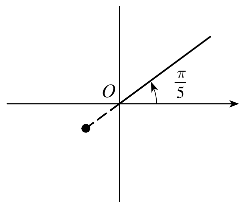
10.3 SOLUÇÕES

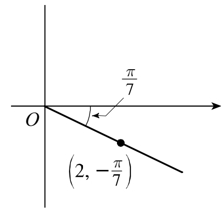
Revisão técnica: Ricardo Miranda Martins – IMECC – Unicamp

1. $\left(-1, \frac{\pi}{5}\right)$



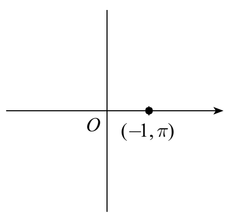
$\left(1, \frac{6\pi}{5}\right)$, $\left(-1, \frac{11\pi}{5}\right)$

2. $\left(2, -\frac{\pi}{7}\right)$



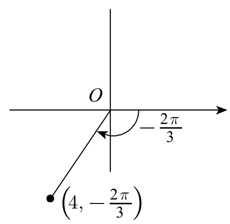
$\left(2, \frac{13\pi}{7}\right)$, $\left(-2, \frac{6\pi}{7}\right)$

3. $(-1, \pi)$



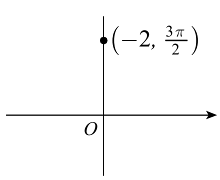
$(-1, 3\pi)$, $(1, 0)$

4. $\left(4, -\frac{2\pi}{3}\right)$



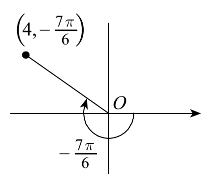
$\left(4, \frac{4\pi}{3}\right)$, $\left(-4, \frac{\pi}{3}\right)$

5. $\left(-2, \frac{3\pi}{2}\right)$



$\left(2, \frac{\pi}{2}\right)$, $\left(-2, -\frac{\pi}{2}\right)$

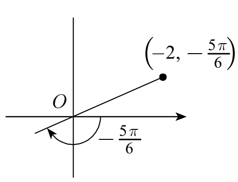
6.



$$x = 4\cos\left(-\frac{7\pi}{6}\right) = 4\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$$

$$y = 4\,\text{sen}\left(-\frac{7\pi}{6}\right) = 4 \cdot \frac{1}{2} = 2$$

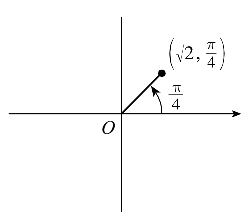
7.



$$x = -4\cos\left(\frac{5\pi}{4}\right) = 2\sqrt{2}$$

$$y = -4\,\text{sen}\left(\frac{5\pi}{4}\right) = 2\sqrt{2}$$

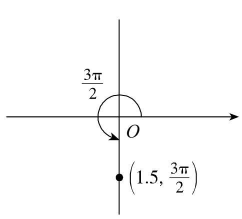
8.



$$x = \sqrt{2}\cos\frac{\pi}{4} = 1,$$

$$y = \sqrt{2}\,\text{sen}\,\frac{\pi}{4} = 1$$

9.



$\left(0, -\frac{3}{2}\right)$

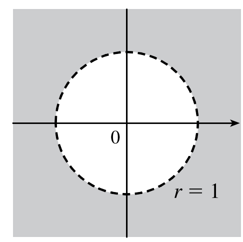
10. $(x, y) = (-1, 1)$, $r = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$,

$\text{tg}\ \theta = y/x = -1$ e (x, y) está no quadrante II, então $\theta = \frac{3\pi}{4}$.

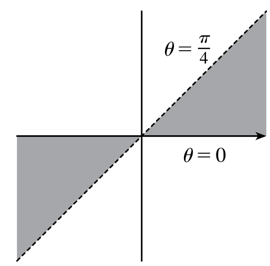
Coordenadas $\left(\sqrt{2}, \frac{3\pi}{4}\right)$.

