Dynamical Gauge Symmetry Breaking

A Collection of Reprints

Edited by

E. Farhi and R. Jackiw

World Scientific

INTRODUCTION

Two reasons explain why spontaneously broken symmetries have been used by theoretical physicists for building models of fundamental particle interactions. First, it is satisfying to suppose that the ultimate laws of Nature enjoy a high degree of symmetry, and that observed asymmetries arise because symmetric quantum field theory equations can have solutions that do not respect the symmetries. In this way the occurrence and also, one hopes, the amount of symmetry breaking can be predicted. Second, as a practical matter, it is important that theories with symmetry are less afflicted with ultraviolet divergences, and this is not lost when the symmetry is spontaneously broken. As a consequence, spontaneous symmetry breaking is an essential component of current models for strong, weak and electromagnetic interactions, as well as of speculative extensions of Einstein's gravity theory.

That a ground state of a quantal system need not possess the Hamiltonian's symmetries, and is therefore degenerate, was first appreciated in non-relativistic many-body physics and is realized in many condensed matter situations. Heisenberg,[1] Goldstone[2] and Nambu[3] made the seminal suggestion that this may also be true for the vacuum state of a relativistic quantum field theory, and that massless excitations — "Goldstone" particles — will necessarily be present. The suggestion came to fruition in the modern theory of chiral symmetry breaking, where the pion is identified as the [approximate] Goldstone Boson.

However, it was soon realized by many people,[4] that there are exceptions to Goldstone's theorem,[2] which is applicable to spontaneous symmetry breaking only when it is unaffected by gauge invariance, *i.e.* when a global symmetry is broken. In the presence of gauge invariant long-range forces, the "Higgs phenomenon"[4] replaces that of Goldstone; massless vector mesons acquire a mass, and the Goldstone particle disappears from the spectrum by combining with the transverse meson to give the third helicity state of a massive spin-1 particle. In many-body physics, this was appreciated earlier in connection with BCS theory,[5] and suggestions were made that such ideas are applicable to relativistic quantum field theory, as is seen from papers [1, 2, 5]. Whether one should describe the Higgs effect as spontaneously violating a gauge symmetry is still debated, but there is no doubt that the association of a massless vector meson with a gauge invariant theory is not essential.

The Higgs mechanism became phenomenologically important after Weinberg and Salam incorporated it in their unified model of electro-weak interactions,[6] where the massive vector mesons are identified with carriers of the weak force. Spontaneous symmetry breaking is achieved by postulating the existence of a "Higgs sector" in the theory. This sector is populated by scalar fields which acquire symmetry violating vacuum expectation values by virtue of the shape of a postulated scalar-field potential, either in the classical[4] or semi-classical[7] approximation. The Goldstone mechanism gives rise to a massless field, which then is shown to decouple from physical processes, since a gauge transformation unites it with the vector meson field.

While the Weinberg-Salam model is recognized to be a theoretical and experimental success, it is frequently believed that the Higgs mechanism, as described above, is an unsatisfactory feature of the theory. There is no experimental evidence for fundamental scalar fields, which are introduced in an *ad hoc* manner with *ad hoc* interactions solely to effect the symmetry breakdown. There is no other compelling theoretical reason for scalar fields [save supersymmetry, for which there is no experimental evidence]; indeed there are theoretical obstacles: asymptotic freedom may be easily spoiled,[8] there may be difficulties with quantum gravity,[9] and there is the "fine tuning" problem about which more will be said later. Fortunately, we may take an alternate route to massive, yet gauge invariant vector mesons.

Independent of the above development and even before discussions of the Higgs mechanism, Schwinger observed that gauge invariance need not preclude a mass for the gauge field, if the vacuum polarization tensor, $\Pi^{\mu\nu}(p)$, possesses a pole at light-like momenta, $p^2 = 0$. [3] The argument is straightforward. Let j^μ be the current which is the source for the gauge field $F^{\mu\nu}$.

$$\partial_\mu F^{\mu\nu} = j^\nu \tag{1}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \qquad \partial_\mu j^\mu = 0$$

The vacuum polarization tensor $\Pi^{\mu\nu}$ is the irreducible contribution to the current correlation function, which is necessarily transverse.

$$\Pi^{\mu\nu}(p) = (g^{\mu\nu}p^2 - p^\mu p^\nu)\Pi(p^2) \tag{2}$$

$$\int dx\, e^{ip\cdot x}\langle 0|Tj^\mu(x)j^\nu(0)|0\rangle = -i(g^{\mu\nu}p^2 - p^\mu p^\nu)\frac{\Pi(p^2)}{1-\Pi(p^2)}$$

It is clear that if $\Pi(p^2)$ has a pole at $p^2 = 0$ with positive residue μ^2, then the two-current correlation function and the gauge field propagator have a pole at $p^2 = \mu^2$. Since in most physical systems bound states are expected to exist and to produce poles at time-like momenta in $\Pi^{\mu\nu}$, one may suppose that for sufficiently strong binding, the mass of such a bound state will be reduced to zero, thus generating a mass for the vector meson without upsetting gauge invariance. Schwinger demonstrated his general ideas in 2-dimensional, massless spinor electrodynamics, which is explicitly solvable. $\Pi^{\mu\nu}$ does indeed have a pole at light-like momenta and the "photon" acquires a mass. [4]

Schwinger's approach has the advantage of making no reference to scalar fields, though the reason for a pole in the vacuum polarization tensor is unspecified. In the 2-dimensional example, the pole occurs "dynamically" without scalar fields; although in the end, after solving the model, it is found that the dynamics are trivial — there are no interactions between the massive vector mesons. The Higgs mechanism is another particular example of the more general Schwinger mechanism. While it is not usually phrased in this way, the Higgs phenomenon can also be seen producing light-like poles in the vacuum polarization tensor, as a consequence of the scalar fields' vacuum expectation value;[10] *i.e.* the poles are due to the Goldstone particles, which in the end decouple from physical processes.

