Fadiga de Materiais Estruturais: Fundamentos e Aplicações
Metodologia S-N
(Stress-based Methodology)

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AGENDA

1. *Definições (Definitions)*
2. *Curvas de Fadiga S-N (Stress versus Life Curves)*
3. *Fatores de Segurança(Safety Factors)*
4. *Tensão Média(Mean Stress)*
   - *Diagramas Amplitude-Valor Médio(Normalized Amplitude-Mean Diagrams)*
     - *Goodman, Gerber and Morrow Equations*
     - *Smith, Watson and Topper (SWT)*
   - *Estimativas de vida à fadiga incluindo efeitos da tensão média (Life Estimates with Mean Stress)*

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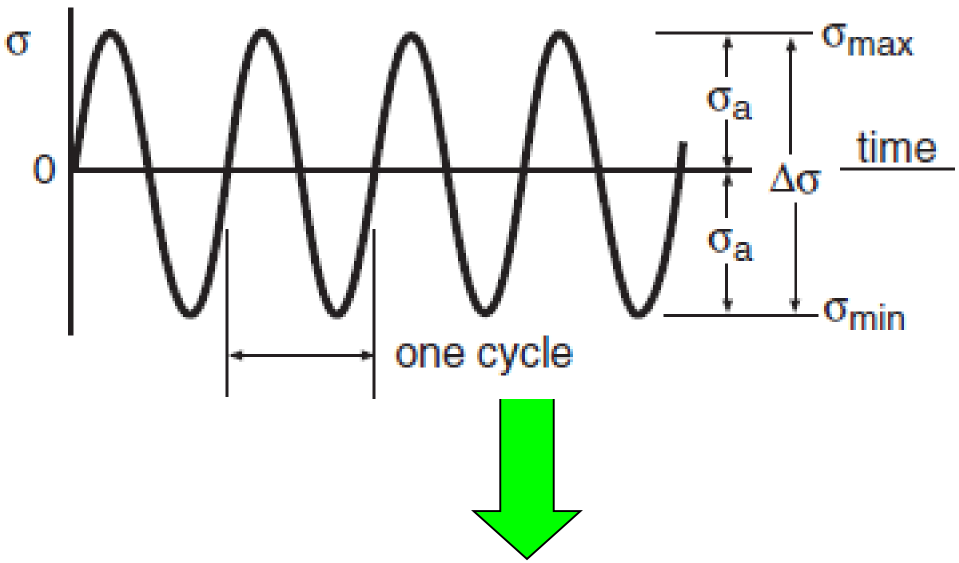
Carregamento Cíclico (Cycling Loading)

- ***Constant Amplitude Loading***



- ***Definitions***

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$ ⟷ Stress range

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$ ⟷ Mean stress

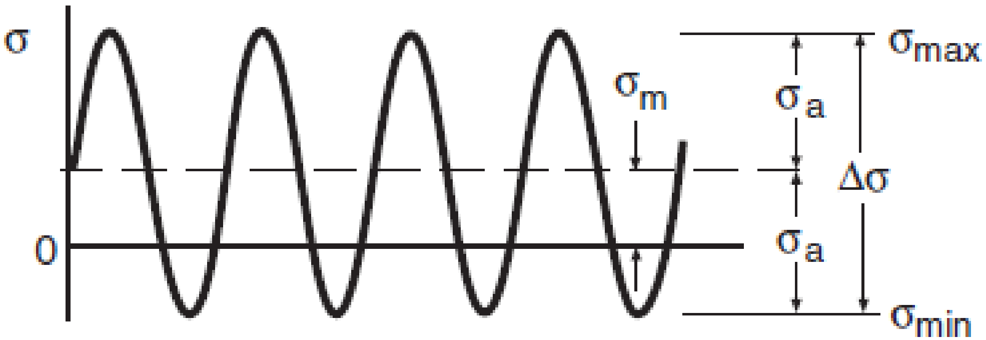
$$\sigma_a = \frac{\Delta\sigma}{2} = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$ ⟷ Stress amplitude

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{P_{min}}{P_{\max}}$$ ⟷ Stress ratio

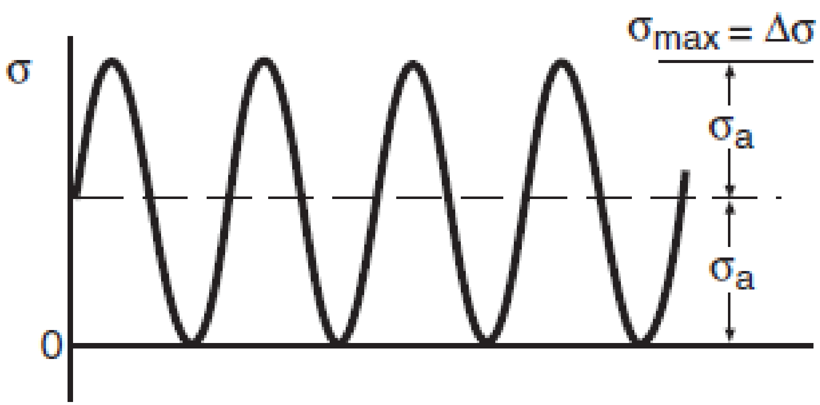
The term alternating stress is used as synonymous for stress amplitude

Carregamento Cíclico (Cycling Loading)

- ***Constant Amplitude Loading***



Nonzero mean stress (σ_m)



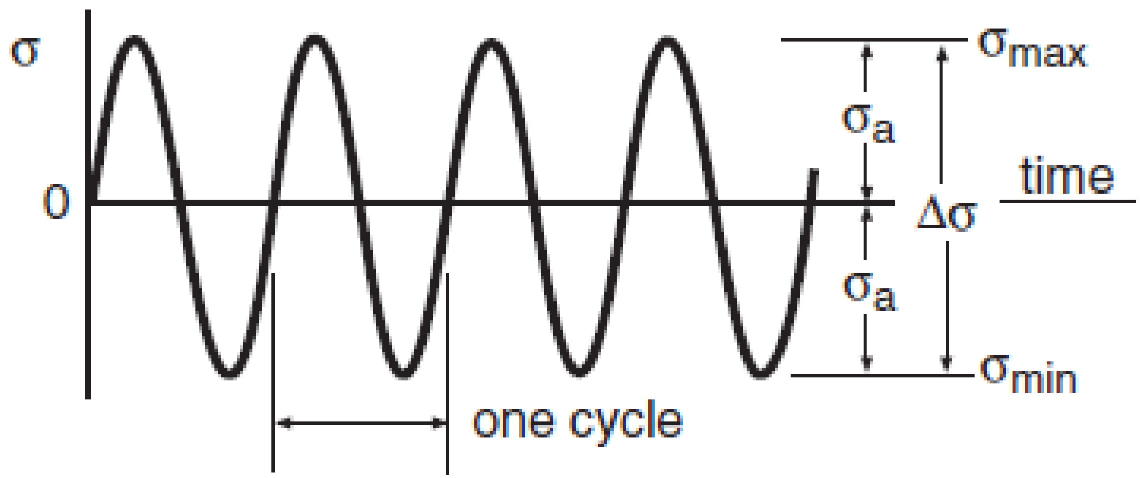
Zero-to-tension loads ($\sigma_{min} = 0$)

If σ_m is not zero, 2 independent values are needed to specify the loading

Carregamento Cíclico (Cycling Loading)

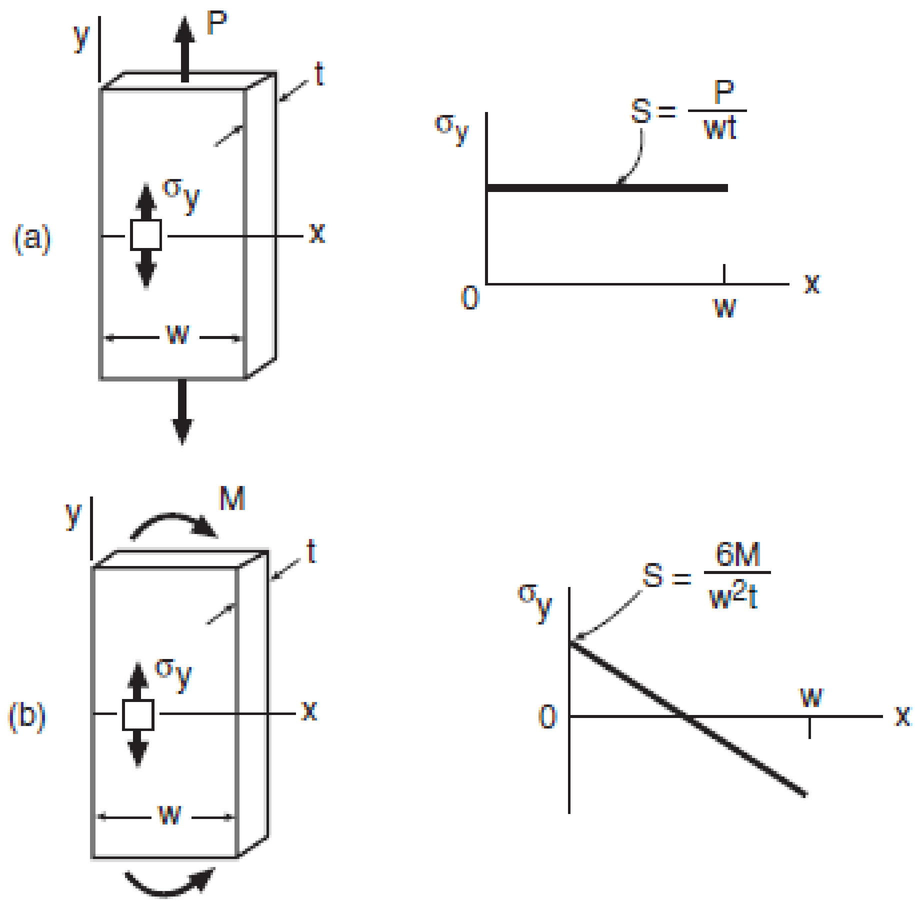
- ***Constant Amplitude Loading***

Completely Reversed Cycling ($\sigma_m = 0$ or $R = -1$)



Same subscripts are used for other variables: Force P, strain ε, Momen M, etc

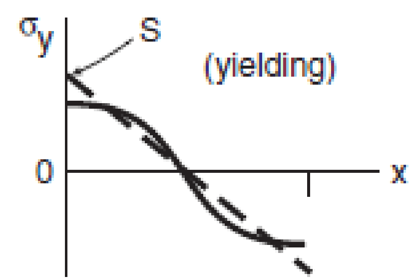
Tensão no ponto versus Tensão Nominal



$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}$$

$$S = \frac{P}{(w \times t)}$$

$$S = \frac{M \times \frac{w}{2}}{\frac{t \times w^3}{12}}$$

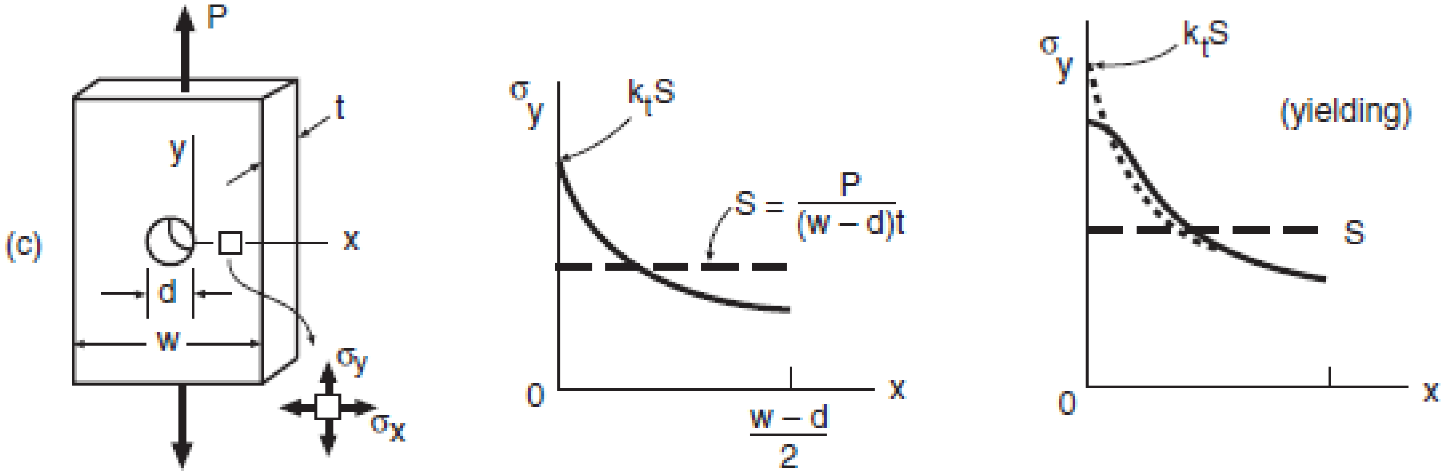


Nominal (average) stress is calculated from force P or moment M or their combinations

Tensão no ponto versus Tensão Nominal

$$\sigma_{ij} = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} \longleftrightarrow S = \frac{P}{((w-d)\times t)}$$

- Notched member
  - Área neta = (w-d)t



Notch= stress raiser (holes, grooves, fillets, etc)

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Stress Life Curves

Specimen subjected to sufficiently severe cyclic stress

Fatigue crack or other damage will develop

After N cycles

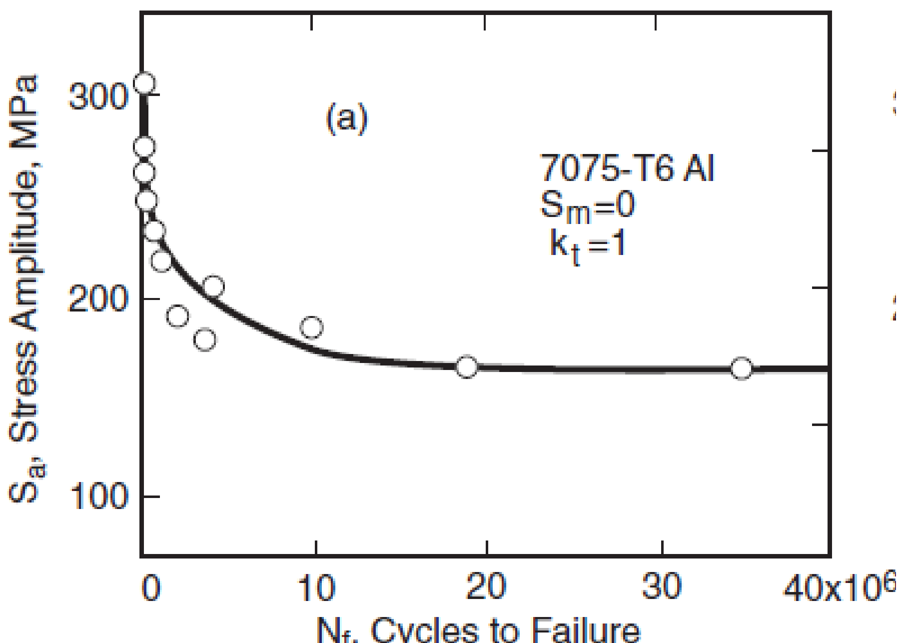
Specimen fails

Change Stress levels
(higher or lower)

Stress Life Curves

The repetition of such procedure is used to obtain a stress-life curve.

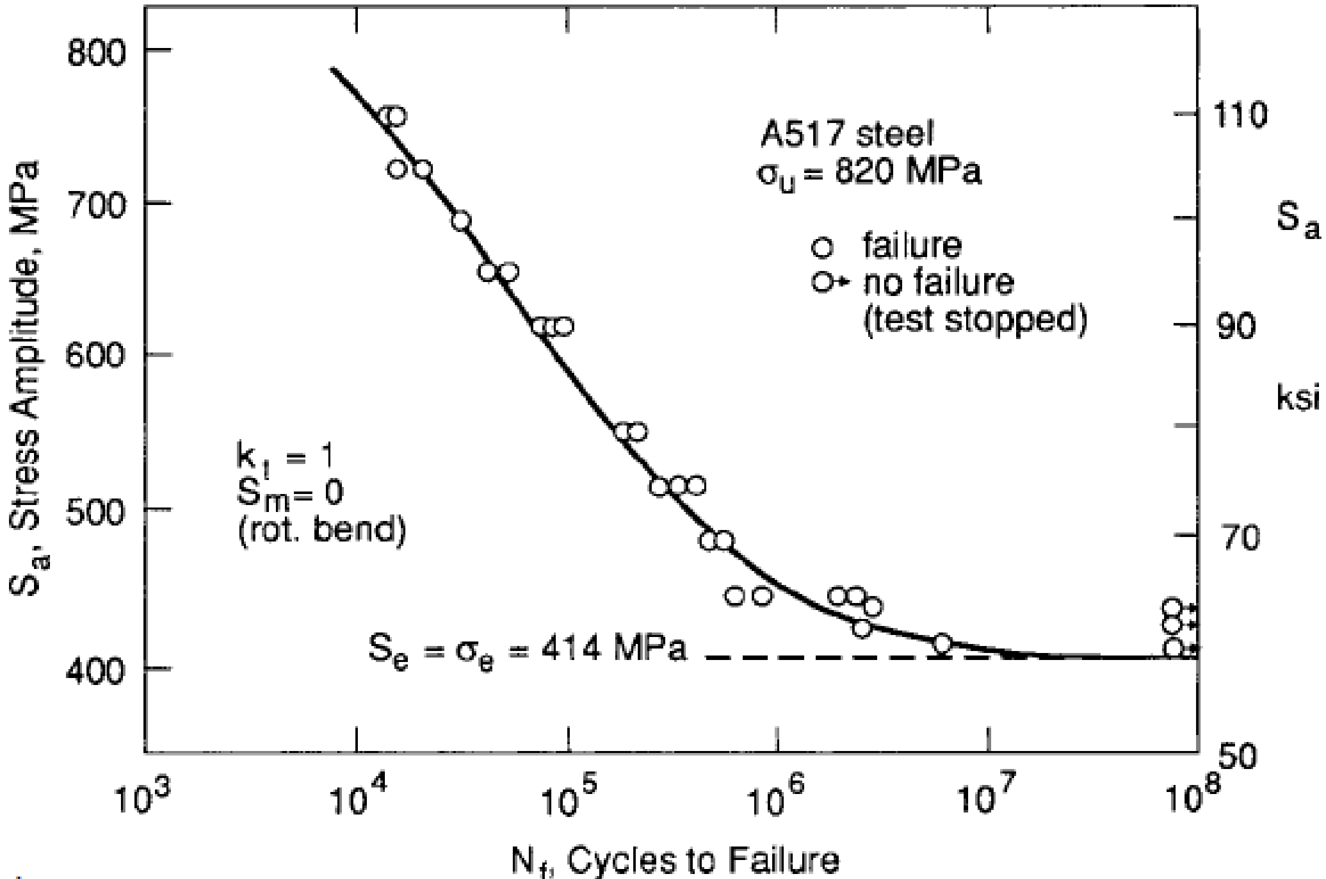
It is also called S-N curve



Data from [MacGregor

Stress Life Curves

S-N curve



Adapted from [Brockenbrough

Stress Life Curves

S-N curve fitting

The cyclic number are plotted on a logaritmic scale.