11. $(x, y) = (3, 4)$, $r = \sqrt{9 + 16} = 5$, $\text{tg}\ \theta = y/x = \frac{4}{3}$, assim $\theta = \text{tg}^{-1}\left(\frac{4}{3}\right) \cdot \left(5, \text{tg}^{-1}\left(\frac{4}{3}\right)\right)$.

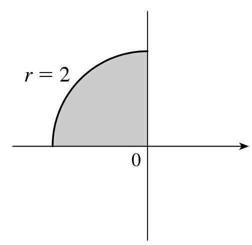
12. $r > 1$



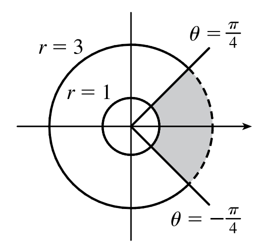
13. $0 \leq \theta < \frac{\pi}{4}$



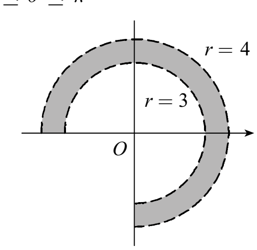
14. $0 \leq r \leq 2$, $\frac{\pi}{2} \leq \theta \leq \pi$



15. $1 \leq r < 3$, $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$



16. $3 < r < 4$, $-\frac{\pi}{2} \leq \theta \leq \pi$



17. Uma vez que $y = r\,\text{sen}\,\theta$, a equação $r\,\text{sen}\,\theta = 2$ se torna $y = 2$.

18. $r = 2\,\text{sen}\,\theta \Rightarrow r^2 = 2r\,\text{sen}\,\theta \Rightarrow x^2 + y^2 = 2y$

19. $r = \dfrac{1}{1 - \cos\theta} \Leftrightarrow r - r\cos\theta = 1$

$r = 1 + r\cos\theta \Leftrightarrow r^2 = (1 + r\cos\theta)^2$

$x^2 + y^2 = (1 + x)^2 = 1 + 2x + x^2 \Leftrightarrow y^2 = 1 + 2x$

20. $r = \dfrac{5}{3 - 4\,\text{sen}\,\theta} \Rightarrow 3r - 4r\,\text{sen}\,\theta = 5 \Rightarrow$

$3r = 5 + 4r\,\text{sen}\,\theta \Rightarrow 9r^2 = (5 + 4r\,\text{sen}\,\theta)^2 \Rightarrow$

$9\left(x^2 + y^2\right) = (5 + 4y)^2 \Rightarrow 9x^2 = 7y^2 + 40y + 25$

21. $r = \dfrac{1}{1 + 2\,\text{sen}\,\theta} \;\Rightarrow\; r + 2r\,\text{sen}\,\theta = 1 \;\Leftrightarrow$

$r = 1 - 2r\,\text{sen}\,\theta \;\Leftrightarrow\; \sqrt{x^2 + y^2} = 1 - 2y \;\Rightarrow$

$x^2 + y^2 = 1 - 4y + 4y^2 \;\Leftrightarrow\; 3y^2 - 4y - x^2 = -1$

$\Leftrightarrow\; 3\left(y^2 - \frac{4}{3}y + \frac{4}{9}\right) - x^2 = \frac{4}{3} - 1 \;\Leftrightarrow$

$3\left(y - \frac{2}{3}\right)^2 - x^2 = \frac{1}{3} \;\Leftrightarrow\; 9\left(y - \frac{2}{3}\right)^2 - 3x^2 = 1 \;\Leftrightarrow$

$\dfrac{\left(y - \frac{2}{3}\right)^2}{\left(\frac{1}{3}\right)^2} - \dfrac{x^2}{\left(\frac{1}{\sqrt{3}}\right)^2} = 1$. Esta é uma hipérbole centralizada em $\left(0, \frac{2}{3}\right)$.

