general account see P. J. Black and J. M. Ogborn, Phys. Bull. **21,** 301 (1970).

[2] F. Reif, *Statistical Physics Berkeley Physics Course. Volume 5* (McGraw–Hill, New York, 1967).

[3] Physical Science Study Committee, *College Physics* (Raytheon, Boston, 1968).

[4] Atlas Computer Laboratory, Chilton, Didcot, Berkshire, Eng. Films were programmed and produced by Dr. F. R. A. Hopgood, using the Atlas Computer and a Stromberg–Carlson 5C-4020 high speed CRT recorder.

[5] Penguin Books Limited, Harmondsworth, Middlesex, are producing these films as a 16-mm film.

[6] H. A. Bent, *The Second Law* (Oxford U. P., New York, 1965).

[7] R. W. Gurney, *Introduction to Statistical Mechanics* (McGraw–Hill, New York, 1949).

[8] C. W. Sherwin, *Basic Concepts of Physics* (Holt, Rinehart and Winston, New York, 1961).

[9] W. Feller, *An Introduction to Probability Theory and its Applications* (Wiley, New York, 1950), Vol. 1.

An Experiment on Electron Emission

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A thermionic emission experiment is described in which the cathode temperature is determined from the energy distribution of the emitted electrons. The experiment can be carried out using a commercially available tube and a millimicroammeter of modest performance and so is quite suitable for undergraduate laboratories at the sophomore and junior level.

I. INTRODUCTION

Thermionic emission experiments contain a lot of good physics but are difficult to carry out in undergraduate laboratories because of the problem of cathode temperature measurement. This problem can be solved by determining the cathode temperature from the energy distribution of the emitted electrons, obtained by measuring the anode current as a function of the strength of a retarding field applied between the anode and the cathode.

The purpose of this paper is to describe such an experiment from which may be obtained verification of the Maxwell–Boltzmann distribution for thermionic electrons, a calibration of cathode temperature as a function of heater power, the work function of the anode, and the work function and temperature coefficient of the work function of the cathode. The only special equipment required is a millimicroammeter. It is possible to use a commercial vacuum tube voltmeter if correction is made for the potential drop across the instrument.

II. THEORY

A. Electron Emission by a Hot Conductor: Determination of the Work Function

Electrons are emitted by a hot metal surface according to the Richardson–Langmuir–Dushman equation:

$$j_0 = AT^2 \exp(-\phi e/kT), \qquad (1)$$

where j_0 is the emitted current density, ϕ the emitting surface work function in volts, and A is a universal constant (120 A cm^{-2} °K^{-2}) for metals.[1] The work function ϕ is usually determined by plotting $\ln j_0/T^2$ vs $1/T$ and computing ϕ from the slope of a straight line through these points. This is not the best method since any temperature dependence of ϕ is thereby concealed and an entirely fictitious value of A may be obtained.[2] Since the value of A for a metal surface or even for a sufficiently hot oxide cathode is known to approximate quite closely the theoretical value, it is better to define an "effective" work function

$$\phi_E \equiv (kT/e) \ln(j_0/AT^2). \qquad (2)$$

As Hensley points out, ϕ_E approximates the "true" work function ϕ_t, which is the difference between the Fermi energy ϵ_F and the surface potential ϵ_s, and would be identical to it for a uniform surface of zero reflection coefficient. Since ϵ_F, the surface potential, and the reflection coefficient are all temperature dependent, so generally will be ϕ_t. However, these dependencies are usually small and masked by much larger effects arising from surface nonuniformities or temperature-dependent surface contaminants. Since in experiments accessible to undergraduates, including this one, the sorting out of all these processes is impracticably difficult, one does better to treat the problem phenomenologically, assuming the theoretical value of 120 A cm^{-2} °K^{-2} for A and determining ϕ_E by the method of Hensley. This does not lead the student to believe that he can get more out of the data than he really can and still avoids the artificiality of the traditional method. The temperature dependence of ϕ_E then follows as an empirical result.

