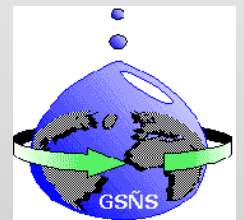


# Resistência ao Cisalhamento

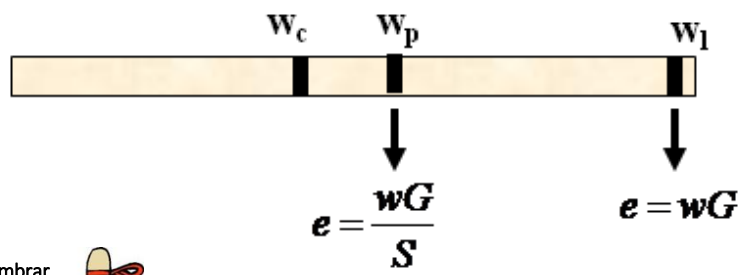
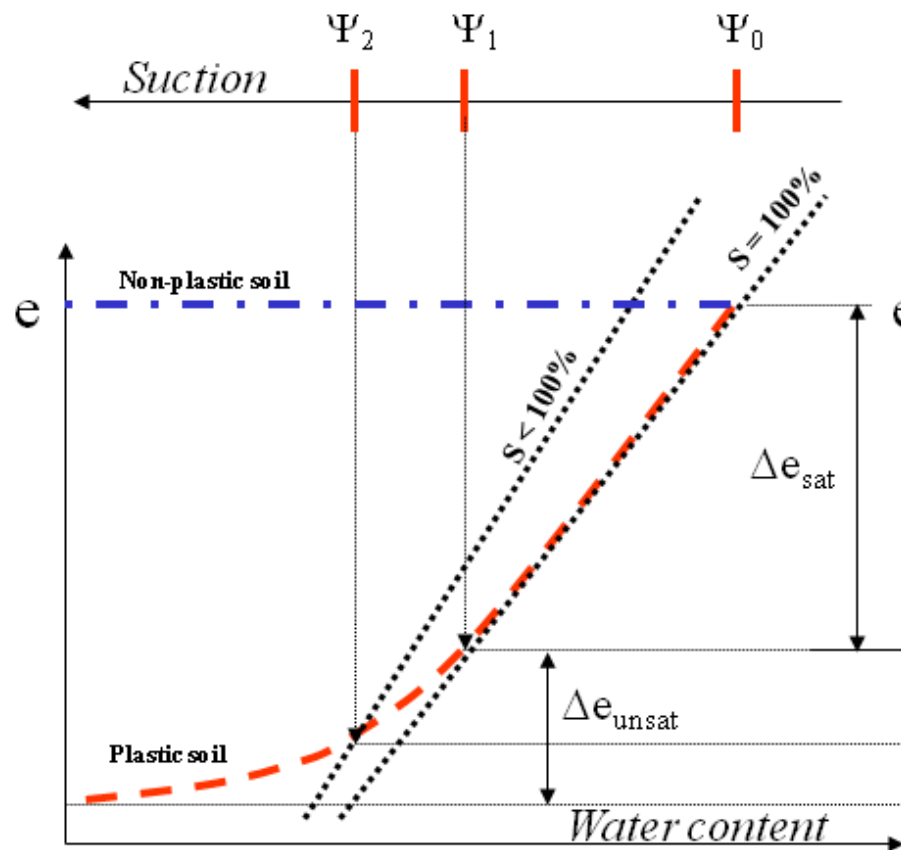
***Prof. Fernando A. M. Marinho***

Universidade de São Paulo

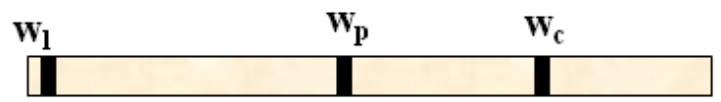
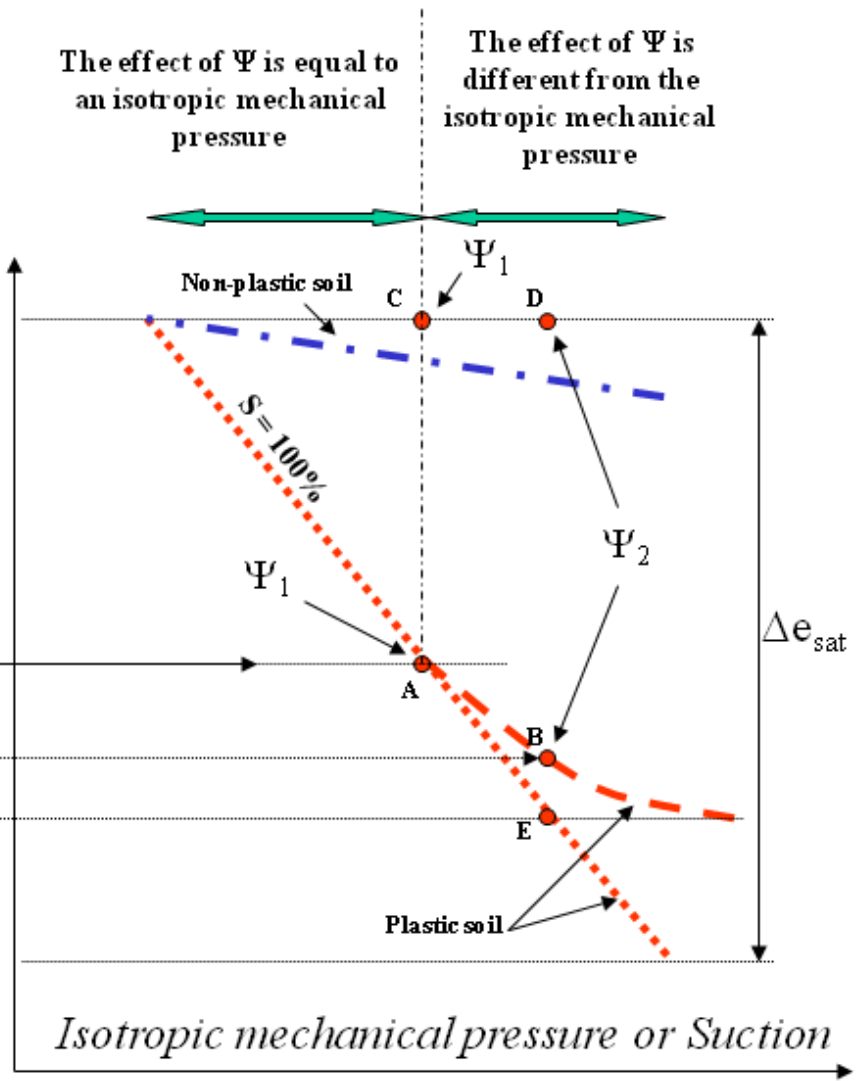
2019



[sns.org.br](http://sns.org.br)



(a)



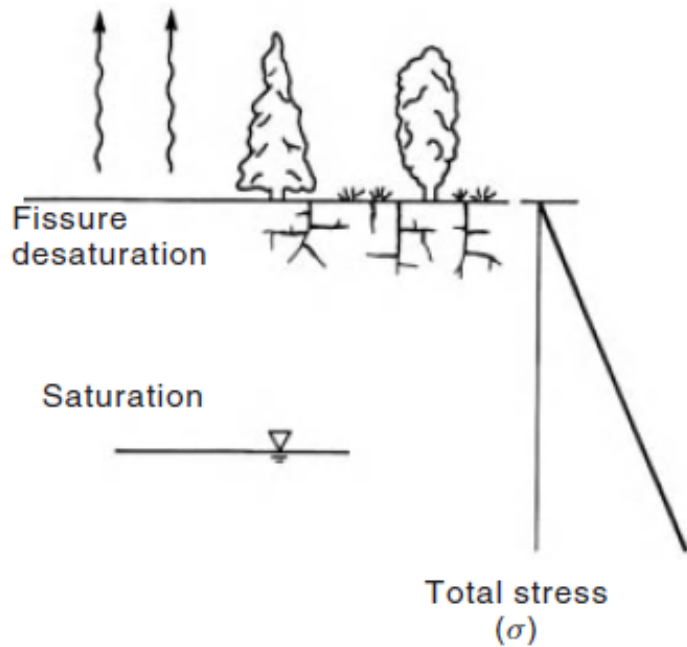
(b)

Para Lembrar



# Total stress, air pressure and pore water pressure distribution for a unsaturated soil

Evaporation      Evapo transpiration



Equilibrium with water table

Flooding of desiccated soli

Excessive evaporation

At time of deposition

Atmospheric

Pore-air pressure ( $u_a$ )

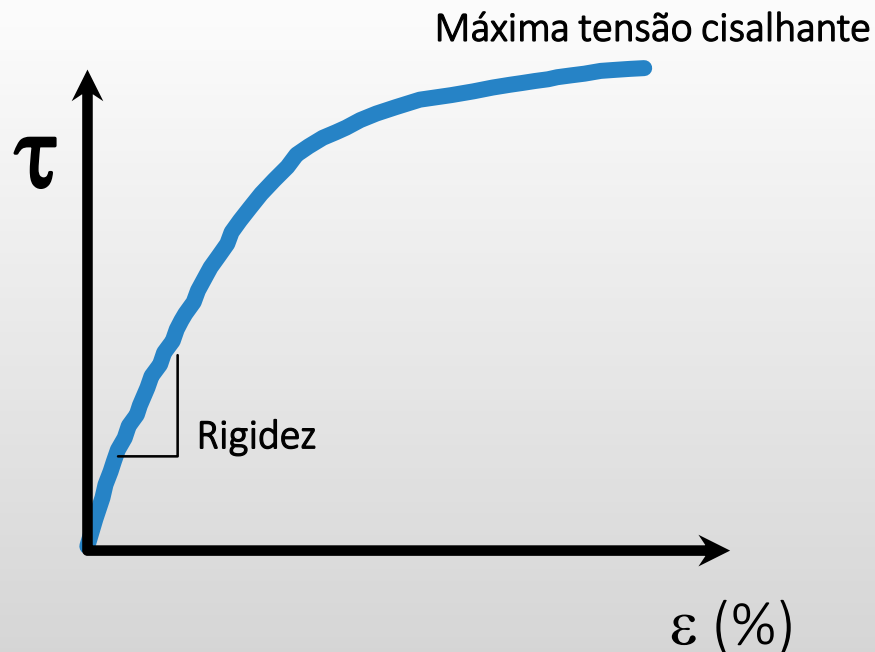
Pore-water pressure ( $u_w$ )

# O que afeta a resistência dos solos?

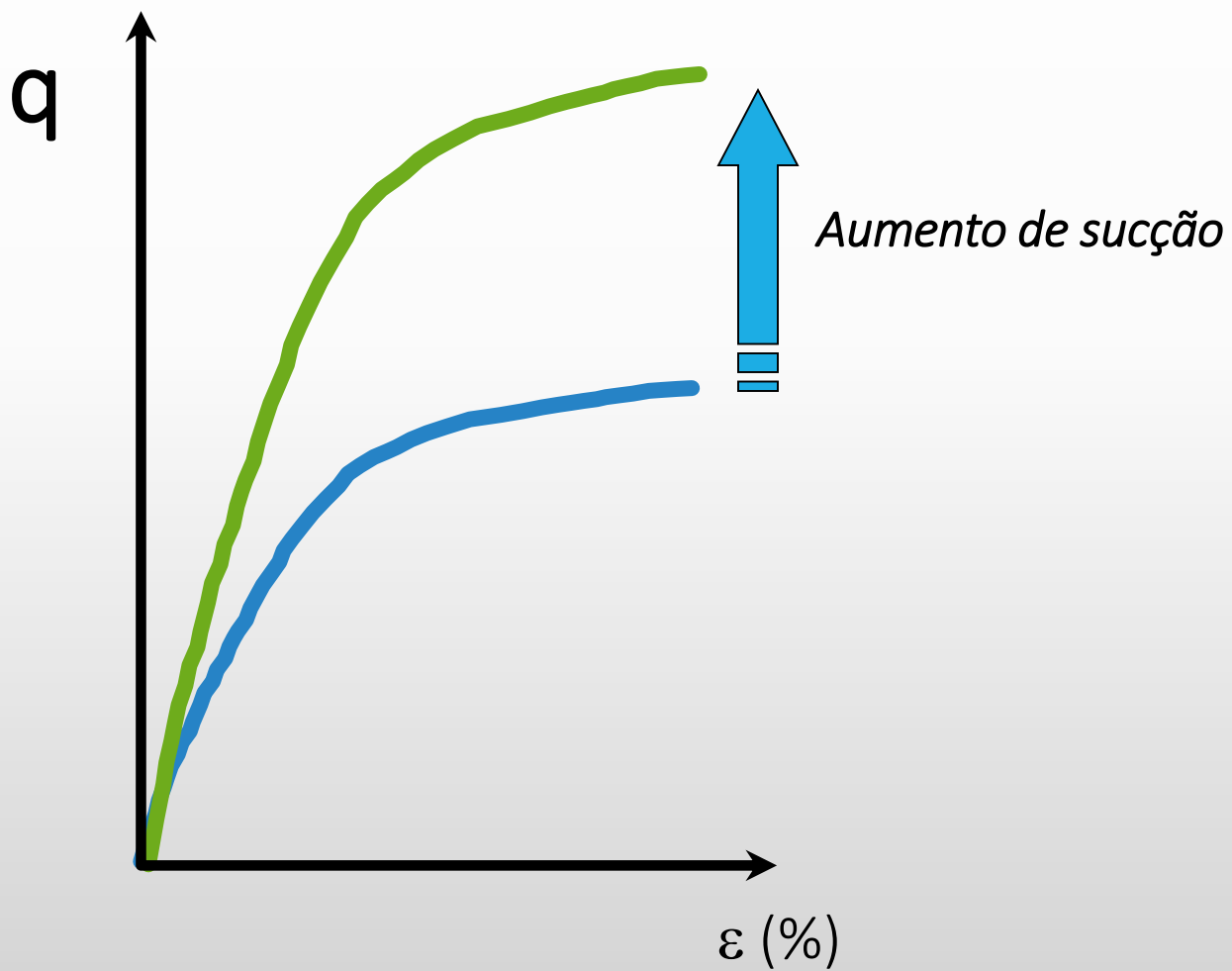
- Tipo de mineral
  - Quantidade do mineral presente
  - Forma e distribuição das partículas
- 
- Teor de umidade
  - Densidade
  - Grau de saturação
  - Temperatura
  - Estrutura
  - Condutividade hidráulica
  - Disponibilidade de água

- Para analisar qualquer tipo de estrutura ou qualquer material sólido é necessário obter a relação entre tensão e deformação.
- Esta relação é chamada de relação constitutiva e pode assumir muitas formas dependendo do material e da forma de carregamento.

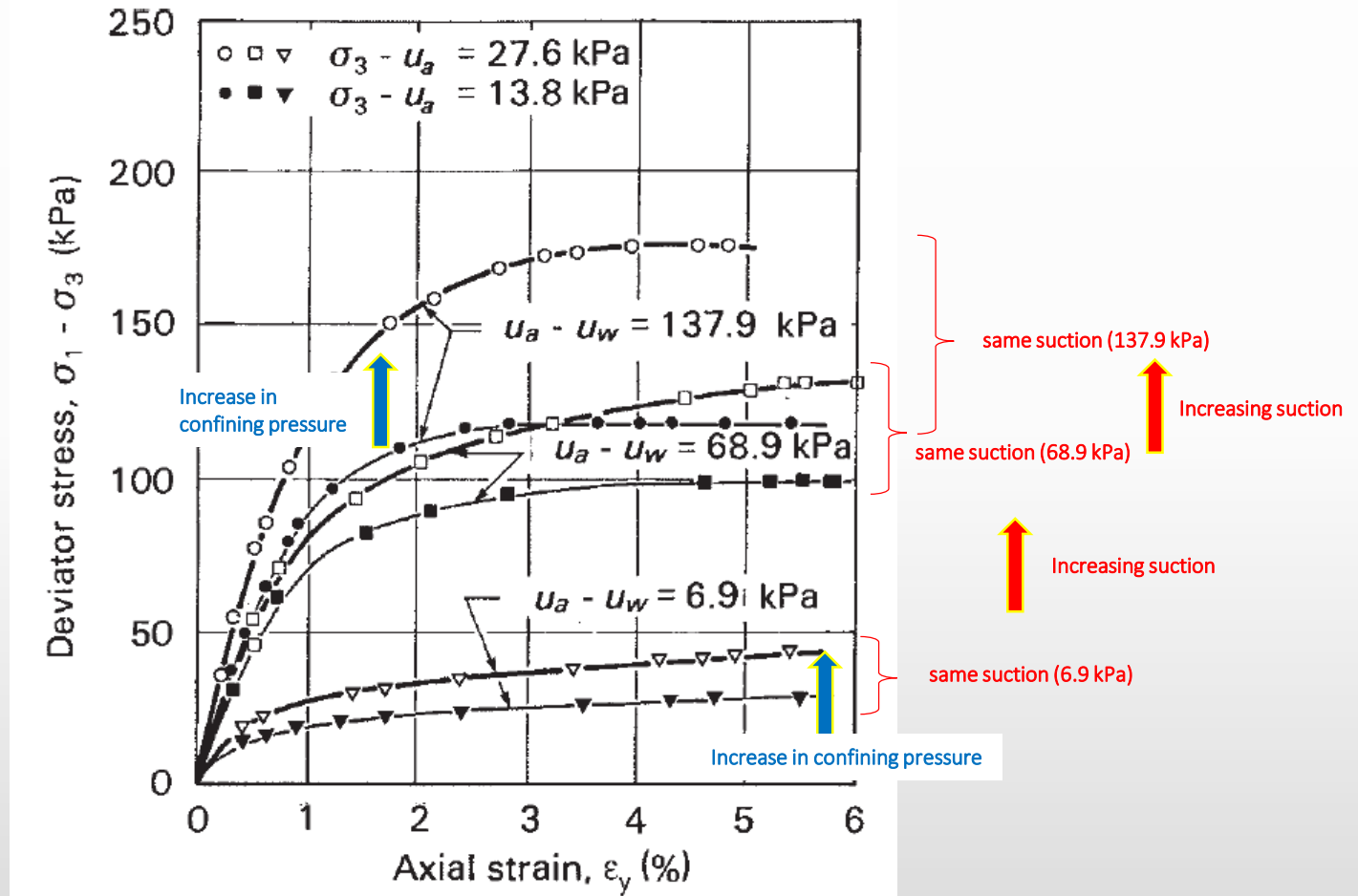
Curva tensão deformação típica de um solo normalmente adensado



## Efeito da não saturação



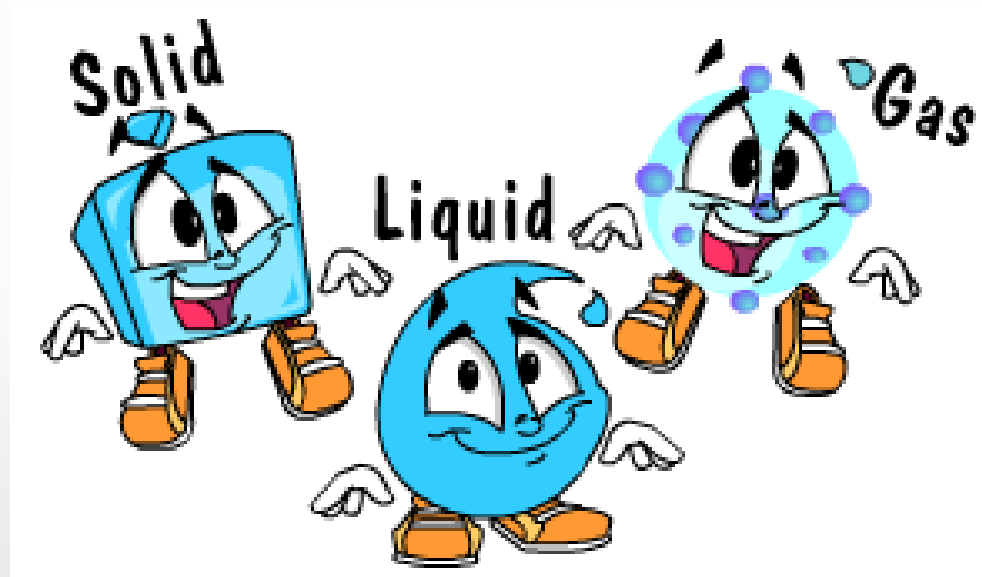
# Efeito da sucção e da pressão confinante na curva tensão - deformação



# Qual o papel da sucção da resistência ao cisalhamento?

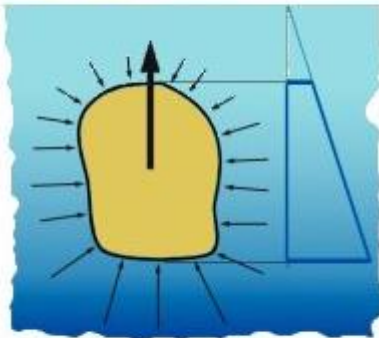
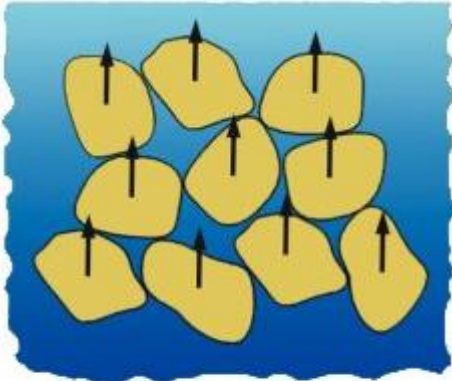
Quando o Sistema está saturado tem-se duas fases.

Quando não está saturado possui três fases



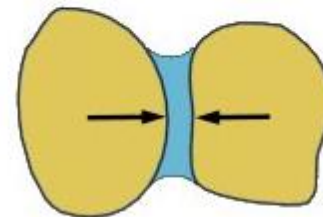
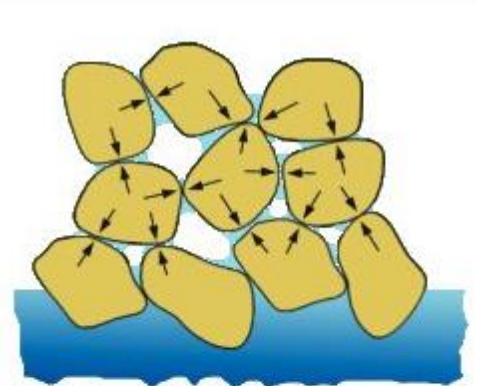


Princípio das tensões efetivas



Water pressure forces in saturated porous media

Qual a efetividade da pressão da água no estado não saturado?



Capillary forces in unsaturated porous media

# Effective Stress Principle (saturated soils)

$$\sigma' = \sigma - u$$

Valid for shear strength and volume change if:

The soil grains are incompressible

The stress controlling the contact area and the inter-granular strength are independent of the confining stress.

In general soils do not meet these two conditions and their mechanical behavior is best described by the expression:

$$\sigma' = \sigma - ku$$

$$k = \left( 1 - \frac{a \tan \psi}{\tan \phi'} \right) \text{ Shear strength}$$

$$k = \left( 1 - \frac{C_s}{C} \right) \text{ Volume change}$$

$a$  – contact area between particles, per unit area of the material.

$\psi$  – Intrinsic friction angle of the grains.

$\phi'$  – Friction angle of the soil.

$C_s$  – Particle compressibility

$C$  – Soil compressibility

An extension for unsaturated soils is:

$$\sigma' = \sigma - k_1 u_w - k_2 u_a$$

$u_w$  – Pore water pressure

$u_a$  – Pore air pressure

Simultaneous changes of the total stress, pore water pressure and air pressure do not cause variations in volume and in shear strength, then:

$$0 = \Delta\sigma - k_1 \Delta u_w - k_2 \Delta u_a$$

e

$$\Delta\sigma = \Delta u_w = \Delta u_a$$

portanto

$$k_2 = 1 - k_1$$

Se

$$k_1 = \chi$$

$$\sigma' = \sigma - u_a + \chi(u_a - u_w)$$

Bishop (1959)

Se  $\chi = 1$  – Saturated soil

Se  $\chi = 0$  – Dry soil

# Effective stress equations for unsaturated soils

Reference	Equation	Description of variables
Bishop (1959)	$\sigma' = \sigma - u_a + \chi(u_a - u_w)$	$\sigma$ = total normal stress $u_w$ = pore-water pressure $\chi$ = parameter related to degree of saturation $u_a$ = the pressure in gas and vapor phase
Croney et al. (1958)	$\sigma' = \sigma - \beta' u_w$	$\beta'$ = holding or bonding factor which is measure of number of bonds under tension effective in contributing to soil strength $\bar{\sigma}$ = mineral interparticle stress
Lambe (1960)	$\sigma = \bar{\sigma} a_m + u_a a_a + u_w a_w + R - A$	$a_m$ = mineral particle contact area $a_w$ = water phase contact area $a_a$ = fraction of total area that is air-air contact $R$ = repulsive pore fluid stress due to chemistry $A$ = attractive pore fluid stress due to chemistry
Aitchison (1961)	$\sigma' = \sigma + \psi p''$	$\psi$ = parameter with values ranging from zero to one $p''$ = pore-water pressure deficiency
Jennings (1960)	$\sigma' = \sigma + \beta p''$	$\beta$ = statistical factor of same type as contact area; should be measured experimentally in each case
Richards (1966)	$\sigma' = \sigma - u_a + \chi_m(h_m + u_a) + \chi_s(h_s + u_a)$	$\chi_m$ = effective stress parameter for matric suction $h_m$ = matric suction $\chi_s$ = effective stress parameter for solute suction $h_s$ = solute suction

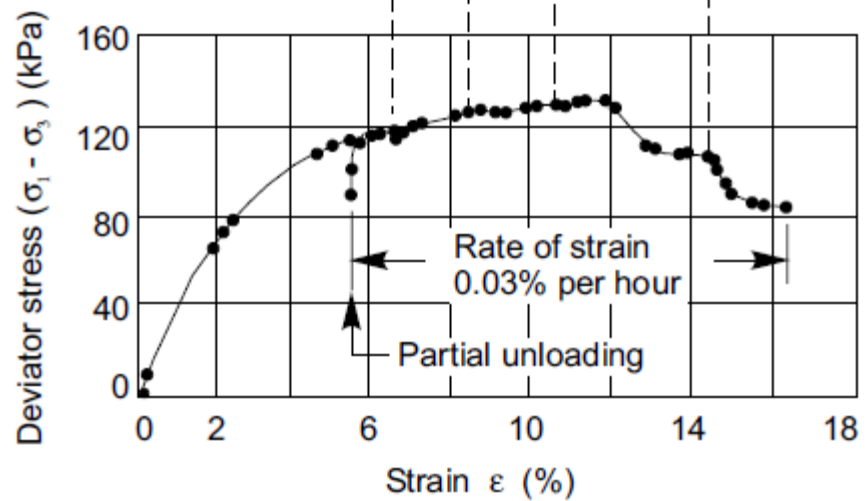
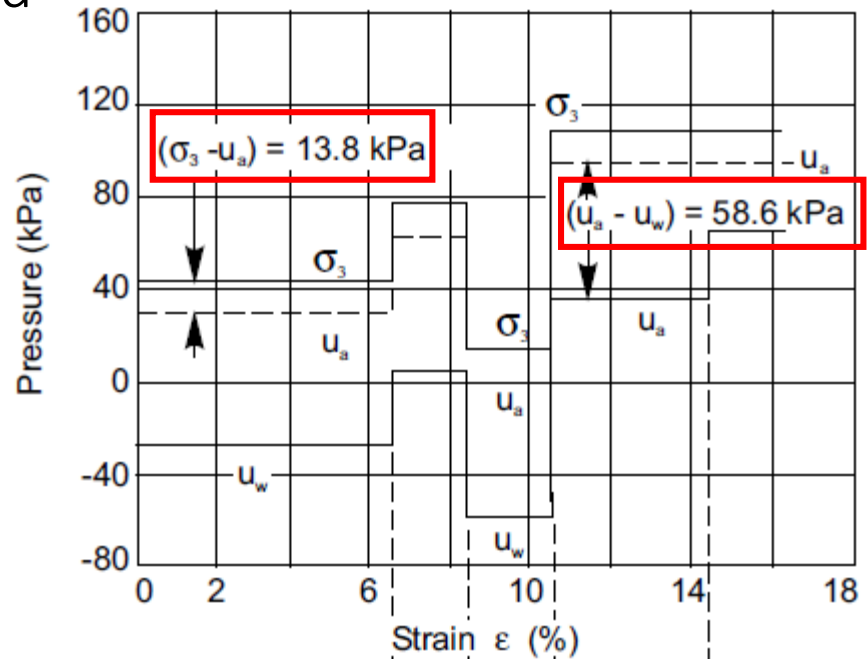
Fredlund and Morgenstern (1977)

# Limitation of the Effective stress equations for unsaturated soils

- Effective stress equations for unsaturated soil requires a soil parameter.
- Experimental results have shown that soil properties measured do not yield a single-valued relationship to the proposed effective stress.
- The soil properties used in the proposed effective stress equations has different magnitudes for different problems (i.e. volume change and shear strength).

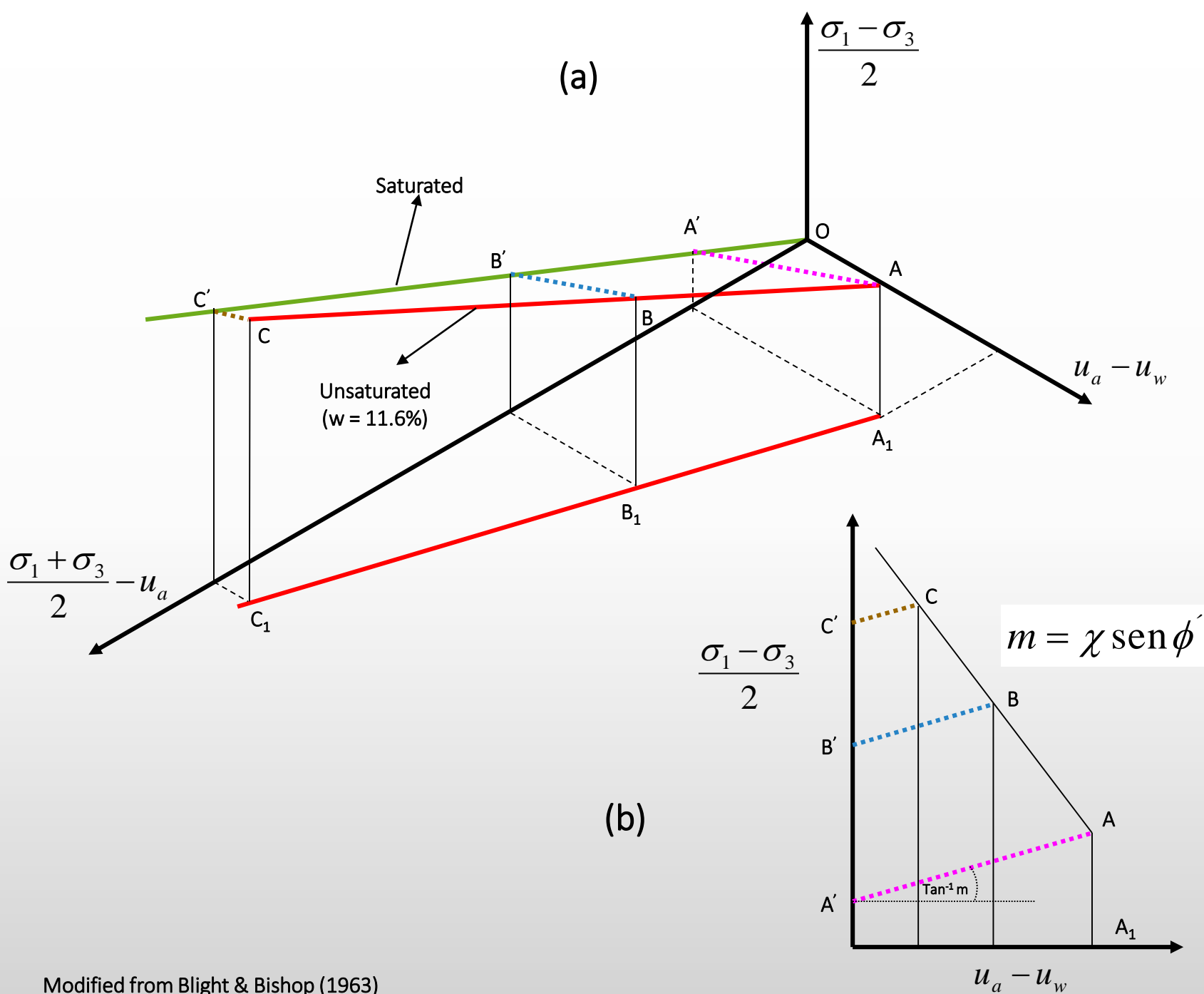
Jennings and Burland (1962)  
Coleman (1962)  
Bishop and Blight (1963)  
Burland (1964)  
Burland (1965)  
Blight (1965)

# Drained test on partially saturated loose silt



$e_{final} = 0.86$   
 $S = 43 \%$

Liquid limit = 29 %  
 Plastic limit = 23 %



Modified from Blight & Bishop (1963)

# Shear strength for Unsaturated Soils

Using Bishop (1954) effective stress principle for unsaturated soil

$$\sigma' = \sigma - u_a + \chi(u_a - u_w)$$

$$\tau_f = c' + [(\sigma - u_a)_f + \chi_f(u_a - u_w)_f] \tan \phi'$$

$$\chi_f = \frac{\tau_f - c' - (\sigma - u_a)_f \tan \phi'}{(u_a - u_w)_f \tan \phi'}$$

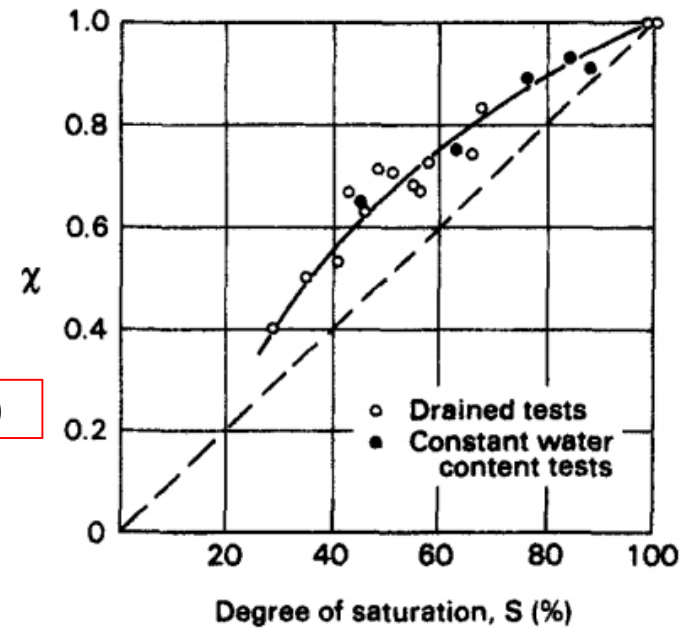
Although it is relatively easy to relate the shear strength of unsaturated soil to a single stress parameter involving suction,  $u_a$  and  $u_w$ , the volumetric behaviour is not controlled by the same stress parameter or by any other single stress variable.

Ng and Manzies (2007)



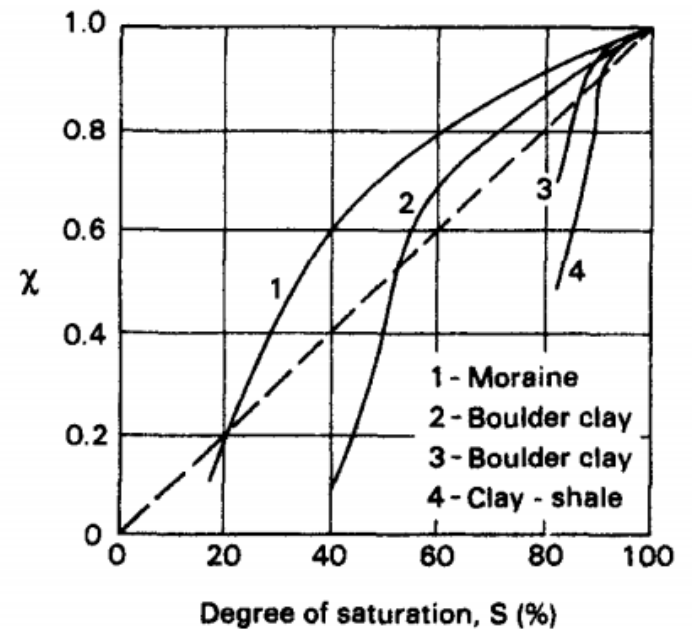
The relationship between the  $\chi$  parameter and the degree of saturation,  $S$ .

