Campo acústico

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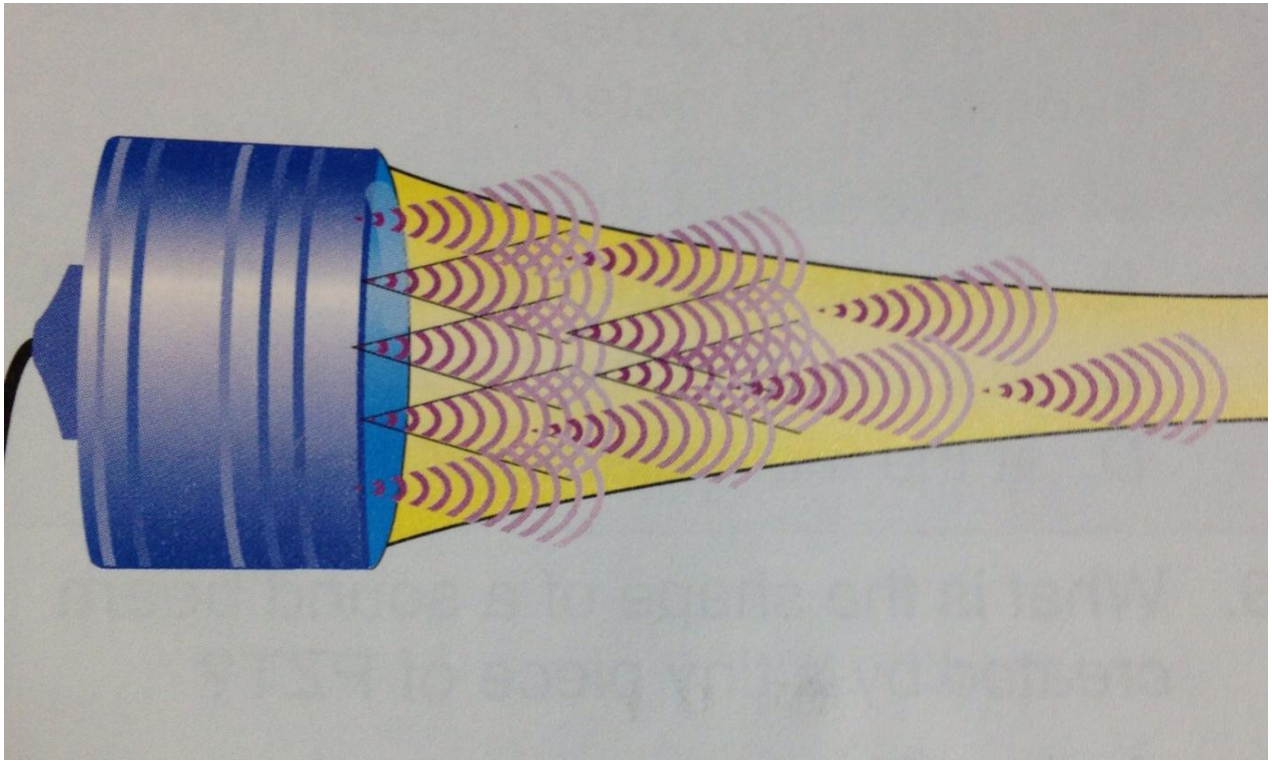
OBS: O Complemento dessa aula está no capitulo 7 do livro Fundamentals of acoustics;

Fonte sonora

- Princípio de Huygen



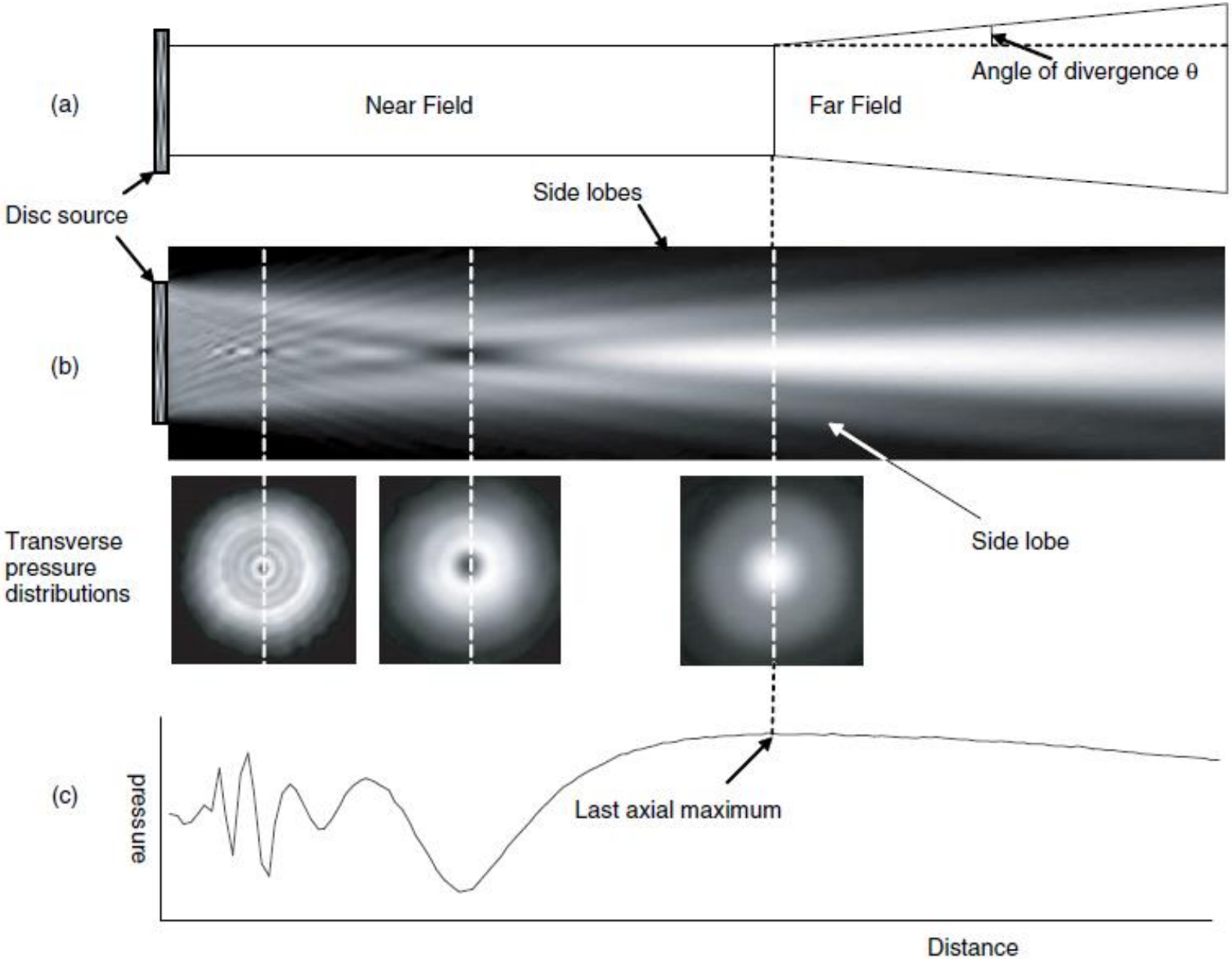
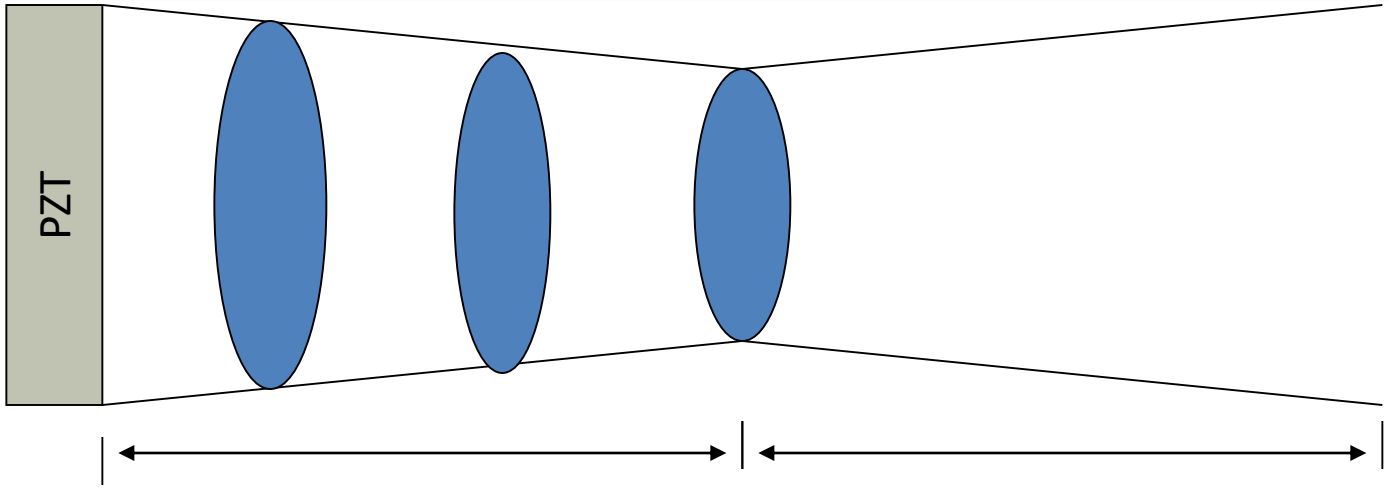


Fig. 2.15 The ultrasound beam from a plane disc source consists of a near field, in which the pressure distribution is complex, and a far field, in which it is more uniform.

