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# Heat, mass and momentum transfer

Chapter 2 considered radiative energy exchange between plants and their environment. Other ways in which plants interact with their aerial environment include the transfer of matter, heat and momentum. The mechanisms involved in mass transfer processes, such as the exchanges of CO<sub>2</sub> and water vapour between plant leaves and the atmosphere, and in heat transfer are very closely related so will be treated together. These can be broadly divided into those operating at a molecular level which do not involve mass movement of the medium (i.e. diffusion of matter and conduction of heat) and those processes, generally termed convection, where the entity is transported by mass movement of the fluid. The forces exerted on plants by the wind are a manifestation of momentum transfer.

Clear discussion of heat and mass transfer processes may be found in Campbell (1977) and in Monteith & Unsworth (1990) with more advanced treatments in Monteith (1975) and Edwards et al. (1979). The physical principles underlying these transfer processes and the analogies between them are outlined in this chapter, and this information is used to analyse transfer between the atmosphere and both single leaves and whole canopies. Although the principles described are applicable to transfer in any fluid, the examples in this chapter will be confined to transfer in air.

### Measures of concentration

Before going into details of the different mechanisms of heat and mass transfer it is necessary to define what is meant by concentration. In general the spontaneous transfer of mass, or other entities such as heat or momentum, occurs from a region of high 'concentration' to one of low 'concentration'. There are, however, many alternative ways in which one can specify the amount or concentration of an entity 'i' in a mixture, each of which may be appropriate for certain purposes, as can be seen in the following discussion.

1. Concentration. A widely used measure of composition is the (mass) concentration  $(c_i)$  or density  $(\rho_i)$  where

$$c_i = \rho_i = \text{mass of i per unit volume of mixture}$$
 (3.1)

Alternatively one can use the molar concentration  $(c_i^m)$ 

$$c_i^{\rm m}$$
 = number of moles of i per unit volume of mixture =  $c_i/M_i$  (3.2)

where  $M_i$  is the molecular weight. Although concentration is often used as a fundamental measure of gas composition, in a closed system concentration changes with temperature or pressure as these factors alter the volume according to the ideal gas laws:

$$PV = n\Re T \tag{3.3}$$

where n is the number of moles present, T is the absolute temperature, P is the pressure, V is the volume and  $\mathcal{R}$  is the universal gas constant. Because liquids are not as compressible as gases, concentration is much less sensitive to pressure or temperature in solutions.

2. Mole fraction. A more conservative measure of composition is the mole fraction  $(x_i)$ , which is the number of moles of i  $(n_i)$  as a fraction of the total number of moles present in the mixture  $(\Sigma n)$ :

$$x_i = n_i / \Sigma n \tag{3.4}$$

In this case alterations in temperature, pressure or volume do not affect the mole fraction as they affect all components equally. A related measure appropriate for gases is partial pressure  $(p_i)$ , which for any component is the pressure that it would exert if allowed to occupy the whole volume available. The equivalence with mole fraction follows from combining the ideal gas law with *Dalton's Law of Partial Pressures* which states that, in a gas mixture of several components, the total pressure equals the sum of the partial pressures of the components, therefore

$$x_{i} = p_{i}/P \tag{3.5}$$

Using the above relationships it may easily be shown that gas concentration is related to partial pressure by

$$c_{i} = \frac{\text{mass}_{i}}{V} = \frac{n_{i} M_{i}}{V} = \frac{p_{i} M_{i}}{\Re T}$$
(3.6)

3. Mass fraction. Another useful term is the mass fraction  $(m_i)$ :

$$m_{\rm i} = {\rm mass~of~i~per~unit~total~mass~of~mixture} = c_{\rm i}/\rho$$
 (3.7)

where  $\rho$  is the density of the mixture. This is also independent of temperature and pressure. The mass fraction is related to mole fraction by

$$m_{\rm i} = x_{\rm i} M_{\rm i}/M \tag{3.8}$$

where M is the average molecular weight of the mixture.

- 4. Volume fraction. For a gas the volume fraction (the volume of i per unit total volume of mixture) is identical to the mole fraction.
- 5. Mixing ratio. A term common in the meteorological literature to describe the composition of air is the mixing ratio  $(w_i)$  where

$$w_i = \text{mass of i per (total mass} - \text{mass of i)}$$
 (3.9)

### Molecular transport processes

Diffusion - Fick's First Law

The rapid thermal motions of the individual molecules in a fluid lead to random rearrangement of molecular position and, in an inhomogeneous fluid, to transfer of mass and heat. This process is called diffusion. For example, in a motionless fluid, mass transfer occurs as a result of the net movement of molecules of one species from any area of high concentration to one of lower concentration. In a one-dimensional system, the flux density or rate of mass transfer  $(J_i)$  of an entity i per unit area through a plane is directly related to the concentration gradient  $(\partial c_i/\partial x)$  of i across the plane by a constant called the diffusion coefficient  $(D_i)$ . This can be written mathematically as

$$\mathbf{J}_{i} = -\mathbf{D}_{i} \frac{\partial c_{i}}{\partial \mathbf{x}} \tag{3.10}$$

This is the one-dimensional form of Fick's First Law of Diffusion. The minus sign is a mathematical convention to show that the flux is in the direction of decreasing concentration. Corresponding equations can be written for transfer in more than one dimension, but in what follows only the one-dimensional case will be treated.

Although it is common to use the concentration gradient as the driving force for diffusion as in equation 3.10, and this will be done in much of what follows, it can be inadequate for precise work when other factors are varying. For example, in solutions that depart significantly from ideal behaviour one needs to replace concentration by activity (see physical chemistry texts, e.g. Atkins 1990). Similarly in gases where there is a temperature gradient between the source and the sink, use of concentration can lead to significant errors (Cowan 1977). This is because the rate of diffusion depends on the rate at which the individual molecules move (a function of temperature) as well as on their concentration. The use of mole fraction, partial pressure or mass fraction takes this effect into account. By using the appropriate substitutions for  $c_i$  (equations 3.7, 3.6 and 3.5)

equation 3.10 may be rewritten in any of the following forms which are more appropriate for non-isothermal gases:

$$\mathbf{J}_{i} = -\mathbf{D}_{i} \rho \frac{\partial m_{i}}{\partial \mathbf{x}} \tag{3.11}$$

$$\mathbf{J}_{i} = -\mathbf{D}_{i} \frac{M_{i}}{\Re T} \frac{\partial p_{i}}{\partial \mathbf{x}} \tag{3.12}$$

$$\mathbf{J}_{i} = -\mathbf{D}_{i} \frac{PM_{i}}{\mathcal{R}T} \frac{\partial x_{i}}{\partial x} \tag{3.13}$$

These equations may appear similar to equation 3.10 since, for example,  $\rho m_i = c_i$ . However,  $\rho \Delta m_i$  is not necessarily identical to  $\Delta c_i$  (where  $\Delta$  represents a finite difference) so they can provide a significant improvement in non-isothermal systems. Unfortunately even these equations still involve some simplification.

#### Heat conduction

Heat transfer by conduction is analogous to diffusion. Conduction is the transfer of heat along a temperature gradient from a region of higher temperature (or kinetic energy) to one of lower temperature, without mass movement of the medium. In solids this energy transfer occurs as a result of molecular collisons transferring kinetic energy between molecules that are not themselves displaced, while in fluids the higher energy molecules themselves may diffuse.

