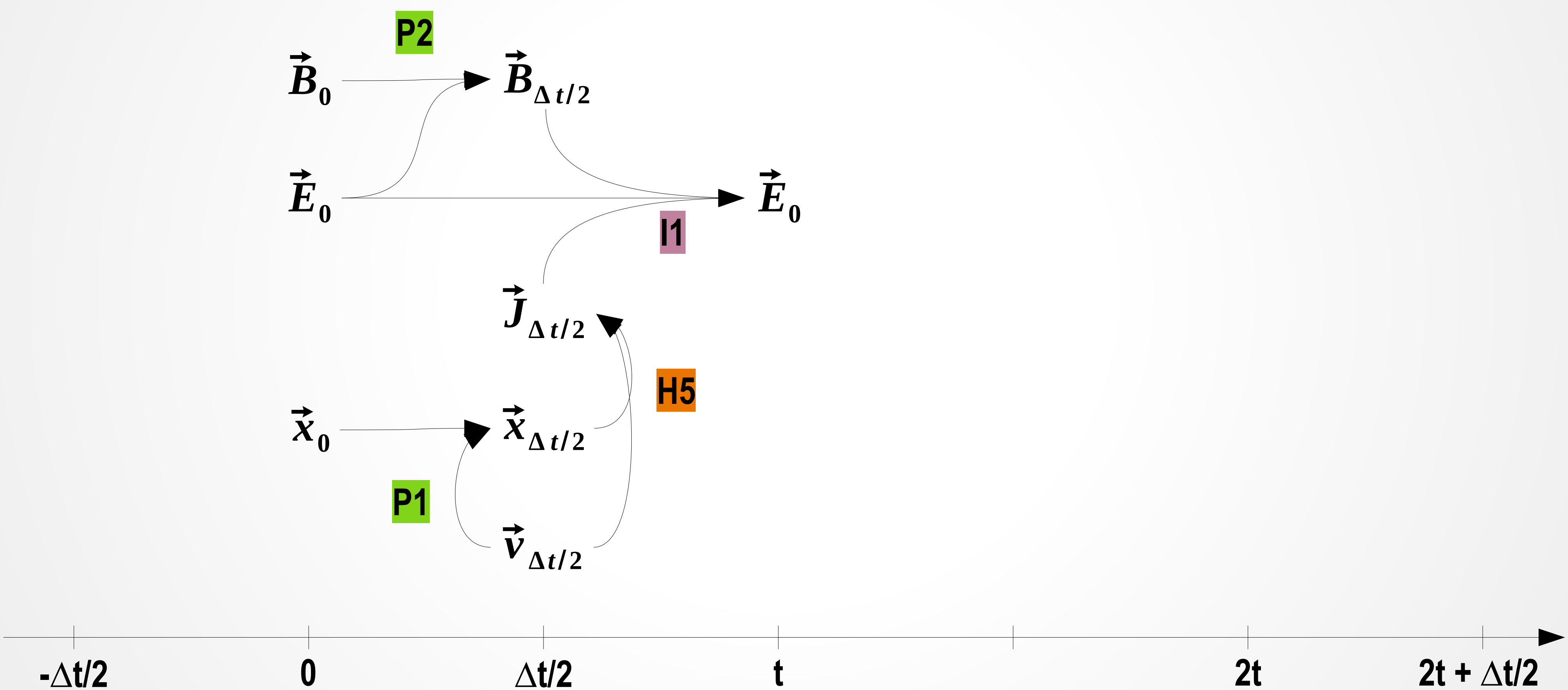
P2 $$-\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} \qquad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$



