i) Onda quadrada

⇝ $n \neq 0$

$$X[n] = \frac{1}{T} \int_T x(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T} \int_0^{T_0/2} e^{-j\frac{2\pi}{T_0} n t}\, dt + \int_{T_0/2}^{T_0} \cancel{0\, e^{-j\frac{2\pi}{T_0} n t}}\, dt$$

$$X[n] = \frac{1}{T_0} \left(\frac{e^{-j\frac{2\pi n}{T_0} t}}{-j\frac{2\pi}{T_0} n} \right) \Bigg|_0^{T_0/2} = \frac{1}{\cancel{T_0}} \frac{e^{-j\pi n} - 1}{-j\frac{2\pi}{\cancel{T_0}} n} = \frac{e^{-j\pi n} - 1}{-j 2\pi n}$$

Como $e^{-j\pi} = -1$,

$$X[n] = \frac{(-1)^n - 1}{-j2\pi n} = \frac{1-(-1)^n}{j2\pi n} \quad \text{para } n \neq 0$$

⇝ $n = 0$

$$X[0] = \frac{1}{T_0} \int_0^{T_0/2} e^{-j0t}\, dt = \frac{1}{T_0} \frac{T_0}{2} = \frac{1}{2}$$

$$\therefore \quad X[n] = \begin{cases} -\frac{1}{\pi n} j, & n \text{ ímpar} \\ 0, & n \text{ par} \\ \frac{1}{2}, & n = 0 \end{cases}$$

Perceba que $X[n] \to 0$ à medida que $n \to \infty$

ii) Onda dente de serra

$x = \frac{A}{T} t, \quad 0 \leq x \leq T$ $\qquad$ $x(t+T) = x(t)$, $\omega_0 = \frac{2\pi}{T}$

⇝ $n \neq 0$

$$X[n] = \frac{1}{T} \int_T x(t)\, e^{-jn\omega_0 t}\, dt$$

$$= \frac{1}{T} \int_T \frac{At}{T}\, e^{-jn\omega_0 t}\, dt = \frac{A}{T^2} \int_0^T t\, e^{-jn\omega_0 t}\, dt$$

Integral por partes

$$uv = \int u\,dv + \int v\,du \qquad u = t \quad du = dt \qquad v = e^{-jn\omega_0 t} = e^{-at} \quad dv = -a e^{-at}$$

$$t e^{-at} = -\int a t e^{-at} dt + \int e^{-at} dt$$

$$\int t e^{-at} dt = -\frac{1}{a}\left[t e^{-at}\Big|_0^T - \int e^{-at} dt \right]$$

$$= -\frac{t e^{-aT}}{a} - \frac{1}{a^2} e^{-at}\Big|_0^T = -\frac{T e^{-aT}}{a} - \frac{1}{a^2}\left(e^{-aT} - e^{0}\right)$$

$$e^{-aT} = e^{-jn\omega_0 T} \cdot e^{-j2\pi n} = \cos 2\pi n - j \operatorname{sen} 2\pi n = 1$$

$$\therefore \int_0^T t\, e^{-jn\omega_0 t} dt = -\frac{T}{a} = -\frac{T}{jn\omega_0} = \frac{T}{n\omega_0} j$$

$$\therefore X[n] = \frac{A}{T^2}\frac{T}{n\omega_0} j = \frac{A}{n \underbrace{T\omega_0}_{2\pi}} j = \frac{A}{2\pi n} j$$

$\rightarrow n = 0$

$$X[0] = \frac{1}{T}\int_0^T \frac{A}{T} t = \frac{A}{T^2}\left(\frac{t^2}{2}\right)\Big|_0^T = \frac{A}{2}$$

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} X[n]\, e^{jn\omega_0 t}$$

$$\therefore \boxed{x(t) = \frac{A}{2} + \frac{A}{2\pi} j \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{e^{jn\omega_0 t}}{n}}$$

Qual o primeiro harmônico?

$$n = \pm 1 \quad z \rightarrow \frac{A}{2\pi} j \left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) = -\frac{A}{\pi}\left(\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}\right) = \boxed{-\frac{A}{\pi} \operatorname{sen} \omega_0 t}$$

Qual o segundo harmônico? ...

