Fenômenos de Transporte III - PAIVA

ADIMENSIONALIZAÇÃO

Expressão das equações de Fenômenos de Transporte na forma adimensionalizada

- **Números ADIMENSIONAIS**

  (Re, Pr, Sc, Pe, Gr, Sh, Nu etc..)

  - **Correlações semi-empíricas generalizadas**
  - **Analogia dos FT's**

- **"SCALING"**

  Identificação das ordens de grandeza

  - **Simplificação das equações (equações aproximadas)**
  - **Resultados semi-quantitativos**

- **CFD _Fluidodinâmica Computacional**

  - **Simplificação das equações (equações aproximadas)**
  - **Elaboração de critérios e modelos**

ADIMENSIONALIZAÇÃO

PARÂMETROS CARACTERÍSTICOS: t_0, ρ_0, v_0, L

ADIMENSIONALIZAÇÃO DAS VARIÁVEIS: $\hat{\rho} = \dfrac{\rho}{\rho_0}$; $\hat{\vec{v}} = \dfrac{\vec{v}}{v_0}$; $\hat{\vec{r}} = \dfrac{\vec{r}}{L}$; $\hat{t} = \dfrac{t}{t_0}$

$$\rho = \rho_0 \hat{\rho} \quad ; \quad \vec{v} = v_0 \hat{\vec{v}} \quad ; \quad \vec{r} = L \hat{\vec{r}} \quad ; \quad t = t_0 \hat{t}$$

ADIMENSIONALIZAÇÃO DOS OPERADORES:

$$\hat{\vec{\text{grad}}}(\ ) = L\,\vec{\text{grad}}(\ ) \quad ; \quad \hat{\text{div}}(\ ) = L\,\text{div}(\ ) \quad ; \quad \hat{\text{lap}}(\ ) = \hat{\text{div}}\,\hat{\vec{\text{grad}}}(\ ) = L^2\,\text{lap}(\ )$$

$$\frac{\partial}{\partial t} = \frac{1}{t_0}\frac{\partial}{\partial \hat{t}}$$

$$\frac{\partial f}{\partial x} = \frac{\partial(\hat{f} f_0)}{\partial(\hat{x} x_0)} = \frac{f_0}{x_0}\frac{\partial \hat{f}}{\partial \hat{x}}$$

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial(\hat{x} x_0)}\left(\frac{\partial(\hat{f} f_0)}{\partial(\hat{x} x_0)}\right) = \frac{f_0}{x_0^{\,2}}\frac{\partial}{\partial \hat{x}}\left(\frac{\partial \hat{f}}{\partial \hat{x}}\right) = \frac{f_0}{x_0^{\,2}}\frac{\partial^2 \hat{f}}{\partial \hat{x}^2}$$

Equação da Continuidade

$$\frac{\partial \rho}{\partial t} = -\operatorname{div} \rho \vec{v}$$

$$\frac{\rho_0}{t_0}\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\frac{\rho_0 v_0}{L}\operatorname{div} \hat{\rho}\hat{\vec{v}} \quad \Rightarrow \quad \frac{L}{t_0 v_0}\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\operatorname{div} \hat{\rho}\hat{\vec{v}}$$

$$\boxed{\frac{1}{Sr}\frac{\partial \hat{\rho}}{\partial \hat{t}} = -\operatorname{div} \hat{\rho}\hat{\vec{v}}}$$

STROUHAL

$$\boxed{Sr = \frac{t_0 v_0}{L}}$$

t_0 : tempo para o escoamento "atingir o regime permanente" ou associado à frequência em processo oscilatório.

Equação de Conservação Generalizada

Balanço microscópico de φ:

$$\rho \frac{D\varphi}{Dt} = \frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\left(\rho\varphi \vec{v}\right) = -\mathrm{div}\, \vec{j}_\phi + \dot{\varphi}_{\forall\Phi}$$

$$\boxed{\frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\left(\rho \vec{v}\varphi + \vec{j}_\Phi\right) = \dot{\varphi}_{\forall_\Phi}}$$

Equação constitutiva de difusão:

$$\boxed{\vec{j}_\Phi = -\rho\Gamma_\varphi\, \mathrm{grad}\, \varphi}$$

φ	Γ_Φ	ν/Γ_Φ
V	ν	1
w_A	D_{AB}	Sc
c_pT	α	Pr

$$\boxed{\frac{\partial \rho\varphi}{\partial t} + \mathrm{div}\, \rho\left(\vec{v}\varphi - \Gamma_\varphi \mathrm{grad}\, \varphi\right) = \dot{\varphi}_{\forall_\Phi}}$$

Equação de Conservação Generalizada

$$\boxed{\frac{\partial \rho\varphi}{\partial t} + \operatorname{div} \rho\left(\vec{v}\varphi - \Gamma_{\varphi}\operatorname{grad}\ \varphi\right) = \dot{\varphi}_{\forall_{\Phi}}}$$

$$\boxed{\rho = \rho_0\,\hat{\rho}\quad;\quad \vec{v} = v_0\,\hat{\vec{v}}\ ;\ \vec{r} = L\,\hat{\vec{r}}\ ;\quad t = t_0\,\hat{t}\quad;\quad \varphi = \hat{\varphi}\,\Delta\varphi + \varphi_0}$$

$$\frac{1}{t_0}\frac{\partial \rho_0\,\hat{\rho}\left(\hat{\varphi}\Delta\varphi + \varphi_0\right)}{\partial \hat{t}} + \frac{1}{L}\hat{\operatorname{div}}\,\hat{\rho}\left(\rho_0 v_0\,\hat{\vec{v}}\left(\hat{\varphi}\Delta\varphi + \varphi_0\right) - \frac{\rho_0}{L}\Gamma_{\varphi}\hat{\operatorname{grad}}\left(\hat{\varphi}\Delta\varphi + \varphi_0\right)\right) = \dot{\varphi}_{\forall_{\Phi}}$$

Reagrupando-se os termos:

$$\varphi_0\frac{\rho_0 v_0}{L}\underbrace{\left[\frac{L}{t_0 v_0}\frac{\partial \hat{\rho}}{\partial \hat{t}} + \hat{\operatorname{div}}\left(\hat{\rho}\hat{\vec{v}}\right)\right]}_{=0\text{, continuidade}} + \frac{\rho_0\Delta\varphi}{t_0}\frac{\partial\left(\hat{\rho}\hat{\varphi}\right)}{\partial \hat{t}} + \frac{\rho_0 v_0\Delta\varphi}{L}\left[\hat{\operatorname{div}}\left(\hat{\rho}\hat{\vec{v}}\hat{\varphi}\right) - \hat{\operatorname{div}}\,\hat{\rho}\left(\frac{\Gamma_{\varphi}}{v_0 L}\hat{\operatorname{grad}}\ \hat{\varphi}\right)\right] = \dot{\varphi}_{\forall_{\Phi}}$$

