CHAPTER 4
AC MACHINES

4.1 Synchronous Machines

4.1.1 Derivation of the Equivalent Circuit for a Synchronous Machine

A synchronous machine can be described as "inside out" compared to a DC generator. In a synchronous machine the field generating windings are on the rotating element — the rotor — and the power generating windings are on the stationary element — the stator.

A DC current, I_f, is fed to the rotor windings producing a magnetic flux from the rotor through the stator and back to the rotor again. A simplified rotor with a single pole pair (two poles) is shown in **Figure 4.1**.

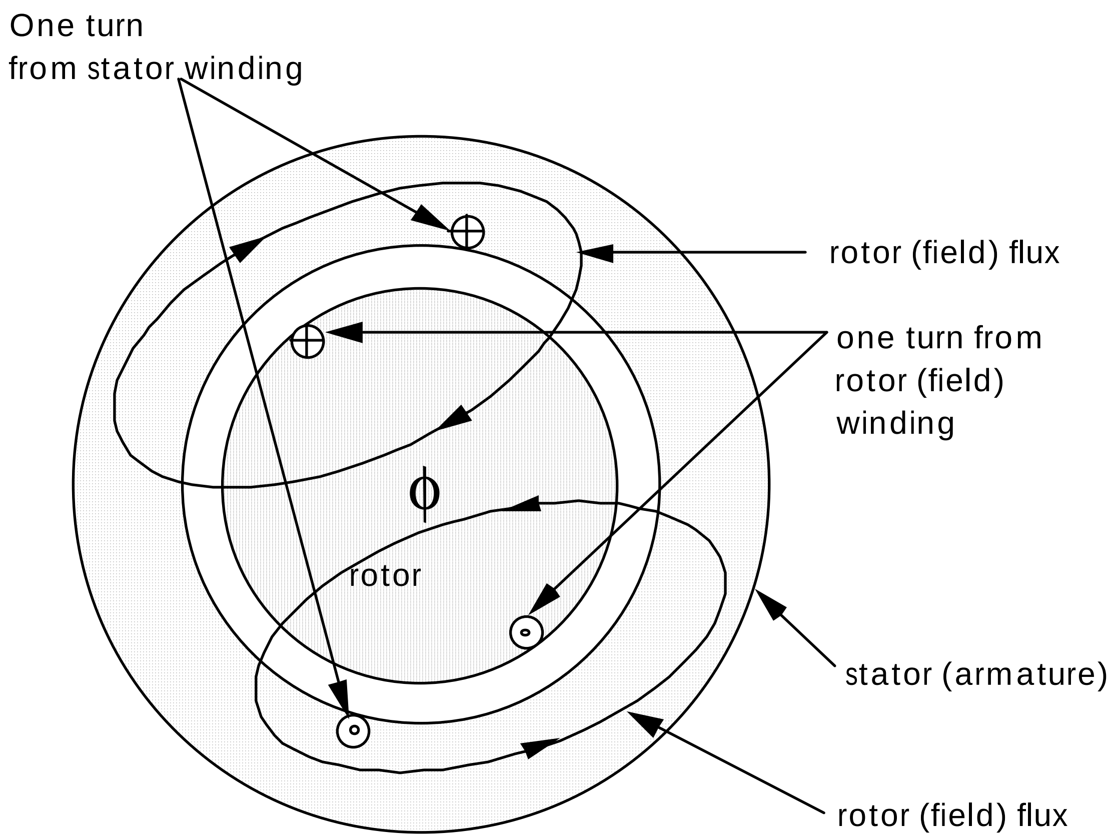


Figure 4.1 Cross section through a two pole AC machine

Rotors with more than one pole pair are common and a two pole pair rotor (4 poles) is shown in **Figure 4.2**.

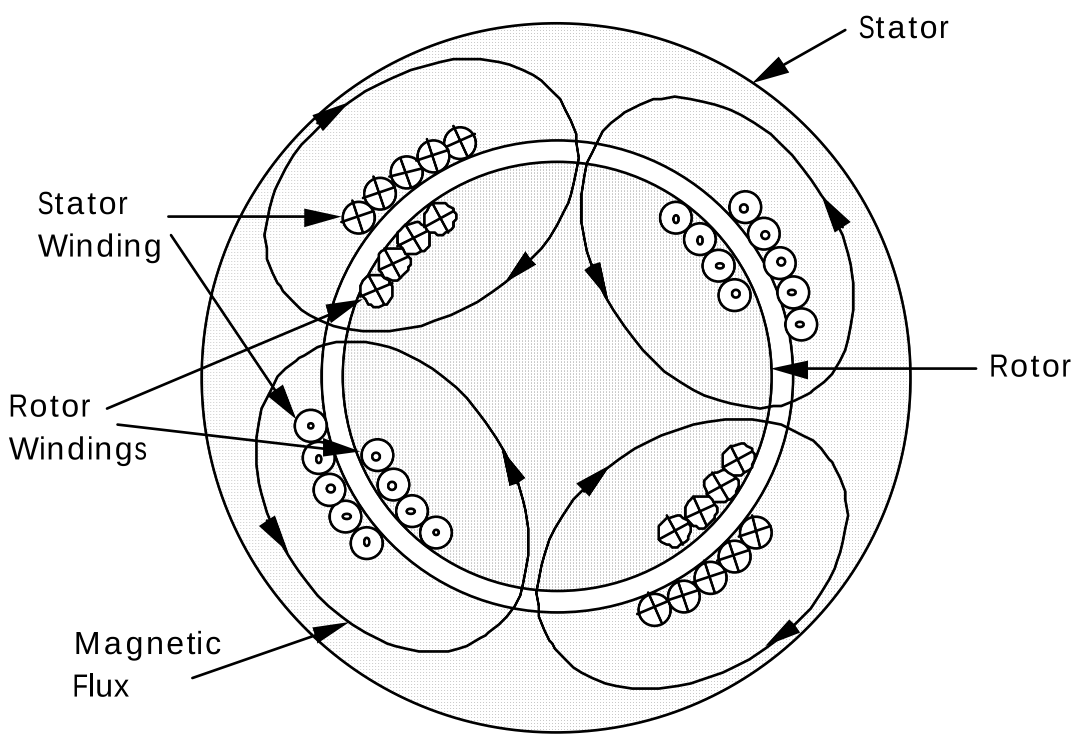
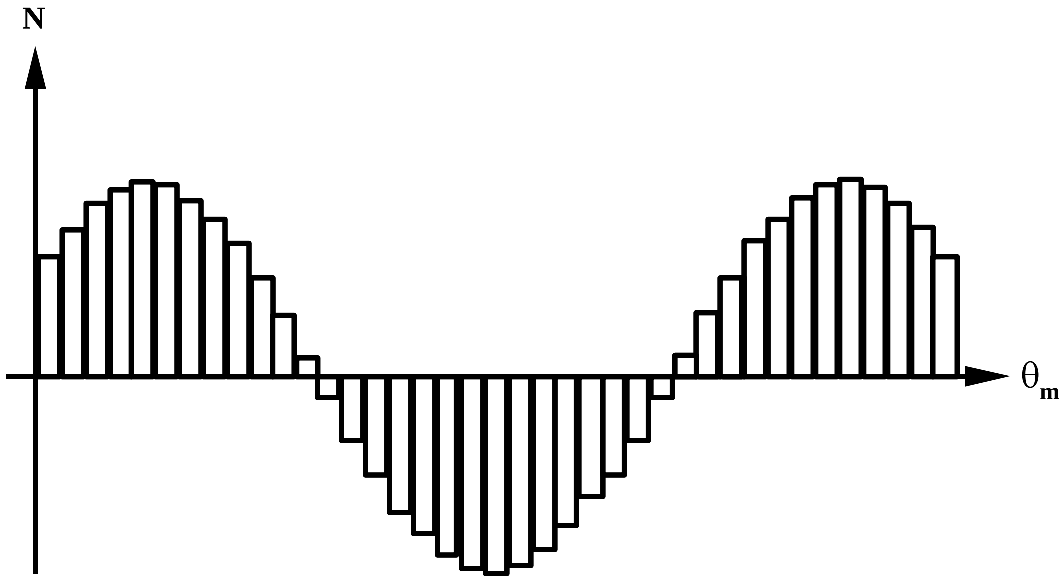
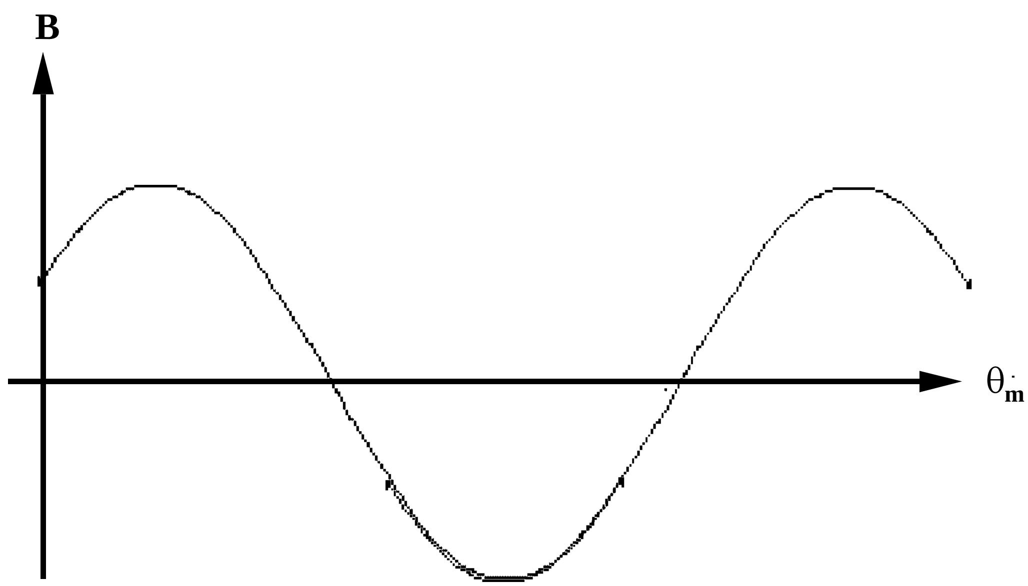


Figure 4.2 Cross section of a 4 pole AC Machine

In both cases the rotor windings are sinusoidally distributed. The number of conductors in each slot varies approximately as a sinusoidal function of the angle θ_m, where θ_m is the angle of rotation along the rotor as shown in **Figure 4.3a**. The radial flux density B can also be plotted as a function of θ_m as shown in **Figure 4.3b**.



a) Sinusoidal distribution of turns, N , in each slot position



b) Resulting radial flux density

Figure 4.3 Sinusoidal turns distribution and resulting radial flux density

The resultant flux density distribution (neglegting saturation effects) can be described as:

$$B = \hat{B}\sin(\frac{p}{2}\theta_m)$$

Where p is the number of poles. It is convenient to define an electrical angle θ_e, where;

$$\theta_e = \frac{p}{2}\theta_m - \delta$$

Thus:

$$B = \hat{B}\sin(\frac{p}{2}\theta_m) = \hat{B}\sin(\theta_e + \delta)$$

The flux enclosed by each turn in the stator can be determined from;

$$\Phi_S = \int B\partial A$$

Where A is the circumferential area enclosed by the turn as shown in **Figure 4.4**.

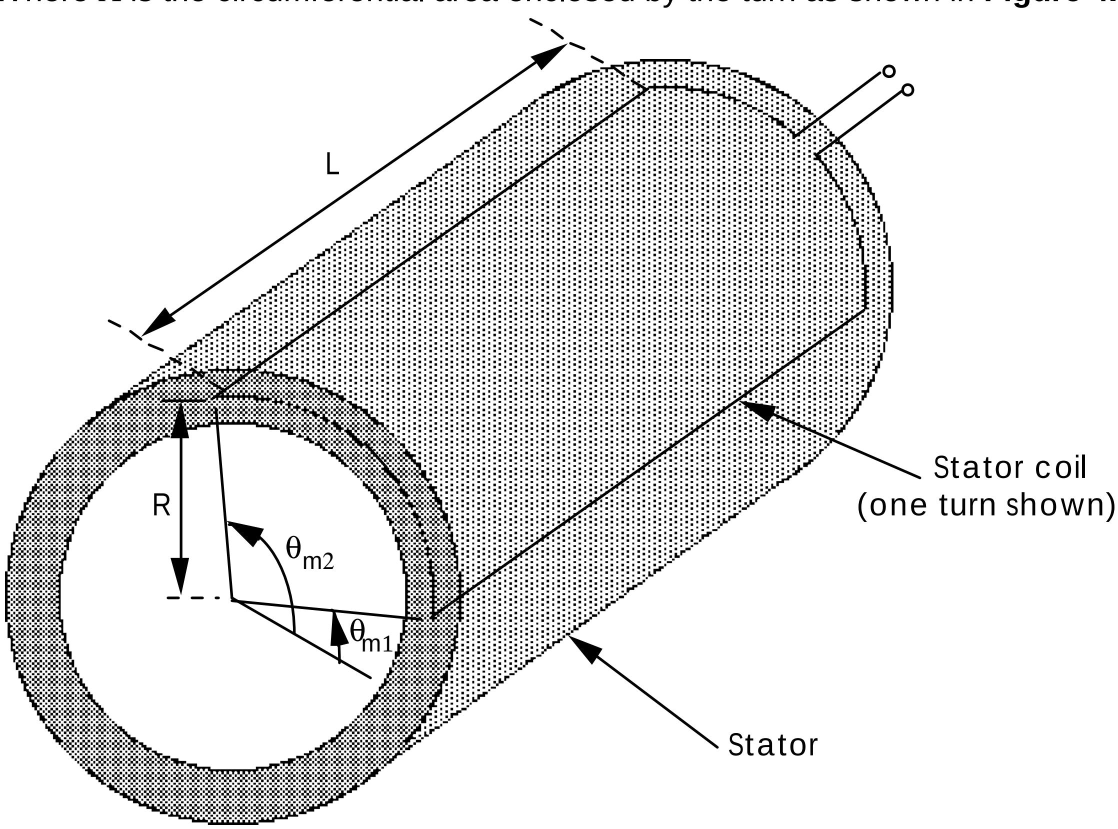


Figure 4.4 Orientation of a single turn of the stator winding (Rotor has been omitted for clarity)

Thus;

$$A = LR(\theta_{m1} - \theta_{m2})$$

And:

$$\partial A = LR\partial\theta_m$$

Substitute for ∂A into the expression for Φ_S and obtain;

$$\Phi_S = \int B\partial A = \int_{\theta_{m1}}^{\theta_{m2}} BLR\partial\theta_m = \int_{\theta_{m1}}^{\theta_{m2}} LR\hat{B}\sin(\theta_e + \partial)\partial\theta_m$$
$$= LR\hat{B}\int\sin(\theta_e + \delta)\partial\theta_m$$

All the stator turns are connected in series in each winding. All will have the same length L and radius R. Therefore the voltage induced in each stator winding, $e_S(t)$, can be determined from:

$$e_S(t) = \frac{\partial\lambda}{\partial t} = N_s\frac{\partial\Phi}{\partial t} = N_s\frac{\partial\Phi}{\partial\theta_m}\frac{\partial\theta_m}{\partial t}$$

Where N_S is the number of turns in each stator phase winding.

