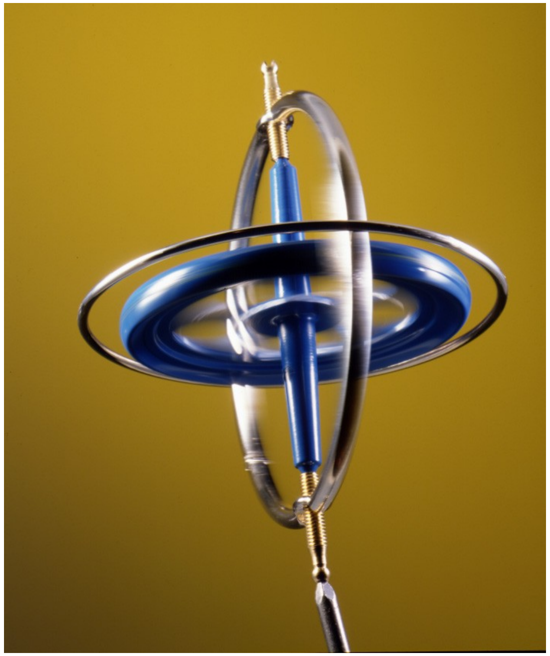
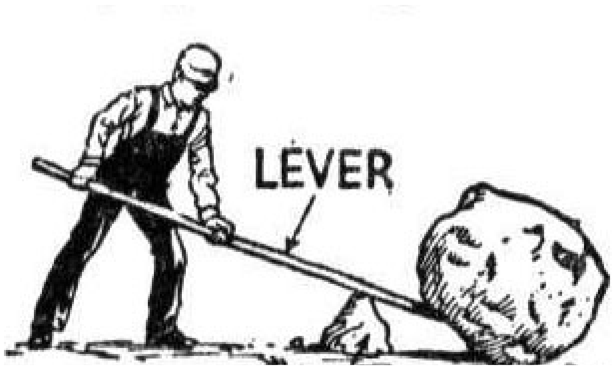
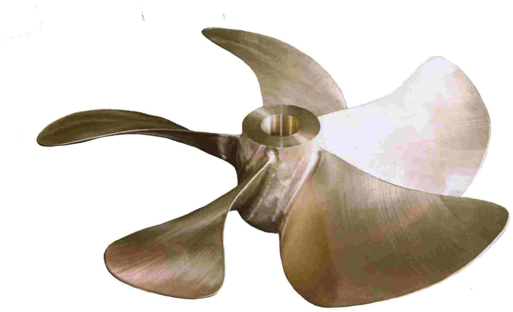
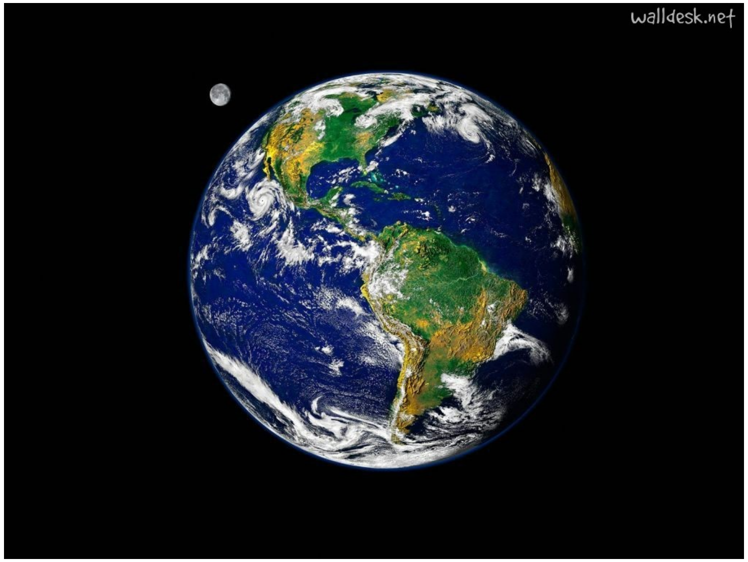
Rotação de Corpos Rígidos (Cap. 9)

