PSI3211 – CIRCUITOS ELÉTRICOS I

Solução da Lista 6: Redes de 1ª Ordem e Excitação Impulsiva

Redes de 1ª Ordem

1 – Temos que:
$$\begin{cases} -4e^{-4t} + 4e^{-4t} + 20 = f_1(t) \\ \\ -4e^{-4t} + 10e^{-10t} + 4e^{-4t} - 4e^{-10t} = f_2(t) \end{cases}$$

$\rightarrow \quad f_1(t) = 20, \; t > 0, \qquad f_2(t) = 6e^{-10t}, \; t > 0.$

2 – $i(t) = i_0 e^{-t/\tau} \;\rightarrow\; i_0 = 10 \text{ A} \qquad \tau = 0{,}2 \text{ s} = 200 \text{ ms}$

$v(t) = Ri_0 e^{-t/\tau} \;\rightarrow\; R = \dfrac{400}{10} = 40\Omega \qquad \tau = \dfrac{L}{R} \;\rightarrow\; L = 40 \times 0{,}2 = 8 \text{ H}$

Energia inicial: $\dfrac{1}{2} L i_0^2 = \dfrac{1}{2} \times 8 \times 100 = 400 \text{ J}$

Em $t = 50 \text{ ms} \;\rightarrow\; i = 7{,}788 \text{ A} \;\rightarrow\; W_L = \dfrac{1}{2} L i^2 = 242{,}61 \text{ J}$

Energia dissipada: $400 - 242{,}61 = 157{,}39 \text{ J}$

3 – a) $i_L(0_+) = i_L(0_-) = 0$ (admitido)

$i_L(\infty) = I$

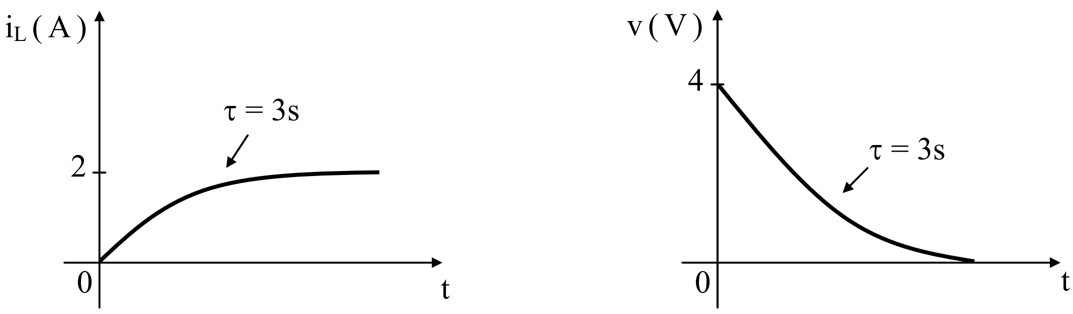
$\rightarrow i_L(t) = I(1 - e^{-t/\tau}), \; t \geq 0, \qquad \tau = L/R$

$v(0_+) = RI$

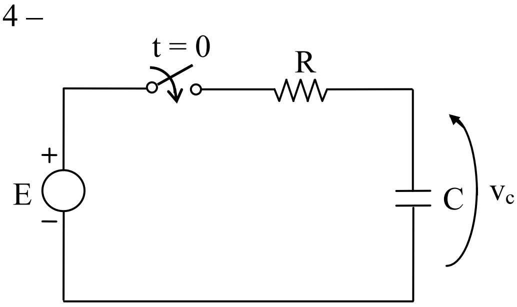
$v(\infty) = 0$

$\rightarrow v(t) = RI\, e^{-t/\tau}, \qquad t > 0, \qquad \tau = L/R$

b)



4 –



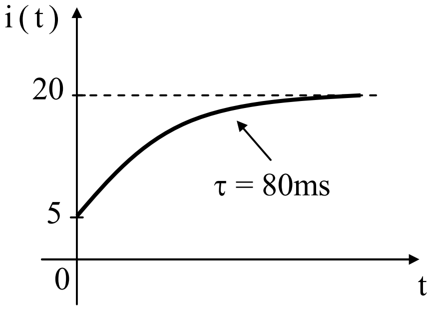
$$\begin{cases} v_c(0_+) = v_c(0_-) = 0 \;\; (\text{admitido}) \\ v_c(\infty) = E \end{cases}$$

