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Proceedings of the 34th
Conference on Decision & Control
New Orleans, LA - December 1995

SLIDING MODE OBSERVERS. TUTORIAL.

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ABSTRACT

Tutorial discusses the problem of designing observers for state estimation using sliding modes. The theory and design principles are presented for linear and nonlinear systems. For linear systems the observers are developed using a block-observable form which is similar to a lower triangular matrix form. Comparing with known approaches such observers have better robustness properties. In case of nonlinear system an equivalent control concept makes possible to develop finite-time observers for a wide class of systems.

Summary

The problem of state estimation for the nonlinear system

$$\dot{x} = f(x) + B(x)u, \tag{1}$$

where $x \in R^n$, by output vector measurements

$$y = h(x) \in R^m \tag{2}$$

has been considered by many authors (see, for example, survey [7]). For exactly known smooth nonlinearities f and h and if $B = 0$, the method of global linearization [9] by using nonlinear state transformation permits to obtain a system in the new variables $z \in R^n$ which is linear with respect to z and has in the right hand side nonlinear function only of the measurements y. Then the standard linear pole-placement technique can be used for designing observer.

For successful application of this method the nonlinear transformation must be determined. The necessary and sufficient conditions of existence for this transformation were given by Krener and Respondek [6]. These conditions are considerably more restrictive than the observability condition of the system 1, 2. On the other hand the problem of integration of a corresponding partial differential equation for obtaining this transformation makes sometimes this method difficult to implement in practical situations.

Observers based on sliding mode approach [1, 2] first were developed for linear case [3, 5, 4, 9, 8].

$$\dot{x} = Ax + Bu, \tag{3}$$

$$y = Cx. \tag{4}$$

By structure they are very similar to the standard full order observers with replacement of the linear innovative term by a discontinuous function:

$$\dot{\hat{x}} = A\hat{x} + Bu + Lsign(y - C\hat{x}), \tag{5}$$

where $sign$ is understood componentwise for vector argument $z = col(z_1, \ldots, z_n)$:

$$sgn(z) = col(sgn(z_1), \ldots, sgn(z_n)). \tag{6}$$

Under a suitable choice of a gain matrix L sliding occurs in the system 5 on the manifold $y - C\hat{x} = 0$, and it becomes equivalent to the reduced order observer.

The observer design can be based on sequential application of the state transformations reducing the system to the block form [3, 5].

Let us illustrate this method for the system 3, 4. By using linear transformation of state variables it can be written in the form:

$$\dot{y} = A_{11}y + A_{12}x_1 + B_1u \tag{7}$$

$$\dot{x}_1 = A_{21}y + A_{22}x_1 + B_2u. \tag{8}$$

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The corresponding sliding mode observer for the first subsystem is

$$\dot{\hat{y}} = A_{11}\hat{y} + A_{12}\hat{x}_1 + B_1 u + L_1 sgn(y - \hat{y}) \quad (9)$$

The system for the error $\bar{y} = y - \hat{y}$ obtained by subtracting 7 from 9, has the form

$$\dot{\bar{y}} = A_{11}\bar{y} + A_{12}\bar{x}_1 - L_1 sgn(\bar{y}) \quad (10)$$

By appropriate choice of the observer gain matrix L_1 sliding occurs in 10 along the manifold $\bar{y} = 0$.

For the second subsystem 8 we can use the observer equation of the form

$$\dot{\hat{x}}_1 = A_{21}y + A_{22}\hat{x}_1 + B_2 u - L_2 L_1 sign(y - \hat{y}). \quad (11)$$

which gives the following equation for the error

$$\dot{\bar{x}}_1 = A_{22}\bar{x}_1 - L_2 L_1 sign(\bar{y}). \quad (12)$$

According to the equivalent control method, the system in sliding mode behaves as if $L_1 sign(\bar{y})$ is replaced by its equivalent value $(L_1 sign(\bar{y}))_{eq}$, which can be calculated from the subsystem 10 assuming $\bar{y} = 0$, $\dot{\bar{y}} = 0$. Hence

$$(L_1 sign(\bar{y}))_{eq} = A_{12}\bar{x}_1. \quad (13)$$

Substituting 13 into 14 we obtain

$$\dot{\bar{x}}_1 = (A_{22} - L_2 A_{12})\bar{x}_1. \quad (14)$$

It can be shown that if the system 3, 4 (the pair (A, C)) is observable then the pair (A_{22}, A_{21}) is also observable and, therefore, by appropriate choice of L_2 the desired rate of convergence $\bar{x}_1 \to 0$ can be obtained.

