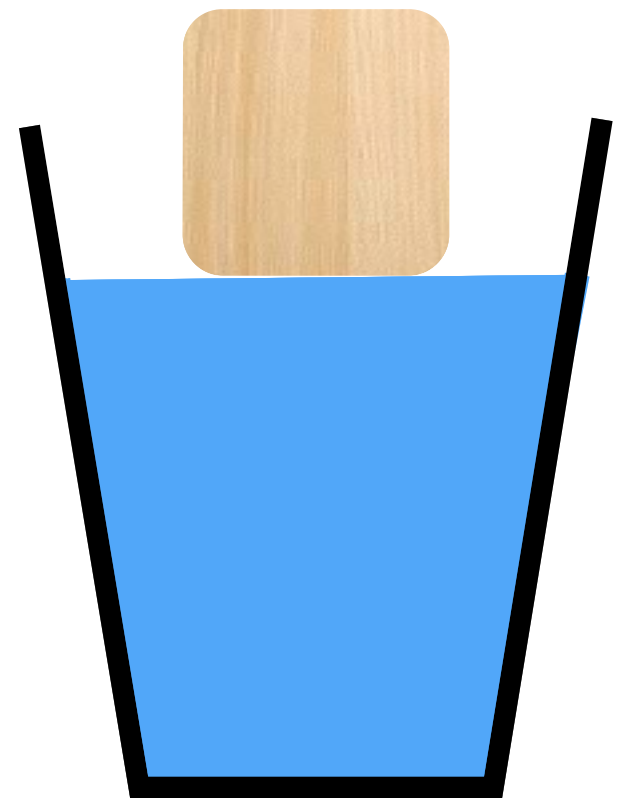
IAG/USP

Modelos quantitativos de bacias sedimentares

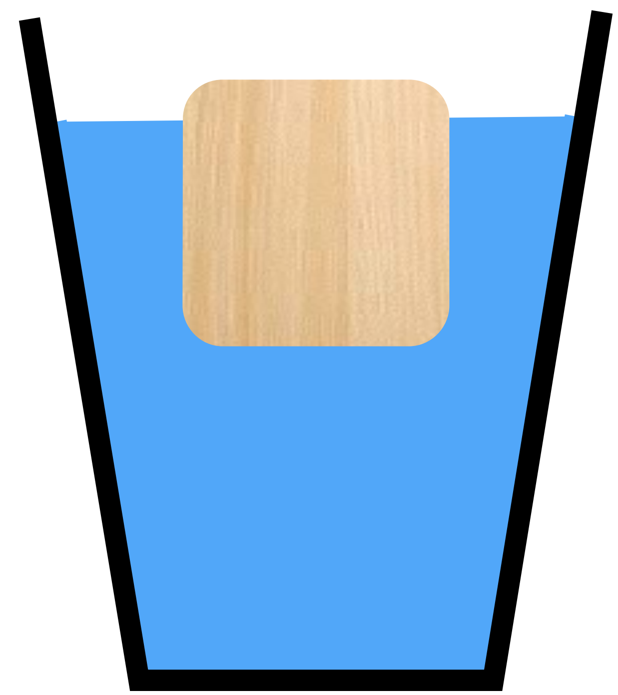
Isostasia e Flexura da Litosfera

Princípio de Arquimedes



Any floating object displaces its own weight of fluid.
— Arquimedes de Siracusa

Princípio de Arquimedes



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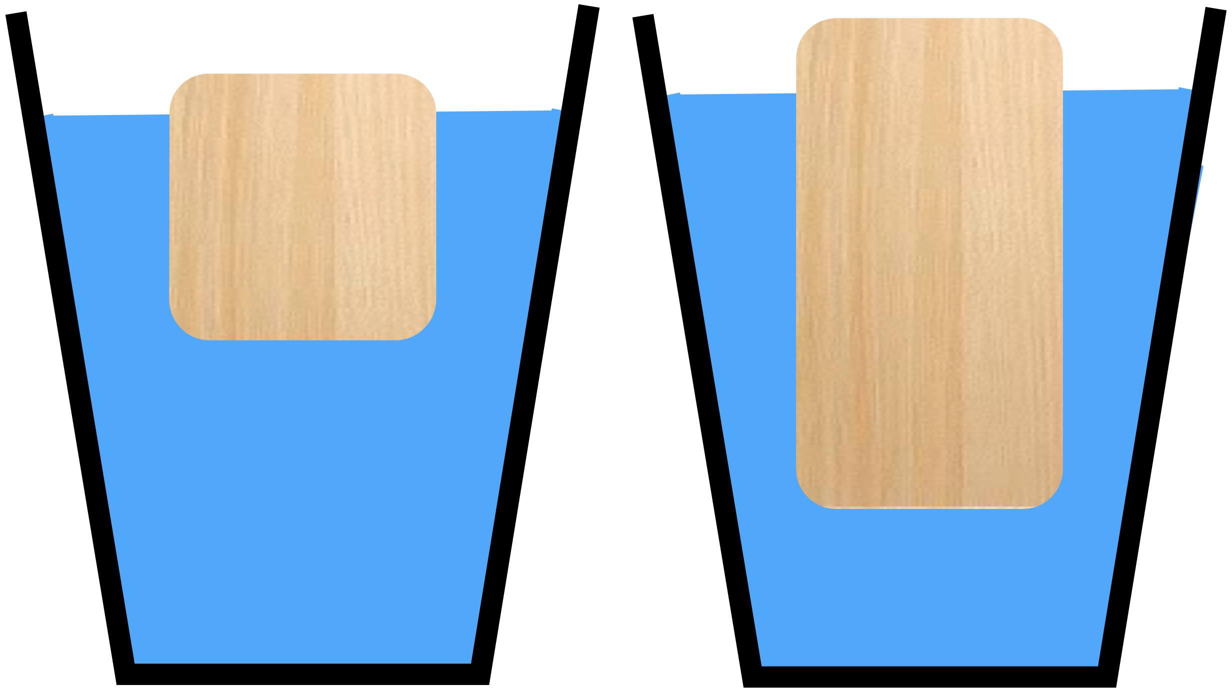

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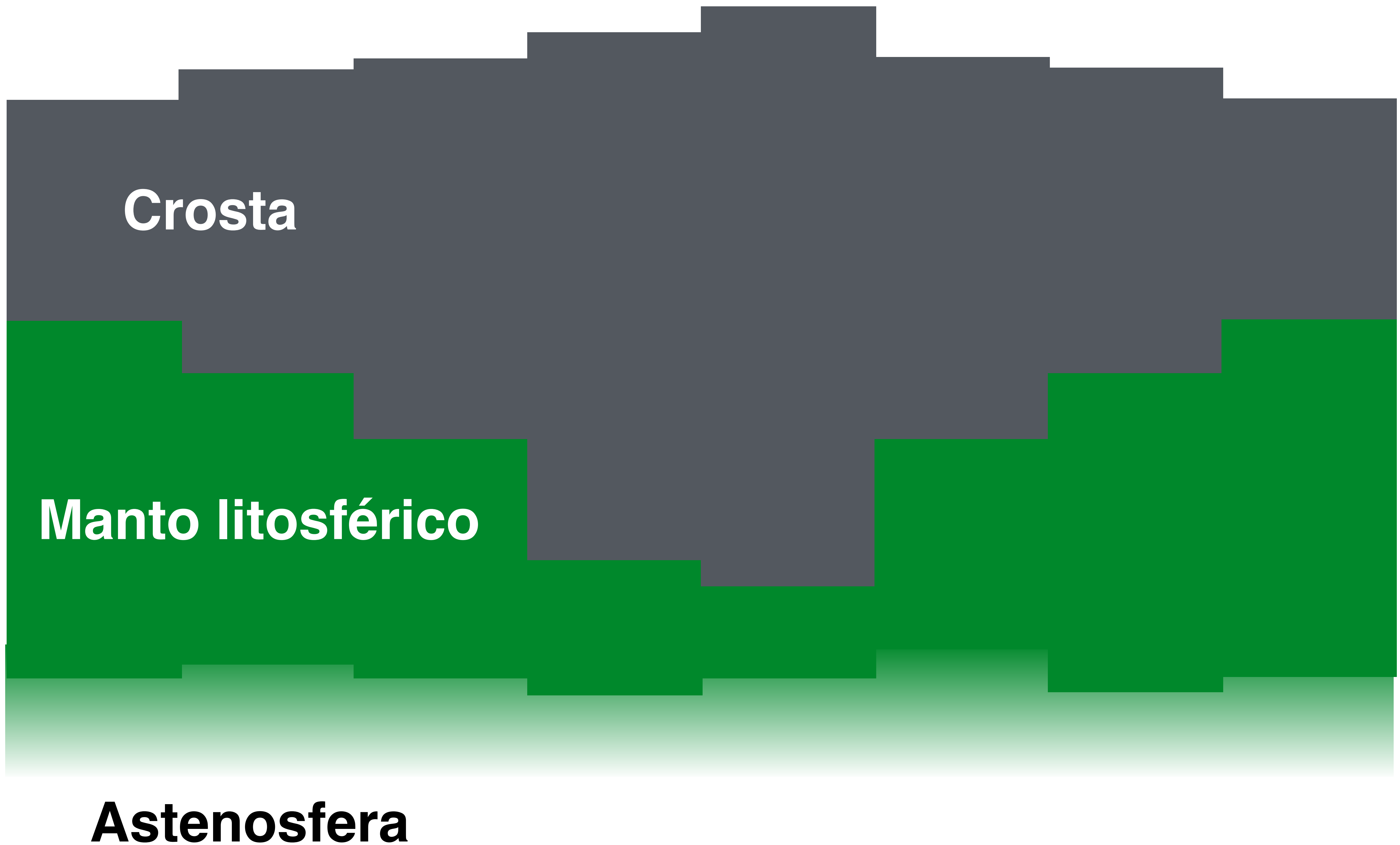
Isostasia da litosfera



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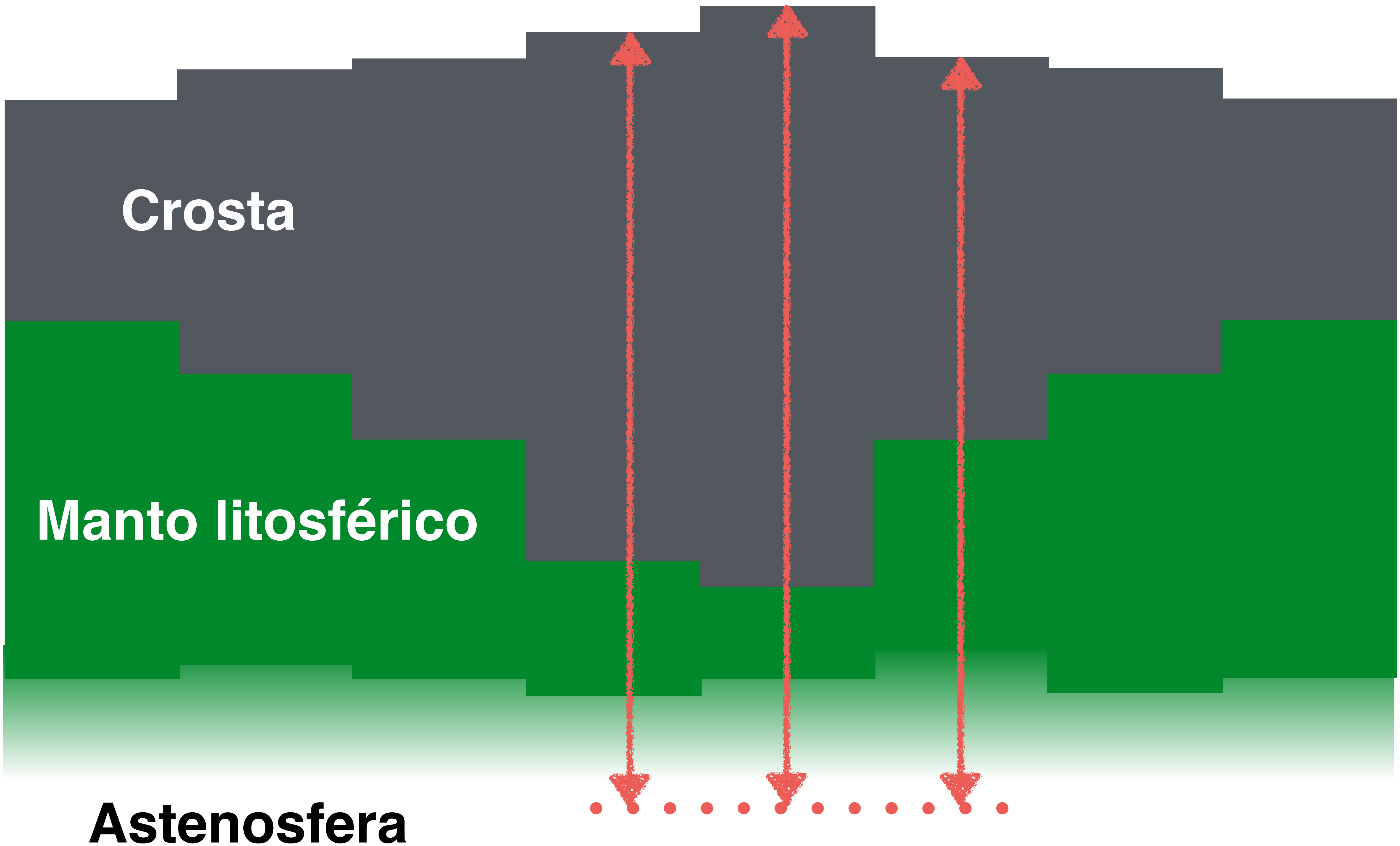
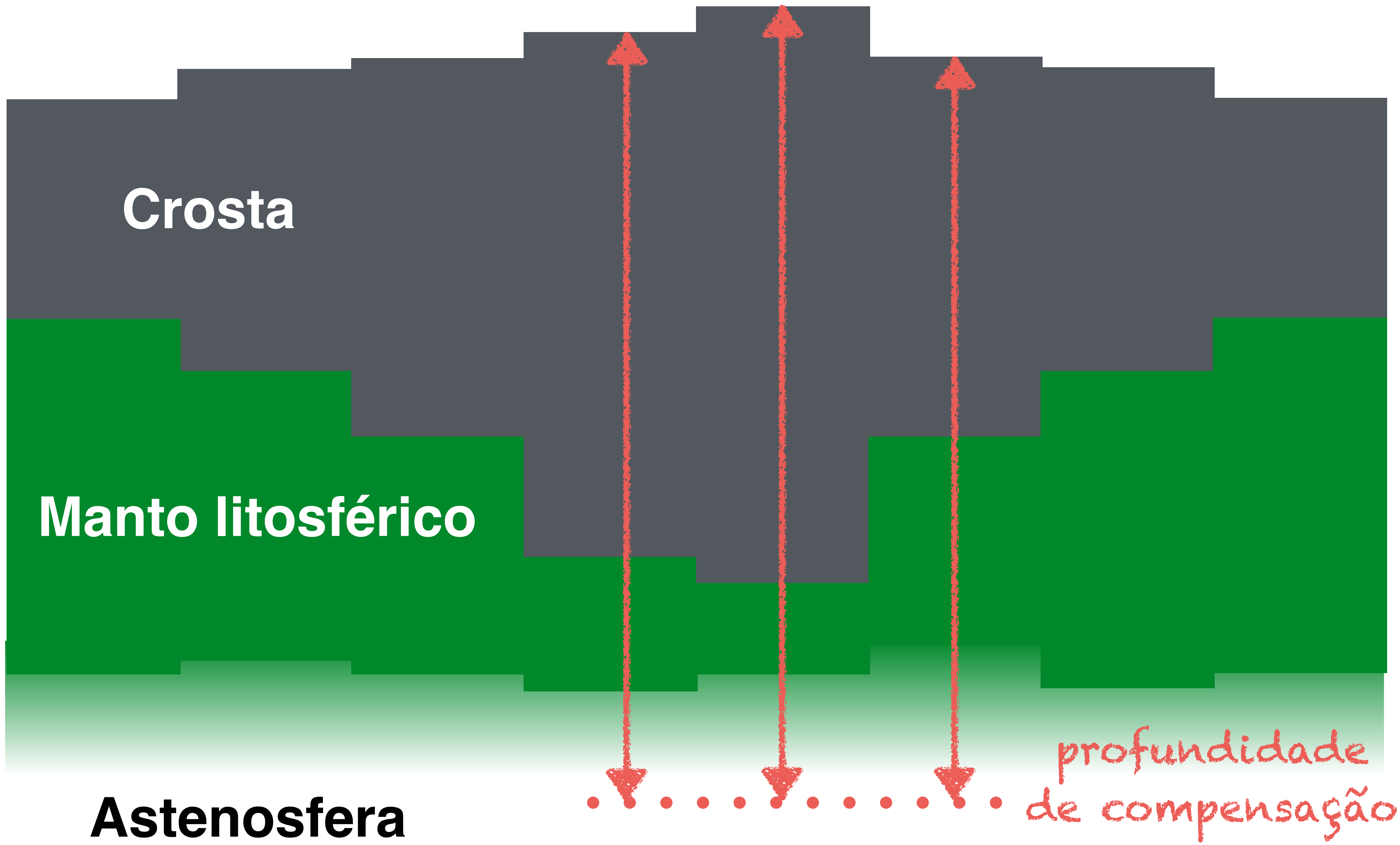
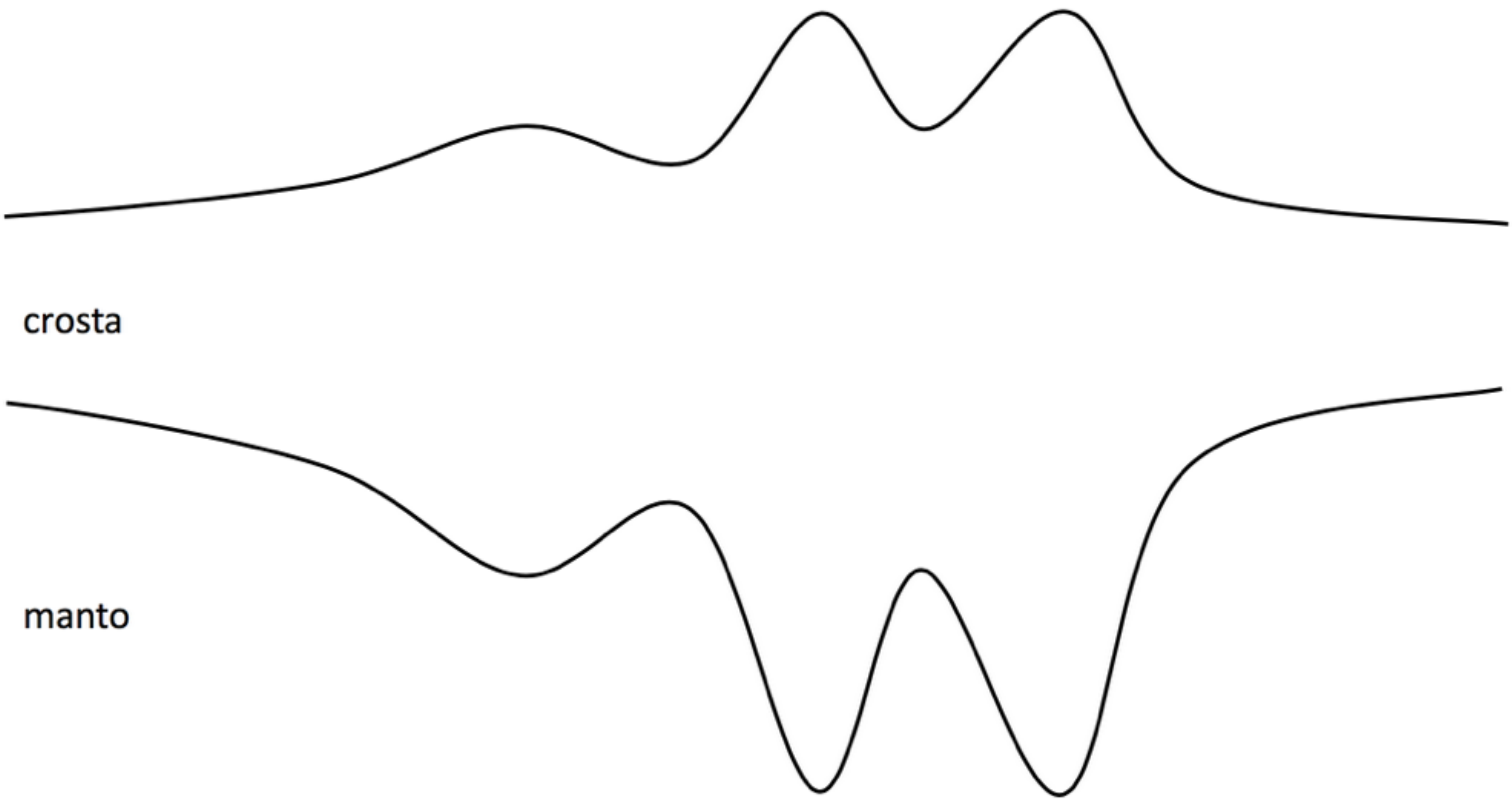


