Particle-in-Cell (PIC) kinetic simulations
02. Random number generation and its application

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www.slido.com code: #P320

Random number generator

RAND

Return a real random number within the range 0 <= x < 1.

RANDOM_NUMBER

Return a single random number or an array of random numbers within the range 0 <= x < 1.

Example: 02_01_random.f90

Random number generator

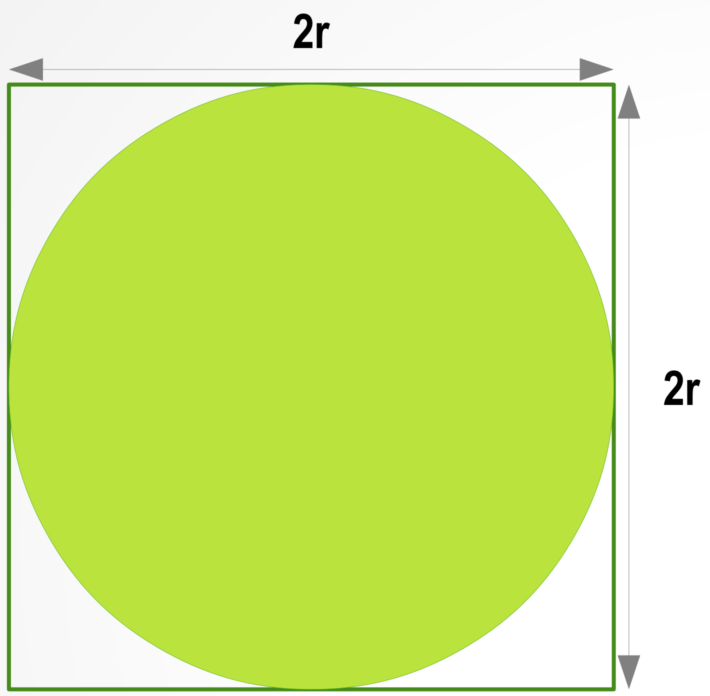
rand1 = RAND()

rand2 = rand1 * 4

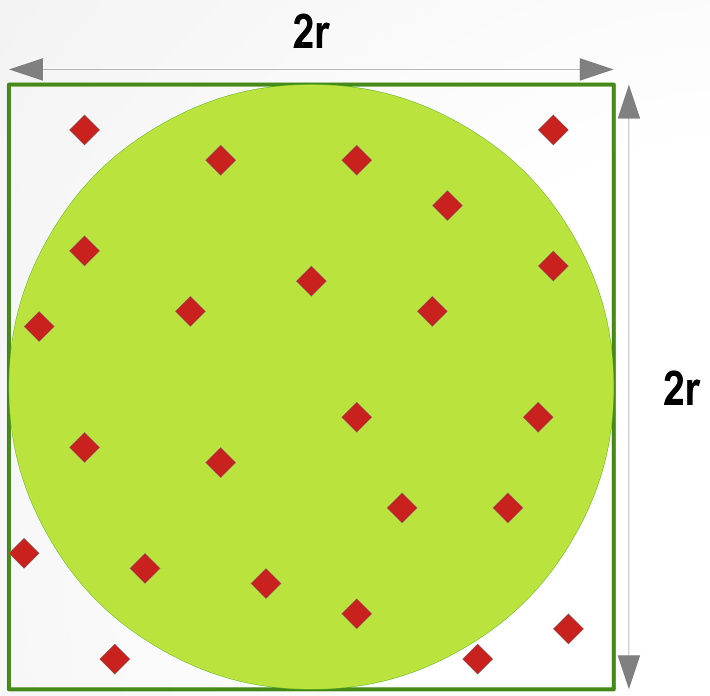
rand3 = rand1 – 2

rand4 = rand * 2 -1

Area and volume



Area and volume



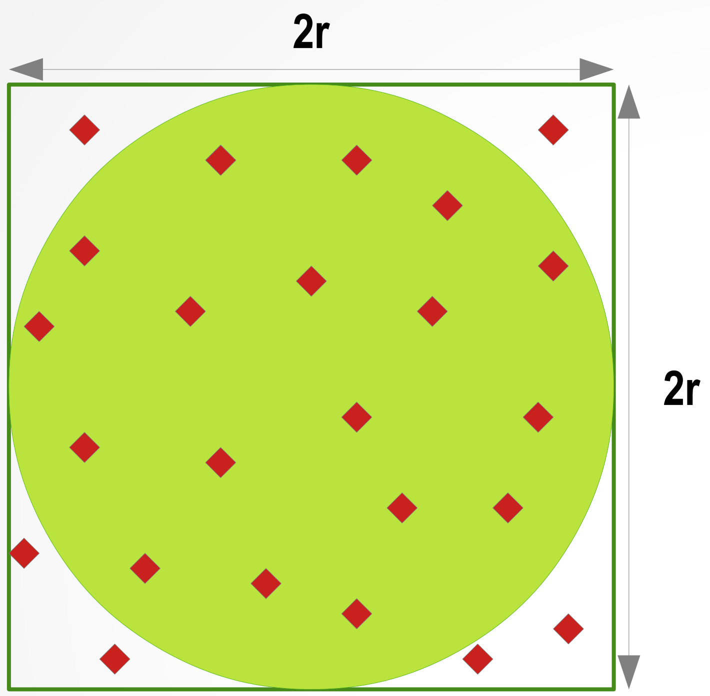
$\mathbf{N}_{\mathbf{total}}$: the total number of spots randomly distributed within the 2r x 2r square

$\mathbf{N}_{\mathbf{circle}}$: the total number of spots *luckily* located in the circle with the radius r.

While $\mathbf{N}_{\mathbf{total}} \longrightarrow \infty$,

$$\mathbf{N}_{\mathbf{circle}} / \mathbf{N}_{\mathbf{total}} \longrightarrow ?$$

Area and volume



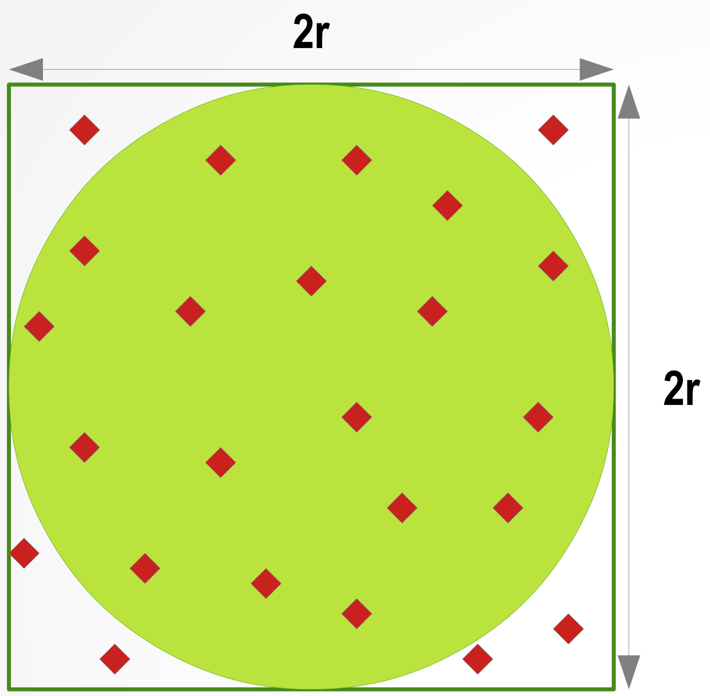
N_{total}: the total number of spots randomly distributed within the 2r x 2r square

N_{lucky}: the total number of spots *luckily* located in the circle with the radius r.

