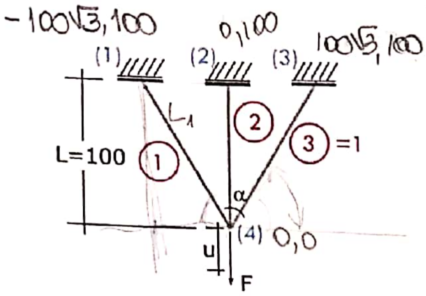
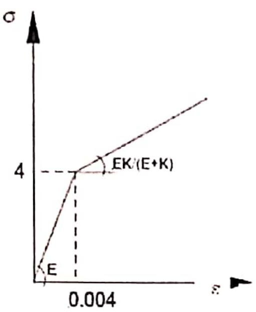




$$E^{\varphi} = \begin{cases} E \quad se \ \Delta\lambda > 0 \\ \dfrac{EK}{(E+K)} \quad se \ \Delta\lambda = 0 \end{cases}$$

$L_1 \cos 60° = 100$

$L_1 = 200 = L_3$

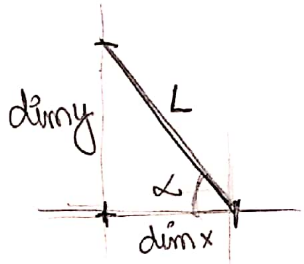
Dados complementares: $\alpha = 60°$; $S = 1$; $E = 1000$; $K = 111.11$; $EK/(E+K) = 100$.

Barras:

1 — (1)(4) — $L_1 = 200$

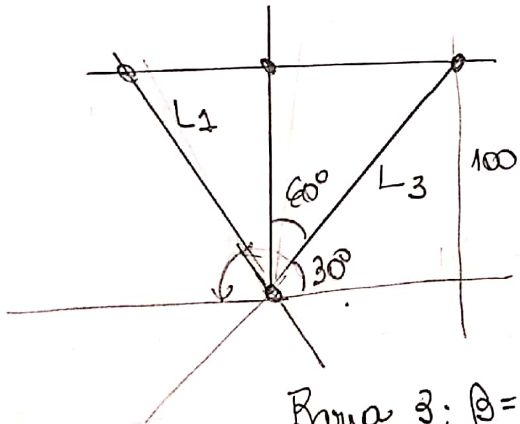
2 — (2)(4) — $L_2 = 100$

3 — (3)(4) — $L_3 = 200$



$L \cos\alpha = \text{dim } x$

$L \operatorname{sen}\alpha = \text{dim } y$



Barra 1:

$L_1 = L_3$

$\beta = 120°$

$C = 0,866$

$S = -0,5$

$L \operatorname{sen} 30° = 100$

$\therefore L_3 = 200$

Barra 3: $\beta = 30°$

$C = -0,866$

$S = +0,5$

Barra 2:

$L_2 = 100$

$C = 0$

$S = 1$

$$K_{elem} = \frac{EA}{L} \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} -3{,}75 & -2.1651 & -3.75 & 2.1651 \\ -2.1651 & 1.25 & 2.1651 & -1.25 \\ -3{,}75 & 2.1651 & 3.75 & -2.1651 \\ 2.1651 & -1.25 & -2.1651 & 1.25 \end{bmatrix}$$

$$K_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & -10 & 0 & 10 \end{bmatrix}$$

$$K_3 = \begin{bmatrix} 3.75 & 2.1651 & -3.75 & -2.1651 \\ 2.1651 & 1.25 & -2.1651 & -1.25 \\ -3.75 & -2.1651 & 3.75 & 2.1651 \\ -2.1651 & -1.25 & 2.1651 & 1.25 \end{bmatrix}$$

