



EPUSP

# COEFICIENTES CONVECTIVOS

PQI 3301 - Fenômenos de Transporte III

# Equação de Conservação Generalizada

Balanço microscópico de  $\phi$ :

$$\rho \frac{D\phi}{Dt} = \frac{\partial \rho \phi}{\partial t} + \text{div}(\rho \phi \vec{v}) = -\text{div} \vec{j}_\phi + \dot{\sigma}_{V\phi}$$

$$\frac{\partial \rho \phi}{\partial t} + \text{div}(\rho \vec{v} \phi + \vec{j}_\phi) = \dot{\sigma}_{V\phi}$$

Equação constitutiva de difusão:

$$\vec{j}_\phi = -\rho \Gamma_\phi \text{grad } \phi$$

$\phi$	$\Gamma_\phi$	$v/\Gamma_\phi$
$V$	$v$	1
$w_A$	$D_{AB}$	Sc
$c_p T$	$\alpha$	Pr

$$\frac{\partial \rho \phi}{\partial t} + \text{div} \rho (\vec{v} \phi - \Gamma_\phi \text{grad } \phi) = \dot{\sigma}_{V\phi}$$

# Equação de Conservação Generalizada - Adimensionalização

$$\frac{\partial \rho \varphi}{\partial t} + \text{div} \rho \left( \vec{v} \varphi - \Gamma_{\varphi} \text{grad} \varphi \right) = \dot{\sigma}_{\nabla_{\varphi}}$$

$$\rho = \rho_0 \hat{\rho} \quad ; \quad \vec{v} = v_0 \hat{\vec{v}} \quad ; \quad \vec{r} = L \hat{\vec{r}} \quad ; \quad t = t_0 \hat{t} \quad ; \quad \varphi = \hat{\varphi} \Delta \varphi + \varphi_0$$

$$\frac{1}{\text{Sr}} \frac{\partial \hat{\rho} \hat{\varphi}}{\partial \hat{t}} + \text{div} \hat{\rho} \left( \hat{\vec{v}} \hat{\varphi} - \frac{1}{\text{Pe}} \text{grâd} \hat{\varphi} \right) = \frac{\dot{\sigma}_{\nabla_{\varphi}} L}{\rho_0 \Delta \varphi v_0}$$

# Equação de Conservação Generalizada - Adimensionalização

$$\varphi = \omega_A \quad \text{fração mássica} \quad \hat{\varphi} = \hat{\omega}_A = \frac{\omega_A - \omega_{A0}}{\omega_{AS} - \omega_{A0}}$$

**Reação química**  
**Equação Cinética:**

$$\dot{\sigma}_{\nu T} = r_A \left( \frac{\text{kg de A}}{\text{m}^3} \right)$$

$$\frac{L}{t_0 v_0} \frac{\partial \hat{\rho} \hat{\omega}_A}{\partial \hat{t}} + \text{div} \hat{v} \hat{\rho} \left( \hat{v} \hat{\omega}_A - \frac{D_{AB}}{v_0 L} \text{grad} \hat{\omega}_A \right) = \frac{r_A L}{\rho_0 \Delta \omega_A v_0}$$

## Equação da Continuidade para espécie A

$$\frac{1}{\text{Sr}} \frac{\partial \hat{\rho} \hat{\omega}_A}{\partial \hat{t}} + \text{div} \hat{\mathbf{v}} \hat{\rho} \left( \hat{\mathbf{v}} \hat{\omega}_A - \frac{1}{\text{Pe}} \text{grad} \hat{\omega}_A \right) = \frac{r_A L}{\rho_0 \Delta \omega_A v_0}$$

**PECLET**

$$\text{Pe} = \frac{v_0 L}{\Gamma_\phi} = \frac{v}{\Gamma_\phi} \frac{v_0 L}{v} = \text{Re} \frac{v}{\Gamma_\phi}$$

$$\text{Pe} = \text{Re} \frac{v}{\Gamma_\phi}$$

# Coeficientes Convectivos

## ADIMENSIONALIZAÇÃO

$$\frac{1}{Sr} \frac{\partial \hat{\rho} \hat{\phi}}{\partial \hat{t}} + \text{div} \hat{\rho} \left( \hat{\mathbf{v}} \hat{\phi} - \frac{1}{Pe} \text{grad} \hat{\phi} \right) = \frac{\dot{\sigma}_{\nabla \phi} L}{\rho_0 \Delta \phi v_0}$$

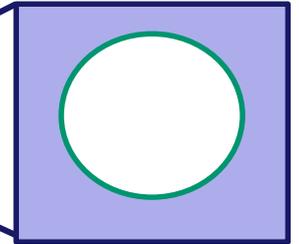
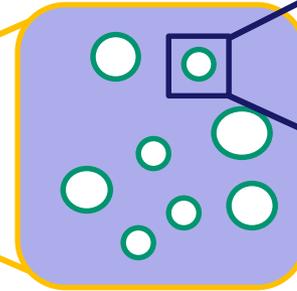
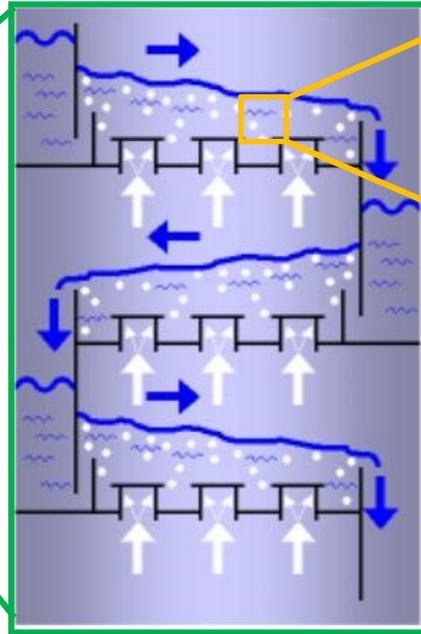
ESCOAMENTO + DIFUSÃO

