# • Radiação Acústica

9- Na adiabática temos

$$\frac{P}{P_0} = \left(\frac{\rho}{\rho_0}\right)^a \quad \text{ou} \quad P = P_0\left(\frac{\rho}{\rho_0}\right)^a \qquad (1)$$

$$P = P\big|_{\rho_0} + \frac{\partial P}{\partial \rho}\Big|_{\rho_0}\Delta\rho + \frac{1}{2}\left(\frac{\partial^2 P}{\partial \rho_0^2}\right)_{\rho_0}\Delta\rho^2 \qquad (2)$$

$$P = P_0 + \frac{a P_0}{\rho_0}\left(\frac{\rho}{\rho_0}\right)^{a-1}\Big|_{\rho_0}\Delta\rho + \frac{1}{2}\frac{a(a-1)P_0}{\rho_0\,\rho_0}\left(\frac{\rho}{\rho_0}\right)^{a-2}\Big|_{\rho_0}\Delta\rho^2 \qquad (3)$$

$$P = P_0 + A\frac{\Delta\rho}{\rho_0} + \frac{B}{2}\frac{\Delta\rho^2}{\rho_0^2}$$

Sendo $A = \underline{aP_0} = \mathcal{B}$ $\qquad \frac{B}{A} = \frac{a^2-a}{2a} = \underline{a-1} \qquad (4)$

$B = \underline{(a^2-a)P_0}$

de (3) temos

$$P = P_0\left(1 + as + \frac{1}{2}a(a-1)s^2\right), \quad \text{sendo } s = \frac{\Delta\rho}{\rho_0} = \frac{\rho-\rho_0}{\rho_0} \qquad (5)$$

Sendo $\mathcal{B} = \rho_0 \frac{\partial P}{\partial \rho}\Big|_{\rho_0}$ então

$$\frac{B}{A} = \frac{\frac{\partial^2 P}{\partial \rho^2}\Big|_{\rho_0}(\rho_0)}{\frac{\partial P}{\partial \rho}(\rho_0)} = \frac{\rho_0}{\mathcal{B}}\frac{\partial \mathcal{B}}{\partial \rho} = a-1 \qquad a = 1 + \frac{\rho_0}{\mathcal{B}}\frac{\partial \mathcal{B}}{\partial \rho}\Big|_{\rho_0}$$

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~~10~~- 1 A equação exata da continuidade diz que

$$\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \frac{\partial \rho}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial \rho}{\partial y}\frac{\partial y}{\partial t} + \frac{\partial \rho}{\partial z}\frac{\partial z}{\partial t}$$

$$\boxed{\frac{d\rho}{dt} = \frac{\partial \rho}{\partial t} + \vec{u}\cdot\vec{\nabla}\rho}$$

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~~11~~- 2 Da equação da continuidade, temos,

a) $\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\vec{u}) = 0$ ou $\frac{\partial \rho}{\partial t} + \rho\vec{\nabla}\cdot\vec{u} + u\cdot\nabla\rho = 0$

Sendo $\frac{\partial \rho}{\partial t} + \vec{u}\cdot\nabla\rho = 0$ então $\boxed{\rho\,\vec{\nabla}\cdot\vec{u} = 0}$

b)

2 ~~11~~.b) A equação de Euler do fluxo compressível é

$$\rho_o \frac{\partial \vec{u}}{\partial t} = -\nabla p \quad \text{eq. de Euler linear}$$

Aplicando o divergente tem-se

$$\rho_o \frac{\partial (\cancel{\nabla \cdot \vec{u}})^{0}}{\partial t} = -\nabla^2 p$$

$$-\nabla^2 p = 0$$

então $\nabla p = \underline{\underline{cte}}$

2 ~~11~~.c) $\nabla^2 p = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = 0$ no fluxo incompressível

isto implica $c \to \infty$

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~~12~~. 3

$P_o = 1$ atm $= 1.01325$ bar $\quad c^2 = B/\rho_o = ?$

$T = 0°C \qquad \rho_o = 0{,}09 \qquad P = 1$ atm $= 1.035 \times 10^5$ Pa

$\gamma = 1.41$

o hidrogênio é um gás monoatômico, logo sua compressão é adiabática

a) $B = \gamma P_o \Rightarrow c^2 = \frac{\gamma P_o}{\rho_o} = \left(\frac{1.41 \times 1.01325 \times 10^5}{0{,}09}\right)$

