MAP 2320 – MÉTODOS NUMÉRICOS EM EQUAÇÕES DIFERENCIAIS II

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MAP2320

Equações Diferenciais Parciais: Uma Introdução (Versão Preliminar)

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2.1 Teorema de Fourier

Teorema 2.1 (Fourier). *Seja L um número real maior que zero. Para toda função $f : [-L, L] \to \mathbb{R}$ contínua por partes tal que a sua derivada f' também seja contínua por partes, a série de Fourier de f*

$$S_f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi t}{L} + \sum_{n=1}^{\infty} b_n \operatorname{sen} \frac{n\pi t}{L},$$

em que

$$\begin{aligned} a_n &= \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi t}{L} dt \quad \text{para } n = 0, 1, 2, \ldots \\ b_n &= \frac{1}{L} \int_{-L}^{L} f(t) \operatorname{sen} \frac{n\pi t}{L} dt, \quad \text{para } n = 1, 2, \ldots \end{aligned}$$

converge para f nos pontos de $(-L, L)$ em que f é contínua. Ou seja, podemos representar f por sua série de Fourier:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi t}{L} + \sum_{n=1}^{\infty} b_n \operatorname{sen} \frac{n\pi t}{L}, \quad \text{para } t \in (-L, L) \text{ em que } f \text{ é contínua.}$$

2.1.4 Tabela de Coeficientes de Séries de Fourier

Coeficientes das Séries de Fourier de Funções Elementares		
$f : [-L, L] \to \mathbb{R}, -1 \le c < d \le 1$	$a_n(f, L) = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}dt$	$b_n(f, L) = \frac{1}{L}\int_{-L}^{L} f(t)\,\mathrm{sen}\,\frac{n\pi t}{L}dt$
$f_{c,d}^{(0)}(t) = \begin{cases} 1, & \text{se } t \in [cL, dL] \\ 0, & \text{caso contrário} \end{cases}$	$a_0(f_{c,d}^{(0)}, L) = d - c$ $a_n(f_{c,d}^{(0)}, L) = \frac{1}{n\pi}\,\mathrm{sen}\, s\Big\vert_{n\pi c}^{n\pi d}$	$b_n(f_{c,d}^{(0)}, L) = -\frac{1}{n\pi}\cos s\Big\vert_{n\pi c}^{n\pi d}$
$f_{c,d}^{(1)}(t) = \begin{cases} t, & \text{se } t \in [cL, dL] \\ 0, & \text{caso contrário} \end{cases}$	$a_0(f_{c,d}^{(1)}, L) = \frac{L}{2}(d^2 - c^2)$ $a_n(f_{c,d}^{(1)}, L) =$ $\frac{L}{n^2\pi^2}(s\,\mathrm{sen}\, s + \cos s)\Big\vert_{n\pi c}^{n\pi d}$	$b_n(f_{c,d}^{(1)}, L) =$ $\frac{L}{n^2\pi^2}(-s\cos s + \mathrm{sen}\, s)\Big\vert_{n\pi c}^{n\pi d}$
$f_{c,d}^{(2)}(t) = \begin{cases} t^2, & \text{se } t \in [cL, dL] \\ 0, & \text{caso contrário} \end{cases}$	$a_0(f_{c,d}^{(2)}, L) = \frac{L^2}{3}(d^3 - c^3)$ $a_n(f_{c,d}^{(2)}, L) =$ $\frac{L^2}{n^3\pi^3}\left((s^2 - 2)\,\mathrm{sen}\, s + 2s\cos s\right)\Big\vert_{n\pi c}^{n\pi d}$	$b_n(f_{c,d}^{(2)}, L) =$ $\frac{L^2}{n^3\pi^3}\left(2s\,\mathrm{sen}\, s + (2 - s^2)\cos s\right)\Big\vert_{n\pi c}^{n\pi d}$

Proposição 2.3. *Sejam* $f, g : [-L, L] \to \mathbb{R}$. *Se*

$$a_n(f, L) = \frac{1}{L}\int_{-L}^{L} f(t)\cos\frac{n\pi t}{L}dt, \;\; b_n(f, L) = \frac{1}{L}\int_{-L}^{L} f(t)\,\mathrm{sen}\,\frac{n\pi t}{L}dt,$$

$$a_n(g, L) = \frac{1}{L}\int_{-L}^{L} g(t)\cos\frac{n\pi t}{L}dt, \;\; b_n(g, L) = \frac{1}{L}\int_{-L}^{L} g(t)\,\mathrm{sen}\,\frac{n\pi t}{L}dt,$$

então para quaisquer números α *e* β,

$$a_n(\alpha f + \beta g, L) = \alpha a_n(f, L) + \beta a_n(g, L) \quad e \quad b_n(\alpha f + \beta g, L) = \alpha b_n(f, L) + \beta b_n(g, L).$$

Series

FOURIER SERIES

Graham S McDonald

A self-contained Tutorial Module for learning the technique of Fourier series analysis

