CHAPTER 4

Sound Intensity and Sound Pressure: The Decibel

We learned in Chapter 1 that sound is a form of wave motion in which a pattern of pressure — or a change in density — is propagated through an elastic medium. **Energy** is transferred through the medium and the transfer of energy occurs at some particular rate. *The rate at which sound energy is transferred through the medium* is called the **acoustic power**.

More generally, **power** is defined as the rate at which work is accomplished, or as the rate at which energy is transformed or transferred:

Power = work/unit time = energy transformed or transferred/unit time.

For example, **power** refers to the amount of work accomplished by an engine per second, or the amount of electrical energy that is transformed into heat energy per second by an electrical heater.

It is important to realize that **power** and **energy** are not equivalent. Consider the case of a person who is engaged in a physical activity such as shoveling snow. The person might be able to shovel for an hour at a rate of two shovelfuls per minute before the amount of energy stored in the muscles is used up and exhaustion results. However, if the person is anxious to complete the job more quickly and picks up the pace to four shovelfuls each minute, exhaustion will occur earlier. Thus, our ability to complete such tasks is limited not only by our **energy**, the capacity to do work, but also by the rate at which the energy is expended, the **power**.

The unit of measure of power, including acoustic power, is the **watt** (**W**), which was named in honor of James Watt, the developer of the steam engine. If we expend energy at a rate of 1 joule per sec (**MKS** system) or 10,000,000 ergs per sec (**cgs** system), we have expended *one watt of power.* Therefore,

$$1 \text{ watt} = 1 \text{ joule/sec} = 10{,}000{,}000 \text{ ergs/sec } (10^7 \text{ ergs/sec}).$$

ABSOLUTE AND RELATIVE MEASURES OF ACOUSTIC POWER

Absolute Measure of Power

If we say that a particular sound wave has an acoustic power of some number of watts, we are referring to the **absolute acoustic power**. In the case of sound waves, we should not expect to encounter large amounts such as 60, 100, or 200 watts, and we certainly will find no acoustic rival to the megawatt electrical power station. Instead, we commonly deal with very small magnitudes of acoustical power such as, for example, 10^{-8} watt or 2.13×10^{-9} watt. Even though the value of acoustic power is small, the absolute measure of acoustic power in watts refers to *the rate at which energy is consumed.*

Relative Measure of Power

We frequently speak of **relative acoustic power** rather than absolute power. In this case, *the absolute power in one sound wave is compared with the absolute power in another (reference) sound wave,* and the two absolute sound powers are used to form a ratio. Thus, we might say, for example, that the acoustic power of wave A is 10 times greater than the acoustic power of wave B.

$$A = 10B.$$

We do not know the absolute power in watts of either wave A or wave B, but we do know the relation between the two. If the absolute power of B is 10^{-6} watt, the power of A must be 10^{-5} watt if we are to preserve the ratio of A = 10B. With the example above, if A = 10B, then

$$B = \frac{A}{10}$$

Thus, if the absolute power of wave A were 6.21×10^{-3} watt, then the power of wave B must be 6.21×10^{-4}.

$$B = \frac{(6.21 \times 10^{-3})}{(1 \times 10^{1})} = 6.21 \times 10^{-4}.$$

That is the only solution in which the 1:10 ratio would be preserved.

Summary

In summary, with relative power we are expressing the **level** of power in a sound wave by forming a ratio of two absolute powers: the ratio of the absolute acoustic power of the sound wave in question (W_x) to the absolute acoustic power of some reference sound wave (W_r).

$$\text{Level} = \frac{W_x}{W_r}. \qquad \textbf{Equation 4.1}$$

Importance of Specifying the Reference Power

Because level of power is simply the ratio of *any* two absolute powers, *the measure of level is generally meaningless unless the value of the reference power is specified.* If we determine that the level of a particular sound wave is 1,000, we know only that its power is 1,000 times greater than the power of some unspecified reference sound wave. It could be that the power of W_x is 10^{-1} relative to (re:) W_r of 10^{-4}. But, of course, a level of 1,000 would apply equally to $W_x = 10^{-13}/W_r = 10^{-16}$. In either case, the ratio of 1,000 is preserved.

A more explicit statement, therefore, would be: "the level of acoustic power is 1,000 re: 10^{-4} watt." When the reference (W_r) is specified, there should be no ambiguity. *If the reference is not known, the concept of level has little meaning.*

SOUND INTENSITY

Imagine that we have what is called an "idealized point source of sound" that operates in a "free, unbounded medium." By an "idealized point source," we mean a very small sphere that is capable of pulsating in and out in a manner similar to the pulsation of the balloon in Figure 2-9. By "free and unbounded medium" we mean that there are no reflecting surfaces or energy-absorbing materials in the medium to affect sound transmission in any way. In such a case, sound energy is transferred uniformly through the medium in all directions.

In essence, the *energy is transferred outward from the point source as an ever-expanding sphere.* To measure the acoustic power in such a wave at some distance from the point source, we would have to integrate over the entire surface of the sphere. However, we generally are more interested in the *amount of power that acts upon or that is dissipated upon, or passes through, some much smaller area.* The area is the square meter (**m²**) in the **MKS** metric system or the square centimeter (**cm²**) in the **cgs** metric system. Thus, instead of referring to the *energy per second* (the **acoustic power**), we speak of the *energy per second per square meter,* and that is called the **intensity** of the sound wave.

Intensity is the *amount of energy transmitted per second over an area of one square meter.*

1. If the unit of measure of **acoustic power** is the **watt** (energy per second); and
2. if **intensity** is the amount of energy per second per square meter, then it follows that
3. the unit of measure for **intensity** must be the **watt per square meter (watt/m²)**.

Absolute and Relative Measures of Sound Intensity

Sound power was expressed in both absolute and relative terms, and the same is true of sound intensity. If we say that a sound has an intensity of 10^{-8} watt/m², we are referring to the **absolute intensity** of the sound wave. Alternatively, we can speak of the **relative intensity**, or the level of intensity (not to be confused with "intensity level," because the label "intensity level" implies a specific reference intensity that will be introduced later), by reference to the same kind of ratio that was used for power.

$$\text{Level} = \frac{I_x}{I_r}. \qquad \text{Equation 4.2}$$

Importance of Specifying the Reference Intensity

It is just as important to specify the reference for level of sound intensity as it is to specify the reference for level of sound power. Consider the examples in Table 4-1 in which each of six absolute sound intensities in **watt/m²** is referenced to two different reference intensities: $I_r = 10^{-10}$ watt/m² and $I_r = 10^{-12}$ watt/m².

For each absolute intensity (I_x), the ratio (I_x/I_r) depends on the reference intensity. Thus, for example, the level of intensity in the first example is 10^2 when referenced to 10^{-10} watt/m², but 10^4 when referenced to 10^{-12} watt/m². In a similar vein, it is not acceptable to say that some sound has a level of intensity of 10^{-1} because, for the examples in Table 4-1, that would apply equally to 10^{-13} re: 10^{-12} and to 10^{-11} re: 10^{-10}.

THE DECIBEL

When we speak of the level of power or the level of intensity in a sound wave, we often are faced with a rather cumbersome number. For example, a conservative estimate of the range of intensities to which the human auditory system can respond is about 10^{12}:1, and the range of acoustic powers of interest in noise measurements is even greater: about 10^{18}:1. Thus, we are faced with a wide range of intensities, and that presents us with awkward numerical notations.

The awkwardness can be minimized, though, by some elementary transformations of the numbers. Look again at Table 4-1. Entries in the second and the third columns are the ratios of the absolute intensities (I_x) to each of the two reference intensities (I_r). Each of those entries is the base 10 raised to some power.

Table 4-1. Ratios I_x/I_r for two different reference intensities

Absolute Intensity in watt/m²	Relative Intensity, I_x/I_r in watt/m²	
I_x	$I_r = 10^{-10}$	$I_r = 10^{-12}$
10^{-8}	10^{2}	10^{4}
10^{-9}	10^{1}	10^{3}
10^{-10}	10^{0}	10^{2}
10^{-11}	10^{-1}	10^{1}
10^{-12}	10^{-2}	10^{0}
10^{-13}	10^{-3}	10^{-1}

It should be obvious that the base is redundant from one example to another in the table; it is always 10. Thus, we could just as well eliminate the base and only list the exponents. We also know that *an exponent is a log* (Chapter 3). In other words: $\log_{10} 10^2 = 2$; $\log_{10} 10^0 = 0$; $\log_{10} 10^{-3} = -3$; and so on. The numbers are simplified, therefore, by *calculating the logarithm to the base ten of the ratio of two intensities.*

$$\log_{10} \frac{I_x}{I_r}.$$

The Bel

For the first example in Table 4-1 where the absolute intensity (I_x) equaled 10^{-8} watt/m^2, we calculated that its level re: 10^{-10} watt/m^2 was 10^2. Now we see that one further transformation, the log of the ratio, yields a value of 2. This unit of measurement of relative intensity is called the **bel** in honor of Alexander Graham Bell, an inventor who patented the first telephone. Thus,

Equation 4.3

$$N \text{ (bels)} = \log_{10} \frac{I_x}{I_r},$$

where $\mathbf{I_x}$ is the absolute intensity of the wave in question in watt/m^2,

$\mathbf{I_r}$ is the absolute intensity of a *reference* sound wave in watt/m^2,

and

the **bel** is a unit of level of intensity.

Thus, the relative intensities of the six sounds in Table 4-1 can be described as 2, 1, 0, −1, −2, and −3 Bels re: 10^{-10} watt/m^2, and as 4, 3, 2, 1, 0, and −1 bels re: 10^{-12} watt/m^2.

Summary

Before proceeding further, it might be helpful to provide a brief review of the transformations that have been accomplished for specifying the intensity of a sound wave. Consider the first example in Table 4-1.

1. The absolute intensity is 10^{-8} watt/m^2.
2. The relative intensity, referenced to 10^{-12} watt/m^2, is 10,000, which can be expressed more simply as 10^4.
3. The base 10 is redundant because the level of intensity will always be described as the base 10 raised to some power. The expression can be simplified further, therefore, by calculating the log of the ratio. Because an exponent is a log, the log of 10 raised to some power (exponent) is the exponent. The level in this example, therefore, is 4 (**bels**).

4. Thus, a sound wave with an absolute intensity (I_x) of 10^{-8} watt/m^2 has a relative intensity of 4 Bels re: 10^{-12} watt/m^2. Because relative intensity varies with the reference that is chosen for comparison, that same sound has a relative intensity of 2 Bels re: 10^{-10} watt/m^2.

From the Bel to the Decibel

The **bel** as a measure of relative intensity is less cumbersome than the ratio of intensities because the wide-range linear scale of intensities has been compressed by transformation to a logarithmic scale. Thus, instead of expressing the relative intensity for various sounds as 0.0001, 0.01, 1,000, 100,000, or 1,000,000,000,000, we could say simply that the relative intensities are −4, −2, 3, 5, and 12 bels respectively.

Although the bel might be less cumbersome than a ratio (a level of 10^{12} is expressed simply as 12 bels), the scale has been compressed so much that fractional values often are required to reflect an appropriate accuracy of measurement. For example, a sound intensity of 2×10^{-12} watt/m^2 has a relative intensity of 4.3 bels re: 10^{-16} watt/m^2.

$$\begin{aligned} \text{Bels} &= \log \frac{(2 \times 10^{-12})}{(1 \times 10^{-16})} \\ &= \log 2 \times 10^4 \\ &= 4.3. \end{aligned}$$

Excessive use of decimals can be minimized if we use **decibels (dB)** rather than bels in the same way that we sometimes use inches rather than feet and centimeters or millimeters rather than meters. A length of 5.5 ft can be expressed as 66 in. because each foot contains 12 in.