22. $r^2 = \text{sen}\,2\theta = 2\,\text{sen}\,\theta\cos\theta \;\Leftrightarrow\; r^4 = 2r\,\text{sen}\,\theta\, r\cos\theta \;\Leftrightarrow$ $\left(x^2 + y^2\right)^2 = 2yx$

23. $r^2 = \theta \;\Rightarrow\; \text{tg}\left(r^2\right) = \text{tg}\,\theta \;\Rightarrow$ $\text{tg}\left(x^2 + y^2\right) = y/x$

24. $y = 5 \;\Leftrightarrow\; r\,\text{sen}\,\theta = 5$

25. $y = 2x - 1 \;\Leftrightarrow\; r\,\text{sen}\,\theta = 2r\cos\theta - 1 \;\Leftrightarrow$

$r\,(2\cos\theta - \text{sen}\,\theta) = 1 \;\Leftrightarrow\; r = \dfrac{1}{2\cos\theta - \text{sen}\,\theta}$.

(Podemos dividir por $2\cos\theta - \text{sen}\,\theta$, pois este termo deve ser diferente de zero para que o produto com r seja igual a 1.)

26. $x^2 + y^2 = 25 \;\Leftrightarrow\; r^2 = 25 \;\Rightarrow\; r = 5$

27. $x^2 = 4y \;\Leftrightarrow\; r^2\cos^2\theta = 4r\,\text{sen}\,\theta \;\Leftrightarrow$

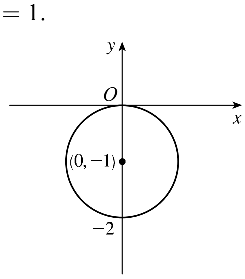
$r\cos^2\theta = 4\,\text{sen}\,\theta \;\Leftrightarrow\; r = 4\,\text{tg}\,\theta\sec\theta$

28. $2xy = 1 \;\Leftrightarrow\; 2r\cos\theta\, r\,\text{sen}\,\theta = 1 \;\Leftrightarrow\; r^2\,\text{sen}\,2\theta = 1$ $\Leftrightarrow\; r^2 = \text{cossec}\,2\theta$

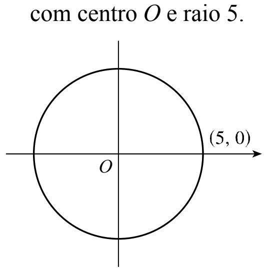
29. $r = -2\,\text{sen}\,\theta \;\Leftrightarrow\; r^2 = -2r\,\text{sen}\,\theta$ (uma vez que a possibilidade $r = 0$ está englobada pela equação $r = -2\,\text{sen}\,\theta$) $\Leftrightarrow$

$x^2 + y^2 = -2y \;\Leftrightarrow\; x^2 + y^2 + 2y + 1 = 1 \;\Leftrightarrow$

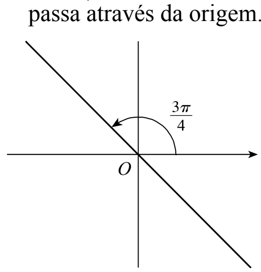
$x^2 + (y + 1)^2 = 1$.



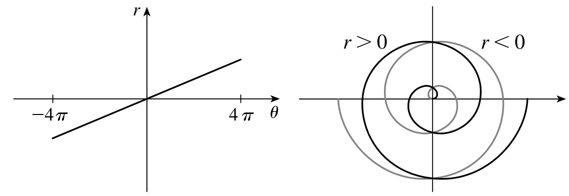
30. $r = 5$ representa o círculo com centro O e raio 5.



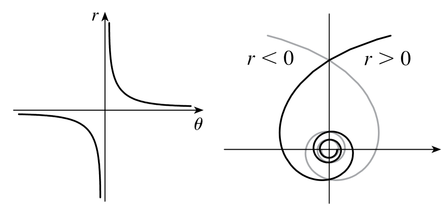
31. $\theta = \frac{3\pi}{4}$ é uma linha que passa através da origem.



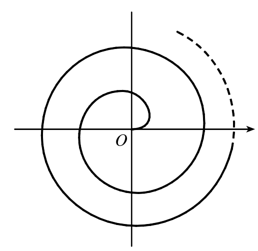
32. $r = \theta/2,\ -4\pi \le \theta \le 4\pi$



33. $r = 1/\theta$



34. $r = \sqrt{\theta}$. A curva é uma espiral.

