EE320 Homework # 9
Synchronous Machines

SOLUTION

Problem 1: A 4-pole, 60Hz, Y-connected, 3-phase generator has a regulated terminal line-to-neutral voltage of $\tilde{V}_{as} = 260V_{rms}\angle 0^{\circ}$, a synchronous reactance of 0.06Ω, a stator resistance of 0.003Ω, and $L_{sf} = 0.02H$. The balanced 3-phase load draws 2MW at a lagging 0.8 power factor.

a. Compute the 3-phase complex power of the load (magnitude and angle)
b. Compute the generator phase current (magnitude and angle; hint: the angle of the current should be negative for the load to have a lagging power factor)
c. Find the required excitation voltage ($\tilde{E}_a$)
d. Calculate the required field current
e. Calculate the required speed of the prime mover

Problem 2: Given the same machine as in problem 13.1. If the excitation voltage is $\tilde{E}_a = 360V_{rms}\angle 14^{\circ}$ and the field current is adjusted to maintain the same terminal voltage as in Problem 13.1,

a. Find the new generator phase current
b. Calculate the 3-phase real power consumed by the load and identify the new power factor
c. Calculate the generator efficiency (consider stator winding losses as the only losses)

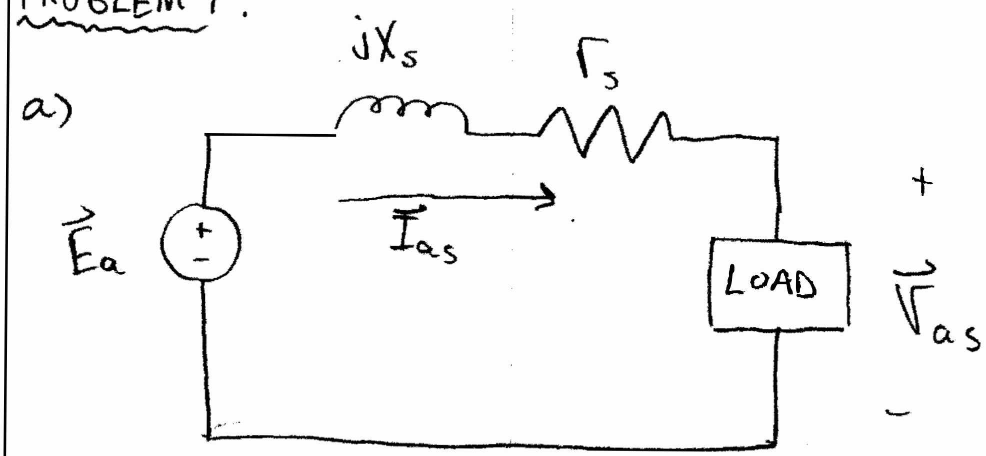
Problem 3: A 3-phase, Y-connected, 6-pole, 60Hz generator has $X_S = 1\Omega$, negligible r_S (stator resistance), and $L_{sf} = 0.2H$. The machine operates at a power angle of 31.48° with a field current of 100A. The line-to-neutral terminal voltage is 2400Vrms.

a. Find the speed of the prime mover in rad/sec
b. Determine the excitation voltage
c. Find the line current
d. Find the power consumed by the load and its power factor
e. Find the developed torque

EE320 ①

PROBLEM 1:

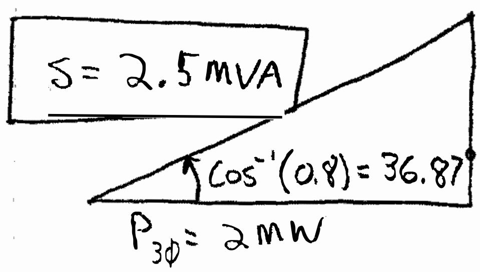
a)