While $N_{total} \rightarrow \infty$,

$$N_{lucky} / N_{total} \rightarrow \pi r^2 / 4r^2$$
$$= \pi / 4$$
$$= 0.785398......$$

Area and volume



If we never know the area of a circle is πr^2…

1. Assume the area of a circle is **A x r x r**

2. We randomly distribute N_{total} spots in the within the 2r x 2r square.

3. We calculate the number of spots, N_{lucky}, which are located in the circle.

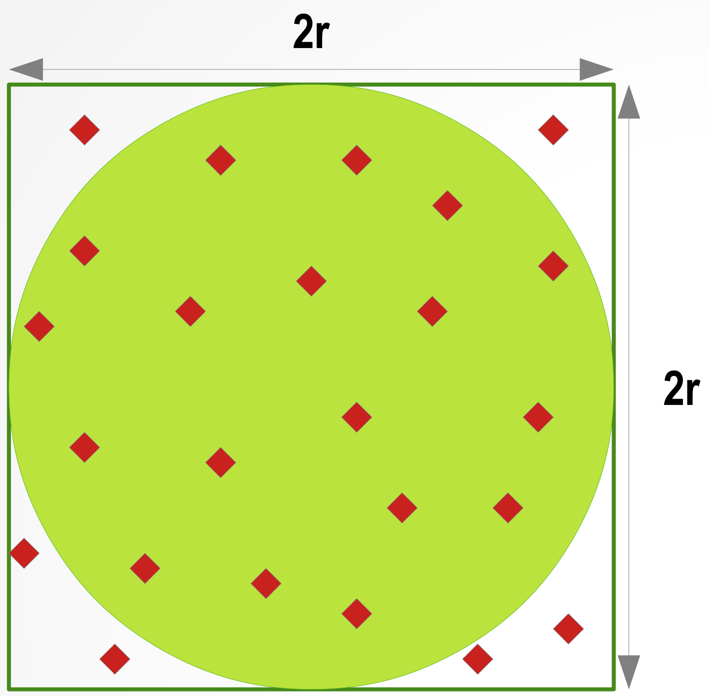
4. While $N_{total} \rightarrow \infty$,

   we will find $N_{lucky} / N_{total} \rightarrow$ **0.785398...**

5. Then we can conclude the area of a circle is

$$S = 4r^2 \times 0.785398$$
$$= 3.141592\, r^2$$

Area and volume



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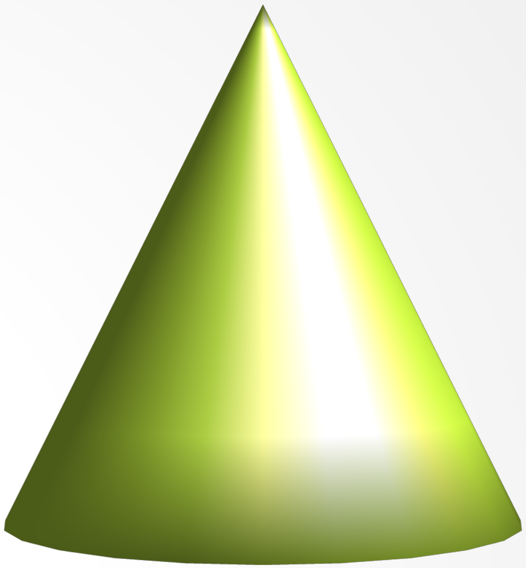
$$= 3.141592\, r^2$$

Example: 02_02_circle.f90

Area and volume



The volume of a sphere

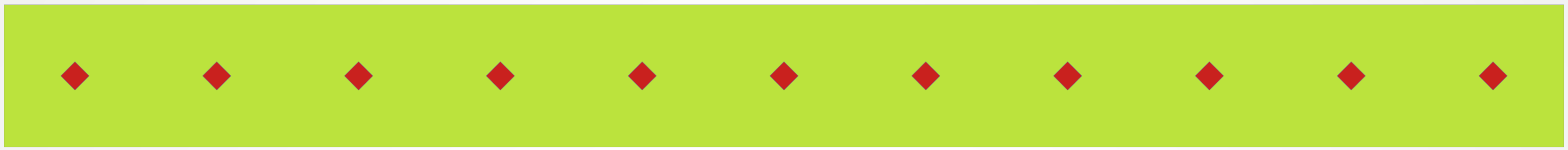
$V = A\,r^3$??
$A =$??

The volume of a cone

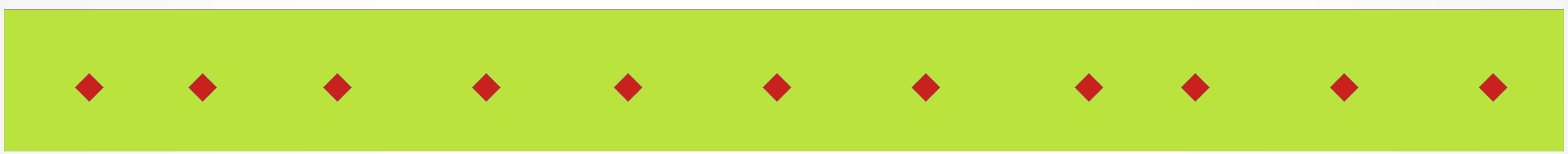
$V = A\,r^2\,h$
$A =$??

