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Particle-in-Cell (PIC) kinetic simulations
10. Relativistic particle pusher

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www.slido.com code: #B194

Boris pusher

$$m\frac{d\vec{v}}{dt}=q(\vec{E}+\vec{v}\times\vec{B})$$

$$\vec{v}^{-}=\vec{v}_{t-\Delta t/2}+\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}[\vec{E}_t+(\frac{\vec{v}_{t+\Delta t/2}+\vec{v}_{t-\Delta t/2}}{2})\times\vec{B}_t]$$

$$\vec{v}\,'=\vec{v}^{-}+\frac{q}{m}\frac{\Delta t}{2}\vec{v}^{-}\times\vec{B}_t$$

$$\vec{v}^{+}=\vec{v}^{-}+\frac{q}{m}\frac{\Delta t}{2}\frac{2}{1+(\frac{q}{m}\frac{\Delta t}{2}\vec{B}_t)^2}\vec{v}\,'\times\vec{B}_t$$

$$\vec{v}_{t+\Delta t/2}=\vec{v}^{+}+\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

Relativistic particle pusher

$$m\frac{d\vec{v}}{dt}=q\left(\vec{E}+\vec{v}\times\vec{B}\right)$$

$$\frac{d}{dt}\left(\gamma m_o \vec{v}\right)=q\left(\vec{E}+\vec{v}\times\vec{B}\right)$$

$$\gamma=\frac{1}{\sqrt{1-(v/c)^2}}$$

Relativistic particle pusher

$$\frac{d}{dt}(\gamma m_o \vec{v}) = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

proper velocity $\vec{u} = \gamma \vec{v}$ $\qquad \vec{u} = \frac{c}{\sqrt{c^2 - |\vec{v}|^2}} \vec{v}$ $\qquad \vec{v} = \frac{c}{\sqrt{c^2 + |\vec{u}|^2}} \vec{u}$

$$\frac{d\vec{u}}{dt} = \frac{q}{m_0}\left(\vec{E} + \frac{c}{\sqrt{c^2 + |\vec{u}|^2}} \vec{u} \times \vec{B}\right)$$

Relativistic particle pusher

$$\frac{d}{dt}\left(\gamma m_o \vec{v}\right) = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \qquad \gamma = \frac{1}{\sqrt{1-\left(v/c\right)^2}}$$

proper velocity $\vec{u} = \gamma \vec{v}$ $\quad \vec{u} = \frac{c}{\sqrt{c^2 - |\vec{v}|^2}} \vec{v}$ $\quad \vec{v} = \frac{c}{\sqrt{c^2 + |\vec{u}|^2}} \vec{u}$

$$\frac{d\vec{u}}{dt} = \frac{q}{m_0}\left(\vec{E} + \frac{c}{\sqrt{c^2+|\vec{u}|^2}} \vec{u} \times \vec{B}\right) \qquad \vec{B}_u = \frac{c}{\sqrt{c^2+|\vec{u}|^2}} \vec{B}$$

$$\frac{d\vec{u}}{dt} = \frac{q}{m_0}\left(\vec{E} + \vec{u} \times \vec{B}_u\right)$$

Relativistic particle pusher

$$\frac{d}{dt}(\gamma m_o \vec{v}) = q(\vec{E} + \vec{v} \times \vec{B}) \qquad \gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$

proper velocity $\vec{u} = \gamma \vec{v}$ $\quad \vec{u} = \frac{c}{\sqrt{c^2 - |\vec{v}|^2}} \vec{v}$ $\quad \vec{v} = \frac{c}{\sqrt{c^2 + |\vec{u}|^2}} \vec{u}$

$$\frac{d\vec{u}}{dt} = \frac{q}{m_0}\left(\vec{E} + \frac{c}{\sqrt{c^2+|\vec{u}|^2}} \vec{u} \times \vec{B}\right) \qquad \vec{B}_u = \frac{c}{\sqrt{c^2+|\vec{u}|^2}} \vec{B}$$

