University of São Paulo
São Carlos School of Engineering
Department of Aeronautical Engineering

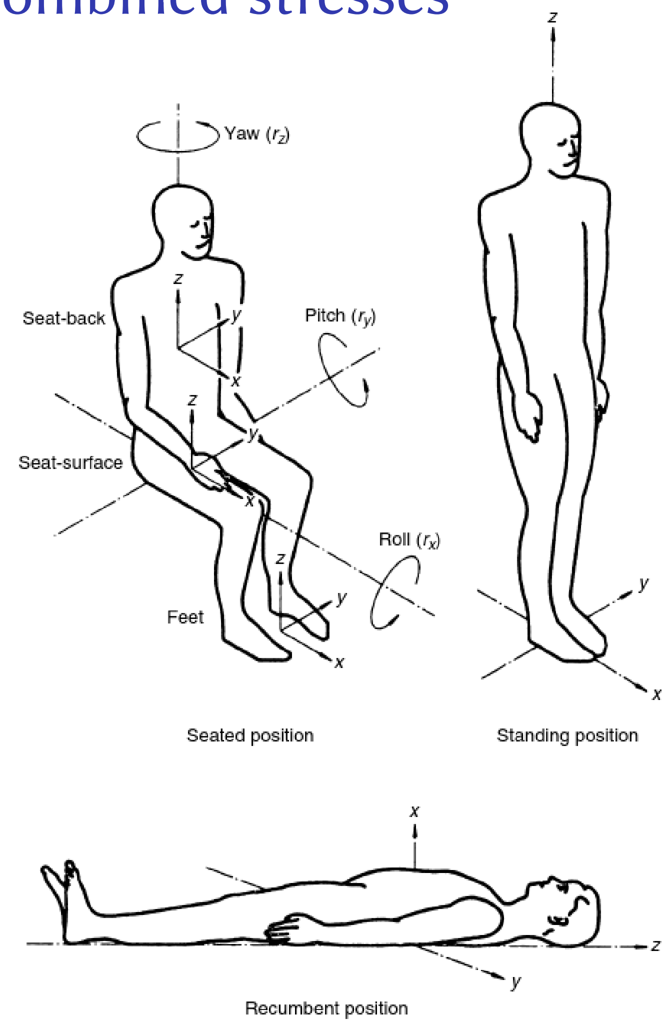
An introduction to flight dynamics

Flight Dynamics II

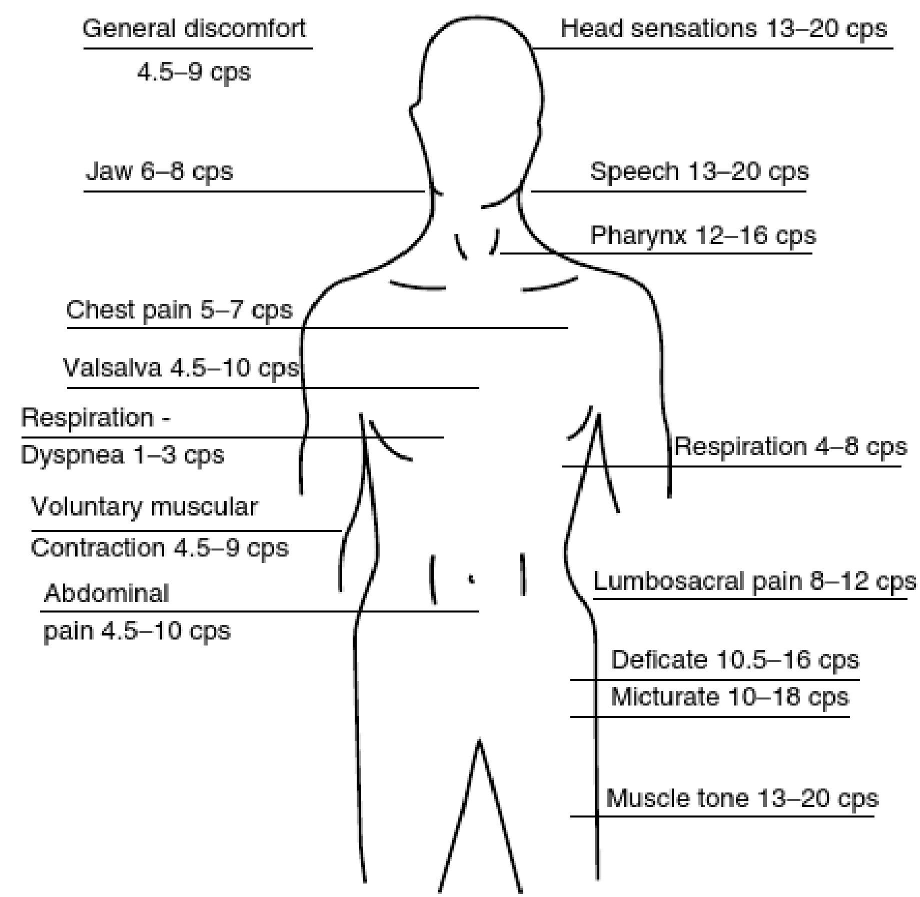
Ricardo Afonso Angélico
ricardoangelico@gmail.com

Human factors

- Acceleration
- Vibration
- Combined stresses



Symptoms 2 < f < 20 Hz



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General equations unsteady motion

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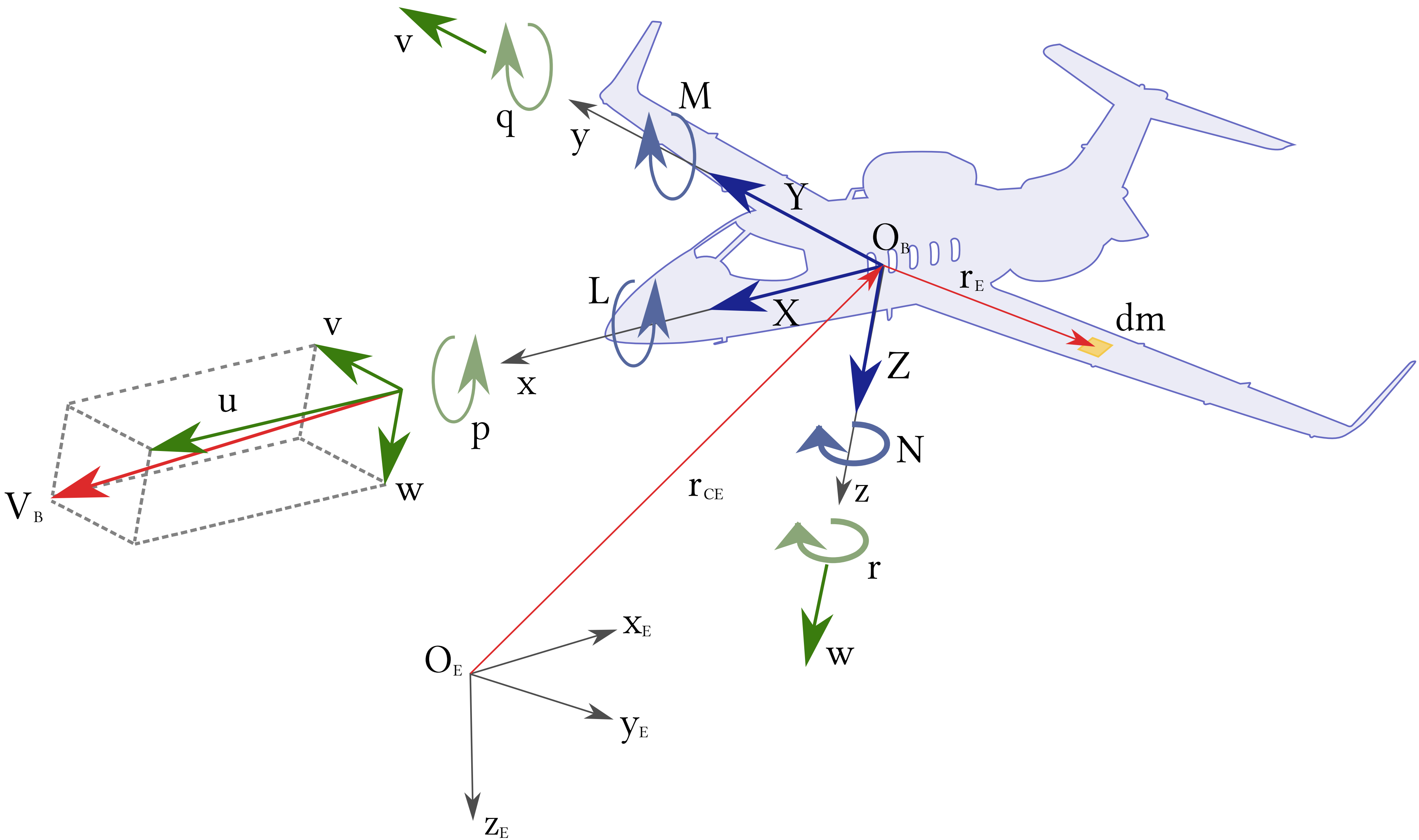
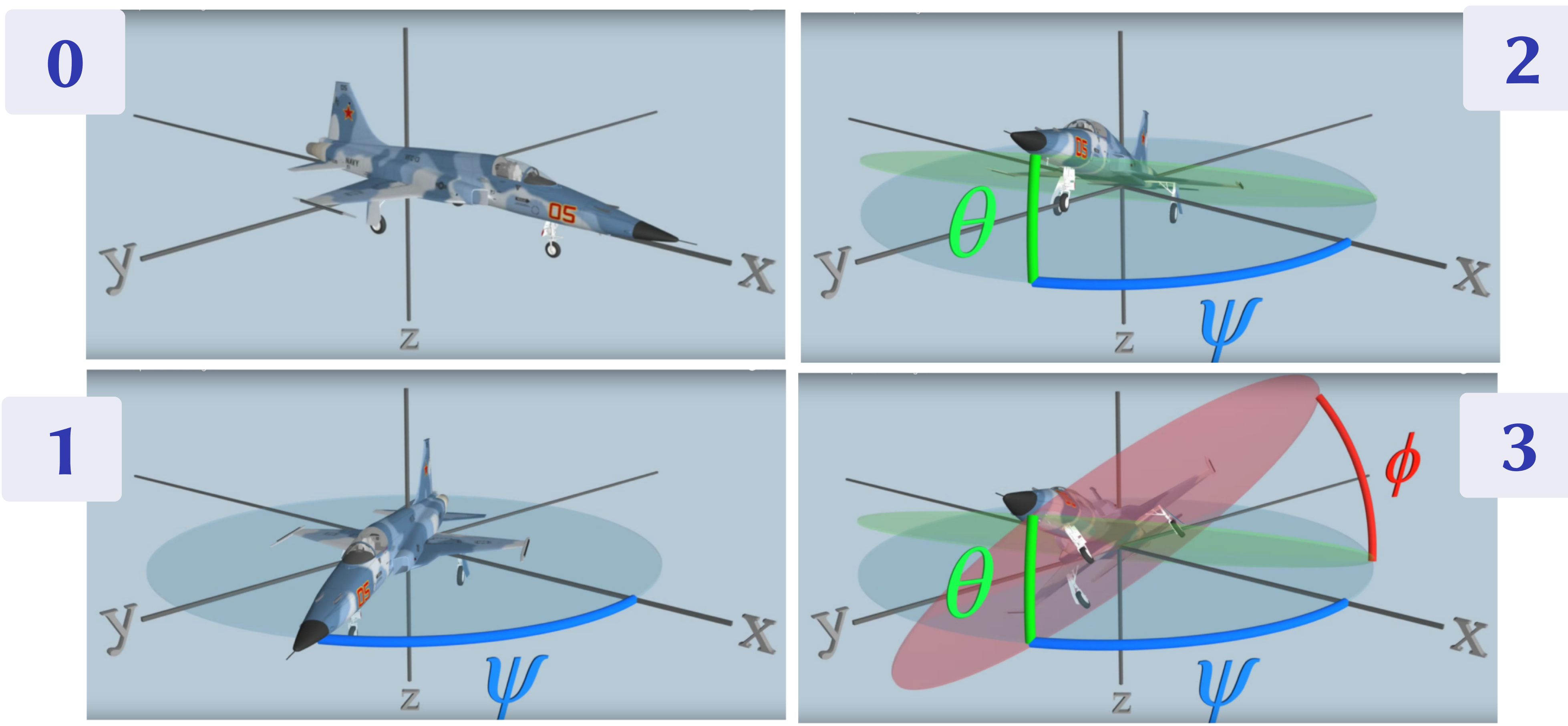
Model



Aircraft orientation - Euler angles



Motion equations

$$\mathbf{f_B} = m(\dot{\mathbf{V}}_\mathbf{B} + \omega_\mathbf{B} \times \mathbf{V_B})$$

$$\mathbf{G_B} = \dot{\mathbf{h}}_\mathbf{B} + \omega_\mathbf{B} \times \mathbf{h_B}$$

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$\mathbf{f_B} = m\mathbf{L_{BE}} \begin{bmatrix} 0 \\ 0 \\ g \end{bmatrix} + \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \qquad \mathbf{G_B} = \begin{bmatrix} L \\ M \\ N \end{bmatrix} \qquad \omega_\mathbf{B} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$\mathbf{h_B} = \mathbf{I}\,\omega_\mathbf{B}$$