The prefix, deci, means 1/10. Therefore, ***a decibel* (dB)** *is 0.1 of a* ***bel.*** Because 1 dB = 0.1 bel and, conversely, 1 bel = 10 dB, Equation 4.3 for bels can be rewritten for decibels as:

$$N\ (\text{dB}) = 10 \log_{10} \frac{I_x}{I_r}. \qquad \textbf{Equation 4.4}$$

Thus, with the preceding example, we can say that the relative intensity is 43 dB rather than 4.3 bels. What, then, is a decibel? To say that "a decibel is one-tenth of a bel" is a true statement, but such a definition is not sufficient unless the mathematical definition of the bel is understood. Instead, the decibel is defined most appropriately by reference to Equation 4.4: *the decibel is ten times the log of an intensity ratio or a power ratio.*[1]

A comment about grammar is in order. The singular forms are **bel, decibel,** and **dB**, but the plural forms are **bels, decibels,** and ***dB*** (not dBs). Only d'birds can rightfully be linked with "dBs."

Intensity Level (dB IL)

The decibel, as with intensity ratios and the bel, is ambiguous unless the reference intensity is specified. How, then, can utter confusion be avoided? First, *no confusion should exist if the reference intensity is specified.* A statement that the relative intensity of a sound is 86 dB re: 10^{-12} watt/m^2 is explicit: we know the relative intensity (86 dB), we know the reference intensity (10^{-12}) to which some absolute intensity has been compared, and we can calculate (with the use of antilogs) that the absolute intensity of the sound is 3.99×10^{-4} watt/m^2.

Second, the possibility of confusion has been lessened by adoption of a *conventional reference intensity of 10^{-12} watt/m^2* (**MKS** system).[2] Any intensity can be used as the reference for expressing relative intensity, but *when the reference is 10^{-12} watt/m^2, the result is called **intensity level (dB IL)*** rather than just "level of intensity." Strictly speaking, the label **intensity level** and its abbreviation **dB IL** should only be used when the reference is 10^{-12} watt/m^2. However, there is still sufficient departure from the convention to warrant one final admonition: *the reference intensity should always be specified when the term **decibel** is used.*

The relation between absolute intensity (I_x) in watt/m^2 and dB IL re: 10^{-12} watt/m^2 is shown in the following illustration.

dB IL		I_x
70	=	10^{-5}
60	=	10^{-6}
50	=	10^{-7}
40	=	10^{-8}
30	=	10^{-9}
20	=	10^{-10}
10	=	10^{-11}
0	**=**	$\mathbf{10^{-12}}$
−10	=	10^{-13}
−20	=	10^{-14}

When $I_x = 10^{-12}$, dB IL = 0 because $I_x = I_r$, the reference intensity. As you move up the scale, I_x increases *multiplicatively* by powers of 10 to the quantities 10^{-11}, 10^{-10}, and so on, to a maximum intensity of 10^{-5} in the illustration. For each tenfold increase in sound intensity, dB IL increases *additively* by 10 dB to a maximum of 70 dB IL when $I_x = 10^{-5}$

watt/m^2. Similarly, if you move down the scale, I_x decreases progressively by powers of 10, and for each power of 10 decrease in sound intensity, dB IL decreases by 10 dB to a minimum of -20 dB IL at a sound intensity of 10^{-14} watt/m^2.

Sample Problems

Two steps should be followed to solve decibel problems for relative intensity (any reference) or **intensity level (dB IL)** (reference = 10^{-12} watt/m^2):

1. *Select the proper equation.* If the problem concerns the **intensity** of a sound wave, use Equation 4.4. Later in this chapter we will focus instead on the **pressure** of a sound wave, and that will necessitate a slight modification of Equation 4.4.
2. *Form a ratio and solve the problem.*

Problem 1: An increase in intensity by a factor of two (2:1) corresponds to how many decibels?

1. Because we are dealing with **intensity**, use Equation 4.4. The ratio is 2:1, which means that some absolute intensity whose value is unknown is twice as large as some unknown reference intensity. It might be that the intensity of one sound wave (A) is twice as great as the intensity of another sound wave (B). Alternatively, it might be that the intensity of one sound wave (A) at one point in time (t_1) has been doubled at another point in time (t_2). In either case, *the ratio is 2:1.*
2. dB = 10 log 2/1
   = 10 × 0.3010
   = 3.01 (which normally is rounded to 3 dB)
3. Because the problem could have involved any absolute intensity and any reference intensity that preserved the ratio of 2:1, we see that *as any sound intensity is doubled (the ratio of absolute to reference = 2:1), the level is increased by 3 dB.* Correspondingly, if intensity were halved (ratio = 1:2), the level is decreased by 3 dB because:
   dB = 10 log 1/2
   = −10 log 2 (**Log Law 4**)
   = −3.

Problem 2: An increase in intensity by a factor of ten corresponds to how many decibels?

1. dB = 10 log 10/1
   = 10 × 1
   = 10.

2. Thus, as intensity increases *multiplicatively* by 10, relative intensity increases *additively* by 10 dB. Similarly, if intensity were decreased by a factor of 10, intensity would decrease by 10 dB.
   $$\begin{aligned} dB &= 10 \log 1/10 \\ &= -10 \log 10 \\ &= -10. \end{aligned}$$

Problem 3: What is the intensity level re: 10^{-12} watt/m^2 of a sound whose absolute intensity is 10^{-6} watt/m^2?

1. Follow the same steps that were used with the first two problems, but note that the ratio is now different. The intensity was not simply increased (or decreased) from some unknown value by a stated ratio such as 2:1 or 10:1, but rather a specific absolute intensity was compared with a specified reference intensity.
   $$\begin{aligned} dB &= 10 \log 10^{-6}/10^{-12} \\ &= 10 \log 10^{6} \\ &= 10 \times 6 \\ &= 60. \end{aligned}$$

Problem 4: What is the intensity level re: 10^{-12} watt/m^2 of a sound whose intensity is 2×10^{-6} watt/m^2?

1. There are two approaches that can be taken. First, you can solve the problem in the same way that was used for Problem 3.
   $$\begin{aligned} dB &= 10 \log (2 \times 10^{-6})/(1 \times 10^{-12}) \\ &= 10 \log 2 \times 10^{6} \\ &= 63. \end{aligned}$$
2. However, a quicker solution is available. We learned in Problem 3 that a sound whose absolute intensity is 10^{-6} has an intensity level of 60 dB. The sound in Problem 4 has an intensity of 2×10^{-6}, which is twice as great as an intensity of 1×10^{-6}. We also know from Problem 1 that any time you increase the intensity by 2:1, relative intensity increases by 3 dB. Therefore, if the intensity level of $1 \times 10^{-6} = 60$ dB, the intensity level of 2×10^{-6} must = 63 dB. It will almost always be a useful shortcut to inspect a problem to see if it involves powers of 2 (3 dB) or powers of 10 (10 dB). For example, see if such shortcuts can be used to solve Problem 5.

Problem 5: What is the intensity level re: 10^{-12} watt/m^2 of a sound whose intensity is 4×10^{-5} watt/m^2?

1. We know from Problem 4 that 2×10^{-6} corresponds to 63 dB. In addition, we should see that 10^{-5} is 10 times greater than 10^{-6} (10 dB), and 4 is 2 times greater than 2 (3 dB). Therefore,

we should expect the answer to be 13 dB (10 + 3) greater than for Problem 4, which is 76 dB.

2. If we solve the problem step-by-step without taking answers to previous problems into account, we must get the same result.
$$\begin{aligned} dB &= 10 \log (4 \times 10^{-5})/(10^{-12}) \\ &= 10 \log 4 \times 10^{7} \\ &= 10 \times 7.6 \\ &= 76. \end{aligned}$$

Problem 6: 3 dB corresponds to what intensity ratio?

1. This is an antilog problem, and the solution requires following the steps suggested in Chapter 3.
2. $3 \text{ dB} = 10 \log X$
   $0.3 = \log X$ (Divide both sides by 10.)
   $X = 2 \times 10^{0}$. (Zero is the characteristic of the log and also the exponent in scientific notation; 0.3 is the mantissa of the log and, with the aid of pocket calculator or a log table, we determine that the multiplier in scientific notation is 2.)

Problem 7: 13 dB corresponds to what intensity ratio?

1. Try to solve the problem initially without paper and pencil. You know that 3 dB corresponds to a ratio of 2:1 and that 10 dB corresponds to a ratio of 10:1. Because 13 dB consists of 3 + 10, the ratio must consist of 2 × 10 = 20:1.
2. Check the answer above by solving the problem step-by-step.
   $13 \text{ dB} = 10 \log X$
   $1.3 = \log X$
   $X = 2 \times 10^{1}$ (which is 20:1).

Problem 8: 14 dB corresponds to what intensity ratio?

1. There might not be an obvious combination of 3 dB (2:1) and 10 dB (10:1) that adds to 14 dB, which means that a shortcut that involves powers of 2 and/or powers of 10 does not *seem* to be available. However, you can use your knowledge of powers of 2 and powers of 10 to at least "bracket your answer." We know that 13 dB corresponds to a ratio of 20:1. We should also know that 16 dB would correspond to a ratio of 40:1 because another 3 dB represents another doubling. Therefore, the answer must lie between 20:1 and 40:1.
2. $14 \text{ dB} = 10 \log X$
   $1.4 = \log X$
   $X = 2.51 \times 10^{1}$ (and 25.1 lies between 20 and 40).
3. Actually, *a shortcut is available,* even if it is not obvious at first glance. We know that 20 dB corresponds to a ratio of 100:1.

Therefore, 17 dB, which is 3 dB less, must correspond to a halving of sound intensity, or a ratio of 50:1. Finally, 14 dB represents another halving of sound intensity, which corresponds to a ratio of 25:1.

Problem 9: An intensity level of 65 dB re: 10^{-12} watt/m^2 corresponds to what intensity?

1. Combinations of 10 dB and 3 dB will not add to 65 (unless you use a long string such as 10 + 10 + 10 + 10 + 10 + 3 + 3 + 3 + 3 + 3 = 65). In this case it might be less cumbersome to solve the problem step-by-step. However, we can determine a range within which the answer must lie by use of powers of 10 and 2, and that will enable us to determine if the answer we calculate is reasonable. We should know that 63 dB consists of 10 + 10 + 10 + 10 + 10 + 10 + 3. Each 10 dB involves a tenfold increase in intensity relative to the reference, which therefore corresponds to 10^{-6} (the reference increased by a factor of 10^6). Similarly, each 3 dB involves a twofold increase in intensity relative to the reference. Thus 63 dB would correspond to an intensity of 2×10^{-6}, and that is a lower boundary of the bracket. To get an upper boundary add another 3 dB. 66 dB would represent another doubling of intensity, which would be 4×10^{-6}. Because 65 dB lies between 63 and 66, we should expect that the intensity corresponding to 65 dB would lie between 2×10^{-6} and 4×10^{-6}.
2.
$$\begin{aligned} 65 \text{ dB} &= 10 \log I_x/10^{-12} \\ 6.5 &= \log I_x/10^{-12} \\ 3.16 \times 10^6 &= I_x/10^{-12} \\ I_x &= 3.16 \times 10^6 \times 10^{-12} \\ &= 3.16 \times 10^{-6}. \end{aligned}$$

SOUND PRESSURE

We often wish to refer to the **pressure** associated with a sound wave rather than its acoustic power or intensity. We saw previously (Equation 1.5) that **pressure** is the *amount of force per unit area.* The unit of measure in the **MKS** system is the **Nt/m²** (newton per square meter) or **Pa** (pascal), and we learned in Chapter 2 that 1 Nt/m^2 = 1 Pa. It has now become more common to use the **μNt/m²** (microNt/m^2) or the **μPa** (microPa) as the unit of measure, and we will use **μPa** in all future computations involving sound pressure.[3]

If we say that a particular sound has a pressure of 200 μPa (2×10^2 μPa), we are referring to the **absolute pressure** of the wave. As with intensity, we also can speak of **relative pressure** in decibels by calculating the log of a pressure ratio. However, Equation 4.4 *cannot be used for decibels of sound pressure.* That equation for decibels of intensity must, therefore, be modified in an appropriate manner. In order that we might understand the reasons for the transformation of Equation 4.4, it will be helpful first to reconsider the concepts of sound intensity and sound pressure in relation to **impedance**.