Spatial distribution

An uniform-uniform spatial distribution

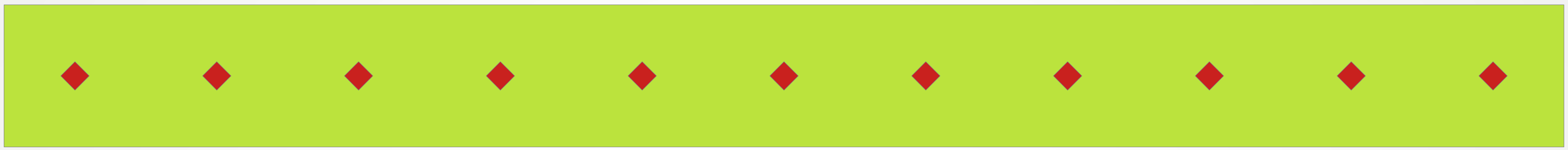


A random-uniform spatial distribution



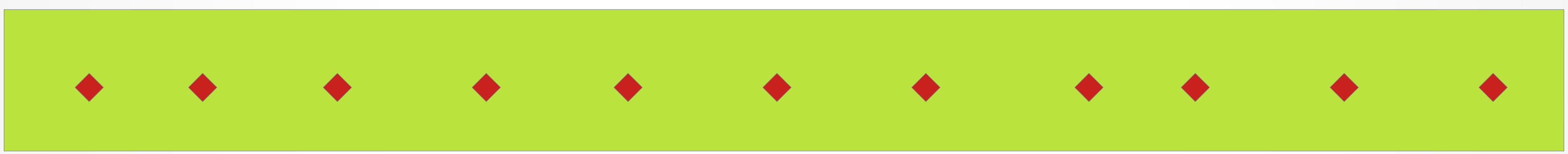
Spatial distribution

An uniform-uniform spatial distribution



$Px_i = \Delta x \text{ x } I$ $\Delta x = L / NP$

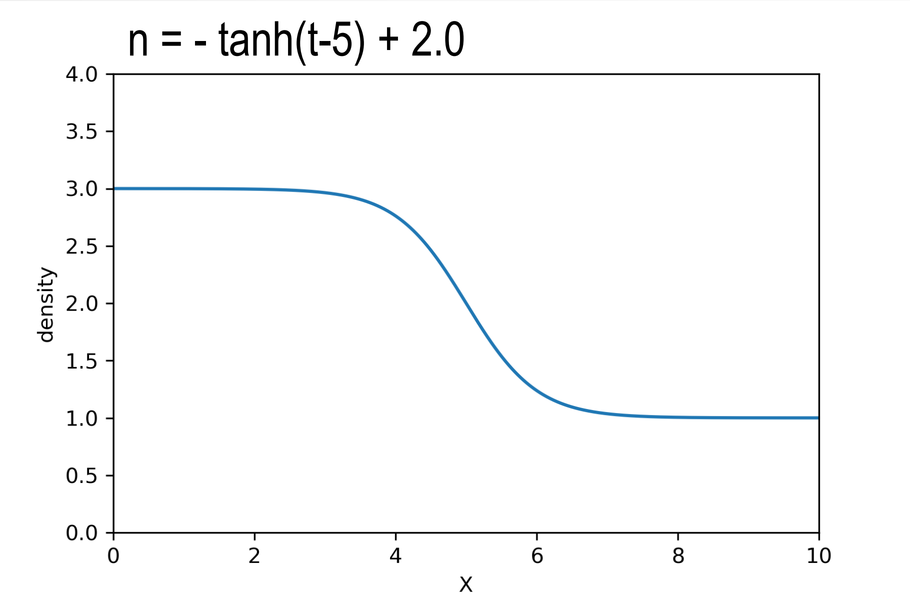
A random-uniform spatial distribution



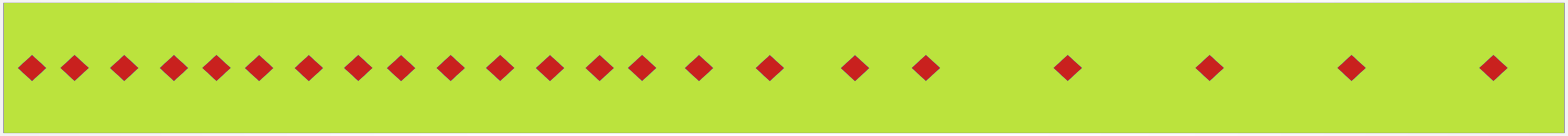
$Px_i = RANDOM\_NUMBER_i \text{ x } L$

L: Length of the system NP: number of particles

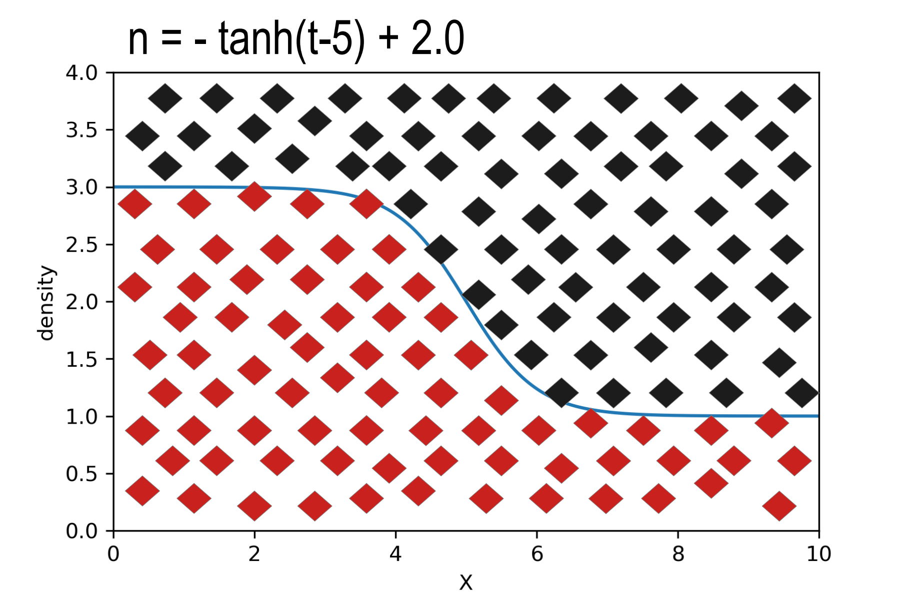
Spatial distribution



If you do the math, you can still get...



Spatial distribution



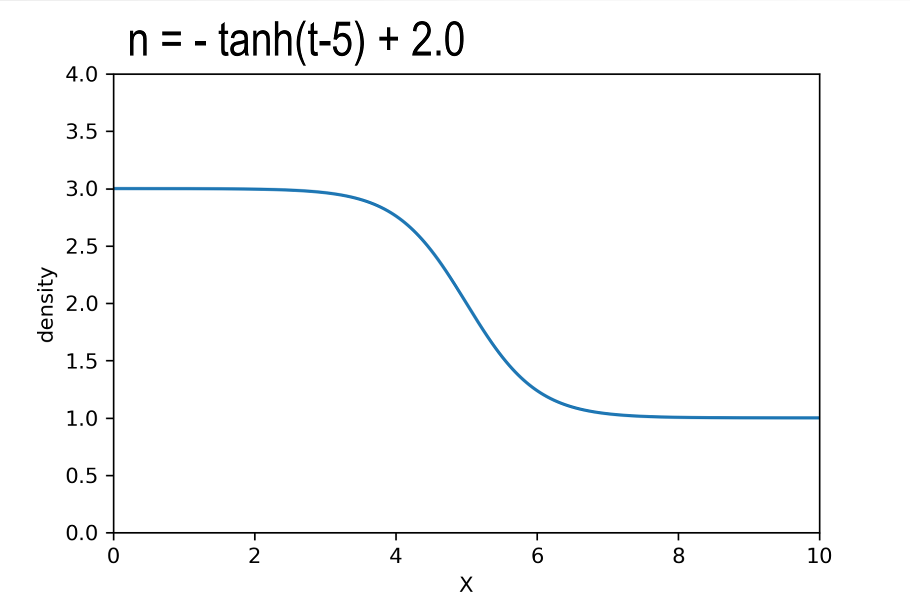
1. For each particle, we pick a random number as the location of the particle.

2. Based on the location of the particle, read the *flag-number* from the density distribution function.

3. Pick another random number as a *lottery.*

4. For the particles that with the lottery-number smaller than the flag-number, we keep them. And we give up the other particles that with the lottery-number larger than the flag-number.

Spatial distribution



For each particle, we

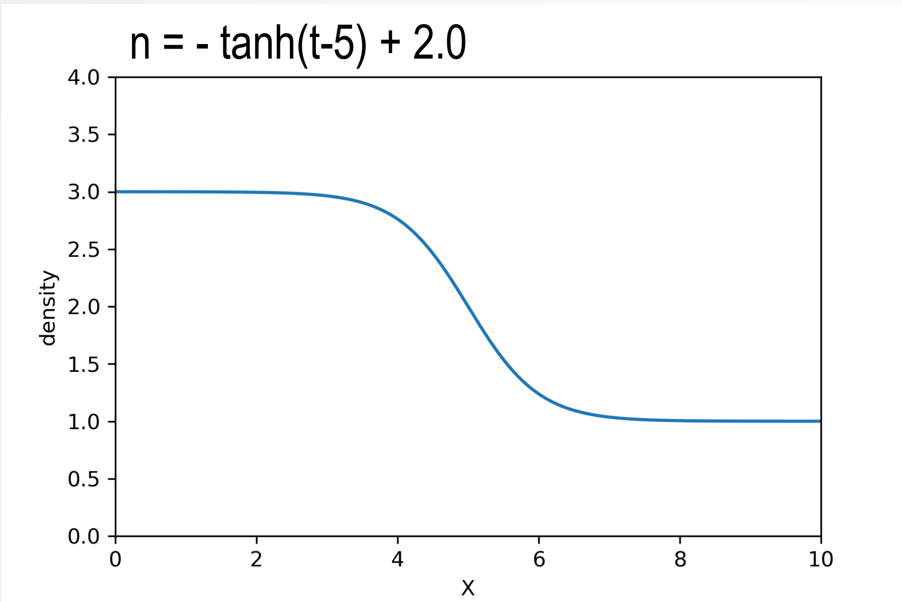
1. Pick a random number as the location of the particle.

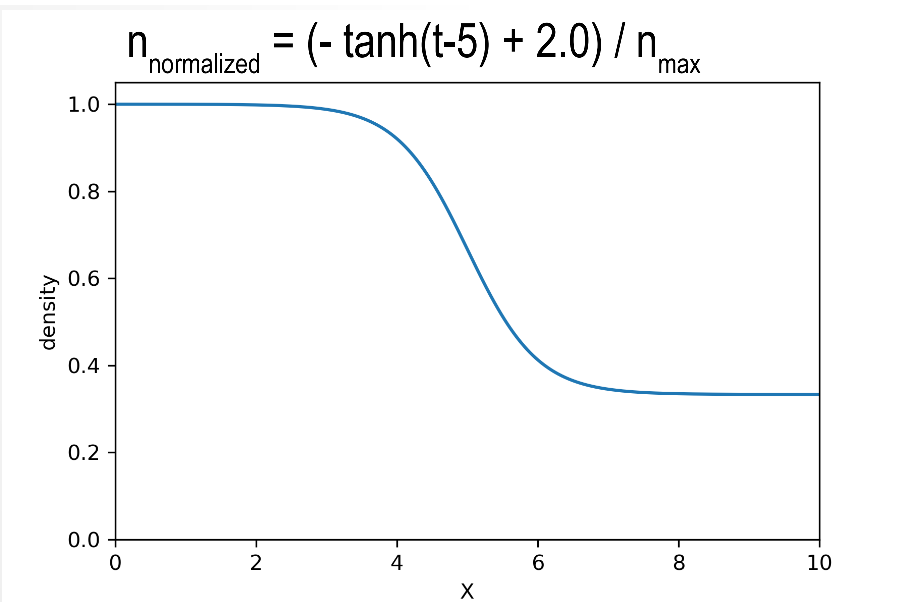
2. Based on the location of the particle, read the *flag-number* from the density distribution function.

3. Pick another random number as a *lottery*, if the lottery-number is smaller than the flag-number. We set down the particle. If not, we go back to step 1 and pick a new location for the same particle.