A logarithmic scale is also often used for the stress axis.

If S-N data are a straight line on a log-linear plot:

$$\sigma_a = C + D \log N_f$$

C and D are fitting constants

Stress Life Curves

S-N curve fitting

If S-N data are a straight line on a log-log plot:

$$\sigma_a = A\left(N_f\right)^B$$

or

$$\boxed{\sigma_a = \sigma_f^{'}\left(2N_f\right)^b}$$

$$A = 2^b \sigma_f^{'}$$

$$b = B$$

Constants for Stress Life Curves

Table 9.1 Constants for Stress–Life Curves for Various Ductile Engineering Metals, From Tests at Zero Mean Stress on Unnotched Axial Specimens

Material	Yield Strength σ_o	Ultimate Strength σ_u	True Fracture Strength $\tilde{\sigma}_{fB}$	$\sigma_a = \sigma'_f(2N_f)^b = AN_f^B$ σ'_f	A	$b = B$
(a) Steels						
SAE 1015 (normalized)	228 (33)	415 (60.2)	726 (105)	1020 (148)	927 (134)	−0.138
Man-Ten (hot rolled)	322 (46.7)	557 (80.8)	990 (144)	1089 (158)	1006 (146)	−0.115
RQC-100 (roller Q & T)	683 (99.0)	758 (110)	1186 (172)	938 (136)	897 (131)	−0.0648
SAE 4142 (Q & T, 450 HB)	1584 (230)	1757 (255)	1998 (290)	1937 (281)	1837 (266)	−0.0762
AISI 4340 (aircraft quality)	1103 (160)	1172 (170)	1634 (237)	1758 (255)	1643 (238)	−0.0977
(b) Other Metals						
2024-T4 Al	303 (44.0)	476 (69.0)	631 (91.5)	900 (131)	839 (122)	−0.102
Ti-6Al-4V (solution treated and aged)	1185 (172)	1233 (179)	1717 (249)	2030 (295)	1889 (274)	−0.104

Notes: The tabulated values have units of MPa (ksi), except for dimensionless $b = B$.

$$\boxed{\sigma_a = \sigma'_f \left(2N_f\right)^b}$$

$$\sigma_m = 0$$

$$\sigma'_f \approx \tilde{\sigma}_f$$

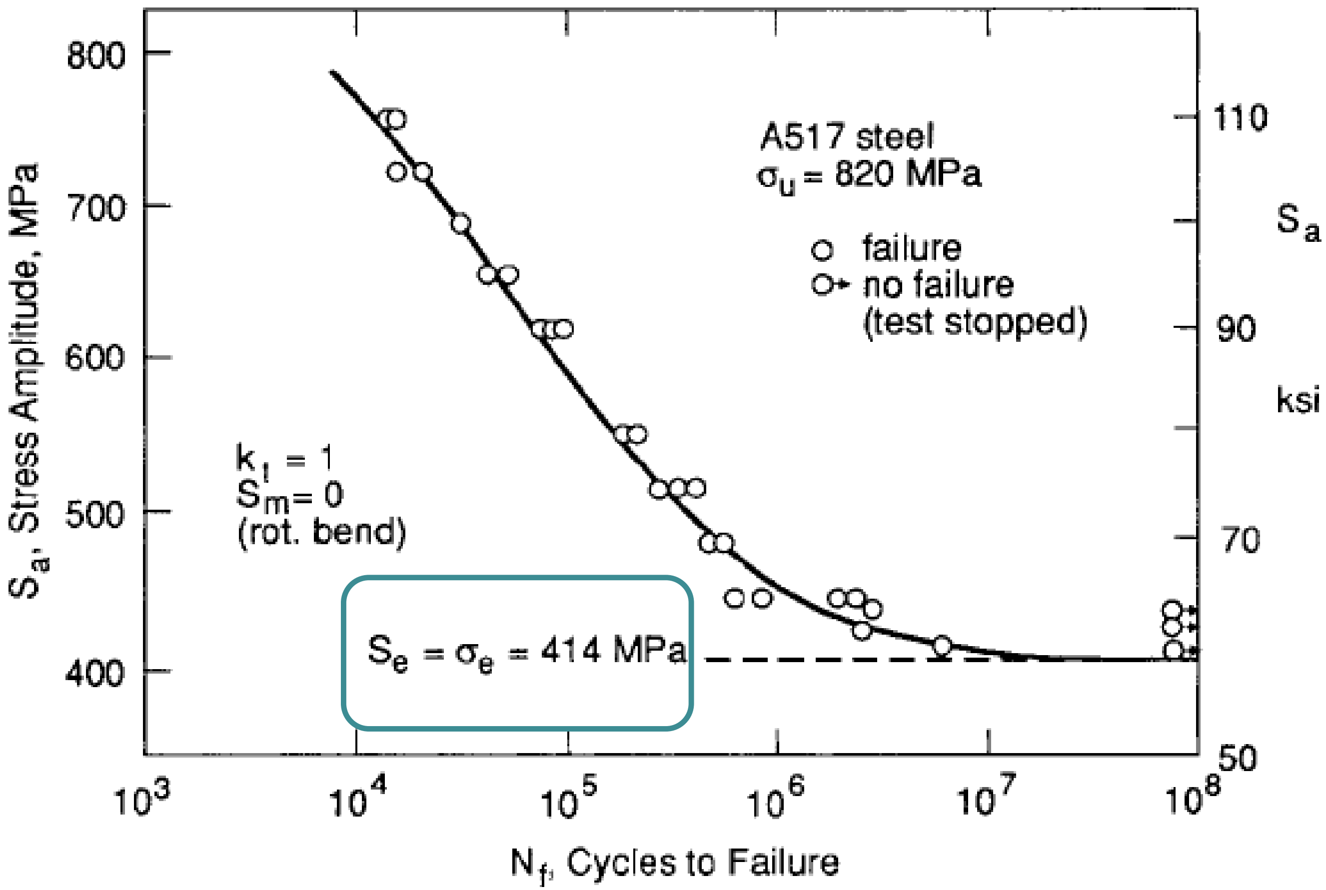
Stress Life Curves

Fatigue limits or endurance

$$S_e = \sigma_e$$

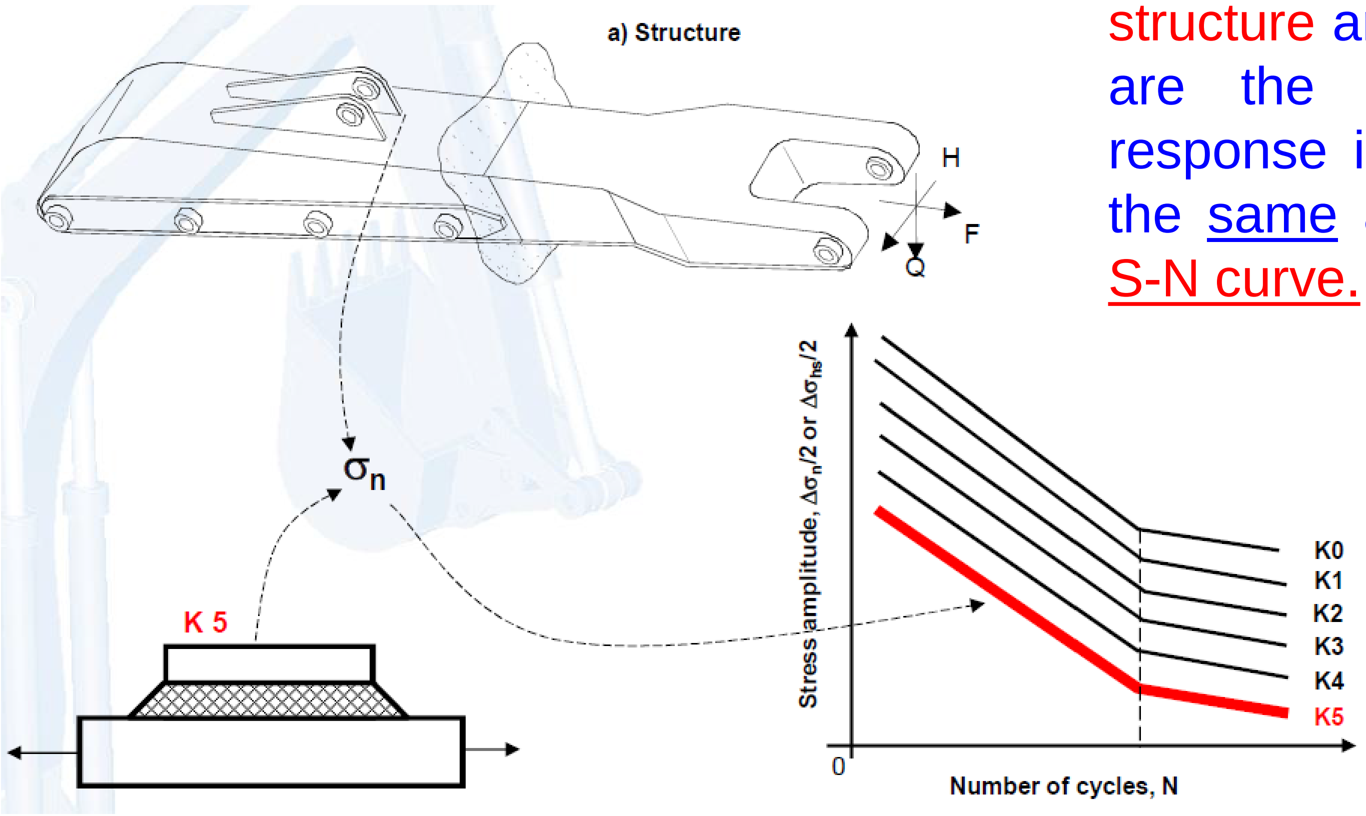
Material Property

- Specimens
  - Smooth surface
  - Unnotched
  - R = -1



Adapted from [Brockenbrough

Princípio de Similaridade (Similitude)



- If nominal stress history in the structure and the test specimen are the same, the fatigue response in each case will be the same and describe by the S-N curve.

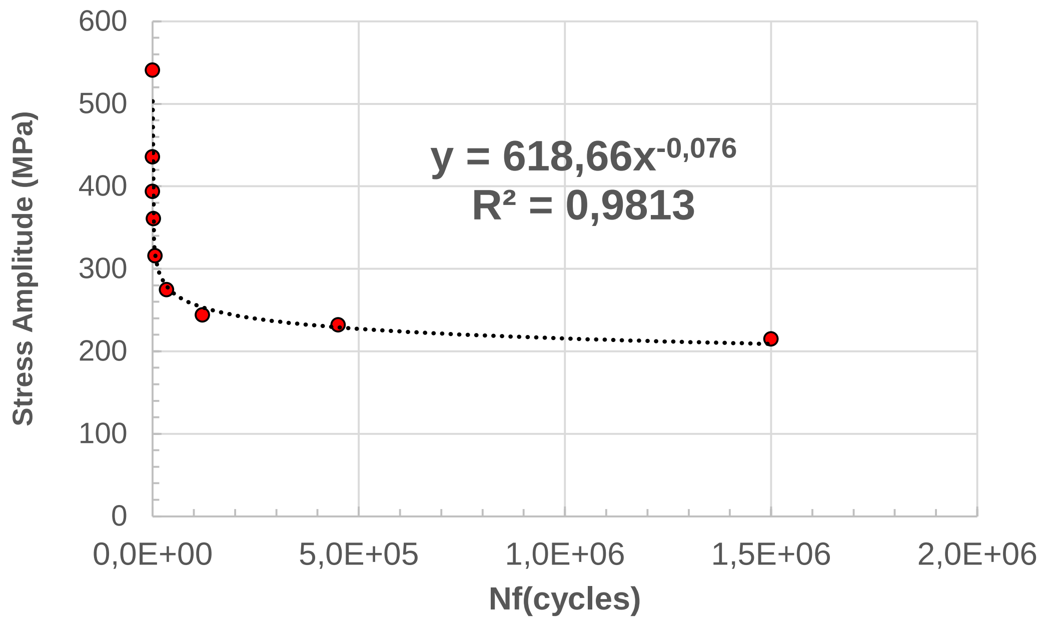
Plot from Glinka's presentation

Example

Obtain the fitting constants for the fatigue data given in table 1

Table 1

σ_a	N_f
MPa	cycles
541	15
436	50
394	200
361	2080
316	5900
275	34100
244	121000
232	450000
215	1500000



$$\sigma_a = A\left(N_f\right)^B$$

Example

Obtain the fitting constants for the fatigue data given in table 1

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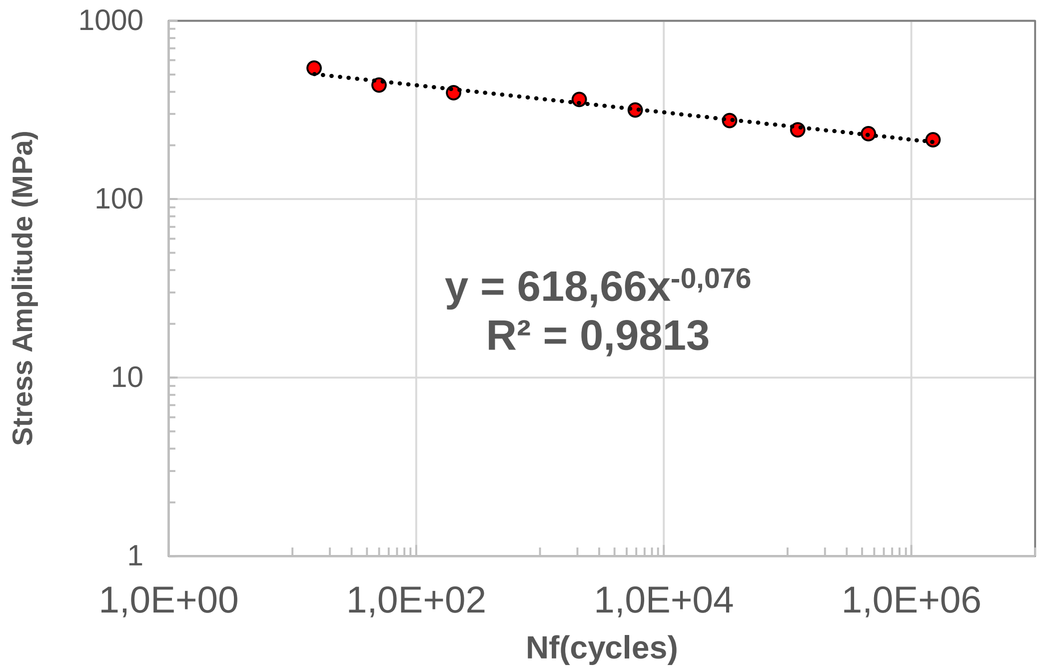