$$K_J = \begin{bmatrix} 3.75 & -2.1651 & 0 & 0 & 0 & 0 & -3.75 & 2.1651 \\ -2{,}1651 & 1.25 & 0 & 0 & 0 & 0 & 2.1651 & -1.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 & 0 & 0 & -10 \\ 0 & 0 & 0 & 0 & 3.75 & 2.1651 & -3{,}75 & -2.1651 \\ 0 & 0 & 0 & 0 & 2.1651 & 1.25 & -2.1651 & -1.25 \\ -3.75 & 2.1651 & 0 & 0 & -3.75 & -2.1651 & 7.5 & 0 \\ 2.1651 & -1.25 & 0 & -10 & -2.1651 & -1.25 & 0 & 12.5 \end{bmatrix}$$

$$K = \begin{bmatrix} 7.5 & 0 \\ 0 & 12.5 \end{bmatrix}$$

1º Passo $\Delta P = 5$

$$\begin{bmatrix} 7.5 & 0 \\ 0 & 12.5 \end{bmatrix} \begin{bmatrix} u_7 \\ u_8 \end{bmatrix} = \begin{bmatrix} 0 \\ -5 \end{bmatrix} \qquad u_8 = \frac{5}{12.5} = -0{,}4 \qquad u_7 = \emptyset$$

Barra 01

$$\varepsilon_1 = \frac{1}{200}\begin{pmatrix} -0{,}866 & 0.5 & 0{,}866 & -0{,}5 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ -0{,}4 \end{pmatrix} = 1e\text{-}3$$

$$\varepsilon_1^{e,t} = 1e\text{-}3$$

$$\sigma_1^t = 1000 \times 1e\text{-}3 = 1$$

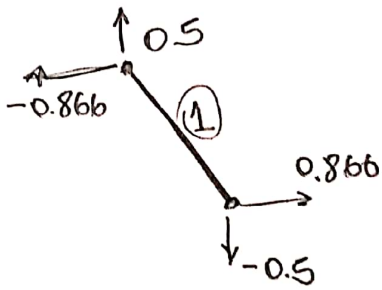
$$\varepsilon = \frac{1}{L}\begin{bmatrix} -c & -s & c & s \end{bmatrix}\begin{Bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \end{Bmatrix}$$

$$f_1 = 1 - 4 = -3 < 0$$

$$\sigma = E\varepsilon$$

$\therefore \sigma_1 = 1$

$$\varepsilon_1 = \varepsilon_1^e = 1e\text{-}3$$



Barra 02

$$\varepsilon_1 = \frac{1}{100}\begin{pmatrix} 0 & 1 & 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ -0{,}4 \end{pmatrix} = 4e\text{-}3$$

$$\varepsilon_1^{e,t} = 4e\text{-}3$$

$$\sigma_1^t = 1000 \times 4e\text{-}3 = 4$$

$$f_1 = 0$$

$\therefore \sigma_1 = 4$

$$\varepsilon_1 = \varepsilon_1^e = 4e\text{-}3$$

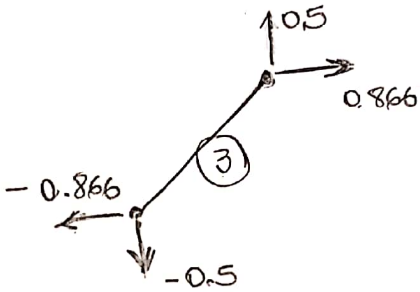
Barra 03:

$$\varepsilon_1 = \frac{1}{200}\begin{pmatrix}0.866 & 0.5 & -0.866 & -0.5\end{pmatrix}\begin{pmatrix}0\\0\\0\\-0,4\end{pmatrix} = 1e-3$$

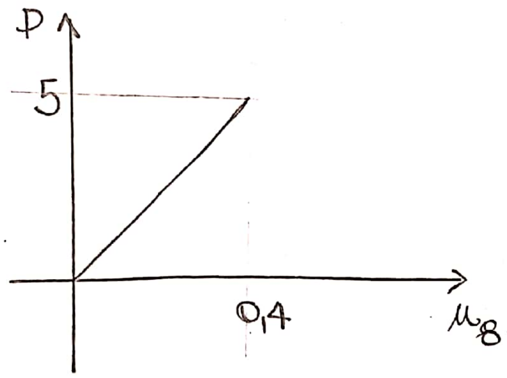
Análogo à barra 1:

$\sigma_1 = 1$

$\varepsilon_1 = \varepsilon_1^e = 1e-3$



∴ Passo elástico.