Assume that the rotor is rotating at a shaft speed of ω_m such that:

$$\theta_m = \omega_m t$$

And

$$\theta_e = \frac{p}{2}\theta_m - \delta = \frac{p}{2}\omega_m t - \delta$$

and substitute for :

$$\frac{\partial\Phi}{\partial\theta_m} = LR\hat{B}\sin(\theta_e + \delta) = LR\hat{B}\sin(\frac{p}{2}\omega_m t + \delta)$$

and

$$\frac{\partial\theta_m}{\partial t} = \omega_m$$

To obtain:

$$e_S(t) = \omega_m N_S LR\hat{B}\sin(\frac{p}{2}\omega_m t + \delta)$$

If the effects of saturation are neglected, then the peak flux density, $\hat{B}$, will be proportional to the field current, I_f;

$$\hat{B} = K_f I_f$$

Substitute for $\hat{B}$ into the expression for $e_S(t)$ to obtain:

$$e_S(t) = \omega_m N_S \mathsf{LR}\, K_f I_f \sin(\frac{p}{2}\omega_m t + \delta)$$

Note that the above expression for e_S is:

a) sinusoidal
b) proportional in amplitude to rotor speed ω_m and field current I_f
c) proportional in frequency to rotor speed ω_m and number of poles p

For AC machines we define ω as the electrical supply frequency;

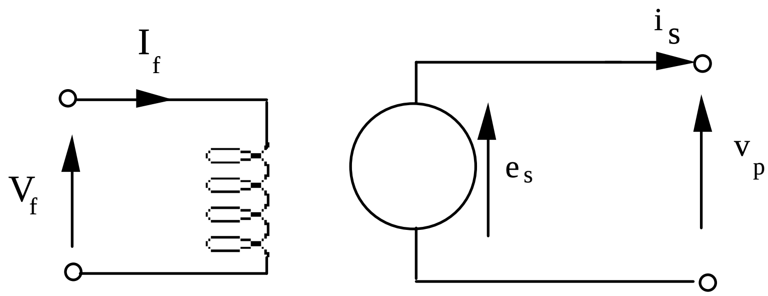
$$\omega = \omega_e = \frac{\partial\theta_e}{\partial t} = \frac{p}{2}\frac{\partial\theta_m}{\partial t} = \frac{p}{2}\omega_m$$

Make the above substitutions into the expression for $e_S(t)$ and obtain;

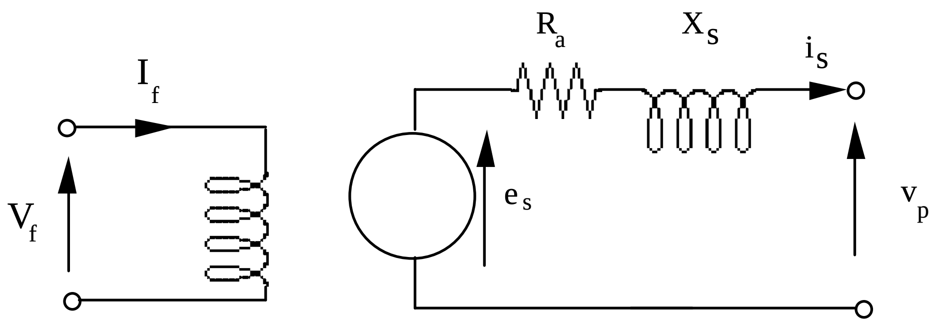
$$e_S(t) = \frac{2}{p}\omega N_S \mathsf{LR}\, K_f I_f \sin(\omega t + \delta) = K_S I_f \sin(\omega t + \delta)$$
$$= \hat{E}_S \sin(\omega t + \delta)$$

This is the expression for the voltage generated by each stator winding of a synchronous machine. For a three phase synchronous machine three such windings will be arranged within the stator at 120° displacement from each other to produce a three phase output voltage.

The per phase equivalent circuit for the above expression is shown in **Figure 4.5**. The basic circuit shown in **Figure 4.5a** does not include stator winding resistance and leakage inductance. The modified circuit shown in **Figure 4.5b** includes stator winding resistance and leakage reactance.



a) per phase equivalent circuit for a synchronous machine



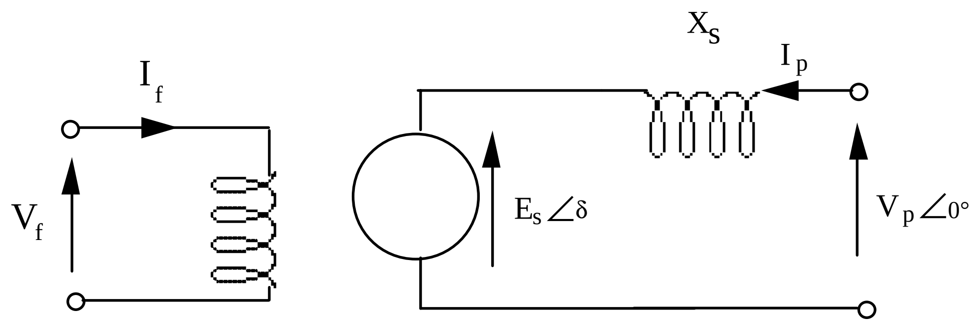
b) equivalent circuit including leakage reactance and winding resistance

Figure 4.5 Equivalent circuits for a synchronous machine

It is important to note that for synchronous machines the electrical frequency $\omega = \omega_e$ is constant and it is more convenient to refer to leakage reactance rather than leakage inductance. Also for most synchronous machine applications the leakage reactance is referred to as the synchronous reactance and it is much larger than the winding resistance and thus as a first approximation it is usually sufficient to ignore the winding resistance.

Example 4.1
A 3 Phase, 11kV, 60 Hz, 72 pole, Y connected synchronous motor has a synchronous reactance of 5Ω/phase.
a) Draw the per phase equivalent circuit.
b) Determine the shaft rpm
c) Determine the synchronous voltage, (back EMF) when it draws 1050A at 11 kV and 0.9 lagging PF.

Solution:
a) Per phase equivalent circuit:



b) Shaft rpm:

$$\omega_m = \frac{2}{p}\omega = \frac{2}{72} \times 60 \times 60 = 100 \text{ rpm}$$

c) Synchronous voltage:
For a Y connected system:

$$V_p = \frac{V_{AB}}{\sqrt{3}} = \frac{11\text{kV}}{\sqrt{3}} = 6.35 \text{ kV}$$

$$I_p = I_A = 1{,}050 \text{ A}$$

$$= 1{,}050\angle^{\cos^{-1} 0.9} = 945 - j458$$

$$E_S = V_p - jI_pX_S = 6350 - [945 - j458] \times [j5.0]$$

$$= 4060 - j4725 = 6{,}229\angle 49.3° \text{ per phase}$$

4.1.2 Power and Torque in a Synchronous Machine

For a balanced three phase system the expression for electrical power, P_e, converted to mechanical power within the machine is given by:

$$P_e = \sum_{1}^{3} \mathrm{Real}\left[e_s i_s^*\right]$$
$$= 3\mathrm{Real}\left[e_s i_s^*\right]$$

Where i_S^* represents the complex conjugate of i_S.
The above expression for P_e can be further elaborated by determining i_S from the equivalent circuit of **Figure 4.5b**. Assume that the per phase voltage at the machine terminals, v_p is;

$$v_p = V_p \angle 0 = V_p$$

and

$$e_S = E_s \angle \delta = E_s\cos(\delta) + jE_S\sin(\delta)$$

Therefore, from the analysis of the circuit of **Figure 4.5b**;

$$i_S = \frac{e_S - v_p}{R_a + jX_S}$$

In most cases however;

$$R_a << jX_S$$

Therefore;

$$i_S = \frac{e_S - v_p}{jX_S} = \frac{1}{X_S}\left[jv_p - je_S\right] = \frac{1}{X_S}\left[jv_p - jE_S\cos(\delta) + E_S\sin(\delta)\right]$$

Substitute for i_S and e_S into the equation for P_e:

$$P_e = 3\mathrm{Real}\left[e_s i_s^*\right]$$
$$= \frac{3}{X_S}\left[E_S\cos(\delta) + jE_S\sin(\delta)\right]\left[E_S\sin(\delta) + jE_S\cos(\delta) - jV_p\right]$$
$$= 3\frac{E_S V_p}{X_S}\sin(\delta) = \hat{P}_e\sin(\delta)$$

Therefore;

$$P_e = \hat{P}_e \sin(\delta)$$

Also the torque generated by the machine from the above electrical power can be determined;

$$T_e = \frac{P_e}{\omega_m} = 3\frac{E_S V_p}{\omega_m X_S}\sin(\delta) = \hat{T}_e \sin(\delta)$$

In analyzing the above equations for P_e and T_e it is important to note the following:

a) The power and torque converted by the machine is dependent on the angle δ which is the angle between the terminal voltage at the machine, V_p, and the internally generated voltage, E_S.

b) Because δ is dependent on phase angles, the power and torque converted by the machine is dependent on load power factor.

c) The power and torque converted by the machine is also dependent on the synchronous reactance, X_S.

d) ω_m is the speed of the rotor, which is often referred to as the shaft speed, in radians per second.

e) The above equations are equally valid for all polarities of voltage, current, torque, power etc. and thus these equations are valid for both synchronous generators and synchronous motors.

f) The machine will be generating for positive values of δ and motoring for negative values of δ.

g) The relationship between torque/power and δ is shown graphically in **Figure 4.6**. It is important to note that for $|\delta| > 90°$ the machine will be unstable and will 'pull out' of synchronization. It will no longer operate as a synchronous machine.

g) The peak attainable torque, $\hat{T}_e$ and power, $\hat{P}_e$ are referred to as the pullout torque and the pullout power, respectively.

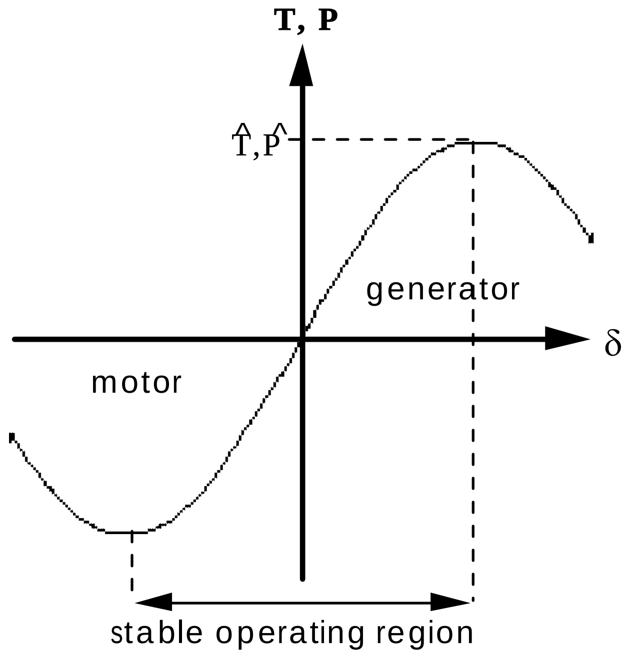


Figure 4.6 Relationship between torque, power and δ for a synchronous machine

Also it is important to note that in a synchronous machine (as in all AC machines) the electrical power is transmitted through the stator windings whereas in DC machines the electrical power is transmitted through the rotor windings. Therefore in AC machines the stator dissipates much more internal heat than the rotor. This is the opposite of the situation in a DC machine where the rotor dissipates much more internal heat than the stator. Since it is much easier to water cool a stationary part than a rotating part it is thus common to find water cooled AC machines, especially at high power levels. Water cooled DC machines are virtually non-existent.

