Spatio-temporal Analysis and Modeling of Earth Surface Processes

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The Plan

Introduction to processes in space and time - Natural and Anthropogenic

Mathematical tools for measuring, describing and simulating processes

Discussion, Lunch & Break Time

Laboratory exercise & Project consultation

Representing Processes with Numbers

Sampling and Measurement

Accuracy - Degree of closeness of measurements of a quantity to the quantity's *true* value.

Precision - Degree to which repeated measurements under unchanged conditions show the same results.



Error - Difference between a computed, estimated or measured value and the *true* value.

Bias - Systematic difference between a measurement or statistical estimate and the quantity intended to be measured or estimated.

Measurement Bias Example; NYC temperature

What is the temperature in NYC?





Time & Space Specific







NYC air temperature 2010



Over one very uncomfortable day



Manhattan The Bronx Queens

NYCCAS Reference Stations 2009-2010







Surface Temperature (°F)



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Resolution

Spatial Resolution – How Small? Spectral Resolution – How Many Colors? Temporal Resolution – How Often?



Strain & Engle, 1992

In theory, resolution is determined by sampling frequency (or, equivalently, sample spacing).

The minimum sampling frequency necessary to resolve a feature is called the *Nyquist Frequency*.

The *Sampling Theorem* states that the minimum sampling frequency is twice the smallest frequency of interest.

Phase of sampling is critical also

Undersampling results in *aliasing*.

In practice, resolutions are determined by trade-offsbetween dwell time, IFOV and spectral bandwidth.



Spatial Aliasing

In the context of this class: *Periodic = Regular = Single Frequency* Periodic sampling of a periodic pattern produces a periodic result

- but the period may not be immediately recognizable.

When a periodic pattern is aliased constructive & destructive interference can produce a *moiré pattern*.

Most natural features in remotely sensed images do not form periodic patterns so most spatial aliasing effects will not be as obvious as moiré patterns.



wikipedia.org





wikipedia.org



Temporal Aliasing

Temporal aliasing is generally even more insidious than spatial aliasing - because temporal sampling by sensors on moving platforms is rarely periodic.

Any temporal change that is not purely *monotonic* can be aliased.

If the objective of your analysis is to identify & quantify the amplitude or frequency of temporal processes, temporal aliasing can be disastrous.

A graphic example of obvious temporal aliasing is the *Stroboscopic Effect* - sometimes referred to as the *Wagon Wheel Effect*.

Aliasing effects occur w/in the human visual system also. See: *http://www.michaelbach.de/ot/mot_strob/index.html*

Most pixels imaged on land are *Mixed Pixels*. Spectrally pure pixels are very rare.



Spatial resolution generally imposes the limiting constraint on feature recognition. Bandwidth & dynamic range further constrain spectral resolution.



Mitchell, 1992





Satellite views of upper Manhattan at different spatial and spectral resolutions. Moderate resolution Landsat imagery resolves seasonal to interannual variations in visible and infrared brightness over the past 30 years. High resolution commercial imagery provides meter scale imagery with revisit times of several days. Details at: www.LDEO.Columbia.edu/~small/Urban.html

Broadband and Hyperspectral Remote Sensing

- Hyperspectral sensors measure radiance at hundreds of intervals ~10 nm.
- Broadband sensors measure radiance in 3 to 10 bands > 100 nm.



• Limited spectral resolution constrains broadband sensors' ability to resolve subtle differences in surface reflectance.

• Limited spatial resolution results in spectral mixing w/in the sensor's field of view - *Mixed Pixels*.



Representing Processes with Numbers

Statistics and Estimates

Location - Sample Mean $\bar{x}_j = \frac{1}{N} \sum_{i=1}^{N} x_{ij}, \quad j = 1, ..., K.$ *alternative* - median $P(X \le m) \ge \frac{1}{2}$ and $P(X \ge m) \ge \frac{1}{2}$ *alternative* - mode = most common value in a data set.

Dispersion - Sample Variance $q_{jk} = \frac{1}{N-1} \sum_{i=1}^{N} (x_{ij} - \bar{x}_j) (x_{ik} - \bar{x}_k),$ alternative - Mean Absolute Deviation $\frac{1}{n} \sum_{i=1}^{n} |x_i - m(X)|.$

Example: for data = $\{2, 2, 3, 4, 14\}$ Mean Absolute Deviationmean = 5|2-5|+|2-5|+|3-5|+|4-5|+|14-5| = 3.6median = 3|2-3|+|2-3|+|3-3|+|4-3|+|14-3| = 2.8mode = 2|2-2|+|2-2|+|3-2|+|4-2|+|14-2| = 3.0

Distributions

A probability distribution assigns a probability to each outcome of an experiment or measurement of a sample.

Usually given in reference to the mean (μ) and standard deviation (σ).



The Gaussian (Normal) Distribution

Three Lognormal Distributions