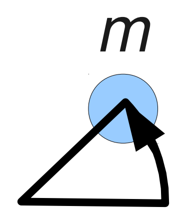








Analogia Translação-Rotação



$s \Longleftrightarrow \theta$

$v \Longleftrightarrow \omega$

$m \Longleftrightarrow I$

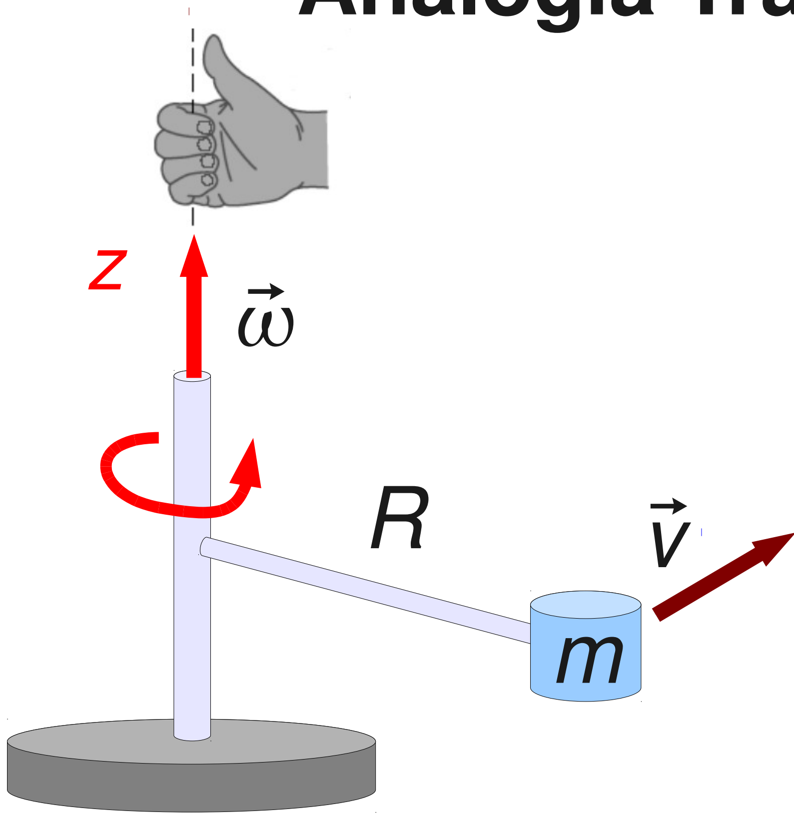
$F \Longleftrightarrow \tau$

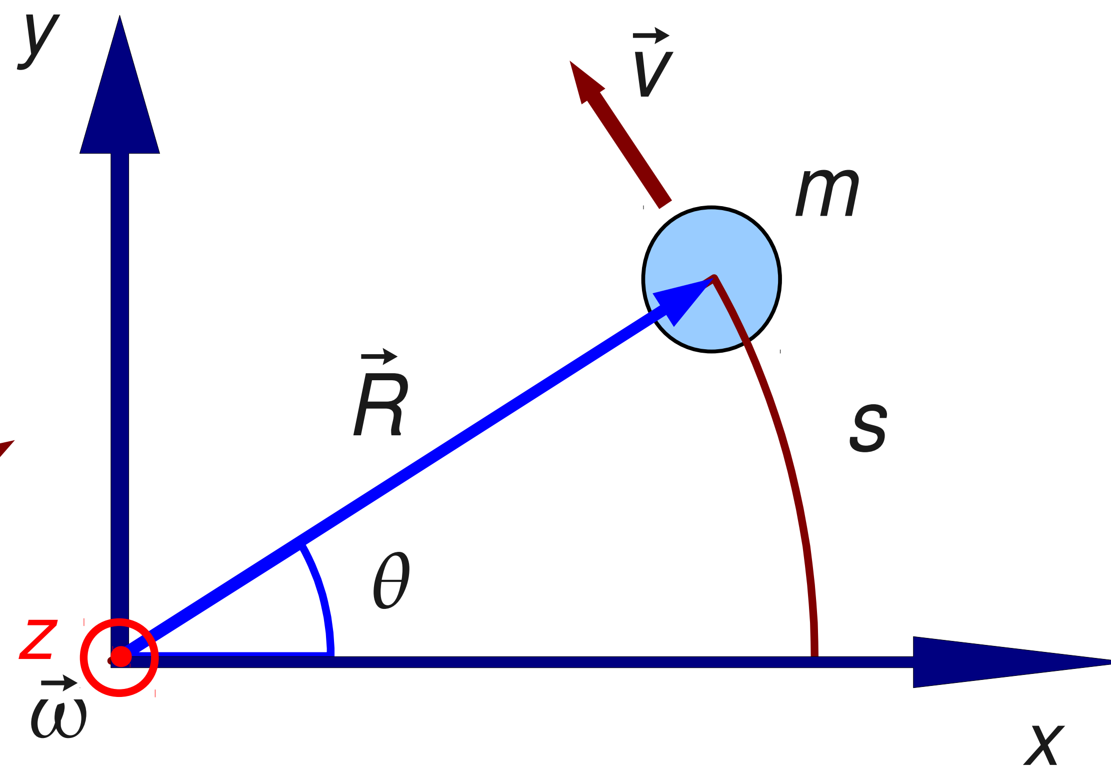
$p \Longleftrightarrow L$

Exemplo: energia cinética

$$E_{ct} = \frac{1}{2} m v^2 \qquad E_{cr} = \frac{1}{2} I \omega^2$$

Analogia Translação-Rotação





$$s = R\theta$$

OBS: θ em radianos

