

01. Introduction to the PIC simulation

02. Random number generation and its application

03. Particle weighting and normalization

04. Particle pusher

05. Poisson's equation

06. One-dimensional electrostatic PIC code

07. Numerical tips and tricks in PIC simulations

08. Visualization

09. Electromagnetic field solver

10. Relativistic particle pusher

Particle-in-Cell (PIC) kinetic simulations

11. One-dimensional electromagnetic PIC code

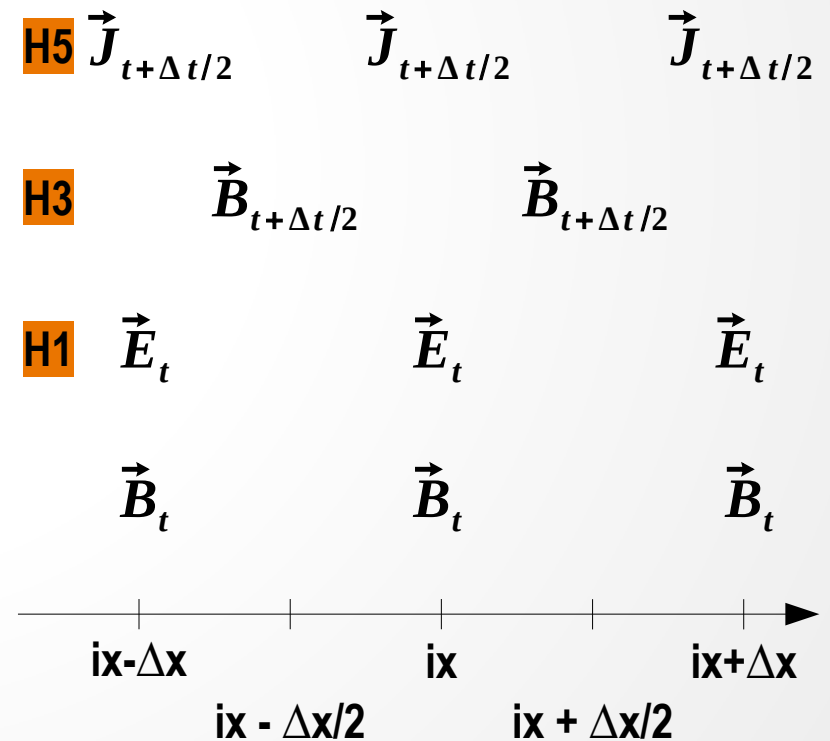
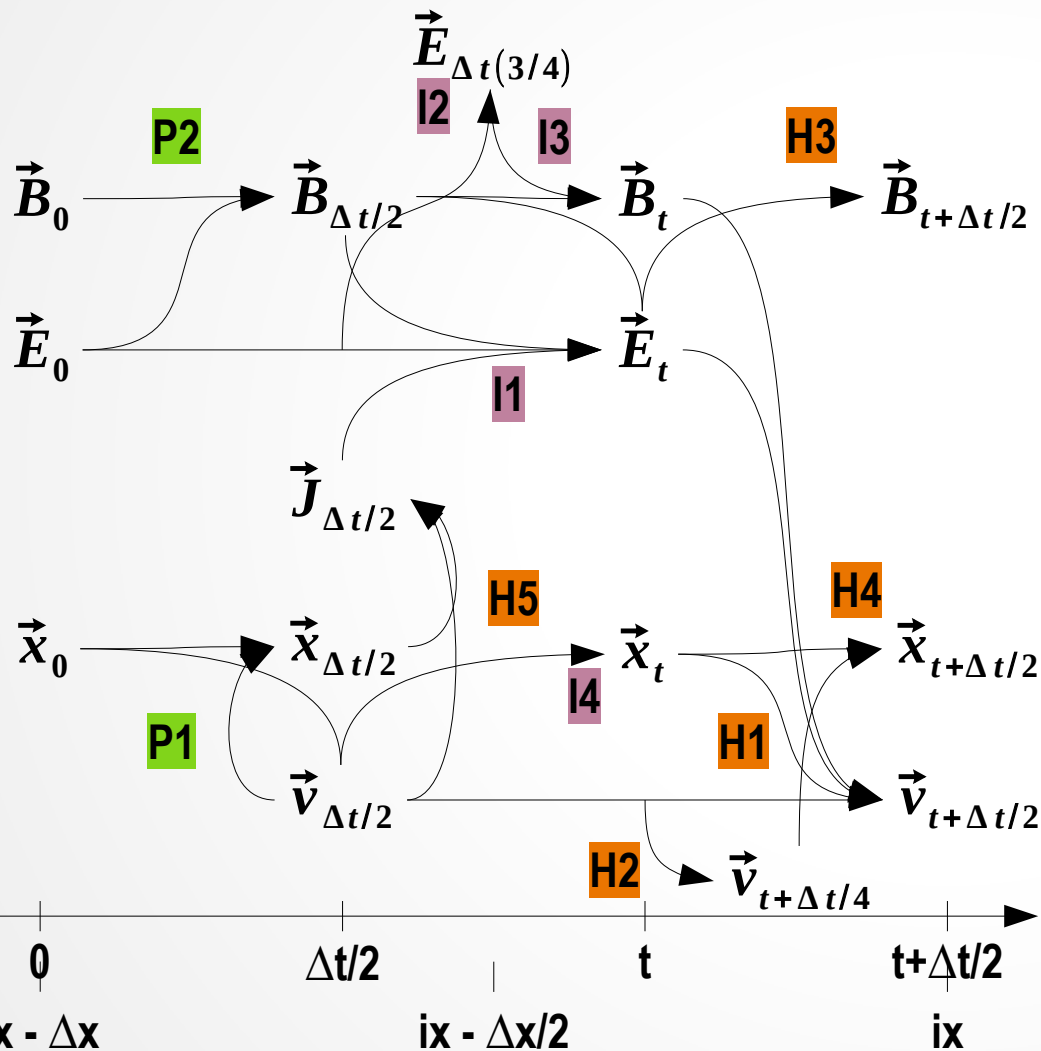
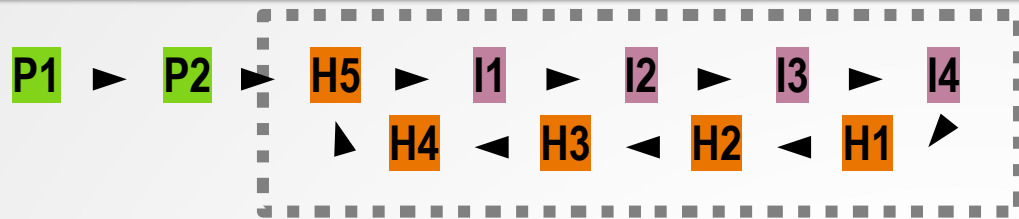
Chun-Sung Jao (饒駿頌)

Assistant Research Scholar,
Institute of Space Science and Engineering,
National Central University, Taiwan

University of São Paulo, 2019.11.25-12.06

www.slido.com code: #B194

Time sequence of electromagnetic code



My first ES code

$$\nabla \cdot \vec{E} = \frac{\rho_c}{\epsilon_0}$$

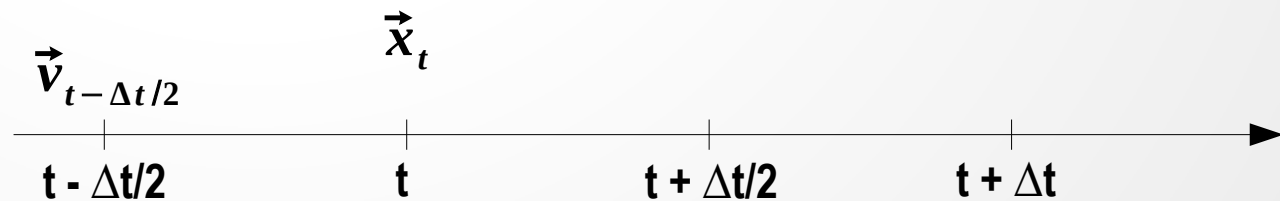
$$\frac{\partial E_x}{\partial x} = \frac{\rho_c}{\epsilon_0}$$

$$m \frac{d\vec{v}}{dt} = q \vec{E}$$

$$\frac{\vec{v}_{t+\Delta t/2} - \vec{v}_{t-\Delta t/2}}{\Delta t} = \frac{q}{m} \vec{E}_t$$

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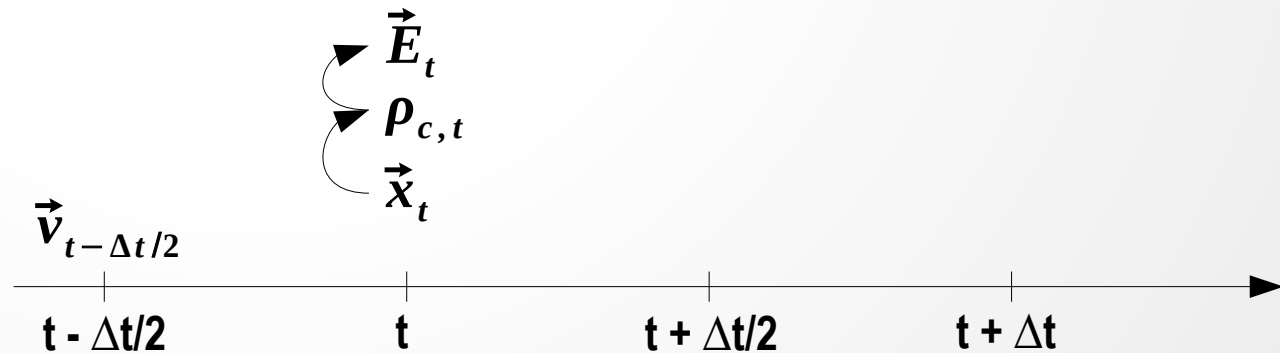
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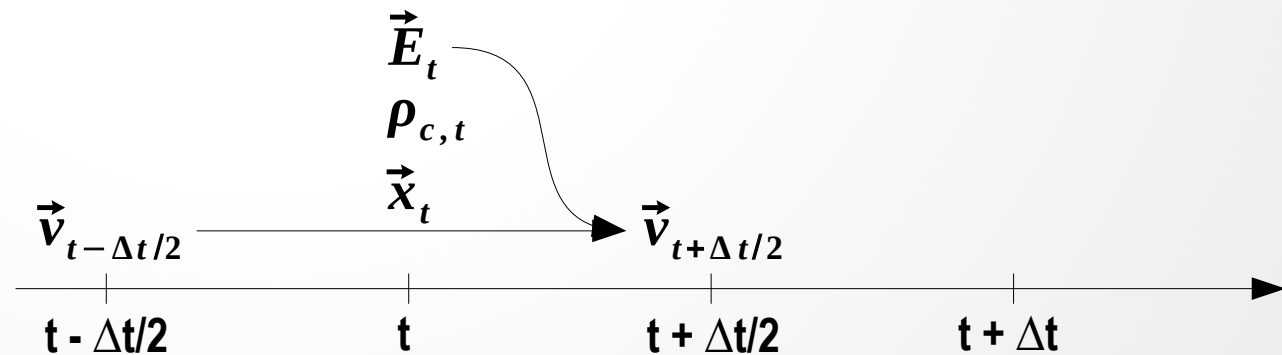
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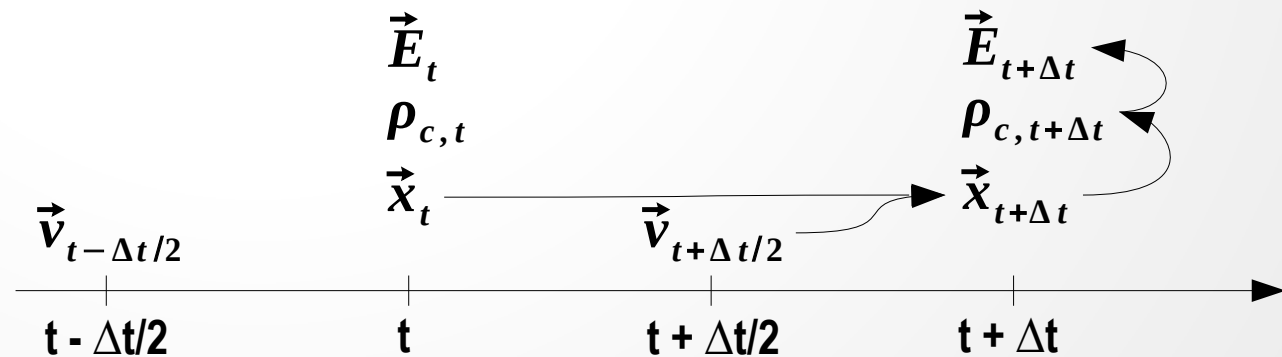
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My first EM code