$\chi$  values for a cohesionless silt (after Donald, 1961)



(a)

$\chi$  values for compacted soils (after Blight, 1961).



(b)

# Shear strength for Unsaturated Soils

In 1977, Fredlund and Morgenstern suggested the use of any two of three possible stress variables,  $\sigma - u_a$ ,  $\sigma - u_w$  and  $u_a - u_w$ , to describe mechanical behaviour of unsaturated soils. The possible combinations are:

1.  $\sigma - u_a$ , and  $u_a - u_w$
2.  $\sigma - u_w$  and  $u_a - u_w$
3.  $\sigma - u_a$  and  $\sigma - u_w$

The most common choice is to use net stress  $\sigma - u_a$  and matric suction  $u_a - u_w$  as the two independent stress state variables.

## Theory of shear strength

The proposed shear strength equation (Fredlund et al. 1978) for an unsaturated soil has the following form:

$$[1] \quad \tau_{ff} = c' + (\sigma_{ff} - u_{af}) \tan \phi' + (u_a - u_w)_f \tan \phi^b$$

where:

$\tau_{ff}$  = shear stress on the failure plane at failure,  
 $c'$  = intercept of the “extended” Mohr-Coulomb failure envelope on the shear stress axis when the net normal stress and the matric suction at failure are equal to zero. It is also referred to as the “effective cohesion”,

$(\sigma_{ff} - u_{af})$  = net normal stress on the failure plane at failure,

$\sigma_{ff}$  = total normal stress on the failure plane at failure,

$u_{af}$  = pore-air pressure at failure,

$\phi'$  = angle of internal friction associated with the net normal stress state variable  $(\sigma_{ff} - u_{af})$ ,

$(u_a - u_w)_f$  = matric suction at failure,

$u_{wf}$  = pore-water pressure at failure, and

$\phi^b$  = angle indicating the rate of change in shear strength relative to changes in matric suction,  $(u_a - u_w)_f$ .

# Shear strength for Unsaturated Soils

Relation between  $\chi$  e  $\phi^b$

$$\sigma'_f = (\sigma - u_a)_f + \chi_f (u_a - u_w)_f$$

$$\tau_f = c' + \sigma'_n \tan \phi'$$

$$\tau_f = c' + [(\sigma - u_a)_f + \chi_f (u_a - u_w)_f] \tan \phi'$$

$$\tau_f = c' + (\sigma - u_a)_f \tan \phi' + \chi_f (u_a - u_w)_f \tan \phi'$$

$$\tau_f = c' + (\sigma - u_a)_f \tan \phi' + (u_a - u_w)_f \tan \phi^b$$

$$\chi_f (u_a - u_w)_f \tan \phi' = (u_a - u_w)_f \tan \phi^b$$

$$\chi_f = \frac{\tan \phi^b}{\tan \phi'}$$

$$m = \chi \sin \phi'$$

Bishop & Blight (1963)

Terzaghi (1925)

$$\tau_f = c' + (\sigma - u_w)_f \tan \phi'$$

Bishop (1959)

$$\tau_f = c' + [(\sigma - u_a)_f + \chi_f (u_a - u_w)_f] \tan \phi'$$

Fredlund et al. (1978)

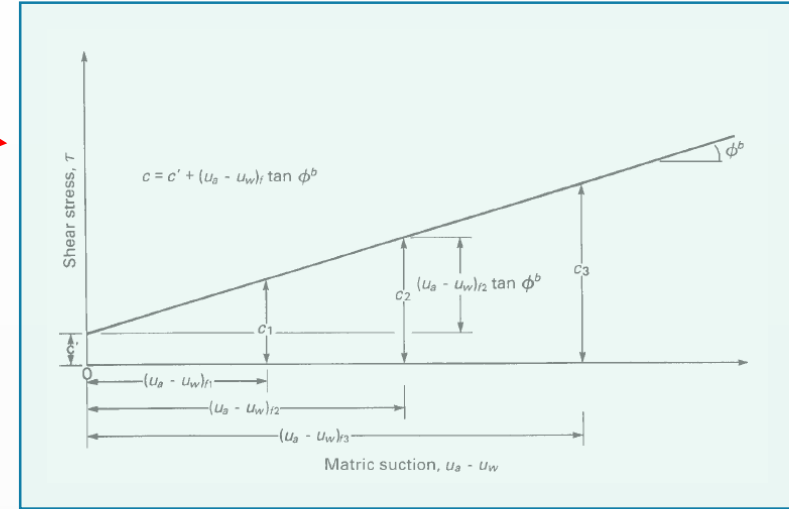
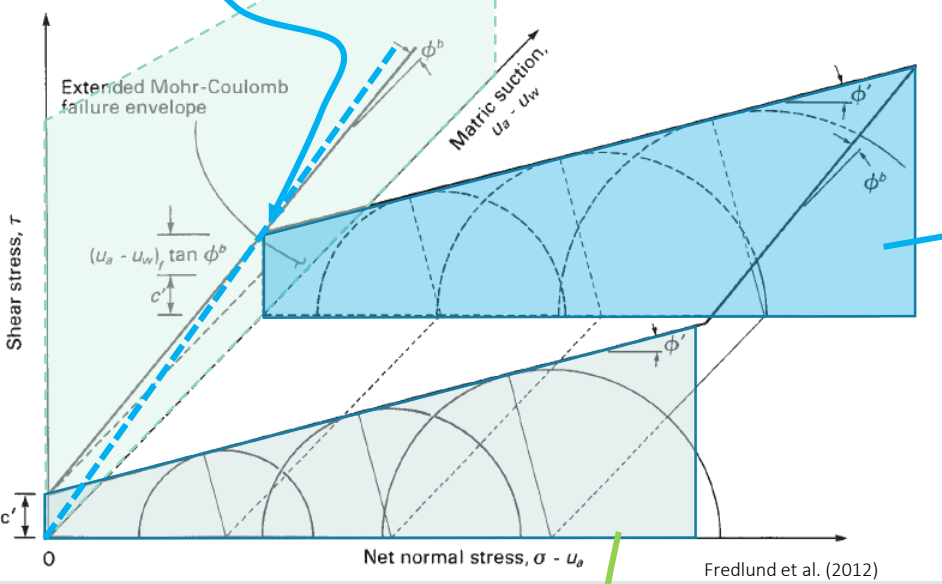
$$\tau_f = c' + (\sigma - u_a)_f \tan \phi' + (u_a - u_w)_f \tan \phi^b$$

$$\tan \phi^b = \chi_f \tan \phi'$$

# Failure envelopes for unsaturated soils

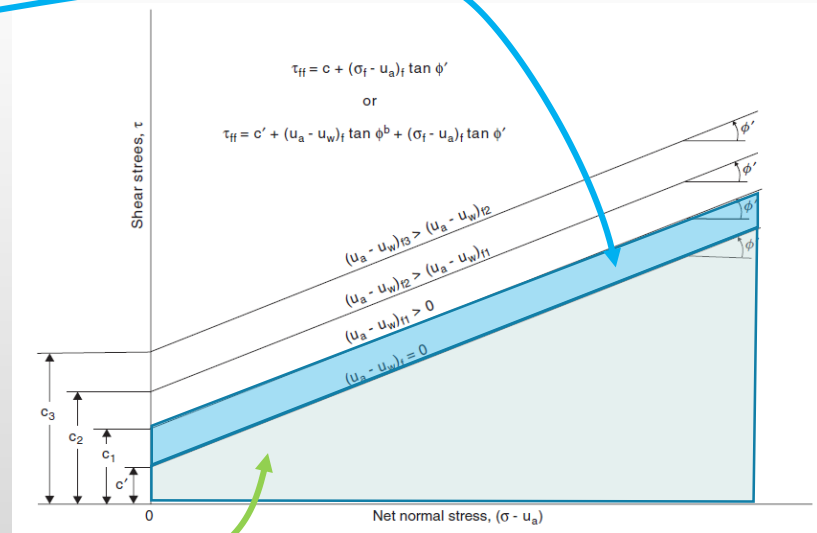


$$u_a - u_w = \text{suction}$$

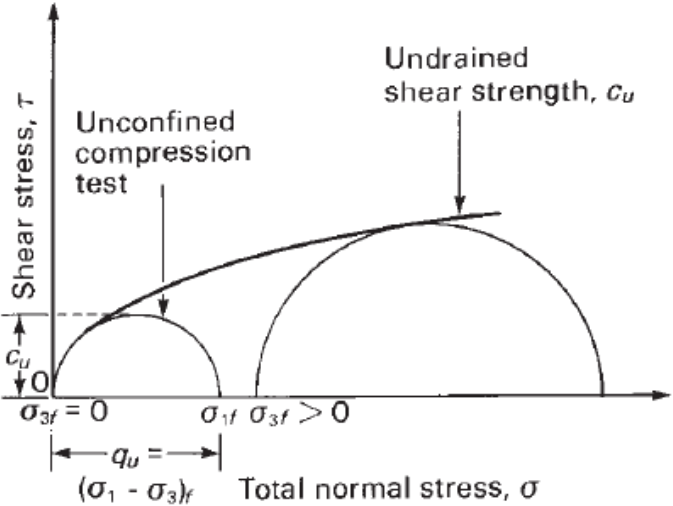


$$c = c' + (\text{suction}) \tan \phi^b$$

$$\tau = c + \sigma * \tan \phi'$$

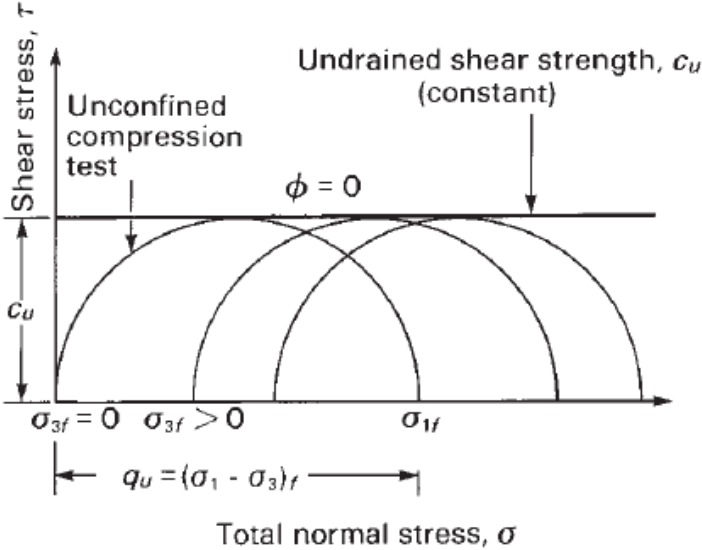


# Undrained



(a)

Initial condition – Unsaturated

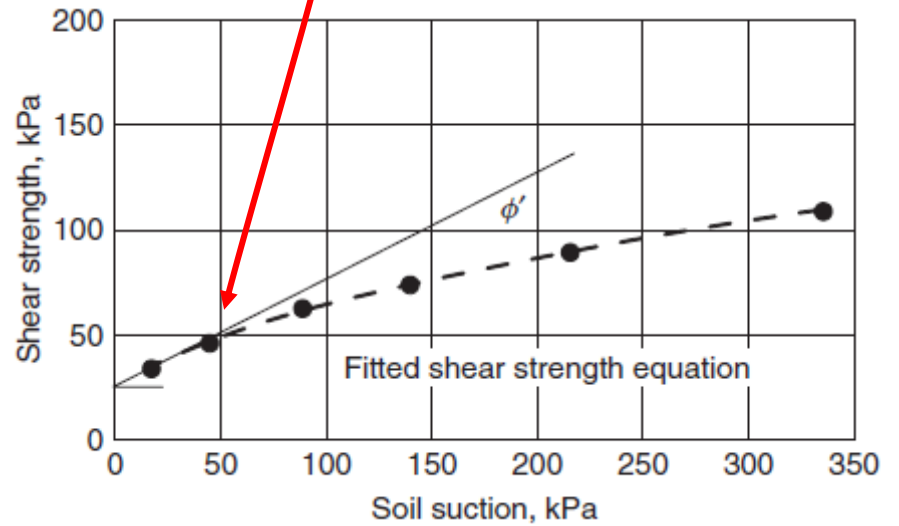
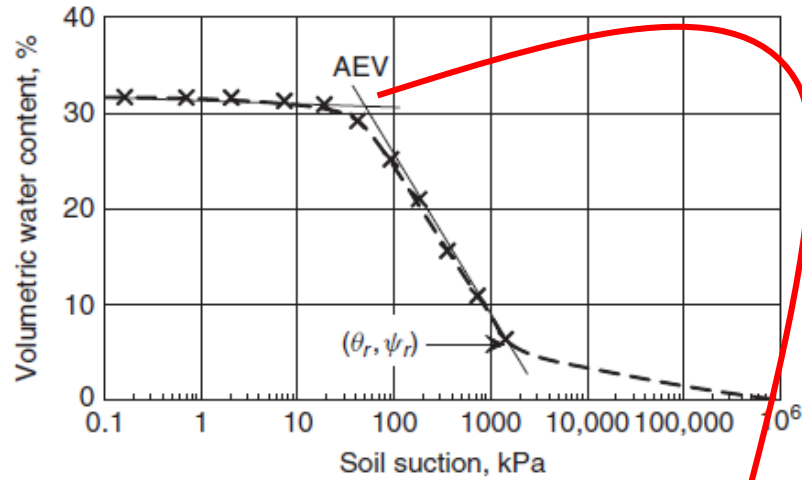


(b)

Initial condition – Saturated

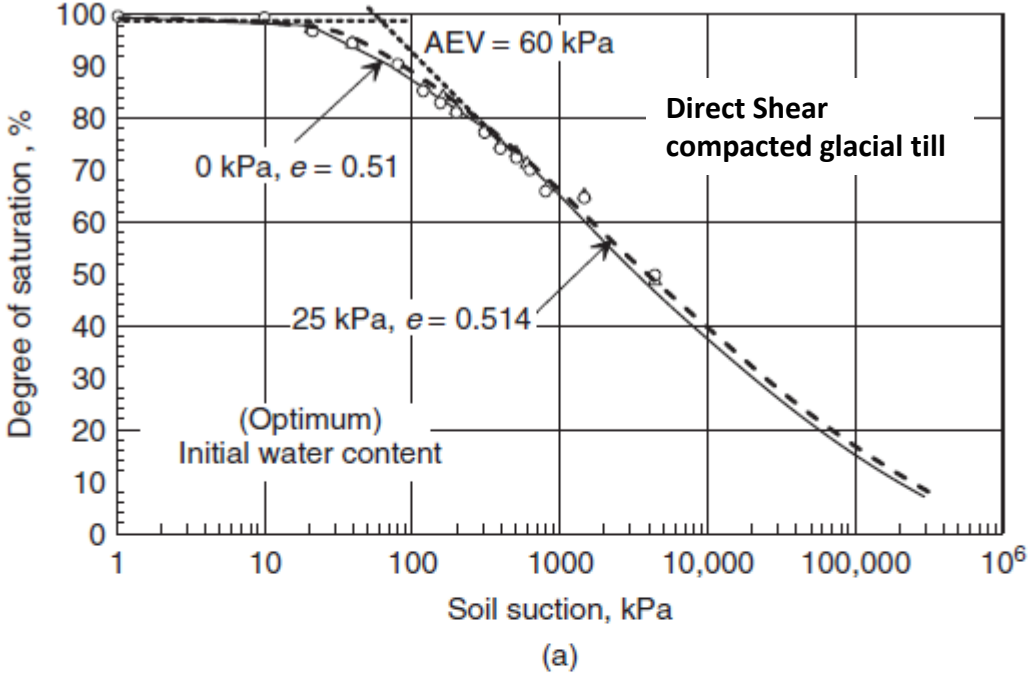
# Curved shear strength envelope in relation to suction

Soil water retention curve (SWRC)

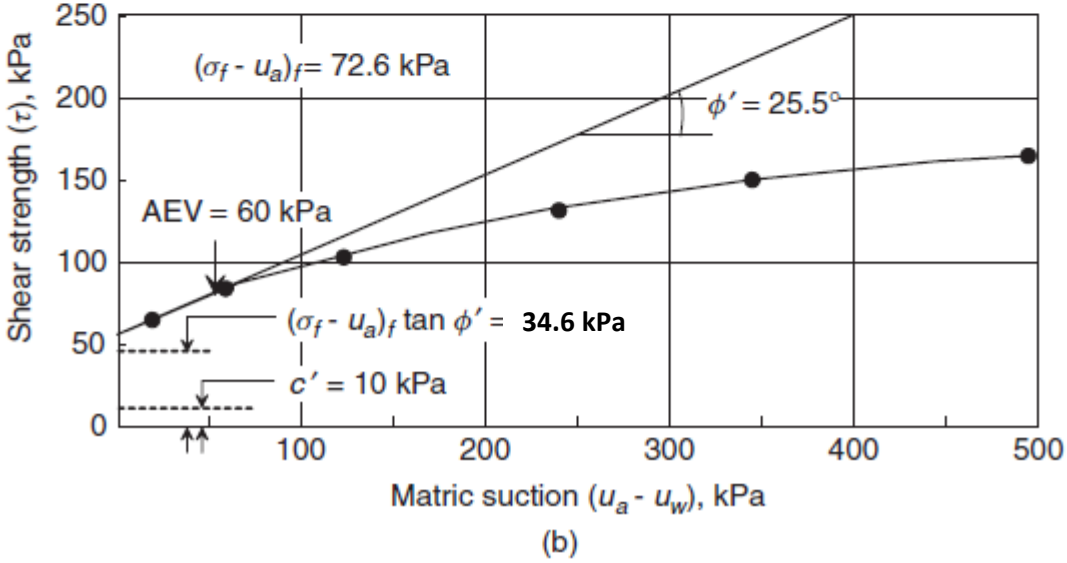




# Curved shear strength envelope in relation to suction

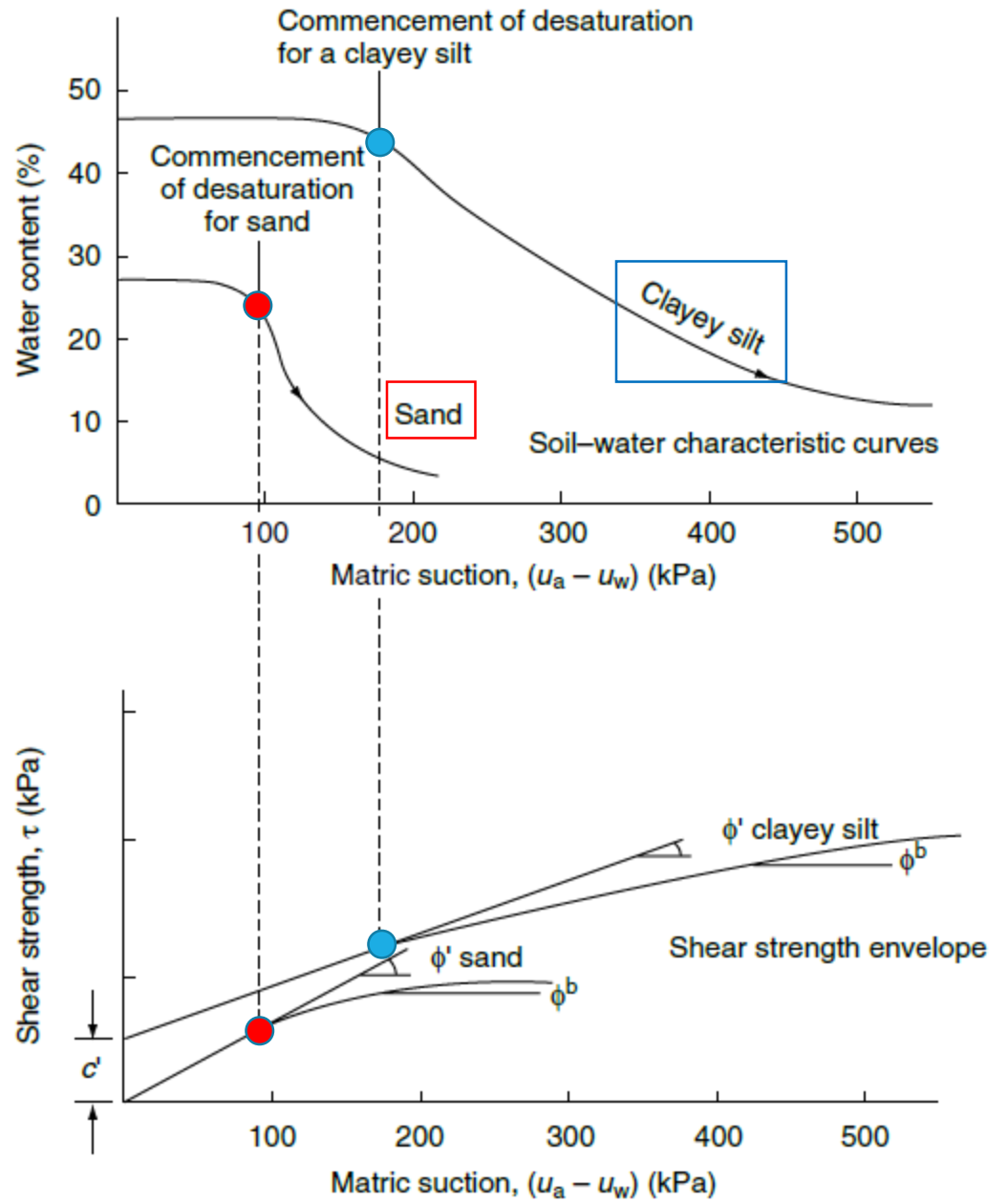


Soil water retention curve (SWRC)



Vanapalli et al.(1996) & Gan et al.(1988)

# Soil type and the relation of shear strength and suction

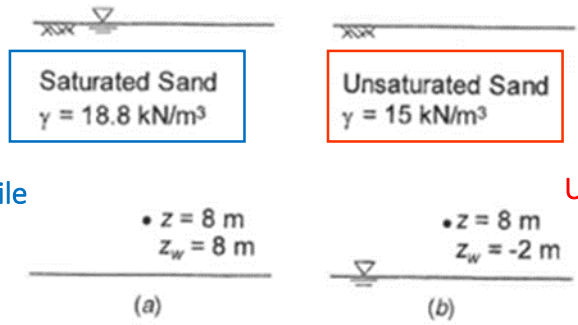


**Table 11.1 Experimental Values Measured for  $\phi^b$**

Soil Type	$c'$ (kPa)	$\phi'$ (deg)	$\phi^b$ (deg)	Test Procedure	Reference
Compacted shale; $w = 18.6\%$	15.8	24.8	18.1	Constant water content triaxial	Bishop et al. (1960)
Boulder clay; $w = 11.6\%$	9.6	27.3	21.7	Constant water content triaxial	Bishop et al. (1960)
Dhanauri clay; $w = 22.2\%$ , $\rho_d = 1580 \text{ kg/m}^3$	37.3	28.5	16.2	Consolidated drained triaxial	Satija (1978)
Dhanauri clay; $w = 22.2\%$ , $\rho_d = 1478 \text{ kg/m}^3$	20.3	29.0	12.6	Constant drained triaxial	Satija (1978)
Dhanauri clay; $w = 22.2\%$ , $\rho_d = 1580 \text{ kg/m}^3$	15.5	28.5	22.6	Consolidated water content triaxial	Satija (1978)
Dhanauri clay; $w = 22.2\%$ , $\rho_d = 1478 \text{ kg/m}^3$	11.3	29.0	16.5	Constant water content triaxial	Satija (1978)
Madrid grey clay; $w = 29\%$	23.7	22.5 <sup>a</sup>	16.1	Consolidated drained direct shear	Escario (1980)
Undisturbed decomposed granite; Hong Kong	28.9	33.4	15.3	Consolidated drained multistage triaxial	Ho and Fredlund (1982a)
Undisturbed decomposed rhyolite; Hong Kong	7.4	35.3	13.8	Consolidated drained multistage triaxial	Ho and Fredlund (1982a)
Tappen-Notch Hill silt; $w = 21.5\%$ , $\rho_d = 1590 \text{ kg/m}^3$	0.0	35.0	16.0	Consolidated drained multistage triaxial	Krahn et al. (1989)
Compacted glacial till; $w = 12.2\%$ , $\rho_d = 1810 \text{ kg/m}^3$	10.0	25.3	7–25.5	Consolidated drained multistage direct shear	Gan et al. (1988)

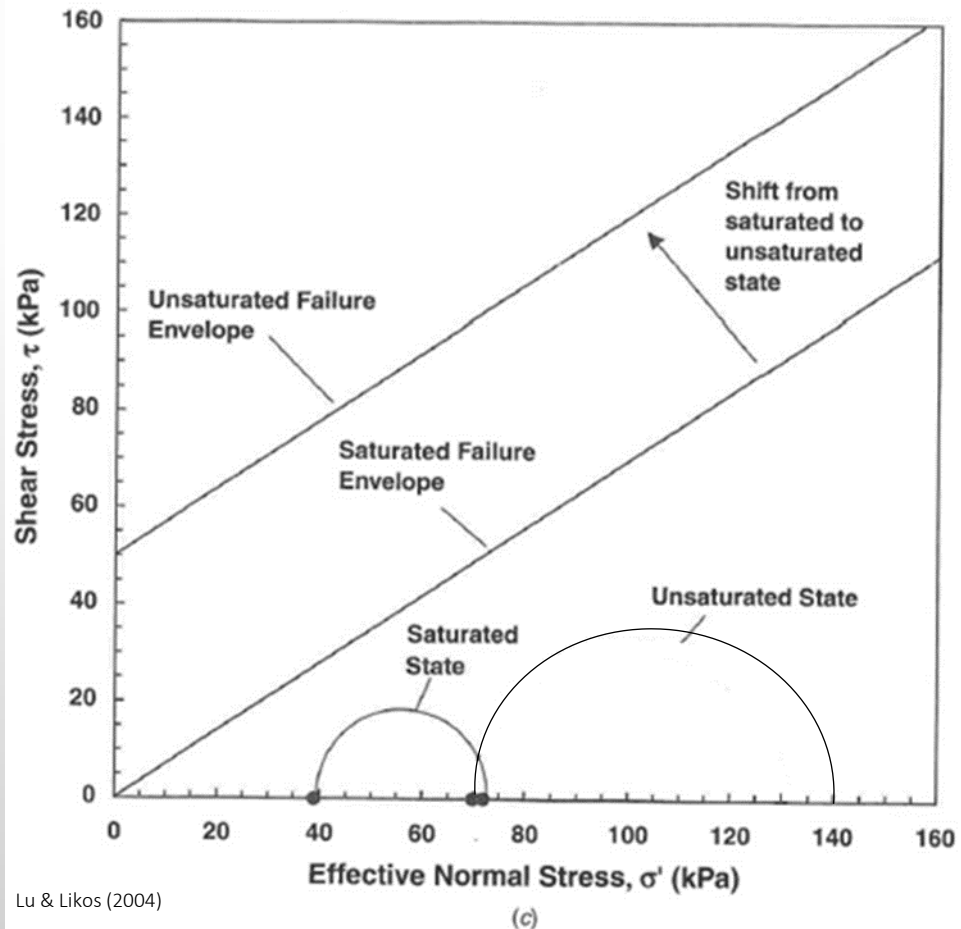
<sup>a</sup> Average value.

# Conceptual stress analysis for Sandy soil



Saturated profile

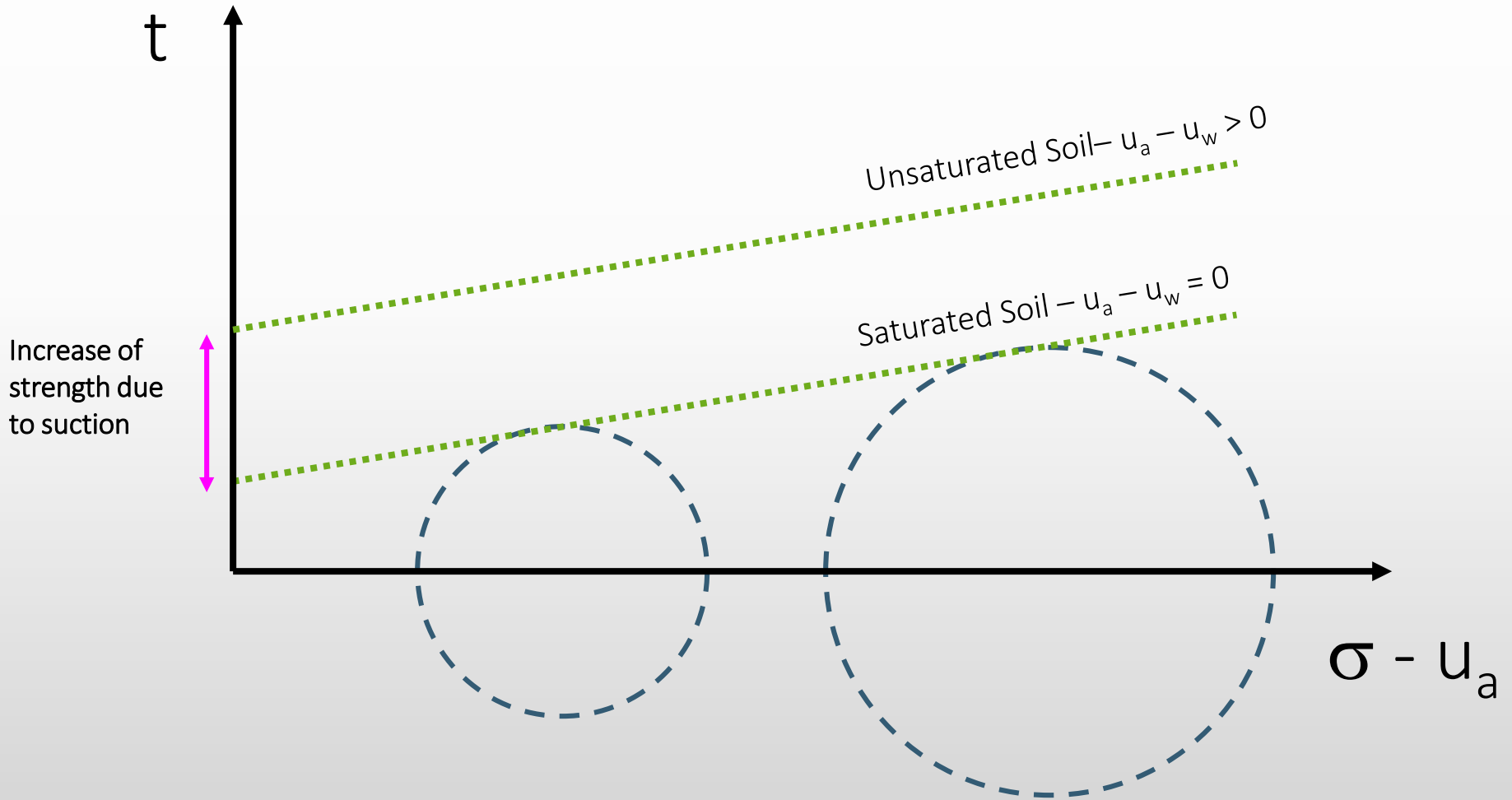
Unaturated profile



States of stress at 8 m and Mohr-Coulomb envelope. (hypothesis – the soil remains saturated at 8m depth)

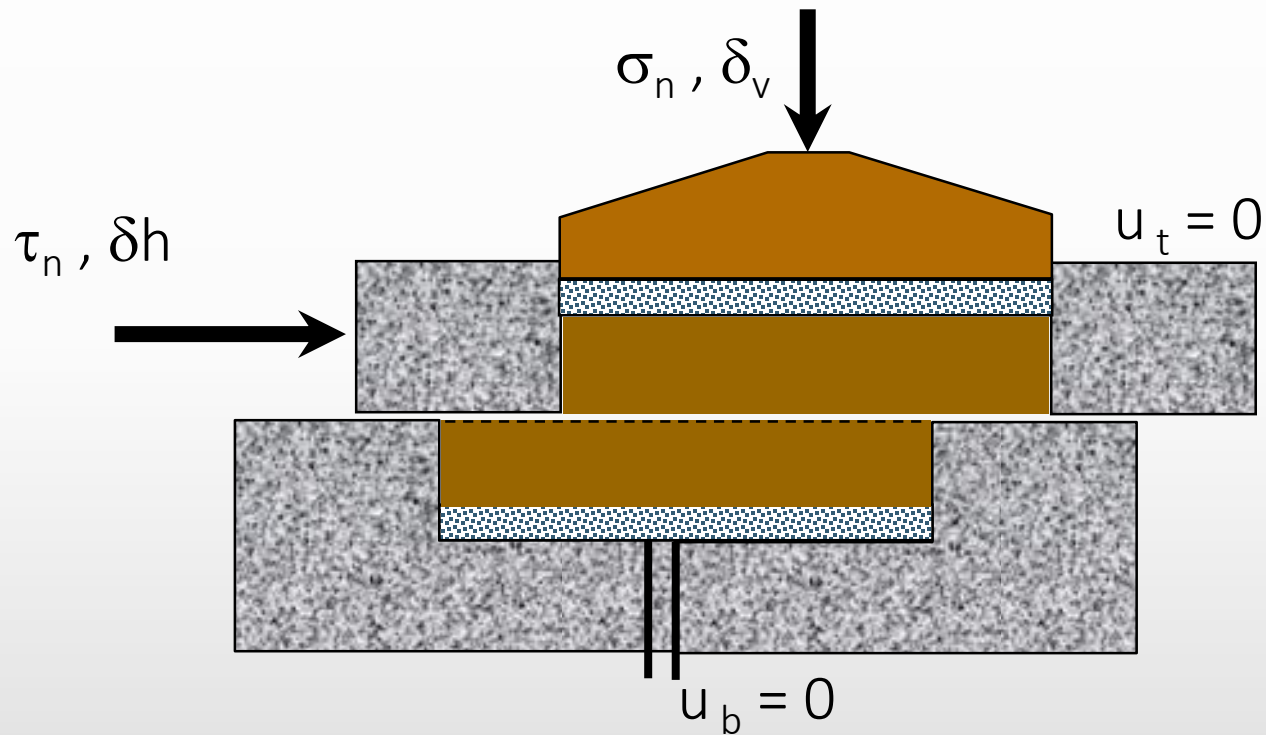
Lu & Likos (2004)