$$v = \frac{ds}{dt} = R\frac{d\theta}{dt} = R\omega$$

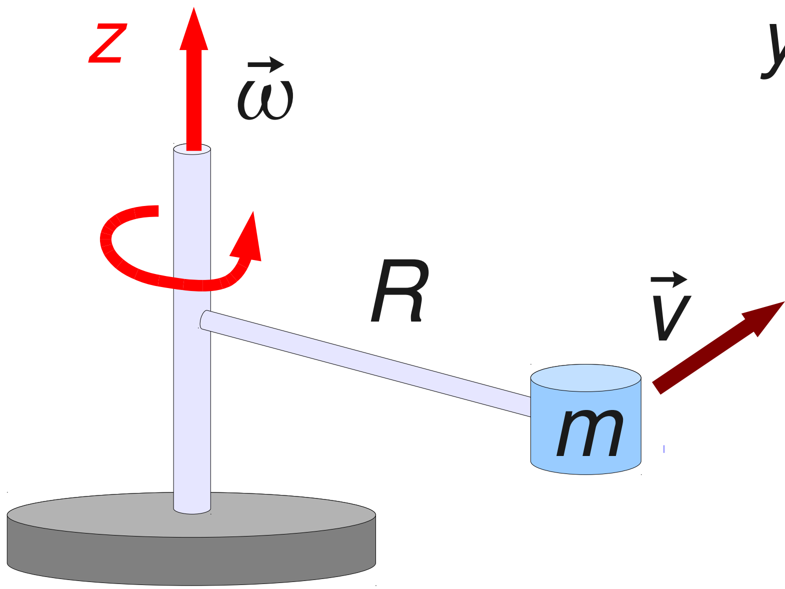
Corpo rígido: R constante

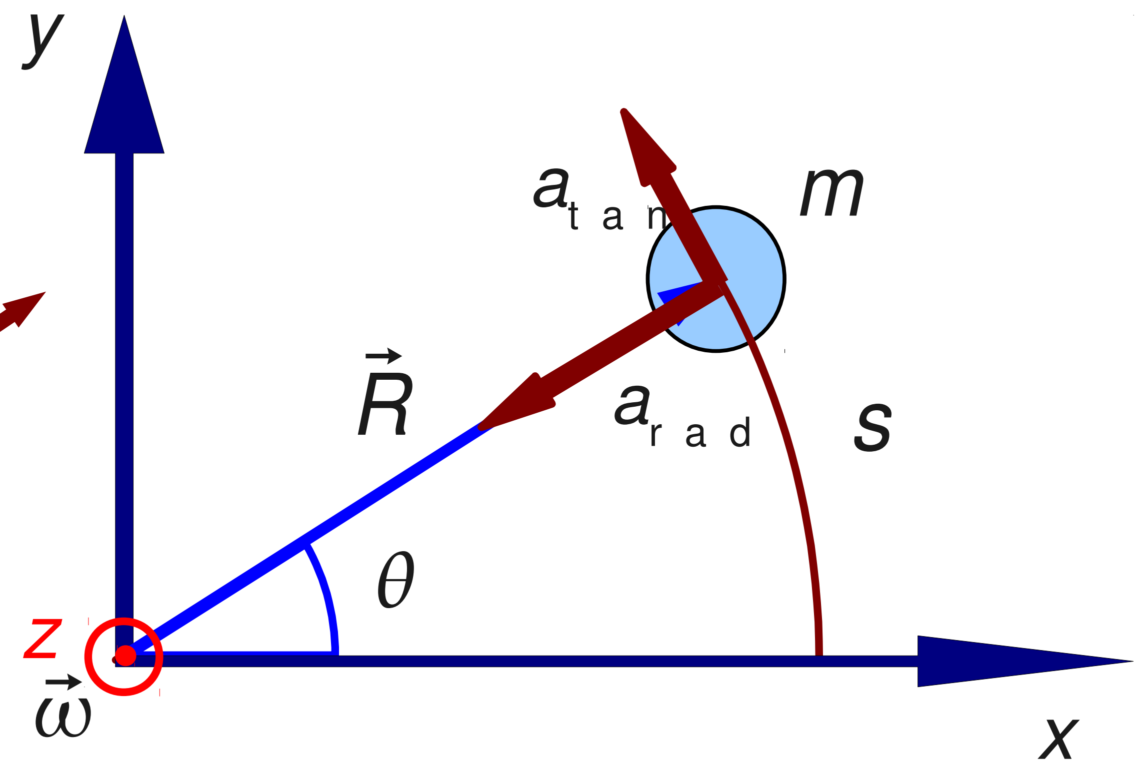
$$\omega = \frac{d\theta}{dt}$$

$$s \Leftrightarrow \theta$$

$$v \Leftrightarrow \omega$$

Aceleração angular





$$v = R\omega$$

$$a_{rad} = \frac{v^2}{R} = R\omega^2$$

$$a_{\tan} = \frac{dv}{dt} = R\frac{d\omega}{dt} = R\alpha_z$$

$$\alpha_z = \frac{d\omega}{dt}$$

Acel. Radial (centrípeta)