Gauge invariant but massive vector fields can be described in yet another way, which is closer to ideas of many-body theory. An immediate consequence of Eq. (1) [in any number of dimensions] is a second order equation,

$$\Box F_{\mu\nu} = \partial_\mu j_\nu - \partial_\nu j_\mu. \tag{3}$$

If the gauge field $F_{\mu\nu}$ is massive, it must satisfy an equation where the d'Alembertian is supplemented by a mass term:

$$(\Box + \mu^2)F_{\mu\nu} = \text{non-linear and non-local terms.} \tag{4}$$

A comparison with (3) shows that the curl of the current must be proportional to the field strength, in order that the gauge field acquires a mass.

$$\partial_\mu j_\nu - \partial_\nu j_\mu = -\mu^2 F_{\mu\nu} + \cdots. \tag{5}$$

The dots stand for non-linear and non-local terms. Equivalently, it must be true that the current j_μ satisfies a London *Ansatz*:

$$j_\mu = -\mu^2 A_\mu + \partial_\mu \Theta + \cdots. \tag{6}$$

Here Θ is a gauge function which balances the gauge non-invariance of A^μ to produce a gauge invariant combination.

The question now arises whether the current-gauge potential proportionality of Eq. (6) is an operator identity, or whether it is true only for the solutions of the theory. In the Higgs mechanism it is the former. The current is constructed from scalar fields; in an Abelian example we have

$$j_\mu = ie\phi^*(\partial_\mu + ieA_\mu)\phi - ie\phi(\partial_\mu - ieA_\mu)\phi^*. \tag{7a}$$

When ϕ has a vacuum expectation value λ, Eq. (7a) also reads

$$j_\mu = -2e^2|\lambda|^2 A_\mu + \cdots, \tag{7b}$$

which verifies (6) with $\mu = \sqrt{2}|e\lambda|$.

For dynamical mass generation, the London *Ansatz* is established only after solving the theory. For example, in the 2-dimensional Schwinger model there is no *a priori* connection between the photon field and the current, which is bilinear in the massless Dirac fields ψ;

$$j_\mu = -e\bar\psi\gamma_\mu\psi. \tag{8}$$

However, by virtue of special properties of 2-dimensional Dirac algebra,

$$\varepsilon^{\mu\nu}\gamma_\nu = i\gamma^\mu\gamma_5, \qquad \gamma_5 = i\gamma^0\gamma^1, \tag{9}$$

the curl of the vector current is also the divergence of the axial vector current,

$$\partial_\mu j_\nu - \partial_\nu j_\mu = -\varepsilon_{\mu\nu}\varepsilon^{\alpha\beta}\partial_\alpha j_\beta = \varepsilon_{\mu\nu}\,\partial_\alpha j_5^\alpha, \qquad j_5^\alpha = e\bar\psi i\gamma^\alpha\gamma_5\psi. \tag{10a}$$

Moreover, just as in four dimensions,[11] the axial vector current is not conserved, even though the Fermions are massless; rather, there is an anomaly.[12]

$$\partial_\alpha j_5^\alpha = \frac{e^2}{2\pi}\varepsilon^{\mu\nu}F_{\mu\nu}. \tag{10b}$$

Thus, the London *Ansatz* is exact here,

$$\partial_\mu j_\nu - \partial_\nu j_\mu = -\frac{e^2}{\pi}F_{\mu\nu}, \tag{10c}$$

but the field-current identity emerges only after the model has been solved.[13]

Another example, in three dimensions, demonstrates the emergence of the London *Ansatz* dynamically, even though the theory is linear. In 3-dimensional space-time it is possible to introduce a gauge invariant mass for the gauge field.[14] The Lagrangian in an Abelian theory is

$$\mathcal{L} = -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \frac{\mu}{4}\varepsilon^{\mu\nu\alpha}F_{\mu\nu}A_\alpha. \tag{11}$$

The gauge invariant field equation

$$\partial_\mu F^{\mu\nu} + \frac{\mu}{2}\varepsilon^{\nu\alpha\beta}F_{\alpha\beta} = 0, \tag{12}$$

may be presented in the form (1) by identifying the second term in (12) with a "current":

$$j^\mu = -\frac{\mu}{2}\,\varepsilon^{\mu\alpha\beta}F_{\alpha\beta}. \tag{13a}$$

The curl of the above is evaluated with the help of (12).

$$\partial_\mu j_\nu - \partial_\nu j_\mu = \varepsilon_{\mu\nu\alpha}\varepsilon^{\alpha\beta\gamma}\partial_\beta\left(-\frac{\mu}{2}\,\varepsilon_{\gamma\delta\varepsilon}F^{\delta\varepsilon}\right)$$
$$= \mu\varepsilon_{\mu\nu\alpha}\,\partial_\beta F^{\beta\alpha} = -\mu^2 F_{\mu\nu}. \tag{13b}$$

The London *Ansatz* is again observed, but only for solutions to the equations, and not as an identity. [This model can be extended to a non-Abelian theory.][14]

For a truly general dynamical model in four dimensions, Schwinger's viewpoint and the London *Ansatz* may be reconciled in the following way. [5] To effect the derivation, we introduce an external gauge potential A_e^μ, coupled to the dynamical current j_μ, and compute the latter's vacuum expectation value. In momentum space, the formula for the induced current is

$$\langle j_\mu(p)\rangle = -(g_{\mu\nu}p^2 - p_\mu p_\nu)\,\frac{\Pi(p^2)}{1-\Pi(p^2)}\,A_e^\nu(p). \tag{14a}$$

We adopt the Landau gauge, and conclude that (14a) corresponds to an induced potential.

$$\langle A^\mu(p)\rangle = \frac{\Pi(p^2)}{1-\Pi(p^2)}\,A_e^\mu(p). \tag{14b}$$

The total potential

$$A_T^\mu = \langle A^\mu\rangle + A_e^\mu \tag{15a}$$

is given by

$$A_T^\mu = \frac{1}{1-\Pi(p^2)}\,A_e^\mu. \tag{15b}$$

Consequently, upon comparing (14a) with (15b), we see that

$$\langle j^\mu(p)\rangle = -p^2\Pi(p^2)A_T^\mu(p). \tag{16}$$

When $\Pi(p^2)$ has a pole at $p^2 = 0$, the current is indeed proportional to the total potential, for light-like momenta.