$$0 = \mu_0 J_x + \frac{1}{c^2} \frac{\partial E_x}{\partial t}$$

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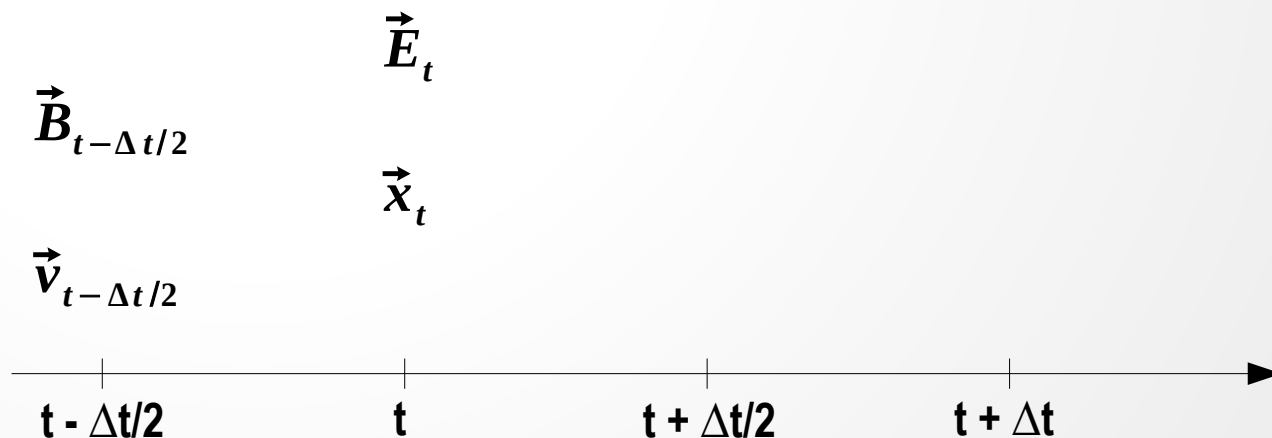
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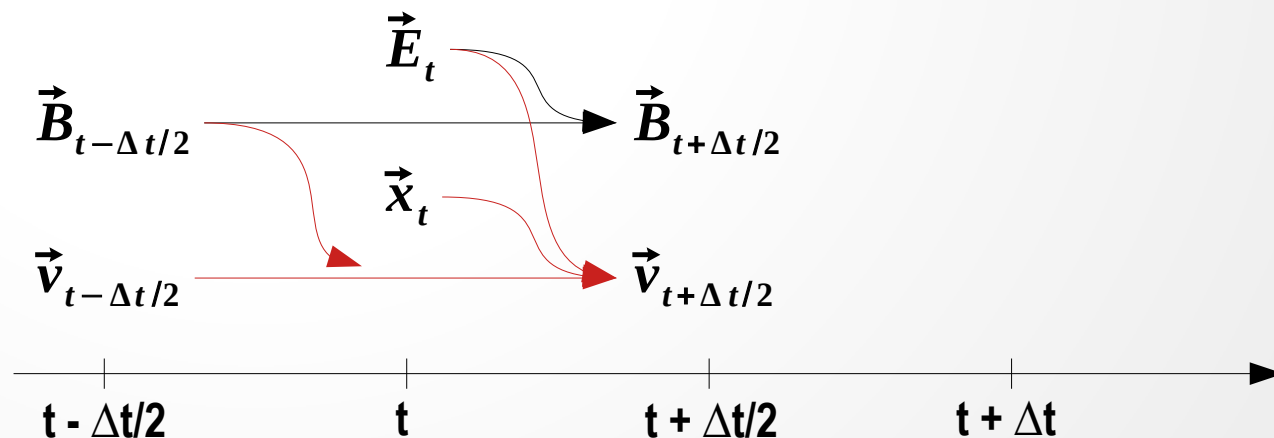
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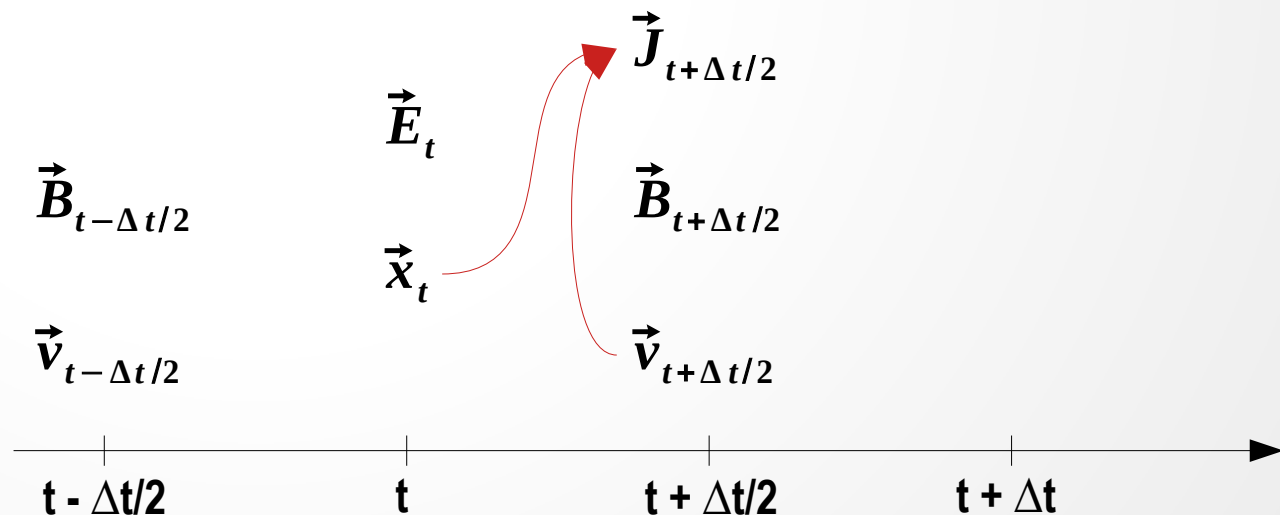
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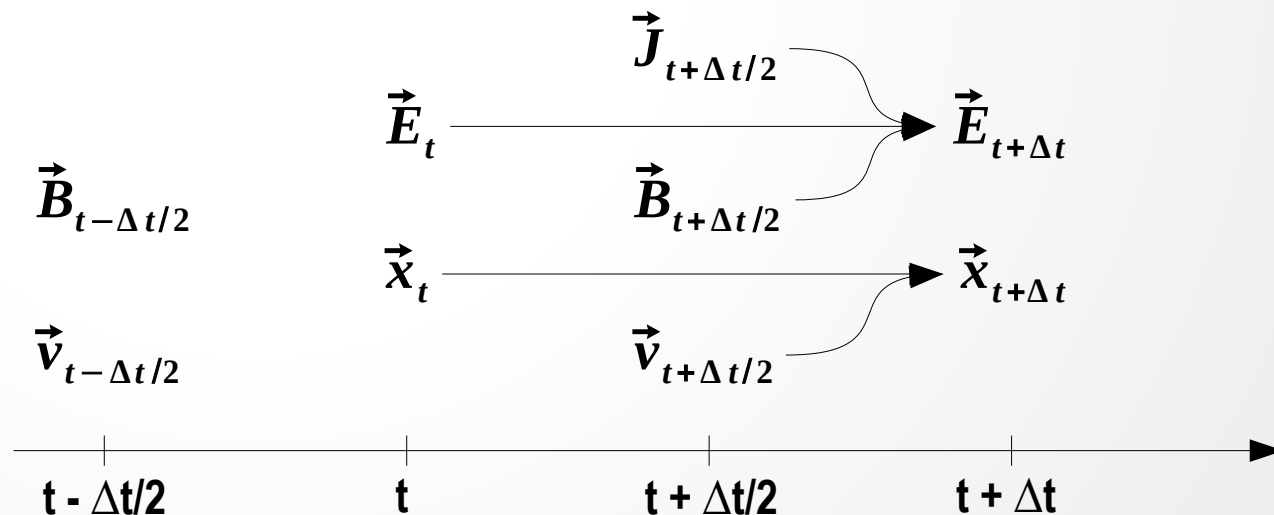
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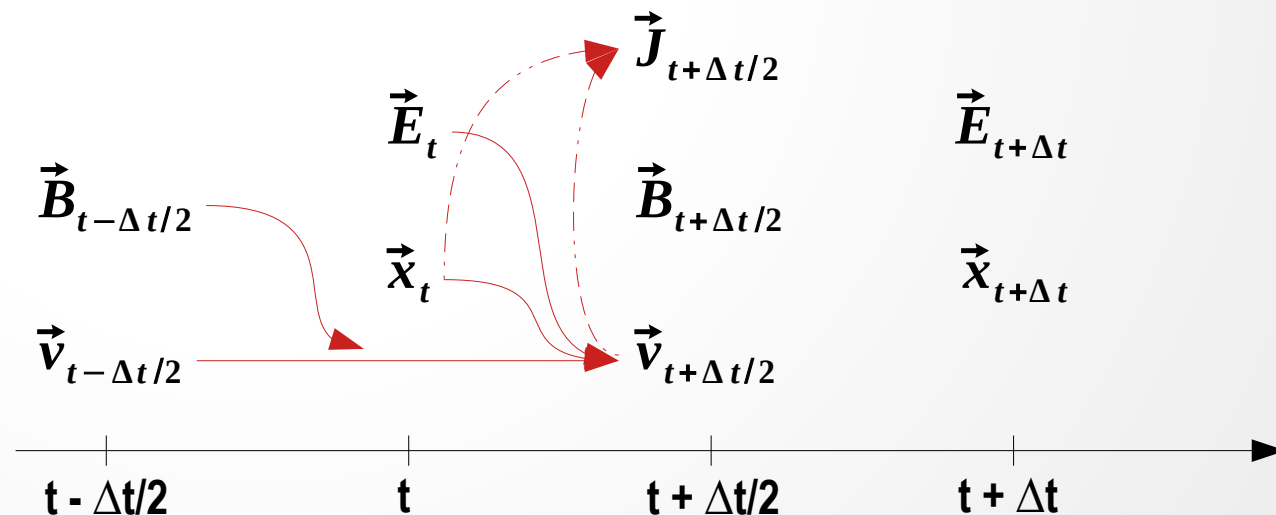
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My second EM code (refereed to KEMPO1)

$$0 = \mu_0 J_x + \frac{1}{c^2} \frac{\partial E_x}{\partial t}$$

$$\frac{d}{dt}(\gamma m_o \vec{v}) = q(\vec{E} + \vec{v} \times \vec{B})$$

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$$\frac{d\vec{x}}{dt} = \vec{v}$$