Na maioria dos casos o critério de Mohr-Coulomb é utilizado

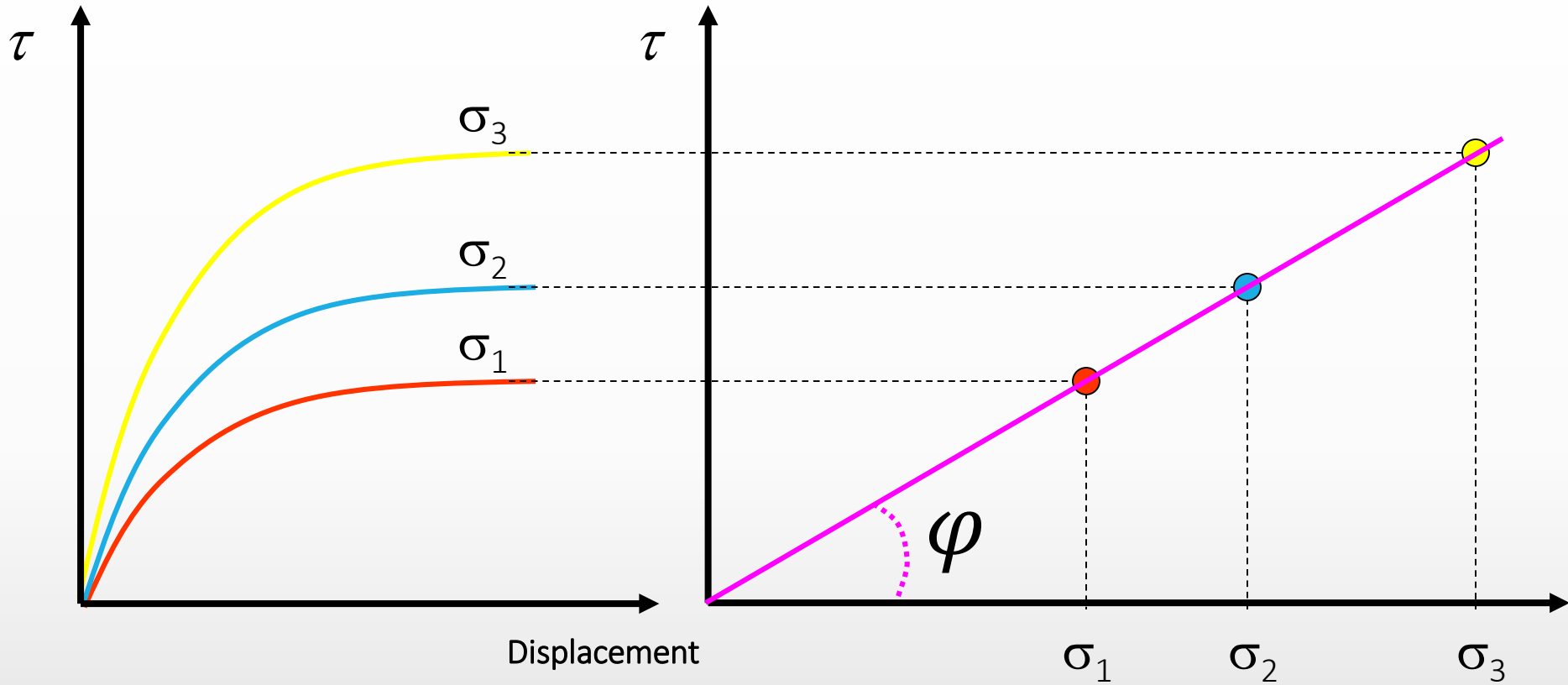


# Shear Strength Test

## Direct Shear



# Saturated Soil

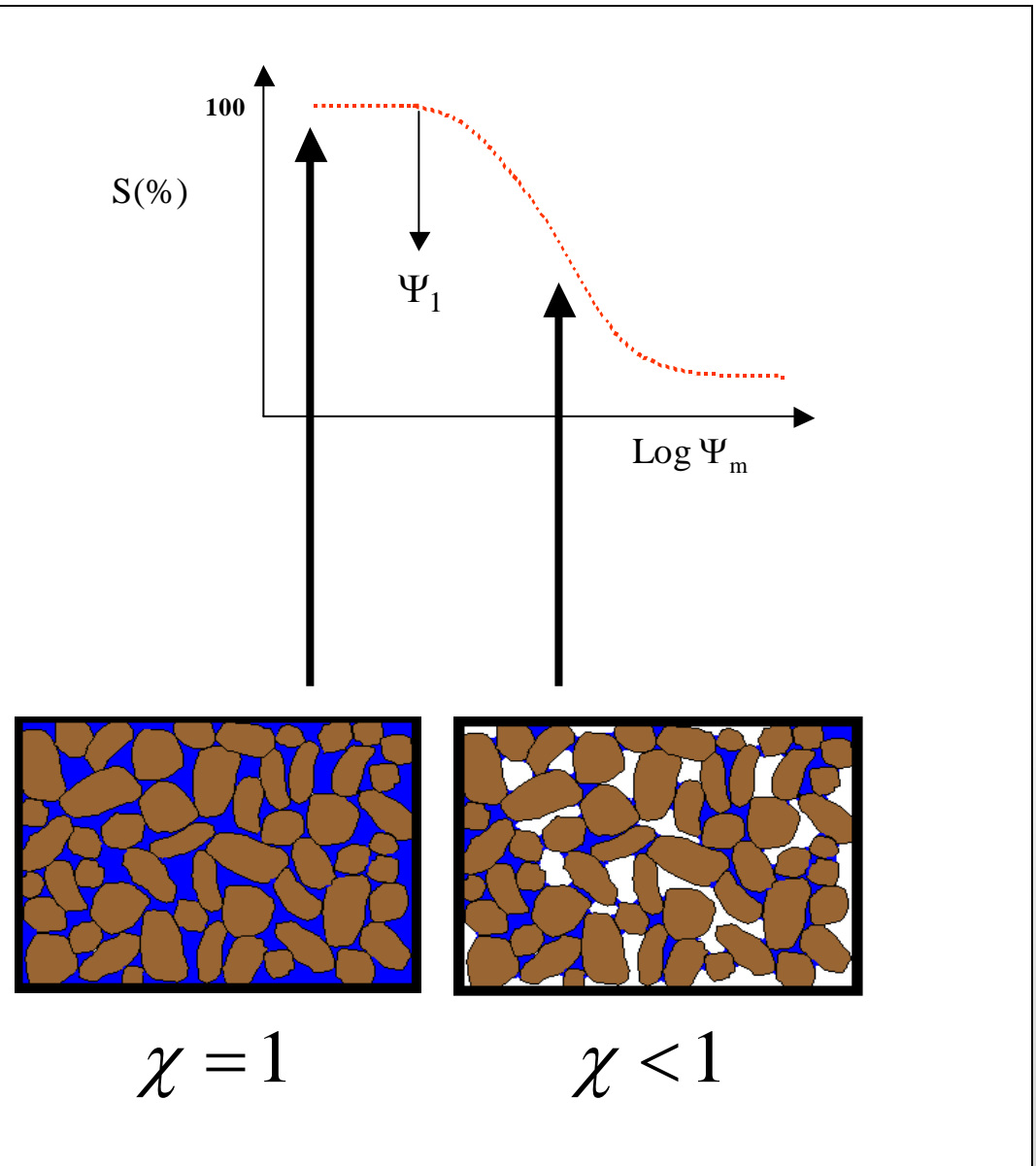


$$\tau = \sigma \tan \varphi$$

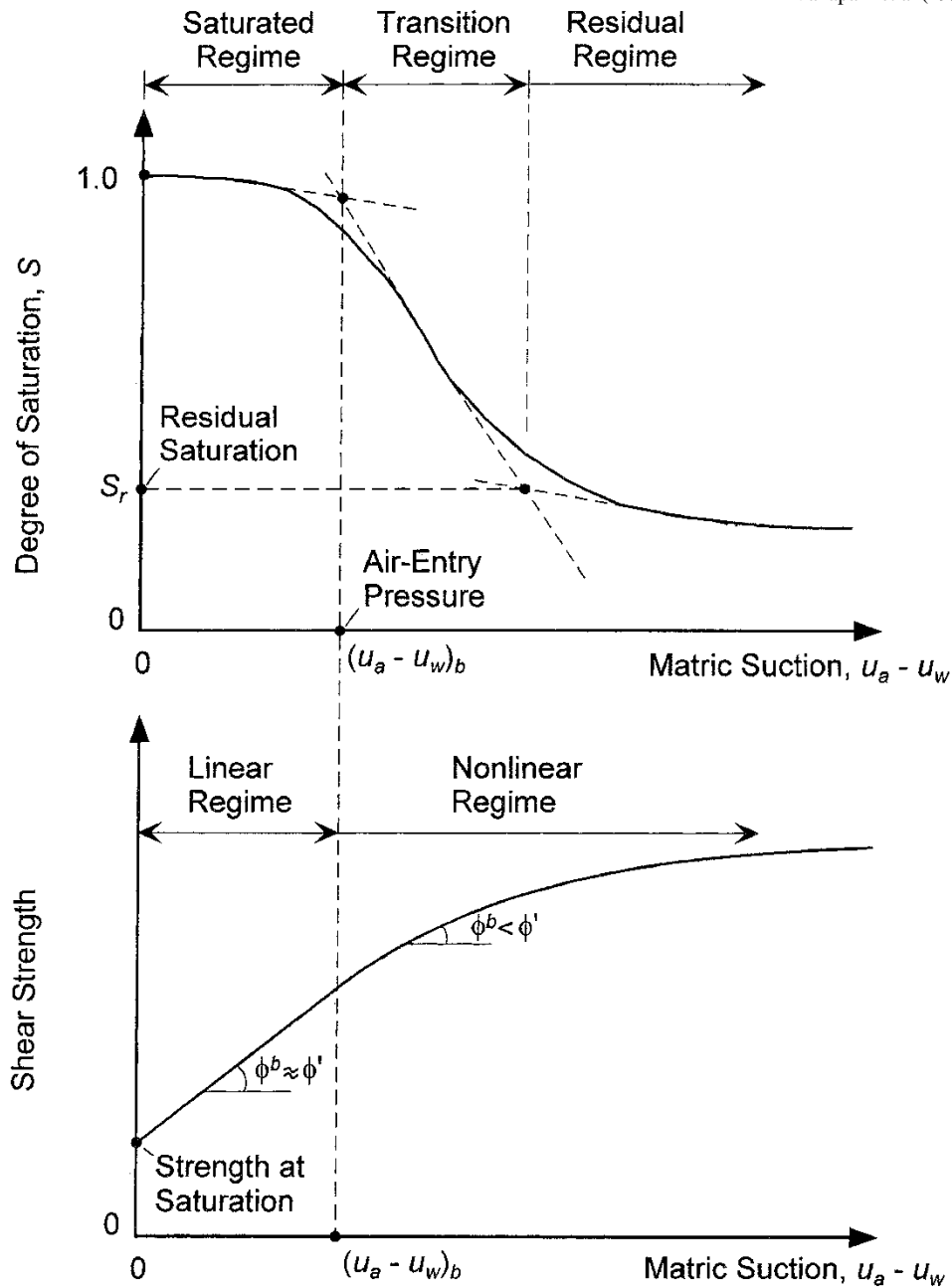
# Unsaturated Soil

$$\sigma' = \sigma - u_a + \chi(u_a - u_w)$$

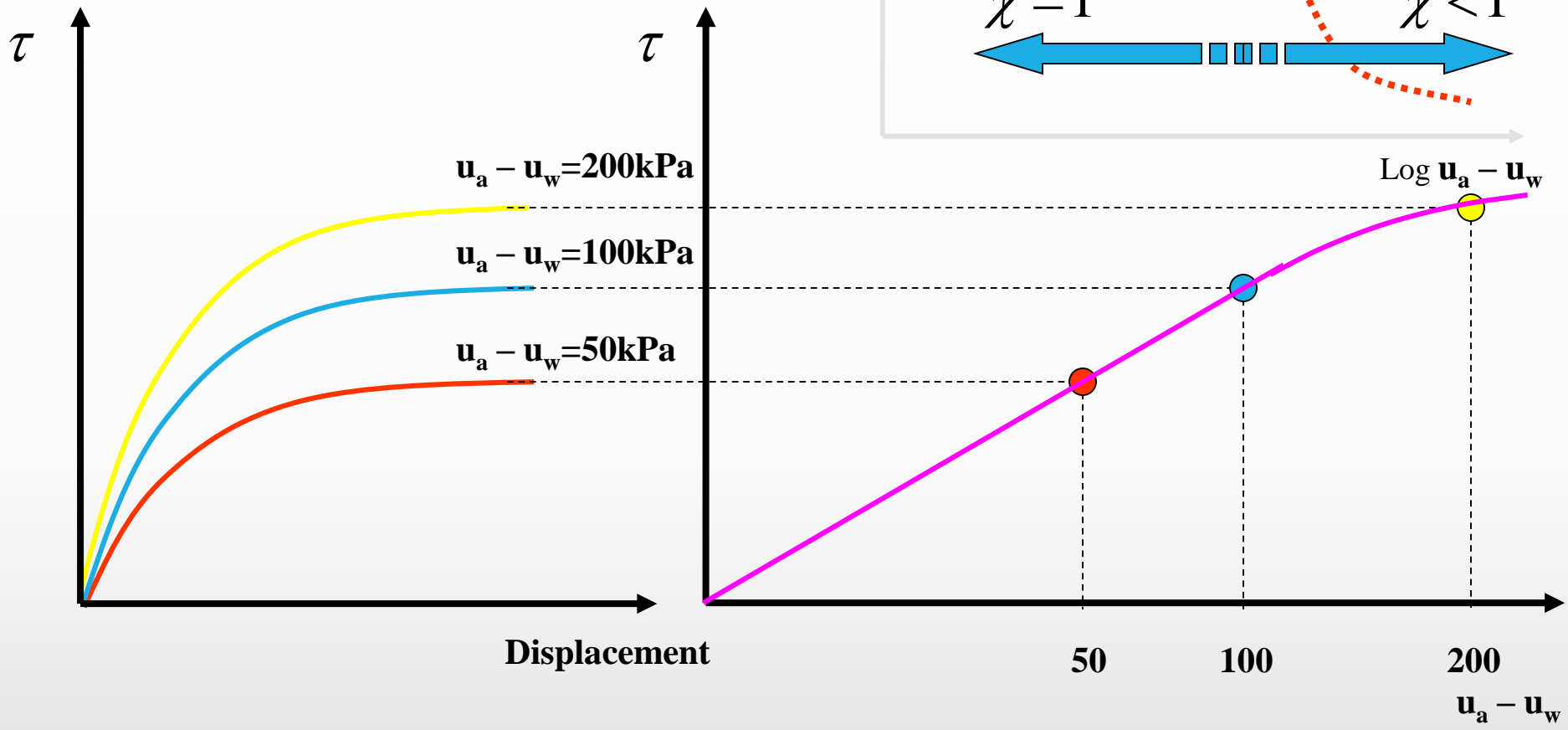
Although the principle of effective stress does not apply to all situations in unsaturated soils, it is useful in the shear strength of the case.







**Conceptual relationship between the retention curve and unsaturated soil strength**



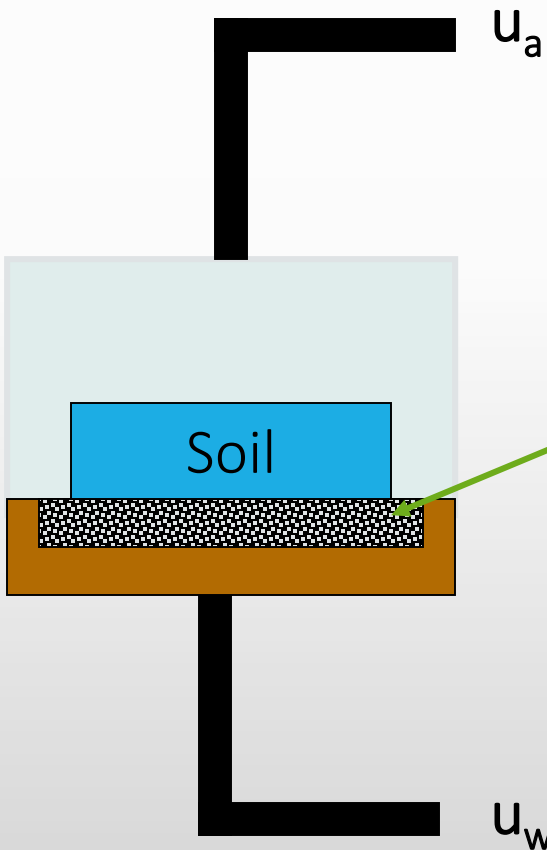
$$\sigma' = \sigma - u_a + \chi(u_a - u_w)$$

# Test Techniques for Shear Strength

- Axis Translation technique (ATT)
- Osmotic Control
- Direct measurement of suction

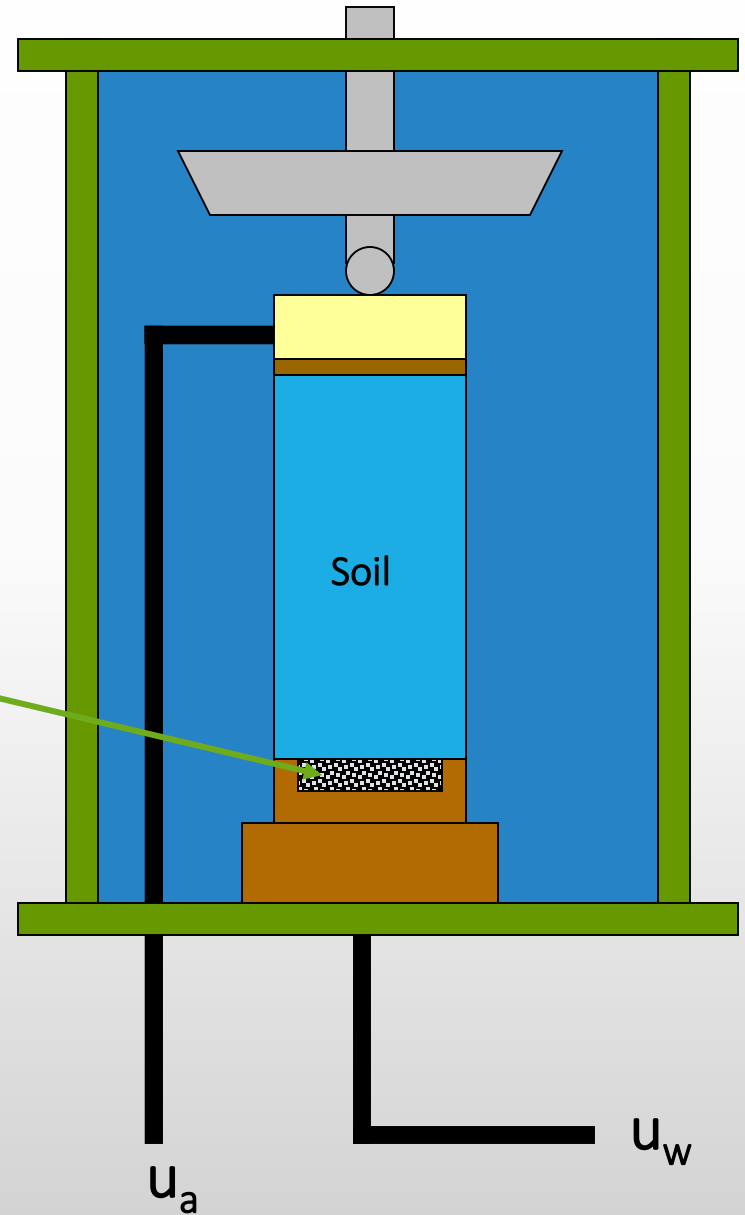
Axis translation technique for applying or control suction

Retention Curve

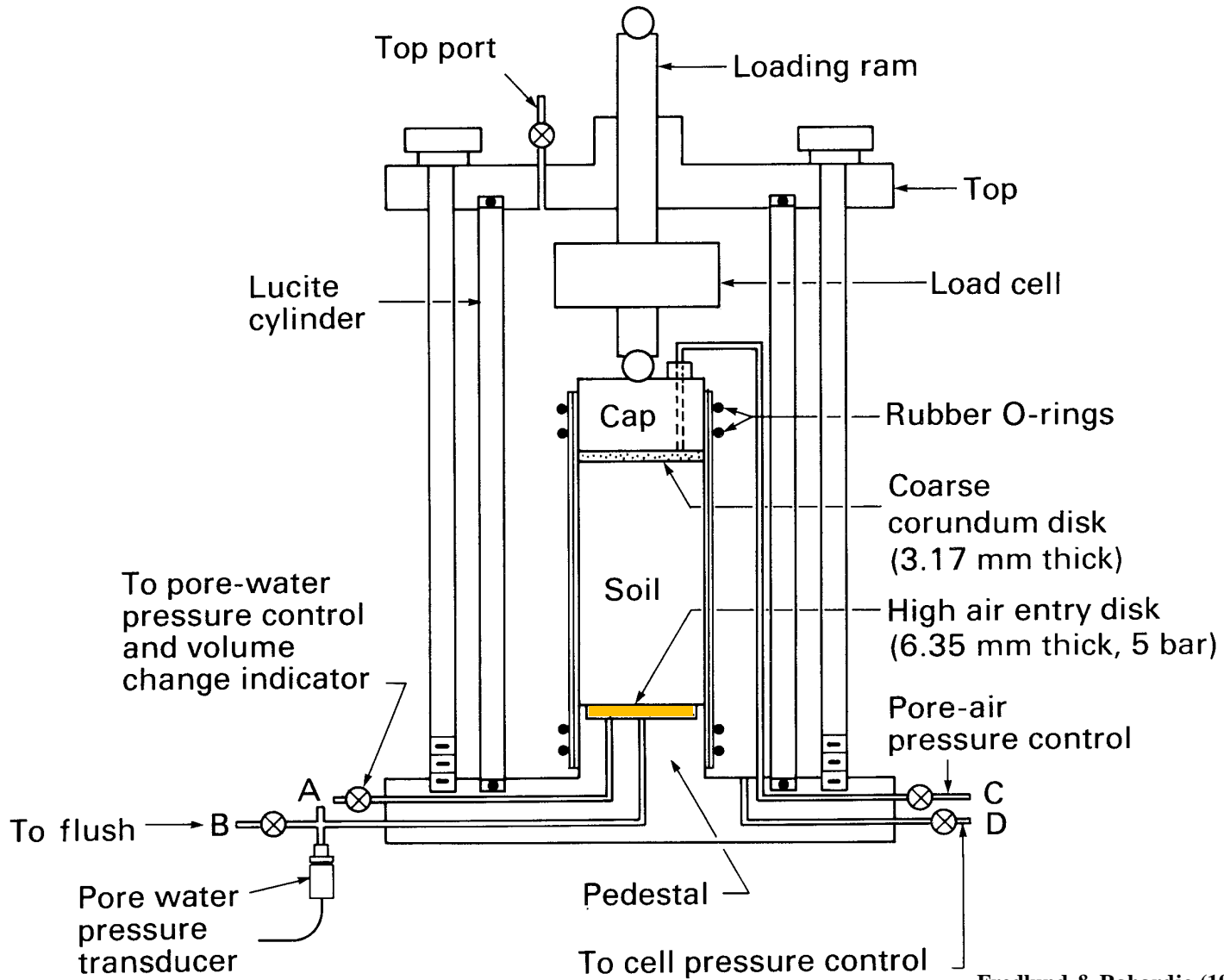


High air entry ceramic disk

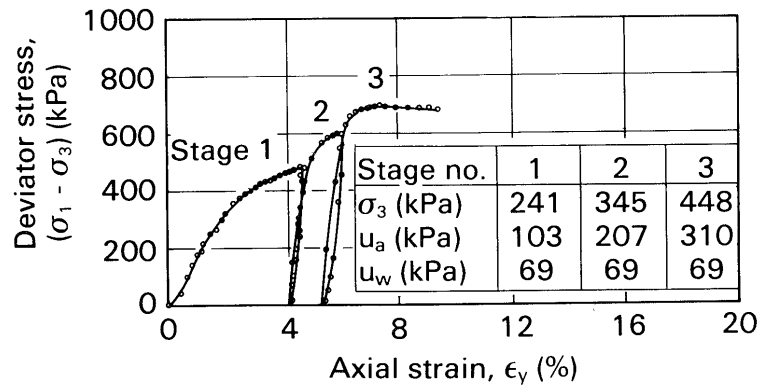
Triaxial Test



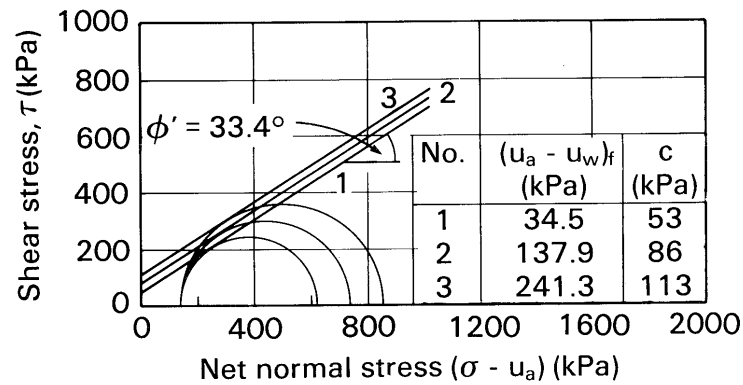
# Triaxial cell for unsaturated soils



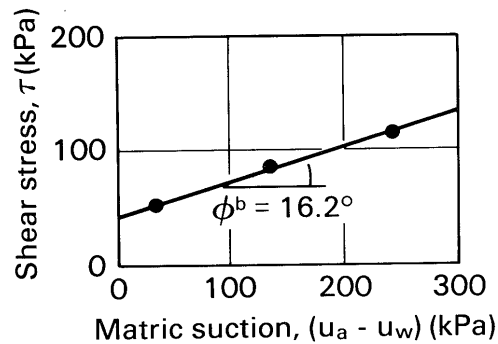
## Results for a residual soil of granite



(a)



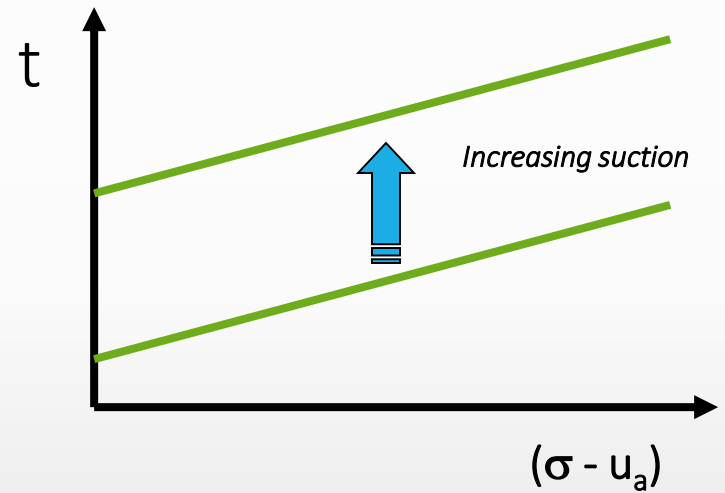
(b)



(c)

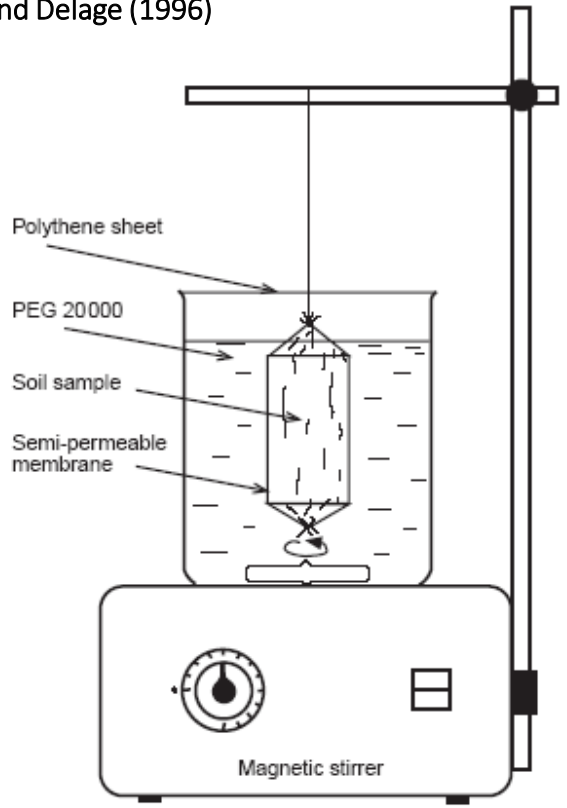
Ho and Fredlund (1982)

Multi-stage test  
(one specimen is used)



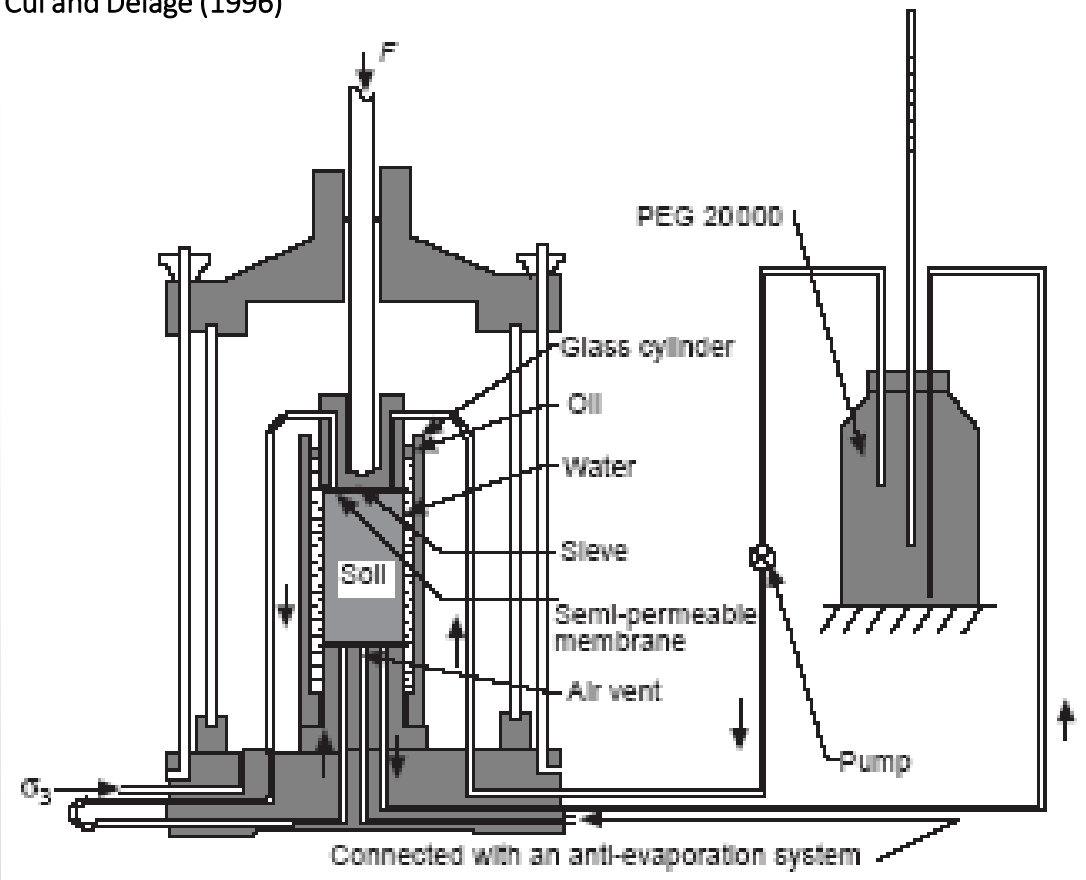
# Osmosis to induce different values of suction

Cui and Delage (1996)



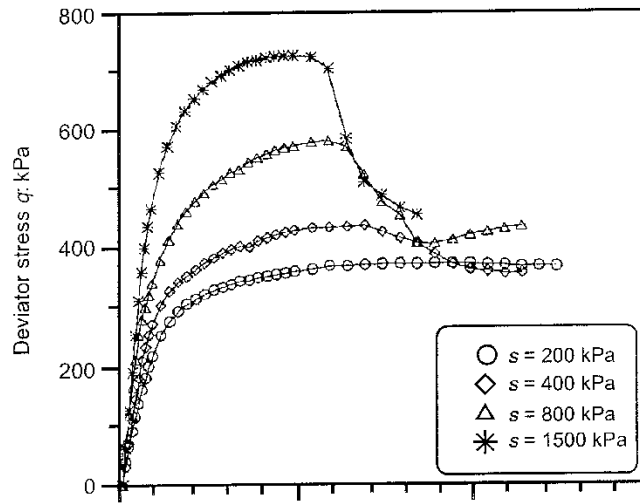
Osmotic control of suction under zero total stress

Cui and Delage (1996)

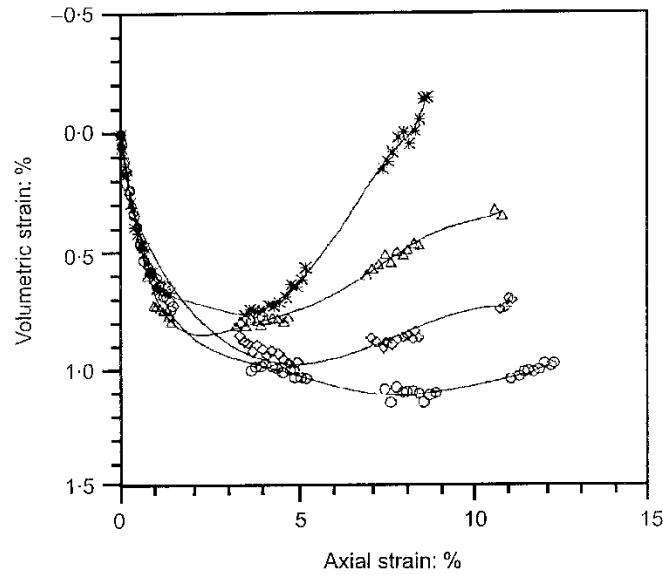


Triaxial cell with osmotic control

# Triaxial Test using Osmosis

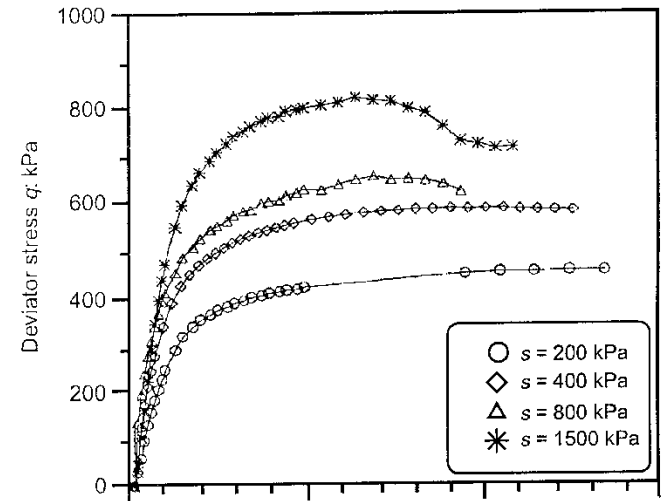


(a)

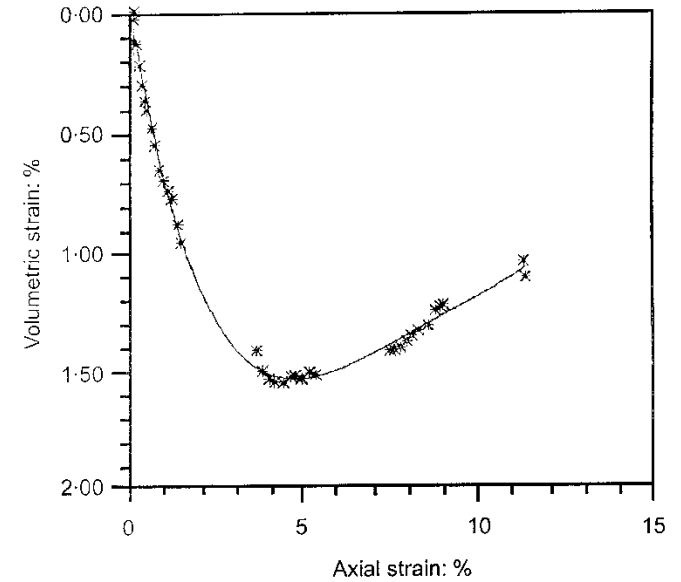


(b)

Tensão – Deformação e variação de volume para  $\sigma_3=50\text{kPa}$  e diversas succões



(a)

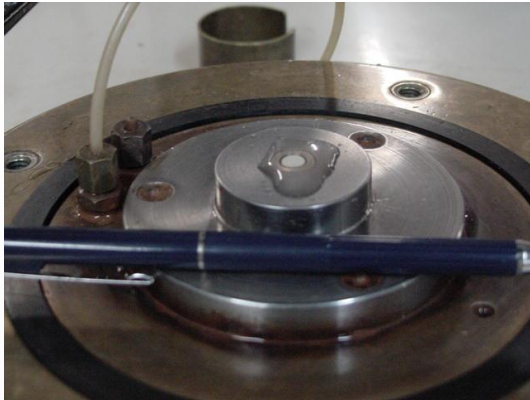


(b)

Tensão – Deformação e variação de volume para  $\sigma_3=100\text{kPa}$  e diversas succões



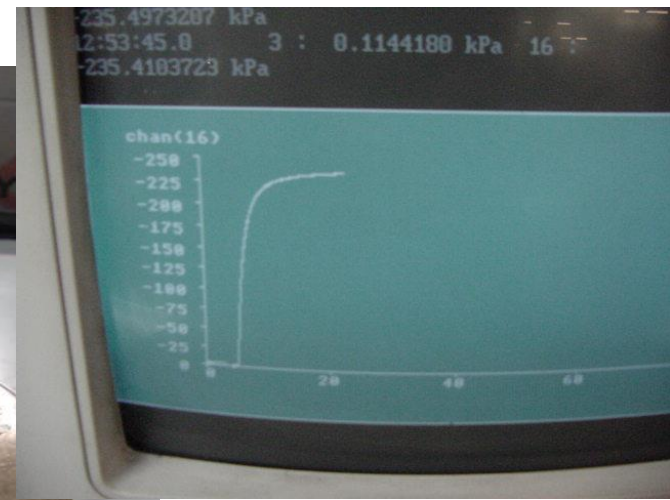
# Triaxial – Direct suction measurement



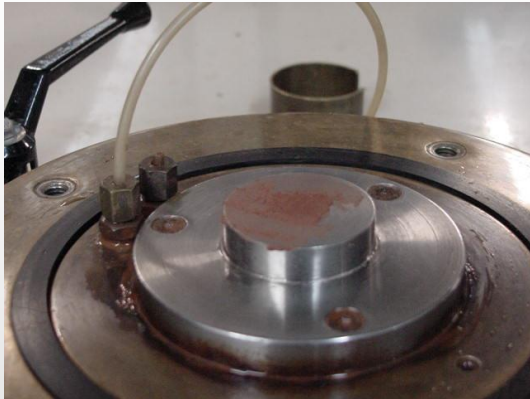
High capacity tensiometer



Initial measurement of suction



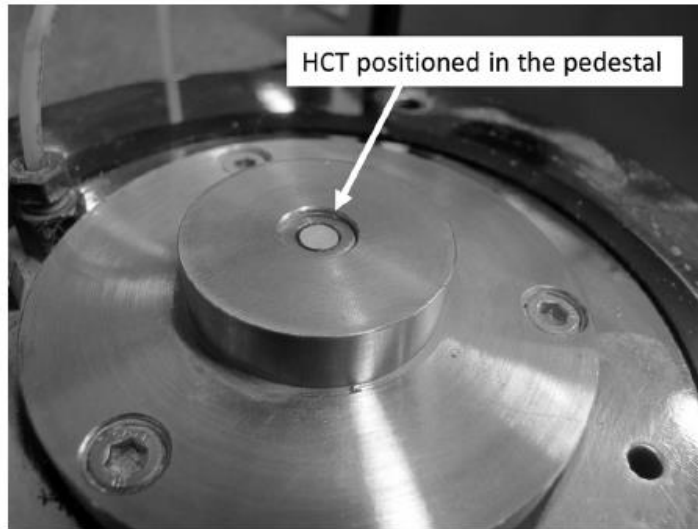
Monitoring the initial soil suction



Paste for improving contact

Specimen prepared for the triaxial test with direct suction measurement.