Conductive heat transfer is described by *Fourier's Law*, where the rate of sensible heat transfer per unit area (C, with units of W  $m^{-2} = J m^{-2} s^{-1}$ ) is given by

$$\mathbf{C} = -k \frac{\partial T}{\partial \mathbf{x}} \tag{3.14}$$

where k is the **thermal conductivity** (W m<sup>-1</sup> K<sup>-1</sup>). Although the driving force for heat transfer is the temperature gradient, it is convenient to make a simple mathematical manipulation so as to obtain the proportionality constant in the same units as were used for mass transfer (Monteith & Unsworth 1990). If T is replaced by a 'heat concentration'  $c_H = \rho c_p T$ , where  $c_p$  is the specific heat capacity of the fluid (J kg<sup>-1</sup>), one obtains an equation analogous to 3.10:

$$C = -D_{\rm H} \rho c_p \frac{\partial T}{\partial x} \tag{3.15}$$

where  $D_{\rm H}$  is a thermal diffusion coefficient (often called a thermal diffusivity). Values of  $D_{\rm H}$  and k for various fluids and solids are given in Appendixes 2 and 5.

# Momentum transfer

When a force is applied tangentially to a surface it tends to cause the surface layer to slide or shear in relation to the underlying material. A rigid solid transmits such a shearing stress, which is given the symbol  $\tau$  and has units of force per unit area (kg m<sup>-1</sup> s<sup>-2</sup>), without undergoing deformation. In a fluid, however, adjacent layers slide relative to each other with any one layer being relatively ineffective at transmitting a shearing stress to the next layer of fluid, so that a velocity gradient develops when a fluid flows over a surface. The viscosity of a fluid is a measure of the internal frictional forces that arise from molecular interactions between adjacent layers, with viscous fluids being more effective at transmitting a shearing stress than non-viscous fluids. This process is described by Newton's Law of Viscosity, which states that the shearing stress at a plane in a fluid is directly proportional to the velocity gradient  $(\partial u/\partial x)$ :

$$\tau = \eta \frac{\partial u}{\partial x} \tag{3.16}$$

where  $\eta$  is called the dynamic viscosity (kg m<sup>-1</sup> s<sup>-1</sup>).

This equation is similar in form to those already introduced for heat and mass transfer. In this case the shearing stress has the dimensions of a momentum flux density, where momentum is mass × velocity (i.e.  $ML^{-1} T^{-2} = MLT^{-1} \cdot L^{-2} \cdot T^{-1}$ ). As for heat transfer, the velocity gradient can be replaced by a gradient of momentum 'concentration'  $(c_M = \text{mass} \times \text{velocity/volume} = \rho u)$  thus giving a proportionality constant with the dimensions of a diffusion coefficient  $(L^2 T^{-1})$ . This diffusion coefficient for momentum  $(D_M)$  is also called the **kinematic viscosity** (v).

The tangential force exerted on a surface by a fluid flowing over it is termed skin friction. In addition to this transfer of momentum to a body across the streamlines of flow, a moving fluid can also exert a force as a consequence of form drag  $(\tau_t)$  where the pressure exerted on the front of an object is greater than that on the rear. This is the main force that causes the bending of trees and other plants in the wind. The magnitude of the form drag, that is the force per unit cross-sectional area normal to the flow (A) for any object is given by

$$\tau_{\rm s} = c_{\rm p} \frac{1}{2} \rho u^2 \tag{3.17}$$

where  $c_D$  is a drag coefficient that relates the actual drag to the maximum potential force that could be exerted if all the air movement was completely

stopped  $(\frac{1}{2}\rho u^2)$ . 'Streamlined' objects, such as aircraft, will have much lower drag coefficients than objects without streamlining (e.g. buildings). The value of  $c_D$  decreases dramatically if the airflow is turbulent. Further discussion of turbulence and its importance may be found in the section on convective and turbulent transfer (see p. 57), while drag and its significance is considered further in Chapter 11.

# Diffusion coefficients

Choice of an appropriate 'concentration' gradient enables us to express the proportionality constant D for a wide range of transfer processes in common units. For example, dimensional analyses of equations 3.10 to 3.13 for mass transfer and of 3.15 for heat transfer all give D with dimensions  $L^2$   $T^{-1}$ , while the same is true for momentum transfer.

The value of D in a binary mixture (e.g.  $CO_2$  and air) is called a mutual diffusion coefficient, where  $D_{CA}$  for  $CO_2$  diffusing into air is the same as  $D_{AC}$  for air diffusing into  $CO_2$ , with very little effect of altering the proportions of air and  $CO_2$ . Values for diffusion coefficients for quantities including various gases, heat and momentum in both air and in water are listed in Appendix 2. When a substance is diffusing within itself, D is called a self-diffusion coefficient; this can be very different from the mutual diffusion coefficient. For example the self-diffusion coefficient for  $CO_2$  ( $D_C$ ) is 5.8 mm<sup>2</sup> s<sup>-1</sup> compared with  $D_{CA}$  of 14.7 mm<sup>2</sup> s<sup>-1</sup> at 20 °C. Plant physiology is often concerned with ternary systems of air,  $CO_2$  and  $CO_2$  and

The relative values of D for different gases are approximately as predicted by  $Graham's \ Law$ , which states that the diffusion coefficients of gases are inversely proportional to the square roots of their densities when pure (i.e.  $D_i \propto (M_i)^{-\frac{1}{2}}$ , since density is proportional to  $M_i$ ). Effects of temperature and pressure on D are given by

$$D = D^{\circ}(T/T^{\circ})^{n} (P^{\circ}/P)$$
(3.18)

where the superscript 'o' refers to a reference value which can be taken as the value at 20 °C (293.15 K) and a pressure of 101.3 kPa (1013 mbar). Although the exponent n depends on the gas, a value of 1.75 predicts  $\mathbf{D}$  over the normal range of environmental temperatures with less than 1% error. In addition  $\mathbf{D}$  is modified when diffusion occurs in a confined physical system where the average distance that a molecule travels between collisions (the mean free path, which for  $\mathrm{CO}_2$  in air at 20 °C is approximately 54 nm) is of the same order as the size of the system. An example of a situation where this effect may be relevant is gaseous diffusion through nearly closed stomata.

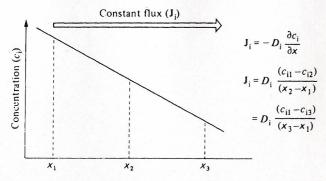


Fig. 3.1. Diffusion down a concentration gradient showing that where  $D_1$  is constant, the flux is constant with distance and concentration drop is proportional to distance.

