

SEL 0449 - Processamento Digital de Imagens Médicas

Aula 7 – Outros Filtros no Domínio da Frequência

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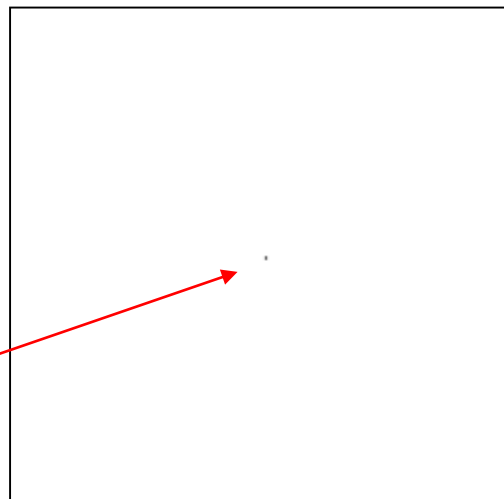
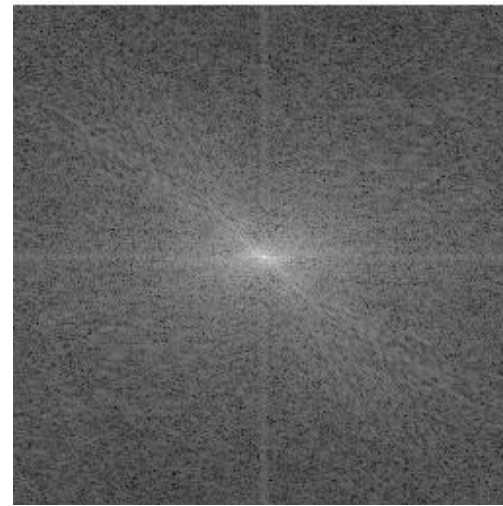
Filtros Notch (seletivos)

Eliminação de frequências
indesejadas - interferências

Filtros *Notch*

- Retira (*reject*) ou mantém (*pass*) na imagem frequências específicas, ou regiões em torno de uma frequência pré-definida na construção do filtro;
- Todas as frequências escolhidas devem vir em pares, devido à simetria da Transformada de Fourier;
- Não há realce de nenhum componente espectral da imagem.
- Usado para Texturas e para remoção de ruídos e interferências
- Podem ser de vários tipos. Os mais comuns são: Ideal, Butterworth e Gaussiano.

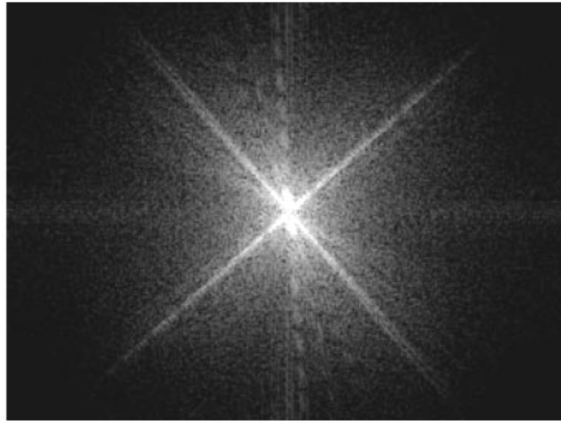
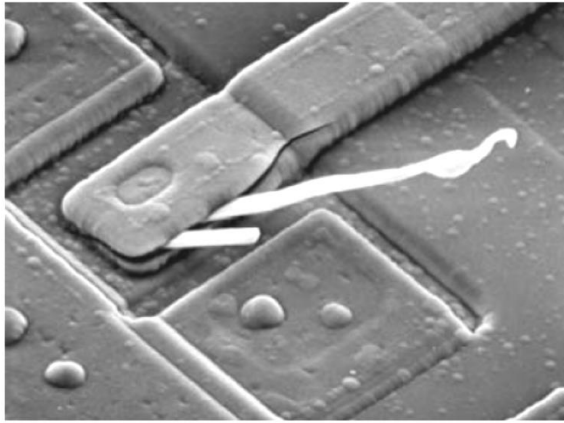
Filtros *Notch Reject*



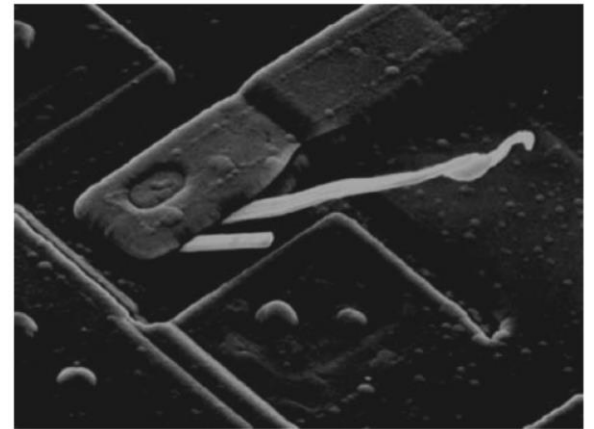
Só a frequência zero
foi retirada



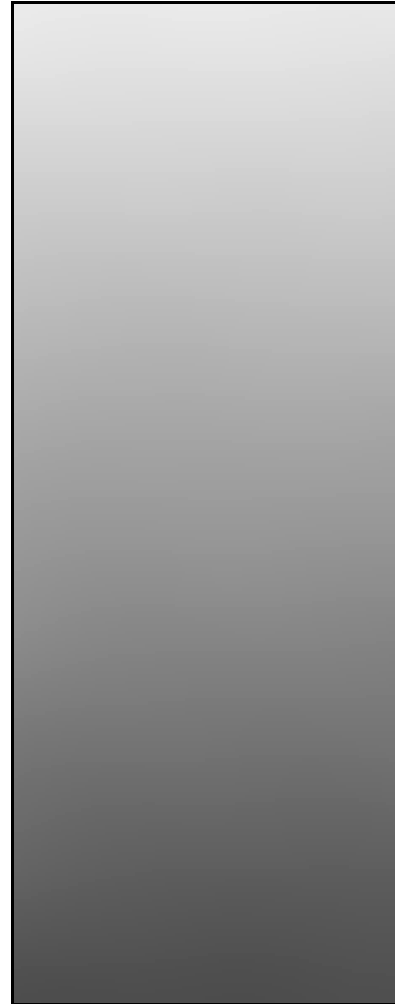
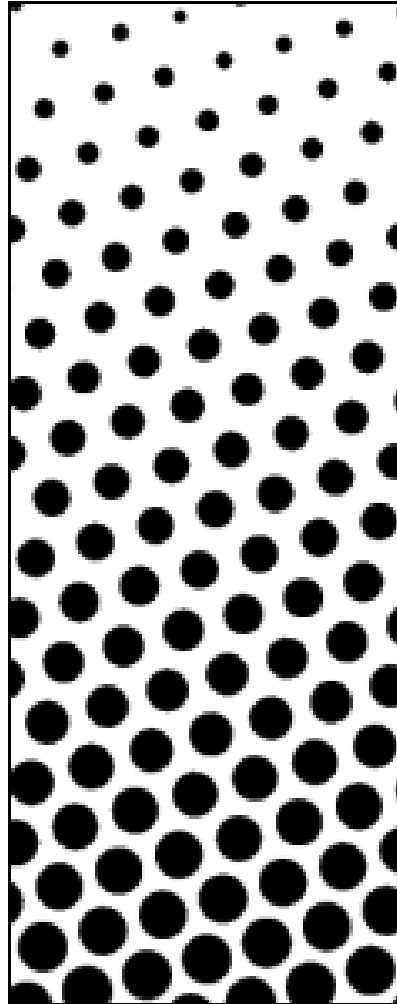
Filtros *Notch Reject*



Só a frequência zero
foi retirada



Impressão em *Halftone*



Impressão em *Halftone*

304 THE DAILY GRAPHIC, NEW YORK, TUESDAY, DECEMBER 1, 1876.



BRADLEY, PRAY & CO.
CARRIAGE MANUFACTURERS.
 555 BROADWAY.
 NEW YORK.

OFFICE OF BRADLEY, PRAY & CO. 555 BROADWAY, NEW YORK.

BRADLEY, PRAY & CO. have on hand a large stock of
 SLEIGHS,
 and carry on their premises a full
 assortment of harness, and other
 articles pertaining to the carriage
 and sleigh business. They also
 carry a large stock of
 H. H. MACY & CO.



TOYS, DOLLS.
HOLIDAY TRAIL.
 Brilliantly Illuminated.

H. H. MACY & CO.
American General Ice Co.
 of ST. LOUIS.
 CAPITAL & RESERVE \$1,000,000
 SOLE AGENTS FOR THE CITY OF NEW YORK

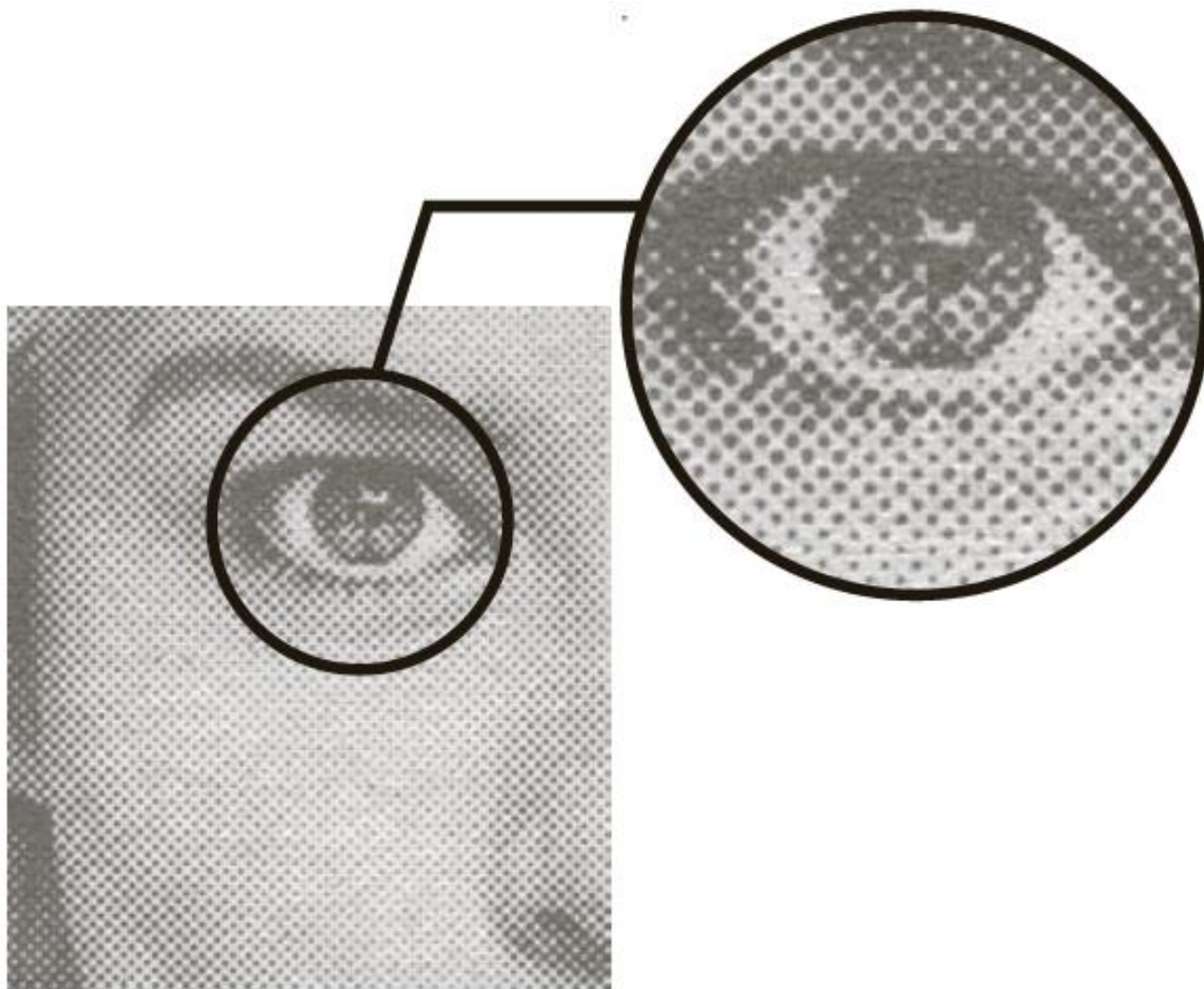
Thomas, Yelverton & Co.
 5 Maiden Lane.
FINE FANCY GOODS.
Elton Jewelry.
Sets & Half Sets
FRENCH CIGARS AND BONGAYS.
MUNN & COBB