- Table of contents
- Begin Tutorial

 g.s.mcdonald@salford.ac.uk

Section 6: Alternative notation

- For a waveform $f(x)$ with period $2L = \frac{2\pi}{k}$, we have that $k = \frac{2\pi}{2L} = \frac{\pi}{L}$ and $nkx = \frac{n\pi x}{L}$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi x}{L} + b_n \sin \frac{n\pi x}{L} \right]$$

The corresponding Fourier coefficients are

STEP ONE

$$a_0 = \frac{1}{L} \int_{2L} f(x)\, dx$$

STEP TWO

$$a_n = \frac{1}{L} \int_{2L} f(x) \cos \frac{n\pi x}{L}\, dx$$

STEP THREE

$$b_n = \frac{1}{L} \int_{2L} f(x) \sin \frac{n\pi x}{L}\, dx$$

and integrations are over a single interval in x of $2L$

EXERCISE 1.

Let $f(x)$ be a function of period 2π such that

$$f(x) = \begin{cases} 1, & -\pi < x < 0 \\ 0, & 0 < x < \pi\,. \end{cases}$$

a) Sketch a graph of $f(x)$ in the interval $-2\pi < x < 2\pi$

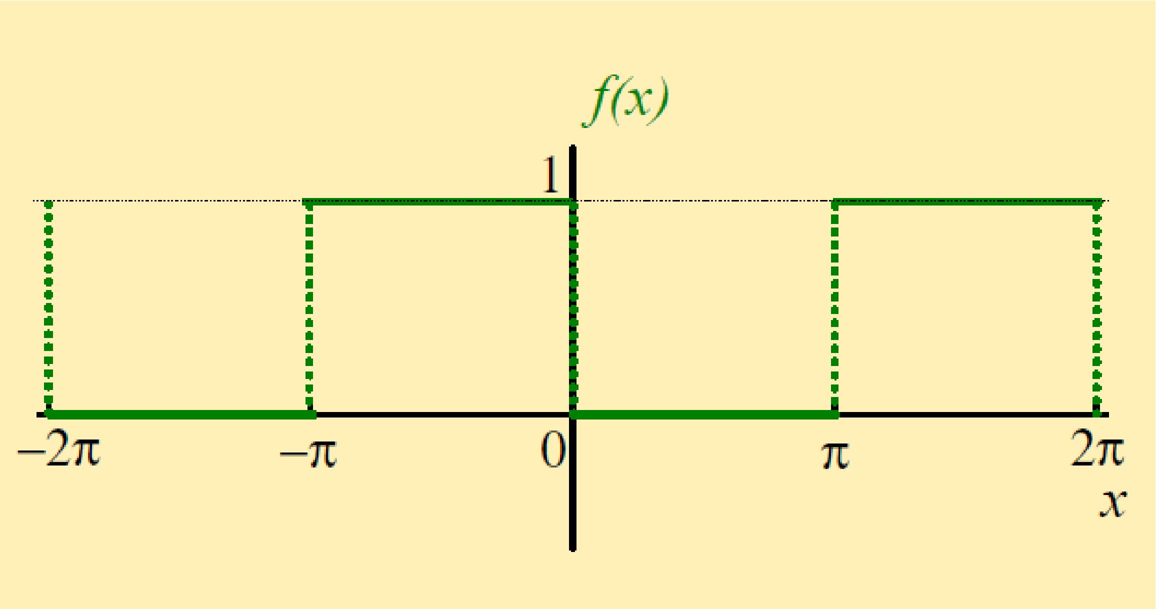
b) Show that the Fourier series for $f(x)$ in the interval $-\pi < x < \pi$ is

$$\frac{1}{2} - \frac{2}{\pi}\left[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots\right]$$

Exercise 1.

$$f(x) = \begin{cases} 1, & -\pi < x < 0 \\ 0, & 0 < x < \pi, \end{cases} \quad \text{and has period } 2\pi$$

a) Sketch a graph of $f(x)$ in the interval $-2\pi < x < 2\pi$



b) Fourier series representation of $f(x)$

STEP ONE

$$
\begin{aligned}
a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\mathrm{d}x &= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} f(x)\mathrm{d}x \\
&= \frac{1}{\pi}\int_{-\pi}^{0} 1\cdot \mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} 0\cdot \mathrm{d}x \\
&= \frac{1}{\pi}\int_{-\pi}^{0} \mathrm{d}x \\
&= \frac{1}{\pi}\left[x\right]_{-\pi}^{0} \\
&= \frac{1}{\pi}\left(0-(-\pi)\right) \\
&= \frac{1}{\pi}\cdot(\pi) \\
\text{i.e. } a_0 &= 1\,.
\end{aligned}
$$

STEP TWO

$$
\begin{aligned}
a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,\mathrm{d}x &= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} f(x)\cos nx\,\mathrm{d}x \\
&= \frac{1}{\pi}\int_{-\pi}^{0} 1\cdot\cos nx\,\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} 0\cdot\cos nx\,\mathrm{d}x \\
&= \frac{1}{\pi}\int_{-\pi}^{0} \cos nx\,\mathrm{d}x \\
&= \frac{1}{\pi}\left[\frac{\sin nx}{n}\right]_{-\pi}^{0} = \frac{1}{n\pi}\left[\sin nx\right]_{-\pi}^{0} \\
&= \frac{1}{n\pi}(\sin 0 - \sin(-n\pi)) \\
&= \frac{1}{n\pi}(0 + \sin n\pi) \\
\text{i.e. } a_n &= \frac{1}{n\pi}(0+0) = 0.
\end{aligned}
$$