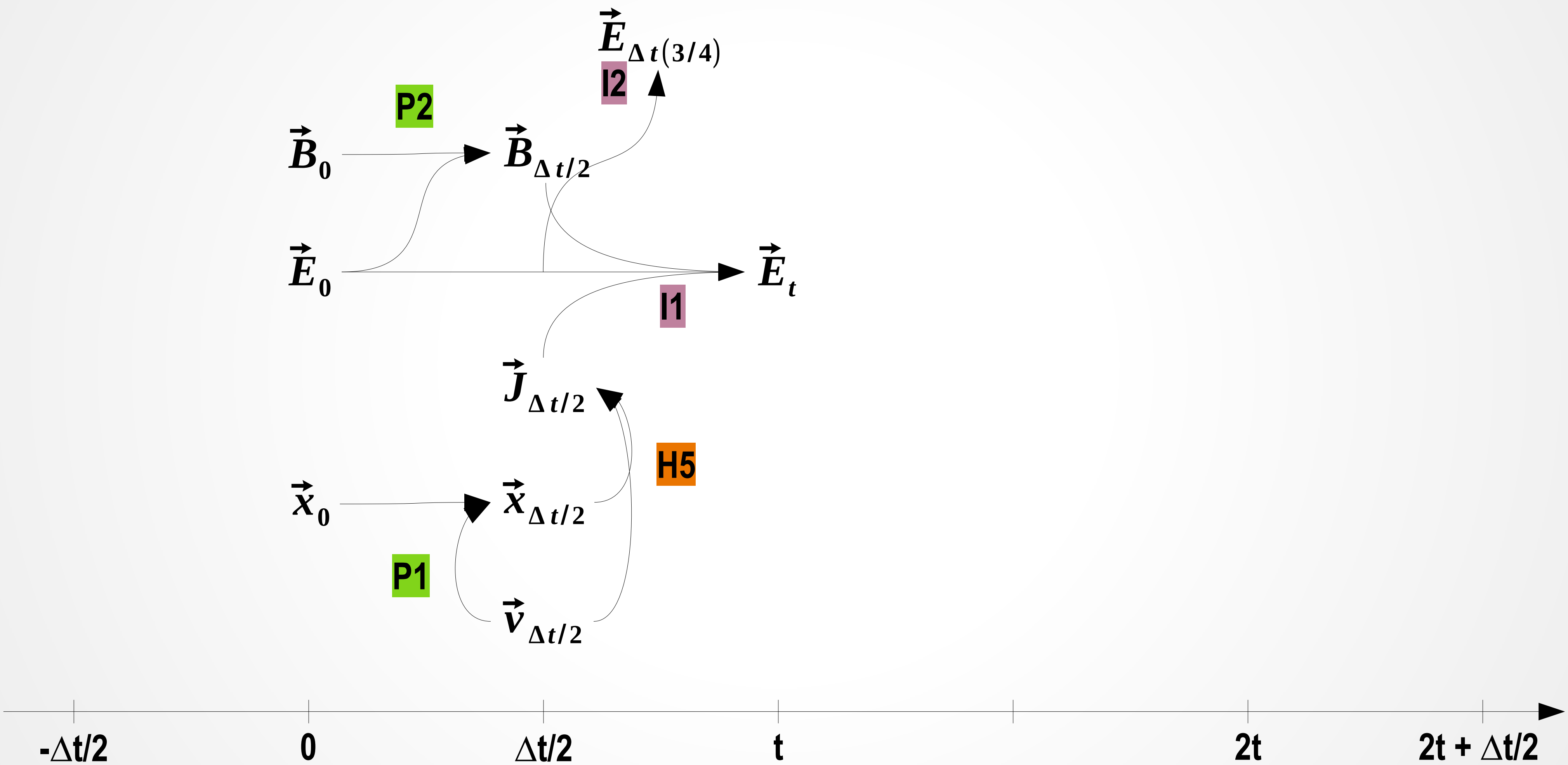
My third EM code

I1

$$0=\mu_0 J_x+\frac{1}{c^2}\frac{\partial E_x}{\partial t} \qquad -\frac{\partial B_z}{\partial x}=\mu_0 J_y+\frac{1}{c^2}\frac{\partial E_y}{\partial t} \qquad \frac{\partial B_y}{\partial x}=\mu_0 J_z+\frac{1}{c^2}\frac{\partial E_z}{\partial t}$$