HOLIDAY ATTRACTIONS.
LORD & TAYLOR,
HAVANA SOTTERY
Where to get Street Lamps.
SOLID SILVER WARE
AT RETAIL.

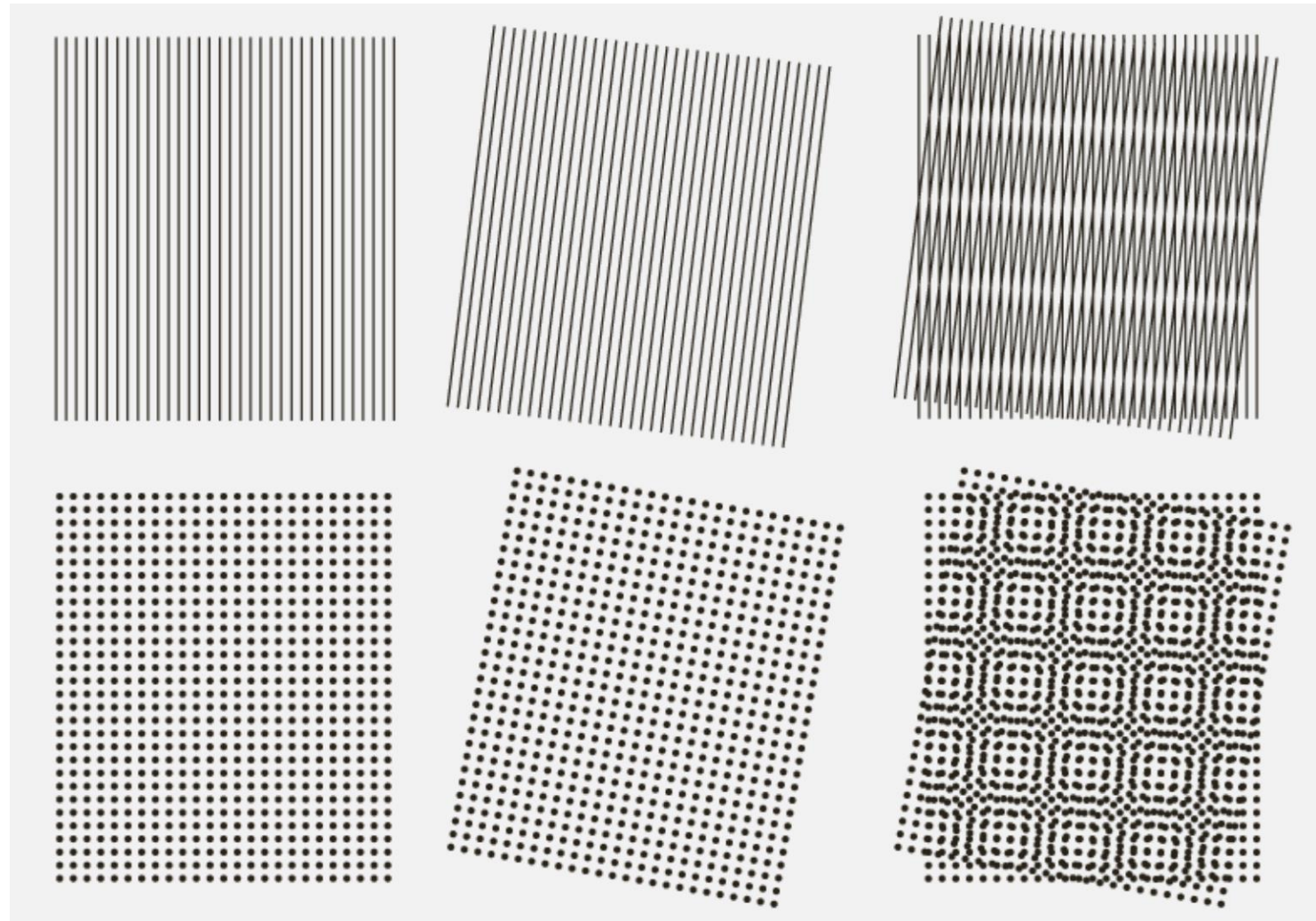
BLOOD'S
PATENT SLEIGH CARBETS
CHAMPAGNES