B. The Electron Energy Distribution Function: Determination of the Cathode Temperature

While most electrons in a conductor are at the degenerate end of the Fermi distribution, those emitted from the surface come from its nondegenerate "hot tail" and so will obey Boltzmann statistics. Thus if a saturation current density j_0 can be drawn from an emitting surface, the current density j that will flow against a retarding potential V is[3]

$$j = j_0 \exp(-Ve/kT). \tag{3}$$

This suggests that by plotting $\ln j$ against V one should obtain a straight line with slope $(-e/kT)$ and so determine T. This is the method used in the experiment described in this paper. Two difficulties can arise in its use, however. Since the space charge between cathode and anode generates a potential hill δ, the cathode–anode potential difference may not always be the determining V in Eq. (3). The second difficulty is simply that the cathode–anode potential difference is in any case equal to the sum of the externally applied retarding potential V_r and the cathode–anode contact potential $\phi_c - \phi_a$. Thus to reach the anode electrons must have energies

$$V_e > V_r + (\phi_c - \phi_a) \tag{4a}$$

or

$$V_e > \delta, \tag{4b}$$

whichever is greater. By keeping the current density small, δ can be kept small and so we will assume that Eq. (4a) can be made the determining condition. Then combining Eqs. (4a), (3), and (1) we find

$$j = AT^2 \exp(-V_r - \phi_a) e/kT. \tag{5}$$

Rather surprisingly the current that flows against a retarding potential is independent of the cathode work function ϕ_c. Since ϕ_a is constant for constant T, it is still true that a plot of $\ln j$ vs V_r will yield a straight line of slope $-e/kT$ from which T may be determined.

C. Determination of the Anode Work Function

An effective anode work function [in the sense of Eq. (2)] may be calculated from Eq. (5) in the form

$$\phi_a = (kT/e) \ln(j_0/AT^2), \tag{6}$$

where j_0 is the (extrapolated) current density at $V=0$. This value of ϕ_a is, however, characteristic of the anode temperature, which is generally not measurable since T in the above equations is the cathode temperature.

D. Determination of the Cathode Work Function

The traditional method of determining the work function of a cathode by measuring the saturation current as a function of temperature does not work well with an oxide cathode; the work function is strongly field dependent and no definite saturation current can be identified. For the measurement to be well defined the effective value of the work function as computed from Eq. (2) should therefore be quoted at a specified cathode field as well as temperature. While the cathode field will generally be less than the vacuum field because of space charge, if the current density is at least an order of magnitude less than

$$j_c = (e/2m) \epsilon_0 V^{3/2} / X_0^2, \tag{7}$$

which is the Childs–Langmuir value for the plane parallel zero cathode field case, then the vacuum field may be assumed.[1]

It should be noted that a theoretical model for the dependence of work function on field proposed a long time ago by Schottky is based on the assumption of a uniform, perfectly conducting surface and does not necessarily apply to an oxide cathode.[4]

III. EXPERIMENTAL

A. Equipment

The type 6X4 rectifier is convenient for this experiment. The geometry is regular and the cathode–anode spacing small enough to minimize the height of the space charge hill. The cathode–anode spacing is small compared with the cathode radius so that plane-parallel geometry may be assumed to a fair approximation.

The circuit used for retarding field measurements is shown in Fig. 1. The potentiometer should be a good one; commercial 10-turn potentiometers with dial are quite satisfactory.

For accelerating field measurements the potentiometer–battery circuit is replaced by a 150 V variable power supply of opposite polarity. Commercial supplies such as Heathkit or Eico are satisfactory if the current is read with a decent meter.

The millimicroammeter *must* be shunted with a capacitor as shown to eliminate 60 Hz pickup from heater–cathode leakage.

B. Determination of Filament Temperature and Anode Work Function

Choose a filament voltage and after equilibrium is reached record V_f and i_f. Then read anode

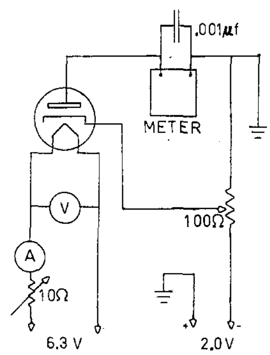

FIG. 1. Circuit for reverse current determination. The potentiometer should be a precision unit.

