

Algoritmos de Ordenação

Problema Computacional: Ordenação

- Problema computacional que surge em diversas situações.
- Definição:
 - ✓ Input: Uma sequência de n números: $\{a_1, a_2, a_3, \dots, a_n\}$
 - ✓ Output: Uma reordenação da sequência em $\{a'_1, a'_2, a'_3, \dots, a'_n\}$
tal que $a'_1 \leq a'_2 \leq a'_3 \leq \dots \leq a'_n$

Bubble Sort

```
def bubble_sort(L):  
    swap = False  
    while not swap:  
        print('bubble sort: ' + str(L))  
        swap = True  
        for j in range(1, len(L)):  
            if L[j-1] > L[j]:  
                swap = False  
                temp = L[j]  
                L[j] = L[j-1]  
                L[j-1] = temp
```

j

0	1	2	3	4	5
12	81	44	23	67	90

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Bubble Sort

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    swap = False  
    while not swap:                                     O(n)  
        print('bubble sort: ' + str(L))  
        swap = True  
        for j in range(1, len(L)):                     O(n)  
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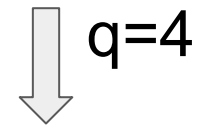
$O(n^2)$ no pior caso para $n = \text{len}(L)$

Merge sort

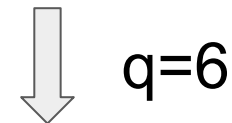
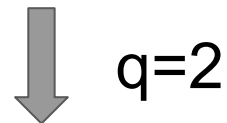
```
def merge_sort(L):  
    print('merge sort: ' + str(L))  
    if len(L) < 2:  
        return L[:]  
    else:
```

```
        middle = len(L)//2  
        left = merge_sort(L[:middle])  
        right = merge_sort(L[middle:])  
        return merge(left, right)
```

p				r			
1	2	3	4	5	6	7	8
50	20	40	70	10	30	20	60



p		r		p		r	
1	2	3	4	5	6	7	8
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1	2	3	4	5	6	7	8
50	20	40	70	10	30	20	60

↓ q=4

p				r			
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50	20	40	70	10	30	20	60

↓ q=2 ↓ q=6

p		r		p		r	
1	2	3	4	5	6	7	8
50	20	40	70	10	30	20	60

↓ q=1 ↓ q=3 ↓ q=6 ↓ q=7

1	2	3	4	5	6	7	8
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1	2	3	4	5	6	7	8
10	20	20	30	40	50	60	70

↑ q=4

p		r		p		r	
1	2	3	4	5	6	7	8
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↑ q=2 ↑ q=6