Acel. tangencial

Acel. angular

Aceleração angular constante

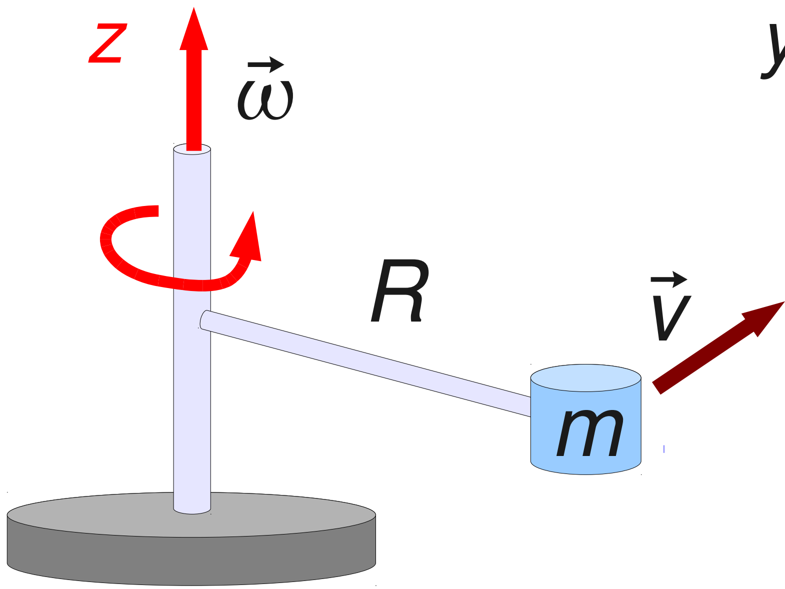
$$\omega = \omega_0 + \alpha_z t$$

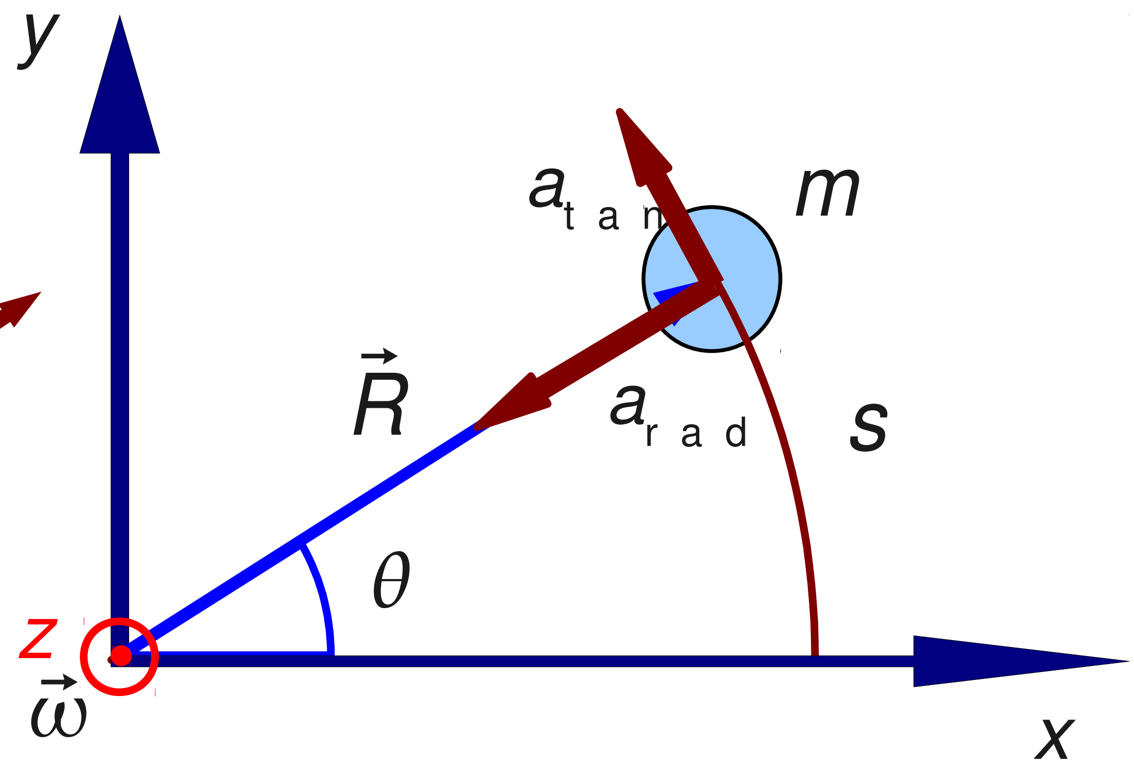
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_z t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha_z(\theta - \theta_0)$$

Análogas a correspondentes lineares

Aceleração angular





$$v = R\omega$$

$$E_{cin} = \frac{1}{2} m v^2 = \frac{1}{2} m R^2 \omega^2 = \frac{1}{2} I \omega^2$$