ON & ON

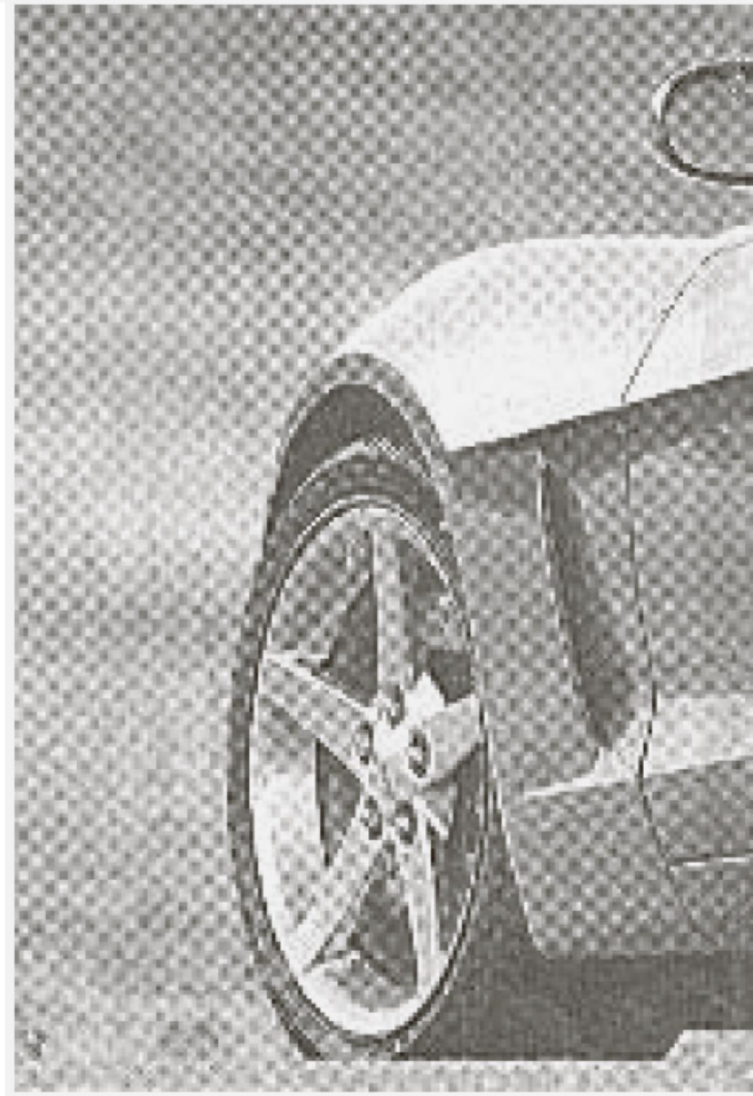
Halftone



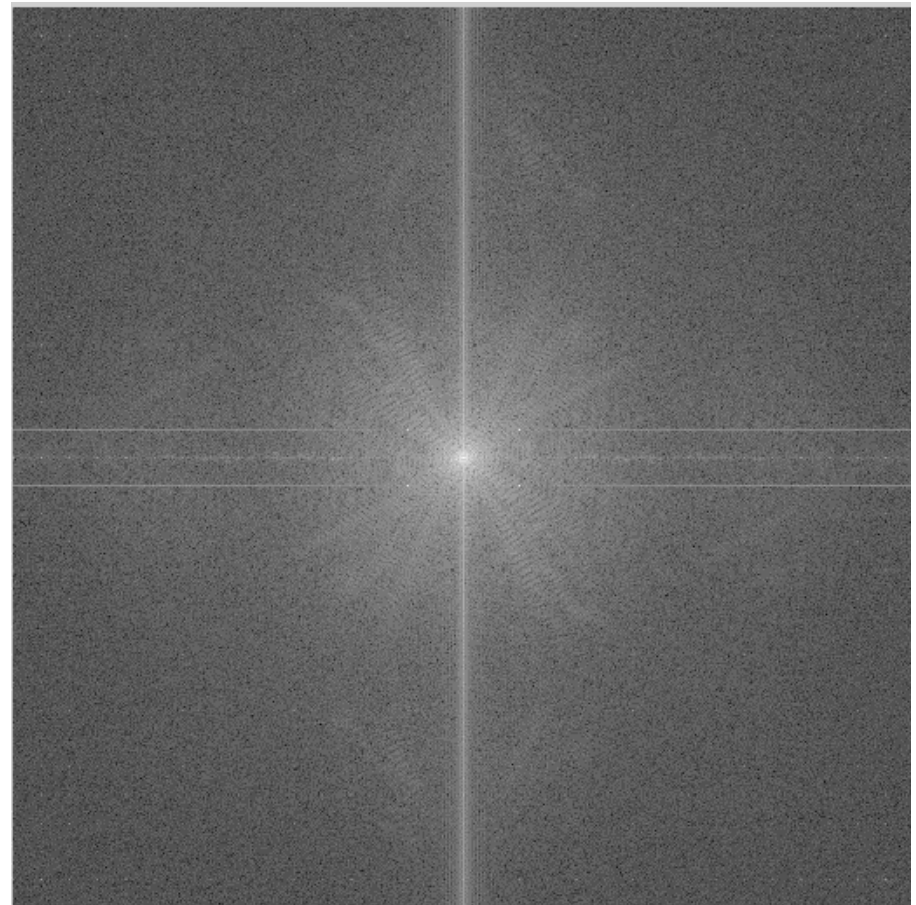
Padrão Moiré



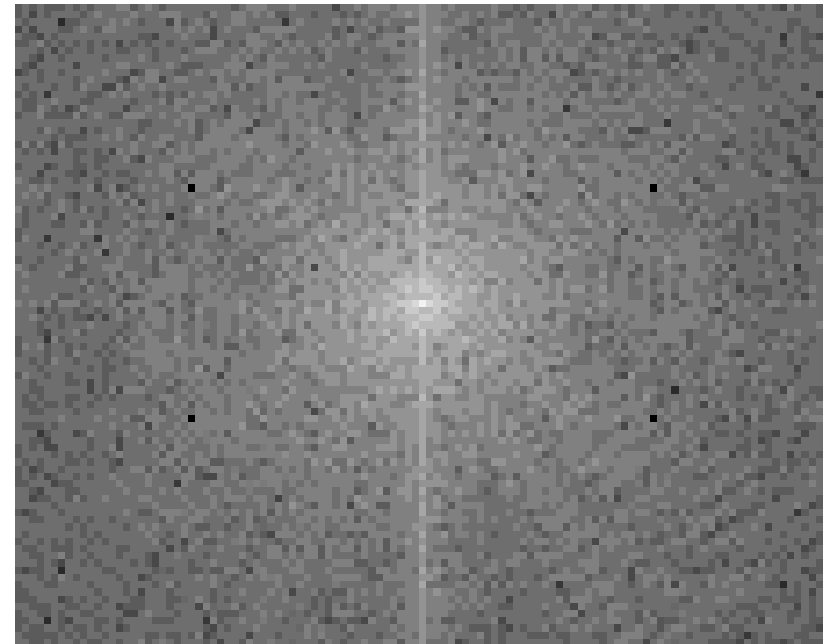
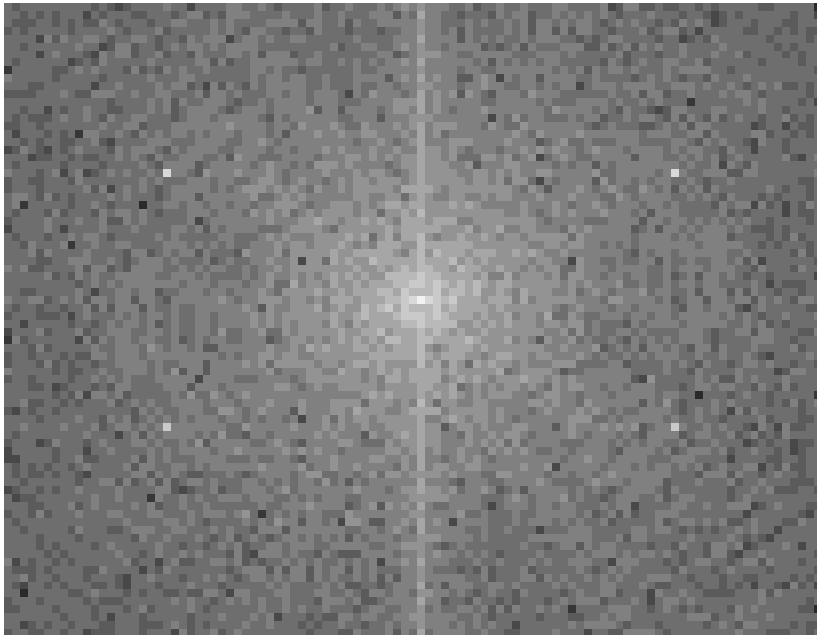
Padrão Moiré - Halftone



Filtros *Notch*



Filtros *Notch*



Filtros *Notch*



Filtro *Notch Reject* Ideal



- A figura mostra apenas um par de regiões sendo retirado, mas o filtro *notch reject* pode retirar quantas frequências forem necessárias;
- A área em torno da frequência escolhida (raio D_0) que pode ser retirada é definida na construção do filtro;

Seja D_0 o raio do filtro *notch reject* centrado na frequência (u_0, v_0) e, por simetria $(-u_0, -v_0)$:

$$D_1(u, v) = \sqrt{(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2}$$

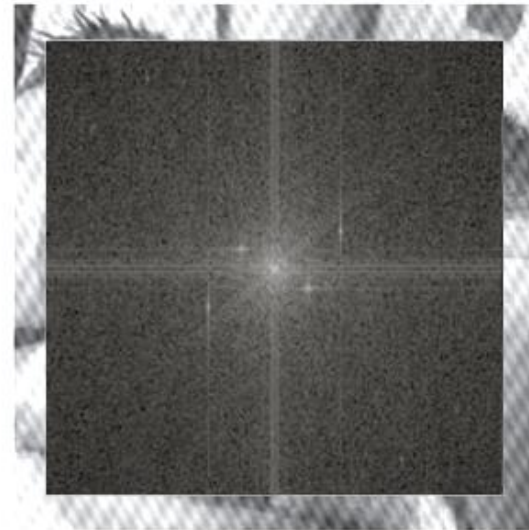
$$D_2(u, v) = \sqrt{(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2}$$

$$H(u, v) = \begin{cases} 0, & \text{se } D_1(u, v) \leq D_0 \text{ ou } D_2(u, v) \leq D_0 \\ 1, & \text{em todas as outras regiões} \end{cases}$$

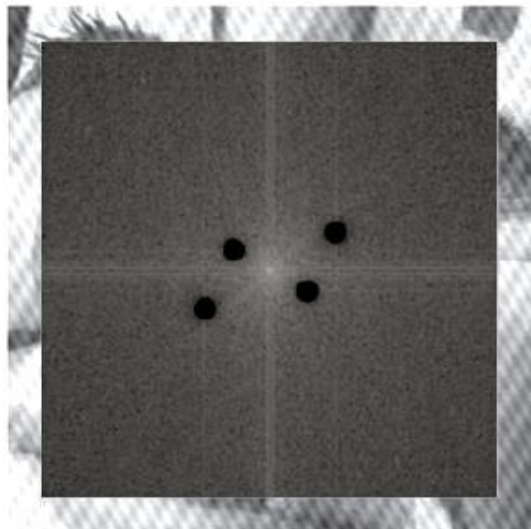
Filtros *Notch Reject*



(a)



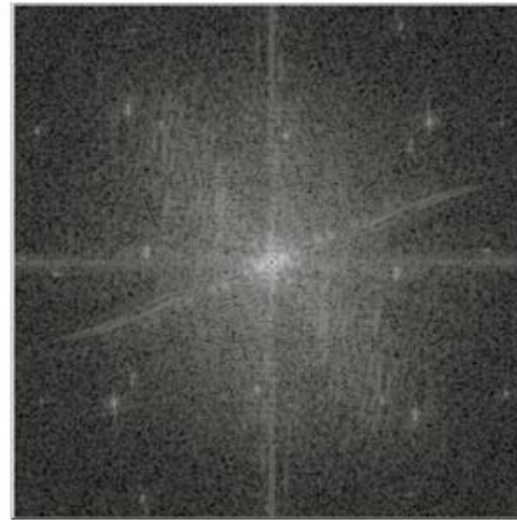
(b)



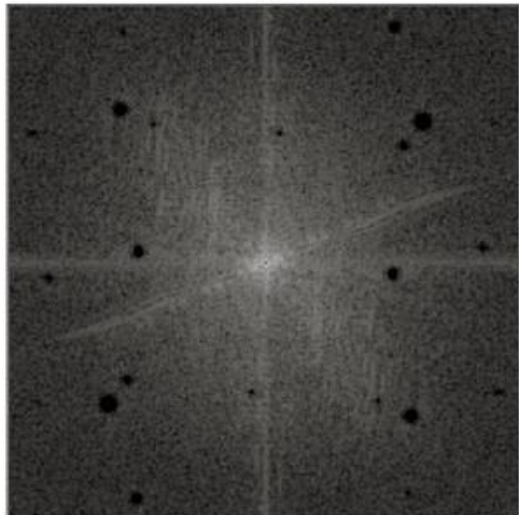
Filtros *Notch Reject*



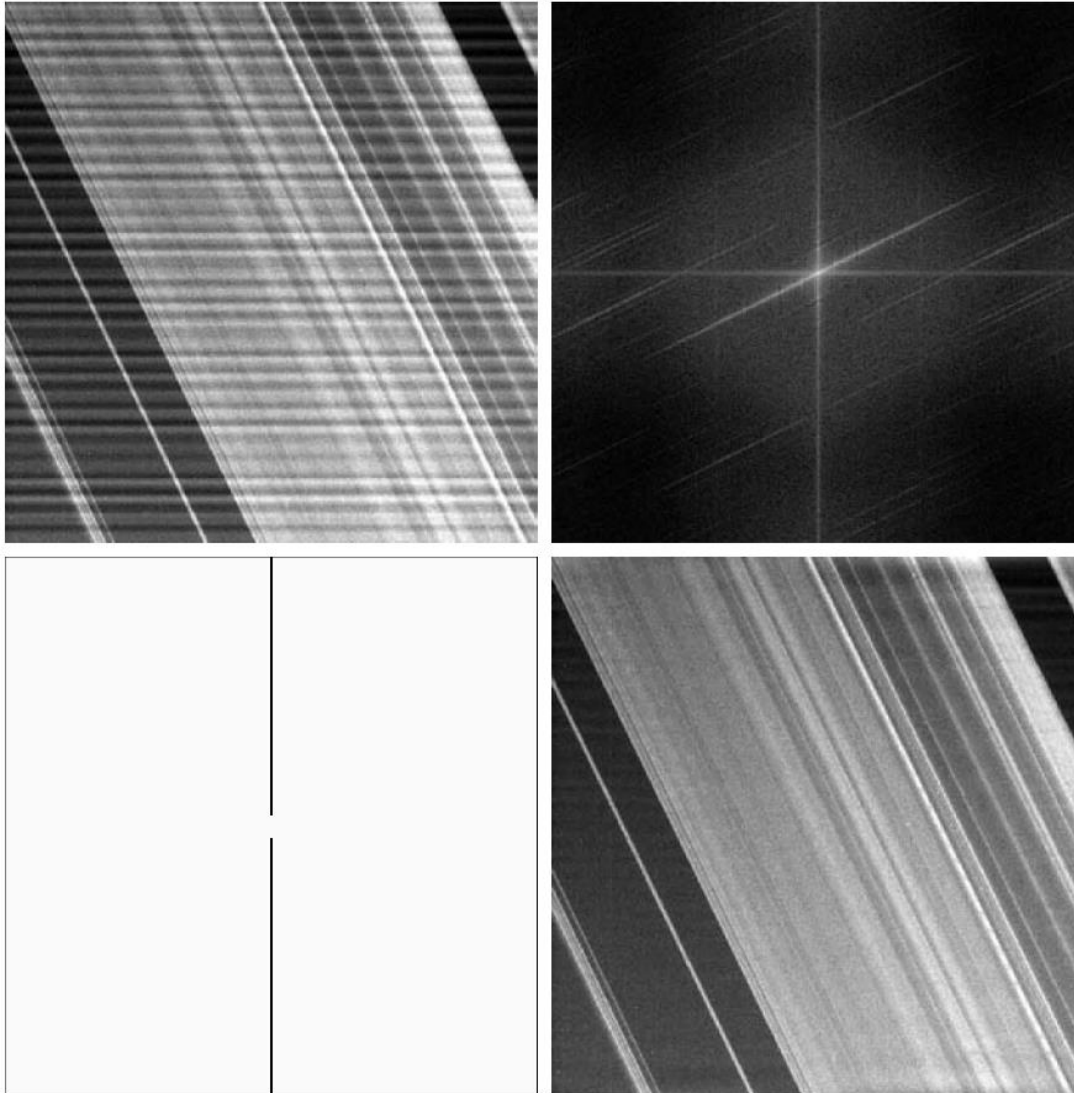
(a)