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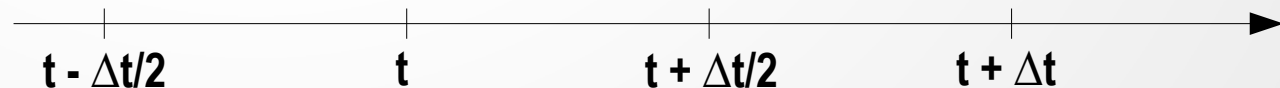
$$\vec{E}_t$$

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$$\vec{v}_{t-\Delta t/2}$$



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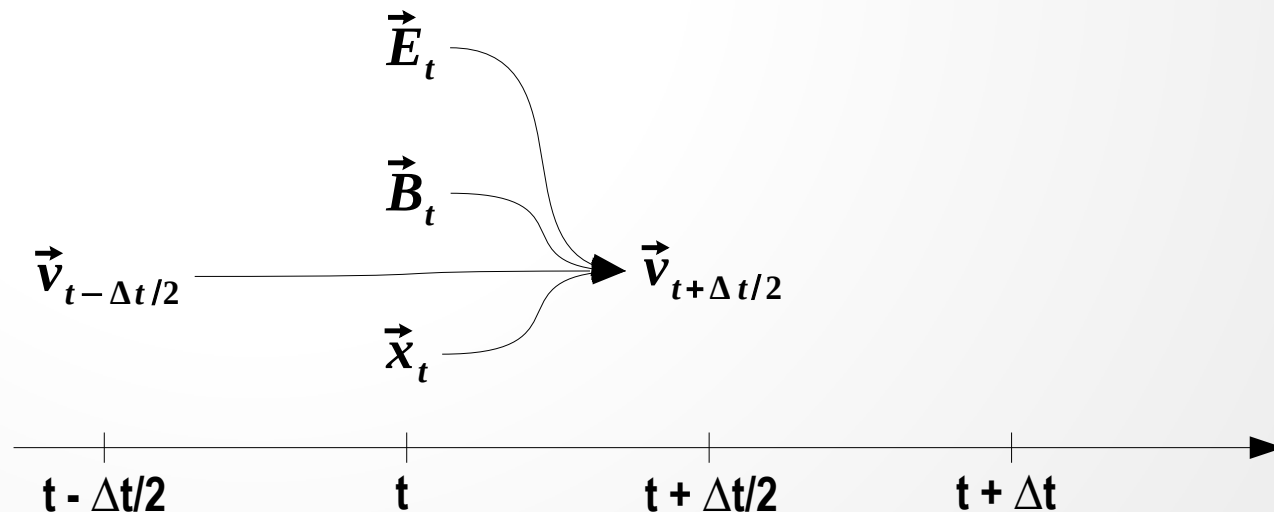
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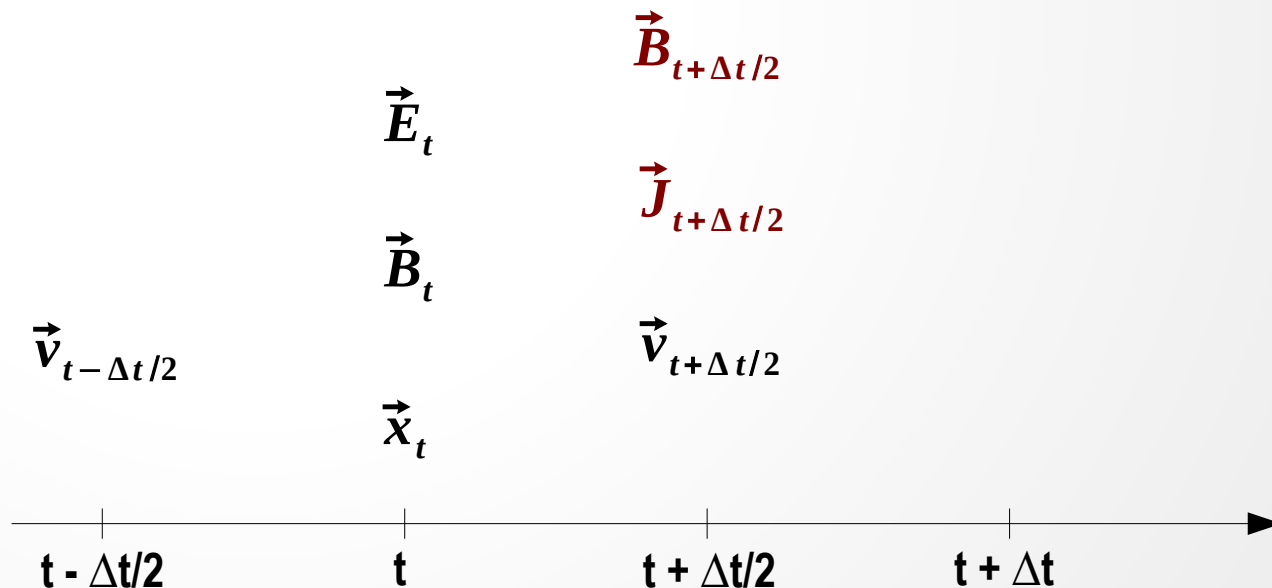
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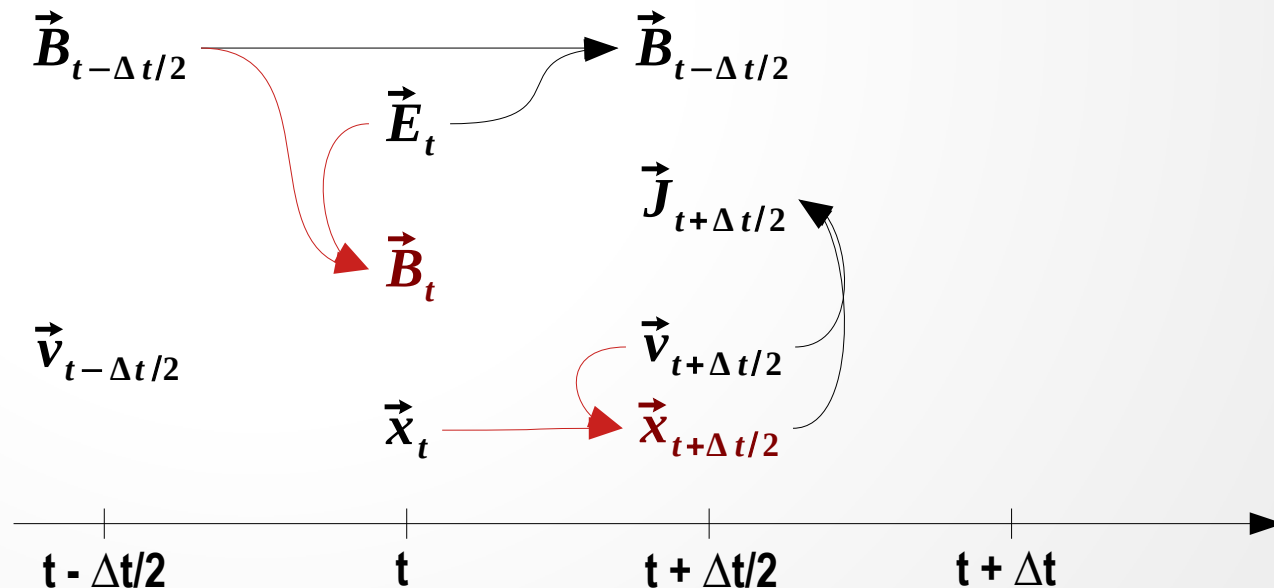
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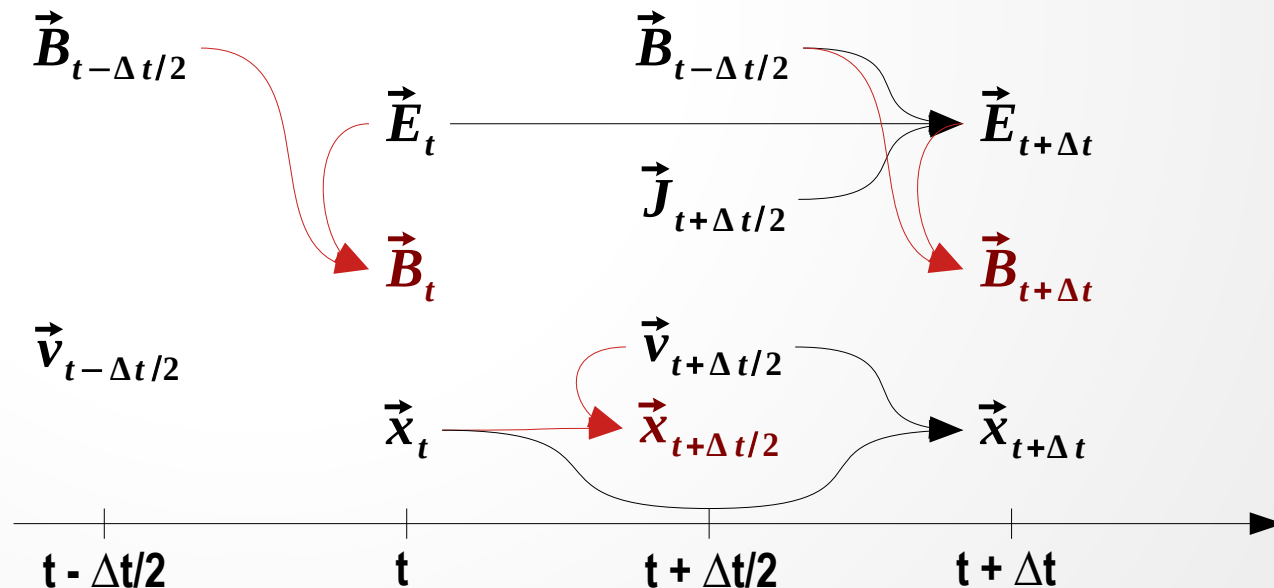
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My third EM code??