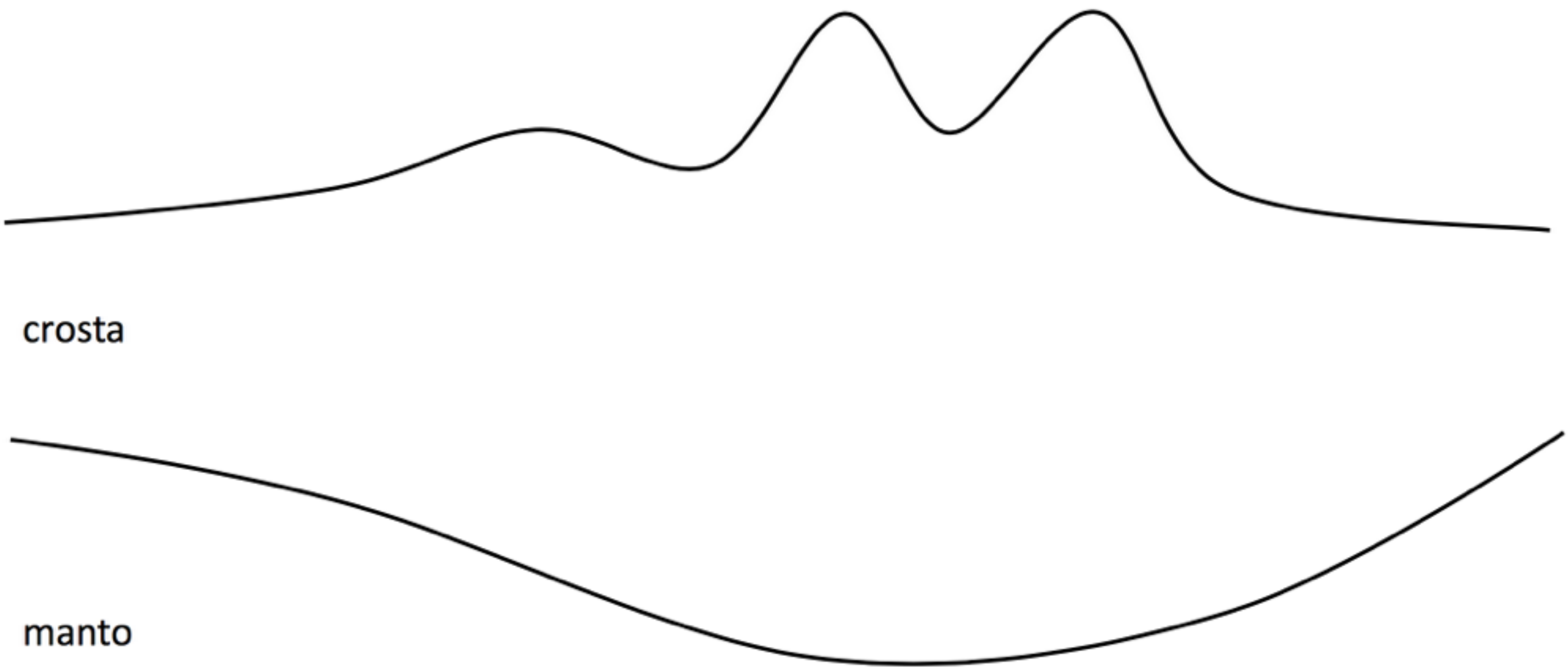
Isostasia da litosfera



Isostasia local



Isostasia regional

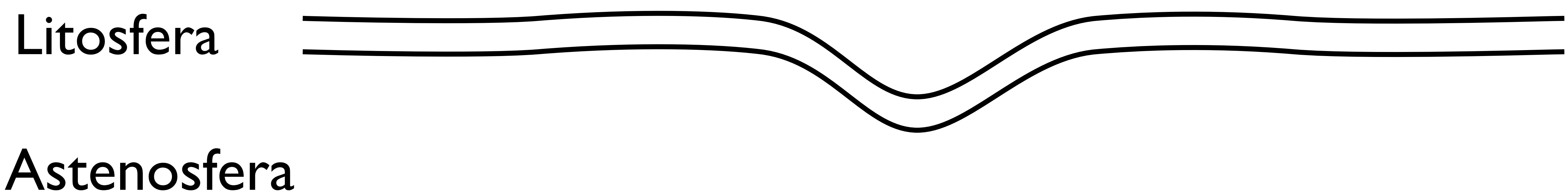


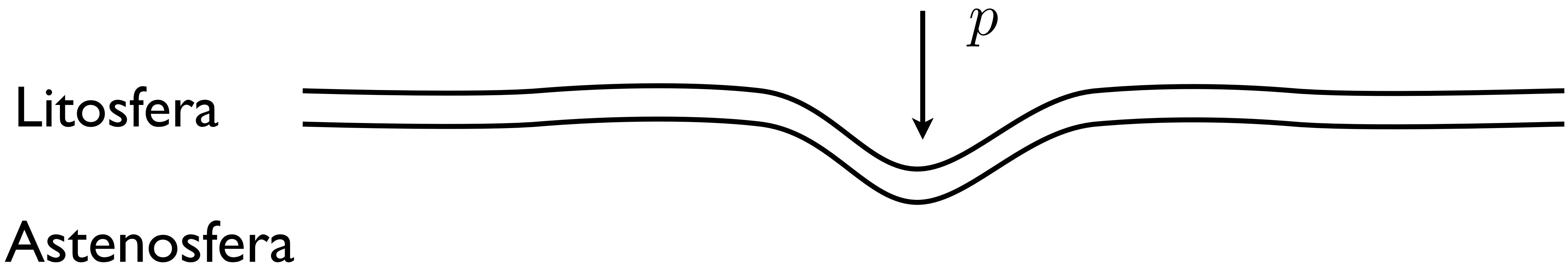
Flexura da litosfera: os primórdios

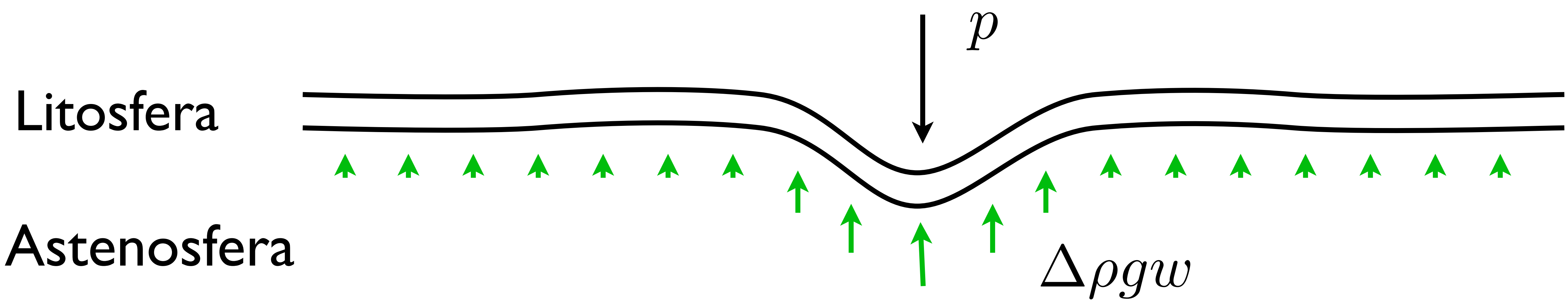
Para grandes blocos – por exemplo, um continente ou uma bacia oceânica inteira – a teoria da isostasia deve ser aceita sem dúvida; mas onde há feições menores, como montanhas individuais, a lei perde a sua validade. Tais feições podem ser sustentadas pela elasticidade do bloco todo.

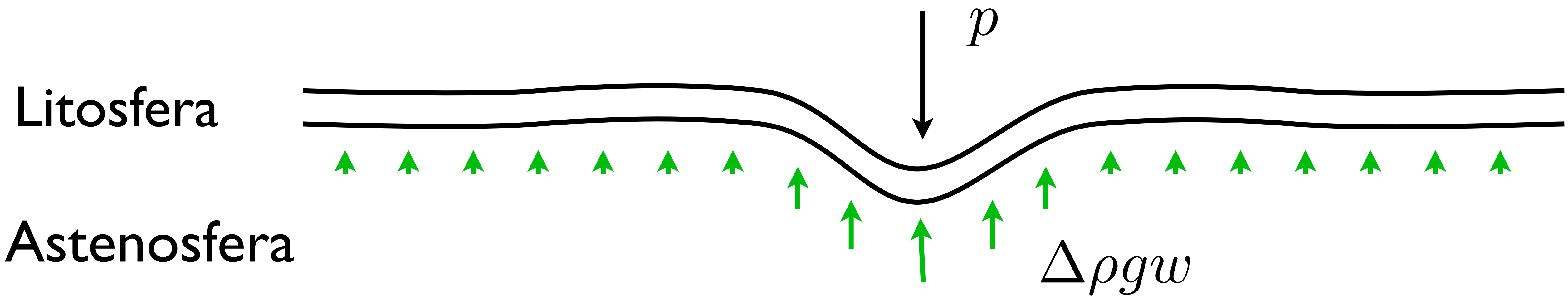
Alfred Wegener (1929) *tradução livre*

https://en.wikipedia.org/wiki/Alfred_Wegener#/media/File:Wegener_Expedition-1930_008.jpg

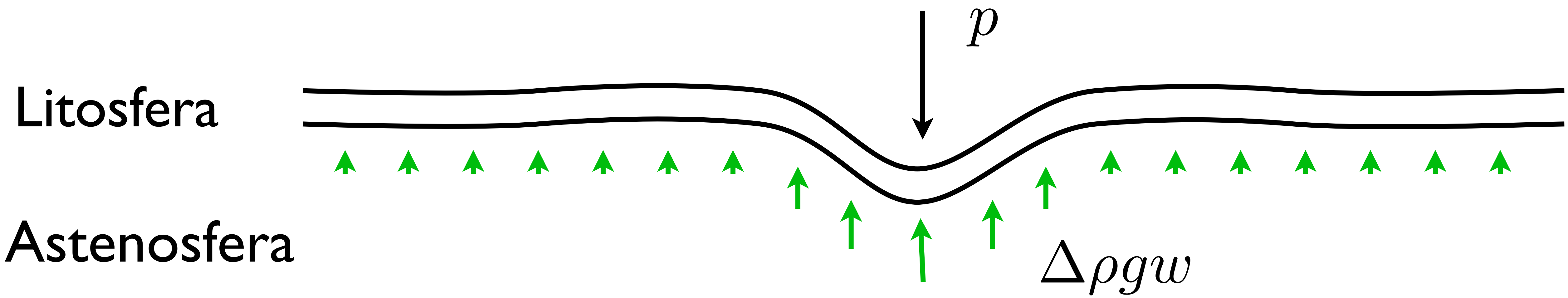

Litosfera
Astenosfera


p
Litosfera
Astenosfera


p
Litosfera
Astenosfera
$\Delta\rho g w$

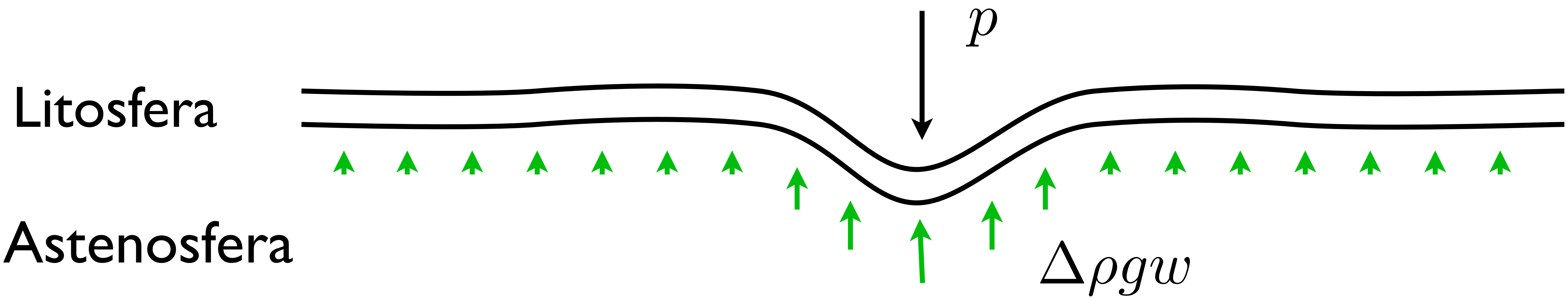


$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$



deslocamento
vertical da placa

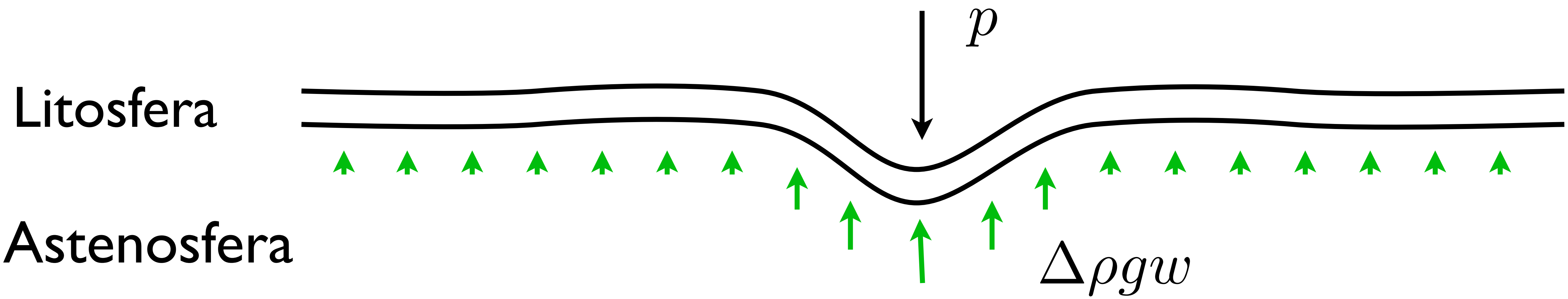
$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$



deslocamento vertical da placa

$$D\frac{d^4 w}{dx^4} + \Delta\rho g w = p$$

Rigidez da placa elástica

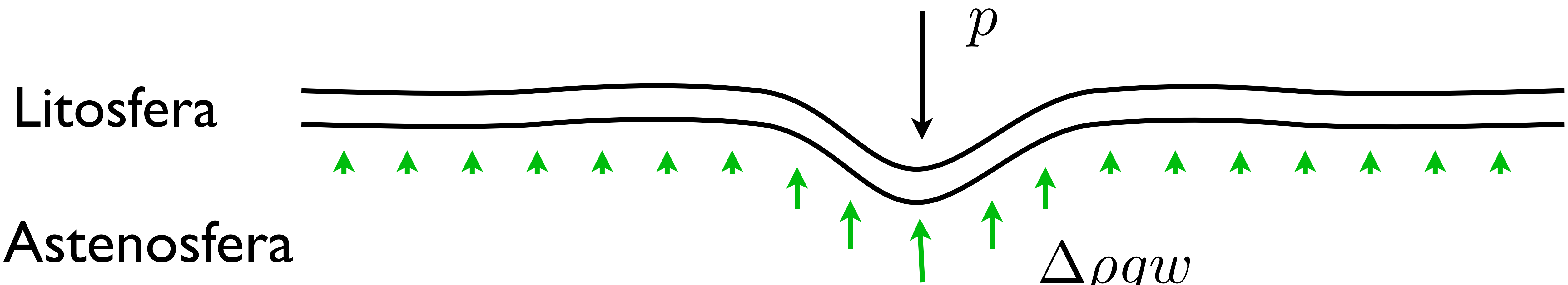


deslocamento
vertical da placa

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

Rigidez da placa
elástica

coordenada



deslocamento
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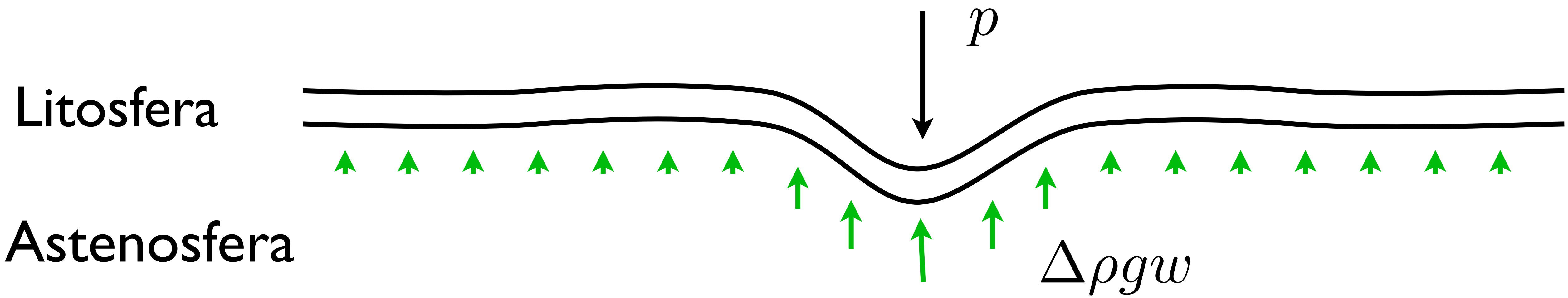
Rigidez da placa
elástica

coordenada

diferença de
densidade



deslocamento vertical da placa

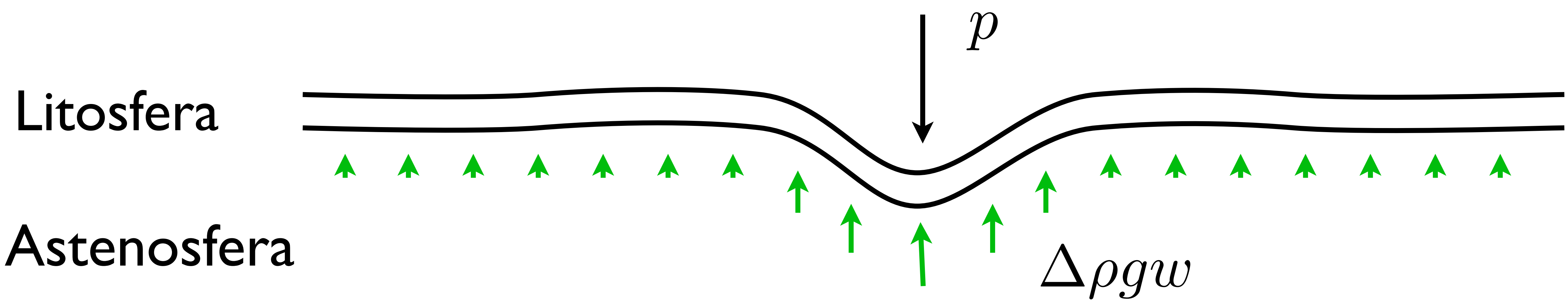
$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

Rigidez da placa elástica

coordenada

diferença de densidade

gravidade



deslocamento vertical da placa

carga

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

Rigidez da placa elástica

coordenada

gravidade

diferença de densidade

deslocamento
vertical da placa

carga

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

Rigidez da placa
elástica

coordenada

diferença de
densidade

gravidade

deslocamento vertical da placa

carga

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Rigidez da placa elástica

coordenada

gravidade

diferença de densidade

$$D = \frac{ET_e^3}{12(1-\nu^2)}$$

deslocamento vertical da placa

carga

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

Rigidez da placa elástica

coordenada

gravidade

diferença de densidade

Módulo de elasticidade

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vertical da placa

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Rigidez da placa
elástica

coordenada

gravidade

diferença de
densidade

Módulo de
elasticidade

Espessura
elástica
efetiva

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deslocamento vertical da placa

