01. Introduction to the PIC simulation

02. Random number generation and its application

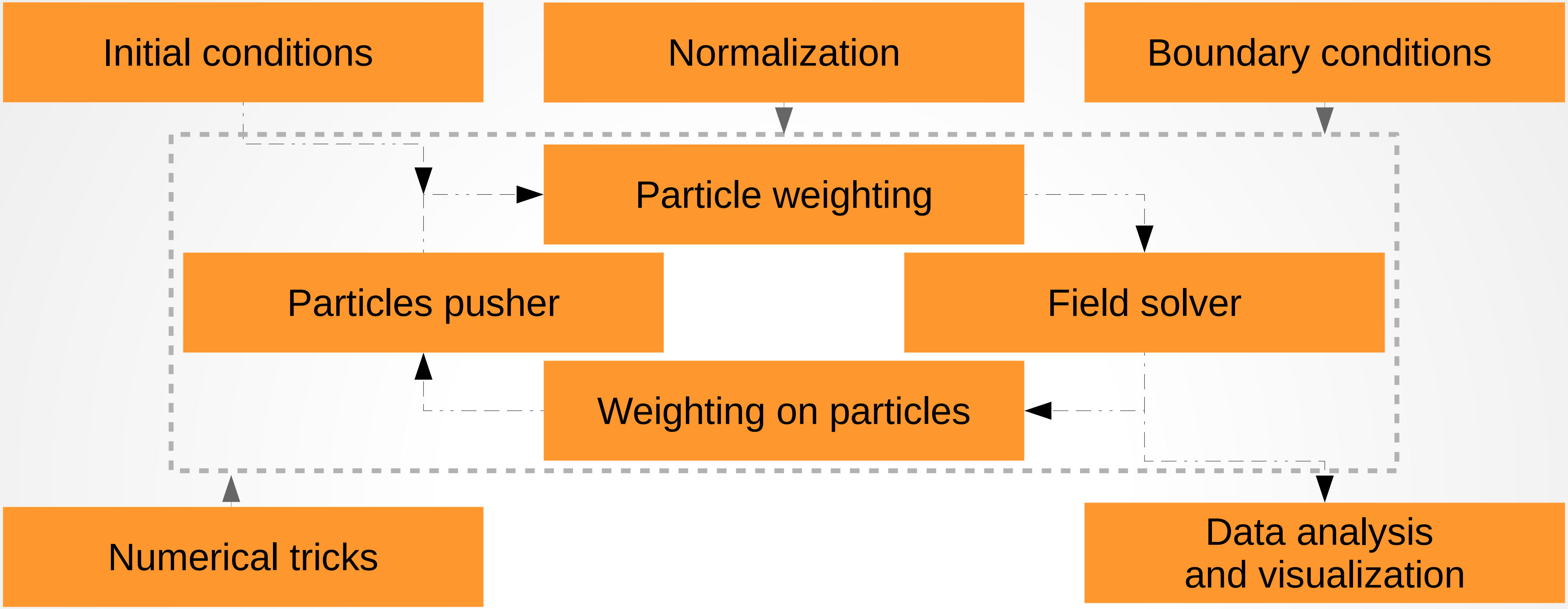
Particle-in-Cell (PIC) kinetic simulations
03. Particle weighting and normalization

Chun-Sung Jao (饒駿頌)
Assistant Research Scholar,
Institute of Space Science and Engineering,
National Central University, Taiwan

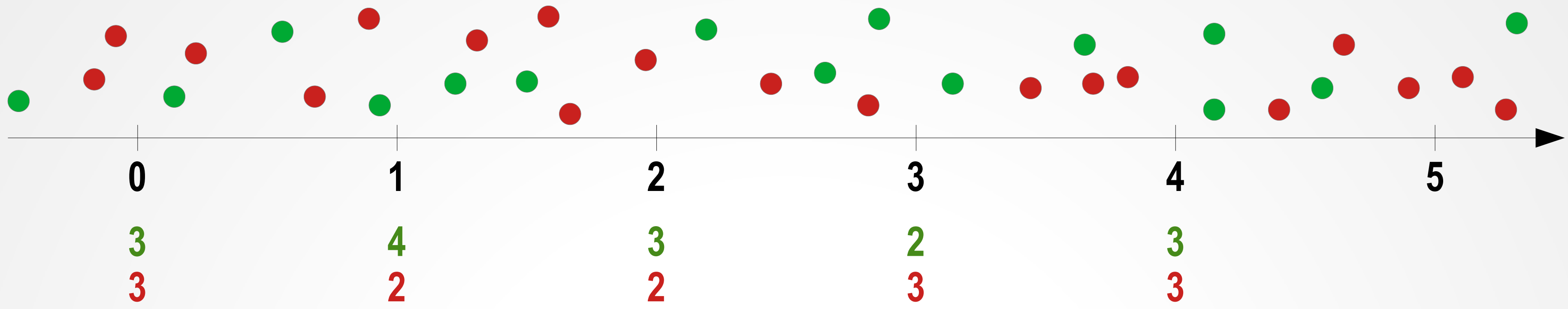
University of São Paulo, 2019.11.25-12.06