$c = 1260$ m/s

b) $C_T = C_o + \Delta C \Rightarrow \boxed{\Delta C = 9{,}5 \text{ m/s}}$

$c^2 = \gamma r T_K \quad$ ou $\quad c = c_o (1 + T_c/273)^{1/2}$

c) $T = \left(\left(\frac{c}{c_o}\right)^2 - 1\right) * 273 = \left(\left(\frac{1269{,}5}{1260}\right)^2 - 1\right) * 273 = 4{,}1\ °C$

$\boxed{T = 4{,}1\ °C}$

~~13~~- 4 $c(P,t) = 1402{,}7 + 4{,}88 \cdot t - 4{,}82\, t^2 + 1{,}35\, t^3 + \ldots$

$+ (15{,}9 + 2{,}8t + 2{,}4t^2)(P_G / 100)$

$P_G = P - P_0 = 1 - 1 = 0$

$P_0 = 1\ atm = 1{,}01325\ 10^5\ Pa.$

$T = 30°C$

$t = \frac{T}{100} = 0{,}3$

$C = 1.505{,}7 + (16{,}55) \cdot \left(\frac{0}{100}\right)$

a) $\boxed{C = 1.505{,}7\ m/s}$

b) $taxa = \frac{\Delta c}{\Delta T} = \frac{1{,}505.7 - 1481}{30 - 20} = 2{,}47\ m/s°c$

ou seja p/ cada 1 °C a velocidade do som na água muda de 2,47 m/s

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~~14~~) 5 $u = U \exp[j(wt - kx)]$

a) $-\nabla p = \rho_0 \frac{\partial u}{\partial t}$ ① e a expressão p/ a pressão será

$p = P \exp[j(wt - kx)]$ ②

$\nabla p = -jk\, p$

$\frac{\partial u}{\partial t} = jw\, u$

- de ① temos

$-(-jk\, p) = \rho_0\, jw\, u$

$\cancel{k} P = \rho_0 \cancel{k} c\, U$ , sendo $w = kc$

$\boxed{U/c = P/\rho_0 c^2}$

b) Da eq. 5.76 do livro Fundamental acústica

$s = p / \rho_0 c^2$ , então

$|s| = P / \rho_0 c^2$ ou seja

$U/c = |s|$

~~15~~ 6 - $p = A e^{j(\omega t - k_x x - k_y y - k_z z)}$

$\phi = \nabla u = ?$

da eq 5.59 do Fundamental acústica, temos

$$p = -\rho_0 \frac{\partial \phi}{\partial t} =$$

$$\phi = -\frac{1}{\rho_0} \int p \, dt = -\frac{1}{\rho_0 (j\omega)} A e^{j(\omega t - k_x x - k_y y - k_z z)}$$

$$\boxed{\phi = \frac{j}{\rho_0 \omega} p}$$

$$u = \nabla \phi = \frac{j}{\rho \omega} \left[ \frac{\partial}{\partial x} p \, \hat{x} + \frac{\partial p}{\partial y} \hat{y} + \frac{\partial}{\partial z} p \, \hat{z} \right]$$

$$u = \frac{j}{\rho \omega} \left[ -jk_x \, p \, \hat{x} - jk_y \, p \, \hat{y} - jk_z \, p \, \hat{z} \right]$$

$$\vec{u} = \left[ k_x \hat{x} + k_y \hat{y} + k_z \hat{z} \right] \frac{p}{\rho \omega} =$$

Como $\vec{k}$ aponta na direção de propagação, então $u$ é paralela a direção de propagação //