Modified magnetic field

$$\frac{d\vec{u}}{dt} = \frac{q}{m_0}(\vec{E} + \vec{u} \times \vec{B}_u) \qquad \frac{\vec{u}_{t+\Delta t/2} - \vec{u}_{t-\Delta t/2}}{\Delta t} = \frac{q}{m_0}\left[\vec{E}_t + \left(\frac{\vec{u}_{t+\Delta t/2} + \vec{u}_{t-\Delta t/2}}{2}\right) \times \vec{B}_{u,t}\right]$$

$$\frac{d\vec{v}}{dt} = \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}) \qquad \frac{\vec{v}_{t+\Delta t/2} - \vec{v}_{t-\Delta t/2}}{\Delta t} = \frac{q}{m}\left[\vec{E}_t + \left(\frac{\vec{v}_{t+\Delta t/2} + \vec{v}_{t-\Delta t/2}}{2}\right) \times \vec{B}_t\right]$$

Relativistic particle pusher

$$\frac{\vec{v}_{t+\Delta t/2}-\vec{v}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m}[\vec{E}_t+(\frac{\vec{v}_{t+\Delta t/2}+\vec{v}_{t-\Delta t/2}}{2})\times\vec{B}_t]$$

$$\frac{\vec{u}_{t+\Delta t/2}-\vec{u}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m_0}[\vec{E}_t+(\frac{\vec{u}_{t+\Delta t/2}+\vec{u}_{t-\Delta t/2}}{2})\times\vec{B}_{u,t}]$$

$$\vec{v}^-=\vec{v}_{t-\Delta t/2}+\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\vec{u}^-=\vec{u}_{t-\Delta t/2}+\frac{q\vec{E}_t}{m_0}\frac{\Delta t}{2}$$

$$\vec{v}'=\vec{v}^-+\frac{q}{m}\frac{\Delta t}{2}\vec{v}^-\times\vec{B}_t$$

$$\vec{u}'=\vec{u}^-+\frac{q}{m_0}\frac{\Delta t}{2}\vec{u}^-\times\vec{B}_{u,t}$$

$$\vec{v}^+=\vec{v}^-+\frac{q}{m}\frac{\Delta t}{2}\frac{2}{1+(\frac{q}{m}\frac{\Delta t}{2}\vec{B}_t)^2}\vec{v}'\times\vec{B}_t$$

$$\vec{u}^+=\vec{u}^-+\frac{q}{m_0}\frac{\Delta t}{2}\frac{2}{1+(\frac{q}{m_0}\frac{\Delta t}{2}\vec{B}_{u,t})^2}\vec{u}'\times\vec{B}_{u,t}$$

$$\vec{v}_{t+\Delta t/2}=\vec{v}^++\frac{q\vec{E}_t}{m}\frac{\Delta t}{2}$$

$$\vec{u}_{t+\Delta t/2}=\vec{u}^++\frac{q\vec{E}_t}{m_0}\frac{\Delta t}{2}$$

Relativistic particle pusher

Modified magnetic field

$$\vec{B}_u = \frac{c}{\sqrt{c^2 + |\vec{u}|^2}} \vec{B}$$

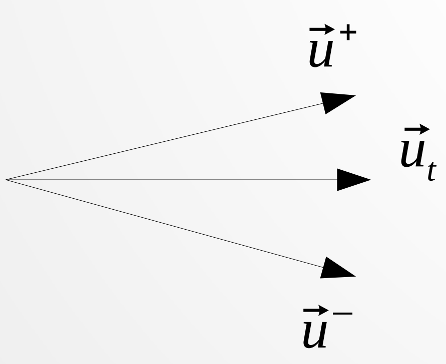
$$\frac{\vec{u}_{t+\Delta t/2} - \vec{u}_{t-\Delta t/2}}{\Delta t} = \frac{q}{m_0} [\vec{E}_t + (\frac{\vec{u}_{t+\Delta t/2} + \vec{u}_{t-\Delta t/2}}{2}) \times \vec{B}_{u,t}]$$

$$|\vec{u}|^2 = |\frac{\vec{u}_{t+\Delta t/2} + \vec{u}_{t-\Delta t/2}}{2}|^2$$

$$= |\frac{\vec{u}^+ + \vec{u}^-}{2}|^2 = |\vec{u}_t|^2$$

$$|\vec{u}^+| = |\vec{u}_t| = |\vec{u}^-|$$

$$\vec{B}_{u,t} = \frac{c}{\sqrt{c^2 + |\vec{u}^-|^2}} \vec{B}_t$$



$$\vec{u}^- = \vec{u}_{t-\Delta t/2} + \frac{q \vec{E}_t}{m_0} \frac{\Delta t}{2}$$

