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Cruising flights (review)

Flight performance

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Model

Equilibrium

$$\sum F_x = 0$$
$$\sum F_z = 0$$

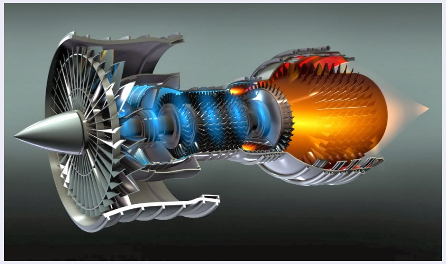
$$\boxed{\begin{aligned} T - D &= 0 \\ L - W &= 0 \end{aligned}}$$

Considerations:
- No accelerations
- Leveled flight
- Trimmed flight
- Weight variation comes only from fuel consumption

Kinematics

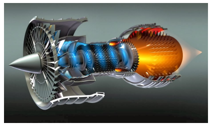
$$\boxed{\frac{dx}{dt} = V}$$

Consumption



$$\boxed{\frac{dW}{dt} = -c\,T}$$

$$\boxed{\frac{dW}{dt} = -\frac{\hat{c}\,P}{\hat{k}\eta_P}}$$



Important relations

Velocities

$$V = \left\{ \frac{T}{S} \frac{1}{\rho C_{D0}} \left[1 \pm \sqrt{1 - \frac{1}{E_m (T/W)^2}} \right] \right\}^{1/2}$$

$$V^4 - \frac{2\hat{k}\eta_P}{\rho} \frac{P_e}{W} \frac{W}{S} V + 4k \left(\frac{W}{S} \right)^2 \frac{1}{\rho^2 C_{D,0}^2} = 0$$

Thrust / power required

$$T_r = \frac{1}{2} \rho V^2 S C_{D0} + 2Sk \left(\frac{W}{S} \right)^2 \frac{1}{\rho V^2}$$

$$P_r = \frac{1}{2} \rho S C_{D0} V^3 + 2Sk \left(\frac{W}{S} \right)^2 \frac{1}{\rho V}$$

Range

$$x = -\frac{1}{c} \int_{W_1}^{W_2} V \left(\frac{C_L}{C_D} \right) \frac{1}{W} \, dW$$

$$x = -\frac{\hat{k}\eta_P}{\hat{c}} \int_{W_1}^{W_2} \left(\frac{C_L}{C_D} \right) \frac{1}{W} \, dW$$