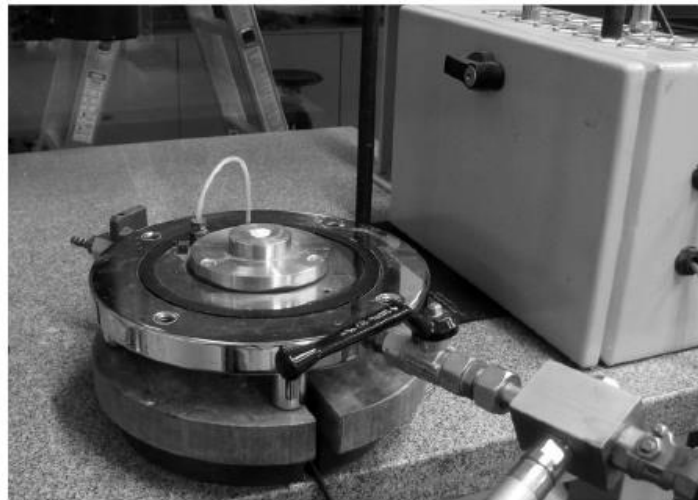




(a)



(b)



(c)

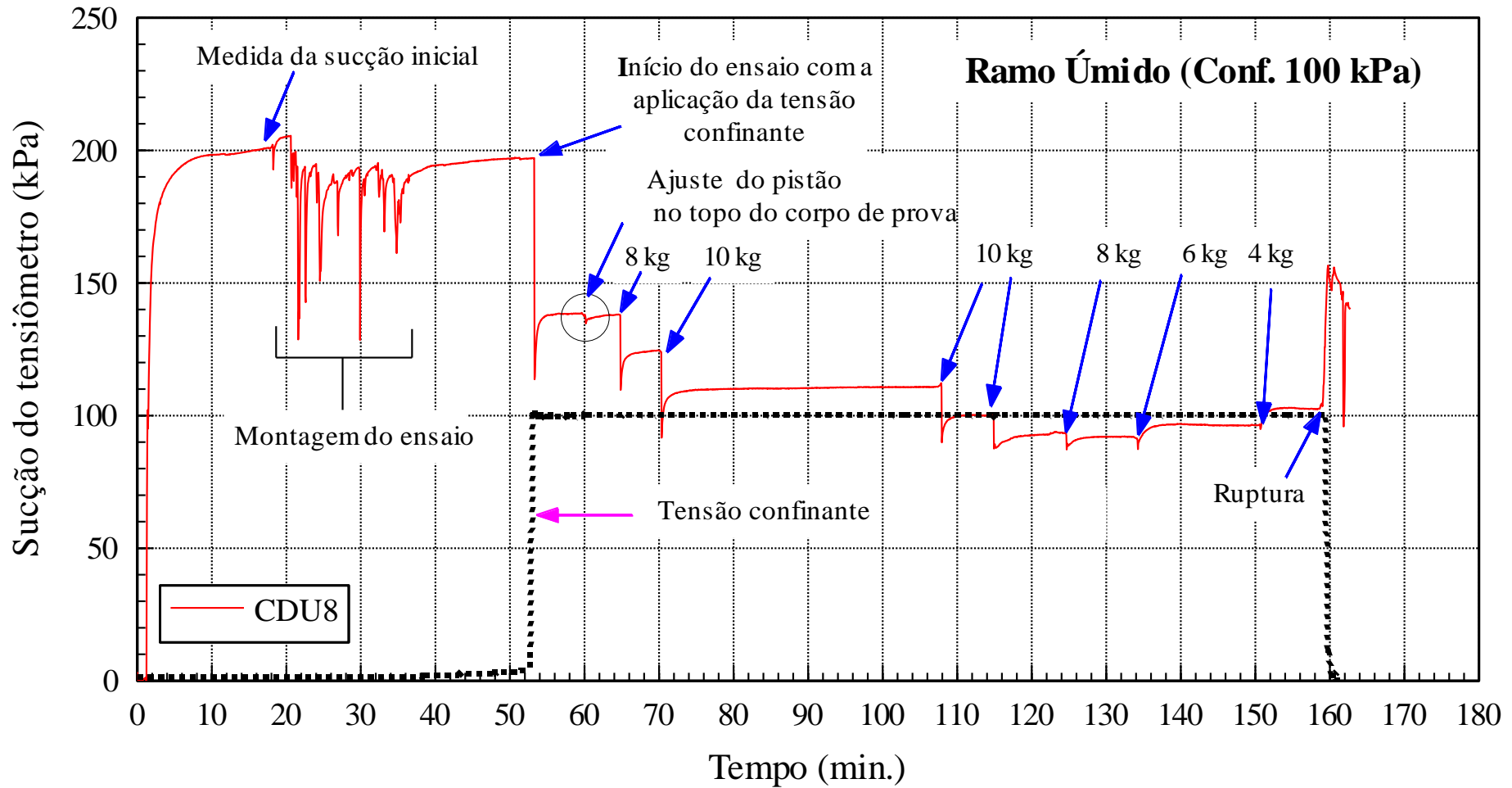


(d)

**Fig. 4.** Setup for the CW triaxial tests with suction measurement: (a) HCT at the base of the triaxial cell; (b) kaolin paste to improve contact with the HCT; (c) general view of the base cell; (d) placement of the specimen for testing

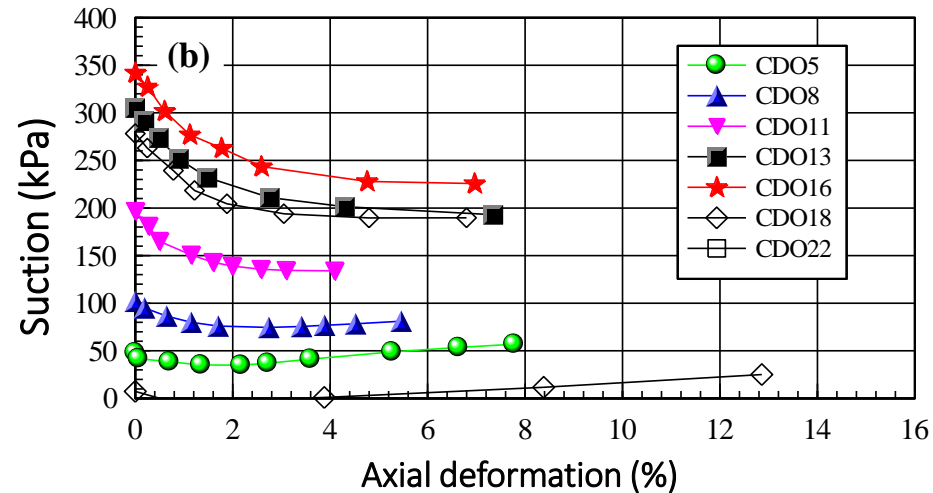
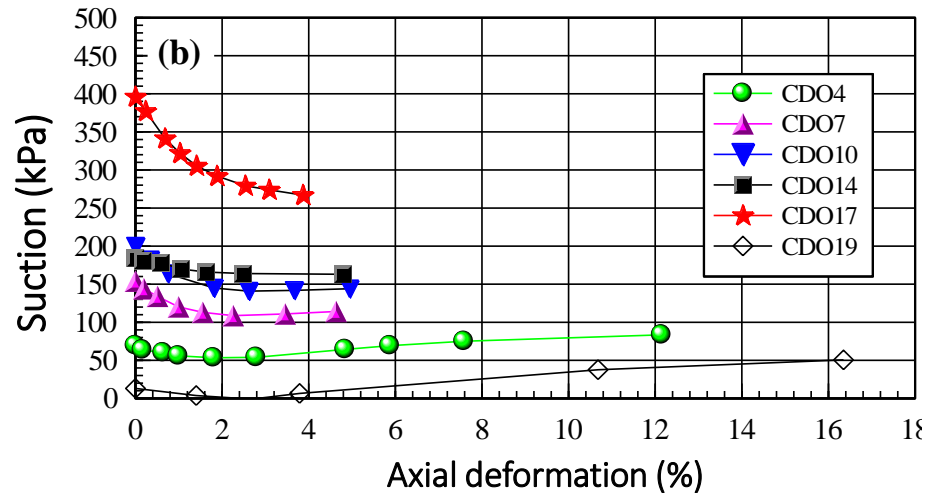
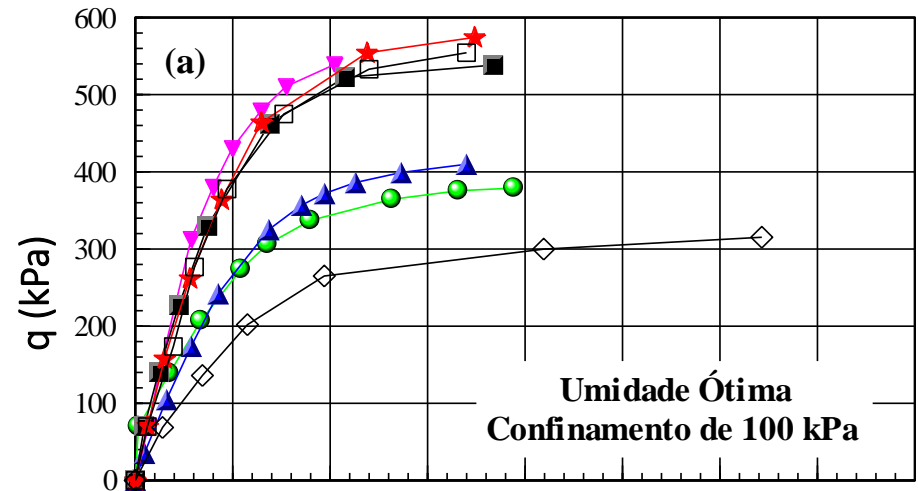
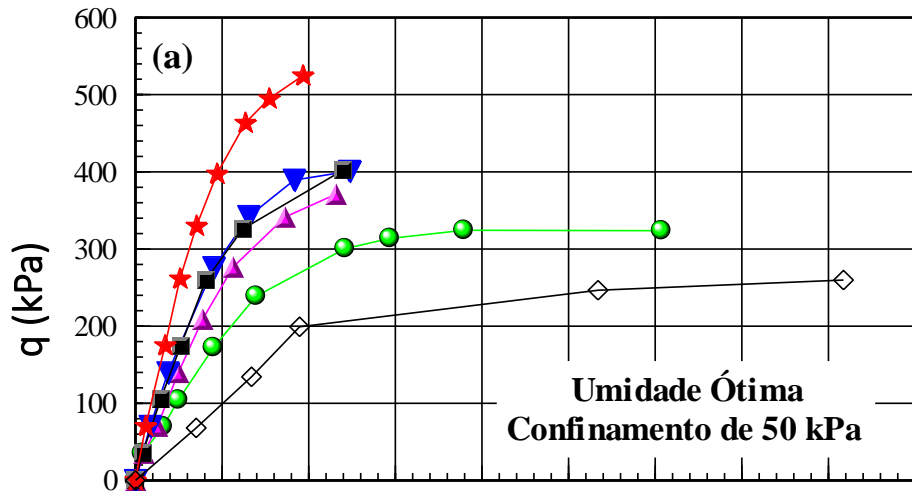
# Following up of a CW test

## Effect of the setup and load application on the suction



# CW Triaxial Test under Stress Control

## Specimens with different initial suction



# CW Triaxial Test under Stress Control Specimens with different initial suction

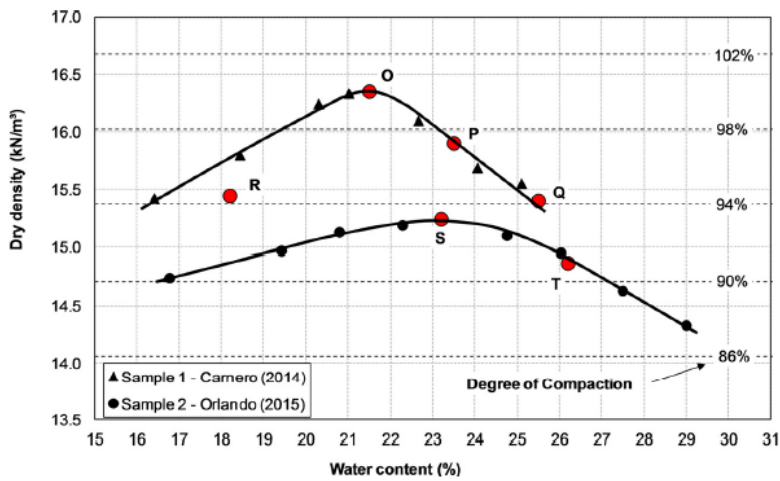


Fig. 2. Compaction curves for the residual soil from USPES

Marinho et al. (2016)

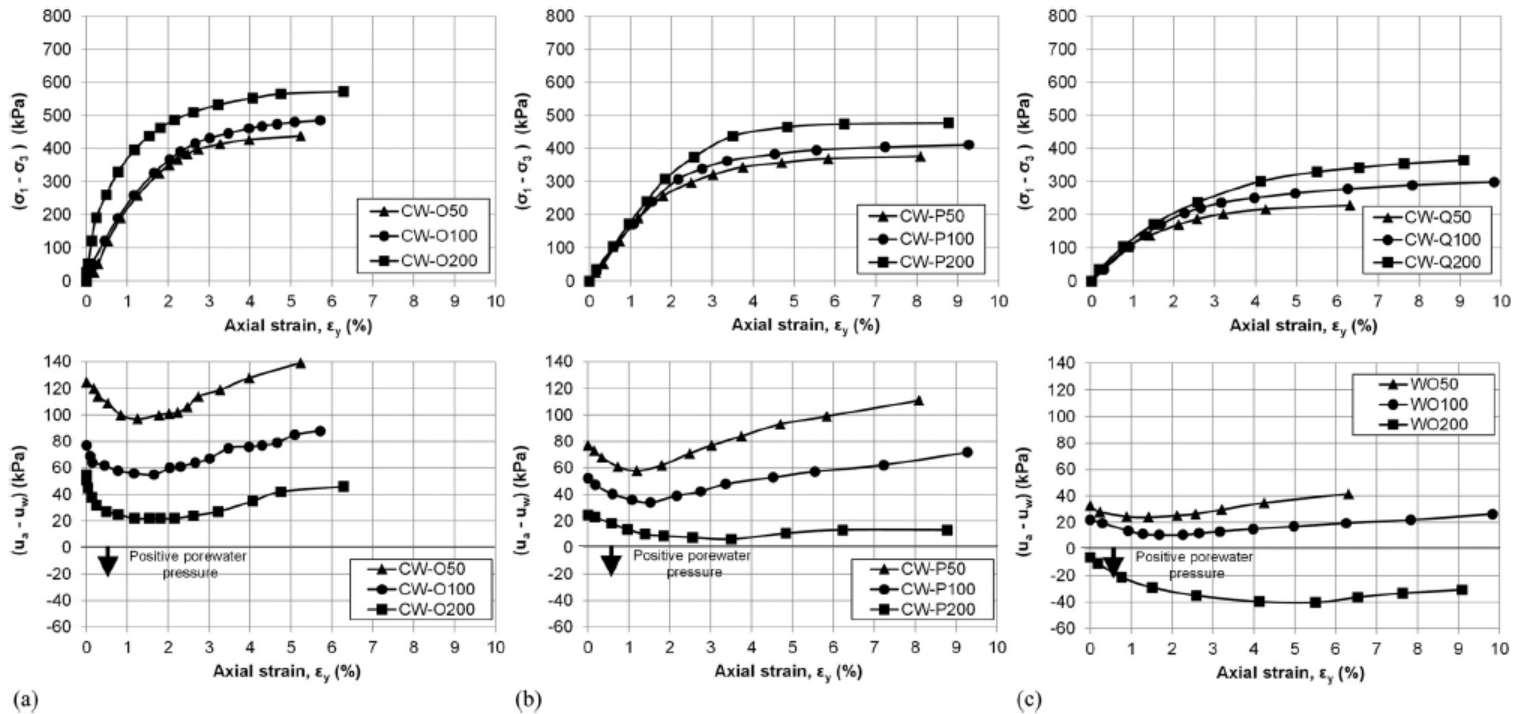


Fig. 9. Results of CW triaxial tests with suction measurement for (a) Point O, (b) Point P, and (c) Point Q

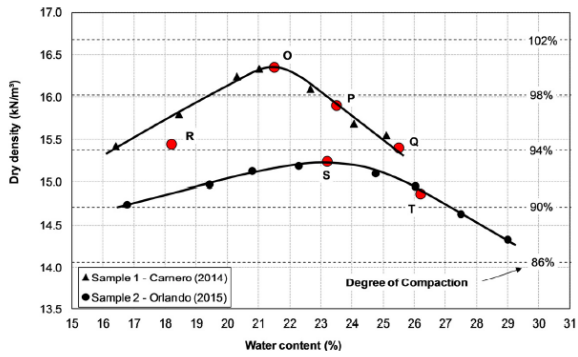


Fig. 2. Compaction curves for the residual soil from USPES

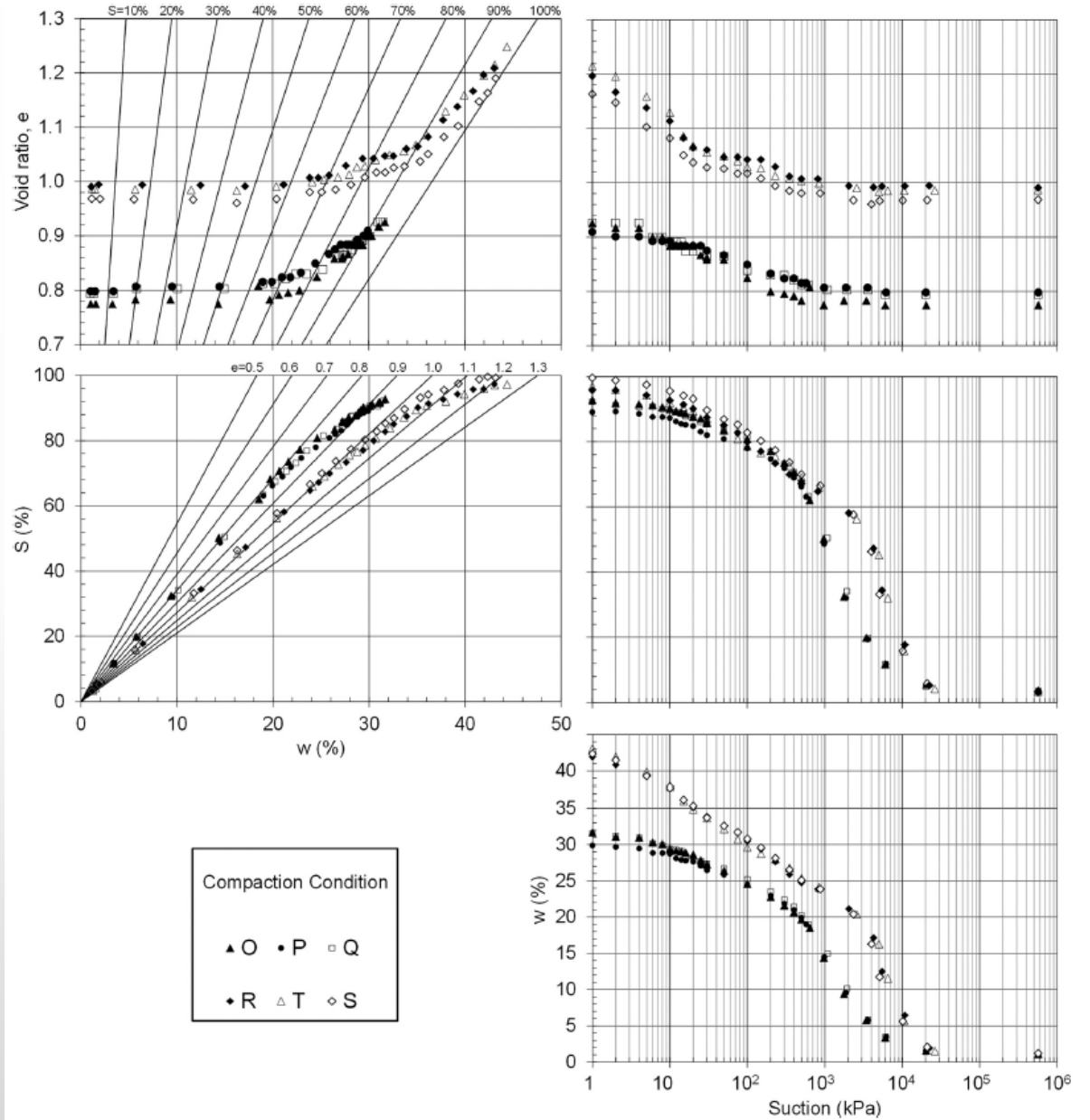
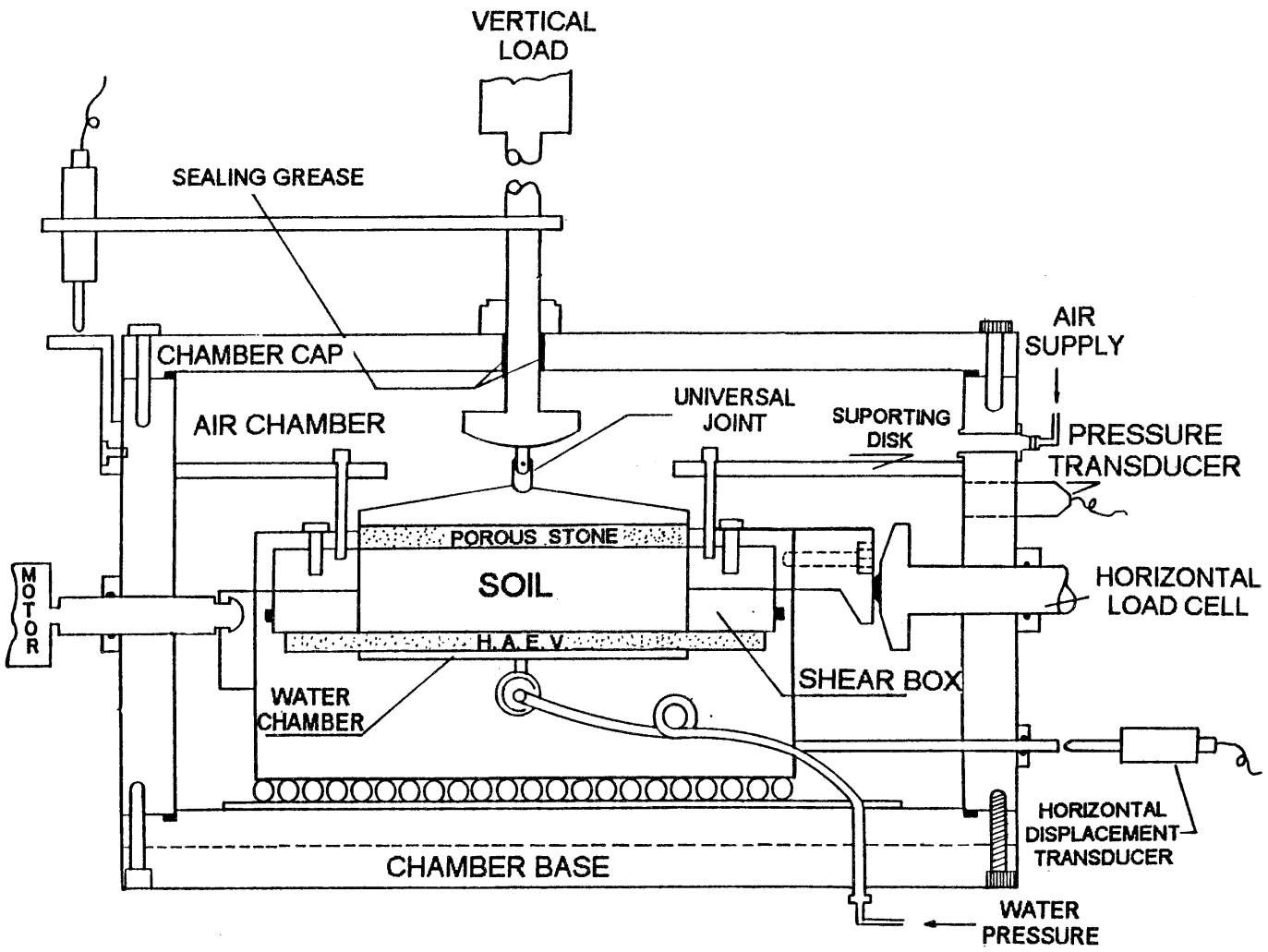


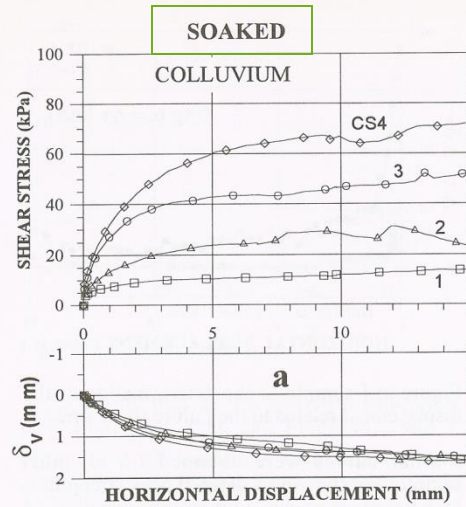
Fig. 3. SWRCs and volume change during drying for the soil tested

# Direct Shear Box for Unsaturated Soils

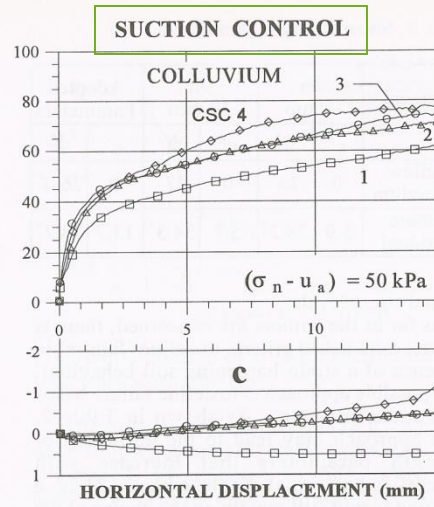
Axis Translation Technique



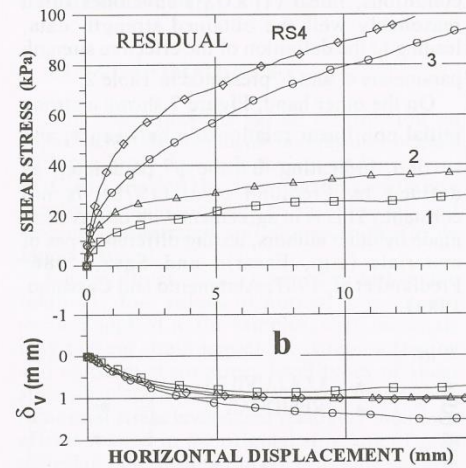
# Colluvium



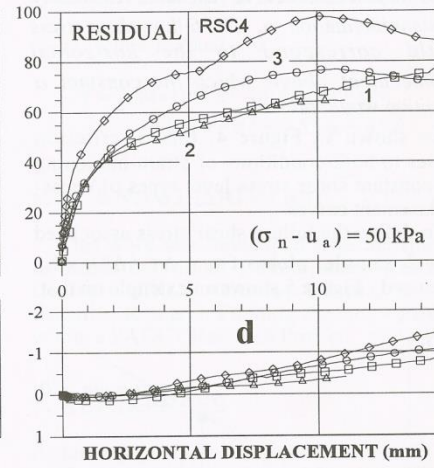
TEST	$\sigma_n$ (kPa)	$\gamma_{nat}$ kN/m <sup>3</sup>	w% initial	e initial	S initial
CS 1	20	13.44	11.59	1.3	25.59
CS 2	50	14.60	23.94	1.3	50.82
CS 3	90	15.54	24.11	1.2	57.12
CS 4	110	14.90	22.32	1.2	50.29



TEST	$u_a - u_w$ (kPa)	$\gamma_{nat_3}$ kN/m <sup>3</sup>	w% initial	e initial	S initial
CSC 1	30	15.57	24.8	1.2	58.4
CSC 2	80	15.61	23.2	1.1	56.2
CSC 3	150	14.93	24.4	1.3	53.5
CSC 4	210	15.67	23.9	1.1	57.7



TEST	$\sigma_n$ (kPa)	$\gamma_{nat}$ kN/m <sup>3</sup>	w% initial	e initial	S initial
RS 1	20	15.82	20.5	1.1	53.1
RS 2	50	19.45	19.9	0.67	81.8
RS 3	90	16.98	15.0	0.84	49.6
RS 4	110	17.10	14.5	0.82	49.1



TEST	$u_a - u_w$ (kPa)	$\gamma_{nat_3}$ kN/m <sup>3</sup>	w% initial	e initial	S initial
RSC 1	30	16.3	16.4	0.94	48.4
RSC 2	80	15.2	16.7	1.09	42.6
RSC 3	150	15.6	16.9	1.04	45.3
RSC 4	210	17.1	17.7	0.87	56.4

# Residual soil of Gnaiss



To remember



## Liquidity Index

$$I_L = \frac{(w - w_p)}{I_P}$$

## Liquidity Index for Compacted Soils

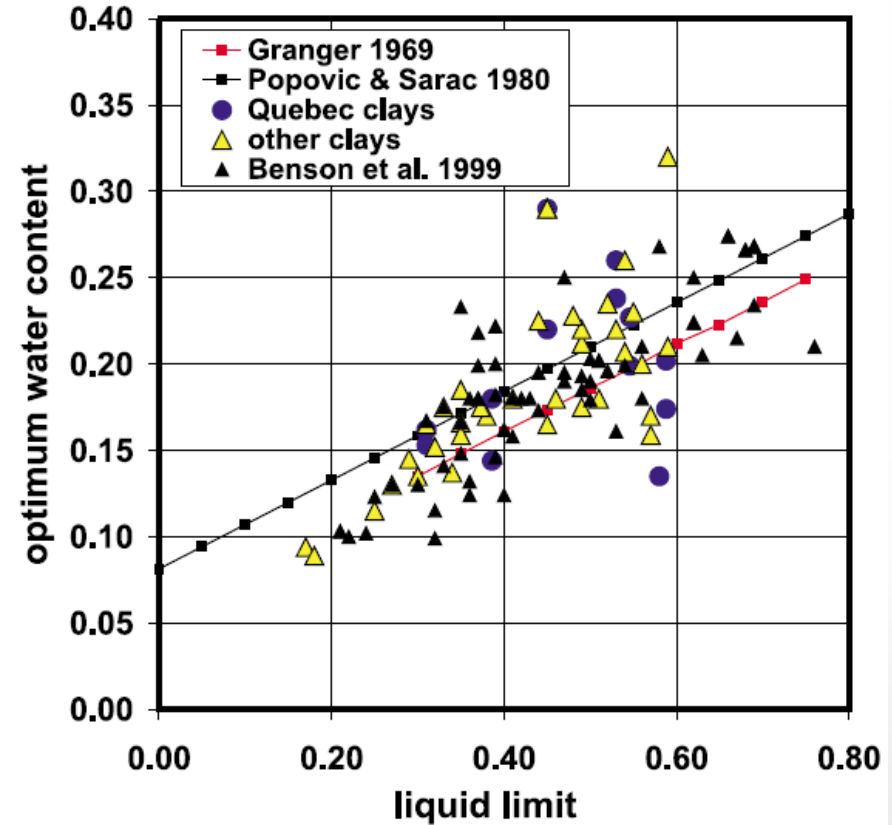
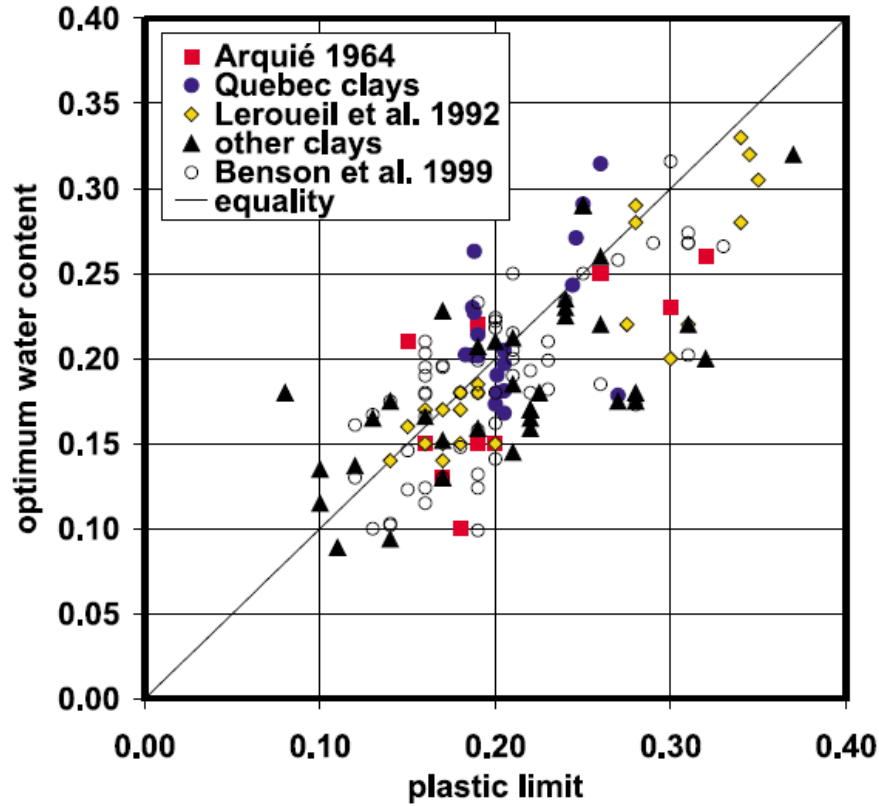
$$I_L^C = \frac{(w - w_{opt})}{I_P}$$