Example 4.2

1. A synchronous motor has the following characteristics:

   6 pole, Y connected,

   Open circuit voltage: 440 V, with $I_f = 12$ A

   Full load voltage: 440 V, with $I_f = 15$ A

   Full load is 100 A at unity power factor.

a) Draw the per phase equivalent circuit for this machine

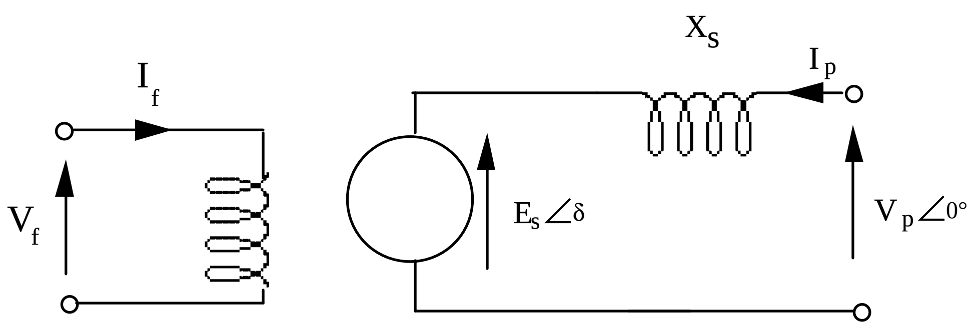
b) Determine the machine parameters: K_S, X_S, and ω_m in rpm.

c) What is the maximum power available from this machine assuming maximum I_f is 24 Amps, V_{AB} is 440 V and there are no losses in the machine.

d) What is the maximum torque in part c).

Solution:

a) Per phase equivalent circuit for a synchronous motor:



b) Determine K_S:

For a Y connected machine:

$$I_p = I_A \quad \text{and} \quad V_p = \frac{V_{AB}}{\sqrt{3}}$$

Under open circuit conditions:

$$E_S = V_p = \frac{V_{AB}}{\sqrt{3}} = K_S I_f$$

Solve for:

$$K_S = \frac{V_{AB}}{\sqrt{3} I_f} = \frac{440}{\sqrt{3}\times 12} = 21.2\ \Omega$$

Determine X_S:

$$I_p = \frac{V_p - E_S}{jX_S} = I_A$$

Solve for:

$$jX_S = \frac{V_p - E_S}{I_A}$$

Substitute for:

$$E_S = K_S I_f \angle\delta = K_S I_f \cos(\delta) + jK_S I_f \sin(\delta)$$

To obtain:

$$jX_S = \frac{V_p - K_S I_f \cos(\delta) - jK_S I_f \sin(\delta)}{I_A} = \frac{V_p - K_S I_f \cos(\delta)}{I_A} - \frac{jK_S I_f \sin(\delta)}{I_A}$$

For real values of X_S the real parts of the above equation must be zero, therefore:

$$V_p - K_S I_f \cos(\delta) = 0$$

Solve for:

$$\delta = \cos^{-1}\left[\frac{V_p}{K_S I_f}\right] = \cos^{-1}\left[\frac{V_{AB}}{\sqrt{3} K_S I_f}\right] = \cos^{-1}\left[\frac{440}{\sqrt{3}\times 21.2\times 15}\right] = -36.9°$$

(Note that δ is negative because the machine is motoring).
The preceding equation for X_S simplifies to:

$$X_S = -\frac{K_S I_f \sin(\delta)}{I_A} = \frac{21.2\times 15\times \sin(-36.9°)}{100} = 1.91\ \Omega$$

Determine ω_m in rpm:
For a synchronous machine:

$$\omega_m = \omega_S = \frac{2}{p}\omega = \frac{2}{6}\times 60\times 60 = 1200 \text{ rpm}$$

c) Maximum power:

$$\hat{P}_e = 3\frac{E_S V_p}{X_S} = 3\frac{K_S I_f V_{AB}}{X_S\sqrt{3}} = \frac{\sqrt{3}\times 21.2\times 24\times 440}{1.91} = 203 \text{ kW}$$

d) Maximum torque:

$$\hat{T}_e = \frac{\hat{P}_e}{\omega_m} = \frac{\hat{P}_e}{\frac{2}{p}\omega} = \frac{203\times 10^{-3}}{\frac{2}{6}\times 60\times 2\pi} = 1{,}615 \text{ n-m}$$

4.1.3 Synchronous Generator

The basic per phase equivalent circuit for a synchronous generator is shown in **Figure 4.5**. By convention, the stator current is shown as going out of the machine when it is in the generating mode.

Synchronous generators (sometimes called alternators) are the most commonly used machines for generating AC and DC* power. Some examples are:

a) Commercial power grids such as Ontario Hydro and Quebec Hydro
b) Automotive "alternators"*
c) Diesel-electric locomotives*
d) Standby emergency generators
e) Special AC power such as 400 Hz and 1000 Hz for aircraft

*In these cases DC power is derived from an AC generator (or alternator) producing AC which is then rectified by diodes to produce DC.

The relevant equations for a synchronous generator are:

Electrical output power, P_{out} is:

$$P_{out} = 3V_p I_p \cos(\theta) = \sqrt{3} V_{AB} I_A \cos(\theta)$$

Where θ is the power factor angle, or the phase angle between V_p and I_p.

Net mechanical power converted by the generator, P_e is

$$P_e = \hat{P}_e \sin(\delta)$$

Where;

$$\hat{P}_e = 3\frac{E_S V_p}{X_S}$$

And

$$E_S = K_S I_f$$

And δ is the angle between E_S and V_p and will vary depending on loading and on the field current I_f.

For a generator the total mechanical input power into the generator, P_m, will also have to include the power required to overcome friction;

$$P_m = P_e + P_f$$

Where P_f is the power required to overcome friction.

Similarly the net mechanical torque converted by the generator, T_e is;

$$T_e = \frac{P_e}{\omega_m} = \hat{T}_e \sin(\delta)$$

Where;

$$\hat{T}_e = 3\frac{E_S V_p}{\omega_m X_S}\sin(\delta)$$

And ω_m is the shaft speed, where;

$$\omega_m = \frac{2}{p}\omega$$

And ω is the electrical supply frequency.
Also for a generator the total mechanical input torque into the generator, T_m, will also have to include the torque required to overcome friction;

$$T_m = T_e + T_f$$

Where T_f is the friction torque.
A synchronous generator will always operate at synchronous speed, ω_s, where;

$$\omega_m = \omega_s = \frac{2}{p}\omega$$

As long as the electrical output power, P_{out} does not exceed the pullout power, $\hat{P}_e$.

For high power generators, (>10MW) major considerations are, low maintenance and high efficiency. These are achieved by using a high number of poles, typically 72 poles. This results in a low mechanical shaft speed;

$$\omega_m = \frac{2}{p}\omega = \frac{2}{72} \times 60 \times 60 \quad \text{rpm}$$
$$= 100 \text{ rpm}$$

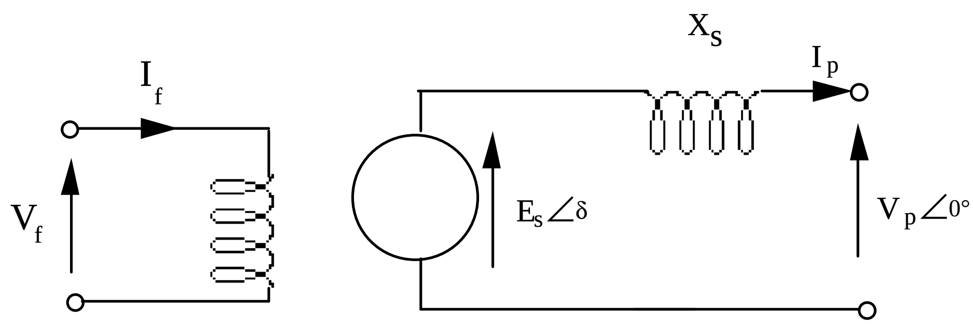
This reduces mechanical bearing wear and friction and windage losses. However, it requires high torque, (because torque is inversely proportional to shaft speed), which results in large shaft and a large, heavy generator.

Example 4.3
A delta connected synchronous generator has the following ratings:
135 MVA, 13.8 KV, 60 Hz, 3Φ
The following test data is available:

Field excitation: 485A
Open circuit voltage: 13.8 KV
Short circuit current: 2000 A

Assume the machine is linear, doesn't saturate, and all power losses are negligible.
a) Draw the equivalent circuit and determine the synchronous reactance, X_S, and field constant K_S.
b) Determine the field excitation required to deliver rated voltage and current to a load of 0.8 PF lagging.
c) If the field current is kept constant at the value in part b) determine the regulation.

Solution:



a) Per phase equivalent circuit, if there are no losses then $R_a = 0$
For a delta connected machine, then:

$$V_p = V_{AB} = E_S\angle^{\delta} - jX_sI_p = K_SI_f\angle^{\delta} - jX_sI_p$$

Solve for:

$$K_S = \frac{V_{AB} + jX_sI_p}{I_f} = \frac{13800 + jX_s \times 0}{485} = 28.45\ \Omega$$

For a delta connected machine:

$$I_p = \frac{I_A}{\sqrt{3}}$$

$$X_S = \frac{E_s}{I_p} = \frac{\sqrt{3}E_s}{I_A} = \frac{\sqrt{3} \times 13800}{2000} = 12.0\ \Omega\ \text{/phase}$$

b) Field excitation required for A rated voltage, current at 0.8 PF lagging:

$$\text{rated } I_p = \frac{VA}{3V_p} = \frac{VA}{3V_{AB}} = \frac{135\times10^6}{3\times13800} = 3{,}260\text{ A at 0.8 PF lagging}$$

$$= 3{,}260\angle{-36.7°} = 2608 - j1956\text{ A}$$

$$E_s\angle^{\delta} = V_p + jX_sI_p = 13800 + j\,12\times3260\angle^{-36.7°} = 48{,}670\angle^{40°}\text{ V}$$

Also

$$|E_s| = K_S I_f$$

Solve for:

$$I_f = \frac{|E_s|}{K_s} = \frac{48670}{28.45} = 1{,}711\text{ A}$$

c) Regulation:

$$VR = \frac{V_{oc} - V_L}{V_L} = \frac{48670 - 13800}{13800} = 253\%$$

This is a very lousy regulation and would be unacceptable for most applications. To make synchronous generators acceptable they usually require a feedback loop that controls the field excitation in order to keep the output voltage constant.

4.1.4 Synchronous Motor

The basic per phase equivalent circuit for a synchronous motor is shown in **Figure 4.7**. By convention, the stator current is shown as going into the machine when it is in the motoring mode.

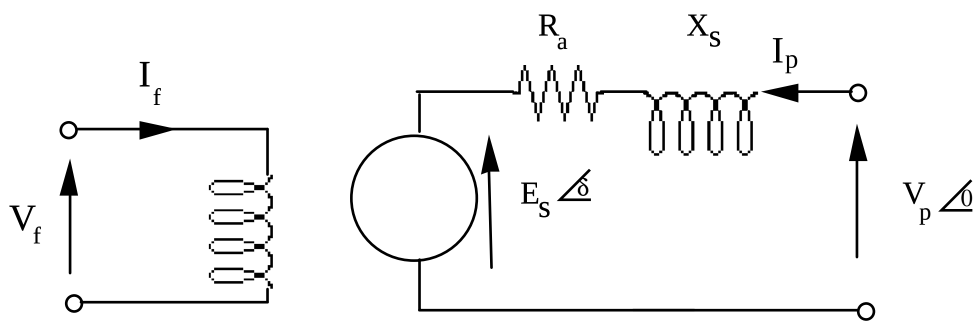


Figure 4.7 Equivalent Circuit for a Synchronous Motor

The relevant equations for a synchronous motor are:

Electrical input power, P_{in} is

$$P_{in} = 3V_p I_p \cos(\theta) = \sqrt{3}\, V_{AB} I_A \cos(\theta)$$

Where θ is the power factor angle, or the phase angle between V_p and I_p .

The total mechanical power produced by the motor, P_e is

$$P_e = \hat{P}_e \sin(\delta)$$

Where;

$$\hat{P}_e = 3\frac{E_s V_p}{X_s}$$

And

$$|E_s| = K_s I_f \quad \text{(assuming no saturation effects)}$$

And δ is the angle between E_s and V_p and will vary depending on loading.

For a motor the net mechanical power available from the motor, P_m , will be reduced by the power required to overcome friction;

$$P_m = P_e - P_f$$

Where P_f is the power required to overcome friction.

Similarly the total mechanical torque produced by the motor, T_e is

$$T_e = \frac{P_e}{\omega_m} = \hat{T}_e \sin(\delta)$$

Where;

$$\hat{T}_e = 3\frac{E_s V_p}{\omega_m X_s} \sin(\delta)$$

And ω_m is the shaft speed, where;

$$\omega_m = \frac{2}{p}\omega$$

And ω is the electrical supply frequency.
Also for a motor the net mechanical torque available from the motor, T_m, will be reduced by the torque required to overcome friction;

$$T_m = T_e - T_f$$

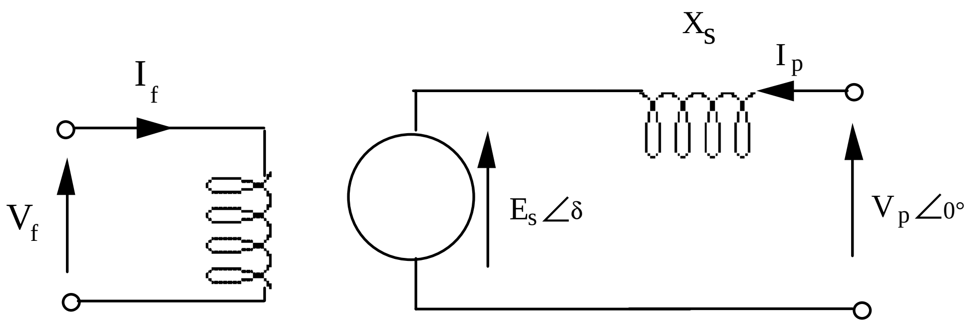
Where T_f is the friction torque.
A synchronous motor will always operate at synchronous speed, ω_s, where;

$$\omega_m = \omega_s = \frac{2}{p}\omega$$

As long as the total mechanical torque, $T_m + T_f$, does not exceed the pullout torque, $\hat{T}$

Example 4.4
A synchronous, Y connected, 6 pole, motor has a synchronous reactance of 2.5 Ω and draws 100 A/line at unity power factor and rated voltage of 460 V line to line. Assuming constant line voltage, determine;
a) The maximum power and torque available from this machine assuming constant field excitation.
b) The line current and power factor if the machine field excitation is increased by 25%, assuming load power is constant.
c) The line current and power factor if the machine field excitation is decreased by 25%, assuming load power is constant.