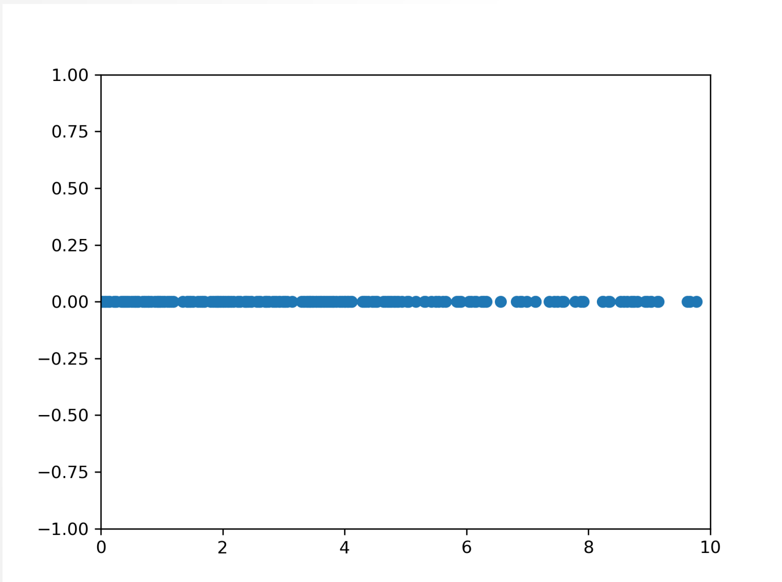
www.slido.com code: #P320

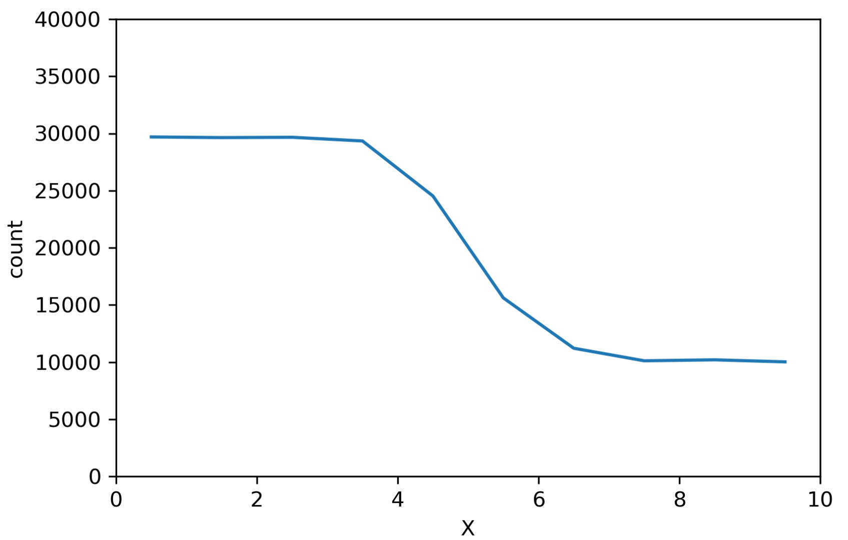
Concept and basic structure



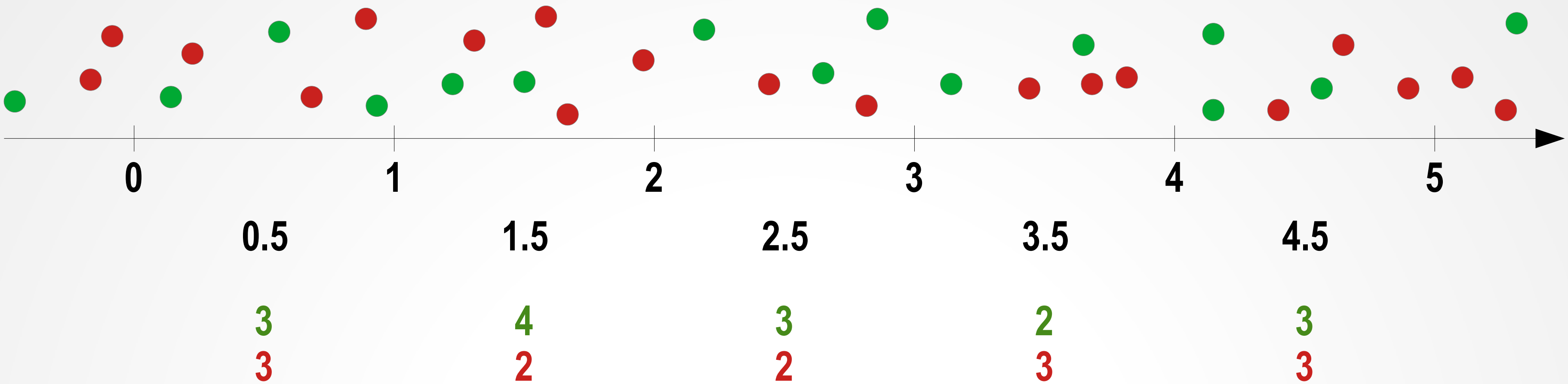
Particle weighting 1D: from continuous data to discrete data

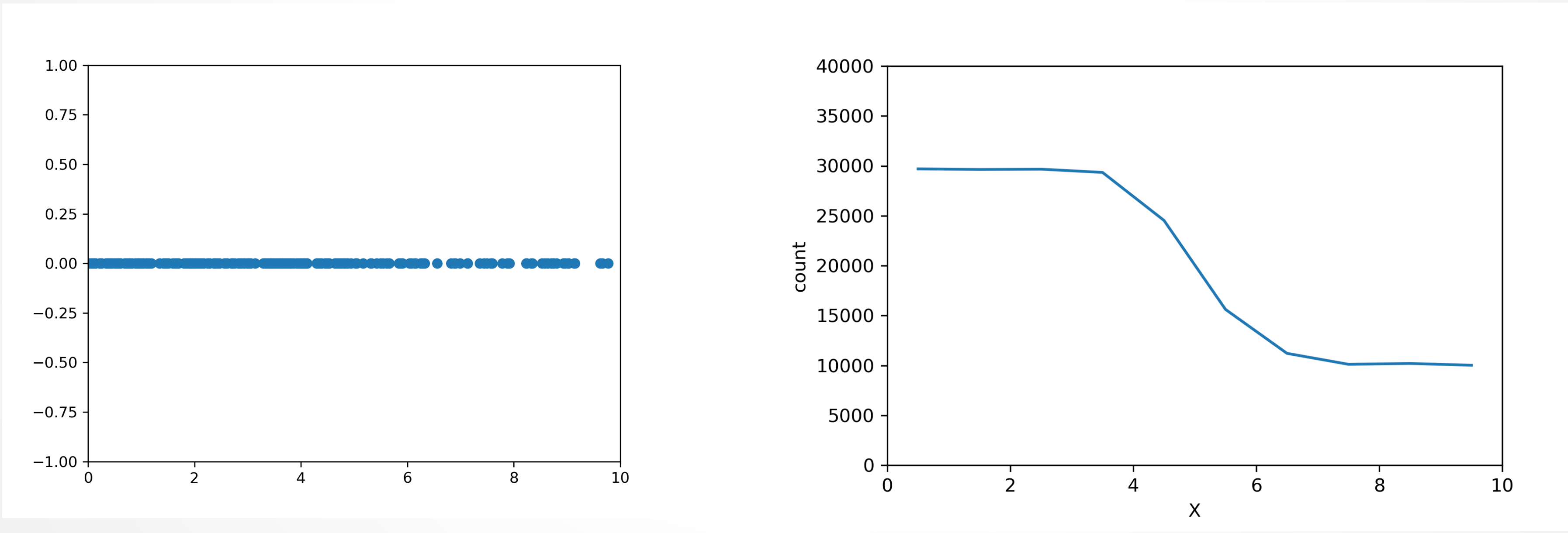




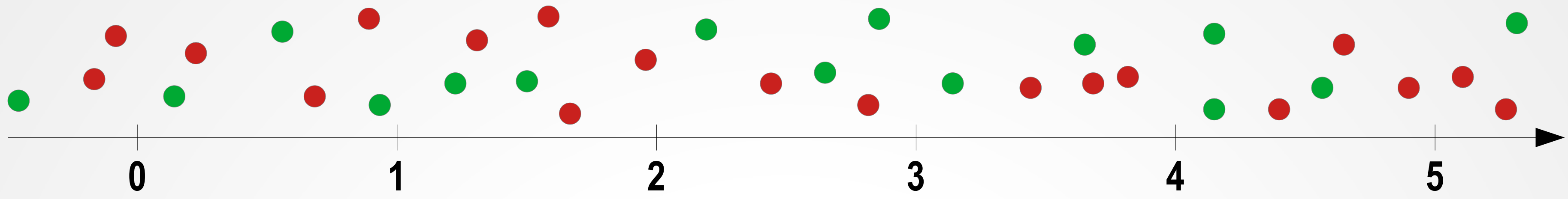


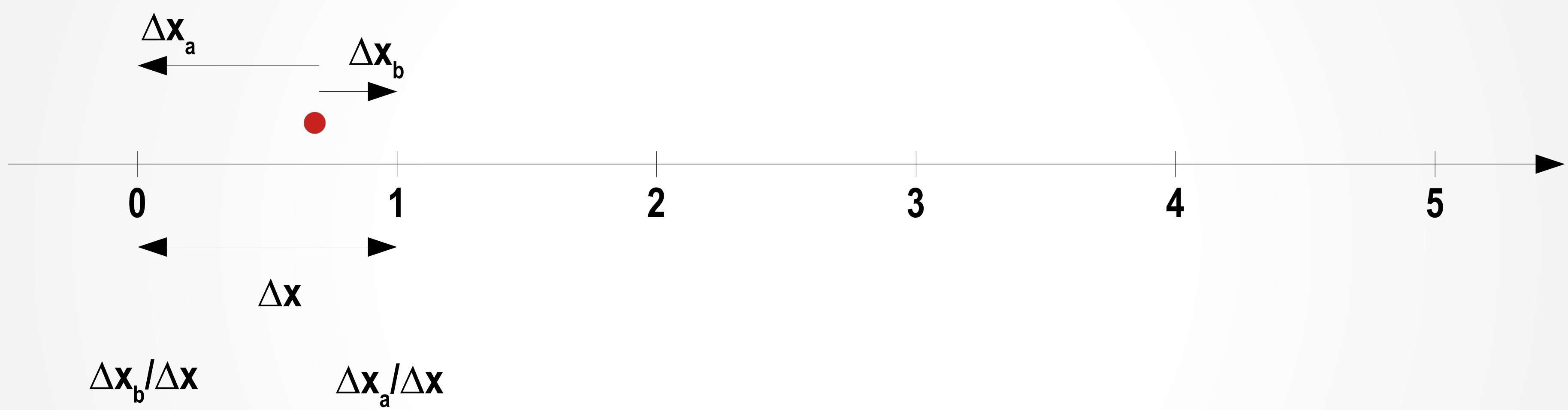
Particle weighting 1D: from continuous data to discrete data





