



Table of Continuous-time Pulsation Fourier Transform Pairs

$x(t) = \mathcal{F}_\omega^{-1} \{X(\omega)\} = \int_{-\infty}^{+\infty} x(t)e^{j\omega t} d\omega$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega) = \mathcal{F}_\omega \{x(t)\} = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt$
transform	$x(t)$	$\xleftrightarrow{\mathcal{F}_\omega} X(\omega)$
time reversal	$x(-t)$	$\xleftrightarrow{\mathcal{F}_\omega} X(-\omega)$ frequency reversal
complex conjugation	$x^*(t)$	$\xleftrightarrow{\mathcal{F}_\omega} X^*(-\omega)$ reversed conjugation
reversed conjugation	$x^*(-t)$	$\xleftrightarrow{\mathcal{F}_\omega} X^*(\omega)$ complex conjugation
	$x(t)$ is purely real	$\xleftrightarrow{\mathcal{F}_\omega} X(\omega) = X^*(-\omega)$ even/symmetry
	$x(t)$ is purely imaginary	$\xleftrightarrow{\mathcal{F}_\omega} X(\omega) = -X^*(-\omega)$ odd/antisymmetry
even/symmetry	$x(t) = x^*(-t)$	$\xleftrightarrow{\mathcal{F}_\omega} X(\omega)$ is purely real
odd/antisymmetry	$x(t) = -x^*(-t)$	$\xleftrightarrow{\mathcal{F}_\omega} X(\omega)$ is purely imaginary
time shifting	$x(t - t_0)$	$\xleftrightarrow{\mathcal{F}_\omega} X(\omega)e^{-j\omega t_0}$
	$x(t)e^{j\omega_0 t}$	$\xleftrightarrow{\mathcal{F}_\omega} X(\omega - \omega_0)$ frequency shifting
time scaling	$x(af)$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{1}{ a } X\left(\frac{\omega}{a}\right)$
	$\frac{1}{ a } x\left(\frac{f}{a}\right)$	$\xleftrightarrow{\mathcal{F}_\omega} X(a\omega)$ frequency scaling
linearity	$ax_1(t) + bx_2(t)$	$\xleftrightarrow{\mathcal{F}_\omega} aX_1(\omega) + bX_2(\omega)$
time multiplication	$x_1(t)x_2(t)$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{1}{2\pi} X_1(\omega) * X_2(\omega)$ frequency convolution
frequency convolution	$x_1(t) * x_2(t)$	$\xleftrightarrow{\mathcal{F}_\omega} X_1(\omega)X_2(\omega)$ frequency multiplication
delta function	$\delta(t)$	$\xleftrightarrow{\mathcal{F}_\omega} 1$
shifted delta function	$\delta(t - t_0)$	$\xleftrightarrow{\mathcal{F}_\omega} e^{-j\omega t_0}$
	1	$\xleftrightarrow{\mathcal{F}_\omega} 2\pi\delta(\omega)$ delta function
	$e^{j\omega_0 t}$	$\xleftrightarrow{\mathcal{F}_\omega} 2\pi\delta(\omega - \omega_0)$ shifted delta function
two-sided exponential decay	$e^{-a t } \quad a > 0$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{2a}{a^2 + \omega^2}$
exponential decay	$e^{-at}u(t) \quad \Re\{a\} > 0$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{1}{a + j\omega}$
reversed exponential decay	$e^{-at}u(-t) \quad \Re\{a\} > 0$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{1}{a - j\omega}$
	$e^{\frac{t^2}{2\sigma^2}}$	$\xleftrightarrow{\mathcal{F}_\omega} \sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$
sine	$\sin(\omega_0 t + \phi)$	$\xleftrightarrow{\mathcal{F}_\omega} j\pi [e^{-j\phi}\delta(\omega + \omega_0) - e^{j\phi}\delta(\omega - \omega_0)]$
cosine	$\cos(\omega_0 t + \phi)$	$\xleftrightarrow{\mathcal{F}_\omega} \pi [e^{-j\phi}\delta(\omega + \omega_0) + e^{j\phi}\delta(\omega - \omega_0)]$
sine modulation	$x(t)\sin(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{j}{2} [X(\omega + \omega_0) - X(\omega - \omega_0)]$
cosine modulation	$x(t)\cos(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{1}{2} [X(\omega + \omega_0) + X(\omega - \omega_0)]$
squared sine	$\sin^2(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega} \pi^2 [2\delta(\omega) - \delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
squared cosine	$\cos^2(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega} \pi^2 [2\delta(\omega) + \delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
rectangular	$\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & t \leq \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega} T \text{sinc}\left(\frac{\omega T}{2}\right)$
triangular	$\text{triang}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leq T \\ 0 & t > T \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega} T \text{sinc}^2\left(\frac{\omega T}{2}\right)$
step	$u(t) = 1_{[0, +\infty]}(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega} \pi\delta(\omega) + \frac{1}{j\omega}$
signum	$\text{sgn}(t) = \begin{cases} 1 & t \geq 0 \\ -1 & t < 0 \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{2}{j\omega}$
sinc	$\text{sinc}(Tt)$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{\pi}{T} \text{rect}\left(\frac{\omega}{2T}\right)$
squared sinc	$\text{sinc}^2\left(\frac{T}{2}t\right)$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{2\pi}{T} \text{triang}\left(\frac{\omega}{2T}\right)$
integration	$\int_{-\infty}^t x(t) dt$	$\xleftrightarrow{\mathcal{F}_\omega} \frac{1}{j\omega} X(\omega) + \pi X(0)\delta(\omega)$
n -th time derivative	$\frac{d^n}{dt^n} f(t)$	$\xleftrightarrow{\mathcal{F}_\omega} (j\omega)^n X(\omega)$
n -th frequency derivative	$t^n f(t)$	$\xleftrightarrow{\mathcal{F}_\omega} j^n \frac{d^n}{df^n} X(\omega)$
time inverse	$\frac{1}{t}$	$\xleftrightarrow{\mathcal{F}_\omega} -j\pi \text{sgn}(\omega)$