Equações de Maxwell aplicado a interface de dois meios dielétricos

1) $\boxed{\text{Mais geral,}}$ mas já aplicado a interface

$$(\vec{D}_2 - \vec{D}_1)\cdot\hat{n} = \sigma$$

$\vec{D} = \varepsilon E$
↳ Deslocamento elétrico
↳ permissividade elétrica

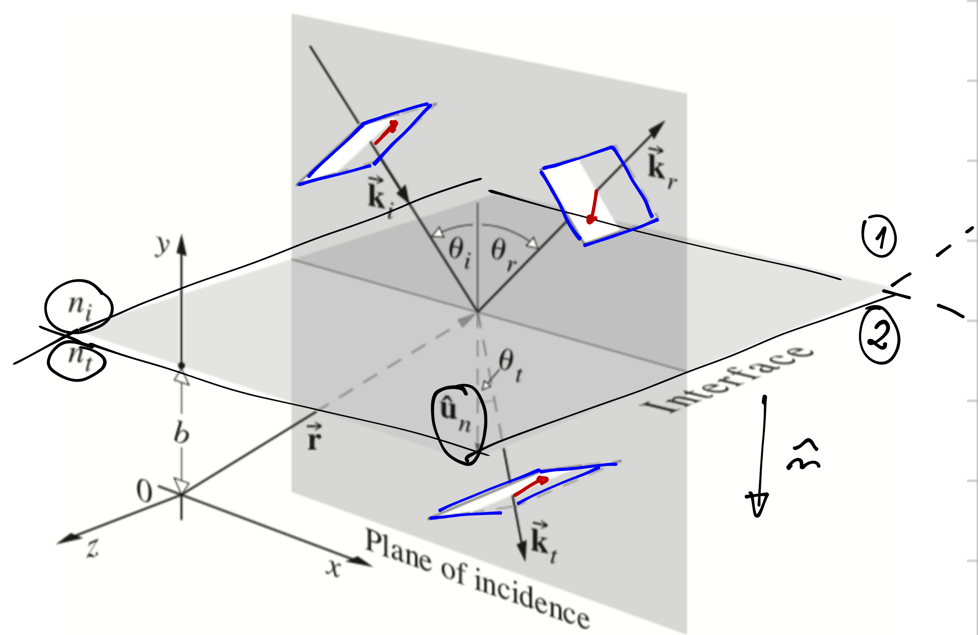

Figure 4.45 Plane waves incident on the boundary between two homogeneous, isotropic, lossless dielectric media.

$$\vec{B}_1\cdot\hat{n} = \vec{B}_2\cdot\hat{n}$$

$$\vec{E}_1\cdot d\vec{l} = \vec{E}_2\cdot dl$$

$$\hat{n}\times(\vec{H}_2 - \vec{H}_1) = \vec{J}$$

$\vec{H} = \dfrac{\vec{B}}{\mu}$ → permeab. magnética

$\vec{J}$ = corrente de deslocamento

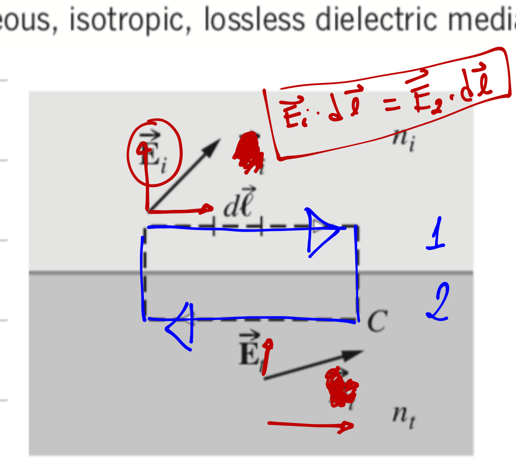

Figure 4.46 Boundary conditions at the interface between two dielectrics

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mais restrito → que não há cargas acumuladas na interface $\sigma = 0$
→ que não há corrente de deslocamento $\vec{J} = 0$

