Teoria de flexura de placas elásticas

Victor Sacek

Flexura cilíndrica

Victor Sacek IAG/USP

Flexura cilíndrica

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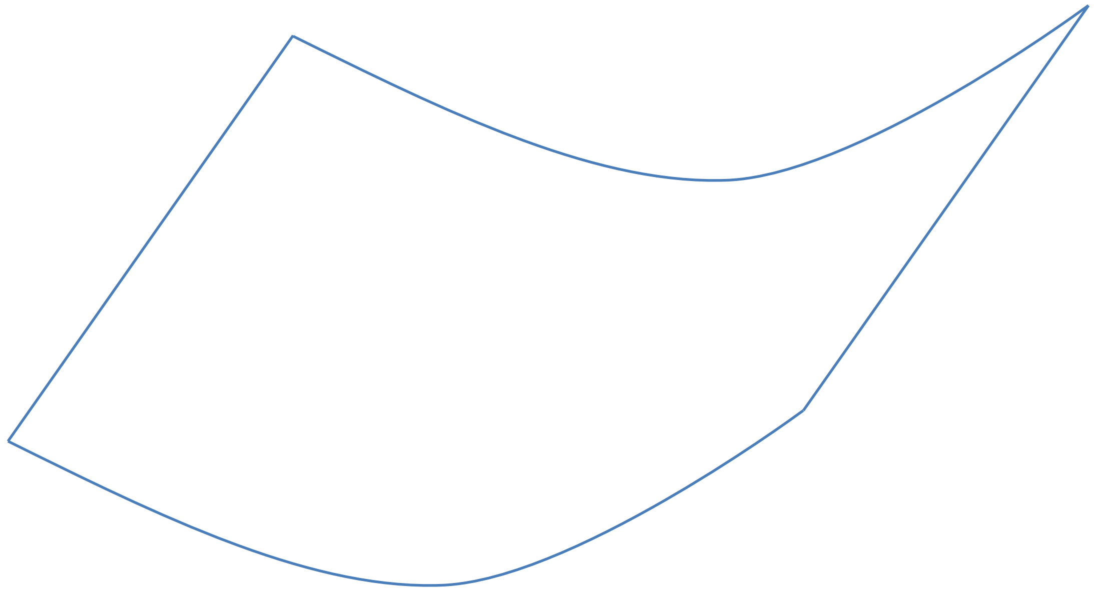
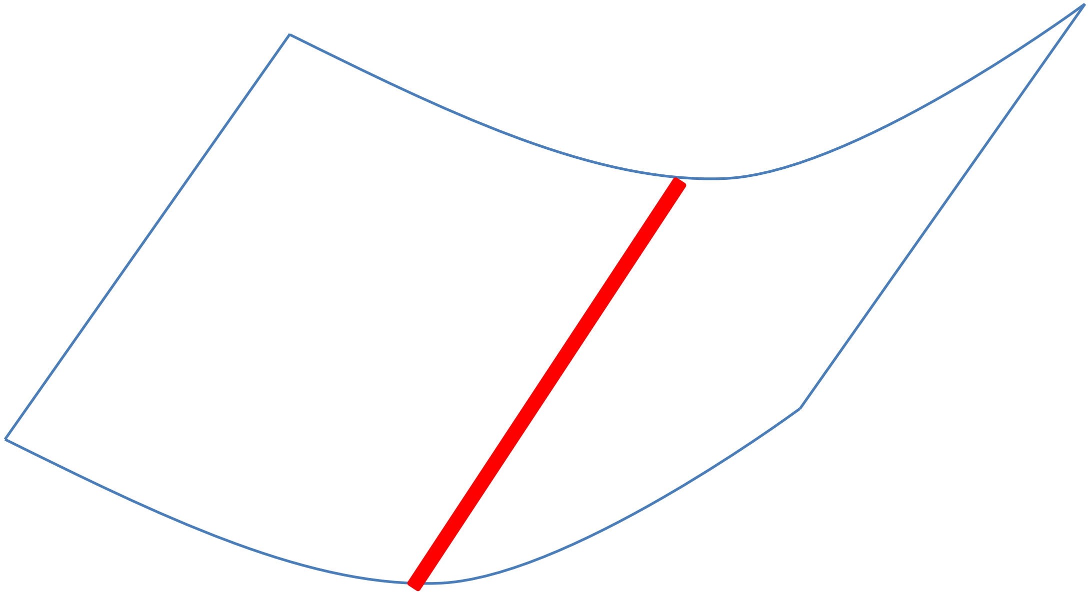


Flexura cilíndrica



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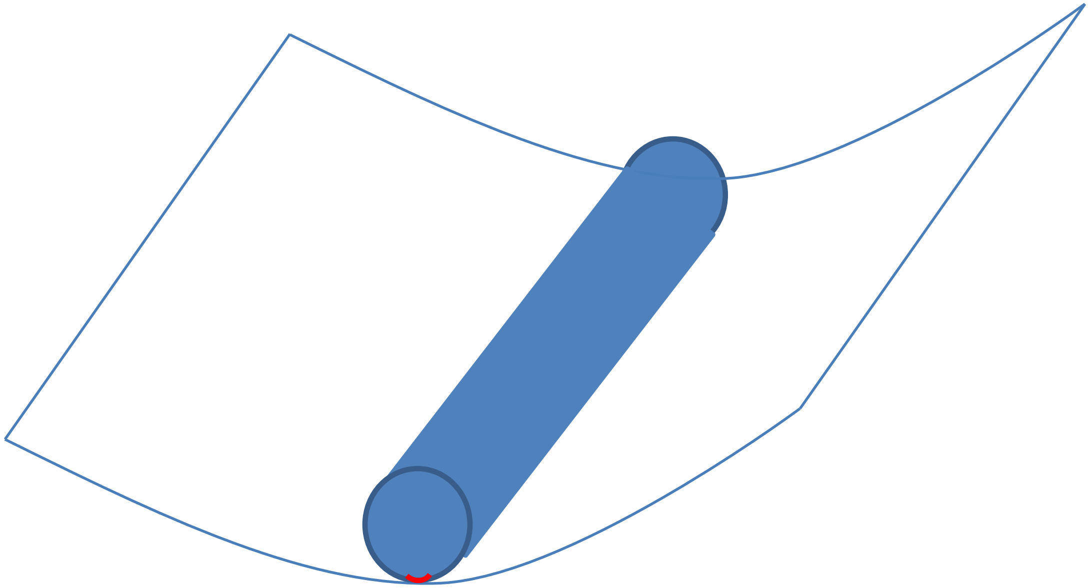

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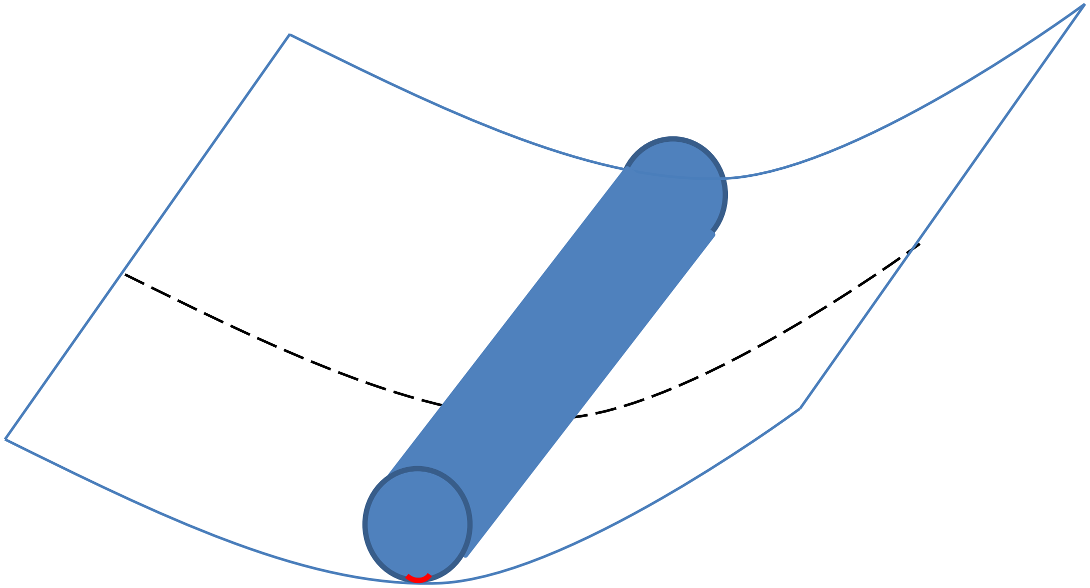
Flexura cilíndrica



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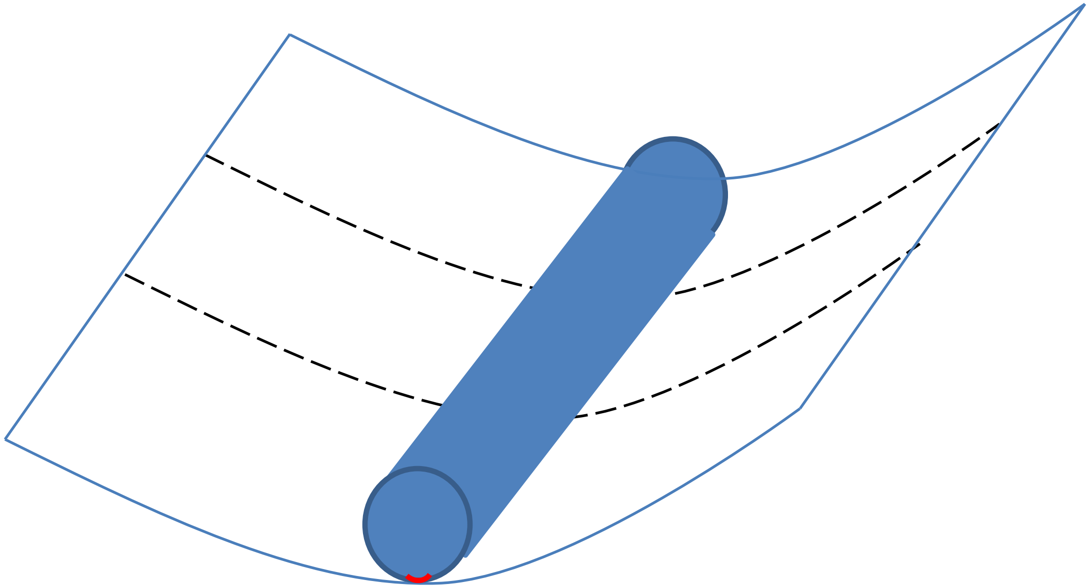
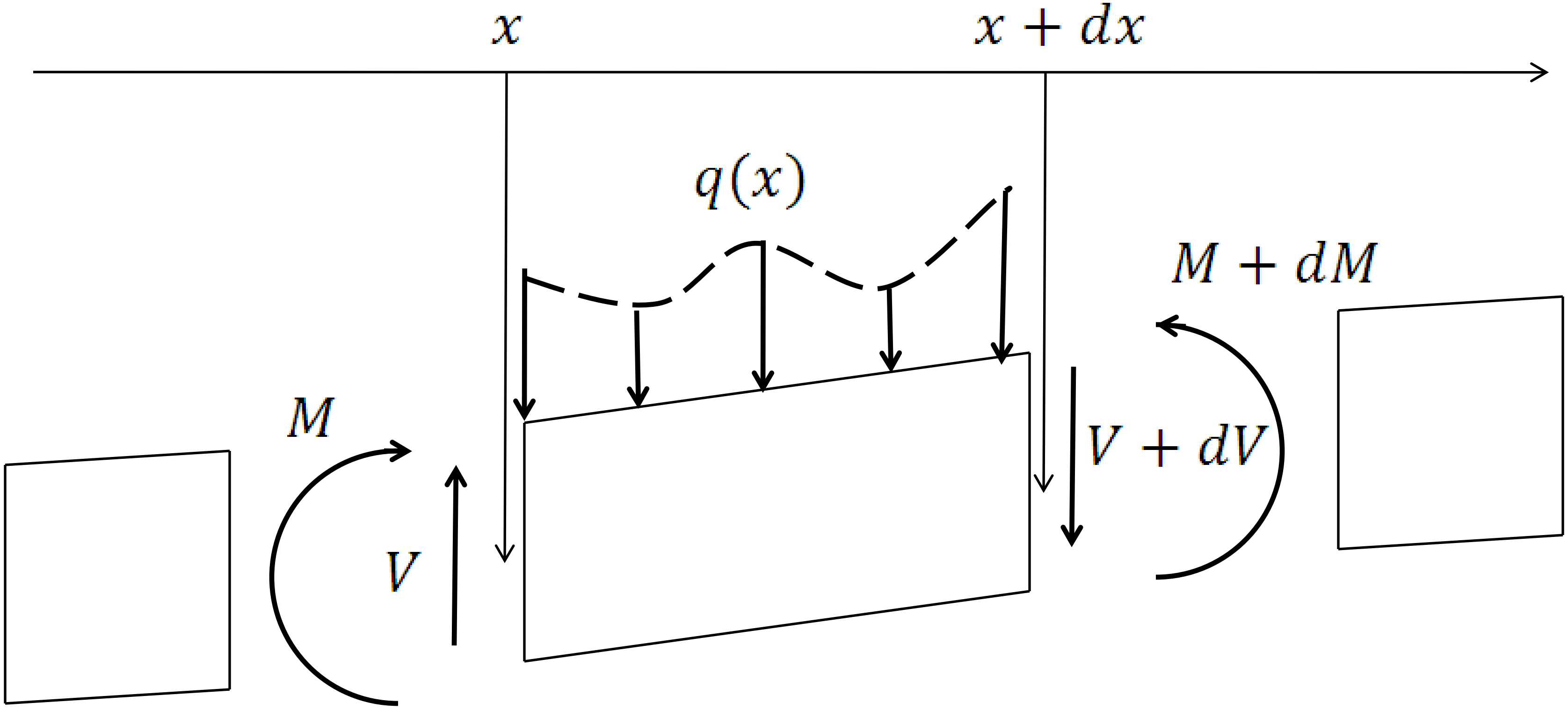


Forças e torques em uma seção da placa



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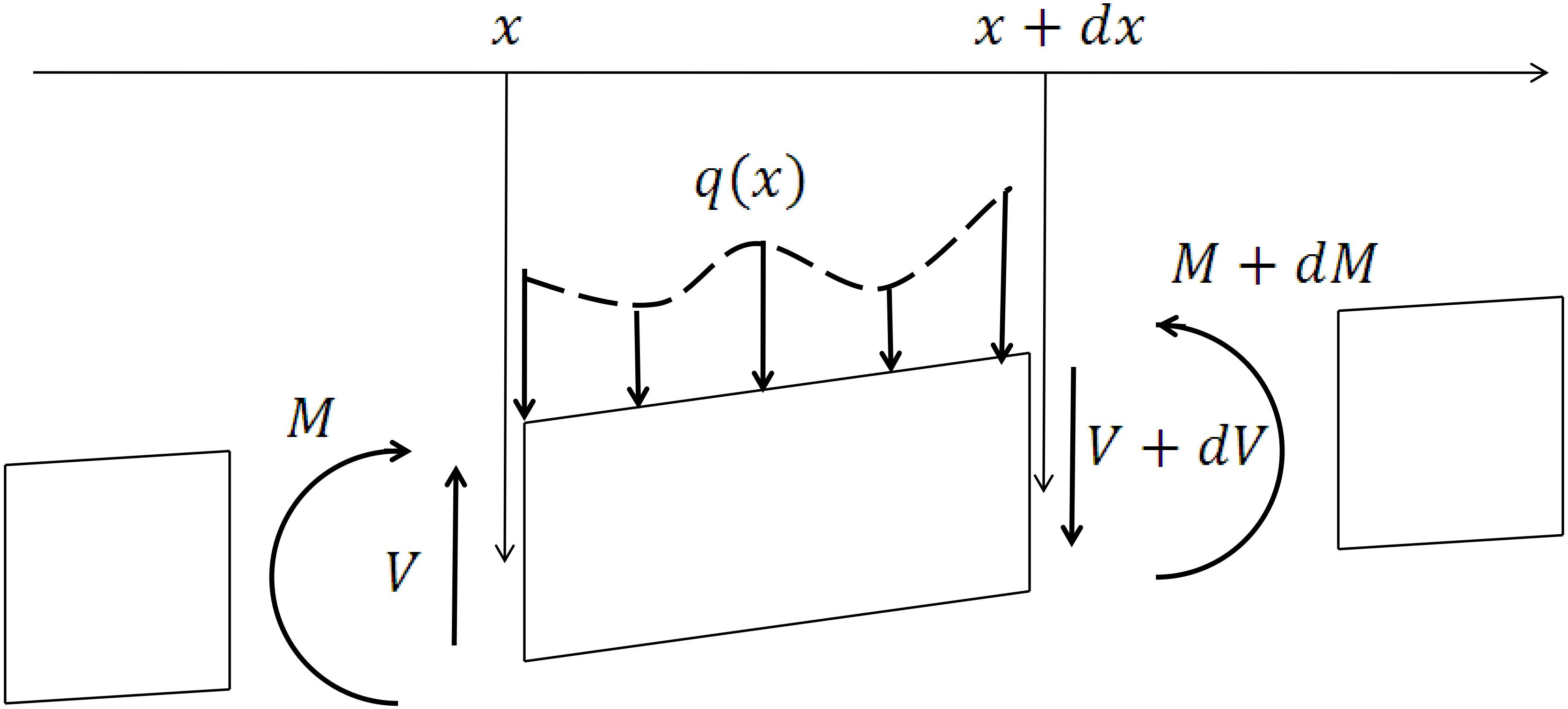
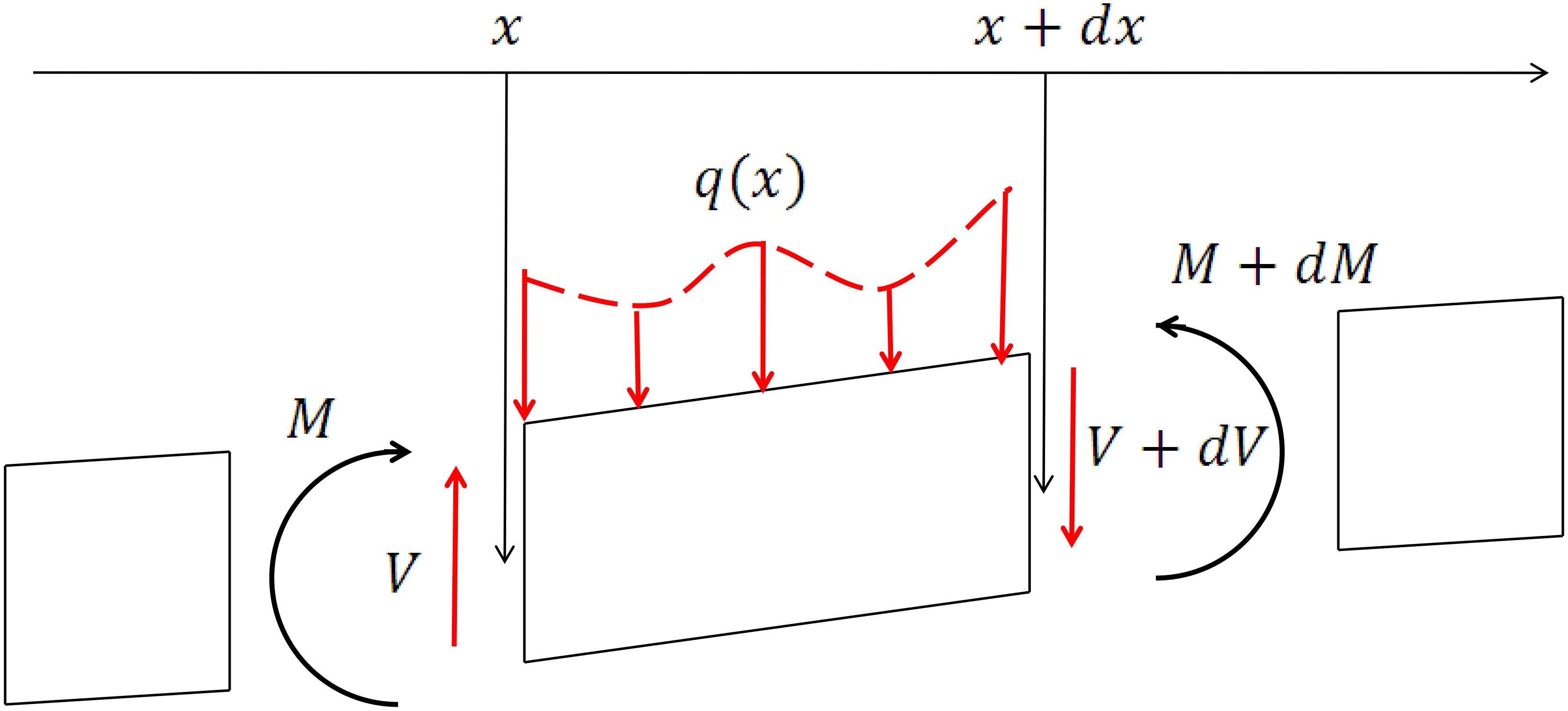
Forças e torques em uma seção da placa



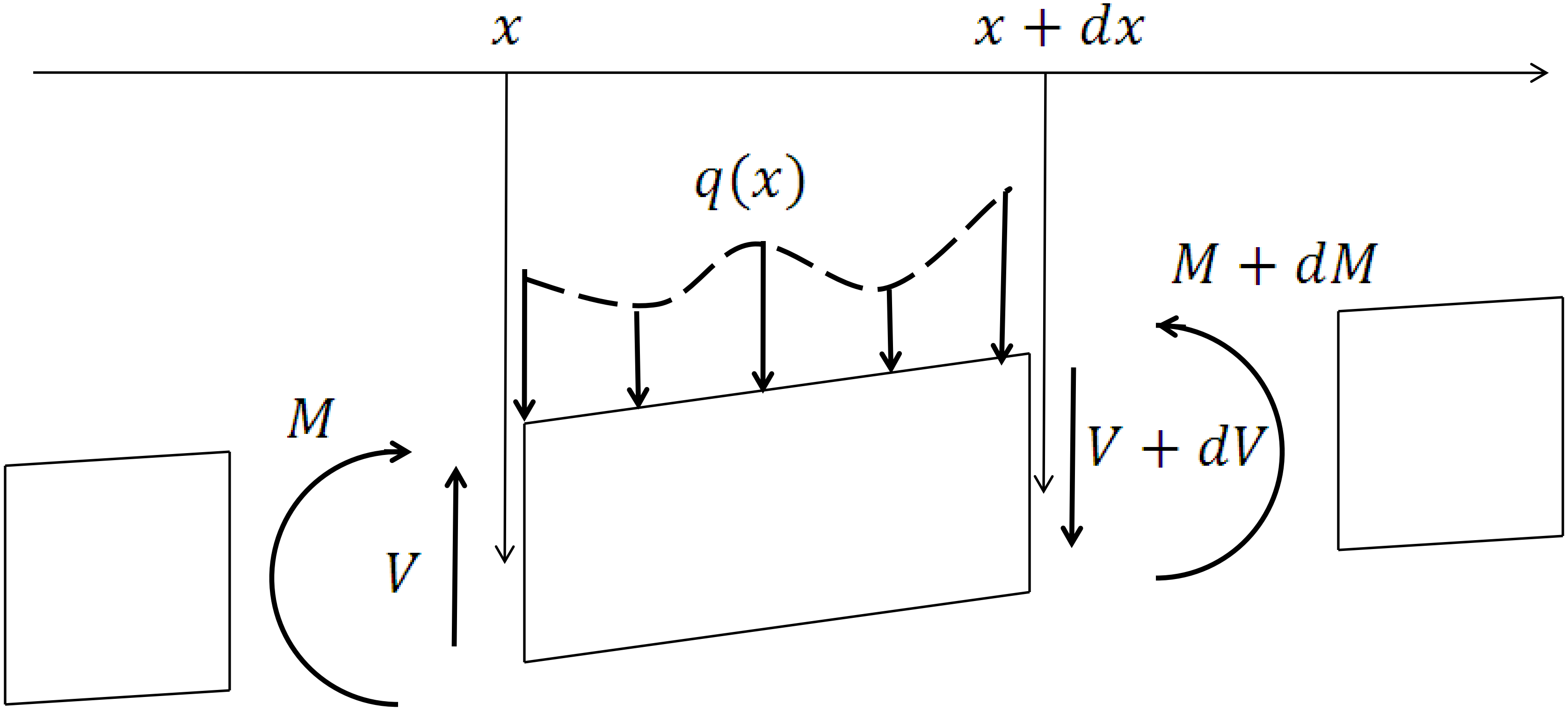
Forças:

Forças e torques em uma seção da placa



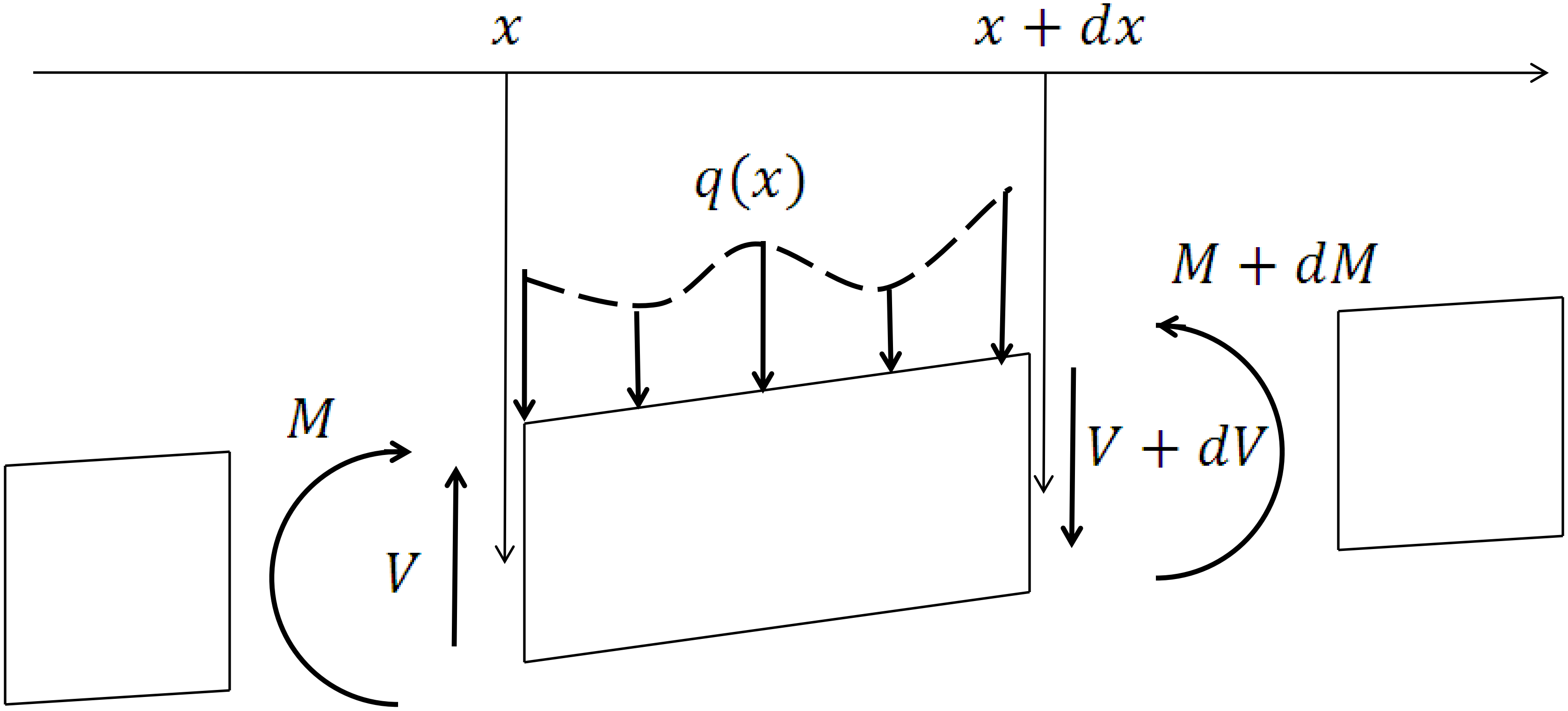
Forças: $q(x)dx + dV = 0$

Forças e torques em uma seção da placa



Forças: $q(x)dx + dV = 0 \rightarrow \frac{dV}{dx} = -q$

Forças e torques em uma seção da placa



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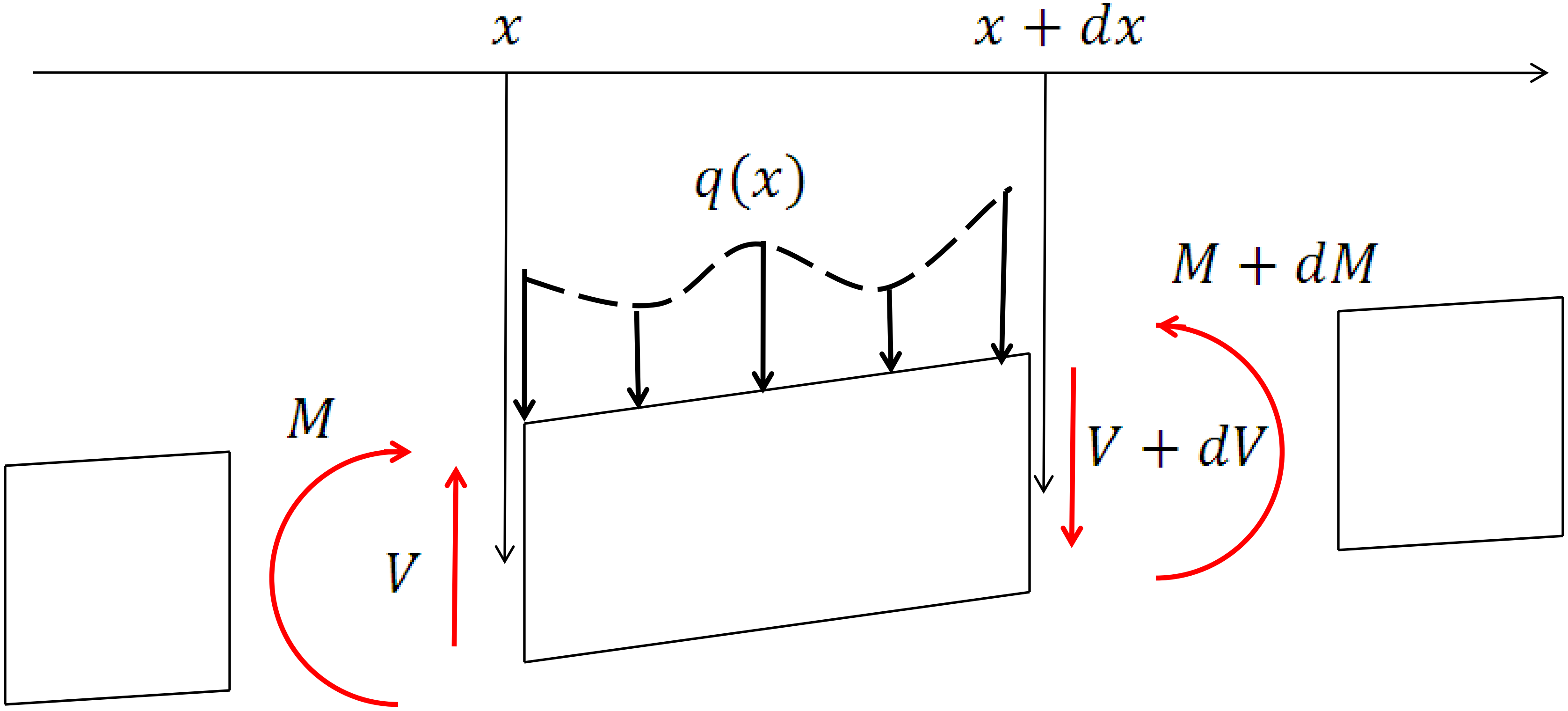
Torques:

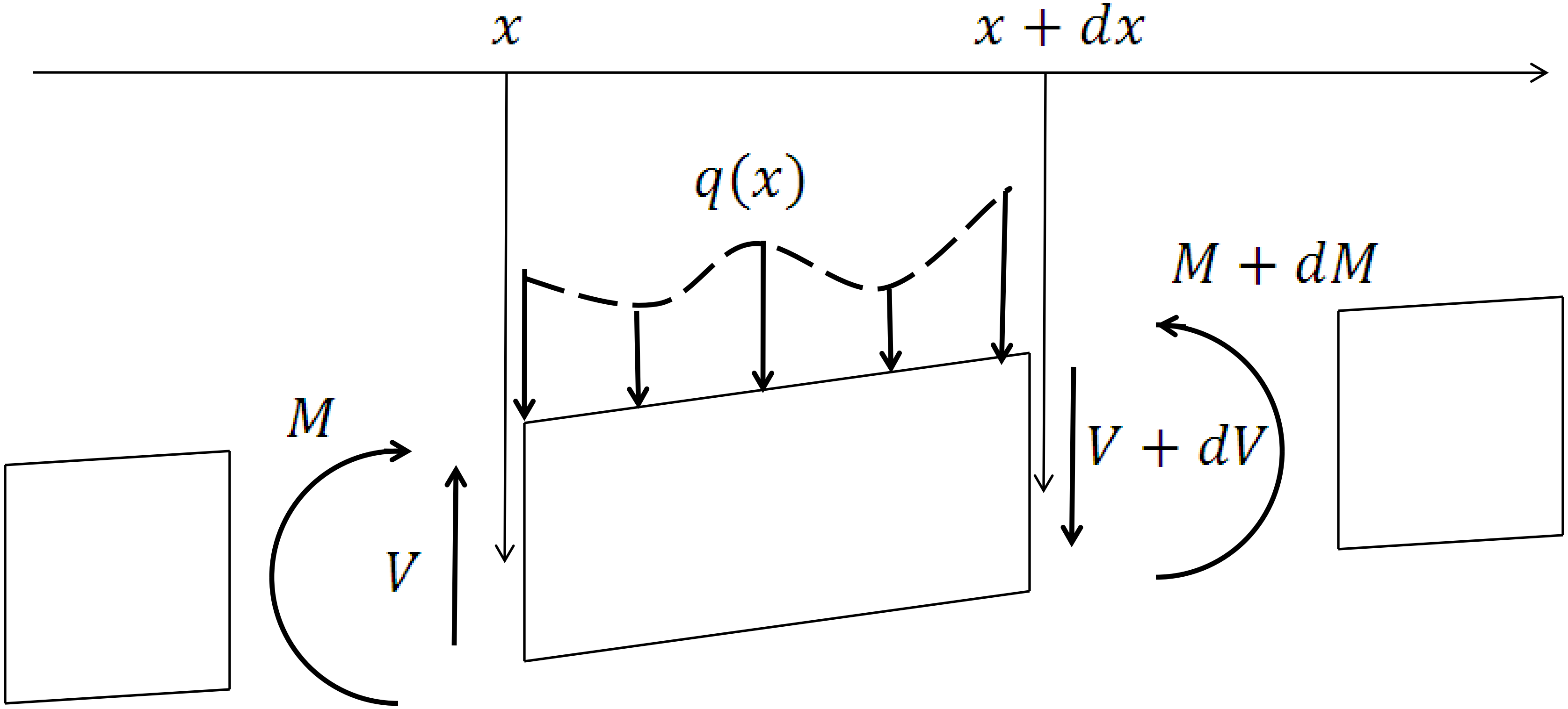
Forças e torques em uma seção da placa



Forças: $q(x)dx + dV = 0 \rightarrow \frac{dV}{dx} = -q$

Torques: $dM = Vdx$

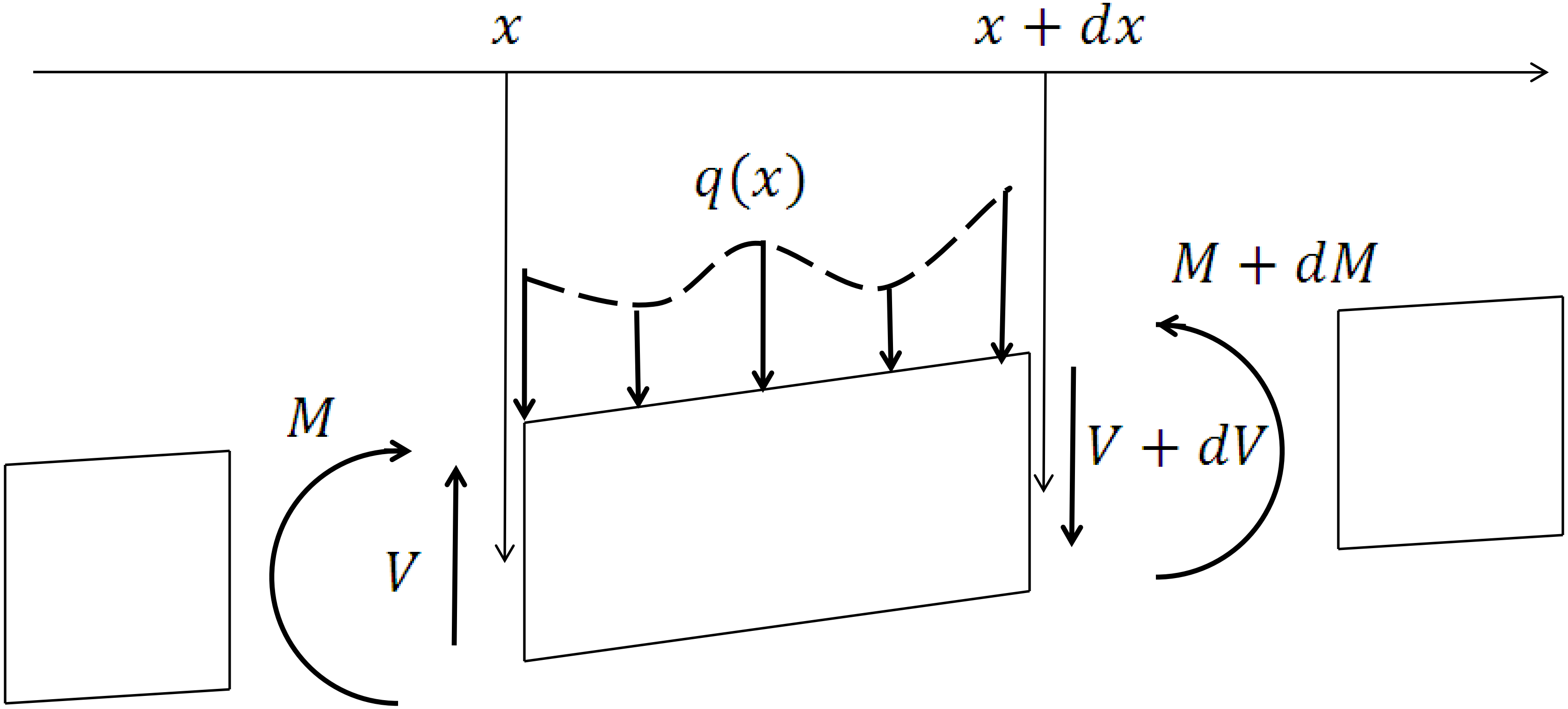
Forças e torques em uma seção da placa



Forças: $q(x)dx + dV = 0 \rightarrow \frac{dV}{dx} = -q$

Torques: $dM = Vdx \rightarrow \frac{dM}{dx} = V$

Forças e torques em uma seção da placa



Forças: $q(x)dx + dV = 0 \rightarrow \frac{dV}{dx} = -q$

Torques: $dM = Vdx \rightarrow \frac{dM}{dx} = V \rightarrow \frac{d^2M}{dx^2} = \frac{dV}{dx}$

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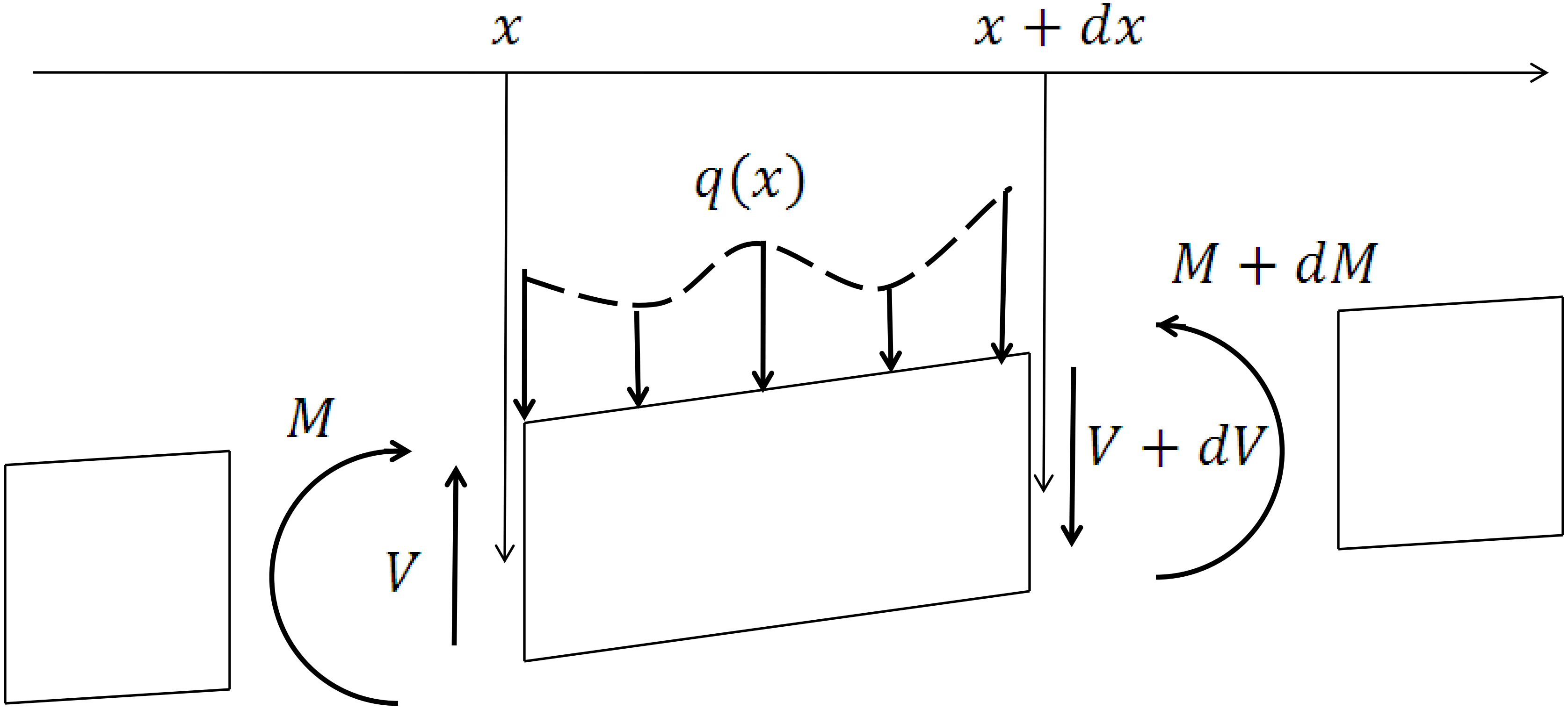
Forças e torques em uma seção da placa



Forças: $q(x)dx + dV = 0 \rightarrow \boxed{\frac{dV}{dx} = -q}$

Torques: $dM = Vdx \rightarrow \frac{dM}{dx} = V \rightarrow \boxed{\frac{d^2M}{dx^2} = \frac{dV}{dx}}$

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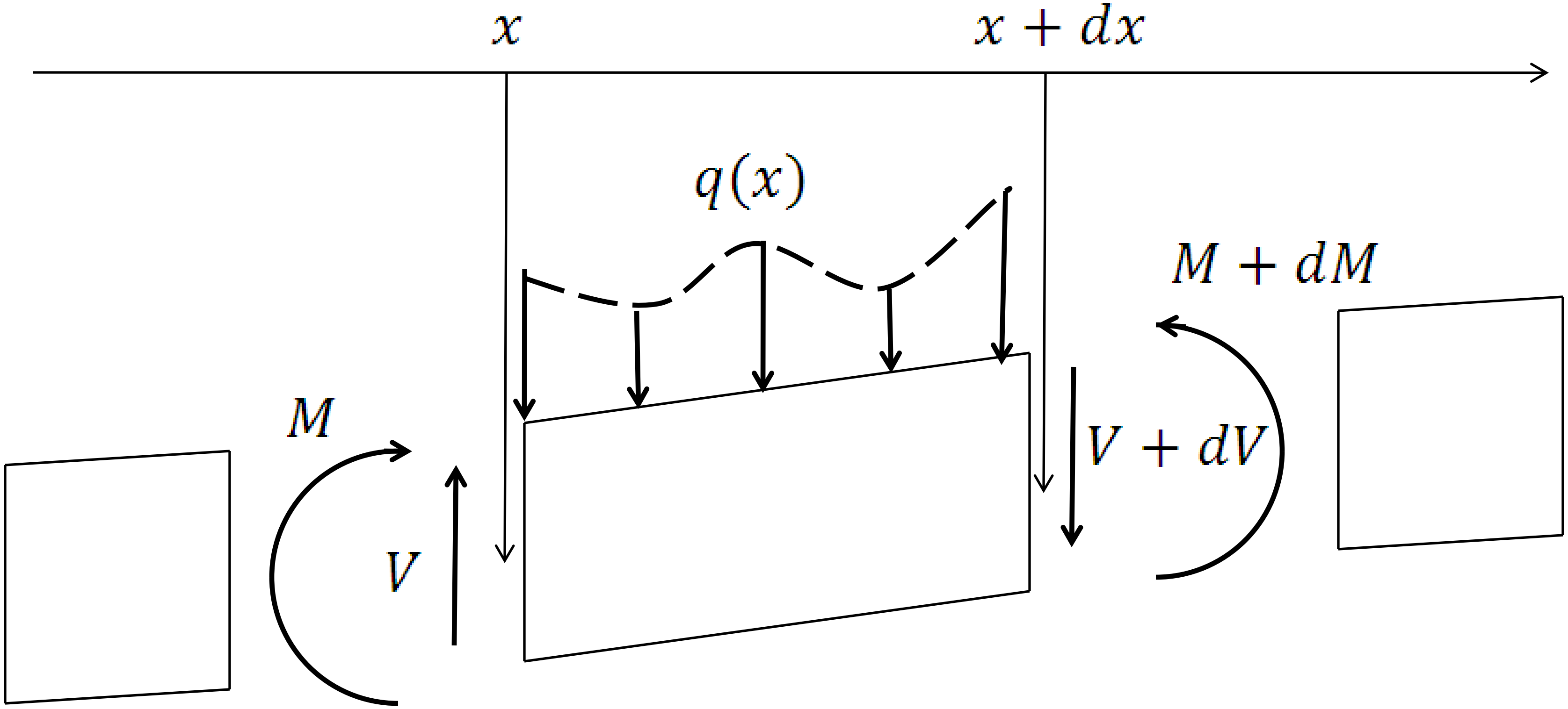
Forças e torques em uma seção da placa



Forças: $q(x)dx + dV = 0 \rightarrow \boxed{\frac{dV}{dx} = -q}$

Torques: $dM = Vdx \rightarrow \frac{dM}{dx} = V \rightarrow \boxed{\frac{d^2M}{dx^2} = \frac{dV}{dx}}$

$$\boxed{\boxed{\frac{d^2M}{dx^2} = -q}}$$

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Elasticidade linear

Elasticidade linear

$$\begin{cases} \varepsilon_1 = \dfrac{1}{E}\sigma_1 - \dfrac{\nu}{E}\sigma_2 - \dfrac{\nu}{E}\sigma_3 \\ \varepsilon_2 = -\dfrac{\nu}{E}\sigma_1 + \dfrac{1}{E}\sigma_2 - \dfrac{\nu}{E}\sigma_3 \\ \varepsilon_3 = -\dfrac{\nu}{E}\sigma_1 - \dfrac{\nu}{E}\sigma_2 + \dfrac{1}{E}\sigma_3 \end{cases}$$

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$\varepsilon_1, \varepsilon_2, \varepsilon_3$

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$\varepsilon_1, \varepsilon_2, \varepsilon_3$ Deformações principais

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$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Deformações principais
$\sigma_1, \sigma_2, \sigma_3$	Tensões principais

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$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Deformações principais
$\sigma_1, \sigma_2, \sigma_3$	Tensões principais
E	

Elasticidade linear

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$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Deformações principais
$\sigma_1, \sigma_2, \sigma_3$	Tensões principais
E	Módulo de elasticidade

Elasticidade linear

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$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Deformações principais
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E	Módulo de elasticidade
ν	

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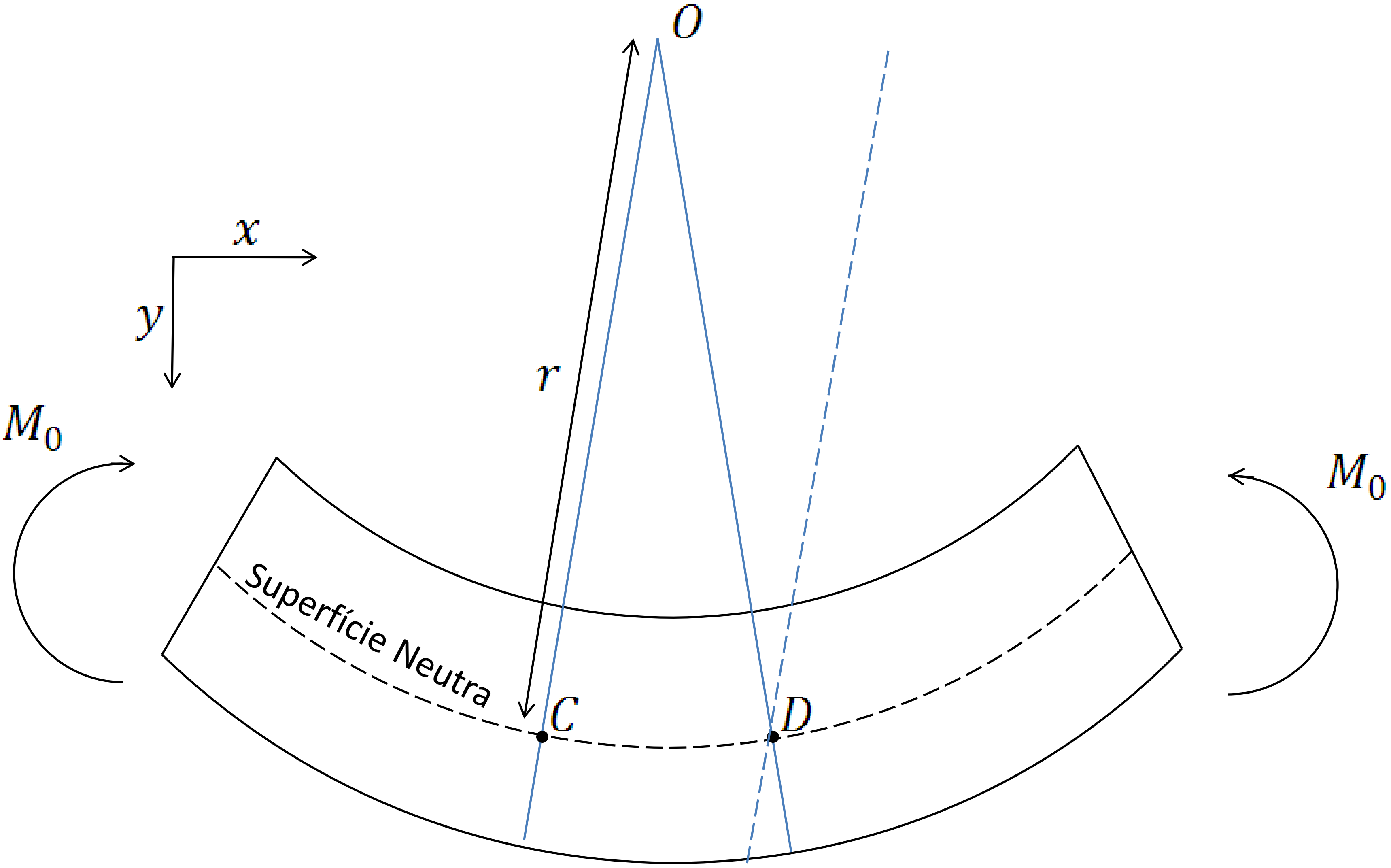
$\varepsilon_1, \varepsilon_2, \varepsilon_3$	Deformações principais
$\sigma_1, \sigma_2, \sigma_3$	Tensões principais
E	Módulo de elasticidade
ν	Coeficiente de Poisson

Deformações no interior da placa



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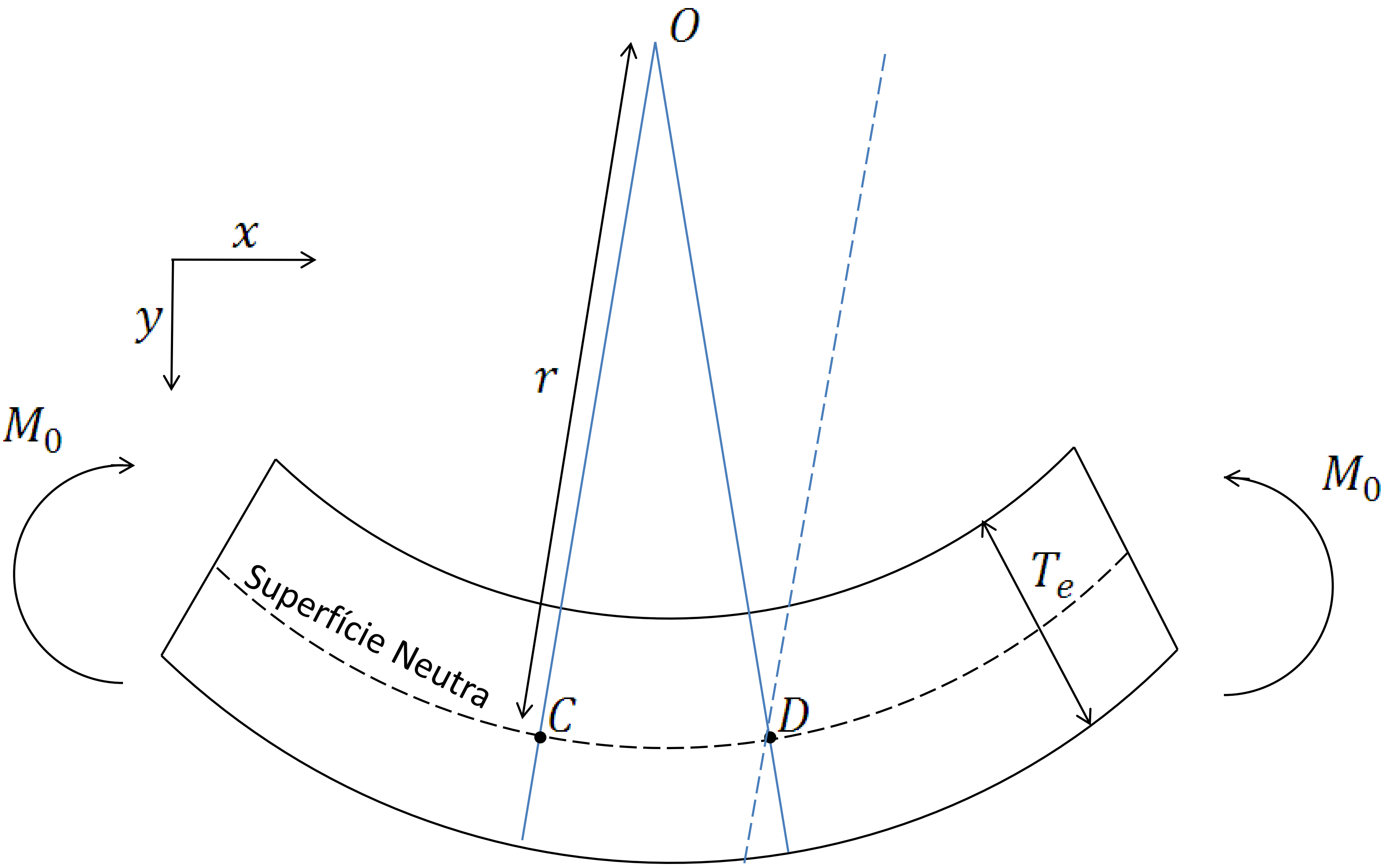
Deformações no interior da placa



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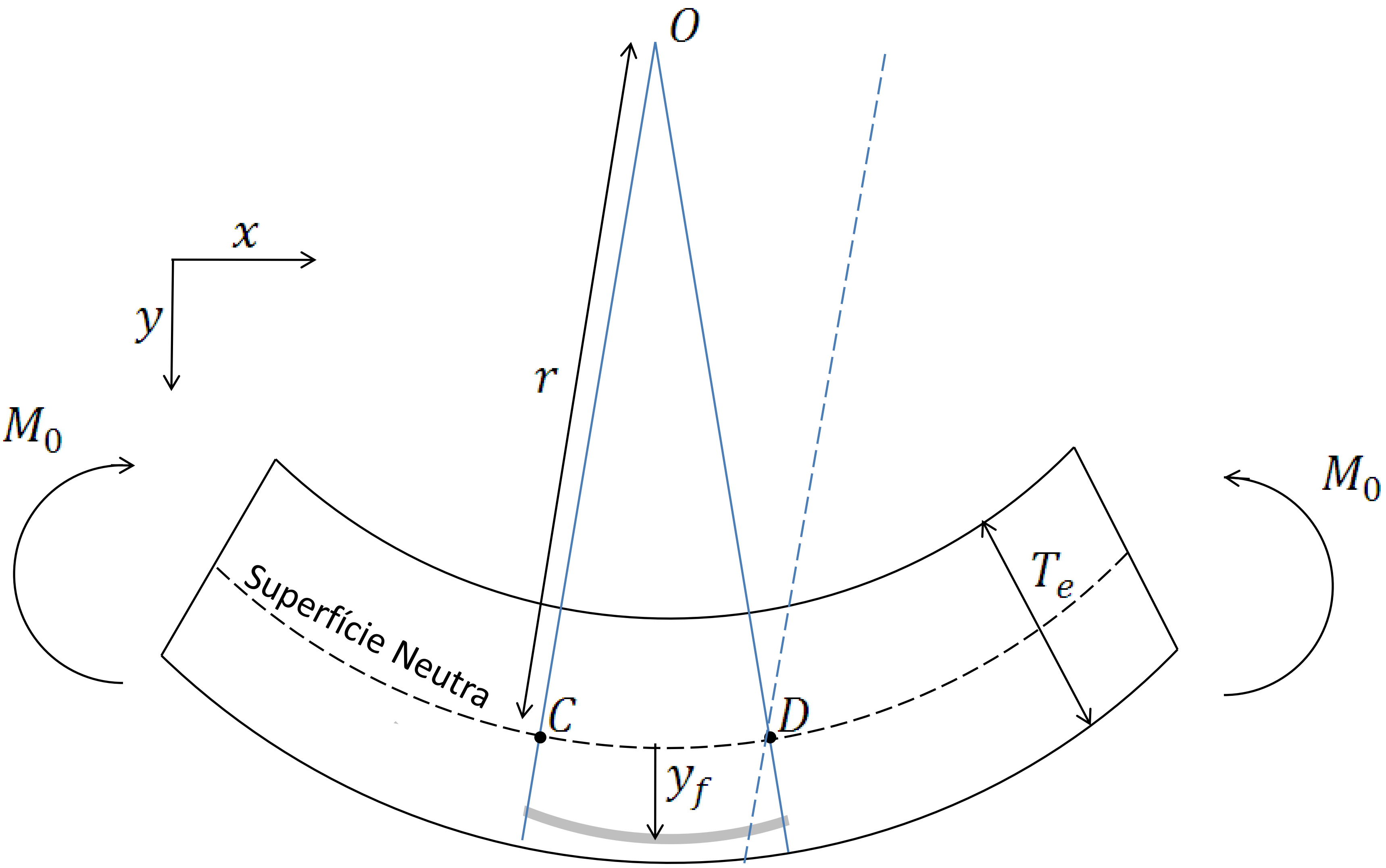

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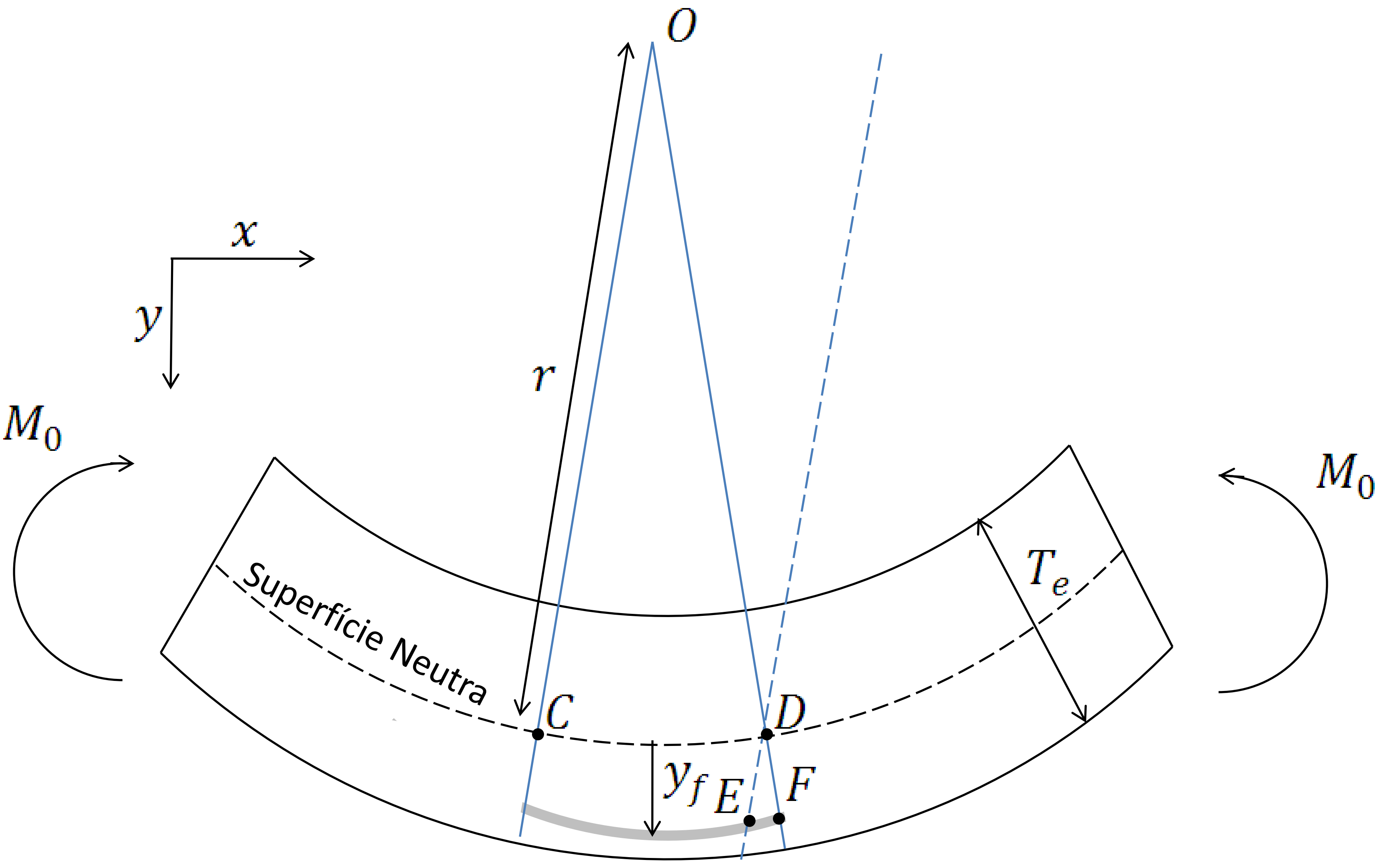