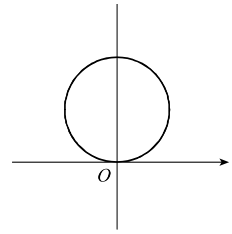


35. $r = 2\,\text{sen}\,\theta \;\Leftrightarrow$

$r^2 = 2r\,\text{sen}\,\theta \;\Leftrightarrow$

$x^2 + y^2 = 2y \;\Leftrightarrow$

$x^2 + (y - 1)^2 = 1$

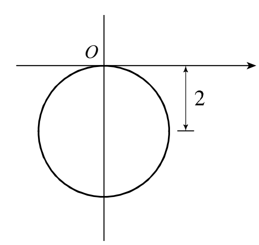


36. $r = -4\,\text{sen}\,\theta \;\Leftrightarrow$

$r^2 = -4r\,\text{sen}\,\theta \;\Leftrightarrow$

$x^2 + y^2 = -4y \;\Leftrightarrow$

$x^2 + (y + 2)^2 = 4$

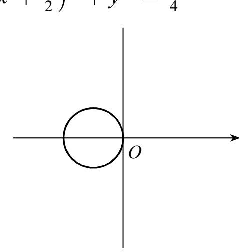


37. $r = -\cos\theta \;\Leftrightarrow$

$r^2 = -r\cos\theta \;\Leftrightarrow$

$x^2 + y^2 = -x \;\Leftrightarrow$

$\left(x + \frac{1}{2}\right)^2 + y^2 = \frac{1}{4}$

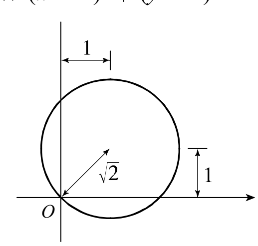


38. $r = 2\,\text{sen}\,\theta + 2\cos\theta \;\Leftrightarrow$

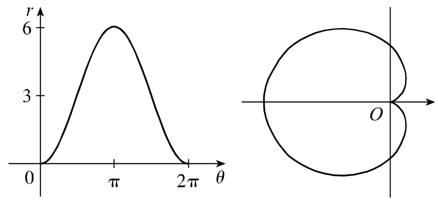
$r^2 = 2r\,\text{sen}\,\theta + 2r\cos\theta$

$\Leftrightarrow\; x^2 + y^2 = 2y + 2x$

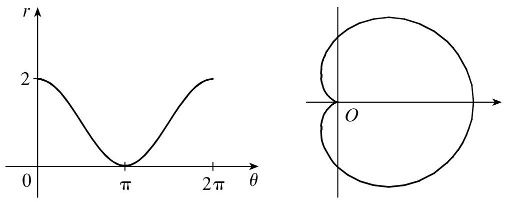
$\Leftrightarrow (x - 1)^2 + (y - 1)^2 = 2$