Motion equations

Equations in expanded form:

$$m\begin{Bmatrix}\dot{u}+wq-vr\\ \dot{v}+ur-pw\\ \dot{w}+vp-uq\end{Bmatrix}=\begin{Bmatrix}X-mg\,sen\,\theta\\ Y+mg\cos\theta\,sen\,\phi\\ Z+mg\cos\theta\cos\phi\end{Bmatrix}$$

$$\begin{Bmatrix}I_x\dot{p}-I_{xy}\dot{q}-I_{xz}\dot{r}\\ -I_{xy}\dot{p}+I_y\dot{q}-I_{yz}\dot{r}\\ -I_{xz}\dot{p}-I_{yz}\dot{q}+I_z\dot{r}\end{Bmatrix}+\begin{Bmatrix}\left(I_z-I_y\right)qr+\left(r^2-q^2\right)I_{yz}+I_{xy}pr-I_{xz}pq\\ \left(I_x-I_z\right)pr+\left(p^2-r^2\right)I_{xz}+I_{yz}pq-I_{xy}qr\\ \left(I_y-I_x\right)pq+\left(q^2-p^2\right)I_{xy}+I_{xz}qr-I_{yz}pr\end{Bmatrix}=\begin{Bmatrix}L\\ M\\ N\end{Bmatrix}$$

$$\begin{Bmatrix}p\\ q\\ r\end{Bmatrix}=\begin{bmatrix}1 & 0 & -sen\,\theta\\ 0 & \cos\phi & \cos\theta\,sen\,\phi\\ 0 & -sen\,\phi & \cos\theta\cos\phi\end{bmatrix}\begin{Bmatrix}\dot{\phi}\\ \dot{\theta}\\ \dot{\psi}\end{Bmatrix}$$

Trimmed flight

$$m\begin{Bmatrix} w_0 q_0 - v_0 r_0 \\ u_0 r_0 - w_0 p_0 \\ v_0 p_0 - u_0 q_0 \end{Bmatrix} = \begin{Bmatrix} X_0 - mg\, sen\,\theta_0 \\ Y_0 + mg\cos\theta_0\, sen\,\phi_0 \\ Z_0 + mg\cos\theta_0 \cos\phi_0 \end{Bmatrix}$$

$$\begin{Bmatrix} \left(I_{zz} - I_{yy}\right) q_0 r_0 - I_{xz} p_0 q_0 \\ \left(I_{xx} - I_{zz}\right) p_0 r_0 + \left(p_0^{\,2} - r_0^{\,2}\right) I_{xz} \\ \left(I_{yy} - I_{xx}\right) p_0 q_0 + I_{xz} q_0 r_0 \end{Bmatrix} = \begin{Bmatrix} L_0 \\ M_0 \\ N_0 \end{Bmatrix}$$

$$\begin{Bmatrix} p_0 \\ q_0 \\ r_0 \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -sen\,\theta_0 \\ 0 & \cos\phi_0 & \cos\theta_0\, sen\,\phi_0 \\ 0 & -sen\,\phi_0 & \cos\theta_0 \cos\phi_0 \end{bmatrix} \begin{Bmatrix} \acute{\phi}_0 \\ \acute{\theta}_0 \\ \acute{\psi}_0 \end{Bmatrix}$$

Small disturbance theory

$$u = u_0 + \Delta u \qquad X = X_0 + \Delta X$$
$$v = v_0 + \Delta v \qquad Y = Y_0 + \Delta Y$$
$$w = w_0 + \Delta w \qquad Z = Z_0 + \Delta Z$$
$$p = p_0 + \Delta p \qquad L = L_0 + \Delta L$$
$$q = q_0 + \Delta q \qquad M = M_0 + \Delta M$$
$$r = r_0 + \Delta r \qquad N = N_0 + \Delta N$$

$$\phi = \phi_0 + \Delta \phi$$
$$\theta = \theta_0 + \Delta \theta$$
$$\psi = \psi_0 + \Delta \psi$$

Procedure:

1) Substituir nas equações de movimento
2) Desprezar o produto de pequenas perturbações
3) Subtrair as equações de voo trimado
4) Utilizar aproximações de senos e cossenos para pequenos ângulos

Small disturbance theory

$$m\begin{Bmatrix} \Delta\dot{u}+w_0\Delta q+q_0\Delta w-v_0\Delta r-r_0\Delta v \\ \Delta\dot{v}+u_0\Delta r+r_0\Delta u-w_0\Delta p-p_0\Delta w \\ \Delta\dot{w}+v_0\Delta p+p_0\Delta v-u_0\Delta q-q_0\Delta u \end{Bmatrix}=\begin{Bmatrix} \Delta X-mg\,\Delta\theta\cos\theta_0 \\ \Delta Y-mg\,\Delta\theta\,sen\,\theta_0 sen\,\phi_0+mg\,\Delta\phi\cos\theta_0\cos\phi_0 \\ \Delta Z-mg\,\Delta\theta\,sen\,\theta_0\cos\phi_0-mg\,\Delta\phi\cos\theta_0 sen\,\phi_0 \end{Bmatrix}$$

$$\begin{Bmatrix} I_x\Delta\dot{p}-I_{xz}\Delta\dot{r} \\ I_y\Delta\dot{q} \\ I_z\Delta\dot{r}-I_{xz}\Delta\dot{p} \end{Bmatrix}+\begin{Bmatrix} (I_z-I_y)(q_0\Delta r+r_0\Delta q)-I_{xz}(p_0\Delta q+q_0\Delta p) \\ (I_x-I_z)(p_0\Delta r+r_0\Delta p)+I_{xz}(2p_0\Delta p-2r_0\Delta r) \\ (I_y-I_x)(p_0\Delta q+q_0\Delta p)+I_{xz}(q_0\Delta r+r_0\Delta q) \end{Bmatrix}=\begin{Bmatrix} \Delta L \\ \Delta M \\ \Delta N \end{Bmatrix}$$

$$\begin{Bmatrix} p \\ q \\ r \end{Bmatrix}=\begin{bmatrix} 1 & 0 & -sen\,\Theta_0 \\ 0 & \cos\Phi_0 & \cos\Theta_0 sen\,\Phi_0 \\ 0 & -sen\,\Phi_0 & \cos\Theta_0\cos\Phi_0 \end{bmatrix}\begin{Bmatrix} \Delta\dot{\phi} \\ \Delta\dot{\theta} \\ \Delta\dot{\psi} \end{Bmatrix}$$

Small disturbance theory

$$m\begin{Bmatrix} \Delta\dot{u} + w_0\Delta q \\ \Delta\dot{v} + u_0\Delta r - w_0\Delta p \\ \Delta\dot{w} - u_0\Delta q \end{Bmatrix} = \begin{Bmatrix} \Delta X - mg\,\Delta\theta\cos\theta_0 \\ \Delta Y + mg\,\Delta\phi\cos\theta_0 \\ \Delta Z - mg\,\Delta\theta\,sen\,\theta_0 \end{Bmatrix}$$

$$\begin{Bmatrix} I_x\Delta\dot{p} - I_{xz}\Delta\dot{r} \\ I_y\Delta\dot{q} \\ I_z\Delta\dot{r} - I_{xz}\Delta\dot{p} \end{Bmatrix} = \begin{Bmatrix} \Delta L \\ \Delta M \\ \Delta N \end{Bmatrix}$$

$$\begin{Bmatrix} \Delta p \\ \Delta q \\ \Delta r \end{Bmatrix} = \begin{bmatrix} 1 & 0 & -sen\,\theta_0 \\ 0 & 1 & 0 \\ 0 & 0 & \cos\theta_0 \end{bmatrix} \begin{Bmatrix} \Delta\dot{\phi} \\ \Delta\dot{\theta} \\ \Delta\dot{\psi} \end{Bmatrix}$$

Voo retilineo

$$v_0 = p_0 = q_0 = r_0 = \phi_0 = 0$$

Longitudinal

Lateral

Motion decomposition: longitudinal / lateral

- Existence of a plane of symmetry
- The absence of rotor gyroscopic effects

Linear motion equations

$$
\begin{aligned}
\Delta X &= X_u \Delta u + X_w w + \Delta X_c \\
\Delta Y &= Y_v v + Y_p p + Y_r r + \Delta Y_c \\
\Delta Z &= Z_u \Delta u + Z_w w + Z_{\dot{w}} \dot{w} + Z_q q + \Delta Z_c \\
\Delta L &= L_v v + L_p p + L_r r + \Delta L_c \\
\Delta M &= M_u \Delta u + M_w w + M_{\dot{w}} \dot{w} + M_q q + \Delta M_c \\
\Delta N &= N_v v + N_p p + N_r r + \Delta N_c
\end{aligned}
$$

Longitudinal – Small disturbance equations

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\boldsymbol{u}$$

$$\mathbf{x} = \begin{Bmatrix} \Delta u \\ w \\ q \\ \Delta\theta \end{Bmatrix} \qquad \boldsymbol{u} = \begin{Bmatrix} \Delta\delta_e \\ \Delta\delta_p \end{Bmatrix}$$

Elevator

Propulsion

$$\mathbf{A} = \begin{bmatrix} \dfrac{X_u}{m} & \dfrac{X_w}{m} & 0 & -g\cos(\theta_0) \\ \dfrac{Z_u}{m - Z_{\dot{w}}} & \dfrac{Z_w}{m - Z_{\dot{w}}} & \dfrac{Z_q + mu_0}{m - Z_{\dot{w}}} & \dfrac{-mg\sin(\theta_0)}{m - Z_{\dot{w}}} \\ \dfrac{1}{I_y}\left(M_u + \dfrac{M_{\dot{w}} Z_u}{m - Z_{\dot{w}}}\right) & \dfrac{1}{I_y}\left(M_w + \dfrac{M_{\dot{w}} Z_w}{m - Z_{\dot{w}}}\right) & \dfrac{1}{I_y}\left[M_w + \dfrac{M_{\dot{w}}(Z_q + mu_0)}{m - Z_{\dot{w}}}\right] & \dfrac{M_{\dot{w}} mg\sin(\theta)}{I_y(m - Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Etkins notation

Lateral – Small disturbance equations

$$\mathbf{A_{LD}} = \begin{bmatrix} \dfrac{Y_v}{m} & \dfrac{Y_p}{m} & \left(\dfrac{Y_r}{m} - u_0\right) & g\cos\theta_0 \\ \left(\dfrac{L_v}{I'_x} + I'_{zx}N_v\right) & \left(\dfrac{L_p}{I'_x} + I'_{zx}N_p\right) & \left(\dfrac{L_r}{I'_x} + I'_{zx}N_r\right) & 0 \\ \left(I'_{xz}L_v + \dfrac{N_v}{I'_z}\right) & \left(I'_{xz}L_p + \dfrac{N_p}{I'_z}\right) & \left(I'_{xz}L_r + \dfrac{N_r}{I'_z}\right) & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix}$$

Etkins notation

$$\dot{\psi} = r\sec\theta_0$$

$$I'_x = (I_x I_z - I_{xz}^2)/I_z$$

$$I'_z = (I_x I_z - I_{xz}^2)/I_x$$

$$I'_{xz} = I_{zx}/(I_x I_z - I_{xz}^2)$$

Control fixed problems

$$\mathbf{x} = \mathbf{x}_0 \exp(\lambda t) \begin{cases} \mathbf{x}_0 & \text{eigenvector} \\ \lambda & \text{eigenvalue} \end{cases}$$

Considering a conjugate pair of eigenvalues

$$\lambda = n \pm i\omega$$

Pair of terms

$$a_1 \exp(n + i\omega t) + a_2 \exp(n - i\omega t)$$

Euler's formula

$$\exp(ix) = \cos x + i \sin x$$

$$\exp(nt)\left(A_1 \cos \omega t + A_2 \sin \omega t\right)$$

$$A_1 = a_1 + a_2$$

$$A_2 = (a_1 - a_2)i$$

Oscillatory mode of period $T = 2\pi/\omega$

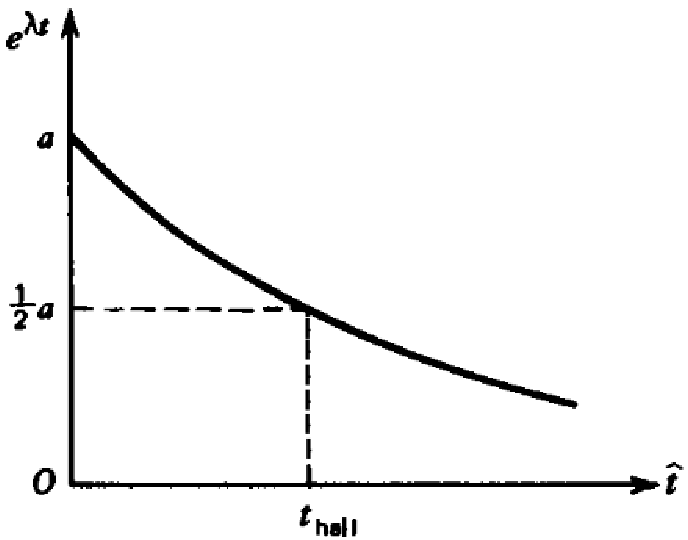
Decays depends on $\operatorname{sign}(n)$