Características do feixe acústico



Campo Próximo

Zona de Fresnel

Campo Distante

Zona de Frauhofer

$$z_0 = \frac{a^2}{\lambda}$$

↑ **Frequência** => ↑ **Zona de Fresnel**

Z_0 → **Dimensão do campo próximo** --- ***a*** → **Raio do disco**

Características do feixe acústico

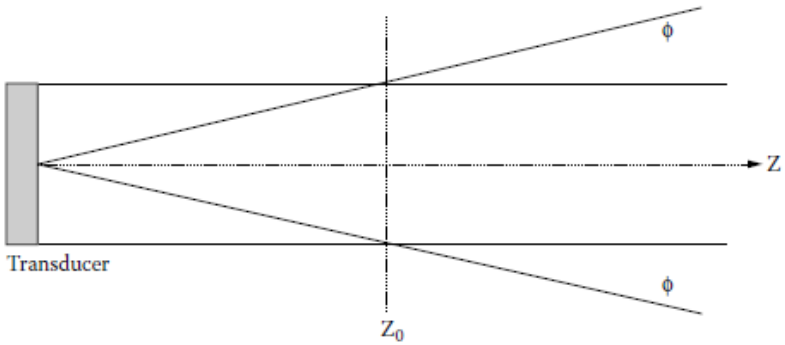
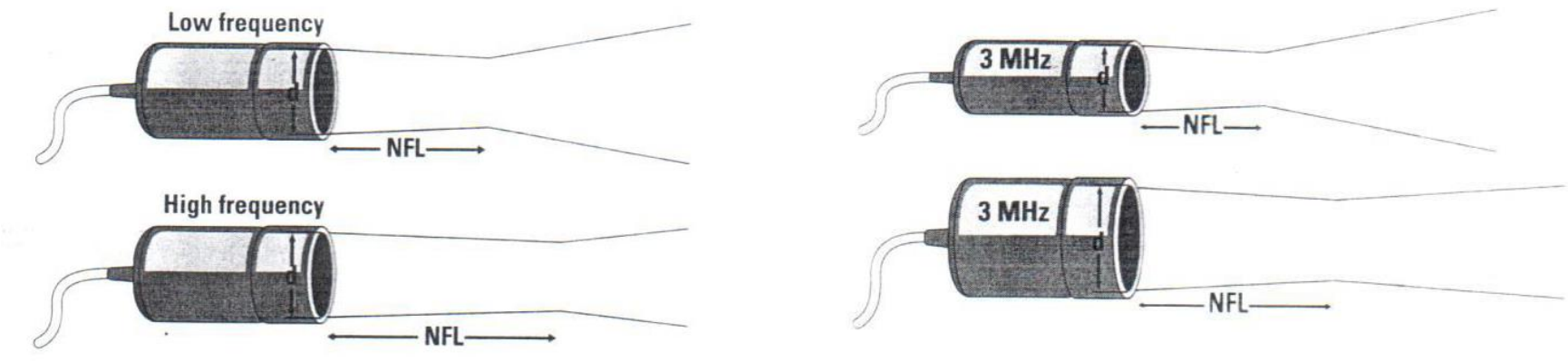


FIGURE 3.16 Idealistic representation of the beam behavior of a single-element piston transducer.

$$\phi = \sin^{-1}(0.61(\lambda/a))$$

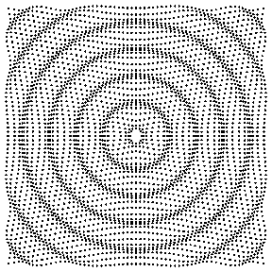
$$z_o = \frac{a^2}{\lambda}$$

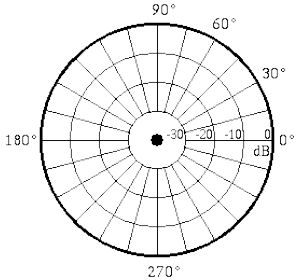


Radiação emitida por uma esfera pulsátil

- A fonte acústica mais simples de se analisar é uma esfera pulsátil.

http://www.acs.psu.edu/drussell/Demos/rad2/mdq.html





- Em um meio homogêneo e isotrópico a esfera pulsátil vai produzir uma onda esférica como:

$$\mathbf{p}(r,t) = (\mathbf{A}/r)e^{j(\omega t - kr)}$$

- Considere essa esfera de raio ***a*** vibrando radialmente com uma velocidade complexa:

$$U_0 \exp(j\omega t)$$

Impedância acústica (*Z*)

- Grandeza análoga à resistência elétrica.

- Sendo a diferença de pressão (***p***) devido a passagem da onda análoga à diferença de potencial elétrico.

- Velocidade adquirida pelas partículas (***u***) no meio é análoga a corrente elétrica.

$$z = \frac{p}{u}$$

Velocidade das partículas para onda plana

Aplicando $$\rho_0 \frac{\partial u_x}{\partial t} = -\frac{\partial p(x,t)}{\partial x}$$

$$p(x,t) = Ae^{j(\omega t - kx)} + Be^{j(\omega t + kx)}$$

$$\vec{u} = u\hat{x} = \frac{A}{\rho_0 c} e^{j(\omega t - kx)} - \frac{B}{\rho_0 c} e^{j(\omega t + kx)}$$

$$\boxed{u_\pm = \pm \frac{p_\pm}{\rho_0 c}}$$

Impedância acústica→ onda plana

- **Para uma onda plana:**

$$z = \frac{p}{u} \qquad u_{\pm} = \pm \frac{p_{\pm}}{\rho_0 c}$$

$$\boxed{z_{\pm} = \pm \rho_0 c}$$

Impedância Acústica – Onda esférica

Ao contrário das ondas planas, a velocidade das partículas não está em fase com a pressão.

Onda Plana

$$u_{\pm} = \pm \frac{p_{\pm}}{\rho_0 c}$$

Onda Esférica

$$\vec{\mathbf{u}} = \frac{\mathbf{p}}{\rho_0 c}\left(1 - \frac{j}{kr}\right)\hat{r}$$

Impedância acústica→ onda esférica

Para uma onda esférica:

$$z = \frac{p}{u}$$

$$\vec{\mathbf{u}} = \frac{\mathbf{p}}{\rho_0 c}\left(1 - \frac{j}{kr}\right)\hat{r}$$

$$\boxed{z = \rho_0 c \frac{(kr)^2}{1 + kr^2} + j\rho_0 c \frac{kr}{1 + kr^2}}$$

Impedância acústica→ onda esférica

$$z = \rho_0 c \frac{(kr)^2}{1 + kr^2} + j\rho_0 c \frac{kr}{1 + kr^2}$$

Para valores **altos de *kr*:**

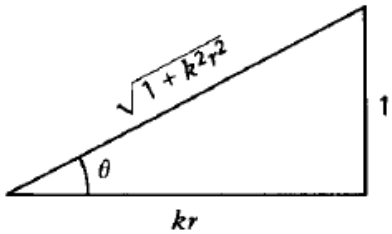
- termo **resistivo se aproxima de $\rho_o c$**.
- termo **reativo se aproxima de zero**.

Representação geométrica

$$z = \rho_0 c \frac{(kr)^2}{1 + kr^2} + j\rho_0 c \frac{kr}{1 + kr^2}$$

$$z = \rho_0 c \frac{kr}{\left[1 + (kr)^2\right]^{1/2}} e^{j\theta}$$

$$\boxed{z = \rho_0 c \cos(\theta) e^{j\theta}}$$



$$\cot\theta = kr$$

Figure 5.11.1 The relationship between θ and kr at a distance r from the source of a spherical wave of wave number k.

Impedância Acústica – Onda esférica

A geometric representation of θ is given in Fig. 5.11.1. As is true with many other acoustic phenomena, the product kr is the determining factor, rather than k or r separately. Since $kr = 2\pi r/\lambda$, the angle θ is a function of the ratio of the source distance to the wavelength. When the distance from the source is only a small fraction of a wavelength, the phase difference between the complex pressure and particle speed is large. At distances corresponding to a considerable number of wavelengths, **p** and **u** are very nearly in phase and the spherical wave assumes the characteristics of a plane wave. This is to be expected, since the wave fronts become essentially planar at great distances from the source.

Separating (5.11.9) into real and imaginary parts, we have

$$\mathbf{z} = \rho_0 c \frac{(kr)^2}{1 + (kr)^2} + j\rho_0 c \frac{kr}{1 + (kr)^2} \tag{5.11.12}$$

The first term is the specific acoustic resistance and the second term the specific acoustic reactance. Both approach zero for very small values of kr, but for very large values of kr the resistive term approaches $\rho_0 c$ and the reactive term approaches zero.

The absolute magnitude z of the specific acoustic impedance is equal to the ratio of the pressure amplitude P of the wave to its speed amplitude U,

$$z = P/U = \rho_0 c \cos\theta \tag{5.11.13}$$

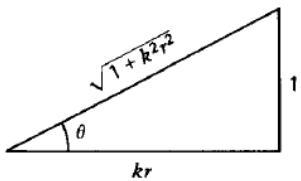


Figure 5.11.1 The relationship between θ and kr at a distance r from the source of a spherical wave of wave number k.

$$\mathbf{z} = \rho_0 c \frac{kr}{[1 + (kr)^2]^{1/2}} e^{j\theta} \tag{5.11.9}$$

$$\mathbf{z} = \rho_0 c \cos\theta \, e^{j\theta} \tag{5.11.10}$$

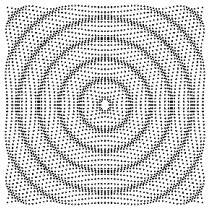
$$\cot\theta = kr \tag{5.11.11}$$

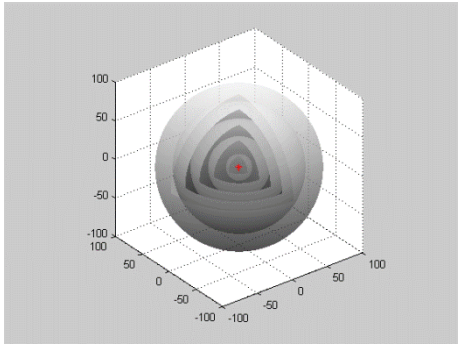
$$z = \frac{p}{u}$$

Para valores **altos de *kr*:**

- termo **resistivo se aproxima de $\rho_o c$.**
- termo **reativo se aproxima de zero.**

Radiação emitida por uma esfera pulsátil





$$\mathbf{p}(r,t) = (\mathbf{A}/r)e^{j(\omega t - kr)}$$

$$U_0 \exp(j\omega t) \qquad z = \frac{p}{u}$$

$$\mathbf{z}(a) = \rho_0 c \cos\theta_a \, e^{j\theta_a}$$

Pressão na superfície da fonte

$$\mathbf{p}(a,t) = \rho_0 c U_0 \cos\theta_a \, e^{j(\omega t \; + \theta_a)}$$

$$\mathbf{A} = \rho_0 c U_0 a \cos\theta_a \, e^{j(ka+\theta_a)}$$

so the pressure at any distance $r > a$ is

$$\mathbf{p}(r,t) = \rho_0 c U_0 (a/r) \cos\theta_a \, e^{j[\omega t - k(r-a)+\theta_a]}$$

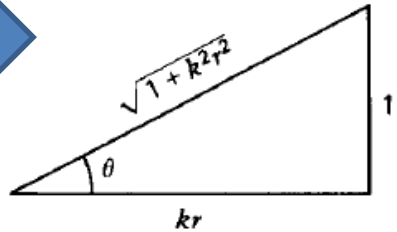
"Spherical Wave" by Jonathan Papa - Own work. Licensed under CC BY-SA 3.0 via Commons - https://commons.wikimedia.org/wiki/File:Spherical_Wave.gif#/media/File:Spherical_Wave.gif