### Integrated form of the transport equation

The close similarity between the equations describing transport of a wide range of different entities including water vapour, carbon dioxide, electric charge, heat and momentum, has led to them being sometimes referred to as particular examples of the general transport equation

Flux density (or flux) = proportionality constant 
$$\times$$
 driving force (3.19)

In many practical situations it is more convenient to measure concentrations at two positions in a system, rather than to determine the concentration gradient at a point: therefore the transport equation is commonly applied in an integrated form. In the simple case where the flux is constant over the path being considered (i.e. there is no absorption or evolution of the transported species in that region) and where D does not change with position (generally true for molecular diffusion), integration of equation 3.10 between planes at  $x_1$  and  $x_2$ , a distance  $\ell$  apart, gives

$$\mathbf{J}_{i} = \mathbf{D}_{i}(c_{i1} - c_{i2})/\ell \tag{3.20}$$

where  $c_{11}$  and  $c_{12}$  are the concentrations at  $x_1$  and  $x_2$  (see Fig. 3.1). In this equation, the driving force is the concentration difference across the path. Therefore the proportionality constant relating any flux density to the appropriate concentration difference is equal to  $D_1/\ell$ . In plant physiology this constant is conventionally called a **conductance** (and given the symbol g for diffusive transfer and for heat transfer). For many purposes, as will be seen later, it is more convenient to replace the conductance by its reciprocal, termed a **resistance** (given the symbol r), so that

$$r_{i} = 1/g_{i} = \ell/D_{i} \tag{3.21}$$

(a)
$$\begin{array}{c}
r_1 \\
R \\
g_1
\end{array}
\qquad \equiv \qquad \begin{array}{c}
R \\
G
\end{array}
\qquad \begin{cases}
G = g_1 + g_2 \\
\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} \\
R = r_1 r_2 / (r_1 + r_2)
\end{cases}$$

Fig. 3.2. The rules for simplifying complex networks of resistors: (a) resistors in parallel; (b) resistors in series and (c) the Delta-Wye transform.

Another transport process that fits the general transport equation is the transfer of electric charge as described by Ohm's Law (when a steady current (1) is flowing through a conductor, the potential difference between its ends (V) is directly proportional to the current, with the constant of proportionality being called a resistance (R) – i.e. V = IR). The close analogy between Ohm's Law and other transport processes has proved extremely useful in the analysis of transfer processes in plants. This is because electrical circuit theory is well developed and is directly applicable to the analysis of the complex networks that occur in plant systems. As a simple example, for a leaf losing water by evaporation from both surfaces, the analogous electrical system is two resistors in parallel with the same potential difference across them. The rules for simplifying complex electrical networks are summarised in Fig. 3.2.

It is often preferable to use conductances rather than resistances in transport studies because the flux across a path with a given driving force is directly proportional to its conductance, but inversely related to the resistance. This inverse relationship to resistance can be misleading in simple systems with only one dominant resistance (see e.g. Chapter 6). It is readily apparent from Fig. 3.2, however, that when a system is predominantly composed of resistors in series, it is more convenient to work with resistances, particularly if one is concerned with the relative limitation imposed by each component (see Chapter 7). A system of

Table 3.1. Analogies between different molecular transfer processes

| General transport equation                          | Flux density  | = (apparent) driving force  | ×conductance   |
|---|---|---|--|
| Fick's Law<br>(mass transfer)                       | <b>J</b> <sub>i</sub> (kg m <sup>-2</sup> s <sup>-1</sup> )   | $= \Delta c_{\rm i} $ (kg m <sup>-3</sup> )                                       | $(\mathbf{m} \mathbf{s}^{-1})^{\ell} (= \mathbf{g}_{\mathbf{i}})$  |
|   | $\begin{array}{l} \boldsymbol{J}_{i}^{m} \\ (\text{mol } m^{-2} \ s^{-1}) \end{array}$                | $= \Delta x_{i}$ (dimensionless)  | $ \times PD_{i}/\ell \mathcal{R} T (=g_{i}^{m}) $ $ (\text{mol } m^{-2} s^{-1}) $  |
|   | $\begin{array}{l} \mathbf{J}_{i}^{m} \\ \text{(mol m}^{-2} \text{ s}^{-1} \text{)} \end{array}$       | $= (P/\Re T) \Delta x_{i}$ (mol m <sup>-3</sup> )                                 | $ \times D_{i}/\ell (=g_{i}) $ (m s <sup>-1</sup> )  |
| Fourier's Law (heat conduction)                     | ${f C} \ (J \ m^{-2} \ s^{-1})$   | $= \Delta T $ (K)   | $\times k/\ell$ (W m <sup>-2</sup> K <sup>-1</sup> )   |
|   | C (J m <sup>-2</sup> s <sup>-1</sup> )  | $= \rho c_p \Delta T (= \Delta c_H)$ (J m <sup>-3</sup> )                         | $ \times \boldsymbol{D}_{\mathrm{H}}/\ell  (= \boldsymbol{g}_{\mathrm{H}}) $ $ (\mathrm{m \ s^{-1}}) $   |
| Newton's Law of<br>Viscosity<br>(momentum transfer) | $ \begin{array}{c} \tau \\ (kg \; m^{-1} \; s^{-2}) \\ \tau \\ (kg \; m^{-1} \; s^{-2}) \end{array} $ | $= \Delta u$ $(m s^{-1})$ $= \rho \Delta u (= \Delta c_{M})$ $(kg m^{-2} s^{-1})$ | $ \begin{array}{l} \times \eta/\ell \\ (\text{kg m}^{-2}\text{s}^{-1}) \\ \times D_{\text{M}}/\ell  (=g_{\text{M}}) \\ (\text{m s}^{-1}) \end{array} $ |
| Poiseuille's Law <sup>a</sup> (flow in pipes)       | $J_v$ $(m^3 m^{-2} s^{-1})$   | $= \Delta P$ (kg m <sup>-1</sup> s <sup>-2</sup> )                                | $ \times r^2/8 \ell\eta  (= L_{\rm p}) $ $ (\text{m}^2  \text{s kg}^{-1}) $  |
| Ohm's Law<br>(electric charge)                      | / (flux)<br>(A)   | $= V $ $(W A^{-1})$   |  |

<sup>&</sup>lt;sup>a</sup> For details of Poiseuille's Law see Chapter 4.

resistors in parallel, on the other hand, is most easily treated using conductances. Either form will be used as appropriate in the following chapters so it is necessary to be familiar with both type of expression and their conversion.

The analogies between different transfer processes are summarised in Table 3.1. It is clear from this that the units for conductance depend on what is chosen as the driving force, it being to some extent arbitrary which factors are included in which term. In each case it is possible to manipulate units to give a conductance in m s<sup>-1</sup> (or mm s<sup>-1</sup>). Note that for electricity the current is a flux rather than a flux density, so that the analogy is not complete.

Diffusion coefficients are fundamental properties of the medium and of the material diffusing rather than of the particular system geometry. This is in direct contrast to conductances or resistances that are basically a property of the whole system in that they vary with geometry (e.g. the distance over which the transport occurs) as well as with the mechanism of transport (e.g. molecular diffusion or the rather more rapid turbulent transport).

# Units for resistance and conductance

Until recently it was normal practice among plant physiologists to express mass and heat transfer resistances in units of s m<sup>-1</sup> (or s cm<sup>-1</sup>) and conductances in mm s<sup>-1</sup> (or m s<sup>-1</sup>). These units arise (see equation 3.20) if the flux is expressed as a mass flux density (e.g. kg m<sup>-2</sup> s<sup>-1</sup>) and the driving force is a concentration difference (kg m<sup>-3</sup>). The same units arise for heat transfer when treated according to equation 3.15, and for momentum transfer (see Table 3.1).