Impedance

We learned in Chapter 2 that the **impedance** of any vibratory system, including a volume of air through which sound is transmitted, is determined by the **resistance, mass reactance,** and **compliant reactance** of the system. Moreover, we learned in Chapter 1 that the speed of sound in a medium such as air also is determined by properties of the medium, namely, the **elasticity** and the **density**. We might reason, therefore, that **impedance** also would be dependent on the speed of sound in a medium.

The **acoustic impedance** for a plane progressive wave is given by the product of the ambient **density** (ρ_o) in kg/m^3 and the **speed of sound** (**s**).[4] Thus,

$$Z_c = \rho_o s. \qquad \text{Equation 4.5}$$

The subscript (**c**) is used in the equation because the impedance of the medium with plane progressive waves is called the **characteristic impedance**.

We previously defined intensity as *energy per second per square meter.* Intensity also can be defined as the *ratio of the square of rms pressure to the characteristic impedance:*

$$I = \frac{P_{rms}^{\ 2}}{\rho_o s}, \qquad \text{Equation 4.6}$$

where **I** is intensity,

$\mathbf{P_{rms}}$ is root mean square pressure,

and

the product $\rho_o \mathbf{s}$ is the characteristic impedance.

Decibels for Sound Pressure

We can see from Equation 4.6 that intensity is proportional to the square of rms pressure, and conversely, **rms pressure** is proportional to the square root of sound intensity.

$$P \propto \sqrt{I},$$

$$I \propto P^2, \text{ and}$$

$$I = \frac{P^2}{Z_c},$$

where **P** refers to rms sound pressure,
I refers to sound intensity,
and
$\mathbf{Z_c}$ is the characteristic impedance.

Thus, if intensity increases by 16:1, pressure increases by only 4:1 (square root of 16); if intensity increases by 10:1, pressure increases by only 3.16 (square root of 10); or if sound intensity is doubled, pressure increases by only 1.414 (square root of 2). It is important to keep in mind that:

1. As sound intensity increases by some factor, rms pressure increases only by the *square root* of the factor.
2. As rms pressure increases by some factor, sound intensity increases by the *square* of that factor.

Equation 4.4 *cannot* be used for decibels of pressure because rms pressure is proportional to the square root of intensity, not to intensity. However, Equation 4.4 can be modified to be appropriate for measures of pressure from our knowledge of the relation between sound intensity and rms pressure.

$$\text{dB} = 10 \log_{10} \frac{I_x}{I_r} \qquad \textbf{(Equation 4.4)}$$

Because

$$I = \frac{P^2}{Z_c}$$

$$\text{dB} = 10 \log \frac{(P_x^2/Z_c)}{(P_r^2/Z_c)} \qquad \text{(by substitution)}$$

$$= 10 \log \frac{(P_x^2)}{(P_r^2)} \qquad \text{(canceling } Z_c\text{)}$$

$$= 10 \log \left(\frac{P_x}{P_r}\right)^2$$

$$= 10 \times 2 \log \frac{P_x}{P_r} \qquad \textbf{(Log Law 3: } \log a^b = b \log a\text{)}$$

Equation 4.7

$$\text{dB} = 20 \log_{10} \frac{P_x}{P_r}.$$

Equation 4.4 should be used to solve problems that involve sound intensity, and Equation 4.7 should be used to solve problems that involve sound pressure. Thus, the decibel now can be defined as *10 times the log of a power or intensity ratio and as 20 times the log of a pressure ratio.* The only difference between the two equations is that a multiplier of 10 is used for intensity problems and a multiplier of 20 is used for pressure problems.

Sound Pressure Level (dB SPL)

Equation 4.7 applies for all instances in which we wish to represent a pressure ratio by decibels, and it is imperative that the reference pressure be specified. We have learned that the standard reference intensity for **dB IL** is 10^{-12} watt/m^2. The standard reference for **sound pressure** in the **MKS** system is 20 μPa (2×10^1 μPa), which is the pressure created in air by a sound wave whose intensity is 10^{-12} watt/m^2 under what are called "standard conditions."

"Standard conditions" generally mean a temperature of 20^0 Centigrade and a barometric pressure of 760 mm of mercury. When the reference is 20 μPa, we refer to decibels **sound pressure level (dB SPL)**[5]. This does not mean that 20 μPa is always the reference for sound pressure level. On the contrary, sound pressure level could be (and on occasion has been) referenced to 1 dyne/cm^2 (**cgs**) or to other values. However, the moral by now should be obvious: *all uses of decibel notation require that the reference be specified.*

In Chapter 1 we emphasized that when an air medium is energized by a vibrating source of sound, *the individual air particles are displaced over a very small distance.* We are now in a position to specify the magnitude of that "small displacement" more precisely to provide some perspective. If the sound wave is a 1000 Hz sinusoid with a sound pressure of 20 μPa (0 dB SPL), the displacement of the air particles is approximately 7.68×10^{-8} m, which is about 1/300 of the diameter of a hydrogen molecule (2.34×10^{-6} m).

Think for a moment about the vastly different magnitudes that we encounter in the study of sound. For the 1000 Hz sine wave, air particles are displaced a nearly infinitesimal distance (7.68×10^{-8} m), whereas the wavelength of that sound wave is about 0.34 m. If we substitute a 100 Hz sine wave at the same sound pressure level, air particle displacement will be unchanged, but now the wavelength will equal about 3.4 m.

The Relation Between Absolute Pressure and Decibels

The relation between absolute pressure (P_x) in μPa and dB SPL re: 20 μPa is illustrated on the next page. When $P_x = 2 \times 10^1$, dB SPL = 0 because $P_x = P_r$. As you move up the scale, P_x increases *multiplicatively* by powers of 10 to the magnitudes 2×10^2, 2×10^3, and so on, to a maximum pressure of 2×10^6 in the illustration. For each tenfold increase in sound pressure, dB SPL increases *additively* by 20 dB to a maximum of 100 dB SPL when $P_x = 2 \times 10^6$ μPa. Similarly, if you move down the scale, P_x decreases progressively by powers of 10, and for each power of 10 decrease in sound pressure, dB SPL decreases by 20 dB to a minimum of −40 dB SPL in the illustration at a sound pressure of 2×10^{-1} μPa.

dB SPL		P_X
100	=	2×10^6
80	=	2×10^5
60	=	2×10^4
40	=	2×10^3
20	=	2×10^2
0	**=**	$\mathbf{2 \times 10^1}$
−20	=	2×10^0
−40	=	2×10^{-1}

Sample Problems

The same two steps that were used to calculate decibels for intensity should be used to calculate decibels for pressure: (1) Select the proper equation, and (2) form a ratio and solve the problem.

Problem 1: An increase in sound pressure by a factor of two (2:1) corresponds to how many decibels?

1. $\text{dB} = 20 \log 2/1$
   $= 20 \times 0.3010$
   $= 6.02$ (which normally is rounded to 6 dB).
2. We learned previously that as sound intensity increases by a factor of 2:1, the level is increased by 3 dB. However, from this problem we see that as sound pressure increases by a factor of 2:1, the *pressure level* is increased by *6 dB.* That difference, of course, occurs because with intensity we multiply the log of 2 (0.3) by 10 and with pressure we multiply the log of 2 (0.3) by 20. Correspondingly, if pressure is *halved* (ratio = 1:2), the pressure level is *decreased* by 6 dB.

Problem 2: An increase in pressure by a factor of ten corresponds to how many decibels?

1. $\text{dB} = 20 \log 10/1$
   $= 20 \times 1$
   $= 20.$
2. As pressure increases *multiplicatively* by 10, relative pressure increases *additively* by 20 dB. If the pressure is decreased by a factor of 10 (1:10), the level is decreased by 20 dB.

Problem 3: What is the sound pressure level re: 2×10^1 μPa of a sound whose absolute pressure is 2×10^4 μPa?

$$\begin{aligned} dB &= 20 \log (2 \times 10^4)/(2 \times 10^1) \\ &= 20 \log 10^3 \\ &= 60. \end{aligned}$$

Problem 4: What is the sound pressure level re: 2×10^1 μPa of a sound whose absolute pressure is 4×10^4 μPa?

1. First you should note that the pressure in this problem (4×10^4) is exactly twice as great as the pressure in Problem 3 (2×10^4). Because a doubling of pressure corresponds to 6 dB, you should expect that the answer would be 6 dB greater than the answer for Problem 3 ($60 + 6 = 66$).
2. $$\begin{aligned} dB &= 20 \log (4 \times 10^4)/(2 \times 10^1) \\ &= 20 \log 2 \times 10^3 \\ &= 66. \end{aligned}$$
3. By now, it should not be necessary to use a calculator or log table for this kind of problem, and you might not even need scratch paper. You know that the log of 10^3 is 3 (a log is an exponent), and you should remember that the log of 2 is 0.3. Law 1 of logarithms stated that the log of a product is equal to the sum of the logs of the factors, so the log of 2×10^3 must equal $3 + 0.3$, which multiplied by 20 yields 66.

Problem 5: 26 dB corresponds to what pressure ratio?

1. Before using paper and pencil, see if you can determine the answer by recalling powers of 2 and 10.
2. $$\begin{aligned} 26 \text{ dB} &= 20 \log X \\ 1.3 &= \log X \\ 2 \times 10^1 &= X \\ &= 20. \end{aligned}$$
3. With powers of 2 and 10 in mind, you might have noticed that 26 dB was composed of 20 dB (10:1) and 6 dB (2:1). Because 26 dB consists of $20 + 6$, the ratio must be $10 \times 2 = 20{:}1$.

Problem 6: A sound pressure level of 65 dB re: 2×10^1 μPa corresponds to what sound pressure?

$$\begin{aligned} 65 \text{ dB} &= 20 \log P_x/(2 \times 10^1) \\ 3.25 &= \log P_x/(2 \times 10^1) \\ 1.78 \times 10^3 &= P_x/(2 \times 10^1) \\ P_x &= 1.78 \times 2 \times 10^3 \times 10^1 \\ P_x &= 3.56 \times 10^4. \end{aligned}$$

THE RELATION BETWEEN dB IL AND dB SPL

In Table 4–2 we see that an *intensity ratio* of 10:1, for example, corresponds to 10 dB, whereas a *pressure ratio* of 10:1 corresponds to 20 dB. Would you conclude, therefore, that 60 dB IL = 120 dB SPL? *You should not!*

It is true that an intensity ratio of 10:1 corresponds to 10 dB and a pressure ratio of 10:1 corresponds to 20 dB. However, if the intensity of a particular sound were increased by a factor of 10:1, the pressure of the same sound wave would be increased only *by the square root of 10:1* (3.1623), and the decibel equivalent would still be 10 (20 log 3.1623 = 10 dB).

Recall that *pressure is proportional to the square root of intensity,* and conversely, *intensity is proportional to the square of pressure.* Other examples of this same relation can be seen in the table. An intensity ratio of 100:1 corresponds to 20 dB and a pressure ratio of 100:1 corresponds to 40 dB. However, if the intensity of a particular wave were increased by 100:1, the pressure would be increased by only 10:1 and, in either case, the decibel equivalent would be 20 dB.