Solution:



Per phase quivalent circuit for a synchronous motor.
a) Maximum power and torque:
Maximum power:

$$\hat{P}_e = 3\frac{E_s V_p}{X_s}$$

$$E_s = V_p - jX_s I_p$$

And for a Y connected machine:

$$V_p = \frac{V_{AB}}{\sqrt{3}}$$

Therefore:

$$E_s = \frac{V_{AB}}{\sqrt{3}} - jX_s I_p = \frac{460}{\sqrt{3}} - j2.5 \times 100 = 266 - j250 = 365\angle{-43.2°}$$

Substitute into:

$$\hat{P}_e = 3\frac{E_s V_p}{X_s} = \sqrt{3}\frac{E_s V_{AB}}{X_s} = \sqrt{3}\frac{365 \times 450}{2.5} = 116 \text{ kW}$$

Maximum torque:

$$\hat{T}_e = \frac{\hat{P}_e}{\omega_m}$$

For a 6 pole synchronous machine:

$$\omega_m = \omega_s = \frac{2}{p}\omega = \frac{2}{6} \times 60 \times 2\pi = 126 \text{ r/s}$$

Substitute to obtain:

$$\hat{T}_e = \frac{\hat{P}_e}{\omega_m} = \frac{116\times10^3}{126} = 921 \text{ n-m}$$

b) The line current and power factor if the machine field excitation is increased by 25%:
For a Y connected machine:

$$I_A = I_p = \frac{V_p - E_s\angle^{\delta}}{jX_s}$$

However E_s has changed in amplitude and phase.

$$|E_s| = K_s I_f$$

Therefore:

$$|E_{s2}| = |E_{s1}|\frac{I_{f2}}{I_{f1}} = 365 \times 1.25 = 456$$

Also since:

$$P_e = 3\frac{E_s V_p}{X_s}\sin\delta$$

and P_e, V_p, X_s are constant then:

$$\sin\delta_2 = \sin\delta_1\frac{E_{s1}}{E_{s2}} = \sin\delta_1\frac{I_{f1}}{I_{f2}} = \sin(-43.2°)\frac{1}{1.25} = -0.547$$

Therefore:

$$\delta_2 = -33.2°$$

Therefore:

$$E_s\angle\delta = 456\angle^{-33.2°}$$

And:
j

$$I_p = \frac{V_p - E_s\angle^{\delta}}{jX_s} = \frac{266 - 456\angle^{-33.2°}}{j2.5} = 100 + j46 = 110\angle^{+24.7°}$$

Therefore:

PF = cos(+24.7°) = 0.91 leading

c) The line current and power factor if the machine field excitation is decreased by 25%, assuming load is constant:

$$|E_{s3}| = |E_{s1}|\frac{I_{f3}}{I_{f1}} = 365 \times 0.75 = 274$$

$$\sin\delta_3 = \sin\delta_1\frac{I_{f1}}{I_{f3}} = \sin(-43.2°)\frac{1}{0.75} = -0.913$$

Therefore:

$$\delta_3 = -65.9°$$

Therefore:

$$E_s \angle\delta = 274\angle -65.9°$$

And:

$$I_p = \frac{V_p - E_s\angle\delta}{jX_s} = \frac{266 - 274\angle -65.9°}{j2.5}$$

$$= 100 - j61.6 = 117.5\angle -31.6°$$

Therefore:

PF = cos(-31.6°) = 0.851 lagging

4.1.5 Synchronous Reactor

The reactive power, Q, drawn by a synchronous motor can be either lagging (inductive) or leading (capacitive) depending on the field excitation. Where Q is determined from, S, the total complex VA drawn by the machine, as follows:

$$S = P + jQ = 3v_p i_s^*$$

Where:

$$v_p = V_p \angle 0°$$

$$i_s = \frac{v_p - e_s}{jX_s} \quad \text{and thus} \quad i_s^* = \left[\frac{v_p - e_s}{jX_s}\right]^*$$

And

$$e_s = E_s \angle \delta$$

Substitute for v_p and e_s into the equation for i_s^* to obtain:

$$i_s^* = \frac{1}{X_s}\left[-E_s \sin(\delta) + j\left(V_p - E_s \cos(\delta)\right)\right]$$

Substitute for i_s^* into the equation for S to obtain;

$$P + jQ = 3v_p i_s^* = \frac{3V_p}{X_s}\left[-E_s \sin(\delta) + j\left(V_p - E_s \cos(\delta)\right)\right]$$

And thus:

$$Q = \frac{3V_p}{X_s}\left[V_p - E_s \cos(\delta)\right]$$

Therefore, a synchronous machine can have a positive or negative reactance, depending on the relative values of E_s cos(δ) and V_p. Of course, E_s, is dependent on the excitation level of the machine, where:

$$|E_s| = K_s I_f$$

And thus:

$$Q = \frac{3V_p}{X_s}\left[V_p - K_s I_f \cos(\delta)\right]$$

For an overexcited machine, with a high field current such that;

$$I_f > \frac{V_p}{K_s \cos(\delta)}$$

Then,

$$E_s \cos(\delta) > V_p$$

And,

$$Q < 0 \text{ (capacitive)}$$

Similarly, for an underexcited machine, with a low field current such that,

$$I_f < \frac{V_p}{K_s \cos(\delta)}$$

Then,

$$E_s \sin(\delta) < V_p$$

And,

$$Q > 0 \text{ (inductive)}$$

Furthermore, if the field excitation is set exactly such that;

$$I_f = \frac{V_p}{K_s \cos(\delta)}$$

Then,

$$E_s \cos(\delta) = V_p$$

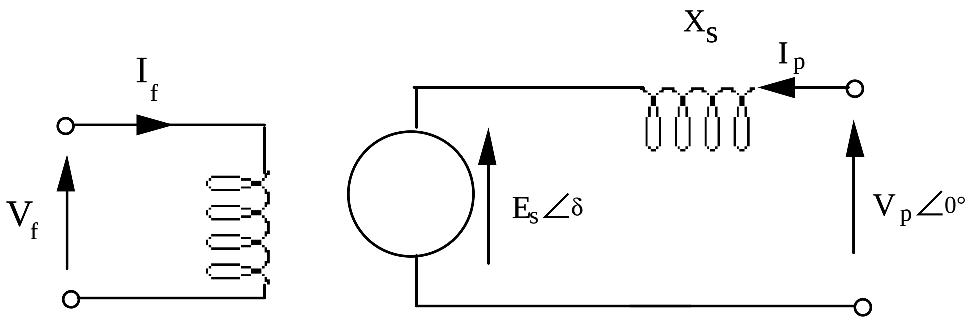
And,

$$Q = 0 \text{ (resistive)}$$

Thus a synchronous motor can be a capacitive or inductive or resistive load depending on field excitation. This makes synchronous motors very attractive for large installations that require unity or capacitive power factor. In some cases utilities will charge customers more per kW for reactive power than for unity power and thus a customer can actually save money on their electricity charges by running their synchronous motors at a high excitation level. Some utilities even use synchronous motors as "synchronous reactors" solely to correct for the inductive power factor that most transmission lines produce.

Example 4.5
A factory has an overall load of 2.4 MVA at 0.707 lagging PF. Determine the MVA rating of a synchronous motor that would bring the total plant to unity power factor under the following conditions:
a) Assuming the motor is used only for power factor correction.
b) Assuming the motor is used to drive a new load of 0.405 MW and the motor is 91% efficient.

Solution:



Per phase equivalent circuit for a synchronous motor.
a) Assuming the motor is used only for power factor correction:
Plant load is:

$$S_p = 2.4 \angle \cos^{-1} 0.707$$
$$= P_p + jQ_p = 1.697 + j1.697 \text{ MVA}$$

Therefore:

$$Q_p = +1.697 \text{ MVA (lagging)}$$

Therefore MVA rating of synchronous reactor is:

$$Q_m = -Q_p = -1.697 \text{ MVA (leading)}$$

b) Assuming the motor is used to drive a new load of 0.405 MW and the motor is 91% efficient:

$$S_m = \frac{P_m}{0.91} + jQ_m = \frac{0.405}{0.91} + jQ_m = 0.5 + jQ_m$$

Also Q_m is unchanged from part a):

$$Q_m = -1.697 \text{ MVA (leading)}$$

Therefore:

$$S_m = 0.5 - j1.697 = 1.769 \angle -73.6$$

$\angle 73.6°$ MVA

4.2 Induction Machines

4.2.1 Derivation of the Equivalent Circuit for an Induction Machine

An induction machine can be considered as a synchronous machine in which the field excitation, I_f , is zero. Thus:

$$E_s = K_s I_f = 0$$

And also the pullout torque, $\hat{T}_e$;

$$\hat{T}_e = 3\frac{E_s V_p}{\omega_m X_S} \sin(\delta) = 0$$

Therefore, the machine pullout torque will always be exceeded and that means the shaft and rotor will not rotate at synchronous speed.

There are several methods of producing zero field excitation but for the purposes of this analysis we shall assume that the rotor windings are shorted at the rotor terminals.

Assume that the shaft speed is ω_m . We can then visualize this machine as a transformer in which the stator is the primary and the rotor is the secondary. However, the secondary winding is rotating at a mechanical frequency of ω_m , which is equivalent to an electrical frequency of ω_e , where:

$$\omega_e = \frac{p}{2}\omega_m$$

Since there is no current in the rotor (secondary) windings, the flux in the machine will be produced by current in the stator (primary) windings. Assume there are N_s turns in the windings of each phase of the stator, (which are the primary windings of the transformer). Also assume that the applied voltage per phase is v_p , where;

$$v_p = \hat{V}_p \sin(\omega t)$$

And therefore;

$$\phi_s = \frac{1}{N_s}\int v_p \partial t = \frac{1}{N_s}\int \hat{V}_p \sin(\omega t)\, \partial t$$

$$= \frac{-\hat{V}_p}{\omega N_s}\cos(\omega t)$$

Assuming negligible leakage flux;

$$|\phi_r| = |\phi_s|$$

And thus;

$$|\phi_r| = \frac{-1}{\omega N_s} V_p$$

If the rotor, (which is the secondary of the transformer), is rotating at a speed of ω_m , then the frequency of the rotor (secondary) flux will be ω_r , where;

$$\omega_r = \omega - \frac{p}{2}\omega_m$$

And;

$$\phi_r = \frac{-1}{\omega N_s}\hat{V}_p \cos(\omega_r t)$$

Then the voltage induced in the rotor, (secondary) windings can be determined:

$$v_r(t) = N_r \frac{\partial \Phi_r}{\partial t} = \frac{\omega_r N_r}{\omega N_S}\hat{V}_p \sin(\omega_r t)$$

It is convenient at this point to refer to the synchronous speed ω_s , which is the speed at which the induction machine would rotate if it were a synchronous machine, i.e;

$$\omega_s = \frac{2}{p}\omega$$

Also because the induction machine does not rotate at synchronous speed, it is convenient to define the slip, s, which is a relative measure of how close the machine speed is to synchronous speed. The slip, s, is defined as:

$$s = \frac{(\omega_s - \omega_m)}{\omega_s} = \frac{(\frac{2}{P}\omega - \omega_m)}{\frac{2}{P}\omega}$$

$$= \frac{(\omega - \frac{P}{2}\omega_m)}{\omega} = \frac{\omega_r}{\omega}$$

Also define, N, as the turns ratio between the stator and rotor windings:

$$N = \frac{N_s}{N_r}$$

Substitute for N and s into the equation for $e_r(t)$ to obtain;

$$v_r(t) = \frac{s}{N} \hat{V}_p \sin(\omega_r t)$$

Assuming that the rotor can be represented by a winding resistance, R_r', and a leakage inductance, L_r', then the rotor current, I_r, can be determined;

$$I_r = \frac{v_r}{R_r' + j\omega_r L_r'} = \frac{sV_p}{N\left[R_r' + j\omega_r L_r'\right]}$$

$$= \frac{V_p}{N} \frac{1}{\left[R_r'/s + j\omega L_r'\right]}$$

The above equation represents an R L circuit operating at a frequency of ω but with a resistance divided by s. Thus the equivalent circuit for the rotor winding can be represented at the stator frequency by the same inductance and the resistance divided by s, as shown in **Figure 4.8**.

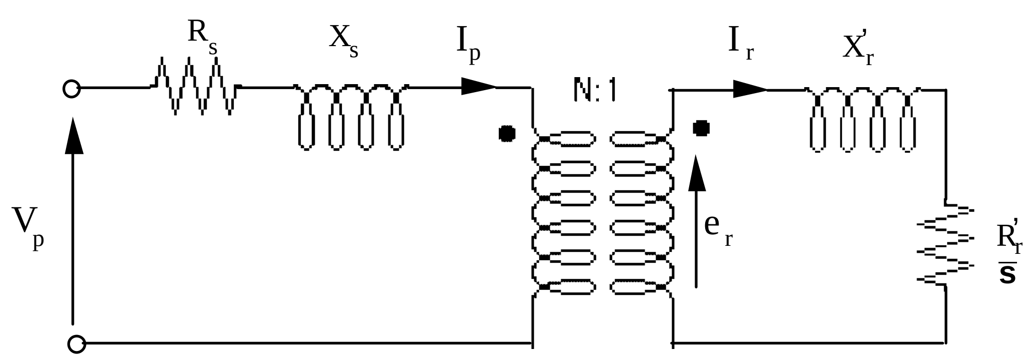


Figure 4.8 Equivalent Transformer Circuit for an Induction Machine

This circuit can now be reflected to the primary (stator) side of the transformer by applying the turns ratio, N. The resultant per phase equivalent circuit for an induction machine is shown in **Figure 4.9**.