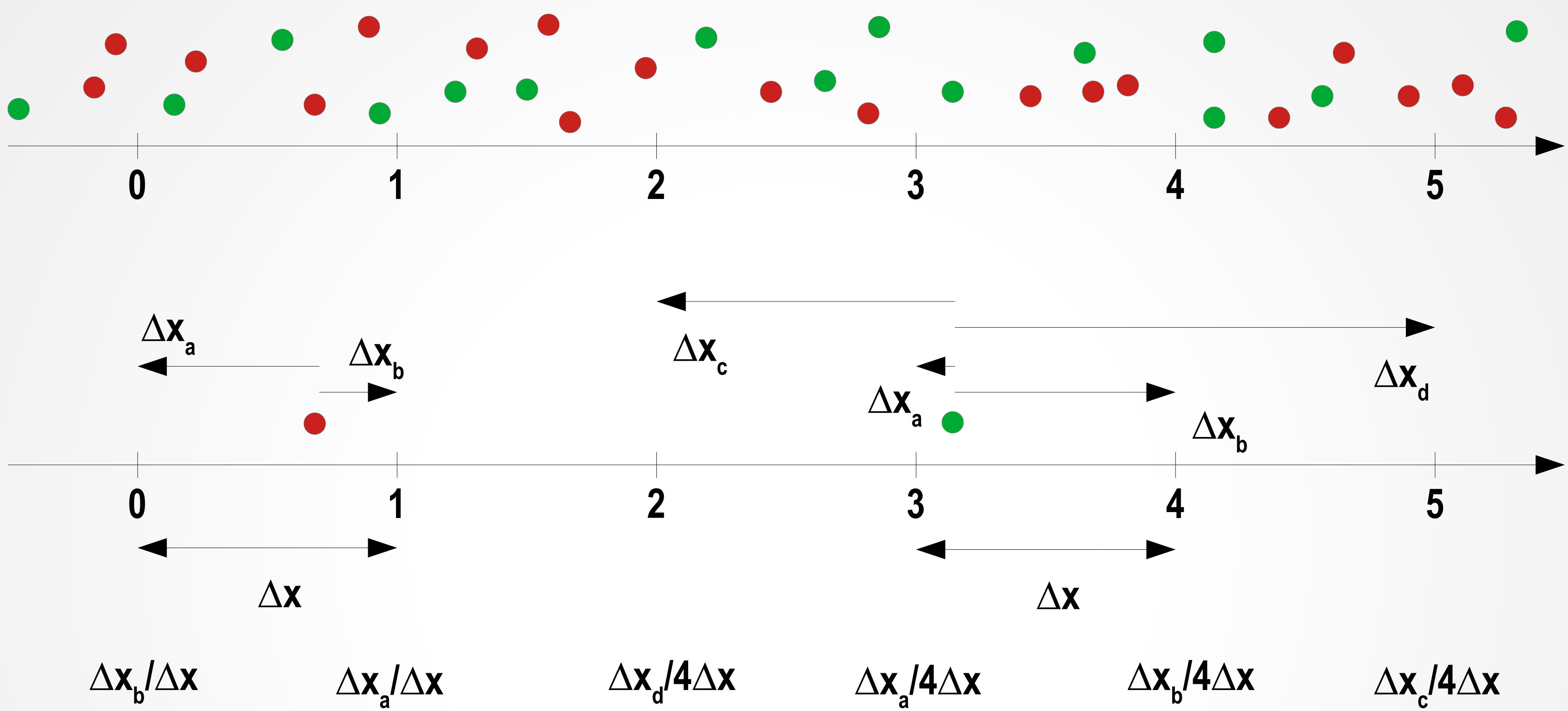
Particle weighting 1D: from continuous data to discrete data



