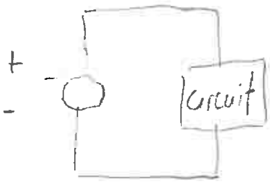
Apresentação : aula substituir Eftore 11/9/18

$$R = \frac{e_R}{i} \qquad C = \frac{q}{e_c} \ [\text{Faraday}] \qquad L = \frac{e_L}{di/dt} \ [\text{Henry}] \longrightarrow i_L = \frac{\int v_L\,dt + i_L(0)}{L}$$

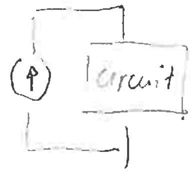
$$\left.\begin{aligned} e_c &= \frac{q}{C} \\ i &= \frac{dq}{dt} \end{aligned}\right\} \rightarrow e_c = \frac{\int i\,dt}{C} + e_c(0)$$

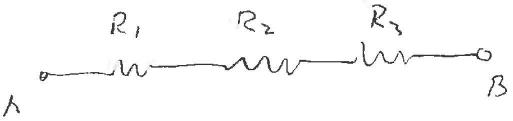
Voltage source



Constant Volt. Source

current source





$$e_{AB} = e_1 + e_2 + e_3$$
$$= R_1 i + R_2 i + R_3 i = R_E i$$
$$R_E = R_1 + R_2 + R_3$$

$$e_{AB} = R_E\, i = R_E\,(i_1 + i_2 + i_3)$$

$$i = i_1 + i_2 + i_3 \qquad i = \left(\frac{V_{AB}}{R_1} + \frac{V_{AB}}{R_2} + \frac{V_{AB}}{R_3}\right) = \frac{V_{AB}}{R_E}$$

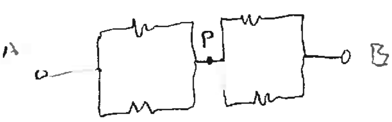
$$\frac{1}{R_E} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)$$

$$R_E = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3}$$

Exemplo

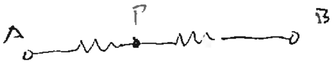
$$R_{AP} = \frac{R_1 R_3}{R_1 + R_3} \qquad R_{PB} = \frac{R_2 R_4}{R_2 + R_4}$$

≡

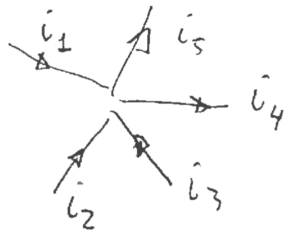


$$R = R_{AP} + R_{PB} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

≡



Kirchhoff's current law



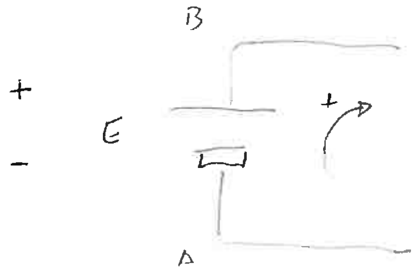
$$i_1+i_2+i_3-i_4-i_5=0$$

Kirchhoff's voltage law : ~~A~~ soma das quedas de tensão é igual a soma das subidas de tensão

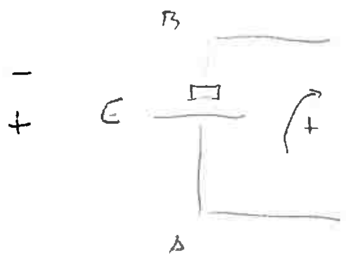
↓

do neg p/ o pos

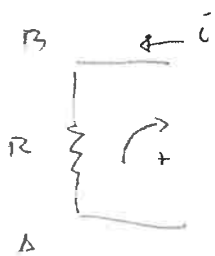
ou através de R em oposição à corrente



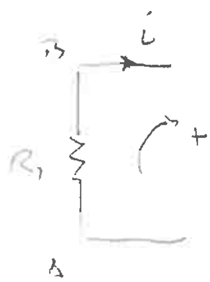
$e_{AB}=E$



$e_{AB}=-E$



$e_{AB}=Ri$

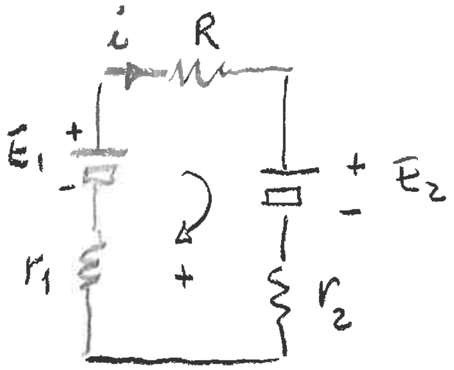


$e_{AB}=-Ri$

EXEMPLOS

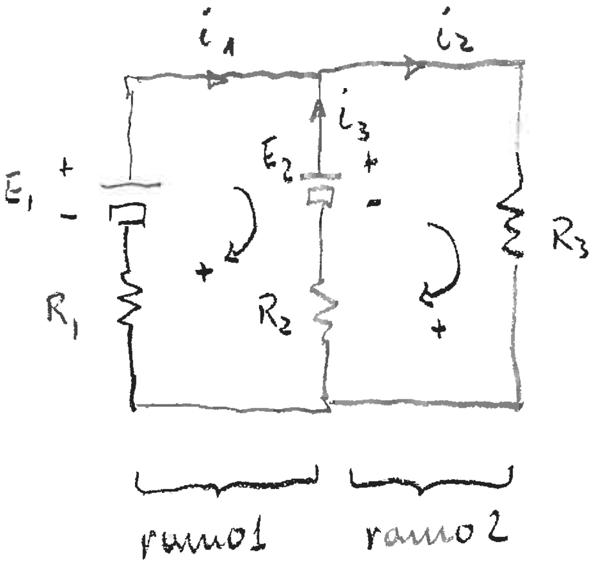
$$E_1 - Ri - ri = 0$$

$$i = \frac{E_1}{R+r}$$



$$E_1 - Ri - E_2 - r_2 i - r_1 i = 0$$

$$i = \frac{E_1 - E_2}{R + r_1 + r_2}$$ if $E_1 - E_2 < 0$ i is counterclockwis



$$i_1 + i_3 = i_2$$

$$E_1 - E_2 + R_2 i_3 - R_1 i_1 = 0$$

$$E_2 - R_3 i_2 - R_2 i_3 = 0$$

$$E_1 - E_2 + R_2 i_2 - (R_1 + R_2) i_1 = 0$$

$$i_1 = \frac{E_1 - E_2 + R_2 i_2}{R_1 + R_2}$$

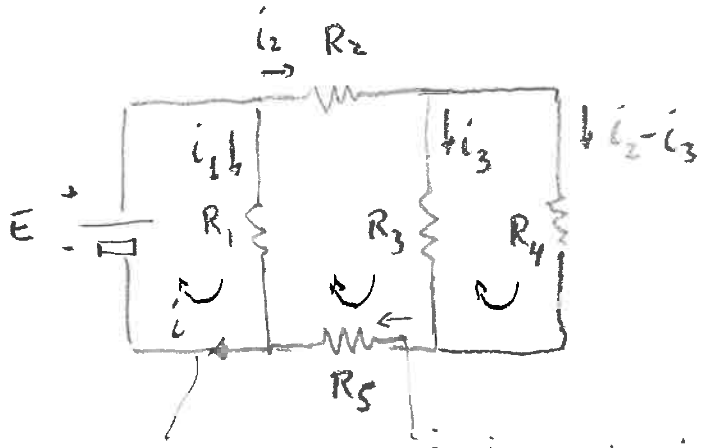
$$E_2 - R_3 i_2 - R_2 i_2 - R_2 \cdot i_1 = 0$$

$$E_2 - (R_2 + R_3) i_2 + \frac{R_2}{R_1 + R_2} (E_1 - E_2 + R_2 i_2) = 0$$

$$i_2 = \frac{E_2 + \frac{R_2}{R_1+R_2}(E_1 - E_2)}{R_2 + R_3 - \frac{R_2 R_2}{R_1 + R_2}} = \frac{E_1 R_2 + E_2 R_1}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

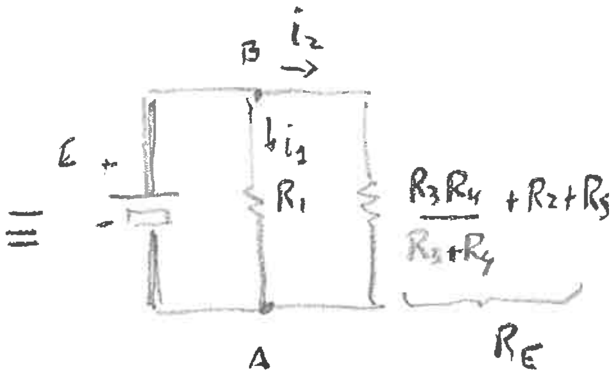
$$i_1 = \frac{E_1 (R_2 + R_3) - E_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$

$$i_3 = i_2 - i_1 = \frac{E_1 R_3 + E_2 R_1 + E_2 R_3}{R_1 R_2 + R_2 R_3 + R_1 R_3}$$