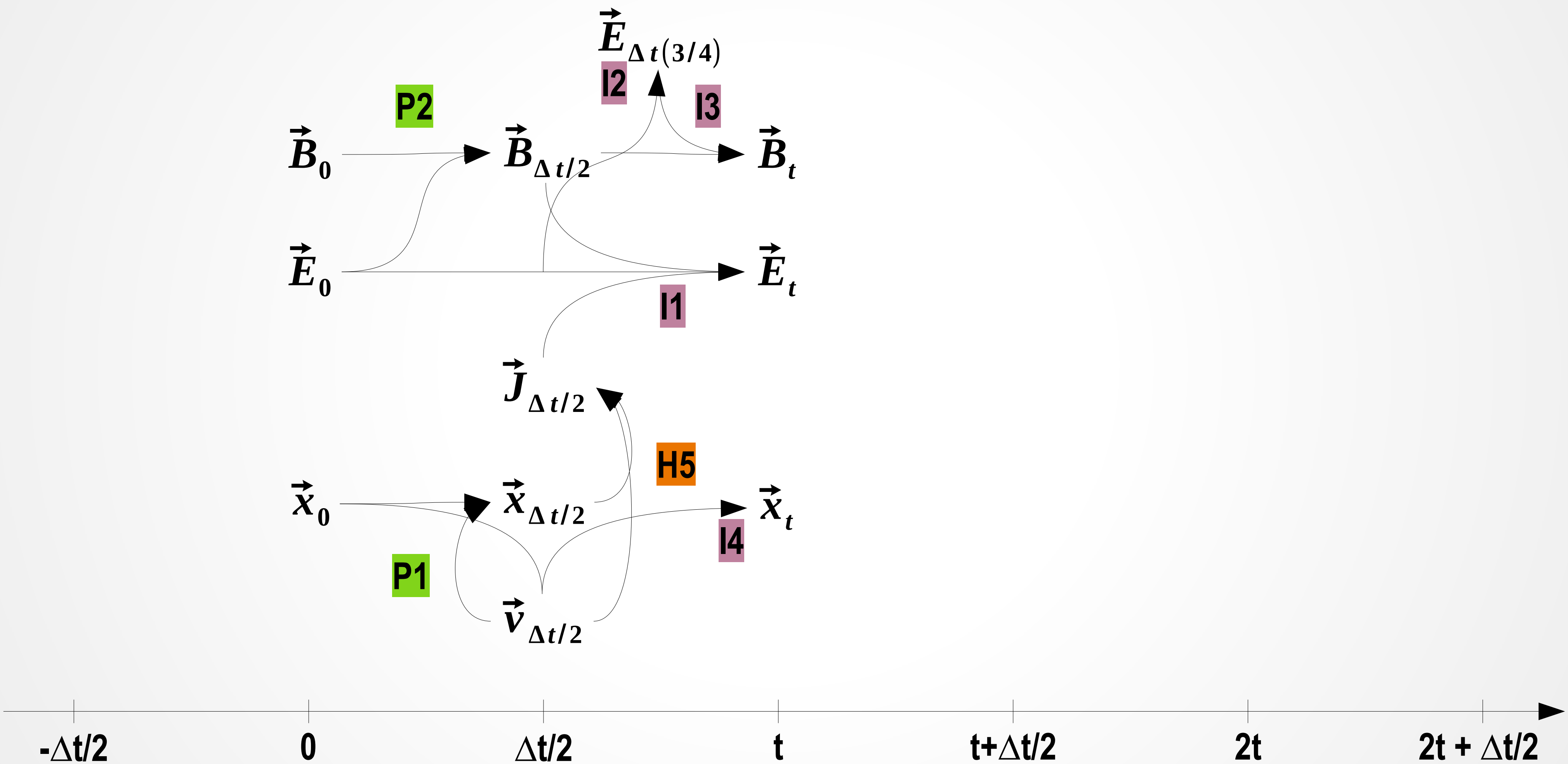
My third EM code



My third EM code

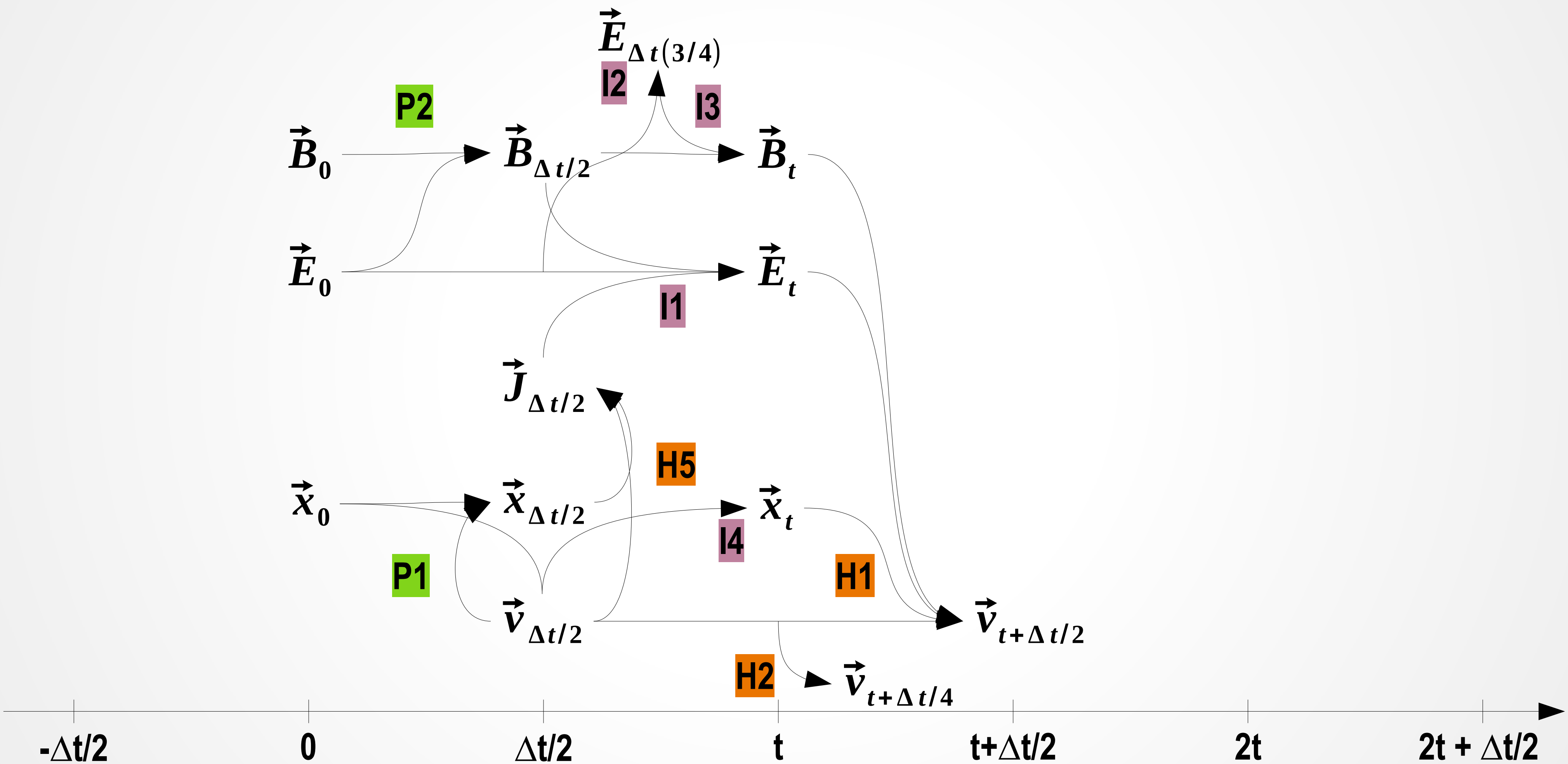
I4 $$\frac{d\vec{x}}{dt}=\vec{v}$$

I3 $$-\frac{\partial E_z}{\partial x}=-\frac{\partial B_y}{\partial t} \qquad \frac{\partial E_y}{\partial x}=-\frac{\partial B_z}{\partial t}$$



My third EM code

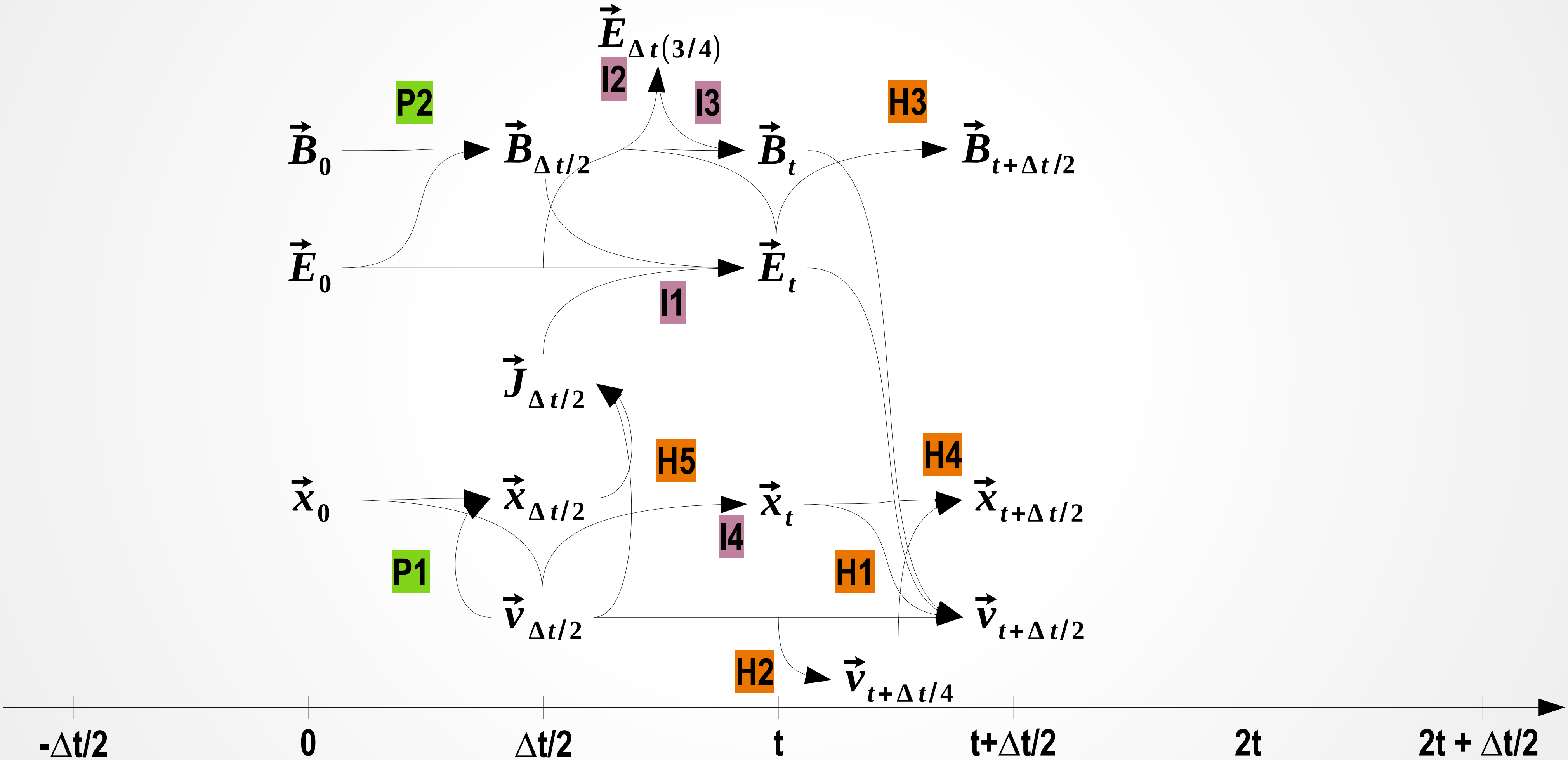
H1 $$\frac{d}{dt}(\gamma m_o \vec{v}) = q(\vec{E} + \vec{v} \times \vec{B})$$