$(\vec{D}_2 - \vec{D}_1)\cdot\hat{n} = 0$ ⟵ $\oint_A \vec{E}\cdot d\vec{s} = \dfrac{1}{\varepsilon_0}\int \sigma\, dV$

$(\vec{B}_2 - \vec{B}_1)\cdot\hat{n} = 0$ ⟵ $\oint_A \vec{B}\cdot d\vec{s} = 0$

$(\vec{E}_2 - \vec{E}_1)\cdot d\vec{l} = 0$ ⟵ $\oint_C \vec{E}\cdot d\vec{l} = -\dfrac{d}{dt}\int_A \vec{B}\cdot d\vec{s}$

$\hat{n}\times(\vec{H}_2 - \vec{H}_1) = 0$

$$\oint_C \vec{B}\cdot d\vec{l} = \mu \int_A \left(\vec{J} + \epsilon \frac{\partial E}{\partial t}\right)\cdot d\vec{S}$$

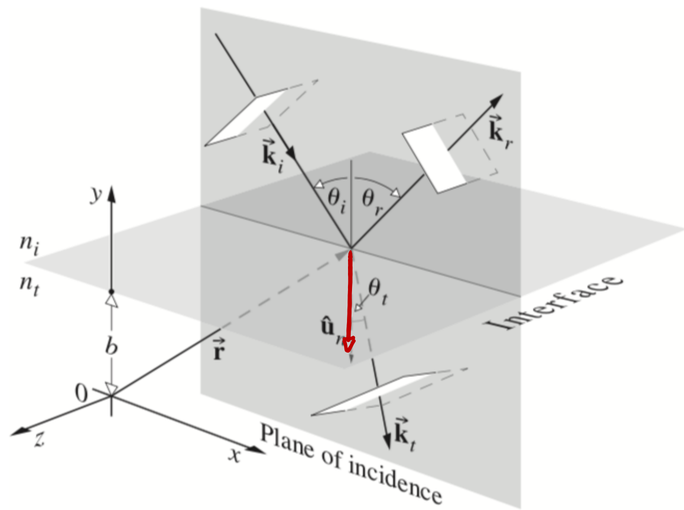


Figure 4.45 Plane waves incident on the boundary between two homogeneous, isotropic, lossless dielectric media.

$$\vec{E}_i = \vec{E}_{0i} \exp\left[i \vec{K}_i \cdot \vec{r} - \omega_i t\right]$$

ou

$$\vec{E}_i = \vec{E}_{0i} \cos\left(\vec{K}_i \cdot \vec{r} - \omega_i t\right)$$

↳ onda incidente

$$\vec{E}_r = \vec{E}_{0r} \cos\left(\vec{K}_r \cdot \vec{r} - \omega_r t + \varepsilon_r\right)$$

↳ onda refletida

$$\vec{E}_t = \vec{E}_{0t} \cos\left(\vec{K}_t \cdot \vec{r} - \omega_t t + \varepsilon_t\right)$$