iii) Sinal dente de serra deslocado $\tau = 2$ no tempo

Da solução anterior, sabe-se que,

$$X[n] = \frac{A}{2\pi n} j \qquad\qquad X[0] = \frac{A}{2}$$

Da propriedade de deslocamento no tempo, sabe-se que:

$$Y[n] = X[n]\, e^{-jn\omega_0 \tau}$$

onde $Y[n]$ é a solução para o sinal dente de serra deslocado τ no tempo

$$X[n] = \frac{4}{2\pi n} j = \frac{2}{\pi n} j$$

$$\omega_0 = \frac{2\pi}{4} = \frac{\pi}{2}$$

Para $\tau = 2$

$$Y[n] = \frac{2}{\pi n} j\, e^{-jn\frac{\pi}{2}\cdot 2} = \frac{2}{\pi n} j\, e^{-jn\pi}$$

$$e^{-jn\pi} = \cos n\pi - j\cancel{\sin n\pi}^{\,0}$$
$$= (-1)^n$$

$$Y[n] = \frac{2(-1)^n}{\pi n} j$$

$$\therefore y(t) = \frac{4}{2} + \frac{2}{\pi} j \sum_{\substack{n=-\infty \\ n\neq 0}}^{\infty} \frac{(-1)^n e^{jn\frac{\pi}{2}t}}{n}$$

Qual o primeiro harmônico?

$$n = 1 \Rightarrow \frac{2}{\pi} j \left[e^{-j\pi/2t} \frac{(-1)^{-1}}{(-1)} + e^{j\pi/2t} \frac{(-1)^1}{1} \right]$$

$$= \frac{2\times 2}{\pi} \left[\frac{e^{j\pi/2t} - e^{-j\pi/2t}}{2j} \right] = \frac{4}{\pi} \sin \frac{\pi}{2} t$$

iv)

$x(t) = e^t \quad -\pi < t < \pi \qquad x(t+2\pi) = x(t)$

$T = 2\pi \qquad \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$

$$X[n] = \frac{1}{T}\int_0^T x(t)\, e^{-jn\omega_0 t}\, dt$$

$$X[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^t e^{-jnt} dt = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{(1-jn)t} dt = \frac{1}{2\pi}\left[\frac{e^{(1-jn)t}}{(1-jn)}\right]_{-\pi}^{\pi}$$

$$= \frac{1}{2\pi(1-jn)}\left(e^{\pi} e^{-jn\pi} - e^{-\pi} e^{jn\pi}\right)$$

Sabe-se que $e^{(1-jn)\pi} = e^{\pi}\left(\cos n\pi - i\,\cancel{\operatorname{sen} n\pi}\right) = e^{\pi}\cos n\pi$

$$\therefore X[n] = \frac{1}{2\pi(1-jn)}\left(e^{\pi} - e^{-\pi}\right)\cos n\pi \qquad \sinh x = \frac{e^x - e^{-x}}{2}$$

$$X[n] = \frac{\operatorname{senh}\pi}{\pi}\frac{(1-jn)}{(1+n^2)}(-1)^n$$

$$X[0] = \frac{\operatorname{senh}\pi}{\pi}$$

$$\therefore \boxed{x(t) = \frac{\operatorname{senh}\pi}{\pi}\sum_{n=-\infty}^{\infty}(-1)^n\frac{(1+jn)}{(1+n^2)}e^{jnt}}$$

$n = \pm 1$

$$-\frac{\operatorname{senh}\pi}{2\pi}e^{jt}(1+j)$$

$$-\frac{\operatorname{senh}\pi}{2\pi}e^{-jt}(1-j)$$

$$-\frac{\operatorname{senh}\pi}{2\pi}\left[\left(e^{jt} - e^{-jt}\right)j + \left(e^{jt} + e^{-jt}\right)\right]$$

$$= -\frac{\operatorname{senh}\pi}{\pi}\left[-\operatorname{sen} t + \cos t\right]$$

$\omega_0 = 1$

$$-\frac{\operatorname{senh}\pi}{2\pi}$$

iv) $T_1 = 0.02$, $T = 0.01$

$$X[n] = \frac{1}{T}\int_0^T \operatorname{sen}(2\pi t/T_1)\, e^{-j\frac{2\pi}{T}nt}\, dt$$

$$= \frac{1}{T}\int_0^T \left(\frac{e^{j\pi/T\, t} - e^{-j\pi/T\, t}}{2j}\right) e^{-j\frac{2\pi}{T}nt}\, dt$$

$$= \frac{1}{2jT}\int_0^T \left(e^{-j\frac{\pi}{T}(2n-1)t} - e^{-j\frac{\pi}{T}(2n+1)t}\right) dt$$

$$= \frac{-\cancel{T}}{2j\cancel{T}\, j\pi}\left(\frac{1}{2n-1} e^{-j\pi/T(2n-1)t} - \frac{1}{2n+1} e^{-j\pi/T(2n+1)t}\right)\Bigg|_0^T$$

$$= \frac{1}{2\pi}\left[\left(\frac{1}{2n-1}\underbrace{\cancel{e^{-j\pi(2n-1)}}}_{-1} - \frac{1}{2n+1}\underbrace{\cancel{e^{-j\pi(2n+1)}}}_{-1}\right) - \left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)\right]$$