My third EM code

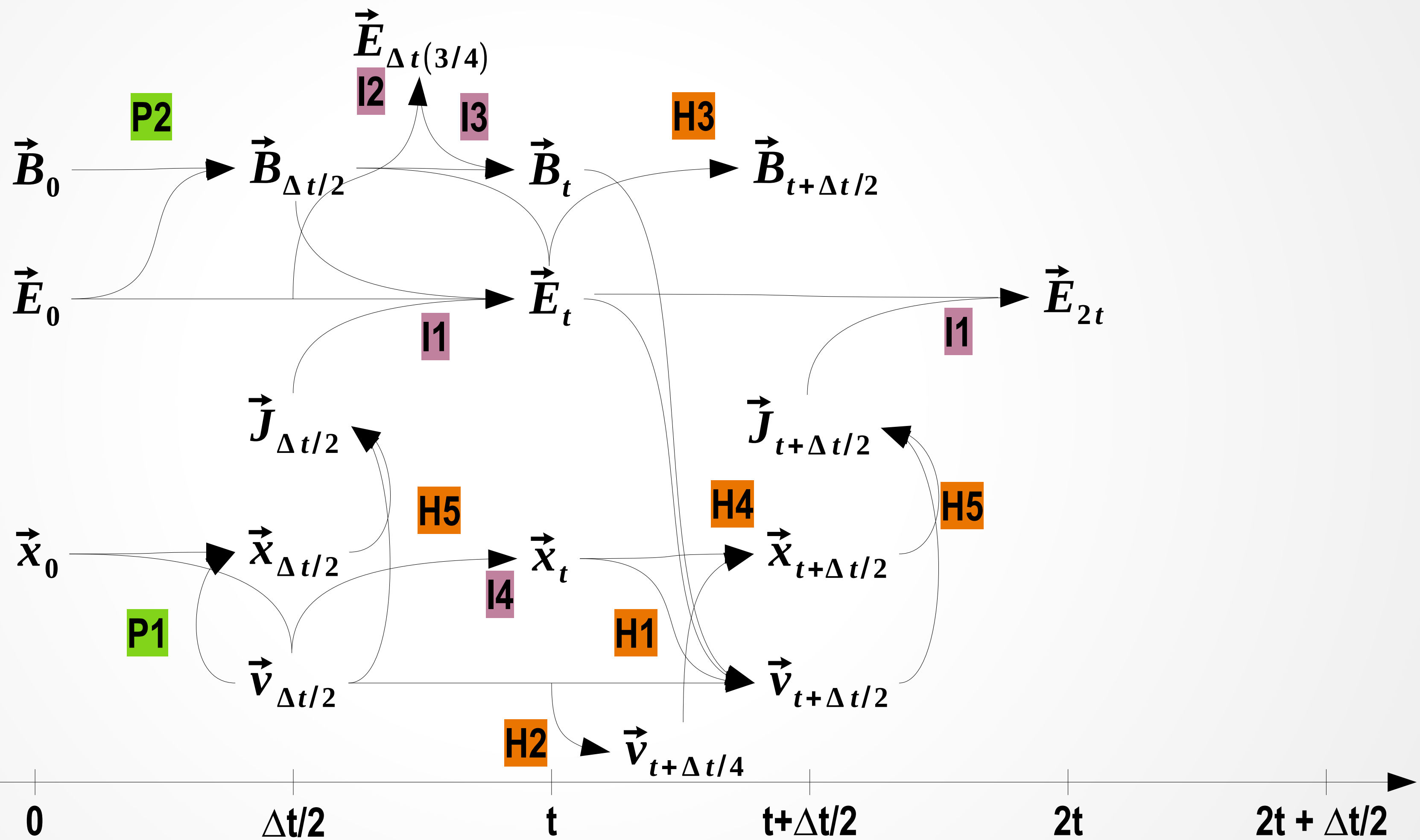
H3 $-\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$ $\quad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$

