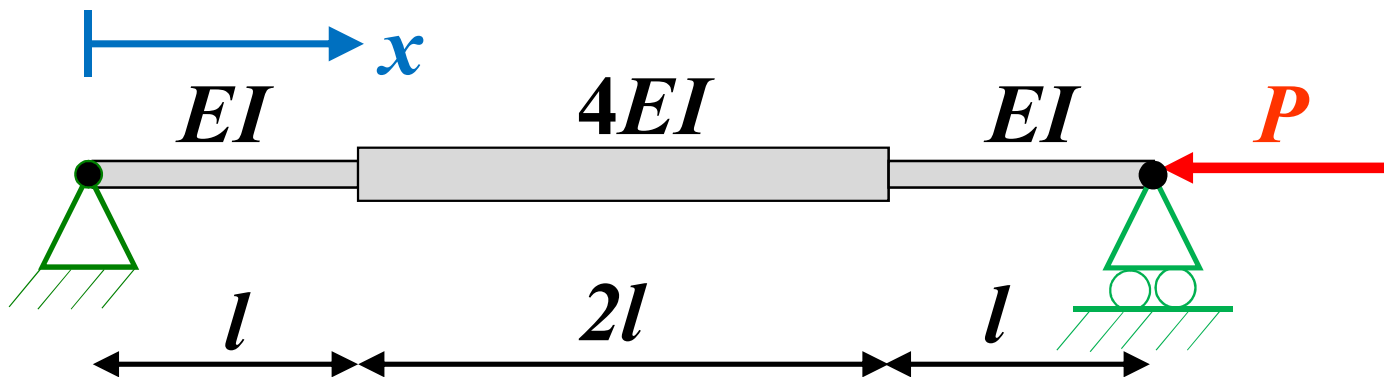
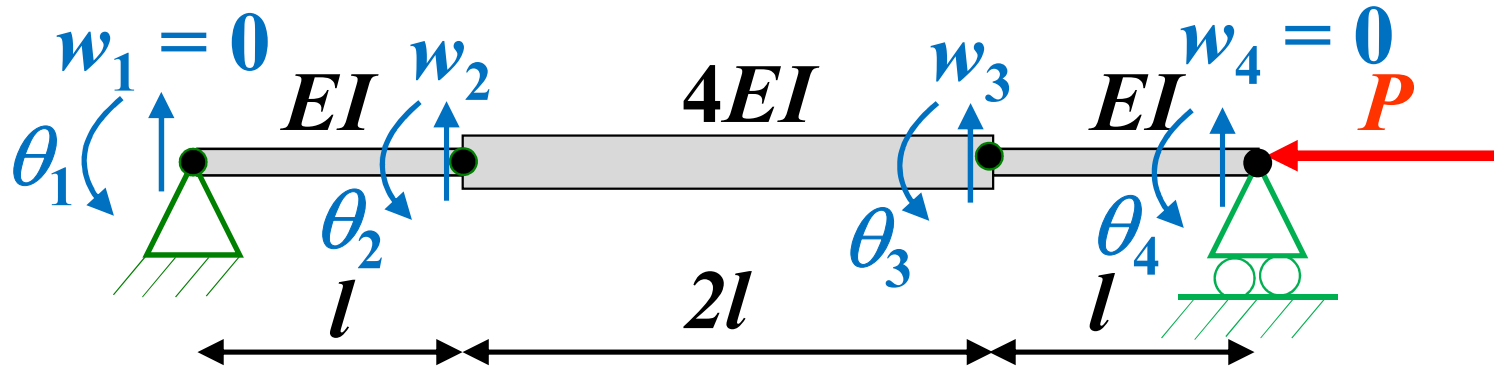


Stability Problems

1. Consider the simply supported beam with the geometry and properties depicted in the figure. The beam is under a compressive load. Assemble a finite element model for the system.