Eigenvalues interpretation

Eigenvalues $\lambda = n \pm i\omega$ $\qquad$ $\exp(nt)\,(A_1 \cos \omega t + A_2 \sin \omega t)$

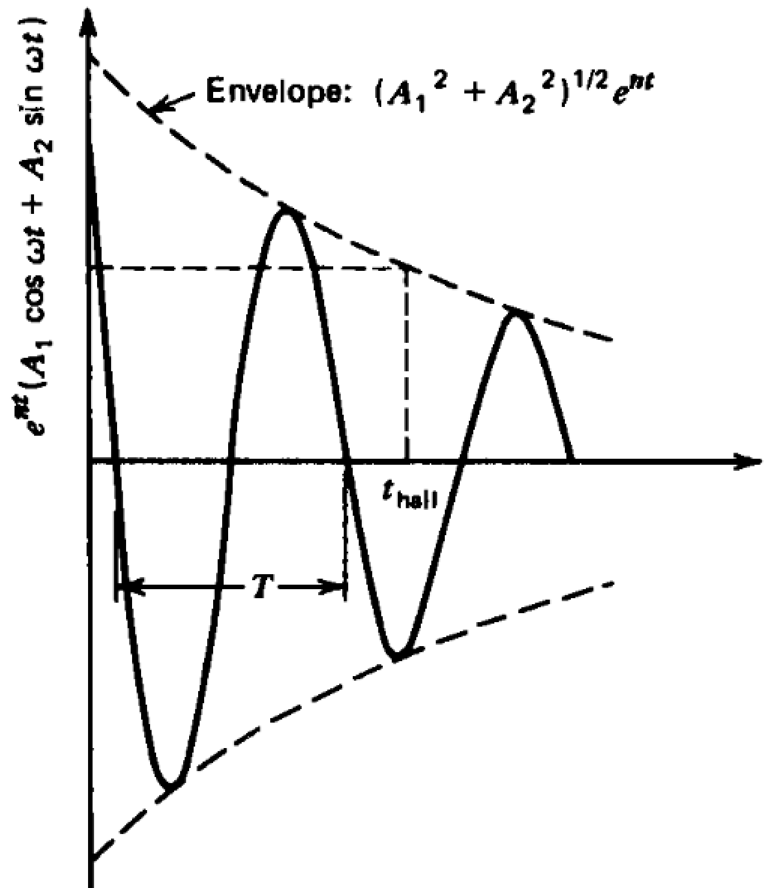
λ real

$\lambda = n < 0$



λ complex

$\lambda = n \pm i\omega, \quad n < 0$

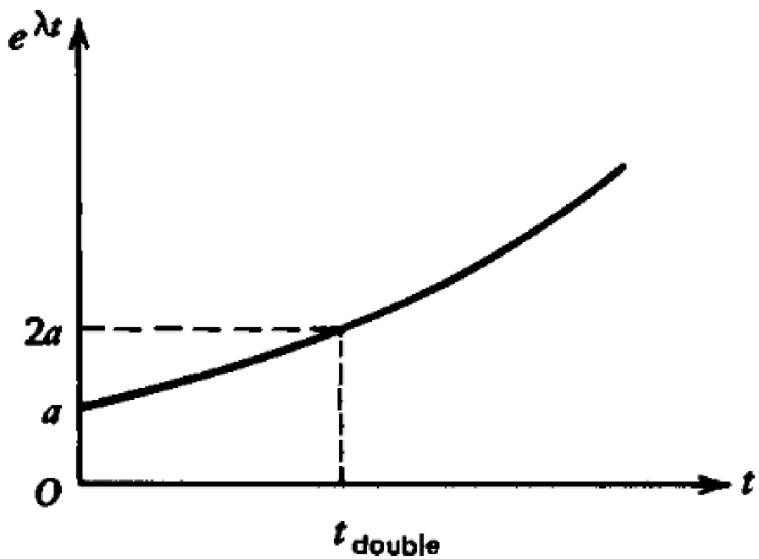


Eigenvalues interpretation

Eigenvalues $\lambda = n \pm i\omega$ $\qquad \exp(nt)\,(A_1 \cos\omega t + A_2 \sin\omega t)$

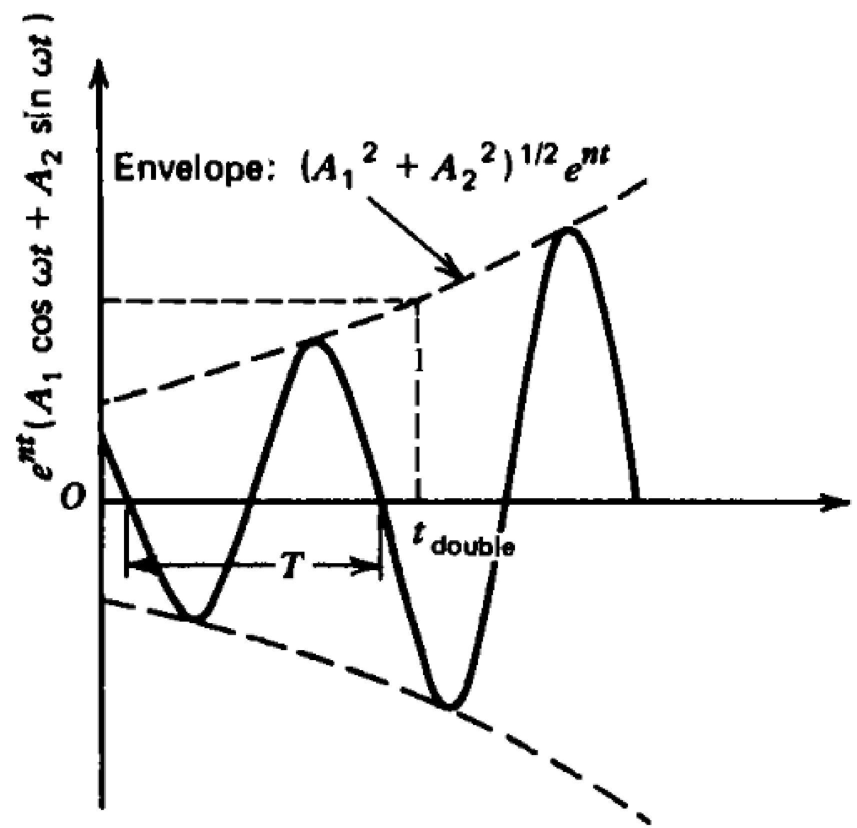
λ real

$\lambda = n > 0$

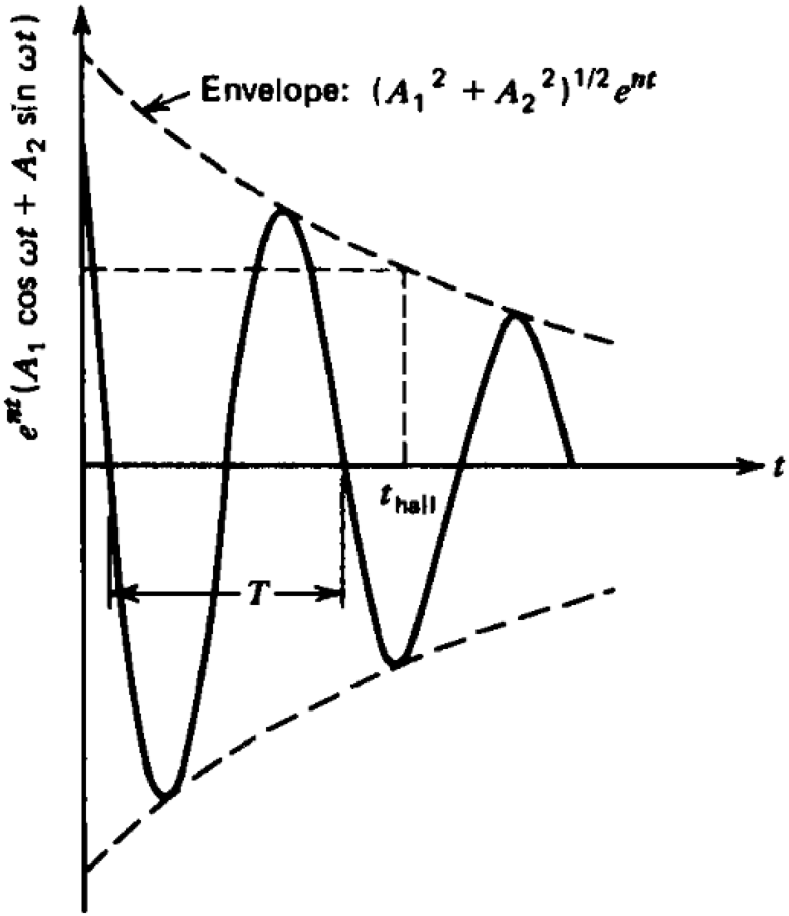


λ complex

$\lambda = n \pm i\omega, \quad n > 0$



Eigenvalues interpretation



Important parameters

- Period
- Time to double of time to half
- Cycles to double or cycles to half

Until now, we cannot infer
about aircraft handling qualities

Eigenvalues interpretation

$$\omega_n = \sqrt{\omega^2 + n^2}$$ "undamped" circular frequency

$$\zeta = -\frac{n}{\omega_n}$$ damping ratio

$$\exp(nt)\left(A_1 \cos \omega t + A_2 \sin \omega t\right)$$

Time to double or half

$$t_{double} \text{ or } t_{half} = \frac{0.693}{|n|} = \frac{0.693}{|\zeta|\omega_n}$$

Cycles to double or half

$$N_{double} \text{ or } N_{half} = 0.110\frac{\omega}{|n|} = 0.110\frac{\sqrt{1-\zeta^2}}{|\zeta|}$$

Logarithmic decrement (log ratio of successive peaks)

$$\delta = \ln\frac{\exp(nt)}{\exp\left[n(t+T)\right]} = -nT = 2\pi\frac{\zeta}{\sqrt{1-\zeta^2}}$$

Longitudinal modes of an aircraft

Boeing 747 at Mach number 0.8 (40000 ft) cruising in horizontal flight

$W = 636{,}636$ lb (2.83176×10^6 N)

$S = 5500$ ft^2 (511.0 m^2)

$\bar{c} = 27.31$ ft (8.324 m)

$b = 195.7$ ft (59.64 m)

$I_x = 0.183 \times 10^8$ slug ft^2 (0.247×10^8 kg m^2)

$I_y = 0.331 \times 10^8$ slug ft^2 (0.449×10^8 kg m^2)

$I_z = 0.497 \times 10^8$ slug ft^2 (0.673×10^8 kg m^2)

$I_{zx} = -.156 \times 10^7$ slug ft^2 ($-.212 \times 10^7$ kg m^2)

$u_0 = 774$ fps (235.9 m/s) $\quad \theta_0 = 0$

$\rho = 0.0005909$ slug/ft^3 (0.3045 kg/m^3)

$C_{L_0} = 0.654$

$C_{D_0} = 0.$

Dimensional Derivatives—B747 Airplane

	X (lb)	Z (lb)	M (ft·lb)
u(ft/s)	-1.358×10^2	-1.778×10^3	3.581×10^3
w(ft/s)	2.758×10^2	-6.188×10^3	-3.515×10^4
q(rad/s)	0	-1.017×10^5	-1.122×10^7
$\dot{w}$(ft/s^2)	0	1.308×10^2	-3.826×10^3

Longitudinal modes of an aircraft

Boeing 747 – Longitudinal matrix

$$\mathbf{A} = \begin{bmatrix} -0.006868 & 0.01395 & 0 & -32.2 \\ -0.09055 & -0.3151 & 733.98 & 0 \\ 0.0001187 & -0.001026 & -0.4285 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Computing eigenvalues in MATLAB

```
[a,b] = eig(A)
```

Mode	Name	Period (s)	t_{half} (s)	N_{half} (cycles)
1	Phugoid	93.4	211	22.5
2	Short-period	7.08	1.86	0.26