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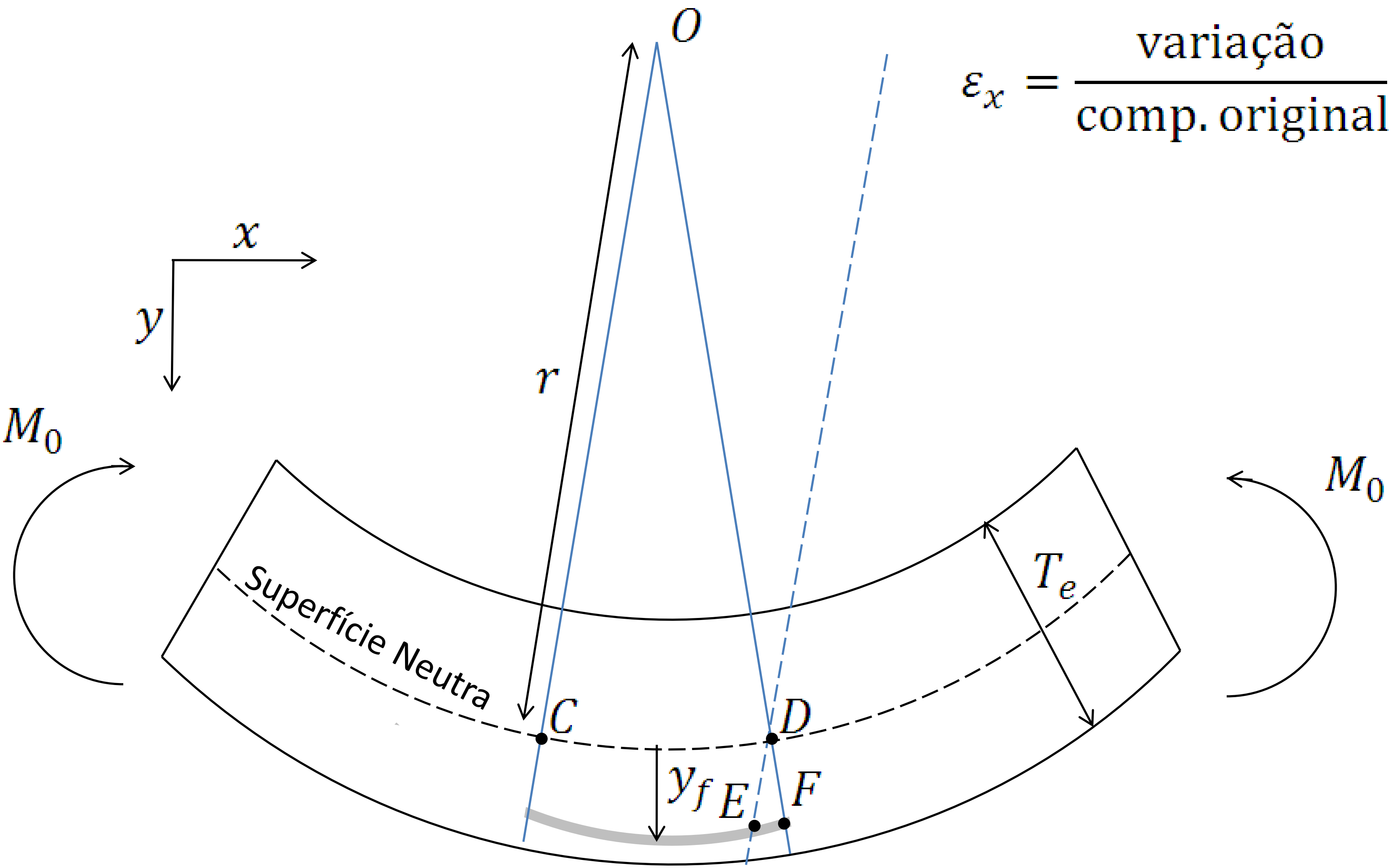
Deformações no interior da placa

$$\varepsilon_x = \frac{\text{variação}}{\text{comp. original}}$$



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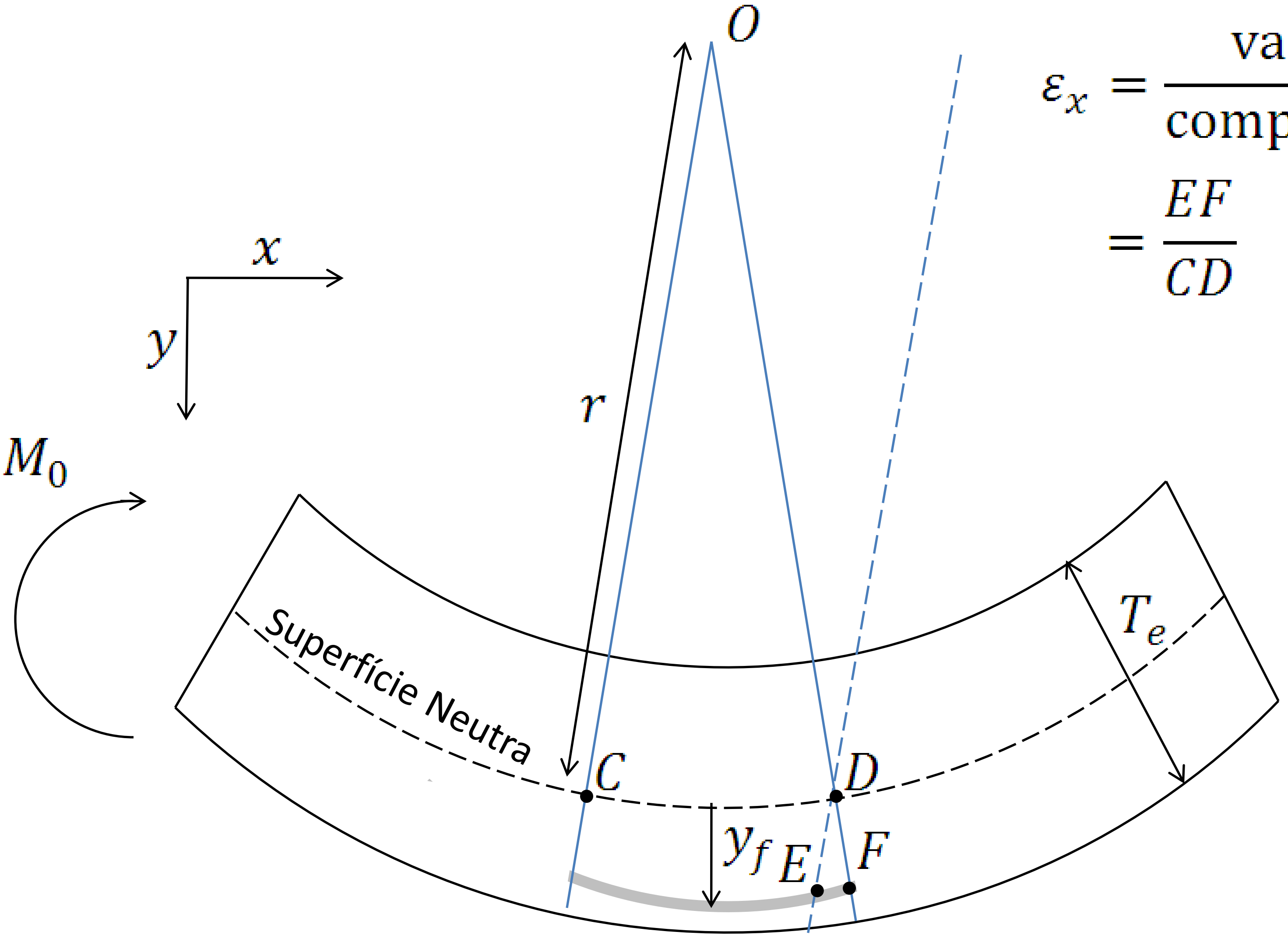
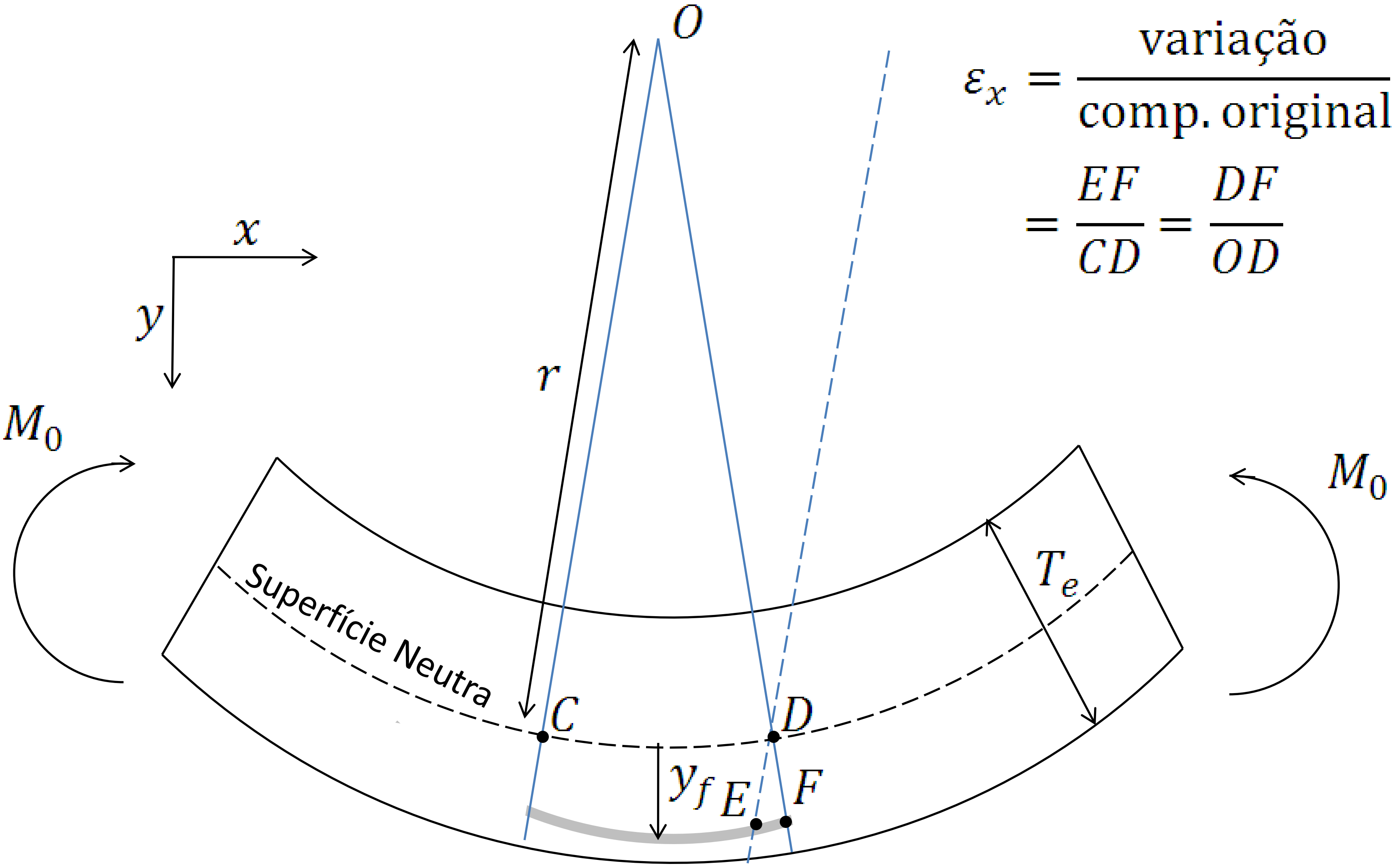


$$\varepsilon_x = \frac{\text{variação}}{\text{comp. original}}$$

$$= \frac{EF}{CD}$$

Deformações no interior da placa



$$\varepsilon_x = \frac{\text{variação}}{\text{comp. original}}$$

$$= \frac{EF}{CD} = \frac{DF}{OD}$$

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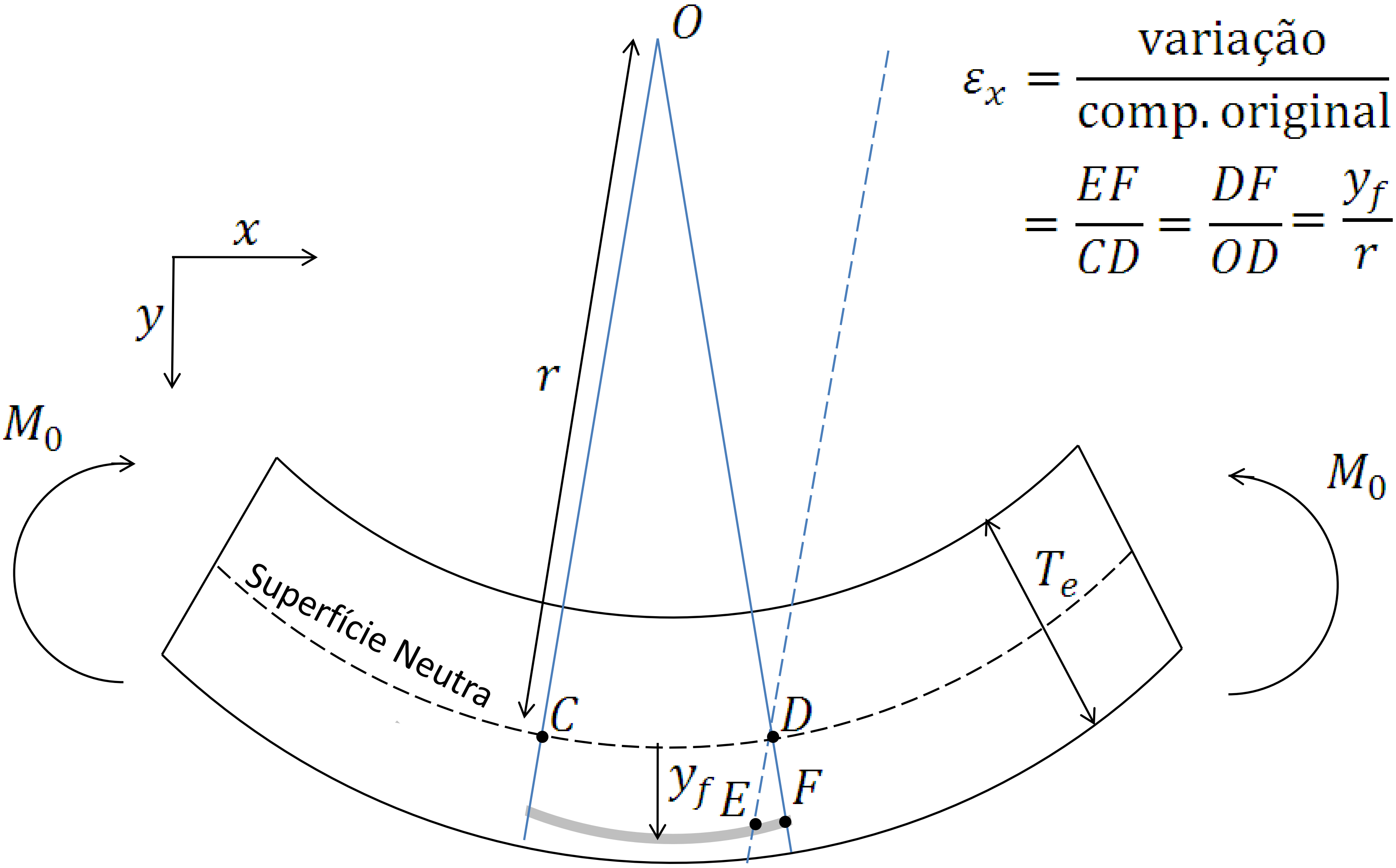
Deformações no interior da placa



$$\varepsilon_x = \frac{\text{variação}}{\text{comp. original}}$$

$$= \frac{EF}{CD} = \frac{DF}{OD} = \frac{y_f}{r}$$

Tensões no interior da placa

Problema de flexura em 2D:

Tensões no interior da placa

Problema de flexura em 2D: $\varepsilon_z = 0, \qquad \sigma_y = 0$

Tensões no interior da placa

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$$\begin{cases} \varepsilon_x = \dfrac{1}{E}\sigma_x - \dfrac{\nu}{E}\sigma_y - \dfrac{\nu}{E}\sigma_z \\ \varepsilon_y = -\dfrac{\nu}{E}\sigma_x + \dfrac{1}{E}\sigma_y - \dfrac{\nu}{E}\sigma_z \\ \varepsilon_z = -\dfrac{\nu}{E}\sigma_x - \dfrac{\nu}{E}\sigma_y + \dfrac{1}{E}\sigma_z \end{cases}$$

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Tensões no interior da placa

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Tensões no interior da placa

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Tensões no interior da placa

Problema de flexura em 2D: $\varepsilon_z = 0, \qquad \sigma_y = 0$

$$\varepsilon_x = \frac{y_f}{r}$$

$$\begin{cases} \varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\cancel{\sigma_y} - \frac{\nu}{E}\sigma_z \\ \varepsilon_y = -\frac{\nu}{E}\sigma_x + \frac{1}{E}\cancel{\sigma_y} - \frac{\nu}{E}\sigma_z \\ \cancel{\varepsilon_z} = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\cancel{\sigma_y} + \frac{1}{E}\sigma_z \end{cases} \rightarrow \begin{cases} \varepsilon_x = \frac{1}{E}\sigma_x - \frac{\nu}{E}\sigma_z \\ \varepsilon_y = -\frac{\nu}{E}\sigma_x - \frac{\nu}{E}\sigma_z \\ 0 = -\frac{\nu}{E}\sigma_x + \frac{1}{E}\sigma_z \end{cases} \rightarrow \begin{cases} \sigma_x = \frac{E\varepsilon_x}{1-\nu^2} \\ \sigma_z = \frac{E\varepsilon_x\nu}{1-\nu^2} \end{cases}$$

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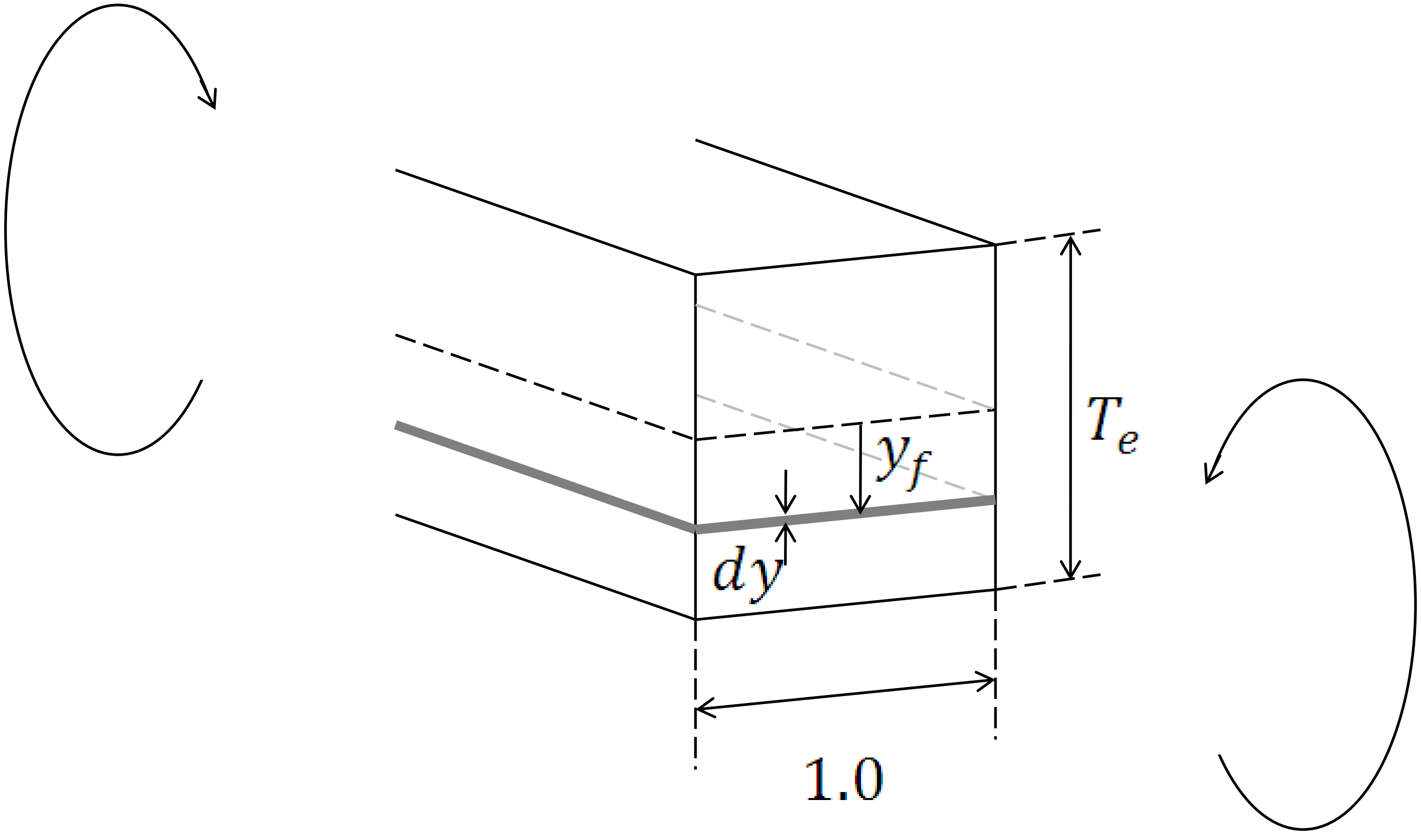
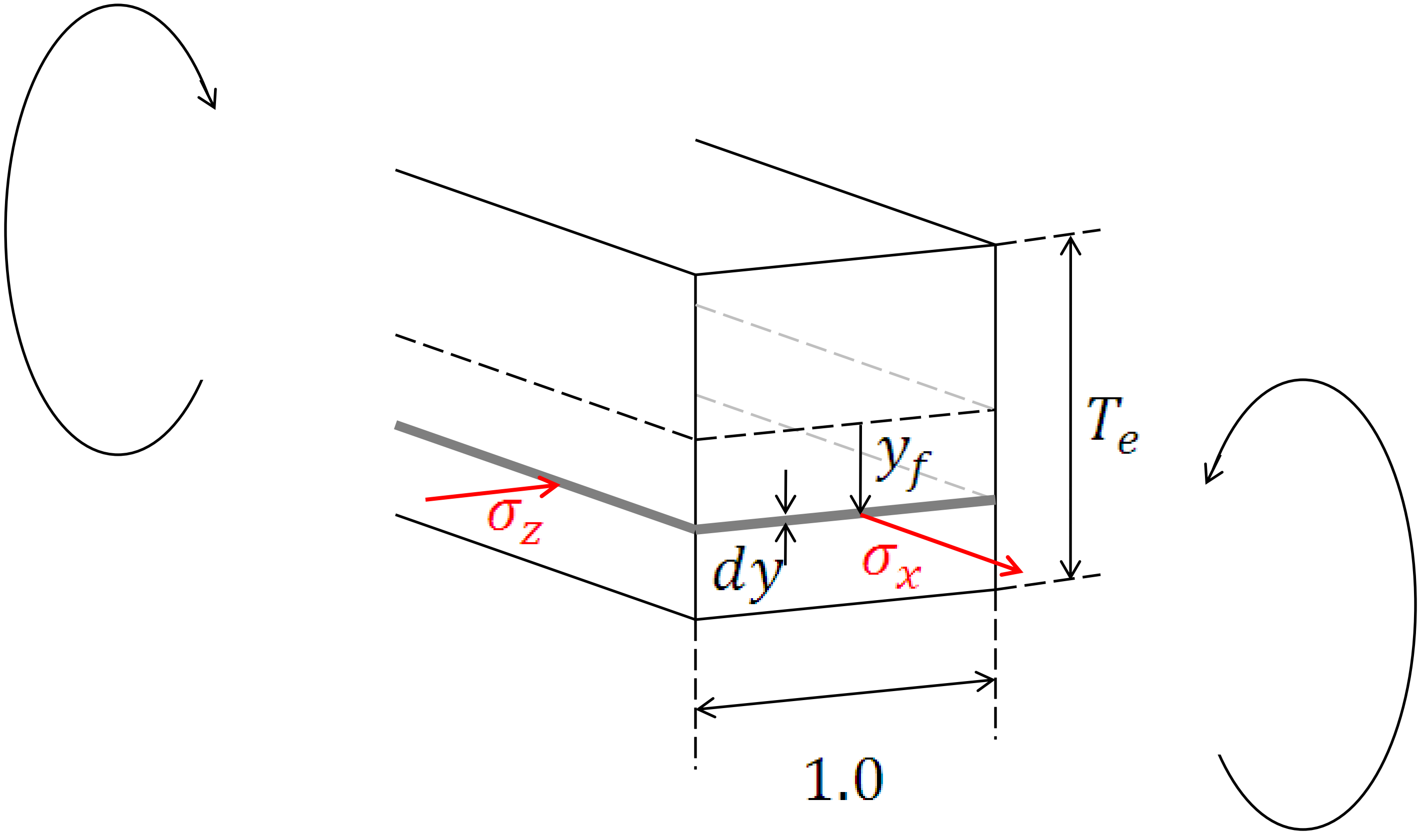
$$\begin{cases} \sigma_x = \frac{E y_f}{r(1-\nu^2)} \\ \sigma_z = \frac{E y_f \nu}{r(1-\nu^2)} \end{cases}$$

Força cisalhante e Torque



Força cisalhante e Torque



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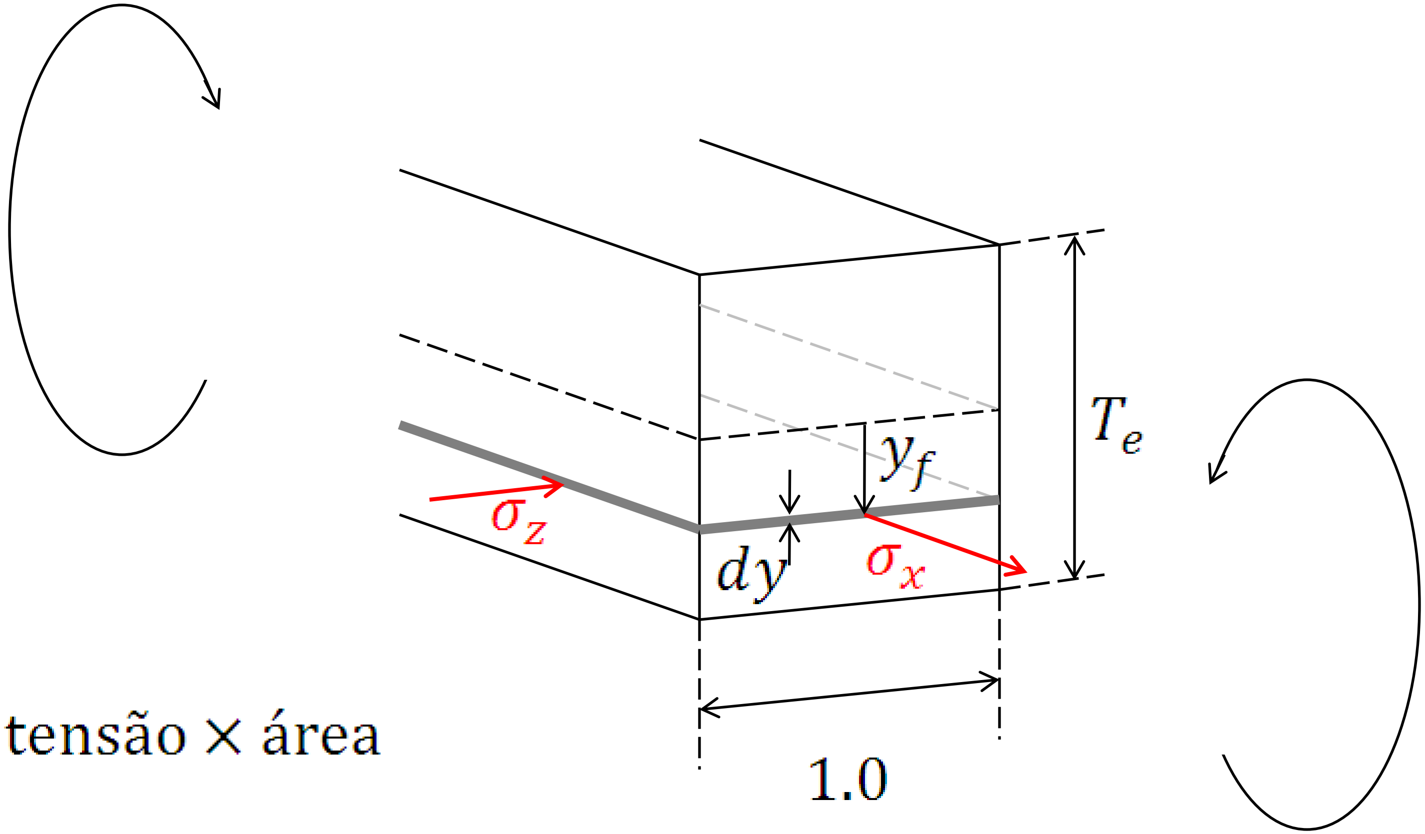
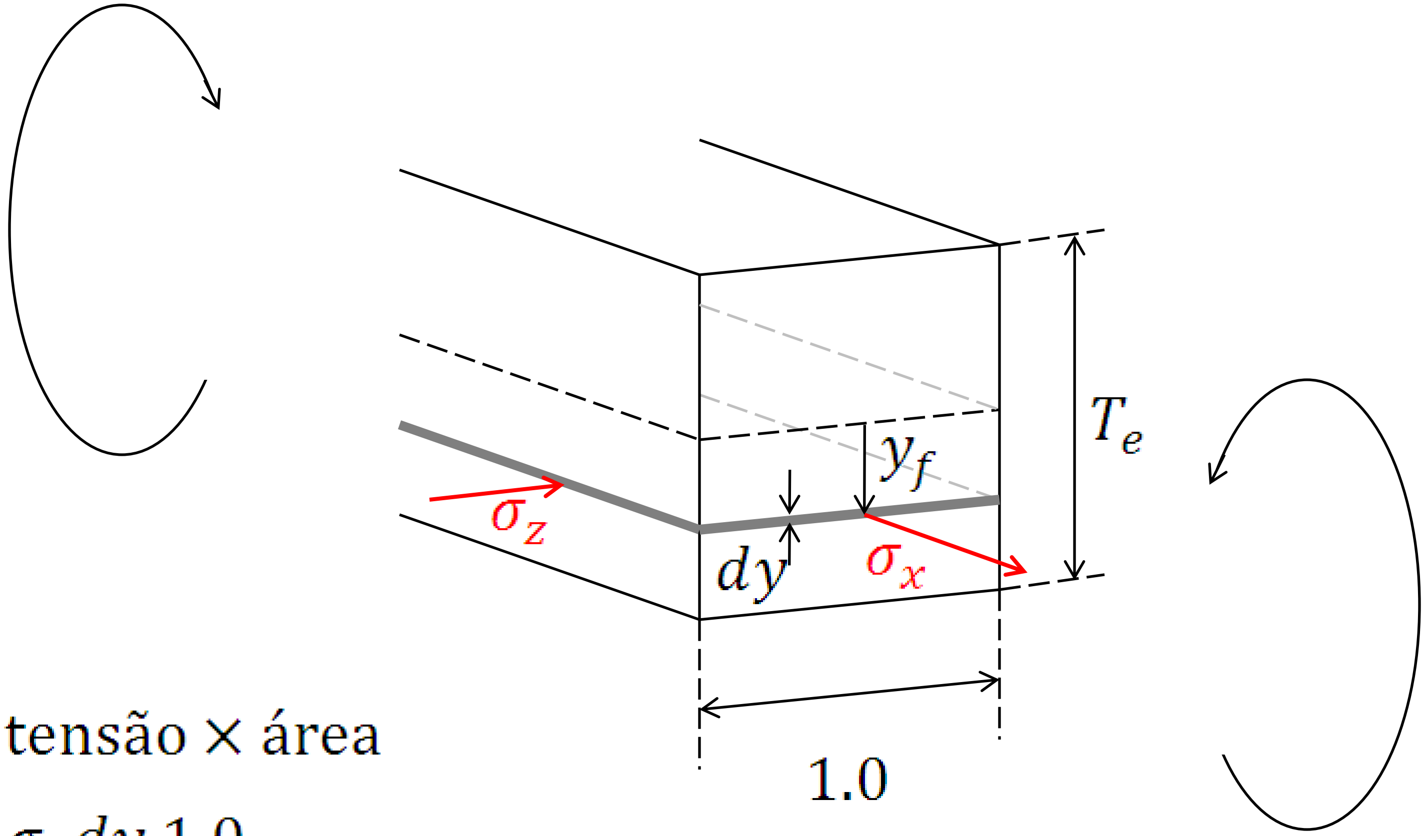