Leroueil, et al. (1992)

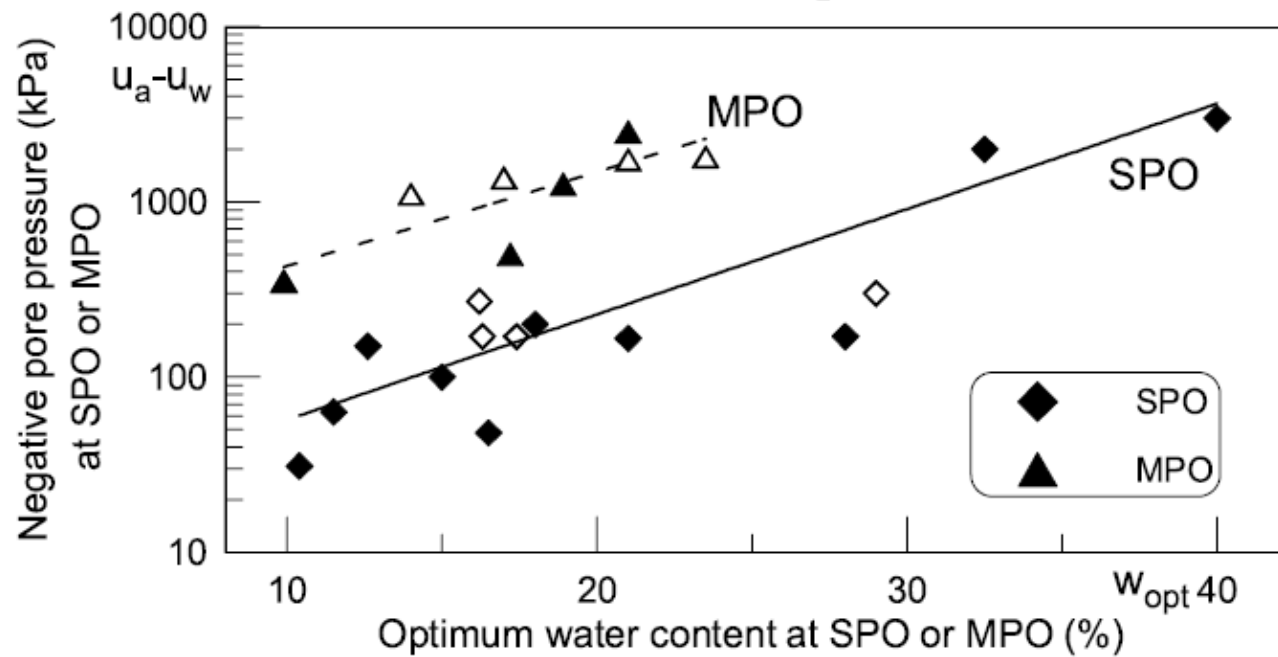
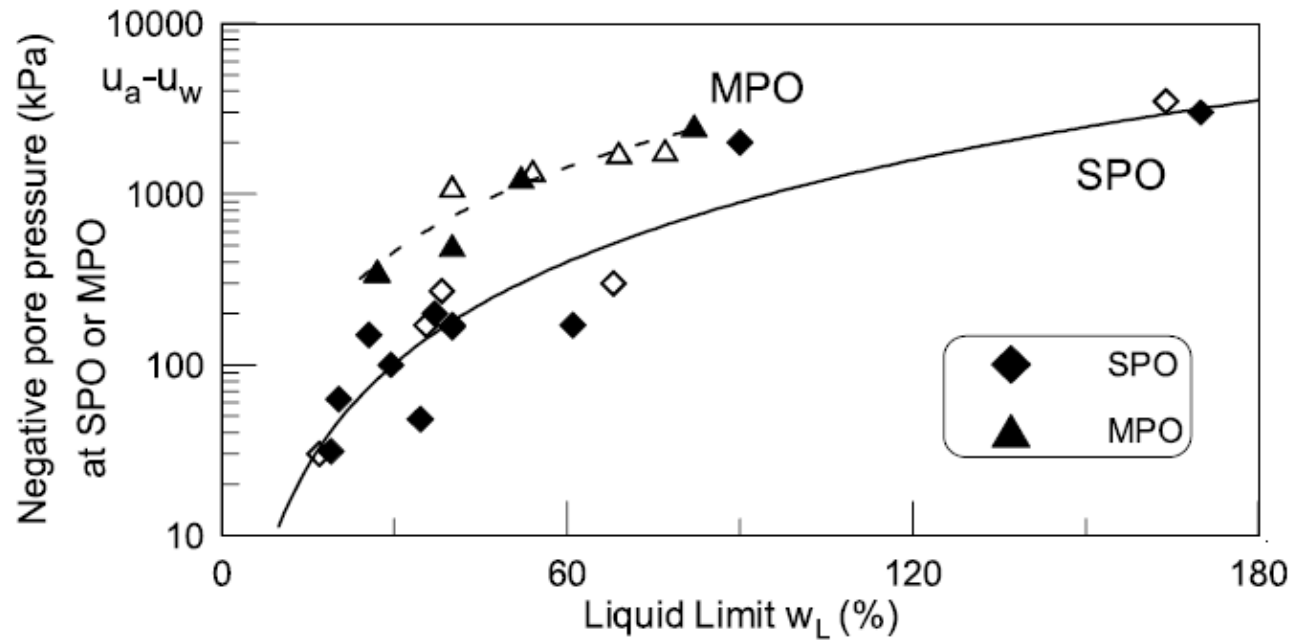
$$I_L = \frac{(w - w_p)}{I_P}$$

Liquidity Index

# Relation Between Optimum w/c and plasticity limit and liquid limit



Chapuis (2002)



Fleureau et al (2002)

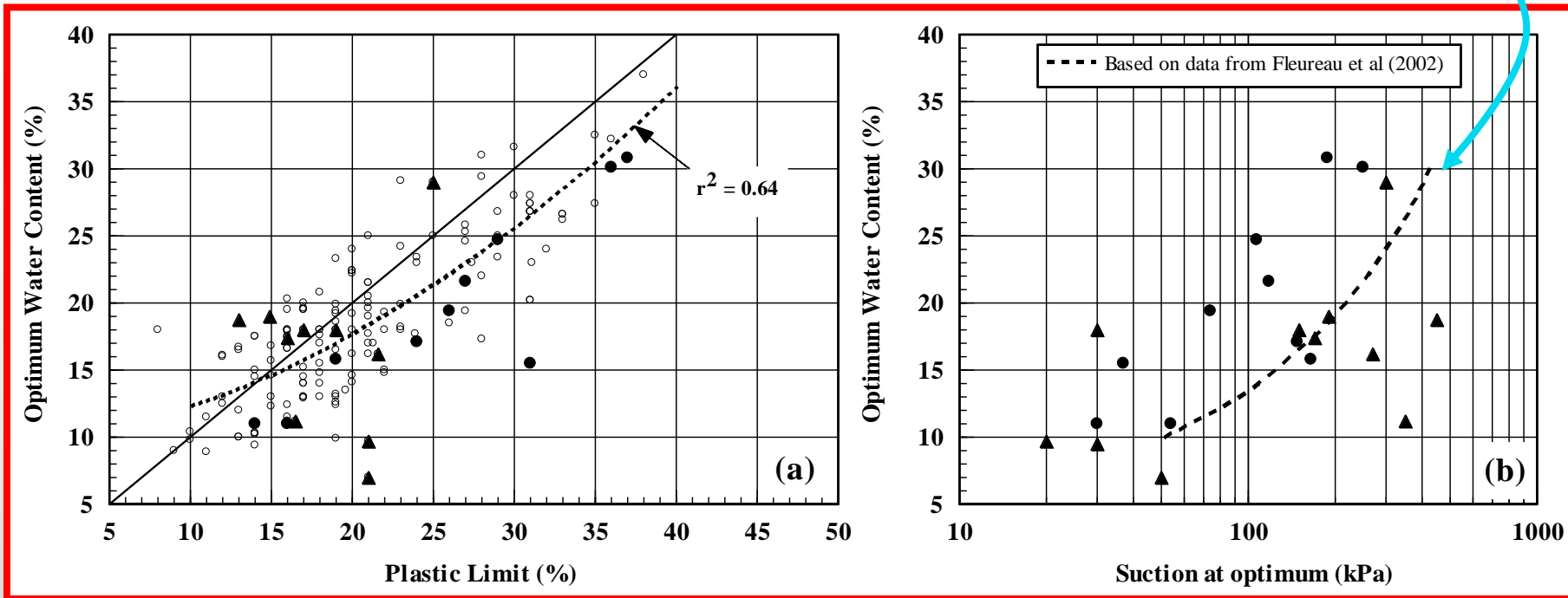
$$w_{opt} = 1.99 + 0.46 * w_l - 0.0012 w_l^2$$

$$suction(kPa) = 0.118 * w_l^{1.98}$$

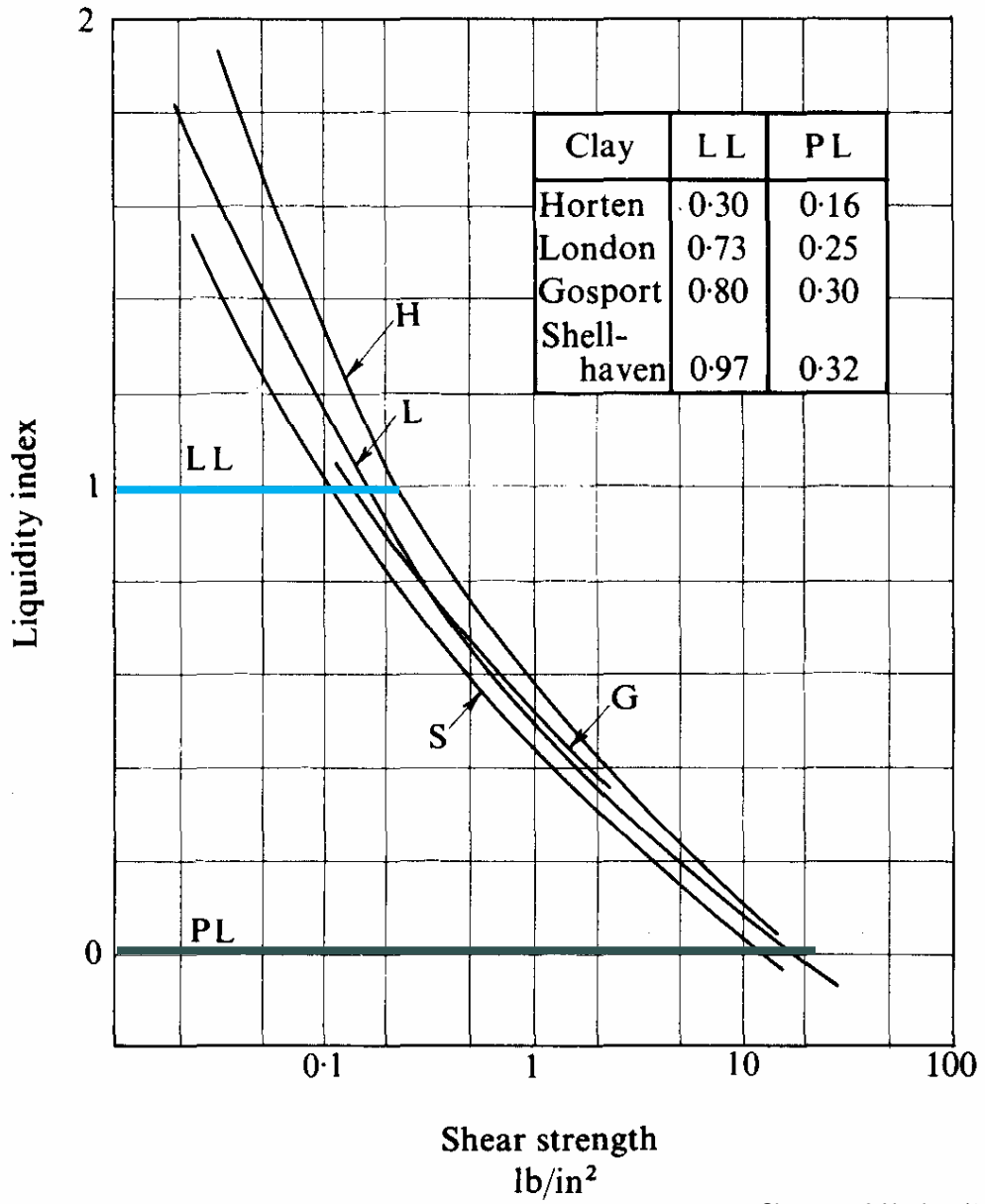


$$suction(kPa) = 0.118[-0.028 * w_{opt}^2 + 3.163 * w_{opt} - 7.47]^{1.98}$$

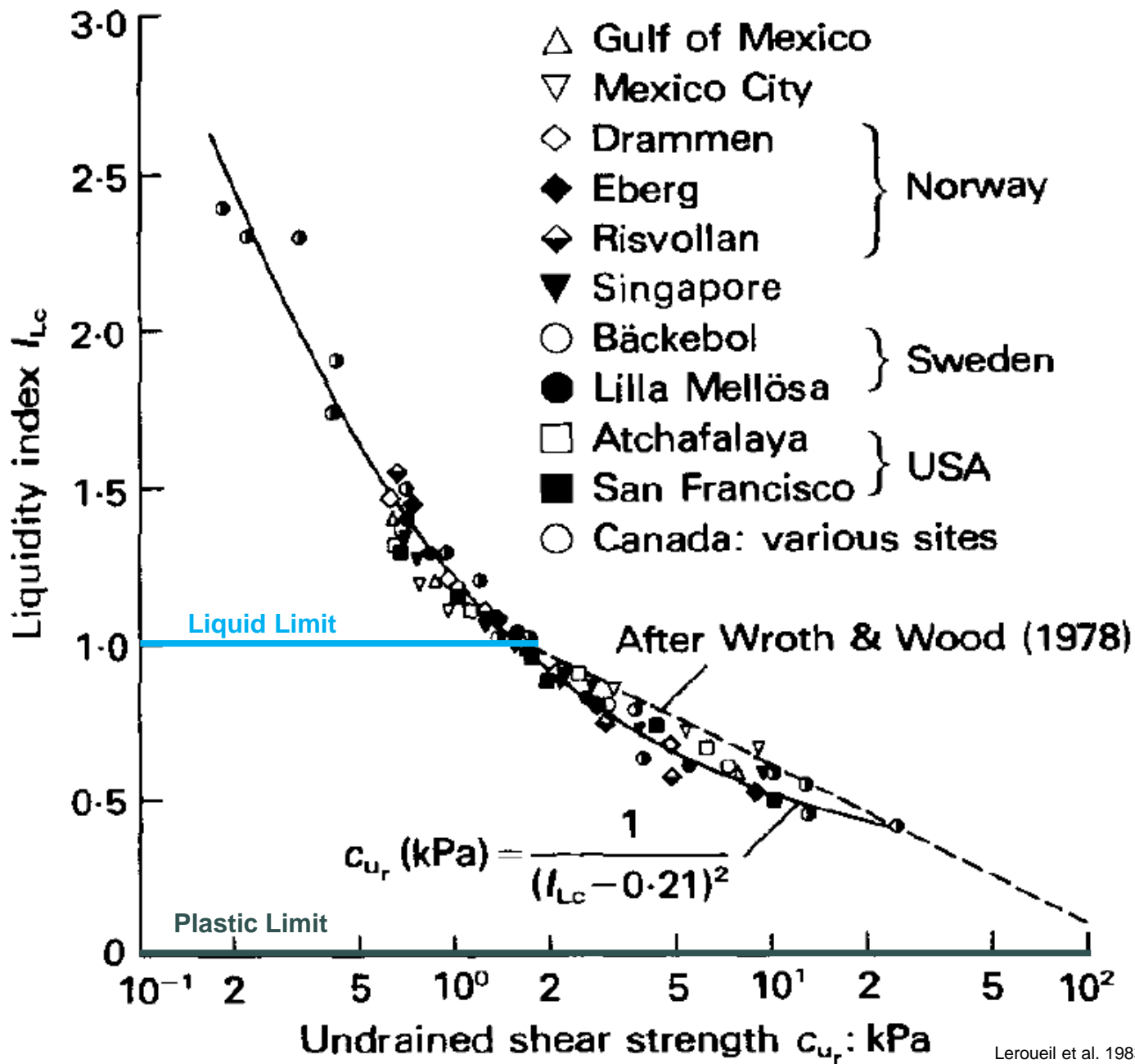
Relation Between Optimum w/c and Plasticity Limit

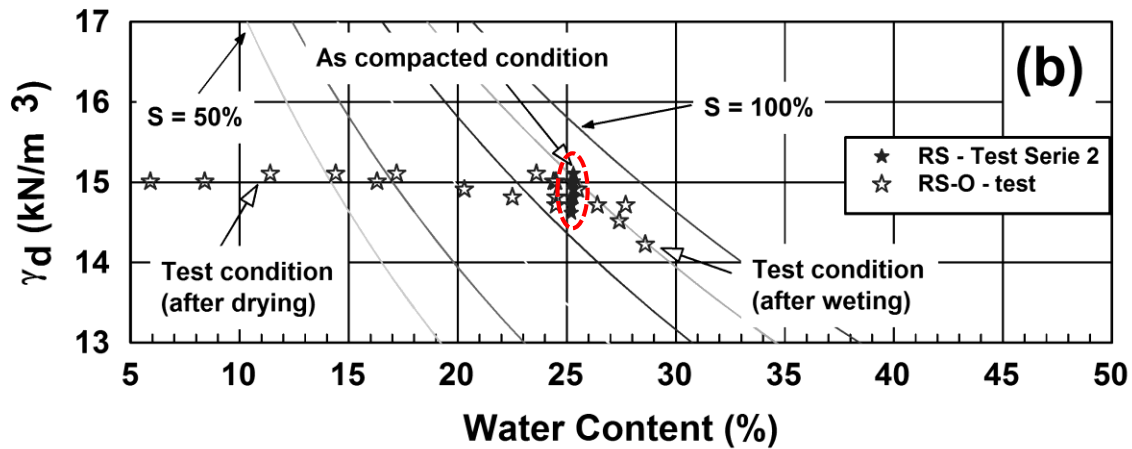
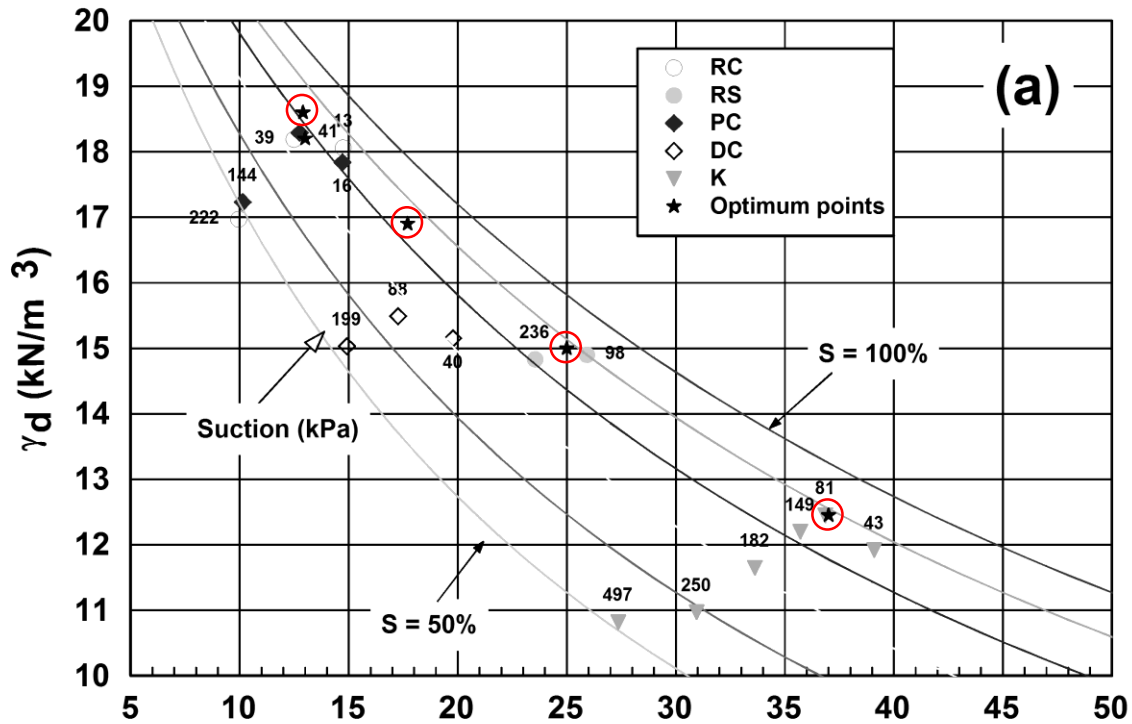


Relation Between Optimum w/c and Suction at Optimum w/c

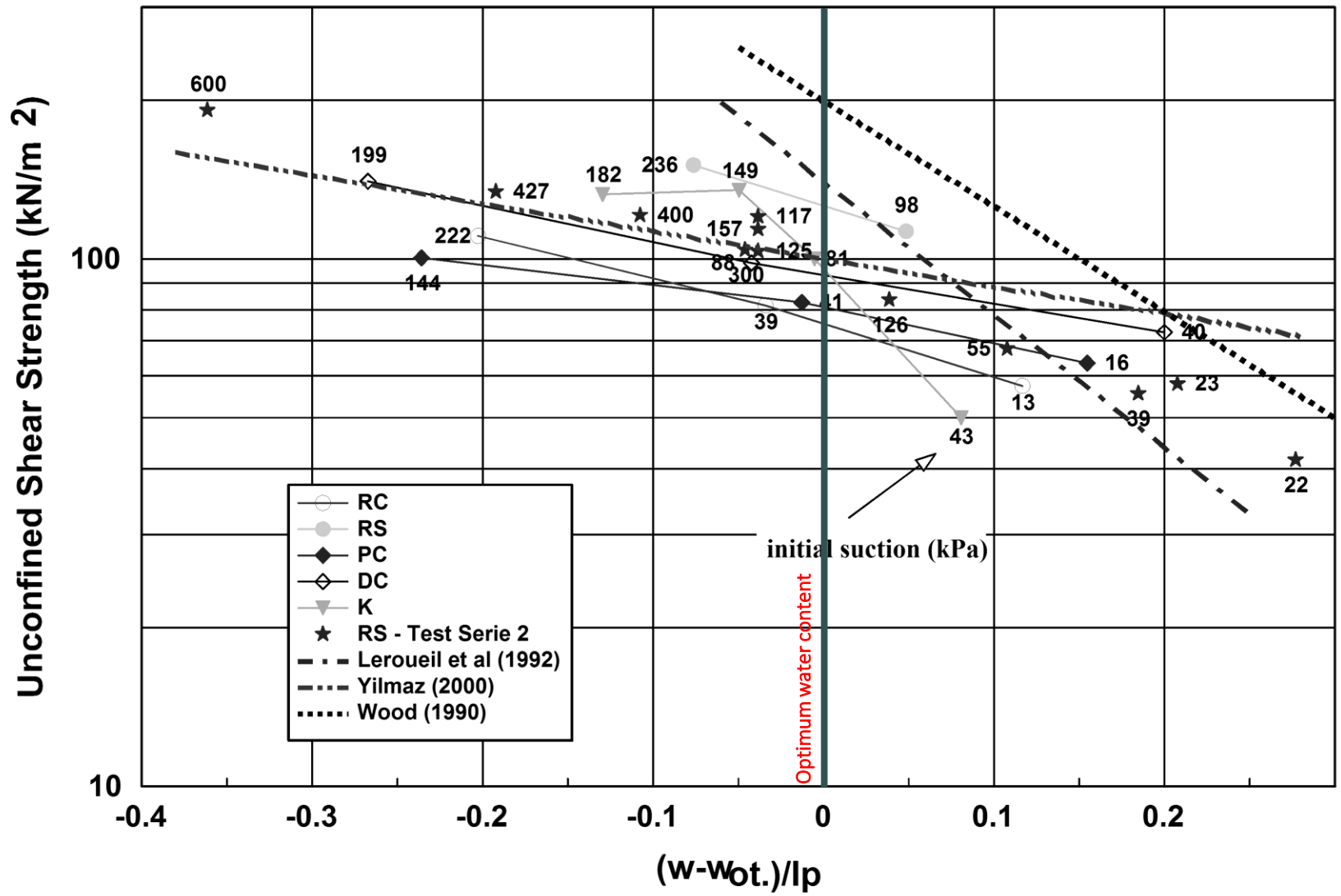


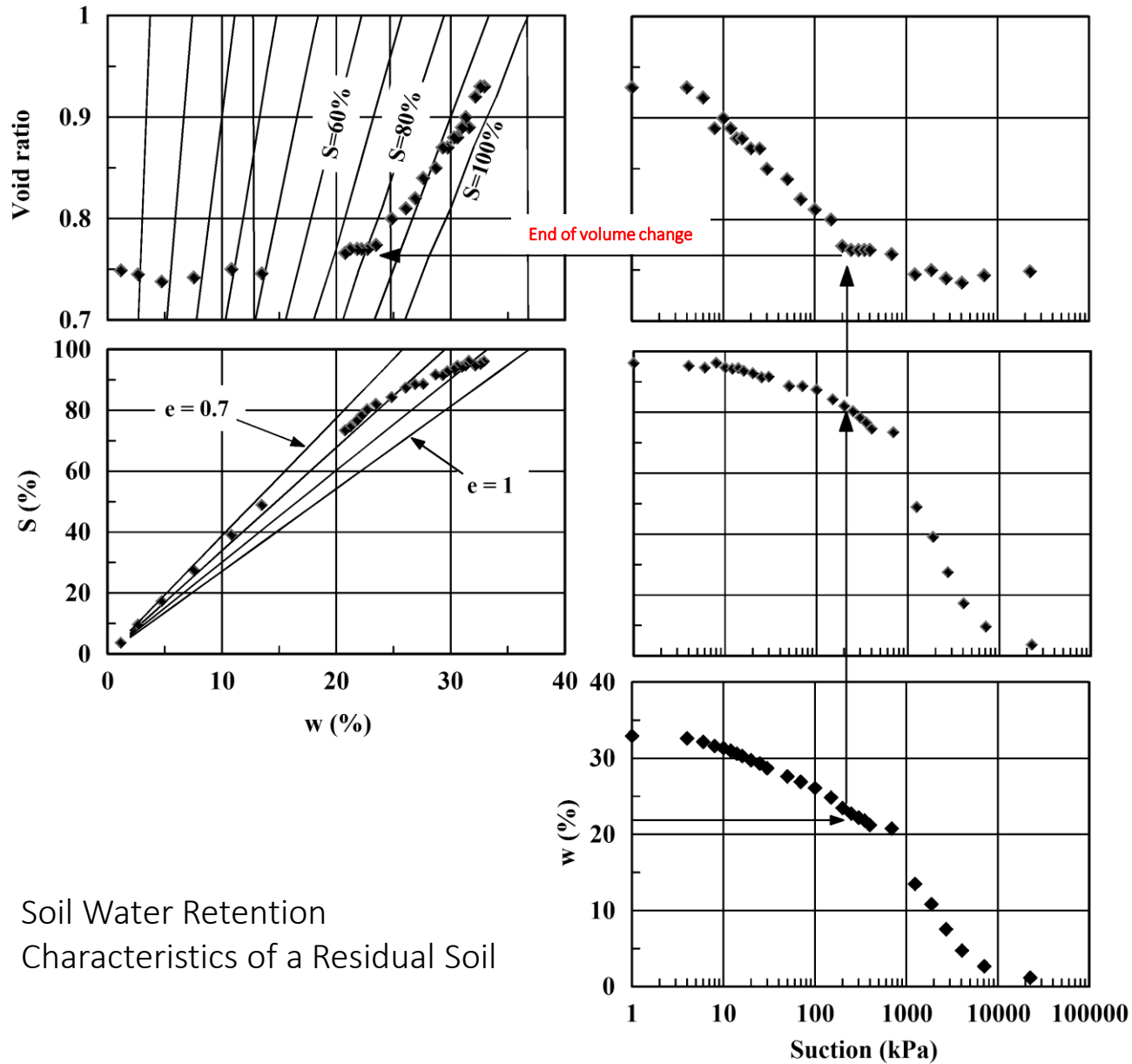
Skempton & Northey (1953)



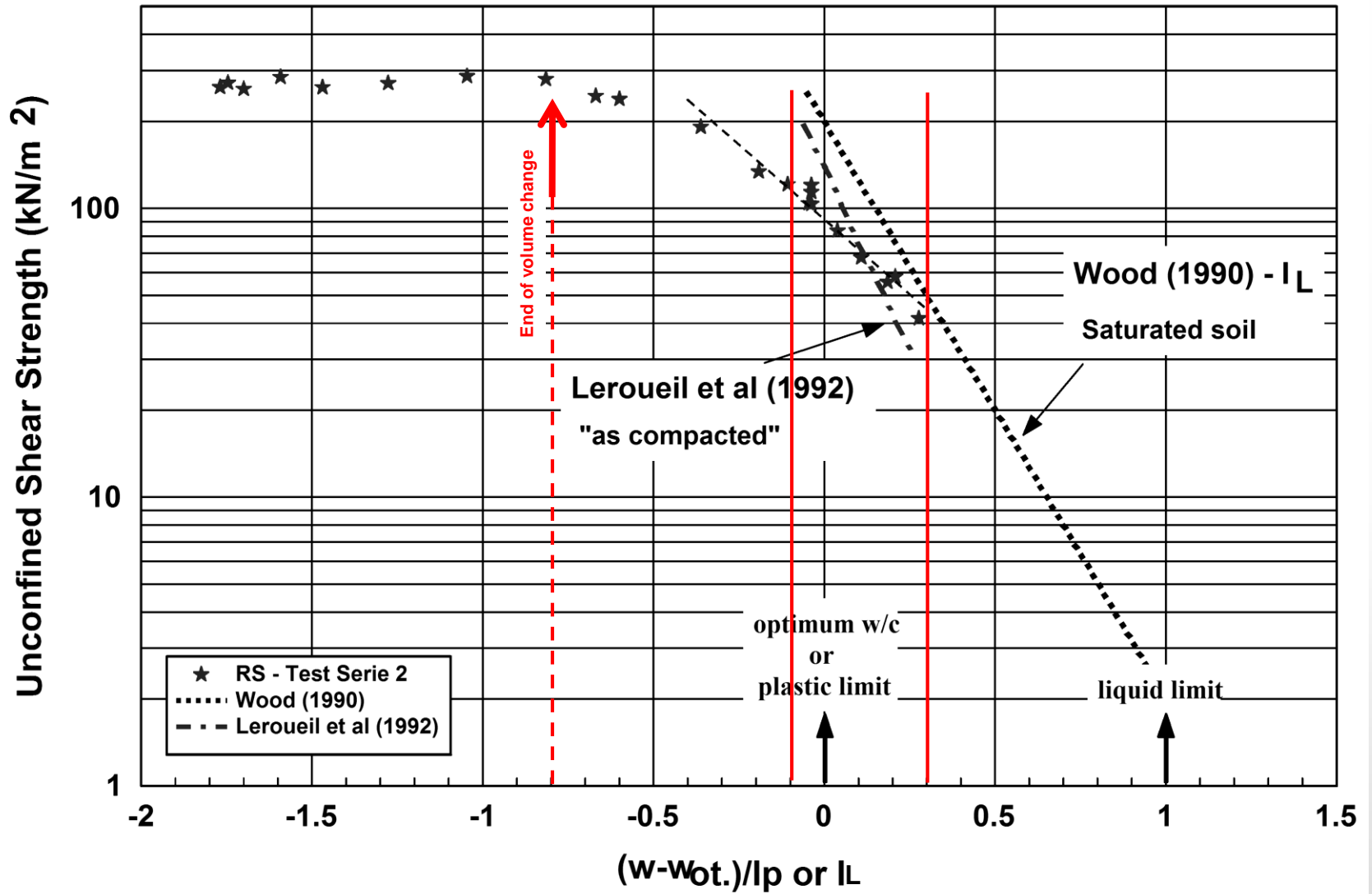




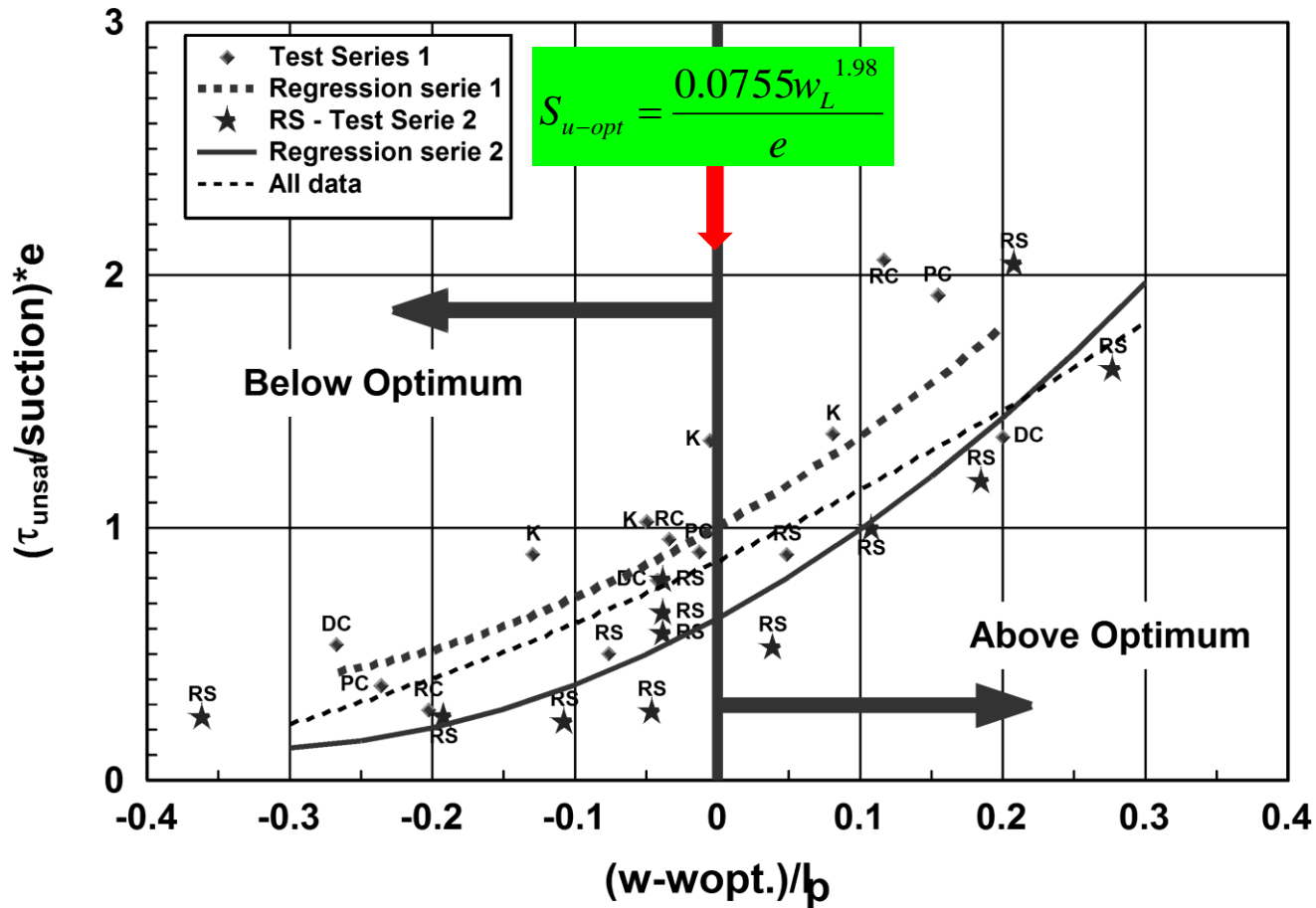




Soil Water Retention  
Characteristics of a Residual Soil

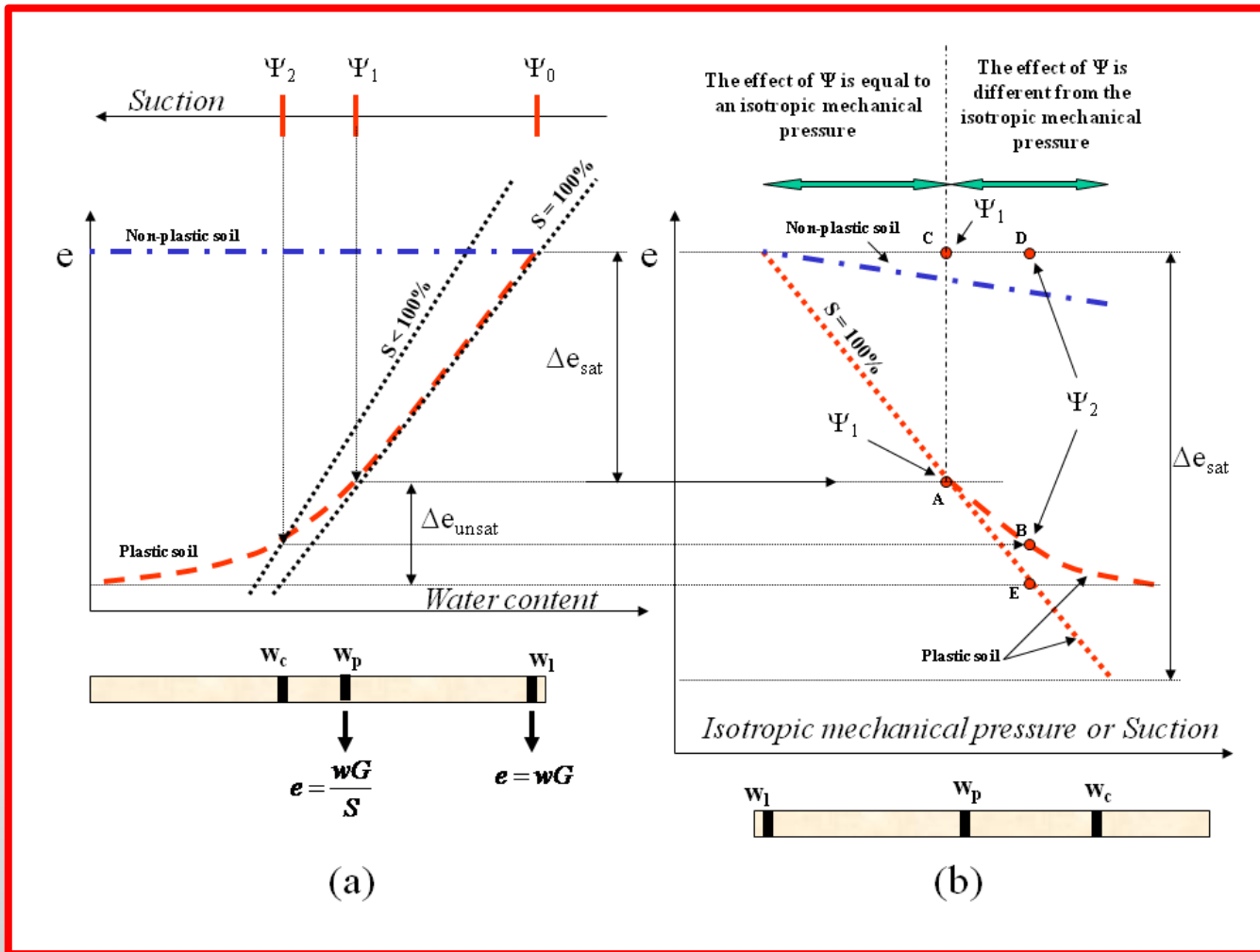


$$S_{u-unsat} (kPa) = 8.42 * 10.8^{(1-I_L^C)}$$



$$\frac{S_{u-unsat}}{suction} e = 4.5 * (I_L^C)^2 + 3.1 * I_L^C + 0.64$$

- It is important to have the unsaturated soil concept in mind when designing or investigating a problem. We have to teach unsaturated soil mechanics for undergraduate students.
- The association of the unsaturated soils behaviour with the Atterberg limits and the unconfined shear strength characteristic of unsaturated soils have been demonstrated using the drying curve as a reference.



