MAP 2210 – Aplicações de Álgebra Linear
1º Semestre - 2019

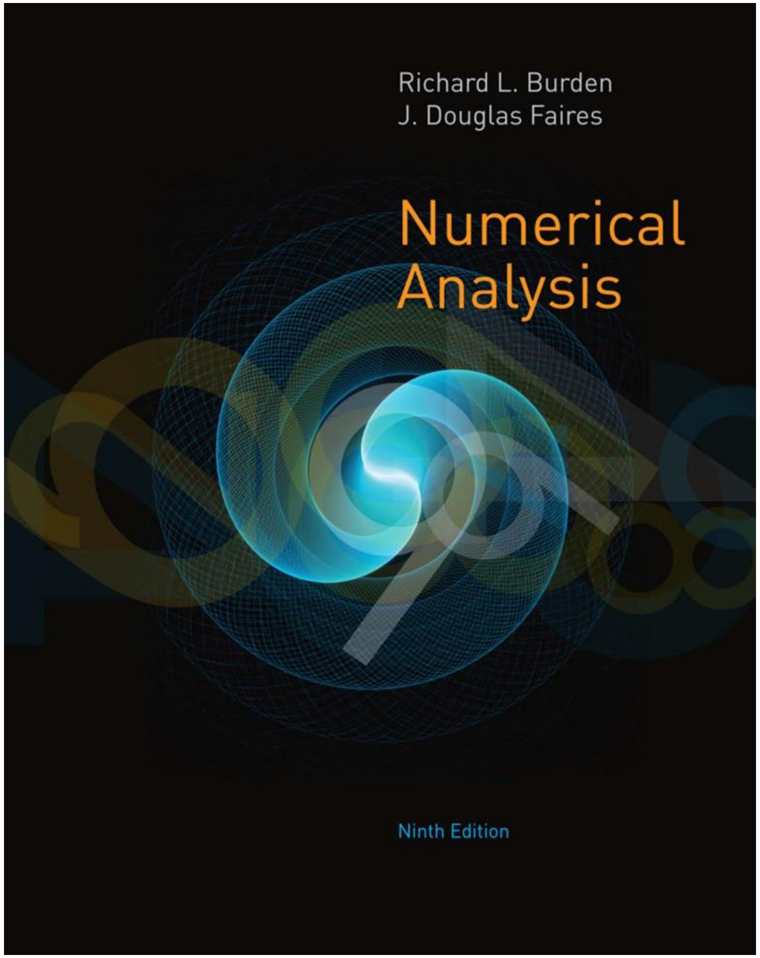
Prof. Dr. Luis Carlos de Castro Santos

lsantos@ime.usp.br

Objetivos

Formação básica de álgebra linear aplicada a problemas numéricos. Resolução de problemas em microcomputadores usando linguagens e/ou software adequados fora do horário de aula.

MAP2210



Numerical Analysis

NINTH EDITION

Richard L. Burden
Youngstown State University

J. Douglas Faires
Youngstown State University

EXERCISE SET 6.1

3. Use Gaussian elimination with backward substitution and two-digit rounding arithmetic to solve the following linear systems. Do not reorder the equations. (The exact solution to each system is $x_1 = 1, x_2 = -1, x_3 = 3$.)

a.
$$\begin{aligned} 4x_1 - x_2 + x_3 &= 8, \\ 2x_1 + 5x_2 + 2x_3 &= 3, \\ x_1 + 2x_2 + 4x_3 &= 11. \end{aligned}$$

b.
$$\begin{aligned} 4x_1 + x_2 + 2x_3 &= 9, \\ 2x_1 + 4x_2 - x_3 &= -5, \\ x_1 + x_2 - 3x_3 &= -9. \end{aligned}$$

10. Given the linear system

$$\begin{aligned} x_1 - x_2 + \alpha x_3 &= -2, \\ -x_1 + 2x_2 - \alpha x_3 &= 3, \\ \alpha x_1 + x_2 + x_3 &= 2. \end{aligned}$$

a. Find value(s) of α for which the system has no solutions.

b. Find value(s) of α for which the system has an infinite number of solutions.

c. Assuming a unique solution exists for a given α, find the solution.