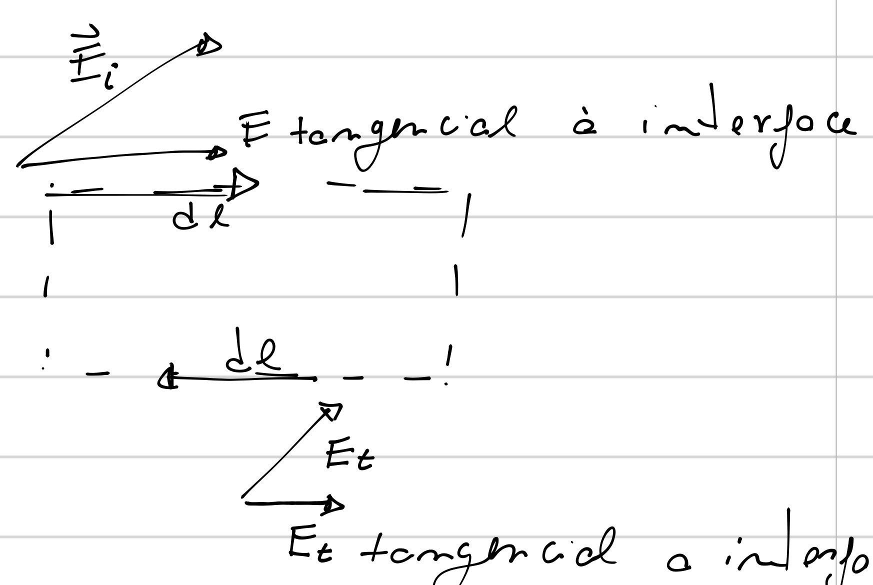
↳ onda transmitida

$\omega_i = \omega_r = \omega_t = \omega$ frequência não muda ao passar do meio 1 → 2 ou $i \to r$ ou $i \to t$

$\boxed{\omega = 2\pi f}$

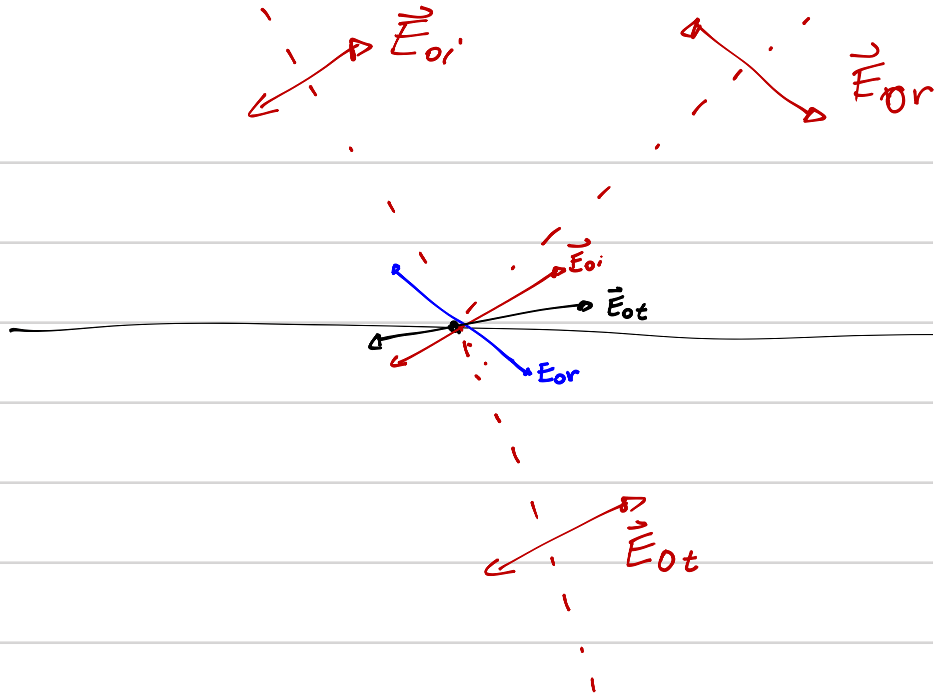
$\boxed{v = \lambda \cdot f}$ → constante

$(\vec{E}_2 - \vec{E}_1)\cdot d\vec{l} = 0 \quad \Rightarrow$



$$\boxed{\vec{E}_{2\,tang} = \vec{E}_{1\,tangencial}}$$

↳ Válido na interface entre o meio 1 e 2



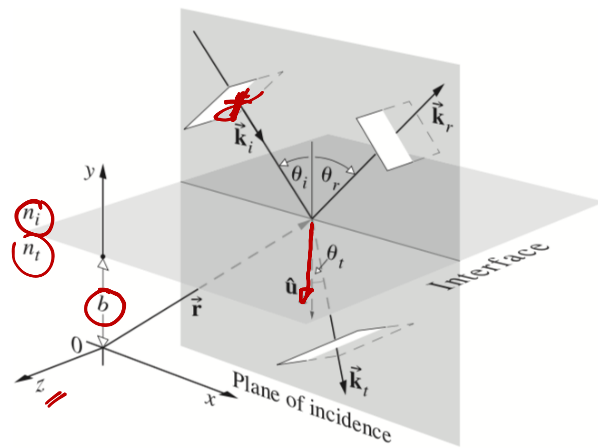


Figure 4.45 Plane waves incident on the boundary between two homogeneous, isotropic, lossless dielectric media.