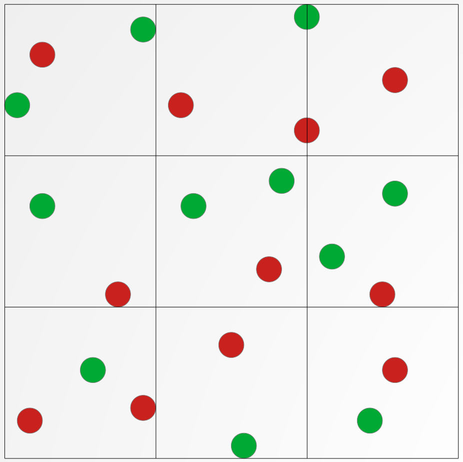


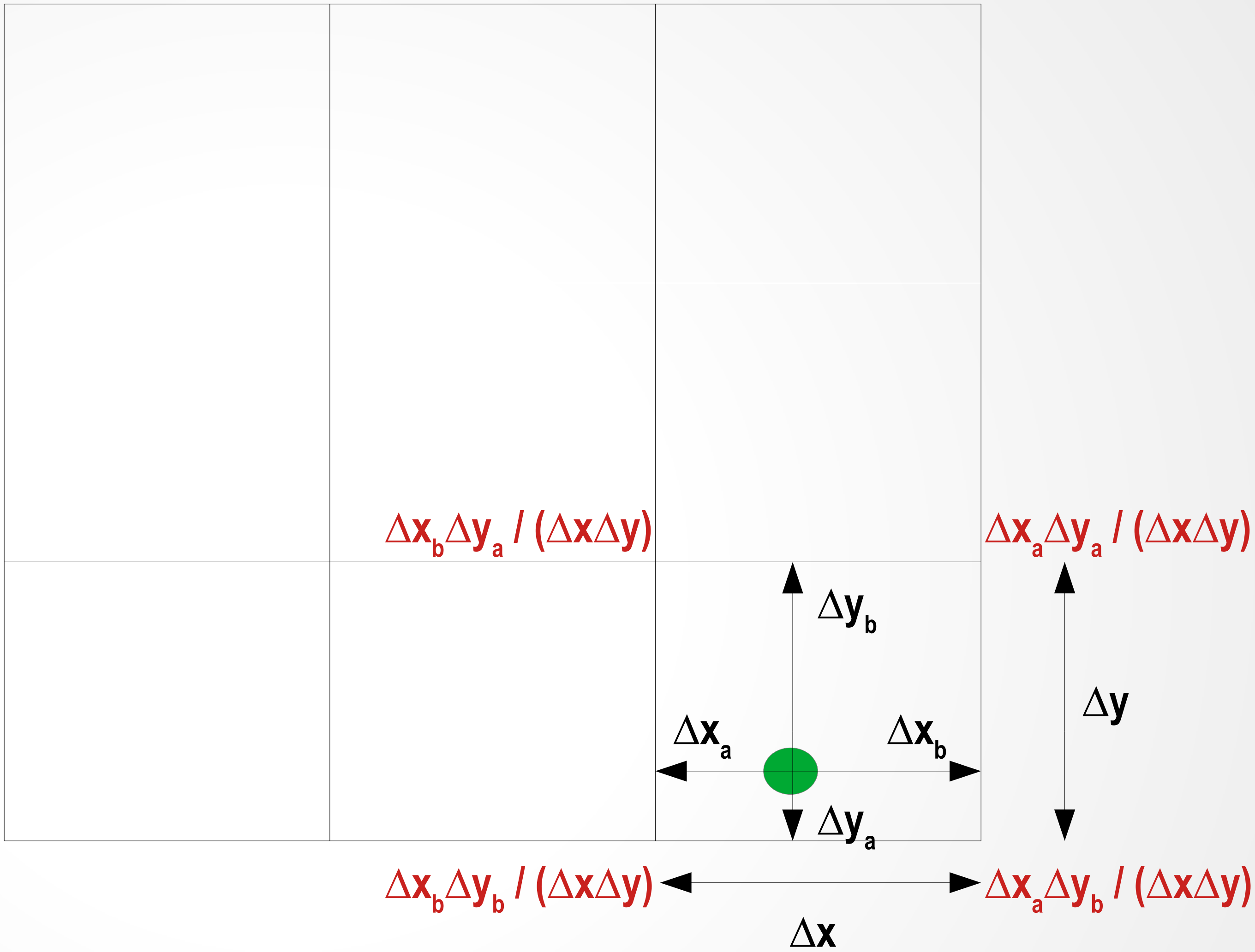
Linear interpolation

Particle weighting 1D: from continuous data to discrete data

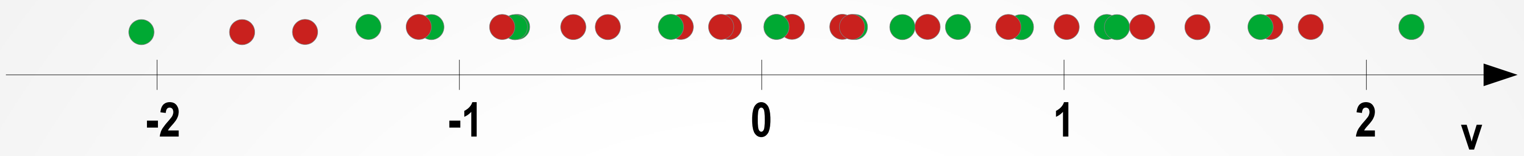


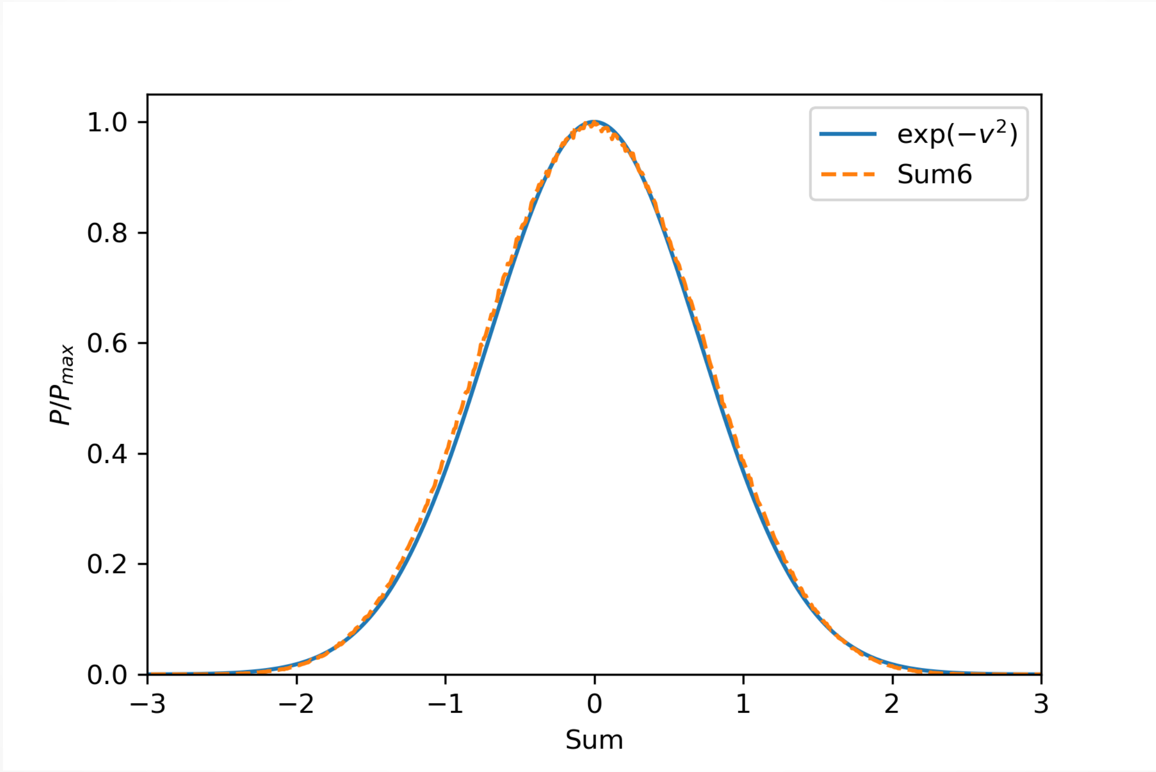
Particle weighting 2D: from continuous data to discrete data



