Multitask Principal–Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design

Bengt Holmstrom
Yale University

Paul Milgrom
Stanford University

1. Introduction

In the standard economic treatment of the principal–agent problem, compensation systems serve the dual function of allocating risks and rewarding productive work. A tension between these two functions arises when the agent is risk averse, for providing the agent with effective work incentives often forces him to bear unwanted risk. Existing formal models that have analyzed this tension, however, have produced only limited results.[1] It remains a puzzle for this theory that employment contracts so often specify fixed wages and more generally that incentives within firms appear to be so muted, especially compared to those of the market. Also, the models have remained too intractable to effectively address broader organizational issues such as asset ownership, job design, and allocation of authority.

In this article, we will analyze a principal–agent model that (i) can account for paying fixed wages even when good, objective output measures are available and agents are highly responsive to incentive pay; (ii) can make recommendations and predictions about ownership patterns even when contracts can take full account of all observable variables and court enforcement is perfect; (iii) can explain why employment is sometimes superior to independent con-

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1. Some of the predictive weaknesses of standard agency models are discussed in the surveys by MacDonald, Hart and Holmstrom, and Baker, Jensen, and Murphy.

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tracting even when there are no productive advantages to specific physical or human capital and no financial market imperfections to limit the agent's borrowings; (iv) can explain bureaucratic constraints; and (v) can shed light on how tasks get allocated to different jobs.

The distinguishing mark of our model is that the principal either has several different tasks for the agent or agents to perform, or the agent's single task has several dimensions to it. Some of the issues raised by this modeling are well illustrated by the current controversy over the use of incentive pay for teachers based on their students' test scores.[2] Proponents of the system, guided by a conception very like the standard one-dimensional incentive model, argue that these incentives will lead teachers to work harder at teaching and to take greater interest in their students' success. Opponents counter that the principal effect of the proposed reform would be that teachers would sacrifice such activities as promoting curiosity and creative thinking and refining students' oral and written communication skills in order to teach the narrowly defined basic skills that are tested on standardized exams. *It would be better, these critics argue, to pay a fixed wage without any incentive scheme than to base teachers' compensation only on the limited dimensions of student achievement that can be effectively measured.*[3]

Multidimensional tasks are ubiquitous in the world of business. As simple examples, production workers may be responsible for producing a *high volume* of *good quality* output, or they may be required both to produce output and to care for the machines they use. In the first case, if volume of output is easy to measure but the quality is not, then a system of piece rates for output may lead agents to increase the volume of output at the expense of quality. Or, if quality can be assured by a system of monitoring or by a robust product design, then piece rates may lead agents to abuse shared equipment or to take inadequate care of it. In general, when there are multiple tasks, incentive pay serves not only to allocate risks and to motivate hard work, it also serves to direct the allocation of the agents' attention *among* their various duties. This represents the first fundamental difference between the multidimensional theory and the more common one-dimensional principal–agent models.

There is a second fundamental difference as well, and it, too, can be illustrated by reference to the problem of teaching basic skills: If the task of teaching basic skills could be separated from that of teaching higher-level thinking, then these tasks could be carried out by different teachers at different times during the day. Similarly, in the example of the production worker, when the care and maintenance of a productive asset can be separated from the use of that asset in producing output, the problem that a piece rate system would lead to inadequate care can be mitigated or even eliminated. In general, in multitask principal–agent problems, *job design is an important instrument*

2. See Hannaway for a discussion of these issues.

3. As a concrete illustration of the distortions that testing can cause, in 1989 a ninth-grade teacher in Greenville, South Carolina was caught having passed answers to questions on the statewide tests of basic skills to students in her geography classes in order to improve her performance rating (*Wall Street Journal*, November 2, 1989).

for the control of incentives. In the standard model, when each agent can engage in only one task, the grouping of tasks into jobs is not a relevant issue.[4]

Our formal modeling of these issues utilizes our linear principal–agent model (Holmstrom and Milgrom, 1987), mainly specialized to the case where the agent's costs depend only on the *total* effort or attention the agent devotes to all of his tasks. This modeling assures that an increase in an agent's compensation in any one task will cause some reallocation of attention away from other tasks. First, we show that an optimal incentive contract can be to pay a fixed wage independent of measured performance, just as the opponents of incentives based on educational testing have argued. More generally, the desirability of providing incentives for any one activity decreases with the difficulty of measuring performance in any other activities that make competing demands on the agent's time and attention. This result may explain a substantial part of the puzzle of why incentive clauses are so much less common than one-dimensional theories would predict.

Second, we specialize our model to the case where the unmeasurable aspect of performance is how the value of a productive asset changes over time. The difficulties of valuing assets are well recognized, and the vast majority of accounting systems value assets using fixed depreciation schedules based on historical costs, deviating from this procedure only in exceptional circumstances. Under these conditions, when the principal owns the returns from the asset, the optimal incentive contract will provide only muted incentives for the agent to produce output, in order to mitigate any abuse of the asset or any substitution of effort away from asset maintenance. However, when the agent owns the asset returns, the optimal incentive contract will provide more intensive incentives to engage in production, in order to alleviate the reverse problem that the agent may use the asset too cautiously or devote too much attention to its care and improvement. This analysis supports Williamson's observation that "high-powered" incentives are more common in market arrangements than within firms, without relying on any assumptions about specific investments. Moreover, it provides a rudimentary theory of ownership, according to which the conditions that favor the agent owning the assets are (i) that the agent is not too risk averse, (ii) that the variance of asset returns is low, and (iii) that the variance of measurement error in other aspects of the agent's performance is low. Thus, it emphasizes measurement cost as an important determinant of integration in contrast to the leading approaches, which stress asset specificity.[5]

4. Riordan and Sappington also analyze an incentive model in which job assignment is central, but for a very different reason. They ask when the principal should do one of two sequential production stages herself in order to reduce the agent's information advantage. In our model, job assignments do not affect the principal's information.

5. Alchian and Demsetz argued that monitoring difficulties account for the formation of firms, but their theory was subsequently rejected in favor of the view that asset specificity and ex post bargaining problems drive integration (Grossman and Hart, Williamson). We are reintroducing measurement cost as a key factor, but in a way that differs from the original Alchian–Demsetz theory. In particular, we do not argue that owners can better monitor the work force. Our approach is more closely related to Barzel's work.

Our prediction fits well with the empirical evidence reported by Anderson and Anderson and Schmittlein. They found that firms in the electronics industry tend to employ their own sales forces rather than independent manufacturer's representatives when some aspects of the representative's performance are hard to measure. Our result can also help explain why franchisees face steep performance incentives, while managers of identical company-owned stores receive no incentive pay at all (Krueger, Brickley and Dark), and why a free-lance writer might be paid for articles by the word, while a staff reporter for the same publication receives a fixed wage.

Third, we explore how a firm might optimally set policies limiting personal business activities on company time. Again, it is not just the characteristics of the "outside activities" themselves that determine whether these activities should be permitted. We find that outside activities should be most severely restricted when performance in the tasks that benefit the firm—the "inside activities"—are hard to measure and reward. Thus, a salesperson whose pay is mostly in the form of commissions will optimally be permitted to engage in more personal activities during business hours than a bureaucrat who is paid a fixed wage, because the commissions direct the salesperson toward inside activities in a way that cannot be duplicated for the bureaucrat. Our theory also predicts that home office work should be accompanied by a stronger reliance on performance-based pay incentives, a prediction that seems to fit casual observation.

Our analysis of restrictions on outside activities underscores the fact that incentives for a task can be provided in two ways: either the task itself can be rewarded or the marginal opportunity cost for the task can be lowered by removing or reducing the incentives on competing tasks. Constraints are substitutes for performance incentives and are extensively used when it is hard to assess the performance of the agent. We believe this opens a new avenue for understanding large-scale organization. It also offers an alternative interpretation of the Anderson–Schmittlein evidence. It is inefficient to let a salesperson, whose performance is poorly measured, divert his time into commission selling of competing products. If the employer has an advantage in restricting the employees' other activities, as both Simon and Coase have argued, then problems with measuring sales performance will lead to employing an in-house sales force.

Finally, we obtain a series of results in the theory of job design, using a model in which the employer can divide responsibility for many small tasks between two agents and can determine how performance in each task will be compensated. The resulting optimization problem is a fundamentally nonconvex one, and we have had to make some extra assumptions to keep the analysis tractable. Nevertheless, the results we obtain seem intriguing and suggestive. First, we find that each task should be made the responsibility of just one agent. To our knowledge, this is the first formal derivation in the incentive literature of the principle of unity of responsibility, which underlies the theory of hierarchy. Second, we find that tasks should be grouped into jobs in such a way that the tasks in which performance is most easily measured are assigned to one worker and the remaining tasks are assigned to the other

worker. This conclusion squares nicely with the intuition that it is the *differences* between the measurability of quantity and quality in production, or of the so-called "basic skills" and "higher-order thinking skills" in education, that make those incentive problems difficult. The theory indicates that even when the agents have identical ex ante characteristics, the principal should still design their jobs to have measurement characteristics that differ as widely as possible. The principal should then provide more intensive incentives and require more work effort from the jobholder whose performance is more easily measured.