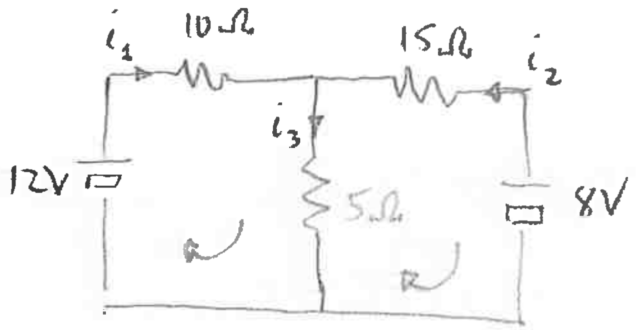
$i = i_3 + i_2 - i_3 = i_2$

$i = i_1 + i_2$



$$E = E_{AB} = R_1 i_1 = R_E i_2$$

$$i = \frac{E_{AB}}{R_1} \qquad i_2 = \frac{E_{AB}}{R_E}$$



$$12 - 10 i_1 - 5 i_3 = 0$$

$$15 i_2 - 8 + 5 i_3 = 0$$

$$i_1 + i_2 = i_3$$

$$\rightarrow \begin{cases} 12 - 10 i_1 - 5 i_1 - 5 i_2 = 0 \\ -8 + 15 i_2 + 5 i_1 + 5 i_2 = 0 \end{cases}$$

$$\begin{cases} 12 - 15 i_1 - 5 i_2 = 0 \\ -8 + 5 i_1 + 20 i_2 = 0 \quad \times 3 + \end{cases}$$

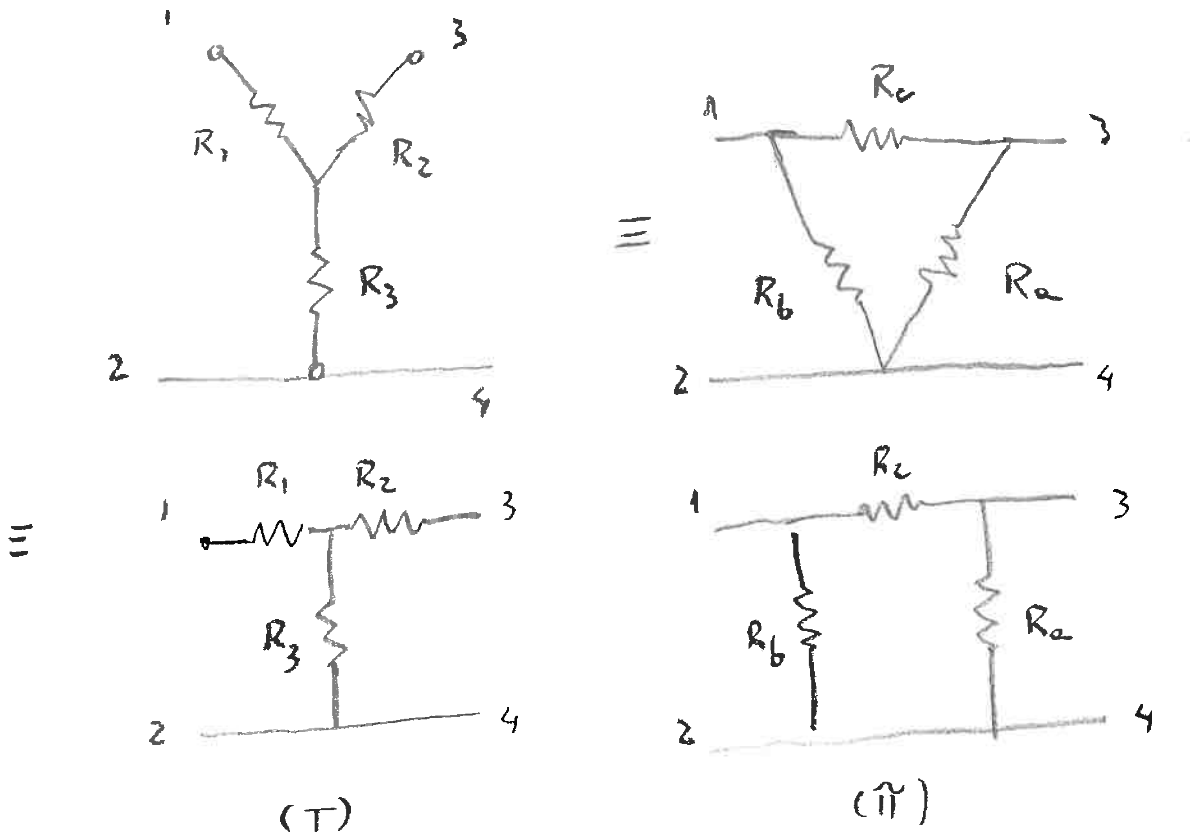
$$12 - 15 i_1 - 5 i_2 - 24 + 15 i_1 + 60 i_2 = 0$$

$$-12 + 55 i_2 = 0 \qquad i_2 = 0.22\,A.$$

$$i_1 = \frac{12 - 5 i_2}{15} = 0.73\,A$$

$$i_3 = 0.95\,A$$

Transformação Y - Δ ou T - Π



Vamos fazer estes circuitos serem equivalentes:

- Entre terminais 1 e 2 temos:

$\rightarrow$ circuito Y $\rightarrow$ $R_{eq_Y} = R_1 + R_3$

$\rightarrow$ circuito ∇ $\rightarrow$ $R_{eq_\nabla} = \dfrac{R_b(R_a+R_c)}{R_a+R_b+R_c}$

$$R_1 + R_3 = \frac{R_b(R_a+R_c)}{R_a+R_b+R_c}$$

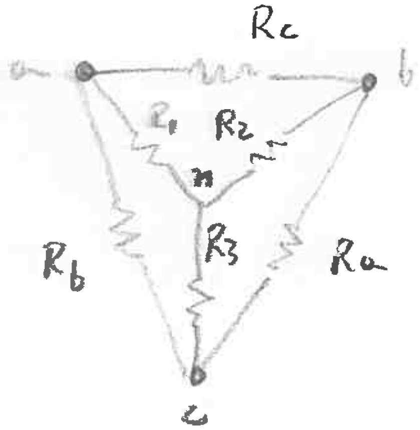
- Igualmente para terminais

1 e 3 $\quad R_1 + R_2 = \dfrac{(R_a+R_b)R_c}{R_a+R_b+R_c}$

3 e 4 $\quad R_2 + R_3 = \dfrac{(R_b + R_c) R_a}{R_a + R_b + R_c}$

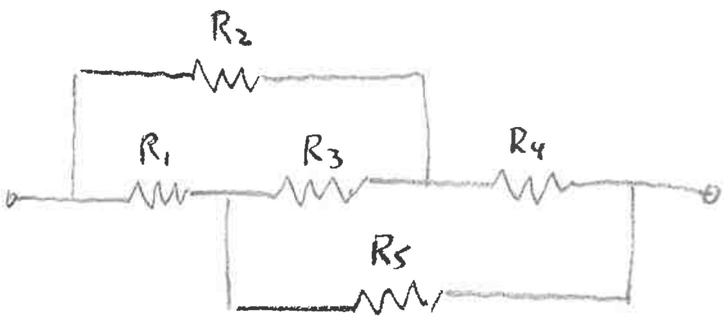
$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} \qquad R_2 = \frac{R_a R_c}{R_a + R_b + R_c} \qquad R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$

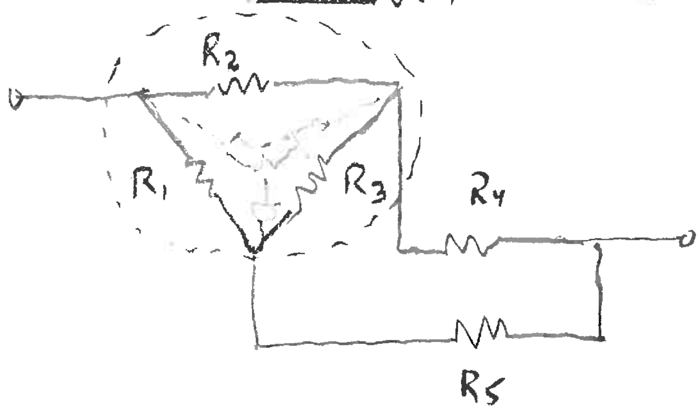
Cada resistor do circuito Y é o produto dos dois resistores adjacentes no circuito ∇ dividido pela soma dos três resistores ∇