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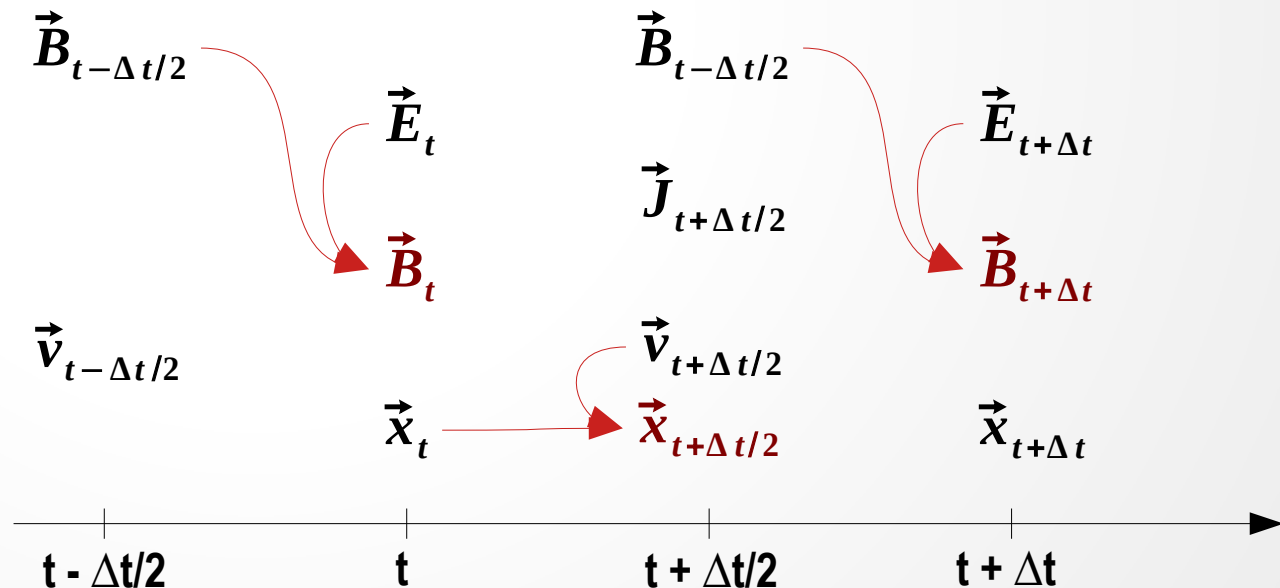
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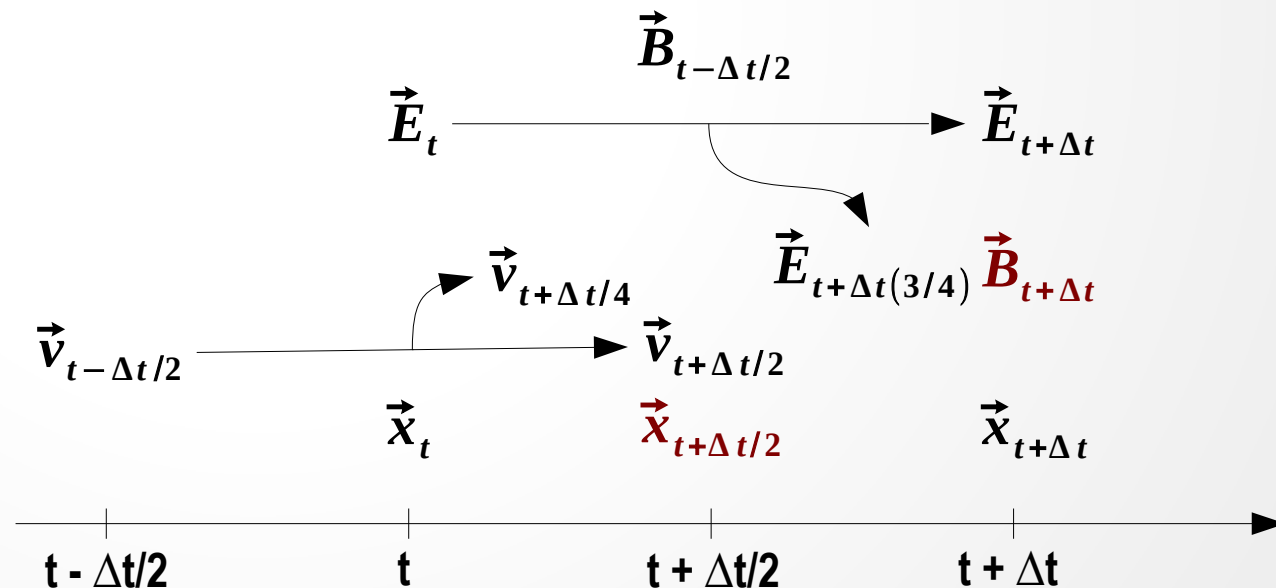
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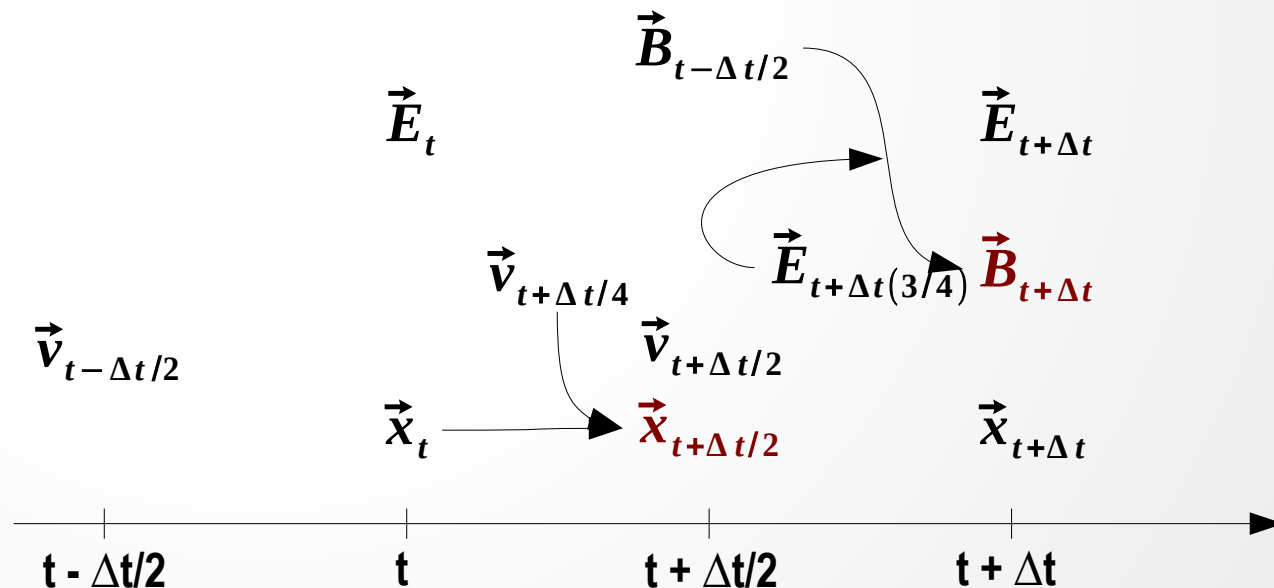
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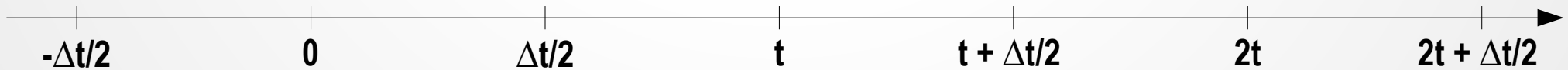
My third EM code

$$\vec{B}_0$$

$$\vec{E}_0$$

$$\vec{x}_0$$

$$\vec{v}_{\Delta t/2}$$

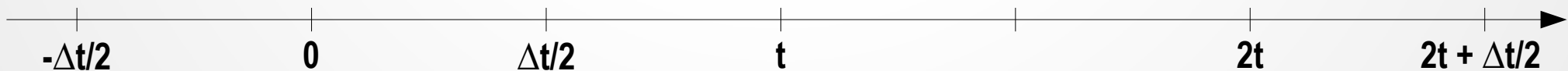
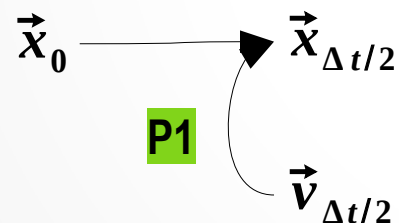
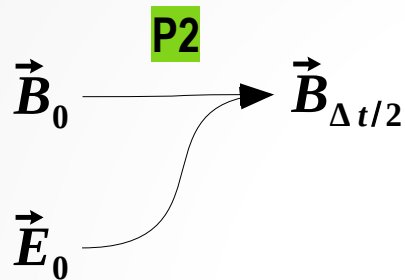


My third EM code

P1 $\frac{d\vec{x}}{dt} = \vec{v}$

P2 $-\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$

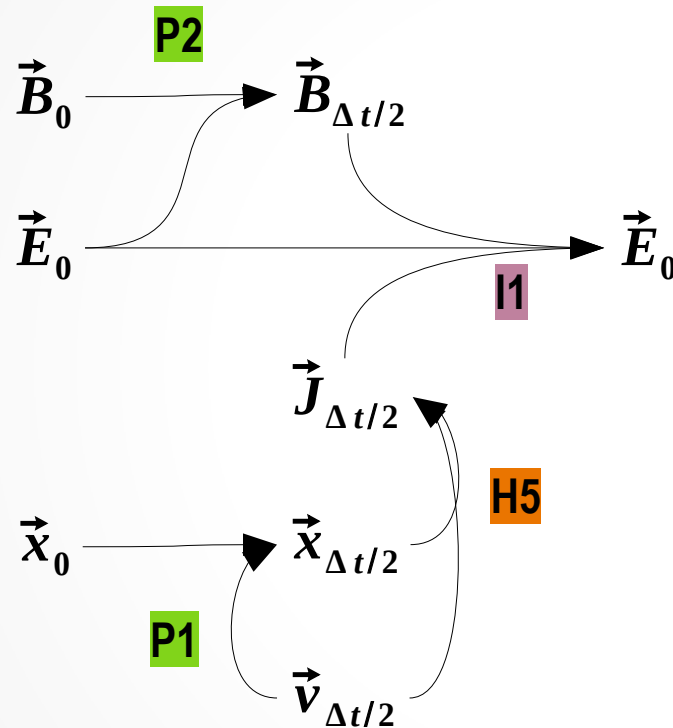
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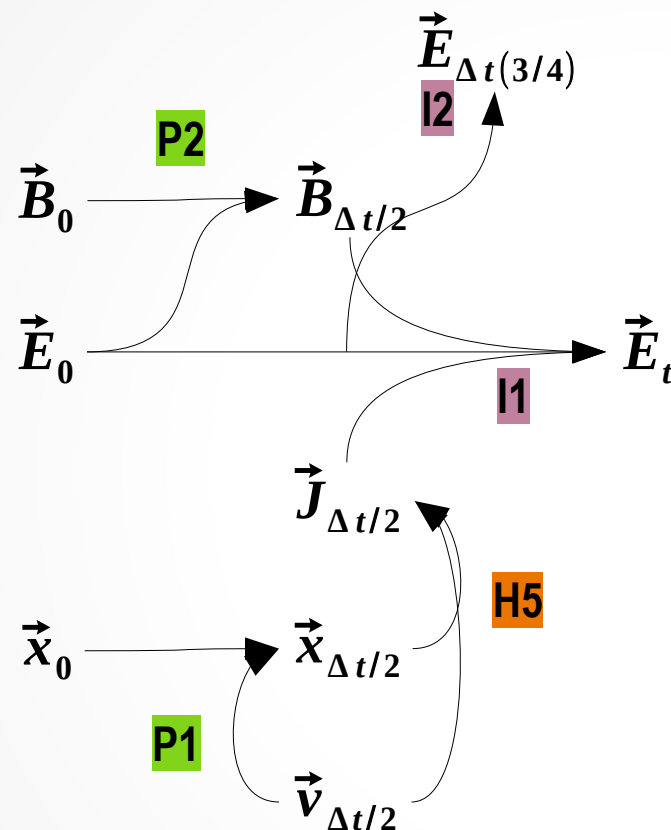
My third EM code