Easier to control
the total number of particles

Spatial distribution





We adjust the density distribution function first.

For each particle, we

1. Pick a random number as the location of the particle.

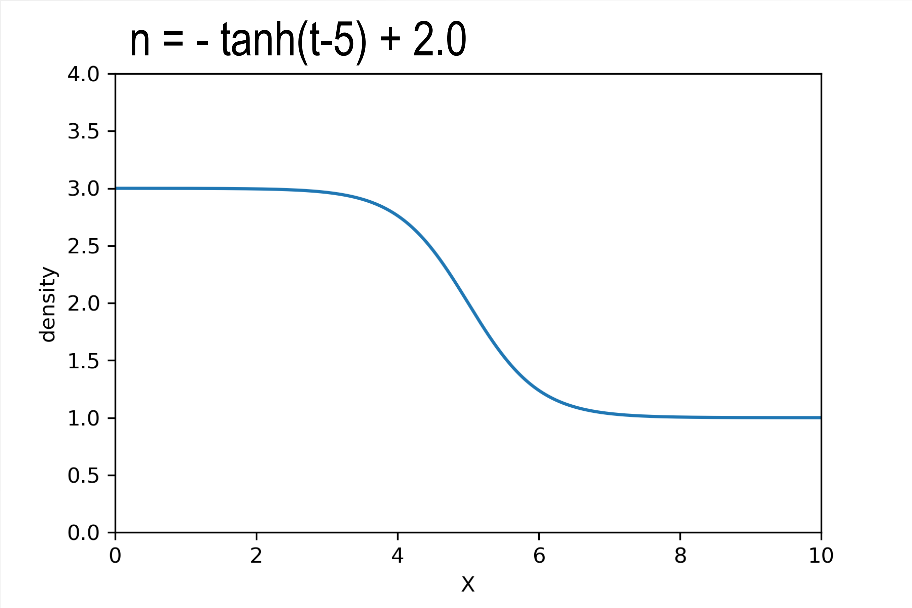
2. Based on the location of the particle, read the *flag-number* from the density distribution function.

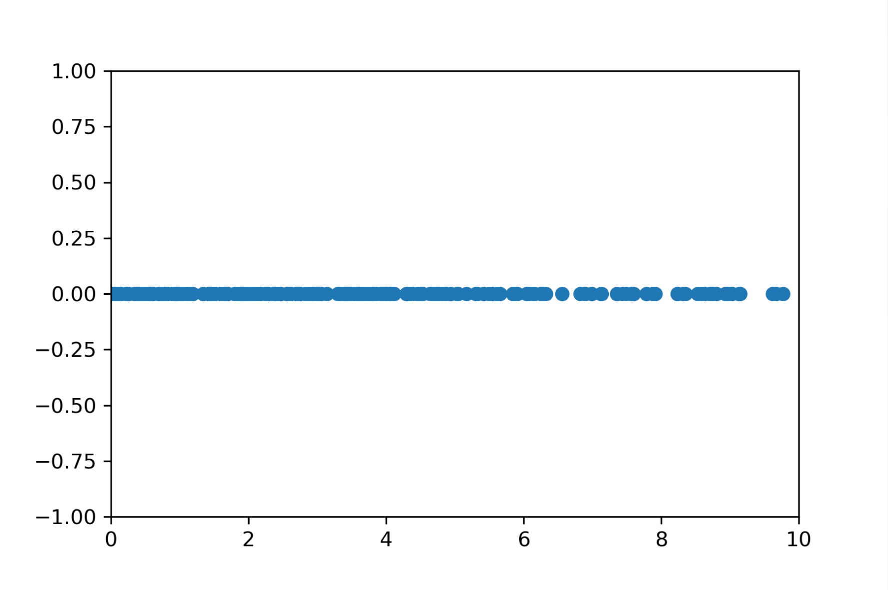
3. Pick another random number as a *lottery*, if the lottery-number is smaller than the flag-number. We set down the particle. If not, we go back to step 1 and pick a new location for the same particle.

More efficient!!