$\hat{u} \times E \Rightarrow$ Seleciono a componente tangencial a interface

$$\boxed{\vec{E}_{1\,tang} = \vec{E}_{2\,tang}}$$

$$\hat{u} \times \vec{E}_i + \hat{u} \times \vec{E}_r = \hat{u} \times \vec{E}_t$$

$$\hat{u} \times \vec{E}_{oi} \cos(\vec{K}_i \cdot \vec{r} - \omega t) + \hat{u} \times \vec{E}_{oi} \cos(\vec{K}_r \cdot \vec{r} - \omega t + \varepsilon_r) =$$
$$= \hat{u} \times \vec{E}_{ot} \cos(\vec{K}_t \cdot \vec{r} - \omega t + \varepsilon_t)$$

$\Rightarrow$ Preciso ser válido em qualquer tempo e em qualquer posição na interface $y = b$

para isto ser válido precisamos ter a igualdade nos argumentos

$$(\vec{K}_i \cdot \vec{r} - \omega t)\Big|_{y=b} = (\vec{K}_r \cdot \vec{r} - \omega t + \varepsilon_r)\Big|_{y=b} = (\vec{K}_t \cdot \vec{r} - \omega t + \varepsilon_t)\Big|_{y=b}$$

$t = 0$ $\quad \varepsilon_r$ e $\varepsilon_t \Rightarrow$ eliminar $= 0$

$$(\vec{K}_i \cdot \vec{r}) = (\vec{K}_r \cdot \vec{r}) = (\vec{K}_t \cdot \vec{r})$$

para um dos casos particulares $\boxed{z = 0}$

$\vec{K} = K_x \hat{i} + K_y \hat{j} + K_z \hat{k}$ $\qquad \vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$

apenas p/ simplificar o valor $\vec{K}$

$$\boxed{(K_{xi}\cdot x + K_{yi}\cdot y) = (K_{xr}\cdot x + K_{yr}\cdot y) = (K_{xt}\cdot x + K_{yt}\cdot y)}$$

p/ um exemplo $y=0$ (linha no plano)
uma condição particular

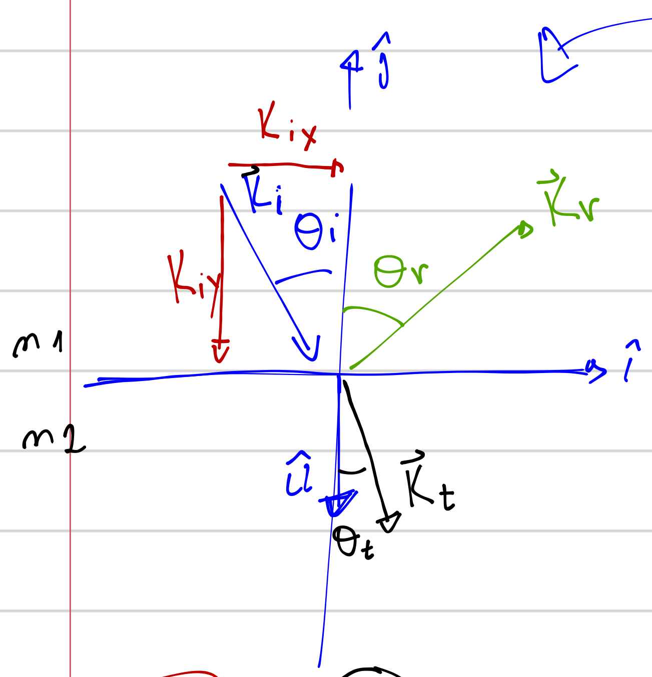
$$K_{xi}\cancel{x} = K_{xr}\cancel{x} = K_{xt}\cancel{x}$$