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~~16~~ 7 - a) $\rho = ?$
b) $u = ?$
c) $\phi = ?$
d) $\varepsilon = ?$
e) $I = ?$

dado $p = P e^{j(\omega t - kx)}$

$$c^2 = \frac{\partial p}{\partial \rho}$$

$$\int_{\rho_0}^{\rho} \partial \rho = \frac{1}{c^2} \partial p$$

$$\rho - \rho_0 = \frac{1}{c^2} \int_{p_0}^{p} \partial p = \frac{1}{c^2} p$$

a) $\boxed{(\rho - \rho_0) = \frac{1}{c^2} p = \frac{P}{c^2} e^{j(\omega t - kx)}}$

b) $\boxed{u = p/\rho_0 c = \frac{P}{\rho_0 c} e^{j(\omega t - kx)}}$

c) $\boxed{\phi = -p/j\omega\rho_0 = \frac{jP}{\omega \rho_0} e^{j(\omega t - kx)}}$

d) $\varepsilon_i = \text{real}\left( \frac{p^2}{\rho_0 c^2} \right) = \frac{P^2}{\rho_0 c^2} \cos^2(\omega t - kx)$

e) $I = \langle pu \rangle_T = \frac{1}{T} \int_0^T pu \, dt$

$$I = \frac{1}{2} \frac{P^2}{\rho_0 c^2}$$

~~17~~ 8

$$p = A_1 e^{-j(\omega_1 t - kx)} + A_2 e^{-j(\omega_2 t - kx)}$$

$$Z = \frac{p}{u} =$$

$$\int \partial \vec{u} = -\frac{1}{\rho_0} \int (\nabla p)\, \partial t$$

$$\vec{u} = \frac{1}{\rho_0} \int \left( \frac{A_1}{(-jk} e^{+j(\omega_1 t - k_1 x)} + (-jk)A_2\, e^{+j(\omega_2 t - k_2 x)} \right) \partial t$$

$$= -\frac{1}{\rho_0} \left[ \frac{A_1(-jk_1)}{+(j\omega_1)} A_1 e^{j(\omega t - k_1 x)} + \frac{A(-jk)}{(j\omega_2)} e^{j(\omega t - k_2 x)} \right]$$

$$= \frac{1}{\rho_0} \left( \underbrace{\cancel{\frac{k_1}{\omega_1}}}_{c} A_1 e^{j(\omega t - k_1 x)} + \underbrace{\cancel{\frac{k_2}{\omega_2}}}_{c} A_2\, e^{j(\omega t - k_2 x)} \right)$$

$$\vec{u} = \frac{1}{\rho_0\, c} (p)$$

$$\boxed{Z = \frac{p}{u} = \rho_0\, c}$$

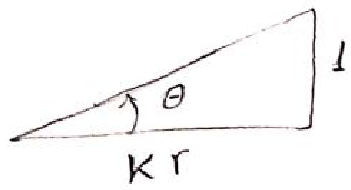
~~18~~ 9

da eq. 3.11.13 do Fundamentals Acoustic

$C_{ar} = 343$ m/s

$$tg\,\theta = \frac{1}{kr}$$

$$k = \frac{\omega}{C_{ar}} = \frac{2\pi f}{C_{ar}} = \frac{2\pi}{343} f$$



a) $f = 10$ Hz

$$\theta = \operatorname{atan}\left(\frac{1}{kr}\right) = \operatorname{atan}\left(\frac{343}{2\pi 10 \times 0{,}1}\right) = 88{,}9^\circ$$

b) $f = 100$ Hz

$\theta = 79{,}6^\circ$

c) $f = 1000$ Hz

$\theta = 28{,}6$

d) $f = 10$ kHz ; $\theta = 3{,}1^\circ$

e) $f = 100$ kHz ; $\theta = 0{,}31^\circ$

e) $f = 1$ MHz

$0{,}031^\circ$

$$Z = \rho_0\, c \cos\theta$$

OBS.: P/ uma onda acústica c/ frequência equivalente ao do diagnóstico ≈ 1 MHz, o ângulo de fase é praticamente zero. e $Z \approx Z_0$