Example: 02_03_spatialdis.f90

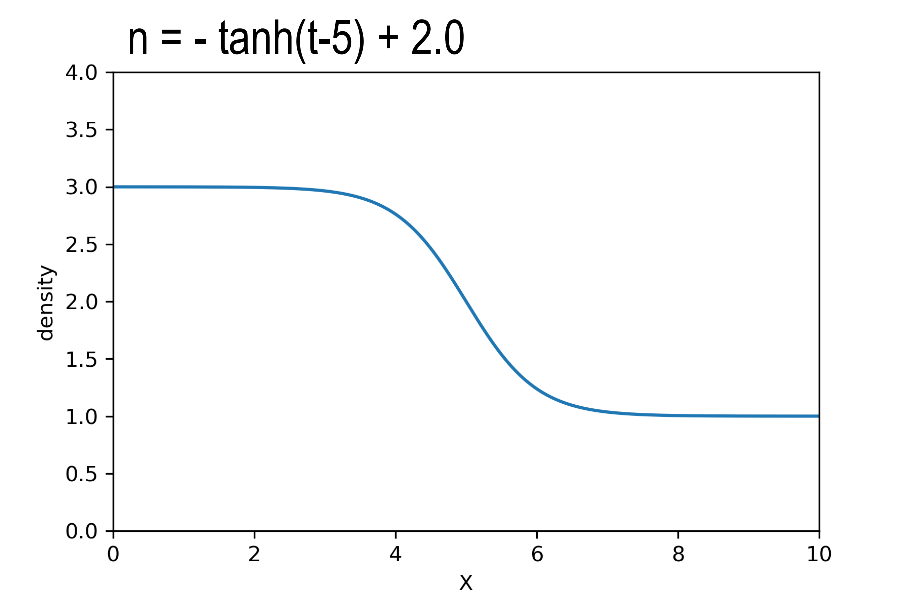
Spatial distribution

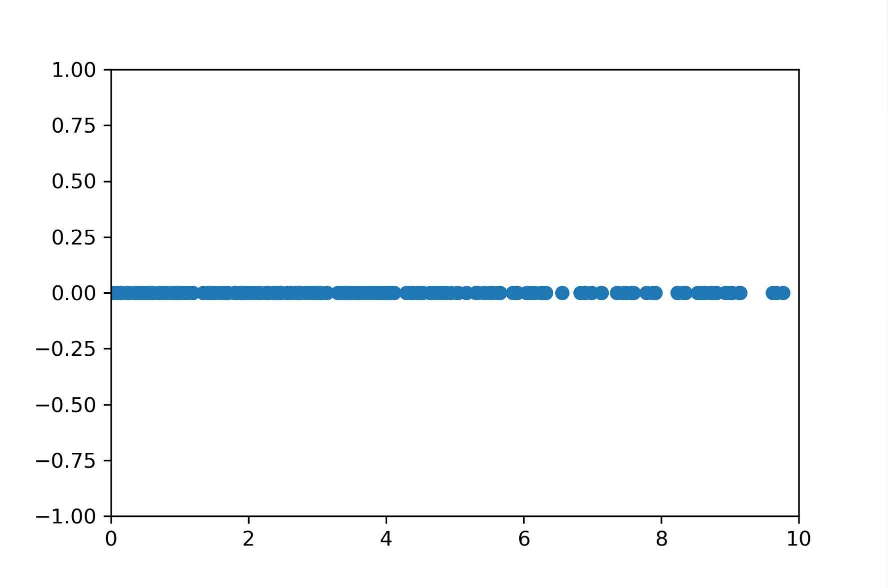




Example: 02_04_spatialdis_n.f90

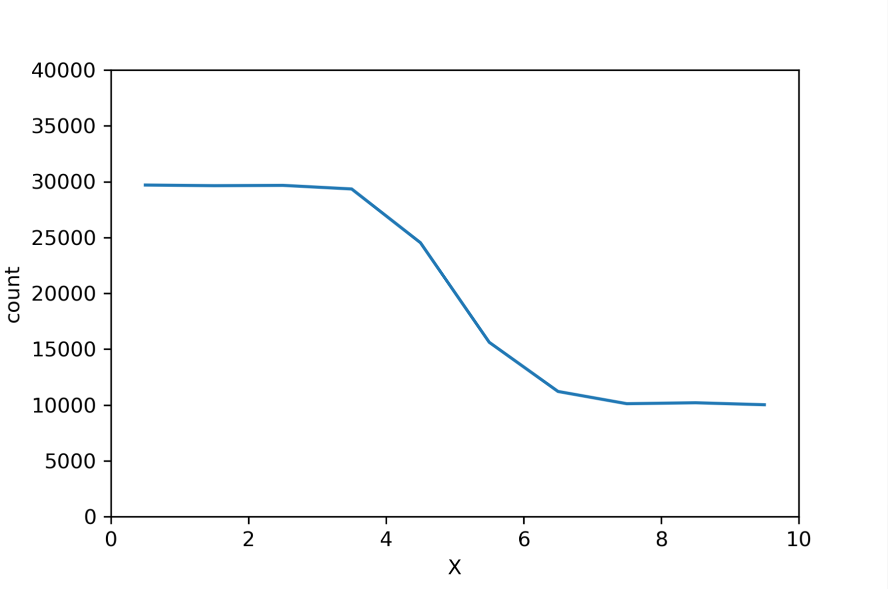
Spatial distribution



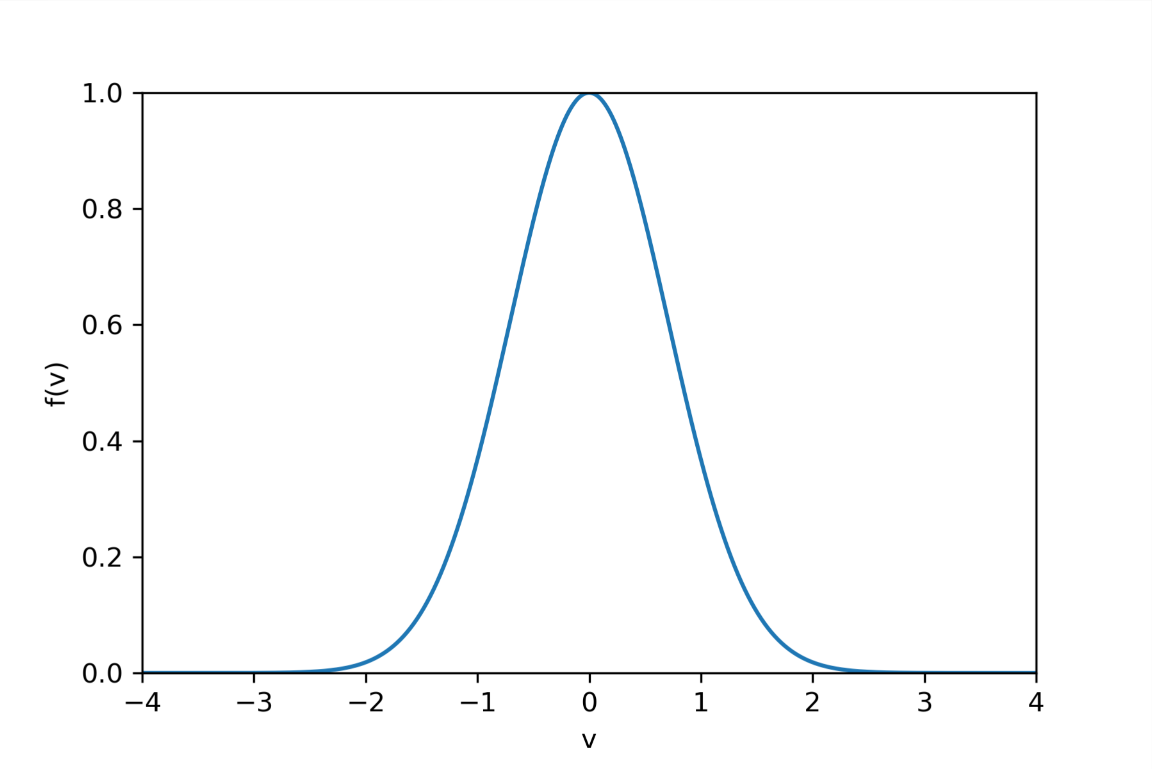


Example: **02_04_spatialdis_n.f90**

Later we will come back to talk about "Particle weighting and normalization"

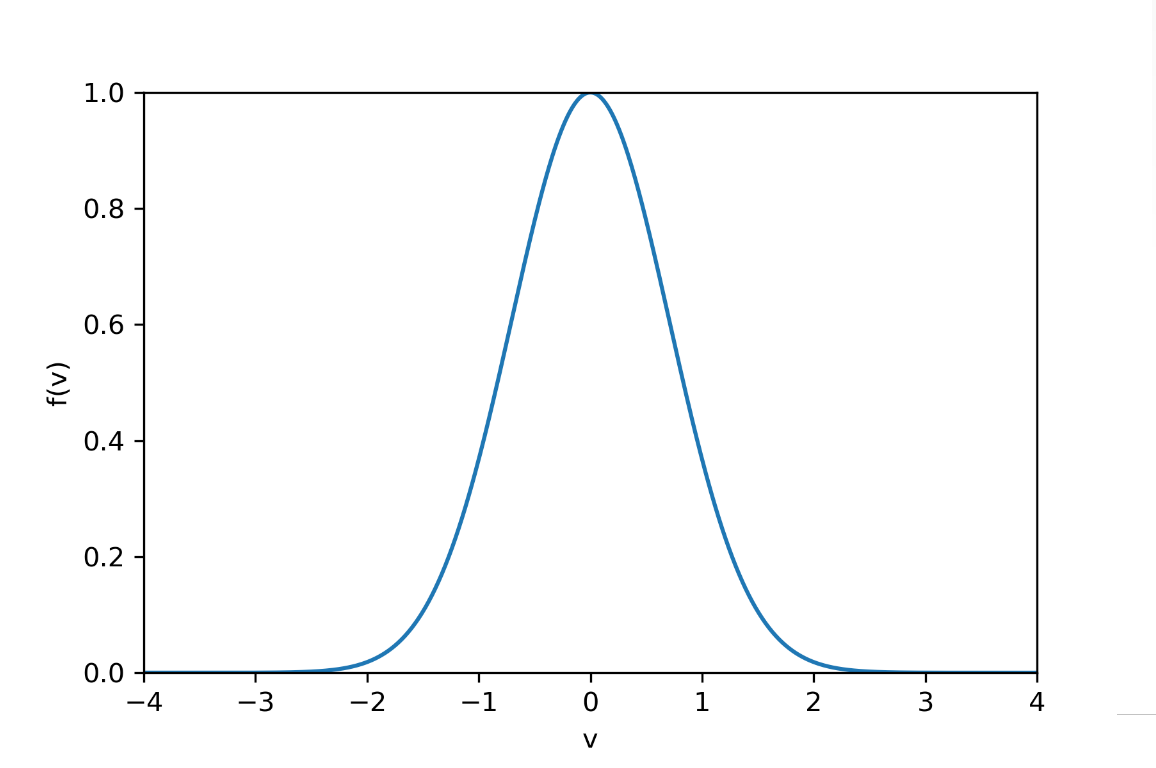


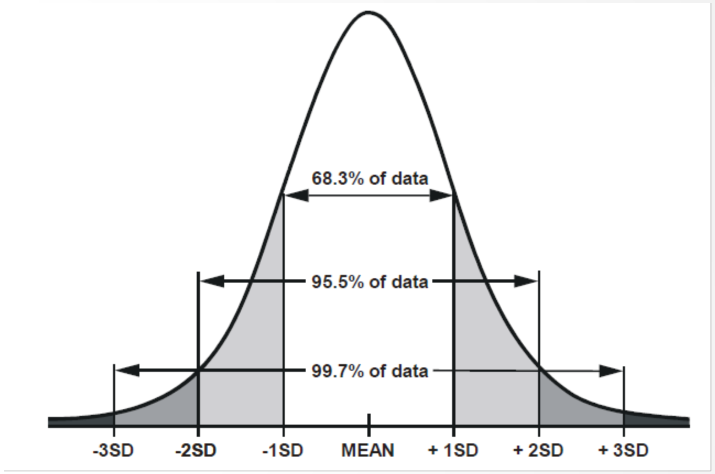
Velocity distribution



Any idea?