To be sure, in a theory with interactions, the above general argument is considerably less sharp than those in the dynamically trivial 2- and 3-dimensional examples, whose simplicity derives presumably from the absence of interactions. Nevertheless, it is interesting to identify a property of the two lower dimensional models, which has thus far not been associated with dynamical mass generation for 4-dimensional gauge fields. In both cases, the emergence of massive vector fields is due to mathematical/topological structures that can be constructed in gauge theories: in two dimensions, the mass arises because the axial vector current has an anomalous divergence proportional to $(1/2\pi)\varepsilon^{\mu\nu}F_{\mu\nu}$, which is the 2-dimensional Pontryagin density; in three dimensions, the action for the mass term is proportional to $\int d^3x\,\varepsilon^{\mu\nu\alpha}F_{\mu\nu}A_\alpha$, which is the 3-dimensional Chern-Simons secondary characteristic class.[15]

Schwinger's [or London's] idea, that a gauge invariant mass is equivalent to a light-like pole in the momentum space vacuum polarization, can be useful for model building only if assurance can be given that such a pole is indeed present. [Realistic models cannot be solved!] As already mentioned, the Higgs mechanism with scalar fields guarantees this in the classical or semi-classical approximation. But one wants to dispense with scalar fields, and to this end, the suggestion was made in papers [6–8] that the *dynamical* Goldstone mechanism be combined with a gauge theory to give *dynamical* gauge invariant masses to vector mesons. The Goldstone phenomenon would create the pole in $\Pi^{\mu\nu}$ and gauge invariance would ensure that no physical massless particles are present.

Although, most frequently, Goldstone's theorem is exemplified by a scalar field acquiring a symmetry breaking vacuum expectation, Nambu in his initial papers on the subject[3] did not use scalar fields. Rather, the symmetry breaking order parameter is a vacuum expectation value of a Fermion bilinear — analogous to the pair condensates of many-body theory — and the Goldstone Boson is a Fermion anti-Fermion bound state. Moreover, the pion — the only physical candidate for a Goldstone Boson — is not an elementary excitation of a fundamental scalar field; rather, it is formed by quark anti-quark binding. Thus, only a dynamical Goldstone Boson is relevant to chiral symmetry breaking, and it is plausible that a similar dynamical effect can replace the elementary scalar fields of the conventional Higgs method.

It remained to be shown that the massless bound state decouples, as in the Higgs model. This was readily done for various Abelian theories in [6–8] and for a non-Abelian theory in [9]. These investigations establish that 4-dimensional gauge fields can acquire masses without violating gauge invariance, and without introducing scalar fields; apparently dynamical gauge symmetry breaking works exactly the same way as in the Higgs model with scalar fields.

Yet it is curious that there remains in lacuna in our understanding of the dynamical mechanism. In the Higgs model, one useful description of symmetry breaking employs the "unitary gauge."[16] Conceptually, this may be the most satisfactory approach since the Goldstone Boson never appears, and no symmetry is broken. Indeed it is also true that the "unitary gauge" is not even a choice of gauge; rather, for any gauge choice, one may [functionally] integrate the gauge degrees of freedom, leaving a unitary Lagrangian whose fields correspond to physical degrees of freedom.[17] Such a description has not been given in the dynamical framework. Indeed much effort is expended on attempts to establish that massless bound states arise from the dynamics; yet in the end these states are not in the physical spectrum. One should expect that questions about binding of decoupled Goldstone particles must be irrelevant to gauge meson mass generation.

With the exception of an important result about the vector meson mass, to be mentioned below, phenomenological applications are not attempted in the early papers; the models studied there are too simple to be physically relevant. Rather, discussion concerns whether a zero mass bound state does indeed occur. A formalism for studying the vacuum expectation value of composite operators is developed in [10], which is based on similar approaches to many-body theory.[18] However, no definite conclusions could be reached, owing to well-known difficulties of arriving at non-perturbative results in quantum field theory. Nevertheless, some useful qualitative observations can be made.[19]

The existence of any bound state, and in particular of a zero-mass bound state, can be examined by a homogeneous integral equation [*e.g.* a Bethe-Salpeter equation],

$$P(p) = g^2 \int dk\, K(p, k) P(k) \tag{17}$$

where P is an amplitude describing the bound state, K the [Bethe-Salpeter] kernel, and g the coupling constant. For (17) to have a non-trivial solution, it is necessary that in some heuristic sense $g^2 \int K = 1$. In general, if the kernel is Fredholm, this implies a quantization condition on the coupling constant; no bound state will exist for arbitrarily weak coupling. However, in field theory the kernel is not likely to be Fredholm; the integral $\int K$ probably diverges, even though $\int KP$ converges. The growth of the kernel may be in the ultraviolet region or in the infrared; *viz*. the bound state may exist because of ultraviolet or infrared instabilities of the theory. [Equivalently, one can absorb the coupling constant in the kernel, and describe the coupling "constant" as growing in the ultraviolet region, or in the infrared.] In this circumstance, (17) will always have a non-trivial solution, and no eigenvalue condition is required.

In the early investigations [6–8, 10], ultraviolet unstable [asymptotically non-free] theories were considered, and some evidence for a bound state, arising from the ultraviolet instability was found. Today, with dynamics provided by non-Abelian gauge theories, one expects that a bound state will arise from the strong infrared forces.