Phugoid mode was first described by Lanchester in 1908

Longitudinal modes of an aircraft

Boeing 747 – Longitudinal matrix

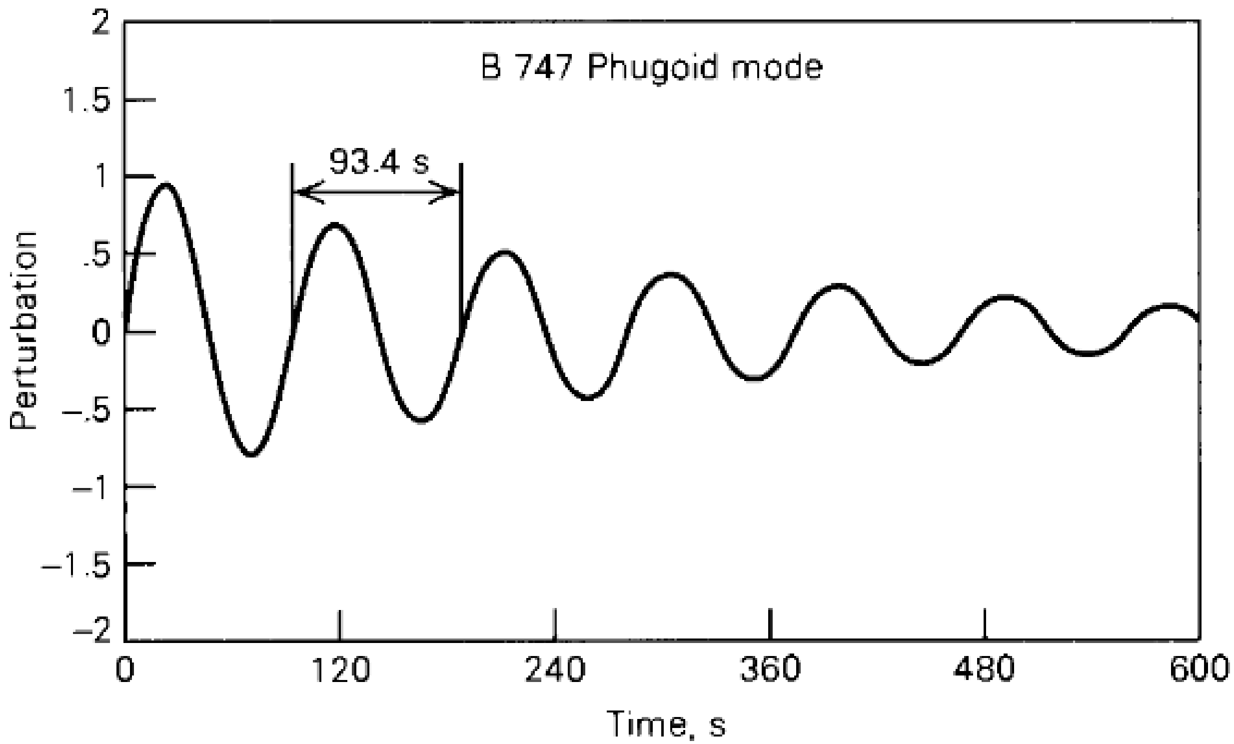
	Phugoid		*Short-Period*	
	Magnitude	*Phase*	*Magnitude*	*Phase*
$\Delta\hat{u}$	0.62	92.4°	0.029	57.4°
$\alpha = \hat{w}$	0.036	82.8°	1.08	19.2°
$\hat{q}$	0.0012	92.8°	0.017	112.7°
$\Delta\theta$	1.0	0°	1.0	0°

Mode	Name	Period (s)	t_{half} (s)	N_{half} (cycles)
1	Phugoid	93.4	211	22.5
2	Short-period	7.08	1.86	0.26

Phugoid mode was first described by Lanchester in 1908

Longitudinal modes of an aircraft

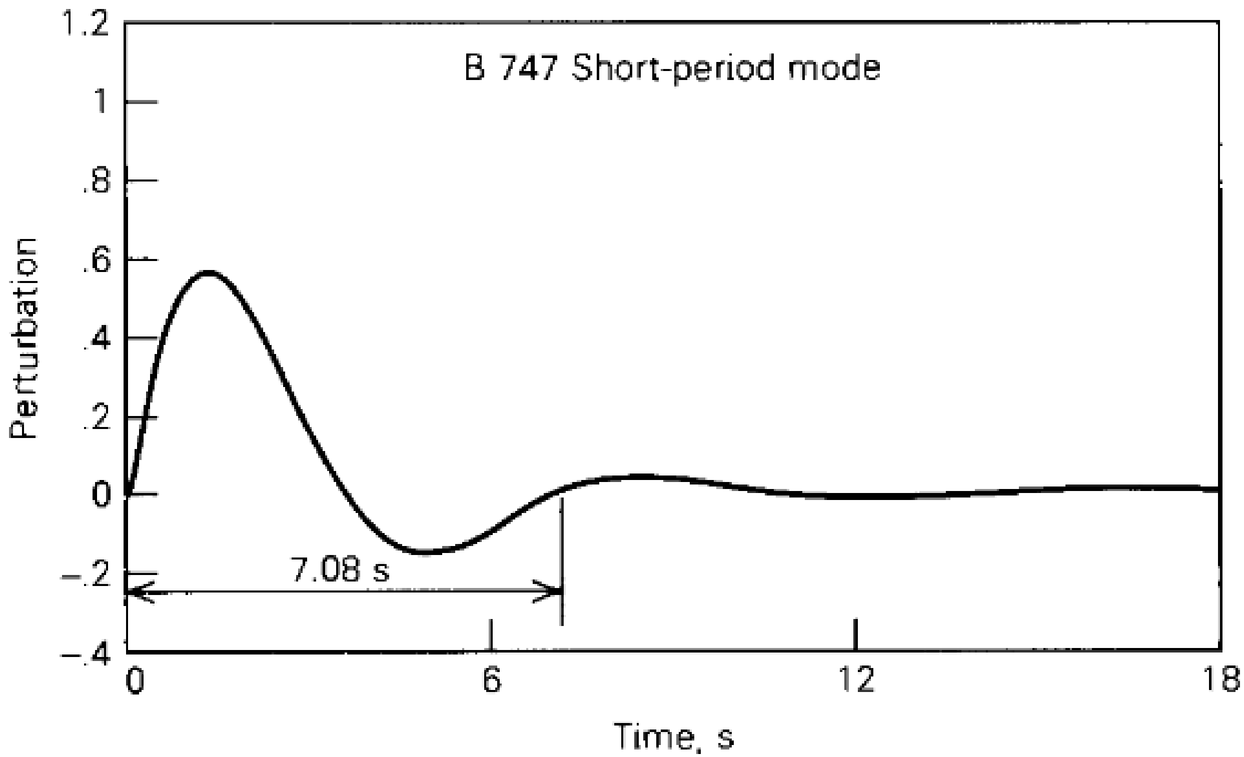
Boeing 747 – Phugoid motion



Mode	Name	Period (s)	t_{half} (s)	N_{half} (cycles)
1	**Phugoid**	**93.4**	**211**	**22.5**
2	Short-period	7.08	1.86	0.26

Longitudinal modes of an aircraft

Boeing 747 – Short period



Mode	Name	Period (s)	t_{half} (s)	N_{half} (cycles)
1	Phugoid	93.4	211	22.5
2	**Short-period**	**7.08**	**1.86**	**0.26**

Reduced order models

Short mode approximation $\quad \mathbf{x} = \{\cancelto{0}{\Delta u} \quad w \quad q \quad \Delta\theta\}^T$

$$\mathbf{A} = \begin{bmatrix} \dfrac{X_u}{m} & \dfrac{X_w}{m} & 0 & -g\cos(\theta_0) \\ \dfrac{Z_u}{m - Z_{\dot{w}}} & \boxed{\dfrac{Z_w}{m - Z_{\dot{w}}}} & \boxed{\dfrac{Z_q + mu_0}{m - Z_{\dot{w}}}} & \dfrac{-mg\sin(\theta_0)}{m - Z_{\dot{w}}} \\ \dfrac{1}{I_y}\left(M_u + \dfrac{M_{\dot{w}} Z_u}{m - Z_{\dot{w}}}\right) & \boxed{\dfrac{1}{I_y}\left(M_w + \dfrac{M_{\dot{w}} Z_w}{m - Z_{\dot{w}}}\right)} & \boxed{\dfrac{1}{I_y}\left[M_q + \dfrac{M_{\dot{w}}(Z_q + mu_0)}{m - Z_{\dot{w}}}\right]} & \dfrac{M_{\dot{w}} mg\sin(\theta)}{I_y(m - Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Rewriting...

$$\mathbf{A_{sp}} = \begin{bmatrix} \dfrac{Z_w}{m - Z_{\dot{w}}} & \dfrac{Z_q + mu_0}{m - Z_{\dot{w}}} \\ \dfrac{1}{I_y}\left(M_w + \dfrac{M_{\dot{w}} Z_w}{m - Z_{\dot{w}}}\right) & \dfrac{1}{I_y}\left[M_q + \dfrac{M_{\dot{w}}(Z_q + mu_0)}{m - Z_{\dot{w}}}\right] \end{bmatrix}$$

Reduced order models

$$\mathbf{x_{sp}} = \begin{Bmatrix} w & q \end{Bmatrix}^T$$

$$\mathbf{A_{sp}} = \begin{bmatrix} \dfrac{Z_w}{m - Z_{\dot{w}}} & \dfrac{Z_q + mu_0}{m - Z_{\dot{w}}} \\ \dfrac{1}{I_y}\left(M_w + \dfrac{M_{\dot{w}} Z_w}{m - Z_{\dot{w}}}\right) & \dfrac{1}{I_y}\left[M_q + \dfrac{M_{\dot{w}}(Z_q + mu_0)}{m - Z_{\dot{w}}}\right] \end{bmatrix}$$

$$Z_{\dot{w}} << m, \quad Z_q << mu_0$$

$$\mathbf{A_{sp}} = \begin{bmatrix} \dfrac{Z_w}{m} & u_0 \\ \dfrac{1}{I_y}\left(M_w + \dfrac{M_{\dot{w}} Z_w}{m}\right) & \dfrac{1}{I_y}\left(M_q + M_{\dot{w}} u_0\right) \end{bmatrix}$$

For Boeing 747:

$$\lambda = -0.371 \pm 0.889$$

Reduced order models

Phugoid mode approximation

$$\mathbf{x} = \left\{\Delta u \quad w \quad q \quad \Delta\theta\right\}^T$$

$$\mathbf{A} = \begin{bmatrix} \dfrac{X_u}{m} & \dfrac{X_w}{m} & 0 & -g\cos(\theta_0) \\ \dfrac{Z_u}{m - Z_{\dot{w}}} & \dfrac{Z_w}{m - Z_{\dot{w}}} & \dfrac{Z_q + mu_0}{m - Z_{\dot{w}}} & \dfrac{-mg\sin(\theta_0)}{m - Z_{\dot{w}}} \\ \dfrac{1}{I_y}\left(M_u + \dfrac{M_{\dot{w}} Z_u}{m - Z_{\dot{w}}}\right) & \dfrac{1}{I_y}\left(M_w + \dfrac{M_{\dot{w}} Z_w}{m - Z_{\dot{w}}}\right) & \dfrac{1}{I_y}\left[M_q + \dfrac{M_{\dot{w}}(Z_q + mu_0)}{m - Z_{\dot{w}}}\right] & \dfrac{M_{\dot{w}} mg\sin(\theta)}{I_y(m - Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{x} = \begin{Bmatrix} \Delta\dot{u} \\ \dot{w} \\ \cancelto{0}{\dot{q}} \\ \Delta\dot{\theta} \end{Bmatrix}$$

$$q \approx 0 \qquad \theta_0 = 0 \qquad Z_q,\ Z_w \approx 0$$

$$\Delta M = M_u + M_w w + \cancelto{0}{M_{\dot{w}}\dot{w}} + \cancelto{0}{M_q q} + \cancelto{0}{\Delta M_c}$$

Reduced order models

Phugoid mode approximation

$$\mathbf{x} = \left\{\Delta u \quad w \quad q \quad \Delta\theta\right\}^T$$

$$\mathbf{A} = \begin{bmatrix} \dfrac{X_u}{m} & \dfrac{X_w}{m} & 0 & -g\cos(\theta_0) \\ \dfrac{Z_u}{m - Z_{\dot{w}}} & \dfrac{Z_w}{m - Z_{\dot{w}}} & \dfrac{Z_q + m u_0}{m - Z_{\dot{w}}} & \dfrac{-mg\sin(\theta_0)}{m - Z_{\dot{w}}} \\ \dfrac{1}{I_y}\left(M_u + \dfrac{M_{\dot{w}} Z_u}{m - Z_{\dot{w}}}\right) & \dfrac{1}{I_y}\left(M_w + \dfrac{M_{\dot{w}} Z_w}{m - Z_{\dot{w}}}\right) & \dfrac{1}{I_y}\left[M_q + \dfrac{M_{\dot{w}}(Z_q + m u_0)}{m - Z_{\dot{w}}}\right] & \dfrac{M_{\dot{w}} m g \sin(\theta)}{I_y(m - Z_{\dot{w}})} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\dot{\mathbf{x}} = \begin{Bmatrix} \Delta\dot{u} \\ \dot{w} \\ \cancelto{0}{\dot{q}} \\ \Delta\dot{\theta} \end{Bmatrix}$$

$$q \approx 0 \qquad \theta_0 = 0 \qquad Z_q,\, Z_w \approx 0$$

$$\Delta M = M_u + M_w w + \cancelto{0}{M_{\dot{w}}\dot{w}} + \cancelto{0}{M_q q} + \cancelto{0}{\Delta M_c}$$

Reduced order models

Phugoid mode approximation

$$q \approx 0 \qquad \theta_0 = 0 \qquad Z_q,\, Z_w \approx 0$$

$$\Delta M = M_u + M_w w + \cancelto{0}{M_{\dot{w}} \dot{w}} + \cancelto{0}{M_q q} + \cancelto{0}{\Delta M_c}$$

$$\hat{\mathbf{B}}\dot{\mathbf{x}} = \hat{\mathbf{A}}\mathbf{x}$$

$$\dot{\mathbf{x}} = \begin{Bmatrix} \Delta \dot{u} \\ \dot{w} \\ \cancelto{0}{\dot{q}} \\ \Delta \dot{\theta} \end{Bmatrix} \qquad \hat{\mathbf{A}} = \begin{bmatrix} \dfrac{X_u}{m} & \dfrac{X_w}{m} & 0 & -g \\ \dfrac{Z_u}{m} & \dfrac{Z_w}{m} & u_0 & 0 \\ M_u & M_w & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad \hat{\mathbf{B}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\hat{\mathbf{B}}\dot{\mathbf{x}} = \hat{\mathbf{A}}\mathbf{x} \rightarrow \left(\hat{\mathbf{B}}\lambda - \hat{\mathbf{A}}\right)\mathbf{x} = \mathbf{0} \qquad \det\left(\hat{\mathbf{B}}\lambda - \hat{\mathbf{A}}\right) = 0$$