Under special conditions sliding mode observer similar to 5 can be used for estimating state of the uncertain system (see [9, 8]).

The other way of designing observer is to continue transforming the system like it was done on the first step instead of using 11. Introducing a variable

$$y_1 = L_1^{-1} A_{12} x_1 \quad (15)$$

the second subsystem 8 can be decomposed to the form

$$\dot{y}_1 = A_{31}y_1 + A_{32}x_2 + B_3 u \quad (16)$$

$$\dot{x}_2 = A_{41}y_1 + A_{42}x_2 + B_4 u. \quad (17)$$

The observer for 16 is

$$\dot{\hat{y}}_1 = A_{31}\hat{y}_1 + A_{32}\hat{x}_2 + B_3 u + L_3 sign(y_1 - \hat{y}_1), \quad (18)$$

which requires the measurements of y_1.

Since according to 13, 15

$$y_1 - \hat{y}_1 = (sign(\bar{y}))_{eq},$$

this equivalent value can be used in the right hand side of the observer 18:

$$\dot{\hat{y}}_1 = A_{31}\hat{y}_1 + A_{32}\hat{x}_2 + B_3 u + L_3 sign((sign(\bar{y}))_{eq}). \quad (19)$$

As it is shown in [1] for real-life system $(sign(\bar{y}))_{eq}$ by its physical meaning is an average value of the chattering with high frequency variable $sign(\bar{y})$. $(sign(\bar{y}))_{eq}$ can be obtained by using low pass filter whose time constant is "sufficiently small to pass the slow component of the motion (which is an equivalent control) but is large enough to eliminate the high frequency component" caused by the presence of nonidealities. Repeating sequentially the described procedure we obtain the observer which converges in finite time.

The idea described above also can be implemented for a nonlinear case [4]. As in linear case the main idea of the design is to use the equivalent control method to obtain additional information from the system.

Let us consider the system 1, 2 with $m = 1, B = 0$ and introduce the following notations $H(x) = col(h_1(x), \ldots, h_n(x))$ where $h_1(x) = h(x)$ and $h_{i+1}(x)$ is ith Lie (directional) derivative of the function $h(x)$ along the trajectories of the system 1

$$h_i(x) = L_f^{i-1} h(x). \quad (20)$$

or

$$h_{i+1}(x) = \frac{\partial h_i(x)}{\partial x} f(x). \quad (21)$$

Let $H(x) = col(h_1(x), \ldots, h_n(x))$, is nondegenerating in X i.e.:

$$\left| \det \frac{\partial H(x)}{\partial x} \right| \geq \delta > 0 \quad (22)$$

for some $\delta > 0$ and every $x \in X$.

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To estimate system 1 state variables by measurements 2 one can use an observer ([4]) of the form:

$$\dot{\hat{x}} = \left(\frac{\partial H(\hat{x})}{\partial x}\right)^{-1} M(\hat{x}) sign(V(t) - H(\hat{x})), \tag{23}$$

where $V(t) = col(v_1(t), \ldots, v_n(t))$

$$v_1(t) = y(t), \tag{24}$$

$$v_{i+1}(t) = (m_i(\hat{x}) sign(v_i(t) - h_i(\hat{x}(t))))_{eq}, \tag{25}$$

$i = 1, \ldots, n-1$ and matrix M is $n \times n$ diagonal matrix with positive elements $m_i(\hat{(x)}$, $i = 1, \ldots, n$

$$M(\hat{x}) = diag(m_1(\hat{x}), \ldots, m_n(\hat{x})). \tag{26}$$

By suitable choice of M, observer 23 converges in **any** prescribed finite time interval.