The distinction between symmetry breaking arising from the ultraviolet or from the infrared has important bearing on symmetry restoring phase transitions at high temperatures, as in the early universe. It is now widely appreciated that the Higgs mechanism is undone at some finite temperature.[20] Finite temperature modifies a field theory in the infrared region, hence a dynamically broken symmetry can be restored only if the forces that bring about the breaking are due to infrared instabilities. On the other hand, ultraviolet dominated dynamical breaking should not be influenced by temperature. This is exemplified in the Schwinger model: the axial vector anomaly — an ultraviolet effect—is here responsible for mass generation. Correspondingly, the massive photon persists in the Schwinger model at all temperatures.[21] Since symmetry breaking in realistic physical models is expected to be an infrared-dominated phenomenon, we may expect transitions to phases with different symmetries as the temperature is varied.

The application of dynamical gauge symmetry breaking to the electro-weak interaction begins with the observation that the relationship between the vector meson M, and the symmetry breaking parameters is the same regardless whether scalar fields or Fermion condensates are responsible for symmetry breaking. [6–8]

$$M = |gF| \tag{18}$$

Here g is the gauge coupling constant and F describes the coupling of the massless Goldstone Boson to the broken gauge current. [In theories with scalar fields, F also equals the vacuum expectation value of the scalar field.] For these vector mesons to mediate the weak interactions, g^2/M^2 must equal the Fermi constant $G_F \approx 10^{-5}$ $(\text{GeV})^{-2}$, so $|F| \approx 300$ GeV, which is about 3000 times larger than $f_\pi \approx 100$ Mev— the chiral symmetry breaking parameter of ordinary hadronic physics. Therefore, the characteristic scale of interactions responsible for a possible dynamical breaking of electro-weak symmetry should be about 3000 times the quantum chromodynamical (QCD) scale.

It is very instructive to see how QCD acting alone would break the weak interaction gauge symmetries.[22] Consider QCD with two quarks, u and d, and imagine that in the Lagrangian the quark mass terms are zero so that the Lagrangian has an exact $SU(2)_{L(EFT)} \times SU(2)_{R(IGHT)}$ symmetry. [There are additional U(1) symmetries which do not concern us here.] It is commonly supposed that this theory spontaneously breaks its chiral symmetries and $SU(2)_L \times SU(2)_R$ is reduced to SU(2), resulting in three massless Goldstone Bosons — the pions. [In the real world the pions are not strictly massless because there are small quark mass terms in the Lagrangian, so the symmetries are not exact.] Now consider the effect of turning on the standard $SU(2)_L \times U(1)$ electro-weak gauge interactions and ignore all other sources of symmetry breaking. The electro-weak gauge Bosons couple to generators of the $SU(2)_L \times SU(2)_R$ symmetry group which are spontaneously broken. The broken generators correspond to currents which couple to the pions with strength f_π. Therefore, the electro-weak gauge Bosons get masses gf_π and the pions are now in the spectrum only as longitudinal components of massive vector Bosons.

Obviously, this cannot be a picture of the real world. As discussed before, the scale is off by a factor of 3000 and physical pions are after all observed. [However, regardless of the source for weak interaction symmetry breaking, the Goldstone Boson which is the longitudinal component of the weak interaction vector meson has a small $\approx f_\pi/300$ GeV admixture of the QCD pion, and the physical pion has a small admixture of the weak-interaction Goldstone Boson.]

Weinberg [11] speculated that a new strong interaction, here called "hypercolor", is in fact responsible for the electro-weak symmetry breaking. The hypercolor force is assumed to have many properties in common with QCD: it acts on a set of hypercolor Fermions and induces chiral symmetry breakdown. A set of Goldstone Bosons is produced and those which couple to the electro-weak gauge fields disappear from the spectrum. Weinberg has a general discussion of the fate of all the Goldstone Bosons, including those which do not disappear, and he also gives a formalism for deciding into which direction in group space will the symmetry breaking parameters point. (The application of these ideas about "vacuum alignment" to a variety of semi-realistic models is carried out in papers [21, 22].[23])

The standard electro-weak model[6] with fundamental scalar fields produces charged vector Bosons $W^\pm$, and a neutral vector Boson Z; their mass ratio is determined by the electro-weak gauge coupling constants.

$$\frac{M_W}{M_Z} = \cos\theta = \frac{g_2}{\sqrt{g_1^2 + g_2^2}} \tag{19}$$

Here g_2 is the $SU(2)_L$ gauge coupling and g_1 is the U(1) hypercharge coupling. The exchange of these vector mesons leads to a low energy effective four-Fermion interaction characterized by the scale factor G_F and one dimensionless parameter, $\cos\theta$. This low energy effective interaction has been very successful in fitting all weak interaction data and within the context of the standard model this success can be seen as verification of the mass ratio (19).

Dynamical symmetry breaking can be a serious alternative to the standard model's Higgs fields only if the mass ratio (19) is guaranteed. Therefore, it was crucial to show that Eq. (19) is true when the hypercolor Fermion sector possesses certain symmetries. [12, 13][24] In fact, this symmetry is found in QCD with two massless quarks; it is the residual SU(2) symmetry previously mentioned. Again, QCD alone cannot account for the weak interaction symmetry breaking, but it would produce a W and Z with the correct mass ratio. But hypercolor, which by assumption parallels QCD at a higher mass scale, would also lead to (19), and this realization [12, 13] ushered intense research and model building. The conclusion of this activity is that a hypercolor interaction, at a scale of approximately 300 GeV, along with a set of hypercolor Fermions having the right symmetry properties, can successfully replace the scalar sector in the standard electro-weak theory.[25] The usual weak interaction phenomenology of quarks, leptons and intermediate vector mesons is reproduced at energies below 300 GeV, with the exception, which we shall soon discuss, that a new source for Fermion masses must be introduced. Above 300 GeV, hypercolor dynamics would have obvious experimental implications[26] and physics would be quite different from what is expected in the standard scalar theory.