Thus, we see that 60 dB IL does *not* equal 120 dB SPL. Recall from the derivation of Equation 4.7 that the multiplier of 10 in decibels for intensity was changed to a multiplier of 20 in decibels for pressure. As long as equivalent references are used in the two equations, dB IL = dB SPL, and as we learned previously, 2×10^1 µPa is the pressure equivalent to an intensity of 10^{-12} watt/m^2.

The equivalence of dB IL and dB SPL is illustrated in Figure 4–1, which shows what might be called isodecibel contours. At the left of the figure, intensity in watt/m^2 increases from 10^{-13} at the bottom to 10^{-4} at the top. At the right of the figure, *corresponding* pressure values in µPa increase from 6.32×10^0 to a maximum of 2×10^5.

Thus, we see that as intensity is increased or decreased by a power of 10, pressure changes only by the square root of 10. Each contour is a line that connects a given intensity at the left with a corresponding pressure at the right. The parameter of the figure, then, is decibels, *either dB IL or dB SPL.* They are equivalent.

Table 4–2. Relation between decibels for intensity and decibels for pressure

Intensity		Pressure	
Ratio I_x/I_r	dB $10 \log_{10} I_x/I_r$	Ratio P_x/P_r	dB $20 \log_{10} P_x/P_r$
1	0	1.0000	0
10	10	3.1623	10
100	20	10.0000	20
1,000	30	31.6228	30
10,000	40	100.0000	40
100,000	50	316.2278	50
1,000,000	60	1,000.0000	60

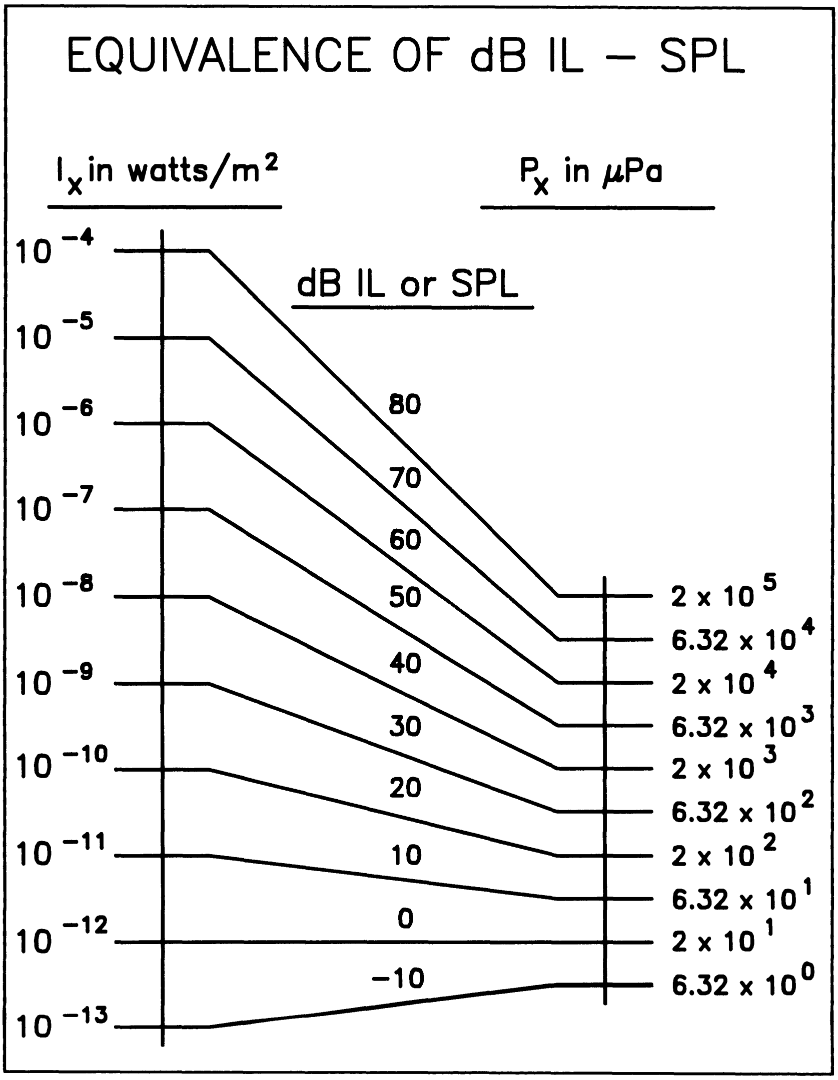

Figure 4–1. Equivalence of dB IL and dB SPL. The reference intensity for dB IL is 10^{-12} watt/m^2. The pressure created in air by a sound wave that has an intensity of 10^{-12} watt/m^2 is 20 µPa. Thus, the reference pressure for dB SPL is equivalent to the reference intensity for dB IL, and dB IL *always* equals dB SPL.

UNITS OF MEASURE FOR PRESSURE

In Chapter 1 we indicated that force is measured in **newtons (MKS)** or **dynes (cgs)** and therefore that pressure (the amount of force per unit area) is measured in **newtons/m^2 (MKS)** or **dynes/cm^2 (cgs)**. The popularity of these and other units of measure for pressure have changed over the past several years, and it is necessary to compare and understand the relations among the various systems if we are to read and understand all of the scientific literature, not just the literature written in the past few years.

Table 4–3. Comparisons of various units of **measure for pressure**

dB SPL	dynes/cm² or microbar	Nt/m² or Pa	μNt/m² or μPa
100	2×10^1	2×10^0	2×10^6
94	10^1	10^0	10^6
80	2×10^0	2×10^{-1}	2×10^5
74	10^0	10^{-1}	10^5
60	2×10^{-1}	2×10^{-2}	2×10^4
54	10^{-1}	10^{-2}	10^4
40	2×10^{-2}	2×10^{-3}	2×10^3
34	10^{-2}	10^{-3}	10^3
20	2×10^{-3}	2×10^{-4}	2×10^2
14	10^{-3}	10^{-4}	10^2
0	$\mathbf{2 \times 10^{-4}}$	$\mathbf{2 \times 10^{-5}}$	$\mathbf{2 \times 10^1}$
−6	10^{-4}	10^{-5}	10^1

Table 4–3 compares, in approximate order of their appearance in the literature, the dyne/cm² (*and its equivalent, the* ***microbar***) with the **Nt/m²** (and its equivalent, the **pascal [Pa]**) and with the **μNt/m²** (and its equivalent, the **μPa**). The first column of Table 4–3 lists various values of dB SPL ranging from 100 to −6. The second column shows the corresponding *absolute* pressure in **dynes/cm²** or **microbar**. Those two units are equal to each other. The third column shows the equivalent absolute pressures in **Nt/m²** and **pascals (Pa)**, which are equal to each other, and the fourth column lists the corresponding absolute pressures in **μNt/m²** and **μPa**, which also are equivalent to one another.

We can make several observations from the entries in Table 4–3, some of which have been stressed previously.

1. The **reference pressures** for dB SPL appear in the row for 0 dB SPL, and *the three entries are all equivalent.* Thus, 2×10^{-4} dyne/cm² (or microbar) = 2×10^{-5} Nt/m² (or Pa) = 2×10^1 μNt/m² (or μPa). In a similar vein, all entries of absolute pressure *in any single row are equivalent.* Thus, for the row corresponding to 74 dB SPL, 10^0 dyne/cm² (or microbar) = 10^{-1} Nt/m² (or Pa) = 10^5 μNt/m² (or μPa).
2. Regardless of which unit of measure of sound pressure is selected, any twofold change in pressure corresponds to 6 dB: +6 dB if pressure increases and −6 dB if pressure decreases. For example, 2×10^{-2} Pa = 60 dB SPL; a pressure of 10^{-2} is only half as great as a pressure of 2×10^{-2}, and it corresponds to 54 dB SPL, which is 6 dB less. Similarly, 10^1 μPa = −6 dB SPL, and 2×10^1 μPa, which is twice as great, corresponds to 0 dB SPL, which is 6 dB more.
3. Any tenfold change in sound pressure corresponds to 20 dB: +20 dB if pressure increases and −20 dB if pressure decreases.

Thus, 10^{-3} microbar = 14 dB SPL and 10^{-2} microbar (change in pressure by 10:1) corresponds to 34 dB SPL (+20 dB).

4. What does 0 dB SPL mean? *It does not mean absence of sound.* It simply means that the pressure in question, P_x, is exactly equal to the reference pressure, P_r. When $P_x = P_r$, the ratio is 1, the log of 1 = 0, and 20 times 0 will always equal 0 dB. Of course, the same is true for dB IL or any other decibel. A decibel is always 10 or 20 times the log of a ratio, and 0 dB always mean that $P_x = P_r$ or that $I_x = I_r$.
5. What do negative decibels mean? It does not matter whether we are dealing with decibels for pressure or decibels for intensity. Negative decibels mean that the value of the reference pressure (P_r) or reference intensity (I_r) exceeds the value of the pressure (P_x) or intensity (I_x) in question, that is, $P_r > P_x$ or $I_r > I_x$. That situation can be seen in the last row of Table 4–3 for all of the units of measure.

CONVERSION FROM ONE REFERENCE TO ANOTHER

The values of absolute pressure listed in Table 4–3 opposite 0 dB SPL are the **standard reference pressures** for decibels sound pressure level. However, there are occasions in which a different reference might be adopted for some reason. In that circumstance, it might be important to compare decibels corresponding to one reference pressure with decibels referenced to a different pressure to determine whether or not they are equivalent.

You are likely to encounter older, but still important, literature in which decibels **sound pressure level** are expressed re: 1 dyne/cm^2 instead of 2×10^{-4} dyne/cm^2 in the **cgs** metric system. Suppose, for example, one author contends that level of some particular noise is 74 dB SPL re: 2×10^{-4} dyne/cm^2. A second investigator describes the level of the same noise as 0 dB re: 1 dyne/cm^2. Are the results equivalent? We shall see that the answer is "yes," and only a few calculations are necessary to demonstrate the equivalence.

You might encounter a "conversion rule" that says: to convert from a reference pressure of 2×10^{-4} dyne/cm^2 to a reference of 1 dyne/cm^2, subtract 74 dB, or, conversely, to convert from 1 dyne/cm^2 to 2×10^{-4} dyne/cm^2, add 74 dB. That is in fact a correct rule — if you can remember when to add and when to subtract — but it only applies to those two values. What we need, then, is a system that will permit us to convert between any two reference pressures expressed in the same metric system. Two alternative approaches can be used.

One choice, albeit more cumbersome, is to calculate P_x (an antilog problem) from knowledge of the original reference pressure and then recalculate decibels (a log problem) using the new reference pressure.

1. 74 dB SPL re: 2×10^{-4} dyne/cm^2 = how many dB SPL re: 1 dyne/cm^2? Calculate the pressure, P_x, corresponding to 74 dB re: 2×10^{-4}.

$$\begin{aligned} 74 &= 20 \log (P_x)/(2 \times 10^{-4}) \\ 3.7 &= \log (P_x)/(2 \times 10^{-4}) \\ 5 \times 10^3 &= (P_x)/(2 \times 10^{-4}) \\ P_x &= 10 \times 10^{-1} \\ &= 1 \times 10^0. \end{aligned}$$

2. From that calculation we know that the sound had an absolute pressure (P_x) = 1 dyne/cm^2 (1×10^0). The next step, then, is to recalculate dB SPL using Equation 4.7, but with the new reference pressure, 1 dyne/cm^2, in the denominator.

$$\begin{aligned} \text{dB} &= 20 \log (1 \times 10^0)/(1 \times 10^0) \\ &= 0 \text{ dB SPL.} \end{aligned}$$

Consider a second example. 20 dB SPL re: 1 dyne/cm^2 = ? dB SPL re: 2 dynes/cm^2?

