Universidade de São Paulo
Escola de Engenharia de São Carlos

SEM0551 Fenômenos de Transporte

ESCOAMENTO SOBRE UMA PLACA PLANA: CAMADA LIMITE

Paulo Seleghim Jr.
Universidade de São Paulo

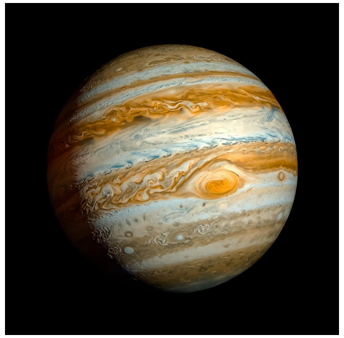
Equações governantes:

Continuidade (massa) → $$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{U}) = 0$$

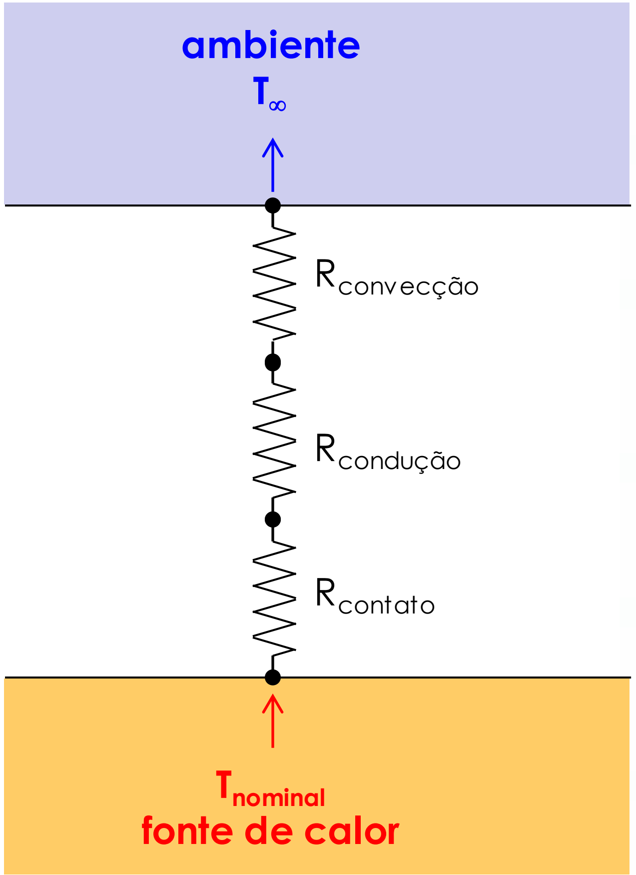
Q. de movimento (Navier-Stokes) → $$\rho \cdot \left(\frac{\partial \vec{U}}{\partial t} + \vec{U} \cdot \vec{\nabla} \vec{U} \right) = -\vec{\nabla} P + \vec{\nabla} \cdot \tilde{T} + \sum \vec{F}_{3D}$$

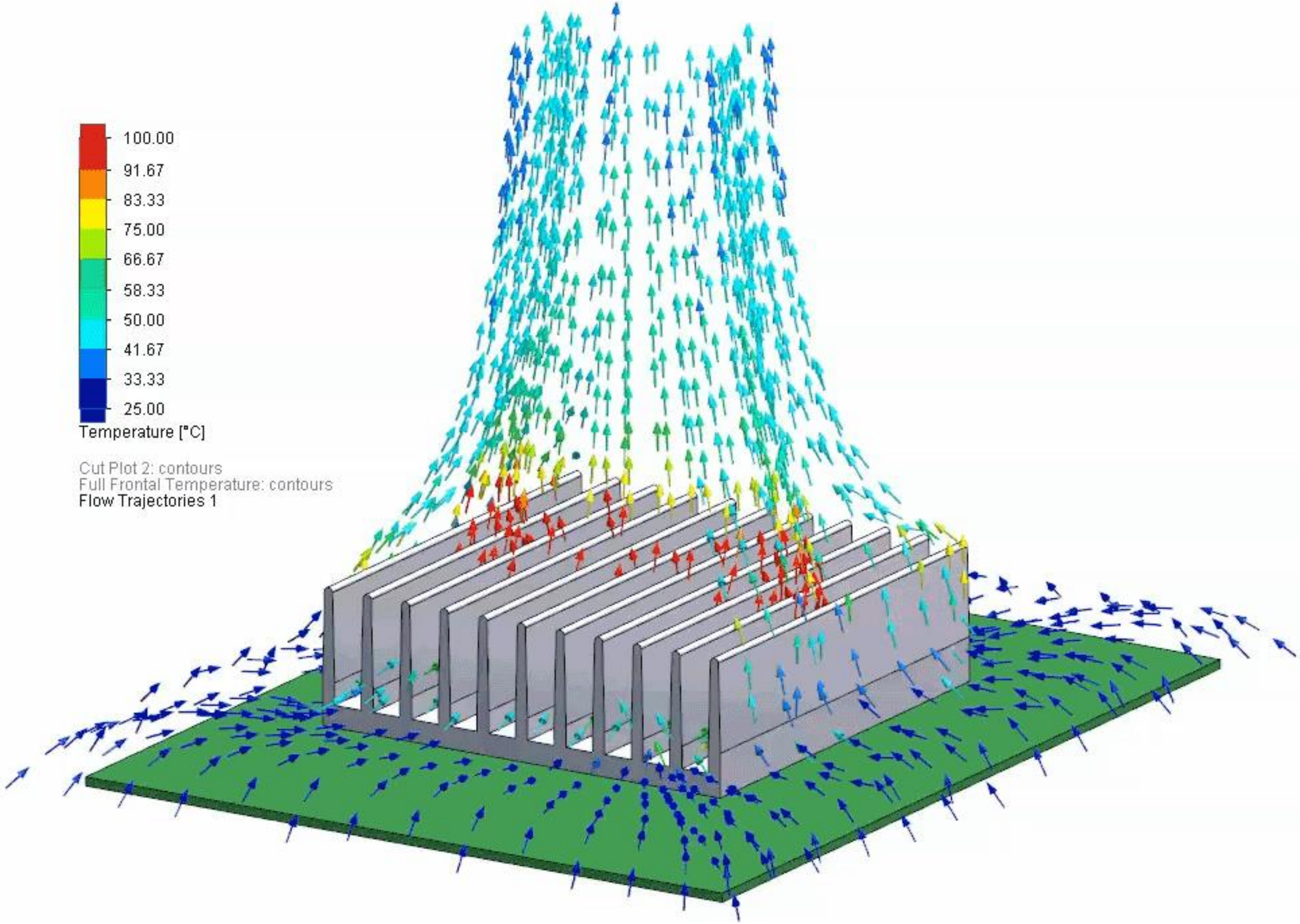
Energia (1ra lei) → $$\rho C_P \left(\frac{\partial T}{\partial t} + \vec{U} \cdot \vec{\nabla} T \right) = \vec{\nabla} \cdot (k \vec{\nabla} T) + \tilde{T} : \tilde{D}$$

Escalas microscópicas (Kolmogorov) a escalas sinóticas...



Resfriamento de componentes eletrônicos...





Escoamento incidindo sobre uma placa plana...

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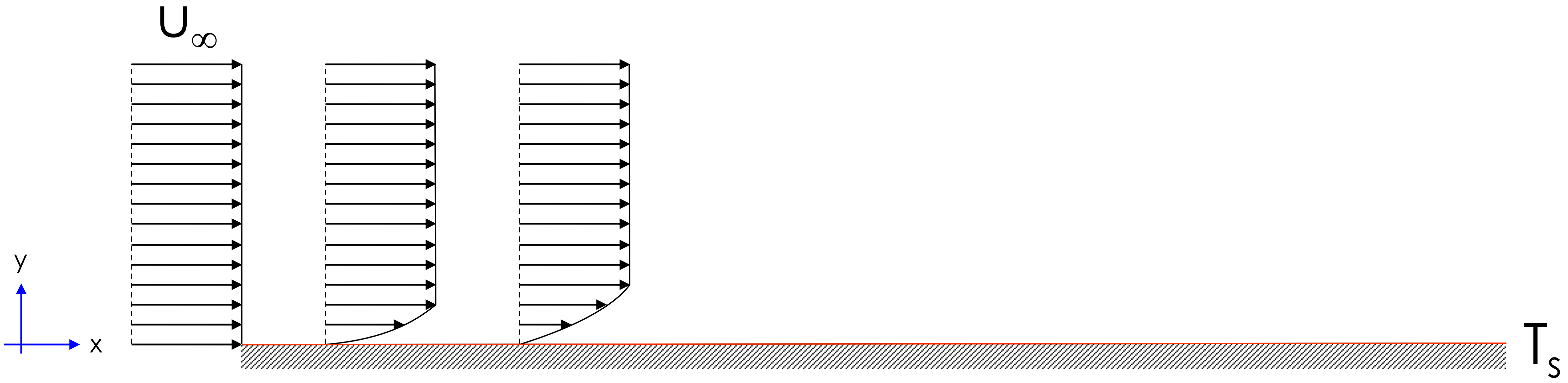

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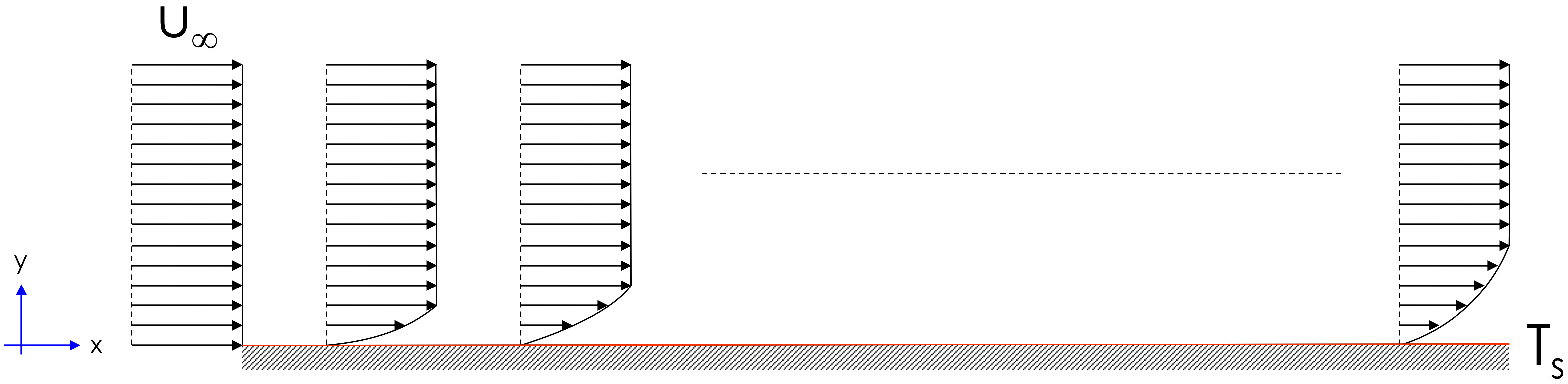

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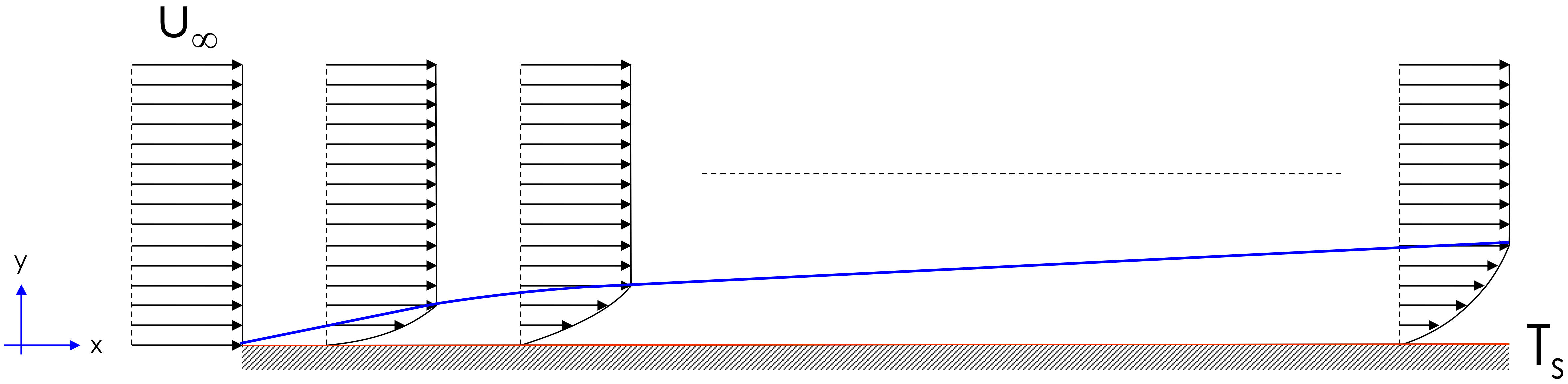