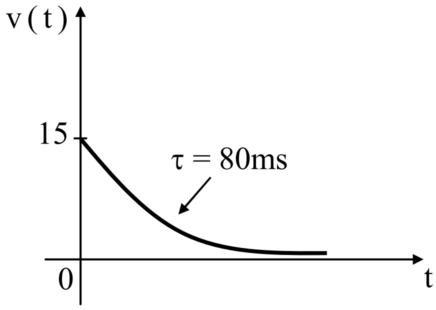
$\longrightarrow \; v_c(t) = E(1 - e^{-t/\tau}), \quad t \geq 0,$

$\tau = RC$

5 –
$$\begin{cases} i(0_+) = i(0_-) = \dfrac{20}{4} = 5\text{A} \\ i(\infty) = \dfrac{20}{1} = 20\text{A} \end{cases} \qquad \begin{cases} v(0_-) = 0 \\ v(0_+) = 20 - (5 \text{ x } 1) = 15\text{V} \\ v(\infty) = 0 \end{cases}$$

$$\tau = \frac{L}{R} = \frac{80 \text{ x } 10^{-3}}{1} = 80 \text{ ms}$$

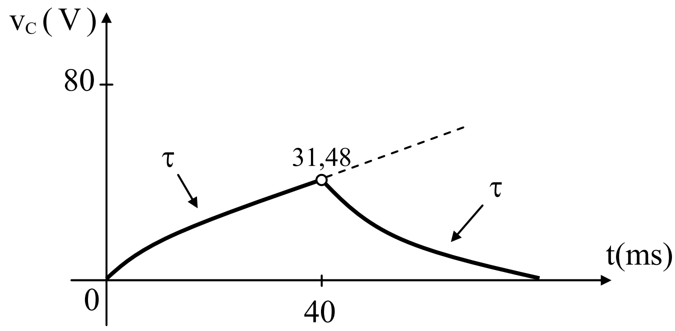


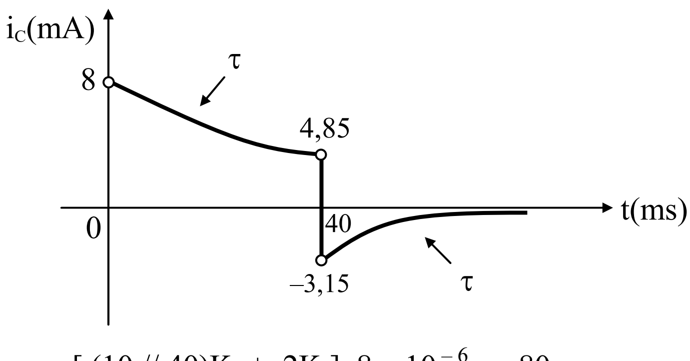


$i(t) = (20 - 15e^{-12,5\,t})$ (A, s)
$t \geq 0$

$v(t) = 15\,e^{-12,5\,t}$ (V, s)
$t > 0$

6 –





$$\tau = [\,(10 \,//\, 40)\text{K} + 2\text{K}\,].\ 8 \text{ x } 10^{-6} = 80 \text{ ms}$$

– Para $0 < t < 40$ ms
$v_c(0_+) = v_c(0_-) = 0$
$v_c(\infty) = (100 \text{ x } 40)/\,50 = 80$ V
$v_c(t) = -80e^{-t/80} + 80$ (V, ms)

$$i_c(0_+) = \frac{v_1(0_+)}{2\,K} = \frac{100(40\,K\,//\,2\,K)}{10\,K + (40\,K\,//\,2\,K)} \text{ x } \frac{1}{2\,K} = 8 \text{ mA}$$

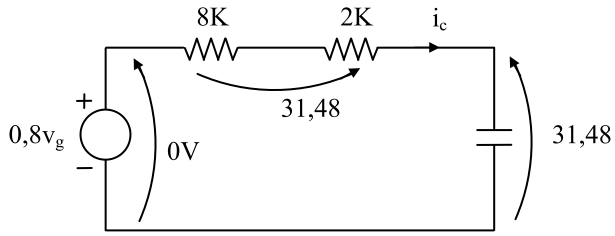
$i_c(\infty) = 0 \rightarrow i_c(t) = 8e^{-t/80}$ (mA, ms)

– Para $t = 40$ ms

$v_c(40_-) = 31{,}48$ V $\qquad$ $i_c(40_-) = 4{,}85$ mA

$v_c(40_+) = v_c(40_-)$ $\qquad$ $i_c(40_+) = -31{,}48/10$ K $= -3{,}15$ mA

(Thévenin equivalente): $t = 40_+$



– Para $t > 40$ ms:

$v_c(t) = 31{,}48e^{-(t-40)/80}$ (V, ms)

$i_c(t) = -3{,}15e^{-(t-40)/80}$ (mA, ms)