Equação de Conservação Generalizada

$$\frac{\partial \rho\varphi}{\partial t} + \operatorname{div} \rho\left(\vec{v}\varphi - \Gamma_\varphi \operatorname{grad} \varphi\right) = \dot{\varphi}_{\forall_\Phi}$$

$$\frac{L}{v_0\, t_0}\frac{\partial(\hat{\rho}\hat{\varphi})}{\partial \hat{t}} + \hat{\operatorname{div}}\,\hat{\rho}\left(\vec{v}\hat{\varphi} - \frac{\Gamma_\varphi}{v_0 L}\,\hat{\operatorname{grad}}\,\hat{\varphi}\right) = \frac{\dot{\varphi}_{\forall_\Phi} L}{\rho_0 v_0 \Delta\varphi} \Rightarrow$$

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\varphi}}{\partial \hat{t}} + \operatorname{div}\hat{\rho}\left(\vec{v}\hat{\varphi} - \frac{1}{Pe}\,\hat{\operatorname{grad}}\,\hat{\varphi}\right) = \frac{\dot{\varphi}_{\forall_\Phi} L}{\rho_0\, \Delta\varphi\, v_0}$$

PECLET

$$Pe = \frac{v_0 L}{\Gamma_\varphi} = \frac{\nu}{\Gamma_\varphi}\frac{v_0 L}{\nu} = Re\,\frac{\nu}{\Gamma_\varphi}$$

$$Pe = Re\,\frac{\nu}{\Gamma_\varphi}$$

Equação de Conservação Generalizada

ADIMENSIONAIS

PECLET

$$\boxed{Pe = Re\,\frac{\nu}{\Gamma_\varphi}}$$

CONVECÇÃO / DIFUSÃO

PECLET MÁSSICO

$$\boxed{Pe = Re\,Sc}$$

PECLET TÉRMICO

$$\boxed{Pe = Re\;Pr}$$

PRANDTL

$$\boxed{Pr = \frac{\nu}{\alpha}}$$

$$\alpha = \frac{k}{\rho Cp}$$

SCHMIDT

$$\boxed{Sc = \frac{\nu}{D_{AB}}}$$

φ	Γ_Φ	ν/Γ_Φ
V	ν	1
w_A	D_{AB}	Sc
c_pT	α	Pr

Equação da Continuidade para espécie A:

$$\boxed{\frac{L}{v_0 t_0}\frac{\partial(\hat{\rho}\hat{\varphi})}{\partial \hat{t}} + \hat{\mathrm{div}}\,\hat{\rho}\left(\hat{v}\hat{\varphi} - \frac{1}{Pe}\,\hat{\mathrm{grad}}\,\hat{\varphi}\right) = \frac{\dot{\varphi}_{\forall_\Phi} L}{\rho_0 v_0 \Delta\varphi} \Rightarrow}$$

$\varphi = \omega_A$ *fração mássica* $\hat{\varphi} = \hat{\omega}_A = \dfrac{\omega_A - \omega_{A0}}{\omega_{AS} - \omega_{A0}}$

Reação química

Equação Cinética:

$$\boxed{\dot{\varphi}_{\forall_T} = r_A \quad \left(\frac{\mathrm{kg\,de\,A}}{\mathrm{m^3.s}}\right)}$$

$$\boxed{\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{\mathrm{div}}\,\hat{\rho}\left(\hat{v}\hat{\omega}_A - \frac{1}{Pe}\,\hat{\mathrm{grad}}\,\hat{\omega}_A\right) = \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}}$$

Equação da Continuidade para espécie A – Escalas de tempos:

$$\boxed{\frac{L}{t_0 v_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{\mathrm{div}}\,\hat{\rho}\left(\hat{\rho}\hat{v}\hat{\omega}_A - \frac{D_{AB}}{v_0 L}\,\hat{\mathrm{grad}}\,\hat{\omega}_A\right) = \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}}$$

$$\frac{L}{t_0 v_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{\mathrm{div}}\,\hat{\rho}\hat{\rho}\hat{v}\hat{\omega}_A = \frac{D_{AB}}{v_0 L}\,\hat{\mathrm{lap}}\,\hat{\omega}_A + \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}$$

$$\frac{1}{t_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \frac{v_0}{L}\hat{\mathrm{div}}\,\hat{\rho}\hat{\rho}\hat{v}\hat{\omega}_A = \frac{D_{AB}}{L^2}\,\hat{\mathrm{lap}}\,\hat{\omega}_A + \frac{r_A}{\rho_0\,\Delta\omega_A}$$

$$\boxed{\frac{1}{t_0}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \frac{1}{t_C}\hat{\mathrm{div}}\,\hat{\rho}\hat{\rho}\hat{v}\hat{\omega}_A = \frac{1}{t_D}\,\hat{\mathrm{lap}}\,\hat{\omega}_A + \frac{1}{t_R}}$$

Convecção

$$\boxed{t_C = \frac{L}{v_0}}$$

Difusão

$$\boxed{t_D = \frac{L^2}{D_{AB}}}$$

Reação

$$\boxed{t_R = \frac{\rho_0\,\Delta\omega_A}{r_A}}$$

Equação da Continuidade para espécie A:

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(\hat{v}\hat{\rho}\hat{\omega}_A - \frac{1}{Pe}\,\hat{grad}\,\hat{\omega}_A\right) = \frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0}$$

$$\frac{1}{Pe} = \frac{L}{v_0}\frac{D_{AB}}{L^2} = \frac{t_C}{t_D}$$

$$\frac{r_A L}{\rho_0\,\Delta\omega_A\,v_0} = \frac{t_C}{t_R} = Da_1$$

DAMKÖHLER 1

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(\hat{v}\hat{\rho}\hat{\omega}_A - \frac{1}{Pe}\,\hat{grad}\,\hat{\omega}_A\right) = Da_1$$

Equação da Continuidade para espécie A:

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(\hat{\vec{v}}\hat{\omega}_A - \frac{1}{Pe}\,\hat{grad}\,\hat{\omega}_A\right) = \frac{r_A L}{\rho_0 \Delta\omega_A v_0}$$

x Pe:

$$\frac{Pe}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(Pe\,\hat{\vec{v}}\hat{\omega}_A - \hat{grad}\,\hat{\omega}_A\right) = \frac{r_A L^2}{\rho_0 \Delta\omega_A D_{AB}}$$