- Empirical relationship between unconfined shear strength for unsaturated and liquidity index specially defined for compacted soil.

$$\frac{S_{u-unsat}}{suction} e = 4.5 * (I_L^C)^2 + 3.1 * I_L^C + 0.64$$

- An specific equation for the unconfined shear strength of a residual soil of gneiss at optimum water content.

$$S_{u-opt} = \frac{0.0755w_L^{1.98}}{e}$$

- It should be emphasized that the empirical expression given here was obtained for compacted soil.

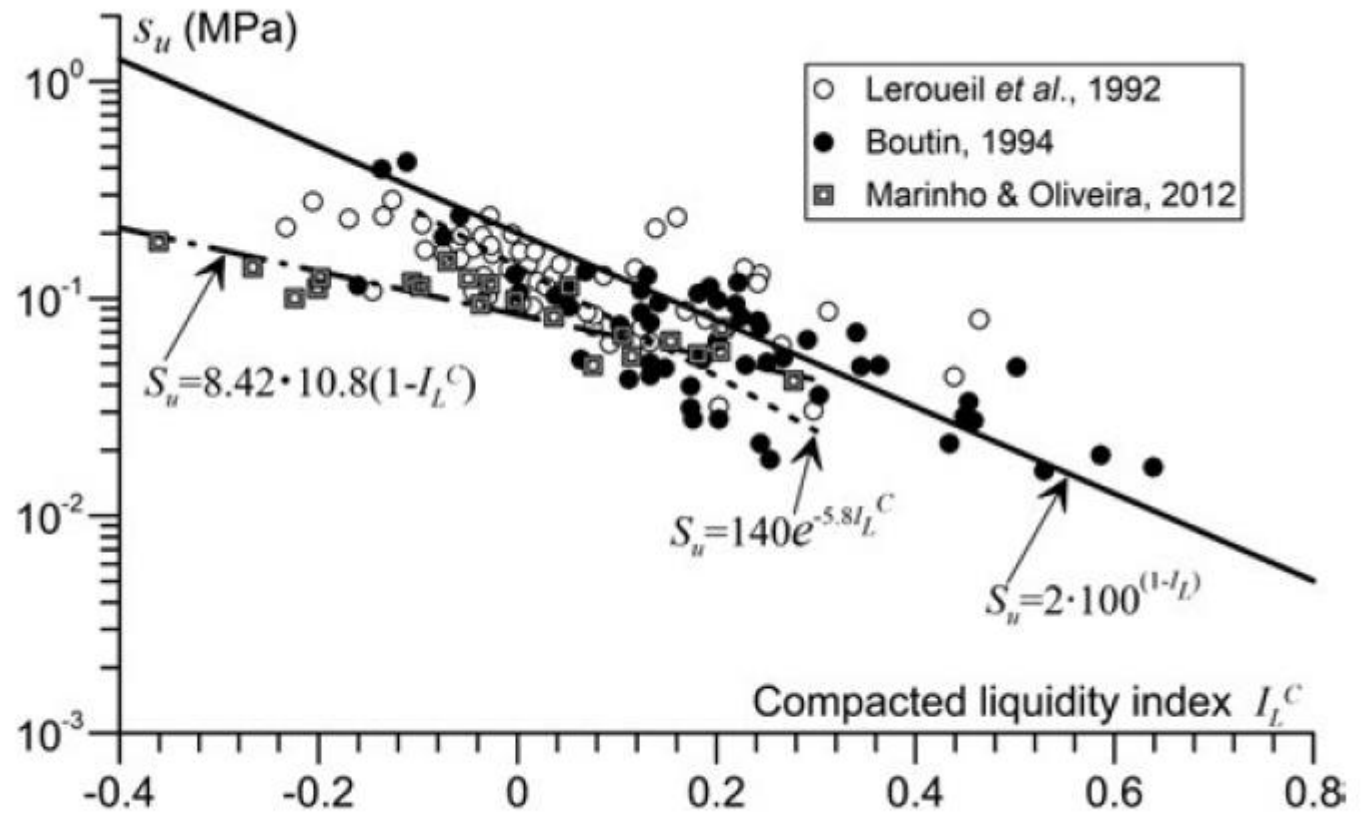


Figure 3.78 Undrained shear strength for compacted soils, data from [259, 260, 277].

Caicedo (2019)

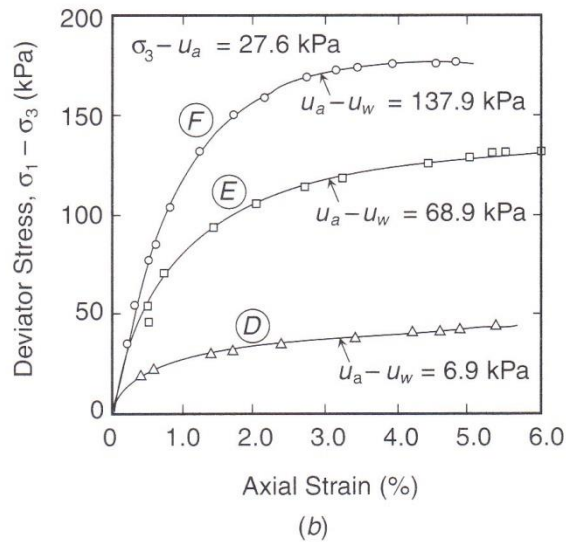
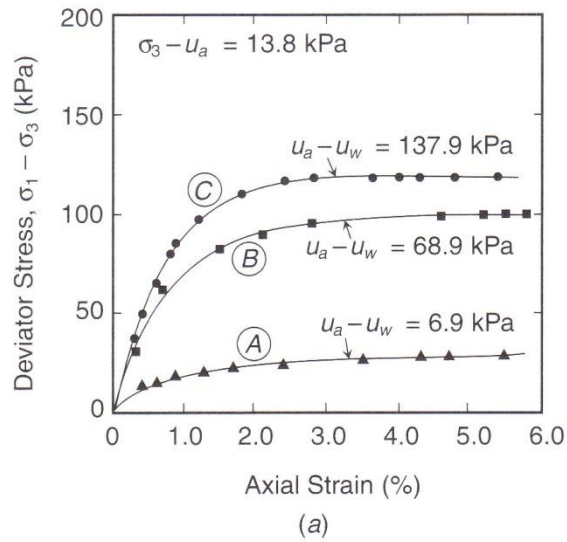
Experimental Observations on  
Unsaturated Soils  
(Strength)



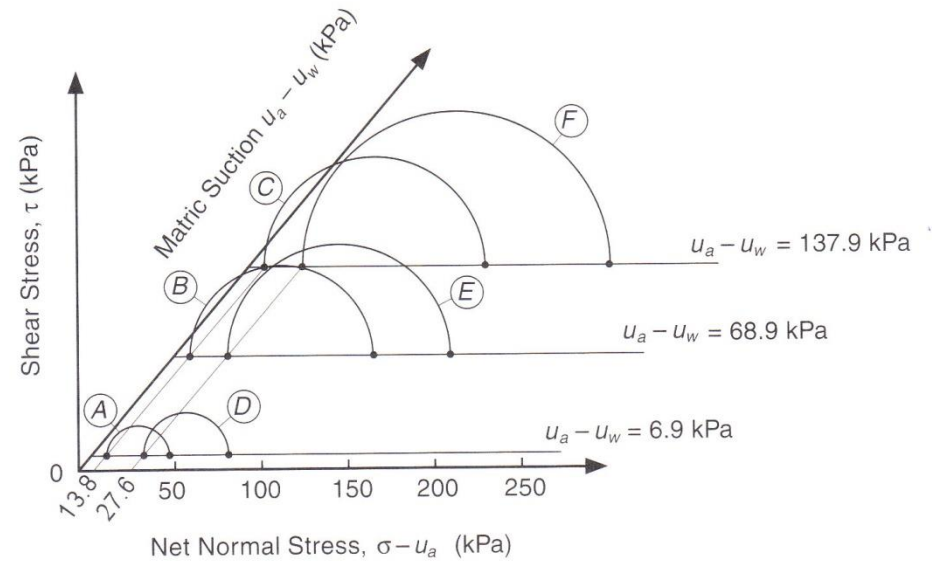


# Shear Strength of Unsaturated Soil

## Results of CD test in unsaturated soil



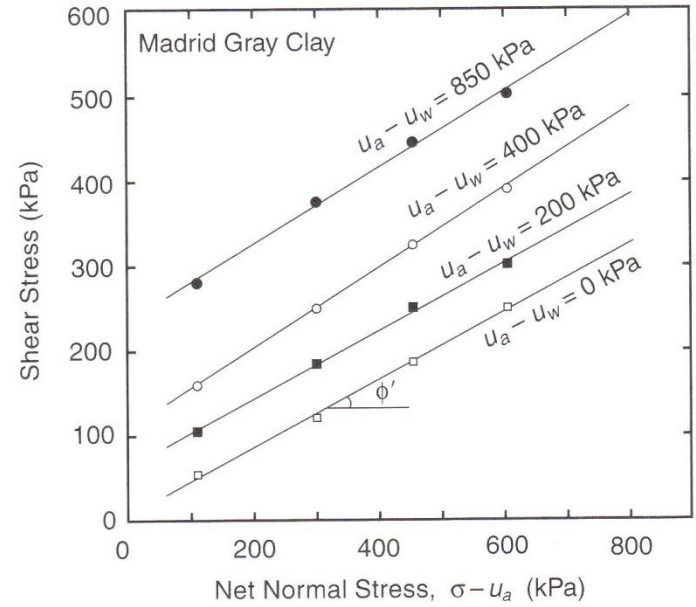
## Extended Mohr-Coulomb diagram



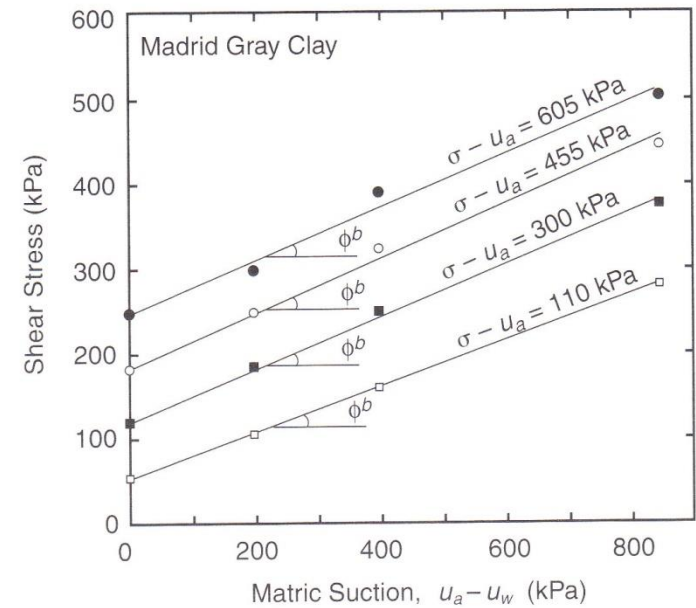
Lu & Likos (2004)  
 Dados de Blight (1967)

# Shear Strength of Unsaturated Soil

## Direct shear test in unsaturated expansive soil



(a)

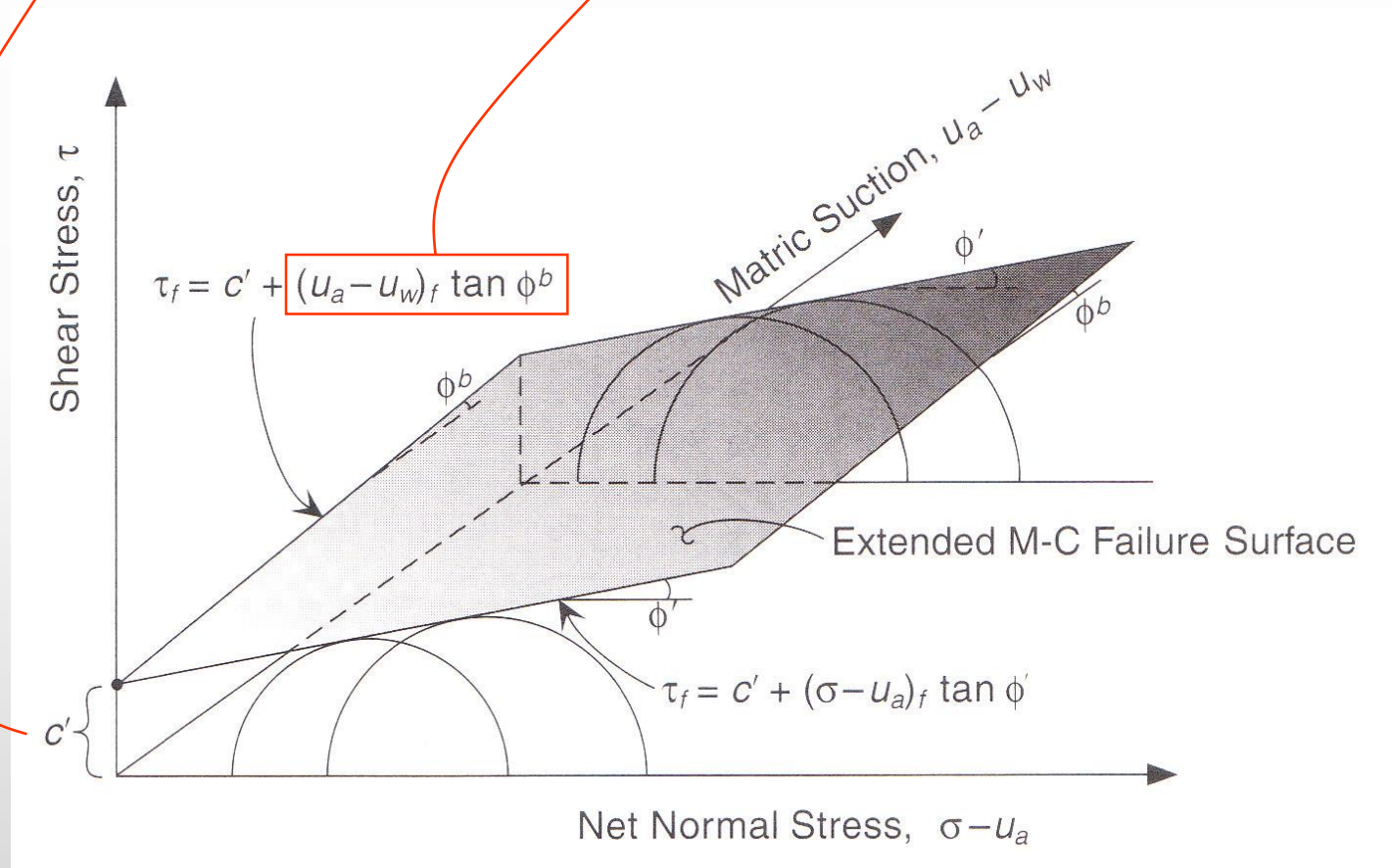


(b)

Lu & Likos (2004)  
 Datos de Escario (1980)

# Shear Strength of Unsaturated Soil

$$\tau_f = c' + (\sigma - u_a)_f \tan \phi' + (u_a - u_w)_f \tan \phi^b$$



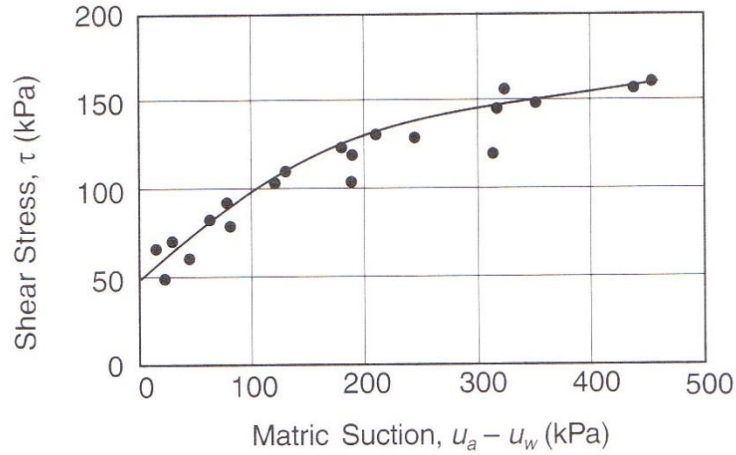
Lu & Likos (2004)

## Parâmetros de resistência ao cisalhamentos para vários solos

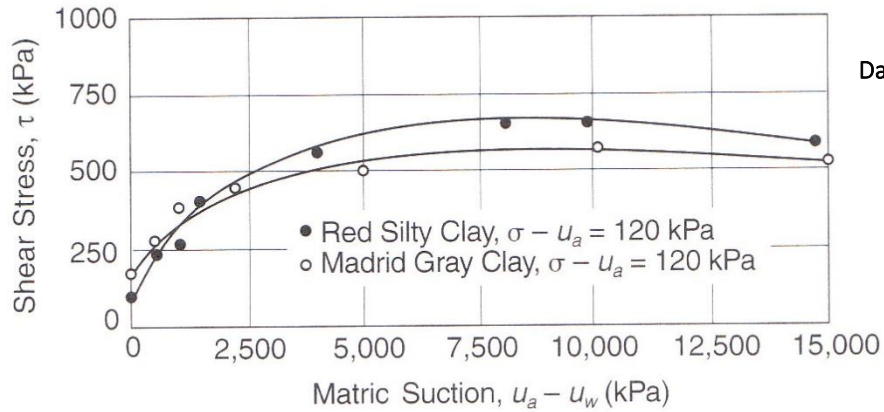
Soil Type	$c'$ (kPa)	$\phi'$ (deg)	$\phi^b$ (deg)	References
Compacted shale; $w = 18.6\%$	15.8	24.8	18.1	Bishop et al. (1960)
Boulder clay; $w = 11.6\%$	9.6	27.3	21.7	Bishop et al. (1960)
Dhanauri clay; $w = 22.2\%$ , $\rho_d = 1580 \text{ kg/m}^3$	37.3	28.5	16.2	Satija (1978)
Dhanauri clay; $w = 22.2\%$ , $\rho_d = 1478 \text{ kg/m}^3$	20.3	29.0	12.6	Satija (1978)
Madrid gray clay; $w = 29\%$	23.7	22.5	16.1	Escario (1980)
Undisturbed decomposed granite	28.9	33.4	15.3	Ho and Fredlund (1982)
Tappen-Notch Hill silt; $w = 21.5\%$ , $\rho_d = 1590 \text{ kg/m}^3$	0.0	35.0	16.0	Krahn et al. (1989)
Compacted glacial till; $w = 12.2\%$ , $\rho_d = 1810 \text{ kg/m}^3$	10.0	25.3	7-25.5	Gan et al. (1988)

Source: Modified from Fredlund and Rahardjo (1993).

## Example of non-linearity of the envelope



Dados de Gan et al. (1988)  
Glacial till

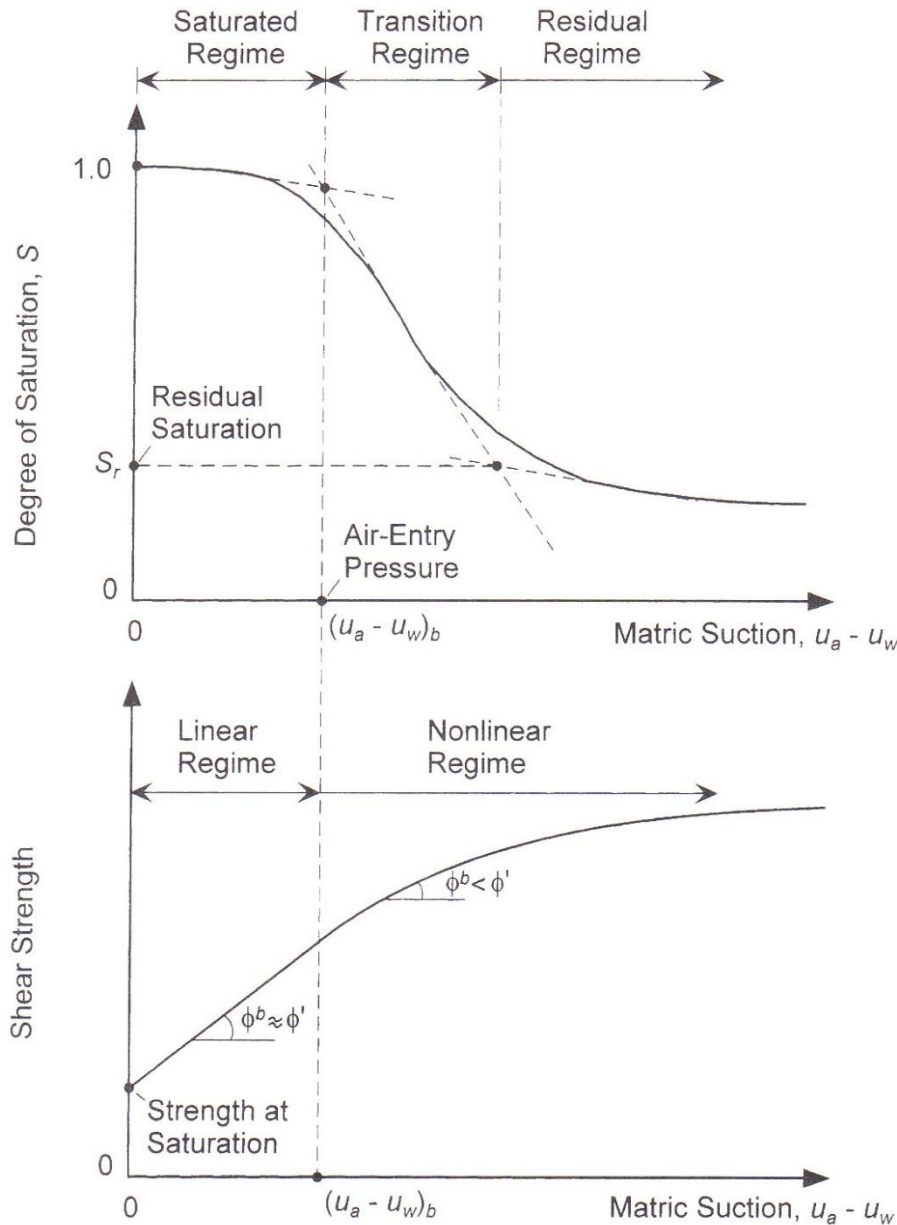


Dados de Escário et al. (1989)

(a)

(b)

# Shear Strength of Unsaturated Soil

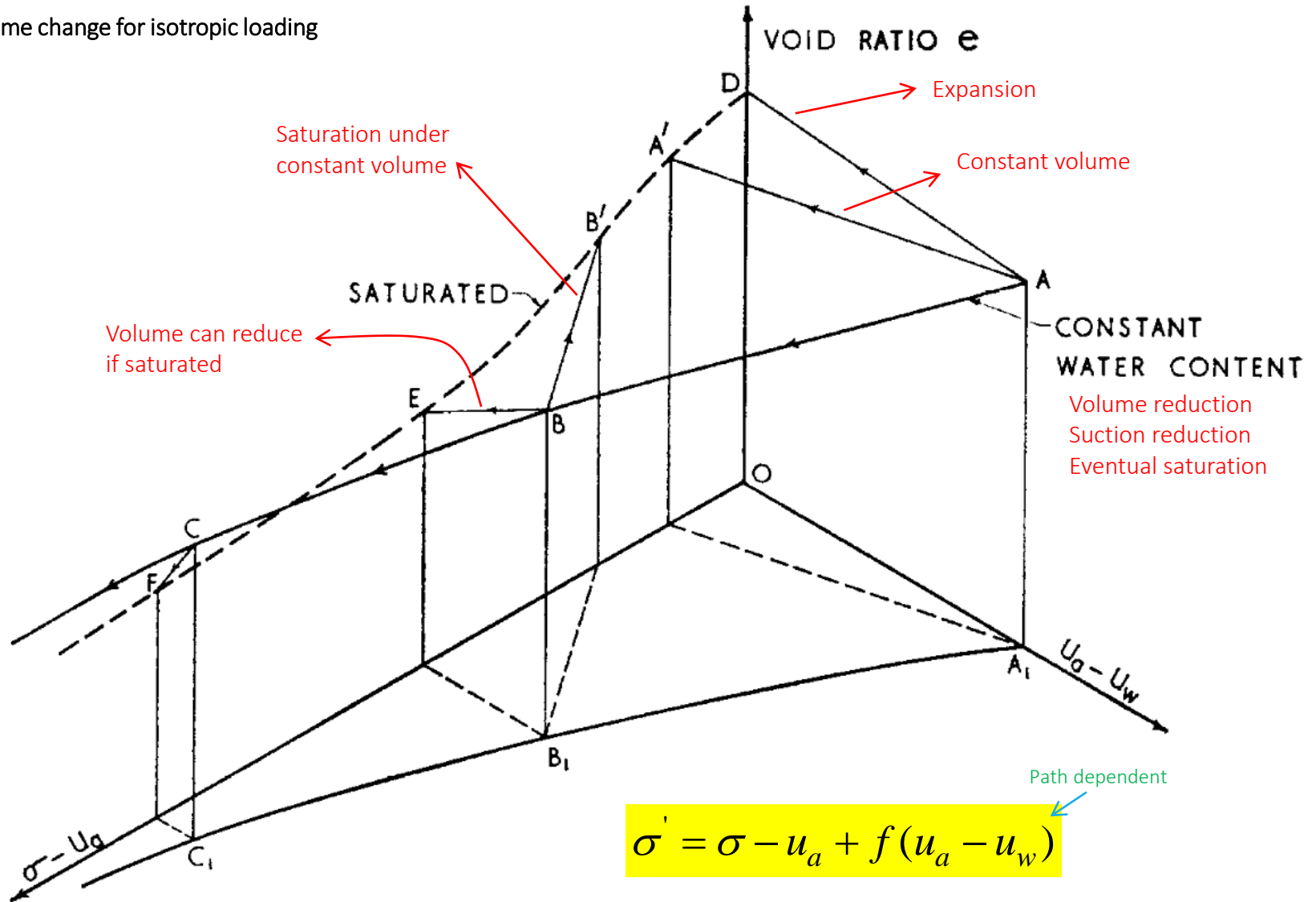


Conceptual relation between SWRC and the failure envelope



# Shear Strength of Unsaturated Soil

Volume change for isotropic loading

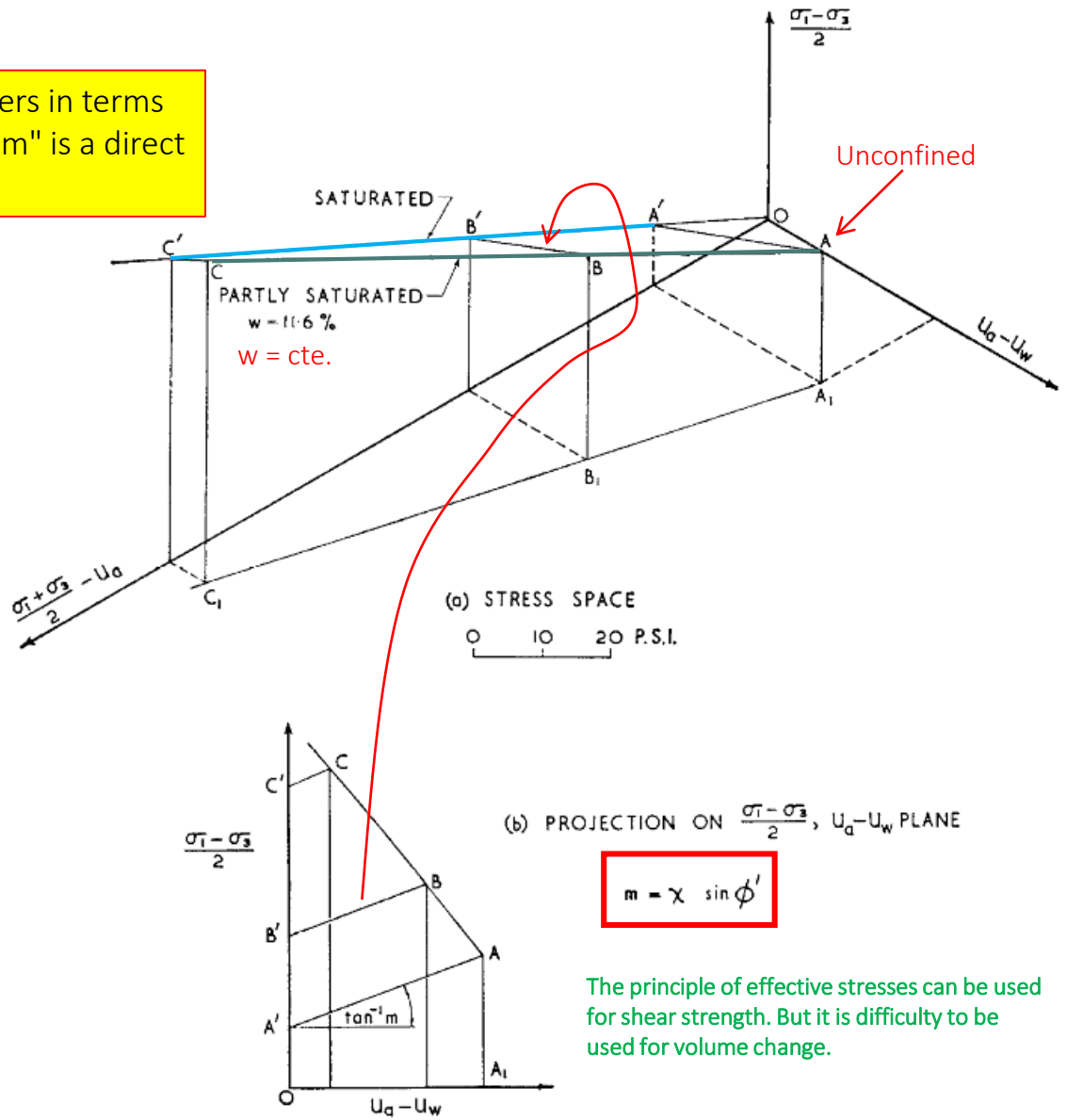




# Shear Strength of Unsaturated Soil

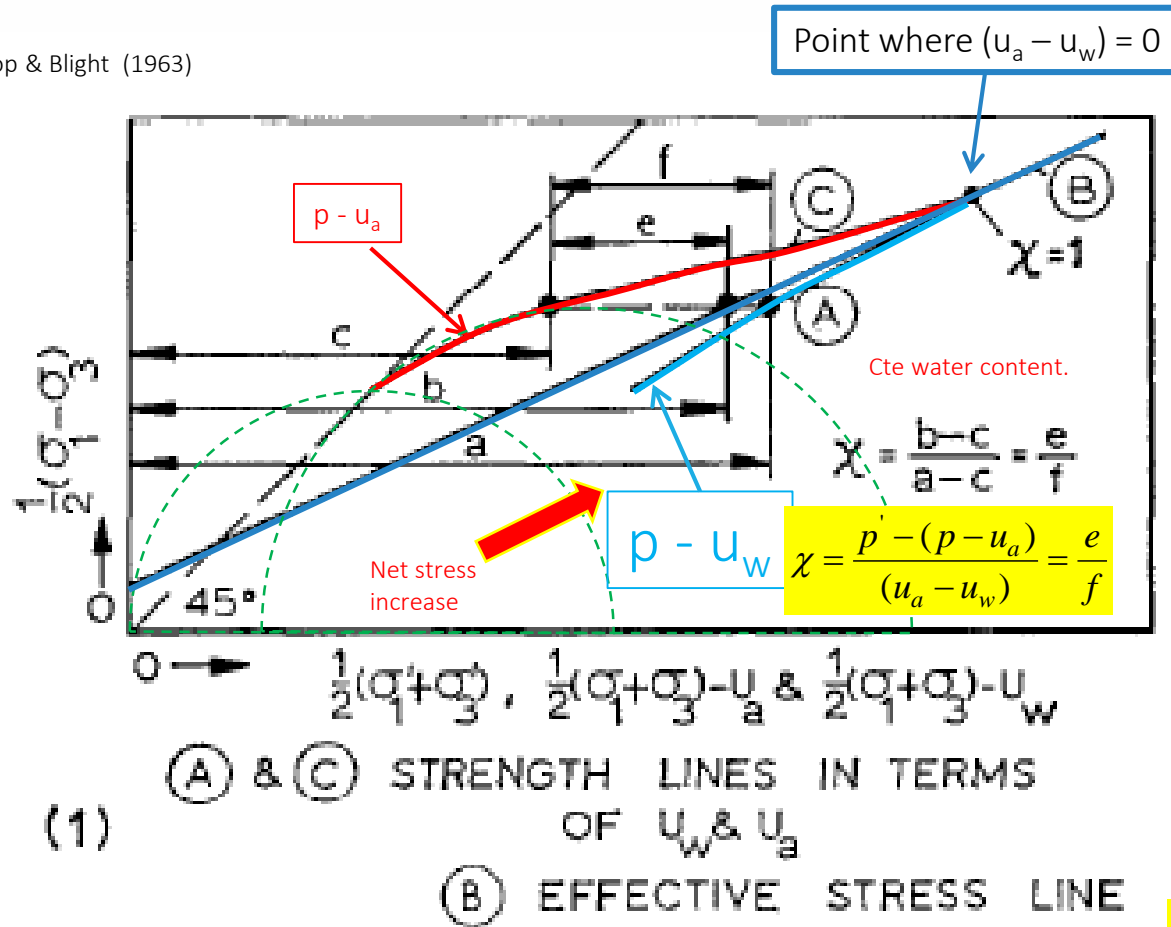
## Results of tests for Selset clay (saturated and unsaturated)

Assuming the same parameters in terms of effective stress the slope "m" is a direct measure of  $\chi$ .



