PSI3211 – CIRCUITOS ELÉTRICOS I

Solução da Lista 7: Redes de 2ª Ordem

1 –

$i_1(0_+) = i_1(0_-) = 2$ A	$v_1(0_+) = 10 - 4 = 6$ V
$i_1(\infty) = 10/R_2 = 10$ A	$v_1(\infty) = 0$ V
$i_2(0_+) = v_1(0_+)/R_1 = 3$ A	$v_2(0_+) = v_1(0_+) = 6$ V
$i_2(\infty) = 0$ A	$v_2(\infty) = v_1(\infty) = 0$ V
$i_3(0_+) = 4\text{V}/R_2 = 4$ A	$v_3(0_+) = 4$ V $= v_4(0_+)$
$i_3(\infty) = i_1(\infty) = 10$ A	$v_3(\infty) = 10$ V
$i_4(0_+) = i_1 + i_2 - i_3 = 1$ A	$v_4(0_+) = 4$ V
$i_4(\infty) = 0$ A	$v_4(\infty) = 10$ V

2 – a) $\alpha = \dfrac{G}{2C} = \dfrac{1}{2RC} = 200\,\text{s}^{-1}$ $\qquad \omega_0 = \dfrac{1}{\sqrt{LC}} = 1000$ rd/s

$$\alpha < \omega_0 \;\rightarrow\; \omega_d = \sqrt{\omega_0^2 - \alpha^2} = 979{,}80 \text{ rd/s}$$

FCP: $s_{1,2} = (-200 \pm j\,979{,}80)$ rd/s

b) Equação dual da Eq. (6.20), p. 178 do livro-texto (Vol. 1):

$$v(t) = \sqrt{v_0^2 + \left(\frac{\alpha}{\omega_d}\,v_0 + \frac{1}{C\,\omega_d}\,i_0\right)^2}\; e^{-\alpha t}\,.\cos\left(\omega_d\,t + \psi\right)$$

$$\psi = \text{arctg}\left(\frac{i_0}{C\,\omega_d\,v_0} + \frac{\alpha}{\omega_d}\right)$$

Para $v_0 = 0$ e $i_0 = -12{,}25$ mA

Temos: $v(t) = 100e^{-200t}\,\text{sen}(979{,}80t), \quad t \geq 0 \quad (\text{V,s})$

3 – a) $i_L(0_+) = i_L(0_-) = 29$ mA $\qquad$ b) $v_L(0_+) = v_c(0_+) = v_0 = 50$ V

mas $\quad v_L(0_+) = L\left.\dfrac{di_L}{dt}\right|_{t=0_+} \;\rightarrow\; \left.\dfrac{di_L}{dt}\right|_{t=0_+} = \dfrac{50}{L} = 2000$ A/s

c) $\alpha = \dfrac{G}{2R} = \dfrac{1}{2RC} = 40000\ \text{s}^{-1}$

$\omega_0 = \dfrac{1}{\sqrt{LC}} = 40000$ rd/s $\quad\rightarrow\quad \alpha = \omega_0 \quad$ amortecimento crítico

$$s_1 = s_2 = -\alpha \quad \rightarrow \quad i_L(t) = \underbrace{A_1 e^{-\alpha t} + A_2 t\, e^{-\alpha t}}_{\text{transitório}} + \underbrace{I}_{\text{permanente}}$$

$$i_L(0_+) = A_1 + I = 29 \text{ mA} \quad \rightarrow \quad A_1 = 29 - 24 = 5 \text{ mA}$$

$$\left.\frac{d\,i_L}{d\,t}\right|_{t=0_+} = A_1.(-\alpha) + A_2 = 2000 \text{ A/s} \rightarrow A_2 = 2200 \text{ A/s}$$

$$i_L(t) = (5e^{-40000t} + 2{,}2.10^6\, t\, e^{-40000t} + 24), \quad t \geq 0 \quad (\text{mA, s})$$

4 – a) $\omega_0 = \dfrac{1}{\sqrt{LC}} = 1000$ rd/s $\qquad Q_0 = \dfrac{\omega_0 L}{R} = \dfrac{1000}{20} = 50$

b) $Z(j\omega) = R + j\omega L - \dfrac{j}{\omega C} = 20 + j\,(\omega - 1 \times 10^6/\omega)$

Máximo de corrente $\rightarrow$ corrente na frequência de ressonância
$\omega = \omega_0 = 1000$ rd/s.

$Z(j\omega_0) = R = 20 \qquad \hat{I} = \dfrac{\hat{E}}{20} \qquad \left|\hat{I}\right| = \dfrac{\left|\hat{E}\right|}{20} = \dfrac{10}{20} = 0{,}5$ A

c) $\left|\hat{V}_c\right| = \left|\dfrac{1}{j\omega}\right| \left|\hat{I}\right|$

Para $\omega = \omega_0$:

$\rightarrow \left|\hat{V}_c\right| = 1000 \left|\hat{I}\right| = 1000.0{,}5 = 500$ V

Tensão eficaz $V_{cef} = \dfrac{\left|\hat{V}_c\right|}{\sqrt{2}} = 353{,}55$ Vef.

5 – a) $B = \dfrac{\omega_0}{Q_0} \rightarrow Q_0 = \dfrac{\omega_0}{B} = \dfrac{1000\pi}{20\pi} = 50$

$Q_0 = \dfrac{R}{\omega_0 L} = \omega_0 RC \rightarrow R = \dfrac{Q_0}{\omega_0 C}$

$R = \dfrac{50}{1000\pi.500.10^{-6}} = \dfrac{100}{\pi} = 31{,}831$ kΩ

(Estão sendo usadas unidades do sistema AF)

$L = \dfrac{1}{\omega_0^2 C} = \dfrac{1}{10^6 \pi^2 .\, 50.10^{-6}} = 202{,}64\,.10^{-6}$ A $\quad$ **ou** $\quad L = 202{,}64\ \mu$H

b) $$\frac{\hat{V}_C}{\hat{I}_s} = Z(j\omega) = \frac{1}{\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)} = \frac{\omega R L}{\omega L + j\left(\omega^2 R L C - R\right)} =$$

$$= \frac{6{,}45\omega}{202{,}64.10^{-6}\,\omega + j\left(3{,}225.10^{-9}\,\omega^2 - 31{,}831.10^3\right)} \quad \textbf{(SI)}$$

c) $f = 500\ \text{kHz} \quad \rightarrow \quad \omega = 2\pi\, 500 = 1000\pi = \omega_0$

Na ressonância, a impedância do circuito é igual a R. Portanto, a tensão eficaz nos terminais do circuito será

$$V = 31{,}831 \,.\, 1 = 31{,}831 \ \ (\text{Vef})$$

d) A impedância na frequência ω é

$$Z(j\omega) = \frac{R}{1 + jQ_0\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Fazendo $\omega_0 = 1000\pi$, $\quad \omega = 2.\pi\, 400 = 800\pi$ resulta

$$Z(j\omega) = \frac{31{,}831}{1 + j\,50\left(\frac{4}{5} - \frac{5}{4}\right)} = 1{,}4133 \angle 87{,}46^\circ \ \ (\text{k}\Omega)$$

de modo que a tensão eficaz será

$$V = 1{,}4133 \,.\, 1 = 1{,}4133 \ \ (\text{Vef})$$