Our results are variations on the general theme of second best, which stresses that when prices cannot allocate inputs efficiently, then optimal incentives will typically be provided by subsidizing or taxing all inputs. For instance, Greenwald and Stiglitz, in a vivid metaphor, point out the value of a government subsidy for home fire extinguishers, since homeowners with fire insurance have too little incentive to invest in all forms of fire prevention and to fight fires once they have started. This mechanism has been most extensively analyzed in the theory of optimal taxation and in welfare theory.

However, the study of interdependencies among incentives and the use of instruments other than compensation to alleviate incentive problems have entered agency analyses more recently. Lazear argues that where cooperation among workers is important, we should expect to see less wage differentiation, that is, "lower-powered" incentives. Holmstrom and Ricart i Costa have observed how a firm's capital budgeting policy, including the hurdle rate and the way the firm assesses idiosyncratic risks, can affect the willingness of risk-averse managers to propose risky investment projects. Milgrom and Milgrom and Roberts have studied how organizational decision processes affect the allocation of effort between politicking and directly productive work. Farrell and Shapiro show that a price clause may be worse than no contract at all, because it reduces incentives to supply quality; this is similar to our result that it may be optimal to provide no quantity incentives when quality is poorly measured.

Some articles containing related ideas have been developed contemporaneously. Itoh (1991), in an analysis complementary to ours, studies conditions under which an employer might induce workers to work separately on their tasks, and those in which it is best for them to spend some effort helping one another. Laffont and Tirole show that concerns for quality help explain the use of cost-plus contracting in procurement. Baker investigates a model in which observable proxies of marginal product are imperfect in a way that causes the agent to misallocate effort across contingencies and therefore leads to incentives that are not as powerful as standard theory would suggest. Minahan reports a result on task separation that suggests a job design similar to ours but based on a different argument, as we will later explain.

The remainder of this article is organized as follows. In Section 2, we recapitulate our basic principal–agent theory, upon which the entire analysis is based. In Section 3, we specialize the analysis to the case where the agent's costs depend only on the total attention supplied and prove the various propositions about the optimality of fixed wages, the factors determining the assignment of ownership, and the optimal limits on outside business activities. In

Section 4, we consider restrictions on private tasks. In Section 5, we offer a summary and suggest directions in which this line of research can be taken.

2. The Linear Principal–Agent Model

2.1 Description of the Model

Consider a principal–agent relationship in which the agent makes a one-time choice of a vector of efforts $t = (t_1,\ldots,t_n)$ at personal cost $C(t)$. The efforts t lead to expected gross benefits of $B(t)$, which accrue directly to the principal. We assume that the function C is strictly convex and that the function B is strictly concave. The agent's efforts also generate a vector of information signals

$$x = \mu(t) + \varepsilon,$$

where we assume that μ: $\Re^n_+ \to \Re^k$ is concave and ε is normally distributed with mean vector zero and covariance matrix Σ. If the compensation contract specifies a wage of $w(x)$, then the agent's expected utility is assumed to take the form

$$u(\mathrm{CE}) = E\{u[w(\mu(t) + \varepsilon) - C(t)]\},$$

where $u(w) = -e^{-rw}$ and CE denotes the agent's "certainty equivalent" money payoff. The coefficient r measures the agent's risk aversion. The principal is risk neutral.

If the compensation rule were linear of the form $w(x) = \alpha^T x + \beta$, then one could utilize the exponential form to deduce that the agent's certainty equivalent is

$$CE = \alpha^T\mu(t) + \beta - C(t) - \tfrac{1}{2}r\alpha^T\Sigma\alpha.$$

That is, the agent's certainty equivalent consists of the expected wage minus the private cost of action and minus a risk premium. The term $\alpha^T\Sigma\alpha$ is the variance of the agent's income under this linear compensation scheme.

The principal's expected profit is $B(t) - E\{w[\mu(t) + \varepsilon]\}$ which, under the linear compensation scheme, is $B(t) - \alpha^T\mu(t) - \beta$. Consequently, the total certainty equivalent of the principal and the agent (their joint surplus) under the linear compensation plan is $B(t) - C(t) - \frac{1}{2}r\alpha^T\Sigma\alpha$. Notice that this expression is independent of the intercept term β; this intercept serves only to allocate the total certainty equivalent between the two parties. This last observation simplifies the principal–agent problem drastically. It implies that, given any technological and incentive constraints on the set of feasible (α,t) pairs, the utility possibility frontier, expressed in certainty equivalent terms, is a line in $\Re^2$ with slope -1. Hence, the incentive-efficient linear contracts are precisely those that maximize the total certainty equivalent subject to the constraints. If (t,α,β) is such a contract, then (t,α) must be a solution to

$$\underset{t,\alpha}{\text{Maximize}}\ B(t) - C(t) - \tfrac{1}{2}\, r\alpha^T\Sigma\alpha, \tag{1}$$

subject to

$$t \text{ maximizes } \alpha^T\mu(t') - C(t'). \tag{2}$$

If the agent's certainty equivalent is CE, then it follows that the intercept is $\beta = \text{CE} - \alpha^T\mu(t) + C(t) + \frac{1}{2}r\alpha^T\Sigma\alpha$. This intercept is equal to the agent's certainty equivalent income, minus the expected compensation from the incentive term, plus compensation for the cost that the agent incurs, plus a compensation for risk.

A central feature of our model is the general way in which we may allow observables to enter. We can study situations in which different activities can be measured with varying degrees of precision, including the important special case in which certain activities cannot be measured at all.[6] We can study cases in which performance measures can be influenced by activities other than those the principal desires the agent to undertake—for instance, the manipulation of accounting figures. We can study cases in which the number of observables is much smaller than the number of activities in t, forcing the contract to be based on aggregate information about the agent's activities. A special case of this, discussed in Holmstrom and Milgrom (1987), occurs when the agent acts on private information (to avoid adverse selection, one assumes that the information is observed after contracting). We can bring in contingent actions explicitly by specializing the model as follows. Let $\lambda^T = (\lambda_1,\ldots,\lambda_m)$ be a vector of probabilities of m possible states. Let t_i be the agent's contingent action in state i and let $B_i(t_i)$, $C_i(t_i)$, $\mu_i(t_i)$, and ε_i represent state-contingent profits, costs, signal functions, and memory errors, respectively. The analysis of that contingent-action model is equivalent to the analysis of our model with the specifications:

$$B(t) = \Sigma\lambda_i B_i(t_i), \qquad C(t) = \Sigma\lambda_i C_i(t_i),$$
$$\mu(t) = \Sigma\lambda_i\mu_i(t_i), \qquad \varepsilon = \Sigma\lambda_i\varepsilon_i.$$

Another important feature of the model is that B need not be part of x (i.e., the returns to the principal may not be observed). This puts B and C in a symmetric role. Indeed, if $B = -C$, the principal and the agent share the same objective and first best can be achieved in (1) and (2) by setting $\alpha = 0$. On the other hand, if B is different from $-C$, (1) and (2) may lead to a nontrivial agency problem even without the agent being risk averse. This occurs when the standard solution of making the agent a residual claimant is rendered infeasible because B is insufficiently well observed—a point made in Baker using a model with state-contingent actions of the type described above. Thus, risk aversion is not essential for the analysis to follow. The cost of measurement error, as expressed in (1), could alternatively arise out of a risk-neutral formulation.[7]

6. Note that if an activity can be measured without error, then a linear scheme allows the principal to set this activity at any desired level costlessly, assuming that the cost function is convex.

7. It is of interest to note that instead of a measurement error the incentive problem could be

2.2 Optimality of Linear Performance Incentives

The model described above involves two seemingly ad hoc assumptions. The more obvious one is that the contract that the parties sign specifies a wage payment that is a linear function of measured performance. The second assumption is more conventional and therefore less likely to be noticed, but it is no less troubling. It is the assumption that the agent is required to make a single, once-and-for-all choice of how he will allocate his efforts during the relationship without regard to the arrival of performance information over time. A remarkable fact, which we established in Holmstrom and Milgrom (1987), is that *these two simplifying assumptions are exactly offsetting in this model.* That is, the solution to the program (1) and (2) coincides with the solution to a principal–agent problem in which (i) the agent chooses efforts continuously over the time interval [0,1] to control the drift vector of a stationary stochastic process (Brownian motion) $\{X(\tau);\ 0 \leq \tau \leq 1\}$, and (ii) the agent can observe his accumulated performance before acting. We show that in this continuous time model an efficient contract specifies that the agent will choose $t(\tau)$ to be constant over time, regardless of the history at time τ, and that the agent's wage will be of the form $w = \alpha^T x + \beta$, that is, it is a linear function of the final outcome x alone, without regard to any intermediate outcomes. The constant t and the slope vector α are the solution to problem (1) and (2).

In view of its underlying assumptions, the model seems especially well suited for representing compensation paid over a short period, like a month, a quarter, or perhaps a year, in environments where profits are the cumulative result of persistent efforts over time. As such, the model seems most appropriate for analyzing the use of piece rates or commission systems; however, because the model is so tractable, we shall not avoid the temptation to stretch its use somewhat further in this article.