$$\boxed{K_{xi} = K_{xr} = K_{xt}}$$

p/ outra condição particular $x=0$

$$\boxed{K_{yi} = K_{yr} = K_{yt}}$$

→ Para qualquer uma das configurações $\left(\begin{smallmatrix} y=0 \\ x=0 \end{smallmatrix}\right)$ é válido obter uma relação para K_i, K_r e K_t



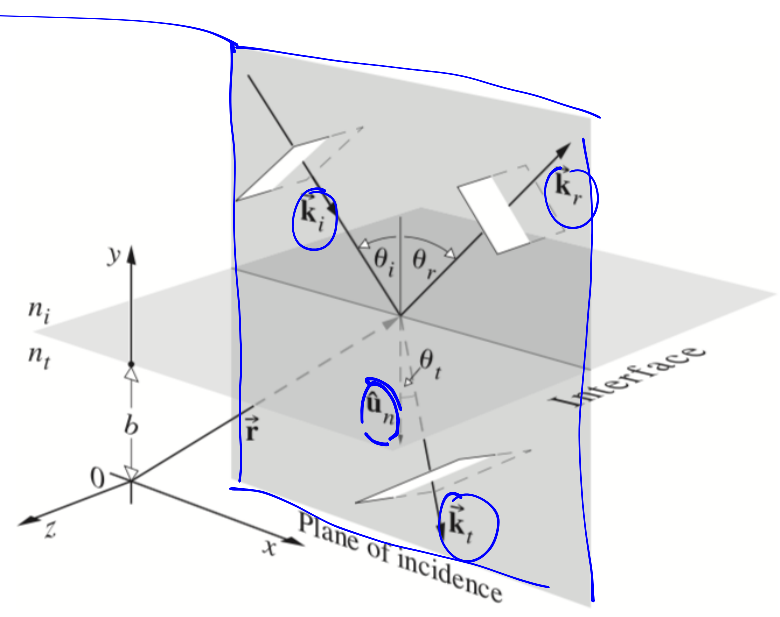


Figure 4.45 Plane waves incident on the boundary between two homogeneous, isotropic, lossless dielectric media.

$$\hat{u}\times\vec{K}_i = K_{xi} = K_i \,\text{Sen}\,\theta_i$$

$$\hat{u}\times\vec{K}_t = K_{xt} = K_t \,\text{Sen}\,\theta_t$$

$$\hat{u}\times\vec{K}_r = K_{xr} = K_r \,\text{Sen}\,\theta_r$$

$$K_i \operatorname{Sen}\theta_i = K_t \operatorname{Sen}\theta_t = K_r \operatorname{Sen}\theta_r$$

$$K = \frac{2\pi}{\lambda} \Rightarrow \lambda = v/f \qquad K = \frac{\overbrace{2\pi f}^{=\omega}}{v} \qquad v = \frac{c}{n}$$

$$K = \frac{\omega}{c/n}$$

$$K_i = n_i\left(\frac{\omega}{c}\right)$$

$$K_r = n_r\left(\frac{\omega}{c}\right)$$

$$K_t = n_t\left(\frac{\omega}{c}\right)$$

$$n_i = n_r = n_i$$

$$\boxed{n_i \operatorname{Sen}\theta_i = n_t \operatorname{Sen}\theta_t}$$

→ refração

→ $\boxed{n_1 \operatorname{Sen}\theta_i = n_2 \operatorname{Sen}\theta_t}$

$$\cancel{K_i} \operatorname{Sen}\theta_i = \cancel{K_r} \operatorname{Sen}\theta_r$$

$$\boxed{\theta_i = \theta_r}$$

lei da reflexão

— x — x — x —