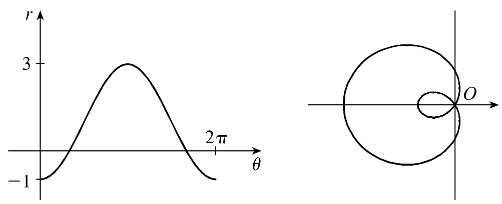
39. $r = 3\,(1 - \cos\theta)$



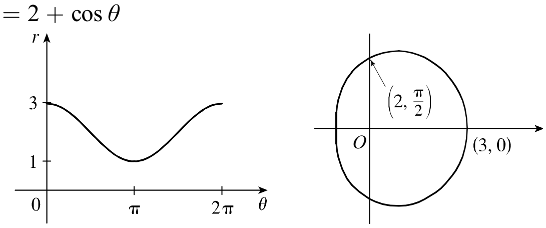
40. $r = 1 + \cos\theta$



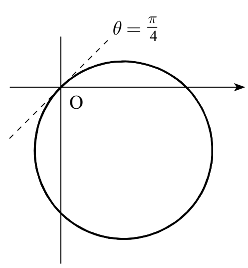
41. $r = 1 - 2\cos\theta$



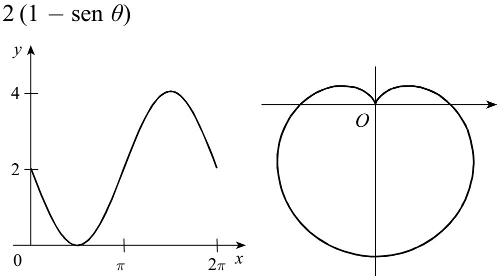
42. $r = 2 + \cos\theta$



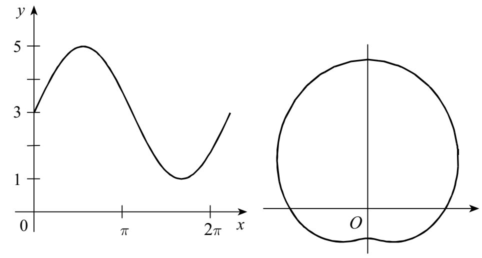
43. $r = \cos\theta - \operatorname{sen}\theta \quad \Leftrightarrow \quad r^2 = r\cos\theta - r\operatorname{sen}\theta \quad \Leftrightarrow$
$x^2 + y^2 = x - y \quad \Leftrightarrow \quad \left(x - \frac{1}{2}\right)^2 + \left(y + \frac{1}{2}\right)^2 = \frac{1}{2}$



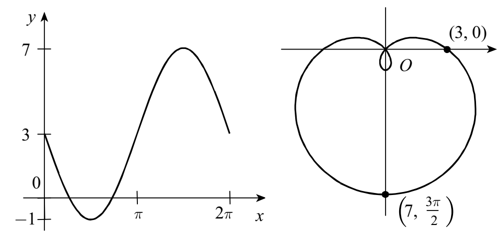
44. $r = 2\,(1 - \operatorname{sen}\,\theta)$



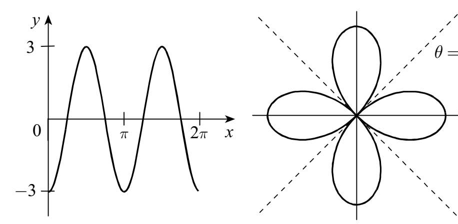
45. $r = 3 + 2\operatorname{sen}\theta$



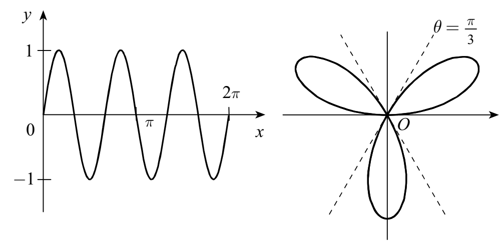
46. $r = 3 - 4\operatorname{sen}\theta$



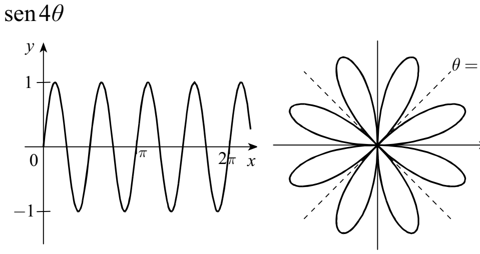
47. $r = -3\cos 2\theta$



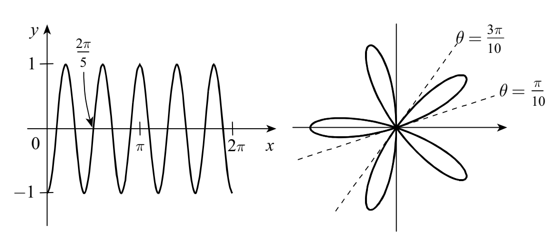
48. $r = \operatorname{sen}\,3\theta$



49. $r = \operatorname{sen}4\theta$



50. $r = -\cos 5\theta$



51. Usando a Equação 3 com $r = 3\cos\theta$, temos

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{(dr/d\theta)(\operatorname{sen}\theta) + r\cos\theta}{(dr/d\theta)(\cos\theta) - r\operatorname{sen}\theta}$$

$$= \frac{-3\operatorname{sen}\theta\operatorname{sen}\theta + 3\cos\theta\cos\theta}{-3\operatorname{sen}\theta\cos\theta - 3\cos\theta\operatorname{sen}\theta}$$

$$= \frac{3\left(\cos^2\theta - \operatorname{sen}^2\theta\right)}{-3\,(2\operatorname{sen}\theta\cos\theta)} = -\frac{\cos 2\theta}{\operatorname{sen}2\theta}$$

$$= -\operatorname{cotg}2\theta = \tfrac{1}{\sqrt{3}} \text{ quando } \theta = \tfrac{\pi}{3}$$