$$\sigma_a = A\left(N_f\right)^B$$

$$A = 2^b \sigma'_f \qquad b = B$$

Example

Obtain the fitting constants for the fatigue data given in table 1

Table 1	
σ_a	N_f
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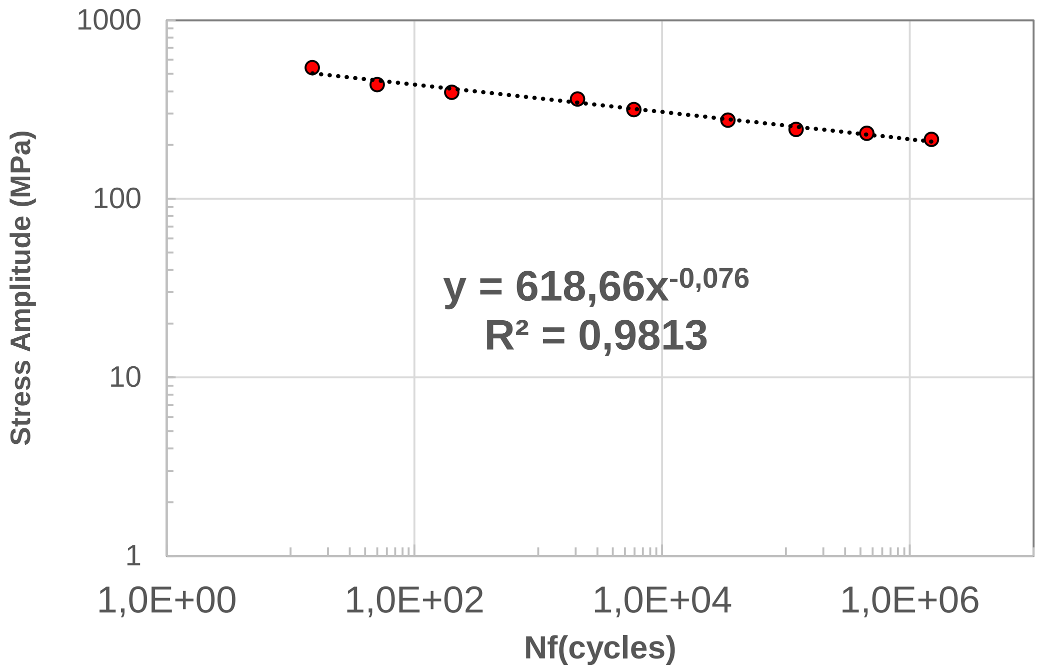
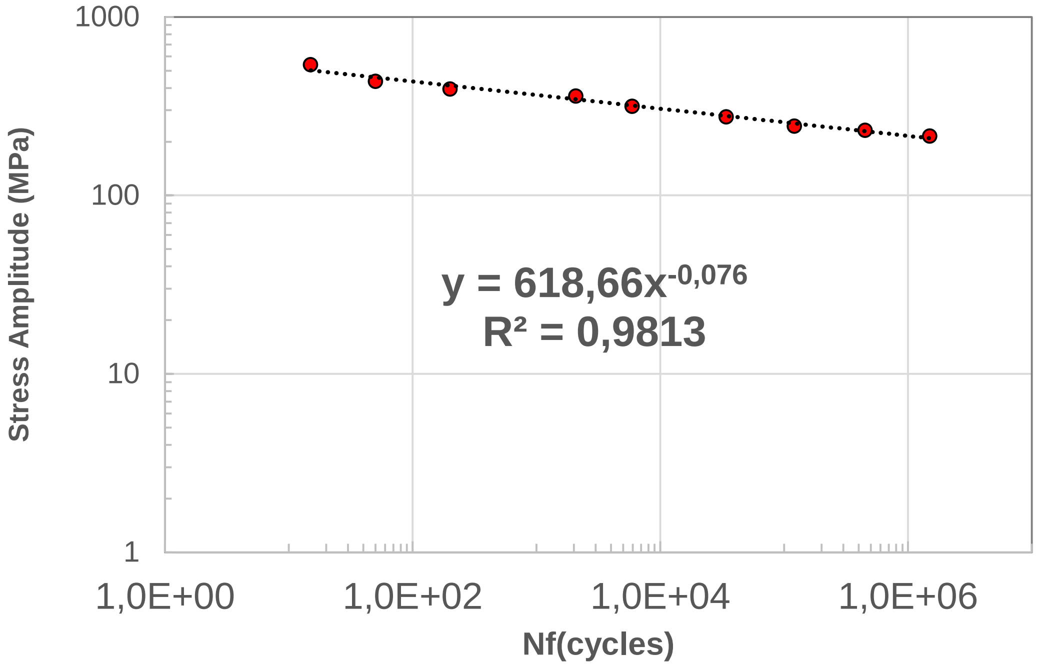


$$\boxed{b = 0,076}$$

$$\sigma_a = A\left(N_f\right)^B \quad 618,66 = 2^{-0,076}\sigma_f^{'} \longrightarrow \boxed{\sigma_f^{'} = 652,12}$$