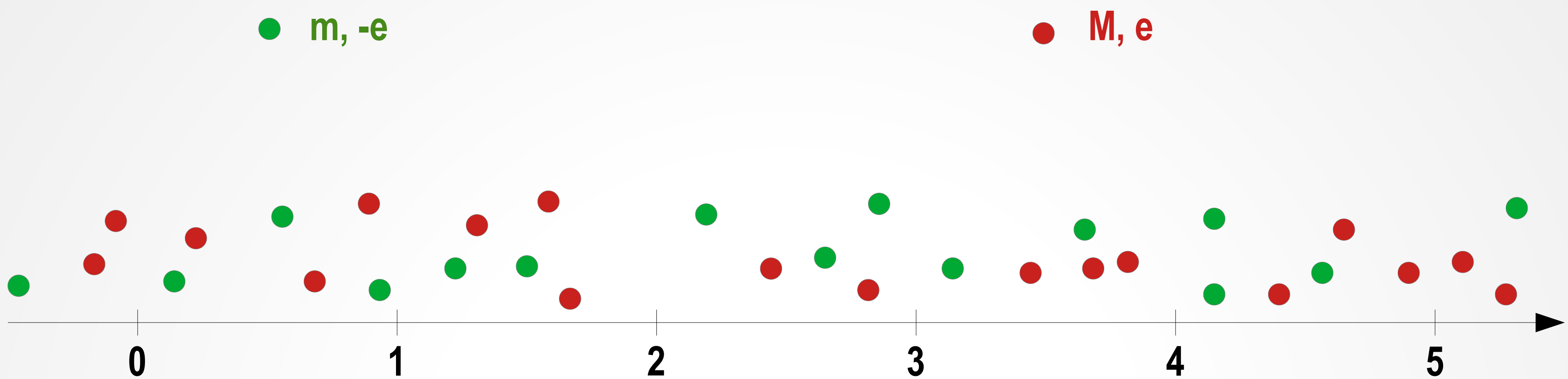


Particle weighting: velocity distribution





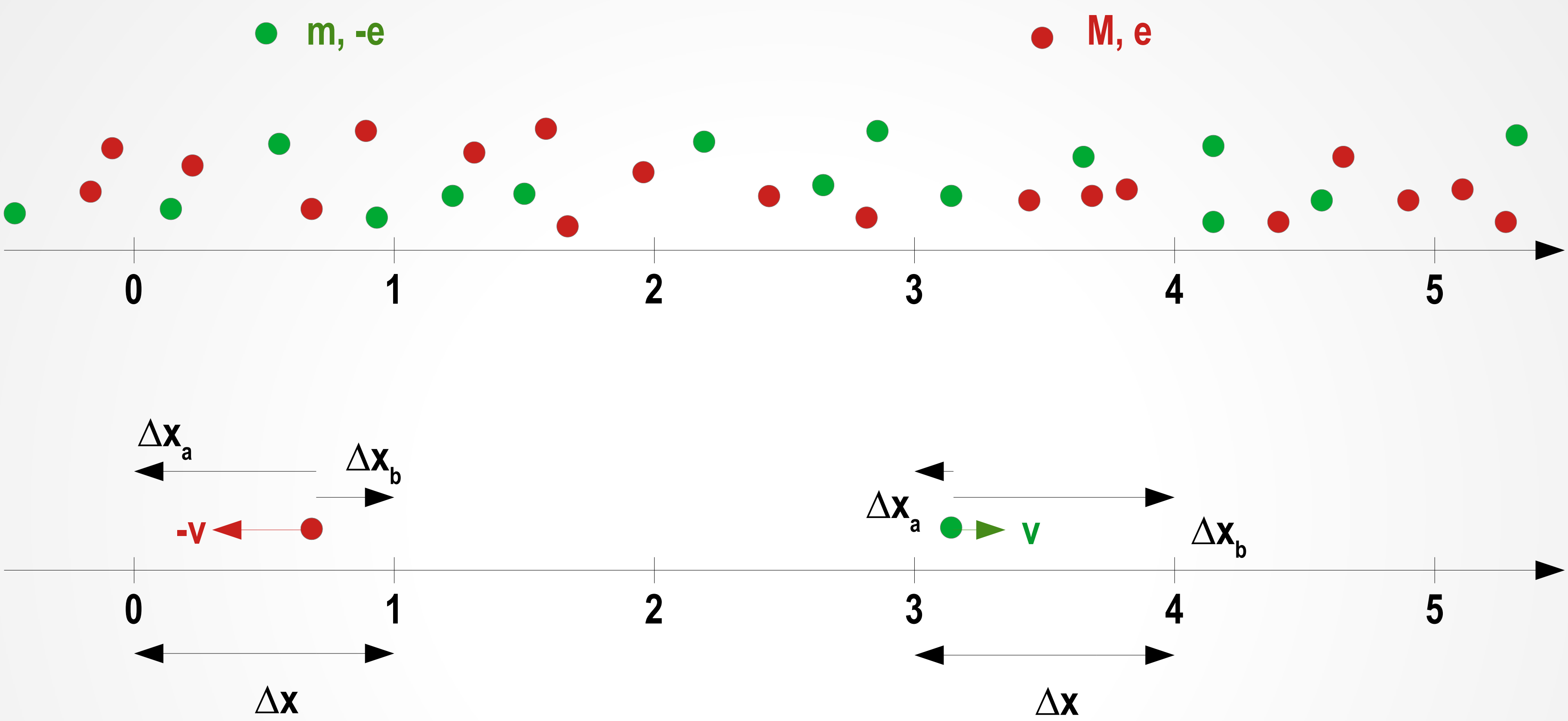
Particle weighting 1D: from continuous data to discrete data



Number density	3 3 6	4 2 6	3 2 5	2 3 5	3 3 6
Density	3m+3M	4m+2M	3m+2M	2m+3M	3m+3M
Charge density	0	-2e	-e	e	0
Current density	??				

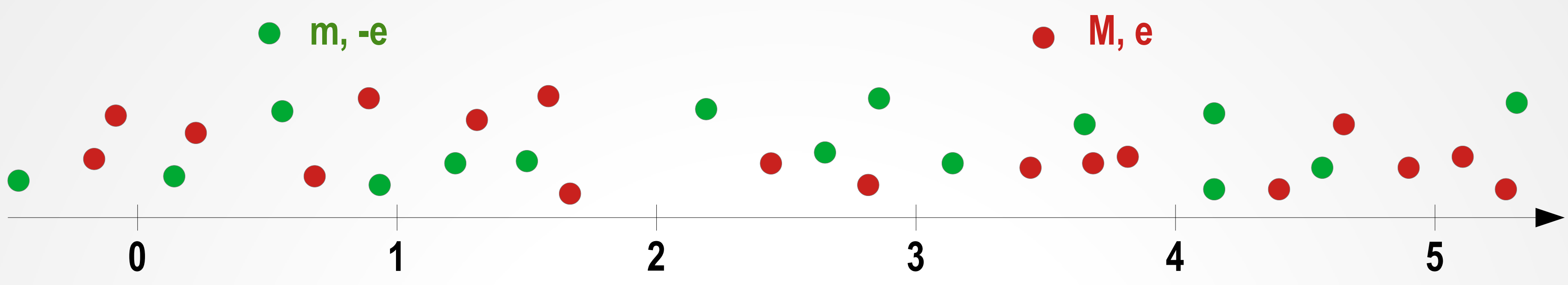
(per grid)

Particle weighting 1D: from continuous data to discrete data



$e(-v)\Delta x_b/\Delta x$ $e(-v)\Delta x_a/\Delta x$ $(-e)v\Delta x_a/\Delta x$ $(-e)v\Delta x_b/\Delta x$

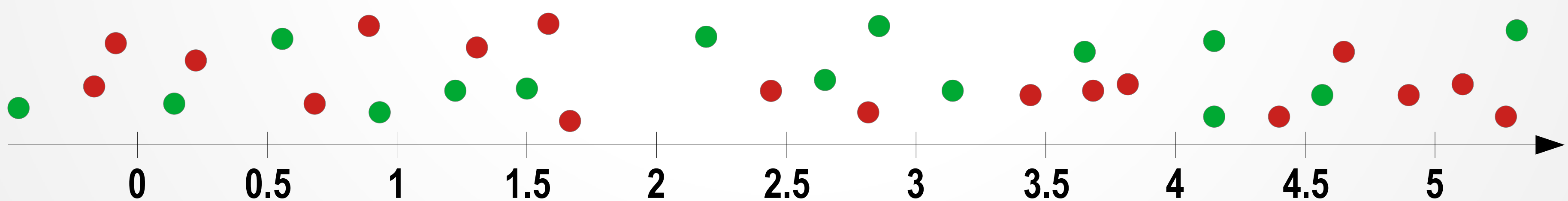
Normalization



Number density						
	3	4	3	2	3	
	3	2	2	3	3	
	6	6	5	5	6	(per grid)

We have to define the unit here!!

km? m? cm? dimensionless units?



Number density											
	2	4	2	3	2	3	2	3	3	3	(per half grid)

Dimensionless units

Length

AU, km, m cm… → Debye length, skin length, ion inertial length...

Time

Yr, s… → inverse of plasma frequency, inverse of Larmor frequency...

Velocity

m/s → speed of light, thermal velocity, Alfven speed...

L=8?? T=120?? v=0.03

Normalization

Length

AU, km, m cm… → Debye length, skin length, **ion inertial length**...

c / ω_{pi}

Time

Yr, s… → inverse of ion plasma frequency, inverse of Larmor frequency...

ω_{pi}

Velocity

m/s → **speed of light**, thermal velocity, Alfven speed.

c

Normalization

Length

AU, km, m cm… → Debye length, skin length, ion inertial length, gyroradius

$mv_{th,e}/eB$

Time

Yr, s… → inverse of plasma frequency, **inverse of Larmor frequency**...

$(eB/m)^{-1}$

Velocity

m/s → speed of light, **thermal velocity**, Alfven speed…

$v_{th,e}$

Normalization

Length

AU, km, m cm… → Debye length, skin length, ion inertial length, gyroradius

mc/eB

Time

Yr, s… → inverse of plasma frequency, **inverse of Larmor frequency**...

$(eB/m)^{-1}$

Velocity

m/s → **speed of light**, thermal velocity, Alfven speed…

c

Normalization

Length

Debye length, **skin length**, ion inertial length...

$c/\omega_{pe} = 1.0$,
$\omega_{pe} = 1.0$,

Time
inverse of plasma frequency, inverse of Larmor frequency...

Velocity
speed of light, thermal velocity, Alfven speed...

In our simulation, the length of simulation system is 1024 c/ω_{pe} and the time interval $\Delta t = 0.125 / \omega_{pe}$ is adopted.

Normalization

Length

Debye length, **skin length**, ion inertial length...

$c/\omega_{pe} = 1.0$,
$\omega_{pe} = 1.0$,
$\omega_{pe}/\omega_{ce} = 0.1 = v_A/c$,

Time
inverse of plasma frequency, inverse of Larmor frequency...

Velocity
speed of light, thermal velocity, Alfven speed...

In our simulation, the length of simulation system is 1024 c/ω_{pe} and the time interval $\Delta t = 0.125 / \omega_{pe}$ is adopted. The frequency ratio $\omega_{pe}/\omega_{ce} = 0.1$.

Normalization

Length

Debye length, **skin length**, ion inertial length...

Time

inverse of plasma frequency, inverse of Larmor frequency...

Velocity

speed of light, thermal velocity, Alfven speed...