Radiação emitida por uma esfera pulsátil

- Para o caso em que o raio da esfera é menor que o comprimento de onda

$$ka \ll 1$$

$$\theta_a \rightarrow \pi/2$$



$$\mathbf{p}(r,t) = \rho_0 c U_0 (a/r) \cos\theta_a \, e^{j[\omega t - k(r-a) + \theta_a]}$$

Temos que a pressão é:

$$\mathbf{p}(r,t) = j\rho_0 c U_0 (a/r) ka \, e^{j(\omega t - kr)} \qquad ka \ll 1$$

Intensidade

Intensidade instantânea de uma onda acústica

$$I(t) = p(t)u(t)$$

Intensidade média temporal

$$I = \frac{1}{T}\int_0^T p\,\mathrm{sen}(\omega t)u\,\mathrm{sen}(\omega t)dt = \frac{1}{2}pu$$

$$u_{\pm} = \pm\frac{p_{\pm}}{\rho_0 c}$$

$$I = \frac{p^2}{2\rho_0 c}$$

Radiação emitida por uma esfera pulsátil

- Para o caso em que o raio da esfera é menor que o comprimento de onda pode-se mostrar que a pressão é:

$$\mathbf{p}(r,t) = j\rho_0 c U_0 (a/r) ka\, e^{j(\omega t - kr)} \qquad ka \ll 1$$

- E a intensidade acústica:

$$I = \frac{1}{2}\rho_0 c U_0^2 (a/r)^2 (ka)^2$$

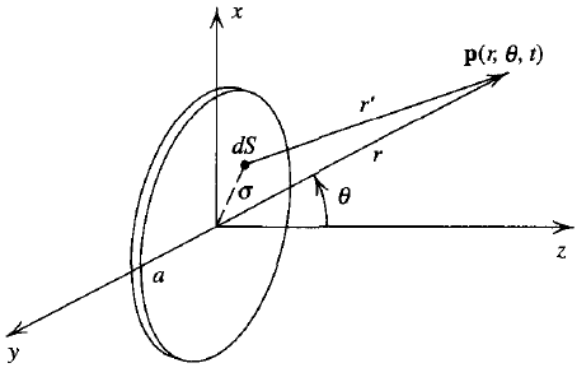
Intensidade depende do quadrado da frequência e da quarta potência do raio da fonte.

Portanto, fontes pequenas em relação ao comprimento de onda são pobre irradiadoras.

Feixe irradiado por um transdutor

- O comportamento desse feixe é importante para determinar a sensibilidade espacial do instrumento de imagem.

- Para analisar o padrão de radiação vamos nos basear no princípio de Huygen, que diz que esse padrão pode ser obtido ao considerar a fonte como uma coleção de fontes pontuais. E cada uma emitindo uma onda esférica.



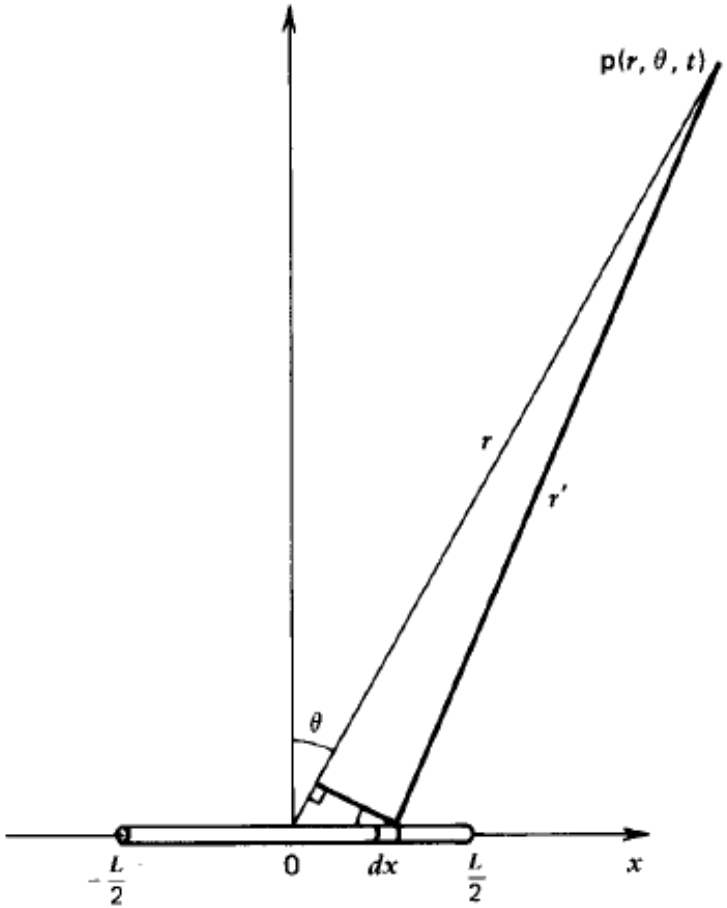
Fonte de intensidade acústica (*Q*)

- $Q = U_0 S$
  - $S \rightarrow$ Área.
- Para uma fonte esférica temos
  - $Q = 4\pi a^2 U_0$

$$\mathbf{p}(r,t) = j\rho_0 c U_0 (a/r) ka\, e^{j(\omega t - kr)}.$$

$$\mathbf{p}(r,t) = \tfrac{1}{2} j\rho_0 c (Q/\lambda r) e^{j(\omega t - kr)}$$

Feixe irradiado por uma linha



Consideremos uma fonte linear de comprimento *L* e raio *a*.

Superfície vibrando radialmente com velocidade $U_0 \exp(j\omega t)$

A fonte é formada por um grande número de pequenos cilindros de comprimento *dx*. Cada um com uma fonte de intensidade acústica:

$$dQ = U_0\, 2\pi a\, dx$$

Cada pequena fonte pontual tem intensidade:

$$dQ = U_0\, dS$$

A pressão total pode então ser escrita como:

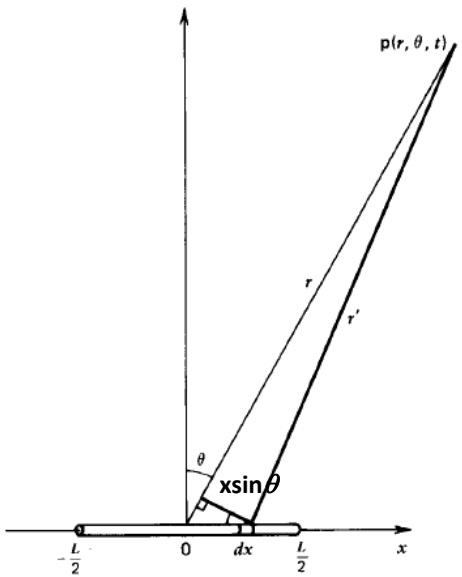
$$\mathbf{p}(r,\theta,t) = \frac{j}{2}\rho_0 c U_0 ka \int_{-L/2}^{L/2} \frac{1}{r'} e^{j(\omega t - kr')} dx$$

$$\mathbf{p}(r,t) = \tfrac{1}{2} j \rho_0 c (Q/\lambda r) e^{j(\omega t - kr)}$$

Feixe irradiado por uma linha

Calcular o campo acústico próximo a fonte (campo próximo) é uma tarefa bem complexa.

Mas para a aproximação de campo distante uma expressão mais simples pode ser obtida.



Para:

$$r \gg L,$$

O denominador (*r'*) pode ser substituído por *r*

No expoente essa simplificação não é válida. Nesse caso, o mais exato é usar: *r'=r - xsinθ*

$$\mathbf{p}(r,\theta,t) = \frac{j}{2}\rho_0 c U_0 ka \int_{-L/2}^{L/2} \frac{1}{r'} e^{j(\omega t - kr')} dx$$

Para estudar

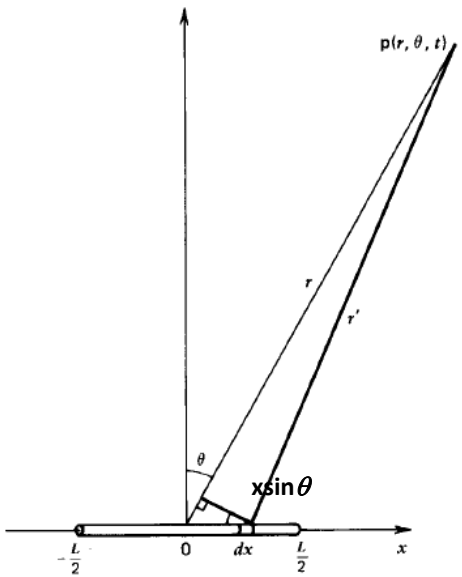
The acoustic field close to the source is complicated, but a simple expression can be obtained in the *far field approximation*. Under the assumption $r \gg L$, the denominator of the integrand can be replaced by its approximate value r, which amounts to making very small errors in the amplitudes of the acoustic fields at (r, θ) generated by each of the simple sources. In the exponent, however, this simplification cannot always be made because the relative phases of the elements will be very strong functions of angle when kL approaches or exceeds unity. Then the more accurate approximation $r' \approx r - x \sin\theta$ must be used, and the integral takes the form

Para:

$$r \gg L,$$

O denominador (*r'*) pode ser substituído por *r*

No expoente essa simplificação não é válida. Nesse caso, o mais exato é usar: *r'=r - xsin*θ



$$\mathrm{p}(r,\theta,t) = \frac{j}{2}\rho_0 c U_0 ka \int_{-L/2}^{L/2} \frac{1}{r'} e^{j(\omega t - kr')} dx$$

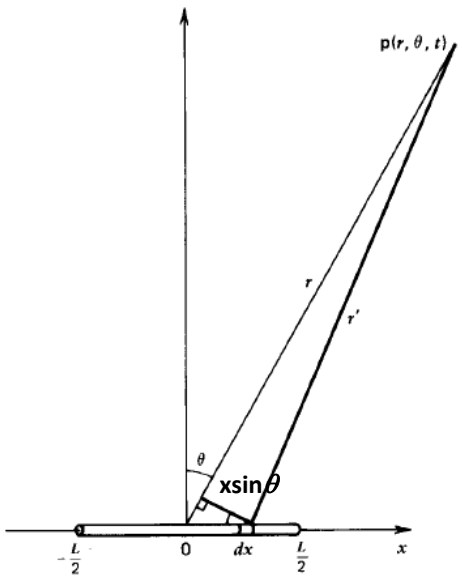
$$\mathrm{p}(r,\theta,t) = \frac{j}{2}\rho_0 c U_0 \frac{ka}{r} e^{j(\omega t - kr)} \int_{-L/2}^{L/2} e^{jkx\sin\theta} dx$$

$$\mathrm{p}(r,\theta,t) = \frac{j}{2}\rho_0 c U_0 \frac{a}{r} kL \left(\frac{\sin\left(\frac{1}{2}kL\sin\theta\right)}{\frac{1}{2}kL\sin\theta} \right) e^{j(\omega t - kr)}$$

Indefinite integral:

$$\int \exp(i\,a\,x)\,dx = -\frac{i\,e^{i\,a\,x}}{a} + \text{constant}$$

Feixe irradiado por uma linha



$$\mathbf{p}(r,\theta,t) = \frac{j}{2}\rho_0 c U_0 \frac{a}{r} kL \left(\frac{\sin\left(\frac{1}{2}kL\sin\theta\right)}{\frac{1}{2}kL\sin\theta} \right) e^{j(\omega t - kr)}$$

A amplitude de pressão acústica no campo distante pode ser escrita como:

$$\boxed{P(r,\theta) = P_{ax}(r)H(\theta)}$$

where

$$H(\theta) = \left| \frac{\sin v}{v} \right| \qquad v = \tfrac{1}{2}kL\sin\theta$$

is the *directional factor* and

$$P_{ax}(r) = \tfrac{1}{2}\rho_0 c U_0 (a/r) kL$$

is the amplitude of the *far field axial pressure*.