Example

Obtain the fitting constants for the fatigue data given in table 1



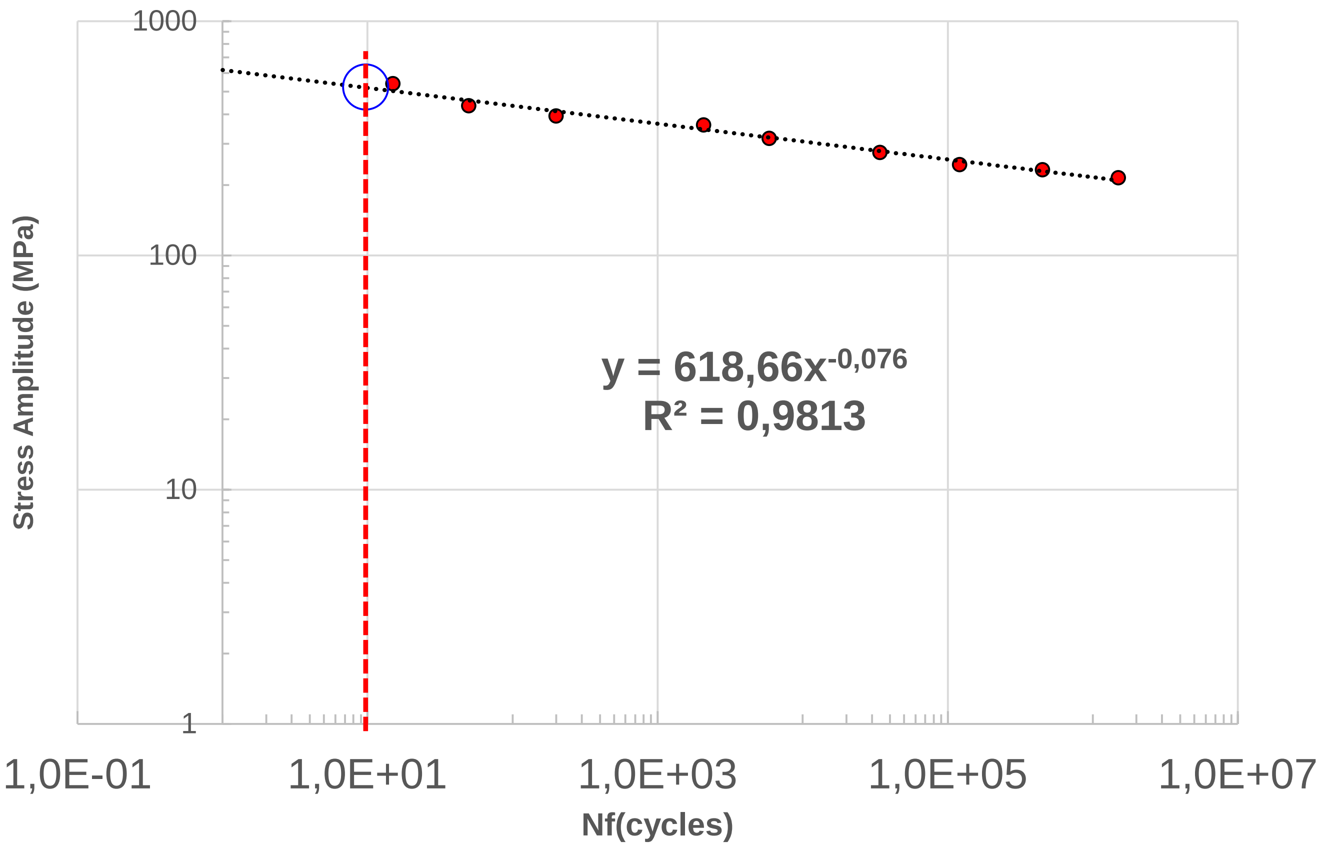
$$\sigma_a = A\left(N_f\right)^B$$

$$N_f = 1$$

A = σ_a

Example

Obtain the fitting constants for the fatigue data given in table 1



$$\sigma_a = A\left(N_f\right)^B$$

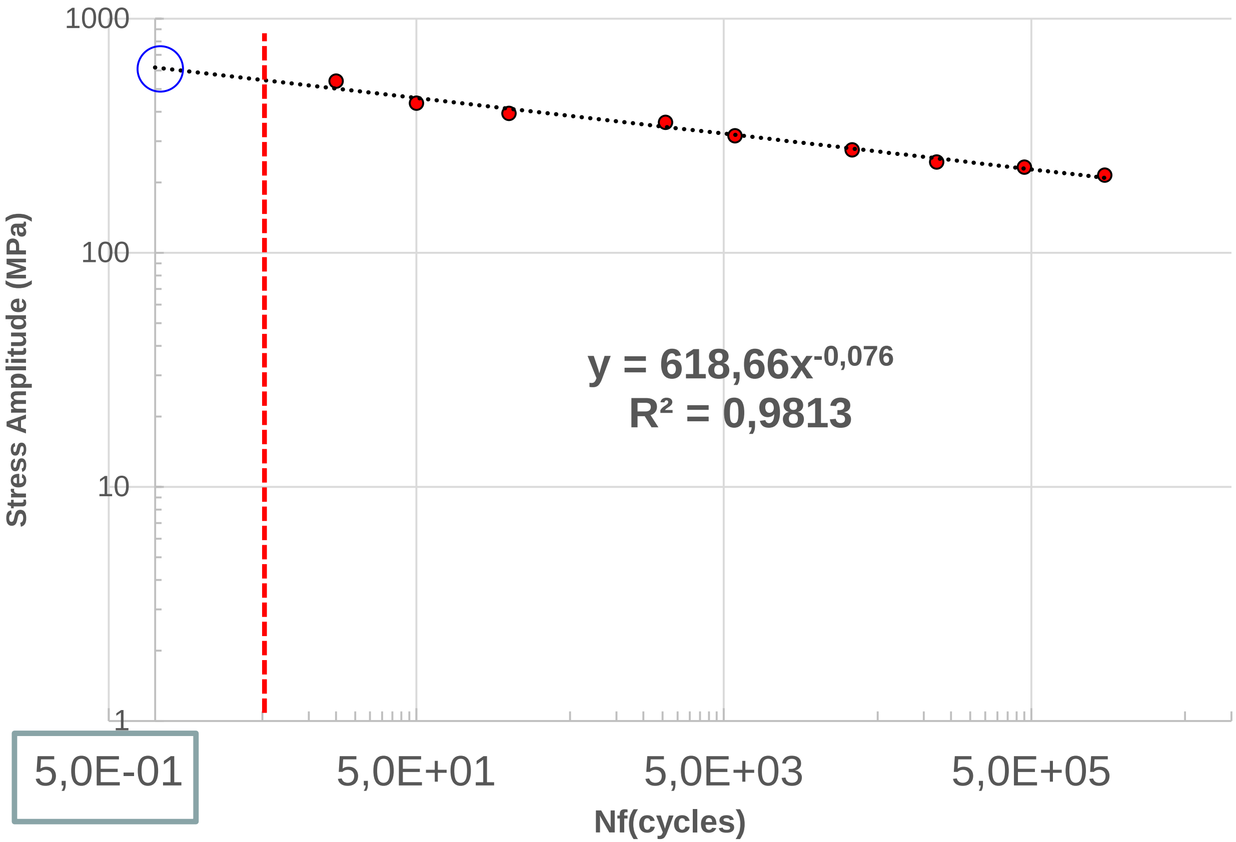
$$N_f = 1$$

$$A = \sigma_a$$

$$A \approx 618$$

Example

Obtain the fitting constants for the fatigue data given in table 1



$$\sigma_a = \sigma_f^{'} \left(2N_f\right)^b$$

$$\downarrow$$

$$N_f = 1/2$$

$$\downarrow$$

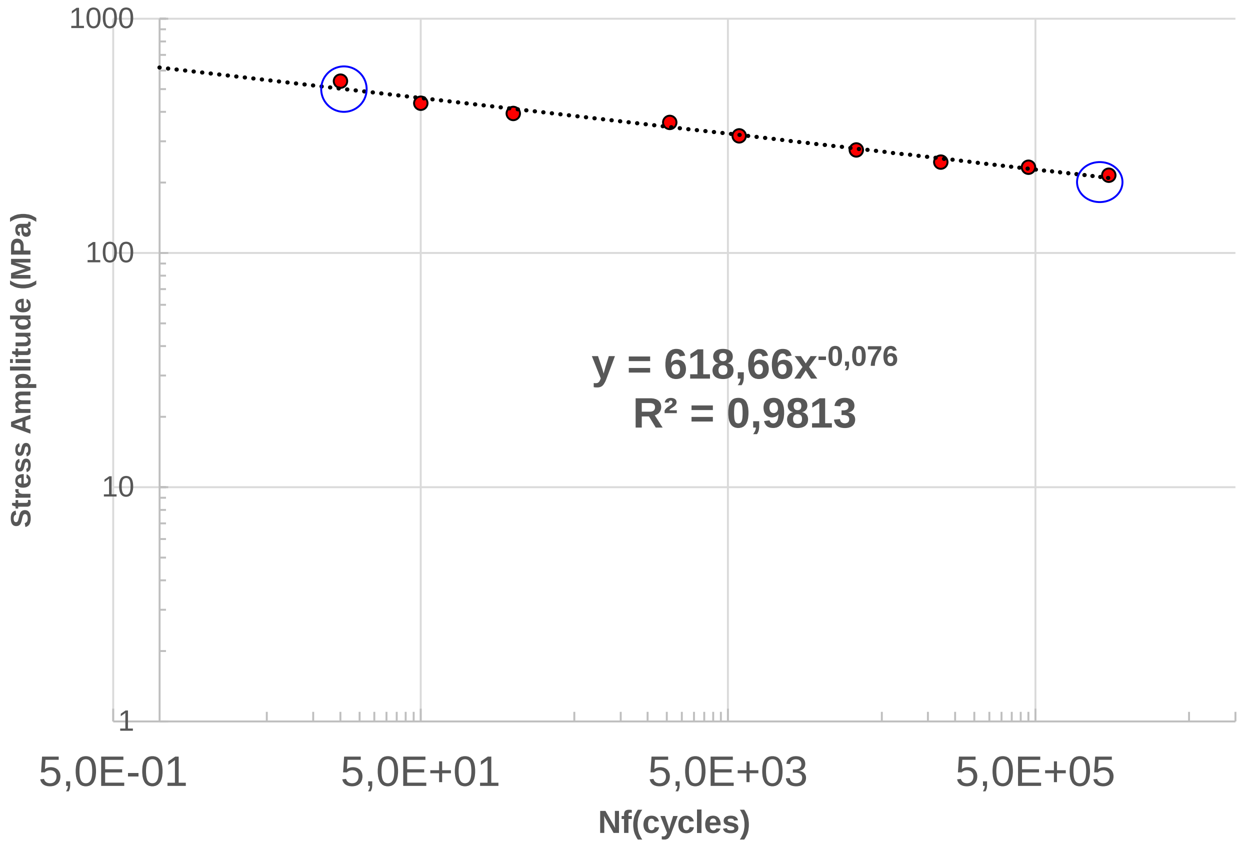
$$\sigma_f^{'} = \sigma_a$$

$$\downarrow$$

$$\sigma_f^{'} \approx 652$$

Example

Obtain the fitting constants for the fatigue data given in table 1



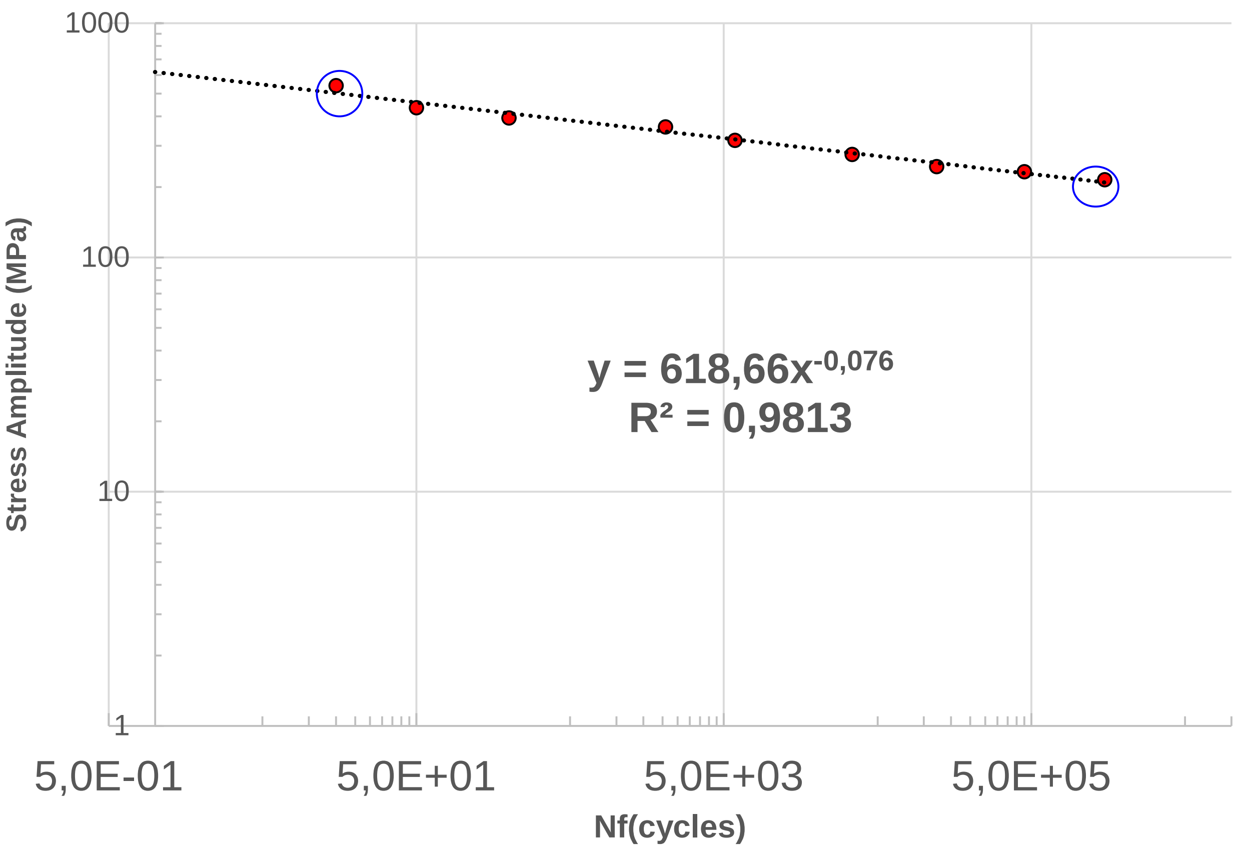
$$\sigma_a = A\left(N_f\right)^B$$

$$\left(\sigma_1, N_1\right)\ \left(\sigma_2, N_2\right)$$

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{N_1}{N_2}\right)^B$$

Example

Obtain the fitting constants for the fatigue data given in table 1



$$\sigma_a = A\left(N_f\right)^B$$

$$\left(\sigma_1, N_1\right) \quad \left(\sigma_2, N_2\right)$$

$$\frac{\sigma_1}{\sigma_2} = \left(\frac{N_1}{N_2}\right)^B$$

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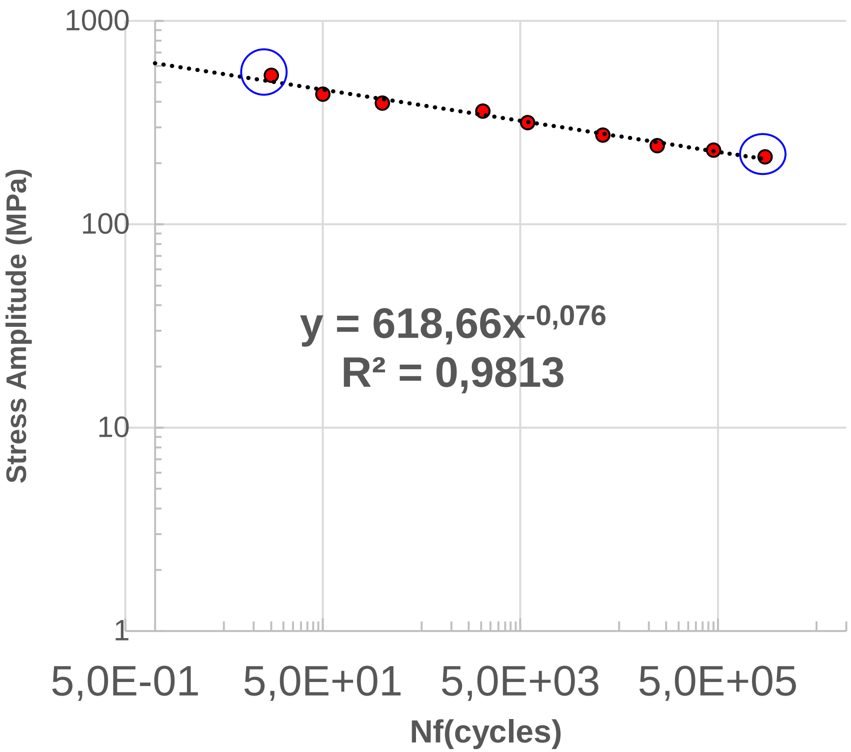


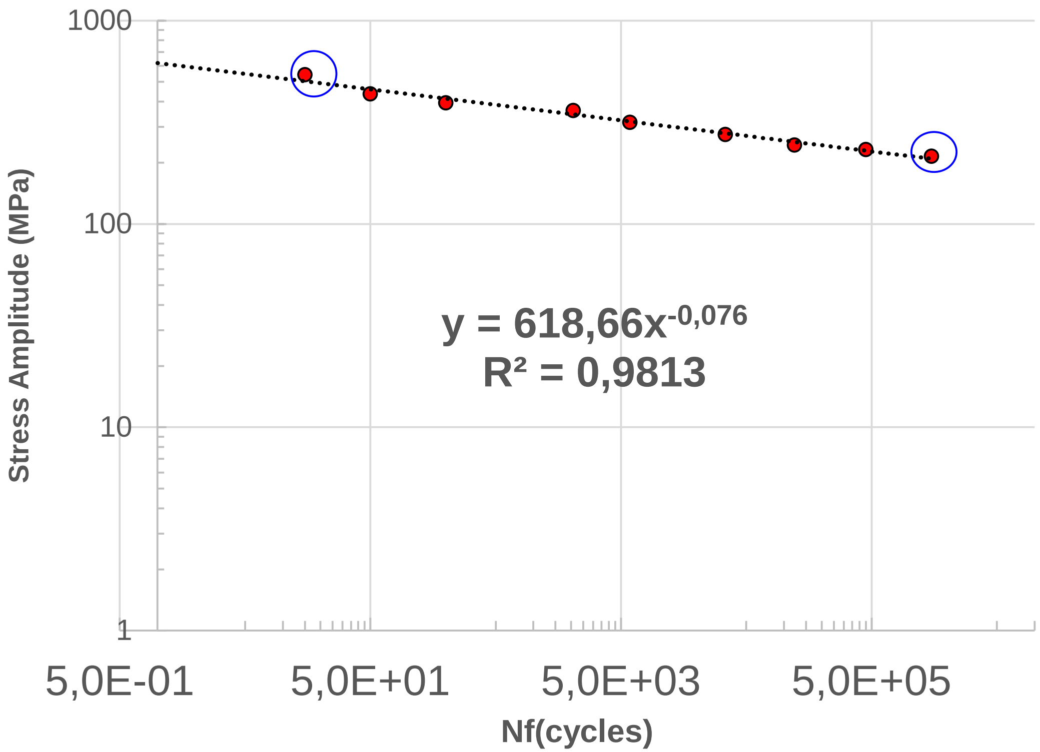
$$\frac{\sigma_1}{\sigma_2} = \left(\frac{N_1}{N_2}\right)^B$$

$$\log\frac{\sigma_1}{\sigma_2} = B\log\frac{N_1}{N_2}$$

$$B = \frac{\log\sigma_1 - \log\sigma_2}{\log N_1 - \log N_2}$$

Example

Obtain the fitting constants for the fatigue data given in table 1



$$\sigma_a = A\left(N_f\right)^B$$

$$541 = A(15)^{-0,0801}$$

$$\boxed{A = 672,14}$$

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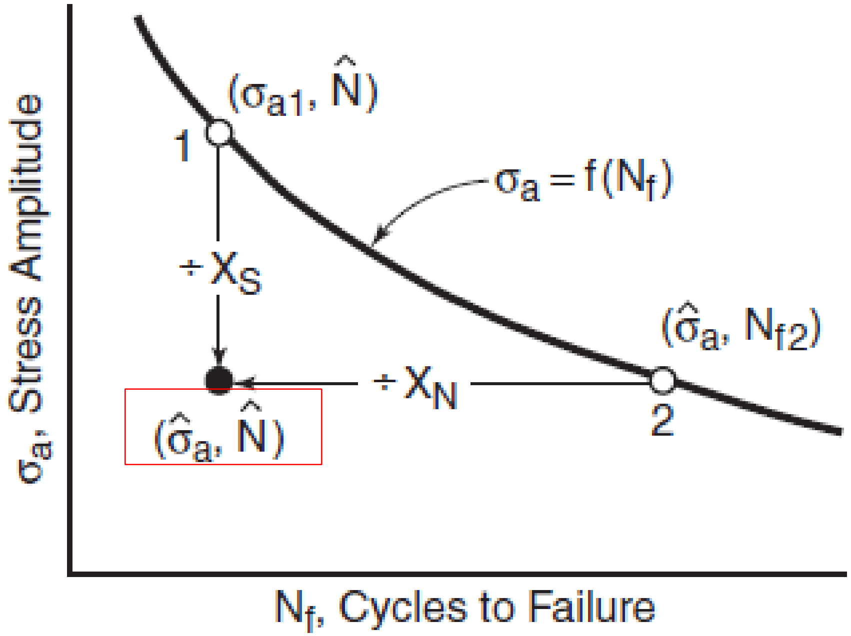
Safety Factors

$\hat{N}$ Number of cycles desired

$\hat{\sigma}_a$ Stress level expected in service

SAFE COMBINATION



$$X_S = \frac{\sigma_{a1}}{\hat{\sigma}_a}$$

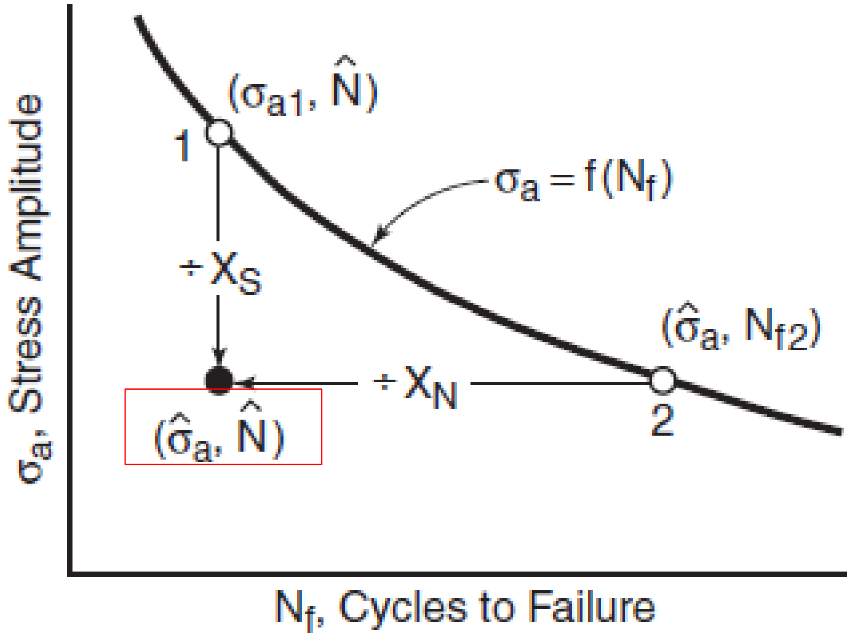
$$X_N = \frac{N_{f2}}{\hat{N}}$$

Fatigue Life sensitive to stress

$$X_N = 5 \text{ to } 20$$

Safety Factors

$$\hat{N}$$
$$\hat{\sigma}_a$$



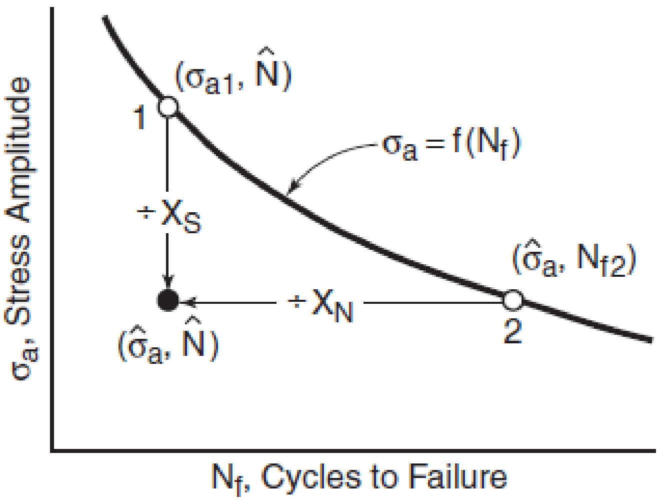
$$\sigma_{a1} = A\hat{N}^B, \qquad \hat{\sigma}_a = A\,N_{f2}^B$$

$$X_S = \frac{A\hat{N}^B}{A\,N_{f2}^B} = \left(\frac{1}{X_N}\right)^B = X_N^{-B}$$

$$X_N = X_S^{-1/B}$$

A given X_S corresponds to a particular X_N and vice versa

Safety Factors



- Notched engineering components B ~ -0.2
- Welded Structural members B~ -0.33

$$X_N = X_S^{-1/B}$$

B	$-1/B$	X_N for $X_S = 2$	X_S for $X_N = 10$
-0.1	10	1024	1.26
-0.2	5	32	1.58
-0.333	3	8	2.15

Large S.F in Life is needed to achieve a modest S.F in Stress

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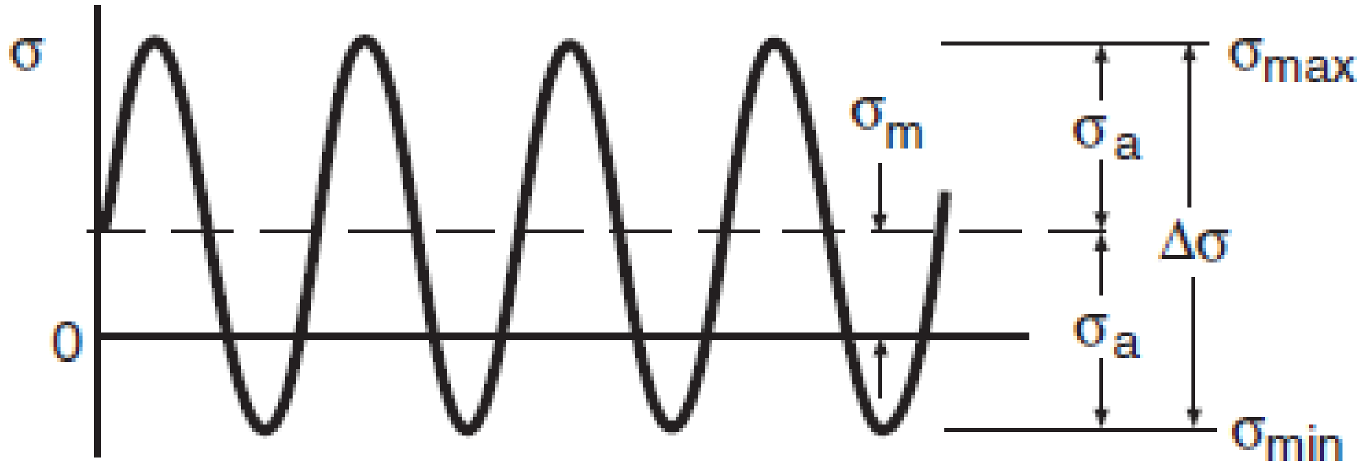
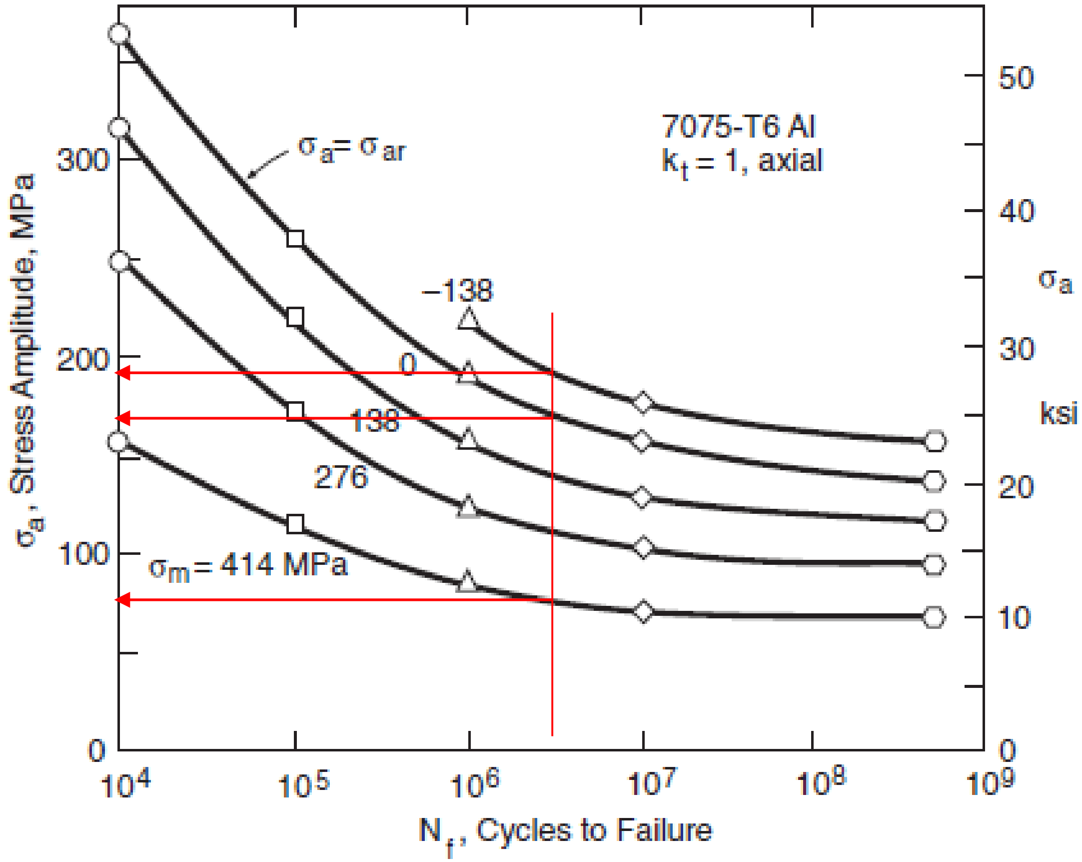
Mean Stress Effects