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Rigidez da placa elástica

coordenada

gravidade

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Módulo de elasticidade

Espessura elástica efetiva

$$D = \frac{E T_e^3}{12(1 - \nu^2)}$$

Coeficiente de Poisson

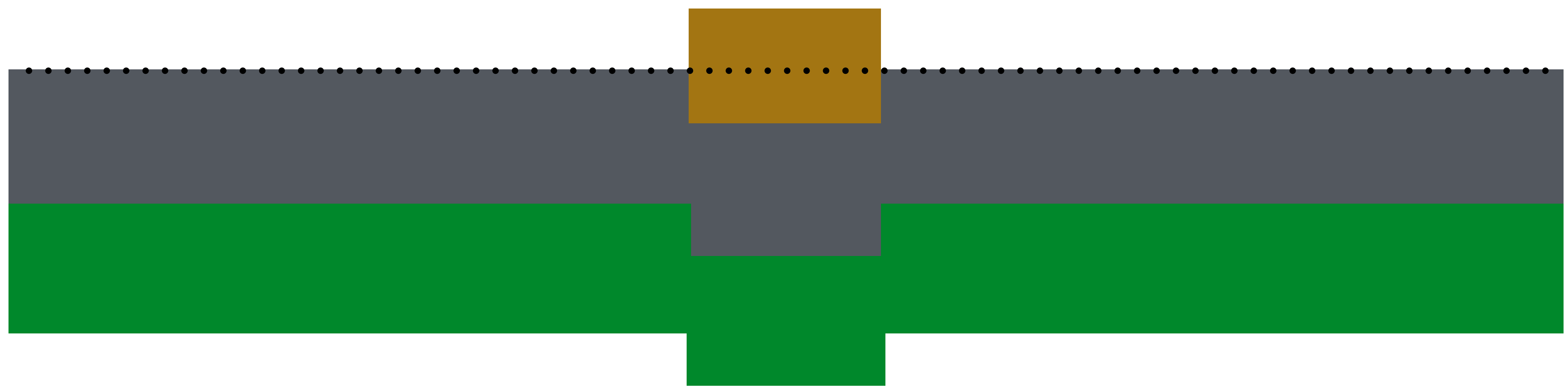
Isostasia e Flexura da Litosfera

Isostasia
Local

Isostasia
Flexural

Isostasia e Flexura da Litosfera

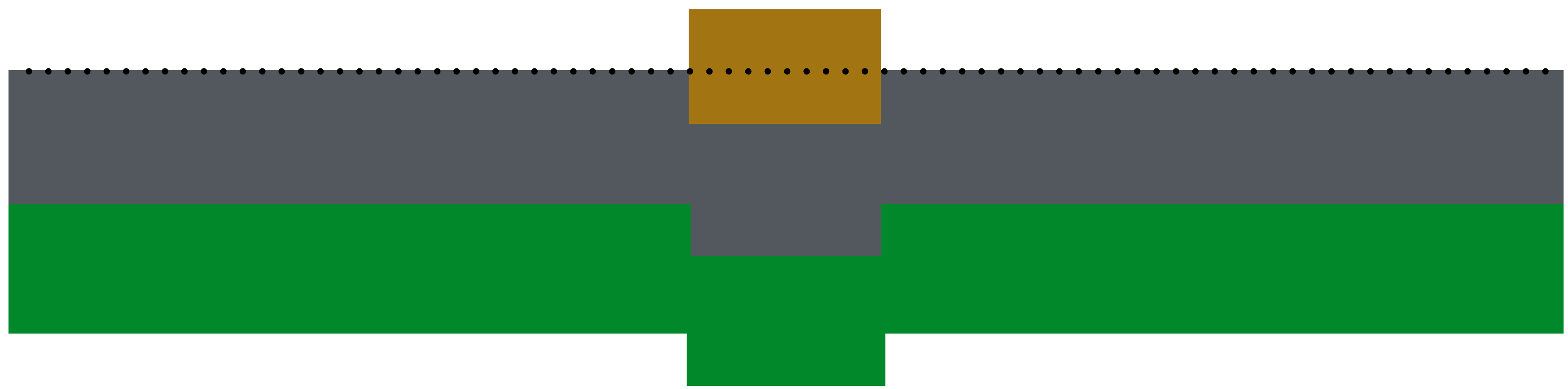
Isostasia Local



Isostasia Flexural

Isostasia e Flexura da Litosfera

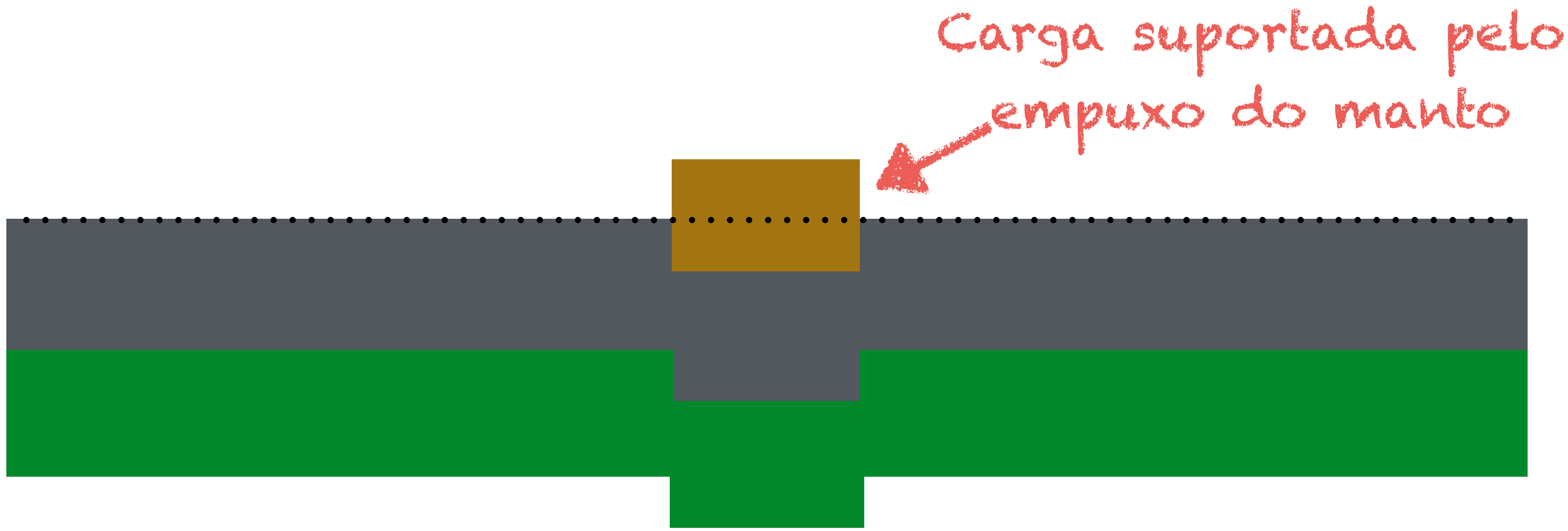
Isostasia
Local



Isostasia
Flexural

Isostasia e Flexura da Litosfera

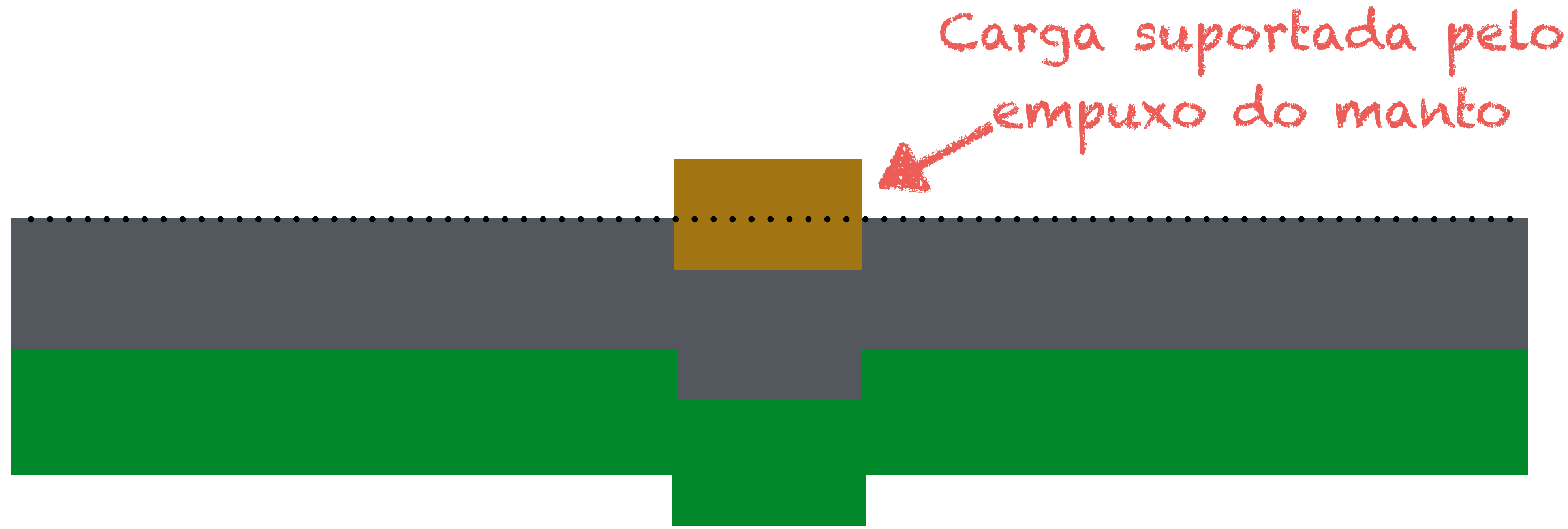
Isostasia Local



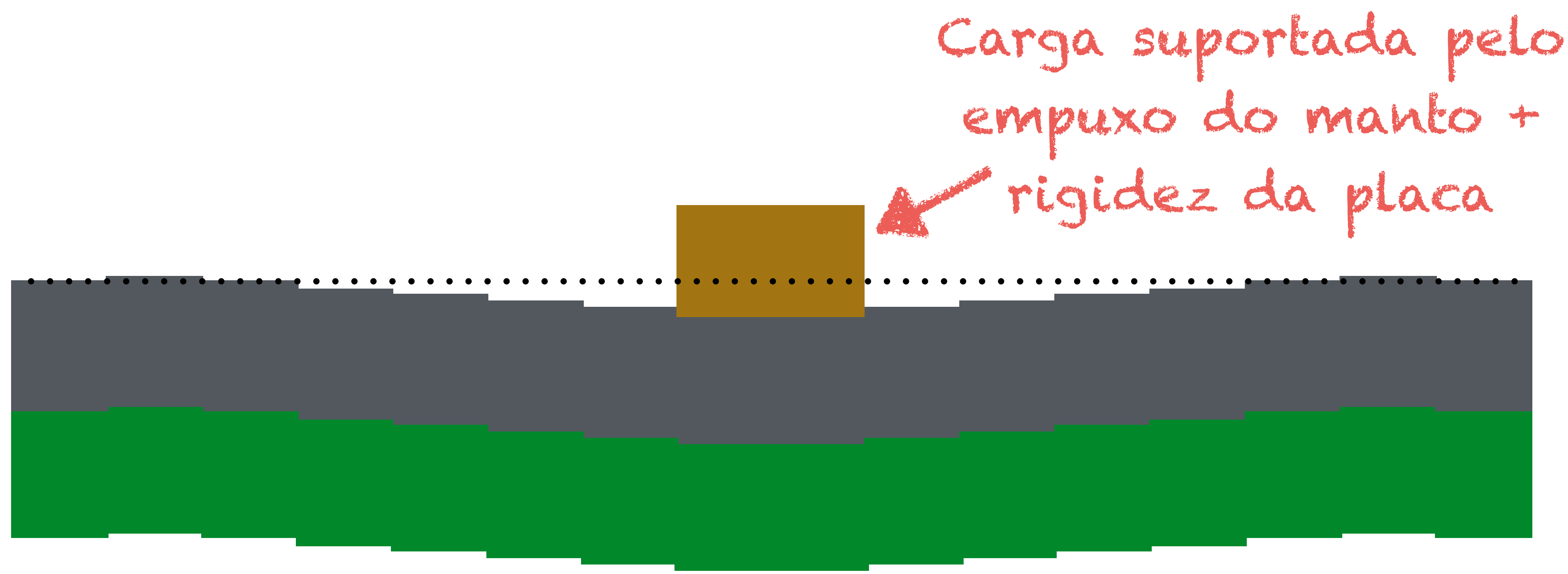
Isostasia Flexural

Isostasia e Flexura da Litosfera

Isostasia Local



Isostasia Flexural



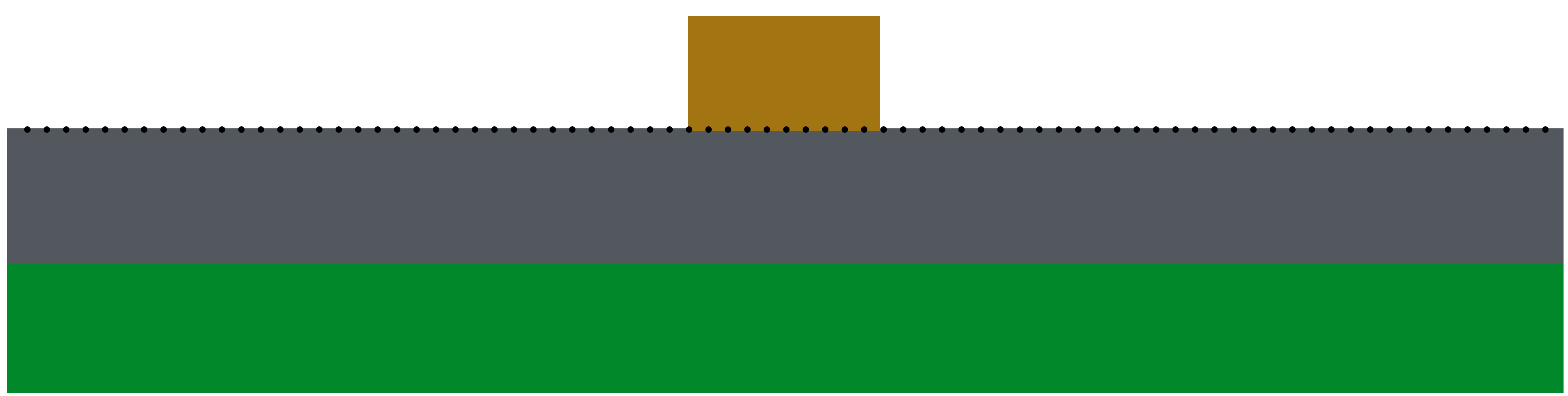
Espessura da placa elástica

T_e : Espessura elástica efetiva

$T_e = 0$

T_e finite

$T_e \to \infty$



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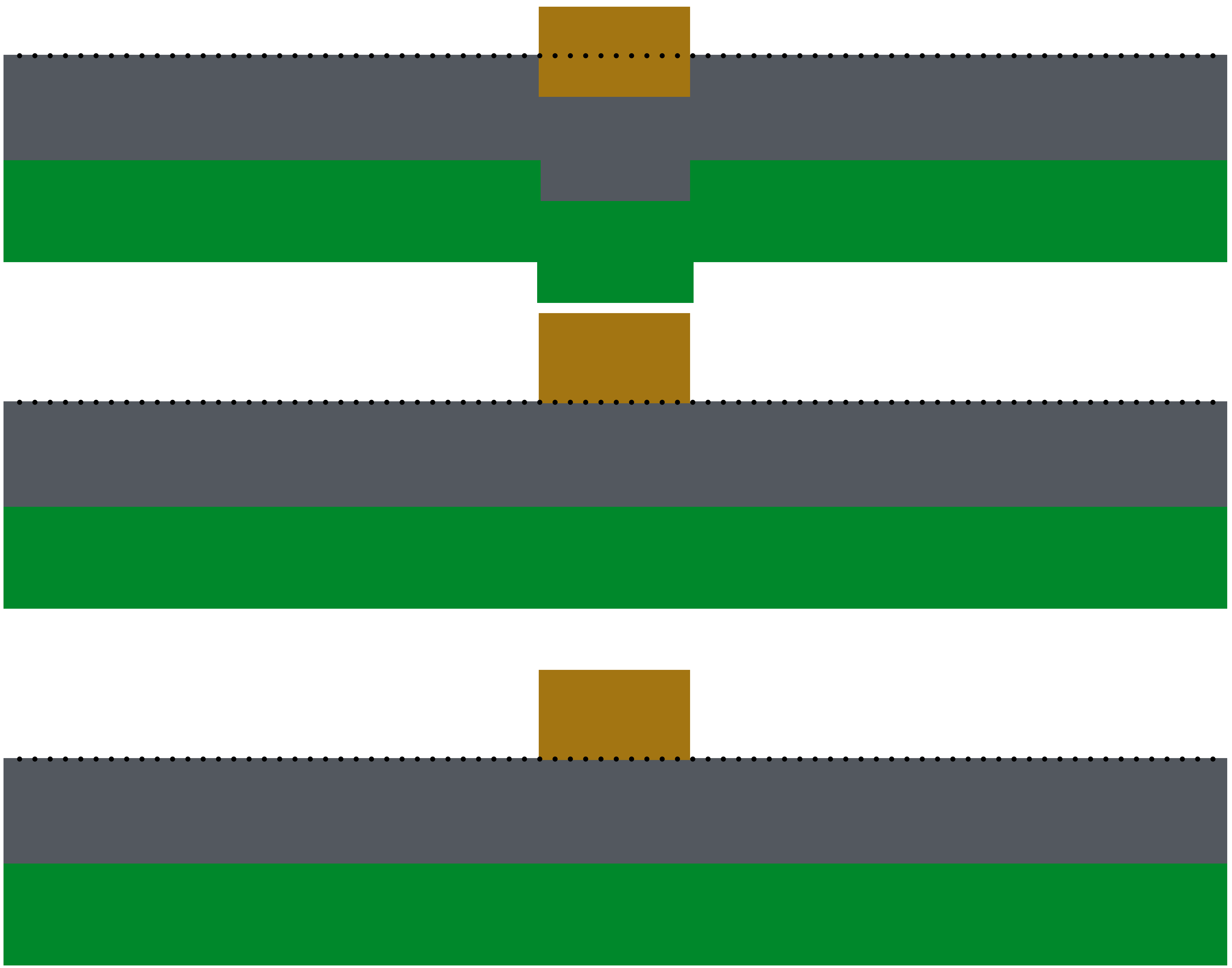