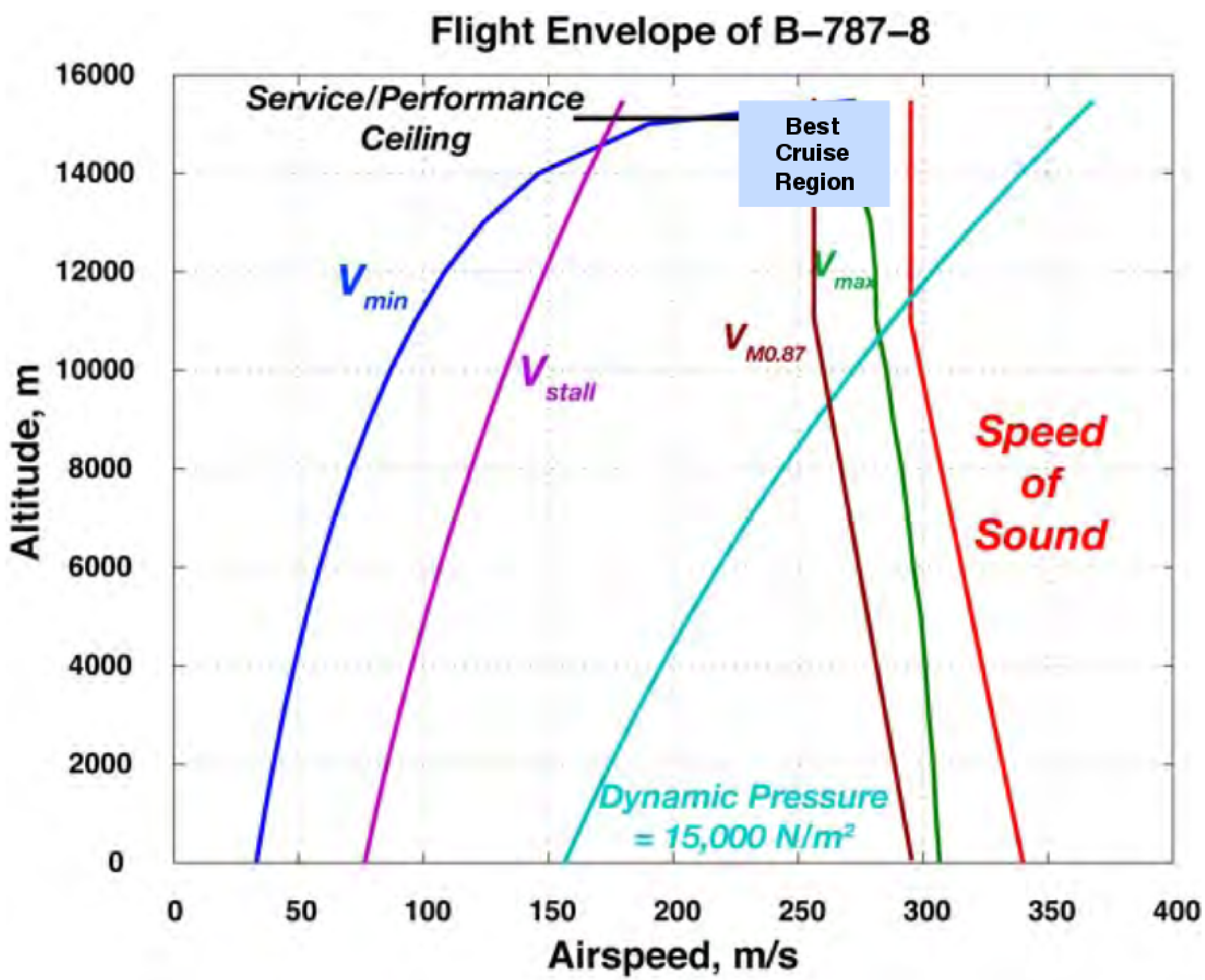
Endurance

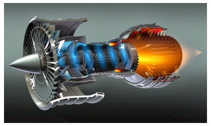
$$t = -\frac{1}{c} \int_{W_1}^{W_2} \left(\frac{C_L}{C_D} \right) \frac{1}{W} \, dW$$

$$t = -\frac{\hat{k}\eta_P}{\hat{c}} \int_{W_1}^{W_2} \left(\frac{C_L}{C_D} \right) \frac{1}{VW} \, dW$$

Jet | Piston-prop

Flight envelope





Range

Cruise-fuel weight consumption $\xi = \dfrac{\Delta W_f}{W_1}$

Case: constant altitude and constant lift coefficient

$$x_{h-C_L} = \frac{2V_1E}{c}\left(1-\sqrt{1-\xi}\right)$$

$$x_{h-C_L} = \frac{\hat{k}\eta_P E}{\hat{c}} ln\left(\frac{1}{1-\xi}\right)$$

Case: constant velocity and constant lift coefficient

$$x_{V-C_L} = \frac{V_1E}{c} ln\left(\frac{1}{1-\xi}\right)$$

$$x_{V-C_L} = \frac{\hat{k}\eta_P E}{\hat{c}} ln\left(\frac{1}{1-\xi}\right)$$

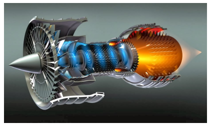
Case: constant velocity and constant height

$$x_{V-h} = \frac{2V_1E_m}{c}\tan^{-1}\left[\frac{E_1\xi}{2E_m}\frac{1}{(1-kC_{L,1}E_1\xi)}\right]$$

$$x_{V-h} = \frac{2\hat{k}\eta_P E_m}{\hat{c}}\tan^{-1}\left[\frac{E_1\xi}{2E_m(1-kC_{L,1}E_1\xi)}\right]$$

Conclusions

$$\left(\frac{C_L^{0.5}}{C_D}\right)_{max} \qquad \left(M\frac{L}{D}\right)_{max} \qquad \left(\frac{C_L}{C_D}\right)_{max}$$



Endurance

Case: constant altitude and constant lift coefficient

$$t_{h-C_L} = \frac{\boxed{E_1}}{c} ln\left(\frac{1}{1-\xi}\right)$$

$$t_{h-C_L} = \frac{2\hat{k}\eta_P \boxed{E}}{\hat{c}\boxed{V_1}}\left(\frac{1}{\sqrt{1-\xi}} - 1\right)$$

Case: constant velocity and constant lift coefficient

$$t_{V-C_L} = \frac{\boxed{E_1}}{c} ln\left(\frac{1}{1-\xi}\right)$$

$$t_{V-C_L} = \frac{\hat{k}\eta_P \boxed{E}}{\hat{c}\boxed{V}} ln\left(\frac{1}{1-\xi}\right)$$