$c/\omega_{pe} = 1.0$,
$\omega_{pe} = 1.0$,
$\omega_{pe}/\omega_{ce} = 0.1 = v_A/c$,
$\omega_{be}/\omega_{pe} = 0.04 = sqrt(n_b/n_e)$,

In our simulation, the length of simulation system is 1024 c/ω_{pe} and the time interval $\Delta t = 0.125 / \omega_{pe}$ is adopted. The frequency ratio $\omega_{pe}/\omega_{ce} = 0.1$. The ratio of plasma frequency between beam electrons and background electrons is $\omega_{be}/\omega_{pe} = 0.04$.

Normalization

Length

Debye length, **skin length**, ion inertial length...

Time

inverse of plasma frequency, inverse of Larmor frequency...

Velocity

speed of light, thermal velocity, Alfven speed...

$c/\omega_{pe} = 1.0$,
$\omega_{pe} = 1.0$,
$\omega_{pe}/\omega_{ce} = 0.1 = v_A/c$,
$\omega_{be}/\omega_{pe} = 0.04 = \mathrm{sqrt}(n_b/n_e)$,

In our simulation, the length of simulation system is 1024 c/ω_{pe} and the time interval $\Delta t = 0.125 / \omega_{pe}$ is adopted. The frequency ratio $\omega_{pe}/\omega_{ce} = 0.1$. The ratio of plasma frequency between beam electrons and background electrons is $\omega_{be}/\omega_{pe} = 0.04$.

Normalization

The speed of light is set as $c = 1$ in our simulation, and the electric constant $\varepsilon_0 = 1$.

$$c = 1.0,\ \varepsilon_0 = 1.0 \quad \rightarrow \quad \mu_0 = 1.0$$

Normalization

The speed of light is set as $c = 1$ in our simulation, and the electric constant $\varepsilon_0 = 1$. The time interval $\Delta t = 0.125 / \omega_{pe}$ is adopted, where ω_{pe} is the plasma frequency of background electrons.

$c = 1.0$, $\varepsilon_0 = 1.0$ → $\mu_0 = 1.0$

$\omega_{pe} = 1.0$, → $c / \omega_{pe} = 1.0$

Normalization

The speed of light is set as $c = 1$ in our simulation, and the electric constant $\varepsilon_0 = 1$. The time interval $\Delta t = 0.125 / \omega_{pe}$ is adopted, where ω_{pe} is the plasma frequency of background electrons. $\Delta x = 0.1\ c/\omega_{pe}$ and 10 particle per cell is used.

$c = 1.0,\ \varepsilon_0 = 1.0 \rightarrow \mu_0 = 1.0$

$\omega_{pe} = 1.0, \rightarrow c / \omega_{pe} = 1.0$

$N_0 / (c/\omega_{pe}) = 100 \rightarrow n_0 = 100$

Normalization

The speed of light is set as $c = 1$ in our simulation, and the electric constant $\varepsilon_0 = 1$. The time interval $\Delta t = 0.125 / \omega_{pe}$ is adopted, where ω_{pe} is the plasma frequency of background electrons. $\Delta x = 0.1\ c/\omega_{pe}$ and 10 particle per cell is used. The mass-to-charge ratio of background electron is set as $m_e/e = 1.0$.

$c = 1.0,\ \varepsilon_0 = 1.0 \quad \rightarrow \quad \mu_0 = 1.0$

$\omega_{pe} = 1.0, \quad \rightarrow \quad c / \omega_{pe} = 1.0$

$N_0 / (c/\omega_{pe}) = 100 \quad \rightarrow \quad n_0 = 100$

$m_e/e = 1.0 \quad \rightarrow \quad e = 1/100$

$\omega_{pe} = \mathrm{sqrt}(n_0 e^2/m\varepsilon_0) = \mathrm{sqrt}(100e) = 1.0$

Parameter table

E10 =SQRT(E8*1.5/(I2*E2*I1))

	A	B	C	D	E	F	H	I
1			electron	H+	O+		ion to electron mass	1836
2	amu			1	16		electron mass me	9.11E-31
3	Numder density	1/m-3	2.00E+11	2.00E+11	2.00E+11		electron charge e	1.60E-19
4		1/cm-3	2.00E+05	2.00E+05	2.00E+05		permittivity of free space epsilon0	8.85E-12
5	plasma frequency Wp	1/s	25200209.81	588122.5319	147030.633		pi	3.14E+00
6	plasma period 2 pi / Wp	s	2.49E-07	1.07E-05	4.27E-05		Boltzmann constant Kb	1.38E-23
7	Temperature	K	2000	2000	2000			
8		J	2.76E-20	2.76E-20	2.76E-20			
9		eV	1.73E-01	1.73E-01	1.73E-01			
10	thermal velocity		213177.3121	4975.132409	1243.783102	m vth ^2 = 3 kb T / 2		
11								
12	Magnetic field B	T	5.00E-05					
13		mu T	5.00E+01					
14	gyro requency Wc	1/s	8.78E+06	4.78E+03	2.99E+02			
15	cyclntron period	s	7.15E-07	1.31E-03	2.10E-02			
16	gyroradius (thermal speed)	m	2.43E-02	1.04E+00	4.16E+00			
17								
18	sound speed	m/s		5744.788071	1436.197018	Cs=sqrt((kTi + kTe) / mi)		
19	Mach number	0.6		3446.872843	861.7182107			
20	Mach number	1		5744.788071	1436.197018			
21	Mach number	1.3		7468.224493	1867.056123			
22		1.3		7468.224493				

Particle weighting and normalization: hand-on

In our simulation system, the Debye length and the plasma frequency of the background electrons are set as λ_{De} = 1.0 and ω_{pe} = 1.0, and the mass and the absolutely charge of electrons are m=1 and |e| = 1. The time interval $\Delta t = 0.125 / \omega_{pe}$ and grid size $\Delta x = 0.25\ \lambda_{De}$ are adopted, and the length of the simulation system is $L = 16\ \lambda_{De}$. Both beam and background electrons have the uniform spatial distribution, and the number density ratio between the beam and background electrons is $n_b/n_e = 1.0$. We use 256 particles per grid for both ions and electrons. The velocity distribution of both background and beam electrons can be described as

$$F_e(v) = A \exp(- v^2 / v_{th,e}^2) \quad \text{and } F_b(v) = B \exp(- (v-v_{d,b})^2 / v_{th,b}^2)$$

, respectively, where v_d is the drift velocity and v_{th} is the thermal velocity. For the electron beam, the drift velocity and thermal velocity are $v_{d,b}/v_{th,e} = 5.0$ and $v_{th,b}/v_{th,e} = 1.0$.

Particle weighting and normalization: hand-on

In our simulation system, the Debye length and the plasma frequency of the background electrons are set as λ_{De} = 1.0 and ω_{pe} = 1.0, and the mass and the absolutely charge of electrons are m=1 and |e| = 1. The time interval $\Delta t = 0.125 / \omega_{pe}$ and grid size $\Delta x = 0.25\ \lambda_{De}$ are adopted, and the length of the simulation system is $L = 16\ \lambda_{De}$. Both beam and background electrons have the uniform spatial distribution, and the number density ratio between the beam and background electrons is $n_b/n_e = 1.0$. We use 256 particles per grid for both ions and electrons. The velocity distribution of both background and beam electrons can be described as

$$F_e(v) = A \exp(- v^2 / v_{th,e}^{2}) \quad \text{and } F_b(v) = B \exp(- (v-v_{d,b})^2 / v_{th,b}^{2})$$

, respectively, where v_d is the drift velocity and v_{th} is the thermal velocity. For the electron beam, the drift velocity and thermal velocity are $v_{d,b}/v_{th,e} = 5.0$ and $v_{th,b}/v_{th,e} = 1.0$.

$$v_{th,e} / \omega_{pe} = \lambda_{De}$$

Particle weighting and normalization: hand-on

In our simulation system, the Debye length and the plasma frequency of the background electrons are set as λ_{De} = 1.0 and ω_{pe} = 1.0, and the mass and the absolutely charge of electrons are m=1 and |e| = 1. **The time interval $\Delta t = 0.125 / \omega_{pe}$ and grid size $\Delta x = 0.25\ \lambda_{De}$ are adopted, and the length of the simulation system is $L = 16\ \lambda_{De}$.** Both beam and background electrons have the uniform spatial distribution, and the number density ratio between the beam and background electrons is n_b/n_e = 1.0. We use 256 particles per grid for both ions and electrons. The velocity distribution of both background and beam electrons can be described as

$$F_e(v) = A \exp(- v^2 / v_{th,e}^2) \quad \text{and } F_b(v) = B \exp(- (v-v_{d,b})^2 / v_{th,b}^2)$$

, respectively, where v_d is the drift velocity and v_{th} is the thermal velocity. For the electron beam, the drift velocity and thermal velocity are $v_{d,b}/v_{th,e}$ = 5.0 and $v_{th,b}/v_{th,e}$ = 1.0.