$$\vec{u}\,' = \vec{u}^- + \frac{q}{m_0} \frac{\Delta t}{2} \vec{u}^- \times \vec{B}_{u,t}$$

$$\vec{u}^+ = \vec{u}^- + \frac{q}{m_0} \frac{\Delta t}{2} \frac{2}{1 + (\frac{q}{m_0} \frac{\Delta t}{2} \vec{B}_{u,t})^2} \vec{u}\,' \times \vec{B}_{u,t}$$

$$\vec{u}_{t+\Delta t/2} = \vec{u}^+ + \frac{q \vec{E}_t}{m_0} \frac{\Delta t}{2}$$

Relativistic particle pusher

proper velocity $\vec{u}=\gamma\vec{v}$

$$\vec{u}=\frac{c}{\sqrt{c^2-|\vec{v}|^2}}\vec{v}$$

$$\vec{u}_{t-\Delta t/2}=\frac{c}{\sqrt{c^2-|\vec{v}_{t-\Delta t/2}|^2}}\vec{v}_{t-\Delta t/2}$$

$$\vec{v}=\frac{c}{\sqrt{c^2+|\vec{u}|^2}}\vec{u}$$

$$\vec{v}_{t+\Delta t/2}=\frac{c}{\sqrt{c^2+|\vec{u}_{t+\Delta t/2}|^2}}\vec{u}_{t+\Delta t/2}$$

$$\frac{\vec{u}_{t+\Delta t/2}-\vec{u}_{t-\Delta t/2}}{\Delta t}=\frac{q}{m_0}[\vec{E}_t+(\frac{\vec{u}_{t+\Delta t/2}+\vec{u}_{t-\Delta t/2}}{2})\times\vec{B}_{u,t}]$$

$$\vec{u}^-=\vec{u}_{t-\Delta t/2}+\frac{q\vec{E}_t}{m_0}\frac{\Delta t}{2}$$

$$\vec{B}_{u,t}=\frac{c}{\sqrt{c^2+|\vec{u}^-|^2}}\vec{B}_t$$

$$\vec{u}'=\vec{u}^-+\frac{q}{m_0}\frac{\Delta t}{2}\vec{u}^-\times\vec{B}_{u,t}$$

$$\vec{u}^+=\vec{u}^-+\frac{q}{m_0}\frac{\Delta t}{2}\frac{2}{1+(\frac{q}{m_0}\frac{\Delta t}{2}\vec{B}_{u,t})^2}\vec{u}'\times\vec{B}_{u,t}$$

$$\vec{u}_{t+\Delta t/2}=\vec{u}^++\frac{q\vec{E}_t}{m_0}\frac{\Delta t}{2}$$

Relativistic particle pusher

$$\frac{d}{dt}(\gamma m_o \vec{v}) = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{u}_{t-\Delta t/2} = \frac{c}{\sqrt{c^2 - |\vec{v}_{t-\Delta t/2}|^2}} \vec{v}_{t-\Delta t/2}$$

$$\vec{u}^- = \vec{u}_{t-\Delta t/2} + \frac{q \vec{E}_t}{m_0} \frac{\Delta t}{2}$$

$$\vec{B}_{u,t} = \frac{c}{\sqrt{c^2 + |\vec{u}^-|^2}} \vec{B}_t$$

$$\vec{u}' = \vec{u}^- + \frac{q}{m_0} \frac{\Delta t}{2} \vec{u}^- \times \vec{B}_{u,t}$$

$$\vec{u}^+ = \vec{u}^- + \frac{q}{m_0} \frac{\Delta t}{2} \frac{2}{1 + \left(\frac{q}{m_0} \frac{\Delta t}{2} \vec{B}_{u,t}\right)^2} \vec{u}' \times \vec{B}_{u,t}$$

$$\vec{u}_{t+\Delta t/2} = \vec{u}^+ + \frac{q \vec{E}_t}{m_0} \frac{\Delta t}{2}$$

$$\vec{v}_{t+\Delta t/2} = \frac{c}{\sqrt{c^2 + |\vec{u}_{t+\Delta t/2}|^2}} \vec{u}_{t+\Delta t/2}$$