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$$\dot{X} = \frac{S_0 C k r}{1+(kr)^2}$$

$$\dot{X}_{max} \Rightarrow \frac{\partial \dot{X}}{\partial r} = 0$$

$$\frac{\partial \dot{X}}{\partial r} = S_0 C \left[\frac{k}{1+(kr)^2} - \frac{2k^3 r^2}{\left(1+(kr)^2\right)^2}\right] = 0$$

$$\Rightarrow \underline{kr = 1}$$

20- $f = 1\,MHz = 10^6\,Hz$ $\quad C_{tec} = 1.540\,m/s$

10 $P_o = 46\,Pa$ $\quad \rho_{tec} \approx \rho_{H_2O} = 10^3\,kg/m^3$

$U_o = 0{,}03\,mm/s = 3\times10^{-5}\,m/s$

$r = ?$

$$Z = \frac{P}{U} = \rho_o\, c \cos\theta = \rho_o\, c \frac{Kr}{\left(1+(Kr)^2\right)^{1/2}}$$

$$\left(\frac{P}{U\rho_o C}\right)^2 = \frac{(Kr)^2}{1+(Kr)^2}$$

$$r = \frac{1}{K}\left\{\frac{P^2}{U^2\rho_o^2 c^2 - P^2}\right\}^{1/2}$$

$$K = \frac{2\pi f}{c} = \frac{2\pi\times10^{+6}}{1540} = \underline{\underline{4{,}8\times10^{3}}}$$

$$r = \frac{1}{4{,}8\times10^3}\left\{\frac{(46\,Pa)^2}{(3\times10^{-5})^2\times(10^3)^2\times(1540)^2-(46)^2}\right\}^{1/2}$$

$$\boxed{r = 2{,}23\ mm}$$

~~21~~ 11

a) $I$ ?

b) $\Delta\xi = \int u\,dt$

c) $U$ ?

d) $P_e$ ?

e) $20 \log \frac{P}{P_0}$

$\begin{cases} f = 100 \text{ Hz} \\ P_0 = 100 \text{ Pa} \end{cases}$ $\begin{cases} c_{te} = 1540 \\ \rho_0 = 10^3 \text{ kg/m}^3 \end{cases}$ ⑦

a) $I = \frac{P^2}{2\rho_0 c}$

$= \frac{(100)^2}{2 \times 10^3 \times 1540} = 3{,}24 \times 10^{-3} \text{ W/m}^2$

$\boxed{a)\ I = 3{,}24 \text{ mW/m}^2}$

b) $= \Delta\xi = \frac{du}{dt}$

$u = U e^{j(\omega t - kx)}$

$\Delta\xi = \int u\,dt = \frac{U}{j\omega} u(t, x) =$

$|\Delta\xi| = \frac{U}{\omega} = \frac{P/\rho_0 c}{\omega} =$

c) $U = \frac{P}{\rho_0 c} =$

# • Propagação de onda

22. $P_e = \frac{P}{\sqrt{2}} = 200\ Pa$

R $P_{t\,tecido}$ ?

ar | tecido
--- | ---
c | c =
$P_e = 200\ Pa$ | $P_e$ ?
$Z_{ar} = 0{,}0004$ | $Z_{tec} = 1{,}63$

$\rho_{tec} \approx 1060\ kg/m^3$

a) $\frac{P_t}{P_i} = T = \frac{P_{tecido}}{P_{ar}} = \frac{2 Z_{tecido}}{Z_{tec} + Z_{ar}}$

$P_{e\,tecido} = P_{e\,ar} \frac{2 \times 1{,}63}{0{,}0004 + 1{,}63} \approx 2\, P_{e\,ar}$

$\boxed{P_{e\,tecido} \approx 400\ Pa\ (rms)}$

b) $I_{tec}$? $I_{ar}$?

P/ uma onda plana progressiva

$I_{tec} = \left(\frac{P_{etec}^2}{\rho_0 c}\right)_{tec} = \left(\frac{(400)^2}{1060 \times 1540}\right) =$

$I_{ar} = \left(\frac{P_e}{\rho_0 c}\right)_{ar} = \left(\frac{(200)^2}{1{,}2 \times 330}\right) =$

c) $T_I = 10 \log\left(\frac{I_t}{I_i}\right) = 20 \log\left(\frac{P_{e\,tec}}{P_{e,ar}}\right) = 10 \log\left(\frac{400}{200}\right)$

$\boxed{T_I = 20 \log(2)\ dB}$

24) $f = 5\ MHz$

água ① | osso ② | água ③

| ① tecido | ② osso |
|---|---|
| $z_1 = 1,63$ | $z_2 = 7,8$ |
| $c_1 = 1540$ | $c_2 = 4050$ |
| $\rho_1 = 1060$ | $\rho_2 = 1912$ |

a) $T_p$? $T_I$?