$X_s = 0.06\,\Omega$

$r_s = 0.003\,\Omega$

$L_{sf} = 0.02\ H$



$Q = P_{3\phi} \tan 36.87°$

$= 1.5\ MVAR$

b) Generator phase current

$$S_{3\phi} = 3\vec{V}_{as}\vec{I}_{as}^{*} \Rightarrow \vec{I}_{as}^{*} = \frac{S_{3\phi}}{3\vec{V}_{as}} = \frac{2.5\times10^6\angle 36.87°}{3(260\angle 0°)}$$

$$\vec{I}_{as}^{*} = 3205\angle 36.87°\ A$$

$$\boxed{\vec{I}_{as} = 3205\angle -36.87°\ A}$$

c) Find $\vec{E}_a$

KVL:

$$\vec{E}_a - \vec{I}_{as}(jX_s) - \vec{I}_{as}(r_s) - \vec{V}_{as} = 0$$

$$\vec{E}_a = (3205\angle -36.87)(j0.06) + (3205\angle -36.87°)(0.003) + 260\angle 0°$$

$$\boxed{\vec{E}_a = 410.7\angle 21.1°\ V}$$

Problem 1 (CONT'D)

d) FIND I_F

$$\vec{E}_a = \frac{L_{sf} I_f \omega_e}{\sqrt{2}} \angle \delta$$

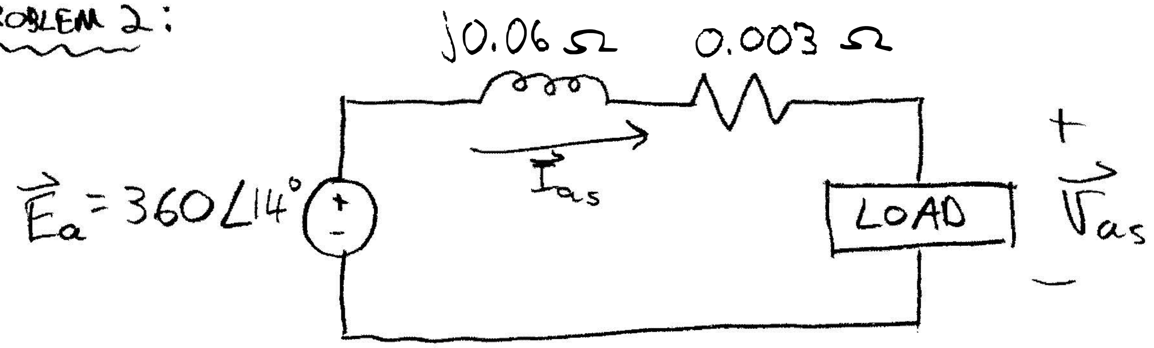
$$I_f = \frac{\sqrt{2}\,|\vec{E}_a|}{L_{sf}\,\omega_e} = \frac{\sqrt{2}\,(410.7)}{(0.02)(2\pi 60)} = \boxed{77.03 \text{ A}}$$

e) find ω_s

$$\omega_s = \omega_e = 2\pi 60 = 377 \frac{\text{rad}}{\text{sec}}$$

$$n_s = \frac{120 f_e}{P} = \frac{120(60)}{4} = \boxed{1800 \text{ rpm}}$$

PROBLEM 2:



a) Find $\vec{I}_{as}$

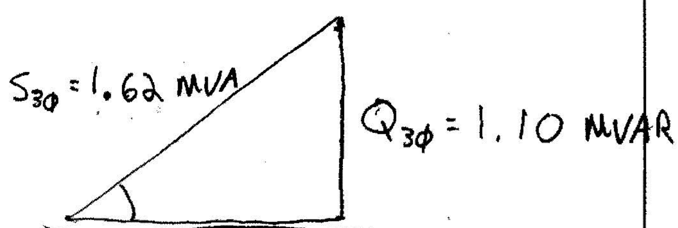
$$\vec{E}_a - j0.06(\vec{I}_{as}) - 0.003(\vec{I}_{as}) - \vec{V}_{as} = 0$$

$$\vec{I}_{as} = \frac{E_a - V_{as}}{(0.003 + j0.06)} = \boxed{2076.4\angle -42.86^\circ \text{ A}}$$

b) Find $P_{3\phi}$

$$S_{3\phi} = 3\,\vec{V}_{as}\,\vec{I}_{as}^{\,*} = 3(260\angle 0^\circ)(2076.4\angle 42.86^\circ)$$