STEP THREE

$$\begin{aligned}
b_n &= \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,\mathrm{d}x \\
&= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\sin nx\,\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} f(x)\sin nx\,\mathrm{d}x \\
&= \frac{1}{\pi}\int_{-\pi}^{0} 1\cdot\sin nx\,\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} 0\cdot\sin nx\,\mathrm{d}x
\end{aligned}$$

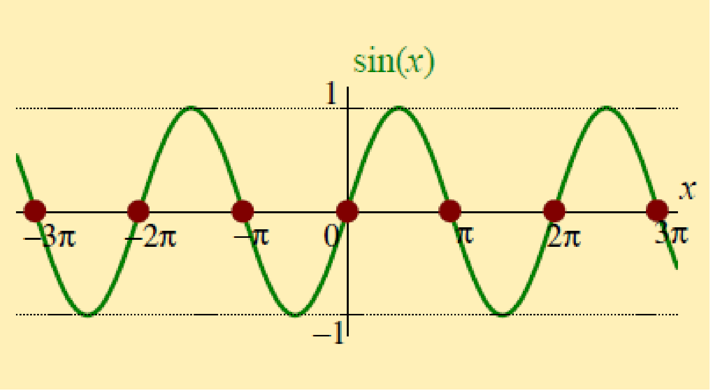
$$\begin{aligned}
\text{i.e.}\quad b_n &= \frac{1}{\pi}\int_{-\pi}^{0}\sin nx\,\mathrm{d}x = \frac{1}{\pi}\left[\frac{-\cos nx}{n}\right]_{-\pi}^{0} \\
&= -\frac{1}{n\pi}\left[\cos nx\right]_{-\pi}^{0} = -\frac{1}{n\pi}\left(\cos 0 - \cos(-n\pi)\right) \\
&= -\frac{1}{n\pi}(1-\cos n\pi) = -\frac{1}{n\pi}\left(1-(-1)^n\right),\ \text{see TRIG}
\end{aligned}$$

$$\text{i.e.}\quad b_n = \begin{cases} 0 & ,\ n \text{ even} \\ -\frac{2}{n\pi} & ,\ n \text{ odd} \end{cases} \quad ,\ \text{since } (-1)^n = \begin{cases} 1 & ,\ n \text{ even} \\ -1 & ,\ n \text{ odd} \end{cases}$$

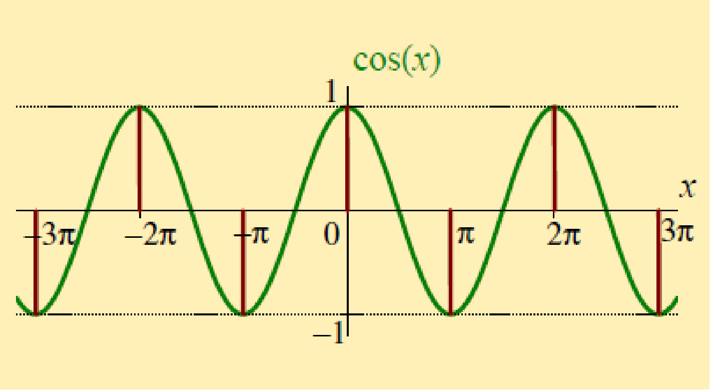
5. Useful trig results

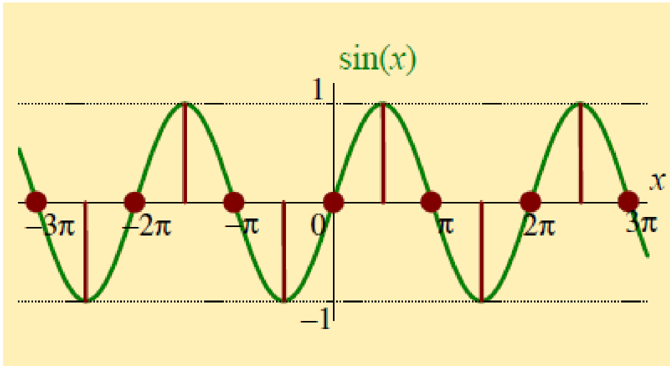
When calculating the Fourier coefficients a_n and b_n, for which $n = 1, 2, 3, \ldots,$ the following trig. results are useful. Each of these results, which are also true for $n = 0, -1, -2, -3, \ldots,$ can be deduced from the graph of $\sin x$ or that of $\cos x$