1. $$\begin{aligned} 20 &= 20 \log (P_x)/(1 \times 10^0) \\ 1 &= \log (P_x)/(1 \times 10^0) \\ 10^1 &= (P_x)/(1 \times 10^0) \\ P_x &= 10^1 \times 1 \times 10^0 \\ &= 1 \times 10^1. \end{aligned}$$

2. $$\begin{aligned} \text{dB} &= 20 \log (1 \times 10^1)/(2 \times 10^0) \\ &= 20 \log .5 \times 10^1 \\ &= 20 \log 5 \times 10^0 \\ &= 20 \times 0.7 \\ &= 14. \end{aligned}$$

The approach just described required two steps: (1) solve an antilog problem to determine the value of P_x; and (2) solve a log problem to express P_x in decibels re: the new reference. An easier alternative is to use Equation 4.8, where only one step is necessary.

Equation 4.8

$$\text{dB}_{Pr(2)} = \text{dB}_{Pr(1)} - 20 \log_{10} \frac{P_{r(2)}}{P_{r(1)}},$$

where $P_{r(1)}$ = the original reference

and

$P_{r(2)}$ = the new reference.

Equation 4.8 tells us that decibels re: a second (new) reference are given by subtracting 20 log of the ratio of the two references from decibels re: the first (original) reference. To illustrate that the two approaches yield identical results, we shall solve the first of the two problems listed above: 74 dB SPL re: 2×10^{-4} dyne/cm^2 = ? dB re: 1 dyne/cm^2.

$$\begin{aligned} dB &= 74 - 20 \log (1 \times 10^0)/(2 \times 10^{-4}) \\ &= 74 - 20 \log .5 \times 10^4 \\ &= 0. \end{aligned}$$

COMBINING SOUND INTENSITIES FROM INDEPENDENT SOURCES

If the intensity of some sound is 10^{-5} watt/m^2, we can calculate with Equation 4.4 that the intensity level is 70 dB IL re: 10^{-12} watt/m^2. Suppose, then, that two independent sound sources are operating simultaneously and that each produces a sound with an intensity of 10^{-5} watt/m^2. We know that each source has an intensity level of 70 dB, but what is the *total intensity level* from the two sources combined? It is *not* 140 dB. Work through the problem and see why 140 dB is not a reasonable answer. To have an intensity level of 140 dB would require an absolute intensity of 10^2 watt/m^2.

$$\begin{aligned} 140 &= 10 \log I_x/10^{-12} \\ 14 &= \log I_x/10^{-12} \\ I_x &= 10^{14} \times 10^{-12} \\ &= 10^2. \end{aligned}$$

It should be apparent that it is clearly *impossible* to have a total intensity of 10^2 watt/m^2. Each independent source produces a finite amount of energy. If the intensity (**energy/sec/m^2**) from each source is an identical 10^{-5} watt/m^2, then the total intensity from the combined sources can only be twice as great, 2×10^{-5} watt/m^2, not 10^2 watt/m^2. If the total **intensity** from the two sources combined is 2×10^{-5} watt/m^2, what is the **intensity level** from the two combined?

$$\begin{aligned} dB &= 10 \log 2 \times 10^{-5}/10^{-12} \\ &= 73. \end{aligned}$$

If the **intensity level** is 73 dB, what is the **sound pressure level** from the combined sources? If the reference pressure is 2×10^1 µPa, *dB SPL also equals 73.* First, a brief calculation should demonstrate the equivalence. The **intensity** from the combined sources was twice as great as for either source alone. A doubling of intensity corresponds to +3 dB, and we saw that the total of 73 dB was 3 dB greater than the intensity level from only one of the sources. However, if the intensity were increased by a factor of 2:1, what would have happened to the pressure? It would have increased by the square root of 2 ($P \propto \sqrt{I}$), or 1.414. From Equation 4.7 we see that sound pressure level also would have increased by 3 dB from 70 dB SPL.

$$\begin{aligned} dB &= 20 \log 1.414 \\ &= 3. \end{aligned}$$

Second, we learned previously (see Figure 4–1) that dB IL always equals dB SPL *when equivalent reference intensities and pressures are used.* Thus, 70 dB IL (the level for either source alone) equals 70 dB SPL, and if the intensity level increases by only 3 dB, the same must be true for the sound pressure level, which produces a total level of 73 dB SPL.

Unfortunately, the equivalence of dB IL and dB SPL is not always understood. Several years ago a person took a national examination that is required for certification by the American Speech-Language-Hearing Association and encountered the following kind of problem: If two sound sources each produce an identical sound pressure level, by how much will the total sound pressure level of the two combined sources exceed the level for either source alone?

By now, you should know that the answer is 3 dB because it is the energies, powers, or intensities that should be added, which requires us to use the "10-log" equation (Equation 4.4). The person wrote a letter-to-the-editor of the journal *Asha* to complain that the examination was unfair because the alternative answers included 3 dB, but not 6 dB, and the writer was certain that 6 dB was the correct choice.

That letter produced some lively (and correct) responses from Mitchell Kramer, who then was a doctoral student at Northwestern University, and from Professors Larry Feth and W. Dixon Ward that were published in *Asha* (March, 1977). Some comments from those three individuals should serve to emphasize the main point that has been stressed in this section:

> Both sound pressure and power change with a change in energy of an acoustic signal, with power changing as the square of pressure. But, a 3 dB change in pressure is equal, identical, and the very same as a 3 dB change in power or intensity. Of course, a doubling of pressure results in a 6 dB increase (of both pressure and intensity or power) but unless two signals are (1) the same frequency, and (2) added in phase, the intensities add, and a 3 dB increase results. This is equivalent to doubling the sound power, or increasing pressure by a factor of 1.41. But a dB is still a dB. (Kramer, p. 225)

> What students often carry away from the lecture on combining sound sources is the rule of thumb: "Double the intensity means a 3 dB increase; double the sound pressure results in a 6 dB increase." That rule by itself is correct. The errors arise in not understanding how sound pressure waveforms are added and in thinking that there is a difference between intensity level (IL) and sound pressure level (SPL) . . . . It must be kept in mind that sound pressure level (SPL) and intensity level (IL) are always synonymous and further are always numerically equal . . . . When two independent sound sources are combined, one must always add their intensities. Two sound pressure waveforms can only be added if their relative phase is known, and it seldom is. (Feth, pp. 225–226)

> What I suspect happened is that the problem went something like this: "If we put two machines side by side, each of which develops an SPL of 80 dB, what will be the SPL when they are both running?" The question is designed to lead the unwary down the primrose path by using SPL instead

of IL, so that he or she is hoodwinked into thinking: "Oh yes, doubling the pressure is a 6-dB increase" — which is correct, but, alas, irrelevant, because the pressure simply does not get doubled in this situation. The correct answer is indeed 83 dB SPL — and, assuming that it is still taking place here on earth, also 83 dB IL. In air at standard temperature and pressure, the zeroes on the SPL and IL scales have been chosen to represent the same acoustic conditions, so X dB SPL is also X dB IL . . . . To double the pressure (still under optimum conditions) would require four equal machines. (Ward, p. 226)

It is important to remember that when the total intensity level or sound pressure level from uncorrelated (independent) sound sources is desired, it is *the* ***energies,*** *or* ***powers,*** *or* ***intensities*** *that should be summed, not the pressures.* As Feth stated in his letter cited above, we cannot add the pressures unless the relative phases are known, and they seldom are.

There are two approaches that can be taken to solving problems such as these.

Equal Source Intensities

We will consider first the easier situation in which all of the contributing sources are characterized by an identical intensity level or sound pressure level. In that case,

$$dB_N = dB_i + 10 \log_{10} N,$$

Equation 4.9

where **i** = dB SPL (or dB IL) from one of the equal sources

and

N = the number of sources combined.

We do not have to determine the **intensity** of any of the sources or of the total. We simply want to know the total **intensity level** in dB.

Example 1

Two snowmobiles each produce 94 dB IL. What is the total level from the two combined?

$$\begin{aligned} dB_N &= 94 + 10 \log 2 \\ &= 97. \end{aligned}$$

Each source produces some finite amount of intensity that we do not need to calculate. Because there are two such sources, *the two sources combined must produce twice the intensity as either one alone.* Anytime we increase intensity by a factor of 2:1, decibels increase by 3: 94 + 3 = 97. Why does Equation 4.9 "apparently" not contain a ratio? Actually it does. We are asking what happened to the intensity level when we increased from one source to two sources. Thus, the reference is 1 and need not be shown in the denominator.

Example 2 What is the total **sound pressure level** for eight sound sources, each of which produces 92 dB when operating alone?

Even though the level in the problem is expressed in SPL rather than IL, it is the intensities that must be added, not the pressures. Thus, we proceed just as we did with Example 1.

$$\begin{aligned} dB_N &= 92 + 10 \log 8 \\ &= 92 + 9 \\ &= 101. \end{aligned}$$

By now you should be able to work this kind of problem mentally by thinking of powers of 2. 8 is 2^3, which means that 8 is the result of the base 2 being used 3 times in multiplication. Each doubling corresponds to 3 dB. Thus, eight sources would produce a sound whose intensity is 9 dB (3 + 3 + 3) greater than just one of the sources alone.

Unequal Source Intensities

When the contributing sources are not all characterized by equal intensity, Equation 4.9 will not apply, but the problem can still be solved by following three steps: (1) calculate the **intensity** of each source, (2) add the intensities, and the sum of the intensities becomes the numerator of the ratio, and (3) calculate decibels with Equation 4.4 with a reference of 10^{-12} watt/m^2. The result will be either dB IL or dB SPL because the two are equivalent.

Example 1 What is the total SPL that results from combining one source that produces 90 dB SPL with a second source that produces 80 dB SPL?

1. $90 = 10 \log I_x/10^{-12}$, Therefore, $I_{x(1)} = 10^{-3}$,
and
$80 = 10 \log I_x/10^{-12}$, Therefore, $I_{x(2)} = 10^{-4}$,

2. $10^{-3} + 10^{-4} = 1.1 \times 10^{-3}$,

3. $dB = 10 \log 1.1 \times 10^{-3}/10^{-12}$
$= 10 \log 1.1 \times 10^{9}$
$= 90.4.$

There are two important observations to make. First, when the levels of two sources differ by 10 dB, the total level is only 0.4 dB greater than the level of the source with the higher intensity. That is true of any two levels that differ by 10 dB: 90 and 80 (90.4), 80 and 70 (80.4), 0 and −10 (0.4), and so on. Second, one must be careful with addition of the intensities in Step 2 because the exponents are different.

$$\begin{aligned} I_{x(1)} &= 1 \times 10^{-3} \\ + \; I_{x(2)} &= \underline{1 \times 10^{-4}} \\ &= \; ? \end{aligned}$$

Before the two intensities can be added, the exponent for one must be converted to be the same as the other exponent. We will illustrate the process by converting $I_{x(2)}$ to have an exponent of -3 rather than -4. To convert from -4 to -3 we *multiply* by 10. To preserve equivalence, therefore, we must *divide* the other term in the product, the coefficient, by 10: Thus,

$$1 \times 10^{-4} = .1 \times 10^{-3}.$$

Now when we add the two intensities, we obtain:

$$\begin{aligned} & 1 \times 10^{-3} \\ + \; & \underline{.1 \times 10^{-3}} \\ = \; & 1.1 \times 10^{-3}. \end{aligned}$$

If the validity of that series of steps is not obvious, the two intensities are added below in conventional notation rather than scientific notation,

$$\begin{aligned} 1 \times 10^{-3} &= .001 \\ + \; 1 \times 10^{-4} &= \underline{.0001} \\ &= .0011 \end{aligned}$$

and .0011 in conventional notation equals 1.1×10^{-3} in scientific notation.