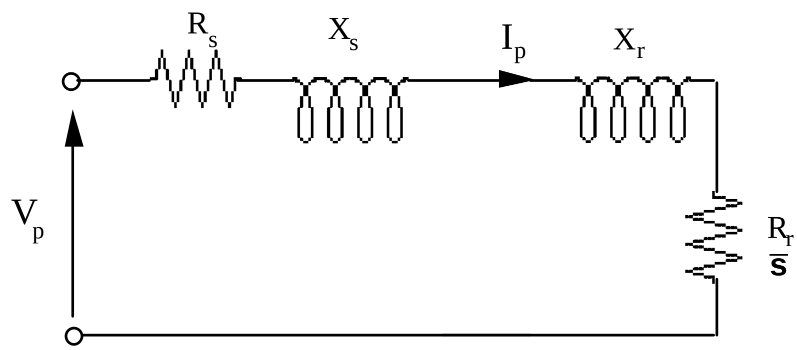
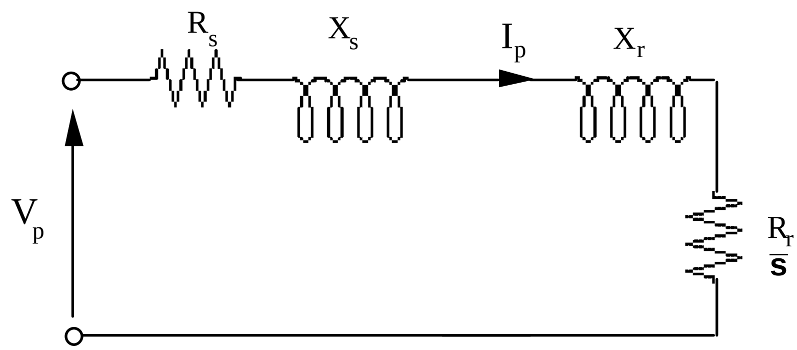


Figure 4.9 Simplified Equivalent Circuit for an Induction Machine with the rotor elements reflected to the stator side

Example 4.6
A 60 Hz induction motor operates at 1746 rpm under full load. Under no load the speed is 1791 rpm. Determine:
a) The slip at no load and at full load.
b) The speed regulation in %.

Solution:
Per phase equivalent circuit for an induction motor:



a) Slip at no load and at full load:

$$\omega = 60 \text{ Hz} = 3600 \text{ rpm}$$

$$\omega_s = \frac{2}{P}\,\omega \approx \omega_m = 1791 \text{ rpm}$$

Solve for:

$$P = \frac{2\omega}{\omega_s} \approx \frac{2\omega}{\omega_m} = \frac{2 \times 3600}{1791} = 4.02$$

Therefore

$$P = 4 \text{ and } \omega_s = 1800 \text{ rpm}$$

$$s = \frac{(\omega_s - \omega_m)}{\omega_s} = \frac{(1800 - 1791)}{1800} = 0.005 \text{ at no load}$$

$$= \frac{(1800 - 1746)}{1800} = 0.030 \text{ at full load}$$

b) Speed regulation:

$$SR = \frac{(\omega_{\text{no load}} - \omega_{\text{full load}})}{\omega_{\text{full load}}} = \frac{(1791 - 1746)}{1746} = 2.6\%$$

4.2.2 Power and Torque in an Induction Motor

The electrical power converted within an induction motor can be determined by analyzing the equivalent circuit of **Figure 4.9**.

The power dissipated in the apparent rotor resistance, $\frac{R_r}{s}$ is given by;

$$P_s = I_p^2 \frac{R_r}{s} \qquad \text{per phase}$$

Where, I_p is the per phase stator current, derived from the per phase equivalent circuit shown in **Figure 4.9;**

$$I_p = \frac{V_p}{R_s + \frac{R_r}{s} + j(X_s + X_r)}$$

The preceding equation represents the power that is dissipated in the rotor resistance as seen from the stator. However, on the rotor side, the actual power dissipated in the rotor winding resistance, R_r' is given by;

$$P_r = I_r^2 R_r' \qquad \text{per phase}$$

Substitute for;

$$I_r = NI_p$$

And

$$R_r' = \frac{R_r}{N^2}$$

Into the equation for P_r to obtain;

$$P_r = I_r^2 R_r \qquad \text{per phase}$$

In the preceding equations P_s represents the power dissipated in the rotor windings as seen from the stator side, whereas P_r represents the power actually dissipated in the rotor windings. The difference between the two is the electrical power "lost" in the machine, P_e. Where;

$$P_e = P_s - P_r \qquad \text{per phase}$$
$$= 3(P_s - P_r) \qquad \text{for a three phase machine}$$

$$= 3I_p^2\left[\frac{R_r}{s} - R_r\right]$$

$$= 3I_p^2\frac{R_r}{s}(1-s)$$

The power, P_e, represents the electrical power converted by the machine into mechanical power, and thus;

$$P_e = T_e\omega_m$$

Where T_e is the electrical torque produced by the machine and ω_m is the shaft speed in radians per second.
Thus;

$$T_e = \frac{P_e}{\omega_m} = 3I_p^2\frac{R_r}{s\omega_m}(1-s)$$

This expression can be plotted as a torque vs. slip curve as shown in **Figure 4.10**.

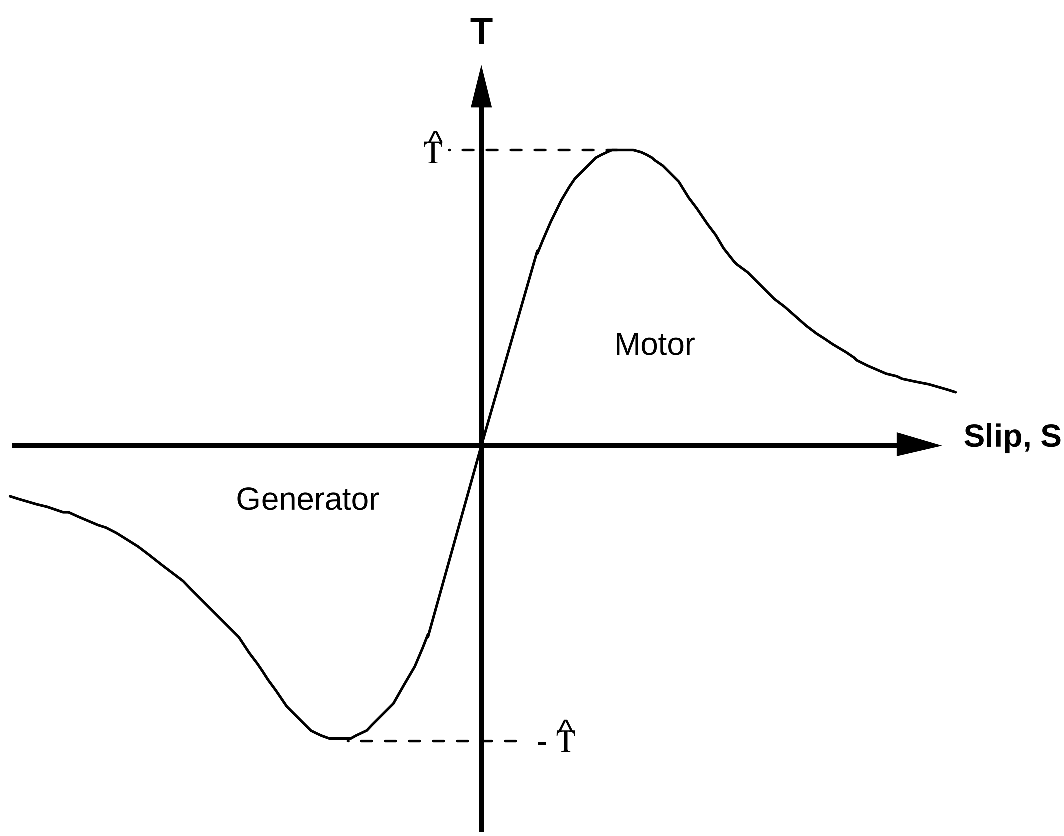


Figure 4.10 Torque vs. Slip curve for an Induction Machine

This expression can also be plotted as a torque vs speed curve as shown in **Figure 4.11**.

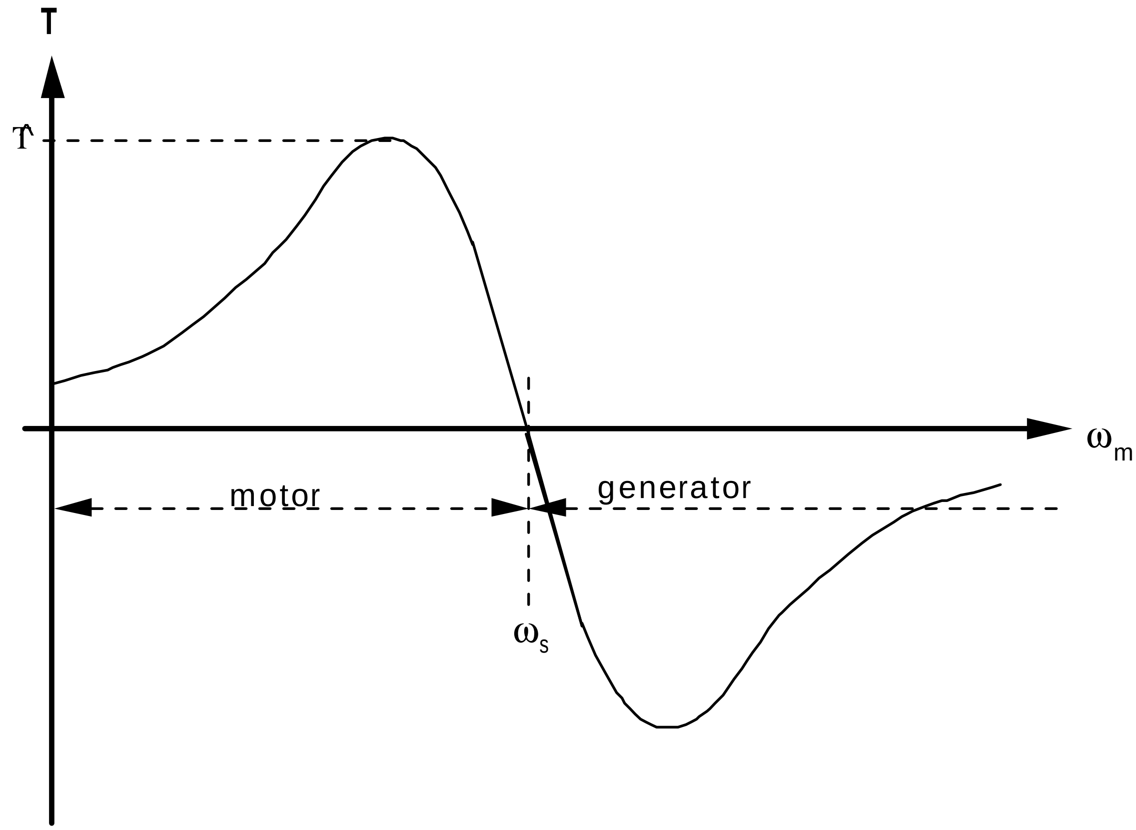


Figure 4.11 Torque vs. Speed Curve for an Induction Machine

Some important characteristics of this curve are;
a) There is a relatively low starting torque, which means that induction motors inherently take relatively long to 'get up to speed'
ω_m

b) There is a characteristic pullout torque, $\hat{T}$, which is the maximum torque the machine can deliver. If this torque is exceeded, even momentarily the machine will 'pull out' and start to decelerate.
c) The torque-speed curve has two regions;
- the region in which the torque increases with speed is unstable and the machine will only pass through this region transiently while accelerating towards the stable region or decelerating to a stop.
- the region in which the torque decreases with speed is the stable operating region.

d) The machine produces zero torque at synchronous speed ($\omega_m = \omega_s$)
e) The machine produces negative torque if the speed exceeds synchronous speed, ($\omega_m > \omega_s$), in which case the machine is acting as a generator.