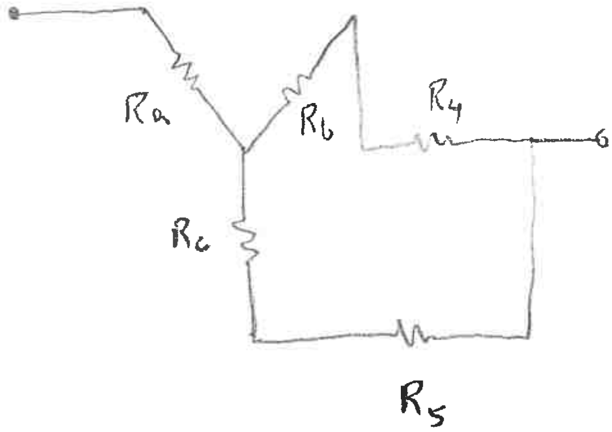


$$R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1} \qquad R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2} \qquad R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

Cada resistor no circuito Δ é a soma dos produtos dos resistores em Y dividido pelo resistor Y oposto



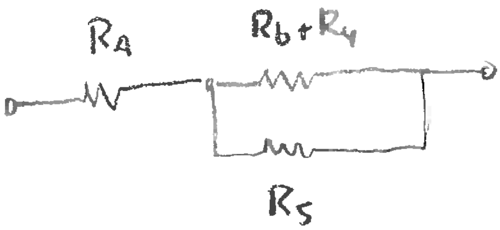


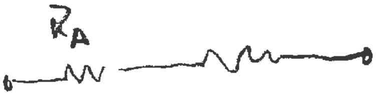


$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

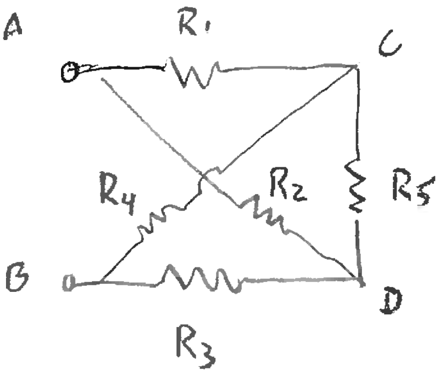
$$R_c = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$



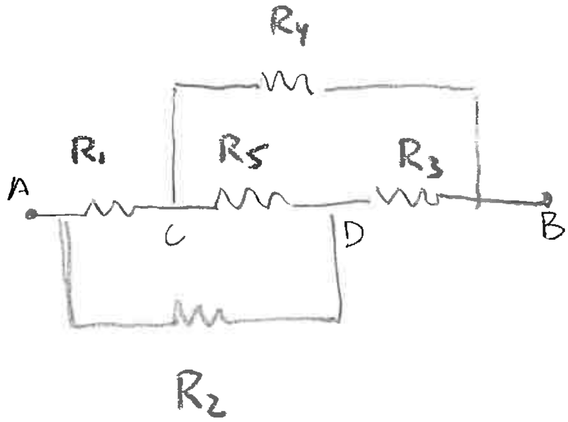


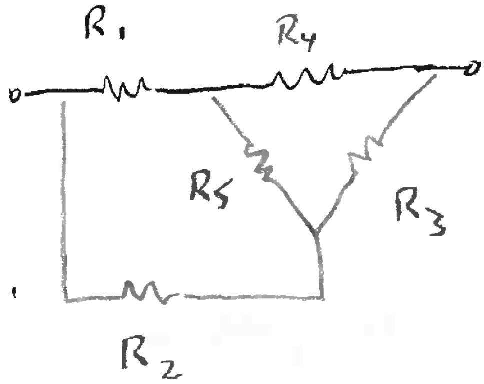
$$\frac{(R_b + R_4) R_5}{R_5 + R_b + R_4}$$

$R_A + (R_b + R_4) R_5$

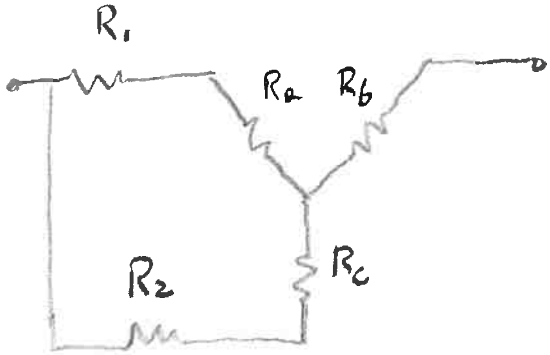


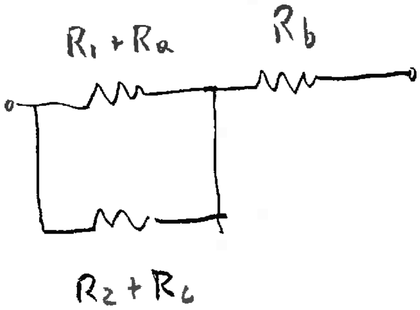
$\equiv$





$\equiv$



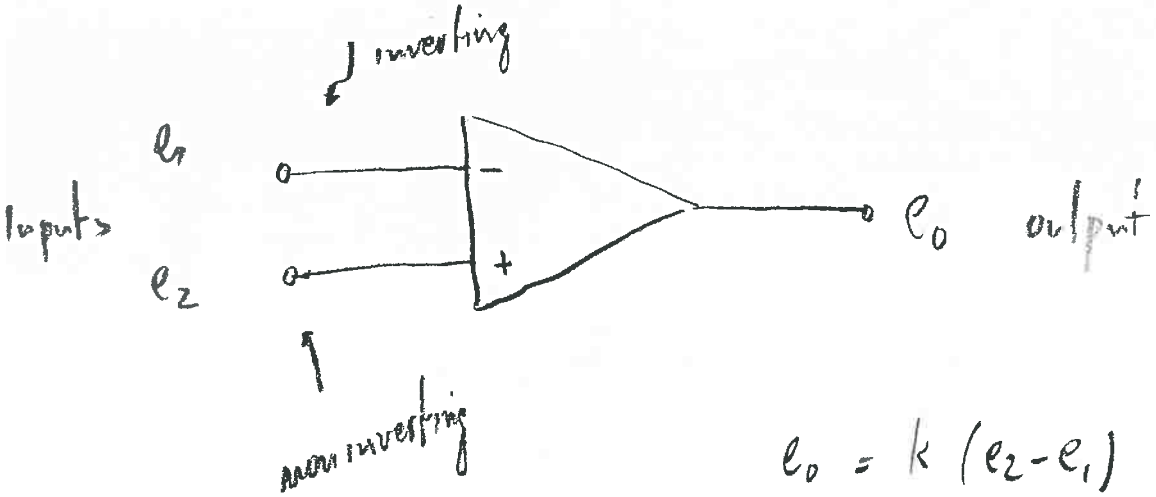


$\equiv$

$$\frac{(R_1 + R_a)(R_2 + R_c)}{R_1 + R_2 + R_a + R_c} + R_b$$

$$R_a = \frac{R_4 R_5}{R_3 + R_4 + R_5} \qquad R_b = \frac{R_3 R_4}{R_3 + R_4 + R_5} \qquad R_c = \frac{R_3 R_5}{R_3 + R_4 + R_5}$$

Modelling of Operational Amplifiers



$e_0 = k\,(e_2 - e_1)$

ganho diferencial

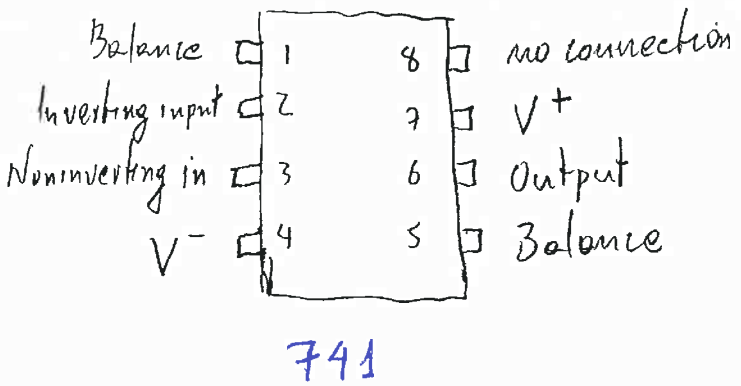
$k = 10^5 \sim 10^6 \quad f < 10\,Hz$

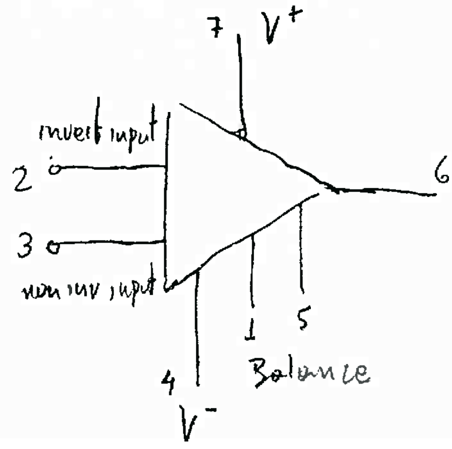
impedance $10^5 - 10^{13}\ \Omega$

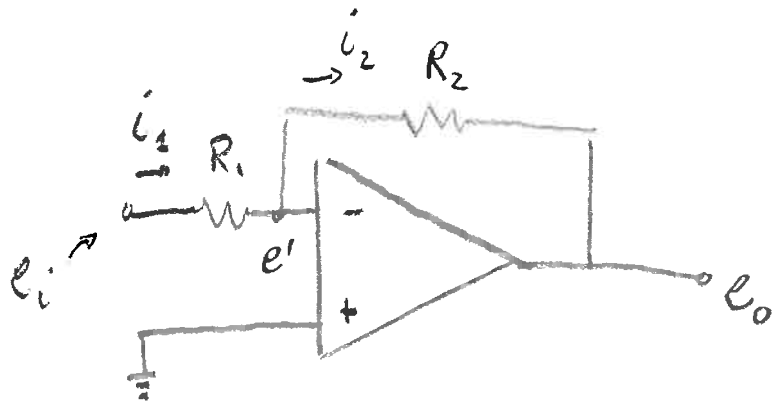
Amplificador ideal $\quad V_{cc}\ \ 5 - 24\,V$