Tempo de Reação: $t_R = \dfrac{\rho_0\,\Delta\omega_A}{r_A}$

Tempo de Difusão: $t_D = \dfrac{L^2}{D_{AB}}$

$$\frac{r_A L^2}{\rho_0 \Delta\omega_A D_{AB}} = \frac{t_D}{t_R} = Da_2$$

DAMKÖHLER 2

$$\frac{Pe}{Sr}\frac{\partial \hat{\rho}\hat{\omega}_A}{\partial \hat{t}} + \hat{div}\,\hat{\rho}\left(Pe\,\hat{\vec{v}}\hat{\omega}_A - \hat{grad}\,\hat{\omega}_A\right) = Da_2$$

Reação/difusão em poros de catalisador sólido –

Módulo de Thiele = Th = $(Da_2)^{0,5}$

Coeficientes Convectivos

ADIMENSIONALIZAÇÃO

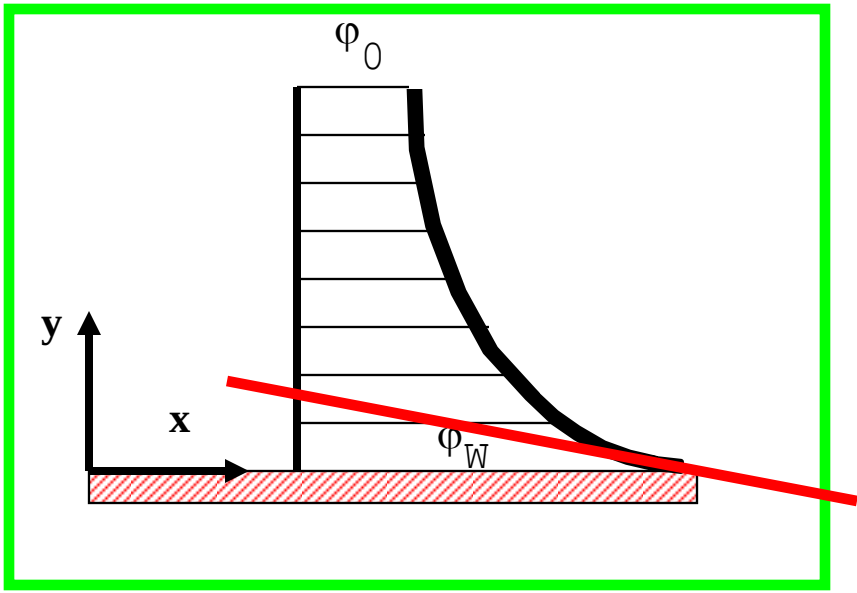
$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\varphi}}{\partial \hat{t}} + \mathrm{div}\,\hat{\rho}\left(\underbrace{\hat{v}\hat{\varphi}}_{1} - \underbrace{\frac{1}{Pe}\,\mathrm{grad}\,\hat{\varphi}}_{2}\right) = \underbrace{\frac{\dot{\varphi}_{\forall_\Phi} L}{\rho_0\,\Delta\varphi\, v_0}}_{3}$$

ESCOAMENTO + DIFUSÃO

INTERFACE / PAREDE

COEFICIENTE DE CONVECÇÃO

$$\dot{j}_{\phi,parede} = CO_{\varphi}\left(\varphi_S - \varphi_0\right)$$

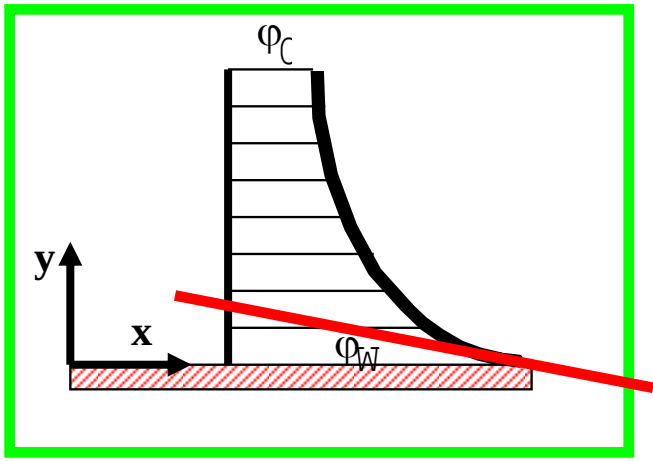


grad φ

Coeficientes Convectivos

ADIMENSIONAIS

$$\frac{1}{Sr}\frac{\partial \hat{\rho}\hat{\varphi}}{\partial \hat{t}} + \operatorname{div}\hat{\rho}\left(\underbrace{\vec{v}\hat{\varphi} - \frac{1}{Pe}\,\widehat{\operatorname{grad}}\,\hat{\varphi}}_{\text{NA PAREDE } \vec{v}=0}\right) = \frac{\dot{\varepsilon}_{\varphi}\, L}{\rho_0\, \Delta\varphi\, v_0}$$



grad φ

NUSSELT

$$Nu = \left(\widehat{\operatorname{grad}}\,\hat{T}\right)_{\hat{r}=0} = \frac{h\,L}{k}$$

CALOR

SHERWOOD

$$Sh = \left(\widehat{\operatorname{grad}}\,\hat{w}_i\right)_{\hat{r}=0} = \frac{k\,L}{D_{AB}}$$

MASSA

FATOR DE ATRITO

$$f = \frac{1}{Re}\left(\widehat{\operatorname{grad}}\,\hat{v}\right)_{\hat{r}=0}$$