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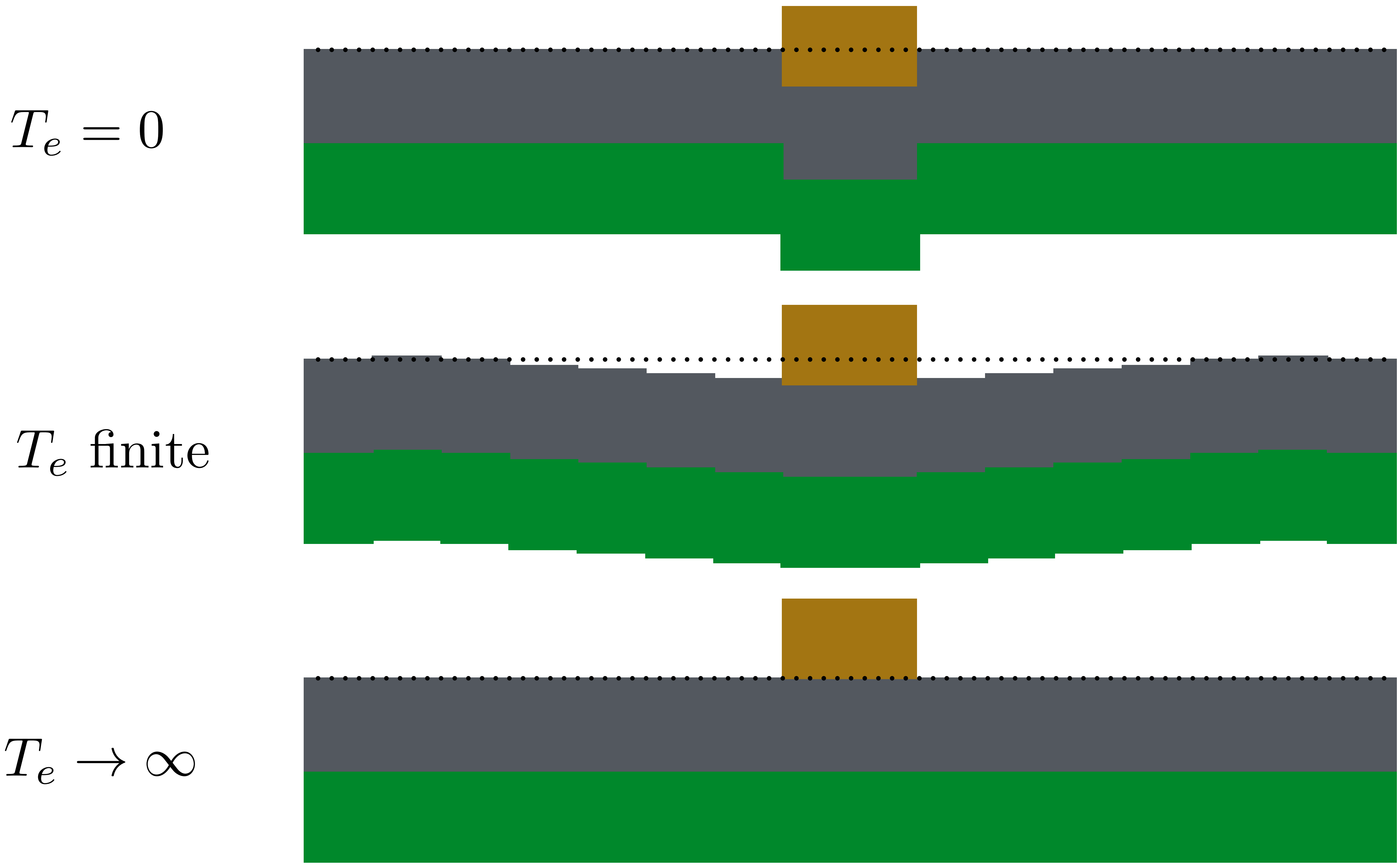

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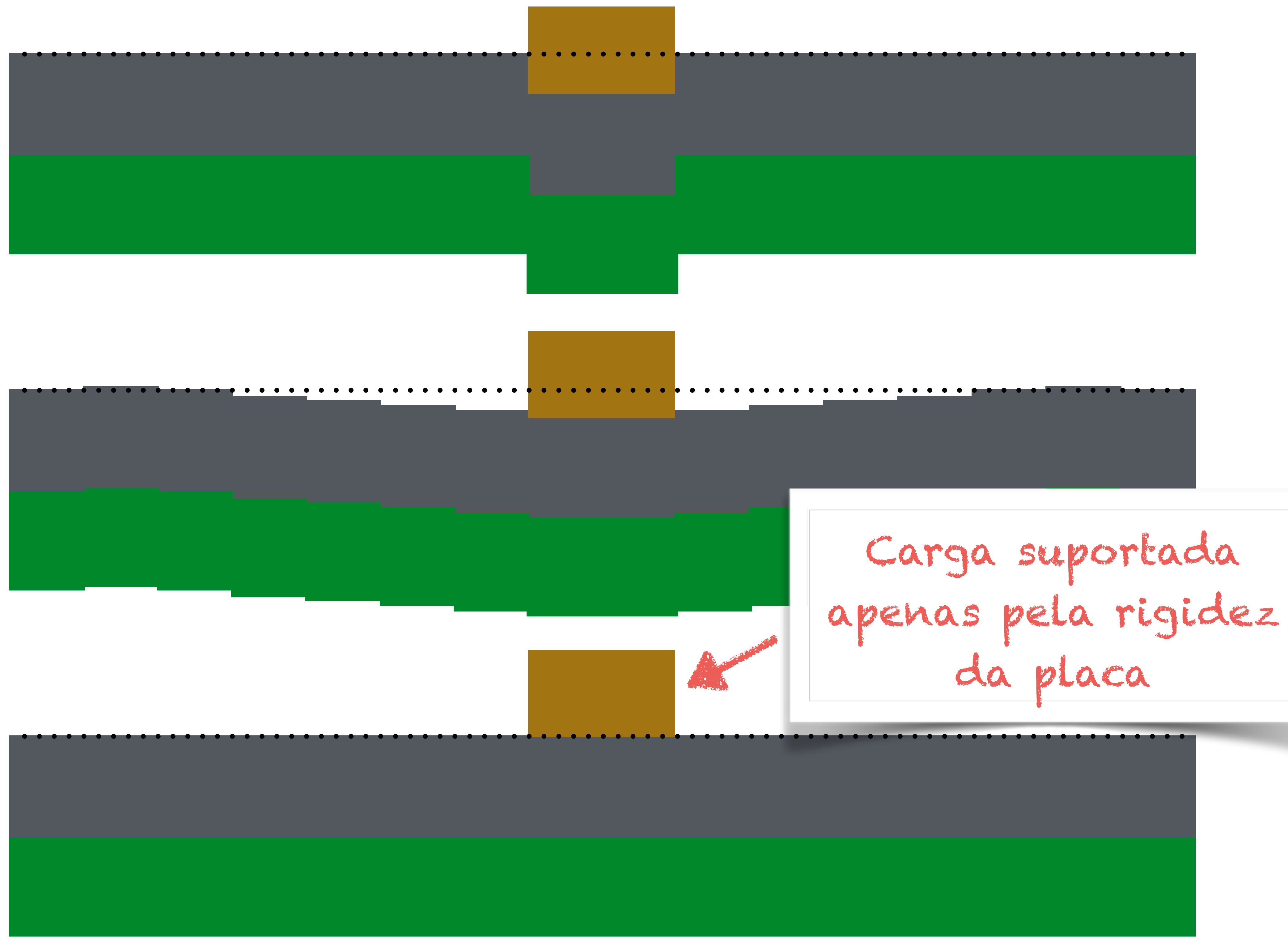


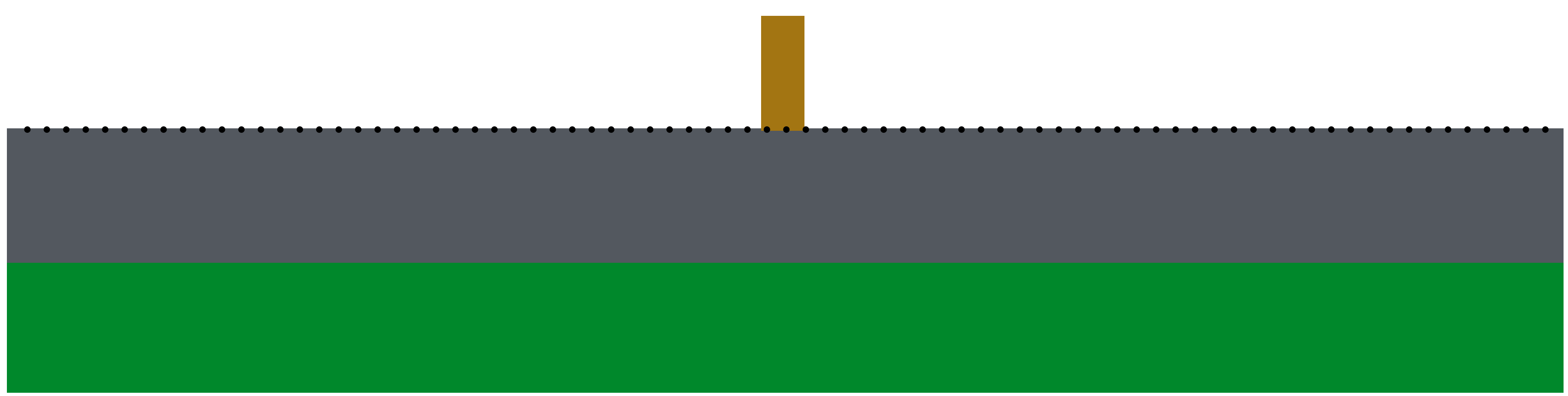
Size of the load x T_e

Para o mesmo T_e :

Carga larga

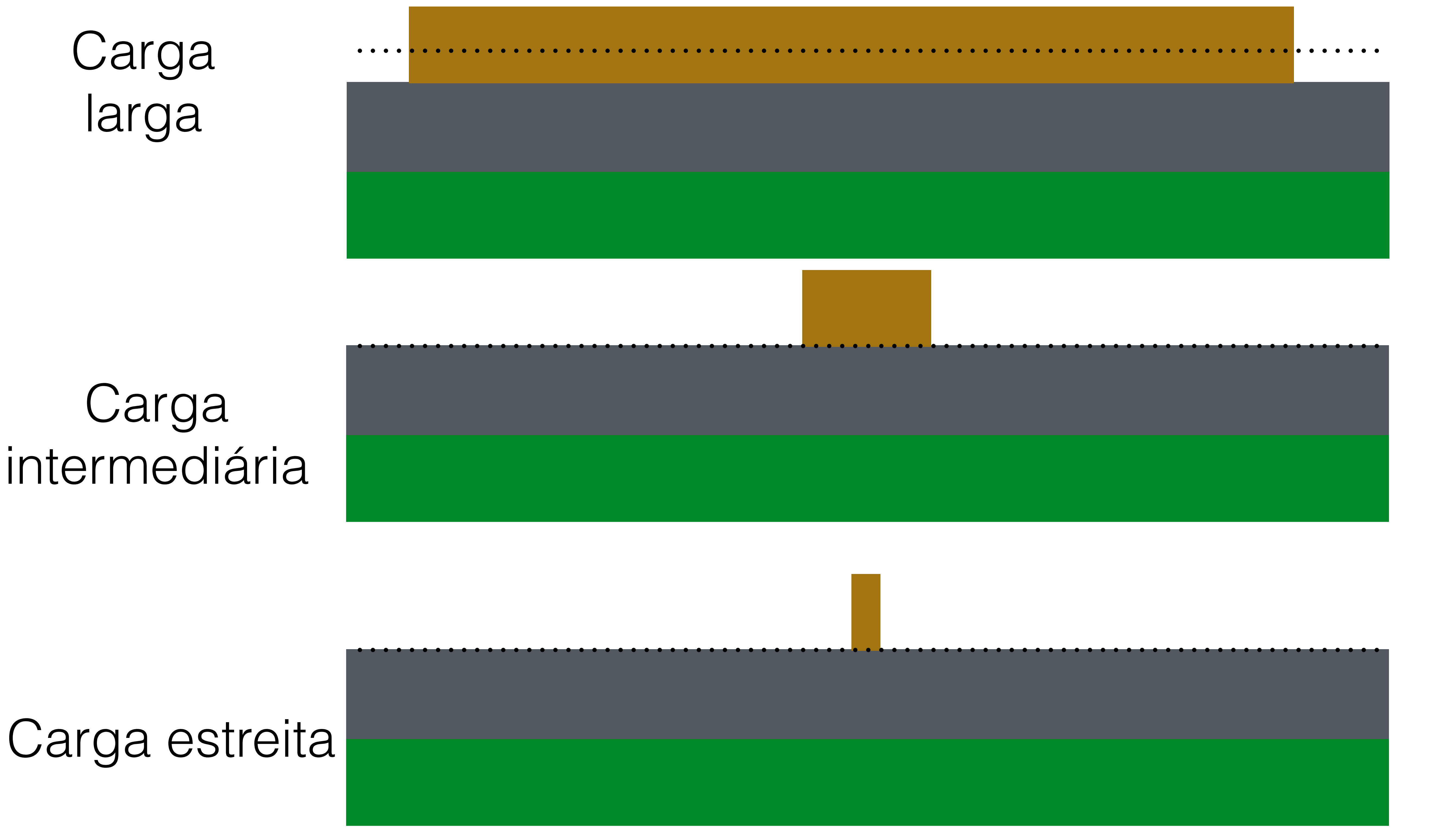
Carga intermediária

Carga estreita



Size of the load x T_e

Para o mesmo T_e :



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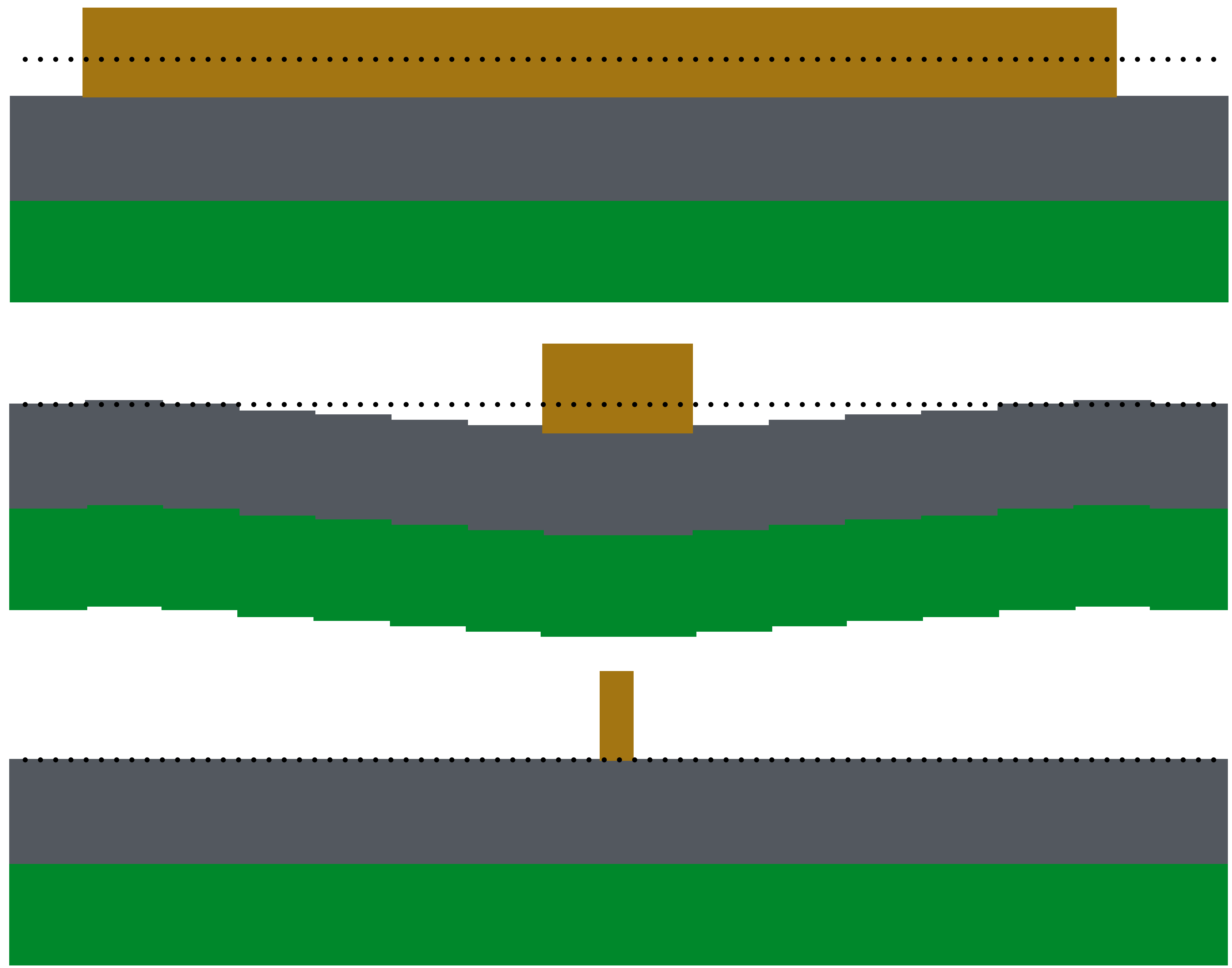


Contribuições da isostasia e flexura para a tectônica de placas

- Existência de um fluido (astenosfera) que permite a compensação isostática e o deslocamento lateral das placas litosféricas.

- A camada externa da Terra (litosfera) preserva a sua rigidez ao longo do tempo geológico.

Cinemática das placas

http://www.bl.uk/voices-of-science/interviewees/dan-mckenzie

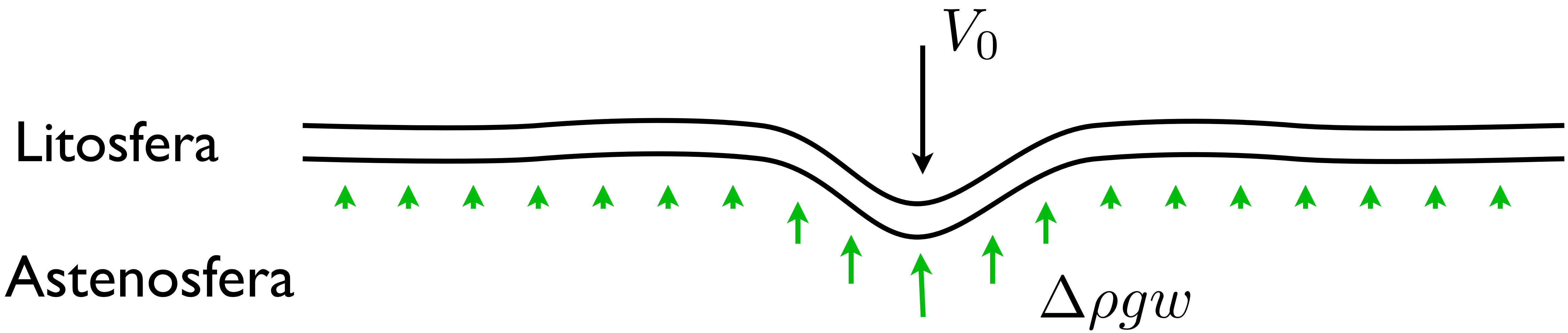
McKenzie & Parker (1967)

https://www.onbeing.org/program/fragility-and-evolution-our-humanity/101

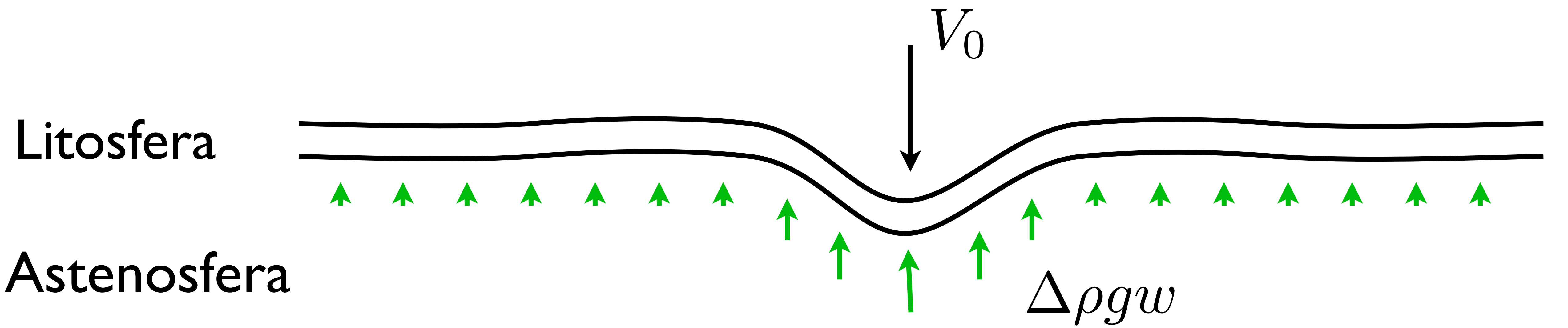
Le Pichon (1968)

https://en.wikipedia.org/wiki/W._Jason_Morgan#/media/File:Morgan,_W._Jason.jpg

Morgan (1968)