Energia cinética

$$I = m R^2$$

Momento de inércia

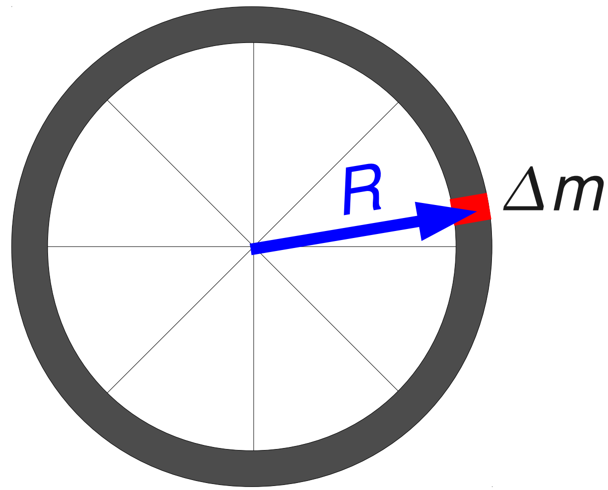
$s \leftrightarrow \theta$ $m \leftrightarrow I$

$v \leftrightarrow \omega$ $F \leftrightarrow \tau$

Momento de inércia

Roda de bicicleta

Modelo (*M*, *R*)



$$M = \sum \Delta m$$

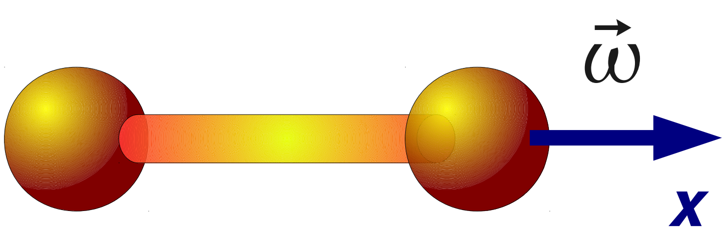
$$\Delta I = \Delta m R^2$$

$$I = \sum \Delta m R^2$$

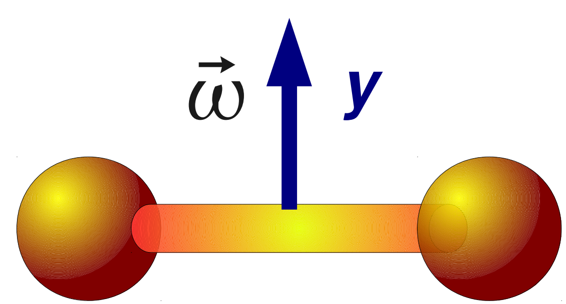
$$I = M R^2$$

Dependência do eixo de rotação

Haltere



$$I_x < I_y$$



Momento de Inércia de distribuição contínua de massa $\rho(x,y,x)$

Densidade no pto. *x,y,z*

$$I_z = \int R^2\, dm = \int\int\int \rho(\vec{r})\, R^2\, dx\, dy\, dz$$

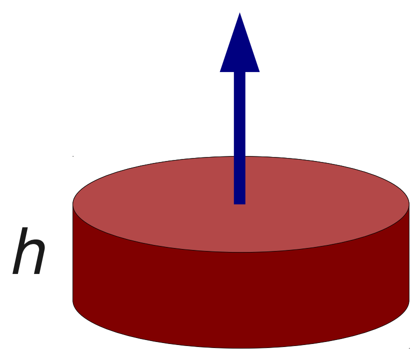
$\rho\, dV$

dV (elem. de volume)

$$R^2 = x^2 + y^2$$

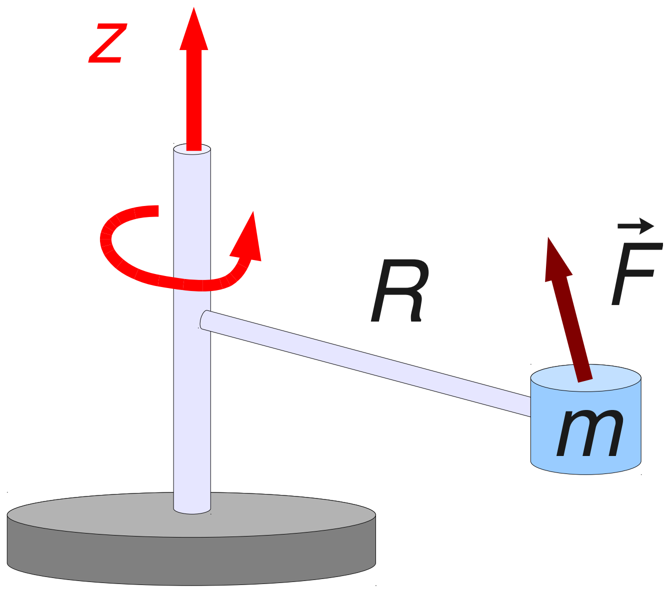
(distância ao eixo de rotação, *z*)