From 22 it follows that the map $H : R^n \mapsto R^n$ is an injection therefore it is sufficient to show that the modified observation error $e(t) = H(x(t)) - H(\hat{x}(t))$ converges to zero in finite time.

From 1, 2 and 23 it follows that

$$\dot{e}(t) = \frac{dH(x(t))}{dt} - M(\hat{x}) sign(V(t) - H(\hat{x}(t))). \tag{27}$$

Therefore from 21, 27

$$\dot{e}_i(t) = h_{i+1}(x(t)) - m_i(\hat{x}) sign(v_i(t) - h_i(\hat{x}(t))) \tag{28}$$

for $i = 1, \ldots, n$.

If $|h_2(x(t))| \leq m_1(\hat{x}(t))$ then in the first equation 28 sliding mode occurs since $v_1(t) = h_1(x(t))$ and hence $v_1(t) - h_1(\hat{x}(t)) = e_1(t)$.

During sliding, $e_1(t) = 0$. According to the equivalent control method $((m_1(\hat{x}) sign(v_1(t) - h_1(\hat{x}(t))))_{eq} = h_2(x(t))$, or, as it follows from 25, $v_2(t) = h_2(x(t))$. Again if the condition $|h_3(x(t))| \leq m_2(\hat{x}(t))$ is satisfied sliding mode occurs in the second equation and we will have $e_2(t) = 0$ and so on.

When the sliding mode occurs in the last equation 28 we have $e(t) = 0$. The conditions for sliding mode to exist in every equation 28 are

$$|h_{i+1}(x(t))| \leq m_i(\hat{x}(t)). \tag{29}$$

Generalization when control enters the right hand side of the equation was also obtained but in contrast with linear case in this situation the observation problem depends on the control.

In conclusion let us note that a wide variety of parameter estimation problems can be solved by means of sliding mode observers. Assuming that these parameters are some of the unknown state variables one can use an observer for nonlinear case described above. The other way is to use Lyapunov approach to derive a sliding mode observer specifically for parameter estimation or simultaneous parameter and state estimation in linear system [3].

References

[1] Utkin V.I., *Sliding Modes and Their Application In Variable Structure Systems.* Moscow: MIR, 1978.

[2] DeCarlo R. A., S. H. Zak, and G.P. Matthews, *Variable Structure Control for Nonlinear Multivariable Systems: A Tutorial,* IEEE Trans. Aut. Contr., Vol. 76, No. 3, pp. 212-232. 1988.

[3] Utkin V.I. *A Method of Separation of Motion for Designing a State Identifier,* in: Problems of Control of multivariable Systems, Nauka, Moscow, 1983.

[4] Drakunov, S.V. *Sliding-Mode Observers Based on Equivalent Control Method,* Proceedings of the 31st IEEE Conference on Decision and Control (CDC), Tucson, Arizona, December 16-18, 1992, pp. 2368-2369.

[5] Drakunov, S.V., Izosimov D.B., Luk'yanov A.G., Utkin V.A. and Utkin V.I., *The Block Control Principle, Part II,* Automation and Remote Control, V. 51, no. 6,1990,pp. 737-746.

[6] Krener, A.J. and Respondek, W., *Nonlinear observers with Linearizable Error Dynamics,* SIAM J. Control and Optimization, V.23,no.2,March 1985.

[7] Misawa, E.A. and Hedrick,J.K., *Nonlinear Observers A State-of-the-art Survey,* ASME J. Dynamic Syst. Measurement Contr.,v.111,No.3,1989.

[8] Walcott, B.L., and Zak, S.H., *Observation of Dynamical Systems in the Presence of Bounded Nonlinearities-Uncertaities,* Proceedings of 25th CDC, pp. 961-966, 1986.

[9] Walcott, B.L., Corless, M.J. and Zak, S.H., *Comparative Study of Nonlinear State Observation Techniques,* Int,J.Control, V.45, no. 6, pp. 2109-2132, 1987.

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