Nonzero mean stress (σ_m)



Mean Stress Effects



(Data from [Howell

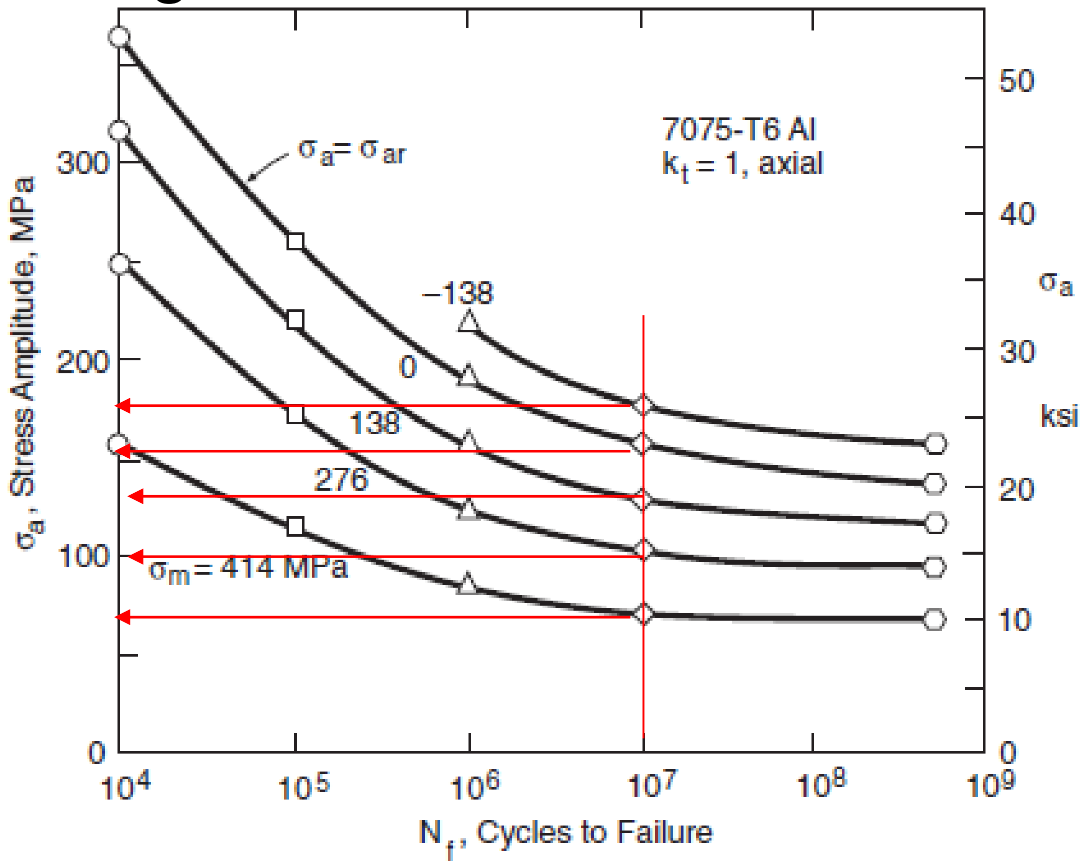
Mean Stress Effects

Constant-life diagram

$$(N)_{Constant}$$

$$\downarrow$$

$$(\sigma_m, \sigma_a)$$



(Data from [Howell

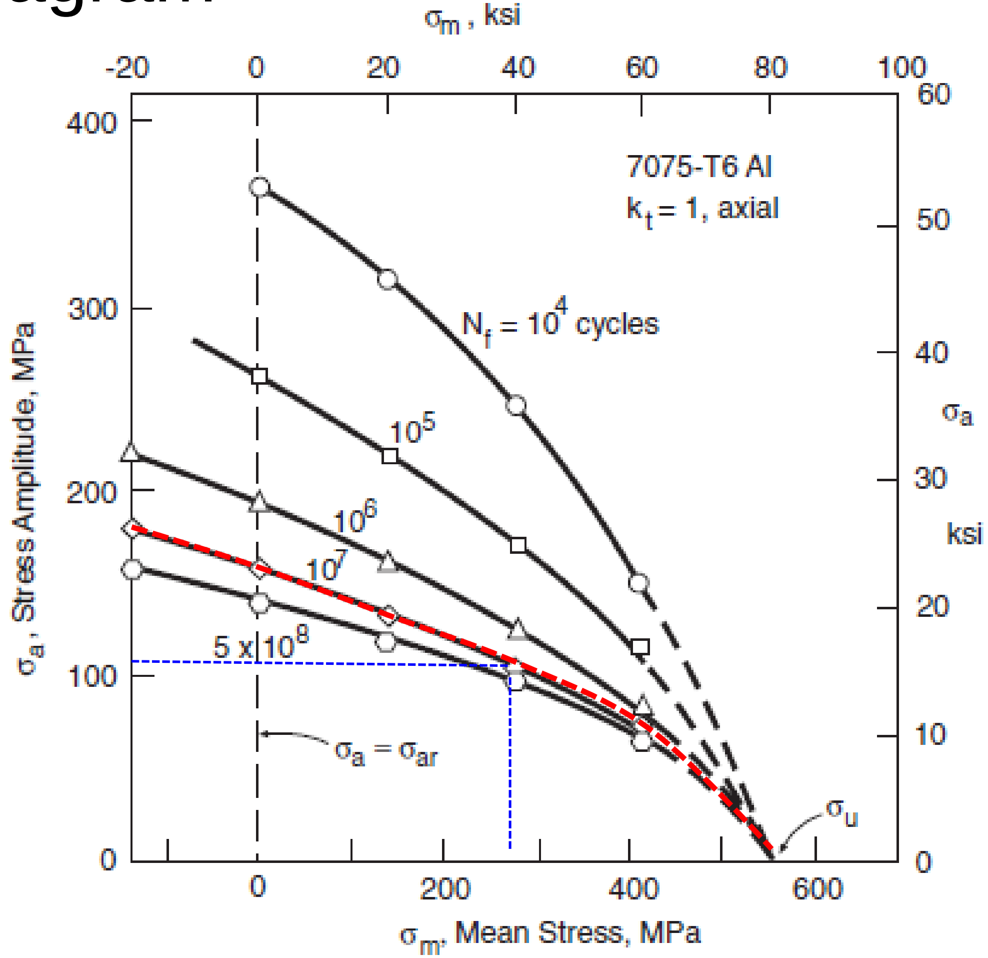
Mean Stress Effects

Constant-life diagram

$$\left(N\right)_{Constant}$$

$$\downarrow$$

$$\left(\sigma_m, \sigma_a\right)$$



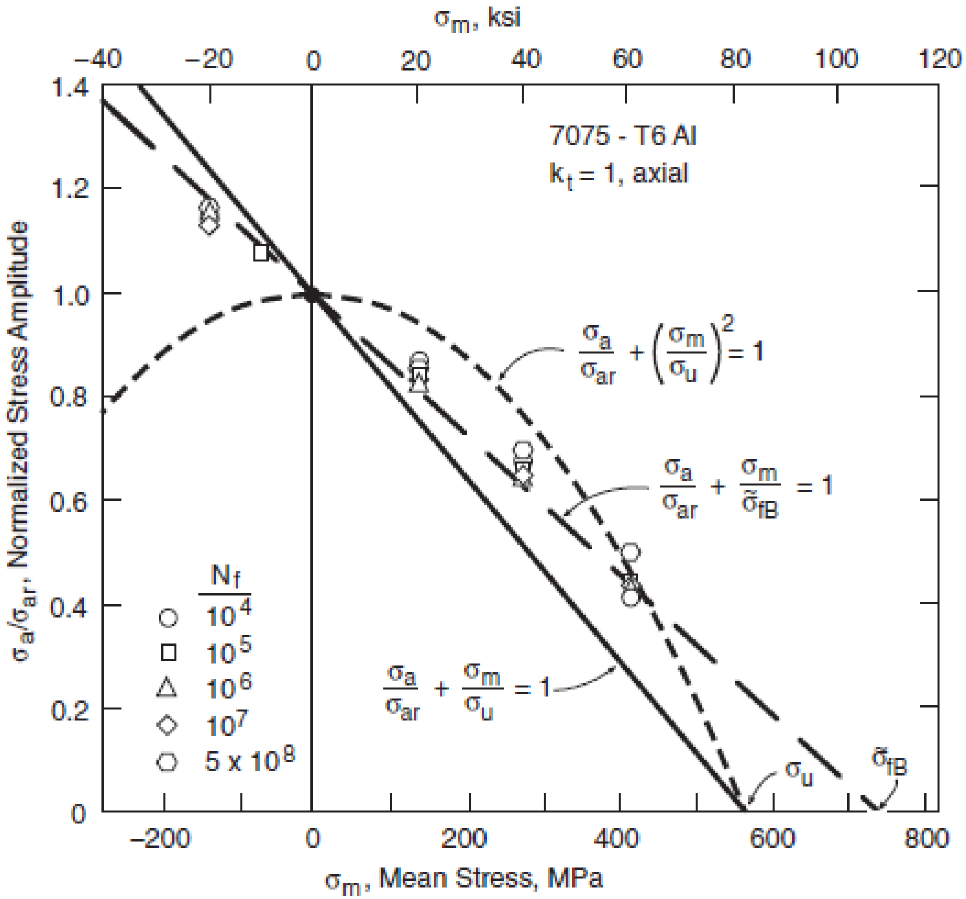
$$\left(\sigma_m\right)\uparrow$$

$$\left(\sigma_a\right)\downarrow$$

(Data from [Howell

Mean Stress Effects

Normalized Amplitude-Mean Diagrams



$$\sigma_m = 0 \rightarrow \quad \sigma_a = \sigma_{ar}$$

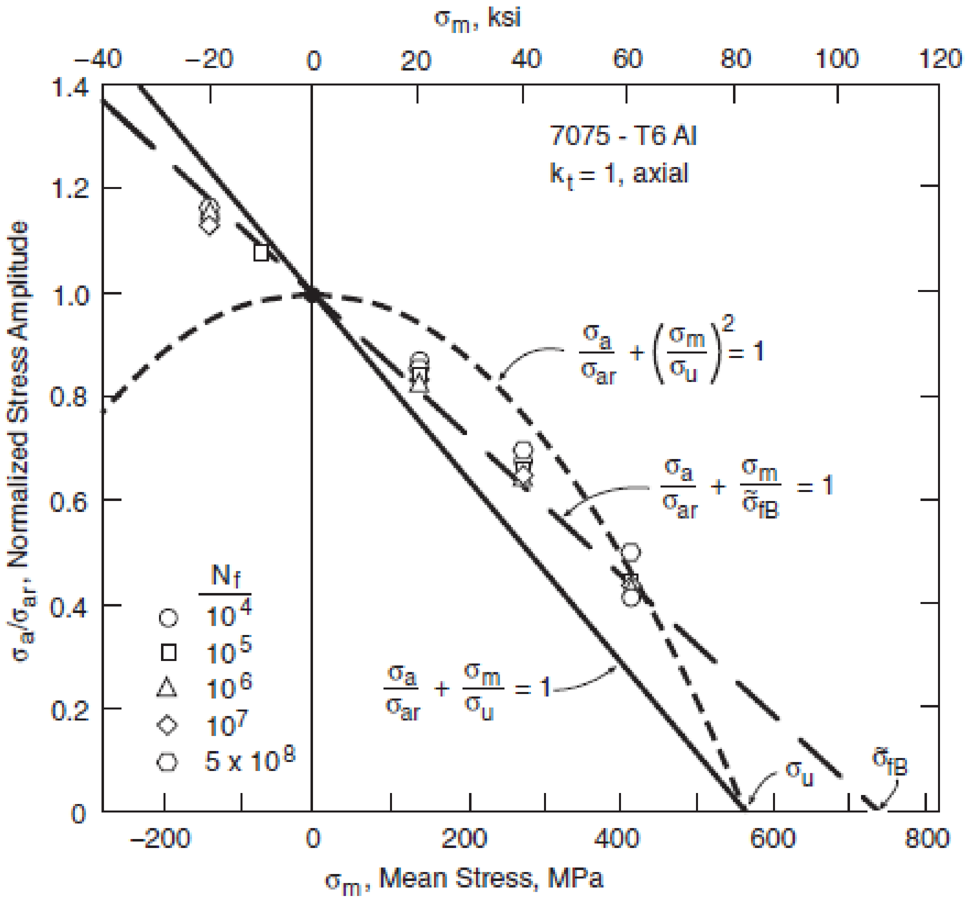
Plot:

$$\frac{\sigma_a}{\sigma_{ar}} \quad \text{vs.} \quad \sigma_m$$

Consolidate data into a ***single curve***

Mean Stress Effects

Normalized Amplitude-Mean Diagrams

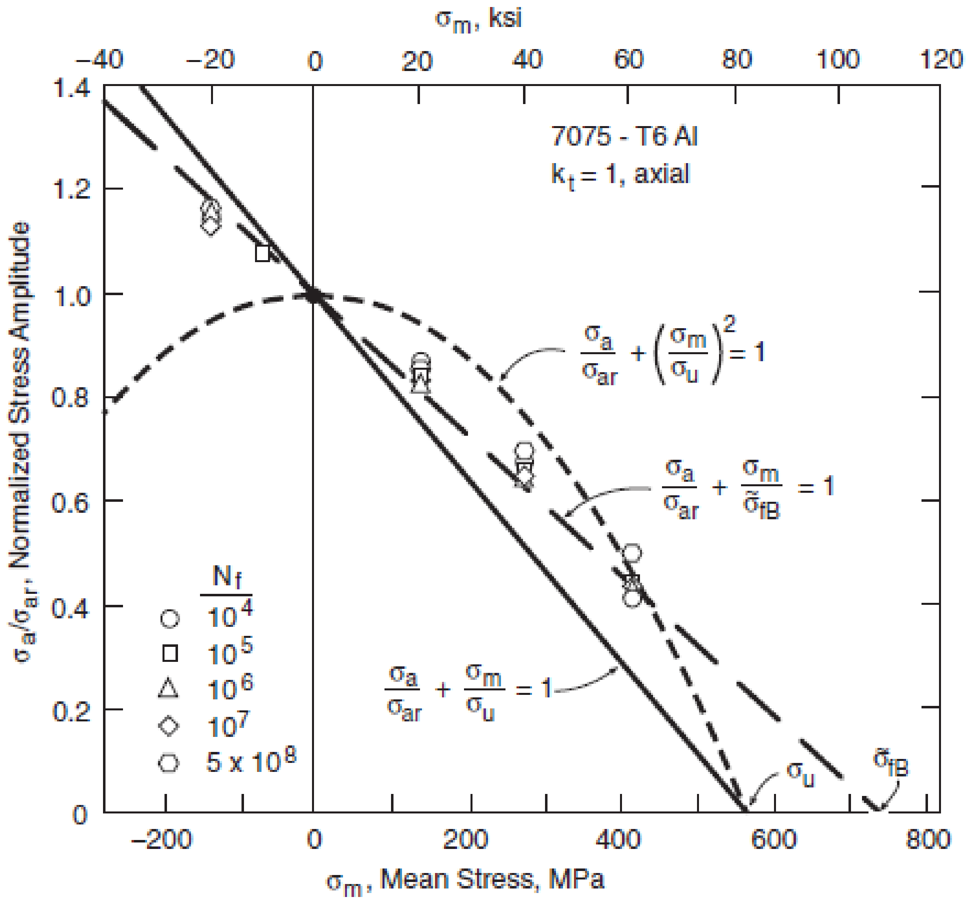


$$\frac{\sigma_a}{\sigma_{ar}} \quad \text{vs.} \quad \sigma_m$$

Fit a single curve to obtain an equation representing the data

Mean Stress Effects

Normalized Amplitude-Mean Diagrams

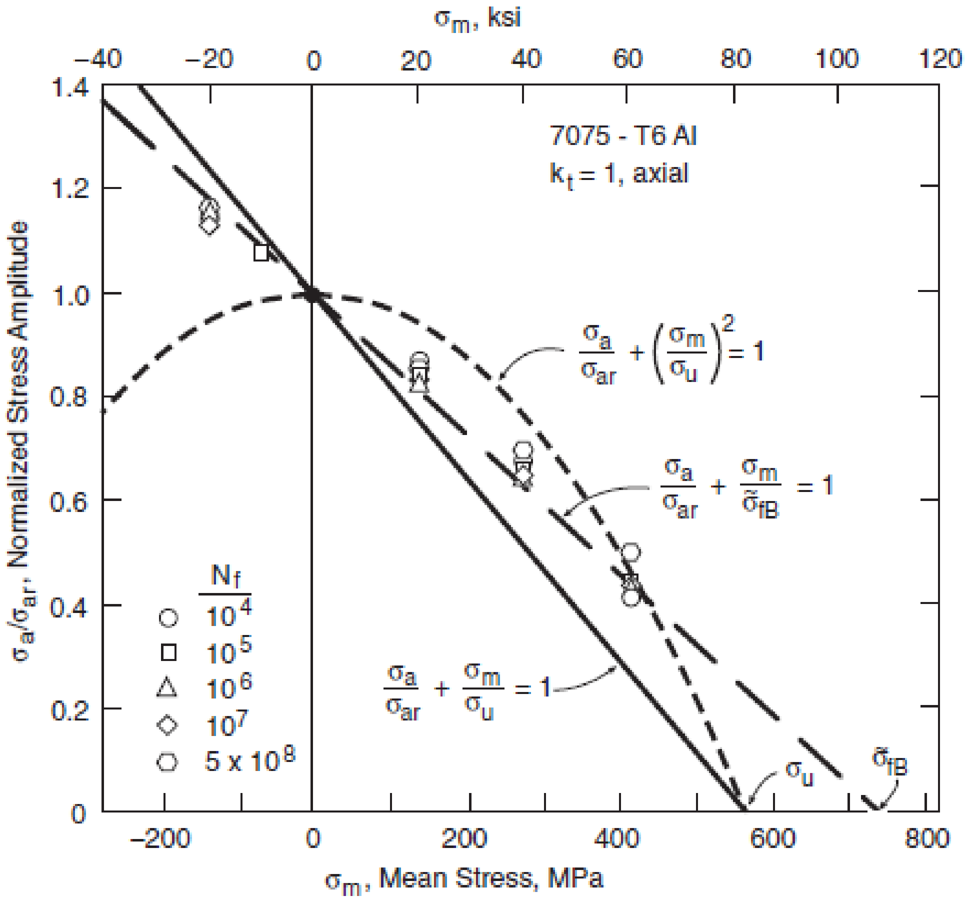


Godman Equation

$$\frac{\sigma_a}{\boxed{\sigma_{ar}}}+\frac{\sigma_m}{\sigma_u}=1$$

Mean Stress Effects

Additional Mean Stress Equations



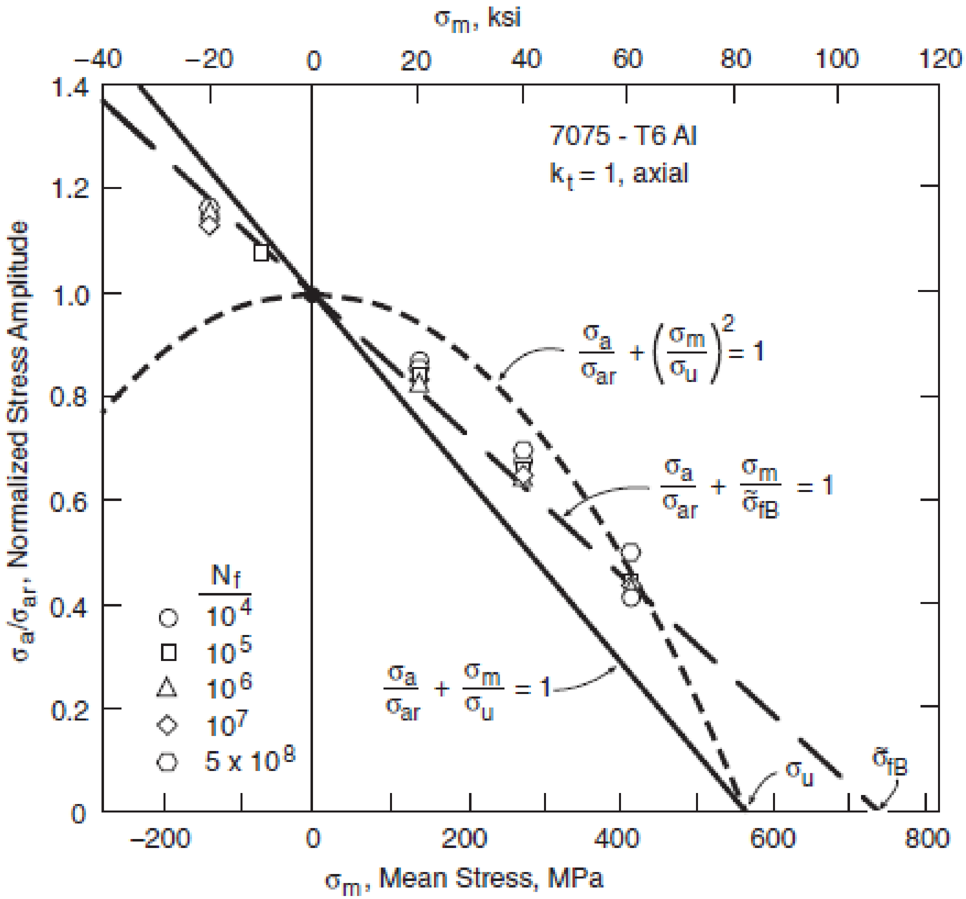
Gerber parabola:

$$\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1$$

$$\sigma_m \geq 0$$

Mean Stress Effects

Additional Mean Stress Equations



Morrow lines:

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma'_f} = 1$$

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\tilde{\sigma}_{fB}} = 1$$

$\tilde{\sigma}_{fB}$: True fracture strength from tension test

Mean Stress Effects

Additional Mean Stress Equations

Morrow line

$$\frac{\sigma_a}{\boxed{\sigma_{ar}}} + \frac{\sigma_m}{\underline{\sigma'_f}} = 1 \longrightarrow$$ It gives reasonable results for steels

σ'_f : Constant from unnotched axial S-N curve for R=-1.

Mean Stress Effects

Additional Mean Stress Equations

Smith, Watson and Topper (STW)

$$\boxed{\sigma_{ar}} = \sqrt{\sigma_{\max} \times \sigma_a} \longrightarrow$$ It gives good results for Alminum alloys

↓

$$\sigma_{\max} > 0$$

$$\sigma_{\max} = \sigma_m + \sigma_a$$

Advantage of not relying on any material constant

Mean Stress Effects

Additional Mean Stress Equations

- Godman often is conservative
- Gerber often is nonconservative
- Morrow is reasonably accurate
  - Fit data very well for steels
  - Should be avoided for aluminum alloys

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f^{'}} = 1$$

Life Estimation including Mean Stress

Let's solve Morrow's equation for σ_{ar}

$$\frac{\sigma_a}{\boxed{\sigma_{ar}}}+\frac{\sigma_m}{\sigma_f'}=1 \Rightarrow \sigma_{ar}=\frac{\sigma_a}{1-\dfrac{\sigma_m}{\sigma_f'}}$$

Substituting (σ_a,σ_m) gives a stress amplitude σ_{ar} that is expected to produce the same fatigue life at zero mean stress as (σ_a,σ_m) combination.

Life Estimation including Mean Stress

Let's solve Morrow's equation for σ_{ar}

$$\frac{\sigma_a}{\boxed{\sigma_{ar}}} + \frac{\sigma_m}{\sigma'_f} = 1 \Rightarrow \sigma_{ar} = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{\sigma'_f}}$$

Therefore, σ_{ar} can be considered as ***an equivalent completely reversed stress amplitude.***

σ_{ar} +stres-life curve for σ_m=0 → life estimation for (σ_a,σ_m) combination

Life Estimation including Mean Stress

Stres-life curve for σ_m=0 → $\sigma_{ar} = \sigma_f^{'}\left(2N_f\right)^b$

$\boxed{+}$

$$\frac{\sigma_a}{1-\frac{\sigma_m}{\sigma_f^{'}}} = \sigma_f^{'}\left(2N_f\right)^b \longleftarrow \sigma_{ar} = \frac{\sigma_a}{1-\frac{\sigma_m}{\sigma_f^{'}}}$$

General stres-life curve for nonzero σ_m

$$\sigma_a = \left(\sigma_f^{'} - \sigma_m\right)\left(2N_f\right)^b$$

Life Estimation including Mean Stress

General stres-life curve for <u>nonzero</u> σ_m

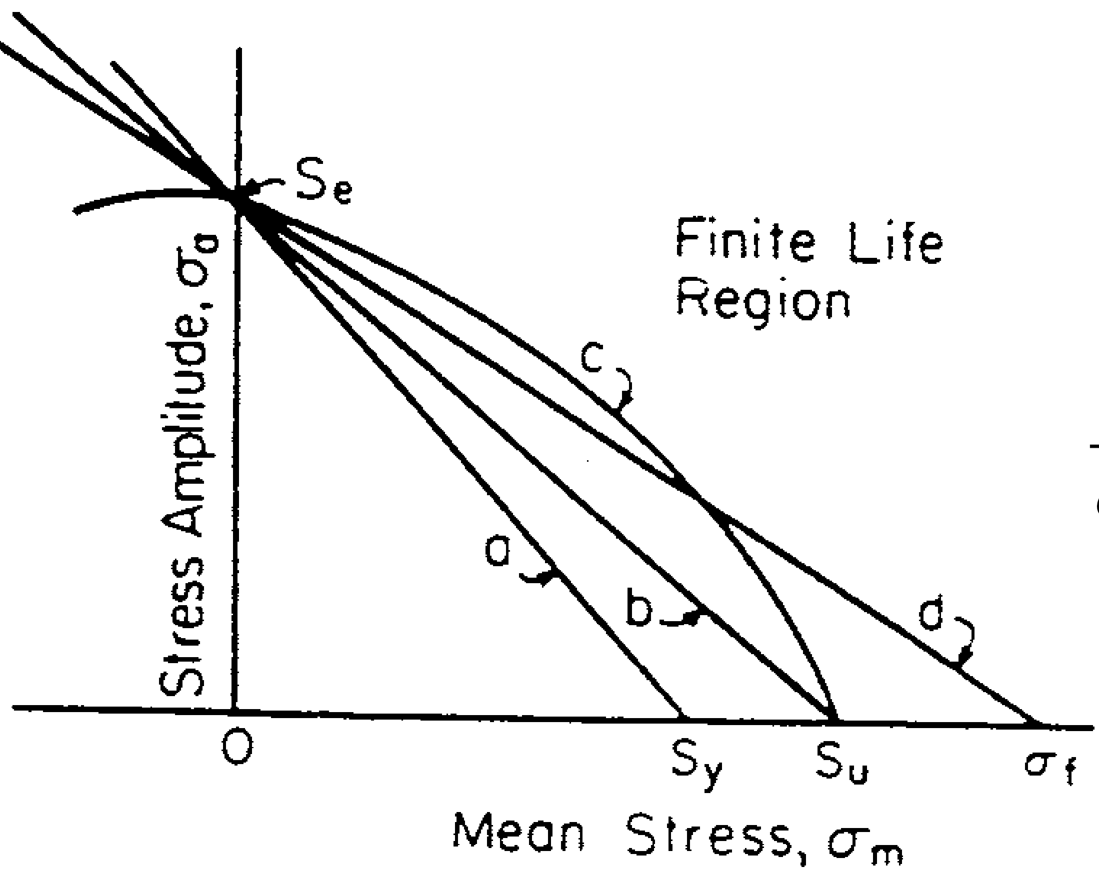
$$\sigma_a = \left(\sigma_f^{'} - \sigma_m\right)\left(2N_f\right)^b$$

- Produce a family of fatigue curves for different σ_m

General stres-life curve for <u>nonzero</u> σ_m using SWT

$$\sqrt{\sigma_{\max} \times \sigma_a} = \sigma_f^{'}\left(2N_f\right)^b$$

Life Estimation including Mean Stress



$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1$$ Godman (1899)

$$\frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1$$ Gerber (1874)

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_y} = 1$$ Soderberg (1930)

$$\frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f'} = 1$$ Morrow (1960)

Life Estimation including Mean Stress

Example

- Um eixo cilindrico com área uniforme possui um raio de 1 in. O eixo está submetido a uma força axial média de 120 kN. Ensaios em corpos de prova sem entalhe mostram que a vida à fadiga do componente para um tensão alternada (σ_a) de 250 MPa é 1 milhão de ciclos com R = -1. Estime a amplitude da força permitida para que o eixo suporte pelo menos um (1) milhão de ciclos. Considere que
  - σ_y=0.55σ_u=228 Mpa (~SAE 1015)

Life Estimation including Mean Stress

Example

Godman (1899)

$$\left(\boxed{\sigma_a},\sigma_m\right) \longrightarrow \frac{\sigma_a}{\sigma_{ar}}+\frac{\sigma_m}{\sigma_u}=1 \Rightarrow \sigma_a=\sigma_{ar}\left(1-\frac{\sigma_m}{\sigma_u}\right)$$

?

$$\sigma_m=\frac{F}{\pi\times r^2} \longrightarrow \sigma_m=\frac{120\times10^3}{\pi\times25.4^2}=59\,MPa$$

Life Estimation including Mean Stress

Example

Godman (1899)

$$\left(\boxed{\sigma_a}, \sigma_m\right) \longrightarrow \frac{\boxed{\sigma_a}}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1 \Rightarrow \sigma_a = \sigma_{ar}\left(1 - \frac{\sigma_m}{\sigma_u}\right)$$

?

$$\sigma_u = \frac{\sigma_y}{0.55} \longrightarrow \sigma_u = \frac{228}{0.55} = 415\,MPa$$

Life Estimation including Mean Stress

Example

Godman (1899)

$$\left(\boxed{\sigma_a}, \sigma_m\right) \longrightarrow \frac{\boxed{\sigma_a}}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_u} = 1 \Rightarrow \sigma_a = 250 \times \left(1 - \frac{59}{415}\right)$$

?

$$\sigma_a = 214\,MPa$$

Life Estimation including Mean Stress

Example

Gerber (1899)

$$\left(\boxed{\sigma_a}, \sigma_m\right) \longrightarrow \frac{\sigma_a}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1 \Rightarrow \sigma_a = 250 \times \left(1 - \left[\frac{59}{415}\right]^2\right)$$

?

$$\sigma_a = 245\ MPa$$

Life Estimation including Mean Stress

Example

Gerber (1874)

$$\left(\boxed{\sigma_a}, \sigma_m\right) \longrightarrow \frac{\boxed{\sigma_a}}{\sigma_{ar}} + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1 \Rightarrow \sigma_a = 250 \times \left(1 - \left[\frac{59}{415}\right]^2\right)$$

?

$$\sigma_a = 245\ MPa$$

Life Estimation including Mean Stress

Example

Soderberg (1930)

$$\left(\boxed{\sigma_a}, \sigma_m\right) \rightarrow \frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_y} = 1 \Rightarrow \sigma_a = 250 \times \left(1 - \frac{59}{228}\right)$$

?

$$\sigma_y = 288\ MPa$$

(SAE 1015)

$$\sigma_a = 185\ MPa$$

Life Estimation including Mean Stress

Example

Morrow (1960)

$$\left(\boxed{\sigma_a}, \sigma_m\right) \longrightarrow \frac{\sigma_a}{\sigma_{ar}} + \frac{\sigma_m}{\sigma_f^{'}} = 1 \Rightarrow \sigma_a = 250 \times \left(1 - \frac{59}{1020}\right)$$

?

$$\sigma_f^{'} = 1020\ MPa$$

(SAE 1015)

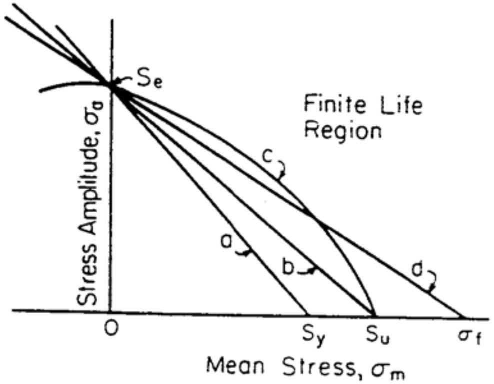
$$\sigma_a = 235\ MPa$$

Life Estimation including Mean Stress

Example

$$\left(\boxed{\sigma_a},\sigma_m\right)$$

?



Morrow (1960)

$$\sigma_a = 235\,MPa$$

Soderberg (1930)

$$\boxed{\sigma_a = 185\,MPa}$$

Gerber (1874)

$$\sigma_a = 245\,MPa$$

Godman (1899)

$$\sigma_a = 214\,MPa$$

Life Estimation including Mean Stress

Example 2

Um componente estrutural é construido com um aço AISI 4340 e está submetido a um carregamento cíclico com tensão média de 150 MPa. Estime a vida à fadiga do componente se a amplitude da tensão aplicada é 400 MPa.

Life Estimation including Mean Stress

Example 2

Um componente estrutural é construido com um aço AISI 4340 e está submetido a um carregamento cíclico com tensão média de 150 MPa. Estime a vida à fadiga do componente se a amplitude da tensão aplicada é 400 MPa.

Dados:

$$\sigma_a = 400\ MPa$$

$$\sigma_m = 150\ MPa$$

Morrow (1960)

Aço AISI 4340

↓

Procurar dados (propriedades) de fadiga na literatura!

Constants for Stress Life Curves

Table 9.1 Constants for Stress–Life Curves for Various Ductile Engineering Metals, From Tests at Zero Mean Stress on Unnotched Axial Specimens

Material	Yield Strength σ_o	Ultimate Strength σ_u	True Fracture Strength $\tilde{\sigma}_{fB}$	$\sigma_a = \sigma_f'(2N_f)^b = AN_f^B$		
				σ_f'	A	$b = B$
(a) Steels						
SAE 1015 (normalized)	228 (33)	415 (60.2)	726 (105)	1020 (148)	927 (134)	−0.138
Man-Ten (hot rolled)	322 (46.7)	557 (80.8)	990 (144)	1089 (158)	1006 (146)	−0.115
RQC-100 (roller Q & T)	683 (99.0)	758 (110)	1186 (172)	938 (136)	897 (131)	−0.0648
SAE 4142 (Q & T, 450 HB)	1584 (230)	1757 (255)	1998 (290)	1937 (281)	1837 (266)	−0.0762
AISI 4340 (aircraft quality)	1103 (160)	1172 (170)	1634 (237)	1758 (255)	1643 (238)	−0.0977
(b) Other Metals						
2024-T4 Al	303 (44.0)	476 (69.0)	631 (91.5)	900 (131)	839 (122)	−0.102
Ti-6Al-4V (solution treated and aged)	1185 (172)	1233 (179)	1717 (249)	2030 (295)	1889 (274)	−0.104

Notes: The tabulated values have units of MPa (ksi), except for dimensionless $b = B$.

$$\boxed{\sigma_a = \sigma_f'\left(2N_f\right)^b}$$

$$\sigma_m = 0$$

$$\sigma_f' \approx \tilde{\sigma}_f$$

Life Estimation including Mean Stress

Example 2

Um componente estrutural é construido com um aço AISI 4340 e está submetido a um carregamento cíclico com tensão média de 150 MPa. Estime a vida à fadiga do componente se a amplitude da tensão aplicada é 400 MPa.