$P = -5$

$\delta = -0{,}4$

$\sigma_1 = \sigma_3 = 1$

$\sigma_2 = 4$

$\varepsilon_i^p = 0$, $i = 1,2,3$

$\Delta\lambda_i = 0$

$\varepsilon_1 = \varepsilon_3 = 1e-3$

$\varepsilon_2 = 4e-3$

2º PASSO

$\Delta P = -1{,}4$

$(P = -6{,}4)$

$$\begin{bmatrix}7{,}5 & 0\\ 0 & 12{,}5\end{bmatrix}\begin{pmatrix}u_7\\u_8\end{pmatrix} = \begin{pmatrix}0\\-1{,}4\end{pmatrix}$$

$u_8 = -0{,}112$

$\delta = -(0{,}4 + 0{,}112)$

$= -0{,}512$

Barra 1: (= Barra 3)

$\Delta\varepsilon_1 = 2{,}8e-4$

$\varepsilon_1^{e,t} = 1e-3 + 2{,}8e-4$

$= 0{,}0013 = \varepsilon_2$

$\sigma_1^t = 1{,}28$

$f_1^t = 1{,}28 - 4 < 0$

Barra 2:

$\Delta\varepsilon_2 = 1.12\,e-3$

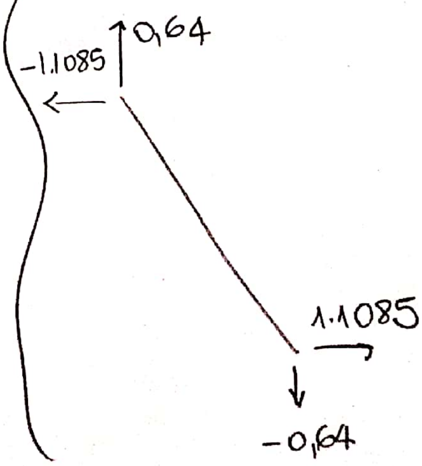
$\varepsilon_2^{e,t} = 4e-3 + 1.12e-3$

$= 5.12e-3 = \varepsilon_2$

$\sigma_2^t = 5{,}12$

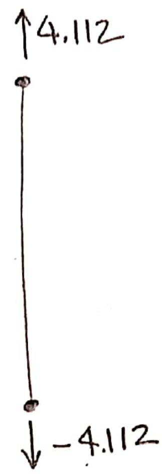
$f_2 = 5{,}12 - 4 > 0$

Barra 3 = Barra 1



Para barra 2 será necessária correção,

$$\Delta\lambda_2 = \frac{1,12}{1111.11} = 0,00101$$



$$\Delta\varepsilon^p_2 = 0,00101 \qquad \varepsilon^e_2 = 5.12e\text{-}3 - 0,00101 = 0.0041$$

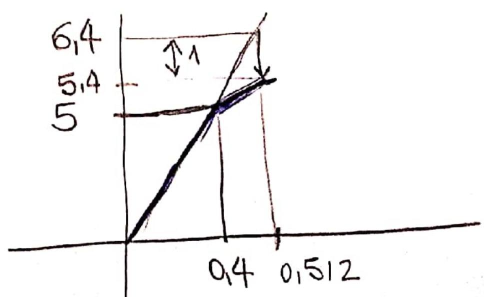
$$\therefore \sigma_2 = 5,12 - 1000 \times 0.00101 = 4.112$$

$$\alpha = 0.00101$$

$$F_{int} = \begin{pmatrix} 4.112 + 0,64 \times 2 \cong 5,4 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -(4.112 + 0.64 \times 2) \cong -5,4 \end{pmatrix} \qquad F_{ext} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -6,4 \end{Bmatrix}$$

$$R = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{Bmatrix}$$

Deve-se proceder uma etapa iterativa aplicando-se o resíduo de modo a retomar ao nível F=6,4



$$\begin{bmatrix} 7,5 & 0 \\ 0 & 12,5 \end{bmatrix} \begin{pmatrix} u_7 \\ u_8 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad \Delta u_8 = -0,08$$

$$\delta = -(0,512 + 0,08) = -0,592$$

Barra 1 (= Barra 3)