Campo distante

Separating the far field pressure amplitude into one factor that depends only on angle and has maximum value of unity on the *acoustic axis* and another that depends only on the distance from the source is common practice in describing the sound fields of complicated sources. Note that in the far field the axial pressure is proportional to $1/r$, as for a simple source. This is a feature common to all acoustic sources.

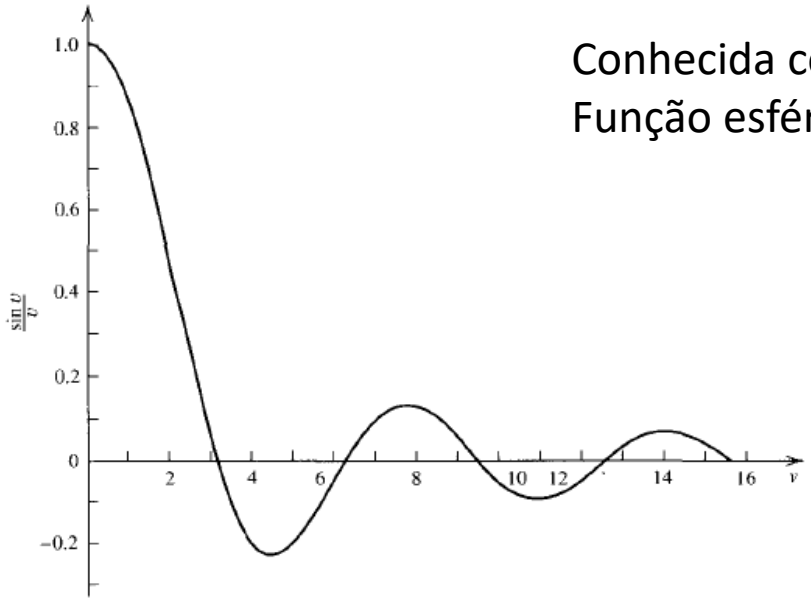
$$P(r,\theta) = P_{ax}(r)H(\theta)$$

$$H(\theta) = \left|\frac{\sin v}{v}\right| \qquad v = \tfrac{1}{2}kL\sin\theta$$

$$P_{ax}(r) = \tfrac{1}{2}\rho_0 c U_0 (a/r)kL$$

Fator dependente da angulação

Fator dependente da distância
Proporcional a 1/*r*.



Conhecida como função **sinc** ou
Função esférica de Bessel de primeira espécie de ordem zero

$$H(\theta) = \left| \frac{\sin v}{v} \right| \qquad v = \tfrac{1}{2} kL \sin\theta$$

Figure 7.3.2 Functional behavior of $(\sin v)/v$.

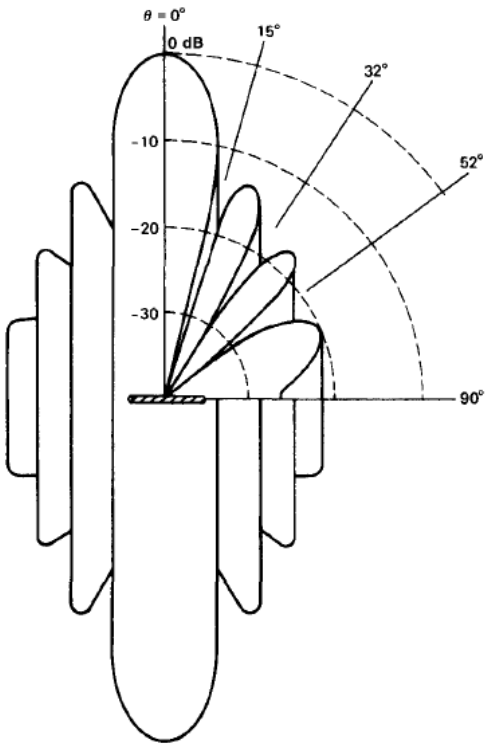
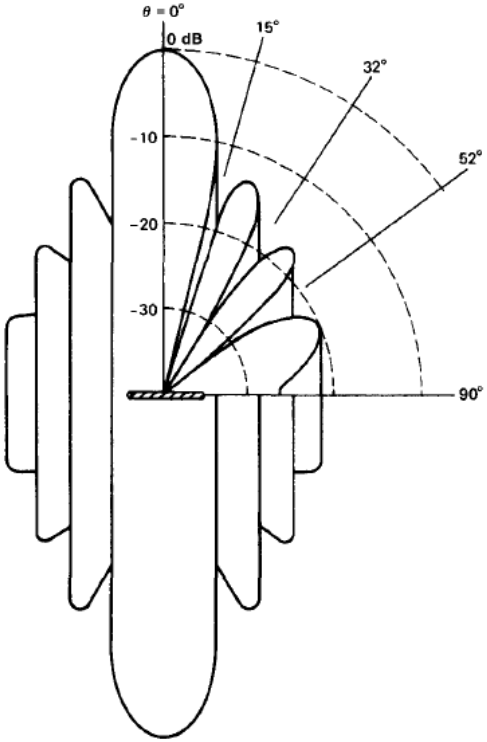


Figure 7.3.3 Beam pattern $b(\theta)$ for a continuous line source of length L radiating sound of wave number k with $kL = 24$.

- **Padrão de campo b(θ)**

- **Nessa figura podemos visualizar que o campo acústico possui um lobo principal e lobos laterais.**

- **Visualizamos que também existem os nós de pressão.**



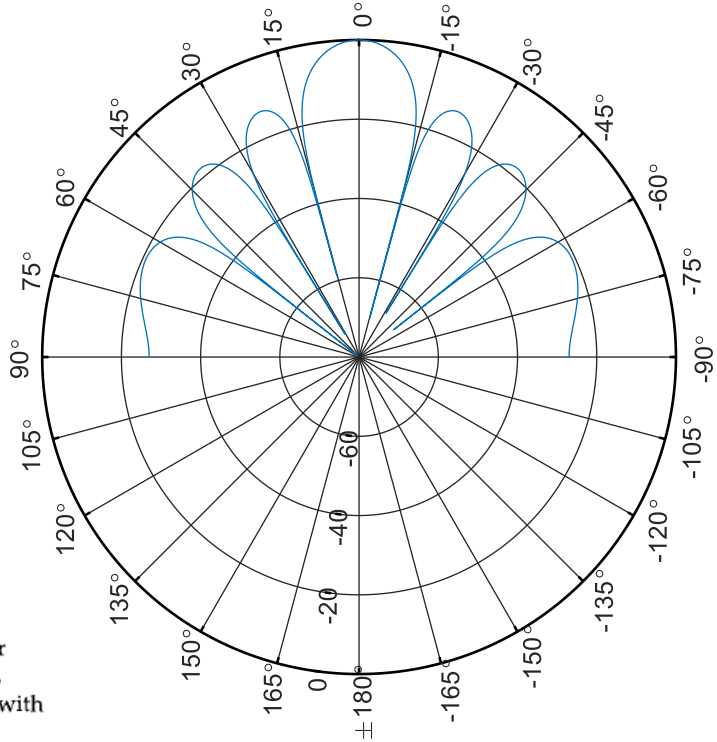
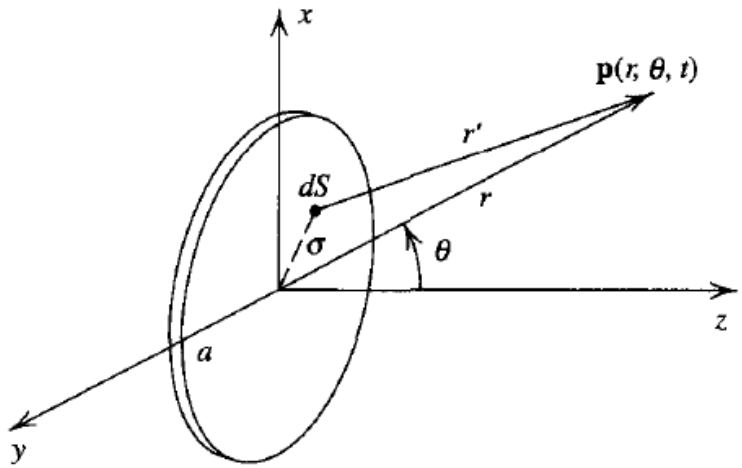


Figure 7.3.3 Beam pattern $b(\theta)$ for a continuous line source of length L radiating sound of wave number k with $kL = 24$.

Campo distante – Para estudar

The behavior of $(\sin v)/v$ is shown in Fig. 7.3.2. This function is known as the *sinc function* or the *zeroth order spherical Bessel function of the first kind*. The corresponding *beam pattern* $b(\theta) = 20 \log H(\theta)$ is plotted in Fig. 7.3.3 for the case $kL = 24$. There are *nodal surfaces* (cones in the present case) at angles where $H(\theta) = 0$, for which $\frac{1}{2}kL \sin\theta_n = \pm n\pi$, with $n = 1, 2, 3, \ldots$. These nodal surfaces are separated by *lobes* where the acoustic energy is nonzero. Most of the acoustic energy is projected in the *major lobe*, contained within the angles given by $n = 1$ and centered on a plane perpendicular to the line source. The amplitudes of the *minor lobes* are less than unity and tend to decrease away from this plane. Clearly, the larger the value of kL the more narrowly directed will be the major lobe and the greater the number of minor lobes.