$$S_{3\phi} = 16.20\angle 42.86^\circ$$



$$pf = \cos^{-1}(42.86^\circ) = \boxed{0.733}$$

$$\boxed{P_{3\phi} = 1.19 \text{ MW}}$$

c) Find η_{gen}

$$P_{SCL} = 3|\vec{I}_{as}|^2(0.003) = 38.8 \text{ kW}$$

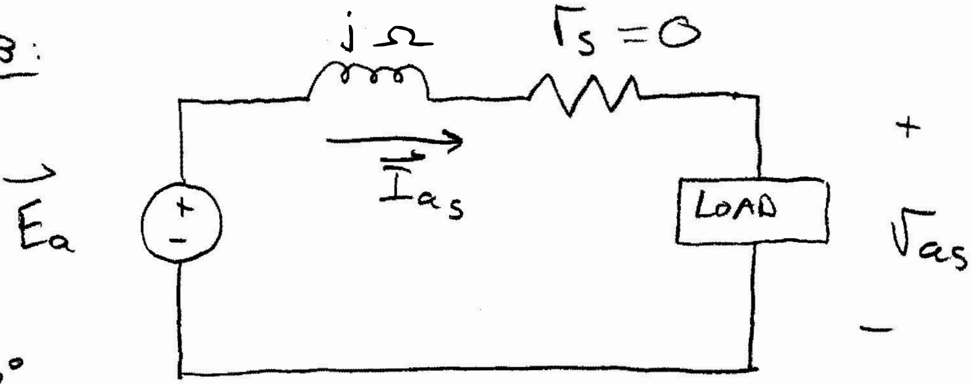
Stator copper losses

$$P_{out} = P_{in} - P_{SCL}$$

$$P_{in} = 1.19 \text{ MW} + 38.8 \text{ kW} = 1.23 \text{ MW}$$

$$\eta = \frac{1.19 \text{ MW}}{1.23 \text{ MW}} = \boxed{96.8\%}$$

PROBLEM 3:



$\delta = 31.48°$

$I_f = 100A$

$\vec{V}_{as} = 2400 \angle 0° \ V$

$L_{sf} = 0.2H$

a) Find ω_m

$$\omega_m = \omega_s = \frac{2}{P}\omega_e = \frac{2}{6}(2\pi 60) = 125.7 \ \frac{rad}{sec}$$

b) Find $\vec{E}_a$

$$|\vec{E}_a| = \frac{L_{sf} I_f \omega_e}{\sqrt{2}} = \frac{(0.2)(100)(2\pi 60)}{\sqrt{2}} = 5331.5 \ V$$

$$\boxed{\vec{E}_a = 5331.5 \angle 31.48° \ V}$$

c) Find $\vec{I}_{as}$

$$\vec{I}_{as} = \frac{\vec{E}_a - \vec{V}_{as}}{j} = \frac{5331.5 \angle 31.48° - 2400 \angle 0°}{j}$$

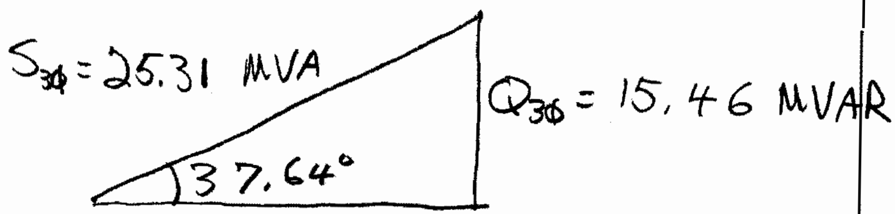
$$\boxed{\vec{I}_{as} = 3515.7 \angle -37.6° \ A}$$

d) Find $P_{3\phi}$ and p.f.

$$\vec{S}_{3\phi} = 3 \vec{V}_{as} \vec{I}_{as}^{*} = 3(2400 \angle 0°)(3515.7 \angle 37.6°)$$
$$= 25.31 \angle 37.64° \ MVA$$

Problem 3: (CONT'D)

d) Find $P_{3\phi}$



$P.f. = \cos^{-1}(37.64°)$

$\boxed{= 0.792}$

$\boxed{P_{3\phi} = 20.05 \text{ MW}}$

e) find $\tau_{dev} = \tau_{ind}$

No losses, $r_s = 0$

$$\tau_{dev} = \frac{P_{mech}}{\omega_m} = \frac{P_{3\phi}}{\omega_m} = \frac{20.05 \times 10^6}{125.7} = \boxed{159.5 \text{ kN}\cdot\text{m}}$$

or you can solve

$$\tau_{dev} = \frac{3 V_{as} E_a \sin\delta}{\omega_m X_s} = \frac{3(2400)(5331.5)\sin(31.48°)}{125.7}$$

$$\boxed{= 159.5 \text{ kN}\cdot\text{m}}$$