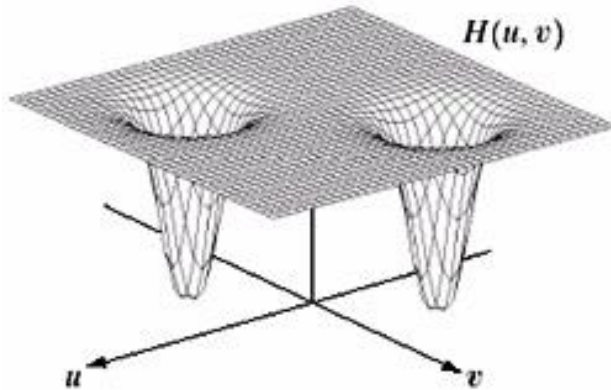
(b)



Filtros *Notch Reject*



Filtro *Notch Reject* Butterworth



- A figura mostra apenas um par de regiões sendo retirado, mas o filtro *notch reject* pode retirar quantas frequências forem necessárias;
- A área em torno da frequência **de corte** escolhida (D_0) que pode ser retirada é definida na construção do filtro;

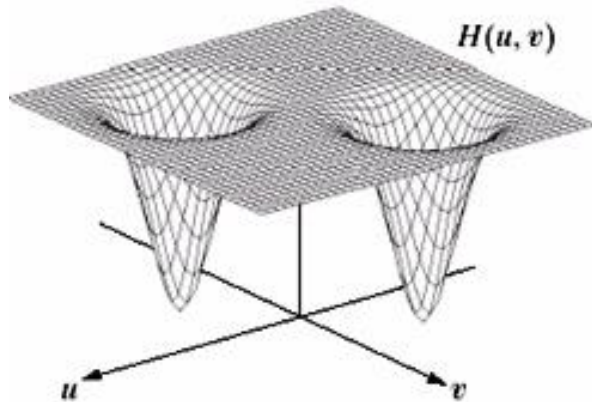
Seja D_0 a frequência de corte do filtro *notch reject* centrado em (u_0, v_0) e, por simetria $(-u_0, -v_0)$:

$$D_1(u, v) = \sqrt{(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2}$$

$$D_2(u, v) = \sqrt{(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2}$$

$$H(u, v) = \frac{1}{1 + \left[\frac{D_0^2}{D_1(u, v) \cdot D_2(u, v)} \right]^n}$$

Filtro *Notch Reject* Gaussiano



- A figura mostra apenas um par de regiões sendo retirado, mas o filtro *notch reject* pode retirar quantas frequências forem necessárias;
- A área em torno da frequência **de corte** (D_0) que pode ser retirada é definida na construção do filtro;

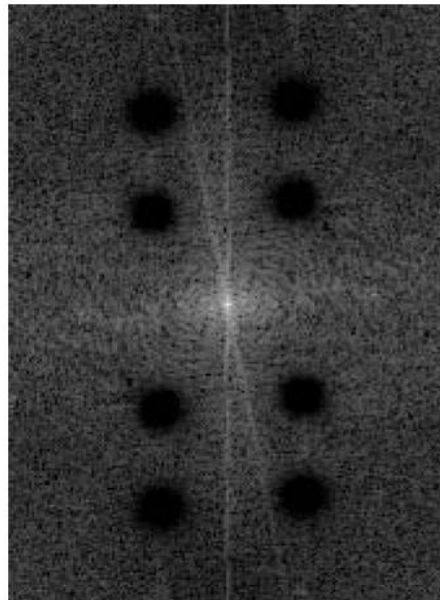
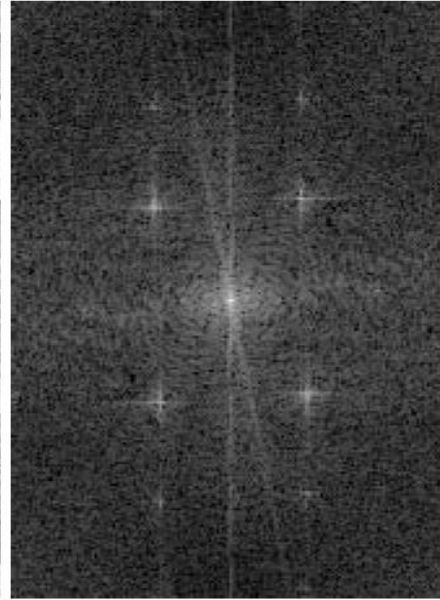
Seja D_0 a frequência de corte do filtro *notch reject* centrado em (u_0, v_0) e, por simetria $(-u_0, -v_0)$:

$$D_1(u, v) = \sqrt{(u - M/2 - u_0)^2 + (v - N/2 - v_0)^2}$$

$$D_2(u, v) = \sqrt{(u - M/2 + u_0)^2 + (v - N/2 + v_0)^2}$$

$$H(u, v) = 1 - e^{-\frac{1}{2} \left[\frac{D_1(u, v) \cdot D_2(u, v)}{D_0^2} \right]}$$

Filtro *Notch Reject* Butterworth

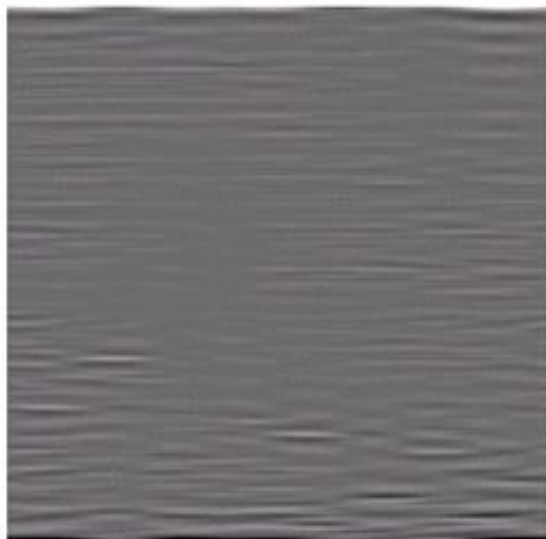
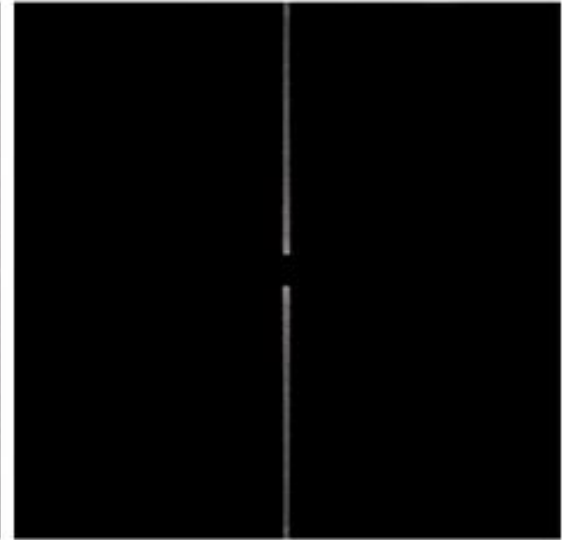
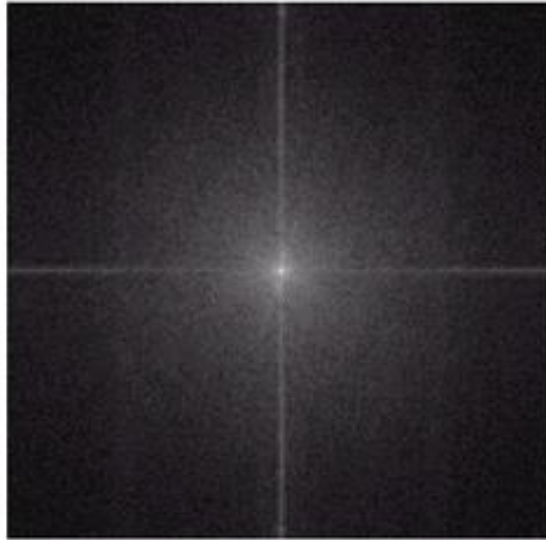
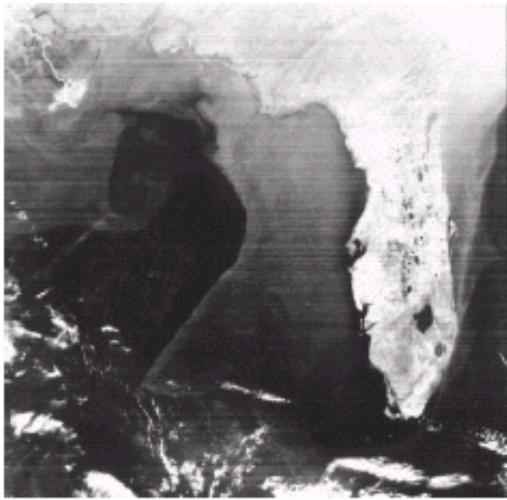