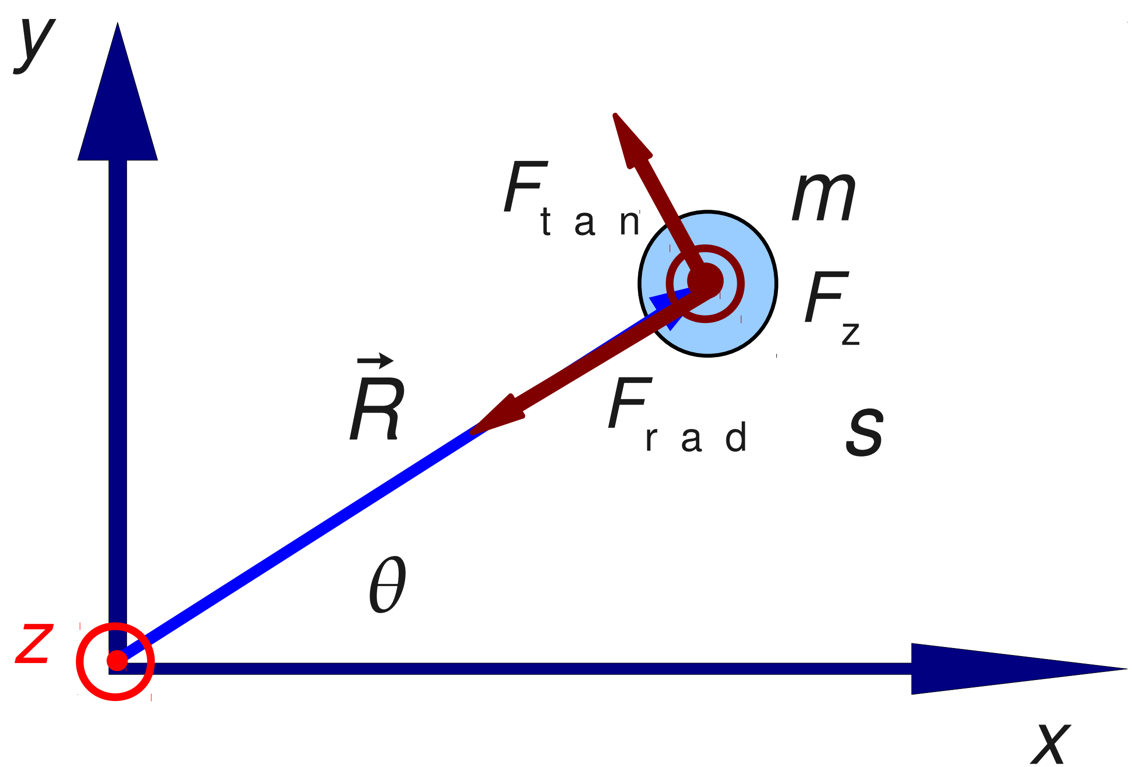
Exemplo: disco de densidade uniforme e raio *R* em rotação em torno do eixo de simetria