$e^{-j\pi n} = \cos\pi n - \operatorname{sen}\overset{0}{\pi n}$
$= (-1)$ se n ímpar
$= (1)$ se n par

$$= \frac{1}{\cancel{2}\pi}\,\frac{\cancel{2n}+\cancel{1}-\cancel{2n}+\cancel{1}}{(4n^2-1)} \times 2 = \frac{2}{\pi(4n^2-1)}$$

$$\therefore\ X[n] = \frac{2}{\pi(4n^2-1)}$$

$$\boxed{x(t) = \frac{2}{\pi}\sum_{n=-\infty}^{\infty}\frac{1}{(4n^2-1)}}$$

vi) $x(t) = \sin^3 3\pi t$

$$x(t) = \left(\frac{e^{j3\pi t} - e^{-j3\pi t}}{2j} \right)^3 = \frac{1}{-8j} \left(e^{j9\pi t} - 3 e^{j6\pi t} e^{-j3\pi t} + 3 e^{j3\pi t} e^{-j6\pi t} - e^{-j9\pi t} \right)$$

$$= \frac{j}{8}\left[e^{j9\pi t} - e^{-j9\pi t} \right] - \frac{3}{8}j \left[e^{j3\pi t} - e^{-j3\pi t} \right]$$

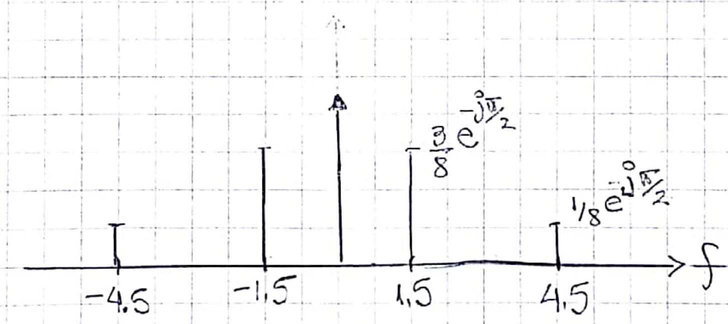
O exemplo mostra que não é sempre necessário resolver a integral para obter os valores dos coeficientes da série de Fourier.

Verifica-se que $x(t)$ é a soma de quatro termos com frequências $\omega = \pm 3\pi$ e $\omega = \pm 9\pi$. A frequência fundamental é $\omega_0 = 3\pi$ rad/s.

Lembre-se que $a_n \cos n\omega_0 t + b_n \operatorname{sen} n\omega_0 t = \frac{a_n}{2}\left(e^{jn\omega_0 t} + e^{-jn\omega_0 t}\right) + \frac{b_n}{2j}\left(e^{jn\omega_0 t} - e^{-jn\omega_0 t}\right)$

$$= e^{jn\omega_0 t} \underbrace{\left(\frac{a_n - b_n j}{2} \right)}_{X[n]} + e^{-jn\omega_0 t} \underbrace{\left(\frac{a_n + b_n j}{2} \right)}_{X[-n]}$$

$$\therefore X[n] = \begin{cases} 0 & \text{se } n = 0 \\ \mp 3j/8 & \text{se } n = \pm 1 \\ 0 & \text{se } n = \pm 2 \\ \pm j/8 & \text{se } n = \pm 3 \\ 0 & \text{c.c. anteriores} \end{cases}$$

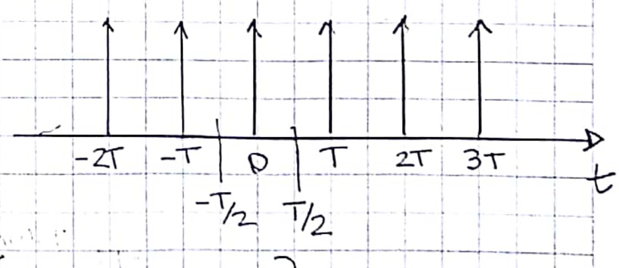


OBS

$$\begin{cases} e^{-j\pi/2} = -j \\ e^{j\pi/2} = j \end{cases}$$

vii) trem de impulsos

$$x(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)$$



$$X[n] = \frac{1}{T}\int_T x(t)\, e^{-jn\omega_0 t}\, dt$$

Seleciona-se o intervalo $-T/2$ e $T/2$, evitando a colocação de impulsos nos limites da integração.

$$X[n] = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, \underbrace{e^{-jn\omega_0 t}}_{1}\, dt$$

$$X[n] = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)\, dt = \frac{1}{T}$$

Em outras palavras, os coeficientes da série de Fourier

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} \frac{1}{T} e^{jn\frac{2\pi}{T}t}$$