I1

$$0 = \mu_0 J_x + \frac{1}{c^2} \frac{\partial E_x}{\partial t} \quad -\frac{\partial B_z}{\partial x} = \mu_0 J_y + \frac{1}{c^2} \frac{\partial E_y}{\partial t} \quad \frac{\partial B_y}{\partial x} = \mu_0 J_z + \frac{1}{c^2} \frac{\partial E_z}{\partial t}$$



My third EM code

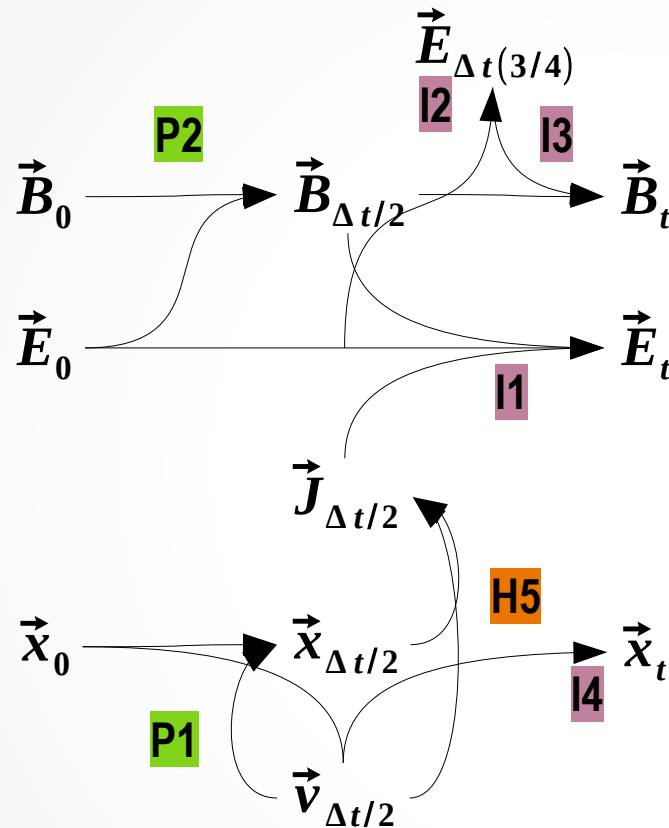


My third EM code

I4 $\frac{d\vec{x}}{dt} = \vec{v}$

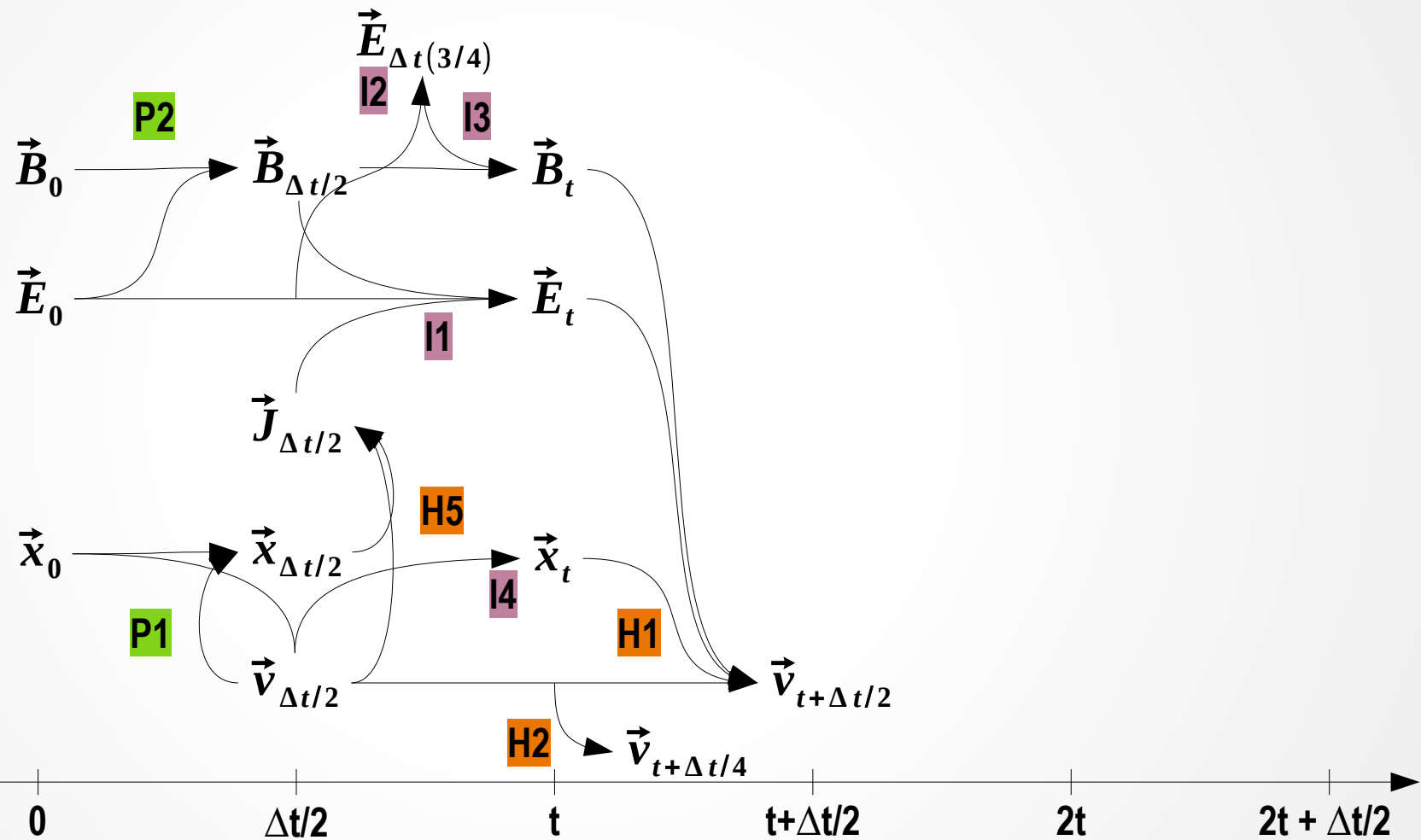
I3 $-\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$