The replacement of the scalar sector by gauge interactions and Fermions can also solve an important problem faced by "grand" unified models of strong and electro-weak forces.[27] In these theories there is a fundamental scale,[28] M_{GUT}, which is extraordinarily large, close or equal to the Planck scale of 10^{19} GeV, at which the strong force becomes unified with the electro-weak forces. The Higgs scalar field used for electro-weak symmetry breaking has couplings to particles interacting at this scale and one expects the dimensionful ϕ^2 coupling to be on the order of M_{GUT}^2. The vacuum expectation value of the scalar field will then be near M_{GUT} and so will the masses of the W and Z. One way of avoiding this problem — the so-called "hierarchy problem"[29]—is to adjust carefully the parameters of the scalar sector so that the relatively small numbers characterizing electro-weak symmetry breaking emerge due to cancellations between certain large numbers of the grand unification scenario. However, this "fine tuning" appears unnatural and is unstable against tiny changes in values of the parameters at the unification scale.

A more acceptable solution is achieved by eliminating the scalar sector and replacing it by bound Fermion states. The scale of symmetry breaking is now the scale at which the dynamics become strong, *i.e.* the scale at which the dimensionless gauge coupling becomes big. There are no dimensionful couplings, as there are in scalar theories, to set the scale of the dynamics. The dimensionless coupling constant of a non-Abelian gauge theory varies with the logarithm of the energy and it may take a huge change of energy before the coupling becomes large. For example, the QCD coupling constant becomes strong near 1 GeV — 15 orders of magnitude below a typical M_{GUT} of 10^{15} GeV — yet this seems natural. For these reasons, dynamical symmetry breaking is much more attractive in grand unified theories than the usual symmetry breaking by scalar fields.

The scalar sector of the standard model has a function beyond generating massive vector mesons. By introducing Yukawa couplings between scalar fields and Fermion bilinears, the Fermions are provided with masses, coming from non-vanishing vacuum expectation values of scalar fields. The dimensionless Yukawa couplings can have arbitrary magnitudes, and it is possible to arrive at mass terms for all observed Fermions. These Yukawa couplings explicitly break Fermion chiral symmetries; we know that *explicit* chiral symmetry breaking is required because the pion is not actually massless, *i.e.* chiral symmetry is not an exact symmetry of the Lagrangian.

In hypercolor theories, there are no Yukawa couplings [no scalars], so we must find a new source for explicit chiral symmetry breaking. The ordinary gauge interactions of SU(3) × SU(2) × U(1), together with hypercolor will not do, since gauge interactions [at the Lagrangian level] preserve chirality.

The most popular method for producing chiral symmetry breaking interactions in dynamical models is to enlarge the hypercolor gauge group into a larger group, called "extended" hypercolor, which at an energy scale $\Lambda > 300$ GeV breaks down to hypercolor. [14, 15] The Fermion representations of the extended hypercolor group are chosen so that ordinary and hypercolored Fermions belong to the same irreducible representations of extended hypercolor. When extended hypercolor breaks down, the massive gauge Bosons corresponding to broken generators mediate transitions between ordinary and hypercolored Fermions. Total chirality is preserved but not the separate chiralities of ordinary and hypercolored Fermions. These new interactions can lead to ordinary Fermion masses on the order of $(300\ \text{GeV})^3/\Lambda^2$.[30] For $\Lambda \approx 10$ TeV reasonable Fermion masses are obtained. The price paid is yet another

interaction above the weak interaction scale.

No satisfactory model of extended hypercolor has been found. One natural and minimal idea is to let the extended hypercolor group unify the hypercolor group with the SU(3) × SU(2) × U(1) gauge group. The hypercolor group breaks off from the other subgroups at the scale Λ and produces the required massive extended hypercolor Bosons which give mass to the ordinary Fermions. These ideas are illustrated in a toy model based on SU(7). [16] The difficulties encountered in this model are typical of the difficulties of the whole approach.[31] The cause of symmetry breaking at the scale Λ is unknown, and the scale Λ itself seems arbitrary. Also, the Fermion masses which are predicted do not agree with experiment.

Even though a realistic model has not been constructed, it is still important to look for low energy [well below 300 GeV] experimental signatures of hypercolor. Many of the models considered have a large number of hypercolor Fermions which typically come in "families." [A "family" is a set of Fermions with the gauge quantum numbers of the up and down quarks, the electron and the neutrino.] Chiral symmetry breaking in the hypercolor sector will lead to many Goldstone Bosons while only three are required to give masses to the $W^{\pm}$, and Z mesons. Usually the remaining Goldstone Bosons are only approximate or "pseudo"-Goldstone Bosons, *i.e.* like the pion their masses are exactly zero only when some parameters are exactly zero. These pseudo-Goldstone Bosons can have masses well below 300 GeV, and might be observable at present accelerators or in the near future. Much effort has gone into estimating the masses, production mechanisms and decay properties of these particles. [15, 17–22][32] Typically the lightest of these states is electrically and color neutral and mass estimates range from a few GeV to tens of GeV's.

Of course, the standard scalar theory also predicts a spin-0 particle whose mass is thought to be somewhere below 300 GeV. If a neutral spin-0 particle is discovered, it will be very important to study its decay characteristics carefully to see whether it is the Higgs scalar, a hypercolor pseudo-Goldstone Boson or something more exotic.

The extended hypercolor scale is roughly 10 TeV. However, any interactions whose scale is less than 100 TeV may lead to tiny but unwanted observable effects in the $K_L - K_S$ mass difference or in rare decay modes of kaons and other particles.[33] Extended hypercolor theories which are complex enough to produce non-trivial Cabibbo angles generally also predict these undesired effects. [15, 23] This is one of the most troublesome difficulties facing hypercolor model builders who try to use extended hypercolor to explain the Fermion masses. In addition, the extended hypercolor theories involve many new particles and interactions with complicated postulated dynamics. This imposing structure must be contrasted with the scalar interactions it is attempting to replace. In models with scalar fields the Fermion mass matrix is simply determined by arbitrary Yukawa couplings. No problems arise but no understanding is gained. Perhaps the origin of Fermion masses is in fact beyond the reach of any weak interaction theory.