força = tensão × área

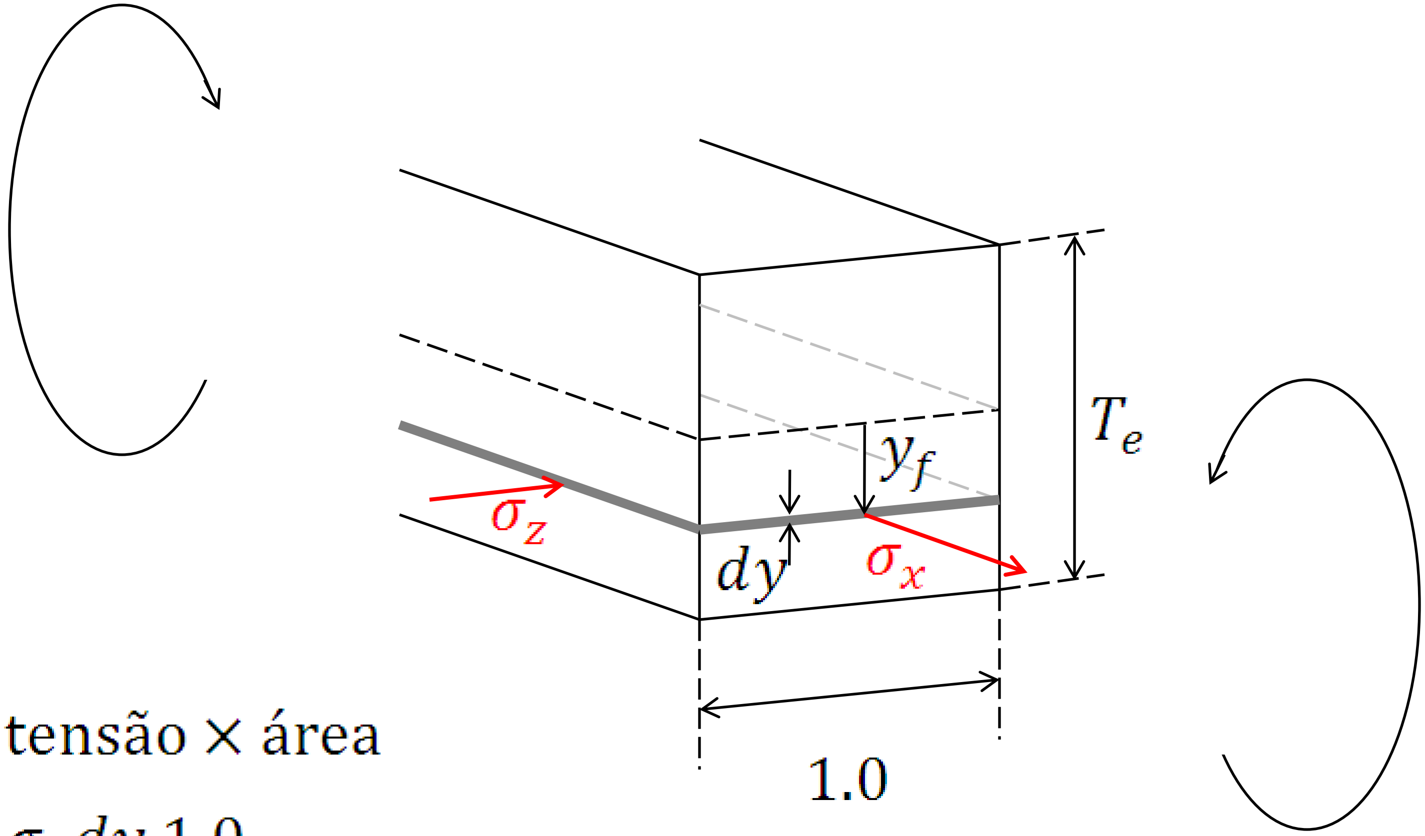
Força cisalhante e Torque



força = tensão × área

$$dF = \sigma_x dy\, 1.0$$

Força cisalhante e Torque

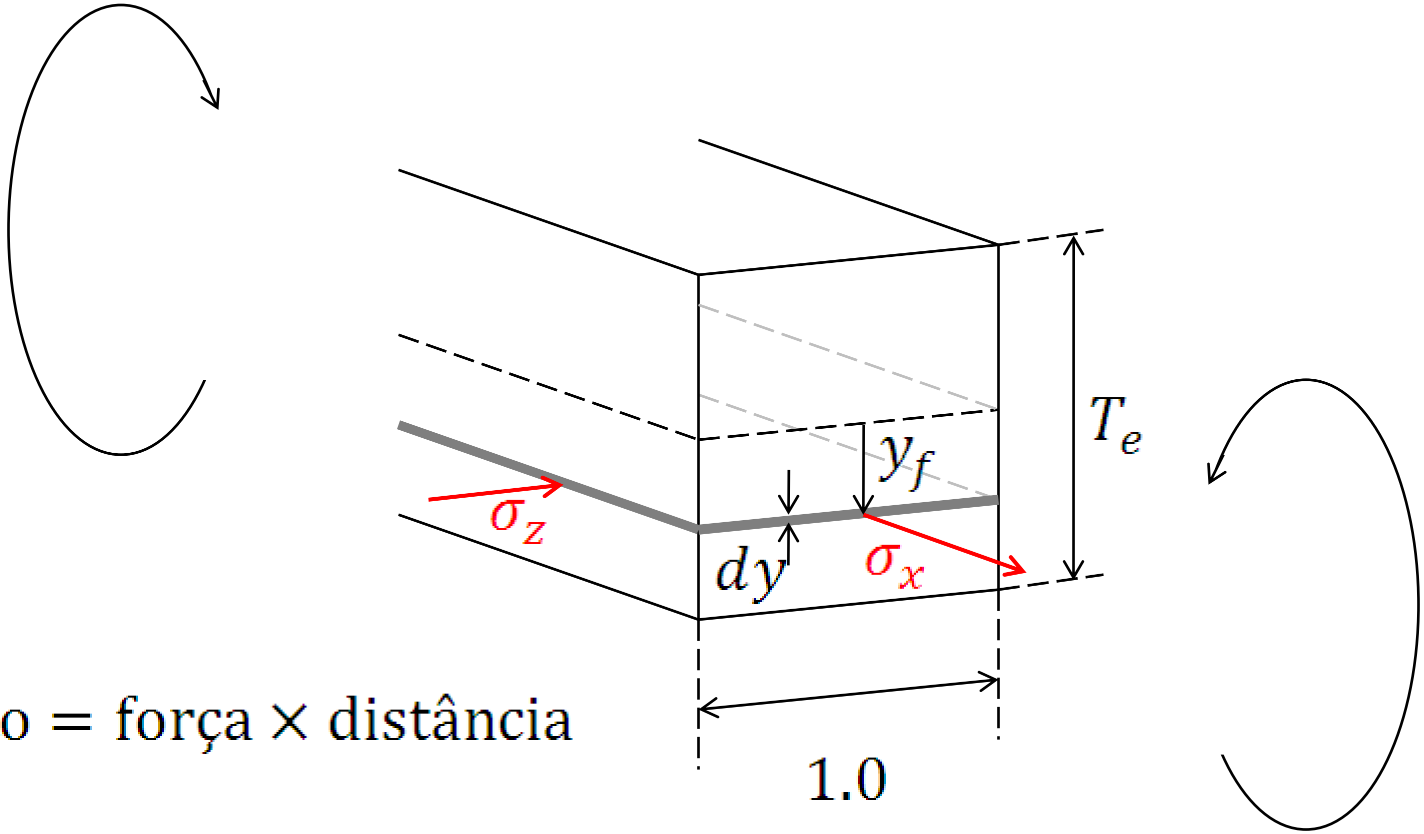


$$\text{força} = \text{tensão} \times \text{área}$$

$$dF = \sigma_x dy \, 1.0$$

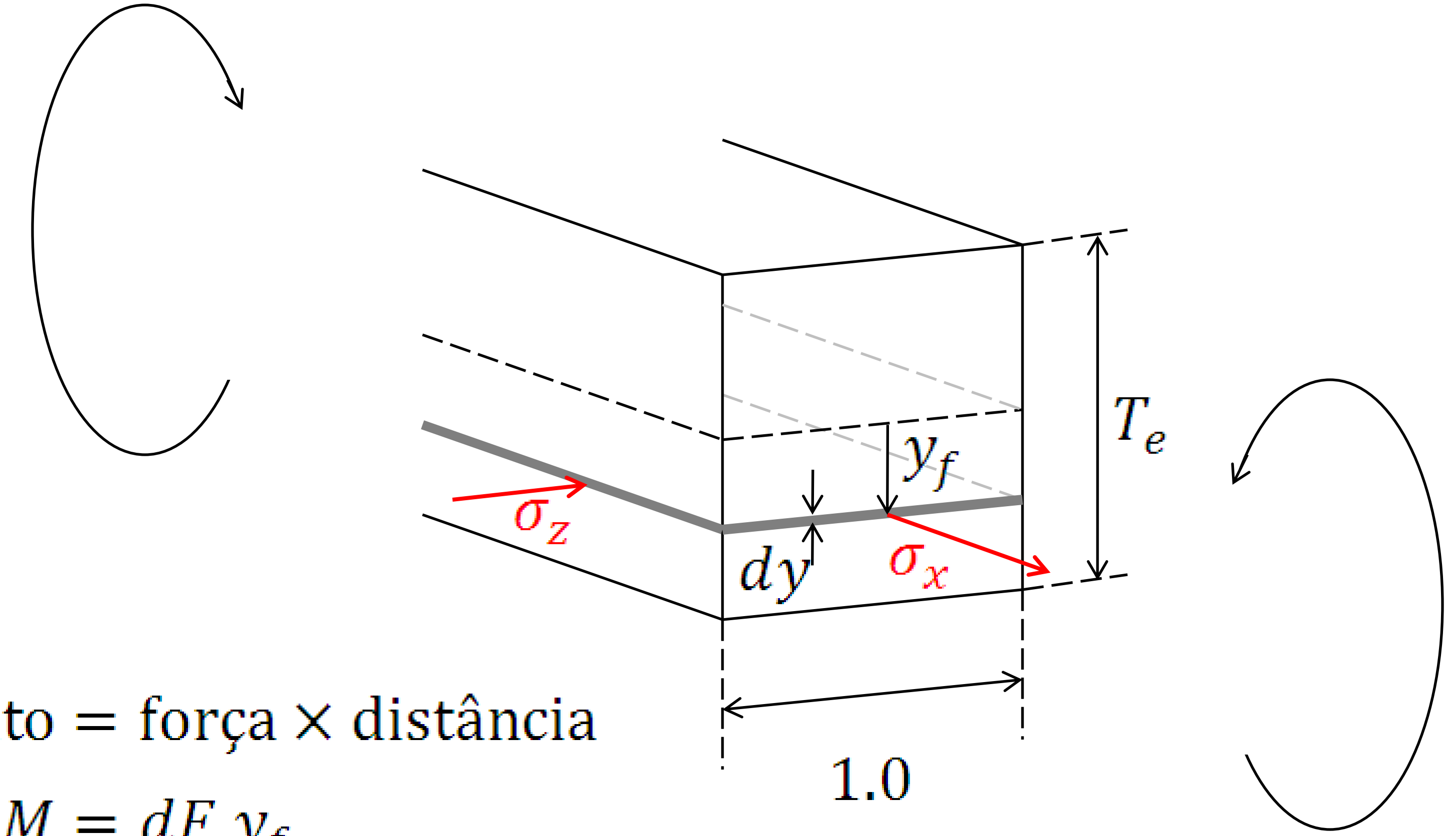
$$= \frac{E y_f}{r(1-\nu^2)} dy$$

Força cisalhante e Torque



momento = força × distância

Força cisalhante e Torque



$$\text{momento} = \text{força} \times \text{distância}$$

$$dM = dF\ y_f$$

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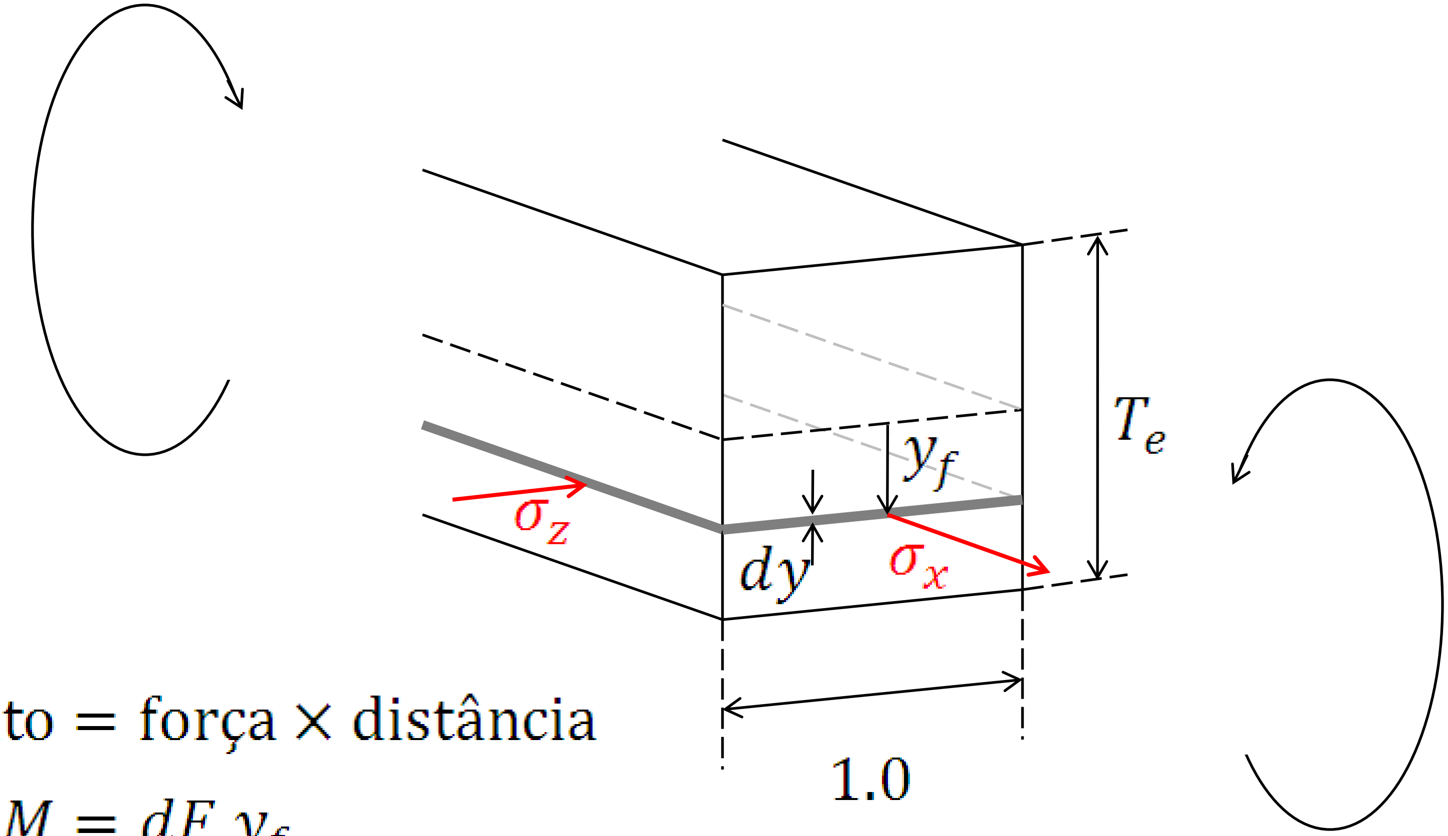
Força cisalhante e Torque



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$$dM = dF\ y_f$$

$$= \frac{E y_f^2}{r(1-\nu^2)} dy$$

Força cisalhante e Torque

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Força cisalhante e Torque

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$M = \int dM$

Força cisalhante e Torque

$$dM = \frac{E y_f^2}{r(1-\nu^2)} dy$$

$$M = \int dM \; = \int_{-\frac{T_e}{2}}^{\frac{T_e}{2}} \frac{E y_f^2}{r(1-\nu^2)} dy$$

Força cisalhante e Torque

$$dM = \frac{E y_f^2}{r(1-\nu^2)} dy$$

$$M = \int dM \ = \int_{-\frac{T_e}{2}}^{\frac{T_e}{2}} \frac{E y_f^2}{r(1-\nu^2)} dy \ = \frac{E}{r(1-\nu^2)} \left[\frac{y^3}{3}\right] \Bigg|_{-\frac{T_e}{2}}^{\frac{T_e}{2}}$$

Força cisalhante e Torque

$$dM = \frac{Ey_f^2}{r(1-\nu^2)}dy$$

$$M = \int dM = \int_{-\frac{T_e}{2}}^{\frac{T_e}{2}} \frac{Ey_f^2}{r(1-\nu^2)}dy = \frac{E}{r(1-\nu^2)}\left[\frac{y^3}{3}\right]\Bigg|_{-\frac{T_e}{2}}^{\frac{T_e}{2}} = \frac{ET_e^3}{12(1-\nu^2)} \cdot \frac{1}{r}$$

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Rigidez flexural

Força cisalhante e Torque

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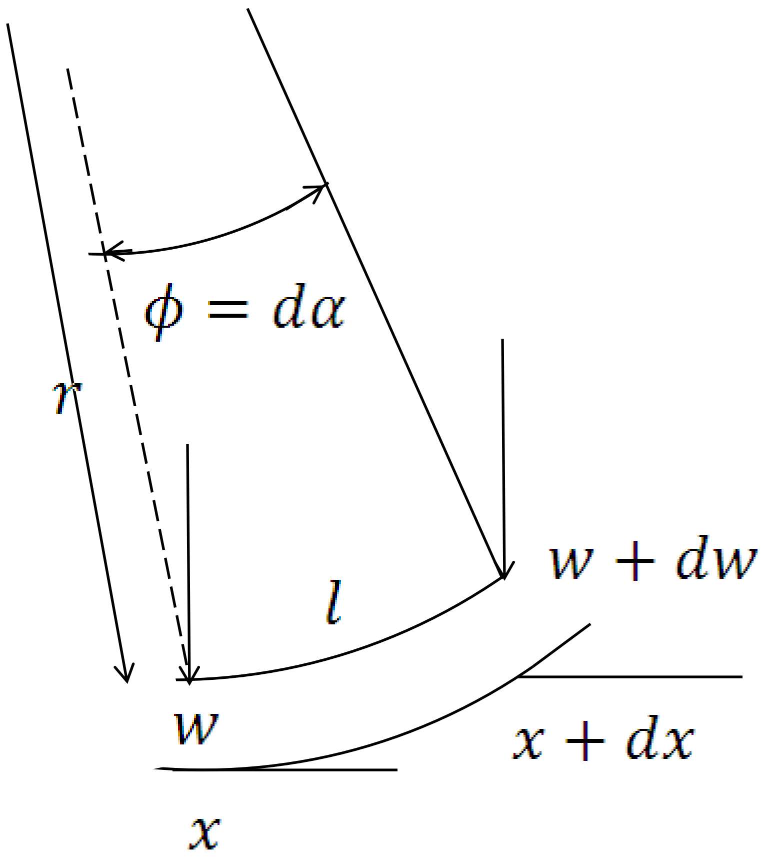
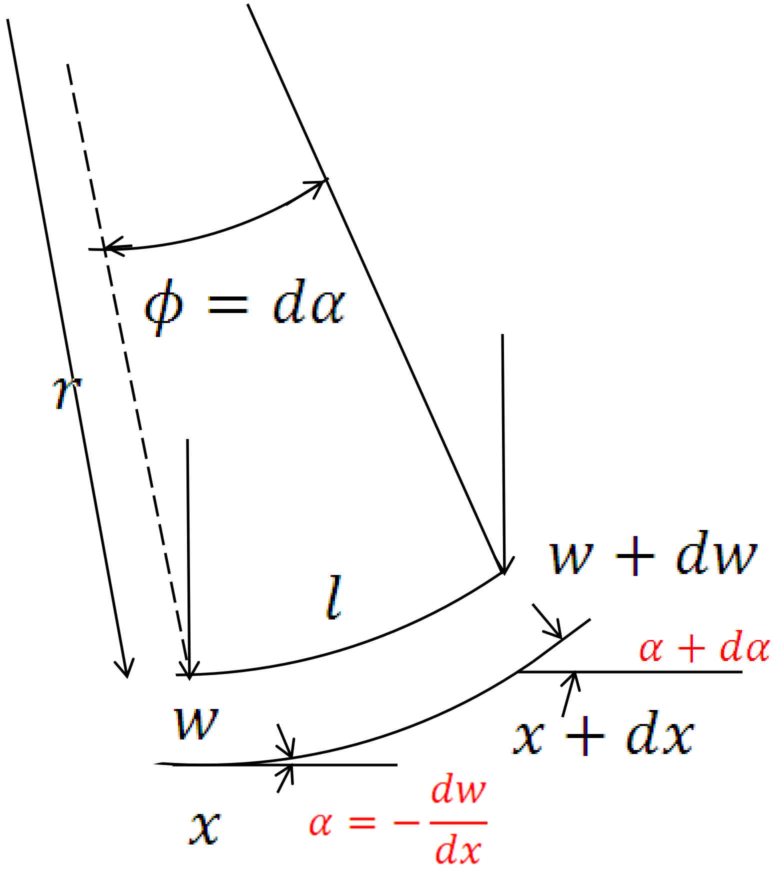
Rigidez flexural

$$\boxed{\boxed{M = \frac{D}{r}}}$$

Determinação do raio de curvatura



Determinação do raio de curvatura



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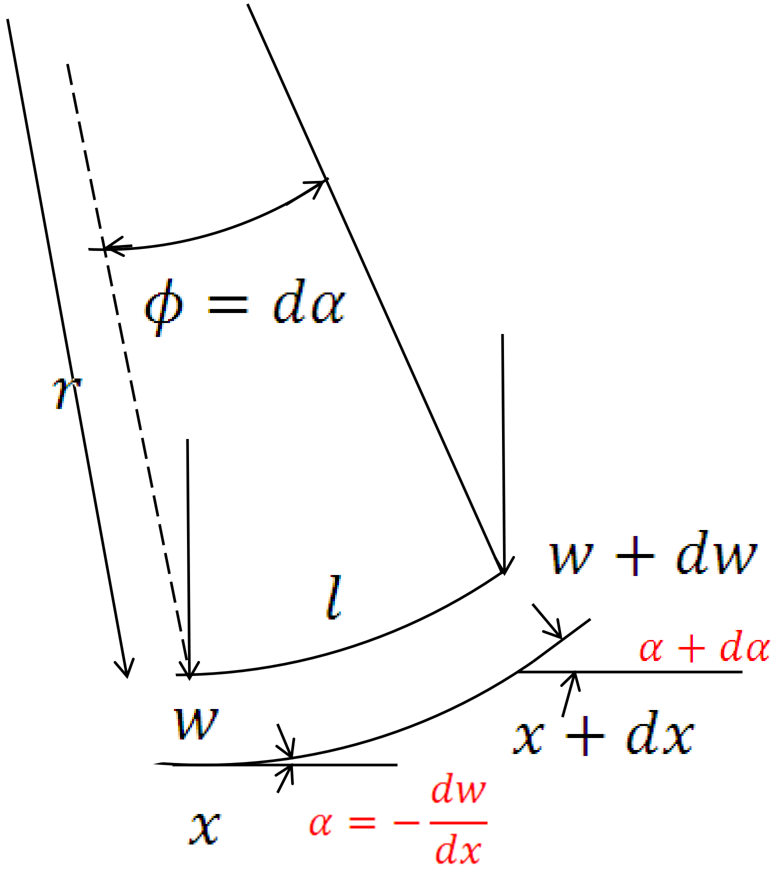
Determinação do raio de curvatura

$\phi = d\alpha$



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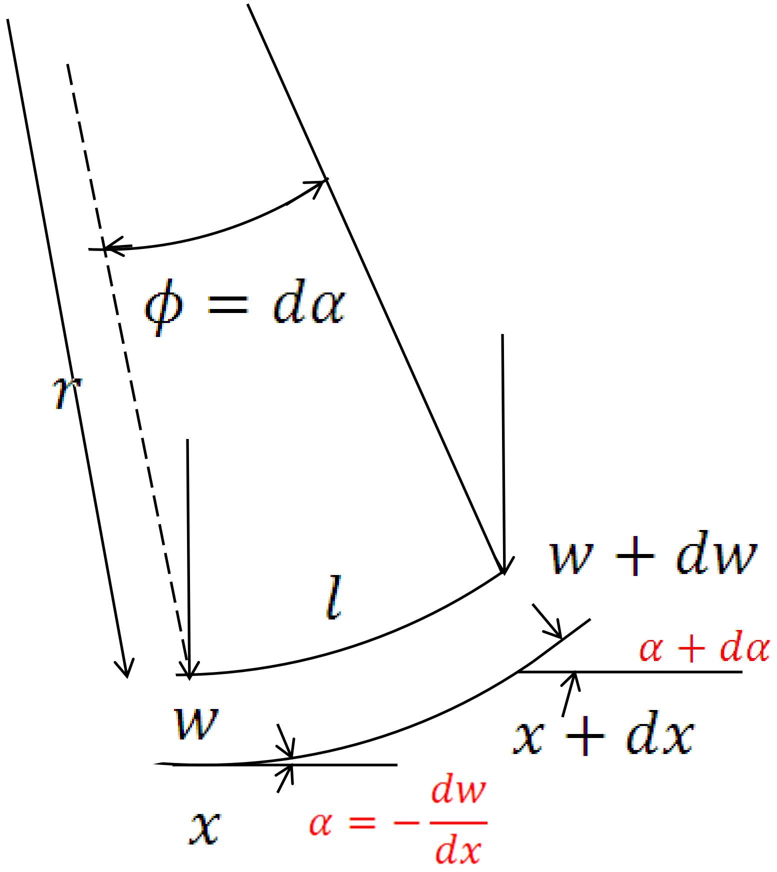
Determinação do raio de curvatura

$$\phi = d\alpha = \frac{d\alpha}{dx}dx$$



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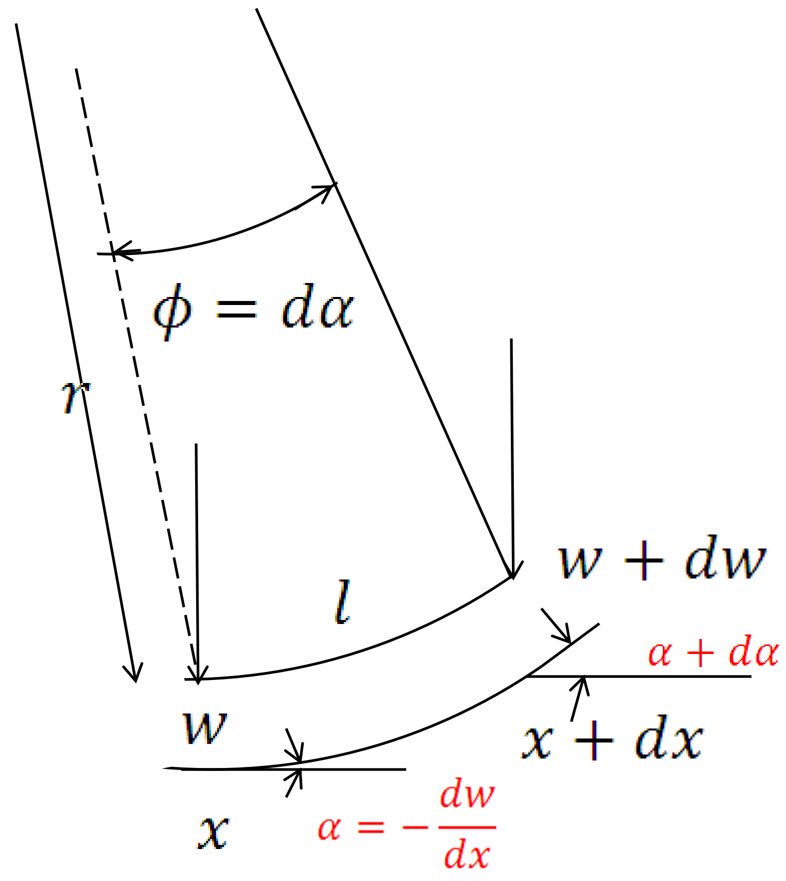
Determinação do raio de curvatura

$$\phi = d\alpha = \frac{d\alpha}{dx}dx$$

$$= \frac{d}{dx}\left(-\frac{dw}{dx}\right)dx$$



Determinação do raio de curvatura

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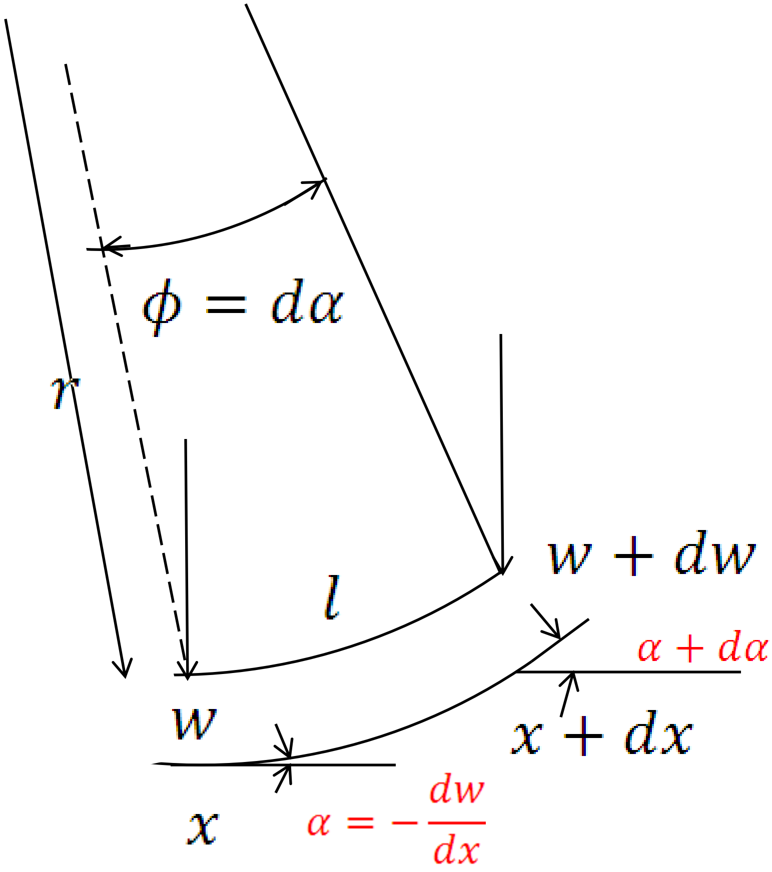

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Determinação do raio de curvatura

$$\phi = d\alpha = \frac{d\alpha}{dx} dx$$

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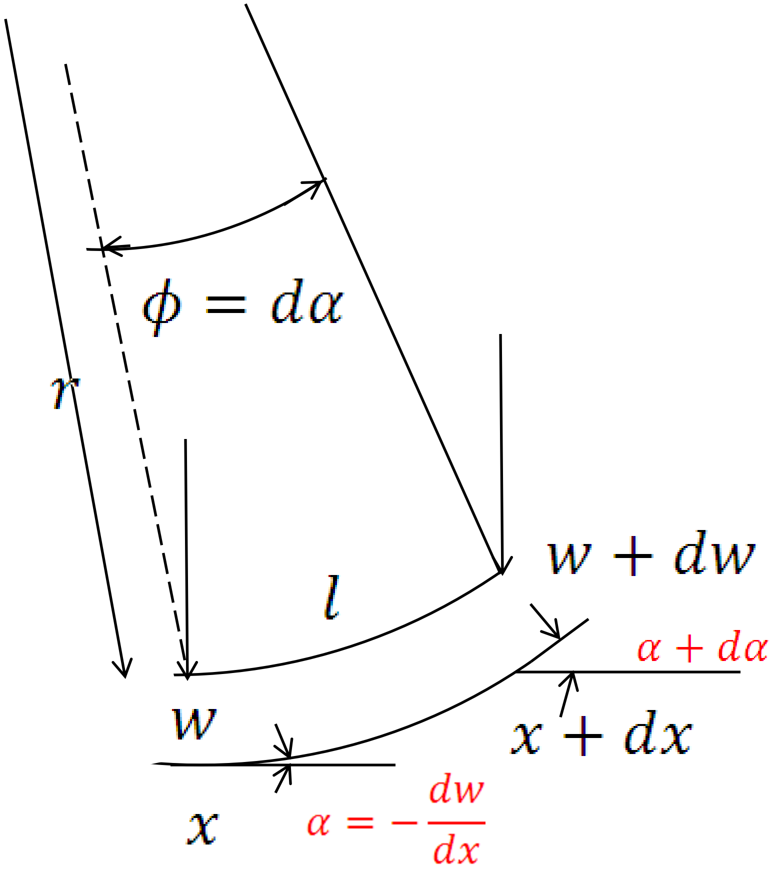
$$\frac{1}{r} = \frac{\phi}{l}$$



Determinação do raio de curvatura

$$\phi = d\alpha = \frac{d\alpha}{dx}dx$$

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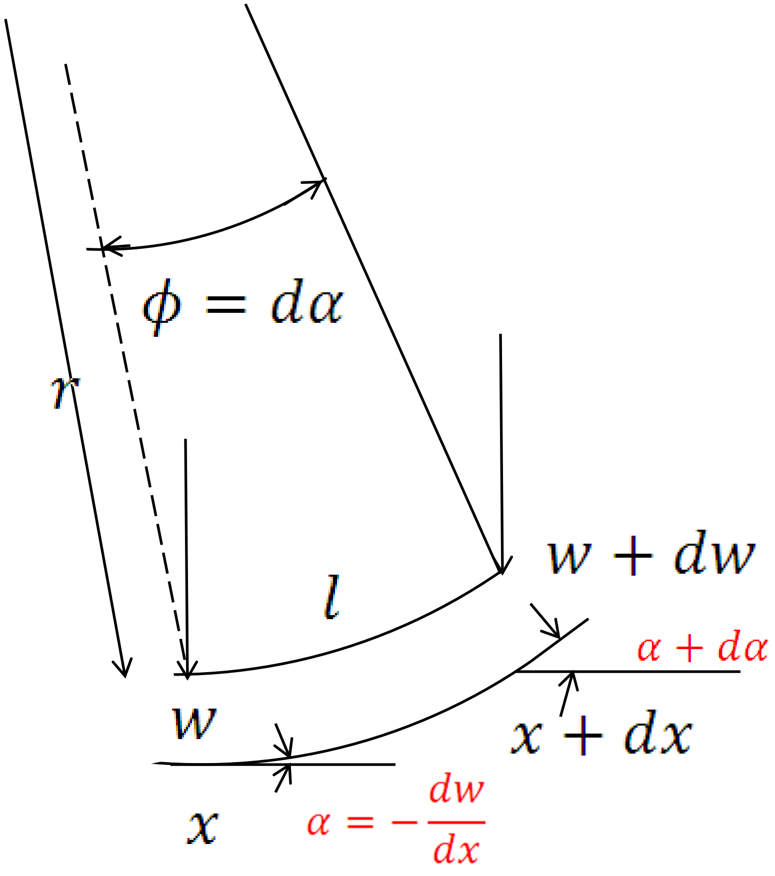
$$\frac{1}{r} = \frac{\phi}{l} \approx \frac{\phi}{dx} = -\frac{d^2w}{dx^2}$$