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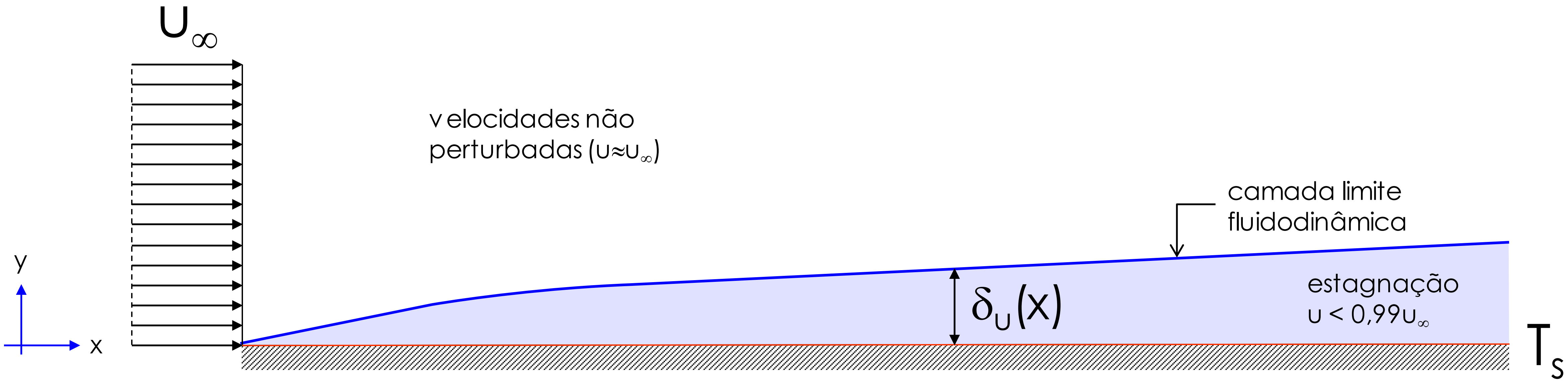

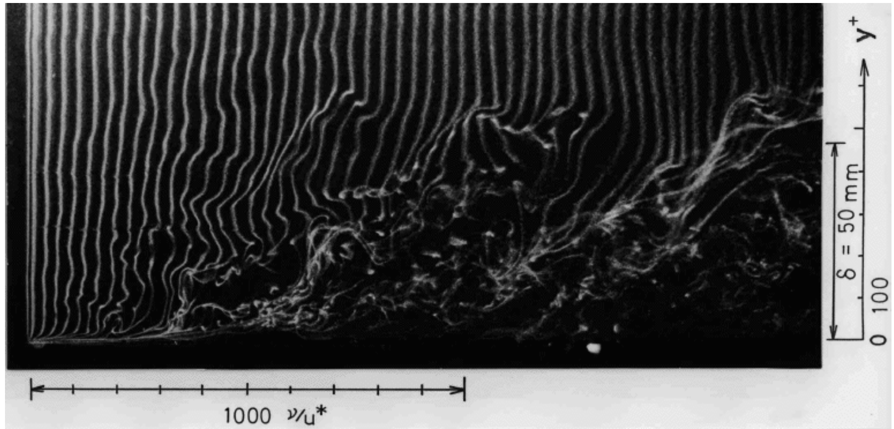

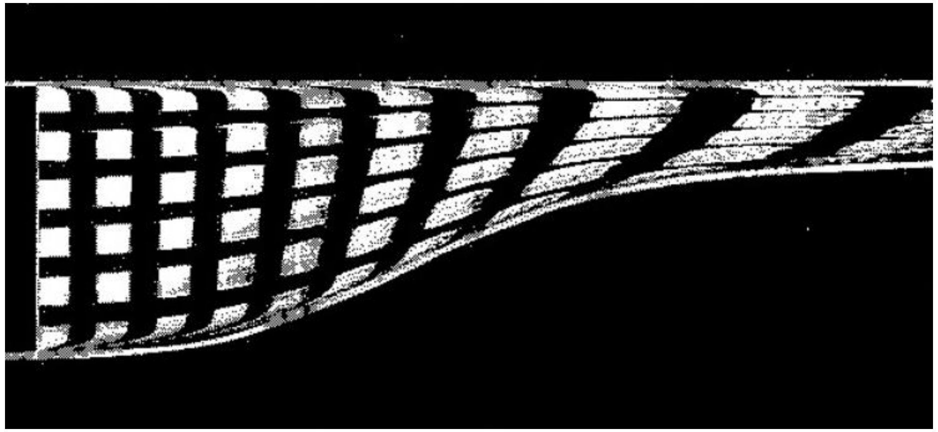

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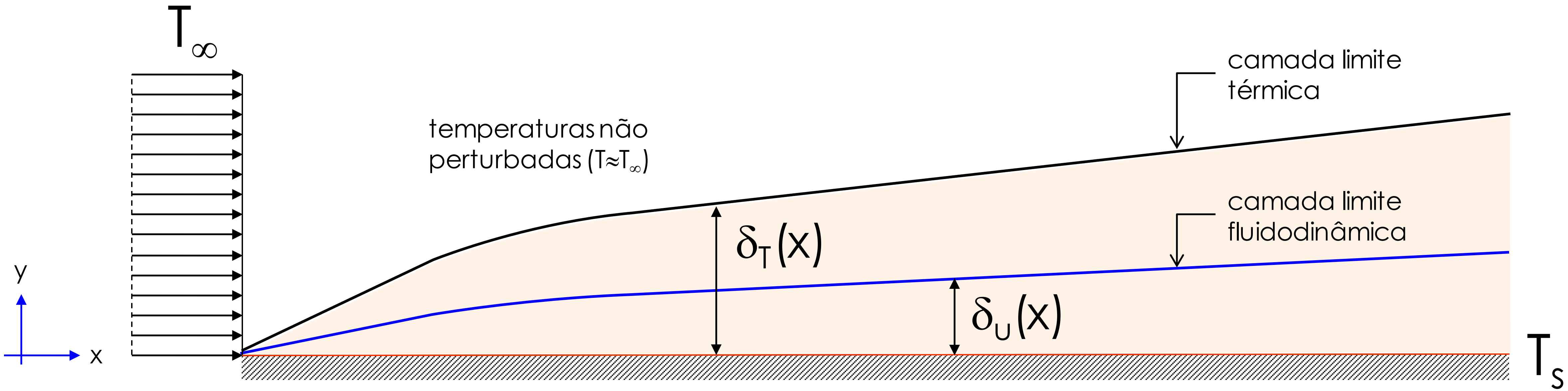

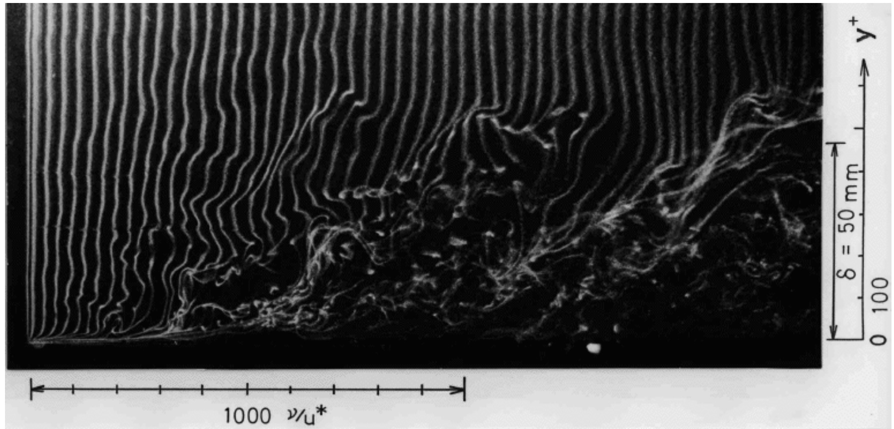

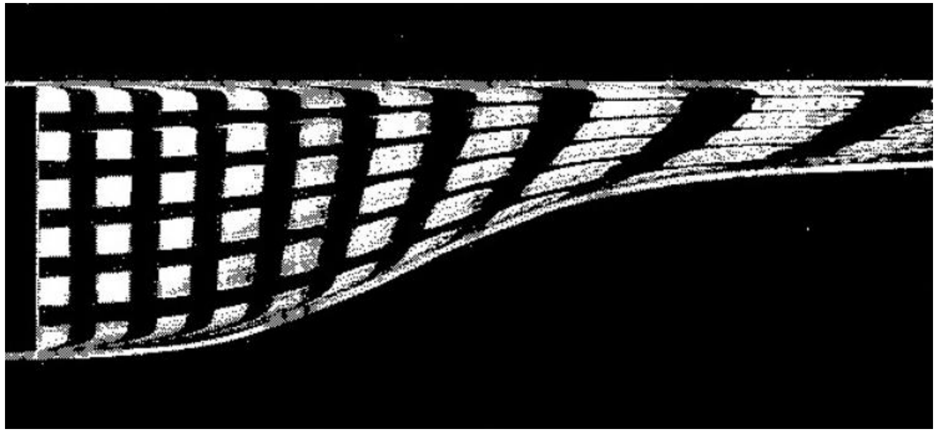

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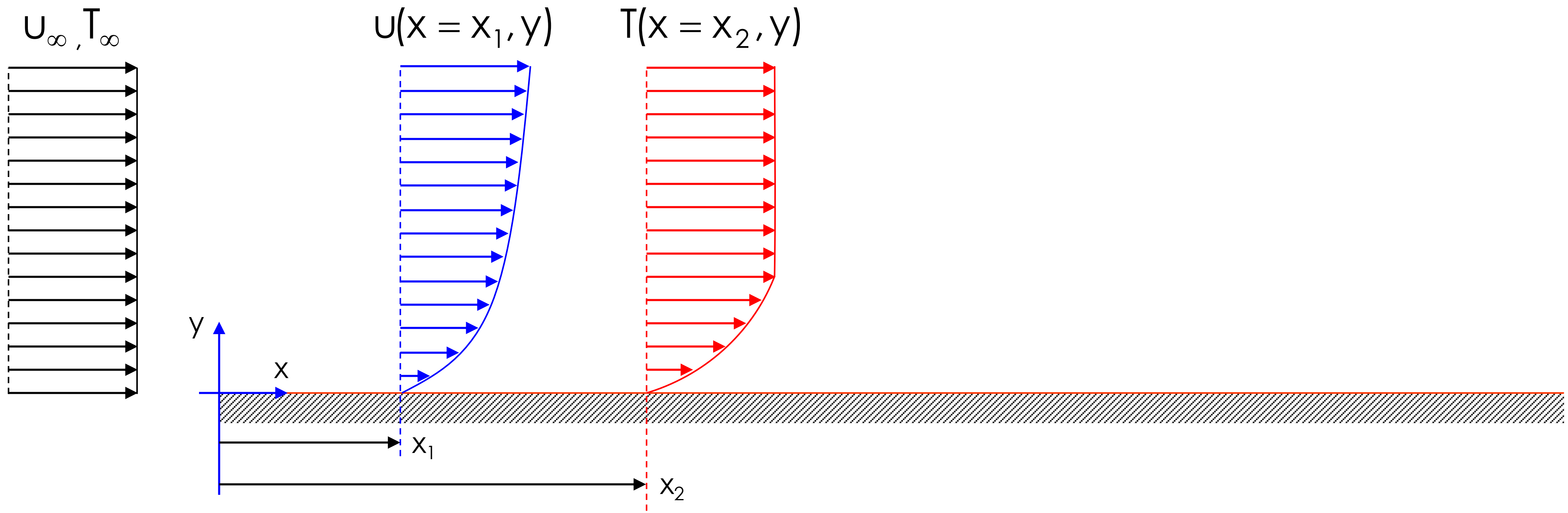
Cálculo das camadas limites hidrodinâmica e térmica...



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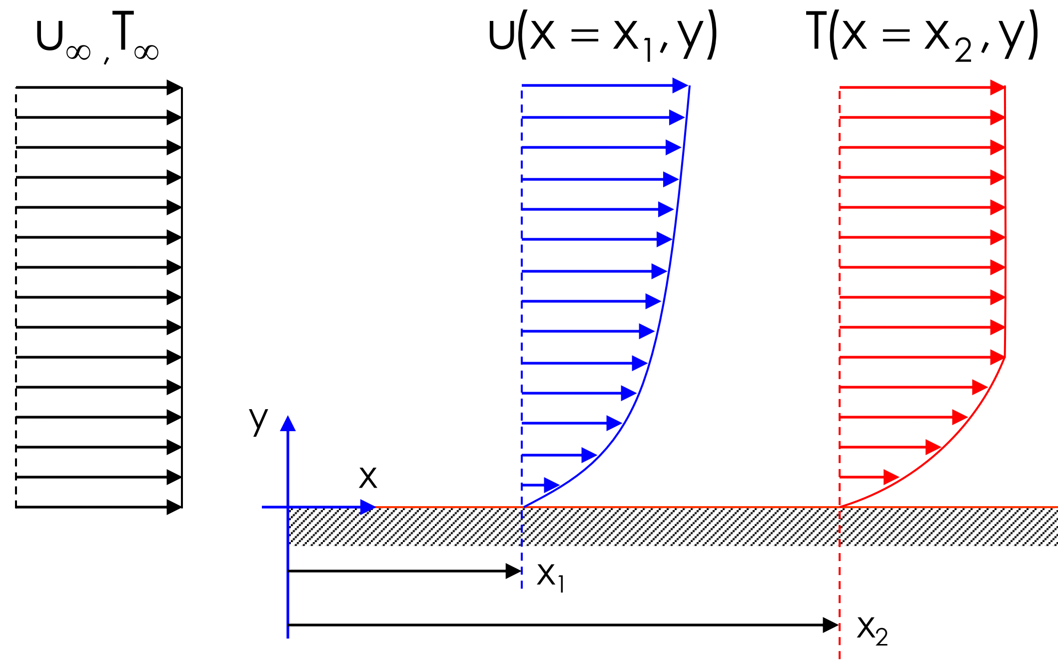

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \leftarrow \text{massa}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \qquad \leftarrow \text{q. de movimento}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2} \qquad \leftarrow \text{energia}$$