It is, however, becoming increasingly common, particularly in the biochemical literature, to express the flux as a molar flux density ( $J^m$ , mol m<sup>-2</sup> s<sup>-1</sup>), because biochemical reactions concern numbers of molecules rather than the mass of material. [More correctly  $J^m$  should be termed a mole flux density because use of the term molar should strictly be limited to the meaning 'divided by moles'.] Similarly, concentrations (e.g. of water vapour and  $CO_2$ ) are usually measured as partial pressures (or the related volume fraction), and as the appropriate driving force for diffusion is the gradient of partial pressure ( $p_1$ ) or mole fraction ( $x_1$ ) (rather than concentration), one can write the integrated form of the transport equation for a molar flux in either of the equivalent forms:

$$J_{i}^{m} = \frac{D_{i}P}{\ell \mathcal{R}T}(x_{i1} - x_{i2})$$

$$= \frac{D_{i}}{\ell \mathcal{R}T}(p_{i1} - p_{i2})$$
(3.22)

If one now follows general usage and defines a molar conductance  $(g^m)$  as  $PD/\ell \mathcal{R}T$ , it has dimensions mol  $L^{-2}T^{-1}$ , giving as appropriate units  $(\text{mol m}^{-2} \text{ s}^{-1})$  and the corresponding molar resistance  $(r^m)$  has units  $(m^2 \text{ s mol}^{-1})$ . These molar units will be used frequently in much of the rest of this book, especially when considering gas exchange through stomata, so the superscript 'm' will often be omitted in what follows so as to simplify presentation of equations, and the type of units will be indicated by the choice of font (i.e.  $g^m = g$  and  $r^m = r$ ).

This alternative definition of conductance has some advantages. In the more usual definition where  $g = D/\ell$ , conductance is approximately proportional to the square of the temperature and inversely proportional to P (see equation 3.18). Where, however,  $g = PD/\ell \Re T$ , it is relatively independent of the properties of the air, being independent of P and

approximately proportional to absolute temperature. The usual formulation is clearly less appropriate if one is considering effects of altitude (and hence total pressure) on gas exchange. A further advantage of using partial pressure is that it obviates the need for correcting for changing temperature and pressure that arises when using concentration. It is particularly important to use partial pressures rather than concentration gradients where the system is non-isothermal (Cowan 1977).

It follows from equations 3.20 and 3.22 that conversion between the two types of units is by means of

$$g = g(P/\Re T) \tag{3.23a}$$

and

$$r = r(\Re T/P) \tag{3.23b}$$

At sea level and 25 °C approximate conversions are, for resistance

$$r(\text{m}^2 \text{ s mol}^{-1}) = 2.5 r (\text{s cm}^{-1}) = 0.025 r (\text{s m}^{-1})$$
 (3.24 a)

and for conductance

$$g \text{ (mol m}^{-2} \text{ s}^{-1}) = 0.04 g \text{ (mm s}^{-1})$$
 (3.24b)

Conversions at other temperatures are given in Appendix 3.

In spite of the advantages of using a molar basis for expression of mass transfer resistances and conductances, the units s m<sup>-1</sup> and mm s<sup>-1</sup> will be retained for some purposes in the following treatment and especially for the analysis of heat and momentum transfer where there are no obvious analogies to molar fluxes. In addition, these are still the most commonly used and appropriate units for most canopy-level studies, especially when considering evaporation, and will normally be used in that context. Of course it should be recognised that for any given conditions of temperature and pressure the two sets of units are directly interconvertible using equations 3.23 and 3.24 as appropriate.

# Fick's Second Law of Diffusion

In many situations where diffusion takes place the flux is not constant with distance because some of the material diffusing goes into changing the concentration at any position. Using the principle of conservation, that is that matter cannot normally be created or destroyed, it is easy to show that, in a one-dimensional system where  $J_i$  is increasing with distance in the x direction, the extra material required must be obtained by decreasing  $c_i$ , so that

$$\frac{\partial \mathbf{J_i}}{\partial \mathbf{x}} = -\frac{\partial c_i}{\partial t} \tag{3.25}$$

This is known as the continuity equation. Substituting for  $J_i$  using Fick's First Law (equation 3.10) leads to

$$\frac{\partial c_{i}}{\partial t} = -\frac{\partial}{\partial x} \left( -D_{i} \frac{\partial c_{i}}{\partial x} \right) = D_{i} \frac{\partial^{2} c_{i}}{\partial x^{2}}$$
(3.26)

which is known as Fick's Second Law. This equation describes the time-distance relationships of concentration when diffusion occurs. The solution of this equation that is appropriate for any particular problem depends on the initial conditions and on the details of the system geometry. Solutions of this equation for a wide range of systems and boundary conditions are presented by Crank (1975). Here I will discuss only one example that can be used to illustrate the scale of diffusive transport in plant systems. This is the case where a finite amount of material is released at time zero in a plane at the origin and allowed to spread by diffusion in one dimension. The shape of the resulting curve relating concentration to distance is that of the Gaussian or normal distribution. For this curve the distance from the origin at which the concentration drops to 37% (= 1/e) of that at the origin is given by

$$x = \sqrt{(4Dt)} \tag{3.27}$$

An alternative explanation of x is that 16% of the material initially placed at the origin will diffuse at least as far as x in time t. The distance over which diffusive transport occurs increases with the square root of time. Substituting a typical value for D in air of 20 mm s<sup>-1</sup>, gives x = 9 mm for t = 1 s (i.e.  $\sqrt{(4 \times 20 \times 10^{-6} \times 1)} \simeq 9 \times 10^{-3}$  m). This illustrates the sort of distance over which gaseous diffusion is an effective transport mechanism.

#### Convective and turbulent transfer

The transfer of mass or heat by diffusion is a consequence of the thermal movements of individual molecules and is the dominant mechanism in still fluids such as the air within the intercellular spaces of plant leaves. For surfaces that are exposed to the atmosphere, such as leaves, air movement over the surface can speed up heat and mass transfer considerably. There are two processes involved.

In the first, the air movement continuously replenishes the air close to the surface with unmodified air, thus maintaining a steep gradient of concentration (the driving force for diffusion) and therefore more rapid tranport than obtains in still air. This is only important for isolated surfaces (such as isolated leaves or plants – see Chapter 5). Where there is an extensive homogeneous surface an equilibrium is achieved such that the air flowing close to the surface has already been modified by passage over an

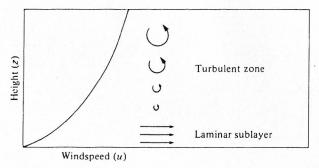


Fig. 3.3. Profile of windspeed moving over a surface showing the laminar sublayer where windspeed changes rapidly with height and a turbulent zone with eddy size increasing with distance.

identical surface upwind. In this case the concentration gradient is the same as would occur in still air unless the airstream is turbulent. Turbulence or random eddies in the airstream provides the second way in which air movement can speed up transfer processes; in this case, materials are transported directly in the moving air currents.

The air in the lower atmosphere is never completely still. Not only is there usually a net horizontal motion or wind, but there are also many random movements of small packets of air. The actual pattern of air movement depends on the type of convection regime that exists. This may be **free convection**, where the air movements are caused by changes in air density, as occur where the air adjacent to a heated surface expands and therefore rises, or where cold air sinks below a cool surface. Or it may be **forced convection**, where the air movement is determined by an external pressure gradient causing wind. Room heating with conventional 'radiators' relies largely on free convection, while fan-assisted radiators use forced convection to transfer heat to the room.