The gird size is 0.25, and the length of simulation system is 16.
There are 64 grids in the simulation system.

Particle weighting and normalization: hand-on

In our simulation system, the Debye length and the plasma frequency of the background electrons are set as λ_{De} = 1.0 and ω_{pe} = 1.0, and the mass and the absolutely charge of electrons are m=1 and |e| = 1. The time interval $\Delta t = 0.125 / \omega_{pe}$ and grid size $\Delta x = 0.25\ \lambda_{De}$ are adopted, and the length of the simulation system is $L = 16\ \lambda_{De}$. **Both beam and background electrons have the uniform spatial distribution, and the number density ratio between the beam and background electrons is n_b/n_e = 1.0.** We use 256 particles per grid for both ions and electrons. The velocity distribution of both background and beam electrons can be described as

$$F_e(v) = A \exp(- v^2 / v_{th,e}^{\ 2}) \quad \text{and } F_b(v) = B \exp(- (v-v_{d,b})^2 / v_{th,b}^{\ 2})$$

, respectively, where v_d is the drift velocity and v_{th} is the thermal velocity. For the electron beam, the drift velocity and thermal velocity are $v_{d,b}/v_{th,e}$ = 5.0 and $v_{th,b}/v_{th,e}$ = 1.0.

We can use the program from the course yesterday.

Particle weighting and normalization: hand-on

In our simulation system, the Debye length and the plasma frequency of the background electrons are set as λ_{De} = 1.0 and ω_{pe} = 1.0, and the mass and the absolutely charge of electrons are m=1 and |e| = 1. The time interval $\Delta t = 0.125 / \omega_{pe}$ and grid size $\Delta x = 0.25\ \lambda_{De}$ are adopted, and the length of the simulation system is $L = 16\ \lambda_{De}$. Both beam and background electrons have the uniform spatial distribution, and the number density ratio between the beam and background electrons is $n_b/n_e = 1.0$. **We use 256 particles per grid for both ions and electrons.** The velocity distribution of both background and beam electrons can be described as

$$F_e(v) = A \exp(- v^2 / v_{th,e}^2) \quad \text{and } F_b(v) = B \exp(- (v-v_{d,b})^2 / v_{th,b}^2)$$

, respectively, where v_d is the drift velocity and v_{th} is the thermal velocity. For the electron beam, the drift velocity and thermal velocity are $v_{d,b}/v_{th,e} = 5.0$ and $v_{th,b}/v_{th,e} = 1.0$.

Since there are 64 grids in the simulation system, now in total we will employed 64x256 particles for beam electrons and another 64x256 particles for background electrons.

Particle weighting and normalization: hand-on

In our simulation system, the Debye length and the plasma frequency of the background electrons are set as λ_{De} = 1.0 and ω_{pe} = 1.0, and the mass and the absolutely charge of electrons are m=1 and |e| = 1. The time interval $\Delta t = 0.125 / \omega_{pe}$ and grid size $\Delta x = 0.25\ \lambda_{De}$ are adopted, and the length of the simulation system is $L = 16\ \lambda_{De}$. Both beam and background electrons have the uniform spatial distribution, and the number density ratio between the beam and background electrons is $n_b/n_e = 1.0$. We use 256 particles per grid for both ions and electrons. **The velocity distribution of both background and beam electrons can be described as**

$$F_e(v) = A \exp(- v^2 / v_{th,e}^2) \quad \text{and } F_b(v) = B \exp(- (v-v_{d,b})^2 / v_{th,b}^2)$$

, respectively, where v_d is the drift velocity and v_{th} is the thermal velocity. For the electron beam, the drift velocity and thermal velocity are $v_{d,b}/v_{th,e} = 5.0$ and $v_{th,b}/v_{th,e} = 1.0$.

We can use the program from the course yesterday.

Particle weighting and normalization: hand-on

In our simulation system, the Debye length and the plasma frequency of the background electrons are set as λ_{De} = 1.0 and ω_{pe} = 1.0, and the mass and the absolutely charge of electrons are m=1 and |e| = 1. The time interval $\Delta t = 0.125 / \omega_{pe}$ and grid size $\Delta x = 0.25\ \lambda_{De}$ are adopted, and the length of the simulation system is L = 16 λ_{De}. Both beam and background electrons have the uniform spatial distribution, and the number density ratio between the beam and background electrons is n_b/n_e = 1.0. We use 256 particles per grid for both ions and electrons. The velocity distribution of both background and beam electrons can be described as

$$F_e(v) = A \exp(- v^2 / v_{th,e}{}^2) \quad \text{and } F_b(v) = B \exp(- (v-v_{d,b})^2 / v_{th,b}{}^2)$$

, respectively, where v_d is the drift velocity and v_{th} is the thermal velocity. For the electron beam, the drift velocity and thermal velocity are $v_{d,b}/v_{th,e}$ = 5.0 and $v_{th,b}/v_{th,e}$ = 1.0.

For initial condition:

1. Create a position array with 16384 elements and a velocity array with 16384 elements to save beam particle information.

2. Create a position array with 16384 elements and a velocity array with 16384 elements to save beam particle information.

Particle weighting and normalization: hand-on

In our simulation system, the Debye length and the plasma frequency of the background electrons are set as λ_{De} = 1.0 and ω_{pe} = 1.0, and the mass and the absolutely charge of electrons are m=1 and |e| = 1. The time interval $\Delta t = 0.125 / \omega_{pe}$ and grid size $\Delta x = 0.25\ \lambda_{De}$ are adopted, and the length of the simulation system is $L = 16\ \lambda_{De}$. Both beam and background electrons have the uniform spatial distribution, and the number density ratio between the beam and background electrons is $n_b/n_e = 1.0$. We use 256 particles per grid for both ions and electrons. The velocity distribution of both background and beam electrons can be described as

$$F_e(v) = A \exp(- v^2 / v_{th,e}^2) \quad \text{and } F_b(v) = B \exp(- (v-v_{d,b})^2 / v_{th,b}^2)$$