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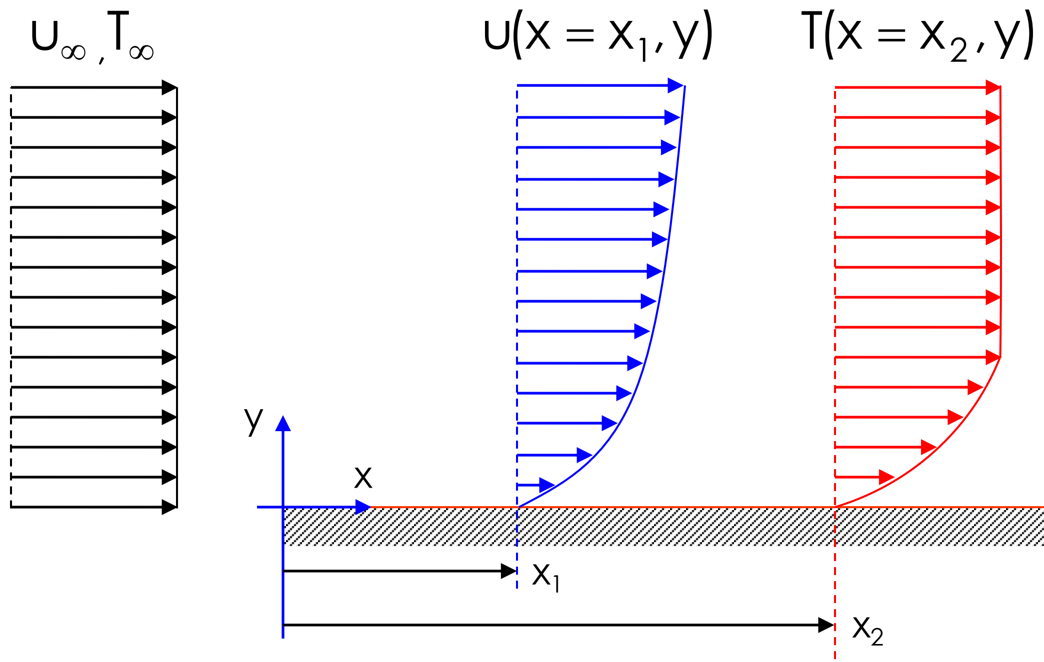

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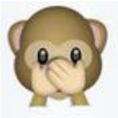
$$p/x = 0 \rightarrow u(0,y) = u_\infty \;\text{ e }\; T(0,y) = T_\infty$$

$$p/y = 0 \rightarrow u(x,0) = 0,\; v(x,0) = 0 \;\text{ e }\; T(x,0) = T_s$$

$$p/y \rightarrow \infty \rightarrow u(x,\infty) = u_\infty \;\text{ e }\; T(x,\infty) = T_\infty$$

Solução de Blasius (1908)... **velocidades**

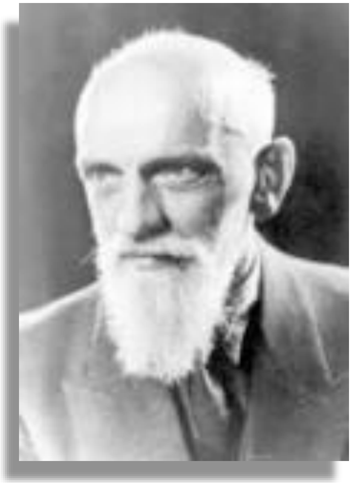


$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

transformação

$$a_0 \cdot f(\eta) + a_1 \frac{f(\eta)}{d\eta} + a_2 \frac{f^2(\eta)}{d\eta^2} + a_3 \frac{f^3(\eta)}{d\eta^3} + \ldots.$$



Paul R.H. Blasius
(1883 – 1970)

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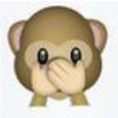

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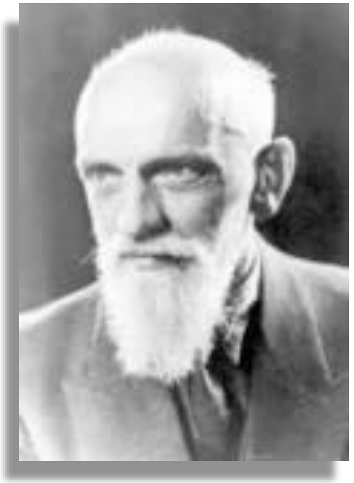

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variável de similaridade

$$\eta \overset{def}{=} y \cdot \sqrt{\frac{U_\infty}{x \cdot \mu / \rho}}$$

$$f(\eta) \overset{def}{=} \frac{\psi}{U_\infty \cdot \sqrt{\dfrac{x \cdot \mu / \rho}{U_\infty}}}$$

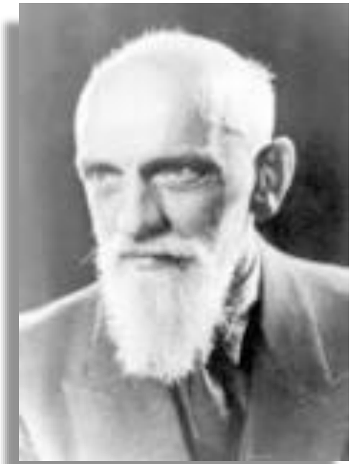
$\leftarrow \psi$ = função de corrente

Solução de Blasius (1908)... **velocidades**

$$\psi(x,y) \overset{\text{def}}{=} \begin{cases} u = +\dfrac{\partial \psi}{\partial y} \\[2ex] v = -\dfrac{\partial \psi}{\partial x} \end{cases} \Rightarrow$$

$$\equiv 0$$

função de corrente



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Solução de Blasius (1908)... **velocidades**

$$\underset{\text{função de corrente}}{\psi(x,y)} \overset{\text{def}}{=} \begin{cases} u = +\dfrac{\partial \psi}{\partial y} \\ v = -\dfrac{\partial \psi}{\partial x} \end{cases} \Rightarrow$$

$$\equiv 0$$

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As componentes da velocidade se escrevem então como:

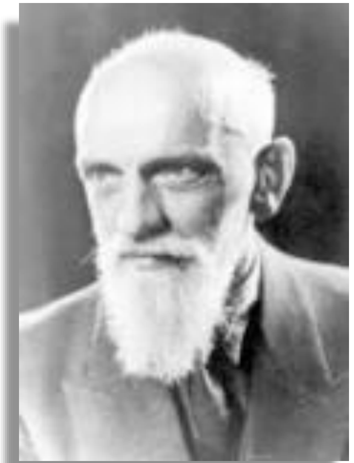
$$\eta = y \cdot \sqrt{\frac{U_\infty}{x \cdot \rho / \mu}}$$

$$\psi = U_\infty \sqrt{x \cdot \frac{\mu / \rho}{U_\infty}} \cdot f(\eta)$$

$$u = +\frac{\partial \psi}{\partial y} = +\left(\frac{\partial \psi}{\partial \eta}\right) \cdot \left(\frac{\partial \eta}{\partial y}\right) = \left(U_\infty \sqrt{\frac{x\nu}{U_\infty}} \cdot \frac{df}{d\eta}\right) \cdot \left(\sqrt{\frac{U_\infty}{x\nu}}\right) = U_\infty \frac{df}{d\eta}$$

$$\nu = \mu / \rho$$

Solução de Blasius (1908)... **velocidades**



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$$v = -\frac{\partial \psi}{\partial x} = -U_\infty \sqrt{\frac{\nu}{U_\infty}} \cdot f(\eta) \cdot \left(\frac{x^{-1/2}}{2}\right) - \left(U_\infty \sqrt{x \cdot \frac{\nu}{U_\infty}} \cdot \frac{\partial f}{\partial x}\right) = \frac{1}{2} \sqrt{\frac{U_\infty \nu}{x}} \cdot \left(\eta \frac{df}{d\eta} - f\right)$$

$$\nu = \mu / \rho$$

Solução de Blasius (1908)... **velocidades**

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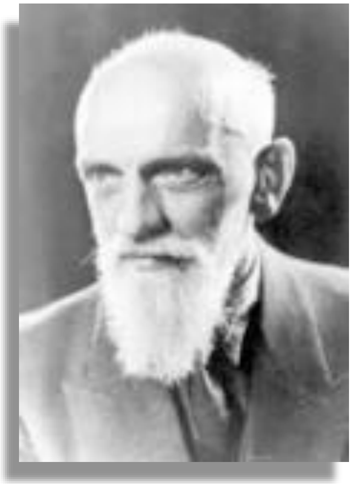
função de corrente



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$$u = \ldots$$
$$v = \ldots$$

$\Rightarrow$ 

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\mu}{\rho} \frac{\partial^2 u}{\partial y^2}$$

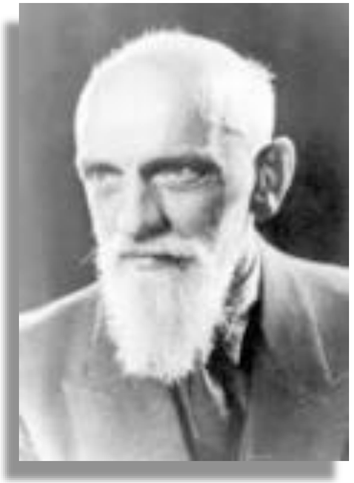
Solução de Blasius (1908)... **velocidades**

$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x}\eta\frac{d^2f}{d\eta^2}$$

$$\frac{\partial u}{\partial y} = u_\infty\sqrt{\frac{u_\infty}{\nu x}}\frac{d^2f}{d\eta^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x}\frac{d^3f}{d\eta^3}$$

$$\Rightarrow \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$



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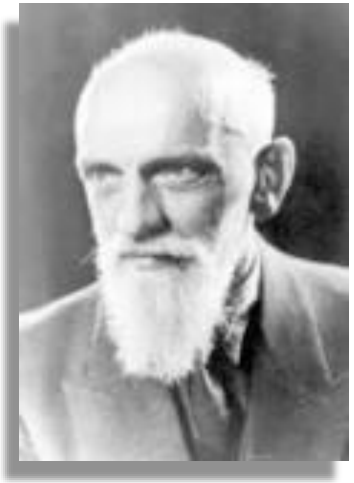
Solução de Blasius (1908)... **velocidades**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$$\Rightarrow \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$\Rightarrow \quad 2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0$$

Solução de Blasius (1908)... **velocidades**

$$\frac{\partial u}{\partial x} = -\frac{U_\infty}{2x}\eta\frac{d^2 f}{d\eta^2}$$

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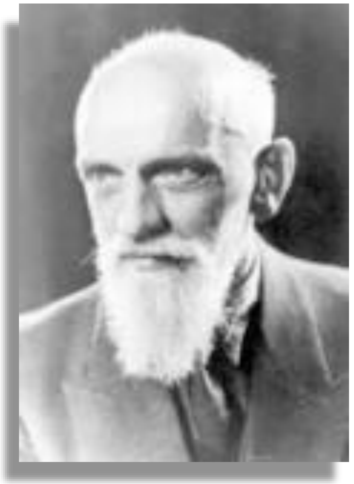

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$$p / x = 0 \rightarrow u(0, y) = U_\infty$$

$$p / y = 0 \rightarrow u(x,0) = 0, \;\; v(x,0) = 0$$

$$p / y \rightarrow \infty \rightarrow u(x, \infty) = U_\infty$$

$$\Rightarrow$$

$$p / \eta = 0 \rightarrow f(\eta) = 0$$

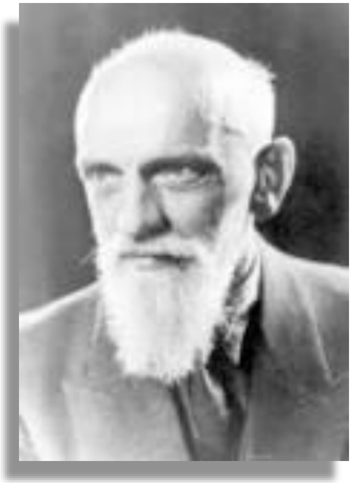
$$p / \eta = 0 \rightarrow df / d\eta = 0$$

$$p / \eta = \infty \rightarrow df / d\eta = 1$$

Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} = 0$$

η	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
∞	∞	1	0



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Solução de Blasius (1908)... **c. limite fluidodinâmica**

$$2\frac{d^3f}{d\eta^3}+f\frac{d^2f}{d\eta^2}=0$$

$$\delta \Rightarrow u/u_\infty = 0.99 \;\Rightarrow \eta = 5.0$$

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Solução de Blasius (1908)... **c. limite fluidodinâmica**

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$$\eta = y\cdot\sqrt{\frac{u_\infty}{x\nu}} \Rightarrow \quad \delta = 5.0\cdot\sqrt{\frac{x\nu}{u_\infty}}$$

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$$\eta = y \cdot \sqrt{\frac{u_\infty}{x\nu}} \Rightarrow \quad \delta = 5.0 \cdot \sqrt{\frac{x\nu}{u_\infty}}$$

$$\Rightarrow \quad \delta = \frac{5.0 \cdot x}{\sqrt{Re_x}} \qquad Re = \frac{\rho u_\infty x}{\mu}$$