Solution 1



Vectors of nodal displacements

Global	$\begin{Bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \\ \theta_4 \\ w_1 = 0 \\ w_4 = 0 \end{Bmatrix}$	Elem. 1	Elem. 2	Elem. 3
		$\begin{Bmatrix} w_1 = 0 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix}$	$\begin{Bmatrix} w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \end{Bmatrix}$	$\begin{Bmatrix} w_3 \\ \theta_3 \\ w_4 = 0 \\ \theta_4 \end{Bmatrix}$

Stiffness matrices

Elem. 1

$$\frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{matrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{matrix}$$

Elem. 3

$$\frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{matrix} w_3 \\ \theta_3 \\ w_4 \\ \theta_4 \end{matrix}$$

Elem. 2

$$\frac{EI}{2l^3} \begin{bmatrix} 12 & 12l & -12 & 12l \\ 12l & 16l^2 & -12l & 8l^2 \\ -12 & -12l & 12 & -12l \\ 12l & 8l^2 & -12l & 16l^2 \end{bmatrix} \begin{matrix} w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \end{matrix}$$

Global

$$\frac{EI}{l^3} \begin{bmatrix} 4l^2 & -6l & 2l^2 & 0 & 0 & 0 \\ -6l & 18 & 0 & -6 & 6l & 0 \\ 2l^2 & 0 & 12l^2 & -6l & 4l^2 & 0 \\ 0 & -6 & -6l & 18 & 0 & 6l \\ 0 & 6l & 4l^2 & 0 & 12l^2 & 2l^2 \\ 0 & 0 & 0 & 6l & 2l^2 & 4l^2 \end{bmatrix} \begin{matrix} \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \\ \theta_4 \end{matrix}$$

Geometric stiffness matrices

Pre-buckling problem: longitudinal force is equal to P

Elem. 1

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix}$$

Elem. 3

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \begin{bmatrix} w_3 \\ \theta_3 \\ w_4 \\ \theta_4 \end{bmatrix}$$

Elem. 2

$$\frac{1}{30} \frac{P}{2l} \begin{bmatrix} 36 & 6l & -36 & 6l \\ 6l & 16l^2 & -6l & -4l^2 \\ -36 & -6l & 36 & -6l \\ 6l & -4l^2 & -6l & 16l^2 \end{bmatrix} \begin{bmatrix} w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \end{bmatrix}$$

Global

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} 4l^2 & -3l & -l^2 & 0 & 0 & 0 \\ -3l & 54 & 0 & -18 & 3l & 0 \\ -l^2 & 0 & 12l^2 & -3l & -2l^2 & 0 \\ 0 & -18 & -3l & 54 & 0 & 3l \\ 0 & 3l & -2l^2 & 0 & 12l^2 & -l^2 \\ 0 & 0 & 0 & 3l & -l^2 & 4l^2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

Buckling problem:

$$\left[\frac{EI}{l^3} \begin{bmatrix} 4l^2 & -6l & 2l^2 & 0 & 0 & 0 \\ -6l & 18 & 0 & -6 & 6l & 0 \\ 2l^2 & 0 & 12l^2 & -6l & 4l^2 & 0 \\ 0 & -6 & -6l & 18 & 0 & 6l \\ 0 & 6l & 4l^2 & 0 & 12l^2 & 2l^2 \\ 0 & 0 & 0 & 6l & 2l^2 & 4l^2 \end{bmatrix} - \frac{1}{30} \frac{P}{l} \begin{bmatrix} 4l^2 & -3l & -l^2 & 0 & 0 & 0 \\ -3l & 54 & 0 & -18 & 3l & 0 \\ -l^2 & 0 & 12l^2 & -3l & -2l^2 & 0 \\ 0 & -18 & -3l & 54 & 0 & 3l \\ 0 & 3l & -2l^2 & 0 & 12l^2 & -l^2 \\ 0 & 0 & 0 & 3l & -l^2 & 4l^2 \end{bmatrix} \right] \begin{Bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \\ \theta_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