INTERFACE / PAREDE

COEFICIENTE DE CONVECÇÃO

$$\vec{j}_{\phi, \text{parede}} = CO_{\phi} (\varphi_S - \varphi_0)$$

# Estágio de Equilíbrio e FT's



# FLUXOS MÁSSICOS E MOLARES: DIFUSIVOS E CONVECTIVOS - BIRD

	Quantity	With Respect to Stationary Axes	With Respect to $v$	With Respect to $v^*$
Basic definitions	Velocity of species $A$ (cm sec <sup>-1</sup> )	$v_A$ (A)	$v_A - v$ (B)	$v_A - v^*$ (C)
	Mass flux of species $A$ (g cm <sup>-2</sup> sec <sup>-1</sup> )	$n_A = \rho_A v_A$ (D)	$j_A = \rho_A(v_A - v)$ (E)	$j_A^* = \rho_A(v_A - v^*)$ (F)
	Molar flux of species $A$ (g-moles cm <sup>-2</sup> sec <sup>-1</sup> )	$N_A = c_A v_A$ (G)	$J_A = c_A(v_A - v)$ (H)	$J_A^* = c_A(v_A - v^*)$ (I)
Relations among the fluxes, for reference only	Sum of mass fluxes (g cm <sup>-2</sup> sec <sup>-1</sup> )	$n_A + n_B = \rho v$ (J)	$j_A + j_B = 0$ (K)	$j_A^* + j_B^* = \rho(v - v^*)$ (L)
	Sum of molar fluxes (g-moles cm <sup>-2</sup> sec <sup>-1</sup> )	$N_A + N_B = cv^*$ (M)	$J_A + J_B = c(v^* - v)$ (N)	$J_A^* + J_B^* = 0$ (O)
Relations among the fluxes, for reference only	Fluxes in terms of $n_A$ and $n_B$	$N_A = \frac{n_A}{M_A}$ (P)	$j_A = n_A - \omega_A(n_A + n_B)$ (Q)	$j_A^* = n_A - x_A \left( n_A + \frac{M_A}{M_B} n_B \right)$ (R)
	Fluxes in terms of $N_A$ and $N_B$	$n_A = N_A M_A$ (S)	$J_A = N_A - \omega_A \left( N_A + \frac{M_B}{M_A} N_B \right)$ (T)	$J_A^* = N_A - x_A(N_A + N_B)$ (U)
	Fluxes in terms of $j_A$ and $v$	$n_A = j_A + \rho_A v$ (V)	$J_A = \frac{j_A}{M_A}$ (W)	$j_A^* = \frac{M}{M_B} j_A$ (X)
	Fluxes in terms of $J_A^*$ and $v^*$	$N_A = J_A^* + c_A v^*$ (Y)	$J_A = \frac{M_B}{M} J_A^*$ (Z)	$j_A^* = J_A^* M_A$ (AA)

# CONCENTRAÇÕES USUAIS EM TRANSPORTE DE MASSA

	mass fraction	mole fraction	absolute humidity	mole ratio
	$\omega$ [-]	$x$ [-]	$H \left[ \frac{\text{kg} - \text{A}}{\text{kg} - \text{B}} \right]$	$X \left[ \frac{\text{mol} - \text{A}}{\text{mol} - \text{B}} \right]$
mass fraction $\omega$ [-]	$\omega$	$\frac{x}{\frac{M_B}{M_A} + \left\{ 1 - \frac{M_B}{M_A} \right\} x}$	$\frac{H}{1 + H}$	$\frac{X}{\frac{M_B}{M_A} + X}$
mole fraction $x$ [-]	$\frac{\omega}{\frac{M_A}{M_B} + \left\{ 1 - \frac{M_A}{M_B} \right\} \omega}$	$x$	$\frac{H}{\frac{M_A}{M_B} + H}$	$\frac{X}{1 + X}$
absolute humidity $H \left[ \frac{\text{kg} - \text{A}}{\text{kg} - \text{B}} \right]$	$\frac{\omega}{1 - \omega}$	$\left( \frac{x}{1 - x} \right) \left( \frac{M_A}{M_B} \right)$	$H$	$\left( \frac{M_A}{M_B} \right) X$
mole ratio $X \left[ \frac{\text{mol} - \text{A}}{\text{mol} - \text{B}} \right]$	$\left( \frac{M_B}{M_A} \right) \left( \frac{\omega}{1 - \omega} \right)$	$\left( \frac{x}{1 - x} \right)$	$\left( \frac{M_B}{M_A} \right) H$	$X$

# RELAÇÕES ENTRE AS DEFINIÇÕES DE CONCENTRAÇÃO

	mass fraction $\omega_A [-]$	mole fraction $x_A [-]$	partial density $\rho_A [\text{kg m}^{-3}]$	molar density $c_A [\text{kmol m}^{-3}]$	partial pressure $p_A [\text{kPa}]$
mass fraction $\omega_A [-]$	$\omega$	$\frac{x_A M_A}{\sum_i x_i M_i}$	$\frac{\rho_A}{\sum_i \rho_i}$	$\frac{c_A M_A}{\sum_i c_i M_i}$	$\frac{p_A M_A}{\sum_i p_i M_i}$
mole fraction $x_A [-]$	$\frac{(\omega_A/M_A)}{\sum_i (\omega_i/M_i)}$	$x_A$	$\frac{\rho_A/M_A}{\sum_i (\rho_i/M_i)}$	$\frac{c_A}{\sum_i c_i}$	$\frac{p_A}{\sum_i p_i}$
partial density $\rho_A [\text{kg m}^{-3}]$	$\rho \omega_A$	$\frac{\rho x_A M_A}{\sum_i x_i M_i}$	$\rho_A$	$c_A M_A$	$\frac{M_A p_A}{RT}$
molar density $c_A [\text{kmol m}^{-3}]$	$\frac{\rho \omega_A}{M_A}$	$c x_A$	$\frac{\rho_A}{M_A}$	$c_A$	$\frac{p_A}{RT}$
partial pressure $p_A [\text{kPa}]$	$\frac{(\omega_A/M_A)P}{\sum_i (\omega_i/M_i)}$	$P x_A$	$\frac{RT \rho_A}{M_A}$	$c_A RT$	$p_A$

Mixture:  $\sum_i x_i = 1$ ,  $\sum_i \omega_i = 1$ ,  $\rho = \sum_i \rho_i$ ,  $c = \sum_i c_i$ ,  $P = \sum_i p_i$