Velocity distribution

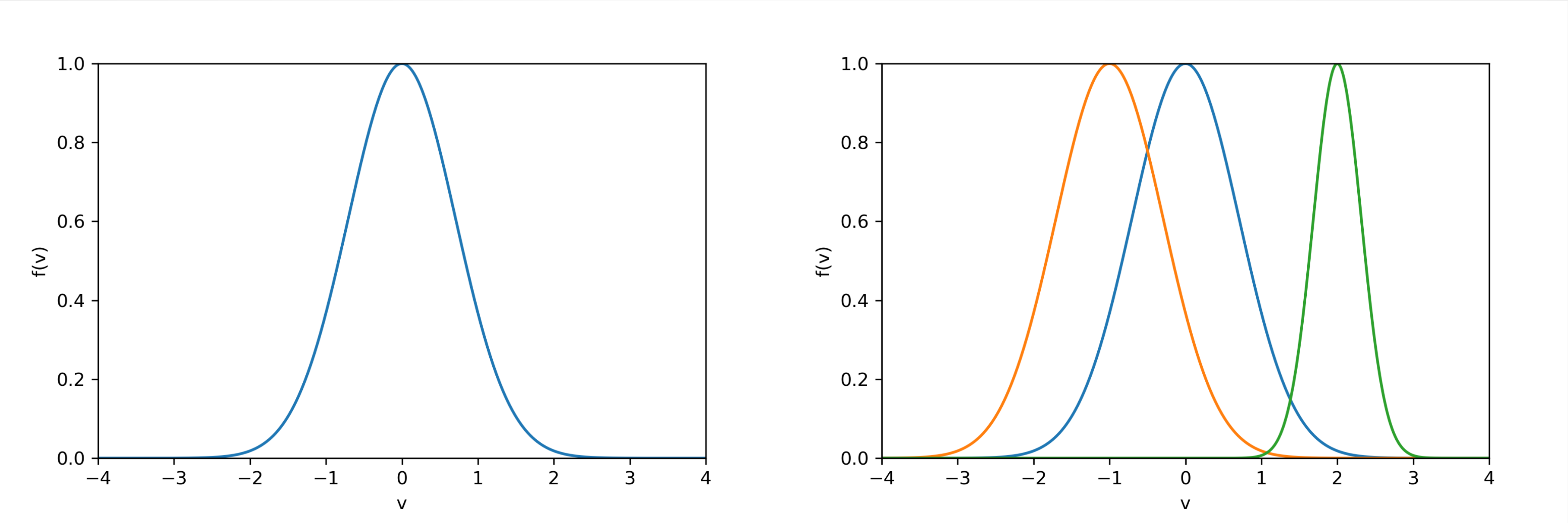




Any idea?

How large of the velocity range we should set?

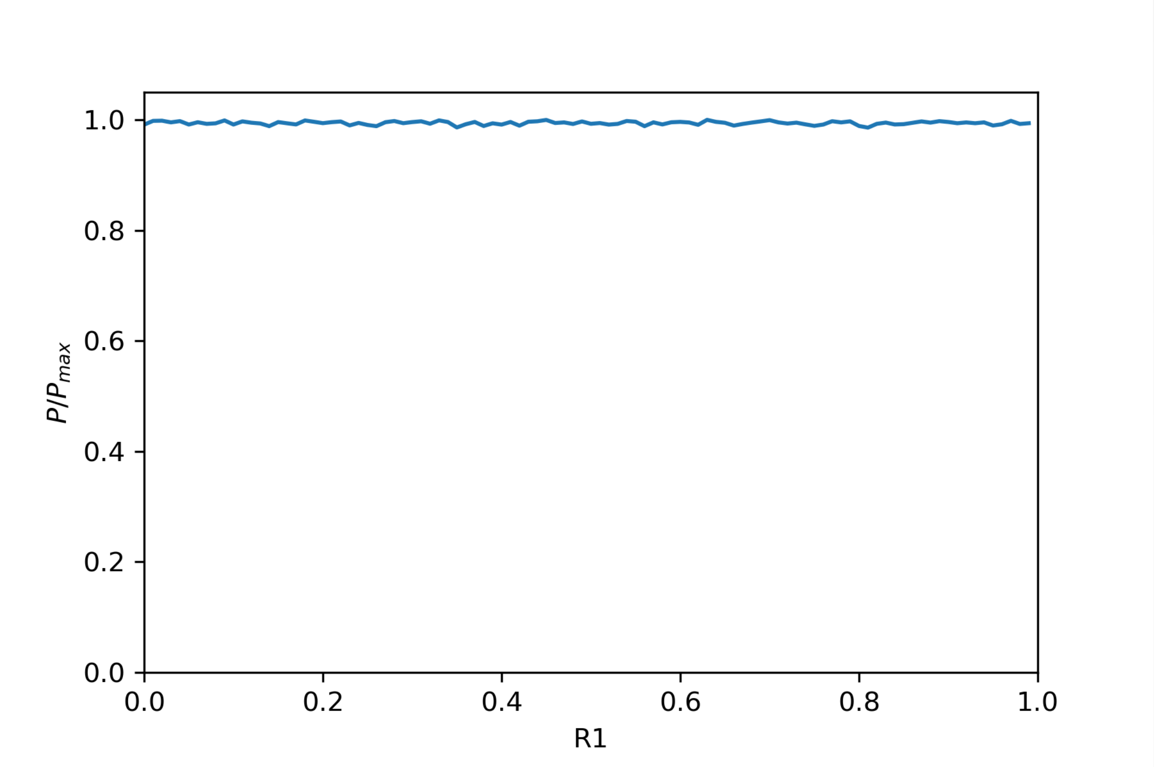
Velocity distribution



Any idea?

Or any kind of velocity distribution functions?

Sums of independent random variables



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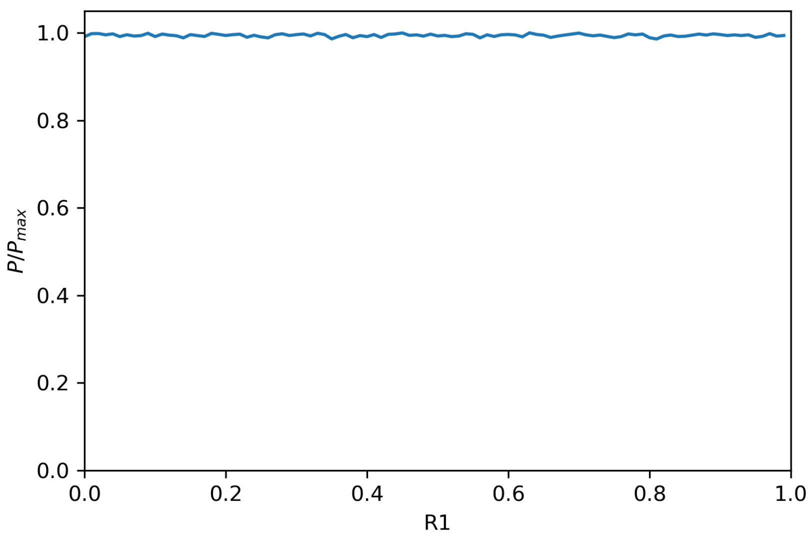

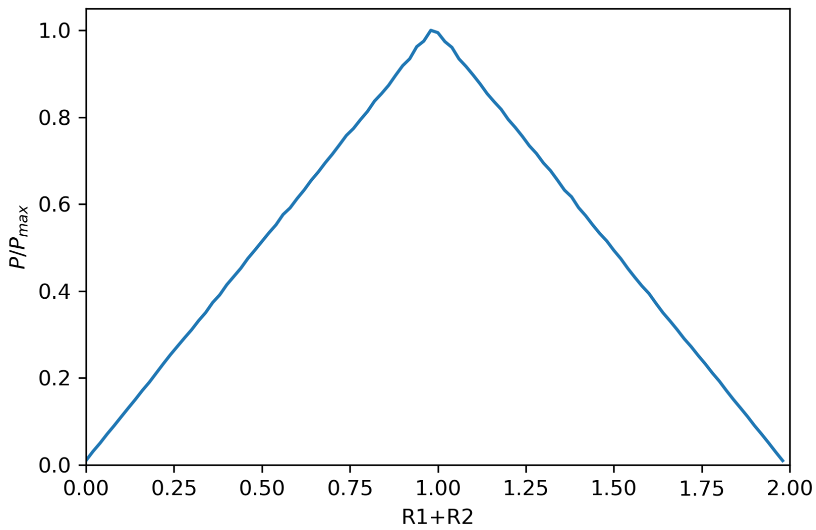

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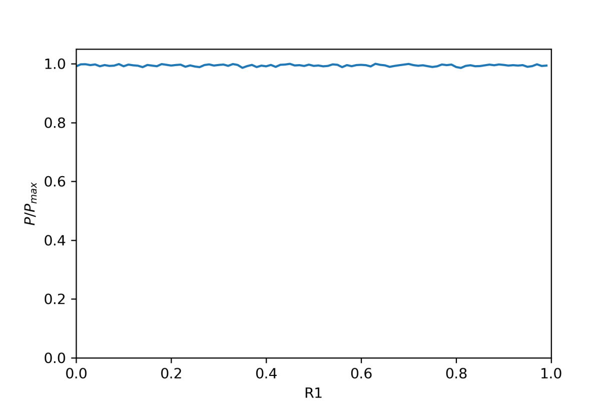