Example 2

A fan is operating at an unknown level in a noisy environment. You wish to determine the sound pressure level of the fan, but it cannot be removed from its location and the surrounding noise cannot be turned off.

1. Measure the sound pressure level of the combined noise produced by the fan and the surrounding equipment. This measurement would be accomplished with the aid of a sound level meter, which will not be discussed in this book. Suppose this total level is 90 dB SPL.
2. Turn off the fan and measure the level produced by the surrounding equipment. Suppose that this new measurement is 86 dB SPL.
3. From those two measurements we know that the surrounding equipment and the fan together produce a noise of 90 dB SPL

and that the surrounding equipment alone produces a noise of 86 dB SPL. The fan, therefore, must be responsible for the difference of 4 dB, but *do not conclude that its level is 4 dB SPL.* Before we solve the problem, try to approximate the answer by mentally using powers of 2. Recall the earlier problem involving the two snowmobiles. We learned that combining two sound sources that have the same intensity produced a 3 dB increase from the level produced by either machine alone. In the present problem, the surrounding noise is 86 dB. Therefore, if the noise from the fan also were 86 dB, the two combined would produce a noise of 89 dB, which is just 1 dB less than the 90 we measured. Therefore, we should expect the true noise level of the fan to be just a little more than 86.

a. (Intensity of fan plus surrounding noise)
$90 = 10 \log I_x/10^{-12}$. Therefore, $I_{x(2)} = 10^{-3}$.

b. (Intensity of surrounding noise alone)
$86 = 10 \log I_x/10^{-12}$. Therefore, $I_{x(1)} = 3.98 \times 10^{-4}$.

c. (Intensities of fan alone — by subtraction)
$(1 \times 10^{-3}) - (3.98 \times 10^{-4}) = 6.02 \times 10^{-4}$.

d. (Intensity level of fan alone)
$$\begin{aligned} \text{dB} &= 10 \log 6.02 \times 10^{-4}/10^{-12} \\ &= 87.8. \end{aligned}$$

Because the preceding computational methods might sometimes be inconvenient, published charts are available to aid with combining decibels. Direct computations are preferable, however, if precise determination of small differences is required.

SUMMARY OF DECIBELS FOR SOUND INTENSITY AND SOUND PRESSURE

Earlier in this chapter two steps were suggested for solution of decibel problems. First, select the proper equation. Second, form a ratio and solve the problem. When the problem involves **intensity or power**, use Equation 4.4 — the so-called "10 log equation." When the problem involves **pressure**, use Equation 4.7 — the "20 log equation."

The distinction between power and pressure applies if decibels are used to express the level of some quantity other than sound. For example, if you have three dollars, and someone gives you three more dollars, that represents an increase of 3 dB. In contrast, if a committee comprises three people, and three more members are added, that represents an increase of 6 dB. We might justify use of the 10 log equation in one case, but the 20 log equation in the other, because "money is power," but "people are pressure."

The only confusion that should remain might stem from problems that involve combining sound levels from independent sources. In those instances it is important to keep in mind that unless the relative phases are known, *it is the energies or powers or intensities that should be added, not the pressures.* Thus, it does not matter whether the problem is stated by reference to IL or SPL. *As long as the references are equivalent, dB IL = db SPL, and you must always use Equation 4.4* — the "10 log equation."

PRACTICE PROBLEMS

Recall the two-step procedure for solving decibel problems. (1) Select the proper equation, and (2) form a ratio and solve the problem.

It will always be useful to inspect a problem to see if it involves powers of 2 (3 dB for intensity; 6 dB for pressure) or powers of 10 (10 dB for intensity; 20 dB for pressure). Even though some problems cannot be solved in this way, you will be able to estimate the answers in many cases by using powers of 2 and powers of 10 to set upper and lower limits that bracket the correct answers reasonably closely. Try to solve as many problems as possible "mentally," using powers of 2 and powers of 10, and then use your calculator to check your computations.

Set 1

Convert each of the following intensity ratios to decibels.

a. 1:1	g. 3:1	m. 9:1	s. 1:7	y. .001:1
b. 10:1	h. 4:1	n. 1:2	t. 1:8	z. 10^{-3}:1
c. 100:1	i. 5:1	o. 1:3	u. 1:9	aa. 20:1
d. 1,000:1	j. 6:1	p. 1:4	v. 1:10	bb. 200:1
e. 10^3:1	k. 7:1	q. 1:5	w. 1:100	cc. 40:1
f. 2:1	l. 8:1	r. 1:6	x. 1:1,000	dd. 400:1

ee. 60:1

ff. 600:1

gg. 2.45×10^0:1

hh. 2.45×10^1:1

ii. $2.45 \times 10^1/10^{-12}$:1

jj. $2.45 \times 10^{-8}/10^{-16}$:1

Set 2

Convert each of the following decibels to intensity ratios.

a. 0	f. 70	k. 23	p. −10	u. 17
b. 10	g. 3	l. 29	q. −20	v. 62
c. 20	h. 6	m. 46	r. −30	w. 91
d. 30	i. 9	n. 56	s. −23	x. 5.4
e. 40	j. 12	o. 76	t. −36	y. 12.6

Set 3

Calculate dB IL re: 10^{-12} watt/m^2 for each of the following values of sound intensity (I_x).

a. 10^{-12}	f. 2×10^{-8}	k. 1×10^{-2}	p. 0.25×10^{-12}
b. 10^{-11}	g. 4×10^{-8}	l. 2×10^{-2}	q. 1.4×10^{-4}
c. 10^{-10}	h. 8×10^{-8}	m. $.5 \times 10^{-2}$	r. 2.8×10^{-4}
d. 10^{-9}	i. 1×10^{-3}	n. $.5 \times 10^{-5}$	s. 1.65×10^{-6}
e. 10^{-8}	j. 4×10^{-3}	o. $.5 \times 10^{-12}$	t. 3.00×10^{-6}

Set 4

Calculate sound intensity (I_x) in watt/m^2 for each of the following values of dB IL re: 10^{-12} watt/m^2.

a. 0	f. 60	k. −10	p. −23	u. 87
b. 10	g. 13	l. −20	q. −26	v. 16.8
c. 20	h. 23	m. −3	r. 41	w. 24.2
d. 30	i. 36	n. −6	s. 62	x. 38
e. 40	j. 49	o. −13	t. 73	y. 47

Set 5

Convert each of the following pressure ratios to decibels.

a. 1:1	g. 3:1	m. 9:1	s. 1:7	y. .001:1
b. 10:1	h. 4:1	n. 1:2	t. 1:8	z. 10^{-3}:1
c. 100:1	i. 5:1	o. 1:3	u. 1:9	aa. 20:1
d. 1,000:1	j. 6:1	p. 1:4	v. 1:10	bb. 200:1
e. 10^{3}:1	k. 7:1	q. 1:5	w. 1:100	cc. 40:1
f. 2:1	l. 8:1	r. 1:6	x. 1:1,000	dd. 400:1

ee. 60:1	kk. 10^{0}:1
ff. 600:1	ll. $10^{-4}/10^{-4}$:1
gg. 10^{-4}:1	mm. $10^{-3}/10^{-4}$:1
hh. 10^{-5}:1	nn. $(2 \times 10^{-4})/(2 \times 10^{-4})$:1
ii. 10^{-2}:1	oo. $(4 \times 10^{-4})/(2 \times 10^{-4})$:1
jj. 10^{-1}:1	pp. $(10^{-4})/(2 \times 10^{-4})$:1

Set 6

Convert each of the following decibels to pressure ratios.

a. 0	g. 6	m. 26	s. 30
b. 20	h. 12	n. 46	t. 50
c. 40	i. 18	o. 72	u. 44
d. 60	j. 24	p. −20	v. 17
e. 80	k. −6	q. −40	w. 62
f. 100	l. −12	r. 10	x. 5.5

Set 7

Calculate dB SPL re: 2×10^1 μPa for each of the following values of sound pressure (P_x) in μPa.

a. 2×10^1	g. 4×10^1	m. 1.05×10^6
b. 2×10^2	h. 8×10^1	n. 1×10^5
c. 2×10^3	i. 8×10^4	o. $.5 \times 10^5$
d. 2×10^4	j. 2×10^0	p. 4×10^5
e. 2×10^5	k. 4×10^0	q. 4.25×10^5
f. 10^5	l. 4×10^3	r. 8.5×10^5

Set 8

Calculate sound pressure (P_x) in μPa for each of the following values of dB SPL re: 2×10^1 μPa.

a. 0	f. 40	k. 30	p. 34
b. 6	g. 60	l. 50	q. 72
c. 12	h. 3	m. 43	r. 16.8
d. −6	i. 9	n. 46	s. −7
e. 20	j. 10	o. 36	t. −8

Set 9

Calculate dB SPL re: 2×10^{-4} dyne/cm² for each of the following values of sound pressure (P_x) in dyne/cm².

a. 0.0002	f. 0.002
b. 0.0004	g. 2×10^{-3}
c. 8×10^{-4}	h. 4×10^{-2}
d. 2×10^{-4}	i. 1×10^{-5}
e. 4×10^{-4}	j. 2×10^0

Set 10

Calculate the sound pressure level that results from combining the following uncorrelated sound sources whose levels are given in dB SPL.

a. 20 + 20

b. 30 + 30

c. 46.2 + 46.2

d. 20 + 20 + 20

e. 30 + 30 + 30

f. 46.2 + 46.2 + 46.2

g. 60 + 70

h. 60 + 66

i. 60 + 70 + 80

Set 11

Calculate the total intensity in watt/m² that results from combining the following intensities from uncorrelated sources.

a. $10^{-8} + 10^{-8}$

b. $10^{-6} + 10^{-6}$

c. $2 \times 10^{-6} + 10^{-6}$

d. $2 \times 10^{-6} + 5 \times 10^{-6}$

e. $2 \times 10^{-6} + 5 \times 10^{-6} + 2 \times 10^{-6}$

f. $2 \times 10^{-6} + 3 \times 10^{-5}$

Set 12

Calculate the intensity level re: 10^{-12} watt/m² that results from combining the intensities in Set 11.

a.

b.

c.

d.

e.

f.

ANSWERS TO PRACTICE PROBLEMS

Set 1

Equation 4.4 should be used to convert the intensity ratios to decibels. In each of these problems, the reference is not specified, but the ratio I_x/I_r is known. For example, if the intensity ratio is 12:1,

$$\begin{aligned} N(dB) &= 10 \log 12 \\ &= 10 \times 1.08 \\ &= 10.8 \text{ dB.} \end{aligned}$$

Set 1 *(continued)*

a. 0	g. 4.8	m. 9.5	s. −8.5	y. −30
b. 10	h. 6	n. −3	t. −9	z. −30
c. 20	i. 7	o. −4.8	u. −9.5	aa. 13
d. 30	j. 7.8	p. −6	v. −10	bb. 23
e. 30	k. 8.5	q. −7	w. −20	cc. 16
f. 3	l. 9	r. −7.8	x. −30	dd. 26

ee. 17.8	gg. 3.9	ii. 133.9
ff. 27.8	hh. 13.9	jj. 83.9

Notes

(a). 0 dB does not mean "silence." It means only that $I_x = I_r$ and, therefore, that the ratio $I_x/I_r = 1.0$ regardless of the absolute value of I_x.

(b, c, d). You should note that each of these involves powers of 10, and each power of 10 corresponds to 10 dB. Thus, in 1-c the ratio is 100, which is a power of 10 twice (10^2). Because each power of 10 corresponds to 10 dB, the answer is given by 10 dB + 10 dB = 20 dB.

(f). A power of 2, which is 3 dB.