Reduced order models

Phugoid mode approximation

$$\hat{\mathbf{B}}\dot{\mathbf{x}} = \hat{\mathbf{A}}\mathbf{x} \rightarrow \left(\hat{\mathbf{B}}\lambda - \hat{\mathbf{A}}\right)\mathbf{x} = \mathbf{0} \qquad \det\left(\hat{\mathbf{B}}\lambda - \hat{\mathbf{A}}\right) = 0$$

Solving the equation:

$$\omega_n^2 = -\frac{g}{mu_0}\left(Z_u - \frac{M_u Z_w}{M_w}\right)$$

$$\zeta = -\frac{1}{2}\left[\frac{g}{mu_0}\left(\frac{M_u}{M_w}Z_w - Z_u\right)\right]^{-1/2}\left[\frac{g}{u_0}\frac{M_u}{M_w} + \frac{1}{m}\left(X_u - \frac{M_u}{M_w}X_w\right)\right]$$

Exercises

$$\mathbf{A} = \begin{bmatrix} X_{tu} + X_u & \dfrac{X_\alpha}{U_1} & 0 & -g\cos(\theta_1) \\ \dfrac{U_1 Z_u}{U1 - Z_{\dot{\alpha}}} & \dfrac{Z_a}{U1 - Z_{\dot{\alpha}}} & \dfrac{U_1^2}{(U_1 - Z_{\dot{\alpha}})} + \dfrac{U_1 Z_q}{U_1 - Z_{\dot{\alpha}}} & -\dfrac{U_1 g \sin(\theta_1)}{U_1 - Z_{\dot{\alpha}}} \\ M_{tu} + M_u + \dfrac{M_{\dot{\alpha}} Zu}{U_1 - Z_{\dot{\alpha}}} & \dfrac{M_a + M_{t\alpha}}{U_1} + \dfrac{M_{\dot{\alpha}} Z_\alpha}{U_1(U_1 - Z_{\dot{\alpha}})} & M_q + \dfrac{M_{\dot{\alpha}} U_1}{U_1 - Z_{\dot{\alpha}}} + \dfrac{M_{\dot{\alpha}} Z_q}{U_1 - Z_{\dot{\alpha}}} & -\dfrac{M_{\dot{\alpha}} g \sin(\theta_1)}{U_1 - Z_{\dot{\alpha}}} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Roskam notation

Airplane A

Airplane B

Tasks

- Compute
  - the eigenvalues
  - Period
  - Time to double of time to half
  - Cycles to double or cycles to half
- Repeat the tasks using a reduced model for short period

Exercises

Airplane A – Cessna 182

Airplane B – Cessna T-37 A

University of São Paulo
São Carlos School of Engineering
Department of Aeronautical Engineering

Lateral motion

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Ricardo Afonso Angélico
ricardoangelico@gmail.com

Control fixed problems

$$\dot{\mathbf{x}}_{\mathbf{LD}} = \mathbf{A}_{\mathbf{LD}}\mathbf{x}_{\mathbf{LD}} + \mathbf{B}_{\mathbf{LD}}\boldsymbol{u}_{\boldsymbol{LD}}$$

No control forces

$$\boldsymbol{u}_{\boldsymbol{LD}} = \mathbf{0}^T$$

First-order differential equation

$$\dot{\mathbf{x}}_{\mathbf{LD}} = \mathbf{A}_{\mathbf{LD}}\,\mathbf{x}_{\mathbf{LD}}$$

Solution

$$\mathbf{x} = \mathbf{x}_0 \exp(\lambda t)$$

Eigenvalues are given by:

$$\det(\mathbf{A}_{\mathbf{LD}} - \lambda \mathbf{I}) = 0$$

Solution: $$\mathbf{x}(t) = \sum_{i=1}^{N} \mathbf{x}_{0i} \exp(\lambda_i t)$$

Lateral motion eigenvalues

Eigenvalues expected:

- Two reals eigenvalues corresponding to spiral mode and rolling convergence
- One complex conjugate pair corresponding to dutch-roll mode

For real eigenvalues, a characteristic time can be computed:

$$T = -\frac{1}{\lambda}$$

λ: real eigenvalues

T_s : characteristic time for spiral mode

T_r : characteristic time for rolling convergence

Lateral motion equations

$$\mathbf{A_{LD}} = \begin{bmatrix} \dfrac{Y_v}{m} & \dfrac{Y_p}{m} & \left(\dfrac{Y_r}{m} - u_0\right) & g\cos\theta_0 \\ \left(\dfrac{L_v}{I'_x} + I'_{zx}N_v\right) & \left(\dfrac{L_p}{I'_x} + I'_{zx}N_p\right) & \left(\dfrac{L_r}{I'_x} + I'_{zx}N_r\right) & 0 \\ \left(I'_{xz}L_v + \dfrac{N_v}{I'_z}\right) & \left(I'_{xz}L_p + \dfrac{N_p}{I'_z}\right) & \left(I'_{xz}L_r + \dfrac{N_r}{I'_z}\right) & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix}$$

Main lateral modes:

- Spiral mode
- Rolling mode
- Dutch roll

$$\dot{\psi} = r\sec\theta_0$$

$$I'_x = (I_x I_z - I_{xz}^2)/I_z$$

$$I'_z = (I_x I_z - I_{xz}^2)/I_x$$

$$I'_{xz} = I_{zx}/(I_x I_z - I_{xz}^2)$$

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Transfer functions

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Ricardo Afonso Angélico
ricardoangelico@gmail.com

Transfer functions

State-space form:

$$\begin{aligned}\dot{\mathbf{x}}(t) &= \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}\,\mathbf{u}(t)\\ \mathbf{y}(t) &= \mathbf{C}\,\mathbf{x}(t) + \mathbf{D}\,\mathbf{u}(t)\end{aligned} \quad \xrightarrow{\mathcal{L}} \quad \begin{aligned}s\,\mathbf{x}(s) &= \mathbf{A}\,\mathbf{x}(s) + \mathbf{B}\,\mathbf{u}(s)\\ \mathbf{y}(s) &= \mathbf{C}\,\mathbf{x}(s) + \mathbf{D}\,\mathbf{u}(s)\end{aligned}$$

$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are matrices of constant coefficients

$$\mathbf{x}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B}\,\mathbf{u}(s)$$

$$\mathbf{y}(s) = \left[\mathbf{C}\,(s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B} + \mathbf{D}\right]\mathbf{u}(s) = \mathbf{G}(s)\,\mathbf{u}(s)$$

$\mathbf{G}(s)$ is the *Transfer Function Matrix*

$$\mathbf{G}(s) = \frac{1}{\Delta(s)}\mathbf{N}(s)$$

$\mathbf{N}(s)$ is the polynomial matrix of function numerators

Transfer functions

$$\mathbf{G}(s) = \frac{1}{\Delta(s)}\mathbf{N}(s) \qquad \mathbf{G}(s) = \mathbf{C}\,(s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B} + \mathbf{D}$$

Considering $\begin{cases} \mathbf{C} = \mathbf{I} \\ \mathbf{D} = \mathbf{0} \end{cases}$

$$\mathbf{G}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\,\mathbf{B} = \frac{\mathrm{Adj}(s\mathbf{I} - \mathbf{A})\,\mathbf{B}}{|s\mathbf{I} - \mathbf{A}|}$$

Numerator matrix $\mathbf{N}(s) = \mathrm{Adj}(s\mathbf{I} - \mathbf{A})\,\mathbf{B}$

Characteristic polynomial $\Delta(s) = |s\mathbf{I} - \mathbf{A}|$

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Pilot ratings and aircraft handling qualities

Flight Dynamics II

Ricardo Afonso Angélico
ricardoangelico@gmail.com

Introduction

Flight Dynamics	⇄	**System control and Control actuation systems**

- Provide some description of requirements on the aircraft dynamics
- VLA – Very light airplanes (JAR-VLA)
- FAR-23 (Wto < 5000 lb)
- FAR-25 (Wto > 5000 lb)
- MIL-F-8785C – military aircraft

Flight envelope

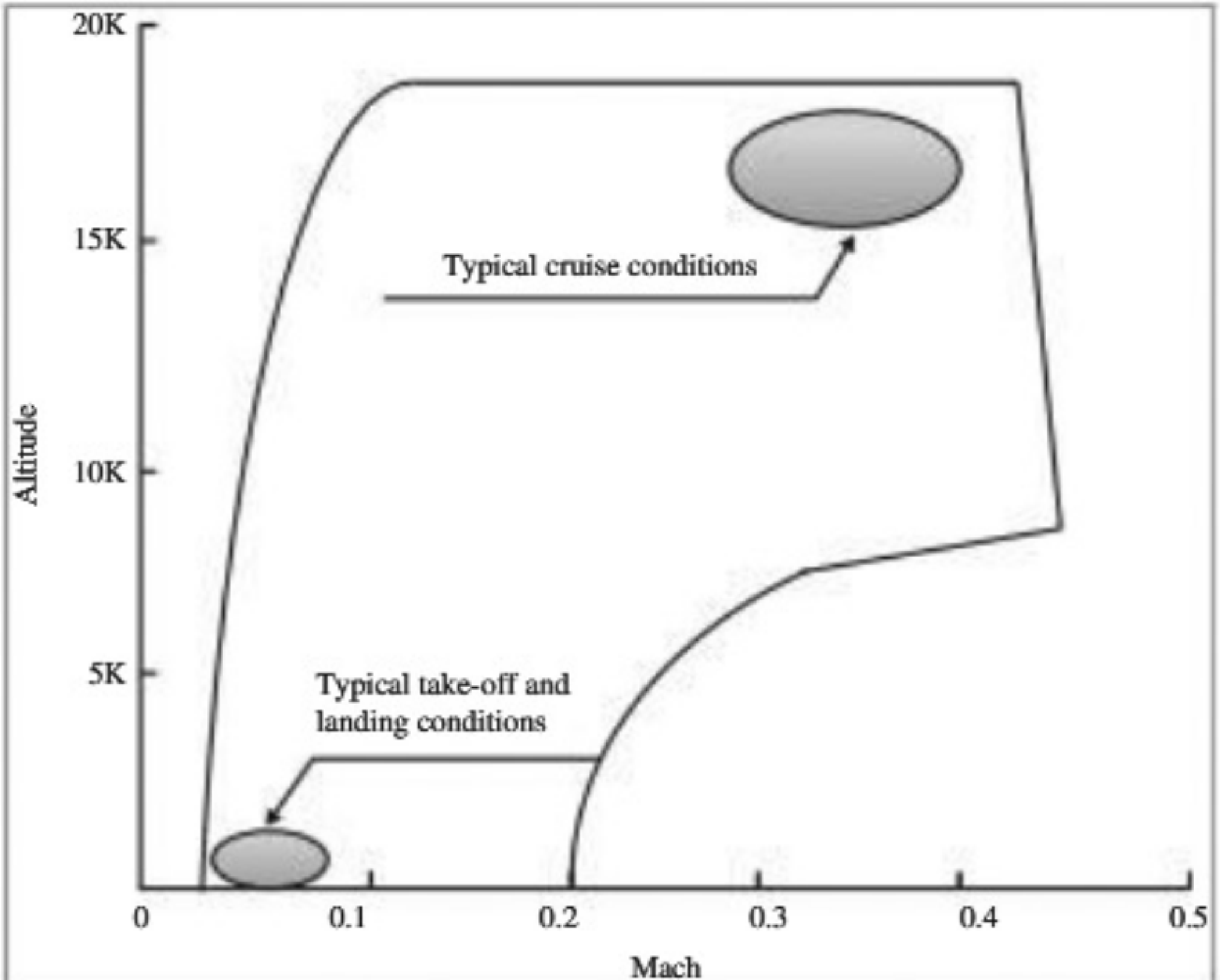


Figure 10.1 Typical Flight Envelope for a General Aviation and Regional Commuter Aircraft