2.3 Simple Interactions Among Tasks

To explore some of the properties of our model, let us now work with the special case in which $\mu(t) = t$.[8] Then, when t is strictly positive in all components ($t \gg 0$), the incentive constraint (2) becomes

$$\alpha_i = C_i(t) \qquad \text{for all } i, \tag{3}$$

where subscripts on C denote partial derivatives. Differentiating (3), we may write

$$\frac{\partial \alpha}{\partial t} = [C_{ij}] \qquad \text{and} \qquad \frac{\partial t}{\partial \alpha} = [C_{ij}]^{-1} \tag{4}$$

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driven by a nonstochastic measurement bias. Suppose the agent can manipulate the performance measure. If this activity wastes resources, then incentives will optimally be set to balance this loss against genuine work incentives. One can specify the cost of manipulation so that optimal incentives come out exactly the same as in the stochastic model we are studying.

8. This is really not a special case, since we can always reformulate the model by redefining the agent's choice variables so that $\mu(t) = t$.

by the inverse function theorem. The second equation in (4) characterizes how changes in the "prices" α affect the level of effort that will be supplied.

Using Equations (3) and (4), one can compute first-order necessary conditions for an optimum in (1) and (2) when $t \gg 0$:

$$\alpha = (I + r[C_{ij}]\Sigma)^{-1}B', \tag{5}$$

where $B' = (B_1,\ldots,B_n)$ is the vector of first derivatives of B. Condition (5) is also sufficient when the expression $C'(t)^T\Sigma C'(t)$ is convex in t.

As a benchmark case, note that when the error terms are stochastically independent (Σ is a diagonal matrix) and the activities are technologically independent (all cross-partials of the cost function are zero), the solution in (5) simplifies to $\alpha_i = B_i(1 + rC_{ii}\sigma_i^2)^{-1}$, for all i. In this case, commissions are set independently of each other since the cost of inducing the agent to perform any given task is independent of the other tasks. As expected, α_i is decreasing in risk aversion (r) and risk (σ_i^2). It is also decreasing in C_{ii}. To interpret this, note from (4) that $\partial t_i/\partial \alpha_i = 1/C_{ii}$. Thus, the above formula says that α_i should be higher, the more responsive the agent is to incentives.

In the general case, notice that the cross-partials of C but not those of B enter into (5). Complementarities in the agent's private cost of generating signals can have an important role in determining optimal incentive pay. To illustrate, consider again the case of motivating teachers to teach both basic skills and higher-order thinking skills, but assuming that higher-order thinking skills cannot be measured. We model this by supposing that there are two activities so that the agent chooses the pair (t_1,t_2), but that only one activity (teaching basic skills) is observable:

$$x = t_1 + \varepsilon. \tag{6}$$

We can apply (5) assuming that σ_2^2 is infinite and σ_{12} is zero. Then, if the optimal solution entails $t \gg 0$, it must satisfy

$$\alpha_1 = (B_1 - B_2C_{12}/C_{22}) \,/\, [1 + r\sigma_1^2(C_{11} - C_{12}^2/C_{22})].^9 \tag{7}$$

When C_{12} is negative, making it more negative leads to a larger optimal value of α_1. That is, when the activities of teaching basic skills and higher-thinking skills are complementary in the agent-teacher's private cost function, the desirability of rewarding achievement in teaching basic skills is enhanced. If

9. We are assuming that teachers are motivated to teach some higher-thinking skills even without explicit financial incentives to do so. In one-dimensional agency models, it is typically assumed that the agent will not work without incentive pay. The reason for this is not that the agent dislikes even small amounts of work, but rather that the level of work the agent would provide without explicit incentives does not affect the optimal solution. In multitask models, however, the fact that agents supply inputs even without incentive pay can be quite consequential as the teacher example and the example in Section 3.2 show.

the two dimensions of teaching are substitutes in the agent's cost function ($C_{12} > 0$), then α_1 is correspondingly reduced, because high values of α_1 cause the teacher to substitute effort away from teaching higher-thinking skills.

In general, when inputs are substitutes, incentives for any given activity t_i can be provided either by rewarding that activity or by reducing its opportunity cost (by reducing the incentives for the other activities). Here, t_2 cannot be measured at all, so the only way to provide incentives for t_2 is to reduce α_1 as (7) shows.

Notice that (7) allows the possibility that it may be optimal to set α_1 negative even if B_1 is positive, provided $B_1 < B_2C_{12}/C_{22}$. If the agent can always reduce measured performance at no cost to himself, then this observation can be used to produce robust examples in which it is optimal to provide zero incentives for a desirable activity even when perfectly reliable performance measures ($\sigma_1^2 = 0$) may exist. A second case in which zero incentives can arise in this example is when effort in the two activities are perfect substitutes in the agent's cost function and the second activity is unobservable—that is, when $C(t_1,t_2) = c(t_1 + t_2)$ and $\sigma_2^2 = +\infty$. Then, if $t \gg 0$, the incentive constraint in (3) implies that $\alpha_1 = \alpha_2$ (intuitively, the agent must equate the marginal return to effort in various tasks). If, as in our teaching example, $\sigma_2^2 = \infty$, it then follows that $0 = \alpha_2 = \alpha_1$. This idea resurfaces in several of the applications in Section 3.

Another important possibility is that (5) does not apply because it is not optimal to set $t \geqslant 0$. Even in the model where t is one dimensional, the cost of providing positive incentives for a small amount of effort is discontinuously higher than the cost of providing no incentive for effort if $C'(0) > 0$. If no effort is required and no incentive is provided, then the risk premium incurred by the agent is zero. If a small amount of effort t is required, then $\alpha = C'(t) > C'(0) > 0$ and the risk premium is therefore at least $\frac{1}{2}r[C'(0)]^2\sigma^2$. Providing incentives for an activity involves an inherent fixed cost, and the size of that cost can be affected by the selection and levels of the agent's other activities. These observations will prove to be important when we apply our theory to issues of employment and job design.

3. Allocation Incentives for Effort and Attention

3.1 The Effort and Attention Allocation Model

We now move to a group of models in which the agent's effort or attention is a homogeneous input that can be allocated among tasks however the agent likes. We shall suppose that effort in the various tasks is perfectly substitutable in the agent's cost function. More formally, we suppose that the agent chooses a vector $t = (t_1,\ldots,t_m)$ at a personal (strictly convex) cost $C(t_1 + \cdots + t_m)$, leading to expected profits $B(t)$ and generating signals $x(t) = \mu(t) + \varepsilon$. Then, if the agent increases the amount of time or attention devoted to one activity, the marginal cost of attention to the other activities will grow larger.

Contrary to most earlier principal–agent models, we shall not suppose that all work is unpleasant (see note 9). A worker on the job may take pleasure in working up to some limit; incentives are only required to encourage work

beyond that limit. Formally, we assume that there is some number $\bar{t} > 0$ such that $C'(t) \leq 0$ for $t \leq \bar{t}$ and $C(\bar{t}) = 0$. This is important, because it means that contracts that provide for fixed wages may still elicit some effort, though more may be elicited by providing positive incentives. It also means that there is a range of effort allocations among which the agent is indifferent and willing to follow the principal's preference.

3.2 Missing Incentive Clauses in Contracts

One of the most puzzling and troubling failures of incentive models has been their inability to account for the paucity of explicit incentive provisions in actual contracts. For example, it is surprisingly uncommon in contracts for home remodeling to incorporate explicit incentives for timely completion of construction, even though construction delays arise frequently and can be profoundly disruptive to the homeowner. There can be little doubt that such clauses could be written into the contracts; similar clauses are common in commercial construction contracts. We shall argue that these facts can best be understood as a result of the greater standardization of commercial construction and the consequent ability of commercial buyers to specify and monitor quality standards. The innovation in our analysis is that our explanation of the presence or absence of the timely completion clause lies in an examination of the principal's ability to monitor *other aspects* of the agent's performance.[10]

Thus, suppose that some desirable attributes of the contractor's performance (such as courtesy, attention to detail, or helpful advice) are unmeasurable but are enhanced by attention t_1 spent on that activity, while other aspects of quality (such as timely completion) are measurable (perhaps imperfectly) and enhanced by attention t_2 devoted to this second activity. Supposing that the measured quality is one dimensional, we may write $\mu(t_1,t_2) = \mu(t_2)$, $x = \mu + \varepsilon$. As we have seen, the agent's efficient compensation contract pays an amount $S = \alpha x + \beta$.

Suppose that the overall value of the job to the homeowner is determined by the function $B(t_1,t_2)$. To model the idea that the first activity is "very important" and that both activities are valuable, we assume that B is increasing and that $B(0,t_2) = 0$, for all $t_2 \geq 0$.

Proposition 1. For the home contractor model specified in the last paragraph, the efficient linear compensation rule pays a fixed wage and contains no incentive component ($\alpha = 0$), even if the contractor is risk neutral.[11]

10. Another plausible explanation is that home construction contracts are frequently changed to reflect design modifications, and timely completion clauses would be nullified by these changes.