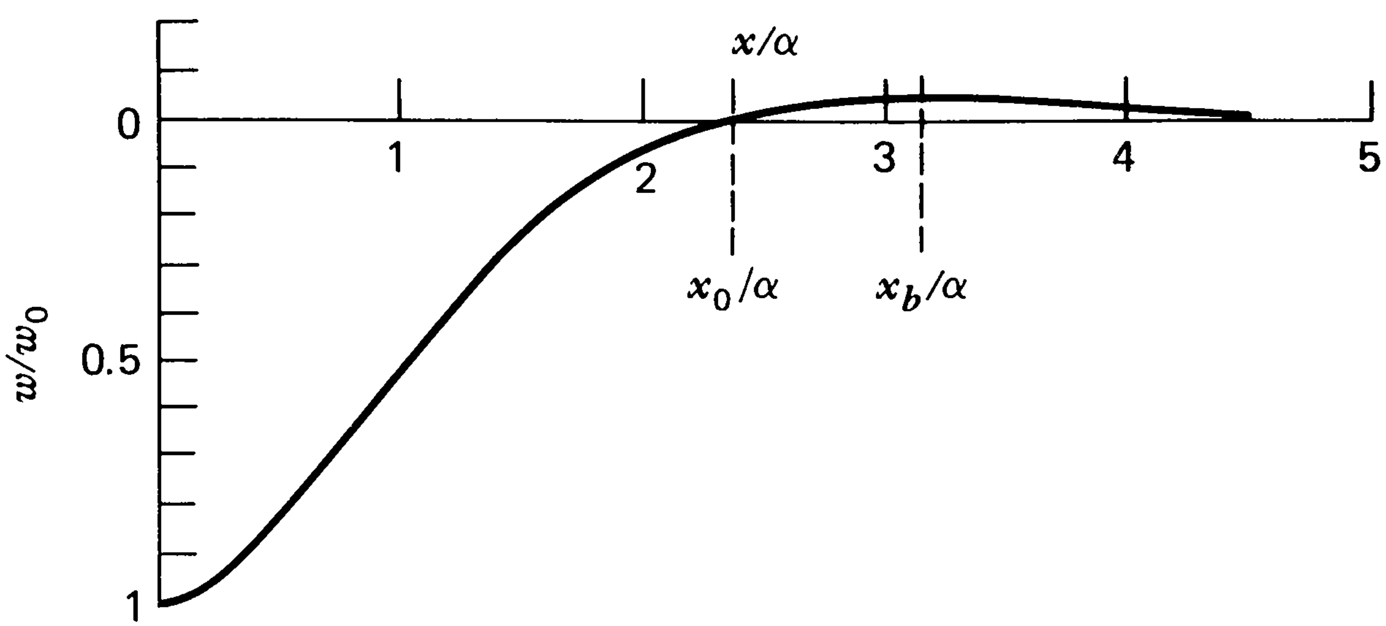
V_0
Litosfera
Astenosfera
$\Delta\rho g w$



$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

Solução analítica para o caso de uma carga pontual

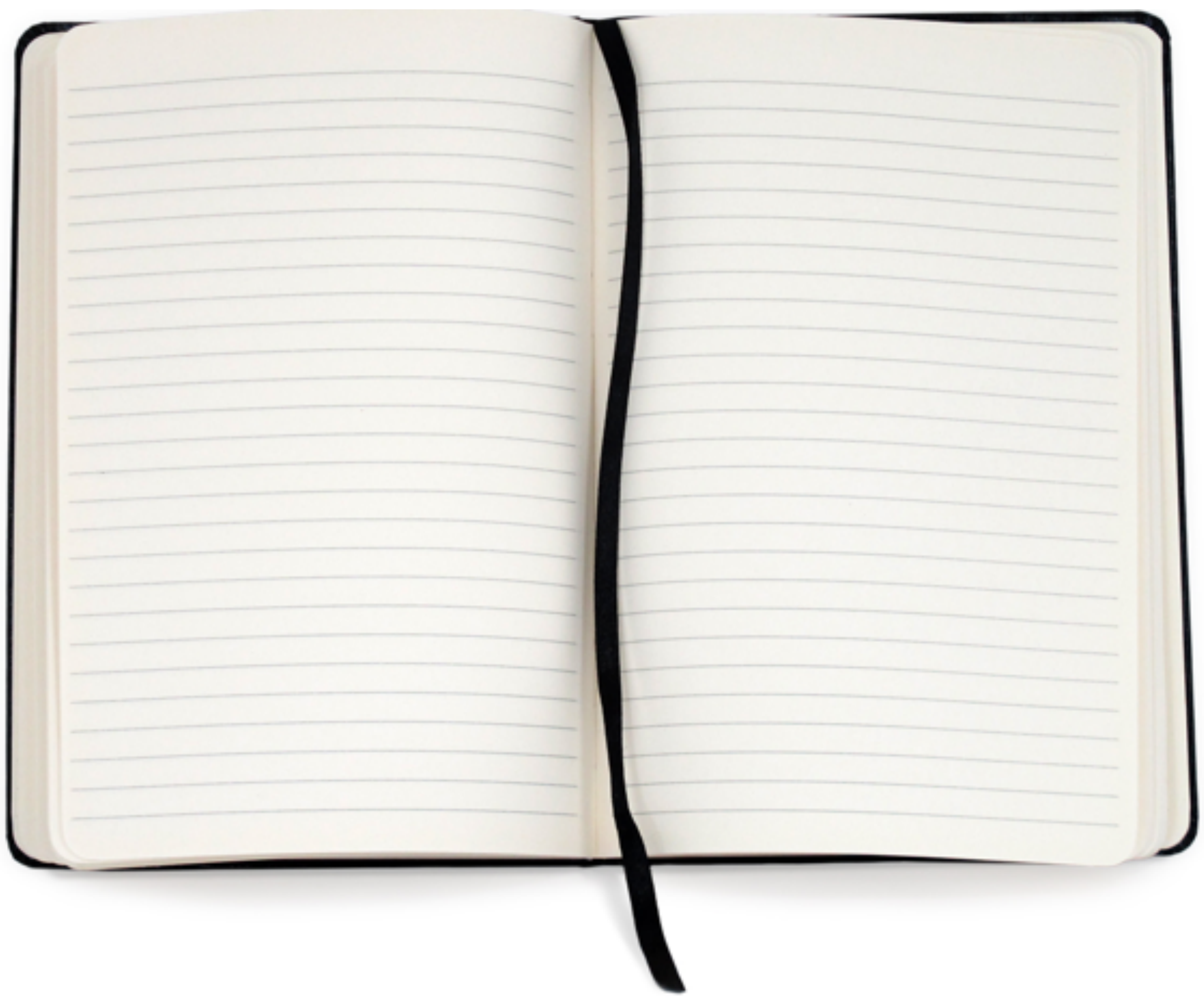
$$w = \frac{V_0 \alpha^3}{8D} e^{-x/\alpha} \left(\cos\frac{x}{\alpha} + \sin\frac{x}{\alpha} \right)$$



$$D = \frac{E T_e^3}{12(1-\nu^2)}$$

$$\alpha = \left[\frac{4D}{(\rho_m - \rho_w) g} \right]^{1/4}$$

Prática: Flex_analitico



Aproximação numérica

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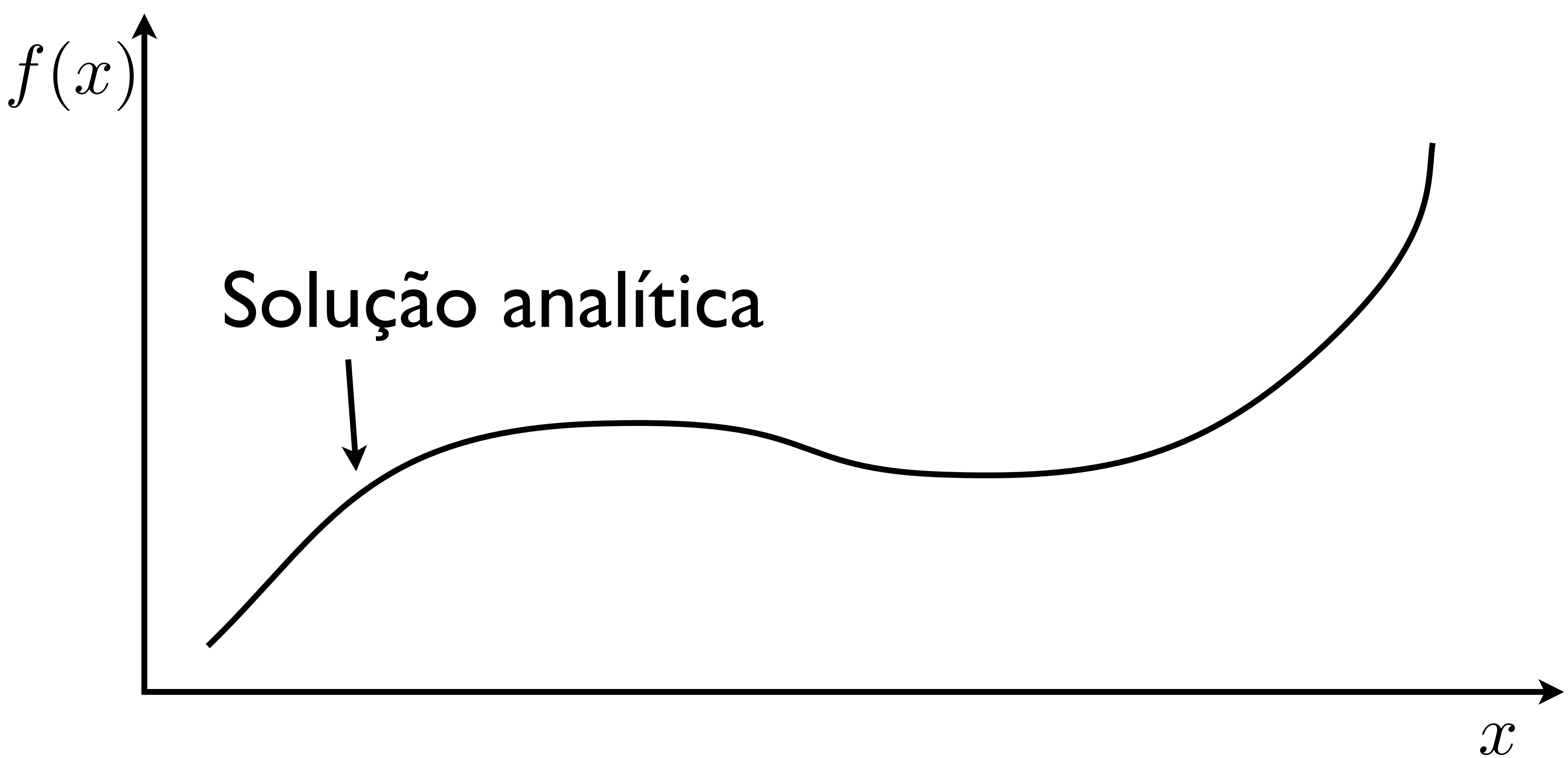
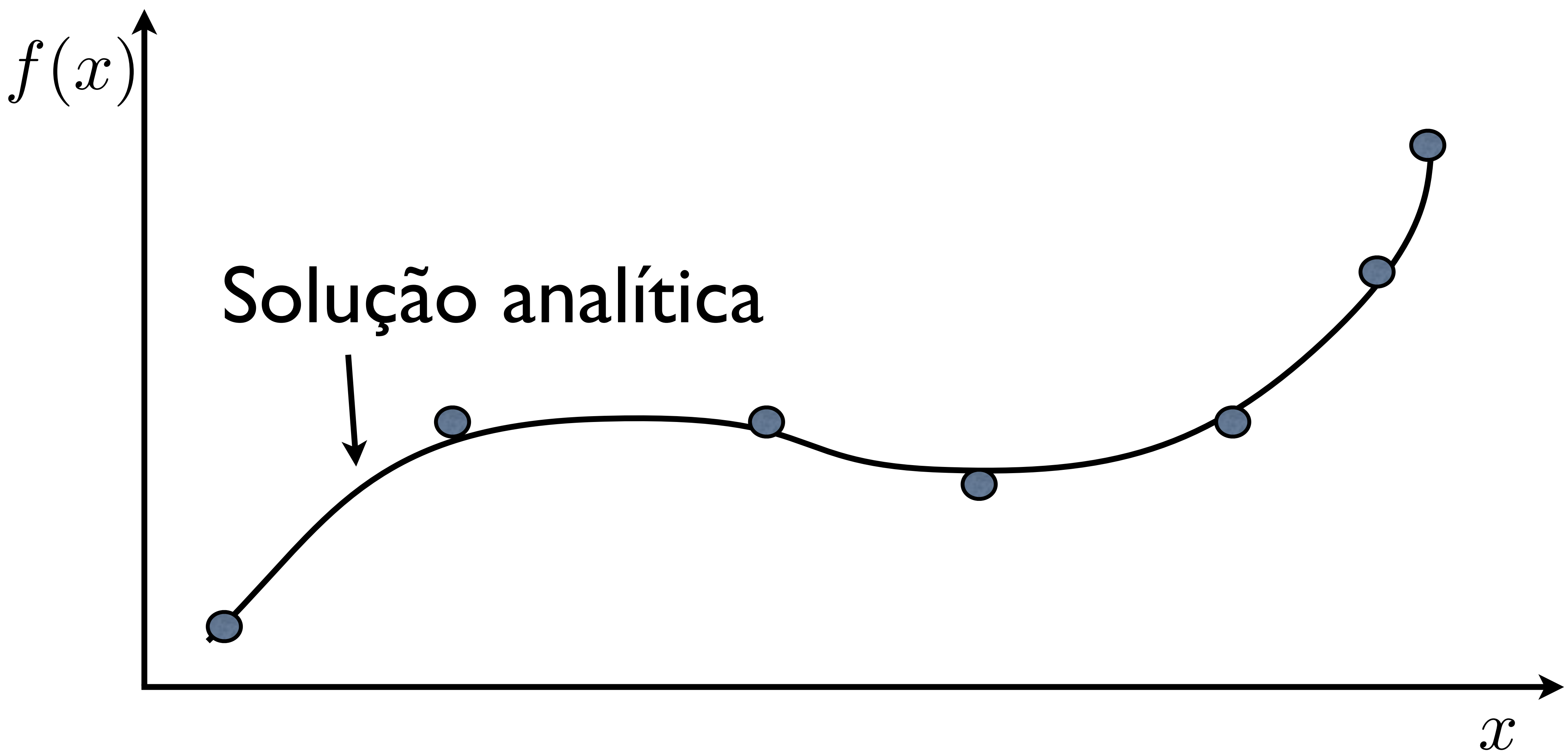
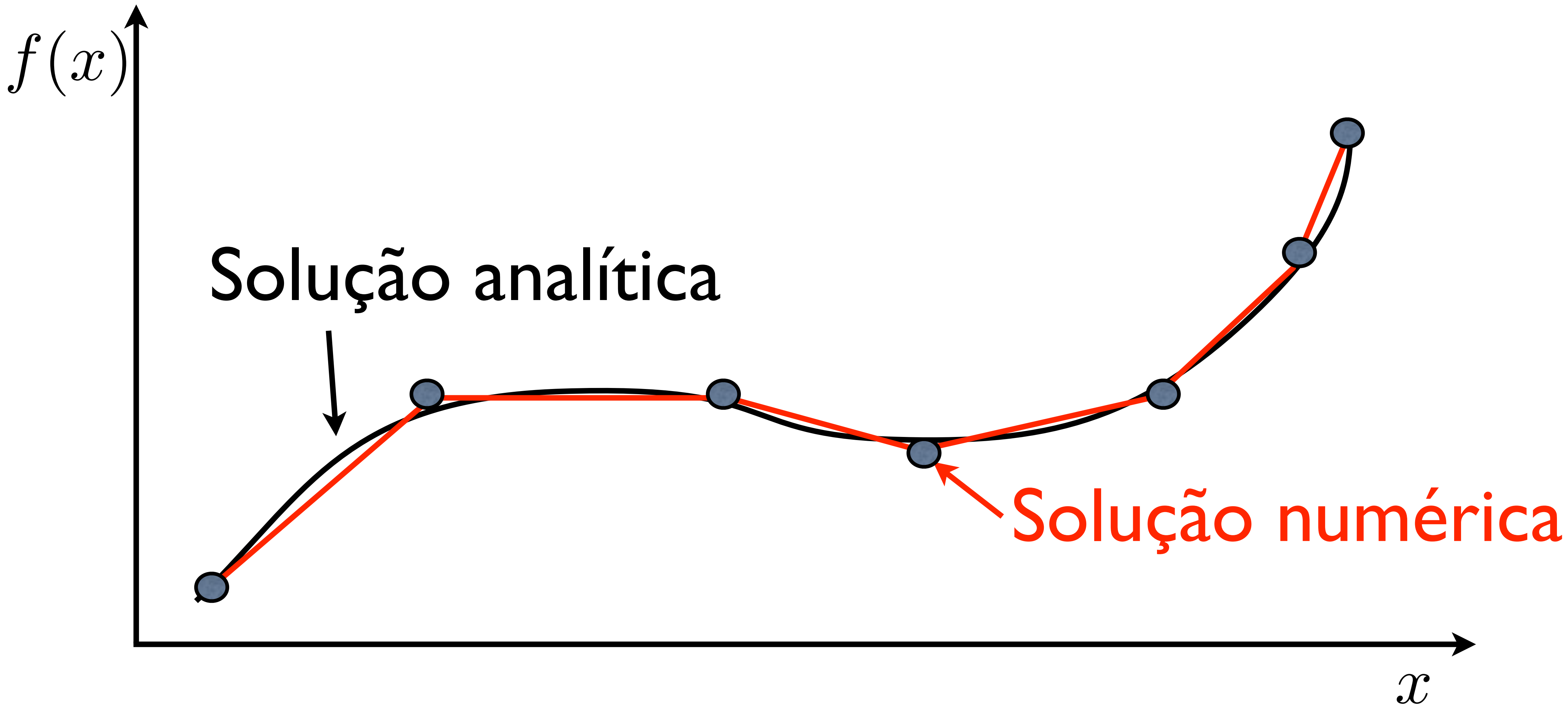


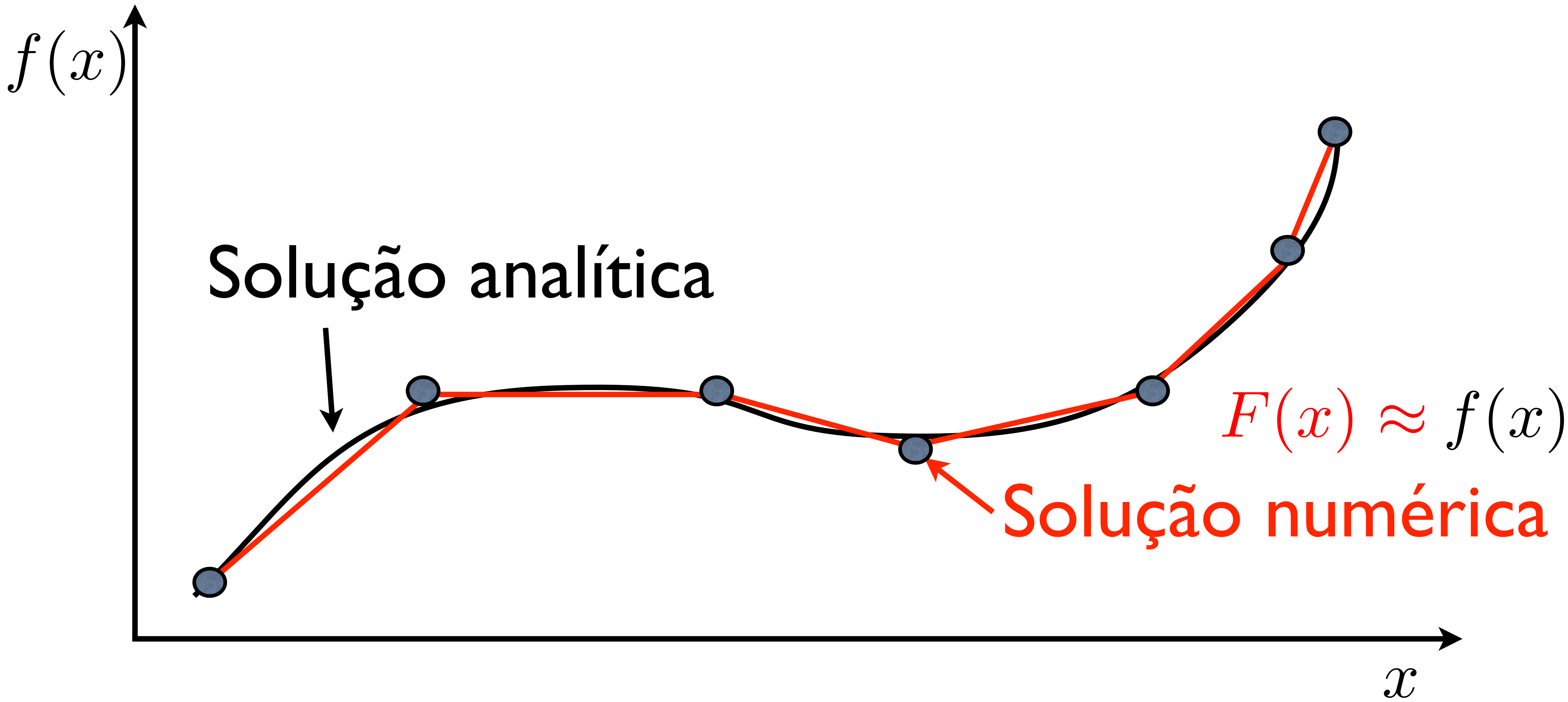
Aproximação numérica



Aproximação numérica



Aproximação numérica



Sol. analítica x Sol. numérica

Sol. analítica x Sol. numérica

- Solução exata

Sol. analítica x Sol. numérica

- Solução exata
- Solução aproximada

Sol. analítica x Sol. numérica

- Solução exata
- Contínua

- Solução aproximada

Sol. analítica x Sol. numérica

- Solução exata
- Contínua

- Solução aproximada
- Discreta

Sol. analítica x Sol. numérica

- Solução exata
- Contínua
- Nada é escondido

- Solução aproximada
- Discreta

Sol. analítica x Sol. numérica

- Solução exata
- Contínua
- Nada é escondido

- Solução aproximada
- Discreta
- Pode ser uma caixa preta

Sol. analítica x Sol. numérica

- Solução exata
- Contínua
- Nada é escondido
- Todos os cenários

- Solução aproximada
- Discreta
- Pode ser uma caixa preta

Sol. analítica x Sol. numérica

- Solução exata
- Contínua
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- Solução aproximada
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- Pode ser uma caixa preta
- 1 simulação por cenário

Sol. analítica x Sol. numérica

- Solução exata
- Contínua
- Nada é escondido
- Todos os cenários
- Não exige validação

- Solução aproximada
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Sol. analítica x Sol. numérica

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Sol. analítica x Sol. numérica

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- **1** simulação por cenário
- Exige validação

Ora bolas! Para que serve a modelagem numérica?