Flight envelope

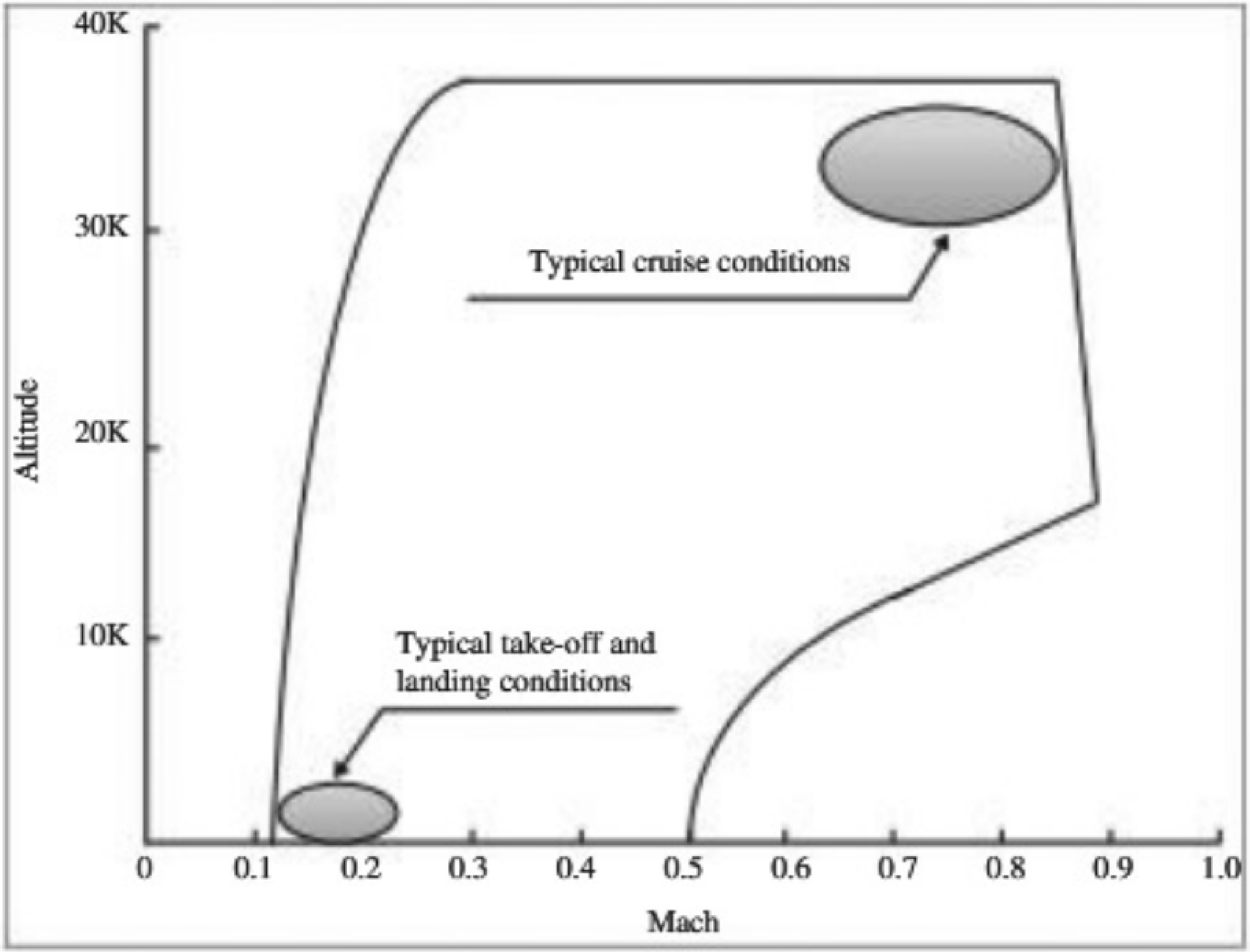


Figure 10.2 Typical Flight Envelope for a Commercial Jetliner

Flight envelope

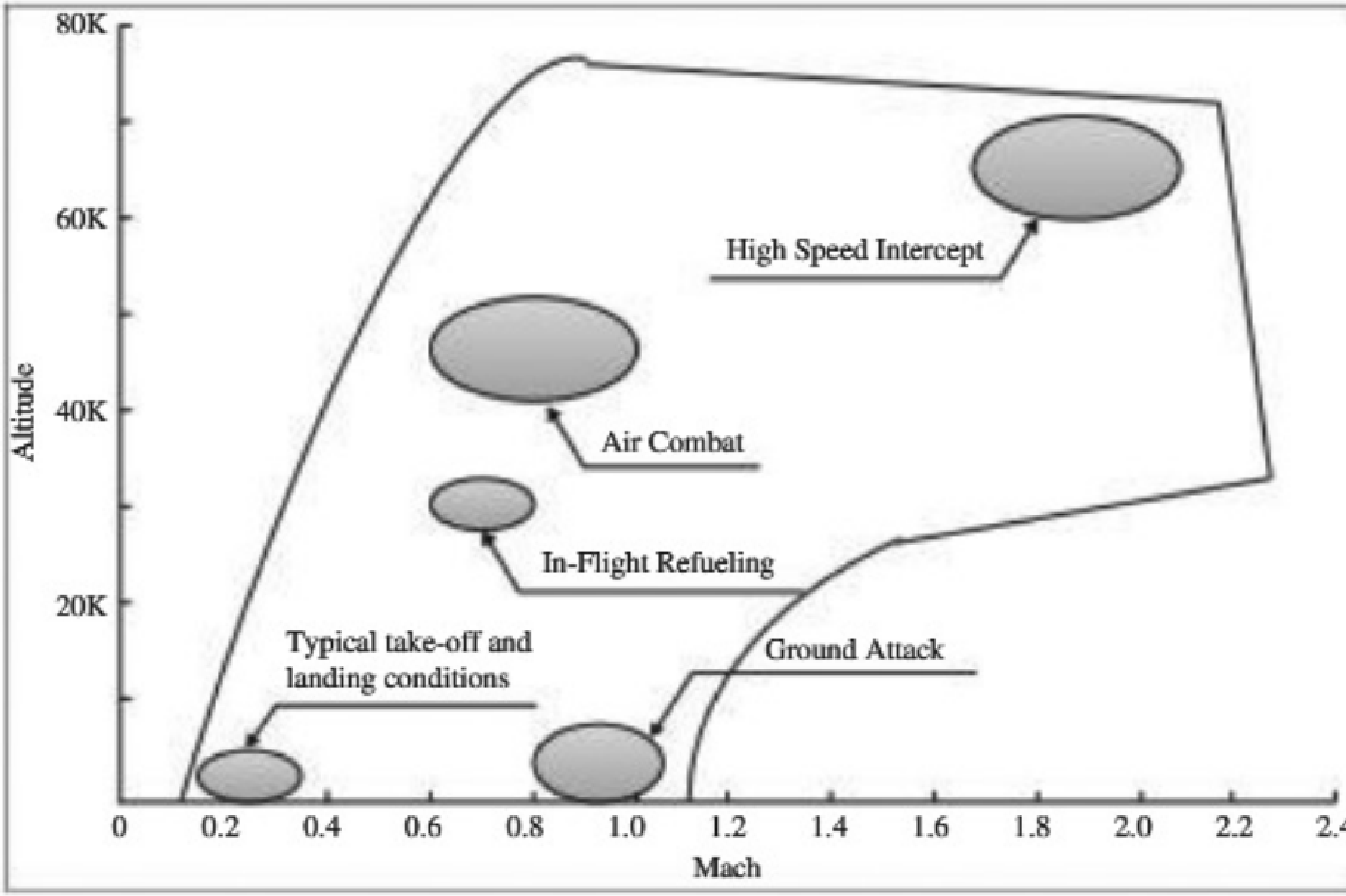


Figure 10.3 Typical Flight Envelope for a Fighter Aircraft

Flight envelope

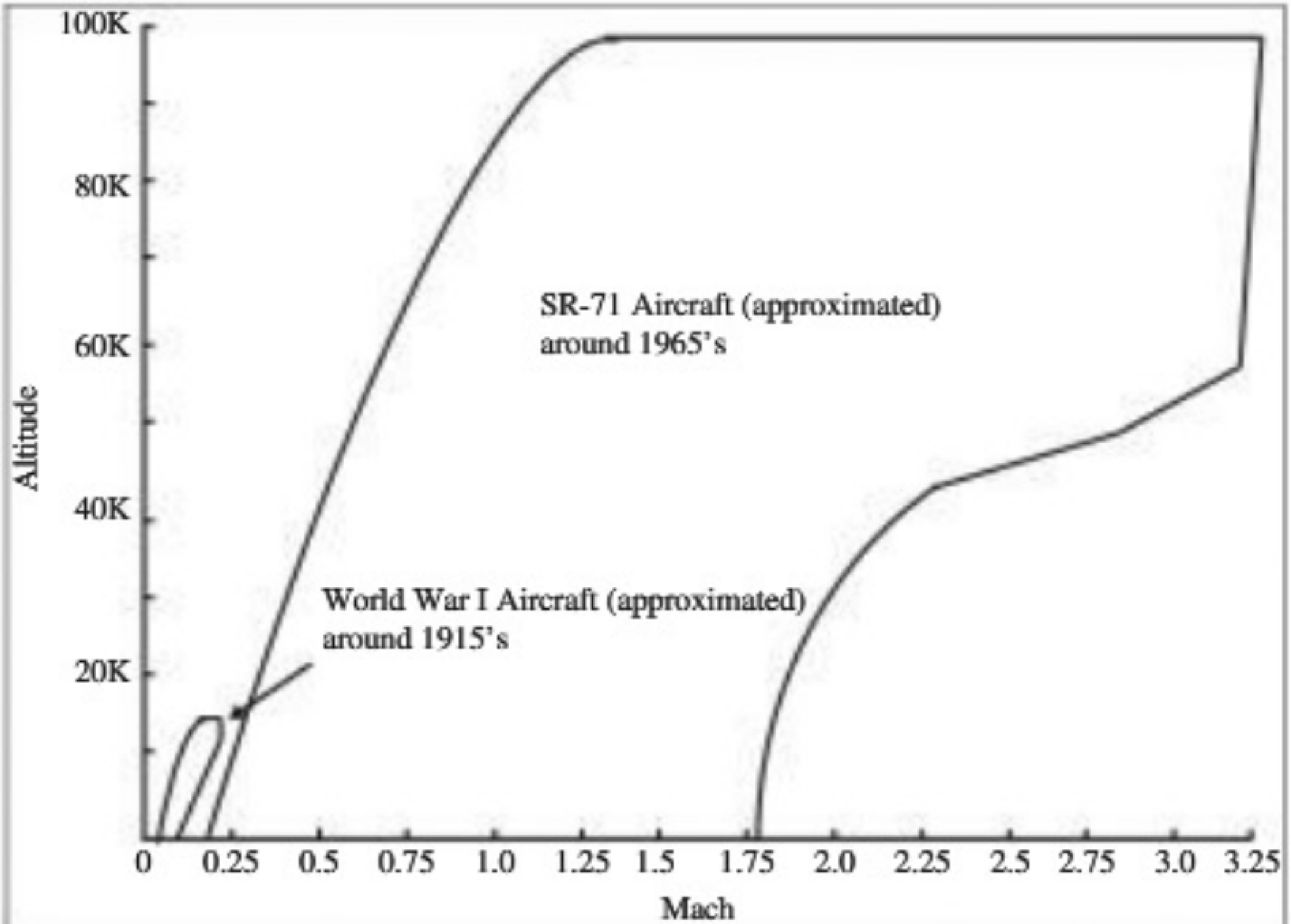


Figure 10.4 Flight Envelopes for World War I Fighter and SR-71 Aircraft

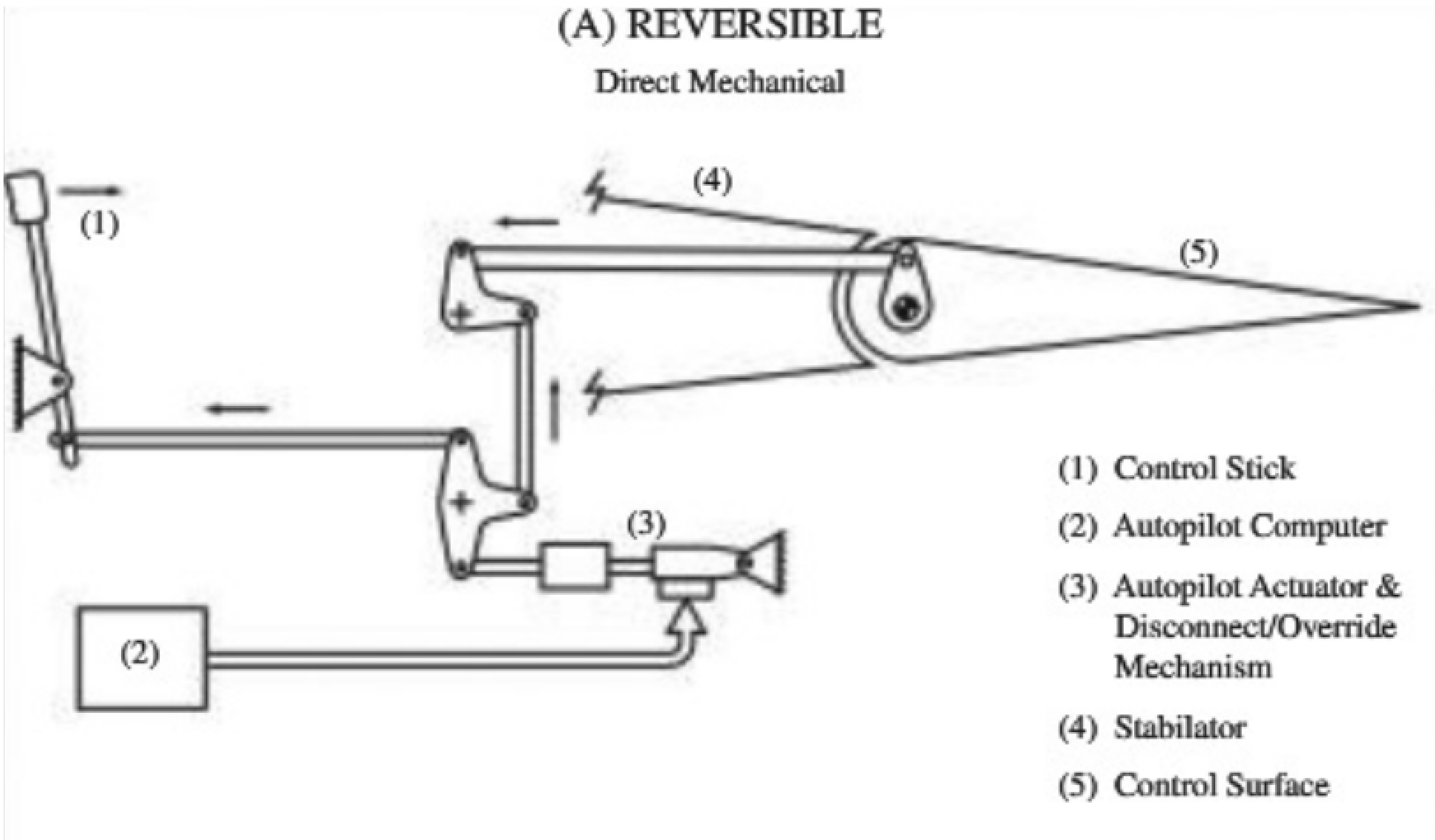
Levels of aircraft qualities

- Aircraft control authority
  - To maintain steady-state rectilinear flight (ex.: cruise conditions)
  - To maintain steady-state maneuvering (ex.: circular holding pattern)
  - To take-off and to climb safely
  - To conduct an approach and to land safely