11. A related conclusion—that incorrect weightings of profit contributions in an accounting system lead the optimizing employer to weaken effort incentives—is derived by Baker. Baker's analysis can be conducted within our model by recognizing that a state-contingent strategy for the agent—that is, making different decisions in different states—is equivalent to a vector effort strategy. Baker's assumption that the principal cannot distinguish the state in which an action is

Proof. If $\alpha = 0$, then the agent can be instructed to spend total time $\bar{t}$ where $C'(\bar{t}) = 0$ and to choose $\bar{t}_1 \in [0,\bar{t}]$ to maximize $B(\bar{t}_1,\bar{t} - \bar{t}_1)$, which is strictly positive because $\bar{t} > 0$. In this case, the cost of risk-bearing by the agent is zero, so the total wealth will be $B(\bar{t}_1,\bar{t} - \bar{t}_1) - C(\bar{t})$. If $\alpha > 0$, then t_1 will be set to zero and the total wealth will be $0 - C(\hat{t}) - r\alpha^2\sigma_\varepsilon^2/2 \leq -C(\hat{t}) < B(\bar{t}_1,\bar{t} - \bar{t}_1) - C(\bar{t})$ because $\bar{t}$ is cost minimizing for the agent. If $\alpha < 0$, then $t_2 = 0$ and $t_1 < \bar{t}$ [because $C'(t_1) < 0 = C'(\bar{t})$] so the total profits are

$$B(t_1,0) - C(t_1) - r\alpha^2\sigma_\varepsilon^2 < B(\bar{t},0) - C(\bar{t}) < B(\bar{t}_1,\bar{t} - \bar{t}_1) - C(\bar{t}).$$

Q.E.D.

The ideas that underlie this analysis have many applications. For example, piece rates are relatively rare in manufacturing and, where they are used, they are frequently accompanied by careful attention to monitoring the quality of the work. Our analysis indicates that if quality were poorly measured, it would be expensive or impossible to maintain good quality while using a piece-rate scheme. Similarly, where individuals spend part of their efforts on individual projects and part on team production, and assuming that individual contributions to the team effort are difficult to assess, it would be dangerous to provide incentives for good performance on the individual projects. The problem, of course, is that individuals may shift their attention from the team activity where their individual contributions are poorly measured to the better measured and well-compensated individual activity. For this reason, piece-rate schemes may be especially dysfunctional in large hierarchies.

3.3 "Low-Powered Incentives" in Firms

A similar model can be used to explain Williamson's observation that the incentives offered to employees in firms are generally "low-powered" compared to the "high-powered" incentives offered to independent contractors. Like Williamson, we distinguish employees from independent contractors by the condition of asset ownership: Employees use and develop assets that are owned by others while contractors use and develop their own assets.

Once again, the heart of our modeling is our assumption that there are multiple activities to be undertaken and that the allocation of time and attention between them is crucial. Thus, let the expected gross profit from the enterprise be the sum of two parts, $B(t_1) + V(t_2)$, where B represents the expected net receipts and V the expected change in the net asset value. We assume that B and V are increasing, concave, and twice continuously differentiable and that $B(0) = V(0) = 0$. The actual change in asset value, $V + \varepsilon_v$, accrues to whoever owns the asset. Assets are notoriously hard to value (that is why accountants generally use historical cost as a valuation basis), so we

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taken is then the same as assuming that the principal cannot distinguish performance along the several dimensions of the vector strategy. The formal mapping from these "hidden information" models to our "hidden action" model is discussed in Holmstrom and Milgrom (1987).

assume that there is no performance indicator for the asset enhancement activity t_2. The primary activity t_1 is to produce output for sale in the current period: its indicator is $x = \mu(t_1) + \varepsilon_x$, where μ is increasing and concave. We assume that ε_x and ε_v are independent.

We consider two alternative organizational modes—*contracting*, in which the change in asset value accrues to the agent, and *employment*, in which the change in asset value accrues to the firm or principal. The crucial difference between these lies in the incentives for the agent to engage in the two kinds of activities. To focus on the most interesting case, we will assume that it is highly desirable to induce the agent to devote a positive amount of effort to both activities. Let

$$\pi^1 = \operatorname*{Max}_{t_1} B(t_1) - C(t_1),$$

$$\pi^2 = \operatorname*{Max}_{t_2} V(t_2) - C(t_2),$$

$$\pi^{12} = \operatorname*{Max}_{t_1} B(t_1) + V(\bar{t} - t_1) - C(\bar{t}).$$

Proposition 2. Assume that $\pi^{12} \geq \text{Max}(\pi^1,\pi^2)$. Then, the optimal employment contract always entails paying a fixed wage ($\alpha = 0$). Whenever the independent contracting relation is optimal, it involves "high-powered incentives" ($\alpha > 0$). Furthermore, there exist values of the parameters r, σ_v^2, and σ_x^2 for which employment contracts are optimal and others for which independent contracting is optimal. If employment contracting is optimal for some fixed parameters $(r,\sigma_v^2,\sigma_x^2)$, then it is also optimal for higher values of these parameters. Similarly, if independent contracting is optimal, then it is also optimal for lower values of these parameters.[12]

Proof. First, consider the case of the employment contract, where the returns $V(t_2)$ accrue to the firm. If the principal sets $\alpha > 0$, the agent will respond by setting t_1 so that $\alpha = C'(t_1)$ and setting $t_2 = 0$. The total certainty equivalent wealth is equal to $B(t_1) - C(t_1) - \frac{1}{2}r\sigma^2\alpha^2 < \pi^1 \leq \pi^{12}$. However, if $\alpha = 0$, the agent is willing to spend time $\bar{t}$ in any proportions and so a total certainty equivalent wealth of π^{12} is obtained. Therefore, it is optimal for the principal to set $\alpha = 0$ in an employment contract.

For the independent contractor, the maximum total certainty equivalent wealth is computed as follows. Let $(\hat{t}_1(\alpha),\hat{t}_2(\alpha))$ maximize $\alpha\mu(t_1) + V(t_2) - C(t_1 + t_2) - r\sigma_v^2/2$; this represents the agent's optimal response to α. The total certainty equivalent wealth for any fixed α is

12. One can derive a similar result with a general quadratic cost function $C(t_1,t_2)$. The only difference is that the commission rate would not necessarily be zero for an employed agent, though it would always be smaller than for an independent contractor.

$$B(\hat{t}_1(\alpha)) + V(\hat{t}_2(\alpha)) - C(\hat{t}_1(\alpha) + \hat{t}_2(\alpha)) - \tfrac{1}{2}r\{\sigma_x^2[C'(\hat{t}_1(\alpha) + \hat{t}_2(\alpha))]^2 + \sigma_v^2\}$$

and the maximum surplus is the maximum of this expression over α. If we fix $\alpha = 0$, then this expression is lower for the independent contractor than for the employment regime. Hence, whenever the independent contractor regime is optimal, it must be optimal to set $\alpha > 0$.

Note that if $\sigma_x^2 = \sigma_v^2 = 0$, first best is achieved by setting $\alpha = 1$ in the independent contractor regime. Since first best never can be achieved in the employment regime, the independent contractor regime is better in this case. Letting $r\sigma_v^2$ grow large makes the payoff to the independent contract regime fall without limit, so there are also some parameters for which the employment regime is better.

The last two sentences of Proposition 2 follow from the observation that the expression for the total certainty equivalent in the independent contractor regime is decreasing in $r\sigma_v^2$ and $r\sigma_x^2$, but these two terms do not affect the total certainty equivalent in the employment regime. Q.E.D.

Proposition 2 is consistent with the evidence reported by Anderson and Anderson and Schmittlein. They attempted to identify the reasons why firms in the electronic components industry have an employed sales force in some districts and independent sales representatives in other districts. (Many, but not all firms, used both forms of sales organization, suggesting that economies of scale play a lesser role.) They found that the perceived difficulty of measuring sales of individual salespeople (due to team selling or costly record keeping) was the best empirical predictor of the use of an in-house sales force. Transaction cost variables—such as specific training, with the exception of confidential information—were not significant either alone or in conjunction with performance measurement. If we suppose that one function of the sales force is to build an asset that is impossible to measure, such as "goodwill" (how satisfied and loyal are the customers?), then our model suggests that the difficulty of measuring sales would lead to the pattern of sales organization that Anderson and Schmittlein observed, and that commission rates would be lower for company-run sales forces.[13]

Anderson also finds that the importance of nonselling activities, such as promoting new products or products with a long selling cycle, is positively related to the use of an in-house sales force. We can analyze this finding by introducing a third activity t_3, which benefits the principal, but not the agent. Since an independent contractor will spend no time on nonselling activities

13. It may be argued that risk aversion cannot be a very relevant factor if the independent sales representative is itself a large firm. Recall, however, that the cost of risk can equivalently be derived from imperfect observability of B, paired with a convex cost C, in a risk-neutral model (see Baker). We can rely on work aversion instead of risk aversion. In this case, to make sure that it is not optimal to transfer B to the independent sales firm (i.e., make the manufacturer a subcontractor), one has to add an imperfectly observed input by the manufacturer. With two equally important, equally costly, and imperfectly observed inputs, the residual return B will be allocated to the party whose input is more difficult to measure.

(just as practitioners claim), it is easy to see that an increase in the value of this activity will work in favor of an owner-run sales force in our model.