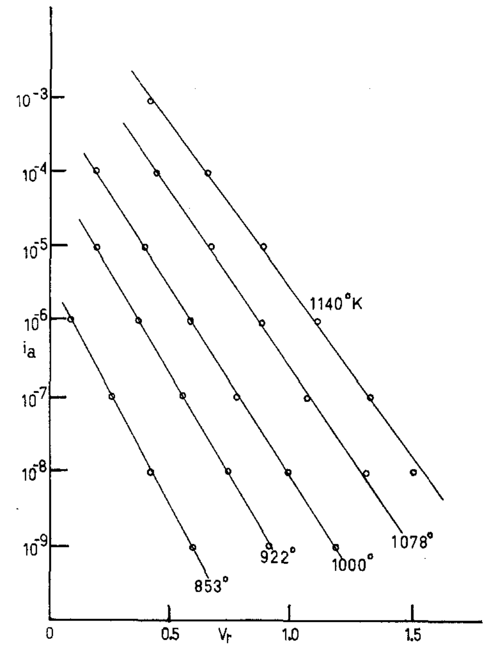

FIG. 2. Reverse current i_a in amperes vs retarding potential V_r in volts for five different cathode temperatures.

current as a function of retarding potential over several decades of current. Repeat for four or five different filament temperatures. Filament voltages in the range of 2.5–6 V give convenient currents.

Then plot ln$i$ vs V_r on five-cycle semilog paper, drawing best straight lines in accordance with Eq. (3). Retarding potentials much above 1.5 V may cause false readings at higher temperatures because of reverse current emitted by low work function patches on the anode, which becomes quite hot due to the small cathode–anode spacing. Sufficiently low retarding potentials—less than a few tenths of a volt—will violate the condition $V+(\phi_c-\phi_a)>\delta$ [Eq. (4)]. The region of valid data will be evident once the points are plotted.

Figure 2 shows ln$i$ vs V_r from a typical run. Cathode temperatures were computed from the slopes of the lines as discussed earlier. By extrapolating these lines through $V_r=0$ the current i_0 was obtained at each temperature. A tube was sacrificed to obtain the physical data necessary to calculate j_0 (see Table I) and ϕ_a, the anode work function, was computed from Eq. (6). This is shown plotted versus cathode temperature in Fig. 3 (triangles) as a convenient means of presentation. Nothing much can be deduced from

TABLE I. Cathode and anode dimensions.

	Length	Diameter	Area
Cathode	51/64 in.	0.073 in.	1.26 cm^2
Anode	50/64 in.	0.111 in.	1.76 cm^2

this because the *anode* temperature, against which ϕ_a ought to be plotted, is unknown.

C. Calibration of Cathode Temperature vs Filament Power

Quite low cathode temperatures must be used in the determination of the cathode work function to keep anode dissipation within reasonable limits so it is convenient to have a simple method of interpolating cathode temperatures. Since filament power losses are almost entirely radiative one should be able to write

$$P_f \propto (T^4 - T_0^4), \tag{8}$$

where T_0 is some effective ambient temperature. It would be room temperature if the emissivity of the anode were temperature independent. This is not the case; emissivities approach zero at absolute zero and are rather low near room temperature. Over a small temperature range Eq. (8) will fit well; however, the intercept T_0^4 will generally lie far above room temperature.

Figure 4 shows P_f vs T^4 for the data reported in this paper. The linearity of this kind of plot is a very sensitive test for the internal consistency of the filament temperature data.

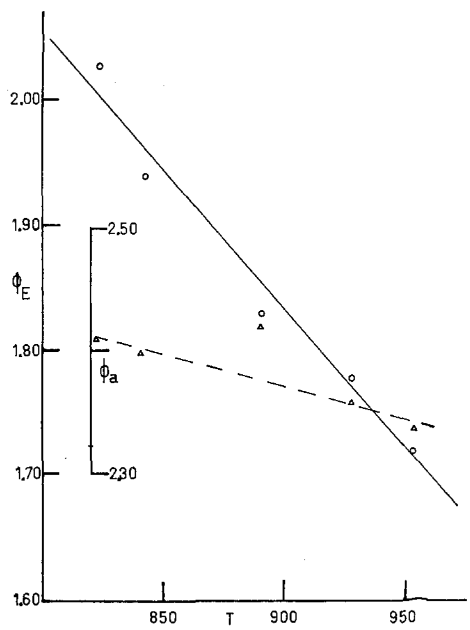

FIG. 3. Cathode and anode work functions vs temperature. Circles and the solid line refer to the effective cathode work function ϕ_E and triangles and the dashed line to the anode work function ϕ_a. Work functions are in volts and temperature in degrees Kelvin.