QUANTIDADE DE MOVIMENTO

ADIMENSIONAIS –ANALOGIA de REYNOLDS
NÚMERO DE STANTON

St = NÚMERO DE STANTON

QUANTIDADE DE MOVIMENTO

$$St = \frac{f}{2}$$

FATOR DE FANNING

CALOR

$$St = \frac{Nu}{Pe} = \frac{Nu}{Re\,Pr} = \frac{h}{\rho c_P V}$$

MASSA

$$St = \frac{Sh}{Pe} = \frac{Sh}{Re\,Sc} = \frac{k}{V}$$

ADIMENSIONAIS –ANALOGIA DE COLBURN

FATOR j

QUANTIDADE DE MOVIMENTO

$$\frac{f}{2} = j_H = j_M$$

FATOR DE FANNING

CALOR

$$j_H = \frac{Nu}{Re\,Pr^{1/3}}$$

MASSA

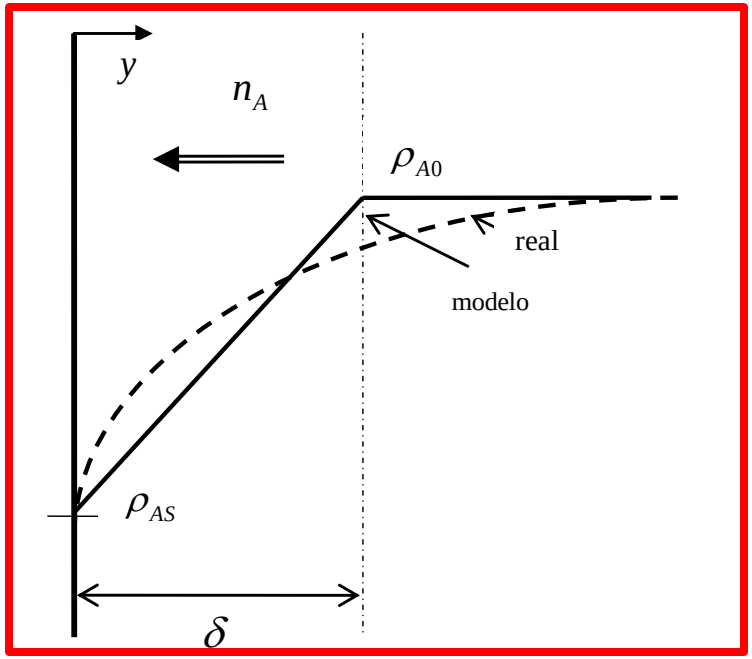
$$j_M = \frac{Sh}{Re\,Sc^{1/3}}$$

Modelo do filme

LEWIS E WHITMAN (1929)

-FILME ESTAGNADO, PRÓXIMO À INTERFACE/SUPERFÍCIE

-TRANSPORTE DE MASSA (NO FILME) POR DIFUSÃO.



$$D_{AB}\frac{d^2\rho_A}{dy^2}=0$$

condições de contorno: $\begin{cases} y=0, & \rho_A=\rho_{AS} \\ y=\delta, & \rho_A=\rho_{A0} \end{cases}$

perfil de concentrações: $\rho_A=\left(\frac{\rho_{A0}-\rho_{AS}}{\delta}\right)y+\rho_{AS}$

No caso diluído: $n_A \cong j_A=-D_{AB}\left(\frac{d\rho_A}{dy}\right)_{y=0}=-D_{AB}\left(\frac{\rho_{A0}-\rho_{AS}}{\delta}\right)$

$$n_A=k_\rho\left(\rho_{A0}-\rho_{AS}\right) \qquad \boxed{k_\rho=\frac{D_{AB}}{\delta}}$$

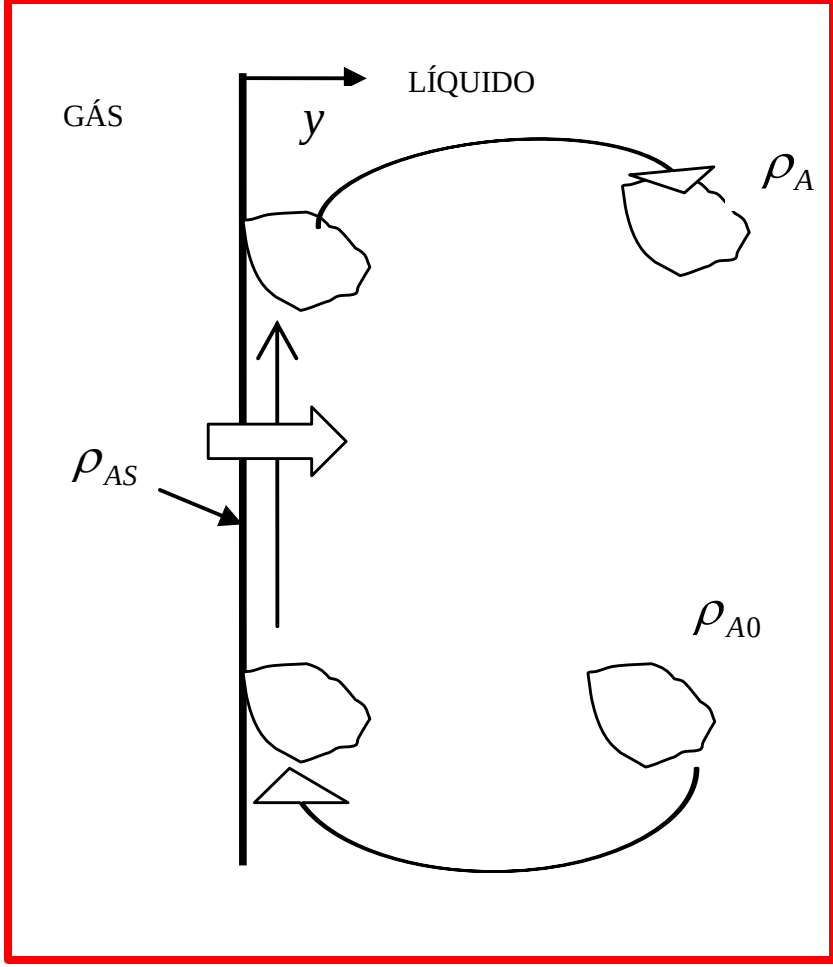
Modelo de Higbie

HIGBIE (1935)

PEQUENAS PORÇÕES DO LÍQUIDO SÃO TRANSPORTADAS CONTINUAMENTE DO SEIO DO LÍQUIDO PARA A INTERFACE,

PERMANECENDO EM CONTATO COM A FASE GÁS, DURANTE UM TEMPO, Θ, NO QUAL OCORRE A TRANSFERÊNCIA DE MASSA POR DIFUSÃO,

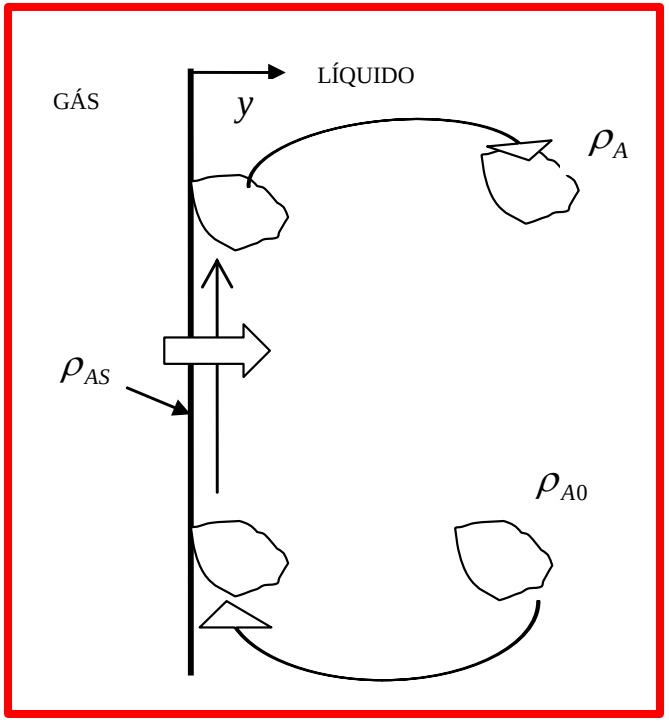
E DEPOIS SÃO TRANSPORTADOS PARA O SEIO DO LÍQUIDO.