Case: constant velocity and constant height

$$t_{V-h} = \frac{2E_m}{c}\tan^{-1}\left[\frac{\boxed{E_1}\xi}{2E_m}\frac{1}{(1-kC_{L,1}E_1\xi)}\right]$$

$$t_{V-h} = \frac{2\hat{k}\eta_{P\,E_m}}{\hat{c}\boxed{V}}\tan^{-1}\left[\frac{\boxed{E_1}\xi}{2E_m}\frac{1}{(1-kC_{L,1}E_1\xi)}\right]$$

Conclusions

$$\left(\frac{C_L}{C_D}\right)_{max}$$

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{max}$$

Important relations

$$C_D = C_{D0} + kC_L^2$$

$$C_L = C_{L\alpha}(\alpha - \alpha_{L=0})$$

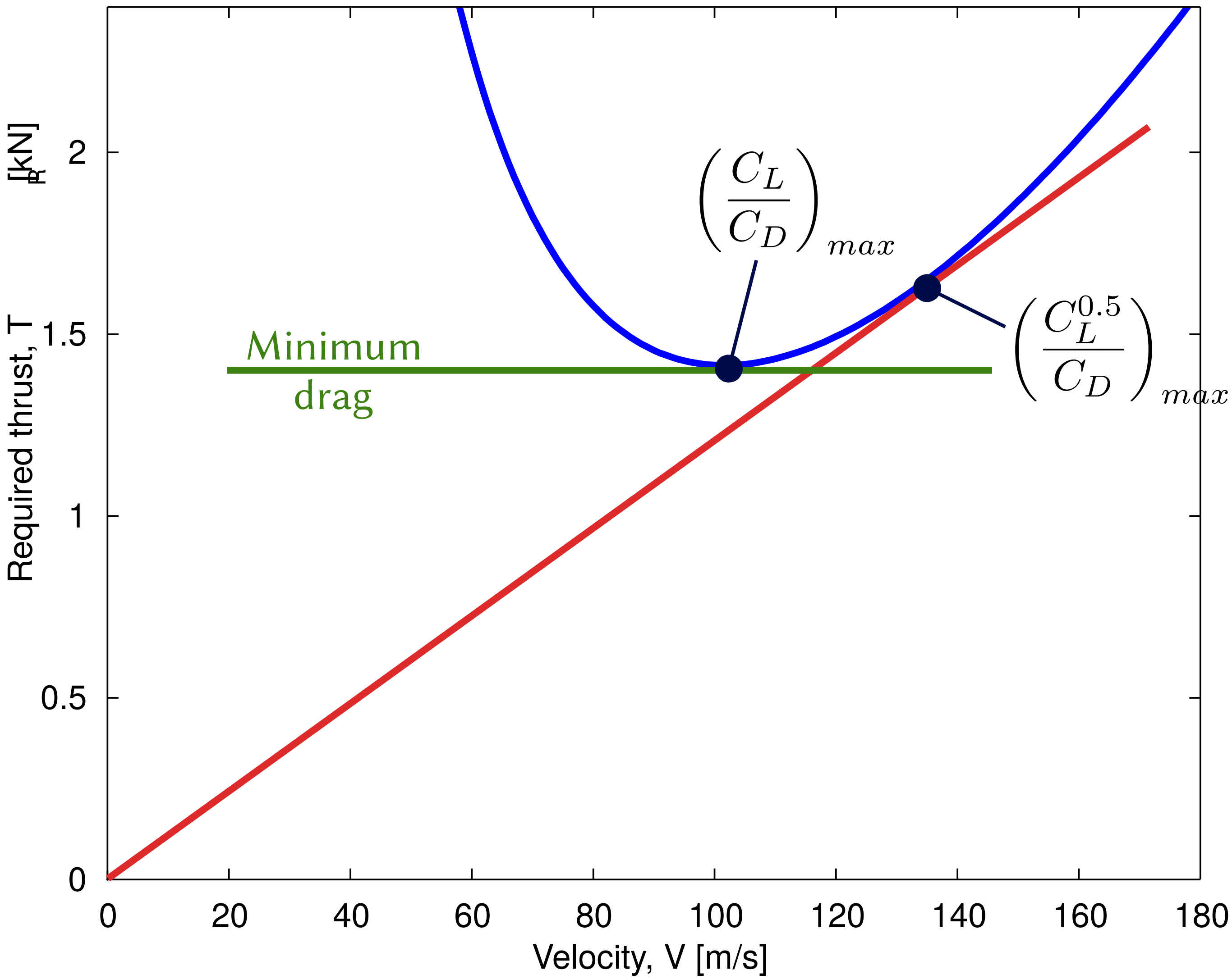
$$\left(\frac{C_L}{C_D}\right)_{max} \rightarrow C_L = \left(\frac{C_{D0}}{k}\right)^{1/2} \; ; \quad C_D = 2C_{D0}$$

$$\left(\frac{C_L^{0.5}}{C_D}\right)_{max} \rightarrow C_L = \left(\frac{C_{D0}}{3k}\right)^{1/2} \; ; \quad C_D = \frac{4C_{D0}}{3}$$