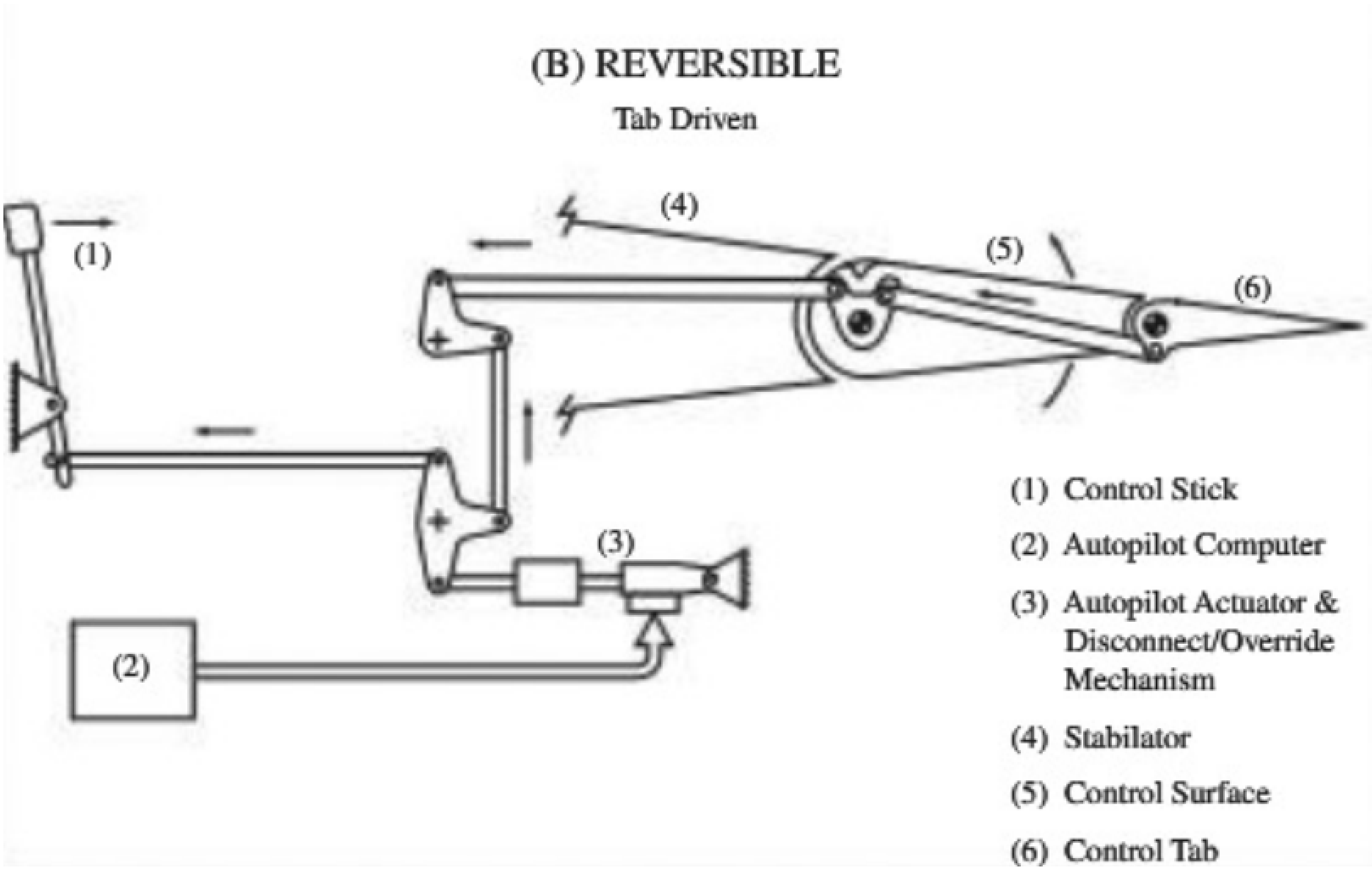
- Pilot compensation
  - Mental effort by the pilot while at the controls of aircraft

- Pilot workload
  - Maximum required stick force
  - Stick force per g
  - Stick force-speed gradient
  - Maximum aileron wheel force
  - Reversible and irreversible control systems

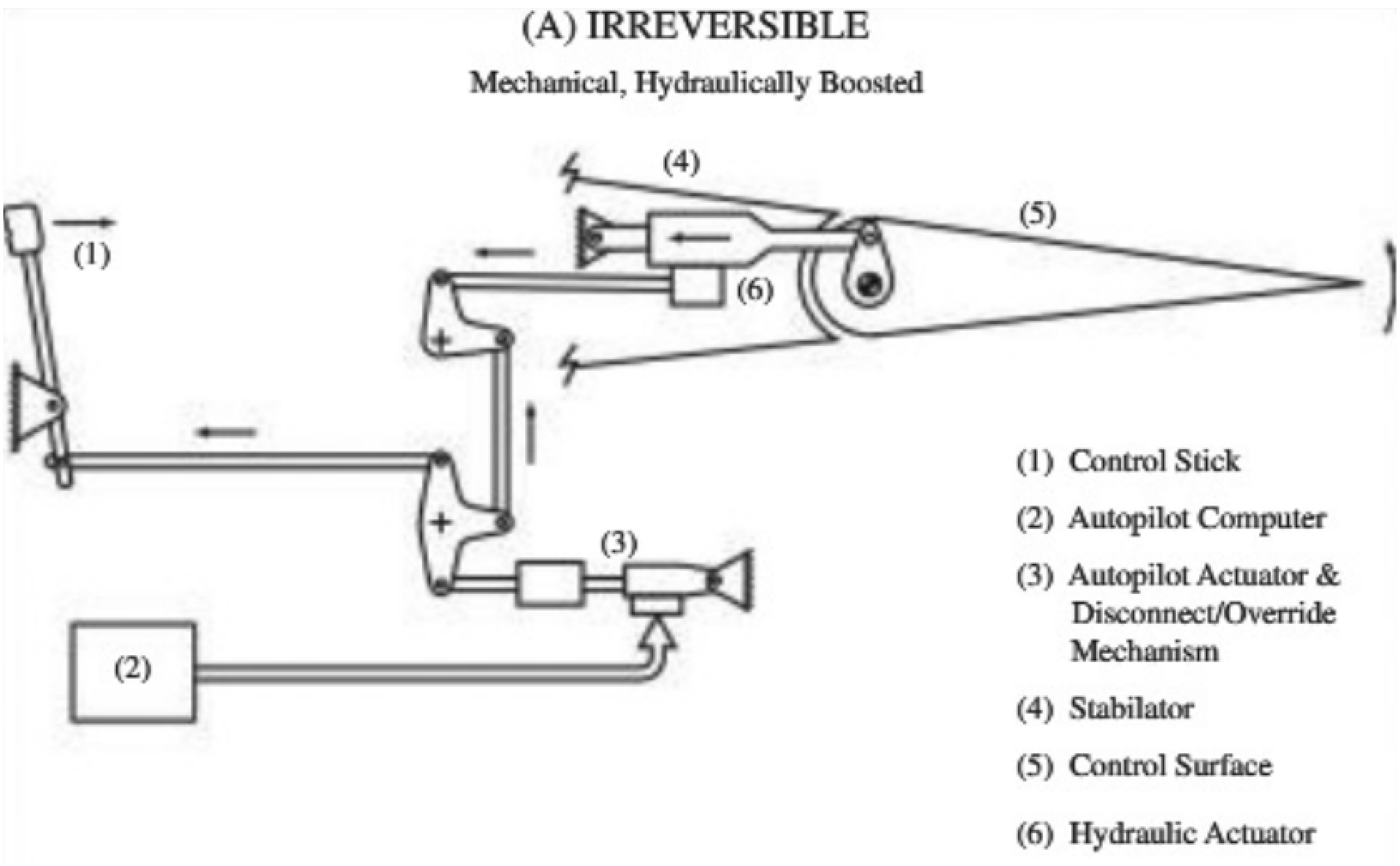
Reversible control system



Reversible control system



Irreversible control system



Levels of flying qualities

- Classification

Level I: Satisfactory and desirable for the given maneuver at the given flight conditions.

Level II: Adequate and sufficient for the given maneuver at the given flight conditions but with an increase in the workload and compensation by the pilot, leading to a mild deterioration of the mission's effectiveness.

Level III: Safe control and guidance of the aircraft but with a large increase in the workload and compensation by the pilot leading to a substantial drop in the mission's effectiveness.

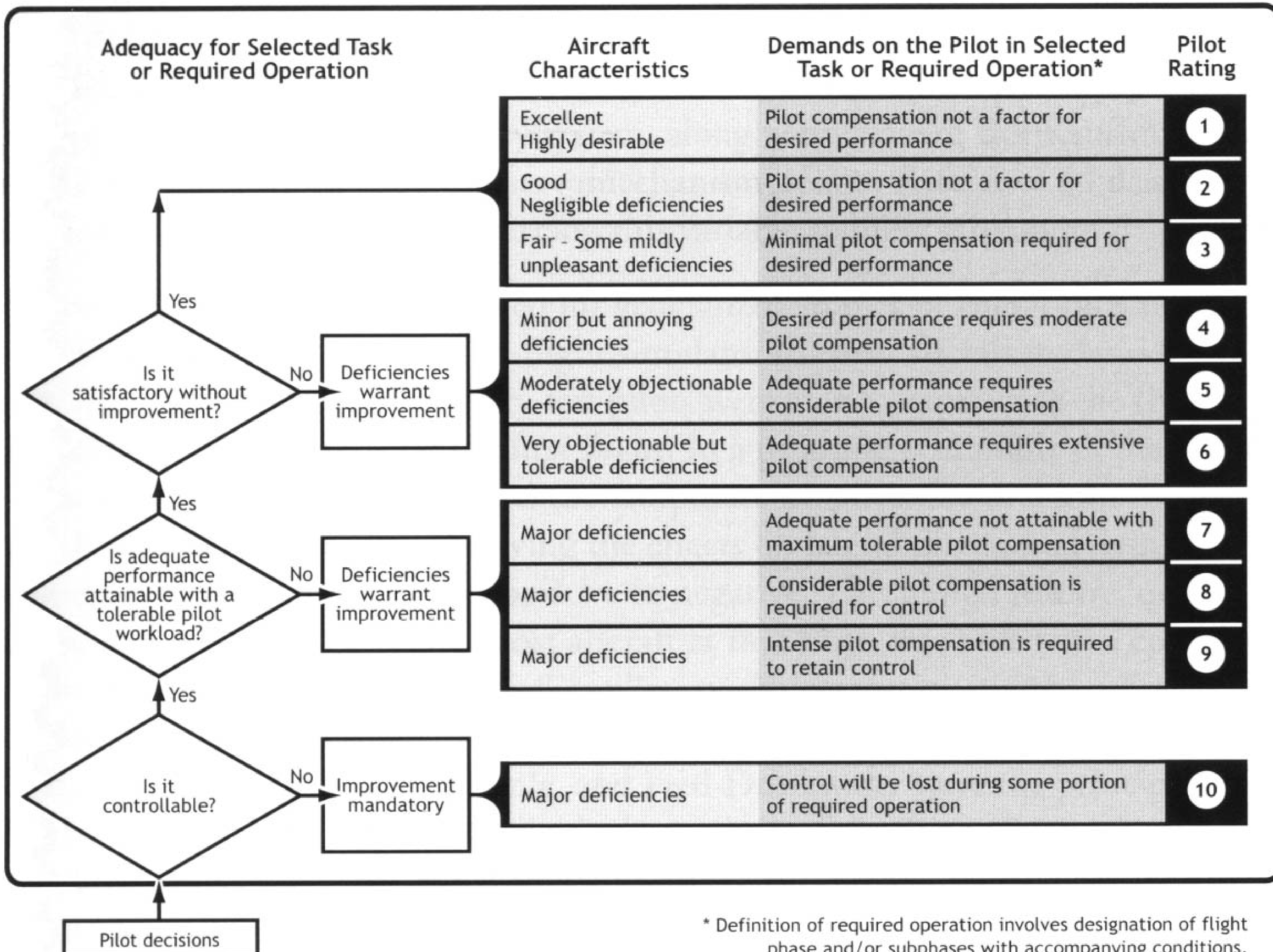
Table 10.1 Flying Quality Levels vs. Probability of Failure Occurrence

Probability of Failure Occurrence Within Operational Flight Envelope	Military Aircraft	Civilian Equivalent
Level II (following failure)	$< 10^{-2}$ per flight	$< 10^{-4}$ per flight
Level III (following failure)	$< 10^{-4}$ per flight	$< 10^{-6}$ per flight

- Sublevels

  - Sublevel #1, #2, and #3 for Level I;
  - Sublevel #4, #5, and #6 for Level II;
  - Sublevel #7, #8, and #9 for Level III.