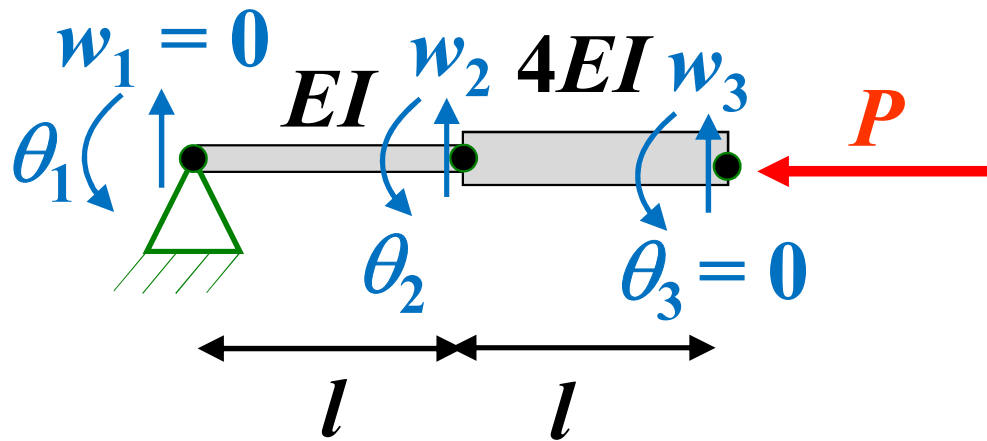
The solution of the above problem yields 6 pairs of eigenvalues / eigenvectors

Both, symmetric and anti-symmetric modes are computed

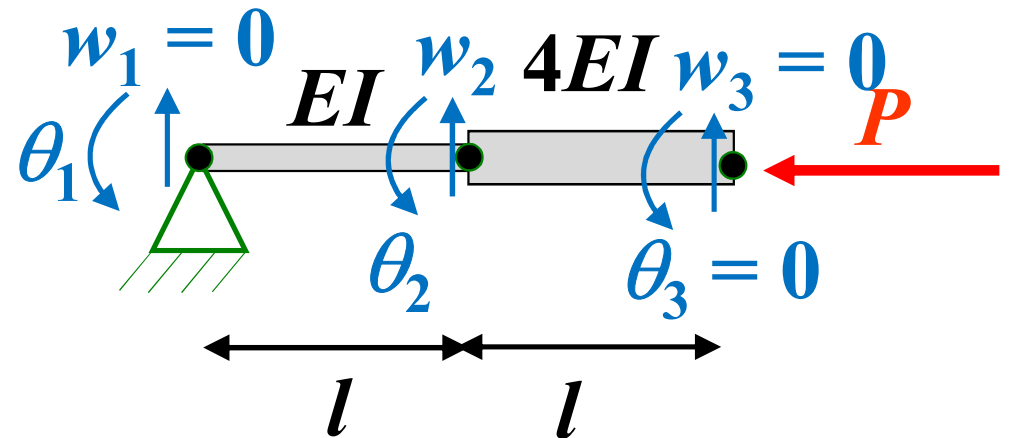
Solution 2

The problem is symmetric but has symmetric and anti-symmetric modes. One alternative is to compute them separately

Symmetric modes



Anti-symmetric modes



Symmetric modes

Vectors of nodal displacements

	Elem. 1	Elem. 2
Global	$\begin{Bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \\ w_1 = 0 \\ \theta_3 = 0 \end{Bmatrix}$	$\begin{Bmatrix} w_2 \\ \theta_2 \\ w_3 \\ \theta_3 = 0 \end{Bmatrix}$

Stiffness matrices

Elem. 1

$$\frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix}$$

Elem. 2

$$\frac{EI}{l^3} \begin{bmatrix} 48 & 24l & -48 & 24l \\ 24l & 16l^2 & -24l & 8l^2 \\ -48 & -24l & 48 & -24l \\ 24l & 8l^2 & -24l & 16l^2 \end{bmatrix} \begin{bmatrix} w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \end{bmatrix}$$

Global

$$\frac{EI}{l^3} \begin{bmatrix} 4l^2 & -6l & 2l^2 & 0 \\ -6l & 60 & 18l & -48 \\ 2l^2 & 18l & 20l^2 & -24l \\ 0 & -48 & -24l & 48 \end{bmatrix} \begin{bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \end{bmatrix}$$

Geometric stiffness matrices

Pre-buckling problem: longitudinal force is equal to P

Elem. 1

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix}$$

Elem. 2

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \begin{bmatrix} w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \end{bmatrix}$$

Global

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} 4l^2 & -3l & -l^2 & 0 \\ -3l & 72 & 0 & -36 \\ -l^2 & 0 & 8l^2 & -3l \\ 0 & -36 & -3l & 36 \end{bmatrix} \begin{bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \end{bmatrix}$$