Outra solução: $r = 3\cos\theta \Rightarrow$

$x = r\cos\theta = 3\cos^2\theta$, $y = r\operatorname{sen}\theta = 3\operatorname{sen}\theta\cos\theta \Rightarrow$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-3\operatorname{sen}^2\theta + 3\cos^2\theta}{-6\cos\theta\operatorname{sen}\theta}$$

$$= \frac{\cos 2\theta}{-\operatorname{sen}2\theta} = -\operatorname{cotg}2\theta = \tfrac{1}{\sqrt{3}} \text{ quando } \theta = \tfrac{\pi}{3}$$

52. Usando a Equação 3 com $r = \cos\theta + \operatorname{sen}\theta$, temos

$$\frac{dy}{dx} = \frac{(dr/d\theta)\operatorname{sen}\theta + r\cos\theta}{(dr/d\theta)\cos\theta - r\operatorname{sen}\theta}$$

$$= \frac{(-\operatorname{sen}\theta + \cos\theta)\operatorname{sen}\theta + (\cos\theta + \operatorname{sen}\theta)\cos\theta}{(-\operatorname{sen}\theta + \cos\theta)\cos\theta - (\cos\theta + \operatorname{sen}\theta)\operatorname{sen}\theta}$$

$$= -1 \text{ quando } \theta = \tfrac{\pi}{4}$$

Outra Solução:

$r = \cos\theta + \operatorname{sen}\theta \Rightarrow x = r\cos\theta = (\cos\theta + \operatorname{sen}\theta)\cos\theta$,

$y = r\operatorname{sen}\theta = (\cos\theta + \operatorname{sen}\theta)\operatorname{sen}\theta \Rightarrow$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta}$$

$$= \frac{\operatorname{sen}\theta\,(-\operatorname{sen}\theta + \cos\theta) + (\cos\theta + \operatorname{sen}\theta)\cos\theta}{\cos\theta\,(-\operatorname{sen}\theta + \cos\theta) - (\cos\theta + \operatorname{sen}\theta)\operatorname{sen}\theta}$$

$$= -1 \text{ quando } \theta = \tfrac{\pi}{4}$$

53. $r = 2 + 4\cos\theta \Rightarrow x = r\cos\theta = (2 + 4\cos\theta)\cos\theta$,

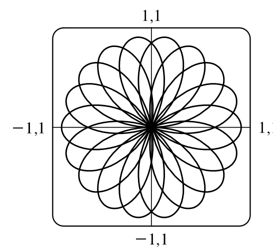
$y = 4\operatorname{sen}\theta = (2 + 4\cos\theta)\operatorname{sen}\theta \Rightarrow$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{-4\operatorname{sen}^2\theta + (2 + 4\cos\theta)\cos\theta}{-4\operatorname{sen}\theta\cos\theta - (2 + 4\cos\theta)\operatorname{sen}\theta}$$

$$= -\frac{2\cos 2\theta + \cos\theta}{2\operatorname{sen}2\theta + \operatorname{sen}\theta} = -\frac{2\left(\frac{1}{2}\right) + \frac{\sqrt{3}}{2}}{2\left(\frac{\sqrt{3}}{2}\right) + \frac{1}{2}}$$

$$= -\frac{4 + 3\sqrt{3}}{11} \text{ quando } \theta = \tfrac{\pi}{6}.$$

54. $r = \operatorname{sen}(9\theta/4)$. O intervalo do parâmetro é $[0, 8\pi]$.



55. $r = 1 + 4\cos(\theta/3)$. O intervalo do parâmetro é $[0, 6\pi]$.