Forced convection may lead to the generation of eddies or turbulence as a result of the frictional forces acting between the wind and the surfaces over which it flows. The size and velocity of the individual eddies depend on a number of factors but they tend to decrease in magnitude as the surface is approached (Fig. 3.3). Evidence for the random spatial distribution and the persistence of these eddies may easily be seen if one looks at the patterns on a field of waving barley. On a smaller scale they can be detected by instruments such as hot-wire anemometers that respond rapidly to changes in air velocity (see Chapter 11). Because the size of eddies in an airstream tends to be similar to the scale of surface irregularities, they are therefore several orders of magnitude larger than the average molecular

movements giving rise to diffusion. For this reason turbulent transfer tends to be much faster than diffusion, typically by between three and seven orders of magnitude.

The relative importance of free and forced convection in heat and mass transfer depends on the balance between the buoyancy forces arising from temperature gradients and the inertial forces arising from air movements that cause turbulence. In most plant environments, heat and mass transfer are rarely determined by free convection alone, though it may be an important component of the transfer mechanism in very light wind.

### Boundary layers

It was pointed out earlier that when a fluid flows over a surface the flow velocity decreases towards that surface as a consequence of the friction between the surface and the fluid and of the viscous forces within the fluid. The zone adjacent to a surface, where the mean velocity is reduced significantly below that of the free stream, is termed the boundary layer. In what follows, the transfer conductances and resistances in the boundary layer will be distinguished by the use of the subscript 'a', so that the boundary layer conductance for heat transfer would be  $g_{aH}$ . One common arbitrary definition of the boundary layer defines its limit as that streamline where the velocity reaches 99% of that in the free airstream. Because the depth of the boundary layer in air tends to be about two orders of magnitude less than the size of the object, mass and heat transfer can be regarded as one-dimensional processes at right angles to the surface.

The pattern of fluid movement within a boundary layer may be either laminar, where all the fluid movement is parallel to the surface, or it may be turbulent. In a turbulent boundary layer the movements of individual molecules rather resemble the movements of commuters going to or from work in a large city: although the individual particles may be moving in a very irregular pattern, the overall motion is regular and predictable. Whether or not a particular boundary layer is laminar or turbulent depends on the balance between inertial forces in the fluid (because of its velocity) and the viscous forces that tend to produce stability and a laminar flow pattern.

Experimentally it has been found that, for a smooth plate, the transition from a laminar to a turbulent boundary layer generally occurs when the value of a group of terms called the **Reynolds number** exceeds a value between  $10^4$  and  $10^5$ . The Reynolds number is a dimensionless group given by ud/v, where u is the free fluid velocity, d is a characteristic dimension of the object and v is the kinematic viscosity (=  $D_{\rm M}$ ). For parallel-sided flat plates (approximately equivalent to grass leaves), d is the downwind width,

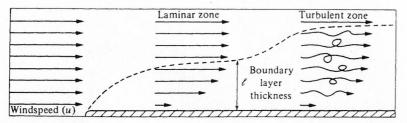


Fig. 3.4. Diagrammatic representation (with much exaggerated vertical scale) of the development of the boundary layer over a smooth flat plate in a laminar airstream, showing the windspeed profiles, the initial laminar zone and the onset of turbulence.

while for circular plates (appropriate for certain other leaves) d is  $0.9 \times \text{diameter}$ . For irregular plates d is the average downwind width, while for spheres or cylinders with their long axis normal to the flow d is equal to the diameter.

The build-up of a boundary layer in an airstream flowing over a flat plate such as a leaf is illustrated in Fig. 3.4. Initially there is a laminar zone that gradually increases in thickness with increasing distance from the leading edge. The laminar layer may then break down to form a turbulent zone when d increases enough to make the local Reynolds number larger than the critical value. There is good evidence that this critical Reynolds number is achieved at values well below 10<sup>4</sup> (i.e. 400-3000: Grace 1981) for plant leaves, because of the tendency of their surface irregularities, such as veins and hairs, to induce turbulence. Turbulence in the boundary layer is also encouraged by any turbulence in the free stream that might be caused by objects such as leaves and stems upwind (see Haseba 1973). Even where the majority of the boundary layer is turbulent there remains a thin zone close to the surface called the laminar sublayer where the flow is laminar, though this may be only a few tens of micrometres thick. As an indication of the sort of conditions under which turbulence may occur with real leaves, a leaf only 1 cm wide would achieve a possibly critical Reynolds number of 500 at a windspeed of only  $0.76 \text{ m s}^{-1}$  (i.e.  $500 \times 15.1 \times 10^{-6}/0.01$ ).

Mass and heat transfer through a boundary layer can be described by the general transport equation in the form already used for molecular diffusion in still air:

$$\mathbf{J}_{i} = g_{i}(c_{i1} - c_{i2}) \tag{3.28}$$

or for heat:

$$C = g_{H} \rho c_{p} (T_{1} - T_{2}) \tag{3.29}$$

As transport within a laminar boundary layer is by diffusion, the

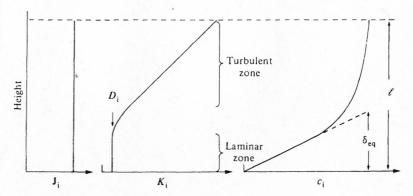


Fig. 3.5. Profiles of  $J_i$ ,  $K_i$  and  $c_i$  across a mixed boundary layer of total depth  $\ell$ , illustrating the equivalent boundary layer thickness  $\delta_{eq}$ .

conductance of a laminar layer with a mean thickness  $\delta$  is given by  $D_i/\delta$  (see equation 3.21). The thickness of a laminar boundary layer over a flat surface increases in proportion to the square root of the distance from the leading edge and in proportion to the reciprocal of the square root of the free fluid velocity. The thickness,  $\delta$ , is also weakly dependent on  $D_i$ , being approximately proportional to  $D_i^{0.33}$ , so that the boundary layer thickness is different for heat, mass and momentum.

Where the flow regime in the boundary layer is turbulent or mixed, the same form of equation applies but mass transfer is more rapid because of the eddies. In this case the boundary layer conductance is increased because D, the molecular diffusion coefficient, is replaced by a larger eddy transfer coefficient, K. The value of this transfer coefficient varies with the size of the eddies and tends to increase with distance from the surface (Fig. 3.3). The value of K may increase from around  $10^{-5}$  m<sup>2</sup> s<sup>-1</sup> near the leaves where the eddies are small to about  $10^{-1}$  m<sup>2</sup> s<sup>-1</sup> at the top of a plant canopy, reaching as much as  $10^2$  m<sup>2</sup> s<sup>-1</sup> well above the canopy.

It has already been noted that the integration of Fick's First Law (equation 3.10) to obtain the integrated form (e.g. equation 3.29) is easiest where the transfer coefficient does not alter with distance. Where it does vary, as in a turbulent boundary layer, the definitions of conductance and resistance in equation 3.21 must be replaced by

$$r_{i} = g_{i}^{-1} = \int_{x_{1}}^{x_{2}} \frac{\mathrm{d}x}{K_{i}} \tag{3.30}$$

Figure 3.5 illustrates how the transfer coefficient and concentration gradient might vary across a typical mixed boundary layer having a laminar sublayer and a turbulent zone.