Buckling problem:

$$\left[\frac{EI}{l^3} \begin{bmatrix} 4l^2 & -6l & 2l^2 & 0 \\ -6l & 60 & 18l & -48 \\ 2l^2 & 18l & 20l^2 & -24l \\ 0 & -48 & -24l & 48 \end{bmatrix} - \frac{1}{30} \frac{P}{l} \begin{bmatrix} 4l^2 & -3l & -l^2 & 0 \\ -3l & 72 & 0 & -36 \\ -l^2 & 0 & 8l^2 & -3l \\ 0 & -36 & -3l & 36 \end{bmatrix} \right] \begin{Bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ w_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

The solution of the above problem yields 4 pairs of eigenvalues / eigenvectors

Only symmetric modes are computed

Since the critical buckling load is symmetric only this analysis have to be performed if the other eigenvalues are of no interest

Anti-symmetric modes

Vectors of nodal displacements

	Elem. 1	Elem. 2
Global	$\begin{Bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ \theta_3 \\ w_1 = 0 \\ w_3 = 0 \end{Bmatrix}$	$\begin{Bmatrix} w_2 \\ \theta_2 \\ w_3 = 0 \\ \theta_3 \end{Bmatrix}$

Stiffness matrices

Elem. 1

$$\frac{EI}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \begin{matrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{matrix}$$

Elem. 2

$$\frac{EI}{l^3} \begin{bmatrix} 48 & 24l & -48 & 24l \\ 24l & 16l^2 & -24l & 8l^2 \\ -48 & -24l & 48 & -24l \\ 24l & 8l^2 & -24l & 16l^2 \end{bmatrix} \begin{matrix} w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \end{matrix}$$

Global

$$\frac{EI}{l^3} \begin{bmatrix} 4l^2 & -6l & 2l^2 & 0 \\ -6l & 60 & 18l & 24l \\ 2l^2 & 18l & 20l^2 & 8l^2 \\ 0 & 24l & 8l^2 & 16l^2 \end{bmatrix} \begin{matrix} \theta_1 \\ w_2 \\ \theta_2 \\ \theta_3 \end{matrix}$$

Geometric stiffness matrices

Pre-buckling problem: longitudinal force is equal to P

Elem. 1

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \begin{bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{bmatrix}$$

Elem. 2

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix} \begin{bmatrix} w_2 \\ \theta_2 \\ w_3 \\ \theta_3 \end{bmatrix}$$

Global

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} 4l^2 & -3l & -l^2 & 0 \\ -3l & 72 & 0 & 3l \\ -l^2 & 0 & 8l^2 & -l^2 \\ 0 & 3l & -l^2 & 4l^2 \end{bmatrix} \begin{bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ \theta_3 \end{bmatrix}$$

Buckling problem:

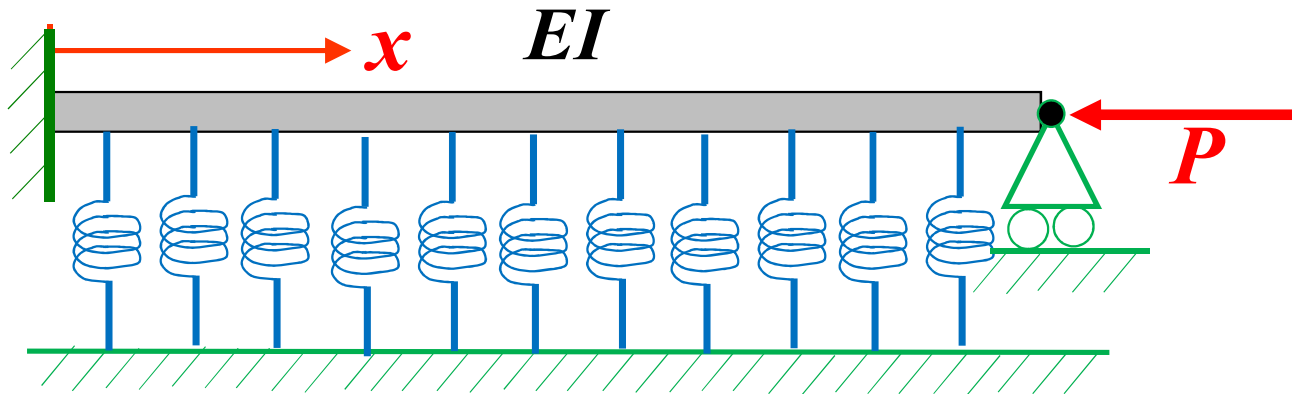
$$\left[\frac{EI}{l^3} \begin{bmatrix} 4l^2 & -6l & 2l^2 & 0 \\ -6l & 60 & 18l & 24l \\ 2l^2 & 18l & 20l^2 & 8l^2 \\ 0 & 24l & 8l^2 & 16l^2 \end{bmatrix} - \frac{1}{30} \frac{P}{l} \begin{bmatrix} 4l^2 & -3l & -l^2 & 0 \\ -3l & 72 & 0 & 3l \\ -l^2 & 0 & 8l^2 & -l^2 \\ 0 & 3l & -l^2 & 4l^2 \end{bmatrix} \right] \begin{Bmatrix} \theta_1 \\ w_2 \\ \theta_2 \\ \theta_3 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