(h). Another power of 2. The ratio 4:1 corresponds to a power of 2 twice (2^2), and each power of 2 is 3 dB. Thus, 3 dB + 3 dB = 6 dB.

(i). Can you see that the ratio 5:1 involves powers of 10 and powers of 2? The ratio 5:1 can be thought of as the ratio 10:1 (+10 dB) divided by the ratio 2:1 (−3 dB), which is 7 dB.

(l). A power of 2 three times (2^3), which therefore involves 3 dB + 3 dB + 3 dB = 9 dB.

(n–z). These are the inverse of the otherwise identical problems that were solved earlier in this set. Because the absolute value of $I_x < I_r$, the answer in decibels will be negative. Solution of such problems is simplified by recalling Log Law 4 ($\log 1/a = -\log a$). Thus, for example:

$$10 \log 1/2 = -10 \log 2 = -3 \text{ dB}.$$

(aa–dd). By now you should quickly see that each involves a combination of powers of 10 (10 dB) and powers of 2 (3 dB). Thus, 400 consists of a power of 10 twice (+20 dB) and a power of 2 twice (+6 dB): $10 \times 10 \times 2 \times 2 = 400$ and the corresponding quantities in decibels are 10 dB + 10 dB + 3 dB + 3 dB = 26 dB.

(ee, ff). These can only be approximated by powers of 2 and 10. To see how the approximation works, consider the ratio 600:1. You know that 400 ($10 \times 10 \times 2 \times 2$) is 26 dB and that 1,000 ($10 \times 10 \times 10$) is 30 dB. So, the answer for 600 must lie between 26 dB and 30 dB. However, if you think back to problem 1–i, you should be able to set the limits even closer. You solved that 5:1 is 7 dB (10/2). If 5:1 is 7 dB, 500 must be another 20 dB for a total of 27 dB. Therefore, because 600 lies between 500 and 1,000, the answer must lie between 27 and 30. See if you can find a way, still using powers of 2 and 10, to lower the upper limit. (Hint: you should be able to set the upper limit to 29 dB.)

(gg, hh). Solution of these problems is made easy by recalling two concepts from Chapter 3. First, both problems involve the log of a product. We learned from Log Law 1 that "log ab = log a + log b." Thus, with Problem 1-gg, we need only to add the logs of the two factors: log 2.45 + log 10^0. You will need a calculator or log table to determine the log of 2.45, but a second concept from Chapter 3 will allow you to determine the log of 10^0 without reference to a log table: "An exponent is a log, and a log is an exponent." Thus, the log of 10 raised to any power is the value of the power. Log $10^0 = 0$; log $10^1 = 1$; log $10^2 = 2; \ldots$; log $10^n = n$.

(ii, jj). These problems represent application of two laws of logarithms and one law of exponents. As a consequence, the problems can be solved in two ways. We will use 1–ii as an example. We can apply Log Law 1, where a = 2.45 and b = $(10^1/10^{-12})$. The log of 2.45 is determined from a calculator or log table, but there are two approaches available for determining the log of the ratio $10^1/10^{-12}$. With the first approach we apply Log Law 2: log a/b = log a − log b. Because we know that an exponent is a log, the log of that ratio is given by the difference between the exponents, $1 - (-12)$, which is 13. From Log Law 1, then the log of the product is $0.4 + 13 = 13.4$, which multiplied by 10 = 134 dB. With the second approach we apply Law 2 of exponents, which is a companion to Log Law 2: $X^m/X^n = X^{m-n}$. Therefore,

$$10^1/10^{-12} = 10^{13}$$
$$\log 10^{13} = 13$$
and
$$\text{dB} = 10\,(0.4 + 13) = 134 \text{ dB}.$$

Why do both approaches give the same answer? With one we use Log Law 2. With the other we use the second law of exponents. *We get the same answer because a log is an exponent.*

Set 2

Equation 4.4 also should be used to convert each of the decibels to intensity ratios. Note that you are not solving for the absolute intensity, I_x, but just the ratio of the two intensities, I_x/I_r. For example, if dB = 5,

$$5 = 10 \log X$$

$$0.5 = \log X \quad \text{(dividing both sides of the equation by 10)}$$

$$X = 3.16 \quad \text{(the ratio of the unknown } I_x \text{ to the unknown } I_r\text{.)}$$

a. 10^0:1 (1:1)	f. 10^7	k. 2×10^2	p. 10^{-1}	u. 5×10^1
b. 10^1	g. 2	l. 8×10^2	q. 10^{-2}	v. 1.58×10^6
c. 10^2	h. 4	m. 4×10^4	r. 10^{-3}	w. 1.26×10^9
d. 10^3	i. 8	n. 4×10^5	s. 5×10^{-3}	x. 3.47×10^0
e. 10^4	j. 16	o. 4×10^7	t. 2.5×10^{-4}	y. 1.82×10^1

Notes

(a–f). Each of the decibel values is divisible evenly by 10, and each 10 dB of intensity corresponds to a power of 10. Thus, for these five problems, the solutions are a power of 10: the powers of 0, 1, 2, 3, 4, and 7, respectively. A second, but not independent, approach is to recall that when you convert decibels to intensity ratios, the first step is to divide by 10. The result is a log, which in these cases is an integer of 0, 1, 2, 3, 4, and 7 followed by 0.0000. The integers are the characteristics, and they indicate the exponents in scientific notation. Thus, the results are 10^0, 10^1, 10^2, 10^3, 10^4, and 10^7.

(g–j). Each of these is a value that is evenly divisible by 3, and each 3 dB corresponds to a power of 2. Thus, the answers are 2, 4, 8, and 16.

(k–o). Each of these involves a combination of powers of 10 and powers of 2. For example, 2-k is 23 dB, and 23 dB consists of 10 + 10 + 3. Each 10 dB is a power of 10, and each 3 dB is a power of 2. Thus, the answer is $10 \times 10 \times 2 = 200 = 2 \times 10^2$.

(p–t). Each of these involves either a power of 10 or a combination of powers of 10 and powers of 2. However, because the decibel is negative, we know that *the exponent will be negative.* Thus, 10 dB corresponds to 10^1, whereas -10 dB corresponds to 10^{-1}.

(u). Although you might not see it on first inspection, this problem also can be solved with powers of 10 and 2. Actually, 17 comprises $10 + 10 - 3$. Thus, 17 dB would correspond to a tenfold increase in intensity twice, and a halving of intensity once: $(10 \times 10)/2 = 50$. If the problem had involved 14 dB, could you have used the same approach? Yes. $14 = (10 + 10 - 3 - 3)$. Thus, 14 dB corresponds to: $(10 \times 10)/(2 \times 2) = 25$.

(v, w). These, too, actually can be worked as combinations of powers of 10 and 2, but it might be quicker to solve them step-by-step with a log table or more quickly with your calculator rather than to spend time seeing if the powers of 10 and 2 rules apply. However, it is surprising how many problems can be solved in that simple way without use of log tables or calculators. Consider 2-v. 62 dB $= 10 + 10 + 10 + 10 + 10 + 3 + 3 + 3 + 3$. So, we have a power of 10 five times (10^5) and a power of 2 four times (2^4). That corresponds to 16×10^5, which is 1.6×10^6 in scientific notation. Can you see how Problem 2-w can be approached in the same way? (Hint: You need to sum 10s and subtract 3s.)

(x, y). There is no quick solution available for these, but you should be able to determine upper and lower limits to check to see if the answers you calculate are reasonable.

General Comment

We have emphasized that reasonably quick solutions to many problems can be realized by employing powers of 10 (10 dB) and powers of 2 (3 dB). However, because the log of 2 = 3.0103, not 3.0000, you will sometimes experience a rounding error that might or might not be tolerable, depending on the accuracy that is required. Thus, 3 dB really corresponds to an intensity ratio of 1.9953:1 rather than 2:1, and that probably will not pose any difficulty most of the time. However, look at problem 2-1. For 29 dB the answer was listed as 800:1, but the correct answer (two decimals) is 794.33:1. When greater accuracy is required, you should use the powers of 10 and powers of 2 shortcut only to estimate the answer and to aid you in determining if the answer you calculated is reasonable.

Set 3

Equation 4.4 also should be used for all of these problems. All problems in the set are conceptually identical to those in Set 1. The only difference is that in Set 1 you dealt only with a ratio of intensities (I_x/I_r) where the absolute values of I_x and I_r were unknown. In Set 3, both I_x and I_r are known. The first step, then, is to solve the ratio by reference to Law 2 of exponents $(X^m/X^n = X^{m-n})$, and the exponent in the result is the log of the ratio. For example,

$$\begin{aligned} \text{dB} &= 10 \log 10^{-7}/10^{-12} \\ &= 10 \log 10^{(-7)-(-12)} \\ &= 10 \log 10^5 \\ &= 10 \times 5 \\ &= 50. \end{aligned}$$

a. 0	f. 43	k. 100	p. −6
b. 10	g. 46	l. 103	q. 81.5
c. 20	h. 49	m. 97	r. 84.5
d. 30	i. 90	n. 67	s. 62.2
e. 40	j. 96	o. −3	t. 64.8

Notes

(a–e). Each involves a power of 10, and by now you should be able to solve these quickly. For example, in 3-c, 10^{-10} is two powers of 10 larger than the reference of 10^{-12}, each power of 10 corresponds to 10 dB, and the answer, therefore, is 20 dB.

(f–h). These are combinations of powers of 10 and powers of 2.

(m). This also is a combination of powers of 10 and powers of 2. Therefore, 10^{-2} involves 10 tenfold increases (100 dB). If 10^{-2} is 100 dB, then 0.5×10^{-2} (which is only half as great) must be 3 dB less, or 97 dB.

(q–r). Problem 3-q cannot be solved by inspection. However, having already solved 3-q, the answer to 3-r must be 3 dB greater than the answer to 3-q because 2.8×10^{-4} is twice as great as 1.4×10^{-4}.

Set 4

Equation 4.4 also should be used for these problems, which are conceptually identical to those in Set 2. The only difference is that in Set 2 you solved only for the intensity ratio (X), whereas in Set 4 you must carry the computation one step further to determine the actual value of I_x.

a. 10^{-12}	f. 10^{-6}	k. 1.00×10^{-13}
b. 10^{-11}	g. 2×10^{-11}	l. 1.00×10^{-14}
c. 10^{-10}	h. 2×10^{-10}	m. 0.50×10^{-12}
d. 10^{-9}	i. 4×10^{-9}	n. 1.25×10^{-12}
e. 10^{-8}	j. 8×10^{-8}	o. 0.50×10^{-13}

p. 0.50×10^{-14}	u. 5.00×10^{-8}
q. 0.25×10^{-14}	v. 4.79×10^{-11}
r. 1.26×10^{-8}	w. 2.63×10^{-10}
s. 1.58×10^{-6}	x. 6.30×10^{-9}
t. 2.00×10^{-5}	y. 5.00×10^{-8}

Notes

(m). If you followed the step-by-step procedures, you probably obtained 5×10^{-13} for your answer rather than 0.5×10^{-12} that is shown above, but the two are numerically identical. The answer of 0.5×10^{-12} came from inspecting for powers of 2 and 10. We know that 0 dB corresponds to an intensity of 10^{-12}, so -3 dB must correspond to only half as much intensity, or 0.5×10^{-12}. The same explanation applies to Problems 4-n, o, p, and q.

(u–y). Did you notice that you could solve these by inspection for powers of 10 and 2?

Set 5

Equation 4.7 should be used to convert the pressure ratios to decibels. Each of these problems is identical in concept to those in Set 1, and problems 5-a through 5-ff are identical numerically. The only difference is that now you are presented with pressure ratios rather than intensity ratios, and therefore the log of the ratio is multiplied by 20 rather than by 10. Solution of the two sets of problems is otherwise identical.