Aço AISI 4340

Dados:

$$\sigma_a = 400\ MPa$$

$$\sigma_m = 150\ MPa$$

Morrow (1960)

$$\sigma'_f = 1758\ MPa$$

$$b = -0.0977$$

Life Estimation including Mean Stress

Example 2

Um componente estrutural é construido com um aço AISI 4340 e está submetido a um carregamento cíclico com tensão média de 150 MPa. Estime a vida à fadiga do componente se a amplitude da tensão aplicada é 400 MPa.

$$\sigma_a = \left(\sigma_f^{'} - \sigma_m\right)\left(2N_f\right)^b$$

$$\downarrow$$

$$400 = \left(1758 - 150\right)\left(2N_f\right)^{-0.0977} \longrightarrow N_f = 764 \times 10^3 \ Cycles$$

Life Estimation including Mean Stress

Example 2

Um componente estrutural é construido com um aço AISI 4340 e está submetido a um carregamento cíclico com tensão média de 150 MPa. Estime a vida à fadiga do componente se a amplitude da tensão aplicada é 400 MPa.

$$\sigma_a = \left(\sigma_f' - \sigma_m\right)\left(2N_f\right)^b$$

$$\downarrow$$

$$400 = \left(1758 - 150\right)\left(2N_f\right)^{-0.0977} \longrightarrow N_f = 764 \times 10^3 \ Cycles$$

Life Estimation including Mean Stress

Example 2

Usando a relação SWT

$$\sqrt{\sigma_{\max} \times \sigma_a} = \sigma_f^{'} \left(2N_f\right)^b$$

$$\sigma_{\max} = \sigma_m + \sigma_a$$

$$\sigma_{\max} = 150 + 400 = 550\ MPa$$

$$\left[\frac{\sqrt{\sigma_{\max} \times \sigma_a}}{\sigma_f^{'}}\right]^{\frac{1}{b}} = \left[\left(2N_f\right)^b\right]^{\frac{1}{b}} \longrightarrow N_f = \frac{1}{2}\left[\frac{\sqrt{\sigma_{\max} \times \sigma_a}}{\sigma_f^{'}}\right]^{\frac{1}{b}}$$

$$N_f = \frac{1}{2}\left[\frac{\sqrt{550 \times 400}}{1728}\right]^{\frac{1}{-0.0977}} \longrightarrow N_f = 313 \times 10^3\ Cycles$$

Life Estimation including Mean Stress

Example 2

SWT

$$\sqrt{\sigma_{\max} \times \sigma_a} = \sigma_f^{'} \left(2N_f\right)^b$$

Morrow

$$\sigma_{ar} = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{\sigma_f^{'}}}$$

$$N_f = 313 \times 10^3 \ Cycles$$

vs.

$$N_f = 764 \times 10^3 \ Cycles$$

Life Estimation including Mean Stress

Example 3

Um componente está submetido a um carregamento cíclico com σ_{max} =110 ksi e σ_{min} =10 ksi. O componente está feito de um aço com limíte de resistência σ_u = 150 ksi, limíte de fadiga S_e= 60 ksi e possui uma resistência à fadiga (fatigue strength), com carregamento reverso, de 110 ksi para 1000 ciclos. Estime a vida à fadiga do componente.

Life Estimation including Mean Stress

Example 3

Um componente está submetido a um carregamento cíclico com σ_{max} =110 ksi e σ_{min} =10 ksi. O componente está feito de um aço com limíte de resistência σ_u = 150 ksi, limíte de fadiga S_e= 60 ksi e possui uma resistência à fadiga (fatigue strength), com carregamento reverso, de 110 ksi para 1000 ciclos. Estime a vida à fadiga do componente.

Dados:

Loading

$$\left.\begin{aligned}\sigma_{\max} &= 758.5\,MPa \\ \sigma_{\min} &= 68.9\,MPa\end{aligned}\right\}$$

Mechanical Properties

$$\sigma_u = 1034\,MPa$$

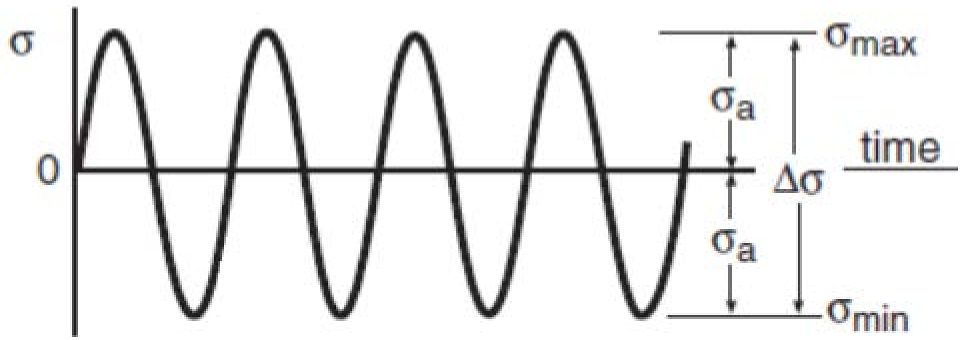
$$\sigma_e = 414\,MPa$$

$$(\sigma_a, \sigma_m) = (758.5, 0) \rightarrow N_f = 1000\,Cycles$$

1 kilopound per square inch (ksi) = 6.895 MPa

Life Estimation including Mean Stress

Example 3



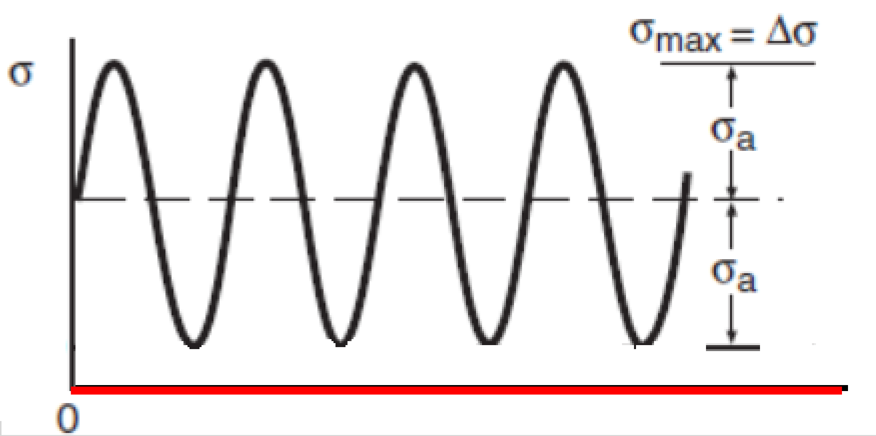
Experimental Data

$$\sigma_e = 414\ MPa \quad \sigma_u = 1034\ MPa$$

$$(\sigma_a, \sigma_m) = (758.5, 0)$$

$$\downarrow$$

$$N_f = 1000\ Cycles$$



Applied Cycle Loading

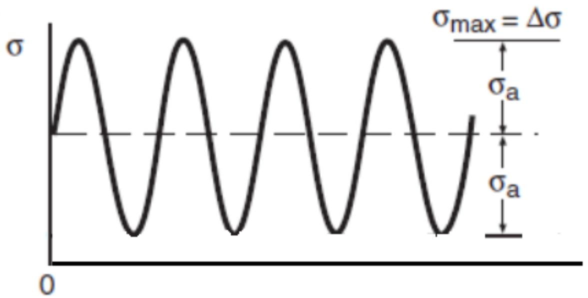
$$\sigma_{\max} = 758.5\ MPa$$

$$\sigma_{\min} = 68.9\ MPa$$

$$\longrightarrow \quad N_f = ?$$

Life Estimation including Mean Stress

Example 3



$$\sigma_{\max} = 758.5\ MPa$$

$$\sigma_{\min} = 68.9\ MPa$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{\min}}{2}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{\min}}{2}$$

$$\sigma_a = \frac{758.5 - 68.9}{2} = 345\ MPa$$

$$\sigma_m = \frac{758.5 + 68.9}{2} = 414\ MPa$$

Life Estimation including Mean Stress

Example 3

Generate Haigh Diagram
(σ_a vs. σ_m)

Loading

$$\sigma_a = 345\ MPa$$

$$\sigma_m = 414\ MPa$$

$$(\sigma_a, \sigma_m) = (758.5, 0)$$
$$\downarrow$$
$$N_f = 1000\ Cycles$$

$$(\sigma_e, \sigma_m) = (414, 0)$$
$$\downarrow$$
$$N_f = 1 \times 10^6\ Cycles$$

Haigh diagram is constructed by connecting the endurance limit, S_e, and the fatigue strength for 1000 cycles (S_{1000}) on the vertical axis to the <u>ultimate strength **σ_u**</u>, Godman criteria, on the horizontal axis (mean stress axis).

Life Estimation including Mean Stress

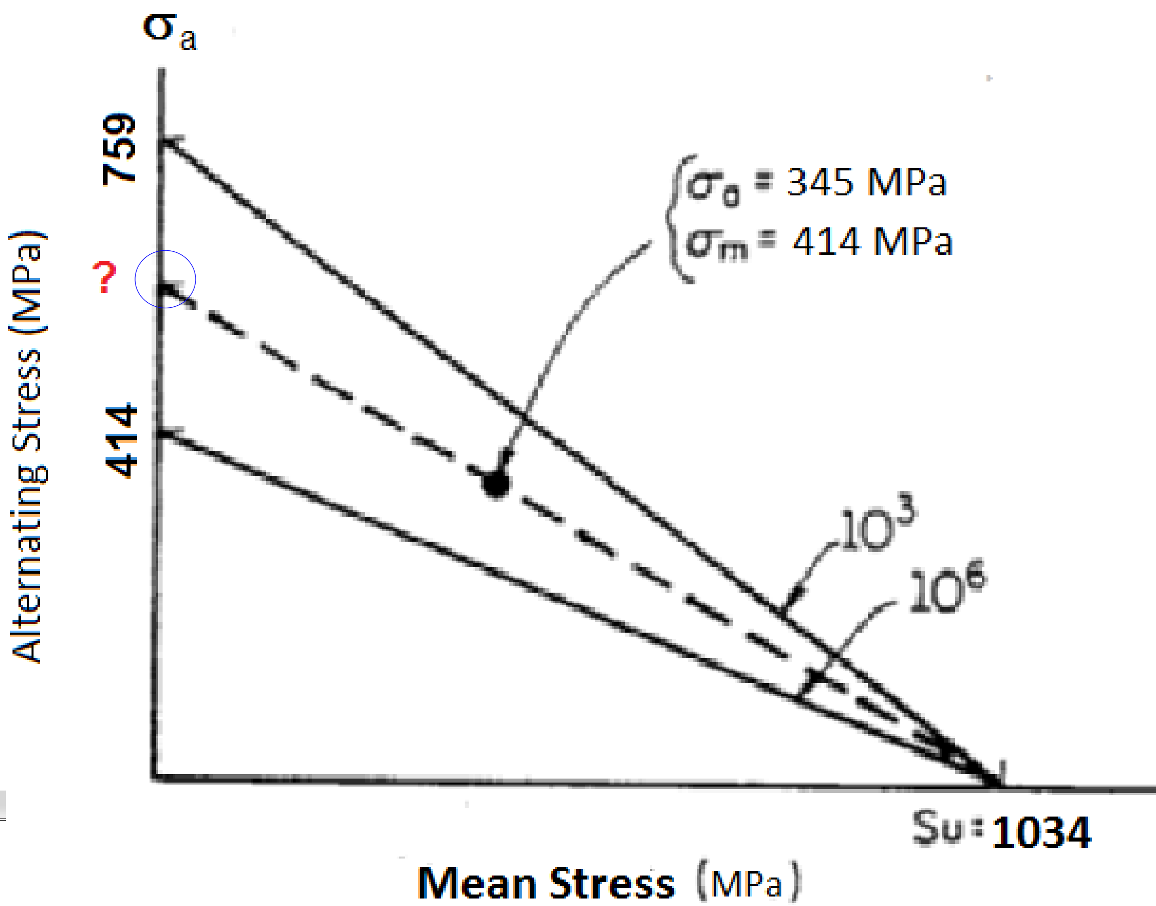
Example 3

Loading

$$\sigma_a = 345\ MPa$$
$$\sigma_m = 414\ MPa$$

Expected life between 10^3 e 10^6 cycles

Generate Haigh Diagram (σ_a vs. σ_m)



Life Estimation including Mean Stress

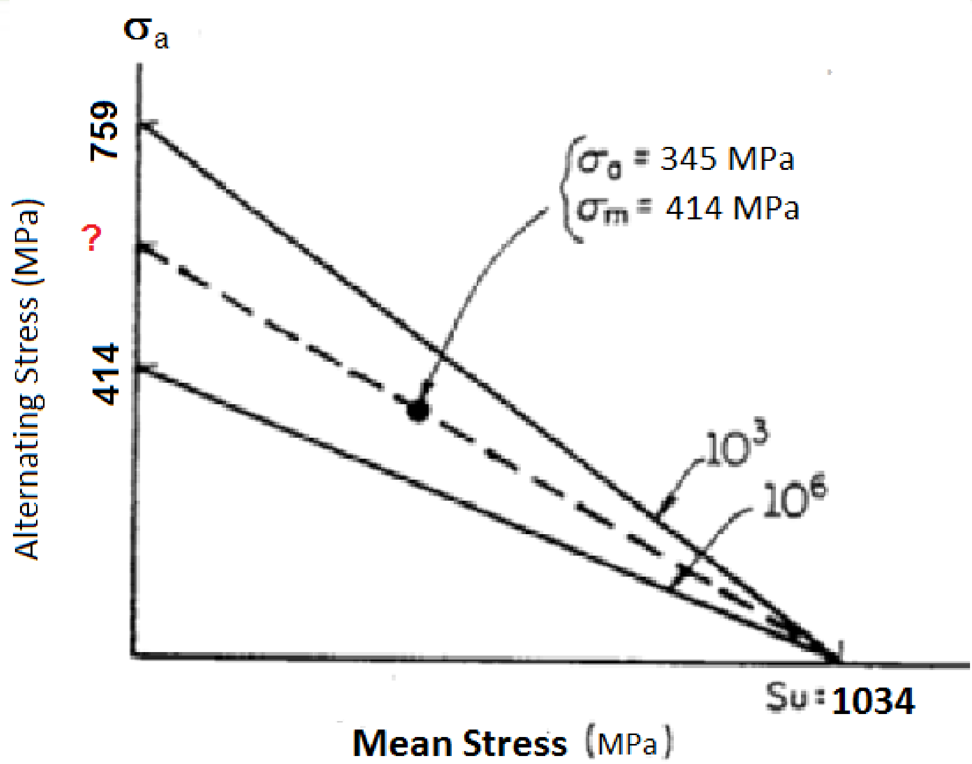
Example 3

Loading

$$\sigma_a = 345\ MPa$$
$$\sigma_m = 414\ MPa$$

Godman $\dfrac{\sigma_a}{\boxed{\sigma_{ar}}} + \dfrac{\sigma_m}{\sigma_u} = 1$

?



Fully reversed stress level corresponding to the same life as that obtained with the stress combination (σ_a, σ_m).

