14.5 A REGRA DA CADEIA

Revisão técnica: Ricardo Miranda Martins – IMECC – Unicamp

1-8 Use a Regra da Cadeia para determinar dz/dt ou dw/dt.

1. $z = x^2 + y^2, \quad x = t^3, \quad y = 1 + t^2$

2. $z = x^2y^3, \quad x = 1 + \sqrt{t}, \quad y = 1 - \sqrt{t}$

3. $z = \ln(x + y^2), \quad x = \sqrt{1 + t}, \quad y = 1 + \sqrt{t}$

4. $z = xe^{x/y}, \quad x = \cos t, \quad y = e^{2t}$

5. $z = 6x^3 - 3xy + 2y^2, \quad x = e^t, \quad y = \cos t$

6. $z = x\sqrt{1 + y^2}, \quad x = te^{2t}, \quad y = e^{-t}$

7. $w = xy^2z^3, \quad x = \operatorname{sen} t, \quad y = \cos t, \quad z = 1 + e^{2t}$

8. $w = \dfrac{x}{y} + \dfrac{y}{z}, \quad x = \sqrt{t}, \quad y = \cos 2t, \quad z = e^{-3t}$

9-14 Use a Regra da Cadeia para determinar $\partial z/\partial s$ e $\partial z/\partial t$.

9. $z = x^2 \operatorname{sen} y, \quad x = s^2 + t^2, \quad y = 2st$

10. $z = \operatorname{sen} x \cos y, \quad x = (s - t)^2, \quad y = s^2 - t^2$

11. $z = x^2 - 3x^2y^3, \quad x = se^t, \quad y = se^{-t}$

12. $z = x \operatorname{tg}^{-1}(xy), \quad x = t^2, \quad y = se^t$

13. $z = 2^{x-3y}, \quad x = s^2t, \quad y = st^2$

14. $z = xe^y + ye^{-x}, \quad x = e^t, \quad y = st^2$

15-22 Use a Regra da Cadeia para determinar as derivadas parciais indicadas.

15. $w = x^2 + y^2 + z^2, \ x = st, \ y = s\cos t, \ z = s \operatorname{sen} t;$
$\dfrac{\partial w}{\partial s}, \dfrac{\partial w}{\partial t}$ quando $s = 1, t = 0$

16. $u = xy + yz + zx, \ x = st, \ y = e^{st}, \ z = t^2;$
$\dfrac{\partial u}{\partial s}, \dfrac{\partial u}{\partial t}$ quando $s = 0, t = 1$

17. $z = y^2 \operatorname{tg} x, \ x = t^2uv, \ y = u + tv^2;$
$\dfrac{\partial z}{\partial t}, \dfrac{\partial z}{\partial u}, \dfrac{\partial z}{\partial v}$ quando $t = 2, u = 1, v = 0$

18. $z = \dfrac{x}{y}, \ x = re^{st}, \ y = rse^t;$
$\dfrac{\partial z}{\partial r}, \dfrac{\partial z}{\partial s}, \dfrac{\partial z}{\partial t}$ quando $r = 1, s = 2, t = 0$

19. $u = \dfrac{x + y}{y + z}, \ x = p + r + t, \ y = p - r + t, \ z = p + r - t;$
$\dfrac{\partial u}{\partial p}, \dfrac{\partial u}{\partial r}, \dfrac{\partial u}{\partial t}$

20. $t = z \sec(xy), \ x = uv, \ y = vw, \ z = wu; \quad \dfrac{\partial t}{\partial u}, \dfrac{\partial t}{\partial v}, \dfrac{\partial t}{\partial w}$

21. $w = \cos(x - y), \ x = rs^2t^3 \operatorname{sen}\theta, \ y = r^2st\cos\theta;$
$\dfrac{\partial w}{\partial r}, \dfrac{\partial w}{\partial s}, \dfrac{\partial w}{\partial t}, \dfrac{\partial w}{\partial \theta}$

22. $u = pq - p^2r^2s, \ p = x + 2y, \ q = x - 2y, \ r = \dfrac{x}{y^4},$
$s = 2xy^{3/2}; \quad \dfrac{\partial u}{\partial x}, \dfrac{\partial u}{\partial y}$

23-26 Use a Equação 6 para determinar dy/dx.

23. $x^2 - xy + y^3 = 8$

24. $y^5 + 3x^2y^2 + 5x^4 = 12$

25. $x\cos y + y\cos x = 1$

26. $2y^2 + \sqrt[3]{xy} = 3x^2 + 17$

27-33 Use as Equações 7 para determinar $\partial z/\partial x$ e $\partial z/\partial y$.

27. $xy + yz - xz = 0$

28. $x^2 + y^2 - z^2 = 2x(y + z)$

29. $xy^2z^3 + x^3y^2z = x + y + z$

30. $y^2ze^{x+y} - \operatorname{sen}(xyz) = 0$

31. $xy^2 + yz^2 + zx^2 = 3$

32. $xe^y + yz + ze^x = 0$

33. $\ln(x + yz) = 1 + xy^2z^3$

34. O raio de um cilindro reto está aumentando em uma taxa de 1,2 cm/s enquanto sua altura está decrescendo em uma taxa de 3 cm/s. Em qual taxa o volume do cilindro está variando quando o raio é 80 cm e a altura é 150 cm?