1. Geometria complicada

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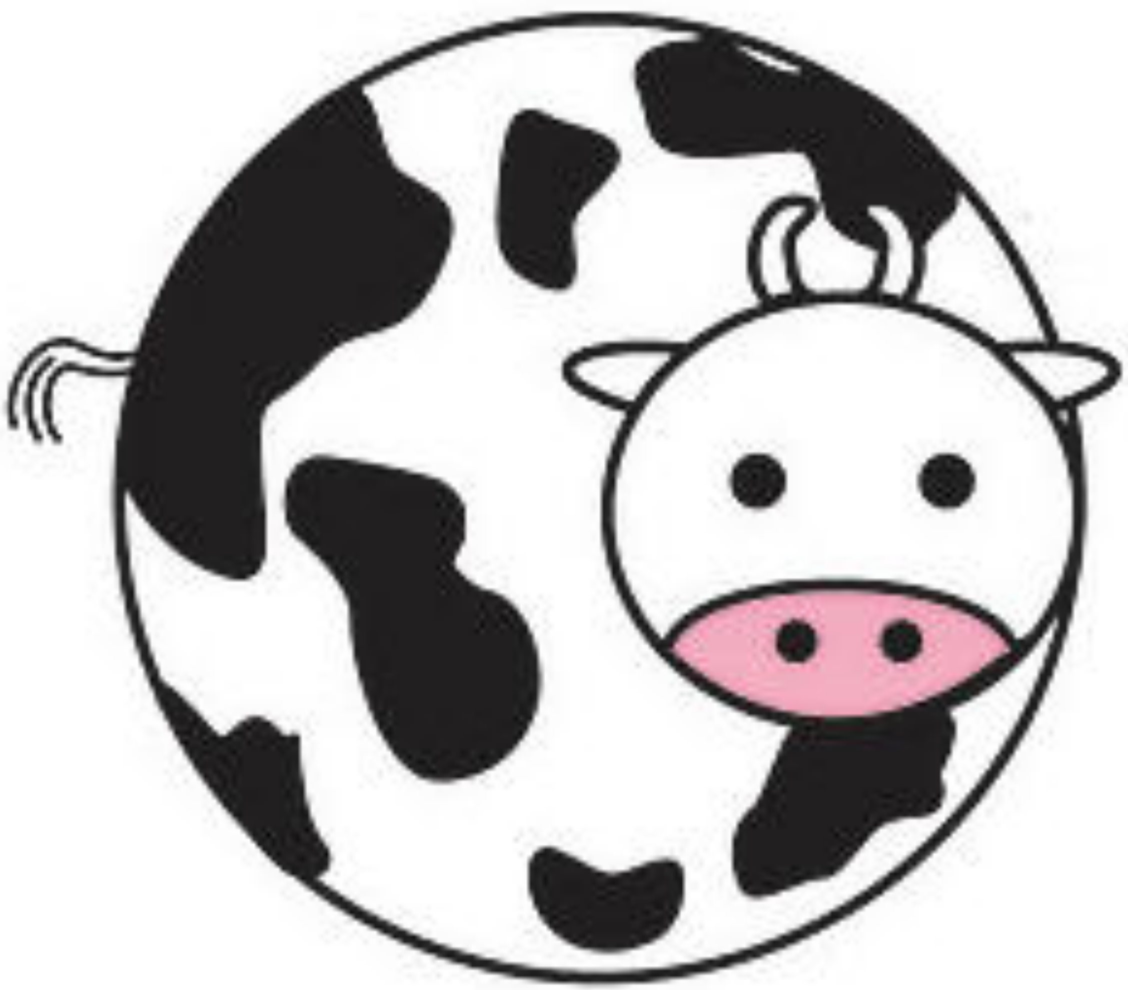

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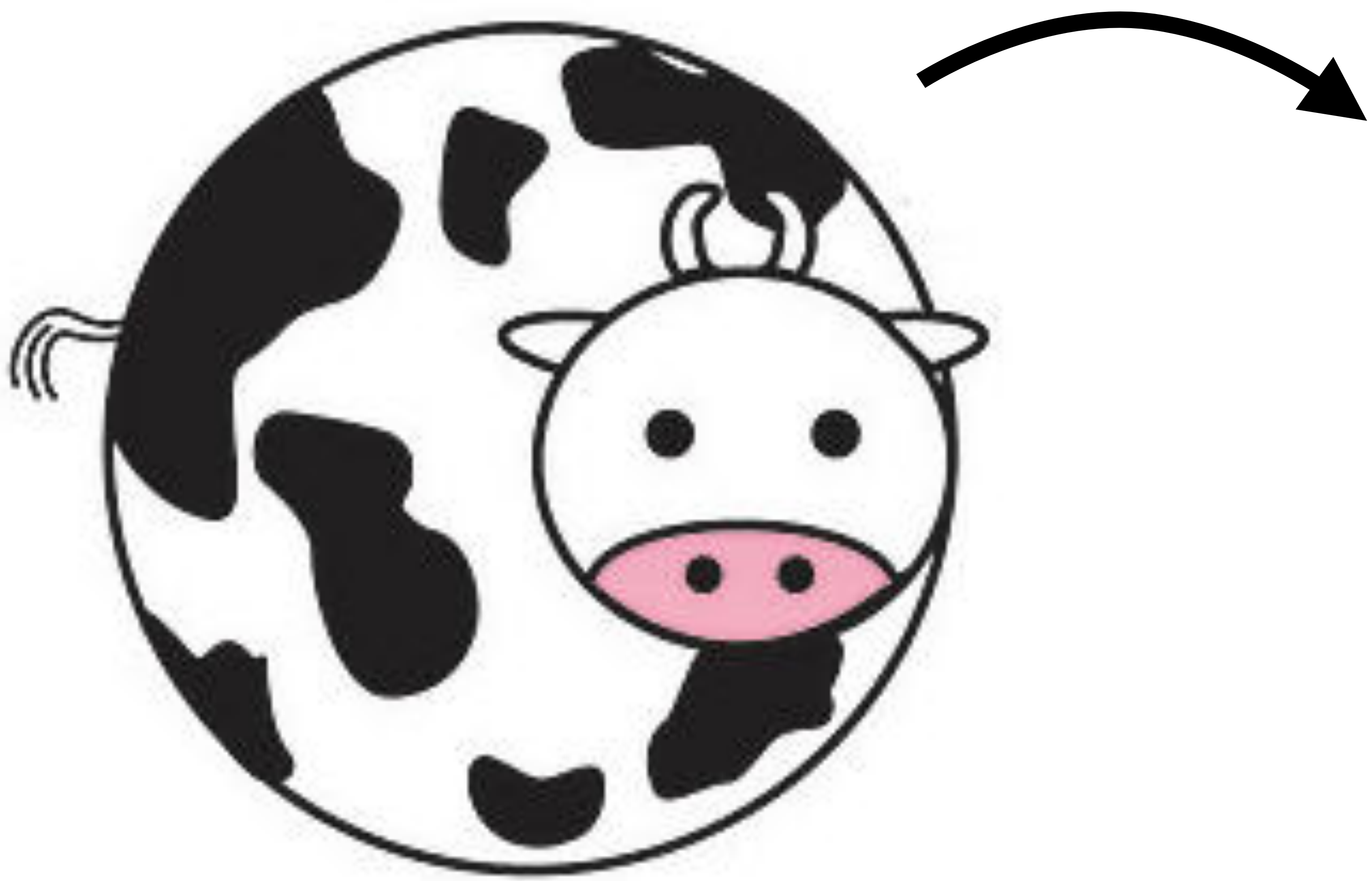

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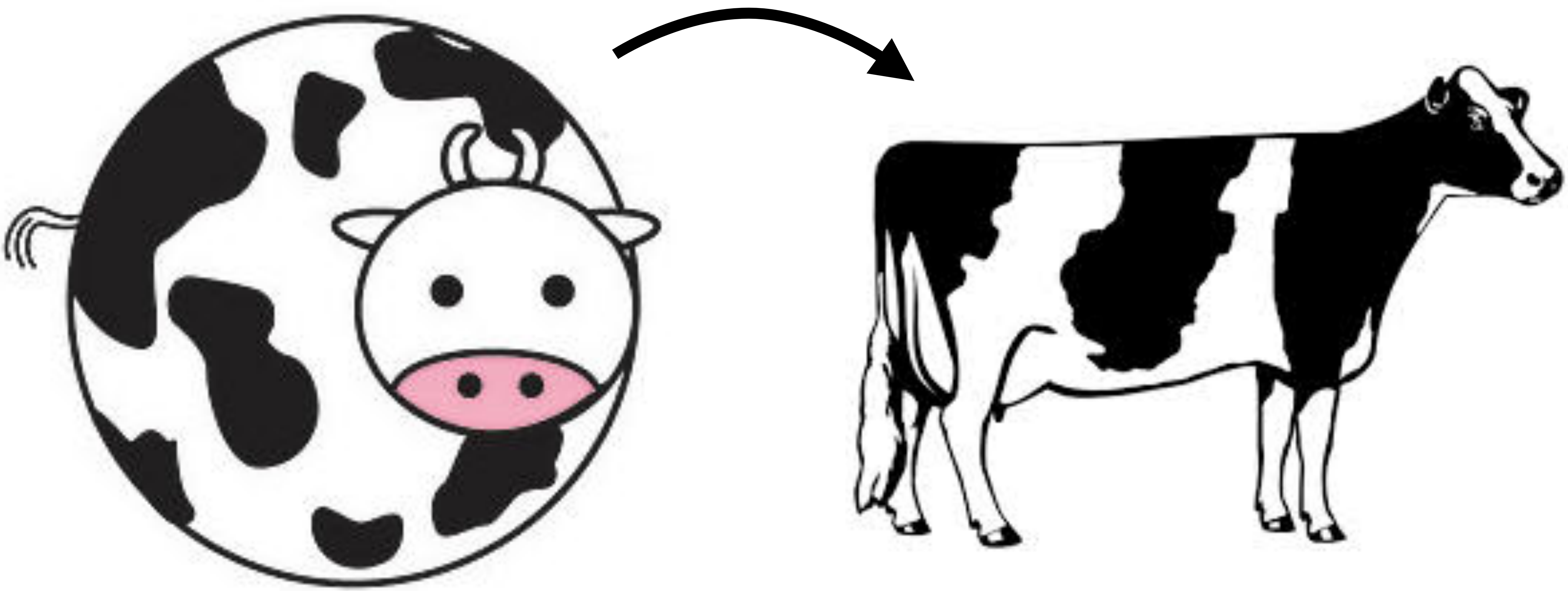

2. Acoplamento entre diferentes processos

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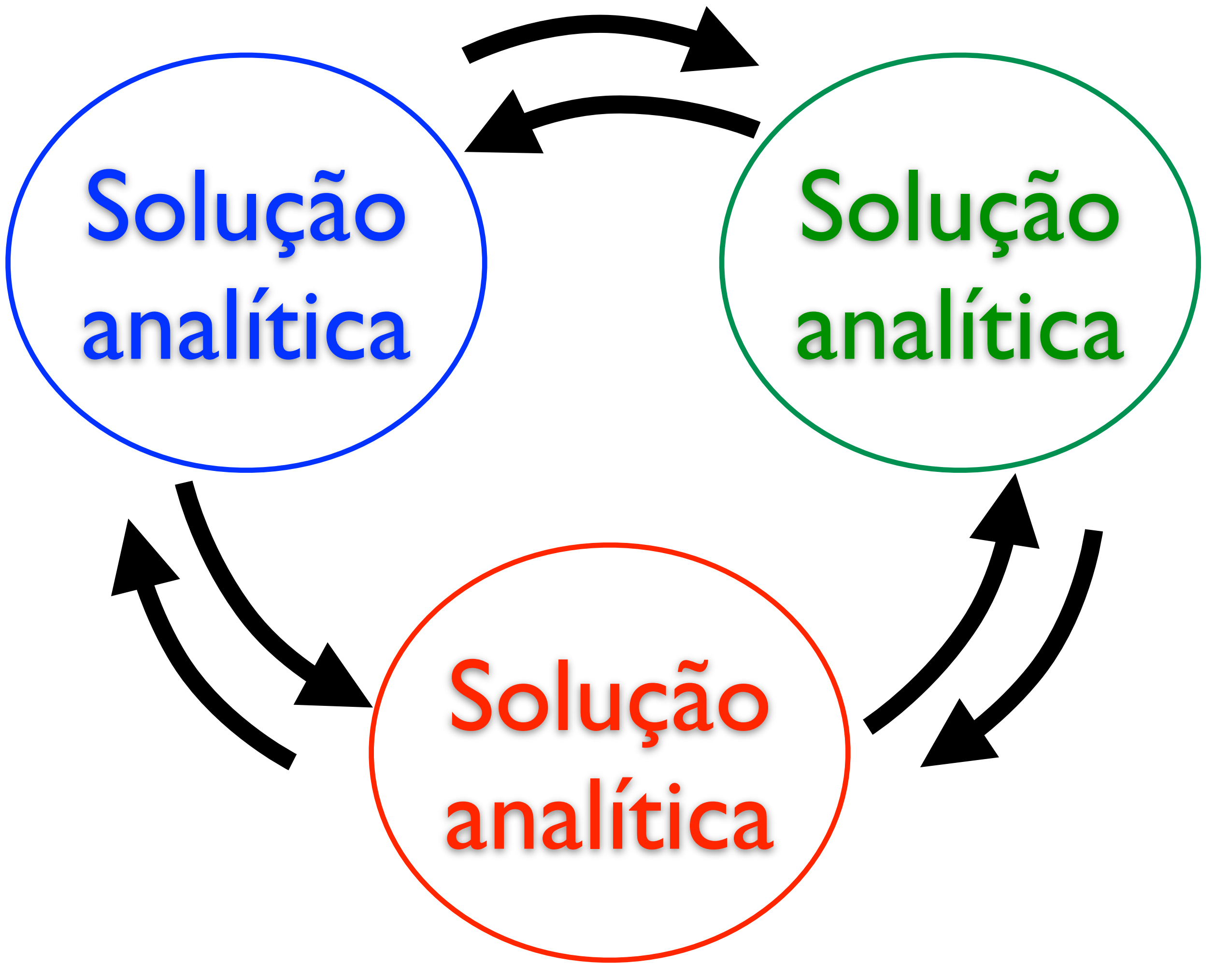

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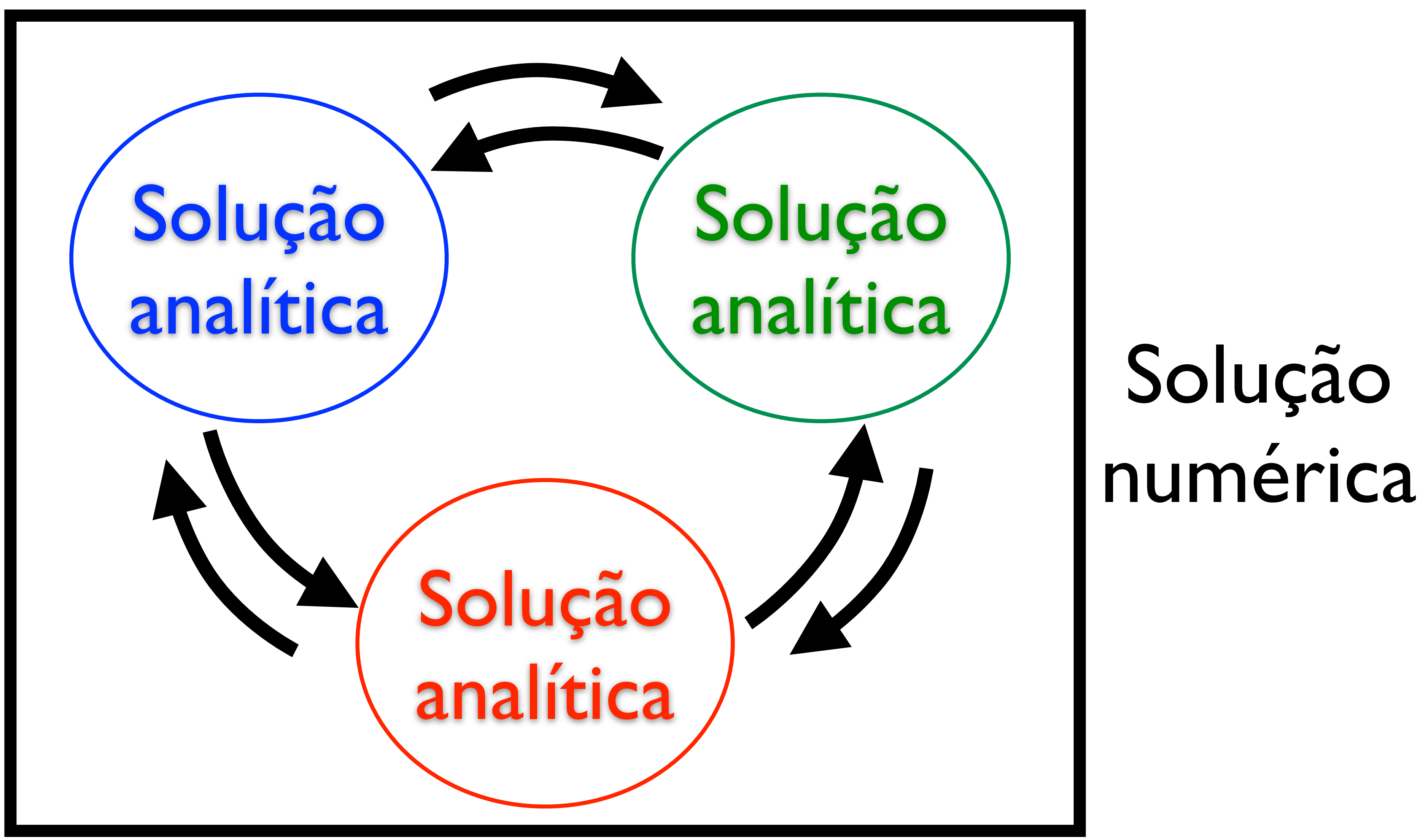
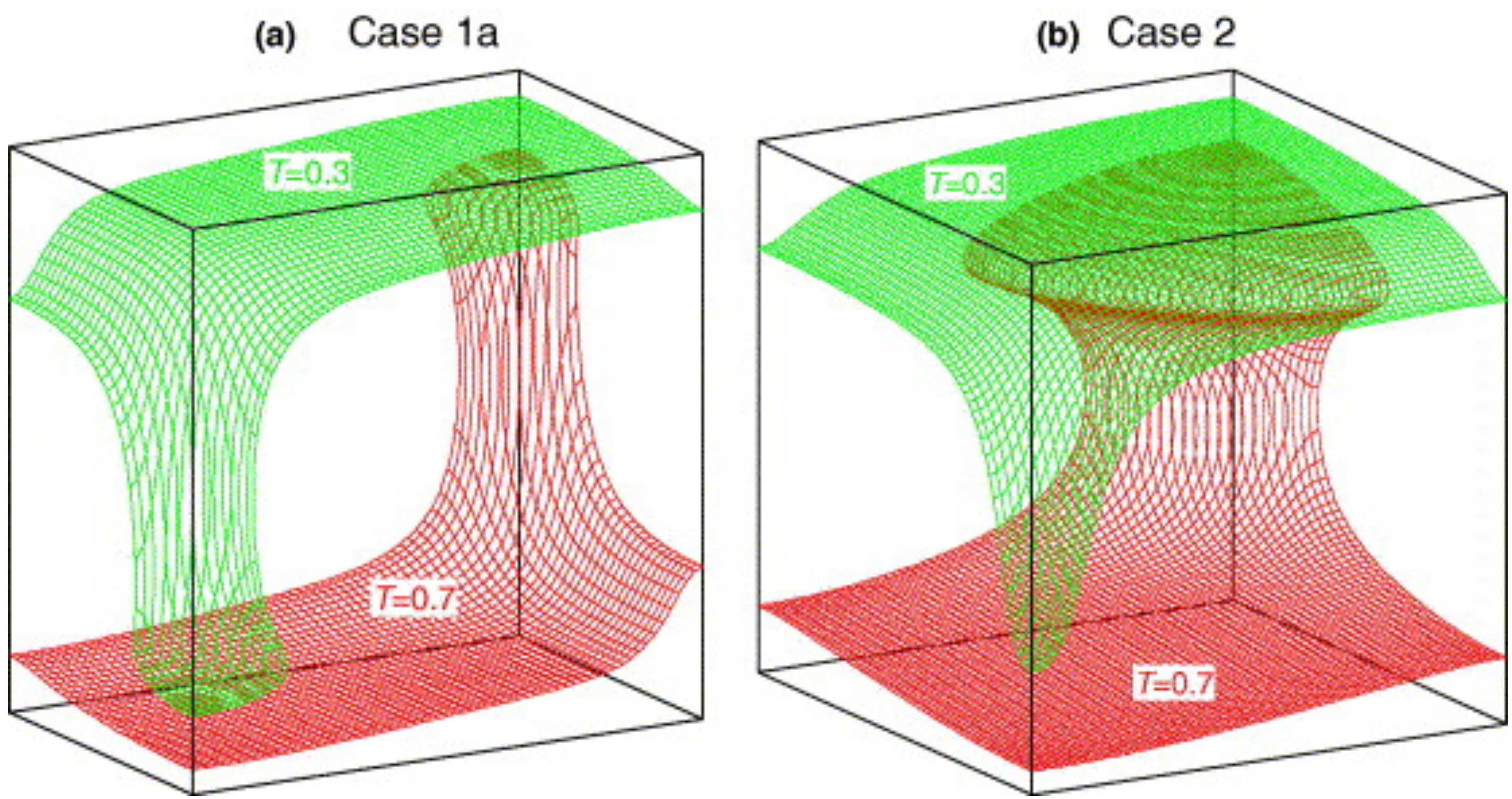