$\Delta\varepsilon_1 = 2e{-4}$ $\quad \Delta\sigma^t = 0,2$

$\varepsilon_1 = 0,0013 + 2e{-4}$

$= 0.0015 = \varepsilon_3$

$\sigma_1^t = 1,28 + 0,2$

$= 1,48$

$f_1^t = 1,48 - 4 < 0$ OK

Barra 2

$\Delta\varepsilon_2 = 8e{-4}$ $\quad \Delta\sigma^t = 0,8$

$\varepsilon_2 = 5.12e{-3} + 8e{-4}$

$\varepsilon_2 = 0,0059$

$\sigma_2^t = 4.112 + 0,8$

$= 4,912$

$f_2 = 4,912 - (4 + 0.00101 \times 111.11)$

$= 0,7998 > 0$

$$\Delta\lambda_2 = \frac{0,7998}{1111.11} = 0,00072$$

$\varepsilon_2^p = 0,00101 + 0,00072 = 0,00173$

$\alpha_2 = 0,00173$

$\sigma_2 = 4,912 - 1000 \times 0,00072 = 4.192$

$\sigma_1 = \sigma_3 = 1,48$ $\quad \varepsilon_1 = \varepsilon_3 = 0,0015$

$\sigma_2 = 4,192$ $\quad \varepsilon_2 = 0,0059$

$\varepsilon_2^p = 0,00173$

$\alpha_2 = 0,00173$

$R_8 = (-4,192 - 2 \times 0,74) + 6,4$

$= -5.672 + 6,4$

$= 0,728$

Deve-se reaplicar o resíduo.

Porém...

Pelo que se percebe, o processo de convergência está lento.

Para acelerarmos, pode-se atualizar a rigidez da estrutura e refazermos o passo anterior.

Para barra 2,

$$E^{ep} = \frac{EK}{E+K} = 100$$

$$K_2 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

, de modo que a matriz de rigidez global, reduzida pelas condições de contorno, é dada por,

$$\begin{bmatrix} 7.5 & 0 \\ 0 & 3.5 \end{bmatrix} \begin{Bmatrix} u_7 \\ u_8 \end{Bmatrix} = \begin{Bmatrix} 0 \\ -1 \end{Bmatrix}$$

$u_8 = -0{,}286$

$\delta = -0{,}512 - 0{,}286$

$\delta \cong -0{,}8$

Barra 1:

$\Delta\varepsilon_1 = 0{,}000715$

$\varepsilon_1 = 0{,}0013 + 0{,}000715$

$\varepsilon_1 = 0{,}002$

$\sigma = 2$

$f = 2 - 4 < 0 \therefore ok$

Barra 2

$\Delta\varepsilon_2 = 0{,}00286$

$\varepsilon_2 = 0{,}00512 + 0{,}00286$
$= 0{,}008$

$\Delta\sigma^t = 1000 \times 0{,}00286 = 2{,}86$

$\sigma^t = 4{,}112 + 2{,}86 = 6{,}972$

$f^t = 6{,}972 - (4 + 111.11 \times 0{,}00101)$
$= 2{,}86 > 0$

$\Delta\lambda_2 = \frac{2{,}86}{1111.11}$

$\Delta\lambda_2 = 0{,}0026$

$\varepsilon^p_2 = 0{,}00101 + 0{,}0026$

$\varepsilon^p_2 = 0{,}00361$

$\sigma_2 = 6{,}972 - 1000 \times 0{,}0026$
$\cong 4{,}4$

8

$$R_1 = (4{,}4 + 2 \times 2) - 6{,}4$$
$$= 6{,}4 - 6{,}4 \approx \emptyset$$

Perceba que a atualização da matriz de rigidez da estrutura permite anular o resíduo em apenas uma iteração, para encruamento linear.