$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$



My third EM code

H1 $\frac{d}{dt}(\gamma m_o \vec{v}) = q(\vec{E} + \vec{v} \times \vec{B})$

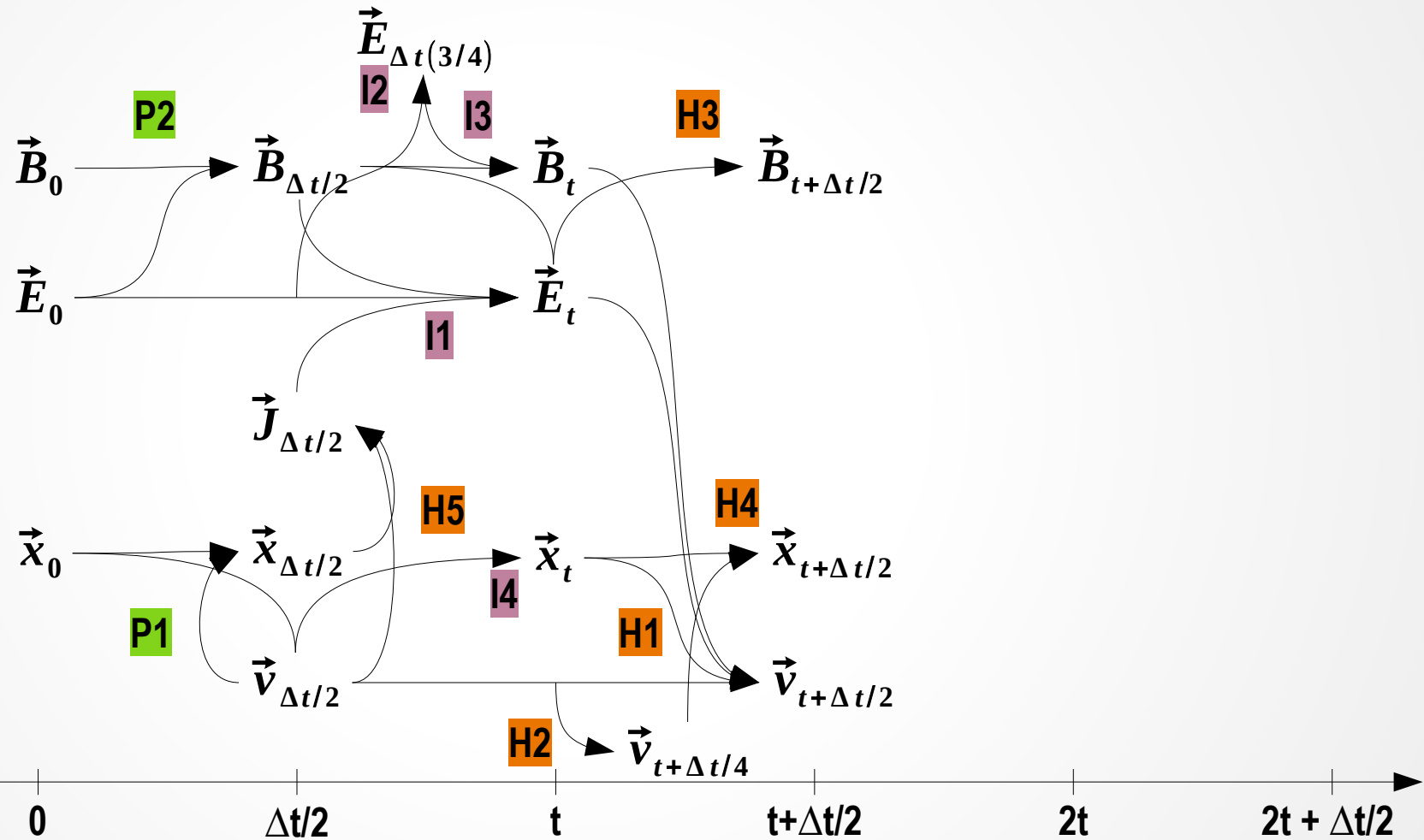


My third EM code

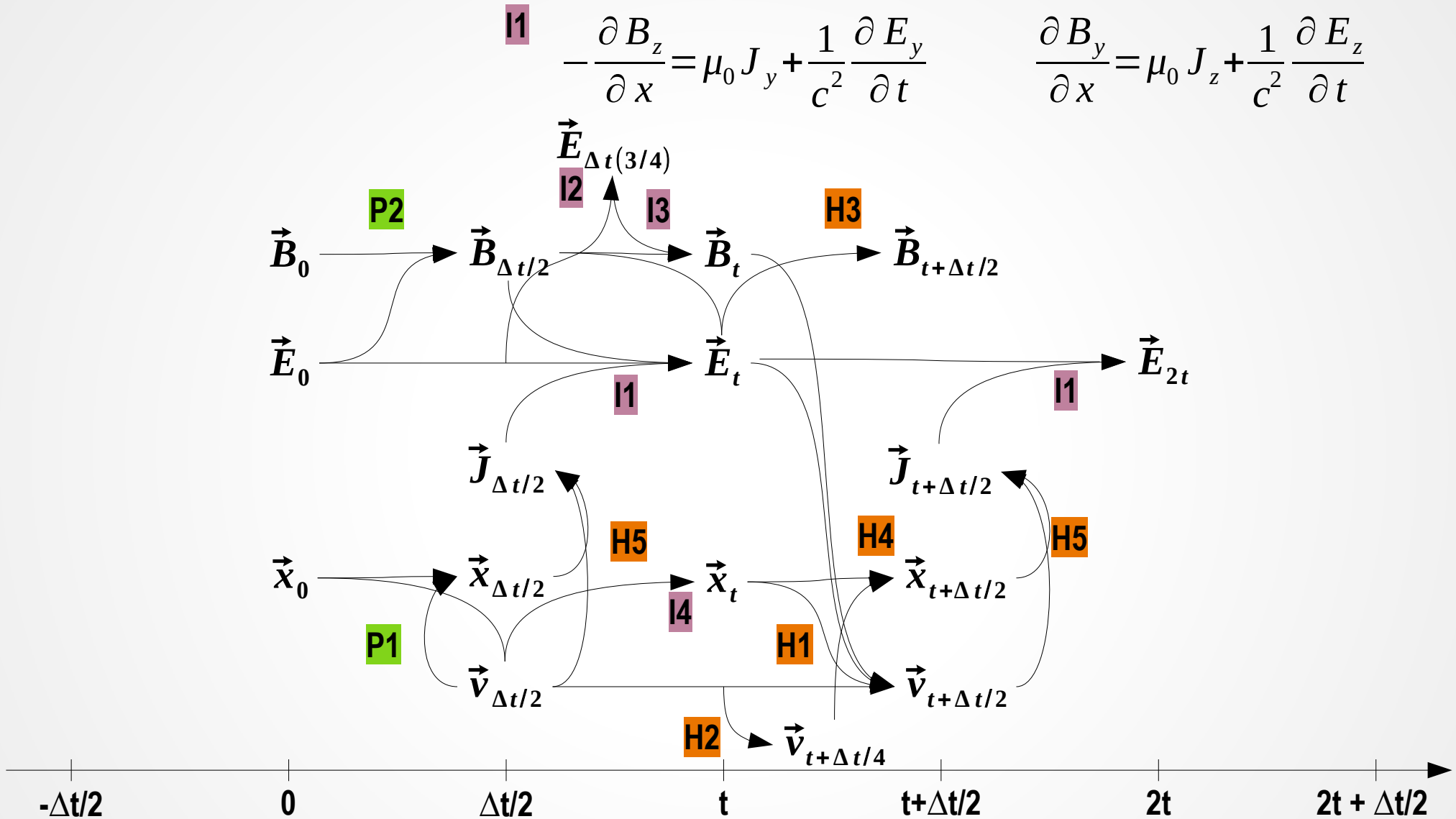
$$\text{H3} \quad -\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t}$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

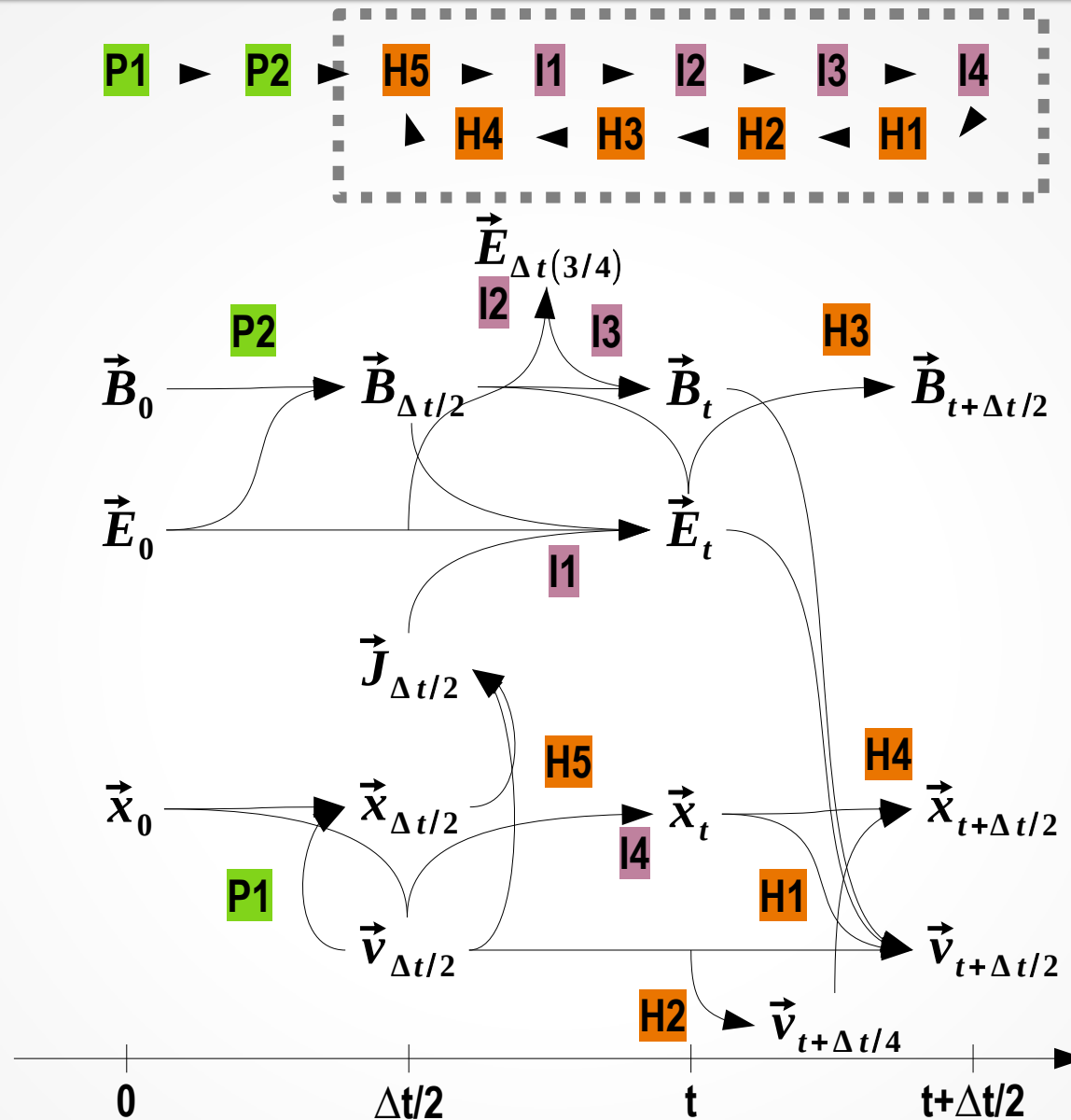
$$\text{H4} \quad \frac{d\vec{x}}{dt} = \vec{v}$$



My third EM code



My third EM code



My third EM code

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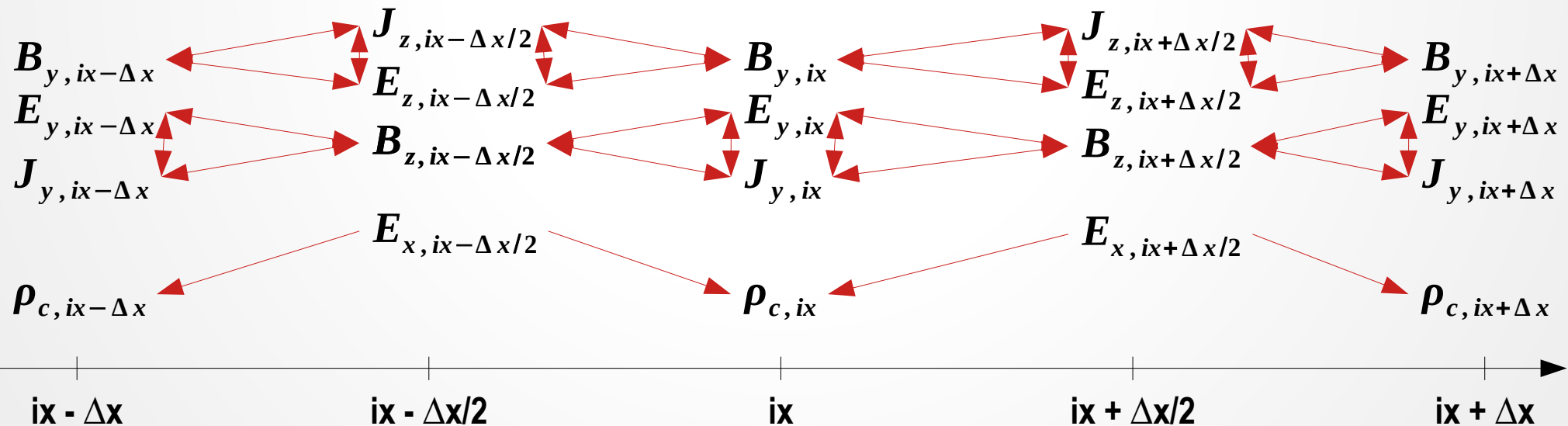
$$\frac{\partial}{\partial y} = \frac{\partial}{\partial z} = 0$$

