

How to interpret the logistic regression with fixed effects

Klaus Pforr

5th ESRA Conference,
Ljubljana, Slovenia, July 15–19, 2013

Outlook

- Fixed-effects logit
 - Advantages
 - Disadvantages
- Interpretation
 - Standard technique
 - Alternative interpretations
 - Alternative model
- Conclusion

Fixed-effects logit (Chamberlain, 1980)

Individual intercepts instead of fixed constants for sample

$$\Pr(y_{it} = 1) = \frac{\exp(\alpha_j + x_{it}\beta)}{1 + \exp(\alpha_j + x_{it}\beta)}$$

Advantages

- Implicit control of unobserved heterogeneity
 - Forgotten or hard-to-measure variables
 - No restriction on correlation with indep. var's
- Reduces problem of self-selection and omitted-variable bias

Fixed-effects logit

Disadvantages

- Panel data
- Only constant heterogeneity controlled
- Neglected heterogeneity weakened, but remains
- Interpretation severely limited
 - Part of index function unspecified
 - No predicted probabilities of outcome
 - No partial/discrete change effects

⇒ How do we interpret fixed-effects logit?

Interpretation alternatives

1. Odds ratio effects

- OR-effect:

$$\frac{\Pr(y_{it} = 1 | x_{it} + 1)}{\Pr(y_{it} = 0 | x_{it} + 1)} \bigg/ \frac{\Pr(y_{it} = 1 | x_{it})}{\Pr(y_{it} = 0 | x_{it})} = \exp(\beta)$$

- ☺ Straightforward

“All else equal with increase of x by 1 unit, odds of $y = 1$ vs. $y = 0$ increase by factor $\exp(\beta)$.”

- ☹ Odds non-intuitive
- ☹ Polytomous DV: not necessarily same sign as change in prob.

Interpretation alternatives

2. Effect on cond. probability (Cameron & Trivedi, 2010)

- Probability to realize sequence of outcomes conditional on the number of occurrence of outcome within person
- Cond. prob. independent of α_j

$$\Pr \left(\mathbf{y}_i \mid \mathbf{x}_i, \sum_{t=1}^{T_i} y_{it} \right) = \frac{\exp \left(\sum_{t=1}^{T_i} y_{it} \mathbf{x}_{it} \beta \right)}{\sum_{\mathbf{d}_i \in B_i} \exp \left(\sum_{t=1}^{T_i} d_{it} \mathbf{x}_{it} \beta \right)}$$

- ☺ Predicted probabilities and average marginal/discrete changes possible
- ☹ Conditional probability non-intuitive

Interpretation alternatives

3. Effect on simplified cond. prob. (Cameron & Trivedi, 2009)

- Only $t = 2$ and $y_{i1} = 0, y_{i2} = 1$

$$\Pr(y_{i1} = 0, y_{i2} = 1 | \mathbf{x}_i, y_{i1} + y_{i2} = 1) = \frac{\exp((x_{i2} - x_{i1})\beta)}{1 + \exp((x_{i2} - x_{i1})\beta)}$$

- ☺ Cond. prob. of reduced case makes sense
- ☹ $T > 2$: Which time points to choose?
- ☹ Assumption on α_j introduced without basis in data

⇒ Iff $T = 2$, this is a reasonable option!

Interpretation alternatives

4. Effect on probabilities for prototypes (Schröder, 2010)

- Assume probability for outcome y_{it} for prototypical unit with $x_{it} \Rightarrow$ Derive α_j

😊 Now intuitive effect on prob's etc. possible

😊 Assumption on α_j based on aggreg. data

😞 Relevance of prototype depends on $\alpha_j | \mathbf{x}_i$

$$E(\alpha_j | y_{it}, \mathbf{x}_i) = \frac{E(y_{it} | \alpha_j, \mathbf{x}_i) E(\alpha_j, \mathbf{x}_i)}{E(y_{it})}$$

\Rightarrow Relevance of estimated effects unknown \Rightarrow Only more intuitive interpretation of OR-effect

Alternative Model

Correlated random effects probit (Mundlak, 1978)

- Estimate random effects probit with across-time-means of covariates

- ☹ Stronger assumptions than full fixed-effects

$$\alpha_j | \mathbf{x}_j \sim \mathbb{N}(\gamma + \bar{\mathbf{x}}_j \delta, \sigma_{\alpha_j}^2)$$

⇒ Simple correlation between α_j and \mathbf{x}_j allowed

- 😊 Effects on probabilities possible
- 😊 Average marginal effects possible

Conclusion

- Standard interpretation of fixed-effects logit limited to odds-ratio effects
 - Other interpretation strategies within fixed-effects:
 - Conditional probability
 - Simplified conditional probability
 - Probability of prototype
 } infeasible for $T > 2$
 - Correlated random effects probit
 - Stricter assumptions
 - Correlation between unobs. heterogeneity and covariates still allowed
 - Effect on probabilities possible
- ⇒ For $T > 2$, either accept odds-ratio effects or one step back with abandoning assumptions

Thank you

Back-up

Fixed-effects logit with person-dummies

- Linear fixed-effects models can be estimated with panel group indicators
- Non-linear fixed-effects models with group-dummies:
 - Person panel data (large N and fixed T)
⇒ Estimates inconsistent for person-level heterogeneity, consistent for period dummies
 - Persons within countries (fixed " N " and large " T ")
⇒ Estimates consistent for country-level heterogeneity, inconsistent for person dummies
- ⊖ Problem of omitted variables at one level remains

Back-up

Linear probability models with fixed-effects

- ☺ Linear probability models (OLS) can include fixed-effects
- ☺ Interpretation of effects on probabilities etc. possible
- ☺ Serial correlation across time can be allowed
- ☺ Neglected heterogeneity problem weakened
- ☹ Predicted probabilities unbounded

⇒ Works for marginal effects, not for predicted probabilities

References

- Cameron, A. Colin, und Pravin K. Trivedi. 2009. *Microeconometrics: Methods and applications*. Cambridge and New York and NY: Cambridge University Press.
- Cameron, A. Colin, und Pravin K. Trivedi. 2010. *Microeconometrics using Stata*. College Station and TX: Stata Press.
- Chamberlain, Gary. 1980. Analysis of covariance with qualitative data. *Review of Economic Studies* 57: 225–238.
- Mundlak, Yair. 1978. On the pooling of time series and cross section data. *Econometrica* 46: 69–85.
- Schröder, Jette. 2010. *Der Zusammenhang zwischen der Erwerbstätigkeit von Frauen und ihrer Fertilität*. Würzburg: Ergon Verlag.