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Solução de Blasius (1908)... **tensões de cisalhamento**

$$2\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} = 0$$

$$\tau_x = \mu\frac{\partial u}{\partial y}\bigg|_{y=0} = \mu U_\infty\sqrt{\frac{U_\infty}{\nu x}}\frac{d^2f}{d\nu^2}\bigg|_{\eta=0}$$

$$\tau_x = 0.332\frac{\rho U_\infty^2}{\sqrt{Re_x}}$$

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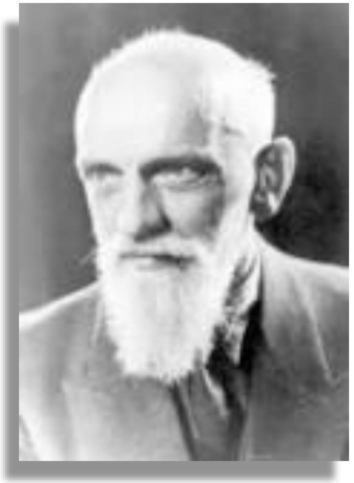
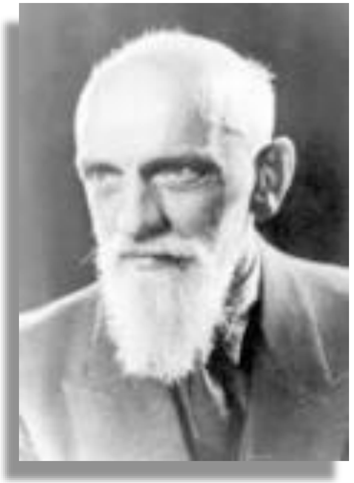


Paul R.H. Blasius
(1883 – 1970)

Solução de Blasius (1908)... **c. limite térmica**

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}$$



Paul R.H. Blasius
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$$\psi = u_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{u_\infty}} \cdot f(\eta)$$

Solução de Blasius (1908)... **c. limite térmica**

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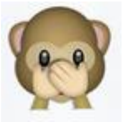
$$\psi(x,y) \overset{\text{def}}{=} \begin{cases} u = +\dfrac{\partial \psi}{\partial y} \\ v = -\dfrac{\partial \psi}{\partial x} \end{cases}$$



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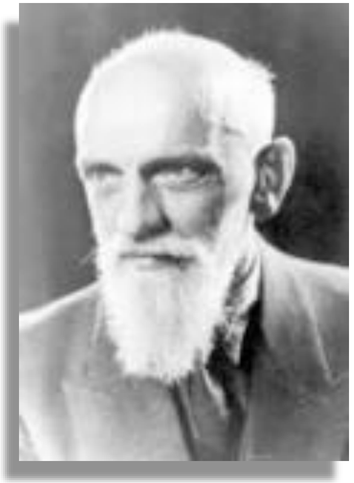

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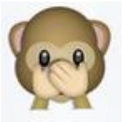


$$\theta(x,y) \overset{def}{=} \frac{T(x,y) - T_s}{T_\infty - T_s}$$

$$\psi = u_\infty \cdot \sqrt{\frac{x \cdot \mu / \rho}{u_\infty}} \cdot f(\eta)$$

Solução de Blasius (1908)... **c. limite térmica**

Paul R.H. Blasius
(1883 – 1970)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}$$

$$\psi(x,y)\overset{\text{def}}{=}\begin{cases} u=+\dfrac{\partial\psi}{\partial y}\\ v=-\dfrac{\partial\psi}{\partial x}\end{cases}$$

$$\theta(x,y)\overset{\text{def}}{=}\frac{T(x,y)-T_s}{T_\infty-T_s}$$

$$\psi=u_\infty\cdot\sqrt{\frac{x\cdot\mu/\rho}{u_\infty}}\cdot f(\eta)$$

$$u_\infty\frac{df}{d\eta}\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x}+\frac{1}{2}\sqrt{\frac{u_\infty\nu}{x}}\left(\eta\frac{df}{d\eta}-f\right)\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial y}=\frac{k}{\rho C_P}\frac{d^2\theta}{d\eta^2}\left(\frac{\partial\eta}{\partial y}\right)^2$$

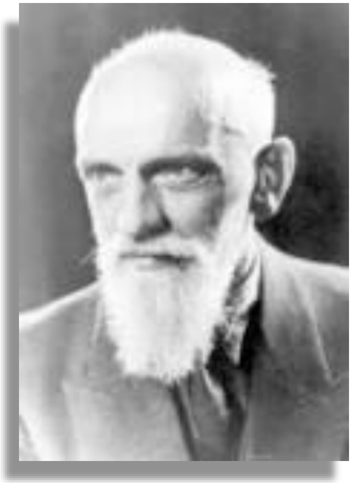
Solução de Blasius (1908)... **c. limite térmica**

$$U_\infty \frac{df}{d\eta}\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial x} + \frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}\left(\eta\frac{df}{d\eta}-f\right)\frac{\partial\theta}{\partial\eta}\frac{\partial\eta}{\partial y} = \frac{k}{\rho C_P}\frac{d^2\theta}{d\eta^2}\left(\frac{\partial\eta}{\partial y}\right)^2$$

$$\vdots$$

$$Pr = \frac{\mu}{k/C_P} \Rightarrow 2\frac{d^2\theta}{d\eta^2} + Pr \cdot f\frac{d\theta}{d\eta} = 0$$

$$\theta(0) = 0 \quad e \quad \theta(\infty) = 1$$



Paul R.H. Blasius
(1883 – 1970)

η	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
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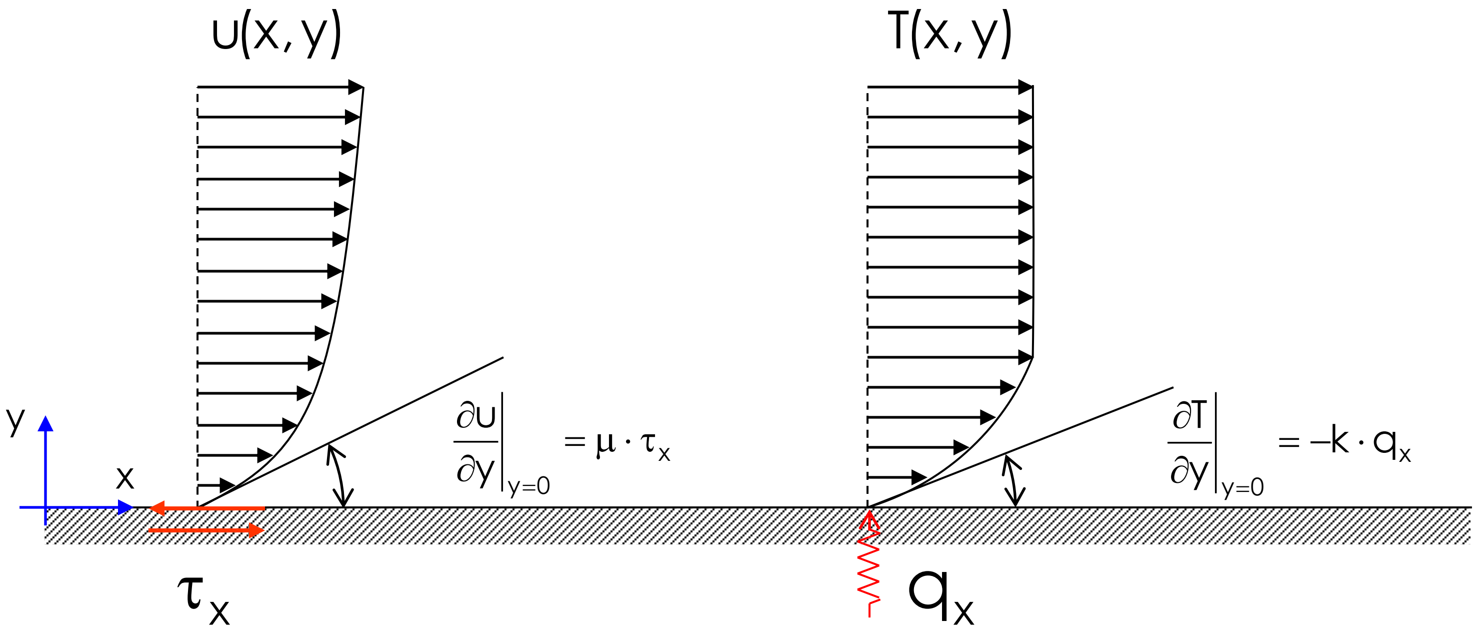
Solução de Blasius (1908)... **c. limite térmica**

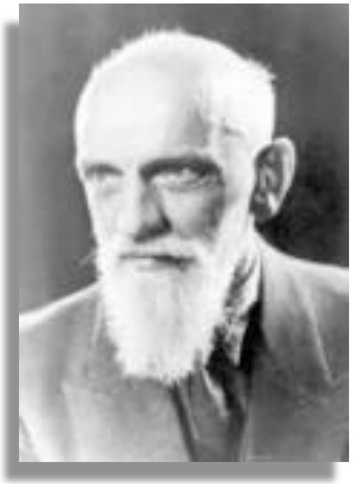


$$\tau_x = 0.332 \frac{\rho u_\infty^2}{\sqrt{Re_x}}$$

$$-\frac{q_x}{k} = 0.332 \cdot Pr^{1/3} (T_\infty - T_s) \sqrt{\frac{u_\infty}{\nu x}}$$

$$\delta_{FD} = \frac{5.0 \cdot x}{\sqrt{Re_x}}$$

$$\delta_{TM} = \frac{5.0 \cdot x}{Pr^{1/3} \sqrt{Re_x}}$$



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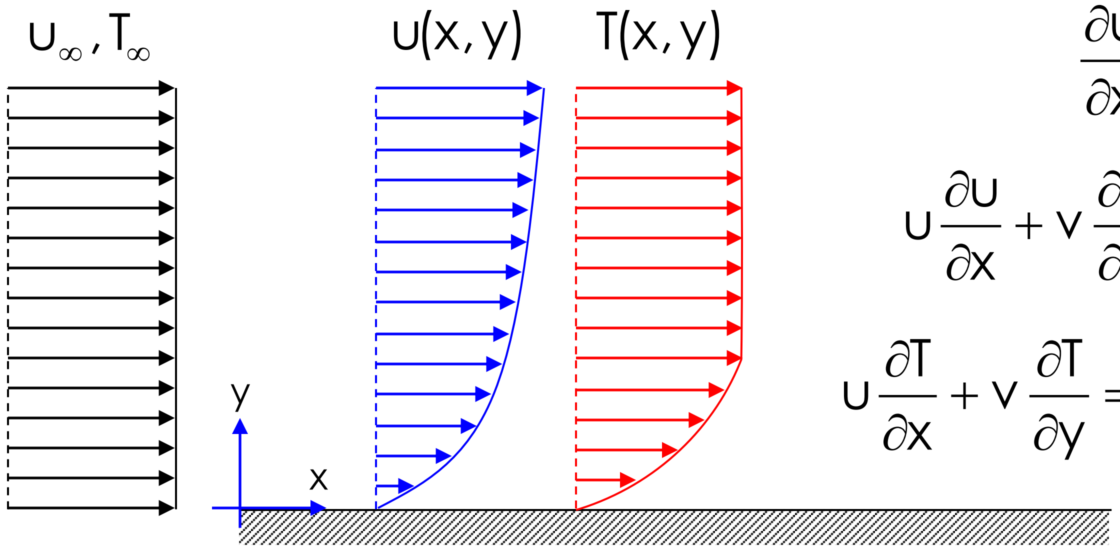
ADIMENSIONALIZAÇÃO DAS EQUAÇÕES GOVERNANTES

Adimensionalização das equações governantes...



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}$$

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L}$$

$$u^* = \frac{u}{u_\infty} \qquad v^* = \frac{v}{u_\infty} \qquad P^* = \frac{P}{\rho u_\infty^2}$$

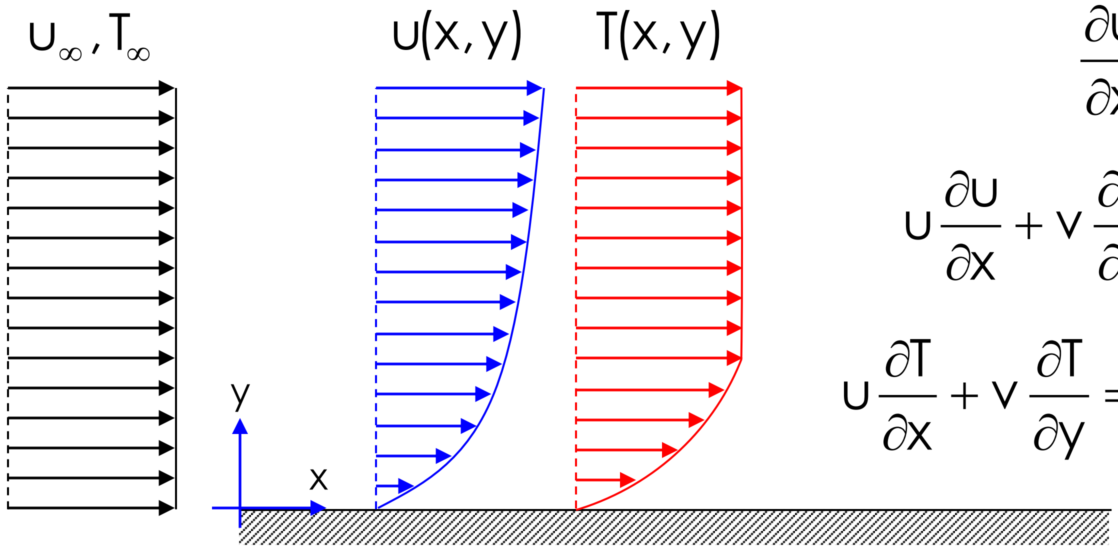
$$T^* = \frac{T - T_2}{T_\infty - T_s}$$

$$\frac{\partial u}{\partial x} = \frac{u_\infty}{L}\frac{\partial u^*}{\partial x^*}$$

$$\frac{\partial v}{\partial y} = \frac{u_\infty}{L}\frac{\partial v^*}{\partial y^*}$$

$$\Rightarrow \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \rightarrow \boxed{\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0}$$

Adimensionalização das equações governantes...