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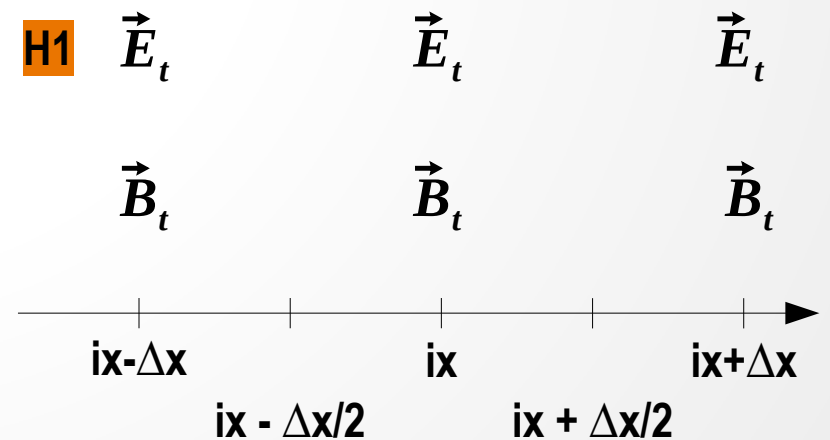
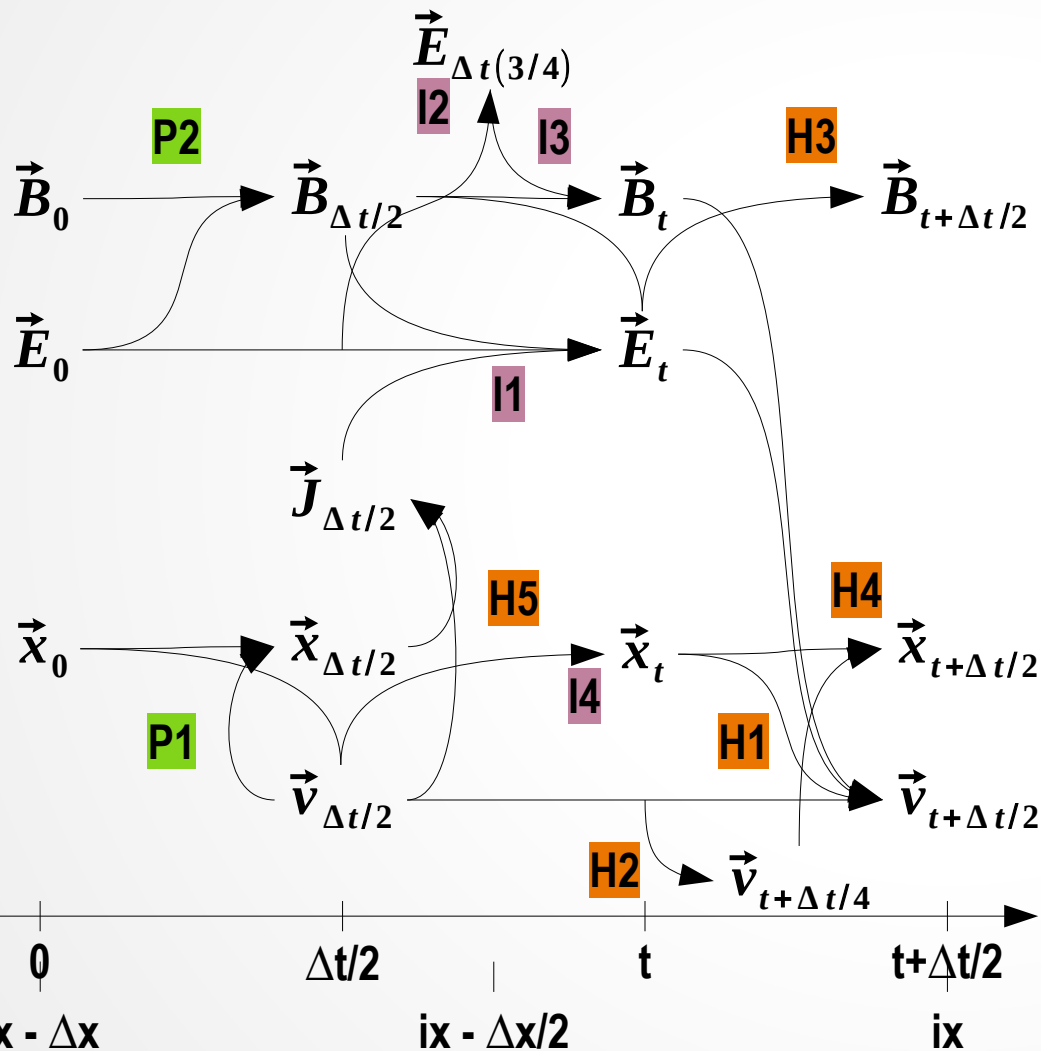
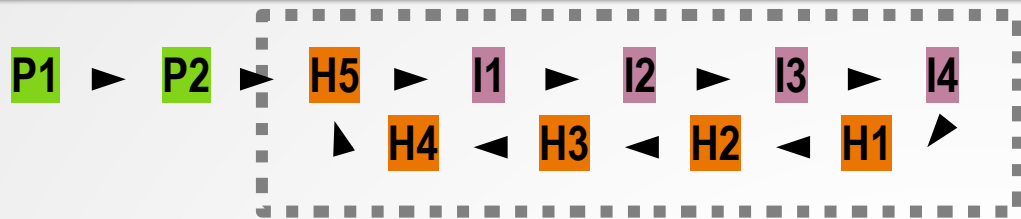
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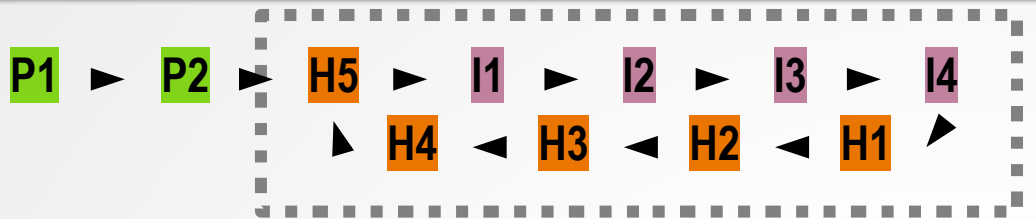
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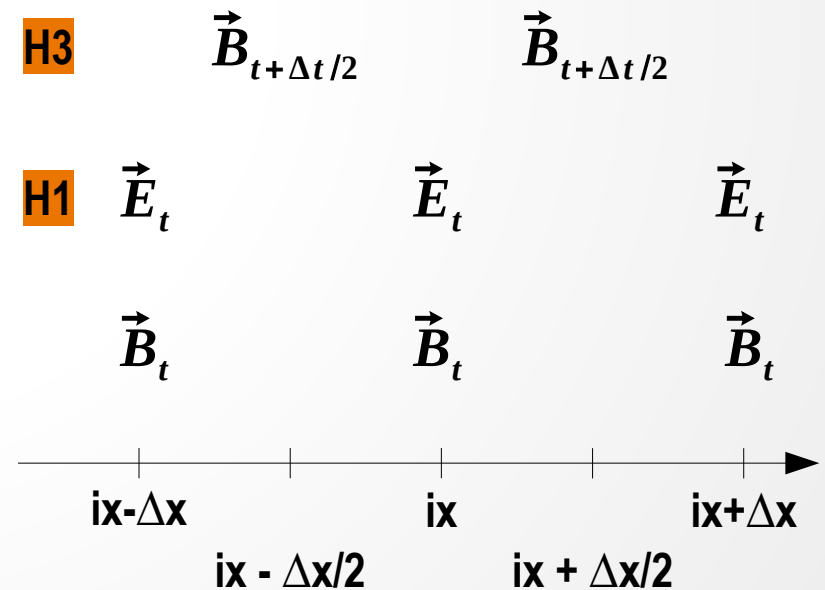
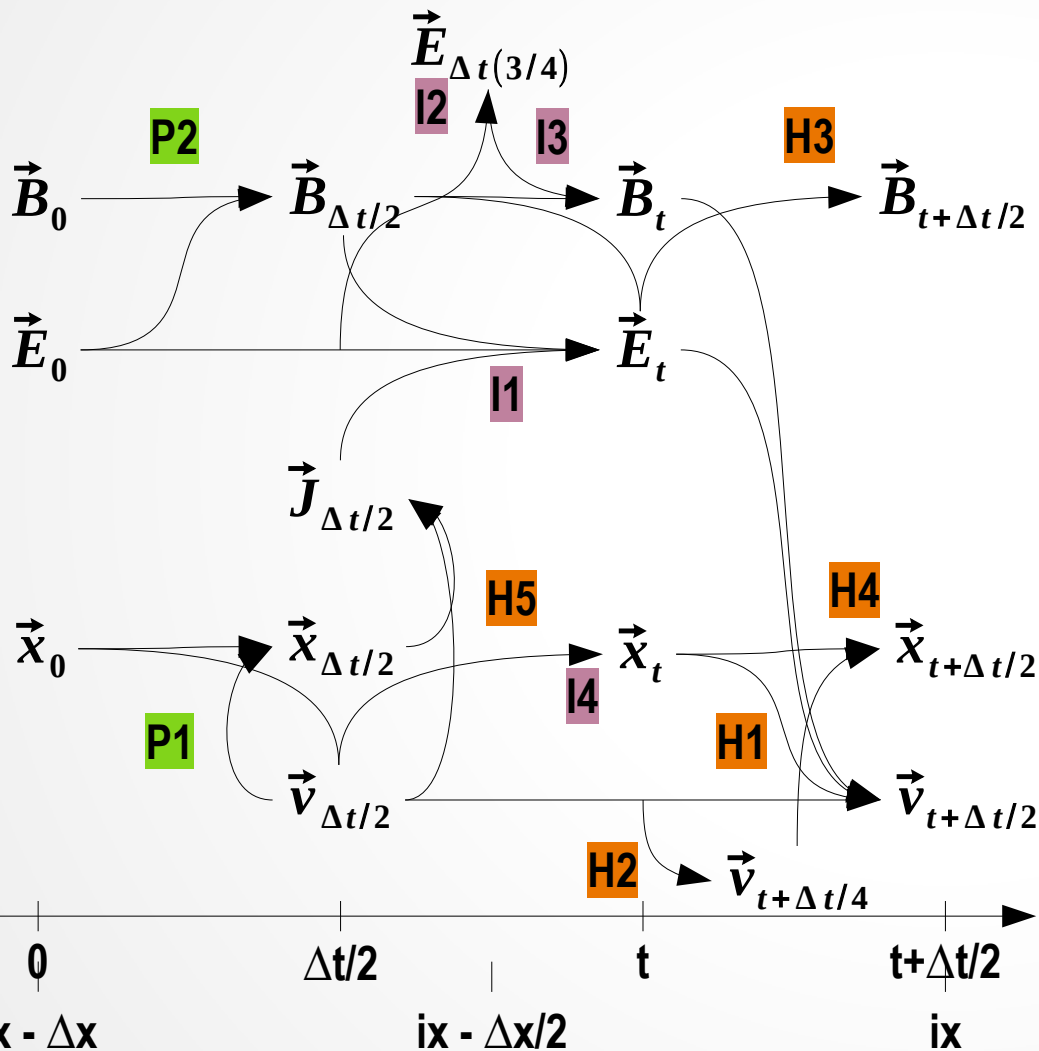
My third EM code