EXERCISE SET 6.2

9. Use Gaussian elimination and three-digit chopping arithmetic to solve the following linear systems, and compare the approximations to the actual solution.

a. $0.03x_1 + 58.9x_2 = 59.2,$

$5.31x_1 - 6.10x_2 = 47.0.$

Actual solution $[10, 1]$.

b. $3.03x_1 - 12.1x_2 + 14x_3 = -119,$

$-3.03x_1 + 12.1x_2 - 7x_3 = 120,$

$6.11x_1 - 14.2x_2 + 21x_3 = -139.$

Actual solution $[0, 10, \frac{1}{7}]$.

13. Repeat Exercise 9 using Gaussian elimination with partial pivoting.

17. Repeat Exercise 9 using Gaussian elimination with scaled partial pivoting.

EXERCISE SET 6.3

8. Consider the four 3×3 linear systems having the same coefficient matrix:

$$
\begin{aligned}
2x_1 - 3x_2 + x_3 &= 2, \\
x_1 + x_2 - x_3 &= -1, \\
-x_1 + x_2 - 3x_3 &= 0;
\end{aligned}
\qquad
\begin{aligned}
2x_1 - 3x_2 + x_3 &= 6, \\
x_1 + x_2 - x_3 &= 4, \\
-x_1 + x_2 - 3x_3 &= 5;
\end{aligned}
$$

$$
\begin{aligned}
2x_1 - 3x_2 + x_3 &= 0, \\
x_1 + x_2 - x_3 &= 1, \\
-x_1 + x_2 - 3x_3 &= -3;
\end{aligned}
\qquad
\begin{aligned}
2x_1 - 3x_2 + x_3 &= -1, \\
x_1 + x_2 - x_3 &= 0, \\
-x_1 + x_2 - 3x_3 &= 0.
\end{aligned}
$$

a. Solve the linear systems by applying Gaussian elimination to the augmented matrix

$$
\left[\begin{array}{rrr:rrrr}
2 & -3 & 1 & 2 & 6 & 0 & -1 \\
1 & 1 & -1 & -1 & 4 & 1 & 0 \\
-1 & 1 & -3 & 0 & 5 & -3 & 0
\end{array}\right].
$$

EXERCISE SET 6.4

7. Find all values of α so that the following linear system has no solutions.

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 5, \\ 4x_1 + 2x_2 + 2x_3 &= 6, \\ -2x_1 + \alpha x_2 + 3x_3 &= 4. \end{aligned}$$

8. Find all values of α so that the following linear system has an infinite number of solutions.

$$\begin{aligned} 2x_1 - x_2 + 3x_3 &= 5, \\ 4x_1 + 2x_2 + 2x_3 &= 6, \\ -2x_1 + \alpha x_2 + 3x_3 &= 1. \end{aligned}$$

EXERCISE SET 6.5

10. Suppose $A = P^t LU$, where P is a permutation matrix, L is a lower-triangular matrix with ones on the diagonal, and U is an upper-triangular matrix.

b. Show that if P contains k row interchanges, then

$$\det P = \det P^t = (-1)^k.$$

c. Use $\det A = \det P^t \det L \det U = (-1)^k \det U$ to count the number of operations for determining $\det A$ by factoring.

d. Compute $\det A$

$$A = \begin{bmatrix} 0 & 2 & 1 & 4 & -1 & 3 \\ 1 & 2 & -1 & 3 & 4 & 0 \\ 0 & 1 & 1 & -1 & 2 & -1 \\ 2 & 3 & -4 & 2 & 0 & 5 \\ 1 & 1 & 1 & 3 & 0 & 2 \\ -1 & -1 & 2 & -1 & 2 & 0 \end{bmatrix}.$$

EXERCISE SET 6.6

3. Use the LDL^t Factorization Algorithm to find a factorizaton of the form $A = LDL^t$ for the following matrices:

a. $A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

b. $A = \begin{bmatrix} 4 & 1 & 1 & 1 \\ 1 & 3 & -1 & 1 \\ 1 & -1 & 2 & 0 \\ 1 & 1 & 0 & 2 \end{bmatrix}$

5. Use the Cholesky Algorithm to find a factorization of the form $A = LL^t$ for the matrices in Exercise 3.

22. Let

$$A = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & \alpha \end{bmatrix}.$$

Find all values of α for which

a. A is singular.

b. A is strictly diagonally dominant.

c. A is symmetric.

d. A is positive definite.

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