3. Não existência de solução analítica



Método numérico: Diferenças finitas

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$$\frac{dw(a)}{dx}$$

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$$\frac{dw(a)}{dx} = \lim_{\Delta x \to 0} \frac{w(a + \Delta x) - w(a)}{\Delta x}$$

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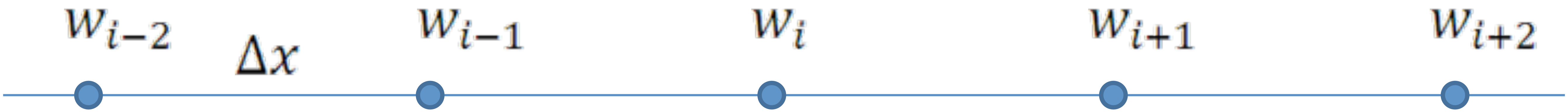
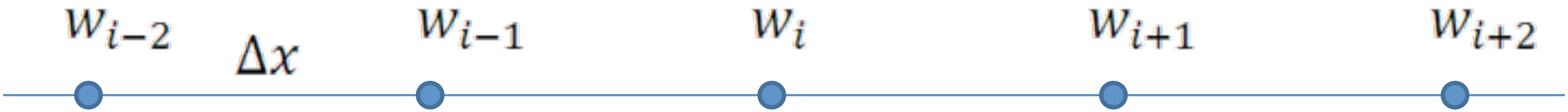
Método numérico: Diferenças finitas

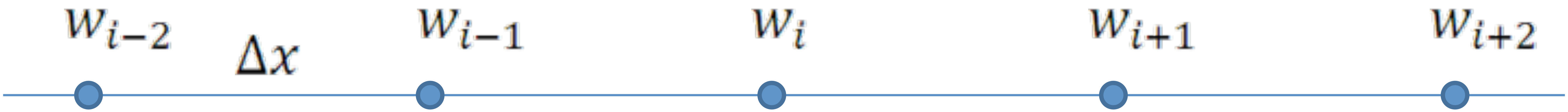
$$\frac{dw(a)}{dx} = \lim_{\Delta x \to 0} \frac{w(a + \Delta x) - w(a)}{\Delta x}$$

$$\approx \frac{w(a + \Delta x) - w(a)}{\Delta x} = \frac{\Delta w}{\Delta x}$$

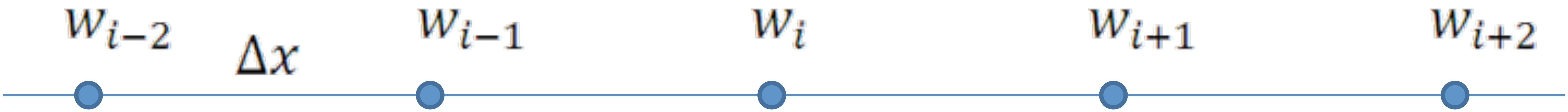

w_{i-2}
Δx
w_{i-1}
w_i
w_{i+1}
w_{i+2}



$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

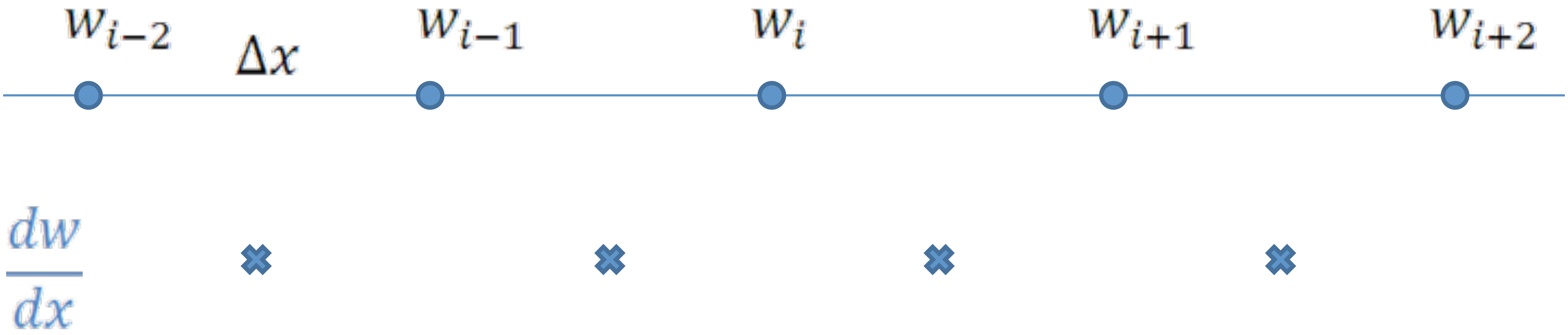


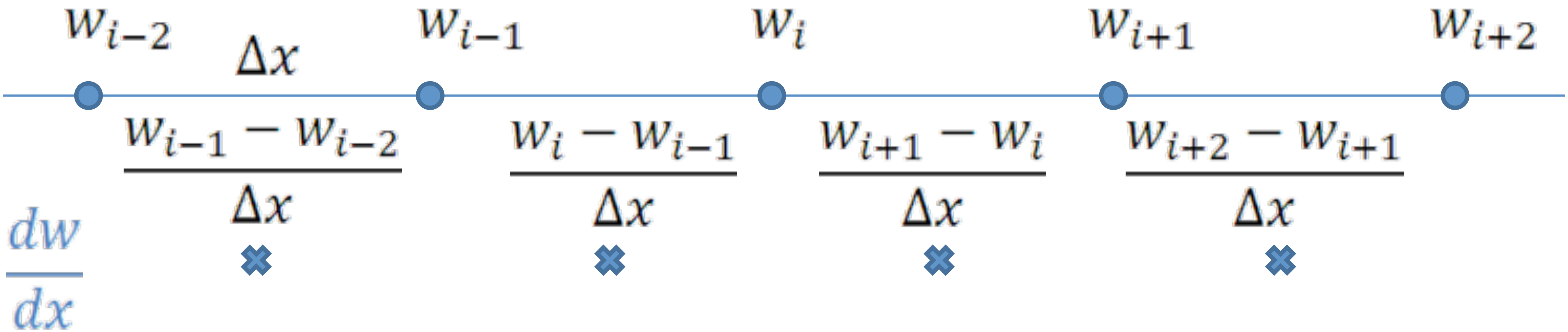
$$D\frac{d^4 w}{dx^4} + \Delta\rho g w = p$$

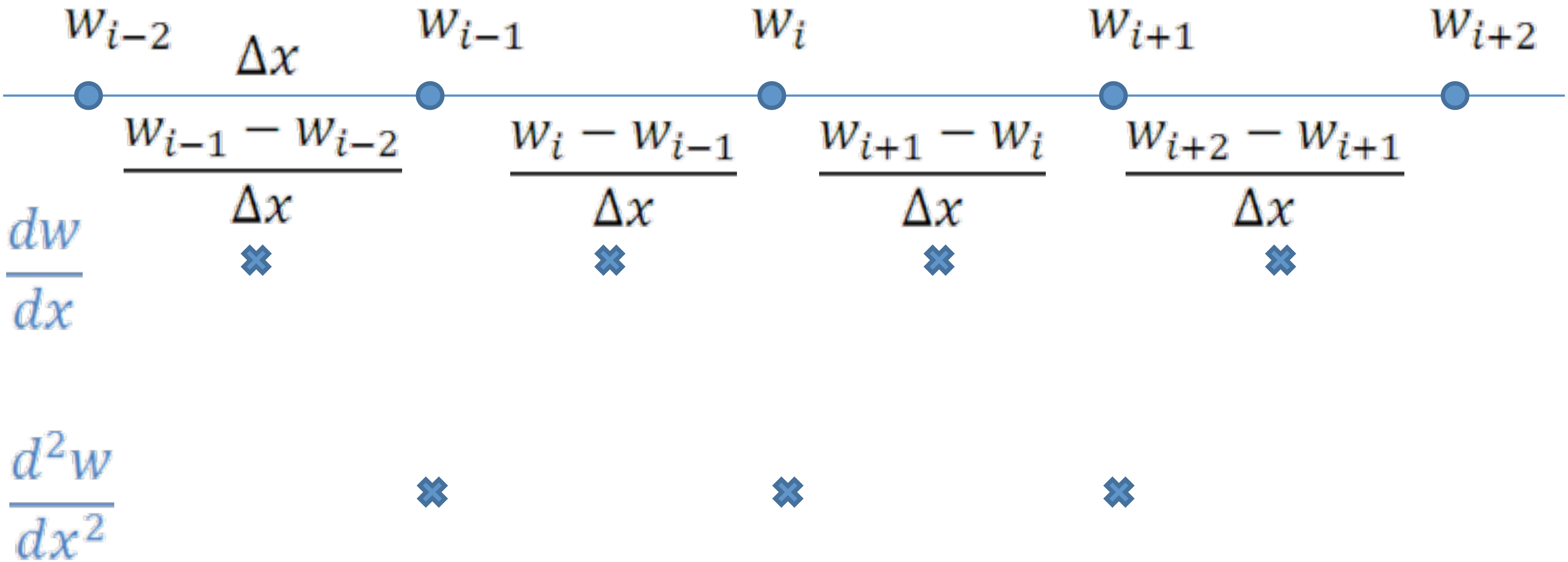


$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

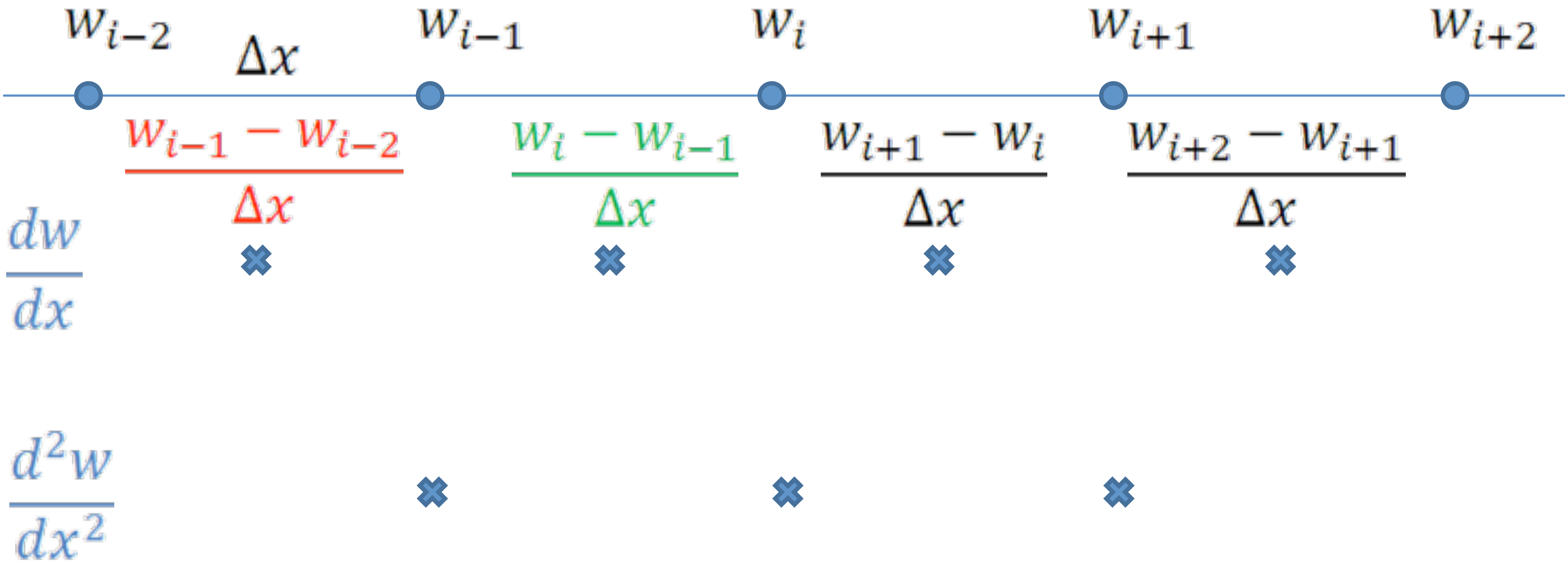
$$? \quad + \Delta\rho g w_i = p_i$$


w_{i-2}
Δx
w_{i-1}
w_i
w_{i+1}
w_{i+2}
$\frac{dw}{dx}$

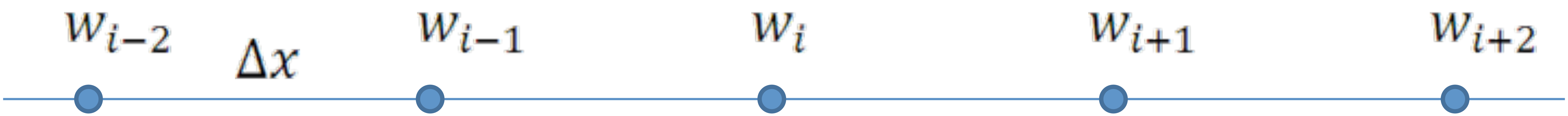

w_{i-2}
w_{i-1}
w_i
w_{i+1}
w_{i+2}
Δx
$\frac{w_{i-1} - w_{i-2}}{\Delta x}$
$\frac{w_i - w_{i-1}}{\Delta x}$
$\frac{w_{i+1} - w_i}{\Delta x}$
$\frac{w_{i+2} - w_{i+1}}{\Delta x}$
$\frac{dw}{dx}$



w_{i-2}
Δx
w_{i-1}
w_i
w_{i+1}
w_{i+2}
$\frac{w_{i-1} - w_{i-2}}{\Delta x}$
$\frac{w_i - w_{i-1}}{\Delta x}$
$\frac{w_{i+1} - w_i}{\Delta x}$
$\frac{w_{i+2} - w_{i+1}}{\Delta x}$
$\frac{dw}{dx}$
$\frac{d^2w}{dx^2}$



w_{i-2}
Δx
w_{i-1}
w_i
w_{i+1}
w_{i+2}
$\frac{w_{i-1} - w_{i-2}}{\Delta x}$
$\frac{w_i - w_{i-1}}{\Delta x}$
$\frac{w_{i+1} - w_i}{\Delta x}$
$\frac{w_{i+2} - w_{i+1}}{\Delta x}$
$\frac{dw}{dx}$
$\frac{d^2w}{dx^2}$



$$\frac{dw}{dx} \qquad \frac{w_{i-1}-w_{i-2}}{\Delta x} \qquad \frac{w_i-w_{i-1}}{\Delta x} \qquad \frac{w_{i+1}-w_i}{\Delta x} \qquad \frac{w_{i+2}-w_{i+1}}{\Delta x}$$

$$\frac{d^2w}{dx^2} \qquad \frac{\frac{w_i-w_{i-1}}{\Delta x}-\frac{w_{i-1}-w_{i-2}}{\Delta x}}{\Delta x}$$

w_{i-2} $\quad$ w_{i-1} $\quad$ w_i $\quad$ w_{i+1} $\quad$ w_{i+2}

Δx

$$\frac{dw}{dx}: \quad \frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

$$\frac{d^2w}{dx^2}: \quad \frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2}$$

w_{i-2} $\quad$ Δx $\quad$ w_{i-1} $\quad$ w_i $\quad$ w_{i+1} $\quad$ w_{i+2}

$\frac{dw}{dx}$

$$\frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

$\frac{d^2w}{dx^2}$

$$\frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2} \qquad \frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2} \qquad \frac{w_{i+2} - 2w_{i+1} + w_i}{\Delta x^2}$$

w_{i-2} $\quad \Delta x \quad$ w_{i-1} $\quad$ w_i $\quad$ w_{i+1} $\quad$ w_{i+2}

$$\frac{dw}{dx}: \quad \frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

$$\frac{d^2w}{dx^2}: \quad \frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2} \qquad \frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2} \qquad \frac{w_{i+2} - 2w_{i+1} + w_i}{\Delta x^2}$$

$$\frac{d^3w}{dx^3}$$

w_{i-2} $\quad \Delta x \quad$ w_{i-1} $\quad$ w_i $\quad$ w_{i+1} $\quad$ w_{i+2}

$$\frac{dw}{dx}: \quad \frac{w_{i-1} - w_{i-2}}{\Delta x} \qquad \frac{w_i - w_{i-1}}{\Delta x} \qquad \frac{w_{i+1} - w_i}{\Delta x} \qquad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

$$\frac{d^2w}{dx^2}: \quad \frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2} \qquad \frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2} \qquad \frac{w_{i+2} - 2w_{i+1} + w_i}{\Delta x^2}$$

$$\frac{d^3w}{dx^3}: \quad \frac{w_{i+1} - 3w_i + 3w_{i-1} - w_{i-2}}{\Delta x^3} \qquad \frac{w_{i+2} - 3w_{i+1} + 3w_i - w_{i-1}}{\Delta x^3}$$

$$w_{i-2} \quad \Delta x \quad w_{i-1} \quad w_i \quad w_{i+1} \quad w_{i+2}$$

$$\frac{dw}{dx}: \quad \frac{w_{i-1} - w_{i-2}}{\Delta x} \quad \frac{w_i - w_{i-1}}{\Delta x} \quad \frac{w_{i+1} - w_i}{\Delta x} \quad \frac{w_{i+2} - w_{i+1}}{\Delta x}$$

$$\frac{d^2w}{dx^2}: \quad \frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2} \quad \frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2} \quad \frac{w_{i+2} - 2w_{i+1} + w_i}{\Delta x^2}$$

$$\frac{d^3w}{dx^3}: \quad \frac{w_{i+1} - 3w_i + 3w_{i-1} - w_{i-2}}{\Delta x^3} \quad \frac{w_{i+2} - 3w_{i+1} + 3w_i - w_{i-1}}{\Delta x^3}$$

$$\frac{d^4w}{dx^4}$$

w_{i-2} $\quad \Delta x \quad$ w_{i-1} $\quad$ w_i $\quad$ w_{i+1} $\quad$ w_{i+2}