The solution of the above problem yields 4 pairs of eigenvalues / eigenvectors

Only anti-symmetric modes are computed

Since the critical buckling load is symmetric this analysis does not have to be performed if the other eigenvalues are of no interest

2. A uniform straight beam of length l and flexural stiffness EI is clamped at $x = 0$ and simply supported at $x = l$. The beam is also attached to an elastic foundation of constant stiffness k per unit length. Derive the finite element formulation for the elastic foundation and estimate the buckling load using a single element.



Solution

The first step is to obtain the stiffness matrix of the elastic foundation.

The infinitesimal stored energy is:

$$dV = \frac{1}{2} k(x) dx w^2(x)$$

The transverse displacements $w(x)$ are given by: $w(\xi) = [N(\xi)] \{w_\theta\}$

Therefore: $w^2(\xi) = \{w_\theta\}^T [N(\xi)]^T [N(\xi)] \{w_\theta\}$

where $[N(\xi)]$ are the interpolation functions and $\{w_\theta\}$ are nodal displacements of the beam:

$$\{w_\theta\}^T = [w_1 \quad l\theta_1 \quad w_2 \quad l\theta_2]$$

The total stored energy is:

$$V = \frac{1}{2} \{w_\theta\}^T \left[k \int_0^l [N(x)]^T [N(x)] dx \right] \{w_\theta\} = \frac{1}{2} \{w_\theta\}^T \left[k \int_0^1 [N(\xi)]^T [N(\xi)] l d\xi \right] \{w_\theta\}$$

In the problem, only θ_2 is different from 0, therefore:

$$V = \frac{1}{2} \{l\theta_2\}^T \left[kl \int_0^1 [N_4(\xi)]^T [N_4(\xi)] d\xi \right] \{l\theta_2\} = \frac{1}{2} \left[kl \int_0^1 N_4^2(\xi) d\xi \right] l^2 \theta_2^2$$

$N_4(\xi)$ is: $N_4(\xi) = -\xi^2 + \xi^3 \longrightarrow \int_0^1 N_4^2(\xi) d\xi = \frac{1}{105}$

Therefore: $V = \frac{1}{2} \{l\theta_2\}^T \left[\frac{kl}{105} \right] \{l\theta_2\}$

Stiffness matrices:

$$\frac{EI}{l^3} \begin{bmatrix} & w_1 & l\theta_1 & & w_2 & l\theta_2 \\ 12 & 6 & -12 & 6 & & \\ 6 & 4 & -6 & 2 & & \\ -12 & -6 & 12 & -6 & & \\ 6 & 2 & -6 & 4 & & \end{bmatrix} \begin{bmatrix} w_1 \\ l\theta_1 \\ w_2 \\ l\theta_2 \end{bmatrix} + \begin{bmatrix} l\theta_2 \\ \frac{kl}{105} l\theta_2 \end{bmatrix}$$