# Shear Strength of Unsaturated Soil

Bishop & Blight (1963)



Determination of  $\chi$  by means of triaxial tests.

The same compaction water content and different values of net stress.

$$e = b - c = \frac{1}{2}(\sigma'_1 + \sigma'_3) - \left\{ \frac{1}{2}(\sigma_1 + \sigma_3) - u_a \right\}$$

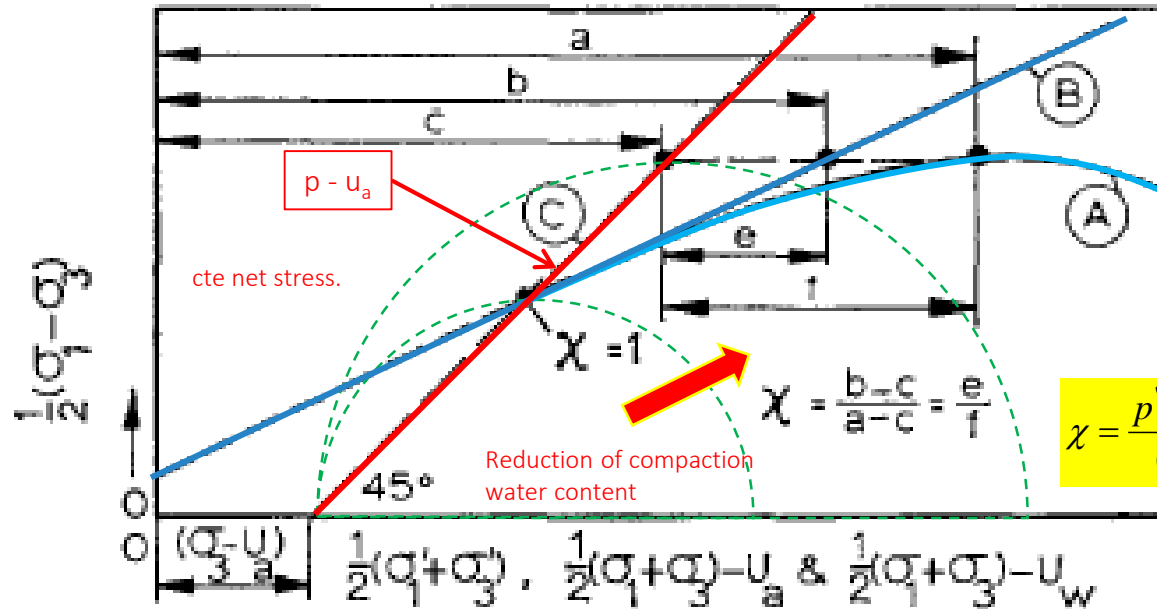
$$f = a - c = (u_a - u_w)$$

$$\sigma' = \sigma - u_a + \chi(u_a - u_w)$$

$$\frac{1}{2}(\sigma'_1 + \sigma'_3) = \frac{1}{2}(\sigma_1 + \sigma_3) - u_a + \chi(u_a - u_w)$$

$$\chi = \frac{\frac{1}{2}(\sigma'_1 + \sigma'_3) - \left\{ \frac{1}{2}(\sigma_1 + \sigma_3) - u_a \right\}}{(u_a - u_w)} = \frac{e}{f}$$

# Shear Strength of Unsaturated Soil



Determination of  $\chi$  by means of triaxial tests.

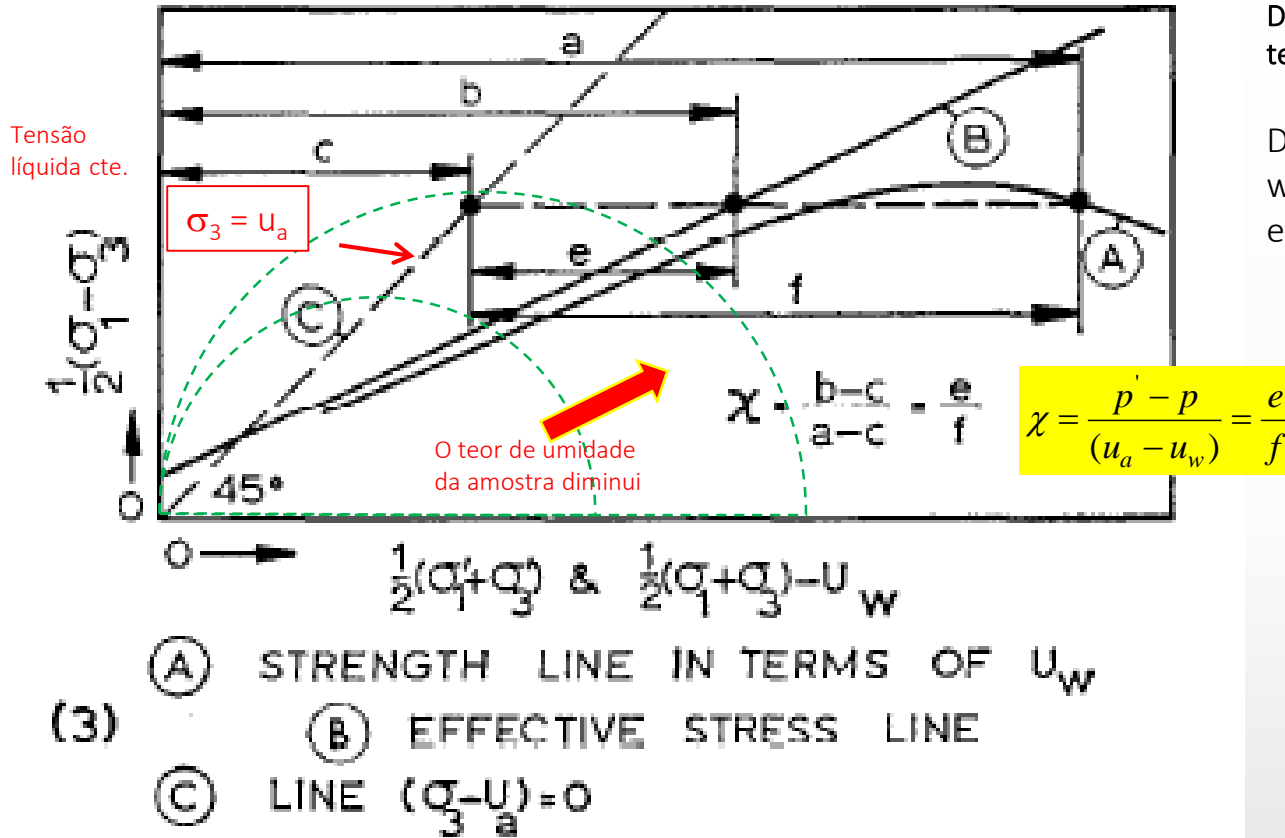
The same net stress and different values of compaction water content.

$$\chi = \frac{p' - (p - u_a)}{(u_a - u_w)} = \frac{e}{f}$$

- (2)
- (A) STRENGTH LINE IN TERMS OF  $U_w$
  - (B) EFFECTIVE STRESS LINE
  - (C) LINE  $(\sigma_3 - U) = \text{CONST.}$

Bishop & Blight (1963)

# Shear Strength of Unsaturated Soil



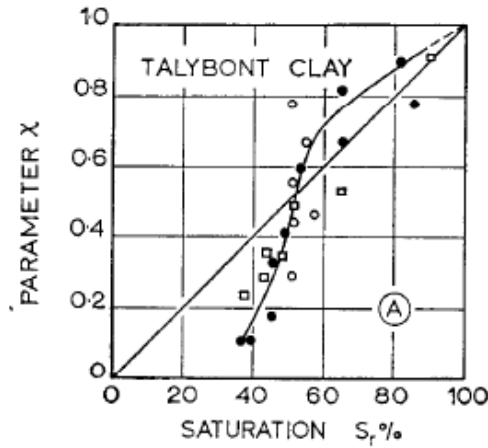
Determination of  $\chi$  by means of triaxial tests.

Different values of compaction water content and net stress equal to zero (unconfined).

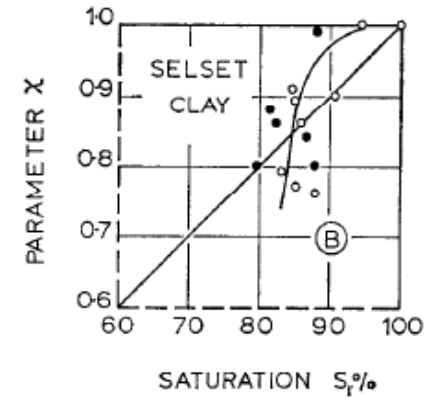
Bishop & Blight (1963)

# Shear Strength of Unsaturated Soil

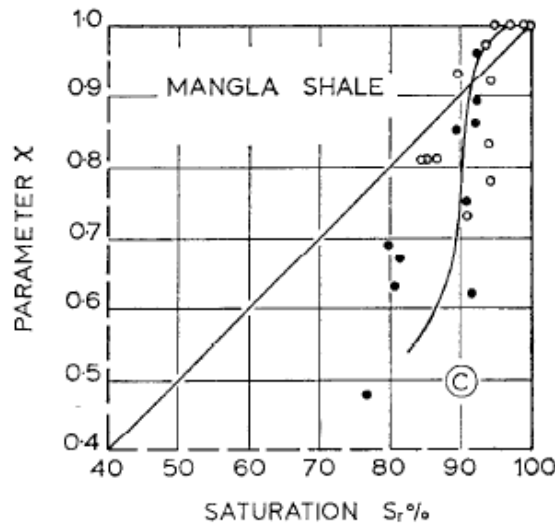
Variation of  $\chi$  with the degree of saturation for compacted soils



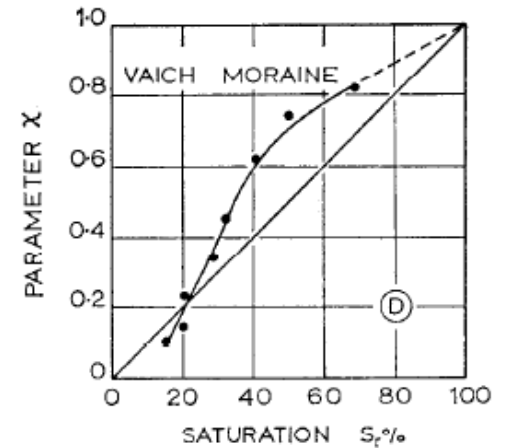
(a)



(b)

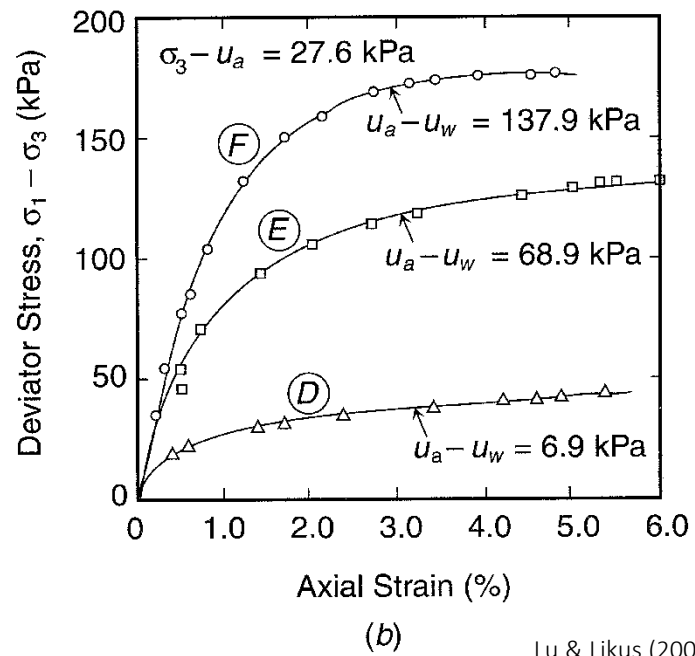
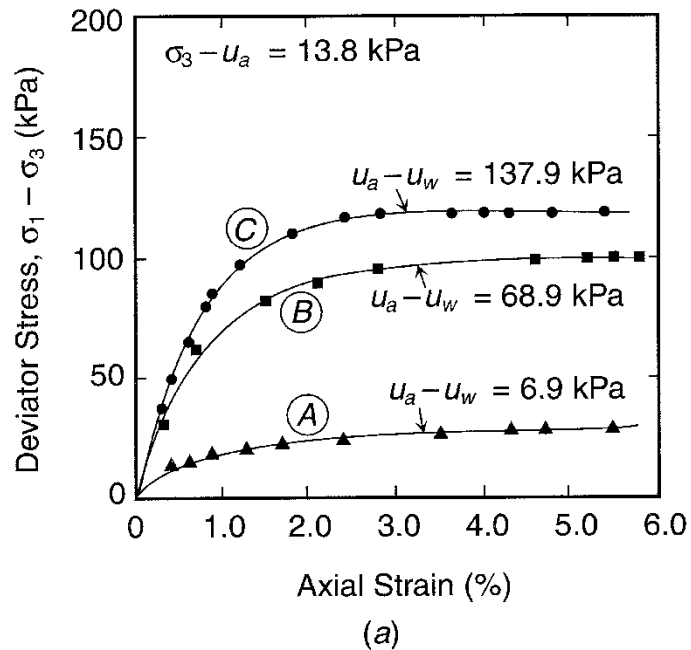


(c)



(d)

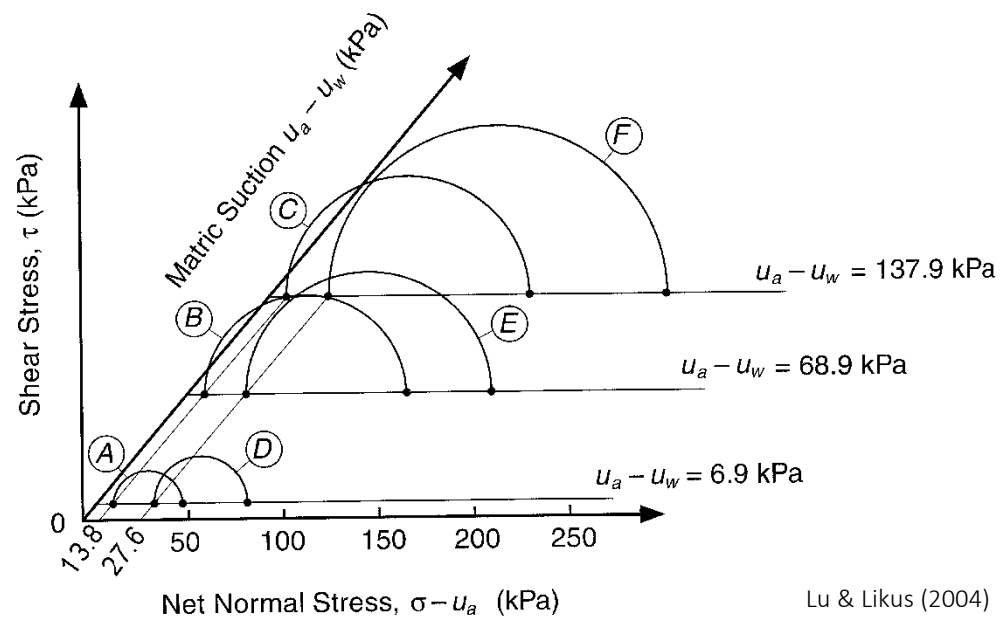
- $w$  CONSTANT, VARIOUS  $(\sigma_3 - U_a)$
- $\sigma_3 - U_a = 0$ , VARIOUS  $w$  VALUES
- $\sigma_3 - U_a = 30$  PSI, VARIOUS  $w$  VALUES



Lu & Likus (2004)

## Results of CD triaxial tests for a silt

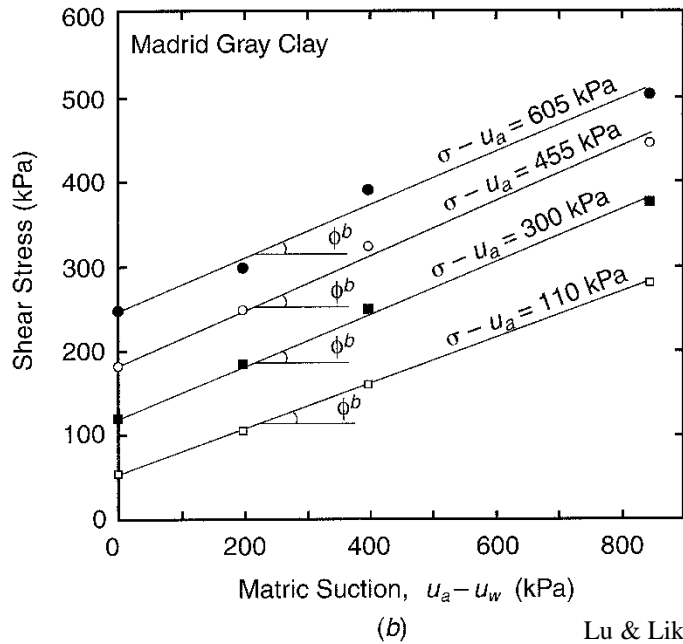
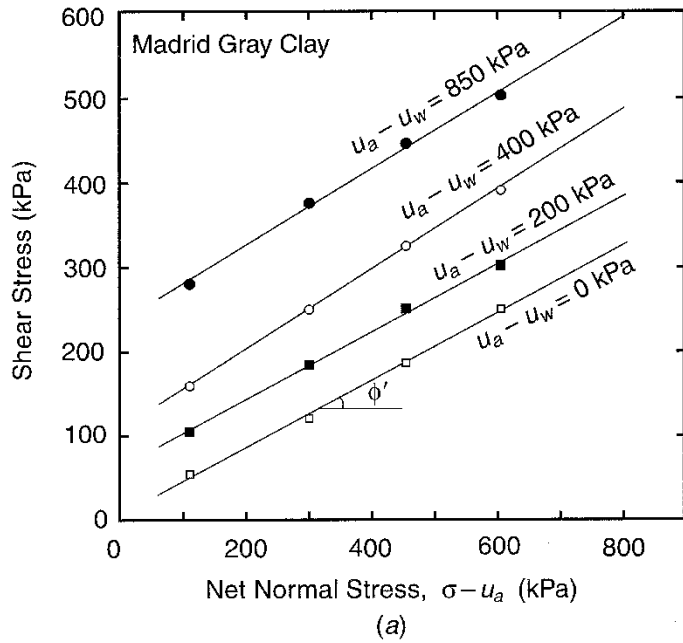
Data from Blight (1967)



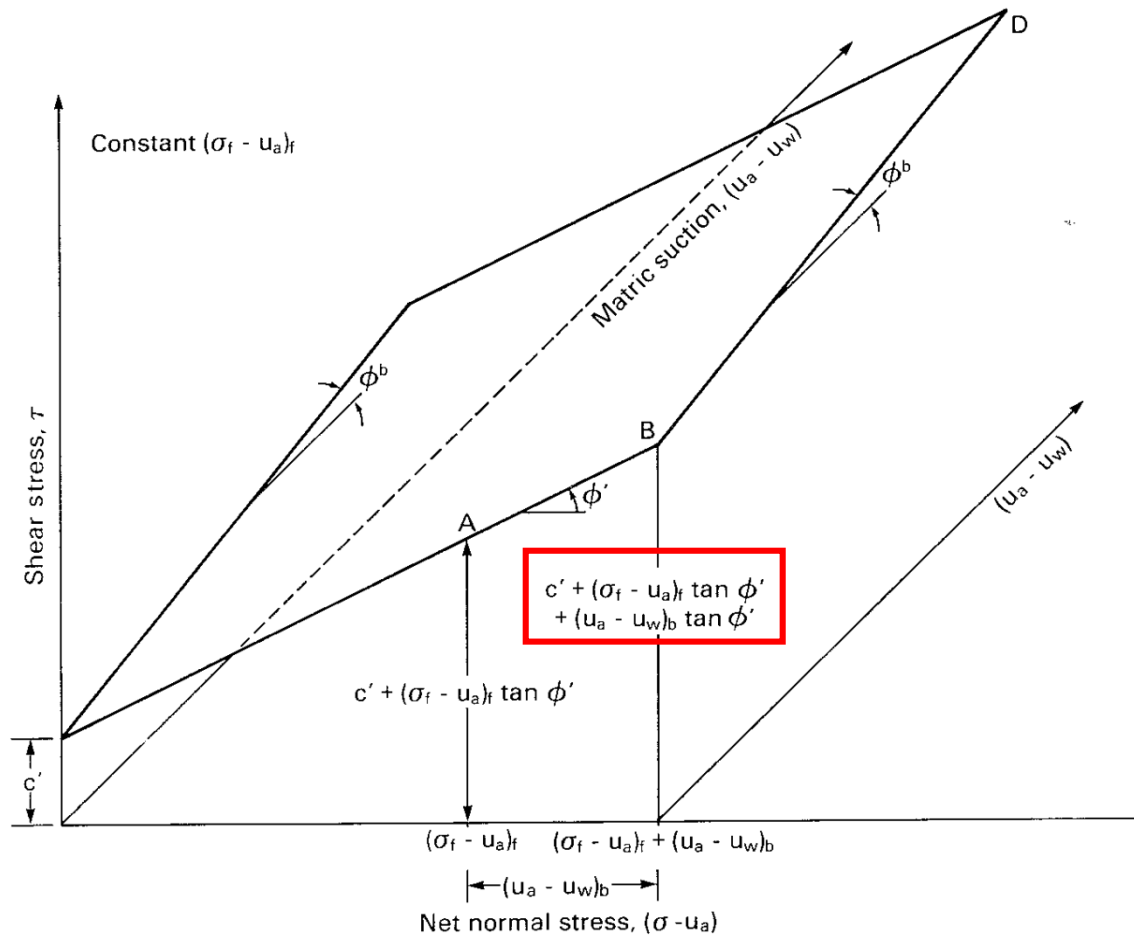
Lu & Likus (2004)

# Results of direct shear test in clay

Data from Escario (1980)



Lu & Likus (2004)



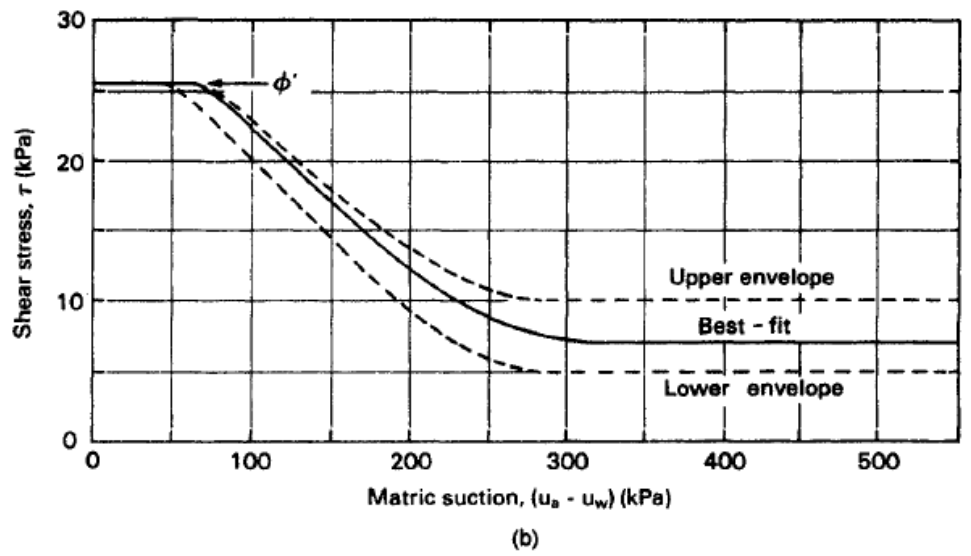
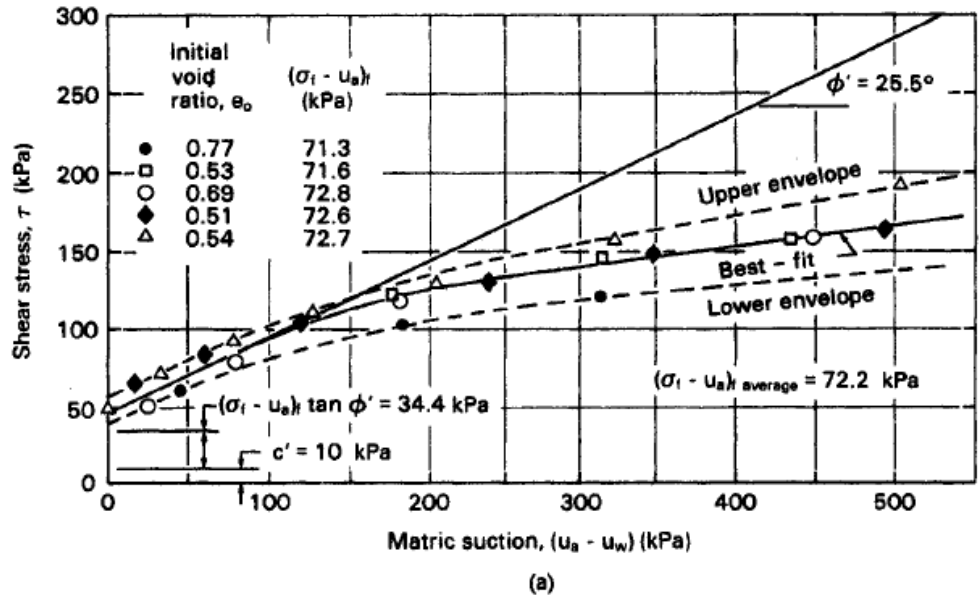
**Figure 9.57** Linearized extended Mohr-Coulomb failure envelope.



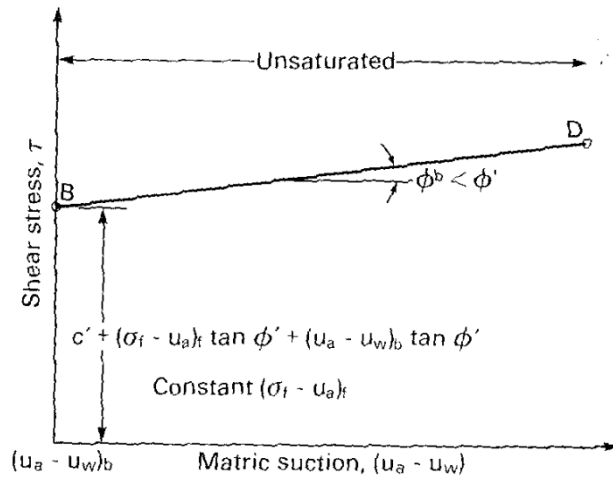


Failure envelopes obtained from unsaturated glacial till specimens.

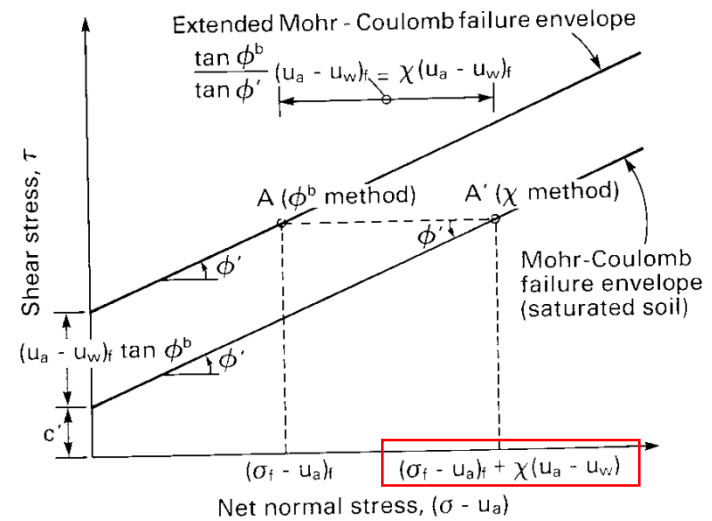
Failure envelopes on the  $\tau$  versus  $(u_a - u_w)$  plane;



the  $\phi^b$  values corresponding to the upper, lower and best-fit failure envelopes



**Figure 9.58** Linearized failure envelope on the shear stress versus matric suction plane.

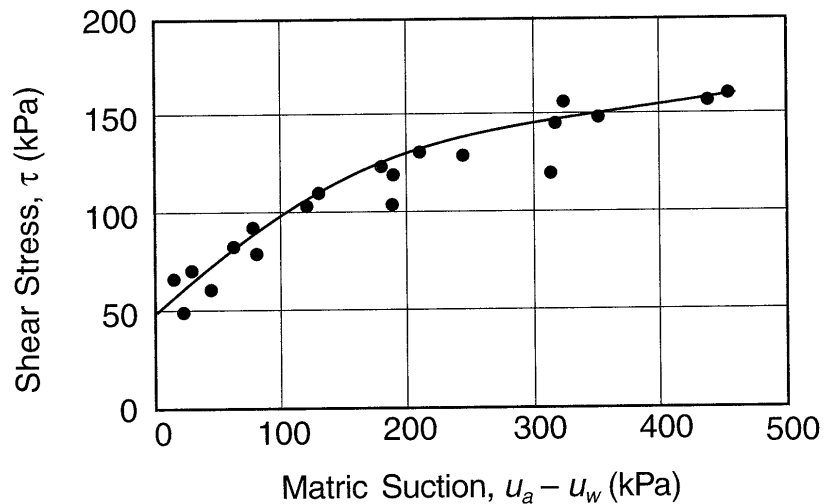


**Figure 9.59** Comparison of the  $\phi^b$  and  $\chi$  methods of designating shear strength.

$$(u_a - u_w)_f \tan \phi^b = \chi(u_a - u_w)_f \tan \phi'$$

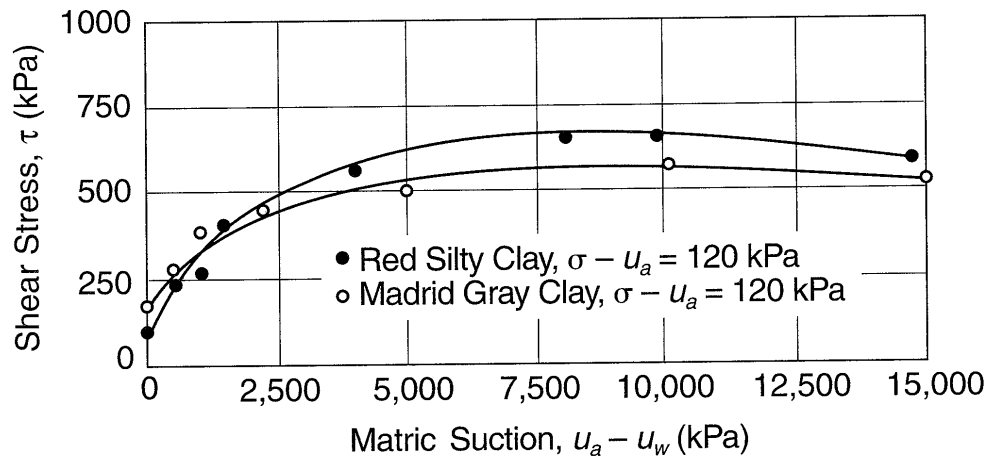
$$\chi = \frac{\tan \phi^b}{\tan \phi'}$$

Data from Gan et al. (1988)



(a)

Data from Escario et al. (1989)



(b)

Lu & Likus (2004)

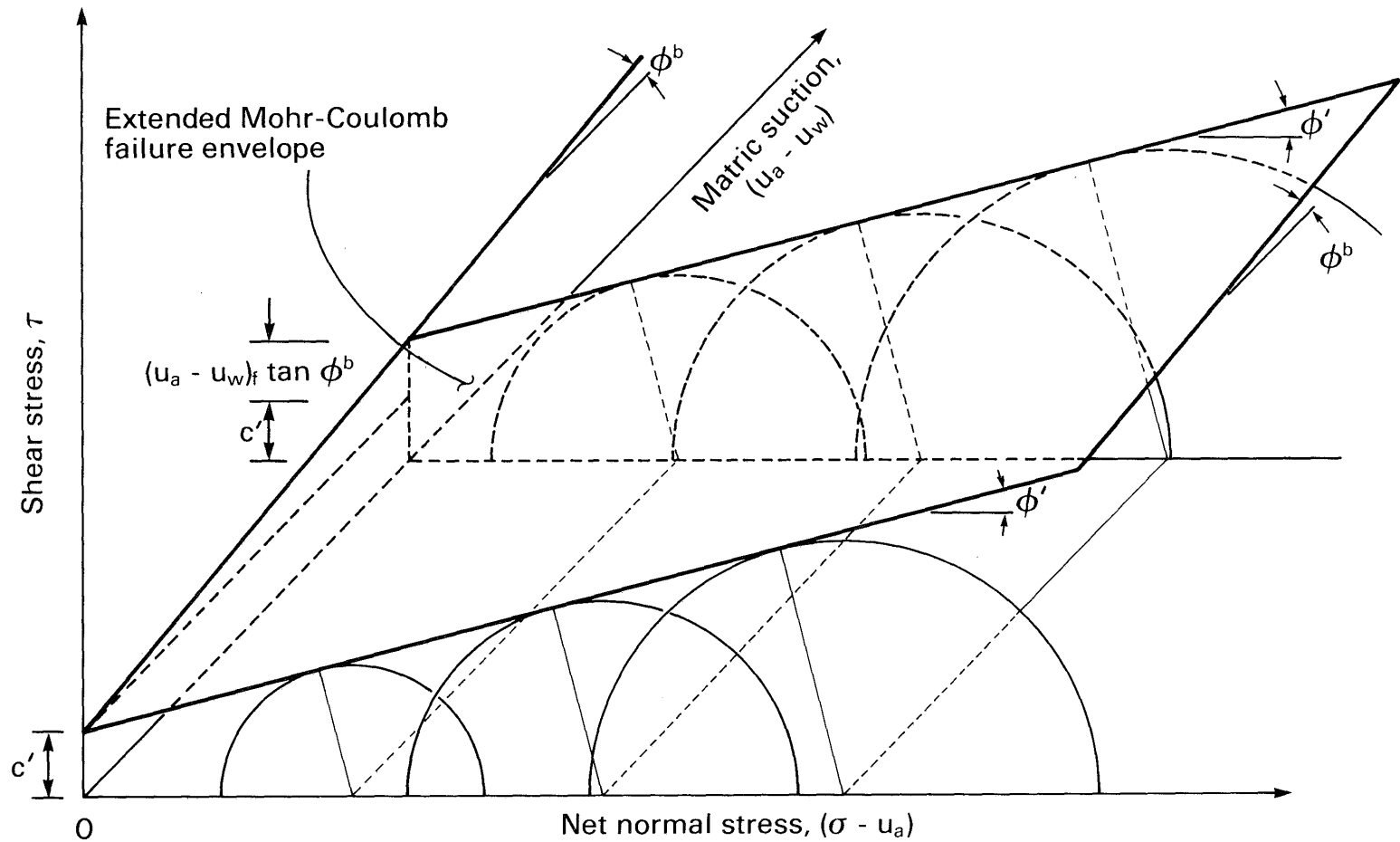
Non linear envelopes

## Triaxial Test for Unsaturated Soils

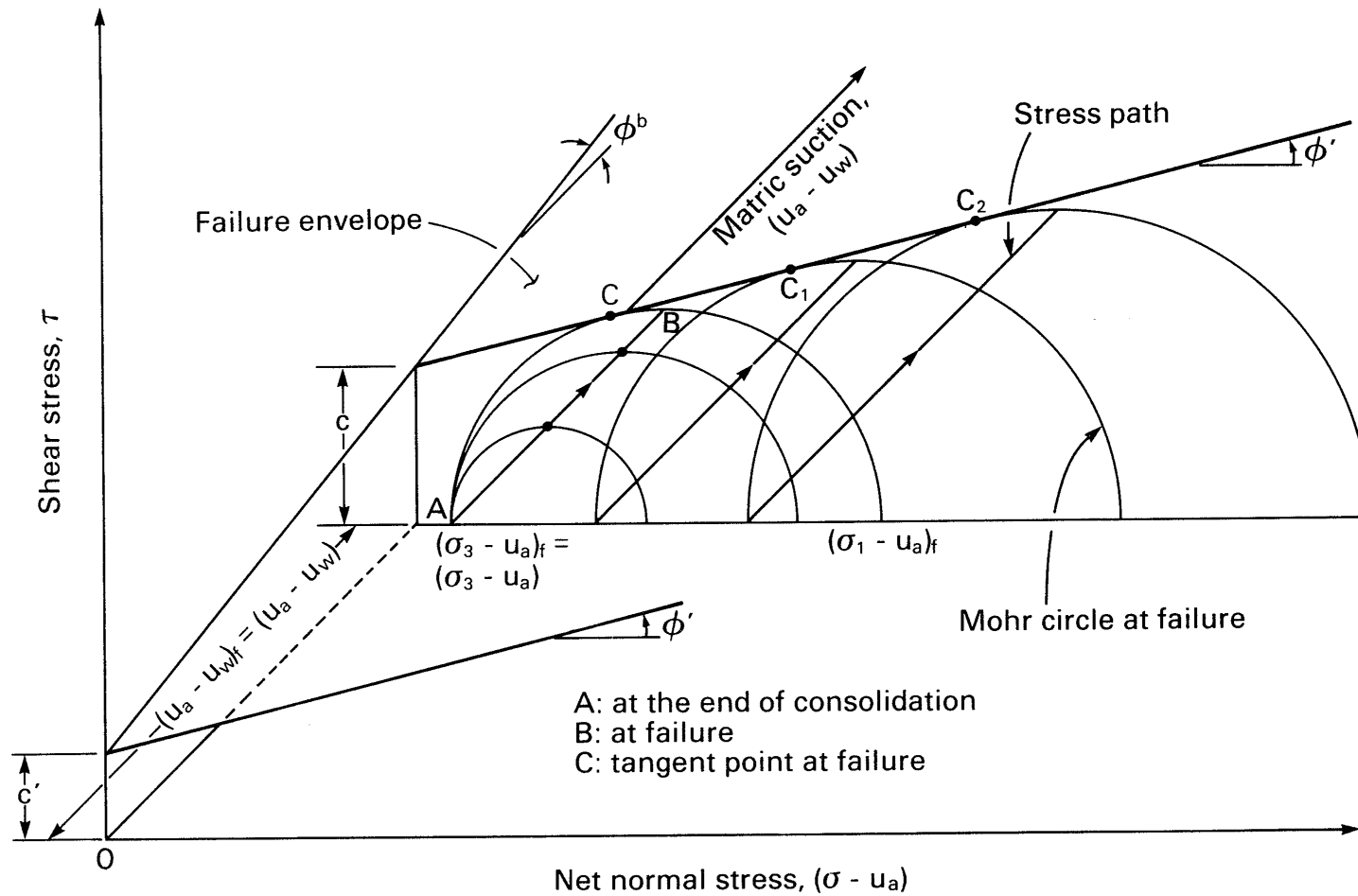
Test Methods	Consolidation	Drainage		Shearing Process		
	Prior to Shearing			Pore-Air	Pore-Water	Soil Volume
	Process	Pore-Air	Pore-Water	Pressure, $u_a$	Pressure, $u_w$	Change, $\Delta V$
Consolidated drained	Yes	Yes	Yes	C	C	M
Constant water content	Yes	Yes	No	C	M	M
Consolidated undrained	Yes	No	No	M	M	
Undrained	No	No	No			
Unconfined compression	No	No	No			

<sup>a</sup>Note: M = measurement, C = controlled.