- it amplifies the difference between e_1 and e_2
- no current flows between −/+ (infinite impedance)
- output impedance equals zero

Typical pin configuration







$$e_i - e' = R_1 i_1 \qquad e' - e_o = R_2 i_2$$

$$e_i = R_1 i_1 + R_2 i_2 + e_o$$

$$e_i - e_o = R_1 i_1 + R_2 i_2$$

$i_1 = i_2$ impedância de entrada infinita

$$e_i - e_o = (R_1 + R_2)\, i_1$$

$$i_1 = i_2$$

$$\frac{e_i - e'}{R_1} = \frac{e' - e_o}{R_2}$$

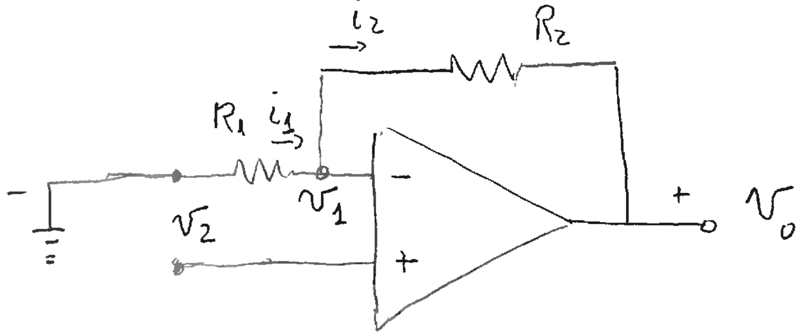
$e' = 0$ pois o "+" está aterrado

$$e_o = -\frac{R_2}{R_1} e_i$$

inversor

se $R_1 = R_2$ inverte sinal

Noninverting amplifier



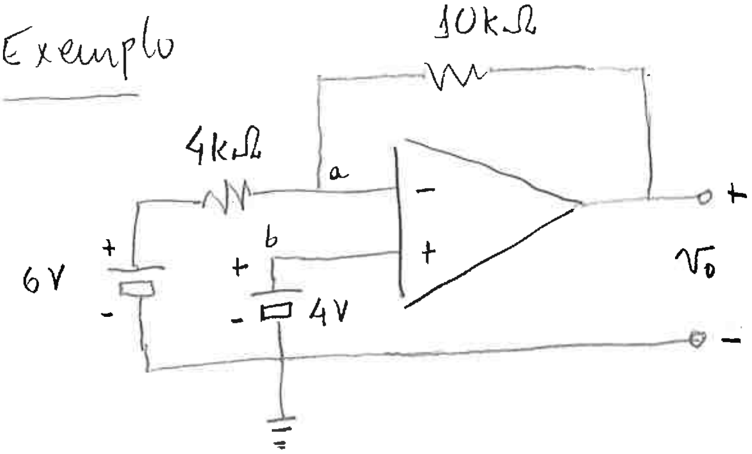
$$i_1 = i_2$$

$$\frac{0 - v_1}{R_1} = \frac{v_1 - v_0}{R_2}$$

$$v_0 = \left(1 + \frac{R_2}{R_1}\right) v_1$$

$$v_0 = \left(1 + R_2/R_1\right) v_2$$

Exemplo

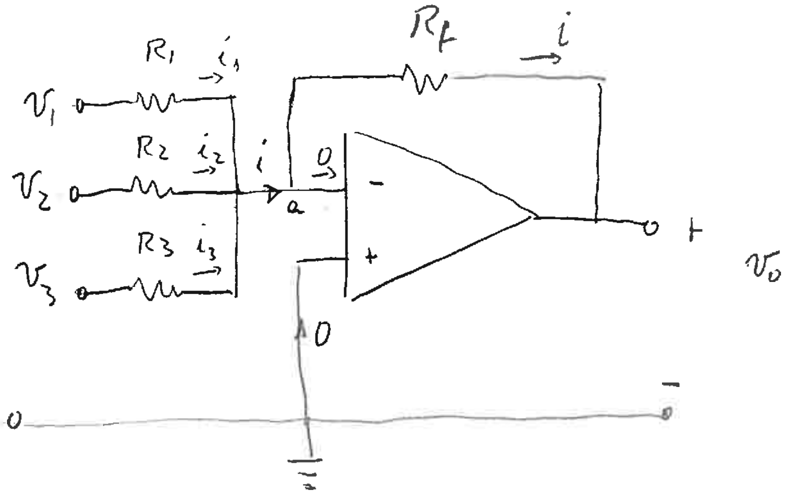


$$\frac{6 - v_a}{4K} = \frac{v_a - v_0}{10K}$$

Mas $v_a = v_b = 4$

$$v_0 = -1\,V$$

Summing amplifier



$i = i_1 + i_2 + i_3$

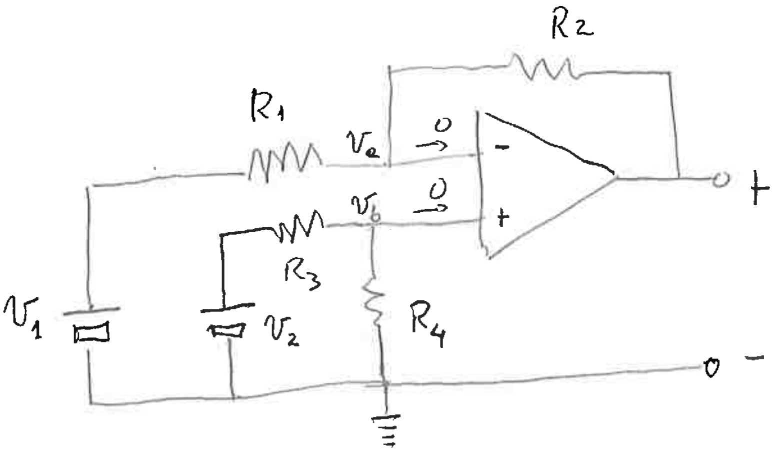
$$\frac{v_a - v_0}{R_f} = \frac{v_1 - v_a}{R_1} + \frac{v_2 - v_a}{R_2} + \frac{v_3 - v_a}{R_3}$$

$v_a = 0$

$$\Downarrow$$

$$v_0 = -\left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 \right)$$

Difference amplifier



node a

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_0}{R_2}$$

$$v_0 = \left(\frac{R_2}{R_1} + 1 \right) v_a - \frac{R_2}{R_1} v_1$$

node b

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

$$v_b = \frac{R_4}{R_3 + R_4} v_2$$

$v_a = v_b$

$$v_o = \frac{R_2(1 + R_3/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1$$

condição de amplificação diferencial: quando $v_1 = v_2$, $v_o = 0$

$$\Rightarrow \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

Se v_2 = sinal 1 + sinal 2

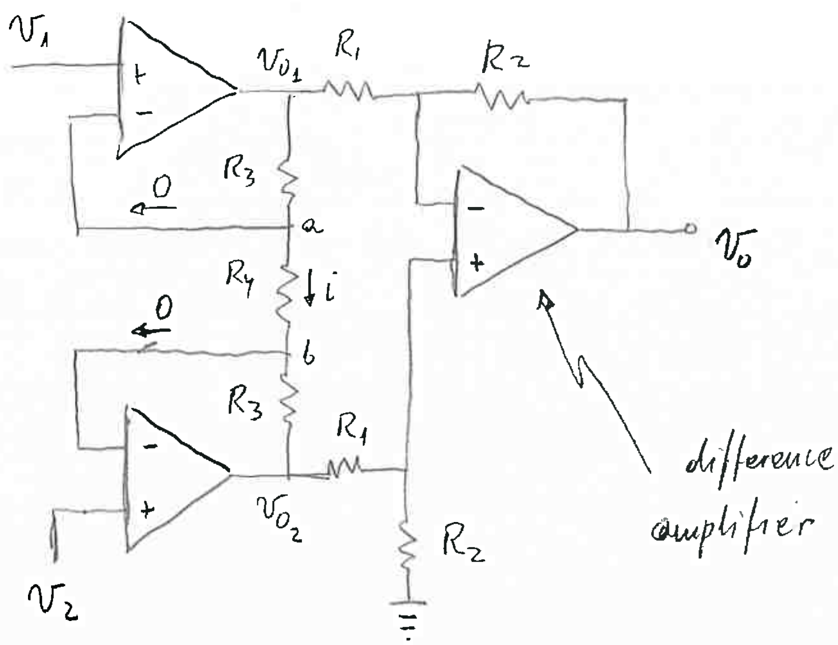
v_1 = sinal 2 = ruido

v_o = sinal 1

$$v_o = \frac{R_2}{R_1}(v_2 - v_1)$$

se $R_2 = R_1$ — $v_o \simeq v_2 - v_1$ "subtractor"