The pre-buckling problem is not affected by the elastic foundation

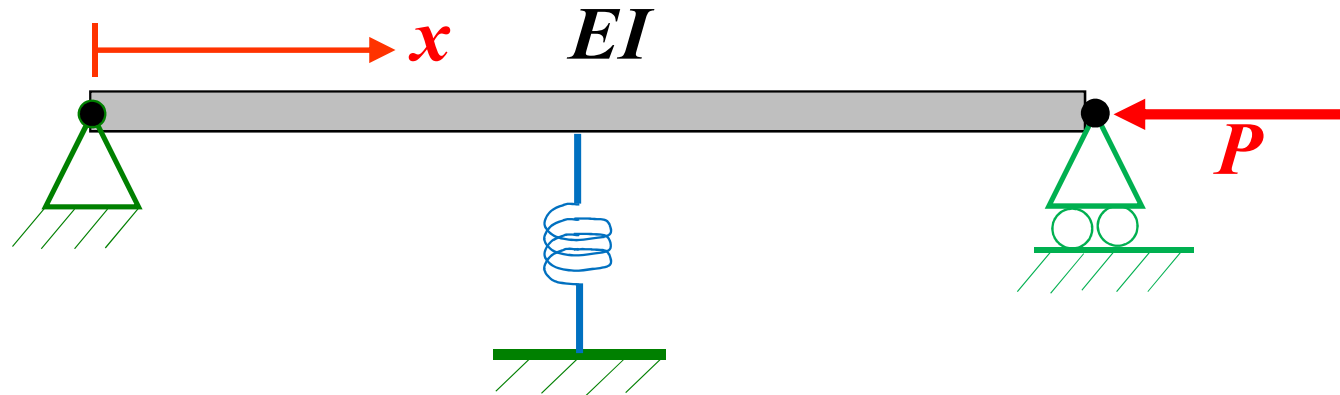
Geometric stiffness matrices:

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} & w_1 & l\theta_1 & & w_2 & l\theta_2 \\ 36 & 3 & -36 & 3 & & \\ 3 & 4 & -3 & -1 & & \\ -36 & -3 & 36 & -3 & & \\ 3 & -1 & -3 & 4 & & \end{bmatrix} \begin{bmatrix} w_1 \\ l\theta_1 \\ w_2 \\ l\theta_2 \end{bmatrix}$$

Keeping only the values for θ_2 :

$$\left(\frac{4EI}{l^3} + \frac{kl}{105} - \frac{4}{30} \frac{P}{l} \right) (l\theta_2) = 0 \quad \Rightarrow \quad \frac{4}{30} \frac{P_{crit}}{l} = \frac{4EI}{l^3} + \frac{kl}{105} \quad \Rightarrow \quad P_{crit} = \frac{30EI}{l^2} + \frac{kl^2}{14}$$

3. A uniform straight beam of length l and flexural stiffness EI is simply supported at both ends and is compressed by a load P . The beam is attached to a spring of stiffness K at its center. Estimate the buckling load for the first symmetric mode using a single element.



Solution

The energy stored at the spring is:

$$V = \frac{1}{2} K w^2 \left(\frac{l}{2} \right) \quad \text{but:} \quad x = \frac{l}{2} \quad \longrightarrow \quad \xi = \frac{1}{2}$$

$$w_1 = w_2 = 0 \quad \longrightarrow \quad w \left(\frac{1}{2} \right) = N_2 \left(\frac{1}{2} \right) l \theta_1 + N_4 \left(\frac{1}{2} \right) l \theta_2 = \frac{1}{8} l \theta_1 - \frac{1}{8} l \theta_2$$

$$\longrightarrow \quad w^2 \left(\frac{l}{2} \right) = \begin{Bmatrix} l \theta_1 \\ l \theta_2 \end{Bmatrix}^T \begin{bmatrix} \frac{1}{8} & -\frac{1}{8} \end{bmatrix}^T \begin{bmatrix} \frac{1}{8} & -\frac{1}{8} \end{bmatrix} \begin{Bmatrix} l \theta_1 \\ l \theta_2 \end{Bmatrix} = \frac{1}{64} \begin{Bmatrix} l \theta_1 \\ l \theta_2 \end{Bmatrix}^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} l \theta_1 \\ l \theta_2 \end{Bmatrix}$$

$$\longrightarrow \quad V = \frac{1}{2} \frac{K}{64} \begin{Bmatrix} l \theta_1 \\ l \theta_2 \end{Bmatrix}^T \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} l \theta_1 \\ l \theta_2 \end{Bmatrix}$$