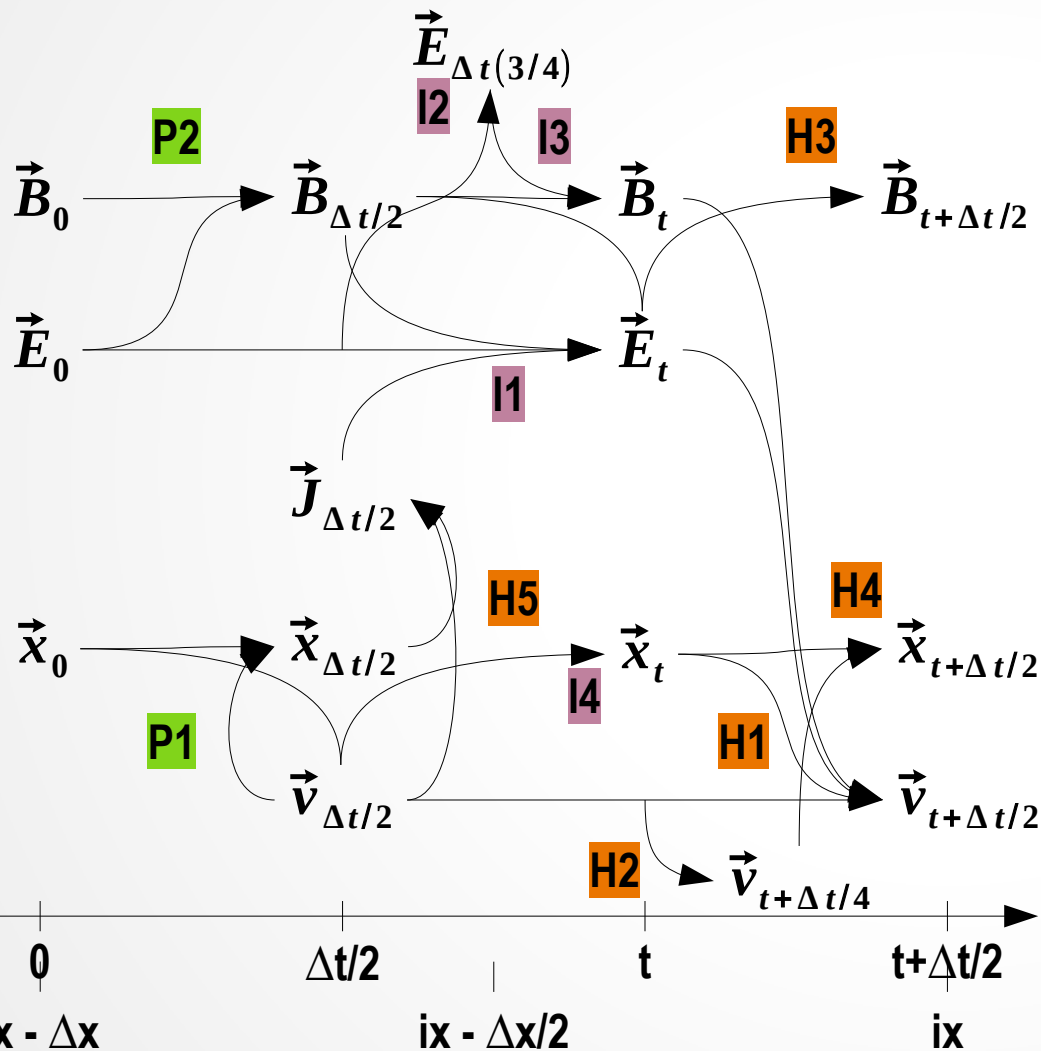
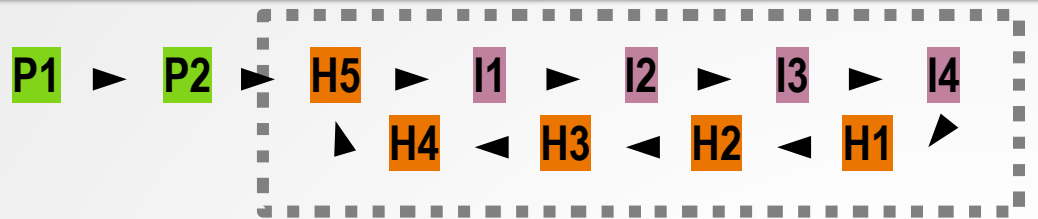
My third EM code



$$\text{H3} \quad -\frac{\partial E_z}{\partial x} = -\frac{\partial B_y}{\partial t} \quad \frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$



My third EM code



$$I1 \quad -\frac{\partial B_z}{\partial x} = \mu_0 J_y + \frac{1}{c^2} \frac{\partial E_y}{\partial t}$$

$$I1 \quad \frac{\partial B_y}{\partial x} = \mu_0 J_z + \frac{1}{c^2} \frac{\partial E_z}{\partial t}$$

$$H5 \quad \vec{J}_{t+\Delta t/2} \quad \vec{J}_{t+\Delta t/2} \quad \vec{J}_{t+\Delta t/2}$$

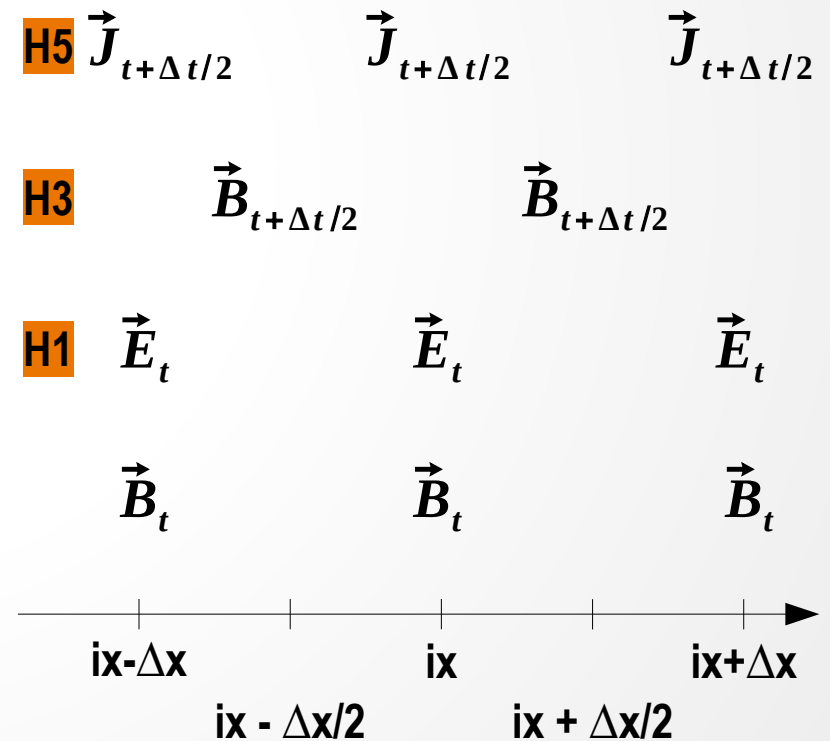
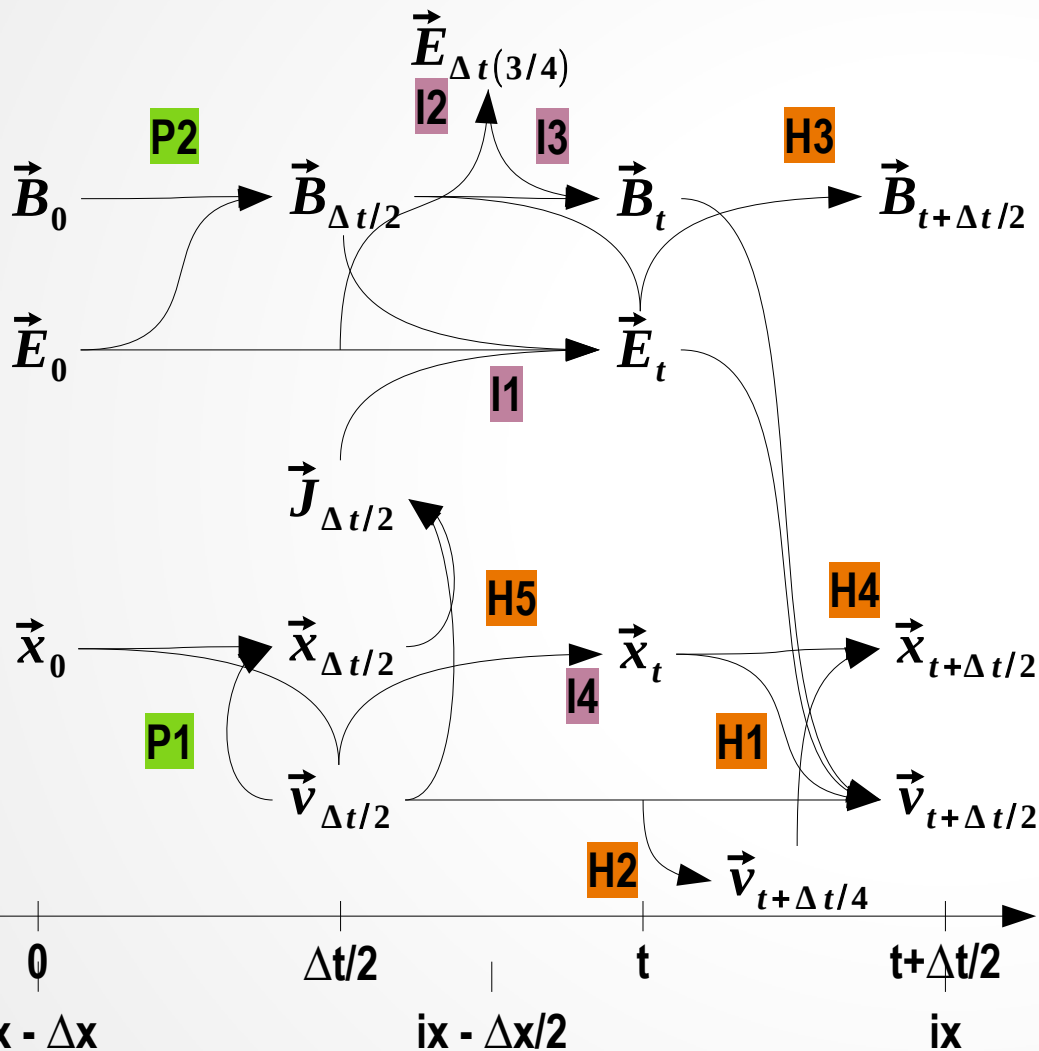
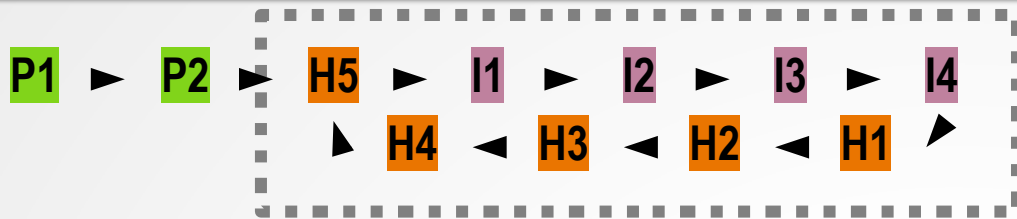
$$H3 \quad \vec{B}_{t+\Delta t/2} \quad \vec{B}_{t+\Delta t/2}$$

$$H1 \quad \vec{E}_t \quad \vec{E}_t \quad \vec{E}_t$$

$$\vec{B}_t \quad \vec{B}_t \quad \vec{B}_t$$

$$ix - \Delta x \quad ix - \Delta x/2 \quad ix \quad ix + \Delta x/2 \quad ix + \Delta x$$

Time sequence of electromagnetic code



Relativistic particle pusher: hand on

$C = 1.0$, $L_x = 20.48$, $\Delta x = 0.01$, $NX = 2048$, $\Delta t = 0.01$, $NT = 6400$
 $BFx = 1.0$, $BFy = 0.0$, $BFz = 0.0$, $EFx = 0.0$, $BFy = 0.0$, $BFz = 0.0$

Ion: $NP = 65536$, $m = 1836$, $e = 1$, $vdx = vdy = vdz = 0.0$, $vthx = vthy = vthz = 0.01$
electron1: $NP = 32768$, $m = 1$, $e = -1$, $vdy = vdz = 0.0$, $vthx = vthy = vthz = 0.01$
electron2: $NP = 32768$, $m = 1$, $e = -1$, $vdy = vdz = 0.0$, $vthx = vthy = vthz = 0.01$

$W_{pe, total} = 1.0$

Example: **10_01_1DES_Re.90 & 11_01_1DEM_Re_subroutine.f90**

1. electron1: **$vdx = 0.1$** , electron2: **$vdx = -0.1$**

2. electron1: **$vdx = 0.8$** , electron2: **$vdx = -0.8$**