Instrumentation amplifier



$$v_o = \frac{R_2}{R_1}(v_{o_2} - v_{o_1})$$

$$v_{o_1} - v_{o_2} = (R_3 + R_4 + R_3)\, i$$

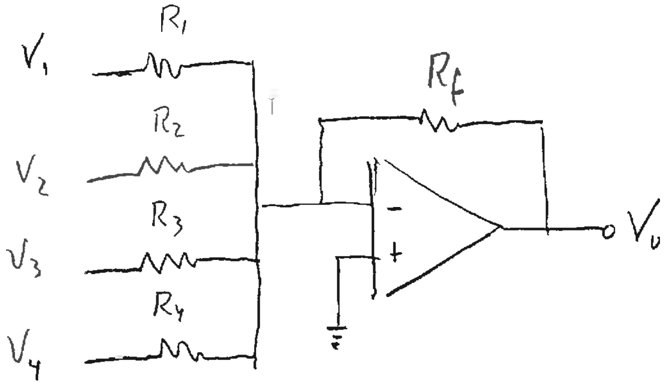
$$i = \frac{v_a - v_b}{R_4}$$

$v_a = v_{o_1}$, $v_b = v_{o_2}$

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

PSpice → design of amp op circuitry

Digital to Analog Converter, DAC



$$-V_0 = \frac{R_f}{R_1}V_1 + \frac{R_f}{R_2}V_2 + \frac{R_f}{R_3}V_3 + \frac{R_f}{R_4}V_4$$

Summing amplifier

$R_f = 10k\Omega$ $R_1 = 10k\Omega$ $R_2 = 20k\Omega$ $R_3 = 40k\Omega$ $R_4 = 80k\Omega$

$$-V_0 = V_1 + 0.5V_2 + 0.25V_3 + 0.125V_4$$

$[V_1\, V_2\, V_3\, V_4] = [0000]^{=0} \rightarrow -V_0 = 0$

$[0001]^{=1} \rightarrow -V_0 = 0.125V$

$[0010]^{=2} \rightarrow -V_0 = 0.250V$

Capacitores em Série e Paralelo

$$C = \epsilon \frac{A}{d} \qquad q = Cv$$

$$i = \frac{dq}{dt} \qquad i = C\frac{dv}{dt} \qquad v = \frac{1}{C}\int_0^t i\,dt + v(0)$$

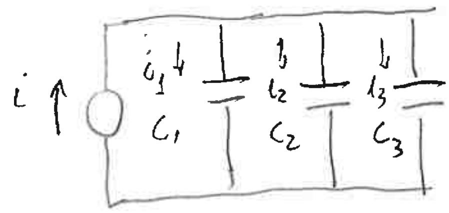
Potência

$$p = vi = vC\frac{dv}{dt}$$

Energia

$$w = \int_{-\infty}^{\omega} p\,dt = \int vC\,dv = \frac{1}{2}Cv^2 = \frac{q^2}{2C}$$

Capacitores em paralelo



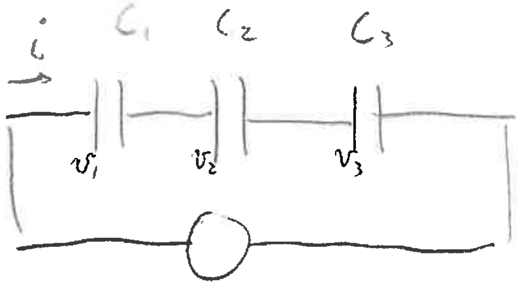
$$i = i_1 + i_2 + i_3$$

$$i = C_1\frac{dv}{dt} + C_2\frac{dv}{dt} + C_3\frac{dv}{dt}$$

$$i = C_{eq}\frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3$$

Capacitores em série



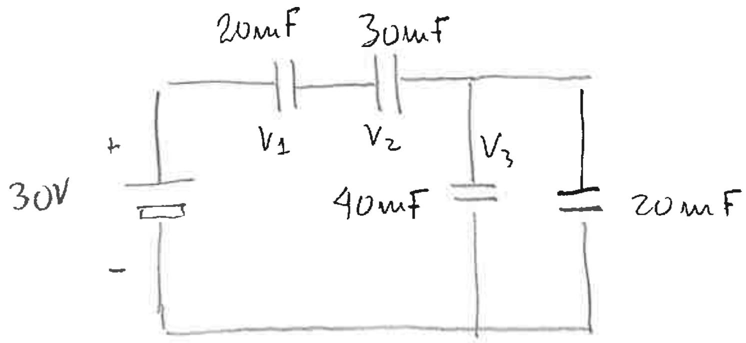
$$v = v_1 + v_2 + v_3$$

$$v = \frac{1}{C_1}\int i_1 dt + \frac{1}{C_2}\int i_2 dt + \frac{1}{C_3}\int i_3 dt$$

$$v = \left[\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right]\int i dt$$

$$C_{eq} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Calcular as voltagens no circuito abaixo



$$\frac{1}{C_{eq}} = \frac{1}{20} + \frac{1}{30} + \frac{1}{40+20} \text{ mF} = 10\text{ mF}$$

$$q = C_{eq} v = 10\text{ mF} \times 30\text{V} = 0.3\text{ C}$$

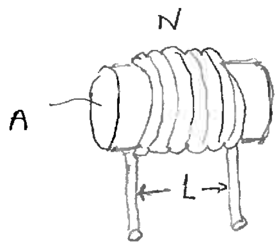
$$v_1 = \frac{q}{C_1} = \frac{0.3}{20 \times 10^{-3}} = 15\text{V}$$

$$v_2 = \frac{q}{C_2} = 10\text{V}$$

$$V_3 = 30 - V_1 - V_2 = 5V$$

ou $$V_3 = \frac{q}{60mF} = \frac{0.3}{60\times10^{-3}} = 5V$$

Indutores ← elemento passivo



$$L = \frac{N^2 \mu A}{L}$$

$$v = L\frac{di}{dt}$$

↓

indutância (Henry)

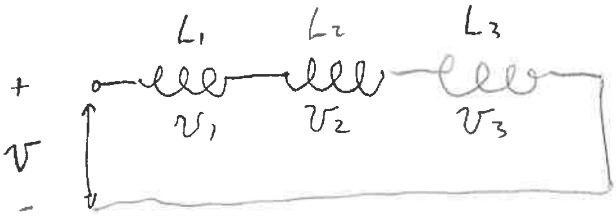
$$1 \frac{V.s}{A} = 1 H$$

$$L = \frac{v}{di/dt}$$ oposição à variação de corrente

$$\frac{di}{dt} = \frac{v}{L} \rightarrow i = \frac{1}{L}\int v\,dt$$

winding resistance and winding capacitance are ignored in the analysis here (significant for high frequencies)

Indutores em série

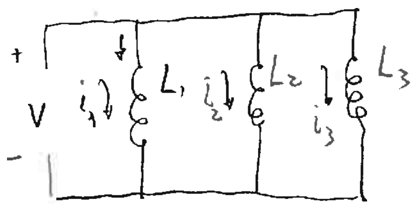


$$v = v_1 + v_2 + v_3$$

$$v = (L_1 + L_2 + L_3)\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3$$

Indutores em paralelo



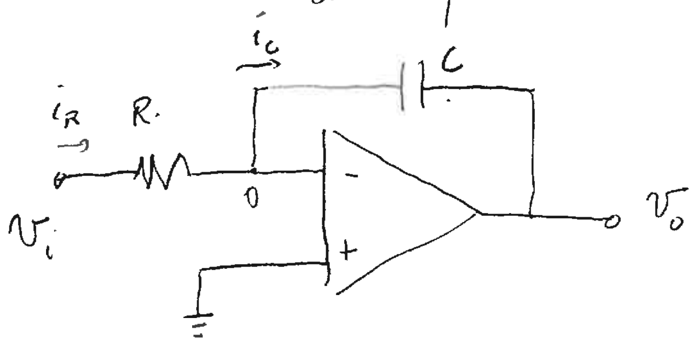
$$i = i_1 + i_2 + i_3$$

$$\int \frac{v}{L_{eq}}\,dt = \frac{1}{L_1}\int v\,dt + \frac{1}{L_2}\int v\,dt + \frac{1}{L_3}\int v\,dt$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