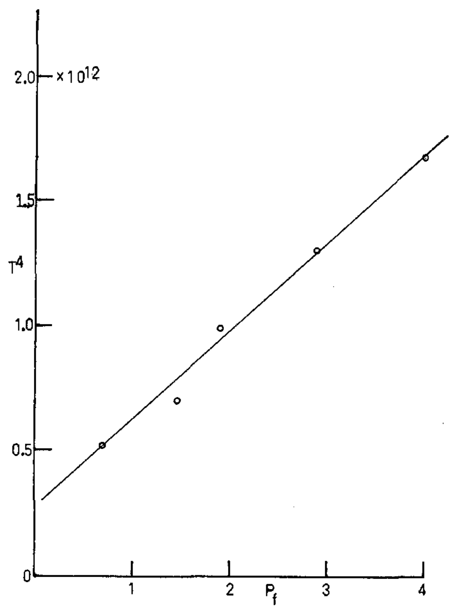

FIG. 4. Cathode temperature in degrees Kelvin to the fourth power vs filament temperature P_f in watts.

D. Determination of the Cathode Work Function

Set the anode potential sufficiently high to make j_c from Eq. (7) at least 10 times the highest current to be drawn. (If a type 6X4 is used a potential of 100 V yields a Childs–Langmuir j_c of $\frac{1}{2}$ A/cm^2 so the maximum allowable current would be about 0.05 A.) Adjust the cathode temperature to yield this value. Then reduce the filament current and record the new anode current. Four or five points may be obtained for cathode temperatures between 800°–950°K. The effective cathode work function ϕ_c may then be calculated using Eq. (2) and plotted as a function of temperature. Figure 3 (circles) shows results obtained for the tube used in this study.

IV. DISCUSSION

The effective cathode work function shown in Fig. 3 may be translated into a traditional Richardson work function. Data which yield a linear Richardson plot will also yield a linear effective work function plot since a linear Richardson plot implies that the work function is at most linearly dependent upon the temperature, that is,

$$\phi_E = \phi_R + \alpha_E T, \tag{9a}$$

where

$$\alpha_E = d\phi_E/dT$$
$$= (k/e)\ \ln(120/A_R), \tag{9b}$$

A_R being the value of A that would be obtained from a Richardson plot. For the study reported here,

$$\alpha_E = -2\times 10^{-3}\ \mathrm{eV/°K},$$

so

$$\phi_R = 3.7\ \mathrm{eV}, \qquad A_R = 6\times 10^{12}\ \mathrm{A\ cm^{-2}\ °K^{-2}}$$

at a field strength of about 2000 V/cm. Alternatively, ϕ_R and A_R may be obtained by making a traditional Richardson plot.

An interpretation of Eq. (9) is that ϕ_R is the effective work function at absolute zero. Such an interpretation depends, however, on a linear temperature dependence of ϕ_E. Considering the number of uncontrolled variables one must deal with when performing this experiment with a commercial vacuum tube, it is probably better to abstain from such extrapolations.

The (Richardson) work function usually quoted for oxide cathodes is $\sim$1 V. Higher values occur at low cathode temperatures because the cathode is not uniform in temperature in commercial tubes, and emission increases with temperature both because of Eq. (1) and because more of the cathode becomes warm enough to emit. This is therefore the weakest part of the experiment; accurate values of the cathode work function cannot usually be obtained. This also shows up in anomalously high values for α_E, the one obtained here being 10 times the usual value of 10^{-4} eV/°K.

We use part of this experiment in the sophomore physics laboratory. It is not easy for a sophomore to get good data, however; meticulous experimental technique is required. The calculations are tedious and since the student has not generally done this kind of work before, he is likely to make mistakes. Nevertheless, it is probably worth the frustration for the student occasionally to face an experiment that taxes his skill. A junior enrolled in an electrical measurements laboratory should find it just right.

V. CONCLUSION

An experiment on thermionic emission has been described in which the cathode temperature is determined from the energy distribution of the emitted electrons. The experiment permits calibration of the cathode temperature vs heater power, the determination of the work functions of both anode and cathode, and the temperature coefficient of the latter. The equipment requirements are modest, and a commercial vacuum tube may be used. The experiment is suitable for an undergraduate electricity and magnetism laboratory.

[1] C. Herring and M. H. Nichols, Rev. Mod. Phys. **21**, 185 (1949).
[2] E. B. Hensley, J. Appl. Phys. **32**, 301 (1961).
[3] H. Moss, J. Electron. Control **II**, 305 (1957). This paper is one of several in *Selected Papers on New Techniques for Energy Conversion*, edited by S. Levine (Dover, New York, 1961).
[4] W. Schottky, Ann. Physik **44**, 1011 (1914).