Feixe irradiado por um transdutor



$$\mathbf{p}(r,t) = \tfrac{1}{2} j\rho_0 c (Q/\lambda r) e^{j(\omega t - kr)}$$

Cada pequena fonte pontual tem fonte de intensidade acústica :

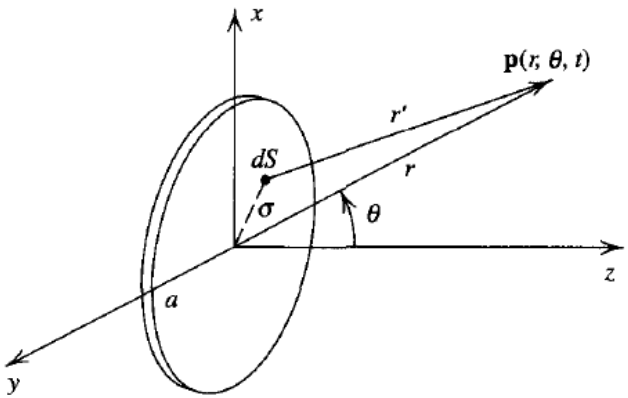
$$dQ = U_0 \, dS$$

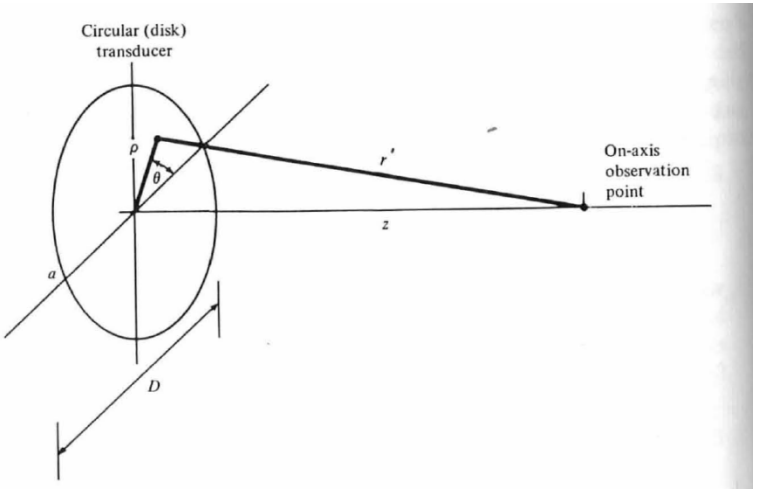
Solução geral

$$\mathbf{p}(r,\theta,t) = j\rho_0 c \frac{U_0}{\lambda} \int_S \frac{1}{r'} e^{j(\omega t - kr')} \, dS$$

Para um transdutor qualquer para um ponto de observação arbitrário, essa equação tem solução bastante complexa e requer solução computacional.

Campo próximo no eixo principal de um transdutor circular





Solução geral

$$\mathbf{p}(r,\theta,t) = j\rho_0 c \frac{U_0}{\lambda} \int_S \frac{1}{r'} e^{j(\omega t - kr')}\, dS$$

Solução no eixo principal

$$\mathbf{p}(r,0,t) = j\rho_0 c \frac{U_0}{\lambda} e^{j\omega t} \int_0^a \frac{\exp\left(-jk\sqrt{r^2+\sigma^2}\right)}{\sqrt{r^2+\sigma^2}} 2\pi\sigma\, d\sigma$$

The integrand is a perfect differential,

$$\frac{\sigma \exp\left(-jk\sqrt{r^2+\sigma^2}\right)}{\sqrt{r^2+\sigma^2}} = -\frac{d}{d\sigma}\left(\frac{\exp\left(-jk\sqrt{r^2+\sigma^2}\right)}{jk}\right)$$

Campo próximo no eixo principal de um transdutor circular

$$\mathbf{p}(r,0,t) = j\rho_0 c\,\frac{U_0}{\lambda}e^{j\omega t}\int_0^a \frac{\exp\left(-jk\sqrt{r^2+\sigma^2}\right)}{\sqrt{r^2+\sigma^2}}2\pi\sigma\,d\sigma$$

The integrand is a perfect differential,

$$\frac{\sigma\exp\left(-jk\sqrt{r^2+\sigma^2}\right)}{\sqrt{r^2+\sigma^2}} = -\frac{d}{d\sigma}\left(\frac{\exp\left(-jk\sqrt{r^2+\sigma^2}\right)}{jk}\right)$$

Exercício: mostrar que a solução dessa integral é:

$$\mathbf{p}(r,0,t) = \rho_0 c\,U_0\left\{1-\exp\left[-jk\left(\sqrt{r^2+a^2}-r\right)\right]\right\}e^{j(\omega t-kr)}$$

A amplitude de pressão no eixo do transdutor é a magnitude da expressão acima.

$$P(r,0) = 2\rho_0 c U_0\left|\sin\left\{\tfrac{1}{2}kr\left[\sqrt{1+(a/r)^2}-1\right]\right\}\right|$$

Campo próximo no eixo principal de um transdutor circular

$$P(r,0) = 2\rho_0 c U_0 \left| \sin\left\{ \tfrac{1}{2}kr\left[\sqrt{1+(a/r)^2} - 1 \right] \right\} \right|$$

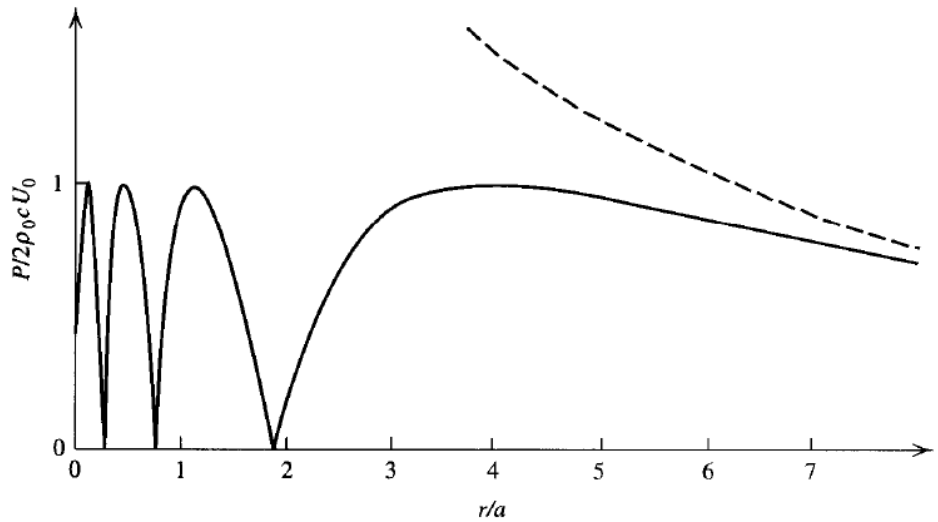


Figure 7.4.2 Axial pressure amplitude for a baffled circular plane piston of radius a radiating sound of wave number k with $ka = 8\pi$. Solid line is calculated from the exact theory. Dashed line is the far field approximation extrapolated into the near field. For this case, the far field approximation is accurate only for distances beyond about seven piston radii.

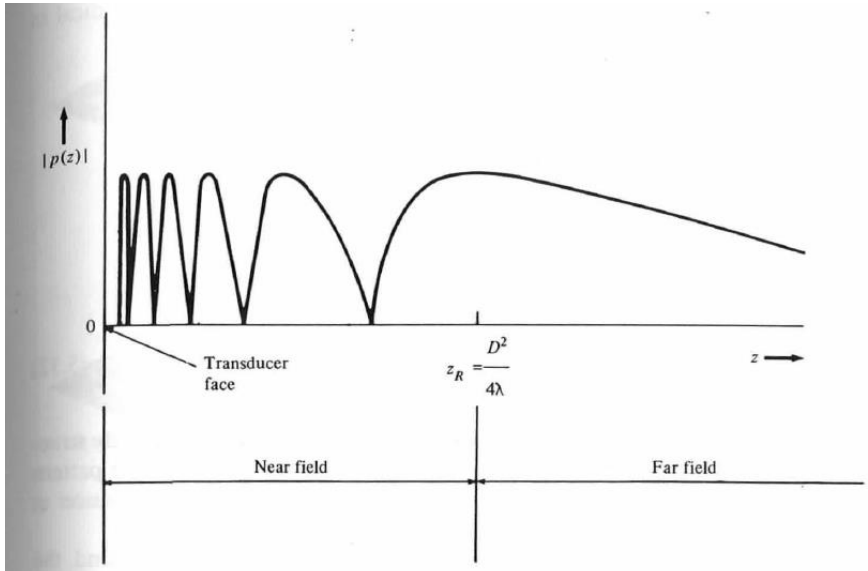
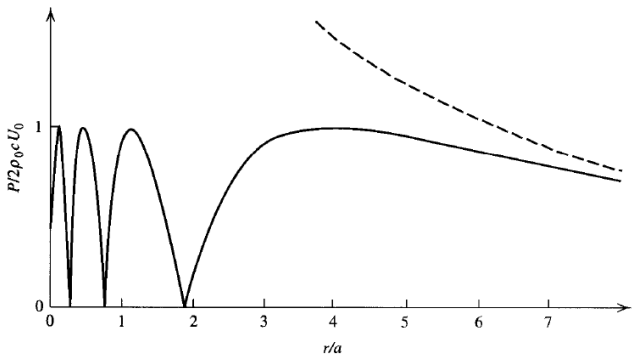


Figure 5.13 Variation of the magnitude of on-axis pressure field from a circular transducer of diameter D. This is a plot of the absolute magnitude of Equation (5.30) at one particular time, $t = 0$, and is the envelope of the oscillating pressure. The transition point from the near field to the far field is defined as the position of the furthest maximum. Beyond that point the field is more uniform.