Life Estimation including Mean Stress

Example 3

Loading

$$\sigma_a = 345\ MPa$$

$$\sigma_{\mathrm{m}} = 414\ MPa$$

Godman

$$\boxed{\sigma_{ar}} = \frac{\sigma_a}{1 - \dfrac{\sigma_m}{\sigma_u}} \longrightarrow \sigma_{ar} = \frac{345}{1 - \dfrac{414}{1034}} = 575.4$$

Life Estimation including Mean Stress

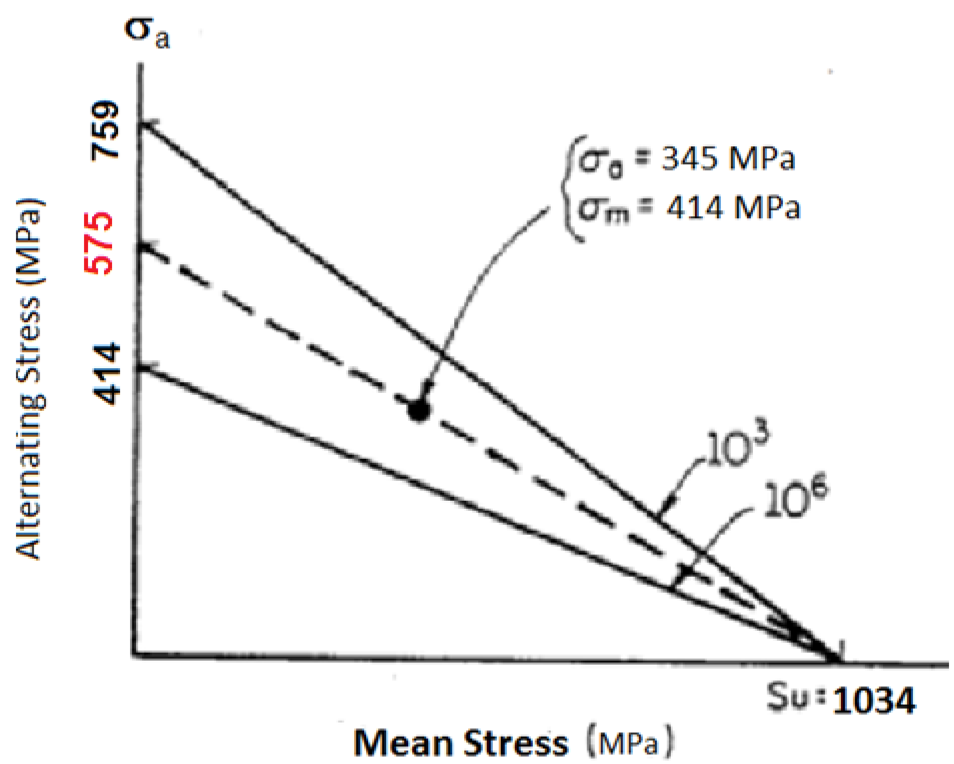
Example 3

Loading

$$\sigma_a = 345\ MPa$$

$$\sigma_{\mathrm{m}} = 414\ MPa$$

$$\boxed{\sigma_{ar} = 575.4}$$



Life Estimation including Mean Stress

Example 3

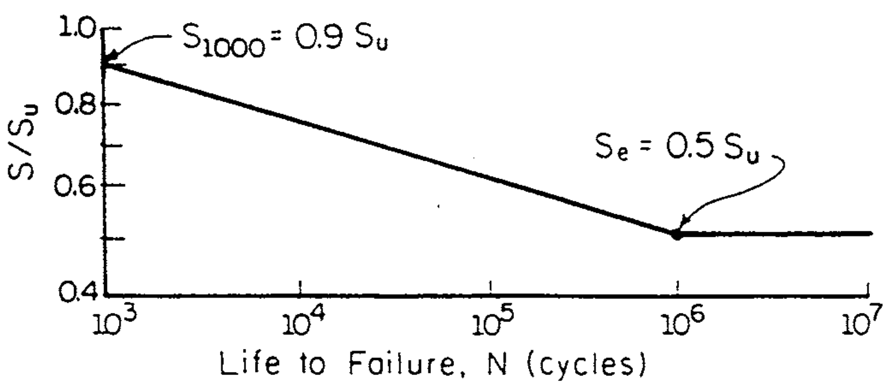
$$\boxed{\sigma_{ar} = 575.4}$$

It is necessary to estimate the stress-life curve for R=-1

Loading

$$\sigma_a = 345\ MPa$$

$$\sigma_{\mathrm{m}} = 414\ MPa$$



First approximation

Life Estimation including Mean Stress

Example 3

$$\boxed{\sigma_{ar} = 575.4}$$

Dados:

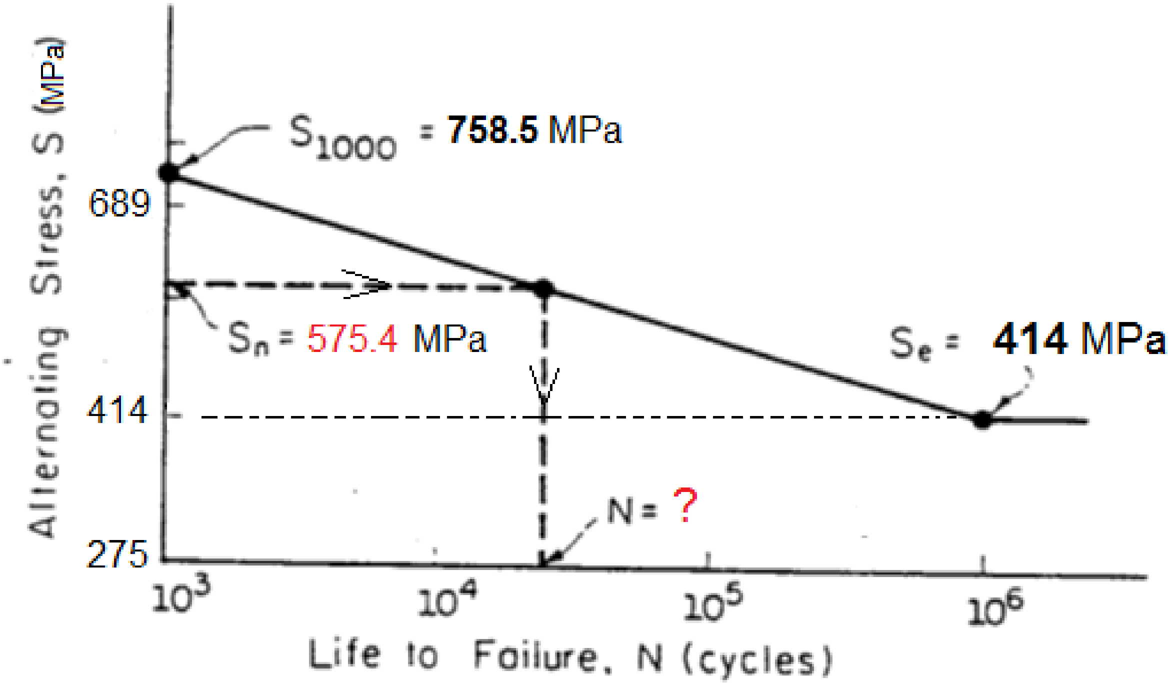
$$\sigma_a = A\left(N_f\right)^B$$

$$\left(758.5, 10^3\right)\left(414, 10^6\right)$$

$$B = -0.08765$$

$$A = 1389\ MPa$$

$$575.4 = 1389\left(\boxed{N_f}\right)^{-0.08765}$$

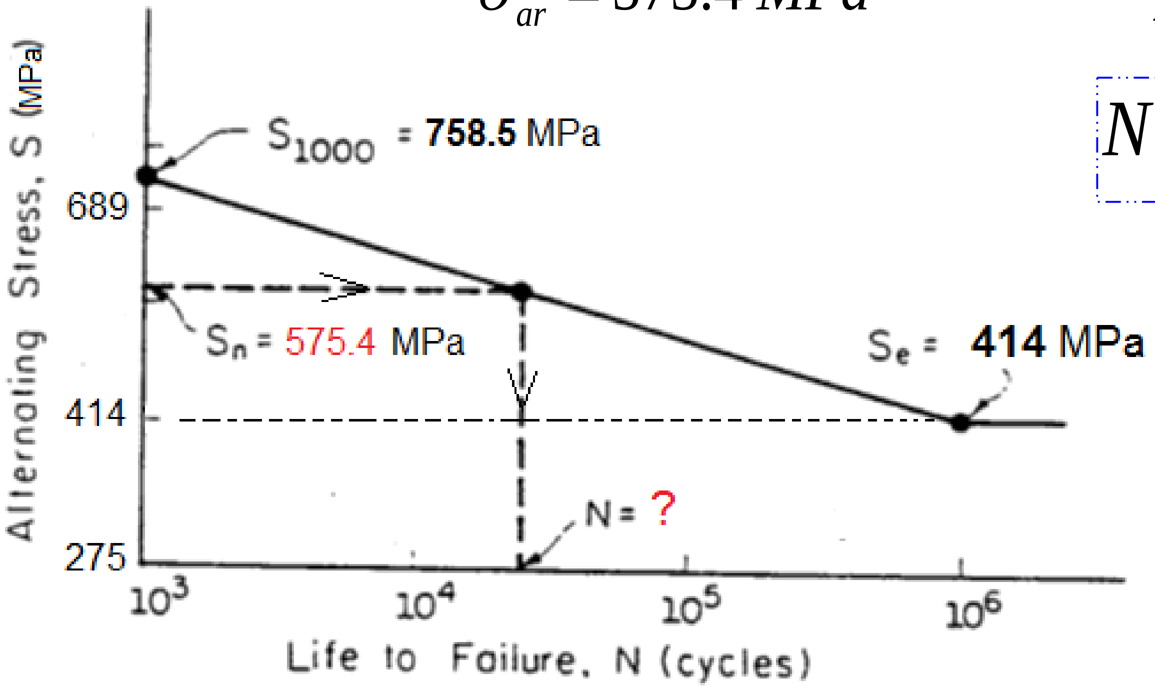


Life Estimation including Mean Stress

Example 3

$$\sigma_{ar} = 575.4\ MPa \longrightarrow N_f = 23382\ cycles$$

$$\boxed{N_f = 2.34 \times 10^4\ cycles}$$



Morrow?
STWP?
Gerber?

Modifying Factors on Baseline S-N

For many years the emphasis of most fatigue testing was to gain an empirical understanding of the effects of various factors on the baseline S–N curves for ferrous alloys in the intermediate to long life ranges. The variables investigated include:

1. Size
2. Type of loading
3. Surface finish
4. Surface treatments
5. Temperature
6. Environment

Modifying Factors on Baseline S-N

The results of these tests have been quantified as modification factors which are applied to the baseline S–N data.

$$S_e' = S_e \times C_{size} \times C_{load} \times C_{surface} \times C_{temperature} \times \ldots$$

- Ci factors are specified for the endurance limite.
- Corrections for the remainder of the S-N curve is not clearly defined.
- At extreme limit of monotonic loading all factors aproach 1.
- Conservative estimate is to use the Ci factors on the entire S-N Curve.

Modifying Factors on Baseline S-N

Size effects

- Fatigue failure is dependent on the interaction of a large stress with a critical flaw.
- Fatigue can be tought to be controlled by the weakest link of the material.
- Probability of a weak link increase with material volume.

TABLE 1.1 Influence of Size on Endurance Limit

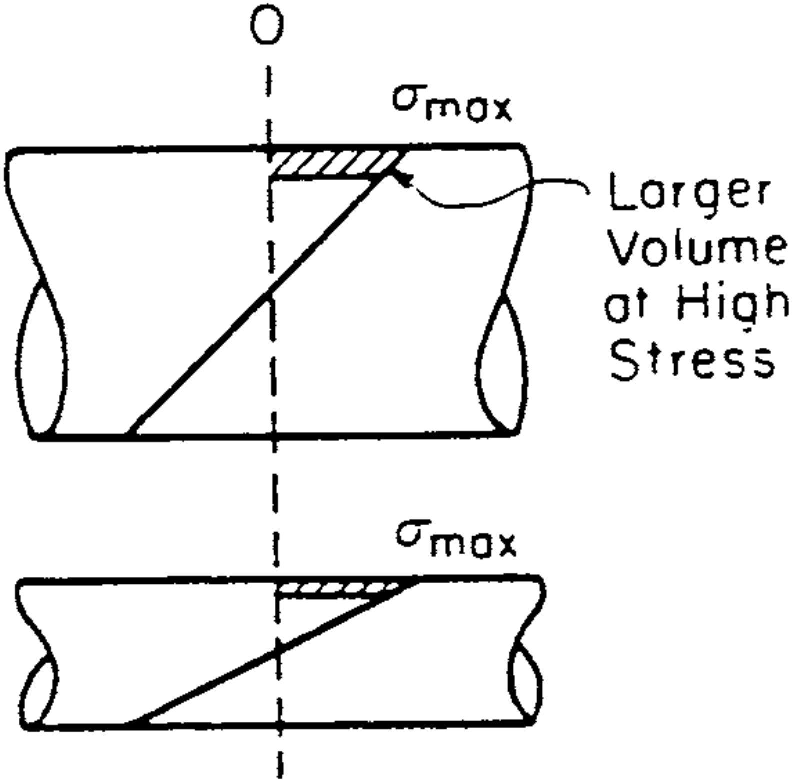
Diameter (in)	Endurance Limit (ksi)
0.3	33.0
1.5	27.6
6.75	17.3

Source: J. H. Faupel and F. E. Fisher, *Engineering Design*, John Wiley and Sons, New York,

Modifying Factors on Baseline S-N

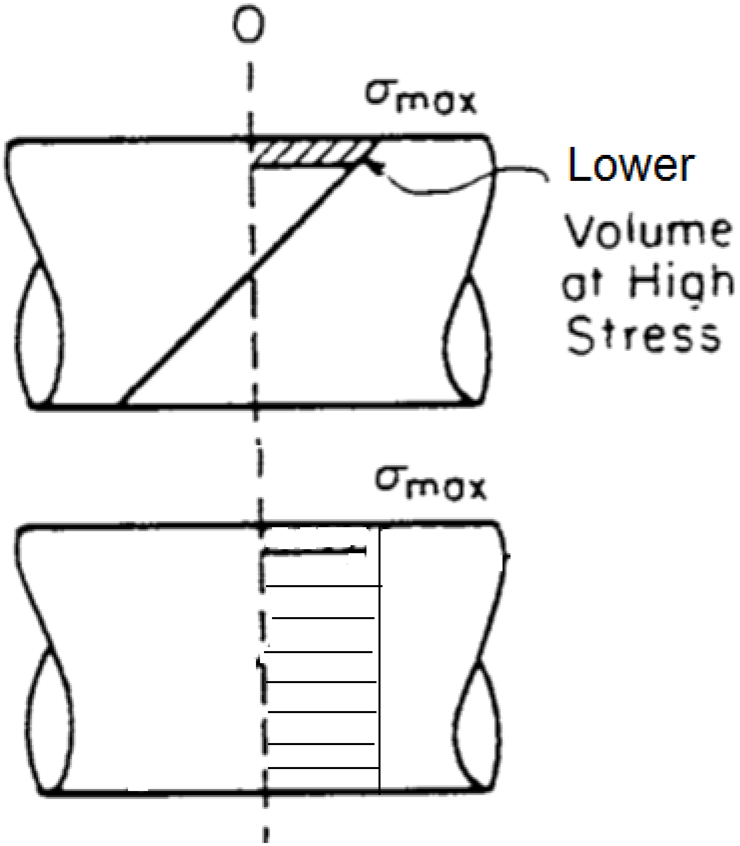
Size effects

$$C_{size} = \begin{cases} 1.0 & \text{if } d \le 8 \text{ mm} \\ 1.189d^{-0.097} & \text{if } 8 \text{ mm} \le d \le 250 \text{ mm} \end{cases}$$



Modifying Factors on Baseline S-N

Loading effects

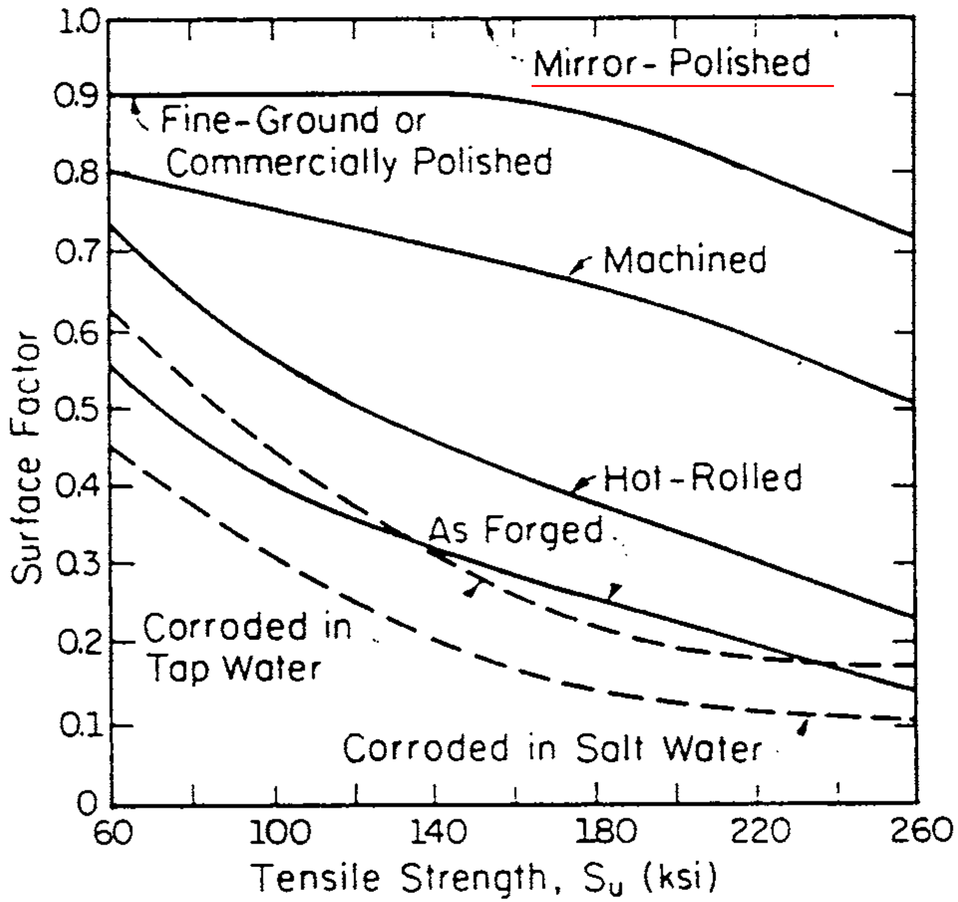


$$S_e(\text{axial}) \approx 0.70 S_e(\text{bending})$$

$$\tau_e(\text{torsion}) \approx 0.577 S_e(\text{bending})$$

Modifying Factors on Baseline S-N

Surface finish effects



OBRIGADO

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