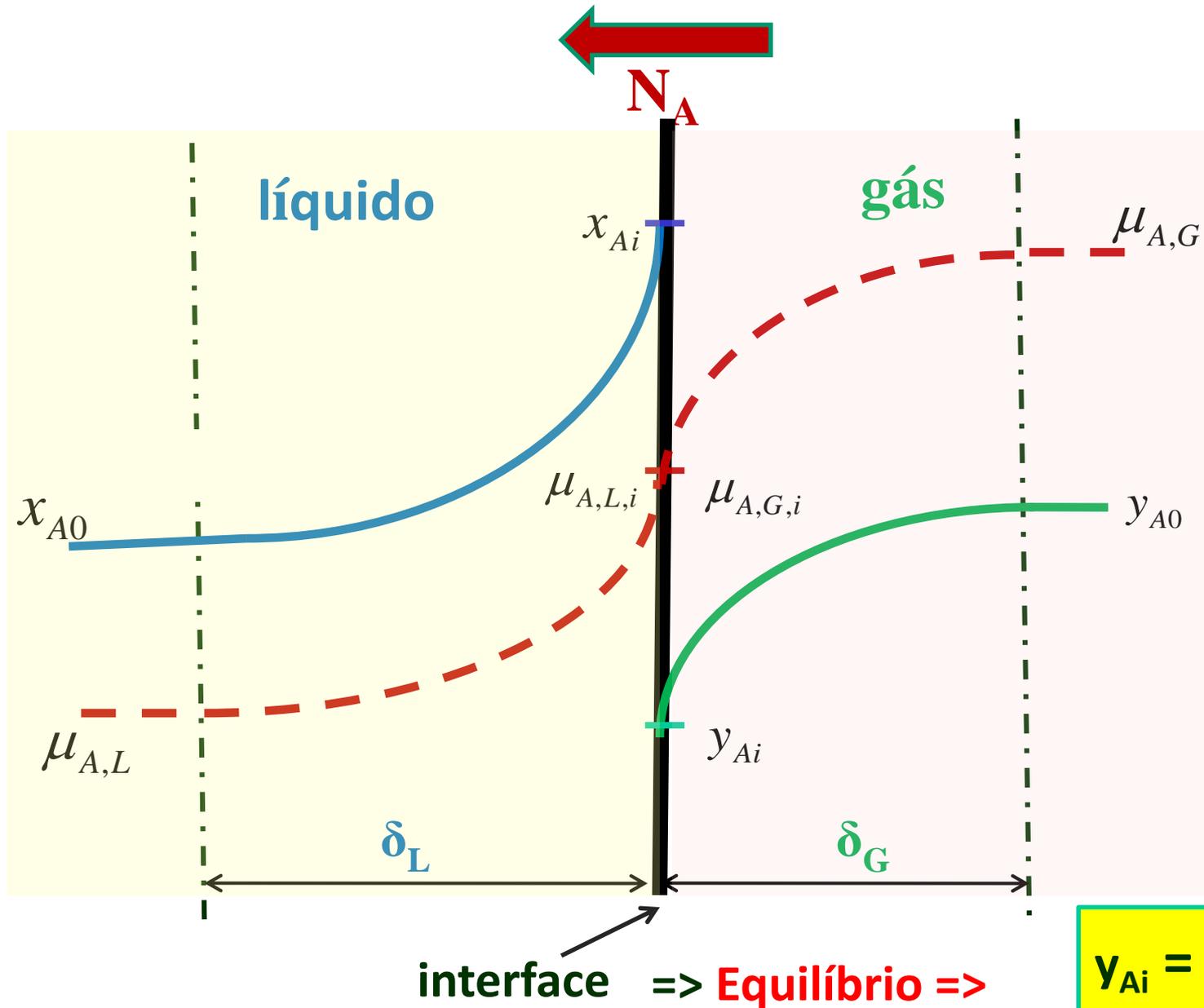
$$M = \sum_i M_i x_i = \left( \sum_i \frac{\omega_i}{M_i} \right)^{-1}$$

# COEFICIENTES DE TRANSPORTE DE MASSA

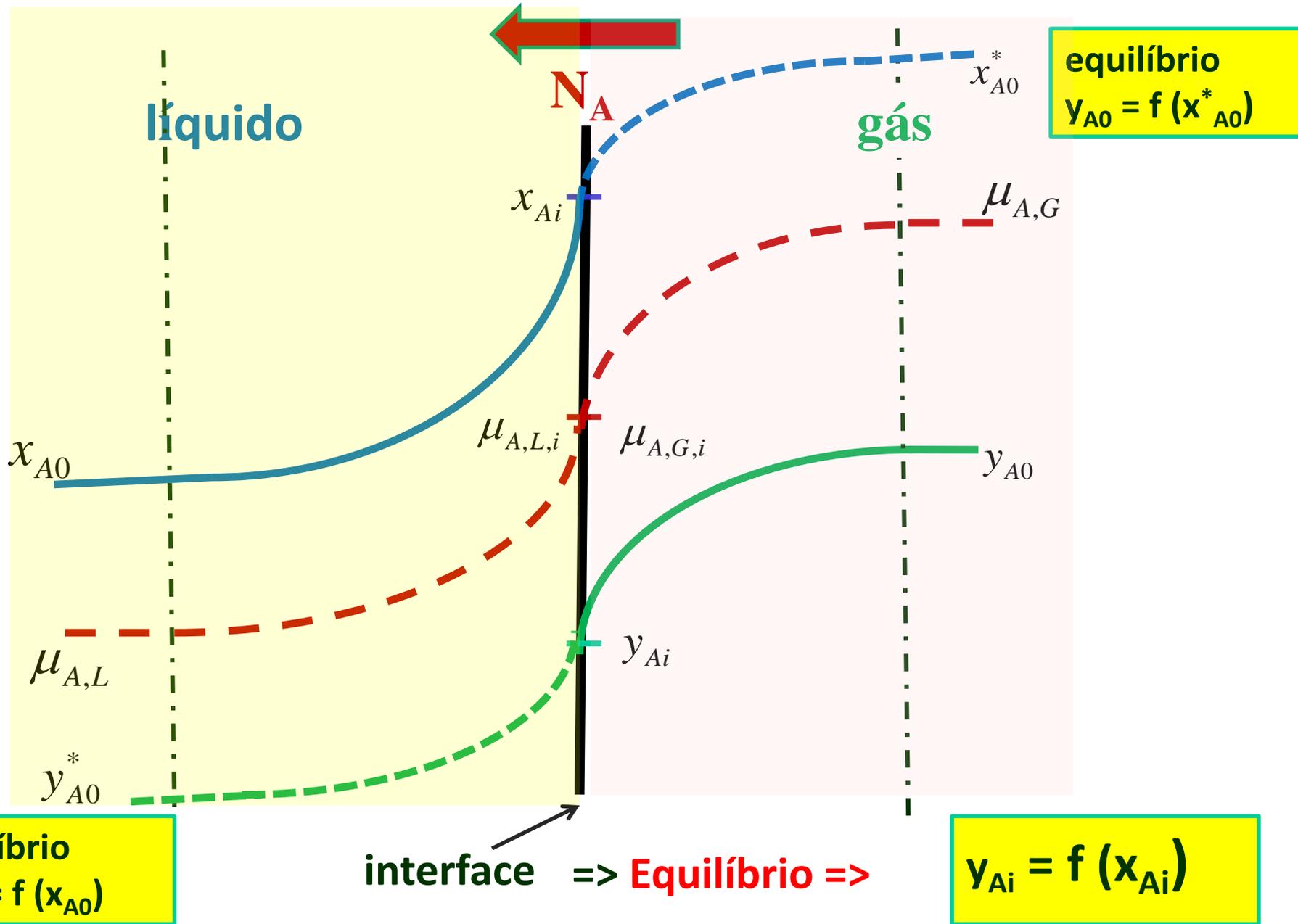
Mass transfer coefficient	Unit	Definition	Driving force	Phase
$k_Y$	$[\text{kmol m}^{-2} \text{s}^{-1}]$	$N_A^* = k_Y (\gamma_s - \gamma_\infty)$	$\Delta\gamma$	Gas phase
$k_G$	$[\text{kmol m}^{-2} \text{s}^{-1} \text{kPa}^{-1}]$	$N_A^* = k_G (p_s - p_\infty)$	$\Delta p$	
$k_Y$	$[\text{kmol m}^{-2} \text{s}^{-1}]$	$N_A^* = k_Y (Y_s - Y_\infty)$	$\Delta Y$	
$k$	$[\text{m s}^{-1}]$	$N_A = \rho_G k (\omega_{Gs} - \omega_{G\infty})$	$\Delta\omega_G$	
$k_H$	$[\text{kg m}^{-2} \text{s}^{-1}]$	$N_A = k_H (H_s - H_\infty)$	$\Delta H$	
$k_L$	$[\text{m s}^{-1}]$	$N_A^* = k_L (c_s - c_\infty)$	$\Delta c$	Liquid phase
$k_x$	$[\text{kmol m}^{-2} \text{s}^{-1}]$	$N_A^* = k_x (x_s - x_\infty)$	$\Delta x$	
$k_X$	$[\text{kmol m}^{-2} \text{s}^{-1}]$	$N_A^* = k_X (X_s - X_\infty)$	$\Delta X$	
$k$	$[\text{m s}^{-1}]$	$N_A = \rho_L k (\omega_{Ls} - \omega_{L\infty})$	$\Delta\omega_L$	