Filtros *Notch Pass*

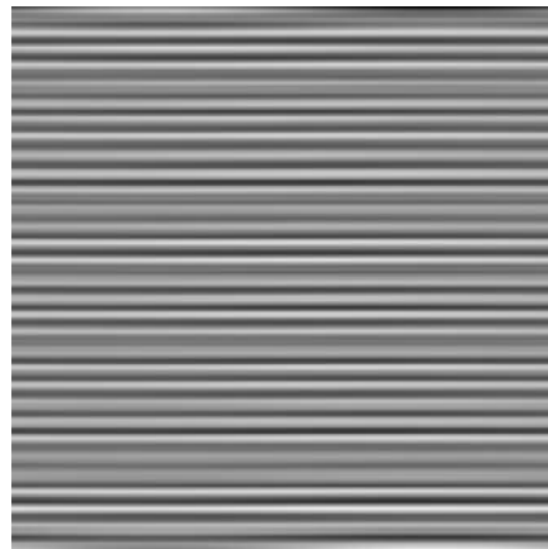
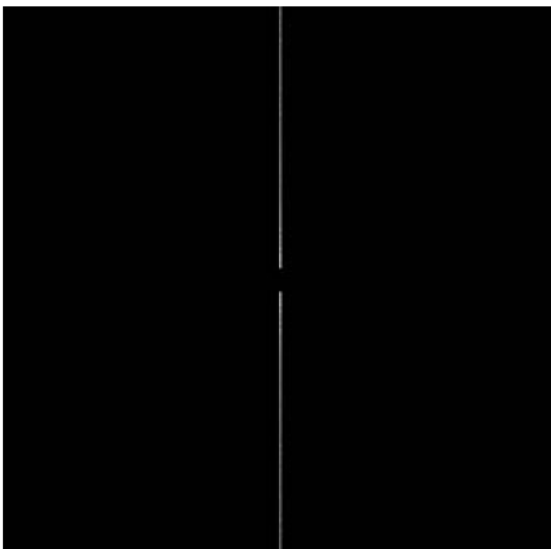
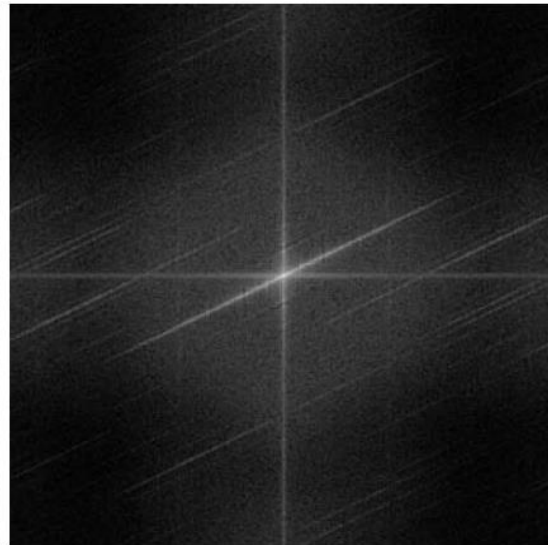
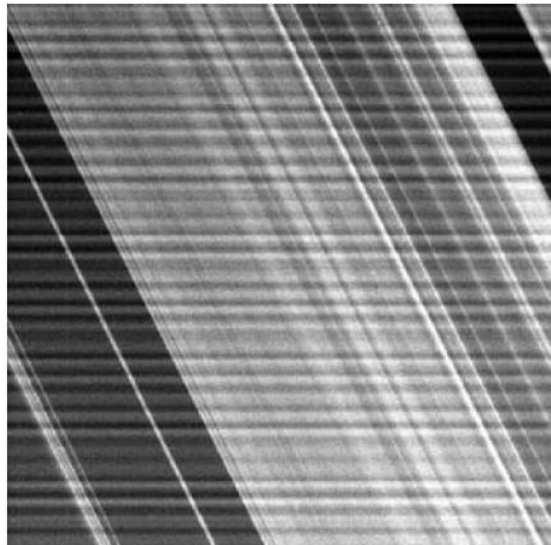
As equações dos filtros *Notch Pass* podem ser obtidos a partir das equações dos filtros *Notch Reject* :

$$H(u, v)_{NP} = 1 - H(u, v)_{NR}$$

Filtros *Notch Pass*



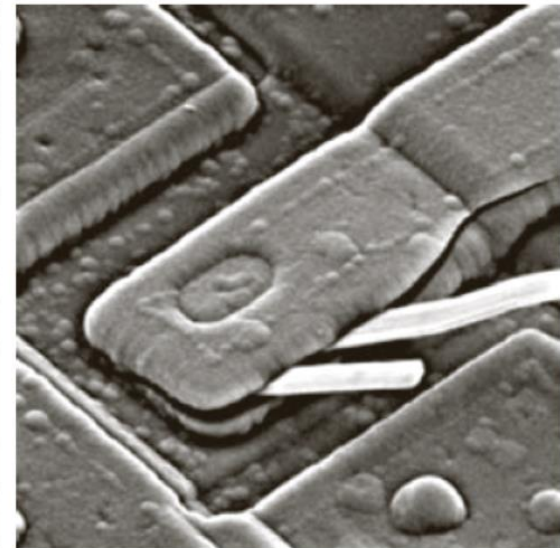
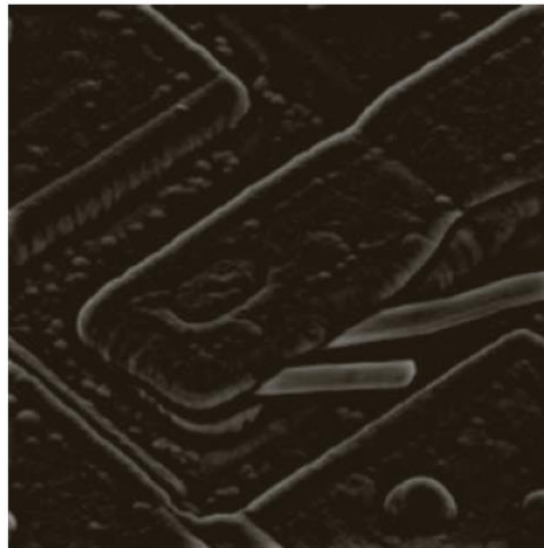
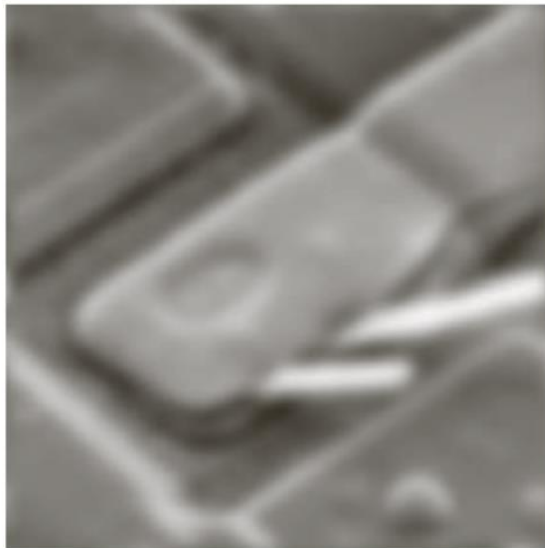
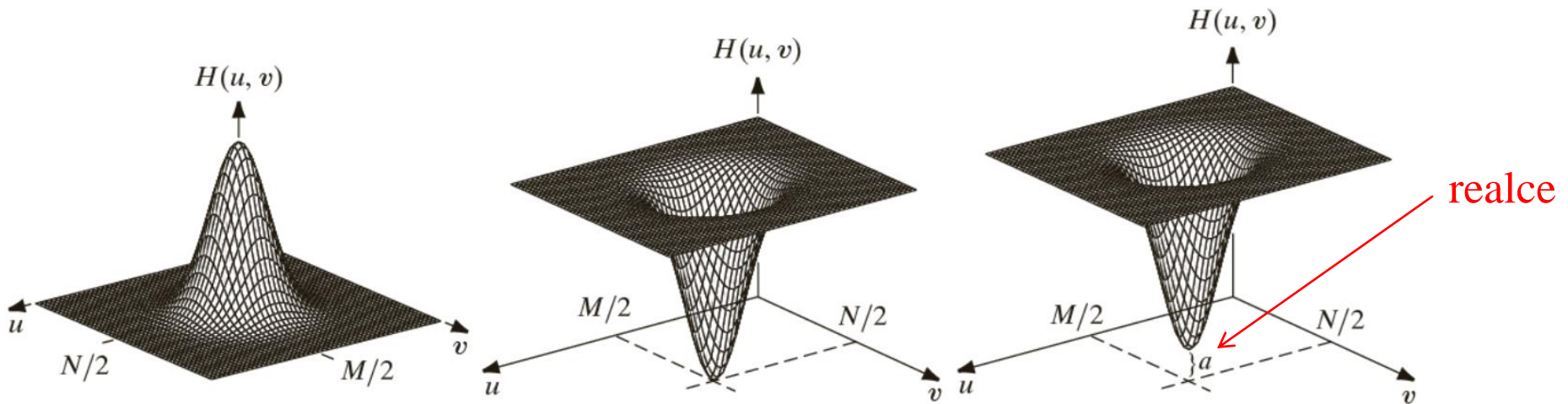
Filtros *Notch Pass*



Filtros de Realce

- Atenua ou mantém os componentes de alguma faixa de frequência da imagem e aumenta (realça) outras faixas de frequências;
- Geralmente mantém as baixas-frequências e realça as altas-frequências;
- Na transição pode-se utilizar qualquer curva, geralmente utiliza-se Butterworth ou Gaussiano.

Filtros de Realce



Filtros de Realce

$$g(x, y) = \mathfrak{S}^{-1}\{[1 + k * H_{\text{HP}}(u, v)]F(u, v)\}$$

k - controla a contribuição das altas frequências (realce)

Filtros de Realce

$$g(x, y) = \mathfrak{F}^{-1}\{[k_1 + k_2 * H_{\text{HP}}(u, v)]F(u, v)\}$$

- k_1 - controla o offset do filtro (brilho da imagem)
- k_2 - controla a contribuição das altas frequências (realce)

Filtro Homomórfico

- Atenua as baixas-frequências e realça as altas baseando-se no modelo de iluminação-reflectância;
- O filtro homomórfico trabalha com a idéia de que a “iluminação” (γ_L) é componente de baixa-frequência e a “reflectância” de alta-freqüência (γ_H);
- Aumenta-se o contraste se a iluminação é diminuída ($\gamma_L < 1$) e a reflectância é aumentada ($\gamma_H > 1$);
- Na transição pode-se utilizar qualquer a curva, geralmente utiliza-se Butterworth ou Gaussiano.