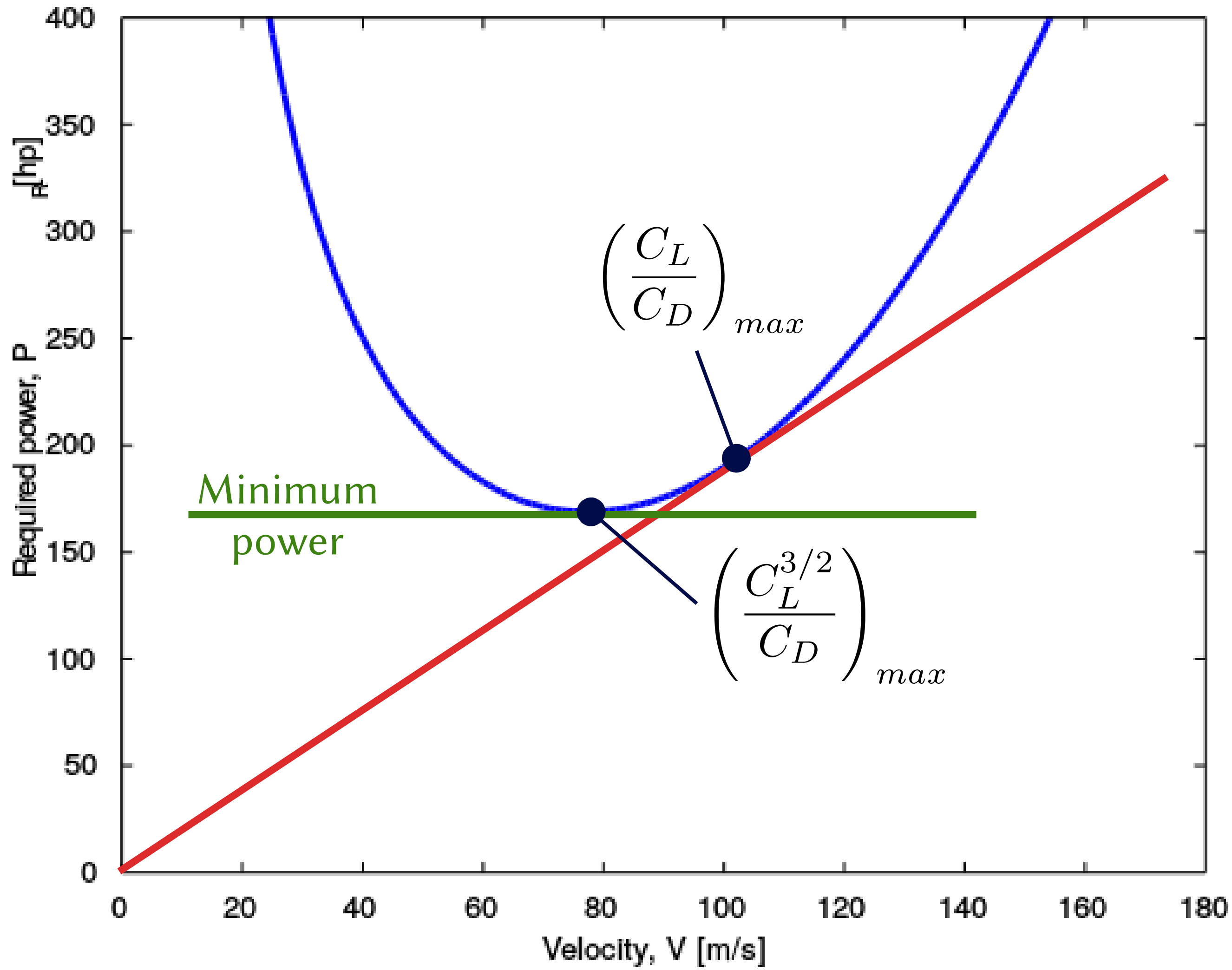
$$\left(\frac{C_L^{1.5}}{C_D}\right)_{max} \rightarrow C_L = \left(\frac{3C_{D0}}{k}\right)^{1/2} \; ; \quad C_D = 4C_{D0}$$

$$\left(\frac{C_L^3}{C_D^2}\right)_{max} \rightarrow C_L = \left(\frac{3C_{D0}}{k}\right)^{1/2} \; ; \quad C_D = 4C_{D0}$$

Thrust required



Power required



Payload vs. range