23

$$T_p = \frac{2 z_2}{z_2 + z_1} = \frac{2 \times 7,8}{7,8 + 1,63} = 1,65$$

13

$$T_I = \left(\frac{z_1}{z_2}\right)|T|^2 = \frac{4 z_2 z_1}{(z_2 + z_1)^2}$$

$$T_I = \frac{4 \times 7,8 \times 1,63}{(7,8 + 1,63)^2} = 0,57$$

b) Nível de pressão SPL?

Nível de intensidade IL?

$P_{ref,osso} = P_{ref,tec}$

$$SPL_{tec} = 20 \log \left(\frac{P_e}{P_{ref}}\right)_{tec}$$

$$SPL_{osso} = 20 \log \left(\frac{P_e}{P_{ref}}\right)_{osso}$$

$$\Delta SPL = SPL_{osso} - SPL_{tec} = 20 \log \left(\frac{P_e}{P_{ref}}\right)_{osso} - 20 \log \left(\frac{P_e}{P_{ref}}\right)_{tec}$$

$$\Delta SPL = 20 \log \left(\frac{P_{osso}}{P_{tec}}\right) = 20 \log (T_p)$$

$\Delta SPL =$

Resolvendo a diferença de nível de intensidade de modo análogo ao nível de pressão temos:

$$\Delta IL = IL_{osso} - IL_{tec} = 10 \log \left(\frac{I_{osso}}{I_{tec}}\right) = 10 \log (T_I)$$

$$\Delta IL = 10 \log (T_I)$$

| (1) água | (2) osso | (3) água |
|---|---|---|
| $\rho_1$ = 1000 | $\rho_2$ = 1912 | $\rho_3 = \rho_1$ |
| $c_1$ = 1 [illegible] | $c_2$ = 4,0[illegible] | $c_3 = c_1$ |
| $Z_1$ = 1,48 | $Z_2$ = 7,[illegible] | $Z_3 = Z_1$ |

~~24~~) $f = 5\ MHz$

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a) $\Delta$ = Perda em dB ?

$$\Delta = 10 \log \frac{I_{T3}}{I_{T1}} = 20 \log \left(\frac{P_3}{P_1}\right)$$

$$T_{1,2} = \frac{P_2}{P_1} = \frac{2Z_2}{Z_1 + Z_2}$$

$$T_{2,3} = \frac{P_3}{P_2} = \frac{2 \times Z_3}{Z_2 + Z_3} = \frac{2 \times Z_1}{Z_2 + Z_1}$$

$$\frac{P_3}{P_1} = \frac{2 \times Z_1}{\cancel{(Z_2 + Z_1)}} \times \frac{\cancel{(Z_1 + Z_2)}}{2 \times Z_2} = \frac{Z_1}{Z_2}$$

$$\Delta = 20 \log \left(\frac{Z_1}{Z_2}\right)$$

$$\boxed{\Delta = 20 \log \left(\frac{1,48}{7,8}\right) [dB]}$$

b) $$R_\pi = |R|^2 = \left(\frac{Z_2 - Z_1}{Z_1 + Z_2}\right)^2 = \left(\frac{7,8 - 1,48}{7,8 + 1,48}\right)^2$$

$$R_\pi =$$

c) $\rho_{bor}$ = 500 Kg/m³ $\qquad$ $\Delta$ ?