Filtro Homomórfico

$$f(x, y) = i(x, y)r(x, y) \quad \Rightarrow \quad \mathfrak{S}[f(x, y)] \neq \mathfrak{S}[i(x, y)]\mathfrak{S}[r(x, y)]$$

$$\begin{aligned} z(x, y) &= \ln f(x, y) \\ &= \ln i(x, y) + \ln r(x, y) \end{aligned} \quad \Rightarrow \quad \begin{aligned} \mathfrak{S}\{z(x, y)\} &= \mathfrak{S}\{\ln f(x, y)\} \\ &= \mathfrak{S}\{\ln i(x, y)\} + \mathfrak{S}\{\ln r(x, y)\} \end{aligned}$$



$$Z(u, v) = F_i(u, v) + F_r(u, v)$$

$$\begin{aligned} S(u, v) &= H(u, v)Z(u, v) \\ &= H(u, v)F_i(u, v) + H(u, v)F_r(u, v) \end{aligned}$$

Filtro Homomórfico

$$\begin{aligned} s(x, y) &= \mathfrak{S}^{-1}\{S(u, v)\} \\ &= \mathfrak{S}^{-1}\{H(u, v)F_i(u, v)\} + \mathfrak{S}^{-1}\{H(u, v)F_r(u, v)\} \end{aligned}$$

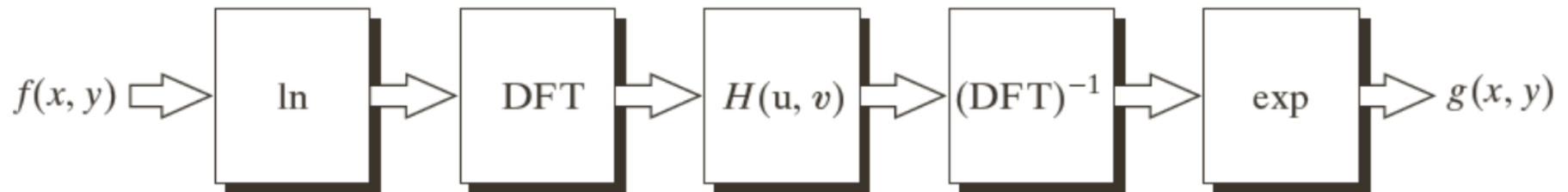
$$\left. \begin{aligned} i'(x, y) &= \mathfrak{S}^{-1}\{H(u, v)F_i(u, v)\} \\ r'(x, y) &= \mathfrak{S}^{-1}\{H(u, v)F_r(u, v)\} \end{aligned} \right\}$$



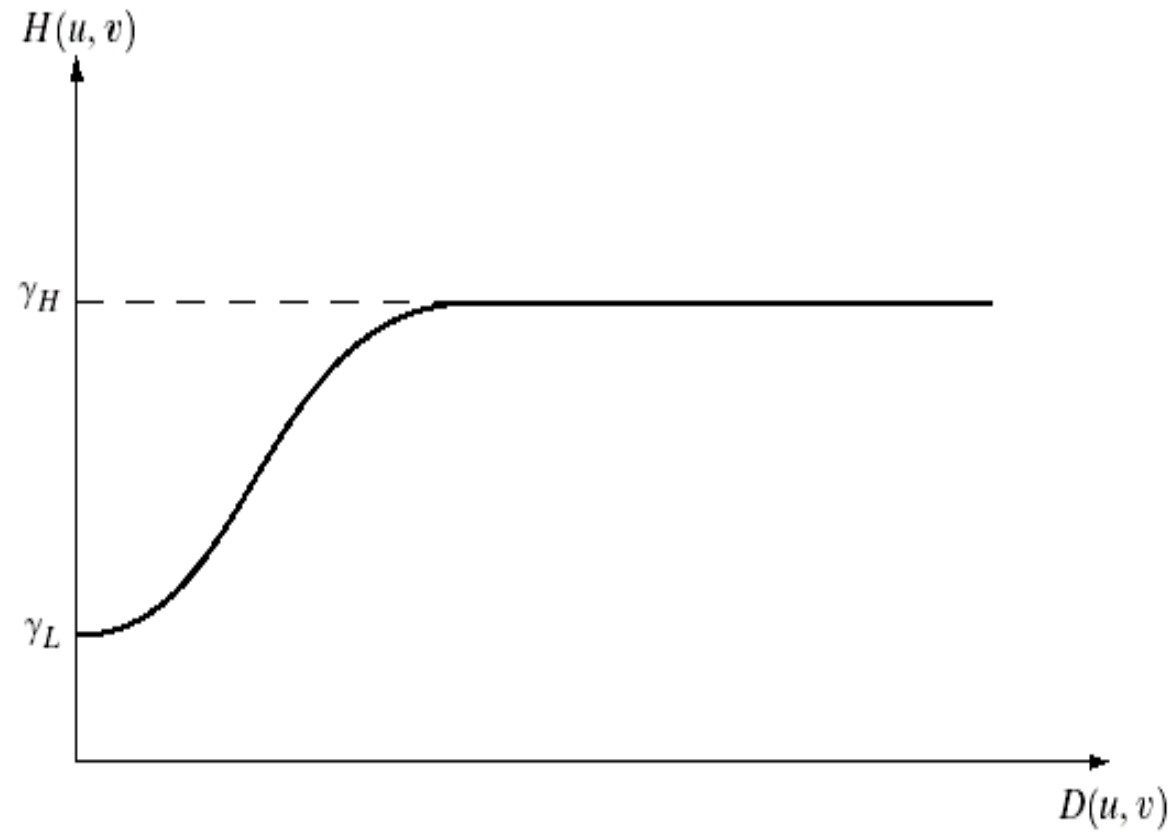
$$s(x, y) = i'(x, y) + r'(x, y) \quad \Longrightarrow$$

$$\begin{aligned} g(x, y) &= e^{s(x, y)} \\ &= e^{i'(x, y)} e^{r'(x, y)} \\ &= i_0(x, y) r_0(x, y) \end{aligned}$$

Filtro Homomórfico



Filtro Homomórfico

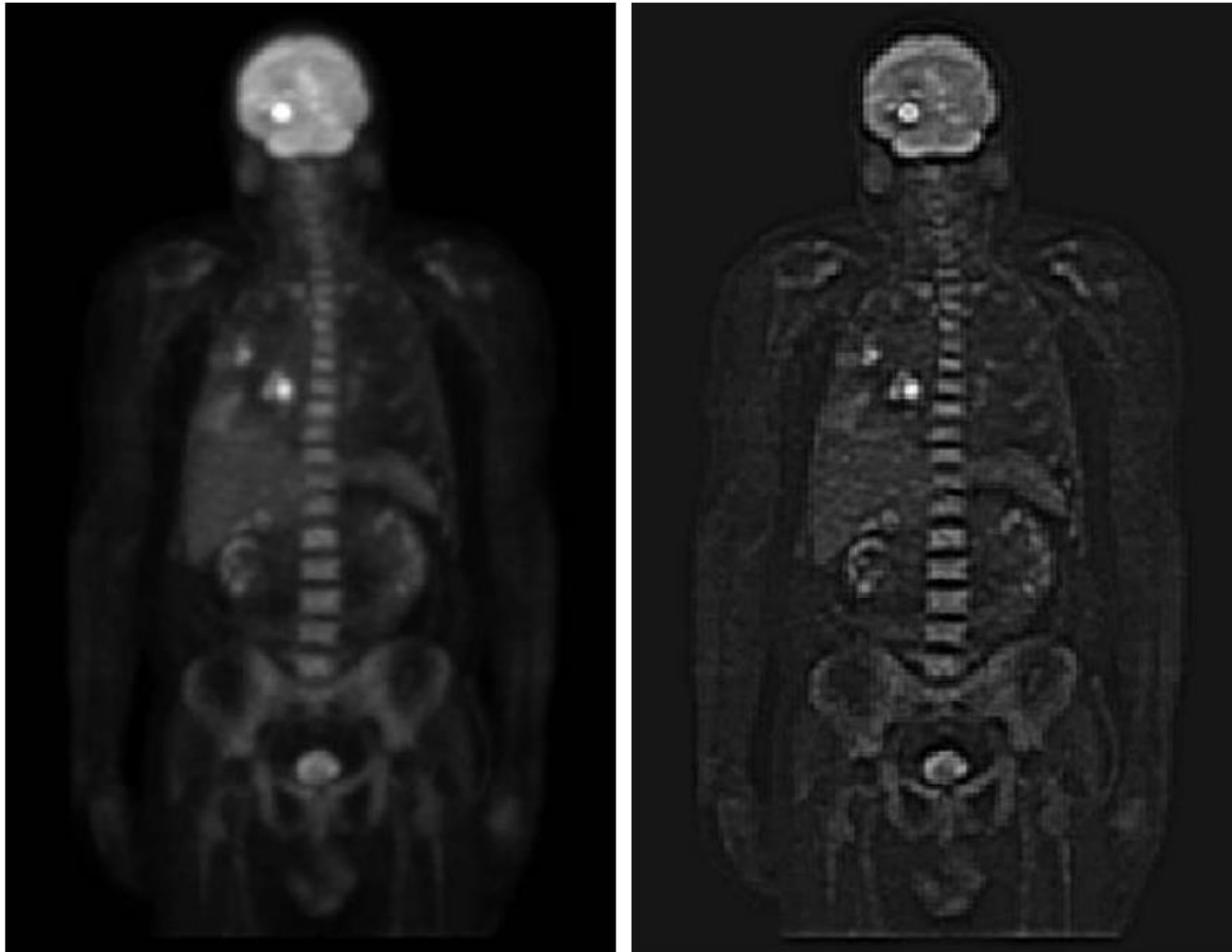


Filtro Homomórfico

Considerando a curva gaussiana

$$H(u, v) = (\gamma_H - \gamma_L) \left[1 - e^{-c \left[\frac{D^2(u, v)}{D_0^2} \right]} \right] + \gamma_L$$

Filtro Homomórfico



Filtro Homomórfico



Relação entre filtros no domínio da frequência e no domínio do espaço

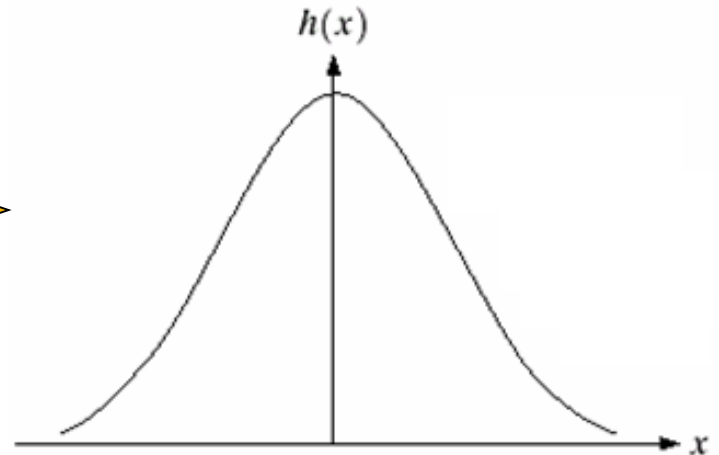
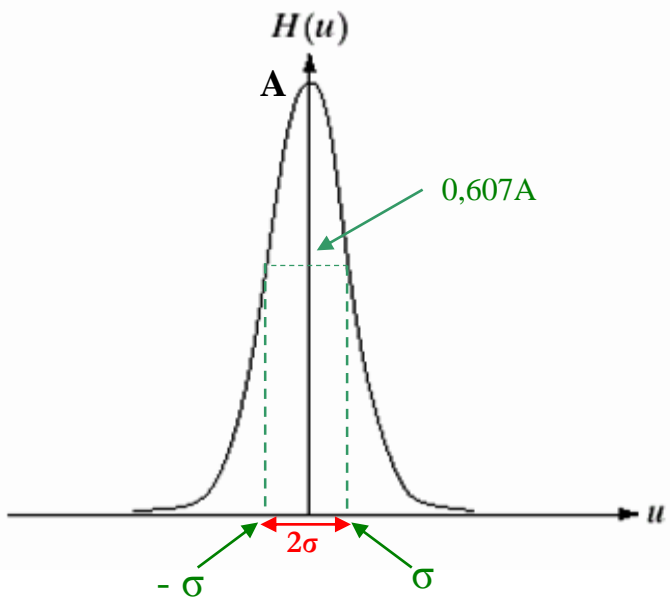
Filtro Passa-Baixa