Região de transição



$$P(r,0) = 2\rho_0 c U_0 \left| \sin\left\{ \tfrac{1}{2}kr\left[\sqrt{1+(a/r)^2} - 1 \right] \right\} \right|$$

Essa análise revela que a pressão axial apresenta muitas flutuações devido a interferências. Os máximos e mínimos de pressão acontecem quando

$$\tfrac{1}{2}kr\left[\sqrt{1+(a/r)^2} - 1 \right] = m\pi/2 \qquad m = 0, 1, 2, \ldots$$

For $r/a \gg 1$, the square root can be simplified to

$$\sqrt{1+(a/r)^2} \approx 1 + \tfrac{1}{2}(a/r)^2$$

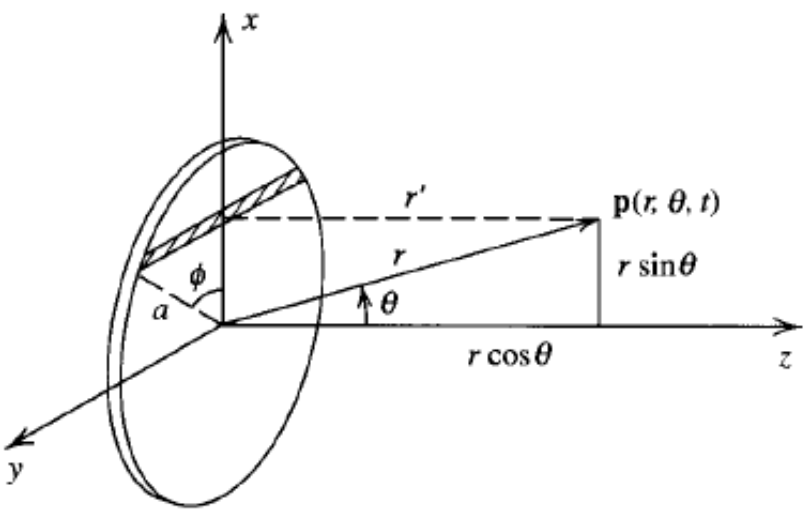
$$\boldsymbol{r_m = \frac{a^2}{m\lambda}}$$

Vindo do infinito em relação a fonte mostrar que o primeiro máximo é:

$$r_1 = \frac{a^2}{\lambda}$$

Campo distante

- No campo distante, considerando, **r>>a.**



$$\mathbf{p}(r,\theta,t) = \frac{j}{2}\rho_0 c U_0 \frac{a}{r} ka \boxed{\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]} e^{j(\omega t - kr)}$$

Dependência angular do feixe

Bastão

$$\mathbf{p}(r,\theta,t) = \frac{j}{2}\rho_0 c U_0 \frac{a}{r} kL \left(\frac{\sin\left(\frac{1}{2}kL\sin\theta\right)}{\frac{1}{2}kL\sin\theta}\right) e^{j(\omega t - kr)}$$

Campo distante

$$|\mathbf{p}(r,\theta)| = P_{ax}(r)H(\theta) \qquad v = ka\sin\theta$$

$$H(\theta) = \left|\frac{2J_1(v)}{v}\right|$$

J_1 → Função de Bessel de 1ª espécie de 1ª ordem

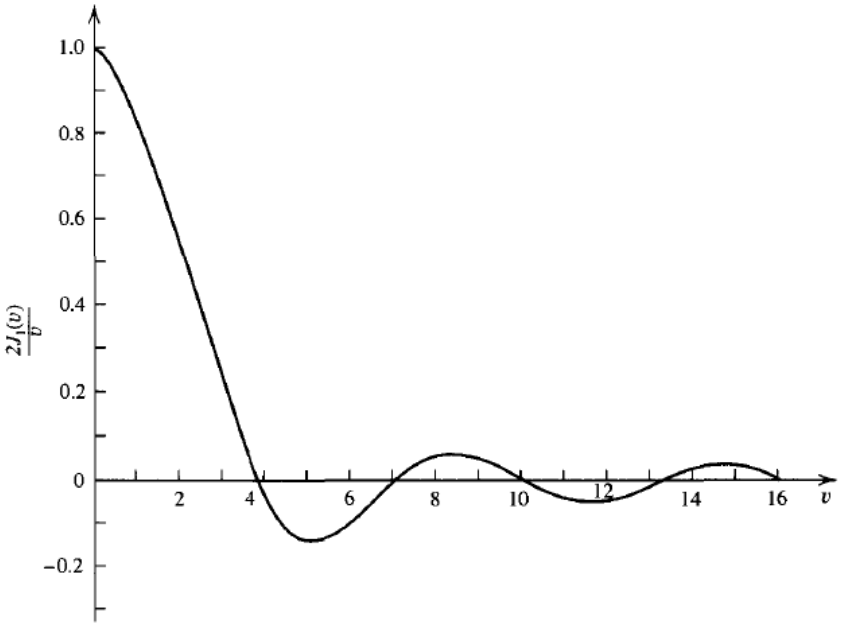


Figure 7.4.4 Functional behavior of $2J_1(v)/v$.

Campo distante

- H → Função de diretividade do campo acústico.

$$|\mathbf{p}(r,\theta)| = P_{ax}(r)H(\theta) \qquad v = ka\sin\theta$$

$$H(\theta) = \left|\frac{2J_1(v)}{v}\right|$$

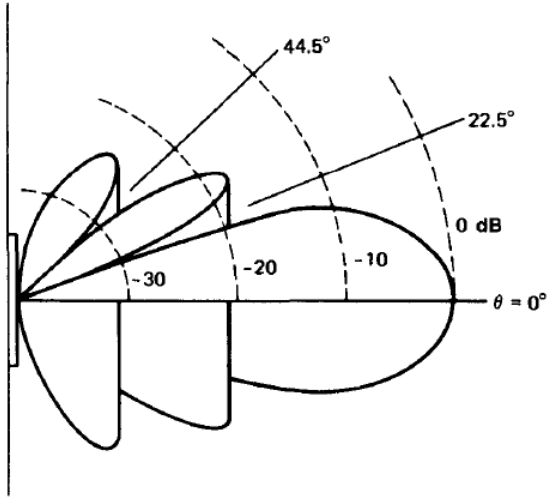


Figure 7.4.5 Beam pattern $b(\theta)$ for a circular plane piston of radius a radiating sound with $ka = 10$.

- Nessa figura podemos visualizar que o campo acústico possui um lobo principal e lobos laterais.

- Visualizamos que também existem os nós de pressão.

Campo distante

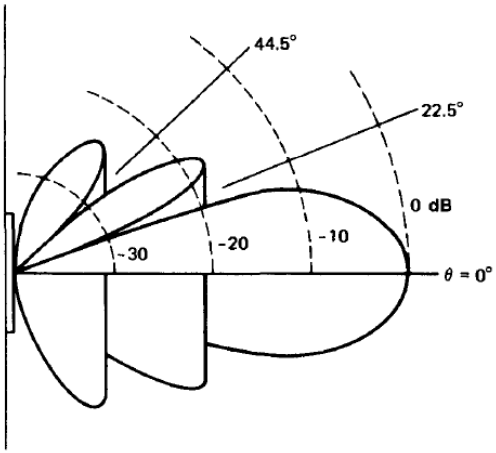


Figure 7.4.5 Beam pattern $b(\theta)$ for a circular plane piston of radius a radiating sound with $ka = 10$.

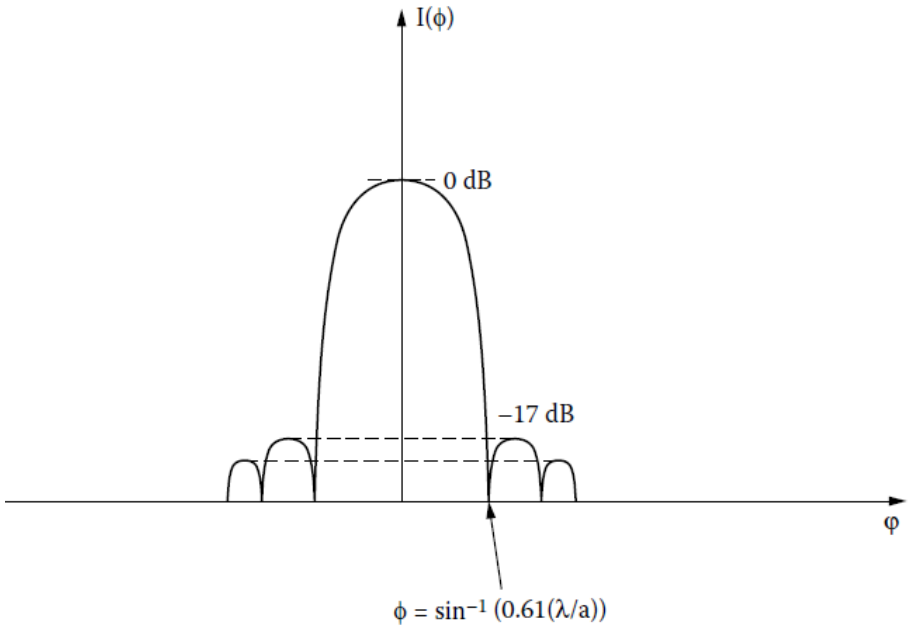


FIGURE 3.15 Angular radiation pattern of a disc piston transducer.

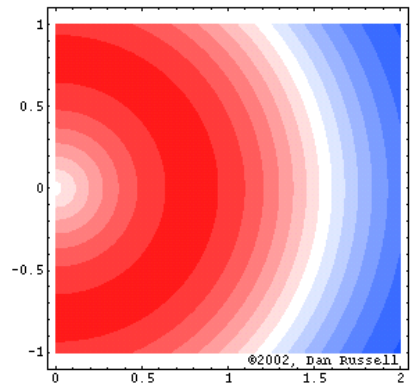
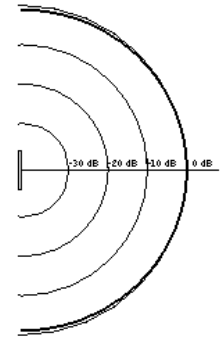
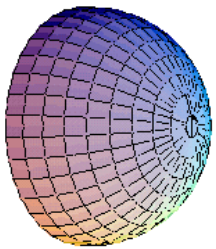
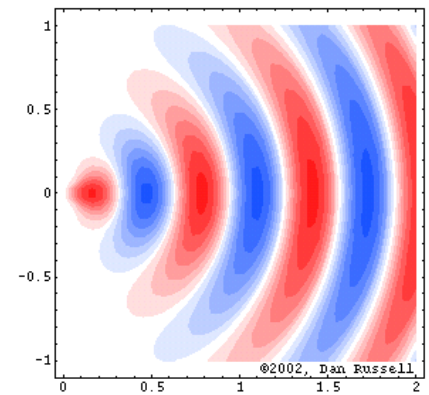
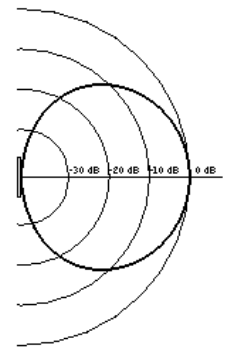
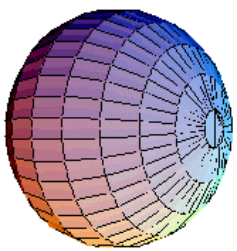
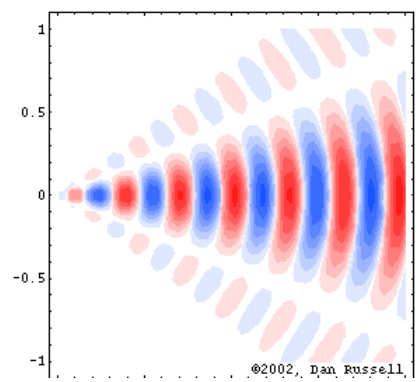
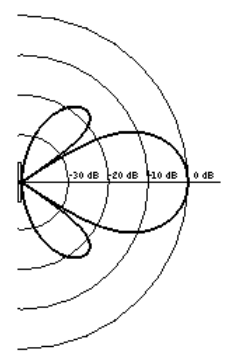
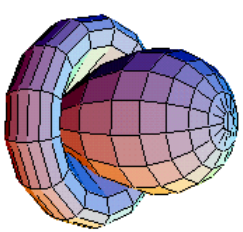
Campo distante

Exercício: ver apêndices A5 e A6 do Kinsler

- Mostre que para comprimentos de onda suficientemente grandes somente o lobo principal estará presente.