Another piece of evidence consistent with our model comes from the fast-food industry. Firms such as McDonald's and Burger King own about 30% of their stores and franchise the rest. The difference in incentives between franchisees and owner-managed firms is striking. Franchisees pay royalties that are at most 10% of sales, corresponding to at least a 90% commission, whereas managers of company-owned stores typically receive no explicit incentives either on profit or sales (Krueger, Brickley and Dark). The difference in incentives is all the more remarkable, considering how similar the two types of stores are in all other aspects. According to our theory, the discontinuous shift in residual returns [$V(t_2)$] associated with franchising and the attendant shift in attention toward long-term asset values and cost containment, forces the franchise contract to increase short-term incentives sharply. Or, looked upon the other way, short-term incentives for employed managers must be muted to prevent them from allocating their attention away from important, but hard to measure, asset values.

4. Limits on Outside Activities

Our previous analysis emphasizes the importance of studying the full range of the agent's activities for analyzing incentives. If activities interact in the agent's cost function, incentive strength can be predicted only once the agent's whole portfolio of tasks is known. An equally important implication is that the principal can influence the agent's incentives by choosing the agent's portfolio of tasks. In the next section, we will study the optimal allocation of tasks between two agents. In this section, we consider how the principal might try to manage the agent's access to outside (private) activities.

Even casual observation makes it clear that the rules governing outside activities depend on the job. It is a commonplace observation that employees in "responsible positions" are allowed more freedom of action than other employees, and that they use that freedom in part to pursue personally beneficial activities. To analyze the issues that this observation raises, we begin with the assumption that it is easier for an employer to exclude an activity entirely than to monitor it and limit its extent. For example, a rule against personal telephone calls during business hours is found in many offices and seems to be motivated in part by its ease of enforcement compared, say, to a rule that limits the percentage of business hours devoted to personal calls to 2%. Although generalizations about employment all seem to have exceptions, a common feature of employment contracts is that the employer has authority to restrict the employee's outside activities during business hours, and sometimes after hours as well.

Assume then that the agent has a finite pool $K = \{1,\ldots,N\}$ of potential activities, which the principal can control only by exclusion. The returns to these tasks, which we will refer to as the agent's *personal business* for short, are assumed nonstochastic and to benefit the agent alone (in principle, these tasks could benefit the principal, too, but the analytics would be more compli-

cated). The principal controls the agent's personal business by allowing the agent to engage only in a subset of tasks $A \subset K$. Within the *set of allowable tasks, A,* the agent can engage in as much or as little personal business as he pleases, but none outside A. To focus on the interactions between the agent's workplace activities and personal business, we represent workplace activities simply as a single task in which performance is imperfectly measured.

Let t denote the attention the agent devotes to the principal's task and t_k the time he devotes to personal business k. We model the personal benefits that the agent derives as an offset against, or deduction from, his personal cost of effort, as follows:

$$c(t,t_1,\ldots,t_N) = C(t + \Sigma_K t_k) - \Sigma_K v_k(t_k). \tag{8}$$

The notation Σ_K stands for summation over k in K. Here C is the agent's private cost of the total attention he devotes to all his (permitted) personal activities. The return from personal activity k is measured by the function $v_k(t_k)$; these functions are assumed to be strictly concave with $v_k(0) = 0$. If $k \notin A$, then $t_k = 0$, so we could replace Σ_K with Σ_A in (8).

We make the simplifying assumptions that there are constant returns to time both in generating profits and in improving measured performance:

$$B(t,t_1,\ldots,t_N) = pt, \qquad x(t,t_1,\ldots,t_N) = t + \varepsilon. \tag{9}$$

The variance of ε is σ^2.

The principal's control instruments are the commission rate α and the allowed set of personal business tasks $A \subset K$. We will study the principal's problem in two stages. First, we fix α and consider the optimal choice of A, denoted $A(\alpha)$, and then we determine the optimal α.

Given the parameters α and A, the agent chooses t and t_k to maximize

$$\alpha t + \Sigma_A v_k(t_k) - C(t + \Sigma_A t_k).$$

Assume for the moment that this problem has an interior solution. Then the first-order conditions that characterize the agent's optimum are

$$\alpha = C'(t + \Sigma_A t_k), \tag{10}$$

$$\alpha = v_k'(t_k). \tag{11}$$

We note from (11) that the amount of time the agent chooses to spend on task k, denoted $t_k(\alpha)$, only depends on α and not on A. Also, the total time spent working, $t + \Sigma_A t_k$, is independent of A. Consequently, if the agent is allowed more personal tasks, without a change in α, all the time for those tasks will be reallocated away from the principal's task; this is the convenience of assuming (9) together with a cost function that only depends on total time. It makes it very simple to determine which personal tasks the agent should be allowed

for a given α. The benefit of allowing the agent to spend time on task k is $v_k(t_k(\alpha))$, while the (opportunity) cost is $pt_k(\alpha)$. Therefore, the optimal set of allowable personal tasks is

$$A(\alpha) = \{k \in K | v_k(t_k(\alpha)) > pt_k(\alpha)\}. \tag{12}$$

Figure 1 shows the determination of $A(\alpha)$. The pt line represents the returns from spending time on the principal's task. The v_1 and v_2 curves represent the returns from two private tasks. Both private tasks are socially valuable in that the v_k curves rise above the pt line on a positive interval $t_k \in [0,\hat{t}_k]$, where $\hat{t}_k$ is defined by the intersection $v_k(\hat{t}_k) = p\hat{t}_k$. However, for the chosen α, only task 1 is worth keeping; it is optimal to exclude task 2 since $t_2(\alpha) > \hat{t}_2$—that is, time $t_2(\alpha)$ yields more in the principal's task than it yields in task 2.

The geometry of Figure 1 makes it evident that $A(\alpha)$ expands as α is increased. This follows because $t_k(\alpha)$ is decreasing as v_k is strictly concave. As α is raised, the agent will spend less time on private business. This brings more projects into the efficient region $t_k(\alpha) \leq \hat{t}_k$, which is characterized by the condition that time $t_k(\alpha)$ in the private task yields more than the same amount of time spent in the principal's task. Furthermore, we see that the critical value of α at which private task k will be excluded is entirely determined by the slope of v_k at the point where v_k intersects the pt line. This follows since $t_k(\alpha) \leq \hat{t}_k$ if and only if $v_k'(\hat{t}_k) \leq \alpha$.

We record these observations in the following proposition.

Proposition 3. Assume that α is such that $t(\alpha) > 0$. Then the following statements hold.

(i) It is optimal to let the agent pursue exactly those private business opportunities that belong to $A(\alpha)$ defined in (12); that is, those tasks k for which the resulting *average product* $v_k(t_k(\alpha))/t_k(\alpha)$ exceeds the *marginal product* p in the principal's task.

(ii) The higher is the agent's marginal reward for performance in the main job, the greater is his freedom to pursue personal business. Formally, if $\alpha \leq \alpha'$, then $A(\alpha) \supset A(\alpha')$.

(iii) If it is optimal to exclude task k, then it is also optimal to exclude all tasks m, for which $v_m'(\hat{t}_m) > v_k'(\hat{t}_k)$, where $\hat{t}_j$ is defined by $v_j(\hat{t}_j) = p\hat{t}_j$.

It is possible that for small enough α it will be optimal to set $t(\alpha) = 0$ and hence $A(\alpha) = K$. In that case, there are no gains from trade and the principal will not employ the agent. Such a solution may be optimal if the cost of bearing risk becomes sufficently large. One could exclude that case by assuming that the agent's private businesses are less productive than working for the principal with zero incentive (as we saw earlier, zero incentive does not preclude productive work), but there is no need to make such a restriction. Obviously, if $t(\alpha) = 0$, then $t(\alpha') = 0$, for all $\alpha' < \alpha$. Therefore, job separation will occur, if at all, below a critical cutoff value for α.