Because of the difficulties in integrating the transport equation in situations where the transfer coefficient varies, it is convenient to define an equivalent boundary layer of thickness  $\delta_{\rm eq}$ . This is the thickness of still air that would have the same conductance or resistance as the turbulent boundary layer of thickness  $\ell$  (see Fig. 3.5). Thus for a turbulent boundary layer where the value of the transfer coefficient, K, is say  $10^3 \times D$ , the thickness of the equivalent boundary layer ( $\delta_{\rm eq}$ ) is  $10^{-3} \times \ell$ . Note that both  $\delta$  and  $\delta_{\rm eq}$  are average thicknesses, since the actual thickness of the boundary layer is less near the leading than the trailing edge.

As it is often difficult to determine the boundary layer thickness it has been found convenient to express heat and mass transfer in terms of the characteristic dimension (d). The ratio  $d/\delta_{\rm eq}$  is often called the Nusselt number when studying heat transfer or the Sherwood number when referring to mass transfer. These two dimensionless groups are among those widely used in the fluid dynamics literature to summarise information on heat and mass transfer. For our purposes it is more convenient to express this information directly in terms of the dependence of boundary layer conductance or resistance on windspeed and leaf dimensions. The application of dimensionless groups is discussed by Monteith & Unsworth (1990) and in textbooks on heat and mass transfer (e.g. Kreith 1973).

# Conductance of leaf boundary layers

The conversions between conductances for different entities depend on the nature of the boundary layer. Both when the air surrounding a plant organ is still and within the intercellular spaces of leaves transfer of heat or mass depends on molecular diffusion. Conductances for different entities (e.g. CO<sub>2</sub>, water vapour or heat) through such a layer of still air would be in the ratio of their molecular diffusion coefficients (Appendix 2). In a laminar boundary layer transport is still by diffusion so that one might expect the conductances to be in the same ratio, but as the effective boundary layer thicknesses for mass and heat transfer are proportional to  $D^{\frac{1}{3}}$ , it follows that conductances are approximately in the ratio of the  $\frac{2}{3}$  power of the diffusion coefficients. As turbulence increases, transport in eddies becomes rapid in relation to molecular diffusion, so that in a fully turbulent boundary layer above a canopy, heat, water vapour and carbon dioxide are all transported equally efficiently and therefore the conductances approach equality. Appropriate factors for converting between conductances for other entities are given in Table 3.2.

The value of the boundary layer conductance for a leaf or other object depends mainly on its shape and size and on the windspeed. It is best determined empirically for leaves of any given dimensions by measuring

Table 3.2. Factors for converting conductances for different entities in different boundary layers relative to the heat transfer conductance  $(g_{aH})$ 

| 4         | Relationship                   | $g_{aH}$ | $g_{aw}$ | $g_{\mathrm{ac}}$ | $g_{aM}$ |
|-----------|--------------------------------|----------|----------|-------------------|----------|
| Still air | $(D_{\rm i}/D_{\rm H})$        | 1.0      | 1.12     | 0.68              | 0.73     |
| Laminar   | $(D_{\rm i}/D_{\rm H})^{0.67}$ | 1.0      | 1.08     | 0.76              | 0.80     |
| Turbulent | $(D_{\rm H})$                  | 1.0      | 1.0      | 1.0               | 1.0      |

water loss from wet surfaces (e.g. blotting paper) of the same size with the same external conditions, or by energy balance measurements. These methods are outlined in Appendix 8. It has been found that conductance may be estimated with adequate precision for many purposes from the wind velocity (u) and the characteristic dimension (d), by making use of relationships that have been derived from a range of experiments and from heat transfer theory (see Monteith 1981b; Monteith & Unsworth 1990). For flat plates in laminar forced convection conditions, the value of the boundary layer conductance to heat transfer (mm s<sup>-1</sup>) is given by

$$g_{\rm aH} = r_{\rm aH}^{-1} = 6.62 \, (u/d)^{0.5}$$
 (3.31)

where d is the characteristic dimension (m) and u is the wind velocity (m s<sup>-1</sup>). Note that this conductance refers to unit projected area of leaf (that is the area of *one* side) but includes heat transfer from both surfaces in parallel. Since mean boundary layer thickness is inversely related to g (i.e.  $\delta = D_{\rm H}/g_{\rm H}$ ) it is easy to calculate the corresponding boundary layer thickness as  $\delta = 2$   $D_{\rm H}(u/d)^{-0.5}/6.62$ , where the factor 2 converts the conductance to that appropriate for exchange from one side of a leaf. For a 1 cm leaf in a wind of 1 m s<sup>-1</sup>, therefore,  $\delta = 0.65$  mm (i.e.  $2 \times 0.215 \times 10^{-4} \times (1/0.01)^{-0.5}/6.62 \times 10^{-3}$  m).

Corresponding expressions for conductances of other shaped objects are: for cylinders with their long axis normal to the flow

$$g_{\rm aH} = r_{\rm aH}^{-1} = 4.03 \, (u^{0.6}/d^{0.4}) \tag{3.32}$$

and for spheres

$$g_{\rm aH} = r_{\rm aH}^{-1} = 5.71 \, (u^{0.6}/d^{0.4}) \tag{3.33}$$

where d is the diameter of the cylinder or sphere. Equations 3.32 and 3.33 both refer to unit *surface* area.

Although equations 3.31–3.33 are really only applicable to smooth isothermal plates or other shapes in laminar flow, they are commonly used to estimate conductances for real leaves and other plant organs. In practice,

however, surface temperatures are not uniform and some degree of turbulence in the leaf boundary layer is common. Where turbulence occurs, equations 3.31–3.33, which apply to laminar conditions, tend to underestimate the true conductance usually by a factor of between 1 and 2, though perhaps by as much as 3 in certain circumstances (see Monteith 1981b and Grace 1981). In addition to factors such as leaf size and windspeed, the turbulent regime in the airstream can be important: Haseba (1973), for example, has shown that the altered turbulence pattern within dense plant canopies can increase the conductance of rigid leaf models independently of windspeed.

The presence of leaf hairs also affects transfer in the leaf boundary layer (see Johnson 1975). Sparse hairs may increase surface roughness and the tendency for turbulence. On the other hand, a dense mat of hairs is likely to increase the effective depth of the boundary layer (certainly for water vapour or  $CO_2$  transfer) by up to the depth of the hair mat. A layer of still air trapped by hairs 1 mm long would have a resistance to water vapour diffusion of  $\ell/D_w = 1 \times 10^{-3}/0.242 \times 10^{-4} = 41 \text{ s m}^{-1}$ . For momentum, however, the hairs would move the effective sink for momentum from the leaf surface to the surface of the hair mat, so hairs would affect the ratio between conductances for heat, mass and momentum.

Because of these complexities it is clear that it is difficult to estimate leaf conductance accurately. Perhaps the best available generalisation is to increase the conductances calculated from equations 3.31-3.33 by a factor of 1.5, giving the dependence of leaf conductance on windspeed and dimensions shown in Fig. 3.6. The characteristic dimensions used in these calculations range from 1 mm (as for narrow-leaved grasses and pine needles) to 30 cm corresponding to very large leaves such as bananas. Variation of leaf size over this range changes  $g_a$  by more than an order of magnitude. Windspeeds at the top of plant canopies can often exceed 1 m s<sup>-1</sup>, but at times (e.g. at night) and deep in the canopy values may fall to 0.1 m s<sup>-1</sup> or less.