Relativistic particle pusher: hand on

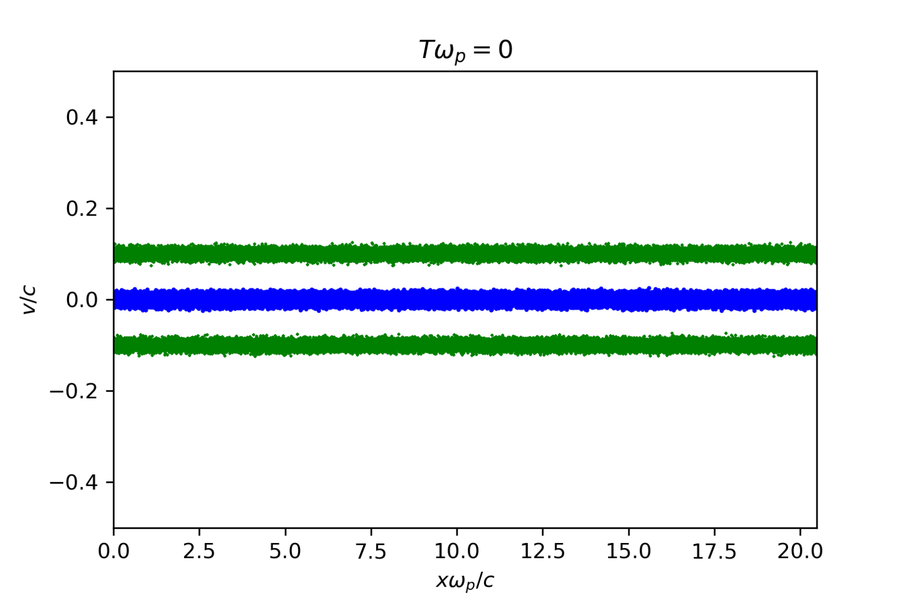
C = 1.0, Lx =20.48, Δx = 0.01, NX = 2048, Δt = 0.01, NT = 6400
BFx=1.0, BFy=0.0, BFz=0.0, EFx=0.0, BFy=0.0, BFz=0.0

Ion: NP = 65536,m = 1836, e = 1, vdx = vdy = vdz = 0.0, vthx = vthy = vthz = 0.01
electron1: NP = 32768,m = 1 , e = -1, vdy = vdz = 0.0, vthx = vthy = vthz = 0.01
electron2: NP = 32768,m = 1 , e = -1, vdy = vdz = 0.0, vthx = vthy = vthz = 0.01