$c$  = molar density  $[\text{mol m}^{-3}]$ ,  $H$  = absolute humidity  $[-]$ ,  $M_A$  = molecular weight  $[\text{kg kmol}^{-1}]$ ,  
 $N_A$  = mass flux  $[\text{kg m}^{-2} \text{s}^{-1}]$ ,  $N_A^* = N_A/M_A$  = molar flux  $[\text{kmol m}^{-2} \text{s}^{-1}]$ ,  $p$  = partial pressure  
 $[\text{kPa}]$ ,  $x, \gamma$  = mole fraction  $[-]$ ,  $X = x/(1 - x)$   $[-]$ ,  $Y = \gamma/(1 - x)$   $[-]$ ,  $\omega$  = mass fraction  $[-]$ .

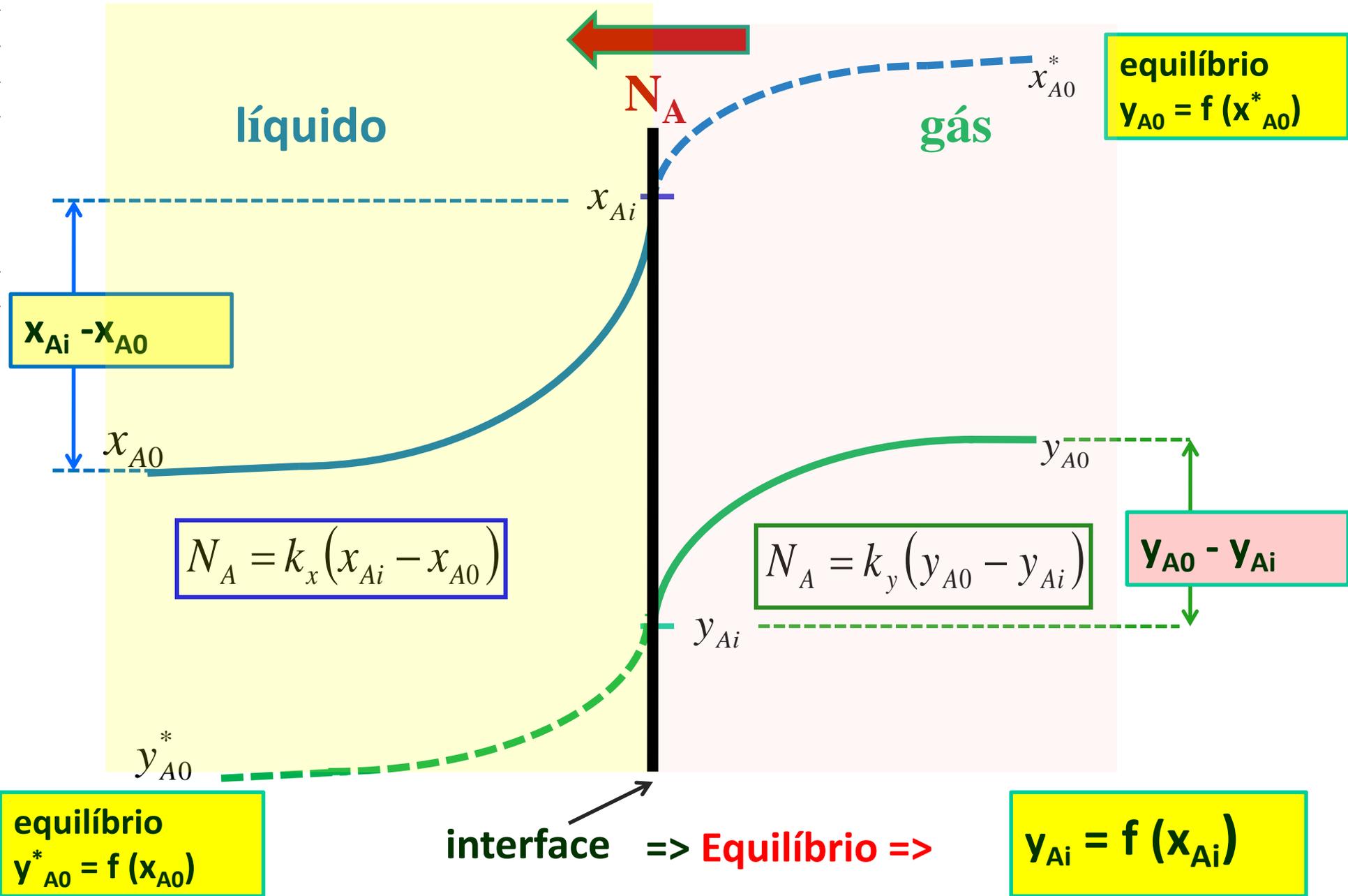
# Transporte de massa - Interface



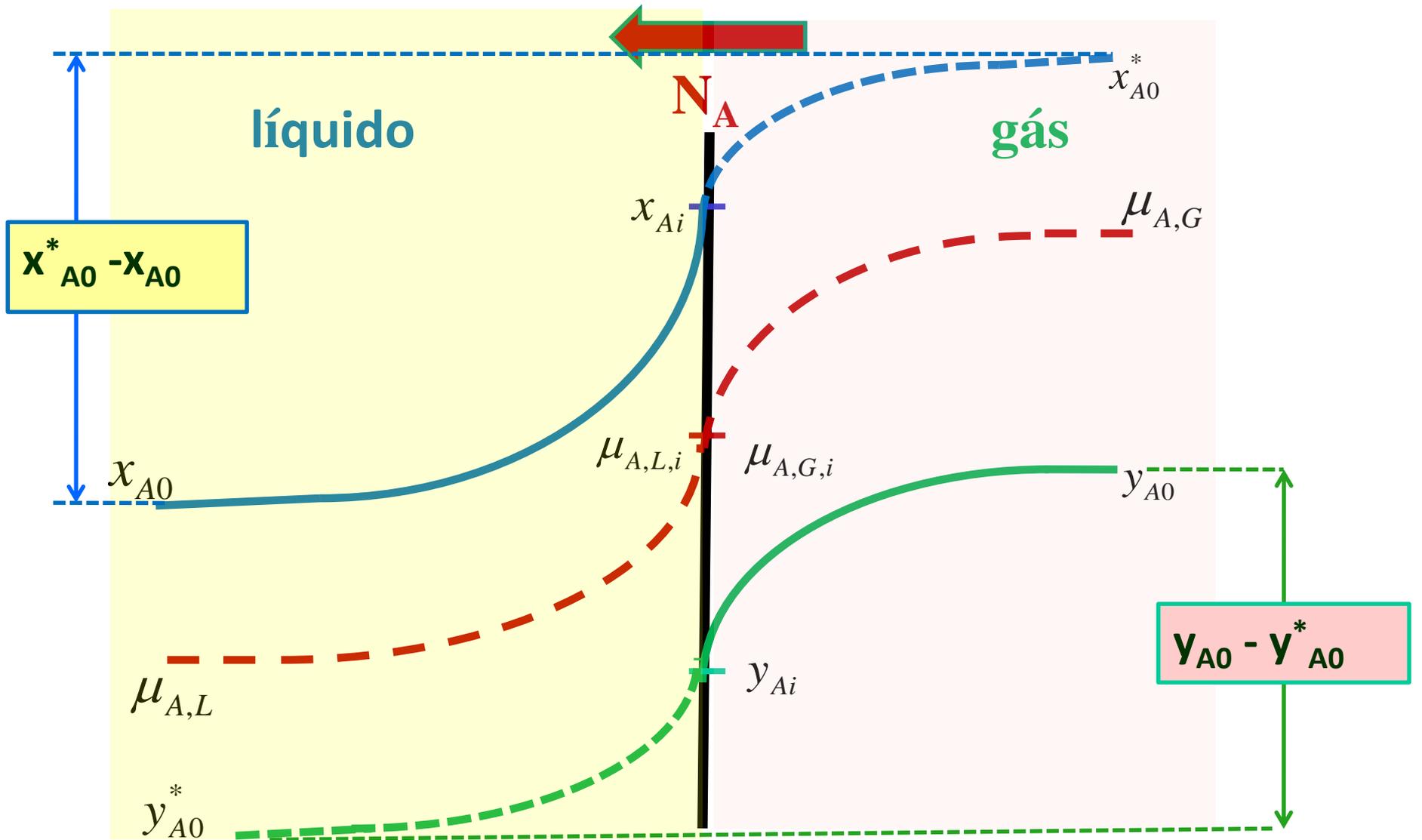
# Transporte de massa - Interface



# Interface – Coeficientes de T. de Massa



# Interface – Coeficientes de T. de Massa



$$N_A = K_x (x_{A0}^* - x_{A0}) = K_y (y_{A0} - y_{A0}^*)$$

$$N_A = k_x (x_{Ai} - x_{A0}) = k_y (y_{A0} - y_{Ai})$$

# Coeficiente Global de Transporte de Massa

$$(\mu_{A0,G} - \mu_{A0,L}) = (\mu_{A0,G} - \mu_{Ai,G}) + (\mu_{Ai,L} - \mu_{A0,L})$$

$$(y_{A0} - y_{A0}^*) = (y_{A0} - y_{Ai}) + \left( \underbrace{mx_{Ai}}_{y_{Ai}} - \underbrace{mx_{A0}^*}_{y_{A0}^*} \right)$$