Modelo de Higbie

$$\frac{D\rho_A}{Dt} = D_{AB}\frac{d^2\rho_A}{dy^2}$$

observação lagrangeana



Condições de contorno $\begin{cases} t = 0, \ \rho_A = \rho_{A0} \\ t > 0 \begin{cases} y = 0, \ \rho_A = \rho_{AS} \\ y = \infty, \ \rho_A = \rho_{A0} \end{cases} \end{cases}$

$$\frac{\rho_{AS} - \rho_A}{\rho_{AS} - \rho_{A0}} = \mathrm{erf}\,\frac{y}{2\sqrt{t\,D_{AB}}}$$

$$\mathrm{erf}\,\frac{y}{2\sqrt{t\,D_{AB}}} = \frac{2}{\sqrt{\pi}}\int_0^{\frac{y}{2\sqrt{t\,D_{AB}}}} e^{-n^2}\,dn$$

Fluxo de transporte de massa instantâneo, no caso diluído:

$$n_{A,y=0} \cong j_{A,y=0} = -D_{AB}\left(\frac{d\rho_A}{dy}\right)_{y=0} = \left(\rho_{AS} - \rho_{A0}\right)\sqrt{\frac{D_{AB}}{\pi\,t}}$$

Fluxo de transporte de massa médio é (tempo de contato θ)

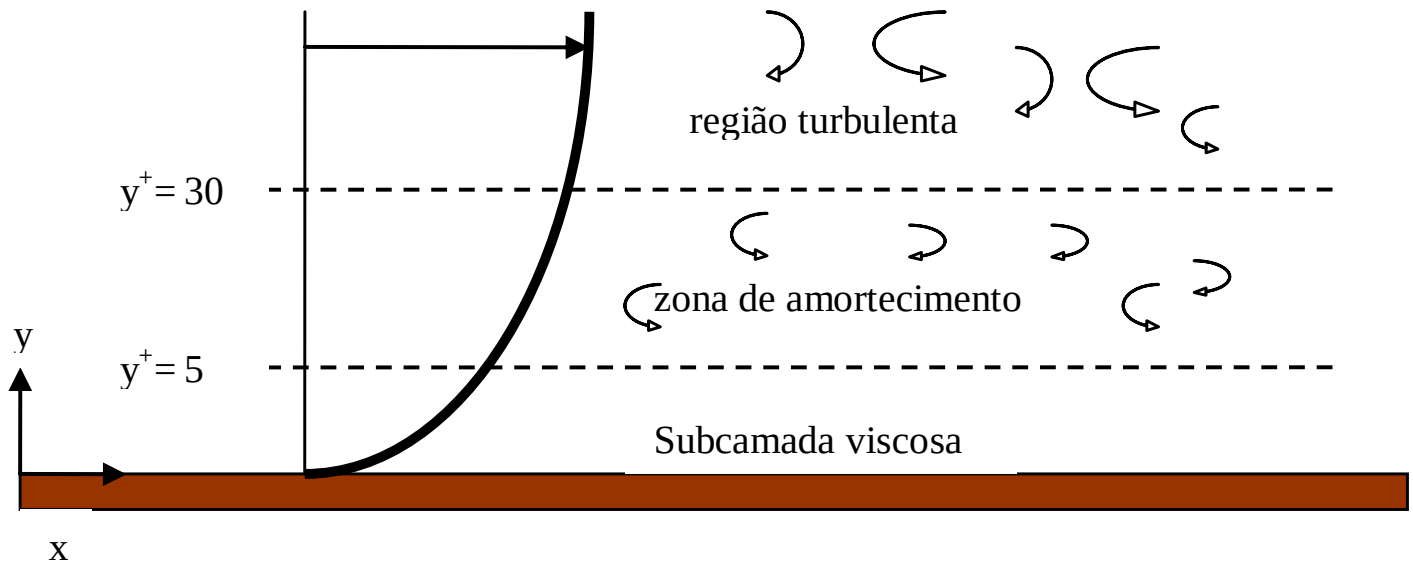
$$\bar{n}_A = \frac{\int_0^{\theta} n_A\,dt}{\theta} = \frac{\left(\rho_{AS} - \rho_{A0}\right)}{\theta}\sqrt{\frac{D_{AB}}{\pi}}\int_0^{\theta}\frac{dt}{\sqrt{t}} = 2\left(\rho_{AS} - \rho_{A0}\right)\sqrt{\frac{D_{AB}}{\theta\,\pi}}$$

$$\boxed{k_\rho = 2\sqrt{\frac{D_{AB}}{\theta\,\pi}}}$$

ANALOGIAS – Escoamento turbulento

$$n_A = -(D_{AB} + \varepsilon_D)\frac{d\rho_A}{dy}$$

$$y^+ = \frac{u_0 y}{\nu} \qquad u^+ = \frac{v_x}{u_0} \qquad u_0 = \sqrt{\frac{\tau_0}{\rho}}$$



subcamada viscosa	$u^+ = y^+$	$0 \langle y^+ \langle 5$
zona de amortecimento	$u^+ = 5 \ln y^+ - 3{,}05$	$5 \langle y^+ \langle 30$
região turbulenta	$u^+ = 2{,}5 \ln y^+ + 5{,}5$	$y^+ \rangle 30$

ANALOGIAS

ANALOGIA de Prandtl-Taylor

$$\mathrm{Sh} = \frac{\left(\mathrm{f}/2\right)\mathrm{Re}\,\mathrm{Sc}}{1+5\sqrt{f/2}\left(Sc-1)\right)}$$

ANALOGIA de Von-Karman

$$\mathrm{Sh} = \frac{\left(\mathrm{f}/2\right)\mathrm{Re}\,\mathrm{Sc}}{1+5\sqrt{f/2}\left\{Sc-1+\ln\left[\left(1+5Sc\right)/6\right]\right\}}$$