Four-dimensional strongly interacting theories have not been solved exactly. Much of our insight comes from the *observed* behavior of hadrons, which we attribute to QCD. Our assumptions about symmetry breakdown due to hypercolor are plausible extensions of what we believe happens in QCD, but all this is still only assumption. For general gauge theories with arbitrary Fermion representations, the interesting dynamical conjecture has been made that the energetics of Fermion condensation is determined by one gluon exchange. This postulate implies that the Fermion condensate which forms is the one with the most negative associated group theory factor — an idea that can be applied to condensates which break global symmetries as well as to condensates which are not gauge singlets. For the latter, the gauge group can be said to "break itself" through its Fermion condensates.[34]

A set of algebraic [rather than dynamical] conditions has been established which must be satisfied by any theory of strongly interacting Fermions not undergoing spontaneous symmetry breakdown. [24] These conditions involve evaluating triangle anomaly diagrams,[11] first using the Fermion fields in the Lagrangian and then using the bound, gauge singlet states which are the particles in the physical spectrum. If there is no symmetry breakdown, some of these bound states will be massless composite Fermions and their contribution to the triangle anomaly must equal that coming from the fundamental Fermions. If the appropriate massless composite Fermions cannot be constructed, the symmetry must be spontaneously broken. These conditions are remarkable because they do not require a solution of the whole theory and they provide a simple algebraic criterion for deciding a dynamical issue.[35]

One example of a theory which does not necessarily break its chiral symmetries and can produce massless composites is QCD with two massless quarks. [24] [Actually the gauge group can be SU(N) with any odd N, not just $N = 3$.] With more than two quark flavors, and with additional assumptions about the decoupling of heavy quarks, the chiral symmetries must be spontaneously broken and this may help explain why chiral symmetry is broken in the real world.[36]

Quarks and leptons may themselves be composites of other particles bound at some scale B. We know that B is probably greater than 500 GeV from the precise agreement of the experimental and theoretical values for the muon's magnetic moment. In terms of the large binding scale B, ordinary quarks and leptons are effectively massless. If we suppose that indeed they are approximately massless bound states of constituents interacting through a force whose characteristic scale is B, then they should satisfy the anomaly conditions. Using Fermions and gauge Bosons as constituents, people have searched for and constructed other solutions to the anomaly conditions which result in massless composite Fermions. [25, 26][37] Unfortunately none of these solutions has a spectrum which resembles that of the real world.

A realistic solution can be constructed, which surprisingly has electro-weak interactions without symmetry breakdown. [27] The constituents are Fermions, gauge Bosons and scalars, although the scalars can be replaced by hypercolor Fermion anti-Fermion bound states.[38] This model uses the same Lagrangian as the standard Weinberg-Salam electro-weak theory but assumes that there is no vacuum expectation value for the scalar field. [If the scalar field is replaced by a bound state the assumption is that no symmetry breakdown is induced by the hypercolor force.] The $SU(2)_L$ interaction is unbroken and becomes strong, binding non-singlet fields into $SU(2)_L$ singlet, massless composite Fermions. The scale of the binding is assumed to be the weak interaction scale $G_F^{-1/2} \approx 300$ GeV. The massless composite Fermions are all left-handed and there is exactly one left-handed bound state for each left-handed quark and lepton. The right-handed fields do not feel the $SU(2)_L$ force and do not participate in the new strong dynamics.

In this model, the weak interactions arise as a consequence of the bound state structure of the left-handed Fermions. By making certain dynamical assumptions, it is possible to show that the low energy effective four-Fermion interaction produced by the new dynamics exactly matches the low energy effective four-Fermion interaction of the standard model and therefore can account for all weak interaction data. This raises the interesting question of what really is the experimental evidence for spontaneously broken gauge theories. The low energy effective interaction of the confining weak interaction theory is mediated by vector Bosons whose masses are not governed by Eqs. (18) and (19). The observation of a W and Z obeying (18) and (19), where the gauge couplings and F have been independently determined, would be the ultimate confirmation of the Weinberg-Salam weak interaction model, based on a spontaneously broken gauge theory.

The anticipated discovery of the $W^{\pm}$ and Z mesons will still not answer the question whether they are associated with dynamical symmetry breakdown or with fundamental scalar fields. Experimentalists must search for evidence of possible new dynamics near or above the weak interaction scale. Experiments are now being done at energies of 500 GeV in the center of mass, and higher energies will soon be reached. We shall then know much more about the nature of weak interactions, and we hope that this experimental information will also advance theoretical ideas about gauge symmetry breaking.

REFERENCES

Numbers in brackets refer to reprinted papers.