$$(y_{A0} - y_{A0}^*) = (y_{A0} - y_{Ai}) + m(x_{Ai} - x_{A0})$$

$$(y_{A0} - y_{A0}^*) = \frac{N_A}{k_y} + m \frac{N_A}{k_x}$$

$$N_A = K_y (y_{A0} - y_{A0}^*)$$

$$\frac{1}{K_{\tilde{y}}} = \frac{1}{k_{\tilde{y}}} + \frac{m}{k_{\tilde{x}}}$$

$$N_A = K_x (x_{A0}^* - x_{A0})$$

$$\frac{1}{K_{\tilde{x}}} = \frac{1}{mk_{\tilde{y}}} + \frac{1}{k_{\tilde{x}}}$$

## Equilíbrio

$$\mu_{Ai,G} = \mu_{Ai,L}$$

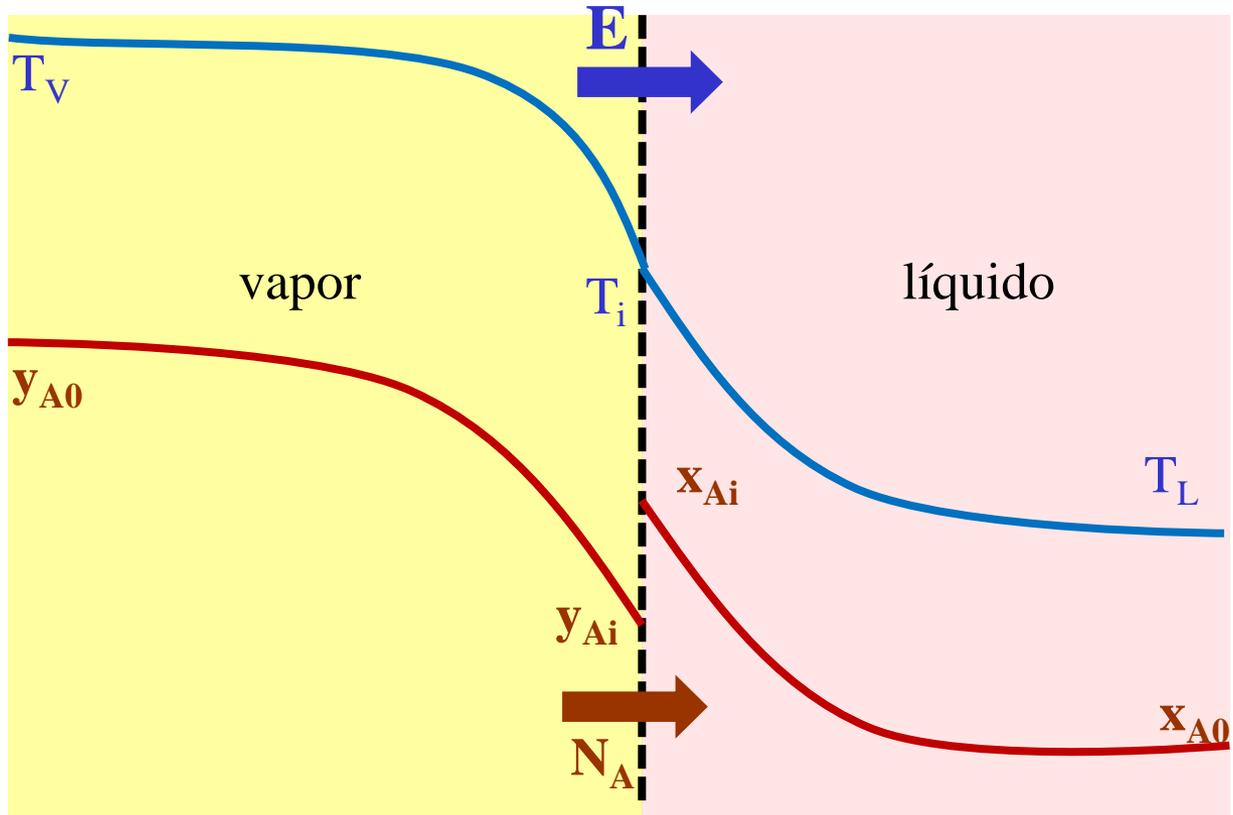
$$y_{Ai} = m x_{Ai}$$

## Fluxos

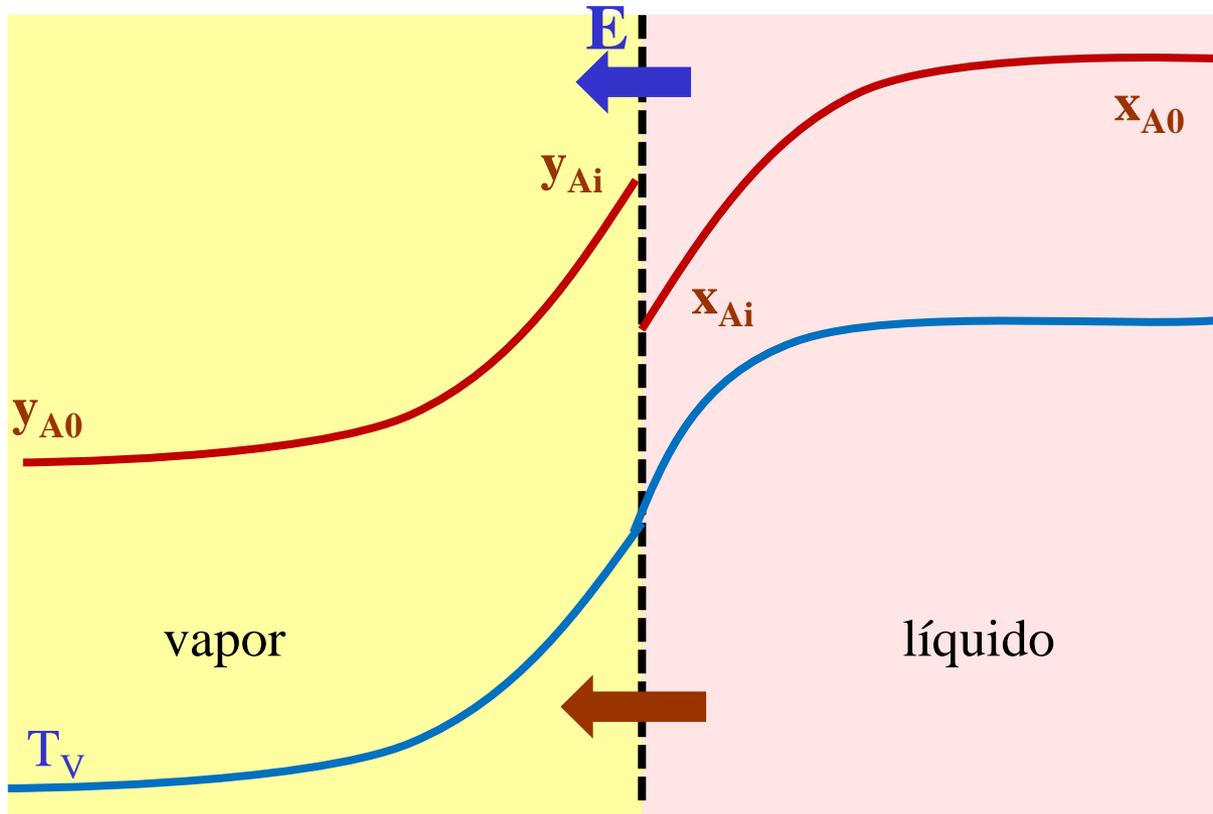
$$N_A = k_y (y_{A0} - y_{Ai})$$

$$N_A = k_x (x_{Ai} - x_{A0})$$

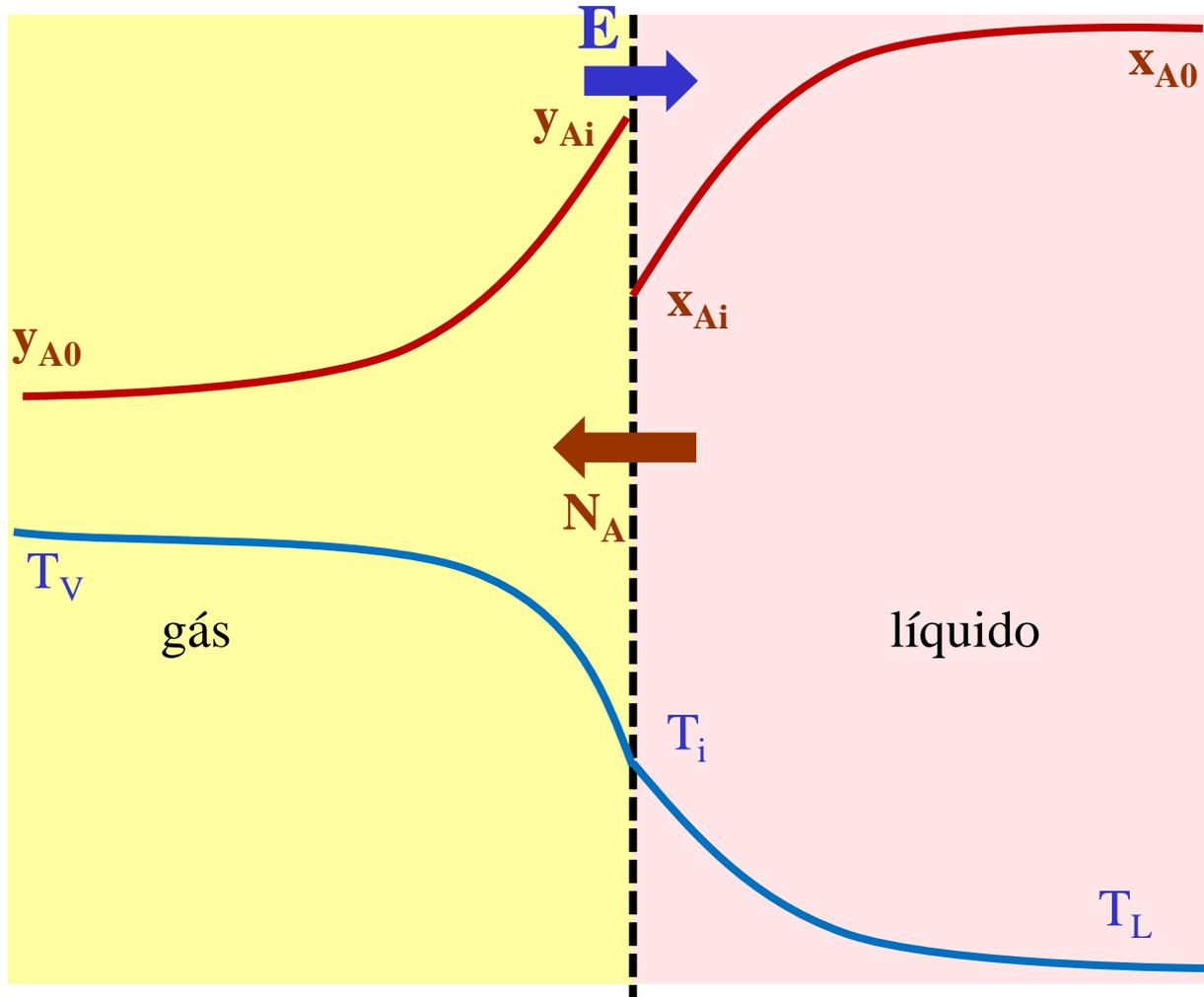
# Condensação



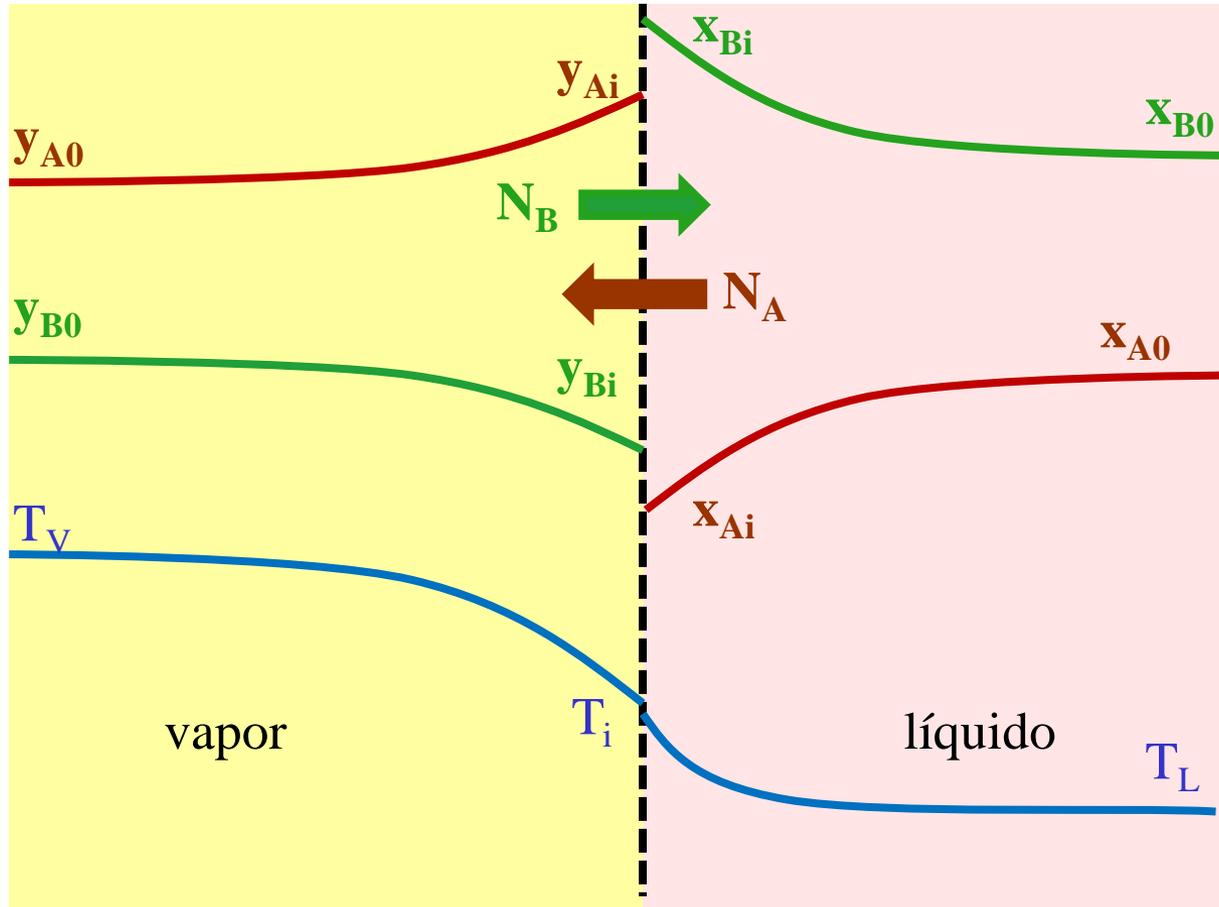
# Evaporação



# Secagem

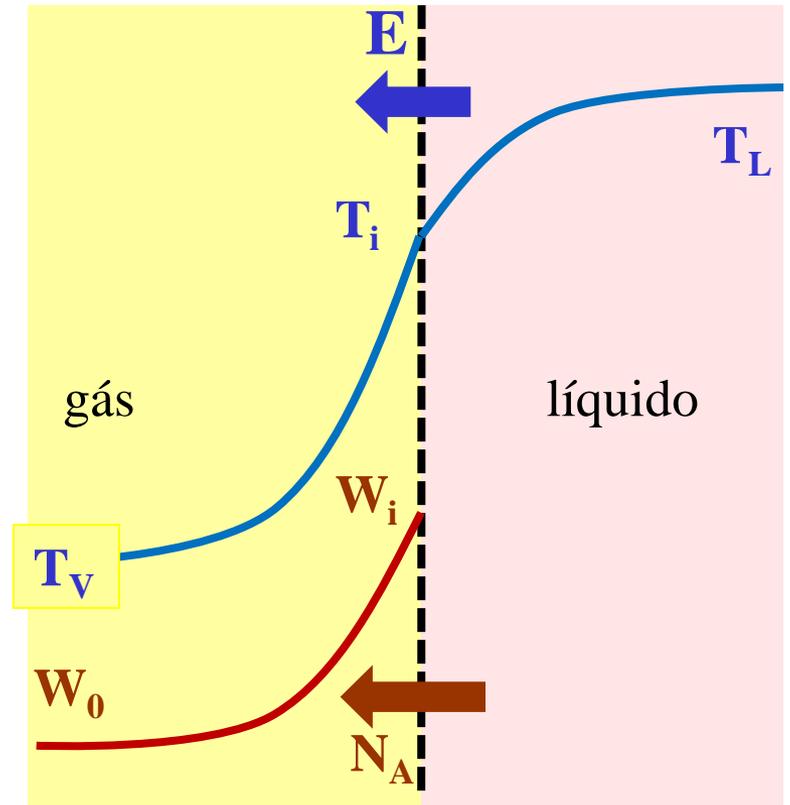
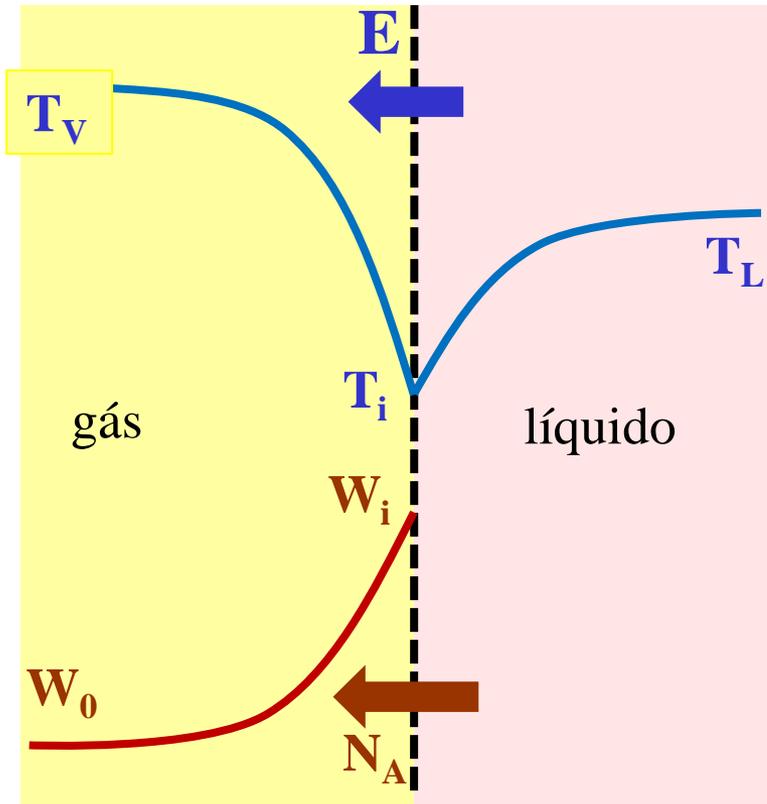