Cooper-Harper Pilot Rating Scale (CHPRS)



Classes of aircraft

Classes of Military Aircraft (according to MIL-F-8785C)	Examples	Civilian Equivalent
Class I: Small & Light Airplanes.		
■ Light utility;	■ Cessna T-41;	■ Cessna 210;
■ Primary trainer;	■ Beech T-34C.	■ Piper Tomahawk.
■ Light observation and/or reconnaissance.		
Class II: Medium weight, low-to-medium maneuverability airplanes.		
■ Heavy utility/search and rescue;	■ Fairchild C-119;	■ Boeing 727;
■ Light or medium transport/cargo/tanker;	■ Grumman E-2C;	■ Boeing 737;
■ Early warning/electronic counter-measures/ airborne command, control or communications relay;	■ Boeing E-3A;	■ McDD DC-9;
	■ Lockheed S-3A;	■ McDD MD-80;
■ Anti-submarine;	■ Lockheed C-130;	■ Airbus A 320.
■ Assault transport;	■ Fairchild A-10;	
■ Reconnaissance;	■ Aeritalia G222;	
■ Tactical Bomber;	■ Douglas B-60;	
■ Heavy Attack;	■ Grumman A-6;	
■ Trainer for Class II aircraft.	■ Beech T-1A.	

Classes of aircraft

Class III: Large, heavy, low-to-medium maneuverability airplanes.

■ Heavy transport/cargo/tanker;	■ McDD C-17;	■ McDD MD-11;
■ Heavy bomber;	■ Boeing B-52H;	■ Boeing 747;
■ Patrol/early warning/electronic counter-measures/ airborne command, control or communications relay;	■ Lockheed P-3;	■ Boeing 777;
■ Trainer for Class III.	■ Lockheed C-5;	■ Airbus 330;
	■ Boeing E-3D;	■ Airbus 340;
	■ Boeing KC-135.	■ Airbus 380.

Class IV: High maneuverability airplanes.

■ Fighter/interceptor;	■ Lockheed F-22;	■ Pitts Special.
■ Attack;	■ McDD F-4;	
■ Tactical reconnaissance;	■ McDD F-15;	
■ Observation;	■ Lockheed SR-71;	
■ Trainer for Class IV aircraft.	■ Northrop T-38.	

Classification of aircraft maneuvers

- Category A

"Non-Terminal" Maneuvers

Category A: Non-terminal maneuvers requiring rapid execution, precision tracking, and/or precise flight path control.

- Air-to-air combat;
- Ground attack;
- Weapon delivery/launch;
- Aerial recovery;
- Reconnaissance;
- In-flight refueling (as receiver aircraft);
- Terrain following;
- Anti-submarine search;
- Close formation flying.

Classification of aircraft maneuvers

- Category B

Category B: Non-terminal maneuvers requiring slow and/or gradual maneuvers, moderately accurate flight path control without extreme precision tracking.

- Climb;
- Cruise;
- Loiter;
- In-flight refueling (as tanker);
- Descent;
- Emergency descent;
- Emergency deceleration;
- Aerial delivery.

- Category C

"Terminal" Maneuvers

Category C: Terminal maneuvers requiring fast or gradual maneuvers and very accurate flight path control.

- Takeoff;
- Carrier and/or catapult takeoff;
- Approach;
- Wave-off/go-around;
- Landing.

Flying qualities for longitudinal dynamics

- Longitudinal control forces
  - Control forces for steady-state flight condition
  - Control forces for maneuvered flight
  - Control forces for take-off and landing
  - Control forces during dives

- Control forces for steady-state flight condition

  - Deployment of landing gears;
  - Deployment of flaps;
  - Deployment of speed brakes;
  - Changing power settings;
  - One engine-out condition;

Force *(lbs)*	VLA	FAR 23	FAR 25	MIL-F-8785C
For **temporary** force application:				
■ Stick Controller;	45.0	60.0	No requirement	No requirement
■ Wheel Controller.	56.2	75.0	75.0	
For **prolonged** application:				
■ Stick and Wheel Controller.	4.5	10.0	10.0	No requirement

Flying qualities for longitudinal dynamics

- Control forces for maneuvered flight
  - Control force vs. load factor gradient

$$\frac{\partial F_s}{\partial n}$$

  - Civil aircraft

Table 10.5 Longitudinal Control Force Limits in Maneuvering Flight (Civilian Aircraft)[4–6]

Civilian Requirements		
VLA	FAR-23	FAR-25
$\partial F_S/\partial n > \frac{15.7}{n_{Limit}}$	For wheel controllers: $\partial F_S/\partial n > \frac{W_{TO}/140}{n_{Limit}}$ and $\frac{15}{n_{Limit}}$ but not more than $\frac{35}{n_{Limit}}$ For stick controllers: $\partial F_S/\partial n > W/140$	No requirement: The use MIL-F-8785C is recommended.

Flying qualities for longitudinal dynamics

- Control forces for maneuvered flight
  - Military aircraft

Military Requirements MIL-F-8785C	Minimum allowable Gradient $\partial F_S/\partial n$ (lbs/g)	Maximum Allowable Gradient $\partial F_S/\partial n$ (lbs/g)
Stick Controller		
Level I	higher of: $\frac{21}{(n_{Limit}-1)}$ and 3.0	$[240/(n/\alpha)]$ but not more than 28.0 nor less than $\frac{56}{(n_{Limit}-1)}$*
Level II	higher of: $\frac{18}{(n_{Limit}-1)}$ and 3.0	$[360/(n/\alpha)]$ but not more than 42.5 nor less than $\frac{85}{(n_{Limit}-1)}$*
Level III	higher of: $\frac{12}{(n_{Limit}-1)}$ and 2.0	56
*For $n_{Limit} < 3.0$, $\partial F_S/\partial n = 28$ for Level I, and $\partial F_S/\partial n = 42.5$ for Level II		
Wheel Controller		
Level I	higher of: $\frac{35}{(n_{Limit}-1)}$ and 6.0	$[500/(n/\alpha)]$ but not more than 120.0 nor less than $\frac{120}{(n_{Limit}-1)}$*
Level II	higher of: $\frac{30}{(n_{Limit}-1)}$ and 6.0	$[775/(n/\alpha)]$ but not more than 182.0 nor less than $\frac{182}{(n_{Limit}-1)}$*
Level III	5.0	240.0

Flying qualities for longitudinal dynamics

- Control forces for take-off and landing

MIL-F-8785C Airplane Class	Takeoff Pull (lbs)	Takeoff Push (lbs)	Airplane Class	Landing Pull only
Nose-wheel and bicycle-gear airplanes			All gear configurations	
Classes I, IV-C	20.0	10.0	Classes I, II-C	35.0
Classes II-C, IV-L	30.0	10.0	Classes II-L	50.0
Classes II-L, III	50.0	20.0		
Tail-wheel airplanes				
Classes I,II-C, IV	20.0	10.0		
Classes II-L, III	35.0	15.0		

- Control forces for dives

Class of Aircraft	Dives Pull (lbs)	Dives Push (lbs)
Stick controller. Class I, II, III, and IV (C and L)	10.0	50.0
Wheel controller. Class I, II, III, and IV (C and L)	75.0	10.0

'Requirements'

- Phugoid mode

Table 10.9 Requirements for the Phugoid Damping[6]

MIL-F-8785C	VAL, FAR 23 and FAR 25
Level I $\zeta_{Ph} \geq 0.04$	Level I No requirement
Level II $\zeta_{Ph} \geq 0$	Level II No requirement
Level III $T_{2_{Ph}} \geq 55$ sec.	Level III No requirement

- Short-period mode

Table 10.10 Requirements for the Short Period Damping (Military Aircraft)[1–3,6]

Category A & C Maneuvers	Category B Maneuvers
Level I $\zeta_{SP} > 0.35$	Level I $\zeta_{SP} > 0.35$
Level II $\zeta_{SP} > 0.25$	Level II $\zeta_{SP} > 0.2$
Level III $\zeta_{SP} > 0.15$	Level III $\zeta_{SP} > 0.15$

Flying qualities for lateral directional

- Lateral directional control forces
  - Control forces on rolling maneuvers
  - Control forces for holding an specific direction (heading) with asymmetric loading configurations
  - Control forces for holding an specific direction (heading) with bank angle with an engine-out condition

- Control forces on rolling maneuvers

Table 10.11 Maximum Forces for Roll Control for Military Aircraft[1−3,6]

Level	Airplane Class	Category of Maneuvers	Maximum Allowable Stick Force (lbs)	Maximum Allowable Wheel Force (lbs)
Level I	I, II-C, IV	A, B	20.0	40.0
		C	20.0	20.0
	II-L, III	A, B	25.0	50.0
		C	25.0	25.0
Level II	I, II-C, IV	A, B	30.0	60.0
		C	20.0	20.0
	II-L, III	A, B	30.0	60.0
		C	30.0	30.0
Level III	All	All	35.0	70.0

Flying qualities for lateral directional

- Control forces on rolling maneuvers

Table 10.12 Maximum Forces for Roll and Directional Control for Civilian Aircraft[4–6]

Force (lbs)	VLA	FAR 23	FAR 25
For TEMPORARY force application:			
■ Rudder Pedal;	90.0	150.0	150.0
■ Wheel Controller.	45.0	60.0	60.0
For PROLONGED force application:			
■ Rudder Pedal;	22.5	20.0	20.0
■ Wheel Controller.	3.5	5.0	5.0

- Control forces for holding heading with asymmetric loading

For military aircraft, according to the MIL-F-8785C, the maximum control forces allowed on the pedals for the directional control of an aircraft with asymmetric loading are by

- $<$ 100 lbs. for Level I and Level II flying qualities;
- $<$ 180 lbs. for Level III flying qualities.

Flying qualities for lateral directional

- Control forces on rolling maneuvers

Table 10.12 Maximum Forces for Roll and Directional Control for Civilian Aircraft[4–6]

Force (lbs)	VLA	FAR 23	FAR 25
For TEMPORARY force application:			
■ Rudder Pedal;	90.0	150.0	150.0
■ Wheel Controller.	45.0	60.0	60.0
For PROLONGED force application:			
■ Rudder Pedal;	22.5	20.0	20.0
■ Wheel Controller.	3.5	5.0	5.0

- Control forces for holding heading with asymmetric loading

For military aircraft, according to the MIL-F-8785C, the maximum control forces allowed on the pedals for the directional control of an aircraft with asymmetric loading are by

- $<$ 100 lbs. for Level I and Level II flying qualities;
- $<$ 180 lbs. for Level III flying qualities.

- Control forces for holding an specific direction (heading) with bank angle with an engine-out condition

'Requirements'

- Dutch-roll mode

Table 10.13 Dutch Roll Requirements (Civilian Aircraft)[4–6]

FAR 23, VLA	$\zeta_{DR} > 0.052$
FAR 25	$\zeta_{DR} > 0$

Table 10.14 Dutch Roll Damping and Natural Frequency Requirements (Military Aircraft)[1–3,6]

Level	Category of Maneuvers	Class of Aircraft	Minimum ζ_{DR}*	Minimum $\zeta_{DR}\,\omega_{n_{DR}}$*	Minimum $\omega_{n_{DR}}$*
Level I	A (Combat and Ground Attack)	IV	0.4	–	1.0
	A (All Others)	I, IV	0.19	0.35	1.0
		II, III	0.19	0.35	0.4
	B	All	0.08	0.15	0.4
	C	I, II-C , IV	0.08	0.15	1.0
		II-L, III	0.08	0.10	0.4
Level II	All	All	0.05	0.05	0.4
Level III	All	All	0	–	0.4

*In case of conflicting regulations, the ruling requirement is the one providing the largest value of ζ_{DR}.

References

- Etkin, B.; Reid, L. D. (1996) *Dynamics of Flight: Stability and Control.* John Wiley & Sons. 3rd ed.
- Roskam, J. (2001) *Airplane Flight Dynamics and Automatic Flight Controls.* DARcorporation.
- Davis, J. R.; Johnson, R.; Stepanek, J.; Fogarty, J. A. (2008) *Fundamental of Aerospace Medicine.* Lippincott Williams & Wilkins. 4th ed.
- Napolitano, M. R. (2011) *Aircraft Dynamics: from modeling to simulation.* John Wiley & Sons. Chapter 10 – Pilot rating and aircraft handling