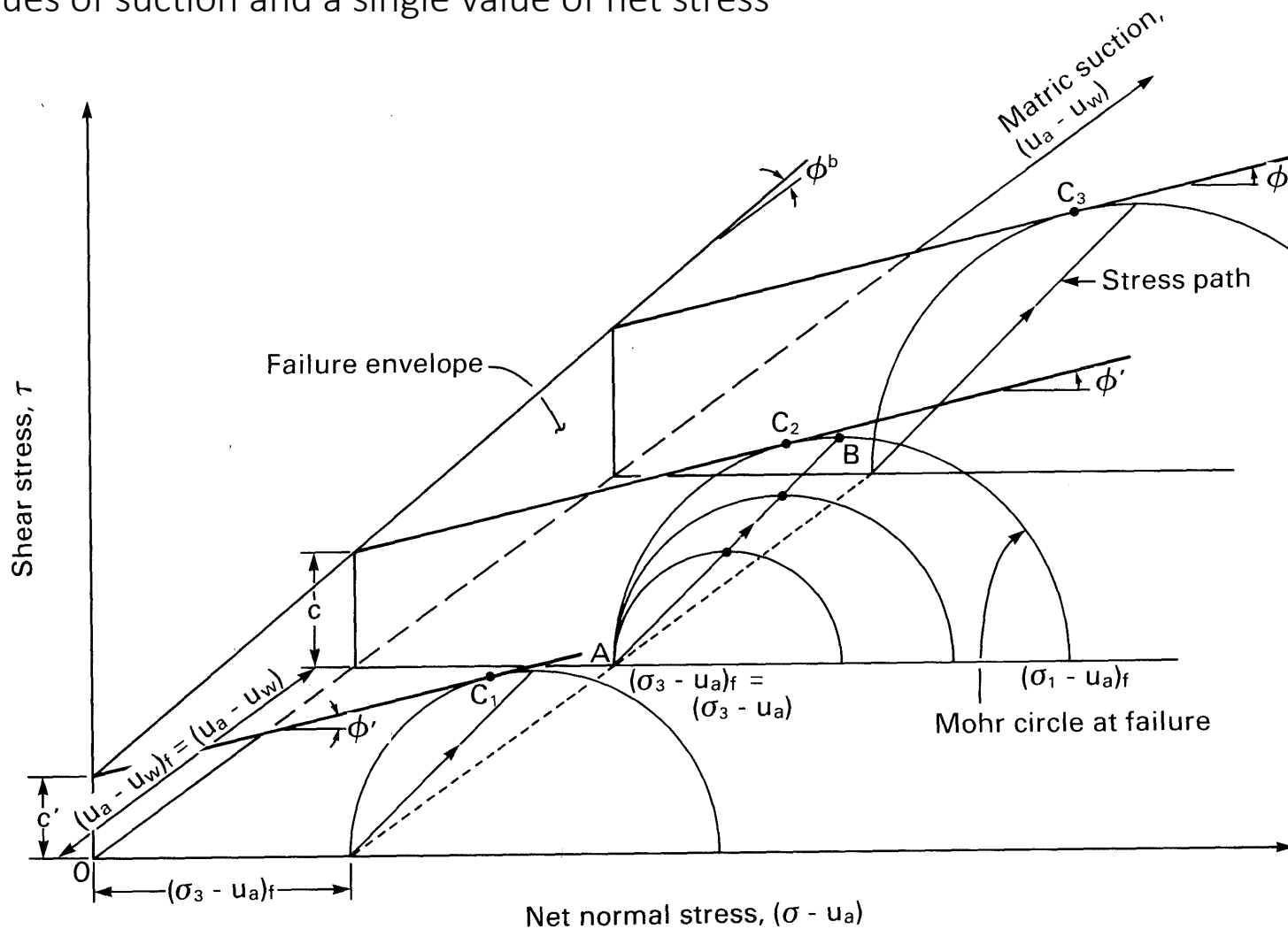
# Extended Mohr-Coulomb envelope for unsaturated soil



# Stress path during CD triaxial test with different net stress and the same suction



Stress path followed during CD test with different values of suction and a single value of net stress



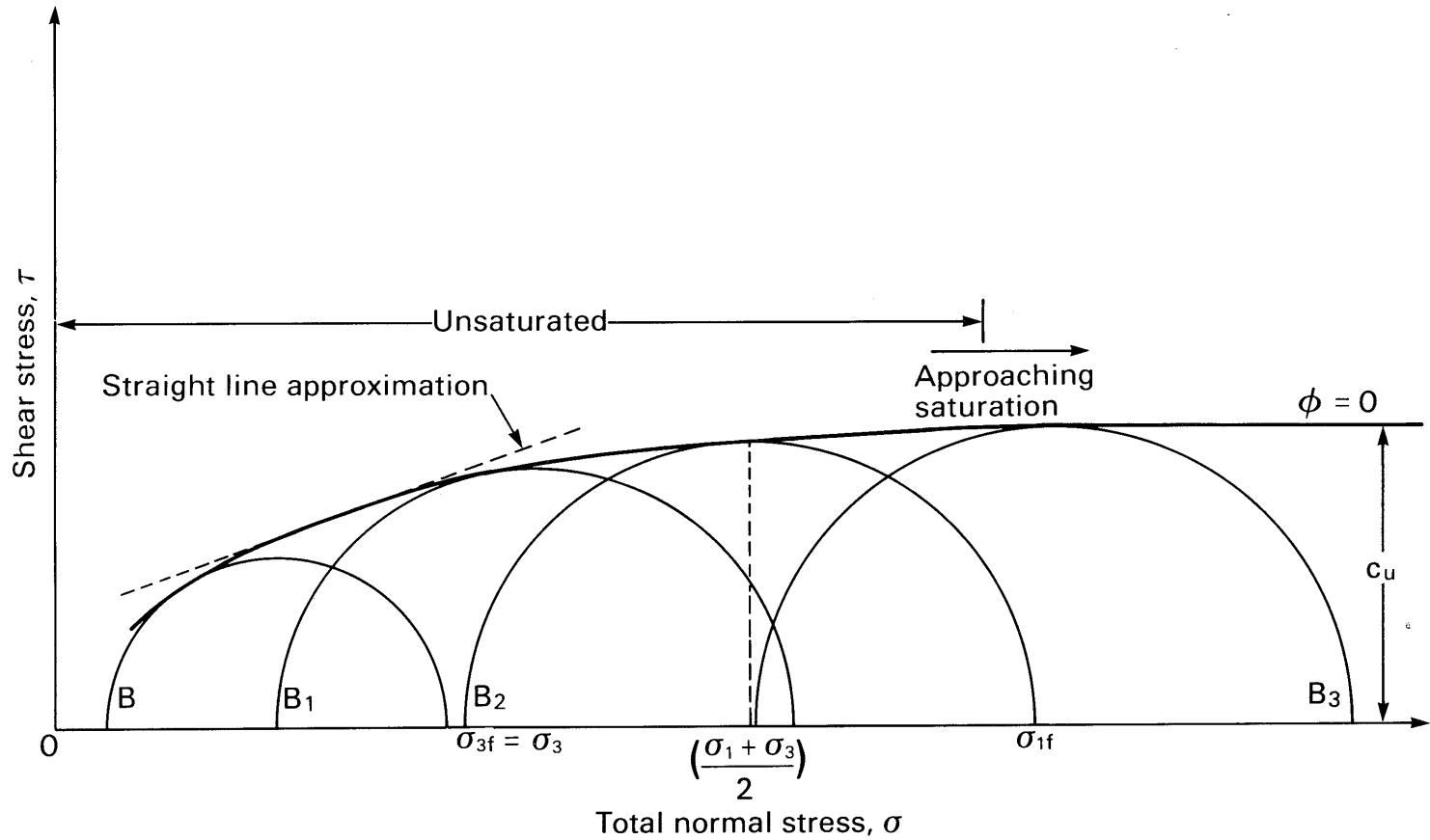




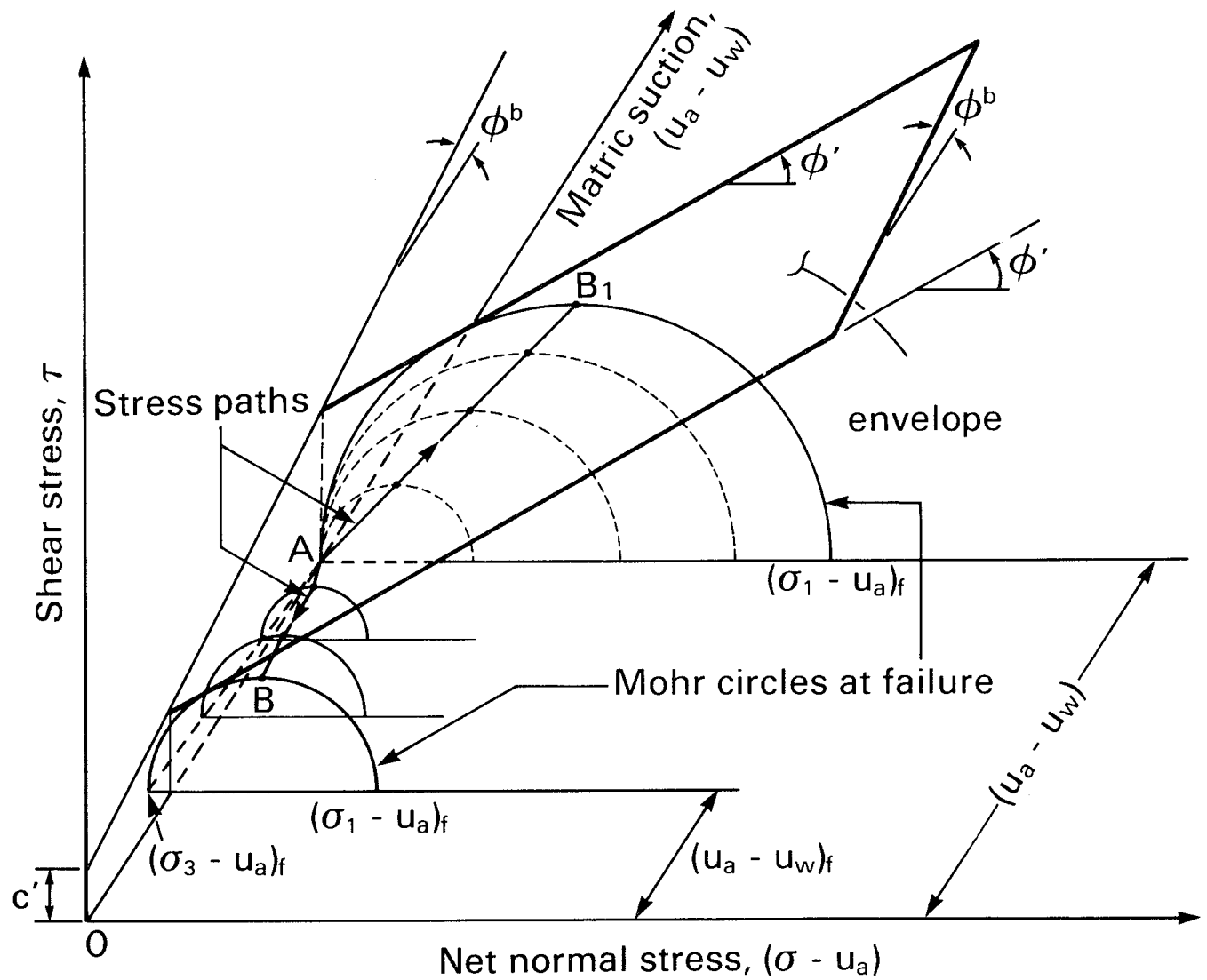




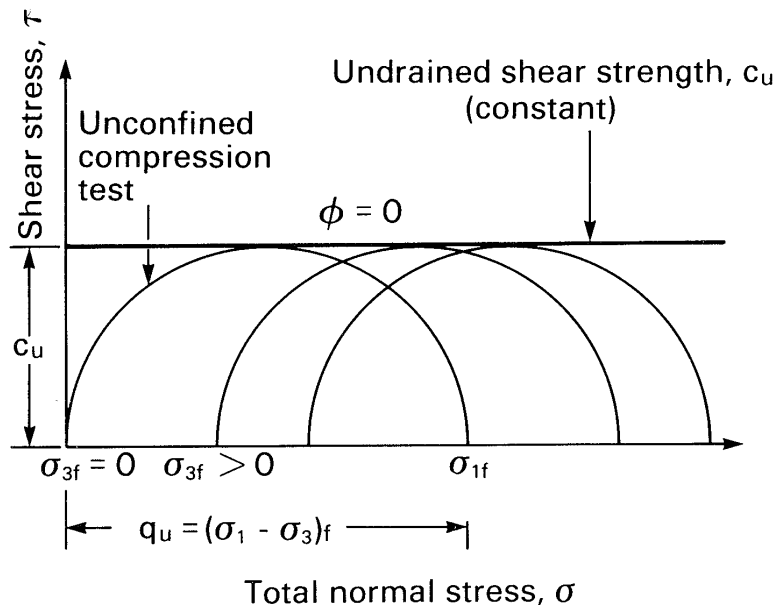
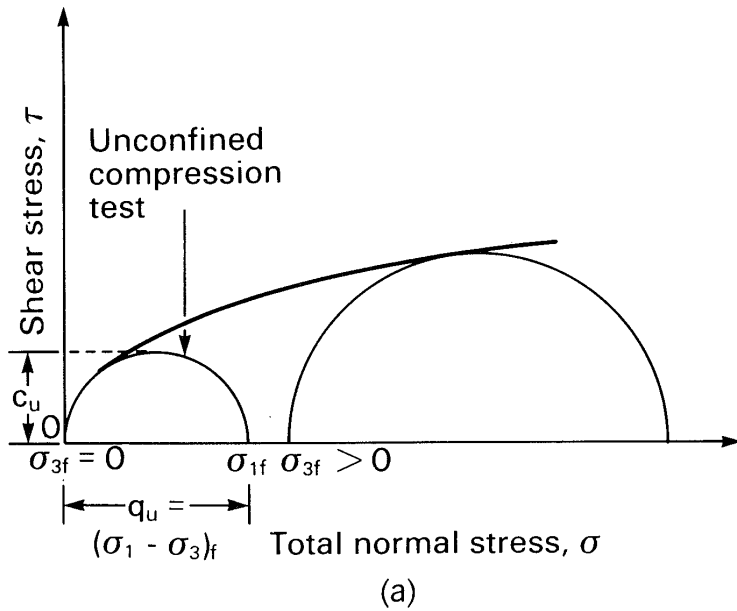
# UU failure envelope



# Possible stress path followed during unconfined compression test



# Unconfined shear strength and the undrained shear strength



(b)

**Table 1.** Experimental values for  $\phi^b$ .

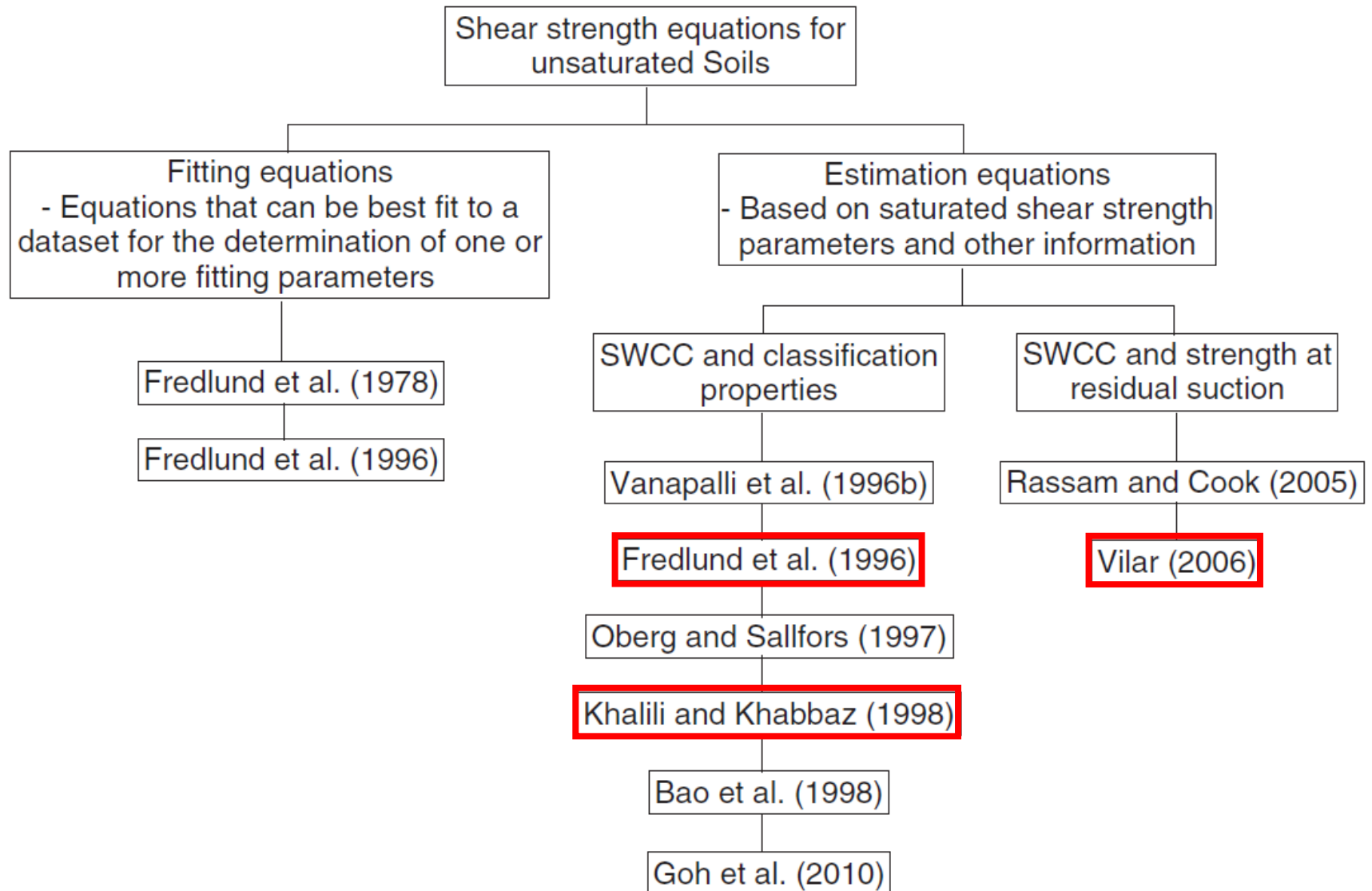
Soil type	$\phi^b$ (degrees)	Test procedure	Reference
Compacted shale; $w = 18.6\%$	18.1	Constant water content Triaxial	Bishop et al. (1960)
Boulder clay; $w = 22.2\%$	21.7	Constant water content Triaxial	Bishop et al. (1960)
Dhanauri clay; $w = 22.2\%$ , $\gamma_d = 15.5 \text{ kN/m}^3$	16.2	Consolidated drained Triaxial	Satija (1978)
Dhanauri clay; $w = 22.2\%$ , $\gamma_d = 14.5 \text{ kN/m}^3$	12.6	Consolidated drained Triaxial	Satija (1978)
Dhanauri clay; $w = 22.2\%$ , $\gamma_d = 15.5 \text{ kN/m}^3$	22.6	Constant water content Triaxial	Satija (1978)
Dhanauri clay; $w = 22.2\%$ , $\gamma_d = 14.5 \text{ kN/m}^3$	16.5	Constant water content Triaxial	Satija (1978)
Madrid Gray clay; $w = 29\%$ , $\gamma_d = 13.1 \text{ kN/m}^3$	16.1	Consolidated drained Direct shear	Escario (1980)
Undisturbed decomposed granite; Hong Kong	15.3	Consolidated drained Multi-stage triaxial	Ho and Fredlund (1982)
Undisturbed decomposed rhyolite; Hong Kong	13.8	Consolidated drained Multi-stage triaxial	Ho and Fredlund (1982)
Cranbrook silt	16.5	Consolidated drained Multi-stage triaxial	Fredlund (unpublished)

Fredlund (1985)

**Table 1.** Shear strength data on unsaturated soils.

Soil type	$c'$ (kPa)	$\phi'$ (deg.)	$\phi^b$ (deg.)	References
Compacted shale $w = 18.6\%$	15.8	24.8	18.1	Bishop et al. (1960)
Boulder Clay $w = 11.6\%$	9.6	27.3	21.7	Bishop et al. (1960)
Madrid grey clay	25.0	22.5	16.1	Escario (1980)
Decomposed granite	28.9	33.4	15.3	Ho and Fredlund (1982a)
Decomposed rhyolite	7.4	35.3	13.8	Ho and Fredlund (1982a)

Fredlund (1989)

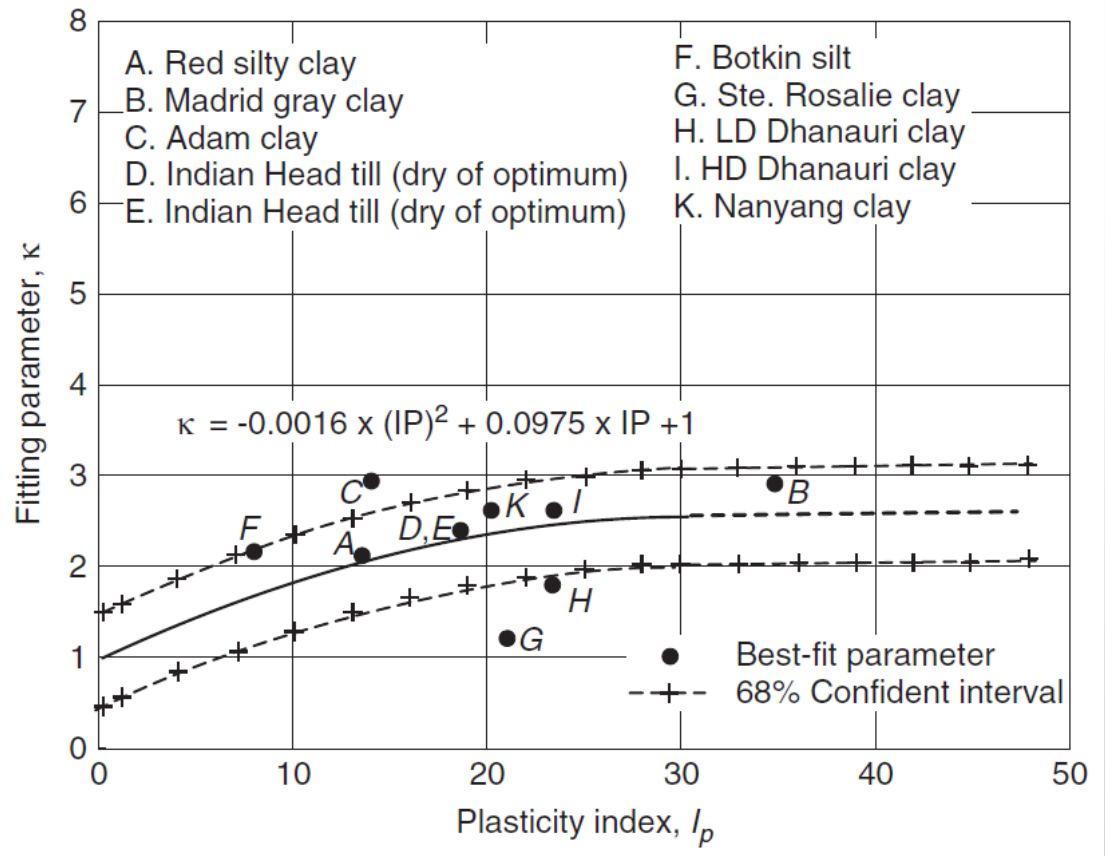




$$\tan \phi^b = \left( \frac{\theta}{\theta_s} \right)^\kappa \tan \phi' = \Theta_d^\kappa \tan \phi' = (S)^\kappa \tan \phi'$$

$$\Theta_d = \theta / \theta_s$$

$$\kappa = -0.0016(\text{PI})^2 + 0.0975(\text{PI}) + 1$$



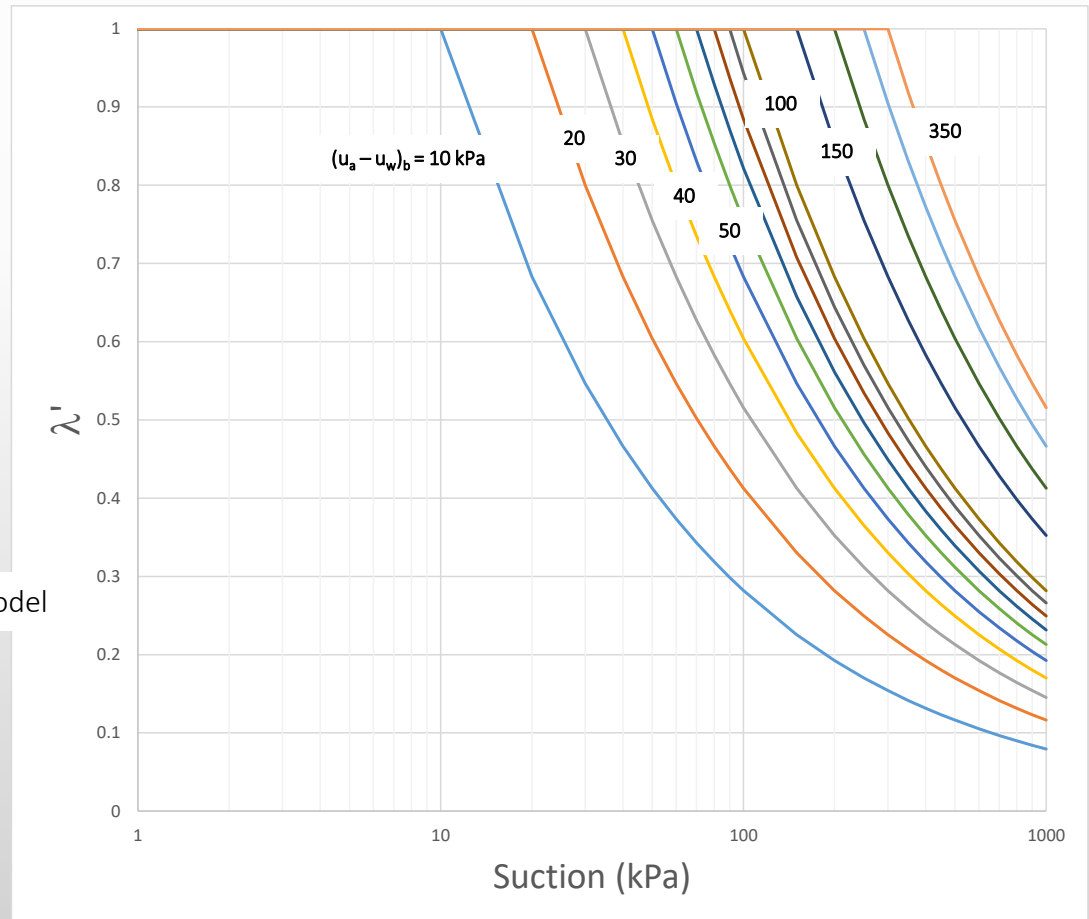
$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w) \tan \phi' \quad (\text{for saturated soil})$$

$$\tau = c' + (\sigma - u_a) \tan \phi' + (u_a - u_w)[\lambda'] \tan \phi'$$

$$\lambda' = \left[ \frac{u_a - u_w}{(u_a - u_w)_b} \right]^{-0.55}$$

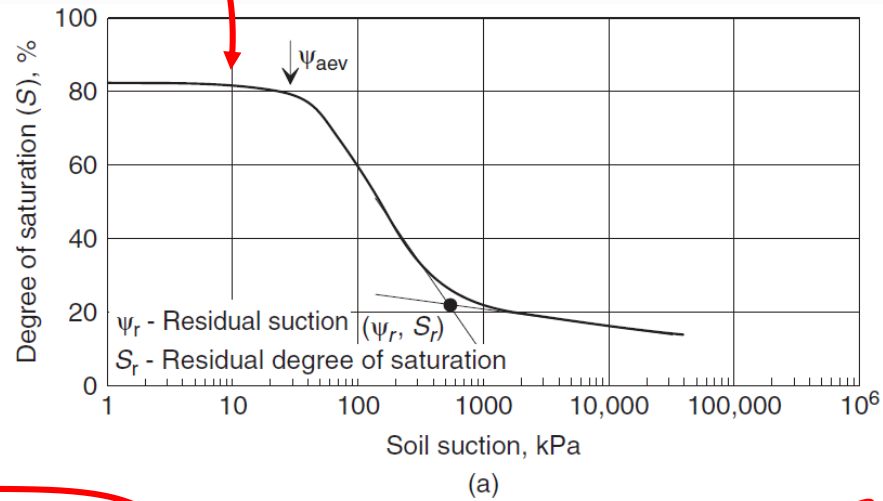
$$\chi = (u_a - u_w)\phi^b = \lambda'$$

The  $\lambda'$  parameter for Khalili and Khabbaz (1998) model

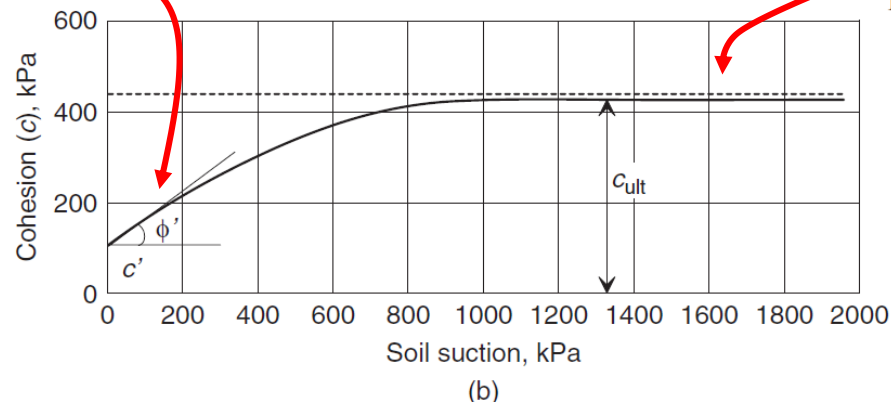


# Vilar (2006) Estimation Shear Strength Equation

$$c = c' + \frac{u_a - u_w}{a + b(u_a - u_w)}$$



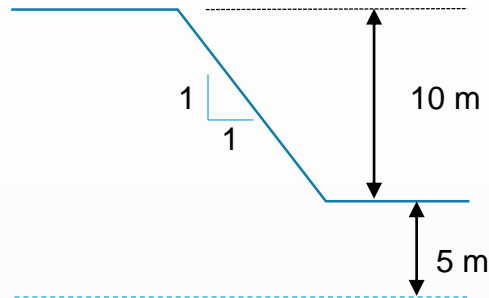
$$\frac{dc}{d(u_a - u_w)} \Big|_{\psi \rightarrow 0} = \frac{1}{a} = \tan \phi'$$



$$\lim_{\psi \rightarrow \infty} c = c_{ultimate} = c' + \frac{1}{b}$$

# Exercício

Considere o declive indicado na figura abaixo e responda as perguntas que seguem e seus aspectos específicos:



1. Suponha que a superfície seja completamente impermeável e estabeleça a distribuição da pressão da água a ser considerada em uma análise de estabilidade de talude.
2. Considere agora que a chuva pode infiltrar e evaporação. Indique a distribuição da pressão da água que seria possível.
3. Discuta o uso da pressão negativa nas seguintes situações:
  - a. Avaliação de uma ruptura,
  - b. projeto de um talude temporário,
  - c. projeto de um talude permanente em áreas urbanas
  - d. projeto de um talude permanente em uma área sem moradia.
4. Que consideração é necessária, além de definir a distribuição da pressão da água na região não saturada, para realizar uma análise de estabilidade do talude. Pense em termos de parâmetros.
5. Discuta a razão pela qual a pressão da água não é totalmente considerada quando o solo está na condição não saturada.
6. Escreva a expressão para a envoltória de resistência ao cisalhamento considerando que o solo está acima da camada freática, mas completamente saturado por capilaridade. O que você conclui?