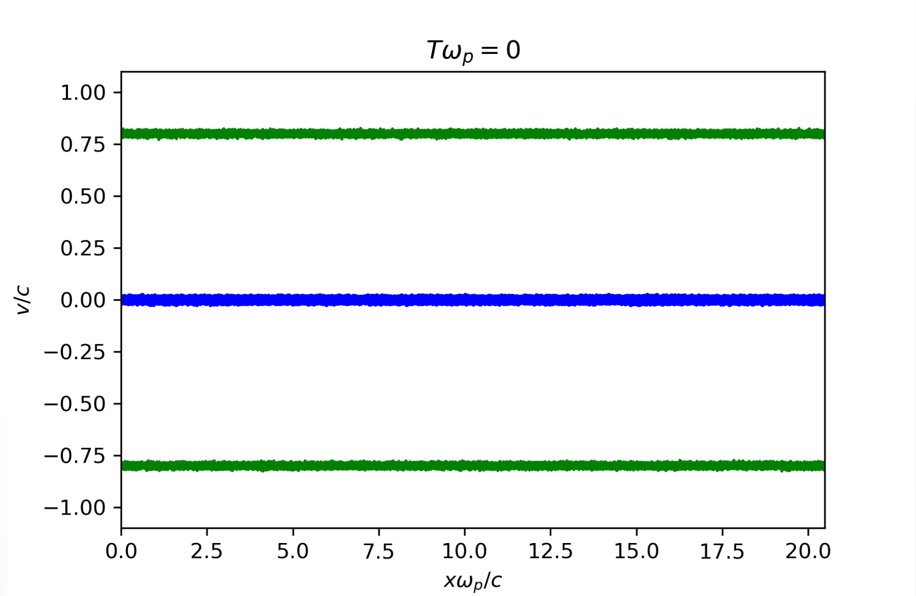
Wpe,total = 1.0

Example:07_01_1DES.f90 & 10_01_Pusher_Re.90

1. electron1: vdx = 0.1, electron2: vdx = -0.1

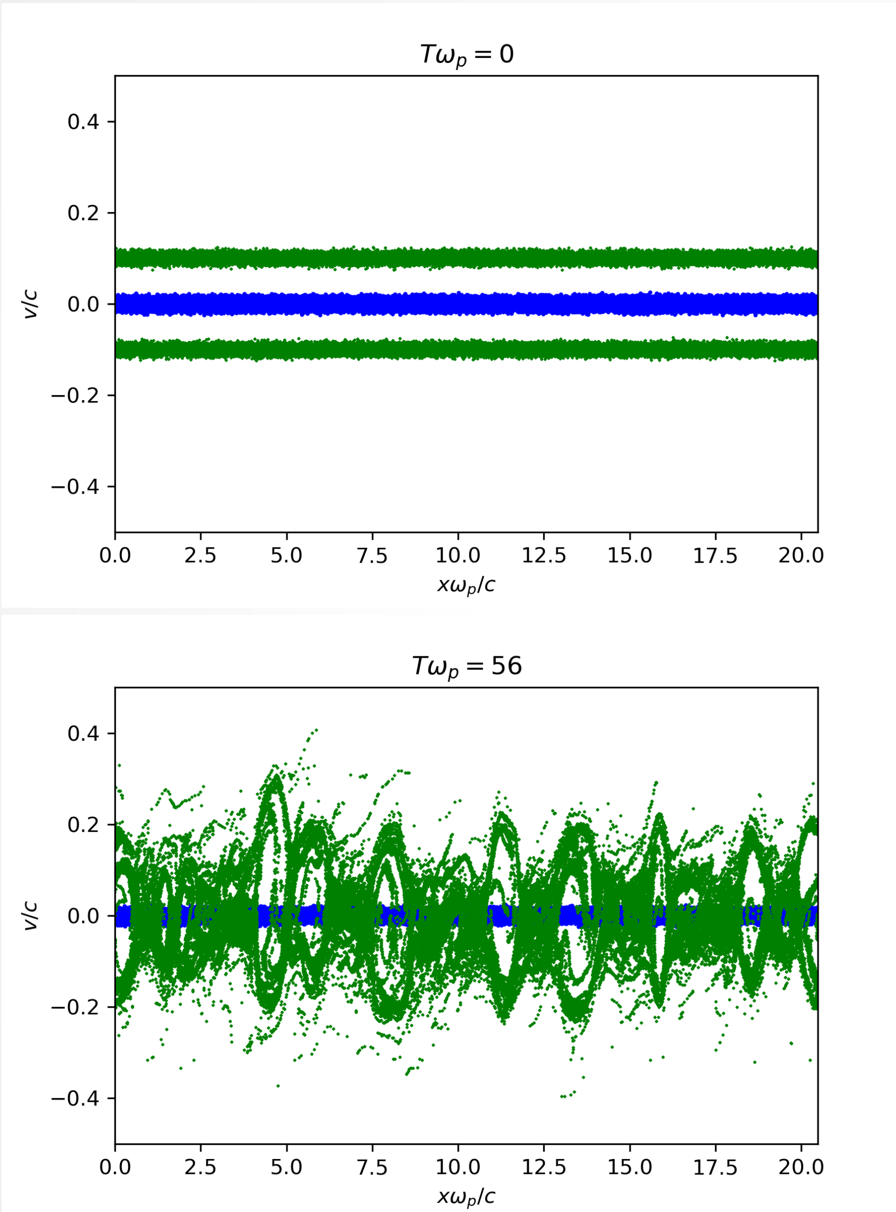


2. electron1: vdx = 0.8, electron2: vdx = -0.8



Relativistic particle pusher: hand on

1. electron1: vdx = 0.1, electron2: vdx = -0.1



2. electron1: vdx = 0.8, electron2: vdx = -0.8