In some applications an induction motor operates at low slip, i.e. small values of s, in which case the equations for power and torque simplify to;

$$P_e = 3I_p^2 \frac{R_r}{s}$$

$$T_e = 3I_p^2 \frac{R_r}{s\omega_m}$$

Furthermore, for very small values of s, the equivalent circuit model is dominated by the R_r /s term as shown in **Figure 4.12.**

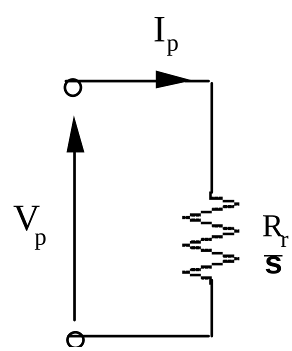


Figure 4.12 Simplified Equivalent Circuit for an Induction Motor for Very Low Values of Slip

In this case a further simplification can be made for I_p ;

$$I_p = V_p /(R_r /s) = sV_p /R_r$$

The equations for power and torque can thus be further simplified:

$$P_e = 3s(1\text{-}s)V_p^2 /R_r \approx 3sV_p^2 /R_r$$

$$T_e = 3s(1\text{-}s)V_p^2 /\omega_m R_r \approx 3sV_p^2 /\omega_s R_r$$

Example 4.7

A 2300V, 800 kW, 60 Hz, 16 pole induction motor has the following parameters, reflected to the stator:

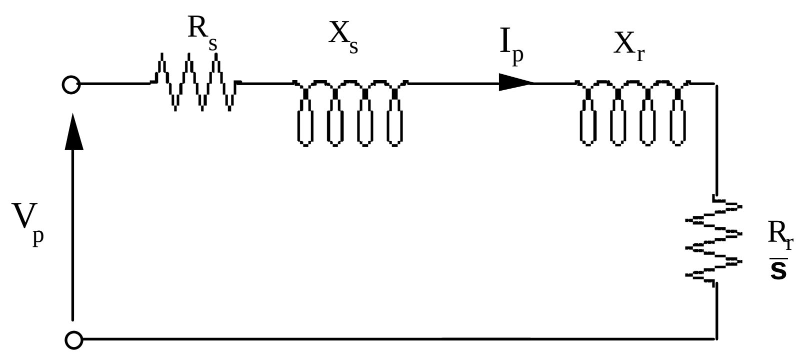
$R_s = 0.0725\ \Omega$/phase, $R_r = 0.1035\ \Omega$ /phase, $X_r = X_s = 0.625\ \Omega$ /phase

The rotor and stator windings are both connected in Y and the speed is 442 rpm at rated voltage. Determine:

a) The torque
b) The shaft power
c) The efficiency
d) The input power factor
e) The stator and rotor losses

Solution:

Per phase equivalent circuit for an induction motor:



a) Torque:

$$T_e = 3I_p^2 \frac{R_r}{s\omega_m}(1 - s)$$

ω = 60 Hz = 3600 rpm

$$\omega_s = \frac{2}{P}\ \omega = \frac{2}{16} \times 3600 = 450 \text{ rpm}$$

$$s = \frac{(\omega_s - \omega_m)}{\omega_s} = \frac{(450 - 442)}{450} = 0.0178$$

$$I_p = \frac{V_p}{Z_p}$$

$$V_p = \frac{V_{AB}}{\sqrt{3}}$$

$$Z_p = R_s + \frac{R_r}{s} + j(X_s + X_r)$$

$$= 0.0725 + \frac{0.1035}{0.0178} + j(0.625 + 0.625) = 5.89 + j1.25$$

$= 6.02\ \angle 11.98°$

Therefore:

$$I_p = \frac{V_p}{Z_p} = \frac{V_{AB}}{\sqrt{3}Z_p} = \frac{2300}{\sqrt{3}\times 6.02\angle 11.98°} = 220.6\ \angle -11.98°$$

Substitute into:

$$T_e = 3I_p^2 \frac{R_r}{s\omega_m}(1 - s) = \frac{3\times 220.6^2 \times 0.1035\times(1-0.0178)}{0.0178\times 442\times\frac{2\pi}{60}}$$

$= 18{,}000$ n-m

b) The shaft power

$$P_e = T_e\,\omega_m = 18000\times 442\times\frac{2p}{60} = 834 \text{ kW}$$

c) The efficiency

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_e}{\sqrt{3}V_{AB}I_A\cos(\theta)}$$

And, for a Y connected machine:

$$I_A = I_p$$

Therefore:

$$\eta = \frac{P_e}{\sqrt{3}V_{AB}I_p\cos(\theta)} = \frac{834\times 10^3}{\sqrt{3}\times 2300\times 220.6\times\cos(-11.98°)} = 97.0\%$$

d) The input power factor:

PF = cos(θ) = cos(-11.98°) = 0.978   lagging

e) The stator and rotor losses:

In the stator:

$$P_{losses} = 3I_p^2 R_s = 3\times 220.6^2 \times 0.0725 = 10.6 \text{ kW}$$

In the rotor:

$$P_{losses} = 3I_p^2 R_r = 3\times 220.6^2 \times 0.1035 = 15.1 \text{ kW}$$

Power Check:

Total input power: $P_{in} = \sqrt{3}\, V_{AB}\, I_A \cos(\theta)$

$= \sqrt{3} \times 2300\times 220.6\times\cos(-11.98°) = 859.7$ kW

Mechanical output power: $P_e = 834$ kW

Total losses: $P_{losses} = 3I_p^2 R_r + 3I_p^2 R_s = 10.6 + 15.1 = 25.7$ kW

Thus $P_{in} = P_e + P_{losses}$

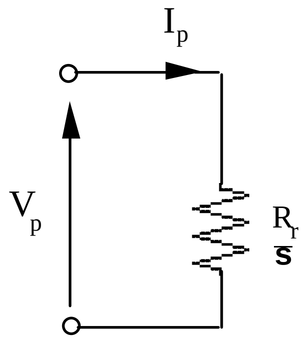
Example 4.8

Assume the motor of example 4.7 can be represented entirely by the rotor resistance. Determine:

a) The torque
b) The shaft power
c) The efficiency
d) The input power factor
e) The stator and rotor losses

Solution:

Simplified per phase equivalent circuit for an induction motor:



a) Torque:

$$T_e = \frac{3sV_p^2}{\omega_m R_r} \approx \frac{3sV_p^2}{\omega_s R_r}$$

However, in a Y connected machine:

$$V_p = \frac{V_{AB}}{\sqrt{3}}$$

Therfore:

$$T_e = \frac{s(1\text{-}s)V_{AB}^2}{\omega_m R_r} = \frac{0.0178 \times (1 - 0.0178) \times 2300^2}{442 \times \frac{2\pi}{60} \times 0.1035}$$

= 19,300 n-m (vs. 18,000 n-m in example 4.7)

b) The shaft power:

$$P_e = \frac{3s(1\text{-}s)V_p^2}{R_r} = \frac{s(1\text{-}s)V_{AB}^2}{R_r}$$

$$= \frac{0.0178 \times (1 - 0.0178) \times 2300^2}{0.1035}$$

= 893.6kW (vs. 834 kW in example 4.7)

c) The efficiency

$$\eta = \frac{P_e}{P_{in}}$$

And for the simplified model:

$$P_e = \frac{s(1\text{-}s)V_{AB}^2}{R_r}$$

$$P_{in} = \frac{3sV_p^2}{R_r} = \frac{sV_{AB}^2}{R_r}$$

Therefore:

$$\eta = \frac{P_e}{P_{in}} = (1 - s) = (1 - 0.0178) = 98.2\% \text{ (vs. 97.0\% in example 4.7)}$$

d) The input power factor
Since the simplified model is entirely resistive,

PF = 1 (vs 0.978 lagging in example 4.7)

e) The stator and rotor losses
The simplified model has no stator losses, therefore:
In the stator:

$$P_{losses} = 0 \text{ (vs. 10.6 kW in example 4.7)}$$

In the rotor:

$$P_{losses} = 3I_p^2 R_r$$

And,

$$I_p = \frac{sV_p}{R_r} = \frac{sV_{AB}}{3R_r}$$

Therefore:

$$P_{losses} = 3I_p^2 R_r = \frac{s^2V_{AB}^2}{R_r} = \frac{0.0178^2 \times 2300^2}{0.1035}$$

$$= 16.2 \text{ kW (vs. 15.1 kW in example 4.7)}$$

Power check:

Input power: $P_{in} = \frac{sV_{AB}^2}{R_r} = \frac{0.0178 \times 2300^2}{0.1035} = 909.8$ kW

Losses: $P_{losses} = 16.2$ kW

Output power: $P_e = 893.6$ kW

4.2.3 Squirrel Cage Induction Motor

In all the preceding analysis for an induction machine there was no rotor excitation. No external voltage or current was applied to the rotor windings. Therefore no external connections are required for the rotor windings. The rotor "windings" can thus be simplified by connecting them internally and removing the external connections entirely. The rotor windings can then consist of a cylindrical arrangement of copper bars shorted at the ends by copper end rings as shown in **Figure 4.13**.

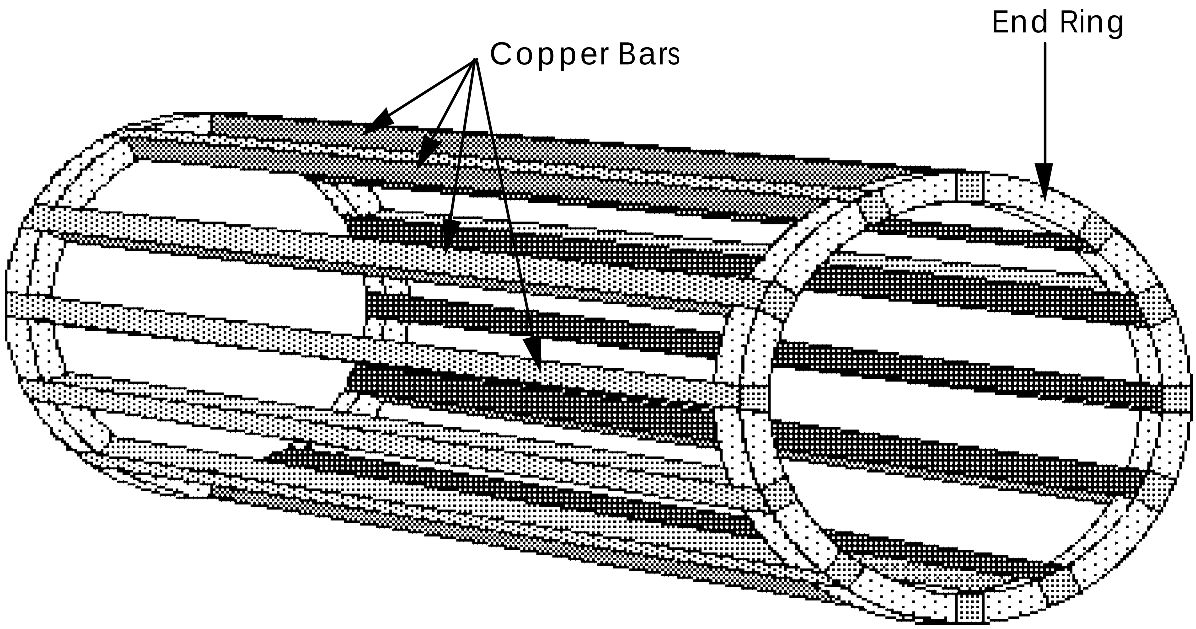


Figure 4.13 "Squirrel Cage" Rotor for a "Squirrel Cage Induction Motor"

It is a very inexpensive yet highly robust and reliable construction and is the most commonly used type of rotor for induction machines. Because this type of 'winding' arrangement resembles a squirrel cage the machines that use it are called squirrel cage induction motors.
The main disadvantage of a squirrel cage induction motor is that there is no access to the rotor windings which reduces the flexibility for speed control.

4.2.4 Wound Rotor Induction Motor

If the rotor of an induction machine has accessible windings it is called a wound rotor. The winding terminals can be used to add external resistance or to apply an external three phase AC voltage, (applying a DC voltage would turn the machine into a synchronous machine). The equivalent circuit is shown in **Figure 4.14**.

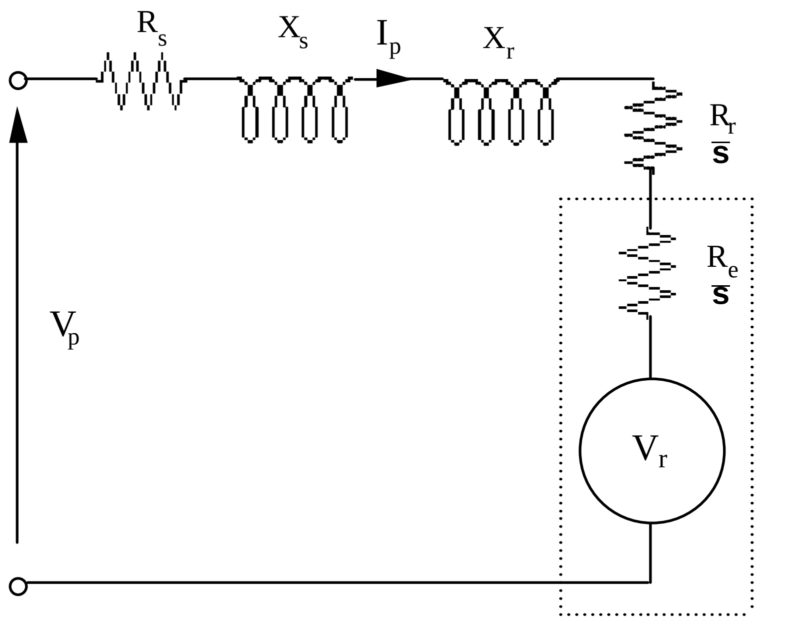


Figure 4.14 Equivalent circuit for a wound rotor induction motor with external rotor resistance and voltage as reflected to the stator.

Assume that the maximum (pullout) torque of the machine is not exceeded and that the external resistance and voltage are R_e , and V_e , respectively where;

$$V_e = \hat{V}_e \sin(\omega_r t)$$

Since the rotor is already rotating at a mechanical shaft speed of ω_m and the external voltage has a frequency of ω_r then it can be shown that the effective frequency of the rotor currents, when reflected to the stator will be ω, where;

$$\omega = \omega_r + \frac{P}{2}\omega_m$$

Thus the external rotor voltage, V_e , can be represented on the stator side by V_r , where;

$$V_r = N\hat{V}_e \sin((\omega_r + \tfrac{p}{2}\omega_m)t) = \hat{V}_r \sin(\omega t)$$

Where:

$$N = \frac{N_s}{N_r}$$

And N_s is the number of turns on the stator and N_r is the number of turns on the rotor.

The external rotor resistance, R_e', has exactly the same effect as increasing the internal rotor resistance, R_r , and therefore can be represented in the stator side equivalent circuit by R_e /s.