Amp op Integrator

- substituir a resistência de retroalimentação por um capacitor

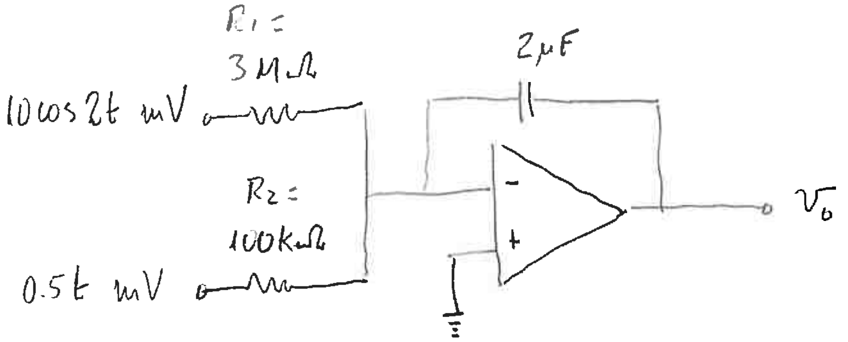


$$i_R = i_c$$

$$\frac{v_i}{R_i} = -C\frac{dv_o}{dt}$$

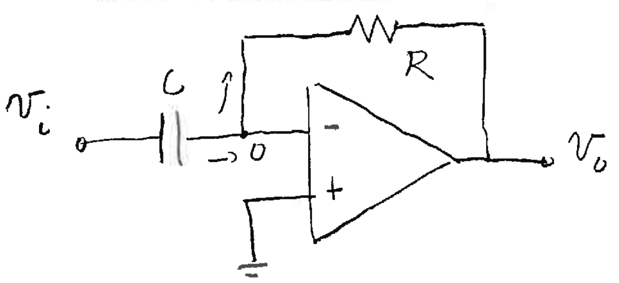
$$v_o = \frac{-1}{RC}\int v_i\,dt$$

Exemplo:



$$v_o = -\frac{1}{R_1 C}\int v_1\,dt - \frac{1}{R_2 C}\int v_2\,dt = -0.83\sin 2t - 1.25t^2 \text{ mV}$$

Amp Op Differentiator

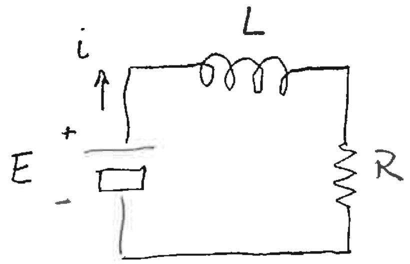


$$i_R = i_c$$

$$\frac{v_o}{R} = -C\frac{dv_i}{dt}$$

$$v_o = -RC\frac{dv_i}{dt}$$

Modelagem de Circuitos Elétricos



$$E - L\frac{di}{dt} - Ri = 0$$

$$L\frac{di}{dt} + Ri = E$$

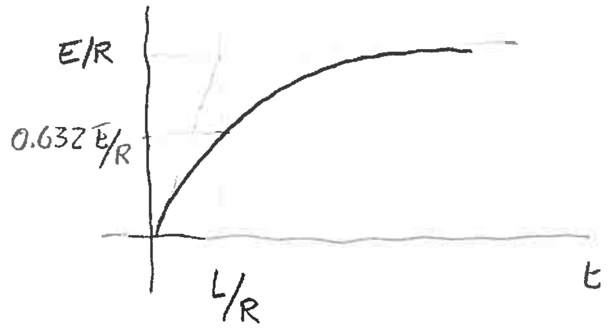
$$\mathcal{L}[\,Li + Ri\,] = \mathcal{L}[E]$$

$$L\,\mathcal{L}[i] + R\,\mathcal{L}[i] = \mathcal{L}[E]$$

$$L s I(s) + R I(s) = \frac{E}{s}$$

$$I(s) = \frac{E}{s(Ls+R)} = \frac{E}{R}\left[\frac{1}{s} - \frac{1}{s+R/L}\right]$$

$$i(t) = \text{inv } I(s) = \frac{E}{R}\left[1 - e^{-R/L\,t}\right]$$



Transformada de Laplace: resuminho

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty e^{-st} dt \, [f(t)] = \int_0^\infty f(t) e^{-st} dt - f(0)$$

$$\mathcal{L}^{-1}[F(s)] = f(t)$$

Exemplos:

→ $f(t) = Ae^{-\alpha t}$ $\quad \mathcal{L}[f(t)] = \int_0^\infty Ae^{-\alpha t} e^{-st} dt = A \int_0^\infty e^{-(\alpha+s)t} dt$

$$\mathcal{L}[f(t)] = A \left[\frac{e^{-(\alpha+s)t}}{-(\alpha+s)} \Big|_0^\infty \right] = A \left[0 - \frac{1}{-(\alpha+s)} \right] = \frac{A}{\alpha+s}$$

→ $f(t) = A$

$$\mathcal{L}[A] = \int_0^\infty A e^{-st} dt = A \frac{e^{-st}}{s} \Big|_0^\infty = \frac{A}{s}$$

→ $f(t) = At$

$$\mathcal{L}[At] = \int_0^\infty Ate^{-\alpha t} dt \qquad u = t \rightarrow du = dt$$

$$dv = e^{-st} dt \rightarrow v = e^{-st}/_{-s}$$

$$\mathcal{L}[At] = A \left(\frac{te^{-st}}{-s} \Big|_0^\infty - \int_0^\infty \frac{e^{-st}}{-s} dt \right)$$

$$\mathcal{L}[At] = \frac{A}{s^2}$$

→ $f(t) = A \operatorname{sen} wt$

$$e^{iwt} = \cos wt + i \operatorname{sen} wt$$

$$e^{-iwt} = \cos wt - i \operatorname{sen} wt$$

$$\sin wt = \frac{1}{2i} (e^{iwt} - e^{-iwt})$$

$$\mathcal{L}[A \operatorname{sen} \omega t] = \frac{A}{2i} \int_0^{\infty} e^{i\omega t} e^{-st}\, dt - \int_0^{\omega} e^{-i\omega t} e^{-st}\, dt$$

$$= \frac{A\omega}{s^2+\omega^2}$$

$$\rightarrow \quad \mathcal{L}[A \cos \omega t] = \frac{As}{s^2+\omega^2}$$

$$\mathcal{L}\left[\frac{d}{dt} f(t)\right] : \quad \underbrace{\int_0^{\infty} f(t) e^{-st}\, dt}_{F(s)} = f(t) \frac{e^{-st}}{-s}\Bigg|_0^{\omega} - \underbrace{\int_0^{\infty}\left[\frac{d}{dt} f(t)\right]\frac{e^{-st}}{-s}\, dt}_{\frac{1}{s}\mathcal{L}\left[\frac{d}{dt} f(t)\right]}$$

$$\frac{f(0)}{s}$$

$$\hookrightarrow \quad = s F(s) - f(0)$$

$$\mathcal{L}\left[\frac{d^2}{dt^2} f(t)\right] = s^2 F(s) - s f(0) - \dot{f}(0)$$

$$\mathcal{L}\left[\int f(t)\, dt\right] = \frac{F(s)}{s} + \frac{f^{-1}(0)}{s}$$

Soluçaõ de $\quad \ddot{x} + 2\dot{x} + 5x = 3 \quad x(0) = 0 \quad \dot{x}(0) = 0$

$$s^2 X(s) + 2s X(s) + 5X(s) = \frac{3}{s}$$

$$X(s) = \frac{3}{s(s^2+2s+5)} = \frac{3}{5s} - \frac{3}{10}\frac{2}{(s+1)^2+2^2} - \frac{3}{5}\frac{s+1}{(s+1)^2+2^2}$$

$$x(t) = \mathcal{L}^{-1}[X(s)] = \frac{3}{5} - \frac{3}{10} e^{-t} \sin 2t - \frac{3}{5} e^{-t} \cos 2t$$

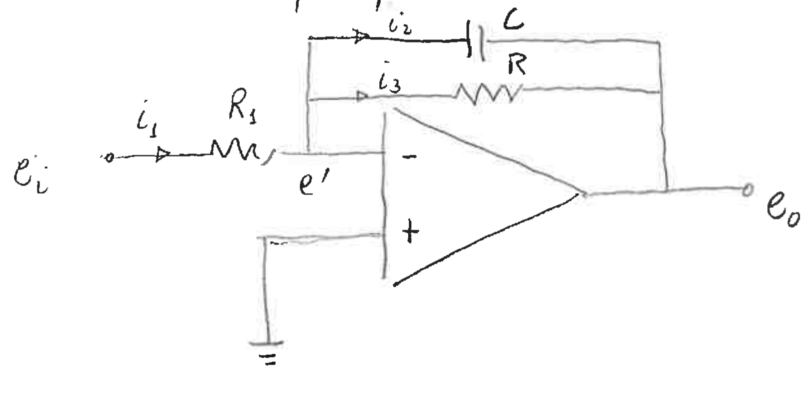
Função de transferência: razão entre a transformada de Laplace da resposta (saída) e da entrada, sendo todas as condições iniciais iguais a zero. e o sistema linear

dinâmica do sistema representada por equações algébricas

$$\text{Transfer function} = G(s) = \left.\frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]}\right|_{\text{zero initial conditions}}$$

$$G(s) = \frac{Y(s)}{X(s)}$$

→ Obtenha a função de transferência do amp-op abaixo.