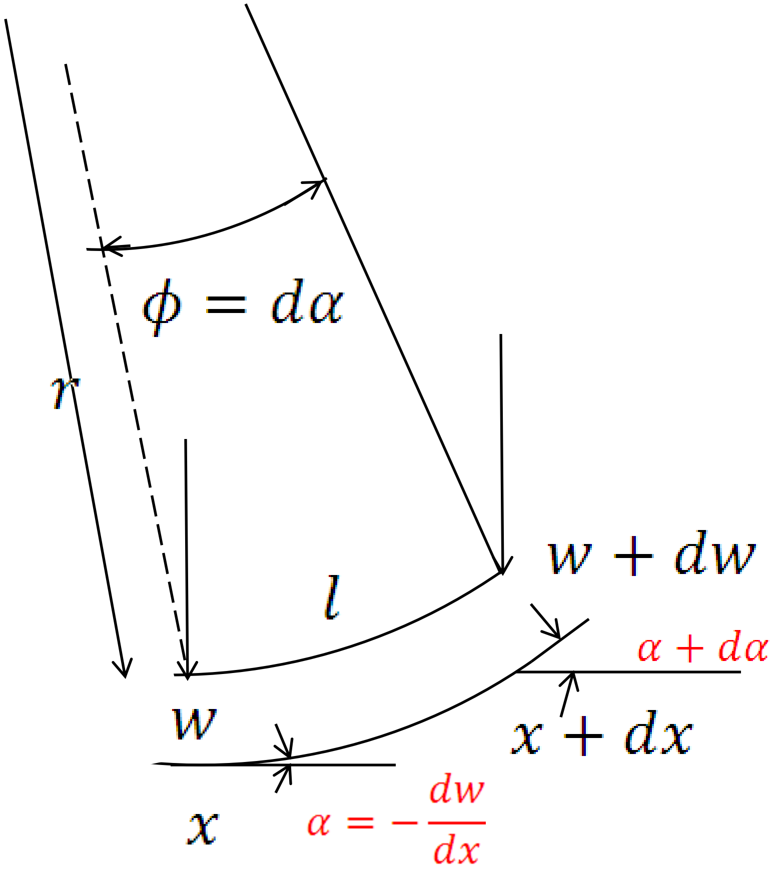
Determinação do raio de curvatura

$$\phi = d\alpha = \frac{d\alpha}{dx}dx$$

$$= \frac{d}{dx}\left(-\frac{dw}{dx}\right)dx$$

$$= -\frac{d^2w}{dx^2}dx$$

$$\boxed{\frac{1}{r} = \frac{\phi}{l} \approx \frac{\phi}{dx} = -\frac{d^2w}{dx^2}}$$



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Flexura de placa elástica fina

Flexura de placa elástica fina

$$\boxed{\frac{d^2M}{dx^2} = -q}$$

$$\boxed{M = \frac{D}{r}}$$

$$\boxed{\frac{1}{r} \rightarrow -\frac{d^2w}{dx^2}}$$

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Flexura de placa elástica fina

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Flexura de placa elástica fina

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$$M = -D\frac{d^2w}{dx^2}$$

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) = q(x)$$

Aplicação para a litosfera

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) = q(x)$$

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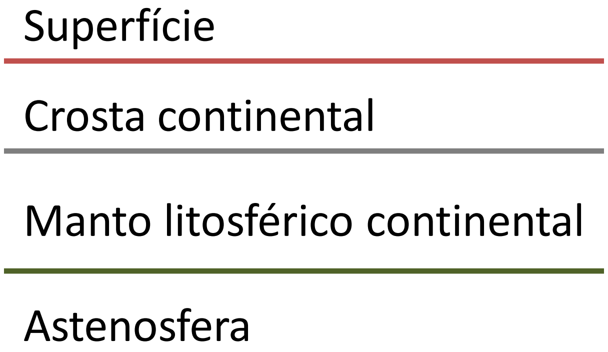

Superfície

Crosta continental

Manto litosférico continental

Astenosfera

Aplicação para a litosfera

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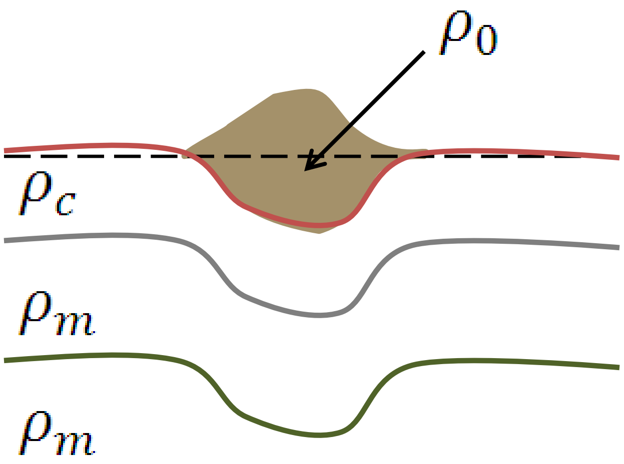

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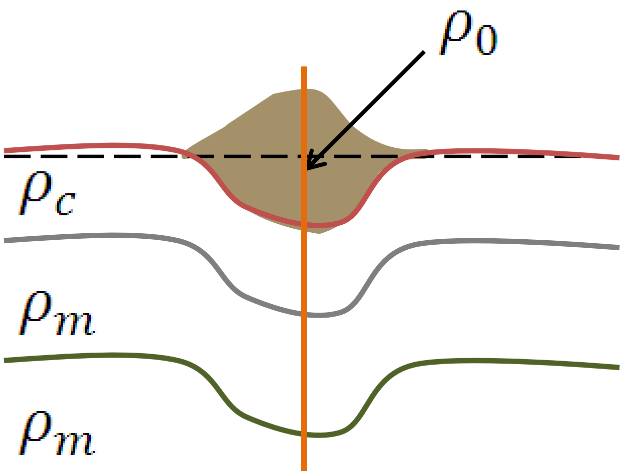

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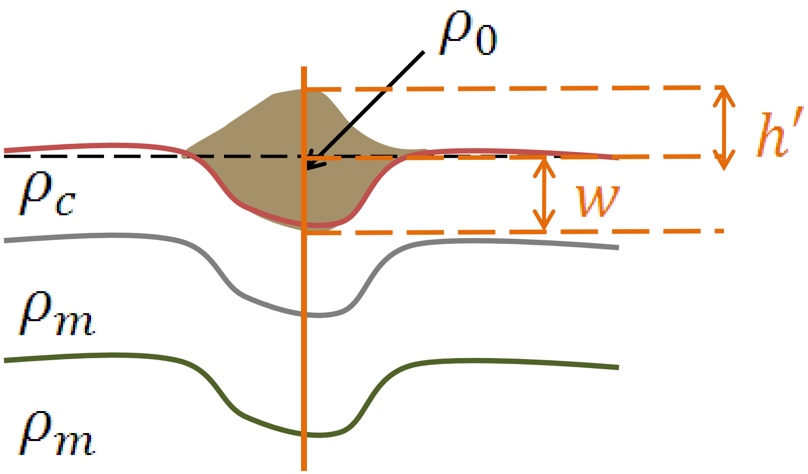

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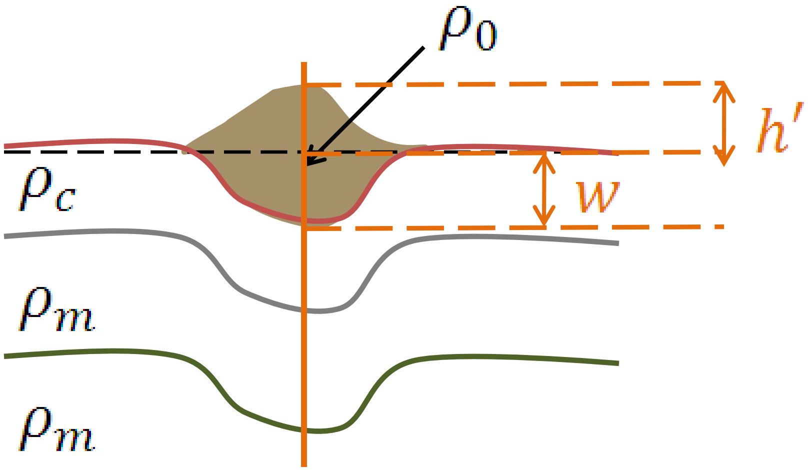

$$q(x) = \text{carga} - \text{empuxo}$$

$$q(x) = \rho_0 g(h' + w) - \rho_m g w$$

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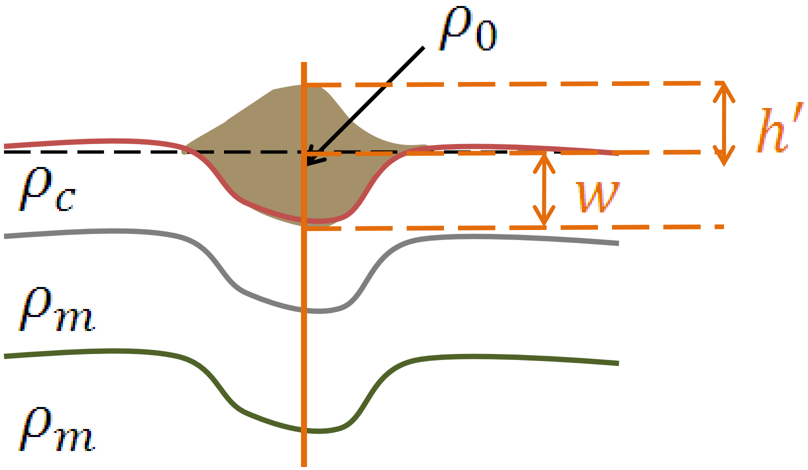

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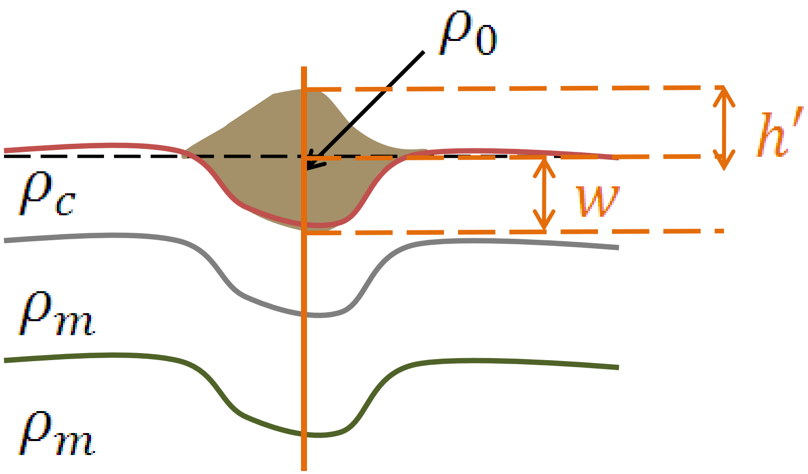

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$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_0) g w = \rho_0 g h'$$

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Aplicação para a litosfera

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Superfície

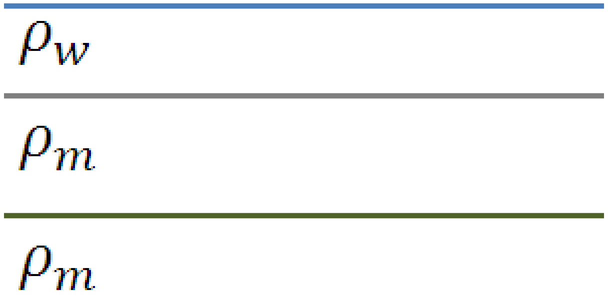
Água

Litosfera oceânica

Astenosfera

Aplicação para a litosfera

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)=q(x)$$



ρ_w

ρ_m

ρ_m

Aplicação para a litosfera

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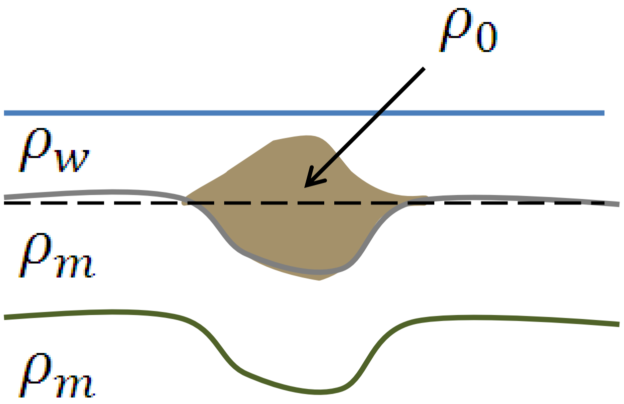

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Aplicação para a litosfera

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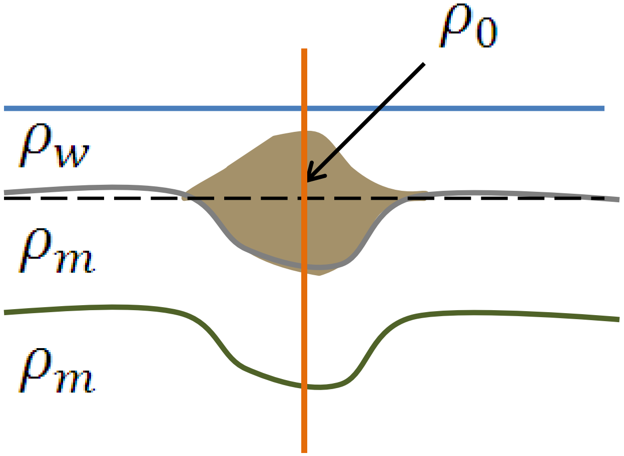

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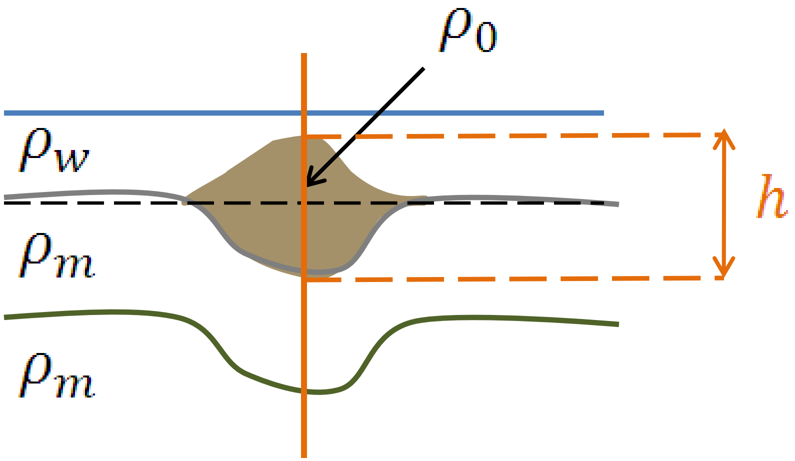

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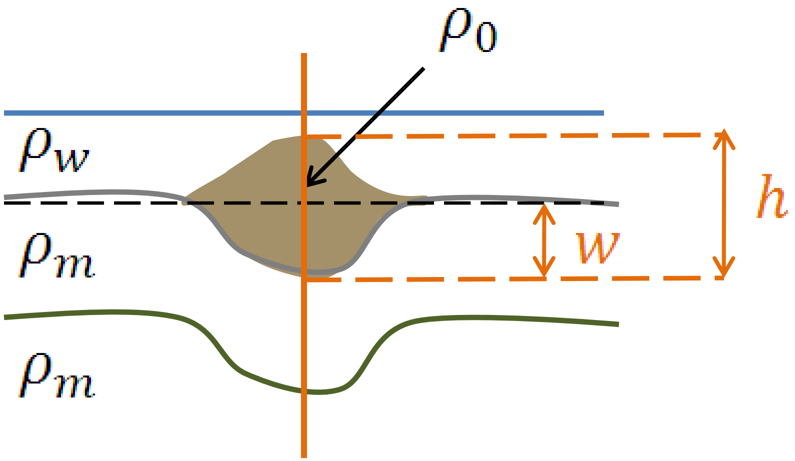

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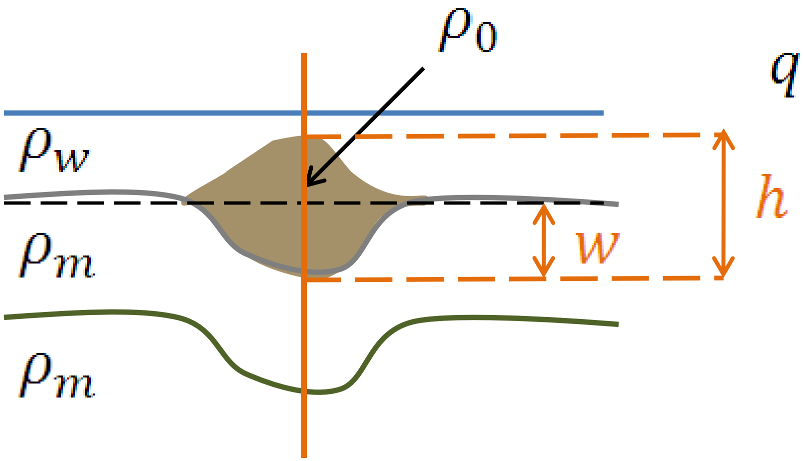
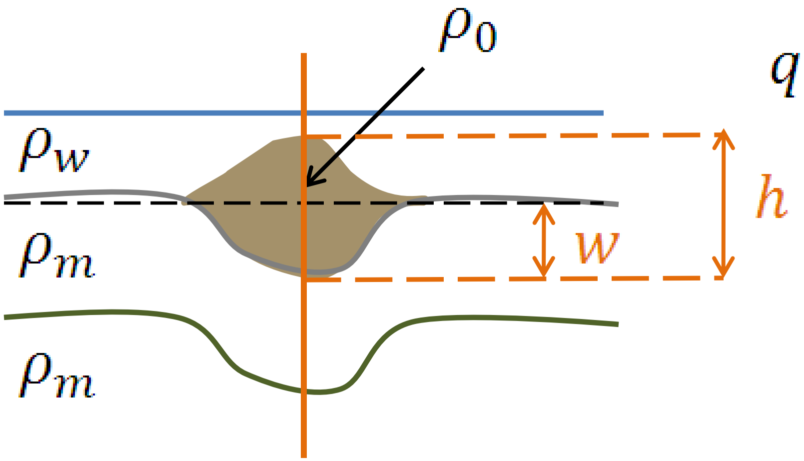


$$q(x) = \text{car.} - \text{água desloc.} - \text{empuxo}$$

$$q(x) = \rho_0 gh - \rho_w g(h-w) - \rho_m gw$$

Aplicação para a litosfera

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) = q(x)$$



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Aplicação para a litosfera

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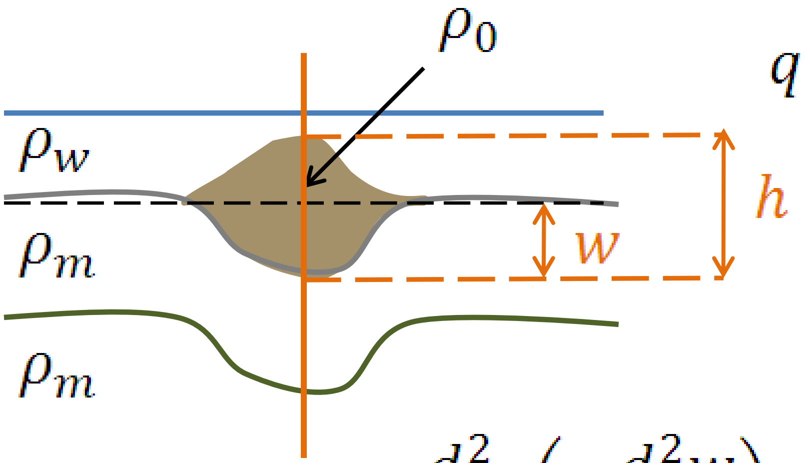

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_w)gw = (\rho_0 - \rho_w)gh$$

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Aplicação para a litosfera

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) = q(x)$$

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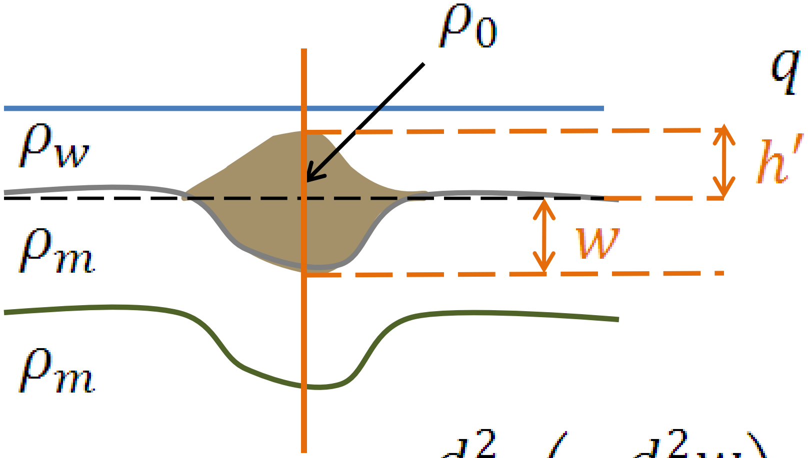

$$q(x) = \rho_0 g(h' + w) - \rho_w gh' - \rho_m gw$$

$$= (\rho_0 - \rho_w)gh' - (\rho_m - \rho_0)gw$$

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_0)gw = (\rho_0 - \rho_w)gh'$$

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Comparação

Caso continental: $\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_0)gw = \rho_0 gh'$

Caso oceânico I: $\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_w)gw = (\rho_0 - \rho_w)gh$

Caso oceânico II: $\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_0)gw = (\rho_0 - \rho_w)gh'$

Comparação

Caso continental: $\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_0)gw = \rho_0 gh'$

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h Espessura total do carregamento

Comparação

Caso continental: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)+(\rho_m-\rho_0)gw=\rho_0 gh'$$

Caso oceânico I: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)+(\rho_m-\rho_w)gw=(\rho_0-\rho_w)gh$$

Caso oceânico II: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)+(\rho_m-\rho_0)gw=(\rho_0-\rho_w)gh'$$

h Espessura total do carregamento

h' Espessura do carregamento acima da paleotopografia/paleobatimetria

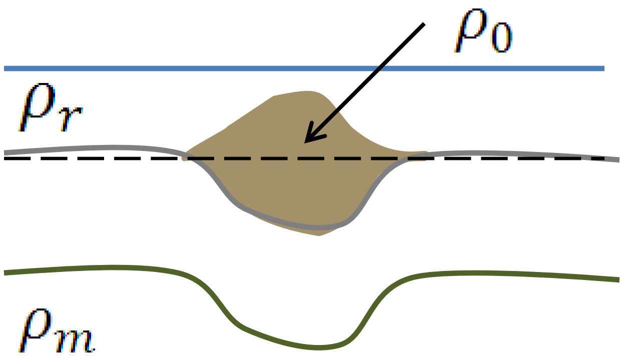
Comparação

→ Caso continental: $\dfrac{d^2}{dx^2}\left(D\dfrac{d^2w}{dx^2}\right)+(\rho_m-\rho_0)gw=\rho_0gh'$

Caso oceânico I: $\dfrac{d^2}{dx^2}\left(D\dfrac{d^2w}{dx^2}\right)+(\rho_m-\rho_w)gw=(\rho_0-\rho_w)gh$

→ Caso oceânico II: $\dfrac{d^2}{dx^2}\left(D\dfrac{d^2w}{dx^2}\right)+(\rho_m-\rho_0)gw=(\rho_0-\rho_w)gh'$

h	Espessura total do carregamento
h'	Espessura do carregamento acima da paleotopografia/paleobatimetria



Comparação

$\rho_m - \rho_0$

→ Caso continental: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + \boxed{(\rho_m - \rho_0)}gw = \rho_0 g h'$$

Caso oceânico I: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_w)gw = (\rho_0 - \rho_w)gh$$

→ Caso oceânico II: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + \boxed{(\rho_m - \rho_0)}gw = (\rho_0 - \rho_w)gh'$$

h Espessura total do carregamento

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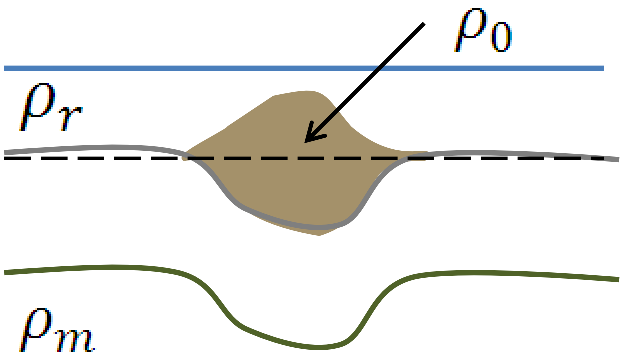

Comparação

→ Caso continental: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)+\overset{\rho_m-\rho_0}{\boxed{(\rho_m-\rho_0)}}gw=\overset{\rho_0-\rho_r}{\boxed{\rho_0}}gh'$$

Caso oceânico I: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)+(\rho_m-\rho_w)gw=(\rho_0-\rho_w)gh$$

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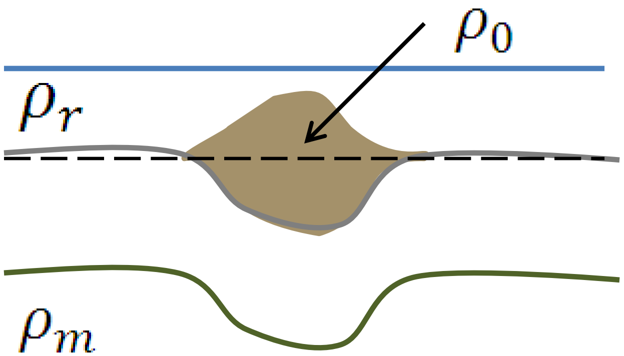

Comparação

Caso continental: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)+(\rho_m-\rho_0)gw=\rho_0gh'$$

→Caso oceânico I: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)+(\rho_m-\rho_w)gw=(\rho_0-\rho_w)gh$$

Caso oceânico II: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)+(\rho_m-\rho_0)gw=(\rho_0-\rho_w)gh'$$

$\boxed{h}$ Espessura total do carregamento

h' Espessura do carregamento acima da paleotopografia/paleobatimetria

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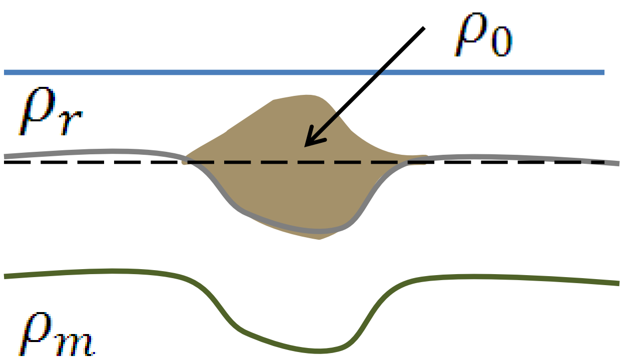
Comparação



Caso continental: $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right)+(\rho_m-\rho_0)gw=\rho_0gh'$$

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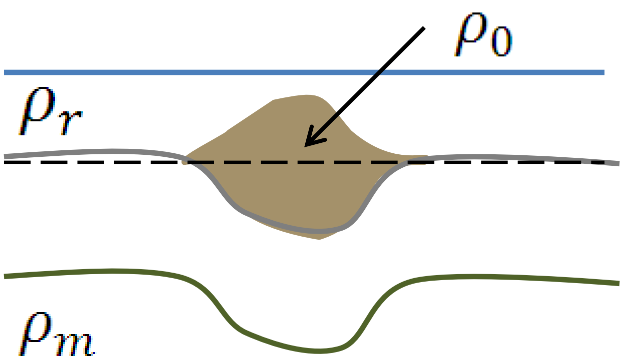

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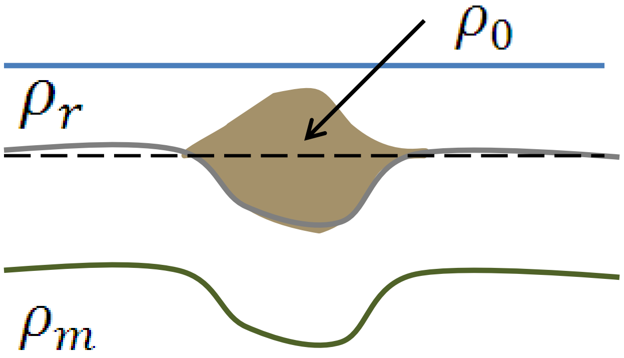

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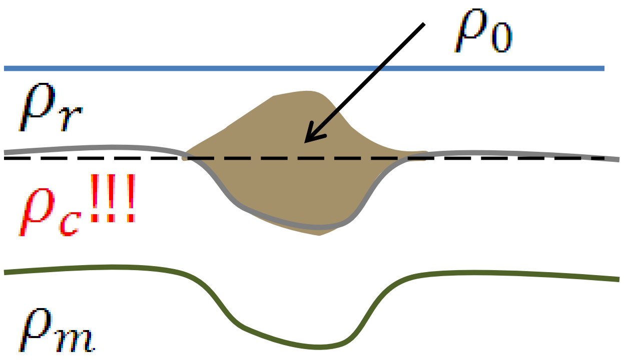