Part (ii) of Proposition 3 articulates a familiar and fundamental principle:

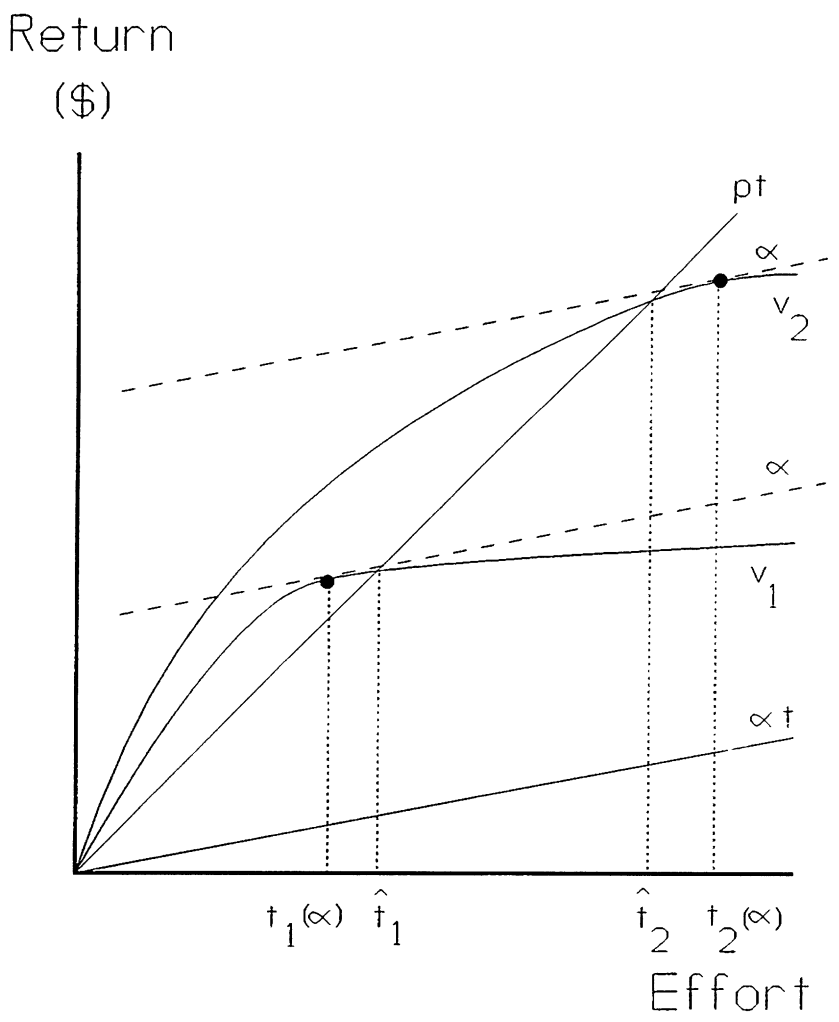

Figure 1. It is optimal to allow task 1 but to exclude task 2, because $t_1(\alpha) < \hat{t}_1$ but $\hat{t}_2 < t_2(\alpha)$. Notice that this is true even though the social returns to task 2 are everywhere higher than those to task 1.

responsibility and authority should go hand in hand. It is optimal to give the agent more freedom to pursue personal business when he is financially more responsible for his performance. In the extreme case, when performance can be measured without error and it is optimal to make the agent a residual claimant ($\alpha = p$), the agent will be free to engage in whatever private business he deems desirable. The responsibility principle again underscores that the agent's incentives can be influenced indirectly by altering the opportunity cost for supplying desired inputs. It is readily seen that the agent's marginal cost (but not total cost) of spending a given amount of time t in the principal's task

is reduced by excluding private tasks. Exclusion will be more extensively used the more costly it is to provide financial rewards.[14]

Part (iii) of the Proposition shows that the social value of a personal activity, and the likelihood that it will be excluded, need bear little relationship to each other. For instance, in Figure 1 the social value of task 2 is higher than that of task 1 for any given amount of time spent on either, yet it is task 2 rather than task 1 that is excluded. The reason is that task 2 more easily invites excess attention.

Before turning to the choice of α, there is a point that deserves to be emphasized. The amount of personal business A that the principal will allow for a fixed α, as characterized in (12), does not depend directly on r or σ^2, nor on the cost function C. These factors affect A only through α. Therefore, given data about r, σ^2, α, C, and A, it is econometrically correct to regress A against the endogenous variable α. Proposition 3 predicts that the extent of agent freedom will be positively related to α, irrespectively of which of the model parameters (other than v_k) are viewed as exogenous. The parameters r and σ^2 are a natural source of cross-sectional variation in α as indicated by the following.

Proposition 4. Assume that the optimal solution features $t(\alpha) > 0$. Then the following statements hold.

(i) The optimal value of α is given by

$$\alpha = p/[1 + r\sigma^2/(dt/d\alpha)],$$

where $dt/d\alpha = 1/C'' + \Sigma_{A(\alpha)}(1/v_k'')$.

(ii) If it becomes easier to measure the agent's performance (σ^2 decreases), or the agent becomes less risk averse (r decreases), then the agent's marginal reward α will be raised and his personal business activities will be less curtailed.

(iii) Any personal task that would be excluded in a first-best arrangement $[v_k'(0) \leq p]$ will also be excluded in a second-best arrangement. For sufficiently high values of $r\sigma^2$, some tasks that would be included at the first best will be excluded at the second best.

Proof. The equation in (i) is a special case of (5); the expression for $dt/d\alpha$ follows from the agent's first-order conditions. Revealed preference paired with Proposition 3 implies (ii). Part (i) implies that $\alpha \leq p$ and that α goes to zero when $r\sigma^2$ goes to infinity; this proves (iii). Q.E.D.

If we assume that the agent's cost and benefit functions are quadratic, we see from (i) that the agent's responsiveness to incentives, $dt/d\alpha$, increases as the set of allowable tasks, $A(\alpha)$, expands. Consequently, viewing $A(\alpha)$ as

14. One can also show that by excluding private tasks, the agent becomes less responsive to increases in the commission α. A less flexible job design is associated with weaker incentives as we mentioned earlier.

exogenous, it is optimal to raise α in response to an increase in the agent's degree of freedom. Of course, $A(\alpha)$ is not exogenous; it expands with an increase in α. We see then that α and $A(\alpha)$ are complementary instruments: increasing either leads to an increase in the other.[15]

Part (ii) is the most interesting one. It predicts that there will be more constraints on an agent's activities in situations where performance rewards are weak because of measurement problems. The rigid rules and limits that characterize bureaucracy, in this view, constitute an optimal response to difficulties in measuring and rewarding performance. Among the "personal business" activities that bureaucracies try to limit are collusion (Tirole; Holmstrom and Milgrom, 1990; Itoh, 1989) and influence activities (Milgrom, Milgrom and Roberts). The restrictions on trade between employees that Holmstrom and Milgrom (1990) recommend and the restrictions on communications that Milgrom and Roberts propose are examples of optimal exclusion of activities that would be permitted or perhaps even encouraged in a first-best world.

The desire to exclude activities provides a second possible explanation of the empirical results of Anderson and Schmittlein. Here, rather than distinguishing the roles of employee and independent contractor on the basis of ownership of productive assets, the focus is on the discretionary authority of the employer to prevent salespeople from outside activities, such as selling the products of other manufacturers during business hours. As the difficulty of monitoring performance (measured by σ^2) rises, Proposition 4 asserts that there is an increasing degree of exclusivity in efficient contracts. If exclusivity is easier to enforce within firms than across firms, then poor sales measurement and employment are positively related.

Our two explanations of the Anderson–Schmittlein evidence are distinct but closely related. In the first, the extra incentive from employment comes from transferring to the firm the return stream associated with the goodwill created by customer satisfaction. In the second, the extra incentive comes from eliminating (rather than transferring) a return stream—that associated with personal business. In each case, eliminating the agent's direct profits from an activity reduces the opportunity cost of work to the employee and lowers the cost of providing incentives.

5. Allocating Tasks between Two Agents

In the single-agent model, the commission rates α_i serve three purposes: they allocate risk, motivate work, and direct the agent's efforts among his various activities. A trade-off arises when these objectives are in conflict with each other: Optimal risk-sharing may be inconsistent with motivating work, and motivating hard work may distort the agent's allocation of efforts across tasks. Among the instruments available to the principal to alleviate these problems

15. Nonlinearities in the principal's task would not alter the conclusion that α is reduced when r or σ^2 is increased; this part is just a revealed preference argument. However, the set $A(\alpha)$ would be harder to characterize as the exclusion of tasks would interact with each other as a result of integer problems. One could even find that a personal task is included when α is reduced.

are job restructuring and relative performance evaluation: The former allows the principal to reduce the distortions in how attention is allocated among activities, while the latter enables the principal to lower the cost of incentives by using a more sensitive measure of actual performance.

5.1 Optimal Groupings of Tasks into Jobs

Here we initiate the study of how incentive considerations might affect the grouping of tasks into jobs. We use a model that eliminates other important effects, such as differences among the agents and complementarities among task assignments. There are two identical agents, indexed $i = 1,2$, who allocate their attention across a continuum of tasks indexed by $k \in [0,1]$. Let $t_i(k)$ denote the attention agent i devotes to task k. We assume that the two agents can share a task and that their labor inputs are perfect substitutes. Thus, profit $B(t)$ is a function of the total time vector $t \equiv \{t(k): k \in [0,1]\}$, where $t(k) = t_1(k) + t_2(k)$. Likewise, the performance signal from task k, $\mu(t(k),k)$, only depends on the total attention $t(k)$ devoted to it. The error variance of task k is $\sigma^2(k) > 0$ and the errors are assumed independent.

Agent i's total labor input is given by

$$\bar{t}_i = \int t_i(k)dk. \tag{13}$$

His private cost is $C(\bar{t}_i)$; the cost function is assumed differentiable and strictly convex.

Since the ex ante specification of the model is symmetric in the roles of the two agents, if the problem entailed a concave objective and convex constraints, we would expect the optimal solution to be symmetric. However, as we shall see, the optimal solution is not symmetric, so we must be careful to deal correctly with the inherent nonconvexities of the problem.