Stiffness matrices:

$$\frac{EI}{l^3} \begin{bmatrix} & w_1 & l\theta_1 & w_2 & l\theta_2 \\ 12 & 6 & -12 & 6 \\ 6 & 4 & -6 & 2 \\ -12 & -6 & 12 & -6 \\ 6 & 2 & -6 & 4 \end{bmatrix} \begin{matrix} w_1 \\ l\theta_1 \\ w_2 \\ l\theta_2 \end{matrix} \quad \frac{K}{64} \begin{bmatrix} & l\theta_1 & l\theta_2 \\ 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{matrix} l\theta_1 \\ l\theta_2 \end{matrix}$$

The pre-buckling problem is not affected by the spring

Geometric stiffness matrices:

$$\frac{1}{30} \frac{P}{l} \begin{bmatrix} & w_1 & l\theta_1 & w_2 & l\theta_2 \\ 36 & 3 & -36 & 3 \\ 3 & 4 & -3 & -1 \\ -36 & -3 & 36 & -3 \\ 3 & -1 & -3 & 4 \end{bmatrix} \begin{matrix} w_1 \\ l\theta_1 \\ w_2 \\ l\theta_2 \end{matrix}$$

Keeping only the values for θ_1 and θ_2 :

$$\left[\frac{EI}{l^3} \begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix} + \frac{K}{64} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} - \frac{1}{30} \frac{P}{l} \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix} \right] \begin{Bmatrix} l\theta_1 \\ l\theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Therefore:

$$\begin{bmatrix} 4 + \alpha - 4\lambda & 2 - \alpha + \lambda \\ 2 - \alpha + \lambda & 4 + \alpha - 4\lambda \end{bmatrix} \begin{Bmatrix} l\theta_1 \\ l\theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad \text{where:} \quad \alpha = \frac{Kl^3}{64EI}$$
$$\lambda = \frac{Pl^2}{30EI}$$

$$\Rightarrow \det \begin{bmatrix} 4 + \alpha - 4\lambda & 2 - \alpha + \lambda \\ 2 - \alpha + \lambda & 4 + \alpha - 4\lambda \end{bmatrix} = 0 \quad \Rightarrow \quad 15\lambda^2 - (36 + 6\alpha)\lambda + 12(1 + \alpha) = 0$$

$$\lambda_{crit} = \frac{36 + 6\alpha - \sqrt{(36 + 6\alpha)^2 - 4(15)12(1 + \alpha)}}{30} = \frac{36 + 6\alpha - \sqrt{576 - 288\alpha + 36\alpha^2}}{30}$$

$$\lambda_{crit} = \frac{36 + 6\alpha - \sqrt{576 - 288\alpha + 36\alpha^2}}{30}$$

$$P_{crit} = 30 \frac{EI \lambda_{crit}}{l^2}$$

The value of the eigenvalue is bounded by: $0 \leq \lambda_{crit} \leq 2$

4. Consider a uniform, simply supported square plate with length l and thickness t . The modulus of elasticity is E and the Poisson ratio is ν . The plate is compressed by a uniaxial load N_x . Estimate the thickness for which the critical buckling load coincides with the load that causes yielding. The yielding stress is σ_y . Note that, for the intended application, wing panels should not buckle or yield.

Solution

Stress in the plate:

$$\sigma_x = \frac{N_x}{t} \leq \sigma_y$$

Critical buckling load:

$$N_{x,crit} = \frac{k_c \pi^2 D}{b^2} \quad \text{where: } a = b = l \text{ and } n = 1$$

$$D = \frac{Et^3}{12(1-\nu^2)} \quad k_c = \left(\frac{mb}{a} + \frac{n^2 a}{mb} \right)^2 = \left(m + \frac{1}{m} \right)^2$$

The minimum value for k_c is for $m = 1$: $k_c = 4$

$$\Rightarrow N_{x,crit} = \frac{4\pi^2 D}{l^2} = \frac{4\pi^2 Et^3}{12(1-\nu^2)l^2}$$

The minimum value for k_c is for $m = 1$:

$$\sigma_{x,crit} = \frac{N_{x,crit}}{t} = \frac{4\pi^2 E t^2}{12(1-\nu^2)l^2} \leq \sigma_y$$

Critical buckling stress = yield stress:

$$\frac{3\pi^2 E}{(1-\nu^2)} \frac{t^2}{l^2} = \sigma_y \quad \Rightarrow \quad t = l \sqrt{\frac{3(1-\nu^2)}{\pi^2} \frac{\sigma_y}{E}}$$