p	r	p	r	p	r	p	r
1	2	3	4	5	6	7	8
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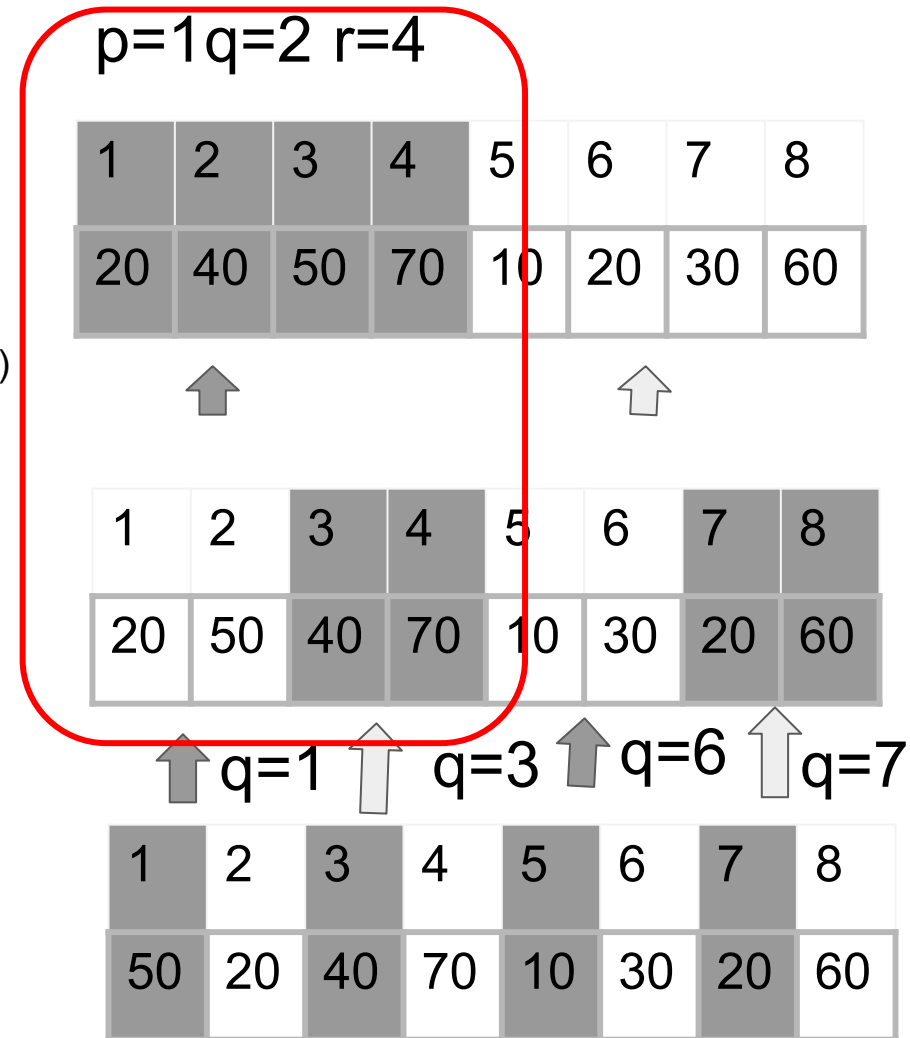
↑ q=1 ↑ q=3 ↑ q=6 ↑ q=7

1	2	3	4	5	6	7	8
50	20	40	70	10	30	20	60

```

def merge(left, right):
    result = []
    i,j = 0,0
    while i < len(left) and j < len(right):
        if left[i] < right[j]:
            result.append(left[i])
            i += 1
        else:
            result.append(right[j])
            j += 1
    while (i < len(left)):
        result.append(left[i])
        i += 1
    while (j < len(right)):
        result.append(right[j])
        j += 1
    print('merge: ' + str(left) + '&' + str(right) + ' to ' +str(result))
    return result

```



Analizando Merge sort

➤ $T(n)$: tempo no pior caso ($n = r - p + 1$).

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        return L[:]  
    else:  
        middle = len(L)//2  
        left = merge_sort(L[:middle])  
        right = merge_sort(L[middle:])  
        return merge(left, right)
```

$D(n)$: Tempo para dividir o problema

$C(n)$: Tempo para combinar o problema

➤ Relação de recorrência:

$$1. T(1) = \Theta(1)$$

$$n=1.$$

$$2. T(n) = 2.T(n/2) + D(n) + C(n)$$

$$n>1.$$

Analizando Merge sort

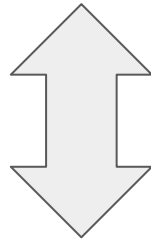
➤ Relação de recorrência:

$$1. T(n) = \Theta(1)$$

$n=1.$

$$2. T(n) = 2T(n/2) + D(n) + C(n)$$

$n>1.$



$$T(n) = \begin{cases} \Theta(1) & n=1 \\ 2T(n/2) + \Theta(n) & n>1 \end{cases}$$

$$T(n) = O(n \lg n)$$

Analizando o Heapsort(A)

```
def heapSort(arr):  
    n = len(arr)  
  
    # Build a maxheap.  
    for i in range(n, -1, -1):  
        heapify(arr, n, i)  $O(n)$   
  
    # One by one extract elements  $\Theta(n)$   
    for i in range(n-1, 0, -1):  $\Theta(n)$   
        arr[i], arr[0] = arr[0], arr[i] # swap  $\Theta(n)$   
        heapify(arr, i, 0)  $nO(\lg n)$ 
```

Pior caso: $O(n \lg n)$