We begin by studying the problem of implementing, at minimum cost, a given vector $t = \{t(k)\}$ of total attention to be devoted to the various tasks, given the constraint that the total attention devoted by agent i is $\bar{t}_i$. Denoting the commission paid to agent i for task k by $\alpha_i(k)$, this problem is described by

$$\underset{\substack{t_1(\cdot),t_2(\cdot)\\ \alpha_1(\cdot),\alpha_2(\cdot)}}{\text{Minimize}}\ C(\bar{t}_1) + C(\bar{t}_2) + \frac{r}{2}\int [\alpha_1^2(k) + \alpha_2^2(k)]\sigma^2(k)dk, \tag{14}$$

subject to (12), (13), and the incentive constraints

$$\begin{aligned} &\alpha_i(k)\mu'(t(k),k) \le C'(\bar{t}_i), && \text{if } t_i(k) = 0,\\ &\alpha_i(k)\mu'(t(k),k) = C'(\bar{t}_i), && \text{if } t_i(k) > 0,\ i=1,2,\ k \in [0,1]. \end{aligned} \tag{15}$$

The incentive constraints can be correctly described by first-order conditions, because the agent's choice problem is a concave maximization problem. As usual, the implementation cost reflects both the direct cost of work as well as the cost of risk-bearing, since both costs are deducted when determining the total certainty equivalent of the parties.

We shall say that the principal makes the two agents *jointly responsible for task k* if $\alpha_1(k) > 0$ and $\alpha_2(k) > 0$. Similarly, agent i is *solely responsible for task k* if $\alpha_i(k) > 0$ and $\alpha_j(k) = 0$, $i \neq j$.

Proposition 5. In the model described above, it is never optimal for the two agents to be jointly responsible for any task k.

Proof. Let k be a set of tasks for which there is joint responsibility—that is, $\alpha_1(k)\alpha_2(k) > 0$, for $k \in K$—and suppose K has positive measure. Let $t_i(K) \equiv \int_K t_i(k)dk$ and choose $K' \subset K$ such that $\int_{K'} t(k)dk = t_1(K)$. Define a new set of attention allocations and commission rates $\{\hat{t}_i(k),\hat{\alpha}_i(k)\}$ so that these coincide with the original specification for $k \notin K$. For $k \in K'$, set $\hat{t}_1(k) = t(k)$, $\hat{\alpha}_1(k) = \alpha_1(k)$, and $\hat{t}_2(k) = \hat{\alpha}_2(k) = 0$; for $k \in K \backslash K'$, set $\hat{t}_1(k) = \hat{\alpha}_1(k) = 0$, $\hat{t}_2(k) = t(k)$, and $\hat{\alpha}_2(k) = \alpha_2(k)$.

The total attention devoted to each task as well as the total attention of each of the two agents is unaltered in the new scheme. By construction, therefore, the first-order conditions (15) hold and the new scheme is feasible. The new scheme strictly improves the objective function as some of the commission rates are lowered to zero for a set of tasks of nonzero measure. Q.E.D.

This proposition reflects our earlier observation that providing incentives for an agent in any task incurs a fixed cost as the agent assumes some nontrivial fraction of the risk associated with that task (or its measurement). Since we have assumed that the tasks are small relative to the agent's capabilities, assigning joint responsibility for any task would incur two fixed costs unnecessarily. As the proof demonstrates, if one begins with an arrangement in which some tasks are shared, it is possible to split the same tasks among the agents without affecting either the total effort required of either agent or the total effort allocated to any task. This rearrangement makes it possible to eliminate some of each agent's responsibilities [setting $\alpha_i(k) = 0$], thereby reducing the risk that the agent must bear and so increasing the total surplus of the three parties.

Having established that each task will be assigned to just one employee, we next turn to the issue of how the tasks will be grouped. With this in mind, it is convenient to redefine our variables. We reinterpret $\alpha_i(k)$ to be the *hypothetical* commission rate that the principal would need to pay in order to elicit the desired level of effort $t(k)$ from agent i if he were assigned task k [see (17) below]. We also define a task assignment variable $I_i(k)$, which is set equal to unity if agent i is assigned task k and is set equal to zero otherwise. Then, the *actual* commission rate paid to agent i for task k is $\alpha_i(k)I_i(k)$; that is, it is $\alpha_i(k)$ if i is assigned the task and it is zero otherwise. Proposition 3 implies that at the optimum, $t_i(k) = I_i(k)t(k)$. We can now state the principal's task assignment problem as follows:

$$\underset{I_1(k),I_2(k)}{\text{Minimize}}\ C(\bar{t}_1) + C(\bar{t}_2) + \frac{r}{2}\int_0^1 [I_1(k)\alpha_1^2(k) + I_2(k)\alpha_2^2(k)]\sigma^2(K)dk, \quad (16)$$

subject to

$$\alpha_i(k)\mu'(t(k),k) = C'(\bar{t}_i), \qquad i = 1,2,\ k \in [0,1], \tag{17}$$

$$\int I_i(k)t(k) = \bar{t}_i, \qquad i = 1,2. \tag{18}$$

$$I_1(k) + I_2(k) = 1, \qquad k \in [0,1], \tag{19}$$

$$I_i(k) = 0 \text{ or } 1, \qquad i = 1,2 \text{ and } k \in [0,1]. \tag{20}$$

Constraint (17) merely defines $\alpha_i(k)$, since $t(k)$ and $\bar{t}_i$ are fixed. If $\bar{t}_1 = \bar{t}_2$, then it is clear from (17) that $\alpha_1(k) = \alpha_2(k)$, and hence that the objective (16) is independent of the task assignment: All feasible assignments then yield the same total certainty equivalent wealth. As we will see below, the important case is the asymmetric one, so let us assume that $\bar{t}_1 < \bar{t}_2$.

To solve program (16)–(20), we first solve the relaxed program in which (20) is replaced by the less restrictive constraint

$$I_i(k) \geq 0, \qquad i = 1,2 \text{ and } k \in [0,1]. \tag{21}$$

In the relaxed problem, the objective and constraints are all linear (hence, convex) in the choice variables $I_i(k)$, so first-order conditions fully characterize the optimum. Let γ_i be the Lagrange multiplier associated with constraint (18). Then, optimizing in the usual way, we find that

$$I_1(k) = 0, \qquad \text{if } (r/2)[\alpha_1^2(k) - \alpha_2^2(k)]\sigma^2(k) + (\gamma_1 - \gamma_2)t(k) > 0$$

and (22)

$$I_1(k) = 1, \qquad \text{if } (r/2)[\alpha_1^2(k) - \alpha_2^2(k)]\sigma^2(k) + (\gamma_1 - \gamma_2)t(k) < 0.$$

By (17), $\alpha_1 < \alpha_2$ as $\bar{t}_1 < \bar{t}_2$; therefore, (22) implies that $\gamma_1 > \gamma_2$. Since $I_i(k)$ takes values 0 and 1 at the optimum of the relaxed program with constraint (21) in place of (20), Equations (22) also characterize the solution to the original problem and identify the marginal tasks. A marginal task is one where the advantage of assigning the task to agent 1, in terms of the lower risk premium required, is just offset by the higher marginal value of agent 1's time. The first of these costs varies with the measurement error attached to the task and the second varies with the amount of time the task requires. These observations suggest an alternative characterization of the optimum assignment policy.

Define the *noise-to-signal ratio* of task k by $n(k) = \sigma^2(k)/\mu'(t(k),k)^2$ and the *information coefficient* by $\varrho(k) = n(k)/t(k)$. Let

$$\varrho \equiv (2/r)(\gamma_1 - \gamma_2)[C'(\bar{t}_2)^2 - C'(\bar{t}_1)^2]^{-1}. \tag{23}$$

We can then restate (22) as follows.

Proposition 6. Suppose that the two agents devote different amounts of total attention to their tasks (i.e., $\bar{t}_1 < \bar{t}_2$). Then, tasks are optimally assigned in this model so that all the hardest-to-monitor tasks are undertaken by agent 1 and all the easiest-to-monitor tasks are undertaken by agent 2. That is, agent 1 is assigned all the tasks k for which $\varrho(k) \geq \varrho$, and agent 2 is assigned all those with $\varrho(k) < \varrho$, where ϱ is defined in (23).

Corollary. Suppose that $\bar{t}_1 < \bar{t}_2$, the required allocation of attention is uniform [i.e., $t(k) = 1$ for all k] and the signal functions are identical [i.e., $\mu(t,k) = \mu(t)$]. Then there exists a $\varkappa$ such that agent 1 will optimally be given all the tasks k for which $\sigma^2(k) \geq \sigma^2(\varkappa)$, and agent 2 all the tasks for which $\sigma^2(k) < \sigma^2(\varkappa)$.