- Mostre que para comprimentos de ondas menores que o raio do irradiador, ka>>1, teremos vários lobos laterais e a largura angular do lobo principal é diminuída.

$$\tilde{\mathbf{p}}(r,\theta,t) = \underbrace{j\rho c\,\frac{Qk}{2\pi r}\,e^{j(\omega t - kr)}}_{\text{baffled simple source}}\ \underbrace{\left[\frac{2J_1(ka\sin\theta)}{ka\sin\theta}\right]}_{\text{directivity}}$$

	Animation of sound field	Directivity pattern	3-D directivity pattern
Low frequency (*ka*<1)			
Medium frequency (*ka*>1)			
High frequency (*ka*>>1)			

http://www.acs.psu.edu/drussell/Demos/BaffledPiston/BaffledPiston.html

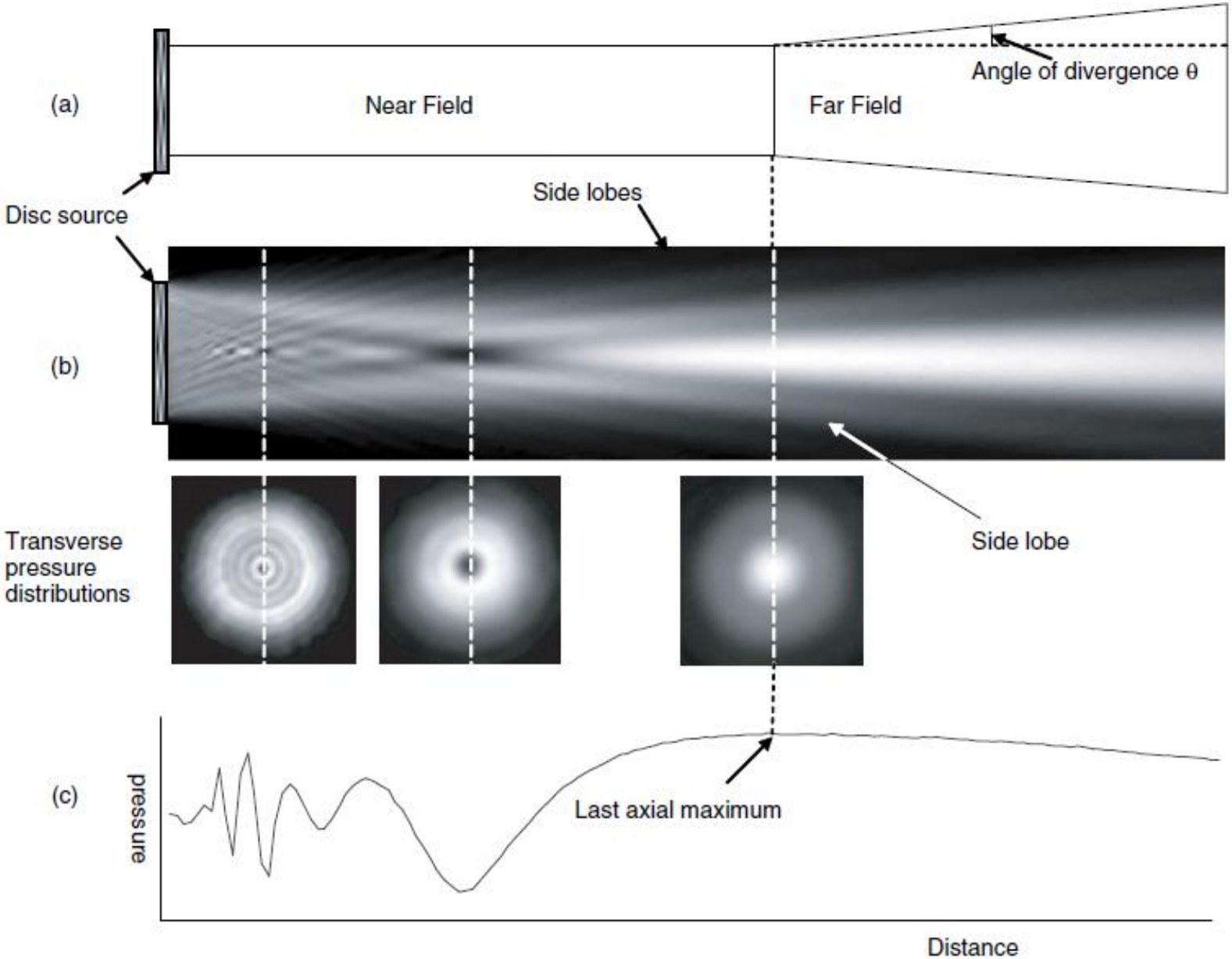


Fig. 2.15 The ultrasound beam from a plane disc source consists of a near field, in which the pressure distribution is complex, and a far field, in which it is more uniform.

Representação do campo acústico

- Podemos então pensar o campo próximo como paralelo a face do transdutor, tendo comportamento complexo.
- E no campo distante, o transdutor poderia ser pensado como uma fonte pontual, irradiando como um cone confinado em um ângulo 2ϕ.

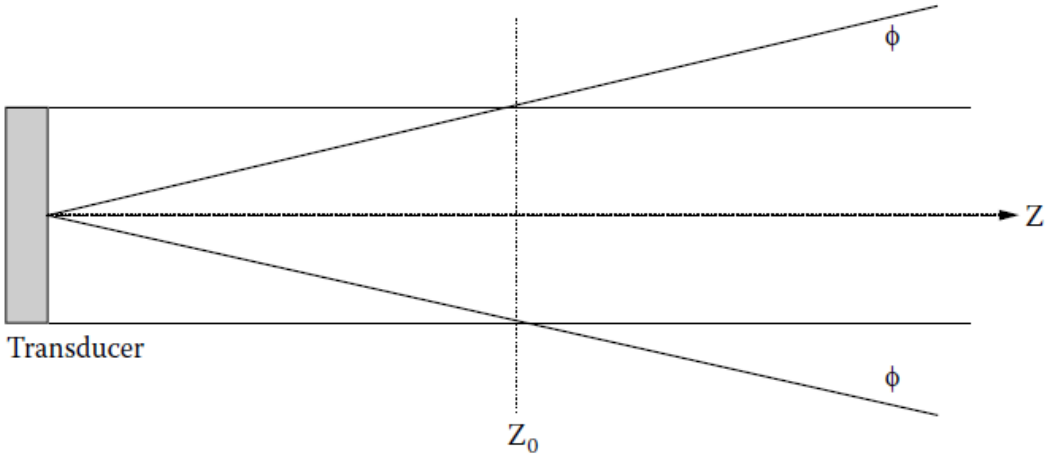


FIGURE 3.16 Idealistic representation of the beam behavior of a single-element piston transducer.

Campo acústico e imagem US

- Lobos laterais são muito indesejáveis em imagens de ultrassom.
- Eles produzem sinais ilegítimos, resultando em artefatos e diminuição no contraste da imagem.
- Ao aumentar a razão entre dimensão do transdutor e o comprimento de onda, teremos feixes mais estreitos, mas com mais lobos laterais.

Campo acústico pulsado

- **A discussão até aqui envolveu excitação do transdutor com ondas contínuas (CW). É possível mostrar que com ondas pulsadas, as características do campo são suavizadas.**

Transdutor circular com 0,5 cm de raio e 5 MHz

CW

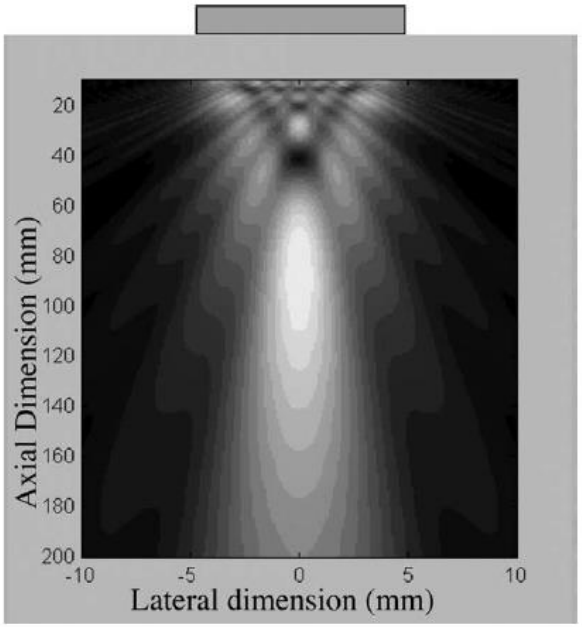


FIGURE 3.17 The pressure amplitude distribution of a single-element transducer with CW excitation.

Pulsada com 3 ciclos

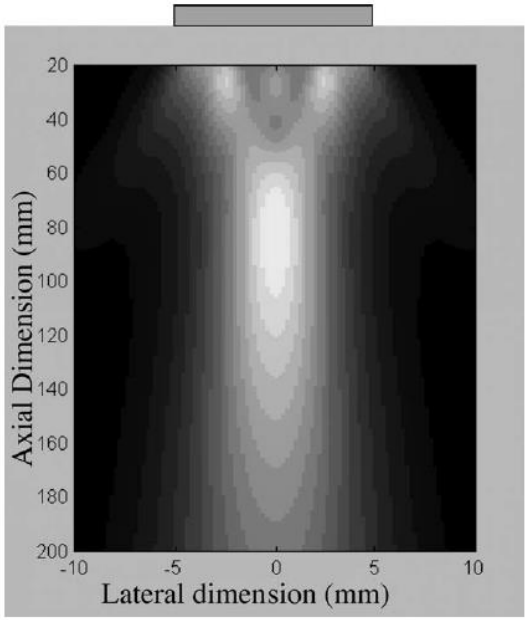


FIGURE 3.19 The pressure amplitude distribution of a single-element transducer with pulsed excitation.