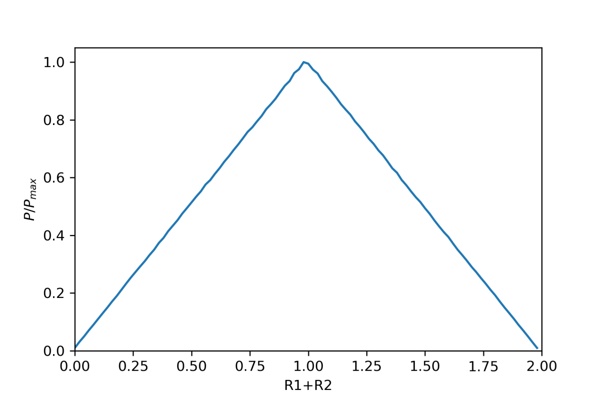

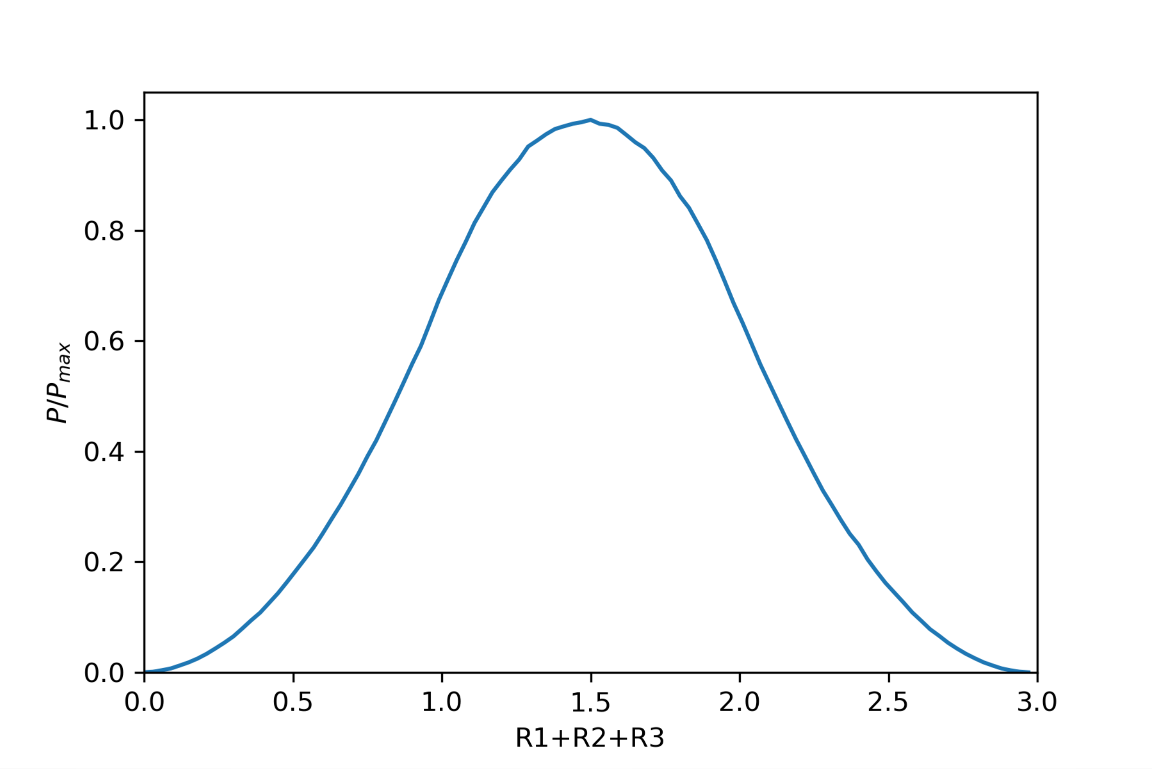

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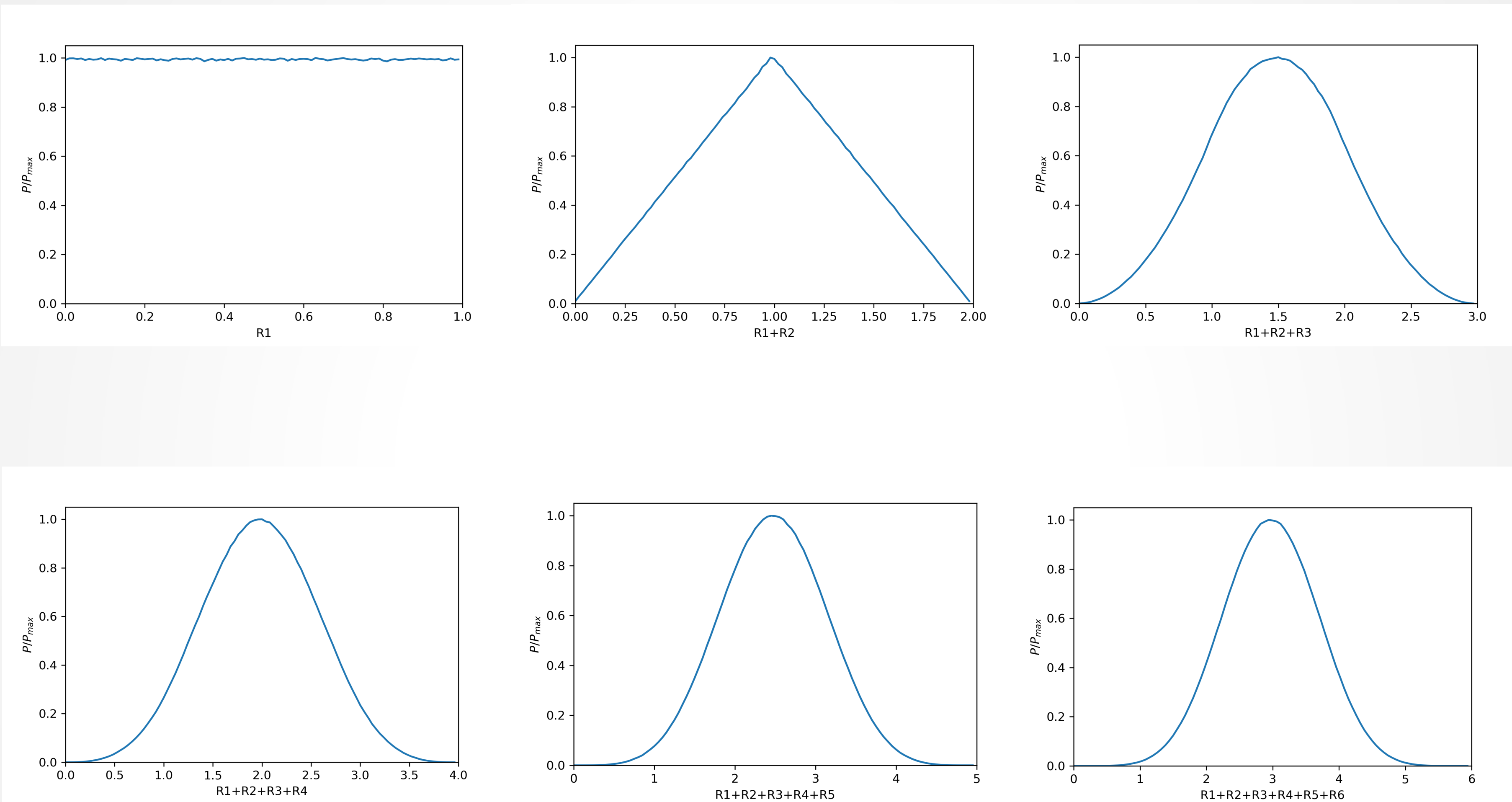
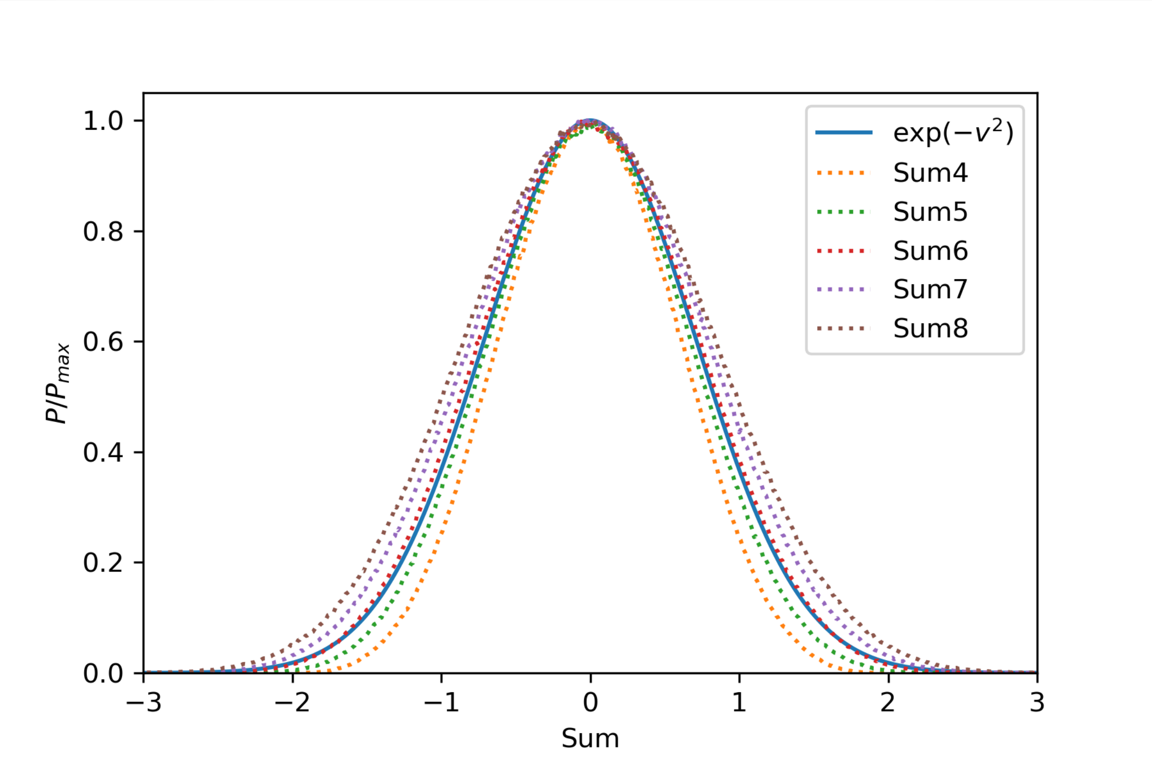
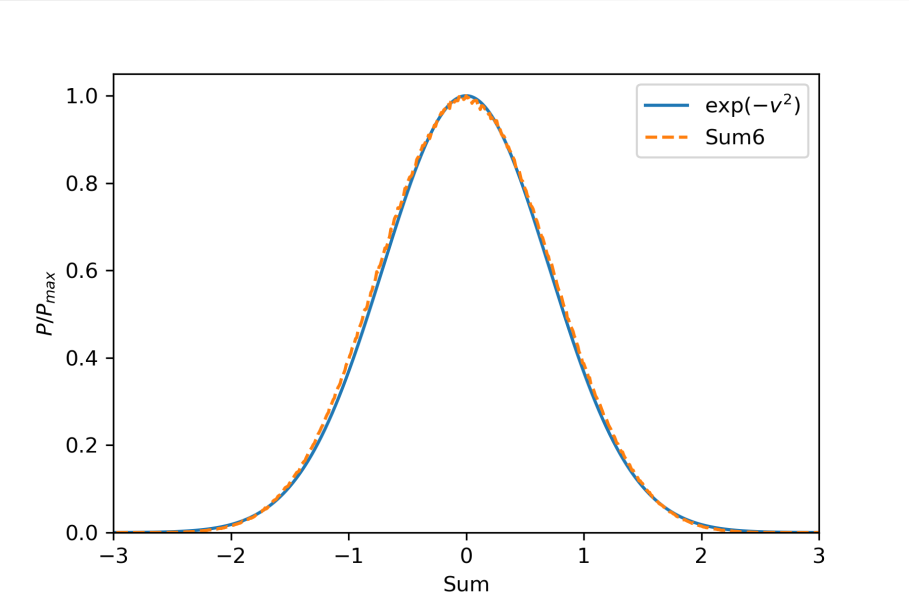


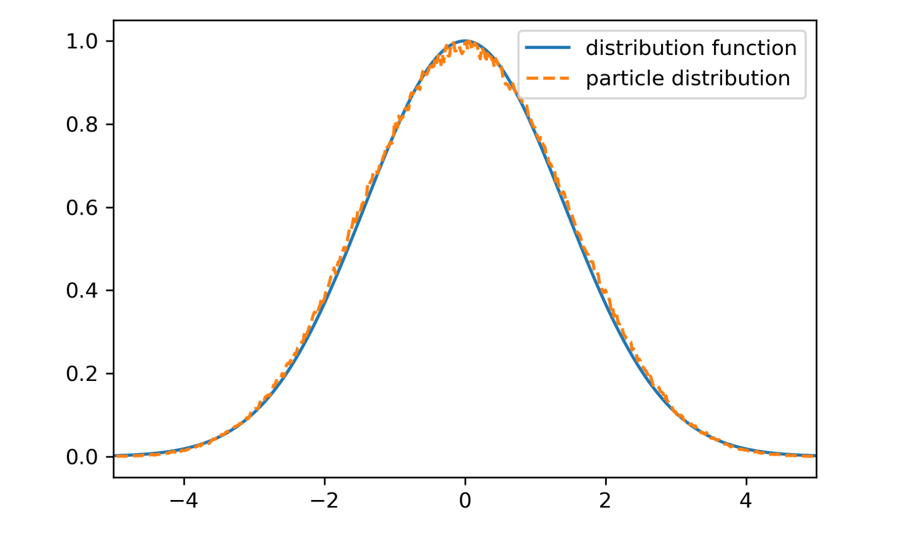
Sums of independent random variables



Sums of the random number [-0.5,0.5]

Sums of independent random variables





For each particle, we

1. Pick six random numbers and adjust the the range as [-0.5,0.5]

2. Sum the six picked numbers as the particle velocity.

3. The central velocity and thermal velocity can be also defined with the varied range of random numbers.

Example: 02_05_sum6rand.f90

Random number generator: hand-on

1. Based on the assumption that the volume of a sphere is $V = A\, r^3$, and use the random number generator to find that $A = \pi * (4/3)$.