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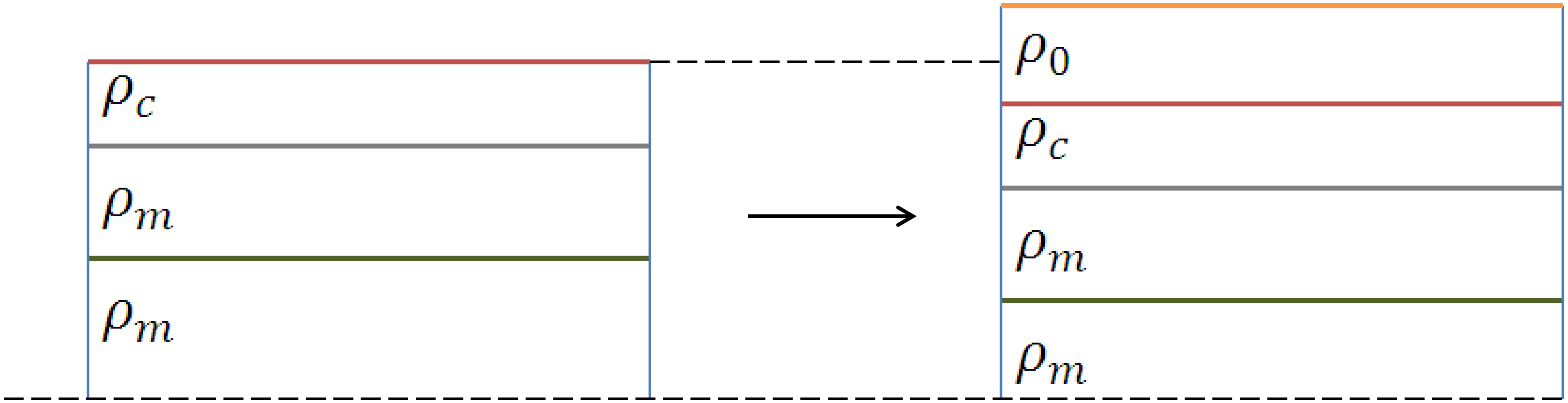
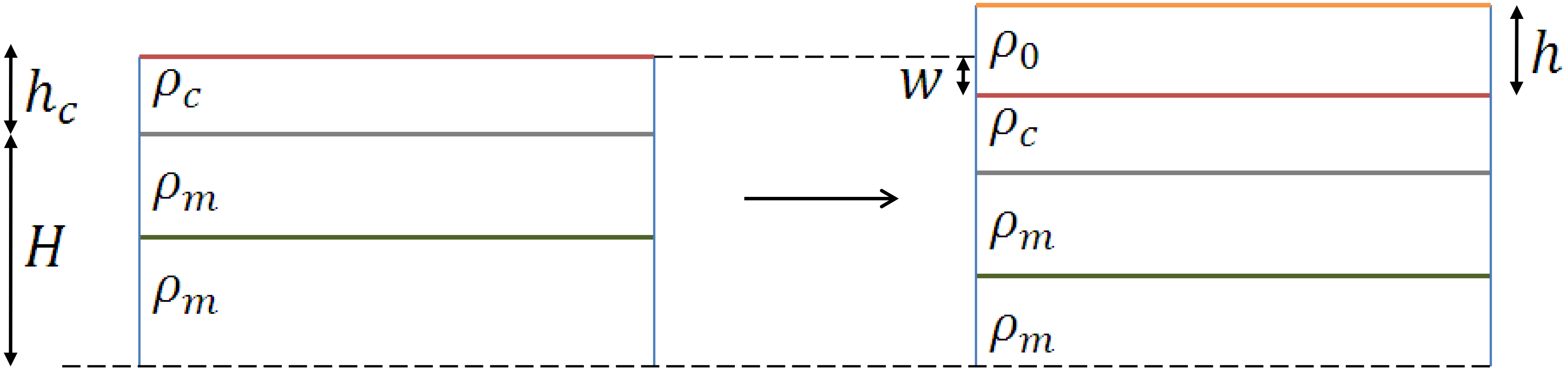
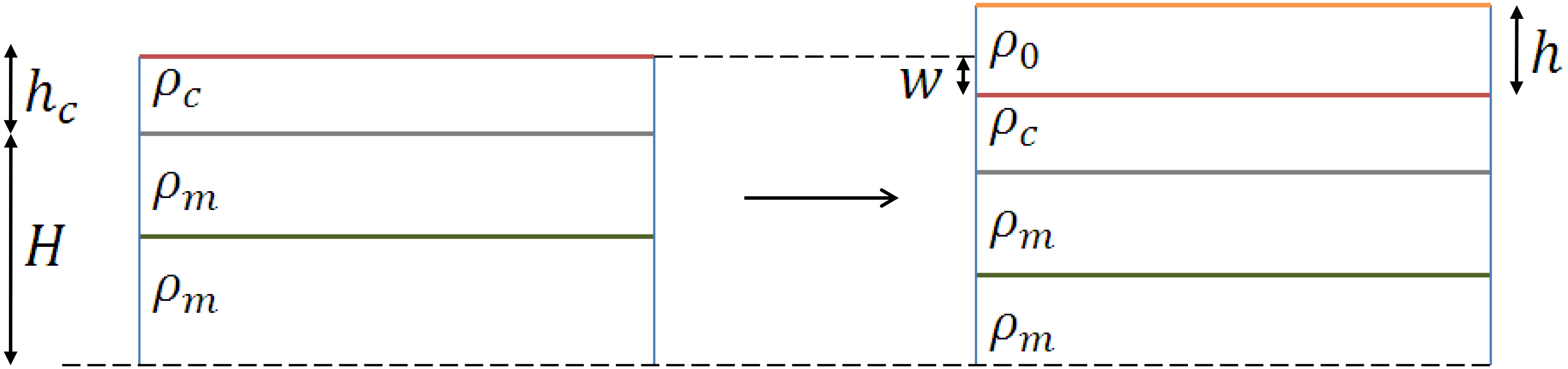
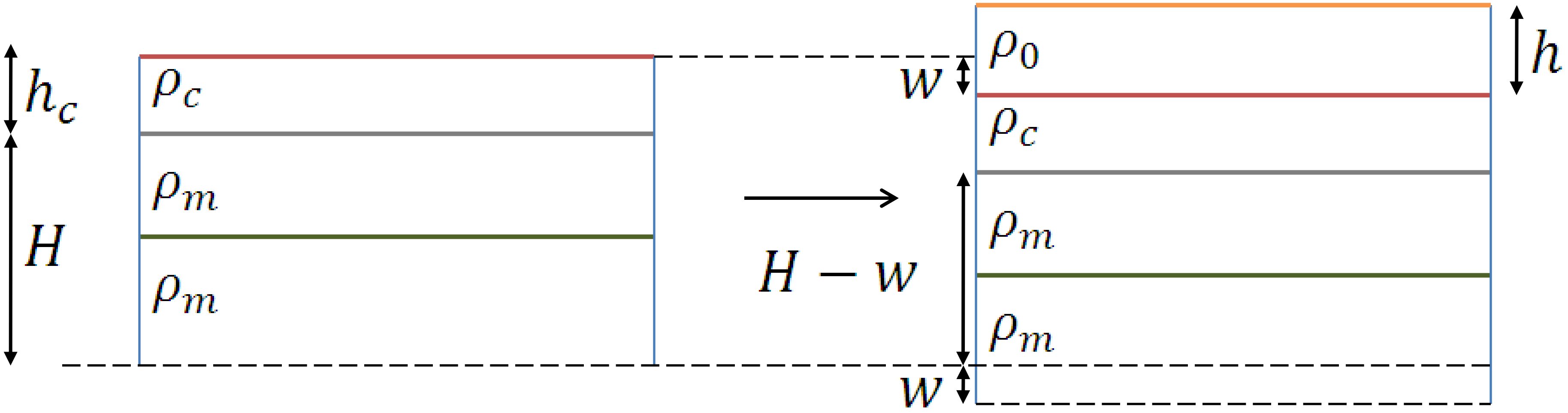

ρ_c
ρ_m
ρ_m
ρ_0
ρ_c
ρ_m
ρ_m

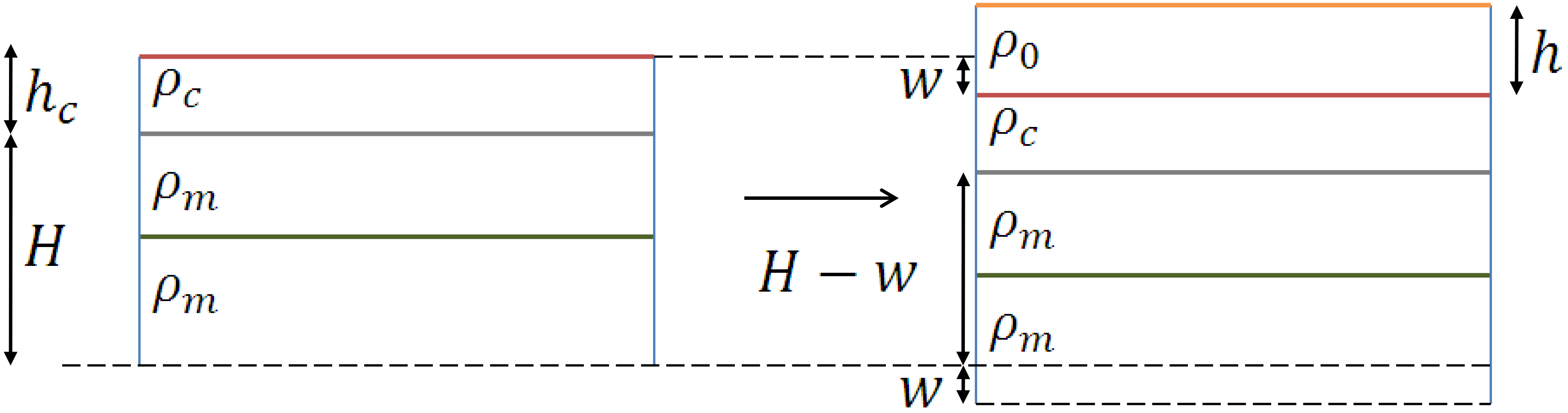

h_c
H
ρ_c
ρ_m
ρ_m
w
ρ_0
ρ_c
ρ_m
ρ_m
h



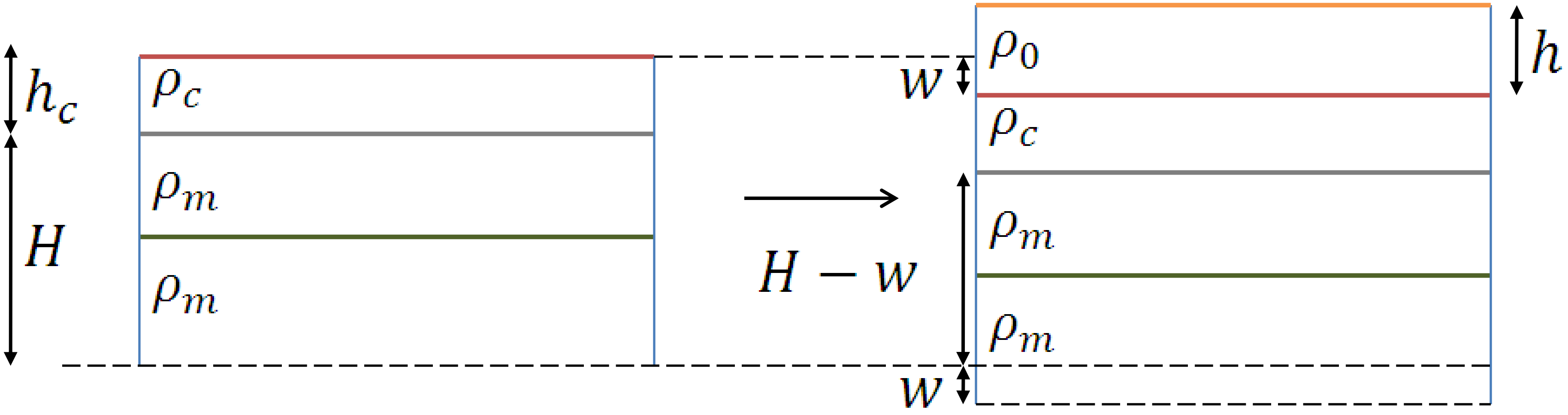
$$\rho_c g h_c + \rho_m g H =$$



$$\rho_c g h_c + \rho_m g H = \rho_0 g h + \rho_c g h_c + \rho_m g (H - w)$$



$$\cancel{\rho_c g h_c} + \rho_m g H = \rho_0 g h + \cancel{\rho_c g h_c} + \rho_m g(H - w)$$



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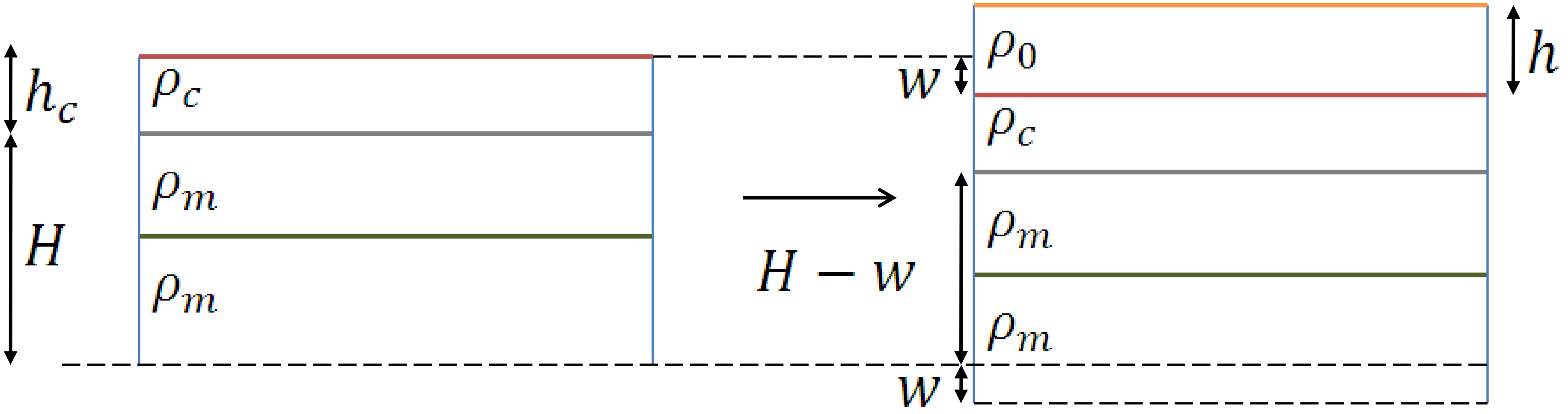
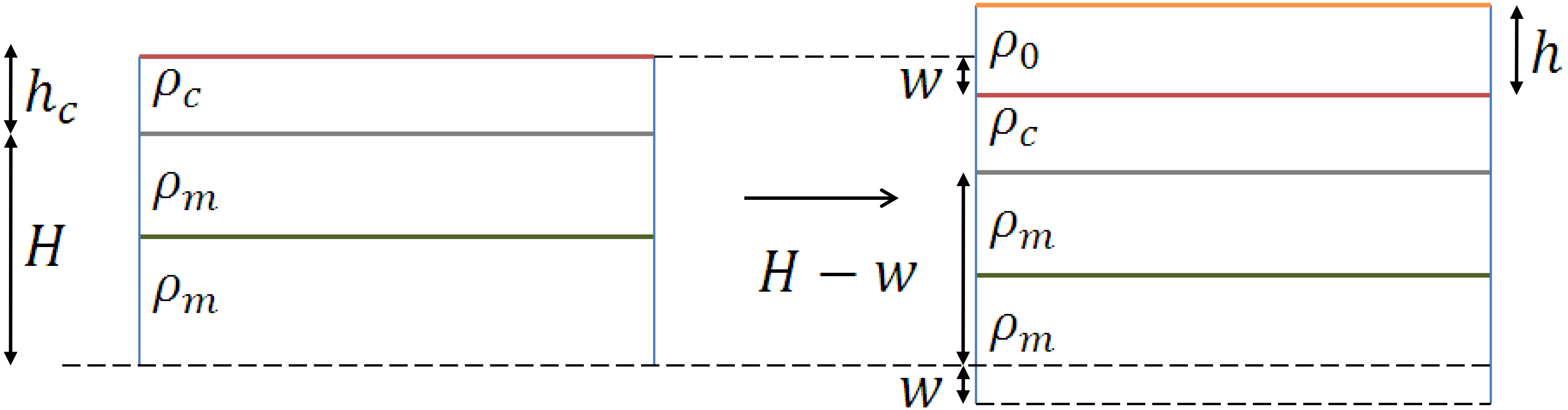


$$\cancel{\rho_c g h_c} + \cancel{\rho_m g H} = \rho_0 g h + \cancel{\rho_c g h_c} + \rho_m g(\cancel{H} - w)$$

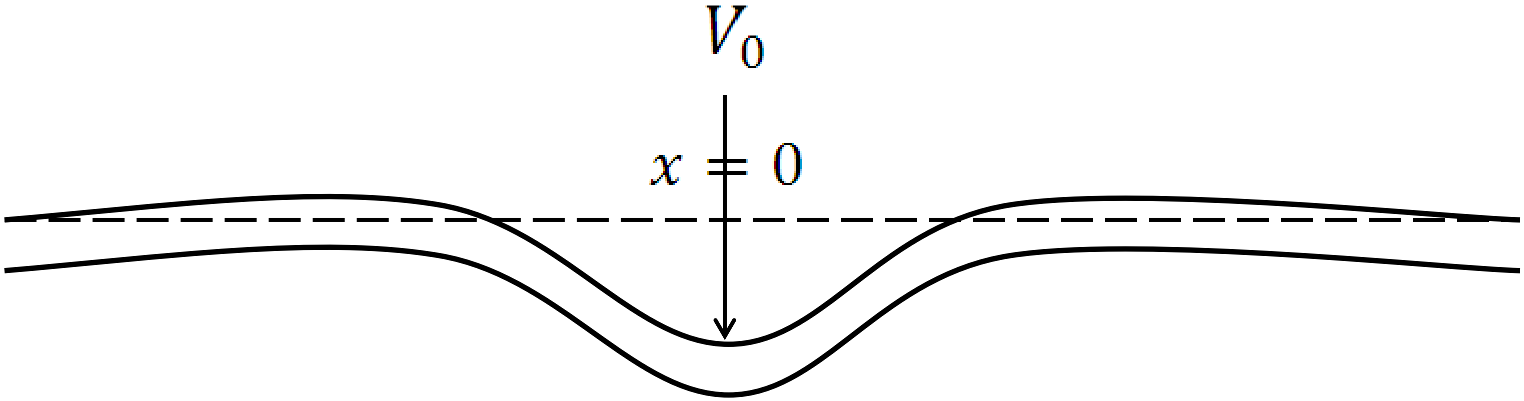
$$0 = \rho_0 g h - \rho_m g w$$



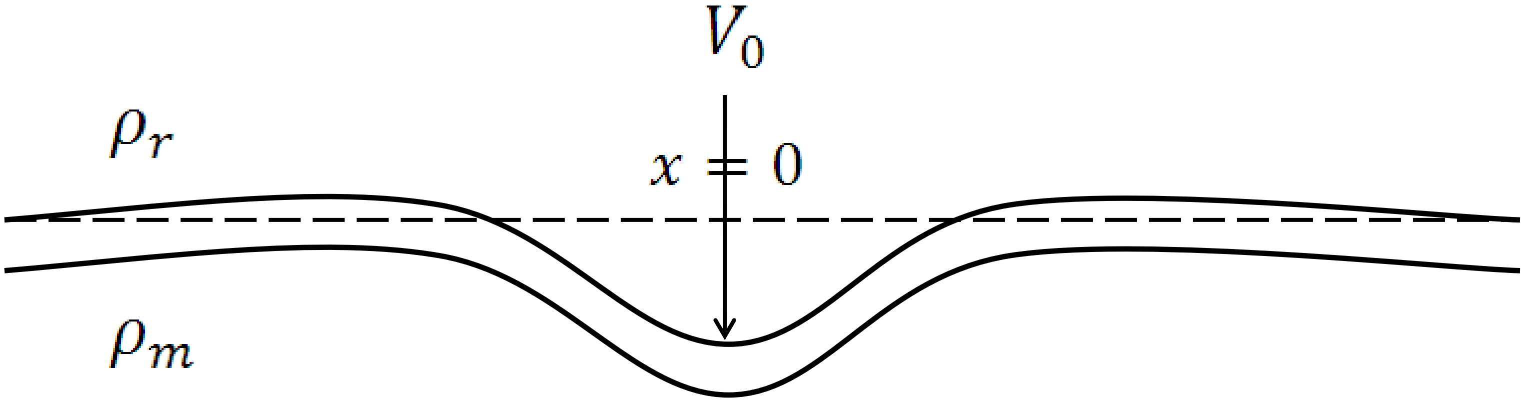
$$\cancel{\rho_c g h_c} + \cancel{\rho_m g H} = \rho_0 g h + \cancel{\rho_c g h_c} + \rho_m g(\cancel{H} - w)$$

$$0 = \enclose{circle}{\rho_0} g h - \enclose{circle}{\rho_m} g w$$

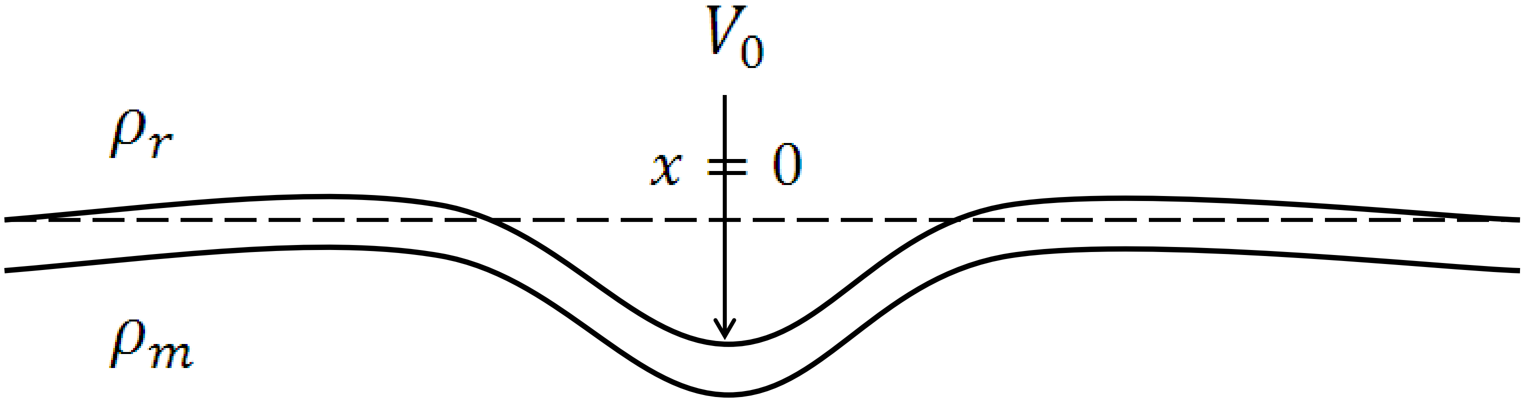
Soluções Analíticas



Soluções Analíticas



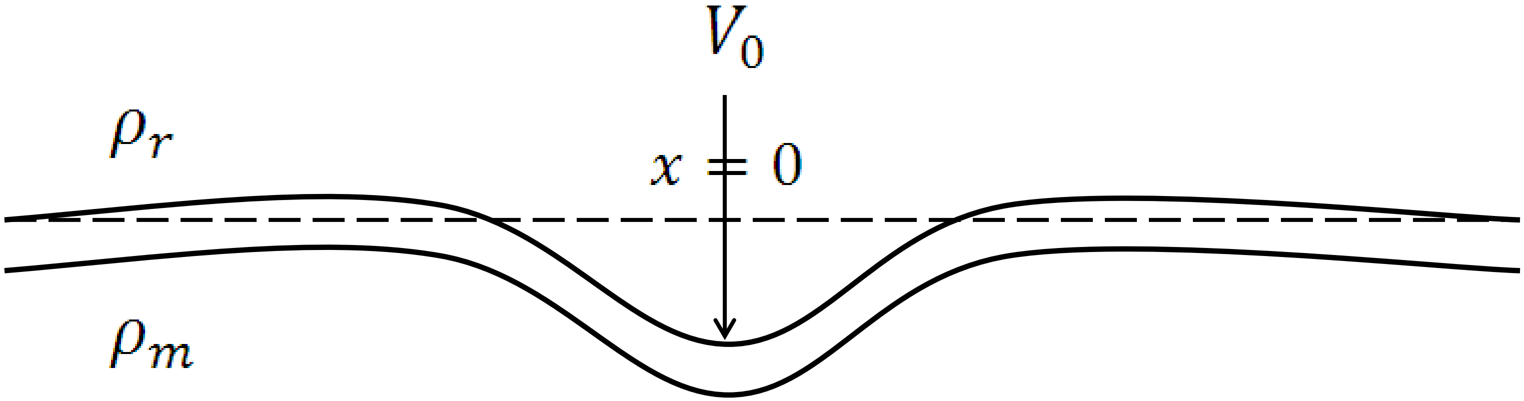
Soluções Analíticas



Para $x \neq 0$

$$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_r)gw = 0$$

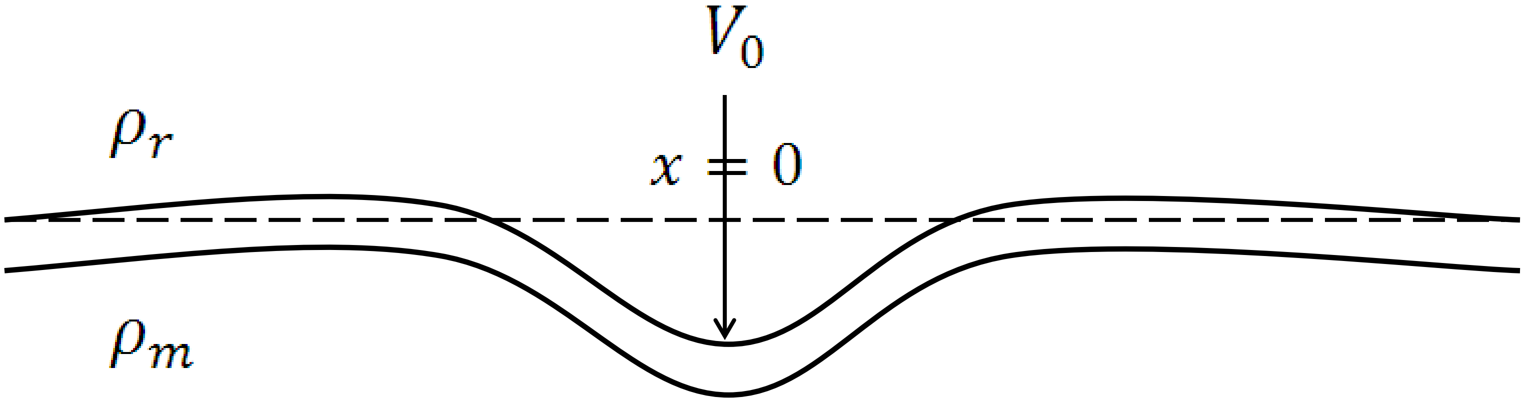
Soluções Analíticas



Para $x \neq 0$ $$\frac{d^2}{dx^2}\left(D\frac{d^2w}{dx^2}\right) + (\rho_m - \rho_r)gw = 0$$

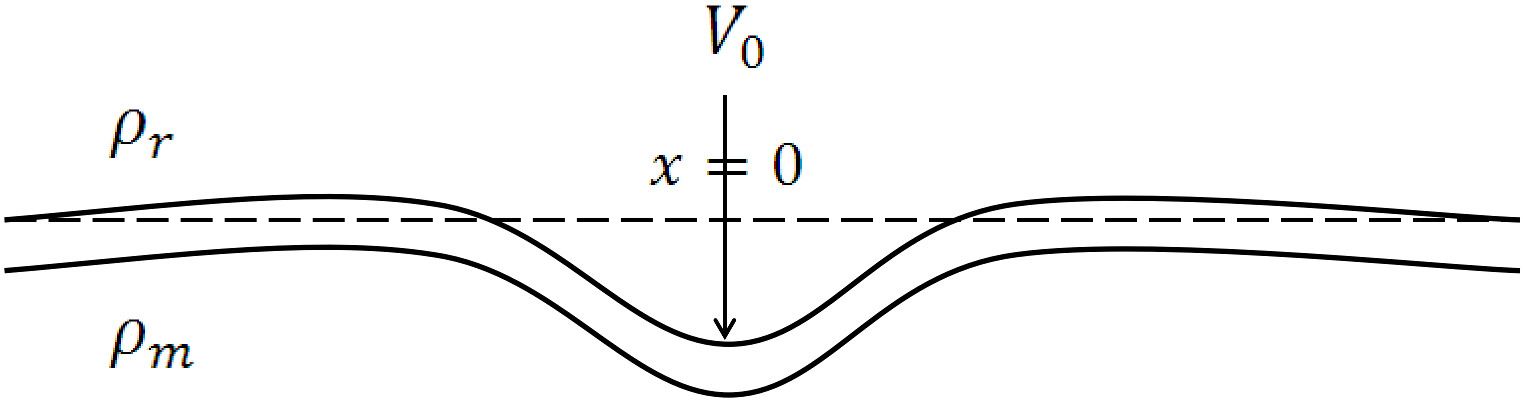
Para D constante $$D\frac{d^4w}{dx^4} + (\rho_m - \rho_r)gw = 0$$

Soluções Analíticas



$$D\frac{d^4w}{dx^4} + (\rho_m - \rho_r)gw = 0$$

Soluções Analíticas

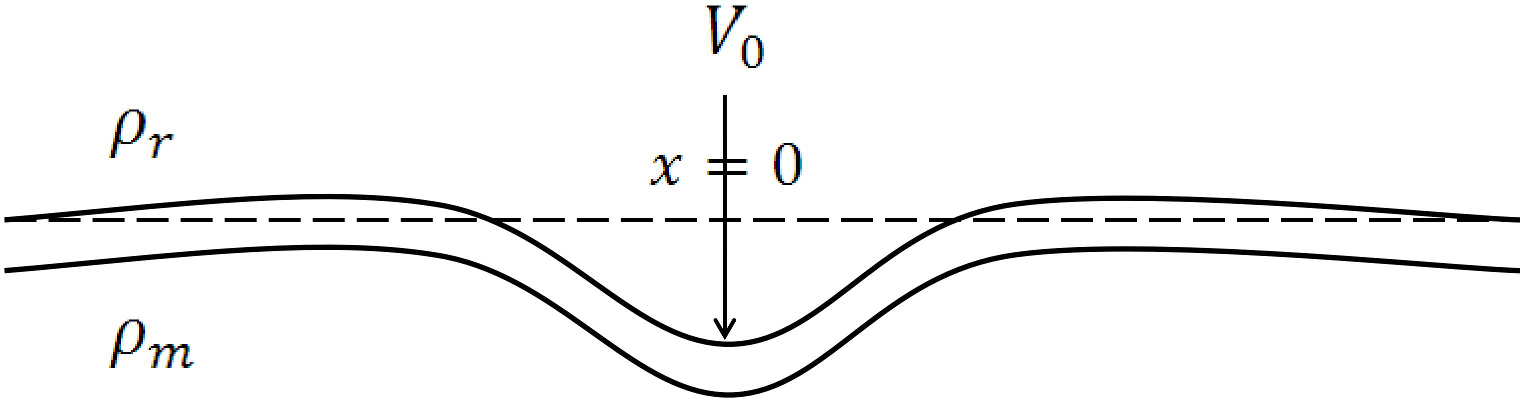


$$D\frac{d^4w}{dx^4} + (\rho_m - \rho_r)gw = 0$$

$$w = e^{\lambda x}(A_c \cos \lambda x + B_c \sin \lambda x) + e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x)$$

$$\lambda = \left[\frac{(\rho_m - \rho_r)g}{4D}\right]^{1/4}$$

Soluções Analíticas



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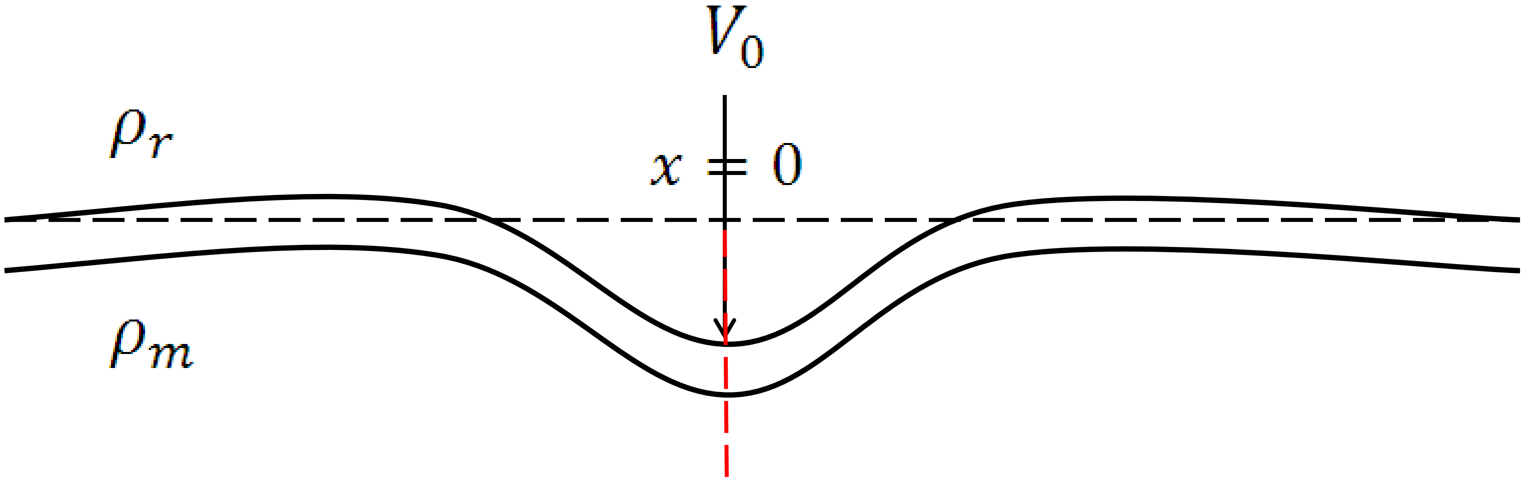
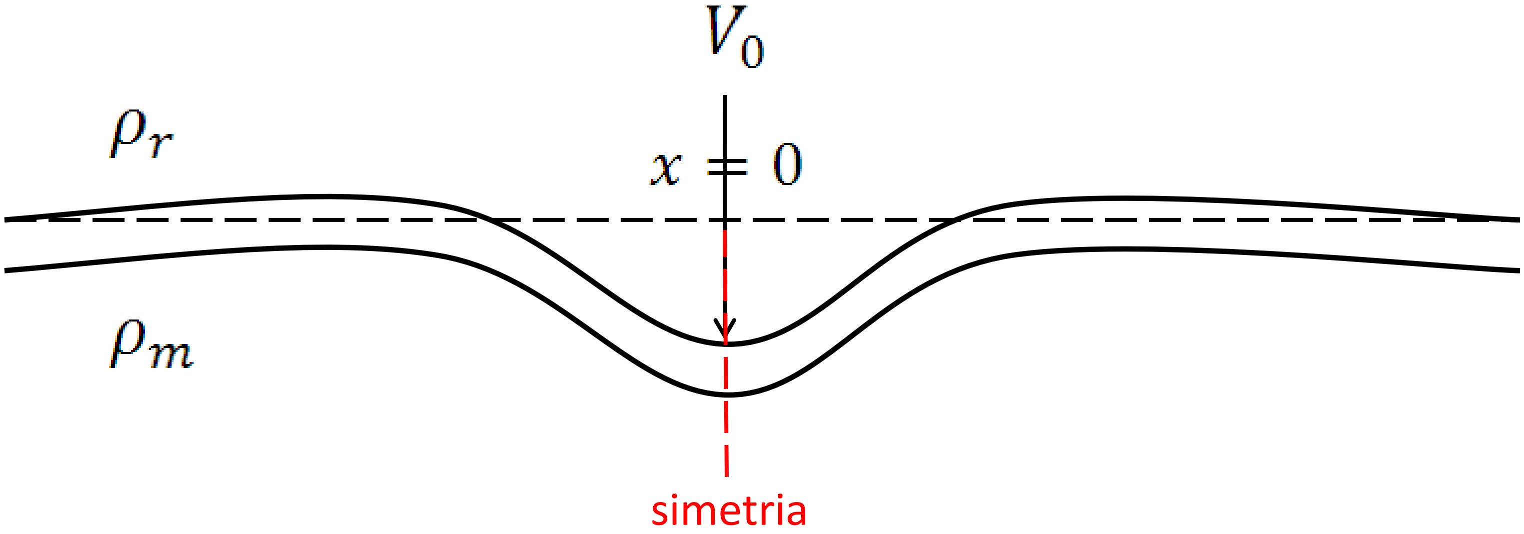
Soluções Analíticas



$$w = e^{\lambda x}(A_c \cos \lambda x + B_c \sin \lambda x) + e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x)$$