Although forced convection is likely to dominate heat and mass transfer from leaves in natural environments, when large leaf-to-air temperature differentials occur, as with large leaves and high irradiances, there may be a significant contribution by free convection. With a 10 °C leaf-air temperature differential, the free convection conductance for heat is likely to be about 3.2 mm s<sup>-1</sup> (Monteith 1981b), so that it is comparable to that arising from forced convection only for the largest leaves at windspeeds less than 0.3 m s<sup>-1</sup>.

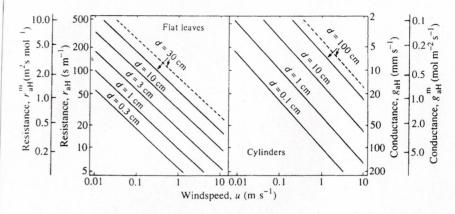


Fig. 3.6. Estimated dependence of  $g_{\rm H}$  or  $r_{\rm aH}$  on characteristic dimension (d) and windspeed for flat leaves or cylindrical leaves or stems in natural environments. The dashed lines give the value of  $g_{\rm aH}$  predicted by equations 3.31 or 3.32 for laminar flow, the solid lines ( $g_{\rm aH}=1.5\times {\rm value}$  predicted by these equations) are for more typical flow conditions.

# Transfer processes within and above plant canopies

Many of the principles that have been applied to heat and mass transfer of individual leaves are also applicable to exchange by large areas of vegetation, but there are a number of important differences and complicating factors. First, the analysis is complicated by the fact that the 'surface' of a plant canopy (i.e. the sources or sinks of heat, water, CO<sub>0</sub> and momentum) is usually distributed over a significant depth of canopy and also the distribution with depth is different for each entity. A second feature that has been of particular value in the development of micrometeorological techniques for the study of transfer processes between vegetation and the atmosphere is the difference in scale, with the boundary layer above a canopy being much deeper than that for a single leaf, so that it is possible to make measurements within the boundary layer and these have been used to infer fluxes. The third feature of transfer within and above canopies is that the crop boundary layer is generally turbulent so that the transfer coefficients (K) for heat and mass transfer are usually assumed equal, though there can be great spatial and temporal heterogeneity. This similarity assumption forms the basis of several of the micrometeorological methods used to study canopy exchange processes.

# Transfer above plant canopies

The theory of transfer processes above plant canopies is outlined by Thom (1975) and well summarised by Monteith & Unsworth (1990). Many examples are presented in Monteith (1976). The analysis of micrometeorological measurements within the crop boundary layer requires that there is no convergence or divergence of flux (i.e. there are no sources or sinks within the boundary layer for the entity being transported) and no advection. In other words, the conservation equation applies and a one-dimensional vertical flux is assumed, the flux being constant at different heights, but the transfer coefficient at any height z,  $K_i(z)$ , varies. The flux of mass or momentum above the crop, therefore, can be described by the standard gradient—diffusion assumption where the flux is proportional to the transfer coefficient multiplied by the driving concentration gradient:

$$\mathbf{J}_{i} = -\mathbf{K}_{i}(\mathbf{z}) \left( \partial c_{i} / \partial \mathbf{z} \right) \tag{3.34}$$

For this equation to hold, measurements must be made entirely within the crop boundary layer that has developed from the 'leading edge' of the field or area of vegetation being studied. The depth of the boundary layer increases with distance or 'fetch' from the leading edge. In general it is assumed that measurements may be made with adequate precision up to a height above the canopy equal to about  $0.01 \times$  fetch. It is also found that measurements need to be made well above the underlying canopy because the erratic turbulence structure near and within canopies leads to such great variability in K that equation 3.34 has little practical value in this zone (see Raupach, 1989). It follows that micrometeorological studies of fluxes through the crop boundary layer require large areas of homogeneous vegetation, the size of which depend on the height above the canopy at which sensors are placed.

# Wind profiles and estimation of conductance

Windspeed increases with height above open ground or above plant communities, with the rate of increase being greatest near the ground, as shown in Fig. 3.7. The shape of this windspeed profile is such that over open ground the logarithm of height ( $\ln z$ ) is linearly related to the windspeed at that height ( $u_z$ ). Expressing  $u_z$  in terms of  $\ln z$  gives

$$u_z = A (\ln z - \ln z_0) = A \ln (z/z_0)$$
 (3.35)

The intercept on the  $\ln z$  axis is  $\ln z_0$ , where  $z_0$  is called the **roughness length**, and is a measure of the aerodynamic roughness of the surface. The slope, A, is usually replaced by the term  $u_*/\ell$ , where  $u_*$  is called the **friction** 

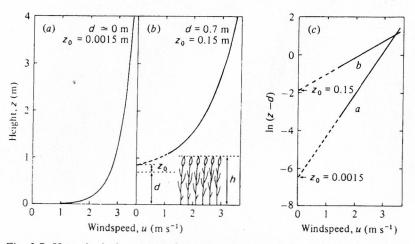


Fig. 3.7. Hypothetical mean profiles of windspeed when windspeed at 4 m equals  $3.5 \text{ m s}^{-1}$  for (a) bare ground and (b) a cereal crop together with (c) corresponding linearising logarithmic transformations.

velocity (having dimensions of velocity) and characterises the turbulent regime, and  $\ell$  is a dimensionless constant (= 0.41) named after von Karman.

Over vegetation, unlike over open ground, the windspeed profile is no longer linear when u is related to  $\ln z$ . Instead u is linearly related to  $\ln (z-d)$ , where d is an apparent reference height, the **zero plane displacement** (Fig. 3.7). As shown in Fig. 3.7 windspeed extrapolates to zero at a height of  $d+z_0$  (though actual windspeed at this height is still finite). Substituting (z-d) for z in equation 3.35 gives

$$u_z = (u_*/\ell) \ln[(z-d)/z_0]$$
 (3.36)

as describing a windspeed profile above vegetation. The plane at a height  $d+z_0$  may be regarded as an apparent sink for momentum.

It has been found that reasonable approximations to d and  $z_o$  for a range of relatively dense vegetation types are (Campbell 1977)

$$d = 0.64 \, h \tag{3.37}$$

and

$$z_0 = 0.13 h$$
 (3.38)

where h is the crop height. More appropriate values for coniferous forest are given by the following equations (Jarvis *et al.* 1976):

$$d = 0.78 h (3.37a)$$

$$z_{o} = 0.075 \, h \tag{3.38a}$$

In practice d and  $z_0$  vary with windspeed and canopy structure in a fairly complex manner (see Monteith 1976; Monteith & Unsworth 1990).