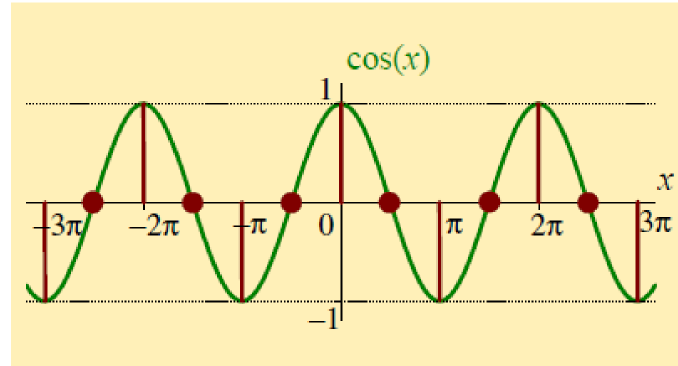
- $\sin n\pi = 0$



- $\cos n\pi = (-1)^n$



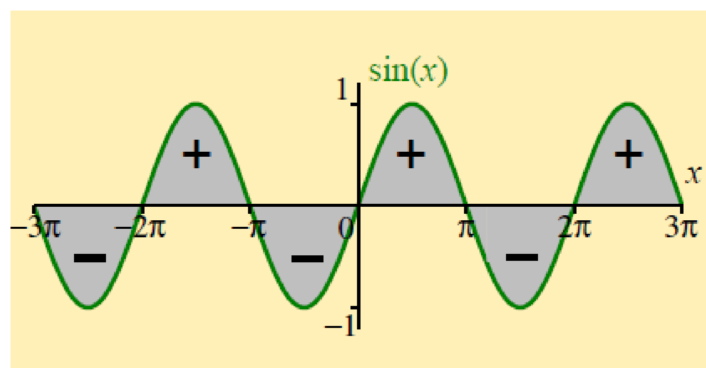




- $\sin n\dfrac{\pi}{2} = \begin{cases} 0 & , n \text{ even} \\ 1 & , n = 1, 5, 9, \ldots \\ -1 & , n = 3, 7, 11, \ldots \end{cases}$
- $\cos n\dfrac{\pi}{2} = \begin{cases} 0 & , n \text{ odd} \\ 1 & , n = 0, 4, 8, \ldots \\ -1 & , n = 2, 6, 10, \ldots \end{cases}$

Areas cancel when when integrating over whole periods

- $\displaystyle\int_{2\pi} \sin nx \, \mathrm{d}x = 0$
- $\displaystyle\int_{2\pi} \cos nx \, \mathrm{d}x = 0$



We now have that

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

with the three steps giving

$$a_0 = 1,\ \ a_n = 0\,,\ \text{and}\ \ b_n = \begin{cases} 0 & , n \text{ even} \\ -\frac{2}{n\pi} & , n \text{ odd} \end{cases}$$

It may be helpful to construct a table of values of b_n

n	1	2	3	4	5
b_n	$-\frac{2}{\pi}$	0	$-\frac{2}{\pi}\left(\frac{1}{3}\right)$	0	$-\frac{2}{\pi}\left(\frac{1}{5}\right)$

Substituting our results now gives the required series

$$f(x) = \frac{1}{2} - \frac{2}{\pi}\left[\sin x + \frac{1}{3}\sin 3x + \frac{1}{5}\sin 5x + \ldots\right]$$

$$f(x) = \begin{cases} 1, & -\pi < x < 0 \\ 0, & 0 < x < \pi, \end{cases}$$

Coeficientes das Séries de Fourier de Funções Elementares		
$f : [-L, L] \to \mathbb{R}, -1 \leq c < d \leq 1$	$a_n(f, L) = \frac{1}{L} \int_{-L}^{L} f(t) \cos \frac{n\pi t}{L} dt$	$b_n(f, L) = \frac{1}{L} \int_{-L}^{L} f(t) \operatorname{sen} \frac{n\pi t}{L} dt$
$f_{c,d}^{(0)}(t) = \begin{cases} 1, & \text{se } t \in [cL, dL] \\ 0, & \text{caso contrário} \end{cases}$	$a_0(f_{c,d}^{(0)}, L) = d - c$ $a_n(f_{c,d}^{(0)}, L) = \frac{1}{n\pi} \operatorname{sen} s \Big\|_{n\pi c}^{n\pi d}$	$b_n(f_{c,d}^{(0)}, L) = -\frac{1}{n\pi} \cos s \Big\|_{n\pi c}^{n\pi d}$