Where;

$$R_e = N^2 R_e'$$

Thus the per phase stator current, I_p , electrical power converted by the machine, P_e , and torque generated by the machine, T_e , will be;

$$I_p = \frac{V_p - V_r}{R_s + \frac{(R_r + R_e)}{s} + j(X_s + X_r)}$$

$$P_e = 3I_p^2 \frac{(R_r + R_e)}{s}(1 - s)$$

$$T_e = \frac{P_e}{\omega_m} = 3I_p^2 \frac{(R_r + R_e)}{s\omega_m}(1 - s)$$

Note that since the rotor frequency, ω_r , is fixed by the external rotor voltage, and the stator frequency, ω, is fixed by the stator voltage then the shaft speed, ω_m , will also be fixed, (as long as the pullout torque is not exceeded), such that;

$$\omega_m = \frac{2}{P}(\omega - \omega_r)$$

Thus, an external rotor voltage, of variable frequency, can be used to control the shaft speed of an induction motor.

4.2.5 Induction Generator

All the preceding analysis can also be applied to describe an induction machine in the generator mode. The per phase equivalent circuit for an induction generator is shown in **Figure 4.15.**

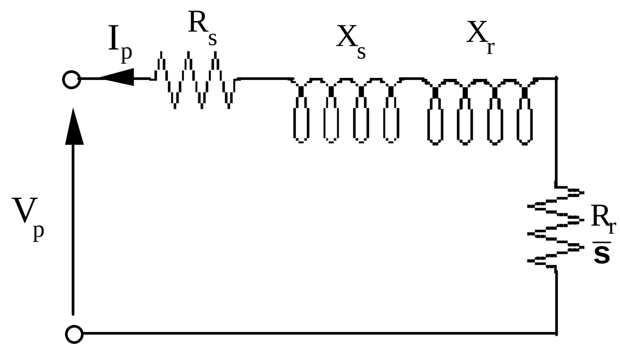


Figure 4.15 Equivalent Circuit for an Induction Generator

In the generator mode the slip, s, is always negative;

$$s = (\omega_s - \omega_m)/\omega_s < 0$$

or

$$\omega_m > \omega_s = \frac{2}{p}\,\omega$$

The reflected rotor resistance, R_r/s, will also be negative and thus act as a source of power for the stator circuit.

The behaviour of an induction generator is much more complex than a sychronous generator. The per phase stator current, I_p, can be determined from;

$$P_e = 3I_p^2\frac{R_r}{s}(1 - s)$$

Solve for I_p;

$$I_p = \sqrt{sP_e/3R_r(1\text{-}s)}$$

The output voltage can then be determined;

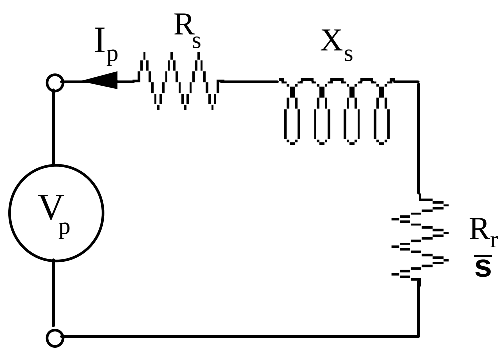
$$V_s = I_p\left[R_s + R_r/s + j(\omega L_r + \omega L_s)\right] \qquad \text{per phase}$$

The frequency of the generated current will be ω, where;

$$\omega = \frac{P}{2}\,\omega_s = \frac{P}{2}\omega_m/(1\text{-s})$$

In general then, the output voltage and frequency of an induction generator, is dependent on the shaft speed, and the available power P_e. This results in a very poor type of generator and therefore induction machines are never used as generators of electrical power.

However, induction machines are often used as generators when regenerating power back into a power system during braking. For such applications the equivalent circuit is shown in **Figure 4.16.**



The voltage source, V_p, is capable of sourcing and sinking power.

Figure 4.16 An induction machine regenerating power to a voltage source

In this case the stator voltage and frequency are determined by the power system. However, the slip, s, is negative for a generator. This means that;

$$s = \frac{\omega_s - \omega_m}{\omega_s} < 0$$

Or;

$$\omega_m > \omega_s$$
$$> \frac{2}{P}\,\omega$$

In other words, an induction machine will only recover braking energy when the shaft speed is above synchronous speed. The machine equations for such a generator become identical to those of an induction motor;

$$I_p = \frac{V_p}{R_s + \frac{R_r}{s} + j(X_s + X_r)}$$

$$P_e = 3I_p^2 \frac{R_r}{s}(1 - s)$$

$$T_e = \frac{P_e}{\omega_m} = 3I_p^2 \frac{R_r}{s\omega_m}(1 - s)$$

Since the slip, s, is negative for an induction generator the stator current can become very high as;

$$R_r / s \Rightarrow -R_s$$

Or;

$$s \Rightarrow -\frac{R_r}{R_s}$$

In which case;

$$I_p \Rightarrow \frac{V_p}{j(X_s + X_r)}$$

The only way the stator current can be controlled is by controlling the slip. The slip, in turn, can only be controlled by controlling the shaft speed ω_m *if the power system voltage and frequency are fixed.* If, however, the power system is a variable frequency, variable voltage power supply then the stator current, braking torque, etc. can be controlled by varying the power system frequency, so as to control the slip. This is discussed further in Section 4.2.6.

Example 4.9

A locomotive has two axles with a 12 pole, three phase, induction motor on each axle. The motors are identical and each have the following parameters, referred to the stator:

R_r = 0.06 Ω/phase, R_s = 0.5 Ω/phase,

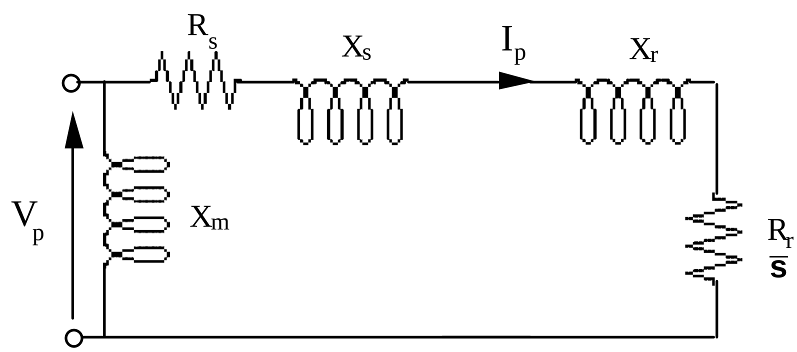
X_r = 0.12 Ω/phase, X_s = 0.2 Ω/phase, X_m = 8.5 Ω/phase

All the above values are in Ω/phase, in Y connection, at 60Hz. Both motors are supplied with 2300Vac, three phase, at 60 Hz. The locomotive is travelling at 112 km/h. Determine:

a) The torque produced by each machine if the both axles have wheels of exactly 1.0 m diameter.

b) The torque produced by each machine if the wheel manufacturer guarantees each wheel radius as 0.5 ± .005 m, and our luck is such that one axle has the largest possible radius and the other axle has the smallest possible radius.

Solution:



Per phase equivalent circuit for an induction motor including magnetizing inductance.

For Y connected machine:

$$V_p = \frac{V_{AB}}{\sqrt{3}}$$

a)Torque produced by each machine:

$$T_e = 3I_p^2 \frac{R_r}{s\omega_m}(1 - s)$$

$$\omega_m = \frac{\text{velocity}}{\text{radius}}$$

$$\text{velocity} = \frac{112\text{ km}}{1\text{ hr}} \times \frac{1000\text{ m}}{1\text{ km}} \times \frac{1\text{ hr}}{3600\text{ sec.}} = 31.11\text{ m/s}$$

radius = 0.5 m

Therefore:

$$\omega_m = \frac{\text{velocity}}{\text{radius}} = \frac{31.11}{0.5} = 62.22\text{ r/s}$$

$$s = \frac{(\omega_s - \omega_m)}{\omega_s}$$

$$\omega_s = \frac{2}{P}\omega = \frac{2}{12} \times 60 \times 2\pi = 62.83 \text{ r/s}$$

Therefore:

$$s = \frac{(\omega_s - \omega_m)}{\omega_s} = \frac{(62.83 - 62.22)}{62.83} = 0.0097$$

From equivalent circuit solve for

$$I_p = \frac{V_p}{R_s + \frac{R_r}{s} + j(X_s + X_r)} = \frac{V_{AB}}{\sqrt{3}\left[R_s + \frac{R_r}{s} + j(X_s + X_r)\right]}$$

$$= \frac{2300}{\sqrt{3}\left[0.05 + \frac{0.06}{0.0097} + j(0.2 + 0.12)\right]} = 207.9 \angle -2.87°$$

Therefore:

$$T_e = 3I_p^2 \frac{R_r}{s\omega_m}(1 - s) = \frac{3 \times 207.9^2 \times 0.06 \times (1 - 0.0097)}{0.0097 \times 62.22}$$

$$= 12{,}754 \text{ n-m}$$

Note that the torque is not affected by the magnetizing reactance.

b) The torque produced by each machine if one wheel radius is 0.5 - .005 m, and the other wheel radius is 0.5 + .005 m.

$$\omega_m = \frac{\text{velocity}}{\text{radius}}$$

Therefore:

$$\omega_{m1} = \frac{31.11}{0.5 - .005} = 62.85 \text{ r/s}$$

$$\omega_{m2} = \frac{31.11}{0.5 + .005} = 61.61 \text{ r/s}$$

And:

$$s_1 = \frac{(\omega_s - \omega_{m1})}{\omega_s} = \frac{(62.83 - 62.85)}{62.83} = -3.18 \times 10^{-4}$$

$$s_2 = \frac{(\omega_s - \omega_{m2})}{\omega_s} = \frac{(62.83 - 61.61)}{62.83} = +0.0195j$$

And:

$$I_{p1} = \frac{V_{AB}}{\sqrt{3}\left[R_s + \frac{R_r}{s_1} + j(X_s + X_r)\right]}$$

$$= \frac{2300}{\sqrt{3}\left[0.05 + \frac{0.06}{-3.18\times10^{-4}} + j(0.2 + 0.12)\right]} = 7.07\angle{-180°}\ \text{A}$$

$$I_{p2} = \frac{V_{AB}}{\sqrt{3}\left[R_s + \frac{R_r}{s_1} + j(X_s + X_r)\right]}$$

$$= \frac{2300}{\sqrt{3}\left[0.05 + \frac{0.06}{0.0195} + j(0.2 + 0.12)\right]} = 422\angle{5.8°}\ \text{A}$$

Substitute into the equation for torque to obtain:

$$T_{e1} = 3I_{p1}^2 \frac{R_r}{s_1\omega_{m1}}(1 - s_1)$$

$$= \frac{3\times7.07^2\times0.06\times(1 - -3.18\times10^{-4})}{-3.18\times10^{-4}\times62.85}$$

$$= -447 \text{ n-m}$$

$$T_{e2} = 3I_{p2}^2 \frac{R_r}{s_2\omega_{m2}}(1 - s_2) = \frac{3\times7.07^2\times0.06\times(1-0.0195)}{0.0195\times62.85}$$

$$= +26{,}187 \text{ n-m}$$

Therefore, the machine with the smaller diameter wheels is acting as a generator.

4.2.6 Variable Speed Control of Induction Motors

The shaft speed of an induction motor, ω_m , will only vary by the slip, s, as long as the stator frequency (and rotor frequency, if applicable) is fixed.

$$\omega_m = \frac{2}{P}\,\omega(1\text{-}s)$$

Since s is typically very small, (0.001 to 0.1), the resultant variation in shaft speed would be 0.1% to 10% below synchronous speed — not much speed variation.

The optimum method of getting wide speed variation is by varying the stator frequency, ω. With a variable stator frequency we can redraw the Torque vs. Speed curve into a family of curves as shown in **Figure 4.17**.

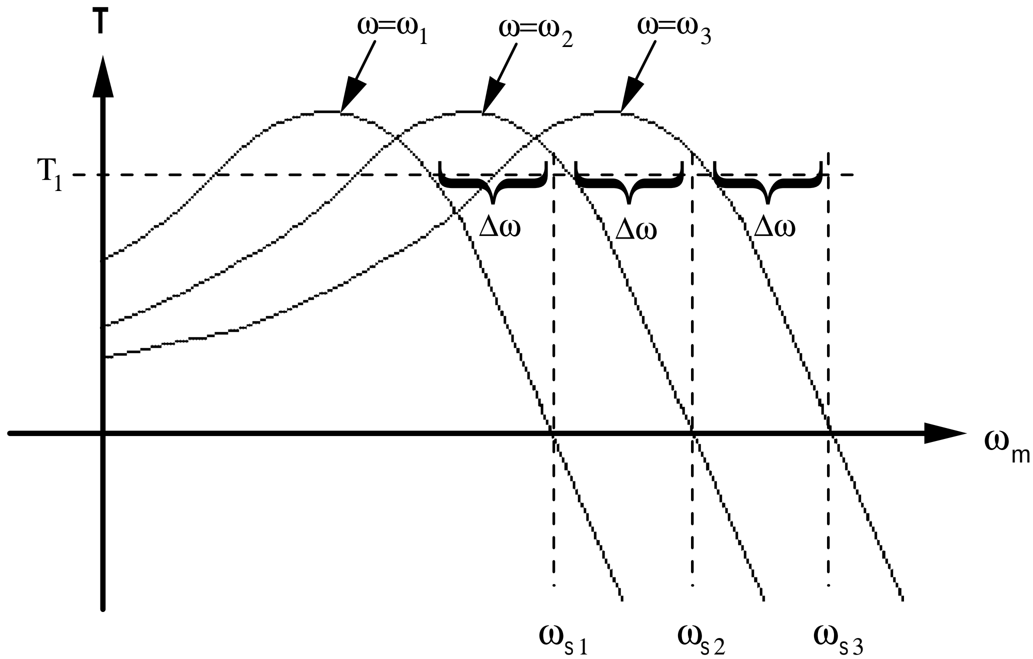


Figure 4.17 Family of Torque vs Speed Curves for an Induction Motor fed from a variable frequency supply

In this case there is a different curve for each stator frequency;

$$\omega = \omega_1 , \omega_2 , \omega_3 ,\ldots , \omega_n , \text{etc.}$$

And the corresponding shaft speed, ω_{mn} , will be;

$$\omega_{mn} = \frac{2}{P}\,\omega_n(1\text{-}s)$$

Thus the machine speed can be controlled by controlling the stator frequency, ω. This mode of operation requires a three phase, variable frequency supply voltage, and this can be produced using power electronics. However the magnetic material within the machine imposes particular constraint on the power supply.