1. H. Dürr, W. Heisenberg, H. Mitter, S. Schlieder and K. Yamazaki, "Zur theorie der elementarteilchen," Zeit. Naturforsch. **14a,** 441 (1959).
2. J. Goldstone, "Field theories with superconductor solutions," Nuovo Cimento **19,** 154 (1961); J. Goldstone, A. Salam and S. Weinberg, "Broken Symmetries," Phys. Rev. **127,** 965 (1962).
3. Y. Nambu and G. Jona-Lasinio, "Dynamical model of elementary particles based on an analogy with superconductivity," Phys. Rev. **122,** 345 (1961).
4. F. Englert and R. Brout, "Broken symmetry and the mass of gauge vector bosons," Phys. Rev. Lett. **13,** 321 (1964); P. Higgs, "Broken symmetries, massless particles and gauge fields," Phys. Lett. **12,** 132 (1964); and "Broken symmetries and the masses of gauge bosons," Phys. Rev. Lett. **13,** 508 (1964); G. Guralnik, C. Hagen and T. Kibble, "Global conservation laws and massless particles," Phys. Rev. Lett. **13,** 585 (1964); P. Higgs, "Spontaneous symmetry breakdown without massless bosons," Phys. Rev. **145,** 1156 (1966); T. Kibble, "Symmetry breaking in non-Abelian gauge theories," Phys. Rev. **155,** 1554 (1967).
5. A historical account is given by P. Anderson, "Uses of solid state analogies in elementary particle theory," in *Gauge Theories and Modern Field Theory*, edited by R. Arnowitt and P. Nath (MIT Press, Cambridge, MA, 1976).
6. S. Weinberg, "A model of leptons," Phys. Rev. Lett. **19,** 1264 (1967); A. Salam, "Weak and electromagnetic interactions," in *Elementary Particle Theory*, edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968).
7. S. Coleman and E. Weinberg, "Radiative corrections as the origin of spontaneous symmetry breaking," Phys. Rev. **D 7,** 1883 (1973).
8. Increasing the number of Higgs fields decreases the strength of asymptotic freedom. Also, if the Higgs-gauge field coupling is sufficiently strong, the ultraviolet unstable Higgs self-couplings can destabilize the high-energy behavior of the Yang-Mills sector.
9. S. Hawking, D. Page and C. Pope, "Quantum gravitational bubbles," Nucl. Phys. **B170 [FS1],** 283 (1980).
10. An analysis of the Higgs effect from Schwinger's point of view is given by R. Jackiw, "Dynamical symmetry breaking," in *Laws of Hadronic Matter*, edited by A. Zichichi (Academic Press, New York, NY, 1975).
11. J. Bell and R. Jackiw, "A PCAC puzzle: $\pi^0 \to 2\gamma$ in the σ model," Nuovo Cimento **60,** 47 (1969); S. Adler, "Axial-vector vertex in spinor electrodynamics," Phys. Rev. **177,** 2426 (1969). For a review, see R. Jackiw, "Field

theoretic investigations in current algebra," in *Lectures on Current Algebra and Its Applications*, by S. Treiman, R. Jackiw and D. Gross (Princeton, Princeton, NJ, 1972).
12. K. Johnson, "γ_5 invariance," Phys. Lett. **5,** 253 (1963).
13. The analysis of the Schwinger model in terms of the axial vector anomaly is given in Ref. 10.
14. R. Jackiw and S. Templeton, "How super-renormalizable interactions cure their infrared divergences," Phys. Rev. **D 23,** 2291 (1981); J. Schonfeld, "A mass term for three-dimensional gauge fields," Nucl. Phys. **B185,** 157 (1981); S. Deser, R. Jackiw and S. Templeton, "Three-dimensional massive gauge theories," Phys. Rev. Lett. **48,** 975 (1982) and "Topologically massive gauge theories," Ann. Phys. (NY) **140,** 372 (1982).
15. Topological mechanisms for mass generation are discussed by R. Jackiw, "Gauge invariance and mass, III," in *Asymptotic Realms of Physics,* edited by A. Guth, K. Huang and R. Jaffe (MIT Press, Cambridge, MA, 1983).
16. The "unitary gauge" was originally used to show that spontaneous symmetry breaking in gauge theories gives rise to massive vector mesons, instead of Goldstone Bosons; see Ref. 4. A general exposition is in S. Weinberg, "General theory of broken symmetries," Phys. Rev. **D 7,** 1068 (1973).
17. L. Dolan and R. Jackiw, "Gauge invariant signal for gauge symmetry breaking," Phys. Rev. **D 9,** 2904 (1974).
18. J. Luttinger and J. Ward, "Ground-state energy of a many fermion system, II," Phys. Rev. **118,** 1417 (1960); C. deDominicis and P. Martin, "Stationary entropy principle and renormalization in normal and superfluid systems, I and II," J. Math. Phys. **5,** 14 and 31 (1964).
19. For reviews of early investigations, together with exhortations that dynamical mechanisms be used for gauge symmetry breaking in electro-weak models, see M. Bég and A. Sirlin, "Gauge theories of weak interactions," Ann. Rev. of Nucl. Sci. **24,** 379 (1974), and Jackiw in Ref. 10.
20. D. Kirzhnits and A. Linde, "Macroscopic consequences of the Weinberg model," Phys. Lett. **42B,** 471 (1972); L. Dolan and R. Jackiw, "Symmetry behavior at finite temperature," Phys. Rev. **D 9,** 3320 (1974); S. Weinberg, "Gauge and global symmetries at high temperature," Phys. Rev. **D 9,** 3357 (1974). For a review see A. Linde, "Phase transitions in gauge theories," Rep. Prog. Phys. **42,** 389 (1979).
21. Dolan and Jackiw, Ref. 20.
22. T. Hagiwara and B. Lee, "Proton neutron mass difference in a unified gauge model of leptons and hadrons," Phys. Rev. **D 7,** 459 (1973); M. Weinstein, "Can all hadronic symmetry breaking be due to weak interactions," Phys. Rev. **D 7,** 1854 (1973) and "Conserved currents, their commutators and the symmetry structure of renormalizable theories of electromagnetic, weak and strong interactions," Phys. Rev. **D 8,** 2511 (1973); also L. Susskind in [13].
23. For earlier work on vacuum alignment see R. Dashen, "Some features of chiral symmetry breaking," Phys. Rev. **D 3,** 1879 (1971); F. Wilczek and A. Zee, "Orientation of the weak interaction with respect to the strong interaction," Phys. Rev. **D 15,** 3701 (1977).
24. In Nature, these symmetries are only approximate. Even before the symmetry was identified, and its relevance to maintaining Eq. (19) was realized, the effect of Fermion mass differences which can modify Eq. (19) was studied by M. Veltman, "Limit on mass differences in the Weinberg model," Nucl. Phys. **B123,** 89 (1977); see also P. Sikivie, L. Susskind, M. Voloshin and V. Zakharov, "Isospin Breaking in technicolor models," Nucl. Phys. **B173,** 189 (1980).