$$I = \frac{1}{2} M R^2$$

Torque τ



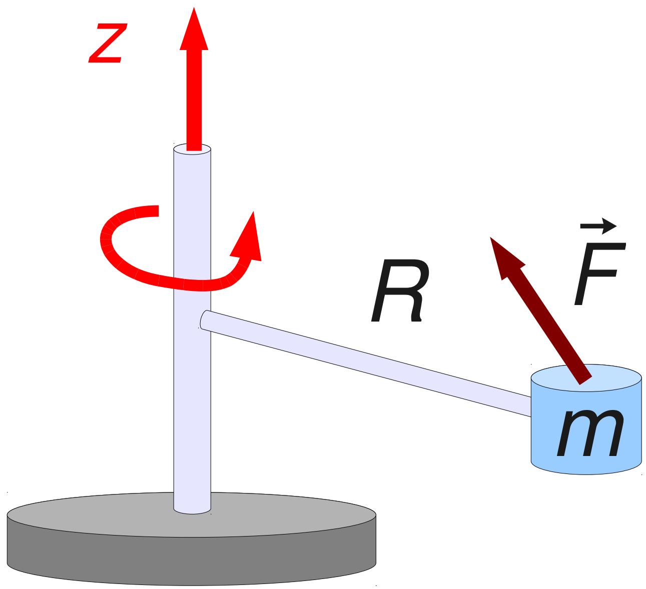


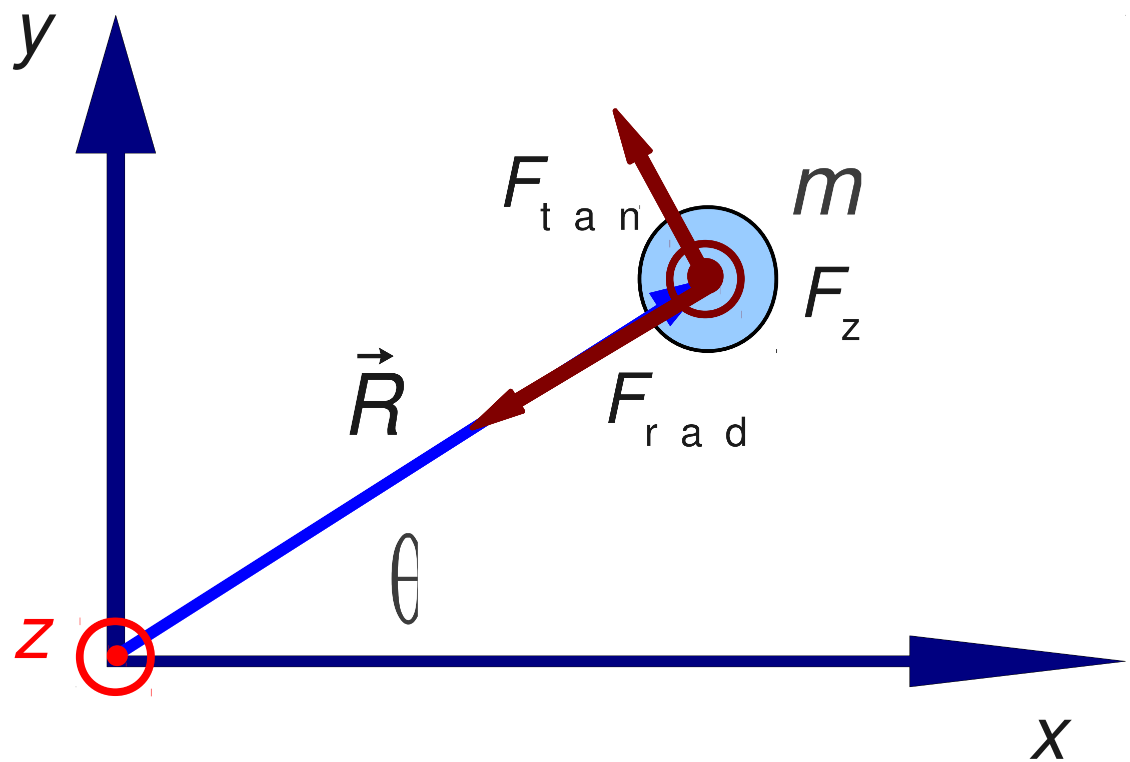
$$\vec{F} = \vec{F}_{rad} + \vec{F}_{\tan} + \vec{F}_z$$

Def.: $$\vec{\tau} = \vec{R} \times \vec{F}$$

$$\tau_z = (\vec{R} \times \vec{F})_z = R\,F_{\tan}$$

Analogia Translação-Rotação $F \Leftrightarrow \tau$





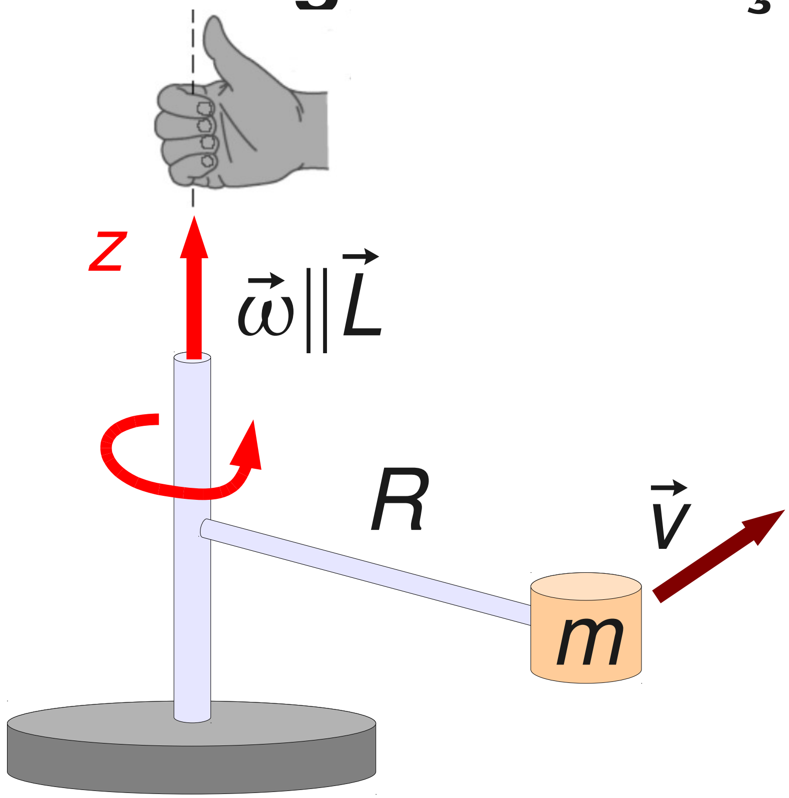
OBS: Corpo Rígido - Forças de vínculo compensam F_{rad} e F_z