Soluções Analíticas



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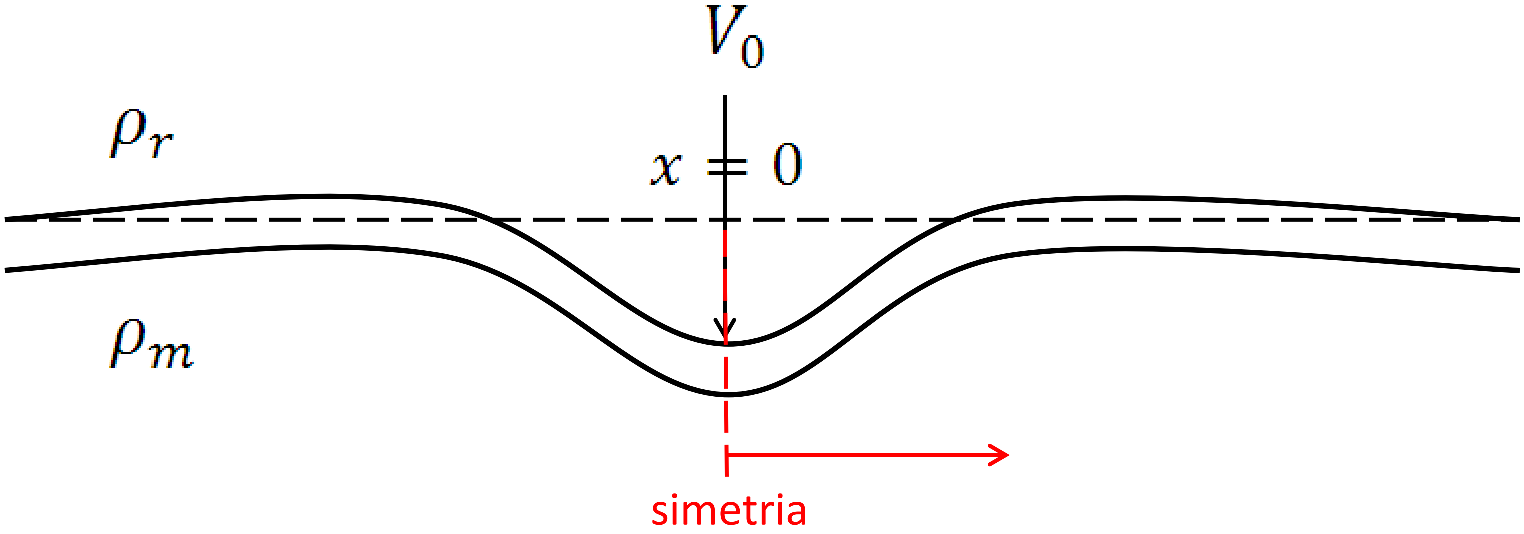
Soluções Analíticas



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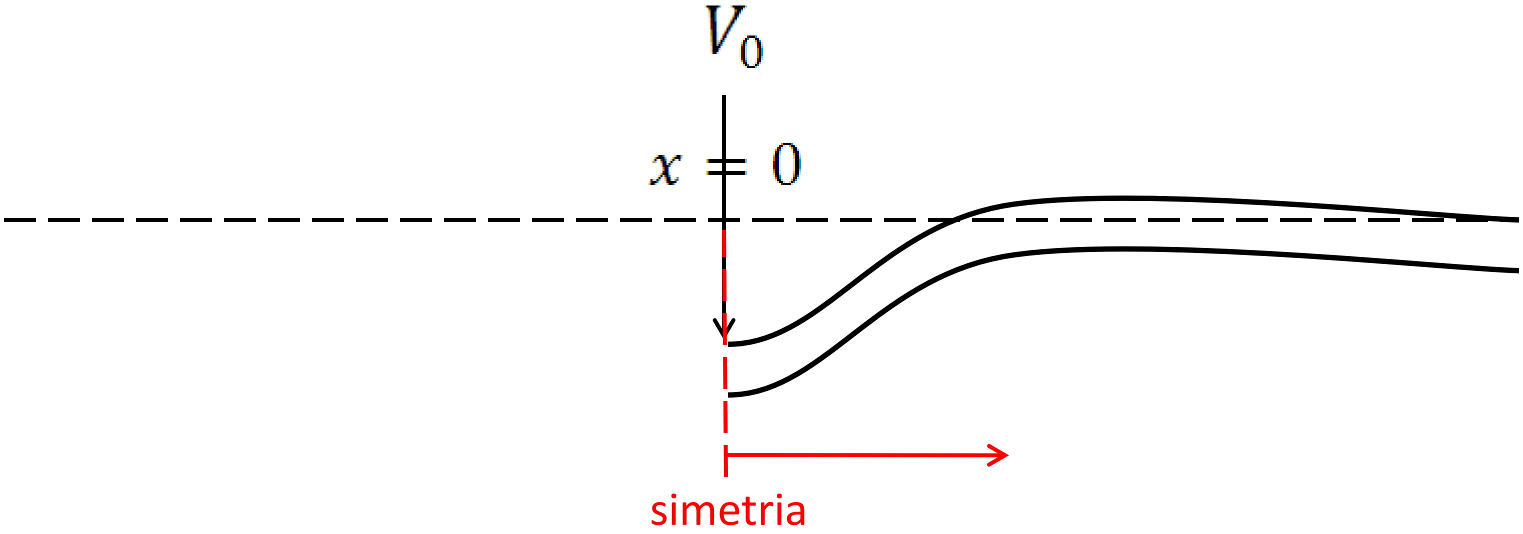
Soluções Analíticas



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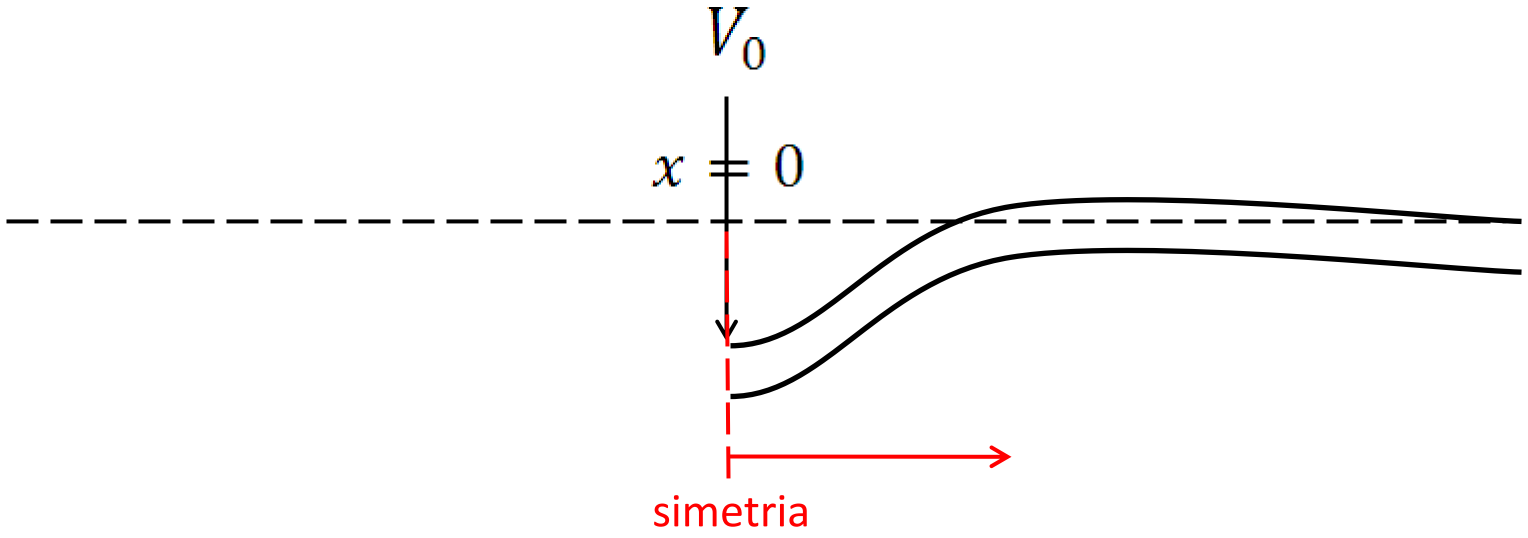
Para $x \to +\infty$

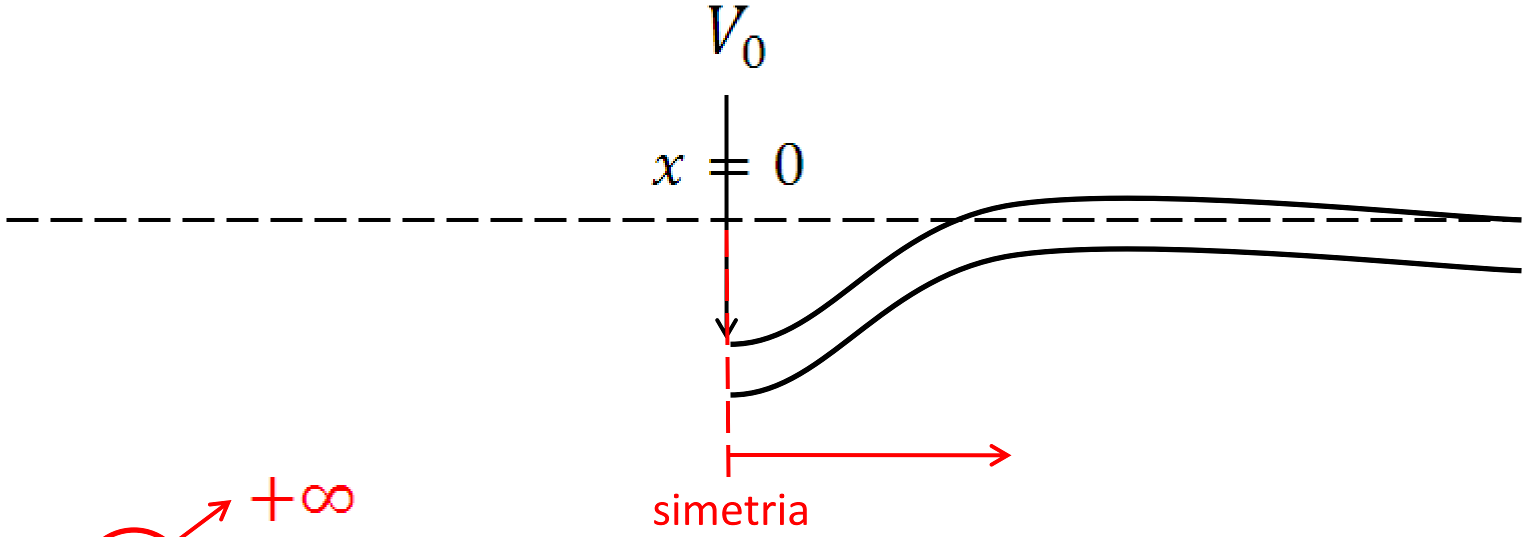
Soluções Analíticas



$$w = e^{\lambda x}(A_c \cos \lambda x + B_c \sin \lambda x) + e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x)$$

Para $x \rightarrow +\infty$ $\quad w \rightarrow 0$

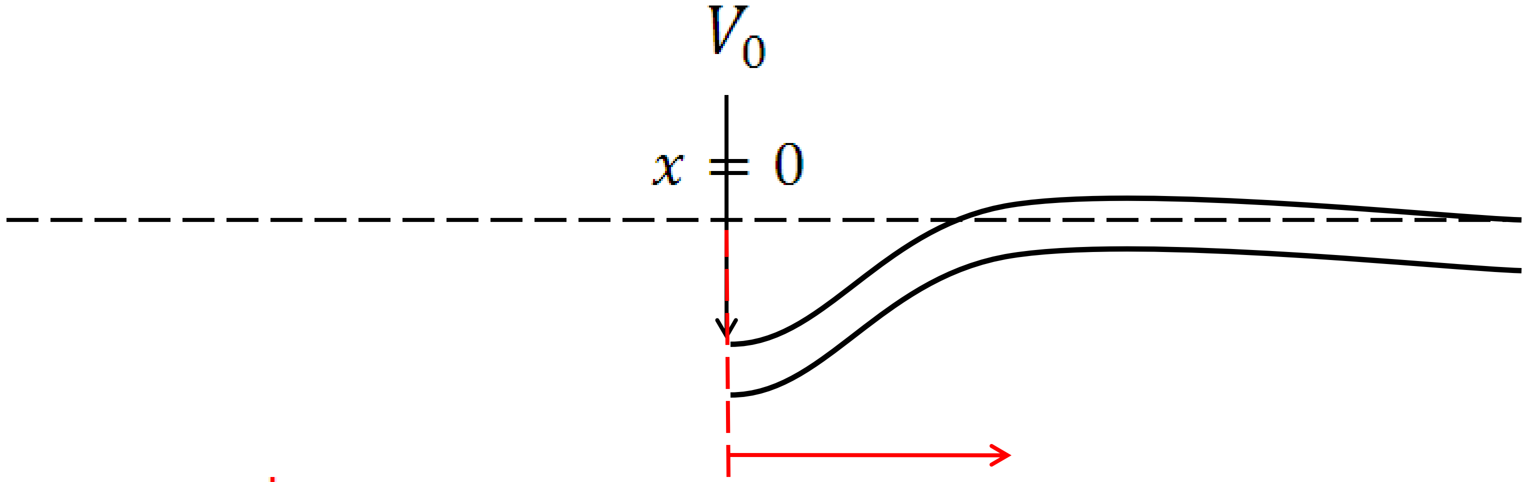
Soluções Analíticas



$$w = \overset{+\infty}{\boxed{e^{\lambda x}}}(A_c \cos \lambda x + B_c \sin \lambda x) + e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x)$$

Para $x \rightarrow +\infty \qquad w \rightarrow 0$

Soluções Analíticas



$$w = \overset{+\infty}{\boxed{e^{\lambda x}}}(A_c \cos \lambda x + B_c \sin \lambda x) + \overset{0}{\boxed{e^{-\lambda x}}}(C_c \cos \lambda x + D_c \sin \lambda x)$$

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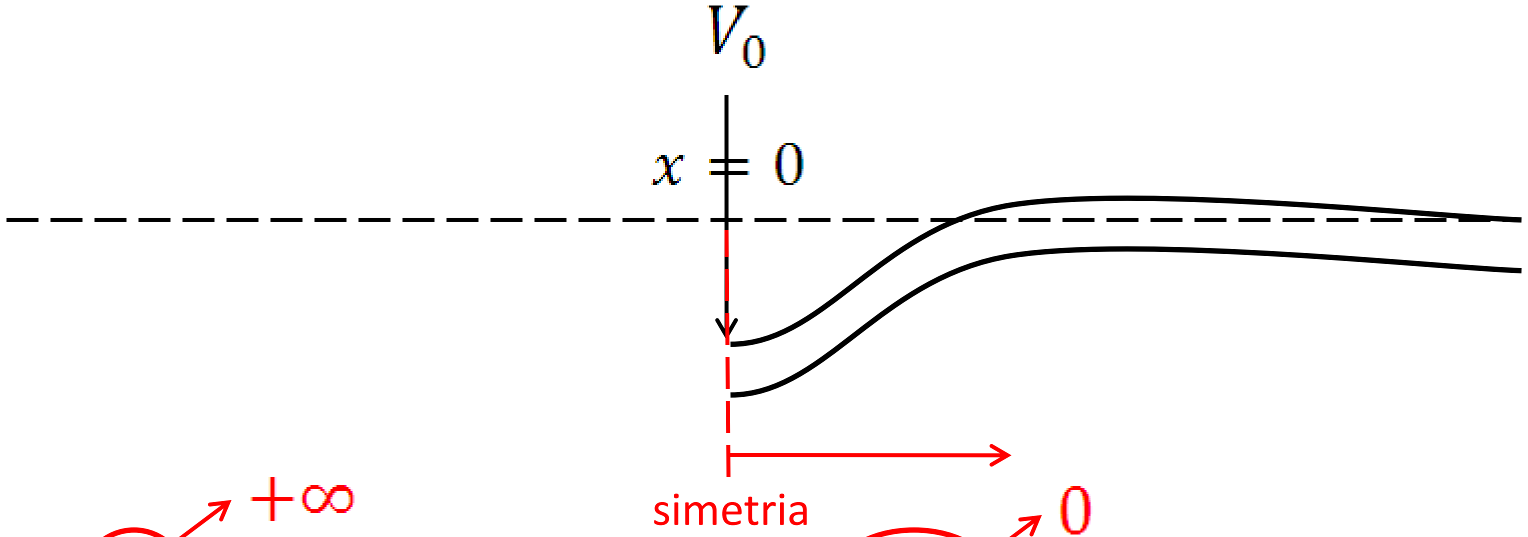
Soluções Analíticas

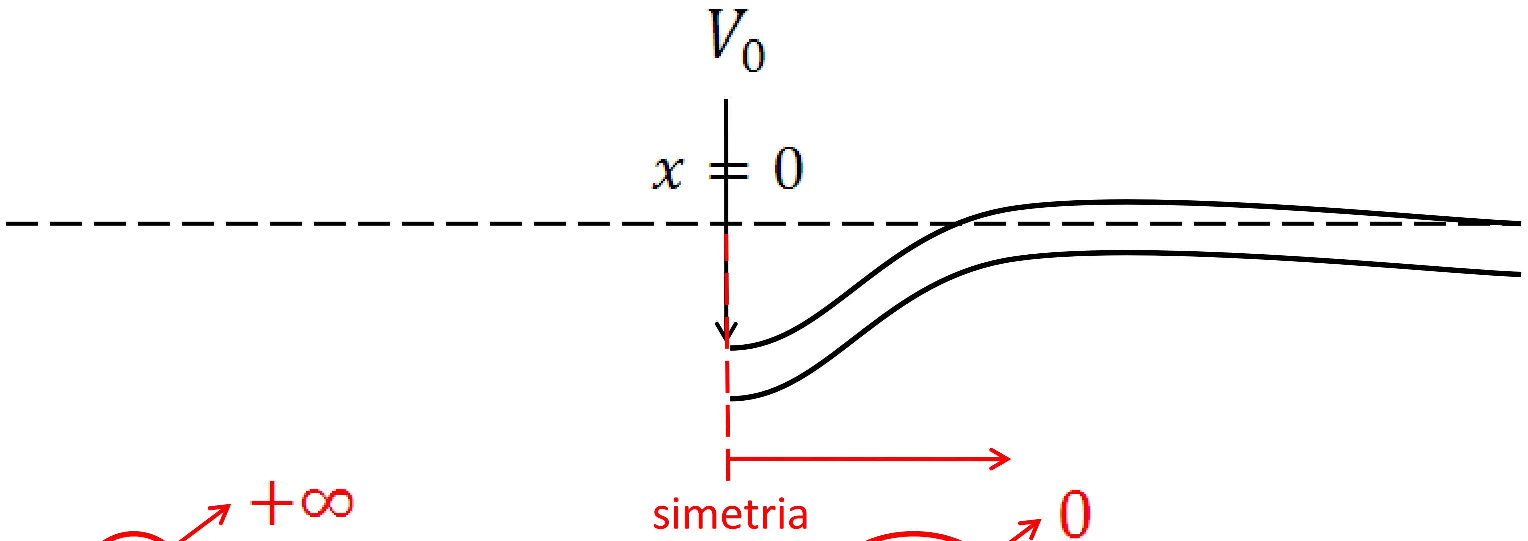


$$w = \overset{+\infty}{\boxed{e^{\lambda x}}}(A_c \cos \lambda x + B_c \sin \lambda x) + \overset{0}{\boxed{e^{-\lambda x}}}(C_c \cos \lambda x + D_c \sin \lambda x)$$

Para $x \to +\infty \qquad w \to 0$

$$A_c = 0, B_c = 0$$

Soluções Analíticas



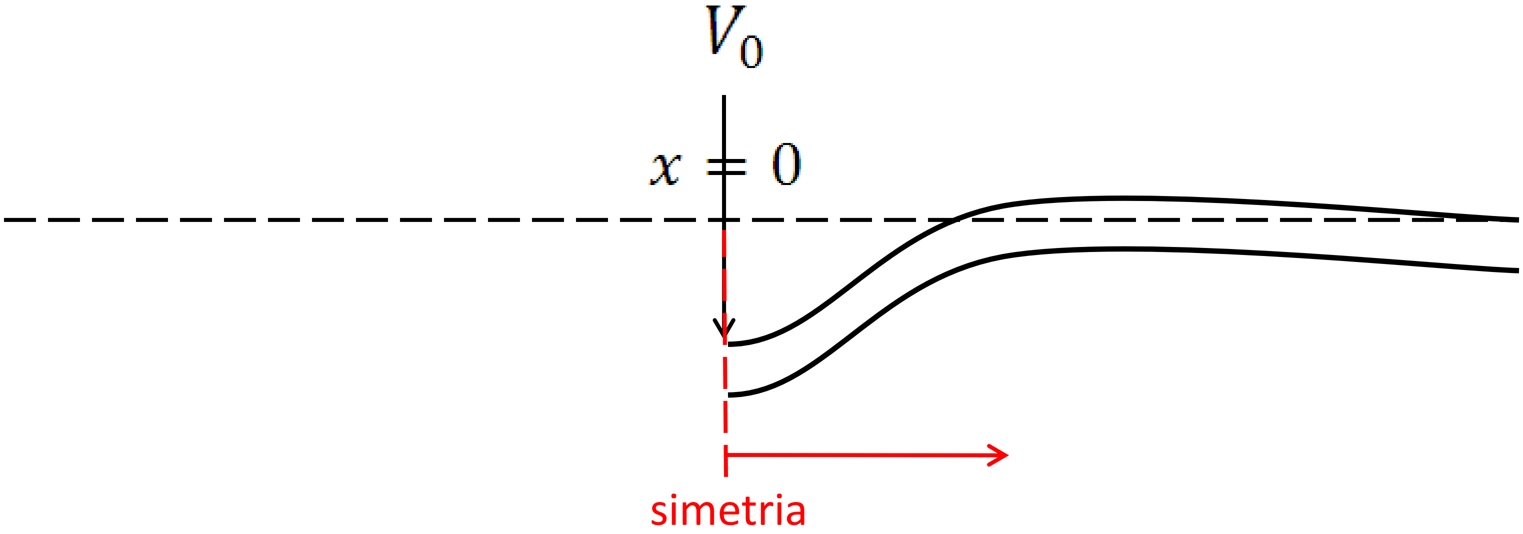
$$w = \overset{\nearrow +\infty}{e^{\lambda x}}(A_c \cos \lambda x + B_c \sin \lambda x) + \overset{\nearrow 0}{e^{-\lambda x}}(C_c \cos \lambda x + D_c \sin \lambda x)$$

Para $x \rightarrow +\infty \qquad w \rightarrow 0$

$$A_c = 0, B_c = 0$$

$$w = e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x)$$

Soluções Analíticas



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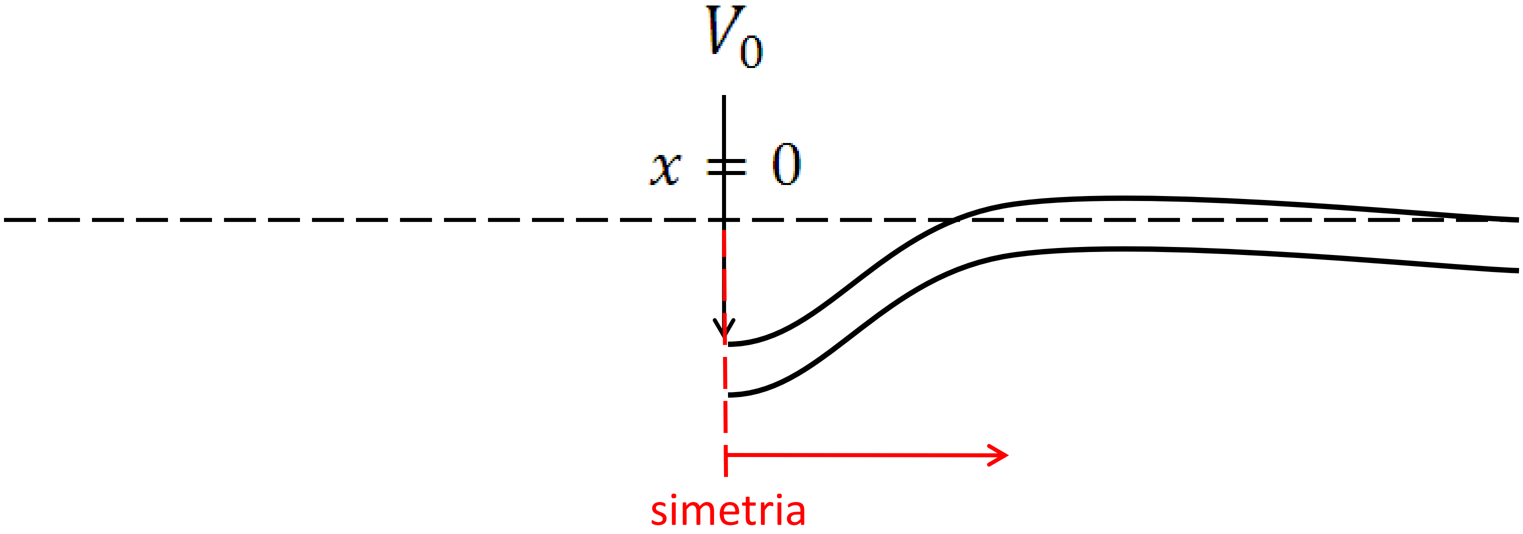
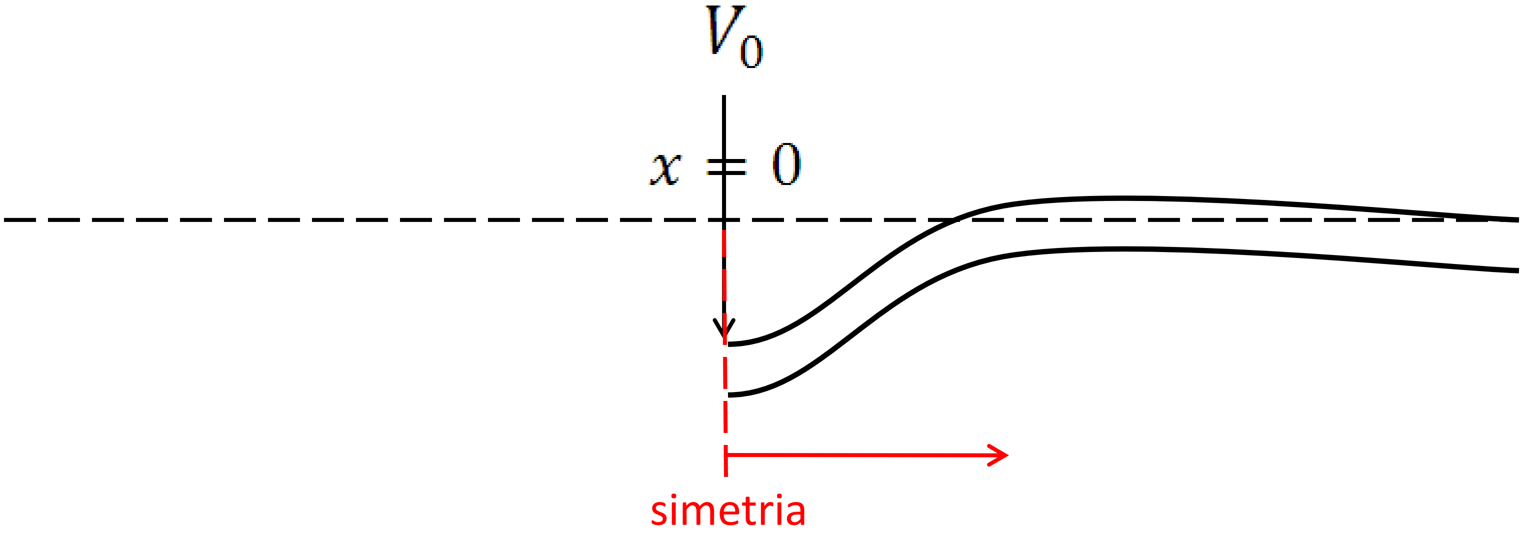
Soluções Analíticas



$w = e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x)$ Para $x \to 0$

Soluções Analíticas



$w = e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x)$ Para $x \rightarrow 0$ $\dfrac{dw}{dx} \rightarrow 0$