25. Recent reviews of hypercolor theories include:
K. Lane and M. Peskin, "An introduction to weak interaction theories with dynamical symmetry breaking," in *XV Rencontre de Moriond, Vol II — Electroweak Interactions and Unified Theories*, edited by T. Thanh Van (Edition Frontières, Gif sur Yvette, France, 1980); P. Sikivie, "An introduction to technicolor," CERN preprint TH 2951 (1980), Proceedings of Varenna Summer School (1980); E. Farhi and L. Susskind, "Technicolor," Phys. Rep. **74,** No.3, 277 (1981).
26. Susskind in [13]; F. Hayot and O. Napoly, "Detecting a heavy colored object at the FNAL tevatron," Zeit. Phys. **C7,** 299, (1981); S. Raby, "The TeV Picture," in *The Second Workshop on Grand Unification*, edited by J. Leveille, L. Sulak and D. Unger (Birkhäuser, Boston, MA, 1981); G. Girardi, P. Mery, P. Sorba, "Heavy quark jets as technicolor signatures in pp and p anti-p collisions," Nucl. Phys. **B195,** 410 (1982).
27. J. Pati and A. Salam, "Unified lepton-hadron symmetry and a gauge theory of the basic interactions," Phys. Rev. **D8,** 1240 (1973); H. Georgi and S. Glashow, "Unity of all elementary particle forces," Phys. Rev. Lett. **32,** 328 (1974).
28. H. Georgi, H. Quinn and S. Weinberg, "Hierarchy of interactions in unified gauge theories," Phys. Rev. Lett. **33,** 451 (1974).
29. For clear expositions see Susskind in [13] and 't Hooft in [24].
30. K. Lane, "Asymptotic freedom and Goldstone realization of chiral symmetry," Phys. Rev. **D 10,** 2605 (1974); H. Politzer, "Effective quark masses in the chiral limit," Nucl. Phys. **B117,** 397 (1976).
31. For a sampling of other models see:
S. Dimopoulos, S. Raby and P. Sikivie, "Problems and virtues of scalarless theories of electroweak interactions," Nucl. Phys. **B176,** 449 (1980); B. Holdom, "A realistic model with dynamically broken symmetries," Phys. Rev. **D 23,** 1637 (1981); M. Dine, W. Fischler and M. Srednicki, "Super symmetric technicolor," Nucl. Phys. **B189,** 575 (1981); S. Dimopoulos and S. Raby, "Super color," Nucl. Phys. **B192,** 353 (1981);
32. A partial list other papers includes:
G. Gounaris and A. Nicolaidis, "Technicolor versus Higgs signatures in e^+e^- collisions," Phys. Lett. **102B,** 144 (1981); and "Discriminating technicolor from Higgs breaking," Phys. Lett. **109B,** 221 (1982); S. Dimopoulos, S. Raby and G. Kane, "Experimental predictions from technicolor theories," Nucl. Phys. **B182,** 77 (1981); S. Chadha and M. Peskin, "Implications of chiral dynamics in theories of technicolor 1. elementary couplings, and 2. the mass of P^+," Nucl. Phys. **B185,** 61 and **B187,** 541 (1981); B. Holdom, "A phenomenological lagrangian from hypercolor," Phys. Rev. **D 24,** 157 (1981); J. Grifols, "Technicolor in gamma gamma processes." Phys. Lett. **102B,** 277 (1981); F. Hayot, "Technicolor pseudo-Goldstone boson couplings to gluon fields," Nucl. Phys. **B191,** 82 (1981); A. Ali, H. Newman and R. Zhu, "Production of charged hyperpions in e^+e^- annihilation," Nucl. Phys. **B191,** 93 (1981); K. Lane, "Hyperpions at the Z," Workshop on Z Physics (Ithaca, NY, 1981).
33. Eichten and Lane in [15]; R. Cahn and H. Harari, "Bounds on the masses of neutral generation-changing gauge bosons," Nucl. Phys. **B176,** 135 (1981).
34. J. Cornwall, "Spontaneous symmetry breaking without scalar mesons. II," Phys. Rev. **D 10,** 500 (1974); Eichten and Feinberg in [9]; H. Georgi, "Towards a Grand Unified Theory of Flavor," Nucl. Phys. **B156,** 126 (1979); S. Raby, S. Dimopoulos and L. Susskind, "Tumbling Gauge Theories," Nucl. Phys. **B169,** 373 (1980). See also some relevant numerical studies by J. Kogut, M. Stone, H.

Wyld, J. Shigemitsu, S. Shenker and D. Sinclair, "Scales of chiral symmetry breaking in quantum chromodynamics," University of Illinois preprint, ILL-(TH)-82-5.

35. A. Zee, "From baryons to quarks through π^0 decay," Phys. Lett. **95B,** 290 (1980); Y. Frishman, A. Schwimmer, T. Banks and S. Yankielowicz, "The axial anomaly and the bound state spectrum in confining theories," Nucl. Phys. **B177,** 157 (1981); S. Coleman and B. Grossman, "'t Hooft consistency condition as a consequence of analyticity and unitarity," Harvard Preprint, HUTP-82/A009.
36. The additional assumptions, which go beyond the strictly algebraic constraints imposed by the axial vector anomaly, have been critically surveyed by J. Preskill and S. Weinberg, "Decoupling constraints on massless composite particles," Phys. Rev. **D 24,** 1059 (1981).
37. Some further examples can be found in:
R. Barbieri, L. Miani and R. Petronzio, "Quarks and leptons as composite states of confined $O(n)$ preons," Phys. Lett. **96B,** 63 (1980); I. Bars and S. Yankielowicz, "Composite quarks and leptons as solutions of anomaly constraints," Phys. Lett. **101B,** 159 (1981); R. Casalbouni and R. Gatto, "Composite description for quarks and leptons from a geometrical picture of complementarity," Phys. Lett. **103B,** 113 (1981); I. Bars, "Spontaneous chiral symmetry breaking in quantum chromodynamics," Phys. Lett. **109B,** 73 (1982).
38. L. Abbott, E. Farhi and A. Schwimmer, "A confining model of the weak interactions in technicolor," MIT Preprint, CTP #962, 1981; Nucl. Phys. (in press).

I. PRE-HISTORY