Efeito de Entrada

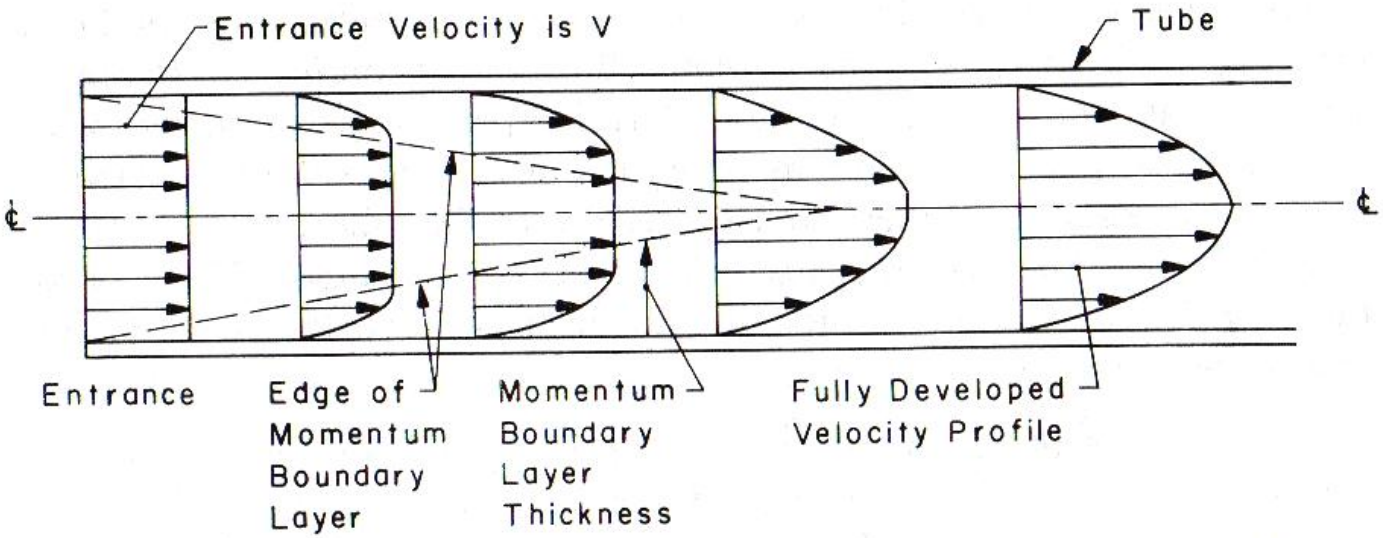


Figure 5.10. Development of the momentum boundary layer and velocity profile for laminar flow in the entrance region of a tube.

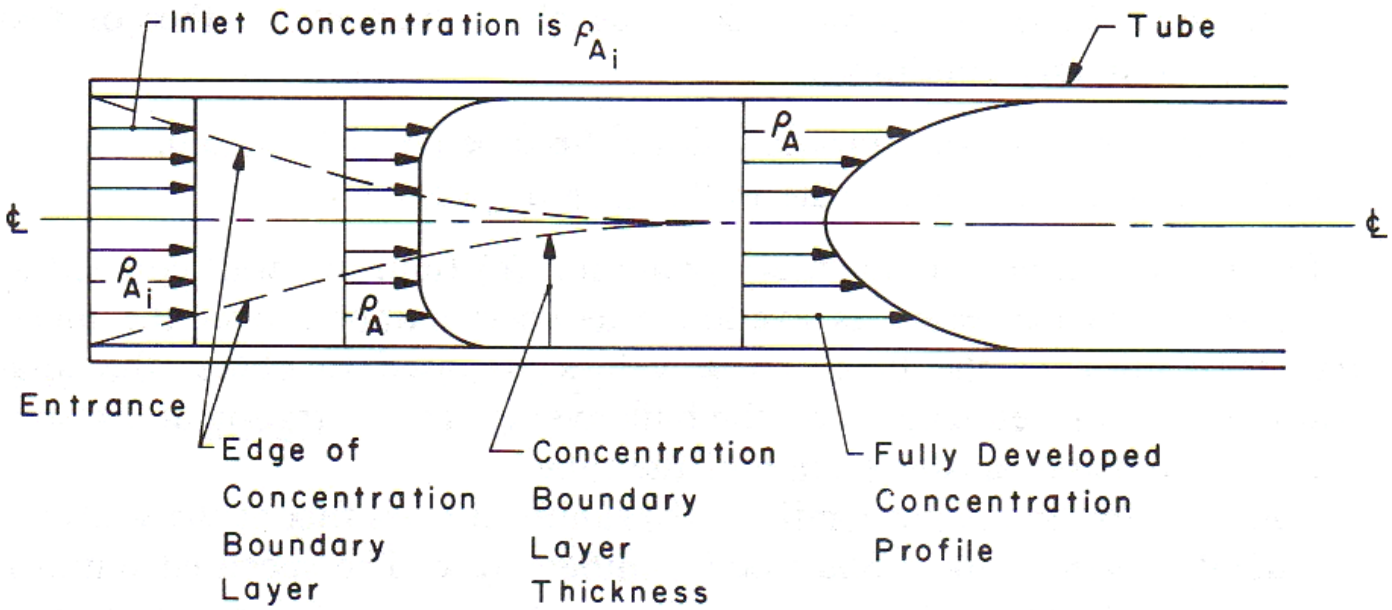
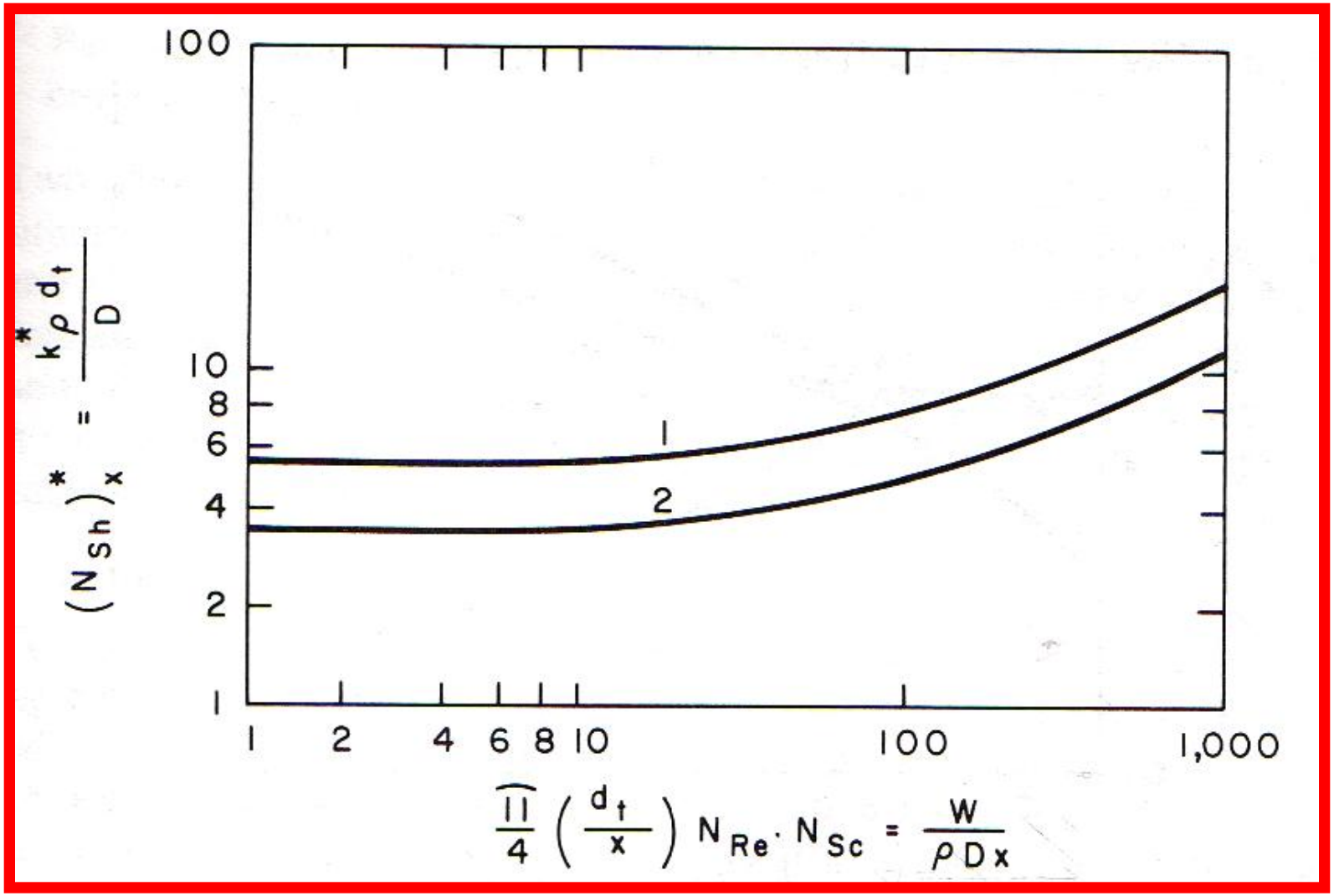