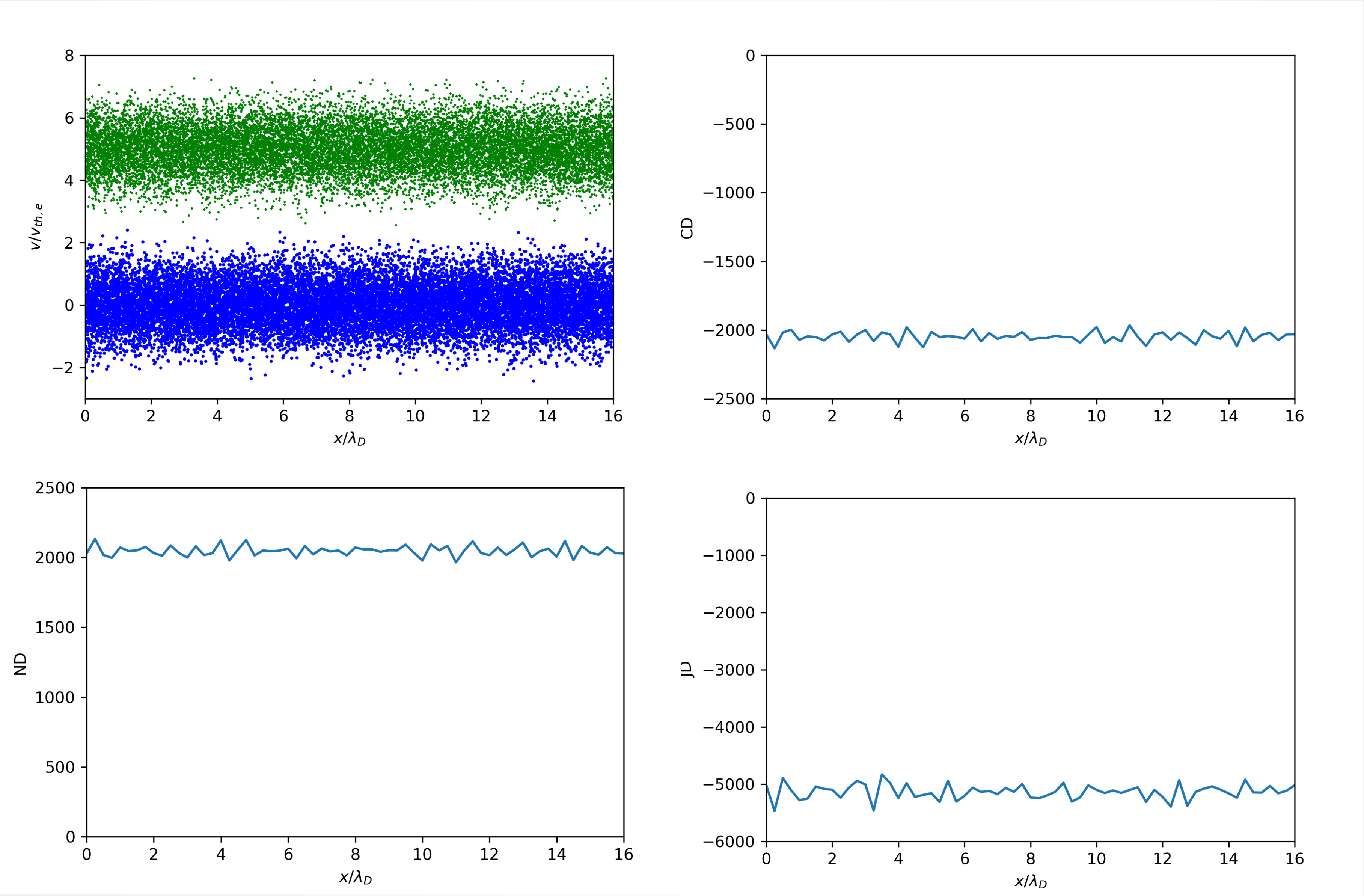
, respectively, where v_d is the drift velocity and v_{th} is the thermal velocity. For the electron beam, the drift velocity and thermal velocity are $v_{d,b}/v_{th,e} = 5.0$ and $v_{th,b}/v_{th,e} = 1.0$.

For visualization:

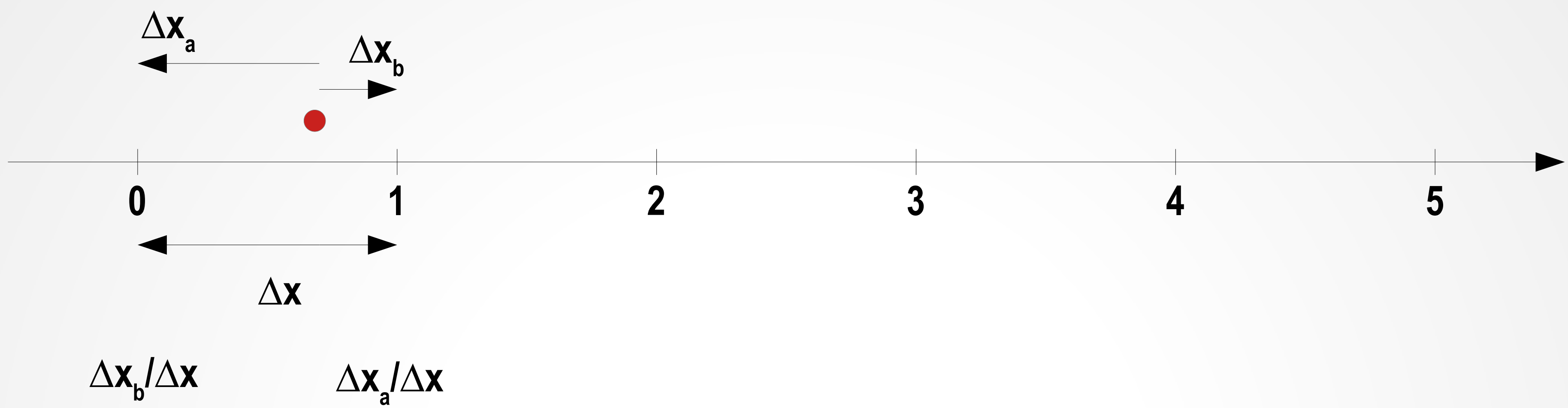
1. Make a x-v phase diagram, just put the particle spots on the plot based on their position and velocity.

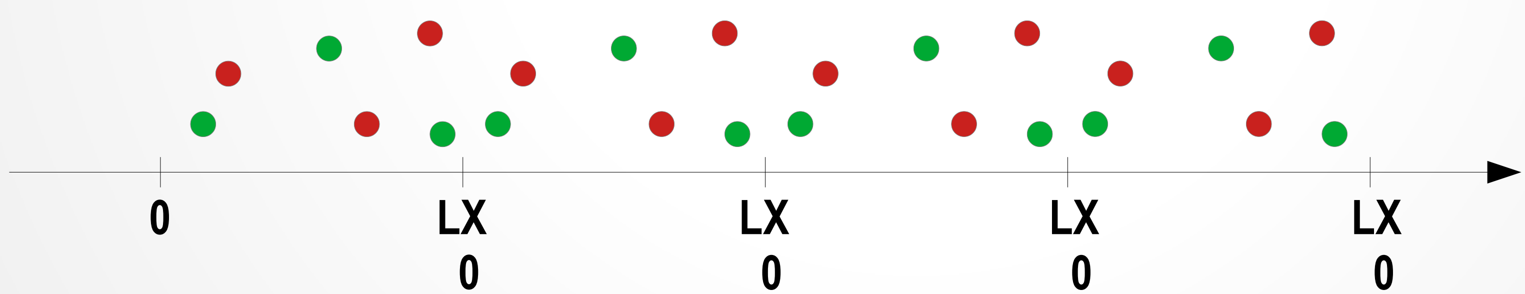
2. Make the spatial distribution plot for the total number density, charge density, and current density.

Particle weighting and normalization: hand-on



Particle weighting and normalization: hand-on





Particle weighting and normalization: hand-on

In our simulation system, the Debye length and the plasma frequency of the background electrons are set as λ_{De} = 1.0 and ω_{pe} = 1.0, and the mass and the absolutely charge of electrons are m=1 and |e| = 1. The time interval $\Delta t = 0.125 / \omega_{pe}$ and grid size $\Delta x = 0.25\ \lambda_{De}$ are adopted, and the length of the simulation system is $L = 16\ \lambda_{De}$. Both beam and background electrons have the uniform spatial distribution, and the number density ratio between the beam and background electrons is $n_b/n_e = 1.0$. We use 256 particles per grid for both ions and electrons. The velocity distribution of both background and beam electrons can be described as

$$F_e(v) = A \exp(- v^2 / v_{th,e}^2) \quad \text{and } F_b(v) = B \exp(- (v-v_{d,b})^2 / v_{th,b}^2)$$

, respectively, where v_d is the drift velocity and v_{th} is the thermal velocity. For the electron beam, the drift velocity and thermal velocity are $v_{d,b}/v_{th,e} = 5.0$ and $v_{th,b}/v_{th,e} = 1.0$.