H4 $\frac{d\vec{x}}{dt} = \vec{v}$

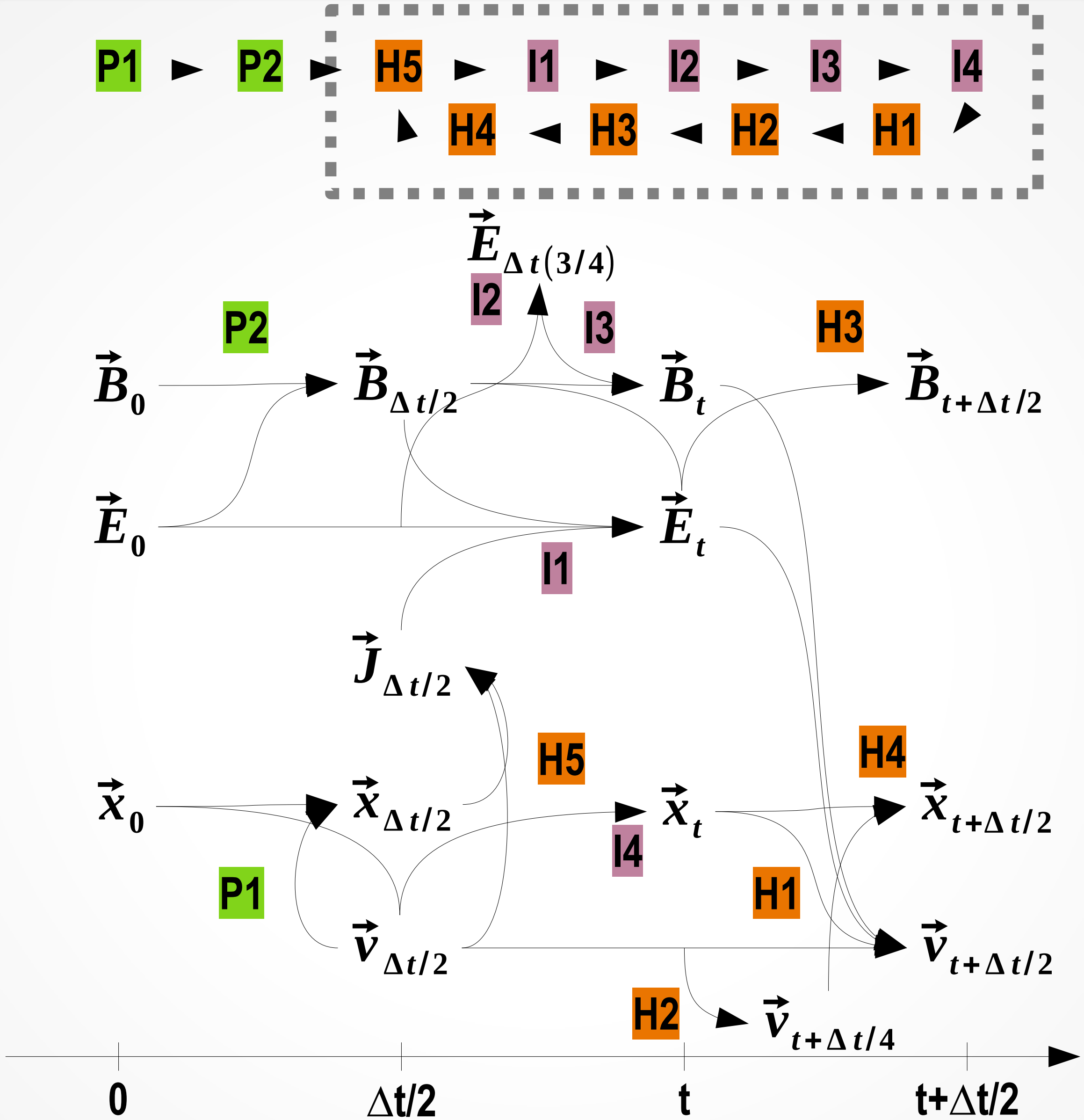


My third EM code

I1 $$-\frac{\partial B_z}{\partial x}=\mu_0 J_y+\frac{1}{c^2}\frac{\partial E_y}{\partial t} \qquad \frac{\partial B_y}{\partial x}=\mu_0 J_z+\frac{1}{c^2}\frac{\partial E_z}{\partial t}$$



My third EM code



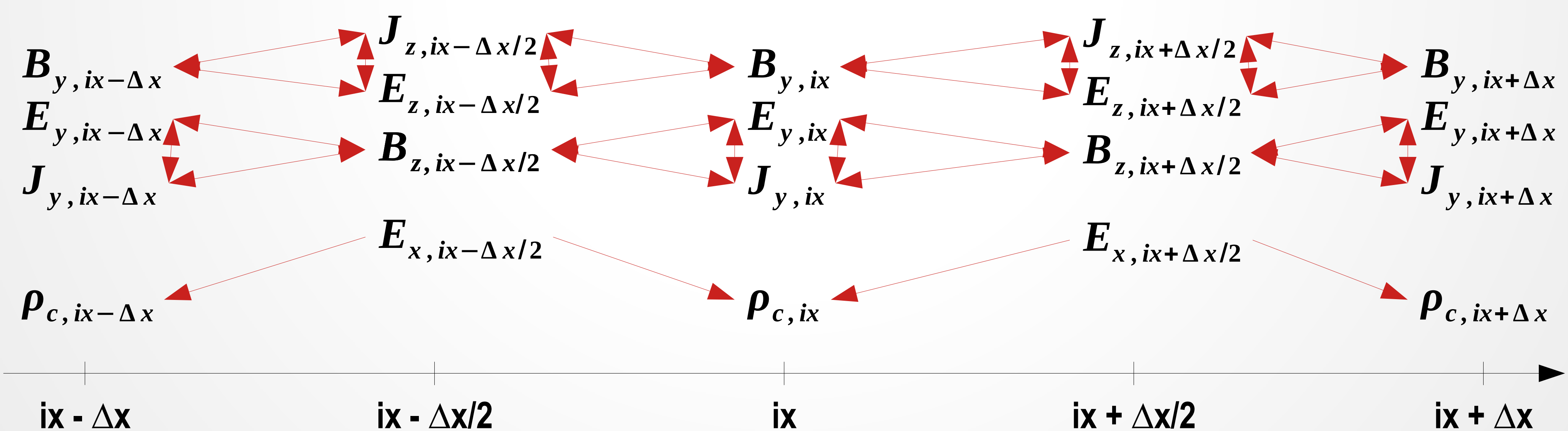
My third EM code

$$\frac{\partial E_x}{\partial x}=\frac{\rho_c}{\varepsilon_0}$$

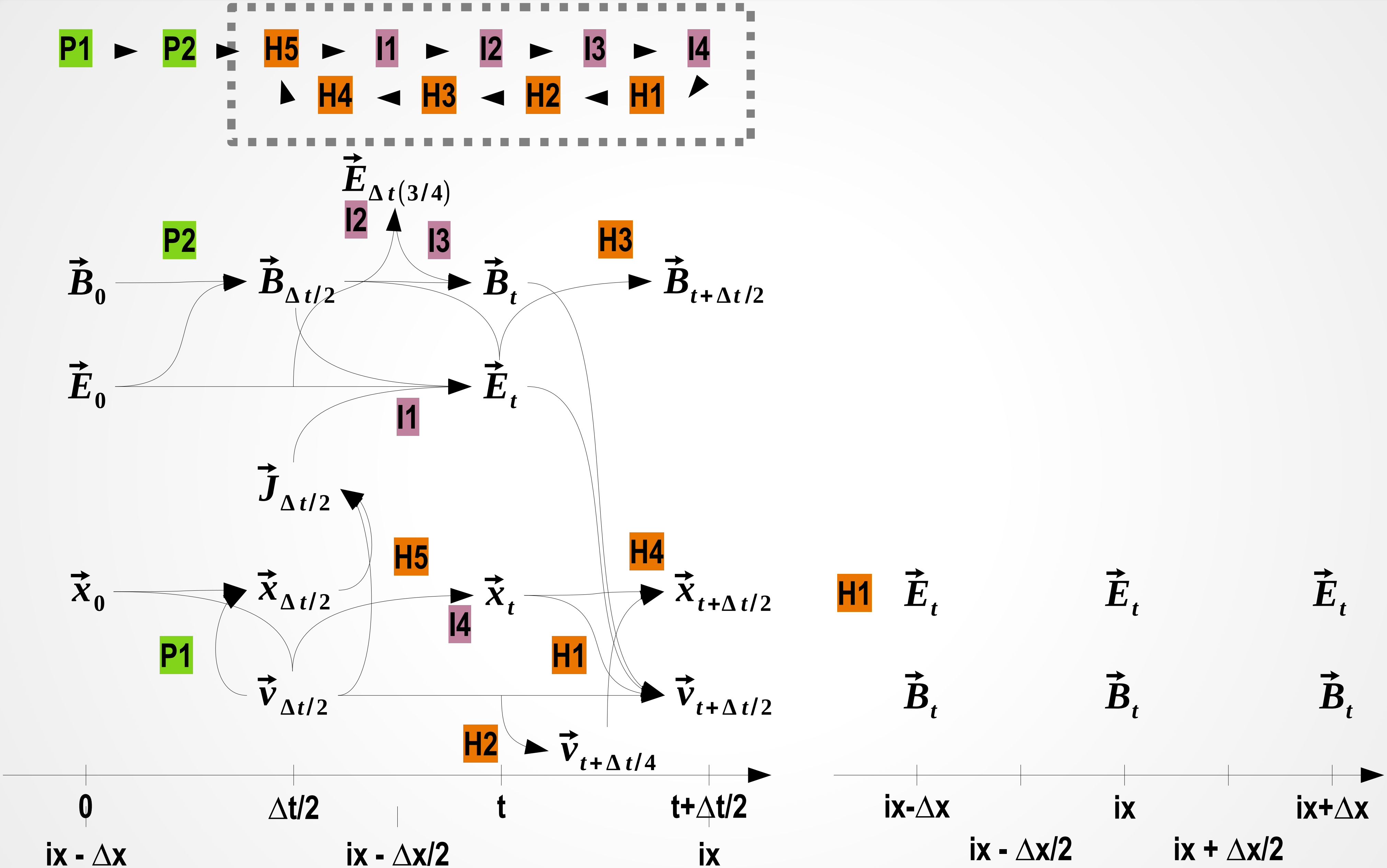
$$\frac{\partial}{\partial y}=\frac{\partial}{\partial z}=0$$