$$i_1 = \frac{e_i - e'}{R_1} \qquad i_2 = \frac{d}{dt}(e' - e_0)\,C \qquad i_3 = \frac{e' - e_0}{R_2} \qquad e' = 0$$

∴

$$i_1 = i_2 + i_3 \rightarrow \frac{e_i}{R_1} = -C\frac{de_0}{dt} - \frac{e_0}{R_2}$$

$$\mathcal{L}\left[\frac{e_i}{R_1}\right] = \mathcal{L}\left[-C\frac{de_o}{dt}\right] - \mathcal{L}\left[\frac{e_o}{R_2}\right]$$

$$\frac{E_i(s)}{R_1} = -C\left[sE_o(s)\right] - \frac{E_o(s)}{R_2}$$

$$\frac{E_i(s)}{R_1} = -\frac{R_2Cs+1}{R_2}E_o(s)$$

$$G(s) = \frac{E_o(s)}{E_i(s)} = -\frac{R_2}{R_1}\frac{1}{R_2Cs+1}$$

Para entrada degrau $e_i(t) = E \quad t>0$

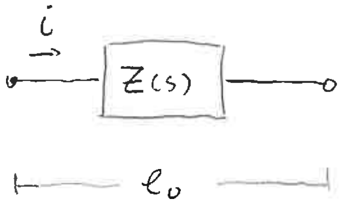
$$E_o(s) = -\frac{R_2}{R_1}\frac{1}{R_2Cs+1}\frac{E}{s}$$

$$E_o(s) = -\frac{R_2E}{R_1}\left[\frac{1}{s} - \frac{1}{s+1/R_2C}\right]$$

$$e_o(t) = -\frac{R_2E}{R_1}\left[1 - e^{-t/R_2C}\right]$$

Impedância Complexa

Usada para escrever diretamente a transformada de Laplace do sistema.



$$E(s) = Z(s)\,I(s)$$

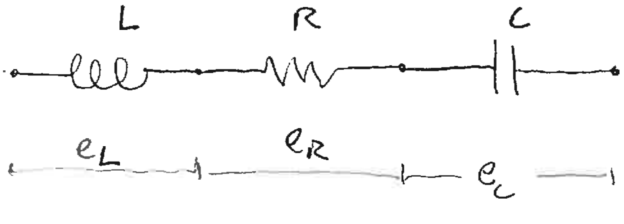
$R \to R$

$C \to 1/Cs$

$L \to Ls$

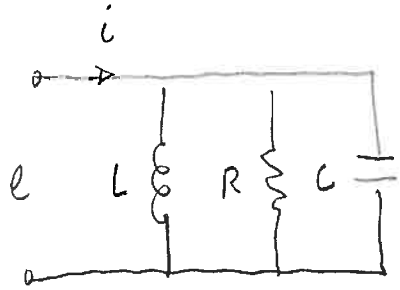
componente passivo → complex impedances

As impedâncias complexas podem ser combinadas em série e em paralelo, como as resistências, R.



Impedâncias em série: $E(s) = E_L(s) + E_R(s) + E_C(s)$

$$E(s) = \underbrace{\left(Ls + R + 1/Cs \right)}_{Z(s)} I(s)$$

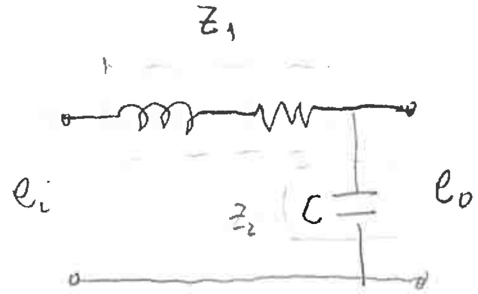


$$\frac{1}{Z(s)} = \frac{1}{Z_L} + \frac{1}{Z_R} + \frac{1}{Z_C}$$

$$\frac{1}{Z(s)} = \frac{1}{Ls} + \frac{1}{R} + \frac{1}{1/Cs}$$

$$E(s) = Z(s)\, I(s)$$

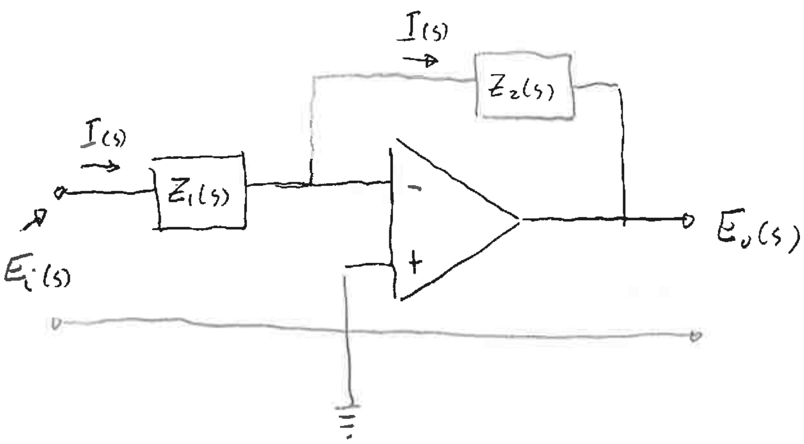
$$I(s) = \frac{E(s)}{Z(s)} = E(s)\left(\frac{1}{Ls} + \frac{1}{R} + Cs\right)$$



$$Z_1(s) = Ls + R$$

$$Z_2(s) = \frac{1}{Cs}$$

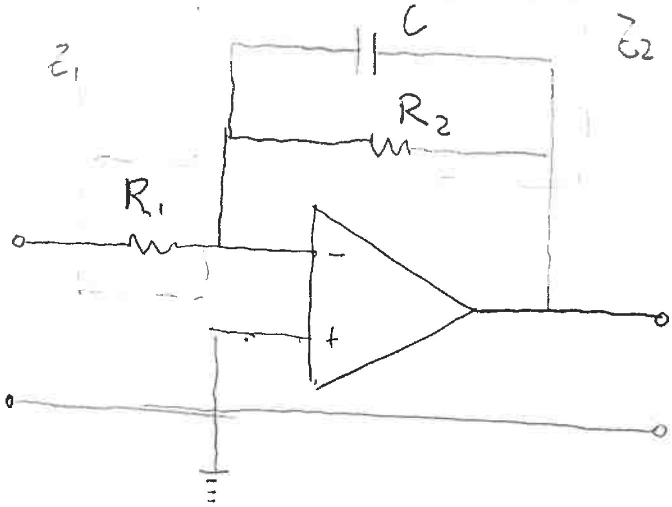
$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)} = \frac{1}{LCs^2 + RCs + 1}$$



$$E_i(s) = Z_1(s)\, I(s)$$

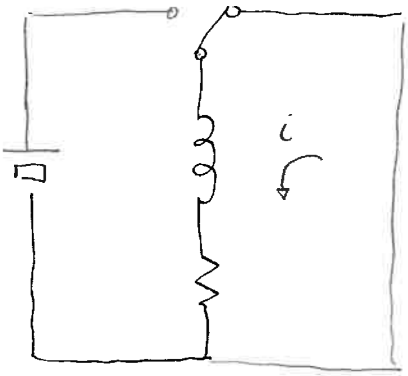
$$E_o(s) = -Z_2(s)\, I(s)$$

$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{-Z_2(s)}{Z_1(s)}$$



$$G(s) = \frac{-Z_2(s)}{Z_1(s)} = \frac{\left(\frac{1}{Cs} + \frac{1}{R_2}\right)^{-1}}{R_1}$$

$$G(s) = \frac{-R_2}{R_1} \frac{1}{R_2 C s + 1}$$



Para $t<0$, fonte está chaveada

Para $t=0$, chave fica como na figura

$t>0 \qquad L\frac{di}{dt} + Ri = 0 \qquad i(0) = \frac{E}{R}$

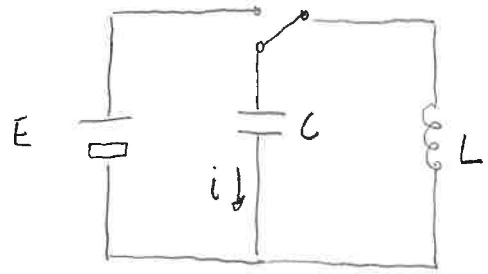
$$\mathcal{L}[L\dot{i}] + \mathcal{L}[Ri] = \mathcal{L}[0]$$

$$L\, s I(s) - L\, i(0) + R I(s) = 0$$

$$[Ls + R] I(s) = L \frac{E}{R}$$

$$I(s) = \frac{E}{R} \frac{L}{Ls+R}$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = \frac{E}{R} e^{-R/L\, t}$$