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C_P}\frac{\partial^2 T}{\partial y^2}$$

$$x^* = \frac{x}{L} \qquad y^* = \frac{y}{L}$$

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$$T^* = \frac{T - T_2}{T_\infty - T_s}$$

$$\frac{\partial u}{\partial x} = \frac{u_\infty}{L}\frac{\partial u^*}{\partial x^*}$$

$$\frac{\partial v}{\partial y} = \frac{u_\infty}{L}\frac{\partial v^*}{\partial y^*}$$

$$\Rightarrow \quad u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \rightarrow u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = \frac{1}{Re}\frac{\partial^2 u^*}{\partial y^{*2}}$$

Adimensionalização das equações governantes...

$$\frac{\partial u}{\partial x} = \frac{u_\infty}{L}\frac{\partial u^*}{\partial x^*}$$

$$\frac{\partial v}{\partial y} = \frac{u_\infty}{L}\frac{\partial v^*}{\partial y^*}$$

$$\frac{\partial P}{\partial x} = \frac{\rho u_\infty^2}{L}\frac{\partial P^*}{\partial x^*}$$

$$\frac{\partial P}{\partial y} = \frac{\rho u_\infty^2}{L}\frac{\partial P^*}{\partial y^*}$$

$\Rightarrow$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{U}) = 0$$

$$\rho\cdot\left(\frac{\partial \vec{U}}{\partial t} + \vec{U}\cdot\vec{\nabla}\vec{U}\right) = -\vec{\nabla}P + \mu\nabla^2\vec{U} + \sum F_k$$

$$\rho C_P\left(\frac{\partial T}{\partial t} + \vec{U}\cdot\vec{\nabla}T\right) = \vec{\nabla}\cdot(k\vec{\nabla}T)$$

Adimensionalização das equações governantes...

$$\frac{\partial u}{\partial x} = \frac{U_\infty}{L}\frac{\partial u^*}{\partial x^*}$$

$$\frac{\partial v}{\partial y} = \frac{U_\infty}{L}\frac{\partial v^*}{\partial y^*}$$

$$\frac{\partial P}{\partial x} = \frac{\rho U_\infty^2}{L}\frac{\partial P^*}{\partial x^*}$$

$$\frac{\partial P}{\partial y} = \frac{\rho U_\infty^2}{L}\frac{\partial P^*}{\partial y^*}$$

$\Rightarrow$

$$\frac{\partial \rho^*}{\partial t^*} + \vec{\nabla}\cdot(\rho^* \vec{U}^*) = 0$$

$$\rho^* \cdot \left(\frac{\partial \vec{U}^*}{\partial t^*} + \vec{U}^* \cdot \vec{\nabla}\vec{U}^*\right) = -\vec{\nabla}P^* + \frac{1}{Re}\nabla^2\vec{U}^* + \sum \frac{1}{R_k}F_k^*$$

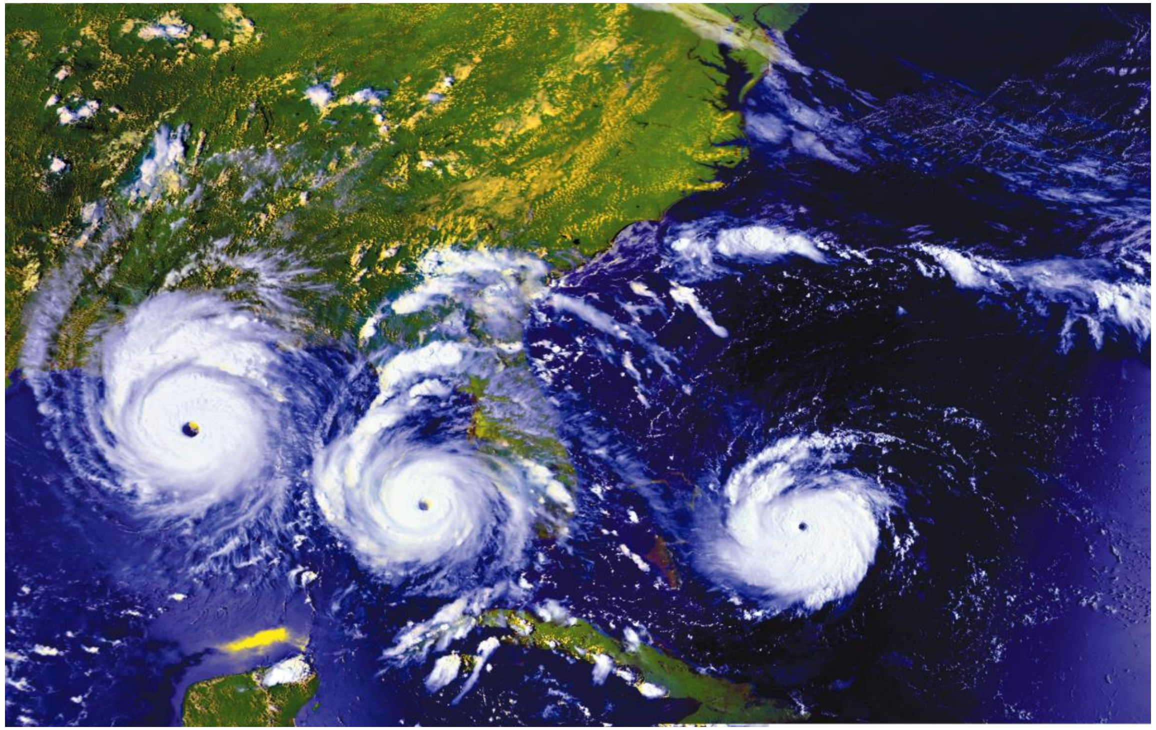
$$\left(\frac{\partial T^*}{\partial t^*} + \vec{U}^* \cdot \vec{\nabla}T^*\right) = \frac{1}{Re\cdot Pr}\vec{\nabla}\cdot\vec{\nabla}T^*$$

$$Re = \frac{\rho U_0 D}{\mu} \rightarrow \frac{\text{inércia}}{\text{d.viscosa}}$$

$$Pr = \frac{\mu}{k/C_P} \rightarrow \frac{\text{d.viscosa}}{\text{d.térmica}}$$

As equações governam o fenômeno via leis de conservação. Os números adimensionais definem o comportamento na escala do problema modelado...

Adimensionalização das equações governantes...

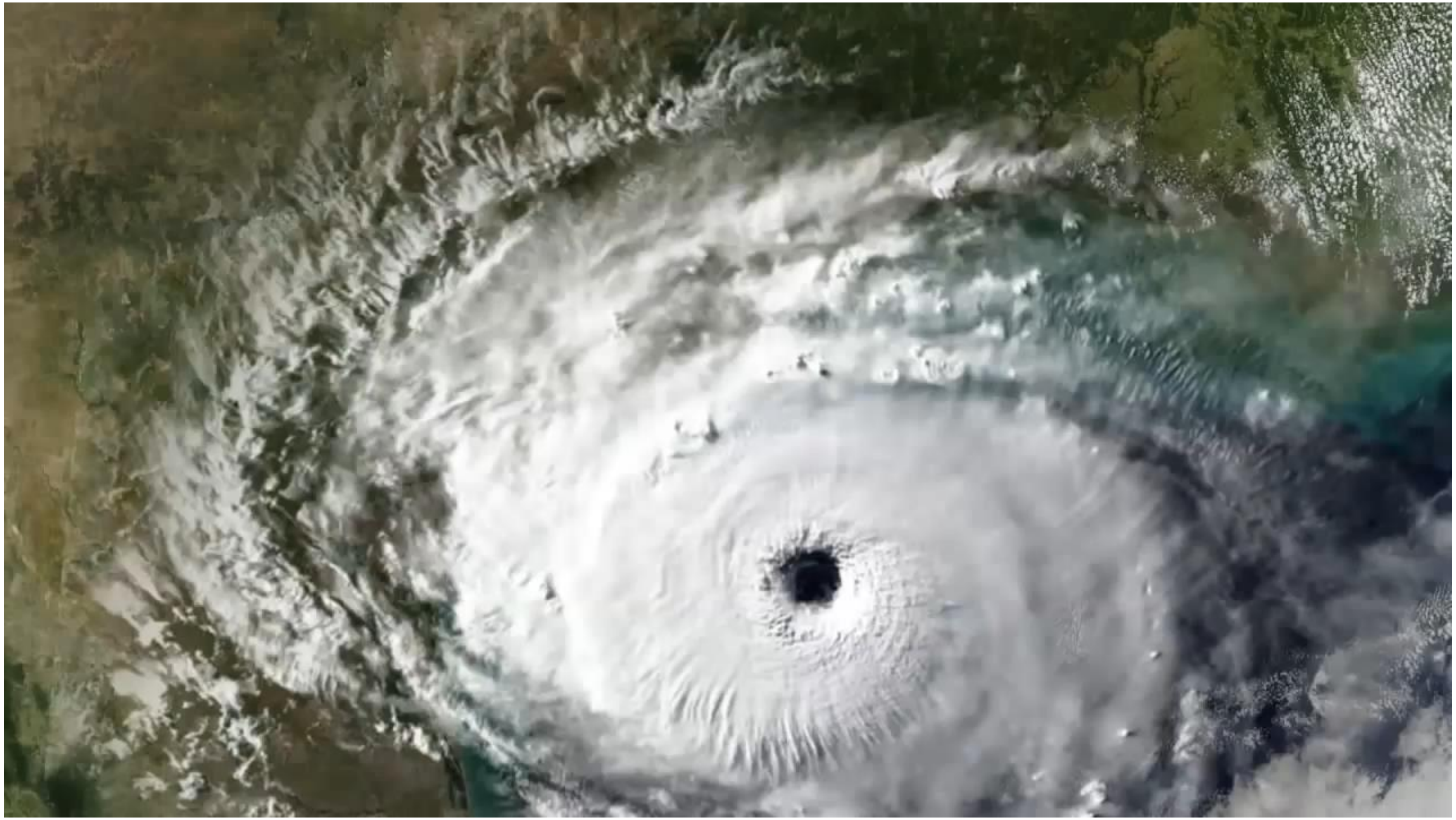


Ω = rotação planetária
φ = latitude

Mesmas equações adimensionais de balanço de massa, q. de movimento e energia porém, com diferentes números adimensionais

$$Re = \frac{\rho u_0 D}{\mu} \quad Pr = \frac{C_p \mu}{k} \quad Eu = \frac{P_0}{\rho u_0^2} \quad Fr = \frac{u_0}{\sqrt{gD}} \quad Ec = \frac{u_0^2}{C_P (T_s - T_\infty)} \quad Ro = \frac{U/L}{2\Omega \sin\varphi}$$

Por que os furacões giram no sentido anti-horário no hemisfério norte e no sentido horário no hemisfério sul ???



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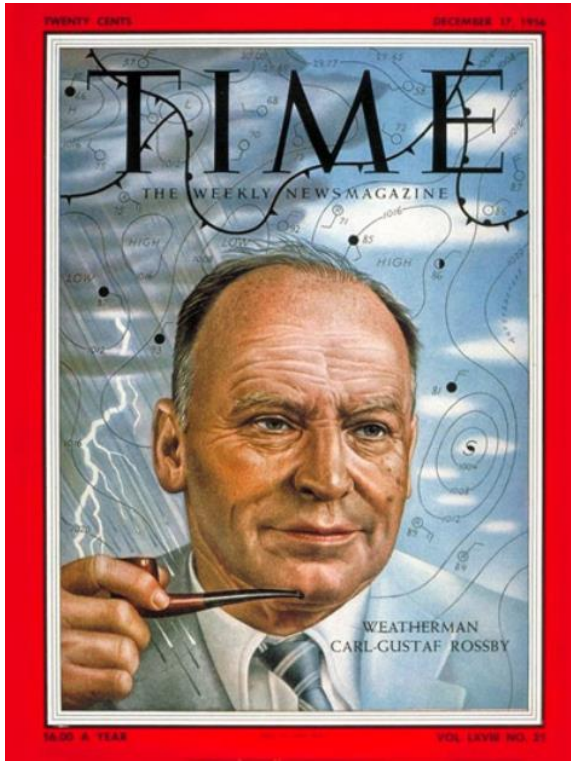
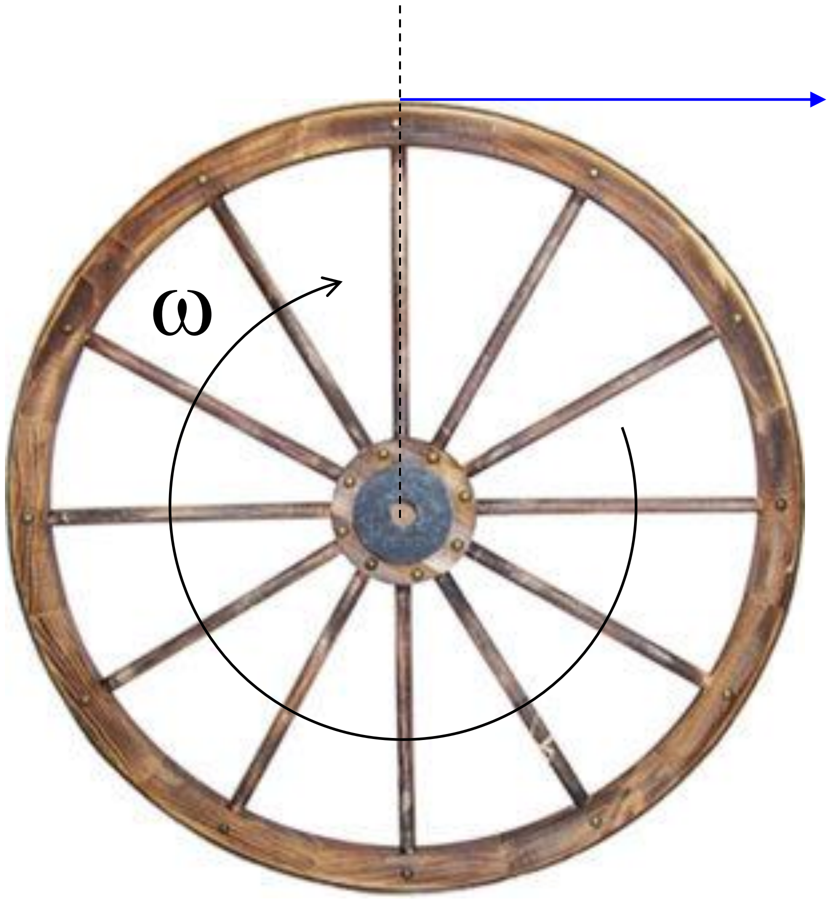
Gaspard-Gustave de Coriolis



Carl-Gustaf Rossby

Fenômenos atmosféricos de grande escala...