$$-\frac{\partial B_z}{\partial x}=\mu_0 J_y+\frac{1}{c^2}\frac{\partial E_y}{\partial t} \qquad \frac{\partial E_y}{\partial x}=-\frac{\partial B_z}{\partial t}$$

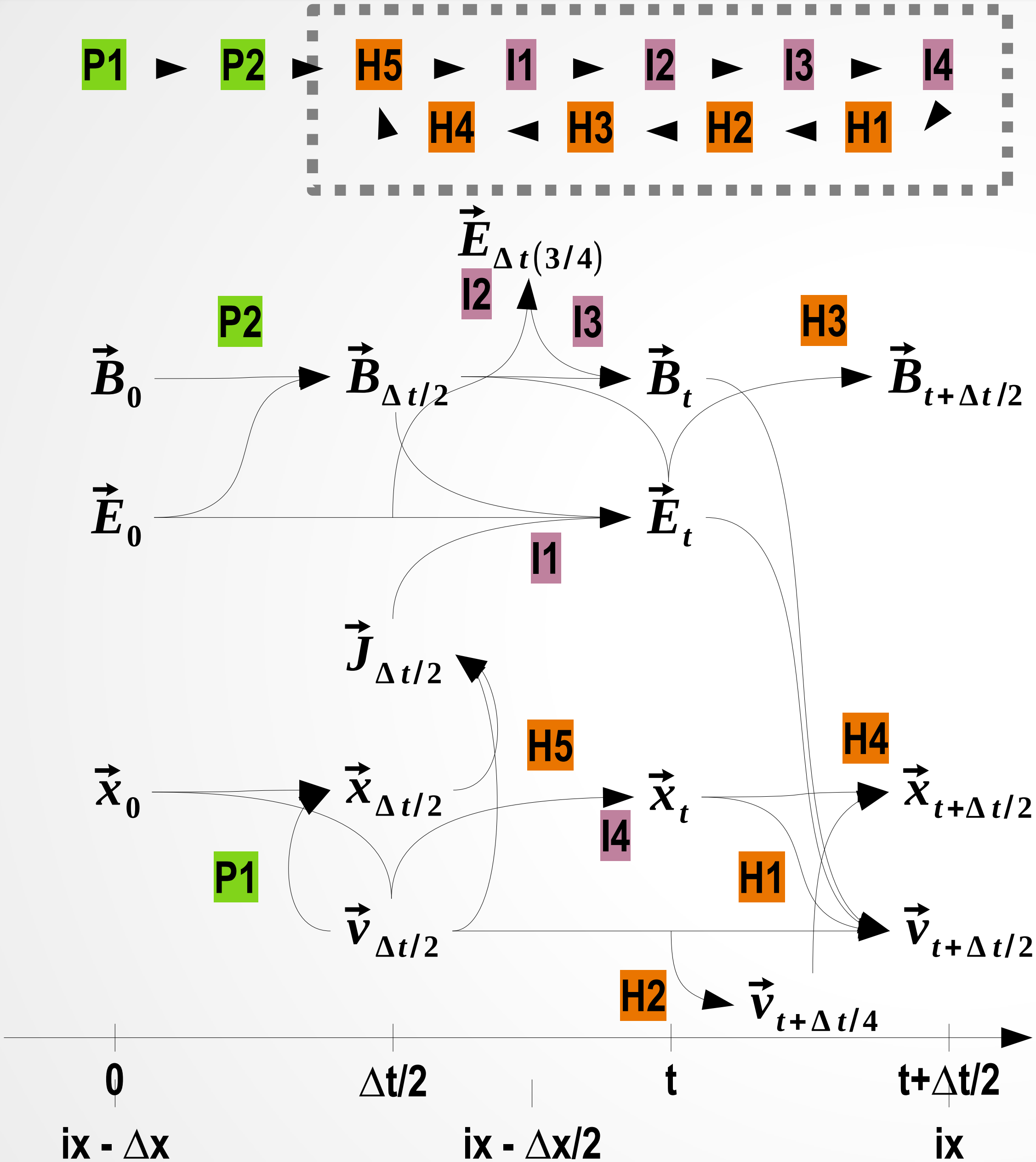
$$\frac{\partial B_y}{\partial x}=\mu_0 J_z+\frac{1}{c^2}\frac{\partial E_z}{\partial t} \qquad -\frac{\partial E_z}{\partial x}=-\frac{\partial B_y}{\partial t}$$