$c_{bor}$ = 1000 m/s $\qquad$ $P_\pi$ ?

$$Z_{bor} = (\rho \cdot c)_{bor} = 0,5 \times 10^6\ Rayl$$

$$\Delta = 20 \log \left(\frac{Z_1}{Z_2}\right) = 20 \log \left(\frac{1,48}{0,5}\right)$$

$$R_\pi = |R|^2 = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2 = \left(\frac{0,5 - 1,48}{0,5 + 1,48}\right)^2$$

$$=$$

OBS: A transmissão e reflexão na placa de 1 cm ocorrem em condições normais, desde que a espessura da placa seja maior que $\frac{\lambda}{2}$

$$\lambda = \frac{c}{f} = \frac{1450}{5 \times 10^6} \approx 3 \times 10^{-4}$$

$\lambda \ll 1$ cm.

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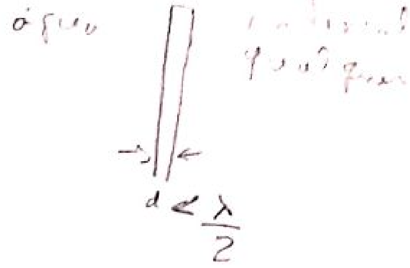
$$T_{I_{1,2}} = \frac{4 Z_1 Z_2}{(Z_1 + Z_2)^2} = 0{,}54$$

$$T_{2,3} = \frac{4 Z_2 Z_1}{(Z_1 + Z_2)^2} = 0{,}54$$

~~25~~) 15

Sendo a placa mais estreita que meio comprimento de onda




Sendo o meio 1 igual ao meio 3, então

$$T_I = \frac{4}{2 + \left(Z_3/Z_1 + Z_1/Z_3\right)\cos^2 K_2 L + \left(Z_2^2/Z_1 Z_3 + \frac{Z_1 Z_3}{Z_2^2}\right)\operatorname{sen}^2 K_2 L}$$

P/ $Z_2^2 = (Z_1 Z_3)$

$$T_I = \frac{4}{2 + \left(\frac{Z_3}{Z_1} + \frac{Z_1}{Z_3}\right)\cos^2 K_2 L + 2\operatorname{sen}^2 K_2 L}$$

P/ $L = \frac{\lambda_2}{4} \rightarrow K_2 L = \frac{2\pi}{\lambda_2} \times \frac{\lambda_2}{4} = \frac{\pi}{2}$

$\cos^2 \frac{\pi}{2} = 0$ e $\operatorname{sen}^2 \frac{\pi}{2} = 1$, então

$$\boxed{T_1 = \frac{4}{2+2} = 1}$$

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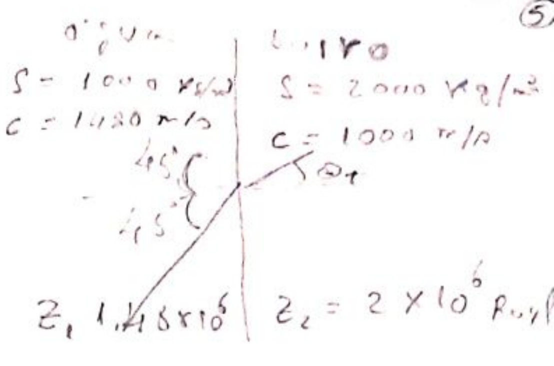


26) $P_0 = 100\ Pa$

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$\theta_t$ ?

a) $\operatorname{sen}\theta_t = \frac{C_2}{C_1}\operatorname{sen}\theta_i$

$$\theta_t = \arcsin\left(\frac{1000}{1430}\operatorname{sen} 45\right)$$

$$\boxed{\theta_t = \arcsin\left(\frac{1000}{1430}\ \frac{\sqrt{2}}{2}\right)}$$

b) $P_t = ?$

$$T = \frac{P_t}{P_i} = 1 + \left(\frac{Z_2/Z_1 - \cos\theta_t/\cos\theta_i}{Z_2/Z_1 + \cos\theta_t/\cos\theta_i}\right)$$

$$P_t = P_i\left(1 + \frac{\frac{2}{1{,}418} - \cos\theta_t/\cos(45°)}{\frac{2}{1{,}418} + \frac{\cos\theta_t}{\cos 45}}\right)$$

c) $P_i + P_r = P_t$

$$\boxed{P_r = P_t - P_i}$$

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