Figure 5.11. Development of the concentration boundary layer and profile for laminar flow in the entrance region of a tube. Transfer is from the tube walls to the fluid.

Sh_x – local – Tubo - laminar



Sh_{LN} e Sh_a – Tubo - Laminar

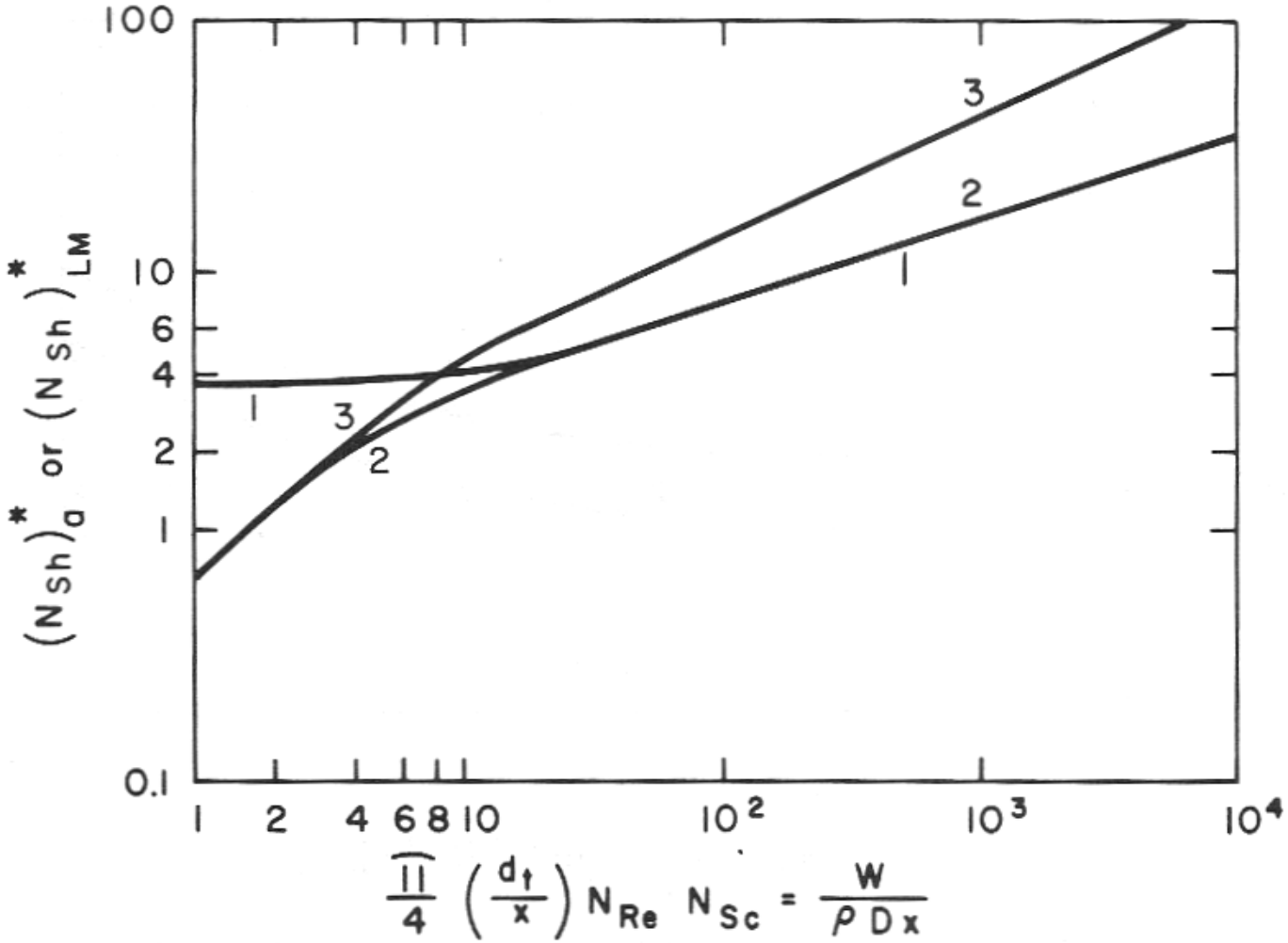


Figure 5.16. Average Sherwood number for mass transfer in laminar flow through a tube with uniform wall concentration. Curve 1: Fully developed parabolic velocity distribution, $(N_{Sh})^*_{LM}$. Curve 2: Fully developed parabolic velocity distribution, $(N_{Sh})^*_a$. Curve 3: Plug flow, $(N_{Sh})^*_a$.