2. Setup 1,000,000 particles in the spatial space X = 0-10 with the uniform distribution.

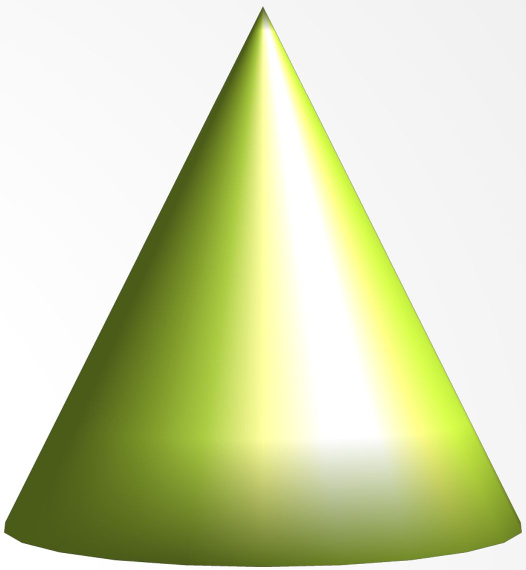
3. Setup 1,000,000 particles with the velocity distribution
   $F(v) = \exp(-v^2)$

4. Setup 1,000,000 particles with the velocity distribution
   $F(v) = \exp(-v^2/4))$

5. Setup 1,000,000 particles with the velocity distribution
   $F(v) = \exp(-(v-3)^2)$

Random number generator: hand-on (advanced)

1. Based on the assumption that the volume of a cone is $V = A\,hr^2$, use the random number generator to find that $A = \pi/3$.



2. Setup 1,000,000 particles in the spatial space X = 0-10 with the uniform distribution. And, in these particles, setup 100,000 particles with the velocity distribution $F(v) = \exp(-v^2/9))$ and 900,000 particles with the velocity distribution $F(v) = \exp(-(v-1)^2)$.