Por que os furacões giram no sentido anti-horário no hemisfério norte e no sentido horário no hemisfério sul ???



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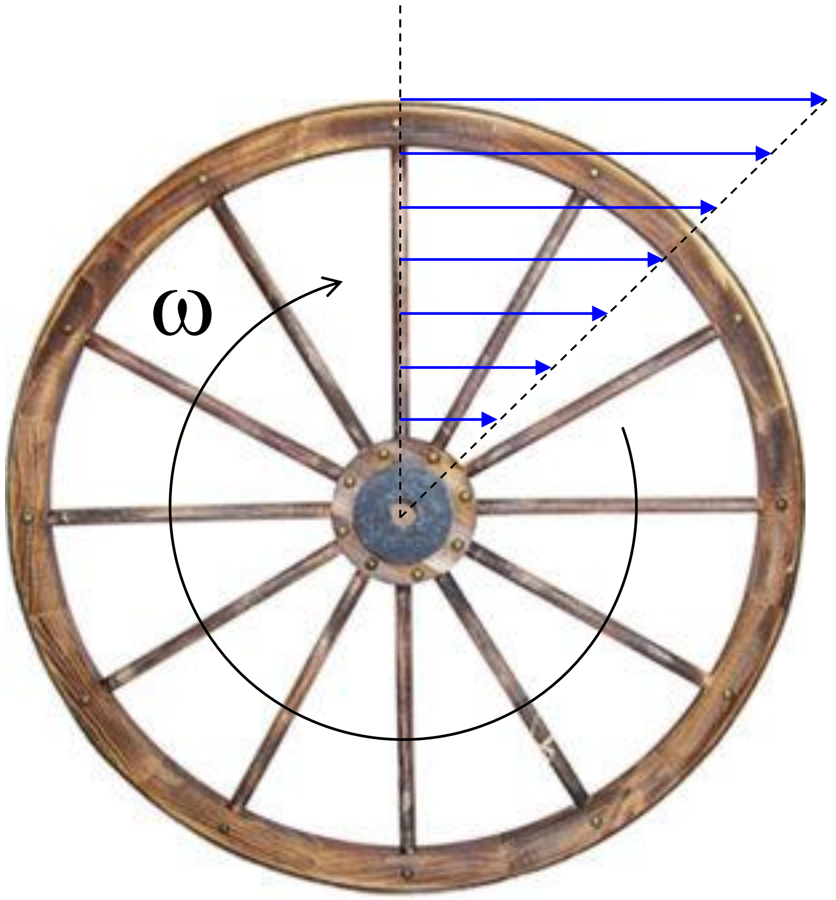
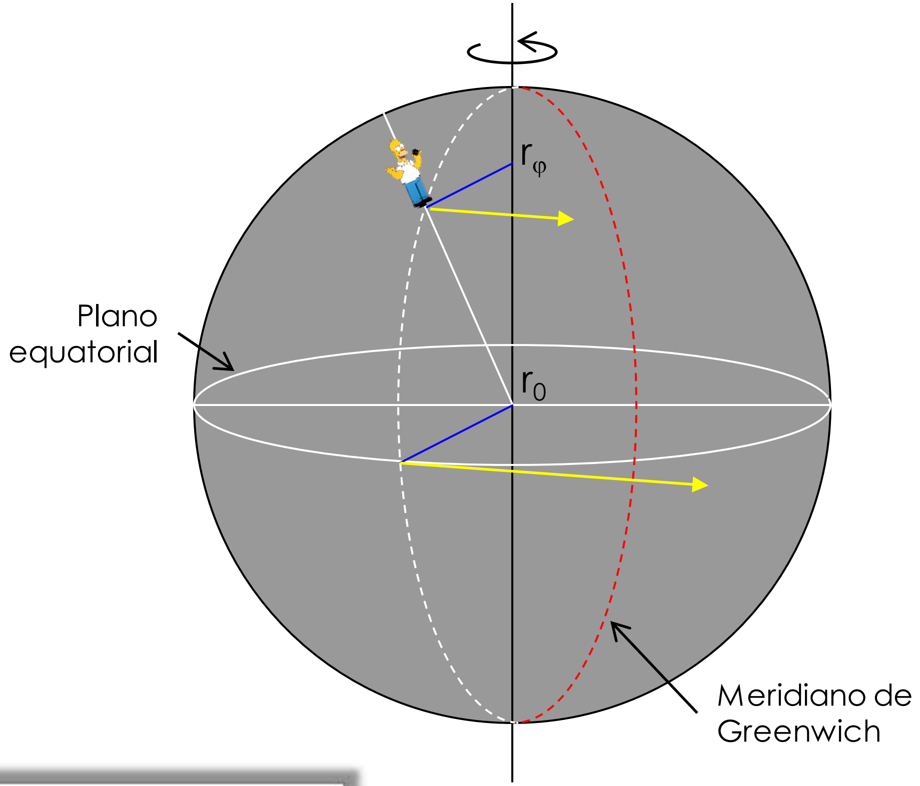


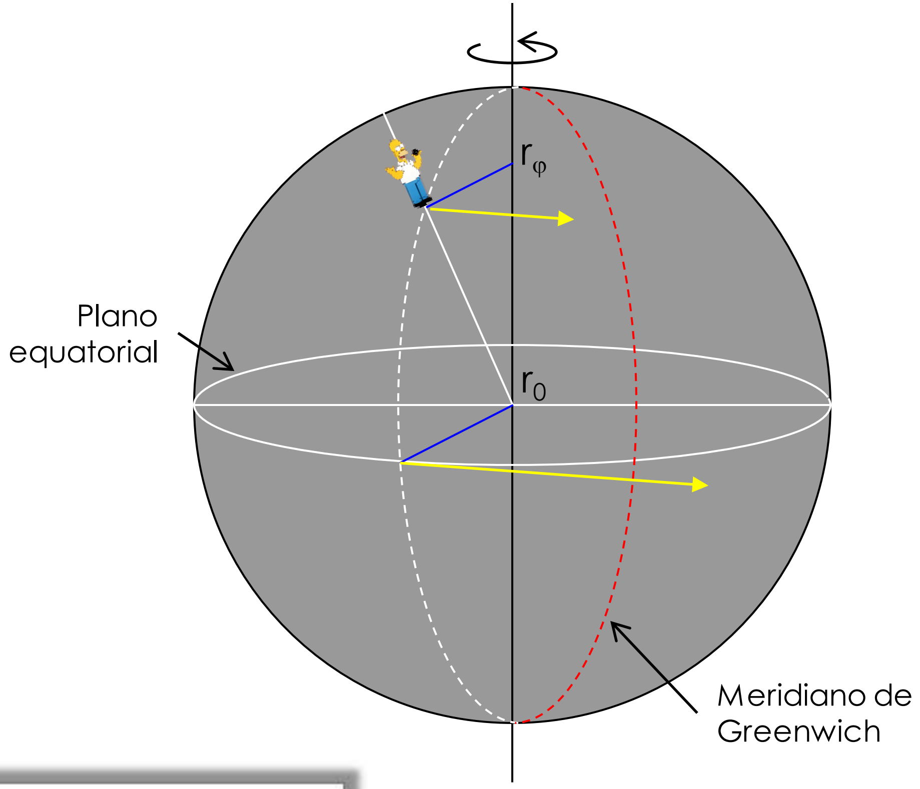
$$V = \omega \cdot r$$

Por que os furacões giram no sentido anti-horário no hemisfério norte e no sentido horário no hemisfério sul ???

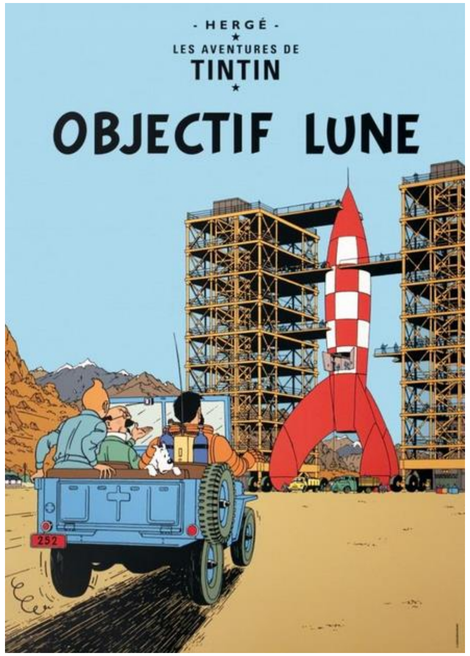


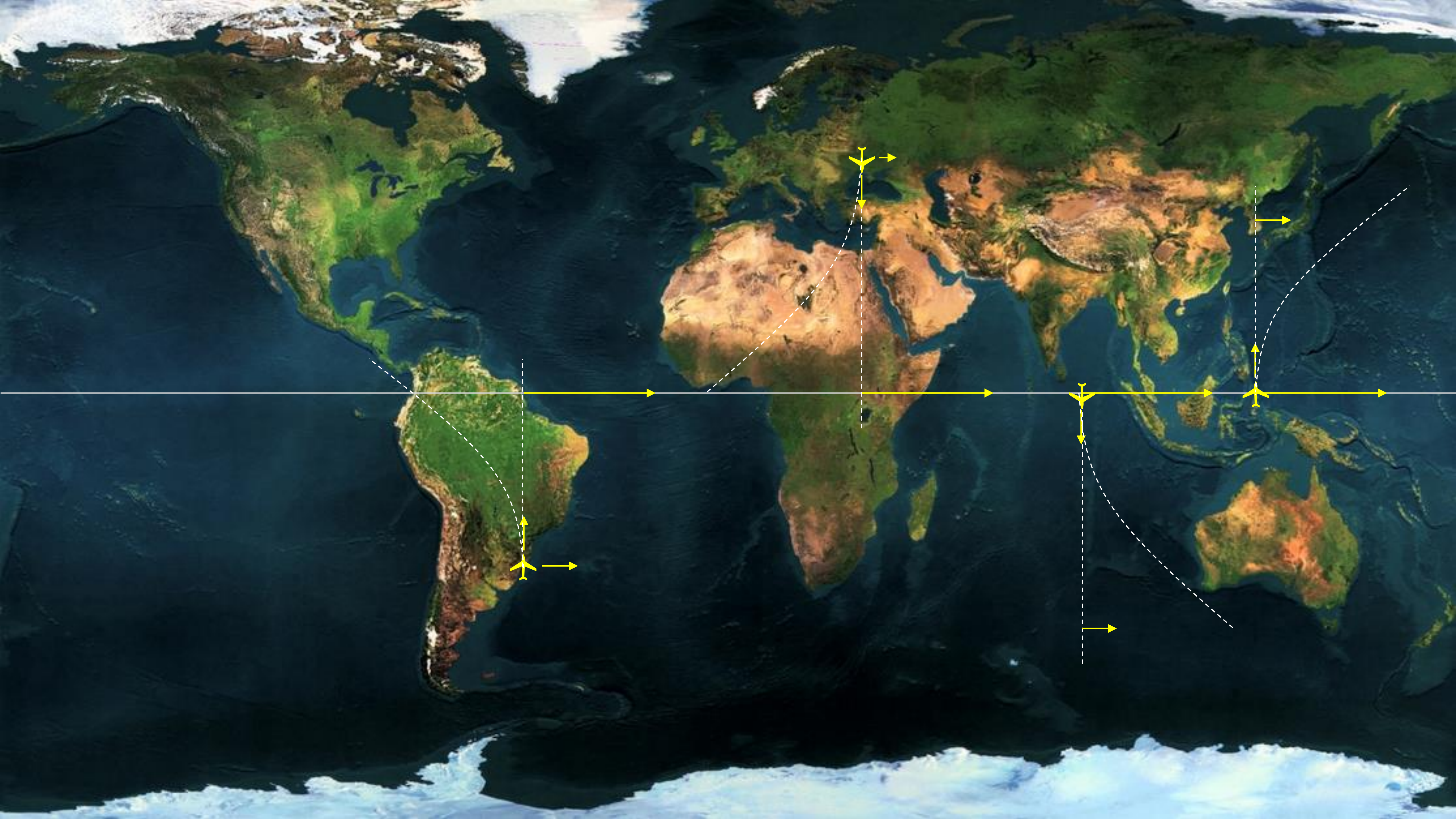
A velocidade tangencial é máxima ao longo do equador e decresce na medida em que a latitude aumenta...