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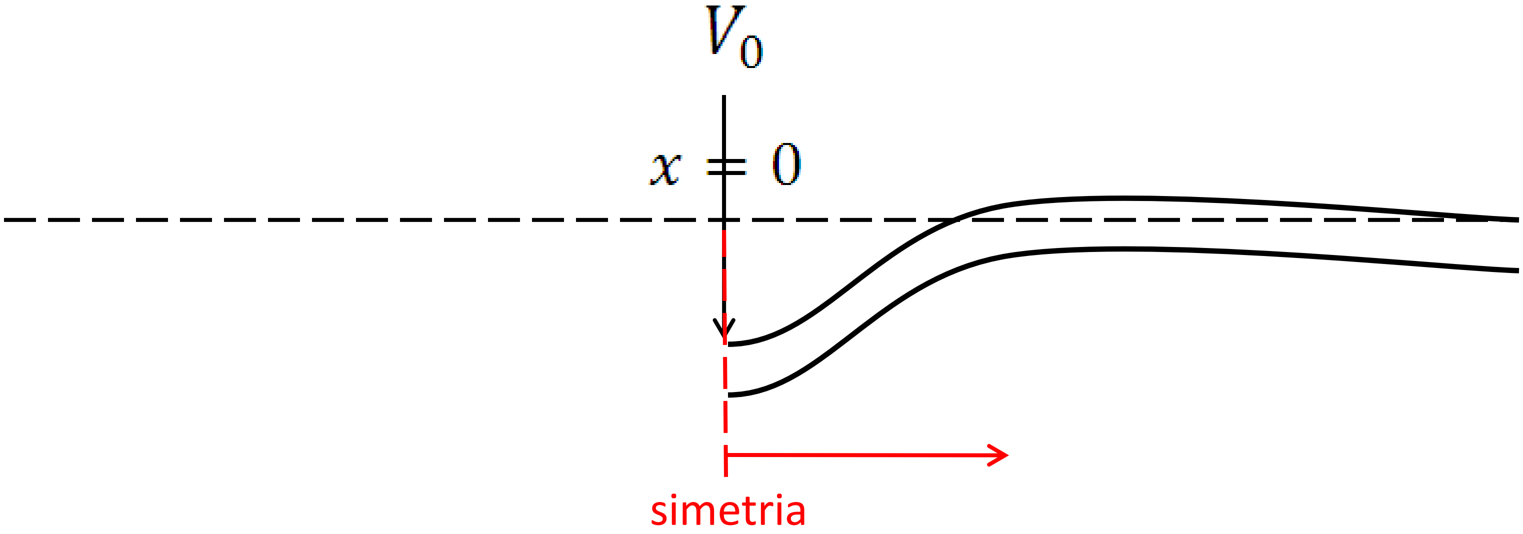
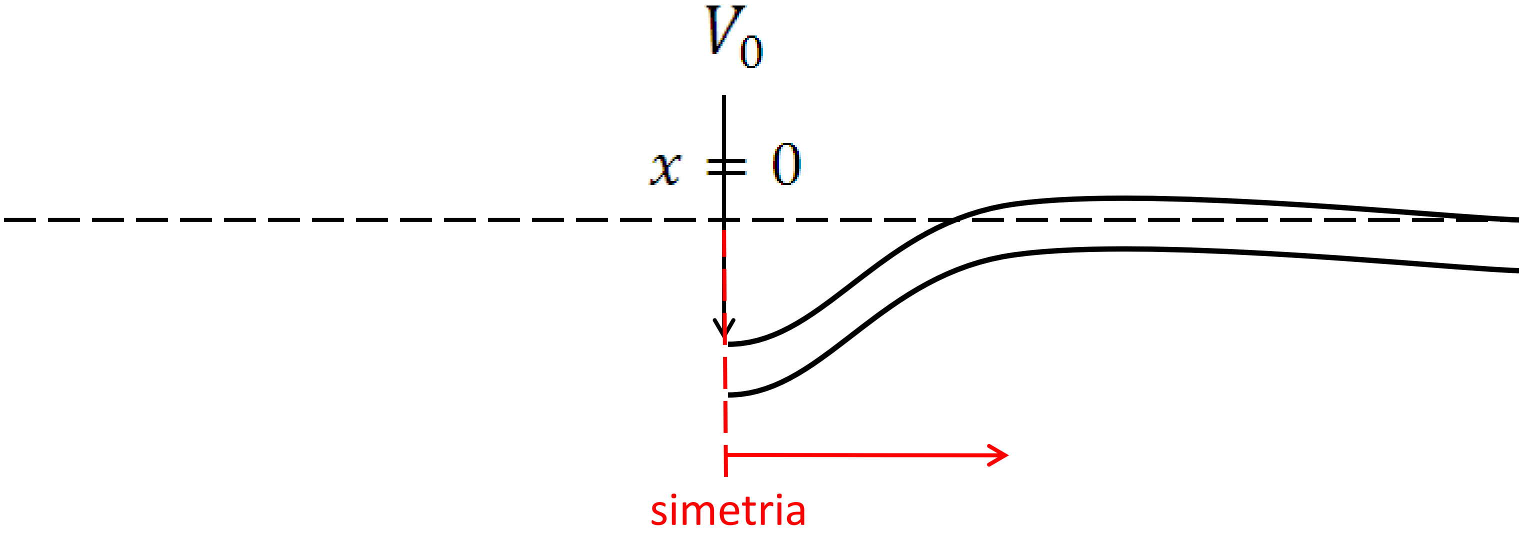
Soluções Analíticas



$$w = e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x) \qquad \text{Para } x \to 0 \qquad \frac{dw}{dx} \to 0$$

$$\frac{dw}{dx} = -\lambda e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x) + \lambda e^{-\lambda x}(-C_c \sin \lambda x + D_c \cos \lambda x)$$

Soluções Analíticas



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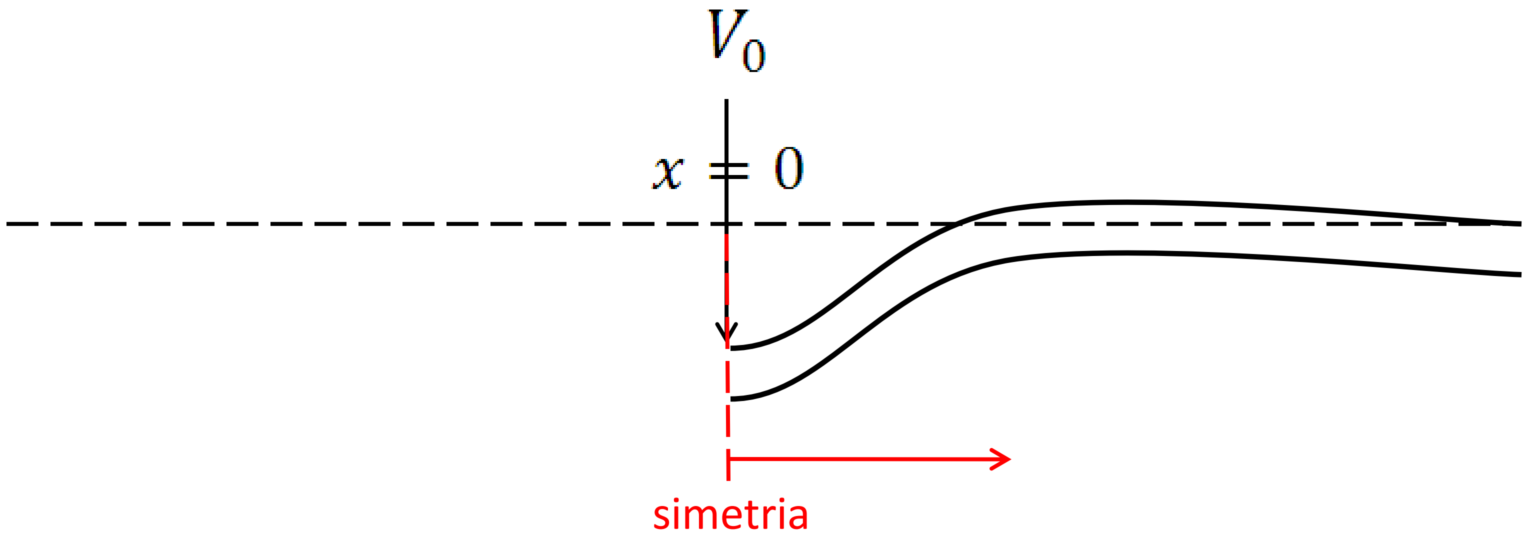
Soluções Analíticas



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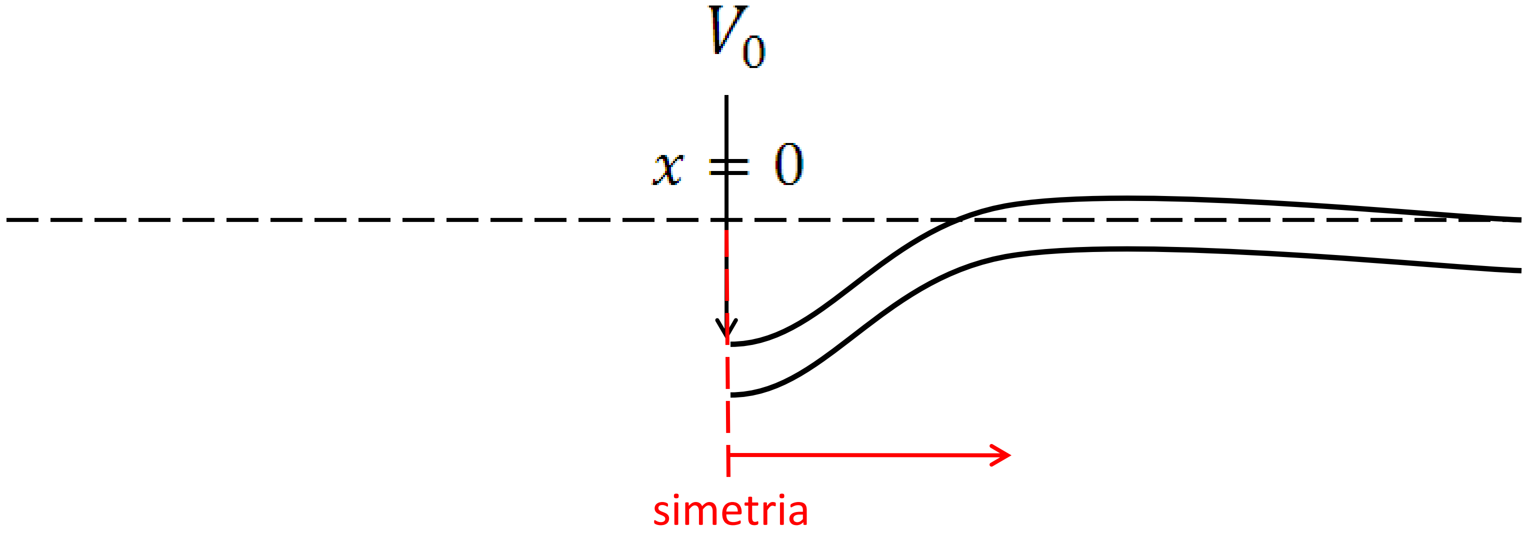
Soluções Analíticas



$$w = e^{-\lambda x}(C_c \cos\lambda x + D_c \sin\lambda x)$$

Para $x \to 0$ $\quad \frac{dw}{dx} \to 0$

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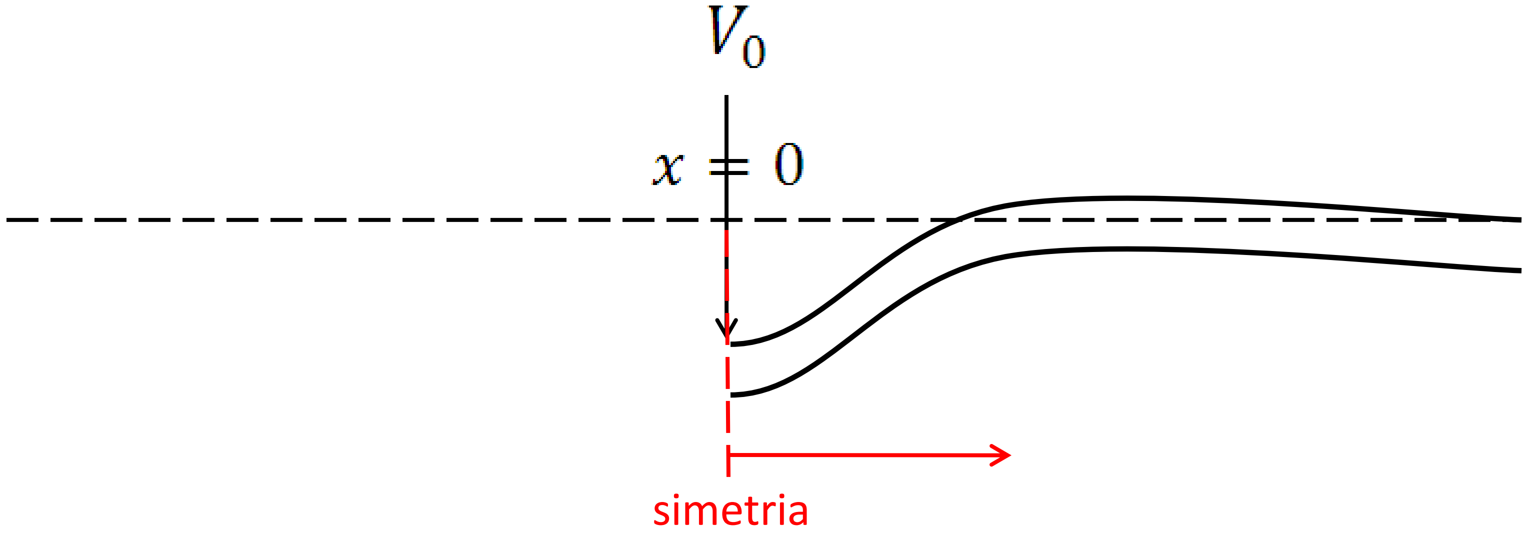
Soluções Analíticas

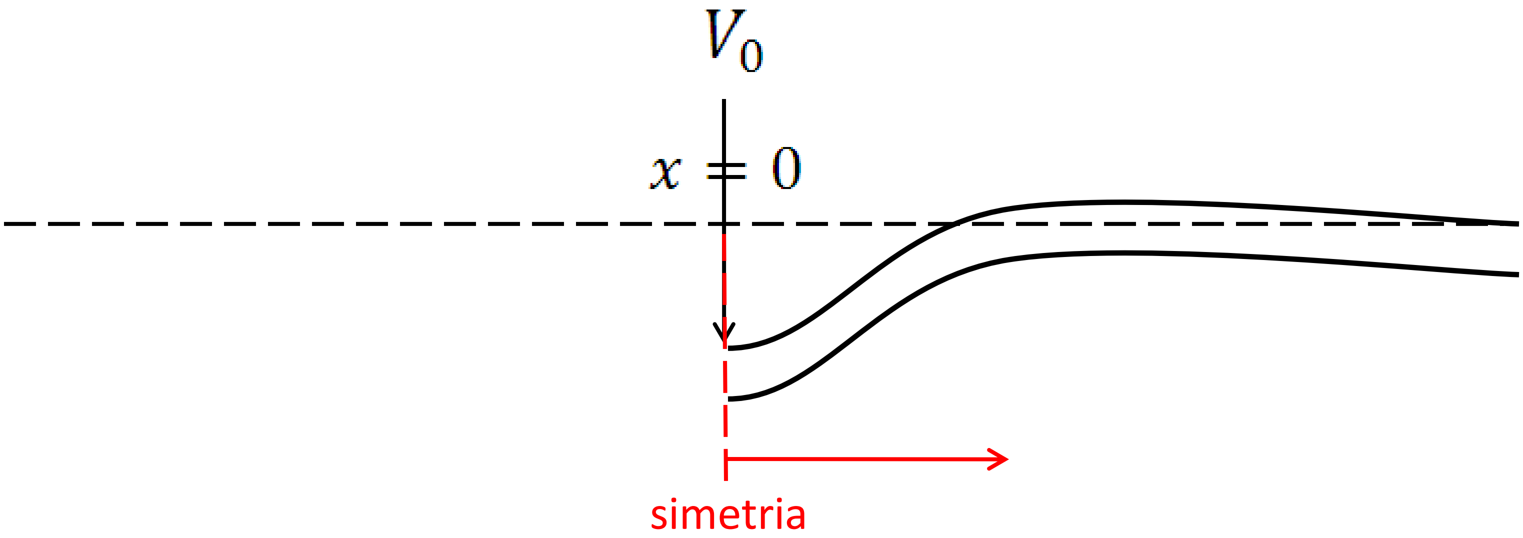


$w = e^{-\lambda x}(C_c \cos\lambda x + D_c \sin\lambda x)$ Para $x \to 0$ $\dfrac{dw}{dx} \to 0$

$$\frac{dw}{dx} = -\lambda \cancelto{1}{e^{-\lambda x}}(C_c \cancelto{1}{\cos\lambda x} + D_c \cancelto{0}{\sin\lambda x}) + \lambda \cancelto{1}{e^{-\lambda x}}(-C_c \cancelto{0}{\sin\lambda x} + D_c \cancelto{1}{\cos\lambda x})$$

$$C_c(-\lambda) + D_c(\lambda) = 0$$

Soluções Analíticas

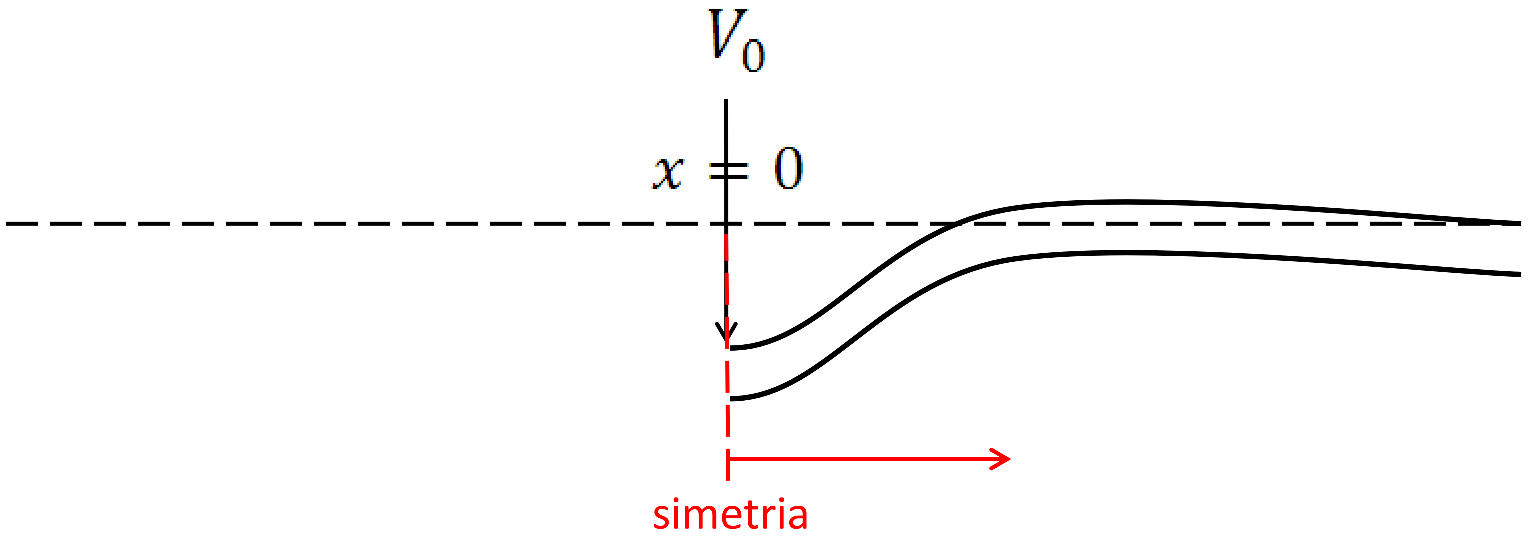


$$w = e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x) \qquad \text{Para } x \rightarrow 0 \qquad \frac{dw}{dx} \rightarrow 0$$

$$\frac{dw}{dx} = -\lambda \cancelto{1}{e^{-\lambda x}}(C_c \cancelto{1}{\cos \lambda x} + D_c \cancelto{0}{\sin \lambda x}) + \lambda \cancelto{1}{e^{-\lambda x}}(-C_c \cancelto{0}{\sin \lambda x} + D_c \cancelto{1}{\cos \lambda x})$$

$$C_c(-\lambda) + D_c(\lambda) = 0 \rightarrow C_c = D_c$$

Soluções Analíticas



$$w = e^{-\lambda x}(C_c \cos \lambda x + D_c \sin \lambda x) \qquad \text{Para } x \to 0 \qquad \frac{dw}{dx} \to 0$$

$$\frac{dw}{dx} = -\lambda \cancelto{1}{e^{-\lambda x}}(C_c \cancelto{1}{\cos \lambda x} + D_c \cancelto{0}{\sin \lambda x}) + \lambda \cancelto{1}{e^{-\lambda x}}(-C_c \cancelto{0}{\sin \lambda x} + D_c \cancelto{1}{\cos \lambda x})$$

$$C_c(-\lambda) + D_c(\lambda) = 0 \to C_c = D_c \to w = e^{-\lambda x} D_c(\cos \lambda x + \sin \lambda x)$$

Soluções Analíticas

$$w = e^{-\lambda x} D_c (\cos \lambda x + \sin \lambda x)$$

Soluções Analíticas

$$w = e^{-\lambda x} D_c (\cos \lambda x + \sin \lambda x)$$

$$V_0 - 2(\rho_m - \rho_r) g \int_0^\infty w dx = 0$$

Soluções Analíticas

$$w = e^{-\lambda x} D_c (\cos \lambda x + \sin \lambda x)$$

$$V_0 - 2(\rho_m - \rho_r) g \int_0^\infty w dx = 0 \rightarrow \int_0^\infty w dx = \frac{V_0}{2(\rho_m - \rho_r) g}$$

Soluções Analíticas

$$w = e^{-\lambda x} D_c (\cos \lambda x + \sin \lambda x)$$

$$V_0 - 2(\rho_m - \rho_r) g \int_0^\infty w dx = 0 \rightarrow \int_0^\infty w dx = \frac{V_0}{2(\rho_m - \rho_r) g}$$

$$\rightarrow D_c \int_0^\infty e^{-\lambda x} (\cos \lambda x + \sin \lambda x) dx = \frac{V_0}{2(\rho_m - \rho_r) g}$$

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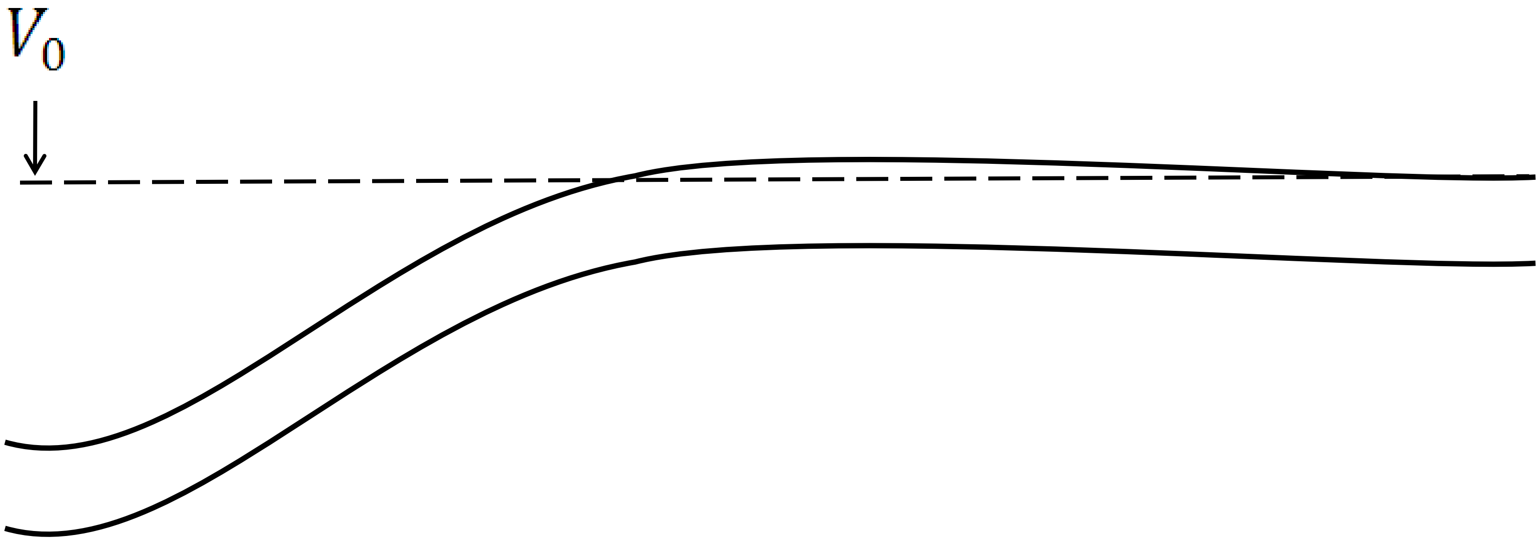
$$\rightarrow D_c \cdot \frac{1}{\lambda} = \frac{V_0}{2(\rho_m - \rho_r)g}$$

Soluções Analíticas

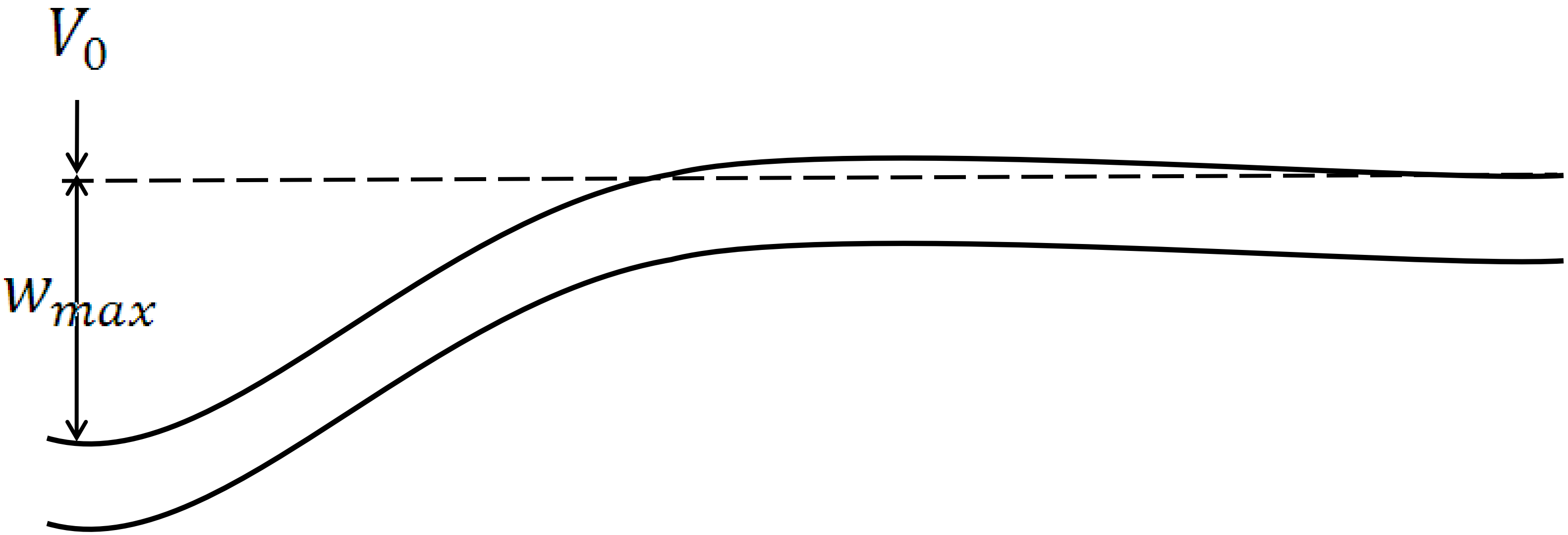
$$w = \frac{V_0 \lambda}{2(\rho_m - \rho_r)g} e^{-\lambda x} (\cos \lambda x + \sin \lambda x)$$



Soluções Analíticas

$$\boxed{w = \frac{V_0\lambda}{2(\rho_m - \rho_r)g} e^{-\lambda x}(\cos \lambda x + \sin \lambda x)}$$

$$x = 0, \quad w = w_{max} = \frac{V_0\lambda}{2(\rho_m - \rho_r)g}$$

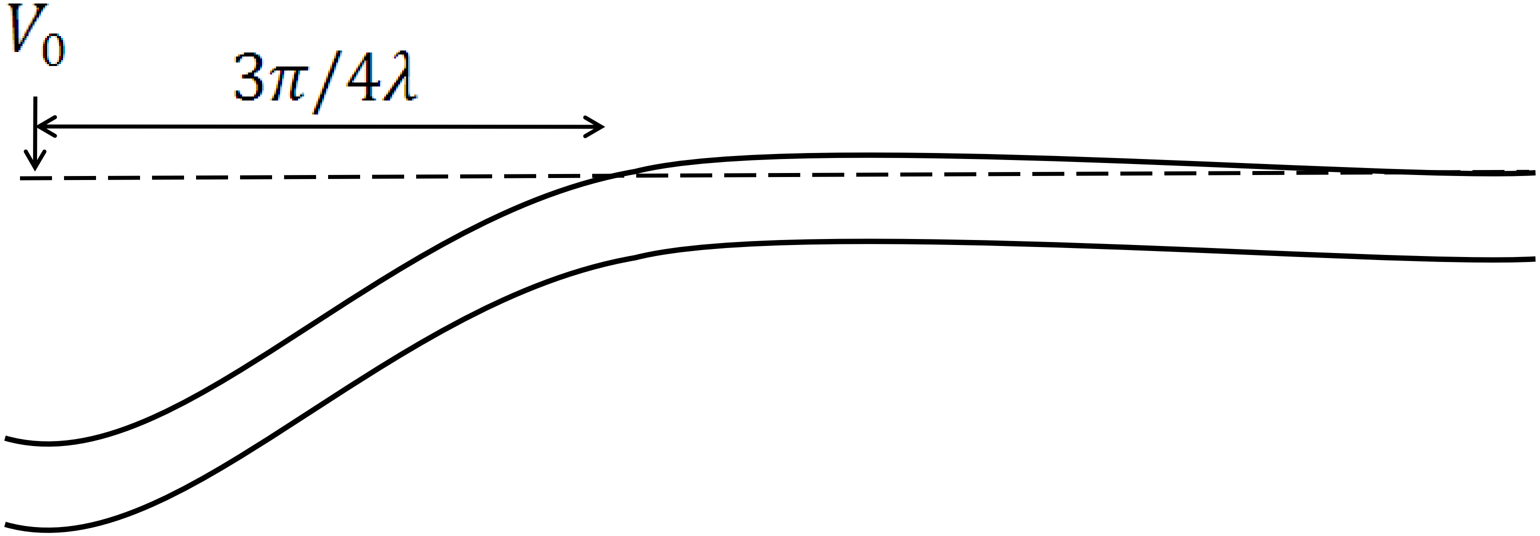


Victor Sacek IAG/USP

Soluções Analíticas

$$w = \frac{V_0 \lambda}{2(\rho_m - \rho_r)g} e^{-\lambda x} (\cos \lambda x + \sin \lambda x)$$

$$w = 0, \quad x = \frac{3\pi}{4\lambda}, \frac{7\pi}{4\lambda}, \cdots$$



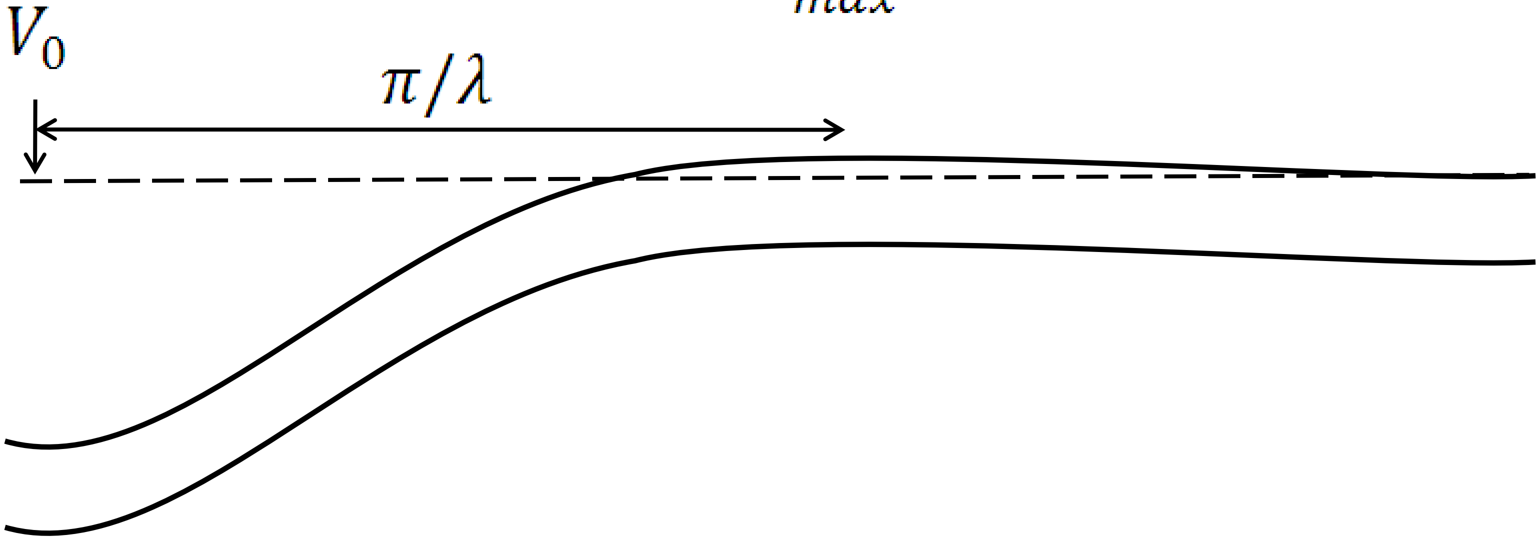
Victor Sacek IAG/USP

Soluções Analíticas

$$\boxed{w = \frac{V_0\lambda}{2(\rho_m - \rho_r)g} e^{-\lambda x}(\cos \lambda x + \sin \lambda x)}$$

$$x = \frac{\pi}{\lambda}, \quad w_b = w_{max} e^{-\pi}(\cos \pi + \sin \pi)$$

$$\approx -w_{max} 0.0432$$



Implementação

$$w = \frac{V_0 \lambda}{2(\rho_m - \rho_r)g} e^{-\lambda x} (\cos \lambda x + \sin \lambda x)$$

$$\lambda = \left[\frac{(\rho_m - \rho_r)g}{4D}\right]^{1/4}$$

$$D = \frac{E T_e^3}{12(1 - \nu^2)}$$

$E = 1 \times 10^{11}\ \mathrm{N/m^2}$ $\nu = 0.25$ $g = 9.8\ \mathrm{m/s^2}$

T_e e V_0 variáveis

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T_e e V_0 variáveis