Excitação Impulsiva

1 – a) $\dot{f}(t) = -e^{-t}H(t) + e^{-t}\delta(t) = -e^{-t}H(t) + \delta(t)$

b) $\dot{f}(t) = \delta(t) + 2e^{-2t}H(t) - e^{-2t}\delta(t) = 2e^{-2t}H(t)$

2 – a) $I = \int_{-2}^{4} 4\delta(t)dt + \int_{-2}^{4} 48\delta(t-2)dt \rightarrow I = 52$

b) $I = \int_{-3}^{4} t^2\delta(t)dt + \int_{-3}^{4} t^2\delta(t+2{,}5)dt + \int_{-3}^{4} t^2\delta(t-5)dt$

$I = 0 + 6{,}25 + 0 = 6{,}25$

c) A integral $\int_{-1/2}^{1/2} (t^2+1)\cos(t)\left[\delta(t) - \delta(t-\pi/3)\right]dt$ pode ser calculada da seguinte forma:

$$\int_{-1/2}^{1/2} (t^2+1)\cos(t)\left[\delta(t)-\delta(t-\pi/3)\right]dt = \int_{-1/2}^{1/2} (t^2+1)\cos(t)\delta(t)dt - \int_{-1/2}^{1/2} (t^2+1)\cos(t)\delta(t-\pi/3)dt$$

Calculando cada uma das integrais da expressão anterior, temos:

$$\int_{-1/2}^{1/2} (t^2+1)\cos(t)\delta(t)dt = \underbrace{(t^2+1)\cos(t)\Big|_{t=0}}_{=1} \underbrace{\int_{-1/2}^{1/2} \delta(t)dt}_{=1} = 1$$

$\int_{-1/2}^{1/2} (t^2+1)\cos(t)\delta\left(t-\pi/3\right)dt = 0$. [Note que $t = \pi/3$ está fora do intervalo de integração]

Assim, a integral $\int_{-1/2}^{1/2} (t^2+1)\cos(t)\left[\delta(t)-\delta\left(t-\pi/3\right)\right]dt$ é igual a 1.

3 – $v(t) = -L\dfrac{di}{dt}$ (convenção do gerador)

$\Rightarrow v(t) = -12\,\delta(t) + 6\,\delta(t-2) \quad (V, s)$

4 – A tensão entre os terminais do capacitor é $e_s(t) = \cos(1000\,t)\,H(t)$ e a corrente que o atravessa é:

$$i(t) = C\frac{d}{dt}e_s(t) = 10^{-4}(-1000)\,\mathrm{sen}(1000t)\,H(t) + 10^{-4}\cos(1000t)\,\delta(t) =$$

$$= -0{,}1\,\mathrm{sen}(1000\,t)\,H(t) + 10^{-4}\,\delta(t)$$

Assim, a potência instantânea recebida pelo capacitor é

$$p(t) = e_s(t)\,.\,i(t) = -0{,}1\,\mathrm{sen}(1000\,t)\cos(1000\,t)\,H(t) + 10^{-4}\cos(1000\,t)\,H(t)\,\delta(t)$$

$$= -\frac{0{,}1}{2}\,\mathrm{sen}(2000t)\,H(t) + 10^{-4}\,\delta(t)$$

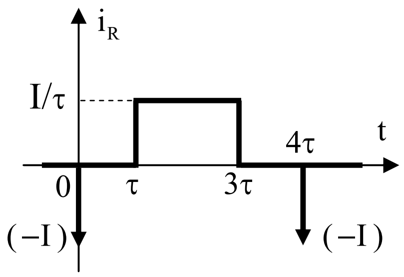
Portanto,

$$p(t) = -0{,}05\,\mathrm{sen}(2000\,t)\,H(t) + 10^{-4}\,\delta(t)$$

5 – A corrente i_R no resistor é dada por

$$i_R(t) = \frac{v_R(t)}{R} = \frac{2v_L(t)}{2} = v_L(t) \Rightarrow i_R(t) = L\frac{di_L(t)}{dt} = \frac{di_L(t)}{dt} = \frac{di_s(t)}{dt}$$

Basta calcular a derivada de $i_s(t)$, que está representada no gráfico a seguir:



6 –

$$i(0_+) = \cancelto{0}{i(0_-)} + \frac{\psi}{L_{eq}} = \frac{\psi}{L_1 + L_2}$$

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{(L_1 + L_2)(R_1 + R_2)}{R_1 R_2}$$

$$i(t) = \frac{\psi}{(L_1 + L_2)} e^{-t/\tau} \qquad t > 0$$