K – esferas

Equation Number	Equation	Range of Variables	Reference
6.134	$N^*_{Sh} = 0.82 N_{Re}^{1/2} N_{Sc}^{1/3}$	$100 \leqslant N_{Re} \leqslant 3500$ $N_{Sc} = 1560$	Aksel'rud (1953)
6.135	$N^*_{Sh} = 0.582 N_{Re}^{1/2} N_{Sc}^{1/3}$	$300 \leqslant N_{Re} \leqslant 7600$ $N_{Sc} \doteq 1210$	Linton and Sutherland (1960)
6.136	$N^*_{Sh} = 0.692 N_{Re}^{0.514} N_{Sc}^{1/3}$	$500 \leqslant N_{Re} \leqslant 5000$	Pasternak and Gauvin (1960)
6.137	$N^*_{Sh} = 0.33 N_{Re}^{0.6} N_{Sc}^{1/3}$	$1500 \leqslant N_{Re} \leqslant 12000$	Evnochides and Thodos (1961)
6.138	$N^*_{Sh} = 0.74 N_{Re}^{1/2} N_{Sc}^{1/3}$	$120 \leqslant N_{Re} \leqslant 6000$ $N_{Sc} \doteq 2.44$	Skelland and Cornish (1963)
With $N^*_{Sh\ nc}$:			
6.139	$N^*_{Sh} = 44 + 0.48 N_{Re}^{1/2} N_{Sc}^{1/3}$	$20 \leqslant N_{Re} \leqslant 850$ $N_{Sc} = 1210$	Garner and Grafton (1954)
6.140	$N^*_{Sh} = N^*_{Sh\,nc} + 0.347 (N_{Re} N_{Sc}^{1/2})^{0.62}$	$1 \leqslant N_{Re} \leqslant 3 \times 10^4$ $0.6 \leqslant N_{Sc} \leqslant 3200$	Steinberger and Treybal (1960)
	$N^*_{Sh\,nc} = 2 + 0.569 (N_{Gr} N_{Sc})^{0.25}$ $N^*_{Sh\,nc} = 2 + 0.0254 (N_{Gr} N_{Sc})^{1/3} N_{Sc}^{0.244}$	$N_{Gr} N_{Sc} < 10^8$ $N_{Gr} N_{Sc} > 10^8$	

K – esferas

Table 6.3. Some experimental correlations of forced-convection mass transfer from single spheres.[a]

Equation Number	Equation	Range of Variables	Reference
With N^*_{Sh0}			
6.127	$N^*_{Sh} = 2 + 0.552 N_{Re}^{1/2} N_{Sc}^{1/3}$	$2 \leqslant N_{Re} \leqslant 800$ $0.6 \leqslant N_{Sc} \leqslant 2.7$	Frössling, (1938, 1940) Maxwell and Storrow (1957)
6.128	$N^*_{Sh} = 2 + 0.60 N_{Re}^{1/2} N_{Sc}^{1/3}$	$2 \leqslant N_{Re} \leqslant 200$ $0.6 \leqslant N_{Sc} \leqslant 2.5$	Ranz and Marshall (1952)
6.129	$N^*_{Sh} = 2 + 0.544 N_{Re}^{1/2} N_{Sc}^{1/3}$	$50 \leqslant N_{Re} \leqslant 350$ $N_{Sc} = 1$	Hsu, Sato, and Sage (1954)
6.130	$N^*_{Sh} = 2 + 0.95 N_{Re}^{1/2} N_{Sc}^{1/3}$	$100 \leqslant N_{Re} \leqslant 700$ $1200 \leqslant N_{Sc} \leqslant 1525$	Garner and Suckling (1958)
6.131	$N^*_{Sh} = 2 + 0.575 N_{Re}^{1/2} N_{Sc}^{0.35}$	$1 < N_{Re}$ $1 \leqslant N_{Sc}$	Griffith (1960)
6.132	$N^*_{Sh} = 2 + 0.79 N_{Re}^{1/2} N_{Sc}^{1/3}$	$20 \leqslant N_{Re} \leqslant 2000$	Rowe, Claxton, and Lewis (1965)
Without N^*_{Sh0}			
6.133	$N^*_{Sh} = 0.43 N_{Re}^{0.56} N_{Sc}^{1/3}$	$200 \leqslant N_{Re} \leqslant 4 \times 10^4$ "air" $\leqslant N_{Sc} \leqslant$ "water"	Williams (1942) Maisel and Sherwood (1950) Linton and Sherwood (1950)

k_C bolha – fase contínua

Table 8.2. Empirical correlations for k^*_{cr}.

Droplet condition	Eq. no.	Correlation	Reference
Stagnant, single, spherical[a]	8.60	$k^*_{cr} = 0.74 \frac{D_c}{d_p} \left(\frac{\rho}{M}\right)_{av} \left(\frac{d_p u_s \rho_c}{\mu_c}\right)^{1/2} \left(\frac{\mu_c}{\rho_c D_c}\right)^{1/3}$	Skelland and Cornish (1963)
Stagnant, single, oblate spheroidal[b]	8.61	$k^*_{cr} = 0.74 \frac{D_c}{d_3} \left(\frac{\rho}{M}\right)_{av} \left(\frac{d_3 u_s \rho_c}{\mu_c}\right)^{1/2} \left(\frac{\mu_c}{\rho_c D_c}\right)^{1/3}$	Skelland and Cornish (1963)
Circulating, single drop stream	8.62	$k^*_{cr} = 0.6 \frac{D_c}{d_p} \left(\frac{\rho}{M}\right)_{av} \left(\frac{d_p u_s \rho_c}{\mu_c}\right)^{1/2} \left(\frac{\mu_c}{\rho_c D_c}\right)^{1/2}$, low σ	Garner and Tayeban (1960)
Circulating, swarms of drops	8.63	$k^*_{cr} = 0.725 \left(\frac{\rho}{M}\right)_{av} \left(\frac{d_p u_s \rho_c}{\mu_c}\right)^{-0.43} \left(\frac{\mu_c}{\rho_c D_c}\right)^{-0.58} u_s(1-\phi_d)$, low σ	Ruby and Elgin (1955), Treybal (1963)
Oscillating, single drop stream	8.64	$k^*_{cr} = \frac{D_c}{d_p} \left(\frac{\rho}{M}\right)_{av} \left[50 + 0.0085 \left(\frac{d_p u_s \rho_c}{\mu_c}\right)^{1.0} \left(\frac{\mu_c}{\rho_c D_c}\right)^{0.7}\right]$, low σ	Garner and Tayeban (1960)