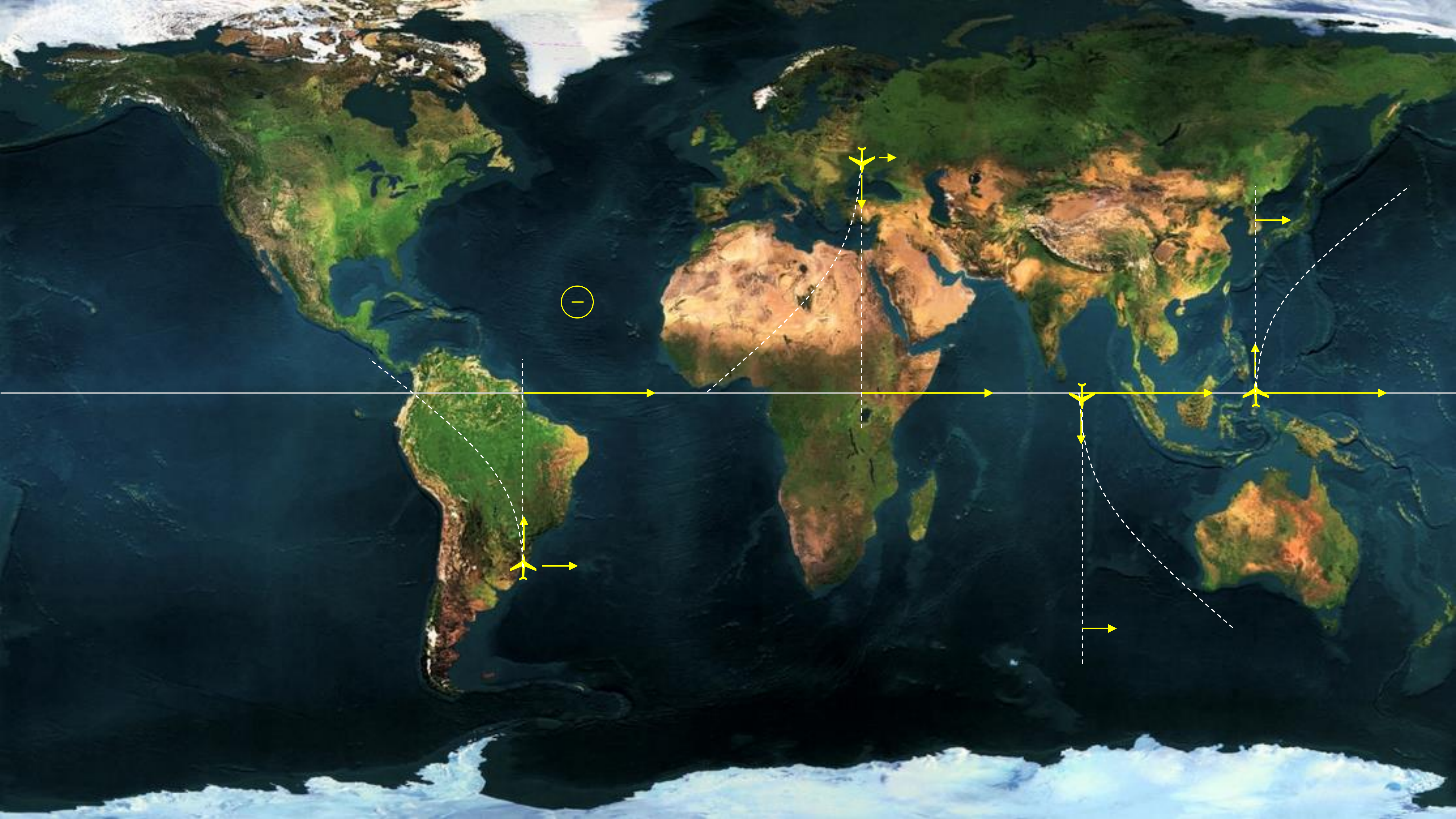
Por que os furacões giram no sentido anti-horário no hemisfério norte e no sentido horário no hemisfério sul ???

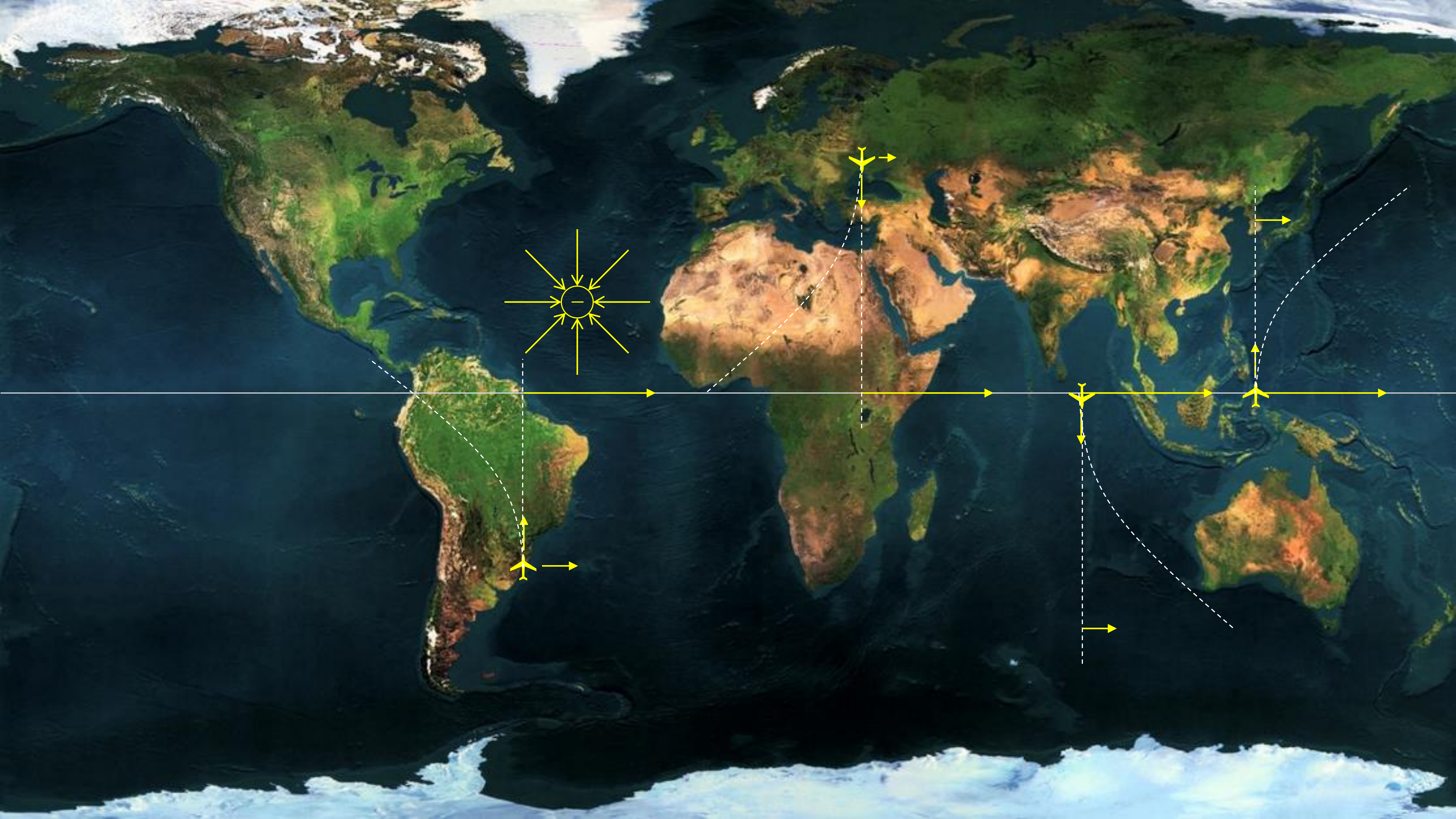


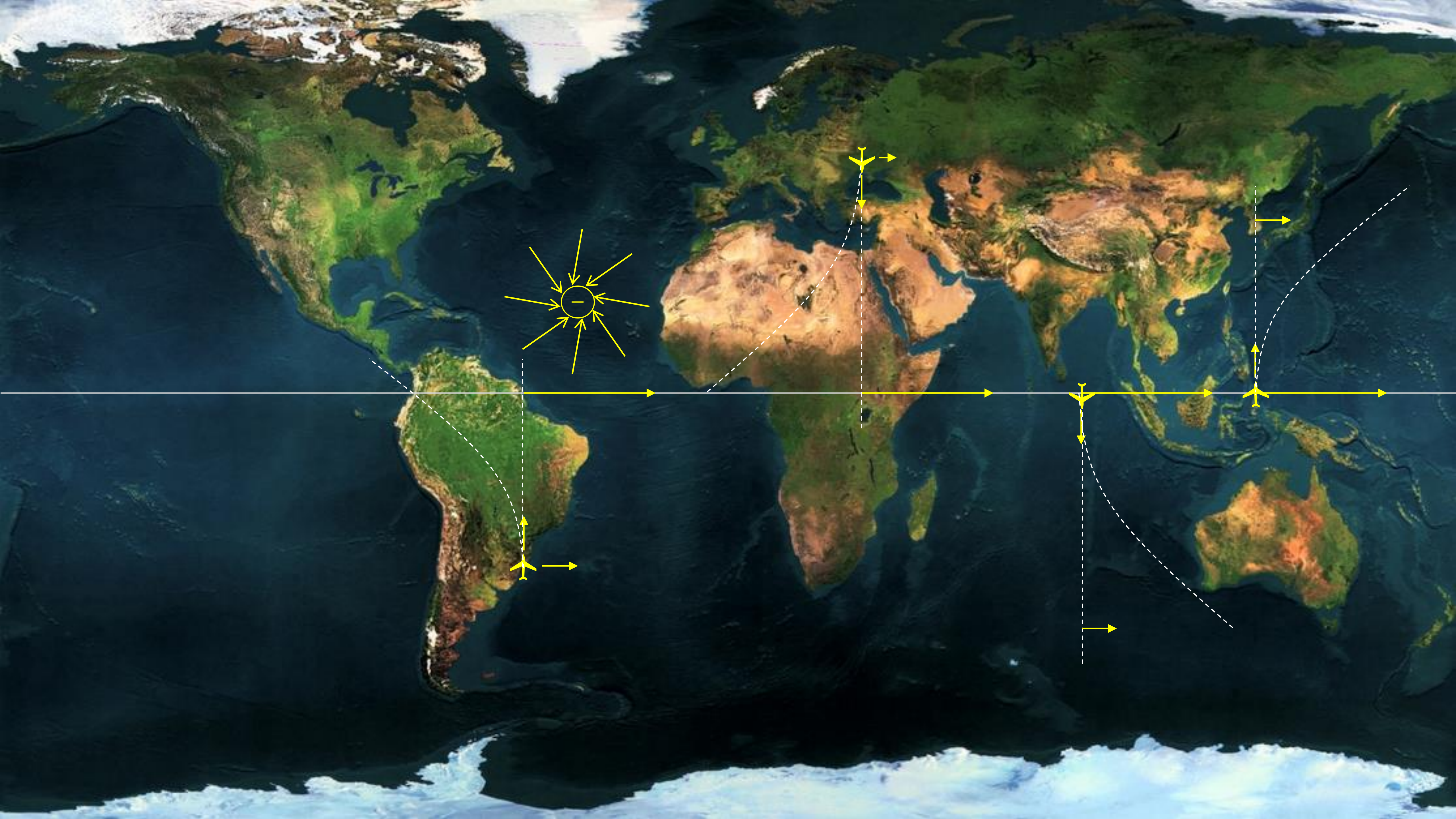
A velocidade tangencial é máxima ao longo do equador e decresce na medida em que a latitude aumenta... ("vitesse de libération...")