Exercício 1

L = π , c = -1 , d = 0

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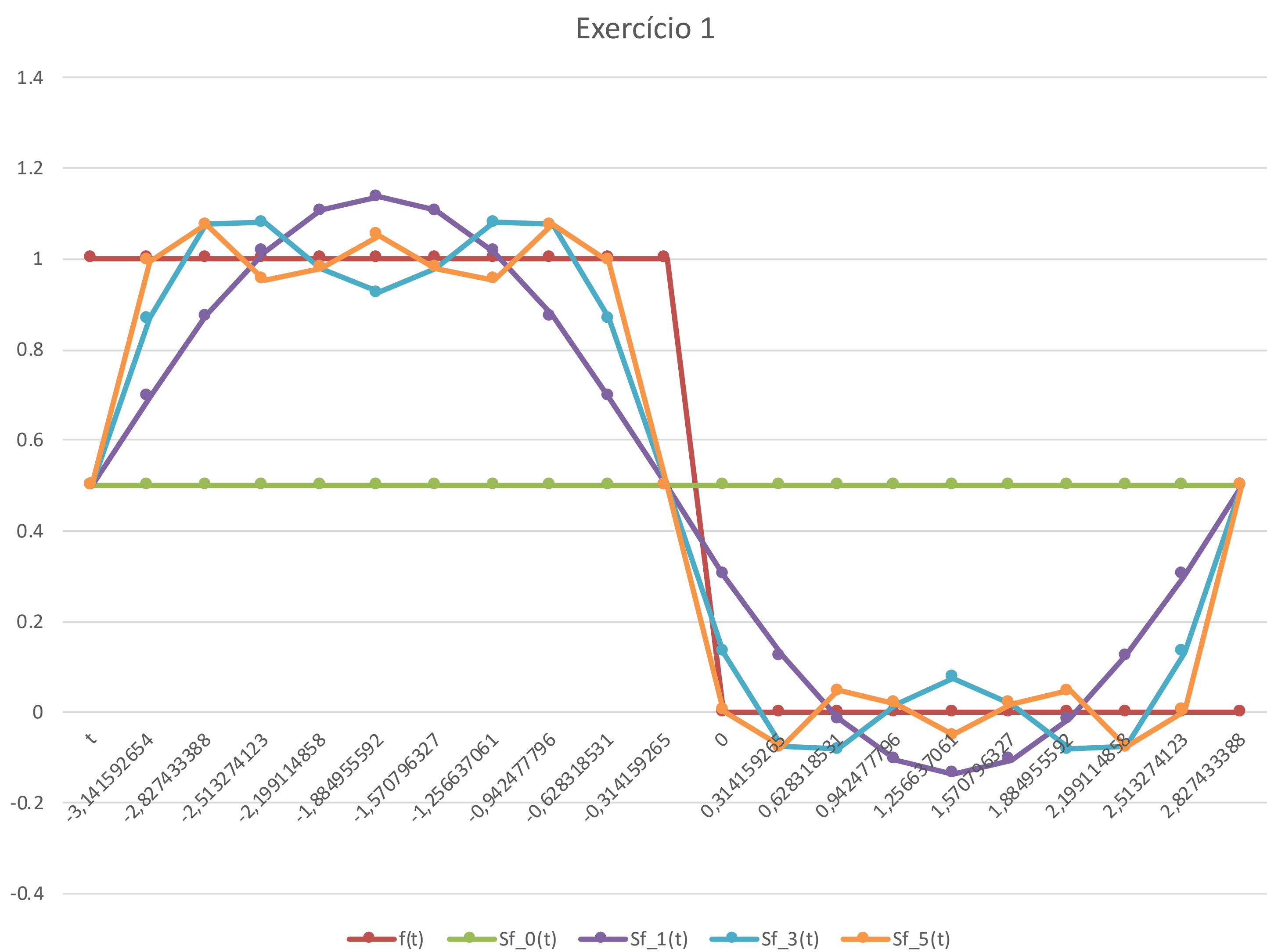
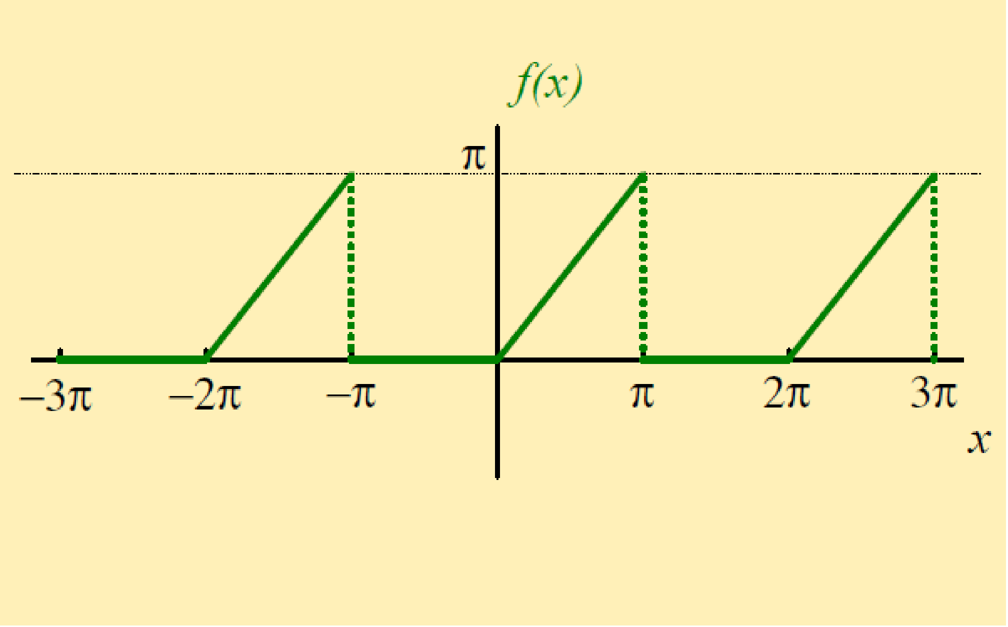