Em $t=0$ C está carregado com q_0

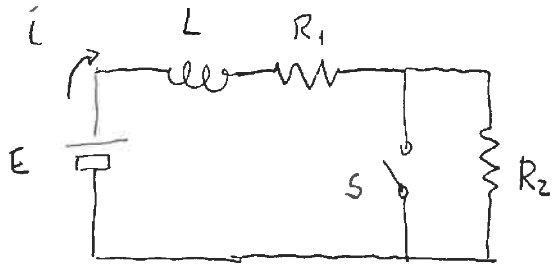
$C = 50\mu F$

Quanto vale L para que $\omega_n = 200Hz$

$$L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0 \qquad i = \frac{dq}{dt}$$

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0 \qquad q(0) = \dot{q}(0) = 0$$

$$\omega_n^2 = \frac{1}{CL} \longrightarrow L = 0.0127\,H$$



$$i(0) = \frac{E}{R_1+R_2}$$ i contínua $\rightarrow$ ∦

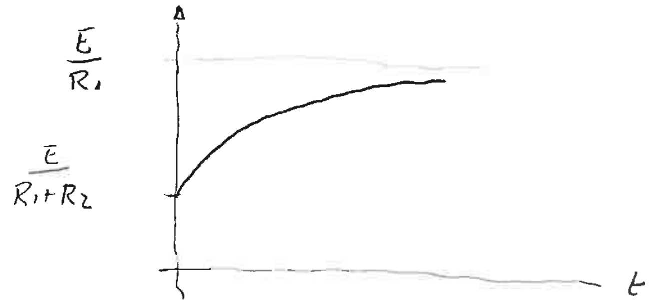
$$L\frac{di}{dt} + R_1 i = E$$

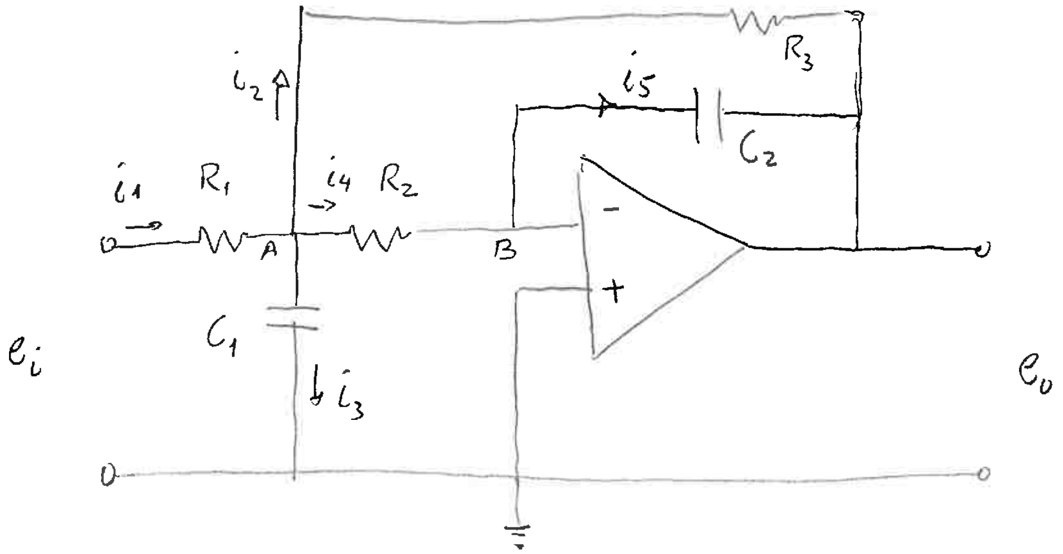
$$\mathcal{L}[L\dot{i}] + \mathcal{L}[R_1 i] = \mathcal{L}[E]$$

$$Ls\,I(s) + R_1 I(s) - L\,i(0) = \frac{E}{s}$$

$$I(s) = \frac{\frac{E}{s} + \frac{EL}{R_1+R_2}}{Ls+R_1} = \frac{E}{s(Ls+R_1)} + \frac{E}{R_1+R_2}\,\frac{L}{Ls+R_1} = \frac{E}{R_1}\left(\frac{1}{s} - \frac{R_2}{R_1+R_2}\,\frac{L}{Ls+R_1}\right.$$

$$i(t) = \mathcal{L}^{-1}[I(s)] = \frac{E}{R_1}\left[1 - \frac{R_2}{R_1+R_2} e^{-\frac{R_1}{L}t}\right]$$





$$i_1 = \frac{e_i - e_A}{R_1} \qquad i_2 = \frac{e_A - e_0}{R_3} \qquad i_3 = C_1 \frac{de_A}{dt}$$

$$i_4 = \frac{e_A}{R_2} \qquad i_5 = C_2 \frac{d}{dt}(0 - e_0)$$

$$i_1 = i_2 + i_3 + i_4$$

$$\frac{e_i - e_A}{R_1} = \frac{e_A - e_0}{R_3} + C_1 \frac{de_A}{dt} + \frac{e_A}{R_2}$$

node B $\quad i_4 = i_5$

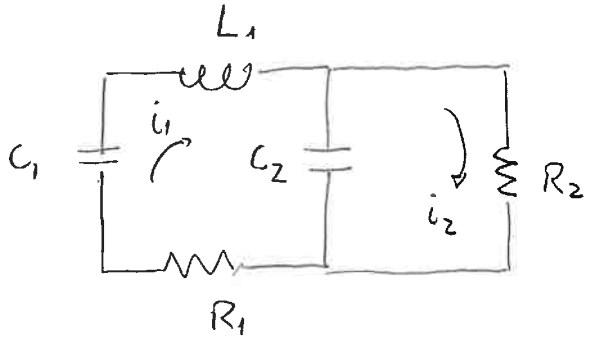
$$\frac{e_A}{R_2} = C_2 \times \frac{-de_o}{dt}$$

$$C_1 \frac{de_A}{dt} + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) e_A = \frac{e_i}{R_1} + \frac{e_o}{R_3}$$

$$C_1\left(-R_2 C_2 \frac{d^2 e_o}{dt^2}\right) + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)(-R_2 C_2) \frac{de_o}{dt} = \frac{e_i}{R_1} + \frac{e_o}{R_3}$$

$$-C_1 C_2 R_2 s^2 E_o(s) + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)(-R_2 C_2)\, s\, E_o(s) - \frac{1}{R_3} E_o(s) = \frac{E_i(s)}{R_1}$$

$$G(s) = \frac{E_o(s)}{E_i(s)} = \frac{1}{R_1 C_1 R_2 C_2 s^2 + [R_2 C_2 + R_1 C_2 + (R_1/R_3) R_2 C_2]\, s + R_1/R_3}$$



$$L_1 \frac{di_1}{dt} + \frac{1}{C_2}\int (i_1 - i_2)\, dt + R_1 i_1 + \frac{1}{C_1}\int i_1\, dt = 0$$

$$R_2 i_2 + \frac{1}{C_2}\int (i_2 - i_1)\, dt = 0$$

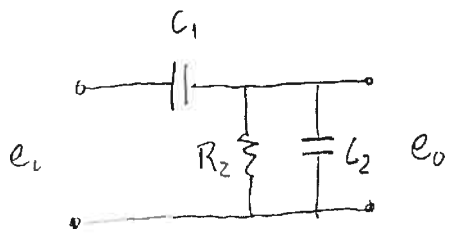
$$i_1 = \dot{q}_1 \qquad i_2 = \dot{q}_2$$

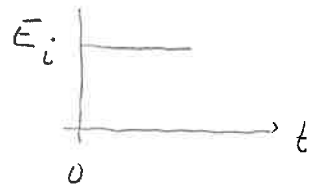
$$L_1 \ddot{q}_1 + R_1 \dot{q}_1 + \frac{1}{C} q_1 + \frac{1}{C_2} (q_1 - q_2) = 0$$

$$R_2 \dot{q}_2 + \frac{1}{C_2} (q_2 - q_1) = 0$$

Problemas

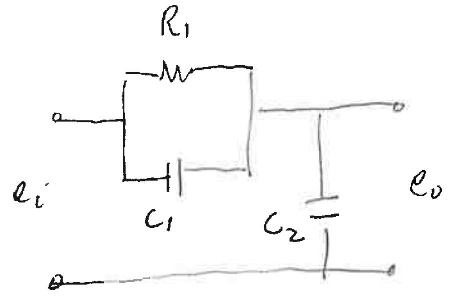
1. Obtenha $e_o(t)$ para o circuito $(e_i(0) = e_o(0) = 0)$





Resp. $e_o(t) = \dfrac{C_1 E_i}{C_1 + C_2} \, e^{\frac{-t}{R_2(C_1+C_2)}}$

2.



$$e_i(t) = 10V \quad 0 \le t \le 5$$
$$= 0 \quad t > 5$$

$$E_i(s) = \frac{10}{s}\left(1 - e^{-5s}\right)$$

$$E_o(s) = \frac{s+1}{2.5s+1} \, \frac{10}{s} \, (1 - e^{-5s}$$

$$e_o(t) = (10 - 6e^{-0.4t}) - \left[10 - 6e^{-0.4(t-5)}\right](t-5)$$

3.