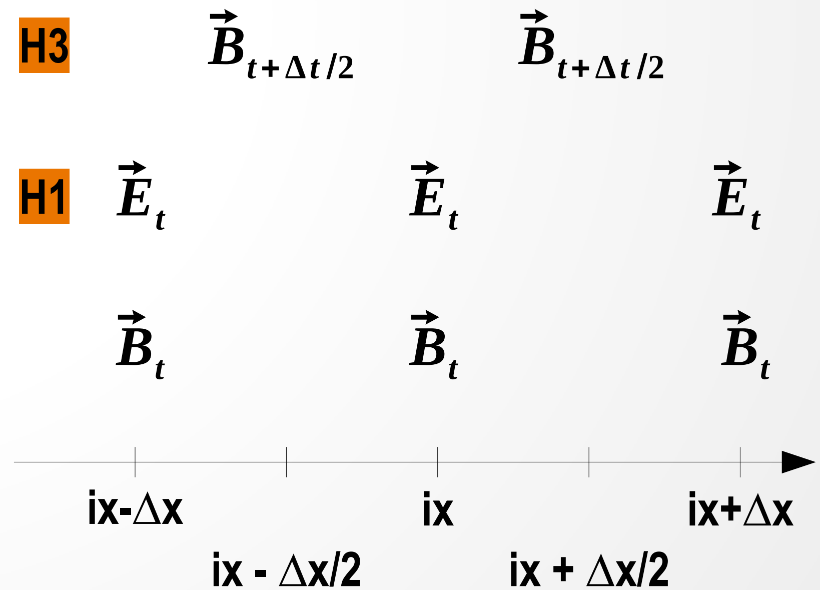
My third EM code



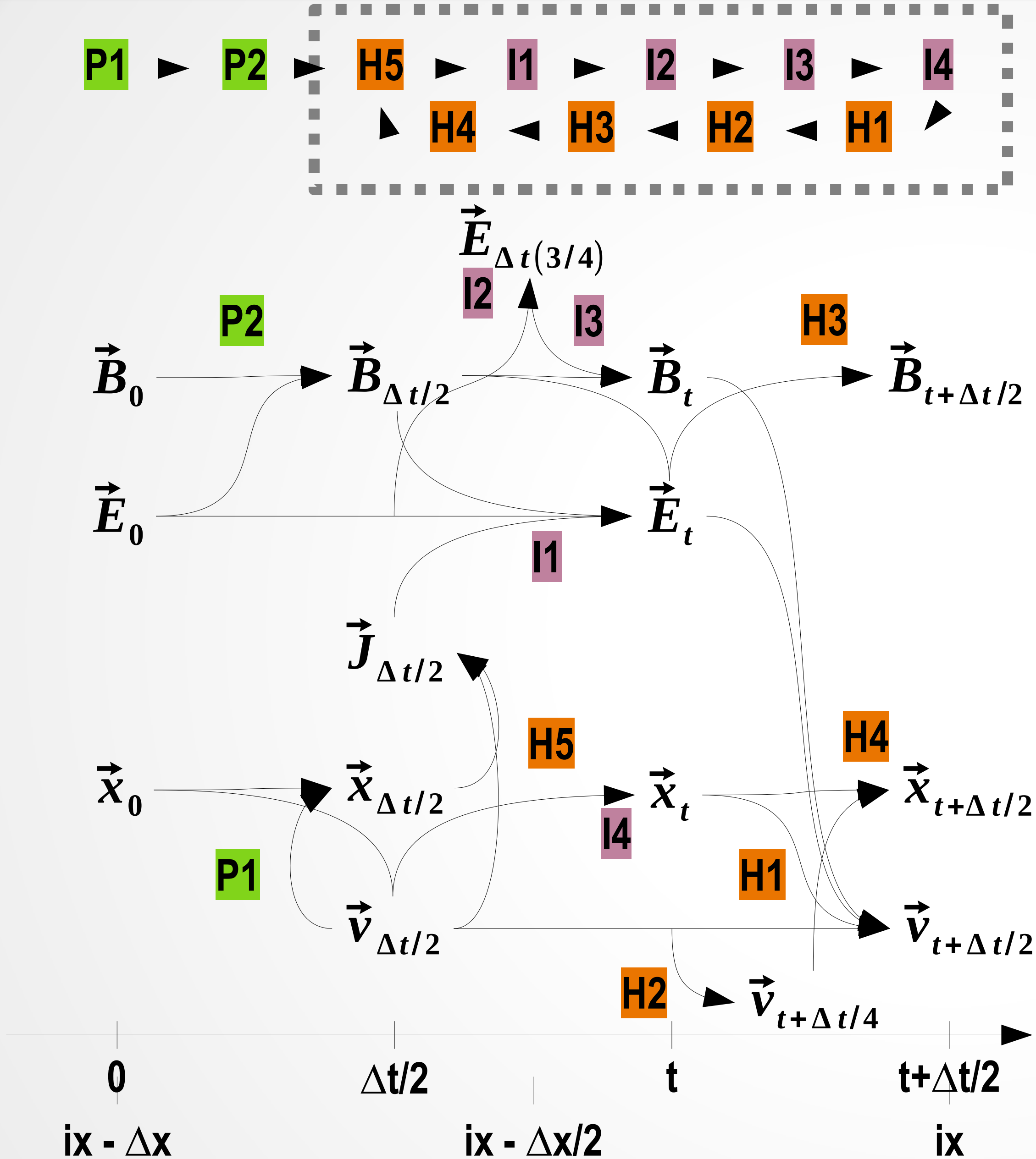
My third EM code



H3 $$-\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} \qquad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$