Exercício 1
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
t
-3,141592654
-2,827433388
-2,513274123
-2,199114858
-1,884955592
-1,570796327
-1,256637061
-0,942477796
-0,628318531
-0,314159265
0
0,314159265
0,628318531
0,942477796
1,256637061
1,570796327
1,884955592
2,199114858
2,513274123
2,827433388
f(t)
Sf_0(t)
Sf_1(t)
Sf_3(t)
Sf_5(t)

Exercise 2.

$$f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x, & 0 < x < \pi, \end{cases} \quad \text{and has period } 2\pi$$

a) Sketch a graph of $f(x)$ in the interval $-3\pi < x < 3\pi$



$$f(x)=\begin{cases} 0, & -\pi<x<0 \\ x, & 0<x<\pi, \end{cases}$$

$f_{c,d}^{(1)}(t)=\begin{cases} t, & \text{se } t\in[cL,dL] \\ 0, & \text{caso contrário} \end{cases}$	$a_0(f_{c,d}^{(1)},L)=\frac{L}{2}(d^2-c^2)$ $a_n(f_{c,d}^{(1)},L)=$ $\frac{L}{n^2\pi^2}\left(s\ \text{sen}\, s+\cos s\right)\Big\vert_{n\pi c}^{n\pi d}$	$b_n(f_{c,d}^{(1)},L)=$ $\frac{L}{n^2\pi^2}\left(-s\cos s+\text{sen}\, s\right)\Big\vert_{n\pi c}^{n\pi d}$

Exercício 2

L = π , c = 0 , d = 1

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b) Fourier series representation of $f(x)$

STEP ONE

$$
\begin{aligned}
a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\mathrm{d}x &= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} f(x)\mathrm{d}x \\
&= \frac{1}{\pi}\int_{-\pi}^{0} 0\cdot\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} x\,\mathrm{d}x \\
&= \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{\pi} \\
&= \frac{1}{\pi}\left(\frac{\pi^2}{2} - 0\right) \\
\text{i.e. } a_0 &= \frac{\pi}{2}\,.
\end{aligned}
$$

STEP TWO

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,\mathrm{d}x = \frac{1}{\pi}\int_{-\pi}^{0} f(x)\cos nx\,\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} f(x)\cos nx\,\mathrm{d}x$$

$$= \frac{1}{\pi}\int_{-\pi}^{0} 0\cdot\cos nx\,\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,\mathrm{d}x$$

$$\text{i.e. } a_n = \frac{1}{\pi}\int_{0}^{\pi} x\cos nx\,\mathrm{d}x = \frac{1}{\pi}\left\{\left[x\frac{\sin nx}{n}\right]_0^{\pi} - \int_0^{\pi}\frac{\sin nx}{n}\,\mathrm{d}x\right\}$$

(using integration by parts)

$$\text{i.e. } a_n = \frac{1}{\pi}\left\{\left(\pi\frac{\sin n\pi}{n} - 0\right) - \frac{1}{n}\left[-\frac{\cos nx}{n}\right]_0^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{(\quad 0 \quad - \ 0) + \frac{1}{n^2}[\cos nx]_0^{\pi}\right\}$$

$$= \frac{1}{\pi n^2}\{\cos n\pi - \cos 0\} = \frac{1}{\pi n^2}\{(-1)^n - 1\}$$

$$\text{i.e. } a_n = \begin{cases} 0 & , n \text{ even} \\ -\frac{2}{\pi n^2} & , n \text{ odd} \end{cases} \ , \text{ see } \textsc{Trig}.$$

STEP THREE

$$
\begin{aligned}
b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,\mathrm{d}x &= \frac{1}{\pi}\int_{-\pi}^{0} f(x)\sin nx\,\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} f(x)\sin nx\,\mathrm{d}x \\
&= \frac{1}{\pi}\int_{-\pi}^{0} 0\cdot\sin nx\,\mathrm{d}x + \frac{1}{\pi}\int_{0}^{\pi} x\sin nx\,\mathrm{d}x \\
\text{i.e. } b_n = \frac{1}{\pi}\int_{0}^{\pi} x\sin nx\,\mathrm{d}x &= \frac{1}{\pi}\left\{\left[x\left(-\frac{\cos nx}{n}\right)\right]_0^{\pi} - \int_0^{\pi}\left(-\frac{\cos nx}{n}\right)\mathrm{d}x\right\} \\
&\qquad \text{(using integration by parts)} \\
&= \frac{1}{\pi}\left\{-\frac{1}{n}\left[x\cos nx\right]_0^{\pi} + \frac{1}{n}\int_0^{\pi}\cos nx\,\mathrm{d}x\right\} \\
&= \frac{1}{\pi}\left\{-\frac{1}{n}(\pi\cos n\pi - 0) + \frac{1}{n}\left[\frac{\sin nx}{n}\right]_0^{\pi}\right\} \\
&= -\frac{1}{n}(-1)^n + \frac{1}{\pi n^2}(0-0), \text{ see TRIG} \\
&= -\frac{1}{n}(-1)^n
\end{aligned}
$$

i.e. $$b_n = \begin{cases} -\frac{1}{n} & , n \text{ even} \\ +\frac{1}{n} & , n \text{ odd} \end{cases}$$