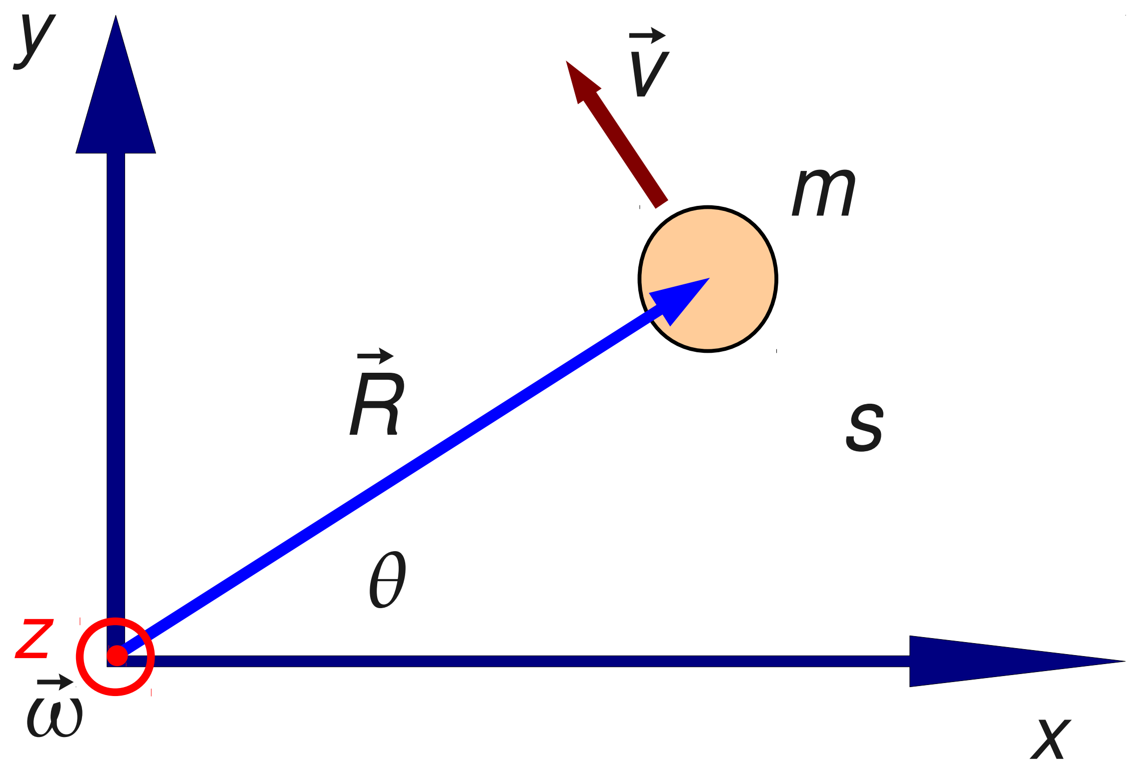
$$F_{tan} = m a_{tan} = m \frac{dv}{dt} = m R \frac{d\omega}{dt} = m R \alpha_z$$

$$\tau_z = R F_{tan} = m R^2 \frac{d\omega}{dt} = m R^2 \alpha_z$$

$$\tau_z = I \alpha_z$$

Analogia Translação-Rotação $p \Longleftrightarrow L$





$$p = m\,v$$

$$\vec{L} = \vec{R} \times \vec{p}$$ Momento Angular

$$L = R\,p = m\,R\,v = m\,R^2\,\omega$$

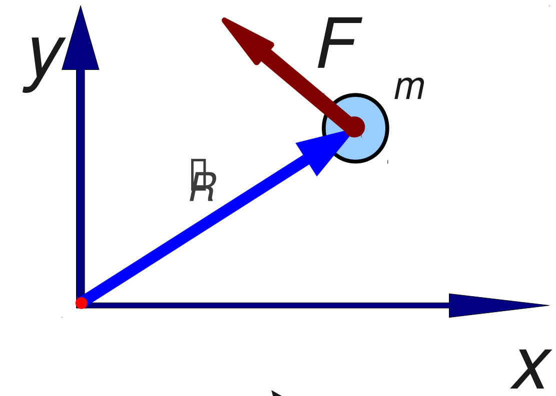
$$(\vec{L} \perp \vec{R})$$

$$L = I\,\omega$$

Analogia Translação-Rotação $\vec{p} \Longleftrightarrow \vec{L}$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$



$$\vec{\tau} = \vec{R} \times \vec{F} = \vec{R} \times \frac{d\vec{p}}{dt} = \frac{d(\vec{R} \times \vec{p})}{dt} = \frac{d\vec{L}}{dt}$$

Pois:

$$\frac{d(\vec{R} \times \vec{p})}{dt} = \frac{d\vec{R}}{dt} \times \vec{p} + \vec{R} \times \frac{d\vec{p}}{dt}$$

e

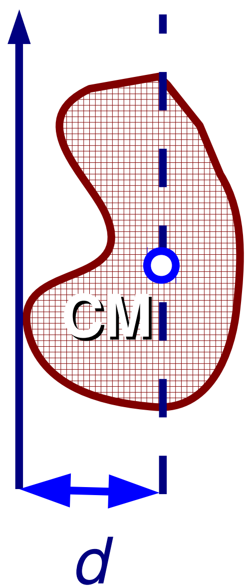
$$\frac{d\vec{R}}{dt} \times \vec{p} = \vec{v} \times m\vec{v} = 0$$

$\vec{F} \Longleftrightarrow \vec{\tau}$

Demonstrações gerais

- Energia cinética de rotação
- Teorema dos Eixos Paralelos

$$I_p = I_{CM} + Md^2$$



- Torque resultante (c. ríg.)
- Energia cinética (K)- eixo móvel (dir. fixa)

$$K = \frac{1}{2} M v_{CM}^2 + \frac{1}{2} I_{CM} \omega^2$$