$\frac{dw}{dx}$: $\quad \frac{w_{i-1} - w_{i-2}}{\Delta x} \quad \frac{w_i - w_{i-1}}{\Delta x} \quad \frac{w_{i+1} - w_i}{\Delta x} \quad \frac{w_{i+2} - w_{i+1}}{\Delta x}$

$\frac{d^2w}{dx^2}$: $\quad \frac{w_i - 2w_{i-1} + w_{i-2}}{\Delta x^2} \quad \frac{w_{i+1} - 2w_i + w_{i-1}}{\Delta x^2} \quad \frac{w_{i+2} - 2w_{i+1} + w_i}{\Delta x^2}$

$\frac{d^3w}{dx^3}$: $\quad \frac{w_{i+1} - 3w_i + 3w_{i-1} - w_{i-2}}{\Delta x^3} \quad \frac{w_{i+2} - 3w_{i+1} + 3w_i - w_{i-1}}{\Delta x^3}$

$\frac{d^4w}{dx^4}$: $\quad \frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4}$

$$D\frac{d^4w}{dx^4}+\Delta\rho g w=p$$

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

$$D\frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta\rho g w_i = p_i$$

$$D \frac{d^4 w}{dx^4} + \Delta\rho g w = p$$

$$D \frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta\rho g w_i = p_i$$

$$D[w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}] + \Delta x^4 \Delta\rho g w_i = \Delta x^4 p_i$$

$$D\frac{d^4 w}{dx^4} + \Delta\rho g w = p$$

$$D\frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta\rho g w_i = p_i$$

$$D[w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}] + \Delta x^4 \Delta\rho g w_i = \Delta x^4 p_i$$

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

$$D\frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta\rho g w_i = p_i$$

$$D[w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}] + \Delta x^4 \Delta\rho g w_i = \Delta x^4 p_i$$

$$Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} =$$
$$= \Delta x^4 p_i$$

$$D\frac{d^4w}{dx^4} + \Delta\rho g w = p$$

$$D\frac{w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}}{\Delta x^4} + \Delta\rho g w_i = p_i$$

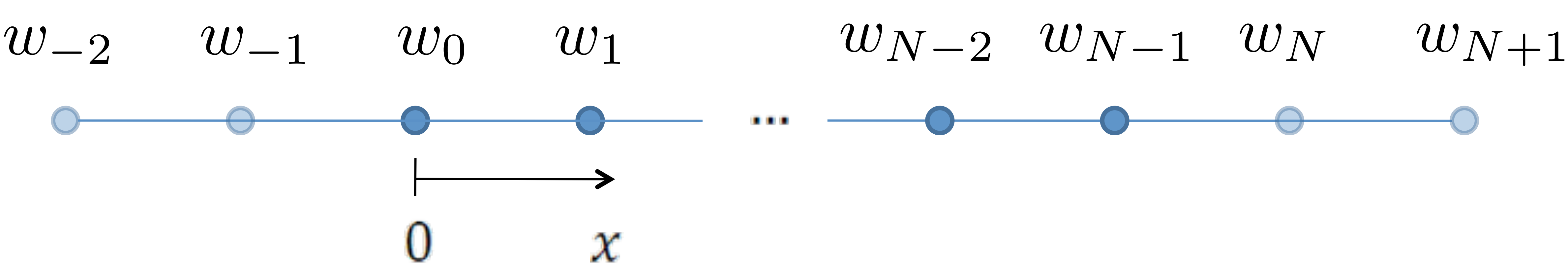
$$D[w_{i+2} - 4w_{i+1} + 6w_i - 4w_{i-1} + w_{i-2}] + \Delta x^4 \Delta\rho g w_i = \Delta x^4 p_i$$

$$\boxed{\begin{aligned} Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} &= \\ = \Delta x^4 p_i \end{aligned}}$$

Condição de contorno

Placa contínua:

$$w \to 0 \;\; \text{para} \;\; x \to 0$$
$$w \to 0 \;\; \text{para} \;\; x \to x_n$$



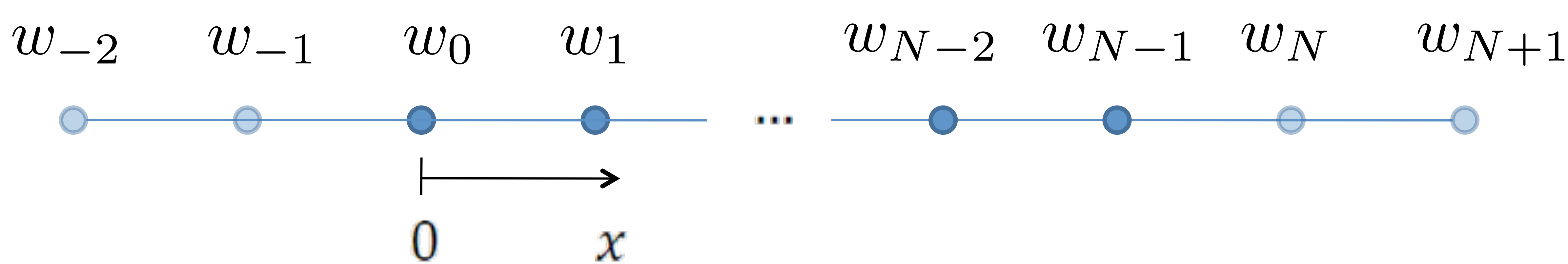
Condição de contorno

Placa contínua:

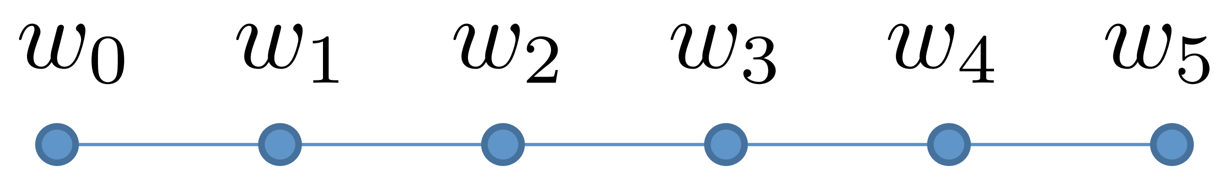
$$w \rightarrow 0 \text{ para } x \rightarrow 0$$

$$w \rightarrow 0 \text{ para } x \rightarrow x_n$$

$$w_{-2}, w_{-1}, w_N, w_{N-1} = 0$$

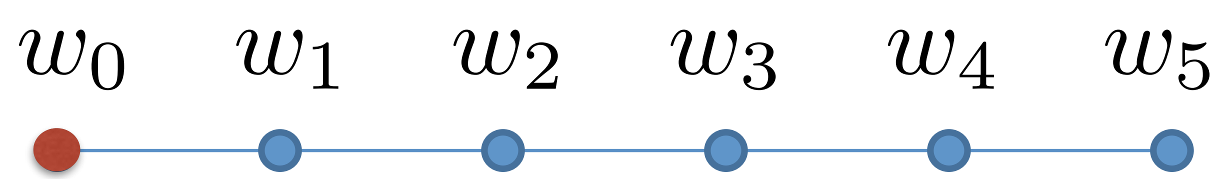


Exemplo



$$Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i$$

Exemplo

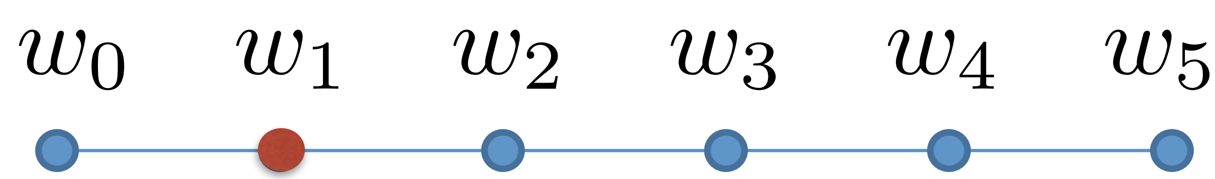


w_0 :

$$\left[6D + \Delta x^4 \Delta \rho g\right] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0$$

$$\boxed{Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i}$$

Exemplo

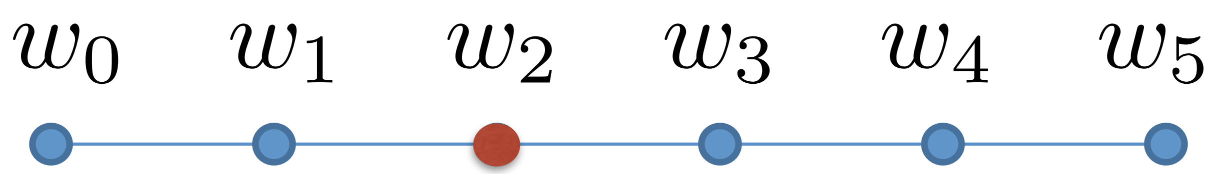


$$w_0: \quad \left[6D + \Delta x^4 \Delta\rho g\right] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0$$

$$w_1: \quad -4Dw_0 + \left[6D + \Delta x^4 \Delta\rho g\right] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

$$\boxed{\begin{aligned} Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} &= \\ = \Delta x^4 p_i \end{aligned}}$$

Exemplo



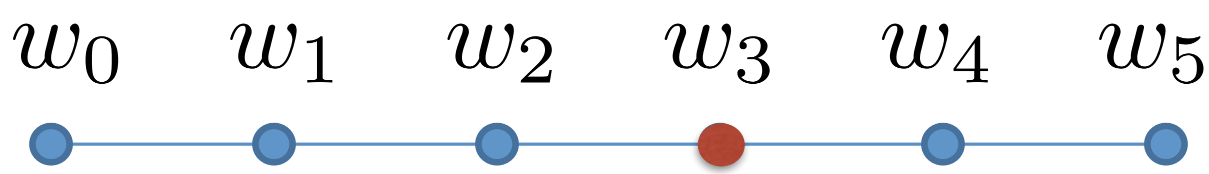
$$w_0: \quad \left[6D + \Delta x^4 \Delta \rho g\right] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0$$

$$w_1: \quad -4Dw_0 + \left[6D + \Delta x^4 \Delta \rho g\right] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

$$w_2: \quad Dw_0 - 4Dw_1 + \left[6D + \Delta x^4 \Delta \rho g\right] w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2$$

$$\boxed{Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta \rho g] w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i}$$

Exemplo



$$w_0: \quad \left[6D + \Delta x^4 \Delta\rho g\right] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0$$

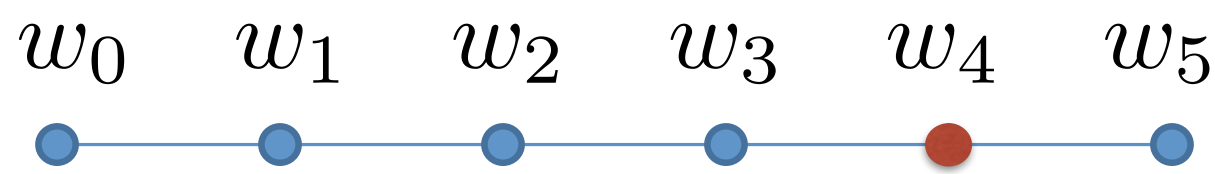
$$w_1: \quad -4Dw_0 + \left[6D + \Delta x^4 \Delta\rho g\right] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1$$

$$w_2: \quad Dw_0 - 4Dw_1 + \left[6D + \Delta x^4 \Delta\rho g\right] w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2$$

$$w_3: \quad Dw_1 - 4Dw_2 + \left[6D + \Delta x^4 \Delta\rho g\right] w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3$$

$$\boxed{Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \Delta x^4 p_i}$$

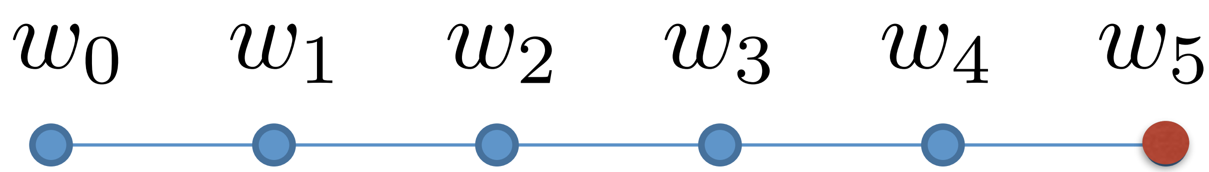
Exemplo



$$
\begin{array}{lr}
w_0: & \left[6D + \Delta x^4 \Delta\rho g\right] w_0 - 4Dw_1 + Dw_2 = \Delta x^4 p_0 \\
w_1: & -4Dw_0 + \left[6D + \Delta x^4 \Delta\rho g\right] w_1 - 4Dw_2 + Dw_3 = \Delta x^4 p_1 \\
w_2: & Dw_0 - 4Dw_1 + \left[6D + \Delta x^4 \Delta\rho g\right] w_2 - 4Dw_3 + Dw_4 = \Delta x^4 p_2 \\
w_3: & Dw_1 - 4Dw_2 + \left[6D + \Delta x^4 \Delta\rho g\right] w_3 - 4Dw_4 + Dw_5 = \Delta x^4 p_3 \\
w_4: & Dw_2 - 4Dw_3 + \left[6D + \Delta x^4 \Delta\rho g\right] w_4 - 4Dw_5 \qquad = \Delta x^4 p_4
\end{array}
$$

$$
\boxed{\begin{aligned} & Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g] w_i - 4Dw_{i+1} + Dw_{i+2} = \\ & \qquad = \Delta x^4 p_i \end{aligned}}
$$

Exemplo



$$
\begin{aligned}
w_0 &: & \left[6D + \Delta x^4 \Delta\rho g\right] w_0 - 4Dw_1 + Dw_2 &= \Delta x^4 p_0 \\
w_1 &: & -4Dw_0 + \left[6D + \Delta x^4 \Delta\rho g\right] w_1 - 4Dw_2 + Dw_3 &= \Delta x^4 p_1 \\
w_2 &: & Dw_0 - 4Dw_1 + \left[6D + \Delta x^4 \Delta\rho g\right] w_2 - 4Dw_3 + Dw_4 &= \Delta x^4 p_2 \\
w_3 &: & Dw_1 - 4Dw_2 + \left[6D + \Delta x^4 \Delta\rho g\right] w_3 - 4Dw_4 + Dw_5 &= \Delta x^4 p_3 \\
w_4 &: & Dw_2 - 4Dw_3 + \left[6D + \Delta x^4 \Delta\rho g\right] w_4 - 4Dw_5 &= \Delta x^4 p_4 \\
w_5 &: & Dw_3 - 4Dw_4 + \left[6D + \Delta x^4 \Delta\rho g\right] w_5 &= \Delta x^4 p_5
\end{aligned}
$$

$$
\boxed{\begin{aligned} Dw_{i-2} - 4Dw_{i-1} + [6D + \Delta x^4 \Delta\rho g]w_i - 4Dw_{i+1} + Dw_{i+2} = \\ = \Delta x^4 p_i \end{aligned}}
$$

$$
\begin{aligned}
\left[6D + \Delta x^4 \Delta\rho g\right] w_0 - 4Dw_1 + Dw_2 &= \Delta x^4 p_0 \\
-4Dw_0 + \left[6D + \Delta x^4 \Delta\rho g\right] w_1 - 4Dw_2 + Dw_3 &= \Delta x^4 p_1 \\
Dw_0 - 4Dw_1 + \left[6D + \Delta x^4 \Delta\rho g\right] w_2 - 4Dw_3 + Dw_4 &= \Delta x^4 p_2 \\
Dw_1 - 4Dw_2 + \left[6D + \Delta x^4 \Delta\rho g\right] w_3 - 4Dw_4 + Dw_5 &= \Delta x^4 p_3 \\
Dw_2 - 4Dw_3 + \left[6D + \Delta x^4 \Delta\rho g\right] w_4 - 4Dw_5 &= \Delta x^4 p_4 \\
Dw_3 - 4Dw_4 + \left[6D + \Delta x^4 \Delta\rho g\right] w_5 &= \Delta x^4 p_5
\end{aligned}
$$

$$
\begin{aligned}
\left[6D + \Delta x^4 \Delta\rho g\right] w_0 - 4Dw_1 + Dw_2 &= \Delta x^4 p_0 \\
-4Dw_0 + \left[6D + \Delta x^4 \Delta\rho g\right] w_1 - 4Dw_2 + Dw_3 &= \Delta x^4 p_1 \\
Dw_0 - 4Dw_1 + \left[6D + \Delta x^4 \Delta\rho g\right] w_2 - 4Dw_3 + Dw_4 &= \Delta x^4 p_2 \\
Dw_1 - 4Dw_2 + \left[6D + \Delta x^4 \Delta\rho g\right] w_3 - 4Dw_4 + Dw_5 &= \Delta x^4 p_3 \\
Dw_2 - 4Dw_3 + \left[6D + \Delta x^4 \Delta\rho g\right] w_4 - 4Dw_5 &= \Delta x^4 p_4 \\
Dw_3 - 4Dw_4 + \left[6D + \Delta x^4 \Delta\rho g\right] w_5 &= \Delta x^4 p_5
\end{aligned}
$$

$$
\begin{bmatrix}
6D + \Delta x^4 \Delta\rho g & -4D & D & 0 & 0 & 0 \\
-4D & 6D + \Delta x^4 \Delta\rho g & -4D & D & 0 & 0 \\
D & -4D & 6D + \Delta x^4 \Delta\rho g & -4D & D & 0 \\
0 & D & -4D & 6D + \Delta x^4 \Delta\rho g & -4D & D \\
0 & 0 & D & -4D & 6D + \Delta x^4 \Delta\rho g & -4D \\
0 & 0 & 0 & D & -4D & 6D + \Delta x^4 \Delta\rho g
\end{bmatrix}
\begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix} =
$$

$$
= \Delta x^4 \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix}
$$

$$\begin{bmatrix} 6D+\Delta x^4\Delta\rho g & -4D & D & 0 & 0 & 0 \\ -4D & 6D+\Delta x^4\Delta\rho g & -4D & D & 0 & 0 \\ D & -4D & 6D+\Delta x^4\Delta\rho g & -4D & D & 0 \\ 0 & D & -4D & 6D+\Delta x^4\Delta\rho g & -4D & D \\ 0 & 0 & D & -4D & 6D+\Delta x^4\Delta\rho g & -4D \\ 0 & 0 & 0 & D & -4D & 6D+\Delta x^4\Delta\rho g \end{bmatrix} \begin{bmatrix} w_0 \\ w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \end{bmatrix} = \Delta x^4 \begin{bmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{bmatrix}$$

$$\mathbf{Aw} = \mathbf{p}$$

$$A(\mathrm{i}, \mathrm{j} = \mathrm{i} - 2) = D$$

$$A(\mathrm{i}, \mathrm{j} = \mathrm{i} - 1) = -4D$$

$$A(\mathrm{i}, \mathrm{j} = \mathrm{i}) = 6D + \Delta x^4 \Delta\rho g$$

$$A(\mathrm{i}, \mathrm{j} = \mathrm{i} + 1) = -4D$$

$$A(\mathrm{i}, \mathrm{j} = \mathrm{i} + 2) = D$$

Prática: Flex_numerico