We now have

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos nx + b_n \sin nx]$$

where $a_0 = \dfrac{\pi}{2}$, $\quad a_n = \begin{cases} 0 & , n \text{ even} \\ -\frac{2}{\pi n^2} & , n \text{ odd} \end{cases}$, $\quad b_n = \begin{cases} -\frac{1}{n} & , n \text{ even} \\ \frac{1}{n} & , n \text{ odd} \end{cases}$

Constructing a table of values gives

n	1	2	3	4	5
a_n	$-\frac{2}{\pi}$	0	$-\frac{2}{\pi}\cdot\frac{1}{3^2}$	0	$-\frac{2}{\pi}\cdot\frac{1}{5^2}$
b_n	1	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{4}$	$\frac{1}{5}$

This table of coefficients gives

$$\begin{aligned} f(x) = \frac{1}{2}\left(\frac{\pi}{2}\right) \quad &+ \left(-\frac{2}{\pi}\right)\cos x + 0\cdot\cos 2x \\ &+ \left(-\frac{2}{\pi}\cdot\frac{1}{3^2}\right)\cos 3x + 0\cdot\cos 4x \\ &+ \left(-\frac{2}{\pi}\cdot\frac{1}{5^2}\right)\cos 5x + \ldots \\ &+ \sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \ldots \end{aligned}$$

$$\begin{aligned} \text{i.e. } f(x) = \frac{\pi}{4} \quad &- \frac{2}{\pi}\left[\cos x + \frac{1}{3^2}\cos 3x + \frac{1}{5^2}\cos 5x + \ldots\right] \\ &+ \left[\sin x - \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x - \ldots\right] \end{aligned}$$

and we have found the required series!

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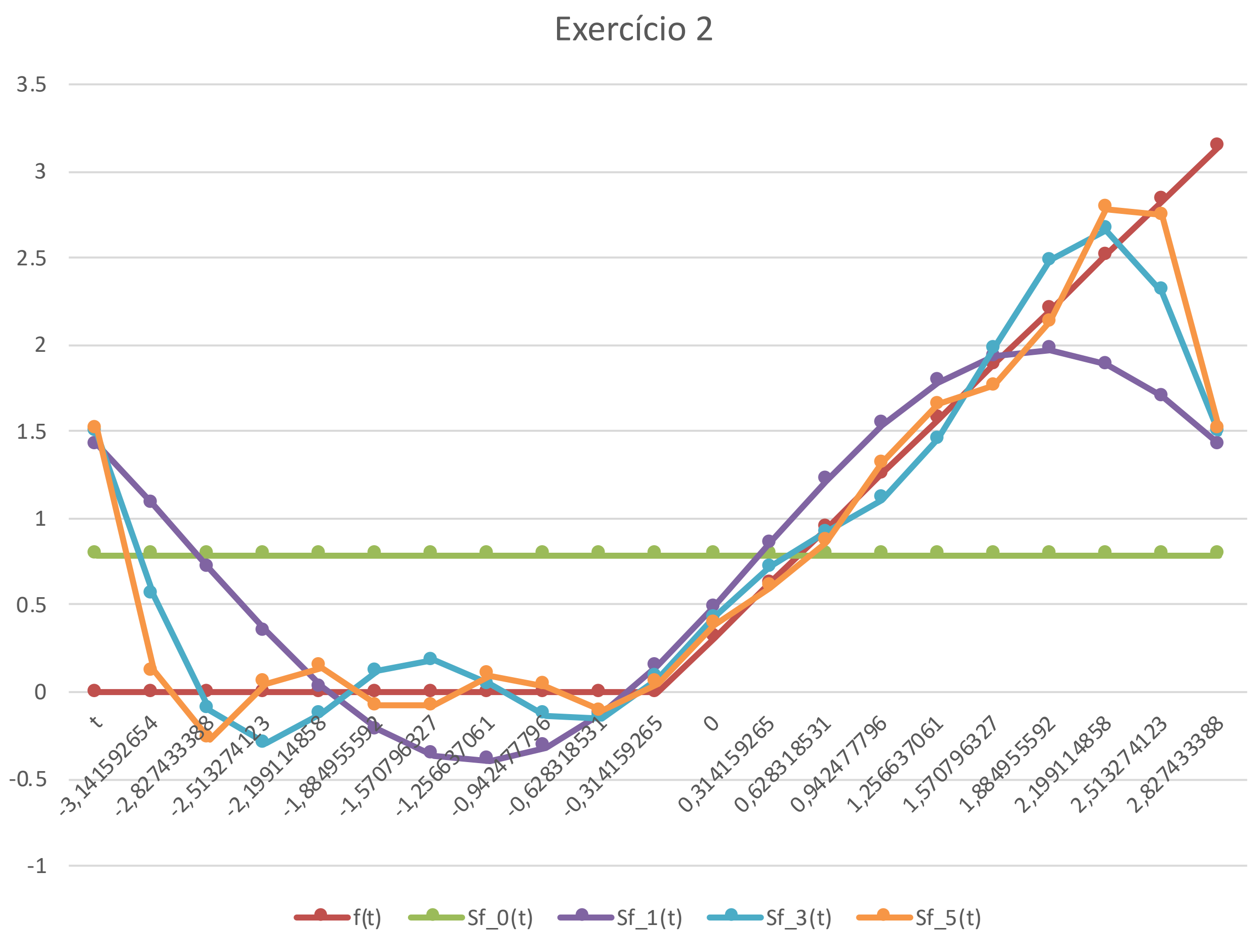
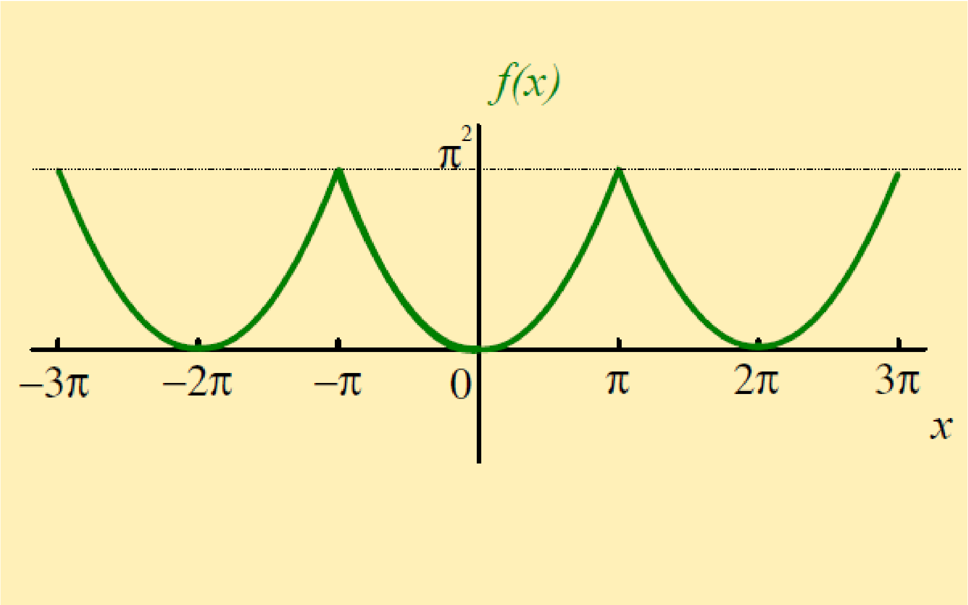