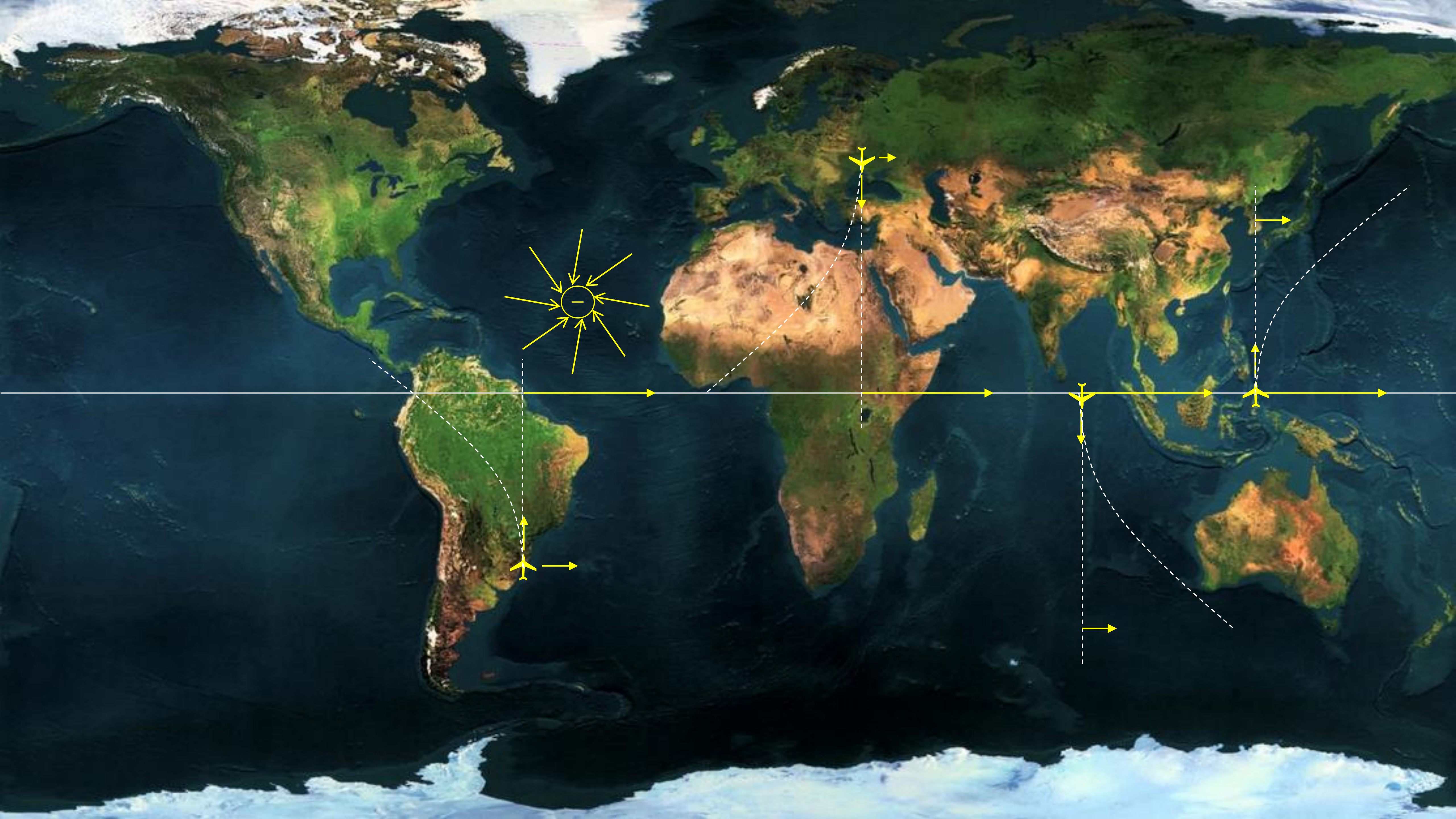


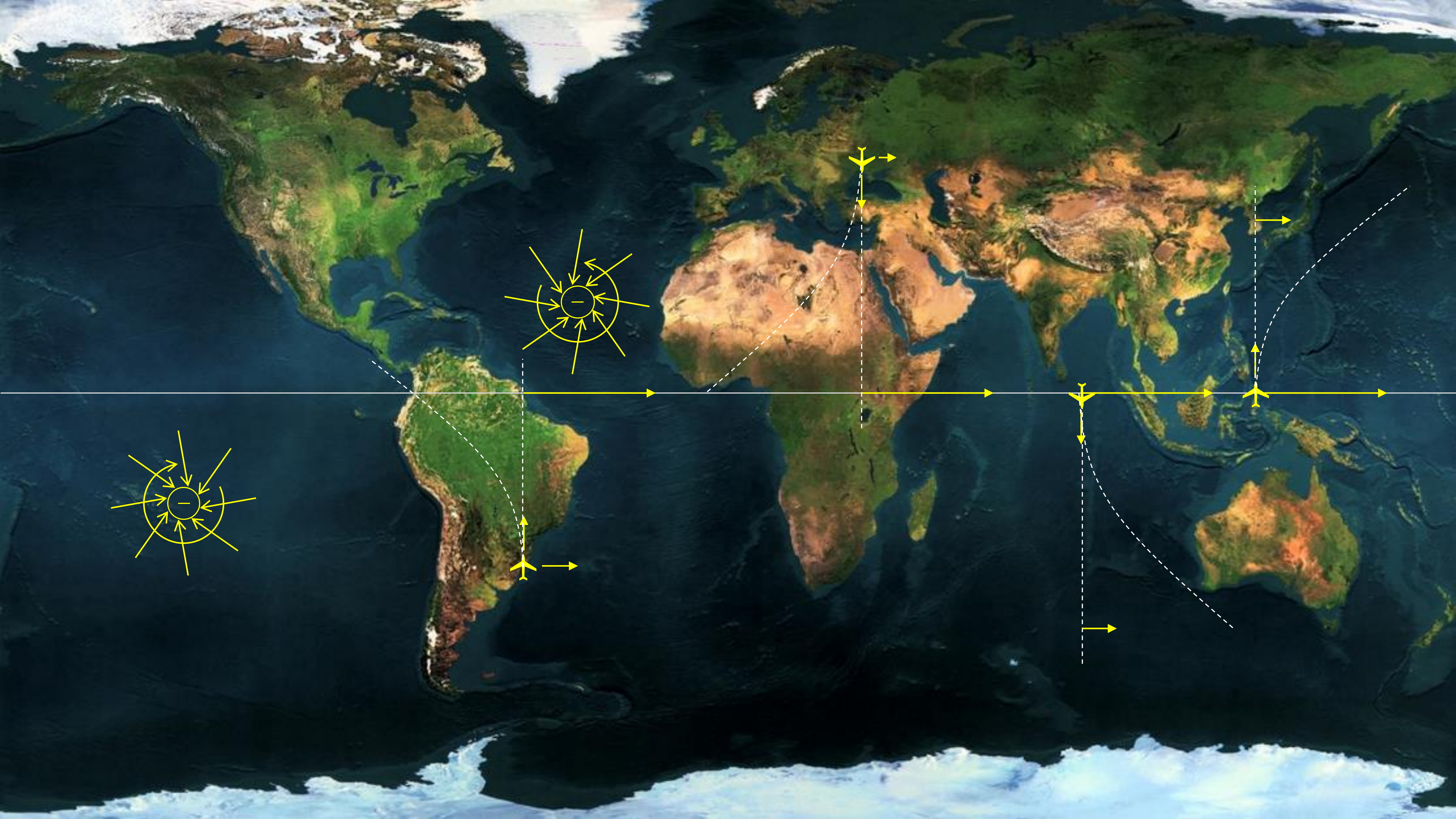


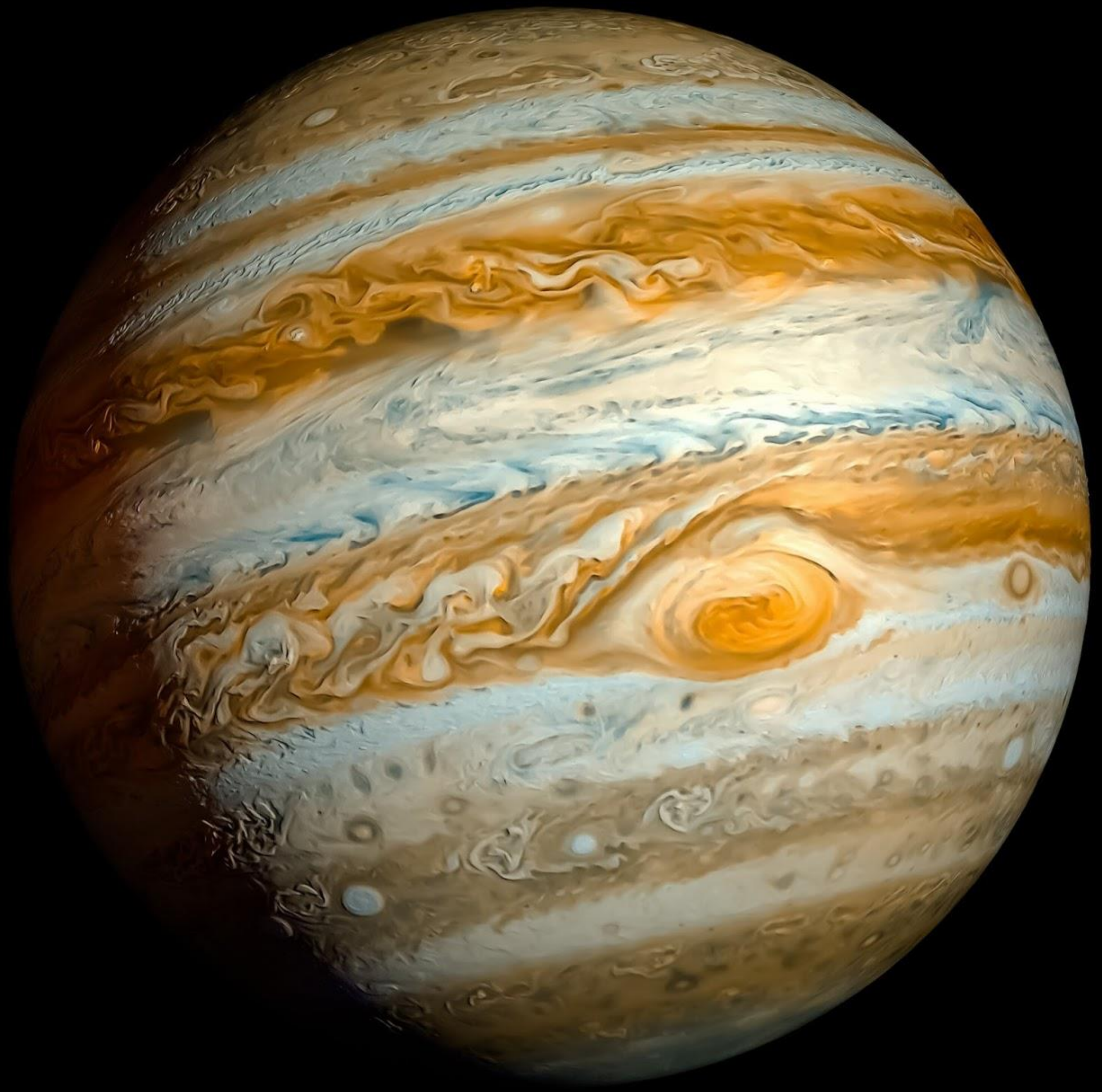




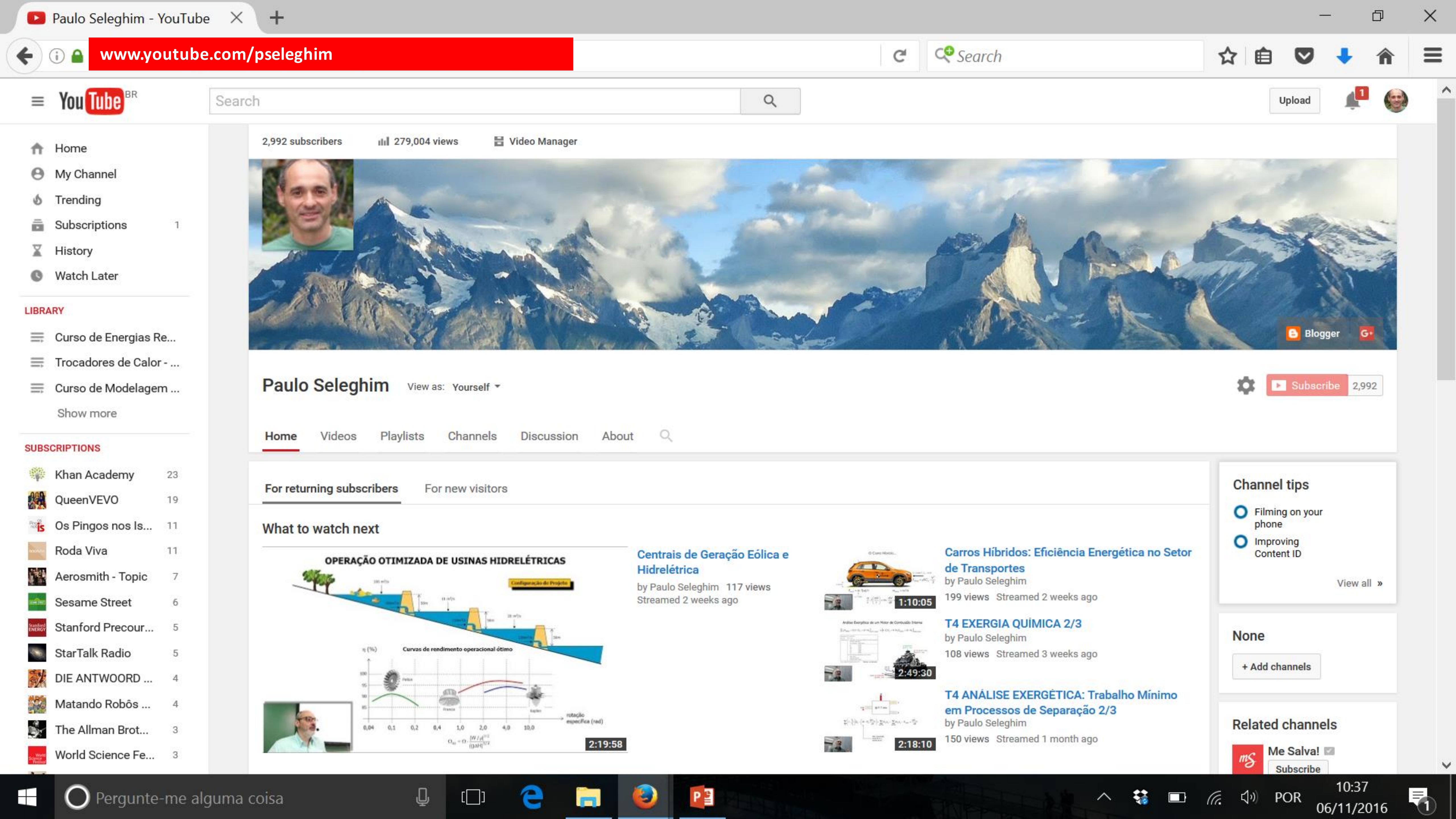








Le pôle sud de Jupiter, photographié par la sonde Cassini...
https://saturn.jpl.nasa.gov



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