It is possible to use the windspeed profile to estimate transfer coefficients and conductances in the crop boundary layer. As momentum transfer is analogous to other transport processes, it is possible to define a conductance for momentum transfer between height z and the reference plane  $(z = d + z_0)$  using the usual transport equation

$$\tau = g_{AM} \rho [u_z - u_{(d+z_o)}] = g_{AM} \rho u_z \tag{3.39}$$

where  $g_{AM}$  is the canopy boundary layer conductance for momentum  $(=r_{AM}^{-1})$ . It can also be shown (see Monteith & Unsworth 1990) that

$$\tau = \rho u_*^2 \tag{3.40}$$

Combining these two equations gives

$$g_{4M} = u_{*}^2 / u_{z} \tag{3.41}$$

which can be expressed in terms of the parameters of the wind profile equation (equation 3.36) to give

$$g_{AM} = \frac{\ell^2 u_z}{\{\ln[(z-d)/z_0]\}^2}$$
 (3.42)

Not only does this equation imply that  $g_{AM}$  increases with windspeed, but it also indicates that conductance tends to increase with crop height (as d and  $z_0$  both increase with height). Substituting, for example, values of u, d and  $z_0$  from Fig. 3.7 into equation 3.42 gives, for a windspeed of 3.5 m s<sup>-1</sup> at 4 m,  $g_{AM} = 9$  mm s<sup>-1</sup> for the bare ground ( $d \approx 0$ ,  $z_0 = 0.0015$ ) and  $g_{AM} = 62$  mm s<sup>-1</sup> for the cereal crop (d = 0.7,  $z_0 = 0.15$ ).

Because the apparent sink for momentum in a canopy is above those for heat or mass exchange, there is a small extra resistance required when converting from  $r_{AM}$  to the corresponding resistances for heat or mass transfer. This extra resistance refers to transfer between the level of the momentum sink  $(d+z_0)$  and the alternate sink (Thom 1975).

Making use of the similarity assumption for the turbulent transfer of different entities in the boundary layer, equation 3.42 can be used as an estimate for the crop boundary layer conductances for other entities such as heat,  $CO_2$  and water vapour. This forms the basis for an important method for estimating fluxes of these quantities. Once the conductance is known (from the wind profile), fluxes may be obtained from measurements of the appropriate concentration differences using equations 3.28 or 3.29. Alternatively fluxes may be estimated directly from equation 3.34 if the concentration gradient and  $K_i$  at any height are known.

A problem with equations 3.36 and 3.42 is that they hold only when the temperature profile in the atmosphere is close to neutrality. At neutrality the temperature decreases with height according to the dry adiabatic lapse rate (0.01 °C m<sup>-1</sup> – see Chapter 11). If temperature decreases more rapidly with height there is a tendency for free convection to occur as a result of 'buoyancy' effects. This makes the atmosphere unstable and turbulent transfer is enhanced. Conversely, when temperature increases with height (a temperature inversion), the atmosphere is stable and transfer is suppressed because the less dense air is above the cooler denser air. In either case the normal profile equations need modification (see Thom 1975; Monteith & Unsworth 1990).

### Transfer within plant canopies

The erratic turbulence structure within plant canopies and the complexities introduced by the distribution of sources and sinks for heat, mass and momentum make the application of the gradient—diffusion analogue to transfer processes within the canopy extremely difficult. Examples of the range of within-canopy wind profiles for different types of plant stand are shown in Fig. 3.8. In some canopies windspeed may be highest near the ground, particularly in forests that have little understorey vegetation.

The microclimate within a canopy depends on the source distributions and concentration fields of heat, water vapour and  $CO_2$ . The variation in source density for an entity i with height (the source density profile,  $S_i(z)$ , where a sink is a negative S), depends on physical and physiological processes particularly at the leaves, while the concentration profile  $c_i(z)$  depends on the turbulent wind flow and the way this distributes the entity under consideration. Since the law of conservation must apply, the source density in any horizontal plane is related to the change in flux across that plane:

$$S_{i}(z) = dJ_{i}/dz \tag{3.43}$$

similarly the flux across the plane at height z is given by the integral from the ground to height z:

$$\mathbf{J}_{i}(z) = \mathbf{J}_{i}(0) + \int_{0}^{z} S_{i}(z) \,dz \tag{3.44}$$

where  $J_i(0)$  is the flux density from the ground at z = 0.

Analysis of the turbulence structure within and above canopies has demonstrated that the strongest turbulent events in a wide range of canopy types are gusts: energetic, downward incursions of air into the canopy space from the faster moving air above (see Raupach, 1989). These gusts tend to be very intermittent but are responsible for most of the momentum transfer

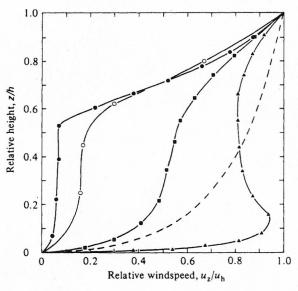


Fig. 3.8. Normalised within canopy windspeed profiles for  $( \bullet )$  a dense stand of cotton,  $( \blacksquare )$  dense hardwood jungle with understorey,  $( \triangle )$  isolated conifer stand with no understorey (see Businger 1975 for original references),  $( \bigcirc )$  a corn crop (data from Lemon 1967) and ( ---- ) a logarithmic profile (equation 3.35) with  $z_0 = 0.01$  h.

(more than 50% of energy may be transferred in events occupying less than 5% of the time). The resulting variability of K means that gradient analogies are not helpful, with counter gradient fluxes (and hence apparently negative K values) having been observed for heat, water vapour and  $CO_2$  fluxes within a pine forest (Denmead & Bradley, 1987). The typical canopy eddies are coherent structures of similar dimensions to the canopy height that persist for long periods: the wind waves across cereal fields provide familiar visual evidence for the persistence of turbulent motions of this scale.

As a result of the common failure of the normal gradient diffusion analogue in plant canopies there has been considerable effort aimed at developing an approach to the analysis of turbulent dispersion in canopies which is applicable to the non-diffusive flow that is found in the presence of persistent, large-scale eddies. One approach is to consider all the individual canopy elements as independent point sources releasing material (for example water vapour) into small parcels of air as they pass and to estimate the statistical probability of independent parcels released into the airstream from all these sources reaching a specific point at a particular time

(see Raupach 1989). It follows that transport depends on the turbulence structure of the airflow.

# Sample problems

- 3.1 Water vapour is diffusing down a 10 cm isothermal tube at 20 °C from a wet surface ( $c_{\rm w}=17.3~{\rm g~m^{-3}}$ ) to a sink consisting of saturated salt solution (equilibrium  $c_{\rm w}=11~{\rm g~m^{-3}}$ ). Calculate (i)  $J_{\rm w}$ , (ii)  $g_{\rm w}$ ; and the equivalent molar values (iii)  $J_{\rm w}^{\rm m}$  and (iv)  $g_{\rm w}^{\rm m}$ .
- 3.2 For a 2 cm diameter circular leaf exposed in a laminar airstream moving at  $1 \text{ m s}^{-1}$ , (i) what are (a)  $g_{aH}$ , (b)  $g_{aW}$ , (c)  $g_{aM}$ , (d) the mean boundary layer thickness for momentum? (ii) What would be the values for these conductances if the leaf was covered in a mat of hairs 1 mm deep? (iii) Is the assumption of a laminar boundary layer likely to be valid?
- 3.3 If the windspeed at 2 m is 4 m s<sup>-1</sup> when blowing over a wheat canopy 80 cm tall, what are (i)  $u_*$ , (ii) the windspeed at the top of the canopy, (iii)  $\tau$  and (iv)  $g_{AM}$  between the reference plane and 2 m?