Transdutor multi-elementos

- O padrão de radiação de um transdutor linear, multi-elementos (N elementos) de dimensão L_a no campo distante: $z > L_a^2/4\lambda$

$$H(u) = \mathrm{sinc}\left(\frac{bu}{\lambda}\right) \cdot \sum_{m=1}^{N} \delta\left(u - m\frac{\lambda}{g}\right) * \mathrm{sinc}\left(\frac{L_a \boldsymbol{u}}{\lambda}\right)$$

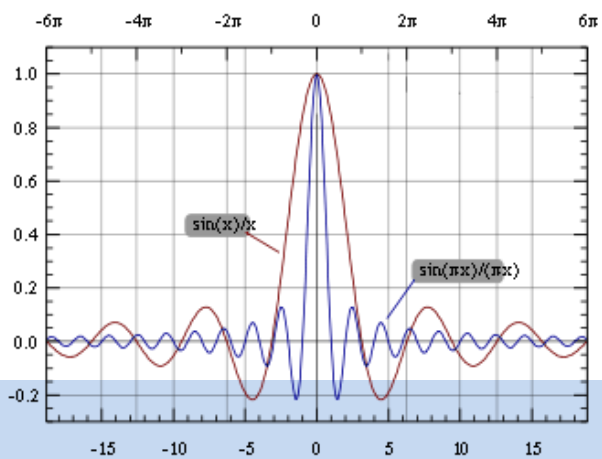
where

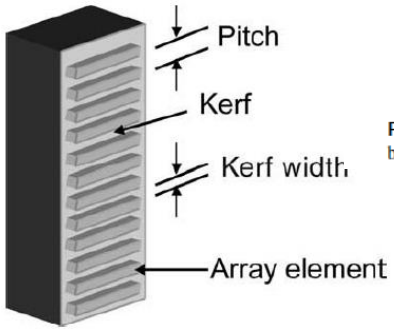
$u = \sin \phi_x$

$H(u)$ is the directivity function at an angle of ϕ_x

b is the width of the element

g is the pitch





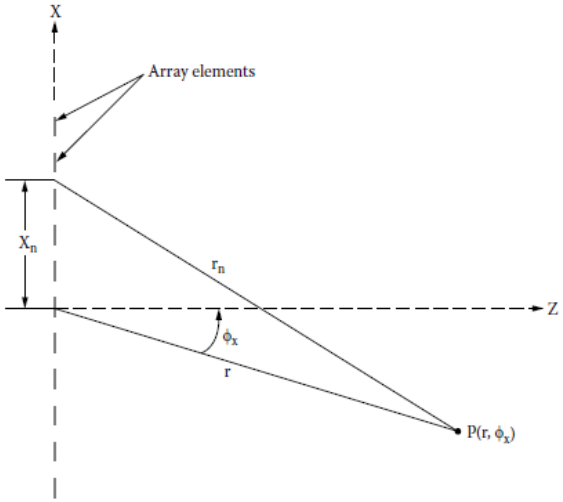
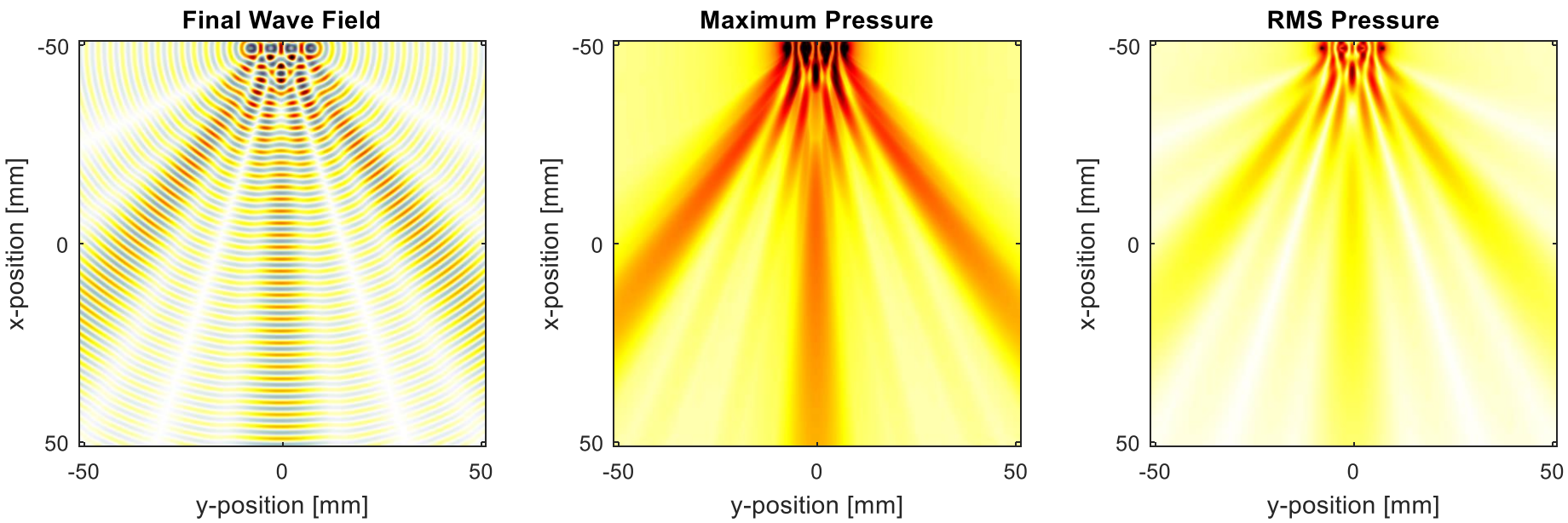


FIGURE 3.27 A two-dimensional coordinate system depicting the difference in path length between the center element of a linear array and the *n*th element.

Lobos Principal, Grandes e Laterais

- Para transdutores com elementos regularmente espaçados, em alguns ângulos, lobos laterais de alta intensidade podem aparecer.
- Esses lobos são conhecidos como grandes lobos (grating lobes).
- Acontecem devido a padrões de interferência construtivas.
- **Angulação em que aparecem depende das distância entre os elementos (pitch).**
- **Magnitude depende do tamanho do elemento.**

Lobos Principal, Grandes e Laterais



Grandes Lobos

Pitch = 1,5 λ

Pitch =0,75 λ

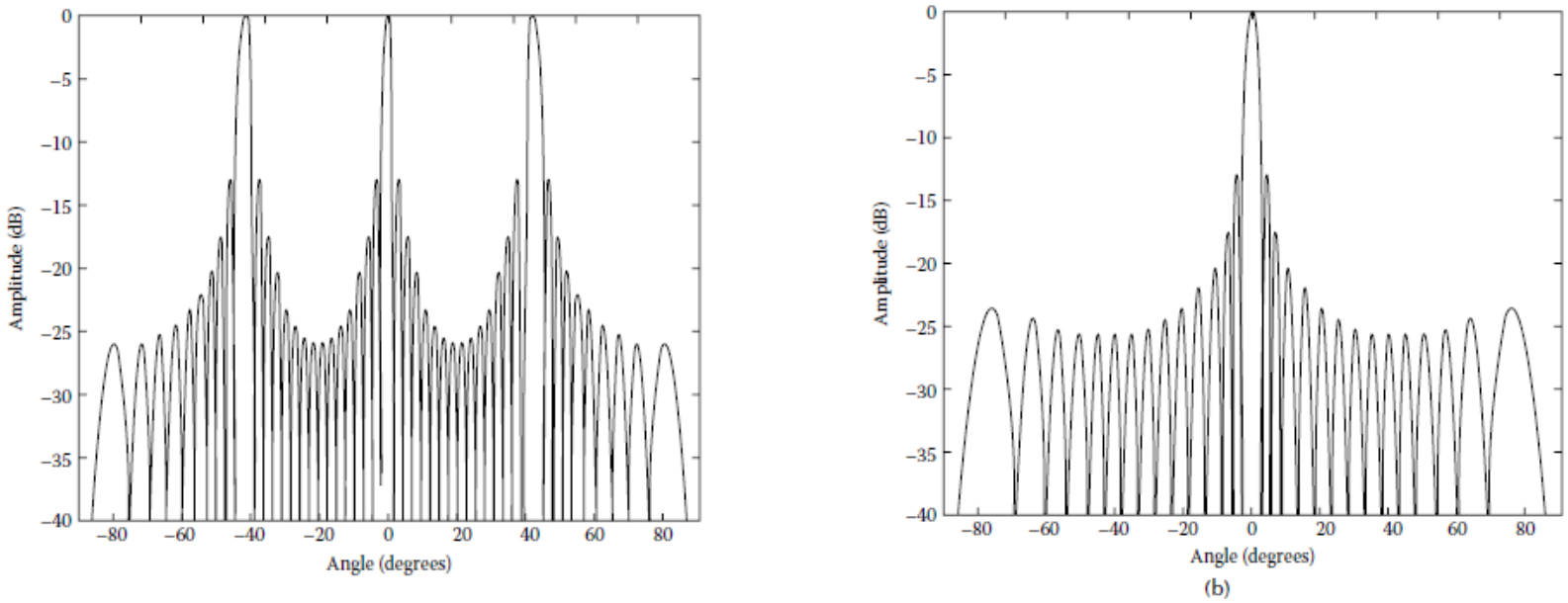
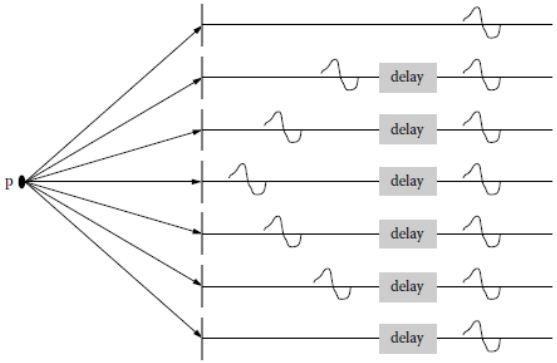


FIGURE 3.29 (a) Large grating lobes are produced at 40 and –40° by a linear array with an undesirable pitch. (b) Grating lobes are moved away when the pitch is reduced to smaller than a wavelength.

Apodização

- Técnica para redução dos lobos laterais e grande lobos. Pode ser obtida por diferentes maneiras.

- Excitando os diferentes elementos do transdutor com diferentes amplitudes de voltagem.

• Aplicando pesos diferentes ao elementos de recepção na técnicas de atraso e soma.