# Destilação



# Resfriamento Evaporativo

## Torre de Resfriamento



# Transporte simultâneo de calor e massa

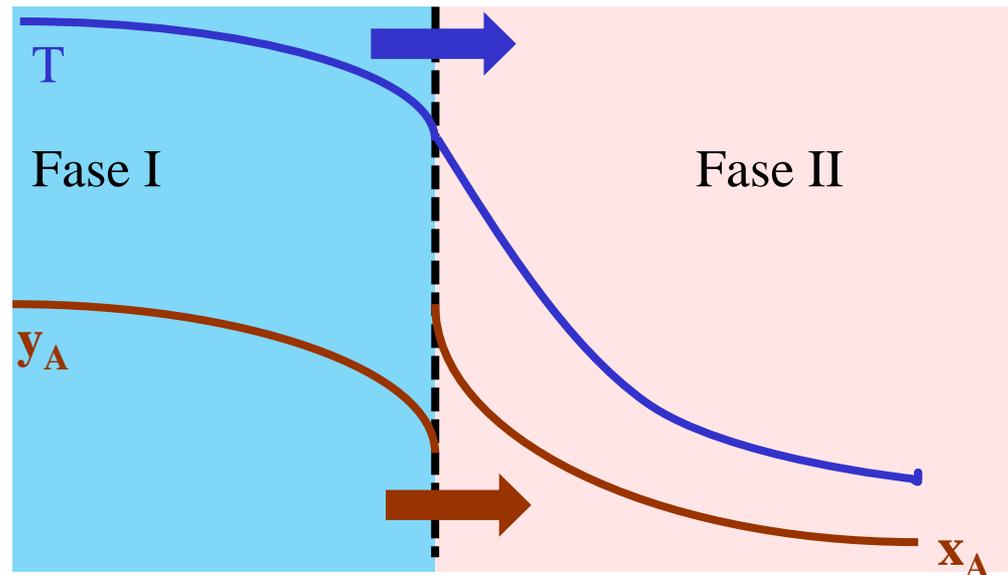
$$\vec{q} = -\vec{k} \cdot \text{grad} T + \sum \vec{j}_i \hat{H}_i + \vec{q}^x$$

**DIFUSÃO DE  
CALOR**

**CONDUÇÃO  
FOURIER**

**DIFUSÃO**

**TERMODIFUSÃO  
DUFOUR**



$$-(k \cdot \text{grad} T)_I + \vec{j}_A \hat{H}_{A,I} = -(k \cdot \text{grad} T)_{II} + \vec{j}_A \hat{H}_{A,II}$$