Exercício 2
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
t
-3,141592654
-2,827433388
-2,513274123
-2,199114858
-1,884955592
-1,570796327
-1,256637061
-0,942477796
-0,628318531
-0,314159265
0
0,314159265
0,628318531
0,942477796
1,256637061
1,570796327
1,884955592
2,199114858
2,513274123
2,827433388
f(t)
Sf_0(t)
Sf_1(t)
Sf_3(t)
Sf_5(t)

Exercise 7.

$$\boxed{f(x) = x^2, \text{ over the interval } -\pi < x < \pi \text{ and has period } 2\pi}$$

a) Sketch a graph of $f(x)$ in the interval $-3\pi < x < 3\pi$



$f_{c,d}^{(2)}(t) = \begin{cases} t^2, & \text{se } t \in [cL, dL] \\ 0, & \text{caso contrário} \end{cases}$	$a_0(f_{c,d}^{(2)}, L) = \frac{L^2}{3}(d^3 - c^3)$ $a_n(f_{c,d}^{(2)}, L) =$ $\frac{L^2}{n^3\pi^3}\left((s^2-2)\operatorname{sen} s + 2s\cos s\right)\Big\vert_{n\pi c}^{n\pi d}$	$b_n(f_{c,d}^{(2)}, L) =$ $\frac{L^2}{n^3\pi^3}\left(2s\operatorname{sen} s + (2-s^2)\cos s\right)\Big\vert_{n\pi c}^{n\pi d}$

MAP2320

$$a_0 = \frac{2\pi^2}{3} \qquad a_n = \begin{cases} \frac{4}{n^2} & , n \text{ even} \\ \frac{-4}{n^2} & , n \text{ odd.} \end{cases} \qquad b_n = 0.$$

$$f(x) = \frac{\pi^2}{3} - 4\left[\cos x - \frac{1}{2^2}\cos 2x + \frac{1}{3^2}\cos 3x - \frac{1}{4^2}\cos 4x + \ldots\right]$$

MAP 2320 – MÉTODOS NUMÉRICOS EM EQUAÇÕES DIFERENCIAIS II

2º Semestre - 2019

Roteiro do curso

- Introdução
- Séries de Fourier
- Método de Diferenças Finitas
- Equação do calor transiente (parabólica)
- Equação de Poisson (elíptica)
- Equação da onda (hiperbólica)