These results provide, in purely incentive-theoretic terms, an account of how activities might be grouped, with some employees specializing in activities that are hard to monitor and others in activities that are easily monitored.[16] Separating tasks according to their measurability characteristics [$\varrho(k)$] allows the principal to give strong incentives for tasks that are easy to measure without fearing that the agent will substitute efforts away from other, harder-to-measure tasks. The present model oversimplifies these issues by assuming that there are no restrictions on how the principal may group tasks. In the case of piece rates discussed in Section 3, it might not be possible to separate the tasks of providing high output from those of providing high quality: The worker might always be able to substitute speed for attention to details. Nevertheless, the results of Proposition 6 are suggestive.[17]

The appearance of $\varrho(k)$ in these results is unfamiliar, and seems worth reviewing in detail. Let realized performance in task k be measured by

$$x_k(t) = \mu(t,k) + \varepsilon_k,$$

where ε_k is distributed normally with zero mean and variance $\sigma^2(k)$. The *normalized performance measure,*

$$\hat{x}_k(t) = [\mu(t,k) + \varepsilon_k]/\mu'(t(k),k),$$

provides the same information and has error variance equal to the noise-to-

16. Minahan derives a result that is related. In his model there are four tasks: two easy to measure and two hard to measure. He shows that it is better not to mix the tasks. The main difference between his model and ours is that in his model the principal cannot provide incentives on individual tasks, just on the sum of the tasks. This would greatly simplify our analysis. On the other hand, Minahan's analysis deals with nonlinear incentives and general utility functions, which adds to the complexity.

17. One manifestation of the task allocation principle may be found in the organization of R&D activities in firms [see Holmstrom (1989)].

signal ratio $n(k)$. If we let $\hat{a}_i(k)$ denote the commission paid based on normalized performance, it follows from (17) that

$$\hat{a}_i(k) = C'(\bar{t}_i) \equiv \hat{a}_i, \qquad \text{for all } k. \tag{24}$$

Thus, normalized commissions $\hat{a}_i(k)$ must all be equal for an agent. This is an implication of the assumption that attention to various tasks are perfect substitutes in the cost function. Since all commissions are equal, the risk cost from allocating task k to agent i is $(r/2)\hat{a}_i^2 n(k)$. Task k requiring attention $t(k)$ will be optimally assigned to the agent with the lowest price per unit effort. The risk cost for agent i per unit effort is $(r/2)\hat{a}_i^2\varrho(k)$ and the value of the agent's attention in its best alternative use is γ_i. Therefore, task k is optimally assigned to agent i if i's total cost of $\gamma_i + (r/2)\hat{a}_i^2\varrho(k)$ per unit effort is less than j's corresponding cost. From this observation and (24), it is evidently optimal to assign the higher $\varrho(k)$ tasks to the agent with the lower $\bar{t}_i$ and to pay that agent a lower "normalized commission rate." This observation is incorporated into the next proposition, but the proposition's main purpose is different: It verifies that even though the two agents in our model are identical ex ante, an optimal solution necessarily treats them asymmetrically, requiring them to specialize in different tasks.

Proposition 7. Suppose that the information coefficients $\varrho(k)$ are not all identical and consider the variant of program (16)–(20) in which the variables $\bar{t}_i$ $(i = 1,2)$ are added to the list of choice variables. This program has no symmetric optimal solution $(\bar{t}_1 \neq \bar{t}_2)$. There is an optimum at which agent 1 is assigned less strenuous work $(\bar{t}_1 < \bar{t}_2)$, takes responsibility for the hard-to-measure tasks [those with $\varrho(k) > \varrho$], and receives lower "normalized commissions" $[\hat{a}_1(k) < \hat{a}_2(k)]$.

Proof. First, we show that there is no optimum with $\bar{t}_1 = \bar{t}_2 = \bar{t}$. If there were, then—in view of (16), (17), and (19)—every feasible allocation of tasks to agents leads to the same total payoff. In particular, there is an optimal solution in which agent 1 is assigned the high $\varrho(k)$ tasks; that is, all tasks for which $\varrho(k) > \varrho$, where ϱ is set to just exhaust the attention $\bar{t}_1$. Agent 2 is then assigned the remaining [low $\varrho(k)$] tasks.

Now consider the family of feasible solutions, parameterized by ε, in which $\bar{t}_1(\varepsilon) = \bar{t} - \varepsilon$, $\bar{t}_2(\varepsilon) = \bar{t} + \varepsilon$, and all the highest $\varrho(k)$ tasks are assigned to agent 1 until the total attention required is $\bar{t}_1(\varepsilon)$. In order for the symmetric solution to be optimal, it is necessary that the derivative of the objective with respect to ε be zero. The following calculation shows that the derivative is negative—that is, that it would be better to specify that the agent who is assigned the hard-to-measure tasks work a bit less than his counterpart. Indeed, the derivative of the objective with respect to ε at $\varepsilon = 0$ is equal to

$$2C'' \frac{\bar{t}}{2} C' \frac{\bar{t}}{2} \frac{r}{2} \int_0^1 (I_2(k) - I_1(k)) \frac{\sigma^2(k)}{\mu'^2(t(k),k)}\, dk$$

$$= rC'' \frac{\bar{t}}{2} C' \frac{\bar{t}}{2} \int_0^1 I_2(k)\, \varrho(k) t(k) dk - \int_0^1 I_1(k)\, \varrho(k) t(k) dk$$

$$< rC'' \frac{\bar{t}}{2} C' \frac{\bar{t}}{2} \varrho(\bar{t}_2 - \bar{t}_1) = 0.$$

The last step uses (18) and the facts that $\varrho(k) \geq \varrho$ when $I_1(k) = 1$ and $\varrho(k) \leq \varrho$ when $I_2(k) = 1$ [and that $\varrho(k)$ is not constant so that the inequality is strict].

The remainder of the proposition is verified in the paragraphs preceding the proposition. Q.E.D.

5.2 Caveats

The model presented in the previous subsection represents merely a first pass at studying the optimal grouping of tasks into jobs. Although it provides some interesting insights, we have omitted so many key elements of the problem and made so many special assumptions to simplify an already complex analysis that it is well to make a preliminary list of these features and omissions and to speculate about how they may have affected our analysis.

First, we had assumed that all tasks are "small" and that the principal has perfect freedom to group them in any way to form a job. Neither of these assumptions is particularly attractive. The assumption that all tasks are small could be replaced by the assumption that there are a finite number of tasks that all required the same amount of time [$t(k)$ constant]; this, however, introduces the possibility that $\bar{t}_1 = \bar{t}_2$, in which case all task assignments are equally good. When tasks require nonnegligible amounts of time and vary in size, then the need to minimize costs borne by the agents by equalizing workloads may reverse some of our conclusions. Moreover, tasks like maintaining quality and producing output cannot always be separated. In short, our model exaggerates the principal's ability to group tasks into homogeneous measurement classes and in so doing caricatures the problem of how jobs are constructed. The main virtue of our model is that it is structured so that incentive considerations alone determine the optimal solution, so that it lends some new insights into the very limited question of how incentive concerns may affect job design.

Second, we had assumed that the errors of measurement in the agent's various tasks are all independent. We know from previous analyses, such as Holmstrom (1982), that when errors are positively correlated, separating the tasks among the two agents allows the use of comparative performance evaluation, which can help to reduce the risk premium incurred in providing incentives. It is not hard to see that even without comparative performance evaluation, separating tasks with positively correlated measurement errors creates a better diversified portfolio of tasks that reduces the risk that the agent must bear. Similarly, grouping tasks in which performance is negatively correlated reduces the agents' risk premium. So, even in the incentive domain, our present model is highly incomplete.

Third, the attention allocation model that we have used throughout is itself a simplification, which forces all activities to be equal substitutes in the agent's cost function and excludes the possibility that some activities may be complementary. In our discussion in Section 2 of the issue of how teachers should be compensated, we found that complementarities in the agent's private cost of attention can have an important effect both on how jobs should be designed and how agents should be compensated, but that complementarities among the same variables in the production function have no similar effect. These are subtle distinctions that our theory, in its attention allocation version, cannot address.

Fourth, the models we have studied assume that the agents focus their attention on the same tasks for all time. As discussed in Section 2, the model we are using is explicitly temporal, and issues of job rotation are an important aspect of real job design. Our preliminary analysis shows that these issues may be susceptible to analysis using an extension of the Section 2 model, in which the players are uncertain about the difficulty of production and use the past performance to learn about it. We hope to be able to discuss these issues more fully in follow-up work.

6. Conclusion

The problem of providing incentives to agents and employees is far more intricate than is represented in standard principal–agent models. The performance measures upon which rewards are based may aggregate highly disparate aspects of performance into a single number and omit other aspects of performance that are essential if the firm is to achieve its goals. Commonly, the principal—agent problem boils down to this: Given a highly incomplete set of performance measures and a highly complex set of potential responses from the agent, how can the agent be motivated to act in the social interest?

Our approach emphasizes that incentive problems must be analyzed in totality; one cannot make correct inferences about the proper incentives for an activity by studying the attributes of that activity alone. Moreover, the range of instruments that can be used to control an agent's performance in one activity is much wider than just deciding how to pay for performance. One can also shift ownership of related assets, vary restrictions on the ways a job can be done, vary limits and incentives for competing activities, group related tasks into a single job, and so on.

In a related article (Holmstrom and Milgrom, 1991), we study the simultaneous use of various instruments for controlling agents to derive new, testable results from the theory of organization. Our emphasis there is on how cross-sectional variations in the parameters that determine the optimal design of jobs, the optimal intensity of incentives, and the optimal allocation of ownership lead to covariations among endogenous variables that are similar to the patterns we find in actual firms.

Most past models of organization focus only on one instrument at a time for determining incentives and a single activity to be motivated. Newer theories,

such as ours, that explicitly recognize connections between instruments and activities, offer new promise to explain the richer patterns of actual practice.

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