$$H(u) = Ae^{-\frac{u^2}{2\sigma^2}}$$

IFFT



$$h(x) = \sqrt{2\pi\sigma} Ae^{-2\pi^2\sigma^2 x^2}$$



Filtros Equivalentes



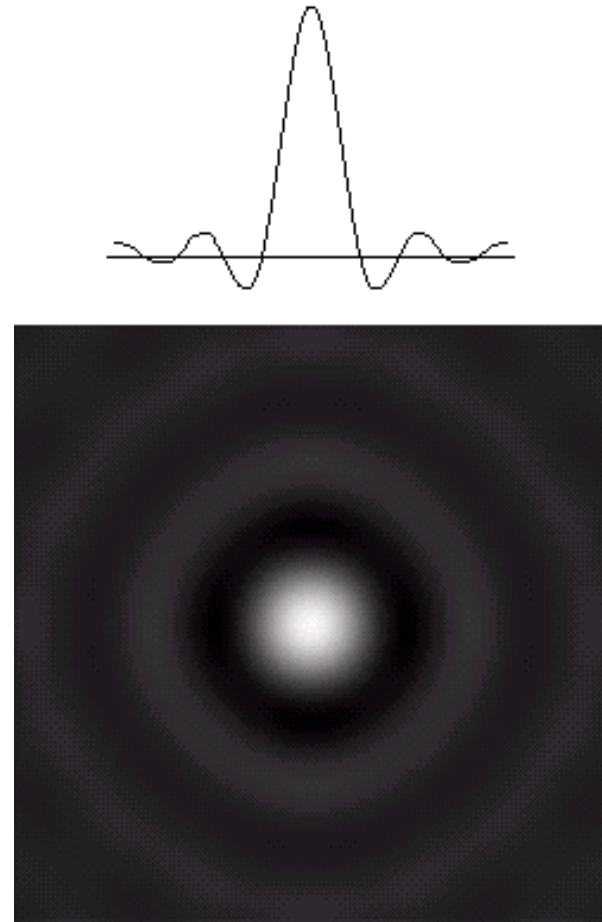
$$\frac{1}{9} \times \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$\frac{1}{16} \times \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$$

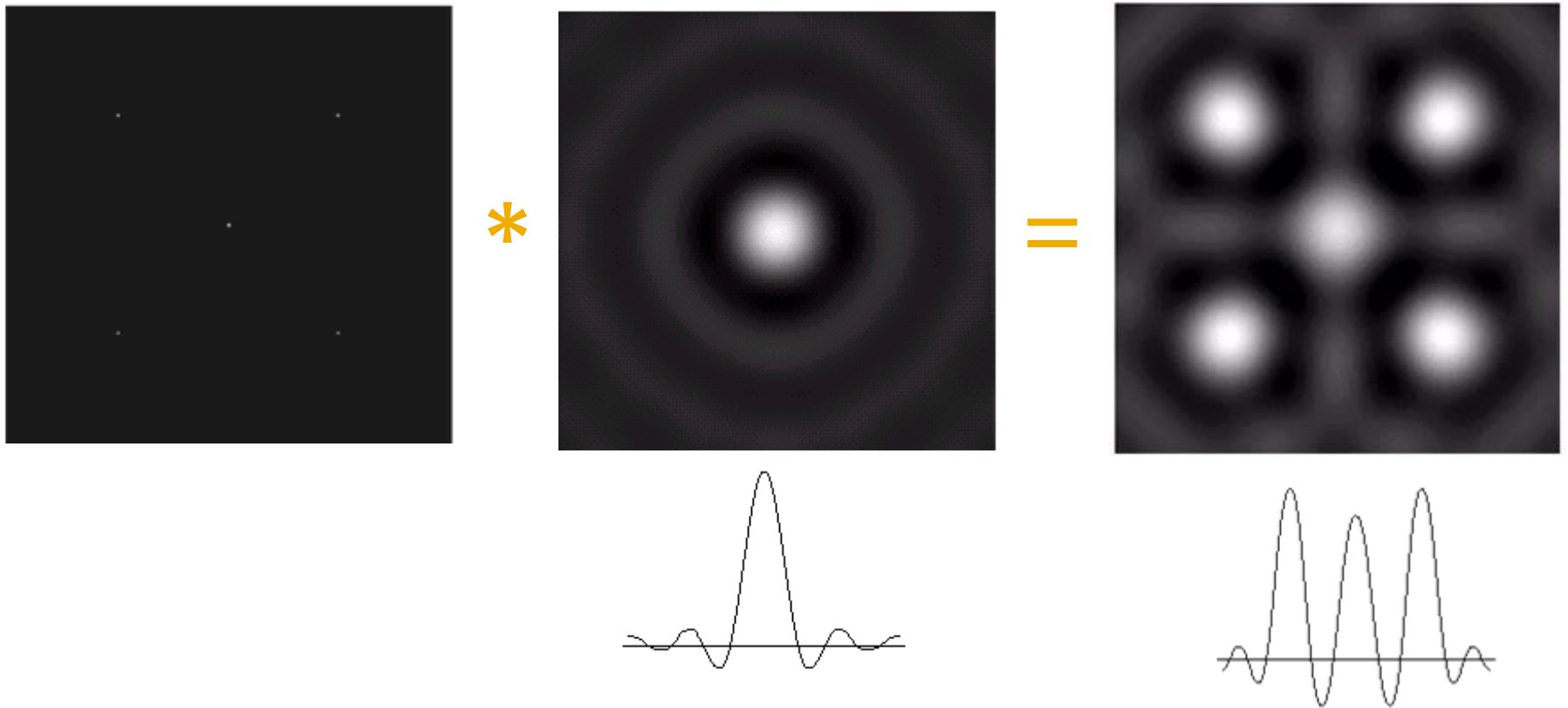
Filtro Passa-Baixa Ideal no domínio da frequência e do espaço



FFT
→



Filtro Passa-Baixa **Ideal** no domínio da frequência e do espaço



Filtro Passa-Baixa **Butterworth** no domínio da frequência e do espaço

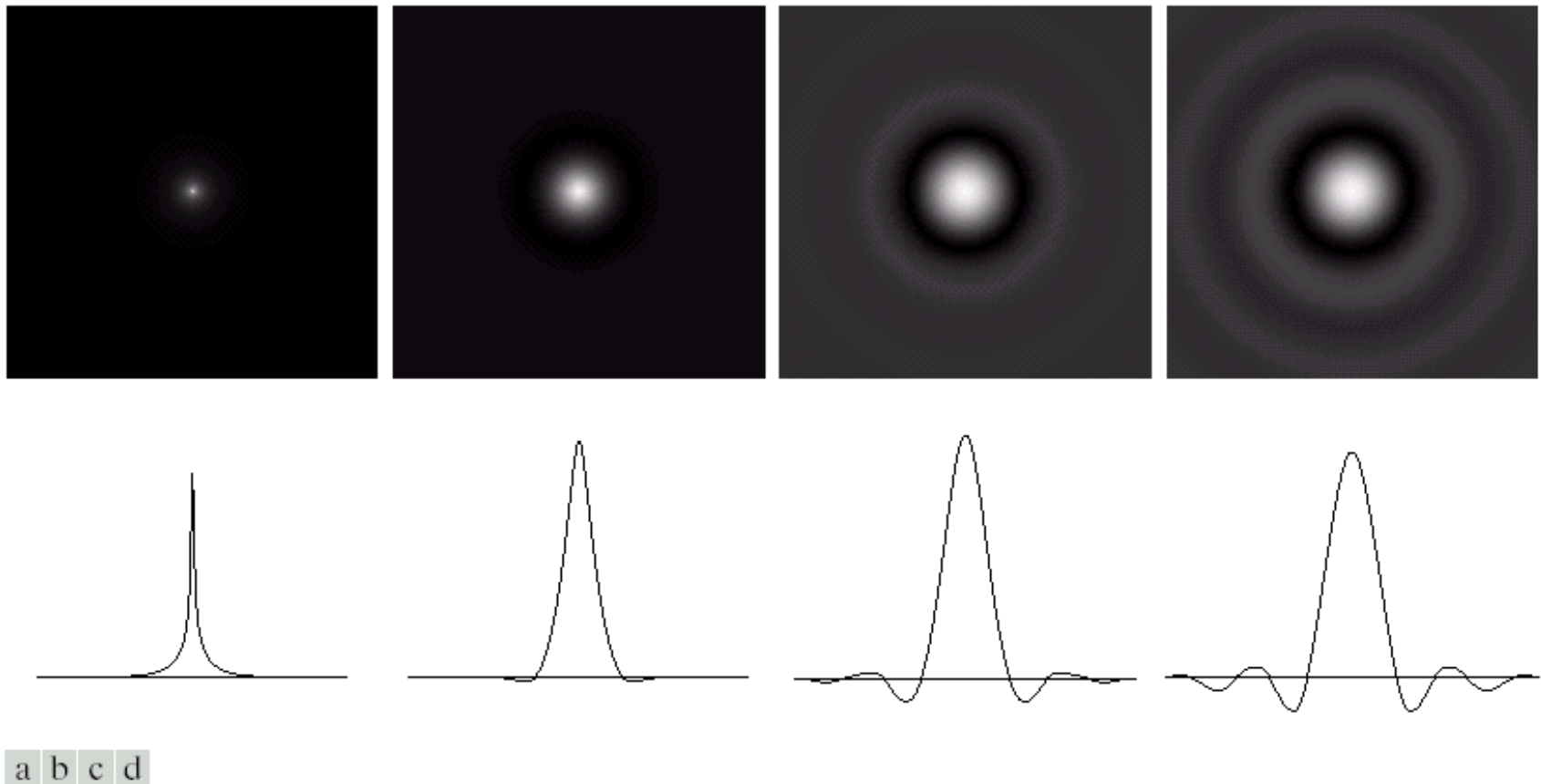
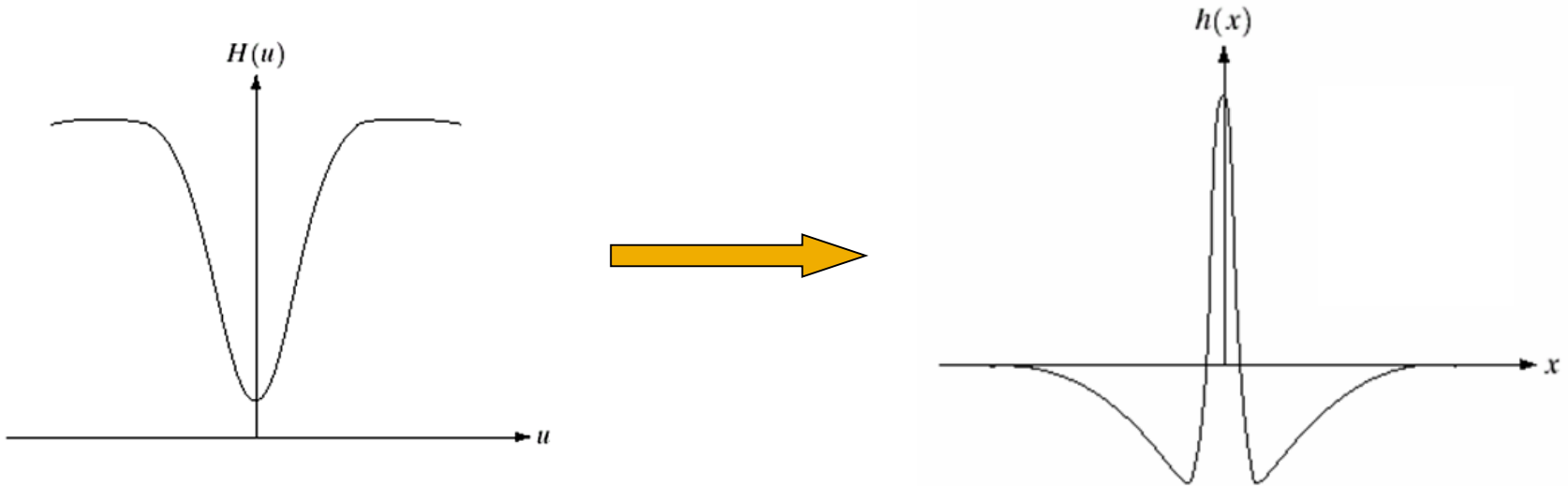