The magnetic flux, ϕ, in the machine is limited by;

$$\phi = \int B \partial A = B_{max} A_{effective}$$

$$= \frac{1}{N} \int_0^p \hat{V}_p \sin(\omega t) \partial t = B_{max} A_{effective}$$

The above equation can be reduced to;

$$\frac{\hat{V}_p}{\omega} = \frac{N}{2} B_{max} A_{effective} = K$$

For a given machine, the values of N, B_{max} , and $A_{effective}$ are constant, and therefore;

$$\frac{\hat{V}_p}{\omega} = K$$

Thus for variable speed drive, the ratio of supply voltage to frequency, (often referred to as the volt-second product) must be kept below some limit to prevent saturation. This is particularly important at low frequencies. Thus variable speed drive of induction machines requires the simultaneous control of frequency and voltage in a three phase power system. This applies to all AC powered magnetic devices, machines and transformers as well.

Varying the voltage and frequency in an induction motor drive always runs the risk of "pullout" and stalling if the load torque exceeds the pullout torque of the machine at any particular operating point. To ensure that this doesn't occur in an induction motor drive requires feedback information on the slip frequency, or rotor frequency, ω_r , where;

$$\omega_r = \omega - \frac{p}{2} \omega_m$$

This information can then be used to determine the actual load torque and take corrective action if load torque increase. Corrective action may be to increase stator voltage, or reduce frequency, etc. This feedback is the weakest link in any induction motor drive system because it requires a mechanical tachometer for shaft speed feedback from which ω_r can be determined. Such a mechanical device usually spoils the otherwise high reliability associated with solid state power electronics and squirrel cage induction motors.

A typical variable speed squirrel cage induction motor drive is shown in **Figure 4.18**.

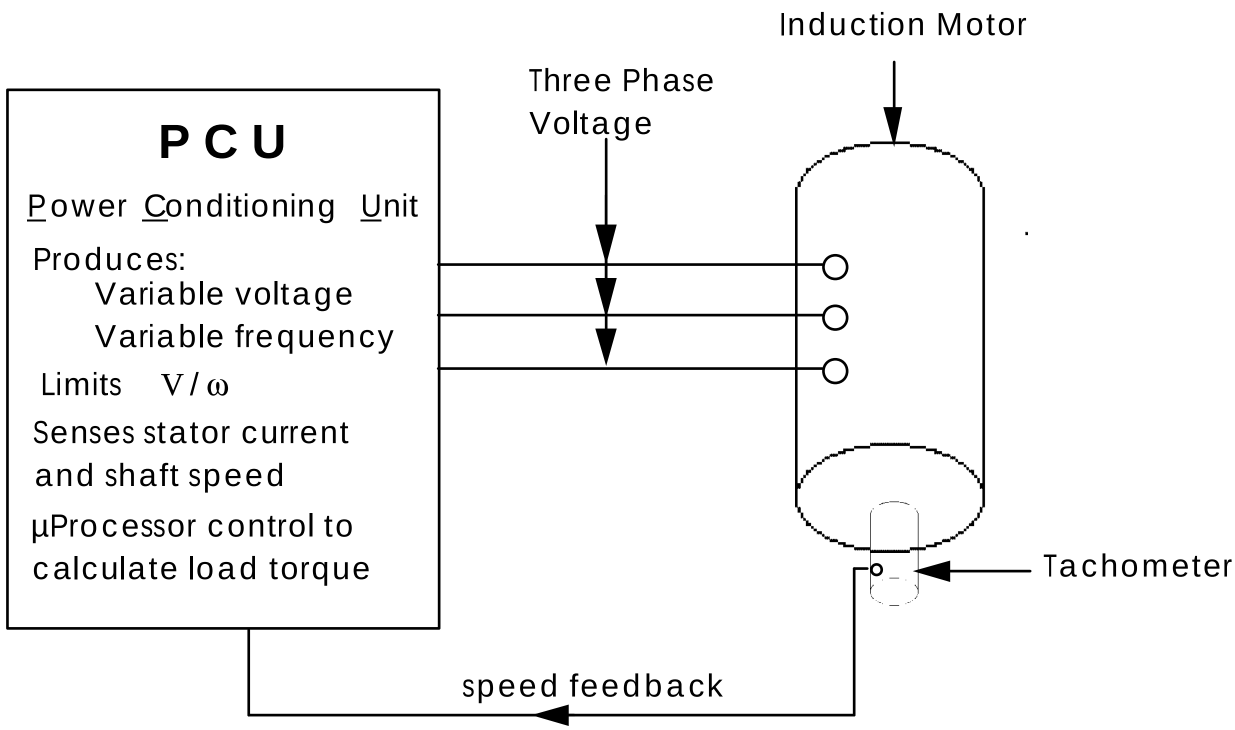


Figure 4.18 A Variable Speed Induction Motor Drive Using a PCU (Power Conditioning Unit)

The critical component in this system is the Power Conditioning Unit, (PCU). It would typically produce a variable three phase voltage, $\hat{E}_s \sin(\omega t)$, at a variable frequency, ω. It would also receive feedback as to shaft speed, ω_m, and machine current, I_s. This information would be used to calculate the load torque which would then be compared to the available pullout torque at the given operating conditions. The PCU would then ensure that the machine never pulled out by adjusting voltage and/or frequency accordingly.

4.3 PROBLEMS

4.3.1 Synchronous Generators

1. A 10 MVA, 13.8 kV, 60 Hz, 2 pole, 3 phase, Y connected alternator has a synchronous reactance of 1.9 Ω/phase and a winding resistance of 0.07 W/phase. The generated EMF, (line-to-line),is related to the field current by;

$$E_s = 60\ I_f$$

a)Determine the field current required to produce rated voltage across rated load at 0.8 PF lagging. (245A)

b)What is the regulation? (6.56%)

2. A three phase, 6000 rpm, 8 pole, synchronous generator produces 140 volts across an ideal inductance of 8 mH/phase, and 100 volts across 4 mH/phase. Assume the generator and inductor are Delta connected, there is no saturation, and no resistive losses in the generator, and the field excitation is kept constant throughout. Determine;

a) The stator frequency, and per phase equivalent circuit. (400 Hz, 13.4 Ω/phase and 233 V/phase)

b) The shaft torque required by this generator when a resistive Delta connected load of 10.0 Ω/phase is connected across the stator terminals. (9.32 n-m)

3. A three phase synchronous generator has the following test data:

Volts L-L	Amps Line	Stator Hz	Shaft RPM
1800	O.C.	60	400
S.C.	600	60	400

Assume it is 'Y' connected, there is no saturation, and no resistive losses in the generator. Determine

a)The per phase equivalent circuit, and the number of poles. (1.73 Ω/phase and 1039 V/phase, 18 poles)

b)The shaft torque required by this generator when a resistive 'Y' connected load of 3.0 Ω/phase is connected across the stator terminals. (19,333 n-m)

4. A three phase synchronous generator has the following test data:

Volts L-L	Amps Line	Stator Hz	Shaft RPM
1400	O.C.	60	600
S.C.	563	60	600

Assume it is Delta connected, there is no saturation, and no resistive losses in the generator. Determine:

a)The number of poles, and the per phase equivalent circuit.

(12 poles, 4.31 Ω/phase and 1400 V/phase) b)The maximum power this machine can produce with an output voltage of 1200 V line to line. (1.169 MW)

5. A 4 pole, 60 Hz alternator (synchronous generator) is Y connected, and has a synchronous reactance of 4 Ω and an open circuit voltage of 2835 V line to line. It is connected to a 2300 V, 3 phase grid until maximum power is drawn from the machine. Determine:

a) The maximum output power. (1.63 MW)

b) The output current. ($527\angle 39°$ A)

c) The torque required by the machine. (8,650 n-m)

4.3.2 Synchronous Motors

6. A 1.8 MVA, 6600 V, 8 pole, 60 Hz, 3 Φ, Y connected, synchronours motor has a synchronous reactance of 20 Ω/phase. The field and load are such that the motor has a lagging power factor of 0.8. The stator current is 130 A/phase. Determine;

a)The shaft torque developed by the motor. (12,616 n-m)

b)The electrical angle δ, between stator and rotor magnetic fields, and the corresponding mechanical angle. (42.75°, 10.7°)

c)The maximum, or pull-out torque. (18,586 n-m)

7. A 250 KVA, 600 V, 8 pole, 60 Hz, 3 phase, Delta connected, synchronous motor has a synchronous reactance of 4 Ω/phase. The field and load are such that the motor has a leading power factor of 0.8. The stator current is 130 A/line. Determine:

a) The shaft torque developed by the motor. (1145 n-m)

b) The maximum, or pull-out torque. (3896 n-m)

8. A synchronous, Y connected, motor has a synchronous reactance of 2.5 Ω and draws 100 A/line at unity power factor and rated voltage of 460 V line to line. Assuming constant line voltage, and constant load on the motor, determine;

a) line current and power factor if the machine field excitation is increased by 50%. ($133.5\angle 41.5°$, 0.749)

b) line current and power factor if the machine field excitation is decreased by 20%.

4.3.3 Wound Rotor Induction Motors

9. A 3 Φ, 60 Hz, 8 pole, Y connected, wound rotor induction motor develops a net mechanical power of 200 KW at a slip of 0.025. The rotor resistance (referred to the stator), is 0.0178 Ω/phase. Friction and windage for this motor are constant at 6750 Watts. The motor reactances and stator resistance are negligible. Determine;

a)The rpm of the machine. (877.5 rpm)
b)The torque this machine is producing. (2,177 n-m)
c)The net available mechanical torque and power if the rotor resistance (per phase), is doubled by connecting appropriate external resistors via the slip rings, and the slip becomes .033. The machine voltage is unchanged from a) and b) above. (1,430 n-m, 130 kW)

10. A 480 V, 3 phase, 60 Hz, 6 pole, Y connected, wound rotor induction motor develops a net mechanical power of 25 kW at 1172 rpm. Friction and windage for this motor are constant at 1500 Watts. The motor reactances and stator resistance are negligible. Determine;

a) The slip, s, of the machine. (0.0233)
b) The electrical torque this machine is producing. (216 n-m)
c) The net available mechanical torque and power if the rotor resistance (per phase), is doubled by connecting appropriate external resistors via slip rings, and the slip becomes 0.033. (140 n-m, 17063 W)

4.3.4 Squirrel Cage Motors

11. A six pole, three phase, Y connected, squirrel cage induction motor has the following per phase parameters, referred to the stator side;

$R_S = 0.11\ \Omega$, $R_r = 0.18\ \Omega$, $X_S = 0.50\ \Omega$, $X_r = 0.40\ \Omega$, $X_m = R_c = 8$

a) Show the per phase equivalent circuit for this motor and determine the stator current if the machine rpm is 1176 and stator voltage is 230 V line to line, 60 Hz. (s=0.02, $I_S = 14.5\ A\angle{-10.8°}$)

b) What is the motor torque and efficiency under the conditions of part a)? (90.3 n-m, 96.8%)

12. A four pole, three phase, 'Y' connected, squirrel cage induction motor has the following per phase parameters, referred to the stator side;

R_S =0.11 W R_r =0.18 W X_S =0.50 W X_r =0.40 W $R_C = X_m = \infty$

a)Show the per phase equivalent circuit for this motor and determine the stator current if the machine RPM is 1782 and stator voltage is 230 V line to line. (s=0.01, $I_S = 7.38\ A\angle{-0.8°}$)

b)What is the motor torque and efficiency under the conditions of part a)? (15.6 n-m, 98.4%)

13. Two identical squirrel cage induction machines are connected to the same conveyer belt and the same three phase, 60Hz, 480V line to line AC voltage. They are each 4 pole, 'Y' connected, and have the following parameters;

$R_S = 2.0\ \Omega$, $R_r = 0.02\ \Omega$, $X_r + X_S = 3.0\ \Omega$, $R_C = X_m = \infty$

Due slight variations in wheel diameter, one machine is operating at 1782 rpm and the other is operating at 1806 rpm. Determine:

a) The equivalent circuit for each machine. (s=0.01, -0.0033)

b) The torque produced by each machine. (97.8 n-m, -293 n-m)

14. Two identical squirrel cage induction machines are connected to the same conveyer belt and the same three phase, 60Hz, 230V line to line AC voltage. They are each 6 pole, Delta connected, and have the following parameters;

$R_S = 0.32\ \Omega$, $R_r = 0.04\ \Omega$, $X_r + X_S = 2.5\ \Omega$, $R_C = X_m = \infty$

Due slight variations in wheel diameter, one machine is operating at 1188 rpm and the other is operating at 1206 rpm. Determine:

a) The equivalent circuit for each machine. (s= 0.01, -0.005)

b) The torque produced by each machine. (202.9 n-m, -155.1 n-m)

15. A 6 pole, three phase, Y connected induction motor is operating at a speed of 1164 rpm, and drawing 50A line current. The input power at this speed is 33 kW and the losses in the stator winding resistance are 1200 W. Assume magnetizing and friction losses are negligible. Determine:

a) the slip. (0.03)

b) the stator resistance and rotor resistance as reflected to the stator side. (0.16 Ω, 0.1272 Ω)

c) the torque developed by the machine. (253 n-m)