Because the two sets of problems are virtually identical, except for the multiplier, there are no explanatory notes to accompany these problems. When in doubt, consult the notes for the corresponding problem in Set 1. However, as a general reminder, the vast majority of the problems can be solved by inspection for powers of 10 and powers of 2, where a power of 10 for pressure corresponds to 20 dB (20 log 10) and a power of 2 for pressure corresponds to 6 dB (20 log 2).

a. 0	g. 9.5	m. 19.1	s. −16.9	y. −60
b. 20	h. 12	n. −6	t. −18	z. −60
c. 40	i. 14	o. −9.5	u. −19.1	aa. 26
d. 60	j. 15.6	p. −12	v. −20	bb. 46
e. 60	k. 16.9	q. −14	w. −40	cc. 32
f. 6	l. 18	r. −15.6	x. −60	dd. 52

ee. 35.6	kk. 0
ff. 55.6	ll. 0
gg. −80	mm. 20
hh. −100	nn. 0
ii. −40	oo. 6
jj. −20	pp. −6

Set 6

Equation 4.7 should be used to convert decibels to pressure ratios. The only difference in solutions of these problems from those encountered with Set 2 is that the first step is to divide by 20 rather than by 10 because the problems involve pressure rather than intensity.

a. 10^0 (1:1)	g. 2	m. 2×10^1	s. 3.16×10^1
b. 10^1	h. 4	n. 2×10^2	t. 3.16×10^2
c. 10^2	i. 8	o. 4×10^3	u. 1.58×10^2

d. 10^3	j. 16	p. 10^{-1}	v. 7×10^0
e. 10^4	k. .5	q. 10^{-2}	w. 1.26×10^3
f. 10^5	l. .25	r. 3.16×10^0	x. 1.88×10^0

Set 7

Equation 4.7 should be used for all of these problems, and all are identical in concept to the problems in Set 5. The only difference is that in Set 5 you dealt only with a pressure ratio (P_x/P_r) where the absolute value of P_x and P_r were not specified. In Set 7, both P_x and P_r are known. The first step, then is to solve the ratio by use of the 2nd Law of exponents ($x^m/x^n = x^{m-n}$), and the exponent in the result is the characteristic in the log of the ratio. For example,

$$\begin{aligned} dB &= 20 \log (3 \times 10^3)/(2 \times 10^1) \\ &= 20 \log 1.5 \times 10^2 \\ &= 20 \times 2.18 \\ &= 43.6. \end{aligned}$$

a. 0	g. 6	m. 94.4
b. 20	h. 12	n. 74
c. 40	i. 72	o. 68
d. 60	j. −20	p. 86
e. 80	k. −14	q. 86.6
f. 74	l. 46	r. 92.6

Notes

Almost all of these problems can be solved by use of powers of 10 (20 dB) and powers of 2 (6 dB).

(a). In this problem $P_x = P_r$, the ratio is therefore 1:1, the log of 1 is 0.0000, and the answer must be 0 dB.

(a–e). As you proceed from 7-a through 7-e, you progressively increase by one power of 10 (10^1), which for pressure corresponds to increases of 20 dB. Thus, the answers are 0, 20, 40, 60, and 80 dB SPL.

(f–h). Problem 7-f is one power of 2 (2^1) *less* than 7-e, which means that the sound pressure level for 7-f must be 6 dB less than the sound pressure level for 7-e. Similarly, you should see powers of 2 relations between 7-g and 7-a, between 7-h and 7-g, between 7-i and 7-d, and so on.

(m). The value of P_x is only fractionally greater than 10^6. By using powers of 10 and 2 you should see that if P_x were 10^6, SPL = 94 dB (2×10^6 would equal 100 dB, so 1×10^6, which is half as much pressure, must be 6 dB less). If SPL = 94 dB when $P_x = 1 \times 10^6$, SPL must be only fractionally greater when $P_x = 1.05 \times 10^6$. Thus, the answer of 94.4 seems reasonable.

Set 8

Equation 4.7 also should be used for these problems, which are identical in concept to those in Set 6. The only difference is that in Set 6 you solved only for the pressure ratio (X), whereas in Set 8 you must carry the computation one step further to determine the actual value of P_x.

a. 2×10^1	f. 2×10^3	k. 6.32×10^2	p. 10^3
b. 4×10^1	g. 2×10^4	l. 6.32×10^3	q. 8×10^4
c. 8×10^1	h. 2.82×10^1	m. 2.82×10^3	r. 1.38×10^2
d. 10^1	i. 5.64×10^1	n. 4×10^3	s. 8.93×10^0
e. 2×10^2	j. 6.32×10^1	o. 1.26×10^3	t. 7.96×10^0

Notes

(a–g). As with many decibel problems, laborious, step-by-step calculations can be avoided with problems 8-a through 8-g by inspecting to determine if powers of 2 (6 dB) or powers of 10 (20 dB) apply. In 8-a, 0 dB *always* means that $P_x = P_r$; therefore the answer must be 2×10^1. The answer to 8-b must be a power of 2 greater (6 dB) than the answer to 8-a, and therefore is 4×10^{-4}; 8-c is one power of 2 greater than 8-b; 8-d is one power of 2 *less* than 8–a; 8-e is one power of 10 (20 dB) greater than 8-a; 8-f is one power of 10 greater than 8-e; and 8-g is one power of 10 greater than 8-f.

(h, i). You need to calculate the answer to 8-h, but having done so, the answer to 8-i must be one power of 2 greater than 8-h because the difference between the two is 6 dB.

(j, k, l). You need to calculate the answer to 8-j, but having done so, you should see that 8-k and 8-l are powers of 10 relative to 8-j.

Set 9

Equation 4.7 should be used for all of these problems. The only difference between these and the problems in Set 7 is that pressure now is expressed in dynes/cm^2 (**cgs** system) rather than µPa, and the reference pressure (P_r) is 2×10^{-4} dyne/cm^2. For example,

$$\begin{aligned} dB &= 20 \log (3 \times 10^{-4})/(2 \times 10^{-4}) \\ &= 20 \log 1.5 \\ &= 20 \times 0.18 \\ &= 3.6. \end{aligned}$$

a. 0	f. 20
b. 6	g. 20
c. 12	h. 46
d. 0	i. −26
e. 6	j. 80

Notes

(a). The answer to 9-a must be 0 dB because $P_x = P_r$.

(b, c). The values of P_x in 9-b and 9-c are, respectively, one and two powers of 2 (each power of 2 corresponds to 6 dB) greater than the value of P_x in 9-a.

(d). 2×10^{-4} dyne/cm^2 is the same as 0.0002 dyne/cm^2 in 9-a.

(e–j). All of the remaining problems in Set 9 involve powers of 2 (6 dB) and/or powers of 10 (20 dB).

Set 10

Even though the uncorrelated noise levels that are being combined are expressed in dB SPL, it is the energies, or powers, or intensities that should be added, not the pressures (See "Combining Sound Intensities from Independent Sources," Chapter 4). If the sources have equal intensity, you can use Equation 4.9:

$$dB_N = dB_i + 10 \log N.$$

For example, if five sources each produce a noise level of 72 dB,

$$\begin{aligned} dB_N &= 72 + 10 \log 5 \\ &= 72 + 10\,(0.7) \\ &= 79. \end{aligned}$$

If the source intensities are not equal, you must execute three calculations.

1. Calculate the *intensity* in watt/m^2 for each source (Equation 4.4).
2. Add the intensities to determine the value of I_x to be used in the third step.
3. Calculate decibels with Equation 4.4 where $I_r = 10^{-12}$ watt/m^2. The result can be expressed as dB IL or dB SPL; they are synonymous.

For example, if two sources have noise levels of 80 dB SPL and 83 dB SPL:

Step 1:

$80 = 10 \log I_x/10^{-12}$, Therefore, $I_x = 10^{-4}$.

$83 = 10 \log I_x/10^{-12}$, Therefore, $I_x = 2 \times 10^{-4}$.

Step 2:

$I_x + I_x = (1 \times 10^{-4}) + (2 \times 10^{-4}) = 3 \times 10^{-4}$.

Step 3:

$$\begin{aligned} dB &= 10 \log (3 \times 10^{-4})/10^{-12} \\ &= 10 \log 3 \times 10^{8} \\ &= 10 \times 8.48 \\ &= 84.8 \text{ (dB IL or dB SPL)}. \end{aligned}$$

a. 23	d. 24.8	g. 70.4
b. 33.0	e. 34.8	h. 67
c. 49.2	f. 51	i. 80.5

Notes

(a–f). Problems 10-a through 10-f involve equal source intensities, which therefore permits you to solve the problems with Equation 4.9. Thus, for 10-a, the answer would be 20 + 10 log 2 = 23 dB. For 10-f, 46.2 + 10 log 3 = 51 dB.

(g, h, i). For each of these problems you should use the three-step procedures described above. The solution for 10-i is shown below.

Step 1:

$60 = 10 \log I_x/10^{-12} = 10^{-6}$

$70 = 10 \log I_x/10^{-12} = 10^{-5}$

$80 = 10 \log I_x/10^{-12} = 10^{-4}$

Step 2:

$$\begin{array}{r l} & 0.01 \times 10^{-4} \\ & 0.1 \quad \times 10^{-4} \\ + & 1.0 \quad \times 10^{-4} \\ \hline = & 1.11 \times 10^{-4}. \end{array}$$

Step 3:

$$\begin{aligned} \text{dB} &= 10 \log (1.11 \times 10^{-4})/10^{-12} \\ &= 10 \log 1.11 \times 10^{8} \\ &= 10 \times 8.05 \\ &= 80.5 \end{aligned}$$

Set 11

In Set 10 you were to calculate the level (dB IL or dB SPL) that resulted from having combined uncorrelated sound sources. In Set 11 you simply execute Steps 1 and 2 of the three-step procedure used in Set 10, because you are asked to determine the *total intensity* in watt/m^2 rather than the *level* in decibels that corresponds to the total intensity.

a. 2×10^{-8} d. 7×10^{-6}

b. 2×10^{-6} e. 9×10^{-6}

c. 3×10^{-6} f. 3.2×10^{-5}

Notes

(f). The only difficulty, if any, that should be encountered is with 11-f. The key is that you must be certain that both intensities have the same exponent before you add. Thus, for example, $2 \times 10^{-6} = 0.2 \times 10^{-5}$, which when added to $3 \times 10^{-5} = 3.2 \times 10^{-5}$.

Set 12

The procedure used to solve these problems should have been sufficiently mastered that no explanation should be necessary.

a. 43 b. 63 c. 64.8 d. 68.5 e. 69.5 f. 75.1

NOTES

1. The proper form for all equations concerning the decibel is "$\log_{10}$ X" preceded by a multiplier of either 10 (intensity) or 20 (pressure), where "X" refers to some ratio. However, because the **base** for all calculations of decibels is **10**, the base will frequently be omitted for convenience.
2. The equivalent reference intensity in the **cgs** metric system is 10^{-16} watt/cm^2.
3. The unit of measure in the **cgs** metric system is the **dyne/cm²** or the **microbar**.
4. In most reference texts, the symbol for the speed of sound is **c** rather than **s**. Thus, the equation for characteristic impedance ordinarily is expressed as $\mathbf{Z_c} = \rho_0 \mathbf{c}$. We have elected to use **s** so that we can consistently distinguish between speed (**s**), which is a scalar quantity, and velocity (**c**), which is a vector quantity.
5. The equivalent reference pressure in the **cgs** metric system is $\mathbf{2 \times 10^{-4}}$ **dyne/cm²** or $\mathbf{2 \times 10^{-4}}$ **microbar**. See Table 4-3 for a more complete description of equivalent measures of pressure in the two metric systems.