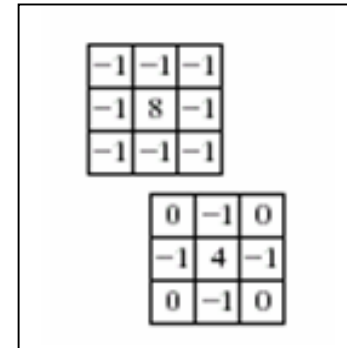
FIGURE 4.16 (a)–(d) Spatial representation of BLPFs of order 1, 2, 5, and 20, and corresponding gray-level profiles through the center of the filters (all filters have a cutoff frequency of 5). Note that ringing increases as a function of filter order.

Filtro Passa-Alta

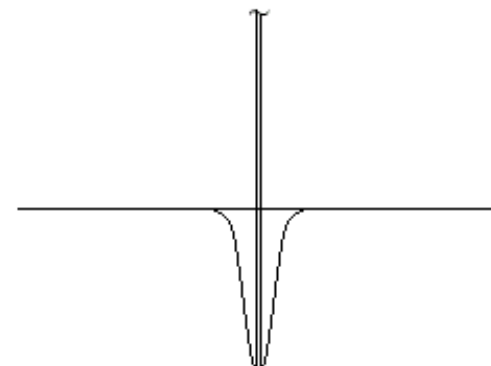
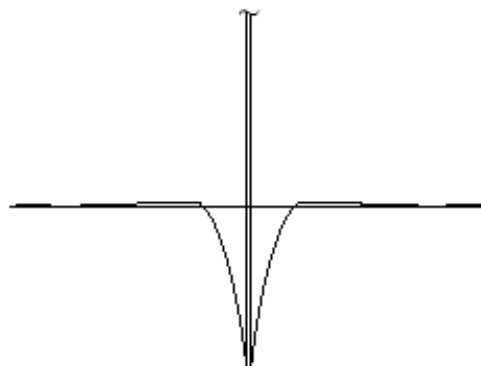
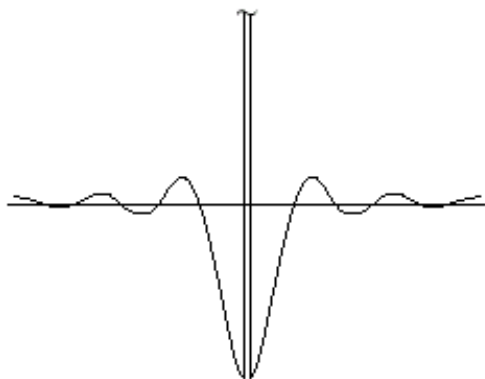
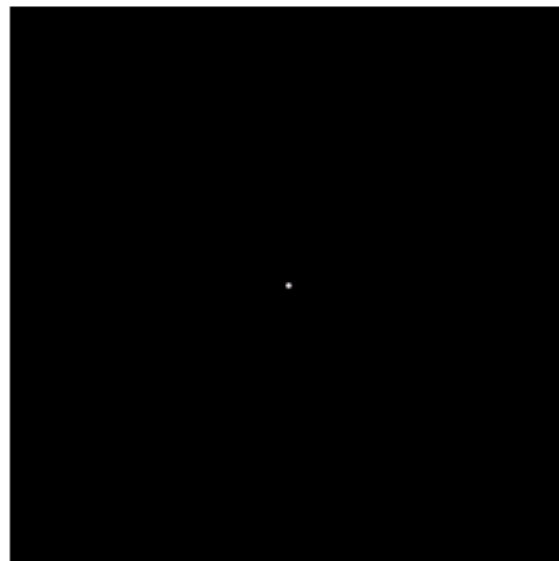
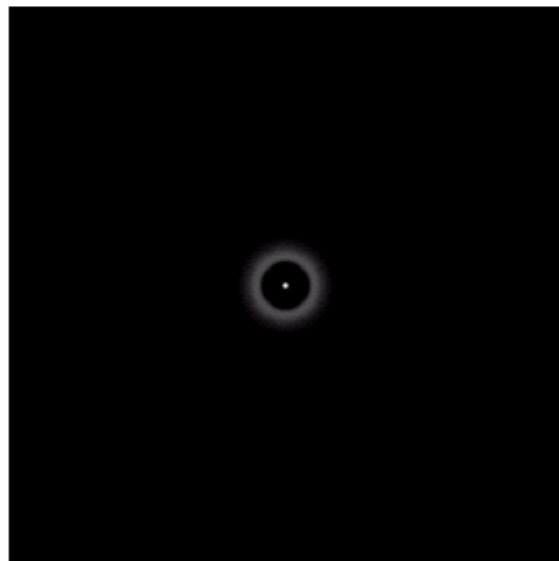
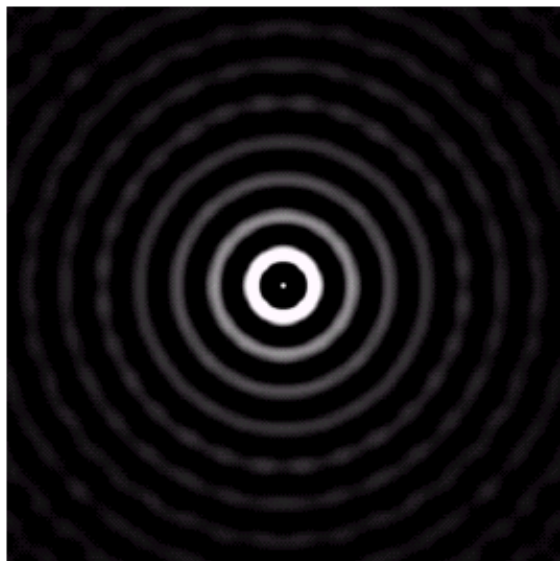
$$H(u) = Ae^{-\frac{u^2}{2\sigma_1^2}} - Be^{-\frac{u^2}{2\sigma_2^2}} \xrightarrow{\text{IFFT}} h(x) = \sqrt{2\pi}\sigma_1 Ae^{-2\pi^2\sigma_1^2 x^2} - \sqrt{2\pi}\sigma_2 Be^{-2\pi^2\sigma_2^2 x^2}$$



Filtros Equivalentes \longrightarrow

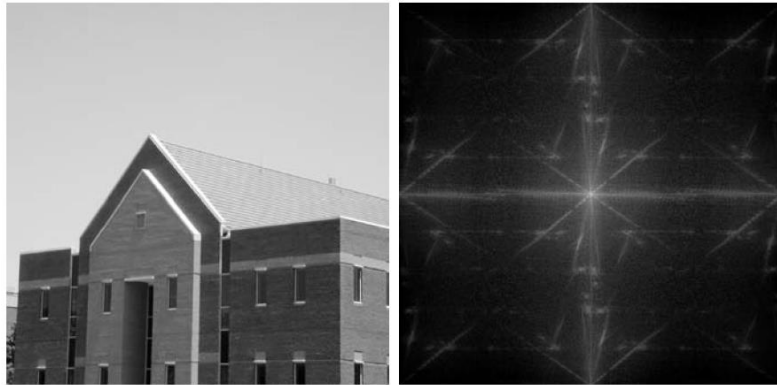


Filtro Passa-Alta Ideal, Butterworth e Gaussiano no domínio do Espaço

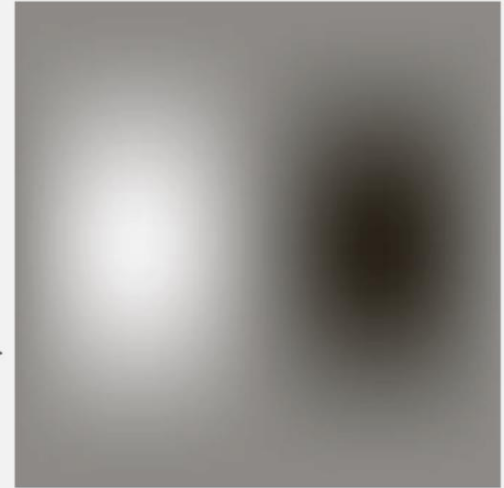
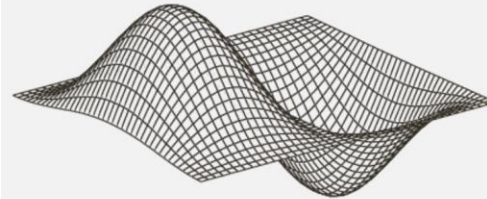


Exemplo de filtro não isotrópico no domínio da frequência

Mudança na fase



-1	0	1
-2	0	2
-1	0	1



FIM