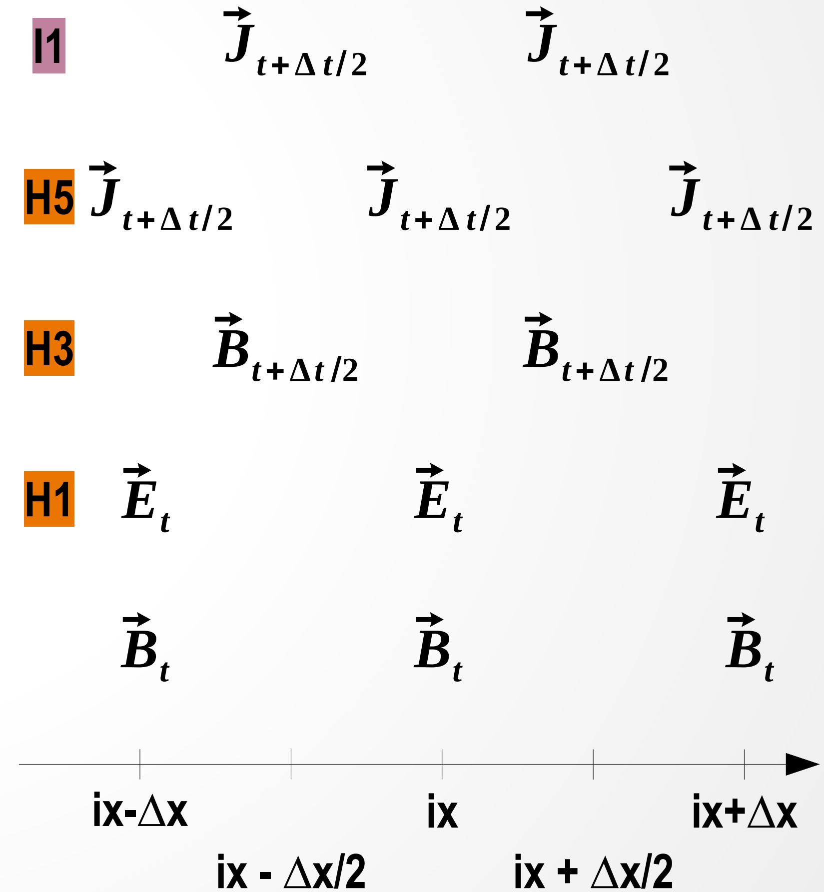
My third EM code



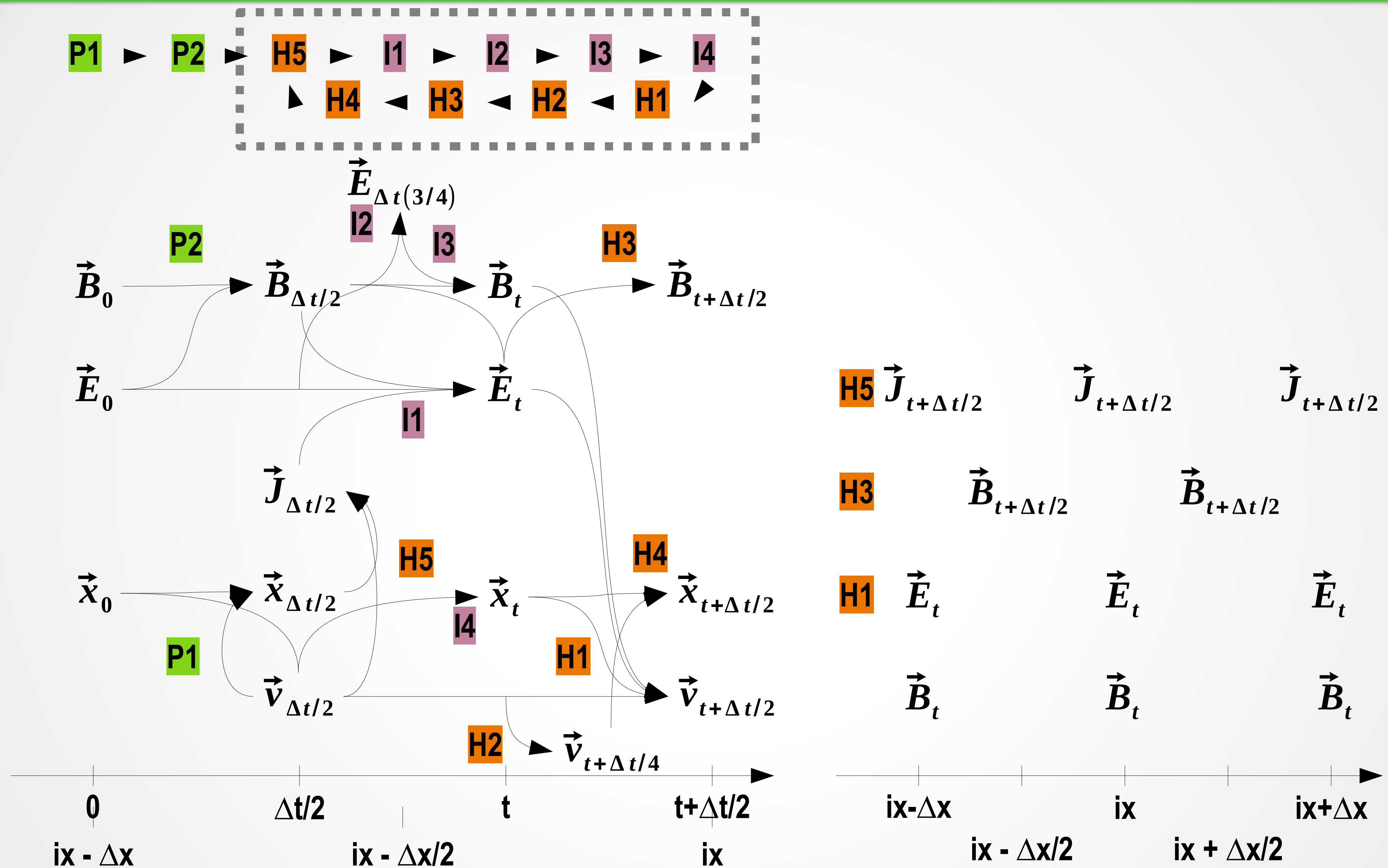
I1

$$-\frac{\partial B_z}{\partial x} = \mu_0 J_y + \frac{1}{c^2}\frac{\partial E_y}{\partial t}$$

$$\frac{\partial B_y}{\partial x} = \mu_0 J_z + \frac{1}{c^2}\frac{\partial E_z}{\partial t}$$



Time sequence of electromagnetic code



Relativistic particle pusher: hand on

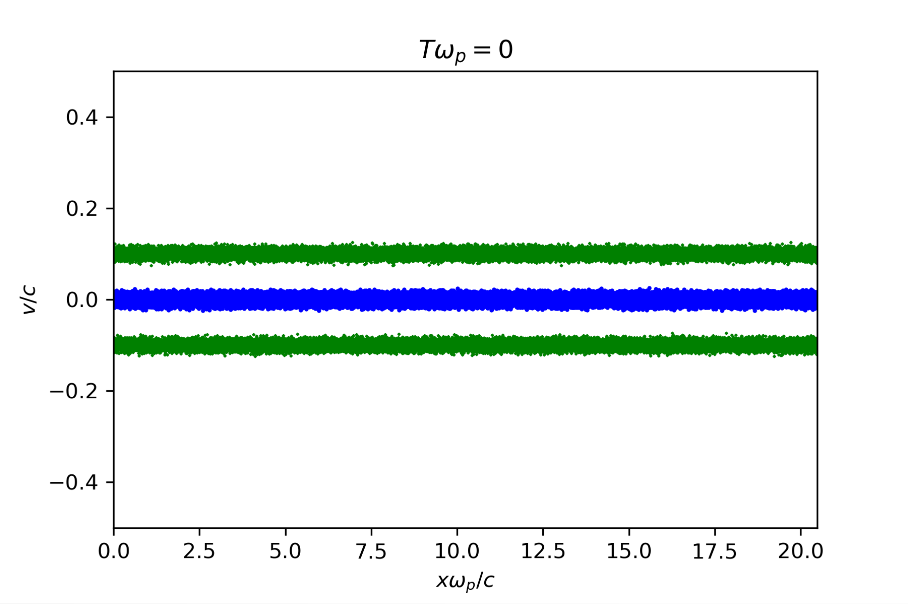
C = 1.0, Lx =20.48, Δx = 0.01, NX = 2048, Δt = 0.01, NT = 6400
BFx=1.0, BFy=0.0, BFz=0.0, EFx=0.0, BFy=0.0, BFz=0.0

Ion: NP = 65536,m = 1836, e = 1, vdx = vdy = vdz = 0.0, vthx = vthy = vthz = 0.01
electron1: NP = 32768,m = 1 , e = -1, vdy = vdz = 0.0, vthx = vthy = vthz = 0.01
electron2: NP = 32768,m = 1 , e = -1, vdy = vdz = 0.0, vthx = vthy = vthz = 0.01

Wpe,total = 1.0

Example:10_01_1DES_Re.90 & 11_01_1DEM_Re_subroutine.f90

1. electron1: vdx = 0.1, electron2: vdx = -0.1



2. electron1: vdx = 0.8, electron2: vdx = -0.8